CHAPTER 3: NUCLEAR PHYSICS

Test Your Concepts-I (Based on Nucleus Properties and Binding Energy)

1. Since, we know that $R = R_0 A^{1/3}$

$$\Rightarrow \quad R = R_0 A^{1/3} = (1.1 \text{ fm})(70)^{1/3}$$

$$\Rightarrow \quad R = (1.1 \text{ fm})(4.12) = 4.53 \text{ fm}$$

2. The radius of a ^{12}C nucleus is $R = R_0 A^{1/3}$

$$\Rightarrow \quad R = (1.1 \text{ fm})(12)^{1/3} = 2.52 \text{ fm}$$

The separation between the centres of the nuclei is $2R = 5.04$ fm. The potential energy of the pair is $U = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r}$.

$$\Rightarrow \quad U = (9\times10^9)\frac{(6\times1.6\times10^{-19})^2}{5.04\times10^{-15}}$$

$$\Rightarrow \quad U = 1.64\times10^{-12} \text{ J} = 10.2 \text{ MeV}$$

3. Average atomic mass A is given by

$$A = \frac{p_1 A_1 + p_2 A_2}{p_1 + p_2}$$

$$\Rightarrow \quad A = \frac{(7.5)(6.0152) + (92.5)(7.016004)}{100}$$

$$\Rightarrow \quad A = \frac{45.39 + 648.98}{100} = 6.94 \text{ amu}$$

4. The surface area S of a sphere is given by

$$S = 4\pi R^2$$

Since, $R = R_0 A^{\frac{1}{3}}$

$$\Rightarrow \quad S = 4\pi R^2 \propto A^{2/3}$$

$$\Rightarrow \quad \frac{S_1}{S_2} = \left(\frac{A_1}{A_2}\right)^{\frac{2}{3}}$$

5. The radius of $^{12}_{6}C$ nucleus is given by

$$R = R_0 A^{\frac{1}{3}}$$

$$\Rightarrow \quad R = 1.2\times10^{-15}\times(12)^{\frac{1}{3}}$$

$$\Rightarrow \quad R = 2.75\times10^{-15} \text{ m}$$

The atomic mass of $^{12}_{6}\text{C}$ is 12 amu. Neglecting the masses and binding energies of the six electrons, the nuclear density is given by

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{12\times1.66\times10^{-27}}{\left(\frac{4}{3}\pi\right)(2.7\times10^{-15})^3}$$

$$\Rightarrow \quad \rho = 2.4\times10^{17} \text{ kgm}^{-3}$$

6. $\Delta m = \dfrac{E}{c^2} = \dfrac{4.2\times10^3}{(3\times10^8)^2}$ kg

$$\Rightarrow \quad \Delta m = 4.7\times10^{-14} \text{ kg}$$

7. The $^{35}_{17}\text{Cl}$ nucleus has 17 protons and 18 neutrons. Therefore, the mass M of constituent nucleons of $^{35}_{17}\text{Cl}$ is

$$M = 17m_p + 18m_n$$

$$\Rightarrow \quad M = 17(1.007825) + 18(1.008665)$$

$$\Rightarrow \quad M = 35.289 \text{ amu}$$

Now, mass defect for the nucleus is

$$\Delta m = \frac{298 \text{ MeV}}{9312 \text{ MeV/amu}} = 0.3200 \text{ amu}$$

Since, mass defect Δm is

$$\Delta m = (17m_p + 18m_n) - m({}_{17}\text{Cl}^{35}) = M - m({}_{17}\text{Cl}^{35})$$

So, atomic mass of $^{35}_{17}\text{Cl}$ is approximately equal to $m({}_{17}\text{Cl}^{35})$, given by

$$m({}_{17}\text{Cl}^{35}) = M - \Delta m = (35.289 - 0.3200) \text{ amu}$$

$$\Rightarrow \quad m({}_{17}\text{Cl}^{35}) = 34.969 \text{ amu}$$

8. The alpha particle contains 2 protons and 2 neutrons. The binding energy is

$$BE = (2\times1.007826 \text{ u} + 2\times1.008665 \text{ u} - 4.00260 \text{ u})c^2$$

$$\Rightarrow \quad BE = (0.03038 \text{ u})c^2$$

$$\Rightarrow \quad BE = 0.03038\times931 \text{ MeV} = 28.3 \text{ MeV}$$

9. The binding energy of $^{8}_{4}\text{Be}$ is determined by the equation

$$BE({}^{8}_{4}\text{Be}) = (\Delta m)c^2$$

where, $\Delta m = \left[4m_n + 4m({}^{1}_{1}\text{H}) - m({}^{8}_{4}\text{Be})\right]$

$\Rightarrow \quad \Delta m = \left(4(1.008665) + 4(1.007825)\right) - 8.005305$

$\Rightarrow \quad \Delta m = 0.06065$ amu

$\Rightarrow \quad BE\left({}^{8}\text{Be}\right) = (0.06065)(931.5) = 56.5$ MeV

Now, let us calculate the binding energy of the decay of ${}^{8}_{4}\text{Be}$ in two α-particles through the reaction

$${}^{8}\text{Be} \to 2\alpha + \Delta E$$

where, $\Delta E = (\Delta m)c^2$ and

$$\Delta m = 2m_{\alpha} - m\left({}^{8}_{4}\text{Be}\right)$$

$\Rightarrow \quad \Delta m = 2(4.002603) - 8.005305 = -0.000099$ amu

$\Rightarrow \quad \Delta E = (-0.000099)(931.5)$ MeV $= -0.092$ MeV

Since, binding energy is negative for this reaction, hence ${}^{8}_{4}\text{Be}$ is unstable against decay to two alpha particles.

10. Mass defect Δm is given by

$$\Delta m = 3(1.007825) + 4(1.008665) - 7.016005$$

$\Rightarrow \quad \Delta m = 0.04213$ amu

$\Rightarrow \quad BE = (0.04213)(931.5)$ MeV $= 39.231$ MeV

Binding energy per nucleon is

$$\frac{BE}{A} = \frac{39.231}{7} = 5.604 \text{ MeV}$$

Test Your Concepts-II (Based on Radioactivity)

1. In 1 second 90% of the nuclei have remained undecayed, so in another 1 second 90% of 90 i.e., 81 nuclei will remain undecayed.

2. Since, $N = \dfrac{10^{-3}}{210} \times 6.02 \times 10^{23} = 2.87 \times 10^{18}$

During one mean life period 63.8% nuclei are decayed. Hence, energy released is

$$E = 0.638 \times 2.87 \times 10^{18} \times 5.3 \times 1.6 \times 10^{-13} \text{ J}$$

$\Rightarrow \quad E = 1.55 \times 10^{6}$ J

3. $N = \dfrac{2}{238} \times 6.02 \times 10^{23} = 5.06 \times 10^{21}$

Since, $R = \lambda N$

$\Rightarrow \quad \lambda = \dfrac{R}{N} = \dfrac{2.5 \times 10^{4}}{5.06 \times 10^{21}} = 4.94 \times 10^{-18} \text{ s}^{-1}$

$\Rightarrow \quad t_{1/2} = \dfrac{0.693}{\lambda} = 1.4 \times 10^{17}$ s

4. $\dfrac{R_1}{R_2} = \dfrac{\lambda_1 N}{\lambda_2 N} = \dfrac{1.2}{98.8}$

$\Rightarrow \quad \lambda_2 = 82.33\lambda_1 \qquad \ldots(1)$

Further, $\lambda = \dfrac{0.693}{21.8} \text{ year}^{-1} = 0.0318 \text{ year}^{-1} \qquad \ldots(2)$

Also, $\left(-\dfrac{dN}{dt}\right) = \left(-\dfrac{dN_1}{dt}\right) + \left(-\dfrac{dN_2}{dt}\right)$

$\Rightarrow \quad \lambda N = \lambda_1 N + \lambda_2 N$

$\Rightarrow \quad \lambda = \lambda_1 + \lambda_2 \qquad \ldots(3)$

Solving equations (1), (2) and (3), we get

$$\lambda_1 = 3.81 \times 10^{-4} \text{ year}^{-1} \text{ and } \lambda_2 = 3.14 \times 10^{-2} \text{ year}^{-1}$$

5. (a) $R = R_0 e^{-\lambda t}$

$\Rightarrow \quad 2700 = 4750 e^{-5\lambda}$

$\Rightarrow \quad \lambda = 0.113 \text{ min}^{-1}$

(b) $t_{1/2} = \dfrac{0.693}{\lambda} = 6.132$ min

6. $R_0 = \lambda N$

$\Rightarrow \quad R_0 = \dfrac{0.693}{14.3 \times 3600 \times 24} \times 6.02 \times 10^{23}$ per sec

$\Rightarrow \quad R_0 = 3.37 \times 10^{17}$ per sec

After 70 hours activity

$$R = R_0 e^{-\lambda t}$$

$\Rightarrow \quad R = \left(3.37 \times 10^{17}\right) e^{-\left(\frac{0.693}{14.3} \times 24\right)(70)} = 2.92 \times 10^{17}$ per sec

In fruits, the activity was observed $1\,\mu$Ci or 3.7×10^{4} per sec.

Therefore, percentage P of activity transmitted from root to the fruit is

$$\left(\begin{array}{c}\text{Percentage of} \\ \text{Activity Transmitted}\end{array}\right) = \frac{3.7 \times 10^{4}}{2.92 \times 10^{17}} \times 100$$

$\Rightarrow \quad P = 1.26 \times 10^{-11}\,\%$

7. (a) $N = \dfrac{1}{109} \times 6.02 \times 10^{23}$

$\Rightarrow \quad R = \lambda N = \dfrac{0.693}{2.7 \times 10^{7}} \times \dfrac{1}{109} \times 6.02 \times 10^{23}$

$\Rightarrow \quad R = 1.42 \times 10^{14}$ per year

(b) After 2 year, $R = R_0 e^{-\lambda t}$

$\Rightarrow \quad R = \left(1.42 \times 10^{14}\right) e^{\left[-0.693/2.7 \times 10^{7}\right](2)}$

$\Rightarrow \quad R = 1.41 \times 10^{14}$ per year

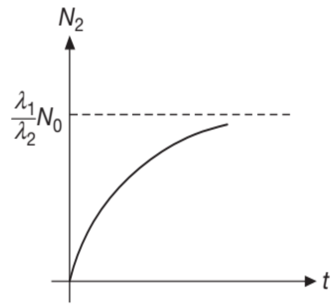
(c) After 2 half lives activity reduces to 25% of the original value, so we have

$$t = 2t_{1/2} = 5.4 \times 10^{7} \text{ years}$$

8. Since, $N_2 = \dfrac{\lambda_1 N_0}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$

When $\lambda_1 \gg \lambda_2$ we have, $N_2 \approx N_0 e^{-\lambda_2 t}$

Physically this means that parent nuclei practically instantly transform into daughter nuclei, which then decay According to the Law of Radioactive Decay with a certain decay constant.



$$N_2 \approx \frac{\lambda_1}{\lambda_2} N_0 e^{-\lambda_1 t}$$

When $\lambda_1 \ll \lambda_2$, then $N_2 \approx \dfrac{\lambda_1}{\lambda_2} N_0 e^{-\lambda_1 t}$

i.e., N_2 versus t graph in this case is as shown in the figure.

9. Probability of a nucleus to decay in time t is

$$\frac{N}{N_0} = 1 - e^{-\lambda t} = 1 - e^{-(1/10)(5)} = 0.39$$

10. (a) $R = R_0 e^{-\lambda t}$

$$\Rightarrow \quad 7.3 = 9.3 e^{-\lambda(4500-2500)}$$

$$\Rightarrow \quad \lambda = 1.21 \times 10^{-4} \text{ year}^{-1}$$

$$\Rightarrow \quad t_{1/2} = \frac{0.693}{\lambda} = 5724 \text{ year}$$

(b) Further applying, $R = R_0 e^{-\lambda t}$

$$\Rightarrow \quad R_0 = \text{Re}^{\lambda t} = (7.3) e^{(1.21\times 10^{-4})(4500)}$$

$$\Rightarrow \quad R_0 = 12.58 \text{ dismin}^{-1}\text{g}^{-1}$$

11. $N = \dfrac{R}{\lambda} = \dfrac{5\times 10^{-3} \times 3.7 \times 10^{10}}{\left(\dfrac{0.693}{138 \times 24 \times 3600}\right)} = 3.2 \times 10^{15}$

$$\Rightarrow \quad m = \left(\frac{3.2 \times 10^{15}}{6.02 \times 10^{23}}\right)(210)\,g$$

$$\Rightarrow \quad m = 1.12 \times 10^{-6} \text{ g}$$

12. Let x the desired ratio, then, mass of Co^{58} in 1 g be x

$$\Rightarrow \quad N = \frac{x}{58} \times 6.02 \times 10^{23}$$

Given, $2.2 \times 10^{12} = \lambda N = \dfrac{0.693}{71.3 \times 24 \times 3600} \times \dfrac{x}{58} \times 6.02 \times 10^{23}$

$$\Rightarrow \quad x = 1.88 \times 10^{-3}$$

13. (a) 13.0 g of ${}^{13}_{7}N$ will contain 6.023×10^{23} nuclei (Avogadro's number).

The number of nuclei N_0 that we have initially in 1.49×10^{-6} g of ${}^{13}_{7}N$, is

$$N_0 = \left(6.023 \times 10^{23}\right)\left(\frac{1.49 \times 10^{-6} \text{ g}}{13.0 \text{ g}}\right),$$

$$\Rightarrow \quad N_0 = 6.90 \times 10^{16} \text{ nuclei.}$$

(b) Since, we have $\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{600 \text{ s}} = 1.16 \times 10^{-3} \text{ s}^{-1}$

So, activity at $t = 0$ is

$$A_0 = \lambda N_0 = \left(1.16 \times 10^{-3} \text{ s}^{-1}\right)\left(6.90 \times 10^{16}\right)$$

$$\Rightarrow \quad A_0 = 8.00 \times 10^{13} \text{ decay/s}$$

(c) 60 minutes is 6 half-lives, so we have

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

$$\Rightarrow \quad A = \frac{A_0}{64} = \frac{8.00 \times 10^{13}}{64} = 1.25 \times 10^{12} \text{ s}^{-1}$$

14. The number of parent nuclei decaying in a short time interval t to $t + dt$ is $\lambda_p N_p dt$. This is also the number of daughter nuclei decaying during the same time interval is $\lambda_d N_d dt$. The number of the daughter nuclei will be constant if

$$\lambda_p N_p dt = \lambda_d N_d dt$$

$$\Rightarrow \quad \lambda_p N_p = \lambda_d N_d$$

15. In one half-life the number of active nuclei reduces to half the original number. Thus, in two half lives the number is reduced to $\left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$ of the original number.

The number of remaining active nuclei is given by

$$N = \frac{N_0}{4} = \frac{6 \times 10^{18}}{4} = 1.5 \times 10^{18}.$$

16. Since the number of ^{206}Pb atoms equal the number of ^{238}U atoms, half of the original ^{238}U atoms have decayed. It takes one half-life to decay half of the active nuclei. Thus, the sample is 4.5×10^{9} years old.

17. 63.5 g of copper has 6×10^{23} atoms. Thus, the number of atoms in 1 μg of Cu is

$$N=\frac{6\times10^{23}\times1\ \mu g}{63.5\ g}=9.45\times10^{15}$$

The activity is $A=\lambda N$

$$A=(1.516\times10^{-5}\ s^{-1})\times(9.45\times10^{15})$$

$$\Rightarrow\quad A=1.43\times10^{11}\ \text{disintegrations/s}$$

$$\Rightarrow\quad A=\frac{1.43\times10^{11}}{3.7\times10^{10}}\ \text{Ci}=3.86\ \text{Ci}$$

18. 40 hours $\Rightarrow$ 2 half lives

Since $A=\dfrac{A_0}{2^n}$

$$\Rightarrow\quad A=\frac{A_0}{(2)^2}=\frac{A_0}{4}$$

$$\Rightarrow\quad \frac{A}{A_0}=\frac{1}{4}$$

So, one fourth of the original activity will remain after 40 hours.

19. Let N_0 be the initial amount of uranium in the rock and let N be the present amount of uranium after 1.5×10^{9} years.

So, the amount of lead present in the rock is (N_0-N)

So, the required ratio is $\dfrac{N_0-N}{N}=\dfrac{1-e^{-\lambda t}}{e^{-\lambda t}}$

Since, $T_{\frac{1}{2}}=\dfrac{\ln 2}{\lambda}$

$$\Rightarrow\quad \lambda=\frac{\ln 2}{T_{\frac{1}{2}}}$$

$$\Rightarrow\quad e^{-\lambda t}=e^{-\left(\frac{\log_e 2}{T_{\frac{1}{2}}}\right)t}=e^{-\log(2)\frac{t}{T_{\frac{1}{2}}}}=2^{-\frac{t}{T_{\frac{1}{2}}}}$$

$$\Rightarrow\quad \frac{N_0-N}{N}=\frac{1-e^{-\left(\frac{\ln 2}{T_{\frac{1}{2}}}\right)t}}{e^{-\left(\frac{\ln 2}{T_{\frac{1}{2}}}\right)t}}=\frac{1-2^{-\left(\frac{t}{T_{\frac{1}{2}}}\right)}}{2^{-\left(\frac{t}{T_{\frac{1}{2}}}\right)}}=\frac{1-2^{-\frac{1}{3}}}{2^{-\frac{1}{3}}}$$

$$\Rightarrow\quad \frac{N_0-N}{N_0}=2^{\frac{1}{3}}-1=0.259$$

20. At maximum, the rate of production of nuclei is equal to the rate of decay

$$\Rightarrow\quad r_{\text{production}}=r_{\text{decay}}$$

$$\Rightarrow\quad 10^{21}=\frac{1}{30}N$$

$$\Rightarrow\quad N=30\times10^{21}$$

Test Your Concepts-III (Based on Nuclear Reactions, Alpha, Beta, Gamma Decay, Fission and Fusion)

1. A reaction is possible (spontaneously) if the binding energy of products is larger than that of $^{238}_{92}U$.

   (a) Total binding energy of $^{238}_{92}U=238\times7.57=1801.66$ MeV and binding energy of $^{206}_{82}Pb=206\times7.83=1612.981$ MeV the binding energy of products is less than that of $^{238}_{92}U$. Note that the protons and neutrons are free and do not have any binding energy. Hence, this reaction is not possible spontaneously. It can take place only if $1801.66-1612.98=188.68$ MeV of energy is supplied from outside.

   (b) In the second case binding energy of products is larger than the binding energy of the parent nucleus. Hence, reaction is possible spontaneously.

2. Since, the masses are given in atomic mass units, so, we shall first calculate the mass difference between reactants and products in the same units and then multiply the mass difference by 931.5 MeVu^{-1}. Thus, we have

$$Q=(14.00307+4.00260-1.00783-16.99913)(931.5)$$

$$\Rightarrow\quad Q=-1.20\ \text{MeV}$$

Q value is negative, so the reaction is endothermic.

Hence, the minimum kinetic energy of α-particle to initiate this reaction is given by

$$K_{\min}=|Q|\left(\frac{m_\alpha}{m_N}+1\right)$$

$$\Rightarrow\quad K_{\min}=(1.20)\left(\frac{4.00260}{14.00307}+1\right)$$

$$\Rightarrow\quad K_{\min}=1.54\ \text{MeV}$$

3. Total energy released is

$$\Delta E=\frac{1}{235}\times6.02\times10^{26}\times185\times1.6\times10^{-13}\ \text{J}$$

$$\Rightarrow\quad \Delta E=7.58\times10^{13}\ \text{J}$$

Power $P = 100 \times 10^6 = 10^8\ \text{Js}^{-1}$

Therefore, time

$$t = \frac{7.58 \times 10^{13}}{10^8} \text{sec} = 8.78 \text{ day}$$

4. ${}_0n^1 \rightarrow {}_1\text{H}^1 + {}_{-1}e^0$

Mass defect $\Delta m = m_n - (m_p + m_e)$

$$\Rightarrow \quad \Delta m = (1.6747 \times 10^{-27}) - (1.6725 \times 10^{-27} + 9 \times 10^{-31})$$

$$\Rightarrow \quad \Delta m = 0.0013 \times 10^{-27} \text{kg}$$

Energy released $Q = (\Delta m)c^2$

$$\Rightarrow \quad Q = (0.0013 \times 10^{-27}) \times (3 \times 10^8)^2 = 1.17 \times 10^{-13} \text{ J}$$

$$\Rightarrow \quad Q = \frac{1.17 \times 10^{-13}}{1.6 \times 10^{-19}} = 0.73 \times 10^6 \text{ eV} = 0.73 \text{ MeV}$$

5. By Law of Conservation of Linear Momentum, we have

$$M_\alpha v_\alpha = mv$$

$$\Rightarrow \quad v = \frac{M_\alpha v_\alpha}{M}$$

Kinetic energy of $T\ell$ atom is

$$(KE)_{T\ell} = \frac{1}{2}Mv^2 = \frac{1}{2}M\left(\frac{M_\alpha^2 v_\alpha^2}{M^2}\right) = \left(\frac{1}{2}M_\alpha v_\alpha^2\right)\left(\frac{M_\alpha}{M}\right)$$

$$\Rightarrow \quad (KE)_{T\ell} = (\text{KE})_\alpha \left(\frac{M_\alpha}{M}\right)$$

$$\Rightarrow \quad (KE)_{T\ell} = 6.082 \times \frac{4}{208} \text{ MeV}$$

$$\Rightarrow \quad (KE)_{T\ell} = 0.1308 \text{ MeV}$$

6. (a) Since the neutron and boron are both initially at rest, so the total momentum both before the reaction and after the reaction is zero. So, we get

$$M_{\text{Li}} v_{\text{Li}} = M_{\text{He}} v_{\text{He}}$$

Please understand that, here we can use the Classical approach rather than the Relativistic approach. Since $v_{He} = 9.30 \times 10^6\ \text{ms}^{-1}$, which is very small compared to the speed of light $c = 3 \times 10^8\ \text{ms}^{-1}$, so we can take kinetic energy as $KE \approx \frac{1}{2}mv^2$. The error in the calculations due to this Classical approach will be negligible or will even be less because $M_{\text{Li}} > M_{\text{He}}$. So, we have kinetic energy of Li as

$$K_{\text{Li}} = \frac{1}{2}M_{\text{Li}} v_{\text{Li}}^2 = \frac{1}{2}M_{\text{Li}}\left(\frac{M_{\text{He}} v_{\text{He}}}{M_{\text{Li}}}\right)$$

$$\Rightarrow \quad K_{\text{Li}} = \frac{M_{\text{He}}^2 v_{\text{He}}^2}{2M_{\text{Li}}}$$

Substituting the values, we get

$$K_{\text{Li}} = \frac{(4.0026)^2 (1.66 \times 10^{-27})^2 (9.30 \times 10^6)^2}{2(7.0160)(1.66 \times 10^{-27})}$$

$$\Rightarrow \quad K_{\text{Li}} = 1.64 \times 10^{-13} \text{ J} = 1.02 \text{ MeV}$$

(b) Now, $Q = K_{\text{Li}} + K_{\text{He}}$

where $K_{\text{He}} = \frac{1}{2}M_{\text{He}} v_{\text{He}}^2$

$$\Rightarrow \quad K_{\text{He}} = \frac{1}{2}(4.0026)(1.66 \times 10^{-27})(9.30 \times 10^6)^2$$

$$\Rightarrow \quad K_{\text{He}} = 2.87 \times 10^{-13} \text{ J} = 1.80 \text{ MeV}$$

$$\Rightarrow \quad Q = 1.02 \text{ MeV} + 1.80 \text{ MeV} = 2.82 \text{ MeV}$$

7. Please observe that here, atomic masses are given (not the nuclear masses), but still we can use them for calculating the mass defect because mass of electrons get cancelled both sides. Thus,

Mass defect $\Delta m = (22.9945 - 22.9898) = 0.0047u$

$$\Rightarrow \quad Q = (0.0047u)(931.5 \text{ MeVu}^{-1})$$

$$\Rightarrow \quad Q = 4.4 \text{ MeV}$$

Hence, the energy of beta particles can range from 0 to 4.4 MeV.

