CHAPTER 3

Nuclear Physics

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Nucleus and Nuclear Structure
(b) Properties of Nuclear Forces
(c) Mass Defect
(d) Binding Energy
(e) Nuclear Stability
(f) Radioactivity
(g) Nuclear Radiations
(h) Successive Disintegration
(i) Nuclear Reactions
(j) Energetics of Nuclear Reactions
(k) Alpha Decay
(l) Beta Decay
(m) Pauli's Neutrino Hypothesis
(n) Beta Decay Spectrum
(o) Gamma Decay
(p) Classification of Nuclear Reactions
(q) Nuclear Fission
(r) Chain Reaction and Nuclear Fusion.

All this is followed by an Exercise Set (fully solved) which contains questions as per the latest JEE pattern. At the end of Exercise Set, a collection of problems asked previously in JEE Main are also given.

NUCLEUS AND NUCLEAR STRUCTURE

Nucleus was discovered by Rutherford. The nucleus of an atom consists of two types of particles, protons and neutrons together called as Nucleons.

A proton has a positive charge equal to $+e = 1.6 \times 10^{-19}$ C and a mass equal to $m_p = 16726231 \times 10^{-27}$ kg. It was discovered by Goldstein.

A neutron has no charge i.e. is a neutral particle and its mass is $m_n = 1.6749286 \times 10^{-27}$ kg. *Thus, a neutron is slightly heavier than a proton.*

However, for problems (unless and until specified), we take

$$m_p \approx m_n = 1.6726231 \times 10^{-27} \text{ kg}$$

The total number of protons in the nucleus is called its atomic number (Z). The total number of nucleons (protons plus neutrons) in the nucleus is called its mass number (A). If N is the number of neutrons in the nucleus, then,

$$A = Z + N$$

No electrons are present inside the nucleus.

If X is the chemical symbol for an element then its nucleus is represented as ${}^{A}_{Z}X$ or as ${}_{Z}X^{A}$

ATOMIC MASS UNIT (u OR amu)

Atomic and nuclear masses are generally expressed in terms of atomic mass unit (a.m.u.). It is the nearest integer value of mass represented in a.m.u. (atomic mass unit).

$$1 \text{ amu} = \frac{1}{12}\left(\begin{array}{c}\text{Mass of C}^{12}\text{ atom at} \\ \text{rest in ground state}\end{array}\right)$$

$$\Rightarrow \quad 1 \text{ amu} = 1 \text{ u} = 1.6603 \times 10^{-27} \text{ kg} = 931.478 \text{ MeV/c}^2$$

In general, we take

Mass of proton (m_p) = mass of neutron (m_n) = 1 a.m.u.

ISOTOPES

Nuclides having the same charge number (Z) but different mass number (A) are called isotopes. All the isotopes are chemically similar and hence they occupy the same position in the periodic table.

ISOBARS

Nuclides having the same mass number (A) but different atomic number (Z) are called isobars.

ISOTONES

Nuclides having the same neutron number $(A-Z)$ but different mass number (A) are called isotones.

ILLUSTRATION 1

The three stable isotopes of neon Ne^{20}, Ne^{21} and Ne^{22} have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u respectively. Obtain the average atomic mass of neon.

SOLUTION

Average atomic mass of neon

$$A_{\text{neon}}=\frac{p_1A_1+p_2A_2+p_3A_3}{p_1+p_2+p_3}$$

$$\Rightarrow \quad A_{\text{neon}}=\frac{90.51\times 19.99+0.27\times 20.99+9.22\times 21.99}{100}$$

$$\Rightarrow \quad A_{\text{neon}}=\frac{1809.29+5.67+202.75}{100}=20.184$$

NUCLEAR RADIUS

Assuming that the nuclei are spherical, their radii are well represented by the empirical formula

$$R=R_0A^{1/3}$$

where $R_0=1\cdot 1\times 10^{-15}\ \text{m}=1\cdot 1\ \text{fermi}(\text{fm})$

ILLUSTRATION 2

Calculate the nuclear radius of ^{125}Fe knowing that the nuclear radius of ^{27}Al is 3.6 fermi.

SOLUTION

$$R=R_0A^{1/3}$$

$$\Rightarrow \quad \frac{R_{\text{Fe}}}{R_{\text{Al}}}=\left(\frac{A_{\text{Fe}}}{A_{\text{Al}}}\right)^{1/3}=\left(\frac{125}{27}\right)^{1/3}$$

$$\Rightarrow \quad R_{\text{Fe}}=\frac{5}{3}R_{\text{Al}}=\frac{5}{3}\times 3.6=6\ \text{fermi}$$

NUCLEAR DENSITY

The density of a nucleus of mass M and mass number A can be written as

$$\rho=\frac{M}{\frac{4}{3}\pi R^3}=\frac{A\ \text{amu}}{\frac{4}{3}\pi\left(R_0A^{1/3}\right)^3}=\frac{\left(A\times 1.67\times 10^{-27}\right)\text{kg}}{\frac{4}{3}\pi\left(1\cdot 1\times 10^{-15}\ \text{m}\right)^3A}$$

$$\Rightarrow \quad \rho\cong 2.9\times 10^{17}\ \text{kgm}^{-3}$$

This comes out to be $\sim 10^{17}\ \text{kgm}^{-3}$, which is extremely large as compared to the density of ordinary matter which is $\sim 10^3\ \text{kgm}^{-3}$.

Conceptual Note(s)

Since density is independent of mass number A, so all nuclei same density. So, whether two nuclei are isobars, or isotopes or isotones. They must possess same density.

THE NUCLEAR FORCE

The force which binds the protons and neutrons inside the nucleus is neither electrical nor gravitational. It is an entirely different kind of force called the strong nuclear force. This force is extremely complex in nature. Some of its main characteristics are mentioned below.

(a) Nuclear forces are attractive in nature. Their magnitude, which depends upon inter nucleon distance is of very high order.

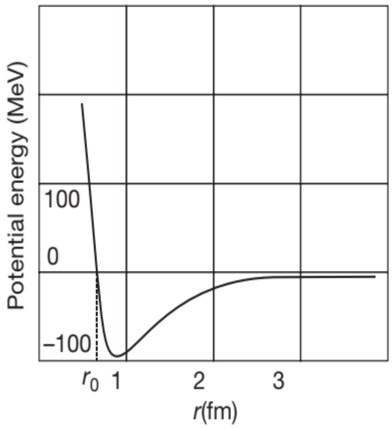
(b) Nuclear forces are charge independent. Nature of force remains the same whether we consider force between two protons, between two neutrons or between a proton and a neutron.

(c) These are short range forces. Nuclear forces operate between two nucleons situated in close neighbourhood only.

(d) Nuclear forces decrease very quickly with distance between two nucleons. Their rate of decrease is much more rapid than that of inverse square law forces (Coulombic forces). The forces become negligible when the nucleons are more than 10^{-14} cm apart.

(e) Nuclear forces are spin dependent. Nucleons having parallel spin are more strongly bound to each other than those having anti-parallel spin.

(f) The nuclear forces are very short-range forces. From a rough plot of the potential energy between two nucleons as a function of their separation is shown in figure.



For a separation greater than r_0, the force is attractive and for separations less than r_0, the force is strongly repulsive. The potential energy is a minimum at a distance r_0 about 0.8 fm from this the force is attractive for distances larger than 0.8 fm and repulsive for distances less than 0.8 fm. Nuclear forces are negligible when distance between nucleons is more than 10 fm.

YUKAWA THEORY OR MESON THEORY OF NUCLEAR FORCES

According to this theory, a nucleon consists of a core surrounded by a cloud of mesons, which may be charged or neutral. The mesons constantly get exchanged, back and forth, between two neighbouring nucleons. In this process the two nucleons remain bound to each other.

Proton-proton Interaction

It is the force between two neighbouring protons. It is due to the exchange of π^0 meson between them. It is represented in the form of a reaction as follows.

Proton P_1 emits π^0 and gets converted into a proton P_1', having different co-ordinates. So,

$$P_1 \longrightarrow P_1' + \pi^0$$

This π^0 is absorbed by P_2 which also gets converted into a new proton P_2'. Hence,

$$P_2 + \pi^0 \longrightarrow P_2'$$

Neutron-Neutron Interaction

It is the force between two neutrons. It is also due to exchange of π^0 between them

$$N_1 \longrightarrow N_1' + \pi^0$$

$$N_2 + \pi^0 \longrightarrow N_2'$$

Proton-Neutron Interaction

It is the force between a proton and a neutron situated close to each other. It can be take place in following two ways.

(a) Due to exchange of π^+ meson.

Proton emits π^+ meson and gets converted to a neutron. Another neutron absorbs this π^+ meson to get converted to a proton. So,

$$P \longrightarrow N' + \pi^+$$

$$N + \pi^+ \longrightarrow P'$$

(b) Due to exchange of π^- meson.

π^- meson is emitted by the neutron which is absorbed by the proton.

$$N \longrightarrow P' + \pi^-$$

$$P + \pi^- \longrightarrow N'$$

Dog-bone Analogy

The above interactions can be explained with the dog bone analogy according to which we consider the two interacting nucleons to be two dogs having a common bone clenched in between their teeth very firmly. Each one of these dogs wants to take the bone and hence they cannot be separated easily. They seem to be bound to each other with a strong attractive force (which is the bone) though the dogs themselves are strong enemies. The meson plays the same role of the common bone in between two nucleons.

MASS ENERGY EQUIVALENCE

According to Einstein the mass and energy are equivalent i.e., mass can be converted into energy and vice-versa. The mass energy equivalence relation is

$$\Delta E = c^2 \Delta m$$

Accordingly, annihilation of 1 kg mass is equivalent to energy given by

$$\Delta E = 1 \times \left(3 \times 10^8\right)^2$$

$$\Rightarrow \quad \Delta E = 9 \times 10^{16}\text{ J}$$

Energy corresponding to annihilation of 1 amu of mass is

$$\Delta E = \left(1.67 \times 10^{-27}\right)\left(9 \times 10^{16}\right)\text{ J}$$

$$\Rightarrow \quad \Delta E = \frac{\left(1.67 \times 10^{-27}\right)\left(9 \times 10^{16}\right)}{1.6 \times 10^{-19}}\text{ eV}$$

$$\Rightarrow \quad \Delta E = 931.5\text{ MeV}$$

MASS DEFECT AND BINDING ENERGY

It has been observed that the mass of a nucleus is always less than the mass of its constituent nucleons (i.e., protons + neutrons). *The difference between the total mass of the nucleons and the mass of the nucleus is called the mass defect* (Δm).

This is due to the fact that when nucleons combine to form a nucleus, the binding energy of nucleons is liberated. *The binding energy is equal to the work that must be done to split the nucleus into particles constituting it.*

Let $m\left({}_Z X^A\right)$ be the mass of nucleus, m_p be the mass of proton and m_n be the mass of neutron, then, the mass defect (Δm) is given by

$$\Delta m = \begin{pmatrix}\text{mass of constituent} \\ \text{nucleons}\end{pmatrix} - \begin{pmatrix}\text{mass of} \\ \text{nucleus}\end{pmatrix}$$

$$\Rightarrow \quad \Delta m = \left[Zm_p + (A-Z)m_n\right] - m\left({}_Z X^A\right)$$

This mass defect exists in the form of binding energy of nucleus, which is responsible for binding the nucleons into a small nucleus. So,

Binding energy of nucleus = $(\Delta m)c^2$ = $(931.5)\Delta m$ (in MeV) and binding energy per nucleon = $\dfrac{(\Delta m)c^2}{A}$

If the masses are taken in atomic mass unit, the binding energy is given by

$$\text{B.E.} = \left[\left(Zm_p + (A-Z)m_n\right) - m\left({}_Z X^A\right)\right] 931.5\text{ MeV}$$

Dividing the binding energy by the number of nucleons A, in the nucleus, we obtain the binding energy per nucleon. The stability of a nucleus is measured by the binding energy per nucleon.

Conceptual Note(s)

(a) It is not the binding energy which accounts for the stability of nucleus.

(b) The stability of nucleus is governed by binding energy per nucleon. The more the binding energy per nucleon, the more stable a nucleus is.

BINDING ENERGY (BE): REVISITED

So, binding energy is the minimum energy required to break the nucleus into its constituent particles **OR** Binding energy is the amount of energy released during the formation of nucleus by its constituent particles and bringing them from infinite separation.

$$\text{Binding Energy } (BE) = \Delta mc^2$$

$$\Rightarrow \quad BE = \Delta m(\text{in amu}) \times 931.5\text{ MeV/amu}$$

$$\Rightarrow \quad BE = \Delta m \times 931.5\text{ MeV}$$

$$\Rightarrow \quad BE = \Delta m \times 931\text{ MeV}$$

Problem Solving Technique(s)

If binding energy per nucleon is more for a nucleus, then it is more stable. So, if $\left(\dfrac{BE_1}{A_1}\right) > \left(\dfrac{BE_2}{A_2}\right)$, then nucleus 1 would be more stable than nucleus 2.

ILLUSTRATION 3

During an experiment, following data is available about three nuclei P, Q and R. Arrange them in the decreasing order of stability.

	P	Q	R
Atomic mass number (A)	10	5	6
Binding energy (MeV)	100	60	66

SOLUTION

For P, we have $\left(\dfrac{BE}{A}\right)_P = \dfrac{100}{10} = 10$

For Q, we have $\left(\dfrac{BE}{A}\right)_Q = \dfrac{60}{5} = 12$

For R, we have $\left(\dfrac{BE}{A}\right)_R = \dfrac{66}{6} = 11$

$\Rightarrow$ Stability order is $Q > R > P$

ILLUSTRATION 4

A nucleus has binding energy of 100 MeV. It further releases 10 MeV energy. Find the new binding energy of the nucleus.

SOLUTION

After releasing 10 MeV, it will become more stable and hence the binding energy of the nucleus will increase. So, new binding energy of the nucleus is

$$(BE)_{\text{new}} = 100 + 10 = 110 \text{ MeV}$$

ILLUSTRATION 5

Find the binding energy of ${}^{56}_{26}\text{Fe}$. Atomic mass of ${}^{56}\text{Fe}$ is 55.9349 u and that of ${}^{1}\text{H}$ is 1.00783 u. Mass of neutron = 1.00867 u.

SOLUTION

The number of protons in ${}^{56}_{26}\text{Fe} = 26$ and the number of neutrons is $(A - Z) = 56 - 26 = 30$.

The binding energy of ${}^{56}_{26}\text{Fe}$ is

$$BE = (26 \times 1.00783\,\text{u} + 30 \times 1.00867\,\text{u} - 55.9349\,\text{u})c^2$$

$$\Rightarrow \quad BE = (0.52878\ \text{u})c^2$$

$$\Rightarrow \quad BE = (0.52878\ \text{u})(931\ \text{MeV/u}) = 492\ \text{MeV}$$

ILLUSTRATION 6

Calculate the binding energy of ${}^{12}_{6}\text{C}$. Also find the binding energy per nucleon. Given that mass of ${}^{1}_{1}\text{H} = 1.0078$ u, mass of ${}^{1}_{0}n = 1.0087$ u and mass of ${}^{12}_{6}\text{C} = 12.00004$ u.

SOLUTION

One atom of ${}^{12}_{6}\text{C}$ consists of 6 protons, 6 electrons and 6 neutrons. The mass of the un-combined protons and electrons is the same as that of six ${}^{1}_{1}\text{H}$ atoms (if we ignore the very small binding energy of each proton-electron pair).

Mass of six ${}^{1}_{1}\text{H}$ atoms $= 6 \times 1.0078 = 6.0468$ u

Mass of six neutrons $= 6 \times 1.0087 = 6.0522$ u

Total mass of component particles $= 12.0990$ u

Mass of ${}^{12}_{6}\text{C}$ atom $= 12.00004$

Mass defect is $\Delta m = 0.0990$ u

Binding energy is $BE = (931)(0.099) = 92$ MeV

Binding energy per nucleon is

$$\frac{BE}{A} = \frac{92}{12} = 7.66 \text{ MeV}$$

ILLUSTRATION 7

Calculate the binding energy per nucleon for ${}^{20}_{10}\text{Ne}$, ${}^{56}_{26}\text{Fe}$ and ${}^{238}_{92}\text{U}$. Given that mass of neutron is 1.008665 amu, mass of proton is 1.007825 amu, mass of ${}^{20}_{10}\text{Ne}$ is 19.992440 amu, mass of ${}^{56}_{26}\text{Fe}$ is 55.93492 amu and mass of ${}^{238}_{92}\text{U}$ is 238.050783 amu.

SOLUTION

Binding energy of nucleus ${}_{Z}X^{A}$ is given by the equation

$$(BE)_{\text{Ne}} = \left[(A - Z)m_n + Zm_p - M\left({}^{A}_{Z}X\right)\right]c^2$$

$$\Rightarrow \quad (BE)_{\text{Ne}} = (\Delta m)c^2 = \left[\left(10m_n + 10m_p\right) - M\left({}^{20}_{10}\text{Ne}\right)\right]c^2$$

where,

$$\Delta m = \left(10(1.008665) + 10(1.007825)\right) - 19.992440$$

$$\Rightarrow \quad \Delta m = 0.17246 \text{ amu}$$

$$\Rightarrow \quad (BE)_{\text{Ne}} = (0.17246)(9315) = 160.64 \text{ MeV}$$

So, binding energy per nucleon is

$$\frac{(BE)_{\text{Ne}}}{A} = 8.03 \text{ MeV/nucleon}$$

Similarly, for $\left({}^{56}_{26}\text{Fe}\right)$, we have

$$(BE)_{\text{Fe}} = (\Delta m)c^2 = \left[\left(30m_n + 26m_p\right) - M\left({}^{56}_{26}\text{Fe}\right)\right]c^2$$

where,

$$\Delta m = \left(30(1.008665) + 26(1.007825)\right) - 55.93492$$

$$\Rightarrow \quad \Delta m = 0.52848 \text{ amu}$$

$$\Rightarrow \quad (BE)_{\text{Fe}} = (0.52848)(931.5) = 492 \text{ MeV}$$

Hence binding energy per nucleon is

$$\frac{(BE)_{\text{Fe}}}{A} = 8.79 \text{ MeV/nucleon}$$

Binding energy for $\left({}^{238}_{92}\text{U}\right)$ is

$$(BE)_{\text{U}} = \left[146m_n + 92m_p - M\left({}^{238}_{92}\text{U}\right)\right]c^2$$

where,

$$\Delta m = \left(146(1.008665) + 92(1.007825)\right) - 238.050783$$

$\Rightarrow \quad \Delta m = 1.934$ amu

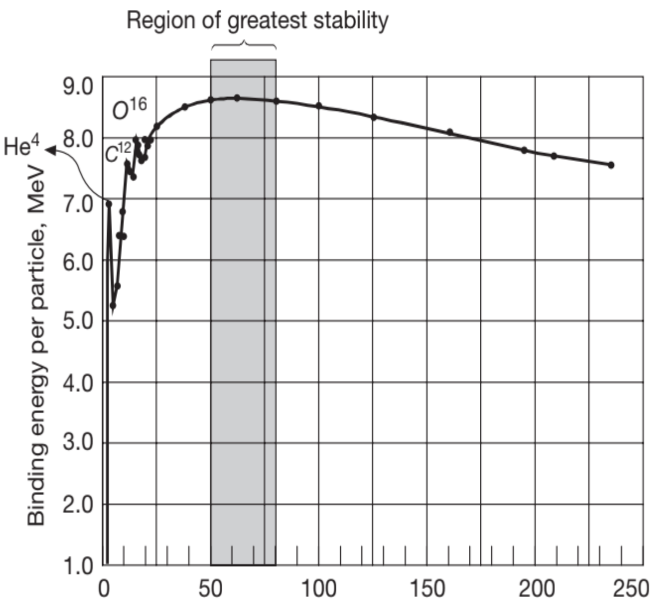
$\Rightarrow \quad (BE)_U = (1.934)(931.5) = 1802$ MeV

Binding energy per nucleon is

$$\frac{(BE)_U}{A} = \frac{1802}{238} = 7.57 \text{ MeV}$$

VARIATION OF BINDING ENERGY PER NUCLEON WITH MASS NUMBER *A*

The graph represents the average binding energy per nucleon in MeV against mass number A. It is observed that the binding energy for nuclei (except ${}_2He^4$, ${}_6C^{12}$ and ${}_8O^{16}$) rises first sharply, reaches a maximum value 8.5 MeV at $A = 50$ and then falls slowly, decreasing to 7.6 MeV for elements of higher mass number $A = 240$. Following facts can be concluded from this curve.



(a) The binding energy per nucleon for light nuclei, such as ${}_1H^2$, is very small $(\simeq 1 \text{ MeV})$.

(b) The binding energy per nucleon increases rapidly for nuclei up to mass number 20 and the curve possesses peaks corresponding to nuclei ${}_2He^4$, ${}_6C^{12}$ and ${}_8O^{16}$. The peaks indicate that these nuclei are more stable than those in their neighbourhood. It confirms the reason for extraordinary stability of α-particle.

(c) After mass number 20, binding energy per nucleon increases gradually and for mass number between 40 and 120, the curve becomes more or less flat. The average value of binding energy per nucleon in this region is about 8.5 MeV. For $A = 56\left({}_{26}Fe^{56}\right)$, the binding energy per nucleon is maximum and it is equal to 8.8 MeV.

(d) After mass number 120, binding energy per nucleon starts decreasing and drops to 7.6 MeV for uranium. This low value of binding energy per nucleon in case of heavy nuclei is unable to have control over the repulsion between the large number of protons. Such nuclei are unstable and are found to disintegrate by emitting α-particles. The emission of α-particle not only decreases repulsive force inside the nucleus but also increases the value of B.E./A of the nucleus due to its extraordinary stable structure (α-particle has large binding energy). It is called α-decay.

Sometimes, the heavy nuclei increase the value of their B.E./A by emitting an electron. It is called β-decay. Inside the nucleus, an electron does not exist. It is created at the time of β-decay due to conversion of a neutron into proton. The β-decay leads to increase in Coulomb's repulsive force, but it increases B.E./A and also improves the neutron-proton ratio.

All such nuclei, which undergo α and β-decay are called radioactive nuclei.

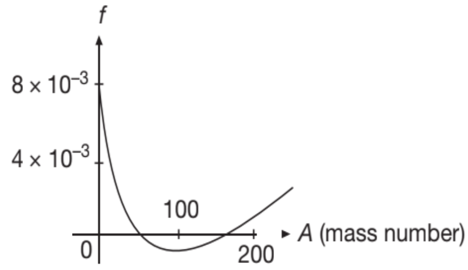
(e) The binding energy per nucleon has a low value for both very light and very heavy nuclei. In order to attain higher value of binding energy per nucleon, the lighter nuclei may unite together to form a heavier nucleus (process of nuclear fusion) or a heavier nucleus may split into lighter nuclei (process of nuclear fission). In both the nuclear processes, the resulting nucleus acquires greater value of binding energy per nucleon along with the liberation of enormous amount of energy.

PACKING FRACTION

Mass defect does not convey much information about nuclear stability and it is misleading to say that higher the mass defect more tightly bound nucleons exist in the nucleus. Packing fraction is a rather arbitrary but convenient and better means of expressing the binding energy. If M is the mass of an atom and A is its mass number then packing fraction

$$f = \frac{\text{Mass defect }(\Delta M)}{\text{Mass number}} = \frac{M - A}{A} = \frac{M - (Z+N)}{(Z+N)}$$

The smaller the value of packing fraction the more stable is the nucleus. From the graph of packing fraction f versus mass number A, following conclusions can be drawn.



(a) The packing fraction for very light nuclei like ${}_1^2H$, ${}_1^3H$ etc., are very large.

(b) As mass number increases packing fraction decreases becomes zero upto $A = 16$ i.e., ${}_8^{16}O$ nucleus.

(c) For nuclei $16 < A < 180$ packing fraction becomes negative. The nucleons in these nuclei are strongly bound in the nucleus.

(d) Beyond $A > 180$ packing fraction is again positive. Thus, most of the nuclei with $A > 235$ are unstable.

NUCLEAR STABILITY

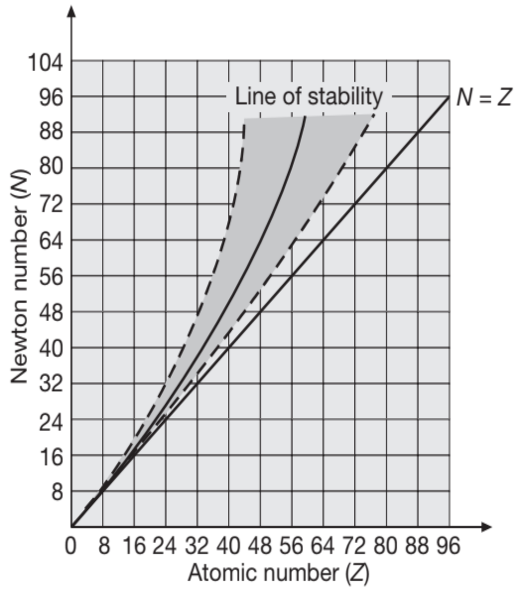
Among about 1500 known nuclides, less than 260 are stable. The others are unstable that decay to form other nuclides by emitting α, β-particles and γ-EM waves. (This process is called radioactivity.) The stability of nucleus is determined by many factors. Few such factors are given below:

(a) Neutron-proton ratio $\left(\frac{N}{Z}\text{ Ratio}\right)$: The chemical properties of an atom are governed entirely by the number of protons (Z) in the nucleus, the stability of an atom appears to depend on both the number of protons and the number of neutrons.

(i) For lighter nuclei, the greatest stability is achieved when the number of protons and neutrons are approximately equal $(N \approx Z)$ i.e., $\frac{N}{Z} = 1$.

(ii) **Heavy nuclei are stable only when they have more neutrons than protons.** Thus, heavy nuclei are neutron rich compared to lighter nuclei (for heavy nuclei, more is the number of protons in the nucleus, greater is the electrical repulsive force between them. Therefore, more neutrons are added to provide the strong attractive forces necessary to keep the nucleus stable.)

(iii) Figure shows a plot of N verses Z for the stable nuclei. For mass number upto about $A = 40$. For larger value of Z the nuclear force is unable to hold the nucleus together against the electrical repulsion of the protons unless the number of neutrons exceeds the number of protons.



At $Bi(Z = 83, A = 209)$, the neutron excess in $N - Z = 43$. There are no stable nuclides with $Z > 83$.

(b) Even or odd numbers of Z or N: The stability of a nuclide is also determined by the consideration whether it contains an even or odd number of protons and neutrons.

(i) It is found that an even-even nucleus (even Z and even N) is more stable (60% of stable nuclide have even Z and even N).

(ii) An even-odd nucleus (even Z and odd N) or odd-even nuclide (odd Z and even N) is found to be lesser sable while the odd-odd nucleus is found to be less stable.

(iii) Only five stable odd-odd nuclides are known: ${}_1H^2$, ${}_3Li^6$, ${}_5Be^{10}$, ${}_7N^{14}$ and ${}_{75}Ta^{180}$

(c) Binding energy per nucleon: The stability of a nucleus is determined by value of it's binding energy per nucleon. In general, higher the value of binding energy per nucleon, the more is the stability of the nucleus.

Test Your Concepts-I

Based on Nucleus Properties and Binding Energy

(Solutions on page H.87)

1. Calculate the nuclear radius of ^{70}Ge.
2. Calculate the electric potential energy of interaction due to the electric repulsion between two nuclei of ^{12}C when they touch each other at the surface. Assume that the potential energy of interaction between two nuclei is given by $U=\frac{q_1q_2}{4\pi\varepsilon_0 r}$.
3. Two stable isotopes of lithium $_3^6Li$ and $_3^7Li$ have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.0152 u and 7.016004 u respectively. Find the atomic weight of lithium.
4. Assuming the nuclei to be spherical in shape, how does the surface area of a nucleus of mass number A_1 compare with that of a nucleus of mass number A_2.
5. Calculate the density of $_6^{12}C$ nucleus. Take atomic mass of $_6^{12}C$ to be 12.000 amu and $R_0=1.2\times10^{-15}$ m.
6. Find the increase in mass of water when 1 kg of water absorbs 4.2×10^3 J of energy to produce a temperature rise of 1 K.
7. The binding energy of $_{17}^{35}Cl$ nucleus is 298 MeV. Calculate its approximate atomic mass. Given that, mass of proton is $m_p=1.007825$ amu and mass of neutron is $m_n=1.008665$ amu.
8. Calculate the binding energy of an alpha particle if mass of $_1^1H$ atom is 1.007826 u, mass of neutron is 1.008665 u, mass of $_2^4He$ atom is 4.00260 u. Take $1\text{ u}=931\text{ MeV}c^{-2}$.
9. Show that the nuclide $_4^8Be$ has a positive binding energy but is unstable with respect to decay into two alpha particles, where masses of neutron, $_1^1H$, $_4^8Be$ and alpha particle are 1.008665 amu, 1.007825 amu, 8.005305 amu and 4.002603 amu respectively.
10. Find the binding energy and the the binding energy per nucleon of the nucleus of lithium isotope $_3^7Li$. Given that mass of $_3^7Li$ atom is 7.016005 amu, mass of $_1H^1$ atom is 1.007825 amu and mass of neutron is 1.008665 amu.

