Units, Dimensions and Vectors

SOLUTIONS

Exercise-I (JEE-Main Pattern)

SECTION–A

1. Ans. (4)

[Surface tension] = [force/length] = $M^1L^0T^{-2}$

Suppose [surface tension] = $E^a\ V^b\ T^c$

$\therefore \quad M^1L^0T^{-2} = [M^1L^2T^{-2}]^a\ [L^1T^{-1}]^b\ [T]^c$

Matching dimensions of $M \Rightarrow a = 1$

Matching dimensions of $L \Rightarrow 2a + b = 0 \Rightarrow b = -2$

Matching dimensions of $T \Rightarrow -2a - b + c = -2 \Rightarrow c = -2$

$\therefore \quad$ [Surface tension] = $EV^{-2}T^{-2}$

2. Ans. (3)

We know that, a dimensionally incorrect equation is always incorrect.

3. Ans. (4)

$\sin 1° = 1 \times \dfrac{\pi}{180}$

$\cos 2° = 1$

4. Ans. (4)

$F = \dfrac{GM^2}{R^2} \qquad \Rightarrow G = [M^{-1}L^3T^{-2}]$

$E = h\nu \qquad \Rightarrow h = [ML^2T^{-1}]$

$C = [LT^{-1}]$

$t \propto G^xh^yC^z$

$[T] = [M^{-1}L^3T^{-2}]^x[ML^2T^{-1}]^y[LT^{-1}]^z$

$[M^0L^0T^1] = [M^{-x+y}L^{3x+2y+z}T^{-2x-y-z}]$

On comparing the powers of M, L, T

$-x + y = 0 \qquad \Rightarrow x = y$

$3x + 2y + z = 0 \qquad \Rightarrow 5x + z = 0 \qquad$ ...(i)

$-2x - y - z = 1 \qquad \Rightarrow 3x + z = -1 \qquad$ ...(ii)

On solving (i) & (ii)

$x = y = \dfrac{1}{2}, z = -\dfrac{5}{2}$

$t \propto \sqrt{\dfrac{Gh}{C^5}}$

5. **Ans. (4)**

$n_1u_1 = n_2u_2$

$$\frac{128kg}{m^3} = n_2\frac{50g}{(25cm)^3}$$

$$\frac{128\times1000g}{(100cm)^3} = n_2\frac{50g}{(25cm)^3}$$

$n_2 = 40$

6. **Ans. (2)**

$$\frac{F}{A} = Y\frac{\Delta\ell}{\ell}$$

$$[Y] = \frac{F}{A}$$

Now from dimension

$[a] = LT^{-2}$

$[V] = LT^{-1}$

$[V]^2 = [L][a]$

$$L^2 = \frac{V^4}{a^2}$$

$$[Y] = \frac{[F]}{[A]} = F^1\,V^{-4}\,a^2$$

7. **Ans. (2)**

$$F = \alpha\beta e^{\left(\frac{-x^2}{\alpha KT}\right)}$$

$$\left[\frac{x^2}{\alpha KT}\right] = M^0L^0T^0$$

$$\frac{L^2}{[\alpha]ML^2T^{-2}} = M^0L^0T^0$$

$\Rightarrow$ $[\alpha] = M^{-1}T^2$

$[F] = [\alpha]\,[\beta]$

$MLT^{-2} = M^{-1}T^2[\beta]$

$\Rightarrow$ $[\beta] = M^2LT^{-4}$

8. **Ans. (4)**

$L = \mu\,F^aA^bT^c$

$\Rightarrow$ $[L] = [MLT^{-2}]^a[LT^{-2}]^b[T]^c$

$\Rightarrow$ $a = 0, b = 1, c = 2$

9. **Ans. (1)**

$n_1u_1 = n_2u_2$

$n_1(10\,gm)(10\,cm)\,(0.1\,sec.)^{-2} = 1\,(kg)(m)\,(sec.)^{-2}$

$\Rightarrow$ $n_1 = 0.1\,N$

10. **Ans. (3)**

Let $x = \mu\, y^a z^b$

$\Rightarrow$ $[ML^2T^{-5}] = [MT^{-1}]^a[LT^{-2}]^b$

11. **Ans. (3)**

If an equation is dimensionally correct then it may or may not be correct.

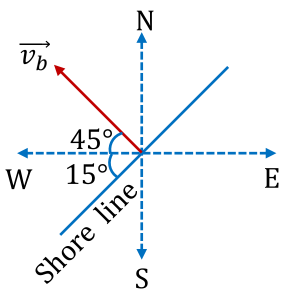
But if an equation is dimensionally incorrect then it will be incorrect for sure.

Ex: $S = ut + \frac{1}{2}at^2$

$[S] = [ut] = [at^2] = [L]$

This equation is dimensionally correct, but by this method we can't say that the exact equation will be $S = ut + \frac{1}{2}at^2$.

12. **Ans. (1)**

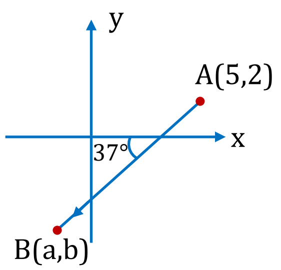


Component = 18 cos 60° = 9 km/hr.

