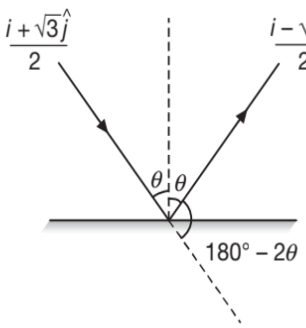
ICON

CHAPTER 1: RAY OPTICS

Test Your Concepts-I (Based on Reflection at Plane Surfaces)

1. The angle between the incident ray and the reflected ray is $180 - 2\theta$, so, we have



$$\cos(180° - 2\theta) = \frac{\left(\frac{i+\sqrt{3}\hat{j}}{2}\right)\cdot\left(\frac{i-\sqrt{3}\hat{j}}{2}\right)}{\left|\frac{i+\sqrt{3}\hat{j}}{2}\right|\left|\frac{i-\sqrt{3}\hat{j}}{2}\right|}$$

$$\Rightarrow \quad -\cos(2\theta) = \frac{\frac{(1-3)}{4}}{1}$$

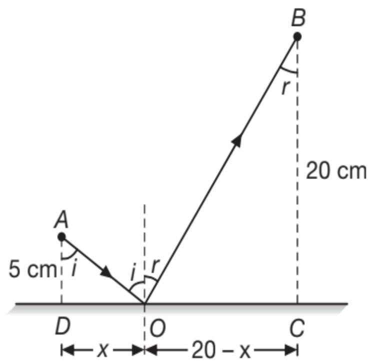
$$\Rightarrow \quad -\cos(2\theta) = -\frac{1}{2}$$

$$\Rightarrow \quad \cos(2\theta) = \frac{1}{2}$$

$$\Rightarrow \quad 2\theta = 60°$$

$$\Rightarrow \quad \theta = 30°$$

2. Drawing the ray diagram and using the Law of Reflection, we get



$$i = r \qquad \ldots(1)$$

$$\Rightarrow \quad \sin i = \sin r$$

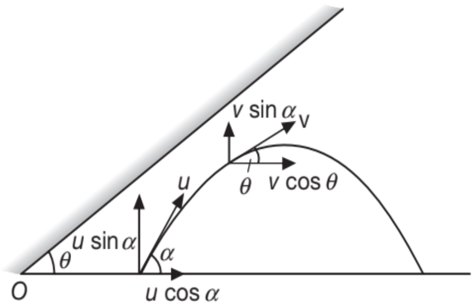
So, we can say that ΔADO and ΔOBC are similar

$$\Rightarrow \quad \frac{x}{5} = \frac{20-x}{20}$$

$$\Rightarrow \quad x = 4 \text{ cm}$$

So, point of incidence of light from A should be at 4 cm from D on mirror.

3. The image will be momentarily at rest when the particle moves parallel to the mirror. Let at the time t the particle has a velocity v parallel to the mirror.



$$v\sin\theta = u\sin\alpha - gt \qquad \ldots(1)$$

and $v\cos\theta = u\cos\alpha$

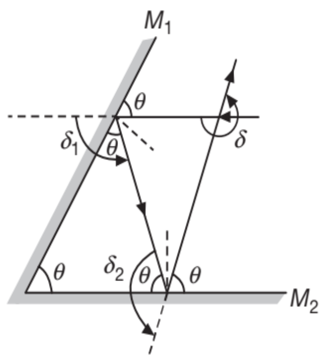
$$\Rightarrow \quad v = \frac{u\cos\alpha}{\cos\theta} \qquad \ldots(2)$$

From (1) and (2)

$$\left(\frac{u\cos\alpha}{\cos\theta}\right)\sin\theta = u\sin\alpha - gt$$

$$\Rightarrow \quad t = \frac{u\cos\alpha(\tan\alpha - \tan\theta)}{g}$$

4.



From the figure, we observe that

$$3\theta = 180°$$

$$\Rightarrow \quad \theta = 60°$$

So, $\delta_1 = 180° - 2(30°) = 120°$ (CCW)

and $\delta_2 = 180° - 2(30°) = 120°$ (CCW)

So, total deviation $\delta = \delta_1 + \delta_2$

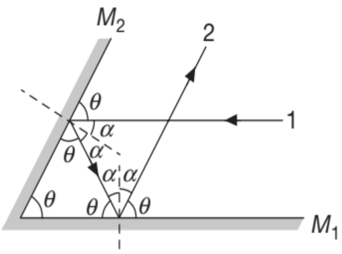
$$\Rightarrow \quad \delta = 240° \text{ (CCW)}$$

Alternatively from the figure, we observe that

$$\delta = 180° + \theta = 240° \text{ (CCW) or } 120° \text{ (CW)}$$

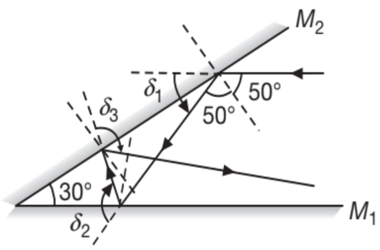
5. Various angles made are as shown in figure. In triangle ABC, we observe that

$$\theta+\theta+\theta=180^\circ$$
$$\Rightarrow \quad \theta=60^\circ$$



6. From the figure, we observe that

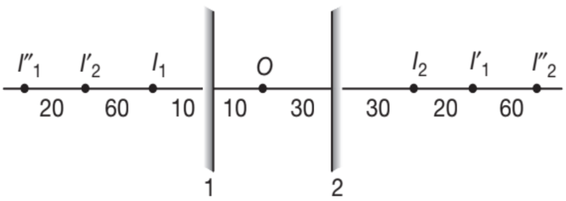
$$\delta_1=180^\circ-2(50^\circ)=100^\circ \text{ (CCW)}$$
$$\delta_2=180^\circ-2(20^\circ)=140^\circ \text{ (CW)}$$
$$\delta_3=180^\circ-2(10^\circ)=160^\circ \text{ (CW)}$$



So, total deviation

$$\delta=100^\circ(\text{CCW})+140^\circ(\text{CW})+160^\circ(\text{CW})$$
$$\Rightarrow \quad \delta=100^\circ \text{ (CW) or } 260^\circ \text{ (CCW)}$$

7. The image is formed as far behind the mirror as the object is in front of it. Also image formed by mirror 1 i.e., I_1, acts as object for mirror 2, so I_1' is formed 50 cm behind the mirror 2 as shown.



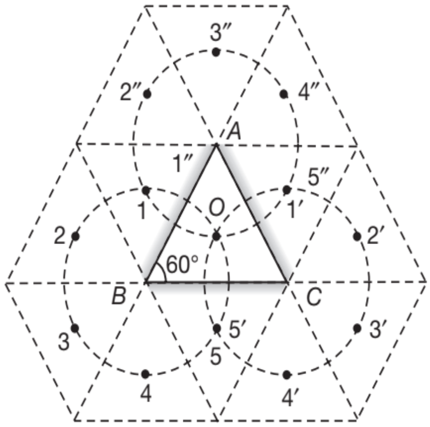
Taking all distances to be in cm and plotting them as shown (but not to scale), we get

$OI_1=20\text{ cm}$	$OI_2=60\text{ cm}$
$OI_1'=80\text{ cm}$	$OI_2'=80\text{ cm}$
$OI_1''=100\text{ cm}$	$OI_2''=140\text{ cm}$

So, the respective distances are 20 cm, 60 cm, 80 cm, 100 cm and 140 cm

8. Let us first consider the mirrors AB and BC, for which we have

$$\frac{360^\circ}{60^\circ}=6$$



So, the number of images formed by the combination is given by

$$N=6-1=5$$

Combination of Mirrors	Images Formed
AB & BC	1, 2, 3, 4, 5
AC & BC	1′, 2′, 3′, 4′, 5′
AB & AC	1″, 2″, 3″, 4″, 5″

These images along with the object must lie on a circle as shown in figure with an angular separation of

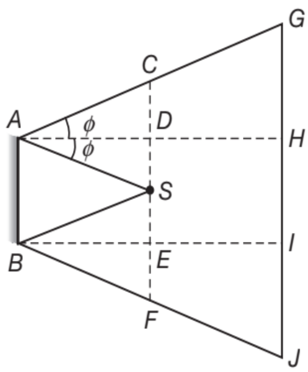
$$\frac{360^\circ}{N}=\frac{360^\circ}{5}=72^\circ$$

Similarly, the other two combination of mirrors also form 5 images each but we find from symmetry that 5 and 5′, 1 and 5″, 1 and 1″ coincide. So the total number of images formed by three mirrors AB, BC and AC is

$$N'=(5)(3)-3=12$$

9. The ray diagram is shown in figure. We observe that

$$HI=AB=d$$
$$DS=CD=\frac{d}{2}$$



Also, $AH=2AD$

$$\Rightarrow \quad GH = 2CD = 2\left(\frac{d}{2}\right) = d$$

Similarly, $IJ = d$

$$\Rightarrow \quad GJ = GH + HI + IJ = d + d + d = 3d$$

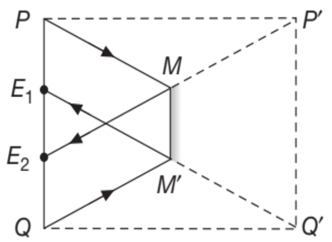
10. (a) For a one eyed man, the required size will be half the each dimension of the face i.e., 12 cm × 8 cm

(b) For a two eyed man, the

Smallest length of the mirror = Half the length of face

$$\Rightarrow \quad \begin{pmatrix}\text{Smallest length}\\ \text{of the Mirror}\end{pmatrix} = \frac{1}{2} \times 24 = 12 \text{ cm}$$

The smallest breadth of the mirror is calculated by using the fact that the rays from extreme part of face should reach one of the eyes after reflection from the mirror. The common overlapping portion is then the required breadth of the mirror. The ray diagram is shown in figure.



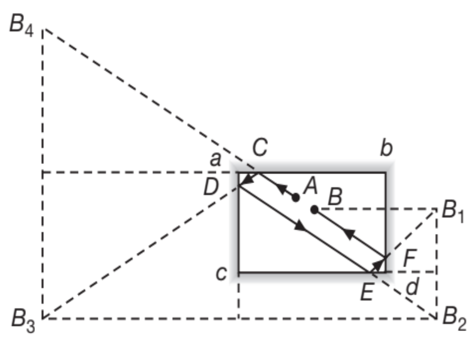
From figure, we get

$$MM' = \frac{1}{2}PQ - \frac{1}{2}E_1E_2$$

$$\Rightarrow \quad \begin{pmatrix}\text{Smallest Breadth}\\ \text{of the Mirror}\end{pmatrix} = \frac{1}{2}(16 - 8) = 4 \text{ cm}$$

So, the shortest size of mirror is 12 cm by 4 cm.

11. Let us first find the image of point B in mirror bd (shown in figure). Let us then construct image B_1 in mirror cd. Also, B_3 is the image of B_2 in mirror ac and B_4 is the image of B_3 in mirror ab. Let us connect points A and B_4. Point C is the point of intersection of ab with line AB_4. Let us now draw line B_3C from B_3 and connect point D at which this line intersects ac with B_2, E with B_1 and F with B.



It can be stated that the line $ACDEFB$ is the sought path of the beam. Further, we observe that since, B_3CB_4 is an isosceles triangle, CD is the reflection of beam AC. Similarly, we can show that DE is the reflection of CD and so on. This solution of the problem is not unique, as the beam will not necessarily always be sent initially to mirror ab.

12. Along x-direction i.e., perpendicular to the mirror, we have

$$\begin{pmatrix}\text{Relative Velocity of}\\ \text{Image w.r.t. mirror}\end{pmatrix} = -\begin{pmatrix}\text{Relative Velocity of}\\ \text{Object w.r.t. mirror}\end{pmatrix}$$

$$\Rightarrow \quad v_I - v_m = -(v_0 - v_m)$$

$$\Rightarrow \quad v_I - (-5\cos 30^\circ) = -(10\cos 60^\circ - (-5\cos 30^\circ))$$

$$\Rightarrow \quad v_I = -5(1 + \sqrt{3}) \text{ ms}^{-1}$$

In the direction parallel to the surface of mirror, i.e., along y-direction we have

$$v_I = v_0$$

$$\Rightarrow \quad v_I = 10\sin(30^\circ) = 5 \text{ ms}^{-1}$$

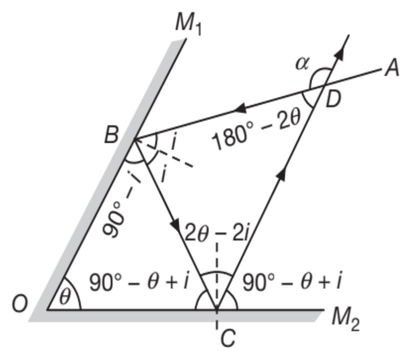
Since $(v_{Im})_{\parallel} = (v_{Om})_{\parallel}$

So, velocity of the image

$$\vec{v}_I = (v_I)_x\hat{i} + (v_I)_y\hat{j}$$

$$\Rightarrow \quad \vec{v}_I = -5(1 + \sqrt{3})\hat{i} + 5\hat{j}$$

13. Let AB be the incident ray and angle of incidence at the mirror M_1 be i, then



$$\angle CBO = 90^\circ - i$$

$$\Rightarrow \quad \angle BCO = 180^\circ - \theta - (90^\circ - i)$$

$$\Rightarrow \quad \angle BCO = 90^\circ - \theta + i$$

Using the Laws of Reflection, we get

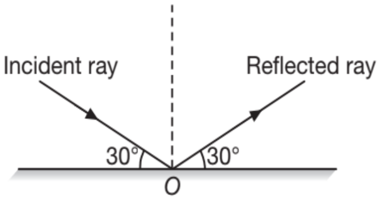
$$\angle DCB = 2\theta - 2i$$

$$\Rightarrow \quad \angle CDB = 180^\circ - 2\theta$$

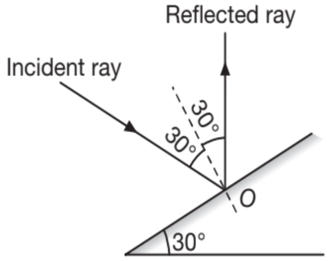
$$\Rightarrow \quad \alpha = 2\theta$$

The angle between incident and emergent ray is 2θ and it is independent of the angle of incidence i.

14. Suppose that a plane mirror is kept horizontal as shown in figure. The reflected ray will make an angle of 30° with horizontal, or an angle of 60° with the vertical.



To make the reflected ray to go vertically upwards, the mirror is required to be rotated about O counterclockwise by 60°. To achieve this, therefore, the plane mirror is required to rotate about O by half the angle, i.e., by 30°, as shown in figure.

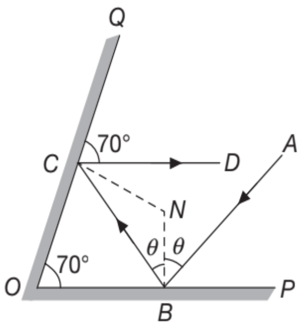


15. Ray AB is incident on mirror OP at an angle θ. The reflected ray BC is incident on second mirror OQ. Finally, the reflected ray CD is parallel to OP. Since CD and OP are parallel, and CO cuts them,

$$\angle QCD = \angle COP = 70^\circ$$

$$\Rightarrow \quad \angle DCN = 90^\circ - \angle QCD = 90^\circ - 70^\circ = 20^\circ$$

$$\Rightarrow \quad \angle NCB = \angle DCN = 20^\circ$$



Further, $\angle OCB = 90^\circ - \angle NCB = 90^\circ - 20^\circ = 70^\circ$

Now, in ΔCOB, we have

$$\angle CBO = 180^\circ - (\angle COB + \angle OCB)$$

$$\Rightarrow \quad \angle CBO = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$$

$$\Rightarrow \quad \theta = \angle NBC = 90^\circ - \angle CBO = 90^\circ - 40^\circ = 50^\circ$$

Test Your Concepts-II (Based on Reflection at Curved Surfaces)

1. Since, $m = \dfrac{f}{f-u}$

Now, $f = -f$ and $u = -1.5\text{ f}$, so

$$m = \frac{-f}{-f + 1.5\text{ f}} \quad \Rightarrow \quad m = \frac{-f}{0.5\text{ f}}$$

$$\Rightarrow \quad m = -2$$

Since $m = \dfrac{h_i}{h_0} = -2$

$$\Rightarrow \quad h_i = -2h_0 = -5\text{ cm}$$

The image is 5 cm long. The minus sign shows that it is inverted.

2. Given: $u = -25\text{ m}$, $m = +4$ (since the image is erect).
Now, the magnification is given by

$$m = -\frac{v}{u}$$

$$\Rightarrow \quad v = -mu = -(+4)\times(-2.5) = 10\text{ m}$$

Using the mirror formula, we get

$$\frac{1}{10} + \frac{1}{-2.5} = \frac{1}{f}$$

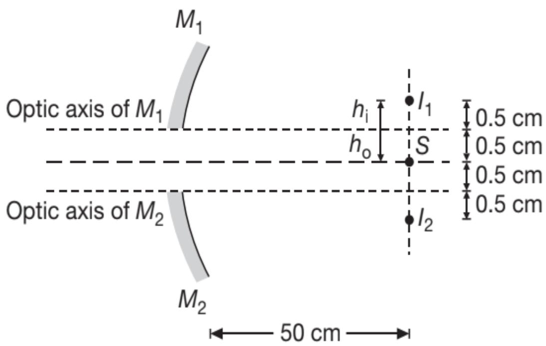
$$\Rightarrow \quad \frac{1}{f} = 0.1 - 0.4 = -0.3$$

$$\Rightarrow \quad f = -\frac{1}{0.3} = -\frac{10}{3}\text{ m}$$

Since f is negative so, the mirror is concave.
The radius of curvature of the mirror is given by

$$R = 2f = 2\left(-\frac{10}{3}\right) = -\frac{20}{3}\text{ m} = -6.67\text{ m}$$

3. According to the mirror formula, we have $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$



$$\Rightarrow \quad \frac{1}{v} - \frac{1}{50} = \frac{1}{-25}$$

$\Rightarrow \quad v = -50$ cm

So, $m = -\frac{v}{u} = -1$

$\Rightarrow \quad \left|\frac{h_i}{h_o}\right| = +1$

$\Rightarrow \quad h_i = h_o = 0.5$ cm

4. Given, $f = -10$ cm. Since a concave mirror can form real as well as virtual image and since the nature of image is not given in the question. So we will consider two possible cases.

CASE-1 (when image is real):

So, $m = -4$

Since $m = \frac{f}{f-u}$

$\Rightarrow \quad -4 = \frac{-10}{-10-u}$

$\Rightarrow \quad u = -12.5$ cm

Please note that here,
$|u| > |f|$ and we know that in case of a concave mirror, image is real when object lies beyond F.

CASE-2 (When image is virtual):

So, $m = +4$

Since $m = \frac{f}{f-u}$

$\Rightarrow \quad 4 = \frac{-10}{-10-u}$

$\Rightarrow \quad u = -7.5$ cm

Please, note that here, $|u| < |f|$, as we know that image is virtual when the object lies between F and P.

5. Since $f = \frac{R}{2} = -12$ cm

Let the object be placed at a distance u from the pole. Since, we know that magnification m is given by

$$m = \frac{f}{f-u}$$

So, now we apply this formula to these situations one by one.

(a) Here, we have $m = +3$

$\Rightarrow \quad 3 = \frac{-12}{-12-u}$

$\Rightarrow \quad -4 = -12 - u$

$\Rightarrow \quad 12 + u = 4$

$\Rightarrow \quad u = -8$ cm

(b) Here, $m = -3$

$\Rightarrow \quad -3 = \frac{-12}{-12-u}$

$\Rightarrow \quad 4 = -12 - u \quad \Rightarrow \quad u = -16$ cm

(c) Here, $m = -\frac{1}{3}$

$\Rightarrow \quad -\frac{1}{3} = \frac{-12}{-12-u}$

$\Rightarrow \quad 36 = -12 - u \quad \Rightarrow \quad u = -48$ cm

6. Distance of image formed by the plane mirror is $(b-a)$ i.e., $(b-5)$ cm and distance of object from mirror is $(b+a)$ i.e., $(b+5)$ cm. Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ we get

$$\frac{1}{(b-5)} - \frac{1}{(b+5)} = \frac{1}{20}$$

Solving this equation, we get

$b = 15$ cm

The coincidence of the images can be established by observing the changes in the relative position of the images when the eye is moved away from the optical axis of the mirror.
When the images are at various distance from the eye the images will be displaced with respect to each other. When the images are at the same distance, they will coincide irrespective of the placement of the eye.

7. (a) At any instant t, we have

$u = -(2f + x) = -(2f + f\cos\omega t)$

Using, the mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v} - \frac{1}{2f + f\cos\omega t} = \frac{-1}{f}$$

$\Rightarrow \quad v = -\left(\frac{2+\cos\omega t}{1+\cos\omega t}\right)f$

i.e., distance of image from mirror at time any instant t is

$$\left(\frac{2+\cos\omega t}{1+\cos\omega t}\right)f$$

(b) Ball coincides with its image at centre of curvature, i.e., at $x = 0$

(c) At $t = \frac{T}{2}$, we have

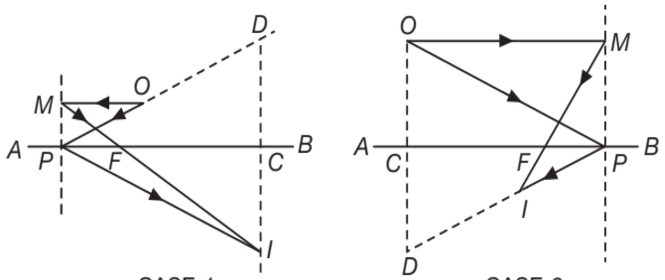
$\omega t = \pi$

$\Rightarrow \quad x = f\cos(\pi) = -f$

So, $u = -f$ i.e., ball is at focus. So, its image is formed at infinity, so $m \to \infty$

8. (a) Since the image is on the opposite side of the principal axis, the mirror is concave. Because convex mirror always forms a virtual and erect image.

(b) The ray diagrams for two different cases are shown in figure.



CASE-1 CASE-2

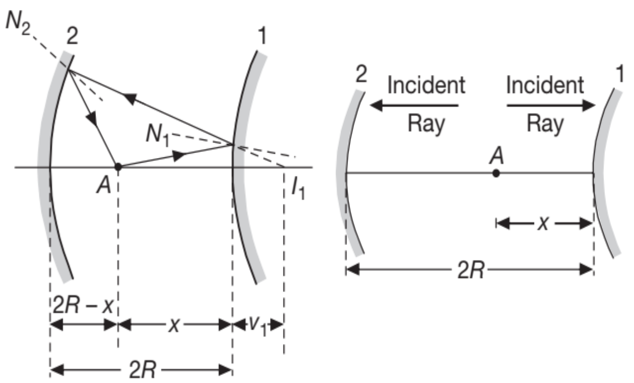
The following steps of construction for drawing the ray diagrams are used.

(i) From I or O drop a perpendicular on principal axis, such that $CI = CD$ or $OC = CD$.

(ii) Draw a line joining D and O or D and I so that it meets the principal axis at P. The point P will be the pole of the mirror as a ray reflected from the pole is always symmetrical about principal axis.

(iii) From O draw a line parallel to principal axis towards the mirror so that it meets the mirror at M. Join M to I, so that it intersects the principal axis at F. F is the focus of the mirror as any ray parallel to principal axis after reflection from the mirror intersects the principal axis at the focus.

9. Let the point A be at a distance x from the convex mirror as shown in mirror, then assuming the origin to be placed at the pole of convex mirror, we get



For convex mirror, we have

$$\frac{1}{v_1} - \frac{1}{x} = \frac{2}{R}$$

$$\Rightarrow \quad v_1 = \frac{xR}{2x+R}$$

For concave mirror, we have

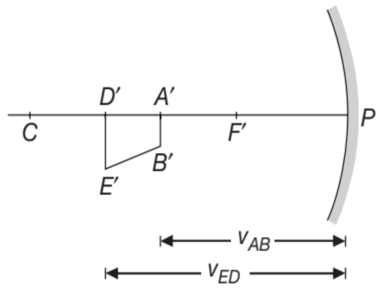
$$\frac{1}{-(2R-x)} - \frac{1}{\left(2R + \dfrac{xR}{2x+R}\right)} = -\frac{2}{R}$$

Solving this equation, we get

$$x = \left(\frac{1+\sqrt{3}}{2}\right)R \text{ and } x = -\left(\frac{\sqrt{3}-1}{2}\right)R$$

Ignoring the negative value, as we have already used a negative sign with x, so the object should be placed at a distance $x = \left(\dfrac{\sqrt{3}+1}{2}\right)R$ from the convex mirror.

10. Object is placed beyond C. Hence, the image will be real and it will lie between C and F. Further u, v and f all are negative, hence the mirror formula becomes



$$-\frac{1}{v} - \frac{1}{u} = -\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{u-f}{uf}$$

$$\Rightarrow \quad v = \frac{f}{1 - \dfrac{f}{u}}$$

Now since, $u_{AB} > u_{ED}$

$\Rightarrow \quad v_{AB} < v_{ED}$

Hence, $|m_{AB}| < |m_{ED}|$ $\qquad \left\{\because m = -\dfrac{v}{u}\right\}$

Therefore, shape of the image will be as shown in figure.

Also note that $v_{AB} < u_{AB}$ and $v_{ED} < u_{ED}$, $|m_{AB}| < 1$ and $|m_{ED}| < 1$

11. Since the image is inverted, so the mirror is concave.

Now, $u = -30$ cm

$$m = -\frac{1}{2} = -\frac{v}{u}$$

$$\Rightarrow \quad v = \frac{u}{2} = -15 \text{ cm}$$

Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\Rightarrow \frac{1}{(-15)}+\frac{1}{(-30)}=\frac{1}{f}$

$\Rightarrow f=-10$ cm

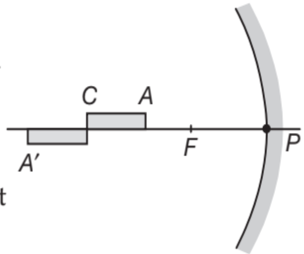
12. Since image touches the rod, the rod must be placed with one end at centre of curvature. However, two cases arise here.

CASE-1: When the other end lies between C and F

For A, we have

$u=-\left(2f-\frac{f}{3}\right)=-\frac{5f}{3}$

$f=-f$



Since, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, so we get

$\frac{1}{v}+\frac{1}{\left(\frac{-5f}{3}\right)}=\frac{1}{(-f)}$

$\Rightarrow v=\frac{5f}{2}$

So, magnification, $m=\frac{\text{Length of Image}}{\text{Length of Object}}$

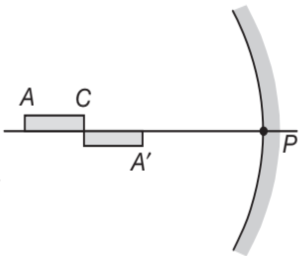
$\Rightarrow m=\frac{v_A-v_C}{u_A-u_C}=\frac{-\frac{5f}{2}-(-2f)}{-\frac{5f}{3}-(-2f)}=-\frac{3}{2}$

CASE-2: When the other end lies beyond C

For A, we have

$u=-\left(2f+\frac{f}{3}\right)=-\frac{7f}{3}$

$f=-f$



Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, so we get

$\frac{1}{v}+\frac{1}{\left(-\frac{7f}{3}\right)}=\frac{1}{-f}$

$\Rightarrow v=-\frac{7f}{4}$

So, magnification, $m=\frac{v_A-v_C}{u_A-u_C}$

$\Rightarrow m=\frac{-\frac{7f}{4}-(-2f)}{-\frac{7f}{3}-(-2f)}=-\frac{3}{4}$

13. Using coordinate convention for mirror formula, we have

$u=-25$ cm, $f=-20$ cm, $v=?$

Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\Rightarrow v=\frac{uf}{u-f}=\frac{-25\times-20}{-5}=-100$ cm

Thus magnification in this case is

$m=-\frac{v}{u}=-\left(\frac{-100}{-25}\right)=-4$

Image velocity along the principal axis is given by

$v_{i(\text{along } PA)}=m^2\times v_{o(\text{along } PA)}$

$\Rightarrow v_{i(\text{along } PA)}=16\times\frac{5}{\sqrt{2}}=40\sqrt{2}$ cm

Image velocity normal to principal axis is given by

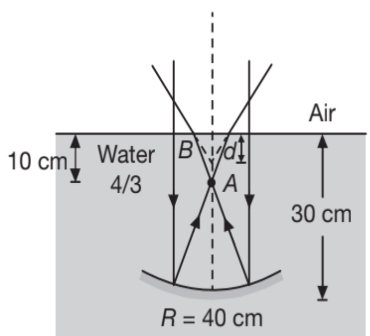
$v_{i(\text{normal to } PA)}=m\times v_{o(\text{normal to } PA)}$

$\Rightarrow v_{i(\text{normal to } PA)}=4\times\frac{5}{\sqrt{2}}=10\sqrt{2}$ cm

Test Your Concepts-III (Based on General Refraction)

1. $d'=\frac{d}{n_{\text{relative}}}=\frac{100}{\frac{4/3}{1}}=75$ cm

2. The incident rays will pass undeviated through the water surface and strike the mirror parallel to its principal axis. Therefore for the mirror, object is at ∞. Its image A (in figure) will be formed at focus which is 20 cm from the mirror. Now for the interface between water and air, $d=10$ cm.



$\Rightarrow d'=\frac{d}{\left(\frac{n_w}{n_a}\right)}=\frac{10}{\left(\frac{4/3}{1}\right)}=7.5$ cm

3. **CASE-1: When No Slab is Inserted**

According to mirror formula, we have

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{v_1}+\frac{1}{-30}=\frac{1}{-10}$$

$$\Rightarrow v_1=-15 \text{ cm}$$

Magnification, $m_1=-\frac{v}{u}=-\frac{1}{2}=-0.5$

CASE-2: When Slab is Inserted

$$\text{Shift}=\left(1-\frac{1}{\mu}\right)t=\left(1-\frac{1}{1.5}\right)6=2 \text{ cm}$$

Again, applying the mirror formula, we get

$$\frac{1}{v_2}\frac{1}{-(30-2)}=\frac{1}{-10}$$

$$\Rightarrow v_2=-15.55 \text{ cm}$$

Magnification, $m_2=-\frac{v}{u}=-0.55$

So, $\Delta v=0.55$ cm

$$\frac{m_2}{m_1}\approx 1.1$$

4. The refractive index of glass,

$$\mu=\frac{v_{\text{air}}}{v_{\text{glass}}}=\frac{c}{v}$$

$$\Rightarrow v=\frac{c}{\mu}=\frac{3\times10^8}{1.5}=2\times10^8 \text{ ms}^{-1}$$

Since the frequency of light remains the same when it passes from one medium to another, so we have

$c=f\lambda_0$ and $v=f\lambda$

Since, $\mu=\frac{c}{v}=\frac{\lambda_0}{\lambda}$

$$\Rightarrow \lambda=\frac{\lambda_0}{\mu}=\frac{6000}{1.5}=4000 \text{ Å}$$

The colour remains yellow, as the colour depends on the frequency and not on the wavelength.

5. Using equation, the total apparent shift is

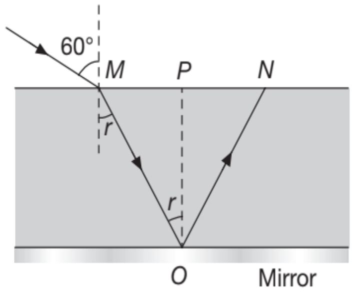
$$\Delta x=h_1\left(1-\frac{1}{\mu_1}\right)+h_2\left(1-\frac{1}{\mu_2}\right)$$

$$\Rightarrow \Delta x=2\left(1-\frac{1}{4/3}\right)+3\left(1-\frac{1}{3/2}\right)=1.5 \text{ cm}$$

Thus, $h=h_1+h_2-\Delta x=2+3-1.5=3.5$ cm

6. $1.8=\frac{\sin(60°)}{\sin r}$

$$\Rightarrow r=28.76°$$



Since, $MP=PO\tan r$

$$\Rightarrow MP=(6)\tan(28.76°)$$

$$\Rightarrow MP=3.3 \text{ cm}$$

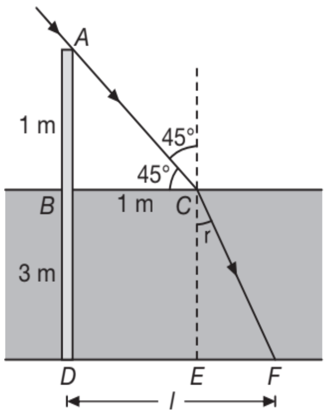
So, $MN=2MP=6.6$ cm

7. From Snell's Law, we have

$$\frac{4}{3}=\frac{\sin(45°)}{\sin r}$$

$$\Rightarrow \sin r=\frac{3}{4}\sin(45°)$$

$$\Rightarrow r=32°$$



Since, $EF=EC\tan r$

$$\Rightarrow EF=(3)\tan 32°=1.88 \text{ m}$$

Length of shadow at the bottom of the lake is

$$\ell=DF=DE+EF=2.88 \text{ m}$$

8. Total deviation suffered by the ray is given by

$$\delta_{\text{Total}}=\delta_P+\delta_Q$$

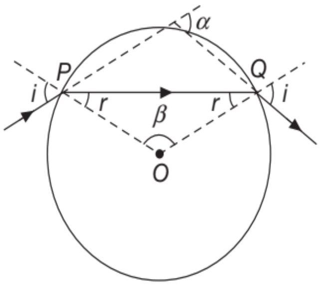
$$\Rightarrow \alpha=(i-r)+(i-r)$$

$$\Rightarrow i-r=\frac{\alpha}{2} \quad \ldots(1)$$

Further, in ΔOPQ, we have

$$r+r+\beta=180°$$

$$\Rightarrow \quad r = 90° - \frac{\beta}{2} \qquad \ldots(2)$$



From equation (1), we get

$$i = r + \frac{\alpha}{2} = 90° + \left(\frac{\alpha - \beta}{2}\right) \qquad \ldots(3)$$

According to Snell's Law, we have

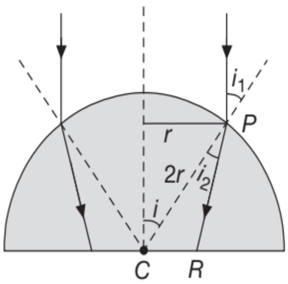
$$\mu = \frac{\sin i}{\sin r}$$

$$\Rightarrow \quad \mu = \frac{\sin\left[90° + \left(\frac{\alpha-\beta}{2}\right)\right]}{\sin\left(90° - \frac{\beta}{2}\right)} = \frac{\cos\left(\frac{\beta-\alpha}{2}\right)}{\cos\left(\frac{\beta}{2}\right)}$$

$$\Rightarrow \quad \cos\left(\frac{\beta-\alpha}{2}\right) = \mu\cos\frac{\beta}{2}$$

9. $\sin i_1 = \frac{r}{2r} = \frac{1}{2}$

$$\Rightarrow \quad i_1 = 30°$$



Applying Snell's Law at P, we get

$$\frac{3}{2} = \frac{\sin i_1}{\sin i_2}$$

$$\Rightarrow \quad i_2 = 19.5°$$

Now, Applying Sine Law (Lami's Theorem), on ΔCPR, we get

$$\frac{2r}{\sin(180° - 60° - 19.5°)} = \frac{CR}{\sin(19.5°)}$$

$$\{\because \angle PCR = 60°\}$$

$$\Rightarrow \quad CR = 0.7r$$

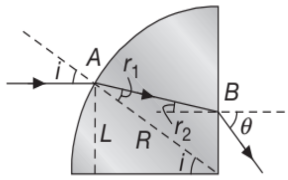
10. Since, $\sin i = \frac{L}{R}$

$$\Rightarrow \quad i = \sin^{-1}\left(\frac{L}{R}\right)$$

According to Snell's Law applied at A, we have

$$\mu = \frac{\sin i}{\sin r_1}$$

$$\Rightarrow \quad r_1 = \sin^{-1}\left(\frac{\sin i}{\mu}\right) = \sin^{-1}\left(\frac{L}{\mu R}\right)$$



Deviation suffered by the ray is

$$\delta = i - r_1 = \sin^{-1}\left(\frac{L}{R}\right) - \sin^{-1}\left(\frac{L}{\mu R}\right)$$

This is also the angle r_2, so we have

$$r_2 = \sin^{-1}\left(\frac{L}{R}\right) - \sin^{-1}\left(\frac{L}{\mu R}\right)$$

From the knowledge of inverse trigonometry, we have

$$\sin^{-1}(C) - \sin^{-1}(D) = \sin^{-1}\left(C\sqrt{1-D^2} - D\sqrt{1-C^2}\right)$$

$$\Rightarrow \quad r_2 = \sin^{-1}\left[\frac{L}{R}\sqrt{1 - \frac{L^2}{\mu^2R^2}} - \frac{L}{\mu R}\sqrt{1 - \frac{L^2}{R^2}}\right]$$

Now, again applying Snell's Law at B, we get

$$\mu = \frac{\sin\theta}{\sin r_2}$$

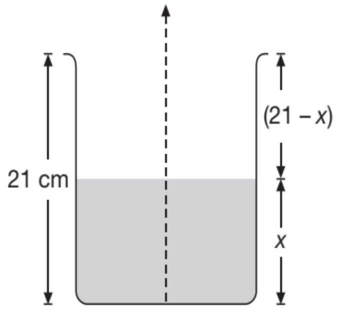
$$\Rightarrow \quad \theta = \sin^{-1}(\mu \sin r_2)$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{\mu L}{R}\sqrt{1 - \frac{L^2}{\mu^2R^2}} - \frac{L}{R}\sqrt{1 - \frac{L^2}{R^2}}\right)$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{L}{R^2}\sqrt{\mu^2R^2 - L^2} - \frac{L}{R^2}\sqrt{R^2 - L^2}\right)$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{L}{R^2}\left(\sqrt{\mu^2R^2 - L^2} - \sqrt{R^2 - L^2}\right)\right)$$

11. Figure shows the container filled with water upto a height x so that when observed from top, it appears to be half filled.



The apparent depth of container is such that it should be equal to the empty length of container for it to appear half filled, so we use

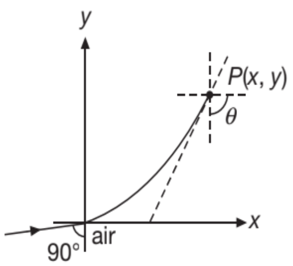
$$\frac{x}{\mu} = 21 - x$$

$$\Rightarrow \frac{3x}{4} + x = 21$$

$$\Rightarrow \frac{7x}{4} = 21$$

$$\Rightarrow x = 12 \text{ cm}$$

12. We draw a tangent at any point (x, y) on the trajectory which makes an angle θ with optical normal parallel to y-axis as shown in figure.



By using Snell's law at the initial point and at the general point of the trajectory of light, we have

$$1\sin 90° = \mu \sin\theta$$

$$\Rightarrow \sin\theta = \frac{1}{\mu} = \frac{1}{\sqrt{1+y}} \quad \ldots(1)$$

Slope of tangent is

$$\frac{dy}{dx} = \tan(90-\theta)$$

$$\Rightarrow \frac{dy}{dx} = \cot\theta \quad \ldots(2)$$

From Equation (1) and (2) we get,

$$\frac{dy}{dx} = y^{\frac{1}{2}}$$

$$\Rightarrow \int_0^y \frac{dy}{y^{\frac{1}{2}}} = \int_0^x dx$$

$$\Rightarrow 2\sqrt{y} = x$$

$$\Rightarrow y = \frac{x^2}{4}$$

Test Your Concepts-IV (Based on Total Internal Reflection (TIR))

1. (a) Critical angle between 2 and 3, is given by

$$\sin C = \frac{1.3}{1.8}$$

Now, applying Snell's Law, we get

$$1.6\sin\theta = 1.8\sin C = (1.8)\left(\frac{1.3}{1.8}\right) = 1.3$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{13}{16}\right) \approx 54.34°$$

(b) If θ is decreased, the angle of incidence at the interface between 2 and 3 gets decreased or $i < C$, so the light will refract into medium 3.

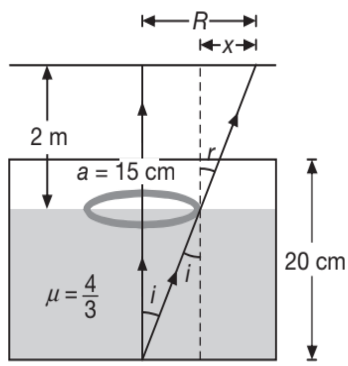
2. (a) Using Snell's Law, we get

$$\mu \sin i = \sin r$$

$$\Rightarrow \frac{4}{3}\left(\frac{15}{\sqrt{15^2+20^2}}\right) = 1\left(\frac{x}{\sqrt{x^2+200^2}}\right)$$

Solving, we get

$$x = \frac{800}{3} \text{ cm}$$



So, the radius of shadow is $R = \left(15 + \frac{800}{3}\right)$ cm

$$\Rightarrow R = \frac{845}{3} \text{ cm} = 2.81 \text{ m}$$

(b) For shadow to be formed, angle of incidence must be less than critical angle.
Using Snell's Law, we get

$$\frac{4}{3}\left(\frac{a_{max}}{\sqrt{a_{max}^2+20^2}}\right) = 1\sin(90°)$$

$$\Rightarrow \quad 16a_{max}^2 = 9a_{max}^2 + 9(20^2)$$

$$\Rightarrow \quad 7a_{max}^2 = 9(20^2)$$

$$\Rightarrow \quad a_{max} = \left(\sqrt{\frac{9}{7}}\right)(20\text{ cm}) = 0.23\text{ m}$$

3. (a) At interface AB, applying Snell's Law, we get

$$1\sin\left(\frac{\pi}{2}\right) = \mu\sin\alpha$$

$$\Rightarrow \quad \sin\alpha = \frac{1}{\mu} \quad \ldots(1)$$

At interface BC, applying Snell's Law again, we get

$$\mu\sin(90-\alpha) = 1(\sin\theta)$$

$$\Rightarrow \quad \sin\theta = \mu\cos\alpha \quad \ldots(2)$$

From equation (1) and (2), we get $\sin\theta = \cot\alpha$

(b) For emergence from BC, we must have

$$90 - \alpha \le C$$

At grazing incidence we have

$$\alpha = C$$

$$\Rightarrow \quad 90 - \alpha \le C$$

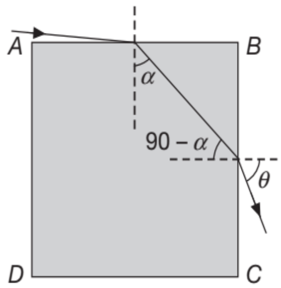
$$\Rightarrow \quad 2C \ge 90°$$

$$\Rightarrow \quad C \ge 45°$$

$$\Rightarrow \quad \sin^{-1}\left(\frac{1}{\mu}\right) \ge 45°$$

$$\Rightarrow \quad \frac{1}{\mu} \ge \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \mu \le \sqrt{2}$$

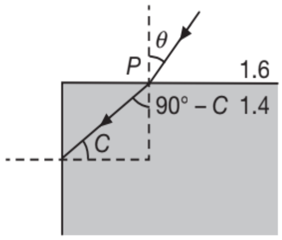


So, the greatest value of refractive index is

$$\mu_{max} = \sqrt{2}$$

4. (a) Critical angle between 2 and 3

$$\sin C = \frac{1.2}{1.4} = \frac{6}{7}$$



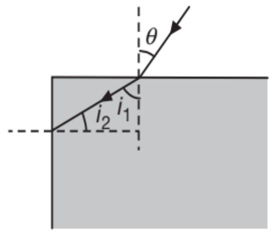
Applying, Snell's Law, at P, we get

$$1.6\sin\theta = 1.4\sin(90° - C) = 1.4\cos C$$

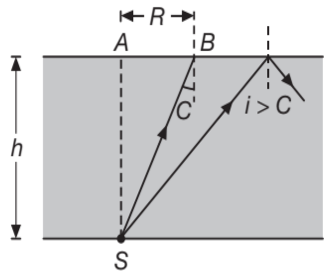
$$\Rightarrow \quad \sin\theta = \frac{1.4\sqrt{1-\frac{36}{49}}}{1.6} = 0.45$$

$$\Rightarrow \quad \theta \approx 26.8°$$

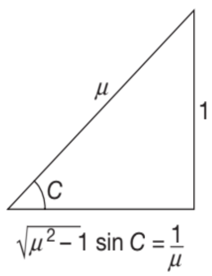
(b) As θ is increased, i_1 will increase or i_2 will decrease or $i_2 < C$ and hence the light will refract in medium 3.



5. As shown in figure, the light from the source will not emerge out of water if $i = C$.



Therefore, minimum radius R corresponds to the situation when $i = C$



In ΔSAB,

$$\frac{R}{h} = \tan C$$

$$\Rightarrow \quad R = h\tan C$$

$$\Rightarrow \quad R = \frac{h}{\sqrt{\mu^2 - 1}}$$

6. (a) Applying Snell's Law, we get

$$1.3\sin\theta_1 = (1)\sin\theta_5$$

$$\Rightarrow \quad \sin\theta_5 = (1.3)\sin(30°) = \frac{1.3}{2} = 0.65$$

$$\Rightarrow \quad \theta_5 \approx 40.54°$$

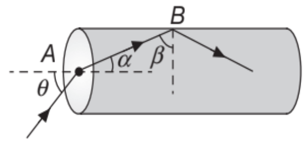
(b) Applying Snell's Law, we get

$$1.3\sin\theta_1 = 1.45\sin\theta_4$$

$$\Rightarrow \quad \sin\theta_4 = \frac{1.3\left(\frac{1}{2}\right)}{1.45} = 0.49$$

$$\Rightarrow \quad \theta_4 \approx 26.6^\circ$$

7. Critical angle, $C = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{1}{1.36}\right) = 47.3^\circ$



For TIR to take place at B, we have

$$\beta > C$$

$$\Rightarrow \quad \beta > 47.3^\circ$$

For this to happen, we have

$$\alpha < 90^\circ - 47.3^\circ$$

$$\Rightarrow \quad \alpha < 42.7^\circ$$

Applying Snell's Law at A, we get

$$\theta < \sin^{-1}(\mu\sin 42.7^\circ)$$

$$\Rightarrow \quad \theta < \sin^{-1}[1.36\sin(42.7^\circ)]$$

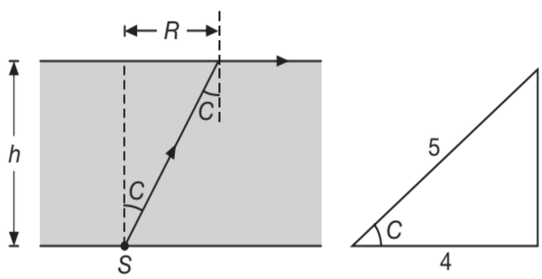
$$\Rightarrow \quad \theta < 67.3^\circ$$

So, the maximum value of θ for TIR to take place at B is 67.3°.

8. $\sin C = \frac{1}{\mu} = \frac{3}{5}$

$$\frac{R}{h} = \tan C = \frac{3}{4}$$

$$\therefore \quad h = \frac{4}{3}R = \frac{4}{3}\text{ cm}$$



9. At the maximum, the ray can enter the glass at the grazing angle, so $(i)_{max} = 90^\circ$

According to Snell's Law

$$\mu_g \sin r_1 = \mu\sin i$$

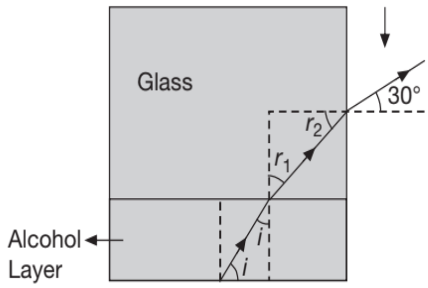
$$\Rightarrow \quad \sin r_1 = \frac{\mu}{1.5}\sin(90^\circ) = \frac{2\mu}{3}$$

$$\Rightarrow \quad (r_1)_{max} = \sin^{-1}\left(\frac{2\mu}{3}\right)$$

Since $r_1 + r_2 = 90^\circ$, so

$$\therefore \quad (r_2)_{min} = \frac{\pi}{2} - \sin^{-1}\left(\frac{2\mu}{3}\right)$$

Again, using Snell's Law, we get $\mu_g = \frac{\sin(30^\circ)}{\sin r_2}$



$$\Rightarrow \quad \frac{3}{2} = \frac{\frac{1}{2}}{\sin\left(\frac{\pi}{2} - r_1\right)} = \frac{1}{2\cos r_1}$$

$$\Rightarrow \quad 3\cos r_1 = 1$$

$$\Rightarrow \quad 3\sqrt{1 - \frac{4\mu^2}{9}} = 1$$

$$\Rightarrow \quad 1 - \frac{4\mu^2}{9} = \frac{1}{9}$$

$$\Rightarrow \quad \frac{4\mu^2}{9} = \frac{8}{9}$$

$$\Rightarrow \quad \mu = \sqrt{2}$$

Now, when the paper is dry then $\mu = 1$

$$\Rightarrow \quad r_1 = \sin^{-1}\left(\frac{2}{3}\right) \approx 42^\circ$$

Since $r_1 + r_2 = 90^\circ$, so we get

$$r_2 = 48^\circ$$

Critical angle at glass air interface, is given by

$$C = \sin^{-1}\left(\frac{1}{1.5}\right) \approx 42^\circ$$

So, we observe that as $r_2 > C$, so it can't be seen.