RADIOACTIVITY

The phenomenon of spontaneous emission of radiations $(\alpha,\ \beta,\ \gamma$ etc.$)$ by certain nuclei is called radioactivity. It is a nuclear phenomenon in which a heavy nucleus disintegrates itself without being forced to do so. It is a statistical probable process. The phenomenon of radioactivity was discovered by Becquerel.

LAWS OF RADIOACTIVE DISINTEGRATION

Rutherford-Soddy Law: Statistical Law

(a) Radioactivity is nuclear disintegration phenomenon. It is independent of all physical and chemical conditions.

(b) The disintegration is random and spontaneous statistical process. It is a matter of chance for any atom to disintegrate first.

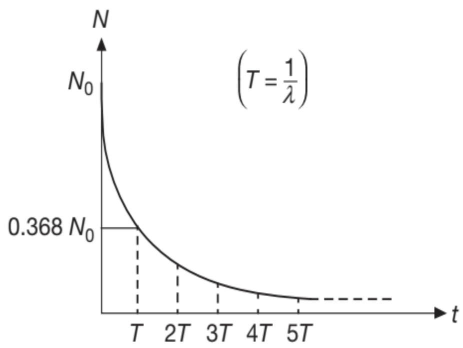
(c) The radioactive substances emit α or β particles along with γ-rays. These rays originate from the nuclei of disintegrating atoms and form fresh radioactive products with different physical and chemical properties.

The rate of decay of nuclei $\left(-\frac{dN}{dt}\right)$ is directly proportional to the number of undecayed nuclei (N) in the sample at time t.

$$\Rightarrow \quad -\frac{dN}{dt}\propto N$$

$$\Rightarrow \quad \frac{dN}{dt}=-\lambda N$$

where λ is constant of proportionality called Decay Constant or Disintegration Constant.



$$\Rightarrow \int_N^N \frac{dN}{N} = -\lambda \int_0^t dt$$

$$\Rightarrow \log_e\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\Rightarrow N = N_0 e^{-\lambda t}$$

where, N is the number of un-decayed nuclei in the sample at time t and N_0 is the number of un-decayed nuclei in the sample at time $t = 0$ (initially).

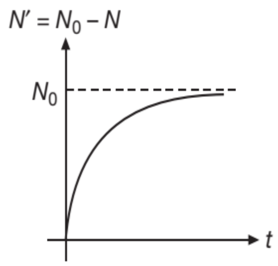
So, we conclude that the number of un-decayed nuclei in the sample decays exponentially with time.

Number of nuclei decayed i.e. the number of nuclei of B formed is given by

$$N' = N_0 - N$$

$$\Rightarrow N' = N_0 - N_0 e^{-\lambda t}$$

$$\Rightarrow N' = N_0\left(1 - e^{-\lambda t}\right)$$



Displacement Laws

(a) When a nuclide emits α-particle, its mass number is reduced by four and atomic number by two, i.e.,

$$_zX^A \longrightarrow {}_{z-2}Y^{A-4} + {}_2\text{He}^4 + \text{Energy}$$

(b) When a nuclide emits a β-particle, its mass number remains unchanged but atomic number increases by one, i.e.,

$$_zX^A \longrightarrow {}_{z+1}Y^A + {}_{-1}\beta^0 + \bar{\nu} + \text{Energy}$$

where $\bar{\nu}$ is the antineutrino

The β-particle is not present initially in the nucleus but is produced due to disintegration of neutron into a proton. i.e.,

$$_0n^1 \longrightarrow {}_1H^1 + {}_{-1}\beta^0 + \bar{\nu} \text{ (antineutrino)}$$

When a proton is converted into a neutron, positive β-particle or positron is emitted.

$$_1H^1 \longrightarrow {}_0n^1 + {}_1\beta^0 + \nu \text{ (neutrino)}$$

(c) When a nuclide emits a gamma photon, neither the atomic number nor the mass number changes.

HALF LIFE ($T_{1/2}$)

The half life period of a radioactive substance is defined as the time in which one-half of the radioactive substance is disintegrated. If N_0 is initial number of radioactive atoms present, then in a half life time $T_{1/2}$, the number of undecayed radioactive atoms will be $\frac{N_0}{2}$ and in next half life $\frac{N_0}{4}$ and so on.

Conceptual Note(s)

So, at $t = T_{1/2}$, $N = \frac{N_0}{2}$

$$N_0 \xrightarrow{1T_{1/2}} \frac{N_0}{2} \xrightarrow{2T_{1/2}} \frac{N_0}{2^2} \xrightarrow{3T_{1/2}} \frac{N_0}{2^3} \longrightarrow \ldots\ldots \xrightarrow{nT_{1/2}} \frac{N_0}{2^n}$$

where $n = \frac{t}{T_{1/2}} = \frac{\text{Time Lapsed}}{\text{Half Life}}$

So, after n half lives, the fraction of undecayed nuclei in the sample is $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$, where $n = \frac{\text{time lapsed}}{T_{1/2}}$.

Since $N = N_0 e^{-\lambda t}$

$$\Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$\Rightarrow \quad e^{-\lambda T_{1/2}} = \frac{1}{2}$

$\Rightarrow \quad \lambda T_{1/2} = \log_e 2 = 0.693$

$\Rightarrow \quad T_{1/2} = \frac{0.693}{\lambda}$

Table 3.1 Fraction of active/decayed atom at different time

Time (t)	Remaining fraction of active atoms (N/N_0) probability of survival	Fraction of atoms decayed $\frac{N_0 - N}{N_0}$ probability of decay
$t = 0$	$1\ (=100\%)$	0
$t = T_{1/2}$	$\frac{1}{2}\ (=50\%)$	$\frac{1}{2}\ (=50\%)$
$t = 2T_{1/2}$	$\frac{1}{4}\ (=25\%)$	$\frac{3}{4}\ (=75\%)$
$t = 3T_{1/2}$	$\frac{1}{8}\ (=12.5\%)$	$\frac{7}{8}\ (=87.5\%)$
$t = 10(T_{1/2})$	$\left(\frac{1}{2}\right)^{10} \approx 0.1\%$	$\approx 99.9\%$
$t = nT_{1/2}$	$\left(\frac{1}{2}\right)^{n}$	$1-\left(\frac{1}{2}\right)^{n}$

ILLUSTRATION 8

At a given instant there are 25% undecayed radioactive nuclei in a sample. After 10 seconds the number of undecayed nuclei reduces to 12.5%. Calculate

(a) mean life of the nuclei

(b) the time in which the number of undecayed nuclei will further reduce to 6.25% of the reduced number.

SOLUTION

(a) In $10\ \text{s}$, number of nuclei has been reduced to half (25% to 12.5%).

Therefore, its half-life is

$$t_{1/2} = 10\ \text{s}$$

Relation between half-life and mean life is

$$t_{\text{mean}} = \frac{t_{1/2}}{\log_e(2)} = \frac{01}{0.693}\ \text{s}$$

$\Rightarrow \quad t_{\text{mean}} = 14.43\ \text{s}$

(b) From initial 100% to reduction till 6.25%, it takes four half lives.

$$100\% \xrightarrow{t_{1/2}} 50\% \xrightarrow{t_{1/2}} 25\% \xrightarrow{t_{1/2}} 12.5\% \xrightarrow{t_{1/2}} 6.25\%$$

$\Rightarrow \quad t = 4t_{1/2} = 4(10)\ \text{s} = 40\ \text{s}$

$\Rightarrow \quad t = 40\ \text{s}$

MEAN LIFE (τ)

The mean life or average life of a radioactive substance is equal to the average time for which the nuclei of atoms of the radioactive substance exist.

The average life of a sample can be calculated by finding the total life of all the nuclei of the substance and then dividing it by the total number of nuclei present in the sample initially. Mathematically

$$\tau = \frac{\int_0^{N_0} t\,dN}{\int_0^{N_0} dN} = \frac{1}{N_0}\int_0^{N_0} t\,dN = \frac{\frac{N_0}{\lambda}}{N_0}$$

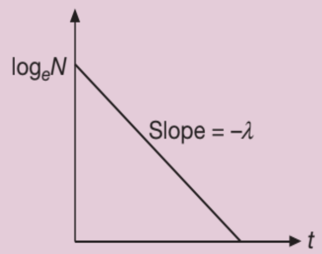
$\Rightarrow \quad \tau = \frac{1}{\lambda}$

Conceptual Note(s)

(a) From $N = N_0 e^{-\lambda t}$, we get

$$\log_e\left(\frac{N}{N_0}\right) = -\lambda t$$

So, the slope of the line shown in the graph i.e. the magnitude of inverse of slope of $\log_e\left(\frac{N}{N_0}\right)$ vs t curve is known as mean life (τ).



(b) From $N = N_0 e^{-\lambda t}$, if $t = \frac{1}{\lambda} = \tau$

$$\Rightarrow \quad N = N_0 e^{-1} = N_0\left(\frac{1}{e}\right) = 0.37N_0 = 37\% \text{ of } N_0.$$

i.e. mean life is the time interval in which number of undecayed atoms (N) becomes $\frac{1}{e}$ times or 0.37 times or 37% of original number of atoms. or

It is the time in which number of decayed atoms $(N_0 - N)$ becomes $\left(1-\frac{1}{e}\right)$ times or 0.63 times or 63% of original number of atoms.

(c) Since, $T_{1/2} = \frac{0.693}{\lambda}$

$$\Rightarrow \quad \frac{1}{\lambda} = \tau = \frac{1}{0.693}(T_{1/2}) = 1.44(T_{1/2})$$

i.e. mean life is about 44% more than that of half life. Which gives us $\tau > T_{(1/2)}$

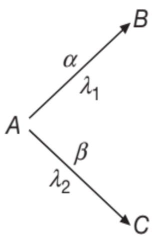
So, we conclude that

$$T_{1/2} = 0.693\tau$$

$$\Rightarrow \quad T_{1/2} < \tau$$

PARALLEL RADIOACTIVE DISINTEGRATION

Let a radioactive nucleus A decay to B and C through a simultaneous process. Assuming that the initial number of nuclei of A is N_0 and after the decay, at time t (say) the number of nuclei of A left is N_A, whereas the number of nuclei of B and C formed are N_B and N_C respectively.



Then at any instant the number of nuclei of A, B and C are given by

$$N_0 = N_A + N_B + N_C$$

$$\Rightarrow \quad \frac{dN_A}{dt} = -\frac{d}{dt}(N_B + N_C) \qquad \ldots(1)$$

Let A disintegrates into B and C by simultaneously emitting α and β particle respectively. Now, the rate of formation of B and C is proportional to the rate at which A decays. So, we have

$$\frac{dN_B}{dt} = +\lambda_1 N_A \text{ and } \frac{dN_C}{dt} = +\lambda_2 N_A$$

$$\Rightarrow \quad \frac{d}{dt}(N_B + N_C) = +(\lambda_1 + \lambda_2)N_A$$

Using equation (1), we get

$$\frac{dN_A}{dt} = -(\lambda_1 + \lambda_2)N_A$$

$$\Rightarrow \quad \lambda_{\text{eff}} = \lambda_1 + \lambda_2$$

$$\Rightarrow \quad t_{\text{eff}} = \frac{t_1 t_2}{t_1 + t_2}$$

Here t_{eff} can be taken as effective half life or effective average life.

ILLUSTRATION 9

The mean lives of a radio-active substances are 1620 years and 405 years for α emission and β emission respectively. Find out the time during which three-fourth of a sample will decay if it is decaying both by α emission and β emission simultaneously.

SOLUTION

Let at some instant of time t, number of atoms of the radioactive substance are N. It may decay either by α emission or by β emission. So, we can write,

$$\left(-\frac{dN}{dt}\right)_{\text{net}} = \left(-\frac{dN}{dt}\right)_{\alpha} + \left(-\frac{dN}{dt}\right)_{\beta}$$

If the effective decay constant is λ, then

$$\lambda N = \lambda_\alpha N + \lambda_\beta N$$

$$\Rightarrow \quad \lambda = \lambda_\alpha + \lambda_\beta = \frac{1}{1620} + \frac{1}{405}$$

$$\Rightarrow \quad \lambda = \frac{1}{324} \text{ year}^{-1}$$

Since, $N = N_0 e^{-\lambda t}$

$$\Rightarrow \quad \frac{N_0}{4} = N_0 e^{-\lambda t}$$

$$\Rightarrow \quad -\lambda t = \log_e\left(\frac{1}{4}\right) = -1.386$$

$$\Rightarrow \quad \left(\frac{1}{324}\right)t = 1.386$$

$$\Rightarrow \quad t = 449 \text{ year}$$

ILLUSTRATION 10

A radioactive nucleus can decay by two different processes, the half life for the first process is t_1 and that for the second process is t_2. Show that the effective half-life t of the nucleus is given by $\frac{1}{t}=\frac{1}{t_1}+\frac{1}{t_2}$.

SOLUTION

Let at any instant t, number of nuclei in radioactive sample be N. Then it may decay by either of two different processes. So,

$$-\frac{dN}{dt}=-\left(\frac{dN}{dt}\right)_1-\left(\frac{dN}{dt}\right)_2$$

$$\Rightarrow \quad \lambda N=\lambda_1 N+\lambda_2 N$$

$$\Rightarrow \quad \lambda=\lambda_1+\lambda_2$$

$$\Rightarrow \quad \frac{1}{t}=\frac{1}{t_1}+\frac{1}{t_2} \qquad \left\{\because t_{1/2}=t=\frac{0.693}{\lambda}\right\}$$

ILLUSTRATION 11

Prove mathematically that mean life or average life of a radioactive substance is $t_{av}=\frac{1}{\lambda}$.

SOLUTION

Let N be the number of atoms that exist at time t. Between t and $t+dt$ let dN atoms are decayed, then

Mean or average life is

$$t_{av}=\frac{\int_{N_0}^{0} t\,dN}{\int_{N_0}^{0} dN}$$

Since, $-\frac{dN}{dt}=\lambda N$

$$\Rightarrow \quad dN=-\lambda N dt$$

$$\Rightarrow \quad t_{av}=\frac{-\int_0^{\infty} t\lambda N dt}{-N_0}$$

Since, $N=N_0e^{-\lambda t}$, so

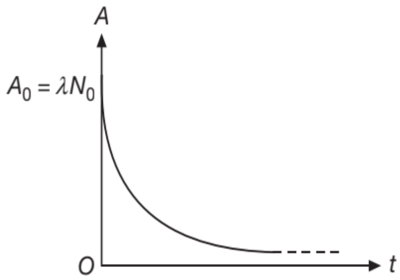
$$t_{av}=\frac{\int_0^{\infty} t\lambda N_0 e^{-\lambda t}dt}{N_0}$$

This integration is done by parts to get the result

$$t_{av}=\frac{1}{\lambda}$$

ACTIVITY OF RADIOACTIVE SUBSTANCE (*A*)

The activity of a radioactive substance means the rate of decay (or the number of disintegration/sec). This is denoted by



$A=-\frac{dN}{dt}=-\frac{d}{dt}\left(N_0e^{-\lambda t}\right)=\lambda N_0 e^{-\lambda t}=A_0e^{-\lambda t}$ where

$A_0=(\lambda N_0)$ is the activity at time $t=0$.

So, activity of a radioactive sample decreases exponentially with time.

SPECIFIC ACTIVITY

The activity per unit mass is called specific activity.

UNITS OF RADIOACTIVITY

Becquerel

In S.I. system the unit of radioactivity is becquerel.

1 becquerel = 1 disintegration/sec = 1 dps.

Rutherford

It is defined as a unit of activity equal to 10^6 dps.

Curie

The traditional unit of activity is curie. It is defined as $3\cdot7\times10^{10}$ dps which is also equal to the radioactivity of 1 g of pure Radium.

ILLUSTRATION 12

The half-life of Cobalt 60 is 5.25 years. How long after its activity have decreased to about one-eight of its original value?

SOLUTION

The activity is proportional to the number of undecayed atoms. In each half-life, half the remaining sample decays.

Since, $A = A_0\left(\frac{1}{2}\right)^n$ and $A = \frac{A_0}{8}$

$$\Rightarrow \quad \frac{A_0}{8} = A_0\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad n = 3$$

Therefore, after three half-lives i.e. after 15.75 yr the sample decays to $\frac{1}{8}$th its original strength.

ILLUSTRATION 13

The half-life of ^{198}Au is 2.7 days. Calculate the

(a) decay constant
(b) average-life and
(c) activity for 1 mg of ^{198}Au.

SOLUTION

(a) The half-life and the decay constant are related as

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

$$\Rightarrow \lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{2.7 \text{ days}}$$

$$\Rightarrow \lambda = \frac{0.693}{2.7 \times 24 \times 3600 \text{ s}} = 2.9 \times 10^{-6} \text{ s}^{-1}$$

(b) The average-life is $t_{av} = \frac{1}{\lambda} = 3.9$ days

(c) The activity is $A = \lambda N$

Since, 198 g of ^{198}Au has 6×10^{23} atoms

The number of atoms in 1 mg of ^{198}Au is

$$N = 6 \times 10^{23} \times \frac{1 \text{ mg}}{198 \text{ g}} = 3.03 \times 10^{18}$$

$$\Rightarrow A = \lambda N = (2.9 \times 10^{-6} \text{ s}^{-1})(3.03 \times 10^{18})$$

$$\Rightarrow A = 8.8 \times 10^{12} \text{ disintegrations/s}$$

$$\Rightarrow A = \frac{8.8 \times 10^{12}}{3.7 \times 10^{10}} \text{ Ci} = 240 \text{ Ci}$$

ILLUSTRATION 14

A count rate-meter is used to measure the activity of a given sample. At one instant the meter shows 4750 counts per minute. Five minutes later it shows 2700 counts per minute.

(a) Find the decay constant
(b) Also, find the half life of the sample

SOLUTION

Initial velocity $A_i = \left.\frac{dN}{dt}\right|_{t=0} = \lambda N_o = 4750$ ...(1)

Final velocity $A_f = \left.\frac{dN}{dt}\right|_{t=5} = \lambda N = 2700$ ...(2)

Dividing (1) by (2), we get

$$\frac{4750}{2700} = \frac{N_0}{N_t} \quad \text{...(3)}$$

The decay constant is given by

$$\lambda = \frac{2.303}{t}\log_e\left(\frac{N_0}{N_t}\right)$$

$$\Rightarrow \quad \lambda = \frac{2.303}{5}\log_e\left(\frac{4750}{2700}\right) = 0.113 \text{ min}^{-1}$$

Half life of the sample is

$$T = \frac{0.693}{\lambda} = \frac{0.693}{0.113} = 6.14 \text{ min}$$

ILLUSTRATION 15

The half lives of radioisotopes P^{32} and P^{33} are 14 days and 25 days respectively. These radio isotopes are mixed in the ratio of 4 : 1 of their atoms. If the initial activity of the mixed sample is 3 mCi, find the activity of the mixed isotopes after 60 days.

SOLUTION

Since, $R = \lambda N = \frac{N \log_e(2)}{t_{1/2}}$

$$\Rightarrow \quad R \propto \frac{N}{t_{1/2}}$$

Given, $\frac{N_1}{N_2} = \frac{4}{1}$ and $\frac{(t_{1/2})_1}{(t_{1/2})_2} = \frac{14}{25}$

$$\Rightarrow \quad \frac{R_1}{R_2}=\frac{N_1}{N_2}\times\frac{\left(t_{1/2}\right)_2}{\left(t_{1/2}\right)_1}=\frac{4}{1}\times\frac{25}{14}=\frac{100}{14}$$

$$\Rightarrow \quad R_1=\frac{100}{114}\times 3=2.63 \text{ mCi}$$

and $R_2=3-2.63=0.37$ mCi

After 60 days, we have

$$R_1'=R_1e^{-\lambda_1 t}=(2.63)e^{-\frac{0.693}{14}\times 60}=0.135 \text{ mCi}$$

$$R_2'=R_2e^{-\lambda_2 t}=(0.37)e^{-\frac{0.693}{25}\times 60}=0.07 \text{ mCi}$$

So, total activity is

$$R'=R_1'+R_2'=0.205 \text{ mCi}$$

ILLUSTRATION 16

A radioactive decay is given by $A\xrightarrow{T_{1/2}=8\text{ yrs}}B$. It is known that only A is present at $t=0$. Find the time at which if we are able to pick one atom out of the sample, then probability of getting B is 15 times of getting A.

SOLUTION

	A	$\rightarrow$	B
at $t=0$	N_0		0
at $t=t$	N		(N_0-N)

Probability of getting A, $P_A=\frac{N}{N_0}$

Probability of getting B, $P_B=\frac{N_0-N}{N_0}$

Since according to the problem, we have $P_B=15P_A$

$$\Rightarrow \quad \frac{N_0-N}{N_0}=15\left(\frac{N}{N_0}\right)$$

$$\Rightarrow \quad N_0=16N$$

$$\Rightarrow \quad N=\frac{N_0}{16}$$

Since, $\frac{N}{N_0}=\left(\frac{1}{2}\right)^n=\frac{1}{16}$

$$\Rightarrow \quad n=4$$

$$\Rightarrow \quad t=4T_{1/2}=4(8)=32 \text{ yr}$$

ILLUSTRATION 17

The half-life of radium is 1620 years. How many radium atoms decay in 1 s in a 1 g sample of radium. The atomic weight of radium is 226 gmol^{-1}.

SOLUTION

Number of atoms in 1 g sample is

$$N=\left(\frac{0.001}{226}\right)\left(6.02\times 10^{26}\right)=2.66\times 10^{21} \text{ atoms}$$

The decay constant is

$$\lambda=\frac{0.693}{T_{1/2}}=\frac{0.693}{(1620)\left(3.16\times 10^7\right)}=1.35\times 10^{-11} \text{ s}^{-1}$$

Taking one year $=3.16\times 10^7$ s

Now,

$$\frac{\Delta N}{\Delta t}=\lambda N=\left(1.35\times 10^{-11}\right)\left(2.66\times 10^{21}\right)$$

$$\Rightarrow \quad \frac{\Delta N}{\Delta t}=3.6\times 10^{10} \text{ s}^{-1}$$

Thus, 3.6×10^{10} nuclei decay in one second.

ILLUSTRATION 18

A certain radionuclide is known to become weaker in activity by 4% every hour. Find the decay constant and the mean life of the radionuclide. Given that $\ln(0.96)=-0.04$

SOLUTION

Suppose initial activity

$$A_0=\lambda N_0$$

Activity after time t

$$A=\lambda N=\lambda N_0 e^{-\lambda t}$$

So, % decrease (η) in activity is $\eta=\frac{A_0-A}{A_0}\times 100\%$

$$\Rightarrow \quad \eta=\frac{\lambda N_0-\lambda N_0 e^{-\lambda t}}{\lambda N_0}\times 100$$

$$\Rightarrow \quad \eta=\left(1-e^{-\lambda t}\right)\times 100$$

According to the problem, $\eta=4\%=\frac{4}{100}$

$$\Rightarrow \quad \frac{4}{100}=1-e^{-\lambda t}$$

$$\Rightarrow \quad e^{-\lambda t}=1-0.04=0.96$$

$$\Rightarrow \quad -\lambda t\log_e e=\log_e(0.96)=-0.04$$

$$\Rightarrow \quad \lambda t=0.04$$

$$\Rightarrow \quad \lambda = \frac{0.04}{t} = \frac{0.04}{3600}\ \text{s}^{-1}$$

$$\Rightarrow \quad \lambda = 1.1\times10^{-5}\ \text{s}^{-1}$$

So, mean life is given by

$$T_{\text{mean}} = \frac{1}{\lambda} = 90000\ \text{s}$$

ILLUSTRATION 19

A F^{32} radio nuclide with half life $T = 14.3$ days is produced in a reactor at a constant rate $q = 2\times10^9$ nuclei per second. How soon after the beginning of production of that radio nuclide will its activity be equal to $R = 10^9$ disintegrations per second?

SOLUTION

$$N = \frac{R}{\lambda} = \frac{10^9}{\dfrac{0.693}{14.3\times3600}} = 7.43\times10^{13}$$

Now, $\dfrac{dN}{dt} = q - \lambda N$

$$\Rightarrow \quad \int_0^N \frac{dN}{q-\lambda N} = \int_0^t dt$$

$$\Rightarrow \quad N = \frac{q}{\lambda}\left(1-e^{-\lambda t}\right)$$

Substituting the values, we get

$$7.43\times10^{13} = \frac{2\times10^9}{\left(\dfrac{0.693}{14.3\times3600}\right)}\left(1-e^{-\left(\frac{0.693}{14.3}\times3600\right)t}\right)$$

Solving this equation, we get $t = 14.3$ hr.

ILLUSTRATION 20

There is a stream of neutrons with a kinetic energy of 0.0327 eV. If the half life of neutrons be 700 s, what fraction of neutrons will decay before they travel a distance of 10 m? Mass of neutron equal to 1.675×10^{-27} kg.

SOLUTION

Since, $K = \dfrac{1}{2}mv^2$

$$\Rightarrow \quad v = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2\times0.0327\times1.6\times10^{-19}}{1.675\times10^{-27}}} = 2.5\times10^3\ \text{ms}^{-1}$$

$$\Rightarrow \quad t = \frac{s}{v} = \frac{10}{2.5\times10^3} = 4\times10^{-3}\ \text{s}$$

Fraction decayed is $\dfrac{N}{N_0} = \dfrac{N_0\left(1-e^{-\lambda t}\right)}{N_0} = 1-e^{-\lambda t}$

$$\Rightarrow \quad \frac{N}{N_0} = 1-e^{-0.6930\times4\times10^{-3}/700}$$

$$\Rightarrow \quad \frac{N}{N_0} = 3.96\times10^{-6}$$

ILLUSTRATION 21

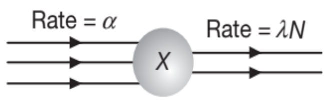
A radionuclide X is produced at constant rate α. At time $t = 0$, number of nuclei of X are zero. Find

(a) the maximum number of nuclei of X.
(b) the number of nuclei at time t.

Decay constant of X is λ.

SOLUTION

(a) Let N be the number of nuclei of X at time t.



Rate of formation of $X = \alpha$ {given}
Rate of disintegration $= \lambda N$
Number of nuclei of X will increase until both the rates will become equal.
Therefore,

$$\alpha = \lambda N_{\max}$$

$$\Rightarrow \quad N_{\max} = \frac{\alpha}{\lambda}$$

(b) Net rate of formation of X at time t is,

$$\frac{dN}{dt} = \alpha - \lambda N$$

$$\Rightarrow \quad \frac{dN}{\alpha-\lambda N} = dt$$

Integrating with proper limits, we have

$$\int_0^N \frac{dN}{\alpha-\lambda N} = \int_0^t dt$$

$$\Rightarrow \quad N = \frac{\alpha}{\lambda}\left(1-e^{-\lambda t}\right)$$

This expression shows that number of nuclei of X are increasing exponentially from 0 to $\dfrac{\alpha}{\lambda}$.

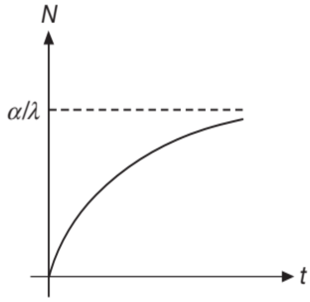


ILLUSTRATION 22

Natural uranium is a mixture of three isotopes ${}^{234}_{92}U$, ${}^{235}_{92}U$ and ${}^{238}_{92}U$ with mass percentage 0.01%, 0.71% and 99.28% respectively. The half life of three isotopes are 2.5×10^5 years. 7.1×10^8 years and 4.5×10^9 years respectively. Determine the share of radioactivity of each isotope into the total activity of the natural uranium.

SOLUTION

Let R_1, R_2 and R_3 be the activities of U^{234}, U^{235} and U^{238} respectively.