13. **Ans. (4)**

$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

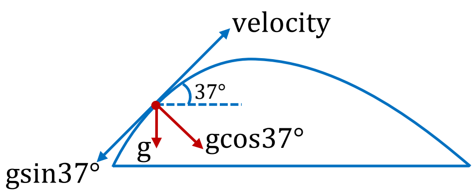
14. **Ans. (1)**



$\sqrt{(5-a)^2 + (2-b)^2} = 10$

$\tan 37° = \frac{2-b}{5-a}$

15. **Ans. (1)**



So, component of acceleration in the direction of velocity = $-g \sin 37° = -6 m/s^2$.

16. **Ans. (1)**

Angle between $\vec{r}$ *and* $\vec{z}$ is not 90°.

17. **Ans. (4)**

$R_1 = \sqrt{(2f)^2 + (3f)^2 + 2 \times 2f \times 3f \cos\theta}$

$R_2 = \sqrt{(2f)^2 + (6f)^2 + 2 \times 2f \times 6f \cos\theta} = 2R_1$

$2\sqrt{4f^2 + 9f^2 + 12f^2\cos\theta} = \sqrt{4f^2 + 36f^2 + 24f^2\cos\theta}$

$4(13f^2 + 12f^2\cos\theta) = (40f^2 + 24f^2\cos\theta)$

$3f^2 = -6f^2\cos\theta$

$\cos\theta = -\dfrac{1}{2}$

$\Rightarrow \quad \theta = 120°$

18. **Ans. (1)**

$(\alpha\vec{A} + \beta\vec{B}) \cdot \vec{C} = 0$

19. **Ans. (3)**

Let's say $\vec{x} = \vec{A} - \vec{B}$

$\vec{A} - \vec{B} = -\hat{\imath} + 5\hat{\jmath} + \hat{k} = \vec{x}$

Unit vector in direction of $(\vec{A} - \vec{B}) = \dfrac{-\hat{\imath} + 5\hat{\jmath} + \hat{k}}{3\sqrt{3}} = \hat{x}$

$\vec{C} = p\hat{\imath} + p\hat{\jmath} + 2p\hat{k}$

$\hat{C} = \dfrac{\hat{\imath} + \hat{\jmath} + 2\hat{k}}{\sqrt{6}}$

$\hat{x}.\hat{C} = \cos\theta$ where θ is angle between $\hat{x}$ and $\vec{c}$

$\dfrac{-1+5+2}{3\sqrt{3} \times \sqrt{6}} = \cos\theta$

$\cos\theta = \dfrac{6}{9\sqrt{2}} = \dfrac{\sqrt{2}}{3}$

$\theta = \cos^{-1}\left(\dfrac{\sqrt{2}}{3}\right)$

20. **Ans. (5)**

Displacement, $\overrightarrow{\Delta r} = \vec{r_f} - \vec{r_i} = (5-2)\hat{\imath} + (4-3)\hat{\jmath} + (3-4)\hat{k} = (3\hat{\imath} + \hat{\jmath} - \hat{k})m$

Net force, $\vec{F} = \vec{F_1} + \vec{F_2} = (2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})N$

Work done, $W = \vec{F} \cdot \overrightarrow{\Delta r} = 6 + 3 - 4 = 5\,J$

SECTION-B

1. **Ans. (4)**

$m = R\, v^x d^y a^z$ (where k is dimensionless)

$[M] = [LT^{-1}]^x [ML^{-3}]^y [LT^{-2}]^z$

Solve for x, y and z.

2. **Ans. (3350)**

$$D \approx r_m\theta = (384000)\left(\frac{0.5}{180/\pi}\right) = 3350km$$

3. **Ans. (3)**

Direction of P, $\hat{v}_1 = \pm\frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \pm\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$

Direction of Q, $\hat{v}_2 = \pm\frac{\vec{A}\times\vec{C}}{|\vec{A}\times\vec{C}|} = \pm\frac{2\hat{k}}{2} = \pm\hat{k}$

Angle between $\hat{v}_1$ *and* $\hat{v}_2$-

$$\cos\theta = \frac{\hat{v}_1.\hat{v}_2}{|\hat{v}_1||\hat{v}_2|} = \frac{\pm 1/\sqrt{3}}{(1)(1)} = \pm\frac{1}{\sqrt{3}}$$

$\Rightarrow \quad x = 3$

4. **Ans. (2)**

$$\left.\begin{array}{l} p \propto m^a \\ p \propto v^b \\ p \propto t_0^c \end{array}\right]$$

$p = k[M]^a\,[LT^{-1}]^b\,[T]^c$

$ML^2T^{-3} = M^aL^bT^{-b+c}$

$a = 1$

$b = 2$

$-b + c = -3$

$-2 + c = -3$

$c = -1$

$p \propto v^2 \Rightarrow p = kv^2$

5. **Ans. (2)**

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\sqrt{7}Q = \sqrt{P^2 + Q^2 + 2PQ\cos 60^0}$

$7Q^2 = P^2 + Q^2 + PQ$

$P^2 - 6Q^2 + PQ = 0$

$P^2 + QP - 6Q^2 = 0$

$$P = \frac{-Q \pm \sqrt{Q^2 + 24Q^2}}{2}$$

$$P = \frac{-Q \pm 5Q}{2} = 2Q$$

$$\frac{P}{Q} = 2$$

6. **Ans. (1)**

$|\vec{F}_1 - \vec{F}_2| = \sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos(1.8^0)}$

$1.8^0 = \frac{1.8}{180}\pi \text{ rad.} = \frac{\pi}{100}\text{rad}$

$\therefore \quad |\vec{F}_1 - \vec{F}_1| = \sqrt{1000 + 1000 - 2000\cos\frac{\pi}{100}}$

