Fluid Mechanics

SOLUTIONS

Fluid Mechanics : RACE-01

1. **Ans. (C)**

$P_0 + \rho g d_1 = P_1$

$P_0 + \rho g d_2 = P_2$

$\rho g(d_2 - d_1) = P_2 - P_1$

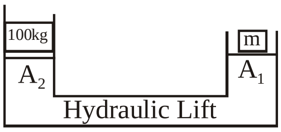
$10^3 \times 10\ (d_2 - d_1) = 3.03 \times 10^6$

$d_2 - d_1 = 303$ m

$\simeq 300m$

2. **Ans. (25600)**

Using Pascals law



$\frac{100 \times g}{A_2} = \frac{mg}{A_1}$ ... (1)

Let m mass can lift M^0 in second case then

$\frac{M_0 g}{16 A_2} = \frac{mg}{A_1 / 16}$ ... (2)

{Since $A = \frac{\pi d^2}{4}$}

From equation (1) and (2) we get

$\frac{M_0}{16.100} = 16$

$\Rightarrow M^0 = 25600\ kg$

3. **Ans. (A)**

$P^1 = \rho g d + P^0 = 3 \times 10^5$ Pa

$\therefore\ \rho g d = 2 \times 10^5$ Pa

$P^2 = 2\rho g d + P^0$

$= 4 \times 10^5 + 10^5 = 5 \times 10^5$ Pa

$\%\text{increase} = \frac{P_2 - P_1}{P_1} \times 100$

$= \frac{5\times10^5 - 3\times10^5}{3\times10^5} \times 100 = \frac{200}{3}\%$

4. **Ans. (C)**

(R) is the statement of Pascal's principle & which explains the assertion (S)

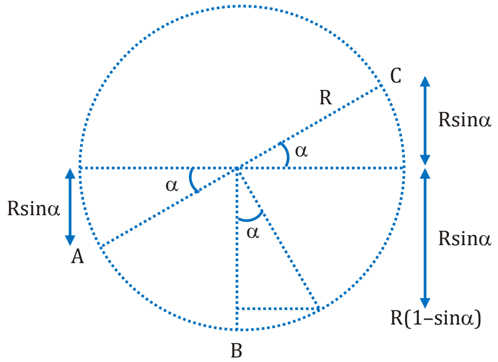
5. Ans. (B)

Pressure in a static liquid will be same at each point on same horizontal level.

$\therefore P = P_{atm} + \rho gh$

As per Pascal law, same pressure applied to enclosed water is transmitted in all directions equally.

6. Ans. (A)



Let Radius of circular tube is R

Also, $PA = PC = P0$

Pressure at point B

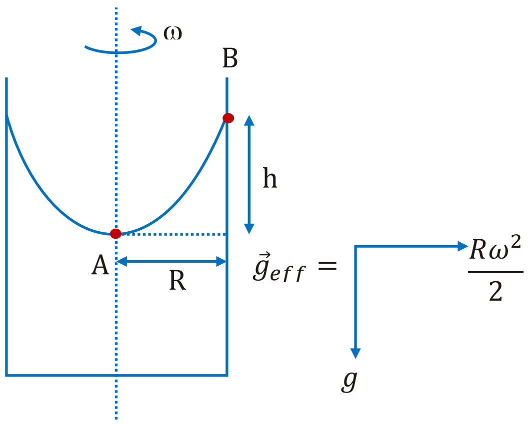
$PB = P0 + d1(R - R\sin\alpha)g$

$= P0 + d2(R\sin a + R\cos\alpha)g$

$\Rightarrow d1(\cos\alpha - \sin\alpha) = d2(\sin\alpha + \cos\alpha)$

$$\frac{d_1}{d_2} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{1 + \tan\alpha}{1 - \tan\alpha}$$

