CHAPTER 2

Wave Optics

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Wave nature of light
(b) Huygen's Principle
(c) Interference
(d) Young's Double Slit Experiment (along with its variations)
(e) Diffraction phenomenon
(f) Resolving power
(g) Fresnel's distance and Polarisation

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

INTRODUCTION

The phenomenon of interference, diffraction and polarisation exhibited by light could not be explained on the basis of Newton's Corpuscular Theory. In 1678, Huygen suggested that light propagates in the form of waves. The first historic experiment in favour of wave theory was done by Focault, who in 1850 found experimentally that velocity of light in denser medium is less than that in the rarer medium which was contrary to Newton's Corpuscular Theory.

NEWTON'S CORPUSCULAR THEORY

Newton proposed that light is made up of tiny, light and elastic particles called **corpuscles** which are emitted by a luminous body. These corpuscles travel with speed equal to the speed of light in all directions in straight lines and carry energy with them. When the corpuscles strike the retina of the eye, they produce the sensation of vision. The corpuscles of different colour are of different sizes (red corpuscles larger than blue corpuscles).

The corpuscular theory explains that light carry energy and momentum, light travels in a straight line, Propagation of light in vacuum, Laws of reflection and refraction. However, it fails to explain the phenomenon of interference, diffraction and polarization.

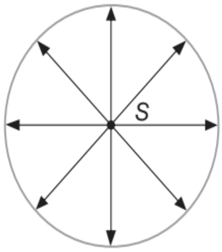
WAVE OPTICS

Wave optics is the study of the wave nature of light. Interference and diffraction are two main phenomena giving convincing evidence that light is a wave.

WAVEFRONTS AND RAYS

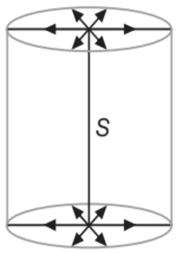
The locus of all the points vibrating in same phase of oscillation is called a **wavefront** (WF) i.e. a wavefront is defined as a surface joining the points vibrating in the same phase. The direction of propagation of light (ray of light) is along the normal to the Wavefront. The speed with which the wavefront moves onwards from the source is called the phase velocity or wave velocity. The energy travels outwards along straight lines emerging from the source, normally to the wavefront, that is, along the radii of the spherical wavefront. These lines are called the **rays**.

For a **point source** in a homogeneous medium the wavefront is **spherical**.



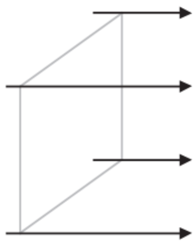
Spherical wavefront

For a **linear** source of light, the wavefront is **cylindrical**.



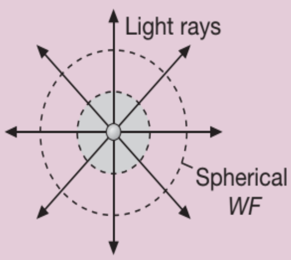
Cylindrical wavefront

A small part of a spherical or cylindrical wavefront from a distant source will appear plane and is, therefore, called a **plane wavefront**.

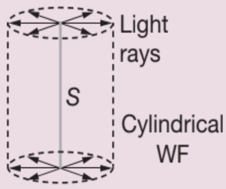
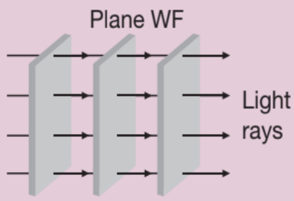


Plane wavefront

Conceptual Note(s)

Types of Wavefront (WF)	Intensity	Amplitude
Spherical WF due to a point source or a spherical source S. 	$I \propto \frac{1}{r^2}$	$A \propto \frac{1}{r}$

(Continued)

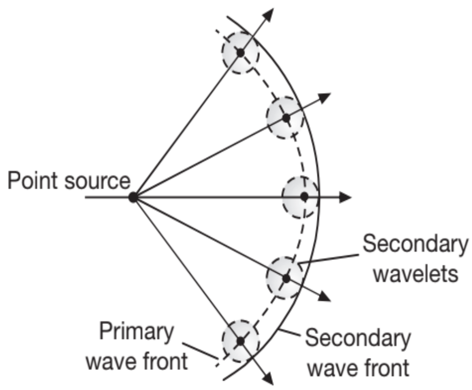
Types of Wavefront (WF)	Intensity	Amplitude
Cylindrical WF due to a line source or a cylindrical source S. 	$I \propto \frac{1}{r}$	$A \propto \frac{1}{\sqrt{r}}$
Plane WF 	$I \propto r^0$	$A \propto r^0$

HUYGEN'S PRINCIPLE

This principle is useful for determining the position of a given wavefront at any further time if its present position is known. The principle may be stated in three parts.

(a) Every point on the given wavefront may be regarded as the source of the new disturbance.

(b) The new disturbances from each point spread out in all directions with the velocity of light in the same manner as the original source of light does and these new disturbances are called **secondary wavelets**.

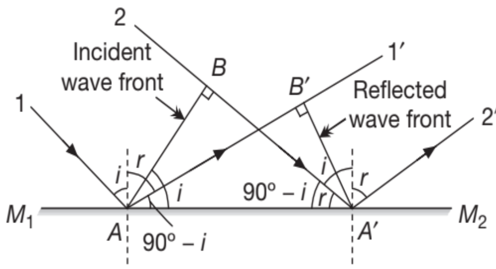


(c) The surface of tangency to the secondary wavelets in forward direction at any time gives the position of the new wavefront at that time. This new wavefront is called the Secondary Wavefront.

This principle explained successfully, reflection, refraction, total internal reflection, interference and diffraction but failed to explain the rectilinear propagation light.

LAWS OF REFLECTION ON THE BASIS OF HUYGEN'S THEORY

Let AB be the plane wave front incident on a plane mirror M_1M_2 at $\angle BAA' = i$, where 1, 2 are the corresponding incident rays perpendicular to AB.



According to Huygen's principle every point on AB is a source of secondary wavelets, so

$BA' = ct$, where c is speed of light

The secondary wavelets from A will travel the same distance ct in the same time. So, $AB' = ct$

Now, $\angle AA'B = 90 - i$, so that $\angle A'AB = i$, $(0 < i < 90°)$

Also, $\angle A'AB' = 90° - r$, so that $\angle AA'B' = r$, $(0 < r < 90°)$

From $\Delta AB'A'$, we have

$$\sin r = \frac{AB'}{AA'} = \frac{ct}{AA'} \quad \ldots(1)$$

From $\Delta A'BA$, we have

$$\sin i = \frac{A'B}{AA'} = \frac{ct}{AA'} \quad \ldots(2)$$

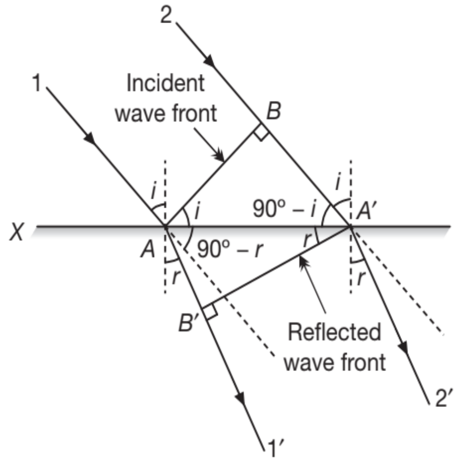
From equation (1) and (2), we get

$$\sin i = \sin r$$

$\angle i = \angle r$ which is the Law of Reflection

LAW OF REFRACTION ON THE BASIS OF HUYGEN'S THEORY

XY is a plane surface that separates a denser medium of refractive index μ from a rarer medium.



If v_1 is velocity of light in rarer medium and v_2 is velocity of light in denser medium, then by definition

$$\mu = \frac{v_1}{v_2} \quad \ldots(1)$$

AB is a plane wave front incident on XY at $\angle BAA' = \angle i$, where 1, 2 are the corresponding incident rays normal to AB

According to Huygen's principle

$$BA' = v_1 t \quad \ldots(2)$$

The secondary wavelets from A travel in the denser medium with a velocity v_2 and would cover a distance $AB' = v_2 t$ in the same time.

So, from $\Delta ABA'$ and $\Delta AB'A'$

$$\sin i = \frac{BA'}{AA'} \text{ and } \sin r = \frac{AB'}{AA'}$$

$$\Rightarrow \quad \frac{\sin i}{\sin r} = \frac{BA'}{AA'} \times \frac{AA'}{AB'} = \frac{BA'}{AB'} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2}$$

So, from equation (1), we get

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \mu$$

which is the Snell's Law of Refraction.

INTERFERENCE

When two waves of same frequency, nearly same amplitude and constant initial phase difference travel in the same direction along same straight line, they superimpose in such a way that in the region of superposition, the intensity is maximum at some points and minimum at some other points. This modification in intensity in the region of superposition is called **Interference**. The sources having the same frequency and constant initial phase difference are called **coherent sources**. The phenomenon of interference is based on the Law of Conservation of Energy.

SUSTAINED INTERFERENCE

The interference pattern in which the positions of maxima and minima remain fixed is called a **sustained interference**.

Conditions for Sustained Interference

(a) The fundamental condition for sustained interference is that the two sources should be coherent i.e., the initial phase difference between the two interfering waves must remain constant with time.

(b) The amplitudes of the two waves should be equal or nearly equal. This will give good contrast between bright and dark fringes.

(c) The two sources should be very closely spaced, otherwise the fringes will be too close for the eye to resolve.

(d) The sources should be monochromatic, otherwise there will be overlapping of interference patterns due to different wavelengths, which will reduce are contrast.

(e) The frequencies of the two interfering waves must equal.

(f) The **sources should be narrow.**

Since two independent sources cannot be coherent, a sustained interference pattern can be obtained only if the two sources simultaneously and, therefore, the phase difference between them remains constant.

COHERENT SOURCES

Two sources which emit light of the same wavelength with zero or a constant phase difference are called **coherent sources**.

Unlike sound waves, two independent sources of light cannot be coherent. Sound is a bulk property of matter. So, two independent sources of sound can produce coherent waves. However, two independent sources of light cannot be coherent. The emission of light from any source is from a very large number of atoms and the emission from each atom is random and independent of each other. Therefore, there is no stable phase relationship between radiations from two independent sources. So, for two sources to be coherent, they must be derived from the same parent source.

In practice, coherent sources are obtained either by dividing the wavefront (as in the case of Young's Double Slit Experiment, Fresnel's biprism, Lloyd mirror, etc.) or by dividing the amplitude (as in the case of thin films, Newton rings, etc.) of the incoming waves from a single source.

A **laser** discovered in 1960, is different from common light sources. Its atoms act in a cooperative manner so as to produce intense, monochromatic, unidirectional and coherent light. Thus, two independent laser beams can produce observable interference on a screen.

METHODS OF PRODUCING COHERENT SOURCES

Division of Wavefront

In this method the wavefront is divided into two parts by the use of mirrors, or lenses or prisms. Well known methods are Young's double slit arrangement, Fresnel's biprism and Lloyd's single mirror.

Division of Amplitude

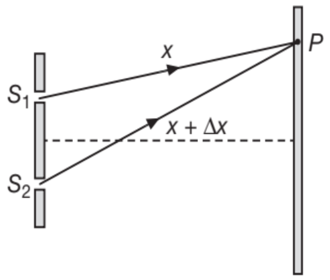
In this method the amplitude of the incoming beam is divided into two parts by means of partial reflection of refraction. These divided parts travel different paths and are finally brought together to produce interference. This class of interference requires broad sources of light. The common examples of such interference of light are the brilliant colours seen when a thin film of transparent material like soap bubble or thin film of kerosene oil spread on the surface of water is exposed to an extended source of light. This kind of interference exists in two types.

(a) Interference due to waves reflected from both the front and back surfaces of the film.

(b) Interference due to transmitted waves.

INTERFERENCE: MATHEMATICAL TREATMENT

Two waves (whether sound or light) of equal frequencies travelling almost in the same direction show interference. Consider two waves coming from sources S_1 and S_2. These reach point P with a path difference Δx, having amplitude A_1 and A_2.



$$y_1 = A_1 \sin(\omega t - kx) \quad \ldots(1)$$

and $y_2 = A_2 \sin[\omega t - k(x + \Delta x)]$

$$\Rightarrow \quad y_2 = A_2 \sin(\omega t - kx - \phi) \quad \ldots(2)$$

where $\phi = k\Delta x = \left(\frac{2\pi}{\lambda}\right)\Delta x$ and Δx is the path difference.

By Principle of Superposition, the resultant wave at P is

$$y = y_1 + y_2 = A_1 \sin(\omega t - kx) + A_2 \sin(\omega t - kx - \phi)$$

$$\Rightarrow \quad y = (A_1 + A_2 \cos\phi)\sin(\omega t - kx) - (A_2 \sin\phi)\cos(\omega t - kx) \quad \ldots(3)$$

Substituting $A_1 + A_2 \cos\phi = A\cos\theta$ and

$A_2 \sin\phi = A \sin\theta$, we get

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos\phi \quad \ldots(4)$$

Equation (3), becomes

$$y = A\sin(\omega t - kx - \theta) \quad \ldots(5)$$

where, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$ and

$$\tan\theta = \frac{A_2 \sin\phi}{A_1 + A_2 \cos\phi}$$

The intensity of the resultant wave is

$$I = \frac{1}{2}\rho v\omega^2 A^2 = KA^2 = K\left[A_1^2 + A_2^2 + 2A_1A_2 \cos\phi\right]$$

$$\Rightarrow \quad I = I_1 + I_2 + \underbrace{2\sqrt{I_1 I_2}\cos\phi}_{\text{Interference term}} \quad \ldots(6)$$

Thus, when interference of two waves of equal intensities occur, the intensity of maxima becomes 4 times that of single wave and that of minima becomes zero.

CONDITION FOR MAXIMA: CONSTRUCTIVE INTERFERENCE

From equation (6), I is maximum, when

$$\cos\phi = +1$$

$$\Rightarrow \quad \phi = 0,\ 2\pi,\ 4\pi, \ldots.$$

$$\Rightarrow \quad \phi = 2n\pi$$

$$\Rightarrow \quad \phi = \left(\frac{2\pi}{\lambda}\right)x = 2n\pi, \text{ where } n = 0, 1, 2, 3, \ldots..$$

$$\Rightarrow \quad \Delta x = (2n)\frac{\lambda}{2}, \text{ where } n = 0, 1, 2, 3, \ldots..$$

So, intensity will be maximum when phase difference ϕ is an even multiple of π or path difference Δx is an even multiple of $\frac{\lambda}{2}$.

$$\Rightarrow \quad I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = (A_1 + A_2)^2$$

CONDITION FOR MINIMA: DESTRUCTIVE INTERFERENCE

Intensity I will be minimum, when

$$\cos\phi = -1$$

$$\Rightarrow \quad \phi = \pi,\ 3\pi,\ 5\pi, \ldots.$$

$$\Rightarrow \quad \phi = (2n+1)\pi$$

$$\Rightarrow \quad \phi = \left(\frac{2\pi}{\lambda}\right)x = (2n+1)\pi, \text{ where } n = 0, 1, 2, 3, \ldots..$$

$$\Rightarrow \quad \Delta x = (2n+1)\frac{\lambda}{2}, \text{ where } n = 0, 1, 2, 3, \ldots..$$

So, intensity will be maximum when phase difference ϕ is an odd multiple of π or path difference Δx is an odd multiple of $\frac{\lambda}{2}$.

$$\Rightarrow \quad I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = K(A_1 \sim A_2)^2$$

The ratios $\dfrac{I_{max}}{I_{min}} = \dfrac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2} = \dfrac{(A_1 + A_2)^2}{(A_1 \sim A_2)^2}$

If $I_1 = I_2 = I_0$ (i.e., $A_1 = A_2$), we have

$$I_{max} = 4I_0 \text{ and } I_{min} = 0$$

Thus, when interference of two waves of equal intensities occur, the intensity of maxima becomes four times that of single wave and that of minima becomes zero.

In **Young's Double Slit Experiment** popularly known as *YDSE*, usually the intensities I_1 and I_2 are equal, so $I_1 = I_2 = I_0$

Since, $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$, so we get

$$I = 2I_0(1+\cos\phi)$$

$$\Rightarrow \quad I = 4I_0\cos^2\left(\frac{\phi}{2}\right)$$

PHASE DIFFERENCE AND PATH DIFFERENCE

If two waves travel different lengths of path to reach a point, they may not be in phase with each other. The phase difference depends on the path difference as $\phi = \frac{2\pi}{\lambda}\Delta x$, where Δx is the difference in length of path traversed by the waves.

The phase difference between two light waves can change if the waves travel through different materials having different refractive indices.

Suppose, we have two waves having identical wavelengths λ, initially in phase, in air. One of the waves travel through medium 1 of refractive index μ_1 and length L and other wave travels through same length L in another medium of refractive index μ_2. As wavelength differs in a medium, the two waves may not remain in phase.

The path difference after crossing through the medium is given by

$$\Delta x = (n_1 - n_2)\lambda$$

where n_1 is number of wavelengths in medium 1 and

n_2 is number of wavelengths in medium 2

$$\Rightarrow \quad \Delta x = \left(\frac{L}{\lambda_1} - \frac{L}{\lambda_2}\right)\lambda = \left(\frac{\lambda}{\lambda_1} - \frac{\lambda}{\lambda_2}\right)L$$

Since $\frac{\lambda}{\lambda_1} = \mu_1$ and $\frac{\lambda}{\lambda_2} = \mu_2$, so

$$\Delta x = (\mu_1 - \mu_2)L$$

ILLUSTRATION 1

Two light rays, initially in phase and having wavelength 6×10^{-7} m, go through different plastic layers of the same thickness, 7×10^{-6} m. The indices of refraction are 1.65 for one layer and 1.49 for the other.

(a) What is the equivalent phase difference between the rays when they emerge?

(b) If those two rays then reach a common point, does the interference result in complete darkness, maximum brightness, intermediate illumination but closer to complete darkness, or intermediate illumination but closer to maximum brightness?

SOLUTION

(a) $\Delta x = (\mu_1 - \mu_2)t = (1.65 - 1.49)(7\times10^{-6})$

$$\Delta x = 1.12\times10^{-6}\text{ m}$$

Since, Phase difference $\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x)$

$$\Rightarrow \phi = \left(\frac{2\pi}{6\times10^{-7}}\right)(1.12\times10^{-6})$$

$$\Rightarrow \phi = 11.72\text{ radian}$$

(b) To discuss this case, two options arise

Option 1: Waves are in phase, then using

$I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$, we get

$$I = I_{max}\cos^2\left(\frac{11.72}{2}\right) = 0.8I_{max}$$

This value is intermediate illumination closer to maximum brightness.

Option 2: Waves are out of phase, then

$$\phi_{net} = 11.72 \pm \pi$$

$$\Rightarrow \phi_{net} = 14.86\text{ rad}$$

$$\Rightarrow I = I_{max}\cos^2\left(\frac{14.86}{2}\right)$$

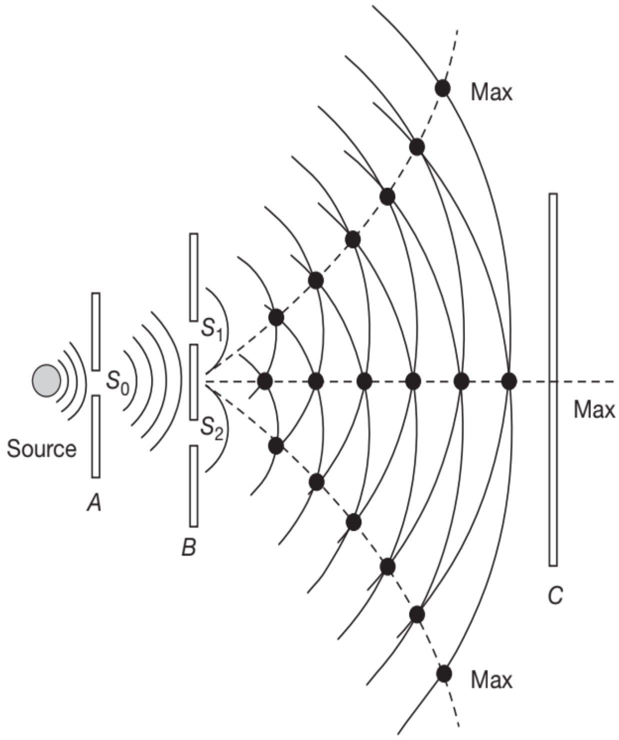
$$\Rightarrow I = 0.17I_0$$

This value is intermediate illumination closer to darkness.

THEORY OF INTERFERENCE: MAXIMA AND MINIMA

Theory of Division of Wavefront: Young's Double Slit Experiment

The phenomenon of interference of light waves arising from two sources was first demonstrated by Thomas Young in 1801.
Light is incident on screen A, which is provided with a narrow slit, S_0.

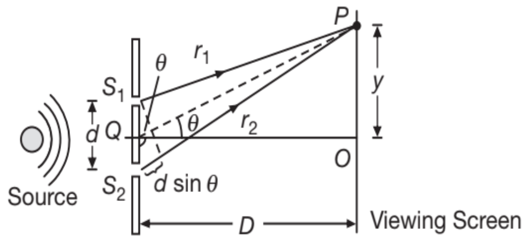


Schematic diagram of young's double-slit experimet. The narrow slits act as sources of cylindrical waves. Slits S_1 and S_2 behave as coherent sources which produce an interference pattern on screen C.

The cylindrical waves emerging from this slit arrive at screen B, which contains two narrow, parallel slits, S_1 and S_2. Light emerges from these two slits as cylindrical waves. In effect, slits S_1 and S_2 act as individual light sources that are in phase as they originate from the same cylindrical wavefront. The light from the two slits produces a visible pattern on screen C. The pattern consists of a series of bright and dark parallel bands called **fringes**. The overall light amplitude at a given point on the screen is the result of the superposition of the two wave amplitudes from S_1 and S_2. Two waves that add constructively give a bright fringe, and any two waves that add destructively produce a dark fringe.

YDSE (QUANTITATIVE TREATMENT): METHOD 1

Consider a point P on the viewing screen located a perpendicular distance D from the two identical slits S_1 and S_2, which are separated by a distance d.



Geometric construction for describing young's double-slit experiment. Note that the path difference between the two rays is $r_2 - r_1 = d\sin\theta$

Let us assume that the source is equidistant from the two slits and is monochromatic, that is, emitting light of a single wavelength λ. Under these assumptions, the waves emerging from slits S_1 and S_2 have the same frequency and amplitude and are in phase. The light intensity on the screen at P is the resultant of light coming from both slits. Note that a wave from the lower slit travels farther than a wave from the upper slit by an amount equal to $d\sin\theta$. This distance is called the **path difference**, x, where

$$x = r_2 - r_1 = d\sin\theta \qquad ...(1)$$

The value of this path difference will determine whether or not the two waves are in phase when they arrive at P. If the path difference is either zero or some integral multiple of the wavelength, the two waves are in phase at P and constructive interference results. Therefore, the condition for bright fringes, or constructive interference, at P is given by

$$x = d\sin\theta = n\lambda \qquad (n = 0, \pm1, \pm2, \pm3...) \qquad ...(2)$$

The index number n is called the **order number** of the **fringe**. The central bright fringe at $\theta = 0 (n = 0)$ is called the **zeroth order maximum**. The first maximum on either side, when $n = \pm1$, is called the **first order maximum**, etc.

Similarly, when the path difference is an odd multiple of $\frac{\lambda}{2}$, the two waves arriving at P will be opposite in phase and will give rise to destructive interference. Therefore, the condition for dark fringes, or destructive interference, at P is given by

$$x = d\sin\theta = (2n+1)\frac{\lambda}{2} \quad (n = 0, \pm 1, \pm 2...) \quad ...(3)$$

It is useful to obtain expressions for the positions of the bright and dark fringes measured vertically from O to P. We shall assume that $D > d$ and consider only points P that are close to O. In this case, θ is small, and so we can use the approximation $\sin\theta \approx \tan\theta$. From the large triangle OPQ in Figure, we see that

$$\sin\theta \approx \tan\theta = \frac{y}{D} \quad ...(4)$$

Using this result together with equation (2), we see that the positions of the bright fringes measured from O are given by

$$y_{\text{bright}} = n\left(\frac{\lambda D}{d}\right) \quad ...(5)$$

From this expression, we find that the separation between any two adjacent bright fringes called **Fringe Width** is equal to $\frac{\lambda D}{d}$, that is,

$$\beta = y_{n+1} - y_n = \frac{\lambda D}{d}(n+1) - \frac{\lambda D}{d}n = \frac{\lambda D}{d} \quad ...(6)$$

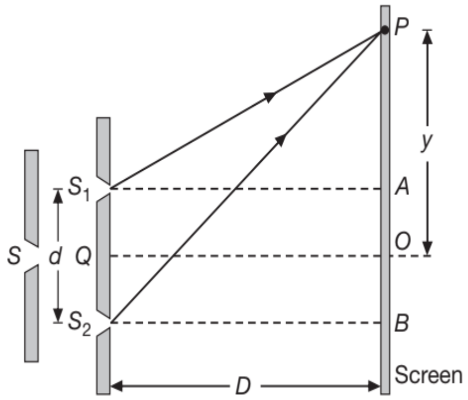
Similarly, using equation (3) and (4), we find that the dark fringes are located at

$$y_{\text{dark}} = (2n+1)\frac{\lambda D}{2d} \quad ...(7)$$

This result shows that the separation between adjacent dark fringes is also equal to $\beta = \frac{\lambda D}{d}$. Since the quantities D and d are both measurable, we see that the double-slit interference pattern, together with equation (6), provides a direct determination of the wavelength λ. Young used this technique to make the first measurement of the wavelength of light.

YDSE (QUANTITATIVE TREATMENT): METHOD 2

Consider that two coherent sources of light S_1 and S_2 are placed at a distance d apart and a screen is placed at a distance D from the plane of the two sources.



Let P be a point on the screen at a distance y from a point O exactly opposite to the centre of the two sources S_1 and S_2. If x is path difference between the light waves reaching point P from the sources S_1 and S_2, then

$$x = S_2P - S_1P$$

In right angled ΔS_2BP, we have

$$S_2P^2 = S_2B^2 + BP^2 = D^2 + \left(y + \frac{d}{2}\right)^2$$

Also, in right angled ΔS_1AP, we have

$$S_1P^2 = S_1A^2 + AP^2 = D^2 + \left(y - \frac{d}{2}\right)^2$$

$$\Rightarrow \quad S_2P^2 - S_1P^2 = \left(D^2 + \left(y + \frac{d}{2}\right)^2\right) - \left(D^2 + \left(y - \frac{d}{2}\right)^2\right)$$

$$\Rightarrow \quad (S_2P + S_1P)(S_2P - S_1P) = \left(y + \frac{d}{2}\right)^2 - \left(y - \frac{d}{2}\right)^2$$

Since $S_2P - S_1P = x$ (the path difference between the two light waves), the above equation becomes

$$(S_2P + S_1P)x = 4y\left(\frac{d}{2}\right) = 2yd$$

$$\Rightarrow \quad x = \frac{2yd}{S_2P + S_1P}$$

In practice, the point P lies very close to the centre of screen, so we have

$$S_2P = S_1P = D$$

$$\Rightarrow \quad x = \frac{2yd}{D + D} = \frac{2yd}{2D}$$

$$\Rightarrow \quad x = \frac{yd}{D} \quad ...(1)$$

For Maxima, we know that path difference x must be an even multiple of $\frac{\lambda}{2}$, so

$$x=(2n)\frac{\lambda}{2}, \text{ where } n=0,1,2, \ldots$$

$$\Rightarrow \frac{yd}{D}=n\lambda, \text{ where } n=0,1,2, \ldots$$

$$\Rightarrow y_n=n\left(\frac{\lambda D}{d}\right), \text{ where } n=0,1,2, \ldots$$

So, $y_{n^{\text{th}}\text{ bright}}=n\left(\frac{\lambda D}{d}\right)$; $n=0,1,2,3,4,5, \ldots$

At $y=0$ (i.e., for $n=0$) we get a Central Bright Fringe.

For Minima, we know that path difference x must be an odd multiple of $\frac{\lambda}{2}$, so

$$x=(2n-1)\frac{\lambda}{2}, \text{ where } n=1,2, \ldots$$

$$\Rightarrow \frac{yd}{D}=(2n-1)\frac{\lambda}{2}, \text{ where } n=1,2, \ldots$$

$$\Rightarrow y_n=(2n-1)\frac{\lambda D}{2d}, \text{ where } n=1,2, \ldots$$

$$\Rightarrow y_n=\left(n-\frac{1}{2}\right)\frac{\lambda D}{d}, \text{ where } n=1,2, \ldots$$

$$y_{n^{\text{th}}\text{ dark}}=\left(n-\frac{1}{2}\right)\frac{\lambda D}{d} \text{ ; } n=1,2,3,4,5, \ldots$$

Conceptual Note(s)

For central bright fringe $n=0$, $y=0$ and $\Delta x=0$

For n^{th} bright fringe the distance from centre of central right fringe is $n\beta=y_{n^{\text{th}}\text{ bright}}$.

For n^{th} dark fringe the distance from centre of central bright fringe is $\left(n-\frac{1}{2}\right)\beta=y_{n^{\text{th}}\text{ bright}}$.

ILLUSTRATION 2

If the maximum intensity in *YDSE* is I_0, find the intensity at a point on the screen where

(a) the phase difference between the two interfering beams is $\frac{\pi}{3}$.

(b) the path difference between the two interfering beams is $\frac{\lambda}{4}$.

SOLUTION

(a) Since, $I=I_{\max}\cos^2\left(\frac{\phi}{2}\right)$

where $I_{\max}$ is I_0 i.e., intensity due to independent sources is $\frac{I_0}{4}$. Therefore, at

$$\phi=\frac{\pi}{3}$$

$$\Rightarrow \frac{\phi}{2}=\frac{\pi}{6}$$

$$\Rightarrow I=I_0\cos^2\left(\frac{\pi}{6}\right)=\frac{3}{4}I_0$$

(b) Phase difference corresponding to the given path difference Δx is given by

$$\phi=\left(\frac{2\pi}{\lambda}\right)\Delta x$$

$$\Rightarrow \phi=\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right)$$

$$\Rightarrow \phi=\frac{\pi}{2}$$

$$\Rightarrow \frac{\phi}{2}=\frac{\pi}{4}$$

$$\Rightarrow I=I_0\cos^2\left(\frac{\pi}{4}\right)=\frac{I_0}{2}$$

ILLUSTRATION 3

In *YDSE*, the interference pattern is found to have an intensity ratio between the bright and dark fringes as 9. Find the ratio of

(a) intensities.

(b) amplitudes of the two interfering waves.

SOLUTION

In case of interference, we have

$$I=I_1+I_2+2\sqrt{I_1I_2}\cos\phi$$

(a) $I_{\max}=I_1+I_2+2\sqrt{I_1I_2}=\left(\sqrt{I_1}+\sqrt{I_2}\right)^2$

and $I_{\min}=I_1+I_2-2\sqrt{I_1I_2}=\left(\sqrt{I_1}\sim\sqrt{I_2}\right)^2$

Since, $\dfrac{I_{max}}{I_{min}} = \dfrac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}\sim\sqrt{I_2}\right)^2} = \dfrac{9}{1}$

$$\Rightarrow \frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}} = \frac{3}{1}$$

Solving, we get

$$\frac{I_1}{I_2} = \frac{4}{1} = 4$$

(b) Since, $I \propto A^2$,

$$\Rightarrow \frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\right)^2$$

$$\Rightarrow \left(\frac{A_1}{A_2}\right)^2 = 4$$

$$\Rightarrow \frac{A_1}{A_2} = 2$$

ILLUSTRATION 4

The intensity of the light coming from one of the slits in $YDSE$ is double the intensity from the other slit. Find the ratio of the maximum intensity to the minimum intensity in the interference fringe pattern observed.

SOLUTION

Since, we know that

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2$$

According to the problem, we have

$$I_1 = 2I_0 \text{ and } I_2 = I_0$$

$$\Rightarrow \frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2 = 34$$

ILLUSTRATION 5

In a Young's double-slit experiment the distance between the slits is 1 mm and the distance of the screen from the slits is 1 m. If light of wavelength 6000 Å is used, find the distance between the second dark fringe and the fourth bright fringe.

SOLUTION

The position of the second dark fringe is given by

$$y_2(\text{dark}) = (2n-1)\frac{\lambda D}{2d} = (4-1)\frac{\lambda D}{2d} = \frac{3}{2}\left(\frac{\lambda D}{d}\right)$$

The position of the 4th bright fringe is given by

$$y_2(\text{bright}) = \frac{n\lambda D}{d} = \frac{4\lambda D}{d}$$

Therefore, the separation is given by

$$\Delta y = y_4(\text{bright}) - y_2(\text{dark}) = \left(4-\frac{3}{2}\right)\frac{\lambda D}{d}$$

$$\Rightarrow \quad \Delta y = \frac{5}{2}\times\frac{6000\times10^{-10}\times1}{10^{-3}} = 1.5\times10^{-3} = 1.5 \text{ mm}$$

ILLUSTRATION 6

In a $YDSE$, the separation between the coherent sources is 6 mm, the separation between coherent sources and the screen is 2 m. If light of wavelength 6000 Å is used, then

(a) find the fringe width.
(b) find the position of the third maxima.
(c) find the position of the second minima.

SOLUTION

(a) Since fringe width is given by $\beta = \dfrac{\lambda D}{d}$, so we have

$$\beta = \frac{\lambda D}{d} = \frac{(6000\times10^{-10})(2)}{6\times10^{-3}} = 0.2 \text{ mm}$$

(b) Position of third maxima is obtained by substituting $n = 3$ in the equation $y_n = n\left(\dfrac{\lambda D}{d}\right)$, so we get

$$y_3 = \frac{3\lambda D}{d} = 3\beta = 3(0.2) = 0.6 \text{ mm}$$

(c) Position of second minima is obtained by putting $n = 2$ in the equation $y_n = (2n-1)\dfrac{\lambda D}{2d}$, so we get

$$y_2 = \left(2-\frac{1}{2}\right)\frac{\lambda D}{d} = \frac{3}{2}\beta = \frac{3}{2}(0.2) = 0.3 \text{ mm}$$

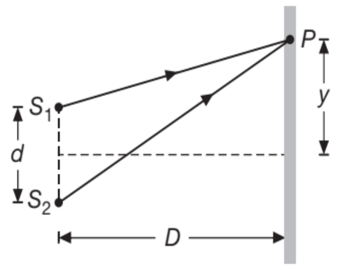
ILLUSTRATION 7

Young's double slit experiment is carried out using microwaves of wavelength $\lambda = 3$ cm. Distance between the slits is $d = 5$ cm and the distance between the plane of slits and the screen is $D = 100$ cm. Find the number of maximas and their positions on the screen.

SOLUTION

$$\begin{pmatrix}\text{The maximum path} \\ \text{difference that} \\ \text{can be produced}\end{pmatrix} = \begin{pmatrix}\text{Distance between the} \\ \text{coherent sources} \\ \text{i.e., 5 cm}\end{pmatrix}$$

Thus, in this case we can have only three maximas, one central maxima and two on its either side (for a path difference of $\lambda = 3$ cm)



For maximum intensity at P, we have

$$S_2P - S_1P = \lambda$$

$$\Rightarrow \sqrt{\left(\frac{y+d}{2}\right)^2 + D^2} - \sqrt{\left(\frac{y-d}{2}\right)^2 + D^2} = \lambda^2$$

Substituting $d = 5$ cm, $D = 100$ cm and $\lambda = 3$ cm and solving the equation, we get

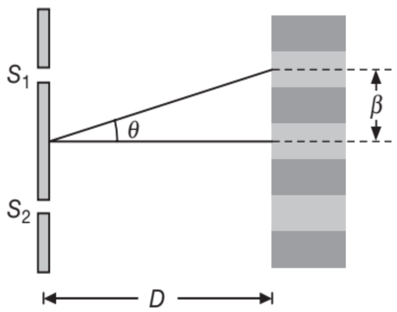
$$y = \pm 75 \text{ cm}$$

Thus, the three maximas will be at

$$y = 0 \text{ and } y = \pm 75 \text{ cm}$$

FRINGE WIDTH AND ANGULAR FRINGE WIDTH

The separation between two consecutive bright (or dark) fingers is called the **fringe width** (β), given by



$$\beta = y_{n+1} - y_n = \frac{\lambda D}{d}$$

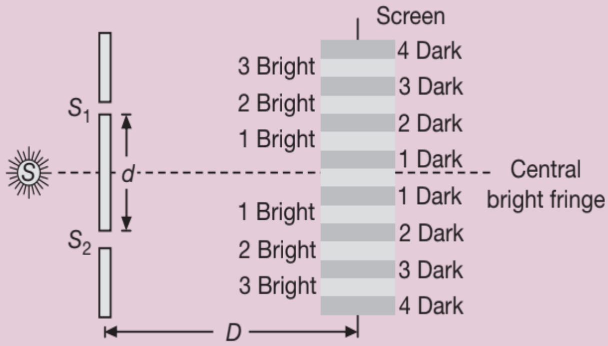
Note that fringe width β is independent of n. That is, the interference fringes have same width throughout.

