ICON

CHAPTER 1

Dual Nature of Radiation and Matter

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Matter waves
(b) de-Broglie's Hypothesis
(c) Photon properties
(d) Radiation pressure
(e) Photoelectric emission
(f) Photoelectric effect and properties.

All this is followed by an Exercise Set (fully solved) which contains questions as per the latest JEE pattern. At the end of Exercise Set, a collection of problems asked previously in JEE Main are also given.

MATTER WAVES

Light possesses dual nature i.e. it behaves both as a wave and as a particle. In some phenomena e.g., interference, diffraction and polarisation, it behaves as waves because they are explained on the basis of Wave theory while in some other phenomena e.g. photoelectric effect, Compton effect, it behaves as particles (photons).

Since nature demands symmetry, therefore de-Broglie thought that matter must have dual nature. The particle nature of matter is well known and hence de-Broglie thought that material particles must possess wave-nature.

de-BROGLIE'S POSTULATE

According to de-Broglie a material particle in motion must have a wave like character and the wavelength associated with it is given by

$$\lambda = \frac{h}{p} \qquad \ldots(1)$$

where, h is the Planck's constant whose value is given by $h = 6.63 \times 10^{-34}$ Js and p is the momentum of the particle.

de-Broglie assumed this expression in analogy with photon because momentum of photon is

$$p = \frac{h}{\lambda}$$

$$\Rightarrow \quad \lambda = \frac{h}{p}$$

If m is the mass of particle and v the velocity, then momentum of particle is $p = mv$.

So, de-Broglie wavelength $\lambda = \dfrac{h}{mv}$ ...(2)

If E_k is kinetic energy of particle, then

$$p = \sqrt{2mE_K} \qquad \left\{\because E_K = \frac{p^2}{2m}\right\}$$

$$\Rightarrow \quad \lambda = \frac{h}{\sqrt{2mE_K}} \qquad \ldots(3)$$

For charged particles accelerated through a potential difference of V volts,

Kinetic energy i.e. $E_K = qV$

$$\Rightarrow \quad \lambda = \frac{h}{\sqrt{2mqV}} \quad \ldots(4)$$

For electrons accelerated through a potential difference of V volts de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2meV}}$.

Substituting $m = 9 \cdot 1 \times 10^{-31}$ kg, $h = 6 \cdot 62 \times 10^{-34}$ Js, $e = 1 \cdot 6 \times 10^{-19}$ C, we get

$$\lambda = \sqrt{\frac{150}{V}} \times 10^{-10} \quad \ldots(5)$$

$$\Rightarrow \quad \lambda = \sqrt{\frac{150}{V}}\ \text{Å} = \frac{12.27}{\sqrt{V}}\ \text{Å}$$

This is expression for de-Broglie wavelength associated with electron accelerated through a potential difference of V. The wave nature is possessed by all particles neutral of charged. The wave nature was first verified by Davisson and Germer for slow electrons.

CHARACTERISTICS OF MATTER WAVES

(a) Matter wave represents the probability of finding a particle in space.

(b) Matter waves are not electromagnetic in nature.

(c) de-Broglie or matter wave is independent of the charge on the material particle. It means, matter wave of de-Broglie wave is associated with every moving particle (whether charged or uncharged).

(d) Practical observation of matter waves is possible only when the de-Broglie wavelength is of the order of the size of the particles.

(e) Electron microscope works on the basis of de-Broglie waves.

(f) The phase velocity of the matter waves can be greater than the speed of the light.

(g) Matter waves can propagate in vacuum, hence they are not mechanical waves.

(h) The number of de-Broglie waves associated with n^{th} orbital electron is n.

(i) Only those circular orbits around the nucleus are stable whose circumference is integral multiple of de-Broglie wavelength associated with the orbital electron.

de-BROGLIE WAVELENGTH ASSOCIATED WITH THE CHARGED PARTICLES

The energy of a charged particle accelerated through potential difference V is $E = \frac{1}{2}mv^2 = qV$

Hence de-Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$$

Using the above formula, we get

$$\lambda_{\text{Electron}} = \frac{12.27}{\sqrt{V}}\ \text{Å}$$

$$\lambda_{\text{Proton}} = \frac{0.286}{\sqrt{V}}\ \text{Å}$$

$$\lambda_{\text{Deutron}} = \frac{0.202}{\sqrt{V}}\ \text{Å}$$

$$\lambda_{\alpha-\text{particle}} = \frac{0.101}{\sqrt{V}}\ \text{Å}$$

ILLUSTRATION 1

Find the ratio of de Broglie wavelength of proton and α-particle which have been accelerated through same potential difference.

SOLUTION

Kinetic energy gained by a charge q after being accelerated through a potential difference V volt is

$$qV = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2qV}{m}}$$

$$\Rightarrow \quad mv = \sqrt{2mqV}$$

So, the de Broglie wavelength is given by

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$$

$$\Rightarrow \quad \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}}$$

$$\Rightarrow \quad \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{4 \times 2}{1 \times 1}} = 2\sqrt{2}$$

de-BROGLIE WAVELENGTH ASSOCIATED WITH UNCHARGED PARTICLES: THERMAL NEUTRONS

Thermal neutrons are generally in thermal equilibrium with the surroundings. For thermal neutrons at temperature T, kinetic energy of most of neutrons i.e. the most probable energy is $E_K = k_B T$, where k_B is the Boltzmann's constant. This energy is 0.025 eV for neutrons at 27 °C. So the de-Broglie wavelength corresponding to most of the thermal neutrons is the wavelength that corresponds to the most probable energy of the thermal neutrons at room temperature. $\lambda = \frac{h}{\sqrt{2mk_BT}}$, where T is the Absolute temperature, k_B is the Boltzmann's constant given by $k_B = 1.38 \times 10^{-23}$ JK^{-1}.

However, the average kinetic energy of thermal neutrons is $E_{av} = \frac{3}{2}k_BT$, hence the de-Broglie wavelength corresponding to this energy is $\lambda = \frac{h}{\sqrt{3mk_BT}}$.

For thermal neutrons de-Broglie wavelength can also be given by the expression

$$\lambda_{\text{Neutron}} = \frac{0.286 \times 10^{-10}}{\sqrt{E(\text{in eV})}} \text{ m} = \frac{0.286}{\sqrt{E(\text{in eV})}} \text{ Å}$$

So, $\lambda_{\text{Thermal neutron}} = \dfrac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1.38 \times 10^{-23} T}}$

$$\Rightarrow \quad \lambda_{\text{Thermal neutron}} = \frac{30.9}{\sqrt{T}} \text{ Å}$$

ILLUSTRATION 2

Obtain the de-Broglie wavelength associated with thermal neutrons at room temperature (27 °C). Hence explain why a fast neutron beam needs to be thermalised with the environment before it can be used for neutron diffraction experiments.

SOLUTION

Kinetic energy of most of the neutrons i.e. the most probable energy of a neutron at temperature T is

$$\frac{1}{2}mv^2 = k_BT$$

$$\Rightarrow \quad \frac{p^2}{2m} = k_BT \qquad \{\because p = mv\}$$

$$\Rightarrow \quad p = \sqrt{2mk_BT}$$

de-Broglie wavelength possessed by most of the neutrons is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mk_BT}}$$

Given $m_n = 1.675 \times 10^{-27}$ kg, $k_B = 1.38 \times 10^{-23}$ Jmol^{-1}K^{-1}

$$T = 27 + 273 = 300 \text{ K}, \; h = 6.63 \times 10^{-34} \text{ Js}$$

$$\Rightarrow \quad \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}}$$

$$\Rightarrow \quad \lambda = 1.78 \times 10^{-10} \text{ m}$$

$$\Rightarrow \quad \lambda \approx 1.8 \text{ Å}$$

Since this wavelength is comparable to interatomic spacing in a crystal, so thermal neutrons can be used for diffraction experiments. A high energy neutron beam should be first thermalised before using it for diffraction experiments.

ILLUSTRATION 3

Find the de Broglie wavelength corresponding to the root-mean-square velocity of hydrogen molecules at room temperature (20 °C).

SOLUTION

Since, $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$

$$v_{\text{rms}} = \sqrt{\frac{3 \times 8.31 \times 293}{2 \times 10^{-3}}} = 1911 \text{ ms}^{-1}$$

Now, $\lambda = \dfrac{h}{p} = \dfrac{h}{mv_{\text{rms}}}$

Mass of one hydrogen molecule is given by

$$m = \frac{2}{6.02 \times 10^{26}} \text{ kg} = 3.32 \times 10^{-27} \text{ kg}$$

$$\Rightarrow \quad \lambda = \frac{6.63 \times 10^{-34}}{3.32 \times 10^{-27} \times 1911} \text{ m}$$

$$\Rightarrow \quad \lambda = 1.04 \times 10^{-10} \text{ m} = 1.04 \text{ Å}$$

HEISENBERG'S UNCERTAINTY PRINCIPLE

According to Heisenberg it is impossible to measure the position and momentum of a particle simultaneously with 100% accuracy. This is called Heisenberg's uncertainty principle. Uncertainty principle successfully explains the

(i) Non-existence of electrons in the nucleus
(ii) Finite size of spectral lines.

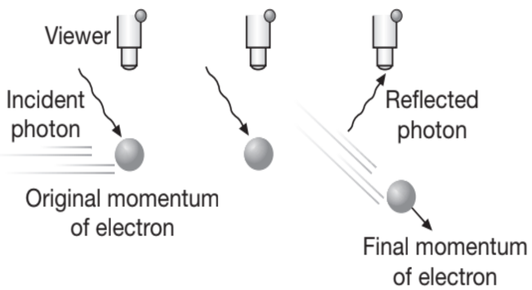
If Δx and Δp are uncertainties in determining the position and momentum of the particle simultaneously, then

$$\Delta x \Delta p \geq \frac{h}{4\pi}, \text{ where } h = 6.63 \times 10^{-34} \text{ Js}$$

$$\Rightarrow \quad m\Delta x \Delta v \geq \frac{h}{4\pi}$$

This principle is universal and holds for all microscopic and macroscopic particles. The principle is also unaffected by experimental techniques.

If $\Delta x = 0$, then $\Delta p \to \infty$ and if $\Delta p = 0$ then $\Delta x \to \infty$,i.e. if we are able to measure the exact position of the particle (say an electron) then the uncertainty in the measurement of the linear momentum of the particle is infinite. Similarly, if we are able to measure the exact linear momentum of the particle i.e., $\Delta p = 0$, then we cannot measure the exact position of the particle at that time.



An electron cannot be observed without changing it's momentum.

This principle is also applicable to energy and time, angular momentum and angular displacement. Hence,

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

$$\Delta L \Delta \theta \geq \frac{h}{4\pi}$$

Problem Solving Technique(s)

(a) For numerical problems, we shall use

$$\Delta x \Delta p \approx h \qquad \Delta E \Delta t \approx h \qquad \Delta L \Delta \theta \approx h$$

(b) If the radius of the nucleus is r then the probability of finding the electron inside the nucleus is $\Delta x = 2r$ and uncertainty in momentum is $\Delta p = \frac{h}{4\pi r}$.

QUANTUM NATURE OF LIGHT AND PLANK'S QUANTUM THEORY

Some phenomena like photoelectric effect, Compton effect, Raman effect could not be explained by Wave theory of light. Therefore, quantum theory of light was proposed by Einstein who extended the Planck's hypothesis to explain Black Body radiation. According to quantum theory of light or radiation, the energy of an electromagnetic wave is not continuously distributes over the wave front (just like the energy possessed by water waves). Instead Plank proposed that an electro-magnetic wave travels in the form of discrete packets or bundles of energy called ***Quanta.***

So, according to Plank, **"light is propagated in bundles of small energy, each bundle being called a photon and possessing energy", given by**

$$E = h\nu = \frac{hc}{\lambda}$$

where, ν is frequency, λ is wavelength of light, h is Planck's constant whose value is $6 \cdot 63 \times 10^{-34}$ Js and $c = 3 \times 10^{8}$ ms^{-1}

ILLUSTRATION 4

An α particle and a proton are fired through the same magnetic fields which is perpendicular to their velocity vectors. The α particle and the proton move such that radius of curvature of their path is same. Find the ratio of their de Broglie wavelengths.

SOLUTION

Magnetic force experienced by a charged particle in a magnetic field is given by,

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$\Rightarrow \quad F = qvB\sin\theta$$

In this case $\theta = 90^\circ$, so we get $F = qvB$ and this magnetic force is responsible for providing the centripetal force to the charged particle to move in a circle.

$$\Rightarrow \quad qvB = \frac{mv^2}{r}$$

$$\Rightarrow \quad mv = qBr$$

Hence the de Broglie wavelength is given by

$$\lambda = \frac{h}{mv} = \frac{h}{qBr}$$

$$\Rightarrow \quad \frac{\lambda_{\alpha\text{-particle}}}{\lambda_{\text{proton}}} = \frac{q_p r_p}{q_\alpha r_\alpha}$$

According to the problem, we have

$$\frac{r_\alpha}{r_p} = 1 \text{ and } \frac{q_\alpha}{q_p} = 2$$

$$\Rightarrow \quad \frac{\lambda_\alpha}{\lambda_p} = \frac{1}{2}$$

ILLUSTRATION 5

What amount of energy should be added to an electron to decrease its de Broglie wavelength from 100 pm to 50 pm?

SOLUTION

According to de Broglie relation, initial momentum of the electron is

$$p_1 = \frac{h}{\lambda_1} = \frac{6.63\times10^{-34}}{10^{-10}}$$

$$\Rightarrow \quad p_1 = 6.63\times10^{-24} \text{ Js}$$

The final momentum of the electron is

$$p_2 = \frac{h}{\lambda_2} = \frac{6.63\times10^{-34}}{0.5\times10^{-10}}$$

$$\Rightarrow \quad p_2 = 13.26\times10^{-24} \text{ Js}$$

Since kinetic energy of the electron is

$$E = \frac{p^2}{2m_e}$$

So, energy added to electron is

$$\Delta E = \frac{p_2^2 - p_1^2}{2m_e}$$

$$\Rightarrow \quad \Delta E = \frac{\left[(13.26)^2 - (6.63)^2\right]\times10^{-48}}{2\times9.1\times10^{-31}\times1.6\times10^{-19}}$$

$$\Rightarrow \quad \Delta E \approx 453 \text{ eV}$$

ILLUSTRATION 6

Two identical nonrelativistic particles move at right angles to each other, possessing de Broglie wavelengths, λ_1 and λ_2. Find the de Broglie wavelength of each particle in the frame of their centre of mass.

SOLUTION

Initial momentum of each particle is

$$\vec{p}_1 = \left(\frac{h}{\lambda_1}\right)\hat{i} \text{ and } \vec{p}_2 = \left(\frac{h}{\lambda_2}\right)\hat{j}$$

$$\Rightarrow \quad \vec{v}_1 = \left(\frac{h}{m\lambda_1}\right)\hat{i} \text{ and } \vec{v}_2 = \left(\frac{h}{m\lambda_2}\right)\hat{j}$$

Velocity of center of mass is gives as

$$\vec{v}_{\text{cm}} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \frac{h}{2m}\left(\frac{1}{\lambda_1}\hat{i} + \frac{1}{\lambda_2}\hat{j}\right)$$

Momentum of particles in frame of center of mass is

$$\vec{p}_{1c} = m(\vec{v}_1 - \vec{v}_{\text{cm}}) = \frac{h}{2}\left(\frac{1}{\lambda_1}\hat{i} - \frac{1}{\lambda_2}\hat{j}\right)$$

and $$\vec{p}_{2c} = m(\vec{v}_2 - \vec{v}_{\text{cm}}) = \frac{h}{2}\left(\frac{1}{\lambda_2}\hat{j} - \frac{1}{\lambda_1}\hat{i}\right)$$

$$|\vec{p}_{1c}| = |\vec{p}_{2c}| = \frac{h}{2}\frac{\sqrt{\lambda_1^2 + \lambda_2^2}}{\lambda_1\lambda_2}$$

de Broglie wavelength of particles in frame of their centre of mass is

$$\lambda_{1c} = \frac{h}{p_{1c}} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$$

and $$\lambda_{2c} = \frac{h}{p_{2c}} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$$

PROPERTIES OF PHOTONS

1. In the interaction of radiation with matter, the radiation behaves as if it is made up of particles called Photons. This fundamental is also called as Corpuscular Theory of Light. So, light behaves both as a particle and a wave.
2. All photons emitted by any source travel through free space with a speed equal to the speed of light i.e. $c = 3\times10^8 \text{ ms}^{-1}$.
3. Each photon has a definite energy depending upon the frequency ν of the radiation and this energy is independent of the intensity. So,

$$E = h\nu = \frac{hc}{\lambda} \text{ (in joule)}$$

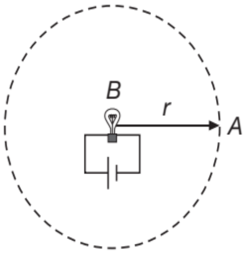
4. If λ is in Å, then $E = \frac{12375}{\lambda}$ eV.

5. If the intensity of the light of given wavelength is increased, then there is an increase in the number of photons incident per second per unit area on a surface. However, energy of the photon remains the same as long as the frequency or the wavelength is unchanged.
6. The speed of the photon changes as it travels through different media due to the change in its wavelength.
7. The frequency of the photon does not change when it goes from one medium to the other.
8. ***In the situations when a photon collides with a material particle, the total energy and momentum remains conserved. However, the number of photons may not be conserved in a collision because during the collision photon(s) may be absorbed or new photon(s) may be created.***
9. A photon is an electrically neutral particle which is not deflected by electric and magnetic field.
10. **Rest mass of photon is zero.**
Since the mass of a particle m moving with a speed v is given by
$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$
where m_0 is the rest mass of the particle
$$\Rightarrow \quad m_0 = m\sqrt{1-\frac{v^2}{c^2}}$$
Now, for a photon, $v = c$, hence
$$m_0 = 0$$
So, from here we can conclude that the rest mass of a photon is always zero, i.e. we cannot have a frame of reference where the photon is at rest.
11. **Dynamic or kinetic mass of a photon,** is determined by using the Einstien's Mass-Energy Equivalence, i.e.,
$$E = h\nu = mc^2$$
$$\Rightarrow \quad m = \frac{h\nu}{c^2} = \frac{h}{c\lambda}$$
12. **The Linear Momentum of a photon** is found by using the de-Broglie relation according to which, we have
$$\lambda = \frac{h}{p}, \text{ where } \lambda = \frac{c}{\nu}$$
$$\Rightarrow \quad p = \frac{h\nu}{c} = \frac{h}{\lambda}$$
However, this result is also obtained by using the fact that the total energy of a subatomic particle of rest mass m_0, moving with a velocity v, having momentum p is given by
$$E^2 = p^2c^2 + m_0^2c^4$$
Now, for a photon, $m_0 = 0$, so we have from above expression that
$$E = pc$$
Since, $E = h\nu = \frac{hc}{\lambda}$
$$\Rightarrow \quad pc = \frac{hc}{\lambda}$$
$$\Rightarrow \quad p = \frac{h\nu}{c} = \frac{h}{\lambda}$$
13. The number of photons N, each of energy E, emitted from a source of monochromatic radiation of wavelength λ and energy W and power P
$$N = \frac{W}{E} = \frac{W}{h\nu} = \frac{Pt}{h\nu}$$
14. **Intensity of light (I)**
Energy crossing per unit area normally per second is called intensity or energy flux, i.e.
$$I = \frac{E}{At} = \frac{P}{A} \qquad \left(\frac{E}{t} = P = \text{radiation power}\right)$$
At a distance r from a point source of power P intensity is given by
$$I = \frac{P}{4\pi r^2}$$
$$\Rightarrow \quad I \propto \frac{1}{r^2}$$
15. **Number of photons falling per second (n)**
If P is the power of radiation and E is the energy of a photon then $n = \frac{P}{E}$.