7. Ans. (A)



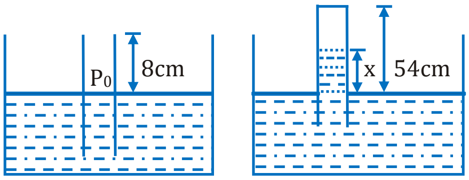
Applying pressure equation from A to B

$$P_0 + \rho.\frac{R\omega^2}{2}.R - \rho gh = P_0$$

$$\frac{\rho R^2\omega^2}{2} = \rho gh$$

$$h = \frac{R^2\omega^2}{2g} = (5)^2\frac{\omega^2}{2g} = \frac{25}{2}\frac{\omega^2}{g}$$

8. **Ans. (C)**



$P0\ A(8) = P'\ A(54 - x)$

$P0\ 8 = P'\ (54 - x)$ ... (i)

$P' = P0 - \rho gx$ ... (ii)

Comparing $(P0)8 = (P0 - \rho gx)\ (54 - x)$

$(76)8 = (76 - x)\ (54 - x)$

Solving we get $x = 38$ cm

Air column = 54 – 38 = 16 cm

9. **Ans. (C)**

$$\tau = \rho g\int_0^h yady(\text{h}-\text{y})$$

10. **Ans. (B)**

Pressure at half the dept $= P_0 + \frac{h}{2}dg$

Pressure at the bottom = $P_0 + hdg$

According to given condition,

$$P_0 + \frac{h}{2}dg = \frac{2}{3}(P_0 + hdg)$$

$$\Rightarrow 3P_0 + \frac{3h}{2}dg = 2P_0 + 2hdg$$

$$\Rightarrow h = \frac{2P_0}{dg} = \frac{2\times10^5}{10^3\times10} = 20m$$

Fluid Mechanics : RACE-02

1. **Ans. (A)**

$F_B = \rho v(g - a)$

$a = g$

$\therefore\ F_B = 0$

2. **Ans. (A)**

Velocity of the ball as it reaches water surface is $\sqrt{2gh}$ i.e. $\sqrt{4g}$

Lets say, ρ is the density of ball and σ is the density of water. Now

Net upward force acting on the body $F = \sigma Vg - \rho Vg$

$\Rightarrow ma = \sigma Vg - \rho Vg$

$\Rightarrow (\rho V)a = Vg(\sigma - \rho)$

$\Rightarrow \; a = g\left(\frac{\sigma}{\rho} - 1\right) = g\left(\frac{1}{0.8} - 1\right) = \frac{g}{4}$ in upward direction

Lets say it sinks to the depth H, at this instant final velocity becomes $v = 0$, so

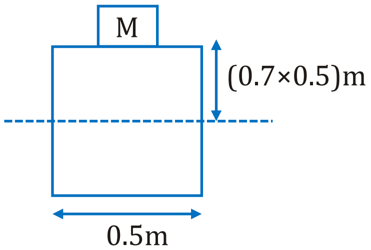
We have

$$0 = \left(\sqrt{4g}\right)^2 - 2 \times \frac{g}{4} \times H \qquad \because \; v^2 = u^2 + 2as$$

$$4g = \frac{2g}{4}H$$

$\Rightarrow H = 8m$

3. Ans. (B)



$M = \rho_L\,[0.5 \times 0.5 \times 0.35]$

$= 10^3\,[0.0875]$

$M = 87.5\ kg$

4. Ans. (B)

$$\frac{4}{3}\pi\left(R^3 - r^3\right)\rho_m g = \frac{4}{3}\pi R^3 \rho_w g$$

$$1 - \left(\frac{r}{R}\right)^3 = \frac{8}{27}$$

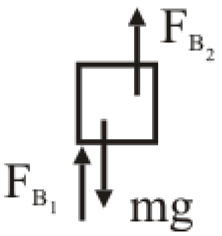
$$\Rightarrow \frac{r}{R} = \left(\frac{19}{27}\right)^{1/3} = \frac{19^{1/3}}{3}$$

$$= 0.88 \simeq \frac{8}{9}$$

5. Ans. (B)

$$F_V = mg - F_B = mg - \left(\frac{m}{d_1} \times d_2\right)g = mg\left(1 - \frac{d_2}{d_1}\right)$$