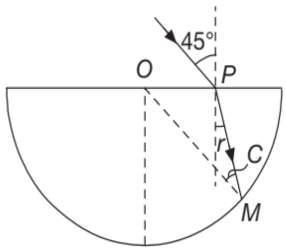
10. According to Snell's Law, we have

$$\sqrt{2} = \frac{\sin(45^\circ)}{\sin r}$$

$$\Rightarrow \quad r = 30^\circ$$

The critical angle C is given by

$$C = \sin^{-1}\left(\frac{1}{\mu}\right) = 45^\circ$$



Applying, Sine Law (i.e., Snell's Law) in ΔOPM, we get

$$\frac{OP}{\sin C} = \frac{OM}{\sin(90^\circ + r)}$$

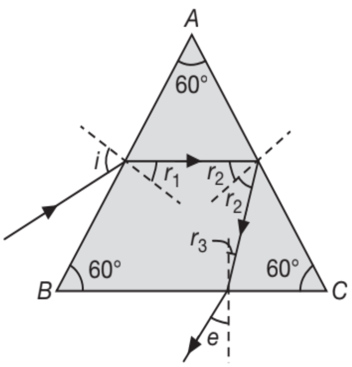
$$\Rightarrow \quad \frac{OP}{\left(\frac{1}{\sqrt{2}}\right)} = \frac{R}{\cos r} \qquad \{R = \text{radius}\}$$

$$\Rightarrow \quad OP = \sqrt{\frac{2}{3}}R$$

As we move away from O, angle PMO will increase. Therefore, $OP \ngtr \sqrt{\frac{2}{3}}R$. Same is the case on left side of O.

Test Your Concepts-V (Based on Prism)

1. The ray diagram for the situation discussed is shown in figure.



$e = 35^\circ, r_2 = C$

(a) From the figure, we observe that

$$r_1 + r_2 = r_2 + r_3 = 60^\circ$$

$$\Rightarrow \quad r_1 = r_3$$

Applying Snell's Law at the faces AB and BC, we get

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin(30^\circ)}{\sin r_3}$$

Since $r_1 = r_3$

$$\Rightarrow \quad i = 30^\circ$$

(b) Since $r_1 + r_2 = A = 60^\circ$

$$\Rightarrow \quad r_1 = 60^\circ - r_2 \cong 60 - C \qquad \{\because r_2 \approx C\}$$

Further, $\mu = \dfrac{\sin i}{\sin r_1} = \dfrac{\sin(30^\circ)}{\sin(60^\circ - C)}$

$$\mu = \frac{0.5}{\sin(60^\circ)\cos C - \cos(60^\circ)\sin C}$$

Since $\sin C = \dfrac{1}{\mu}$

$$\Rightarrow \quad \cos C = \sqrt{1 - \frac{1}{\mu^2}}$$

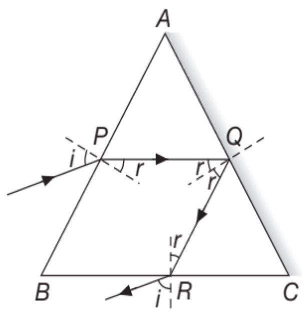
$$\Rightarrow \quad \mu\left(\frac{\sqrt{3}}{2}\right)\sqrt{1 - \frac{1}{\mu^2}} - \frac{1}{2} = 0.5$$

$$\Rightarrow \quad \left(\frac{\sqrt{3}}{2}\right)\sqrt{\mu^2 - 1} = 1$$

$$\Rightarrow \quad \mu = \sqrt{\frac{7}{3}}$$

2. At minimum deviation, we have

$$r = \frac{A}{2} = 30^\circ$$



(a) Applying Snell's Law at AB, we get

$$1.5 = \frac{\sin i}{\sin(30^\circ)}$$

$$\Rightarrow \quad i = 48.6^\circ$$

Since, $\delta_{\text{Total}} = \delta_P + \delta_Q + \delta_R$

$$\Rightarrow \quad \delta_{\text{Total}} = (i - r) + (180^\circ - 2r) + (i - r)$$

$$\Rightarrow \quad \delta_{\text{Total}} = 180^\circ + 2i - 4r$$

$$\Rightarrow \quad \delta_{\text{Total}} = 157.2^\circ$$

(b) Again applying Snell's Law for water-glass interface, we get

$$\frac{4}{3}\sin i' = \frac{3}{2}\sin(30^\circ)$$

$\Rightarrow \quad i' = 34.2°$

$\Rightarrow \quad \delta_{Total} = 180° + 2i' - 4r$

$\Rightarrow \quad \delta_{Total} = 128.4°$

3. Since, the condition for no emergence is

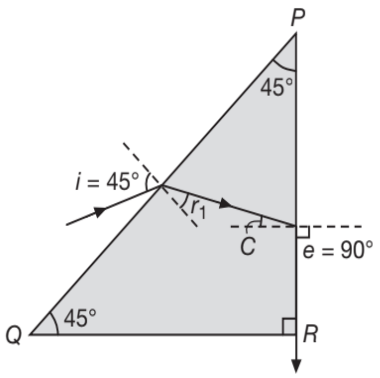
$A > 2C$

$\Rightarrow \quad A > 2\sin^{-1}\left(\frac{1}{\mu}\right)$

$\Rightarrow \quad A > 2\sin^{-1}\left(\frac{1}{1.5}\right) > 83.62°$

Therefore, $A_{max} = 83.62°$, for escaping of the ray through the adjacent face.

4. The situation is shown in figure



(a) Since, $e = 90°$

Also, $r_2 = C = \sin^{-1}\left(\frac{1}{\mu}\right)$

Now, $i = A = 45°$ and $r_1 = A - r_2 = 45° - C$

Applying Snell's Law at AB, we get

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin(45°)}{\sin(45° - C)}$$

$$\Rightarrow \quad \mu = \frac{\sin(45°)}{\sin(45°)\cos C - \cos(45°)\sin C}$$

Since $\sin C = \frac{1}{\mu}$

$$\Rightarrow \quad \cos C = \sqrt{1 - \frac{1}{\mu^2}}$$

$$\Rightarrow \quad \mu\sqrt{1 - \frac{1}{\mu^2}} - 1 = 1 \qquad (\sin 45° = \cos 45°)$$

$$\Rightarrow \quad \mu^2 - 1 = 4$$

$$\Rightarrow \quad \mu = \sqrt{5}$$

(b) At minimum deviation, we have

$$r_1 = r_2 = \frac{A}{2} = 22.5°$$

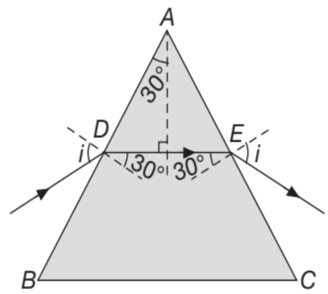
Since, $\sqrt{5} = \frac{\sin i}{\sin(22.5°)}$

$\Rightarrow \quad i = 58.8°$

5. (a) Applying Snell's law at D, we get

$$\left(\frac{4}{3}\right)\sin i = \left(\frac{3}{2}\right)\sin 30°$$

$\Rightarrow \quad i = 34.2°$



(b) Total deviation suffered by the ray is

$$\delta = \delta_D + \delta_E = 2\delta_D$$

$$\Rightarrow \quad \delta = 2(i - 30°) = 8.4°$$

6. At near normal incidence, $i \approx r_1 = 0°$

Since $r_1 + r_2 = A$

$\Rightarrow \quad r_2 = \alpha$

From Snell's Law applied at the face from where the refracted ray emerges, we get

$$\mu = \frac{\sin e}{\sin r_2}$$

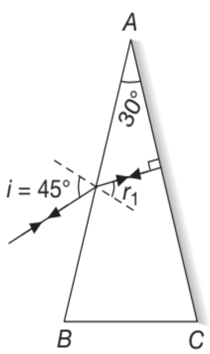
$\Rightarrow \quad e = \sin^{-1}(\mu \sin\alpha)$

Now, deviation $\delta = i + e - A = \sin^{-1}(\mu\sin\alpha) - \alpha$

$\Rightarrow \quad \delta = \sin^{-1}(\mu\sin\alpha) - \alpha \qquad \{\because i = 0°\}$

7. For the ray to retrace its path, it must be incident normally to the face AC. So, we have

$r_2 = 0°$



Since $r_1 + r_2 = A$

$\Rightarrow \quad r_1 = A = 30°$

From Snell's Law, we have

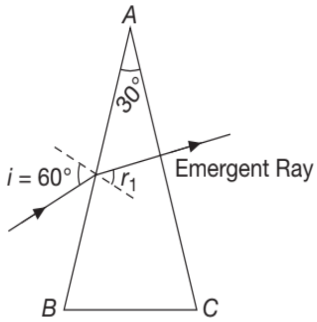
$$\mu = \frac{\sin i}{\sin r_1} = \sqrt{2}$$

8. From the statement of the problem, we gather the information that

$$i = 60°,\ A = 30°,\ \delta = 30°$$

Since, $\delta = i + e - A$

$\Rightarrow \quad e = \delta - i + A = 30° - 60° + 30° = 0°$



i.e., the emergent ray is perpendicular to the face through which it emerges.

Further, $r_2 = 0$ and $r_1 + r_2 = A$

$\Rightarrow \quad r_1 = A = 30° \qquad \{\text{as } e = 0\}$

From Snell's Law applied at face AC, we get

$$\mu = \frac{\sin i}{\sin r_1} = \sqrt{3}$$

9. **For Violet Light:** According to Snell's Law, applied at the plane of incidence, we get

$$\mu = \frac{\sin i}{\sin r_1}$$

$\Rightarrow \quad 1.66 = \dfrac{\sin(50°)}{\sin r_1}$

$\Rightarrow \quad r_1 = 27.5°$

Since, $r_1 + r_2 = A$

$\Rightarrow \quad r_2 = A - r_1 = 32.5°$

Applying Snell's Law at the plane of emergence, we get

$$\mu = \frac{\sin e}{\sin r_2}$$

$\Rightarrow \quad 1.66 = \dfrac{\sin e}{\sin(32.5°)}$

$\Rightarrow \quad e = 63.1°$

Since, $\delta_V = (i + e) - A$

$\Rightarrow \quad \delta_V = 53.1°$

For Red Light: According to Snell's Law, applied at the plane of incidence, we get

$$1.62 = \frac{\sin(50°)}{\sin r_1}$$

$\Rightarrow \quad r_1 = 28.2°$

Since, $r_1 + r_2 = A$

$\Rightarrow \quad r_2 = A - r_1 = 31.8°$

Applying Snell's Law at the plane of emergence, we get

$$1.62 = \frac{\sin e}{\sin(31.8°)}$$

$\Rightarrow \quad e = 58.6°$

Since, $\delta_R = (i + e) - A = 48.6°$

So, angular dispersion is given by

$$\delta_V - \delta_R = 4.5°$$

10. For minimum deviation, we have

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$\Rightarrow \quad 1.3 \sin(45°) = \sin(45° + \delta_m)$

Solving, this we get

$$\delta_m = 22°$$

For maximum deviation, we have the emergent ray to be grazing on the surface of emergence. So,

$$e = 90°,\ r_2 = C \text{ and } r_1 = 90° - r_2 = 90° - C$$

$\Rightarrow \quad \delta_{max} = i + e - A$

We can find i by using $\mu = \dfrac{\sin i}{\sin r_1}$ and $\sin C = \dfrac{1}{\mu}$.

Substituting the values, we get

$$\delta_{max} = 56°$$

11. Given that, $i = 60°$, $A = 30°$ and $\delta = 30°$

Since, $\delta = i + e - A$

Substituting the values we get, $e = 0°$

Now, $e = 0°$, means that the emergent ray is normal to the face through which it emerges.

12. Given that $A = 30°$ and $i = 0°$, so $r_1 = 0°$

Since, $r_1 + r_2 = A$

$\Rightarrow \quad r_2 = A = 30°$

Further applying Snell's Law at the plane of emergence, we get

$$1.5 = \frac{\sin e}{\sin r_2}$$

Substituting the values, we get

$e = 49°$

$\Rightarrow \quad \delta = i + e - A = 19°$

13. For no total internal reflection, when the ray leaves the prism,

$r_2 = C$

But $\sin C = \dfrac{1}{\mu} = \dfrac{1}{1.6}$

$r_2 = C = \sin^{-1}\left(\dfrac{1}{1.6}\right) = 38.7°$

Further $r_1 + r_2 = A = 45°$

$\Rightarrow \quad r_1 = 45° - 38.7° = 6.3°$

Now, $i = \sin^{-1}(\mu \sin r_1) = \sin^{-1}[1.6 \times \sin(6.3°)] = 10.1°$

14. For red ray, we have

$e = 0° = r_2$

Since, $r_1 + r_2 = A$

$\Rightarrow \quad r_1 = A = 45°$

Now, according to Snell's Law, we have

$$1.37 = \frac{\sin i}{\sin(45°)}$$

$\Rightarrow \quad i = 75.6°$

$\Rightarrow \quad \delta_{red} = i + e - A = 30.6°$

For violet ray, we have

$$1.42 = \frac{\sin(75.6°)}{\sin r_1}$$

$\Rightarrow \quad r_1 = 43°$

$\Rightarrow \quad r_2 = A - r_1 = 2° \qquad \{\because r_1 + r_2 = A\}$

Again, applying Snell's Law at the emerging face, for violet rays, we get

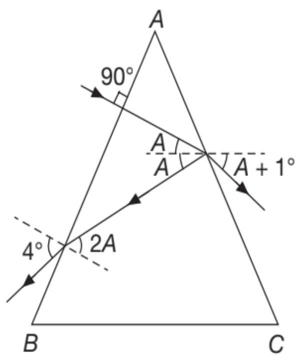
$$1.42 = \frac{\sin e}{\sin r_2}$$

$\Rightarrow \quad e = 2.84°$

$\Rightarrow \quad \delta_{violet} = i + e - A = 33.4°$

15. As the angles are small, we can take,

$\sin\theta \approx \theta$



Applying Snell's Law for the two emerging rays at AC and AB, we get

$$\mu = \frac{\sin(A + 1°)}{\sin A} = \frac{\sin(4°)}{\sin(2A)}$$

$\Rightarrow \quad \mu \approx \dfrac{A + 1°}{A} = \dfrac{4°}{2A}$

$\Rightarrow \quad A = 1°$ and $\mu = 2$

Test Your Concepts-VI
(Based on Refraction at Curved Surfaces)

1. Let us see where do the parallel rays converge (or diverge) on the principal axis. Let us call it the focus and the corresponding length the focal length f. Using $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$ with appropriate values and signs, we get

$$\frac{\frac{4}{3}}{f} - \frac{1}{\infty} = \frac{\frac{4}{3} - 1}{+10}$$

$\Rightarrow \quad f = 40 \text{ cm} = 0.4 \text{ m}$

Since, the rays are converging, its power should be positive. Hence,

$$P \text{ (in dioptre)} = \frac{1}{f(\text{metre})} = \frac{1}{0.4}$$

$\Rightarrow \quad P = 2.5 \text{ dioptre} = 2.5 \text{ D}$

2. For first refraction at the unsilvered surface, we have

$$\frac{1.5}{v_1} - \frac{1}{(-2r)} = \frac{1.5 - 1}{r}$$

$\Rightarrow \quad v_1 \to \infty$

i.e., rays become parallel to the principal axis.

Hence the image formed by the curved mirror will lie at the focus of the mirror i.e., a distance $\dfrac{r}{2}$ from pole of mirror.

So, $v_2 = \dfrac{r}{2}$ {from pole of the mirror}

For second refraction at the unsilvered surface, we have

$$\frac{1}{v_3} - \frac{1.5}{\left(-\frac{3r}{2}\right)} = \frac{1 - 1.5}{(-r)}$$

$\Rightarrow \quad v_3 = -2R$

i.e., final image is formed at pole of the mirror.

3. For the image of object O to be formed at O, the light should fall normally on mirror. First image I_1 (after refraction from the plane surface) will be formed at a distance of $\frac{2R+x}{\mu}$ from plane surface, because $d_{app.} = \frac{d_{actual}}{\mu}$.

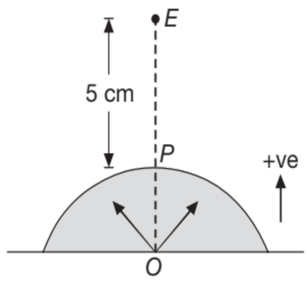
Now applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ with the idea that I_2 is formed at C, because light falls normally on the mirror.

$$\Rightarrow \quad \frac{\mu}{-(R+x)} - \frac{1}{-\left(\frac{2R+x}{\mu}+R\right)} = \frac{\mu-1}{-R}$$

Solving this equation for $\mu = 1.5$, we get

$$x \approx 0.75R$$

4. Applying, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$, we get



$$\frac{1}{PI} - \frac{1.6}{(-3)} = \frac{1-1.6}{-5}$$

$\Rightarrow \quad PI = -2.42$ cm

$\Rightarrow \quad EI = (5+2.42)$ cm

$\Rightarrow \quad EI = 7.42$ cm

5. (a) Applying, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$, we get

$$\frac{1.5}{v} - \frac{1}{-1} = \frac{1.5-1}{6}$$

$\Rightarrow \quad v = -90$ cm

So, the distance between object and its image is 80 cm

(b) Again applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\Rightarrow \quad \frac{1.5}{v} - \frac{1}{(-u)} = \frac{1.5-1}{6}$$

$$\Rightarrow \quad v = \frac{18}{\left(1 - \frac{12}{u}\right)}$$

v is negative when $\frac{12}{u} > 1$

$\Rightarrow \quad u < 12$ cm

6. Applying, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ twice, we get

$$\frac{1.5}{v_1} - \frac{1}{(-2.5)} = \frac{1.5-1}{+10}$$

$$\Rightarrow \quad v_1 = -\frac{30}{7} \text{ cm}$$

Further, $\frac{1}{v_2} - \frac{1.5}{-\left(20 + \frac{30}{7}\right)} = \frac{1-1.5}{-10}$

$\Rightarrow \quad v_2 = -85$ cm

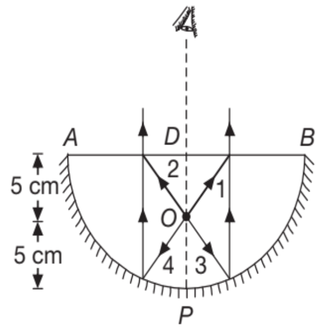
i.e., final image is formed at 65 cm from first face on the same side of the object.

7. First image will be formed by direct rays 1 and 2, etc.

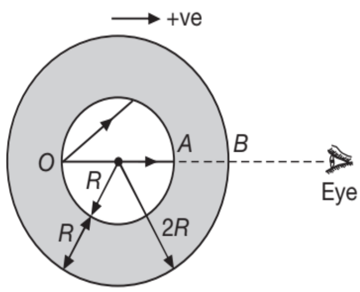
$$DI_1 = \frac{DO}{\mu} = \frac{5}{1.5} = 3.33 \text{ cm}$$

Second image will be formed by reflected rays 3 and 4, etc.

Object is placed at the focus of the mirror. Hence, I_2 is formed at infinity.



8. We have to see the image of O from the other side



Applying, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ twice, we get

$$\frac{\mu}{AI_1} - \frac{1}{(-2R)} = \frac{\mu-1}{-R}$$

$$\Rightarrow \quad AI_1 = \frac{2\mu R}{1-2\mu}$$

Further, $\frac{1}{BI_2} - \frac{\mu}{(AI_1 - R)} = \frac{1-\mu}{-R}$

Solving this equation, we get

$$BI_2 = -\frac{2R(4\mu-1)}{3\mu-1}$$

So, the distance between the final image and the object is

$$d = 3R - \frac{2R(4\mu-1)}{3\mu-1} = \frac{(\mu-1)R}{(3\mu-1)}$$

9. For refraction formula, we use

$$u = +15 \text{ cm} ;\ \mu_1 = \frac{4}{3} ;\ \mu_2 = 1 \text{ and } R = +10 \text{ cm}$$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Substituting the values, we get

$$\frac{1}{v} - \frac{4}{3\times 15} = \frac{1-\frac{4}{3}}{10} = -\frac{1}{30}$$

$$\Rightarrow \frac{1}{v} = \frac{4}{45} - \frac{1}{30} = \frac{1}{18}$$

$$\Rightarrow v = +18 \text{ cm}$$

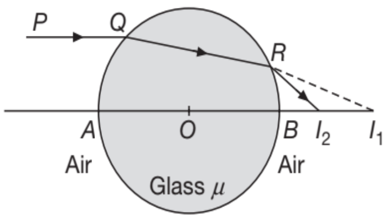
Velocity magnification along optic axis is given as

$$m = \frac{\mu_1 v^2}{\mu_2 u^2} = \frac{\frac{4}{3}\times(18)^2}{1\times(15)^2} = 1.92$$

Velocity of image of fish is

$$\Rightarrow V_{fish} = 1.92 \times 2 = 3.84 \text{ ms}^{-1}$$

10. Figure shows the refraction of a parallel beam on a solid glass sphere at near normal incidence. At first refraction image I_1 is produced which acts as an object for second refraction at other surface of sphere at which after refraction final image I_2 is produced.



Using refraction formula at first surface we have

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u_1} = \frac{\mu_1 - \mu_2}{R} \quad \ldots(1)$$

Here we use $\mu_1 = 1$, $\mu_2 = \mu$, $u_1 = \infty$ and $R = +R$

Substituting the values in refraction formula we get

$$v_1 = \frac{\mu R}{(\mu-1)} \quad \ldots(2)$$

For the refraction at the second surface, we use

$$\mu_1 = \mu,\ \mu_2 = 1,\ R = -R \text{ and}$$

$$u_2 = +\frac{\mu R}{(\mu-1)} - 2R = \frac{2R - \mu R}{(\mu-1)} = \frac{R(2-\mu)}{(\mu-1)}$$

For the second surface,

$$\frac{1}{v_2} - \frac{\mu}{u_2} = \frac{(1-\mu)}{-R}$$

$$\Rightarrow \frac{1}{v_2} - \frac{\mu(\mu-1)}{R(2-\mu)} = \frac{(1-\mu)}{-R}$$

$$\frac{1}{v_2} = \frac{\mu(\mu-1)}{R(2-\mu)} - \frac{(1-\mu)}{R}$$

Simplifying, we get

$$v_2 = \frac{R(2-\mu)}{2(\mu-1)}$$

11. For refraction formula, we use

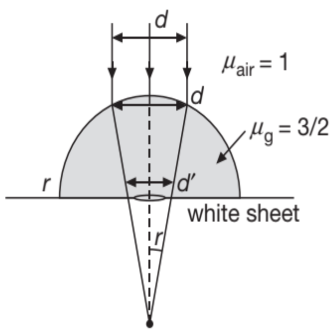
$$u = \infty ;\ \mu_1 = 1 ;\ \mu_2 = \frac{3}{2} \text{ and } R = +r$$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Substituting the values, we have

$$\frac{3}{2v} = \frac{\frac{3}{2} - 1}{r} = \frac{1}{2r}$$

$$\Rightarrow v = +3r$$



by similarity, we can write

$$\frac{d}{d'} = \frac{3r}{2r}$$

$$\Rightarrow d' = \frac{2d}{3}$$

12. We consider refractive index of glass to be μ and after first refraction, image is produced at v then by refraction formula, we use

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u_1} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \quad \frac{\mu}{v} - \frac{1}{\infty} = \frac{\mu - 1}{R}$$

$$\Rightarrow \quad v = \frac{\mu R}{\mu - 1}$$

For second refraction using refraction formula, we have

$$u_1 = \frac{\mu R}{\mu - 1} - R = +\frac{R}{\mu - 1}; v_1 = +R; R = +\frac{R}{2}; \mu_1 = \mu$$

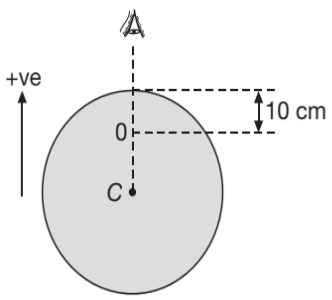
and $\mu_2 = 1$

$$\Rightarrow \quad \frac{1}{R} - \frac{\mu(\mu - 1)}{R} = \frac{2(1 - \mu)}{R}$$

$$\Rightarrow \quad \mu^2 - 3\mu + 1 = 0$$

Solving we get $\mu = \frac{3 + \sqrt{5}}{2}$

13. Figure below shows the situation described in the question.



We apply refraction formula

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Here we use $u = -10$ cm ; $R = -15$ cm ; $\mu_1 = 1.5$ and $\mu_2 = 1$

Substituting the values, we have

$$\frac{1}{v} - \frac{1.5}{-10} = \frac{1 - 1.5}{-15}$$

Solving we get $v = -8.57$ cm

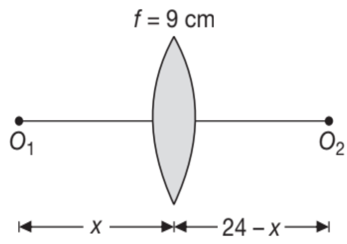
Thus, the apparent depth of bubble will be 8.57 cm from the top of the sphere.

Test Your Concepts-VII (Based on Lens Formula)

1. When the images of both the sources are formed at the same point, then v will be same for both (in value). However, for one case the image will be real and for the other case it will be virtual. So,

For O_1, we have $\frac{1}{v} + \frac{1}{x} = \frac{1}{9}$ ...(1)

For O_2, we have $-\frac{1}{v} + \frac{1}{24 - x} = \frac{1}{9}$ ...(2)



Adding equations (1) and (2), we get

$$\frac{1}{x} + \frac{1}{24 - x} = \frac{2}{9}$$

Solving we get $x = 6$ cm

Hence, the lens should be kept at a distance of 6 cm from either of the object.

2. Here the image formed can be virtual as well as real, so the value of m should be +3 in one case (virtual image) and −3 in the other (real image). Magnification of +3 can be obtained only when the object is placed within F (i.e., for smaller value of the object distance). The magnification of −3 is obtained when the object is kept between F and $2F$ (i.e., for greater value of object distance). So,

$m = +3$, for $u = -8$ cm

Therefore, from the definition of magnification, we have

$$m = \frac{v}{u}$$

$$\Rightarrow \quad +3 = \frac{v}{-8}$$

$$\Rightarrow \quad v = 3 \times (-8) = -24 \text{ cm}$$

Using lens formula, we get

$$\frac{1}{-24} - \frac{1}{-8} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{-1 + 3}{24} = \frac{1}{f}$$

$$\Rightarrow \quad f = 12 \text{ cm}$$

Note that you would get the same answer by considering the other case ($m = -3$, for $u = -16$ cm)

3. The Lens Maker's Formula is given by

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $f = 30$ cm, $R_1 = 10$ cm, $R_2 = \infty$

$$\Rightarrow \quad \frac{1}{30} = (\mu - 1)\left(\frac{1}{10} - \frac{1}{\infty}\right) = \frac{\mu - 1}{10}$$

$\Rightarrow \quad 30\mu - 30 = 10$

$\Rightarrow \quad \mu = \dfrac{4}{3}$

4. Applying Lens Maker's Formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $\mu = 1.5$, $f = 60$ cm, $R_1 = +R$, $R_2 = -R$

$$\Rightarrow \quad \frac{1}{60} = (1.5 - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$\Rightarrow \quad R = 60$ cm

Therefore, the focal length of the spherical silvered surface, is given by

$$f_m = \frac{R}{2} = \frac{60}{2} = +30 \text{ cm}$$

(Positive, because it is a converging mirror)

The equivalent focal length of the lens-mirror combination is then given by

$$-\frac{1}{F} = \frac{2}{f_l} - \frac{1}{f_m} = \frac{2}{60} - \frac{1}{(-30)}$$

$\Rightarrow \quad F = -15$ cm

The negative sign indicates that the combination behaves as a concave mirror.

5. According to the Lens Maker's Formula, we have

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \text{ where } f = 10 \text{ cm} \qquad \ldots(1)$$

When placed in medium 1, then

$$\frac{1}{f_1} = \left(\frac{\mu}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots(2)$$

When placed in medium 2, then

$$\frac{1}{f_2} = \left(\frac{\mu}{\mu_2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots(3)$$

From equations (1) and (2), we get

$$\frac{f_1}{f} = \frac{(\mu - 1)}{\left(\frac{\mu}{\mu_1} - 1\right)} = \frac{1.6 - 1}{\left(\frac{1.6}{1.5} - 1\right)} = 9$$

$\Rightarrow \quad f_1 = 9f = 90$ cm

i.e., still it behaves as a converging lens

From equations (1) and (3), we get

$$\frac{f_2}{f} = \frac{(\mu - 1)}{\left(\frac{\mu}{\mu_2} - 1\right)} = \frac{1.6 - 1}{\left(\frac{1.6}{1.7} - 1\right)} = -10.2$$

$\Rightarrow \quad f_2 = -10.2f = -102$ cm

i.e., it now behaves like a diverging lens.

6. Here $R_1 = +25$ cm, $R_2 = -25$ cm, $\mu_1 = 1$ and $\mu_2 = \dfrac{3}{2}$

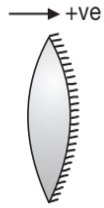


Image coincides with object, hence, $u = v = -x$ (say)

$$\frac{1}{-x} - \frac{1}{x} = \frac{2(3/2)}{-25} - \frac{2(3/2 - 1)}{25}$$

$$\Rightarrow \quad \frac{2}{x} = \frac{3}{25} + \frac{1}{25} = \frac{4}{25}$$

$\Rightarrow \quad x = 12.5$ cm

Hence, the object should be placed at a distance 12.5 cm in front of the silvered lens.

7. Given, $f_1 = +5$ cm and $f_2 = -10$ cm

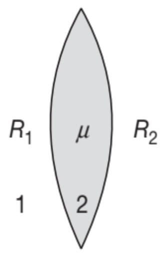
The combined focal length F is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{5} - \frac{1}{10} = +\frac{1}{10}$$

$\Rightarrow \quad F = +10$ cm

i.e., the combination behaves as a converging lens of focal length 10 cm.

8. According to Lens Maker's Formula, we have



$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{10} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{-10}\right)$$

$\Rightarrow \quad R_1 = +10$ cm

Now, using, $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{2(\mu_2/\mu_1)}{R_2} - \dfrac{2(\mu_2/\mu_1 - 1)}{R_1}$

Substituting the values, we get

$$\frac{1}{v} - \frac{1}{-15} = \frac{2(1.5)}{-10} - \frac{2(1.5 - 1)}{+10}$$

$\Rightarrow \quad v = -2.14$ cm

9. Using lens formula, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$, we get

$$\frac{1}{36} - \frac{1}{-45} = \frac{1}{f}$$

$\Rightarrow \quad f = 20$ cm

In the second case, let μ be the refractive index of the liquid, then

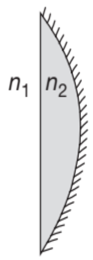
$$\frac{1}{48}-\frac{1}{-\left(5+\frac{40}{\mu}\right)}=\frac{1}{20}$$

Solving, we get

$\mu = 1.37$

10. The system behaves like a mirror of focal length given by,

$$\frac{1}{F}=\frac{2(\mu_2/\mu_1)}{R_2}-\frac{2(\mu_2/\mu_1-1)}{R_1}$$



Substituting the values with appropriate signs, we get

$$\frac{1}{F}=\frac{2\left(\frac{4}{3}\right)}{-20} \qquad \{\because R_1 \to \infty\}$$

$\Rightarrow \quad F = -7.5$ cm

So, the system behaves as a concave mirror of focal length 7.5 cm .

11. For a convex lens, the distance between an object and its real image is minimum when $u = 2f_1$ and $v = 2f_1$
When concave lens is placed in contact, then we have

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_{comb}}$$

$$\frac{1}{v'}+\frac{1}{2f_1}=\frac{1}{f_1}-\frac{1}{f_2}=\frac{f_2-f_1}{f_1f_2}$$

$$\Rightarrow \quad v'=\frac{2f_1f_2}{f_2-2f_1}$$

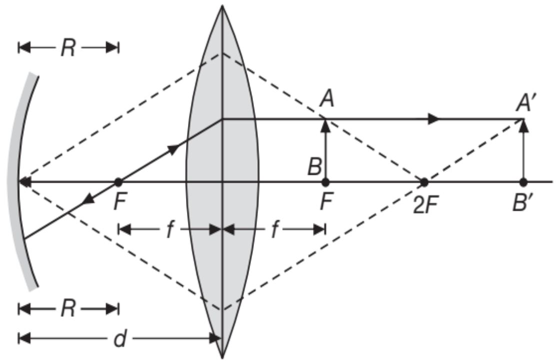
Shift of image is

$$\Delta v = v'-v=\frac{2f_1f_2}{f_2-2f_1}-2f_1=\frac{4f_1^2}{f_2-2f_1}$$

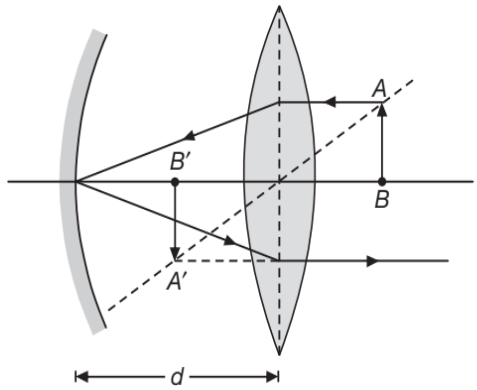
Since, $f_2 \gg f_1$, so the shift of image is

$$\Delta v \approx \frac{4f_1^2}{f_2}$$

12. The following two cases are possible.
CASE-1: The mirror is at a distance of $d = f + R = 2$ m from the lens. The path of the beam parallel to the optical axis of the system and the image of object AB are shown in figure. Image $A'B'$ (direct and real) is obtained to full scale with the object in any position.



CASE-2: The mirror is at a distance of $d = f = R = 1$ m from the lens. The image of object $A'B'$, also full scale, will be inverted and virtual with the object in any position.



13. For a convex lens, distance between an object and its real image is minimum when $u = 2f_1$. Hence,

(a) $3f_1-\left(1-\frac{1}{\mu}\right)t=2f_1$

$\Rightarrow \quad \left(1-\frac{1}{\mu}\right)t=f_1$

$\Rightarrow \quad \frac{1}{\mu}=1-\frac{f_1}{t}=\frac{t-f_1}{t}$

$\Rightarrow \quad \mu=\frac{t}{t-f_1}$

(b) If a concave lens of very large focal length f_2 (i.e., very small power) is placed in contact with the convex lens, then its power and hence the focal length are almost unaltered. Therefore, there will be no shifting of the image.

14. Using the lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we get for the first lens

$$\frac{1}{v_1}+\frac{1}{30}=\frac{1}{20}$$

$\Rightarrow \quad v_1 = 60$ cm

For the second lens

$$\frac{1}{v_2}-\frac{1}{30}=\frac{1}{10}$$

$\Rightarrow \quad v_2=7.5$ cm

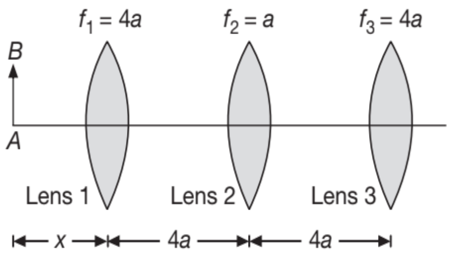
15. Since, $\frac{1}{F}=\frac{2\left(\frac{\mu_2}{\mu_1}\right)}{R_2}-\frac{2\left(\frac{\mu_2}{\mu_1}-1\right)}{R_1}=\frac{2\mu}{-r}-\frac{2(\mu-1)}{r}$

$\Rightarrow \quad \frac{1}{F}=-\left(\frac{4\mu-2}{r}\right)$

$\Rightarrow \quad F=-\left(\frac{r}{4\mu-2}\right)$

i.e., the lens is equivalent to a concave mirror of focal length $\frac{r}{4\mu-2}$

16. Let x be the distance between first lens and the object AB. Applying the lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ thrice, we get for lens 1,



$$\frac{1}{v_1}+\frac{1}{x}=\frac{1}{4a} \quad \ldots(1)$$

$\Rightarrow \quad v_1=\frac{4ax}{x-4a}$

$\Rightarrow \quad m_1=\frac{v}{u}=\frac{v_1}{-x}=\frac{4a}{4a-x} \quad \ldots(2)$

Similarly, for Lens 2, we get

$$\frac{1}{v_2}-\frac{1}{v_1-4a}=\frac{1}{a}$$

$\Rightarrow \quad \frac{1}{v_2}-\frac{x-4a}{16a^2}=\frac{1}{a} \quad \ldots(3)$

$\Rightarrow \quad v_2=\frac{16a^2}{x+12a}$

$\Rightarrow \quad m_2=\frac{\left(\frac{16a^2}{x+12a}\right)}{\left(\frac{16a^2}{x-4a}\right)}=\frac{x-4a}{x+12a} \quad \ldots(4)$

Similarly, for lens 3, we get

$$\frac{1}{v_3}-\frac{1}{v_2-4a}=\frac{1}{4a} \quad \ldots(5)$$

$\Rightarrow \quad v_3=-(x+8a)$

$\Rightarrow \quad m_3=\frac{x+12a}{4a} \quad \ldots(6)$

So, we observe from (2), (4) and (6) that

$$m_1m_2m_3=-1$$

Also, $v_3=-(x+8a)$ means that the object and its image lie at the same place and $m_1m_2m_3=-1$ means final image is of the same size but inverted.

17. According to the lens formula, we have, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

Since $u=-mf$, so we get

$$\frac{1}{v}+\frac{1}{mf}=\frac{1}{(-f)}$$

$\Rightarrow \quad v=-\left(\frac{m}{m+1}\right)f$

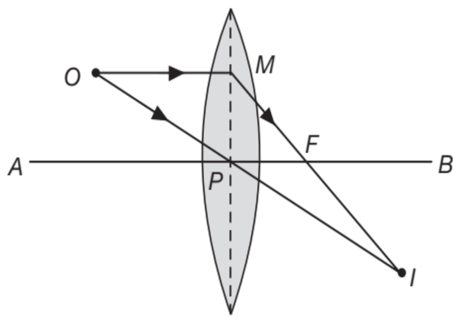
So, lateral magnification is given by

$$m_L=\frac{v}{u}=\frac{1}{m+1}$$

Hence, the image will be $(m+1)$ times smaller than the object.

18. (a) Since a concave lens always forms an erect image whereas the given image I is on the other size of the optic axis, so the lens is convex.

(b) Join O with I. Line OI cuts the optic axis AB at pole (P) of the lens. The dotted line shows the position of lens.



From point O, draw a line parallel to AB which after refraction must pass through the focus F of the lens.

19. Let parallel rays be incident on first lens, then

$$v_1=f_1=7\text{ cm}$$

For the second lens, we have

$$\frac{1}{v_2}-\frac{1}{(7-3)}=\frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{v_2} - \frac{1}{4} = \frac{1}{6}$$

$$\Rightarrow \quad v_2 = 2.4 \text{ cm}$$

20. (a) From Lens Maker's Formula, we get

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{f_1}{f_2} = \frac{\mu_2 - 1}{\mu_1 - 1} = \frac{1.7 - 1}{1.5 - 1} = \frac{7}{5} = 1.4$$

(b) In this liquid the first lens will be a diverging (as refractive index of liquid > 1.5) and the second a converging one (as refractive index of liquid < 1.7).

21. Since the incident beam is parallel, so we have

$$v_1 = f_1 = 10 \text{ cm}$$

For the second lens, we have

$$\frac{1}{v_2} - \frac{1}{-5} = \frac{1}{-20}$$

$$\Rightarrow \quad v_2 = -4 \text{ cm}$$

For the third lens, we have

$$\frac{1}{v_3} - \frac{1}{-9} = \frac{1}{9}$$

$$\Rightarrow \quad v_3 \to \infty$$

i.e., rays will become parallel to the optic axis.

22. Since the focal length is equal to two times the radius of curvature, so $f = -2r$

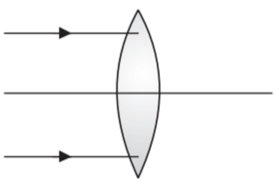
$$\frac{1}{(-2r)} = (\mu_0 + a\mathrm{I} - 1)\left(\frac{1}{\infty} - \frac{1}{r}\right)$$

$$\Rightarrow \quad \mathrm{I} = \frac{3 - 2\mu_0}{2a}$$

23. The rays will first get refracted, then n-times reflected and finally again refracted. So, using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ for first refraction, we get

$$\frac{\mu}{v_i} - \frac{1}{\infty} = \frac{\mu - 1}{R}$$

$$\Rightarrow \quad v_i = \left(\frac{\mu}{\mu - 1}\right)R$$



For first reflection, let us use the mirror formula, i.e.,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{R}$$

$$\Rightarrow \quad \frac{1}{v_1} + \left(\frac{\mu - 1}{\mu R}\right) = \frac{-2}{R}$$

$$\Rightarrow \quad \frac{1}{v_1} = -\left(\frac{3\mu - 1}{\mu R}\right)$$

For second reflection, similarly we get

$$\frac{1}{v_2} + \frac{3\mu - 1}{\mu R} = \frac{-2}{R}$$

$$\Rightarrow \quad \frac{1}{v_2} = -\left(\frac{5\mu - 1}{\mu R}\right)$$

Similarly, after nth reflections, we get

$$\frac{1}{v_n} = -\left[\frac{(2n+1)\mu - 1}{\mu R}\right]$$

Finally, again using the refraction formula, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$, applied at the second curved surface, we get

$$\frac{1}{v_f} - \left\{\frac{(2n+1)\mu - 1}{R}\right\} = \frac{1 - \mu}{-R}$$

$$\Rightarrow \quad v_f = \frac{R}{2(\mu n + \mu - 1)}$$

24.

$$\frac{1}{f_1} = \frac{2\left(\frac{\mu_2}{\mu_1}\right)}{R_2} - \frac{2\left(\frac{\mu_2}{\mu_1} - 1\right)}{R_1}$$

$$\Rightarrow \quad \frac{1}{f_1} = \frac{2\mu}{\infty} - \frac{2(\mu - 1)}{R}$$

$$\Rightarrow \quad f_1 = \frac{-R}{2(\mu - 1)}$$

So, the system behaves as a concave mirror of focal length $\frac{R}{2(\mu - 1)}$. The object will coincide with image when the object is placed at centre of curvature. So, we get

$$x_1 = 2|f_1| = \frac{R}{\mu - 1} \quad \ldots(1)$$

In the second case, $\frac{1}{f_2} = \frac{-2\mu}{R}$

$$\Rightarrow \quad f_2 = -\frac{R}{2\mu}$$

CHAPTER 1

$$\Rightarrow \quad x_2 = 2|f_2| = \frac{R}{\mu} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$\mu = \frac{x_1}{x_1 - x_2}$$

and $R = \dfrac{x_1 x_2}{x_1 - x_2}$

Now, according to Lens Maker's Formula, we have

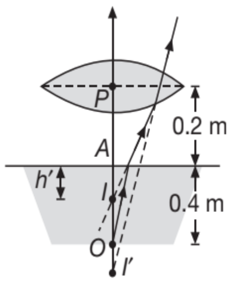
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R}\right) = \frac{\mu - 1}{R} = \frac{1}{x_1} \quad \text{\{from equation (1)\}}$$

$$\Rightarrow \quad f = x_1$$

25. The apparent position of the object O from the surface of water is

$$h' = \frac{h}{\mu} = \frac{0.4}{\frac{4}{3}}$$

$$\Rightarrow \quad h' = 0.3$$



The distance $PI = 0.3 + 0.2 = 0.5$ m

For convex lens, we use

$$u = -0.5 \text{ m}, \; f = +3 \text{ m}$$

By lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \text{ we have}$$

$$\frac{1}{v} - \frac{1}{-0.5} = \frac{1}{3}$$

$$\Rightarrow \quad v = -0.6 \text{ m}$$

26. Focal length of an equiconvex lens is given as

$$f = \frac{R}{2(\mu - 1)}$$

For lenses A and B, we use

$$\frac{R_A}{2(\mu_A - 1)} = \frac{R_B}{2(\mu_B - 1)}$$

$$\Rightarrow \quad \frac{\mu_A - 1}{0.9} = \mu_B - 1$$

$$\Rightarrow \quad \frac{0.63}{0.9} = \mu_B - 1$$

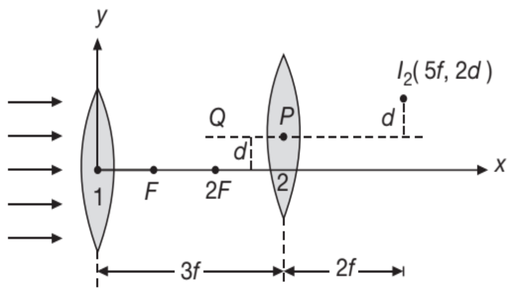
$$\Rightarrow \quad \mu_B = 1 + 0.7 = 1.7$$

27. The x-axis is the principal axis for first lens and PQ is the principal axis for second lens. First lens produces image at its focal point at a distance f on x-axis. Now for second lens, we use lens formula as

$$\frac{1}{v} - \frac{1}{-2f} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{2f}$$

$$\Rightarrow \quad v = 2f$$



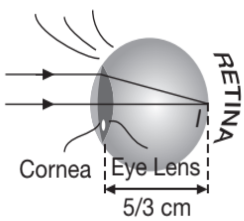
Magnification for the second lens is given as

$$m = \frac{h_i}{h_0} = +\frac{v}{u} = \frac{+2f}{-2f} = -1$$

Here negative sign indicates that the image will be formed above the principal axis PQ for the second lens. Thus, final coordinates of image are $(5f, 2d)$

Test Your Concepts-VIII (Based on Aberrations, Human Eye and Optical Instruments)

1. The human eye has a cornea, an eye lens and retina as shown in figure.



The cornea has fixed focal length whereas the eye lens has variable focal length. Since

$$P_{\text{eye}} = P_{\text{cornea}} + P_{\text{eye lens}} = 40 + 20 = 60 \text{ D}$$

If f_{eye} be the focal length of human eye (cornea + eye lens), then

$$f_{eye} = \frac{1}{P} = \frac{1}{60}\text{ m} = \frac{100}{60}\text{ cm} = \frac{5}{3}\text{ cm}$$

Therefore, the distance between eye lens and retina is $\frac{5}{3}$ cm.

So, for rays coming from infinity, the eye will make the image at retina i.e. at $\frac{5}{3}$ cm from the lens.

For object lying at distance of distinct vision i.e. $D = 25$ cm, we have $u = -25$ cm, $v = \frac{5}{3}$ cm

Applying lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{f'} = \frac{1}{5/3} - \frac{1}{(-25)} = \frac{3}{5} + \frac{1}{25} = \frac{16}{25}$$

$$\Rightarrow \quad f' = \frac{25}{16}\text{ cm}$$

$$\Rightarrow \quad P' = \frac{16}{25} \times 100 = 64\text{ D}$$

Since $P' = P_{cornea} + P'_{eye\ lens}$

$$\Rightarrow \quad 64 = 40 + P'_{eye\ lens}$$

$$\Rightarrow \quad P'_{eye\ lens} = 24\text{ D}$$

Therefore, power of accommodation of eye lens is between 20 D and 24 D.

Hence the power of accommodation of human eye (cornea + eye lens) is between 60 D (when object is at infinity) and 64 D (when object is at 25 cm)

2. Chromatic aberration is given by

$$\Delta f = f_r - f_b = (1.0 - 0.968)\text{ m} = 0.032\text{ m}$$

Since dispersive power of material of lens is given by

$$\omega = \frac{f_r - f_b}{f}, \text{ where } f = \sqrt{f_r f_b}$$

$$\Rightarrow \quad \omega = \frac{f_r - f_b}{\sqrt{f_r f_b}} = \frac{0.032}{\sqrt{1 \times 0.968}} = 0.0325$$

3. If f_e be the accommodate focal length of the eye-lenses for 2 m, then by lens formula, we have

$$\frac{1}{v} - \frac{1}{(-2)} = \frac{1}{f_e} \quad \ldots(1)$$

where v is the distance between eye lens and retina.

If f be the focal length of the correcting lens for seeing distant objects then by lens formula for combination, we have

$$\frac{1}{v} - \frac{1}{\infty} = \frac{1}{f_e} + \frac{1}{f} \quad \ldots(2)$$

Subtracting Equation (1) from Equation (2), we get

$$f = -2\text{ m}$$

Thus, power of the lens required is

$$P = \frac{1}{f} = \frac{1}{(-2)} = -0.5\text{ dipotre}$$

4. (i) We are given that the focal length of the lens is $f = 10$ cm and distance between the screen and lens is $v = 500$ cm.