NUMBER OF PHOTONS EMITTED PER SECOND BY A SOURCE

Consider a light bulb B having power P watt as shown in figure. If the wavelength of light emitted by the bulb is λ, then energy of each photon emitted by the bulb is

$$E_{\text{single photon}} = \frac{hc}{\lambda}$$



Since the power of bulb is P watt, so we can say that bulb is emitting light of energy P joule in one second in the form of photons (assuming the efficiency of the bulb to be 100%). Then the number of photons emitted per second (i.e. n) by the source (bulb) will be

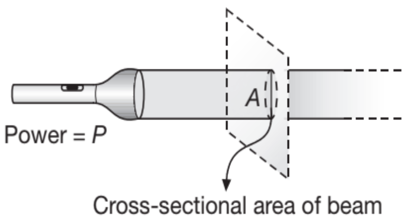
$$n = \frac{N}{t} = \frac{P}{E_{\text{single photon}}} = \frac{P\lambda}{hc}$$

where N is the total number of photons emitted by the source in time t.

These all photons are assumed to be emitted uniformly in all directions if bulb B is assumed to be a point isotropic source of light due to which we can consider that all the light energy emitted by the source is uniformly distributed in the spherical region with centre at the source i.e. bulb B. Also, it is observed that the radius of this sphere will uniformly increase at a rate of $c = 3\times10^8\ \text{ms}^{-1}$ in free space, because the light propagates in free space at a speed c.

INTENSITY OF LIGHT DUE TO A LIGHT SOURCE

Consider a torch that emits a uniform cylindrical beam of light having power P, cross-sectional area A as shown in figure.

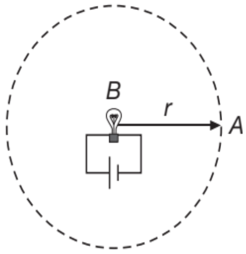


Since intensity (I) is defined as the energy emitted per second per unit area of the source normal to the direction of propagation of light, so we have

$$I = \frac{E_{\text{photon beam}}}{At} = \frac{P}{A}$$

Since the cross-sectional area of the beam is constant throughout, so the beam intensity at every point remains constant.

Similarly, we can find the intensity of light due to a point isotropic source. Consider a light bulb B having power P watt emitting light in all directions uniformly as shown in figure.



If we wish to find light intensity at a point A, at a distance r from the bulb B, then it can be given as

$$I = \frac{E_{\text{photon source}}}{At} = \frac{P}{4\pi r^2} \qquad \ldots(1)$$

Here we have assumed that the entire power P of the spherical source of light is incident normally on the hypothetical spherical surface of radius r with bulb at the centre of the sphere.

PHOTON FLUX IN A LIGHT BEAM

Photon flux ϕ_N is defined as the number of photons incident on a surface per second per unit area of the surface held normally to the direction of propagation of the light beam. If a light beam of intensity I having wavelength λ is incident on a surface, then the number of photons per second per unit area (i.e. photon flux ϕ_N) is given by

$$\phi_N = \frac{N}{At} = \frac{1}{A}\left(\frac{N}{t}\right) = \frac{n}{A}$$

Since, we know that the number of photons emitted by the source per second is given by $n = \frac{P\lambda}{hc}$. So, we have

$$\phi_N = \frac{n}{A} = \frac{1}{A}\left(\frac{P\lambda}{hc}\right) = \left(\frac{P}{A}\right)\left(\frac{\lambda}{hc}\right) = \frac{I\lambda}{hc}$$

If we consider a point source of power P which emits light in all directions then the number of photons emitted per second by this point source is given by

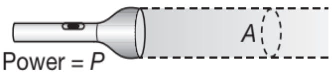
$$n = \frac{P\lambda}{hc}$$

Since these all photons are distributed in the three-dimensional spherical space around the source, so the photon flux at a distance r from the point source, is given by

$$\phi_N = \frac{n}{4\pi r^2}$$

PHOTON DENSITY IN A LIGHT BEAM

When photons are emitted by a light source, they move away from the source with speed of light. Consider a uniform cylindrical beam of light as shown in figure.



If a torch having a power P is producing a uniform light beam of cross-sectional area A, then the intensity of light beam is given by

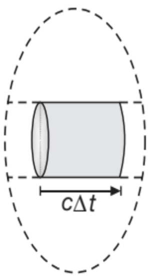
$$I = \frac{P}{A}$$

The photon flux at the cross-sectional area A of light beam is

$$\phi_N = \frac{I\lambda}{hc}$$

where λ is the wavelength of light.

The distance covered by these photons in time Δt is $c\Delta t$ and all the photons will lie in the shaded volume $A(c\Delta t)$ as shown in the figure.



Thus, photon density in the light beam is defined as the number of photons per unit volume.

$$\rho_{\text{ph}} = \frac{N}{A(c\Delta t)} = \left(\frac{N}{\Delta t}\right)\frac{1}{Ac} = \frac{1}{c}\left(\frac{n}{A}\right) = \frac{\phi_N}{c}$$

Since, $\phi_N = \frac{n}{A} = \frac{I\lambda}{hc}$

$$\Rightarrow \quad \rho_{\text{ph}} = \frac{\phi_N}{c} = \frac{I\lambda}{hc^2}$$

As the beam is uniform and cylindrical, the photon density throughout the beam remains, constant and at any point in space photon density can be given as

$$\rho_{\text{ph}} = \frac{\phi_N}{c} = \frac{\text{photon flux}}{\text{speed of light}}$$

Similarly, for a point isotropic source of light we can say that as the emitted photons move away from the source, the distance between photons increases and the photon density decreases.

If we wish to find photon density at a distance r from a point source of light of power P watt, then we first find the photon flux at a distance r from the source is given by

$$\phi_N = \frac{n}{A} = \frac{\left(\frac{P\lambda}{hc}\right)}{4\pi r^2}$$

So, the photon density at a distance r from the point source is given by

$$\rho_{\text{ph}} = \frac{\phi_N}{c} = \frac{P\lambda}{4\pi r^2 hc^2}$$

ILLUSTRATION 7

Find the energy, the mass and the momentum of a photon of ultraviolet radiation of 280 nm wavelength.

SOLUTION

Given, $\lambda = 280 \times 10^{-9}$ m

Since, $E = \frac{hc}{\lambda}$

$$\Rightarrow \quad E = \frac{(4.316 \times 10^{-15}\text{ eV sec})(3 \times 10^8\text{ ms}^{-1})}{(280 \times 10^{-9}\text{ m})} = 4.6\text{ eV}$$

Mass of photon is $m = \frac{E}{c^2}$

$$\Rightarrow \quad m = \frac{4.6 \times 1.6 \times 10^{-19}}{(3 \times 10^8)^2} = 8.2 \times 10^{-36}\text{ kg}$$

Momentum of a photon is

$$p = \frac{E}{c}$$

$$\Rightarrow \quad p = \frac{4.6 \times 1.6 \times 10^{-19}}{3 \times 10^8} = 2.45 \times 10^{-27}\text{ kg ms}^{-1}$$

ILLUSTRATION 8

The intensity of sunlight on the surface of earth is $1400\ \text{Wm}^{-2}$. Assuming the mean wavelength of sunlight to be $6000\ \text{Å}$, calculate

(a) the photon flux arriving at $1\ \text{m}^2$ area on earth perpendicular to light radiations, and
(b) the number of photons emitted from the sun per second assuming the average radius of Earth's orbit is 1.49×10^{11} m.

SOLUTION

(a) Energy of a photon

$$E=\frac{hc}{\lambda}=\frac{12400}{6000}=2.06\ \text{eV}=3.3\times0^{-19}\ \text{J}$$

$$\text{Photon flux}=\frac{IA}{E}=\frac{(1400)(1)}{3.3\times10^{-19}}=4.22\times10^{21}$$

(b) $n=\dfrac{P}{E}=\dfrac{I(4\pi R^2)}{E}$

$$\Rightarrow\quad n=\frac{(1400)(4\pi)(1.49\times10^{11})^2}{3.3\times10^{-19}}=1.18\times10^{45}$$

ILLUSTRATION 9

A small plate of a metal is placed at a distance of 2 m from a monochromatic light source of wavelength 4.8×10^{-7} m and power 1 watt. The light falls normally on the plate. Find the number of photons striking the metal plate per square meter per second.

SOLUTION

$$E=\frac{hc}{\lambda}=\frac{(6.6\times10^{-34})(3\times10^{8})}{(4.8\times10^{-7})}=4.125\times10^{-19}\ \text{J}$$

Number of photons striking the metal plate per square meter per second is

$$n=\left(\frac{P}{E}\right)\left(\frac{1}{4\pi r^2}\right)$$

$$\Rightarrow\quad n=\left(\frac{1}{4.125\times10^{-19}}\right)\frac{1}{(4\pi)(2)^2}=4.82\times10^{16}\ \text{m}^{-2}\text{s}^{-1}$$

ILLUSTRATION 10

Find the number of photons entering the pupil of our eye per second corresponding to the minimum intensity of white light that we humans can perceive $(-10^{-10}\ \text{Wm}^{-2})$. Take the area of the pupil to be about $0.4\ \text{cm}^2$ and the average frequency of white light to be about 6×10^{14} Hz.

SOLUTION

Minimum intensity, $I=10^{-10}\ \text{Wm}^{-2}$

Area of pupil, $A=0.4\ \text{cm}^2=0.4\times10^{-4}\ \text{m}$

Average frequency, $\nu=6\times10^{-14}$ Hz

Energy of one photon

$$E=h\nu=6.63\times10^{-34}\times6\times10^{14}\ \text{J}=4\times10^{-19}\ \text{J}$$

Let n be the number of photons crossing per square metre area per second.

Now Intensity = Energy incident per square metre area per second

$\Rightarrow\quad I=\text{Total Energy of } n \text{ photons}$

$\Rightarrow\quad I=n\times\text{Energy of one photon}$

$$\Rightarrow\quad n=\frac{\text{Intensity}}{\text{Energy of one photon}}$$

$$\Rightarrow\quad I=\frac{10^{-10}\ \text{Wm}^{-2}}{4\times10^{-19}\ \text{J}}=2.5\times10^{8}\ \text{m}^{-2}\ \text{s}^{-1}$$

So, the number of photons entering the pupil of our eye per second is

$$N=n(\text{Area of the pupil})$$

$$\Rightarrow\quad N=2.5\times10^{8}\times0.4\times10^{-4}\ \text{s}^{-1}=10^{4}\ \text{s}^{-1}$$

ILLUSTRATION 11

Find the number of photons emitted per second by a 25 W source of monochromatic light of wavelength 6600 Å. What is the photoelectric current assuming 3% efficiency for photoelectric effect? Given $h=6.6\times10^{-34}$ Js.

SOLUTION

Energy of each photon is

$$E=\frac{hc}{\lambda}=\frac{6.6\times10^{-34}\times3\times10^{8}}{6600\times10^{-10}}=3\times10^{-19}\ \text{J}$$

Total energy emitted per second by 25 W source is

$$E=25\ \text{J}$$

Number of photons emitted per second is

$$n=\frac{25}{3\times10^{-19}}=8.33\times10^{19}$$

Photoelectric current (I) is

$$I=\begin{pmatrix}\text{3\% of photons}\\ \text{emitted per second}\end{pmatrix}\times\begin{pmatrix}\text{charge on}\\ \text{electron}\end{pmatrix}$$

$$\Rightarrow \quad I=\frac{3}{100}\times\frac{25}{3\times10^{-19}}\times1.6\times10^{-19}=0.4\text{ A}$$

ILLUSTRATION 12

A source emits monochromatic light of frequency 5.5×10^{14} Hz at a rate of 0.1 W. Of the photons given out, 0.15% fall on the cathode of a photocell which gives a current of $6\ \mu\text{A}$ in an external circuit.

(a) Find the energy of a photon.
(b) Find the number of photons leaving the source per second.
(c) Find the percentage of the photons falling on the cathode which produce photoelectrons.

SOLUTION

(a) Since,

$$E=h\nu=(6.6\times10^{-34})(5.5\times10^{14})=36.3\times10^{-20}\text{ J}$$

$$\Rightarrow \quad E=2.27\text{ eV}$$

(b) Number of photons leaving the source per second is

$$n=\frac{P}{E}=\frac{0.1}{36.3\times10^{-20}}=2.75\times10^{17}$$

(c) Number of photons falling on cathode per second is

$$n_1=\frac{0.15}{100}\times2.75\times10^{17}=4.125\times10^{14}$$

Number of photoelectrons emitting per second is

$$n_2=\frac{6\times10^{-6}}{1.6\times10^{-19}}=3.75\times10^{13}$$

So, $$\begin{pmatrix}\text{\%age of Photons}\\ \text{falling on Cathode}\end{pmatrix}=\frac{n_2}{n_1}\times100$$

$$\Rightarrow \quad \begin{pmatrix}\text{\%age of Photons}\\ \text{falling on Cathode}\end{pmatrix}=\frac{3.75\times10^{13}}{4.125\times10^{14}}\times100=9\%$$

ILLUSTRATION 13

A cylindrical rod of some laser material 5×10^{-2} m long and 10^{-2} m in diameter contains 2×10^{25} ions per m^3. If on excitation all the ions are in the upper energy level and de-excite simultaneously emitting photons in the same direction, calculate the maximum energy contained in a pulse of radiation of wavelength 6.6×10^{-7} m. If the pulse lasts for 10^{-7} second, calculate the average power of the laser during the pulse.

SOLUTION

Total number of ions in the rod is

$$N=\begin{pmatrix}\text{Number of ions}\\ \text{per unit volume}\end{pmatrix}\times\begin{pmatrix}\text{Volume of}\\ \text{the rod}\end{pmatrix}$$

$$\Rightarrow \quad N=(2\times10^{25}\text{ m}^{-3})\times(3.14\times(0.005)^2\times5\times10^{-2}\text{ m}^3)$$

$$\Rightarrow \quad N=7.85\times10^{19}$$

As all the ions de-excite simultaneously, the number of photons emitted in the same direction is also 7.85×10^{19}.
So, the energy contained in a pulse of radiation of wavelength 6.6×10^{-7} m is

$$E=n\left(\frac{hc}{\lambda}\right)$$

$$\Rightarrow \quad E=\frac{hc}{\lambda}\times7.85\times10^{19}$$

$$\Rightarrow \quad E=\frac{6.6\times10^{-34}\times3\times10^{8}}{6.6\times10^{-7}}=23.55\text{ J}$$

Average power $P=\dfrac{\text{Energy}}{\text{Time}}$

$$P=\frac{23.55\text{ J}}{10^{-7}\text{ s}}=23.55\times10^{7}\text{ W}=235.5\text{ MW}$$

ILLUSTRATION 14

A 100 watt sodium lamp is radiating light of wavelength 5890 Å, uniformly in all directions,

(i) At what rate, photons are emitted from the lamp?
(ii) At what distance from the lamp the average flux is $1\text{ photon/cm}^2\text{s}$?
(iii) At what distance the average density is 1 photon/cm^3?
(iv) What are the photon flux and photon density at 2 m from the lamp?

SOLUTION

(i) The energy of each photon is given by

$$E=h\nu=\frac{hc}{\lambda}=\frac{12375}{5890}=2.1\text{ eV}$$

$$\Rightarrow \quad E=3.376\times10^{-25}\text{ J}$$

Since the lamp is emitting energy at the rate of $100\ \text{Js}^{-1}$ i.e. $P = 100\ \text{W}$. Hence the number of photons emitted per second n is given by

$$n = \frac{100}{3.376\times10^{-25}} \approx 3\times10^{20}\ \text{photons/sec}$$

(ii) If we consider the lamp to be a point source of light, then at a distance r from the lamp, the light energy is uniformly distributed over the surface of sphere of radius r. So, photon flux at a distance r is given by

$$\phi_N = \frac{n}{4\pi r^2}$$

For photon flux to have a value of $1\ \text{photon/cm}^2\text{s}$, we have

$$1 = \frac{n}{4\pi r^2}$$

$$\Rightarrow \quad r = \sqrt{\frac{n}{4\pi}}$$

$$\Rightarrow \quad r = \sqrt{\frac{3\times10^{20}}{4\times3.14}}\ \text{cm} = 4.9\times10^{9}\ \text{cm}$$

$$\Rightarrow \quad r = 4.9\times10^{4}\ \text{km}$$

So, at this distance, on the average, one photon will cross through $1\ \text{cm}^2$ area normal to radial direction.

(iii) Since, we know that the photon density is given by

$$\rho_{\text{ph}} = \frac{1}{c}\left(\frac{n}{A}\right) = \frac{\phi_N}{c}$$

So, the photon density at a distance r from the point source is given by

$$\rho_{\text{ph}} = \frac{1}{c}\left(\frac{n}{A}\right) = \frac{n}{4\pi r^2 c}$$

For $\rho = 1\ \text{photon/cm}^3 = 10^6\ \text{photons/m}^3$, we get

$$r = \sqrt{\frac{N}{4\pi\rho_{\text{ph}}c}} = \sqrt{\frac{3\times10^{20}}{4\times3.14\times10^{6}\times10^{8}}}$$

$$\Rightarrow \quad r = 282\ \text{m}$$

Thus, at a distance of $282\ \text{m}$ from $100\ \text{W}$ lamp, there is on the average only $1\ \text{photon/cm}^3$ at any moment.

(iv) Since photon flux is $\phi_N = \frac{n}{4\pi r^2}$, so at $r = 2\ \text{m}$ we calculate the value of photon flux as

$$\phi_N = \frac{3\times10^{20}}{4\pi(200\ \text{cm})^2}$$

$$\Rightarrow \quad \phi_N \approx 6\times10^{18}\ \text{photons/m}^2$$

Average density of photons at $r = 2\ \text{m}$ is given by

$$\rho_{\text{ph}} = \frac{3\times10^{20}}{4\times3.14\times(200)^2\times(3\times10^{10})}$$

$$\Rightarrow \quad \rho_{\text{ph}} = 2\times10^{10}\ \text{photons/m}^3$$

MOMENTUM OF A PHOTON

According to relativistic theory, the total relativistic energy E of a particle having rest mass m_0, momentum p is

$$E^2 = p^2c^2 + m_0^2c^4$$

For a photon, its rest mass is zero i.e. $m_0 = 0$, so we get

$$E = pc = h\nu = \frac{hc}{\lambda}$$

where λ is the wavelength of the photon

So, an electromagnetic wave consists of photons capable of transporting linear momentum. The linear momentum p possessed by an electromagnetic wave is related to the energy E it transports according to the relation

$$p = \frac{E}{c} = \frac{h}{\lambda}$$

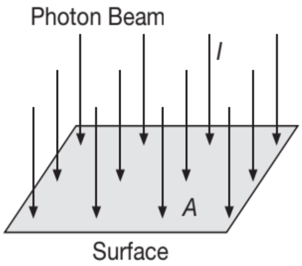
i.e. a photon is a particle with zero rest mass but finite momentum.

FORCE AND RADIATION PRESSURE DUE TO A PHOTON BEAM INCIDENT NORMALLY ON A SURFACE

Since we know that each photon has a definite energy and a definite linear momentum. Every photon in the light beam of a particular wavelength λ has the same energy i.e. $\frac{hc}{\lambda}$ and the same momentum $\frac{h}{\lambda}$. When light of intensity I falls on a surface, it exerts force on that surface. This is because, the incident light may be reflected (partially or completely) due to which there will be a change in its momentum and hence a force is exerted on the surface. Assuming that no transmission takes place from the surface, reflection coefficient of surface to be r, absorption coefficient of the surface to be a, then we have

$$a + r = 1$$

Let us assume that light beam of intensity I, wavelength λ, energy E, having N photons, be incident normally on the surface having area A as shown in figure.



Each incident photon of the light beam has an energy given by $E_{\text{each photon}} = \frac{hc}{\lambda}$ and momentum $p = \frac{h}{\lambda}$. Energy (E) of the photon beam is related to energy of each photon as

$$E_{\text{photon beam}} = E = N\left(E_{\text{each photon}}\right) = N\left(\frac{hc}{\lambda}\right)$$

For calculating the force exerted by beam on surface, we consider following cases.

For Perfectly Absorbing Surface

Let the surface on which light beam is incident be perfectly absorbing, then we have $r = 0$ and $a = 1$.