6. Ans. (A)



$$F_{B_1} + F_{B_2} = mg$$

$V_1\, 1000\, g + V_2\, 800g$

$= (V_1 + V_2)\, 900\, g$

$\frac{V_1}{V_2}.10+8=\frac{V_1}{V_2}.9+9$

$\frac{V_1}{V_2}=1$

7. **Ans. (C)**

In 1st situation

$V_b\rho_b g = V_s\rho_w g$

$\frac{V_S}{V_b} = \frac{\rho_b}{\rho_w} = \frac{4}{5}$ ...(i)

here V_b is volume of block

V_s is submerged volume of block

ρ_b is density of block

ρ_w is density of water

& Let ρ_o is density of oil

finally in equilibrium condition

$V_b\rho_b\, g = \frac{V_b}{2}\rho_0 g + \frac{V_b}{2}\rho_w g$

$2\rho_b = \rho_0 + \rho_w$

$\Rightarrow \frac{\rho_O}{\rho_w} = \frac{3}{5} = 0.6$

8. **Ans. (C)**

When steel balls in boat, volume of water displaced $V_1=(M+m)/\delta_\omega$

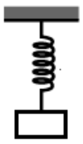
When steel ball drinks $V_2=\frac{M}{\delta_\omega}+\frac{M}{\sigma_s}$

$\sigma_s > \sigma_\omega,\; V_2 < V_1$ So level will falls.

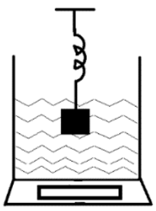
9. **Ans. (C)**

$W_m = 20N$

$W_w = 40N$



$F_s = 20$



$F_s = 20 - F_B = 16$

$F_B = 4N$

Now block exerts same buoyant force on water.

∴ Net force on water

$= Mg + F_B = 40 + 4 = 44N$

10. **Ans. (D)**

In case of minimum density of liquid, sphere will be floating while completely submerged

So $mg = B$

$m = \int_0^R \rho(4\pi r^2 dr) = B$

$= \rho_0 \int_0^R \left(1 - \frac{r^2}{R^2}\right) 4\pi r^2 dr = \frac{4}{3}\pi R^3 \rho_\ell g$

On Solving

$\rho_\ell = \frac{2\rho_0}{5}$

Fluid Mechanics : RACE-03

1. **Ans. (B)**

In flow volume = outflow volume

$\Rightarrow \frac{0.74}{60} = (\pi \times 4 \times 10^{-4}) \times \sqrt{2gh}$

$\Rightarrow \sqrt{2gh} = \frac{74 \times 100}{240\pi}$

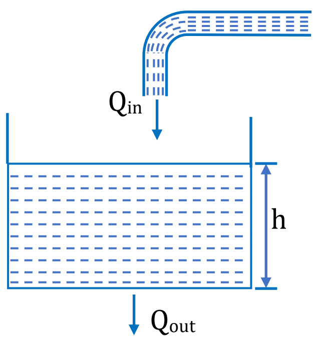
$\Rightarrow \sqrt{2gh} = \frac{740}{24\pi}$

$\Rightarrow 2gh = \frac{740 \times 740}{24 \times 24 \times 10} (\pi^2 = 10)$

$\Rightarrow h = \frac{74 \times 74}{2 \times 24 \times 24}$

$\Rightarrow h \approx 4.8m$

2. **Ans. (D)**



Since height of water column is constant therefore, water inflow rate (Q_{in})