Since, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{u} = \frac{1}{500} - \frac{1}{10} = -\frac{49}{500}$$

$$\Rightarrow \quad u = -\frac{500}{49}\text{ cm}$$

The linear magnification is given by

$$m = \frac{\text{Size of Image}}{\text{Size of Object}} = \frac{v}{u}$$

$$\Rightarrow \quad \text{Image size} = \frac{v}{u} \times \text{Object size}$$

$$\Rightarrow \quad \text{Image size} = \frac{500 \times 49}{500} \times 2 = 98\text{ cm}$$

$$\Rightarrow \quad \text{Size of the slide image} = 98\text{ cm} \times 98\text{ cm}$$

(ii) If the illuminating power of the source is P, then

Illumination of slide is $\frac{P}{2 \times 2}$

So, Illumination of slide image picture is $\frac{P}{98 \times 98}$

$$\Rightarrow \quad \frac{\text{Illumination of slide}}{\text{Illumination of picture}} = \frac{P}{2 \times 2} \times \frac{98 \times 98}{P}$$

$$\Rightarrow \quad \frac{\text{Illumination of slide}}{\text{Illumination of picture}} = 2401$$

5. Since, $M = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$

In first case, $M_1 = \frac{(v_o)_1}{u_o}\left(1 + \frac{D}{f_e}\right)$

where $f_e = 0.03\text{ m} = 3$ cm and $D = 25$ cm

$$\Rightarrow \quad M_1 = \frac{(v_o)_1}{u_o}\left(1 + \frac{25}{3}\right)$$

$$\Rightarrow \quad M_1 = \frac{(v_o)_1}{u_o} \times \frac{28}{3}$$

Similarly in second case

$$M_2 = \frac{(v_o)_2}{u_o}\left(1+\frac{25}{4}\right)$$

$$\Rightarrow \quad M_2 = \frac{(v_o)_2}{u_o}\times\frac{29}{4}$$

According to the problem $M_1 = M_2$

$$\Rightarrow \quad \frac{(v_o)_1}{u_o}\times\frac{28}{3} = \frac{(v_o)_2}{u_o}\times\frac{29}{4}$$

$$\Rightarrow \quad (v_o)_2 = \frac{112}{87}(v_o)_1$$

Let the distance of the object from eyepiece be x in the first arrangement. Then by lens formula for eyepiece, we get

$$-\frac{1}{25}+\frac{1}{x}=\frac{1}{3}$$

$$\Rightarrow \quad x = \frac{75}{28}\text{ cm}$$

$$\Rightarrow \quad (v_o)_1 = 20 - x = 20 - \frac{75}{28} = \frac{485}{28}\text{ cm}$$

and $(v_o)_2 = \frac{112}{87}\times\frac{485}{28} = \frac{1940}{87}$ cm

Let the distance of the object from eyepiece by y in the second arrangement. Then by lens formula, we get

$$-\frac{1}{25}+\frac{1}{y}=\frac{1}{4}$$

$$\Rightarrow \quad y = \frac{100}{29}$$

So, the distance between lenses in the second case is

$$L = (v_o)_2 + y$$

$$\Rightarrow \quad L = \frac{1940}{87}+\frac{100}{29} = \frac{2240}{87} = 25.75\text{ cm} = 0.2575\text{ m}$$

6. Let v_0 be the distance of the image formed by the objective alone so by lens formula for objective, we use

$$\frac{1}{v_0}-\frac{1}{u_0}=\frac{1}{f_0}$$

$$\Rightarrow \quad \frac{1}{v_0}=\frac{1}{u_0}+\frac{1}{f_0}$$

$$\Rightarrow \quad \frac{1}{v_0}=-\frac{1}{3}+\frac{1}{2}=\frac{1}{6}$$

$$\Rightarrow \quad v_0 = +6\text{ cm}$$

The image is formed at 6 cm behind the objective. If f_0' be the new focal length of the objective when a lens of focal length $0.1\text{ m}(=10\text{ cm})$ is removed from it, then for combination, we can use

$$\frac{1}{f_0'}=\frac{1}{f_0}-\frac{1}{10}=\frac{1}{2}-\frac{1}{10}=\frac{2}{5}$$

$$\Rightarrow \quad f_0' = +\left(\frac{5}{2}\right)\text{ cm}$$

Let v_0' be the new distance of the image formed by the objective then by using lens formula again, we have

$$\frac{1}{v_0'}=\frac{1}{u_0}+\frac{1}{f_0'}=-\frac{1}{3}+\frac{2}{5}=\frac{1}{15}$$

$$v_0' = +15\text{ cm}$$

Thus, the image is shifted from the objective through a distance $15\text{ cm} - 6\text{ cm} = 9\text{ cm}$. So, the eyepiece should be moved away from the objective by 9 cm to refocus the image at same position.

7. Magnification of microscope is given as

$$M = \frac{v_0}{u_0}\left(\frac{D}{f_e}\right)$$

Here $v_0 = 16 - 2.5 = 13.5$ cm

and $f_0 = +4\text{ mm} = +0.4$ cm

Using the lens formula, we have

$$\frac{1}{v_0}-\frac{1}{u_0}=\frac{1}{f_0}$$

$$\Rightarrow \quad u_0 = -\left(\frac{54}{131}\right)\text{ cm}$$

$$\Rightarrow \quad M = -\frac{13.5}{\left(\frac{54}{131}\right)}\times\frac{25}{2.5} = -327.5$$

8. For objective, by lens formula, we have

$$\frac{1}{v_0}-\frac{1}{-1.5}=\frac{1}{+0.25}$$

$$\Rightarrow \quad \frac{1}{v_0}=\frac{1}{0.25}-\frac{1}{1.5}=4-0.6667=3.3333$$

$$\Rightarrow \quad v_0 = +0.3\text{ m}$$

For eyepiece, by lens formula we have

$$\frac{1}{-0.25}-\frac{1}{-u_e}=\frac{1}{+0.02}$$

$$\Rightarrow \quad \frac{1}{u_e}=\frac{1}{0.25}+\frac{1}{0.02}=4+50=54$$

$$\Rightarrow \quad u_e = +0.01852\text{ m}$$

The tube length of the telescope is given as

$$L = 0.3 + 0.01852 = 0.31852 \text{ m}$$

Magnification by objective is

$$m_o = \frac{v_o}{u_o} = \frac{0.3}{1.5} = 0.2$$

Magnification by eyepiece is

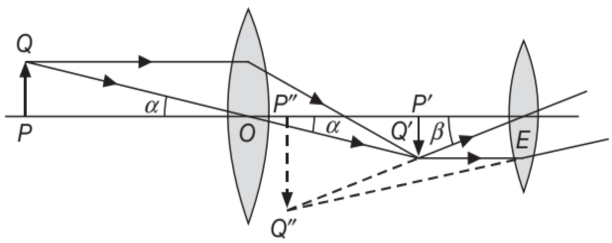
$$m_e = \frac{v_e}{u_e} = \frac{0.25}{0.001852} = 136.98$$

Total Magnification $m_T = m_1 \times m_2 = 27.39$

9. When the telescope is focussed on a distant object and the final image is also at infinity, the distance between lenses

$$L = f_0 + f_e = 50 + 5 = 55 \text{ cm}$$

When it is turned on a nearer object, the image formed by the objective will not be at its focus, but a little away from it. The eye-piece has to be shifted exactly by the same distance through which the image is shifted from focus of objective because the final image is at the same distance without change in accommodation of eye. Figure below shows the ray diagram of this situation.



Using lens formula for the objective, we have

$$\frac{1}{v_0} - \frac{1}{-10} = \frac{1}{+0.5}$$

$$\Rightarrow \quad v_0 = +\frac{50}{95} \text{ m} = +52.63 \text{ cm}$$

The distance by which the eye-piece is to be shifted is

$$s = 52.63 - 50$$

$$\Rightarrow \quad s = 2.63 \text{ cm}$$

When the object is at a finite but large distance then the magnifying power of telescope is given as

$$M = \frac{\beta}{\alpha} = \frac{v_0}{u_e}$$

For the case of focussing the telescope in both cases, using lens formula for eye-piece, we have

$$\frac{1}{\infty} - \frac{1}{-u_e} = \frac{1}{+5}$$

$$\Rightarrow \quad u_e = 5 \text{ cm}$$

Thus, magnification of telescope for the case of focussing on object located at a distance of 10 m is

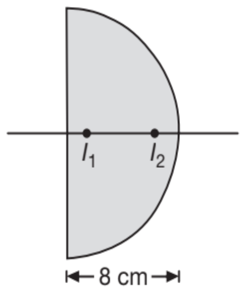
$$M = \frac{52.63}{5} = 10.52$$

Previous magnification when telescope is focussed on Moon, we know magnification is given as

$$M' = \frac{f_0}{f_e} = \frac{50}{5} = 10$$

Single Correct Choice Type Questions

1.



Distance of image from the plane surface is

$$x_1 = \frac{4}{1.6} = 2.5 \text{ cm} \qquad \left\{ \because d_{app} = \frac{d_{actual}}{\mu} \right\}$$

For the curved surface, we have

$$\frac{1.6}{4} + \frac{1}{x_2} = \frac{1 - 1.6}{-8}$$

$$\Rightarrow \quad x_2 \approx -3 \text{ cm}$$

The minus sign means the image is on the side where the object lies. So,

$$I_1I_2 = (8 - 2.5 - 3) \text{ cm} = 2.5 \text{ cm}$$

Hence, the correct answer is (C).

2. Area of object $= 9 \text{ cm}^2$
Also, we know that

$$\text{Areal Magnification} = m_{ar} = \frac{A_I}{A_0} = \frac{v^2}{u^2} = \left(\frac{f}{f - u} \right)^2$$

$$\Rightarrow \quad \frac{A_I}{9} = \left[\frac{-10}{-25 - (-10)} \right]^2$$

$$\Rightarrow \quad A_I = \left(\frac{2}{3} \right)^2 \times 9$$

$$\Rightarrow \quad A_I = 4 \text{ cm}^2$$

Hence, the correct answer is (C).

3. The similar thing is extended and applied here too. Here the answer fabricated by the MISCONCEPTION is 1 (but we must know this is the answer only for a Concave Mirror (or Convex Lens). For Convex Mirror we have

$$m=\frac{f}{f-u}$$

$$\Rightarrow \quad m=\frac{f}{f-(-2f)}$$

$$\Rightarrow \quad m=\frac{1}{3}$$

Hence, the correct answer is (A).

4. Only one image will be formed by this lens system, because the optic axis of both the parts coincide. Two images would have been formed if their optic axis would had been different.

Hence, the correct answer is (D).

5. **MISCONCEPTION**

Maximum students are gripped by the misconception that when object lies at focus then image is formed at infinity. This is true but only for a Concave Mirror (Convex Lens) and is absolutely wrong when applied to Convex Mirror (Concave Lens)

How To Proceed Then?

If object placed at focus then just check out the MIRROR. If CONCAVE then image is formed at infinity and if Convex then apply $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{20}=\frac{1}{+20}$$

$$\Rightarrow \quad v=10 \text{ cm}$$

Hence, the correct answer is (B).

6. Image will be formed at infinity, when the object is placed at focus of the lens i.e., at 20 cm from the lens. So, we have

$$\text{Shift } \Delta x=25-20=\left(1-\frac{1}{\mu}\right)t$$

$$\Rightarrow \quad 5=\left(1-\frac{1}{1.5}\right)t$$

$$\Rightarrow \quad t=\frac{5\times 1.5}{0.5}=15 \text{ cm}$$

Hence, the correct answer is (C).

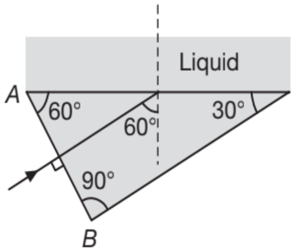
7. Critical angle between glass and liquid interface is

$$\sin C=\frac{\mu}{3/2}=\frac{2\mu}{3}$$

Angle of incidence at face AC is 60°

For TIR to take place, we have

$$i>C$$



$$\Rightarrow \quad \sin(60^\circ)>\frac{2\mu}{3}$$

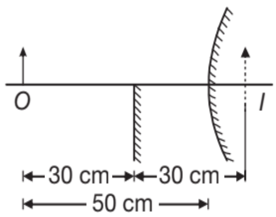
$$\Rightarrow \quad \mu<\frac{3\sqrt{3}}{4}$$

Hence, the correct answer is (C).

8. Since no parallax exists between the images formed by two mirrors (convex and plane) hence the images for both coincide. But for a plane mirror an image is as far behind the mirror as the object is in front of it. Hence the image for plane mirror should be 30 cm behind it or 10 cm behind the convex mirror. So for convex mirror.

$$u=-50\ cm,\ v=+10\ cm$$

Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$



$$\Rightarrow \quad \frac{1}{10}+\frac{1}{-50}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{f}=\frac{5-1}{50}$$

$$\Rightarrow \quad f=\frac{50}{4}$$

$$\Rightarrow \quad f=12.5 \text{ cm}$$

Since $R=2f$

$$\Rightarrow \quad R=25 \text{ cm}$$

Hence, the correct answer is (D).

9. This case is the refering to the setup of displacement method experiment as product of the two magnifications is unity.

Since $f=\frac{x}{m_1-m_2}=\frac{x}{\eta-\frac{1}{\eta}}=\frac{\eta x}{\eta^2-1}$

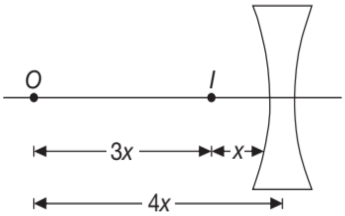
$$\Rightarrow \quad \frac{x}{f}=\frac{\eta^2-1}{\eta} \text{ for } \eta>1$$

Hence, the correct answer is (A).

10. Concave lens forms the virtual image of a real object. So, we have

$$m=-\frac{v}{u}=\frac{1}{4}.\text{ Now, if}$$

$u = -4x$, then $v = -x$ then $3x = 10$ cm



$\Rightarrow \quad x = \frac{10}{3}$ cm

$\Rightarrow \quad u = -\frac{40}{3}$ cm

and $v = -\frac{10}{3}$ cm

Substituting in $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, we get

$$\frac{1}{f} = \frac{-3}{10} + \frac{3}{40}$$

$\Rightarrow \quad f = -\frac{40}{9}$

$\Rightarrow \quad f = -4.4$ cm

Hence, the correct answer is (D).

11. $|m_{real}| = 2|m_{virtual}|$

$\Rightarrow \quad m_{real} = -2m_{virtual}$

$\Rightarrow \quad \frac{f}{f-(-15)} = -2\frac{f}{f-(-20)}$

$\Rightarrow \quad f + 20 = 2f + 30$

$\Rightarrow \quad f = -10$ cm

Hence, the correct answer is (C).

12. $m = +\frac{1}{2}$

$\Rightarrow \quad \frac{1}{2} = \frac{20}{20-u}$

$\Rightarrow \quad 40 = 20 - u$

$\Rightarrow \quad u = -20$ cm

Hence, the correct answer is (B).

13. Focal length of mirror is independent of the refractive index of medium in which it is placed.
Hence, the correct answer is (B).

14. Since image formed is erect, hence it must be virtual. So,

$$m = -\frac{v}{u} = 3$$

Also $|u| + |v| = D$

$|u| + 3|u| = 80$

$|u| = 20$

Since object always lies on negative side. So,

$u = -20$ cm

$\Rightarrow \quad 3 = \frac{f}{f-(-20)}$

$\Rightarrow \quad 3f + 60 = f$

$\Rightarrow \quad f = -30$ cm

Negative sign indicates the mirror is concave.

An Advice

I would always advice you to write $|u| + |v| = D$ wherever you are given the distance between object and image as no error will creep in these because the MOD signs prevent the errors.
Hence, the correct answer is (A).

16. Since, $\delta = (\mu - 1)A = (1.5 - 1)(4) = 2°$

$\Rightarrow \quad i = \delta = 2°$

Let the mirror be rotated by an angle θ, then

$i' = (2° + \theta)$

Since, $\delta_{total} = 180°$

$\Rightarrow \quad \delta + 180° - 2i' = 180°$

$\Rightarrow \quad \delta = 2i'$

$\Rightarrow \quad 2° = 2(2 + \theta)$

$\theta = -2°$

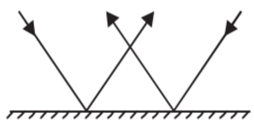
Here, negative sign implies that i gets decreased or $i' = 0$. i.e., light should fall normally on mirror.
Hence, the correct answer is (B).

17. The minimum length of the mirror required for the purpose is half the height of the person.
Hence, the correct answer is (B).

18. When mirror is turned, about an axis perpendicular to plane of mirror, then there will be no change in incident angle and reflected angle so angle between incident and reflected rays after rotation will be same as before.
Hence, the correct answer is (D).

19. A divergent beam appears to converge behind the mirror thus giving a virtual image. So, a convergent beam will give a real image.

Convergent Beam



Hence, the correct answer is (B).

21. Applying $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we get

$$\frac{1}{b}+\frac{1}{a}=\frac{1}{f}$$

$$\Rightarrow \quad f=\frac{ab}{a+b} \qquad \ldots(1)$$

Further in right triangle ACB, we have

$$AC^2+BC^2=AB^2$$

$$\Rightarrow \quad (a^2+c^2)+(b^2+c^2)=(a+b)^2$$

$$\Rightarrow \quad a^2+b^2+2c^2=a^2+b^2+2ab$$

$$\Rightarrow \quad ab=c^2$$

Substituting this in equation (1), we get

$$f=\frac{c^2}{a+b}$$

Hence, the correct answer is (C).

22. If the mirror approaches the object or the object approaches the stationary mirror with speed v then image approaches object with speed 2v.
Hence, the correct answer is (C).

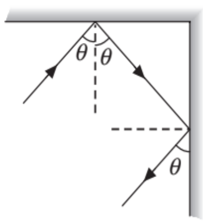
23. $m_{real}=-n=\frac{f}{f-u}$

$$\Rightarrow \quad -nf+nu=f$$

$$\Rightarrow \quad u=\left(\frac{n+1}{n}\right)f$$

Hence, the correct answer is (C).

24. The incident and the second reflected ray make the same angle θ with vertical. Hence, they are parallel for any value of θ.



Hence, the correct answer is (D).

25. $\frac{1}{n}=\frac{f}{f-u}$

$$f-u=nf$$

$$\Rightarrow \quad u=(1-n)f=-(n-1)f$$

According to the sign convention used u must always be negative.
Hence, the correct answer is (D).

26. $m_{real}=-4.5$

$$m=\frac{f}{f-u}$$

$$-4.5=\frac{f}{f-(-20)}$$

$$\Rightarrow \quad -4.5f-90=f$$

$$\Rightarrow \quad 90=-5.5f$$

$$\Rightarrow \quad f=-\frac{90}{5.5}$$

$$\Rightarrow \quad f=-\frac{900}{55}=-\frac{180}{11}\text{ cm}$$

Negative sign with focal length implies that the mirror is a concave mirror.
Hence, the correct answer is (D).

27. $f=+20$ cm

$$u=-10\text{ cm}$$

Since, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{10}=\frac{1}{20}$$

$$\Rightarrow \quad v=\frac{20}{3}\text{ cm}$$

Hence, the correct answer is (B).

28. Taking leftwards as negative, we have acceleration of block AB is $a_1=-\frac{3mg}{3m+m}=-\frac{3}{4}g$

Acceleration of block CD is $a_2=\frac{2mg}{2m+m}=\frac{2g}{3}$

Acceleration of image in mirror AB is twice acceleration of the mirror

$$(a_1)_I=2\left(-\frac{3g}{4}\right)=-\frac{3}{2}g$$

Acceleration of image in mirror CD is

$$(a_2)_I=2\left(\frac{2g}{3}\right)=\frac{4g}{3}$$

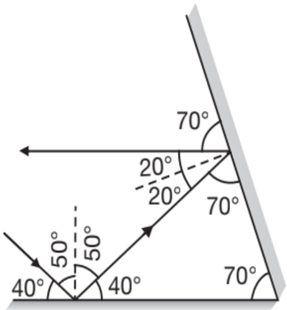
$\Rightarrow$ Acceleration of the two images w.r.t. each other is

$$a_{rel}=\frac{4g}{3}-\left(-\frac{3g}{2}\right)=\frac{17g}{6}$$

Hence, the correct answer is (D).

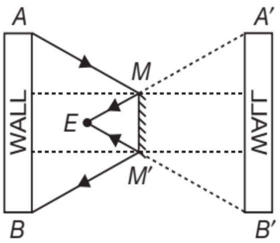
29. Covering the lower half will just make the image less bright (not blurred) as less number of rays will be reflected as compared to the previous case.
Hence, the correct answer is (A).

30.



Hence, the correct answer is (B).

31.



From symmetry we observe that length of mirror is one third of height of wall.

$\Rightarrow \quad \ell = \frac{H}{3}$

Hence, the correct answer is (C).

32. $n = \frac{360}{0} \to \infty$

Hence, the correct answer is (D).

33. $\frac{360}{\theta} = 6$ which is Even

$\Rightarrow \quad n = \frac{360}{\theta} - 1 = 5$

Hence, the correct answer is (A).

34. Ray's after reflections from two perpendicular mirrors are always parallel to incident ray irrespective of angle of incidence.
Hence, the correct answer is (B).

35. Power of concave lens must be less than that of the convex lens to form a real image. So, net power will decrease or focal length will increase. For real image v is positive, u is negative and f is positive. Applying lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ (substituting all values with sign), we get

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

u is constant, f is increasing. So v will also increase.

Hence, the correct answer is (C).

37. Using lens formula we can see that first image after refraction through lens is obtained 10 cm behind the lens from which when light rays are reflected from plane mirror next image will be obtained at a distance 20 cm behind the plane mirror which will act as an object for the next refraction at lens so we use

$u = +30$ cm and $f = +10$ cm

Using lens formula, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

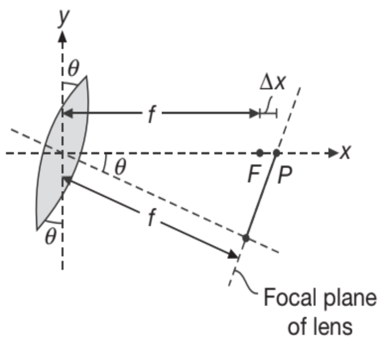
$$\Rightarrow \quad \frac{1}{v} - \frac{1}{30} = \frac{1}{10}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{10} + \frac{1}{30} = \frac{4}{30}$$

$$\Rightarrow \quad v = +7.5 \text{ cm}$$

Thus final image is obtained 7.5 cm to the right of lens.
Hence, the correct answer is (D).

38. When the lens is tilted by θ, the image is formed at the intersection point P of focal plane of lens in tilted position and x-axis.

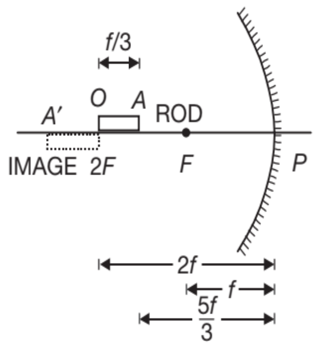


As the lens oscillates, the image shifts on x-axis in between F and P. Thus distance between two extreme positions of the oscillating image is

$$\Delta x = PF = \frac{f}{\cos\theta} - f = f(\sec\theta - 1)$$

Hence, the correct answer is (C).

39. Since an elongated image is formed and it touches one end of the rod, so the rod must lie with one end at $2F$ and other end between $2F$ and F (shown in figure).



For end A, $u=-\frac{5f}{3}$

Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\Rightarrow \frac{1}{v}-\frac{3}{5f}=\frac{1}{-f}$

$\Rightarrow \frac{1}{v}=\frac{3}{5f}-\frac{5}{5f}$

$\Rightarrow \frac{1}{v}=-\frac{2}{5f}$

$\Rightarrow v=-\frac{5f}{2}$

$\Rightarrow PA'=\frac{5f}{2}$

$\Rightarrow OA'=\frac{5f}{2}-2f$

$\Rightarrow OA'=\frac{f}{2}$

Hence, the correct answer is (B).

40. First consider two adjacent walls (not the ceiling). If n_1 is the number of images formed due to these perpendicular walls, then

$$n_1=\frac{360}{90}-1$$

$\Rightarrow n_1=3$

Now, when we consider the mirror on the ceiling then it will make a total of 4 images (one of the original object and three images of the previous arrangement) so, total images formed equals 7.
However, if the object were the observer himself then total number of images is $7-1=6$
Hence, the correct answer is (B).

41. Velocity of light is always normal to the wavefront.
Hence, the correct answer is (C).

42. Both concave and convex mirror give virtual image but a concave mirror gives a magnified virtual image (when object placed between F and P). Since the boy sees his image of diminished size, so the mirror must be convex.
Hence, the correct answer is (B).

43. Let object be placed at a distance x from mirror.

$$u=-x,\ v=-(x+10),\ f=-12 \text{ cm}$$

Since, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\Rightarrow \frac{1}{-(x+10)}+\frac{1}{-x}=\frac{1}{-12}$

$\Rightarrow x=20$ cm

$\Rightarrow u=-20$ cm,

$v=-30$ cm

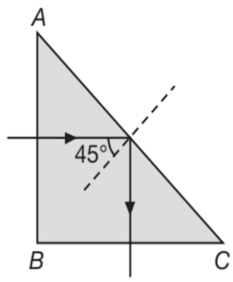
So, $m=-\frac{v}{u}=\frac{-(-30)}{-20}=-1.5$

Negative sign with magnification indicates image is real.
Hence, the correct answer is (A).

44. At face AB, the ray of light suffers no deviation, so applying Snell's Law at face AC, we get

$$\mu=\frac{1}{\sin C}=\frac{1}{\sin(45°)}$$

$\Rightarrow \mu_{\min}=\sqrt{2}$



Hence, the correct answer is (B).

45. $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\Rightarrow v^{-1}+u^{-1}=f^{-1}=\text{constant}$

Take derivative w.r.t. time on both sides

$$\frac{d}{dt}(v^{-1})+\frac{d}{dt}(u^{-1})=0$$

$\Rightarrow (-1)v^{-2}\frac{dv}{dt}+(-1)u^{-2}\frac{du}{dt}=0$

$\Rightarrow \frac{dv}{dt}=-\frac{v^2}{u^2}\left(\frac{du}{dt}\right)$

$\Rightarrow \text{Image speed}=-\frac{v^2}{u^2}(\text{object speed})$

when object moves towards mirror u decreases with passage of time and hence $\frac{du}{dt}=-9\text{ cms}^{-1}$

$\Rightarrow \frac{dv}{dt}=-\left(\frac{f}{f-u}\right)^2\frac{du}{dt}$

$\Rightarrow \frac{dv}{dt}=-\left(\frac{-24}{-24+60}\right)^2\frac{du}{dt}$

$\Rightarrow \frac{dv}{dt}=-\left(\frac{2}{3}\right)^2\frac{du}{dt}$

$\Rightarrow \quad \frac{dv}{dt} = -\frac{4}{9}(-9)$

$\Rightarrow \quad \frac{dv}{dt} = +4 \text{ cms}^{-1}$

Positive value of $\frac{dv}{dt}$ indicates that v increases with the passage of time i.e. image must be going away from mirror.

Hence, the correct answer is (C).

46. $A = 60°$ for equilateral prism.

$i = e = \frac{3}{4}A$

Since, $i + e = A + D$

$\Rightarrow \quad 2i = A + D$

$\Rightarrow \quad 2\left(\frac{3}{4}A\right) = A + D$

$\Rightarrow \quad D = \frac{A}{2} = 30°$

Hence, the correct answer is (D).

47. As the object moves from infinity to centre of curvature, the distance between object and image reduces from infinity to zero.
When the object moves from centre of curvature to focus, the distance between object and image increases from zero to infinity.
When the object moves from focus to pole, the distance between object and its image reduces from infinity to zero. Hence the distance between object and its image shall be 40 cm three times.

Hence, the correct answer is (C).

48. Since, $u_{ab} > u_{cd}$

$\Rightarrow \quad v_{ab} < v_{cd}$ $\quad \left\{\because \frac{1}{v} + \frac{1}{u} = \frac{1}{f}\right\}$

Since, $m = -\frac{v}{u}$

$\Rightarrow \quad |m_{ab}| < |m_{cd}|$

Hence, the correct answer is (B).

49. $\mu = \frac{c}{v}$

$\Rightarrow \quad 2 = \frac{3 \times 10^8}{v}$

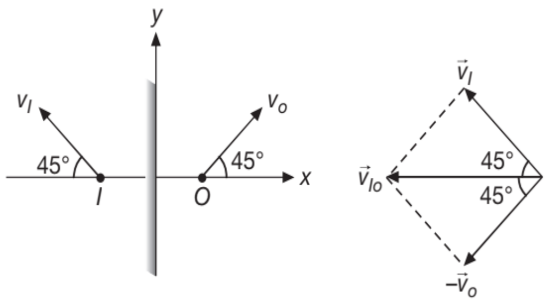
$\Rightarrow \quad v = 1.5 \times 10^8 \text{ ms}^{-1}$

$\Rightarrow \quad v = 1.5 \times 10^{10} \text{ cms}^{-1}$

Hence, the correct answer is (D).

50. $|\vec{v}_O| = |\vec{v}_I| = \sqrt{(2)^2 + (2)^2} = 2\sqrt{2} \text{ ms}^{-1}$

Relative velocity of image with respect to object is in negative x-direction as shown in figure.



Hence, the correct answer is (C).

51. $i = 2r$

Since, $n = \frac{\sin i}{\sin r}$

$\Rightarrow \quad n = \frac{\sin i}{\sin\left(\frac{i}{2}\right)} = \frac{2\sin\left(\frac{i}{2}\right)\cos\left(\frac{i}{2}\right)}{\sin\left(\frac{i}{2}\right)}$

$\left\{\because 2\sin\left(\frac{i}{2}\right)\cos\left(\frac{i}{2}\right) = \sin i\right\}$

$\Rightarrow \quad n = 2\cos\left(\frac{i}{2}\right)$

$\Rightarrow \quad \cos\left(\frac{i}{2}\right) = \frac{n}{2}$

$\Rightarrow \quad i = 2\cos^{-1}\left(\frac{n}{2}\right)$

Hence, the correct answer is (C).

52. The source cannot be seen if angle of incidence in the denser medium is greater than the critical angle.
Since,

$r = \frac{d}{\sqrt{\mu^2 - 1}} = \frac{h}{\sqrt{\mu^2 - 1}}$

$\Rightarrow \quad h = (1)\sqrt{\left(\frac{5}{3}\right)^2 - 1}$

$\Rightarrow \quad h = \frac{4}{3} \text{ cm}$

Hence, the correct answer is (B).

53. $\sin C = \frac{1}{\mu}$

TIR will take place at AC if $i > C$ i.e. $45^o > C$ for

Red Colour

$\mu_R = 1.39$

$\Rightarrow \quad \sin C_R = \frac{1}{1.39} > \frac{1}{1.41} = \sin 45$

$\Rightarrow \quad \sin C_R > \sin 45$

$\Rightarrow \quad C_R > 45^o$ (No TIR will take place)

Green Colour

$\mu_G = 1.44$

$\Rightarrow \quad \sin C_G = \frac{1}{1.44} < \frac{1}{1.41} = \sin 45$

$\Rightarrow \quad \sin C_G < \sin 45$

$\Rightarrow \quad C_G < 45°$ (TIR will take place for Green Colour)

Blue Colour

$\mu_B = 1.47$

$\Rightarrow \quad \sin C_B = \frac{1}{1.47} < \frac{1}{1.41} = \sin 45$

$\Rightarrow \quad \sin C_B < \sin 45$

$\Rightarrow \quad C_B < 45°$ (TIR will take place for Blue Colour)

So, Red is separated from Green and Blue.
Hence, the correct answer is (A).

54. Equivalent focal length of the system of three lenses in contact is given as

$$\frac{1}{f_{eq}} = \frac{1}{f_G} + \frac{1}{f_W} + \frac{1}{f_G} \quad \ldots(1)$$

where focal length of glass lenses is given as

$$f_G = \frac{R}{\mu - 1} = 2R$$

$\Rightarrow \quad R = 5$ cm

Focal length of water lens is given as

$$f_W = \frac{R}{2(\mu - 1)} = \frac{3R}{2} = 7.5 \text{ cm}$$

Now from Equation (1), we have

$$\frac{1}{f_{eq}} = 2\left(\frac{1}{10}\right) + \left(-\frac{2}{15}\right) = \frac{1}{15}$$

Equivalent optical power of the given system is given as

$$P_e = \frac{1}{15} \times 100 = 6.67 \text{ D}$$

Hence, the correct answer is (A).

56. $\sin C = \frac{\mu_{\text{rarer}}}{\mu_{\text{denser}}} = \frac{v_{\text{denser}}}{v_{\text{rarer}}}$

$\Rightarrow \quad \sin\theta = \frac{v_{\text{denser}}}{v_{\text{rarer}}} = \frac{v_A}{v_B}$

$\Rightarrow \quad \sin\theta = \frac{v}{v_B}$

$\Rightarrow \quad v_B = \frac{v}{\sin\theta}$

Hence, the correct answer is (A).

57. Since, $\mu v = \text{constant}$

$\Rightarrow \quad \mu_g v_g = \mu_w v_w$

$\Rightarrow \quad \frac{3}{2}\left(2 \times 10^8\right) = \frac{4}{3} v_w$

$\Rightarrow \quad v_w = 2.25 \times 10^8 \text{ ms}^{-1}$

Hence, the correct answer is (C).

58. After two reflections from two mirrors placed at right angles, the emergent ray will always be parallel to the incident ray for any value of i.
Hence, the correct answer is (D).

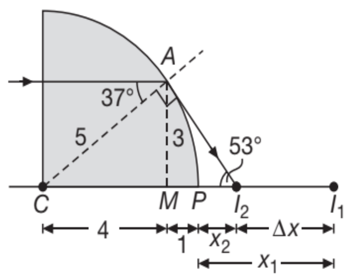
59. The rays just above x-axis (i.e. close to principal axis) are paraxial and after refraction at curved surface intersect the x axis at point I_1 which is at a distance x_1 from P.

Since $\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$

where $u \to \infty$, $v = +x_1$, $R = 5$ cm

$\Rightarrow \quad \frac{1}{x_1} = \frac{\mu - 1}{R}$

$\Rightarrow \quad x_1 = \frac{R}{\mu - 1} = \frac{15}{2}$ cm



The critical angle for air-glass interface is

$$C = \sin^{-1}\left(\frac{3}{5}\right) = 37°.$$

The rays above the ray incident on curved surface at $i = 37°$ will suffer total internal reflection and will not be considered.

The light ray which incident on curved surface at $C = 37°$, after refraction, intersects curved surface at point I_2, a distance x_2 from P, the location of which can be obtained from ΔI_2AM. Since

$$\frac{3}{1+x_2} = \tan(53°) = \frac{4}{3}$$

$$\Rightarrow \quad x_2 = \frac{5}{4}$$

So, required width of region I_1I_2 is

$$\Delta x = x_1 - x_2 = \frac{15}{2} - \frac{5}{4} = \frac{25}{4} \text{ cm}$$

Hence, the correct answer is (D).

60. Since light has to travel from denser to raser medium so, it must be made incident in the denser medium at an angle less than critical angle.
Hence, the correct answer is (C).

61. $C \propto \lambda$

Since $\lambda_{Red} > \lambda_{Violet}$

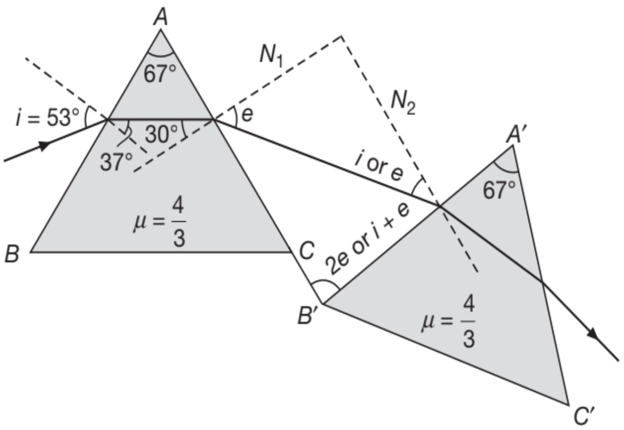
$$\Rightarrow \quad C_{Red} > C_{Violet}$$

Hence, the correct answer is (D).

63. If the angle of emergence for the first prism is e, then applying Snell's law, we get

$$e = \sin^{-1}\left(\frac{2}{3}\right)$$

For net deviation to be double, the incident ray on side $A'B'$ of second prism should make angles i or e with normal.



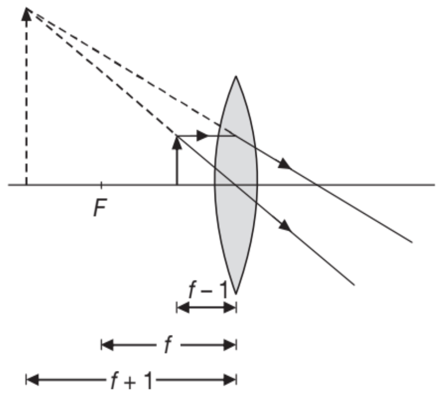
Thus the angle between CA and $A'B'$ will be $2e$ or $(i+e)$.

Hence, the correct answer is (A).

64. According to the problem, we have, $u = -(f-1)$

$$v = -(f+1)$$

$$f = +f$$



Applying $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{-(f+1)} + \frac{1}{(f-1)} = \frac{1}{f}$$

$$\Rightarrow \quad f^2 - 2f - 1 = 0$$

$$\Rightarrow \quad f = (\sqrt{2}+1) \text{ cm}$$

Hence, the correct answer is (B).

66. Apparent shift in the object O due to three slabs S_1, S_2 and S_3 with respect to the medium of $\mu = \frac{4}{3}$ is given by

$$\text{Shift} = 45\left(1 - \frac{1}{\frac{3/2}{4/3}}\right) + 24\left(1 - \frac{1}{\frac{1}{4/3}}\right) + 54\left(1 - \frac{1}{\frac{3/2}{4/3}}\right)$$

$$\Rightarrow \quad \text{Shift} = 45\left(1 - \frac{8}{9}\right) + 24\left(1 - \frac{4}{3}\right) + 54\left(1 - \frac{8}{9}\right)$$

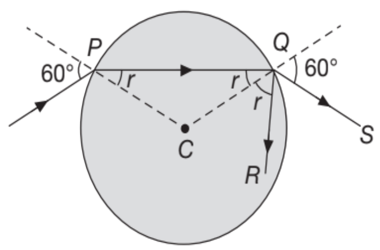
$$\Rightarrow \quad \text{Shift} = 5 + (-8) + 6 = 3 \text{ cm}$$

$\Rightarrow$ Object distance $u = 150$ cm $= R.O.C.$ of mirror

So, image will be formed on the object itself because light rays fall on mirror normally (as the object appears to be located at its centre of curvature).
Hence, the correct answer is (B).

67. Applying Snell's Law $\mu = \frac{\sin i}{\sin r}$, we get

$$\Rightarrow \quad \sin r = \frac{\sin i}{\mu}$$



$$\Rightarrow \quad \sin r = \frac{\sin 60°}{\sqrt{3}} = \frac{1}{2}$$

$\Rightarrow \quad r = 30°$

Since, $PC = QC$

$\Rightarrow \quad \angle CPQ = \angle PQC = \angle r = 30°$

Angle between reflected ray QR and refracted ray QS at the other face is $180° - r - 60° = 90° \quad \{\because r = 30°\}$

Hence, the correct answer is (D).

68. According to Snell's Law

$$\frac{\sin i}{\sin r} = \frac{\mu_w}{\mu_a}$$

$$\Rightarrow \quad \sin r = \frac{1}{\mu_w}\sin i$$

$$\Rightarrow \quad \sin r = \frac{1}{4/3}\sin 60$$

$$\Rightarrow \quad r = \sin^{-1}\left(\frac{3\sqrt{3}}{8}\right)$$

Hence, the correct answer is (A).

69. $\angle DBM = 45°$

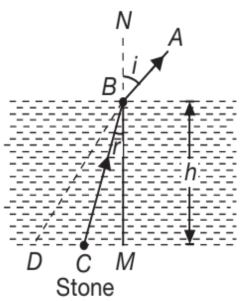
$$\Rightarrow \quad DM = h\tan 45° = h = 32 \text{ cm}$$

$$\mu = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 45°}{4/3} = \frac{3}{4\sqrt{2}}$$

$$\Rightarrow \quad \tan r = \frac{\sin r}{\cos r} = \frac{\sin r}{\sqrt{1-\sin^2 r}}$$

$$\Rightarrow \quad \tan r = \frac{\frac{3}{4\sqrt{2}}}{\sqrt{1-\frac{9}{32}}} = \frac{3}{\sqrt{23}}$$



$$\Rightarrow \quad CM = h\tan r = 32 \times \frac{3}{\sqrt{23}} \simeq 20 \text{ cm}$$

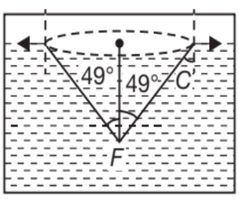
$$\Rightarrow \quad CD = DM - CM = 32 - 20 = 12 \text{ cm}$$

Hence, the correct answer is (B).

70. $\sin C = \frac{1}{\mu_w} = \frac{3}{4}$

$\Rightarrow \quad C = 49°$

Apex angle of cone is $2C = 98°$

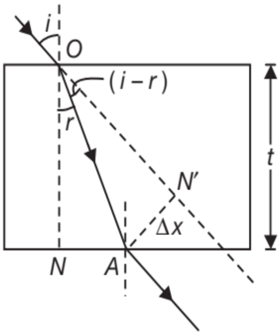


Hence, the correct answer is (C).

71. In $\Delta OAN'$

$$\sin(i-r) = \frac{\Delta x}{OA}$$

$$\Rightarrow \quad \Delta x = OA\sin(i-r)$$



In ΔOAN

$$\cos r = \frac{ON}{OA}$$

$$\Rightarrow \quad OA = \frac{t}{\cos r}$$

$$\Rightarrow \quad \Delta x = \frac{t\sin(i-r)}{\cos r}$$

Hence, the correct answer is (D).

74. For grazing incidence and emergence, we have

$$i = e = 90°, \; r_1 = r_2 = \frac{A}{2} = 30°$$

According to Snell's Law, we get

$$\mu = \frac{\sin i}{\sin r_1} = 2$$

Hence, the correct answer is (C).

75. $\sin C = \frac{\mu_r}{\mu_d} = \frac{4/3}{3/2}$

$$\Rightarrow \quad C = \sin^{-1}\left(\frac{8}{9}\right)$$

and light must go from denser to rarer medium.

Hence, the correct answer is (A).

76. According to Snell's Law

$$\mu\lambda = \text{constant}$$

Since $\lambda_{\text{Red}} > \lambda_{\text{Violet}}$

$$\Rightarrow \quad \mu_{\text{Red}} < \mu_{\text{Violet}}$$

Hence, the correct answer is (B).

78. Critical angle $C = \sin^{-1}\left(\frac{1}{\mu}\right)$

$$\Rightarrow \quad C = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

When $A > 2C$, the ray does not emerge from the prism. So, maximum refracting angle can be $60°$.
Hence, the correct answer is (C).

80. According to Snell's Law

$$\mu v = \text{constant}$$

Hence, the correct answer is (A).

81. This is a case of total internal reflection at B. For TIR at Q, we use

$$\theta > C \text{, where } C = \sin^{-1}\left(\frac{1}{\mu}\right)$$

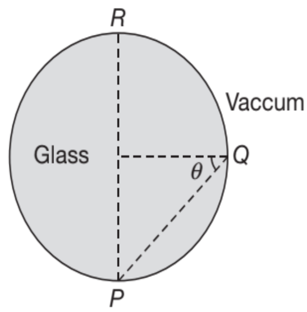
$$\Rightarrow \quad \frac{1}{\mu} < \sin\theta$$

As at point Q angle of incidence of light ray is maximum, we use

$$\frac{1}{\mu} < \sin(45°)$$

$$\Rightarrow \quad \mu > \frac{1}{\sin(45°)}$$

$$\Rightarrow \quad \mu > \sqrt{2}$$



Since $v = \frac{c}{\mu}$

$$\Rightarrow \quad \mu = \frac{c}{v}$$

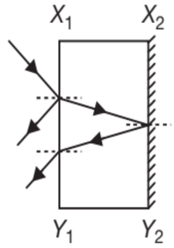
$$\Rightarrow \quad \frac{c}{v} > \sqrt{2}$$

$$\Rightarrow \quad v < \frac{c}{\sqrt{2}} = \frac{3\times10^8}{\sqrt{2}}$$

$$\Rightarrow \quad v < 2.1\times10^8 \text{ ms}^{-1}$$

So, only (B) is not possible.
Hence, the correct answer is (B).

82. This happens due to multiple refractions and reflections. The first image is formed due to reflection at X_1Y_1 and is fainter. The second image is formed due to reflection at X_2Y_2 and is brightest all other images formed further are faint.



Hence, the correct answer is (B).

83. In both A and B, the refracted ray is parallel to the base of prism.
Hence, the correct answer is (C).

85. By refraction formula, we use

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1}$$

where $u = -x$, $R_1 = +10$ cm, $\mu_1 = 1$ and $\mu_2 = \frac{3}{2}$

$$\Rightarrow \quad \frac{\mu_2}{\infty} - \frac{1}{-x} = \frac{1.5-1}{10}$$

$$\Rightarrow \quad x = -20 \text{ cm}$$

Since the second surface is flat so parallel rays after first refraction will not suffer any deviation when falling normally on the flat surface.
Hence, the correct answer is (C).

87. $$\mu = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \quad \sqrt{2}\sin 30 = \sin\left(\frac{A+D_m}{2}\right)$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \sin\left(\frac{A+D_m}{2}\right)$$

$$\Rightarrow \quad \frac{60+D_m}{2} = 45$$

$$\Rightarrow \quad \frac{D_m}{2} = 15$$

$$\Rightarrow \quad D_m = 30°$$

Further we know that at minimum deviation $i = \frac{A+D_m}{2}$

$$\Rightarrow \quad i = 45°$$

Hence, the correct answer is (A).

88. Normal Emergence implies

$$e = 0$$

Since $i + e = A + D$

For prism with small A,

$$D=(\mu-1)A$$

$$\Rightarrow \quad i=A+(\mu-1)A$$

$$\Rightarrow \quad i\cong \mu A$$

Hence, the correct answer is (C).

89. Since, $^{1}\mu_2\times{}^{2}\mu_3\times{}^{3}\mu_4\times{}^{4}\mu_1=1$

$$\Rightarrow \quad {}^{4}\mu_3\times{}^{3}\mu_2\times{}^{2}\mu_1\times{}^{1}\mu_4=1$$

$$\Rightarrow \quad {}^{4}\mu_3\times{}^{3}\mu_2\times{}^{2}\mu_1=\frac{1}{{}^{1}\mu_4}$$

Hence, the correct answer is (C).

90. Let thickness of slab be t and real depth from first side be x. Then

$$\mu=\frac{x}{6} \qquad \text{(when viewed from first side)}$$

$$\mu=\frac{t-x}{4} \qquad \text{(when viewed from second side)}$$

$$\Rightarrow \quad x=6\left(\frac{3}{2}\right) \qquad \left\{\because \mu=\frac{3}{2}\right\}$$

$$\Rightarrow \quad x=9 \text{ cm}$$

$$\Rightarrow \quad \frac{3}{2}=\frac{t-9}{4}$$

$$\Rightarrow \quad t=15 \text{ cm}$$

Hence, the correct answer is (C).

91. Since, $\mu=\dfrac{\sin\left(\dfrac{A+D_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$

$$\Rightarrow \quad \cot\left(\frac{A}{2}\right)=\frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \quad \cos\left(\frac{A}{2}\right)=\sin\left(\frac{A+D_m}{2}\right) \qquad \left\{\because \cot\left(\frac{A}{2}\right)=\frac{\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)}\right\}$$

$$\Rightarrow \quad \sin\left(\frac{\pi}{2}-\frac{A}{2}\right)=\sin\left(\frac{A}{2}+\frac{D_m}{2}\right)$$

$$\Rightarrow \quad D_m=\pi-2A=180-2A$$

Hence, the correct answer is (D).

92. $i+e=A+D$

$$\Rightarrow \quad 60+e=30+30$$

$$\Rightarrow \quad e=0^\circ$$

Hence, the correct answer is (A).

93. $\sin(45)=\dfrac{1}{\mu}$

$$\Rightarrow \quad \mu=\sqrt{2}$$

Hence, the correct answer is (B).

94. $\sin C_1=\dfrac{\mu_r}{\mu_d}=\dfrac{\mu_a}{\mu_g}=\dfrac{1}{\mu_g}$

$$\sin C_2=\frac{\mu_w}{\mu_g}>\sin C_1$$

$$C_1<C_2$$

Hence, the correct answer is (B).

95. Since $r=\dfrac{d}{\sqrt{\mu^2-1}}$

$$\Rightarrow \quad r=\frac{4}{\sqrt{\left(\frac{5}{3}\right)^2-1}}$$

$$\Rightarrow \quad r=\frac{4\times 3}{4}=3 \text{ m}$$

Diameter $=2r=6$ m

Hence, the correct answer is (B).

96. Apparent Depth $=\dfrac{h}{n}$

Hence, the correct answer is (A).

98. From mirror formula, we get

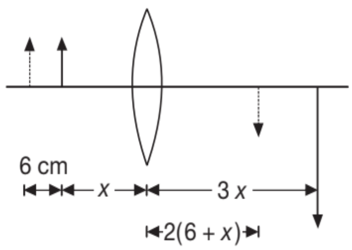
$$\frac{1}{3x}+\frac{1}{x}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{4}{3x}=\frac{1}{f} \qquad \ldots(1)$$

From mirror formula, we get

$$\frac{1}{2(6+x)}+\frac{1}{6+x}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{3}{2(6+x)}=\frac{1}{f} \qquad \ldots(2)$$



Equations (1) and (2) give

$$\frac{4}{3x}=\frac{3}{2(6+x)}$$

$\Rightarrow \quad 9x = 48 + 8x$

$\Rightarrow \quad x = 48$ cm

Shift of screen $= 3x - 2(6 + x)$

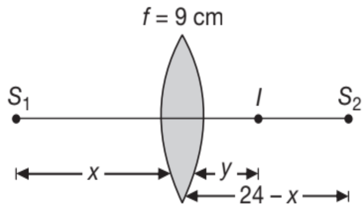
Shift of screen $= 3 \times 48 - 2(6 + 48) = 36$ cm

Hence, the correct answer is (C).