Total activity $R = R_1 + R_2 + R_3$

Share of U^{234}, $\dfrac{R_1}{R} = \dfrac{\lambda_1 N_1}{\lambda_1 N_1 + \lambda_2 N_2 + \lambda_3 N_3}$

Let m be the total mass of natural uranium.

Then $m_1 = \dfrac{0.01}{100}m$, $m_2 = \dfrac{0.71}{100}m$ and $m_3 = \dfrac{99.28}{100}m$

Now, $N_1 = \dfrac{m_1}{M_1}$, $N_2 = \dfrac{m_2}{M_2}$ and $N_3 = \dfrac{m_3}{M_3}$

where M_1, M_2 and M_3 are atomic weights.

$$\Rightarrow \quad \frac{R_1}{R} = \frac{\left(\dfrac{m_1}{M_1}\right)\dfrac{1}{T_1}}{\dfrac{m_1}{M_1}\dfrac{1}{T_1} + \dfrac{m_2}{M_2}\dfrac{1}{T_2} + \dfrac{m_3}{M_3}\dfrac{1}{T_3}}$$

$$\Rightarrow \quad \frac{R_1}{R} = \frac{\dfrac{0.01/100}{234} \times \dfrac{1}{2.5\times10^5 \text{ years}}}{\left[\left(\dfrac{\frac{0.01}{100}}{234}\right)\left(\dfrac{1}{2.5\times10^5}\right) + \left(\dfrac{\frac{0.71}{100}}{235}\right)\left(\dfrac{1}{7.1\times10^8}\right) + \left(\dfrac{\frac{99.28}{100}}{238}\right)\left(\dfrac{1}{4.5\times10^9}\right)\right]}$$

$$\Rightarrow \quad \frac{R_1}{R}\times100\% = 0.648 \approx 64.8\%$$

Similarly, share of U^{235} is 0.016% and of U^{238} is 35.184%

ILLUSTRATION 23

Uranium ores on the earth at the present time typically have a composition consisting of 99.3% of the isotope ${}_{92}U^{238}$ and 0.7% of the isotope ${}_{92}U^{235}$. The half lives of these isotopes are $4.47\times10^9 y$ and $7.04\times10^8 y$ respectively. If these isotopes were equally abundant when the earth was formed, estimate the age of the earth.

SOLUTION

Let N_0 be number of atoms of each isotope at the time of formation of the earth $(t = 0)$, let N_1 and N_2 be the number of atoms at present $(t = t)$. Then

$$N_1 = N_0 e^{-\lambda_1 t} \quad \dots(1)$$

and $$N_2 = N_0 e^{-\lambda_2 t} \quad \dots(2)$$

$$\Rightarrow \quad \frac{N_1}{N_2} = e^{(\lambda_2 - \lambda_1)t} \quad \dots(3)$$

Further it is given that

$$\frac{N_1}{N_2} = \frac{99.3}{0.7} \quad \dots(4)$$

Equating (3) and (4) and taking log both sides, we get

$$(\lambda_2 - \lambda_1)t = \log_e\left(\frac{99.3}{0.7}\right)$$

$$\Rightarrow \quad t = \left(\frac{1}{\lambda_2 - \lambda_1}\right)\log_e\left(\frac{99.3}{0.7}\right)$$

Substituting the values, we get

$$t = \frac{1}{\dfrac{0.693}{7.04\times10^8} - \dfrac{0.693}{4.47\times10^9}}\log_e\left(\frac{99.3}{0.7}\right)$$

$$\Rightarrow \quad t = 5.97\times10^9 \text{ y}$$

ILLUSTRATION 24

In the chemical analysis of a rock the mass ratio of two radioactive isotopes is found to be $100:1$. The mean lives of the two isotopes are 4×10^9 years and

2×10^9 years respectively. If it is assumed that at the time of formation the atoms of both the isotopes were in equal proportional, calculate the age of the rock. Ratio of the atomic weights of the two isotopes is $1.02:1$.

SOLUTION

At the time of observation i.e., at time t, we have

$$\frac{m_1}{m_2}=\frac{100}{1} \qquad \{\text{given}\}$$

Further it is given that

$$\frac{A_1}{A_2}=\frac{1.02}{1}$$

Number of atoms $N=\dfrac{m}{A}$

$$\Rightarrow \quad \frac{N_1}{N_2}=\frac{m_1}{m_2}\times\frac{A_2}{A_1}=\frac{100}{1.02} \qquad \ldots(1)$$

Let N_0 be the number of atoms of both the isotopes at the time of formation, then

$$\frac{N_1}{N_2}=\frac{N_0e^{-\lambda_1 t}}{N_0e^{-\lambda_2 t}}=e^{(\lambda_2-\lambda_1)t} \qquad \ldots(2)$$

Equating (1) and (2), we get

$$e^{(\lambda_2-\lambda_1)t}=\frac{100}{1.02}$$

$$\Rightarrow \quad (\lambda_2-\lambda_1)t=\log_e(100)-\log_e(1.02)$$

$$\Rightarrow \quad t=\frac{\log_e(100)-\log_e(1.02)}{\left(\dfrac{1}{2\times10^9}-\dfrac{1}{4\times10^9}\right)}$$

Substituting the values, we have

$$t=1.834\times10^{10} \text{ years}$$

ILLUSTRATION 25

The mass of carbon in an animal bone fragment found in an archaeological site is 200 g. If the bone registers an activity of 16 decays/s, what is its age? Assume that, when the animal was alive, the ratio of ${}_6C^{14}$ to ${}_6C^{12}$ in its bone was 1.3×10^{-12}.

SOLUTION

The 200 g of carbon is nearly all ${}^{12}_{6}C$

Since 12.0 g of ${}^{12}_{6}C$ contains 6.02×10^{23} atoms, so 200 g contains

$$\left(\frac{6.02\times10^{23}\text{ atoms}}{12\text{ g}}\right)(200\text{ g})=1.00\times10^{25}\text{ atoms}$$

When the animal was alive, the ratio of ${}^{14}_{6}C$ to ${}^{12}_{6}C$ in the bone was 1.3×10^{-12}. The number of ${}^{14}_{6}C$ nuclei at that time was

$$N_0=(1.00\times10^{25}\text{ atoms})(1.3\times10^{-12})$$

$$\Rightarrow \quad N_0=1.3\times10^{13}\text{ atoms.}$$

The magnitude of the activity when the animal was alive $(t=0)$ was

$$A_0=\left(\frac{dN}{dt}\right)_0=\lambda N_0$$

where $\lambda=3.83\times10^{-12}\text{ s}^{-1}$ as we calculated in one of previous example. So, the original activity was

$$A_0=\left(\frac{dN}{dt}\right)_0=\lambda N_0$$

$$\Rightarrow \quad A_0=(3.83\times10^{-12}\text{ s}^{-1})(1.3\times10^{13})=50\text{ s}^{-1}$$

Since, $A=\dfrac{dN}{dt}=\left(\dfrac{dN}{dt}\right)_0 e^{-\lambda t}=A_0e^{-\lambda t}$

where $\dfrac{dN}{dt}=A$ is given to be 16 s^{-1}.

$$\Rightarrow \quad 16=(50)e^{-\lambda t}$$

$$\Rightarrow \quad e^{\lambda t}=\frac{50}{16}$$

Taking natural logarithm of both sides, we get

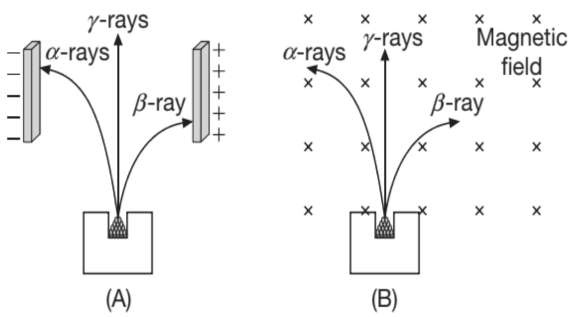
$$t=\frac{1}{\lambda}\ln\left(\frac{50}{16}\right)=\frac{1}{3.83\times10^{-12}\text{s}^{-1}}\ln\left(\frac{50}{16}\right)$$

$$\Rightarrow \quad t=2.98\times10^{11}\text{ s}=9400\text{ yr}$$

So, the time elapsed since the death of the animal is about 9400 yr.

NUCLEAR RADIATIONS

According to Rutherford's experiment when a sample of radioactive substance is put in a lead box and allow the emission of radiation through a small hole only. When the radiation enters into the external electric field, they splits into three parts (α-rays, β-rays and γ-rays)



Alpha Decay

Nearly 90% of the 2500 known nuclides are radioactive; they are not stable but decay into other nuclides

(i) When unstable nuclides decay into different nuclides, they usually emit alpha (α) or beta (β) particles.

(ii) Alpha emission occurs principally with nuclei that are too large to be stable. When a nucleus emits an alpha particle, its N and Z values each decrease by two and A decreases by four.

(iii) Alpha decay is possible whenever the mass of the original neutral atom is greater than the sum of the masses of the final neutral atom and the neutral helium- atom.

Beta Decay

There are different simple type of β-decay β^-, β^+ and electron capture.

(i) A beta minus particle (β^-) is an electron. Emission of β^- involves transformation of a neutron into a proton, an electron and a third particle called an antineutrino $(\bar{\nu})$.

(ii) β^- decay usually occurs with nuclides for which the neutron to proton ratio $\left(\frac{N}{Z}\text{ ratio}\right)$ is too large for stability.

(iii) In β^- decay, N decreases by one, Z increases by one and A doesn't change.

(iv) β^- decay can occur whenever the neutral atomic mass of the original atom is larger than that of the final atom.

(v) Nuclides for which $\frac{N}{Z}$ is too small for stability can emit a positron, the electron's antiparticle, which is identical to the electron but with positive charge. The basic process called beta plus β^+ decay

$$p \to n + \beta^+ + \nu \quad (\nu = \text{neutrino})$$

(vi) β^+ decay can occur whenever the neutral atomic mass of the original atom is at least two electron masses larger than that of the final atom

(vii) The mass of ν and $\bar{\nu}$ is zero. The spin of both is $\frac{1}{2}$ in units of $\frac{h}{2\pi}$. The charge on both is zero. The spin of neutrino is antiparallel to its momentum while that of antineutrino is parallel to its momentum.

(viii) There are a few nuclides for which β^+ emission is not energetically possible but in which an orbital electron (usually in the k-shell) can combine with a proton in the nucleus to form a neutron and a neutrino. The neutron remains in the nucleus and the neutrino is emitted.

$$p + \beta^+ \to n + \nu$$

Gamma Decay

The energy of internal motion of a nucleus is quantized. A typical nucleus has a set of allowed energy levels, including a ground state (state of lowest energy) and several excited states. Because of the great strength of nuclear interactions, excitation energies of nuclei are typically of the order of $1\,\text{MeV}$, compared with a few eV for atomic energy levels. In ordinary physical and chemical transformations, the nucleus always remains in its ground state. When a nucleus is placed in an excited state, either by bombardment with high-energy particles or by a radioactive transformation, it can decay to the ground state by emission of one or more photons called gamma rays or gamma-ray photons, with typical energies of $10\,\text{keV}$ to $5\,\text{MeV}$. This process is called gamma (γ) decay. All the known conservation laws are obeyed in γ-decay.

The intensity of γ-decay after passing through a thickness x of a material is given by $I = I_0 e^{-\mu x}$ (μ = absorption co-efficient)

Table 3.2 Properties of α, β and γ-rays

Features	α-particles	β-particles	γ-rays
1. Identity	Helium nucleus or doubly ionised helium atom $\left({}_2He^4\right)$	Fast moving electron $(-\beta^0$ or $\beta^-)$	Photons (E.M. waves)
2. Charge	$+2e$	$-e$	zero
3. Mass $4m_p$ (m_p = mass of proton $= 1.87 \times 10^{-27}$	$4m_p$	m_e	Massless
4. Speed	$\approx 10^7\ ms^{-1}$	1% to 99% of speed of light	Speed of light
5. Range of kinetic energy	4 MeV to 9 MeV	All possible values between a minimum certain value to 1.2 MeV	Between a minimum value to 2.23 MeV
6. Penetration power (γ, β, α)	1 (Stopped by a paper)	100 (100 times of α)	10,000 (100 times of β upto 30 cm of iron (or Pb) sheet
7. Ionisation power ($\alpha > \beta > \gamma$)	10,000	100	1
8. Effect of electric or magnetic field	Deflected	Deflected	Not deflected
9. Energy spectrum	Line and discrete	Continuous	Line and discrete
10. Mutual interaction with matter	Produces heat	Produces heat	Produces, photo-electric effect, Compton effect, pair production
11. Equation of decay	${}_ZX^A \xrightarrow{\alpha-\text{decay}}$ ${}_{Z-2}Y^{A-4} + {}_2He^4$ ${}_ZX^A \xrightarrow{n\alpha} {}_{Z'}Y^{A'}$ $\Rightarrow \quad n_\alpha = \dfrac{A-A'}{4}$	${}_ZX^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\nu}$ ${}_ZX^A \xrightarrow{n_\beta} {}_{Z'}X^A$ $\Rightarrow n_\beta = (2n_\alpha - Z + Z')$	${}_ZX^A \rightarrow {}_ZX^a + \gamma$

RADIOACTIVE SERIES

(a) If the isotope that results from a radioactive decay is itself radioactive then it will also decay and so on.

(b) The sequence of decays is known as radioactive decay series. Most of the radio-nuclides found in nature are members of four radioactive series. These are as follows

Table 3.3 Four radioactive series

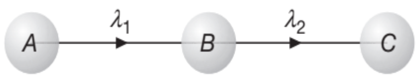
Mass number	Series (Nature)	Parent	Stable end product	Integer n
$4n$	Thorium (natural)	$_{90}Th^{232}$	$_{82}Pb^{208}$	52
$4n+1$	Neptunium (Artificial)	$_{93}Np^{237}$	$_{83}Bi^{209}$	52
$4n+2$	Uranium (Natural)	$_{92}U^{238}$	$_{82}Pb^{206}$	51
$4n+3$	Actinium (Natural)	$_{89}Ac^{227}$	$_{82}Pb^{207}$	51

(c) The $4n+1$ series starts from $_{94}Pu^{241}$ but commonly known as neptunium series because neptunium is the longest lived member of the series.

(d) The $4n+3$ series actually starts from $_{92}U^{235}$.

SUCCESSIVE DISINTEGRATION AND RADIOACTIVE EQUILIBRIUM

Suppose a radioactive element A disintegrates to form another radioactive element B which intern disintegrates to still another element C; such decays are called successive disintegration.



Rate of disintegration of A is $\frac{dN_1}{dt}=-\lambda_1 N_1$ (which is also the rate of formation of B)

Rate of disintegration of B is $\frac{dN_2}{dt}=-\lambda_2 N_2$

$$\therefore \quad \begin{pmatrix}\text{Net rate of}\\ \text{formation}\\ \text{of } B\end{pmatrix}=\begin{pmatrix}\text{Rate of}\\ \text{decay}\\ \text{of } A\end{pmatrix}-\begin{pmatrix}\text{Rate of}\\ \text{decay}\\ \text{of } B\end{pmatrix}$$

$$\Rightarrow \quad (\text{Net rate of formation of } B)=\lambda_1 N_1-\lambda_2 N_2$$

EQUILIBRIUM

In radioactive equilibrium, the rate of decay of any radioactive product is just equal to its rate of production from the previous member.

i.e., $\lambda_1 N_1=\lambda_2 N_2$

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2}=\frac{N_2}{N_1}=\frac{\tau_2}{\tau_1}=\frac{(T_{1/2})_2}{(T_{1/2})_1}$$

ILLUSTRATION 26

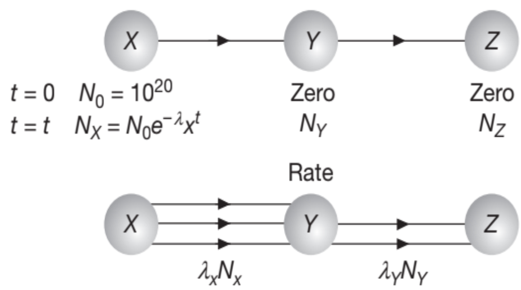
A radioactive nucleus X decays to a nucleus Y with a decay constant $\lambda_X=0.1\ s^{-1}$. Y further decays to a stable nucleus Z with a decay constant $\lambda_Y=\frac{1}{30}\ s^{-1}$. Initially there are only X nuclei and their number is $N_0=10^{20}$. Setup the rate equations for the populations of X, Y and Z. The population of the Y nucleus as a function of time is given by

$$N_Y(t)=\left\{\frac{N_0\lambda_X}{(\lambda_X-\lambda_Y)}\right\}\{\exp(-\lambda_Y t)-\exp(-\lambda_X t)\}$$

Find the time at which N_Y is maximum and determine the population of X and Z at that instant.

SOLUTION

Let at time $t=t$, number of nuclei of Y and Z are N_Y and N_Z. Then



Rate equation of the populations of X, Y and Z are

$$\left(\frac{dN_X}{dt}\right)=-\lambda_X N_X \quad \ldots(1)$$

$$\left(\frac{dN_Y}{dt}\right)=\lambda_X N_X-\lambda_Y N_Y \quad \ldots(2)$$

and $$\left(\frac{dN_Z}{dt}\right)=\lambda_Y N_Y \quad \ldots(3)$$

Given $N_Y(t) = \frac{N_0\lambda_X}{\lambda_X - \lambda_Y}[e^{-\lambda_Y t} - e^{-\lambda_X t}]$

For N_Y to be maximum $\frac{dN_Y(t)}{dt} = 0$

i.e., $\lambda_X N_X = \lambda_Y N_Y$ {from equation (2)} ...(4)

$$\Rightarrow \quad \lambda_X\left(N_0 e^{-\lambda_X t}\right) = \lambda_Y \frac{N_0\lambda_X}{\lambda_X - \lambda_Y}[e^{-\lambda_Y t} - e^{-\lambda_X t}]$$

$$\Rightarrow \quad \frac{\lambda_X - \lambda_Y}{\lambda_Y} = \frac{e^{-\lambda_Y t}}{e^{-\lambda_X t}} - 1$$

$$\Rightarrow \quad \frac{\lambda_X}{\lambda_Y} = e^{(\lambda_X - \lambda_Y)t}$$

$$\Rightarrow \quad (\lambda_X - \lambda_Y)t\log_e(e) = \log_e\left(\frac{\lambda_X}{\lambda_Y}\right)$$

$$\Rightarrow \quad t = \frac{1}{(\lambda_X - \lambda_Y)}\log_e\left(\frac{\lambda_X}{\lambda_Y}\right)$$

Substituting the values of λ_X and λ_Y, we have

$$t = \frac{1}{\left(0.1 - \frac{1}{30}\right)}\log_e\left(\frac{0.1}{\frac{1}{30}}\right) = 15\log_e(3)$$

$$\Rightarrow \quad t = 16.48 \text{ s}$$

The population of X at this moment

$$N_X = N_0 e^{-\lambda_X t} = (10^{20})e^{-(0.1)(16.48)}$$

$$\Rightarrow \quad N_X = 1.92 \times 10^{19}$$

Since, $N_Y = \frac{N_X\lambda_X}{\lambda_Y}$ {From equation (4)}

$$\Rightarrow \quad N_Y = (1.92 \times 10^{19})\frac{(0.1)}{\frac{1}{30}} = 5.76 \times 10^{19}$$

$$\Rightarrow \quad N_Z = N_0 - N_X - N_Y$$

$$\Rightarrow \quad N_Z = 10^{20} - 1.92 \times 10^{19} - 5.76 \times 10^{19}$$

$$\Rightarrow \quad N_Z = 2.32 \times 10^{19}$$

ILLUSTRATION 27

A factory produces a radioactive substance A at a constant rate R which decays with a decay constant λ to form a stable substance. Find

(i) the number of nuclei of A and

(ii) number of nuclei of B, at any time t assuming the production of A starts at $t = 0$.

(iii) also find out the maximum number of nuclei of A present at any time during its formation.

SOLUTION

(i) Factory $\xrightarrow[\text{constant rate}]{R} A \xrightarrow[\text{decay}]{\lambda} B$

Let N be the number of nuclei of A at any time t

$$\Rightarrow \quad \frac{dN}{dt} = R - \lambda N$$

$$\Rightarrow \quad \int_0^N \frac{dN}{R - \lambda N} = \int_0^t dt$$

On solving we will get

$$N = \frac{R}{\lambda}(1 - e^{-\lambda t})$$

(ii) Number of nuclei of B at any time t,

$$N_B = Rt - N_A = Rt - \frac{R}{\lambda}(1 - e^{-\lambda t}) = \frac{R}{\lambda}(\lambda t - 1 + e^{-\lambda t}).$$

(iii) Maximum number of nuclei of A present at any time during its formation $= \frac{R}{\lambda}$.

ILLUSTRATION 28

Consider a radioactive disintegration according to the equation $A \to B \to C$. Decay constant of A and B is same and equal to λ. Number of nuclei of A, B and C are N_0, 0, 0 respectively at $t = 0$. Find

(a) Number of nuclei of B as function of time t.

(b) Time t at which the activity of B is maximum and the value of maximum activity of B.

SOLUTION

(a) Let the number of nuclei at any instant be shown in the table.

	A	B	C
At $t = 0$	N_0	0	0
At t	N_1	N_2	N_3

where, $N_1 = N_0 e^{-\lambda t}$ ...(1)

Now, $\frac{dN_2}{dt} = \lambda(N_1 - N_2)$

$\Rightarrow \quad \frac{dN_2}{dt} = \lambda N_0 e^{-\lambda t} - \lambda N_2$

$\Rightarrow \quad dN_2 + \lambda N_2 dt = \lambda N_0 e^{-\lambda t}$

$\Rightarrow \quad e^{\lambda t} dN_2 + \lambda N_2 e^{\lambda t} dt = \lambda N_0 dt$

$\Rightarrow \quad d(N_2 e^{\lambda t}) = \lambda N_0 dt$

Integrating, we get

$$\int_0^{N_2} d(N_2 e^{\lambda t}) = \int_0^t \lambda N_0 dt$$

$\Rightarrow \quad N_2 = \lambda N_0 t e^{-\lambda t}$

(b) Activity of B is, $R_2 = \lambda N_2 = \lambda^2 N_0 t e^{-\lambda t}$

For maximum activity, we have

$\frac{dR_2}{dt} = 0$

$\Rightarrow \quad t = \frac{1}{\lambda}$

$\Rightarrow \quad R_{max} = \frac{\lambda N_0}{e}$

Test Your Concepts-II

Based on Radioactivity

(Solutions on page H.88)

1. At time $t = 0$, number of nuclei of a radioactive substance are 100. At $t = 1$ s these numbers become 90. Find the number of undecayed nuclei at $t = 2$ s.
2. Find the amount of heat generated by 1 mg of Po^{210} preparation during the mean life period of these nuclei if the emitted alpha particles are known to possess kinetic energy 5.3 MeV and practically all daughter nuclei are formed directly in the ground state.
3. The radioactivity of a uranium specimen with mass number 238 is 2.5×10^4 s^{-1}, the specimen's mass is 2 g. Find the half-life.
4. Ac^{227} has a half life of 21.8 years with respect to radioactive decay. The decay follows two parallel paths, one leading the Th^{227} and the other leading to Fr^{223}. The percentage yields of these two daughters nuclides are 1.2% and 98.8% respectively. What is the rate constant in yrs^{-1}, for each of the separate paths?
5. The disintegration rate of a certain radioactive sample at any instant is 4750 disintegrations per minute. Five minutes later the rate becomes 2700 per minute. Calculate
   (a) decay constant and
   (b) half-life of the sample.
6. In an agricultural experiment, a solution containing 1 mole of a radioactive material $(T_{1/2} = 14.3 \text{ days})$ was injected into the roots of a plant. The plant was allowed 70 hours to settle down and then activity was measured in its fruit. If the activity measured was 1 μCi, what percentage of activity is transmitted from the root to the fruit in steady state?
7. A sample of 1 g of $^{109}_{83}Bi$ with a half life of 2.7×10^7 year decays into a stable isotope of thallium by emitting α particles.
   (a) What is the activity of the sample?
   (b) What will be the activity of the sample after 2 years?
   (c) After what time does the activity reduces to 25% of the original activity?
8. A number N_0 of atoms of a radioactive element are placed inside a closed volume. The radioactive decay constant for the nuclei of this element is λ_1. The daughter nuclei that form as a result of the decay process are assumed to be radioactive, too, with a radioactive decay constant λ_2. Determine the time variation of the number of such nuclei. Consider two limiting cases, when $\lambda_1 \gg \lambda_2$ and $\lambda_1 \ll \lambda_2$.
9. Calculate the probability that a radioactive atom having a mean life of 10 days decays during the fifth day.
10. Old wood from an Egyptian tomb, 4500 years old has C-14 activity of 7.3 dis. min^{-1} g^{-1}. Old wood

known to be 2500 years old has a C-14 activity 9.3 dis. $\text{min}^{-1}\ \text{g}^{-1}$.

(a) What is half life for C-14?

(b) What is the activity of fresh wood?

11. Determine the amount of $_{84}Po^{210}$ (polonium) necessary to provide a source of α particles of 5 millicurie strength. If half life of polonium is 138 days, given 1 curie $= 3.7 \times 10^{10}$ disintegrations/sec.

12. The specific activity of a preparation consisting of radioactive Co^{58} and non-radioactive Co^{59} is 2.2×10^{12} dps/g. The half life of Co^{58} is 71.3 days. Find the ratio of the mass of radioactive cobalt in that preparation to the total mass of the preparation.

13. A laboratory has 1.49 μg of pure $^{13}_{7}N$, which has a half-life of 10.0 min.

(a) How many nuclei are present initially?

(b) What is the activity initially?

(c) What is the activity after 1.00 h?

14. Suppose, the daughter nucleus in a nuclear decay is itself radioactive. Let λ_p and λ_d be the decay constants of the parent and the daughter nuclei. Also, let N_p and N_d be the number of parent and daughter nuclei at time *t*. Find the condition for which the number of daughter nuclei becomes constant.

15. A radioactive sample has 6×10^{18} active nuclei at a certain instant. How many of these nuclei will still be in the same active state after two half-lives?

16. The number of ^{238}U atoms in an ancient rock equals the number of ^{206}Pb atoms. The half-life of decay of ^{238}U is 4.5×10^9 *y*. Estimate the age of the rock assuming that all the ^{206}Pb atoms are formed from the decay of ^{238}U.

17. The decay constant for the radioactive nuclide ^{64}Cu is $1.516 \times 10^{-5}\ s^{-1}$. Find the activity of a sample containing 1 μg of ^{64}Cu. Atomic weight of copper = 63.5 g/mole. Neglect the mass difference between the given radioisotope and normal copper.

18. The half-life of a radioactive nuclide is 20 hours. What fraction of original activity will remain after 40 hours?

19. The age of a rock containing lead and uranium is equal to 1.5×10^9 years. The uranium is decaying into lead with half life equal to 4.5×10^9 years. Find the ratio of lead to uranium present in the rock, assuming that initially no lead was present in the rock. Given that $2^{\frac{1}{3}} = 1.259$.

20. In a radioactive disintegration process shown, *A* is continuously produced at the rate of 10^{21} per second.

$$\xrightarrow{10^{21}\ \text{per second}} A \xrightarrow{\lambda = \frac{1}{30}} B$$

Find the maximum number of nuclei of *A*.

ARTIFICIAL TRANSMUTATION – NUCLEAR REACTIONS

A radio-active substance breaks up by emitting radiations. The daughter nucleus, left behind, has different physical and chemical properties and is assigned a new place in the periodic chart. Thus radio-activity is the phenomenon by which a substance gets converted into another one. This change can be brought about by artificial method, by bombarding a given nucleus with some radiation. The particles constituting the incident radiation must possess sufficient kinetic energy so as to penetrate into the given nucleus. As they enter the given nucleus, a compound nucleus (an intermediate state) is formed which is generally unstable. The compound nucleus then breaks up to produce product nucleus by emitting radiation. The process is, schematically, represented as

$$\underset{\begin{pmatrix}\text{Projectile}\\ \text{particle}\end{pmatrix}}{P} + \underset{\begin{pmatrix}\text{Target}\\ \text{nucleus}\end{pmatrix}}{T} \rightarrow \underset{\begin{pmatrix}\text{Compound}\\ \text{nucleus}\end{pmatrix}}{C^*} \rightarrow \underset{\begin{pmatrix}\text{Residual}\\ \text{nucleus}\end{pmatrix}}{R} + \underset{\begin{pmatrix}\text{Outgoing}\\ \text{particle or}\\ \text{radiation}\end{pmatrix}}{O}$$

This is a reaction in which only the nuclei take part. Orbital electrons have no contribution to it. Such reactions are known as nuclear reactions.