$= \sqrt{2000\left(1 - \cos\frac{\pi}{100}\right)} = \sqrt{2000 \times 2\sin^2\frac{\pi}{200}}$

$= \sqrt{4000\left(\frac{\pi^2}{40000}\right)} = 1$

7. **Ans. (4)**

$(\hat{\imath} + 2\hat{\jmath} - n\hat{k}).(4\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) = 0$

$\Rightarrow \quad 4 + 4 - 2n = 0$

$\Rightarrow \quad n = 4$

8. **Ans. (5)**

Displacement vector $(\vec{d}) = (2-1)\hat{\imath} + (0-1)\hat{\jmath} + (3-1)\hat{k}$

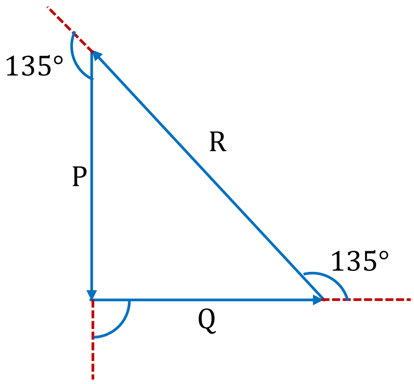
$\Rightarrow \quad \vec{d} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\vec{F} = 5\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\therefore$ Work done $= \vec{F}.\vec{d} = (5\hat{\imath} + 2\hat{\jmath} + \hat{k}).(\hat{\imath} - \hat{\jmath} + 2\hat{k})$

$= 5 - 2 + 2 = 5\text{J}$

9. **Ans. (3)**



10. **Ans. (6)**

Given

$\vec{F} = \vec{F}_1 + \vec{F}_2$

$F_1 + F_2 = 16$

$F = 8$

$F^2 = F_2^2 - F_1^2$

$\Rightarrow \quad 8^2 = (F_2 + F_1)(F_2 - F_1)$

$\Rightarrow \quad 64 = 16\,(F_2 - F_1)$

$\Rightarrow \quad F_2 - F_1 = 4$

$\Rightarrow \quad 16 - F_1 - F_1 = 4$

$\Rightarrow \quad F_1 = 6\ N$

Exercise-II (JEE-Main PYQs)

1. Ans. (3)

$|\vec{A}| = |\vec{B}| = x$ ...(i)

$|\vec{A}+\vec{B}| = n|\vec{A}-\vec{B}|$

$A^2 + B^2 + 2AB\cos\theta = n^2(A^2 + B^2 - 2AB\cos\theta)$ ...(ii)

$2x^2(1+\cos\theta) = n^2 \times 2x^2(1-\cos\theta)$

$1 + \cos\theta = n^2 - n^2\cos^2\theta$

$(1+n^2)\cos\theta = n^2 - 1$

So, $\cos\theta = \frac{n^2-1}{n^2+1}$

$\theta = \cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$

2. Ans. (2)

$\vec{r}_G = \frac{a}{2}\hat{\imath} + \frac{a}{2}\hat{k}$

$\vec{r}_H = \frac{a}{2}\hat{\jmath} + \frac{a}{2}\hat{k}$

$\vec{r}_H - \vec{r}_G = \frac{a}{2}(\hat{\jmath} - \hat{\imath})$

3. Ans. (3)

$|\vec{A}_1| = 3$; $|\vec{A}_2| = 5$; $|\vec{A}_1 + \vec{A}_2| = 5$

$|\vec{A}_1 + \vec{A}_2| = \sqrt{|\vec{A}_1|^2 + |\vec{A}_2|^2 + 2|\vec{A}_1||\vec{A}_2|\cos\theta}$

$5 = \sqrt{9 + 25 + 2\times 3\times 5\cos\theta}$

$\cos\theta = -\frac{9}{2\times 3\times 5} = -\frac{3}{10}$

$\Rightarrow$ $(2\vec{A}_1 + 3\vec{A}_2).(3\vec{A}_1 - 2\vec{A}_2) = 6|\vec{A}_1|^2 + 9\vec{A}_1.\vec{A}_2 - 4\vec{A}_1\vec{A}_2 - 6|\vec{A}_2|^2$

$= 54 + 5 \times 3 \times 5\left(-\frac{3}{10}\right) - 6 \times 25$

$= 54 - 150 - \frac{45}{2} = -118.5$

4. Ans. (3)

$[h] = M^1L^2T^{-1}$

$[C] = L^1T^{-1}$

$[G] = M^{-1}L^3T^{-2}$

$[f] = \sqrt{\frac{M^1L^2T^{-1}\times L^5T^{-5}}{M^{-1}L^3T^{-2}}} = M^1L^2T^{-2}$