The angular fringe width is given by

$$\Delta\phi = \frac{\beta}{D} = \frac{\lambda}{d}$$

Conceptual Note(s)

In YDSE alternate bright and dark bands obtained on the screen. These bands are called Fringes.



d = Distance between slits
D = Distance between slits and screen
λ = Wavelength of monochromatic light emitted from source

(a) Central fringe is always bright, because at central position the path difference, $x = 0$ and hence the phase difference, $\phi = 0°$ and hence the Central Bright Fringe is also called the zeroth maxima.
(b) The nth minima come before the nth maxima.
(c) The fringe pattern obtained due to a slit is more bright than that due to a point.
(d) If the slit widths are unequal, the minima will not be complete dark. For very large width uniform illumination occurs.
(e) If one slit is illuminated with red light and the other slit is illuminated with blue light, no interference pattern is observed on the screen.
(f) If the two coherent sources consist of object and it's reflected image, the central fringe is dark instead of bright one.
(g) $y_{n^{th}\text{ bright}} = n\beta$ and $y_{n^{th}\text{ dark}} = \left(n - \frac{1}{2}\right)\beta.$

ILLUSTRATION 8

In a *YDSE* bi-chromatic light of wavelengths **400 nm** and **560 nm** are used. The distance between the slits is **0.1 mm** and the distance between the plane of the slits and the screen is **1 m**. Find the minimum distance between two successive regions of complete darkness.

SOLUTION

Let n_1 minima of 400 nm coincides with n_2 minima of 560 nm, then

$$(2n_1 - 1)400 = (2n_2 - 1)560$$

$$\Rightarrow \quad \frac{2n_1 - 1}{2n_2 - 1} = \frac{7}{5} = \frac{14}{10} = \frac{21}{15}$$

i.e., 4th minima of 400 nm coincides with 3rd minima of 560 nm

The location of this minima is

$$y_4 = \frac{7(1000)(400 \times 10^{-6})}{2 \times 0.1} = 14 \text{ mm}$$

Next, 11th minima of 400 nm will coincide with 8th minima of 560 nm

Location of this minima is

$$y_{11} = \frac{21(1000)(400 \times 10^{-6})}{2 \times 0.1} = 42 \text{ mm}$$

So, required separation is

$$\Delta y = 42 - 14 = 28 \text{ mm}$$

ILLUSTRATION 9

Light from a source consists of two wavelength $\lambda_1 = 6500$ Å and $\lambda_2 = 5200$ Å. If the separation between the sources from each other is 6.5 mm and that from the screen is **2 m**, find the minimum value of $y(\neq 0)$ where the maxima of both the wavelengths coincide.

SOLUTION

Let n_1 maxima of λ_1 coincides with n_2 maxima of λ_2. Then, $y_{n_1} = y_{n_2}$

$$\Rightarrow \quad \frac{n_1 \lambda_1 D}{d} = \frac{n_2 \lambda_2 D}{d}$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{5200}{6500} = \frac{4}{5}$$

$$\Rightarrow \quad 4\lambda_1 = 5\lambda_2$$

Thus, fourth maxima of λ_1 coincides with fifth maxima of λ_2. The minimum value of $y(\neq 0)$ is given by

$$y = \frac{4\lambda_1 D}{d} = \frac{4(0.65 \times 10^{-6})(2)}{6.5 \times 10^{-3}} = 0.8 \text{ mm}$$

ILLUSTRATION 10

Two coherent sources are 0.3 mm apart. They are 0.9 m away from the screen. The second dark fringe is at a distance of 0.3 cm from the centre. Find the distance of fourth bright fringe from the centre. Also find the wavelength of light used.

SOLUTION

Second dark fringe $n = 2$ will be obtained at

$$y_n = \left(n - \frac{1}{2}\right)\beta$$

$$\Rightarrow \quad y = (2n - 1)\frac{\lambda D}{2d} = \frac{3\lambda D}{2d} \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{\lambda D}{d} = \frac{2}{3}y$$

$$\Rightarrow \quad \beta = \frac{\lambda D}{d} = \frac{2}{3}(0.3) = 0.2 \text{ m}$$

Fourth bright fringe from the centre will be obtained at

$$y_4 = 4\beta = 0.8 \text{ cm}$$

From equation (1), we get

$$\lambda = \frac{2yd}{3D} = \frac{2 \times 0.3 \times 10^{-3} \times 0.3 \times 10^{-2}}{3 \times 0.9}$$

$$\Rightarrow \quad \lambda = 6.67 \times 10^{-7} \text{ m}$$

ILLUSTRATION 11

In a Young's double slit experiment, two narrow slits **0.8 mm** apart are illuminated by a source of light of wavelength 4000 Å. How far apart are the adjacent bright bands in the interference pattern observed on a screen 2 m away?

SOLUTION

Since, $d = 0.8 \times 10^{-3}$ m, $\lambda = 4000$ Å $= 4000 \times 10^{-10}$ m, $D = 2$ m

The distance between the adjacent bright bands or the fringe width is given by

$$\beta = \frac{\lambda D}{d} = \frac{4000 \times 10^{-10} \times 2}{0.8 \times 10^{-3}} \text{ m} = 1 \text{ mm}$$

ILLUSTRATION 12

A beam of light consisting of two wavelengths, 6500 Å and 5200 Å, is used to obtain interference fringes in a Young's double slit experiment. The distance between the slits is 2 mm and the distance between the plane of the slits and the screen is 120 cm.

(a) Find the distance of the third bright fringe on the screen from the central maximum for the wavelength 6500 Å

(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

SOLUTION

According to the problem, we have

$$\lambda_1 = 6500 \text{ Å} = 6500 \times 10^{-10} \text{ m}$$

$$\lambda_2 = 5200 \text{ Å} = 5200 \times 10^{-10} \text{ m}$$

$$d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$

$$D = 120 \text{ cm} = 1.2 \text{ m}$$

(a) $y_n = \dfrac{n\lambda D}{d}$

$$\Rightarrow y_3 = \frac{3 \times 6500 \times 10^{-10} \times 1.2}{2 \times 10^{-3}}$$

$$\Rightarrow y_3 = 1.17 \times 10^{-3} \text{ m} = 1.17 \text{ mm}$$

(b) The least distance from the central maximum where the bright fringes due to both the wavelength coincide corresponds to that value of n for which

$$\frac{n\lambda_1 D}{d} = \frac{(n+1)\lambda_2 D}{d}$$

$$\Rightarrow n\lambda_1 = (n+1)\lambda_2$$

$$\Rightarrow \frac{n+1}{n} = \frac{6500}{5200} = \frac{5}{4}$$

$$\Rightarrow n = 4$$

$$\Rightarrow y_4 = \frac{n\lambda D}{d} = \frac{4 \times 6500 \times 10^{-10} \times 1.2}{2 \times 10^{-3}}$$

$$\Rightarrow y_4 = 1.56 \times 10^{-3} \text{ m} = 1.56 \text{ mm}$$

ILLUSTRATION 13

A convergent lens with a focal length of $f = 10$ cm is cut into two halves that are then moved apart to a distance of $d = 0.5$ mm (a double lens). Find the fringe width on screen at a distance of 60 cm behind the lens if a point source of monochromatic light $(\lambda = 5000 \text{ Å})$ is placed in front of the lens at a distance of $a = 15$ cm from it.

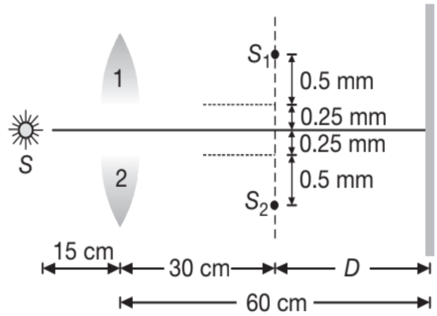
SOLUTION

Applying lens formula, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$, we get

$$\frac{1}{v} + \frac{1}{15} = \frac{1}{10}$$

$$\Rightarrow \quad v = 30 \text{ cm}$$

Since, $m = \dfrac{v}{u} = \dfrac{30}{-15} = -2$



Distance between two slits is $d = 1.5$ mm

Distance between slits and screen is $D = 30$ cm

Fringe width $\beta = \dfrac{\lambda D}{d} = \dfrac{(5 \times 10^{-7})(0.3)}{(1.5 \times 10^{-3})} = 10^{-4}$ m

$$\beta = 0.1 \text{ mm}$$

INTERFERENCE EXPERIMENT IN WATER

In water (liquid), of refractive index μ the wavelength decreases from λ to $\lambda' = \dfrac{\lambda}{\mu}$. Therefore, if interference experiment is performed in water the fringe width decreases from β to β', such that

$$\beta = \frac{\lambda D}{d} \text{ and } \beta' = \frac{\lambda' D}{d} = \frac{\lambda D}{\mu d}$$

$$\Rightarrow \quad \beta' = \frac{\beta}{\mu}$$

ILLUSTRATION 14

A Young's double slit arrangement produces interference fringes for sodium light $(\lambda = 5890\ \text{Å})$ that are 0.20° apart. What is the angular fringe separation if the entire arrangement is immersed in water? Refractive index of water is $\frac{4}{3}$.

SOLUTION

The wavelength of light in water is $\lambda_w = \frac{\lambda}{\mu}$

Angular fringe-width in air, $\theta_a = \frac{\lambda}{d}$

Angular fringe-width in water, $\theta_w = \frac{\lambda_w}{d}$

So, $\theta_w = \frac{\lambda_w}{d} = \frac{\lambda}{\mu d} = \frac{\theta_a}{\mu} = \frac{0.20^\circ}{\frac{4}{3}} = 0.15^\circ$

FRINGE VISIBILITY (V)

With the help of the concept of visibility, the knowledge about coherence, fringe contrast and interference pattern is obtained. Fringe visibility V is defined as

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$$

If $I_{min} = 0$, $V = 1$ (maximum) i.e., fringe visibility will be the best.

Also, if $I_{max} = 0$ then $V = -1$

and if $I_{max} = I_{min}$, then $V = 0$

INTENSITY DISTRIBUTION

When two coherent light waves of intensity I_1 and I_2 with a constant phase difference ϕ superimpose, then the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

In $YDSE$, usually the intensities I_1 and I_2 are equal, so $I_1 = I_2 = I_0$

For maxima, $\phi = 2n\pi$ i.e., $\cos\phi = +1$

$\Rightarrow \quad I_{max} = 4I_0$

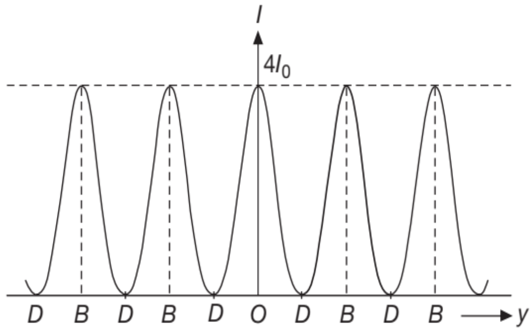
For minima, $\phi = (2n+1)\pi$ i.e., $\cos\phi = -1$

$\Rightarrow \quad I_{min} = 0$

Since, $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$, so we get

$$I = 2I_0(1+\cos\phi)$$

$$\Rightarrow \quad I = 4I_0\cos^2\left(\frac{\phi}{2}\right) = 4I_0\cos^2\left(\frac{\pi x}{\lambda}\right) = I_{max}\cos^2\left(\frac{\pi x}{\lambda}\right)$$



Intensity distribution on the screen as a function of y in YDSE $I_{max} = 4I_0$ for bright fringe and $I_{min} = 0$ for dark fringe.

Problem Solving Technique(s)

(a) Interference occurs due to Law of Conservation of Energy. Actually, redistribution of energy takes place.

(b) If w_1 and w_2 are the widths of the slits and I_1 and I_2 is the intensity of light (with respective amplitudes a_1 and a_2) passing through slits, then

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{w_1}{w_2}$$

(c) $\frac{I_{min}}{I_{max}} = \left(\frac{a_1 - a_2}{a_1 + a_2}\right)^2 = \left(\frac{\sqrt{w_1} - \sqrt{w_2}}{\sqrt{w_1} + \sqrt{w_2}}\right)^2$

$$\Rightarrow \quad \frac{I_{min}}{I_{max}} = \left(\frac{\sqrt{I_1} - \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}}\right)^2$$

(d) If point source is used to illuminate the two slits, the intensity emerging from the slit is proportional to area of exposed part of slit. In case of identical slits.

$I_1 = I_2$

$\Rightarrow a_1 = a_2$

(e) When white light is used to illuminate the slit, we obtain an interference pattern consisting of a central white fringe having on both sides symmetrically a few coloured fringes and then a uniform illumination.

(f) If ϕ is the phase difference between two waves of intensities I_1 and I_2, then

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi, \text{ where } \phi = \Delta\phi = \phi_2 - \phi_1$$

(g) If x is the path difference, then

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(\frac{2\pi}{\lambda}x\right)$$

where $x = \Delta x = x_2 - x_1$

(h) In *YDSE*, if n_1 fringes are visible in a field of view with light of wavelength λ_1, while n_1 with light of wavelength λ_2 in the same field, then $n_1\lambda_1 = n_2\lambda_2$.

(i) Separation (Δx) between fringes

(a) Between n^{th} bright and m^{th} bright fringes $(n > m)$

$$\Delta x = (n - m)\beta$$

(b) Between n^{th} bright and m^{th} dark fringe

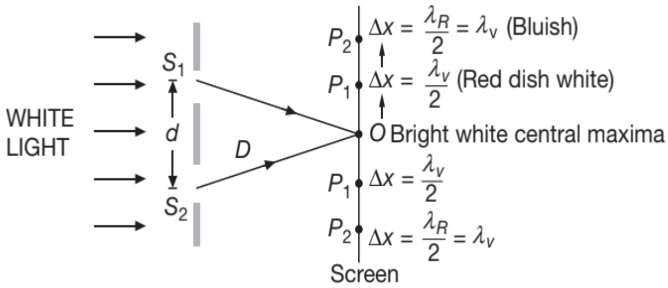
(i) If $n > m$ then $\Delta x = \left(n - m + \frac{1}{2}\right)\beta$

(ii) If $n < m$ then $\Delta x = \left(m - n - \frac{1}{2}\right)\beta$

(j) **Identification of central bright fringe:** To identify central bright fringe, monochromatic light is replaced by white light. Due to overlapping central maxima will be white with red edges. On the other side of it we shall get a few coloured band and then uniform illumination.

USE OF WHITE LIGHT IN YDSE

Let us discuss the effect of using white light in YDSE setup. The slits S_1 and S_2 are illuminated by white light. White light is made of seven colours where approximate wavelength of violet colour is $\lambda_v \cong 4000$ Å and approximate wavelength of red colour is $\lambda_R \cong 8000$ Å (actually $\lambda_v = 4200$ Å and $\lambda_R = 7900$ Å), so $\lambda_R \cong 2\lambda_v$ when white light illuminates two slits, then both the slits act as source of white light. At the centre of slits on the screen, all colours travel equal path from S_1 to O and from S_2 to O, so that they interfere at O with zero path difference so as to give constructive interference at O and hence a bright white fringe is obtained at O.



Now as we start moving away from centre of screen, a path difference is introduced and since λ_v is minimum, so at some point (say P_1) we see that the path difference is $\frac{\lambda_v}{2}$ and at this point destructive interference of violet light occurs (i.e. violet colour is absent but all others present). At this point, the white light has violet colour absent and due to which a slightly reddish in colour or reddish white (but not red because it is white minus violet colour).

So, this point being closest to central maxima, the closest edge of this bright white central maxima will appear reddish in colour.

Now as we move further away from this point, various colours will be absent and there will be a point where path difference will be equal to $\frac{\lambda_{red}}{2} = \frac{\lambda_R}{2} = \lambda_v$ (because $\lambda_R \cong 2\lambda_v$).

So, at this point, (say P_2) not only destructive interference of red colour is obtained but constructive interference of violet is also obtained. In this region, light red colour is absent from white light and violet is interfering constructively. So, at this point the bluish fringe is obtained (not blue, because in white light red colour is absent and violet colour is dominating) and then onwards there is no point where any prominent colour is obtained (because all colours will mix on screen). So in the outer region, we can say whitish fringes merged into each other are obtained and we cannot distinguish between these fringes.

To conclude, we observe that at the centre, a permanent bright white central fringe is obtained whose closer edges are reddish white and farther edges are bluish and then onwards whitish fringes are obtained. The same effect is observed in the pattern below central maxima.

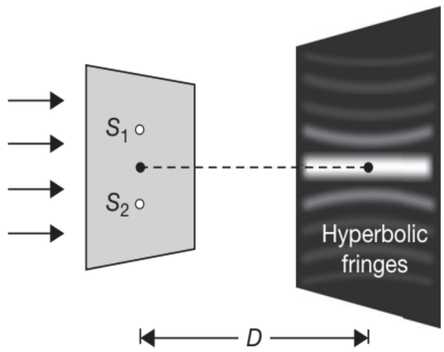
SHAPE OF INTERFERENCE FRINGES DUE TO DIFFERENT TYPES OF SOURCES (IN YDSE SETUP)

In *YDSE* setup, it is observed that the fringes obtained are straight and parallel to the slits. This is because, at all the points on screen where the path difference of the light waves from the two sources is same, will have same intensity and hence fringes of same intensity are obtained.

Let us now consider different cases of two light sources which produce interference pattern on a screen and analyse the shape of fringes obtained in the resulting interference pattern.

WHEN TWO POINT SOURCES IN A LINE ARE PLACED PARALLEL TO SCREEN

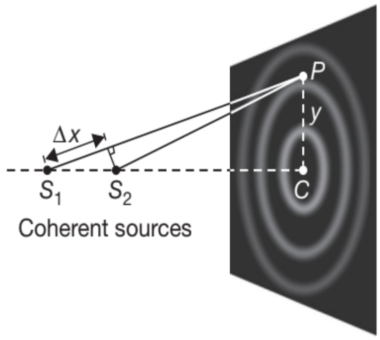
Let us consider a cardboard with two holes in a line and a screen placed in front of the cardboard and parallel to it as shown in figure.



When a light beam incident on the board illuminates the two holes, then these two holes will act like point sources S_1 and S_2 due to which interference pattern is obtained on the screen. We observe that the shape of fringes obtained on the screen is hyperbolic because a hyperbola is the locus of all the points on a plane which are at a constant distance from two points in space.

TWO POINT SOURCES IN A LINE PLACED NORMAL TO THE SCREEN

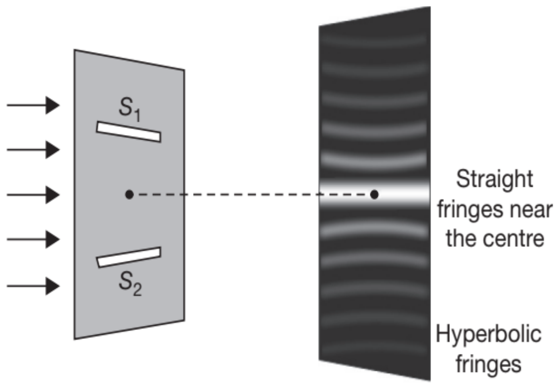
Let us consider two coherent point sources S_1 and S_2 along a line normal to the screen placed at some distance from the screen as shown in figure.



In this situation, if we consider a point P on the screen at a distance y from centre C of the screen, then due to the point sources, the path difference in light waves reaching at P remains constant and hence alternate bright and dark circular fringes are obtained.

TWO RECTANGULAR SLIT SOURCES IN A PLANE PARALLEL TO SCREEN

The fringe pattern obtained in *YDSE* setup when the light waves from two rectangular slit sources S_1 and S_2 interfere on screen is shown in figure.



Due to length of the rectangular slits, we observe that close to the middle region of the screen, fringes are straight and parallel. However, after moving away from the centre of the screen, the shape of fringes will be approximately hyperbolic (along the length of slits) because as discussed earlier, a hyperbola is the locus of all the points on a plane which are at a constant distance from two points in space.

ILLUSTRATION 15

In $YDSE$, light of wavelength 60 nm is used. The separation between the sources is 6 mm and between the sources and the screen is 2 m. Find the positions of a point lying between third maxima and third minima where the intensity is three-fourth of the maximum intensity on the screen.

SOLUTION

Since, $I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$

where $I = \frac{3}{4}(4I_0) = 3I_0$

$$\Rightarrow \quad \cos\left(\frac{\phi}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \frac{\phi}{2} = n\pi \pm \frac{\pi}{6}$$

$$\Rightarrow \quad \phi = 2n\pi \pm \frac{\pi}{3}$$

Since, $\phi = \frac{2\pi}{\lambda}\Delta x$ where $\Delta x = \frac{y_n d}{D}$

$$\Rightarrow \quad \frac{2\pi}{\lambda}\frac{y_n d}{D} = 2n\pi \pm \frac{\pi}{3}$$

$$\Rightarrow \quad y_n = \left(n \pm \frac{1}{6}\right)\frac{\lambda D}{d}$$

For the point lying between third minima and third maxima, we have

$$n = 3$$

$$\Rightarrow \quad y_3 = \left(3 - \frac{1}{6}\right)\frac{\lambda D}{d}$$

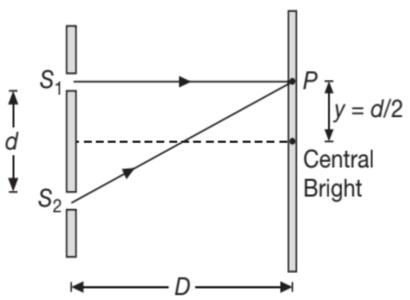
$$\Rightarrow \quad y_3 = \frac{17}{6}\left(\frac{\lambda D}{d}\right)$$

Substituting $\lambda = 0.6 \times 10^{-6}$ m, $D = 2$ m, $d = 6$ mm, we get

$$y_3 = \frac{17}{6}\frac{(0.6 \times 10^{-6})(2)}{6 \times 10^{-3}} = 5.67 \text{ mm}$$

MISSING WAVELENGTH(S) IN FRONT OF ANY ONE SLIT IN YDSE

Suppose P is a point of observation in front of slit S_1 as shown.



The wavelengths missing are the ones obtained by using the condition of destructive interference, i.e.,

$$\Delta x = (2n-1)\frac{\lambda}{2}, \text{ where } n = 1, 2, 3, \ldots\ldots$$

Now $\Delta x = \sqrt{D^2 + d^2} - D$

$$\Rightarrow \quad \Delta x = D\left[\left(1 + \frac{d^2}{D^2}\right)^{\frac{1}{2}} - 1\right] \qquad \ldots(1)$$

Since, $D \gg d$, so $\left(1 + \frac{d^2}{D^2}\right)^{\frac{1}{2}} \cong 1 + \frac{d^2}{2D^2}$

So, (1) becomes

$$\Delta x = \frac{d^2}{2D} = (2n-1)\frac{\lambda}{2}$$

We can also find the missing wavelengths by using the following steps.

Since $d\sin\theta = \left(n - \frac{1}{2}\right)\lambda$, where $\sin\theta = \frac{y}{D}$

$$\Rightarrow \quad \frac{d\left(\frac{d}{2}\right)}{D} = \left(n - \frac{1}{2}\right)\lambda$$

$$\Rightarrow \quad \lambda = \frac{d^2}{(2n-1)D}, \; n = 1, 2, 3, 4, 5, \ldots\ldots$$

$$\Rightarrow \quad \lambda_{\text{missing}} = \frac{d^2}{(2n-1)D}, \text{ where } n = 1, 2, 3, \ldots$$

By putting $n = 1, 2, 3\ldots.$, the missing wavelengths are

$$\lambda = \frac{d^2}{D}, \frac{d^2}{3D}, \frac{d^2}{5D} \ldots.$$

ILLUSTRATION 16

White light is used in a $YDSE$ with separation between the sources to be 0.9 m and separation between the sources and the screen to be 1 m. Light reaching the screen at position $y = 1$ mm is passed through a prism and its spectrum is obtained. Find the missing lines in the visible region of this spectrum.

SOLUTION

Path difference is given by

$$\Delta x = \frac{yd}{D} = (9 \times 10^{-4})(1 \times 10^{-3}) = 900 \text{ nm}$$

For minima $\Delta x = (2n-1)\dfrac{\lambda}{2}$, where $n = 1, 2, 3, \ldots$

$$\Rightarrow \quad \lambda = \frac{2\Delta x}{(2n-1)} = \frac{1800}{(2n-1)}$$

$$\Rightarrow \quad \lambda_{\text{missing}} = \frac{1800}{1}, \frac{1800}{3}, \frac{1800}{5}, \frac{1800}{7} \ldots\ldots$$

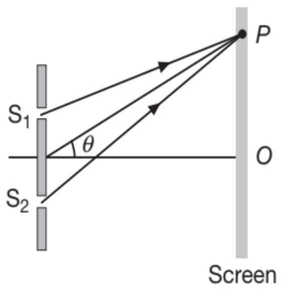
Of these, 600 nm and 360 nm lie in the visible range, so these will be missing lines in the visible spectrum.

ORDER OF FRINGES

When Slits are Vertical

If the slits are vertical, as shown in figure, path difference is,

$$\Delta x = d\sin\theta$$

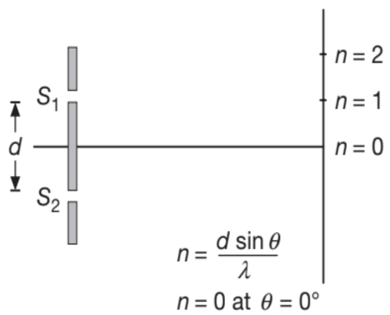


This path difference increases as θ increases.
The order of fringe n is given by

$$d\sin\theta = n\lambda$$

$$\Rightarrow \quad n = \frac{d\sin\theta}{\lambda}$$

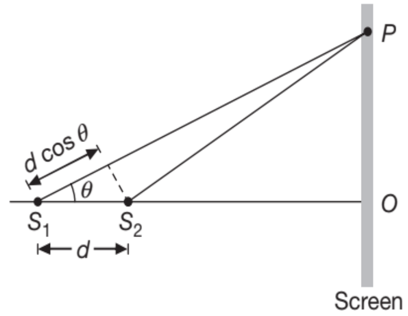
The order of fringe increases as we move away from point O on the screen.



When Slits are Horizontal

If the slits are horizontal, as shown in figure, then the path difference is

$$\Delta x = d\cos\theta$$



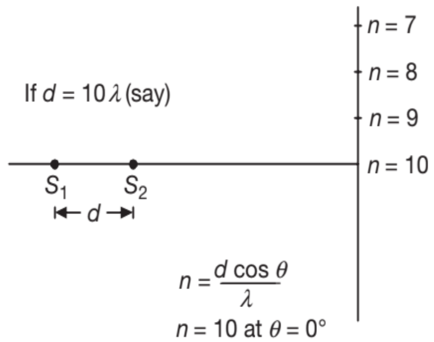
This path difference decreases as θ increases
The order of fringe n is given by

$$d\cos\theta = n\lambda$$

$$\Rightarrow \quad n = \frac{d\cos\theta}{\lambda}$$

The order of fringe decreases as we move away from point O.

Central maxima $(\text{i.e., } \Delta x = 0)$ obtained when $\theta \to \dfrac{\pi}{2}$ i.e., point of central maxima at far off distance from S_1 and S_2.



Conceptual Note(s)

To calculate the number of maximas or minimas that can be obtained on the screen, we use the fact that value of $\sin\theta$ (or $\cos\theta$) can never be greater than 1. For example in the first case when the slits are vertical.

$$\sin\theta = \frac{n\lambda}{d} \qquad \{\text{for maximum intensity}\}$$

Since, $\sin\theta \not> 1$

$$\Rightarrow \quad \frac{n\lambda}{d} \not> 1$$

$$\Rightarrow \quad n \not> \frac{d}{\lambda}$$

Suppose in some question $\frac{d}{\lambda}$ comes out say 3.6 then total number of maximas on the screen will be 7. Corresponding to $n = 0, \pm1, \pm2, \pm3$.
So, highest order of interference maxima is

$$n_{max} = \left[\frac{d}{\lambda}\right]$$

where [ ] represents the greatest integer function. So, total number of maximas obtained are

$$N = 2n_{max} + 1$$

Similarly, highest order of interference minima is

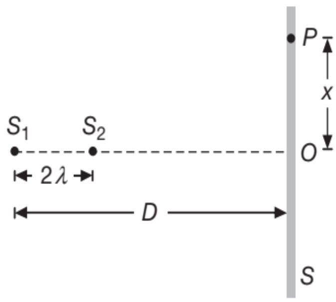
$$n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right]$$

where [ ] represents the greatest integer function. So, total number of minimas obtained are

$$N = 2n_{min}$$

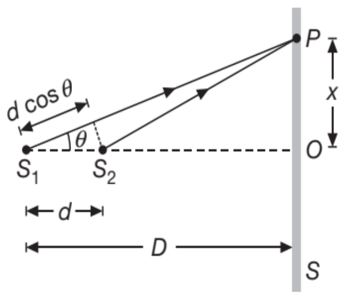
ILLUSTRATION 17

Two coherent narrow slits emitting light of wavelength λ in the same phase are placed parallel to each other at a small separation of 2λ. The light is collected on a screen S which is placed at a distance $D(\gg \lambda)$ from the slit S_1 as shown in figure. Find the finite distance x such that the intensity at P is equal to intensity at O.



SOLUTION

Path difference for waves reaching at O is $S_1O - S_2O = 2\lambda$ i.e., maximum intensity is obtained at O.



Next maxima will be obtained at point P where,

$$S_1P - S_2P = \lambda$$

$$\Rightarrow \quad d\cos\theta = \lambda$$

$$\Rightarrow \quad (2\lambda)\cos\theta = \lambda$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 60^\circ$$

Now in ΔS_1PO

$$\frac{PO}{S_1O} = \tan\theta$$

$$\Rightarrow \quad \frac{x}{D} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow \quad x = \sqrt{3}D$$

Conceptual Note(s)

At point O, path difference is 2λ i.e. we obtain second order maxima. At point P, where path difference is λ (so, $x = \sqrt{3}D$) we get first order maxima. The next, i.e., zero order maxima (central maxima) will be obtained where path difference, $d\cos\theta = 0$

$$\Rightarrow \quad \theta = 90^\circ$$

$$\Rightarrow \quad x \to \infty$$

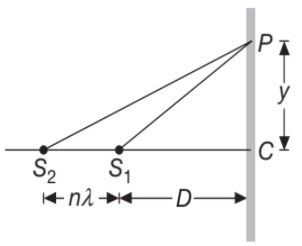
So, our answer, i.e., finite distance of x should be $x = \sqrt{3}D$, corresponding to first order maxima.

ILLUSTRATION 18

Two point sources are $d = n\lambda$ apart. A screen is held at right angles to the line joining the two sources at a distance D from the nearest source. Calculate the distance of the point on the screen, where the first bright fringe (excluding the centre one) is observed. Assume $D \gg d$.

SOLUTION

At point C path difference is $n\lambda$. Therefore, nth bright fringe will be observed.



Next bright fringe is observed where path difference is $(n-1)\lambda$, so

$$S_2P - S_1P = (n-1)\lambda \quad \ldots(1)$$

Now, $S_1P = \left(D^2+y^2\right)^{1/2} = D\left(1+\frac{y^2}{D^2}\right)^{1/2} \approx D + \frac{y^2}{2D} \quad \ldots(2)$

Similarly, $S_2P = (D+d) + \frac{y^2}{2(D+d)} \quad \ldots(3)$

Substituting (2) and (3) in (1) i.e., $S_2P - S_1P = (n-1)\lambda$, we get

$$d - \frac{y^2 d}{2D(D+d)} = (n-1)\lambda$$

Since, $d = n\lambda$

$$\Rightarrow \quad n\lambda - \frac{y^2(n\lambda)}{2D(D+n\lambda)} = n\lambda - \lambda$$

$$\Rightarrow \quad \frac{ny^2}{2D(D+n\lambda)} = 1$$

$$\Rightarrow \quad y = \sqrt{\frac{2D(D+n\lambda)}{n}}$$

OPTICAL PATH

It is defined as distance travelled by light in vacuum in the same time in which it travels a given path length in a medium. If light travels a path length d in a medium at speed v, the time taken by it will be $t = \frac{d}{v}$. So, the optical path length is

$$OPL = ct = c\left(\frac{d}{v}\right) = \left(\frac{c}{v}\right)d = \mu d \qquad \left\{\because \mu = \frac{c}{v}\right\}$$

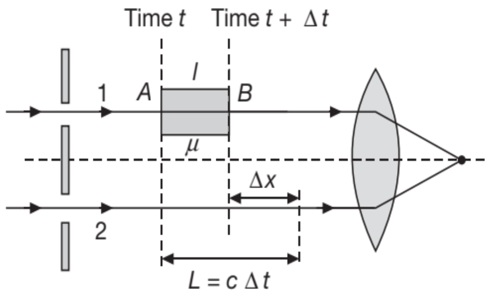
Since, for all media $\mu > 1$, optical path length is always greater than geometrical path length.

When two light waves arrive at a point by travelling different distances in different media, the phase difference between the two is related by their optical path difference instead of simple path difference. So,

$$\text{Phase Difference} = \frac{2\pi}{\lambda}(OPL)$$

PATH DIFFERENCE BETWEEN TWO PARALLEL WAVES DUE TO A DENSER MEDIUM IN PATH OF ONE BEAM

Consider two coherent light rays (thin beams) from a single source of light travelling in same direction parallel to each other. If in path of first beam a glass slab of refractive index μ having width l is placed as shown in figure.



The light ray 1 will slow down after it enters in slab at point A and its speed will reduce to $\frac{c}{\mu}$. When this ray 1 comes out of the slab at point B, then in this duration, the ray 2 which was travelling in space would have travelled a longer path because the ray 2 travels at a speed c.

The time taken by ray 1 in travelling through the glass slab is

$$\Delta t = \left(\frac{l}{\frac{c}{\mu}}\right) = \frac{\mu l}{c}$$

Path length covered by ray 2 in space while ray 1 was travelling in slab is

$$L = c\Delta t = c\left(\frac{\mu l}{c}\right) = \mu l$$

Thus, path difference between the two rays is given by

$$\Delta x = L - l = \mu l - l = l(\mu - 1)$$

If these two light waves (rays) are brought to focus on a converging lens as shown, then the two waves will interfere with a phase difference given as

$$\phi = \Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\left[l(\mu-1)\right]$$

If each of the light wave in the two thin light beams (rays) have intensity I_0 then at the focal point of the lens the resulting intensity of light is given as

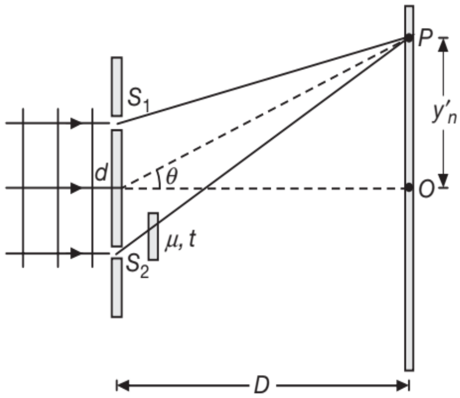
$$I_R = 4I_0\cos^2\left(\frac{\phi}{2}\right) = 4I_0\cos^2\left(\frac{\pi l(\mu-1)}{\lambda}\right)$$

DISPLACEMENT OR SHIFTING OF FRINGE PATTERN IN YDSE

When a transparent film of thickness t and refractive index μ is introduced in front of one of the slits, the fringe pattern shifts in the direction where the film is placed.