So, initial momentum of the photon is $p_i = \frac{h}{\lambda}$

and since the photon gets completely absorbed by the surface, so final momentum of the photon is $p_f = 0$. Taking the direction of incidence of photon as positive, the change in momentum of each incident photon is

$$\Delta p_{\text{each photon}} = \left|0 - \frac{h}{\lambda}\right| = \frac{h}{\lambda} \quad \text{(upwards)}$$

So, total change in momentum of the incident photon beam having N photons is

$$\Delta p = N\left(\Delta p_{\text{each photon}}\right) = N\left(\frac{h}{\lambda}\right)$$

$$\Rightarrow \quad \Delta p = \frac{N}{c}\left(\frac{hc}{\lambda}\right) = \left(\frac{N}{c}\right)E_{\text{each photon}} = \frac{NE_{\text{each photon}}}{c} \quad \ldots(1)$$

Since we know that the photon beam has energy E, so

$$E = N\left(E_{\text{each photon}}\right) = N\left(\frac{hc}{\lambda}\right) \quad \ldots(2)$$

So, from equation (1) and (2), we get

$$\Delta p = \frac{E}{c} \quad \ldots(3)$$

Since, by definition, we know that the intensity I of the light beam is the energy incident per second per unit area of a surface held normally to the direction of propagation of light, so we have

$$I = \frac{E}{A\Delta t} = \frac{P}{A}$$

where P is the power of the light source.

$$\Rightarrow \quad E = IA\Delta t \quad \ldots(4)$$

Substituting the value of equation (4) in equation (3), we get

$$\Delta p = \left(\frac{IA}{c}\right)\Delta t$$

$$\Rightarrow \quad F = \frac{\Delta p}{\Delta t} = \frac{IA}{c} = \frac{P}{c}$$

Hence the radiation pressure $\wp$ due to the light beam incident on a perfectly absorbing surface is

$$\wp = \frac{F}{A} = \frac{I}{c}$$

For Perfectly Reflecting Surface

Let the surface on which light beam is incident be perfectly reflecting, then we have $r = 1$ and $a = 0$.

So, initial momentum of the photon is $p_i = \frac{h}{\lambda}$

and since the photon gets completely reflected by the surface, so final momentum of the photon is $p_f = -\frac{h}{\lambda}$. Taking the direction of incidence of photon as positive, the change in momentum of each incident photon is

$$\Delta p_{\text{each photon}} = \left|-\frac{h}{\lambda} - \frac{h}{\lambda}\right| = \frac{2h}{\lambda} \quad \text{(upwards)}$$

So, total change in momentum of the incident photon beam having N photons is

$$\Delta p = N\left(\Delta p_{\text{each photon}}\right) = N\left(\frac{2h}{\lambda}\right)$$

$$\Rightarrow \quad \Delta p = \frac{N}{c}\left(\frac{2hc}{\lambda}\right) = \left(\frac{2N}{c}\right)E_{\text{each photon}}$$

$$\Rightarrow \quad \Delta p = \frac{2\left(NE_{\text{each photon}}\right)}{c} \qquad \ldots(1)$$

Since we know that the photon beam has energy E, so

$$E = N\left(E_{\text{each photon}}\right) = N\left(\frac{hc}{\lambda}\right) \qquad \ldots(2)$$

So, from equation (1) and (2), we get

$$\Delta p = \frac{2E}{c} \qquad \ldots(3)$$

Since, by definition, we know that the intensity I of the light beam is the energy incident per second per unit area of a surface held normally to the direction of propagation of light, so we have

$$I = \frac{E}{A\Delta t} = \frac{P}{A}$$

where P is the power of the light source.

$$\Rightarrow \quad E = IA\Delta t \qquad \ldots(4)$$

Substituting the value of equation (4) in equation (3), we get

$$\Delta p = \left(\frac{2IA}{c}\right)\Delta t$$

$$\Rightarrow \quad F = \frac{\Delta p}{\Delta t} = \frac{2IA}{c} = \frac{2P}{c}$$

Hence the radiation pressure due to the light beam incident on a perfectly absorbing surface is

$$\wp = \frac{F}{A} = \frac{2I}{c}$$

For Partially Reflecting Surface

For partially reflecting or partially absorbing surface, some fraction of the incident light is reflected and some fraction of it is absorbed such that $a + r = 1$.
In this case, we have

$$F = F_{\text{absorbed}} + F_{\text{reflected}} = a\left(\frac{IA}{c}\right) + r\left(\frac{2IA}{c}\right)$$

$$\Rightarrow \quad F = a\left(\frac{IA}{c}\right) + r\left(\frac{2IA}{c}\right) = \frac{IA}{c}(a + 2r) = \frac{IA}{c}(a + r + r)$$

Since $a + r = 1$, so we get

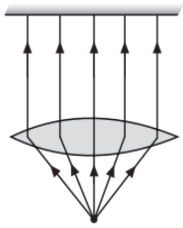
$$F = \frac{IA}{c}(1 + r) = \frac{P}{c}(1 + r)$$

Hence the radiation pressure due to the light beam incident on a partially reflecting surface having reflection coefficient r is

$$\wp = \frac{F}{A} = \frac{I}{c}(1 + r)$$

ILLUSTRATION 15

A totally reflecting, small plane mirror placed horizontally faces a parallel beam of light as shown in the figure. The mass of the mirror is 20 g. Assume that there is no absorption in the lens and that 30% of the light emitted by the source goes through the lens. Calculate the power of the source needed to support the weight of the mirror.



SOLUTION

Since 30% of the total light is incident on the mirror and the same amount of light is reflected by the mirror, so the force due to reflection of light on mirror is given by

$$F = 2\left(\frac{0.3P}{c}\right) = \frac{0.6P}{c}$$

To support the weight of mirror, this force must balance the weight of the mirror i.e.

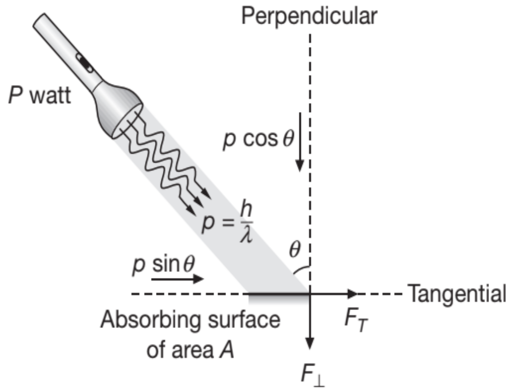
$$\frac{0.6P}{c} = mg$$

$$\Rightarrow \quad P = \frac{mgc}{0.6} = \frac{20 \times 10^{-3} \times 10 \times 3 \times 10^{8}}{0.6}$$

$$\Rightarrow \quad P = 10^{8} \text{ watt}$$

FORCE AND RADIATION PRESSURE DUE TO A PHOTON BEAM INCIDENT ON A PERFECTLY ABSORBING SURFACE AT AN ANGLE OF INCIDENCE

Let a light beam of intensity I, wavelength λ, energy E, power P, having N photons, be incident on the surface having area A at an angle of incidence θ as shown in figure.



Each incident photon of the light beam has an energy given by $E_{\text{each photon}} = \dfrac{hc}{\lambda}$ and momentum $p = \dfrac{h}{\lambda}$. Energy (E) of the photon beam is related to energy of each photon as

$$E_{\text{photon beam}} = E = N\left(E_{\text{each photon}}\right) = N\left(\frac{hc}{\lambda}\right)$$

Assuming that no transmission takes place from the surface, reflection coefficient of surface to be r, absorption coefficient of the surface to be a, then we have

$$a + r = 1$$

Since, the surface on which light beam is incident is perfectly absorbing, so we have $r = 0$ and $a = 1$.

So, initial momentum of the photon is $p_i = p = \dfrac{h}{\lambda}$ and since the photon gets completely absorbed by the surface, so final momentum of the photon is $p_f = 0$ (both tangentially and normally to the surface). Change in momentum of each incident photon is

$$\Delta p_{\text{each photon}} = \left|0 - \frac{h}{\lambda}\right| = \frac{h}{\lambda}$$

This change in momentum is in the direction opposite to the direction of the incident photon.

Also note that when the photon is completely absorbed by the surface, it suffers a change in momentum both along the normal (perpendicular) and the tangential (parallel) to the surface.

$$(\Delta p)_{\text{normal}} = (\Delta p)_{\perp} = \left|0 - \frac{h}{\lambda}\cos\theta\right| = \frac{h}{\lambda}\cos\theta$$

$$(\Delta p)_{\text{tangential}} = (\Delta p)_{T} = \left|0 - \frac{h}{\lambda}\sin\theta\right| = \frac{h}{\lambda}\sin\theta$$

Change in momentum of each incident photon is

$$\Delta p_{\text{each photon}} = \left|0 - \frac{h}{\lambda}\right| = \frac{h}{\lambda}$$

So, change in momentum of the incident photon beam (having N photons) normal/perpendicular to the surface is

$$\Delta p_{\perp} = N\left(\Delta p_{\text{each photon}}\right)_{\perp} = N\left(\frac{h}{\lambda}\cos\theta\right)$$

$$\Rightarrow \quad \Delta p_{\perp} = \frac{N}{c}\left(\frac{hc}{\lambda}\cos\theta\right)$$

$$\Rightarrow \quad (\Delta p)_{\perp} = \frac{1}{c}\left(NE_{\text{each photon}}\right)\cos\theta = \frac{E}{c}\cos\theta$$

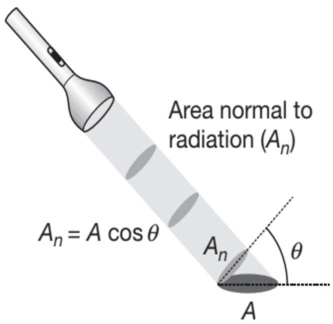
Also, change in momentum of the incident photon beam (having N photons) tangential/parallel to the surface is

$$\Delta p_{T} = N\left(\Delta p_{\text{each photon}}\right)_{T} = N\left(\frac{h}{\lambda}\sin\theta\right)$$

$$\Rightarrow \quad \Delta p_{T} = \frac{N}{c}\left(\frac{hc}{\lambda}\sin\theta\right)$$

$$\Rightarrow \quad (\Delta p)_{T} = \frac{1}{c}\left(NE_{\text{each photon}}\right)\sin\theta = \frac{E}{c}\sin\theta$$

Since, by definition, we know that the intensity I of the light beam is defined as the energy incident per second per unit area of a surface held normally to the direction of propagation of light.



So, we have,

$$I=\frac{E}{A_n\Delta t}=\frac{E}{(A\cos\theta)\Delta t}$$

$$\Rightarrow \quad E=(IA\Delta t)\cos\theta$$

So, we have, $(\Delta p)_\perp=\left(\frac{E}{c}\right)\cos\theta=\left(\frac{IA\Delta t\cos\theta}{c}\right)\cos\theta$

$$\Rightarrow \quad (\Delta p)_\perp=\left(\frac{IA}{c}\cos^2\theta\right)\Delta t$$

$$\Rightarrow \quad F_\perp=\frac{(\Delta p)_\perp}{\Delta t}=\left(\frac{IA}{c}\right)\cos^2\theta$$

Also, we see that,

$$(\Delta p)_T=\left(\frac{E}{c}\right)\sin\theta=\left(\frac{IA\Delta t\cos\theta}{c}\right)\sin\theta$$

$$\Rightarrow \quad F_T=\frac{(\Delta p)_T}{\Delta t}=\left(\frac{IA}{c}\right)\sin\theta\cos\theta$$

The net force F (due to the absorbed photon beam) acts on the surface in the direction of the incident photon beam and has a value

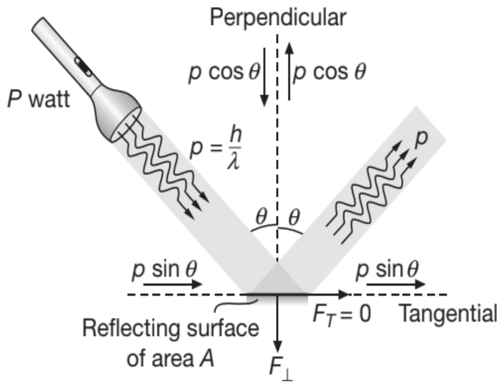
$$F=\sqrt{F_\perp^2+F_T^2}=\frac{IA}{c}\sqrt{\cos^4\theta+\sin^2\theta\cos^2\theta}=\frac{IA}{c}\cos\theta$$

Hence the radiation pressure $\wp_a$ due to the light beam incident on a perfectly absorbing surface is the normal force per unit area.

$$\Rightarrow \quad \wp_a=\frac{F_\perp}{A}=\left(\frac{I}{c}\right)\cos^2\theta$$

FORCE AND RADIATION PRESSURE DUE TO A PHOTON BEAM INCIDENT ON A PERFECTLY REFLECTING SURFACE AT AN ANGLE OF INCIDENCE

Let a light beam of intensity I, wavelength λ, energy E, power P, having N photons, be incident on the surface having area A at an angle of incidence θ as shown in figure.



Each incident photon of the light beam has an energy given by $E_{\text{each photon}}=\frac{hc}{\lambda}$ and momentum $p=\frac{h}{\lambda}$.

Energy (E) of the photon beam is related to energy of each photon as

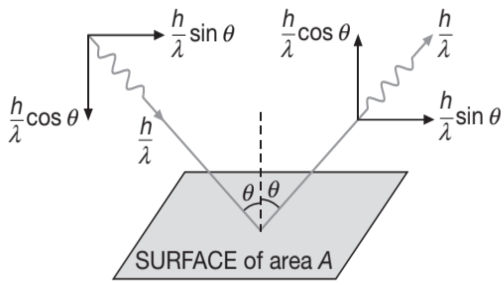
$$E_{\text{photon beam}}=E=N\left(E_{\text{each photon}}\right)=N\left(\frac{hc}{\lambda}\right)$$

Assuming that no transmission takes place from the surface, reflection coefficient of surface to be r, absorption coefficient of the surface to be a, then we have

$$a+r=1$$

Since the surface is perfectly reflecting, so we have $r=1$ and $a=0$.

So, initial momentum of the photon is $p_i=\frac{h}{\lambda}$ and since the photon gets completely reflected by the surface, so final momentum of the photon is $p_f=\frac{h}{\lambda}$.



Change in momentum of each incident photon normal to the surface is

$$(\Delta p_\perp)_{\text{each photon}}=\left|-\frac{h}{\lambda}\cos\theta-\frac{h}{\lambda}\cos\theta\right|=\frac{2h}{\lambda}\cos\theta \text{ and}$$

$$(\Delta p_T)_{\text{each photon}}=\left|\frac{h}{\lambda}\sin\theta-\frac{h}{\lambda}\sin\theta\right|=0$$

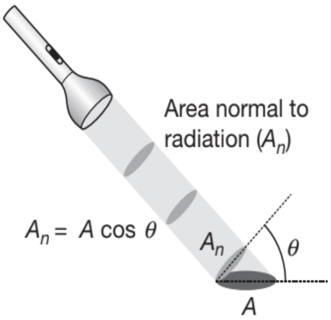
So, total change in momentum of the incident photon beam having N photons is

$$\Delta p_\perp=N\left(\Delta p_{\text{each photon}}\right)=N\left(\frac{2h}{\lambda}\cos\theta\right)$$

$$\Rightarrow \quad \Delta p_\perp=\frac{N}{c}\left(\frac{2hc}{\lambda}\cos\theta\right)=\frac{2E}{c}\cos\theta$$

$$\Rightarrow \quad (\Delta p)_\perp=\left(\frac{2E}{c}\right)\cos\theta \left\{\because E_{\text{photon beam}}=E=N\left(\frac{hc}{\lambda}\right)\right\}$$

Since, by definition, we know that the intensity I of the light beam is the energy incident per second per unit area of a surface held normally to the direction of propagation of light.



So, we have,

$$I = \frac{E}{A_n \Delta t} = \frac{E}{(A\cos\theta)\Delta t}$$

$\Rightarrow$ $E = (IA\Delta t)\cos\theta$

$\Rightarrow$ $(\Delta p)_\perp = \left(\frac{2E}{c}\right)\cos\theta = \left(\frac{2IA}{c}\cos^2\theta\right)\Delta t$

$\Rightarrow$ $F_\perp = \frac{(\Delta p)_\perp}{\Delta t} = \left(\frac{2IA}{c}\right)\cos^2\theta$

However, the force tangential to the surface is

$$F_T = \frac{(\Delta p)_T}{\Delta t} = 0$$

The net force F (due to the reflected photon beam) acts on the surface in the vertically downward direction or in the direction perpendicular to the surface and has a value given by

$$F = \sqrt{F_\perp^2 + F_T^2} = \frac{2IA}{c}\sqrt{\cos^4\theta + 0} = \left(\frac{2IA}{c}\right)\cos^2\theta$$

Hence the radiation pressure $\wp_a$ due to the light beam incident on a perfectly absorbing surface is the normal force per unit area.

$\Rightarrow$ $\wp_a = \frac{F_\perp}{A} = \left(\frac{2I}{c}\right)\cos^2\theta$

FORCE AND RADIATION PRESSURE DUE TO A PHOTON BEAM INCIDENT ON A PARTIALLY REFLECTING SURFACE AT AN ANGLE OF INCIDENCE

For partially reflecting or partially absorbing surface, some fraction of the incident light is reflected and some fraction of it is absorbed such that $a + r = 1$.

In this case, we have to do the calculation very carefully as the direction of the force on the surface is different for absorbed and reflected parts of the incident radiation as discussed earlier.

Change in momentum of the absorbed photon is $\frac{h}{\lambda}$, in the direction opposite to the incident beam.

Change in momentum of the reflected photon is $\frac{2h}{\lambda}\cos\theta$, vertically downwards.

Force on the plate due to the absorbed photons is along the direction of the incident beam i.e. making an angle θ with the vertical and has a value given by

$$F_{\text{absorbed}} = F_a = a\left(\frac{IA}{c}\right)\cos\theta = (1-r)\left(\frac{IA}{c}\right)\cos\theta$$

Force on the plate due to the reflected photons is vertically downwards and has a value given by

$$F_{\text{reflected}} = F_r = r\left(\frac{2IA}{c}\right)\cos^2\theta$$

Both these forces, F_r and F_a are inclined to each other at an angle θ. So, the resultant force F is given by

$$F = \sqrt{F_a^2 + F_r^2 + 2F_rF_a\cos\theta}$$

$\Rightarrow$ $F = \left(\frac{IA\cos\theta}{c}\right)\sqrt{(1-r)^2 + 4r^2\cos^2\theta + 4r(r-1)\cos^2\theta}$

Pressure on the surface due to the radiation is given by

$$P = \frac{F_a\cos\theta + F_r}{A}$$

$\Rightarrow$ $P = \frac{F_a}{A}\cos\theta + \frac{F_r}{A} = \frac{I}{c}(1-r)\cos^2\theta + \frac{I}{c}(2r)\cos^2\theta$

$\Rightarrow$ $P = \frac{I}{c}\cos^2\theta(1 - r + 2r)$

$\Rightarrow$ $P = \frac{I}{c}\cos^2\theta(1+r)$

ILLUSTRATION 16

A light of intensity $2\ \text{kWm}^{-2}$ falls on a plane mirror of reflective power $r = 0.8$ at an angle of incidence is 30°. Calculate the pressure exerted by the light on the mirror.

SOLUTION

Since, we know that the radiation pressure P in this case is given by the relation

$$P = \frac{I}{c}(1+r)\cos^2\theta$$

$$\Rightarrow \quad P=\frac{2000}{3\times10^8}(1+0.8)\cos^2(30°)=\frac{2000}{3\times10^8}(1.8)\left(\frac{3}{4}\right)$$

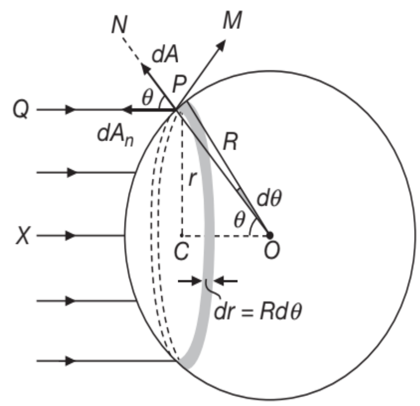
$$\Rightarrow \quad P=9\times10^{-6}\ \text{Nm}^{-2}$$

ILLUSTRATION 17

A perfectly reflecting solid sphere of radius R is placed in the path of a uniform beam of large cross-sectional area and intensity I. Calculate the force exerted on this sphere due to the light beam. Also calculate the force on the sphere if it would have been perfectly absorbing. Support the result obtained by an analytical argument.