99. Apparent Depth $= \dfrac{d}{2\mu_1} + \dfrac{d}{2\mu_2}$

Hence, the correct answer is (D).

100. In this case, one of the image will be real and the other will be virtual. Let us assume that image of S_1 is real and that of S_2 is virtual. Then, applying $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$



for S_1, we get

$$\frac{1}{y} + \frac{1}{x} = \frac{1}{9} \qquad \ldots(1)$$

for S_2, we get

$$-\frac{1}{y} + \frac{1}{24 - x} = \frac{1}{9} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$x = 6$ cm

Hence, the correct answer is (A).

Conceptual Note(s)

This question may have following answer 6 cm from S_1 or 18 cm from S_2 and 18 cm from S_1 or 6 cm from S_2.

101. Optical path length $= nt$

So time taken $= \dfrac{nt}{c}$

Hence, the correct answer is (C).

102. To a fish the outer world is seen in a circle of radius $\dfrac{d}{\sqrt{\mu^2 - 1}}$, d is the depth at which the fish swims.

$\Rightarrow \quad r = \dfrac{12}{\sqrt{\dfrac{16}{9} - 1}}$

$\Rightarrow \quad r = \dfrac{36}{\sqrt{7}}$ cm

Hence, the correct answer is (C).

103. For the diver

$$\sin C = \frac{1}{\mu_w} = \frac{1}{(4/3)} \quad \left(\text{Remember that } \mu_{\text{water}} = \frac{4}{3}\right)$$

$\Rightarrow \quad C = 49°$ with vertical

$\Rightarrow \quad \theta = 90 - 49° = 41°$ with the horizon.

Hence, the correct answer is (A).

104. Using refraction formula, we have

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

where, $v = 2R$, $u \to \infty$, $\mu_1 = 1$ and $\mu_2 = \mu$

$\Rightarrow \quad \dfrac{\mu}{2R} - \dfrac{1}{\infty} = \dfrac{\mu - 1}{R}$

$\Rightarrow \quad \mu = 2\mu - 2$

$\Rightarrow \quad \mu = 2$

Hence, the correct answer is (D).

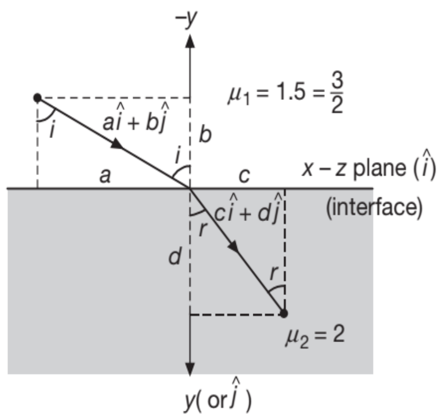
105. $\sin C = \dfrac{\mu_{\text{rarer}}}{\mu_{\text{denser}}} = \dfrac{\mu_w}{\mu_g}$

$\Rightarrow \quad \sin C = \dfrac{4/3}{5/3} = \dfrac{4}{5}$

Hence, the correct answer is (C).

106. Applying Snell's Law at the interface separating two media, we get

$$\mu_1 \sin i = \mu_2 \sin r$$



From the figure, we get

$$\left(\frac{3}{2}\right)\left(\frac{a}{\sqrt{a^2 + b^2}}\right) = 2\left(\frac{c}{\sqrt{c^2 + d^2}}\right) \qquad \ldots(1)$$

Since $a\hat{i} + b\hat{j}$ and $c\hat{i} + d\hat{j}$ are unit vectors, so we get

$$\sqrt{a^2 + b^2} = \sqrt{c^2 + d^2} = 1$$

Substituting in equation (1), we get

$$\frac{a}{c}=\frac{4}{3}$$

Hence, the correct answer is (B).

107. $\sin C=\frac{\mu_{\text{rarer}}}{\mu_{\text{denser}}}=\frac{\mu_B}{\mu_A}$

Since, $\mu_A v_A=\mu_B v_B$

$$\Rightarrow \quad \sin C=\frac{v_A}{v_B}=\frac{2}{2.4}$$

$$\Rightarrow \quad \sin C=\frac{5}{6}$$

Hence, the correct answer is (B).

108. According to Snell's Law

$$\mu v=\text{constant}$$

$$\Rightarrow \quad \mu_g v_g=\mu_\ell v_\ell$$

$$\Rightarrow \quad (1.5)\times\left(2\times10^8\right)=\mu_\ell\left(2.5\times10^8\right)$$

$$\Rightarrow \quad \mu_\ell=\frac{3}{2.5}=\frac{30}{25}=1.2$$

Hence, the correct answer is (C).

109. $\sin C=\frac{1}{\sqrt{2}}$

$$\Rightarrow \quad C=45^\circ$$

For $i=C$ in denser medium angle of refraction in rarer medium is 90°.

Hence, the correct answer is (D).

110. $\text{Apparent Depth}=\frac{\text{Real Depth}}{\mu}$

$$\Rightarrow \quad \text{Apparent Depth}=\frac{24}{4/3}$$

$$\Rightarrow \quad \text{Apparent Depth}=18\text{ cm}$$

Hence, the correct answer is (C).

111. Given that $A=\delta_m=60^\circ$

At minimum deviation, we have

$$i=\left(\frac{A+\delta_m}{2}\right)=60^\circ$$

Hence, the correct answer is (C).

112. $r+i=90$

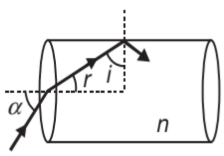
$i=90-r$

For ray not to emerge from curved surface

$$i>C$$

$$\Rightarrow \quad \sin i>\sin C$$

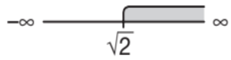
$$\Rightarrow \quad \sin(90-r)>\sin C$$



$$\Rightarrow \quad \cos r>\sin C$$

$$\Rightarrow \quad \sqrt{1-\sin^2 r}>\frac{1}{n} \qquad \left\{\because \sin C=\frac{1}{n}\right\}$$

$$\Rightarrow \quad 1-\frac{\sin^2 i}{n^2}>\frac{1}{n^2}$$



$$\Rightarrow \quad 1>\frac{1}{n^2}\left(1+\sin^2 i\right)$$

$$\Rightarrow \quad n^2>1+\sin^2 i$$

$$\Rightarrow \quad n>\sqrt{2} \qquad \{\sin i\to 1\}$$

$$\Rightarrow \quad \text{least value}=\sqrt{2}$$

Hence, the correct answer is (B).

113. Dispersive Power $=\frac{d\mu}{d\lambda}$

$$\Rightarrow \quad \frac{d\mu}{d\lambda}=-\frac{2B}{\lambda^3}$$

So, as B increases, dispersive power increases.

Hence, the correct answer is (B).

114. Maximum separation will be $4A$ when A is the amplitude which is given as

$$A\omega=v$$

$$\Rightarrow \quad A\sqrt{\frac{k}{m}}=v$$

$$\Rightarrow \quad A=v\sqrt{\frac{m}{k}}$$

Therefore, the maximum separation is $4v\sqrt{\frac{m}{k}}$

Hence, the correct answer is (D).

115. For the near end of the rod applying $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$

Since, here u and f are negative, so

$$|v|=\frac{uf}{u-f}$$

The farther end of the rod is at infinity, so its image will be formed at focus. Hence,

Length of the image is $\ell=|v|-f$

$$\Rightarrow \quad \ell=\frac{uf}{u-f}-f=\frac{f^2}{u-f}$$

Hence, the correct answer is (C).

116. For A

$$\text{Total number of waves}=\frac{(1.5)t}{\lambda} \qquad \ldots(1)$$

$$\because \quad \left(\begin{matrix}\text{Total number}\\ \text{of waves}\end{matrix}\right)=\left(\frac{\text{Optical path length}}{\text{wavelength}}\right)$$

For B and C

Total number of waves

$$= \frac{n_B\left(\frac{t}{3}\right)}{\lambda} + \frac{(1.6)\left(\frac{2t}{3}\right)}{\lambda} \quad \ldots(2)$$

Equating (1) and (2)

$\Rightarrow \quad n_B = 1.3$

Hence, the correct answer is (C).

117. Focal length of curved mirrors is independent of the refractive index of the medium in which the mirror is placed.

Hence, the correct answer is (C).

118. $n = \frac{c}{v}$

$\Rightarrow \quad n = \frac{1/\sqrt{\mu_0\varepsilon_0}}{1/\sqrt{\mu\varepsilon}}$

$\Rightarrow \quad n = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$

Hence, the correct answer is (B).

119. $\sin\theta_1 = \frac{1}{\mu_g}$ and $\sin\theta_2 = \frac{1}{\mu_w}$

Since, $\mu_g > \mu_w$, $\theta_1 < \theta_2$

The critical angle θ between glass-water interface is given by

$$\sin\theta = \frac{\mu_w}{\mu_g}$$

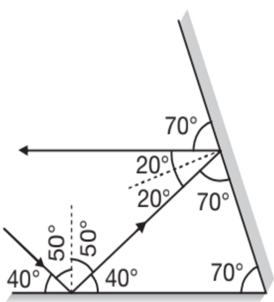
$\Rightarrow \quad \theta > \theta_2$

Hence, the correct answer is (D).

Conceptual Note(s)

Critical angle increases as the relative refractive index is decreased.

120.



Hence, the correct answer is (B).

121. $u = -20$ cm, $f = +20$ cm, $v = ?$

Since $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

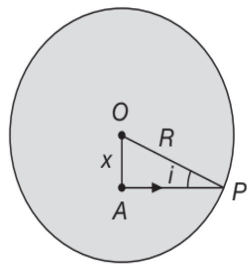
$\Rightarrow \quad \frac{1}{v} + \frac{1}{-20} = \frac{1}{20}$

$\Rightarrow \quad v = 10$ cm

Students generally give (A) as the answer to this problem because they have in their mind that if an object is placed at centre of curvature C, then image is also formed at C, but this is true only for the case of a convex lens or a concave mirror. So correct answer is **(B)**.

Hence, the correct answer is (B).

123. If O is the centre and A is the point source for the light ray incident at any point P on surface of sphere to angle have maximum angle of incidence i, then situation of maximum i is shown in figure.



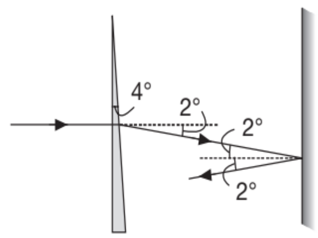
It happens when OA and AP are perpendicular, for which we have

$$i = \sin^{-1}\left(\frac{OA}{OP}\right) = \sin^{-1}\left(\frac{x}{R}\right)$$

Hence, the correct answer is (B).

124. Since, $\delta_{\text{prism}} = (\mu - 1)A = (1.5 - 1)4° = 2°$

$\Rightarrow \quad \delta_{\text{total}} = \delta_{\text{prism}} + \delta_{\text{mirror}}$



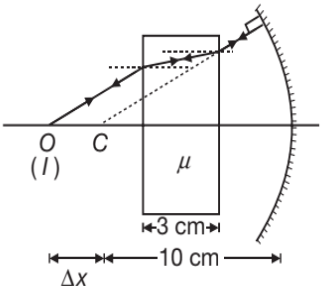
$\Rightarrow \quad \delta_{\text{total}} = (\mu - 1)A + (180 - 2i)$

$\Rightarrow \quad \delta_{\text{total}} = (1.5 - 1)4° + (180 - 2 \times 2°)$

$\Rightarrow \quad \delta_{\text{total}} = 2° + 176° = 178°$

Hence, the correct answer is (C).

125.



$$\Delta x = 3\left[1 - \frac{1}{\left(\frac{3}{2}\right)}\right]$$

$\Rightarrow$ $\Delta x = 1$ cm

$\Rightarrow$ Object distance $= 10 + 1 = 11$ cm

Hence, the correct answer is (C).

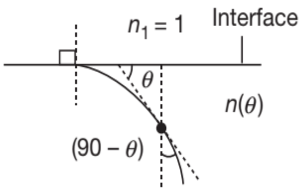
126. Apparent depth $= \frac{d}{n}$

Hence apparent separation $= \frac{2d}{n}$

Hence, the correct answer is (C).

127. According to Snell's Law for anisotropic medium

$$n(\theta)\sin(90 - \theta) = \text{constant}$$



or $n_1 \sin i = n(\theta)\cos\theta$

$\Rightarrow$ $(1)\sin 90 = n(\theta)\cos\theta$

$\Rightarrow$ $n(\theta)\cos\theta = 1$

Hence, the correct answer is (B).

128. Velocity of approach of man towards the bicycle $= (u - v)$

Hence velocity of approach of image towards bicycle is $2(u - v)$.

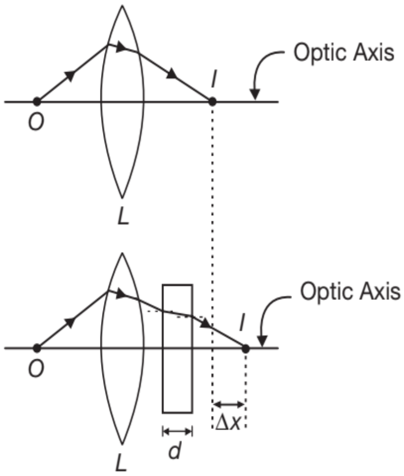
Hence, the correct answer is (D).

129. Optical path length $= \mu t$

Time taken $= \frac{\mu t}{c}$

Hence, the correct answer is (C).

130.



$$\Delta x = d\left(1 - \frac{1}{\mu}\right)$$

Hence, the correct answer is (C).

131. $\sin C = \frac{4000}{6000}$

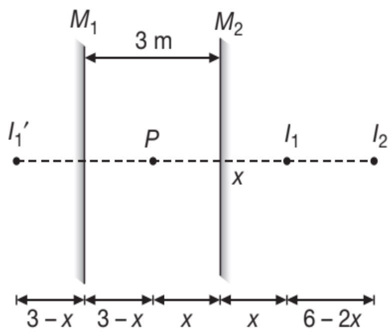
$\Rightarrow$ $C = \sin^{-1}\left(\frac{2}{3}\right)$

Hence, the correct answer is (C).

132. A virtual, erect image is obtained by using the mirror. The mirror can be both concave and convex. But a virtual image obtained by a concave mirror is always magnified and hence the mirror must be convex as we are getting diminished image.

Hence, the correct answer is (B).

133.



Distance between images I_1 and I_2 is given to be 4 m

$\Rightarrow$ $I_1I_2 = 6 - 2x = 4$ m

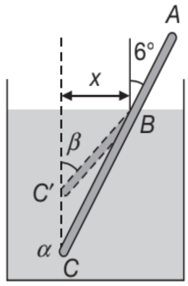
$\Rightarrow$ $x = 1$ m

Hence, the correct answer is (C).

134. Since rays after passing through the glass slab just suffer lateral displacement hence, we have angle between the emergent rays as α.

Hence, the correct answer is (B).

136. From figure, we get



$$\alpha = \frac{x}{OC} \text{ and } \beta = \frac{x}{OC'}$$

$$\Rightarrow \frac{\alpha}{\beta} = \frac{OC'}{OC} = \frac{1}{\mu}$$

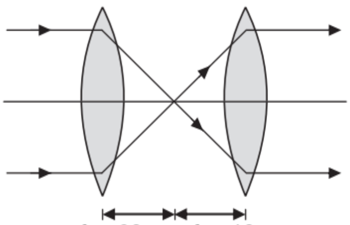
For small angles, we have

$$\beta = \mu\alpha = \left(\frac{4}{3}\right)(6°) = 8°$$

$\Rightarrow$ Bending angle $= \beta - \alpha = 2°$

Hence, the correct answer is (A).

137. As shown in figure the distance between the lenses should be 30 cm .



Hence, the correct answer is (A).

138. Two plano convex lens of focal length f, when combined using optical glue will give rise to a convex lens of focal length $\left(\frac{f}{2}\right)$.

i.e. size of image = size of object.

Object at $2F$ i.e. at a distance f from optical centre.

Hence, the correct answer is (C).

139. $f_{\text{combination}} \to \infty$

Since $\frac{1}{f_{\text{combination}}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$

$$\Rightarrow 0 = \frac{1}{-10} + \frac{1}{f_2} - \frac{10}{(-10)f_2}$$

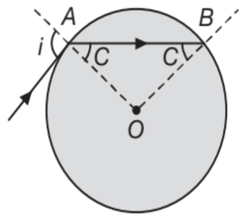
$$\Rightarrow 0 = \frac{1}{-10} + \frac{1}{f_2} + \frac{1}{f_2}$$

$$\Rightarrow \frac{1}{10} = \frac{2}{f_2}$$

$$\Rightarrow f_2 = 20 \text{ cm}$$

Hence, the correct answer is (B).

140. Since, $\angle ABO = \angle OAB = C$



Further, by definition, we have

$$\sin C = \frac{1}{\mu} = \frac{2}{3}$$

Applying Snell's Law at A, we get

$$\frac{\sin i}{\sin C} = \frac{3}{2}$$

$$\Rightarrow \sin i = \left(\frac{3}{2}\right)\sin C = \left(\frac{3}{2}\right)\left(\frac{2}{3}\right) = 1$$

$$\Rightarrow i = 90°$$

Hence, the correct answer is (D).

141. For combination to produce dispersion without deviation.

$$(n_1 - 1)A_1 = (n_2 - 1)A_2$$

$$\Rightarrow A_2 = \left(\frac{n_1 - 1}{n_2 - 1}\right)A_1$$

$$\Rightarrow A_2 = \left(\frac{1.54 - 1}{1.72 - 1}\right)4°$$

$$\Rightarrow A_2 = \frac{3}{4}(4°)$$

$$\Rightarrow A_2 = 3°$$

Hence, the correct answer is (C).

142. $\frac{\mu}{\infty} + \frac{1}{-u} = \frac{\mu - 1}{R}$

$\Rightarrow u = -\frac{R}{\mu - 1}$ from the surface

$\Rightarrow$ Total distance from centre

$$\Rightarrow x = |u| + R = R + \frac{R}{\mu - 1}$$

$$x = R + 2R$$
$$x = 3R$$

Hence, the correct answer is (C).

143. An air bubble in water always behaves as a concave lens and hence is always incapable to form a real image.
Hence, the correct answer is (D).

144. $m_{\text{virtual}} = m_{\text{real}}$

$$\Rightarrow \quad \frac{f}{f+(-12)} = -\frac{f}{f+(-20)}$$

$$\Rightarrow \quad \frac{f}{f-12} = -\frac{f}{f-20}$$

$$\Rightarrow \quad f - 20 = -f + 12$$

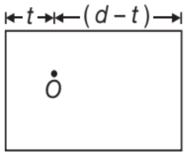
$$\Rightarrow \quad 2f = 32$$

$$\Rightarrow \quad f = 16 \text{ cm}$$

Hence, the correct answer is (B).

145. $4 = \dfrac{t}{\mu}$ $\qquad \left\{\because \ \mu = \dfrac{\text{Real Depth}}{\text{Apparent Depth}}\right\}$

and $6 = \dfrac{d-t}{\mu}$



$$\Rightarrow \quad 6\mu = d - 4\mu$$

$$\Rightarrow \quad d = 10\mu$$

$$\Rightarrow \quad d = 15 \text{ cm}$$

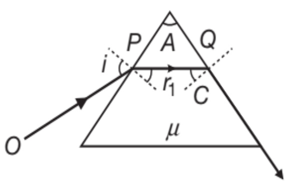
Hence, the correct answer is (C).

146. $\dfrac{P_A}{P_B} = \left(\dfrac{r_B}{r_A}\right)^2 = \dfrac{4}{9} = 0.44$

Hence, the correct answer is (A).

147. An 'achromatic combination' is made from a concave lens and a convex lens with greater power of convex lens so as to make the rays converge at a point.
Hence, the correct answer is (C).

148.



$$\frac{\sin i}{\sin r_1} = \mu \ (\text{at } P) \qquad \ldots(1)$$

and $r_1 + C = A \qquad \ldots(2)$

$$\Rightarrow \quad \sin i = \mu \sin(A - C)$$

$$\Rightarrow \quad \sin i = \mu(\sin A \cos C - \cos A \sin C)$$

$$\Rightarrow \quad \sin i = \mu\left(\sin A\sqrt{1-\frac{1}{\mu^2}} - \cos A\frac{1}{\mu}\right) \quad \left\{\because \sin C = \frac{1}{\mu}\right\}$$

$$\Rightarrow \quad i = \sin^{-1}\left\{\left(\sqrt{\mu^2-1}\right)\sin A - \cos A\right\}$$

Hence, the correct answer is (C).

149. In first case, the apparent depth of the liquid is $(b-a)$

$\Rightarrow$ Real depth $= \mu(b-a)$

In second case, apparent depth $= (d-c)$

$\Rightarrow$ Real depth $= \mu(d-c)$

The difference in depth of liquid is

$$\mu(d-c) - \mu(b-a)$$

Here we use $\mu(d-c) > \mu(b-a)$

From experimental data, the difference is equal to $(d-b)$

$$\Rightarrow \quad \mu(d-c) - \mu(b-a) = d - b$$

$$\Rightarrow \quad \mu = \frac{d-b}{(a+d-c-b)}$$

Hence, the correct answer is (A).

150. At $u = f$ $\qquad$ {focal length}

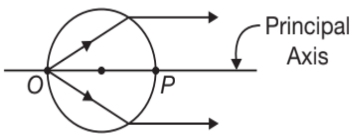
$v \to \infty$

At $u = 0$ $\qquad$ {i.e., object is at pole}

$v = 0$ $\qquad$ {image is also at pole}

Satisfying these two conditions, only OPTION (D) is correct.
Hence, the correct answer is (D).

151.



Considering pole at P, we have

$$\frac{\mu}{-2R} + \frac{1}{\infty} = \frac{\mu - 1}{-R} \qquad \left\{\because \frac{\mu_1}{\mu} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}\right\}$$

$$\Rightarrow \quad \frac{\mu}{2} = \mu - 1$$

$$\Rightarrow \quad \frac{\mu}{2} = 1$$

$$\Rightarrow \quad \mu = 2$$

Hence, the correct answer is (C).

152. The first image is formed due to the reflection from concave mirror M_2.

$$\Rightarrow \quad \frac{1}{v_1} + \frac{1}{(-2R)} = \frac{2}{-R}$$

$$\Rightarrow \quad \frac{1}{v_1} = \frac{1}{2R} - \frac{4}{2R}$$

$$\Rightarrow \quad \frac{1}{v_1} = \frac{-3}{2R}$$

$$\Rightarrow \quad v_1 = \frac{-2R}{3}$$

$$\Rightarrow \quad m_1 = -\left(\frac{\frac{-2R}{3}}{2R}\right) = -\frac{1}{3}$$

$\Rightarrow$ Radius of image circle $= \frac{1}{3}a$, (in magnitude).

Now the second image is formed by convex mirror M_1. The second image will be formed because the image formed by the first acts as object for it.

$\Rightarrow$ Object distance $= 2R - \frac{2R}{3} = \frac{4R}{3}$.

$$\Rightarrow \quad \frac{1}{v_2} + \frac{1}{\left(-\frac{4R}{3}\right)} = \frac{1}{\left(\frac{R}{2}\right)}$$

$$\Rightarrow \quad \frac{1}{v_2} = \frac{2}{R} + \frac{3}{4R}$$

$$\Rightarrow \quad v_2 = \frac{4R}{11}$$

$$\Rightarrow \quad m_2 = \frac{-v_2}{u_2} = \frac{-\frac{4R}{11}}{-\frac{4R}{3}} = \frac{3}{11}$$

$$\Rightarrow \quad m_2 = \frac{3}{11}$$

So, radius of second image $= a_2 = \frac{3}{11} \cdot \frac{a}{3} = \frac{a}{11}$

Similarly, radius of third image is $a_3 = \frac{a}{41}$

Hence, the correct answer is (C).

154. The two slabs will shift the image by a distance

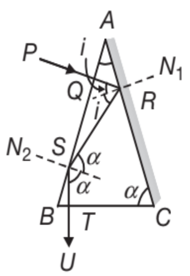
$$\Delta x = 2\left(1 - \frac{1}{\mu}\right)t$$

$$\Rightarrow \quad \Delta x = 2\left(1 - \frac{1}{1.5}\right)(1.5) = 1 \text{ cm}$$

Therefore, final image will be 1 cm above point P.

Hence, the correct answer is (D).

155.



Since

$$PR \parallel N_2$$

$$\Rightarrow \quad \alpha = 2i$$

Also $A + 2\alpha = 180°$

$$\Rightarrow \quad A + 2(2i) = 180$$

$$\Rightarrow \quad i + 4i = 180$$

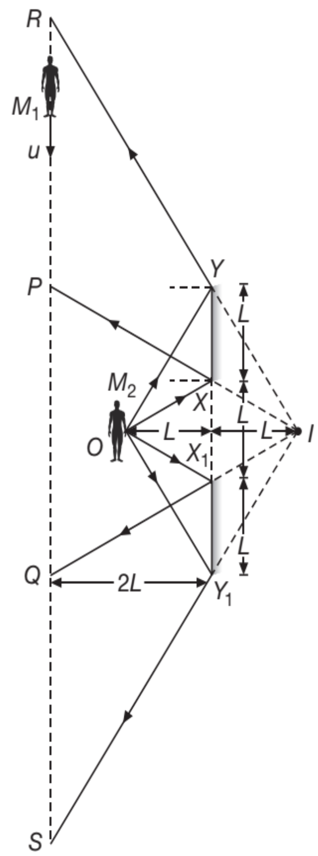
$$\Rightarrow \quad 5i = 180$$

$$\Rightarrow \quad i = 36°$$

$$\Rightarrow \quad A = 36°$$

Hence, the correct answer is (C).

156. When the man M_1 is in the region RP and SQ, then man M_1 sees the image of M_2



From the similar triangle PIQ and XIX_1

$$\Rightarrow \quad \frac{PQ}{XX_1} = \frac{3L}{L}$$

$$\Rightarrow \quad PQ = 3L$$

From the similar triangle, RIS and PIQ

$$\Rightarrow \quad \frac{RS}{YY_1} = \frac{3L}{L}$$

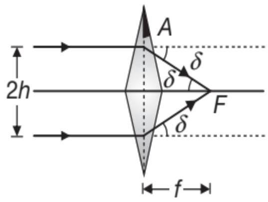
$$\Rightarrow \quad RS = 9L$$

Then $RP + QS = RS - PQ = 9L - 3L = 6L$

$\Rightarrow$ Time $= \dfrac{6L}{u}$

Hence, the correct answer is (C).

157. Since $\tan\delta = \dfrac{h}{f}$



$\Rightarrow \quad f = \dfrac{h}{\tan\delta}$

Further

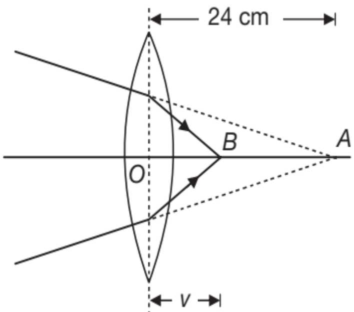
$$\delta = (\mu - 1)A$$

$\Rightarrow \quad f = \dfrac{h}{(\mu-1)A}$

Hence, the correct answer is (C).

158. If no lens had been there the rays would have met at A. On inserting the lens the rays meet at B. Thus A acts as an object for the lens.

Since $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$



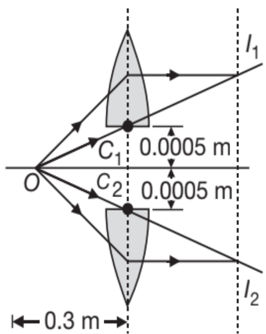
$$u = +24 \text{ cm},\ v = ?,\ f = +24 \text{ cm}$$

$\Rightarrow \quad v = 12$ cm

$\Rightarrow$ Distance $AB = OA - OB = 24 - 12 = 12$ cm

Hence, the correct answer is (A).

159. $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$



$\Rightarrow \quad \dfrac{1}{v} - \dfrac{1}{(-0.3)} = \dfrac{1}{0.2}$

$\Rightarrow \quad v = 0.6$ m

Δ's OC_1C_2 and OI_1I_2 are similar

Hence

$$\frac{0.001}{0.3} = \frac{I_1I_2}{0.3 + 0.6}$$

$\Rightarrow \quad I_1I_2 = 0.003$ m

Hence, the correct answer is (D).

160. Let the critical angle of interface between medium 1 and 2 is C_1 and between 1 and 3 is C_2

Then $\sin C_1 = \dfrac{\mu_2}{\mu_1}$ and $\sin C_2 = \dfrac{\mu_3}{\mu_1}$

For TIR to take place at second interface, we have

$$90 - C_1 > C_2$$

Taking sine of both sides, we get

$$\sin(90 - C_1) > \sin C_2$$

$\Rightarrow \quad \cos C_1 > \sin C_2$

$\Rightarrow \quad \sqrt{1 - \left(\dfrac{\mu_2}{\mu_1}\right)^2} > \dfrac{\mu_3}{\mu_1}$

$\Rightarrow \quad \mu_1^2 - \mu_3^2 > \mu_2^2$

Hence, the correct answer is (B).

161. A convex mirror can never form a real image.
Hence, the correct answer is (D).

162. Given, $\mu = \sqrt{2}$, $A = 60°$, $\delta = 30°$

For the minimum deviation, we have

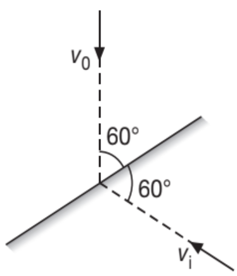
$$\mu = \frac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$$

Substituting the values of A and μ we get

$$\delta_m = 30°$$

i.e., the given deviation $\delta = 30°$ is actually the minimum deviation δ_m. At minimum deviation the ray inside the prism is parallel to the base of the prism (in case of an equilateral prism).
Hence, the correct answer is (A).

164.



$$v_0 = v_i = \omega A$$

$$\Rightarrow \quad v_0 = \left(\sqrt{\frac{k}{M}}\right)A$$

$$v_{\text{rel}} = \sqrt{v_0^2 + v_i^2 - 2v_0\cos(120^\circ)}$$

$$\Rightarrow \quad v_{\text{rel}} = \sqrt{3}A\sqrt{\frac{k}{M}}$$

Hence, the correct answer is (C).

165. Here, PQ is the length of reflected light from the mirror. From the similar triangles, PMA and MXS'

$$\Rightarrow \quad \frac{PA}{XS'} = \frac{3H}{H}$$

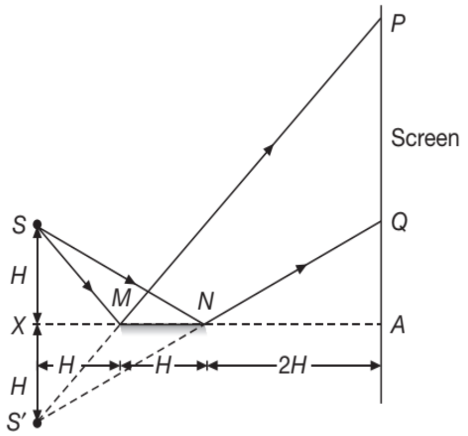
Then, $PA = 3(XS') = 3H$

From the similar triangles QNA and XNS'

$$\frac{QA}{XS'} = \frac{2H}{2H}$$

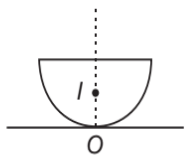
$$\Rightarrow \quad QA = H$$

$$\Rightarrow \quad PQ = PA - QA = 3H - H = 2H$$



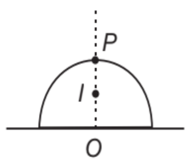
Hence, the correct answer is (A).

166. CASE-1:



$$n = \frac{\text{Real Depth}}{\text{Apparent Depth}} = \frac{4}{3}$$

CASE-2:



Consider origin to be at the pole P.

$$\frac{n_1}{-u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

$$\Rightarrow \quad \frac{\frac{4}{3}}{-(-4)} + \frac{1}{\left(-\frac{25}{8}\right)} = \frac{1-\left(\frac{4}{3}\right)}{R}$$

$$\Rightarrow \quad R = -25 \text{ cm}$$

According to Lens Maker's Formula

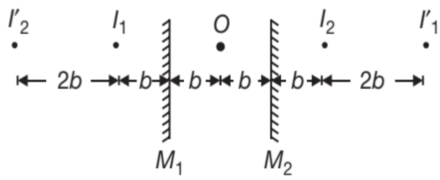
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{\infty} - \frac{1}{-25}\right)$$

$$\Rightarrow \quad f = 75 \text{ cm}$$

Hence, the correct answer is (B).

167.



I_1 is the direct image of O by mirror M_1

I_2 is the direct image of O by mirror M_2

I_1 acts as object for mirror M_2 and gives an image I_1' and similarly I_2 acts as object for mirror M_1 and gives image I_2'.

Since I_1 is located at a distance $3b$ from M_2.

So, I_1' must be at a distance $3b$ from M_2.

Similarly, I_2' must be at a distance $3b$ from M_1.

From above we observe that the first image is at a distance $2b$ from O.

Second image is at a distance $4b$ from O and so on.

So, nth image is at a distance $2nb$ from O.

Hence, the correct answer is (C).

168. $\theta = \frac{1}{2}(2\pi)\left(\frac{1}{4}\right)^2 \qquad \left\{\because \theta = \frac{1}{2}\alpha t^2\right\}$

$$\Rightarrow \quad \theta = \frac{\pi}{16}$$

Reflected ray turns through an angle $2\theta = 22.5^\circ$

Hence, the correct answer is (C).

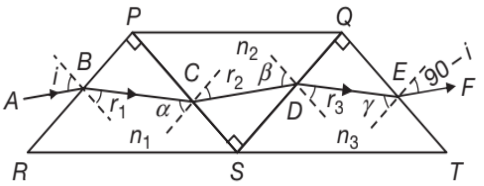
169. Distance of first image (I_1) formed due to refraction from the plane surface of water is

$$d_{\text{app}} = \frac{d_{\text{real}}}{\mu} = \frac{10}{4/3} = 7.5 \text{ cm}$$

Now distance of this image from the plane mirror is $x = 5 + 7.5 = 12.5$ cm. Therefore, distance of second image (I_2) from the mirror will also be equal to 12.5 cm from it.

Hence, the correct answer is (C).

170.



$\alpha = 90 - r_1 \quad \beta = 90 - r_2 \quad \gamma = 90 - r_3$

At B

$$\sin i = n_1 \sin r_1$$

$$\Rightarrow \sin^2 i = n_1^2 \sin^2 r_1 \quad \ldots(1)$$

At C

$$n_1 \sin(90 - r_1) = n_2 \sin r_2$$

$$\Rightarrow n_2^2 \sin^2 r_2 = n_1^2 \cos^2 r_1 \quad \ldots(2)$$

At D

$$n_2 \sin(90 - r_2) = n_3 \sin r_3$$

$$\Rightarrow n_2^2 \cos^2 r_2 = n_3^2 \sin^2 r_3 \quad \ldots(3)$$

At E

$$n_3 \sin(90 - r_3) = (1)\sin(90 - i)$$

$$\Rightarrow \cos^2 i = n_3^2 \cos^2 r_3 \quad \ldots(4)$$

Adding (1), (2), (3) & (4)

$$\Rightarrow 1 + n_2^2 = n_1^2 + n_3^2$$

Hence, the correct answer is (D).

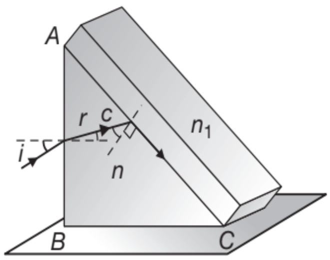
171. $P = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Hence, the correct answer is (A).

173. $(90 - r) + (90 - C) + 45 = 180$

$$\Rightarrow r + C = 45^\circ$$

$$\Rightarrow r = 45 - C$$



Since $\frac{\sin i}{\sin r} = n$

$$\Rightarrow \sin i = n \sin r$$

$$\Rightarrow \sin i = n \sin(45 - C)$$

$$\Rightarrow \sin i = n(\sin 45 \cos C - \cos 45 \sin C)$$

$$\Rightarrow \sin i = \frac{n}{\sqrt{2}}(\cos C - \sin C)$$

Further, $\sin C = \frac{n_1}{n}$

$$\Rightarrow \sin i = \frac{n}{\sqrt{2}}\left[\sqrt{1 - \frac{n_1^2}{n^2}} - \frac{n_1}{n}\right]$$

$$\Rightarrow \sin i = \frac{1}{\sqrt{2}}\left[\sqrt{n^2 - n_1^2} - n_1\right]$$

$$\Rightarrow i = \sin^{-1}\left[\frac{\sqrt{n^2 - n_1^2} - n_1}{\sqrt{2}}\right]$$

Hence, the correct answer is (D).

174. Here nose of the boy is the object and fish is observer. Using refraction formula, we have

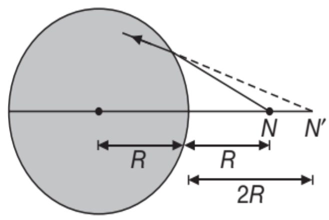
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

where $u = +R$, $\mu_1 = 1$, $\mu_2 = \frac{4}{3}$, $R = -R$

$$\Rightarrow \frac{4}{3v} - \frac{1}{R} = \frac{\frac{4}{3} - 1}{-R}$$

$$\Rightarrow v = +2R$$

Thus, the image of child's nose will appear at a distance $3R$ from the centre of the bowl. The ray diagram is shown in below figure.



Hence, the correct answer is (C).

175. For image of centre of mass of A and B, we have

$$(\vec{v}_{cm})_x = \frac{m_1 v_1}{m_1 + m_2}(-\hat{i})$$

$$(\vec{v}_{cm})_y = \frac{m_2 v_2}{m_1 + m_2}(+\hat{j})$$

Since, $\vec{v}_{cm} = (\vec{v}_{cm})_x + (\vec{v}_{cm})_y$

$$\Rightarrow \vec{v}_{cm} = \frac{1}{m_1 + m_2}\left(-m_1 v_1 \hat{i} + m_2 v_2 \hat{j}\right)$$

$$\Rightarrow \quad |\vec{v}_{cm}| = \sqrt{\frac{m_1^2 v_1^2}{(m_1+m_2)^2} + \frac{m_2^2 v_2^2}{(m_1+m_2)^2}}$$

$$\Rightarrow \quad |\vec{v}_{cm}| = \frac{1}{m_1+m_2}\sqrt{m_1^2 v_1^2 + m_2^2 v_2^2}$$

Hence, the correct answer is (D).

176. $\frac{n_1}{v} + \frac{n_2}{-u} = \frac{n_1 - n_2}{R}$

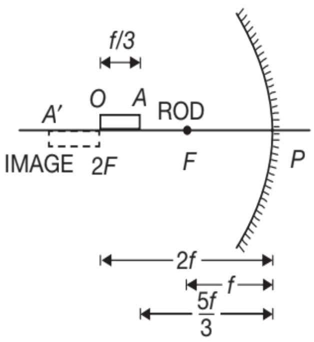
$$\Rightarrow \quad \frac{1}{v} - \frac{2}{(-15)} = \frac{1-2}{-10}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{10} - \frac{2}{15} = -\frac{1}{30}$$

$$\Rightarrow \quad v = -30 \text{ cm}$$

Hence, the correct answer is (B).

177. Since an elongated image is formed and it touches one end of the rod, so the rod must lie with one end at $2F$ and other end between $2F$ and F (shown in figure).



For end A, $u = -\frac{5f}{3}$

Since $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v} - \frac{3}{5f} = \frac{1}{-f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{3}{5f} - \frac{5}{5f}$$

$$\Rightarrow \quad \frac{1}{v} = -\frac{2}{5f}$$

$$\Rightarrow \quad v = -\frac{5f}{2}$$

$$\Rightarrow \quad PA' = \frac{5f}{2}$$

$$\Rightarrow \quad OA' = \frac{5f}{2} - 2f$$

$$\Rightarrow \quad OA' = \frac{f}{2}$$

$$\Rightarrow \quad m = \frac{OA'}{OA} = \frac{\frac{f}{2}}{\frac{f}{3}} = \frac{3}{2}$$

Hence, the correct answer is (C).

178. For a plane refracting surface, the lateral magnification is 1. So, the image of the coin will be of the same size as the coin itself.
Hence, the correct answer is (D).

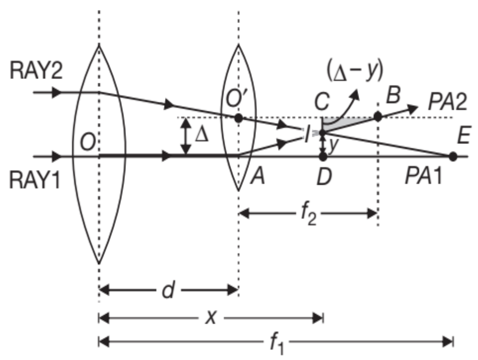
179. $\delta = \frac{A}{2} = \frac{60}{2} = 30$

$\Rightarrow$ The ray passes symmetrically through the prism parallel to its base.

Hence $\theta = 0°$

Hence, the correct answer is (D).

180.



For drawing the above ray diagram, we must consider two rays RAY 1 and RAY 2 in such a manner that RAY 1 passes through optical centre of first convex lens and is parallel to the Principal Axis for the second convex lens $(PA2)$.
So, wherever the RAY 1 cuts $PA2$ is the focal length f_2 of second convex lens. Similarly we have RAY 2 parallel to $PA1$ and let this pass through O' so as to cut $PA1$ at a distance equal to focal length (f_1) for the first convex lens.

ΔIDE and $\Delta O'AE$ are similar

$$\Rightarrow \quad \frac{y}{\Delta} = \frac{f_1 - x}{f_1 - d} \qquad \ldots(1)$$

ΔICB and $AO'B$ are similar

$$\Rightarrow \quad \frac{\Delta - y}{\Delta} = \frac{(f_2 + d) - x}{f_2}$$

$$\Rightarrow \quad -\frac{y}{\Delta} = \frac{d}{f_2} - \frac{x}{f_2}$$

$$\Rightarrow \quad \frac{y}{\Delta} = \frac{x}{f_2} - \frac{d}{f_2} \qquad \ldots(2)$$

From (1) and (2)

$$\frac{x}{f_2}-\frac{d}{f_2}=\frac{f_1-x}{f_1-d}$$

$$\Rightarrow \quad \frac{x}{f_2}-\frac{d}{f_2}=\frac{f_1}{f_1-d}-\frac{x}{f_1-d}$$

$$\Rightarrow \quad x\left(\frac{1}{f_2}+\frac{1}{f_1-d}\right)=\frac{f_1}{f_1-d}+\frac{d}{f_2}$$

$$\Rightarrow \quad x=\frac{f_1 f_2+d(f_1-d)}{f_1+f_2-d}$$

Similarly, we can calculate value of y.

OBJECTIVE TRICK

If $\Delta=0$, y must be zero (satisfied by A, B, C)

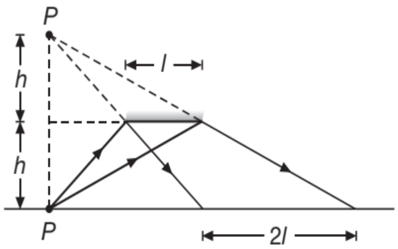
If $d\to f_1+f_2$, then $x\to\infty$ (satisfied by B, C, D)

If $d\to f_1$, then $x\to f_1$ (satisfied by C, D)

So, we observe that OPTION **(C)** is the only option satisfying both the x and y in special situations mentioned above.

Hence, the correct answer is (C).

181. As mirror is moving parallel to itself, it does not affect the position of image so length of spot on ground will remain same. This can also be seen by similar triangles in figure.



Hence, the correct answer is (B).

182. For the minimum angle of deviation produced by the prism we have $i=e$ and $r_1=r_2=\frac{A}{2}=\frac{90^\circ}{2}=45^\circ$

Applying Snell's Law at 1st interface, we get

$$1\sin i=\mu\sin r$$

$$\Rightarrow \quad \sin i=\sqrt{\frac{3}{2}}\times\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad i=60^\circ,$$

Then $D_{\min}=(i-r)+(e-r)=(60^\circ-45^\circ)+(60^\circ-45^\circ)$,

$$\Rightarrow \quad D_{\min}=30^\circ$$

Hence, the correct answer is (C).

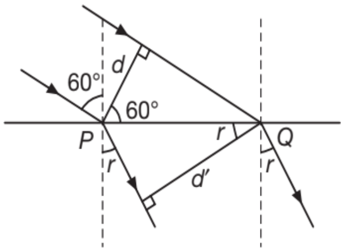
183. Let d' be the diameter of refracted beam. Then

$$d=PQ\cos(60^\circ)$$

and $d'=PQ\cos r$

$$\Rightarrow \quad \frac{d'}{d}=\frac{\cos r}{\cos 60^\circ}=2\cos r$$

$$\Rightarrow \quad d'=2d\cos r$$



Since, $\sin r=\frac{\sin i}{\mu}=\frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}}=\frac{1}{\sqrt{3}}$

Also, $\cos r=\sqrt{1-\sin^2 r}$

$$\Rightarrow \quad \cos r=\sqrt{\frac{2}{3}}$$

$$\Rightarrow \quad d'=(2)(2)\sqrt{\frac{2}{3}}=4\sqrt{\frac{2}{3}}\text{ cm}$$

$$\Rightarrow \quad d'\approx 3.26\text{ cm}$$

Hence, the correct answer is (C).

184. $10=\frac{1}{f_1}+\frac{1}{f_2}$ ...(1)

Since $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{x}{f_1 f_2}$

$$\Rightarrow \quad 6=\frac{1}{f_1}+\frac{1}{f_2}-\frac{0.25}{f_1 f_2} \quad ...(2)$$

$$\Rightarrow \quad 6=10-\frac{0.25}{f_1 f_2}$$

$$\Rightarrow \quad 4=\frac{1}{4f_1 f_2}$$

$$\Rightarrow \quad f_1 f_2=\frac{1}{16} \quad ...(3)$$

Put (3) in (1), we get

$$f_1+f_2=10\left(\frac{1}{16}\right)$$

$$\Rightarrow \quad f_1+f_2=\frac{5}{8} \quad ...(4)$$

Since $(f_1 - f_2)^2 = (f_1 + f_2)^2 - 4f_1f_2$

$$\Rightarrow \quad (f_1 - f_2)^2 = \frac{25}{64} - \frac{1}{4}$$

$$\Rightarrow \quad (f_1 - f_2)^2 = \frac{9}{64}$$

$$\Rightarrow \quad f_1 - f_2 = \frac{3}{8} \qquad \ldots(5)$$

Adding (4) and (5), we get

$$2f_1 = 1$$

$$f_1 = \frac{1}{2} = 0.5 \text{ m}$$

$$\Rightarrow \quad f_2 = \frac{1}{8} = 0.125 \text{ m}$$

Hence, the correct answer is (A).

185. Since, $\frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt}$

Using $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, we get

$$-\frac{1}{12} = -\frac{1}{20} + \frac{1}{v}$$

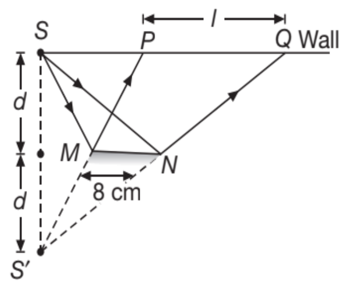
$$\Rightarrow \quad v = -30 \text{ cms}^{-1}$$

$$\Rightarrow \quad \frac{dv}{dt} = -\left(\frac{30}{20}\right)^2 \times 4 = -9 \text{ cms}^{-1}$$

i.e., 9 cms^{-1} away from the mirror.

Hence, the correct answer is (C).

186. Here, PQ is the size of spot formed on the wall. From the similar triangles $\Delta PS'Q$ and $\Delta MS'N$, we have



$$\frac{PQ}{MN} = \frac{2d}{d}$$

$$\Rightarrow \quad PQ = 2 \times 8 = 16 \text{ cm}$$

Hence, the correct answer is (C).

187. After projection of the particle its velocity component in vertical direction is given as

$$u_y = \sqrt{2}\sin(45°) = 1 \text{ ms}^{-1}$$

In vertical direction if we consider y_1 as the displacement of particle and y_2 the displacement of mirror in time t then we use

$$y_1 = (1)(0.5) - \frac{1}{2}gt^2 = 0.5 - \frac{1}{2}gt^2$$

and $y_2 = -\frac{1}{2}gt^2$

Vertical distance of particle from mirror is given by

$$y = y_1 - y_2 = 0.5 \text{ m}$$

Thus, distance between particle and its image is

$$2y = 2(0.5) = 1 \text{ m}$$

Hence, the correct answer is (B).