How much is the fringe shift?

Consider the $YDSE$ arrangement shown in the figure.



A film of thickness t and refractive index μ is placed in front of the lower slit.

The optical path difference is given by

$$x = [(S_2P - t) + \mu t] - S_1P$$

$$\Rightarrow \quad x = (S_2P - S_1P) + t(\mu-1)$$

Since $S_2P - S_1P = d\sin\theta$

$$\Rightarrow \quad x = d\sin\theta + t(\mu-1)$$

Since $\sin\theta \approx \tan\theta = \dfrac{y'_n}{D}$

$$\Rightarrow \quad x = \frac{dy'_n}{D} + t(\mu-1)$$

The maxima will be obtained when the path difference is an even multiple of $\dfrac{\lambda}{2}$ i.e.,

$$x = (2n)\frac{\lambda}{2}$$

$$\Rightarrow \quad (2n)\frac{\lambda}{2} = \frac{dy'_n}{D} + t(\mu-1)$$

$$\Rightarrow \quad y'_n = \frac{n\lambda D}{d} - (\mu-1)\frac{tD}{d}$$

In the absence of film, the position of the nth maxima is given by equation

$$y_n = \frac{n\lambda D}{d}$$

Therefore, the fringe shift (FS) is given by

$$FS = y_n - y'_n = \frac{D}{d}(\mu-1)t = \frac{\beta}{\lambda}(\mu-1)t \quad \left\{\because \beta = \frac{\lambda D}{d}\right\}$$

Note that the shift is in the direction where the film is introduced.

Conceptual Note(s)

(a) The entire pattern shifts towards the side where the plate is introduced and there is no other change in the pattern.

(b) To measure this shift white light must be used because with monochromatic light all the fringes will exactly be similar and hence no shift can be observed.

(c) The effective path in air is increased by an amount $(\mu-1)t$ due to introduction of the plate i.e., the additional path difference is $(\mu-1)t$.

(d) If shift is equivalent to n fringes then $n = \dfrac{(\mu-1)t}{\lambda}$ or $t = \dfrac{n\lambda}{(\mu-1)}$

(e) The shift Δx is independent of the order of fringe n, i.e. Shift of zero order maxima = Shift of n^{th} order maxima.

(f) Shift is independent of wavelength.

ILLUSTRATION 19

Interference fringes are produced by a double slit arrangement and a piece of plane parallel glass of refractive index 1.5 is interposed in one of the interfering beam. If the fringes are displaced through 30 fringe widths for light of wavelength 6×10^{-5} cm, find the thickness of the plate.

SOLUTION

Path difference due to the introduction of glass slab is

$$\Delta x = (\mu-1)t$$

Thirty fringes are displaced due to the introduction of slab. So,

$$\Delta x = 30\lambda$$

$$\Rightarrow \quad (\mu - 1)t = 30\lambda$$

$$\Rightarrow \quad t = \frac{30\lambda}{\mu - 1} = \frac{30 \times 6 \times 10^{-5}}{1.5 - 1}$$

$$\Rightarrow \quad t = 3.6 \times 10^{-3} \text{ cm}$$

ILLUSTRATION 20

In Young's double slit experiment using monochromatic light the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 1.964 microns is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of the monochromatic light used in the experiment.

SOLUTION

Shifting of fringes due to introduction of slab in the path of one of the slits is given by

$$\Delta y = \frac{(\mu - 1)tD}{d} \qquad \ldots(1)$$

Now, the distance between the screen and slits is doubled.
Hence, the new fringe width will become

$$\beta' = \frac{\lambda(2D)}{d} \qquad \ldots(2)$$

Given, $\Delta y = \beta'$

$$\Rightarrow \quad \frac{(\mu - 1)tD}{d} = \frac{\lambda(2D)}{d}$$

$$\Rightarrow \quad \lambda = \frac{(\mu - 1)t}{2} = \frac{(1.6 - 1)(1.964 \times 10^{-6})}{2}$$

$$\Rightarrow \quad \lambda = 0.5892 \times 10^{-6} \text{ m} = 5892 \text{ Å}$$

ILLUSTRATION 21

In a $YDSE$, the two coherent sources are separated from each other by 6 mm and from the screen by 2 m. A light of wavelength 6000 Å is used. A film of refractive index 1.5 is introduced in front of the lower slit such that the third maxima shifts to the origin.

(a) Find the thickness of the film.
(b) Find the positions of the fourth maxima.

SOLUTION

(a) Since third minima shifts to the origin, therefore, the fringe shift (FS) is equal to three fringe widths i.e., 3β, so we have

$$FS = y_3 = 3\left(\frac{\lambda D}{d}\right)$$

Since we know that the fringe shift (FS) is given by

$$FS = (\mu - 1)\frac{tD}{d}$$

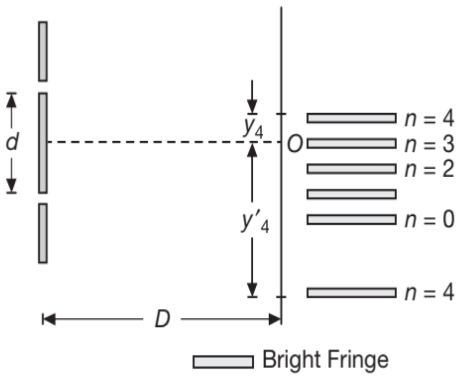
$$\Rightarrow \quad (\mu - 1)\frac{tD}{d} = 3\frac{\lambda D}{d}$$

$$\Rightarrow \quad t = \frac{3\lambda}{\mu - 1}$$

Since, $\lambda = 0.6 \times 10^{-6} \text{ m}$, $\mu = 1.5$

$$\Rightarrow \quad t = \frac{(3)(0.6 \times 10^{-6})}{1.5 - 1} = 3.6 \ \mu\text{m}$$

(b) There are two positions of fourth maxima, one above and the other below the origin. So, we have



$$y_4 = 1\beta = \frac{\lambda D}{d} = 0.2 \text{ mm} \text{ and}$$

$$y_4' = -7\beta = -7\left(\frac{\lambda D}{d}\right) = -1.4 \text{ mm}$$

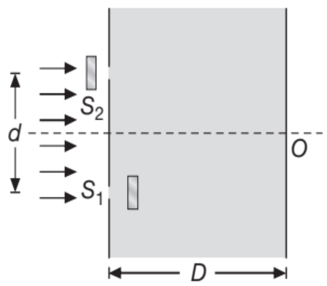
ILLUSTRATION 22

A Young double slit apparatus is immersed in a liquid of refractive index μ_1. The slit plane touches the liquid surface. A parallel beam of monochromatic light of wavelength λ (in air) is incident normally on the slits.

(a) Find the fringe width

(b) If one of the slits (say S_2) is covered by a transparent slab of refractive index μ_2 and thickness t as shown, find the new position of central maxima.

(c) Now the other slit S_1 is also covered by a slab of same thickness and refractive index μ_3 as shown in figure due to which the central maxima recovers its position find the value of μ_3.



(d) Find the ratio of intensities at O in the three conditions (a), (b) and (c).

SOLUTION

(a) Fringe width is given by

$$\beta = \frac{\lambda' D}{d} = \frac{\lambda D}{\mu_1 d} \qquad \left\{\because \lambda' = \frac{\lambda}{\mu}\right\}$$

(b) Position of central maximum is shifted upwards by a distance

$$\Delta y = \frac{(\mu_2 - 1)tD}{d} \qquad \ldots(1)$$

(c) Downward shift is now given by

$$\Delta y' = \frac{\left(\frac{\mu_3}{\mu_1} - 1\right)tD}{d} \qquad \ldots(2)$$

Since the central maxima recovers its position, so

$$\Delta y = \Delta y'$$

So, from (1) and (2), we get

$$\frac{(\mu_1 - 1)tD}{d} = \frac{\left(\frac{\mu_3}{\mu_1} - 1\right)tD}{d}$$

$$\Rightarrow \frac{\mu_3}{\mu_1} = \mu_2$$

$$\Rightarrow \mu_3 = \mu_1 \mu_2$$

(d) Since, $I = I_{max} \cos^2\left(\frac{\phi}{2}\right)$

where $\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x$

$$\Rightarrow \frac{\phi}{2} = \left(\frac{\pi}{\lambda}\right)\Delta x$$

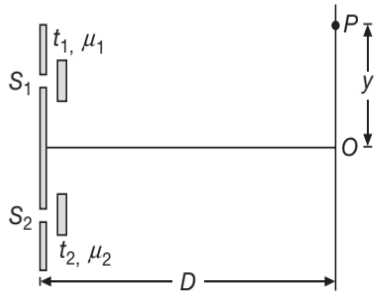
$$\Rightarrow I \propto \cos^2\left(\frac{\phi}{2}\right)$$

In the first and third case, $\Delta x = 0$ while in second case, $\Delta x = (\mu_2 - 1)t$. Therefore, the desired ratio is,

$$I_1 : I_2 : I_3 = 1 : \cos^2\left(\frac{\pi(\mu_2 - 1)t}{\lambda}\right) : 1$$

ILLUSTRATION 23

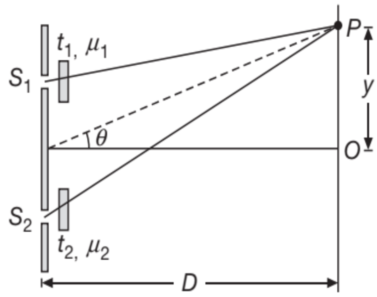
Two transparent sheets of thickness t_1 and t_2 and refractive indexes μ_1 and μ_2 are placed in front of the slits in $YDSE$ setup as shown in figure.



If D is the distance of the screen from the slits, then find the distance of zero order maxima from the centre of the screen. What is the condition that zero order maxima is formed at the centre O?

SOLUTION

The two waves from the two slits reaching the point P on the screen are shown in figure.



If this point is the position of zero order maxima and the distance of P from the centre O of the screen is

y_0, so the optical path of light waves from source S_1 is given as

$$x_1 = S_1P + (\mu_1 - 1)t_1$$

The optical path of light waves from source S_2

$$x_2 = S_2P + (\mu_2 - 1)t_2$$

The path difference between the two waves reaching at P is

$$\Delta x = x_2 - x_1$$

$$\Delta x = (S_2P - S_1P) + (\mu_2 - 1)t_2 - (\mu_1 - 1)t_1$$

Physically the path difference from sources to a point P on the screen is given by

$$S_2P - S_1P = d\sin\theta \approx \frac{yd}{D}$$

$$\Rightarrow \quad \Delta x = \frac{yd}{D} + (\mu_2 - 1)t_2 - (\mu_1 - 1)t_1$$

For zero order maxima, $\Delta x = 0$

$$\Rightarrow \quad 0 = \frac{yd}{D} + (\mu_2 - 1)t_2 - (\mu_1 - 1)t_1$$

$$\Rightarrow \quad y = \frac{D[(\mu_1 - 1)t_1 - (\mu_2 - 1)t_2]}{d}$$

For zero order maxima to be form at the centre O, we have

$$y = 0$$

$$\Rightarrow \quad 0 = \frac{D[(\mu_1 - 1)t_1 - (\mu_2 - 1)t_2]}{d}$$

$$\Rightarrow \quad (\mu_1 - 1)t_1 = (\mu_2 - 1)t_2$$

YDSE FOR SOURCE NOT PLACED AT THE CENTRAL LINE

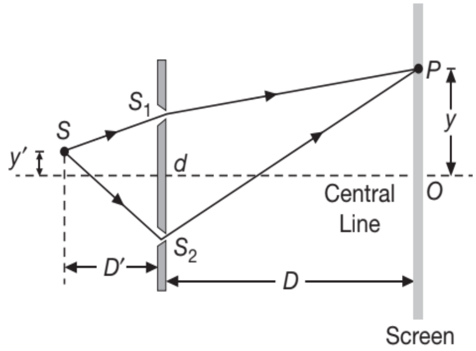
If the source of light is not placed at the central line but a little beyond (slightly up or down) the central line, then the waves reaching S_1 and S_2 from S will already have an initial path difference.

Path difference of waves meeting at the point P is given by

$$\Delta x = (SS_2 + S_2P) - (SS_1 + S_1P)$$

$$\Rightarrow \quad \Delta x = (SS_2 - SS_1) + (S_2P - S_1P)$$

$$\Rightarrow \quad \Delta x = \frac{y'd}{D'} + \frac{yd}{D} = \left(\frac{y'}{D'} + \frac{y}{D}\right)d$$



where, y' is the distance of the source S above or below the central line and D' is the distance of S from S_1 and S_2.

Similarly, here once we calculated the path difference, then

FOR MAXIMA, $\Delta x = (2n)\frac{\lambda}{2}$, $n = 0, 1, 2, 3$ .....

FOR MINIMA, $\Delta x = (2n+1)\frac{\lambda}{2}$, $n = 0, 1, 2, 3$ .....

Problem Solving Technique(s)

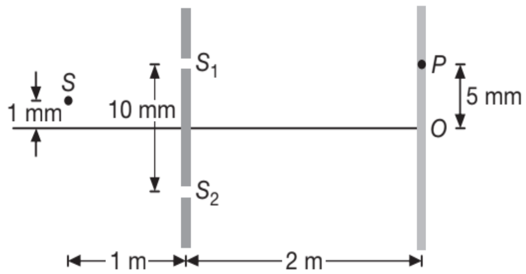
At the position of central maxima, we have

$$\Delta x = 0 \text{ i.e., } \frac{y'}{D'} + \frac{y}{D} = 0$$

i.e., if S is above central line, then central maxima is below the central line and vice versa.

ILLUSTRATION 24

In the Young's Double Slit experiment a point source of $\lambda = 5000$ Å is placed slightly off the central axis as shown in the figure.



(a) Find the nature and order of the interference at the point P.

(b) Find the nature and order of the interference at O.

(c) Where should we place a film of refractive index $\mu = 1.5$ and what should be its thickness so that a maxima of zero order is placed at O.

SOLUTION

(a) The optical path difference between the two waves arriving at P is

$$\Delta x = \frac{y_1 d}{D_1} + \frac{y_2 d}{D_2} = \frac{(1)(10)}{10^3} + \frac{(5)(10)}{2\times 10^3}$$

$$\Rightarrow \Delta x = 3.5\times 10^{-2}\ \text{mm} = 0.035\ \text{mm}$$

To calculate the order of interference, we shall calculate

$$n = \frac{\Delta x}{\lambda}$$

$$\Rightarrow n = \frac{0.035\times 10^{-3}\ \text{m}}{5000\times 10^{-10}\ \text{m}}$$

$$\Rightarrow n = 70$$

$$\Rightarrow \Delta x = 70\lambda$$

So, 70^{th} order maxima is obtained at P.

(b) At O, $\Delta x = \frac{y_1 d}{D_1} = 10^{-2}\ \text{mm} = 0.01\ \text{mm}$

Now, we observe that $\Delta x = 20\lambda$
So, 20^{th} order maxima is obtained at O.

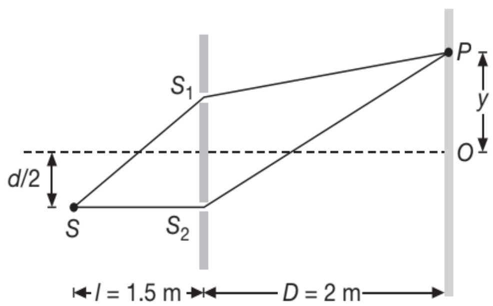
(c) $(\mu - 1)t = 0.01\ \text{mm}$

$$\Rightarrow t = \frac{0.01}{1.5-1} = 0.02\ \text{mm} = 20\ \mu\text{m}$$

Since the pattern has to be shifted upwards, therefore, the film must be placed in front of S_1.

ILLUSTRATION 25

In $YDSE$, the monochromatic source of wavelength λ is placed at a distance $\frac{d}{2}$ from the central axis (as shown in the figure), where d is the separation between the two slits S_1 and S_2.



(a) Find the position of the central maxima.
(b) Find the order of interference formed at O.
(c) Find the minimum thickness of the film of refractive index $\mu = 1.5$ to be placed in front of S_2 so that intensity at O becomes $\frac{3}{4}$th of the maximum intensity.

Given $\lambda = 6000$ Å and $d = 6$ mm.

SOLUTION

(a) $(\Delta x)_{\text{net}} = 0$

$$\Rightarrow \frac{y_1 d}{D_1} = \frac{y_2 d}{D_2}$$

$$\Rightarrow \frac{d/2}{1.5} = \frac{y}{2}$$

$$\Rightarrow y = \frac{d}{1.5} = \frac{6}{1.5} = 4\ \text{mm}$$

(b) At O, net path difference is given by

$$\Delta x = \frac{y_1 d}{D_1} = \frac{\left(\frac{d}{2}\right)(d)}{D_1} = \frac{(6\times 10^{-3})^2}{2\times 1.5} \quad \{\because S_1O = S_2O\}$$

$$\Rightarrow \Delta x = 12\times 10^{-6}\ \text{m}$$

$$\Rightarrow \Delta x = 120\times 10^{-7}\ \text{m}$$

Since, $\lambda = 6000\ \text{Å} = 6\times 10^{-7}\ \text{m}$

$$\Rightarrow \Delta x = 20\lambda$$

So at O, the bright fringe of order 20 will be obtained.

(c) $I = I_{\max}\cos^2\left(\frac{\phi}{2}\right)$

$$\Rightarrow \frac{3}{4}I_{\max} = I_{\max}\cos^2\left(\frac{\phi}{2}\right)$$

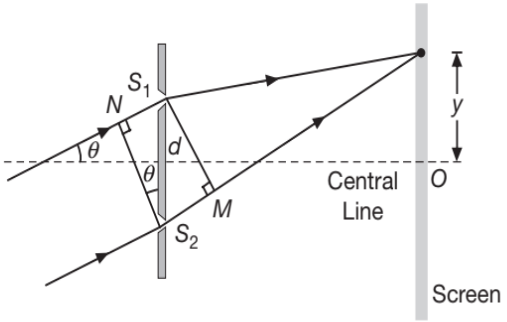
$$\Rightarrow \frac{\phi}{2} = \frac{\pi}{6}$$

$$\Rightarrow \phi = \frac{\pi}{3} = \left(\frac{2\pi}{\lambda}\right)(\mu-1)t$$

$$\Rightarrow t = \frac{\lambda}{6(\mu-1)} = \frac{6000}{6(1.5-1)} = 2000\ \text{Å}$$

YDSE WHEN INCIDENT RAYS ARE NOT PARALLEL TO CENTRAL LINE

In this case, the rays reaching S_1 and S_2 already have an initial path difference.



So, net path difference between the rays reaching the point P is given by

$$\Delta x = (NS_1 + S_1P) - S_2P$$

$$\Rightarrow \quad \Delta x = |NS_1 - (S_2P - S_1P)| = |NS_1 - MS_2|$$

Now, $NS_1 = d\sin\theta$ and

$$MS_2 = S_2P - S_1P = \frac{yd}{D} \text{ (as done earlier)}$$

$$\Rightarrow \quad \Delta x = \left| d\sin\theta - \frac{yd}{D} \right|$$

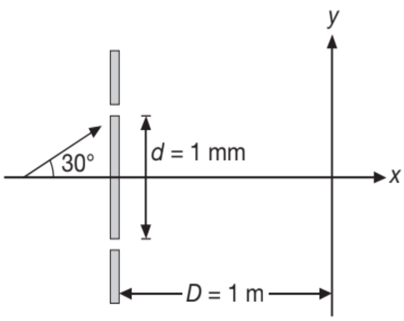
Once the path difference Δx is known, then

FOR MAXIMA, $\Delta x = (2n)\frac{\lambda}{2}, n = 0, 1, 2, 3, \ldots$

FOR MINIMA, $\Delta x = (2n+1)\frac{\lambda}{2}, n = 0, 1, 2, 3, \ldots$

ILLUSTRATION 26

A coherent parallel beam of microwaves of wavelength $\lambda = 0.5$ mm falls on a Young's double slit apparatus. The separation between the slits is 1.0 mm. The intensity of microwaves is measured on a screen placed parallel to the plane of the slits at a distance of 1.0 m from it as shown in the figure.

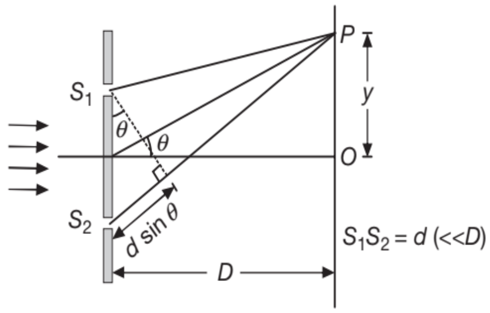


(a) If the incident beam falls normally on the double slit apparatus, find the y-coordinates of all the interference minima on the screen.

(b) If the incident beam makes an angle of 30° with the x-axis (as in the dotted arrow shown in figure), find the y-coordinates of the first minima on either side of the central maximum.

SOLUTION

(a) Given $\lambda = 0.5$ mm, $d = 1$ mm, $D = 1$ m



When the incident beam falls normally, path difference between the two rays S_2P and S_1P is

$$\Delta x = S_2P - S_1P \approx d\sin\theta$$

For minimum intensity,

$$d\sin\theta = (2n-1)\frac{\lambda}{2}, \quad n = 1, 2, 3, \ldots$$

$$\Rightarrow \quad \sin\theta = \frac{(2n-1)\lambda}{2d} = \frac{(2n-1)0.5}{2\times 1} = \frac{2n-1}{4}$$

Since, $\sin\theta \le 1$

$$\Rightarrow \quad \frac{(2n-1)}{4} \le 1$$

$\Rightarrow \quad n \le 2.5$

So, n can be either 1 or 2

When $n = 1$, we have $\sin\theta_1 = \frac{1}{4}$

$$\Rightarrow \quad \tan\theta_1 = \frac{1}{\sqrt{15}}$$

When, $n = 2$, we have $\sin\theta_2 = \frac{3}{4}$

$$\Rightarrow \quad \tan\theta_2 = \frac{3}{\sqrt{7}}$$

Since, $y = D\tan\theta = \tan\theta \ (D = 1 \text{ m})$

So, the position of minima will be

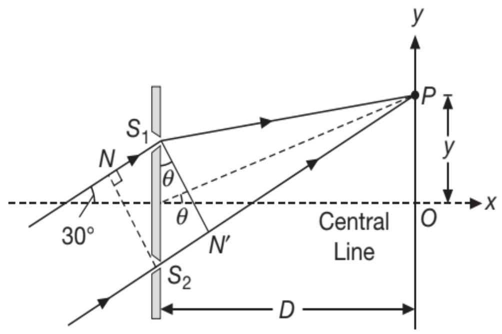
$$y_1 = D\tan\theta_1 = \frac{1}{\sqrt{15}} \text{ m}$$

$$y_2 = D\tan\theta_2 = \frac{3}{\sqrt{7}} \text{ m}$$

Since, minima can be on either side of centre O, so there will be four minimas at positions $\pm\frac{1}{\sqrt{15}}$ m and $\pm\frac{3}{\sqrt{7}}$ m on the screen.

(b) Path difference between the rays before entering the slits S_1 and S_2 is

$$NS_1 = \Delta x_1 = d\sin(30^\circ) = \frac{d}{2} \quad \ldots(1)$$



Path difference between the rays after passing through the slits S_1 and S_2 is

$$S_2P - S_1P = \Delta x_2 = d\sin\theta$$

So, net path difference is given by

$$\Delta x = |NS_1 + S_1P - S_2P| = |NS_1 - (S_2P - S_1P)|$$

$$\Rightarrow \Delta x = |\Delta x_1 - \Delta x_2| = \left|\frac{d}{2} - d\sin\theta\right|$$

For first minima, we have

$$\Delta x = \frac{\lambda}{2}$$

$$\Rightarrow \left|\frac{d}{2} - d\sin\theta\right| = \frac{\lambda}{2}$$

$$\Rightarrow \frac{d}{2} - d\sin\theta = \pm\frac{\lambda}{2}$$

$$\Rightarrow \frac{1}{2} - \sin\theta = \pm\frac{\lambda}{2d}$$

$$\Rightarrow \sin\theta = \frac{1}{2} \mp \frac{\lambda}{2d}$$

Since $d = 1$ mm, $\lambda = 0.5$ mm

$$\Rightarrow \sin\theta = \frac{1}{2} \mp \frac{0.5}{2}$$

$$\Rightarrow \sin\theta = \frac{1}{2} \mp \frac{1}{4}$$

$$\Rightarrow \sin\theta = \frac{1}{4} \text{ and } \sin\theta = \frac{3}{4}$$

Since the position of first minima on either side of central maxima is

$$\tan\theta = \frac{y}{D}$$

$$\Rightarrow y = D\tan\theta$$

For $\sin\theta = \frac{1}{4}$, we have $\tan\theta = \frac{1}{\sqrt{15}}$

$$\Rightarrow y = \frac{1}{\sqrt{15}} \text{ m}$$

For $\sin\theta = \frac{3}{4}$, we have $\tan\theta = \frac{3}{\sqrt{7}}$

$$\Rightarrow y = \frac{3}{\sqrt{7}} \text{ m}$$

Problem Solving Technique(s)

If two thin plates are also inserted just after S_1 and S_2, then our first task is to find the path difference. In the figure shown, path of ray 1 is more than path of ray 2 by a distance,

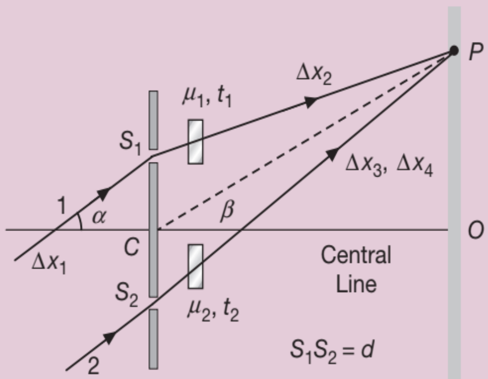
$$\Delta x_1 = d\sin\alpha \text{ and } \Delta x_2 = (\mu_1 - 1)t_1$$

and path of ray 2 is greater than path of ray 1 by a distance.

$$\Delta x_3 = d\sin\beta \text{ and } \Delta x_4 = (\mu_2 - 1)t_2$$

Therefore, net path difference is,

$$\Delta x = (\Delta x_1 + \Delta x_2) \sim (\Delta x_3 + \Delta x_4)$$



Once, we know the path difference Δx, then

FOR MAXIMA, $\Delta x = (2n)\frac{\lambda}{2}, n = 0, 1, 2, \ldots$

FOR MINIMA, $\Delta x = (2n+1)\frac{\lambda}{2}, n = 0, 1, 2, \ldots$

ILLUSTRATION 27

A large opaque sheet placed parallel to the yz plane at $x = 0.03$ m. The region $x \geq 0$ is filled with a transparent liquid of refractive index $\frac{3}{2}$. A wide monochromatic beam of light of wavelength 900 nm falls on the yz-plane at $x = 0$ making an angle of 30° with the x-axis. The sheet has two slits parallel to z-axis at $y = \pm 0.9$ mm. The intensity of the wave is measured on a screen placed at $x = 1.03$ m parallel to the sheet.

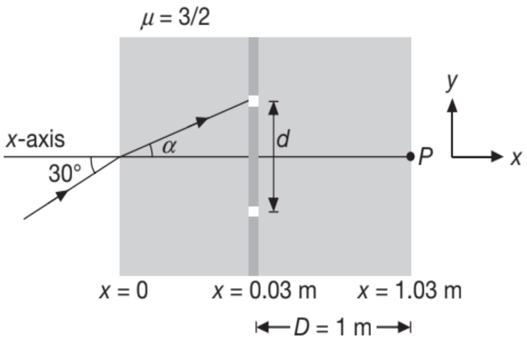
(a) Find the intensity at a point P on the screen where $y = z = 0$.

(b) The lower slit is covered by a transparent strip of refractive index 1.4 and thickness 4.2 mm. Now find the intensity at point P.

Given that $\tan(20°) = \frac{1}{2\sqrt{2}}$

SOLUTION

(a) The situation in the problem is shown in figure.



We observe that separation between the sources is

$d = 2 \times 0.9 = 1.8$ mm

Since, we know that $\lambda' = \frac{\lambda}{\mu} = \frac{900}{\frac{3}{2}} = 600$ nm

Applying Snell's Law at the $x = 0$ interface, we get

$$\frac{3}{2} = \frac{\sin(30°)}{\sin\alpha}$$

$$\Rightarrow \sin\alpha = \frac{1}{3}$$

$$\Rightarrow \alpha = 20° \qquad \text{\{Using Trigonometry\}}$$

Initial path difference is

$$\Delta x = d\sin\alpha = (1.8)\left(\frac{1}{3}\right) = 0.6 \text{ mm}$$

$$\Rightarrow \phi = \frac{2\pi}{\lambda'}\Delta x = \frac{2\pi}{600 \times 10^{-9}} \times 0.6 \times 10^{-3} = 2000\pi$$

$$\Rightarrow I = I_{max}\cos^2\left(\frac{\phi}{2}\right) = I_{max}$$

(b) Net path difference at P is now

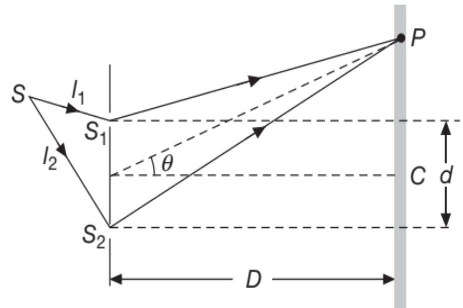
$$\Delta x = (0.6 \text{ mm}) + \left(\frac{1.5}{1.4} - 1\right)(4.2 \text{ mm}) = 0.3 \text{ mm}$$

$$\Rightarrow \phi = 1000\pi$$

$$\Rightarrow I = I_{max}$$

ILLUSTRATION 28

In a Young's Double slit Experiment, the light source is at distance $l_1 = 20$ μm and $l_2 = 40$ μm from the slits. The light of wavelength $\lambda = 500$ nm is incident on slits separated at a distance 10 μm. A screen is placed at a distance $D = 2$ m away from the slits as shown in figure.



(a) Find the values of θ relative to the central line where maxima appear on the screen?

(b) How many maxima will appear on the screen?

(c) What should be minimum thickness of a slab of refractive index 1.5 be placed on the path of one of the ray so that minima occurs at C?

SOLUTION

(a) The optical path difference between the beams arriving at P is given by

$$\Delta x = (l_2 - l_1) + d\sin\theta$$

The condition for maximum intensity is,

$\Delta x = n\lambda$, where $n = 0, \pm1, \pm2, \ldots\ldots$

$$\Rightarrow \sin\theta = \frac{1}{d}\left(\Delta x - (l_2 - l_1)\right) = \frac{1}{d}\left(n\lambda - (l_2 - l_1)\right)$$

$$\Rightarrow \sin\theta = \frac{1}{10 \times 10^{-6}}\left(n \times 500 \times 10^{-9} - 20 \times 10^{-6}\right)$$

$$\Rightarrow \sin\theta = 2\left(\frac{n}{40} - 1\right)$$

$$\Rightarrow \sin\theta = \frac{n}{20} - 2$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{n}{20} - 2\right)$$

(b) Since we know that

$$|\sin\theta| \le 1$$

$$\Rightarrow -1 \le \left(\frac{n}{20} - 2\right) \le 1$$

$$\Rightarrow -20 \le (n - 40) \le 20$$

$$\Rightarrow 20 \le n \le 60$$

Hence, number of maxima obtained is

$$N = 60 - 20 = 40$$

(c) At C, phase difference, $\phi = \frac{2\pi}{\lambda}\Delta x = \left(\frac{2\pi}{\lambda}\right)(l_2 - l_1)$

$$\Rightarrow \phi = \left(\frac{2\pi}{500\times 10^{-9}}\right)(20\times 10^{-6}) = 80\pi$$

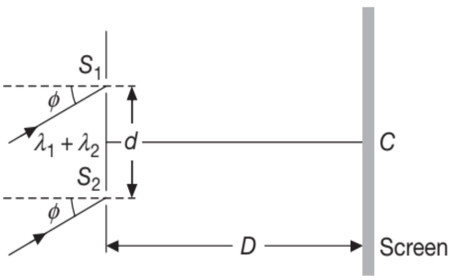
Hence, maximum intensity will appear at C.
For minimum intensity at C, we have

$$(\mu - 1)t = \frac{\lambda}{2}$$

$$\Rightarrow t = \frac{\lambda}{2(\mu - 1)} = \frac{500\times 10^{-9}}{2\times 0.5} = 500 \text{ nm}$$

ILLUSTRATION 29

In a Young's double slit experiment a parallel beam containing wavelengths $\lambda_1 = 4000$ Å and $\lambda_2 = 5600$ Å incident at an angle $\phi = 30°$ on a diaphragm having narrow slits at a separation $d = 2$ mm.



The screen is placed at a distance $D = 40$ cm from slits. A mica slab of thickness $t = 5$ mm is placed in front of one of the slits and whole the apparatus is submerged in water. If the central bright fringe is observed at C, calculate

(a) the refractive index of the slab.

(b) the distance of the first black line from C. Both wavelengths are in air. Take $\mu_w = \frac{4}{3}$.

SOLUTION

(a) To observe bright fringe at C, the mica slab should be placed in front of S_2. In that case, net path difference at C is,

$$\Delta x = d\sin\phi - (\mu_r - 1)t \qquad \left\{\because \mu_r = \frac{\mu_{\text{slab}}}{\mu_w}\right\}$$

For central bright at C we have

$$\Delta x = 0$$

$$\Rightarrow d\sin\phi = (\mu_r - 1)t$$

$$\Rightarrow (\mu_r - 1) = \frac{d\sin\phi}{t} = \frac{(2\times 10^{-3})\sin(30°)}{5\times 10^{-3}} = 0.2$$

$$\Rightarrow \mu_r = 1.2$$

$$\Rightarrow \frac{\mu_{\text{slab}}}{\mu_w} = 1.2$$

$$\Rightarrow \mu_{\text{slab}} = 1.2\mu_w = 1.2\left(\frac{4}{3}\right) = 1.6$$

(b) A black line is formed at the position where both the wavelengths interfere destructively. Distance of nth dark fringe from C is given by

$$y = \frac{(2n-1)\lambda D}{2d}$$

For black line, $\frac{(2n_1 - 1)\lambda_1' D}{2d} = \frac{(2n_2 - 1)\lambda_2' D}{2d}$

where λ_1' and λ_2' are wavelengths in water.

$$\Rightarrow \frac{\lambda_1'}{\lambda_2'} = \frac{\frac{\lambda_1}{\mu_w}}{\frac{\lambda_2}{\mu_w}} = \frac{\lambda_1}{\lambda_2} = \frac{4000}{5600}$$

Substituting these values in equation (1), we get

$$\frac{2n_1 - 1}{2n_2 - 1} = \frac{7}{5}$$

For minimum value $n_1 = 4$ and $n_2 = 3$

Hence, distance of first black line is given by

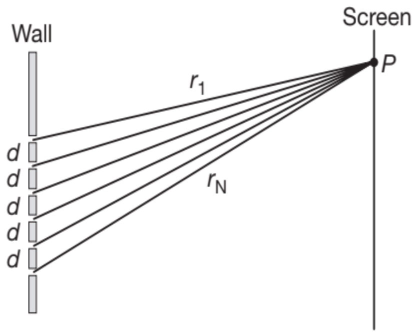
$$y = \frac{(2\times 4 - 1)(4000\times 10^{-10})40\times 10^{-2}\times 3}{2\times 2\times 10^{-3}\times 4}$$

$$\Rightarrow y = 2.1\times 10^{-4} \text{ m}$$

$$\Rightarrow y = 0.21 \text{ mm}$$

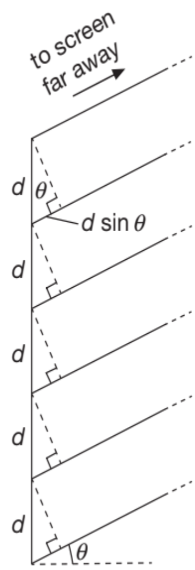
MULTIPLE SLIT INTERFERENCE PATTERN

Let us now look at the case where we have a general number N of equally spaced slits, instead of two equally spaced slits. As an assumption, we have shown in figure a set-up of six equally spaced slits.