LAWS GOVERNING NUCLEAR REACTIONS

(a) Law of Charge: The electric charge involved in nuclear reactions must be same before and after the reaction. So, charge number is conserved in nuclear reactions.

(b) Law of Number of Nucleons: The total number of nucleons involved in a nuclear reaction must be same before and after the reaction. So, mass number is also conserved in nuclear reactions.

(c) Law of Conservation of Energy: The total energy (rest mass energy + K.E.) of the reacting particles must be equal to the total energy of the product particles.

(d) Law of Conservation of Linear Momentum: The total linear momentum of the reacting particles must be equal to the total linear momentum of the product particles.

(e) Law of Conservation of Angular Momentum: Total angular momentum of nuclei before and after reaction must be the same.

Q-VALUE OF A NUCLEAR REACTION

Consider a nuclear reaction, schematically represented by equation

$$P+T \longrightarrow R+O$$

Let K_P, K_R and K_O be the kinetic energies of P (projectile particle), R (residual nucleus) and O (outgoing particle or radiation) respectively, while T (target nucleus) is taken to be at rest initially. Q-value of a nuclear reaction is defined as

$$Q=(K_R+K_O)-K_P$$

Let m_P, m_T, m_R, m_O, respectively, be the masses of P, T, R and O.

Since, for nor-relativistic approach, the total energy of a subatomic particle is equal to the sum of rest mass energy and its kinetic energy, so energy of P, T, R and O are respectively given by

$$E_P=m_Pc^2+K_P$$

$$E_T=m_Tc^2+0=m_Tc^2$$

$$E_R=m_Rc^2+K_R$$

$$E_O=m_Oc^2+K_O$$

Total initial energy is

$$\Sigma E_{\text{initial}}=(m_Pc^2+K_P)+m_Tc^2 \qquad \ldots(1)$$

Total final energy is

$$\Sigma E_{\text{final}}=(m_Rc^2+K_R)+(m_Oc^2+K_O) \qquad \ldots(2)$$

By Law of Conservation of Energy, we have

$$\Sigma E_{\text{initial}}=\Sigma E_{\text{final}}$$

$$\Rightarrow \quad (m_Pc^2+K_P)+m_Tc^2=(m_Rc^2+K_R)+(m_Oc^2+K_O)$$

$$\Rightarrow \quad Q=(K_R+K_O)-K_P=\left[(m_P+m_T)-(m_R+m_O)\right]c^2$$

$$\Rightarrow \quad Q=(\Delta m)c^2$$

where, $\Delta m=(m_P+m_T)-(m_R+m_O)$ is the mass defect between initial and final particles.

Conceptual Note(s)

Since the Q factor of a reaction is given by

$$Q=(K_R+K_O)-K_P$$

$$\Rightarrow K_R+K_O=Q+K_P$$

Also we know that, the kinetic energy is a positive quantity and hence (K_R+K_O) must also be positive.

$$\Rightarrow K_R+K_O>0$$

$$\Rightarrow Q+K_P>0$$

So, a necessary but not sufficient condition for the occurrence of a nuclear reaction is that for the reaction

$$Q+K_P>0$$

$$\Rightarrow K_P>-Q$$

CASE-I: EXOERGIC REACTION

A reaction is said to be exoergic if Q is positive.

$$\Rightarrow \quad (m_R+m_O)<(m_P+m_T)$$

The part of mass which disappears gets converted into the energy in accordance with Einstein's Mass-Energy equivalence.

Also, is positive if $(BE)_{\text{final}}>(BE)_{\text{initial}}$.

CASE-II: ENDOERGIC REACTION

A reaction is said to be endoergic if Q is negative.

$$\Rightarrow \quad (m_R+m_O)>(m_P+m_T) \text{ i.e. } (BE)_{\text{final}}<(BE)_{\text{initial}}$$

i.e., the sum of the masses of product particles is greater than that of reactant particles. So, energy is required, by the kinetic energies of the initial reactants, to make the reaction or transformation "go".

The minimum amount of energy that the bombarding particle or the projectile particle must have in order to initiate an endoergic reaction, is called Threshold Energy E_{Th}.

$$E_{Th} = -Q\left(\frac{m_P}{m_T}+1\right)$$

where, m_P is the mass of the projectile i.e. the nucleus used to hit the target and m_T is the mass of the target nucleus

Problem Solving Technique(s)

For a nuclear reaction $A+B \to C+D$, if the binding energies of A, B, C and D are given as B_1, B_2, B_3 and B_4, then the energy released in the reaction is

$$\Delta E = (B_3+B_4)-(B_1+B_2)$$

ILLUSTRATION 29

(a) A particle having mass m, kinetic energy K_{Lab} measured in the Lab Frame collides head-on with a stationary nucleus of mass M. If $K_{\text{wrt cm}}$ is the total kinetic energy of the system with respect to the centre of mass frame/coordinate system (i.e. the energy available to cause the reaction), then show that

$$K_{\text{wrt cm}} = \left(\frac{M}{M+m}\right)K_{Lab}$$

(b) If K_{cm} is the total kinetic energy of the centre of mass in the lab frame, then show that

$$K_{cm} = \left(\frac{m}{M+m}\right)K_{Lab}$$

(c) If K_{th} is the minimum kinetic energy (called Threshold energy) to cause an endoergic reaction in the above situation, then show that

$$K_{th} = -Q\left(1+\frac{m}{M}\right)$$

Try to prove this result, both using the concept of lab frame (L-Frame) and the centre of mass frame.

SOLUTION

(a) The velocity of the centre of mass of the system is

$$v_{cm} = \frac{mv+M(0)}{m+M} = \frac{mv}{m+M}$$

Let v'_m be the velocity of m with respect to the centre of mass and v'_M be the velocity of M with respect to the centre of mass, then

$$v'_m = v - v_{cm} = v - \left(\frac{mv}{m+M}\right) = \frac{Mv}{m+M} \text{ and}$$

$$v'_M = 0 - \frac{mv}{m+M} = -\frac{mv}{m+M}$$

In the centre of mass frame, the kinetic energy K_{cm} is given by

$$K_{\text{wrt cm}} = \frac{1}{2}mv'^2_m + \frac{1}{2}Mv'^2_M$$

$$\Rightarrow \quad K_{\text{wrt cm}} = \frac{m}{2}\frac{M^2v^2}{(m+M)^2} + \frac{M}{2}\frac{m^2v^2}{(m+M)^2}$$

$$\Rightarrow \quad K_{\text{wrt cm}} = \frac{1}{2}\left(\frac{Mm}{m+M}\right)\left(\frac{M}{m+M}+\frac{m}{m+M}\right)v^2$$

$$\Rightarrow \quad K_{\text{wrt cm}} = \frac{1}{2}\left(\frac{Mm}{m+M}\right)v^2 = \frac{1}{2}\mu v^2 \quad \ldots(1)$$

where μ is the reduced mass of the system given by

$$\mu = \frac{mM}{m+M}$$

Since, $K = K_{Lab} = \frac{1}{2}mv^2$, so equation (1) becomes

$$K_{\text{wrt cm}} = K_{Lab}\left(\frac{M}{m+M}\right) \quad \ldots(2)$$

(b) The kinetic energy of centre of mass K_{cm} is

$$K_{cm} = \frac{1}{2}(m+M)v_{cm}^2$$

$$\Rightarrow \quad K_{cm} = \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2$$

$$\Rightarrow \quad K_{cm} = \frac{1}{2}\left(\frac{m^2v^2}{m+M}\right) = \frac{1}{2}mv^2\left(\frac{m}{m+M}\right)$$

$$\Rightarrow \quad K_{cm} = K_{Lab}\left(\frac{m}{m+M}\right) \quad \ldots(3)$$

(c) From the Lab Frame (L-Frame)

As we have seen that in the lab frame, the kinetic energy of centre of mass is

$$K_{cm} = K_{Lab}\left(\frac{m}{m+M}\right)$$

This energy being possessed by the centre of mass gives us a solid argument that the kinetic energy being carried by the projectile particle in the lab frame i.e. $K_{\text{Lab}} = \frac{1}{2}mv^2$ is not fully available to be dissipated in the reaction, because a part of this energy K_{cm} will be carried away by the centre of mass of the system. So, the available energy to be dissipated is

$$K_0 = K_{\text{Lab}} - K_{\text{cm}} = K_{\text{Lab}}\left(\frac{M}{m+M}\right) \quad \ldots(4)$$

Conceptual Note(s)

As already studied in Colisions, the loss in kinetic energy of the system, when a mass m having a velocity u collides head-on with a stationary mass M is given by

$$\text{Loss} = -\Delta K = \frac{1}{2}\left(\frac{mM}{m+M}\right)u^2$$

If K is the initial kinetic energy of the colliding particle in the lab frame, then the above expression can be re-witten as

$$\text{Loss} = -\Delta K = \left(\frac{M}{m+M}\right)K = K_{Lab} - K_{cm}$$

This loss in kinetic energy is actually calculated above in equation (4) as the difference of K_{Lab} and K_{cm}.

In addition to K_0, there is also the rest mass energy available for the the reaction as Q value of the reaction. So, the total energy available for the nuclear reaction is $(Q+K_0)$. To make the reaction go, the neessary and sufficient condition is that this sum $(Q+K_0)$ must be greater than zero. Hence

$$Q + K_0 > 0$$

$$\Rightarrow \quad K_0 > -Q$$

$$\Rightarrow \quad K_{\text{Lab}}\left(\frac{M}{m+M}\right) > -Q$$

$$\Rightarrow \quad K_{\text{Lab}} > -Q\left(\frac{m+M}{M}\right)$$

$$\Rightarrow \quad K_{\text{Lab}} > -Q\left(1+\frac{m}{M}\right)$$

This minimum energy to be possessed by the projectile particle to initiate an endoergic reaction is called as the Threshold Energy K_{Th}. So

$$K_{\text{Th}} = -Q\left(\frac{m+M}{M}\right) = -Q\left(1+\frac{m}{M}\right)$$

From the Centre of Mass Frame

We understand that in the centre of mass frame, the total momentum of the particles is zero both before and after the collision. Also, in this frame, the sum of kinetic energy of the particles with respect to the centre of mass i.e. $K_{\text{wrt cm}}$ and the Q value of the reaction should be positive. So, for an endoergic reaction, at threshold, we have

$$K_{\text{wrt cm}} + Q > 0\text{, where } (Q<0)$$

$$\Rightarrow \quad K_{\text{wrt cm}} > -Q$$

$$\Rightarrow \quad K_{\text{Lab}}\left(\frac{M}{m+M}\right) > -Q$$

$$\Rightarrow \quad K_{\text{Lab}} > -Q\left(\frac{m+M}{M}\right)$$

$$\Rightarrow \quad K_{\text{Lab}} > -Q\left(1+\frac{m}{M}\right)$$

This minimum energy to be possessed by the projectile particle to initiate an endoergic reaction is called as the Threshold Energy K_{Th}. So

$$K_{\text{Th}} = -Q\left(\frac{m+M}{M}\right) = -Q\left(1+\frac{m}{M}\right)$$

ILLUSTRATION 30

How much energy must a bombarding proton possess to cause the reaction ${}_3^7\text{Li} + {}_1^1\text{H} \to {}_4^7\text{Be} + {}_0^1n$. Given that mass of Li atom is 7.01600 u, mass of Be atom is 7.01693 u, mass of H atom is 1.0783 u and mass of neutron is 1.0866 u.

SOLUTION

Since the mass of an atom also includes the masses of the atomic electrons, so to get the mass of the nucleus, the appropriate electron masses must be subtracted from the given masses of atoms.

Reactants	Products
$m({}_3^7\text{Li}) = 7.01600 - 3m_e$	$m({}_4^7\text{Be}) = 7.01693 - 4m_e$
$m({}_1^1\text{H}) = 1.0783 - 1m_e$	$m({}_0^1n) = 1.0866$
Total $= 8.02383 - 4m_e$	Total $= 8.02559 - 4m_e$

The Q-value of the reaction is given by

$$Q = -0.00176 \text{ u} = -1.65 \text{ MeV}$$

Since the energy is supplied as kinetic energy of the bombarding particle i.e. proton, so the incident proton must have kinetic energy more than this energy, because the system must possess some kinetic energy even after the reaction, so that momentum is conserved.

With momentum conservation taken into account, the minimum kinetic energy that the incident particle is given by

$$E_{\text{th}} = -\left(1+\frac{m}{M}\right)Q = -\left(1+\frac{1}{7}\right)(-1.65) = 1.89 \text{ MeV}$$

ENERGETICS OF NUCLEAR REACTIONS

Consider a nuclear reaction, schematically represented by equation

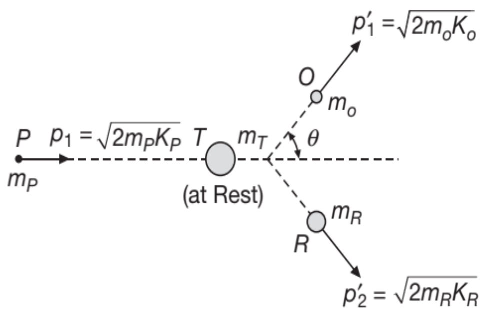
$$P + T \longrightarrow R + O$$

Let K_P, K_R and K_O be the kinetic energies of P (projectile particle), R (residual nucleus) and O (outgoing particle or radiation) respectively, while T (target nucleus) is taken to be at rest initially. Q-value of a nuclear reaction is defined as

$$Q = (K_R + K_O) - K_P$$

Let m_P, m_T, m_R, m_O, respectively, be the masses of P, T, R and O.

It can be shown that, by using Conservation of momentum, if we know the kinetic energy K_O of the outgoing paticle (O) and the angle θ of the outgoing particle with respect to the direction of motion of the projectile particle (P), then we can easily calculate the Q value of the reaction.



For the sake of convenience, let the momentum of projectile particle (P) before collision be p_1, then

$$p_1 = \sqrt{2m_pK_P} \qquad \ldots(1)$$

Since the target nucleus (T) is at rest, so momentum and kinetic energy of target nucleus both are zero.

Similarly, let the momentum of the outgoing particle (O) be p_1', then

$$p_1' = \sqrt{2m_OK_O} \qquad \ldots(2)$$

Similarly, let the momentum of the residual nucleus (R) be p_2', then

$$p_2' = \sqrt{2m_RK_R} \qquad \ldots(3)$$

All the above momenta and kinetic energies are expressed in the Lab Frame (i.e. in the laboratory frame).

Applying Conservation of Linear Momentum, we get

$$p_1' + p_2' = p_1$$

$$\Rightarrow \quad p_2' = p_1 - p_1' \qquad \ldots(4)$$

Squaring (4), we get

$$p_2'^2 = (p_1 - p_1')^2 = p_1^2 + p_1'^2 - 2p_1p_1'\cos\theta$$

where θ is the angle between p_1 and p_1'.

Applying definition of Q value, we get

$$Q = (K_R + K_O) - K_P = \left(\frac{p_2'^2}{2m_R} + \frac{p_1'^2}{2m_O}\right) - \frac{p_1^2}{2m_P}$$

$$\Rightarrow \quad Q = \left(\frac{p_1^2 + p_1'^2 - 2p_1p_1'\cos\theta}{2m_R} + \frac{p_1'^2}{2m_O}\right) - \frac{p_1^2}{2m_P}$$

$$\Rightarrow \quad Q = \frac{p_1'^2}{2}\left(\frac{1}{m_R} + \frac{1}{m_O}\right) + \frac{p_1^2}{2}\left(\frac{1}{m_R} - \frac{1}{m_P}\right) - \frac{p_1p_1'}{m_R}\cos\theta$$

$$\Rightarrow \quad Q = \frac{p_1'^2}{2m_O}\left(1 + \frac{m_O}{m_R}\right) + \frac{p_1^2}{2m_P}\left(\frac{m_P}{m_R} - 1\right) - \frac{p_1p_1'}{m_R}\cos\theta \qquad \ldots(5)$$

Using equations (1), (2) and (3) we can express the above result as

$$Q = K_O\left(1 + \frac{m_O}{m_R}\right) - K_P\left(1 - \frac{m_P}{m_R}\right) - \frac{2\sqrt{m_Pm_OK_PK_O}}{m_R}\cos\theta$$

So, we observe thar, if we know the kinetic energy K_O of the outgoing paticle (O) and the angle θ of the outgoing particle with respect to the direction of motion of the projectile particle (P), then we can easily calculate the Q value of the reaction.

ILLUSTRATION 31

Consider a body at rest in the Lab Frame. If the body explodes in two fragments of masses m_1 and m_2, calculate the final kinetic energies of the fragments in terms of Q value and the masses of the fragments.

SOLUTION

Since the body is initially at rest, hence its total initial momentum is zero. So, due to the explosion, the two fragments will fly off in opposite directions. If p_1 and p_2 be the momentum of the fragments after explosion, then

$$p_1 + p_2 = 0$$

$$\Rightarrow \quad p_1 = -p_2$$

So, the magnitude of momentum will of the fragments is same after the explosion i.e. $|p_1| = |p_2| = p$ (say).

So, the final kinetic energy after explosion is

$$K_{final} = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} = \frac{p^2}{2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$$

whereas, the initial kinetic energy of the system is zero.

Since, $Q = \Sigma K_{final} - \Sigma K_{initial}$

$$\Rightarrow \quad Q = \Sigma K_{final} = \frac{p^2}{2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$$

$$\Rightarrow \quad p = \sqrt{\left(\frac{2m_1m_2}{m_1+m_2}\right)Q} = \sqrt{2\mu Q} \qquad \ldots(1)$$

where μ is the reduced mass of the system given by

$$\mu = \frac{m_1m_2}{m_1+m_2}$$

The kinetic energies of the fragments are

$$K_1 = \frac{p_1^2}{2m_1} = \frac{p^2}{2m_1} \text{ and } K_2 = \frac{p_2^2}{2m_2} = \frac{p^2}{2m_2}$$

Substituting the value of p from equation (1), we get

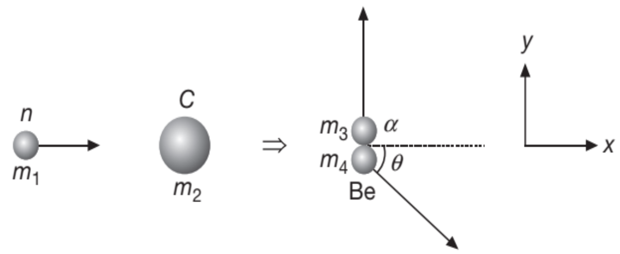
$$K_1 = \frac{p^2}{2m_1} = \left(\frac{m_2}{m_1+m_2}\right)Q \text{ and}$$

$$K_2 = \frac{p^2}{2m_2} = \left(\frac{m_1}{m_1+m_2}\right)Q$$

ILLUSTRATION 32

A neutron with kinetic energy $K = 10$ MeV activates a nuclear reaction $n + {}^{12}C \longrightarrow {}^{9}Be + \alpha$. Find the kinetic energy of the alpha particles outgoing at right angle to the direction of incoming neutrons. Take $u = 931.5$ MeV and threshold energy of reaction $(E_{th}) = 6.17$ MeV.

SOLUTION



Since, $Q + K_1 = K_3 + K_4 \qquad \ldots(1)$

Applying Law of Conservation of Linear Momentum Along x-axis

$$\sqrt{2m_1K_1} = \sqrt{2m_4K_4}\cos\theta \qquad \ldots(2)$$

Along y-axis

$$\sqrt{2m_3K_3} = \sqrt{2m_4K_4}\sin\theta \qquad \ldots(3)$$

Squaring and adding equations (2) and (3), we get

$$m_1K_1 + m_3K_3 = m_4K_4$$

$$\Rightarrow \quad K_4 = \frac{m_1}{m_4}K_1 + \frac{m_3}{m_4}K_3$$

Substituting value of K_4 in equation (1) and rearranging, we get

$$Q + \left(1 - \frac{m_1}{m_4}\right)K_1 = \left(1 + \frac{m_3}{m_4}\right)K_3$$

where, $Q = \dfrac{-E_{th}}{\left(1+\dfrac{m_1}{m_2}\right)} = \dfrac{-6.17}{1+\dfrac{1}{12}} = -5.69$ MeV

$$\Rightarrow \quad \left(1+\frac{4}{9}\right)K_3 = -5.69 + \left(1-\frac{1}{9}\right)(10)$$

$$\Rightarrow \quad K_3 = 2.21 \text{ MeV}$$

ALPHA DECAY

Alpha decay is a process in which an unstable nucleus transforms itself into a new nucleus by emitting an alpha particle (a helium nucleus, ${}^{4}_{2}\text{He}$).

Since an α-particle has two protons and two neutrons, so after an α-decay, the parent nucleus is transformed into a daughter nucleus with mass number smaller by 4 and atomic number smaller by 2.

An alpha decay can be expressed by the equation

$$ {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\text{He} + Q $$

Here Q is the energy released in the process and is determined from Einstein's mass-energy relation which gives

$$ Q = \left(m_X - m_Y - m_{He}\right)c^2 $$

where m_X, m_Y and m_{He} are the masses of the parent nucleus X, daughter nucleus Y and the α-particle respectively. The energy Q is shared by the daughter nucleus X and the α-particle. As the parent nucleus is at rest before its α-decay, the α-particles are emitted with fixed energy and hence are mono-energetic. This energy can be determined by Applying the Laws of Conservation of Energy and Momentum.

For example, uranium (238) on emitting an α-particle changes into thorium (234) as

$$ {}^{238}_{92}\text{U} \longrightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He} + Q $$

Similarly, polonium (208) is transmuted into lead (204) as

$$ {}^{208}_{84}\text{Po} \longrightarrow {}^{204}_{82}\text{Pb} + {}^{4}_{2}\text{He} + Q $$

Generally, the nuclei with mass number 210 or more undergo α-decay. In such nuclei, the long range repulsive forces between the protons dominate over the short range nuclear forces which bind the various nucleons together. By emitting α-particles, these nuclei achieve greater stability. An α-particle has a high value of binding energy $(\approx 28\text{ MeV})$. After the emission of an α-particle, the binding energy per nucleon of the emitting nucleus increases and the residual nucleus becomes more stable.

Speed of Emitted α-particles

Consider the alpha decay process equation i.e.,

$$ {}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\text{He} + Q $$

The speed of the emitted α-particles can be calculated by using the Laws of Conservation of Energy and Momentum.

Suppose the parent nucleus ${}^{A}_{Z}X$ be at rest before decay. Let v_α and v_Y be the velocities of the α-particle and the daughter nucleus. Applying the Law of Conservation of Momentum, we get

$$ m_Y v_Y = m_\alpha v_\alpha \qquad \ldots(1) $$

As the energy Q released in the decay process appears in the form of kinetic energy of α-particle and the daughter nucleus, so we have

$$ \frac{1}{2}m_\alpha v_\alpha^2 + \frac{1}{2}m_Y v_Y^2 = Q $$

Substituting the value of v_Y from equation (1), we get

$$ \frac{1}{2}m_\alpha v_\alpha^2 + \frac{1}{2}\frac{m_\alpha^2 v_\alpha^2}{m_Y^2} = Q $$

$$ \Rightarrow \quad \frac{1}{2}m_Y m_\alpha v_\alpha^2 + \frac{1}{2}m_\alpha^2 v_\alpha^2 = m_Y Q $$

$$ \Rightarrow \quad \frac{1}{2}\left(m_Y + m_\alpha\right)m_\alpha v_\alpha^2 = m_Y Q $$

$$ \Rightarrow \quad K_\alpha = \frac{1}{2}m_\alpha v_\alpha^2 = \left(\frac{m_Y}{m_Y + m_\alpha}\right)Q $$

Since, $m_Y = (A-4)$ amu and $m_\alpha = 4$ amu, so we have

$$ K_\alpha = \frac{1}{2}m_\alpha v_\alpha^2 = \left(\frac{A-4}{A}\right)Q $$

$$ \Rightarrow \quad v_\alpha = \sqrt{\frac{2K_\alpha}{m_\alpha}} = \sqrt{\frac{2(A-4)Q}{Am_\alpha}} $$

For example, in the α-decay of a random nucleus ${}^{222}_{86}\text{Rn}$, we have

$$ Q = 5.587\text{ MeV} $$

$$ \Rightarrow \quad K_\alpha = \left(\frac{A-4}{A}\right)Q = \frac{(222-4)}{222} \times 5.587\text{ MeV} $$

$$ \Rightarrow \quad K_\alpha = 5.486\text{ MeV} = 5.486 \times 1.6 \times 10^{-19}\text{ J} $$

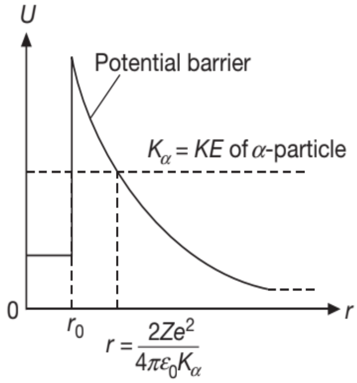
Since, $m_\alpha = 4\text{ amu} = 4 \times 1.66 \times 10^{-27}\text{ kg}$

$$ \Rightarrow \quad v_\alpha = \sqrt{\frac{2 \times 5.486 \times 1.6 \times 10^{-19}}{4 \times 1.66 \times 10^{-27}}}\text{ ms}^{-1} $$

$$ \Rightarrow \quad v_\alpha = 1.62 \times 10^{7}\text{ ms}^{-1} $$

Theory of α-decay (Tunnelling Effect)

The α-particles emitted by different radioactive nuclei have kinetic energy ranging from 4 MeV to 9 MeV. The nucleus of an α-emitter possesses a barrier of height about 25 MeV. Figure shows a plot of the potential energy U of the system consisting of the α-particle and the residual nucleus.



Plot of potential energy U of an α-particle as a function of distance r from the centre of the residual nucleus.

The α-particles are short of about 16 to 25 MeV of energy, needed for the emission. Therefore, classically, we cannot explain the emission of α-particles by radioactive nuclei.

In 1928, Gamow, Congdon and Gurney explained the emission of α-particles in terms of the penetration of the nuclear potential barrier on the basis of Quantum Theory. According to this theory, we have

(a) An α-particle may exist as an entity (already formed) inside a nucleus before it escapes from the nucleus.

(b) The α-particle is in a state of constant motion inside the nucleus with a speed of about $10^7\ \text{ms}^{-1}$.

(c) Quantum mechanically, even an α-particle having insufficient kinetic energy has a small but finite probability p of its crossing the potential barrier.

As the size of the nucleus $\approx 10^{-14}$ m and speed of α-particle $\approx 10^7\ \text{ms}^{-1}$, the α-particle takes about 10^{-21} s to move across the nucleus. Thus α-particle presents itself before the potential barrier 10^{21} times in a second. The probability P of escape of an α-particle from a nucleus will be

$$P = p\nu$$

As ν is large $\left(10^{21}\ \text{s}^{-1}\right)$, so P is sufficiently large and the α-particle can tunnel through the energy barrier which is classically impossible. Hence α-decay occurs as a result of barrier tunnelling.

The barrier tunnelling explains why every $^{238}_{92}\text{U}$ nuclide in a sample of $^{238}_{92}\text{U}$ atoms does not decay at once, even when its decay process has a positive Q value. Consequently, the half-lives for α-decay of most of the alpha unstable nuclei are very long. For example, the half-life of $^{238}_{92}\text{U}$ for α-decay is 4.5×10^9 year.

ILLUSTRATION 33

The nucleus of an atom is $^{235}_{92}Y$, initially at rest, decays by emitting an α-particle as per the equation

$$^{235}_{92}Y \rightarrow {}^{231}_{90}X + {}^{4}_{2}\text{He} + \text{Energy}$$

It is given that the binding energies per nucleon of the parent and the daughter nuclei are 7.8 MeV and 7.835 MeV respectively and that of α-particle is 7.07 MeV/nucleon. Assuming the daughter nucleus to be formed in the unexcited state and neglecting its share in the energy of the reaction, calculate the speed of the emitted α-particle. Take mass of α-particle to be 6.68×10^{-27} kg.