SOLUTION

Let O be the centre of the sphere and QP be the light incident on the sphere of radius R. Let I be the intensity of the incident light. Consider a circular element of radius r, thickness dr, subtending an angle $d\theta$ at the centre of the sphere as shown in figure.



Then we observe that $r=R\sin\theta$ and $dr=Rd\theta$. If dA be the area of this circular element then

$$dA=(2\pi r)dr=2\pi(R\sin\theta)(Rd\theta)=2\pi R^2\sin\theta d\theta$$

Since intensity I is defined as the energy incident per second per unit area of a surface held normal to the direction of propagation of light, hence

$$I=\frac{dE}{(dA_n)(dt)}$$

where, $dA_n=dA\cos\theta$ is the area of the element normal to the direction of propagation of light.

$$\Rightarrow \quad I=\frac{dE}{(dA\cos\theta)(dt)}$$

So, the energy of the light falling on this element of area dA in time dt is

$$dE=Idt(dA\cos\theta)$$

The momentum (dp) of light falling on this element of area dA is $\frac{dE}{c}$ along QP and since light incident at P on this element is reflected by the sphere along PM. So, the change in momentum of the photon beam incident on the element is directed along PN (or OP) and is given by

$$dp=2\left(\frac{dE}{c}\right)\cos\theta=\frac{2}{c}(Idt)(dA\cos^2\theta)$$

The force (f) on dA due to the light falling on it is directed along PQ is given by

$$f=\frac{dp}{dt}=2\left(\frac{IdA}{c}\right)\cos^2\theta$$

It is observed that due to symmetry, the net force on the ring element as well as the sphere is directed along the line XO. The component of this force (directed along the line XO) acting on the element of area dA is

$$dF=f\cos\theta=\left(2\left(\frac{IdA}{c}\right)\cos^2\theta\right)\cos\theta$$

$$\Rightarrow \quad dF=f\cos\theta=2\left(\frac{IdA}{c}\right)\cos^3\theta$$

$$\Rightarrow \quad dF=\left(\frac{2I}{c}\right)(2\pi R^2\sin\theta d\theta)\cos^3\theta$$

The force on the entire sphere is $F=\int dF$

$$\Rightarrow \quad F=\int_0^{\frac{\pi}{2}}\left(\frac{2I}{c}\right)(2\pi R^2\sin\theta d\theta)\cos^3\theta$$

$$\Rightarrow \quad F=\left(\frac{4\pi R^2I}{c}\right)\int_0^{\frac{\pi}{2}}\cos^3\theta\sin\theta d\theta$$

$$\Rightarrow \quad F=-\left(\frac{4\pi R^2I}{c}\right)\int_0^{\frac{\pi}{2}}\cos^3\theta d(\cos\theta)$$

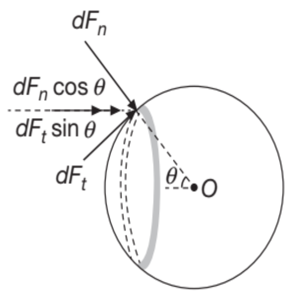
$$\Rightarrow \quad F=-\left(\frac{4\pi R^2I}{c}\right)\left(\frac{\cos^4\theta}{4}\right)\Bigg|_0^{\frac{\pi}{2}}=-\frac{4\pi R^2I}{c}\left(0-\frac{1}{4}\right)$$

$$\Rightarrow \quad F=\frac{\pi R^2I}{c}$$

However, when the sphere is perfectly absorbing, then the normal force (dF_n) and the tangential force (dF_t) on the infinitesimal element of area $dA = 2\pi R^2 \sin\theta d\theta$ are given by

$$dF_n = \left(\frac{I}{c}\cos^2\theta\right)dA \text{ and } dF_t = \left(\frac{I}{c}\sin\theta\cos\theta\right)dA.$$

The net force F on the perfectly absorbing sphere acts along the direction of incident light beam as shown in figure.



$$\Rightarrow \quad F = \int dF = \int_0^{\frac{\pi}{2}} (dF_n\cos\theta + dF_t\sin\theta)$$

$$\Rightarrow \quad F = \left(\frac{I}{c}\right)\int_0^{\frac{\pi}{2}}\left(\cos^3\theta + \sin^2\theta\cos\theta\right)\left(2\pi R^2\sin\theta d\theta\right)$$

$$\Rightarrow \quad F = \left(\frac{I}{c}\right)\int_0^{\frac{\pi}{2}}\cos\theta\left(\cos^2\theta + \sin^2\theta\right)\left(2\pi R^2\sin\theta d\theta\right)$$

$$\Rightarrow \quad F = \left(\frac{\pi R^2 I}{c}\right)\int_0^{\frac{\pi}{2}} 2\sin\theta\cos\theta d\theta = \left(\frac{\pi R^2 I}{c}\right)\int_0^{\frac{\pi}{2}}\sin(2\theta)d\theta$$

$$\Rightarrow \quad F = \left(\frac{\pi R^2 I}{c}\right)\left(-\frac{\cos(2\theta)}{2}\bigg|_0^{\frac{\pi}{2}}\right)$$

$$\Rightarrow \quad F = -\left(\frac{\pi R^2 I}{2c}\right)(\cos\pi - \cos 0)$$

$$\Rightarrow \quad F = -\left(\frac{\pi R^2 I}{2c}\right)(-1-1) = -\left(\frac{\pi R^2 I}{2c}\right)(-2) = \frac{\pi R^2 I}{c}$$

It is observed that, for a sphere placed in the path of a light beam the force exerted on sphere is independent of the nature of the surface of the sphere. This is because, the perfectly reflecting sphere reflects the incoming radiation in the backward sense for $\theta < \frac{\pi}{4}$ but in the forward sense for $\frac{\pi}{4} < \theta < \frac{\pi}{2}$. Due to this, the net momentum of the reflected radiation turns out to be zero and hence the net momentum exchange between the photon beam and the sphere is same in both the cases.

Conceptual Note(s)

For a perfectly absorbing sphere, the wave momentum is completely transferred to the sphere. The easiest approach in this case is to observe that the sphere presents the cross-sectional area πR^2 to the incoming radiation, and therefore the total force is simply given by

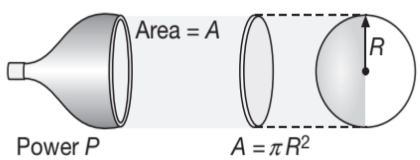
$$F = \frac{IA_{\text{effectice}}}{c} = \frac{I\left(\pi R^2\right)}{c}$$

which is the same result as the perfectly reflecting sphere.

FORCE EXERTED ON ANY OBJECT IN THE PATH OF A LIGHT BEAM

Consider a light source such as a lamp of power P watt. Let this lamp produce a uniform parallel beam of light of cross-sectional area A which is incident on a sphere of radius R as shown in figure. If I be the intensity of this light beam emitted by the lamp. Then

$$I = \frac{P}{A}$$



In this case, only those photons will be incident on the sphere which pass through the cross-sectional area $A = \pi R^2$ which is the projection of sphere on a cross-sectional plane. This area is also called as projected area. Thus, the power incident on sphere is

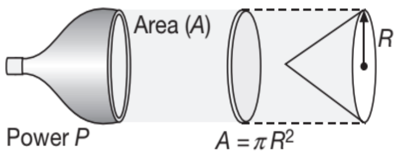
$$P_i = IA_{\text{Projected}} = IA = I\left(\pi R^2\right)$$

So, force exerted on sphere will be

$$F = \frac{P_i}{c} = \frac{I(\pi R^2)}{c}$$

This result will remain the same for perfectly absorbing as well as perfectly reflecting sphere.

Similarly, when a perfectly absorbing cone is placed in front of the light beam as shown in figure, then also the projection of cone along a cross-sectional plane of beam i.e. the projected area is $A = \pi R^2$.



Hence the force exerted on the perfectly absorbing cone due to the light beam will be

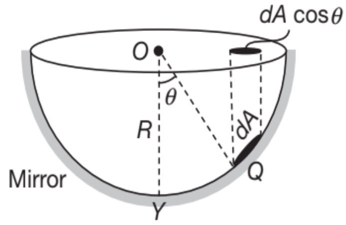
$$F = \frac{P}{c} = \frac{IA_{\text{Projected}}}{c} = \frac{I(\pi R^2)}{c}$$

ILLUSTRATION 18

A point source of light O is placed at the centre of curvature of a hemispherical perfectly reflecting surface radius of curvature R. Find the force on the hemisphere due to the light falling on it if the source emits a power P.

SOLUTION

The energy emitted by the source per unit time, i.e., P falls on an area $4\pi R^2$ at a distance R in unit time. Thus, the energy falling per unit area per unit time is $\frac{P}{(4\pi R^2)}$. Consider a small area dA at the point Q of the hemisphere as shown in figure.



The energy falling per unit time on it is $\frac{PdA}{4\pi R^2}$. The corresponding momentum incident on this area per unit time is $\frac{PdA}{4\pi R^2 c}$. As the light is reflected back, the change in momentum per unit time, i.e., the force on dA is

$$dF = \frac{2PdA}{4\pi R^2 c}$$

Suppose the radius OQ through the area dA makes an angle θ with the symmetry axis OY. The force on dA is along this radius. By symmetry, the resultant force on the hemisphere is along OY. The component of dF along OY is

$$dF\cos\theta = \frac{2PdA}{4\pi R^2 c}\cos\theta$$

If we project the area dA on the plane containing the rim, the projection is $dA\cos\theta$. Thus, the component of dF along OY is,

$$dF\cos\theta = \frac{2P}{4\pi R^2 c}(\text{Projection of } dA)$$

The net force along OY is

$$F = \frac{2P}{4\pi R^2 c}(\Sigma\,\text{Projection of } dA)$$

When all the small areas dA are projected, we get the area enclosed by the rim which is πR^2. Thus,

$$F = \left(\frac{2P}{4\pi R^2 c}\right)(\pi R^2) = \frac{P}{2c}$$

Test Your Concepts-I

Based on Photon Properties and De Broglie Phenomenon

(Solutions on page H.3)

1. The intensity of direct sunlight before it passes through the earth's atmosphere is $1.4\ \text{kWm}^{-2}$. If it is completely absorbed find the corresponding radiation pressure.
2. According to the Maxwell theory of electrodynamics an electron going in a circle should emit radiations of frequency equal to its frequency of revolution. What would be the wavelength of the radiation emitted by a hydrogen atom in ground state if this rule is followed?
3. An electron is accelerated by a potential difference of 25 volt. Find the de-Broglie wavelength associated with it.
4. Find the number of photons emitted per second by a *MW* transmitter of 10 kW power emitting radio waves of wavelength 500 m.
5. If 5% of the energy supplied to an incandescent light bulb is radiated as visible light, how many visible light photons are emitted by 100 watt bulb. Assume wavelength of all visible photons to be 5600 Å. Given $h = 6.625 \times 10^{-34}$ Js.
6. Calculate the number of photons in 6.62 J of radiation energy of frequency 10^{12} Hz. Given $h = 6.62 \times 10^{-34}$ Js.
7. Monochromatic light of frequency 6×10^{14} Hz is produced by a laser. The power emitted is 2×10^{-3} W.
   (a) What is the energy of each photon in the light?
   (b) How many photons per second, on the average, are emitted by the source?
8. Show that a free electron at rest cannot absorb a photon and thereby acquire kinetic energy equal to the energy of the photon. Would the conclusion change if the free electron was moving with a constant velocity?
9. An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de-Broglie wavelength associated with the electrons. If other factors (such as numerical aperture etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?
10. An electron and proton are possessing the same amount of kinetic energy. Which of the two have greater wavelength?
11. An electron and a photon have same de Broglie wavelength (say 1 Å). Which one possesses more kinetic energy?
12. An electron and a proton have same wavelength. Which one possesses more energy?
13. It is desired to move a small space vehicle of mass 50 kg at rest, by a lamp of 100 Watt emitting blue light of wavelength 4700 Å. If the vehicle if in free space, calculate its acceleration. Assume all the emitted photons to be incident on the body of space vehicle.
14. With what velocity must an electron travel so that its momentum is equal to that of photon with a wavelength of $\lambda = 5200$ Å.
15. A parallel beam of monochromatic light of wavelength 496 nm is incident normally on a perfectly absorbing surface. The power through any cross-section of the beam is 10 W. Find (a) the number of photons absorbed per second by the surface and (b) the force exerted by the light beam on the surface. Take $hc = 1240$ eVnm.
16. A monochromatic source of light operating at 200 W emits 4×10^{20} photons per second. Find the wavelength of the light.
17. How many photons are emitted per second by a 5 mW laser source operating at 663 nm?
18. A hydrogen atom moving at a speed *v* absorbs a photon of wavelength 122 nm and stops. Find the value of *v*. Mass of a hydrogen atom = 1.67×10^{-27} kg

EMISSION OF ELECTRONS

As we are aware of the fact that metals have free electrons (negatively charged particles) which are responsible for their conductivity. However, the free electrons cannot normally escape out of the metal surface. If an electron attempts to come out of the metal, the metal surface acquires a positive charge and pulls the electron back to the metal. The free electron is thus held inside the metal surface by the attractive forces of the ions. Consequently, the electron can come out of the metal surface only if it is supplied some minimum energy to overcome the attractive pull of the metal.

This minimum energy required by an electron to escape from the metal surface is called the *work function* of the metal. It is generally denoted by ϕ_0 or sometimes W and is measured in eV (electron volt). One electron volt is the energy gained by an electron when it has been accelerated by a potential difference of 1 volt, so

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

This unit of energy is commonly used in atomic and nuclear physics.

The work function, ϕ_0 depends on the properties of the metal and the nature of its surface. The work function for platinum is the highest $(\phi_0)_{Pt} = 5.65$ eV, whereas it is the lowest for caesium i.e., $(\phi_0)_{Cs} = 2.14$ eV.

The minimum energy required for the electron emission from the metal surface can be supplied to the free electrons by any one of the following physical processes.

Thermionic Emission

By suitably heating a metal, sufficient thermal energy can be imparted to the free electrons to enable them to come out of the metal. The free electrons so emitted are called as ***Thermions.***

Field Emission

When the metal surface is subjected to very strong electric field of the order ranging from 10^3 Vm^{-1} to 10^8 Vm^{-1}, the electrons (beyond a certain limit) start coming out of the metal surface. This method of emission is dangerous and less efficient. This method of emission is also called the ***Cold Cathode Emission.***

Photo-electric Emission

When light of certain minimum energy (or minimum frequency or maximum wavelength) illuminates or falls on a metal surface, electrons are emitted from the metal surface. The emitted electrons are called ***photo-electrons.*** In case of Photoelectric emission, the rate of emission of photoelectrons is very low.

Secondary Emission

When fast moving electrons strike a metal surface, then some of their energy is transferred to the free electrons of the metal. Due to this, when free electrons gain energy more than the work function, then they are emitted from the metal surface. These emitted electrons are called the **secondary electrons.**

PHOTOELECTRIC EFFECT

The phenomenon of emission of electrons from a metallic surface by the use of light (or radiant) energy of certain minimum frequency (or maximum wavelength) is called **photoelectric effect**. The emitted electrons are called as **photoelectrons**. The phenomenon was discovered by **Hallwach** in 1888. For photoelectric emission the metal used must have **low work function** e.g., alkali metals. **Cesium** is assumed to be the best metal for photoelectric effect. To escape from the surface, the electron must absorb enough energy from the incident radiation to overcome the attraction of nucleus of the atom of the metal surface. The explanation to the photoelectric effect given by Einstein is based on the Law of Conservation of Energy. Before discussing the effect further, we must understand the following terms.

Work Function (or Threshold Energy)

The minimum energy of incident radiation, required to eject the electrons from metallic surface is defined as work function of that surface. It is the characteristic of a metal surface and is denoted by ϕ_0 or W.

$$\phi_0 = h\nu_0 = \frac{hc}{\lambda_0} \text{ (in joule), where}$$

ν_0 = Threshold frequency and

λ_0 = Threshold wavelength

Work function in electron volt is given by

$$\phi_0(\text{eV}) = \frac{hc}{e\lambda_0} = \frac{12375}{\lambda_0(\text{in Å})} \approx \frac{12400s}{\lambda_0(\text{in Å})}$$

It is the minimum for Caesium. It is relatively less for alkali metals.

Work functions of some photosensitive metals

Metal	Work function (eV)	Metal	Work function
Caesium	1.9	Calcium	3.2
Potassium	2.2	Copper	4.5
Sodium	2.3	Silver	4.7
Lithium	2.5	Platinum	5.6

Threshold Frequency (ν_0)

The minimum frequency of incident radiations required to eject the electron from metal surface is defined as threshold frequency. If incident frequency $v < v_0$, then no photoelectron emission. For most metals the threshold frequency is in the ultraviolet (corresponding to wavelengths between 200 and 300 nm), but for potassium and caesium oxides it is in the visible spectrum i.e. λ between 400 and 700 nm.

Threshold Wavelength (λ_0)

The maximum wavelength of incident radiations required to eject the electrons from a metallic surface is defined as threshold wavelength. If incident wavelength $\lambda > \lambda_0$, then no photoelectron emission will take place.

SOME IMPORTANT TERMS

(a) Photoelectrons: The electrons emitted in the process of photoelectric effect are called photoelectrons.

(b) Photoelectric Current (i): If current flow in a circuit is due to photoelectric effect then that current due to the photoelectrons is called as photoelectric current.

(c) Stopping Potential (V_s or V_0): It is the minimum value of negative potential of anode or collector (with respect to cathode or emitter) for which photoelectric current is zero is called stopping potential. It can also be defined as that value of negative potential for which no photoelectron reaches the anode. This is also known as cut off voltage. This voltage is independent of intensity.

(d) Saturation Current: When all photoelectrons emitted by cathode reach the anode, then the current flowing in the circuit at that instant is called as saturated current. This is the maximum value of photoelectric current.

PLANK'S QUANTUM THEORY

The light energy from any source is always an integral multiple of a smaller energy value called quantum of light. Hence energy of a photon beam or a light sample is

$$E_{\text{photon beam}} = NE_{\text{each photon}} = N(h\nu), \text{ where}$$

$N = 1, 2, 3, ...$ is the number of photons in the photon beam or the light sample and $E_{\text{each photon}} = h\nu = \frac{hc}{\lambda}$.

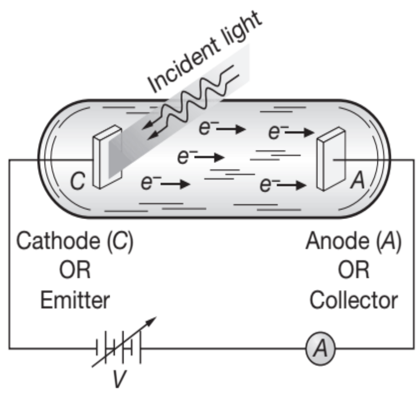
So, for a photon beam or a light sample, energy is quantized. Also, we can say that $h\nu$ is the quantum of energy and this small packet of energy is called as **photon**. This smallest energy is

$$E = h\nu = \frac{hc}{\lambda}$$

where $hc = 12375 \text{ eVÅ} \approx 12400 \text{ eVÅ}$

EXPERIMENTAL SETUP FOR PHOTOELECTRIC EFFECT

It consists of two conducting electrodes, the cathode (C) also called as emitter and anode (A) also called as collector which are enclosed in an evacuated glass tube as shown in figure.



The battery or some other source of potential difference creates an electric field in the direction from anode to cathode. Light of certain wavelength or frequency falling on the surface of cathode causes a current to flow in the external circuit. This current is called the **photoelectric current.**

When the potential difference increases, the photo electric current also increases till saturation is reached. As the polarity of battery is reversed (i.e. plate A is at negative potential w.r.t. plate C) the electrons start moving back towards the cathode. It is observed that at a particular negative potential of plate A, no electron reaches the plate A i.e. the current becomes zero. This negative potential for which the photo-electric current is zero is called the **stopping potential** denoted by V_0. Maximum kinetic energy (in eV) of photo electrons in terms of stopping potential will therefore be $K_{max} = |V_0|$ eV.