Set up for 6 equally spaced slits

Similar to the $N=2$ case discussed already, we will make the far-field assumption that the distance of the sources from the screen is much larger than the total span of the slits, which is $(N-1)d$. We can then say, as we did in the $N=2$ case, that all the paths to a given point P on the screen have approximately the same length in a multiplicative (but not additive) sense, which implies that the amplitudes of the interfering waves are all essentially equal and we can also say that all the paths are essentially parallel (because of far-field assumption). A close-up version near the slits is shown in figure. Also, each path length is $d\sin\theta$ longer than the one just above it. So the lengths take the form of $r_n = r_1 + (n-1)d\sin\theta$.



To find the total wave at a given point at an angle θ on the screen, we need to add up the N individual waves. The procedure is the same as in the $N = 2$ case, except that now we simply have more terms in the sum. If a be the amplitude due to an individual source, then the equations of waves interfering at the point P' are given by

$$y_1 = a\sin(\omega t)$$

$$y_2 = a\sin(\omega t+\phi)$$

$$y_3 = a\sin(\omega t+2\phi)$$

$$y_4 = a\sin(\omega t+3\phi)$$

$$\vdots$$

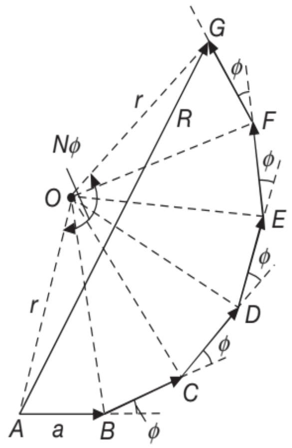
$$y_n = a\sin[\omega t+(N-1)\phi]$$

At angle θ, the path difference between any two successive slits is $\Delta x = d\sin\theta$. So, the corresponding phase difference ϕ is given by

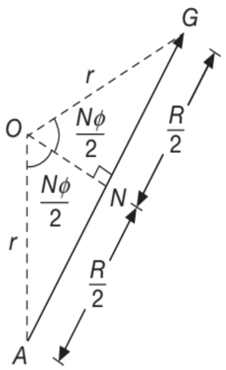
$$\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x = \frac{2\pi}{\lambda}(d\sin\theta)$$

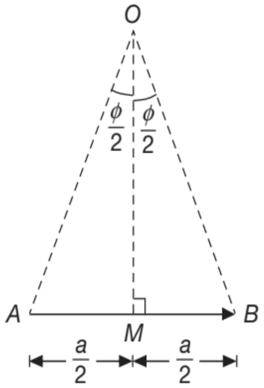
RESULTANT WAVE AMPLITUDE (USING PHASORS)

The above set of equations can be represented by phasor diagram shown in figure (for a set of six sources generalised to N sources).



If R be the amplitude of the resultant of N interfering waves, then from above phasor diagram, on extracting triangles OAG and OAB, we get following figures to be used for evaluation of R.





OAG and OAB are isosceles triangles so for triangle OAG, we have

$$\frac{R}{2} = r\sin\left(\frac{N\phi}{2}\right) \qquad \ldots(1)$$

Triangle OAB, we have

$$\frac{a}{2} = r\sin\left(\frac{\phi}{2}\right) \qquad \ldots(2)$$

Dividing (1) and (2), we get

$$\frac{R}{a} = \frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}$$

So, the resultant amplitude R is given by

$$R = a\left[\frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\right] \qquad \ldots(3)$$

If I_R be the resultant intensity, then

$$I_R = R^2$$

$$\Rightarrow \quad I_R = a^2\left[\frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\right]^2$$

If I_0 be the intensity due to an individual source, then

$$I_R = I_0\left[\frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\right]^2 \qquad \ldots(4)$$

CHECK POINT

For $N = 2$, we get

$$I_R = I_0\left[\frac{\sin\phi}{\sin\left(\frac{\phi}{2}\right)}\right]^2$$

Since $\sin\phi = 2\sin\left(\frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right)$

$$\Rightarrow \quad I_R = 4I_0\cos^2\left(\frac{\phi}{2}\right) = 4a^2\cos^2\left(\frac{\phi}{2}\right)$$

RESULTANT WAVE EQUATION

To find the resultant wave equation, we shall be using the concept of complex numbers. From our knowledge of complex numbers, we know that

$$e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$$

$$\Rightarrow \quad \mathrm{Im}\left(e^{i\omega t}\right) = \sin(\omega t) \text{ and } \mathrm{Re}\left(e^{i\omega t}\right) = \cos(\omega t)$$

Since the resultant wave equation is obtained by adding individual waves, so we get

$$y = a\left[\sin(\omega t) + \sin(\omega t + \phi) + \ldots.. + \sin(\omega t + (N-1)\phi)\right]$$

$$\Rightarrow \quad y = a\,\mathrm{Im}\left[e^{i\omega t} + e^{i(\omega t + \phi)} + \ldots.. + e^{i(\omega t + (N-1)\phi)}\right]$$

$$\Rightarrow \quad y = a\,\mathrm{Im}\left[e^{i\omega t}\left(1 + e^{i\phi} + \ldots.. + e^{i(N-1)\phi}\right)\right]$$

Since, $1 + r + r^2 + \ldots.. + r^{N-1} = 1\left(\frac{1 - r^n}{1 - r}\right)$

$$\Rightarrow \quad y = a\,\mathrm{Im}\left[e^{i\omega t}\left(\frac{1 - e^{iN\phi}}{1 - e^{i\phi}}\right)\right]$$

$$\Rightarrow \quad y = a\,\mathrm{Im}\left[e^{i\omega t}\left(\frac{1 - e^{i\left(\frac{N\phi}{2}\right)}e^{i\left(\frac{N\phi}{2}\right)}}{1 - e^{i\left(\frac{\phi}{2}\right)}e^{i\left(\frac{\phi}{2}\right)}}\right)\right]$$

$$\Rightarrow \quad y = a\,\mathrm{Im}\left[\left(e^{i\omega t}\right)\left(\frac{e^{i\left(\frac{N\phi}{2}\right)}}{e^{i\left(\frac{\phi}{2}\right)}}\right)\left(\frac{e^{i\left(\frac{N\phi}{2}\right)} - e^{-i\left(\frac{N\phi}{2}\right)}}{e^{i\left(\frac{\phi}{2}\right)} - e^{-i\left(\frac{\phi}{2}\right)}}\right)\right]$$

$$\Rightarrow \quad y = a\,\mathrm{Im}\left[e^{i\left(\omega t + (N-1)\frac{\phi}{2}\right)}\left(\frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\right)\right]$$

But $\text{Im}\left[e^{i\left(\omega t+(N-1)\frac{\phi}{2}\right)}\right]=\sin\left(\omega t+(N-1)\frac{\phi}{2}\right)$

$$\Rightarrow \quad y=a\left[\frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\right]\sin\left(\omega t+(N-1)\frac{\phi}{2}\right)$$

Again here, we have directly come across the amplitude of the resultant wave given by

$$R=a\left[\frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\right]$$

$$\Rightarrow \quad I_R=I_0\left[\frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\right]^2, \text{ where } I_0=a^2$$

We observe that at the centre of screen, I_R is indeterminate. So, the maximum intensity at the midpoint of the screen i.e., at $\theta=0°$ is obtained by taking the limit when $\phi\to 0°$. (Please note that since $\phi=\frac{2\pi}{\lambda}(d\sin\theta)$, so $\phi\to 0°$ when $\theta\to 0°$). So,

$$I_{max}=\lim_{\phi\to 0}\left[I_0\frac{\sin^2\left(\frac{N\phi}{2}\right)}{\sin^2\left(\frac{\phi}{2}\right)}\right]$$

For $\phi\to 0$, we have $\sin\phi\approx\phi$

$$\Rightarrow \quad \sin^2\left(\frac{N\phi}{2}\right)\approx\frac{N^2\phi^2}{4} \text{ and } \sin^2\left(\frac{\phi}{2}\right)=\frac{\phi^2}{4}$$

$$\Rightarrow \quad I_{max}=I_0\left[\frac{\left(\frac{N^2\phi^2}{4}\right)}{\left(\frac{\phi^2}{4}\right)}\right]$$

$$\Rightarrow \quad I_{max}=N^2I_0$$

LOCATION OF SECONDARY MINIMA(S)

I_R has zero values, when $\sin^2\left(\frac{N\phi}{2}\right)=0$

$$\Rightarrow \quad \frac{N\phi}{2}=\text{Integral Multiple of } \pi$$

$$\Rightarrow \quad \frac{N\phi}{2}=m\pi, \text{ where } m=0, 1, 2, 3, \ldots$$

$$\Rightarrow \quad \phi=(2m)\frac{\pi}{N}$$

$$\Rightarrow \quad \phi=\text{Even Multiple of } \frac{\pi}{N} \qquad \ldots(5)$$

However, one exception to this is when $\frac{\phi}{2}$ is also an integral multiple of π, because the denominator in equation (4) is also zero.

So, $\frac{\phi}{2}=m'\pi$, where $m'=0, 1, 2, 3, \ldots$

$$\Rightarrow \quad \phi=(2m')\pi$$

$$\Rightarrow \quad \phi=\text{Even multiple of } \pi \qquad \ldots(6)$$

So, from (5) and (6), we conclude that $I_R=0$, when

$$\boxed{\phi=(2m)\frac{\pi}{N} \text{ excluding } \underbrace{\phi=0, 2\pi, 4\pi, 6\pi, \ldots}_{\text{Positions of Primary Maxima}}}$$

i.e., $\phi=(\text{Even Multiple})\frac{\pi}{N}$, excluding Positions of Primary Maxima (located at 0, 2π, 4π, 6π, ...)

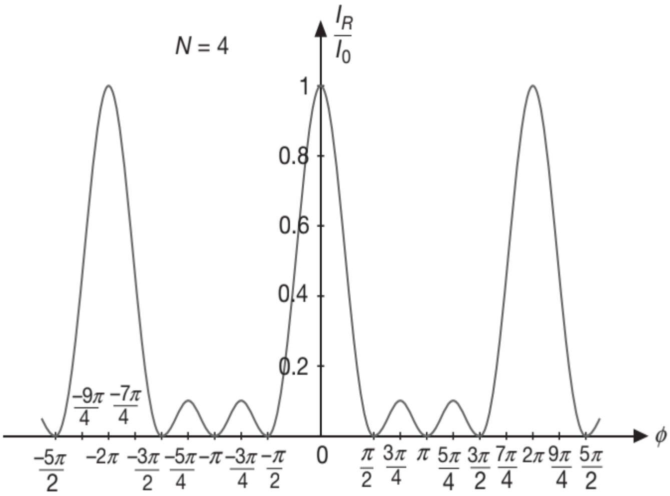
LOCATION OF SECONDARY MAXIMA(S)

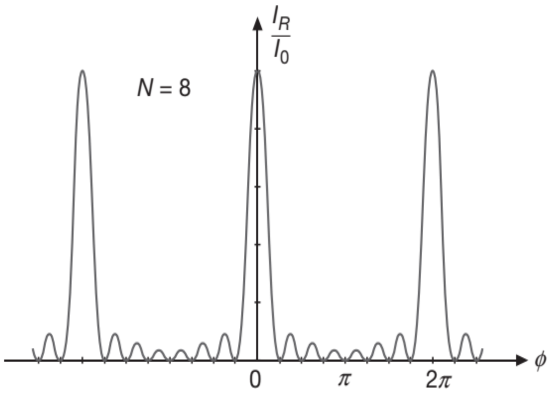
To find the locations of the secondary maxima (i.e., small bumps) we have to find the local maxima of I_R by taking the derivative I_R w.r.t. $\phi\left(\text{i.e., } \frac{dI_R}{d\phi}\right)$ and then equating it to zero.

So, $\frac{dI_R}{d\phi}=0$

$$\Rightarrow \quad N\tan\left(\frac{\phi}{2}\right)=\tan\left(\frac{N\phi}{2}\right)$$

This equation has to be solved numerically. However, for large N, the solutions of ϕ are generally very close to odd multiples of $\frac{\pi}{N}$ excluding the values of $\phi=2\pi\pm\frac{\pi}{N}$, because these values will be lying well within the primary maxima region. Just to make you understand, we are plotting $\frac{I_R}{I_0}$ (with ϕ for $N=4$ and $N=8$).





Conceptual Note(s)

(a) It is customary not to deal with the resultant intensity alone, but rather to deal with the resultant intensity relative to the maximum intensity i.e., $\frac{I_R}{I_{max}}$

$$\Rightarrow \quad \frac{I_R}{I_{max}} = \frac{I_R}{I_0} = \left[\frac{\sin\left(\frac{N\phi}{2}\right)}{N\sin\left(\frac{\phi}{2}\right)}\right]^2 \qquad \text{...(1)}$$

(b) $\lim_{\theta \to 0^\circ}\left(\frac{I_R}{I_{max}}\right) = \lim_{\phi \to 0}\left(\frac{I_R}{I_{max}}\right) = 1$

(c) $\frac{I_R}{I_{max}}$ has a periodicity of 2π in ϕ i.e., repeats itself for integral multiples of 2π.

(d) The number of zeros between the main peaks is $(N - 1)$, where N is the number of slits used.

(e) The number of secondary maxima (little bumps) between the main peaks is $(N - 2)$, where N is the number of slits used.

For $N = 2$ (for two slits), we get ZERO Secondary Maxima.

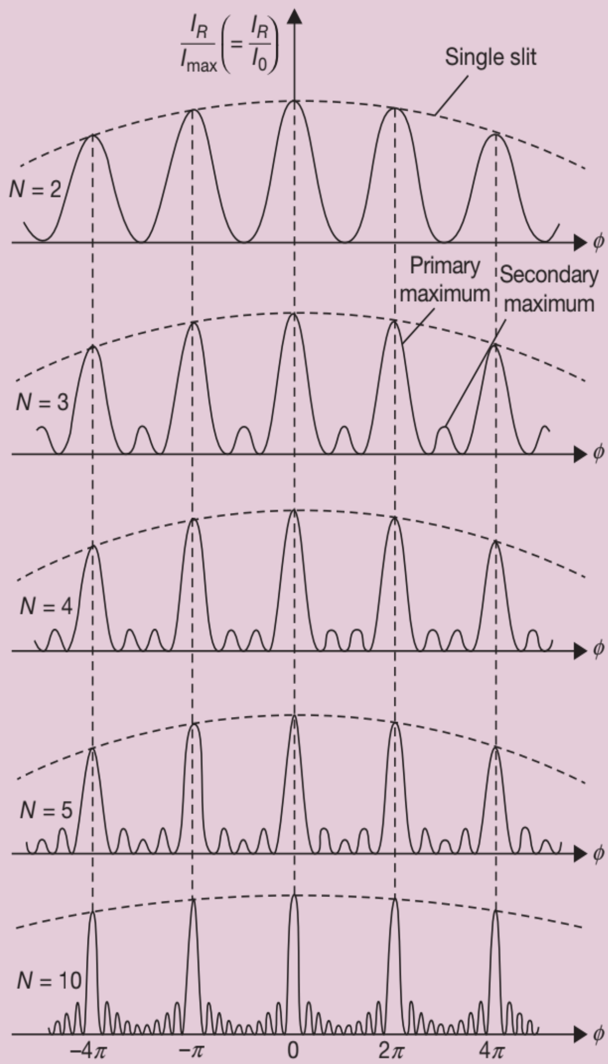
For $N = 3$ (for 3 slits), we get One Secondary Maxima.

For $N = 4$ (for 4 slits), we get Two Secondary Maxima and so on.

(f) A point worth noting here is that the height of the secondary maxima (little bumps) i.e., the bump sizes are symmetric around $\phi = \pi$ (or in general any multiple of π). Also, we know that since

$$I_R = I_0\left[\frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}\right]^2$$

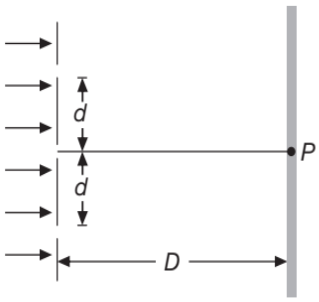
Hence, the bump size is shortest at $\phi = \pi$, because then the denominator in the equation (1), will be having a maximum value at $\phi = \pi$, due to which I_R becomes the least at $\phi = \pi$. Furthermore, the bump size grows as they get closer to the main peaks, as shown for various slits taken.



(g) As N, the number of slits, is increased, the primary maxima (the tallest peaks in each graph) become narrower but remain fixed in position and the number of secondary maxima increases. For any value of N, the decrease in intensity in maxima to the left and right of the central maximum, indicated by the blue dashed arcs, is due to diffraction patterns from the individual slits, which can be neglected here.

ILLUSTRATION 30

A light wave of wavelength 500 nm falls upon three slits a distance 0.5 mm each from one another. A screen is placed at a distance 2 m from slits. Find

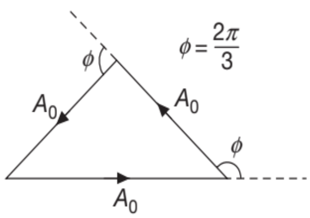


(a) the distances from P where intensity reduces to zero.
(b) the distances from P where next bright fringe are observed.
(c) the ratio of intensities of bright fringes observed on the screen.

SOLUTION

(a) In case of three slits, intensity becomes zero, when phase difference between any two waves is,

$$\phi = 2n\pi + \frac{2\pi}{3}, \text{ where } n = 0, 1, 2, ...$$



The corresponding path difference, $\Delta x = \left(\frac{\lambda}{2\pi}\right)\phi$

$$\Rightarrow d\sin\theta = \left(\frac{\lambda}{2\pi}\right)\left(\frac{2\pi}{3} + 2\pi n\right) = n\lambda + \frac{\lambda}{3}$$

$$\Rightarrow \sin\theta = \frac{\lambda}{d}\left(n + \frac{1}{3}\right)$$

For small angles, $\sin\theta \approx \tan\theta = \frac{y}{D}$

$$\Rightarrow \frac{y}{D} = \frac{\lambda}{d}\left(n + \frac{1}{3}\right)$$

$$\Rightarrow y = \frac{\lambda D}{d}\left(n + \frac{1}{3}\right)$$

Substituting the values, we have

$$y = \frac{500 \times 10^{-9} \times 2}{0.5 \times 10^{-3}}\left(n + \frac{1}{3}\right) = 2 \times 10^{-3}\left(n + \frac{1}{3}\right) \text{ m}$$

$$\Rightarrow y = 2\left(n + \frac{1}{3}\right) \text{ mm, where } n = 0, 1, 2, ...$$

$$\Rightarrow y = \frac{2}{3} \text{ mm for } n = 0,\ y = \frac{8}{3} \text{ mm for } n = 1 \text{ etc.}$$

(b) Bright fringes are obtained on the screen where

(i) $\phi = 2n\pi$, $n = 1, 2, 3, ...$

$$\Delta x = \left(\frac{\lambda}{2\pi}\right)(\phi) = n\lambda$$

$$\Rightarrow d\sin\theta = n\lambda$$

$$\Rightarrow \sin\theta = \frac{n\lambda}{d}$$

For small angles,

$$\sin\theta \approx \tan\theta = \frac{y}{D}$$

$$\Rightarrow \frac{y}{D} = \frac{n\lambda}{d}$$

$$\Rightarrow y = \frac{n\lambda D}{d} = \frac{n(500 \times 10^{-9})(2)}{(0.5 \times 10^{-3})}$$

$$\Rightarrow y = 2n \times 10^{-3} \text{ m}$$

$$\Rightarrow y = (2n) \text{ mm } \{\text{where } n = 1, 2, 3, ... \text{ etc.}\}$$

There are called **primary** maximas.

(ii) $\phi = (2n+1)\pi$, $n = 1, 2, ...$

Proceeding in the similar manner, we get

$$y = 2\left(n + \frac{1}{2}\right) \text{ mm} \quad \{\text{where } n = 1, 2, 3, ...\}$$

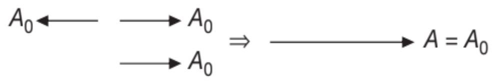
These are called **secondary** maximas.
Note that $y = 0$ is also a secondary maxima because at P, $\phi = \pi$.

(c) At principal maximas, we have $\phi = 2\pi, 4\pi, ...,$ etc.

Resultant amplitude $R = 3A_0$

$\Rightarrow \; I_R = 9I_0 \qquad \{\because I \propto A^2\}$

While at secondary maximas, $(\phi = \pi, \; 3\pi, \; 5\pi.....)$



Resultant amplitude, $R' = A_0$

$\Rightarrow \; I'_R = I_0$

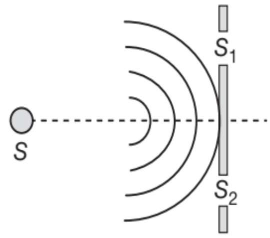
So, the desired ratio is therefore, $9:1$

COHERENT SOURCES BY DIVISION OF WAVEFRONT

When two or more waves travel through a medium simultaneously, the resultant intensity at any point, in the medium depends on whether they interfere constructively or destructively which, in turn, depends upon the phase difference between them. Resultant intensity, at any point, remains constant with time if the phase difference between them does not change. Two independent sources can never have same phase or a constant phase difference, because if we try to have interference with two independent sources, then net intensity at any point undergoes a continuous change due to a change in the phase difference between them. As a result of this no fixed interference pattern can be observed. The interference pattern of such sources is so short-lived that its photograph with the fastest available camera cannot be obtained. To obtain a fixed interference pattern we must have two sources which either have no phase difference or have a constant difference of phase. These sources are called **coherent sources**. It has been generally observed that coherent sources are obtained when they are derived from the same parent source. The methods for obtaining coherent sources (derived from the same parent source) are given below.

Double Slit Method

Light from a source S is limited to a narrow beam with the help of a slit. The emergent light is made to fall upon a screen containing two slits S_1 and S_2 placed symmetrically with respect to the slit.

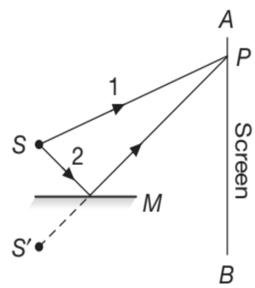


Coherent sources by double slit method

Here, both S_1 and S_2 are illuminated by the same wavefront. Therefore, the beams of light coming out from S_1 and S_2 have no phase difference. Thus S_1 and S_2 can be treated as the coherent sources. Young used this technique in his famous Young's double slit experiment.

A Source and its Own Virtual Image

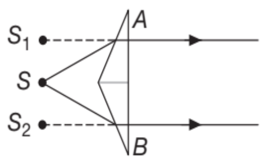
Light from a source S is made to fall on a plane mirror M. Point of observation P on a screen AB receives direct light as well as light reflected from M.



To an observer, reflected light appears to come from a source S' (virtual image of S). So, interference at P takes place between waves coming from S and S'. Since S' is not an independent source, being the virtual image of S, it will have the same phase as S. Hence the two are taken to be coherent sources. Lloyd made use of this arrangement in Lloyd single mirror experiment.

Biprism Method

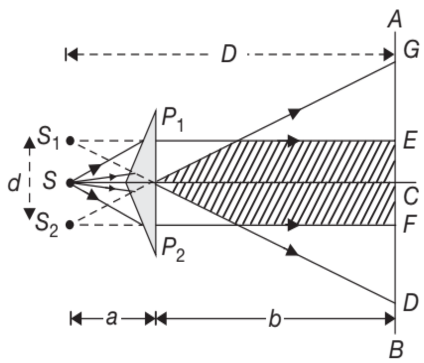
Light from a source S is made to fall on an assembly of two right angled prisms A and B joined base to base as shown in Figure.



S_1 and S_2 are the virtual image of S produced by refraction through prisms A and B respectively. Being virtual images of the same source, S_1 and S_2 have same phase and hence can be treated as the coherent sources. This type of arrangement is made use of in Fresnel's biprism experiment.

FRESNEL'S BIPRISM

It is one of the convenient laboratory arrangements for producing interference fringes. It consists of a combination of two right angled prisms with their bases joined together so that their faces are inclined to each other at angle of $179° 20'$. Source of light is taken in the form of a narrow slit S, illuminated by the monochromatic light and is held symmetrically at a distance of about 5 cm from the biprism.



Light from S gets refracted by prism P_1 and P_2, thereby, producing virtual images S_1 and S_2, which can be taken as two coherent sources producing interference. Light beams from S_1 and S_2 strike the screen in the regions ED and FG respectively. EF is the common region where both the beams can be found. Therefore, interference pattern can be observed in the region EF.

The separation between these sources may be found by using the formula for deviation caused by a thin prism. If α is the small angle of biprism, μ refractive index of material of biprism and a the separation of source S from biprism, then deviation caused by prism.

$$\delta = (\mu - 1)\alpha$$

From Figure, $d = 2a\delta = 2a(\mu - 1)\alpha$

DETERMINATION OF λ

Biprism method can be used to determine the wave length of light. The fringe width β for the interference pattern obtained is given by,

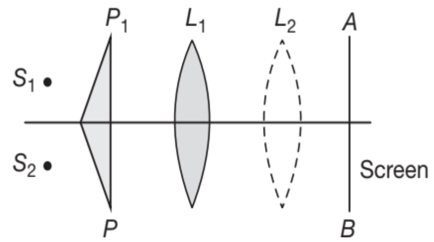
$$\beta = \frac{\lambda D}{d}$$

$$\Rightarrow \quad \lambda = \frac{\beta d}{D} \quad \ldots(1)$$

(a) Determination of D: It is the distance between source and screen. It can be measured with an ordinary metre rod.

(b) Determination of β: A low power travelling microscope is used to find the total separation x between a number of fringes, say 20 and hence $\beta = \frac{x}{20}$.

(c) Determination of d: d can be calculated by using displacement method. A convex lens is placed in between the biprism and the screen.



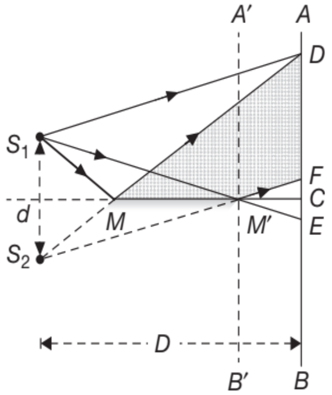
It is observed that for two positions L_1 and L_2 of the lens, the images of S_1 and S_2 can be focussed on the screen AB. Let x and y be the distances between these images when the lens is at L_1 and L_2 respectively. Then,

$$d = \sqrt{xy}$$

Substituting for β, D and d in equation (1), λ can be calculated.

LLOYD'S SINGLE MIRROR

This experimental set-up for producing interference fringes, was devised by Dr. Lloyd in 1834. Light from a source S_1 in the form of narrow slit is held in such a way that the light is incident, at almost grazing incidence, upon a mirror MM' which is blackened at the back to avoid internal reflections. S_2 is the virtual image of source S_1 obtained after reflection from MM'.



Experimental set up for Lloyd's single mirror

Screen AB is placed to receive light coming directly from S_1 as well as that reflected from the mirror. Reflected light can be supposed to be coming from source S_2. DF is the common region on the screen where both the beams are received and hence interference is obtained in region DF.

The point C lies symmetrically w.r.t. S_1 and S_2 and also lies outside the interference region, zero order fringe is not visible. It can be seen by moving the screen to position $A'B'$ so that it just touches the mirror. It will be observed that the zero order fringe at M' is dark instead of being bright as demanded by the theory of interference fringes since at M' path difference is zero. This indicates that the beam which suffers reflection from MM' undergoes in phase of π-radian. Llyod's single mirror can be used to determine the wave length of light.

If, a is the height of source S_1 above MM', then

$$d = 2a$$

If, D is the distance of source S_1 from screen AB, then fringe width β is given by

$$\beta = \frac{\lambda D}{d}$$

$$\Rightarrow \quad \lambda = \frac{\beta d}{D} = \frac{\beta(2a)}{D}$$

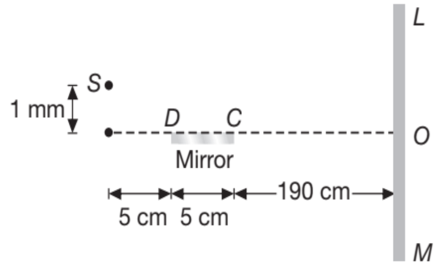
β can be determined, experimentally, by using a low power microscope. Knowing β, a and D value of λ can be calculated.

Conceptual Note(s)

Central spot, in case of Lloyd's single mirror is a dark one instead of being bright. This proves that there is a phase change of π-radian when a transverse wave (light) is reflected from a denser medium.

ILLUSTRATION 31

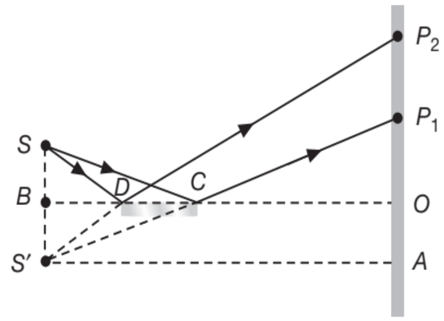
The arrangement for a mirror experiment is shown in the figure. S is a point source of frequency 6×10^{14} Hz. D and C represent the two ends of a mirror placed horizontally and LOM represents the screen.



Determine the position of the region where the fringes will be visible and calculate the number of fringes.

SOLUTION

Fringes will be observed in the region between P_1 and P_2 because the reflected rays lie only in this region.



From similar triangles BDS' and $S'P_2A$, $\frac{AP_2}{BS'} = \frac{AS'}{BD}$

$$\Rightarrow \quad AP_2 = \frac{(AS')(BS')}{BD} = \frac{(190+5+5)(0.1)}{5} = 4 \text{ cm}$$

Similarly, in triangles BCS' and $S'P_1A$, we have

$$\frac{AP_1}{BS'} = \frac{AS'}{BC}$$

$$\Rightarrow \quad AP_1 = \frac{(AS')(BS')}{BC} = \frac{(190+5+5)(0.1)}{10} = 2 \text{ cm}$$

$$\Rightarrow \quad P_1P_2 = AP_2 - AP_1 = 2 \text{ cm}$$

Wavelength of the light $\lambda = \frac{c}{f} = \frac{3\times10^8}{6\times10^{14}} = 5\times10^{-7}$ m

Fringe width $\beta = \frac{\lambda D}{d}$

Since, $D = S'A = (190+5+5) = 200 \text{ cm} = 2 \text{ cm}$,

$$d = SS' = 2 \text{ mm} = 2\times10^{-3} \text{ m}$$

$$\Rightarrow \quad \beta = \frac{(5\times10^{-7})(2)}{2\times10^{-3}} = 5\times10^{-4}\ \text{m} = 0.05\ \text{cm}$$

Number of fringes is

$$N = \frac{P_1P_2}{\beta} = 40$$

ILLUSTRATION 32

A narrow slit S is transmitting light of wavelength λ and it is placed at a distance d above a large plane mirror as shown in figure.

The light coming directly from the slit and that coming after reflection interfere at a screen placed at a distance $D(D \gg d)$ from the slit.

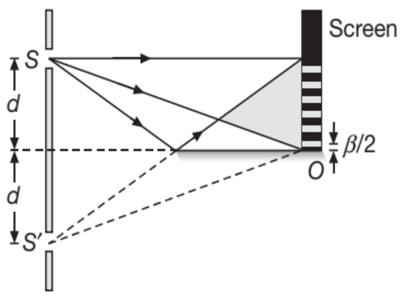
(a) What will be the intensity at a point just above the mirror at point O?

(b) At what distance from O does the first maximum will occur?

SOLUTION

(a) Just above the point O, direct waves from source and reflected waves have a phase difference π (due to reflection from mirror). So these waves interfere destructively due to which a dark fringe is obtained at O. Hence intensity of light at O will be zero.

(b) From the figure, we can see that the distance of first maximum (first bright fringe) will be located at half fringe width above O.

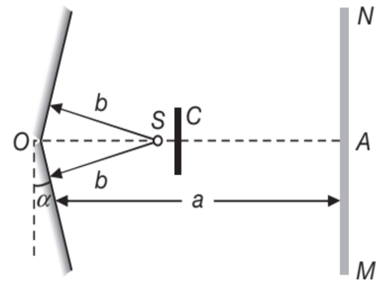


So we have $y = \dfrac{\beta}{2} = \left(\dfrac{\frac{D\lambda}{2d}}{2}\right)$

$$\Rightarrow \quad y = \frac{D\lambda}{4d}$$

ILLUSTRATION 33

Two flat mirrors form an angle close to $180°$. A source of light S is placed at equal distances b from the mirrors. Find the interval between adjacent interference bands on screen MN at a distance $OA = a$ from the point of intersection of the mirror. The wavelength of the light wave is known and equal to λ. Shield C does not allow the light to pass directly from the source to the screen.

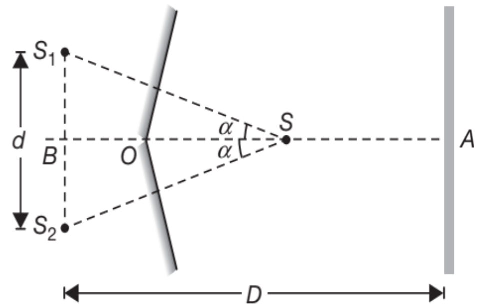


SOLUTION

Fringe width is given by

$$\beta = \frac{\lambda D}{d}$$

where $D = AB \approx a + b$ and $d = S_1S_2$



In ΔS_1SB, we have

$$\frac{d}{2} = 2b\frac{\alpha}{2}$$

$$\Rightarrow \quad d = 2b\alpha$$

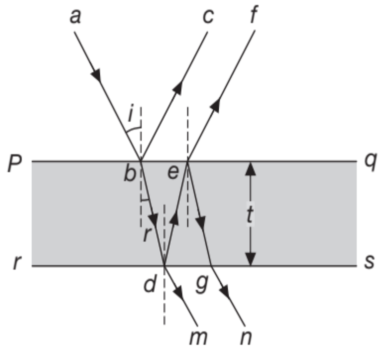
$$\Rightarrow \quad \beta = \frac{\lambda(a+b)}{2b\alpha}$$

THEORY OF DIVISION OF AMPLITUDE

Reflected Light

If μ is the refractive index of material of film of thickness t, then path difference between the waves abc and $abdef$ is $2\mu t \cos r$

Additional path difference due to reflection at denser medium $(\text{at } b)$ is $\dfrac{\lambda}{2}$



So, effective path difference is

$$x = 2\mu t\cos r + \frac{\lambda}{2}$$

For maxima or constructive interference to take place, we have

$$2\mu t\cos r + \frac{\lambda}{2} = (2n)\frac{\lambda}{2}$$

$$\Rightarrow \quad 2\mu t\cos r = (2n-1)\frac{\lambda}{2},\ n = 1, 2, 3, \ldots$$

For minima or destructive interference to take place

$$2\mu t\cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow \quad 2\mu t\cos r = n\lambda,\ n = 1, 2, 3, \ldots$$

Transmitted Light

In transmitted light system there is no phase difference or path difference due to reflection or transmission as all reflections take place from rarer medium. So, the effective path difference is

$$x = 2\mu t\cos r$$

For maxima

$$2\mu t\cos r = n\lambda$$

and **for minima**

$$2\mu t\cos r = (2n-1)\frac{\lambda}{2},\ n = 1, 2, 3, \ldots$$

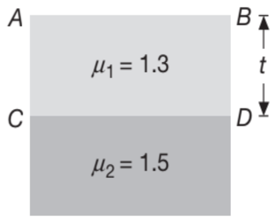
Obviously, the conditions of interference in reflected and transmitted lights are opposite to each other, therefore if the film appears dark in reflected light, it will appear bright in transmitted light and vice versa. With the use of white light, the colours visible in reflected light will be complementary to those visible in transmitted light, i.e., the colours absent in one system will be present in the other system; the sum of two constituting the white light.