188. For the first medium, $\sin\theta_1 = \frac{a}{\sqrt{a^2 + b^2}}$

and for the second medium, $\sin\theta_2 = \frac{c}{\sqrt{c^2 + d^2}}$

By Snell's law, we have $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$

$$\Rightarrow \quad \frac{\mu_1 a}{\sqrt{a^2 + b^2}} = \frac{\mu_2 a}{\sqrt{c^2 + d^2}}$$

Hence, the correct answer is (B).

189. For refraction at plane surface

$$\frac{1}{-(-mR)} + \frac{1.5}{v} = \frac{1.5 - 1}{\infty}$$

$$\Rightarrow \quad v = -1.5\, mR$$

For refraction at the curved surface

$$u' = -(1.5\, mR + R), \; v' \to \infty$$

$$\Rightarrow \quad \frac{1}{\infty} - \frac{1.5}{-(1.5\, mR + R)} = \frac{1 - 1.5}{-R}$$

$$\Rightarrow \quad \frac{3}{(1.5\, m + 1)} = 1$$

$$\Rightarrow \quad 3 = 1.5\, m + 1$$

$$\Rightarrow \quad 1.5\, m = 2$$

$$\Rightarrow \quad m = \frac{4}{3}$$

Hence, the correct answer is (D).

190. Ray diagram of the grazing emergence is shown in figure.
Here we have $r + C = A$

$$\Rightarrow \quad r + C = 60$$

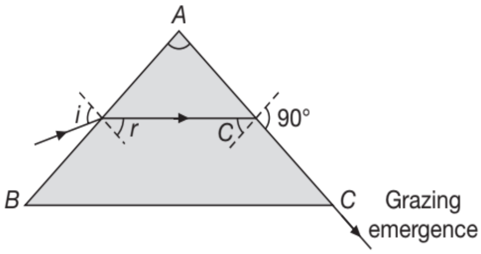
Using snell's law at the second surface, we have

$$\sqrt{2}\sin C = 1\sin(90)$$

$\Rightarrow \quad \sin C = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad C = 45°$

Then, we have $r = A - C = 15°$



Using Snell's law at surface AB, we have

$$1 \sin i = \sqrt{2} \sin r$$

and $\sin i = \sqrt{2}\sin(15°) = \sqrt{2}\sin(45° - 30°)$

$\Rightarrow \quad i = \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$

Hence, the correct answer is (A).

191. Deviation by a sphere is $2(i - r)$

Here, deviation $\delta = 60° = 2(i - r)$

$\Rightarrow \quad i - r = 30°$

$\Rightarrow \quad r = i - 30° = 60° - 30° = 30°$

According to Snell's Law, we have $\mu = \frac{\sin i}{\sin r}$

$\Rightarrow \quad \mu = \frac{\sin 60°}{\sin 30°} = \sqrt{3}$

Hence, the correct answer is (C).

192. Incidence angle on face BC is

$i = 90 - \theta$

$i = A = 90 - \theta > C \qquad$ (for light not to cross BC)

$\Rightarrow \quad 90 - \theta > C$

$\Rightarrow \quad \sin(90 - \theta) > \sin C$

$\Rightarrow \quad \cos\theta > \sin C = \frac{\frac{6}{5}}{\frac{3}{2}} = \frac{4}{5}$

$\Rightarrow \quad \theta < \cos^{-1}\left(\frac{4}{5}\right) = 37°$

Hence, the correct answer is (B).

193. Equivalent power of the system is given as

$$P_{eq} = 2(P_w + P_g) + P_m$$

where P_w and P_g are the respective powers of the water and glass lenses and P_m is the power of the mirror. These are given by

$$P_w = \frac{1}{f_w} = (\mu_w - 1)\left(\frac{1}{\infty} - \frac{1}{-60}\right)$$

$\Rightarrow \quad P_w = \frac{1}{180}\text{ cm}^{-1}$

Also, $P_g = \frac{1}{f_g} = (\mu_g - 1)\left(\frac{1}{-60} - \frac{1}{-20}\right)$

$\Rightarrow \quad P_g = \frac{1}{60}\text{ cm}^{-1}$

and $P_m = \frac{1}{f_m} = \frac{2}{20}\text{ cm}^{-1}$

Equivalent power of the system is given by

$$P_{eq} = 2(P_w + P_g) + P_m,$$

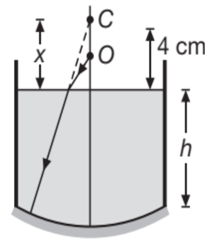
$\Rightarrow \quad P_{eq} = 2\left(\frac{1}{180} + \frac{1}{60}\right) + \frac{2}{20}$

$\Rightarrow \quad P_{eq} = \frac{13}{90}$

$\Rightarrow \quad f_e = \frac{90}{13}\text{ cm}$

Hence, the correct answer is (A).

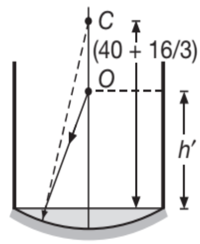
194. The image coincides with object O if the ray starting from O is incident normally on mirror as shown in figure.



Here the image is located at $x = \mu(4)$

$\Rightarrow \quad x = 4 \times \frac{4}{3} = \frac{16}{3}$

In second case, the image will coincides with object O if ray starting from O is incident normally on mirror as shown in figure.



Now, $40 + \frac{16}{3} = \frac{136}{3} = h' \times \frac{4}{3}$

$$\Rightarrow \quad h' = \frac{136}{3} \times \frac{3}{4} = 34 \text{ cm}$$

Hence, the correct answer is (A).

195. Let R be the radius of curvature of each surface. Then applying Lens Maker's Formula, we get

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$$\Rightarrow \quad R = f$$

For the water lens, again applying the Lens Maker's Formula, we get

$$\frac{1}{f'} = \left(\frac{4}{3} - 1\right)\left(-\frac{1}{R} - \frac{1}{R}\right) = \frac{1}{3}\left(-\frac{2}{f}\right)$$

$$\Rightarrow \quad \frac{1}{f'} = -\frac{2}{3f}$$

Since, we know that for lenses placed in contact, we have

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

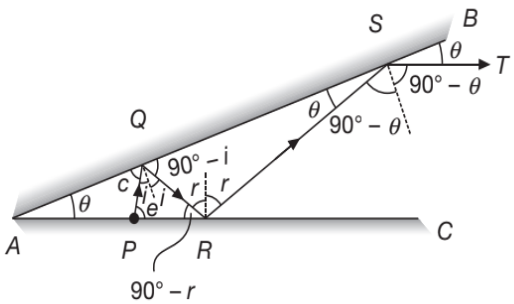
$$\Rightarrow \quad \frac{1}{F} = \frac{1}{f} + \frac{1}{f} + \frac{1}{f'}$$

$$\Rightarrow \quad \frac{1}{F} = \frac{2}{f} - \frac{2}{3f} = \frac{4}{3f}$$

$$\Rightarrow \quad F = \frac{3f}{4}$$

Hence, the correct answer is (B).

196. In triangle QRS, we have



$$90° - i + \theta + 2r = 180°$$

$$\Rightarrow \quad r = \left(\frac{90° + i - \theta}{2}\right)$$

$$\Rightarrow \quad \angle QPR = 180° - (90° - r) - 2i$$

$$\Rightarrow \quad \angle QPR = 180° - 90° + \left(\frac{90° + i - \theta}{2}\right) - 2i$$

$$\Rightarrow \quad \angle QPR = \frac{180° - 4i + 90° + i - \theta}{2}$$

$$\Rightarrow \quad \angle QPR = \left(\frac{270° - 3i - \theta}{2}\right)$$

Since $\angle QPR = \theta + \angle AQP$

$$\Rightarrow \quad \angle AQP = \angle QPR - \theta = 180° - (90° - i) - 2i$$

$$\Rightarrow \quad \left(\frac{270° - 3i - \theta}{2}\right) - \theta = 90 - i$$

Since, $\theta = 20°$, so we get

$$i = 30°$$

Hence, the correct answer is (B).

197. Focal length of lens does not depend on the aperture of lens. It depends on the refractive index of lens, surroundings and the radius of curvature of its surfaces. For intensity of image, we have

$$I \propto (\text{aperture diameter})^2$$

$$\Rightarrow \quad I_{\text{blocked}} \propto \left(\frac{d}{2}\right)^2 \text{ and } I_0 \propto d^2$$

Hence, $I_{\text{blocked}} = \dfrac{I_0}{4}$

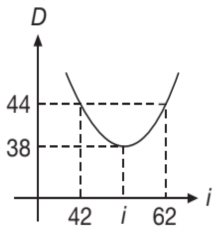
Final image intensity

$$I_f = I_{\text{original}} - I_{\text{blocked}}$$

$$\Rightarrow \quad I_f = I_0 - \frac{I_0}{4} = \frac{3I_0}{4}$$

Hence, the correct answer is (D).

198. D vs i Graph for the prism is shown in figure.



Since, $D = i + e - A$

$$\Rightarrow \quad 44° = 42° + 62° - A$$

$$\Rightarrow \quad A = 60°$$

For the condition of minimum deviation, we have

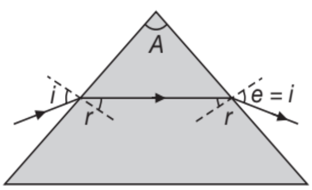
$$i = e \text{ and } r_1 = r_2 = \frac{A}{2}$$

$$\Rightarrow \quad r = 30°$$

$$\Rightarrow \quad D_{\min} = 2i - A$$

$$\Rightarrow \quad 38° = 2i - 60$$

$$\Rightarrow \quad i = 49°$$



Hence, the correct answer is (B).

199. $A_{max} = 2C$

$\Rightarrow \quad A_{max} = 2(36^o)$

$\Rightarrow \quad A_{max} = 72^o$

Hence, the correct answer is (C).

200. $f = \dfrac{D^2 - x^2}{4D}$

$\Rightarrow \quad f = \dfrac{(90)^2 - (20)^2}{4(90)}$

$\Rightarrow \quad f = 21.4$ cm

Hence, the correct answer is (C).

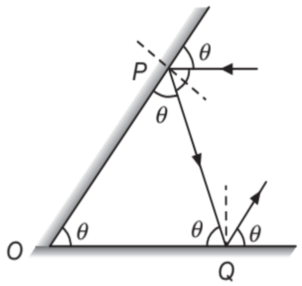
201. Since $\mu = \dfrac{\sin i}{\sin r}$

$\Rightarrow \quad \sqrt{2} = \dfrac{\sin i}{\sin 30}$

$\Rightarrow \quad i = 45°$

Hence, the correct answer is (C).

202.



$\angle OPQ = \angle OQP = \theta$

$\Rightarrow \quad 3\theta = 180°$

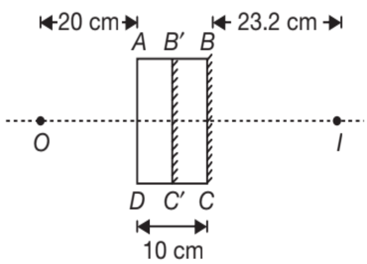
$\Rightarrow \quad \theta = 60°$

Hence, the correct answer is (C).

203. $AB' =$ Apparent Depth $= x$

$AB = 10$ cm

The face BC appear's to be shifted to $B'C'$. If x be the apparent depth, then $AB' = x$ and $B'B = (10 - x)$.



Also, according to Laws of Reflection an image is formed as far behind the mirror as the object is in front of it

$\Rightarrow \quad 20 + x = 23.2 + (10 - x)$

$\Rightarrow \quad 2x = 13.2$

$\Rightarrow \quad x = 6.6$ cm

Since $\mu = \dfrac{\text{Real Depth}}{\text{Apparent Depth}}$

$\Rightarrow \quad \mu = \dfrac{10}{6.6} = 1.51$

Hence, the correct answer is (C).

204. Since $\mu_2 t = \mu_1 t'$

$\Rightarrow \quad t' = \dfrac{\mu_2}{\mu_1} t$

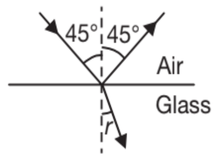
$\Rightarrow \quad \Delta x = t' - t$

$\Rightarrow \quad \Delta x = \left(\dfrac{\mu_2}{\mu_1} - 1\right) t = \left(\dfrac{\frac{3}{2}}{\frac{4}{3}} - 1\right) t = \dfrac{t}{8}$

Hence, the correct answer is (A).

205. By Laws of Reflection

$i = r$



Also

$i + r = 90$

$\Rightarrow \quad i = 45$

According to Snell's Law

$\mu = \dfrac{\sin i}{\sin r}$

$\Rightarrow \quad 1.5 = \dfrac{3}{2} = \dfrac{\sin 45}{\sin r}$

$\Rightarrow \quad r = \sin^{-1}\left(\dfrac{\sqrt{2}}{3}\right)$

Hence, the correct answer is (B).

206. Shift $= t\left(1 - \dfrac{1}{\mu}\right)$ away

Hence, the correct answer is (A).

207. According to Newton's Formula, we have

$x_1 x_2 = f^2$

$\Rightarrow \quad (10)(40) = f^2$

$\Rightarrow \quad f^2 = 400$

$\Rightarrow \quad f = 20$ cm

Hence, the correct answer is (B).

208. $A = 60°$

$i = 55°$, $e = 46°$

Since $i + e = A + D$

$\Rightarrow \quad 55 + 46 = 60 + D$

$\Rightarrow \quad D = 41°$

So, $D_m < D$

Hence, the correct answer is (B).

209. At $D = D_m$

$i = e$

Hence, the correct answer is (A).

210. Areal Magnification $= 9$

$$\Rightarrow \quad m_{ar} = 9 = \left(\frac{f}{f+u}\right)^2$$

$$\Rightarrow \quad \frac{f}{f+u} = -3 \qquad \text{(Since we get a real image)}$$

$$\Rightarrow \quad \frac{f}{f-40} = -3$$

$\Rightarrow \quad -3f + 120 = f$

$\Rightarrow \quad 4f = 120$

$\Rightarrow \quad f = 30$ cm

Hence, the correct answer is (A).

211. Axial Magnification is given by

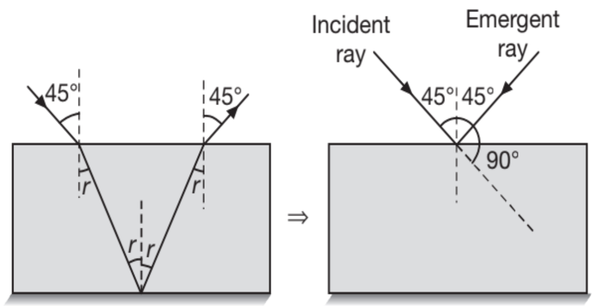
$$m_{ax} = \frac{I \text{ along PA}}{O \text{ along PA}} = \frac{v^2}{u^2}$$

$$\Rightarrow \quad m_{ax} = \frac{I}{b} = \left(\frac{f}{f-u}\right)^2$$

$$\Rightarrow \quad I = b\left(\frac{f}{f-u}\right)^2$$

Hence, the correct answer is (D).

212. From the figure it is clear that the angle between incident ray and the emergent ray is $90°$.



Hence, the correct answer is (B).

213. According to Lens Maker's Formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad f \propto \frac{1}{(\mu - 1)}$$

Since $\mu_{\text{Red}} < \mu_{\text{Violet}}$

$\Rightarrow \quad f_{\text{Red}} > f_{\text{Violet}}$

$\Rightarrow \quad f_{\text{Red}} > f_{\text{Blue}}$

Always keep in mind that whenever you are asked to compare (greater than or less than) u, v or f you must not apply sign conventions for comparison.

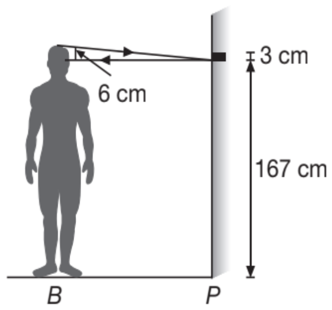
Hence, the correct answer is (D).

214. Since, the refractive index is increasing linearly from top to the bottom, so the light cannot travel in a straight line in the liquid as shown in options (A) and (B).

Initially it will bend towards normal and after reflecting from the bottom it will bend away from the normal as shown below.

Hence, the correct answer is (D).

215. To see the top of his head, the light ray from head after reflection should come to his eyes as shown in figure.



The person will not see the top of his head when the point from which these light rays are getting reflected, is replaced by a hole. Thus, this point is located 167 cm from P.

Hence, the correct answer is (A).

216. $u = -(f + x)$

$$m = \frac{f}{f+u}$$

$$\Rightarrow \quad m = \frac{f}{f - f - x}$$

$$\Rightarrow \quad m = -\frac{f}{x}$$

Negative sign confirms that real magnification is negative.

Hence, the correct answer is (A).

217. For a convex mirror, both (A) and (B) are incorrect.

Hence, the correct answer is (D).

218. $u = -(f + x_1)$

$v = (f + x_2)$

Since $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{f + x_2} + \frac{1}{f + x_1} = \frac{1}{f}$$

$$\Rightarrow \quad f(2f + x_1 + x_2) = f^2 + (x_1 + x_2)f + x_1x_2$$

$$\Rightarrow \quad f^2 = x_1x_2$$

$$\Rightarrow \quad f = \sqrt{x_1x_2}$$

Hence, the correct answer is (D).

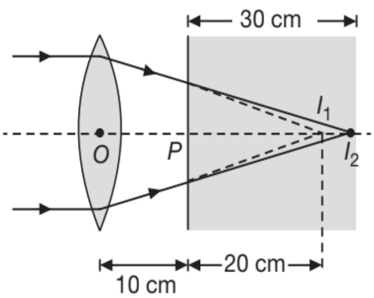
219. Two plano-convex lenses of focal length f on combining give a convex lens of focal length $\frac{f}{2}$. To obtain a real image of magnification unity the object must be at $2F(\text{or } C)$ and so image is also formed at $2F(\text{or } C)$. Hence $u = -2f = -40$ cm

Hence, the correct answer is (C).

220. The lens will converge the rays at its focus i.e., 30 cm from the lens or 20 cm from the refracting surface, so we have

$PI_1 = 20$ cm

Now, $PI_2 = \mu(PI_1) = \frac{3}{2} \times 20 = 30$ cm



Hence, the rays will converge at a distance of 40 cm from the lens.

Hence, the correct answer is (D).

221. $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$

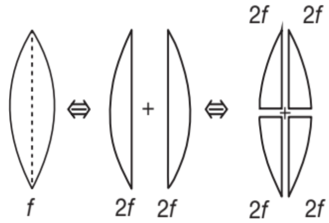
$$\Rightarrow \quad \frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right) \qquad \ldots(1)$$

On cutting $\frac{1}{f'} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$

$$\Rightarrow \quad f' = 2f$$

Hence, the correct answer is (C).

222. When a lens is cut parallel to principal axis its focal length remains the same. Hence each part will have a focal length 2*f*.



Hence, the correct answer is (C).

223. According to Lens Maker's Formula

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

The lens is made of two materials, so for a single object distance, two different image distances are obtained i.e. two images are formed.

Hence, the correct answer is (B).

225. Let f_1 and f_2 be the focal lengths of the lenses of refractive indices μ_1 and μ_2 in water, then

$$\frac{1}{f_1} = \left(\frac{\mu_1}{\mu_w} - 1\right)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$$\Rightarrow \quad \frac{1}{f_1} = \left(\frac{\mu_1}{\mu_w} - 1\right)\left(\frac{2}{R}\right) \qquad \ldots(1)$$

$$\frac{1}{f_2} = \left(\frac{\mu_2}{\mu_w} - 1\right)\left(-\frac{1}{R} - \frac{1}{R}\right)$$

$$\Rightarrow \quad \frac{1}{f_2} = \left(\frac{\mu_2}{\mu_w} - 1\right)\left(\frac{-2}{R}\right) \qquad \ldots(2)$$

Adding equations (1) and (2), we get

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{2(\mu_1 - \mu_2)}{\mu_w R}$$

$$\Rightarrow \quad \frac{1}{30} = \frac{2(\mu_1 - \mu_2)}{\mu_w R}$$

$$\Rightarrow \quad (\mu_1 - \mu_2) = \frac{\mu_w R}{60}$$

Substituting the values, we get

$$\left(\mu_1-\mu_2\right)=\frac{4\times 15}{3\times 60}=\frac{1}{3}$$

Hence, the correct answer is (B).

226. Since image is formed on the screen, hence it must be real. So, Lens used must be convex (and cannot be concave)

$$u=-45\text{ cm},\ v=+90\text{ cm}$$

Since, for a lens

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow\quad \frac{1}{90}+\frac{1}{45}=\frac{1}{f}$$

$$\Rightarrow\quad f=+30\text{ cm}$$

Further again the positive sign with focal length indicates the lens is convex.

Hence, the correct answer is (A)

227. Size of Image $=\frac{v}{u}(\text{Size of Object})$

$$\Rightarrow\quad I=\frac{90}{-45}(5)=-10\text{ cm}$$

Here again the negative sign with I indicates that it is formed below origin i.e. inverted i.e. real.

Hence, the correct answer is (B).

228. $\frac{\mu_1}{-\mu}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}$

$$\Rightarrow\quad \frac{1}{-(-50)}+\frac{1.5}{v}=\frac{1.5-1}{20}$$

$$\Rightarrow\quad \frac{1}{50}+\frac{1.5}{v}=\frac{0.5}{20}$$

$$\Rightarrow\quad \frac{1.5}{v}=\frac{1}{40}-\frac{1}{50}$$

$$\Rightarrow\quad \frac{1.5}{v}=\frac{10}{2000}$$

$$\Rightarrow\quad v=\frac{15\times 200}{10}=+300\text{ cm}$$

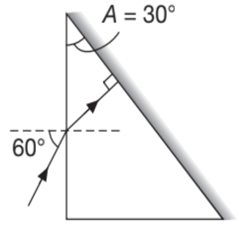
Positive side indicates that image is real.

Hence, the correct answer is (C).

229. $r_2=0°$

$$\Rightarrow\quad r_1=A=30°$$

and $i_1=60°$



So, from Snell's Law, we get

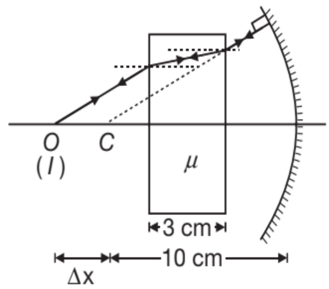
$$\mu=\frac{\sin i_1}{\sin r_i}=\frac{\sin 60°}{\sin 30°}=\sqrt{3}$$

Hence, the correct answer is (C).

230. $\Delta x=3\left[1-\frac{1}{\left(\frac{3}{2}\right)}\right]$

$$\Rightarrow\quad \Delta x=1\text{ cm}$$

$$\Rightarrow\quad \text{Object distance}=10+1=11\text{ cm}$$



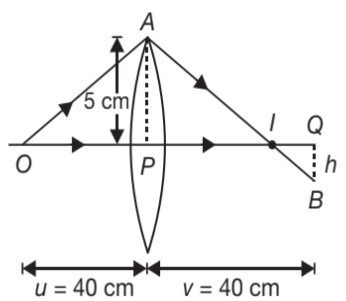
Hence, the correct answer is (C).

231. $f_\ell=\frac{\mu_g-1}{\frac{\mu_g}{\mu_\ell}-1}\times f_a=\frac{1.5-1}{\frac{1.5}{1.6}-1}\times 20$

$$\Rightarrow\quad f_\ell=-\frac{0.5\times 1.6}{0.1}\times 20=-160\text{ cm}$$

Hence, the correct answer is (C).

232.



$f=20\text{ cm},\ u=-40\text{ cm}$

$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$ gives

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{20}-\frac{1}{40}$$

$\Rightarrow \quad v = 40$ cm

The situation is shown in figure.

From similar triangles PAI and IQB

$$\frac{5}{h}=\frac{40}{20}$$

$\Rightarrow \quad h=\frac{5}{2}=2.5$ cm

Hence, the correct answer is (C).

233. Since $O^2 = I_1 I_2$

$\Rightarrow \quad 4=(0.5)I_2$

$\Rightarrow \quad I_2 = 8$ cm

Hence, the correct answer is (C).

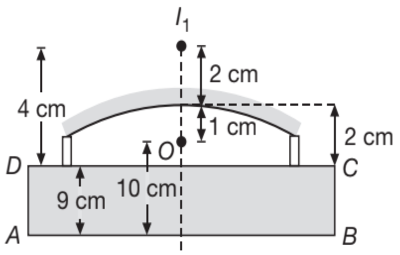
234. According to Newton's Formula

$$x_1x_2 = f^2$$

$\Rightarrow \quad f=\sqrt{16\times 25}=20$ cm

Hence, the correct answer is (C).

235. For reflection from mirror, we have



$u=-1$ cm

$f=-2$ cm

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we get

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\left(-\frac{1}{2}\right)-\left(-\frac{1}{1}\right)=1-\frac{1}{2}$$

$\Rightarrow \quad v = 2$ cm

For the slab, image I_1 can be taken as the object. Here we use

$$h_{app}=\frac{d_1}{\left(\frac{\mu_1}{\mu_3}\right)}+\frac{d_2}{\left(\frac{\mu_2}{\mu_3}\right)}$$

$$\Rightarrow \quad h_{app}=\left(\frac{d_1}{\mu_1}+\frac{d_2}{\mu_2}\right)\mu_3$$

where $d_1 = 9$ cm, $d_2 = 4$ cm, $\mu_1=\frac{3}{2}$, $\mu_2 = 1$, $\mu_3 = 1$

$$\Rightarrow \quad h_{app}=\left(\frac{9}{\frac{3}{2}}+\frac{4}{1}\right)(1)=10 \text{ cm}$$

i.e. the final image formed at 10 cm from side AB of the slab i.e. at the object itself as shown in figure. Here the image formed is virtual.

Hence, the correct answer is (D).

236. Since we know that, along the normal, the velocity of image is double the velocity of mirror and opposite to the velocity of image and along the surface of mirror, image velocity components are same as that of object, so velocity of image is given as $\left(3\hat{i}+4\hat{j}+11\hat{k}\right)$.

Hence, the correct answer is (B).

237. For the mango located at a distance x from the water surface, apparent distance of the image of mango appears to tortoise as

$$x_{app} = \mu x$$

$$\Rightarrow \quad \frac{d^2x_{app}}{dt^2}=\mu\frac{d^2x}{dt^2}$$

$\Rightarrow \quad a_{app} = \mu g$

So, the acceleration of falling mango with respect to tortoise is given as

$$a_{relative} = a + a_{app} = g+\mu g = g(1+\mu)=\frac{7g}{3}$$

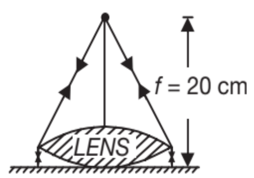
Hence, the correct answer is (D).

238. Focal Length of Lens is $f_\ell = 20$ cm.

Focal length of combination is

$$f_c = 20+5 = 25 \text{ cm}$$

when the space between lens and mirror is filled with a liquid then



$$\frac{1}{f_c}=\frac{1}{f_\ell}+\frac{1}{f}$$

$$\frac{1}{25}=\frac{1}{f_\ell}+\frac{1}{20}$$

$$\Rightarrow \quad \frac{1}{f_\ell}=\frac{1}{25}-\frac{1}{20}$$

$\Rightarrow \quad f_\ell = -100$ cm

For this liquid lens

$$\frac{1}{f_\ell} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad -\frac{1}{100} = (\mu - 1)\left(\frac{1}{-33} - \frac{1}{\infty}\right)$$

$$\Rightarrow \quad \mu - 1 = 0.33$$

$$\Rightarrow \quad \mu = 1.33 = \frac{4}{3}$$

Hence, the correct answer is (A).

240. For real image

$$u = -u_1,\ v = -2u_1,\ f = -20 \text{ cm}$$

Substituting in $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{-2u_1} - \frac{1}{u_1} = -\frac{1}{20}$$

$$\Rightarrow \quad u_1 = 30 \text{ cm}$$

For virtual image $u = -u_2$, $v = 2u_2$, $f = -20$ cm

Again applying $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{2u_2} - \frac{1}{u_2} = -\frac{1}{20}$$

$$\Rightarrow \quad u_2 = 10 \text{ cm}$$

So, the distance between two positions of the object is $u_1 - u_2$

$$\Rightarrow \quad u_1 - u_2 = 30 \text{ cm} - 10 \text{ cm} = 20 \text{ cm}$$

Hence, the correct answer is (D).

241. Since we know that a light ray after successive reflections from two mutually perpendicular mirrors reverses its direction. So the final reflected ray must have same slope as that of the given incident light ray. Hence only OPTION (D) can be the correct answer for a given set of mirrors.
Hence, the correct answer is (D).

242. $P = P_1 + P_2$

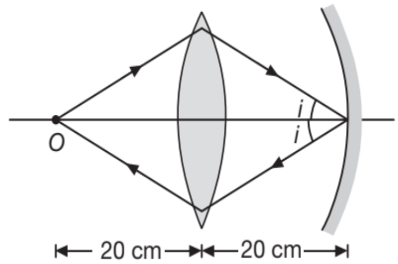
$$\Rightarrow \quad P = 10\ D$$

$$f = \frac{1}{P} = 0.1 \text{ m}$$

$$\Rightarrow \quad f = 10 \text{ cm}$$

Hence, the correct answer is (A).

243. Object is placed at a distance of $2f$ from the lens of focal length f i.e., the image formed by the lens will be at a distance of $2f$ or 20 cm from the lens. So, if the concave mirror is placed in this position, the first image will be formed at its pole and it will reflect all the rays symmetrically to other side as shown below and the final image will coincide with the object.



Hence, the correct answer is (B).

244. $\text{Shift} = d = t\left(1 - \frac{1}{n}\right)$

$$\Rightarrow \quad t = \frac{nd}{n-1}$$

Hence, the correct answer is (D).

245. Focal length of plano convex lens using Lens Maker's Formula is

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{10} - \frac{1}{\infty}\right)$$

$$\Rightarrow \quad f = 20 \text{ cm}$$

If point object O is placed at a distance of 20 cm from the plano convex lens rays become parallel and final image is formed at second focus or 20 cm from concave lens which is independent of y.

Hence, the correct answer is (A).

246. Focal length remains unchanged.

$$I \propto \text{Area of Aperture}$$

$$I = kD^2$$

$$I' = k\left[D^2 - \left(\frac{D}{2}\right)^2\right]$$

$$\Rightarrow \quad I' = \frac{3I}{4}$$

Hence, the correct answer is (D).

247. $\lambda = \frac{v}{f}$

In moving from air to glass, f remains unchanged while v decreases. Hence, λ should decrease.

Hence, the correct answer is (B).

248. $f = \dfrac{x}{m_1 - m_2}$

(See 'Displacement Method' in SYNOPSIS)

Hence, the correct answer is (B).

249. $f = \dfrac{D^2 - x^2}{4D}$

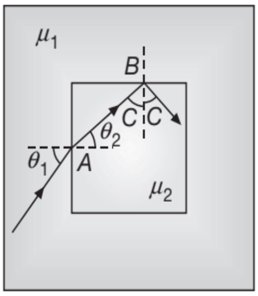
$\Rightarrow \quad f = \dfrac{(0.9)^2 - (0.2)^2}{4(0.9)}$

$\Rightarrow \quad f = 0.214 \text{ m}$

$\Rightarrow \quad f = 21.4 \text{ cm}$

Hence, the correct answer is (B).

250.



Applying Snell's Law at A, we get

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2 \qquad \ldots(1)$$

and angle of incidence at B is

$$\theta_2 = 90° - C$$

for total internal reflection at top face we use

$$\theta_2 > C$$

$$\Rightarrow \quad \cos\theta_2 = \sin C = \frac{\mu_1}{\mu_2} \qquad \ldots(2)$$

Eliminating θ_2 between (1) and (2), we get

$$\sin\theta_1 = \sqrt{\left(\frac{\mu_2}{\mu_1}\right)^2 - 1}$$

Hence, the correct answer is (A).

251. When plane surface is silvered, we use

$$P_{eq} = 2P_L = 2\left(\frac{\mu - 1}{R}\right)$$

Then in case when curved surface is silvered, we use

$$P_{eq}' = 2P_L + P_m$$

$$\Rightarrow \quad P_{eq}' = 2\left(\frac{\mu - 1}{R}\right) + \frac{2}{R} = \frac{2\mu}{R}$$

Since, $f_{eq} = 28$ cm

$$\Rightarrow \quad 28 = \frac{R}{2(\mu - 1)} \qquad \ldots(1)$$

When curved surface is silvered, we have

$$f_{eq}' = 10 = \frac{R}{2\mu} \qquad \ldots(2)$$

From Equations (1) and (2) we get,

$$\mu = \frac{14}{9}$$

Hence, the correct answer is (B).

253. $\dfrac{1}{F} = \dfrac{2}{f_\ell} + \dfrac{1}{f_m}$

$\Rightarrow \quad \dfrac{1}{F} = \dfrac{2}{f_\ell} + \dfrac{1}{\infty}$

$\Rightarrow \quad F = \dfrac{f_\ell}{2}$

Also

$$\frac{1}{f_\ell} = (\mu - 1)\left(\frac{1}{-r} - \frac{1}{\infty}\right)$$

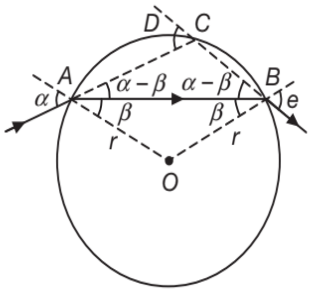
$$\Rightarrow \quad f_\ell = \frac{-r}{\mu - 1}$$

$$\Rightarrow \quad F = -\frac{r}{2(\mu - 1)}$$

Negative sign indicates that mirror is concave.

Hence, the correct answer is (B).

254.



Since angles opposite to equal sides are equal. So, in ΔAOB we have $\angle ABO = \beta$. In ΔCAB, external angle equals the sum of internal opposite angles. So,

$$D = (\alpha - \beta) + (\alpha - \beta)$$

$$D = 2(\alpha - \beta)$$

Hence, the correct answer is (B).

255. When magnification is $m = +2$, then we have

$$u = -x$$

$$v = -2x$$

$$f = +20$$

Applying Mirror Formula, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$, we get

$$\frac{1}{-2x}+\frac{1}{x}=\frac{1}{20}$$

$\Rightarrow \quad x = 10$ cm

To have a magnification of $m' = -2$, we have

$u = -y$

$v = +2y$

and $f = +20$

$$\Rightarrow \quad \frac{1}{2y}+\frac{1}{y}=\frac{1}{20}$$

$\Rightarrow \quad y = 30$ cm

So, the object has to be moved by

$y - x = 20$ cm

Hence, the correct answer is (C).

256. When water is filled in the mirror, a plano convex lens is formed, so now combination contains a plano-convex lens and a mirror. The effective focal length of combination is less than the focal length of above mirror, so image is shifted downwards.
Hence, the correct answer is (D).

258. The given lens is a convex lens. Let the magnification be m, then for real image, we have

$$\frac{1}{mx}+\frac{1}{x}=\frac{1}{f} \qquad \ldots(1)$$

and for virtual image, we have

$$\frac{1}{-my}+\frac{1}{y}=\frac{1}{f} \qquad \ldots(2)$$

From equation (1) and (2), we get

$$f=\frac{x+y}{2}$$

Hence, the correct answer is (D).

259. Using refraction formula for spherical surface (taking B as object, if its image is formed ∞), we have

$$\frac{\mu_2}{\infty}-\frac{\mu_1}{(-2R)}=\frac{\mu_2-\mu_1}{-R}$$

$$\Rightarrow \quad \frac{\mu_1}{\mu_2}=2$$

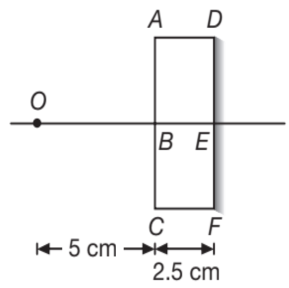
Hence, the correct answer is (A).

261. Let I_1, I_2 and I_3 be the images formed due to

(i) refraction from ABC

(ii) reflection from DEF and

(iii) again refraction from ABC



Then $BI_1 = (5)\mu_g = (5)(1.5) = 7.5$ cm

Now $EI_1 = (7.5 + 2.5) = 10$ cm

$\Rightarrow \quad EI_2 = 10$ cm behind the mirror

Now $BI_2 = (10 + 2.5) = 12.5$ cm

$$\Rightarrow \quad BI_3=\frac{12.5}{\mu_g}=\frac{12.5}{1.5}=\frac{25}{3}\text{ cm}$$

Hence, the correct answer is (B).

262. $f=\dfrac{D^2-x^2}{4D}$ (By Displacement Method)

$$\Rightarrow \quad P=\frac{4D}{D^2-x^2}$$

$$\Rightarrow \quad P=\frac{4(1)}{1-0.16}$$

$$\Rightarrow \quad P=\frac{4}{0.84}$$

$\Rightarrow \quad P = 4.76$ D

Hence, the correct answer is (C).

263. $\text{Shift}=d\left(1-\dfrac{1}{\mu}\right)=7\left(1-\dfrac{1}{1.4}\right)$

$\Rightarrow \quad$ Shift $= 2$ cm (downwards)

Hence, the correct answer is (A).

264. $\dfrac{1}{F}=\dfrac{2}{f_\ell}+\dfrac{1}{f_m}$

$$\frac{1}{f_\ell}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f_\ell}=(1.5-1)\left(\frac{1}{20}-\frac{1}{60}\right)$$

$\Rightarrow \quad f_\ell = 60$ cm and

$$f_m=\frac{20}{2}=10\text{ cm}$$

$$\Rightarrow \quad \frac{1}{F}=\frac{2}{60}+\frac{1}{10}$$

$\Rightarrow \quad F = 7.5$ cm

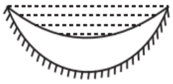
For image to be formed at the same place where object is situated we have $u = 2F = 15$ cm

Hence, the correct answer is (A).

265. $\frac{1}{F} = \frac{2}{f_\ell} + \frac{2}{f_w} + \frac{1}{f_m}$

(Because here we have two refractions at the concave surface of lens and two refraction for water lens). Since

$$\frac{1}{f_w} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{60} - \frac{1}{\infty}\right)$$



$\Rightarrow \quad f_w = 180$ cm

$\Rightarrow \quad \frac{1}{F} = \frac{2}{60} + \frac{2}{180} + \frac{1}{10}$

$\Rightarrow \quad \frac{1}{F} = \frac{13}{90}$ cm

$\Rightarrow \quad F = \frac{90}{13}$ cm

So, required answer to get asked condition is 2F. Hence

$$u = \frac{180}{13} \text{ cm}$$

Hence, the correct answer is (D).

266. For equiconvex lens, we have

$$|R_1| = |R_2| = f = 10 \text{ cm}$$

Now $P = 2P_L + P_M$

$\Rightarrow \quad -\frac{1}{F} = 2(\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - \frac{2}{R_2}$

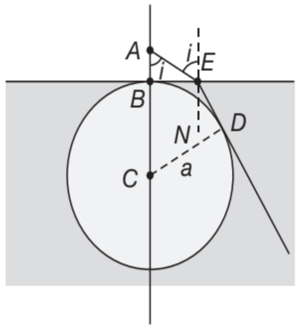
$\Rightarrow \quad -\frac{1}{F} = 2(1.5 - 1)\left(\frac{1}{10} - \frac{1}{-10}\right) - \frac{2}{-10}$

$\Rightarrow \quad F = -2.5$ cm

Therefore, the system will behave like a concave mirror of focal length 2.5 cm.

Hence, the correct answer is (C).

267. The situation described in the problem is shown in the ray diagram.



Since, we are given that

$$\angle NED = 30^\circ$$

$\Rightarrow \quad \angle BED = 120^\circ$

Also, $BCDE$ is a cyclic quadrilateral

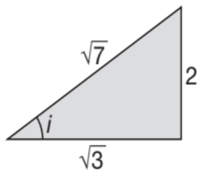
$\Rightarrow \quad \angle BCD = 60^\circ$

The line CE will be angle bisector of $\angle BCD$

$\Rightarrow \quad BE = a\tan(30^\circ) = \frac{a}{\sqrt{3}}$

Since $\tan i = \frac{BE}{AB} = \frac{a\sqrt{3}}{\frac{a}{2}} = \frac{2}{\sqrt{3}}$

$\Rightarrow \quad \sin i = \frac{2}{\sqrt{7}}$



Using Snell's law, we get

$$1 \sin i = n \sin r$$

$\Rightarrow \quad \frac{2}{\sqrt{7}} = n \times \frac{1}{2}$

$\Rightarrow \quad n = \frac{4}{\sqrt{7}}$

Hence, the correct answer is (D).

268. $\frac{1}{F} = \frac{2}{f_\ell} + \frac{1}{f_m}$

Since $f_m \to \infty$

$\Rightarrow \quad F = \frac{f_\ell}{2} = 15$ cm

Further

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$$

$\Rightarrow \quad \frac{1}{v}-\frac{1}{(-40)}=\frac{1}{15}$

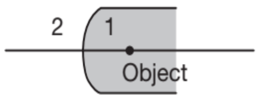
$\Rightarrow \quad \frac{1}{v}=\frac{1}{15}-\frac{1}{40}$

$\Rightarrow \quad v=24$ cm

Hence, the correct answer is (B).

269. Applying $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$, we get

$\frac{1}{v}-\frac{1.5}{(-u)}=\frac{1-(1.5)}{-R}$



$\Rightarrow \quad \frac{1}{v}+\frac{3}{2u}=\frac{1}{2R}$

For v to be positive, we have

$\frac{1}{2R}>\frac{3}{2u}$

$\Rightarrow \quad u>3R$

Hence, the correct answer is (D).

270. $f_{\text{combination}} \to \infty$

Since $\frac{1}{f_{\text{combination}}}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{x}{f_1 f_2}$

$\Rightarrow \quad 0=\frac{1}{-10}+\frac{1}{f_2}-\frac{10}{(-10)f_2}$

$\Rightarrow \quad 0=\frac{1}{-10}+\frac{1}{f_2}+\frac{1}{f_2}$

$\Rightarrow \quad \frac{1}{10}=\frac{2}{f_2}$

$\Rightarrow \quad f_2=20$ cm

Hence, the correct answer is (B).

271. $\frac{f}{f+(-12)}=-\frac{f}{f+(-20)}$

$f-12=-(f-20)$

$f-12=-f+20$

$2f=32$

$f=16$ cm

272. $5=(0.5)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$\Rightarrow \quad \frac{1}{R_1}-\frac{1}{R_2}=10$

$-1=\left({}^{\ell}\mu_g-1\right)10$

$\Rightarrow \quad \left(\frac{\mu_g}{\mu_\ell}-1\right)=-0.1$

$\Rightarrow \quad \frac{1.5}{\mu_\ell}=0.9$

$\Rightarrow \quad \mu_\ell=\frac{1.5}{0.9}=\frac{5}{3}$

Hence, the correct answer is (C).

273. $|u|+|v|=D$

$m=\frac{v}{u}$

$\Rightarrow \quad v=mu$

$|u|+m|u|=D$

$\Rightarrow \quad |u|=\frac{D}{m+1}$

$\Rightarrow \quad u=-\left(\frac{D}{m+1}\right)$

Since image formed is real, so it must be on positive side. Hence

$|v|=+v=\frac{mD}{m+1}$

For a lens

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$\Rightarrow \quad \frac{m+1}{mD}+\frac{m+1}{D}=\frac{1}{f}$

$\Rightarrow \quad f=\frac{mD}{(m+1)^2}$

Hence, the correct answer is (D).

274. $\frac{1}{f}=(1.5-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)$

$\Rightarrow \quad \frac{1}{16}=0.5\,\frac{1}{R}$

$\Rightarrow \quad R=8$ cm

Hence, the correct answer is (A).

275. Since $m=\frac{f}{f+u}$

$\Rightarrow \quad -1=\frac{f}{f+u}$

$\Rightarrow \quad \frac{1}{f}=-\frac{1}{10}$

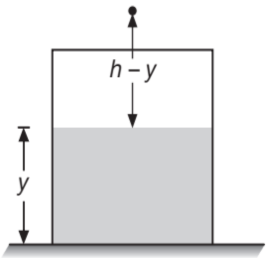
$\Rightarrow \quad f=-10$ cm

Negative sign indicates diverging nature of lens in the liquid.

Hence, the correct answer is (B).

278. Since, we have, at time t

$$d = y + \mu(h - y) = \mu h - (\mu - 1)y$$



If A is the area of the tank, then we have

$$y = \frac{\alpha t}{A} \qquad \{\because Ay = \alpha t\}$$

$$\Rightarrow \quad d = \mu h - \frac{(\mu - 1)\alpha t}{A}$$

i.e., $d - t$ graph is a straight line with negative slope and positive intercept. But d becomes constant once $y = H$.

Hence, the correct answer is (C).

279. On immersing in water f increases and hence P decreases.

Hence, the correct answer is (B).

280. $$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{0.5} - \frac{1}{-0.5}\right)$$

$$\Rightarrow \quad \frac{1}{f} = 0.5\left(\frac{2}{0.5}\right)$$

$$\Rightarrow \quad P = 2\ D$$

Hence, the correct answer is (D).

281. $O = \sqrt{I_1 I_2}$

$O = \sqrt{4 \times 16} = 8$ cm

Hence, the correct answer is (A).

282. For a lens making real image

$$-m = \frac{f}{f + u}$$

$$\Rightarrow \quad -mf - mu = f$$

$$\Rightarrow \quad -mu = f(1 + m)$$

$$\Rightarrow \quad u = -f\left(1 + \frac{1}{m}\right)$$

$$\Rightarrow \quad |u| = f\left(1 + \frac{1}{m}\right)$$

For $|u|$ to be MINIMUM m must be MAXIMUM i.e. $m \to \infty$

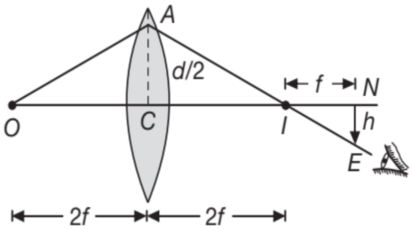
$$\Rightarrow \quad |u|_{\min} = f$$

For $|u|$ to be MAXIMUM m must be MINIMUM i.e. $m = 1$

$$\Rightarrow \quad |u|_{\max} = 2f$$

Hence, the correct answer is (A).

283. The ray diagram is as shown in figure



Since triangles CAI and NEI are similar, so we have

$$\frac{h}{d/2} = \frac{f}{2f}$$

$$\Rightarrow \quad \frac{2h}{d} = \frac{1}{2}$$

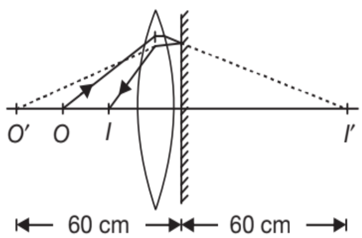
$$\Rightarrow \quad h = \frac{d}{4}$$

Hence, the correct answer is (A).

287. If mirror would have been absent then image is formed on the other side of lens (at I').
So

$$\frac{1}{v} - \frac{1}{-20} = \frac{1}{15}$$

$$\Rightarrow \quad v = 60 \text{ cm}$$



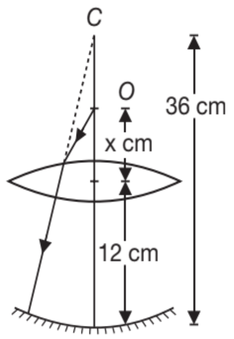
Since the mirror reflects the ray back, so O' serves as a virtual object and forms a real image I in front of mirror.

$$\frac{1}{v} + \frac{1}{-60} = \frac{1}{15}$$

$\Rightarrow \quad v = +12$ cm

Hence, the correct answer is (A).

288. Rays from O must fall normally on the mirror, only then the lens forms a virtual image at C (the centre of curvature).



$$\Rightarrow \quad \frac{1}{x} + \frac{1}{-(36-12)} = \frac{1}{40}$$

$\Rightarrow \quad x = 15$ cm

Hence, the correct answer is (B).

289. Let the length of rod be a.

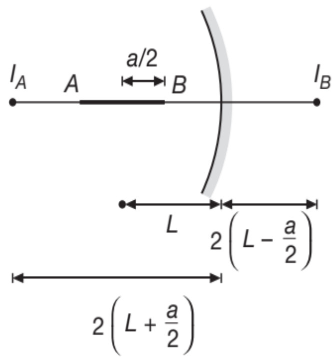
The object has a velocity V but the velocity of image of the end A and B is $4V$. Since

$$(\text{Image Velocity}) = m^2(\text{Object Velocity})$$

$\Rightarrow \quad 4V = m^2 V$

$\Rightarrow \quad m = \pm 2$

This simply means that the magnitude of transverse magnification of images of ends A and B is 2 each. However, the image of B is virtual and that of A is real.



Applying mirror formula to end A, we get

$$-\frac{1}{2\left(L-\frac{a}{2}\right)} - \frac{1}{\left(L-\frac{a}{2}\right)} = -\frac{1}{f} \quad \ldots(1)$$

Applying mirror formula to B, we get

$$-\frac{1}{2\left(L+\frac{a}{2}\right)} - \frac{1}{\left(L+\frac{a}{2}\right)} = -\frac{1}{f} \quad \ldots(2)$$

Solving equations (1) and (2), we get

$a = L$

Hence, the correct answer is (C).