LAWS OF PHOTOELECTRIC EMISSION

We thus have the following laws of photoelectric emission, derived from the experimental observations.

(a) For each emitting metal, there is a certain minimum frequency ν_0 (or maximum wavelength λ_0), called the threshold frequency of the incident radiation, below (above which) which no emission of photoelectron takes place, no matter how great is the intensity. The value of ν_0 (or λ_0) is different for different emitting surfaces.

(b) The process of emission of photoelectrons is an instantaneous process. There is no time lag $(< 10^{-8}\text{ s})$ between the incidence of radiation and the emission of photoelectrons.

(c) Photoelectric effect is a one photon-one electron phenomenon i.e. even if photon has an energy sufficient to strike off 3 electrons (say) it can only strike off one electron with the excess energy being imparted to the struck off electron as kinetic energy.

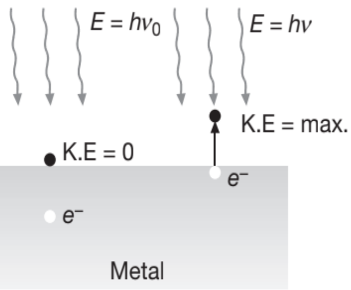
(d) The number of photoelectrons emitted per second, that is, photoelectric current is directly proportional to the intensity of the incident radiation but is independent of the frequency (or wavelength) of light.

(e) The velocities (or the energies) of the emitted photoelectrons vary between zero and a definite maximum (v_{max}). The proportion of photoelectrons having a particular velocity is independent of the light intensity.

(f) The maximum velocity, v_{max}, and hence the maximum kinetic energy is independent of the intensity of the incident light, but depends on its frequency, increasing linearly with the increase of the frequency of the incident light.

EINSTEIN'S EXPLANATION OF PHOTO-ELECTRIC EFFECT

The wave theory of light could not explain the observed characteristics of photoelectric effect. Einstein extended Planck's quantum idea for light to explain photo-electric effect.



According to his idea, the energy of electromagnetic radiation is not continuously distributed over the wave front like the energy of water waves but remains concentrated in packets of energy content $h\nu$, where ν is frequency of radiations and h is universal Planck's constant $(= 6{\cdot}625 \times 10^{-34}\text{ Js})$. Each packet of energy moves with the speed of light. The assumptions of Einstein's theory are

(a) The photoelectric effect is the result of collision of two particles, one of which is a photon of incident light and the other is an electron of photo-metal.

(b) The electron of photo-metal is bound with the nucleus by Coulomb attractive forces. The minimum energy required to free an electron from its bondage is called work function, $W = \phi_0 = h\nu_0$.

(c) The incident photon interacts with a single electron and loses its energy in two parts

  (i) Firstly, in getting the electron released from the bondage of the nucleus.

  (ii) Secondly, to impart kinetic energy to emitted electron.

(d) The efficiency of photoelectric effect is less than 1%, i.e. less than 1% of photons are capable of ejecting photoelectrons.

Accordingly, if $h\nu$ is the energy of incident photon, then

$$h\nu = \phi_0 + K_{max}$$

$$\Rightarrow \quad K_{max} = h\nu - \phi_0 \quad \ldots(1)$$

This is Einstein's photoelectric equation, where W is work function and

$$K_{max} = \frac{1}{2}mv_{max}^2 = eV_s$$

is the maximum kinetic energy of photoelectrons emitted.

Equation (1) is referred as Einstein's photo electric equation that explains all experimental results of photo-electric effect and is based on the Law of Conservation of Energy.

(e) Einstein's photoelectric equation says that when a single photon carrying an energy $h\nu$ falls onto a metal surface (where it is absorbed by a single electron), then a part of this energy ϕ_0 (called the work function of the metal surface) is utilized in causing the electron to escape from the metal surface and the excess energy $(h\nu - \phi_0)$ becomes the electron kinetic energy. If the electron (while coming out of the metal surface) does not lose energy by internal collisions, then it escapes from the metal with a maximum kinetic energy K_{max}.

So, K_{max} represents the maximum kinetic energy that the photoelectron can have outside the surface. This happens to be in complete agreement with the quantum theory of the photon theory with experiment.

Since, we know that the number of photons incident per unit time (n) on a surface held normally to an incident light of intensity I is given by

$$n = \frac{IA}{h\nu}$$

So, for a particular frequency, if we double the light intensity, then we also double the number of photons and hence the photoelectric current is also doubled.

(f) The second objection (the frequency problem) is met when K_{max} equals zero and we have

$$h\nu_{th} = \phi_0$$

This asserts that the photon has just enough energy to eject the photoelectron but has no extra energy to give to the photoelectron as kinetic energy. If ν is reduced below ν_{th}, then $h\nu$ will be smaller than ϕ_0 and the individual photons, no matter how many of them there are (that is, no matter how intense the illumination), will not have enough energy to eject photoelectrons.

(g) The third objection (the time delay problem) also follows from the photon theory because the required energy is supplied to the electron in a concentrated bundle. It is not spread uniformly over the beam cross section as in the wave theory. Hence Einstein's equation for photoelectric effect is given by

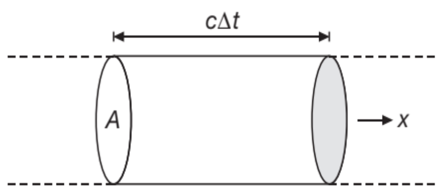
$$h\nu = h\nu_{th} + K_{max}$$

$$\Rightarrow \quad K_{max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_{th}}$$

PHOTOELECTRIC EFFECT CANNOT BE EXPLAINED USING CLASSICAL WAVE THEORY OF LIGHT

Before we try to explain the drawbacks of Classical Wave Theory when applied to Photoelectric effect, we need to understand the concept of intensity of light.

Intensity of light is the energy crossing per second per unit area of a surface held normally to the direction of propagation of the wave. Let us consider a cylindrical volume with area of cross section A and length $c\Delta t$ along the x axis.



The energy U contained in a cylindrical cross-section of area A in time Δt when a wave propagates at speed c is given by $U = u_{av}(c\Delta t)A$, where u_{av} is the average energy density of the electromagnetic wave or the light wave. So, the intensity of the light beam is $I = \frac{U}{A\Delta t} = u_{av}c$. If E_0 is the amplitude of the electric field, then in terms of maximum electric field, the intensity is given by $I = \frac{1}{2}\varepsilon_0 E_0^2 c$.

The Intensity Problem: Wave theory cannot explain why kinetic energy of emitted photoelectrons is independent of intensity.

Since wave theory suggests that the oscillating electric field vector E of the light wave increases in amplitude when the intensity of the light beam is increased. Also, we know that the force applied to the electron in the presence of electric field E is eE, which simply suggests that the kinetic energy of the photoelectrons should also increase when the light beam is made more intense.
However, observations show that maximum kinetic energy is independent of the light intensity.

The Frequency Problem: Wave theory cannot explain the existence of a minimum frequency above which photoelectric effect takes place.

Wave theory also suggests that the photoelectric effect should occur for any frequency of the light but the light should be intense enough to supply the energy required to eject the photoelectrons.

However, observations show that there exists for each surface a characteristic cut off frequency or threshold frequency, $\nu_{\text{threshold}} = \nu_0$, above which photoelectric effect takes place and for frequencies less than the threshold frequency (ν_0), the photoelectric effect does not occur, no matter how intense is light beam.

The Time Delay Problem: Wave theory cannot explain the immediate ejection of photoelectrons from a metal surface.

In accordance with the wave theory, when the energy acquired by a photoelectron is absorbed directly from the wave incident on the metal plate, then the effective target area for an electron in the metal is very limited (probably not much more than that of a circle of diameter roughly equal to that of an atom). As per the classical theory, the light energy is uniformly distributed over the wavefront. So, if the light is feeble enough, then there should be a measurable time lag, between the falling of the light on the surface and the ejection of the photoelectron from the surface. During this interval, the electron should be absorbing sufficient amount of energy from the beam until it accumulates enough energy to escape.

If we consider light as a wave, then the intensity depends upon electric field. If we take work function of metal to be as W or ϕ_0, then we have

$$W = \phi_0 = IAt$$

$$\Rightarrow \quad t = \frac{\phi_0}{IA}$$

So, according to wave theory of light applied on photoelectric effect, there should be time lag between the falling of a photon and emission of a photoelectron, because the metal has work function.

However, experiments show that the photoelectric effect is an instantaneous process. Hence, light is not of wave nature.

*So, **Quantum Theory of Light** solves these problems and provides a correct interpretation of the photoelectric effect phenomenon.*

ILLUSTRATION 19

In an experiment on photoelectric effect light of wavelength 400 nm is incident on a caesium plate at the rate of 5 W. The potential of the collector plate is made sufficiently positive with respect to emitter so that the current reaches the saturation value. Assuming that on the average one out of every 10^6 photons is able to eject a photoelectron, find the photocurrent in the circuit.

SOLUTION

$$E = \frac{12375}{4000} = 3.1 \text{ eV}$$

Number of photoelectrons emitted per second

$$n = \left(\frac{1}{10^6}\right)\left(\frac{5}{3.1 \times 1.6 \times 10^{-19}}\right) = 1 \times 10^{13} \text{ per second}$$

Since, $i = \frac{q}{t} = \frac{Ne}{t} = \left(\frac{N}{t}\right)e = ne$

$$\Rightarrow \quad i = (ne) = 1 \times 10^{13} \times 1.6 \times 10^{-19}$$

$$\Rightarrow \quad i = 1.6 \times 10^{-6} \text{ A} = 1.6\ \mu\text{A}$$

ILLUSTRATION 20

Ultraviolet light of wavelength 2000 Å causes photoemission from a surface. The stopping potential is 2 V.

(a) Find the work function in eV
(b) Find the maximum speed of the photoelectrons.

SOLUTION

(a) Using Einstein relation

$$\phi_0 = \frac{hc}{\lambda} - \text{eV}_0$$

$$\Rightarrow \quad \phi_0 = \frac{12400}{2000} - 2 = 4.2 \text{ eV}$$

(b) Since $\frac{1}{2}mv_{\max}^2 = eV_0$

$$\Rightarrow \quad v_{\max} = \sqrt{\frac{2\,eV_0}{m}} = \sqrt{\frac{2(1.6\times10^{-19})(2)}{9.1\times10^{-31}}}$$

$$\Rightarrow \quad v_{\max} = 8.4\times10^5 \text{ ms}^{-1}$$

ILLUSTRATION 21

When a beam of 10.6 eV photons of intensity 2 Wm^{-2} falls on a platinum surface of area $1\times10^{-4}\text{ m}^2$ and work function 5.6 eV, 0.53% of the incident photons eject photo electrons. Calculate the number of photoelectrons emitted per second and their minimum and maximum energies (in eV). Given that $1\text{ eV} = 1.6\times10^{-19}$ J.

SOLUTION

Number of photoelectrons emitted per second

$$\frac{N}{t} = \frac{(\text{Intensity})(\text{Area})}{(\text{Energy of each photon})}\times\frac{0.53}{100}$$

$$\Rightarrow \quad \frac{N}{t} = \frac{(2)(1\times10^{-4})}{(10.6\times1.6\times10^{-19})}\times\frac{0.53}{100} = 6.25\times10^{11}\text{ s}^{-1}$$

Minimum kinetic energy of photoelectrons is

$$K_{\min} = 0$$

and maximum kinetic energy is,

$$K_{\max} = E - \phi_0 = (10.6 - 5.6)\text{ eV} = 5\text{ eV}$$

ILLUSTRATION 22

One milliwatt of light of wavelength 4560 Å is incident on a caesium surface. Calculate the photoelectric current, assuming a quantum efficiency of 0.5%.
Given Planck's constant $h = 6.62\times10^{-34}$ Js and velocity of light $c = 3\times10^8\text{ ms}^{-1}$.

SOLUTION

The energy of each photon of incident light is

$$E = h\nu = \frac{hc}{\lambda} = \frac{(6.62\times10^{-34})(3\times10^8)}{4560\times10^{-10}}$$

$$\Rightarrow \quad E = 4.35\times10^{19}\text{ J}$$

Number of photons emitted per second by a one milliwatt source is

$$n = \frac{P_{\text{source}}}{E_{\text{single photon}}} = \frac{10^{-3}}{4.35\times10^{-19}} \approx 2.3\times10^{15}\text{ s}^{-1}$$

Number of photons emitted for a quantum efficiency of 0.5% is

$$N = (2.3\times10^{15})\times\frac{0.5}{100}$$

$$\Rightarrow \quad N = 1.15\times10^{13}\text{ s}^{-1}$$

Thus photo-electric current

$$I = \frac{q}{t} = \frac{Ne}{t} = ne = (1.15\times10^{13})(1.6\times10^{-9})$$

$$\Rightarrow \quad I = 1.84\times10^{-6}\text{ A} = 1.84\ \mu\text{A}$$

ILLUSTRATION 23

The surface of a metal of work function ϕ_0 is illuminated by light whose electric field component varies with time as $E = E_0[1+\cos(\omega t)]\sin(\omega_0 t)$. Calculate the maximum kinetic energy of photoelectrons emitted from the surface.

SOLUTION

The given electric field component is

$$E = E_0\sin(\omega_0 t) + E_0\sin(\omega_0 t)\cos(\omega t)$$

$$\Rightarrow \quad E = E_0\sin(\omega_0 t) + \frac{E_0}{2}\left[\sin(\omega_0+\omega)t + \sin(\omega_0-\omega)t\right]$$

So, the given light sample that comprises three different frequencies i.e. ω, $(\omega_0+\omega)$, $(\omega_0-\omega)$
The maximum kinetic energy will be due to most energetic photon i.e. of frequency $\left(\frac{\omega+\omega_0}{2\pi}\right)$.
Since $K_{\max} = h\nu - \phi_0$

$$\Rightarrow \quad K_{\max} = \frac{h(\omega+\omega_0)}{2\pi} - \phi_0$$

ILLUSTRATION 24

A monochromatic light source of frequency illuminates a metallic surface and ejects photoelectrons. The photo electrons having maximum energy are just able to ionize the hydrogen atoms in ground state. When the entire experiment is repeated with an incident radiation of frequency $\frac{5}{6}f$, the photoelectrons so emitted are able to excite the hydrogen atom beam which then emits a radiation of wavelength 1215 Å.

(a) What is the frequency of radiation?
(b) Find the work function of the metal.

SOLUTION

(a) Using Einstein's equation of photoelectric effect i.e.

$$K_{max} = hf - \phi_0$$

where $K_{max} = 13.6 \text{ eV}$

$$\Rightarrow \quad hf - \phi_0 = 13.6 \text{ eV} \qquad \ldots(1)$$

So, when the experiment is repeated, then

$$h\left(\frac{5}{6}f\right) - \phi_0 = \frac{12375}{1215} = 10.2 \text{ eV} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$\frac{hf}{6} = 3.4 \text{ eV}$$

$$\Rightarrow \quad f = \frac{(6)(3.4)(1.6\times10^{-19})}{(6.63\times10^{-34})} = 4.92\times10^{15} \text{ Hz}$$

(b) From equation (1), we have

$$\phi_0 = hf - 13.6$$

$$\Rightarrow \quad \phi_0 = 6(3.4) - 13.6$$

$$\Rightarrow \quad \phi_0 = 6.8 \text{ eV}$$

ILLUSTRATION 25

A photon with an energy of 4.9 eV ejects photoelectrons from tungsten. When the ejected electron enters a constant magnetic field of strength $B = 2.5 \text{ mT}$ at an angle of $60°$ with the field direction, the maximum pitch of the helix described by electron is found to be 2.7 mm. Find the work function of the metal in electron-volts. Given that specific charge of electron is $1.76\times10^{11} \text{ Ckg}^{-1}$.

SOLUTION

Pitch of helical path is

$$p = (v\cos\theta)T = \frac{vT}{2} \qquad \{\because \theta = 60°\}$$

$$\text{where, } T = \frac{2\pi m}{qB} = \frac{2\pi}{B\alpha} \qquad \left\{\because \alpha = \frac{q}{m}\right\}$$

$$\Rightarrow \quad p = \frac{\pi v}{B\alpha}$$

$$\Rightarrow \quad v = \frac{B\alpha p}{\pi} \qquad \ldots(1)$$

$$\Rightarrow \quad v = \frac{(2.5\times10^{-3})(1.76\times10^{11})(2.7\times10^{-3})}{3.14}$$

$$\Rightarrow \quad v = 0.38\times10^{6} \text{ ms}^{-1}$$

Since, $\text{KE} = \frac{1}{2}mv^2 = E - \phi_0$

$$\Rightarrow \quad \phi_0 = E - \frac{1}{2}mv^2 \qquad \ldots(2)$$

Substituting value of v from equation (1) in equation (2), we get

$$\phi_0 = 4.9 - \frac{1}{2}\frac{(9.1\times10^{-31})(0.38\times10^{6})^2}{1.6\times10^{-19}}$$

$$\Rightarrow \quad \phi_0 = (4.9 - 0.4) \text{ eV}$$

$$\Rightarrow \quad \phi_0 = 4.5 \text{ eV}$$

ILLUSTRATION 26

If the wavelength of the incident radiation is increased from 3000 Å to 3010 Å, find the corresponding change in the stopping potential V.

SOLUTION

According to Einstein's Photo-electric equation, we have

$$eV_1 = E_1 - \phi_0 \qquad \ldots(1)$$

$$eV_2 = E_2 - \phi_0 \qquad \ldots(2)$$

Subtracting (2) from (1), we get

$$e(V_1 - V_2) = (E_1 - E_2)$$

$$\Rightarrow \quad V_1 - V_2 = \frac{hc}{e}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$\Rightarrow \quad V_1 - V_2 = \frac{6.6\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times10^{-10}}\left(\frac{1}{3000} - \frac{1}{3010}\right)$$

$$\Rightarrow \quad V_1 - V_2 = 0.012 \text{ V}$$

ILLUSTRATION 27

When light of wavelength λ is incident on a metal surface, stopping potential is found to be V_0. When light of wavelength $n\lambda$ is incident on the same metal surface, stopping potential is found to be $\frac{V_0}{n+1}$. Calculate the threshold wavelength of the metal.

SOLUTION

Let λ_0 be the threshold wavelength so that the work function is $\phi = \frac{hc}{\lambda_0}$. Now, by photoelectric equation, we get

$$eV_0 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} \quad \ldots(1)$$

$$\frac{eV_0}{n+1} = \frac{hc}{n\lambda} - \frac{hc}{\lambda_0} \quad \ldots(2)$$

From (1) and (2), we get

$$\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = (n+1)\frac{hc}{n\lambda} - (n+1)\frac{hc}{\lambda_0}$$

$$\Rightarrow \quad \frac{nhc}{\lambda_0} = \frac{hc}{n\lambda}$$

$$\Rightarrow \quad \lambda_0 = n^2\lambda$$

ILLUSTRATION 28

A light beam of wavelength 400 nm is incident on a metal of work function 2.2 eV. A particular electron absorbs a photon and makes 2 collisions before coming out of the metal

(a) Assuming that 10% of extra energy is lost to the metal in each collision find the final kinetic energy of this electron as it comes out of the metal.

(b) Under the same assumptions find the maximum number of collisions the electron should suffer before it becomes unable to come out of the metal.

SOLUTION

(a) Since, $E(\text{in eV}) = \frac{12375}{\lambda(\text{in Å})}$

$$\Rightarrow \quad E = \frac{12375}{4000} = 3.1\text{ eV}$$

Energy of electron after first collision is

$$E_1 = (90\% \text{ of } E) = 2.79\text{ eV} \quad \{\because 10\% \text{ is lost}\}$$

Energy of electron after second collision

$$E_2 = (90\% \text{ of } E_1) = 2.51\text{ eV}$$

Hence, KE of this electron after emitting from the metal surface $= (2.51 - 2.2)\text{ eV} = 0.31\text{ eV}$

(b) Energy after third collision,

$$E_3 = (90\% \text{ of } E_2) = 2.26\text{ eV}$$

Similarly, $E_4 = (90\% \text{ of } E_3) = 2.03\text{ eV} < W$

So, after four collisions the electron will not be able to come out of the metal.