ILLUSTRATION 34

A thick glass slab $(\mu = 1.5)$ is to be viewed in reflected white light. It is proposed to coat the slab with a thin layer of a material having refractive index 1.3 so that the wavelength 6000 Å is suppressed. Find the minimum thickness of the coating required.

SOLUTION

Optical path difference for the reflected light from coating and slab is $\Delta x = 2\mu t$



For minimum intensity, $2\mu_1 t = \dfrac{\lambda}{2}$

$$\Rightarrow \quad t = \frac{\lambda}{4\mu_1} = \frac{6000}{4\times 1.3}$$

$$\Rightarrow \quad t = 1154\ \text{Å}$$

Conceptual Note(s)

Both reflected rays (one from *AB* and the another from *CD*) get a phase change of π.

ILLUSTRATION 35

A parallel beam of white light falls on a thin film whose refractive index is equal to $\dfrac{4}{3}$. The angle of incidence $i = 53°$. What must be the minimum film thickness if the reflected light is to be coloured yellow (λ of yellow $= 0.6\ \mu m$) most intensively? Given $\tan 53° = \dfrac{4}{3}$.

SOLUTION

According to Snell's Law, we have $\mu = \frac{\sin i}{\sin r}$

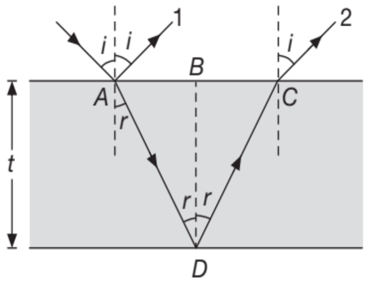
Given that $\tan 53^\circ = \frac{4}{3}$, so $\sin 53^\circ = \frac{4}{5}$

$\Rightarrow \quad \frac{4}{3} = \frac{\sin(53^\circ)}{\sin r} = \frac{\frac{4}{5}}{\sin r}$

$\Rightarrow \quad \sin r = \frac{3}{5}$

$\Rightarrow \quad r = 37^\circ$

From Figure (a):



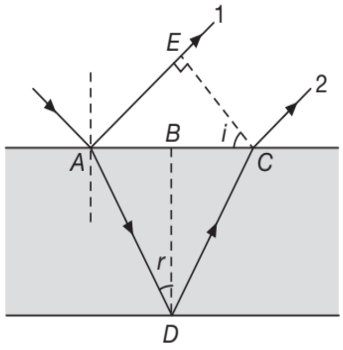
(a)

Path difference between 2 and 1 is $\Delta x_1 = 2(AD)$

$\Rightarrow \quad \Delta x_1 = 2BD\sec r = 2t\sec r$

Their optical path corresponding to Δx_1 is $2\mu t\sec r$

From Figure (b):



(b)

Path difference between 1 and 2 is given by

$\Delta x_2 = AC\sin i = (2t\tan r)\sin i$

$\Rightarrow \quad (\Delta x)_{net} = \Delta x_1 - \Delta x_2 = 2\mu t\sec r - 2t(\tan r)(\sin i)$

$\Rightarrow \quad \Delta x_{net} = 2\times\frac{4}{3}\times t\times\frac{5}{4} - 2\times t\times\frac{3}{4}\times\frac{4}{5}$

$\Rightarrow \quad \Delta x_{net} = \frac{32}{15}t$

Since reflection takes place at the surface of denser medium, so phase difference between 1 and 2 is π. So, for constructive interference, we have

$\frac{32}{15}t = \frac{\lambda}{2}$

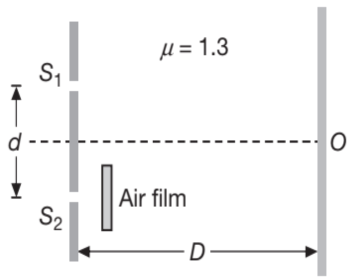
$\Rightarrow \quad t = \frac{15\lambda}{64} = \frac{15\times 0.6}{64} = 0.14\ \mu\text{m}$

Test Your Concepts-I

Based on Interference

(Solutions on page H.121)

1. In a Young's Double Slit Experiment carried out in a liquid of refractive index $\mu = 1.3$, a thin film of air is formed in front of the lower slit as shown in the figure. If a maxima of third order is formed at the origin O, find the
   (a) thickness of the air film.
   (b) positions of the fourth maxima.
   The wavelength of light in air is $\lambda_0 = 0.78\ \mu$m and $\frac{D}{d} = 1000$.



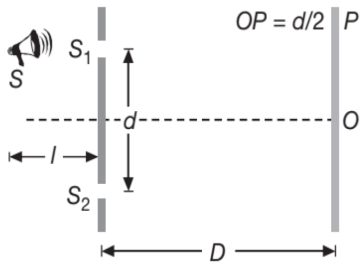
2. In *YDSE*, if light of wavelength 5000 Å is used, find the thickness of a glass slab ($\mu = 1.5$) which should be placed before the upper the upper slit S_1 so that

the central maximum now lies at a point where 5^{th} bright fringe was lying earlier (before inserting the slab).

3. A source S of wavelength λ is kept directly behind the slit S_1 in a double slit apparatus. Find the phase difference at a point O which is equidistant from S_1 and S_2. If $D \gg d$, what will be the phase difference at P if a liquid of refractive index μ is filled

(a) between the screen and the slits?

(b) between the slits and the source S?



4. In solar cells, a silicon solar cell (μ = 3.5) is coated with a thin film of silicon monoxide $SiO(\mu = 1.45)$ to minimize reflective losses from the surface. Determine the minimum thickness of SiO that produces the least reflection at a wavelength of 550 nm, near the centre of the visible spectrum.

5. A parallel beam of green light of wavelength 546 nm passes through a slit of width 0.4 mm. The transmitted light is collected on a screen 40 cm away. Find the distance between the two first order minima.

6. Calculate the minimum thickness of a soap bubble film (μ = 1.33) that results in constructive interference in the reflected light if the film is illuminated with light whose wavelength in free space is $\lambda = 600$ nm.

7. Monochromatic light of wavelength 5000 Å is used in *YDSE*, with slit separation 1 mm, distance between screen and slits 1 m. If intensity at the two slits are, $I_1 = 4I_0$, $I_2 = I_0$, find

(a) fringe width β.

(b) distance of 5^{th} minima from the central maxima on the screen.

(c) intensity at $y = \frac{1}{3}$ mm.

(d) distance of the 1000^{th} maxima.

(e) distance of the 5000^{th} maxima.

8. S_1 and S_2 are two point sources of radiation that are radiating waves in phase with each other of wavelength 400 nm. The sources are located on x-axis at $x = 6.5$ μm and $x = -6$ μm, respectively.

(a) Determine the phase difference (in radian) at the origin between the radiation from S_1 and the radiation from S_2.

(b) Suppose a slab of transparent material with thickness 1.5 μm and index of refraction $\mu = 1.5$ is placed between $x = 0$ and $x = 1.5$ μm. What then is the phase difference (in radian) at the origin between the radiation from S_1 and the radiation from S_2?

9. In a Young's double slit experiment using monochromatic light the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 1.964 micron is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of the monochromatic light used in the experiment.

10. In Young's experiment a thin glass plate is placed in the path of one of the interfering rays. This causes the central light band to shift into a position which was initially occupied by the fifth bright band (not counting the central one). The ray falls onto the plate perpendicularly. The refractive index of the plate is 1.5. The wavelength is 6×10^{-7} m. What is the thickness of the plate?

11. In a double slit pattern ($\lambda = 6000$ Å), the first order and tenth order maxima fall at 12.50 mm and 14.75 mm from a particular reference point. If λ is changed to 5500 Å, find the position of zero order and tenth order fringes, other arrangements remaining the same.

12. In *YDSE*, the two slits are separated by 0.1 mm and they are 0.5 m from the screen. The wavelength of light used is 5000 Å. Find the distance between 7^{th} maxima and 11^{th} minima on the screen.

13. What is the effect on the interference fringes in a YDSE due to each of the following operations?

(a) The screen is moved away from the plane of the slits

(b) The (monochromatic) source is replaced by another (monochromatic) source of shorter wavelength

(c) The separation between the two slits is increased

(d) The monochromatic source is replaced by source of white light

(e) The whole experiment is carried out in a medium of refractive index μ

14. In a Young's double slit experiment, the slits are 2 mm apart and are illuminated with a mixture of two wavelengths $\lambda_1 = 750$ nm and $\lambda_2 = 900$ nm. At what minimum distance from the common central bright fringe on a screen 2 m from the slits will a bright fringe from one interference pattern coincide with a bright fringe from the other?

15. Bi-chromatic light is used in *YDSE* having wavelengths $\lambda_1 = 400$ nm and $\lambda = 700$ nm. Find minimum order of λ_1 which overlaps with λ_2.

16. In Young's double slit experiment set-up with light of wavelength $\lambda = 6000$ Å, distance between the two slits is 2 mm and distance between the plane of slits and the screen is 2 m. The slits are of equal intensity. When a sheet of glass of refractive index 1.5 (which permits only a fraction η of the incident light to pass through) and thickness 8000 Å is placed in front of the lower slit, it is observed that the intensity at a point *P*, 0.15 mm above the central maxima does not change. Find the value of η.

17. In a Young's double slit experiment set up, the wavelength of light used is 546 nm. The distance of screen from slits is 1 metre. The slit separation is 0.3 mm.

(a) Compare the intensity at a point *P* distant 10 mm from the central fringe where the intensity is I_0.

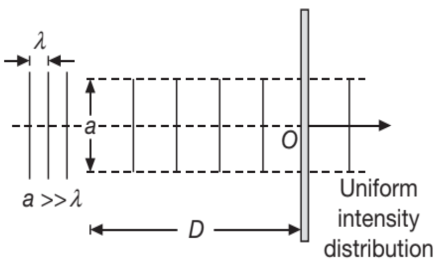
(b) Find the number of bright fringes between *P* and the central fringe.

18. In a Young's double slit experiment, two wavelengths of 500 nm and 700 nm were used. What is the minimum distance from the central maximum where their maximas coincide again? Take $\dfrac{D}{d} = 10^3$. Symbols have their usual meanings.

19. When a thin sheet of a transparent material of thickness 7.2×10^{-4} cm is introduced in the path of one of the interfering beams, the central fringe shift to a position occupied by the sixth bright fringe. If $\lambda = 6 \times 10^{-5}$ cm, find the refractive index of the sheet.

DIFFRACTION: INTRODUCTION AND CLASSIFICATION

When light waves pass through a small aperture, an interference pattern is observed rather than a sharp spot of light cast by the aperture. This shows that light spreads in various directions beyond the aperture into regions where a shadow would be expected if light travelled in straight lines.

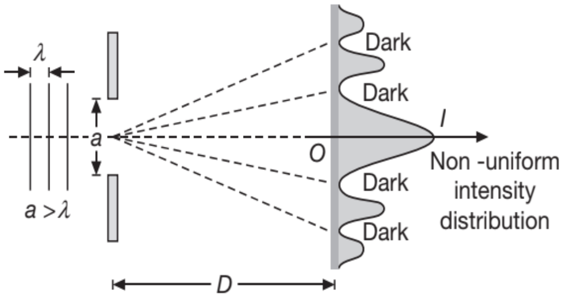


Light passing through two slits does not produce two distinct bright areas on a screen. Instead, an interference pattern is observed on the screen which shows that the light has deviated from a straight-line path and has entered the otherwise shadowed region. Other waves, such as sound waves and water waves, also have this property of being able to bend around corners.

This deviation of light from a straight-line path is called **diffraction**. Diffraction results from the interference of light from many coherent sources. In principle, the intensity of a diffraction pattern at a given point in space can be computed using Huygens' principle, where each point on the wavefront acts as the source emitting waves as the original source does.

*The phenomenon of bending of light around the corners of an obstacle/aperture of the size of the wave length of light is called **diffraction**.*

The phenomenon resulting from the superposition of secondary wavelets originating from different parts of the same wave front is define as diffraction of light. Diffraction is the characteristic of all types of waves. Greater the wave length of wave higher will be it's degree of diffraction.



Conceptual Note(s)

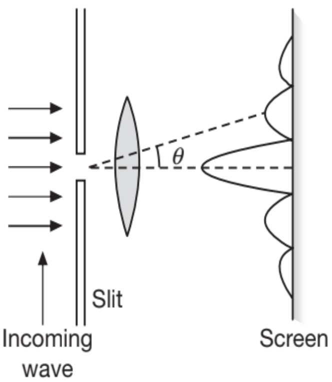
Diffraction, can be regarded as a consequence of interference from many coherent wave sources. In other words, the phenomena of diffraction and interference are basically equivalent.

TYPES OF DIFFRACTION

Diffraction phenomena are usually classified as being one of two types, which are named after the men who first explained them. The first type is called **Fraunhofer Diffraction** and the second is called **Fresnel's Diffraction**.

Fraunhofer Diffraction

This occurs when the rays reaching a point are approximately parallel i.e. when both the source and screen are effectively at infinite distance from the diffracting device. In this case, the incident light is a plane wave so that the phase of the light at each point in the aperture is the same. This can be achieved experimentally either by placing the observing screen at a large distance from the aperture or by using a converging lens to focus parallel rays on the screen, as in Figure.



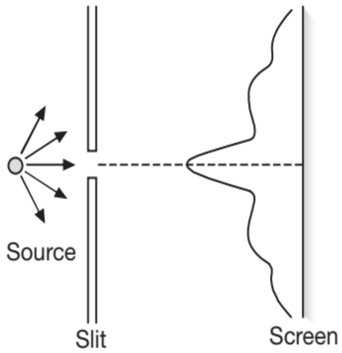
Note that a bright fringe is observed along the axis at $\theta = 0$, with alternating bright and dark fringes on either side of the central bright fringe.

Common examples: Diffraction at single slit, double slit and diffraction grating.

Fresnel's Diffraction

When the observing screen is placed at a finite distance from the slit and no lens is used to focus parallel rays, the observed pattern is called a **Fresnel Diffraction Pattern**. Fresnel diffraction is rather complex to treat quantitatively.

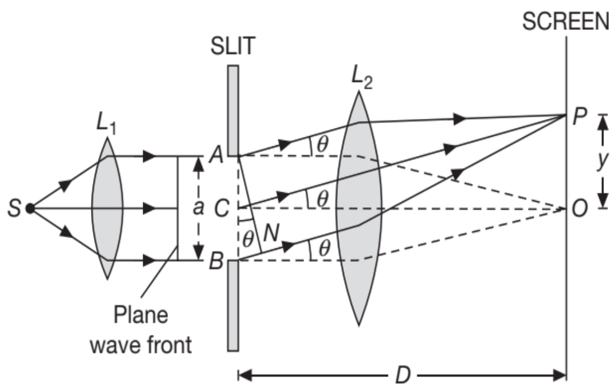
Common examples: Diffraction at a straight edge, narrow wire or small opaque disc etc.



A fresnel diffraction pattern of a single slit is observed when the incident rays are not parallel and the observing screen is at a finite distance from the slit.

FRAUNHOFER DIFFRACTION AT A SINGLE SLIT

Consider that a monochromatic source of light S, emitting light waves of wavelength λ, is placed at the principal focus of the convex lens L_1. A parallel beam of light i.e., a plane wavefront gets incident on a narrow slit AB of width a as shown in figure.



The diffraction pattern is obtained on a screen lying at a distance D from the slit and at the focal plane of the convex lens L_2.

According to rectilinear propagation of light, a bright image of the slit is expected at the centre O of the screen. But in practice, we get a diffraction pattern i.e., a central maximum at the centre O flanked by a number of dark and bright fringes called secondary maxima and minima on either side of the point O.

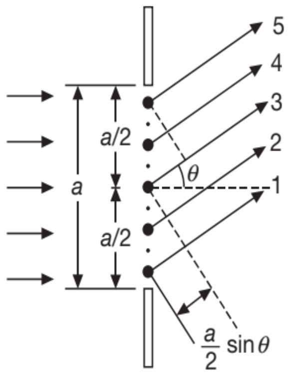
The diffraction pattern is obtained on the screen, which lies at the focal plane of the convex lens L_2. It is found that

(i) the width of the central maximum is twice as that of a secondary maximum and

(ii) the intensity of the secondary maxima goes on decreasing with the order of maxima. These observations are explained on the basis of the phenomenon of diffraction using the following mathematical treatment.

EXPLANATION AND MATHEMATICAL TREATMENT

Consider Fraunhofer diffraction by a single slit as shown in Figure. Important features of this problem can be deduced by examining waves coming from various portions of the slit. According to Huygens' principle, each portion of the slit acts as a source of waves. Hence, light from one portion of the slit can interfere with light from another portion, and the resultant intensity on the screen will depend on the direction θ.



Diffraction of light by a narrow slit of width a. Each portion of the slit acts as a point source of waves. The path difference between rays 1 and 3 or between rays 2 and 4 is equal to $(a/2)\sin\theta$

To analyze the resultant diffraction pattern, it is convenient to divide the slit in two halves as in Figure. All the waves that originate from the slit are initially in phase. Consider waves 1 and 3, which originate from the bottom and center of the slit, respectively. Wave 1 travels farther than wave 3 by an amount equal to the path difference $\left(\frac{a}{2}\right)\sin\theta$, where a is the width of the slit. Similarly, the path difference between waves 2 and 4 is also equal to $\left(\frac{a}{2}\right)\sin\theta$.

If this path difference is exactly one half of a wavelength (corresponding to a phase difference of $180°$), the two waves cancel each other and destructive interference results. This is true, in fact, for any two waves that originate at points separated by half the slit width, since the phase difference between two such points is $180°$. Therefore, waves from the upper half of the slit interfere destructively with waves from the lower half of the slit. when

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

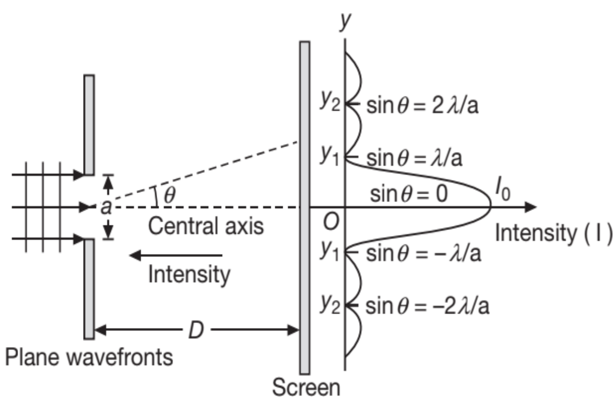
$$\Rightarrow \quad \sin\theta = \frac{\lambda}{a}$$

Similarly, destructive interference (minima) occurs when the path difference $\left(\frac{a}{2}\right)\sin\theta$ equals $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$, etc. These points occur at progressively larger values of θ. Therefore, the general condition for destructive interference is

$$\sin\theta = n\frac{\lambda}{a} \qquad (n = \pm 1, \pm 2, \pm 3, \ldots) \qquad \ldots(1)$$

where $|n| \le \frac{a}{\lambda}$ $\qquad \{\because |\sin\theta| \le 1\}$

Equation (1) gives the values of θ for which the diffraction pattern has zero intensity. However, it tells us nothing about the variation in intensity along the screen. The general features of the intensity distribution along the screen are shown in Figure.



Position of the various minima for the Fraunhofer diffraction pattern of a single slit of width a.

A broad central bright fringe is observed, flanked by much weaker alternating maxima. The central bright fringe corresponds to those points opposite the slit for which the path difference is zero, or $\theta = 0$. All waves originating from the slit reach this region in phase, hence constructive interference results. The various dark fringes (points of zero intensity) occur at the values of θ that satisfy equation (1). The positions of the weaker maxima lie approximately halfway between the dark fringes. Note that the central bright fringe is twice as wide as the weaker maxima (which are narrower).

Angular width of central maxima is $\frac{2\lambda}{a}$ and width of central maxima is $\frac{2\lambda D}{a}$, where D is the distance of the screen from the slit.

The intensity distribution of the **diffraction pattern** is quite different from the interference pattern produced due to superposition of light from two coherent sources. The point O on the central axis is the brightest. The **angular position** (θ) of n^{th} diffraction **minima** is given by

$$a\sin\theta = n\lambda \qquad n = 1, 2, 3, 4, \ldots\ldots$$

For secondary maxima, we have

$$a\sin\theta = (2n+1)\frac{\lambda}{2}, \text{ where } n = 1, 2, 3, 4, \ldots\ldots$$

$$\Rightarrow \quad \theta = (2n+1)\frac{\lambda}{2a};\ n = 1, 2, 3, 4, \ldots\ldots$$

$$\Rightarrow \quad y = (2n+1)\frac{\lambda D}{2a},\ n = 1, 2, 3, 4, \ldots\ldots$$

i.e., angular position of secondary maxima is

$$\frac{3\lambda}{2a}, \frac{5\lambda}{2a}, \frac{7\lambda}{2a}, \ldots\ldots$$

Position of secondary maxima is

$$\frac{3\lambda D}{2a}, \frac{5\lambda D}{2a}, \frac{7\lambda D}{2a}, \ldots\ldots$$

For small angle θ, we have $\sin\theta \approx \theta$. Thus, as shown in the figure, the angular position of the $1^{st}, 2^{nd}, 3^{rd}, \ldots$ minima are $\frac{\lambda}{a}, \frac{2\lambda}{a}, \frac{3\lambda}{a}, \ldots..$ respectively on either side of the central axis. A maximum is approximately halfway between two adjacent minima.

Note that as the slit width a increases, the width of the central diffraction maximum decreases. That is, there is less spreading out of the light by the slit. The secondary maxima also decreases in width and becomes weaker. When a becomes much greater than λ, the secondary maxima disappear.

The **intensity** I of the diffraction pattern as a function of θ is given as

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$$

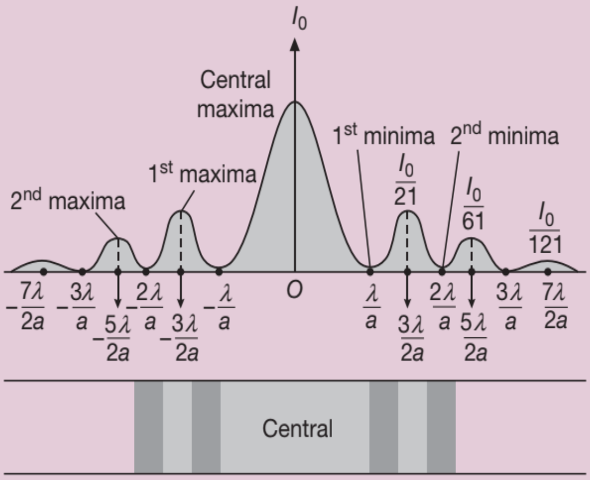
where $\alpha = \frac{\pi a \sin\theta}{\lambda}$

Since $\sin\theta \approx \frac{y}{D}$, so we get $\alpha \approx \frac{\pi}{\lambda}\left(\frac{ya}{D}\right)$

The intensity of secondary maxima is much less. Compared to the intensity of central maximum (I_0), the intensity of the first of the secondary maxima is only 4.5%, of the second is only 1.6%, of the third is merely 0.83%...... The successive secondary maxima decrease rapidly in intensity.

Conceptual Note(s)

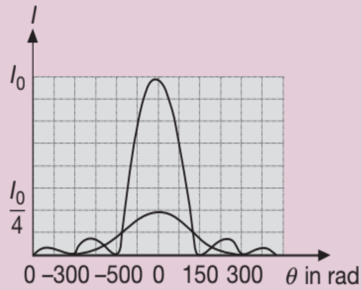
(a) If the intensity of the central maxima is I_0 then the intensity of the first and second secondary maxima are found to be $\frac{I_0}{21}$ and $\frac{I_0}{61}$. Thus, diffraction fringes are of unequal width and unequal intensities. Hence the ratio of the intensities of secondary maxima to central maxima are $1 : \frac{1}{21} : \frac{1}{61} : \frac{1}{121}$....



(b) As the slit width increases (relative to wavelength) the width of the control diffraction maxima decreases, that is, the light undergoes less flaring by the slit. The secondary maxima also decrease in width (and becomes weaker).

(c) If $a \gg \lambda$, the secondary maxima due to the slit disappear; we then no longer have single slit diffraction.

(d) When the slit width is reduced by a factor of 2, the amplitude of the wave at the centre of the screen is reduced by a factor of 2, so the intensity at the centre is reduced by a factor of 4.



DIFFRACTION MAXIMA DUE TO SINGLE SLIT

The angular positions of diffraction minima can be given by the equation

$$\sin\theta = n\frac{\lambda}{a} \qquad (n = \pm 1, \pm 2, \pm 3, \ldots)$$

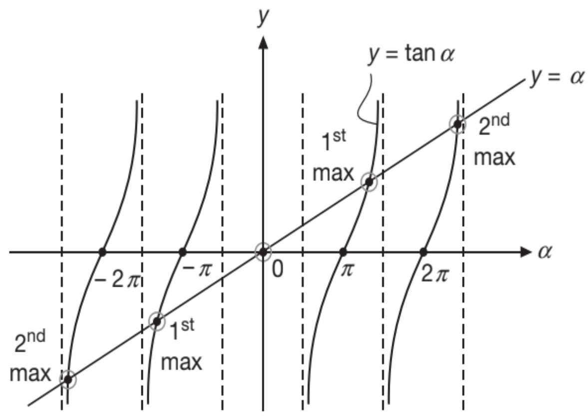
However, to find the angular positions of diffraction maxima other than central maxima we differentiate equation $I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$, w.r.t. α, where $\alpha = \frac{\pi a \sin\theta}{\lambda}$ and equate to zero.

$$\Rightarrow \quad \frac{dI(\theta)}{d\alpha} = \frac{\alpha^2(\alpha\sin\alpha\cos\alpha) - (\sin\alpha)(2\alpha)}{\alpha^4} = 0$$

$$\Rightarrow \quad \tan\alpha = \alpha \qquad \ldots(1)$$

In equation (1), $\alpha = 0$ corresponds to central maxima.

All other values of α satisfying equation (1) will correspond to higher order diffraction maxima in the diffraction pattern which can be calculated graphically by finding the intersection points of the curves $y = \tan\alpha$ and $y = \alpha$ as shown in figure below.



In above figure we can see that successive higher order maxima are not located at the mid points of all the minima's.

ILLUMINATION PATTERN DUE TO DIFFRACTION BY A SINGLE SLIT

Since the first minima in a single slit diffraction pattern is obtained at an angular position given by

$$\theta = \sin^{-1}\left(\frac{\lambda}{a}\right) \qquad \ldots(1)$$

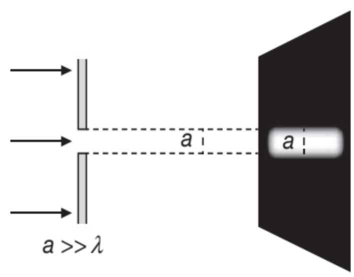
The above angle in equation (1) gives the edges of central diffraction maxima which is most prominent in the illumination pattern of single slit diffraction pattern. Now for different wavelengths and slit widths let us discuss the following cases.

CASE-1: When $a \gg \lambda$

When slit width a is very large compared to wavelength λ of light, then from equation (1) we get

$$\theta = \sin^{-1}\left(\frac{\lambda}{a}\right) \to 0$$

Which simply shows the rectilinear propagation of light because the light being does not flare out of the region beyond $\theta = 0$. This is shown in figure below in which the central maxima will just be the projection of light on screen which of width equal to that of the slit.



CASE-2: When $a > \lambda$

When slit width a is more than the wavelength of light, then first minima and other higher order minima and maxima can also be obtained as discussed earlier.

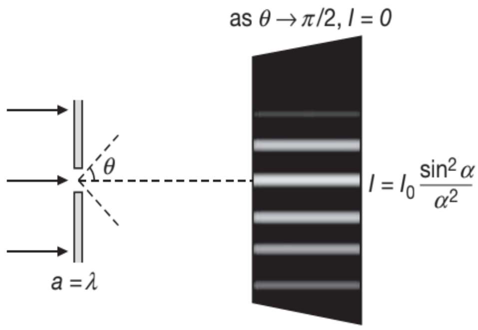
CASE-3: When $a = \lambda$

In this case we can see from equation (1) we get

$$\theta = \sin^{-1}(1) = \frac{\pi}{2}$$

Thus, the central maxima will spread on the whole screen as shown in figure and as we move away from centre of screen the intensity of light gradually decreases with the function given by equation

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2, \text{ w.r.t. } \alpha, \text{ where } \alpha = \frac{\pi a\sin\theta}{\lambda}$$



CASE-4: When $a < \lambda$

When slit width is less than the wavelength of light, then equation (1) is not valid and we observe that no minima is obtained anywhere and on screen and hence there will be almost uniform illumination near of the centre of screen.

ILLUSTRATION 36

A slit of width a is illuminated by white light.

(a) For what value of a, will the first minimum for red light of $\lambda = 6500$ Å be at $\theta = 15°$?

(b) What is the wavelength λ' of the light whose first side-maximum is at $\theta = 15°$, thus coinciding with the first minimum for the red light?

SOLUTION

(a) The angular position θ_n of n^{th} minimum is given by

$$a\sin\theta_n = n\lambda$$

Here, $n = 1$, $\lambda = 6500\times10^{-10}$ m, $\theta = 15°$

$$\Rightarrow a = \frac{n\lambda}{\sin\theta_n} = \frac{1\times6500\times10^{-10}}{\sin 15°} \approx 2.5\ \mu\text{m}$$

(b) This maximum is approximately halfway between the first and second minima produced with light of wavelength λ'. Thus, by putting $n = 1.5$, we get

$$a\sin\theta = 1.5\lambda'$$

$$\Rightarrow \lambda' = \frac{a\sin\theta}{1.5} = \frac{2.5\times10^{-6}\times\sin(15°)}{1.5}$$

$$\Rightarrow \lambda' = 430\text{ mm} = 4300\text{ Å}$$

This is the wavelength of violet light. Note that the first side-maximum for light of $\lambda' = 4300$ Å will always coincide with the first minimum for light of $\lambda = 6500$ Å, no matter what the slit-width is.

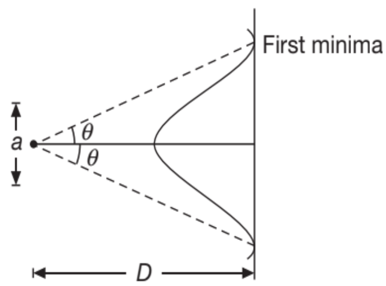
ILLUSTRATION 37

Angular width of central maximum in the Fraunhofer diffraction pattern of a slit is measured. The slit is illuminated by light of wavelength 6000 Å. When the slit is illuminated by light of another wavelength, the angular width decreases by 30%. Calculate the wave-length of this light. The same decrease in the angular width of central maximum is obtained when the orig-inal apparatus is immersed in a liquid. Find refractive index of the liquid.

SOLUTION

(a) Given $\lambda = 6000$ Å

Let a be the width of slit and D the distance between screen and slit.



First minima is obtained at $a\sin\theta = \lambda$

$$\Rightarrow a\theta = \lambda \sin\theta \approx \theta$$

$$\Rightarrow \theta = \frac{\lambda}{a}$$

Angular width of first maxima $= 2\theta = \dfrac{2\lambda}{a} \propto \lambda$

Angular width will decrease by 30% when λ is also decreased by 30%.

Therefore, new wavelength

$$\lambda' = \left\{(6000) - \left(\frac{30}{100}\right)6000\right\} = 4200 \text{ Å}$$

(b) When the apparatus is immersed in a liquid of refractive index μ, the wavelength is decreased μ times.

$$\Rightarrow \quad 4200 \text{ Å} = \frac{6000 \text{ Å}}{\mu}$$

$$\Rightarrow \quad \mu = \frac{6000}{4200}$$

$$\Rightarrow \quad \mu = 1.429 \approx 1.43$$

ILLUSTRATION 38

Angular width of central maximum in the Fraunhofer diffraction pattern of a slit is measured. The slit is illuminated by another wavelength, the angular width decreases by 30%. Calculate the wavelength of this light. The same decrease in angular width of central maximum is obtained when the original apparatus is immersed in a liquid. Find the refractive index of the liquid.

SOLUTION

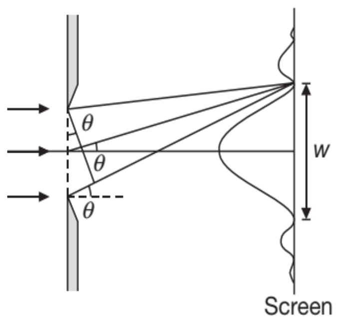
For diffraction minima on screen, we use

$$a\sin\theta = n\lambda, \text{ where } n = 1, 2, 3, \ldots$$

Angular width of central maxima is 2θ for $n = 1$

$$\Rightarrow \quad a\sin\theta = \lambda$$

For small θ, we have $\sin\theta \approx \theta$, so $a\theta = \lambda$



So, angular width of central maxima is given by

$$w = 2\theta = \frac{2\lambda}{a}$$

$$\Rightarrow \quad w = \frac{2\times 6000\times 10^{-10}}{a}$$

When the wavelength is changed the angular width of central maxima is reduced by 30%. Thus new angular width is given by

$$w' = w - 0.3w = 0.7w$$

$$\Rightarrow \quad 0.7w = \frac{2\lambda'}{a}$$

$$\Rightarrow \quad \lambda' = (0.7w)\left(\frac{a}{2}\right) = 0.7\left(\frac{2\times 6000\times 10^{-10}}{a}\right)\left(\frac{a}{2}\right)$$

$$\Rightarrow \quad \lambda' = 0.7\times 6000\times 10^{-10} = 4200\times 10^{-10} \text{ m}$$

$$\Rightarrow \quad \lambda' = 4200 \text{ Å}$$

When the setup is submerged in a liquid, then also the angular width of central maxima decreases by 30%, which indicates that the wavelength of light decreases by 30%, so

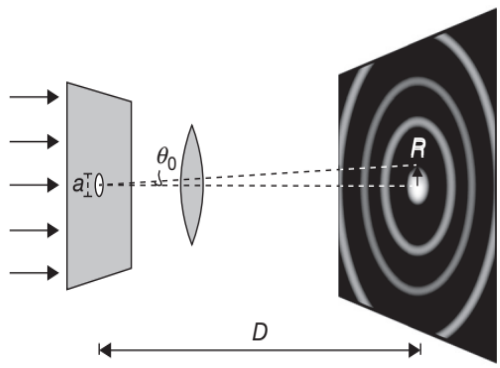
$$\lambda' = \frac{\lambda}{\mu}$$

$$\Rightarrow \quad 4200 = \frac{6000}{\mu}$$

$$\Rightarrow \quad \mu = \frac{6000}{4200} = \frac{10}{7} \approx 1.43$$

FRAUNHOFER DIFFRACTION AT A CIRCULAR APERTURE

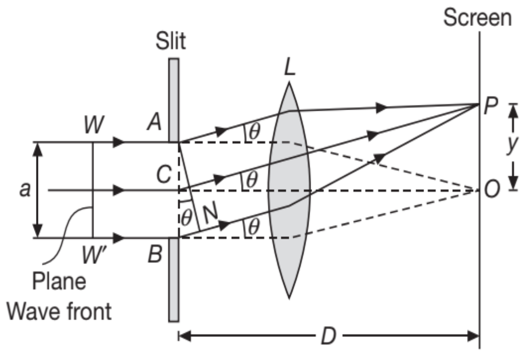
In Fraunhofer diffraction at a circular aperture or disc, the diffraction pattern has intermediate dark and bright fringes with a central bright circular spot.