290. $\frac{1}{F} = \frac{1}{(1/2)} + \frac{1}{(1/3)} = 5$

$\Rightarrow \quad F = 0.2$ m $= 20$ cm

Since

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{(-30)} = \frac{1}{20}$$

$\Rightarrow \quad v = 60$ cm

Hence, the correct answer is (D).

291. $P_{comb} = P_1 + P_2 - xP_1P_2$

$$0 = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$$

$\Rightarrow \quad x = f_1 + f_2$

Hence, the correct answer is (D).

293. Apparent Separation $= 2(\text{Apparent Depth})$

$$\Rightarrow \quad \text{Apparent Separation} = \frac{2h}{\mu}$$

Hence, the correct answer is (D).

294. For distant vision

$u = -\infty$

$$\Rightarrow \quad \frac{1}{-2} - \frac{1}{-\infty} = \frac{1}{f}$$

$\Rightarrow \quad f = -2$ m

$\Rightarrow \quad P = -0.5\ D$

For near vision

$u = -D = -25$ cm

$$\Rightarrow \quad \frac{1}{-1} - \frac{1}{-0.25} = \frac{1}{f}$$

$\Rightarrow \quad P = +3\ D$

Hence he must use bifocal lenses with

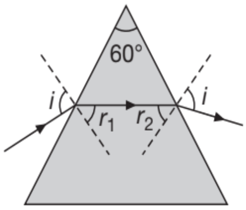
$P = -0.5\ D$ and additional $+3.5$ D

(to give $+3\ D$ net)

Hence, the correct answer is (A).

295. The slab does not affect the minimum deviation by prism. Thus for minimum deviation, we use

$r_1 = r_2 = 30°$ as shown in figure



$\Rightarrow \quad \sin i = \sqrt{2}\sin(30^\circ)$

$\Rightarrow \quad i = 45^\circ$

Thus minimum deviation is

$$2i - A = 90^\circ - 60^\circ = 30^\circ$$

$\Rightarrow \quad \sin i = \sqrt{2}\sin(30^\circ)$

$\Rightarrow \quad i = 45^\circ$

Hence, the correct answer is (A).

297. In the first case. Let x be the distance of object from the mirror. Then

$$u = -x,\ v = +2x \text{ and } f = -f$$

Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{2x} - \frac{1}{x} = -\frac{1}{f}$$

$\Rightarrow \quad x = \frac{f}{2}$

In the second case, let y be the distance of object from the mirror. Then

$$u = -y,\ v = -2y \text{ and } f = -f$$

$\Rightarrow \quad \frac{1}{-2y} - \frac{1}{y} = -\frac{1}{f}$

$\Rightarrow \quad y = \frac{3}{2}f$

So, object will have to be moved by a distance of $y - x = f$.

Hence, the correct answer is (C).

298. $\frac{f_0}{f_e} = -5$

Separation $= f_0 + f_e$

$$36 = -4f_e$$

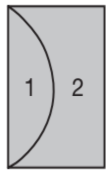
$\Rightarrow \quad f_e = -9 \text{ cm}$

$\Rightarrow \quad f_0 = 45 \text{ cm}$

Hence, the correct answer is (A).

299. For lenses placed in contact, we have

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$



$\Rightarrow \quad \frac{1}{F} = (\mu_1 - 1)\left(\frac{1}{\infty} + \frac{1}{R}\right) + (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$

$\Rightarrow \quad \frac{1}{F} = \frac{\mu_1 - \mu_2}{R}$

$\Rightarrow \quad F = \frac{R}{\mu_1 - \mu_2}$

Hence, the correct answer is (D).

300. $v_e = -D = -25 \text{ cm},\ f_e = 6.25 \text{ cm}$

Since

$$\frac{1}{f_e} = \frac{1}{v_e} + \frac{1}{u_e}$$

$\Rightarrow \quad \frac{1}{u_e} = \frac{1}{f_e} - \frac{1}{v_e}$

$\Rightarrow \quad \frac{1}{u_e} = \frac{1}{6.25} - \frac{1}{(-25)}$

$\Rightarrow \quad u_e = 5 \text{ cm}$

$\Rightarrow \quad v_0 = 15 - 5 = 10 \text{ cm}$

Also, $f_0 = 2 \text{ cm}$ and

$$\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$$

$\Rightarrow \quad \frac{1}{2} = \frac{1}{10} - \frac{1}{u_0}$

$\Rightarrow \quad u_0 = -2.5 \text{ cm}$

Hence, the correct answer is (B).

301. $m = m_0 \times m_e = \frac{v_0}{u_0} \times \frac{v_e}{u_e} = \frac{10}{2.5} \times \frac{25}{5}$

$\Rightarrow \quad m = 20$

Hence, the correct answer is (C).

302. $(\text{Shift})_1 = t\left(1 - \frac{1}{\mu_g}\right) = 9\left(1 - \frac{1}{3/2}\right) = 3 \text{ cm}$

$(\text{Shift})_2 = t\left(1 - \frac{\mu_w}{\mu_g}\right) = 9\left(1 - \frac{8}{9}\right) = 1 \text{ cm}$

So, distance between two images is given by

$$(\text{Shift})_1 - (\text{Shift})_2 = 2 \text{ cm}$$

Hence, the correct answer is (B).

303. Both blocks loose contact immediately after the release because the springs are different and net forces on the two blocks are also not equal. The time period of oscillations of the two blocks is given by

$$T_P = 2\pi\sqrt{\frac{m}{4K}} \text{ and } T_Q = 2\pi\sqrt{\frac{m}{K}}$$

$$\Rightarrow \quad T_Q = 2T_P$$

Now, Q comes at lowest position at time $\frac{T_Q}{2}$ after travelling a distance $\frac{2mg}{K}$ downwards. In this time $\frac{T_Q}{2}$, which is also the time period of block P, the block P will come back to original position.
So, the distance between Q and its image will be

$$x = \frac{2mg}{K} \times 2 = \frac{4mg}{K}$$

Hence, the correct answer is (B).

304. $m = m_O \times m_e = m_O\left(1+\frac{D}{f_e}\right)$

$$\Rightarrow \quad m_O = \frac{m}{\left(1+\frac{D}{f_O}\right)}$$

$$\Rightarrow \quad m_O = \frac{30}{1+5} = 5$$

Hence, the correct answer is (A).

305. $m = 1 + \frac{D}{f}$

$$\Rightarrow \quad m = 1 + \frac{25}{5} = 6$$

Hence, the correct answer is (C).

306. $m = \frac{f_o}{f_e} = 10$

$$\Rightarrow \quad f_o = 50 \text{ cm}$$

Separation $= f_o + f_e = 55$ cm

Hence, the correct answer is (C).

307. $m = \frac{f_o}{f_e} = 5$

Hence, the correct answer is (A).

308. Separation $= f_0 + f_e$

Separation $= 0.3 + 0.05 = 0.35$ m

Hence, the correct answer is (A).

310. For an equilateral prism, we have

$$A = 60°$$

Since, the ray inside the prism is parallel to its base, so we have the condition of minimum deviation. So, $i = e = 60°$ and $\delta_m = i + e - A = (60° + 60°) - 60° = 60°$

Since, $\mu = \dfrac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$$\Rightarrow \quad \mu = \frac{\sin(60°)}{\sin(30°)} = \sqrt{3}$$

Hence, the correct answer is (D).

311. $\frac{f_0}{f_e}$ must be maximum

Hence, the correct answer is (A).

313. There is no effect on the spot. Rotating the glass slab will shift a ray parallel to axis. The direction of a ray before and after the glass rotates is unaffected. All parallel rays are focussed at the focal point of the lens according to ray optics, and there is no effect on the focussed spot.
Hence, the correct answer is (A).

315. Linear magnification $m = \frac{-2 \text{ cm}}{1 \text{ cm}} = -2$

So, the image is real and inverted

$$|v| = 2|u|$$

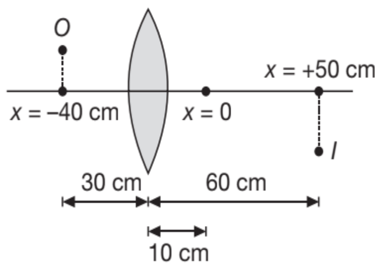
Let $|u| = x$ then $|v| = 2x$

Now $|u| + |v| = 50 - (-40) = 90$

$$\Rightarrow \quad x + 2x = 90$$

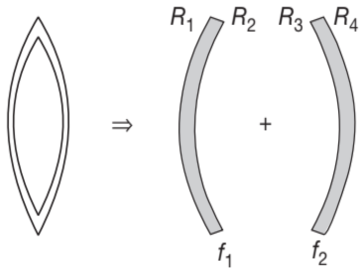
$$\Rightarrow \quad x = 30 \text{ cm}$$

So, the distance of object from the lens is 30 cm and of object is 60 cm, i.e., the lens must be located at $x = -10$ cm as shown in figure.



Hence, the correct answer is (B).

318. Hollow convex lens is as shown in figure. Applying Lens Maker's Formula, we get



Hollow glass lens

$$\frac{1}{f_1}=(\mu_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=0 \qquad \{\because R_1=R_2\}$$

$\Rightarrow \quad f_1 \to \infty$

Similarly, $f_2 \to \infty$

So, a hollow, convex lens of any material will behave like a glass plate.

Hence, the correct answer is (D).

320. If distance of image is v from lens and $u \to -\infty$, so we use

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{f}+\frac{1}{f}+\left(\frac{2}{R_1}+\frac{2}{R_2}\right)$$

Because the ray of light passes through lens thrice and reflected twice from the two spherical surfaces of lens.

$$\Rightarrow \quad \frac{1}{v}=\frac{3}{f}+2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

By Lens Maker's formula, we have

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

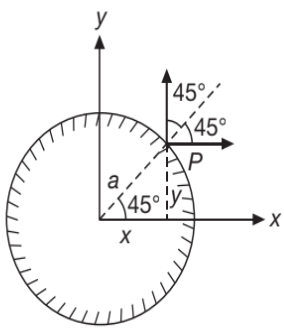
$$\Rightarrow \quad \frac{1}{v}=\left(\frac{1}{R_1}+\frac{1}{R_2}\right)(3\mu-3+2)$$

Since $\mu=1.5$

$$\Rightarrow \quad v=\frac{f(\mu-1)}{3\mu-1}=\frac{f}{7}$$

Hence, the correct answer is (A).

322. The ray diagram is as shown below



Since, $x=\dfrac{a}{\sqrt{2}}$

and $y=\dfrac{a}{\sqrt{2}}$

$$\Rightarrow \quad P \equiv \left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$$

Hence, the correct answer is (D).

324. $$-\frac{f}{f-16}=\frac{f}{f-6}$$

$\Rightarrow \quad -f+16=f-6$

$\Rightarrow \quad 2f=22$

$\Rightarrow \quad f=11 \text{ cm}$

Hence, the correct answer is (C).

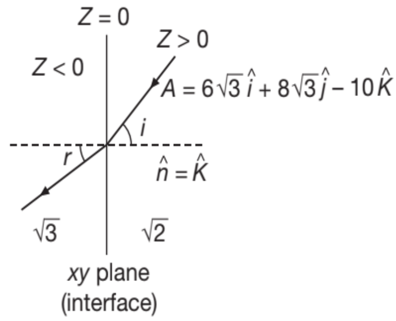
Multiple Correct Choice Type Questions

1. The normal to the interface is along $\hat{k}$.

$$\therefore \quad \cos i=\frac{\vec{A}\cdot(-\hat{k})}{|\vec{A}||\hat{k}|}=\frac{+10}{20}=+\frac{1}{2}$$

$\Rightarrow \quad i=60^\circ$

{i is the angle which incident ray makes with –Z-axis}



But according to Snell's Law

$$\sqrt{2}\sin 60=\sqrt{3}\sin r$$

$$\Rightarrow \quad \sin r=\frac{\sqrt{2}}{\sqrt{3}}\frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \sin r=\frac{1}{\sqrt{2}}$$

$\Rightarrow \quad r=45^\circ$

Hence, (B) and (C) are correct.

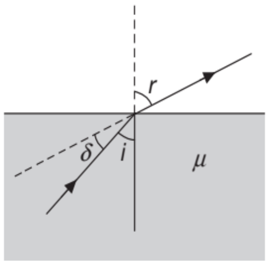
2. **The correct answer is (A).**

3. **The correct answer is (A, D).**

Combined Solution to 2 & 3

For $i<C$, no TIR will take place, so we have deviation (δ) given by

$\delta = r - i$



Now, according to Snell's Law, we have

$\mu \sin i = \sin r$

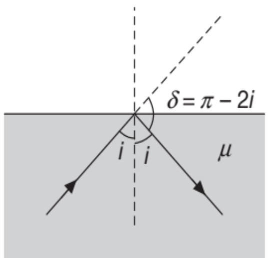
$\Rightarrow \quad r = \sin^{-1}(\mu \sin i)$

$\Rightarrow \quad \delta = \sin^{-1}(\mu \sin i) - i \qquad \ldots(1)$

This is a non-linear variation of δ with i. Also, we have δ to be maximum when $i = C$ and hence $r = \frac{\pi}{2}$. So

$\delta_{max} = \delta_1 = \frac{\pi}{2} - C \qquad \ldots(2)$

Further when $i > C$, then TIR takes place and the incident ray is reflected back in the denser medium as shown in the figure.



So, $\delta = \pi - 2i$

i.e., δ decreases linearly with i. So,

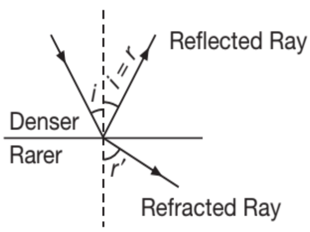
$\delta_{max} = \delta_2 = \pi - 2C \qquad \ldots(3)$

From (2) and (3), we get

$\delta_2 = 2\delta_1$

4. $\sin C = \frac{\mu_{rarer}}{\mu_{denser}} = \frac{\mu_r}{\mu_d}$

$i + r' = 90°$



According to Snell's Law

$\mu_d \sin i = \mu_r \sin(r')$

$\Rightarrow \quad \frac{\sin i}{\sin(r')} = \frac{\mu_r}{\mu_d} = \sin C$

$\Rightarrow \quad \frac{\sin i}{\sin(90 - i)} = \sin C$

$\Rightarrow \quad \sin C = \tan i$

$\Rightarrow \quad C = \sin^{-1}(\tan i)$

$\Rightarrow \quad C = \sin^{-1}(\tan r) \qquad \{\text{Because } i = r\}$

Hence, (A) and (B) are correct.

6. According to Snell's Law, we have

$\mu_1 \lambda_1 = \mu_2 \lambda_2$

$\Rightarrow \quad (1)(6000) = (1.5)\lambda_2$

$\Rightarrow \quad \lambda_2 = 4000 \text{ Å}$

Since frequency does not change when light goes from one medium to another, so

$v = \frac{c}{\lambda_1}$

$\Rightarrow \quad v = \frac{3 \times 10^8}{6000 \times 10^{-10}} = 5 \times 10^{14} \text{ Hz}$

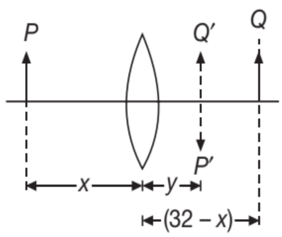
Hence, (A) and (C) are correct.

9. See Displacement Method.

Hence, (B), (C) and (D) are correct.

12. Since images are formed at the same place, So one image must be real and other must be virtual.

$|x| + |y| = 32$



For P

$\frac{1}{y} - \frac{1}{-x} = \frac{1}{15}$

$\Rightarrow \quad \frac{1}{y} + \frac{1}{x} = \frac{1}{15} \qquad \ldots(1)$

For Q

$\frac{1}{-y} - \frac{1}{-(32 - x)} = \frac{1}{15}$

$\Rightarrow \quad \frac{1}{-y}+\frac{1}{32-x}=\frac{1}{15}$ ...(2)

Adding (1) and (2), we get

$$\frac{1}{x}+\frac{1}{32-x}=\frac{2}{15}$$

$$\frac{32}{x(32-x)}=\frac{2}{15}$$

$\Rightarrow \quad 32x-x^2=240$

$\Rightarrow \quad x^2-32x+240=0$

$\Rightarrow \quad x^2-20x-12x+240=0$

$\Rightarrow \quad x(x-20)-12(x-20)=0$

$\Rightarrow \quad (x-12)(x-20)=0$

$\Rightarrow \quad x=12\text{ cm}, 20\text{ cm}$

Hence, (A) and (D) are correct.

13. $m=\frac{f-v}{f}$

$\Rightarrow \quad m=1-\frac{v}{f}$

$\Rightarrow \quad \text{Slope}=-\frac{1}{f}=\frac{b}{c}$

$\Rightarrow \quad f=\frac{c}{b}$

Also at $m=0$, $v=a$

$\Rightarrow \quad 0=f-a$

$\Rightarrow \quad f=a$

Hence, (B) and (C) are correct.

14. All are the consequences of the "DISPLACEMENT METHOD TO FIND FOCAL LENGTH OF A CONVEX LENS"

Hence, (A), (B), (C) and (D) are correct.

15. $\frac{1}{f}=\frac{1}{-15}+\frac{1}{30}$

$\Rightarrow \quad f=-30\text{ cm}$ (Diverging in nature) {OPTION (D)}

Since red deviates the least and violet deviates the maximum. So a coloured pattern with red on the outerside is observed.

Hence, (B) and (D) are correct.

16. Optical path length in passing from 1st medium is n_1s_1.

Optical path length in passing from 2nd medium is n_2s_2 and so on.

So total optical path length $=\sum_{i=1}^{m} n_i s_i$ {OPTION (C)}

$\Rightarrow \quad$ Total time of flight $t=\frac{1}{c}\sum_{i=1}^{m} n_i s_i$ {OPTION (A)}

For inhomogeneous media optical path length is $O.P.L.=\int_A^B n(s)ds$ and the ray must travel along a path in which time taken to go from A to B is minimum. Such paths are called stationary pathways and this is the statement of Fermat's Least Action Principle or Fermat's Principle of Least Time.

Hence, (A), (C) and (D) are correct.

17. The final image is formed at infinity when the combined focal length of the two lenses (in contact) is 30 cm i.e.,

$$\frac{1}{30}=\frac{1}{20}+\frac{1}{f}$$

$$f=-60\text{ cm}$$

So, when another concave lens of focal length 60 cm is kept in contact with the first lens.
Similarly, if μ be the refractive index of a liquid in which focal length of the given convex lens becomes 30 cm. Then from Lens Maker's Formula, we have

$$\frac{1}{20}=\left(\frac{3}{2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad ...(1)$$

$$\frac{1}{30}=\left(\frac{3/2}{\mu}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad ...(2)$$

From equations (1) and (2), we get

$$\mu=\frac{9}{8}$$

Hence, (A) and (D) are correct.

18. $m=+2$, means image is virtual, erect and magnified. A virtual and magnified image can be formed only by a concave mirror and that too when object lies between pole and focus.
Hence, (A) and (C) are correct.

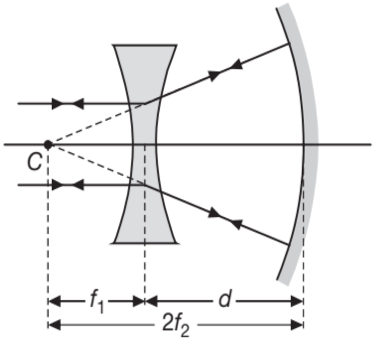
19. When passing from vacuum to a medium, frequency remains unchanged while speed and wavelength decreases μ times.

Hence, (B), (C) and (D) are correct.

20. For convex mirror (having positive focal length) the image is always smaller in size. For concave mirror (having negative focal length) the image is smaller when the object lies beyond $2f$.

Hence, (A), (B) and (C) are correct.

21.



Hence, (A) and (B) are correct.

22. The tube length of an astronomical telescope, in normal adjustment, is $(f_o + f_e)$ and that of Galilean telescope, in normal adjustment is $(f_o - f_e)$ where f_o and f_e are focal lengths of objective and eye piece respectively. In this case, $f_e = f$, so difference in tube lengths is

$$\ell_1 - \ell_2 = (f_o + f_e) - (f_o - f_e) = 2f_e = 2f$$

Hence, (B) and (C) are correct.

25. A concave mirror and a convex lens give virtual magnified image for a particular object position (i.e. when object lies between F and P (or C))

Hence, (B) and (C) are correct.

26. A concave mirror can give both real and virtual magnified images. Since nothing is specified, so

$$m = \pm 3$$

$$\Rightarrow \quad \pm 3 = \frac{-15}{-15 - u}$$

$$+u = -10 \text{ cm}$$

$$-u = -20 \text{ cm}$$

Hence, (B) and (D) are correct.

27. When the image is virtual, then

$u = -x$, so $v = +2x$

From the mirror formula, we get

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{2x} - \frac{1}{x} = \frac{1}{-f}$$

$$\Rightarrow \quad x = \frac{f}{2}$$

When the image is real, then

$u = -y$, so $v = -2y$

Again applying the mirror formula, we get

$$\frac{1}{-2y} - \frac{1}{y} = -\frac{1}{f}$$

$$\Rightarrow \quad \frac{3}{2y} = \frac{1}{f}$$

$$\Rightarrow \quad y = \frac{3}{2}f$$

Hence, (C) and (D) are correct.

28. When the object moves from infinity to the pole of the mirror, the virtual image moves from focus to the pole. **Hence, (A) and (C) are correct.**

29. Since, for refraction at a plane surface, we have

$$\frac{\mu_1}{u} = \frac{\mu_2}{v} \quad \ldots(1)$$

If x be the height of the bird above the water surface, then for the light travelling from the bird to the fish, we have

$$\mu_1 = 1,\ \mu_2 = \mu \text{ and } u = -x$$

So, from (1), we get

$$\frac{1}{(-x)} = \frac{\mu}{v}$$

$$\Rightarrow \quad v = -\mu x$$

$$\Rightarrow \quad |v| = \mu x$$

Now speed of the bird is $\frac{dx}{dt}$

So, apparent speed of the bird is

$$\left|\frac{dv}{dt}\right| = \mu \frac{dx}{dt}$$

$$\Rightarrow \quad \left|\frac{dv}{dt}\right| > \frac{dx}{dt} \quad \{\because \mu > 1\}$$

Hence, (B) and (D) are correct.

30. TIR takes place when ray of light travels from denser to rarer medium.

Further, $\sin C_{12} = \frac{\mu_2}{\mu_1}$ and $\sin C_{13} = \frac{\mu_3}{\mu_1}$

Since, $\frac{\mu_2}{\mu_1} > \frac{\mu_3}{\mu_1}$

$$C_{12} > C_{13}$$

Smaller the value of critical angle, more are the chances of TIR.

Hence, (A), (C) and (D) are correct.

31. Since, $\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}}$

For $\mu = \sqrt{2}$ and $A = 60°$, we get

$$\delta_m = 30°$$

Further, at minimum deviation, we have

$$r_1 = r_2 = \frac{A}{2} = 30°$$

Applying Snell's Law, we get

$$\sin i_1 = \mu \sin r_1$$

$$\Rightarrow \quad \sin i_1 = (\sqrt{2})\sin(30°) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad i_1 = 45°$$

Hence, (A) and (C) are correct.

32. Using, $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$, we get

$$\frac{1.5}{v} - \frac{1}{\infty} = \frac{1.5-1}{20}$$

$$\Rightarrow \quad v = +60 \text{ cm}$$

Since v is positive, the rays actually meet.

Hence, (B) and (C) are correct.

33. In air, $\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$ ...(1)

When immersed in a liquid of refractive index $\dfrac{\mu}{2}$,

$$\frac{1}{f_1} = \left(\frac{\mu}{\mu/2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f_1} = \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(2)}$$

From (1) and (2), we get

$$\frac{1}{f} = \frac{\mu - 1}{f_1}$$

$$\Rightarrow \quad f_1 = \frac{\mu - 1}{f}$$

When immersed in a liquid of refractive index 2μ,

$$\frac{1}{f_2} = \left(\frac{\mu}{2\mu} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f_2} = -\frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(3)}$$

From (1) and (3), we get

$$\frac{1}{f} = (\mu - 1)\left(-\frac{2}{f_2}\right)$$

$$\Rightarrow \quad f_2 = -\frac{2(\mu - 1)}{f}$$

Hence, the correct answer is (B).

34. When in contact, we have

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\Rightarrow \quad 10 = \frac{1}{f_1} + \frac{1}{f_2} \qquad \left\{\because P = \frac{1}{F} = 10\right\}$$

$$\Rightarrow \quad f_1 + f_2 = 10 f_1 f_2 \quad \text{...(1)}$$

When at a separation of 0.25 m, we have

$$\frac{1}{F'} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$$

$$\Rightarrow \quad 6 = \frac{1}{f_1} + \frac{1}{f_2} - \frac{0.25}{f_1 f_2}$$

$$\Rightarrow \quad 6 = 10 - \frac{0.25}{f_1 f_2}$$

$$\Rightarrow \quad f_1 f_2 = \frac{1}{16} \quad \text{...(2)}$$

From (1), we get

$$f_1 + f_2 = \frac{10}{16} = \frac{5}{8}$$

Now, from (2) we get

$$f_1\left(\frac{5}{8} - f_1\right) = \frac{1}{16}$$

$$\Rightarrow \quad 16 f_1^2 - 10 f_1 + 1 = 0$$

$$\Rightarrow \quad 16 f_1^2 - 8 f_1 - 2 f_1 + 1 = 0$$

$$\Rightarrow \quad 8 f_1 (2 f_1 - 1) - 1(2 f_1 - 1) = 0$$

$$\Rightarrow \quad (8 f_1 - 1)(2 f_1 - 1) = 0$$

$$\Rightarrow \quad f_1 = \frac{1}{8} \text{ or } f_1 = \frac{1}{2}$$

$$\Rightarrow \quad P_1 = 8D \text{ or } P_1 = 2D$$

Hence, the correct answer is (B).

35. Since, $\frac{\mu_1}{-u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}$

where $\mu_1=\mu$, $\mu_2=1$, $u=-R$, $R=-R$

$$\Rightarrow \frac{1}{v}-\frac{\mu}{(-R)}=\frac{1-\mu}{-R}$$

$$\Rightarrow \frac{1}{v}=\frac{1}{R}$$

$$\Rightarrow v=R$$

Hence, (A) and (D) are correct.

36. Real image is smaller in size if object lies beyond $2f$ and it is larger if object lies between f and $2f$.
Hence, (A) and (D) are correct.

37. A ray can pass undeviated when $\mu_1=\mu_2$ or the ray is incident normally i.e., angle of incidence is $0°$.
Hence, (A) and (C) are correct.

38. Applying Lens Maker's Formula, we get

$$\frac{1}{f_{air}}=\left(\frac{3}{2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f_{water}}=\left(\frac{3/2}{4/3}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

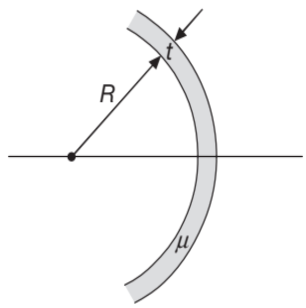
From these two equations, we get

$$f_{water}=4f_{air}=4f$$

In air, the image was inverted, real and magnified i.e., the object must be lying between f and $2f$. Now the focal length has changed to $4f$. Therefore, the object now lies between pole and focus and so the new image formed will be virtual and magnified.

Hence, (A) and (C) are correct.

40. Let t be the thickness of the watch glass, R be the radius of curvature of inner surface, so $(R+t)$ is radius of curvature of outer surface. If μ be the refractive index of this thin lens (watch glass), from Lens Maker's Formula, we get



$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

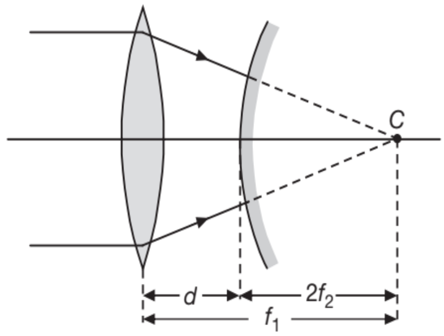
$$\Rightarrow \frac{1}{v}=\frac{1}{f}=(\mu-1)\left[\frac{1}{-R}-\frac{1}{-(R+t)}\right]$$

$$\Rightarrow \frac{1}{f}=(\mu-1)\left(\frac{1}{R+t}-\frac{1}{R}\right)=-\frac{(\mu-1)t}{R(R+t)}<0$$

So, the watch glass will have a diverging nature.

Hence, the correct answer is (C).

41.



Hence, (A) and (B) are correct.

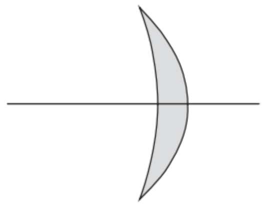
42. A concave mirror can give a virtual image (object lies between F and P) but the image is always magnified and a convex mirror can never give a real image.

Hence, (B) and (C) are correct.

43. Since, the lights used are of different colours, so they have different frequencies and hence (B) is also correct.

Hence, (A), (B) and (C) are correct.

44. $\frac{1}{f}=(\mu-1)\left(\frac{1}{-2R}-\frac{1}{-R}\right)$



$$\Rightarrow \frac{1}{f}=(\mu-1)\left(\frac{1}{R}-\frac{1}{2R}\right)$$

$$\Rightarrow \frac{1}{f}=\frac{\mu-1}{2R}$$

$$\Rightarrow f=\frac{2R}{\mu-1}$$

The focal length of lens has nothing to do with the direction from which the light is incident on it.

Hence, (B) and (C) are correct.

49. For a concave mirror, when object is placed at 2F (or C) real image of the same size as that object is formed at 2F (or C).

$$\Rightarrow \quad m_{\text{concave}} = \frac{-f}{-f-(-2f)} = -1$$

For convex mirror

$$\Rightarrow \quad m_{\text{convex}} = \frac{f}{f-(-2f)} = \frac{1}{3}$$

Hence, (B) and (C) are correct.

Reasoning Based Questions

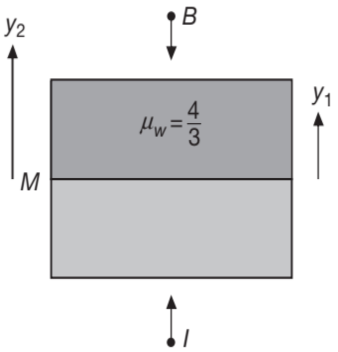
1. Lateral displacement, $\Delta x = t \sin i \left(1 - \frac{1}{\mu}\right) < t$.

Hence, the correct answer is (D).

2. Both Statements are true but Statement-2 is not the correct explanation to Statement-1.

Hence, the correct answer is (B).

6.



$$\frac{y_{I/M}}{\frac{4}{3}} = \frac{y_2 - y_1}{1} + \frac{y_1}{\frac{4}{3}}$$

$$\Rightarrow \quad y_{I/M} = y_1 + \frac{4}{3}(y_2 - y_1) \neq y_2$$

But $y_{I/B} = 2\left[\frac{3y_1}{4} + (y_2 - y_1)\right]$

$$\Rightarrow \quad \frac{dy_{I/B}}{dt} = 2\frac{dy_2}{dt} = 2v_0$$

Hence, the correct answer is (C).

7. When the object is virtual, a real image can be formed by a plane or convex mirror.

Hence, the correct answer is (D).

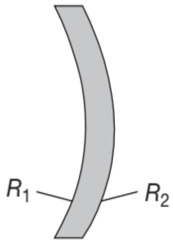
8. Both Statements are correct but Statement-2 is not correct explanation of Statement-1.

Hence, the correct answer is (B).

10. For a mirror, $m = \frac{f}{f-u}$.

Hence, the correct answer is (D).

11.



According to Lens Maker's Formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Since, $R_1 = R_2$

$$\Rightarrow \quad \frac{1}{f} = 0$$

So, power $= \frac{1}{f} = 0$

Hence, the correct answer is (A).

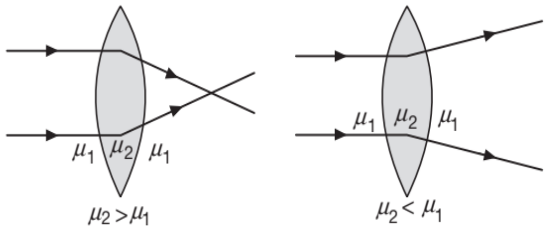
16. When light goes from one medium to another, its frequency remains unchanged.

Hence, the correct answer is (B).

19. Both the Statements are true, and Statement-2 is the correct explanation to Statement-1.

Hence, the correct answer is (A).

20. Statement-1 is true & Statement-2 is false.



Hence, the correct answer is (C).

21. $A = 60^\circ$

$\mu = \sqrt{2}$

$$i = \frac{A + D_m}{2} = \frac{60 + 30}{2} = 45^\circ$$

Since, $r_2 = \frac{A}{2} = 30^\circ$

So, According to Snell's Law, we have

$$\frac{\sin i}{\sin r} = \frac{\sin(45^\circ)}{\sin(30^\circ)} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \sqrt{2}$$

Both Statement-1 and Statement-2 are true but Statement-2 is not the correct explanation of Statement-1.

Hence, the correct answer is (B).

22. Both Statements are true and Statement-2 is correct explanation to Statement-1.

Hence, the correct answer is (A).

23. Both Statements are true and Statement-2 is correct explanation of Statement-1.

Hence, the correct answer is (B).

24. Statement-1 false and Statement-2 is true.

Hence, the correct answer is (D).

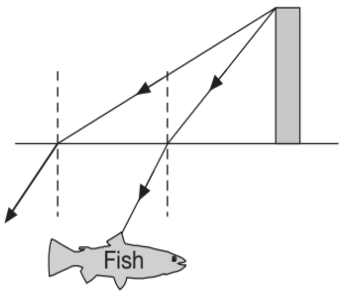
25. When the object is virtual, the convex mirror will give a real image.

Hence, the correct answer is (D).

26. After refraction at two parallel faces of a glass slab, a ray of light emerges in a direction parallel to the direction of incidence of white light on the slab. As rays of all colours emerge in the same direction (of incidence of white light), hence there is no dispersion, but only lateral displacement.

Hence, the correct answer is (B).

28.



Hence, the correct answer is (C).

29. Angle of incidence at any location should be greater than critical angle.

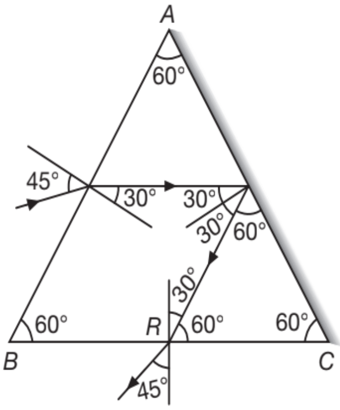
Hence, the correct answer is (C).

30. In search light, we need intense parallel beam of light. When source of light is placed at focus of concave mirror, only paraxial rays are rendered parallel due to large aperture of mirror. Marginal rays give a divergent beam but in case of parabolic mirror, when source is at focus, beam of light produced over the entire cross-section of mirror is a parallel beam.

Hence, the correct answer is (C).

Linked Comprehension Type Questions

1. Different angles by geometry and the given conditions are shown in figure.



$$\mu = \frac{\sin 45°}{\sin 30°} = \sqrt{2}$$

Hence, the correct answer is (B).

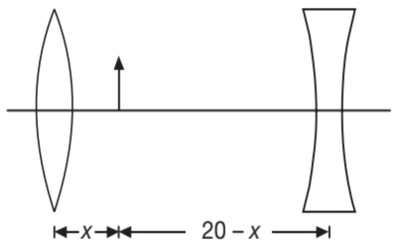
2. $S_{Total} = S_P + S_Q + S_R$

$$S_{Total} = (45° - 30°) + (180° - 2 \times 30°) + (45° - 30°)$$

$$\Rightarrow \quad S_{Total} = 150°$$

Hence, the correct answer is (C).

3. Image formed by concave lens is virtual for all positions of object i.e., image by concave lens lies between the two lenses. For both the images to coincide, image by convex lens should also lie in between the two lenses the two lens or image by convex lens should also be virtual.



For convex lens:

$$\frac{1}{-v} - \frac{1}{-x} = \frac{1}{20}$$

$$\Rightarrow \quad \frac{1}{x} - \frac{1}{v} = \frac{1}{20} \qquad \ldots(1)$$

For concave lens:

$$\frac{1}{-(20-v)} - \frac{1}{-(20-x)} = \frac{1}{-10}$$

$$\Rightarrow \quad \frac{1}{20-x} - \frac{1}{20-v} = \frac{1}{-10}$$

Solving this equation, we get

$$x = \frac{20(\sqrt{3}-1)}{\sqrt{3}} \text{ cm} \quad \ldots(2)$$

$\Rightarrow \quad v = 20(\sqrt{3}-1)$ cm

Hence, the correct answer is (B).

4. Magnification by convex lens is given by

$$m_1 = \frac{v_1}{u_1} = \frac{-v}{-x} = \frac{v}{x} = \frac{20(\sqrt{3}-1)}{20(\sqrt{3}-1)/\sqrt{3}} = \sqrt{3}$$

Magnification by concave lens is given by

$$m_2 = \frac{v_2}{u_2} = \frac{-(20-v)}{-(20-x)} = \frac{20-v}{20-x}$$

$$m_2 = \frac{40-20\sqrt{3}}{20-\frac{20(\sqrt{3}-1)}{\sqrt{3}}} = (2\sqrt{3}-3)$$

Hence, the correct answer is (D).

8. Since refractive index decreases with increase of λ and velocity decreases with increase of refractive index, so we have

$$v_{\text{red}} > v_{\text{orange}} > v_{\text{yellow}}$$

Hence, the correct answer is (A).

9. Colour and frequency remain unchanged as it is property of source.
Hence, the correct answer is (D).

10. Dispersion depends on wavelength
Hence, the correct answer is (B).

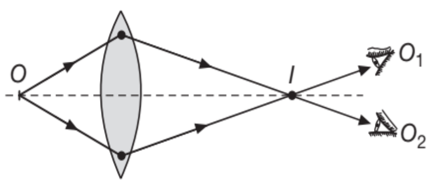
11. Since light takes shortest path, so

$$t = \frac{nx_0}{c}$$

Hence, the correct answer is (D).

12. Speed of light changes with refractive index.
Hence, the correct answer is (C).

13. Since, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$



$\Rightarrow \quad \frac{1}{v} + \frac{1}{20} = \frac{1}{15}$

$\Rightarrow \quad \frac{1}{v} = \frac{1}{15} - \frac{1}{20}$

$\Rightarrow \quad \frac{1}{v} = \frac{1}{60}$

$\Rightarrow \quad v = 60$ cm

Hence, the correct answer is (D).

14. Observer O_1 cannot see the image because light will be absorbed by the blackened portion of the lens.
Hence, the correct answer is (A).

16. According to Snell's Law, we have

$$\frac{\sin i}{\sin r} = \frac{\mu_1}{\mu_2} = K \; i = 60°$$

$\Rightarrow \quad \sin(60°) = K_1 \; r = 90°$

$\Rightarrow \quad K_1 = \frac{\sqrt{3}}{2}$

Hence, the correct answer is (D).

17. $\theta_1 = \frac{\pi}{6}$

Hence, the correct answer is (C).

18. Since, $\theta = 0°$

$\Rightarrow \quad i = r$

$\Rightarrow \quad K_2 = 1$

Hence, the correct answer is (A).

19. Since, $f = R - \frac{R}{2}\sec\theta$

So, for $\theta > 0$, $f < \frac{R}{2}$

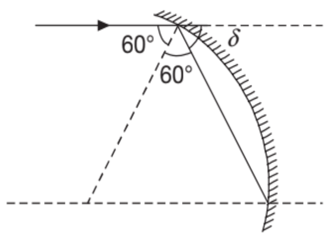
$\Rightarrow \quad f_p < f_m$

Hence, the correct answer is (C).

20. $f_m = R - \frac{R}{2}\sec(60°) = 0$

Hence, the correct answer is (D).

21.



$$\delta = 180 - 120 = 60°$$

Laws of reflection is universally true

Hence, the correct answer is (A).

22. $A = \sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + (-10)^2}$

$A = 20$ units

Hence, the correct answer is (D).

23. $\cos i = \dfrac{\vec{A}\cdot(-\hat{k})}{|\vec{A}|} = \dfrac{\left(6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\hat{k}\right)\cdot(-\hat{k})}{20}$

$\cos i = \dfrac{10}{20} = \dfrac{1}{2};\ i = 60^\circ$

Hence, the correct answer is (C).

24. $\dfrac{\sin i}{\sin r} = \dfrac{\sqrt{3}}{\sqrt{2}},\ \sin r = \dfrac{\sqrt{2}}{\sqrt{3}}\sin i$

$\dfrac{\sin i}{\sin r} = \dfrac{\sqrt{2}}{\sqrt{3}}\sin 60^\circ = \dfrac{\sqrt{2}}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

$\Rightarrow\ r = 45^\circ$

Hence, the correct answer is (B).

25 Let the vector representing the refracted ray be

$\vec{A}' = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} + C\hat{k}$

Since, $\cos r = \dfrac{\vec{A}'\cdot(-\hat{k})}{|\vec{A}'|}$

$\Rightarrow\ \cos r = \dfrac{\left(6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} + C\hat{k}\right)\cdot(-\hat{k})}{\sqrt{\left(6\sqrt{3}\right)^2 + \left(8\sqrt{3}\right)^2 + C^2}}$

$\Rightarrow\ \cos r = \dfrac{-C}{\sqrt{108 + 192 + C^2}}$

$\Rightarrow\ \cos 45^\circ = \dfrac{-C}{\sqrt{300 + C^2}};\ \dfrac{1}{\sqrt{2}} = \dfrac{-C}{\sqrt{300 + C^2}}$

$\Rightarrow\ C = \pm 10\sqrt{3}$

Since the refracted ray travels downwards,

$\Rightarrow\ C = -10\sqrt{3}$

$\Rightarrow\ \vec{A}' = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\sqrt{3}\hat{k}$

Hence, the correct answer is (D).

26. $|A'| = \sqrt{\left(6\sqrt{3}\right)^2 + \left(8\sqrt{3}\right)^2 + \left(-10\sqrt{3}\right)^2} = 10\sqrt{6}$

Hence, the correct answer is (C).

27. $\hat{n} = \dfrac{\vec{A}'}{|\vec{A}'|} = \dfrac{3}{5\sqrt{2}}\hat{i} + \dfrac{4}{5\sqrt{2}}\hat{j} - \dfrac{1}{\sqrt{2}}\hat{k}$

Hence, the correct answer is (A).

28. Optical path length $(OPL) = \int n\,dx$

$\Rightarrow\ OPL = \int_0^1 (1 + x^2)\,dx$

$\Rightarrow\ OPL = \left.\left(x + \dfrac{x^3}{3}\right)\right|_0^1 = \dfrac{4}{3}$ m

Hence, the correct answer is (A).

31. $P_{\text{eq}} = 2P_{\text{lens}} + P_{\text{mirror}}$

where $P_{\text{lens}} = \dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

and $P_{\text{mirror}} = -\dfrac{1}{f} = -\dfrac{2}{R}$

$\dfrac{1}{f_1} = 2(\mu - 1)\left(\dfrac{1}{R} - \dfrac{1}{(-R)}\right) - \dfrac{2}{(-R)}$

$\Rightarrow\ \dfrac{1}{f_1} = \dfrac{4(\mu - 1)}{R} + \dfrac{2}{R}$

$\Rightarrow\ \dfrac{1}{f_1} = \dfrac{4\mu}{R} - \dfrac{4}{R} + \dfrac{2}{R}$

$\Rightarrow\ \dfrac{1}{f_1} = \dfrac{4\mu}{R} - \dfrac{2}{R}$

$\Rightarrow\ \dfrac{1}{f_1} = \dfrac{2}{R}(2\mu - 1)$

$\Rightarrow\ f_1 = \dfrac{R}{2(2\mu - 1)}$

Hence, the correct answer is (C).

32. For a plano-convex lens, we have

$\dfrac{1}{f_2} = (\mu - 1)\left(\dfrac{1}{R} - \dfrac{1}{\infty}\right)$

$\Rightarrow\ \dfrac{1}{f_2} = \dfrac{\mu - 1}{R}$

$\Rightarrow\ f_2 = \dfrac{R}{\mu - 1}$

Hence, the correct answer is (C).

33. When plane surface is silvered, then

$\dfrac{1}{f_3} = \dfrac{2}{f_\ell} - \dfrac{1}{\infty}$ $\quad \{\because f_m \to \infty\}$

$\dfrac{1}{f_3} = \dfrac{2(\mu - 1)}{R}$

$\Rightarrow\ f_3 = \dfrac{R}{2(\mu - 1)}$

Hence, the correct answer is (B).

34. When curved surface is silvered, then

$$\frac{1}{f_4}=\frac{2}{f_\ell}-\frac{1}{f_m}$$

$$\Rightarrow \quad \frac{1}{f_4}=2(\mu-1)\left(\frac{1}{\infty}-\frac{1}{-R}\right)-\frac{2}{(-R)}$$

$$\Rightarrow \quad \frac{1}{f_4}=\frac{2(\mu-1)}{R}+\frac{2}{R}$$

$$\Rightarrow \quad \frac{1}{f_4}=\frac{2\mu}{R}$$

$$\Rightarrow \quad f_4=\frac{R}{2\mu}$$

Hence, the correct answer is (D).

35. Focal length of lens L_1 and L_2 is given by using the Lens Maker's Formula, so

$$\frac{1}{f_1}=(\mu_1-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)$$

$$\Rightarrow \quad f_1=50 \text{ cm}$$

Similarly, $\frac{1}{f_2}=(\mu_2-1)\left(\frac{1}{\infty}-\frac{1}{(-R)}\right)$

$$\Rightarrow \quad f=40 \text{ cm}$$

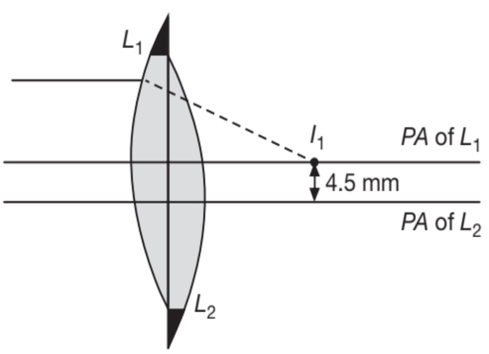
So, equivalent focal length is given by

$$\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$\Rightarrow \quad f_{eq}=\frac{200}{9} \text{ cm}$$

Hence, the correct answer is (D).

36. Image formed by L_1 lies 50 cm behind it and on principal axis of L_1. This will act as an object for L_2. So, for L_2,



we have

$$u=+50 \text{ cm},\ f_2=+40 \text{ cm}$$

Applying Len's Formula, we get

$$\frac{1}{v}-\frac{1}{50}=\frac{1}{40}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{50}+\frac{1}{40}$$

$$\Rightarrow \quad v=\frac{200}{9} \text{ cm}$$

Magnification produced by L_2 is $m_2=\frac{v}{u}=\frac{4}{9}$

However, for L_2, the image I_1 is at a distance of 4.5 mm above its principal axis i.e., (PA of L_2). So, distance of image I_2 from PA of L_2 is

$$y=m_2(4.5 \text{ mm})$$

$$\Rightarrow \quad y=\frac{4}{9}(4.5 \text{ mm})=2 \text{ mm}$$

Hence I_2 is at a distance of $(4.5-2)$ mm $=2.5$ mm from PA of L_1.

Hence, the correct answer is (C).

37. Applying refraction at curved surface formula, i.e.,

$$\frac{\mu_1}{-u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}, \text{ we get}$$

$$\frac{1}{x}+\frac{2}{v_1}=\frac{1}{R}$$

$$\Rightarrow \quad v_1=\frac{2xR}{x-R} \qquad \{\text{from pole of curved surface}\}$$

This image formed will be at a distance $v'=(v_1-R)$ from the plane surface. Now again applying the above formula, but at the plane surface, we get

$$\frac{1}{v}-\frac{2}{v'}=0 \qquad \{\because \text{ for plane surface } R\to\infty\}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{2}{v_1-R}$$

Substituting $v_1=\frac{2xR}{x-R}$, we get

$$\frac{1}{v}=\frac{2(x-R)}{R(x+R)}$$

For the image to be virtual, we have

$$\frac{1}{v}<0$$

$$\Rightarrow \quad \frac{2(x-R)}{R(x+R)}<0$$

$\Rightarrow \quad x < R$

This condition is only satisfied by $x = \frac{R}{3}$.

Hence, the correct answer is (C).

38. Since, $v_1 = \frac{(2x)R}{x - R}$

So, for $x = 2R$, we get

$v_1 = 4R$

Also, $u = -2R$

Since, $m_1 = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{v}{u}\right)$

$\Rightarrow \quad m_1 = \left(\frac{1}{2}\right)\left(\frac{4R}{-2R}\right)$

$\Rightarrow \quad m_1 = -1$

However, for plane mirror, we have

$m_2 = 1$

$m_{net} = m_1 m_2 = -1$

Since, $m_{net} = \ominus$, so the final image formed is real, inverted and of same size.

Hence, the correct answer is (D).