= water outflow rate

$Q_{in} = 10^{-4}$ m^{3}s^{-1}

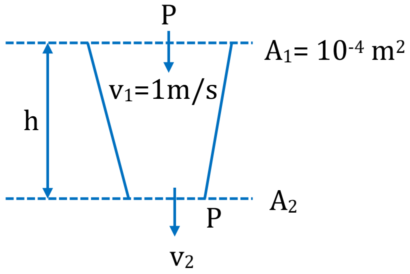
$Q_{out} = Au = 10^{-4} \times \sqrt{2gh}$

$10^{-4} = 10^{-4} \sqrt{20 \times h}$

$h = \frac{1}{20} m$

$h = 5$cm

3. Ans. (B)



$A_1v_1 = A_2v_2$

$10^{-4} \times 1 = A_2v_2$

$A_2v_2 = 10^{-4}$ ...(1)

$P + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho gh = P$

$v_2^2 = v_1^2 + 2gh$

$v_2 = \sqrt{v_1^2 + 2gh}$

$= \sqrt{1 + 2 \times 10 \times 0.15}$

$\frac{10^{-4}}{A_2} = 2$

$A_2 = 5 \times 10^{-5}$ m²

4. Ans. (C)

Rate of flow of water = $A_AV_A = A_BV_B$

$(40)V_A = (20)V_B$

$V_B = 2V_A$ .... (1)

Using Bernoulli's theorem

$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$

$P_A - P_B = \frac{1}{2}\rho(V_B^2 - V_A^2)$

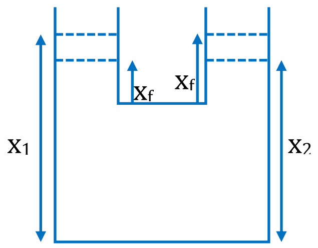
$700 = \frac{1}{2} \times 1000(4V_A^2 - V_A^2)$

$V_A = 0.68$ m/s = 68 cm/s

Rate of flow = A_AV_A

= (40) (68) = 2720 cm³/s

5. Ans. (C)



$U_i = (\rho Sx_1)g \cdot \frac{x_1}{2} + (\rho Sx_2)g \cdot \frac{x_2}{2}$

$U_f = (\rho S x_f) g . \frac{x_f}{2} \times 2$

By volume conservation

$Sx_1 + Sx_2 = S(2x_f)$

$x_f = \frac{x_1 + x_2}{2}$

$$\Delta U = \rho S g \left[\left(\frac{x_1^2}{2} + \frac{x_2^2}{2}\right) - x_f^2\right]$$

$$= \rho S g \left[\frac{x_1^2}{2} + \frac{x_2^2}{2} - \left(\frac{x_1 + x_2}{2}\right)^2\right]$$

$$= \frac{\rho S g}{2} \left[\frac{x_1^2}{2} + \frac{x_2^2}{2} - x_1 x_2\right]$$

$$= \frac{\rho S g}{4} (x_1 - x_2)^2$$

6. **Ans. (B)**

Applying Bernoulli's Equation

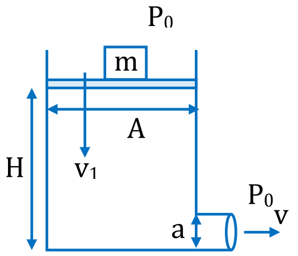
$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$P + \frac{1}{2}\rho v^2 = \frac{P}{2} + \frac{1}{2}\rho V^2$$

$$\frac{2P}{2\rho} + \frac{1}{2}\frac{\rho v^2}{\rho} \times 2 = V^2$$

$$\sqrt{\frac{P}{\rho} + v^2} = V$$

7. **Ans. (3)**



$m = 24\ kg$

$A = 0.4\ m^2$

$a = 1\ cm^2$

$H = 40 cm$

Using Bernoulli's equation

$$\Rightarrow \left(P_0 + \frac{mg}{A}\right) + \rho g H + \cancel{\frac{1}{2}\rho v_1^2}^{\,0}$$