8. $Q = \left| m_c' - (m_B' + m_p) \right| c^2$

where m_c' and m_B' are the nuclear mass of ${}^{11}\text{C}$ and ${}^{11}\text{B}$, so

$$m_B = m_B' + 5m_e$$

$$m_c = m_c' + 6m_e$$

$$\Rightarrow \quad Q = \left[(m_c - 6m_e) - (m_B - 5m_e + m_p)\right]c^2$$

Since $m_{\text{electron}} = m_{\text{positron}}$

$$\Rightarrow \quad Q = \left[m_c - m_B - 2m_p\right]c^2$$

$$\Rightarrow \quad Q = (11.011434 - 11.009305 - 2 \times 0.0005486)931$$

$$\Rightarrow \quad Q = 0.961 \text{ MeV}$$

The disintegration energy is equal to the maximum energy of the emitted photon.

9. $\Delta m = 2(\text{mass of } {}_1\text{H}^2) - (\text{mass of } {}_2\text{He}^4) = 0.0256 \text{ u}$

$$\Rightarrow \quad \Delta E = 0.0256 \times 931.5 \text{ MeV} = 23.85 \text{ MeV}$$

Total energy required per day is

$$E = 200 \times 10^6 \times 24 \times 3600 \text{ J} = 1.728 \times 10^{13} \text{ J}$$

Let m be the mass of deuterium required. Then energy required for reactor is

$$E' = \left(\frac{25}{100}\right)\left(\frac{m/2}{2}\right)(6.02\times10^{23})(23.85\times1.6\times10^{-13})$$

This should be equal to 1.728×10^{13} J

$$\Rightarrow \quad m = \frac{4\times100\times1.728\times10^{13}}{25\times6.02\times10^{23}\times23.85\times1.6\times10^{-13}} \text{ g}$$

$$\Rightarrow \quad m = 120.35 \text{ g}$$

10. (a) Total number of atoms in 1 kg of U^{238} is

$$N = \frac{1}{238}\times6.02\times10^{26} = 2.53\times10^{24}$$

$\Rightarrow$ Total energy released,

$$\Delta E = (200\times2.53\times10^{24}) \text{ MeV}$$

$$\Rightarrow \quad \Delta E = 8.09\times10^{13} \text{ J}$$

(b) $m = \dfrac{8.09\times10^{13}}{30\times10^{3}} \text{ g} = 2.7\times10^{9} \text{ g}$

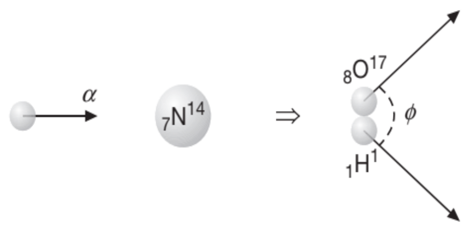
$$\Rightarrow \quad m = 2.7\times10^{6} \text{ kg}$$

11. (a) $Q\text{-value} = (\Delta m)931.5 \text{ MeV} = -1.18 \text{ MeV}$

(b) By Law of Conservation of Linear Momentum, we have

$$P_\alpha^2 = P_0^2 + P_H^2 + 2P_0P_H\cos\phi$$

$$\Rightarrow \quad 2m_\alpha K_\alpha = 2m_0K_0 + 2m_HK_H + 2\sqrt{(2m_0K_0)(2m_HK_H)}\cos\phi \quad \ldots(1)$$



By Law of Conservation of Energy, we have

$$K_0 = Q + K_\alpha - K_H = (-1.18 + 7.7 - 5.5) \text{ MeV}$$

$$\Rightarrow \quad K_0 = 1.02 \text{ MeV}$$

$$\Rightarrow \quad \cos\phi = -\left(\frac{m_0K_0 - m_\alpha K_\alpha - m_HK_H}{2\sqrt{m_\alpha m_H K_\alpha K_H}}\right) = 0.73$$

$$\Rightarrow \quad \phi \approx 43°18'$$

12. The corresponding decay equations can be

(i) ${}^{230}_{92}\text{U} \to n + {}^{229}_{92}\text{U}$ and

(ii) ${}^{230}_{92}\text{U} \to p + {}^{229}_{91}\text{Pa}$

In first equation the energy released is

$$Q = (230.033927 - 229.033496 - 1.008665)(931.5)$$

$$\Rightarrow \quad Q = -7.7 \text{ MeV}$$

Since $Q < 0$, so a spontaneous neutron decay is not possible.

Similarly, for the second reaction, the energy released is

$$Q = (230.033927 - 229.032089 - 1.007825)(931.5)$$

$$\Rightarrow \quad Q = -5.6 \text{ MeV}$$

Since $Q < 0$, so a spontaneous proton decay is also not possible.

13. Since, $\Delta E = (8m_p + 8m_n - m_o)\times931.5$ MeV

$$\Rightarrow \quad \Delta E = 127.6 \text{ MeV}$$

14. The endothermic reaction can be written as

$${}^1_1\text{H} + {}^{13}_6\text{C} \to {}^{13}_7\text{N} + {}^1_0n + Q$$

where $Q = (m_{\text{final}} - m_{\text{initial}})c^2$

$$\Rightarrow \quad Q = (m_{\text{final}} - m_{\text{initial}})\times931.5 \text{ MeV}$$

Now, $m_{\text{final}} = 13.0067384 + 1.0086654 = 14.0154038$ u

$m_{\text{initial}} = 13.0033554 + 1.0078254 = 14.0111808$ u

$$\Rightarrow \quad \Delta m = 0.004223 \text{ u}$$

$$\Rightarrow \quad Q = (0.004223)(931.5) \approx 4.00 \text{ MeV}$$

The minimum amount of energy that a bombarding particle must possess in order to initiate the reaction is

Threshold energy $E_{\text{th}} = Q\left(\dfrac{m_1}{m_2} + 1\right)$

where m_1 is mass of the projectile, m_2 is mass of target

$$\Rightarrow \quad E_{\text{th}} = 4.00\left(\frac{1.0078254}{13.0033554} + 1\right) = 4.31 \text{ MeV}$$

15. $P = 1 \text{ MW} = 10^6 \text{ W} = 10^6 \text{ Js}^{-1}$

$t = 1 \text{ day} = 24\times60\times60 = 86400$ s

So, energy released in one day is $E = Pt = 86400\times10^6$ J

Since energy released per fusion is given to be 20 MeV

$$\Rightarrow \quad (\Delta E)_{\text{per fusion}} = 20\times1.6\times10^{-13} = 3.2\times10^{-12} \text{ J}$$

Mass of ${}_1H^2$ consumed in one fusion $({}_1H^2 + {}_1H^2)$ is $4u$

$$\Rightarrow \quad (\Delta m)_{\text{per fusion}} = 4\times1.66\times10^{-27} \text{ kg} = 6.64\times10^{-27} \text{ kg}$$

$$\Rightarrow \quad (\Delta m)_{1\text{ day}} = \frac{6.64\times10^{-27}}{3.2\times10^{-12}}\times86400\times10^6 = 1.79\times10^{-4} \text{ kg}$$

Single Correct Choice Type Questions

1. When sample is 1000 m above ground i.e. it has fallen by 2000 m. Since $s=\frac{1}{2}gt^2$

$\Rightarrow \quad 2000=\frac{1}{2}(10)t^2$

$\Rightarrow \quad t=20\text{ s}$

$\Rightarrow \quad \frac{N}{N_0}=\left(\frac{1}{2}\right)^n=\left(\frac{1}{2}\right)^{\frac{20}{10}}=\frac{1}{4}$

$\Rightarrow \quad N=\frac{1000}{4}=250$

Hence, the correct answer is (B).

2. Let N be the number of nuclei at any time t. Then

$$\frac{dN}{dt}=200-\lambda N$$

$\Rightarrow \quad \int_0^N \frac{dN}{200-\lambda N}=\int_0^t dt$

$\Rightarrow \quad N=\frac{200}{\lambda}\left(1-e^{-\lambda t}\right)$

Given that $N=100$ and $\lambda=1\text{ s}^{-1}$

$\Rightarrow \quad 100=200\left(1-e^{-t}\right)$

$\Rightarrow \quad e^{-t}=\left(\frac{1}{2}\right)$

$\Rightarrow \quad t=\log_e(2)\text{ s}$

Hence, the correct answer is (B).

3. Since, $A_1=\lambda N_0 e^{-\lambda t_1}$

$\Rightarrow \quad t_1=\frac{1}{\lambda}\log_e\left(\frac{\lambda N_0}{A_1}\right) \quad \ldots(1)$

Since, $A_2=(\lambda)(2N_0)e^{-\lambda t_2}$

$\Rightarrow \quad t_2=\frac{1}{\lambda}\log_e\left(\frac{2\lambda N_0}{A_2}\right) \quad \ldots(2)$

$\Rightarrow \quad t_1-t_2=\frac{1}{\lambda}\log_e\left(\frac{A_2}{2A_1}\right)$

$\Rightarrow \quad t_1-t_2=\frac{T}{\log_e(2)}\log_e\left(\frac{A_2}{2A_1}\right)$

Hence, the correct answer is (C).

4. Using $10.81=10x+11(1-x)$

$\Rightarrow \quad 10.81=11-x$

$\Rightarrow \quad x=0.19$

$\Rightarrow \quad 1-x=0.81$

$\Rightarrow \quad \frac{x}{1-x}=\frac{19}{81}$

Hence, the correct answer is (A).

6. Since, $N=N_0e^{-\lambda t}$

$\Rightarrow \quad \frac{2N_0}{3}=N_0e^{-\lambda t_0}$

$\Rightarrow \quad e^{-\lambda t_0}=\frac{3}{2} \quad \ldots(1)$

Taking log both sides, we get

$$\lambda t_0=\log_e\left(\frac{3}{2}\right)$$

$\Rightarrow \quad \lambda=\frac{1}{t_0}\log_e\left(\frac{3}{2}\right)$

Also $\frac{N_0}{3}=N_0e^{-\lambda t_1}$

$\Rightarrow \quad \lambda t_1=\log_e(3)$

$\Rightarrow \quad t_1=\frac{1}{\lambda}\log_e(3)=\frac{t_0\log_e(3)}{\log_e\left(\frac{3}{2}\right)}$

Hence, the correct answer is (C).

7. $t=1\text{ week}=(7)(24\text{ hr})$

Since $T_{1/2}=12\text{ hr}$

$\Rightarrow \quad n=\frac{t}{T_{1/2}}=14$ and $A_0=1\text{ Ci}$

$\Rightarrow \quad A=\frac{A_0}{2^n}=\frac{1}{2^{14}}Ci=\frac{1}{16384}$

$\Rightarrow \quad A\approx 60\ \mu\text{Ci}$

Hence, the correct answer is (C).

8. $\frac{\lambda_A}{\lambda_B}=\frac{1}{2}$

Probabilities of getting α and β-particles are same. So, rate of disintegration are equal.

$\Rightarrow \quad \lambda_A N_A=\lambda_B N_B$

$\Rightarrow \quad \frac{N_A}{N_B}=\frac{\lambda_B}{\lambda_A}=2$

Hence, the correct answer is (C).

9. $\frac{m}{m_0} = \frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{540}{270}} = \frac{1}{4}$

$\Rightarrow \quad m = \frac{m_0}{4} = \frac{100 \text{ mg}}{4} = 25 \text{ mg}$

Hence, the correct answer is (C).

10. $N = N_0 e^{-\lambda t}$

$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$

$\Rightarrow \quad \lambda = \frac{0.693}{3.8}$

Since $\frac{N}{N_0} = \frac{1}{10}$

$\Rightarrow \quad \frac{1}{10} = e^{-\frac{0.693}{3.8}t}$

$\Rightarrow \quad \log 10 = \frac{0.693}{3.8}t$

$\Rightarrow \quad 2.3 = \frac{0.693}{3.8}t$

$\therefore \quad t = 12.62$ days

Hence, the correct answer is (B).

11. $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

Hence, the correct answer is (C).

12. Mass of product should be less than the mass of reactant because energy is released in a fission reaction.

Hence, the correct answer is (A).

14. Decay process is a probable process and hence nothing can be said for a small sample.

Hence, the correct answer is (D).

15.

	X	$\longrightarrow$	Y
$t = 0$	N_0		0
$t = t$	N		$(N_0 - N)$

$\Rightarrow \quad N_Y = N_0 - N$

Since, $N = N_0 e^{-\lambda t}$

$\Rightarrow \quad N_Y = N_0(1 - e^{-\lambda t})$

Rate of formation of Y is

$$\frac{dN_y}{dt} = N_0 \lambda e^{-\lambda t}$$

The above curve is decreasing with time.

Hence, the correct answer is (C).

17. The decay constant λ is the reciprocal of the mean life τ. For a simultaneous decay process, we have

$\lambda = \lambda_\alpha + \lambda_\beta$, where $\lambda = \frac{1}{\tau}$

Since, $\lambda_\alpha = \frac{1}{1620}$ per year and $\lambda_\beta = \frac{1}{405}$ per year

So, total decay constant is

$\lambda = \lambda_\alpha + \lambda_\beta$

$\Rightarrow \quad \lambda = \frac{1}{1620} + \frac{1}{405} = \frac{1}{324}$ per year

Since, $N = N_0 e^{-\lambda t}$

When $\frac{3}{4}$th part of the sample has disintegrated, then the sample left undecayed is

$N = \frac{N_0}{4}$

$\Rightarrow \quad \frac{N_0}{4} = N_0 e^{-\lambda t}$

$\Rightarrow \quad e^{\lambda t} = 4$

Taking logarithm of both sides, we get

$\lambda t = \log_e 4$

$\Rightarrow \quad t = \frac{1}{\lambda}\log_e (2)^2 = \frac{2}{\lambda}\log_e 2$

$\Rightarrow \quad t = 2 \times 324 \times 0.693 = 449$ year

Hence, the correct answer is (B).

18. λ remains uncharged.

Hence, the correct answer is (A).

19. Three half-lives of A is equivalent to six half-lives of B. Hence, we have

$N_A\left(\frac{1}{2}\right)^3 = N_B\left(\frac{1}{2}\right)^6$

$\Rightarrow \quad \frac{N_A}{N_B} = \frac{1}{8}$

Hence, the correct answer is (D).

20. Since $E=(\Delta m)c^2=(1)(3\times10^8)^2$

$\Rightarrow\ E=9\times10^{16}$ J

$\Rightarrow\ E=\dfrac{9\times10^{16}}{1.6\times10^{-13}}$ MeV

$\Rightarrow\ E=\dfrac{9}{1.6}\times10^{29}$ MeV

$\Rightarrow\ E=5.625\times10^{29}$ MeV

Hence, the correct answer is (D).

21. Positron emission means β^+ decay and electron capture is a reverse β decay process.
Hence, the correct answer is (C).

22. Fraction of nuclei which remain undecayed is

$$f=\frac{N}{N_0}=\frac{N_0e^{-\lambda t}}{N_0}=e^{-\lambda t}$$

$$\Rightarrow\ f=e^{-\left(\frac{\log_e(2)}{T}\right)\left(\frac{T}{2}\right)}=e^{-\frac{1}{2}\log_e(2)}$$

$$\Rightarrow\ f=\frac{1}{e^{\log_e\sqrt{2}}}=\frac{1}{\sqrt{2}}$$

Hence, the correct answer is (A).

23. The number of undecayed nuclei in sample at time t is

$$N=N_0e^{-\lambda t}$$

So, number of decayed nuclei of sample is

$$N_0-N=N_0\left(1-e^{-\lambda t}\right)$$

$$\Rightarrow\ 10^5=N_0\left(1-e^{-36\lambda}\right)\quad\ldots(1)$$

$$\Rightarrow\ 1.11\times10^5=N_0\left(1-e^{-108\lambda}\right)\quad\ldots(2)$$

$$\Rightarrow\ \frac{1-e^{-108\lambda}}{1-e^{-36\lambda}}=1.11=\frac{10}{9}$$

If $e^{-36\lambda}=x$ (say), then

$$\frac{1-x^3}{1-x}=\frac{10}{9}$$

$$\Rightarrow\ \frac{(1-x)(1+x^2+x)}{(1-x)}=\frac{10}{9}$$

$$\Rightarrow\ 9+9x^2+9x=10$$

$$\Rightarrow\ 9x^2+9x-1=0$$

$$\Rightarrow\ x=\frac{-9\pm\sqrt{117}}{18}=\frac{-9+10.81}{18}$$

$$\Rightarrow\ x=\frac{1.81}{18}\approx0.1$$

$$\Rightarrow\ e^{-36\lambda}=0.1$$

$$\Rightarrow\ e^{36\lambda}=10$$

$$\Rightarrow\ \lambda=\frac{\ell n(10)}{36}\text{s}^{-1}$$

$$\Rightarrow\ T_{1/2}=\frac{\ell n2}{\lambda}=\frac{36\ell n2}{\ell n10}\approx10.8\text{ s}$$

Hence, the correct answer is (B).

24. For radioactive equilibrium, we have

$$\lambda_1N_1=\lambda_2N_2$$

$$\Rightarrow\ \frac{N_1}{T_1}=\frac{N_2}{T_2}$$

$$\Rightarrow\ T_2=\frac{N_2}{N_1}\times T_1$$

$$\Rightarrow\ T_2=1.8\times10^4\times2.5\times10^5$$

$$\Rightarrow\ T_2=4.5\times10^9\text{ year}$$

Hence, the correct answer is (A).

25. ${}^{27}_{12}\text{Mg}\rightarrow{}^{27}_{13}\text{Al}+e^-+\overline{v}$

This is example of beta decay in which isotope ${}^{27}_{12}\text{Mg}$ is converted to an isotope of aluminium ${}^{27}_{13}\text{Al}$.
Hence, the correct answer is (C).

26. **The correct answer is (C).**

27. **Method-I**

$$\frac{N_1}{N_0}=90\%=0.9=\left(\frac{1}{2}\right)^{\frac{5}{T}}$$

$$\frac{N_2}{N_0}=\left(\frac{1}{2}\right)^{\frac{20}{T}}=(0.9)^4=0.6561\equiv65.61\%$$

So, percentage decayed is $(100-65.61)\approx34.4\%$

Method-II

$$\underset{\text{(Initial undecayed)}}{100\%}\xrightarrow{5\text{ yrs}}90\%\xrightarrow{5\text{ yrs}}81\%\xrightarrow{5\text{ yrs}}$$

$$72.9\%\xrightarrow{5\text{ yrs}}\underset{\text{Final undecayed}}{65.61\%}$$

So, percentage decayed is $(100-65.61)\approx34.4\%$

Hence, the correct answer is (D).

29. Energy released ΔE is

$$\Delta E=(\text{Final BE})-(\text{Initial BE})$$

$$\Rightarrow\ \Delta E=110\times8.2+90\times8.2-200\times7.4$$

CHAPTER 3

$\Rightarrow \quad \Delta E = 160 \text{ MeV}$

Hence, the correct answer is (D).

30. $A = 232 - 4 = 228$

From conservation of momentum

$$p_\alpha = p_Y$$

$$\Rightarrow \quad \sqrt{2m_\alpha K_\alpha} = \sqrt{2m_Y K_Y}$$

$$\Rightarrow \quad \frac{K_\alpha}{K_Y} = \frac{m_Y}{m_\alpha} = \frac{228}{4}$$

$$\Rightarrow \quad K_\alpha = \left(\frac{228}{228+4}\right)K_{\text{Total}}$$

$$\Rightarrow \quad K_\alpha = \left(\frac{228}{232}\right)K_{\text{Total}}$$

Hence, the correct answer is (B).

31. $E = mc^2$

$$\Rightarrow \quad E = m_0c^2\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

Since $\frac{v}{c} \ll 1$

$$\Rightarrow \quad E \simeq m_0c^2\left(1 + \frac{v^2}{2c^2}\right)$$

$$\Rightarrow \quad E \simeq m_0c^2 + \frac{1}{2}m_0v^2$$

Hence, the correct answer is (C).

32. Since we know that $\tau = \frac{1}{\lambda}$

$$\Rightarrow \quad A = A_0 e^{-\frac{t}{\tau}}$$

$$\Rightarrow \quad y = xe^{-\frac{(t_2 - t_1)}{\tau}}$$

Number of atoms disintegrated during $t_2 - t_1$ is

$$\Delta N = (x - y)\tau$$

Hence, the correct answer is (D).

33. Efficiency of 50% means that we are getting only 100 MeV of energy by the fission of one uranium nucleus.

Number of nuclei per second is

$$\frac{N}{t} = \frac{\text{Energy required per second}}{\text{Energy obtained in one fission}}$$

$$\Rightarrow \quad \frac{N}{t} = \frac{16 \times 10^6}{100 \times 1.6 \times 10^{-13}} = 10^{18}$$

Hence, the correct answer is (D).

34. Let R_0 be the initial activity. Then

$$R_1 = R_0 e^{-\lambda t_1} \text{ and } R_2 = R_0 e^{-\lambda t_2}$$

$$\Rightarrow \quad \frac{R_2}{R_1} = e^{\lambda(t_1 - t_2)}$$

$$\Rightarrow \quad R_2 = R_1 e^{\lambda(t_1 - t_2)}$$

Hence, the correct answer is (D).

35. $R_1 = \lambda N_1$

$$\Rightarrow \quad N_1 = \frac{R_1}{\lambda}$$

and $R_2 = \lambda N_2$

$$\Rightarrow \quad N_2 = \frac{R_2}{\lambda}$$

So, number of atoms decayed is

$$N = N_1 - N_2$$

$$\Rightarrow \quad N = \left(\frac{R_1 - R_2}{\lambda}\right)$$

Hence, the correct answer is (D).

36. Since $N = \frac{N_0}{2^n}$, where $n = \frac{t}{T_{1/2}}$

For A, $N_A = \frac{N_0}{2^{\frac{80}{20}}} = \frac{N_0}{2^4}$

For B, $N_B = \frac{N_0}{2^{\frac{80}{40}}} = \frac{N_0}{2^2}$

$$\Rightarrow \quad \frac{N_A}{N_B} = \frac{1}{4}$$

Hence, the correct answer is (C).

37. ${}_{92}U^{238} \to {}_{82}Pb^{206} + n_1\left({}_2\alpha^4\right) + n_2\left({}_{-1}e^0\right)$

$$\Rightarrow \quad 4n_1 + 0 \times n_2 + 206 = 238$$

$$\Rightarrow \quad n_1 = 8$$

Also, $92 = 82 + 2n_1 - n_2$

$$\Rightarrow \quad 10 = 2 \times 8 - n_2$$

$$\Rightarrow \quad n_2 = 6$$

Hence, the correct answer is (C).

38. Speed of the neutrons in the beam is

$$\frac{1}{2}mv^2 = E_K = 5 \text{ eV}$$

$$\Rightarrow \quad v = \sqrt{\frac{2(5) \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27}}}$$

$$\Rightarrow \quad v = 31 \text{ kms}^{-1}$$

During a time of $T_{1/2} = 12.8$ min, half the neutrons may have decayed from the beam. The distance travelled by the undecayed during this time is

$$x = vt = (31\ \text{kms}^{-1})(12.8\ \text{min})(60\ \text{s/min})$$

$$\Rightarrow \quad x = 23800\ \text{km}$$

Hence, the correct answer is (B).

39. For, successive disintegration, we have

$$A \xrightarrow[\lambda_1]{} B \xrightarrow[\lambda_2]{} C$$

So, rate of decay of B is

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B$$

In equilibrium, $\frac{dN_B}{dt} = 0$ i.e. rate of production of B equals the rate of disintegration of B.

Hence, the correct answer is (A).

41. Count rate for $100\ cc$ volume is c. If some volume is discarded and left out volume is V, then new count rate should be $\left(\frac{c}{100}\right)V$. After 3 half life of remaining liquid, count rate is $\frac{c}{10}$.

$$\Rightarrow \quad \frac{c}{100} = \frac{cV}{100}\left(\frac{1}{2}\right)^3$$

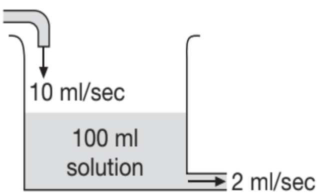
$$\Rightarrow \quad V = 80\ \text{cc}$$

Hence, the correct answer is (D).

42. Rate of disintegration does not change with temperature, pressure or humidity.

Hence, the correct answer is (D).

43.



The volume of water added in time t is $8t$ and the volume of liquid ejected in t seconds is $2t$. Hence, the volume of liquid in beaker at any instant of time t is

$$V = 100 + 8t$$

Number of active atoms being taken out is

$$-dN = \frac{N}{V}(2dt)$$

$$\Rightarrow \quad -\frac{dN}{dt} = \frac{2N}{V} = \frac{2N}{100 + 8t}$$

Multiplying both sides with disintegration constant λ, we get

$$-\lambda dN = \lambda N\left(\frac{2dt}{V}\right) \qquad \ldots(1)$$

Since, $\lambda dN = dA$ and $\lambda N = A$, so equation (1) becomes

$$-dA = A\left(\frac{2dt}{V}\right)$$

where A is activity of the solution.