5. Ans. (2)

Let $[E] = [P]^x [A]^y [T]^z$

$ML^2T^{-2} = [MLT^{-1}]^x [L^2]^y [T]^z$

$ML^2T^{-2} = M^x L^{x+2y} T^{-x+z}$

$\rightarrow \quad x = 1$

$\rightarrow \quad x + 2y = 2$

$1 + 2y = 2$

$y = \frac{1}{2}$

$\rightarrow \quad -x + z = -2$

$-1 + z = -2$

$z = -1$

$[E] = [PA^{1/2} T^{-1}]$

6. Ans. (4)

$$S = \frac{P}{A} = \frac{ML^2T^{-3}}{L^2} = MT^{-3}$$

7. Ans. (4)

$m \propto t^a v^b \ell^c$

$m \propto [T]^a [LT^{-1}]^b [ML^2T^{-1}]^c$

$M^1L^0T^0 = M^c L^{b+2c} T^{a-b-c}$

Comparing powers

$c = 1, b = -2, a = -1$

$m \propto t^{-1} v^{-2} \ell^1$

8. Ans. (2)

$[A] = [MLT^{-2}]$

$[B] = [L^{-1}]$

$[D] = [T^{-1}]$

$$\left[\frac{AD}{B}\right] = \frac{[MLT^{-2}][T^{-1}]}{[L^{-1}]}$$

$$\left[\frac{AD}{B}\right] = [ML^2T^{-3}]$$

9. Ans. (4)

$E = ML^2T^{-2}$

$L = ML^2T^{-1}$

$m = M$

$G = M^{-1}L^{+3}T^{-2}$

$$P = \frac{EL^2}{M^5G^2}$$

$$[P] = \frac{(ML^2T^{-2})(M^2L^4T^{-2})}{M^5(M^{-2}L^6T^{-4})} = M^0L^0T^0$$

10. Ans. (1)

Density = $[F^a L^b T^c]$

$[ML^{-3}] = [M^a L^a T^{-2a} L^b T^c]$

$[M^1 L^{-3}] = [M^a L^{a+b} T^{-2a+c}]$

$a = 1$; $a + b = -3$; $-2a + c = 0$

$1 + b = -3$; $c = 2a$

$b = -4$; $c = 2$

So, density = $[F^1 L^{-4} T^2]$.

11. Ans. (1)

Torque $\tau \rightarrow ML^2T^{-2}$ (III)

Impulse $I \Rightarrow MLT^{-1}$ (I)

Tension force $\Rightarrow MLT^{-2}$ (IV)

Surface tension $\Rightarrow MT^{-2}$ (II)

12. Ans. (2)

$[M] = [F]^a [T]^b [V]^c$

$[M^1] = [M^1L^1T^{-2}]^a [T^1]^b [L^1T^{-1}]^c$

$a = 1, b = 1, c = -1$

$\therefore \quad [M] = [FTV^{-1}]$

13. Ans. (180)

$\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$

$PQ \sin\theta = -PQ \sin\theta$

$\Rightarrow \quad \sin\theta = 0$

$\Rightarrow \quad \theta = 0, 180, 360.......$

Given $0° < \theta < 360°$

$\Rightarrow \quad \theta = 180°$

14. Ans. (2)

We know,

$\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}+\overrightarrow{OE}+\overrightarrow{OF}+\overrightarrow{OG}+\overrightarrow{OH}=\vec{0}$

By triangle law of vector addition, we can write

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$; $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$; $\overrightarrow{AE} = \overrightarrow{AO} + \overrightarrow{OE}$

$\overrightarrow{AF} = \overrightarrow{AO} + \overrightarrow{OF}$; $\overrightarrow{AG} = \overrightarrow{AO} + \overrightarrow{OG}$

$\overrightarrow{AH} = \overrightarrow{AO} + \overrightarrow{OH}$

Now $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} + \overrightarrow{AG} + \overrightarrow{AH}$

$= (7\overrightarrow{AO}) + \overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}+\overrightarrow{OE}+\overrightarrow{OF}+\overrightarrow{OG}+\overrightarrow{OH}$

$= (7\overrightarrow{AO}) + (\vec{0} - \overrightarrow{OA})$

$= (7\overrightarrow{AO}) + \overrightarrow{AO}$

$= 8\overrightarrow{AO} = 8(2\hat{\imath} + 3\hat{\jmath} - 4\hat{k})$

$= 16\hat{\imath} + 24\hat{\jmath} - 32\hat{k}$

15. Ans. (4)

$\vec{A}.\vec{B} = |\vec{A} \times \vec{B}|$

$AB \cos \theta = AB \sin \theta \Rightarrow \theta = 45°$

$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB \cos 45°}$

$= \sqrt{A^2 + B^2 - \sqrt{2}AB}$

16. Ans. (4)

$(A \cos \theta)\hat{B} = A\left(\frac{\vec{A}.\vec{B}}{AB}\right)\hat{B} = \frac{\vec{A}.\vec{B}}{B}\hat{B}$

$= \frac{2}{\sqrt{2}}\left(\frac{\hat{\imath} + \hat{\jmath}}{\sqrt{2}}\right) = \hat{\imath} + \hat{\jmath}$

17. Ans. (1)

$\overrightarrow{AD} = 2\overrightarrow{AO}$ ...(i)

A : $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} = (\overrightarrow{AO} + \overrightarrow{OB}) + (\overrightarrow{AO} + \overrightarrow{OC}) + 2\overrightarrow{AO}$

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} = 4\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}$ ...(ii)

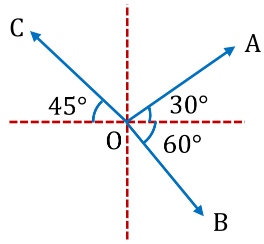
R: $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD}$

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{AD} = 2\overrightarrow{AD} = 4\overrightarrow{AO}$ ...(iii)

So, A is correct but R is not correct.

18. Ans. (1)

Let magnitude be equal to λ.