SOLUTION

$$Q = \left[(7.835\times231)+(7.07\times4)-(7.8\times235)\right] \text{MeV}$$

$$\Rightarrow \quad Q = 5.18 \text{ MeV}$$

$$\Rightarrow \quad Q = 5.18\times1.6\times10^{-13} \text{ J}$$

This entire kinetic energy is taken by the α-particle, so

$$\frac{1}{2}m_\alpha v_\alpha^2 = 5.18\times1.6\times10^{-13}$$

$$\Rightarrow \quad \frac{1}{2}\times6.68\times10^{-27}v_\alpha^2 = 5.18\times1.6\times10^{-13}$$

$$\Rightarrow \quad v_\alpha = 1.57\times10^7\ \text{ms}^{-1}$$

BETA DECAY

The process of spontaneous emission of an electron $\left(e^-\right)$ or a positron $\left(e^+\right)$ from a nucleus is called **beta decay**.

Like α-decay, β-decay is a spontaneous process, with a definite disintegration energy and half-life. It is also a statistical process, obeying the Law of radioactive Decay.

In beta minus (β^-) decay, the mass number of the radioactive nucleus remains unchanged but its atomic number increases by one. An electron and a new particle antineutrino $(\bar{\nu})$ are emitted from the nucleus, as in the decay.

$$^{32}_{15}\text{P} \longrightarrow ^{32}_{16}\text{S} + e^- + \bar{\nu}$$

In general, the **beta minus decay** may be represented as

$$^{A}_{Z}X \longrightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}e + \bar{\nu}$$

OR $$^{A}_{Z}X \longrightarrow {}_{Z+1}^{A}Y + \beta^- + \bar{\nu}$$

The electron emitted from the nucleus is called a beta particle, denoted by β^-.

In beta plus (β^+) decay, the mass number of the radioactive nucleus remains unchanged but its atomic number decreases by one. A positron (e^+) and a new particle neutrino (ν) are emitted from the nucleus, as in the decay.

$$^{22}_{11}\text{Na} \longrightarrow ^{22}_{10}\text{Ne} + e^+ + \nu$$

In general, the **beta plus decay** may be represented as

$$^{A}_{Z}X \longrightarrow {}_{Z-1}^{A}Y + {}_{+1}^{0}e + \nu$$

OR $$^{A}_{Z}X \longrightarrow {}_{Z-1}^{A}Y + \beta^+ + \nu$$

The positron so emitted is called a beta plus particle (β^+)

The positron is an antiparticle of electron. It has a positive charge equal in magnitude to the charge on an electron and has a mass equal to the mass of an electron.

Similarly, neutrino and antineutrino are antiparticles of each other. Both are massless, chargeless particles having spins $\pm\frac{1}{2}$.

Although a nucleus contains no electrons, positrons and neutrinos, yet can it eject these particles. It is believed that electrons, positrons and neutrinos are created during the process of beta decay. If the unstable nucleus has excess neutrons than needed for stability, a neutron converts itself into a proton. So, in a beta-minus decay, an electron and an antineutrino are created and emitted from the nucleus via the reaction given by

$$n \rightarrow p + e^- + \bar{\nu}$$

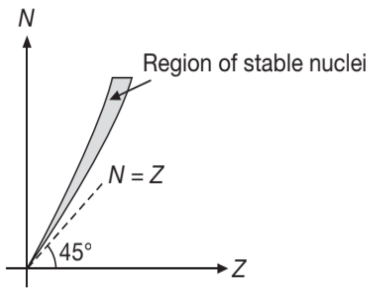
If the unstable nucleus has excess protons than that needed for stability, a proton converts itself into a neutron. So, in a beta-plus decay, a positron and a neutrino are created and emitted from the nucleus via the reaction given by

$$p \rightarrow n + e^+ + \nu$$

Clearly, a beta decay process involves the conversion of a neutron into a proton or vice versa. These nucleons have nearly equal masses. That is why the mass number A of a nuclide undergoing beta decay does not change.

TYPES OF BETA DECAY

In a beta decay process, the $\left(\frac{N}{Z}\right)$ ratio of nucleus is changed. This decay is shown by unstable nuclei. In beta decay, either a neutron is converted into proton or proton is converted into neutron. For better understanding of beta decay let us first draw the N vs Z graph.



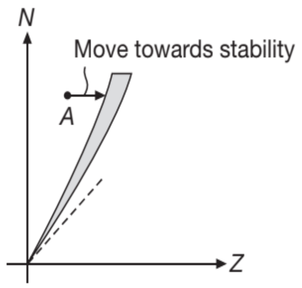
From the N vs Z graph, we observe that there are two types of unstable nuclides.

Negative Beta or β^- Decay or A-Type Beta Decay

The nuclides showing this type of decay are also called as A type nuclides. For A type nuclides, we have

$$\left(\frac{N}{Z}\right)_A > \left(\frac{N}{Z}\right) \text{ stable}$$

To achieve stability, these A type nuclides increase their Z by converting a neutron inside them to a proton by emitting a β^- particle. Due to this, the nucleus now attains an $\left(\frac{N}{Z}\right)$ ratio that takes it towards the line of stability as shown.



$$_0n^1 \rightarrow _1p^1 + e^{-1} + \bar{\nu}$$

$$_ZX^A \rightarrow _{Z+1}Y^A + \underset{(\beta^- \text{ particle})}{e^{-1}} + \bar{\nu}$$

This decay is called β^{-1} decay. Kinetic energy available for β^{-1} and $\bar{\nu}$ is Q

$$Q = K_\beta + K_{\bar{\nu}}$$

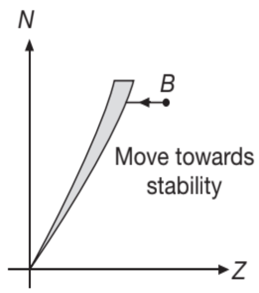
Kinetic energy of the emitted β particles satisfies the condition $0 < K_\beta < Q$

Positive Beta or β^+ Decay or B-Type Beta Decay

The nuclides showing this type of decay are also called as B type nuclides. For B type nuclides, we have

$$\left(\frac{N}{Z}\right)_B > \left(\frac{N}{Z}\right) \text{stable}$$

To achieve stability, these B type nuclides decrease their Z by converting a proton inside them to a neutron by emitting a β^+ particle. Due to this, the nucleus now attains an $\left(\frac{N}{Z}\right)$ ratio that takes it toward the line of stability as shown.



$$p \rightarrow n + \underset{(\text{positron})}{e^+} + \underset{(\text{neutrino})}{\nu}$$

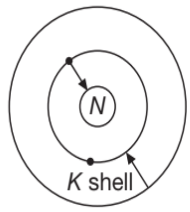
$$_ZX^A \rightarrow _{Z-1}Y^A + \underset{(\beta^+ \text{ particle})}{e^+} + \nu$$

Electron Capture or *K*-capture or Reverse Beta Decay Process

Besides β^{-1} and β^+ emission, there is a third related process. This process is called Electron Capture (abbreviated as EC) or K-capture or a reverse beta decay process. It is a rare process which is found only in few proton rich nuclei. In this process, the proton rich nucleus captures one of the atomic electrons from the K shell and so is called as K-Capture process. A proton in the nucleus combines with this electron and converts itself into a neutron. A neutrino is also emitted in the process and is emitted from the nucleus.

$$p + _{-1}e^0 \rightarrow n + \nu$$

The vacancy created in the K shell is filled by transition of electrons from the outer shells. This results in the emission of **characteristic X-rays.** This process is inferred experimentally by detection of emitted X-rays (due to other electrons jumping down to fill the empty state) of just the proper energy.



If X and Y are atoms then reaction is written as

$$_ZX^A \rightarrow _{Z-1}Y^A + \nu + Q + \text{characteristic X-rays of atom } Y$$

If X and Y are taken as nucleus, then reaction is written as

$$_ZX^A + _{-1}e^0 \rightarrow _{Z-1}Y^A + \nu$$

This process is called **Electron Capture** or **K-Capture** or **Reverse Beta Decay process.**

An example is ${}^7_4\text{Be}$, which as a result becomes ${}^7_3\text{Li}$. The process is written

$${}^7_4\text{Be} + e^- \rightarrow {}^7_3\text{Li} + \nu,$$

Or, in general,

$${}^A_ZN + e^- \rightarrow {}^A_{Z-1}N' + \nu \qquad \{\text{Electron Capture}\}$$

In β decay, it is the weak nuclear force that plays the crucial role. The neutrino is unique because it interacts with matter only via the weak force, which is why it is so hard to detect.

Conceptual Note(s)

(a) Please note that the electron emitted in β decay is not an orbital electron. Instead, the electron is created within the nucleus itself. Actually, inside the nucleus, one of the neutrons changes to a proton and in the process (to conserve charge) emits an electron. Indeed, free neutrons actually decay in this fashion

$$n \to p + e^{-} + a \text{ neutrino}$$

Since the emitted electrons, originate in the nucleus, so the electrons emitted in β decay are often referred to as β particles, rather than as electrons (just to remind us of their origin). However, these electrons or β-particles are indistinguishable from the orbital electrons.

(b) If during the process of beta decay, we do not consider the emission of neutrino or antineutrino, then we observe that the Laws of Conservation of Energy, Conservation of Linear Momentum and Conservation of Angular momentum are not applicable for the beta decay process.

(c) To solve this absurdity, Pauli assumed that in the beta decay process, no doubt the beta particles are emitted but at the same time another particle called neutrino or antineutrino is also emitted along with the beta particle.

Due to Pauli's Neutrino Hypothesis, the Laws of Conservation of Energy, Conservation of Linear Momentum and Conservation of Angular momentum become applicable for the beta decay process.

PAULI'S NEUTRINO HYPOTHESIS AND NEUTRINO PROPERTIES

Pauli assumed that during the beta decay process, emission of beta particle is accompanied by the emission of another particle called neutrino or antineutrino. Due to Pauli's Neutrino Hypothesis, the conservation laws like Conservation of Energy, Conservation of Linear Momentum and Conservation of Angular momentum become applicable for the beta decay process.

According to Pauli, the particle called neutrino or antineutrino has the following properties. These particles have few of their properties matching with a photon.

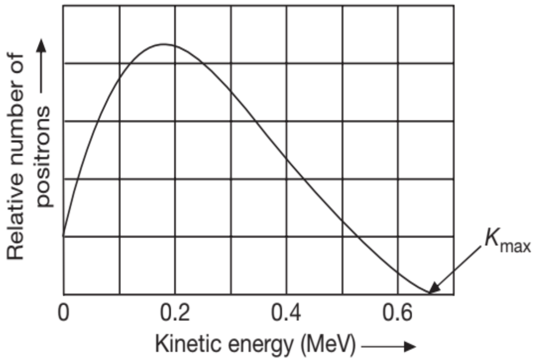
(a) Both have zero rest mass like a photon.
(b) Both have zero charge i.e. both are neutral.
(c) Both particles are energy particles just like photons.
(d) Both particles have linear momentum just like a photon.
(e) Like other nucleons, neutrinos too possess spin. They have spin quantum number $s = \pm\frac{1}{2}$.
(f) They carry angular momentum of $\pm\frac{h}{2\pi}$ just like other nucleons.
(g) Since neutrinos (or antineutrinos) are massless and chargeless, they interact so weakly with matter that it becomes very difficult to detect them. They can penetrate through earth without being absorbed. They can penetrate about 10^{12} km in lead, the most dense material, without interacting with it. By ingenious experiments, neutrinos have been detected and their mass and spin or intrinsic angular momentum have been measured.
(h) The β^{+} decay process is accompanied by the emission of neutrino ν, whereas the β^{-} decay process is accompanied by the emission of an antineutrino $\bar{\nu}$.

CONTINUOUS ENERGY SPECTRUM FOR BETA RAYS

In both α-decay and β-decays processes, the disintegration energy Q depends on the nature of the radionuclide. In the α-decay of a particular radionuclide, every emitted α-particle has a definite amount of kinetic energy.

However, in β-decay, the disintegration energy is shared in all proportions between the three particles i.e., daughter nucleus, electron (or positron) and antineutrino (or neutrino). As a result, the kinetic energy of the electrons (or positrons) is not fixed.

Their energy varies from zero to a maximum value K_{max}. Thus β- rays have a continuous energy spectrum, as shown in Figure.



The distribution of the kinetic energies of positrons emitted in the decay of $^{64}_{29}Cu$

The maximum kinetic energy or end point energy K_{max} must be equal to disintegration energy Q. When the electron (or positron) has maximum energy, the energy carried by the daughter nucleus and neutrino is nearly zero.

Q-VALUE FOR BETA DECAY PROCESS

We have already discussed the three fundamental beta decay processes. Generally, the masses used in the calculation of mass defect involved in a nuclear reaction are the masses of the atoms in which we have neglected the mass of the electrons which is already included in the mass of the atom. However, for the beta decay process, we know that the mass of the beta particle is same as the mass of the electron and hence it is not possible for us to neglect the mass of the orbital electrons of the atom.

Q-value for β^- Decay Process

Considering the emission of antineutrino, the equation of β^--decay can be written as

$$_ZX^A \longrightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + Q + \bar{\nu}$$

Production of antineutrino along with the electron helps to explain the continuous spectrum because the energy is distributed randomly between electron and $\bar{\nu}$ and it also helps to explain the spin quantum number balance (because p, n and $\pm e$ each has spin quantum number $\pm\frac{1}{2}$). During β^- decay, inside the nucleus a neutron is converted to a proton with emission of an electron and antineutrino.

$$n \rightarrow p + {}_{-1}e^0 + \bar{\nu}$$

Let M_X be the mass of atom $_ZX^A$, M_Y be the mass of atom $_{Z+1}Y^A$ and m_e be the mass of electron, then Q value is given by

$$Q = \left[(M_X - Zm_e) - \left\{(M_Y - (Z+1)m_e) + m_e\right\}\right]c^2$$

$$\Rightarrow \quad Q = (M_X - M_Y)c^2$$

Here also, one can see that by applying the law of conservation of momentum and energy, we will get

$$T_e = \frac{m_Y}{m_e + m_Y}Q \text{ and } T_Y = \frac{m_e}{m_e + m_Y}Q$$

Since, $m_e \ll m_Y$, we can consider that almost all the energy is taken away by the electron.

Q-value for β^+ Decay Process

Considering the emission of antineutrino, the equation of β^--decay can be written as

$$_ZX^A \longrightarrow {}_{Z-1}Y^A + {}_{+1}e^0 + Q + \nu$$

During β^+ decay, inside the nucleus a proton is converted to a neutron with emission of a positron and a neutrino.

$$p \rightarrow n + {}_{+1}e^0 + \nu$$

Let M_X be the mass of atom $_ZX^A$, M_Y be the mass of atom $_{Z-1}Y^A$ and m_e be the mass of electron, then Q value is given by

$$Q = \left[(M_X - Zm_e) - \left\{(M_Y - (Z-1)m_e) + m_e\right\}\right]c^2$$

$$\Rightarrow \quad Q = (M_X - M_Y - 2m_e)c^2$$

Q-value for *K*-capture Process

Considering the K-capture or the electron capture by an element $_ZX^A$, written by the equation

$$_ZX^A \longrightarrow {}_{Z-1}Y^A + \bar{\nu}$$

Let M_X be the mass of atom $_ZX^A$, M_Y be the mass of atom $_{Z-1}Y^A$ and m_e be the mass of electron, then

$$Q \text{ value} = \left[(M_X - Zm_e) - (M_Y - (Z-1)m_e)\right]c^2$$

$$\Rightarrow \quad Q \text{ value} = (M_X - M_Y - m_e)c^2$$

ILLUSTRATION 34

Neon - 23 beta decays in the following way:

$$^{23}_{10}\text{Ne} \to {}^{23}_{11}\text{Na} + {}^{0}_{-1}e + \bar{\nu}$$

Find the minimum and maximum kinetic energy that the beta particle ${}^{0}_{-1}e$ can have. The atomic masses of ^{23}Ne and ^{23}Na are $22.9945u$ and $22.9898u$, respectively.

SOLUTION

Reactants	Products
$m\left({}^{23}_{10}\text{Ne}\right) = 22.9945 - 10m_e$	$m\left({}^{23}_{11}\text{Na}\right) = 22.9898 - 11\,m_e$
	$m\left({}^{0}_{-1}e\right) = m_e$
$\text{Total} = 22.9945 - 10m_e$	$\text{Total} = 22.9898 - 10m_e$

Mass defect $= 22.9945 - 22.9898 = 0.0047\text{ u}$

$$Q = (0.0047)(931) = 4.4\text{ MeV}$$

The β-particle and neutrino share this energy. Hence the energy of the β-particle can range from 0 to 4.4 MeV.

ILLUSTRATION 35

Consider the beta decay $^{198}\text{Au} \to {}^{198}\text{Hg}^* + \beta^- + \bar{\nu}$ where $^{198}\text{Hg}^*$ represents a mercury nucleus in an excited state at energy 1.088 MeV above the ground state. What can be the maximum kinetic energy of the electron emitted? The atomic mass ^{198}Au is 197.968233 u and that of ^{198}Hg is 197.966760 u.

SOLUTION

If the product nucleus ^{198}Hg is formed in its ground state, the kinetic energy available to the electron and the antineutrino is

$$Q = \left[m\left({}^{198}\text{Au}\right) - m\left({}^{198}\text{Hg}\right)\right]c^2$$

As $^{198}\text{Hg}^*$ has energy 1.088 MeV more than ^{198}Hg in ground state, the kinetic energy actually available is

$$Q = \left[m\left({}^{198}\text{Au}\right) - m\left({}^{198}\text{Hg}\right)\right]c^2 - 1.088\text{ MeV}$$

$$\Rightarrow \quad Q = (197.968233 - 197.966760)(931)\text{ MeV} - 1.088\text{ MeV}$$

$$\Rightarrow \quad Q = 1.3686\text{ MeV} - 1.088\text{ MeV} = 0.2806\text{ MeV}$$

This is also the maximum possible kinetic energy of the emitted electron.

ILLUSTRATION 36

Calculate the Q-value in the following decays

(a) $^{19}\text{O} \to {}^{19}\text{F} + e^- + \bar{\nu}$

(b) $^{25}\text{Al} \to {}^{25}\text{Mg} + e^+ + \nu$

The atomic masses needed are as follows:

^{19}O	^{19}F	^{25}Al	^{25}Mg
19.003576 u	18.998403 u	24.990432 u	24.985839 u

SOLUTION

(a) The Q-value of β^--decay is

$$Q = \left[m\left({}^{19}\text{O}\right) - m\left({}^{19}\text{F}\right)\right]c^2$$

$$\Rightarrow \quad Q = (19.003576\text{ u} - 18.998403\text{ u})(931\text{ MeV/u})$$

$$\Rightarrow \quad Q = 4.816\text{ MeV}$$

(b) The Q-value of β^+-decay is

$$Q = \left[m\left({}^{25}\text{Al}\right) - m\left({}^{25}\text{Mg}\right) - 2m_e\right]c^2$$

$$\Rightarrow \quad Q = [24.99032\text{ u} - 24.985839\text{ u} - 2 \times 0.511\text{ MeV}_c^{-2}]c^2$$

$$\Rightarrow \quad Q = (0.004593\text{ u})(931\text{ MeV/u}) - 1.022\text{ MeV}$$

$$\Rightarrow \quad Q = 4.276\text{ MeV} - 1.022\text{ MeV} = 3.254\text{ MeV}$$

GAMMA DECAY

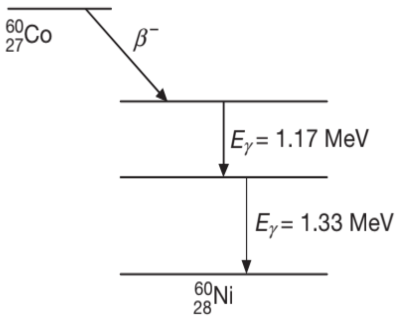
The process of emission of a γ-ray photon during the radioactive disintegration of a nucleus is called **gamma decay**.

As the emitted γ-ray photons have zero rest mass and carry no charge, so in a γ-decay the mass number and atomic number of the nucleus remain unchanged and no new element is formed. A γ-decay can be expressed as

$$\underset{\text{(Excited state)}}{{}^{A}_{Z}X} \longrightarrow \underset{\text{(Ground state)}}{{}^{A}_{Z}X} + \gamma$$

A nucleus does not contain photons, yet it can emit photons. These photons are created during the emission process. We know that a nucleus can exist in different energy states. After an α or a β-decay, the daughter nucleus is usually left in the excited state. It attains the ground state by single or successive transitions by emitting one or more photons. As the nuclear states have energies of the order of MeV, therefore, the photons emitted by the nuclei have energy of the order of several MeV. The wavelength of such high energy photons is a fraction of an angstrom. The short wavelength electromagnetic waves emitted by nuclei are called γ- rays.

An example of γ-decay is shown through an energy level diagram shown in Figure. Here an unstable $^{60}_{27}Co$ nucleus is transformed via a β-decay into an excited $^{60}_{28}Ni$ nucleus, which in turn reaches the stable ground state by emitting photons of energies 1.17 MeV and 1.33 MeV, in two successive γ-decay processes.

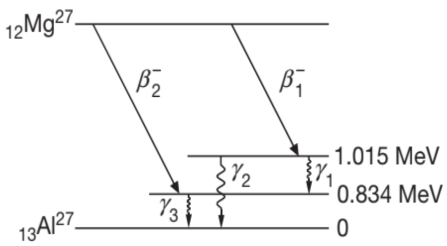


Energy-level diagram showing the emission of γ-rays by a $^{60}_{27}Co$ nucleus subsequent to beta decay

Usually, γ- rays are emitted after α- or β- decay, but there are long lived radioactive nuclei that emit only γ- rays.

ILLUSTRATION 37

Calculate the kinetic energy of β-particles and the radiation frequencies corresponding to the γ-decays shown in figure



Given, mass of $_{12}Mg^{27}$ atom $= 26.991425$ amu and mass of $_{13}Al^{27}$ atom $= 26.990080$ amu

SOLUTION

Energy of photon corresponding to frequency ν_1 is

$$h\nu_1 = E_3 - E_2$$

$$\Rightarrow \quad \nu_1 = \frac{E_3 - E_2}{h}$$

$$\Rightarrow \quad \nu_1 = \frac{(1.015 - 0.834)\text{ MeV}}{6.62\times 10^{-34}\text{ Js}}$$

$$\Rightarrow \quad \nu_1 = \frac{0.181\times 1.6\times 10^{-13}\text{ J}}{6.62\times 10^{-34}\text{ Js}}$$

$$\Rightarrow \quad \nu_1 = 4.37\times 10^{19}\text{ s}^{-1}$$

Energy of photon corresponding to frequency ν_2 is

$$h\nu_2 = E_3 - E_1$$

$$\Rightarrow \quad \nu_2 = \frac{E_3 - E_1}{h}$$

$$\Rightarrow \quad \nu_2 = \frac{(1.015 - 0)\text{ MeV}}{6.62\times 10^{-34}\text{ Js}}$$

$$\Rightarrow \quad \nu_2 = 2.45\times 10^{20}\text{ s}^{-1}$$

Energy of photon corresponding to frequency ν_3 is

$$h\nu_3 = E_2 - E_1$$

$$\Rightarrow \quad \nu_3 = \frac{E_2 - E_1}{h}$$

$$\Rightarrow \quad \nu_3 = \frac{(0.834 - 0)\text{ MeV}}{6.62\times 10^{-34}}$$

$$\Rightarrow \quad \nu_3 = 2.0\times 10^{20}\text{ s}^{-1}$$

Now emission of β_1^--particle is given by

$$_{12}Mg^{27} \to _{13}Al^{27} + \beta^- + \nu_2 + Q_1$$

$$\Rightarrow \quad Q_1 = \left[m\left(_{12}Mg^{27}\right) - m\left(_{13}Al^{27}\right) - E(\nu_2)\right]$$

$$\Rightarrow \quad Q_1 = [26.991425 - 26.990080]u - (E_3 - E_1)\text{ MeV}$$

$$\Rightarrow \quad Q_1 = 0.001345\times 931 - 1.015\text{ MeV} = 0.237\text{ MeV}$$

$$\Rightarrow \quad K.E.\text{ of } \beta_1^- \text{ particle is } 0.237\text{ MeV}$$

Emission of β_2^- particle is given by

$$_{12}Mg^{27} \to _{13}Al^{27} + \beta_2^- + \nu_3 + Q_2$$

$$\Rightarrow \quad Q_2 = \{[26.991425 - 26.990080]931 - 0.834\}\text{ MeV}$$

$$\Rightarrow \quad Q_2 = 0.418\text{ MeV}$$

$\Rightarrow$ KE of β_2^- particle is 0.418 MeV

CLASSIFICATION OF NUCLEAR REACTIONS

Nuclear reactions can be classified into the following categories.

Elastic Scattering

The incident particle gets deflected without any change in its energy, i.e.,

$$^4_2\text{He} + ^{197}_{79}\text{Au} \longrightarrow ^{197}_{79}\text{Au} + ^4_2\text{He}$$

The bombarding particle passes sufficiently at large distance away from the target nucleus so as to get repulsion which changes its direction of motion without any change in its energy.

Inelastic Scattering

If the bombarding particle passes close to target it gets deflected. Due to strong repulsion, the target particle also acquires some energy. So, the energy left with the scattered particle is less than that it had initially.

$$^1_1\text{H} + ^7_3\text{Li} \longrightarrow ^7_3\text{Li} + ^1_1\text{H}$$

^7_3Li means existence of ^7_3Li in one of its excited states.

Simple Capture

The incoming particle is captured by the target nucleus. The product nucleus which is generally in the form of excited state decays to the ground state by emitting γ-ray of energy $h\nu$.

$$^1_1\text{H} + ^{12}_6\text{C} \longrightarrow ^{13}_7\text{N} \longrightarrow ^{13}_7\text{N} + h\nu$$

Disintegration (Nuclear Transmutations)

The intermediate compound nucleus breaks up and results in a product nucleus and an outgoing particle. The product nucleus has different chemical properties as compared to the target particle. Majority of nuclear reactions belong to this category. Such nuclear disintegrations are called Nuclear Transmutations.

(a) Disintegration by α-particles

(i) (α, p) reactions

$$^A_Z\text{X} + ^4_2\text{He} \longrightarrow ^{A+4}_{Z+2}\text{C}^* \longrightarrow ^{A+3}_{Z+1}\text{Y} + ^1_1\text{H}$$

$\{C^*$ is Compound Nucleus$\}$

EXAMPLES

The historical experiment of Rutherford is an α induced transmutation, an (α, p) reaction.

$$_7\text{N}^{14} + _2\text{He}^4 \longrightarrow _9\text{F}^{18*} \longrightarrow ^{17}_8\text{O} + _1\text{H}^1$$

and is exoergic in nature $(Q > 0)$. Other useful (α, p) reactions are

$$^{10}_5\text{B} + ^4_2\text{He} \longrightarrow ^{14}_7\text{N}^* \longrightarrow ^{13}_6\text{C} + ^1_1\text{H}$$

$$^{23}_{11}\text{Na} + ^4_2\text{He} \longrightarrow ^{27}_{13}\text{Al}^* \longrightarrow ^{26}_{12}\text{Mg} + ^1_1\text{H}$$

$$^{27}_{13}\text{Al} + ^4_2\text{He} \longrightarrow ^{31}_{15}\text{P}^* \longrightarrow ^{30}_{14}\text{Si} + ^1_1\text{H}$$

$$^{45}_{21}\text{Sc} + ^4_2\text{He} \longrightarrow ^{49}_{23}\text{V}^* \longrightarrow ^{48}_{22}\text{Ti} + ^1_1\text{H}$$

(ii) (α, n) reactions

$$^A_Z\text{X} + ^4_2\text{He} \longrightarrow ^{A+4}_{Z+2}\text{C}^* \longrightarrow ^{A+3}_{Z+2}\text{Y} + ^1_0n$$

$\{C^*$ is Compound Nucleus$\}$

EXAMPLES

$$^7_3\text{Li} + ^4_2\text{He} \longrightarrow ^{11}_5\text{B}^* \longrightarrow ^{10}_5\text{B} + ^1_0n$$

$$^9_4\text{Be} + ^4_2\text{He} \longrightarrow ^{13}_6\text{C}^* \longrightarrow ^{12}_6\text{C} + ^1_0n$$

$$^{19}_9\text{F} + ^4_2\text{He} \longrightarrow ^{23}_{11}\text{Na}^* \longrightarrow ^{22}_{11}\text{Na} + ^1_0n$$

$$^{23}_{11}\text{Na} + ^4_2\text{He} \longrightarrow ^{27}_{13}\text{Al}^* \longrightarrow ^{26}_{13}\text{Al} + ^1_0n$$

(b) Disintegration by protons

(i) (p, α) reactions: When the reactions yield α-particles. The (p, α) reactions are usually exoergic and have the general form

$$^A_Z\text{X} + ^1_1\text{H} \longrightarrow ^{A+1}_{Z+1}\text{C}^* \longrightarrow ^{A-3}_{Z-1}\text{Y} + ^4_2\text{He}$$

$\{C^*$ is Compound Nucleus$\}$

EXAMPLES

$$^6_3\text{Li} + ^1_1\text{H} \longrightarrow ^7_4\text{Be}^* \longrightarrow ^3_2\text{He} + ^4_2\text{He}$$

$(Q = 4\text{ MeV})$

$$^7_3\text{Li} + ^1_1\text{H} \longrightarrow ^8_4\text{Be}^* \longrightarrow ^4_2\text{He} + ^4_2\text{He}$$

$(Q = 17.35\text{ MeV})$

$$^{11}_5\text{B} + ^1_1\text{H} \longrightarrow ^{12}_6\text{C}^* \longrightarrow ^8_4\text{Be} + ^4_2\text{He}$$

$(Q - 8.59\text{ MeV})$

$\{$The product nucleus ^8_4Be is highly ustable and decays almost immediately as $^8_4\text{Be} \longrightarrow ^4_2\text{He} + ^4_2\text{He}$, so that the final reaction gives three α-particles$\}$

$${}^{19}_{9}F + {}^{1}_{1}H \longrightarrow {}^{20}_{10}Ne^* \longrightarrow {}^{16}_{8}O + {}^{4}_{2}He$$

$(Q = 812\ \text{MeV})$

$${}^{23}_{11}Na + {}^{1}_{1}H \longrightarrow {}^{24}_{12}Mg^* \longrightarrow {}^{20}_{10}Ne + {}^{4}_{2}He$$

$(Q = 2.38\ \text{MeV})$

$${}^{63}_{29}Cu + {}^{1}_{1}H \longrightarrow {}^{64}_{30}Zn^* \longrightarrow {}^{60}_{28}Ni + {}^{4}_{2}He$$

$(Q = 3.76\ \text{MeV})$

(ii) (p, n) reactions: When the reactions yield neutrons. The general equation of this type of reactions is

$${}^{A}_{Z}X + {}^{1}_{1}H \longrightarrow {}^{A+1}_{Z+1}C^* \longrightarrow {}_{Z+1}^{A}Y + {}^{1}_{0}n$$

$\{C^*$ is Compound Nucleus$\}$

the product nucleus being isobaric with the target nucleus.