Since the time taken for 10 ml solution to come out is 5 s

$$\Rightarrow \quad \int_{A_0}^{A} \frac{dA}{A} = \int_0^5 \frac{-2t}{100 + 8t} dt$$

$$\Rightarrow \quad A = A_0\left(\frac{5}{7}\right)^{\frac{1}{4}}$$

So, required activity of the ejected solution is

$$A - A_0 = A_0\left[1 - \left(\frac{5}{7}\right)^{\frac{1}{4}}\right]$$

Hence, the correct answer is (C).

44. $N = \frac{N_0}{(2)^{t/T_{1/2}}}$

$$\Rightarrow \quad (2)^{t/T_{1/2}} = \frac{N_0}{N}$$

$$\Rightarrow \quad (2)^{t/T_{1/2}} = \frac{8}{1}$$

$$\Rightarrow \quad t = 3T_{1/2} = 4.2 \times 10^9\ \text{year}$$

Hence, the correct answer is (C).

45. Given $T_{1/2} = 30$ min

At $t = 2\ hr$, $A = 5\ \text{sec}^{-1}$ and $A_0 = ?$

Since, $n = \frac{t}{T_{1/2}} = \frac{2 \times 80}{30} = 4$

Since, $\frac{A}{A_0} = \left(\frac{1}{2}\right)^n$

$$\Rightarrow \quad A_0 = 2^n A = 2^4 \times 5$$

$$\Rightarrow \quad A_0 = 16 \times 5 = 80\ \text{sec}^{-1}$$

Hence, the correct answer is (B).

46. $\frac{7}{8}$ decays $\Rightarrow$ $\frac{1}{8}$ left undecayed

$\Rightarrow$ since $\frac{1}{8} = \left(\frac{1}{2}\right)^n$

$\Rightarrow \quad x = 3$

$\Rightarrow \quad \dfrac{t}{T_{1/2}} = 3$

Since $t = 12$ day

$\Rightarrow \quad T_{1/2} = 4$ day

So, fraction undecayed after 24 days is

$$\frac{N'}{N_0} = \left(\frac{1}{2}\right)^{\frac{24}{4}} = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

Hence, the correct answer is (C).

47. ${}_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0 + \bar{\nu}$

The electron comes out with a spectrum of energies. The energy released is shared between electron and neutrino.
Hence, the correct answer is (D).

49. $A_2 = A_1\left(\dfrac{90}{100}\right)^{\frac{20}{5}}$

$\Rightarrow \quad \dfrac{A_2}{A_1} \times 100\% = \left(\dfrac{9}{10}\right)^4 100\% = 65.61\%$

Hence, the correct answer is (B).

51. When the rate production is equal to the rate of disintegration then the number of nuclei is maximum.

$\Rightarrow \quad \lambda N = A$

$\Rightarrow \quad \dfrac{\ln 2}{T} N = A$

$\Rightarrow \quad N = \dfrac{AT}{\ln 2} = \text{maximum}$

Hence, the correct answer is (D).

52. $\dfrac{dN_2}{dt} = \lambda N_1 = 2\lambda N_2$

For N_2 to be maximum, we have

$$\frac{dN_2}{dt} = 0$$

$\Rightarrow \quad \lambda N_1 = 2\lambda N_2$

$\Rightarrow \quad \dfrac{N_1}{N_2} = 2$

Hence, the correct answer is (B).

54. ${}_ZX^A \rightarrow {}_{Z-2}Y^{A-4} + {}_2\alpha^4$

Hence, the correct answer is (C).

55. Charge number and mass number for γ-ray is zero.
Hence, the correct answer is (C).

56. In the mixture, initially we have

$$N = N_1 + N_2$$

At any time t, we have

$$N(t) = N_1(t) + N_2(t)$$

$\Rightarrow \quad N(t) = N_1 e^{-\lambda_1 t} + N_2 e^{-\lambda_2 t} \qquad \ldots(1)$

The decay rate $-\dfrac{dN}{dt}$ at time t is obtained by differentiating equation (1)

$\Rightarrow \quad -\dfrac{dN}{dt} = N_1\lambda_1 e^{-\lambda_1 t} + N_2\lambda_2 e^{-\lambda_2 t}$

Hence, the correct answer is (C).

58. $R_0 = 15 \times 200 = 3000$ decay/min from 200 g carbon.

Using, $R = R_0\left(\dfrac{1}{2}\right)^n$

$\Rightarrow \quad 375 = 3000\left(\dfrac{1}{2}\right)^n$

$\Rightarrow \quad \dfrac{1}{8} = \left(\dfrac{1}{2}\right)^n$

$\Rightarrow \quad n =$ number of half lives $= 3$

$\Rightarrow \quad t = 5730 \times 3 = 17190$ yr

Hence, the correct answer is (C).

59. Since $\dfrac{A}{A_0} = \dfrac{3}{5} = \left(\dfrac{1}{2}\right)^{\frac{t}{T}}$

$\Rightarrow \quad \ell n\left(\dfrac{3}{5}\right) = \dfrac{t}{T}\ell n\left(\dfrac{1}{2}\right)$

$\Rightarrow \quad t = \dfrac{T(\ell n5 - \ell n3)}{\ell n2} \approx 4000$ year

Also, we can think that, $\dfrac{3}{5} = 60\%$ and 50% decay takes 5570 year.

So 40% decay takes a little less number of year.
Hence, the correct answer is (D).

60. Since, $\frac{1}{4}=\frac{1}{2^n}$

$\Rightarrow \quad n=2$

$\Rightarrow \quad t=nT_{1/2}=n\left(\frac{T_1\times T_2}{T_1+T_2}\right)=2\left(\frac{1620\times 810}{1620+810}\right)$

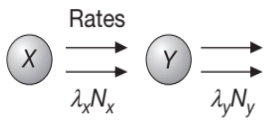
$\Rightarrow \quad t=1080$ yr

Hence, the correct answer is (A).

61. Number of radionuclei become constant, when

$X=\lambda N$

$\Rightarrow \quad N=\frac{x}{\lambda}$



$\Rightarrow \quad N=\frac{X}{\frac{\log_e(2)}{Y}}=\frac{XY}{\log_e(2)}$

Hence, the correct answer is (C).

62. $T=\frac{4\times 12}{16}=3$ year

$\frac{N}{N_0}=\left(\frac{1}{2}\right)^{\frac{12}{3}}=\frac{1}{16}=6.25\%$

Hence, the correct answer is (A).

63. Since decay is to be regarded as a statistical spontaneous process, hence α decay can be regarded as an Adiabatic process.
Hence, the correct answer is (C).

64. $R=R_0A^{1/3}$

$\Rightarrow \quad \log R=\log R_0+\frac{1}{3}\log A$

$\Rightarrow \quad \log\left(\frac{R}{R_0}\right)=\frac{1}{3}\log A$

Hence, the correct answer is (D).

65. Total initial binding energy is 236×7.6

$\Rightarrow \quad U_i=1793.6$ MeV

Total final binding energy is $2(117)(8.5)$

$\Rightarrow \quad U_f=1989$ MeV

$\Rightarrow \quad \Delta U=195.4$ MeV

Hence, the correct answer is (B).

66. $t=0,\ N_0=8\times 10^4$

$N=1\times 10^4$

at $t=?$

Since, $T_{1/2}=3$ year

and $\frac{N}{N_0}=\frac{1\times 10^4}{8\times 10^4}=\frac{1}{8}=\frac{1}{2^n}$

$\Rightarrow \quad n=3$

Since $t=nT_{1/2}$

$\Rightarrow \quad t=3\times 3=9$ year

Hence, the correct answer is (A).

67. $R=R_0e^{-\lambda t}$

$\Rightarrow \quad \lambda=\frac{1}{t}\log_e\left(\frac{R_0}{R}\right)=\frac{2.3}{t}\log_{10}\left(\frac{R_0}{R}\right)$

$\therefore$ Decay constant

$\lambda=\frac{1}{5}\times 2.3\log_{10}\frac{9750}{975}=\frac{2.3}{5}=0.461\ \text{min}^{-1}$

Hence, the correct answer is (C).

68. $N=N_0e^{-\lambda t}$

$\Rightarrow \quad N=N_0e^{-\frac{\ell n2}{T_{1/2}}t}$, where $t=6$ months and $T_{1/2}=1$ year

$\Rightarrow \quad N=N_0e^{-\frac{\ell n2}{2}}=N_0e^{-\ell n\sqrt{2}}$

$\Rightarrow \quad N=\frac{N_0}{\sqrt{2}}$

So, amount decayed is $(N_0-N)=N_0\left(1-\frac{1}{\sqrt{2}}\right)$

$\Rightarrow \quad (N_0-N)=N_0(1-0.707)=0.293N_0$

$\Rightarrow \quad \frac{N_0-N}{N_0}\times 100\%=29.3\%$

So, percentage amount decayed is 29.3%

Hence, the correct answer is (A).

70. $R=\frac{dN}{dt}=\lambda N$

$\Rightarrow \quad \frac{R}{N}=\lambda=$ Decay Constant

Hence, the correct answer is (D).

71. $\frac{N}{N_0}=\left(\frac{1}{2}\right)^4$

$\Rightarrow \frac{N}{N_0}=\frac{1}{16}$

Hence, the correct answer is (D).

72. Decay probability per second i.e. $\left|\frac{dN}{N}\frac{1}{dt}\right|$ is just the decay constant (λ), because $\frac{dN}{dt}=-\lambda N$

$\Rightarrow \left|\frac{dN}{N}\frac{1}{dt}\right|=\lambda$

$\Rightarrow \lambda=\frac{0.693}{t_{1/2}}=\frac{0.693}{2.7\text{ days}}$

$\Rightarrow \lambda=\frac{0.693}{2.7\times24\times60\times60}$

$\Rightarrow \lambda=2.97\times10^{-6}\text{ s}^{-1}$

Hence, the correct answer is (B).

73. Since nuclear density is independent of mass of nucleus, hence all possess equal density.

Hence, the correct answer is (C).

74. Probability that a nucleus will not decay is given by

$$p=\frac{N}{N_0}=\left(\frac{1}{2}\right)^n=\left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

When n is the number of half lives lapsed, so $n=4$

$\Rightarrow p=\frac{N}{N_0}=\left(\frac{1}{2}\right)^4=\frac{1}{16}$

Probability that a nucleus will decay is $q=1-p$

$\Rightarrow q=1-p=1-\frac{1}{16}=\frac{15}{16}$

Hence, the correct answer is (C).

76. For A, $N_A=\frac{N_{0A}}{(2)^{\frac{16}{4}}}=\frac{N_{0A}}{16}=\frac{10^{-2}}{16}$ kg

$\Rightarrow N_A=0.625$ mg

For B, $N_B=\frac{N_{0B}}{(2)^{\frac{16}{8}}}=\frac{N_{0B}}{4}=\frac{10^{-2}}{4}$ kg

$\Rightarrow N_B=2.5$ mg

Hence, the correct answer is (B).

77. If R_0 be the initial activity of the sample, then $R_1=R_0e^{\lambda t_1}$ and $R_2=R_0e^{-\lambda t_2}$

where $\lambda=\frac{1}{T}$ $\{\because \text{Mean life }=T=\frac{1}{\lambda}\}$

$\Rightarrow \frac{R_2}{R_1}=\frac{e^{-\lambda t_2}}{e^{-\lambda t_1}}=e^{\lambda(t_1-t_2)}$

$\Rightarrow R_2=R_1\exp\left(\frac{t_1-t_2}{T}\right)$

Hence, the correct answer is (C).

78. For x, $A_x=A_0\left(\frac{1}{2}\right)^{\frac{48}{24}}=\frac{A_0}{4}$

For y, $A_y=A_0\left(\frac{1}{2}\right)^{\frac{48}{16}}=\frac{A_0}{8}$

When mixed, total activity is

$$A=A_x+A_y=\frac{A_0}{4}+\frac{A_0}{8}=\frac{3A_0}{8}$$

Hence, the correct answer is (D).

79. Rate of decay of sample 1, $R_1=\lambda_1N_1e^{-\lambda_1t}$ and for sample 2, $R_2=\lambda_2N_2e^{-\lambda_2t}$.

When they are mixed, we have $R=R_1R_2$

$\Rightarrow R=\lambda_1N_1e^{-\lambda_1t}\lambda_2N_2e^{-\lambda_2t}$

$\Rightarrow R=\lambda_1N_1\lambda_2N_2e^{-(\lambda_1+\lambda_2)t}$

Hence, the correct answer is (D).

80. Speed of light and γ-rays is c while $v_\beta<c$

Hence, the correct answer is (C).

81. $(T_{1/2})_R=\frac{\log_e(2)}{4.5\times10^{-3}}\text{ yr}^{-1}$

$(T_{1/2})_S=\frac{\log_e(2)}{3\times10^{-3}}\text{ yr}^{-1}$

$\Rightarrow \frac{(T_{1/2})_R}{(T_{1/2})_S}=\frac{2}{3}$

$\Rightarrow 3(T_{1/2})_R=2(T_{1/2})_S$

So, after three half life of R, we see that two half life of S have lapsed.

$$N_R=(N_0)_R\left(\frac{1}{2}\right)^3$$

$$N_S=(N_0)_S\left(\frac{1}{2}\right)^2$$

$$\Rightarrow \frac{N_R}{N_S} = \left[\frac{(N_0)_R}{(N_0)_S}\right]\left(\frac{1}{2}\right)$$

Since $\frac{(N_0)_R}{(N_0)_S} = 2$

$$\Rightarrow \frac{N_R}{N_S} = 2\left(\frac{1}{2}\right) = 1$$

Hence, the correct answer is (C).

82. $240 \to 120 \to 60 \to 30$

Since, 3 half lives = 60 minute

$\Rightarrow$ 1 half life = 20 minute

Hence, the correct answer is (A).

83. Activity $R = \lambda N$

Number of nuclei (N) per mole are equal for both the substances, so we have

$$R \propto \lambda$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{\lambda_1}{\lambda_2} = \frac{4}{3}$$

Hence, the correct answer is (C).

84. $\frac{2}{3} = \left(\frac{1}{2}\right)^{\frac{t_1}{T_{1/2}}}$ ...(1)

$\frac{1}{3} = \left(\frac{1}{2}\right)^{\frac{t_2}{T_{1/2}}}$ ...(2)

Divide (1) by (2) and rearrange, we get

$$2 = 2^{\frac{(t_2 - t_1)}{T_{1/2}}}$$

$$\Rightarrow \frac{t_2 - t_1}{T_{1/2}} = 1$$

$$\Rightarrow \Delta t = T_{1/2} = 20 \text{ min}$$

Hence, the correct answer is (B).

85. Net rate of formation of Y at any time t is

$$\frac{dN_y}{dt} = \lambda_x N_x - \lambda_y N_y$$

N_y is maximum when $\frac{dN_y}{dt} = 0$

$$\Rightarrow \lambda_x N_x = \lambda_y N_y$$

Hence, the correct answer is (D).

86. $\lambda = \lambda_1 + \lambda_2$

$$\Rightarrow \frac{\ln 2}{T} = \frac{\ln 2}{T_1} + \frac{\ln 2}{T_2} \qquad (T = \text{Half life})$$

$$\Rightarrow T = \frac{T_1 T_2}{T_1 + T_2} = 20 \text{ yr}$$

$\frac{1}{4}$th sample remains after 2 half lives or 40 yr

Hence, the correct answer is (C).

88. By Conservation of Linear Momentum, we have

$$p_D = p_\alpha$$

$$\Rightarrow \sqrt{2m_D K_D} = \sqrt{2m_\alpha K_\alpha}$$

$$\Rightarrow K_D = \frac{m_\alpha}{m_D} K_\alpha = \frac{4}{214} \times 6.7 \text{ MeV} = 0.125 \text{ MeV}$$

Hence, the correct answer is (A).

89. Activity of $S_1 = \frac{1}{2}$ (activity of S_2)

$$\Rightarrow \lambda_1 N_1 = \frac{1}{2}(\lambda_2 N_2)$$

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{N_2}{2N_1}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{2N_1}{N_2} \qquad \left\{T = \text{half life} = \frac{\log_e 2}{\lambda}\right\}$$

Given $N_1 = 2N_2$

$$\Rightarrow \frac{T_1}{T_2} = 4$$

Hence, the correct answer is (A).

90. For A, $m_1 = m_0\left(\frac{1}{2}\right)^{\frac{6}{1}} = \frac{m_0}{64}$

For B, $m_2 = m_0\left(\frac{1}{2}\right)^{\frac{6}{2}} = \frac{m_0}{8}$

$$\Rightarrow \frac{A_1}{A_2} = \frac{\lambda_1 N_1}{\lambda_2 N_2} = \left(\frac{T_2}{T_1}\right)\left(\frac{m_1}{m_2}\right)$$

$$\Rightarrow \frac{A_1}{A_2} = \frac{2}{1} \times \frac{m_0/64}{m_0/8} = \frac{16}{64} = \frac{1}{4}$$

Hence, the correct answer is (A).

91. Rest mass of parent nucleus should be greater than the rest mass of daughter nuclei.

Hence, the correct answer is (A).

92. $A_0 = \lambda N_0 = \lambda\left(\frac{m}{M}\right)N_A$

Since $A = A_0 e^{-\lambda t}$

$\Rightarrow \quad A=\left(\dfrac{\lambda m N_A}{M}\right)e^{-\lambda t}$

Hence, the correct answer is (C).

93. Total energy radiated per minute from sun is

$$E_{\text{radiated}}=\sigma\left(4\pi R_{se}^2\right)$$

Energy radiated annually is given by

$$E_{\text{total}}=24\times 60\times 365\times E_{\text{radiated}}$$

$\Rightarrow$ Annual loss of mass $=\Delta m=\dfrac{E_{\text{total}}}{c^2}=1.38\times 10^{17}$ kg

Hence, the correct answer is (C).

94. $\dfrac{N_0}{N}=25\%=\dfrac{1}{4}=\dfrac{1}{2^n}$

$\Rightarrow \quad n=$ Number of half lifes lapsed is 2

$\Rightarrow \quad t=nT_{1/2}$

$\Rightarrow \quad T_{1/2}=\dfrac{t}{n}=\dfrac{16}{2}=8$ day

Hence, the correct answer is (B).

95. Combining two given equations, we have

$$3\,{}_1H^2={}_2He^4+p+n$$

Now, $\quad \Delta m=3\times 2.014-4.001-1.007-1.008$

$\Rightarrow \quad \Delta m=0.026u$

Energy released by 3 deutrons is

$$\Delta E=0.026\times 931.5\times 1.6\times 10^{-13}\text{ J}$$

$\Rightarrow \quad \Delta E=3.9\times 10^{-12}$ J

$\Rightarrow \quad \left(10^{16}\times t\right)=\left(\dfrac{10^{40}}{3}\right)\left(3.9\times 10^{-12}\right)$

Solving we get,

$$t\approx 1.3\times 10^{12}\text{ s}$$

Hence, the correct answer is (C).

96. $\dfrac{m}{m_0}=\dfrac{1\text{ g}}{16\text{ g}}=\left(\dfrac{1}{2}\right)^4=\left(\dfrac{1}{2}\right)^{\frac{t}{T_{1/2}}}$

$\Rightarrow \quad t=4T_{1/2}=4\times 140\text{ day}=560$ day

Hence, the correct answer is (B).

97. $A\longrightarrow B\longrightarrow C$

Let b be the number of nuclei of B, then

$$\frac{dB}{dt}=x-\lambda b$$

$\Rightarrow \quad y=x-\lambda b$

$\Rightarrow \quad \lambda b=x-y$

Activity of B is $A_B=-\lambda b=y-x$

Hence, the correct answer is (B).

98. $\tau=\dfrac{\tau_\alpha\tau_\beta}{\tau_\alpha+\tau_\beta}=\dfrac{1620\times 520}{1620+520}=394$ year

$\Rightarrow \quad \dfrac{1}{\lambda}=394$

Time of decay, $t=\dfrac{2.303}{\lambda}\log_{10}\left(\dfrac{N_0}{N}\right)$

$\Rightarrow \quad t=2.303(394)\left(\log_{10}(4)\right)$

$\Rightarrow \quad t=540$ year

Hence, the correct answer is (A).

99. $f=\dfrac{\Delta m}{A}=\dfrac{M-A}{A}$

Hence, the correct answer is (D).

100. Since $N=N_0e^{-\lambda t}$

$\Rightarrow \quad P=\dfrac{N}{N_0}=e^{-\lambda t}$

Hence, the correct answer is (A).

102. Since $\dfrac{N}{N_0}=\dfrac{1}{64}=\dfrac{1}{2^n}$

$\Rightarrow \quad n=\dfrac{t}{T_{1/2}}=6$

$\Rightarrow \quad t=nT_{1/2}$

$\Rightarrow \quad T_{1/2}=\dfrac{t}{n}=\dfrac{60}{6}=10$ sec

Hence, the correct answer is (B).

103. $\dfrac{N}{N_0}=\left(\dfrac{1}{2}\right)^n=\left(\dfrac{1}{2}\right)^{\frac{1}{2}}=\dfrac{1}{\sqrt{2}}$

Hence, the correct answer is (C).

104. When 75% decays, 25% is left undecayed. This required a time $t=2T_{1/2}$, where $T_{1/2}=\dfrac{\ell n2}{\lambda}$ and also, $\lambda=\dfrac{1}{T}$

$\Rightarrow \quad t=2\left(\dfrac{\ell n2}{\lambda}\right)=2T\ell n(2)$

Hence, the correct answer is (D).

105. $E=mc^2$

$\Rightarrow \quad E=\left(10^{-6}\right)\left(9\times 10^{16}\right)$

$\Rightarrow \quad E=9\times 10^{10}$ J

Hence, the correct answer is (A).

106. $\frac{A}{A_0}=\frac{128}{1024}=\frac{1}{8}=\left(\frac{1}{2}\right)^3=\left(\frac{1}{2}\right)^{\frac{3\text{ min}}{T_{1/2}}}$

$\Rightarrow T_{1/2}=1\text{ min}$

$\Rightarrow A'=A_0\left(\frac{1}{2}\right)^{\frac{5}{1}}=\frac{A_0}{32}=\frac{1024}{32}=32$

Hence, the correct answer is (D).

107. Since $\frac{N}{N_0}=\left(\frac{1}{2}\right)^n$

where $n=\frac{\text{Time lapsed}}{\text{Half life}}$

According to the problem $n=\frac{\frac{T_{1/2}}{2}}{T_{1/2}}=\frac{1}{2}$

$\Rightarrow \frac{N}{N_0}=\left(\frac{1}{2}\right)^{1/2}=\frac{1}{\sqrt{2}}$

Hence, the correct answer is (C).

108. Energy released is $\Delta E=(BE)_{\text{final}}-(BE)_{\text{initial}}$

$\Rightarrow \Delta E=(7.0-1.1)4$

$\Rightarrow \Delta E=23.6\text{ MeV}$

Hence, the correct answer is (B).

110. $\frac{\lambda_1}{\lambda_2}=\frac{\lambda_1 N}{\lambda_2 N}$

$\Rightarrow \frac{\lambda_1}{\lambda_2}=\frac{\text{decay rate of }\alpha\text{ decay}}{\text{decay rate of }\beta\text{ decay}}$

$\Rightarrow \frac{\lambda_1}{\lambda_2}=\frac{\text{probability of }\alpha\text{ decay}}{\text{probability of }\beta\text{ decay}}$

$\Rightarrow \frac{\lambda_1}{\lambda_2}=\frac{\frac{75}{100}}{\frac{25}{100}}=3$

Hence, the correct answer is (A).

111. ${}_yA^x \rightarrow {}_{y-2}B^{x-4}+{}_2\text{He}^4$

${}_{y-2}B^{x-4}\rightarrow {}_yC^{x-4}+2\,{}_{-1}e^0$

Hence, the correct answer is (C).

112. $\Delta A=235-219=16$

$\Rightarrow$ Number of α-particles emitted $=\frac{16}{4}=4$

Hence, the correct answer is (A).