$\overrightarrow{OA} = \lambda[\cos 30° \hat{\imath} + \sin 30° \hat{\jmath}] = \lambda\left[\frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}\right]$

$\overrightarrow{OB} = \lambda[\cos 60° \hat{\imath} - \sin 60° \hat{\jmath}] = \lambda\left[\frac{1}{2}\hat{\imath} - \frac{\sqrt{3}}{2}\hat{\jmath}\right]$

$\overrightarrow{OC} = \lambda[\cos 45° (-\hat{\imath}) + \sin 45° \hat{\jmath}] = \lambda\left[-\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right]$

$\therefore \quad \overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC} = \lambda\left[\left(\frac{\sqrt{3}+1}{2} + \frac{1}{\sqrt{2}}\right)\hat{\imath} + \left(\frac{1-\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right)\hat{\jmath}\right]$

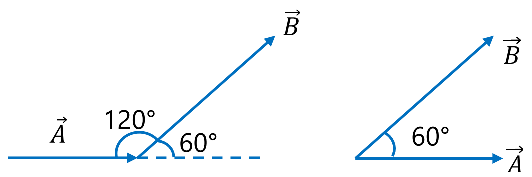
$\therefore$ Angle with x-axis

$$\tan^{-1}\left[\frac{\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{\sqrt{2}}}\right] = \tan^{-1}\left[\frac{1 - \sqrt{3} - \sqrt{2}}{\sqrt{3} + 1 + \sqrt{2}}\right]$$

19. Ans. (3)

Angle between $\vec{A}$ and $\vec{B}$, θ = 60°

Angle between $\vec{A}$ and $\vec{A} - \vec{B}$



$\tan \alpha = \frac{B \sin \theta}{A - B \cos \theta} = \frac{B\sqrt{\frac{3}{2}}}{A - B \times \frac{1}{2}}$

$\tan \alpha = \frac{\sqrt{3}B}{2A - B}$

20. Ans. (4)

$S = \frac{Q}{m\Delta T} = \frac{J}{Kg°C}$

$L = \frac{Q}{m} = \frac{J}{Kg}$

21. Ans. (2)

$[ML^{-3}] = [MLT^{-2}]^a[LT^{-1}]^b[T]^c$

$= [M^aL^{a+b}T^{-2a-b+c}]$

$a = 1, a + b = -3,$

$\Rightarrow \quad b = -4,$

also $\quad -2a - b + c = 0$

$c = -2$

22. Ans. (1)

$v = \lambda^a g^b \rho^c$

Using dimension formula

$\Rightarrow \quad [M^0L^1T^{-1}] = [L^1]^a [L^1T^{-2}]^b [M^1L^{-3}]^c$

$\Rightarrow \quad [M^0L^1T^{-1}] = [M^c L^{a+b-3c} T^{-2b}]$

$\therefore \quad c = 0, a + b - 3c = 1, -2b = -1 \Rightarrow b = \frac{1}{2}$

Now $a + b - 3c = 1$

$\Rightarrow \quad a + \frac{1}{2} - 0 = 1$

$\Rightarrow \quad a = \frac{1}{2}$

$\therefore \quad a = \frac{1}{2}, b = \frac{1}{2}, c = 0$

23. Ans. (4)

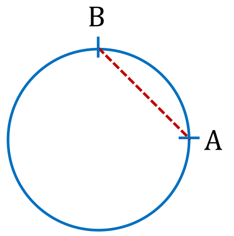
$$\left|\langle\vec{v}\rangle\right| = \frac{\left|\vec{r}_f - \vec{r}_i\right|}{\Delta t} = \frac{2R\cos\left[\frac{\pi-\theta}{2}\right]}{\frac{2\pi R}{3v}} = 3\cos 30^0$$

$1.5\sqrt{3}\ m/s$

Correct option is (4)

24. Ans. (1)

Let instantaneous velocity be v. time,



$AB = R\sqrt{2}$

$$t = \frac{\text{Arc Length}}{v} = \frac{2\pi\frac{R}{4}}{v} = \frac{\pi R}{2v}$$

Average velocity,

$$\langle v\rangle = \frac{AB}{t} = \frac{R\sqrt{2}(2v)}{\pi R} = \frac{2\sqrt{2}v}{\pi}$$

$$\Rightarrow \quad \frac{V}{\langle V\rangle} = \frac{\pi}{2\sqrt{2}}$$

Exercise-III (JEE Advanced Pattern)

SECTION-I

1. Ans. (B)

In ΔABO

$AO = 20\sqrt{3}\ \sin 30 = 10\sqrt{3}$

Comparing $Z = 20$

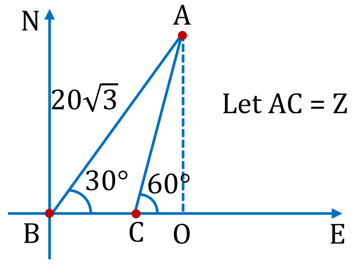
$CO = Z\cos 60 = 20 \times \frac{1}{2} = 10$

$BO = 20\sqrt{3}\ \cos 30 = 30$

$BC = BO - CO = 20$

In ΔAOC

$AO = Z\sin 60$

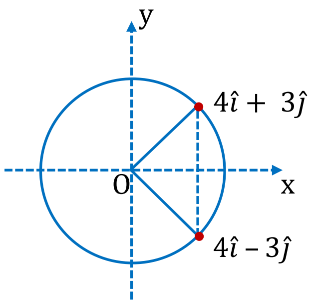


$\vec{r}_A = 30\hat{\imath} + 10\sqrt{3}\hat{\jmath}$

$\vec{r}_C = 20\hat{\imath}$

$\vec{r}_{C/A} = \vec{r}_C - \vec{r}_A = 20\hat{\imath} - (30\hat{\imath} + 10\sqrt{3}\hat{\jmath}) = (-10\hat{\imath} - 10\sqrt{3}\hat{\jmath})$