This circular spot formed at the centre is known as Airy disc which is the description of best spot of light that a perfect lens of circular aperture can make.

Nearby, the circular patterns formed are those known as Airy patterns. These are named after George Biddle Airy. The concentric circular rings will get fainter as it moves from the central spot.

The problem of diffraction at a circular aperture was first solved by Airy in 1835. A circular aperture of diameter a is shown as AB in figure.



Fraunhofer diffraction at a circular aperture

A plane wave front WW' is incident normally on this aperture. Every point on the plane wave front in the aperture acts as a source of secondary wavelets. The secondary wavelets spread out in all directions as diffracted rays in the aperture. These diffracted secondary wavelets are converged on the screen by placing a convex lens L (of focal length f) between the aperture and the screen. The screen is at the focal plane of the convex lens. The diffracted rays travelling normal to the plane of aperture i.e. along CO get converged at O.

All these waves travel some distance to reach the point O and there is no path difference between these rays. Hence a bright spot is formed at O. This bright spot is known as Airy's disc. The point O corresponds to the central maximum.

Next consider the secondary waves travelling at an angle θ with respect to the direction of CO. All these secondary waves travel in the form of a cone and hence, they form a diffracted ring on the screen. The radius of that ring is y and its centre is at O. Now consider a point P on the ring at a distance y from O. The intensity of light at P depends on the path difference between the waves at A and B to reach P. The path difference between the waves from A and B arriving at the point P is

$$BN = AB\sin\theta = a\sin\theta$$

The diffraction due to a circular aperture is similar to the diffraction due to a single slit. Hence, the intensity at P depends on the path difference $a\sin\theta$.

If the path difference is an integral multiple of λ then intensity at P is minimum. On the other hand, if the path difference is an odd multiple of $\frac{\lambda}{2}$, then the intensity is maximum. So,

for minima, $a\sin\theta = n\lambda$ ...(1)

for maxima, $a\sin\theta = (2n-1)\frac{\lambda}{2}$ ...(2)

where $n = 1, 2, 3,........$ and $n = 0$ corresponds to the central maximum.

The Airy disc is surrounded by alternate bright and dark concentric rings called the Airy's rings.

The intensity of the dark ring is zero and the intensity of the bright ring decreases as we go radially from O on the screen. If the collecting lens L is placed very near to the circular aperture (or the screen is at a large distance from the lens), then, $D \approx f$. So, we have

$$\sin\theta \approx \theta \approx \frac{y}{D} \approx \frac{y}{f} \quad ...(3)$$

where f is the focal length of the lens.

Also, from the condition for first secondary minimum i.e. from equation (1), we get

$$\sin\theta \approx \theta \approx \frac{\lambda}{a} \quad ...(4)$$

Equating (3) and (4), we get

$$\frac{y}{f} = \frac{\lambda}{a} \quad ...(5)$$

But according to Airy, the exact value of radius or Airy disc is R given by

$$R = \frac{1.22 f\lambda}{a} \quad ...(6)$$

Using equation (6), the radius of Airy's disc can be obtained. Also, from this equation, we observe that the radius of Airy's disc is inversely proportional to the diameter a of the aperture. Hence on decreasing the diameter of aperture, the size of Airy's disc increases.

The examples for circular apertures are the eyes and optical instruments like camera, microscope, telescope and so on. These are diffraction limited. Diffraction limited is the ability to produce images with angular resolution limited by aperture resolution. Rayleigh creation is used to calculate this resolution.

ILLUSTRATION 39

A convex lens of diameter 8.0 cm is used to focus a parallel beam of light of wavelength 6200 Å. If the light be focused at a distance of 20 cm from the lens, what would be the radius of the central bright spot formed?

SOLUTION

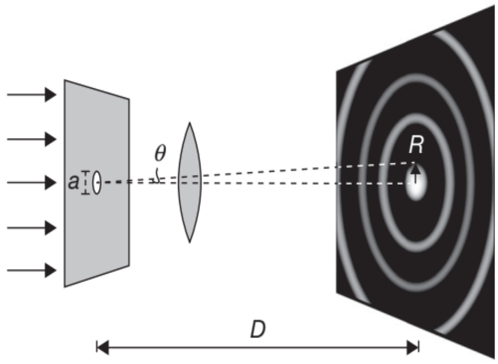
The angular spread of the central bright spot is given by

$$\sin\theta = \frac{1.22\lambda}{a}$$

$$\Rightarrow \quad \sin\theta = \frac{1.22\times 620\times 10^{-9}}{0.08}$$

$$\Rightarrow \quad \sin\theta = 9.455\times 10^{-6} \text{ rad}$$

Since θ is small, so $\sin\theta \approx \theta = 9.455\times 10^{-6}$ rad



Also, $\theta \approx \frac{R}{D}$

$$\Rightarrow \quad \frac{R}{D} = 9.45\times 10^{-6}$$

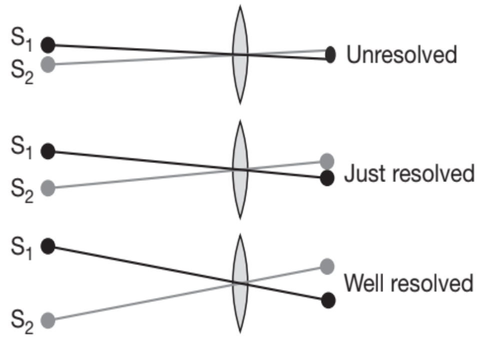
$$\Rightarrow \quad R = 9.45\times 10^{-6}\times 0.20$$

$$\Rightarrow \quad R = 1.89\times 10^{-6} \text{ m}$$

RESOLVING POWER AND RAYLEIGH'S CRITERION

The image of a point object, formed by a converging lens, is not a point image. Rather, it is a diffraction disc surrounded by a few alternate bright and dark fringes of sharply decreasing intensity. The size of the disc depends on the aperture of the lens and the wavelength of light used.

It two bright point objects S_1 and S_2 lying very close to each other are seen with a naked eye or with the help of an optical instrument (a microscope or a telescope), then they may or may not be seen as two separate distinct objects due to the overlapping of their diffraction patterns.



The smallest separation (linear or angular) between two point objects at which they appear just separated is called **the limit of resolution of an optical instrument** and the reciprocal of the limit of resolution is called its **resolving power**.

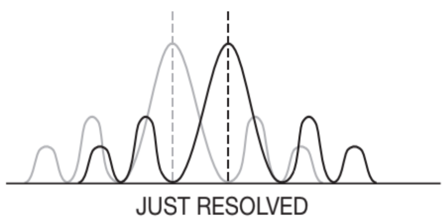
Whether the two objects are seen as two separate point objects or not, depends on the separation between the centres of the bright discs of the images of the two objects. Therefore, when two objects are seen with a naked eye or with the help of an optical instrument the two objects may be just resolved, well resolved or unresolved as explained by Rayleigh's Criterion.

RAYLEIGH'S CRITERION

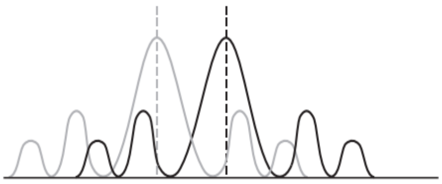
(a) Two objects are said to be **just resolved**, if the separation between the central maxima of the objects is just equal to the distance between the central maximum and the first minimum of any of the two. In other words, two images are said to be just resolved when central maxima of one diffraction pattern falls on first minima of other.

Limit of resolution of a telescope is

$$\theta = \frac{1.22\lambda}{a}$$

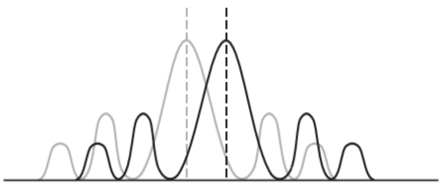


(b) Two objects are said to **well resolved**, if the separation between the central maxima of the objects is greater than the distance between the central maximum and the first minimum of either of them.



WELL RESOLVED

(c) Two objects are said to be **unresolved**, if separation between the central maxima of the objects is less than the distance between the central maximum and the first minimum of either of them.

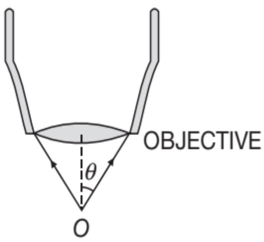


UNRESOLVED RESOLVED

These results are called **Rayleigh's Criterion of Limiting Resolution**. Therefore, it follows that diffraction limits the resolving power of an optical instrument.

RESOLVING POWER OF A MICROSCOPE

Resolving power of a microscope is defined as the reciprocal of the least separation between two close objects, so that they appear just separated, when seen through the microscope.



Consider a point object to be illuminated with the light of wavelength λ and seen through a microscope. The rays of light scattered from the object enter the objective of the microscope in a cone of semi-vertical angle θ. The least separation between the two objects, so that they appear just separated is given by

$$d = \frac{\lambda}{2\mu \sin\theta} \qquad \dots(1)$$

where μ is the refractive index of the medium between the objective of the microscope and the object.

This least separation between the two objects is called the **limit of resolution** of the microscope.

From definition, the resolving power of the microscope is given by

$$\frac{1}{d} = \frac{2\mu \sin\theta}{\lambda} \qquad \dots(2)$$

Following conclusions can be drawn from the above expression.

(a) The resolving power of a microscope increases with decrease in the value of the wavelength of the light used to illuminate the object.

(b) The resolving power of a microscope increases with increase in the value of the refractive index of the medium between its objective and the object. For this reason, **oil immersion objective microscopes** are used to achieve high resolving power.

Since wavelength of ultraviolet light is less than that of the visible light, the microscopes employing ultraviolet light for illuminating the objects are used to achieve high resolving power. Such microscopes are called **ultra microscopes**. Still higher resolving power can be obtained in an **electron microscope.**

RESOLVING POWER OF A TELESCOPE

Resolving power of a telescope is the reciprocal of the smallest angular separation between two distant objects, so that they appear just separated, when seen through the telescope.

Let two distant objects be observed through a telescope, whose objective is of diameter a. Let λ be the wavelength of the light, in which objects are observed. The smallest angular separation between the two objects, so that they appear just separated is found to be

$$d\theta = \frac{1.22\lambda}{a} \qquad \dots(3)$$

$\Rightarrow$ Resolving Power of Telescope $= \dfrac{1}{d\theta} = \dfrac{a}{1.22\lambda}$ ...(4)

So, we observe that the resolving power of a telescope increases, when objective of larger diameter is used or light of smaller wavelength is used to see the objects.

HUMAN EYE

In case of the human eye, two points can be seen distinctly if angle subtended by them at the eye is about one minute. This is the angular limit of resolution of the eye and the reciprocal of this is the resolving power.

The limit of resolution of eye lens is nearly $\left(\frac{1}{60}\right) = 1'$

VALIDITY OF GEOMETRICAL OPTICS AND FRESNEL'S DISTANCE (Z_F)

When a slit of width a is illuminated by a parallel beam of light, the angular spread of diffracted light is approximately $\frac{\lambda}{a}$. Therefore, after travelling a distance D, the diffracted beam acquires a width $\frac{D\lambda}{a}$. Geometrical optics is based on rectilinear propagation of light, which is just an approximation. We can say that geometrical optics is valid, if the width $\frac{D\lambda}{a}$ of the diffracted beam is less than the size of the slit, that is

$$\frac{D\lambda}{a} < a$$

$$\Rightarrow \quad D < \frac{a^2}{\lambda}$$

This distance from a slit or an obstacle upto which the spreading of light due to diffraction can be ignore (i.e. light goes straight and hence ray optics or geometrical optics can be applied) is called Fresnel's distance $Z_F = \frac{a^2}{\lambda}$.

Since λ is very small so Z_F is fairly large (in most of the cases) and so diffraction spreading can be neglected up to a fairly large distance.

Therefore, geometrical optics is valid for $Z_F < \frac{a^2}{\lambda}$ i.e. beyond Z_F spreading of light becomes significance and ray optics cannot be applied. Theoretically when $\lambda \to 0$, then $Z_F \to \infty$.

FRESNEL'S ZONE

If we expect a beam to travel a distance D without too much broadening by diffraction, we must have

$$D < Z_F$$

$$\Rightarrow \quad D < \frac{a^2}{\lambda}$$

$$\Rightarrow \quad a > \sqrt{\lambda D}$$

$\sqrt{\lambda D}$ is called the **size of the Fresnel's zone**, a_F

$$\Rightarrow \quad a_F = \sqrt{\lambda D}$$

ILLUSTRATION 40

For what distance is the ray optics a good approximation, if the slit is 3 mm wide and the wavelength of light is 5000 Å?

SOLUTION

For ray optics to be valid $D < Z_F \left(= \frac{a^2}{\lambda}\right)$

$$\Rightarrow \quad D < \frac{a^2}{\lambda} = \frac{(3\times10^{-3})^2}{5000\times10^{-10}} = 18 \text{ m}$$

Thus, upto a distance of 18 m, we can assume rectilinear propagation of light to a good approximation.

INTERFERENCE AND DIFFRACTION: A COMPARISON

INTERFERENCE	DIFFRACTION
It results from interaction of light coming from two different wave fronts originating from two coherent sources.	It results from interaction of light coming from different parts of the same wavefront.
Here, the fringes are of the same width.	Here the fringes are always of varying width.
The fringes of minimum intensity are dark (or perfectly dark when waves are of same amplitude).	The fringes of minimum intensity are not perfectly dark.
All bright fringes possess the same intensity.	The intensity of all the bright fringes is not same. It is maximum for central fringe and decreases sharply for first, second etc. bright fringes.
An interference pattern consists a good contrast between the dark and bright fringes.	In diffraction pattern the contrast between the bright and dark fringes is comparatively poor.

Test Your Concepts-II

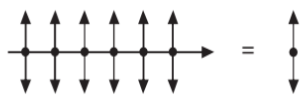
Based on Diffraction

(Solutions on page H.124)

1. A slit of width 0.025 m is placed in front of a lens of focal length 50 cm. The slit is illuminated with light of wavelength 5900 Å. Calculate the distance between the centre and first dark band of diffraction pattern obtained on a screen placed at the focal plane of the lens.
2. Two spectral lines of sodium D_1 and D_2 have wavelengths of approximately 5890 Å and 5896 Å. A sodium lamp sends incident plane wave onto a slit of width 2 micrometre. A screen is located 2 m from the slit. Find the spacing between the first maxima of two sodium lines as measured on the screen.
3. In Young's double slit experiment, the distance d between the slits S_1 and S_2 is 1 mm. What should the width of each slit be so as to obtain 10 maxima of the double slit pattern within the central maximum of the single slit pattern?
4. Estimate the distance for which ray optics is a good approximation for an aperture of 4 mm and wavelength 400 nm.
5. Two towers on the top of two hills are 40 km apart. The line joining them passes 50 m above a hill half way between the towers. What is the longest wavelength of radio waves which can be sent between the towers without appreciable diffraction effects?
6. A slit of width d is illuminated by white light. For what value of d will the first minimum for red light ($\lambda = 6500$ Å) fall at an angle $\theta = 30°$?
7. A screen is placed 2 m away from a single narrow slit. Calculate the slit width if the first minimum lies 5 mm on either side of central maximum. Incident plane waves have a wavelength of 5000 Å.
8. Determine the angular separation between central maximum and first order maximum of the diffraction pattern due to a single slit of width 0.25 mm when light of wavelength 5890 Å is incident on it normally.
9. Parallel light of wavelength 5000 Å falls normally on a single slit. The central maximum spreads out to 30° on either side of the incident light. Find the width of the slit. For what width of the slit the central maximum would spread out to 90° from the direction of the incident light?
10. A laser light beam of power 20 mW is focused on a target by a lens of focal length 0.05 m. If the aperture of the laser be 1 mm and the wavelength of its light 7000 Å, calculate the angular spread of the laser, the area of the target hit by it and the intensity of the impact on the target.

POLARIZATION OF LIGHT

According to Maxwell, light possesses electromagnetic nature. An electromagnetic wave consists of varying electric and magnetic fields, such that the two fields are mutually perpendicular to each other and to the direction of propagation of waves. The optical phenomena i.e., phenomena concerning light may primarily be attributed to the vibrations of electric field vector in a direction perpendicular to the direction of propagation of light. In ordinary or unpolarised light, the vibrations of electric field vector are regularly or symmetrically distributed in a plane perpendicular to the direction of the propagation of the light.

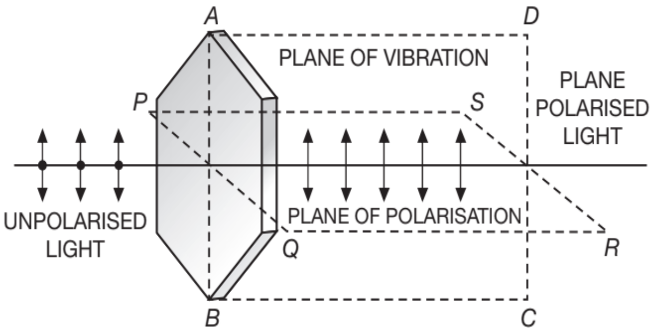


UNPOLARISED LIGHT (REPRESENTATION)

In an ordinary ray of light, the electric vibrations are in all the directions but perpendicular to the direction of propagation of the light. Such a ray of light is called a **ray of ordinary or unpolarised light**. It is schematically represented as shown. The arrows represent vibrations in the plane of the paper, while the dots represent vibrations in a direction perpendicular to the plane of the paper.

The phenomenon, due to which the vibrations of light are restricted to a particular plane, is called the **polarisation of light**.

When ordinary light i.e. unpolarised light passes through a tourmaline crystal, out of all the vibrations which are symmetrical about the direction of propagation, only those passes through it, which are parallel to its crystallographic axis AB. Therefore, on emerging through the crystal, the vibrations no longer remain symmetrical about the direction of propagation but are confined to a single plane (see Figure).



PLANE OF VIBRATION

The plane $(ABCD)$, which contains the vibrations of plane polarised light, is called the **plane of vibration**.

PLANE OF POLARISATION

The plane $(PQRS)$ perpendicular to the plane of vibrations is called the **plane of polarisation**.

PLANE POLARISED LIGHT

It may be defined as the light, in which the vibrations of the light (vibrations of the electric vector) are restricted to a particular plane.

In a plane polarised light, the vibrations are restricted to a fixed plane, so that vibrations are perpendicular to direction of propagation of light. Figure (a) represents plane polarised light having vibrations in the plane of the paper and Figure (b) represents the plane polarised light having vibrations in a plane perpendicular to the plane of the paper.

POLARISED LIGHT (REPRESENTATION)

Problem Solving Technique(s)

The vibrations in plane polarised light are perpendicular to the plane of polarisation.

POLARIZATION BY REFLECTION

Polarized light may also be obtained by the process of reflection. When an unpolarized light beam is reflected, light is completely polarized, partially polarized, or unpolarized, depending on the angle of incidence. If the angle of incidence is either 0 or 90° (normal or grazing angles), the reflected beam is unpolarized. However, for intermediate angles of incidence, the reflected light is polarized to some extent.

Suppose an unpolarized light beam is incident on a surface as in figure. The beam can be described by two electric field components, one parallel to the surface (the dots) and the other perpendicular to the first and to the direction of propagation (the arrows). It is found that the parallel component reflects more strongly than the other component, and this results in a partially polarized beam. Furthermore, the refracted ray is also partially polarized.

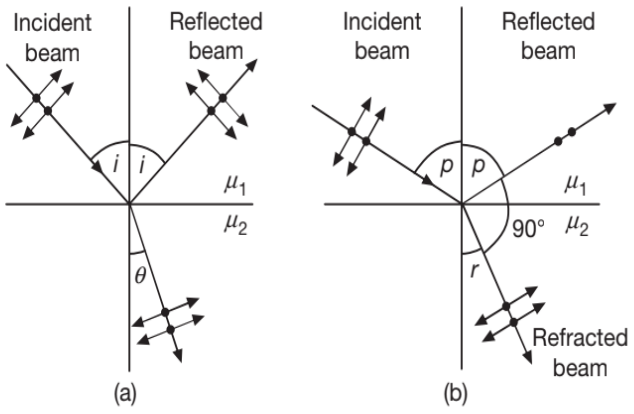
Now suppose the angle of incidence, i, is varied until the angle between the reflected and refracted beams is 90°. At this particular angle of incidence, the reflected beam is completely polarized with its electric field vector parallel to the surface, while the refracted beam is partially polarized. The angle of incidence at which this occurs is called the **polarizing angle,** p.

From figure, we see that at the polarizing angle, $p+90^\circ+r=180^\circ$, so that $r=90^\circ-p$. Using Snell's Law, we have

$$\mu=\frac{\sin p}{\sin r}$$

Since $\sin r=\sin(90^\circ-p)=\cos p$, the expression for μ can be written

$$\mu=\frac{\sin p}{\cos p}=\tan p$$



(a) When unpolarized light is incident on a reflecting surface, the reflected and refracted beams are partially polarized. (b) The reflected beam is completely polarized when the angle of incidence equals the polarizing angle θ_p

This expression is called **Brewster's Law**, and the polarizing angle p is sometimes called **Brewster's Angle**, after its discoverer, Sir David Brewster (1781–1868). For example, the Brewster's angle for crown glass $(\mu = 1.52)$ is $p = \tan^{-1}(1.52) = 56.7°$. Since μ varies with wavelength for a given substance, the Brewster's angle is also a function of the wavelength.

Polarization by reflection is a common phenomenon. Sunlight reflected from water, glass, snow and metallic surfaces is partially polarized. If the surface is horizontal, the electric field vector of the reflected light will have a strong horizontal component. Sunglasses made of polarizing material reduce the glare of reflected light. The transmission axes of the lenses are oriented vertically so as to absorb the strong horizontal component of the reflected light.

ILLUSTRATION 41

A ray of light strikes a glass plate at an angle of 60°. If the reflected and refracted rays are perpendicular to each other, find the refractive index of glass.

SOLUTION

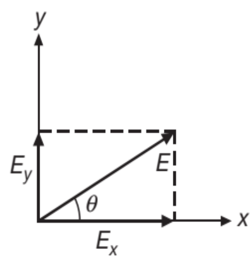
Reflected and refracted rays are mutually perpendicular only when the angle of incidence is equal to polarising angle. So, $i_p = 60°$. Hence the refractive index is given by

$$\mu = \tan i_p = \tan 60° = \sqrt{3} = 1.732$$

LINEARLY, CIRCULARLY AND ELLIPTICALLY POLARISED LIGHT

A wave is said to be linearly polarized if only one of these directions of vibration of E exists at a particular point. (Sometimes such a wave is described as plane-polarized, or simply polarized).

Suppose a light beam travelling in the z direction has an electron field vector that is at an angle θ with the x axis at some instant, as in figure. The vector has components E_x and E_y as shown. Obviously, the light is linearly polarized if one of these components is always zero or if the angle θ remains constant in time. However, if the tip of the vector E rotates in a circle with time, the wave is said to be circularly polarized. This occurs when the magnitudes of E_x and E_y are equal, but differ in phase by $90°$. On the other hand, if the magnitudes of E_x and E_y are not equal, but differ in phase by $90°$, the tip of E moves in an ellipse. Such a wave is said to be elliptically polarized. Finally, if E_x and E_y are, on the average, equal in magnitude, but have a randomly varying phase difference the light beam is unpolarized.



A linearly polarized wave with E at an angle θ to x has components $E_x = E\cos\theta$ and $E_x = E\sin\theta$

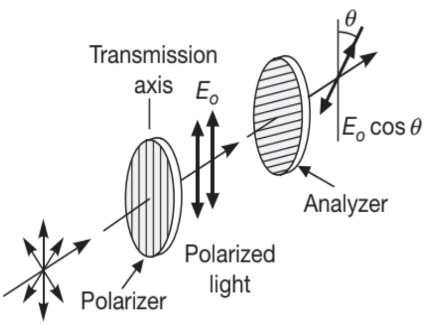
It is possible to obtain a linearly polarized beam from an unpolarized beam by removing all waves from the beam except those whose electric field vectors oscillate in a single plane. Four different physical processes of producing polarized light from unpolarized light are

(a) selective absorption (or dichroism)
(b) reflection
(c) double refraction
(d) scattering

POLARIZATION BY SELECTIVE ABSORPTION

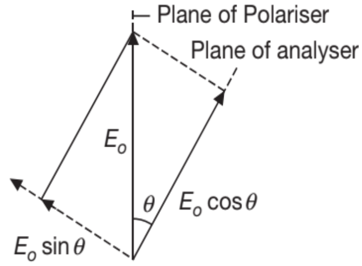
The most common technique for obtaining polarized light is to use a material that will transmit waves whose electric field vectors are parallel to a certain direction and will absorb most other directions of polarization. Any substance that has the property of transmitting light with the electric field vector vibrating in only one direction is called a **dichroic substance**. In 1938, E.H. Land discovered a material, which he called **Polaroid**, that polarizes light through selective absorption by oriented molecules. Long chain hydrocarbon molecules (such as polyvinyl alcohol) in thin-sheet form are aligned in one direction when the sheet is stretched during fabrication. After the sheet is dipped into a solution containing iodine, the molecules become conducting. However, the conduction takes place primarily along the hydrocarbon chains since the valence electrons of the molecules readily absorb light whose electric field vector is parallel to their length and transmit light whose electric field vector is perpendicular to their length. It is common to refer to the direction perpendicular to the molecular chains as the transmission axis. In an ideal polarizer, all light with E parallel to the transmission axis is transmitted, and all light with E perpendicular to the transmission axis is absorbed.

Figure represents an unpolarized light beam incident on the first polarizing sheet, called the **polarizer**, where the transmission axis is indicated by the straight lines on the polarizer.



The light that is passing through this sheet is polarized vertically as shown, where the transmitted electric field vector is E_0. A second polarizing sheet, called the **analyzer**, intercepts this beam with its transmission axis at an angle θ to the axis of the polarizer.

The component of E_0 perpendicular to the axis of the analyzer is completely absorbed. The component of E_0 parallel to the axis of the analyzer is $E_0 \cos\theta$



Two polarizing sheets whose transmission axes make an angle θ with each other. Only a fraction of the polarized light incident on the analyzer is transmitted.

Since the transmitted intensity varies as the square of the transmitted amplitude, we conclude that the transmitted intensity is given by

$$I = I_0 \cos^2\theta$$

where I_0 is the intensity of the polarized wave incident on the analyzer.

This expression, known as **Malus law**, applies to any two polarizing materials whose transmission axes are at an angle θ to each other. From this expression, note that the transmitted intensity is a maximum when the transmission axes are parallel $(\theta = 0 \text{ or } 180^\circ)$. In addition, the transmitted intensity is zero (complete absorption by the analyzer) when the transmission axes are perpendicular to each other.

Conceptual Note(s)

(a) If polarised light of intensity I_0 is passed through an analyser, the intensity of light transmitted is

$$I = I_0 \cos^2\theta \qquad \text{\{Malus Law\}}$$

(b) If incident light is unpolarized (or ordinary light) of intensity I_0, then

$$I = \frac{I_0}{2}\cos^2\theta$$

(c) Analysis of a Given Light Beam

Let a given light beam is made incident on a polaroid (or Nicol) and the polaroid/Nicol is gradually rotated.

(i) If light beam shows no variation in intensity, then given beam is unpolarised.

(ii) If light beam shows variation in intensity but the minimum intensity is non-zero, then given beam is partially polarised.

(iii) If light beam shows variation in intensity and intensity becomes zero twice in a rotation, then given beam of light is plane polarised.

LAW OF MALUS

When a plane polarised light is seen through an analyser, the intensity of transmitted light varies as the analyser is rotated in its own plane about the incident direction. In 1809, E.N. Malus discovered that when a beam of completely plane polarised light is passed through analyser, the intensity I of transmitted light varies directly as the square of the cosine of the angle θ between the transmission directions of polariser and analyser. This statement is known as the **Law of Malus**.

Mathematically, according to Malus Law, we have

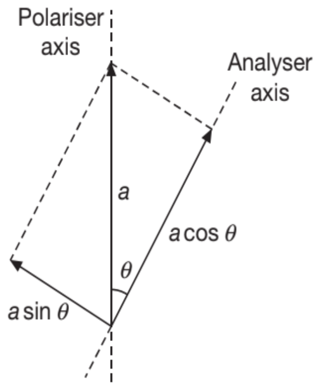
$$I \propto \cos^2\theta$$

$$\Rightarrow \quad I = I_0\cos^2\theta$$

where I_0 is the maximum intensity of transmitted light. **It may be noted that I_0 is equal to half the intensity of unpolarised light incident on the polariser.**

EXPLANATION OF THE LAW

Let the planes of polariser and analyser are inclined to each other at an angle θ as shown in figure. Let I_0 be the intensity and a the amplitude of the plane polarised light transmitted by the polariser.



Law of malus

The amplitude a of the light incident on the analyser has two rectangular components,

(i) $a\cos\theta$, parallel to the plane of transmission of the analyser, and
(ii) $a\sin\theta$, perpendicular to the plane of transmission of the analyser.

So only the component $a\cos\theta$ is transmitted by the analyser. The intensity of light transmitted by the analyser is

$$I = k(a\cos\theta)^2$$

$$\Rightarrow \quad I = ka^2\cos^2\theta$$

$$\Rightarrow \quad I = I_0\cos^2\theta$$

where $I_0 = ka^2$, is the maximum intensity of light transmitted by the analyser (when $\theta = 0°$). The above equation is the **Law of Malus or Malus Law.**

Conceptual Note(s)

(a) When $\theta = 0°$ **or** $180°$, $\cos\theta = \pm 1$

$\Rightarrow \quad I = I_0$

So, when the transmission directions of polariser and analyser are parallel or antiparallel to each other, the maximum intensity of plane polarised light is transmitted by the analyser and is equal to the intensity emerging from the polariser.

(b) When $\theta = 90°$, $\cos\theta = 0$

$\Rightarrow \quad I = 0$

So, when the transmission directions of polariser and analyser are perpendicular to each other, the intensity of light transmitted through the analyser is zero.

(c) When a beam of unpolarised light is incident on the polariser, then

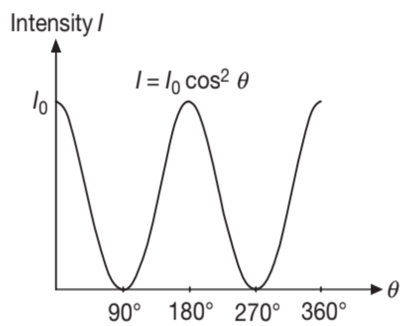
$$I = I_0\langle\cos^2\theta\rangle$$

Since, $\langle\cos^2\theta\rangle = \dfrac{1}{2}$

$$\Rightarrow \quad I = \frac{I_0}{2}$$

INTENSITY CURVE

As the angle θ between the transmission directions of polariser and analyser is varied, the intensity I of the light transmitted by the analyser varies as a function of $\cos^2\theta$, as shown in figure.



POLARISATION BY SCATTERING

When we look at the blue portion of the sky through a polaroid and rotate the polaroid, the transmitted light shows rise and fall of intensity. This shows that the light from the blue portion of the sky is plane polarised. This is because sunlight gets scattered (i.e., its direction is changed) when it encounters the molecules of the earth's atmosphere. The scattered light seen in a direction perpendicular to the direction of incidence is found to be plane polarised.

Explanation. Figure shows the unpolarised light incident on a molecule. The dots show vibrations perpendicular to the plane of paper and double arrows show vibrations in the plane of paper. The electrons in the molecule begin to vibrate in both of these directions. The electrons vibrating parallel to the double arrows cannot send energy towards an observer looking at 90° to the direction of the sun because their acceleration has no transverse component. The light scattered by the molecules in this direction has only dots. It is polarised perpendicular to the plane of paper. This explains the polarisation of light scattered from the sky.

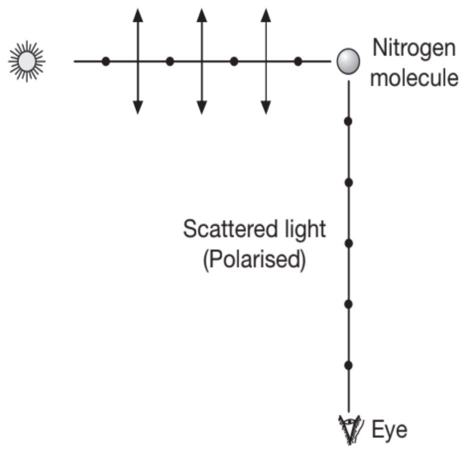


ILLUSTRATION 42

Two 'crossed' polaroids A and B are placed in the path of a light-beam. In between these, a third polaroid C is placed whose polarisation axis makes an angle θ with the polarisation axis of the polaroid A. If the intensity of light emerging from the polaroid A is I_0, then show that the intensity of light emerging from polaroid B will be $\frac{1}{4}I_0\sin^2(2\theta)$.

SOLUTION

By Malus Law, the intensity of light emerging from the middle polaroid C will be

$$I_1 = I_0\cos^2\theta$$

This intensity I_1 falls on the polaroid B whose polarisation axis makes an angle of $(90°-\theta)$ with the polarisation axis of the polaroid C. Therefore, the intensity of light emerging from B will be

$$I_2 = I_1\cos^2(90°-\theta) = \left(I_0\cos^2\theta\right)\cos^2(90°-\theta)$$

$$\Rightarrow \quad I_2 = I_0\cos^2\theta\sin^2\theta = \frac{I}{4}I_0(2\sin\theta\cos\theta)^2$$

$$\Rightarrow \quad I_2 = \frac{1}{4}I_0\sin^2(2\theta)$$

ILLUSTRATION 43

Two polaroids are placed 90° to each other. What happens when $N-1$ more polaroids are inserted between two crossed polaroids (at 90° to each other). Their axes are equally spaced. How does the transmitted intensity behave for large N?

SOLUTION

Transmitted intensity through first polaroid is

$$I_1 = I_0\cos^2\theta$$

where I_0 is the original intensity. Similarly, the transmitted intensity through second polaroid will be

$$I_2 = I_1\cos^2\theta = I_0\cos^4\theta$$

If N polaroids are used, then

$$I_N = I_0(\cos\theta)^{2N}$$

As the optic axes of the polaroids are equally inclined, so angle of rotation θ is same for each polaroid. Thus

$$\frac{I_N}{I_0} = (\cos\theta)^{2N}$$

Since, angle between successive polaroids is given by

$$\theta = \frac{90°}{N} = \frac{\pi}{2N} \text{ radian}$$

For large N, θ becomes small, so we get

$$\left(\cos\frac{\pi}{2N}\right)^{2N} = \left(1-\frac{\pi^2}{8N^2}+....\right)^{2N} \simeq \left(1-\frac{2N\pi^2}{8N^2}+...\right)$$

which approaches 1 for large N. So, fractional intensity, is

$$\frac{I_N}{I_0} = 1$$

$$\Rightarrow \quad I_N = I_0$$

ILLUSTRATION 44

A beam of plane-polarised falls normally on a polariser (cross-sectional area $3\times10^{-4}\ \text{m}^2$) which rotates about the axis of the ray with an angular velocity of $31.4\ \text{rads}^{-1}$. Find the energy of light passing through the polariser per revolution and the intensity of the emergent beam if the flux of energy of the incident ray is 10^{-3} W.