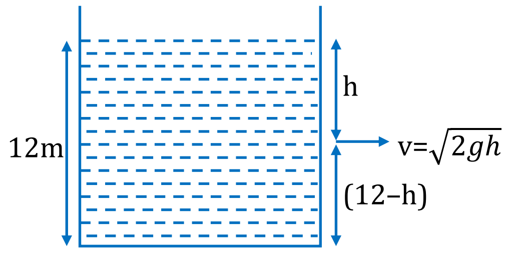
$$= P_0 + 0 + \frac{1}{2}\rho v^2 \qquad \text{... (1)}$$

$\Rightarrow$ Neglecting v_1

$$V = \sqrt{2gH + \frac{2mg}{A\rho}} \Rightarrow v = \sqrt{8 + 1.2}$$

$\Rightarrow$ v = 3.033 m/s $\quad \Rightarrow v \simeq \sim 3 m/s$

8. **Ans. (6)**



$R = \sqrt{2gh} \times \sqrt{\frac{(12-h)\times 2}{g}}$

$\sqrt{4h(12-h)} = R$

For maximum R

$\frac{dR}{dh} = 0$

$\Rightarrow h = 6\text{m}$

9. **Ans. (C)**

Apply Bernoulli's theorem between Piston and hole $P_A + \rho gh = P_0 + \frac{1}{2}\rho v_e^2$

Assuming there is no atmospheric pressure on piston

$\frac{5\times10^5}{\pi} + 10^3 \times 10 \times 10 = 1.01 \times 10^5 + \frac{1}{2} \times 10^3 \times v_e^2$

$v_e = 17.8$ m/s

10. **Ans. (2)**

$A_1V_1 = A_2 V_2$

$750 \times 10^{-4} V_1 = 500 \times 10^{-6} \times 0.3$

$V_1 = \frac{500\times3\times10^{-3}}{750} m/s$

$= 2 \times 10^{-3}$ m/s

$\frac{dh}{dt} = -2 \times 10^{-3} m/s$

Fluid Mechanics : RACE-04

1. **Ans. (C)**

Height of liquid rise in capillary tube

$h = \frac{2T\cos\theta_C}{\rho rg}$

$\Rightarrow h \propto \frac{1}{r}$

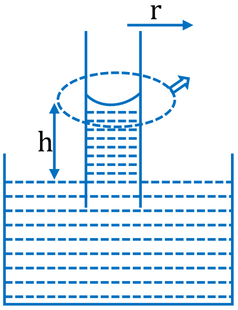
when radius becomes double height become half

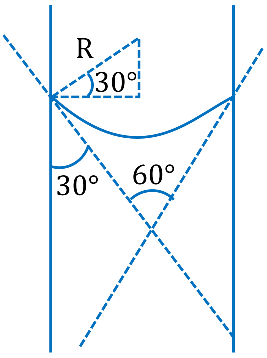
$\therefore h' = \frac{h}{2}$

Now, $M = \pi r^2 h \times \rho$

and $M' = \pi (2r)^2 (h/2) \times \rho = 2M$

2. Ans. (C)





$r \rightarrow$ radius of capillary

$R \rightarrow$ Radius of meniscus.

From figure, $\frac{r}{R} = \cos 30°$

$$R = \frac{2r}{\sqrt{3}} = \frac{2 \times 0.15 \times 10^{-3}}{\sqrt{3}}$$

$$= \frac{0.3}{\sqrt{3}} \times 10^{-3} m$$

Height of capillary

$$h = \frac{2T}{\rho g R} = 2\sqrt{3}\ T$$

$$h = \frac{2 \times 0.05}{667 \times 10 \times \left(\frac{0.3 \times 10^{-3}}{\sqrt{3}}\right)}$$

h = 0.087 m

3. Ans. (D)

We have

$$V_T = \frac{2}{9}\frac{r^2}{\eta}(\rho_0 - \rho_\ell)g \Rightarrow v_T \propto r^2$$