2. Ans. (C)



3. Ans. (B)

Displacement $\vec{r} = (14\hat{\imath} + 13\hat{\jmath} + 9\hat{k}) - (3i + 2\hat{\jmath} - 6\hat{k})$

$\vec{r} = 11\hat{\imath} + 11\hat{\jmath} + 15\hat{k}$

Work done $= \vec{F}\cdot\vec{r} = (4\hat{\imath} + \hat{\jmath} + 3\hat{k})\cdot(11\hat{\imath} + 11\hat{\jmath} + 15\hat{k})$

$= 44 + 11 + 45 = 100\,J$

SECTION-II

4. Ans. (BC)

$2|\hat{\imath} + x\hat{\jmath} + 3\hat{k}| = |4\hat{\imath}(4x-2)\hat{\jmath} + 2\hat{k}|$

$4(1 + x^2 + 9) = (16 + (4x-2)^2 + 4)$

$12x^2 - 16x - 16 = 0$

$x = \dfrac{4 \pm \sqrt{16+48}}{6}$

$x = 2, -\dfrac{2}{3}$

5. Ans. (BD)

$(\vec{A} + \vec{B} - \vec{C} + \vec{D})$ can not be zero if sum of magnitude of three smallest vectors is less than the magnitude of bigger vector.

6. **Ans. (CD)**

$$\left.\begin{aligned}\vec{A}+\vec{B}&=5\hat{\imath}\\ \vec{A}-\vec{B}&=\hat{\imath}+4\hat{\jmath}\end{aligned}\right\}\quad \text{Case} - \text{I} \qquad \text{find } \vec{A}\ \&\ \vec{B}$$

$$\left.\begin{aligned}\vec{A}+\vec{B}&=\hat{\imath}+4\hat{\jmath}\\ \vec{A}-\vec{B}&=5\hat{\imath}\end{aligned}\right\}\quad \text{Case} - \text{II} \qquad \text{find } \vec{A}\ \&\ \vec{B}$$

SECTION-III

7. **Ans. (B)**

8. **Ans. (C)**

9. **Ans. (A)**

(Solution for Q. No. 7 to 9)

$10\ m/s^2 = 5\dfrac{L}{T^2}$

$\dfrac{1}{2}\times(500kg)(400m/s)^2 = 2000\ ML^2T^{-2}$

$(500\ kg)\ (400\ m/s) = 100\ MLT^{-1}$

Solve above equation to get L, T and M.

10. **Ans. (C)**

Fever of a boy can't be measured. Temperature of his body is measured quantity.

11. **Ans. (A)**

$[R] = k[V_x]^a[V_y]^b[g]^c$

$\Rightarrow \quad L_x = k\left[\dfrac{L_x}{T}\right]^a\left[\dfrac{L_y}{T}\right]^b\left[\dfrac{L_y}{T^2}\right]^c \quad \Rightarrow \quad a = 1, b = 1, c = -1$

$\therefore \quad R = k\dfrac{V_xV_y}{g}$

12. **Ans. (B)**

From directed dimension concept,

$[S] = k[U]^a[V]^b[D]^c$

$L_x = k\left(\dfrac{L_y}{T}\right)^a\left(\dfrac{L_x}{T}\right)^b(L_y)^c$

$\therefore \quad a = -1, b = 1$ and $c = 1$

$\therefore \quad S = k\dfrac{VD}{U}$

SECTION-IV

13. **Ans. (BC)**

$F = 10^6\ dyne = 10\ N$

$M = 1\ tonne = 10^3\ kg$

$L = 0.1\ mm = 10^{-4}\ m$

$T = 0.1\ sec. = 10^{-1}\ sec.$ (III) (iv) (P)

$F = (MLT^{-2}) = 10^3 \times 10^{-4}(10^{+2})$

$= 10\ N$

14. Ans. (BD)

$E = 10^9$ erg = 100 Joule

$L = 10\ m$

$M = 10\ gm = 10^{-2}\ kg$

$T = 0.1\ sec. = 10^{-1}\ sec.$ (I) (iii) (P)

$E = ML^2T^{-2} = (10^{-2})\ (10^2)\ (10^2) = 100$ Joule

15. Ans. (B)

$L = 10\ cm = 10^{-1}\ m$

$M = 100\ gm = 10^{-1}\ kg$

$T = 10\ ms = 10^{-2}$ sec. (II) (i) (Q)

$\eta = ML^{-1}T^{-1} = 10^{-1}\ (10)\ (10^2) = 100$ Poiseuille

SECTION-V

16. Ans. (A-Q), (B-S), (C-R), (D-R)

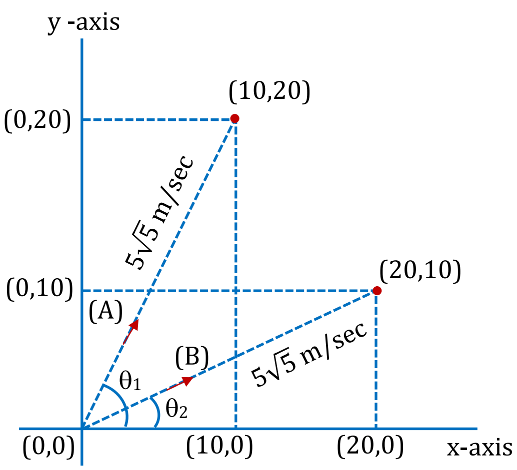
Use $n_1u_1 = n_2u_2$

(A) 1 kg.$m.\ sec^{-2} = n_2$(marg)(retem) $(pal)^{-2}$

(B) $1kg.m^2.sec^{-2} = n_2$(marg)(retem)^2 $(pal)^{-2}$

(C) $1kg.\ m^2.sec^{-3} = n_2$(marg)(retem)^2 $(pal)^{-3}$

17. Ans. (A-Q), (B-R), (C-T), (D-P)



(A) Velocity of particle $A = 5\sqrt{5}\ (\cos\theta_1\,\hat{\imath} + \sin\theta_1\,\hat{\jmath})$