Since two isobars differing in Z by unity cannot both be stable, the product nucleus is β^+ active, decaying by β^+ emission (or electron capture) into ${}^{A}_{Z}X$ (the same as target nucleus):

$${}_{Z+1}^{A}Y \xrightarrow[\text{electron capture}]{\beta^+} {}^{A}_{Z}X$$

EXAMPLES

$$\begin{cases} {}^{11}_{6}B + {}^{1}_{1}H \longrightarrow {}^{12}_{6}C^* \longrightarrow {}^{11}_{6}C + {}^{1}_{0}n \\ {}^{11}_{6}C \xrightarrow{\beta^+} {}^{11}_{5}B \end{cases}$$

$(Q = -1.763\ \text{MeV})$ (Half-life $= 2.5$ min)

$$\begin{cases} {}^{23}_{11}Na + {}^{1}_{1}H \longrightarrow {}^{24}_{12}Mg^* \longrightarrow {}^{23}_{12}Mg + {}^{1}_{0}n \\ {}^{23}_{12}Mg \xrightarrow{\beta^+} {}^{23}_{11}Na \end{cases}$$

$(Q = -4.84\ \text{MeV})$ (Half-life $= 12.3$ s)

$$\begin{cases} {}^{54}_{24}Cr + {}^{1}_{1}H \longrightarrow {}^{55}_{25}Mn^* \longrightarrow {}^{54}_{25}Mn + {}^{1}_{0}n \\ {}^{54}_{25}Mn \xrightarrow{\beta^+} {}^{54}_{24}Cr \end{cases}$$

$(Q = -2.16\ \text{MeV})$ (Half-life $= 310$ days)

$$\begin{cases} {}^{63}_{29}Cu + {}^{1}_{1}H \longrightarrow {}^{64}_{30}Zn^* \longrightarrow {}^{63}_{30}Zn + {}^{1}_{0}n \\ {}^{63}_{30}Zn \xrightarrow{\beta^+} {}^{63}_{29}Cu \end{cases}$$

$(Q = -4.15\ \text{MeV})$ (Half-life $= 38.5$ min)

The (p, n) reaction is always endoergic.

(iii) (p, γ) reactions: When the reactions yield γ-photons. The compound nucleus formed by absorption of proton by the target nucleus does not emit any nuclear particle but goes down to the ground state emitting one or more γ-photons. The (p, γ) reaction is the radiative capture of proton.

The general equation of (p, γ) reaction is

$${}^{A}_{Z}X + {}^{1}_{1}H \longrightarrow {}^{A+1}_{Z+1}C^* \longrightarrow {}^{A+1}_{Z+1}C + \gamma$$

$\{C^*$ is Compound Nucleus$\}$

EXAMPLES

$${}^{7}_{3}Li + {}^{1}_{1}H \longrightarrow {}^{8}_{4}Be^* \longrightarrow {}^{8}_{4}Be + \gamma$$

$${}^{14}_{7}N + {}^{1}_{1}H \longrightarrow {}^{15}_{8}O^* \longrightarrow {}^{15}_{8}O + \gamma$$

$${}^{24}_{12}Mg + {}^{1}_{1}H \longrightarrow {}^{25}_{13}Al^* \longrightarrow {}^{25}_{13}Al + \gamma$$

$${}^{27}_{13}Al + {}^{1}_{1}H \longrightarrow {}^{28}_{14}Sr^* \longrightarrow {}^{28}_{14}Sr + \gamma$$

(iv) (p, d) reactions: When the reactions yield deuterons. The general equation of this type of reaction is

$${}^{A}_{Z}X + {}^{1}_{1}H \longrightarrow {}^{A-1}_{Z}X + {}^{2}_{1}H$$

This is an example of direct reaction without any formation of the compound nucleus.

EXAMPLES

$${}^{7}_{3}Li + {}^{1}_{1}H \longrightarrow {}^{6}_{3}Li + {}^{2}_{1}H$$

$${}^{9}_{4}Be + {}^{1}_{1}H \longrightarrow {}^{8}_{4}Be + {}^{2}_{1}H$$

(c) Disintegration by neutrons

(i) (n, α) reactions: When the reactions yield α-particles. The general equation for (n, α) reactions is

$${}^{A}_{Z}X + {}^{1}_{0}n \longrightarrow {}^{A+1}_{Z}C^* \longrightarrow {}^{A-3}_{Z-2}Y + {}^{4}_{2}He$$

$\{C^*$ is Compound Nucleus$\}$

The (n, α) reactions are usually exoergic, i.e. Q is positive, particularly for medium heavy nuclei.

EXAMPLES

$${}^{6}_{3}Li + {}^{1}_{0}n \longrightarrow {}^{7}_{3}Li^* \longrightarrow {}^{3}_{1}H + {}^{4}_{2}He$$

$(Q = 4.785\ \text{MeV})$

followed by $^3_1\text{H} \xrightarrow{\beta^-} {}^3_2\text{He} + {}^{-1}_{0}e + \bar{\nu}_e$

$$^{10}_5\text{B} + {}^1_0n \longrightarrow {}^{11}_5\text{B}^* \longrightarrow {}^7_3\text{Li} + {}^4_2\text{He}$$

$$(Q = 2.79 \text{ MeV})$$

$$^{35}_{17}\text{Cl} + {}^1_0n \longrightarrow {}^{36}_{17}\text{Cl}^* \longrightarrow {}^{32}_{15}\text{P} + {}^4_2\text{He}$$

$$(Q = 0.935 \text{ MeV})$$

The first two reactions are utilised in the construction of neutron detectors as they have fairly large cross-sections. The first reaction also gives a method of producing tritium which is useful in nuclear fusion.

(ii) (n, p) reactions: When the reactions yield protons. The general equation for (n, p) reactions is

$$^A_ZX + {}^1_0n \longrightarrow {}^{A+1}{}_ZC^* \longrightarrow {}_{Z-1}^{A}Y + {}^1_1H$$

$$\{C^* \text{ is Compound Nucleus}\}$$

The product nucleus Y is an isobar of the target nucleus X with Z-value one unit lower and is thus β^- active decaying to the target nucleus. So,

$$_{Z-1}^{A}Y \xrightarrow{\beta^-} {}^A_ZX$$

So, the (n, p) reactions are such that the initial and final nuclides are identical. The process therefore appears to be a conversion of neutron into a proton and an electron.

EXAMPLES

$$^{14}_7\text{N} + {}^1_0n \longrightarrow {}^{15}_7\text{N}^* \longrightarrow {}^{14}_6\text{C} + {}^1_1\text{H}$$

$$(Q = 0.627 \text{ MeV})$$

followed by $^{14}_6\text{C} \xrightarrow[5568 \text{ yr}]{\beta^-} {}^{14}_7\text{N} + {}^{-1}_{0}e + \bar{\nu}_e$

$$^{27}_{13}\text{Al} + {}^1_0n \longrightarrow {}^{28}_{13}\text{Al}^* \longrightarrow {}^{27}_{12}\text{Mg} + {}^1_1\text{H}$$

$$(Q = -1.83 \text{ MeV})$$

followed by $^{27}_{12}\text{Mg} \xrightarrow[10 \text{ min}]{\beta^-} {}^{27}_{13}\text{Al} + {}^{-1}_{0}e + \bar{\nu}_e$

Only the first reaction is induced by thermal neutrons.

(iii) (n, γ) reactions: When the reactions yield γ-photons. This is the most important neutron-induced transmutation, known as **radiative capture** of neutrons, and has the following general equation.

$$^A_ZX + {}^1_0n \longrightarrow {}^{A+1}{}_ZC^* \longrightarrow {}^{A+1}{}_ZY + \gamma$$

$$\{C^* \text{ is Compound Nucleus}\}$$

The product nucleus is thus the same as the compound nucleus in the ground state. The (n, γ) reaction is always exoergic $(Q > 0)$ and can be induced by almost zero energy neutrons.

The radiative capture raises the target nucleus to an excited isomeric state and by releasing the excitation energy as γ photons, the product nucleus becomes an isotope of the target nucleus. The isotopic product nuclei are generally β^- active, as it has a higher neutron-proton ratio compared to the original one. In fact, this method of inducing β-activity is used extensively with copious supply of neutrons from reactors.

EXAMPLES

$$^2_1\text{H} + {}^1_0n \longrightarrow {}^3_1\text{H}^* \longrightarrow {}^3_1\text{H} + \gamma$$

followed by $^3_1\text{H} \xrightarrow[12.4 \text{ yr}]{\beta^-} {}^3_2\text{He} + {}^{-1}_{0}e + \bar{\nu}_e$

(d) Disintegration by deutrons

(i) (d, α) reactions: When the reactions yield α-particles. The general equation of (d, α) reactions is

$$^A_ZX + {}^2_1H \longrightarrow {}^{A+2}_{Z+1}C^* \longrightarrow {}^{A-2}_{Z-1}Y + {}^4_2He$$

$$\{C^* \text{ is Compound Nucleus}\}$$

The Q-values are usually positive and the reactions exoergic. Some example of (d, α) reactions are given as under.

EXAMPLES

$$^6_3\text{Li} + {}^2_1\text{H} \longrightarrow {}^8_4\text{Be}^* \longrightarrow {}^4_2\text{He} + {}^4_2\text{He}$$

$$(Q = 22.4 \text{ MeV})$$

$$^{14}_7\text{N} + {}^2_1\text{H} \longrightarrow {}^{16}_8\text{O}^* \longrightarrow {}^{12}_6\text{C} + {}^4_2\text{He}$$

$$(Q = 13.57 \text{ MeV})$$

$$^{16}_8\text{O} + {}^2_1\text{H} \longrightarrow {}^{18}_9\text{F}^* \longrightarrow {}^{14}_7\text{N} + {}^4_2\text{He}$$

$$^{23}_{11}\text{Na} + {}^2_1\text{H} \longrightarrow {}^{25}_{12}\text{Mg}^* \longrightarrow {}^{21}_{10}\text{Ne} + {}^4_2\text{He}$$

$$(Q = 6.9 \text{ MeV})$$

$$^{27}_{13}\text{Al} + {}^2_1\text{H} \longrightarrow {}^{29}_{14}\text{Si}^* \longrightarrow {}^{25}_{12}\text{Mg} + {}^4_2\text{He}$$

$$(Q = 6.7 \text{ MeV})$$

$$^{24}_{12}\text{Mg} + ^{2}_{1}\text{H} \longrightarrow ^{26}_{13}\text{Al}^* \longrightarrow ^{22}_{11}\text{Na} + ^{4}_{2}\text{He}$$

$$(Q = 1.96\text{ MeV})$$

Since the α-particles ejected from the compound nucleus are to cross high potential barrier, the (d, α) reactions occur at fairly high energy of deuteron and for low Z target nuclei.

(ii) (d, p) reactions: When the reactions yield protons. The general equation of (d, p) reactions is

$$^{A}_{Z}X + ^{2}_{1}\text{H} \longrightarrow ^{A+2}_{Z+1}C^* \longrightarrow ^{A+1}_{Z}X + ^{1}_{1}\text{H}$$

$\{C^*$ is Compound Nucleus$\}$

The Q-values are usually positive and the reactions exoergic. For some light nuclei, however, Q may be negative. Some examples of (d, p) reactions are as under.

EXAMPLES

$$^{7}_{3}\text{Li} + ^{2}_{1}\text{H} \longrightarrow ^{9}_{4}\text{Be}^* \longrightarrow ^{8}_{3}\text{Li} + ^{1}_{1}\text{H}$$

$$(Q = -0.193\text{ MeV})$$

$$^{12}_{6}\text{C} + ^{2}_{1}\text{H} \longrightarrow ^{14}_{7}\text{N}^* \longrightarrow ^{13}_{6}\text{C} + ^{1}_{1}\text{H}$$

$$(Q = 2.72\text{ MeV})$$

$$^{23}_{11}\text{Na} + ^{2}_{1}\text{H} \longrightarrow ^{25}_{12}\text{Mg}^* \longrightarrow ^{24}_{11}\text{Na} + ^{1}_{1}\text{H}$$

$$(Q = 4.74\text{ MeV})$$

$$^{31}_{15}\text{P} + ^{2}_{1}\text{H} \longrightarrow ^{33}_{16}\text{S}^* \longrightarrow ^{32}_{15}\text{P} + ^{1}_{1}\text{H}$$

$$(Q = 5.71\text{ MeV})$$

$$^{109}_{47}\text{Ag} + ^{2}_{1}\text{H} \longrightarrow ^{111}_{48}\text{Cd}^* \longrightarrow ^{110}_{47}\text{Ag} + ^{1}_{1}\text{H}$$

$$(Q = 4.6\text{ MeV})$$

The products of (d, p) reactions are usually radioactive

(iii) (d, n) reactions: When the reactions yield neutrons. The general equation of (d, n) reaction is

$$^{A}_{Z}X + ^{2}_{1}H \longrightarrow ^{A+2}_{Z+1}C^* \longrightarrow ^{A+1}_{Z+1}Y + ^{1}_{0}n$$

$\{C^*$ is Compound Nucleus$\}$

With some exceptions, the (d, n) reactions are exoergic and the Q-values are positive. The product nucleus Y is an isotope of the compound nucleus. Some examples of (d, n) reactions are given below.

EXAMPLES

$$^{7}_{3}\text{Li} + ^{2}_{1}\text{H} \longrightarrow ^{9}_{4}\text{Be}^* \longrightarrow ^{8}_{4}\text{Be} + ^{1}_{0}n$$

$$(Q = 15.024\text{ MeV})$$

$$^{9}_{4}\text{Be} + ^{2}_{1}\text{H} \longrightarrow ^{11}_{5}\text{B}^* \longrightarrow ^{10}_{5}\text{B} + ^{1}_{0}n$$

$$(Q = 4.36\text{ MeV})$$

$$^{12}_{6}\text{C} + ^{2}_{1}\text{H} \longrightarrow ^{14}_{7}\text{N}^* \longrightarrow ^{13}_{7}\text{N} + ^{1}_{0}n$$

$$(Q = -0.283\text{ MeV})$$

$$^{16}_{8}\text{O} + ^{2}_{1}\text{H} \longrightarrow ^{18}_{9}\text{F}^* \longrightarrow ^{17}_{9}\text{F} + ^{1}_{0}n$$

$$(Q = -1.625\text{ MeV})$$

$$^{35}_{17}\text{Cl} + ^{2}_{1}\text{H} \longrightarrow ^{37}_{18}\text{Ar}^* \longrightarrow ^{36}_{18}\text{Ar} + ^{1}_{0}n$$

$$(Q = 6.28\text{ MeV})$$

When deuterons bombard deuterons both (d, p) and (d, n) reactions may be observed because of the two alternative decay schemes for the compound nucleus.

$$^{2}_{1}\text{H} + ^{2}_{1}\text{H} \longrightarrow ^{4}_{2}\text{He}^* \longrightarrow ^{3}_{1}\text{H} + ^{1}_{1}\text{H}$$

$$(Q = 4.03\text{ MeV})$$

$$^{2}_{1}\text{H} + ^{2}_{1}\text{H} \longrightarrow ^{4}_{2}\text{He}^* \longrightarrow ^{3}_{2}\text{He} + ^{1}_{0}n$$

$$(Q = 3.26\text{ MeV})$$

The product $^{3}_{1}\text{H}$ is tritium, an isotope of hydrogen. Its nucleus is called triton which is β-active.

$$^{3}_{1}\text{H} \xrightarrow{\beta^-} ^{3}_{2}\text{He} + \bar{\nu}_e \qquad (\text{Half-life} = 12.4\text{ yr})$$

The other product $^{3}_{2}\text{He}$ is a stable isotope of helium.

The tritium may be bombarded with deuterons to produce (d, n) reaction

$$^{3}_{1}\text{H} + ^{2}_{1}\text{H} \longrightarrow ^{5}_{2}\text{He}^* \longrightarrow ^{4}_{2}\text{He} + ^{1}_{0}n$$

$$(Q = 17.6\text{ MeV})$$

The reactions with beryllium and deuterium serve as sources of neutrons. A thick beryllium target if bombarded with 1 MeV deuterons (accelerated in a cyclotron or Van de Graaff generator) yields about 10^8 neutrons per sec per μA deuteron-current absorbed in the target.

(iv) (d, t) reactions: When the reactions yield tritium. The general equation of (d, t) reactions is

$$^{A}_{Z}X + ^{2}_{1}\text{H} \longrightarrow ^{A+2}_{Z+1}C^* \longrightarrow ^{A-1}_{Z}Y + ^{3}_{1}\text{H}$$

$\{C^*$ is Compound Nucleus$\}$

The product nucleus Y is an isotope of the target nucleus X. The cross-section of such reactions is low. Some examples are as under.

$$^{7}_{3}\text{Li}+^{2}_{1}\text{H}\longrightarrow ^{9}_{4}\text{Be}^{*}\longrightarrow ^{6}_{3}\text{Li}+^{3}_{1}\text{H} \quad (Q=-0.996\text{ MeV})$$

$$^{9}_{4}\text{Be}+^{2}_{1}\text{H}\longrightarrow ^{11}_{5}\text{B}^{*}\longrightarrow ^{8}_{4}\text{Be}+^{3}_{1}\text{H} \quad (Q=4.59\text{ MeV})$$

NOTE: At higher energies $(>20\text{ MeV})$ of deuterons the $(d, 2n)$, $(d, 2p)$, $(d, 3n)$ etc. reactions in which more than one particle (two or more) is emitted from the compound nucleus become important, e.g.

$$^{107}_{47}\text{Ag}+^{2}_{1}\text{H}\longrightarrow ^{109}_{48}\text{Cd}^{*}\longrightarrow ^{107}_{48}\text{Cd}+2\,^{1}_{0}n$$

(e) Photo disintegration (γ-induced transmutations)
This type of reactions, called photo disintegrations or photonuclear reactions, occur when sufficiently high-energy photons enter into a nucleus. The energy of the incident photon must be greater than the binding energy of a nuclear particle (separation energy) like neutron, proton, α-particle etc. to produce (γ, n), (γ, p), (γ, α) etc. reactions.
The photo disintegration of deuteron, discovered by Chadwick and Goldhaber, deserves special mention, for it was from this reaction that they evaluated neutron mass.

$$^{2}_{1}\text{H}+\gamma\longrightarrow ^{2}_{1}\text{H}^{*}\longrightarrow ^{1}_{1}\text{H}+^{1}_{0}n \quad (\gamma, n \text{ reaction})$$

Another example of (γ, n) reaction is

$$^{9}_{4}Be+\gamma\longrightarrow ^{9}_{4}Be^{*}\longrightarrow ^{8}_{4}Be+^{1}_{0}n$$

This reaction is used for preparing the source for photo-neutrons.
Other γ-induced reactions are:

$$^{12}_{6}\text{C}+\gamma\longrightarrow ^{12}_{6}\text{C}^{*}\longrightarrow ^{8}_{4}\text{Be}+^{4}_{2}\text{He} \quad (\gamma, \alpha \text{ reaction})$$

$$^{11}_{5}\text{B}+\gamma\longrightarrow ^{11}_{5}\text{B}^{*}\longrightarrow ^{8}_{4}\text{Be}+^{3}_{1}\text{H} \quad (\gamma, t \text{ reaction})$$

$$^{10}_{5}\text{B}+\gamma\longrightarrow ^{10}_{5}\text{B}^{*}\longrightarrow ^{8}_{4}\text{Be}+^{2}_{1}\text{H} \quad (\gamma, d \text{ reaction})$$

$$^{9}_{4}\text{Be}+\gamma\longrightarrow ^{9}_{4}\text{Be}^{*}\longrightarrow ^{8}_{4}\text{Be}+^{1}_{0}n \quad (\gamma, n \text{ reaction})$$

$$^{9}_{4}\text{Be}+\gamma\longrightarrow ^{9}_{4}\text{Be}^{*}\longrightarrow ^{8}_{3}\text{Li}+^{1}_{1}\text{H} \quad (\gamma, p \text{ reaction})$$

The last (γ, n) reaction serves as a convenient source of neutrons.
Neutral π-mesons can be artificially produced by the interaction of γ-rays with hydrogen and deuterium.

$$^{1}_{1}\text{H}+\gamma\longrightarrow \pi^{0}+^{1}_{1}\text{H}$$

$$^{2}_{1}\text{H}+\gamma\longrightarrow \pi^{0}+^{1}_{0}n+^{1}_{1}\text{H}$$

$$^{2}_{1}\text{H}+\gamma\longrightarrow \pi^{0}+^{2}_{1}\text{H}$$

The first reaction can occur only if the energy of the γ-photon is not less than the threshold value equal to the mass-energy of the neutral pion mass. Steinberg could produce π^{0}-mesons by bombarding light targets such as hydrogen or beryllium with high energy x-radiation from an electron synchrotron.

$$\underset{\text{(Photon)}}{\gamma}+^{9}_{4}\text{Be}\longrightarrow ^{9}_{4}\text{B}^{*}\longrightarrow ^{8}_{4}\text{Be}+^{1}_{0}n$$

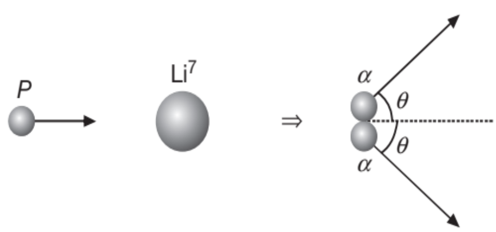
ILLUSTRATION 38

A proton is bombarded on a stationary lithium nucleus. As a result of the collision two α particles are produced. If the direction of motion of the α particles with the initial direction of motion makes an angle $\cos^{-1}\left(\frac{1}{4}\right)$, find the kinetic energy of the striking proton. Given binding energies per nucleon of Li^{7} and He^{4} are 5.60 and 7.06 MeV respectively. (Assume mass of proton ≈ mass of neutron).

SOLUTION

Q value of the reaction is given by

$$Q=(2\times4\times7.06-7\times5.6)\text{ MeV}=17.28\text{ MeV}$$



Applying Law of Conservation of Energy for Collision, we get

$$K_p + Q = 2K_\alpha \quad \ldots(1)$$

where K_p and K_α are the kinetic energies of proton and α particle respectively.
Applying Law of Conservation of Linear Momentum, we get

$$\sqrt{2m_pK_p} = 2\sqrt{2m_\alpha K_\alpha}\cos\theta \quad \ldots(2)$$

$$\Rightarrow \quad K_p = 16K_\alpha\cos^2\theta = (16K_\alpha)\left(\frac{1}{4}\right)^2$$

$$\{\because m_\alpha = 4m_p\}$$

$$\Rightarrow \quad K_\alpha = K_p \quad \ldots(3)$$

Solving equations (1) and (3) with $Q = 17.28\text{ MeV}$, we get

$$K_p = 17.28\text{ MeV}$$

NUCLEAR FISSION

The splitting of heavy nucleus into two or more fragments of comparable masses, with an enormous release of energy is called nuclear fission.

In nuclear fission, heavy nuclei having mass number A greater than 200, break up into two or more fragments of comparable masses. The most suitable fission material, from a practical point of view, to achieve energy from nuclear fission is ${}_{92}U^{236}$. The technique is to hit a uranium sample by sample by slow moving neutrons (kinetic energy $\approx 0.04\text{ eV}$, also called thermal neutrons). A ${}_{92}U^{235}$ nucleus has large probability of absorbing a slow neutron and forming ${}_{92}U^{236}$ nucleus. This nucleus then fissions into two parts. A variety of combinations of the middle-weight nuclei may be formed due to the fission. For example, one may have

$${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236} \rightarrow X + Y + 2\,{}_0n^1,$$

OR $${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236} \rightarrow X' + Y' + 3\,{}_0n^1$$

and a number of other combinations.
In the nuclear fission reaction

$${}^{235}_{92}U + {}_0n^1 \rightarrow {}^{236}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}_0n^1 + \text{energy}$$

The mass defect is given by

$$\Delta m = m_{\text{reactants}} - m_{\text{products}}, \text{ where}$$

$$m_{\text{reactants}} = (M_U - 92m_e) + m_n \text{ and}$$

$$m_{\text{products}} = (M_{\text{Ba}} - 56m_e) + (M_{\text{Kr}} - 36m_e) + 3m_n$$

$$\Rightarrow \quad \Delta m = (M_{\text{U}} + m_n) - (M_{\text{Ba}} + M_{\text{Kr}} + 3m_n)$$

The Q value is given by

$$Q = \left[(M_{\text{U}} + m_n) - (M_{\text{Ba}} + M_{\text{Kr}} + 3m_n)\right]c^2$$

In a nuclear fission reaction, we observe that

(a) on an average, 2.5 neutrons are emitted in each fission event.
(b) mass lost per reaction is ≈ 0.2 a.m.u.
(c) in nuclear fission reaction, the total B.E. increases and excess energy is released.
(d) in each fission event, about 200 MeV of energy is released. A large part of this released energy appears in the form of kinetic energies of the two fragments. Neutrons take away about 5 MeV.

Conceptual Note(s)

When slow neutrons are bombarded on ${}_{92}U^{235}$, the fission takes place according to reaction

$${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3({}_0n^1) + 200\text{ MeV}$$

(a) In nuclear fission the sum of masses before reaction is greater than the sum of masses after reaction, the difference in mass being released in the form of fission energy.
(b) The phenomenon of nuclear fission was discovered by Otto Hans and F. Strassman in 1939 and was explained by N. Bohr and J.A. Wheeler on the basis of liquid drop model of nucleus.
(c) It may be pointed out that it is not necessary that in each fission of uranium, the two fragments Ba^{141} and Kr^{92} are formed but they may be any stable isotopes of middle weight atoms. The most probable division is into two fragments containing about 40% and 60% of the original nucleus

with the emission of 2 or 3 neutrons per fission. So, average number of neutrons produced per fission is 2.5.

(d) Most of energy released appears in the form of kinetic energy of fission fragments.

(e) The fission of U^{238} takes place by fast neutrons.

CHAIN REACTION

If on the average more than one of the neutrons produced in each fission are capable of causing further fission, the number of fissions taking place at successive stages goes on increasing at a rapid rate, giving rise to self sustained sequence of fission known as chain reaction. The chain reaction takes place only if the size of the fissionable material is greater than a certain size called the critical size.
There are two types of chain reactions.