ILLUSTRATION 29

Calculate the velocity of the emitted photoelectrons, if the work function of the target material is 1.24 eV and the wavelength of incident light is 4000 Å. What retarding potential is necessary to stop the emission of the electrons? Take $hc = 1240\text{ eVnm}$

SOLUTION

Energy of incident photons in eV on metal surface is

$$E = \frac{12400}{4000}\text{ eV} = 3.1\text{ eV}$$

According to Einstein's photo electric equation, we have

$$E = \phi_0 + K_{max}$$

$$\Rightarrow \quad K_{max} = E - \phi_0$$

$$\Rightarrow \quad K_{max} = (3.1 - 1.24)\text{ eV} = 1.86\text{ eV}$$

The stopping potential for these ejected electrons is given by

$$V_s = \frac{K_{max}}{e} = \frac{1.86\text{ eV}}{e}$$

$$\Rightarrow \quad V_0 = 1.86\text{ volt}$$

ILLUSTRATION 30

When a beam of 10.6 eV photon of intensity 2.0 Wm^{-2} falls on a platinum surface of area 1.0 cm^2 and work function 5.6 eV, 0.53% of the incident photons eject photoelectrons. Find the number of photoelectrons emitted per second and their minimum and maximum kinetic energies (in eV). Take $1\text{ eV} = 1.6 \times 10^{-19}\text{ J}$.

SOLUTION

Since intensity of the photon beam is defined as the energy incident per second per unit area of a surface, so

$$I = \frac{E}{At} = \frac{P}{A}$$

$\Rightarrow \quad P = IA$

$\Rightarrow \quad P = (2.0\ \text{Wm}^{-2})(1.0\times10^{-4}\ \text{m}^2) = 2\times10^{-4}\ \text{W}$

According to the problem, energy carried by each photon is

$$E_{\text{single photon}} = 10.6\ \text{eV} = (10.6)(1.6\times10^{-19})\ \text{J}$$

$\Rightarrow \quad E_{\text{single photon}} = 16.96\times10^{-19}\ \text{J}$

So, the number of photons striking the metal surface per second is

$$n = \frac{P_{\text{photon beam}}}{E_{\text{single photon}}} = \frac{2.0\times10^{-4}}{16.96\times10^{-19}}$$

$\Rightarrow \quad n = 1.18\times10^{14}$ photons/s

Since, only 0.53% of the incident photons are able to eject photoelectrons, so the number of photoelectrons ejected per second is

$$n_{\text{photons}} = \left(\frac{0.53}{100}\right)n = \left(\frac{0.53}{100}\right)(1.18\times10^{14})$$

$\Rightarrow \quad n_{\text{photons}} = 6.254\times10^{11}$

So, minimum kinetic energy is zero and the maximum energy of the emitted photoelectron is

$$E_{\max} = 10.6\ \text{eV} - 5.6\ \text{eV} = 5.0\ \text{eV}$$

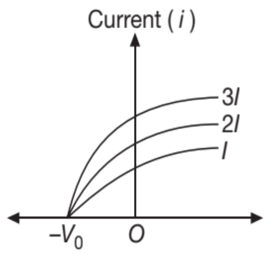
CHARACTERISTICS OF PHOTO ELECTRIC EFFECT

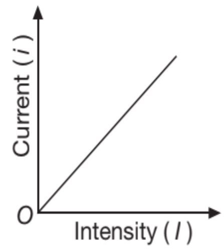
The following observations were made to study the effect of changes in various factors while studying the Photo Electric Effect.

Effect of Intensity

Intensity of light means the energy incident per second per unit area. For a given frequency, if intensity of incident light is increased, then photoelectric saturation current increases by the same factor and with decrease of intensity, the photoelectric saturation current also decreases by the same factor, but the stopping potential remains the same, so maximum value of kinetic energy is not effected.

In photoelectric effect **current** (i) **is directly proportional to intensity** (I) **of incident light.**



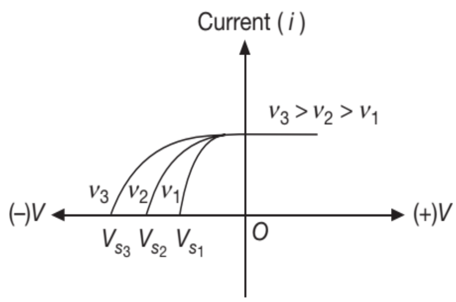


This means that the intensity of incident light affects the photoelectric current but leaves the maximum kinetic energy of photoelectrons unchanged.

Effect of Frequency

When the intensity of incident light is kept fixed and frequency is increased, the photoelectric current remains the same but the stopping potential increases.

If the frequency is decreased, the stopping potential decreases and at a particular frequency of incident light, the stopping potential becomes zero. This value of frequency of incident light for which the stopping potential is zero is called **threshold frequency** ν_0. If the frequency of incident light (ν) is less than the threshold frequency (ν_0), no photoelectric emission takes place.



Thus, the increase of frequency increases maximum kinetic energy of photoelectrons but leaves the photoelectric current unchanged.

Effect of Photo-metal

When frequency and intensity of incident light are kept fixed and photo-metal is changed, we observe that stopping potential (V_s) versus frequency (ν) graphs are parallel straight lines, cutting, frequency axis at different points. This shows that threshold frequency are different for different metals, the slope $\left(\frac{V_s}{\nu}\right)$ for all the metals is same and hence universal constant.

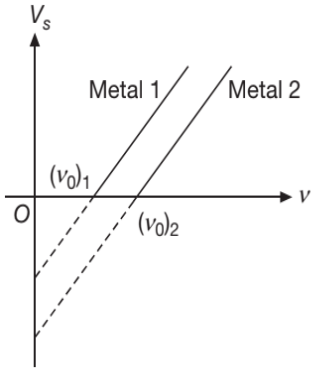
Since we know that

$$eV_s = h\nu - \phi_0$$

$$\Rightarrow \quad V_s = \left(\frac{h}{e}\right)\nu - \left(\frac{W}{e}\right)$$

Comparing with the line $y = mx + c$, where m is the slope and c is the intercept on the y axis.

Then, we observe that the graph is a straight line with slope $\frac{h}{e}$ (a universal constant) and negative intercept $\frac{\phi_0}{e}$ (depending on the nature of the metal). In figure threshold frequency and work function are greater for Metal 2 as compared to Metal 1.



Effect of Time

There is no time lag between incidence of light and the emission of photo-electrons.

Problem Solving Technique(s)

FORMULAE FOR WORKING THE PROBLEMS ON PHOTO-ELECTRIC EFFECT

Maximum Kinetic Energy of photo-electrons

$$E_K = eV_0 = \frac{1}{2}mv_{max}^2$$

If λ_0 is the threshold wavelength and ν_0 the threshold frequency,

Work function of photo-metal,

$$\phi_0 = h\nu_0 = \frac{hc}{\lambda_0}$$

Threshold frequency is minimum frequency and **Threshold wavelength** is maximum wavelength of incident light to cause photoelectric effect.

Einstein's Photo-electric Equation may be expressed as or $E_K = h\nu - \phi_0$

$$E_K = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

The condition for photoelectric emission is $h\nu \geq \phi_0$ or equivalently $\nu \geq \nu_0$ or equivalently $\lambda \leq \lambda_0$.

ILLUSTRATION 31

(a) If the wavelength of the light incident on a photoelectric cell is reduced from λ_1 Å to λ_2 Å, then calculate the change in the cut-off potential?

(b) Light is incident on the cathode of a photocell and the stopping voltages are measured for light of two different wavelengths. From the data given below, calculate the work function of the metal of the cathode in eV and the value of the universal constant $\frac{hc}{e}$.

Wavelength (Å)	Stopping voltage (volt)
4000	1.3
4500	0.9

SOLUTION

(a) Let the work function of the surface be ϕ_0. If ν be the frequency of the light falling on the surface, then according to Einstein's photoelectric equation, the maximum kinetic energy K_{max} of the emitted photoelectron is given by

$$K_{max} = h\nu - \phi_0 = \frac{hc}{\lambda} - \phi_0$$

Since, we know that the maximum kinetic energy of the photoelectrons emitted and the stopping potential are related to each other as $K_{max} = eV_s$

$$\Rightarrow \quad eV_s = \frac{hc}{\lambda} - \phi_0$$

$$\Rightarrow \quad V_s = \frac{hc}{e\lambda} - \frac{\phi_0}{e}$$

Now, $\Delta V_s = V_{s2} - V_{s1}$

$$\Rightarrow \quad \Delta V_s = \left(\frac{hc}{e\lambda_2} - \frac{\phi_0}{e}\right) - \left(\frac{hc}{e\lambda_1} - \frac{\phi_0}{e}\right)$$

$$\Rightarrow \quad \Delta V_s = \frac{hc}{e}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

$$\Rightarrow \quad \Delta V_s = \frac{hc}{e}\left(\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2}\right)$$

(b) From Equation (1), we have

$$\frac{hc}{e} = \frac{\Delta V_s(\lambda_1 \lambda_2)}{\lambda_1 - \lambda_2}$$

$$\Rightarrow \frac{hc}{e} = \frac{(1.3-0.9)\left[(4000\times10^{-10})\times(4500\times10^{-10})\right]}{500\times10^{-10}}$$

$$\Rightarrow \frac{hc}{e} = 1.44\times10^{-6}\ \text{Vm}^{-1}$$

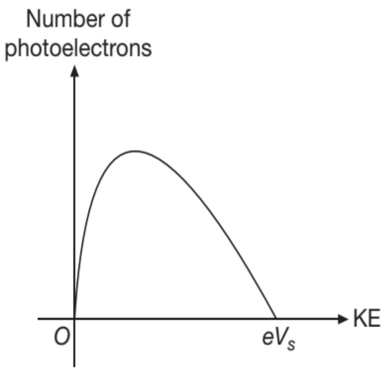
Also, we have $V_s = \frac{hc}{e\lambda} - \frac{\phi_0}{e}$

$$\Rightarrow \frac{\phi_0}{e} = \frac{hc}{e\lambda} - V_s = \frac{1.44\times10^{-6}}{4000\times10^{-10}} - 1.3$$

$$\Rightarrow \phi_0 = 2.3\ \text{eV}$$

GRAPH BETWEEN K_{max} AND ν

Whenever photoelectric effect takes place, electrons are ejected out with kinetic energies ranging from zero to K_{max} i.e. $0 \le K_{max} \le eV_s$. The energy distribution of photoelectrons is shown in figure.



Let us plot a graph between the maximum kinetic energy K_{max} and the frequency of the falling photon ν or the incident light. According to Einstein's Photo-Electric equation, we have

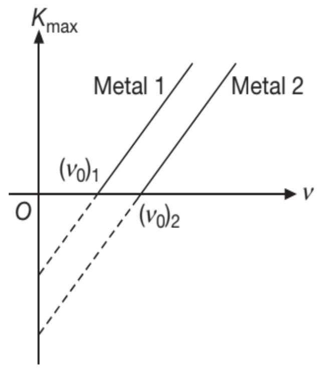
$$h\nu = \phi_0 + E_K$$

$$\Rightarrow \quad K_{max} = h\nu - \phi_0$$

Comparing with the line $y = mx + c$, where m is the slope and c is the intercept on the y axis.

Then, we observe that the graph is a straight line with slope h (a universal constant) and negative intercept ϕ (depending on the nature of the metal). For Metal 2, we observe that

$$\phi_2 > \phi_1 \text{ and hence } (\nu_0)_2 > (\nu_0)_1$$



Also, we observe that when $\nu = \nu_0$, the threshold frequency, then, $K_{max} = 0$

ILLUSTRATION 32

Calculate the value of the Planck's constant h if photoelectrons emitted from a surface of a certain metal by light of frequency 2.2×10^{15} Hz are fully retarded by a reverse potential of 6.6 V and those ejected by light of frequency 4.6×10^{15} Hz by a reverse potential of 16.5 eV.

SOLUTION

According to Einstein's photo electric equation, we have

$$h\nu_1 = \phi_0 + eV_1 \qquad \ldots(1)$$

$$h\nu_2 = \phi_0 + eV_2 \qquad \ldots(2)$$

Subtracting Equation (1) from Equation (2), we get

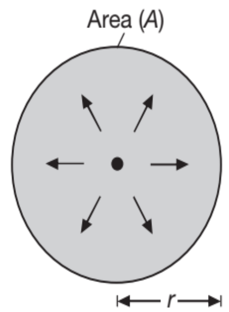
$$h(\nu_2 - \nu_1) = e(V_2 - V_1)$$

$$\Rightarrow \quad h = \frac{(16.5-6.6)(1.6\times10^{-19})}{(4.6-2.2)\times10^{15}}$$

$$\Rightarrow \quad h = 6.6\times10^{-34}\ \text{Js}$$

DETERMINATION OF PHOTOELECTRIC CURRENT

Let P be the power of a point source of electromagnetic radiations as shown.



The intensity I at a distance r from the source is given by $I=\dfrac{P}{4\pi r^2}\text{ Wm}^{-2}$. If A is the area of a metal surface on which radiations are incident, then the power received by the plate is

$$P'=IA=\left(\frac{P}{4\pi r^2}\right)A(\text{W})$$

If ν is the frequency of radiation, then the energy of photon is given by

$$E=h\nu$$

The number of photons incident on the plate per second is given by

$$n=\frac{P'}{E}=\left[\frac{\frac{P}{4\pi r^2}\times A}{h\nu}\right]$$

If $\nu>\nu_0$ (threshold frequency) and photon efficiency of the metal plate is $\eta\%$, then the actual number of photoelectrons emitted per second is given by

$$n'=\left(\frac{\eta}{100}\right)n=\left[\frac{\frac{P}{4\pi r^2}\times A}{h\nu}\right]\frac{\eta}{100}$$

Finally, the photocurrent i is given by

$$i=n'e$$

where e is the charge of an electron $(e=1.6\times10^{-19}\text{ C})$

ILLUSTRATION 33

Light of wavelength 180 nm ejects photoelectrons from a plate of a metal whose work function is 2 eV. If a uniform magnetic field of 50 μT is applied parallel to the plate, what would be the radius of the path followed by electrons ejected normally from the plate with maximum energy.

SOLUTION

$$\lambda=180\text{ nm}=1800\text{ Å}$$

$$\Rightarrow\quad E=\frac{12375}{1800}=6.875\text{ eV}$$

Since, $K_{\max}=E-\phi_0=4.875\text{ eV}$

Since, $r=\dfrac{mv}{qB}=\dfrac{\sqrt{2mK}}{qB}$

Substituting the values, we get

$$r=\frac{\sqrt{2\times4.875\times1.6\times10^{-19}\times9.1\times10^{-31}}}{5\times10^{-5}\times1.6\times10^{-19}}$$

$$\Rightarrow\quad r=0.15\text{ m}=15\text{ cm}$$

ILLUSTRATION 34

A small metal plate, having work function ϕ_0, is kept at a distance d from a singly ionized, fixed ion. A monochromatic light beam is incident on the metal plate and photoelectrons are emitted. Find the maximum wavelength of the light beam so that some of the photoelectrons may go round the ion along a circle.

SOLUTION

For circular motion of electrons around the ion, the electrostatic force between the electron and the positively charged ion must provide the necessary centripetal force to the electron to revolve in a circle of radius d.

$$\Rightarrow\quad \frac{1}{4\pi\varepsilon_0}\frac{e^2}{d^2}=\frac{mv^2}{d}$$

$$\Rightarrow\quad \frac{e^2}{8\pi\varepsilon_0 d}=\frac{1}{2}mv^2=\frac{hc}{\lambda}-\phi_0$$

$$\Rightarrow\quad \lambda=\frac{8\pi\varepsilon_0 hcd}{e^2+8\pi\varepsilon_0\phi_0 d}$$

ILLUSTRATION 35

A small plate of a photosensitive metal having work function 1.1 eV is placed at a distance of 2 m from a monochromatic light source of wavelength 496 nm and power 1 watt. The light falls normally on the plate. Calculate the number of photons striking the metal plate per second per square meter. If a constant magnetic field of strength 10^{-4} T is applied parallel to the metal surface, calculate the radius of the largest circular path followed by the emitted photoelectrons. Given $hc=12400\text{ eVÅ}$

SOLUTION

Energy of each incident photon in eV is

$$E_{\text{single photon}} = \frac{12400}{4960}\ \text{eV} = 2.5\ \text{eV}$$

$$\Rightarrow\quad E_{\text{each photon}} = 2.5\times1.6\times10^{-19}\ \text{J} = 4\times10^{-19}\ \text{J}$$

The rate of emission of photons i.e. the number of photons emitted per second by the source is

$$n = \frac{P_{\text{source}}}{E_{\text{each photon}}} = \frac{1\ \text{Js}^{-1}}{4\times10^{-19}\ \text{J}} = 2.5\times10^{18}\ \text{photons/s}$$

Hence, the number of photons striking the plate per second per square meter (also called as photon flux) is given by

$$\phi_N = \frac{n}{A} = \frac{n}{4\pi r^2} = \frac{2.5\times10^{18}}{4(3.14)(2)^2} \approx 5\times10^{16}\ \text{s}^{-1}\text{m}^{-2}$$

The maximum kinetic energy of the photo-electrons emitted from the plate having work function $\phi_0 = 1.1$ eV is given by

$$K_{\max} = E - \phi_0 = 2.5 - 1.1 = 1.4\ \text{eV}$$

$$\Rightarrow\quad \frac{1}{2}mv_{\max}^2 = 1.4\ \text{eV}$$

The maximum velocity of the ejected photoelectrons is

$$v_{\max} = \sqrt{\frac{2\times1.4\times1.6\times10^{-19}}{9.1\times10^{-31}}}$$

$$\Rightarrow\quad v_{\max} \approx 7.0\times10^{5}\ \text{ms}^{-1}$$

The maximum radius of the circle traversed by photoelectron in magnetic field $B = 10^{-4}$ T is given by

$$r = \frac{mv}{qB} = \frac{(9.1\times10^{-31})(7\times10^{5})}{(1.6\times10^{-19})(10^{-4})} = 0.0398\ \text{m}$$

$$\Rightarrow\quad r \approx 0.04\ \text{metre} = 4.0\ \text{cm}$$

Test Your Concepts-II

Based on Photoelectric Effect

(Solutions on page H.5)

1. What will be the maximum kinetic energy of the photoelectrons ejected from magnesium (for which the work function $W = 3.7$ eV) when irradiated by ultraviolet light of frequency $1.5\times10^{15}\ \text{sec}^{-1}$.
2. A 40 W ultraviolet light source of wavelength 2480 Å illuminates a photosensitive metal surface placed 2 m away. Calculate the number of photons emitted from the source per second and the number incident on unit area of the metal surface per second. The photo-electric work function for the metal is 3.7 eV. Calculate the kinetic energy of the fastest electrons ejected from the surface. Also calculate the maximum wavelength of light that can produce the photoelectric effect from the given metal surface.
   Given $hc = 12400$ eVÅ
3. The hydrogen atom in its ground state is excited by means of monochromatic radiation. Its resulting spectrum has six different lines. These radiations are incident on a metal plate. It is observed that only two of them are responsible for photoelectric effect. If the ratio of maximum kinetic energy of photoelectrons in the two cases is 5 then find the work function of the metal.
   [Take ground state energy of H-atom $= -13.6$ eV].
4. A metallic surface is irradiated with monochromatic light of variable wavelength. Above a wavelength of 5000 Å, no photoelectrons are emitted from the surface. With an unknown wavelength, stopping potential of 3 V is necessary to eliminate the photocurrent. Find the unknown wavelength.
5. A light source, emitting three wavelengths 5000 Å, 6000 Å and 7000 Å, has a total power of 10^{-3} W and a beam diameter of 2 mm. The power density is distributed equally amongst the three wavelengths. The beam shines normally on a metallic surface of area on $10^{-4}\ \text{m}^2$ and having a work function of 1.9 eV. Assuming that each photon liberates an electron, calculate the charge emitted per second from the metal surface.
6. A beam of light consists of four wavelength 4000 Å, 4800 Å, 6000 Å and 7000 Å, each of intensity $1.5\times10^{-3}\ \text{Wm}^{-2}$. The beam falls normally on an area $10^{-4}\ \text{m}^2$ of a clean metallic surface of work

function 1.9 eV. Assuming no loss of light energy calculate the number of photoelectrons liberated per second.