113. $\frac{N_0}{16}=N_0\left(\frac{1}{2}\right)^n$

$\Rightarrow n=4$

So, $3t$ times is equivalent to four half lives. Hence one half life is equal to $\frac{3t}{4}$.

The given time $\frac{11}{2}t-t=\frac{9}{2}t=6\left(\frac{3t}{4}\right)$ is equivalent to 6 half lives

$\Rightarrow N=N_0\left(\frac{1}{2}\right)^6=\frac{N_0}{64}$

Hence, the correct answer is (B).

114. $A=A_0e^{-\lambda t}$

$\Rightarrow 100=800e^{-\lambda(6\times 60)}$

$\Rightarrow e^{-360\lambda}=\frac{1}{8}$

$\Rightarrow -360\lambda=\ell n\left(\frac{1}{8}\right)=-\ell n 8$

$\Rightarrow \lambda=\frac{\ell n(2^3)}{360}=\frac{\ell n 2}{120}$

$\Rightarrow T_{\frac{1}{2}}=\frac{\ell n 2}{\lambda}=\frac{\ell n 2}{\left(\frac{\ell n 2}{120}\right)}=120\text{ min}$

$\Rightarrow T_{\frac{1}{2}}=2\text{ hrs}$

Hence, the correct answer is (B).

115. Charge number decreases by 4, but actually it must decrease by 8 (due to emission of 4α-particles). Hence 4β particles must have also been emitted.

Hence, the correct answer is (A).

116. ${}_{86}A^{222}\longrightarrow {}_{84}B^{210}$

$\Rightarrow n_\alpha=\frac{A-A'}{4}=\frac{222-210}{4}=3$

and $n_\beta=2n_\alpha-Z+Z'$

$\Rightarrow n_\beta=2\times 3-86+84$

$\Rightarrow n_\beta=4$

Hence, the correct answer is (B).

117. $N=N_0\left(\frac{1}{2}\right)^{\frac{15}{5}}=\frac{N_0}{8}$

So, amount of substance undecayed is

$\left(N_0-\frac{N_0}{8}\right)=\frac{7N_0}{8}$

Hence, the correct answer is (C).

118. $\dfrac{dN}{dt}=t^2-\lambda N$

For $\dfrac{dN}{dt}$ to be minimum, we have $\dfrac{d^2N}{dt^2}=0$

$$\Rightarrow \left.\frac{d^2N}{dt^2}\right|_{t=t_0}=2t-\lambda\frac{dN}{dt}=2t_0-\lambda\left(t_0^2-\lambda N\right)=0$$

$$\Rightarrow N=\frac{2t_0-\lambda t_0^2}{\lambda^2}$$

Hence, the correct answer is (A).

119. Probability of decay is $p=1-\dfrac{N}{N_0}=1-\left(\dfrac{1}{2}\right)^n$

where $n=\dfrac{\text{Time lapsed}}{T_{1/2}}=\dfrac{10}{5}=2$

$$\Rightarrow p=1-\frac{1}{2^2}=\frac{3}{4}=75\%$$

Hence, the correct answer is (B).

120. $A_P=A_0e^{-\lambda t_1}$ ...(1)

$A_Q=A_0e^{-\lambda t_2}$ ...(2)

From (1), $\lambda_{t_1}=\ln\left(\dfrac{A_0}{A_P}\right)$

$$\Rightarrow t_1=\frac{1}{\lambda}\ln\left(\frac{A_0}{A_P}\right)=T\ln\left(\frac{A_0}{A_P}\right)$$

Similarly, from (2), $t_2=T\ln\left(\dfrac{A_0}{A_Q}\right)$

$$\Rightarrow t_1-t_2=T\ln\left(\frac{A_Q}{A_P}\right)$$

Hence, the correct answer is (B).

121. $\dfrac{4}{3}\pi R^3\propto A$

$$\Rightarrow R\propto A^{\frac{1}{3}}$$

Hence, the correct answer is (B).

122. After $t=9$ year

$$\frac{A_0}{3}=A_0(2)^{-\frac{9}{T}}$$

$$\Rightarrow T=9\frac{\log(2)}{\log(3)}$$

Nine years further i.e. at $t=18$ year, we have

$$A=A_0(2)^{-\frac{18}{T}}$$

$$\Rightarrow 18\left[\frac{\log(2)}{\log\left(\frac{A_0}{A}\right)}\right]=9\left[\frac{\log(2)}{\log(3)}\right]$$

$$\Rightarrow \left(\frac{A_0}{A}\right)=(9)$$

$$\Rightarrow A=\frac{A_0}{9}$$

Hence, the correct answer is (C).

123. Power of Sun is

$$P_S=1.4\times10^3\times4\pi\left(1.5\times10^{11}\right)^2$$

energy released by Sun per day is

$$E_{\text{sun/day}}=P_S\times86400$$

Mass lost by sun per day is

$$\Delta m=\frac{E_{\text{sun/day}}}{c^2}$$

$$\Rightarrow \Delta m=\frac{1.4\times10^3\times4\times3.14\times\left(1.5\times10^{11}\right)^2\times86400}{\left(3\times10^8\right)^2}$$

$$\Rightarrow m=8.79\times10^{14}\text{ kg}$$

Hence, the correct answer is (D).

124. The nuclear force of interaction between any pair of nucleons is identical i.e. force between two neutrons (F_2) equals the force between neutron and proton (F_3). However, between two protons net force is equal to the resultant of nuclear force between them (attractive in nature) and electrostatic force between them (repulsive in nature). Hence F_1 is a value lesser than F_2 and F_3. So $F_1<F_2=F_3$.

Hence, the correct answer is (C).

125. $N=N_0e^{-\lambda t}$

$$\Rightarrow D=N_0-N=N_0\left(1-e^{-\lambda t}\right)$$

Since $R=R_0e^{-\lambda t}$

$$\Rightarrow \frac{R}{N}=\frac{R_0e^{-\lambda t}}{N_0e^{-\lambda t}}=\frac{R_0}{N_0}=\lambda=\text{constant}$$

Hence, the correct answer is (D).

126.

	A	B
$T_{1/2}$	1 year	2 year
M_0	10 g	1 g
$t = ?$	M	M

For A, $M = \frac{10}{2^{t/1}}$ ...(1)

For B, $M = \frac{1}{2^{t/2}}$ ...(2)

Equating (1) and (2), we get

$$\frac{10}{2^t} = \frac{1}{2^{t/2}}$$

$$\Rightarrow \quad 2^{t/2} = 10$$

Taking log both sides, we get

$$\log_{10}(2^{t/2}) = \log_{10} 10$$

$$\Rightarrow \quad \frac{t}{2}\log_{10}(2) = \log_{10}(10) = 1$$

$$\Rightarrow \quad t = \frac{2}{0.3010} = 6.62 \text{ year}$$

Hence, the correct answer is (A).

128. From given graph we have

$$\ell nA = -\left(\frac{2.5}{25}\right)t + 2.5$$

$$\Rightarrow \quad A = e^{\left(-\frac{t}{10}+2.5\right)}$$

$$\Rightarrow \quad A = e^{2.5}e^{-0.1t}$$

$$\Rightarrow \quad A = 12e^{-0.1t}$$

Hence, the correct answer is (D).

129. $\frac{dN}{dt} = n - \lambda N$

Because the population N is simultaneously increasing at rate n and decreasing due to decay at rate λN.

$$\Rightarrow \quad \int_{N_0}^{N} \frac{dN}{n - \lambda N} = \int_0^t dt$$

$$\Rightarrow \quad \frac{1}{\lambda}\ell n\left(\frac{n - \lambda N_0}{n - \lambda N}\right) = t$$

$$\Rightarrow \quad N = \frac{n}{\lambda} + \left(N_0 - \frac{n}{\lambda}\right)e^{-\lambda t}$$

OBJECTIVE TRICK

At $t = 0$, $N = N_0$ which is satisfied by **(C)** only.

Hence, the correct answer is (C).

130. Since $\frac{N}{N_0} = \frac{1}{2^n}$, where $n = \frac{t}{T_{1/2}} = \frac{1}{2}$

$$\Rightarrow \quad \frac{N}{10000} = \frac{1}{(2)^{\frac{10}{20}}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad N = 7070$$

Hence, the correct answer is (A).

131. From momentum conservation,

$$p_{\text{photon}} = p_{\text{nucleus}}$$

$$\Rightarrow \quad \frac{E}{c} = \sqrt{2mK}$$

$$\Rightarrow \quad K = \frac{E^2}{2mc^2}$$

$$\Rightarrow \quad K = \frac{(7 \times 1.6 \times 10^{-13})^2}{2(24)(1.67 \times 10^{-27})(3 \times 10^8)^2}(1.6 \times 10^{-16})\text{keV}$$

$$\Rightarrow \quad K = 1.1 \text{ keV}$$

Hence, the correct answer is (B).

132. Mass of product should be less than the mass of reactant because energy is released in a fission reaction.

Hence, the correct answer is (A).

133. $R_1 = R_2$

$$\Rightarrow \quad R_{01}e^{-\lambda_1 t} = R_{02}e^{-\lambda_2 t}$$

$$\Rightarrow \quad \lambda_1 N_0 e^{-\lambda_1 t} = \lambda_2 N_0 e^{-\lambda_2 t}$$

where, $\lambda_1 = \frac{\ln 2}{t_1} = \frac{0.693}{t_1}$

and $\lambda_2 = \frac{0.693}{t_2}$

Substituting these values of λ_1 and λ_2 in Equation (1), we get

$$t = \frac{t_1 t_2}{0.693(t_2 - t_1)}\ln\left(\frac{t_2}{t_1}\right)$$

Hence, the correct answer is (A).

134. $R_1 = \lambda N_1$ and $R_2 = \lambda N_2$ (λ is same for a given sample)

As $N_2 < N_1$

Number of atoms disintegrated in time $(t_2 - t_1)$ is

$$N_1 - N_2 = \frac{R_1}{\lambda} - \frac{R_2}{\lambda} = \left(\frac{R_1 - R_2}{\lambda}\right)$$

$$N_1 - N_2 = \frac{(R_1 - R_2)T}{0.693} \propto (R_1 - R_2)T$$

Hence, the correct answer is (D).

138. Expected mass of Cu must be less than that of zinc. So it is unstable and radioactive, decaying to Zn through β decay.

Hence, the correct answer is (D).

139. f_1 is the fraction decayed in one half life $T_{1/2}$ and f_2 is the fraction decayed in one mean life (T).

$$T_{1/2} = \frac{0.693}{\lambda} = 0.693T$$

$\Rightarrow \quad T > T_{1/2}$

Further, since the fraction decayed is $(1 - e^{-\lambda t})$

So, $f_1 = 1 - e^{-\lambda T_{1/2}} = (1 - e^{-0.693}) = \frac{1}{2}$ and

$$f_2 = 1 - e^{-\lambda T} = 1 - e^{-1} = 0.632$$

$\Rightarrow \quad f_2 > f_1$

However, if we had been asked about the fraction of sample undecayed, then we must give our answer in the light of formula $\frac{N}{N_0} = e^{-\lambda t}$ where N is number of undecayed nuclei in sample at time t.

Hence, the correct answer is (B).

142. Q-value = Final binding energy

Initial binding energy is

$$(BE)_{\text{initial}} = E_2N_2 + E_3N_3 - E_1N_1$$

Hence, the correct answer is (B).

143. $t = \frac{0.693}{\lambda}$

$T = \frac{1}{\lambda}$

$\Rightarrow \quad t = 0.693$ T

$\Rightarrow \quad T > t$

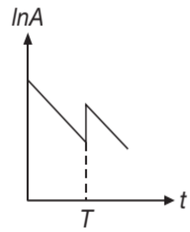
Hence, the correct answer is (C).

145. $80\% \xrightarrow{T} 40\% \xrightarrow{T} 20\%$

$\Rightarrow \quad t = 2T = 40$ minutes

Hence, the correct answer is (A).

146. $A = A_0 e^{-\lambda t}$



Taking log both sides, we get

$$\ell n A = \ell n A_0 - \lambda t$$

Hence, the correct answer is (B).

147. Activity of a radioactive substance $R = \lambda N$

$\Rightarrow \quad \lambda = \frac{R}{N}$

Since, $R = N_2$ particles per second

and $N = N_1$

$\Rightarrow \quad \lambda = \frac{N_2}{N_1}$

Hence, the correct answer is (A).

148. ${}_{92}U^{238} \xrightarrow{\alpha} {}_{90}X^{234} \xrightarrow{\beta^-} {}_{91}Y^{234}$

$\Rightarrow \quad Z = 91,\ A = 234$

Hence, the correct answer is (D).

149. Radioactivity is a probable phenomenon that is not influenced by physical conditions like temperature, pressure, humidity, electronic configuration etc.

Hence, the correct answer is (D).

150. Since, $\tau = \frac{\tau_1\tau_2}{\tau_1 + \tau_2} = \frac{1620 \times 660}{2280} = 469$

$\Rightarrow \quad T_{1/2} = \frac{0.693}{\lambda} = 0.693\tau$

$\Rightarrow \quad N = \frac{N_0}{4} = \frac{N_0}{2^{t/T_{1/2}}}$

$\Rightarrow \quad t = 2T_{1/2} = 2(0.693)(469)$

$\Rightarrow \quad t = 650$ year

Hence, the correct answer is (C).

151. During fusion, binding energy of daughter nucleus is always greater than the total binding energy of the parent nuclei. The difference of binding energies is the energy released. Hence,

$$Q = E_2 - 2E_1$$

Hence, the correct answer is (C).

152. Train moving with uniform speed implies that $F_{ext} = 0$

$\Rightarrow \quad x' = x = (v_{234} + v_\alpha)t$

Hence, the correct answer is (C).

153. Maximum number of nuclei will be present when

Rate of decay = Rate of formation

$\Rightarrow \quad \lambda N = \alpha$

$\Rightarrow \quad N = \dfrac{\alpha}{\lambda}$

Hence, the correct answer is (A).

154. At time $t = 0$, A gets converted to B which has a higher activity. Therefore, activity increases.

Hence, the correct answer is (B).

155. $P(\text{survival}) = \dfrac{N(t)}{N_0} = \dfrac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$

For, $t = \dfrac{1}{\lambda}$, we have

$P(\text{survival}) = \dfrac{1}{e}$

Hence, the correct answer is (B).

156. Energy released is given by

$$\Delta E = \begin{pmatrix}\text{Total Binding} \\ \text{Energy of } {}_2He^4\end{pmatrix} - 2\begin{pmatrix}\text{Total Binding} \\ \text{Energy of } {}_1H^2\end{pmatrix}$$

$\Rightarrow \quad \Delta E = (4)(7) - 2(1.1)(2) = 23.6 \text{ MeV}$

Hence, the correct answer is (D).

157. Let $\lambda_A = \lambda$ and $\lambda_B = 2\lambda$

Initially rate of disintegration of A is λN_0 and that of B is $(2\lambda)N_0$.

After one half-life of A, rate of disintegration of A will becomes $\dfrac{\lambda N_0}{2}$ and that of B would also be $\dfrac{\lambda N_0}{2}$, because

$(\text{Half-life of B}) = \dfrac{1}{2}(\text{Half-life of A})$

So, after one half-life of A or two half-lives of B, we have

$$\left(-\frac{dN}{dt}\right)_A = \left(-\frac{dN}{dt}\right)_B$$

$\Rightarrow \quad n = 1$

Hence, the correct answer is (D).

158. On the last day we have 100% decay i.e. on the ninth day 50% decay must be there or 50% must be left.

Hence, the correct answer is (D).

160. In a beta decay process, mass number cannot increase.

Hence, the correct answer is (C).

164. The mean free path in vacuum is the distance travelled by a neutron in its lifetime i.e., from generation to decay which is about 10^3 s. Since, its energy i.e. 5 eV is much less than its rest energy 940 MeV so, non-relativistic approximation can be used and hence its velocity is given by using the formula for kinetic energy i.e.

$E_K = \dfrac{1}{2}mv^2$

$\Rightarrow \quad v = \sqrt{\dfrac{2E_K}{m}} = c\sqrt{\dfrac{2E_K}{mc^2}} = \sqrt{\dfrac{2 \times 5 \times 10^{-6}}{940}} \times 3 \times 10^8$

$\Rightarrow \quad v = 10^4 \text{ ms}^{-1}$

$\Rightarrow \quad x = vt = 10^4 \text{ km}$

Hence, the correct answer is (D).

165. $N = N_0 e^{-\lambda t}$

For $t = T_{av} = \dfrac{1}{\lambda}$, we have

$N = \dfrac{N_0}{e}$

Hence, the correct answer is (B).

167. $\dfrac{m}{t} = 4 \times 10^9 \text{ kgs}^{-1}$

$E = mc^2$

$\Rightarrow \quad \dfrac{E}{t} = \left(\dfrac{m}{t}\right)c^2$

$\Rightarrow \quad \dfrac{E}{t} = 4 \times 10^9 \times 9 \times 10^{16}$

$\Rightarrow \quad \dfrac{E}{t} = 3.6 \times 10^{26} \text{ Js}^{-1}$

$\Rightarrow \quad \dfrac{E}{t} = 3.6 \times 10^{26} \text{ W}$.

Hence, the correct answer is (A).

168. Activity $= \left|\dfrac{dN}{dt}\right| = \lambda N = \lambda\left(\dfrac{m}{M}\right)L$

Hence, the correct answer is (B).

CHAPTER 3

169. After time t, sample left undecayed is 90% i.e. $0.9N_0$

After time $2t$, sample left undecayed is

$$N = N_0(0.9)^2$$

$$\Rightarrow \quad N = 0.81N_0$$

So, 81% of initial value is left

Hence percentage of the initial sample decayed is $(100-81) = 19\%$

Hence, the correct answer is (B).

170. $$92 \xrightarrow{\alpha} 90 \xrightarrow{2\beta^-} 92 \xrightarrow{5\alpha} 82 \xrightarrow{2\beta^-} 84 \xrightarrow{\alpha}$$

$$82 \xrightarrow{2\beta^+} 80 \xrightarrow{\alpha} 78$$

Hence, the correct answer is (C).

171. Since $\dfrac{A}{A_0} = \dfrac{1}{128} = \dfrac{1}{2^n}$

$$\Rightarrow \quad n = 7$$

Since $t = nT_{1/2}$

$$\Rightarrow \quad t = 7 \times 1 = 7 \text{ hr}$$

Hence, the correct answer is (B).

173. $${}_1H^2 + {}_1H^2 \longrightarrow {}_2He^4 + Q$$

$$\Rightarrow \quad \Delta m = m({}_2He^4) - 2m({}_1H^2)$$

$$\Rightarrow \quad \Delta m = 4.0024 - 2(2.0141)$$

$$\Rightarrow \quad \Delta m = -0.0258 \text{ u}$$

Since, $Q = c^2\Delta m$

$$\Rightarrow \quad Q = (0.0258)(931.5) \text{ MeV}$$

$$\Rightarrow \quad Q \simeq 24 \text{ MeV}$$

Hence, the correct answer is (C).

174. Since, $\dfrac{dN}{dt} = \lambda N$

$$\Rightarrow \quad \lambda dt = \frac{dN}{N}$$

Hence, the correct answer is (A).

175. Let N_0 be the number of atoms of X at time $t = 0$. Then at $t = 4$ h (two half lives), we have

$$N_x = \frac{N_0}{4} \text{ and } N_y = \frac{3N_0}{4}$$

$$\Rightarrow \quad \frac{N_x}{N_y} = \frac{1}{3}$$

and at $t = 6h$ (three half-lives), we have

$$N_x = \frac{N_0}{8} \text{ and } N_y = \frac{7N_0}{8}$$

$$\Rightarrow \quad \frac{N_x}{N_y} = \frac{1}{7}$$

The given ratio $\dfrac{1}{4}$ lies between $\dfrac{1}{3}$ and $\dfrac{1}{7}$

Therefore, t lies between 4 h and 6 h

Hence, the correct answer is (C).

176. $$\frac{N}{N_0} = \frac{1}{8} = \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{\frac{6}{T}}$$

$$\Rightarrow \quad T = \frac{6}{3} = 2 \text{ days}$$

$$\frac{N'}{N_0} = \left(\frac{1}{2}\right)^{\frac{10}{2}} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

$$\Rightarrow \quad \text{Fraction Decayed} = 1 - \frac{N'}{N_0} = 1 - \frac{1}{32} = \frac{31}{32}$$

Hence, the correct answer is (C).

177. $${}^A_Z X \xrightarrow{\alpha} {}^{A-4}_{Z-2} Y \xrightarrow{2\beta^-} {}^{A-4}_{Z} X$$

Hence, the correct answer is (B).

178. In K-capture process, the nucleus captures the K shell electron. So,

$${}^A_Z X + {}_{-1}e^0 \rightarrow {}^A_{Z-1} Y, \text{ where } {}_{-1}X^0 \text{ is } {}_{-1}e^0$$

Hence, the correct answer is (A).

179. A and B can be isotopes if number of β-decays is two times the number of α-decays.

Hence, the correct answer is (B).

180. $N = N_0 e^{-\lambda t}$

Number of nuclei decayed $= (N_0 - N)$

Percentage of initial nuclei decayed is

$$\%\text{age} = \frac{N_0 - N}{N_0} \times 100 = \left(1 - \frac{N}{N_0}\right) \times 100$$

$$\%\text{age} = (1 - e^{-\lambda t}) \times 100 = (1 - 0.37) \times 100$$

$$\%\text{age} = 63\%$$

Hence, the correct answer is (C).

181. In $2s$ only 90% nuclei are left behind. So, in the next $2s$ 90% of 900 i.e., 810 nuclei will be left.

Hence, the correct answer is (D).

182. Let number of α-decays be x and number of β-decays be y. Then

$$92-2x+y=85$$

$$\Rightarrow \quad 2x-y=7 \qquad \ldots(1)$$

and $238-4x=210$

$$\Rightarrow \quad x=7$$

Substituting this value in equation (1), we get

$$y=7$$

Hence, the correct answer is (C).

Multiple Correct Choice Type Questions

1. We have 6.25% left undecayed

$$\Rightarrow \frac{N}{N_0}=\frac{6.25}{100}=\frac{1}{16}$$

$$\Rightarrow \frac{N}{N_0}=\left(\frac{1}{2}\right)^n=\frac{1}{16}$$

$$\Rightarrow n=\frac{t}{T_{1/2}}=4$$

$$\Rightarrow T_{1/2}=\frac{t}{4}$$

$$\Rightarrow T_{1/2}=\frac{4}{4}=1\text{ hr}$$

Since $T_{1/2}=\frac{\ln 2}{\lambda}$ and mean life is $T_m=\frac{1}{\lambda}$

After further 4 hours, the amount left over would be

$$\frac{N}{N_0}=\left(\frac{1}{2}\right)^8=0.39\%$$

Hence, (A), (B), (C) and (D) are correct.

2. By Law of Conservation of energy, we get

$$m_xc^2+K_x=m_Yc^2+K_Y+m_yc^2+K_y$$

$$Q=\left(m_xc^2-m_Yc^2-m_yc^2\right)=K_y+K_Y-K_X$$

Since, X is at rest, so the energy released Q is

$$Q=\left(m_xc^2-m_Yc^2-m_yc^2\right)=K_Y+K_y$$

The Q value i.e., energy released is the rest energy of X minus rest energy of Y and y.
When binding energy of products is more than binding energy of parent nucleus, then energy is released.

Hence, (B) and (C) are correct.

4. $\frac{N}{N_0}$ is the fraction of nuclei that will not decay

$\left(1-\frac{N}{N_0}\right)$ is the fraction of nuclei that will decay

$$\Rightarrow 1-\frac{N}{N_0}=1-e^{-\lambda t}=\left(\begin{array}{l}\text{Probability that a}\\ \text{nucleus will decay}\end{array}\right)$$

Also, $\frac{N}{N_0}=\left(\frac{1}{2}\right)^n$, where $n=\frac{t}{T_{1/2}}$

$$\Rightarrow \frac{N}{N_0}=\left(\frac{1}{2}\right)^4=\frac{1}{16}$$

$$\Rightarrow 1-\frac{N}{N_0}=1-\frac{1}{16}=\frac{15}{16}$$

Probability that a nucleus will decay is

$$\frac{N}{N_0}=\left(\frac{1}{2}\right)^2=\frac{1}{4}$$

So, fraction of nuclei that will remain after two half lives is $\frac{1}{4}$ or 25%

Hence, (A), (B) and (D) are correct.