Uncontrolled Chain Reaction

In this process the number of fissions in a given interval on the average goes on increasing and the system will have the explosive tendency. This forms the principle of atom bomb. If a nuclear reaction is uncontrolled then in about $1\,\mu s$, energy of order of 2×10^3 J is released.

Controlled Chain Reaction (As in a Nuclear Reactor)

In this process the number of fissions in a given interval is maintained constant by absorbing a desired number of neutrons. This forms the principle of nuclear reactor, consisting of the following parts:

(a) Fuel: The fuel is U^{235} or U^{233} or Pu^{239}

(b) Moderator: A moderator is a suitable material to slow down neutrons produced in fission. The best choice as moderators are heavy water (D_2O) and graphite (C).

(c) Controller: To maintain the steady rate of fission, the neutron absorbing material known as controller is used. The control rods are made of Cadmium or Boron-steel.

(d) Coolant: To remove the considerable amount of heat produced in the fission process, suitable cooling fluids known, as coolants are used. The usual coolants are water, carbon-dioxide, air etc.

(e) Reactor Shield: The intense neutrons and gamma radiation produced in nuclear reactors are harmful for human body. To protect the workers from such radiations, the reactor core is surrounded by concrete wall, called the reactor shield.

Critical Mass

If the amount of uranium is too small, then the liberated neutrons have large scope to escape from the surface and the chain reaction may stop before enough energy is released for explosion. Therefore, in order for explosion to occur, the mass uranium has to be greater than some minimum value, called the critical mass.

Reproduction Factor

It is the ratio of the rate of neutron production and the rate at which the neutrons disappear.

Whether a mass of active material will sustain a chain reaction or not is determined by the reproduction factor (K). If $K\geq1$, the chain reaction will be sustained. If $K=1$, the mass is said to be critical.

ILLUSTRATION 39

Polonium $\left({}^{210}_{84}Po\right)$ emits ${}^{4}_{2}He$ particles and is converted into lead $\left({}^{206}_{82}Pb\right)$. This reaction is used for producing electric power in a space mission. Po^{210} has half life of 138.6 days. Assuming an efficiency of 10% for the thermoelectric machine, how much ${}^{210}Po$ is required to produce 1.2×10^7 J of electric energy per day at the end of 693 days. Also find the initial activity of the material.

Given: Masses of nuclei

${}^{210}Po = 209.98264$ amu, ${}^{206}Pb = 205.97440$ amu,

${}^{4}_{2}He = 4.00260$ amu.

$1\text{ amu} = 931\text{ MeV}$ and

Avogadro number $= 6\times10^{23}\text{ mol}^{-1}$

SOLUTION

Since, ${}^{210}_{84}Po \longrightarrow {}^{206}_{82}Pb + {}^{4}_{2}He$

$\Rightarrow\quad \Delta m = 0.00564$ amu

Energy liberated per reaction is

$$\Delta E = (\Delta m)931\text{ MeV} = 8.4\times10^{-13}\text{ J}$$

Electrical energy produced is 10% of ΔE i.e., $= 8.4\times10^{-14}$ J

Let m g of ^{210}Po is required to produce the desired energy, then

$$N=\frac{m}{210}\times6\times10^{23}$$

Also, $\lambda=\frac{0.693}{t_{1/2}}=0.005$ per day

$$\Rightarrow\quad\left(-\frac{dN}{dt}\right)=\lambda N=\frac{(0.005)(6\times10^{23})m}{210}\text{ per day}$$

So, electrical energy produced per day is

$$E=\frac{(0.005)(6\times10^{23})m}{210}\times8.4\times10^{-14}\text{ J}$$

Since, $E=1.2\times10^{7}$ {given}

$$\Rightarrow\quad m=10\text{ g}$$

Activity at the end of 693 days is

$$R=\frac{0.005\times6\times10^{23}\times10}{210}=\frac{10^{21}}{7}\text{ per day}=R_0\left(\frac{1}{2}\right)^n$$

where, n is the number of half lives

$$\Rightarrow\quad n=\frac{693}{138.6}=5$$

$$\Rightarrow\quad R_0=R(2)^5=32\times\frac{10^{21}}{7}=4.57\times10^{21}\text{ per day}$$

ILLUSTRATION 40

In a nuclear reactor, fission is produced in 1 g for U^{235} (235.0439 u) in 24 hours by a slow neutron (1.0087 u). Assume that $_{35}Kr^{92}$ (91.8973 u) and $_{56}Ba^{141}$ (140.9139 amu) are produced in all reactions and no energy is lost.

(a) Write the complete reaction

(b) Calculate the total energy produced in kilowatt hour. Given $1\text{ u}=931$ MeV.

SOLUTION

The nuclear fission reaction is

$$_{92}U^{235}+{}_0n^1\rightarrow{}_{56}Ba^{141}+{}_{36}Kr^{92}+3\,{}_0n^1$$

Mass defect $\Delta m=\left[(m_u+m_n)-(m_{Ba}+m_{Kr}+3m_n)\right]$

$$\Delta m=256.0526-235.8373=0.2153\text{ u}$$

Energy released, $\alpha=0.2153\times931=200$ MeV

Number of atoms in 1 g $=\frac{6.02\times10^{23}}{235}=2.56\times10^{21}$

Energy released in fission of 1 g of U^{235} is

$$Q=200\times2.56\times10^{21}=5.12\times10^{23}\text{ MeV}$$

$$\Rightarrow\quad Q=(5.12\times10^{23})\times(1.6\times10^{-13})=8.2\times10^{10}\text{ J}$$

$$\Rightarrow\quad Q=\frac{8.2\times10^{10}}{3.6\times10^{6}}\text{ kWh}=2.28\times10^{4}\text{ kWh}$$

NUCLEAR FUSION

The phenomenon of combination of two or more light nuclei to form a heavy nucleus with release of enormous amount of energy is called the nuclear fusion. The sum of masses before fusion must be greater than the sum of masses after fusion, the difference in mass appearing as fusion energy. The fusion of two deuterium nuclei into helium is expressed as

$$_1H^2+{}_1H^2\longrightarrow{}_2He^4+23\cdot8\text{ MeV}$$

It may be pointed out that this fusion reaction does not actually occur. Due to huge quantity of energy release, the helium nucleus $_2He^4$ has got such a large value of excitation energy that it breaks up by the emission of a proton or a neutron as soon as it is formed, giving rise to the following reactions.

$$_1H^2+{}_1H^2\longrightarrow{}_2He^3+{}_0n^1+Q(=3\cdot26\text{ MeV})$$

$$_1H^2+{}_1H^2\longrightarrow{}_1H^3+{}_1H^1+Q(=4\cdot04\text{ MeV})$$

The fusion process occurs at extremely high temperature and high pressure just as it takes place at sun where temperature is 10^7 K. So, fusion reactions are also called Thermo-nuclear reactions.

Nuclear fusion has the possibility of being a much better source of energy than fission due to the following reasons.

(a) In fusion there is no radiation hazard as no radioactive material is used.

(b) The fuel needed for fission (U-235 etc.) is not available easily whereas hydrogen needed for fusion can be obtained in huge quantity.

(c) The energy released per nucleon is much more in fusion than in fission.

However, the very high temperature and pressure required for fusion cannot be easily created and maintained and as such it has not been possible as yet to use fusion for power generation.

Conceptual Note(s)

(a) For the fusion to take place, the component nuclei must be brought to within a distance of 10^{-14} m. For this they must be imparted high energies to overcome the repulsive force between nuclei. This is possible when temperature is enormously high.

(b) The principle of hydrogen bomb is also based on nuclear fusion. To start a fusion bomb very high temperature is required. This is achieved by incorporating an atom bomb within the nuclear bomb.

(c) The source of energy of sun and other stars is nuclear fusion (or thermo-nuclear reactions). There are two possible cycles:

(i) Proton-Proton cycle

In 1938, Hans Bethe suggested that the stellar energy is produced by thermonuclear reactions in which protons are combined and transformed into helium nuclei. This is known as proton-proton cycle and is applicable for relatively low stellar temperature. The cycle is

$$_1H^1 + {_1H^1} \longrightarrow {_1H^2} + {_1\beta^0} + \nu + 0.42 \text{ MeV} \quad ...(1)$$

$$_1H^2 + {_1H^1} \longrightarrow {_2He^3} + \gamma + 5.5 \text{ MeV} \quad ...(2)$$

$$_2He^3 + {_2He^3} \longrightarrow {_2He^4} + {_1H^1} + {_1H^1} + 12.8 \text{ MeV} \quad ...(3)$$

The reactions (1) and (2) occur twice to yield two $_2He^3$ nuclei.

Net result is

$$_1H^1 + {_1H^1} + {_1H^1} + {_1H^1} \longrightarrow {_2He^4} + 2\,{_1\beta^0} + 2\nu + 2\gamma + \text{Energy } (24\cdot 6 \text{ MeV})$$

(ii) Carbon-Nitrogen cycle

For the main sequence stars with extremely high temperatures, Bethe suggested an alternative to proton-proton cycle called the Carbon-Nitrogen cycle. The cycle is

$$_1H^1 + {_6C^{12}} \longrightarrow {_7N^{13*}} + \gamma + 1.95 \text{ MeV}$$

$$_7N^{13*} \longrightarrow {_6C^{13}} + {_1\beta^0} + \nu + 2.22 \text{ MeV}$$

$$_6C^{13} + {_1H^1} \longrightarrow {_7N^{14}} + \gamma + 7.54 \text{ MeV}$$

$$_7N^{14} + {_1H^1} \longrightarrow {_8O^{15*}} + \gamma + 7.35 \text{ MeV}$$

$$_8O^{15*} \longrightarrow {_7N^{15}} + {_1\beta^0} + \nu + 2.7 \text{ MeV}$$

$$_7N^{15} + {_1H^1} \longrightarrow {_6C^{12}} + {_2He^4} + 4.96 \text{ MeV}$$

Net result is

$$_1H^1 + {_1H^1} + {_1H^1} + {_1H^1} \longrightarrow {_2He^4} + 2\,{_1\beta^0} + 2\nu + 3\gamma \text{ (Energy} = 26\cdot 7 \text{ MeV)}$$

For sun, both the cycles occur with equal probability. Stars with masses between 0.4 to 2.5 solar mass produce energy mainly by C-N cycle rather than P-P cycle. Stars with masses 0.4 solar mass or lower (which constitute the bulk of stellar population in our galaxy) mainly derive their energy from P-P cycle.

ILLUSTRATION 41

It is proposed to use the nuclear fusion reaction:

$$_1H^2 + {_1H^2} = {_2He^4}$$

in a nuclear reactor of 200 MW rating. If the energy from above reaction is used with a 25% efficiency in the reactor, how many grams of deuterium will be needed per day. (The masses of $_1H^2$ and $_2He^4$ are 2.0141 and 4.0026 u respectively).

SOLUTION

Energy released in the nuclear fusion is

$$Q = \Delta mc^2 = \Delta m(931) \text{ MeV}$$

$$\Rightarrow \quad Q = (2\times 2.0141 - 4.0026)\times 931 \text{ MeV} = 23.834 \text{ MeV}$$

$$\Rightarrow \quad Q = 23.834 \times 10^6 \text{ eV}$$

Since efficiency of reactor is 25%

So effective energy used is

$$\Delta E=\frac{25}{100}\times 23.834\times 10^{6}\times 1.6\times 10^{-19}\text{ J}$$

$$\Rightarrow \quad \Delta E=9.534\times 10^{-13}\text{ J}$$

Since the two deuterium nucleus are involved in a fusion reaction, therefore, energy released per deuterium is $\dfrac{9.534\times 10^{-13}}{2}$

For $200\text{ MW}_{\text{power per day}}$ number of deuterium nuclei required is

$$N=\frac{200\times 10^{6}\times 86400}{\frac{9.534}{2}\times 10^{-13}}=3.624\times 10^{25}$$

Since 2 g of deuterium constitute 6×10^{23} nuclei, therefore amount of deuterium required per day is

$$m=\frac{2\times 3.624\times 10^{25}}{6\times 10^{23}}=120.83\text{ g/day}$$

ILLUSTRATION 42

In the fusion reaction $^{2}_{1}\text{H}+^{2}_{1}\text{H}\longrightarrow ^{3}_{2}\text{He}+^{1}_{0}n$, the masses of deuteron, helium and neutron expressed in amu are 2.015, 3.017 and 1.009 respectively. If 1 kg of deuterium undergoes complete fusion, find the amount of total energy released $1\text{ amu}=931.5\text{ MeVc}^{-2}$.

SOLUTION

$$\Delta m=2(2.015)-(3.017+1.009)=0.004\text{ amu}$$

So, energy released is

$$\Delta E=(0.004\times 931.5)\text{ MeV}=3.726\text{ MeV}$$

Energy released per deuteron $=\dfrac{3.726}{2}=1.863\text{ MeV}$

Number of deuterons in 1 kg $=\dfrac{6.02\times 10^{26}}{2}=3.01\times 10^{26}$

So, energy released per kg of deuterium fusion is

$$E=(3.01\times 10^{26}\times 1.863)=5.6\times 10^{26}\text{ MeV}$$

$$\Rightarrow \quad E\approx 9\times 10^{13}\text{ J}$$

NUCLEAR HOLOCAUST

The estimate of after effects of the atomic (or nuclear) explosion is termed as nuclear holocaust. If a fusion bomb explodes, then a nuclear holocaust will not only destroy every form of life on earth but will also make this planet unfit for life for all times. The radioactive waste will hang like a cloud in earth's atmosphere and will absorb sun's radiations, thus causing a long nuclear winter. One can imagine this only by the mathematical figures quoted, according to which energy liberated by fission of 50 kg of U^{235} is equal to $4\times 10^{15}\text{ J}$ which is the energy available from 20,000 tons of Trinitrotoluene (TNT).

USE OF RADIOISOTOPES

In Medicine

Radioisotopes are extensively used in medicine:

(a) Radio iodine is used to determine the condition of human thyroid gland. Iodine-131 is administered orally to the patient. After a sufficient time, the activity is measured. From the observations it can be interpreted whether the gland is overactive, normal or under-active.

(b) Amounts of sodium and potassium in the body is measured by using Na-24 and K-42 as tracers.

(c) Radioactive isotopes are used to locate the position and extent of cancer.

(d) Radioisotopes are used in locating tumors within the brain.

(e) Radioactive Cr^{51} is used to locate the exact position where the hemorrhage might have taken place inside the body.

(f) Water contents of the body are measured by using deuterium and tritium as tracers.

(g) Radio gold is being used for the treatment of leukemia.

In Industry

(a) Radioactive Carbon-14 is used to study wear and tear of the position of an engine. C-14 is mixed with the ring. After some time, the engine oil is analysed to detect the presence of any radiation in it. In case of wear and tear the radiations are found to be there.

(b) Radio Cobalt is used for testing fields and castings by taking their photographs with γ-rays.

As Tracers

The radioactive isotope has identical chemical properties as another stable isotope. Therefore, by mixing it with stable isotope we can trace the presence or distribution of the element in a biological or physical system by detecting the radiation emitted by radio-isotope of that element. The radioisotope in such a case is said to be a Tracer.

Thus, a radioactive tracer is a radioisotope which, when mixed with a chemically similar element or artificially attached to a biological system, can be traced by radiation detecting devices.

Following are the few examples of radioisotopes acting as tracers:

(a) Phosphorous-32 mixed with phosphorous manure has been used to study the process of extracting food from soil by various plants.

(b) Radio carbon is being used for research in photo-synthesis in plants.

(c) The progress and absorption of sodium chloride in the body can be studied by feeding the person with radio-isotope sodium-24 along with sodium chloride.

(d) With the aid of radioisotopes, the rate, place and sequence of formation of the organic constituents of a living body can be studied.

Test Your Concepts-III

Based on Nuclear Reactions, Alpha, Beta, Gamma Decay, Fission and Fusion

(Solutions on page H.90)

1. Consider two decay reactions.

(a) $^{238}_{92}\text{U} \longrightarrow {}^{206}_{82}\text{Pb} + 10 \text{ protons} + 22 \text{ neutrons}$

(b) $^{238}_{92}\text{U} \longrightarrow {}^{206}_{82}\text{Pb} + 8\,{}^{4}_{2}\text{He} + 6 \text{ elctrons}$

Are both the reactions possible?
Given: Average binding energy of $^{238}_{92}\text{U} = 7.57$ MeV, that of $^{206}_{82}\text{Pb} = 7.83$ MeV and that of $^{4}_{2}\text{He} = 7$ MeV per nucleon.

2. Find the minimum kinetic energy of an α-particle to cause the reaction $^{14}\text{N} + {}^{4}\text{He} \to {}^{17}\text{O} + {}^{1}\text{H}$. Given that, the masses of ^{14}N, ^{4}He, ^{1}H and ^{17}O are respectively 14.00307 u, 4.00260 u, 1.00783 u and 16.99913 u.

3. In a neutron induced fission of $_{92}\text{U}^{235}$ nucleus, usable energy of 185 MeV is released. If a $_{92}\text{U}^{235}$ reactor is continuously operating it at a power level of 100 MW, find the time it takes for 1 kg of uranium to be consumed in this reactor.

4. A neutron breaks into a proton and electron. Calculate the energy produced in this reaction in MeV. Mass of an electron is 9×10^{-31} kg, mass of proton is 1.6725×10^{-27} kg, mass of neutron is 1.6747×10^{-27} kg and speed of light 3×10^{8} ms^{-1}.

5. It is observed that $^{212}_{83}\text{Bi}$ decays as per following equation.

$$^{212}_{83}\text{Bi} \longrightarrow {}^{208}_{81}\text{Tl} + {}^{4}_{2}\text{He}$$

The kinetic energy of α particle emitted is 6.802 MeV. Calculate the kinetic energy of Tl recoil atoms.

6. The nuclear reaction, $n + {}^{10}_{5}\text{B} \to {}^{7}_{3}\text{Li} + {}^{4}_{2}\text{He}$ is observed to occur even when very slow-moving neutrons $(M_n = 1.0087 \text{ amu})$ strike a boron atom at rest. For a particular reaction in which $K_n = 0$, the helium $(M_{\text{He}} = 4.0026 \text{ amu})$ is observed to have a speed of 9.30×10^{6} ms^{-1}. Determine

(a) the kinetic energy of the lithium $(M_{\text{Li}} = 7.0160 \text{ amu})$ and

(b) the Q-value of the reaction.

7. Neon-23 decays in the following way

$$^{23}_{10}\text{Ne} \longrightarrow {}^{23}_{11}\text{Na} + {}^{0}_{-1}e + \bar{\nu}$$

Find the minimum and maximum kinetic energy possessed by the beta particle $\left({}^{0}_{-1}e\right)$. The atomic masses of ^{23}Ne and ^{23}Na are 22.9945 u and 22.9898 u, respectively.

8. The radionuclide ^{11}C decays according to the reaction.

$$^{11}\text{C} \longrightarrow {}^{11}\text{B} + e^{+} + \nu.$$

The maximum energy of emitted positrons is 0.961 MeV. Given that atomic mass of ^{11}C is $m_c = 11.011434$ u, atomic mass of ^{11}B is $m_B = 11.009305$ u, and the mass of positron is $m_p = 0.0005486$ u, calculate disintegration energy Q and compare it with the maximum energy of the emitted positron given above. $(1\text{u} = 931 \text{ MeV})$.

9. It is proposed to use the nuclear fusion reaction $_{1}\text{H}^{2} + {}_{1}\text{H}^{2} \longrightarrow {}_{2}\text{He}^{4}$ in a nuclear reactor of

200 MW rating. If the energy from the above reaction is used with a 25% efficiency in the reactor, how many gram of deuterium fuel will be needed per day. Given that the masses of ${}_1H^2$ and ${}_2He^4$ are 2.0141 atomic mass units and 4.0026 atomic mass unit respectively.

10. Assuming the splitting of U^{235} nucleus liberates 200 MeV energy, find

(a) the energy liberated in the fission of 1 kg of U^{235} and

(b) the mass of coal with calorific value of 30 $kJgm^{-1}$ which is equivalent to 1 kg of U^{235}.

11. In a nuclear reaction $\alpha + {}_7N^{14} \longrightarrow {}_8O^{17} + p$ when α-particles of kinetic energy 7.7 MeV were bombarded on nitrogen atom protons were ejected with a kinetic energy of 5.5 MeV.

(a) Find the Q-value of the reaction

(b) Find the angle ϕ between the direction of motion of proton and α-particle.

Given that atomic mass of ${}_1H^1 = 1.00814$ amu, atomic mass of ${}_7N^{14} = 14.00752$ amu, Atomic mass of ${}_8O^{17} = 17.00453$ and atomic mass of ${}_2He^4 = 4.00388$ amu.

12. Show that ${}^{230}_{92}U$ does not decay by emitting a neutron or proton. Given masses are $m\left({}^{230}_{92}U\right)$ = 230.033927 amu; $m\left({}^{229}_{92}U\right)$ = 229.033496 amu; $m\left({}^{229}_{91}Pa\right)$ = 229.032089 amu; $m(n)$ = 1.008665 amu, $m(p) = 1.007825$ amu.

13. 8 protons and 8 neutrons are separately at rest. How much energy will be released if we form ${}^{16}_{8}O$ nucleus? Given that mass of ${}^{16}_{8}O$ atom is 15.994915 u, mass of neutron is 1.008665 u and mass of hydrogen atom is 1.007825 u

14. Calculate the minimum kinetic energy of protons incident on C^{13} nuclei at rest in the laboratory that will produce the endothermic reaction ${}^{13}C(p, n){}^{13}N$. Given that

$$m\left({}^{13}C\right) = 13.0033554,\ m_n = 1.0086654$$

$$m\left({}^{1}H\right) = 1.0078254,\ m\left({}^{13}N\right) = 13.0067384$$

15. It is observed that 20 MeV energy is released per fusion in the reaction ${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + {}_0n^1$. Calculate the mass of ${}_1H^2$ consumed in a fusion reactor of power 1 MW in 1 day.

SOLVED PROBLEMS

PROBLEM 1

A radionuclide with half life 1620 sec is produced in a reactor at a constant rate 1000 nuclei per second. During each decay energy 200 MeV is released. If production of radio nuclides started at $t = 0$, calculate the rate of release of energy at 3240 second and the total energy released upto 405 second.

SOLUTION

Let N be the number of nuclei at time t, then net rate of increase of nuclei at instant t is,

$$\frac{dN}{dt} = \alpha - \lambda N$$

{where α = rate of production of nuclei}

$$\Rightarrow \int_0^N \frac{dN}{\alpha - \lambda N} = \int_0^t dt$$

$$\Rightarrow N = \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right) \quad \ldots(1)$$

Rate of decay at this instant

$$R = \lambda N = \alpha\left(1 - e^{-\lambda t}\right)$$

Hence, the rate of release of energy i.e. $\frac{dE}{dt}$ is given by

$$\frac{dE}{dt} = R\begin{pmatrix}\text{Energy released}\\ \text{in each decay}\end{pmatrix}$$

$$\Rightarrow \frac{dE}{dt} = \alpha(1 - e^{-\lambda t})(200)\text{ MeV sec}^{-1}$$

Substituting the values, we get

$$\frac{dE}{dt} = 1000\left(1 - e^{\frac{0.693}{1620}\times 3240}\right)(200)$$

$$\Rightarrow \begin{pmatrix}\text{Rate of Release}\\ \text{of Energy}\end{pmatrix} = 1.5\times 10^5 \text{ MeVsec}^{-1}$$

Total number of nuclei decayed upto time t is $\alpha t - N$

$$\Rightarrow \begin{pmatrix}\text{Total number of}\\ \text{decayed nuclei}\end{pmatrix} = \alpha t - \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)$$

Hence, total energy released upto this instant is

$$E = \left[\alpha t - \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)\right](200)\text{ MeV}$$

Substituting the values, we get

$$E = \left[1000\times 405 - \frac{1000}{\frac{0.693}{1620}}\left(1 - e^{-\frac{0.693}{1620}\times 405}\right)\right]200\text{ MeV}$$

$$\Rightarrow \quad E = 6.63\times 10^6 \text{ MeV}$$

PROBLEM 2

The energy received from the sun by earth and its surrounding atmosphere is $2\text{ cal cm}^{-2}\text{ min}^{-1}$ on a surface normal to the rays of sun. Calculate the

(a) total energy received in joules by earth and its atmosphere.

(b) total energy radiated in J min^{-1} by sun to the universe? Distance of sun to earth is 1.49×10^8 km.

(c) rate in mega-grams per minute) at which the hydrogen must be consumed in the fusion reaction to provide the sun with the energy it radiates.

Take mass of hydrogen atom to be 1.008145 amu and mass of He atom to be 4.003874 amu.

SOLUTION

(a) Let D be the diameter of earth. Then effective area of earth receiving radiation normally is

$$A_{\text{eff}} = \pi R^2 = \frac{\pi D^2}{4}$$

$$\Rightarrow \quad A_{\text{eff}} = \frac{\pi\left(1.27\times 10^4\right)^2}{4}\text{ km}^2$$

$$\Rightarrow \quad A_{\text{eff}} = \frac{\pi}{4}(1.27)^2\times 10^{18}\text{ cm}^2$$

Energy received by the earth per minute is

$$E_{\text{received}} = \left\{\frac{\pi}{4}(1.27)^2\times 10^{18}\right\}\times(2\times 4.2)\text{ Jmin}^{-1}$$

$$\Rightarrow E_{\text{received}} = 10.645\times 10^{18}\text{ Jmin}^{-1}$$

(b) The area of the surface surrounding the sun at a distance equal to earth-sun separation is $4\pi d^2$.

$$\Rightarrow \quad A = 4\pi\left(1.49\times 10^8\right)^2\times 10^6\times 10^4\text{ cm}^2$$

$\Rightarrow \quad A = 4\pi(1.49)^2 \times 10^{26}\ \text{cm}^2$

Since the energy is received at the rate of $2\ \text{cal cm}^{-2}\ \text{min}^{-1}$ on this surface, so the energy radiated by sun in Jmin^{-1} is given by

$$E = 2\times 4.2\times\left[4\pi(1.49)^2\times 10^{26}\right]\ \text{Jmin}^{-1}$$

$\Rightarrow \quad E = 2.3444\times 10^{28}\ \text{Jmin}^{-1}$

(c) Since, we know that the fusion reaction in the sun is

$$4\text{H} \rightarrow \text{He} + Q$$

The mass defect for this reaction is given by

$$\Delta m = 4(1.008145) - 4.003874$$

$\Rightarrow \quad \Delta m = 0.028706\ \text{amu}$

The energy released in one reaction is given by

$$Q = (0.028706)(931.5)\ \text{MeV} = 26.74\ \text{MeV}$$

So, mass of hydrogen required for the purpose is

$$m = \frac{(4.032580)\left(1.66\times 10^{-24}\right)\left(2.3444\times 10^{28}\right)}{(0.028706)(931.5)\left(1.6\times 10^{-19}\right)}\text{gmin}^{-1}$$

$\Rightarrow \quad m = 3.6673\times 10^{22}\ \text{gmin}^{-1}$

$\Rightarrow \quad m = 3.6673\times 10^{16}\ \text{megagram/min}$

PROBLEM 3

It is proposed to use nuclear reaction ${}_{84}\text{Po}^{210} \longrightarrow {}_{82}\text{Pb}^{206} + {}_2\text{He}^4$ to produce 2 kW electric power in a generator. The half life of polonium $\left(\text{Po}^{210}\right)$ is 138.6 days. Assuming efficiency of the generator be 10%, calculate

(a) how many grams of $\left(\text{Po}^{210}\right)$ are required per day at the end of 1386 days.