39. $\frac{\sin(90°)}{\sin r} = 2$

$\Rightarrow \quad \sin r = \frac{1}{2}$

$\Rightarrow \quad r = 30°$

In triangle OAC, we have

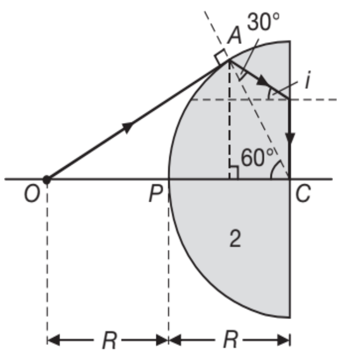
$OC = 2R$, $AC = R$, so, by Pythagora's Theorem, we get

$OA = \sqrt{3}R$

$\Rightarrow \quad \angle ACP = 60°$

Using geometry, we get

$i = 30°$



Since, refractive index of this is 2, so the critical angle C is given by

$C = \sin^{-1}\left(\frac{1}{2}\right) = 30°$

Since, $i = C$, so the ray grazes the plane surface.

Hence, the correct answer is (C).

40. Since yellow is the mean colour, so

$\mu_y = \frac{\mu_b + \mu_r}{2} = \frac{1.51 + 1.49}{2} = 1.50$

Hence, the correct answer is (C).

41. Again yellow is the mean colour, so

$\mu'_y = \frac{\mu'_b + \mu'_r}{2} = \frac{1.77 + 1.73}{2} = 1.75$

Hence, the correct answer is (D).

42. For no deviation to take place, deviation (given by $\delta = (\mu - 1)A$ by one must be cancelling the deviation due to the other. For this

(a) the prisms must be arranged upside down

(b) $\frac{A'}{A} = -\left[\frac{\mu_y - 1}{\mu'_y - 1}\right]$

$\Rightarrow \quad A' = -\left(\frac{1.50 - 1}{1.75 - 1}\right)6° = -4°$

Hence, the correct answer is (D).

43. Net dispersion $= (\delta_b - \delta_r) + (\delta'_b - \delta'_r)$

$= (\mu_b - \mu_r)A + (\mu'_b - \mu'_r)A'$

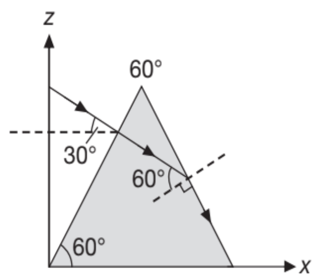
$= (1.51 - 1.49)6° - (1.77 - 1.73)4°$

$= 0.02 \times 6° - 0.04 \times 4°$

$= 0.12° - 0.16° = -0.04°$

Hence, the correct answer is (D).

44.



Since, $\sqrt{3}z + x = 10$

$\Rightarrow \quad z = \frac{-x}{\sqrt{3}} + \frac{10}{\sqrt{3}}$

$\Rightarrow \quad \text{Slope} = \frac{-1}{\sqrt{3}}$

$\Rightarrow \quad \tan\theta = \frac{-1}{\sqrt{3}}$

CHAPTER 1

$\Rightarrow \quad \theta = -30^\circ$

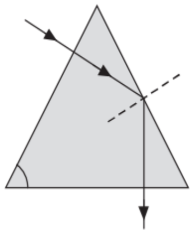
So, the ray is incident normally on the face AB. Thus, angle of incidence on face AC is 60°.
For grazing the face AC, we have

$$\mu \sin(60^\circ) = 1 \times \sin(90^\circ)$$

$$\Rightarrow \quad \mu = \frac{2}{\sqrt{3}}$$

Hence, the correct answer is (C).

45. For $\mu = \frac{3}{2}$ the ray will be internally reflected, so, the ray is normal at face BC. Hence finally, the refracted ray is parallel to z-axis.



Hence, the correct answer is (B).

46. If BC is silvered, the ray will retrace its path. Hence equation of ray coming out of the prism is $\sqrt{3}z + x = 10$.
Hence, the correct answer is (B).

47. The objective lens must form a real image for eyepiece to magnify it.
Hence, the correct answer is (C).

48. The image formed by the objective must lie within the focus of the eyepiece.
Hence, the correct answer is (A).

49. To obtain best magnification, the object must be placed just beyond the focus of the objective lens. In this case, the first image distance from the objective is very large.
Hence, the correct answer is (A).

50. $i + \theta = 90^\circ$, $\theta = 90^\circ - i$,

Slope of tangent $= \tan\theta = \tan(90^\circ - i) = \cot i$

Hence, the correct answer is (C).

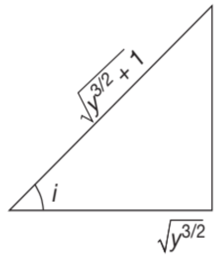
51. But $\tan\theta = \frac{dy}{dx}$

$$\therefore \quad \frac{dy}{dx} = \cot i \qquad \ldots(1)$$

Applying Snell's Law at A and B

$$n_A \sin i_A = n_B \sin i_B$$

$$n_A = 1 \text{ because } y = 0$$



$$\sin i_A = 1 \text{ because } i_A = 90^\circ \quad (\text{Grazing incidence})$$

$$n_B = \sqrt{Ky^{3/2} + 1} = \sqrt{y^{3/2} + 1}$$

because $K = 1.0(m)^{-3/2}$

$$\therefore \quad (1)(1) = \sqrt{(y^{3/2} + 1)} \sin i$$

$$\Rightarrow \quad \sin i = \frac{1}{\sqrt{y^{3/2} + 1}}$$

$$\therefore \quad \cot i = \sqrt{y^{3/2}} \text{ or } y^{3/4} \qquad \ldots(2)$$

Equating equations (1) and (2), we get

$$\frac{dy}{dx} = y^{3/4}$$

$$\Rightarrow \quad y^{-3/4} dy = dx$$

$$\Rightarrow \quad \int_0^y y^{-3/4} dy = \int_0^x dx$$

$$\Rightarrow \quad 4y^{1/4} = x \qquad \ldots(3)$$

The required equation of trajectory is $4y^{1/4} = x$

$$\Rightarrow \quad y = \frac{x^4}{4^4} = \frac{x^4}{256}$$

Hence, the correct answer is (D).

52. At the point of intersection on the upper surface,

$$y = 1 \text{ m}$$

$$\Rightarrow \quad x = (256)^{1/4} = 4 \text{ m}$$

So the co-ordinates are $(4 \text{ m}, 1 \text{ m})$

Hence, the correct answer is (D).

53. As $n_A \sin i_A = n_P \sin i_P$ and as $n_A = n_P = 1$

Therefore, $i_P = i_A = 90^o$ i.e., the ray will emerge parallel to the boundary at P i.e., at grazing emergence.

Hence, the correct answer is (A).

54. $-\frac{1}{F} = \frac{2}{f_\ell} - \frac{1}{f_m}$

$$\Rightarrow \quad \frac{1}{-F} = 2(1.5-1)\left(\frac{1}{20} - \frac{1}{60}\right) - \frac{2}{(-20)}$$

$$\Rightarrow \frac{1}{-F}=\frac{1}{20}-\frac{1}{60}+\frac{2}{20}=\frac{8}{60}$$

$$\Rightarrow F=-\frac{60}{8}=-7.5 \text{ cm}$$

Hence, the correct answer is (C).

55. Since $F=\ominus$, so combination behaves like a concave mirror.

Hence, the correct answer is (B).

56. $m=\frac{f}{f-u}=\frac{-7.5}{-7.5+30}=\frac{-7.5}{22.5}=-\frac{1}{3}$

Hence, the correct answer is (A).

57. **The correct answer is (C).**

58. **The correct answer is (B).**

59. **The correct answer is (B).**

Combined solution of 57, 58, & 59

Each part will work as a separate lens and will form its own image. For any part, we have $u=-0.3$ m, $f=+0.2$ m.

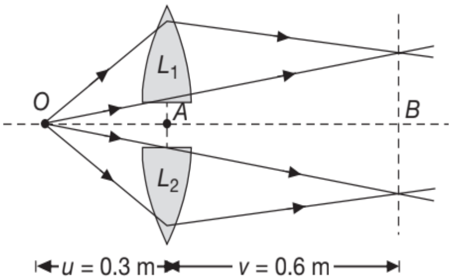
Therefore, from lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{v}-\frac{1}{0.3}=\frac{1}{0.2}$$

$$\Rightarrow v=0.6 \text{ m}$$

So, each part forms a real image of the point object O at 0.6 m from the lens, as shown in figure.



Since the triangles OL_1L_2 and OI_1I_2 are similar. So, we have

$$\frac{I_1I_2}{L_1L_2}=\frac{OB}{OA}=\frac{u+v}{u}$$

$$\Rightarrow \frac{I_1I_2}{L_1L_2}=\frac{0.3+0.6}{0.3}=\frac{0.9}{0.3}=3$$

$$\Rightarrow I_1I_2=3(L_1L_2)=3(2\times 0.0005)=0.003 \text{ m}$$

60. $\frac{1}{f_1}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$\frac{1}{f_1}=\left(\frac{3}{2}-1\right)\left(\frac{1}{R}-\frac{1}{-R}\right)=\frac{1}{2}\times\frac{2}{R}=\frac{1}{R}$$

$$\Rightarrow f_1=R$$

Hence, the correct answer is (A).

61. $\frac{1}{f_2}=\left(\frac{4}{3}-1\right)\left(\frac{1}{-R}-\frac{1}{\infty}\right)=-\frac{1}{3R}$

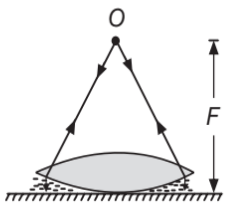
$$\Rightarrow f_2=-3R$$

Hence, the correct answer is (C).

62. Now, $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{R}-\frac{1}{3R}=\frac{2}{3R}$

$$\Rightarrow F=\frac{3R}{2}$$

Since, image coincides with the object so, clearly rays of light must have retraced their path after reflection. This is possible only when rays of light must have fallen normally on the plane mirror. For this, the object is at the focus of the lens system.



$$\Rightarrow F=\text{distance of object}=15 \text{ cm}$$

$$\Rightarrow \frac{3R}{2}=15 \text{ cm}$$

$$\Rightarrow R=10 \text{ cm}$$

Hence, the correct answer is (C).

63 If f is the focal length of the liquid concave lens, then

$$\frac{1}{25}=\frac{1}{10}+\frac{1}{f} \text{ or } \frac{1}{f}=\frac{-3}{50}$$

$$\Rightarrow f=-\frac{50}{3} \text{ cm}$$

Hence, the correct answer is (C).

64. Using Lens-Maker's Formula for the liquid concave lens,

$$\frac{1}{-\frac{50}{3}}=(\mu-1)\left(\frac{1}{-R}-\frac{1}{\infty}\right)=-\frac{\mu-1}{R}=-\frac{\mu-1}{10}$$

$\Rightarrow \quad \dfrac{3}{50} = \dfrac{\mu - 1}{10}$

$\Rightarrow \quad \mu = 1 + 0.6 = 1.6$

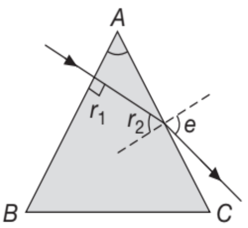
Hence, the correct answer is (D).

65. Since the ray is incident normally, so

$r_1 = 0$

Further, $r_1 + r_2 = A = 60°$

$\Rightarrow \quad r_2 = A = 60°$



Applying Snell's Law at AC, we get

$\mu \sin r_2 = \mu_1 \sin e$

$\Rightarrow \quad \mu_1 \sin e = \dfrac{4}{\sqrt{3}} \times \sin 60°$

$\Rightarrow \quad \sin e = \dfrac{4}{\sqrt{3}\mu_1} \times \dfrac{\sqrt{3}}{2} = \dfrac{2\sqrt{3}}{\sqrt{3}} \times \dfrac{1}{\mu_1} = \dfrac{2}{\mu_1} \qquad \ldots(1)$

Further, deviation is given by

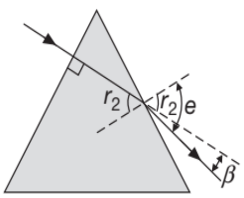
$\beta = e - r_2$

From the graph, we observe that when $\mu_1 = k_2$, $\beta = 0$

So, $\beta = e - r_2 = 0$

$\Rightarrow \quad e = r_2 = 60°$

So, equation (1) becomes



$\sin(60°) = \dfrac{2}{\mu_1}$

$\Rightarrow \quad \dfrac{\sqrt{3}}{2} = \dfrac{2}{\mu_1}$

$\Rightarrow \quad \mu_1 = \dfrac{4}{\sqrt{3}}$

$\Rightarrow \quad k_2 = \dfrac{4}{\sqrt{3}}$

Hence, the correct answer is (B).

66. When $\mu_1 < k_1$ light will not emerge, it can be seen from graph, so we have

$r_2 = C$ when $\mu_1 = k_1$

$\Rightarrow \quad \sin r_2 = \sin C = \dfrac{\mu}{\mu_1}$

$\Rightarrow \quad \sin(60°) = \dfrac{\mu}{\mu_1} \qquad \{\because r_2 = 60°\}$

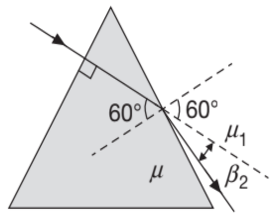
$\Rightarrow \quad \dfrac{\sqrt{3}}{2} = \dfrac{4}{\sqrt{3}} \times \dfrac{1}{\mu_1}$

$\Rightarrow \quad \mu_1 = \dfrac{8}{3}$

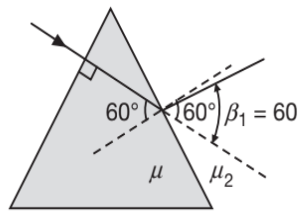
$\Rightarrow \quad k_1 = \dfrac{8}{3}$

Hence, the correct answer is (C).

67. From the graph, we observe that β_1 is the maximum deviation and β_2 is the minimum deviation



So, $\beta_2 = \dfrac{\pi}{2} - 60° = 30°$ and $\beta_1 = 60°$



$\Rightarrow \quad \beta_1 - \beta_2 = 60° - 30° = 30°$

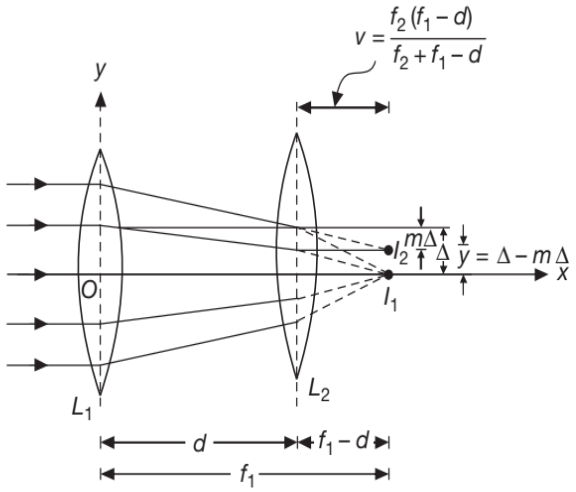
Hence, the correct answer is (B).

68. From the first lens parallel beam of light is focussed at its focus i.e., at a distance f_1 from it. This image I_1 acts as virtual object for second lens L_2. Therefore, for L_2

$u = +(f_1 - d)$, $f = +f_2$

$\Rightarrow \quad \dfrac{1}{v} = \dfrac{1}{f} + \dfrac{1}{u} = \dfrac{1}{f_2} + \dfrac{1}{f_1 - d}$

$\Rightarrow \quad v = \dfrac{f_2(f_1 - d)}{f_2 + f_1 - d}$



Therefore, x-coordinate of its focal point will be

$$x = d + v = d + \frac{f_2(f_1 - d)}{f_2 + f_1 - d}$$

$$\Rightarrow \quad x = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$$

Hence, the correct answer is (A).

69. Linear magnification for L_2, is given by

$$m = \frac{v}{u} = \left(\frac{f_2(f_1 - d)}{f_2 + f_1 - d}\right)\left(\frac{1}{f_1 - d}\right) = \frac{f_2}{f_2 + f_1 - d}$$

Therefore, second image will be formed at a distance of $m\Delta$ or $\left(\frac{f_2}{f_2 + f_1 - d}\right)(\Delta)$ below its optic axis.

Therefore, y-coordinate of the focus of system is given by

$$y = \Delta - \left(\frac{f_2 \Delta}{f_2 + f_1 - d}\right)$$

$$\Rightarrow \quad y = \frac{(f_1 - d)\Delta}{f_2 + f_1 - d}$$

Hence, the correct answer is (A).

72. For both the halves, position of object and image is same, however the only difference is of magnification. Magnification for one of the halves is given as $2(>1)$. This can be for the first one, because for this, $|v| > |u|$. Therefore, magnification, $|m| = \left|\frac{v}{u}\right| > 1$. So, for the first half, we have

$$\left|\frac{v}{u}\right| = 2$$

$$\Rightarrow \quad |v| = 2|u|$$

Let $u = -x$, then $v = +2x$

and $|u| + |v| = 1.8$ m

$\Rightarrow \quad 3x = 1.8$ m

$\Rightarrow \quad x = 0.6$ m

Hence, $u = -0.6$ m and $v = +1.2$ m

Using $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{1.2} - \frac{1}{-0.6} = \frac{1}{0.4}$

$\Rightarrow \quad f = 0.4$ m

For the second half, we have

$$\frac{1}{f} = \frac{1}{1.2 - d} + \frac{1}{-(0.6 + d)}$$

$$\Rightarrow \quad \frac{1}{0.4} = \frac{1}{1.2 - d} + \frac{1}{(0.6 + d)}$$

Solving this, we get $d = 0.6$ m

Magnification for the second half will be

$$m_2 = \frac{v}{u} = \frac{0.6}{-(1.2)} = -\frac{1}{2}$$

and magnification for the first half is

$$m_1 = \frac{v}{u} = \frac{1.2}{-(0.6)} = -2$$

Hence, the correct answer is (D).

73. For direct observation, the eye E appears to be farther that it actually is. So,

$$x = \mu H + \frac{H}{2} = H\left(\mu + \frac{1}{2}\right)$$

Hence, the correct answer is (A).

74. The distance of eye from mirror is

$$\mu H + H$$

So, distance of eye as seen by the fish in the mirror is

$$x = (\mu H + H) + \frac{H}{2}$$

$$\Rightarrow \quad x = \frac{3H}{2} + \mu H$$

$$\Rightarrow \quad x = H\left(\mu + \frac{3}{2}\right)$$

Hence, the correct answer is (B).

75. For direct observation by the eye E, the fish appears to be closer than it actually is. So, we have

$$x = H + \frac{H}{2\mu}$$

$$\Rightarrow \quad x = H\left(1 + \frac{1}{2\mu}\right)$$

Hence, the correct answer is (C).

76. The distance of the mirror as observed by the eye is $H+\dfrac{H}{\mu}$. So, the distance of the fish as seen by the eye in the mirror is

$$x = H+\frac{H}{\mu}+\frac{H}{2\mu}$$

$$\Rightarrow \quad x = H\left(1+\frac{3}{2\mu}\right)$$

Hence, the correct answer is (D).

Matrix Match/Column Match Type Questions

1. A → (r, s)
B → (p, q, r, s, t)
C → (q, r, s, t)
D → (p, r, s)

(A) $m<0$, means real image, possible for concave mirror and convex lens. So, (A) → (r, s).

(B) $m>0$, means virtual image, possible for all i.e., plane mirror, convex mirror (always), concave mirror (when object lies between focus and pole), concave lens (always) and convex lens (when object lies between optical centre and focus). So, (B) → (p, q, r, s, t).

(C) $|m|<1$, means diminished image, possible for concave mirror, convex lens, convex mirror and concave lens. So, (C) → (q, r, s, t).

(D) $|m|\geq 1$, means magnified (>1) and same sized $(=1)$ image, possible for concave mirror/convex lens (both $|m|>1$) and plane mirror $(|m|=1)$. So, (D) → (p, r, s).

2. A → (s, t)
B → (p, t)
C → (s, t)
D → (q, t)

$$\vec{v}_A = \hat{i}+\vec{a}t = \hat{i}+\left(2\hat{i}+\hat{j}\right)(2) = 5\hat{i}+2\hat{j}$$

$$\Rightarrow \quad \vec{v}_{A'} = -5\hat{i}+2\hat{j}$$

$$\Rightarrow \quad \vec{v}_{A'A} = \vec{v}_{A'}-\vec{v}_A = -10\hat{i}$$

Similarly, $\vec{v}_B = \left(-\hat{i}+3\hat{j}\right)$

$$\Rightarrow \quad \vec{v}_{B'} = \left(\hat{i}+3\hat{j}\right)$$

$$\Rightarrow \quad \vec{v}_{B'B} = \vec{v}_{B'}-\vec{v}_B = 2\hat{i}$$

4. A → (r)
B → (q)
C → (p)
D → (p)

(A) Velocity of fish in air $=8\times\dfrac{3}{4}=6\uparrow$

Velocity of fish w.r.t. bird $=6+6=12\uparrow$

(B) Velocity of image of fish after reflection from mirror in air $=8\times\dfrac{3}{4}=6\downarrow$

w.r.t. bird $=-6+6=0$

(C) Velocity of bird as seen from water $=6\times\dfrac{4}{3}=8\downarrow$

Velocity of bird w.r.t. fish $=8+8=16\downarrow$

(D) Velocity of bird in water after reflection from mirror $=8\downarrow$

w.r.t. fish $=8-8=0$

11. A → (p, s)
B → (p, q, r, s)
C → (p, q, r, s)
D → (q, s)

(A) For convex mirror:

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$

$$\Rightarrow \quad v=\left(\frac{uf}{u-f}\right)$$

Now, v may be positive and negative, depending on values of u and f.

Since, $|m|=\left|\dfrac{v}{u}\right|=\left(\dfrac{f}{u-f}\right)=\left(\dfrac{1}{\dfrac{u}{f}-1}\right)$

Again $|m|$ can be greater than or less than 1.

(B) For concave mirror:

$$\frac{1}{v}+\frac{1}{+u}=\frac{1}{-f}$$

$$\Rightarrow \quad v=-\left(\frac{fu}{f+u}\right)$$

So, v is always negative i.e., image is always real.

Further, $|m|=\left|\dfrac{v}{u}\right|=\dfrac{1}{\dfrac{u}{f}+1}$

i.e., m is always less than 1 or image is always diminished.

12. A → (s)
B → (p)
C → (q)
D → (r)

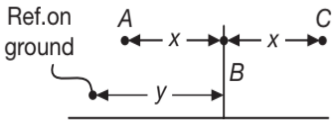
$A \rightarrow$ when object lies between pole and focus image is virtual, magnified and erect.

$B \rightarrow$ when object lies between focus and centre of curvature, image is real, inverted and magnified.

$C \rightarrow$ when object lies at centre of curvature, image is real, inverted and of equal size.

$D \rightarrow$ when object lies beyond centre of curvature, image is real, inverted and smaller in size.

13. A → (s)
B → (q)
C → (p)
D → (q)



(A) $\frac{d}{dt}(x+y) = \frac{dx}{dt} + \frac{dy}{dt} = 5 + 2 = 7 \text{ ms}^{-1}$

(B) $\frac{dx}{dt} = 3 + 2 = 5 \text{ ms}^{-1}$

(C) $\frac{d(2x)}{dt} = 2\frac{dx}{dt} = 10 \text{ ms}^{-1}$

(D) $\frac{dx}{dt} = 5 \text{ ms}^{-1}$

Integer/Numerical Answer Type Questions

1. Forward shift of point of incidence due to a single reflection $x = 0.2\tan(30°) = \frac{0.2}{\sqrt{3}}$.

Hence required number of reflections required is

$$N = \frac{\text{Mirror Length}}{\text{Forward Shift}} = \frac{2\sqrt{3}}{\left(\frac{0.2}{\sqrt{3}}\right)} = 30$$

2. Since, for a concave mirror, we can have both virtual and real image, so
for real image, $m = -5$ and
for virtual image, $m = +5$

Since we know that $m = \frac{f}{f-u}$

where $f = -30$ cm and $m = \pm 5$ because the image can be real or virtual.
For real image, $m = -5$

$\Rightarrow \quad -5 = \frac{-30}{-30-u}$

$\Rightarrow \quad u = -36$ cm

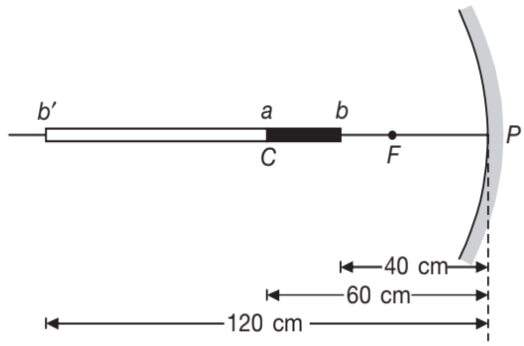
For virtual image, $m = +5$

$\Rightarrow \quad 5 = \frac{-30}{-30-u}$

$\Rightarrow \quad u = -24$ cm

Hence, the object must be placed at 24 cm or 36 cm in front of the concave mirror

3. The image of the end a of the rod which lies at the centre of curvature C is formed at C.



The position of the image of end b can be obtained using mirror formula, according to which we have

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

where $u = -40$ cm, $f = -30$ cm

$\Rightarrow \quad \frac{1}{v} + \frac{1}{-40} = \frac{1}{-30}$

$\Rightarrow \quad v = -120$ cm

Length of the image (rod) is given by

$$\ell_i = 120 - 60 = 60 \text{ cm}$$

So, magnification $m = -\frac{dv}{du} = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right)$

$\Rightarrow \quad m = -\left(\frac{120-60}{60-40}\right)$

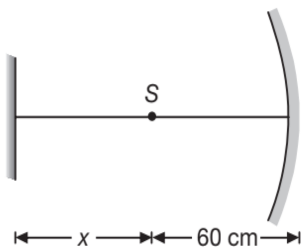
$\Rightarrow \quad m = -3$

$\Rightarrow \quad |m| = 3$

4. Applying mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ for concave mirror, we get

$$\frac{1}{v} - \frac{1}{60} = \frac{1}{(-40)}$$

$\Rightarrow \quad v = -120$ cm



Now, for the rays to again converge at S, (after reflection from the plane mirror)

$$\begin{pmatrix}\text{Distance of } S \\ \text{from Plane Mirror}\end{pmatrix} = \begin{pmatrix}\text{Distance of Image} \\ \text{formed by} \\ \text{Concave Mirror}\end{pmatrix}$$

$\Rightarrow \quad x = 120 - (x + 60)$

$\Rightarrow \quad x = 30 \text{ cm}$

So, the desired distance is 90 cm

From reversibility principle, it hardly matters whether the ray of light is first reflected from the concave mirror or plane mirror.

5. Since the image formed is real, so for the first case, we have

$m = -3$

$\Rightarrow \quad -\dfrac{v_1}{u_1} = -3$

$\Rightarrow \quad v_1 = 3u_1$

Since $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$\Rightarrow \quad \dfrac{1}{(-3u_1)} + \dfrac{1}{(-u_1)} = \dfrac{1}{-f}$

$\Rightarrow \quad f = \dfrac{3u_1}{4} \qquad \ldots(1)$

Similarly, for the second case, we have

$m = -2$

$\Rightarrow \quad -\dfrac{v_2}{u_2} = -2$

$\Rightarrow \quad v_2 = 2u_2$

Since $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$\Rightarrow \quad \dfrac{1}{-(2u_2)} + \dfrac{1}{(-u_2)} = \dfrac{1}{-f}$

$\Rightarrow \quad f = \dfrac{2u_2}{3} \qquad \ldots(2)$

From (1) and (2), we get

$\dfrac{3u_1}{4} = \dfrac{2u_2}{3}$

$\Rightarrow \quad u_2 = \dfrac{9u_1}{8} \qquad \ldots(3)$

But according to the problem, we are given that the shift of the object is 6 cm, so we get

$u_2 - u_1 = 6 \text{ cm}$

$\Rightarrow \quad \dfrac{9}{8}u_1 - u_1 = 6$

$\Rightarrow \quad u_1 = 48 \text{ cm}$

Since, $v_1 = 3u_1$

$\Rightarrow \quad v_1 = 144 \text{ cm}$

So, $f = \dfrac{3u_1}{4} = \dfrac{3}{4}(48) = 36 \text{ cm}$

Now $u_2 = \dfrac{9u_1}{8} = \dfrac{9}{8}(48) = 54 \text{ cm}$

and $v_2 = 2u_1 = 2(54) = 108 \text{ cm}$

So, shift of the screen is $(v_1 - v_2)$

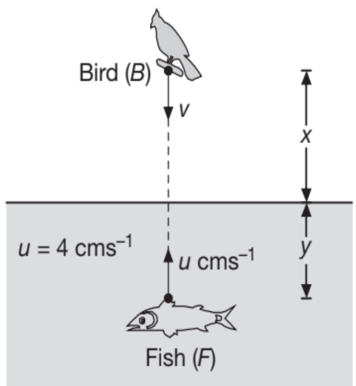
$\Rightarrow \quad (v_1 - v_2) = 144 - 108 = 36 \text{ cm}$

6. Let at some instant bird is at a height of x from the water surface and it is diving downwards with $v \text{ cms}^{-1}$

At this instant fish is at a depth y below water surface. Then the distance between fish and image of bird at this instance will be,

$s = \mu x + y$

$\Rightarrow \quad s = \dfrac{4}{3}x + y$



Differentiating w.r.t. time, we get

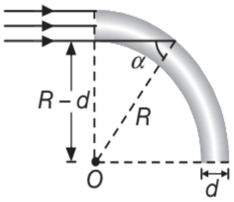
$$\left(-\frac{ds}{dt}\right) = \frac{4}{3}\left(-\frac{dx}{dt}\right) + \left(-\frac{dy}{dt}\right)$$

$\Rightarrow \quad 16 = \dfrac{4}{3}(v) + u$, where $u = 4 \text{ cms}^{-1}$

$\Rightarrow \quad v = 9 \text{ cms}^{-1}$

7. The necessary and sufficient condition for all the rays to pass around the arc is that the ray with least angle of incidence should get internally reflected i.e., should suffer TIR.

From the figure, it becomes obvious that the ray with least angle of incidence is the one which is incident almost grazingly with the inner wall.



For this ray, let α be the angle of incidence, then we observe that

$$\sin\alpha = \frac{R-d}{R} \text{ where } d \text{ is the diameter of tube}$$

For TIR, $\alpha \ge C$

$\Rightarrow \quad \sin\alpha \ge \sin C$

$$\Rightarrow \quad \frac{R-d}{R} \ge \frac{1}{\mu}$$

$$\Rightarrow \quad R \ge \frac{\mu d}{\mu - 1}$$

Since, $\mu = 1.5$ and $d = 4$ cm, so we get

$R = 12$ cm

Hence, the least radius required is 12 cm.

8. Since, $\theta = 90° - i$

$\Rightarrow \quad \tan\theta = \cot i$

$$\Rightarrow \quad \frac{dy}{dx} = \cot i \qquad \ldots(1)$$

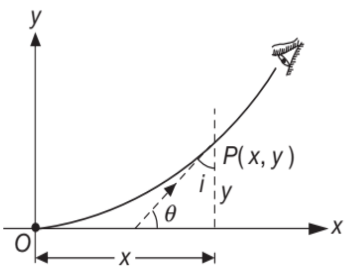
According to Snell's Law at O and P, we have

$$\mu_0 \sin i_0 = \mu_P \sin i_P$$

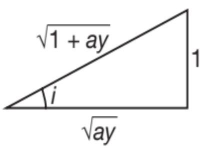
Since $\mu = \sqrt{1+ay}$

$\Rightarrow$ At $y = 0$, $\mu = 1$

$\Rightarrow \quad \sin(90°) = \left(\sqrt{1+ay}\right)\sin i$



$$\Rightarrow \quad \sin i = \frac{1}{\sqrt{1+ay}}$$



$$\Rightarrow \quad \cot i = \sqrt{ay} = \frac{dy}{dx}$$

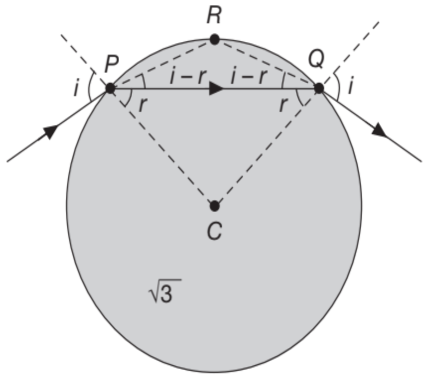
$$\Rightarrow \quad \int_0^y \frac{dy}{\sqrt{ay}} = \int_0^x dx$$

$$\Rightarrow \quad x = 2\sqrt{\frac{y}{a}}$$

Substituting $y = 2$ m and $a = 2\times10^{-6}$ m^{-1}, we get

$x_{max} = 2000$ m $= 2$ km

9. The ray diagram for the situation is shown in figure.



Since, $\angle PCQ = \pi - 2r$, $\angle PRQ = \pi - 2(i-r)$

From the property of a circle, we get

$$2(\angle PRQ) + \angle PCQ = 2\pi$$

$$\Rightarrow \quad 2\pi - 4(i-r) + \pi - 2r = 2\pi$$

$$\Rightarrow \quad r = 2i - \frac{\pi}{2}$$

According to Snell's Law, we have

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin\left(2i - \frac{\pi}{2}\right)}$$

$$\Rightarrow \quad \sqrt{3} = \frac{\sin i}{-\cos 2i} = \frac{\sin i}{2\sin^2 i - 1}$$

$$\Rightarrow \quad 2\sqrt{3}\sin^2 i - \sin i - \sqrt{3} = 0$$

$$\Rightarrow \quad \sin i = \frac{1 \pm \sqrt{1+24}}{4\sqrt{3}} = \frac{1\pm5}{4\sqrt{3}}$$

Rejecting the negative value, we get

$$\sin i = \frac{\sqrt{3}}{2}$$

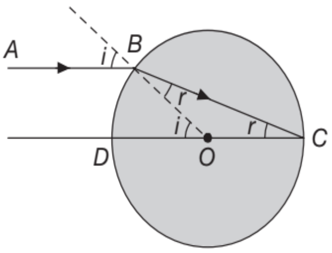
$\Rightarrow \quad i = 60°$

10. From the figure, we observe that

$BO = OC$

So, if $\angle OBC = \angle BCO = r$ (say) and i be the angle of incidence, then

$i = r + r = 2r$

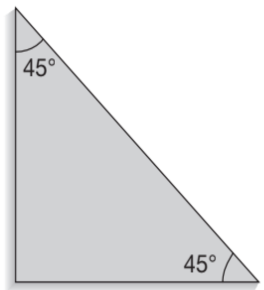


Since, we know that

$$\mu = \frac{\sin i}{\sin r}$$

$$\Rightarrow \quad \mu = \frac{\sin 2r}{\sin r} \approx \frac{2r}{r} = 2$$

11. After reflecting twice from two plane mirrors at right angles, a ray of light gets deviated by 180°, irrespective the angle of incidence, so the emergent ray is antiparallel to incident ray and hence angle of deviation is 180°.



12. From Archimedes Principle, we know that

$$\frac{V_{\text{immersed}}}{V_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{liquid}}}$$

$$\Rightarrow \quad \frac{V_{\text{immersed}}}{V} = \frac{\rho_{\text{sphere}}}{\rho_{\text{liquid}}} = \frac{\rho}{2\rho}$$

$$\Rightarrow \quad \frac{V_{\text{immersed}}}{V} = \frac{1}{2}$$

i.e., half the sphere is inside the liquid. For the image to coincide with the object, light should fall normally on the sphere. Using, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ twice, we have

$$\frac{\frac{3}{2}}{v_1} - \frac{1}{(-8)} = \frac{\left(\frac{3}{2}\right) - 1}{+2}$$

$$\Rightarrow \quad v_1 = 12 \text{ cm}$$

Further, $\frac{\frac{4}{3}}{h-10} - \frac{\frac{3}{2}}{8} = \frac{\frac{4}{3} - \frac{3}{2}}{-2}$

Solving this equation, we get

$$h = 15 \text{ cm}$$

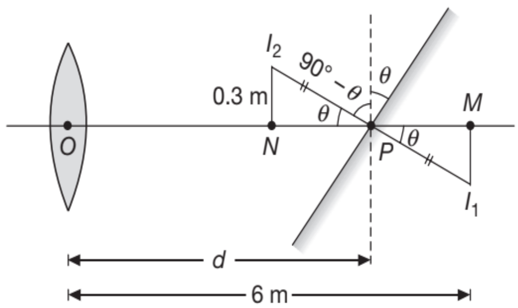
13. For the lens, $u = -2 \text{ m}$, $f = +1.5 \text{ m}$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{-2} = \frac{1}{1.5}$$

$$\Rightarrow \quad v = 6 \text{ m}$$

Since, $m = \frac{6}{(-2)} = -3$

$I_1P = I_2P$ and
I_1PI_2 is $\perp$ to mirror.



Therefore, y co-ordinate of image formed by the lens is given by

$$y = m(0.1) = -0.3 \text{ m}$$

In triangle PNI_2, we have

$$\tan\theta = \frac{I_2N}{NP}$$

$$\Rightarrow \quad \tan\theta = \frac{0.3}{NP} = 0.3 \qquad \{\because \tan\theta = 0.3\}$$

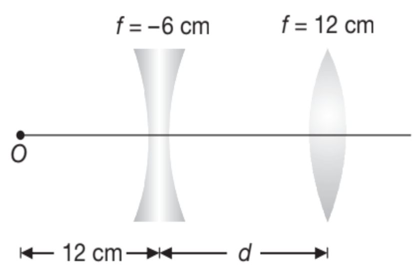
$$\Rightarrow \quad NP = MP = 1 \text{ m}$$

$$\Rightarrow \quad d = 6 - 1 = 5 \text{ m}$$

and x co-ordinate of final image I_2 is,

$$x = d - 1 = 4 \text{ m}$$

14. Applying lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ twice we get



$$\frac{1}{v_1} - \frac{1}{-12} = \frac{1}{-6} \qquad \ldots(1)$$

$$\frac{1}{\infty} - \frac{1}{v_1 - d} = \frac{1}{12} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$v_1 = -4 \text{ cm}$$

and $d = 8$ cm

15. Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v_1} + \frac{1}{40} = \frac{1}{20}$$

$\Rightarrow$ $v_1 = 40$ cm

Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v_2} + \frac{1}{10} = \frac{1}{-10}$$

$\Rightarrow$ $v_2 = -5$ cm

so, the final image is formed at a distance of 5 cm from the mirror towards lens.

16. From Lens Maker's Formula, we get

$$\frac{1}{20} = (1.5-1)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

$\Rightarrow$ $R = 20$ cm

Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ twice with the condition that rays must fall normally on the concave mirror, we get

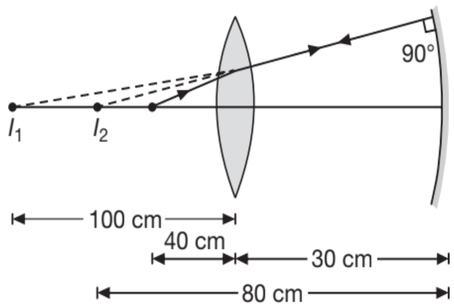
$$\frac{1.5}{v_1} - \frac{1.2}{-40} = \frac{1.5-1.2}{+20} \quad \ldots(1)$$

$$\frac{2}{d-80} - \frac{1.5}{v_1} = \frac{2-1.5}{-20} \quad \ldots(2)$$

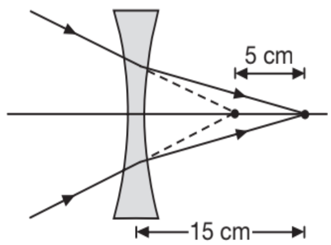
Solving equations (1) and (2), we get

$d = 30$ cm and $v_1 = -100$ cm

The ray diagram is as shown in figure.



17. Using the lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get



$$\frac{1}{f} = \frac{1}{15} - \frac{1}{10} = -\frac{1}{30}$$

$\Rightarrow$ $f = -30$ cm

18. Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v_1} - \frac{1}{(-40)} = \frac{1}{30}$$

$\Rightarrow$ $v_1 = 120$ cm

Shift due to the slab $\Delta x = \left(1 - \frac{1}{\mu}\right)d = \left(1 - \frac{1}{1.8}\right)9 = 4$ cm

So, $u' = -(40 - \Delta x) = -36$ cm

$$\Rightarrow \frac{1}{v_2} - \frac{1}{(-36)} = \frac{1}{30}$$

$\Rightarrow$ $v_2 = 180$ cm

Therefore, the screen has to be shifted away from the lens by a distance

$$x = v_2 - v_1 = 60 \text{ cm}$$

19. Lateral magnification in first case is -3, so

if $u = -x$ then $v = +3x$

Since $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \frac{1}{+3x} + \frac{1}{x} = \frac{1}{f}$$

$$\Rightarrow \frac{4}{3x} = \frac{1}{f}$$

$$\Rightarrow x = \frac{4f}{3} \quad \ldots(1)$$

In the second case magnification is -2, so now we have

CHAPTER 1

$u = -(x+1.5)$, $v = 2(x+1.5)$

Since $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \frac{1}{2(x+1.5)} + \frac{1}{(x+1.5)} = \frac{1}{f}$$

$$\Rightarrow \frac{3}{2(x+1.5)} = \frac{1}{f}$$

$$\Rightarrow f = \frac{2}{3}(x+1.5) \quad \ldots(2)$$

Solving equations (1) and (2), we get

$f = 9$ cm

20. Using Lens Maker's Formula for both the cases, we get

$$\frac{1}{f_{\text{air}}} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots(1)$$

and $$\frac{1}{f_{\text{water}}} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots(2)$$

Dividing equation (1) by (2), we get

$$\frac{f_{\text{water}}}{f_{\text{air}}} = \frac{(\mu_g - 1)}{\left(\frac{\mu_g}{\mu_w} - 1\right)}$$

Substituting the values, we get

$$f_{\text{water}} = \frac{\left(\frac{3}{2} - 1\right)}{\left(\frac{\frac{3}{2}}{\frac{4}{3}} - 1\right)} f_{\text{air}}$$

$$\Rightarrow f_{\text{water}} = 4 f_{\text{air}} = 4 \times 10 = 40 \text{ cm}$$

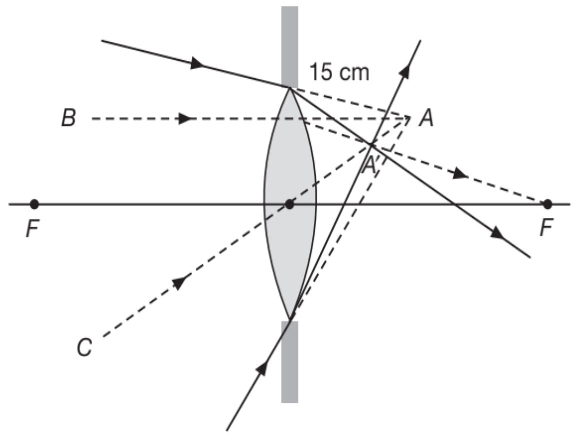
21. Using the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$u = +15$ cm, $f = +30$ cm

$$\Rightarrow \frac{1}{v} - \frac{1}{15} = \frac{1}{30}$$

$$\Rightarrow v = +10 \text{ cm}$$

Therefore, the focus of the rays will move 5 cm closer to the screen. The ray diagram is as shown in figure.



22. Since we know that

$$f = \frac{D^2 - x^2}{4D}$$

Substituting $f = 16$ cm and $x = 60$ cm, we get

$$16 = \frac{D^2 - (60)^2}{4D}$$

$$\Rightarrow D^2 - 3600 = 64D$$

$$\Rightarrow D^2 - 64D - 3600 = 0$$

$$\Rightarrow D^2 - 100D + 36D - 3600 = 0$$

$$\Rightarrow D(D-100) + 36(D-100) = 0$$

$$\Rightarrow (D-100)(D+36) = 0$$

But $D \neq 36$ cm, because it happens to be less than 60 cm. So,

$D = 100$ cm

23. For the first flare spot, the lens acts as if its right face is silvered. The equivalent focal length is given by

$$\frac{1}{F} = \frac{2\left(\frac{\mu_2}{\mu_1}\right)}{R_2} - \frac{2\left(\frac{\mu_2}{\mu_1} - 1\right)}{R_1}$$

$$\Rightarrow \frac{1}{F} = \frac{2 \times 1.5}{-60} - \frac{2(1.5-1)}{+30}$$

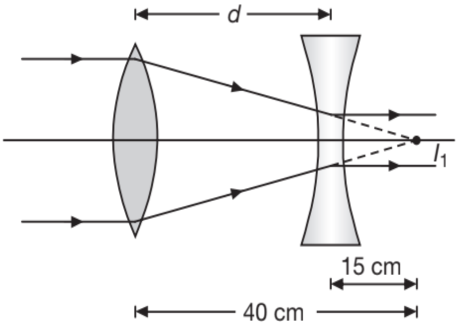
$$\Rightarrow F = -12 \text{ cm}$$

Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{F}$ with $u \to \infty$, we get

$$\frac{1}{v} + \frac{1}{\infty} = \frac{1}{-12}$$

$$\Rightarrow v = -12 \text{ cm}$$

24. **Method 1:** Ray diagram is as shown



$d = 25$ cm

Method 2: Since this combination just behaves as a plane glass plate, so the power of the combination is zero. Since

$$P_{\text{comb}} = \frac{1}{f_{\text{comb}}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$$

$$\Rightarrow \quad O = \frac{1}{40} + \frac{1}{(-15)} - \frac{x}{(40)(-15)}$$

$$\Rightarrow \quad \frac{x}{(40)(15)} = \frac{1}{15} - \frac{1}{40}$$

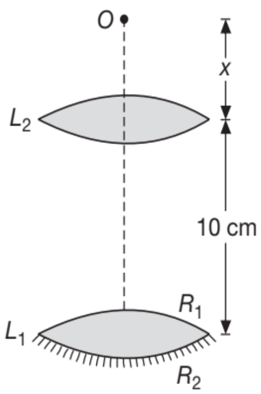
$$\Rightarrow \quad \frac{x}{(40)(15)} = \frac{40-15}{(40)(15)}$$

$$\Rightarrow \quad x = 25 \text{ cm}$$

25. Applying Lens Maker's Formula, we get

$$\frac{1}{40} = (1.5-1)\left(\frac{1}{120} + \frac{1}{R_1}\right)$$

$$\Rightarrow \quad R_1 = 24 \text{ cm}$$



Applying lens formula, for L_2, we get

$$\frac{1}{v_1} + \frac{1}{x} = \frac{1}{20} \qquad \ldots(1)$$

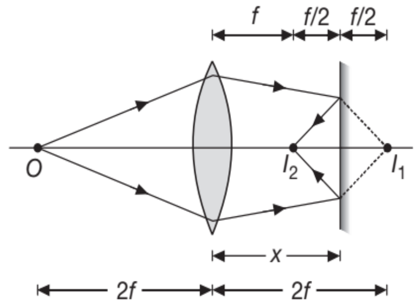
Using, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ for unsilvered side of L_1, we get

$$\frac{1.5}{(-120)} - \frac{1}{(v_1 - 10)} = \frac{1.5-1}{24} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$x = 10 \text{ cm}$$

26. The ray diagram is as shown in figure for first two steps. If the rays reflected from the mirror are parallel after passing through the lens for the second time, then I_2 must lie at first focus of lens. So, the desired distance is given by



$$x = (2f + 2f) - \frac{f}{2}$$

$$\Rightarrow \quad x = \frac{3f}{2} = \frac{3(30)}{2} = 45 \text{ cm}$$

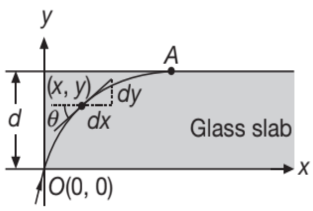
27. $\mu \sin\theta = 1 \times \sin 90°$

$$\Rightarrow \quad \sin\theta = \frac{1}{\mu}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{\mu^2 - 1}} = \frac{1}{\sqrt{1 + e^{\frac{x}{d}} - 1}} = e^{-\frac{x}{2d}}$$

$$\Rightarrow \quad \frac{dy}{dx} = e^{-\frac{x}{2d}}$$

$$\Rightarrow \quad y = 2d\left(1 - e^{-\frac{x}{2d}}\right) \qquad \ldots(1)$$



At A, $y = d$, so from equation (1), we get

$$\frac{1}{2} = 1 - e^{\frac{x}{2d}}$$

$$\Rightarrow \quad e^{\frac{x}{2d}} = 2$$

$\Rightarrow \quad x = 2d\ln 2 = d\ln(4)$

$\Rightarrow \quad \alpha = 4$

$\Rightarrow \quad \mu = \sqrt{1+e^{\frac{2d\ln 2}{d}}} = \sqrt{1+e^{\ln 4}} = \sqrt{1+4} = \sqrt{5}$

$\Rightarrow \quad \beta = 5$

28. For refraction formula, we use

$u \to \infty$, $R = +10$ cm, $v = 2R = +20$ cm

$\mu_1 = 1$ and $\mu_2 = \mu$

Since, $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

$\Rightarrow \quad \dfrac{\mu}{20} = \dfrac{\mu - 1}{10}$

$\Rightarrow \quad \mu = 2\mu - 2$

$\Rightarrow \quad \mu = 2$

29. Let AB be the object placed at a distance of 15 cm from the lens as shown in figure. We shall calculate the position of image formed by this lens in absence of plane mirror.