6. For $^{20}_{10}\text{Ne}$ nucleus to exist

$$M_1<10\,m_p+10\,m_n$$

$$M_1<10\left(m_p+m_n\right)$$

Further since $^{40}_{20}\text{Ca}$ has 20 more nucleons and thus it requires more energy to hold all of them together and hence $M_1\neq 2M_2$ instead $M_1>2M_2$ because some additional mass defect must occur to provide an additional B.E. to $^{40}_{20}\text{Ca}$ nucleus.

Hence, (C) and (D) are correct.

7. $R=R_0A^{\frac{1}{3}}$

$$\Rightarrow R\propto A^{\frac{1}{3}}$$

Hence, (B) and (C) are correct.

9. $N=N_0e^{-\lambda t}$

$N=$ Number of undecayed nuclei in the sample at time t.

Total number of undecayed nuclei equals $\left(N_0-N\right)$

$$\Rightarrow \left(N_0-N\right)=N_0\left(1-e^{-\lambda t}\right)$$

which is growing exponentially with time.

Activity $R = -\lambda N = \frac{dN}{dt}$

Hence, (A), (B) and (D) are correct.

11. **(B)** $R \propto \sqrt[3]{A}$

$\Rightarrow$ Volume $\propto A$

$\Rightarrow$ Mass $\propto A$

$\Rightarrow$ Density is independent of mass number A

(D) $v \propto \frac{1}{n}$

$r \propto n^2$

$\Rightarrow \frac{v^2}{r} \propto \frac{1}{n^4}$

Hence, (B) and (C) are correct.

13. α **decay:** ${}_{Z}^{A}X \rightarrow {}_{Z-2}^{A-4}\gamma + {}_{2}^{4}\text{He}$

β^+ **decay:** ${}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}\gamma + \beta^+ + \nu$

β^- **decay:** ${}_{Z}^{A}X \rightarrow {}_{Z+1}^{A}\gamma + \beta^- + \bar{\nu}$

In γ decay process, only the quantum state of nucleons change

Hence, (A) and (B) are correct.

15. Due to the emission of an α-particle, atomic number decreases by 2 and due to the emission of two beta particles atomic number increases by 2. Hence net atomic number remains unchanged.

Hence, (A), (B) and (C) are correct.

16. Since Bohr's radius is given by

$$r = \frac{n^2 h^2 \varepsilon_0}{\pi m e^2 Z}$$

$\Rightarrow r \propto \frac{1}{m}$

$\Rightarrow \frac{r}{r_0} = \frac{m_e}{m_\mu}$

$\Rightarrow r = \frac{r_0}{212} = \frac{0.53\ \text{Å}}{212} = 250\ \text{fm}$

Since, $F \propto m$

$\Rightarrow E = E_0\left(\frac{m_\mu}{m_e}\right)$

$\Rightarrow E = (13.6\ \text{eV})(212)$

$\Rightarrow E = 2883\ \text{eV}$

Angular momentum is given as

$$L = mvr = \frac{nh}{2\pi}$$

For ground state $n = 1$, so we have

$$L = \frac{h}{2\pi}$$

Hence, (A), (B) and (C) are correct.

18. $y = \lambda x = \left(\frac{\ln 2}{T}\right) \cdot x$

$\Rightarrow \frac{x}{y} = \frac{1}{\lambda} = \text{constant}$

$\Rightarrow \frac{x}{y} = \frac{T}{\ln 2}$

$\Rightarrow \frac{x}{y} > T$ $\quad \{\text{as } \ln 2 = 0.693\}$

Further, $xy = x(\lambda x) = \lambda x^2$

After one half life, x remains half. Hence, x^2 remains $\frac{1}{4}$th.

Hence, (A), (B) and (D) are correct.

19. ${}_{7}^{14}\text{N} + {}_{0}^{1}n \longrightarrow {}_{3}^{7}\text{Li}$

Difference in mass number $= 8$

Difference in charge number $= 4$

(A) ${}_{2}^{14}\text{N} + {}_{0}^{1}n \longrightarrow {}_{3}^{7}\text{Li} + 4\,{}_{1}^{1}\text{H} + 4\,{}_{0}^{1}n$

This reaction is balanced hence (A) is correct.

(B) ${}_{7}^{14}\text{N} + {}_{0}^{1}n \longrightarrow {}_{3}^{7}\text{Li} + 5\,{}_{1}^{1}\text{H} + 1\,{}_{-1}^{0}e$

This reaction is not balanced

(C) ${}_{7}^{14}\text{N} + {}_{0}^{1}n \longrightarrow 2\,{}_{2}^{4}\text{He} + 2\gamma + {}_{3}^{7}\text{Li}$

This reaction is balanced

(D) ${}_{7}^{14}\text{N} + {}_{0}^{1}n \longrightarrow {}_{3}^{7}\text{Li} + {}_{2}^{4}\text{He} + 4\,{}_{1}^{1}\text{H} + 2\,{}_{-1}^{0}\beta$

This reaction is balanced.

Hence, (A), (C) and (D) are correct.

21. $Q = \left[M(\text{Po}^{210}) - M(\text{Pb}^{206}) - M(\alpha)\right]931\ \text{MeV}$ and $\lambda = \frac{0.693}{T_{1/2}}$

Hence, (A), (B) and (C) are correct.

23. In a beta decay process, a neutron decays to a proton

$${}_0^1n \rightarrow p^+ + e^-$$

Spin of p^+, e^- and ${}_0^1n$ is $\frac{1}{2}$

Therefore spin (R.H.S.) is either 0 or 1 whereas spin (L.H.S.) is $\frac{1}{2}$

Since, Spin (R.H.S.) $\neq$ Spin (L.H.S.)

Hence spin angular momentum is not conserved.

However, total energy, mass number and charge is conserved in the process.

Hence, (A), (B) and (C) are correct.

25. **In ground frame**

Kinetic energy, $K_x = |-Q|\left[1 + \frac{m_{\text{projectile}}}{m_{\text{target}}}\right]$

$\Rightarrow \quad K_x > Q$

In centre of mass frame

Kinetic energy, $K_x = Q$

In nuclear reactions, linear momentum is conserved.

Hence, (A), (B) and (C) are correct.

26. In fusion two or more lighter nuclei combine to make a comparatively heavier nucleus.

In fission, a heavy nucleus breaks into two or more comparatively lighter nuclei.

Further, energy will be released in a nuclear process if total binding energy increases.

Hence, (B) and (D) are correct.

27. At $t = 4T$

Number of half lives of first is $n_1 = 4$ and number of half lives of second is $n_2 = 2$

Since $N = N_0\left(\frac{1}{2}\right)^n$

$$\Rightarrow \quad x = \frac{N_1}{N_2} = \frac{N_0\left(\frac{1}{2}\right)^4}{N_0\left(\frac{1}{2}\right)^2} = \frac{1}{4}$$

Since $R = R_0\left(\frac{1}{2}\right)^n = \lambda N_0\left(\frac{1}{2}\right)^n$

$$\Rightarrow \quad y = \frac{R_1}{R_2} = \frac{\lambda_1 N_0\left(\frac{1}{2}\right)^4}{\lambda_2 N_0\left(\frac{1}{2}\right)^2}$$

$$\Rightarrow \quad y = \frac{\lambda_1}{4\lambda_2} = \frac{T_2}{4T_1}$$

$$\Rightarrow \quad y = \frac{2T}{4T} = \frac{1}{2}$$

Hence, (B) and (C) are correct.

Reasoning Based Questions

1. $(1\text{ amu})c^2 = 931.48$ MeV

$\Rightarrow \quad 1\text{ amu} = 931.5\ \text{MeV}/c^2$

Hence, the correct answer is (D).

3. By the emission of one α-particle atomic number decreases by 2 and mass number by 4. But by the emission of one β-particle, atomic number increases by 1 and mass number remains unchanged.

Hence, the correct answer is (B).

5. α-particles are heaviest. Hence, their ionising power is maximum.

Hence, the correct answer is (B).

6. In moving from lower energy state to higher energy state, electromagnetic waves are absorbed.

Hence, the correct answer is (C).

8. Antineutrino is also produced during β-decay.

Hence, the correct answer is (D).

10. After 200 days the number of un-decayed nuclei in the sample will be $\frac{1}{4}$ the initial number of un-decayed nuclei in the sample initially.

Hence, the correct answer is (D).

12. In binding energy per nucleon versus mass number graph, the binding energy per nucleon of daughter nuclei should increase (for release of energy) or, the daughter nuclei should lie towards the peak of the graph.

Hence, (A) and (B) are correct.

15. A very large amount of energy is involved in any nuclear process, which cannot be increased or decreased by pressure or temperature.

    Hence, the correct answer is (B).

17. Total binding energy per nucleon is more important for stability.

    Hence, the correct answer is (D).

19. Some lighter nuclei are also radioactive.

    Hence, the correct answer is (D).

Linked Comprehension Type Questions

1. $\frac{dN_X}{dt} = \alpha - \lambda N_X$

$$\Rightarrow \quad N_X = \frac{1}{\lambda}\left[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}\right]$$

At $t = T_{1/2} = \frac{\log_e 2}{\lambda}$, we have

$$N_x = \frac{1}{\lambda}\left(\alpha - (\alpha - \lambda N_0)e^{-\lambda\left(\frac{\log_e 2}{\lambda}\right)}\right)$$

$$\Rightarrow \quad N_x = \frac{1}{\lambda}\left[\alpha - (\alpha - \lambda N_0)e^{-\log_e 2}\right]$$

Since, $e^{\log_e x} = x$

$$\Rightarrow \quad e^{-\log_e 2} = e^{\log_e\left(\frac{1}{2}\right)} = \frac{1}{2}$$

$$\Rightarrow \quad N_x = \frac{1}{\lambda}\left[\alpha - \left(\frac{\alpha - \lambda N_0}{2}\right)\right]$$

$$\Rightarrow \quad N_x = \frac{\alpha + \lambda N_0}{2\lambda}$$

Hence, the correct answer is (D).

2. Further, due to the decay of X to the stable nucleus Y, we have

$$\frac{dN_Y}{dt} = \lambda N_X$$

Since, $N_X = \frac{1}{\lambda}\left[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}\right]$

Substituting the value of N_x and solving, we get

$$\Rightarrow \quad N_Y = \alpha t + \left(\frac{\alpha - \lambda N_0}{\lambda}\right)e^{-\lambda t} - \left(\frac{\alpha - \lambda N_0}{\lambda}\right)$$

Hence, the correct answer is (B).

3. Since, $N_Y = \alpha t + \left(\frac{\alpha - \lambda N_0}{\lambda}\right)e^{-\lambda t} - \left(\frac{\alpha - \lambda N_0}{\lambda}\right)$

Substituting the value of $T_{1/2} = \frac{\log_e(2)}{\lambda}$ in this equation, we get

$$N_Y = \frac{\alpha}{\lambda}\log_e(2) + \left(\frac{\alpha - \lambda N_0}{2\lambda}\right) - \left(\frac{\alpha - \lambda N_0}{\lambda}\right)$$

$$\Rightarrow \quad N_Y = \frac{\alpha}{\lambda}\log_e(2) - \frac{1}{2}\left(\frac{\alpha - \lambda N_0}{\lambda}\right)$$

Hence, the correct answer is (C).

4. $m\left({}_1^2\text{H}\right) + m\left({}_2^4\text{He}\right) = 2.014102 + 4.002603$

$$\Rightarrow \quad m\left({}_1^2\text{H}\right) + m\left({}_2^4\text{He}\right) = 6.016705\ \text{u}$$

Since, $m\left({}_3^6\text{Li}\right) = 6.015123\ \text{u}$

$$\Rightarrow \quad m_1 + m_2 > M$$

So, (A) is incorrect.

$$m\left({}_1^1\text{H}\right) + m\left({}_{83}^{209}\text{Bi}\right) = 1.007825 + 208.980388$$

$$m\left({}_1^1\text{H}\right) + m\left({}_{83}^{209}\text{Bi}\right) = 209.988213\ \text{u}$$

Since, $m\left({}_{84}^{210}\text{Po}\right) = 209.982876\ \text{u}$

$$\Rightarrow \quad m_1 + m_2 > M$$

So, B is incorrect

$$m\left({}_1^2\text{H}\right) + m\left({}_2^4\text{He}\right) = 2.014102 + 4.002603$$

$$\Rightarrow \quad m\left({}_1^2\text{H}\right) + m\left({}_2^4\text{He}\right) = 6.016705\ \text{u}$$

Since, ${}_3^6\text{Li} = 6.015123\ \text{u}$

$$\Rightarrow \quad (m_3 + m_4) > M'$$

So, (C) is correct and hence deuteron and alpha particle can go complete fusion.

$$m\left({}_{30}^{70}\text{Zn}\right) + m\left({}_{34}^{82}\text{Se}\right) = 69.925325 + 81.916709$$

$$\Rightarrow \quad m\left({}_{30}^{70}\text{Zn}\right) + m\left({}_{34}^{82}\text{Se}\right) = 151.842034\ \text{u}$$

Since, ${}_{64}^{152}\text{Gd} = 151.919803\ \text{u}$

$$\Rightarrow \quad m_3 + m_4 < M'$$

So, (D) is incorrect.
Hence, the correct answer is (C).

5. ${}_{84}^{210}\text{Po} = {}_{82}^{206}\text{Pb} + {}_2^4\text{He} + \Delta E$

$$m\left({}_{82}^{206}\text{Pb}\right) = 205.974455\ \text{u}$$

$$m\left({}_2^4\text{He}\right) = 4.002603\ \text{u}$$

$$\Rightarrow \quad m\left({}_{82}^{206}\text{Pb}\right) + m\left({}_2^4\text{He}\right) = 209.977058\ \text{u}$$

Now, $\Delta m = 209.977058 - 209.982876$

$\Rightarrow \Delta m = 0.005818$ u

$\Rightarrow Q = \Delta E = 0.005818 \times 931.5$

$\Rightarrow Q = 5.419467$ MeV $= 5419.467$ keV

$\Rightarrow Q = 5419.5$ keV

By Law of Conservation of Momentum, we have

$$0 = p_\alpha - p_{\text{lead}}$$

$$\Rightarrow p_\alpha = p_{\text{lead}}$$

$$\Rightarrow \sqrt{2m_\alpha E_\alpha} = \sqrt{2m_{Pb}E_{Pb}}$$

$$\Rightarrow 4E_\alpha = 206E_{Pb}$$

$$\Rightarrow E_\alpha = \frac{103}{2}E_{Pb}$$

Now, since $E_\alpha = \left(\frac{m_{Pb}}{m_{Pb} + m_\alpha}\right)Q$

$$\Rightarrow E_\alpha = \left(\frac{206}{206+4}\right)Q$$

$$\Rightarrow E_\alpha = \frac{103}{105}(5.422) = 5319 \text{ MeV}$$

Hence, the correct answer is (A).

6. Since, $\frac{dN}{dt} = P - \lambda N$

$$\Rightarrow \frac{dN}{P - \lambda N} = dt, \text{ where } \lambda = \frac{\log_e 2}{T}$$

$$\Rightarrow N = \frac{PT}{\log_e 2}\left(1 - e^{-\frac{t \log_e 2}{T}}\right)$$

Hence, the correct answer is (A).

7. Since, $A = \lambda N = P\left(1 - e^{-\frac{t \log_e 2}{T}}\right)$

$\Rightarrow$ Rate of energy release is $AE_0 = PE_0\left(1 - e^{-\frac{t \log_e 2}{T}}\right)$

Hence, the correct answer is (B).

8. Energy released upto time t is

$$E_{\text{released}} = (Pt - N)E_0$$

Hence, the correct answer is (A).

9. $Z_1 - (2)(2) + (3)(1) = Z_2 - (2)(1) + (5)(1) = Z_c$

$$\Rightarrow Z_1 - Z_2 = 4$$

$$A_1 - (4)(2) = A_2 - (1)(4) = A_c$$

$$\Rightarrow A_1 - A_2 = 4$$

Hence, the correct answer is (B).

10. For A

$$4N_0 \xrightarrow{1\text{ min}} 2N_0 \xrightarrow{1\text{ min}} N_0 \xrightarrow{1\text{ min}} \frac{N_0}{2} \xrightarrow{1\text{ min}} \frac{N_0}{4}$$

For B

$$N_0 \xrightarrow{2\text{ min}} \frac{N_0}{2} \xrightarrow{2\text{ min}} \frac{N_0}{4}$$

After 4 minute, we have

$$N_A = N_B = \frac{N_0}{4}$$

$$\Rightarrow N_C = (4N_C + N_0) - \left(\frac{N_0}{4} + \frac{N_0}{4}\right) = \frac{9N_0}{2}$$

Hence, the correct answer is (C).

11. Given $R_A = R_B$

$$\Rightarrow \lambda_A N_A = \lambda_B N_B$$

$$\Rightarrow \left(\frac{\log_e 2}{T_A}\right)\left(4N_0 e^{-\lambda_A t_0}\right) = N_0\left(\frac{\log_e 2}{T_B}\right)e^{-\lambda_B t_0}$$

$$\Rightarrow t_0 = 6 \text{ min}$$

Hence, the correct answer is (B).

12. If $x + X \longrightarrow Y + y$

For the above nuclear reaction, threshold energy is given as

$$E_{th} = -Q\left(1 + \frac{m_x}{m_X}\right)$$

The Q-value of reaction is given by

$$Q = (1.007825 + 3.016049 - 2 \times 2.014102) \times 931.5$$

$$\Rightarrow Q = -4.033 \text{ MeV}$$

When protons are incident on ${}_1^3\text{H}$, then

$$x = {}_1^1\text{H} \text{ and } X = {}_1^3\text{H}$$

$$K_{th} = 4.033 \text{ MeV}\left(1 + \frac{1.007825}{3.016049}\right)$$

$$\Rightarrow K_{th} = 5.381 \text{ MeV}$$

Hence, the correct answer is (A).

13. When ${}_1^3H$ is incident on protons

$$x = {}_1^3H \text{ and } X = {}_1^1H$$

$$\Rightarrow K_{th} = (4.033 \text{ MeV})\left(1 + \frac{3.016049}{1.007825}\right)$$

$$\Rightarrow K_{th} = 16.10 \text{ MeV}$$

Hence, the correct answer is (C).

14. From above calculations we observe that less energy is required for a nuclear reaction, which a light particle is incident on a heavy target than if a heavy particle is incident on a light target.
Hence, the correct answer is (C).

16. B.E. per nucleon for intermediate nucleus is more than lighter or heavier nuclei.
Hence, the correct answer is (C).

17. $V_{Nu} = \frac{4}{3}\pi r_{Nu}^3$

Since, $r_{Nu} = r_0 A^{\frac{1}{3}}$

$\Rightarrow \quad V_{Nu} = \left(\frac{4}{3}\pi r_0^3\right)A$

$\Rightarrow \quad V_{Nu} \propto A$

Hence, the correct answer is (D).

18. Mass defect in the reaction is

$$\Delta m = \left(2(2.014102) - 3.016049 - 1.007825\right) \text{ u}$$

$\Rightarrow \quad \Delta m = 0.0043 \text{ u}$

So, energy released in the reaction is

$$\Delta E = (\Delta m)(931.5) \text{ MeV}$$

$\Rightarrow \quad \Delta E = (0.0043)(931.5) \text{ MeV}$

$\Rightarrow \quad \Delta E \approx 4 \text{ MeV}$

Hence, the correct answer is (C).

19. Let N number of fusion reactions be required for the purpose. Then

$$N\left(4 \times 1.6 \times 10^{-13}\right) = 10^3 \times 3600$$

$\Rightarrow \quad N = 5.625 \times 10^{18}$

Hence, the correct answer is (B).

20. In one fusion reaction two ${}_1^2\text{H}$ nuclei are used. Hence total number of ${}_1^2\text{H}$ nuclei are

$$N' = 2N = 1.125 \times 10^{19}$$

So, mass m required in kg is

$$m = \left(\frac{1.125 \times 10^{19}}{6.02 \times 10^{26}}\right)(2) \text{ kg}$$

$\Rightarrow \quad m = 3.7 \times 10^{-8} \text{ kg}$

Hence, the correct answer is (D).

21. $Q = (N_2E_2 + N_3E_3) - N_1E_1$

Hence, the correct answer is (B).

24. Since, $1 \text{ kW} \equiv 3.1 \times 10^{13} \ \frac{\text{fission}}{\text{sec}}$

$\Rightarrow \quad 1.6 \times 10^3 \text{ kW} \equiv 1.6 \times 10^3 \times 3.1 \times 10^{13}$

$= 5 \times 10^{16}$ fission per second

Hence, the correct answer is (D).

26. The reaction for the second stage is given by

$${}_1T^3 + {}_1D^2 \longrightarrow {}_2\text{He}^4 + {}_0n^1 + E_2$$

$\Rightarrow \quad \Delta m = (3.016049 + 2.014102) - (4.002603 + 1.008665)$

$\Rightarrow \quad \Delta m = 0.01888 \text{ u}$

$\Rightarrow \quad E_2 = 0.01888 \times 931 = 17.587 \text{ MeV}$

Hence, the correct answer is (B).

27. Total energy released $E = E_1 + E_2 = 4.033 + 17.587$

$\Rightarrow \quad E = 21.62 \text{ MeV}$

So, energy released/deuteron is $\frac{21.62}{3} = 7.207 \text{ MeV}$

Hence, the correct answer is (D).

28. $\left(\begin{array}{c}\text{\% of Rest Mass of} \\ \text{deuterium released}\end{array}\right) = \frac{7.207 \times 100}{2.014102 \times 931.4} = 0.384\%$

Hence, the correct answer is (C).

29. $T_M = \frac{1}{\lambda}$ and $T_H = \frac{0.693}{\lambda}$

$\Rightarrow \quad T_M > T_H$

Hence, the correct answer is (A).

30. $A = \frac{dN}{dt} = \lambda N$

$\Rightarrow \quad n = \lambda N = \frac{0.693}{T} \text{ N}$

$\Rightarrow \quad T = \frac{0.693}{n}$

Hence, the correct answer is (C).

31. $R_1 = R_0 e^{-\lambda t}$ and $R_2 = R_0 e^{-\lambda_2 t}$

$\Rightarrow \quad \frac{R_1}{R_2} = e^{\lambda(t_2 - t_1)}$

$\Rightarrow \quad R_2 = R_1 e^{-\lambda(t_2 - t_1)}$

Hence, the correct answer is (C).

32. $N_0 = \frac{mN_A}{M}$

Hence, the correct answer is (C).

33. The number of undecayed nuclei after time t is $N = N_0 e^{-\lambda t}$

So, the number of decayed nuclei is

$$N' = N_0 - N = N_0(1 - e^{-\lambda t})$$

Hence, the correct answer is (C).

34. Activity $= A = -\frac{dN}{dt}$

$$\Rightarrow \quad A = A_0 e^{-\lambda t} = \lambda N_0 e^{-\lambda t}$$

Hence, the correct answer is (A).

36. From conservation of mechanical energy, we have

$$U_i + K_i = U_f + K_f$$

$$\Rightarrow \quad 0 + 2(1.5 \text{ kT}) = \frac{1}{4\pi\varepsilon_0}\frac{(e)(e)}{d} + 0$$

Substituting the values, we get

$$T = 1.4 \times 10^9 \text{ K}$$

Hence, the correct answer is (A).

37. As given in the paragraph, a reactor is termed successful, if

$$nt_0 > 5 \times 10^{14} \text{ s cm}^{-3}$$

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

2. A → (p)
B → (p)
C → (p)
D → (s)

When the daughter nuclei lie towards peak of this graph then energy is released. Hence the binding energy per nucleon or total binding energy in the nuclear process increases.