SOLUTION

Cross-sectional area of polaroid, $A = 3\times10^{-4}\ \text{m}^2$

Angular velocity, $\omega = 3.14\ \text{rads}^{-1}$

Time taken to complete one revolution,

$$T = \frac{2\pi}{\omega} = \frac{2\times3.14}{31.4} = 0.2\ \text{s}$$

$$(\text{Energy incident/sec}) = 10^{-3}\ \text{W}$$

So, intensity of incident polarised beam is given by

$$I_0 = \frac{\text{Energy incident/sec}}{\text{Area}} = \frac{10^{-3}}{3\times10^{-4}} = \frac{10}{3}\ \text{Wm}^{-2}$$

Since, $I = I_0\cos^2\theta$ where $\langle\cos^2\theta\rangle = \frac{1}{2}$

So, average intensity transmitted is

$$I_{av} = \frac{I_0}{2} = \frac{10}{3\times2} = 1.67\ \text{Wm}^{-2}$$

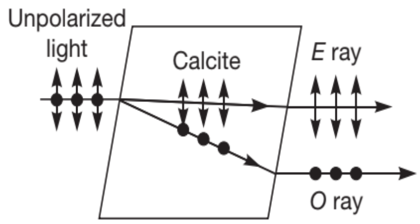
Light energy passing through polariser per revolution is given by

$$E = I_{av}AT = \frac{10}{6}(3\times10^{-4})(0.2) = 10^{-4}\ \text{J}$$

POLARIZATION BY DOUBLE REFRACTION

When light travels through an isotropic medium, such as glass, it travels with a speed that is the same in all directions. Such isotropic materials are characterized by a single index of refraction. However, in certain crystals, such as calcite and quartz, the speed of light is not the same in all directions. The fundamental reason for this phenomenon is associated with the complex arrangement of the crystalline structures. Such optically anisotropic materials are characterized by two indices of refraction. Hence, they are often referred to as **double refracting or birefringent materials**.

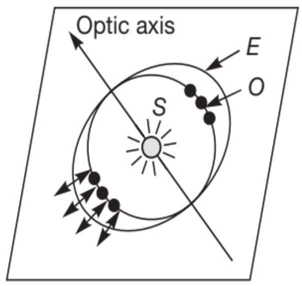
When an unpolarized beam of light enters a calcite crystal, it splits into two plane-polarized rays which travel with different velocities, corresponding to two different angles of refraction, as shown.



(a) When an unpolarized light beam is incident on a calcite crystal, it splits into an ordinary (O) ray and an extraordinary (E) ray. The rays are polarized in mutually perpendicular directions.

The two rays are polarized in two mutually perpendicular directions, as indicated by the dots and arrows. One ray called the **Ordinary** (O) **ray**, is characterized by an index of refraction μ_O that is the same in all directions, hence the ordinary ray has a spherical wavefront. The second ray, called the **Extraordinary** (E) **ray**, travels with different speeds in different directions and hence is characterized by an index of refraction μ_E that varies with the direction of propagation.

The wavefronts for the extraordinary ray are ellipsoids of revolution. Figure (b) illustrates the wavefronts associated with the ordinary and extraordinary rays, assuming a point source within the material.



(b) A point source S inside a doubly refracting crystal produces a spherical wavefront corresponding to the O ray and an elliptical wavefront corresponding to the E ray. The two waves propagate with the same velocity along the optic axis.

Note that there is one direction, called the **Optic axis**, along which the O and E rays have the same velocity, corresponding to the direction for which $\mu_O = \mu_E$. The difference in velocity for the two rays is a maximum in the direction perpendicular to the optic axis. For example, in calcite $\mu_O = 1.658$ at a wavelength of 589.3 nm, while μ_E varies from 1.658 along the optic axis to 1.486 perpendicular to the optic axis.

QUARTER WAVE PLATE

Quarter wave plate is a plate of a doubly refracting crystal, whose refracting faces are cut parallel to direction of optic axis and which produces a path difference of $\frac{\lambda}{4}$ between ordinary (O) and extraordinary (E) rays. If t is thickness of such plate, then

$$(\mu_O - \mu_E)t = \frac{\lambda}{4}$$

$$\Rightarrow \quad t = \frac{\lambda}{4(\mu_O - \mu_E)}$$

When a plane polarised light is incident on a quarter wave plate with its vibrations, making an angle of $45°$ with optic axis, the emergent light is circularly polarised. But if the vibrations of incident polarised light do not make an angle of $45°$ with optic axis, the emergent light is elliptically polarised.

HALF WAVE PLATE

Half wave plate is a plate of doubly refracting crystal (quartz or calcite), whose refracting faces are cut parallel to optic axis and whose thickness is such that it produces a path difference of $\frac{\lambda}{2}$ between (ordinary) O and E (extra ordinary) rays. If t is thickness of half wave plate, then

$$(\mu_O - \mu_E)t = \frac{\lambda}{2}$$

$$\Rightarrow \quad t = \frac{\lambda}{2(\mu_O - \mu_E)}$$

When a plane polarised light falls on a half wave plate, the emergent light is also plane polarised, but its direction of vibration is rotated through an angle 2θ with respect to incident light.

Conceptual Note(s)

(a) Polarisation is the property of transverse waves only. It is not shown by longitudinal waves.
(b) Plane of polarisation does not contain any component of vibrations in it.
(c) Extent of polarisation by reflection is maximum if the light is incident at polarising angle.
(d) Tangent of polarising angle is equal to the refractive index of medium upon which the light is incident.
(e) In double refraction, the two beams are polarised in mutually perpendicular planes.

ILLUSTRATION 45

The faces of a half wave plate are parallel to the optical axis of the crystal.

(i) What is the thinnest possible plate that would serve to put the ordinary and extra-ordinary rays of $\lambda = 5890$ Å, a half wave apart on their exist?
(ii) What multiples of this thickness would give the same result? The indices of refraction of quartz are $\mu_E = 1.553$, $\mu_0 = 1.544$.

SOLUTION

Given that $\lambda = 5890$ Å $= 5890 \times 10^{-8}$ cm, $\mu_E = 1.553$, $\mu_0 = 1.544$

(i) The thickness of the half wave plate is given by

$$t_{1/2} = \frac{\lambda}{2|\mu_0 - \mu_E|} = \frac{5890 \times 10^{-8}}{2|1.544 - 1.553|}$$

$$\Rightarrow \quad t_{1/2} = 32.7 \times 10^{-4} \text{ cm}$$

(ii) The other thicknesses which will give the same result are t, $3t$, $5t$, ….., $(2n+1)t$, where n is an integer.

ILLUSTRATION 46

A beam of linearly polarised is changed into circularly polarised light by passing it through a slice of crystal 0.003 cm thick. Calculate the difference in the refractive index of the two rays in the crystal assuming this to be minimum thickness that will produce the effect and that the wavelength is 6×10^{-5} cm.

SOLUTION

To convert a linearly polarised light into a circularly polarised light, a thickness equal to that of a quarter-wave plate is required which is given by

$$t_{1/2} = \frac{\lambda}{4(\mu_0 - \mu_e)}$$

Since $t = 0.003$ cm , $\lambda = 6\times10^{-5}$ m

$$\Rightarrow \quad \mu_0 - \mu_e = \frac{\lambda}{4t} = \frac{6\times10^{-5}}{4\times0.003} = 0.005$$

OPTICAL ACTIVITY AND SPECIFIC ROTATION (α)

The optical activity of pure liquids and solutions is measured by specific rotation α, which is defined as the rotation produced in the plane of vibration of plane polarised light by a decimeter length of solution having unit concentration of optically active substance i.e.,

$$\alpha(\lambda, T) = \frac{\theta}{lc}$$

where c is concentration, (in kgm^{-3}), l is length of solution tube (in metre) and θ is angle of rotation (in radian).

If length of solution tube is in cm, then

$$\alpha(\lambda, T) = \frac{10\theta}{lc}$$

The measured angle of rotation (θ) depends upon

(a) the type or nature of the sample e.g. sugar solution.

(b) concentrations (c) of optical active components.

(c) length (l) of sample tube.

(d) wavelength (λ) of light source.

(e) temperature (T) of the sample.

SI unit of α is $\text{radm}^2\text{kg}^{-1}$. However practically concentration c is measured in gcm^{-3}, length of solution tube l in decimetre (where 1 decimetre = 1 dm = 10 cm) and angle of rotation θ in degree. Therefore, the practical unit of α can also be written as degree cubic centimetre per gram per decimetre shortly written as $^\circ\text{cm}^3\text{g}^{-1}(\text{dm})^{-1}$.

ILLUSTRATION 47

Calculate the thickness of quartz plate cut with its faces perpendicular to the optic axis, which will produce the same rotation as that of a 0.1 m long solution of concentration 400 kg m^{-3}. Given specific rotation of quartz 380 rad m^{-1} and that of sugar 0.011 $\text{radm}^2\text{kg}^{-1}$.

SOLUTION

Let t be the required thickness of the quartz plate. Then rotation produced by quartz in the plane of polarisation

$$\theta = 380t \qquad \ldots(1)$$

For sugar solution, we have

$$l = 0.1 \text{ m}, s = 0.011 \text{ rad m}^{-1} \text{ kg}^{-1} \text{ m}^3,$$

$$c = 400 \text{ kg m}^{-3}$$

$$\Rightarrow \quad \theta = slc = 0.011\times0.1\times400 \qquad \ldots(2)$$

From (1) and (2), we get

$$380t = 0.011\times0.1\times400$$

$$\Rightarrow \quad t = \frac{0.011\times0.1\times400}{380} = 1.6\times10^{-3} \text{ m}$$

Test Your Concepts-III

Based on Polarisation

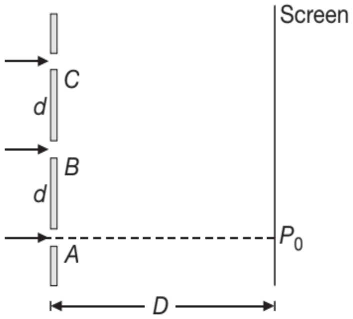
(Solutions on page H.126)

1. Two polarising sheets have their polarising directions parallel so that the intensity of the transmitted light is maximum. Through what angle must the either sheet be turned if the intensity is to drop by one-half?
2. A polariser and an analyser are oriented so that the maximum light is transmitted. What is the fraction of maximum light transmitted when analyser is rotated through (a) 30° (b) 60°?
3. Two polaroids are crossed to each other. If one of them is rotated through 60°, then what percentage of the incident unpolarised light will be transmitted by the polaroids?
4. Two polaroids are placed at 90° to each other and the transmitted intensity is zero. What happens when one more polaroid is placed between these two bisecting the angle between them?
5. A polaroid examines two adjacent plane-polarised light beams A and B whose planes of polarisation are mutually at right angles. In one position of the polaroid, the beam B shows zero intensity. From this position a rotation of 30° shows the two beams of equal intensities. Find the intensity ratio $\frac{I_A}{I_B}$ of the two beams.
6. Show that when a ray of light is incident on the surface of a transparent medium at the polarising angle, the reflected and transmitted rays are perpendicular to each other.
7. Unpolarised light of intensity 32 Wm^{-2} passes through three polarisers such that the transmission axis of the last polariser is crossed with the first, if the intensity of the emerging light is 3 Wm^{-2}, what is the angle between the transmission axes of the first two polarisers? At what angle will the transmitted intensity be maximum?
8. Yellow light is incident on the smooth surface of a block of dense flint glass for which the refractive index is 1.6640. Find the polarising angle. Also find the angle of refraction.
9. Calculate the thickness of (a) a quarter wave plate (b) a half wave plate, given that $\mu_e = 1.533$, $\mu_0 = 1.544$ and $\lambda = 5000$ Å.
10. Calculate the specific rotation if the plane of polarization is turned through 13.2°, traversing a length of 20 cm of 10% sugar solution.
11. A 5% solution taken in a decimetre tube produces an optical rotation of 20°. How much length of 10% solution of the same substance will produce a rotation of 30°?

SOLVED PROBLEMS

PROBLEM 1

Three equidistant slits of equal width being illuminated by a monochromatic parallel beam of light as shown in figure. A point P_0 is taken on the screen directly in front of A. If in this situation path difference $BP_0 - AP_0 = \frac{\lambda}{3}$ and $D \gg \lambda$. Show that the



(a) slit separation is given by $d = \sqrt{\frac{2\lambda D}{3}}$.

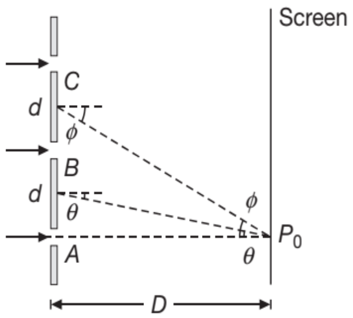
(b) intensity at P_0 is three times the intensity due to any of the three slits individually.

SOLUTION

(a) For calculating the path difference in the given situation, we redraw the arrangement as shown in figure. Since the path difference in the waves reaching from slit A and B to point P_0 is given by $BP_0 - AP_0 = \frac{\lambda}{3}$

Also, $BP_0 - AP_0 = d\sin\theta$

$$\Rightarrow \; d\sin\theta = \frac{\lambda}{3}$$



For small θ, we can use $\sin\theta \approx \theta$

$$\Rightarrow \; \theta d = \frac{\lambda}{3} \qquad \ldots(1)$$

From figure, for $D \gg d$, θ is given by

$$\theta = \frac{\frac{d}{2}}{D}$$

So, from equation (1), we get

$$d\left(\frac{d}{2D}\right) = \frac{\lambda}{3}$$

$$\Rightarrow \; d = \sqrt{\frac{2\lambda D}{3}} \qquad \ldots(2)$$

(b) If we consider Δx_{AB} as the path difference between waves coming from A and B which is given as $\Delta x_{AB} = \frac{\lambda}{3}$. If ϕ_{AB} is the corresponding phase difference, then

$$\phi_{AB} = \frac{2\pi}{\lambda}\Delta x_{AB} = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{3}\right) = \frac{2\pi}{3}$$

So, amplitude of resultant wave obtained at P_0 due to sources A and B (each of amplitude a) is

$$a_{AB} = \sqrt{a^2 + a^2 + 2a^2\cos\left(\frac{2\pi}{3}\right)} = a$$

Similarly, for waves coming from slits B and C to point P_0, we use

$$\Delta x_{BC} = d\sin\phi, \text{ where } \sin\phi \approx \frac{d + \frac{d}{2}}{D} = \frac{3d}{2D}$$

$$\Rightarrow \; \Delta x_{BC} = d\left(\frac{3d}{2D}\right) = \frac{3d^2}{2D} \qquad \ldots(3)$$

From equation (2), we have $d = \sqrt{\frac{2\lambda D}{3}}$

Substituting this value of d in equation (3), we get

$$\Delta x_{BC} = \frac{3d^2}{2D} = \frac{3}{2D}\left(\frac{2\lambda D}{3}\right) = \lambda$$

Thus the waves from slits B and C will reach point P_0 in same phase, so the resulting amplitude due to superposition of the waves at P_0 from slits B and C will become $a_{BC} = 2a$

Phase difference between waves arriving at P_0 from sources AB and BC is

$$\Delta\phi = \phi_{BC} - \phi_{AB} = 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$$

So the resultant wave amplitude of the waves arriving at the point P_0 is given by

$$a_r = \sqrt{(a_{AB})^2 + (a_{BC})^2 + 2(a_{AB})(a_{AB})\cos\left(\frac{4\pi}{3}\right)}$$

where, $a_{AB} = a$, $a_{BC} = 2a$ and $\cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$

$$\Rightarrow \quad a_r = \sqrt{a^2 + (2a)^2 + 2(a)(2a)\left(-\frac{1}{2}\right)} = \sqrt{3}a$$

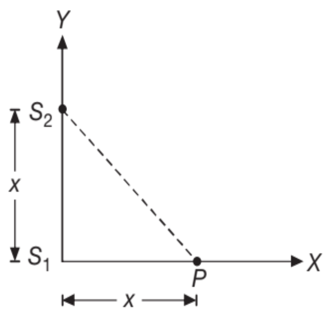
Since intensity of light is directly proportional to the square of amplitude, so we can conclude that intensity at point P_0 will be three times the intensity due to any of the three slits individually.

PROBLEM 2

An interference pattern is observed due to two coherent sources S_1 placed at origin and S_2 placed at $(0, 3\lambda, 0)$, where λ is the wavelength of the sources. A detector D is moved along the positive x-axis. Find the coordinates on the x-axis (excluding $x = 0$ and ∞) where maximum intensity is observed.

SOLUTION

At $x = 0$, path difference is 3λ. Hence, third order maxima will be obtained. At $x \to \infty$, path difference is zero. Hence, zero order maxima is obtained. In between first and second order maximas will be obtained.



For First order maxima, we have

$$S_2P - S_1P = \lambda$$

$$\Rightarrow \quad \sqrt{x^2 + 9\lambda^2} - x = \lambda$$

$$\Rightarrow \quad \sqrt{x^2 + 9\lambda^2} = x + \lambda$$

Squaring both sides, we get $x^2 + 9\lambda^2 = x^2 + \lambda^2 + 2x\lambda$

Solving this, we get

$$x = 4\lambda$$

For Second order maxima, we have

$$S_2P - S_1P = 2\lambda$$

$$\Rightarrow \quad \sqrt{x^2 - 9\lambda^2} - x = 2\lambda$$

$$\Rightarrow \quad \sqrt{x^2 + 9\lambda^2} = (x + 2\lambda)$$

Squaring both sides, we get

$$x^2 + 9\lambda^2 = x^2 + 4\lambda^2 + 4x\lambda$$

Solving, we get

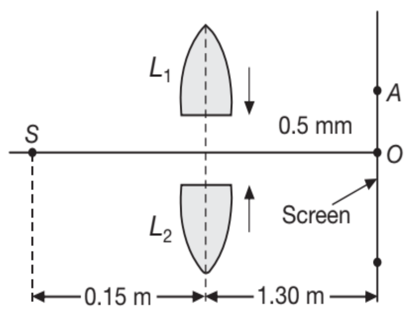
$$x = \frac{5}{4}\lambda = 1.25\lambda$$

Hence, the desired x coordinates are

$$x = 1.25\lambda \text{ and } x = 4\lambda$$

PROBLEM 3

In given figure, S is a monochromatic point source emitting light of wavelength $\lambda = 500$ nm. A thin lens of circular shape and focal length 0.10 m is cut into two identical halves L_1 and L_2 by a plane passing through a diameter. The two halves are placed symmetrically about the central axis SO with a gap of 0.5 mm. The distance along the axis from S to L_1 and L_2 is 0.15 m while that from L_1 and L_2 to O is 1.30 m. The screen at O is normal to SO.



(i) If the third intensity maximum excluding central maximum, occurs at the point A on the screen, find the distance OA.

(ii) If the gap between L_1 and L_2 is reduced from its original value of 0.5 mm, will the distance OA increase, decrease, or remain the same.

SOLUTION

(i) For the lens, $u = -0.15$ m, $f = +0.10$ m

Therefore, using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we get

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{1}{(-0.15)} + \frac{1}{(0.10)}$$

$\Rightarrow v = 0.3$ m

Linear magnification, $m = \frac{v}{u} = \frac{0.3}{-0.15} = -2$

Hence, two images S_1 and S_2 of S will be formed at 0.3 m from the lens as shown in figure. Image S_1 due to part 1 will be formed at 0.5 mm above its optics axis $(m = -2)$. Similarly, S_2 due to part 2 is formed 0.5 mm below the optic axis of this part as shown.

Hence, distance between S_1 and S_2 is $d = 1.5$ mm

Also, $D = 1.30 - 0.30 = 1.0 \text{ m} = 10^3$ mm

and $\lambda = 500 \text{ nm} = 5 \times 10^{-4}$ mm

So, fringe width is given by

$$\beta = \frac{\lambda D}{d} = \frac{(5\times10^{-4})(10^3)}{(1.5)} \text{ mm} = \frac{1}{3} \text{ mm}$$

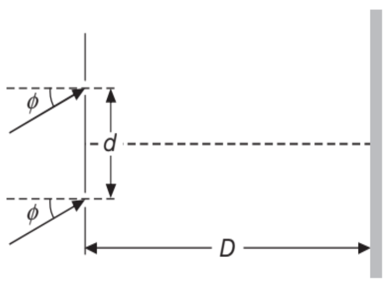
Now, as the point A is at the third maxima

$$\Rightarrow OA = 3\beta = 3\left(\frac{1}{3}\right) = 1 \text{ mm}$$

(ii) If the gap between L_1 and L_2 is reduced, d will decrease. Hence, the fringe width β will increase or the distance OA will increase.

PROBLEM 4

Light of wavelength $\lambda = 500$ nm falls on two narrow slits placed a distance $d = 50\times10^{-4}$ cm apart, at an angle $\phi = 30°$ relative to the slits shown in figure. ON the lower slit a transparent slab of thickness 0.1 nm and refractive index $\frac{3}{2}$ is placed. The interference pattern is observed on a screen at a distance $D = 2$ m from the slits.



(a) Locate the position of the central maxima.

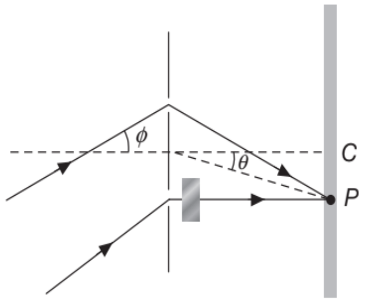
(b) Find the order of minima closest to centre C of screen.

(c) How many fringes will pass over C, if we remove the transparent slab from the lower slit?

SOLUTION

(a) Path difference is given by

$$\Delta x = d\sin\phi + d\sin\theta - (\mu - 1)t$$



For central maxima, $\Delta x = 0$

$$\Rightarrow \sin\theta = \frac{(\mu-1)t}{d} - \sin\phi$$

$$\Rightarrow \sin\theta = \frac{\left(\frac{3}{2}-1\right)(0.1)}{50\times10^{-3}} - \sin(30°) = \frac{1}{2}$$

$\Rightarrow \theta = 30°$

(b) At C, $\theta = 0°$, so we get

$$\Delta x = d\sin\phi - (\mu-1)t$$

$$\Rightarrow \Delta x = (50\times10^{-3})\left(\frac{1}{2}\right) - \left(\frac{3}{2}-1\right)(0.1)$$

$\Rightarrow \Delta x = 0.025 - 0.05 = -0.025$ mm

Substituting, $\Delta x = n\lambda$, we get

$$n = \frac{\Delta x}{\lambda} = \frac{-0.025}{500\times10^{-6}} = -50$$

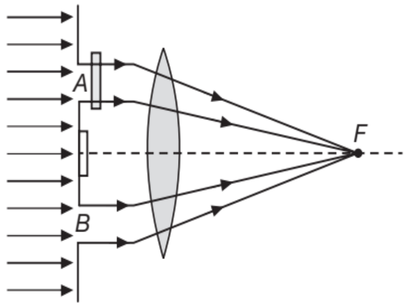
Hence, at C there will be maxima. Therefore the order of minima closest to the C are -49.

(c) Number of fringes shifted upwards is

$$N = \frac{(\mu-1)t}{\lambda} = \frac{\left(\frac{3}{2}-1\right)(0.1)}{500\times10^{-6}} = 100$$

PROBLEM 5

In a modified Young's double slit experiment, a monochromatic uniform and parallel beam of light of wavelength 6000 Å and intensity $\left(\frac{10}{\pi}\right)$ Wm^{-2} is incident normally on two apertures A and B of radii 0.001 m and 0.002 m respectively. A perfectly transparent film of thickness 2000 Å and refractive index 1.5 for the wavelength of 6000 Å is placed in front of aperture A (shown in figure).



Calculate the power $(\text{in } W)$ received at the focal spot F of the lens. The lens is symmetrically placed with respect to the apertures. Assume that 10% of the power received by each aperture goes in the original direction and is brought to the focal spot.

SOLUTION

Power received by aperture A is given by

$$P_A = I(\pi r_A^2) = \frac{10}{\pi}(\pi)(0.001)^2 = 10^{-5}\text{ W} \quad \left\{\because I = \frac{P}{A}\right\}$$

Power received by aperture B is given by

$$P_B = I(\pi r_B^2) = \frac{10}{\pi}(\pi)(0.002)^2 = 4\times10^{-5}\text{ W}$$

Only 10% of P_A and P_B goes to the original direction, so

Portion of P_A going to original direction is $P_1 = 10^{-6}\text{ W}$

Portion of P_B going to original direction is $P_2 = 4\times10^{-6}\text{ W}$

Path difference created by slab is given by

$$\Delta x = (\mu-1)t = (1.5-1)(2000) = 1000\text{ Å}$$

Corresponding phase difference is given by

$$\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x = \frac{2\pi}{6000}\times1000 = \frac{\pi}{3}$$

Now, resultant power at the focal point is given by

$$P = P_1 + P_2 + 2\sqrt{P_1P_2}\cos\phi$$

$$\Rightarrow \quad P = 10^{-6} + 4\times10^{-6} + 2\sqrt{(10^{-6})(4\times10^{-6})}\cos\left(\frac{\pi}{3}\right)$$

$$\Rightarrow \quad P = 7\times10^{-6}\text{ W}$$

PROBLEM 6

A central portion with a width of $d = 0.5\text{ mm}$ is cut out of a convergent lens having a focal length of $f = 10\text{ cm}$, as shown in figure. Both halves are tightly fitted against each other. The lens receives monochromatic light $(\lambda = 5000\text{ Å})$ from a point source at a distance of 5 cm from it.

(a) At what distance should a screen be fixed on the opposite side of the lens to observe three interference bands on it?

(b) What is the maximum possible number of interference bands that can be observed in this installation?

SOLUTION

Applying the lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

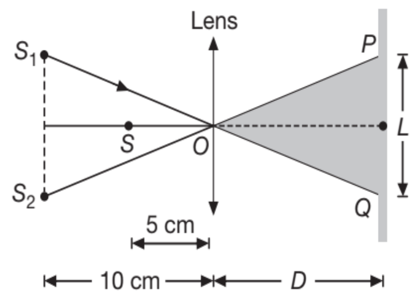
$$\frac{1}{v} + \frac{1}{5} = \frac{1}{10}$$

$$\Rightarrow \quad v = -10\text{ cm}$$

$$\Rightarrow \quad m = \frac{v}{u} = \frac{-10}{-5} = 2$$

i.e., two virtual sources are formed with distance between them

$$d = 0.5\text{ mm}$$



Fringe width $\beta = \frac{\lambda(D+10)}{d}$

Fringes are observed between the region P and Q (waves interfere in this region only), where

$$\frac{L}{D} = \frac{d}{10}$$

$$\Rightarrow \quad L = \frac{Dd}{10}$$

Number of fringes that can be observed on the screen is given by

$$N = \frac{L}{\beta} = \frac{d^2 D}{10\lambda(D+10)} \quad \ldots(1)$$

Substituting the values, we get

$$3 = \frac{(0.05)^2 D}{10 \times 5 \times 10^{-5}(D+10)}$$

Solving this equation, we get

$$D = 15 \text{ cm}$$

From equation (1), we have

$$N = \frac{d^2}{10\lambda\left(1+\frac{10}{D}\right)}$$

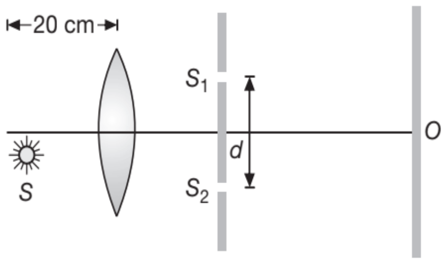
N will be maximum when $D \to \infty$

$$\Rightarrow \quad \frac{10}{D} \to 0$$

$$\Rightarrow \quad N_{max} = \frac{d^2}{10\lambda} = \frac{(0.05)^2}{10 \times 5 \times 10^{-5}} = 5$$

PROBLEM 7

A point source is placed at a distance $\frac{d}{2}$ below the principal axis of an equiconvex lens of refractive index $\frac{3}{2}$ and radius 20 cm. The emergent light from lens having wavelength $\lambda = 5000$ Å falls on the slits S_1 and S_2 separated by $d = 1$ mm which are placed symmetrically along the principal axis. The resulting interference pattern is observed on the screen kept at a distance $D = 1$ m from the slit plane.



(a) Find the position of central maxima and its width

(b) Find the intensity at point O.

SOLUTION

Using Lens Maker's Formula, we get

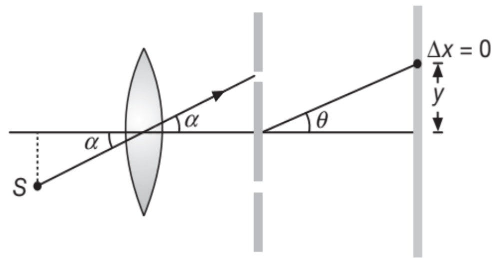
$$\frac{1}{f} = \left(\frac{3}{2}-1\right)\left(\frac{1}{20}-\frac{1}{-20}\right)$$

$$\Rightarrow \quad f = 20 \text{ cm}$$

Since source lies in focal plane of lens. So, all the emergent rays will be parallel. So,

$$\tan\alpha = \frac{d/2}{20} = \frac{d}{40} = \frac{1}{400} \approx \sin\alpha$$

Initial path difference $(\Delta x)_{initial} = d\sin\alpha$



Let the central maxima is obtained at angle θ. Then

$$d\sin\theta = d\sin\alpha$$

$$\Rightarrow \quad \sin\theta = \sin\alpha$$

$$\Rightarrow \quad \tan\theta = \tan\alpha$$

$$\Rightarrow \quad \frac{y}{D} = \frac{d}{40}$$

$$\Rightarrow \quad y = \frac{Dd}{40}$$

$$\Rightarrow \quad y = \frac{100}{40}d = 2.5d = 2.5 \text{ mm}$$

At O, net path difference is given by

$$\Delta x = d\sin\alpha = \frac{1}{400} \text{ mm}$$

Since, $\phi = \frac{2\pi}{\lambda}\Delta x$

$$\Rightarrow \quad \phi = \frac{2\pi}{5000 \times 10^{-10}} \times \frac{1}{400} \times 10^{-3}$$

$$\Rightarrow \quad \phi = 10\pi$$

Since, $I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$

$$\Rightarrow \quad I = I_{max}$$

PROBLEM 8

In a given *YDSE* setup, the upper slit is covered by a thin glass plate of refractive index 1.4 while the lower slit is covered by another glass plate having the same thickness as the first one but having refractive index 1.7. Interference pattern is observed using light of wavelength 5400 Å. It is found that the point P on the screen where the central maximum $(n=0)$ fell before the glass plates were inserted now has $\frac{3}{4}$ the original intensity. It is further observed that what used to be the 5^{th} maximum earlier, lies below the point P while the 6^{th} minimum lies above P. Calculate the thickness of the glass plate. Absorption of light by glass plate may be neglected.

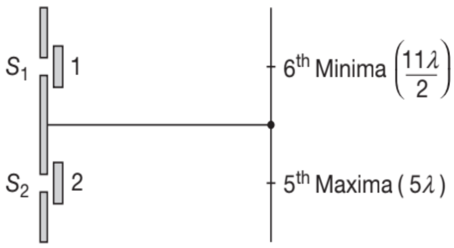
SOLUTION

As the given refractive indices of the glass plates are $\mu_1 = 1.4$ and $\mu_2 = 1.7$ and if t be the thickness of each glass plate then the path difference at the screen centre O, due to insertion of glass plates will be

$$\Delta x = (\mu_2 - \mu_1)t = (1.7 - 1.4)t$$

$$\Rightarrow \quad \Delta x = 0.3t \qquad \dots(1)$$

As 5^{th} maxima (earlier) lies below point O and 6^{th} minima lies above point O, this path difference must lie between 5λ and 5.5λ. This situation is shown in figure.



Due to the path difference Δx, the phase difference at O will be

$$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}(5\lambda + \Delta_1)$$

$$\Rightarrow \quad \phi = \left(10\pi + \frac{2\pi}{\lambda}\Delta_1\right) \qquad \dots(2)$$

In above equation Δ_1 is considered less than $\frac{\lambda}{2}$.

Intensity at point O is given $\frac{3}{4}I_{max}$ so we use the intensity at a point where the phase difference between the two waves is ϕ is given as

$$I(\phi) = I_{max}\cos^2\left(\frac{\phi}{2}\right)$$

$$\Rightarrow \quad \frac{3}{4}I_{max} = I_{max}\cos^2\left(\frac{\phi}{2}\right)$$

$$\Rightarrow \quad \frac{3}{4} = \cos^2\left(\frac{\phi}{2}\right)$$

$$\Rightarrow \quad \cos\left(\frac{\phi}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \frac{\phi}{2} = \frac{\pi}{6}$$

$$\Rightarrow \quad \phi = \frac{\pi}{3} \qquad \dots(3)$$

From equation, (2) and (3), we get

$$\Delta_1 = \frac{\lambda}{6}$$

So, path difference at O is given by

$$\Delta x = 5\lambda + \frac{\lambda}{6} = \frac{31}{6}\lambda \qquad \dots(4)$$

Since from (1), $\Delta x = 0.3t$

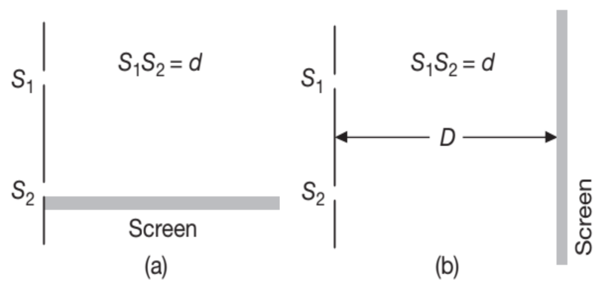
$$\Rightarrow \quad 0.3t = \frac{31\lambda}{6}$$

$$\Rightarrow \quad t = \frac{31\lambda}{6(0.3)} = \frac{(31)(5400 \times 10^{-10})}{1.8}\text{ m}$$

$$\Rightarrow \quad t = 9.3 \times 10^{-6}\text{ m} = 9.3\ \mu\text{m}$$

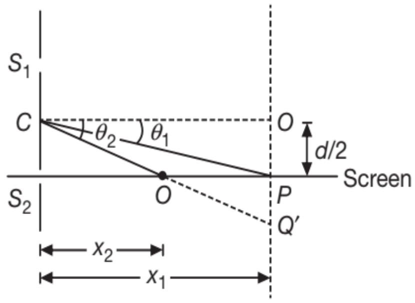
PROBLEM 9

The interference pattern of a Young's double slit experiment is observed in two ways by placing the screen in two possible ways as shown in figure (a) and (b). The distance between two consecutive right most minima on the screen of figure (a) using light of wavelength $\lambda_1 = 4000$ Å is observed to be 600 times the fringe width in the screen of figure (b) using the wavelength $\lambda_2 = 6000$ Å. If D (as shown in figure) is 1 m then find the separation between the coherent sources S_1 and S_2. Given that $d > \frac{3\lambda_1}{2}$.



SOLUTION

Had the screen been perpendicular to S_2P, then P and Q' would have been the positions of first and second minima (last two).



Since, the angular positions of minima do not depend on the position of the screen, so the second minima is formed at Q on the screen.
For right most minima at P, we have

$$d\sin\theta_1 = \frac{\lambda_1}{2} \quad \ldots(1)$$

For small angles, we have

$$\sin\theta_1 \approx \tan\theta_2 = \frac{d/2}{x_2}$$

Substituting in equation (1), we get

$$x_2 = \frac{d^2}{\lambda_1} \quad \ldots(2)$$

For next minima at Q, we have

$$d\sin\theta_2 = \frac{3}{2}\lambda_1 \quad \ldots(3)$$

For small angles, we have

$$\sin\theta_2 \approx \tan\theta_2 = \frac{\frac{d}{2}}{x_2}$$

Substituting in equation (3), we get

$$x_2 = \frac{d^2}{3\lambda_1} \quad \ldots(4)$$

$$\Rightarrow \quad PQ = x_1 - x_2 = \frac{2d^2}{3\lambda_1} \quad \ldots(5)$$

In the second case, fringe width is given by

$$\beta = \frac{\lambda_2 D}{d} \quad \ldots(6)$$

Since it is given that $PQ = 600\beta$

$$\Rightarrow \quad \frac{2d^2}{3\lambda_1} = 600\frac{\lambda_2 D}{d}$$

$$\Rightarrow \quad d^3 = 900\lambda_1\lambda_2 D$$

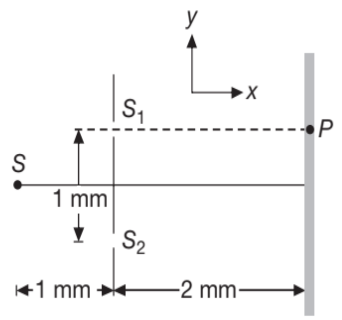
$$\Rightarrow \quad d^3 = 900\times 4000\times 6000\times 10^{-20}\times 1$$

$$\Rightarrow \quad d^3 = 216\times 10^{-12}$$

$$\Rightarrow \quad d = 6\times 10^{-4}\text{ m} = 0.6\text{ mm}$$

PROBLEM 10

In a Young's double slit experiment set-up source S of wavelength 5000 Å illuminates two slits S_1 and S_2, which act as two coherent sources. The source S oscillates about its shown position according to the equation $y = 0.5\sin(\pi t)$, where y is in millimetres and t in seconds.