(b) initial activity of the material

Mass of nuclei: $\text{Po}^{210} = 209.98264$ amu,

$\text{Pb}^{206} = 205.97440$ amu,

${}_2\text{He}^4 = 4.00260$ amu

1 amu = 931 MeV

SOLUTION

(a) $\Delta m = 0.00564\ \text{amu} \equiv 5.25\ \text{MeV} = 8.4\times 10^{-13}\ \text{J}$

Since, $\lambda = \dfrac{0.693}{t_{1/2}} = 0.005$ per day

Let m g of Po^{210} are required per day for the reactor, then

$$n = \frac{\left(6.02\times 10^{23}\right)m}{210}$$

$$\Rightarrow \quad \left(-\frac{dN}{dt}\right) = \lambda N = \frac{0.005\times 6.02\times 10^{23}\times m}{210}\ \text{per day}$$

So, energy produced per day is

$$E = \frac{0.005\times 6.02\times 10^{23}\times m}{210}\times 8.4\times 10^{-13}\ \text{J}$$

$\Rightarrow \quad E = \left(12\times 10^6\right)m\text{J}$

Now, 10% of $\left(12\times 10^6\right)m$ equals $2\times 10^3\times 24\times 3600\ \text{J}$

$$\Rightarrow \quad m = \frac{2\times 10^3\times 24\times 3600}{1.2\times 10^6} = 144\ \text{g}$$

(b) $R = \lambda N = (0.005)\left(\dfrac{144}{210}\right)\left(6.02\times 10^{23}\right)$

$\Rightarrow \quad R = 2.064\times 10^{21}$ per day

Now, $R = R_0\left(\dfrac{1}{2}\right)^{10}$ $\qquad \left\{\because\ n = \dfrac{1386}{138.6} = 10\right\}$

$\Rightarrow \quad R_0 = (2)^{10}R = 2.11\times 10^{24}$ per day

PROBLEM 4

Suppose a nucleus initially at rest undergoes α decay according to equation

$${}^{225}_{92}X \longrightarrow Y + \alpha$$

At $t = 0$, the emitted α particle enters in a region of space where a uniform magnetic field $\vec{B} = B_0\hat{i}$ and electric field $\vec{E} = E_0\hat{i}$ exist. The α particle enters in the region with velocity $\vec{v} = v_0\hat{j}$ from $x = 0$. At time $t = \sqrt{3}\times 10^7\dfrac{m_\alpha}{q_\alpha E_0}$ sec, the particle was observed to have speed twice the initial speed v_0, then find

(a) the velocity of α particle at time t.

(b) the initial velocity v_0 of the α particle

(c) the binding energy per nucleon of α particle.

Given that

$m(Y) = 221.03$ u, $m(\alpha) = 4.003$ u,

$m(n) = 1.009$ u, $m(p) = 1.008$ u,

$m_\alpha = \dfrac{2}{3}\times 10^{-26}$ kg, $q_\alpha = 3.2\times 10^{-19}$ C and

$1\ \text{u} = 931\ \text{MeVc}^{-2}$

SOLUTION

(a) Magnetic force on α particle, (at $t=0$)

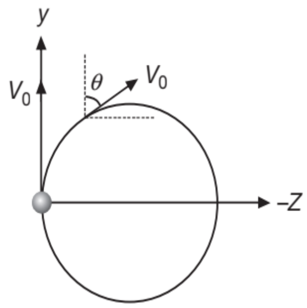
$$\vec{F}_m = q(\vec{v}\times\vec{B}) = q_\alpha\left[(v_0\hat{j})\times(B_0\hat{i})\right]$$

$$\Rightarrow \quad \vec{F}_m = -q_\alpha v_0 B_0\hat{k}$$

Force due to electric field (at any time t)

$$\vec{F}_e = q\vec{E} = q_\alpha E_0\hat{i}$$

Hence, the particle will move in a circular path in y-z plane due to magnetic field and at the same time it will move along x-direction. The resultant path is therefore, a helix with increasing pitch.



Hence, velocity of particle at any time t can be written as,

$$\vec{v} = \left(\frac{q_\alpha E_0}{m_\alpha}t\right)\hat{i} + v_0\cos\theta\hat{j} - v_0\sin\theta\hat{k}$$

where $\theta = \omega t = \dfrac{B_0 q_\alpha}{m_\alpha}t$

(b) Speed of particle at any time t is

$$v = \sqrt{\left(\frac{q_\alpha E_0 t}{m_\alpha}\right)^2 + v_0^2} \quad \{\because \sin^2\theta + \cos^2\theta = 1\}$$

Given $v = 2v_0$ at $t = (\sqrt{3}\times 10^7)\dfrac{m_\alpha}{q_\alpha E_0}$, so, we get

$$(2v_0)^2 = (\sqrt{3}\times 10^7)^2 + v_0^2$$

$$\Rightarrow \quad v_0 = 10^7 \text{ ms}^{-1}$$

(c) When an α-particle is emitted with velocity v_0 from a stationary nucleus X, decay product (nucleus Y) recoils. Then by Law of Conservation of Linear Momentum, we have

$$m_y v_y = m_\alpha v_0$$

$$\Rightarrow \quad v_y = \frac{m_\alpha v_0}{m_y} = \left(\frac{4.003}{221.03}\right)(10^7)$$

$$\Rightarrow \quad v_y = 1.81\times 10^5 \text{ ms}^{-1}$$

Total energy released during α-decay of nucleus X is

$$E = \text{K.E. of nucleus } Y + \text{K.E. of } \alpha\text{-particle}$$

$$\Rightarrow \quad E = \frac{1}{2}m_y v_y^2 + \frac{1}{2}m_\alpha v_0^2$$

$$\Rightarrow \quad E = \frac{1.66\times 10^{-27}}{2\times 1.6\times 10^{-13}}\left[(221.03)(1.81\times 10^5)^2 + (4.003)(10^7)^2\right]$$

$$\Rightarrow \quad E = 2.11 \text{ MeV}$$

Hence, Mass lost during α-decay is $\dfrac{2.11}{931.5}\text{u} = 0.0023\,\text{u}$

Mass of nucleus X is

$$m_x = (m_y + m_\alpha + 0.0023)\text{u}$$

$$\Rightarrow \quad m_x = 225.0353\,\text{u}$$

Mass defect in nucleus X is

$$\Delta m = 92m_p + (225-92)m_n - m_x = 1.898\,\text{u}$$

So, binding energy per nucleon is

$$\frac{BE}{A} = \frac{1.898\times 931.5}{225}\text{ MeV} = 7.86 \text{ MeV}$$

PROBLEM 5

Checkout from the following data, whether alpha decay or any of the beta decay are allowed for ${}^{226}_{89}\text{Ac}$.

$$m({}^{226}_{89}\text{Ac}) = 226.028356 \text{ amu},$$

$$m({}^{222}_{87}\text{Fr}) = 222.017415 \text{ amu},$$

$$m({}^{226}_{90}\text{Th}) = 226.017388 \text{ amu},$$

$$m({}^{226}_{88}\text{Ra}) = 226.025406 \text{ amu},$$

$$m({}^{4}_{2}\text{He}) = 4.002603 \text{ amu}.$$

SOLUTION

Let us first write the reaction for the corresponding decays and then find the disintegration energy Q. If $Q>0$, then the decay is allowed.

For alpha decay

$${}^{226}_{89}\text{Ac} \rightarrow {}^{222}_{87}\text{Fr} + \alpha$$

$$\Rightarrow \quad Q = \left[m\left({}^{226}_{89}\text{Ac}\right) - m\left({}^{222}_{87}\text{Fr}\right) - m\left({}^{4}\text{He}\right)\right]c^2$$

$$\Rightarrow \quad Q = 5.50 \text{ MeV} \qquad \{\text{Alpha decay is allowed}\}$$

For β^- decay

$${}^{226}_{89}\text{Ac} = {}^{226}_{90}\text{Th} + \beta^- + \bar{\nu}$$

$$\Rightarrow \quad Q = \left[M\left({}^{226}_{89}\text{Ac}\right) - M\left({}^{226}_{90}\text{Th}\right)\right]c^2$$

$$\Rightarrow \quad Q = 1.12 \text{ MeV} \qquad \{\beta^- \text{ decay is allowed}\}$$

For β^+ decay

$${}^{226}_{89}\text{Ac} \rightarrow {}^{226}_{88}\text{Ra} + \beta^+ + \nu$$

$$Q = \left[m\left({}^{226}_{89}\text{Ac}\right) - m\left({}^{226}_{88}\text{Ra}\right) - 2m_e\right]c^2$$

$$\Rightarrow \quad Q = -0.38 \text{ MeV} \qquad \{\beta^+ \text{ decay is not allowed}\}$$

For electron capture

$${}^{226}_{89}\text{Ac} + e^- \rightarrow {}^{226}_{88}\text{Ra} + \nu$$

$$\Rightarrow \quad Q = \left[m\left({}^{226}_{89}\text{Ac}\right) - m\left({}^{226}_{88}\text{Ra}\right)\right]c^2$$

$$\Rightarrow \quad Q = 0.64 \text{ MeV} \qquad \{\text{Electron capture is allowed}\}$$

From the above analysis, it is clear that during α-decay the Q-value is maximum and hence chances of α-decay are maximum.

PROBLEM 6

A radionuclide consists of two isotopes. One of the isotopes decays by α-emission and other by β-emission with half lives $T_1 = 405$ s, $T_2 = 1620$ s, respectively. At $t = 0$, probabilities of getting α and β-particles from the radionuclide are equal. Calculate their respective probabilities at $t = 1620$ s. If at $t = 0$, total number of nuclei in the radio nuclide are N_0. Calculate the time t when total number of nuclei remained undecayed becomes equal to $\dfrac{N_0}{2}$.

Given, $\log_{10} 2 = 0.3010$, $\log_{10} 5.94 = 0.7742$ and $x^4 + 4x - 2.5 = 0$, $x = 0.594$.

SOLUTION

At $t = 0$, probabilities of getting α and β particles are same. This implies that initial activity of both is equal, say R_0.

Activity after $t = 1620$ s is

$$R_1 = R_0\left(\frac{1}{2}\right)^{\frac{1620}{405}} = \frac{R_0}{16}$$

and $$R_2 = R_0\left(\frac{1}{2}\right)^{\frac{1620}{1620}} = \frac{R_0}{2}$$

Total activity $R = R_1 + R_2 = \dfrac{9}{16}R_0$

Probability of getting α particles is $\dfrac{R_1}{R} = \dfrac{1}{9}$ and

Probability of getting β particles is $\dfrac{R_2}{R} = \dfrac{8}{9}$

Since, $R_{01} = R_{02}$

$$\Rightarrow \quad \frac{N_{01}}{T_1} = \frac{N_{02}}{T_2}$$

$$\Rightarrow \quad \frac{N_{01}}{N_{02}} = \frac{1}{4}$$

Let N_0 be the total number of nuclei at $t = 0$ then,

$$N_{01} = \frac{N_0}{5} \text{ and } N_{02} = \frac{4N_0}{5}$$

Given, that $N_1 + N_2 = \dfrac{N_0}{2}$

$$\Rightarrow \quad \frac{N_0}{5}\left(\frac{1}{2}\right)^{\frac{t}{405}} + \frac{4N_0}{5}\left(\frac{1}{2}\right)^{\frac{t}{1620}} = \frac{N_0}{2} \qquad \ldots(1)$$

Let $\left(\dfrac{1}{2}\right)^{\frac{t}{1620}} = x$, then the equation (1) becomes

$$x^4 + 4x - 2.5 = 0$$

$$\Rightarrow \quad x = 0.594$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^{\frac{t}{1620}} = 0.594$$

Solving, we get

$$t = 1215 \text{ s}$$

PROBLEM 7

A radioactive element decays by β-emission. A detector records n beta particles in 2 seconds and in next 2-seconds it records $\dfrac{3}{4}n$ beta particles. Find mean life correct to nearest whole number. Given $\log_e|2| = 0.6931$, $\log_e|3| = 1.0986$.

SOLUTION

Let n_0 be the number of radioactive nuclei at time $t=0$. Number of nuclei decayed in time t are given by $n_0\left(1-e^{-\lambda t}\right)$, which is also equal to the number of beta particles emitted during the same interval of time. For the given condition.

$$n=n_0\left(1-e^{-2\lambda}\right) \quad \ldots(1)$$

$$\left(n+\frac{3}{4}n\right)=n_0\left(1-e^{-4\lambda}\right) \quad \ldots(2)$$

Dividing equation (2) by (1), we get

$$1.75=\frac{1-e^{-4\lambda}}{1-e^{-2\lambda}}$$

$$\Rightarrow \quad 1.75-1.75e^{-2\lambda}=1-e^{-4\lambda}$$

$$\Rightarrow \quad 1.75e^{-2\lambda}-e^{-4\lambda}=\frac{3}{4} \quad \ldots(3)$$

Let us take $e^{-2\lambda}=x$

Then the above equation becomes

$$x^2-1.75x+0.75=0$$

$$\Rightarrow \quad x=\frac{1.75\pm\sqrt{(1.75)^2-(4)(0.75)}}{2}$$

$$\Rightarrow \quad x=1 \text{ and } \frac{3}{4}$$

$$\Rightarrow \quad e^{-2\lambda}=1 \text{ or } e^{-2\lambda}=\frac{3}{4}$$

But $e^{-2\lambda}=1$ is not accepted because which means $\lambda=0$. Hence, $e^{-2\lambda}=\frac{3}{4}$

$$\Rightarrow \quad -2\lambda\log_e(e)=\log_e(3)-\log_e(4)$$

$$\Rightarrow \quad -2\lambda=\log_e(3)-2\log_e(2)$$

$$\Rightarrow \quad \lambda=\log_e(2)-\frac{1}{2}\log_e(3)$$

Substituting the given values, we get

$$\lambda=0.6931-\frac{1}{2}\times(1.0986)=0.14395 \text{ s}^{-1}$$

So, mean life

$$t_{av}=\frac{1}{\lambda}=6.947 \text{ s}$$

PROBLEM 8

A nuclear reactor generates power at 50% efficiency by fission of $^{235}_{92}\text{U}$ into two equal fragments of $^{116}_{46}\text{Pd}$ with the emission of two gamma rays of 5.2 MeV each and three neutrons. The average binding energies per particle of $^{235}_{92}\text{U}$ and $^{116}_{46}\text{Pd}$ are 7.2 MeV and 8.2 MeV respectively. Calculate the energy released in one fission event. Also estimate the amount to ^{235}U consumed per hour to produce 1600 megawatt power.

SOLUTION

$$\begin{pmatrix}\text{Energy}\\\text{released}\\\text{in}\\\text{one}\\\text{fission}\end{pmatrix}=\begin{pmatrix}\text{Binding}\\\text{energy}\\\text{of two}\\{}^{116}_{46}Pd\\\text{nuclei}\end{pmatrix}-\begin{pmatrix}\text{Binding}\\\text{energy}\\\text{of}\\{}^{235}_{92}U\\\text{nucleus}\end{pmatrix}-\begin{pmatrix}\text{Energy}\\\text{of two}\\\text{emitted}\\\text{gamma}\\\text{rays}\end{pmatrix}$$

Here binding energy of $^{235}_{92}\text{U}$ nucleus is

$$(\Delta E)_{\text{U}}=72\times235=1692 \text{ MeV}$$

Binding energy of two $^{116}_{46}\text{Pd}$ nuclei

$$2(\Delta E)_{\text{Pd}}=2\times8.2\times116=1902.4 \text{ MeV}$$

Energy of two emitted gamma rays is

$$2E_\gamma=2\times5.2=10.4 \text{ MeV}$$

Total energy released in one event is

$$E=(1902.4-1692-10.4) \text{ MeV}$$

$$\Rightarrow \quad E=200 \text{ MeV}$$

$$\Rightarrow \quad E=200\times\left(1.6\times10^{-13}\right)$$

$$\Rightarrow \quad E=3.2\times10^{-11} \text{ J}$$

So, the number of fission per second required to produce 1600×10^6 J of energy per second i.e. 1600 MW will be

$$N=\frac{1600\times10^6}{3.2\times10^{-11}}=5\times10^{19} \text{ s}^{-1}$$

So, 5×10^{19} nuclei of ^{235}U per second are required for this purpose. The mass of these nuclei is

$$m=\frac{235}{6.02\times10^{23}}\times\left(5\times10^{19}\right)$$

$$\Rightarrow \quad m=195.2\times10^{-4} \text{ gs}^{-1}$$

Thus, amount of ^{235}U consumed per hour is

$$m=(195.2\times10^{-4})\times3600$$

$$\Rightarrow \quad m=70.27\ \text{ghr}^{-1}$$

Since reactor efficiency is 50%, hence the consumption of ^{235}U per hour is given by

$$m=70.27\times2=140.5\ \text{g}$$

PROBLEM 9

Nuclei of a radioactive element A are being produced at a constant rate α. The element has a decay constant λ. At time $t=0$, there are N_0 nuclei of the element.

(a) Calculate the number N of nuclei of A at time t.

(b) if $\alpha=2N_0\lambda$, calculate the number of nuclei of A after one half-life of A and also the limiting value of N as $t\to\infty$.

SOLUTION

(a) Let at time t, number of radioactive nuclei are N. Net rate of formation of nuclei of A

$$\frac{dN}{dt}=\alpha-\lambda N$$

$$\Rightarrow \quad \frac{dN}{\alpha-\lambda N}=dt$$

$$\Rightarrow \quad \int_{N_0}^{N}\frac{dN}{\alpha-\lambda N}=\int_0^t dt$$

Solving this equation, we get

$$N=\frac{1}{\lambda}\left(\alpha-(\alpha-\lambda N_0)e^{-\lambda t}\right) \qquad \dots(1)$$

(b) (i) Substituting $\alpha=2\lambda N_0$ and $t=t_{1/2}=\dfrac{\log_e(2)}{\lambda}$ in equation (1), we get,

$$N=\frac{3}{2}N_0$$

(ii) Substituting $\alpha=2\lambda N_0$ and $t\to\infty$ in equation (1), we get

$$N=\frac{\alpha}{\lambda}=2N_0$$

$$\Rightarrow \quad N=2N_0$$

PROBLEM 10

A nucleus X-initially at rest, undergoes alpha-decay, according to the equation

$${}^{A}_{92}X\longrightarrow {}^{228}_{z}Y+\alpha$$

(a) Find the value of A and z in the above process.

(b) The α-particle in the above process is found to move in a circular track of radius 0.11 m in a uniform magnetic field of 3 T. Find the energy (in MeV) released during the process and binding energy of the parent nucleus X.

Given: $m_y=228.03$ amu $\qquad m_\alpha=4.003$ amu

$m({}^1_0n)=1.009$ amu $\qquad m({}^1_1\text{H})=1.008$ amu

$1\ \text{amu}=1.66\times10^{-27}\ \text{kg}=931.5\ \text{MeVc}^{-2}$

SOLUTION

(a) The given equation is,

$${}^{A}_{92}X\longrightarrow {}^{228}_{z}Y+{}^{4}_{2}\text{He}$$

$$A=228+4=232$$

and $92=z+2$

$$\Rightarrow \quad z=90$$

(b) $$\frac{m_\alpha v_\alpha^2}{r}=qv_\alpha B$$

$$\Rightarrow \quad r=\frac{m_\alpha v_\alpha}{qB}=\frac{\sqrt{2m_\alpha K_\alpha}}{qB}$$

$$\Rightarrow \quad K_\alpha=\frac{q^2B^2r^2}{2m_\alpha}$$

$$\Rightarrow \quad K_\alpha=\frac{(1.6\times10^{-19})^2(3)^2(0.11)^2}{2(4.003)(1.66\times10^{-27})(1.6\times10^{-13})}\text{MeV}$$

$$\Rightarrow \quad K_\alpha=5.21\ \text{MeV}$$

Applying Law of Conservation of Linear Momentum, we get

$$0=m_\alpha v_\alpha+m_Y v_Y$$

$$\Rightarrow \quad |p_y|=|p_\alpha|$$

$$\Rightarrow \quad \sqrt{2m_YK_Y}=\sqrt{2m_\alpha K_\alpha}$$

$$\Rightarrow \quad K_Y=\left(\frac{m_\alpha}{m_Y}\right)K_\alpha=\left(\frac{4.003}{228.03}\right)(5.21)=0.09\ \text{MeV}$$

Therefore, energy released during the process is

$$E=\frac{1}{2}\left(m_\alpha v_\alpha^2+m_Y v_Y^2\right)-0=K_\alpha+K_Y$$

$$\Rightarrow \quad E=K_\alpha+K_Y=5.21+0.09=5.3 \text{ MeV}$$

Now, mass of ${}^{232}_{92}X$ is

$$m\left({}^{232}_{92}X\right)=m_Y+m_\alpha+0.000365=232.033365 \text{ u}$$

So, mass defect is given by

$$\Delta m=92(1.008)+(232-92)(1.009)-232.033365$$

$$\Rightarrow \quad \Delta m=1.962635 \text{ amu}=1.962635 \text{ u}$$

$$\Rightarrow \quad \text{Binding Energy}=1.962635\times 931.5 \text{ MeV}$$

$$\Rightarrow \quad \text{Binding Energy}=1828.2 \text{ MeV}$$

PROBLEM 11

A radioactive source in the form of metal sphere of diameter 10^{-3} m emits beta particles at a constant rate of 6.25×10^{10} particles per second. If the source is electrically insulated, how long will it take for its potential to rise by 1 volt. Assume that 80% of the emitted beta particles escape from the source?

SOLUTION

Let t be the time for the potential of metal sphere to rise by one volt. Then up to this time, β-particles emitted from sphere are

$$N=\left(6.25\times 10^{10}\right)t$$

Number of β-particles that escape in this time are

$$N_e=\left(\frac{80}{100}\right)\times\left(6.25\times 10^{10}\right)t$$

$$\Rightarrow \quad N_e=5\times 10^{10}t$$

Since, the emission of a β-particle leads to a charge e on metal sphere, so charge acquired by the sphere in time t sec is

$$Q=\left(5\times 10^{10}t\right)\times\left(1.6\times 10^{-19}\right)$$

$$\Rightarrow \quad Q=8\times 10^{-9}t \text{ coulomb} \qquad \ldots(1)$$

The capacitance C of the metal sphere is given by

$$C=4\pi\varepsilon_0 r$$

$$\Rightarrow \quad C=\left(\frac{1}{9\times 10^9}\right)\times\left(\frac{10^{-3}}{2}\right)$$

$$\Rightarrow \quad C=\frac{10^{-12}}{18} \text{ farad} \qquad \ldots(2)$$

Since $Q=CV$ where $V=1$ volt

$$\Rightarrow \quad \left(8\times 10^{-9}\right)t=\left(\frac{10^{-12}}{18}\right)\times 1$$

$$\Rightarrow \quad t=6.95\ \mu\text{s}$$

PROBLEM 12

A nucleus at rest undergoes a decay emitting an α-particle of de-Broglie wavelength, $\lambda=5.76\times 10^{-15}$ m. If the mass of the daughter nucleus is 223.610 amu and that of the α-particle is 4.002 amu. Determine the total kinetic energy in the final state. Hence obtain the mass of the parent nucleus in amu. $\left(1 \text{ amu}=931.470 \text{ MeVc}^{-2}\right)$

SOLUTION

Given mass of α-particle, $m=4.002$ amu and mass of daughter nucleus,

$$M=223.610 \text{ amu}$$

de-Broglie wavelength of α-particle,

$$\lambda=5.76\times 10^{-15} \text{ m}$$

So, momentum of α-particle is

$$p=\frac{h}{\lambda}=\frac{6.63\times 10^{-34}}{5.76\times 10^{-15}} \text{ kgms}^{-1}$$

$$\Rightarrow \quad p=1.151\times 10^{-19} \text{ kgms}^{-1}$$

By Law of Conservation of Linear Momentum, this should also be equal to the linear momentum of the daughter nucleus (in opposite direction).

Let K_1 and K_2 be the kinetic energies of α-particle and daughter nucleus. Then total kinetic energy in the final state is

$$K=K_1+K_2=\frac{p^2}{2m}+\frac{p^2}{2M}$$

$$\Rightarrow \quad K=\frac{p^2}{2}\left(\frac{1}{m}+\frac{1}{M}\right)$$

$$\Rightarrow \quad K=\frac{p^2}{2}\left(\frac{M+m}{Mm}\right)$$

Since, $1 \text{ amu}=1.67\times 10^{-27}$ kg

Substituting the values, we get

$$K=10^{-12} \text{ J}$$

$$\Rightarrow \quad K=\frac{10^{-12}}{1.6\times 10^{-13}}=6.25 \text{ MeV}$$

$\Rightarrow \quad K = 6.25\text{ MeV}$

Mass defect, $\Delta m = \dfrac{6.25}{931.470} = 0.0067\text{ amu}$

$$\therefore \begin{pmatrix}\text{mass of}\\\text{parent}\\\text{nucleus}\end{pmatrix} = \begin{pmatrix}\text{mass}\\\text{of}\\\alpha\text{-particle}\end{pmatrix} + \begin{pmatrix}\text{mass of}\\\text{daughter}\\\text{nucleus}\end{pmatrix} + \begin{pmatrix}\text{mass}\\\text{defect}\\(\Delta m)\end{pmatrix}$$

$\Rightarrow \quad m_{parent} = (4.002 + 223.610 + 0.0067)\text{ amu}$

$\Rightarrow \quad m_{parent} = 227.62\text{ amu}$

Hence, mass of parent nucleus is 227.62 amu.

PROBLEM 13

A small quantity of solution containing Na_{24} radio nuclide (half-life = 15 hour) of activity 1.0 microcurie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5 hour shows an activity of 296 disintegrations per minute. Determine the total volume of the bond in the body of the person. Assume that the radioactive solution mixes uniformly in the blood of the person.
(1 curie $= 3.7\times10^{10}$ disintegrations per second)

SOLUTION

λ is the disintegration constant, then

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{15}\text{ h}^{-1}$$

$\Rightarrow \quad \lambda = 0.0462\text{ h}^{-1}$

Let R_0 be the initial activity then

$$R_0 = 1\text{ microcurie} = 3.7\times10^4\text{ dps}$$

Let r be the activity in 1 cm^3 of blood at $t = 5\text{ hr}$, then

$$r = \frac{296}{60}\text{ disintegration per second}$$

$\Rightarrow \quad r = 4.93$ disintegration per second, and

R be the activity of whole blood at time $t = 5\text{ hr}$

Total volume of blood should be

$$V = \frac{R}{r}$$

$$\Rightarrow \quad V = \frac{R_0 e^{-\lambda t}}{r}$$

Substituting the values, we get

$$V = \left(\frac{3.7\times10^4}{4.93}\right)e^{-(0.0462)(5)}\text{ cm}^3$$

$\Rightarrow \quad V = 5.95\times10^3\text{ cm}^3$

$\Rightarrow \quad V = 5.95\text{ lt.}$

PROBLEM 14

The element curium ${}^{248}_{96}\text{Cm}$ has a mean life of 10^{13} seconds. Its primary decay modes are spontaneous fission and α-decay, the former with a probability of 8% and the latter with a probability of 92%. Each fission releases 200 MeV of energy. If the mass of ${}^{248}_{96}\text{Cm}$ is 248.072220 u, ${}^{244}_{94}\text{Pu}$ is 244.064100 u and ${}^{4}_{2}\text{He}$ is 4.002603 u. Calculate the power output from a sample of 10^{20} Cm atoms. $(1\text{ u} = 931\text{ meVc}^{-2})$

SOLUTION

The reaction involved in α-decay is

$${}^{248}_{96}\text{Cm} \rightarrow {}^{244}_{94}\text{Pu} + {}^{4}_{2}\text{He}$$

Mass defect

$$\Delta m = m\left({}^{248}_{96}\text{Cm}\right) - \left[m\left({}^{244}_{94}\text{Pu}\right) + m\left({}^{4}_{2}\text{He}\right)\right]$$

$\Rightarrow \quad \Delta m = (248.072220 - 244.064100 - 4.002603)\text{u}$

$\Rightarrow \quad \Delta m = 0.005517\text{ u}$

Therefore, energy released in α-decay will be

$$E_\alpha = (0.005517\times931)\text{ MeV} = 5.136\text{ MeV}$$

Similarly, $E_{fission} = 200\text{ MeV}$ {given}

Mean life is

$$t_{mean} = 10^{13}\text{ s} = \frac{1}{\lambda}$$

So, disintegration constant $\lambda = 10^{-13}\text{ s}^{-1}$

Rate of decay at the moment when number of nuclei are 10^{20}

$$-\frac{dN}{dt} = \lambda N = (10^{-13})(10^{20})$$

$$\Rightarrow \quad -\frac{dN}{dt} = 10^7 \text{ disintegration per second}$$

Of these disintegrations, 8% are in fission and 92% are in α -decay.

Therefore, energy released per second is

$$P = \frac{E}{t}$$

$$\Rightarrow \quad P = \left(0.08 \times 10^7 \times 200 + 0.92 \times 10^7 \times 5.136\right) \text{ MeV}$$

$$P = \frac{E}{t} = 2.074 \times 10^8 \text{ MeV}$$

So, power output (in watt) is

$$P = \text{energy released per second } \left(\text{Js}^{-1}\right)$$

$$\Rightarrow \quad P = \left(2.074 \times 10^8\right)\left(1.6 \times 10^{-13}\right)$$

So, power output is $P = 3.32 \times 10^{-5}$ W