3. A → (p)
B → (q)
C → (r)
D → (q)

$$\gamma \rightarrow e^- + e^+$$

For pair production, we have

$$E = 2m_e c^2 = 2 \times 0.51 \text{ MeV} = 1.02 \text{ MeV}$$

Inverse photoelectric effect is X-ray production and energy involved in it is of order of tens of KeV

For de-excitation of Be^{+4} from first excited state, we have

$$E = \frac{Z^2}{n^2} \times 13.6 = \frac{4^2}{Z^2} \times 13.6$$

$$\Rightarrow \quad E \approx 54.4 \text{ eV} = 55 \text{ eV}$$

For K_α X-ray photon of molybdenum, we have

$$E(K_\alpha) = h\nu = \frac{h \times 3}{4} cR(Z-1)^2$$

$$\Rightarrow \quad E(K_\alpha) = \frac{3}{4}(hcR)(Z-1)^2 = \frac{3}{4}E_0(Z-1)^2$$

$$\Rightarrow \quad E(K_\alpha) = \frac{3}{4} \times 13.6 \times (42-1)^2 = 17.146 \times 10^3 \text{ eV}$$

$$\Rightarrow \quad E(K_\alpha) \approx 17 \text{ KeV}$$

5. A → (p, r)
B → (q, r, s)
C → (q, r)
D → (p, r)

For α-decay, we have

$${}^A_Z X \longrightarrow {}^{A-4}_{Z-2}Y + {}^4_2\text{He}$$

$$Q = (KE)_Y + (KE)_\alpha$$

Since, $(KE)_Y \ll (KE)_\alpha$, we have

$$Q = (KE)_\alpha$$

So, kinetic energy of all the emitted α- particles is equal to Q i.e., mono-energetic α- particles are emitted.
Angular momentum is conserved in α- decay

For β-Decay, we have

a neutron decaying to a proton, so

$${}^A_Z X \longrightarrow {}^{A}_{Z+1}Y + \bar{\nu} + e^-$$

$$Q = (KE)_Y + (KE)_e + E_{\bar{\nu}}$$

Since, $(KE)_Y \ll (KE)_e$

$$\Rightarrow \quad Q \approx (KE)_e + E_{\bar{\nu}}$$

Also note that,

(a) $E_{\bar{\nu}}$ is the energy of anti-neutrino and $E_{\bar{\nu}}$ takes on values from zero to maximum. Hence poly-energetic particles are emitted. i.e., poly-energetic antineutrinos are emitted.

(b) Due to emission of antineutrinos spin angular momentum is conserved.

(c) Neutron can decay in free space i.e., outside nucleus

For Positron emission, we have

a proton decaying to a neutron, so

$$ {}_Z^AX \longrightarrow {}_{Z-1}^{A}Y + \nu + e^{+} $$

Same explanation as above in case of Beta decay can be applied here too.

Proton cannot decay in free space i.e., outside nucleus because rest mass of proton is less than that of neutron.

For Electron capture, we have

$$ {}_Z^AX + e^{-} \longrightarrow {}_{Z-1}^{A}Y + \nu $$

$$ Q = (KE)_Y + E_\nu $$

Since, $(KE)_Y \ll E_\nu$

$$ \Rightarrow \quad Q \approx E_\nu $$

All the neutrinos emitted are of equal energies and their energies are approximately equal to Q. That is mono-energetic neutrinos are emitted. So, for electron capture we have that

(a) angular momentum is conserved.

(b) it cannot take place outside nucleus i.e., in free space.

7. A $\rightarrow$ (q)
B $\rightarrow$ (s)
C $\rightarrow$ (p)
D $\rightarrow$ (s)

Let N_0 be the total initial number of nuclei of A, B and C. So, $N_0 = N_A + N_B + N_C$. Now, when A decreases, then C increases, so (A + B) will continuously decrease, because C is formed only from A and B. So, A is continuously decreasing and hence $(C+B)$ will continuously increase.

9. A $\rightarrow$ (s)
B $\rightarrow$ (p, r)
C $\rightarrow$ (s)
D $\rightarrow$ (q, r)

(a) $Z' = (Z-2)+1 = Z-1$ and

$A' = A-4$

(b) $Z' = 2(Z-2)+1 = Z-3$ and

$A' = A-(2)(4) = A-8$

(c) $Z' = (Z-2)+(2)(1) = Z$ and

$A' = A-4$

(d) $Z' = 2(Z-2)+(2)(2) = Z-2$ and

$A' = A-2\times 4 = A-8$

12. A $\rightarrow$ (q)
B $\rightarrow$ (p)
C $\rightarrow$ (s)
D $\rightarrow$ (r)

In alpha decay, charge number decreases by 2 and mass number decreases by 4.

In β^{+}- decay, charge number decreases by 1 and mass number remains same.

In proton emission, charge no, decreases by 1 and mass no, decreases by 1.

13. A $\rightarrow$ (s)
B $\rightarrow$ (p)
C $\rightarrow$ (r)
D $\rightarrow$ (s)

Since, $\left(-\dfrac{dN}{dt}\right) = \lambda N$

$$ \Rightarrow \quad y = \lambda x $$

$$ \Rightarrow \quad \lambda = \frac{y}{x} $$

$$ \Rightarrow \quad T_{1/2} = \frac{\ln 2}{\lambda} = (\ln 2)\left(\frac{x}{y}\right) $$

Also, $R = R_0 e^{-\lambda t}$

$$ \Rightarrow \quad R = y e^{-\lambda\left(\frac{1}{\lambda}\right)} = \frac{y}{e} $$

$$ \Rightarrow \quad R = \frac{y}{e} = \lambda N $$

$$ \Rightarrow \quad N = \frac{y}{e\lambda} = \frac{y}{e\left(\frac{y}{x}\right)} = \frac{x}{e} $$

Integer/Numerical Answer Type Questions

1. $A = \left|\dfrac{dN}{dt}\right| = \lambda N_0$

$$ \Rightarrow \quad A = \frac{\ell n 2}{T_{\frac{1}{2}}} N_0 = \left(\frac{0.693}{T_{\frac{1}{2}}}\right) N_0 $$

$$ \Rightarrow \quad A = \frac{0.693 \times 0.5 \times 10^{-3} \times 6.02 \times 10^{23}}{3.8 \times (24 \times 3600) \times 222 \times (3.7 \times 10^{10})} $$

$\Rightarrow \quad A = 77.35 \text{ Ci}$

$\Rightarrow \quad A \approx 77 \text{ Ci}$

2. $N = N_0\left(\frac{1}{2}\right)^n$

$$20 = 80\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \quad n = 2$$

$$\Rightarrow \quad t = nT_{\frac{1}{2}} = 2 \times 4 = 8 \text{ minutes}$$

3. Let rate of production be R

$$\Rightarrow \quad \frac{dN}{dt} = R - \lambda N$$

$$\Rightarrow \quad \frac{dN}{dt} + \lambda N = R$$

Multiplying both sides by $e^{\lambda t}$, we get

$$e^{\lambda t}\frac{dN}{dt} + \lambda N e^{\lambda t} = \text{R}e^{\lambda t}$$

$$\Rightarrow \quad \frac{d\left(Ne^{\lambda t}\right)}{dt} = \text{R}e^{\lambda t}$$

$$\Rightarrow \quad d\left(Ne^{\lambda t}\right) = \text{R}e^{\lambda t}\, dt$$

Integrating, we get

$$Ne^{\lambda t} = \frac{\text{R}e^{\lambda t}}{\lambda} + C$$

where C is a constant of integration

Now, at $t = 0$, $N = 0$

$$\Rightarrow \quad C = -\frac{R}{\lambda}$$

$$\Rightarrow \quad N = \frac{R}{\lambda}\left(1 - e^{-\lambda t}\right)$$

At equilibrium, we have $N = \frac{R}{\lambda}$, for $t \to \infty$

For 50% of the equilibrium quantity, we have $N = \frac{R}{2\lambda}$

$$\Rightarrow \quad \frac{R}{2\lambda} = \frac{R}{\lambda}\left(1 - e^{-\lambda t}\right)$$

$$\Rightarrow \quad e^{-\lambda t} = \frac{1}{2}$$

$$\Rightarrow \quad t = \frac{\ell n 2}{\lambda} = T_{\frac{1}{2}} = 100 \text{ years}$$

4. From our knowledge of Collision Theory, the fraction of kinetic energy lost by neutron is

$$\frac{\Delta K}{K_i} = \frac{4m_1 m_2}{\left(m_1 + m_2\right)^2} = \frac{4(1)(2)}{(1+2)^2} = \frac{8}{9}$$

where K_i is the initial kinetic energy of neutron and ΔK is the energy loss.

After first collision $\Delta K_1 = \frac{8}{9}K_0$

After second collision $\Delta K_2 = \frac{8}{9}K_1$ and so on

So, total energy loss is

$$\Delta K = \Delta K_1 + \Delta K_2 + \ldots + \Delta K_n = \frac{8}{9}\left(K_0 + K_1 + \ldots. + K_{n-1}\right)$$

where, $K_1 = K_0 - \Delta K_1 = \frac{K_0}{9}$

$$K_2 = \frac{K_1}{9} = \left(\frac{1}{9}\right)^2 K_0$$

$$\Rightarrow \quad K_{n-1} = \left(\frac{1}{9}\right)^{n-1} K_0$$

$$\Rightarrow \quad \Delta K = \frac{8}{9}K_0\left[1 + \frac{1}{9} + \ldots\left(\frac{1}{9}\right)^{n-1}\right]$$

$$\Rightarrow \quad \Delta K = \frac{8}{9}K_0\left(\frac{1 - \frac{1}{9^n}}{1 - \frac{1}{9}}\right) = 1 - \frac{1}{9^n}$$

Since, $K_0 = 10^6$ eV and $\Delta K = \left(10^6 - 0.025\right)$ eV

$$\Rightarrow \quad \frac{1}{9^n} = \frac{K_0 - \Delta K}{K_0} = \frac{0.025}{10^6}$$

$$\Rightarrow \quad 9^n = 4 \times 10^7$$

Taking log both sides, we get

$$n = 8$$

5. Given, $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$

Since the probabilities of getting α and β particles are equal. So, rate of disintegration is equal for both.

$\Rightarrow \lambda_A N_A = \lambda_B N_B$

$\Rightarrow \frac{N_A}{N_B} = \frac{\lambda_B}{\lambda_A} = 2$

6. At time t, let say there are N atoms of ^{7}Be (radioactive). Then net rate of formation of ^{7}Be nuclei at this instant is,

$$\frac{dN}{dt} = \frac{10^{-4}}{1.6\times10^{-19}\times1000} - \lambda N$$

$$\Rightarrow \frac{dN}{dt} = 6.25\times10^{11} - \lambda N$$

$$\Rightarrow \int_0^{N_0} \frac{dN}{6.25\times10^{11} - \lambda N} = \int_0^{3600} dt$$

where N_0 are the number of nuclei at $t = 1$ hr or 3600 second.

$$\Rightarrow -\frac{1}{\lambda}\log_e\left(\frac{6.25\times10^{11} - \lambda N_0}{6.25\times10^{11}}\right) = 3600$$

Since, $\lambda N_0 =$ Activity of ^{7}Be at $t = 1\text{ hr} = 1.8\times10^8$ disintegrations/second

$$\Rightarrow -\frac{1}{\lambda}\log_e\left(\frac{6.25\times10^{11} - 1.8\times10^8}{6.25\times10^{11}}\right) = 3600$$

$\Rightarrow \lambda = 8\times10^{-8}\text{ sec}^{-1}$

Therefore, half life

$$t_{1/2} = \frac{0.693}{8\times10^{-8}} = 8.66\times10^6\text{ sec} = 100.26\text{ days}$$

$\Rightarrow t_{1/2} \cong 100$ days

7. In 2 sec only 90% of nuclei are left. Thus, in next 2 seconds 90% of 900 i.e. 810 nuclei will be left.

8. Since, $R = R_0\left(\frac{1}{2}\right)^n$

where, n is the number of half lives.

Given, $R = \frac{R_0}{16}$

$\Rightarrow \frac{R_0}{16} = R_0\left(\frac{1}{2}\right)^n$

$\Rightarrow n = 4$

Four half lives are equivalent to 8 s. Hence, 2 s is equal to one half life. So in one half life activity will remain half of 1600 Bq i.e., 800 Bq

9. ${}_6C^{13} \longrightarrow {}_6C^{12} + {}_0n^1$

Energy required is equal to difference in binding energy of parent nucleus and daughter nucleus.

10. For a simultaneous decay process, we have

$$\frac{1}{T} = \frac{1}{T_\alpha} + \frac{1}{T_\beta}$$

$$\Rightarrow T = \frac{T_\alpha T_\beta}{T_\alpha + T_\beta} = 324\text{ years}$$

Since $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$, where $n = \frac{t}{T_{\frac{1}{2}}}$

According to problem, we have

$$\frac{N}{N_0} = \frac{1}{4}$$

$$\Rightarrow \frac{t}{T_{\frac{1}{2}}} = 2$$

$$\Rightarrow t = 2T_{\frac{1}{2}}$$

$$\Rightarrow t = 2(0.693T) \approx 449\text{ yr}$$

11. The activity of a sample is

$$A = \lambda N = \left(\frac{0.693}{T_{\frac{1}{2}}}\right)N$$

$$\Rightarrow N = \frac{AT_{\frac{1}{2}}}{0.693} = \frac{6\times10^6\times30.2\times3.16\times10^7}{0.693}$$

$$\Rightarrow N = 3.1\times10^{26}$$

One mole of ^{137}Cs has a mass 137 g and one mole possesses 6.02×10^{23} nuclei. So, the mass of ^{137}Cs released was

$$m = \left(\frac{137}{6.02\times10^{23}}\right)(3.1\times10^{26}) = 70\times10^3\text{ g}$$

$\Rightarrow m = 70$ kg

$\Rightarrow x = 7$

12. $\frac{N}{N_0 - N} = \frac{3}{1}$

$$\Rightarrow \frac{N}{N_0} = \frac{3}{4}$$

$$\Rightarrow \frac{N}{N_0} = \frac{3}{4} = e^{-\lambda t} \qquad \left\{\because \frac{N}{N_0} = e^{-\lambda t}\right\}$$

$$\Rightarrow \lambda t = \log_e\left(\frac{4}{3}\right)$$

$$\Rightarrow t = \left(\frac{\log_e\left(\frac{4}{3}\right)}{0.693}\right)T_{\frac{1}{2}}$$

$$\Rightarrow \quad t = 4.5\times10^9\left[\frac{0.1249}{0.3010}\right]$$

$$\Rightarrow \quad t = 1.8678\times10^9 \text{ years} \approx 2\times10^9 \text{ years}$$

13. At time $t = t$

$$N_1 = N_0 e^{-\lambda_1 t}$$

and $N_2 = \dfrac{N_0\lambda_1}{\lambda_2-\lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$

$$\Rightarrow \quad N_3 = N_0 - N_1 - N_2$$

$$\Rightarrow \quad N_3 = N_0\left[1 - e^{-\lambda_1 t} - \frac{\lambda_1}{\lambda_2-\lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)\right]$$

$$\Rightarrow \quad \frac{N_3}{N_0} = 1 - e^{-\lambda_1 t} - \frac{\lambda_1}{\lambda_2-\lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

Since, $\lambda_1 = \dfrac{0.693}{30} = 0.0231 \text{ min}^{-1}$

$$\lambda_2 = \frac{0.693}{45} = 0.0154 \text{ min}^{-1}$$

and $t = 60$ minutes

$$\Rightarrow \quad \frac{N_3}{N_0} = 1 - e^{-0.0231\times60} - \frac{0.0231}{0.0154-0.0231}\left(e^{-0.0231\times60} - e^{-0.0154\times60}\right)$$

$$\Rightarrow \quad \frac{N_3}{N_0} = 1 - 0.25 + 3(0.25-0.4) = 0.31$$

$$\Rightarrow \quad \%\text{age} = \frac{N_3}{N_0}\times100\% = 31\%$$

14. $\lambda = \dfrac{h}{p}$ for α-particle

Kinetic energy of α-particle is

$K_\alpha = \dfrac{p^2}{2m_\alpha}$ and kinetic energy of nucleus is

$$K_{\text{nucleus}} = \frac{p^2}{2m_n}$$

$$\Rightarrow \quad E = \frac{p^2}{2}\left(\frac{1}{m_\alpha} + \frac{1}{m_n}\right) = 6.25 \text{ MeV}$$

So, mass of parent nucleus is $M = m_n + m_\alpha + \dfrac{E}{c^2}$

$$\Rightarrow \quad M = 227.62 \text{ amu}$$

15. Charge on capacitor is $Q = Q_0 e^{-\frac{t}{RC}}$ and activity of sample is $A = A_0 e^{-\lambda t}$

$$\Rightarrow \quad \frac{Q}{A} = \frac{Q_0 e^{-\frac{t}{RC}}}{A_0 e^{-\lambda t}} = \frac{Q_0}{A_0} e^{\left(\lambda - \frac{1}{RC}\right)t}$$

For $\dfrac{Q}{A}$ to be independent of time, we have

$$\lambda - \frac{1}{RC} = 0$$

$$\Rightarrow \quad \lambda = \frac{1}{RC}$$

$$\Rightarrow \quad R = \frac{1}{C\lambda} = \frac{T_m}{C} = \frac{20\times10^{-3}}{100\times10^{-6}} = 200\ \Omega$$

16. Cross-sectional area of the torus is

$$A = \pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4} \text{ m}^2$$

Circumference of torus is

$$\ell = 2\pi\left(\frac{3}{2}\right) = 3\pi \text{ metre}$$

So, volume of the torus is

$$V = A\ell = \left(\frac{\pi}{4}\right)(3\pi) = \left(\frac{3\pi^2}{4}\right) \text{ m}^3$$

$$\Rightarrow \quad V = \left(\frac{3\pi^2}{4}\right) \text{ m}^3 = 7.5 \text{ m}^3$$

Pressure of the gas is

$$P = 10^{-5}\times13.6\times10^3\times10 = 1.36 \text{ Nm}^{-2}$$

According to ideal gas equation, we have

$$PV = Nk_BT_0$$

where $T_0 = 27 + 273 = 300$ K

$$\Rightarrow \quad Nk_B = \frac{PV}{T_0} = \frac{(1.36)(7.5)}{300} = 0.034$$

The energy obtained from the discharge is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(1200\times10^{-6}\right)\left(4\times10^4\right)^2$$

$$\Rightarrow \quad U = 9.6\times10^5 \text{ J}$$

This energy is transferred to the plasma in the form of kinetic energy. So, we have

$$U = KE = 9.6\times10^5 \text{ J}$$

Since we know that, at temperature T, the average kinetic energy associated with the gas molecule is

$$E = \frac{3}{2}Nk_BT$$

So, the kinetic energy gained by the plasma molecules is $E_K = \left(\dfrac{10}{100}\right)U = 9.6\times10^4$ J

Further it is given that, each deuterium molecule produces two ions and two electrons, hence the total energy of the plasma at temperature T is given by

$$E_{Total} = 4\left(\frac{3}{2}Nk_BT\right) = 6Nk_BT = 9.6\times 10^4$$

$$T = 4706\times 10^2 \text{ K}.$$

17. Let $\lambda_A = \lambda$ and $\lambda_B = 2\lambda$

Initially rate of disintegration of A is λN_0 and that of B is $(2\lambda)N_0$.

After one half life of A, rate of disintegration of A will becomes $\frac{\lambda N_0}{2}$ and that of B would also be $\frac{\lambda N_0}{2}$.

So, after one half life of A or two half life of B.

$$\left(-\frac{dN}{dt}\right)_A = \left(-\frac{dN}{dt}\right)_B$$

$\Rightarrow \quad n = 1$

18. $BE = (\Delta m)c^2$

$\Rightarrow \quad BE = 0.0302\times 930$

$\Rightarrow \quad BE = 28.086$

$\Rightarrow \quad \frac{BE}{4} = \frac{28.086}{4} \approx 7 \text{ MeV}$

19. 3 half lives of A is equivalent to 6 half lives of B.

$\Rightarrow \quad N_A\left(\frac{1}{2}\right)^3 = N_B\left(\frac{1}{2}\right)^6$

$\Rightarrow \quad \frac{N_B}{N_A} = 8$

20. Since, $\Delta E = (m_n - m_p)\times 931.5 \text{ MeV} = 1.4 \text{ MeV}$

Also, $\Delta E = E_1 + E_2$ ...(1)

where $E_1 = m_0c^2 + K_1 =$ Energy of electron

$E_2 =$ Energy of antineutrino (having zero rest mass)

By conservation of momentum, we have

$p_1 = -p_2$

$\Rightarrow \quad p_1^2 = p_2^2$

Since $E = \sqrt{m_0^2c^4 + p^2c^2}$

$\Rightarrow \quad p = \frac{1}{c}\sqrt{E^2 - m_0^2c^4}$

$\Rightarrow \quad \frac{1}{c}\sqrt{E_1^2 - m_0^2c^4} = \frac{1}{c}\sqrt{E_2^2 - 0}$

$\Rightarrow \quad E_1^2 = m_0^2c^4 = E_2^2$ Using (1), we get

$E_1^2 - m_0^2c^4 = (\Delta E - E_1)^2$

$\Rightarrow \quad E_1 = \frac{(\Delta E)^2 + m_0^2c^4}{2\Delta E} = 0.7937 \text{ MeV}$

$\Rightarrow \quad E_2 = 1.4 - 0.7937 = 0.6068 \text{ MeV}$

$\Rightarrow \quad E_2 \approx 6\times 10^5 \text{ eV}$

21. Let ${}^A_Z X \longrightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu}$

$\Rightarrow \quad {}^{23}_{10}\text{Ne} \longrightarrow {}^{23}_{11}\text{Na} + e^- + \bar{\nu}$

$Q = \left[m({}^{23}\text{Ne}) - m({}^{23}\text{Na})\right]\times 931.5 \text{ MeV}$

$\Rightarrow \quad Q = 4.375 \text{ MeV} = 4.4 \text{ MeV}$

$\Rightarrow \quad Q \approx 4 \text{ MeV}$

Since, $Q = (KE)_Y + (KE)_e + E(\bar{\nu})$

As, $(KE)_Y$ is very very small, so

$Q \approx (KE)_e + E(\bar{\nu})$

When $(KE)_e$ is maximum, then $E(\bar{\nu})$ is negligible

$\Rightarrow \quad (KE)_e \approx Q = 4 \text{ MeV}$

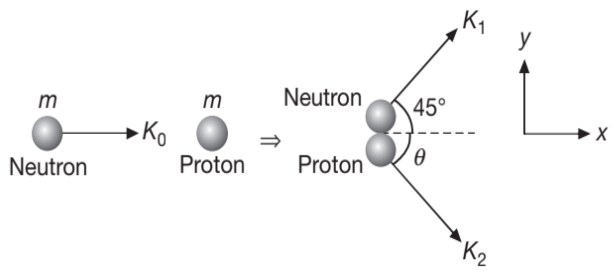
22. ${}_{92}U^{238} \longrightarrow {}_2He^4 + {}_{90}Th^{234} + Q$

$\Delta m = 238.05081 - (4.00260 + 234.04363)$

$\Rightarrow \quad \Delta m = 0.00458 \text{ u}$

$\Rightarrow \quad Q = \Delta m \times 931.5 \text{ MeV} = 4.26 \text{ MeV}$

23. Mass of neutron $\approx$ mass of proton $= m$



Applying Law of Conservation of Linear Momentum along y-direction, we get

$\sqrt{2mK_1}\sin 45° = \sqrt{2mK_2}\sin\theta$ ...(1)

along x-direction, we get

$\sqrt{2mK_0} - \sqrt{2mK_1}\cos 45° = \sqrt{2mK_2}\cos\theta$ ...(2)

Squaring and adding equations (1) and (2), we get

$K_2 = K_1 + K_0 - \sqrt{2K_0K_1}$ ...(3)

By Law of Conservation of Energy, we get

$K_2 = K_0 - K_1$ ...(4)

Solving equations (3) and (4), we get

$K_1 = \frac{K_0}{2}$

$\Rightarrow \quad \Delta K = K_0 - K_1 = \frac{K_0}{2}$

i.e., after each collision energy remains half. Therefore, after n collisions, we have

$$K_n = K_0\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad 0.23 = \left(4.6\times 10^6\right)\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad 2^n = \frac{4.6\times 10^6}{0.23}$$

Taking log both sides, we get

$$n \approx 24$$

24.