$= 5\sqrt{5} \times \frac{10}{\sqrt{100+400}}\hat{\imath} + 5\sqrt{5} \times \frac{20}{\sqrt{100+400}}\hat{\jmath}$

$= 5\hat{\imath} + 10\hat{\jmath}$

(B) Velocity of particle B

$= 5\sqrt{5}\cos\theta_2\,\hat{\imath} + 5\sqrt{5}\ \sin\theta_2\,\hat{\jmath}$

$= 5\sqrt{5} \times \frac{20}{\sqrt{100+400}}\hat{\imath} + \frac{5\sqrt{5}\times 10}{\sqrt{100+400}}\hat{\jmath} = 10\hat{\imath} + 5\hat{\jmath}$

(C) $\vec{v}_B = 2\sqrt{2}\frac{(-10\hat{\imath} + 10\hat{\jmath})}{10\sqrt{2}}$

(D) Velocity = –(initial velocity vector of A) $= -(5\hat{\imath} + 10\hat{\jmath})$

18. Ans. (A-S), (B-P), (C-R), (D-T)

By Triangle law of vector addition.

(A) $\overrightarrow{PS} = -2(\vec{B} + \vec{A})$

(B) $\overrightarrow{PT} = -2\vec{B} - 3\vec{A}$

(C) $\overrightarrow{RS} = -\vec{B} - 2\vec{A}$

(D) $\overrightarrow{TS} = \vec{A}$

Exercise-IV (JEE Advanced PYQs)

1. Ans. (C)

(P) by $E = \frac{f}{2}kT$ $\Rightarrow$ $[K] = [ML^2T^{-2}K^{-1}]$

(Q) by $F = \eta A\frac{\Delta v}{\Delta x}$ $\Rightarrow$ $[\eta] = [ML^{-1}T^{-1}]$

(R) by $E = h\nu$ $\Rightarrow$ $[h] = [ML^2T^{-1}]$

(S) by $Q = \frac{kA(\Delta\theta)t}{\ell}$ $\Rightarrow$ $[k] = [MLT^{-3}K^{-1}]$

2. Ans. (3)

$[d] = (S)^x(\rho)^y(f)^z$

$L = (M^1L^0T^{-3})^x(M^1L^{-3})^y(T^{-1})^z$

$L = M^{x+y}L^{-3y}T^{-3x-z}$

$-3y = 1$ $\quad$ $x + y = 0$ $\quad$ $3x + z = 0$

$y = -\frac{1}{3}$ $\quad$ $x - \frac{1}{3} = 0$ $\quad$ $z = -3x$

$x = \frac{1}{3}$ $\quad$ $z = -1$

$L = (S)^{1/3}(\rho)^{-1/3}(f)^{-1}$

$n = 3$

3. Ans. (AC)

Using $C = \frac{1}{\sqrt{\mu_0\epsilon_0}}$ and $R = \sqrt{\frac{\mu_0}{\epsilon_0}}$ we can check the correctness.

(A) $\mu_0 I^2 = \varepsilon_0 V^2$

$\frac{\mu_0}{\epsilon_0} = \frac{V^2}{I^2} = R^2$

$\therefore$ $R^2 = R^2$ correct

(B) $\varepsilon_0 I = \mu_0 V$

$\frac{\epsilon_0}{\mu_0} = \frac{V}{I}$

$\frac{1}{R^2} = R$ not correct

(C) $I = \varepsilon_0 cV$

$\frac{I}{V} = \epsilon_0\, c = \frac{\epsilon_0}{\sqrt{\mu_0\,\epsilon_0}}$

$\frac{1}{R} = \sqrt{\frac{\epsilon_0}{\mu_0}} = \frac{1}{R}$ correct

(D) $\mu_0 cI = \varepsilon_0 V$

$\frac{\mu_0 C}{\epsilon_0} = \frac{V}{I}$

$\frac{\mu_0}{\epsilon_0}\frac{1}{\sqrt{\mu_0\,\epsilon_0}} = R$

$\therefore$ $\frac{1}{\epsilon_0}\sqrt{\frac{\mu_0}{\epsilon_0}} = R \Rightarrow \frac{R}{\epsilon_0} = R$ incorrect

4. Ans. (C)

Let vector from point P to point S be $\vec{C}$

$\Rightarrow \quad \vec{C}=b|\vec{R}|\hat{R} = b|\vec{R}|\left(\frac{\vec{R}}{|\vec{R}|}\right) = b\vec{R} = b(\vec{Q}-\vec{P})$

From triangle rule of vector addition

$$\vec{P}+\vec{C}=\vec{S}$$

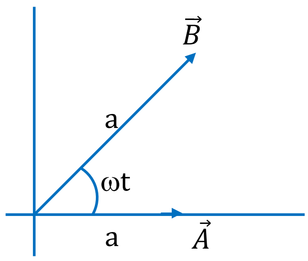
$$\vec{P}+b(\vec{Q}-\vec{P})=\vec{S}$$

$\Rightarrow \quad \vec{S}=(1-b)\vec{P}+b\vec{Q}$

5. Ans. (2 [1.99, 2.01])

$|\vec{A}+\vec{B}| = 2a\cos\frac{\omega t}{2}$

$|\vec{A}-\vec{B}| = 2a\sin\frac{\omega t}{2}$



So, $\quad 2a\cos\frac{\omega t}{2} = \sqrt{3}\left(2a\sin\frac{\omega t}{2}\right)$

$$\tan\frac{\omega t}{2} = \frac{1}{\sqrt{3}}$$

$$\frac{\omega t}{2} = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6} \ldots$$

$$t = 2, 14, 26 \ldots \qquad \left(\because \omega = \frac{\pi}{6}\right)$$

So, the given condition occurs for the first time at $t = 2$ seconds.