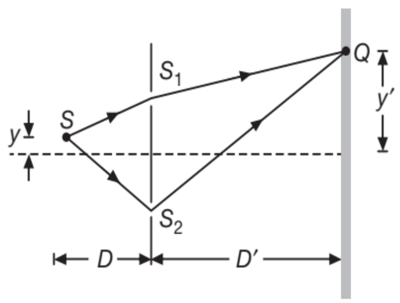
(a) Locate the position of the central maxima as a function of time.

(b) Calculate the minimum value of t for which the intensity at point P on the screen exactly in front of the upper slit becomes maximum.

SOLUTION

(a) Net path difference of the waves reaching at Q, is

$$\Delta x = \frac{yd}{D} + \frac{y'd}{D'}$$



For central maximum, $\Delta x = 0$

$$\Rightarrow \quad y' = -\frac{D'}{D}y'$$

$$\Rightarrow \quad y' = -\left(\frac{2}{1}\right)(0.5\sin(\pi t))$$

$\Rightarrow\ y' = -\sin(\pi t)$ mm

(b) $y' = \dfrac{d}{2}$, at point P exactly in front of S_1, so we have

$$\Delta x = \left(\frac{yd}{D}\right) + \frac{\left(\frac{d^2}{2}\right)}{D'}$$

For maximum intensity, we have path difference to be an even multiple of $\dfrac{\lambda}{2}$, so

$$\Delta x = (2n)\frac{\lambda}{2} = n\lambda$$

Substituting the values, we get

$$0.5\sin(\pi t) + 0.25 = 0.5n$$

$$\Rightarrow \sin(\pi t) = \frac{0.5n - 0.25}{0.5}$$

For minimum value of t, we have $n = 1$

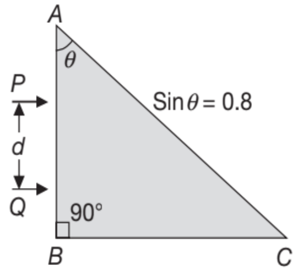
$$\Rightarrow \sin(\pi t) = 0.5$$

$$\Rightarrow \pi t = \frac{\pi}{6}$$

$$\Rightarrow t = \frac{1}{6} = 0.167 \text{ s}$$

PROBLEM 11

Two parallel beams of light P and Q (separation d) containing radiations of wavelengths 4000 Å and 5000 Å (which are mutually coherent in each wavelength separately) are incident normally on a prism as shown in figure. The refractive index of the prism as a function of wavelength is given by the relation, $\mu(\lambda) = 1.20 + \dfrac{b}{\lambda^2}$ where λ is in Å and b is positive constant. The value of b is such that the condition for total reflection at the face AC is just satisfied for one wavelength and is not satisfied for the other.



(a) Find the value of b

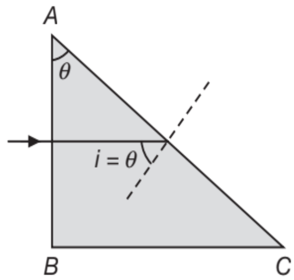
(b) Find the deviation of the beams transmitted through the face AC

(c) A convergent lens is used to bring these transmitted beams into focus. If the intensities of the upper and the lower beams immediately after transmission from the face AC, are $4I$ and I respectively, find the resultant intensity at the focus.

SOLUTION

(a) Total internal reflection (TIR) will take place first for those wavelength for which critical angle is small or μ is large.

From the given expression of μ, it is more for the wavelength for which value of λ is less.



Thus, condition of TIR is just satisfied for 4000 Å

$\Rightarrow\ i = C$ for 4000 Å

$\Rightarrow\ \theta = C$

$\Rightarrow\ \sin\theta = \sin C$

Since $\sin\theta = 0.8$ and $\sin C = \dfrac{1}{\mu}$, so we get

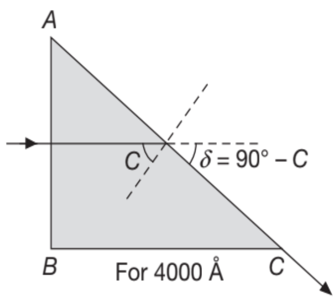
$$\Rightarrow 0.8 = \frac{1}{\mu} \quad \text{(for 4000 Å)}$$

$$\Rightarrow 0.8 = \frac{1}{1.20 + \frac{b}{(4000)^2}}$$

Solving this equation, we get

$$b = 8.0 \times 10^5\ (\text{Å})^2$$

(b) For, 4000 Å condition of TIR is just satisfied. Hence, it will emerge from AC, just grazingly.

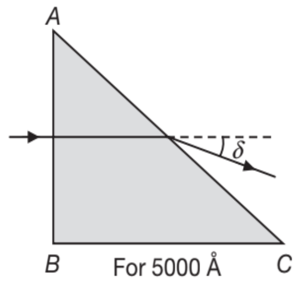


So, deviation for 4000 Å is given by

$$\Rightarrow \delta_{4000\text{ Å}} = 90 - i = 90 - \sin^{-1}(0.8) \approx 37°$$

For 5000 Å, we have

$$\mu = 1.2 + \frac{b}{\lambda^2} = 1.2 + \frac{8\times10^5}{(5000)^2} = 1.232$$



Applying, $\mu = \dfrac{\sin i_{air}}{\sin i_{medium}}$

$$\Rightarrow 1.232 = \frac{\sin i_{air}}{\sin\theta} = \frac{\sin i_{air}}{0.8}$$

$$\Rightarrow i_{air} = 80.26°$$

So, deviation for 5000 Å is given by

$$\delta_{5000\text{ Å}} = i_{air} - i_{medium} = 80.26° - \sin^{-1}(0.8) = 27.13°$$

(c) The intensity of the upper beam (4000 Å) and lower beam (5000 Å) after transmission are $4I$ and I respectively, then

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Since no phase change takes place for the waves refracted from the lens, so $\phi = 0°$.

$$\Rightarrow I_R = 4I + I + 2\sqrt{(4I)I}\cos(0°)$$

$$\Rightarrow I_R = 9I$$

PROBLEM 12

A glass plate of refractive index 1.5 is coated with a thin layer of thickness t and refractive index 1.8. Light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and the two reflected rays interfere. Write the condition for their constructive interference. If $\lambda = 648$ nm, obtain the least value of t for which the rays interfere constructively.

SOLUTION

Incident ray AB is partly reflected as ray 1 from the upper surface and partly reflected as ray 2 from the lower surface of the layer of thickness t and refractive index $\mu_1 = 1.8$ as shown in figure. Path difference between the two rays would be

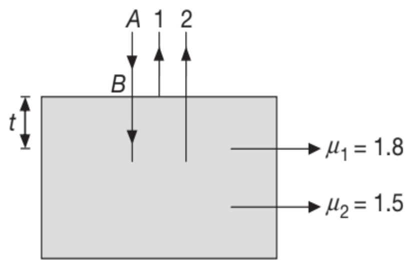
$$\Delta x = 2\mu_1 t = 2(1.8)t = 3.6t$$

Ray 1 is reflected from a denser medium, therefore, it undergoes a phase change of π, whereas the ray 2 gets reflected from a rarer medium, therefore, there is no change in phase of ray 2.

Hence, phase difference between rays 1 and 2 would be $\Delta\phi = \pi$. Therefore, condition of constructive interference will be

$$\Delta x = \left(n - \frac{1}{2}\right)\lambda \qquad \text{where } n = 1, 2, 3, \ldots$$

$$\Rightarrow 3.6t = \left(n - \frac{1}{2}\right)\lambda$$



Least values of t is corresponding to $n = 1$ or

$$t_{min} = \frac{\lambda}{2\times3.6}$$

$$\Rightarrow t_{min} = \frac{648}{7.2}\text{ nm}$$

$$\Rightarrow t_{min} = 90\text{ nm}$$

Conceptual Note(s)

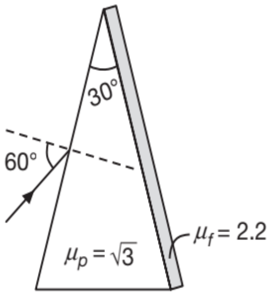
(a) For a wave (whether it is sound or electromagnetic), a medium is denser or rarer is decided from the speed of wave in that medium. In denser medium speed of wave is less. For example, water is rarer for sound, while denser for light compared to air because speed of sound in water is more than in air, while speed of light is less.

(b) In transmission/refraction, no phase change takes place. In reflection, there is a change of phase of π when it is reflected by a denser medium and phase change is zero if it is reflected by a rarer medium.

(c) If two waves in phase interfere having a path difference of Δx; then condition of maximum intensity would be $\Delta x = n\lambda$, $n = 0, 1, 2, ...$
But if two waves, which are already out of phase (a phase difference of π) interfere with path difference Δx, then condition of maximum intensity will be $\Delta x = \left(n - \frac{1}{2}\right)\lambda$, $n = 1, 2, ...$

PROBLEM 13

Shown in the figure is a prism of refracting angle $30°$ and refractive index $\mu_p (= \sqrt{3})$. The face AC of the prism is covered with a thin film of refractive index $\mu_f (= 2.2)$. A monochromatic light of wavelength $\lambda = 550$ nm falls on the face AB at an angle of incidence of $60°$. Calculate



(a) the angle of emergence.

(b) the minimum value of thickness t of the coated film so that the intensity of the emergent ray is maximum.

SOLUTION

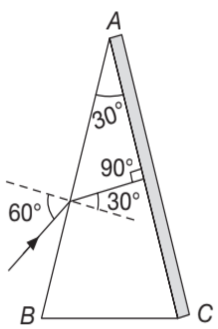
(a) Applying Snell's Law, we get

$$\sin i_1 = \mu \sin r_1$$

$$\Rightarrow \sin(60°) = \sqrt{3} \sin r_1$$

$$\Rightarrow \sin r_1 = \frac{1}{2}$$

$$\Rightarrow r_1 = 30°$$



Since, $r_1 + r_2 = A$

$$\Rightarrow r_2 = A - r_1 = 30° - 30° = 0°$$

Therefore, ray of light falls normally on the face AC and angle of emergence $e = i_2 = 0°$.

(b) Multiple reflection occurs between surfaces of film. Intensity will be maximum if interference takes place in the transmitted wave.

For maximum thickness, we have

$$\Delta x = 2\mu t = \lambda$$

where t is the thickness of coated film

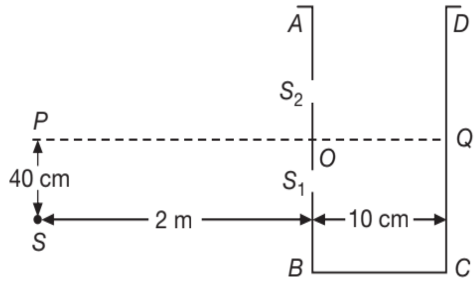
$$\Rightarrow t = \frac{\lambda}{2\mu} = \frac{550}{2 \times 2.2} = 125 \text{ nm}$$

PROBLEM 14

A vessel $ABCD$ of 10 cm width has two small slits S_1 and S_2 sealed with identical glass plates of equal thickness. The distance between the slits is 0.8 mm. POQ is the line perpendicular to the plane AB and passing through O, the middle point of S_1 and S_2. A monochromatic light source is kept at S, 40 cm below P and 2 m from the vessel, to illuminate the slits as shown in the figure alongside.

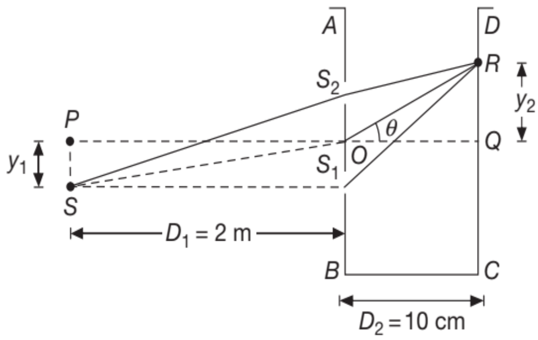
(a) Calculate the position of the central bright fringe on the other wall CD with respect to the line OQ.

(b) Now, a liquid is poured into the vessel and filled upto OQ. The central bright fringe is found to be at Q. Calculate the refractive index of the liquid.



SOLUTION

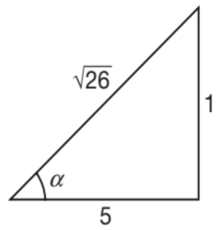
(a) Given $y_1 = 40$ cm, $D_1 = 2$ m $= 200$ cm, $D_2 = 10$ cm



$$\tan \alpha = \frac{y_1}{D_1} = \frac{40}{200} = \frac{1}{5}$$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{1}{5}\right)$$

$$\Rightarrow \sin\alpha = \frac{1}{\sqrt{26}} \approx \frac{1}{5} = \tan\alpha$$



Path difference between SS_1 and SS_2 is

$$\Delta x_1 = SS_1 - SS_2$$

$$\Rightarrow \Delta x_1 = d\sin\alpha = (0.8 \text{ mm})\left(\frac{1}{5}\right)$$

$$\Rightarrow \Delta x_1 = 0.16 \text{ mm} \quad \ldots(1)$$

Now, let at point R on the screen, central bright fringe is observed (i.e., net path difference $=0$). Path difference between S_2R and S_1R is

$$\Delta x_2 = S_2R - S_1R$$

$$\Rightarrow \Delta x_2 = d\sin\theta \quad \ldots(2)$$

Central bright fringe will be observed when net path difference is zero.

$$\Rightarrow \Delta x_2 - \Delta x_1 = 0$$

$$\Rightarrow \Delta x_2 = \Delta x_1$$

$$\Rightarrow d\sin\theta = 0.16$$

$$\Rightarrow (0.8)\sin\theta = 0.16$$

$$\Rightarrow \sin\theta = \frac{0.16}{0.8} = \frac{1}{5}$$

$$\Rightarrow \tan\theta = \frac{1}{\sqrt{24}}$$

$$\Rightarrow \sin\theta \approx \frac{1}{5}$$

$$\Rightarrow \tan\theta \approx \sin\theta = \frac{y_2}{D_2} = \frac{1}{5}$$

$$\Rightarrow y_2 = \frac{D_2}{5} = \frac{10}{5} = 2 \text{ cm}$$

Therefore, central bright fringe is observed at 2 cm above point Q on side CD.

Alternate solution for (a)

Δx at R will be zero if $\Delta x_1 = \Delta x_2$

$$\Rightarrow d\sin\alpha = d\sin\theta$$

$$\Rightarrow \alpha = \theta$$

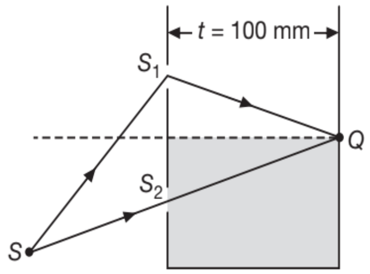
$$\Rightarrow \tan\alpha = \tan\theta$$

$$\Rightarrow \frac{y_1}{D_1} = \frac{y_2}{D_2}$$

$$\Rightarrow y_2 = \left(\frac{D_2}{D_1}\right)y_1 = \left(\frac{10}{200}\right)(40) \text{ cm}$$

$$\Rightarrow y_2 = 2 \text{ cm}$$

(b) The central bright fringe will be observed at point Q, if the path difference created by the liquid slab of thickness $t = 10$ cm or 100 mm is equal to Δx_1, so that the net path difference at Q becomes zero.



$$\Rightarrow (\mu - 1)t = \Delta x_1$$

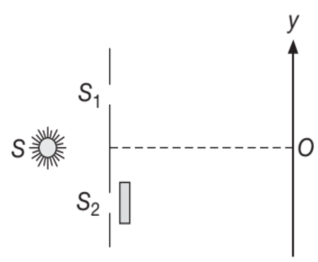
$$\Rightarrow (\mu - 1)(100) = 0.16$$

$$\Rightarrow \mu - 1 = 0.0016$$

$$\Rightarrow \mu = 1.0016$$

PROBLEM 15

The Young's double slit experiment is done in a medium of refractive index $\frac{4}{3}$. A light of 600 nm wavelength is falling on the slits having 0.45 mm separation. The lower slit S_2 is covered by a thin glass sheet of thickness 10.4 μm and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slits as shown in the figure.



(a) Find the location of central maximum (bright fringe with zero path difference) on the y-axis.

(b) Find the light intensity of point O relative to the maximum fringe intensity.

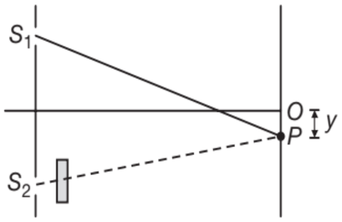
(c) Now, if 600 nm light is replaced by white light of range 400 to 700 nm, find the wavelengths of the light that form maxima exactly at point O.

[All wavelengths in the problem are for the given medium of refractive index $\frac{4}{3}$. Ignore dispersion]

SOLUTION

Given $\lambda = 600\ \text{nm} = 6\times10^{-7}\ \text{m}$,

$$d = 0.45\ \text{mm} = 0.45\times10^{-3}\ \text{m} \text{ and } D = 1.5\ \text{m}$$



Thickness of glass sheet, $t = 10.4\ \mu\text{m} = 10.4\times10^{-6}\ \text{m}$

Refractive index of the medium, $\mu_m = \frac{4}{3}$

And refractive index of glass sheet, $\mu_g = 1.5$

(a) Let central maximum is obtained at a distance y below point O.

$$\Rightarrow \Delta x_1 = S_1P - S_2P = \frac{yd}{D}$$

Path difference due to glass sheet is given by

$$\Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right)t$$

Net path difference will be zero, when we have

$$\Delta x_1 = \Delta x_2$$

$$\Rightarrow \frac{yd}{D} = \left(\frac{\mu_g}{\mu_m} - 1\right)t$$

$$\Rightarrow y = \left(\frac{\mu_g}{\mu_m} - 1\right)t\frac{D}{d}$$

Substituting the values, we get

$$y = \left(\frac{1.5}{4/3} - 1\right)\frac{10.4\times10^{-6}(1.5)}{0.45\times10^{-3}}$$

$$\Rightarrow y = 4.33\times10^{-3}\ \text{m}$$

$$\Rightarrow y = 4.33\ \text{mm}$$

(b) At O, $\Delta x_1 = 0$ and $\Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right)t$

So, net path difference is

$$\Delta x = \Delta x_2$$

Corresponding phase difference, $\Delta\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x$

Substituting the values, we get

$$\phi = \Delta\phi = \frac{2\pi}{6\times10^{-7}}\left(\frac{1.5}{4/3} - 1\right)(10.4\times10^{-6})$$

$$\Rightarrow \phi = \left(\frac{13}{3}\right)\pi$$

Now, $I(\phi) = I_{\max}\cos^2\left(\frac{\phi}{2}\right)$

$$\Rightarrow I = I_{\max}\cos^2\left(\frac{13\pi}{6}\right)$$

$$\Rightarrow I = \frac{3}{4}I_{\max}$$

(c) At O, path difference is $\Delta x = \Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right)t$

For maximum intensity at O, we have

$$\Delta x = n\lambda \text{, where } n = 1, 2, 3, \ldots\ldots$$

$$\Rightarrow \lambda = \frac{\Delta x}{1}, \frac{\Delta x}{2}, \frac{\Delta x}{3}, \ldots\ldots \text{ and so on}$$

$$\Rightarrow \Delta x = \left(\frac{1.5}{4/3} - 1\right)(10.4\times10^{-6}\ \text{m})$$

$$\Rightarrow \Delta x = \left(\frac{1.5}{4/3} - 1\right)(10.4\times10^{3}\ \text{nm}) = 1300\ \text{nm}$$

So, maximum intensity will be corresponding to

$$\lambda = 1300\ \text{nm}, \frac{1300}{2}\ \text{nm}, \frac{1300}{3}\ \text{nm}, \frac{1300}{4}\ \text{nm}, \ldots$$

$$\Rightarrow \lambda = 1300\ \text{nm}, 650\ \text{mm}, 433.33\ \text{nm}, 325\ \text{nm}, \ldots.$$

The wavelength in the range $400\ \text{nm}$ to $700\ \text{nm}$ are $650\ \text{nm}$ and $433.33\ \text{nm}$.

PROBLEM 16

In Young's experiment, the source is red light of wavelength $7\times10^{-7}\ \text{m}$. When a thin glass plate of refractive index 1.5 at this wavelength is put in the path of one of the interfering beams, the central bright fringe shifts by $10^{-3}\ \text{m}$ to the position previously occupied by the 5th bright fringe. Find the thickness of the plate. When the source is now changed to green light of wavelength 5×10^{-7} m, the central fringe shifts to a position initially occupied by the 6th bright fringe due to red light. Find the refractive index of glass for green light. Also estimate the change in fringe width due to the change in wavelength.

SOLUTION

Path difference due to the glass slab,

$$\Delta x=(\mu-1)t=(1.5-1)t=0.5t$$

Due to this slab, 5 red fringes have been shifted upwards. So, we have

$$\Delta x=5\lambda_{red}$$

$$\Rightarrow\quad 0.5t=(5)(7\times10^{-7}\text{ m})$$

$$\Rightarrow\quad t=\text{ thickness of glass slab }=7\times10^{-6}\text{ m}$$

Let μ' be the refractive index for green light, then

$$\Delta x'=(\mu'-1)t$$

Now the shifting is of 6 fringes of red light. So, we have

$$\Delta x'=6\lambda_{red}$$

$$\Rightarrow\quad (\mu'-1)t=6\lambda_{red}$$

$$\Rightarrow\quad (\mu'-1)=\frac{(6)(7\times10^{-7})}{7\times10^{-6}}=0.6$$

$$\Rightarrow\quad \mu'=1.6$$

Since the shifting of 5 bright fringes was equal to 10^{-3} m

$$\Rightarrow\quad 5\beta_{red}=10^{-3}\text{ m},\text{ where } \beta \text{ is the Fringe width}$$

$$\Rightarrow\quad \beta_{red}=\frac{10^{-3}}{5}\text{ m}=0.2\times10^{-3}\text{ m}$$

Now since $\beta=\dfrac{\lambda D}{d}$

$$\Rightarrow\quad \beta\propto\lambda$$

$$\Rightarrow\quad \frac{\beta_{green}}{\beta_{red}}=\frac{\lambda_{green}}{\lambda_{red}}$$

$$\Rightarrow\quad \beta_{green}=\beta_{red}\frac{\lambda_{green}}{\lambda_{red}}=(0.2\times10^{-3})\left(\frac{5\times10^{-7}}{7\times10^{-7}}\right)$$

$$\Rightarrow\quad \beta_{green}=0.143\times10^{-3}\text{ m}$$

$$\Rightarrow\quad \Delta\beta=\beta_{green}-\beta_{red}=(0.143-0.2)\times10^{-3}\text{ m}$$

$$\Rightarrow\quad \Delta\beta=-5.71\times10^{-5}\text{ m}$$

PROBLEM 17

A double slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and distance between the plane of slits and screen is 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 6300 Å.

(i) Calculate the fringe width.

(ii) One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the smallest thickness of the sheet to bring the adjacent minimum as the axis.

SOLUTION

Given $\mu=1.33$, $d=1$ mm, $D=1.33$ m and $\lambda=6300$ Å

(i) Wavelength of light in the given liquid is

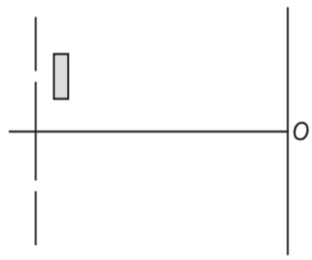
$$\lambda'=\frac{\lambda}{\mu}=\frac{6300}{1.33}\text{ Å}\approx4737\text{ Å}=4737\times10^{-10}\text{ m}$$

$$\Rightarrow\text{ Fringe width, }\beta=\frac{\lambda' D}{d}$$

$$\Rightarrow\quad \beta=\frac{(4737\times10^{-10}\text{ m})(1.33\text{ m})}{(1\times10^{-3}\text{ m})}$$

$$\Rightarrow\quad \beta=6.3\times10^{-4}\text{ m}=0.63\text{ mm}$$

(ii) Let t be the thickness of the glass slab.



Path difference due to glass slab at centre O is given by

$$\Delta x=\left(\frac{\mu_{glass}}{\mu_{liquid}}-1\right)t=\left(\frac{1.53}{1.33}-1\right)t$$

$$\Rightarrow\quad \Delta x=0.15t$$

Now, for the intensity to be minimum at O, this path difference should be equal to $\dfrac{\lambda'}{2}$

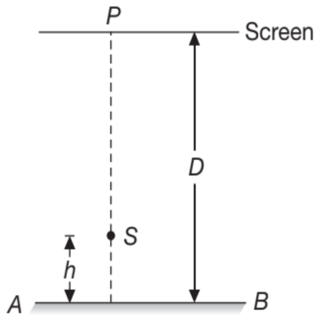
$$\Rightarrow \Delta x = \frac{\lambda'}{2}$$

$$\Rightarrow 0.15t = \frac{4737}{2} \text{ Å}$$

$$\Rightarrow t = 15790 \text{ Å} = 1.579 \ \mu\text{m}$$

PROBLEM 18

A point source S emitting light of wavelength 600 nm is placed at a very small height h above a flat reflecting surface AB (shown in figure). The intensity of the reflected light is 36% of the incident intensity. Interference fringes are observed on a screen placed parallel to the reflecting surface at a very large distance D from it.



(a) What is the shape of the interference fringes on the screen?

(b) Calculate the ratio of the minimum to the maximum intensities in the interference fringes formed near the point P (shown in the figure).

(c) If the intensity at point P corresponds to a maximum, calculate the minimum distance through which the reflecting surface AB should be shifted so that the intensity at P again becomes maximum.

SOLUTION

(a) Since there is symmetry about the line SP, so the shape of the interference fringes will be circular.

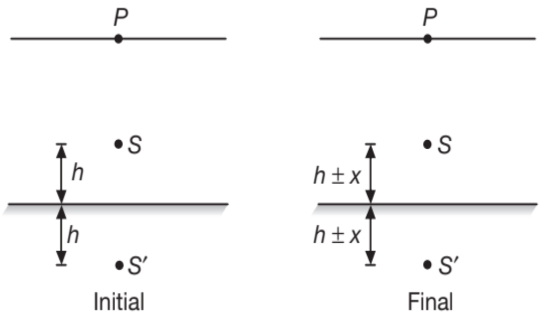
(b) Intensity of light reaching on the screen directly from the source $I_1 = I_0$ (say) and intensity of light reaching on the screen after reflecting from the mirror is $I_2 = 36\%$ of $I_0 = 0.36 I_0$.

$$\Rightarrow \frac{I_1}{I_2} = \frac{I_0}{0.36 I_0} = \frac{1}{0.36}$$

$$\Rightarrow \sqrt{\frac{I_1}{I_2}} = \frac{1}{0.6}$$

$$\Rightarrow \frac{I_{min}}{I_{max}} = \frac{\left(\sqrt{\frac{I_1}{I_2}} - 1\right)^2}{\left(\sqrt{\frac{I_1}{I_2}} + 1\right)^2} = \frac{\left(\frac{1}{0.6} - 1\right)^2}{\left(\frac{1}{0.6} + 1\right)^2} = \frac{1}{16}$$

(c) Initially path difference at P between two waves reaching from S and S' is as shown.



Since the ray is reflected from the surface of a denser medium, so it suffers an additional path change of $\frac{\lambda}{2}$ or a phase change of π.

For maximum at P, path difference equals $n\lambda$.

If AB is shifted by x, then this will cause an additional path difference of $2\left(x - \frac{\lambda}{2}\right)$ (for object and its image taken as coherent sources). Since reflection takes place at surface of denser medium, so this will produce an additional phase change of π or a path change of $\frac{\lambda}{2}$. So, we get

$$2\left(x - \frac{\lambda}{2}\right) = n\lambda$$

$$2x - \lambda = n\lambda$$

$$\Rightarrow 2x = (n+1)\lambda$$

$$\Rightarrow x = (n+1)\frac{\lambda}{2} \text{ where } n = 0, 1, 2, 3, \ldots$$

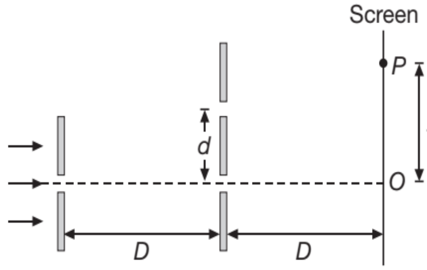
Now, to get minimum value of x, n must be minimum i.e., $n = 0$

$$\Rightarrow x = \frac{\lambda}{2}$$

$$\Rightarrow x = \frac{600}{2} = 300 \text{ nm}$$

PROBLEM 19

Consider the arrangement shown in figure. The distance D is large compared to the separation d between the slits.



(a) Find the minimum value of d so that there is a dark fringe at O.

(b) Suppose d has this value. Find the distance x at which the next bright fringe is formed.

(c) Find the fringe width.

SOLUTION

(a) The path difference at O is given as

$$\Delta x = 2\sqrt{D^2 + d^2} - 2D$$

For the dark fringe at O, this path difference should be

$$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots$$

For minimum value of d, we have

$$\Delta x = 2\sqrt{D^2 + d^2} - 2D = \frac{\lambda}{2}$$

$$\Rightarrow (D^2 + d^2)^{\frac{1}{2}} - D = \frac{\lambda}{4}$$

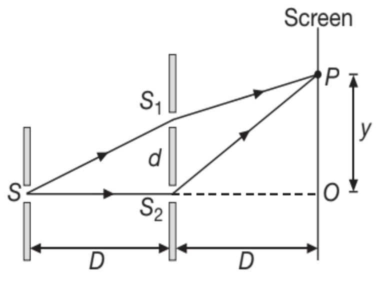
$$\Rightarrow D\left(1 + \frac{d^2}{D^2}\right)^{\frac{1}{2}} - D = \frac{\lambda}{4}$$

$$\Rightarrow D\left(1 + \frac{d^2}{2D^2}\right) - D = \frac{\lambda}{4}$$

$$\Rightarrow D + \frac{d^2}{2D} - D = \frac{\lambda}{4}$$

$$\Rightarrow d = \sqrt{\frac{D\lambda}{2}}$$

(b) At the above calculated value of d, first bright fringe will be obtained at a position where the path difference between the two waves will be λ.



For the situation shown in figure, the path difference in waves from S_1 and S_2 at point P is given as

$$\Delta x = (SS_1 + S_1P) - (SS_2 + S_2P)$$

$$\Rightarrow \Delta x = \left[\sqrt{D^2 + d^2} + \sqrt{(y-d)^2 + D^2}\right] - \left[D + \sqrt{D^2 + y^2}\right]$$

For the next bright fringe after first dark fringe,

$$\Delta x = \lambda$$

$$\Rightarrow \left[\sqrt{D^2 + d^2} + \sqrt{(y-d)^2 + D^2}\right] - \left[D + \sqrt{D^2 + y^2}\right] = \lambda$$

$$\Rightarrow D\left[\left(1 + \frac{d^2}{D^2}\right)^{\frac{1}{2}} + \left(1 + \frac{(y-d)^2}{D^2}\right)^{\frac{1}{2}}\right] - \left[D + D\left(1 + \frac{y^2}{D^2}\right)^{\frac{1}{2}}\right] = \lambda$$

$$\Rightarrow \left(D + \frac{d^2}{2D}\right) + D + \frac{(y-d)^2}{2D} - \left(D + D + \frac{y^2}{2D}\right) = \lambda$$

$$\Rightarrow \frac{d^2 + (y-d)^2 - y^2}{2D} = \lambda$$

$$\Rightarrow d^2 + y^2 + d^2 - 2yd - y^2 = 2\lambda D$$

$$\Rightarrow 2d^2 - 2yd = 2\lambda D$$

For $d = \sqrt{\frac{D\lambda}{2}}$, we get

$$2\left(\sqrt{\frac{D\lambda}{2}}\right)^2 - 2y\sqrt{\frac{D\lambda}{2}} = 2\lambda D$$

$$\Rightarrow 2\frac{D\lambda}{2} - 2y\sqrt{\frac{D\lambda}{2}} = 2\lambda D$$

$$\Rightarrow 2y\sqrt{\frac{D\lambda}{2}} = -D\lambda$$

Squaring both sides, we get

$$\left(2y\sqrt{\frac{D\lambda}{2}}\right)^2 = (-D\lambda)^2$$

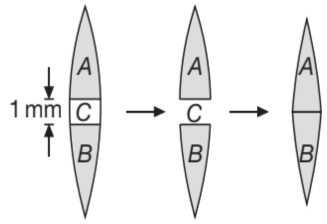
$$\Rightarrow y = \sqrt{\frac{D\lambda}{2}} = d$$

(c) Fringe width on screen can be given by the relation we studied in YDSE, given as

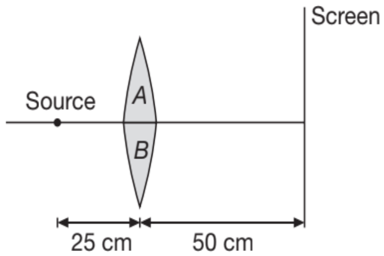
$$\beta = \frac{D\lambda}{d}$$

PROBLEM 20

In Billet's Lens Arrangement, a convex lens of focal length 50 cm is cut along the diameter into two identical halves A and B and in the process a layer C of the lens thickness 1 mm is lost. Then the two halves A and B are put together to form a composite lens as shown in figure.



Now, in front of this new composite lens a source of light emitting wavelength $\lambda = 6000\text{ Å}$ is placed at a distance of 25 cm. Behind the lens there is a screen at a distance 50 cm from it as shown in the figure.



Calculate the fringe width of the interference pattern obtained on the screen.

SOLUTION

Applying lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ for $u=-25\text{ cm}$, we get

$$\frac{1}{v}-\frac{1}{(-25)}=\frac{1}{50}$$

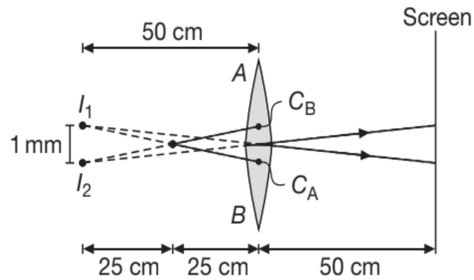
$$\Rightarrow \quad \frac{1}{v}=-\frac{1}{50}$$

$$\Rightarrow \quad v=-50\text{ cm}$$

The two parts A and B of the lens produce two virtual images (of the source) at I_1 and I_2 at a distance 50 cm behind the lens. Figure shows the locations of I_1 and I_2 which are obtained by joining the source with the optic centres of the two lenses.

Please note that the optical centre of the original lens (from which the new composite lens is made) lies at the centre of the original lens.

On cutting the lens, the region C is lost and when the portions A and B are joined, then the optical centre for A will lie in the region of B and that for B will lie in the region of A.



Now the interference pattern is obtained on the screen due to the interference of light waves from the sources I_1 and I_2 which are separated by a distance 1 mm and are at a distance 1 m from the screen. So, fringe width of the fringes obtained on screen is given by

$$\beta = \frac{\lambda D}{d}$$

$$\Rightarrow \quad \beta = \frac{6\times10^{-7}\times1}{10^{-3}} = 6\times10^{-4} = 0.6\text{ mm}$$