	A^{236}	:	A^{234}
Nuclei	$4N_0$	:	N_0
Half life	30	:	60

Activity is $A = \lambda N = \lambda N_0 e^{-\lambda t}$

$$\lambda_1(4N_0)e^{-\frac{0.693}{30}t} = \lambda_2 N_0 e^{-\frac{0.693}{60}t}$$

$$\Rightarrow \quad \frac{0.693}{30}(4N_0)e^{-\frac{0.693}{30}t} = \frac{0.693}{60}(N_0)e^{-\frac{0.693}{60}t}$$

$$\Rightarrow \quad 8 = e^{0.693t\left(\frac{1}{30}-\frac{1}{60}\right)}$$

$$\Rightarrow \quad 8 = e^{+\frac{0.693}{60}t}$$

Taking natural log both sides, we get

$$3\times 0.693 = \frac{0.693t}{60}$$

$$\Rightarrow \quad t = 180 \text{ min}$$

25. When a substance decays by α and β emission simultaneously, the average disintegration constant λ_{av} is given by

$$\lambda_{av} = \lambda_\alpha + \lambda_\beta$$

where λ_α = disintegration constant for α-emission only

λ_β = disintegration constant for β-emission only

Mean life is given by $T_m = \dfrac{1}{\lambda_{av}}$

$$\Rightarrow \quad \lambda_{av} = \lambda_\alpha + \lambda_\beta$$

$$\Rightarrow \quad \frac{1}{T_m} = \frac{1}{T_\alpha} + \frac{1}{T_\beta} = \frac{1}{1200} + \frac{1}{600}$$

$$\Rightarrow \quad T_m = 400 \text{ yr}$$

Since, $t = \dfrac{1}{\lambda}\log_e\left(\dfrac{N_0}{N_t}\right) = T_m \log_e\left(\dfrac{N_0}{N_t}\right)$

$$\Rightarrow \quad t = 400\log_e\left(\frac{100}{25}\right) = 400\log_e(4)$$

$$\Rightarrow \quad t = 400\times 1.4 = 560 \text{ year}$$

26. 235 kg contains 6.023×10^{26} atoms

So, 1.5 kg contains 0.384×10^{25} atoms

Energy released is $\Delta E = 0.384\times 10^{25}\times 200$ MeV

$$\Rightarrow \quad \Delta E = 7.7\times 10^{26} \text{ MeV}$$

$$\Rightarrow \quad \Delta E = 12.32\times 10^{13} \text{ J}$$

$$\Rightarrow \quad \Delta E \approx 1.23\times 10^{14} \text{ J}$$

27. Let $\lambda_A = \lambda$ and $\lambda_B = 2\lambda$

Initially rate of disintegration of A is λN_0 and that of B is $(2\lambda)N_0$.

After one half life of A, rate of disintegration of A will becomes $\dfrac{\lambda N_0}{2}$ and that of B would also be $\dfrac{\lambda N_0}{2}$.

So, after one half life of A or two half life of B.

$$\left(-\frac{dN}{dt}\right)_A = \left(-\frac{dN}{dt}\right)_B$$

$$\Rightarrow \quad n = 1$$

28.

$$\begin{array}{cccc} {}^{200}X & \longrightarrow & {}^{80}Y & + & {}^{120}Z \\ 6.5 & & 7 & & 8 \end{array}$$

So, energy released is

$$\Delta E = 80\times 7 + 120\times 8 - 200\times 6.5$$

$$\Rightarrow \quad \Delta E = 220 \text{ MeV} = 2200\times 10^5 \text{ eV}$$

29. Energy per fission

$$E_0 = 200\times 10^6\times 1.6\times 10^{-19} \text{ J}$$

$$\Rightarrow \quad E_0 = 3.2\times 10^{-11} \text{ J}$$

$$P = 4 \text{ MW} = 4\times 10^6 \text{ Js}^{-1}$$

In time $t(=1 \text{ year})$, the energy delivered is $E = Pt$

Number of fission required is $N = \dfrac{Pt}{E_0}$

$$\Rightarrow \quad N = \frac{4\times 10^6\times 365\times 24\times 3600}{3.2\times 10^{-11}}$$

$$\Rightarrow \quad N = 3.942\times 10^{24}$$

So, mass of uranium required is

$$m = \frac{3.942\times 10^{24}\times 235}{6.02\times 10^{23}} \text{ g}$$

$$\Rightarrow \quad m = 1538.8 \text{ g}$$

$$\Rightarrow \quad m \approx 1539 \text{ g}$$

30. $N_1 = N_0 e^{-\lambda(10)}$

Let age of sample be x, them

$$N_2 = N_0 e^{-\lambda x}$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{4}{100} = e^{\lambda(10-x)}$$

$$e^{\lambda(x-10)} = \frac{100}{4}$$

$$\Rightarrow \lambda(x-10) = \ell n(100) - \ell n 4 = 2\ell n(10) - 2\ell n(2)$$

$$\Rightarrow \lambda(x-10) = 2(2.3) - 2(0.693)$$

$$\Rightarrow \lambda(x-10) = 3.22$$

Since $\lambda = \frac{0.693}{12.5}\ \text{yr}^{-1}$

$$\Rightarrow x - 10 = \left(\frac{12.5}{0.693}\right)(3.22) = 58.08$$

$$\Rightarrow x \approx 68 \text{ years}$$

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1. Densities of nucleus happens to be constant, irrespective of mass number.

Hence, the correct answer is (B).

2. Number of nuclei present at any time t

$$N = N_0 e^{-\lambda t}$$

Given that $\frac{N_A}{N_B} = \frac{1}{e}$

$$\Rightarrow \frac{N_A}{N_B} = e^{(\lambda_B - \lambda_A)t} = \frac{1}{e}$$

$$\Rightarrow (\lambda_A - \lambda_B)t = 1$$

$$\Rightarrow t = \frac{1}{-\lambda + 10\lambda} = \frac{1}{9\lambda}$$

Hence, the correct answer is (C).

3. At time t, $N_x = N_0 e^{-5\lambda t}$

At time t, $N_y = N_0 e^{-\lambda t}$

Since $\frac{N_x}{N_y} = \frac{1}{e^2} = e^{-4\lambda t}$

$$\Rightarrow 4\lambda t = 2$$

$$\Rightarrow t = \frac{2}{4\lambda} = \left(\frac{1}{2\lambda}\right)$$

Hence, the correct answer is (A).

4. $N_A = \frac{N_0}{(2)^{\frac{60}{10}}} = \frac{N_0}{64}$

$$N_B = \frac{N_0}{(2)^{\frac{60}{20}}} = \frac{N_0}{8}$$

So, number of decayed nuclei in A is $N'_A = N_0 - N_A$ and number of decayed nuclei in B is $N'_B = N_0 - N_B$

So, required ratio is $\frac{N'_A}{N'_B} = \frac{N_0 - \frac{N_0}{64}}{N_0 - \frac{N_0}{8}}$

$$\Rightarrow \frac{N'_A}{N'_B} = \frac{63 \times 8}{64 \times 7} = \frac{9}{8}$$

Hence, the correct answer is (A).

5. $10 \text{ mCi} = \lambda_A N_A(t)$ ...(1)

$20 \text{ mCi} = \lambda_B N_B(t)$ ...(2)

Since $N_A(t) = 2N_B(t)$

$$\Rightarrow \frac{1}{2} = \frac{\lambda_A N_A(t)}{\lambda_B N_B(t)}$$

$$\Rightarrow \frac{1}{2} = \left(\frac{\lambda_A}{\lambda_B}\right)2$$

$$\Rightarrow \lambda_B = 4\lambda_A$$

$$\Rightarrow (t_B)_{\frac{1}{2}} = \frac{(t_A)_{\frac{1}{2}}}{4}$$

$$\Rightarrow (t_A)_{\frac{1}{2}} = 4(t_B)_{\frac{1}{2}}$$

Hence, the correct answer is (B).

6. $R_A = R_0 e^{-\lambda_A t}$

$$R_B = R_0 e^{-\lambda_B t}$$

Since $\frac{R_B}{R_A} = e^{-3t}$

$$\Rightarrow \frac{R_B}{R_A} = e^{-(\lambda_B - \lambda_A)t} = e^{-3t}$$

$$\Rightarrow \lambda_B - \lambda_A = 3$$

$$\Rightarrow \frac{\ln 2}{T_2} - \frac{\ln 2}{\ln 2} = 3$$

$$\Rightarrow T_2 = \frac{\ln 2}{4}$$

Hence, the correct answer is (C).

7. $(2)^N = \left(\frac{1600}{100}\right)$

$N = 4$

$4t = 8$

$t = 2$ s

Count Rate $= \frac{1600}{(2)^3} = 200$

Hence, the correct answer is (D).

8. Amount of energy released is

$$\Delta E = 2(\text{BE of He}^4) + (\text{BE of C}^{12}) - (\text{BE of Ne}^{20})$$

$$\Rightarrow \quad \Delta E = (2)(4)(7.07) + (12)(7.86) - (20)(8.03)$$

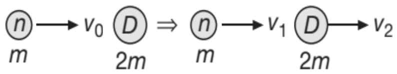
$$\Rightarrow \quad \Delta E = -9.72 \approx -9.7 \text{ MeV}$$

So, 9.7 MeV of energy will be supplied.

Hence, the correct answer is (D).

9. Let initial speed of neutron is v_0 and kinetic energy is K.

For 1st collision



Using momentum conservation principle,

$$mv_0 = mv_1 + 2mv_2$$

$$\Rightarrow \quad v_1 + 2v_2 = v_0 \quad \ldots(1)$$

As, $e = 1$, so

$$v_2 - v_1 = v_0 \quad \ldots(2)$$

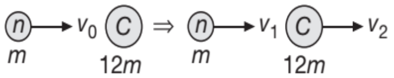
From equations (1) and (2), we get

$$v_2 = \frac{2v_0}{3}, \; v_1 = -\frac{v_0}{3}$$

Fractional loss of energy is $p_d = \dfrac{\frac{1}{2}mv_0^2 - \frac{1}{2}m\left(-\frac{v_0}{3}\right)^2}{\frac{1}{2}mv_0^2}$

$$\Rightarrow \quad p_d = \frac{8}{9} \approx 0.89$$

For 2nd collision



Using momentum conservation principle,

$$mv_0 = mv_1 + 12mv_2$$

$$\Rightarrow \quad v_1 + 12v_2 = v_0 \quad \ldots(1)$$

As $e = 1$;

$$\Rightarrow \quad v_2 - v_1 = v_0 \quad \ldots(2)$$

From equations (1) and (2), we get

$$v_2 = \frac{2v_0}{13}; \; v_1 = \frac{-11v_0}{13}$$

Now fraction loss of energy

$$P_c = \frac{\frac{1}{2}mv_0^2 - \frac{1}{2}m\left(-\frac{11v_0}{13}\right)^2}{\frac{1}{2}mv_0^2} = \frac{48}{169} \approx 0.28$$

Hence, the correct answer is (A).

10. Let total volume of blood is V

Initial activity $A_0 = 0.8\ \mu$Ci

Its activity at time t, $A = A_0 e^{-\lambda t}$

Activity of solution of volume x is

$$A_1 = \left(\frac{A}{V}\right)x = x\left(\frac{A_0}{V}\right)e^{-\lambda t}$$

$$\Rightarrow \quad V = x\left(\frac{A_0}{A_1}\right)e^{-\lambda t}$$

$$\Rightarrow \quad V = (1 \text{ cm}^3)\left(\frac{8 \times 10^{-7} \times 3.7 \times 10^{10}}{\frac{300}{60}}\right)(0.84)$$

$$\Rightarrow \quad V = 4.97 \times 10^3 \text{ cm}^3 = 4.97 \text{ litres} \approx 5 \text{ litres}$$

Hence, the correct answer is (C).

11. As, $\frac{v_1}{v_2} = \frac{8}{27}$; $\frac{r_1}{r_2} = ?$

Using law of conservation of linear momentum,

$$0 = m_1v_1 - m_2v_2$$

(As both are moving in opposite directions.)

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{27}{8}$$

$$\Rightarrow \quad \frac{\rho\left(\frac{4}{3}\pi r_1^3\right)}{\rho\left(\frac{4}{3}\pi r_2^3\right)} = \frac{27}{8}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{3}{2}$$

Hence, the correct answer is (A).

12. As per question, $N_1 = 2N_2$

Also $A_1 = 5\ \mu$Ci, $A_2 = 10\ \mu$Ci

As, $A = \lambda N = \frac{\ln 2}{T_{\frac{1}{2}}} N$

CHAPTER 3

$$\Rightarrow \frac{A_1}{A_2} = \frac{\left(T_{\frac{1}{2}}\right)_2}{\left(T_{\frac{1}{2}}\right)_1} \times \frac{N_1}{N_2}$$

$$\frac{\left(T_{\frac{1}{2}}\right)_1}{\left(T_{\frac{1}{2}}\right)_2} = \frac{N_1}{N_2} \times \frac{A_2}{A_1} = 2 \times 2 = 4$$

Hence, the correct answer is (C).

13. $\frac{N_B}{N_A} = \frac{N_0 - N_0 e^{-\lambda t}}{N_0 e^{-\lambda t}} = 0.3$

$$\Rightarrow e^{\lambda t} = 1.3$$

$$\Rightarrow \lambda t = \ln 1.3$$

$$\Rightarrow \left(\frac{\ln 2}{T}\right) t = \ln(1.3)$$

$$\Rightarrow t = T \cdot \frac{\ln(1.3)}{\ln 2}$$

$$\Rightarrow t = T \frac{\log(1.3)}{\log 2}$$

Hence, the correct answer is (B).

14. ${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4$

Energy released is $\Delta E = 4\left(B.E.\left({}_1^2H\right)\right) - 4\left(B.E.\left({}_2^4He\right)\right)$

$$\Rightarrow \Delta E = 4 \times 7 - 4 \times 1.1 = 23.6 \text{ MeV}$$

Hence, the correct answer is (D).

15. Here, $P = 10^9$ W, $c = 3 \times 10^8$ ms^{-1}, $\frac{\Delta m}{\Delta t} = ?$

We know, $P = \frac{E}{\Delta t} = \frac{\Delta mc^2}{\Delta t}$

$$\Rightarrow \frac{\Delta m}{\Delta t} = \frac{P}{c^2} = \frac{10^9}{(3 \times 10^8)^2} = \frac{10^{-7}}{9} \text{ kgs}^{-1}$$

$$\Rightarrow \frac{\Delta m}{\Delta t} = \frac{10^{-7}}{9} \times 1000 \times 3600 \text{ gh}^{-1} = 4 \times 10^{-2} \text{ gh}^{-1}$$

Hence, the correct answer is (A).

16. $t = 80 \text{ min} = 4T_A = 2T_B$

So, number of nuclei of A decayed $= N_0 - \frac{N_0}{2^4} = \frac{15N_0}{16}$

So, number of nuclei of B decayed $= N_0 - \frac{N_0}{2^2} = \frac{3N_0}{4}$

$\Rightarrow$ Required ratio $= \frac{5}{4}$

Hence, the correct answer is (A).

17. Using conservation of linear momentum,

$$\begin{pmatrix}\text{Total momentum}\\ \text{before collision}\end{pmatrix} = \begin{pmatrix}\text{Total momentum}\\ \text{after collision}\end{pmatrix}$$

$$\Rightarrow mv = (m+m)v'$$

$$\Rightarrow v' = \frac{v}{2}$$

Loss in kinetic energy during the process,

$$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2$$

For minimum kinetic energy of neutron, lost kinetic energy should be used by the electron to jump from first orbit to second orbit.

$$\Rightarrow \frac{1}{4}mv^2 = (13.6 - 3.4)\text{eV} = 10.2 \text{ eV}$$

$\Rightarrow \frac{1}{2}mv^2 = 20.4 \text{ eV} = K.E.$ of neutron for inelastic collision.

Hence, the correct answer is (A).

18. Half life $= 15 \text{ hrs} = \frac{0.693}{\lambda}$

$$\Rightarrow \lambda = 0.0462 \text{ hr}^{-1}$$

$$N_0 = \frac{1}{24} \text{moles of Na}, \ t = 7.5 \text{ hrs}$$

Number of β-particles disintegrated, $N_\beta = N_0\left(1 - e^{-\lambda t}\right)$

$$\Rightarrow N_\beta = \left(\frac{1}{24}\text{moles}\right)\left(1 - e^{-(0.0462 \times 7.5)}\right)$$

$$\Rightarrow N_\beta = \left(\frac{1}{24}\text{moles}\right)\left(1 - e^{-0.35}\right)$$

$$\Rightarrow N_\beta = 0.0122 \text{ moles} = 0.0122 \times 6.023 \times 10^{23}$$

$$\Rightarrow N_\beta = 7.4 \times 10^{21}$$

Hence, the correct answer is (B).

19. Since, $r = \frac{mv}{qB} = \frac{mv}{eB}$

$$\Rightarrow KE = \frac{1}{2}mv^2 = \frac{e^2B^2r^2}{2m}$$

is the KE of the ejected photoelectrons. Now, according to Law of Photo-electricity, we have

$$hv = hv_0 + \frac{1}{2}mv^2$$

$$\Rightarrow hv = \phi_0 + \frac{1}{2}mv^2 \quad \ldots(1)$$

Now $\frac{1}{2}mv^2 = \frac{e^2B^2r^2}{2m} = 0.8 \text{ eV}$

$$hv = 13.6\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{68}{36} = 1.9 \text{ eV}$$

So, from (1), we get

$$1.9 = \phi_0 + 0.8$$

$$\Rightarrow \quad \phi_0 = 1.1 \text{ eV}$$

Hence, the correct answer is (B).

20. Mass defect, $\Delta m = m_p + m_e - m_n$

$$\Rightarrow \quad \Delta m = \left(1.6725\times10^{-27} + 9\times10^{-31} - 1.6725\times10^{-27}\right)\text{kg}$$

$$\Rightarrow \quad \Delta m = 9\times10^{-31} \text{ kg}$$

Energy released is $\Delta E = \Delta mc^2 = 9\times10^{-31}\times\left(3\times10^8\right)^2$ J

$$\Rightarrow \quad \Delta E = \frac{9\times10^{-31}\times9\times10^{16}}{1.6\times10^{-13}} \text{ MeV} = 0.51 \text{ MeV}$$

Hence, the correct answer is (D).

21. Number of undecayed atoms after time t_2,

$$\frac{N_0}{3} = N_0 e^{-\lambda t_2} \quad \ldots(1)$$

Number of undecayed atoms after time t_1,

$$\frac{2}{3}N_0 = N_0 e^{-\lambda t_1}$$

Dividing (2) by (1), we get $2 = e^{\lambda(t_2 - t_1)}$

$$\Rightarrow \quad \ln 2 = \lambda(t_2 - t_1)$$

$$\Rightarrow \quad (t_2 - t_1) = \frac{\ln 2}{\lambda}$$

As per question, $t_{\frac{1}{2}}$ = half life time = 20 min

$$\Rightarrow \quad t_2 - t_1 = 20 \text{ min} \qquad \left\{\because t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}\right\}$$

Hence, the correct answer is (C).

22. When a radioactive nucleus emits an alpha particle, its mass number decreases by 4 while the atomic number decreases by 2.
When a radioactive nucleus, emits a β^+ particle (or positron (e^+)) its mass number remains unchanged while the atomic number decreases by 1.

$$\Rightarrow \quad {}^{A}_{Z}X \xrightarrow{3\alpha} {}^{A-12}_{Z-6}Y \xrightarrow{2e^+} {}^{A-12}_{Z-8}W$$

In the final nucleus,

Number of protons, $N_p = Z - 8$

Number of neutrons, $N_n = A - 12 - (Z - 8) = A - Z - 4$

$$\Rightarrow \quad \frac{N_n}{N_p} = \frac{A - Z - 4}{Z - 8}$$

Hence, the correct answer is (C).

23. Mass defect, $\Delta M = \left[(M + \Delta m) - \left(\frac{M}{2} + \frac{M}{2}\right)\right] = \Delta m$

Energy released, $Q = \Delta Mc^2 = \Delta mc^2 \quad \ldots(1)$

According to law of conservation of momentum, we get

$$(M + \Delta m)\times 0 = \frac{M}{2}\times v_1 - \frac{M}{2}\times v_2$$

$$\Rightarrow \quad v_1 = v_2$$

Also, $Q = \frac{1}{2}\left(\frac{M}{2}\right)v_1^2 + \frac{1}{2}\left(\frac{M}{2}\right)v_2^2 - \frac{1}{2}(M + \Delta m)\times(0)^2$

$$\Rightarrow \quad Q = \frac{M}{2}v_1^2 \qquad \{\because v_1 = v_2\} \quad \ldots(2)$$

Equating equations (1) and (2), we get $\left(\frac{M}{2}\right)v_1^2 = \Delta mc^2$

$$v_1^2 = \frac{2\Delta mc^2}{M}$$

$$\Rightarrow \quad v_1 = c\sqrt{\frac{2\Delta m}{M}}$$

Hence, the correct answer is (C).

24. After decay, the daughter nuclei will be more stable, hence binding energy per nucleon of daughter nuclei is more than that of their parent nucleus.
Hence, $E_2 > E_1$.

Hence, the correct answer is (D).

25. When two nucleons combine to form a third one, and energy is released, one has fusion reaction. If a single nucleus splits into two, one has fission. The possibility of fusion is more for light elements and fission takes place for heavy elements. Out of the choices given for fusion, only A and B are light elements and D and E are heavy elements. Therefore $A + B \rightarrow C + \varepsilon$ is correct. In the possibility of fission is only for F and not C. Therefore $F \rightarrow D + E + \varepsilon$ is the correct choice.

Hence, the correct answer is (A).

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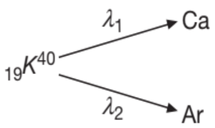
Single Correct Choice Type Problems

1. Since, $\frac{dN}{dt} = -\lambda_1 N - \lambda_2 N$

$\Rightarrow \quad \frac{dN}{dt} = -(\lambda_1 + \lambda_2)dt$

$\Rightarrow \quad N = N_0 e^{-(\lambda_1+\lambda_2)t}$



Since $N = N_0 - 99\%$ of N_0 i.e., $N = 0.01N_0$, we have

$$t = \frac{\ln(100)}{\lambda_1 + \lambda_2} = \frac{2.3 \times 2}{5 \times 10^{-10}}$$

$\Rightarrow \quad t = 9.2 \times 10^9$ year

Hence, the correct answer is (B).

2. Required activity $= \frac{\text{Initial activity}}{64} = \frac{\text{Initial activity}}{2^6}$

Time required $=$ 6 half lives

$\Rightarrow \quad t = 6 \times 18$ days

$\Rightarrow \quad t = 108$ days

Hence, the correct answer is (C).

3. For $^{15}_{8}\text{O}$, $E_0 = \frac{3}{5} \times \frac{8 \times 7}{R} \times \frac{e^2}{4\pi\varepsilon_0} = \frac{3}{5} \times \frac{8 \times 7}{R} \times 1.44$ MeV

For $^{15}_{7}\text{N}$, $E_N = \frac{3}{5} \times \frac{7 \times 6}{R} \times \frac{e^2}{4\pi\varepsilon_0} = \frac{3}{5} \times \frac{7 \times 6}{R} \times 1.44$ MeV

$\Rightarrow \quad |E_0 - E_N| = \frac{3}{5} \times \frac{1.44}{R} \times 7(2)$ ...(1)

Mass defect of N atom is

$\Delta m_N = 8 \times 1.008665 + 7 \times 1.007825 - 15.000109$

$\Rightarrow \quad \Delta m_N = 0.1239864\ u$

$\Rightarrow$ Binding energy is $B_N = 0.1239864 \times 931.5$ MeV

Mass defect of O atom is

$\Delta m_0 = 7 \times 1.008665 + 8 \times 1.007825 - 15.003065$

$\Rightarrow \quad \Delta m_0 = 0.12019044\ u$

$\Rightarrow$ Binding energy $B_0 = 0.12019044 \times 931.5$ MeV

So $|B_0 - B_N| = 0.0037960 \times 931.5$ MeV ...(2)

Equating (1) and (2), we get

$R = 3.42$ fm

Hence, the correct answer is (C).

4. From conservation laws of mass number and atomic number, we can say that $x = n$, $y = n$

$\left(x = {}^1_0 n,\ y = {}^1_0 n\right)$

$\Rightarrow$ Only (A) and (D) options may be correct.

From conservation of momentum, $|P_{xe}| = |P_{sr}|$

From $K = \frac{P^2}{2m}$

$\Rightarrow \quad K \propto \frac{1}{m}$

$\frac{K_{sr}}{K_{xe}} = \frac{m_{xe}}{m_{sr}}$

$\Rightarrow \quad K_{sr} = 129$ MeV, $K_{xe} = 86$ MeV

Note: There is no need of finding total energy released in the process.

Hence, the correct answer is (A).

5. Activity of $S_1 = \frac{1}{2}$ (activity of S_2)

$\Rightarrow \quad \lambda_1 N_1 = \frac{1}{2}(\lambda_2 N_2)$

$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{N_2}{2N_1}$

$\Rightarrow \quad \frac{T_1}{T_2} = \frac{2N_1}{N_2}$ $\quad \left\{T = \text{half life} = \frac{\log_e 2}{\lambda}\right\}$

Given $N_1 = 2N_2$

$\Rightarrow \quad \frac{T_1}{T_2} = 4$

Hence, the correct answer is (A).