7. In an experiment on photo electric emission, following observations were made:
(i) Wavelength of the incident light $= 1.98 \times 10^{-7}$ m
(ii) Stopping potential = 2.5 V.
Find the
(a) threshold frequency.
(b) work function and
(c) energy of photo electrons with maximum speed.

8. Radiation of wavelength 5461 Å falls on a photo cathode and electrons with a maximum kinetic energy of 0.18 eV are emitted. When radiation of wavelength 1849 Å falls on the same surface a (negative) potential of 4.6 V has to be applied to the collector electrode to reduce the photoelectric current is zero. Find the value of h and cutoff wavelength.

9. Illuminating the surface of a certain metal alternately with light of wavelengths $\lambda_1 = 0.35\ \mu$m and $\lambda_2 = 0.54\ \mu$m, it was found that the corresponding maximum velocities of photo electrons differ by a factor $\eta = 2$. Find the work function of that metal.

10. When a surface is irradiated with light of $\lambda = 4950$ Å a photocurrent appears which vanishes if a retarding potential 0.6 V is applied. When a different source of light is used it is found that critical retarding potential is changed to 1.1 V. Find the work function of emitting surface and wavelength of second source. If photoelectrons after emission from surface are subjected to a magnetic field of 10 T, what changes will be observed in the above two retarding potentials?

11. The photoelectric work function of potassium is 2.3 eV. If light having a wavelength of 2800 Å falls on potassium, find
(a) the kinetic energy in electron volt of the most energetic electrons ejected.
(b) the stopping potential in volt.

12. Electrons with a maximum kinetic energy of 3 eV are ejected from a metal surface by ultraviolet radiation of wavelength 1500 Å. Calculate the work function of the metal, the threshold wavelength of metal and the stopping potential difference required to stop the emission of photoelectrons.

13. In an experiment tungsten cathode which has a threshold 2300 Å is irradiated by ultraviolet light of wavelength 1800 Å. Calculate the
(a) work function for tungsten in eV.
(b) maximum energy of emitted photoelectron in eV.
Given that the Planck's constant is $h = 6.6 \times 10^{-34}$ Js, $1\text{ eV} = 1.6 \times 10^{-19}$ J and velocity of light $c = 3 \times 10^{8}\ \text{ms}^{-1}$

14. A low intensity ultraviolet light of wavelength 2250 Å irradiates a photocell made of molybdenum metal. If the stopping potential is 1.5 V, calculate the work function of the metal. Will the photocell work if it is irradiated by a high intensity light of wavelength 6875 Å?

15. The photoelectric threshold of the photo electric effect of a certain metal is 2750 Å. Calculate the
(a) work function of emission of an electron from this metal,
(b) maximum kinetic energy of these electrons,
(c) maximum velocity of the electrons ejected from the metal by light with a wavelength 1800 Å.
Take $hc = 1243$ eVnm

16. Light quanta with an energy 4.9 eV eject photoelectrons from metal with work function 4.5 eV. Find the maximum impulse transmitted to the surface of the metal when each electron flies out.

SOLVED PROBLEMS

PROBLEM 1

A uniform monochromatic beam of light of wavelength 365 nm and intensity 10^{-8} Wm^{-2} falls on a photosensitive metal surface having absorption coefficient 0.8 and work function 1.6 eV. Calculate the rate of number of electrons emitted per m^2, power absorbed per m^2 and the maximum kinetic energy of emitted photoelectrons.

SOLUTION

The rate of number of electrons emitted per m^2 i.e. the number of photons crossing unit area per unit time i.e. the incident photon flux ϕ_i is given by

$$\phi_{\text{incident}} = \frac{I\lambda}{hc} = \frac{10^{-8} \times 365 \times 10^{-9}}{6.62 \times 10^{-34} \times 3 \times 10^{8}} = 18.35 \times 10^{9}$$

The number of photons absorbed by the surface per second per unit area is given by

$$\phi_{\text{absorbed}} = (0.8)\phi_{\text{incident}} = (0.8)\left(8.35 \times 10^{9}\right)$$

$$\Rightarrow \quad \phi_{\text{absorbed}} \approx 1.5 \times 10^{10}\ \text{s}^{-1}\text{m}^{-2}$$

Now assuming that each photon ejects only one electron, then the rate of electrons emitted per second per unit area is also

$$\phi_{\text{absorbed}} = 1.5 \times 10^{10}\ \text{s}^{-1}\text{m}^{-2}$$

Since power absorbed per square metre is the absorption coefficient times the power incident per square metre, so

$$P_{\text{absorbed per square metre}} = (0.8)P_{\text{incident per square metre}}$$

$$\Rightarrow \quad P_{\text{absorbed per square metre}} = (0.8)\left(10^{-8}\right) = 8 \times 10^{-9}\ \text{Wm}^{-2}$$

From Einstein's equation, maximum kinetic energy of the emitted photoelectrons from the metal with work function W is given by

$$K_{\max} = h\nu - W_0 = \frac{hc}{\lambda} - W_0$$

$$\Rightarrow \quad K_{\max} = \frac{\left(6.62 \times 10^{-34}\right)\left(3 \times 10^{8}\right)}{365 \times 10^{-9}} - 1.6 \times 1.6 \times 10^{-19}$$

$$\Rightarrow \quad K_{\max} = 2.89 \times 10^{-19}\ \text{joule} = 1.80\ \text{eV}$$

PROBLEM 2

In a photocell the plates P and Q have a separation of 10 cm, which are connected through a galvanometer without any cell. Bi-chromatic light of wavelengths 4000 Å and 6000 Å are incident on plate Q whose work function is 2.39 eV. If a uniform magnetic field B exists parallel to the plates, find the minimum value of B for which the galvanometer shows zero deflection.

SOLUTION

Energy of photons corresponding to light of wavelength $\lambda_1 = 4000$ Å is

$$E_1 = \frac{12375}{4000} = 3.1\ \text{eV}$$

and that corresponding to $\lambda_2 = 6000$ Å is,

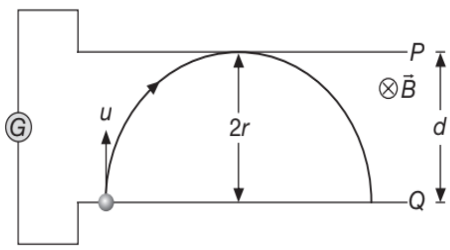
$$E_2 = \frac{12375}{6000} = 2.06\ \text{eV}$$

Given that the work function for the metal is $W = 2.39$ eV, so we observe that

$$E_2 < W \text{ and } E_1 > W$$

Hence photoelectric emission is possible with λ_1 only.

Photoelectrons experience magnetic force and move along a circular path.



The galvanometer will indicate zero deflection when the photoelectrons complete semi-circular path before reaching the plate P.

Thus, $d = 2r = 10$ cm

$$\Rightarrow \quad r = 5\ \text{cm} = 0.05\ \text{m}$$

Further $r = \dfrac{mv}{Bq} = \dfrac{\sqrt{2Km}}{Bq}$

$$\Rightarrow \quad B_{min} = \frac{\sqrt{2Km}}{rq}$$

where $K = E_1 - W = (3.1 - 2.39) = 0.71$ eV

Substituting the values, we have

$$B_{min} = \frac{\sqrt{2 \times 0.71 \times 1.6 \times 10^{-19} \times 9.109 \times 10^{-31}}}{(0.05)(1.6 \times 10^{-19})}$$

$$\Rightarrow \quad B_{min} = 5.68 \times 10^{-5} \text{ Tesla}$$

PROBLEM 3

When a metal surface is irradiated with light of wavelength 4950 Å, a photo current appears. This photo current vanishes, if a retarding potential greater than 0.6 volt is applied across the photo tube. However, when a different source of light is used, it is found that the critical retarding potential is changed to 1.1 volt. Calculate the work function of the emitting metal surface and the wavelength of second source.

If the photo electrons (after emission from the surface) are subjected to a magnetic field of 10 tesla, what changes will be observed in the above two retarding potentials.

SOLUTION

In the first case, energy of incident photon in eV is

$$E_1 = \frac{12375}{4950} \text{ eV} = 2.5 \text{ eV}$$

The maximum kinetic energy of ejected electrons is

$$(K_{max})_1 = eV_1 = 0.6 \text{ eV}$$

According to Einstein's photo electric equation, we have

$$E = \phi_0 + K_{max}$$

$$\Rightarrow \quad \phi_0 = E_1 - (K_{max})_1$$

$$\Rightarrow \quad \phi_0 = (2.5 - 0.6) \text{ eV} = 1.9 \text{ eV}$$

In second case, the maximum kinetic energy of ejected electrons is

$$(K_{max})_2 = eV_2 = 1.1 \text{ eV}$$

According to Einstein's photo electric equation, we have

$$E = \phi_0 + K_{max}$$

$$\Rightarrow \quad E_2 = (1.9 + 1.1) \text{ eV} = 3.0 \text{ eV}$$

So, the wavelength of incident photons in second case is

$$\lambda_2 = \frac{12375}{3.0} \text{ Å} = 4125 \text{ Å}$$

Since work done by magnetic force in moving a charged particle is zero, so a magnetic field can never speed up or slow down a charged particle and hence there will be no effect on the stopping potentials, because the kinetic energy of the emitted photoelectrons remains the same.

PROBLEM 4

In a photoelectric effect set-up, a point source of light of power 3.2×10^{-3} W emits mono energetic photons of energy 5 eV. The source is located at a distance of 0.8 m from the centre of a stationary metallic sphere of work function 3 eV and of radius 8×10^{-3} m. The efficiency of photoelectron emission is one for every 10^6 incident photons. Assume that the sphere is isolated and electrons are instantly swept away after emission.

(a) Calculate the number of photoelectrons emitted per second.

(b) Find the ratio of the wavelength of incident light to the de Broglie wavelength of the fastest photoelectrons emitted.

(c) It is observed that the photoelectron emission stops at a certain time t after the light source is switched on. Why?

(d) Evaluate the time t.

SOLUTION

(a) Number of photoelectrons emitted per second is

$$n = \left(\frac{1}{10^6}\right)\left(\frac{P}{E}\right)\left(\frac{\pi r^2}{4\pi R^2}\right)$$

$$\Rightarrow n = \left(\frac{1}{10^6}\right) \times \left(\frac{3.2 \times 10^{-3}}{5 \times 1.6 \times 10^{-19}}\right) \times \left(\frac{1}{4\pi \times 0.8 \times 0.8}\right) \times (\pi \times 8 \times 10^{-3} \times 8 \times 10^{-3})$$

$$\Rightarrow n = 10^5 \text{ sec}^{-1}$$

(b) $K_{max} = E - \phi_0 = 2$ eV

Since, for an electron, we have

$$\lambda_2 = \sqrt{\frac{150}{KE(\text{in eV})}} \text{ Å}$$

$$\Rightarrow \quad \lambda_2 = \sqrt{\frac{150}{2}} = 8.66 \text{ Å}$$

Further, Wavelength of incident photon is

$$\lambda_1 = \frac{12375}{5} = 2475\ \text{Å}$$

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} \approx 286$$

(c) Photoemission will stop when potential on the sphere becomes equal to the stopping potential.

(d) $K_{max} = 2\ eV_0$. Therefore, the stopping potential V_0 is 2 volt. Let t be the desired time. Then

$$V_0 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \frac{Ne}{4\pi\varepsilon_0 r} = \frac{(nt)e}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad t = \frac{V_0 r}{\left(\frac{1}{4\pi\varepsilon_0}\right)(ne)} = \frac{2\times8\times10^{-3}}{9\times10^{9}\times10^{5}\times1.6\times10^{-19}}$$

$$\Rightarrow \quad t = 111\ \text{s}$$

PROBLEM 5

When photons of energy 4.25 eV strike the surface of a metal A, the ejected photoelectrons have maximum kinetic energy, T_A expressed in eV and de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photons of energy 4.70 eV is $T_B = (T_A - 1.50\ \text{eV})$. If the de-Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$, then find

(a) the work function W_A of metal A and the work function W_B of metal B.

(b) the maximum kinetic energy T_A of the electrons ejected from metal A.

SOLUTION

$$K_{max} = E - W$$

Therefore, $T_A = 4.25 - W_A$ ...(1)

$$T_B = (T_A - 1.50) = 4.70 - W_B \quad ...(2)$$

Equations (1) and (2) gives,

$$W_B - W_A = 1.95\ \text{eV} \quad ...(3)$$

de-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2Km}}$$

$$\Rightarrow \quad \lambda \propto \frac{1}{\sqrt{K}} \qquad K = \text{KE of electron}$$

$$\Rightarrow \quad \frac{\lambda_B}{\lambda_A} = \sqrt{\frac{K_A}{K_B}}$$

$$\Rightarrow \quad 2 = \sqrt{\frac{T_A}{T_A - 1.5}}$$

$$\Rightarrow \quad T_A = 2\ \text{eV}$$

From equation (1), we get

$$W_A = 4.25 - T_A = 2.25\ \text{eV}$$

From equation (3), we get

$$W_B = W_A + 1.95\ \text{eV} = (2.25 + 1.95)$$

$$\Rightarrow \quad W_B = 4.20\ \text{eV}$$

$$\Rightarrow \quad T_B = 4.70 - W_B = 4.70 - 4.20 = 0.50\ \text{eV}$$

PROBLEM 6

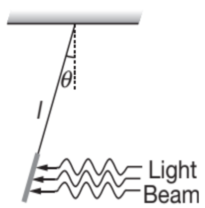
A small, plane strip is suspended from a fixed support through a string of length l as shown. A continuous beam of monochromatic light is incident horizontally on the strip and is completely absorbed. The energy falling on the strip per unit time is P.

(a) Find the deflection of the string from the vertical if the mirror stays in equilibrium.

(b) If the strip is deflected slightly from its equilibrium position in the plane of the figure, what will be the time period of the resulting oscillations?

SOLUTION

(a) The linear momentum of the light falling per unit time on the strip is $\frac{P}{c}$. As the light is incident on the strip, its momentum is absorbed by the mirror. The change in momentum imparted to the strip per unit time is thus $\frac{P}{c}$. This is equal to the force on the strip by the light beam. In equilibrium, the force by the light beam, the weight of the strip and the force due to tension add to zero. If the string makes an angle θ with the vertical,



$$T\cos\theta = mg$$

and $T\sin\theta = \dfrac{P}{c}$

Thus, $\tan\theta = \dfrac{P}{mgc}$

(b) In equilibrium, the tension is

$$T = \left[(mg)^2 + \left(\frac{P}{c}\right)^2\right]^{\frac{1}{2}}$$

$$\Rightarrow \quad \frac{T}{m} = \left[g^2 + \left(\frac{P}{mc}\right)^2\right]^{\frac{1}{2}}$$

This plays the role of effective g. The time period of small oscillations is

$$t = 2\pi\sqrt{\frac{l}{T/m}} = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \left(\frac{P}{mc}\right)^2}}}$$

PROBLEM 7

Ultraviolet light of wavelengths $800\ \text{Å}$ and $700\ \text{Å}$ when allowed to fall on hydrogen atoms in their ground state is found to liberate electrons with kinetic energy $1.8\ \text{eV}$ and $4.0\ \text{eV}$ respectively. Find the value of Planck's constant.

SOLUTION

When $800\ \text{Å}$ wavelength falls on hydrogen atom (in ground state) $13.6\ \text{eV}$ energy is used in liberating the electron. The rest is given as kinetic energy to the electron.

Hence, $K = E - 13.6(\text{in eV})$

$$\Rightarrow \quad (1.8\times1.6\times10^{-19}) = \frac{hc}{800\times10^{-10}} - 13.6\times1.6\times10^{-19} \quad \ldots(1)$$

Similarly, for the second wavelength, we have

$$(4\times1.6\times10^{-19}) = \frac{hc}{700\times10^{-10}} - 13.6\times1.6\times10^{-19} \quad \ldots(2)$$

Solving these two equations, we get

$$h = 6.6\times10^{-34}\ \text{Js}$$

PROBLEM 8

A beam of light has three wavelength 4144 Å, 4972 Å and 6216 Å with a total intensity of $3.6\times10^{-3}\ \text{Wm}^{-2}$ equally distributed amongst the three wavelengths. The beam falls normally on $1.0\ \text{cm}^2$ area of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photoelectrons emitted in two seconds.

SOLUTION

Threshold wavelength for the metal having a work function of $2.3\ \text{eV}$ is

$$\lambda_{\text{th}} = \frac{12375}{2.3}\ \text{Å} = 5380\ \text{Å}$$

So, only the wavelengths $4144\ \text{Å}$ and $4972\ \text{Å}$ will be able to emit electrons from the metal surface because they are lesser than the threshold wavelength.

Since the intensity is equally distributed amongst the three incident wavelengths, so we have

$$I = \left(\frac{I_{\text{total}}}{3}\right) = 1.2\times10^{-3}\ \text{Wm}^{-2}$$

The energy incident per second (i.e. power incident) on the surface for each wavelength is

$$P = IA = \left(\frac{I_{\text{total}}}{3}\right)A$$

$$\Rightarrow \quad P = (1.2\times10^{-3}\ \text{Wm}^{-2})(1.0\ \text{cm}^2)$$

$$\Rightarrow \quad P = (1.2\times10^{-3})\times(10^{-4})\ \text{W} = 1.2\times10^{-7}\ \text{W}$$

Energy incident on surface for each wavelength in an interval of 2 seconds is

$$E = Pt = (1.2\times10^{-7})(2) = 2.4\times10^{-7}\ \text{J}$$

The number of photons (N) in a light beam of energy E having photons of wavelength λ are given by

$$N = \frac{E_{\text{light beam}}}{E_{\text{single photon}}} = \frac{E}{\left(\frac{hc}{\lambda}\right)} = \frac{E\lambda}{hc}$$

Number of photons N_1 due to wavelength $4144\ \text{Å}$ is

$$N_1 = \frac{(1.2\times10^{-7})(4144\times10^{-10})}{(6.63\times10^{-34})(3\times10^{8})} = 0.5\times10^{12}$$

Number of photons N_2 due to wavelength 4972 Å is

$$N_2 = \frac{(2.4\times10^{-7})(4972\times10^{-10})}{(6.63\times10^{-34})(3\times10^{8})} = 0.575\times10^{12}$$

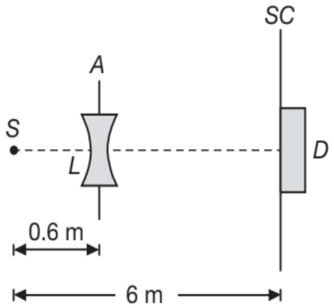
So, total number of photons (N) is given by

$$N = N_1 + N_2 = 0.5\times10^{12} + 0.575\times10^{12}$$

$$\Rightarrow \quad N = 1.075\times10^{12}$$

PROBLEM 9

A monochromatic point source S radiating wavelength 6000 Å, with power 2 watt, an aperture A of diameter 0.1 m and a large screen SC are placed as shown in figure. A photoemissive detector D of surface area $0.5\ \text{cm}^2$ is placed at the centre of the screen. The efficiency of the detector for the photoelectron generation per incident photon is 0.9.



(a) Calculate the photon flux at the centre of the screen and the photocurrent in the detector.

(b) If the concave lens L of focal length 0.6 m is inserted in the aperture as shown, find the new values of photon flux and photocurrent. Assume a uniform average transmission of 80% from the lens.

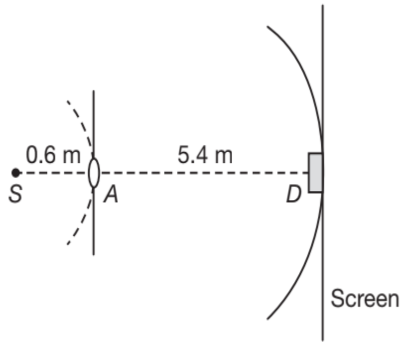
(c) If the work function of the photo emissive surface is 1 eV, calculate the values of the stopping potential in the two cases (without and with the lens in the aperture).