6. Ans. (C)

We have $\frac{E}{C} = B$

$\therefore \quad [B] = \frac{[E]}{[C]} = [E]L^{-1}T^{1}$

$\Rightarrow \quad [E] = [B]\,[L][T^{-1}]$

7. Ans. (D)

We have,

$$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$\therefore \quad [C^2] = \left[\frac{1}{\mu_0 \epsilon_0}\right]$

$\Rightarrow \quad L^2T^{-2} = \frac{1}{[\mu_0][\epsilon_0]}$

$\Rightarrow \quad [\mu_0] = [\epsilon_0]^{-1}[L]^{-2}[T]^{2}$

8. Ans. (ABD)

Mass = $M^0L^0T^0$

$MVr = M^0L^0T^0$

$M^0\frac{L^1}{T^1}.L^1 = M^0L^0T^0$

$L^2 = T^1$ ...(1)

Force = $M^1L^1T^{-2}$ (in SI)

$= M^0\,L^1\,L^{-4}$ (In new system from equation (1))

$= L^{-3}$

Energy = $M^1L^2T^{-2}$ (In SI)

$= M^0L^2L^{-4}$ (In new system from equation (1))

$= L^{-2}$

Power = $\frac{\text{Energy}}{\text{Time}}$

$= M^1L^2T^{-3}$ (in SI)

$= M^0L^2L^{-6}$ (In new system from equation (1))

$= L^{-4}$

Linear momentum = $M^1L^1T^{-1}$ (in SI)

$= M^0L^1L^{-2}$ (In new system from equation (1))

$= L^{-1}$

9. Ans. (AB)

Given $L = x^\alpha$ ...(1)

$LT^{-1} = x^\beta$ ...(2)

$LT^{-2} = x^p$ ...(3)

$MLT^{-1} = x^q$ ...(4)

$MLT^{-2} = x^r$ ...(5)

$\Rightarrow \frac{(1)}{(2)}\ T = x^{\alpha-\beta}$

From (3)

$\frac{x^\alpha}{x^{2(\alpha-\beta)}} = x^p$

$\Rightarrow \alpha + p = 2\beta \Rightarrow$ (A)

From (4)

$M = x^{q-\beta}$

From (5)

$\Rightarrow x^q = x^r\ x^{\alpha-\beta}$

$\Rightarrow \alpha + r - q = \beta$ ...(6)

Replacing value of 'α' in equation (6) from (A)

$2\beta - p + r - q = \beta$

$\Rightarrow p + q - r = \beta \Rightarrow$ (B)

Replacing value of 'β' in equation (6) from (A)

$2\alpha + 2r - 2q = \alpha + p$

$\alpha = p + 2q - 2r$

Answer is (AB).

10. Ans. (BD)

$\vec{S} = [\vec{E} \times \vec{B}]\frac{1}{\mu_0}$

$\vec{S}$ is poynting vector which denotes flow of energy per unit area per unit time.

$S = \frac{\text{watt}}{m^2}$

Hence B, D are correct.

11. Ans. (4)

$B = e^{\alpha}(m_e)^{\beta}h^{\gamma}k^{\delta}$

$[B] = [e^{\alpha}][m_e]^{\beta}[h]^{\gamma}[k^{\delta}]$

$[M^1T^{-2}A^{-1}] = [AT]^{\alpha}[M]^{\beta}[ML^2T^{-1}]^{\gamma}[ML^3A^{-2}T^{-4}]^{\delta}$

$M^1T^{-2}A^{-1} = M^{\beta+\gamma+\delta}L^{2\gamma+3\delta}T^{\alpha-\gamma-4\delta}A^{\alpha-2\delta}$

Compare : $\beta + \gamma + \delta = 1$; $2\gamma + 3\delta = 0$, $\alpha - \gamma - 4\delta = -2$, $\alpha - 2\delta = -1$

On solving $\alpha = 3$, $\beta = 2$, $\gamma = -3$, $\delta = 2$

$\alpha + \beta + \gamma + \delta = 4$

12. Ans. (A)

$Y = c^{\alpha}h^{\beta}G^{\gamma}$

$ML^{-1}T^{-2} = (LT^{-1})^{\alpha}(ML^2T^{-1})^{\beta}(M^{-1}L^3T^{-2})^{\gamma}$

$1 = \beta - \gamma$ ...(1)

$-1 = \alpha + 2\beta + 3\gamma$ ...(2)

$-2 = -\alpha - \beta - 2\gamma$ ...(3)

$-3 = \beta + \gamma$

$1 = \beta - \gamma$

$-2 = 2\beta$ $\Rightarrow \beta = -1, \gamma = -2$

$-1 = \alpha - 2 - 6$ $\therefore \alpha = 7$