6. Rest mass of parent nucleus should be greater than the rest mass of daughter nuclei. Therefore, option (A) will be correct.

Hence, the correct answer is (A).

7. After two half lives $\frac{1}{4}$th fraction of nuclei will remain undecayed. Or, $\frac{3}{4}$th fraction will decay. Hence, the probability that a nucleus decays in two half lives is $\frac{3}{4}$.

Hence, the correct answer is (B).

8. $4\left({}_2\text{He}^4\right) = {}_8\text{O}^{16}$

Mass defect, $\Delta m = (4(4.0026) - 15.9994) = 0.011$ amu

Energy released per oxygen nuclei is

$\Delta E = (0.011)(931.48)\text{MeV} = 10.24$ MeV

Hence, the correct answer is (C).

9. Activity reduces from 6000 dps to 3000 dps in 140 days. It implies that half-life of the radioactive sample is 140 days. In 280 days (or two half-lives) activity will remain $\frac{1}{4}$th of the initial activity. Hence, the initial activity of the sample is

$$4 \times 6000 \text{ dps} = 24000 \text{ dps}$$

Hence, the correct answer is (D).

10. Given that $K_1 + K_2 = 55$ MeV ...(1)

From Conservation of Linear Momentum,

$$P_1 = P_2$$

$$\Rightarrow \sqrt{2K_1(216m)} = \sqrt{2K_2(4m)}$$

as $P = \sqrt{2Km}$

$$\Rightarrow K_2 = 54K_1 \quad \text{...(2)}$$

Solving equations (1) and (2), we get $K_2 = KE$ of α-particle $= 5.4$ MeV.

Hence, the correct answer is (B).

11. Nuclear density is constant hence, mass $\propto$ volume or $m \propto V$.

Hence, the correct answer is (A).

12. $(r_m) = \left(\frac{m^2}{z}\right)(0.53 \text{ Å}) = (n \times 0.53)$ Å

$$\Rightarrow \frac{m^2}{z} = n$$

$m = 5$ for ${}_{100}\text{Fm}^{257}$ (the outermost shell) and $z = 100$

$$\Rightarrow n = \frac{(5)^2}{100} = \frac{1}{4}$$

Hence, the correct answer is (D).

13. During γ-decay atomic number (Z) and mass number (A) does not change. So, the correct option is (C) because in all other options either Z, A or both is/are changing.

Hence, the correct answer is (C).

14. $\frac{1}{16} = \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{\left(\frac{\text{Time lapsed}}{T_{1/2}}\right)}$

$$\Rightarrow \frac{\text{Time lapsed}}{T_{1/2}} = 4$$

$$\Rightarrow \text{Time lapsed} = 400\ \mu s$$

Hence, the correct answer is (A).

15. The total number of atoms can neither remain constant (as in option A) nor can ever increase (as in options B and C). They will continuously decreases with time. Therefore, (D) is the appropriate option.

Hence, the correct answer is (D).

16. During β-decay, a neutron is transformed into a proton and an electron. This is why atomic number ($Z =$ number of protons) increases by one and mass number ($A =$ number of protons + neutrons) remains unchanged during beta decay.

Hence, the correct answer is (C).

17. $N_1 = N_0 e^{-10\lambda t}$

$$N_2 = N_0 e^{-\lambda t}$$

$$\Rightarrow \frac{N_1}{N_2} = \frac{1}{e} = e^{(-10\lambda + \lambda)t}$$

$$\Rightarrow 9\lambda t = 1$$

$$\Rightarrow t = \frac{1}{9\lambda}$$

Hence, the correct answer is (D).

18. $\frac{0.693}{\lambda_X} = \frac{1}{\lambda_Y}$

$$\Rightarrow \lambda_Y > \lambda_X$$

Since, $\left(-\frac{dN}{dt}\right)_X = \lambda_X N$

And $\left(-\frac{dN}{dt}\right)_Y = \lambda_Y N$

$$\Rightarrow \left(-\frac{dN}{dt}\right)_Y > \left(-\frac{dN}{dt}\right)_X$$

Decay rate of $Y >$ Decay rate of X

Hence, the correct answer is (C).

19. Both the beta rays and the cathode rays are made up of electrons. So, only option (A) is correct.

(B) Gamma rays are electromagnetic waves.

(C) Alpha particles are doubly ionized helium atoms and

(D) Protons and neutrons have approximately the same mass.

Therefore, (B), (C) and (D) are wrong options.

Hence, the correct answer is (A).

20. ${}^{22}_{10}\text{Ne} \longrightarrow 2\,{}^{4}_{2}\text{He} + {}^{A}_{Z}X$

$$\Rightarrow A = 14$$

$$\Rightarrow Z = 6$$

i.e. ${}^{14}_{6}\text{C}$ is the unknown nucleus.

Hence, the correct answer is (B).

21. $\rho = \dfrac{A(1.67\times10^{-27})}{\frac{4}{3}\pi R^3}$

Since, $R = R_0 A^{\frac{1}{3}}$ where $R_0 = 1.1$ fm

$\Rightarrow \quad \rho = \dfrac{A(1.67\times10^{-27})}{\frac{4}{3}\pi R_0^3 A}$

$\Rightarrow \quad \rho = 2.9\times10^{17}\ \text{kgm}^{-3}$

Hence, the correct answer is (B).

22. **If it was A**

$Y \to 2Z$

Reactant: $R = 60\times8.5 = 510$ MeV

Product: $P = 2\times30\times5 = 300$ MeV

$\Delta E = -210$ MeV

ENDOTHERMIC

If it was B

$W \to X + Z$

$R = 120\times7.5 = 900$ MeV

$P = 90\times8 + 30\times5 = 870$ MeV

$\Delta E = -30$ MeV

ENDOTHERMIC

If it was C

$W \to 2Y$

$R = 120\times7.5 = 900$ MeV

$P = 2\times60\times8.5 = 1020$ MeV

$\Delta E = 120$ MeV

EXOTHERMIC

If it was D

$X \to Y + Z$

$R = 90\times8.0 = 720$ MeV

$P = 60\times8.5 + 30\times5.0 = 660$ MeV

$\Delta E = -60$ MeV

ENDOTHERMIC

Hence, the correct answer is (C).

23. Number of nuclei decreases exponentially

$N = N_o e^{-\lambda t}$ and

Rate of decay $\left(-\dfrac{dN}{dt}\right) = \lambda N$

Therefore, decay process lasts upto $t \to \infty$ Therefore, a given nucleus may decay at any time after $t = 0$.

Hence, the correct answer is (D).

24. In beta decay, atomic number is increased by 1 whereas the mass number remains the same. Therefore, following equation can be possible

$${}^{64}_{29}\text{Cu} \to {}^{64}_{30}\text{Zn} + {}_{-1}e^{o}$$

Hence, the correct answer is (D).

25. Penetrating power is maximum for $\gamma-$rays, then of $\beta-$particles and then $\alpha-$particles because basically it depends on the velocity. However, ionization power is in reverse order.

Hence, the correct answer is (A).

26. Heavy water is used as moderators in nuclear reactors to slow down the neutrons.

Hence, the correct answer is (B).

27. ${}_1\text{H}^2 + {}_1\text{H}^2 \longrightarrow {}_1\text{H}^3 + p$

${}_1\text{H}^2 + {}_1\text{H}^3 \longrightarrow {}_2\text{He}^4 + n$

$\Rightarrow \quad 3\,{}_1\text{H}^2 \longrightarrow {}_2\text{He}^4 + p + n$

$\Delta m = m({}_2\text{He}^4) + m(p) + m(n) - 3m({}_1\text{H}^2)$

$\Rightarrow \quad \Delta m = [4.001 + 1.007 + 1.008 - 3(2.014)]\,u$

$\Rightarrow \quad \Delta m = -0.026\,u$

$\Rightarrow \quad |\Delta E| = c^2|\Delta m|$

$\Rightarrow \quad \Delta E = (9\times10^{16})(0.026\times1.67\times10^{-27})$

$\Rightarrow \quad \Delta E = (931.5)(0.026)$ MeV

$\Rightarrow \quad \Delta E = 3.87\times10^{-12}$ J

As each reaction involves 3 deutrons, so total number of reactions involved in the process $= \dfrac{10^{40}}{3}$. If each reaction produces an energy ΔE, then

$$E_{\text{total}} = \frac{10^{40}}{3}\Delta E = 1.29\times10^{28}\ \text{J}$$

$$E_{\text{total}} = Pt$$

Time of exhaustion of the star

$$t = \frac{1.29\times10^{28}}{10^{16}}$$

$\Rightarrow \quad t = 1.29\times10^{12}$ s

Hence, the correct answer is (C).

29. From $R = R_o\left(\dfrac{1}{2}\right)^n$

We have, $1 = 64\left(\dfrac{1}{2}\right)^n$

or $n = 6$ = number of half-lives.

$\therefore \quad t = n \times t_{1/2} = 6 \times 2 = 12$ h

Hence, the correct answer is (B).

32. Following nuclear reaction takes place

$${}_0n^1 \rightarrow {}_1\text{H}^1 + {}_{-1}e^o + \bar{v}$$

$\bar{v}$ is antineutrino.

Hence, the correct answer is (C).

33. β and $\gamma -$ decay take place from a radioactive nucleus.
Hence, options (A) and (B) are wrong. During fusion process two or more lighter nuclei combine to form a heavy nucleus.
Hence, the correct answer is (C).

34. Beta particles are fast moving electrons which are emitted by the nucleus.
Hence, the correct answer is (C).

35. Using $N = N_0 e^{-\lambda t}$ where $\lambda = \frac{\log_e 2}{t_{1/2}} = \frac{\log_e(2)}{3.8}$

$$\Rightarrow \frac{N_0}{20} = N_0 e^{-\frac{\log_e(2)}{3.8}t}$$

Solving this equation with the help of given data we find $t = 16.5$ days.

Hence, the correct answer is (B).

Multiple Correct Choice Type Problems

1. ${}_zX^A \xrightarrow{\alpha\text{-Decay}} {}_{Z-2}Y^{A-4}$

${}_zX^A \xrightarrow{\beta^-\text{-Decay}} {}_{Z+1}X_1^A$

${}_{90}\text{Th}^{232} \longrightarrow {}_{82}\text{Pb}^{212}$

Number of α-particles emitted $= \frac{232-212}{4} = 5$

Since Z decreases by $(90-82) = 8$ only

Hence number of β^- decay $= 2$

Hence, (A) and (C) are correct.

2. In fusion two or more lighter nuclei combine to make a comparatively heavier nucleus.
In fission, a heavy nucleus breaks into two or more comparatively lighter nuclei.
Further, energy will be released in a nuclear process if total binding energy increases.
Hence, (B) and (D) are correct.

3. Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less than the sum of masses of its constituent particles.

${}^{20}_{10}\text{Ne}$ is made up of 10 protons plus 10 neutrons.

Therefore, mass of ${}^{20}_{10}\text{Ne}$ nucleus,

$$M_1 < 10(m_p + m_n)$$

Also, heavier the nucleus, more is the mass defect.

Thus, $20(m_n + m_p) - M_2 > 10(m_p + m_n) - M_1$

$\Rightarrow \quad 10(m_p + m_n) > M_2 - M_1$

$\Rightarrow \quad M_2 < M_1 + 10(m_p + m_n)$

Now since, $M_1 < 10(m_p + m_n)$

$\Rightarrow \quad M_2 < 2M_1$

Hence, (C) and (D) are correct.

4. In nuclear fusion two or more lighter nuclei are combined to form a relatively heavy nucleus and thus, releasing the energy.
Hence, (A) and (D) are correct.

5. Cut off voltage is independent of intensity and hence remains the same. Since distance becomes 3 times, so I becomes $\frac{I}{9}$. Hence photocurrent also decreases by this factor i.e. becomes $\frac{18}{9} = 2$ mA

Hence, (B) and (D) are correct.

6. In case of ${}_1\text{H}^1$, mass number and atomic number are equal and in case of ${}_1\text{H}^2$, mass number is greater than its atomic number.
Hence, (C) and (D) are correct.

7. In fusion reaction, two or more lighter nuclei combine to form a comparatively heavier nucleus.
Hence, (B) and (C) are correct.

Comprehension Type Questions

1. $r = \frac{1-a}{1+a}$ ...(1)

$\Rightarrow \quad \ln r = \ln(1-a) - \ln(1+a)$

$$\Rightarrow \quad \frac{\Delta r}{r} = \frac{\Delta a}{1-a} + \frac{\Delta a}{1+a}$$

$$\Rightarrow \quad \frac{\Delta r}{r} = \frac{2\Delta a}{1-a^2}$$

Substituting value of r from equation (1), we get

$$(1+a)\frac{\Delta r}{(1-a)}=\frac{2(\Delta a)}{(1-a^2)}$$

$$\Rightarrow \quad \Delta r=\frac{2\Delta a}{(1+a)^2}$$

Hence, the correct answer is (B).

2. Let number of nuclei decayed be N

$$N=N_0\left(1-e^{-\lambda t}\right)$$

$$\lambda t=\ln\left(\frac{N_0}{N_0-N}\right)$$

$$\lambda t=\ln N_0-\ln(N_0-N)$$

$$(\Delta\lambda)t=\frac{dN}{(N_0-N)}$$

$$(\Delta\lambda)=\frac{40}{(3000-1000)}=0.02\ \text{s}^{-1}$$

Hence, the correct answer is (C).

3. $m\left({}_1^2\text{H}\right)+m\left({}_2^4\text{He}\right)=2.014102+4.002603$

$$\Rightarrow \quad m\left({}_1^2\text{H}\right)+m\left({}_2^4\text{He}\right)=6.016705\ \text{u}$$

Since, $m\left({}_3^6\text{Li}\right)=6.015123\ \text{u}$

$$\Rightarrow \quad m_1+m_2>M$$

So, (A) is incorrect.

$$m\left({}_1^1\text{H}\right)+m\left({}_{83}^{209}\text{Bi}\right)=1.007825+208.980388$$

$$m\left({}_1^1\text{H}\right)+m\left({}_{83}^{209}\text{Bi}\right)=209.988213\ \text{u}$$

Since, $m\left({}_{84}^{210}\text{Po}\right)=209.982876\ \text{u}$

$$\Rightarrow \quad m_1+m_2>M$$

So, B is incorrect

$$m\left({}_1^2\text{H}\right)+m\left({}_2^4\text{He}\right)=2.014102+4.002603$$

$$\Rightarrow \quad m\left({}_1^2\text{H}\right)+m\left({}_2^4\text{He}\right)=6.016705\ \text{u}$$

Since, ${}_3^6\text{Li}=6.015123\ \text{u}$

$$\Rightarrow \quad (m_3+m_4)>M'$$

So, (C) is correct and hence deuteron and alpha particle can go complete fusion.

$$m\left({}_{30}^{70}\text{Zn}\right)+m\left({}_{34}^{82}\text{Se}\right)=69.925325+81.916709$$

$$\Rightarrow \quad m\left({}_{30}^{70}\text{Zn}\right)+m\left({}_{34}^{82}\text{Se}\right)=151.842034\ \text{u}$$

Since, ${}_{64}^{152}\text{Gd}=151.919803\ \text{u}$

$$\Rightarrow \quad m_3+m_4<M'$$

So, (D) is incorrect.

Hence, the correct answer is (C).

4. ${}_{84}^{210}\text{Po}={}_{82}^{206}\text{Pb}+{}_2^4\text{He}+\Delta E$

$$m\left({}_{82}^{206}\text{Pb}\right)=205.974455\ \text{u}$$

$$m\left({}_2^4\text{He}\right)=4.002603\ \text{u}$$

$$\Rightarrow \quad m\left({}_{82}^{206}\text{Pb}\right)+m\left({}_2^4\text{He}\right)=209.977058\ \text{u}$$

Now, $\Delta m=209.977058-209.982876$

$$\Rightarrow \quad \Delta m=0.005818\ \text{u}$$

$$\Rightarrow \quad Q=\Delta E=0.005818\times 931.5$$

$$\Rightarrow \quad Q=5.419467\ \text{MeV}=5419.467\ \text{keV}$$

$$\Rightarrow \quad Q=5419.5\ \text{keV}$$

By Law of Conservation of Momentum, we have

$$0=p_\alpha-p_{\text{lead}}$$

$$\Rightarrow \quad p_\alpha=p_{\text{lead}}$$

$$\Rightarrow \quad \sqrt{2m_\alpha E_\alpha}=\sqrt{2m_{Pb}E_{Pb}}$$

$$\Rightarrow \quad 4E_\alpha=206E_{Pb}$$

$$\Rightarrow \quad E_\alpha=\frac{103}{2}E_{Pb}$$

Now, since $E_\alpha=\left(\dfrac{m_{Pb}}{m_{Pb}+m_\alpha}\right)Q$

$$\Rightarrow \quad E_\alpha=\left(\frac{206}{206+4}\right)Q$$

$$\Rightarrow \quad E_\alpha=\frac{103}{105}(5.422)=5319\ \text{MeV}$$

Hence, the correct answer is (A).

5. Maximum energy of the antineutrino will be nearly 0.8×10^6 eV

Hence, the correct answer is (C).

6. Minimum kinetic energy of electron can be zero or greater than zero. But maximum kinetic energy will be less than 0.8×10^6 eV.

Hence, the correct answer is (D).

8. From conservation of mechanical energy, we have

$$U_i+K_i=U_f+K_f$$

$$\Rightarrow \quad 0+2(1.5\ \text{kT})=\frac{1}{4\pi\varepsilon_0}\frac{(e)(e)}{d}+0$$

Substituting the values, we get

$$T=1.4\times10^9\ \text{K}$$

Hence, the correct answer is (A).

9. As given in the paragraph, a reactor is termed successful, if

$$nt_0 > 5\times10^{14}\ \text{s cm}^{-3}$$

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

3. A → (q)
B → (p)
C → (s)
D → (r)

(p) In α-decay mass number decrease by 4 and atomic number decreases by 2.

(q) In β^+-decay mass number remains unchanged while atomic number decreases by 1.

(r) In fission, parent nucleus breaks into all most two equally fragments.

(s) In proton emission both mass number and atomic number decreases by 1.

Integer/Numerical Answer Type Questions

1. $\Delta m = (226.005 - 222 - 4) = 0.005$ amu

$\Rightarrow\ Q = \Delta mc^2$

$\Rightarrow\ Q = 931.5\times0.005 = 4.655$ MeV

Since momentum is conserved, kinetic energy is in inverse ratio of masses.

$$K_T = 4.44 + K_{Rn}$$

$$\Rightarrow\ K_{Rn} = \frac{4.44\times4}{222} = 0.08\ \text{MeV}$$

$$E_{\gamma\text{-photon}} = 4.655 - 4.520$$

$$\Rightarrow\ E_{\gamma\text{-photon}} = 0.135\ \text{MeV} = 135\ \text{keV}$$

2. $$\text{I}^{131} \xrightarrow[T_{\frac{1}{2}}=8\text{ Days}]{} \text{Xe}^{131} + \beta$$

$A_0 = 2.4\times10^5\ \text{Bq} = \lambda N_0$

Let the volume is V,

$t = 0 \qquad A_0 = \lambda N_0$

$t = 11.5\text{ h} \qquad A = \lambda N$

$$115 = \lambda\left(\frac{N}{V}\times2.5\right)$$

$$\Rightarrow\ 115 = \frac{\lambda}{V}\times2.5\times\left(N_0e^{-\lambda t}\right)$$

$$\Rightarrow\ 115 = \frac{(N_0\lambda)}{V}\times(2.5)\times e^{-\frac{\ln2}{8\text{ day}}(11.5\text{ h})}$$

$$\Rightarrow\ 115 = \frac{(2.4\times10^5)}{V}\times(2.5)\times e^{-\frac{1}{24}}$$

$$\Rightarrow\ V = \frac{2.4\times10^5}{115}\times2.5\left[1-\frac{1}{24}\right]$$

$$\Rightarrow\ V = \frac{2.4\times10^5}{115}\times2.5\left[\frac{23}{24}\right]$$

$$\Rightarrow\ V = \frac{10^5\times23\times25}{115\times10^2} = 5\times10^3\ \text{ml} = 5\ \text{L}$$

3. $$\text{Q value} = \left[12.014u - \left(12u + 4.041\frac{\text{MeV}}{c^2}\right)\right]c^2$$

$$\Rightarrow\ Q = (0.014u\times931.5)\text{MeV} - (4.041)\text{MeV} = 9\ \text{MeV}$$

Hence, β particle will have a maximum KE of 9 MeV

4. Let initial numbers are N_1 and N_2.

$$\frac{\lambda_1}{\lambda_2} = \frac{\tau_2}{\tau_1} = \frac{2\tau}{\tau} = 2 = \frac{T_2}{T_1} \qquad (T = \text{Half life})$$

$$A = \frac{-dN}{dt} = \lambda N$$

Initial activity is same

$\Rightarrow\ \lambda_1N_1 = \lambda_2N_2$

Activity at time t,

$$A = \lambda N = \lambda N_0e^{-\lambda t}$$

$$A_1 = \lambda_1N_1e^{-\lambda_1t}$$

$$\Rightarrow\ R_1 - = \frac{dA_1}{dt} = \lambda_1^2N_1e^{-\lambda_1t}$$

Similarly, $R_2 = \lambda_2^2N_2e^{-\lambda_2t}$

After $t = 2\tau$

$$\lambda_1t = \frac{1}{\tau_1}(t) = \frac{1}{\tau}(2\tau) = 2$$

$$\lambda_2t = \frac{1}{\tau_2}(t) = 1 = \frac{1}{2\tau}(2\tau) = 1$$

$$\frac{R_P}{R_Q} = \frac{\lambda_1^2N_1e^{-\lambda_1t}}{\lambda_2^2N_2e^{-\lambda_2t}}$$

$$\Rightarrow\ \frac{R_P}{R_Q} = \frac{\lambda_1}{\lambda_1}\left(\frac{e^{-2}}{e^{-1}}\right) = \frac{2}{e}$$

5. Let initial power available from the plant is P_0. After time $t = nT$ or n half lives, this will become $\left(\frac{1}{2}\right)^n P_0$. Now, it is given that, $\left(\frac{1}{2}\right)^n P_0 = 12.5\%$ of $P_0 = (0.125)P_0$

Solving this equation we get, $n = 3$

6. Since, $N = N_0 e^{-\lambda t}$

$$\Rightarrow \frac{N}{N_0} = e^{-\lambda t}$$

$$\Rightarrow \frac{N}{N_0} = e^{-\frac{\log_e 2}{1386}\times 80}$$

$$\Rightarrow \frac{N}{N_0} = e^{-\frac{0.693\times 80}{1386}}$$

$$\Rightarrow \frac{N}{N_0} = e^{-0.04}$$

$$\Rightarrow \frac{N}{N_0} = \left(\frac{1}{e}\right)^{0.04}$$

Fraction of nuclei decayed is

$$1 - \frac{N}{N_0} = 1 - \left(\frac{1}{e}\right)^{0.04} = 0.04 = 4\%$$

7. $\frac{dN}{dt} = -\lambda N = -10^{10}$

$$\Rightarrow N = 10^{10} \times \left(\frac{1}{\lambda}\right)$$

$$\Rightarrow m_{\text{total}} = N \times m_1$$

$$\Rightarrow m_{\text{total}} = 10^{10} \times 10^9 \times 10^{-25} \times 10^6 \text{ mg}$$

$$\Rightarrow m_{\text{total}} = 1 \text{ mg}$$

8. $\left|\frac{dN}{dt}\right| = |\text{Activity of radioactive substance}|$

$$= \lambda N = \lambda N_0 e^{-\lambda t}$$

Taking log both sides

$$\ln\left|\frac{dN}{dt}\right| = \ln(\lambda N_0) - \lambda t$$

Hence, $\ln\left|\frac{dN}{dt}\right|$ versus t graph is a straight line with slope $-\lambda$.

From the graph we can see that,

$$\lambda = \frac{1}{2} = 0.5 \text{ yr}^{-1}$$

Now applying the equation,

$$N = N_0 e^{-\lambda t} = N_0 e^{-0.5\times 4.16}$$

$$\Rightarrow N = N_0 e^{-2.08} = 0.125 N_0 = \frac{N_0}{8}$$

i.e., nuclei decrease by a factor of 8.

Hence, the answer is 8.