SOLUTION

(a) Energy of one photon,

$$E = \frac{hc}{\lambda} = \frac{(6.6\times10^{-34})(3\times10^{8})}{6000\times10^{-10}}$$

$$\Rightarrow \quad E = 3.3\times10^{-19}\ \text{J}$$



Power of the source is $2\ \text{W} = 2\ \text{Js}^{-1}$. Therefore, number of photons emitting per second is

$$n_1 = \frac{2}{3.3\times10^{-19}} = 6.06\times10^{18}\ \text{s}^{-1}$$

At distance 0.6 m, number of photons incident per unit area per unit time is

$$n_2 = \frac{n_1}{4\pi(0.6)^2} = 1.34\times10^{18}\ \text{m}^{-2}\text{s}^{-1}$$

Area of aperture is,

$$S_1 = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.1)^2 = 7.85\times10^{-3}\ \text{m}^2$$

So, total number of photons incident per unit time on the aperture,

$$n_3 = n_2S_1 = (1.34\times10^{18})(7.85\times10^{-3})\ \text{s}^{-1}$$

$$\Rightarrow \quad n_3 = 1.052\times10^{16}\ \text{s}^{-1}$$

This aperture will become new source of light. Now these photons are further distributed in all directions. Hence, at the location of the detector, photons incident per unit area per unit time is

$$n_4 = \frac{n_3}{4\pi(6-0.6)^2} = \frac{1.052\times10^{16}}{4\pi(5.4)^2}$$

$$\Rightarrow \quad n_4 = 2.87\times10^{13}\ \text{m}^{-2}\text{s}^{-1}$$

This is the photon flux at the centre of the screen. Area of detector is $0.5\ \text{cm}^2$ or $0.5\times10^{-4}\ \text{m}^2$. Therefore, total number of photons incident on the detector per unit time is

$$n_5 = (0.5\times10^{-4})(2.87\times10^{13}d) = 1.435\times10^{9}\ \text{s}^{-1}$$

The efficiency of photoelectron generation is 0.9. Hence, total photoelectrons generated per unit time is

$$n_6 = 0.9n_5 = 1.2915\times10^{9}\ \text{s}^{-1}$$

Hence, photocurrent in the detector is

$$i=(e)n_6=(1.6\times10^{-19})(1.2915\times10^9)$$

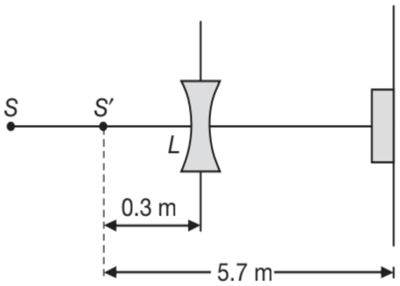
$$\Rightarrow \quad i=2.07\times10^{-10}\text{ A}$$

(b) Using the lens formula, we get

$$\frac{1}{v}-\frac{1}{-0.6}=\frac{1}{-0.6}$$

$$\Rightarrow \quad v=-0.3\text{ m}$$

i.e., image of source (say S', is formed at 0.3 m) from the lens.



Total number of photons incident per unit time on the lens are still n_3 or $1.052\times10^{16}\text{ s}^{-1}$. Since, 80% of it transmits to second medium, therefore, at a distance of 5.7 m from S' number of photons incident per unit area per unit time will be

$$n_7=\frac{\left(\frac{80}{100}\right)(1.052\times10^{16})}{(4\pi)(5.7)^2}$$

$$\Rightarrow \quad n_7=2.06\times10^{13}\text{ m}^{-2}\text{s}^{-1}$$

This is the photon flux at the detector.
New value of photocurrent is given by

$$i'=(2.06\times10^{13})(0.5\times10^{-4})(0.9)(1.6\times10^{-19})$$

$$\Rightarrow \quad i'=1.483\times10^{-10}\text{ A}$$

(c) Energy of incident photons (in both the cases) is

$$E(\text{in eV})=\frac{12375}{\lambda(\text{in Å})}$$

$$\Rightarrow \quad E=\frac{12375}{6000\text{ Å}}=2.06\text{ eV}$$

Work function $W=1\text{ eV}$
Maximum kinetic energy of photoelectrons in both cases,

$$K_{max}=E-W=1.06\text{ eV}$$

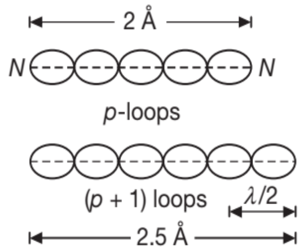
or the stopping potential will be 1.06 V.

PROBLEM 10

Assume that the de-Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one-dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance d between the atoms of the array is 2 Å. A similar standing wave is again formed if d is increased to 2.5 Å but not for any intermediate value of d. Find the energy of the electron in eV and the least value of d for which the standing wave of the type described above can form.

SOLUTION

From the figure it is clear that



So, $p\cdot\left(\frac{\lambda}{2}\right)=2\text{ Å}$ and

$$(p+1)\cdot\frac{\lambda}{2}=2.5\text{ Å}$$

$$\Rightarrow \quad \frac{\lambda}{2}=(2.5-2)\text{ Å}=0.5\text{ Å}$$

$$\Rightarrow \quad \lambda=1\text{ Å}=10^{-10}\text{ m}$$

de-Broglie wavelength is given by

$$\lambda=\frac{h}{p}=\frac{h}{\sqrt{2mK}}$$

where K is the kinetic energy of electron

$$\Rightarrow \quad K=\frac{h^2}{2m\lambda^2}$$

$$\Rightarrow \quad K=\frac{(6.63\times10^{-34})^2}{2(9.1\times10^{-31})(10^{-10})^2}$$

$$\Rightarrow \quad K=2.415\times10^{-17}\text{ J}$$

$$\Rightarrow \quad K=\left(\frac{2.415\times10^{-17}}{1.6\times10^{-19}}\right)\text{eV}$$

$$\Rightarrow \quad K=150.8\text{ eV}$$

The least value of d will be, when only one loop is formed. So, we have

$$d_{\min} = \frac{\lambda}{2}$$

$$\Rightarrow \quad d_{\min} = 0.5 \text{ Å}$$

PROBLEM 11

When a beam of 10.6 eV photons of intensity 2 Wm^{-2} falls on a platinum surface of area $1\times10^{-4} \text{ m}^2$ and work function 5.6 eV. 0.53% of the incident photons eject photoelectrons. Find the number of photoelectrons emitted per second and their minimum and maximum energies (in eV). Take $1 \text{ eV} = 1.6\times10^{-19} \text{ J}$.

SOLUTION

Energy of incident photons,

$$E_i = 10.6 \text{ eV}$$

$$\Rightarrow \quad E_i = 10.6\times1.6\times10^{-19} \text{ J}$$

$$\Rightarrow \quad E_i = 16.96\times10^{-19} \text{ J}$$

Energy incident per unit area per unit time $(\text{intensity}) = 2 \text{ J}$

So, number of photons incident on unit area in unit time is

$$\frac{n}{A} = \frac{2}{16.96\times10^{-19}} = 1.18\times10^{18}$$

Therefore, number of photons incident per unit time on given area $(1\times10^{-4} \text{ m}^2)$ is

$$n = (1.18\times10^{18})(1\times10^{-4})$$

$$\Rightarrow \quad n = 1.18\times10^{14}$$

But only 0.53% of incident photons emit photoelectrons, so number of photoelectrons emitted per second (n) is

$$n = \left(\frac{0.53}{100}\right)(1.18\times10^{14})$$

$$\Rightarrow \quad n = 6.25\times10^{11}$$

$$K_{\min} = 0$$

and $\quad K_{\max} = E_i - W$

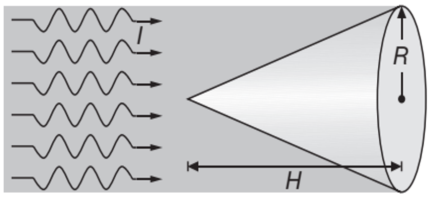
$$\Rightarrow \quad K_{\max} = (10.6 - 5.6) \text{ eV} = 5 \text{ eV}$$

$$\Rightarrow \quad K_{\max} = 5 \text{ eV}$$

and $\quad K_{\min} = 0$

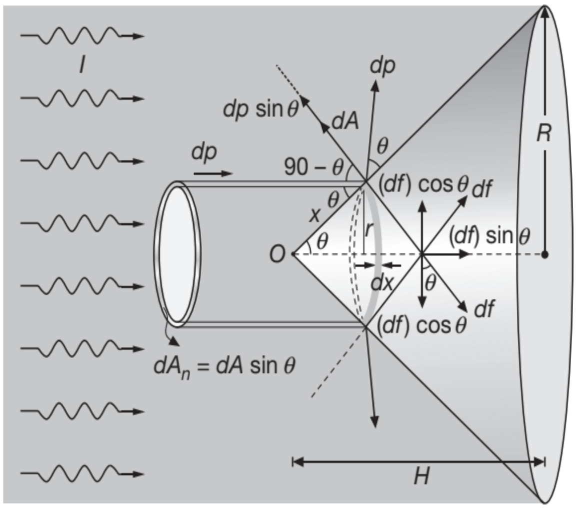
PROBLEM 12

A cone of radius R and height H with perfectly reflecting lateral surface, is placed in the path of a light beam of intensity I as shown. Calculate the force exerted by the light beam on this cone.



SOLUTION

To find the force on cone, we consider an elemental strip of width dx on the lateral surface of cone at a distance x from the vertex O of cone as shown in figure.



If the radius of the strip is r, then surface area of the strip is

$$dA = (2\pi r)dx \qquad \ldots(1)$$

Since for the cone, we have

$$\sin\theta = \frac{r}{x} = \frac{R}{\sqrt{R^2+H^2}} \qquad \ldots(2)$$

$$\Rightarrow \quad r = x\sin\theta$$

So, the area of strip is

$$dA = 2\pi(x\sin\theta)dx$$

If dA_n be the projection of area dA of the slant strip along the cross-sectional plane of the light beam i.e. normal to the light beam, then we have

$$dA_n = dA\sin\theta \qquad \ldots(3)$$

If dP is the power of light beam incident on this infinitesimal strip, then

$$dP = IdA_n \quad \ldots(4)$$

So, the force df on this infinitesimal strip element acts along the normal at the point of incidence and is given by

$$df = \left(\frac{2dP}{c}\right)\sin\theta = \left(\frac{2IdA_n}{c}\right)\sin\theta = \frac{2IdA\sin^2\theta}{c} \quad \ldots(5)$$

On resolving this infinitesimal force, we observe that the vertical components of the forces cancel, whereas the net force F is only obtained by integrating the component $df\sin\theta$ acting along the beam of light.

$$F = \int df\sin\theta$$

$$\Rightarrow \quad F = \int\left(\frac{2IdA}{c}\sin^2\theta\right)\sin\theta$$

$$\Rightarrow \quad F = \int\left(\frac{2I}{c}\sin^3\theta\right)dA = \int\left(\frac{2I}{c}\sin^3\theta\right)\left(2\pi\left(x\sin\theta\right)dx\right)$$

$$\Rightarrow \quad F = \left(\frac{4\pi I}{c}\sin^4\theta\right)\int_0^{\sqrt{R^2+h^2}} xdx$$

$$\Rightarrow \quad F = \left(\frac{4\pi I}{c}\sin^4\theta\right)\left(\left.\frac{x^2}{2}\right|_0^{\sqrt{R^2+h^2}}\right)$$

$$\Rightarrow \quad F = \left(\frac{2\pi I}{c}\right)\left(\frac{R^4}{\left(R^2+H^2\right)^2}\right)\left(R^2+H^2\right)$$

$$\Rightarrow \quad F = \left(\frac{2\pi I}{c}\right)\left(\frac{R^4}{R^2+H^2}\right) = \frac{2\pi IR^4}{c\left(R^2+H^2\right)}$$

PROBLEM 13

A small plate of a metal having work function of 1.17 eV is placed at a distance of 2 m from a monochromatic light source of wave length $4.8\times10^{-7}\text{ m}$ and power 1 W. The light falls normally on the plate. Find the number of photons striking the metal plate per m^2 per sec. If a constant uniform magnetic field of strength 10^{-4} T is applied parallel to the metal surface, find the radius of the largest circular path followed by the emitted photo electrons. Given $h = 6.6\times10^{-34}\text{ Js}$, $c = 3\times10^8\text{ ms}^{-1}$, $e = 1.6\times10^{-19}\text{ C}$ and electron mass $m = 9.1\times10^{-31}\text{ kg}$.

SOLUTION

Energy of photons of wavelength $4.8\times10^{-19}\text{ J}$ is

$$E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^8}{4.8\times10^{-7}} = 4.125\times10^{-19}\text{ J}$$

Power of source $= 1\text{ W} = 1\text{ Js}^{-1}$

So, rate of emission of photons from the source is

$$n = \frac{1\text{ Js}^{-1}}{4.125\times10^{-19}\text{ J}} = 2.424\times10^{18}\text{ s}^{-1}$$

These photons move in all directions randomly. At a distance r from the source, the photons fall normally over a spherical surface of area $4\pi r^2$. The plate is at a distance $r = 2\text{ m}$. Hence the number of photons striking the surface per m^2 per second is

$$n = \frac{2.424\times10^{18}}{4\times3.14\times(2)^2} = 4.82\times10^{16}$$

The maximum KE of a photoelectron emitted from the plate is

$$K_{\max} = \frac{hc}{\lambda} - W_0$$

$$\Rightarrow \quad K_{\max} = 4.125\times10^{-19} - 1.17\times1.6\times10^{-19}$$

$$\Rightarrow \quad K_{\max} = 2.253\times10^{-19}\text{ J}$$

Hence the maximum velocity of the photoelectron is

$$v_{\max} = \sqrt{\frac{2K_{\max}}{m}}$$

$$\Rightarrow \quad v_{\max} = \sqrt{\frac{2\times2.153\times10^{-19}}{9.1\times10^{-31}}} = 7.03\times10^5\text{ ms}^{-1}$$

Radius of the largest circular path of the photoelectrons in the magnetic field is

$$r = \frac{mv_{\max}}{eB} = \frac{9.1\times10^{-31}\times7.036\times10^5}{1.6\times10^{-19}\times10^{-4}}$$

$$\Rightarrow \quad r = 4\times10^{-2}\text{ m} = 4\text{ cm}$$

PROBLEM 14

Two metallic plates A and B each of area $5\times10^{-4}\text{ m}^2$, are placed parallel to each other at separation of 1 cm. Plate B carries a positive charge of $33.7\times10^{-12}\text{ C}$.

A monochromatic beam of light, with photons of energy 5 eV each, starts falling on place A at $t=0$ so that 10^6 photons fall on it per square meter per second. Assume that the photoelectron is emitted for every 10^6 incident photons. Also assume that all the emitted photoelectrons are collected by plate B and the work function of plate A remains constant at the value 2 eV.

(a) the number of photoelectrons emitted up to $t=10$ s

(b) the magnitude of the electric field between the plates A and B at $t=10$ s and

(c) The kinetic energy of the most energetic photo-electrons emitted at t = 10 s when it reaches plate B.

Neglect the time taken by the photoelectron to reach plate B.

(Take $\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$)

SOLUTION

Area of plates $A = 5\times10^{-4}\ \text{m}^2$

Distance between the plates $d = 1\ \text{cm} = 10^{-2}\ \text{m}$

(a) Number of photoelectrons emitted upto $t=10$ s are

$$n = \frac{\begin{pmatrix}\text{number of photons}\\ \text{falling in unit}\\ \text{area in unit time}\end{pmatrix}\times(\text{area}\times\text{time})}{10^6}$$

$$\Rightarrow \quad n = \frac{1}{10^6}\left[(10)^{16}\times(5\times10^{-4})\times(10)\right]$$

$$\Rightarrow \quad n = 5\times10^7$$

(b) At time $t=10$ s, charge on plate A

$$q_A = +ne = (5\times10^7)(1.6\times10^{-19})$$

$$\Rightarrow \quad q_A = 8\times10^{-12}\ \text{C}$$

and charge on plate B,

$$q_B = (33.7\times10^{-12} - 8\times10^{-12})$$

$$\Rightarrow \quad q_B = 25.7\times10^{-12}\ \text{C}$$

Since, electric field between the plates is

$$E = \frac{(q_B - q_A)}{2A\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{(25.7-8)\times10^{-12}}{2\times(5\times10^{-4})(8.85\times10^{-12})} = 2\times10^3\ \text{NC}^{-1}$$

(c) Energy of photoelectrons at plate A is

$$K = E - W = (5-2)\ \text{eV} = 3\ \text{eV}$$

Increase in kinetic energy of photoelectrons when they reach B is

$$\Delta K = (eEd)\ \text{joule} = (Ed)\ \text{eV}$$

$$\Rightarrow \quad \Delta K = (2\times10^3)(10^{-2})\text{eV} = 20\ \text{eV}$$

Energy of photoelectrons at plate B is

$$K_B = (20+3)\ \text{eV} = 23\ \text{eV}$$

PROBLEM 15

A mercury arc lamp provides 100 mW of UV radiation at a wavelength of 2480 Å (all other wavelengths having been absorbed by filters). The cathode of photoelectric device (a photo-tube) consists of potassium and has an effective area of $4\ \text{cm}^2$. The anode is located at a distance of 1 m from radiation source. The work function (ϕ_0) for potassium is 2.25 eV.

(a) According to classical theory, the radiation from the arc spreads out uniformly in space as spherical wave. Calculate the time of exposure of the metal to the radiation so that a potassium atom (radius 2 Å) in the anode accumulates sufficient energy to eject a photo-electron.

(b) Calculate energy of a single photon from the source.

(c) Calculate the number of photons striking the cathode per second. Also calculate the saturation current, if the photo-conversion efficiency is 5% (i.e., if each photon has a probability of 0.05 of ejecting an electron).

(d) Calculate the cut off potential V_0.

Given $hc = 12400$ eVÅ

SOLUTION

(a) The energy emitted per second per unit area (i.e. intensity) of the UV lamp at a distance of one metre is

$$I = \frac{P}{4\pi r^2} = \frac{100\times10^{-3}}{4\pi(1)^2} = \frac{0.1}{4\pi}\ \text{Wm}^{-2}$$

The cross-sectional area of atom i.e. effective area of the atom exposed to radiation is $A_{\text{eff}} = \pi r^2$

$$\Rightarrow \quad A_{\text{eff}} = \pi(2\times10^{-10}\ \text{m})^2 = 4\pi\times10^{-20}\text{m}^2$$

Energy required to eject photo-electron from the metal surface is

$$\phi_0 = (2.25)(1.6\times10^{-19})\ \text{J} = 3.6\times10^{-19}\ \text{J}$$

Since, $\phi_0 = IAt$, hence the exposure time is given by

$$t = \frac{\phi_0}{IA_{\text{eff}}} = \frac{3.6\times10^{-19}}{\left(\frac{0.1}{4\pi}\right)(4\pi\times10^{-20})} = 360\ \text{s}$$

(b) Incident photon energy in eV is

$$E = \frac{12400}{2480}\ \text{eV} = 5\ \text{eV}$$

$$\Rightarrow \quad E = 5\times1.6\times10^{-19}\ \text{J} = 8\times10^{-19}\ \text{J}$$

(c) Since, we know that the intensity

$$I = \frac{E_{\text{photon source}}}{At} = \left(\frac{N}{t}\right)\left(\frac{E_{\text{each photon}}}{A}\right)$$

$$\Rightarrow \quad I = n\left(\frac{E_{\text{each photon}}}{A}\right)$$

$$\Rightarrow \quad n = \frac{IA}{E_{\text{each photon}}}$$

So, at the cathode (area $4\times10^{-4}\,\text{m}^2$), the number of photons striking per second is

$$n = \frac{IA}{E_{\text{each photon}}} = \left(\frac{0.1}{4\pi}\right)\frac{4\times10^{-4}}{8\times10^{-19}}$$

$$\Rightarrow \quad n \approx 4\times10^{12}\ \text{photons/s}$$

With an efficiency of 5%, the photo-current is

$$I = \left(\frac{5}{100}\right)ne = (0.05)(4\times10^{12})(1.6\times10^{-19})$$

$$\Rightarrow \quad I = 32\times10^{-9}\ \text{A} = 32\ \text{nA}$$

(d) The stopping potential for ejected electrons is

$$V_s = \frac{h\nu - \phi_0}{e} = \frac{(5-2.25)\ \text{eV}}{e} = 2.75\ \text{V}$$