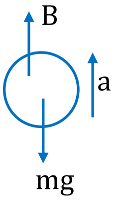
since mass of the sphere will be same

$$\therefore \quad \rho \frac{4}{3}\pi R^3 = 27.\frac{4}{3}\pi r^3 \rho \Rightarrow r = \frac{R}{3}$$

$$\therefore \quad \frac{v_1}{v_2} = \frac{R^2}{r^2} = 9$$

4. Ans. (C)

Volume $V = \frac{4\pi}{3} r^3 = \frac{4\pi}{3} \times (1)^3 = 4.19 cm^3$



a = 9.8 cm/s^2

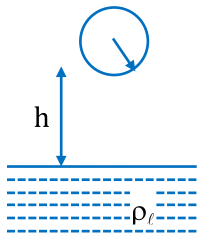
$B - mg = ma$

$$m = \frac{B}{g+a}$$

$$m = \frac{(V\rho_\omega g)}{g+a} = \frac{V\rho_\omega}{1+\frac{a}{g}}$$

$$= \frac{(4.19) \times 1}{1+\frac{9.8}{980}} = \frac{4.19}{1.01} = 4.15 gm$$

5. Ans. (B)



After falling through h, the velocity be

equal to terminal velocity

$$\sqrt{2gh} = \frac{2}{9}\frac{r^2 g}{\eta}(\rho_\ell - \rho)$$

$$\Rightarrow h = \frac{2}{81}\frac{r^4 g(\rho_\ell - \rho)^2}{\eta^2}$$

$$\Rightarrow h \propto r^4$$

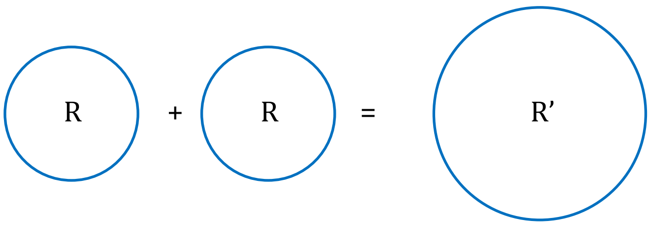
6. Ans. (A)

Excess pressure at common surface is given by

$$P_{ex} = 4T\, P_{ex} = 4T\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{4T}{r}$$

$$\therefore \frac{1}{r} = \frac{1}{a} - \frac{1}{b};\ r = \frac{ab}{b-a}$$

7. Ans. (A)



$$\frac{4}{3}\pi R^2 + \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R'^3$$

$$R' = 2^{\frac{1}{3}}R \qquad \text{... (i)}$$

$$A_i = 2[4\pi R^2]$$

$$A_f = 4\pi R'^2$$

$$\frac{U_i}{U_f} = \frac{A_i}{A_f} = \frac{2R^2}{2^{2/3}R^2} = 2^{1/3}$$

8. **Ans. (C)**

At terminal speed

$a \quad = 0$

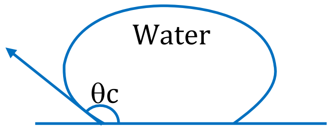
$F_{net} = 0$

$mg = F_v = 6\pi\,\eta Rv$

$v = \dfrac{mg}{6\pi\eta Rv}$

$$\frac{\rho_W \frac{4\pi}{3} R^3 g}{6\pi\eta R} = \frac{2\rho_W R^2 g}{9\eta} = \frac{400}{81} m/s = 4.94\ m/s$$

9. **Ans. (A)**



$\theta c > 90^o$

For water oil interface

10. **Ans. (A)**

Work done = $P\Delta V$

= $3 \times 10^5 \times 1600 \times 10^{-6}$

$= 480\ J$

Only 10% of heat is used in work done.

Hence $\Delta Q = 4800\ J$

The rest goes in internal energy, which is 90% of heat.

Change in internal energy = $0.9 \times 4800 = 4320\ J$