For lens formula we use $\mu_1 = -15$ cm, and $f = +10$ cm

$\Rightarrow \quad \dfrac{1}{v_1} = \dfrac{1}{10} - \dfrac{1}{15} = \dfrac{3-2}{30} = \dfrac{1}{30}$

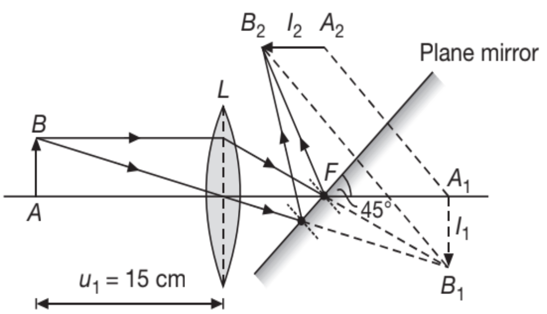
$\Rightarrow \quad v_1 = +30$ cm

So the image would be formed at 30 cm from the lens to the right of it

Magnification by lens is $m_1 = \dfrac{v_1}{u_1} = \dfrac{30}{15} = 2$

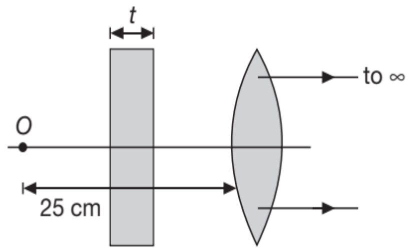
$\Rightarrow$ Size of the image $= 2 \times 4 = 8$ cm

The image produced by lens is I_1 which is shown by A_1B_1 in figure.



The image I_1 acts as the object for the plane mirror and after reflection of light rays from the plane mirror final image produced is A_2B_2. In a plane mirror, the image formed is at same distance at which object is kept from it and size remain same. So the final image is produced at a distance $30 - 10 = 20$ cm as shown in figure at an angle $90°$ to the principal axis as mirror rotates the reflected rays by twice the angle at which mirror is rotated.

30. For final image at infinity object must at focal point of lens i.e., 20 cm.



As actual object is kept at 25 cm so 5 cm is the shift due to glass slab, gives as

$$\text{Shift} = t\left(1 - \frac{1}{\mu}\right)$$

$\Rightarrow \quad 5 = t\left(1 - \dfrac{1}{1.5}\right)$

$\Rightarrow \quad t = 15$ cm

31. Using lens formula for objective, we have

$$\frac{1}{v_0} - \frac{1}{-5} = \frac{1}{+0.25}$$

$\Rightarrow \quad v_0 = -0.2632$ m

Now using lens formula for eyepiece, we have

$$\frac{1}{(-0.25)} - \frac{1}{-u_e} = \frac{1}{+0.025}$$

$\Rightarrow \quad \dfrac{1}{u_e} = 4 + 40 = 44$

$\Rightarrow \quad u_e = 0.0227$ m

Tube length of the telescope is

$$L = 0.2632 + 0.0227 = 0.2859 \text{ m} = 28.59 \text{ cm}$$

$\Rightarrow \quad L \approx 29$ cm

Magnifying power $M = \dfrac{v_o}{u_e} = \dfrac{0.2632}{0.0227} = 11.6$

$\Rightarrow \quad 10M = 116$

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1. $1 \times \sin(40°) = 1.31 \sin\theta$

$\Rightarrow \sin\theta = \dfrac{0.64}{1.31}$

$\Rightarrow \theta \approx 30°$

$\ell = (20\ \mu m)\cot\theta$

$\Rightarrow N = \dfrac{2}{20 \times 10^{-6} \times \cot\theta}$

$\Rightarrow N = \dfrac{2 \times 10^6}{20 \times \sqrt{3}} = 57735$

$\Rightarrow N \approx 57000$

Hence, the correct answer is (D).

2. $\dfrac{1}{v_1} - \dfrac{1}{u_1} = \dfrac{1}{f_1}$

$\Rightarrow v_1 = \dfrac{40 \times 20}{(40-20)} = 40$ cm

Now $u_2 = 60 - 40 = 20$ cm

Since $\dfrac{1}{v_2} + \dfrac{1}{u_2} = \dfrac{1}{f_2}$

$\Rightarrow v_2 = \dfrac{20 \times 10}{(20-10)} = 20$ cm

So, image traces back to object itself because image formed by the lens is at the centre of curvature of mirror.

Hence, the correct answer is (C).

3. $\theta = \dfrac{1.22\lambda}{D}$

$\Rightarrow \theta = \dfrac{1.22 \times 500 \times 10^{-9}}{200 \times 10^{-2}} = \dfrac{1.22 \times 500 \times 10^{-9}}{2}$

$\Rightarrow \theta = 305 \times 10^{-9}$ radian

Hence, the correct answer is (B).

4.

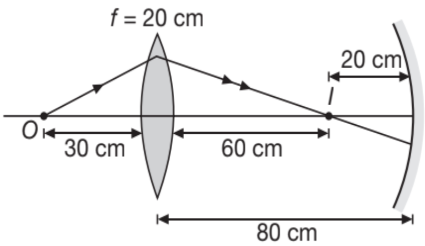


Image formed by lens

$\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$\Rightarrow \dfrac{1}{v} + \dfrac{1}{30} = \dfrac{1}{20}$

$\Rightarrow v = +60$ cm

If image position does not change even when mirror is removed, it means that image formed by lens is formed at centre of curvature of spherical mirror. Radius of curvature of mirror is

$R = 80 - 60 = 20$ cm

So, focal length of mirror is $f = 10$ cm

For virtual image, object is to be kept between focus and pole and hence maximum distance of object from spherical mirror for which virtual image is formed, is 10 cm

Hence, the correct answer is (D).

5. $\dfrac{1}{v} + \dfrac{1}{u} = -\dfrac{1}{40}$

Given, $\dfrac{v}{u} = -5$

$\Rightarrow -\dfrac{1}{5u} + \dfrac{1}{u} = -\dfrac{1}{40}$

$\Rightarrow u = -32$ cm

Hence, the correct answer is (A)

6. Since, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

Given that $v = \pm 2u$

$\Rightarrow \dfrac{1}{2u} + \dfrac{1}{u} = \dfrac{1}{20}$

$\Rightarrow \dfrac{3}{2u} = \dfrac{1}{20}$

$\Rightarrow u_1 = x_1 = 30$ cm

And $\dfrac{1}{u} - \dfrac{1}{2u} = \dfrac{1}{20}$

$\Rightarrow \dfrac{1}{2u} = \dfrac{1}{20}$

$\Rightarrow u_2 = x_2 = 10$

$\Rightarrow \dfrac{x_1}{x_2} = \dfrac{30}{10} = 3$

Hence, the correct answer is (A).

7. $\theta = \dfrac{1.22\lambda}{D}$

$\Rightarrow \quad \theta = \dfrac{1.22 \times 600 \times 10^{-9}}{250} \times 100 = 2.92 \times 10^{-7}$

$\Rightarrow \quad \theta = 3 \times 10^{-7}$ rad

Hence, the correct answer is (B).

8. $f_L = 18$ cm

$\Rightarrow \quad \dfrac{1}{18} = 0.5 \times \dfrac{2}{R}$

$\Rightarrow \quad R = 18$ cm

Since, $\dfrac{1}{f_2} = (\mu_\ell - 1)\left(-\dfrac{1}{18}\right)$ and $OA' = 27$ cm

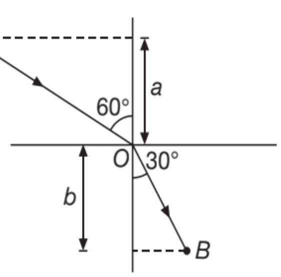
$\Rightarrow \quad \dfrac{1}{27} = \dfrac{1}{18} - \dfrac{(\mu_\ell - 1)}{18} = \dfrac{1 - \mu_\ell + 1}{18}$

$\Rightarrow \quad 2 = 3(2 - \mu_\ell) = 6 - 3\mu_\ell$

$\Rightarrow \quad \mu_\ell = \dfrac{4}{3}$

Hence, the correct answer is (B).

9. From the given figure



$\dfrac{a}{AO} = \cos(60°)$

$\Rightarrow \quad AO = 2a$

$\dfrac{b}{BO} = \cos 30°$

$\Rightarrow \quad BO = \dfrac{2b}{\sqrt{3}}$

So, length of optical path is

$L = AO + BO \times \sqrt{3}$

$\Rightarrow \quad L = 2a + 2b$

Hence, the correct answer is (D).

10. Focal length of plano-convex lens is

$f_1 = \dfrac{R}{(\mu_1 - 1)}$

Focal length of plano concave lens is

$f_2 = \dfrac{-R}{(\mu_2 - 1)}$

For the combination of two lenses, we have

$\dfrac{1}{f_{eq}} = \dfrac{1}{f_1} + \dfrac{1}{f_2} = \dfrac{\mu_1 - 1}{R} - \dfrac{\mu_2 - 1}{R} = \dfrac{\mu_1 - \mu_2}{R}$

$\Rightarrow \quad f_{eq} = \dfrac{R}{\mu_1 - \mu_2}$

Hence, the correct answer is (C).

11. As the graph between magnification (m) and image distance (v) varies linearly, then

$m = k_1 v + k_2$

$\Rightarrow \quad \dfrac{v}{u} = k_1 v + k_2$

$\Rightarrow \quad \dfrac{1}{u} = k_1 + \dfrac{k_2}{v}$

$\Rightarrow \quad \dfrac{k_2}{v} - \dfrac{1}{u} = k_1$

Clearly, from above, we have

$k_1 = \dfrac{1}{f}$ and $k_2 = 1$

$\Rightarrow \quad f = \dfrac{1}{\text{slope of m-v graph}} = \dfrac{b}{c}$

Hence, the correct answer is (D).

12. Numerical aperature (NA) of the microscope is

$NA = \dfrac{0.61\lambda}{d}$

where d is the minimum separation between two points to be seen as distinct.

$\Rightarrow \quad d = \dfrac{0.61\lambda}{NA} = \dfrac{(0.61) \times (5000 \times 10 \text{ m}^{-10})}{1.25}$

$\Rightarrow \quad d = 2.4 \times 10^{-7}$ m

$\Rightarrow \quad d = 0.24\ \mu m$

Hence, the correct answer is (A).

13. $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$\Rightarrow \quad \dfrac{1}{v} + \dfrac{1}{(-5)} = \dfrac{1}{(-20)}$

$\Rightarrow \quad \dfrac{1}{v} = -\dfrac{1}{20} + \dfrac{1}{5}$

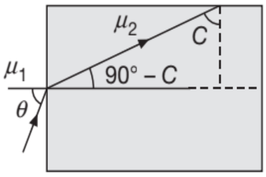
$$\Rightarrow \quad \frac{1}{v} = \frac{3}{20}$$

$$\Rightarrow \quad d = \frac{\left(\frac{20}{3} + 5\right)}{\mu} = \frac{\frac{35}{3}}{\frac{4}{3}}$$

$$\Rightarrow \quad d = \frac{35}{4}$$

$$\Rightarrow \quad d = 8.8 \text{ cm}$$

Hence, the correct answer is (D).

14. 

$$\sin C = \frac{\mu_1}{\mu_2} \qquad \ldots(1)$$

$$\mu_1 \sin\theta = \mu_2 \sin(90° - C) = \mu_2 \cos C$$

From (1), we get $\cos C = \sqrt{1 - \sin^2 C} = \sqrt{1 - \frac{\mu_1^2}{\mu_2^2}}$

$$\Rightarrow \quad \sin\theta = \frac{\mu_2\sqrt{1 - \frac{\mu_1^2}{\mu_2^2}}}{\mu_1}$$

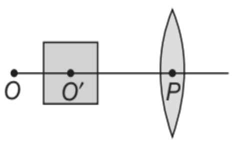
$$\theta = \sin^{-1}\left(\sqrt{\frac{\mu_2^2 - \mu_1^2}{\mu_1^2}}\right) = \sin^{-1}\left(\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}\right)$$

For TIR,

$$\theta < \sin^{-1}\left(\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}\right)$$

Hence, the correct answer is (B).

15. Since $2f = 10$ cm



$$\Rightarrow \quad f = 5 \text{ cm}$$

Shift $= OO' = 1.5\left(1 - \frac{2}{3}\right) = 0.5$ cm

$$\Rightarrow \quad O'P = (10 - 0.5) \text{ cm} = 9.5 \text{ cm}$$

Since, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{(-9.5)} = \frac{1}{5}$$

$$\Rightarrow \quad \frac{1}{v} + \frac{1}{9.5} = \frac{1}{5}$$

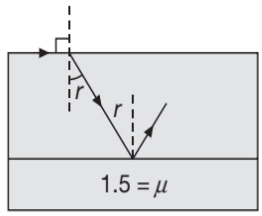
$$\Rightarrow \quad v = 10.55 \text{ cm}$$

Hence shift is $(10.55 - 10)$ cm $= 0.55$ cm away from lens.

Hence, the correct answer is (D).

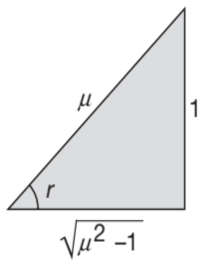
16. For air-medium interface maximum angle of incidence can be $90°$

$$\Rightarrow \quad \frac{\sin(90°)}{\sin r} = \mu$$



$$\Rightarrow \quad \sin r = \frac{1}{\mu}$$

For light reflected from liquid glass interface to be never completely polarised, we must have angle of incidence (r) at the liquid glass interface to be less than the polarising angle p at the interface



$$\Rightarrow \quad r < p$$

$\Rightarrow \quad \tan r < \tan p$, where

$$\tan p = \frac{3}{2\mu} \qquad \text{\{from Brewster's law\}}$$

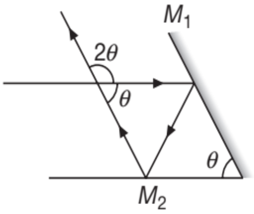
$$\Rightarrow \quad \frac{1}{\sqrt{\mu^2 - 1}} < \frac{3}{2\mu}$$

$$\Rightarrow \quad \mu \geq \frac{3}{\sqrt{5}}$$

$$\Rightarrow \quad \mu_{min} = \frac{3}{\sqrt{5}}$$

Hence, the correct answer is (C).

17. The ray diagram for the situation asked in the problem is shown in figure.



From above, we see that

$3\theta = 180°$

$\Rightarrow \quad \theta = 60°$

Hence, the correct answer is (D).

18. $\dfrac{1}{f_1} = (\mu_1 - 1)\dfrac{1}{R}$

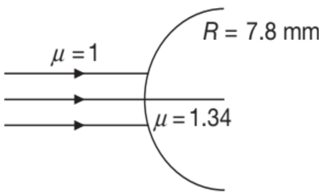
$\dfrac{1}{f_2} = (\mu_2 - 1)\dfrac{1}{R}$

$\Rightarrow \quad 2(\mu_1 - 1)\dfrac{1}{R} = (\mu_2 - 1)\dfrac{1}{R}$

$\Rightarrow \quad 2\mu_1 - \mu_2 = 1$

Hence, the correct answer is (A).

19. $\dfrac{1.34}{v} - \dfrac{1}{u} = \dfrac{1.34 - 1}{(7.8)}$



Since $u \to -\infty$

$\Rightarrow \quad v = f$

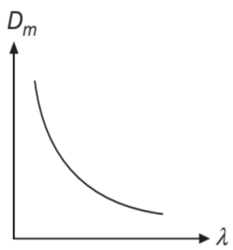
$\Rightarrow \quad \dfrac{1.34}{f} = \dfrac{0.34}{7.8 \text{ mm}}$

$\Rightarrow \quad f = \left(\dfrac{1.34 \times 7.8}{0.34}\right) \text{mm}$

$\Rightarrow \quad f = 3.07 \text{ cm} \approx 3.1 \text{ cm}$

Hence, the correct answer is (C).

20. Since for a thin prism, we have $D_m = (\mu - 1)A$



Hence, the correct answer is (A).

21. $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

Since, $u = -20 \text{ m}$, $f = 0.3$

$\Rightarrow \quad \dfrac{1}{v} = \dfrac{1}{0.3} - \dfrac{1}{20}$

$\Rightarrow \quad \dfrac{1}{v} = \dfrac{10}{3} - \dfrac{1}{20}$

$\Rightarrow \quad v = \dfrac{60}{197} \text{ m}$

$\Rightarrow \quad m = \dfrac{v}{u} = \dfrac{3}{197}$

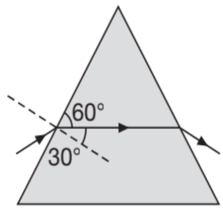
Since $(\text{velocity})_{\text{image}} = m^2 (\text{velocity})_{\text{object}}$

$\Rightarrow \quad v_{\text{image}} = \left(\dfrac{3}{197}\right)^2 \times 5$

$\Rightarrow \quad v_{\text{image}} = 1.16 \times 10^{-3} \text{ ms}^{-1}$ toward the lens.

Hence, the correct answer is (C).

22. For minimum deviation the ray passes symmetrically



$\Rightarrow \quad r = 30°$

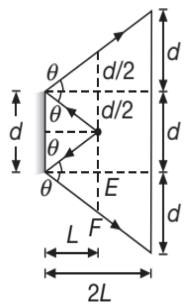
$\sin i = \sqrt{3} \sin 30° = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad i = 60°$

Hence, the correct answer is (D).

23. $\dfrac{d}{d/2} = \dfrac{y}{2L}$

$\Rightarrow \quad y = d$



Hence, the distance over which the image can be seen is $d + d + d = 3d$.

Hence, the correct answer is (B).

24. For lens A

$\dfrac{1}{v} - \dfrac{1}{(-20)} = \dfrac{1}{5}$

$\Rightarrow \quad v = \dfrac{20}{3} \text{ cm}$

For lens B

$u = \dfrac{20}{3} - 2$

$$u = \frac{14}{3} \text{ cm}$$

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{\frac{14}{3}} = -\frac{1}{5}$$

$$\Rightarrow \quad v = 70 \text{ cm}$$

Image is real and right of B.

Hence, the correct answer is (B).

25. Using Lens Maker's Formula, we get

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\frac{2}{R}$$

$$\Rightarrow \quad f = R$$

In water $\left(\mu = \frac{4}{3}\right)$, we get the new focal length f' as

$$\frac{1}{f'} = \left(\frac{3/2}{4/3} - 1\right)\frac{2}{R}$$

$$\Rightarrow \quad f' = 4f \qquad \{\because R = f\}$$

Now since the object in placed is between focus and lens, so no real image on the screen is obtained.

Hence, the correct answer is (C).

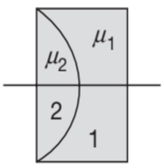
26. For plano convex lens, we have

$$\frac{1}{f_2} = \frac{\mu_2 - 1}{R} \qquad \ldots(1)$$

For plano concave lens, we have

$$\frac{1}{f_1} = -\left(\frac{\mu_1 - 1}{R}\right) \qquad \ldots(2)$$

Now for combination of both as shown, we have



For 2, $\frac{\mu_2}{v_1} - \frac{1}{\infty} = \frac{\mu_2 - 1}{\infty}$

For 1, $\frac{\mu_1}{v_1'} - \frac{\mu_2}{v_1} = \frac{\mu_2 - \mu_1}{R}$

For combination, $\frac{1}{f} - \frac{\mu_1}{v_1'} = \frac{1 - \mu_1}{\infty}$

$$\Rightarrow \quad \frac{1}{f} = \frac{(\mu_2 - \mu_1)}{R} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\Rightarrow \quad f = \frac{R}{\mu_2 - \mu_1}$$

Hence, the correct answer is (B).

27. When object is at 8 cm, image distance

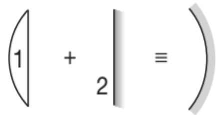
$$v_1 = \frac{f \times u}{u - f} = \frac{(-5)(-8)}{-8 + 5} = -\frac{40}{3} \text{ cm}$$

When object is at 12 cm, image distance $v_2 = -\frac{60}{7}$ cm

$$\Rightarrow \quad \text{Separation } = |v_1 - v_2| = \frac{100}{21} \text{ cm}$$

Hence, the correct answer is (C).

28. For figure (A),



$$\frac{1}{f_\ell} = \left(\frac{\mu - 1}{R}\right) \text{ and } f_{eq} = -28 \text{ cm}$$

$$P = 2P_\ell + P_m = 2P_\ell + 0$$

$$\Rightarrow \quad -\frac{1}{28} = -2\left(\frac{\mu - 1}{R}\right) \qquad \ldots(1)$$

For figure (B),

$$\frac{1}{f_\ell} = \left(\frac{\mu - 1}{R}\right), \; f_2 = -\frac{R}{2} \text{ and } f_{eq} = -10 \text{ cm}$$

Since, $P = 2P_\ell + P_m$

$$\Rightarrow \quad -\frac{1}{10} = -2\left(\frac{\mu - 1}{R}\right) - \frac{2}{R}$$

$$\Rightarrow \quad \frac{1}{10} = \frac{1}{28} + \frac{2}{R}$$

$$\Rightarrow \quad \frac{2}{R} = \frac{1}{10} - \frac{1}{28} = \frac{18}{280}$$

$$\Rightarrow \quad R = \frac{280}{9} \text{ cm}$$

Substituting R in equation (1), we get

$$\frac{1}{28} = 2\left(\frac{\mu - 1}{280}\right)9$$

$$\Rightarrow \quad \mu - 1 = \frac{5}{9}$$

$$\Rightarrow \quad \mu = 1 + \frac{5}{9} = \frac{14}{9} = 1.55$$

Hence, the correct answer is (C).

29. Since, $\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$

$$\Rightarrow \quad \frac{1}{10} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{2}{f_1 f_2}$$

Substituting the values of f_1 and f_2 option wise, we see that

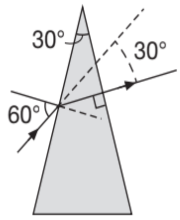
$$f_{eq} = 10 \text{ cm for } f_1 = 18 \text{ cm and } f_2 = 20 \text{ cm}.$$

Hence, the correct answer is (A).

30. As $\delta = i + e - A$

$\Rightarrow \quad 30^\circ = 60^\circ + e - 30^\circ$

$\Rightarrow \quad e = 0$



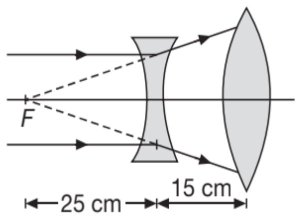
So, required angle with face of prism is 90°

Hence, the correct answer is (C).

31. Given that focal length of concave lens is $f = -25$ cm

Focal length of convex lens is $f' = 20$ cm

The formation of image is shown here.



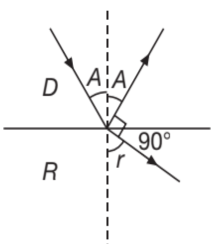
The image for diverging lens will form at F. i.e. at focal length of concave lens.
Now, this image will serve as the object for convex lens. It is at twice the focal length of the convex lens (i.e. $2f$). Hence, the final image will also form at $2f$, which is at a distance of 40 cm from the convergent lens. Also, the image formed is real.

Hence, the correct answer is (A).

32. Refractive index of denser medium with respect to rarer medium, $n_{12} = \dfrac{n_D}{n_R} = \dfrac{1}{\sin\theta_C}$

$$\Rightarrow \quad \frac{n_R}{n_D} = \sin\theta_C \qquad \ldots(1)$$

Using Snell's law at the interface of two media,



$$n_D \sin A = n_R \sin r$$

$$\Rightarrow \quad \frac{n_R}{n_D} = \frac{\sin A}{\sin(90 - A)} = \frac{\sin A}{\cos A} = \tan A$$

$$\Rightarrow \quad \tan A = \sin\theta_C$$

$$\Rightarrow \quad A = \tan^{-1}(\sin\theta_C)$$

Hence, the correct answer is (C).

33. Telescope resolves and brings the objects closer which is far away from the telescope. Hence for telescope with magnifying power 20, the tree appears 20 times nearer.

Hence, the correct answer is (D).

34. For convex lens, $u_1 = -60$ cm, $f_1 = 30$ cm

$$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{30} - \frac{1}{60} = \frac{1}{60}$$

$$\Rightarrow \quad v_1 = 60 \text{ cm}$$

For concave lens

$$u_2 = 60 - 20 = 40 \text{ cm}, \; f_2 = -120 \text{ cm}$$

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{120} + \frac{1}{40} = \frac{2}{120}$$

$$\Rightarrow \quad v_2 = 60 \text{ cm}$$

For plane mirror, virtual object is 10 cm behind the mirror.
Hence, real image is formed 10 cm in front of the mirror.
Now, again for concave lens, $u_2 = 40$ cm i.e., light rays from the object retrace their path after striking the plane mirror, Hence the final image is formed at the object itself.

Hence, the correct answer is (A).

35. The refractive index is given by

$$\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$$

$$\Rightarrow \quad \mu = \frac{\text{Reading 3} - \text{Reading 1}}{\text{Reading 3} - \text{Reading 2}}$$

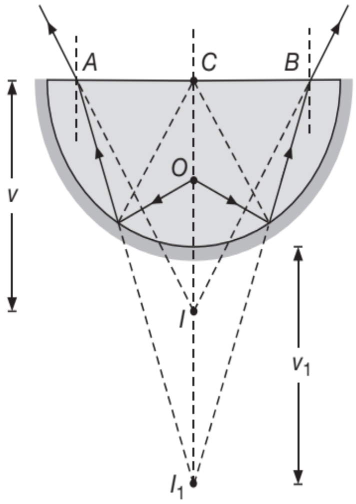
Therefore, the minimum of three readings are required.

Hence, the correct answer is (C).

36. Since, $R = -10$ cm

So, object distance from mirror is

$$u = -(10 - 6) = -4 \text{ cm}$$



Focal length of mirror is

$$f = -\frac{R}{2} = -\frac{10}{2} = -5 \text{ cm}$$

Image distance $= v = ?$

Using mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$\Rightarrow \quad \frac{1}{v_1} + \frac{1}{-4} = \frac{1}{-5}$$

$$\Rightarrow \quad \frac{1}{v_1} = \frac{1}{4} - \frac{1}{5} = \frac{1}{20}$$

$$\Rightarrow \quad v_1 = 20 \text{ cm}$$

Now, I_1 acts as object for plane glass surface AB

$$\Rightarrow \quad \text{Apparent depth } v = \frac{R + v_1}{\mu} = \frac{30}{1.5} = 20 \text{ cm}$$

Hence, the position of the image of the air bubble made by the mirror is seen 20 cm below the flat surface.

Hence, the correct answer is (B).

37. Here, $\lambda = 600 \text{ nm} = 6 \times 10^{-7}$ m

$D = 30 \text{ cm} = 30 \times 10^{-2} \text{ m} = 0.3$ m

$R = 10 \text{ ly} = 10 \times 9.46 \times 10^{15}$ m

$\ell = ?$

The limit of resolution of a telescope $\Delta\theta = \frac{1.22\lambda}{D} = \frac{\ell}{R}$

$$\ell = \frac{1.22\lambda R}{D} = \frac{1.22 \times 6 \times 10^{-7} \times 10 \times 9.46 \times 10^{15}}{0.3}$$

$$\Rightarrow \quad \ell = 2.31 \times 10^{8} \text{ km}$$

Hence, the correct answer is (A).

38. Given, wavelength of light, $\lambda = 500 \text{ nm} = 500 \times 10^{-9}$ m

Least distance of distinct vision, $D = 25 \text{ cm} = 25 \times 10^{-2}$ m

Radius of pupil, $r = 0.25$ cm

$\Rightarrow$ Diameter of pupil, $d = 2r = 0.50 \text{ cm} = 0.50 \times 10^{-2}$ m

Resolving power of eye, $\Delta\theta = \frac{1.22\lambda}{d}$

$$\Rightarrow \quad \Delta\theta = \frac{1.22 \times 500 \times 10^{-9}}{0.50 \times 10^{-2}} = 1.22 \times 10^{-4} \text{ rad}$$

So, minimum separation that eye can resolve is

$$x = D\Delta\theta = 1.22 \times 10^{-4} \times 25 \times 10^{-2}$$

$$\Rightarrow \quad x = 30.5 \times 10^{-6} \text{ m} = 30\ \mu\text{m}$$

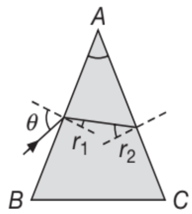
Hence, the correct answer is (D).

39. According to Snell's law

$$\sin\theta = \mu \sin r_1$$

$$\Rightarrow \quad \sin r_1 = \frac{\sin\theta}{\mu}$$

$$\Rightarrow \quad r_1 = \sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$$



Now, $A = r_1 + r_2$

$$\Rightarrow \quad r_2 = A - r_1 = A - \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) \quad \ldots(1)$$

For the ray to get transmitted through the face AC, r_2 must be less than critical angle, i.e., $r_2 < \sin^{-1}\left(\frac{1}{\mu}\right)$

$$\Rightarrow \quad A - \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) < \sin^{-1}\left(\frac{1}{\mu}\right) \quad \text{(using (1))}$$

$$\Rightarrow \quad \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) > A - \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\Rightarrow \quad \frac{\sin\theta}{\mu} > \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)$$

$$\Rightarrow \quad \theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$$

Hence, the correct answer is (C).

CHAPTER 1

40. Here, $u = -10$ cm , $v = +15$ cm

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{15}-\frac{1}{10}=\frac{1}{f}=\frac{2}{R}$$

$$\Rightarrow \quad -\frac{5}{150}=\frac{2}{R}$$

$$R=-\frac{300}{5}=-60 \text{ cm}$$

Hence, the correct answer is (C).

41. Angular magnification $m=\frac{f_0}{f_e}=\frac{150}{5}=30$

$$\Rightarrow \quad \frac{\tan\beta}{\tan\alpha}=30$$

$$\Rightarrow \quad \tan\beta=\tan\alpha\times 30=\left(\frac{50}{1000}\right)\times 30=\frac{15}{10}=\frac{3}{2}$$

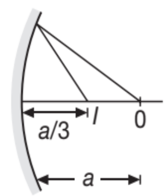
$$\Rightarrow \quad \beta=\tan^{-1}\left(\frac{3}{2}\right)$$

$$\Rightarrow \quad \theta=\beta\approx 60^\circ$$

Hence, the correct answer is (D).

42. This combination will behave like a mirror of power,

$$P_{eq}=2P_L+P_M$$



$$\Rightarrow \quad P_{eq}=2\frac{1}{f}+0$$

$$\Rightarrow \quad f_{eq}=\frac{f}{2}$$

So, the behaviour of the system, will be like a mirror of focal length $-\frac{f}{2}$

Using mirror equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f_{eq}}$, we get

Since, $u=-a$, $v=-\frac{a}{3}$, $f_{eq}=-\frac{f}{2}$

$$\Rightarrow \quad \frac{1}{-\frac{a}{3}}+\frac{1}{-a}=\frac{-1}{\frac{f}{2}}$$

$$\Rightarrow \quad \frac{4}{a}=\frac{2}{f}$$

$$\Rightarrow \quad a=2f$$

Hence, the correct answer is (B).

43. Given $\mu=\frac{3}{2}$ (crown glass) and focal length $= f$

Let focal length be f_1 when lens is placed in liquid of refractive index $\mu_1=\frac{4}{3}$

Let focal length be f_2 when lens is placed in liquid of refractive index $\mu_2=\frac{5}{3}$

According to Lens Makers Formula, we have

$$\frac{1}{f}=\left(\frac{3}{2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Using Lens maker's formula $\frac{1}{f_1}=\left(\frac{\mu}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$, we get

$$\frac{1}{f_1}=\left(\frac{\frac{3}{2}}{\frac{4}{3}}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{8}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Similarly, $\frac{1}{f_2}=\left(\frac{\mu}{\mu_2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$\Rightarrow \quad \frac{1}{f_2}=\left(\frac{\frac{3}{2}}{\frac{5}{3}}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{-1}{10}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

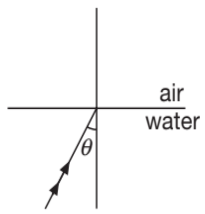
$$\Rightarrow \quad f_1=4f \text{ and } f_2=-5f$$

Hence, the correct answer is (C).

44. Since, $\sin C=\sin\theta=\frac{1}{\mu}$ and $\mu\lambda=$ constant

$$\Rightarrow \quad \sin\theta\propto\lambda$$

Also, refractive index (μ) of the medium depends on the wavelength of the light. μ is less for the greater wavelength (i.e. lesser frequency).



So, θ will be more for lesser frequency of light. Hence, the spectrum of visible light whose frequency is less than that of green light will come out to the air medium.

Hence, the correct answer is (C).

45. According to Lens Maker's Formula, we have

$$\frac{1}{f}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

Since the lens is plano-convex, so

$$R_1 = R,\ R_2 \to \infty$$

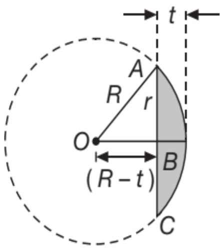
$$\Rightarrow \frac{1}{f} = \frac{(\mu-1)}{R}$$

$$\Rightarrow f = \frac{R}{(\mu-1)} \quad \ldots(1)$$

The speed of light in the medium of lens is $2\times10^8\ \text{ms}^{-1}$

$$\Rightarrow \mu = \frac{c}{v} = \frac{3\times10^8\ \text{ms}^{-1}}{2\times10^8\ \text{ms}^{-1}} = \frac{3}{2} \quad \ldots(2)$$

If r is the radius and t is the thickness of lens (at the centre), the radius of curvature R of its curved surface in accordance with figure will be given by using Pythagoras Theorem.



$$R^2 = r^2 + (R-t)^2$$

$$\Rightarrow R^2 = r^2 + R^2 + t^2 - 2Rt$$

Since $r \gg t$

$$\Rightarrow 2Rt = r^2 + t^2$$

$$\Rightarrow R = \frac{r^2}{2t} \quad \ldots(3)$$

Since, $r = 3$ cm, $t = 3$ mm $= 0.3$ cm

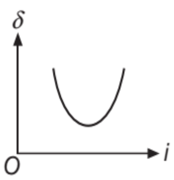
$$\Rightarrow R = \frac{(3)^2}{2\times0.3} = 15 \text{ cm}$$

On substituting the values of μ and R from Equations (2) and (3) in (1), we get

$$f = \frac{15 \text{ cm}}{(1.5-1)} = 30 \text{ cm}$$

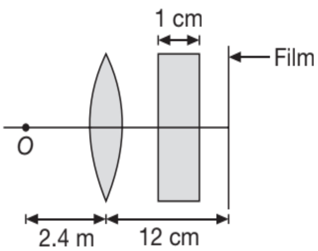
Hence, the correct answer is (D).

46. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is as shown in the adjacent figure.



Hence, the correct answer is (D).

47.



According to thin lens formula, we have

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Since, $u = -2.4$ m $= -240$ cm, $v = 12$ cm

$$\Rightarrow \frac{1}{f} = \frac{1}{12} - \frac{1}{(-240)} = \frac{1}{12} + \frac{1}{240}$$

$$\Rightarrow \frac{1}{f} = \frac{21}{240}$$

$$\Rightarrow f = \frac{240}{21} \text{ cm}$$

When a glass plate is interposed between lens and film, so shift produced by the glass plate will be

$$\text{Shift} = t\left(1-\frac{1}{\mu}\right) = 1\left(1-\frac{1}{1.5}\right) = 1\left(1-\frac{2}{3}\right) = \frac{1}{3} \text{ cm}$$

To get image at film, lens should form an image at a distance

$$v' = 12 - \frac{1}{3} = \frac{35}{3} \text{ cm}$$

Again, using lens formula, we get

$$\frac{21}{240} = \frac{3}{35} - \frac{1}{u'}$$

$$\Rightarrow \frac{1}{u'} = \frac{3}{35} - \frac{21}{240} = \frac{1}{5}\left[\frac{3}{7} - \frac{21}{48}\right]$$

$$\Rightarrow \frac{1}{u'} = \frac{1}{5}\left[\frac{144-147}{336}\right]$$

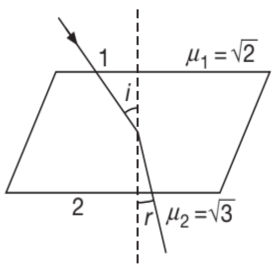
$$\Rightarrow \frac{1}{u'} = -\frac{3}{1680}$$

$$\Rightarrow u' = -560 \text{ cm} = -5.6 \text{ m}$$

$$\Rightarrow |u'| = 5.6 \text{ m}$$

Hence, the correct answer is (C).

48.



Since, $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$

$$\Rightarrow \cos i = \frac{10}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + (-10)^2}} = \frac{10}{20}$$

$$\Rightarrow \cos i = \frac{1}{2}$$

$$\Rightarrow i = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

Using Snell's Law, $\mu_1 \sin i = \mu_2 \sin r$, we get

$$\sqrt{2}\sin(60^\circ) = \sqrt{3}\sin r$$

$\Rightarrow \quad r = 45^\circ$

Hence, the correct answer is (B).

49. Since $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$$

$$\Rightarrow \quad \frac{dv}{dt} = -\frac{v^2}{u^2}\left(\frac{du}{dv}\right)$$

where $\frac{dv}{dt}$ is velocity of image and $\frac{du}{dt}$ is velocity of object.

Given that, $f = 20$ cm

$$\Rightarrow \quad \frac{1}{u} + \frac{1}{(-280)} = \frac{1}{20}$$

$$\Rightarrow \quad v = \frac{280}{15}\text{ cm}$$

So, velocity of image is given by

$$\frac{dv}{dt} = -\left(\frac{280}{15\times 280}\right)^2 \times 15$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{1}{15}\text{ ms}^{-1}$$

Hence, the correct answer is (B).

50. As the beam is initially parallel, the shape of wave-front is planar.

Hence, the correct answer is (A).

51. Given $\mu = \mu_0 + \mu_2 I$

$$\text{As } \mu = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$$

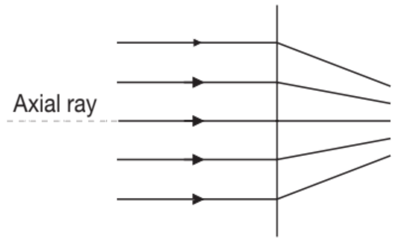
$$\mu = \frac{c}{v}$$

$$\Rightarrow \quad v = \frac{c}{\mu} = \frac{c}{\mu_0 + \mu_2 I}$$

As the intensity is maximum on the axis of the beam, therefore v is minimum on the axis of the beam.

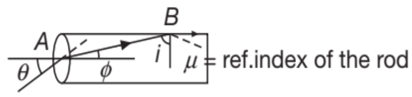
Hence, the correct answer is (B).

52. When the beam enters the medium, the axial ray will travel slowest and hence it will lag behind. To compensate for the path, the rays will bend towards the axis.



Hence, the correct answer is (C).

53. For light ray to graze along the wall of rod, $i = C$



where, C is the critical angle given by

$$C = \sin^{-1}\left(\frac{1}{\mu}\right)$$

Since $C = 90^\circ - \phi$

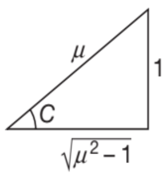
$\Rightarrow \quad \phi = 90^\circ - C$

Applying Snell's Law at A, we get

$$\frac{\sin\theta}{\sin\phi} = \mu = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{\sin\theta}{\cos C} = \mu \qquad \{\because \phi = 90 - C\}$$

Since, $\cos C = \frac{\sqrt{\mu^2 - 1}}{\mu}$



$$\Rightarrow \quad \sin\theta = \mu\frac{\sqrt{\mu^2 - 1}}{\mu} = \sqrt{\mu^2 - 1}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\sqrt{\frac{4}{3} - 1}\right) = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Hence, the correct answer is (D).

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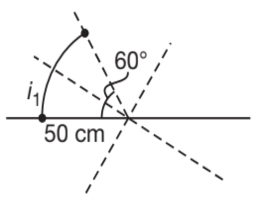
Single Correct Choice Type Problems

1. Image formed by the lens is at $(75, 0)$

This acts as a virtual image for the mirror



Assuming that the mirror is not tilted, then for mirror

$u = +25$, $v = ?$, $f = +50$

$\Rightarrow \frac{1}{v} + \frac{1}{25} = \frac{1}{50}$

$\Rightarrow v = -50$ cm ...(1)

This image is formed on x-axis

Now, when the mirror is rotated clockwise by $30°$, image rotates by $60°$ clockwise. Since the ray strikes the pole of the mirror, so this ray rotates by $60°$ and image lies on this ray. Hence image rotates by $60°$.

New co-ordinates of image will be

$x = 50 - 50\cos(60°) = 25$ cm

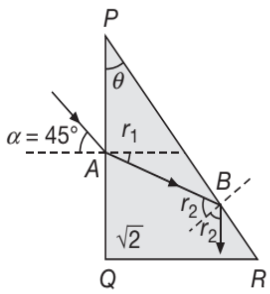
$y = 50\sin(60°) = \frac{50\sqrt{3}}{2}$ cm $= 25\sqrt{3}$ cm

Hence, the correct answer is (A).

2. For Refraction at A, we have

$\frac{\sin(45°)}{\sin r_1} = \sqrt{2}$

$\Rightarrow \sin r_1 = \frac{1}{2}$



$\Rightarrow r_1 = 30°$

For TIR at B, we have

$r_2 \geq C$

Since, $\sin C = \frac{1}{\mu} = \frac{1}{\sqrt{2}}$

$\Rightarrow C = 45°$

$\Rightarrow r_2 \geq 45°$

In ΔPAB, $(90° + r_1) + \theta + (90° - r_2) = 180°$

$\Rightarrow r_2 = r_1 + \theta$

$\Rightarrow 45° = 30° + \theta$

$\Rightarrow \theta = 15°$

Hence, the correct answer is (A).

3. $R = 10$ cm

Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\frac{1}{v} - \frac{1.5}{-50} = \frac{1 - 1.5}{-10}$

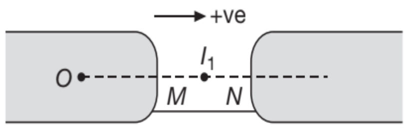
$\Rightarrow \frac{1}{v} + \frac{1.5}{50} = \frac{0.5}{10}$

$\Rightarrow \frac{1}{v} = \frac{0.5}{10} - \frac{1.5}{50} = \frac{2.5 - 1.5}{50}$

$\Rightarrow v = 50$

Now, $MN = d$, $MI_1 = 50$ cm,

$NI_1 = (d - 50)$ cm



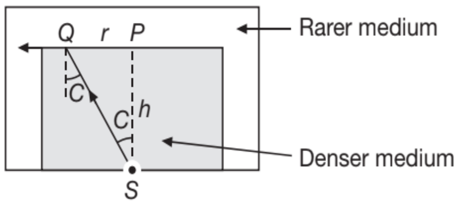
Again, $\frac{1.5}{\infty} - \frac{1}{-(d-50)} = \frac{1.5 - 1}{10}$

$\frac{1}{d - 50} = \frac{1}{20}$

$d = 70$

Hence, the correct answer is (B).

4. At point Q angle of incidence is critical angle C, where



$\sin C = \frac{\mu_\ell}{\mu_{block}}$

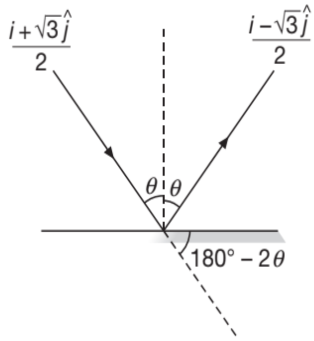
In ΔPQS, $\sin C = \frac{r}{\sqrt{r^2 + h^2}}$

$\Rightarrow \frac{\mu_\ell}{\mu_{block}} = \frac{r}{\sqrt{r^2 + h^2}}$

$\Rightarrow \mu_\ell = \frac{r}{\sqrt{r^2 + h^2}} \times 2.72 = \frac{5.77}{11.54} \times 2.72 = 1.36$

Hence, the correct answer is (C).

5. $$\cos(180^\circ - 2\theta) = \frac{\left(\frac{i+\sqrt{3}\hat{j}}{2}\right)\left(\frac{i-\sqrt{3}\hat{j}}{2}\right)}{\left|\frac{i+\sqrt{3}\hat{j}}{2}\right|\left|\frac{i-\sqrt{3}\hat{j}}{2}\right|}$$



$$\Rightarrow \quad -\cos 2\theta = \frac{\frac{(1-3)}{4}}{1}$$

$$\Rightarrow \quad -\cos 2\theta = \frac{-1}{2}$$

$$\Rightarrow \quad \cos 2\theta = \frac{1}{2}$$

$$\Rightarrow \quad 2\theta = 60^\circ$$

$$\Rightarrow \quad \theta = 30^\circ$$

Hence, the correct answer is (A).

6. $$\mu = \frac{c}{v_{\text{med}}} = \frac{f\lambda_{\text{air}}}{f\lambda_{\text{med}}} = \frac{3}{2}$$

Now, $v = +8\text{ m}$, $m = -\frac{1}{3} = \frac{v}{u}$

$$\Rightarrow \quad u = -24\text{ m}$$

Since, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{8} + \frac{1}{24} = \frac{4}{24}$$

$$\Rightarrow \quad f = 6\text{ m}$$

For a plano-convex lens, we have

$$f = \frac{R}{\mu - 1}$$

$$\Rightarrow \quad 6 = \frac{R}{0.5}$$

$$\Rightarrow \quad R = 3\text{ m}$$

Hence, the correct answer is (C).

7. $$\frac{1}{f} = (1.5-1)\left(\frac{1}{14} - \frac{1}{\infty}\right) + (1.2-1)\left(\frac{-1}{\infty} - \frac{1}{-14}\right)$$

$$\Rightarrow \quad f = 20\text{ cm}$$

Since, object distance $= 40\text{ cm} = 2f$

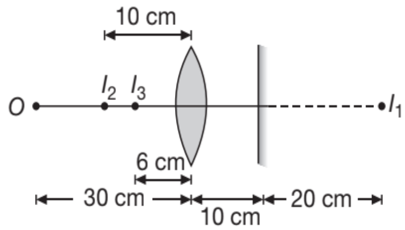
$\Rightarrow$ Image distance $= 40\text{ cm}$

Hence, the correct answer is (B).

8. When $\theta < C$ partial transmission and reflection will occur. When $\theta > C$, only reflection takes place.

Hence, the correct answer is (C).

9. Since object is placed at distance $2f$ from the lens, so first image I_1 will be formed at distance $2f$ on other side. This image I_1 will behave like a virtual object for mirror. The second image I_2 will be formed at distance 20 cm in front of the mirror, or at distance 10 cm to the left hand side of the lens.



Now, applying lens formula, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

where, $v = ?$, $u = +10\text{ cm}$, $f = +15\text{ cm}$

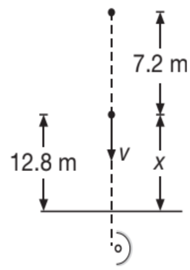
$$\Rightarrow \quad \frac{1}{v} - \frac{1}{+10} = \frac{1}{+15}$$

$$\Rightarrow \quad v = 6\text{ cm}$$

So, the final image is at distance 16 cm from the mirror. But, this image will be real, because the ray of light is travelling from right to left.

Hence, the correct answer is (B).

10. $$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 7} = 12\text{ ms}^{-1}$$



Position of ball with respect to fish is

$$x_{\text{app}} = \mu x$$

$$\Rightarrow \quad \frac{dx_{\text{app}}}{dt} = \mu\left(\frac{dx}{dt}\right)$$

$$\Rightarrow \quad v_{\text{app}} = \mu v = \frac{4}{3} \times 12 = 16\text{ ms}^{-1}$$

Hence, the correct answer is (C).

11. At minimum deviation $(\delta = \delta_m)$

$$r_1 = r_2 = \frac{A}{2} = \frac{60^\circ}{2} = 30^\circ \qquad \text{(For both colours)}$$

Hence, the correct answer is (A).

12. Critical angle from Region III to Region IV

$$\sin C = \frac{\frac{n_0}{8}}{\frac{n_0}{6}} = \frac{3}{4}$$

Now, applying Snell's law in Region I and Region III, we get

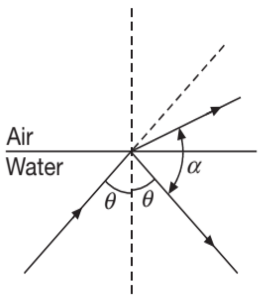
$$n_0 \sin\theta = \frac{n_0}{6}\sin C$$

$$\Rightarrow \quad \sin\theta = \frac{1}{6}\sin C = \frac{1}{6}\left(\frac{3}{4}\right) = \frac{1}{8}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{1}{8}\right)$$

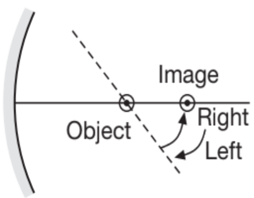
Hence, the correct answer is (B).

13. Since $\theta < C$, both reflection and refraction will take place. From the figure we can see that angle between reflected and refracted rays α is less than $180^\circ - 2\theta$.



Hence, the correct answer is (C).

14. Since object and image move in opposite directions, the positioning should be as shown in the figure.



Object lies between focus and centre of curvature $f < x < 2f$.

Hence, the correct answer is (B).

15. For refraction from lens, we have $\frac{1}{v_1} - \frac{1}{(-20)} = \frac{1}{15}$

$$\Rightarrow \quad v_1 = 60 \text{ cm}$$

i.e., first image is formed at 60 cm to the right of lens system.

Reflection from mirror

After reflection from the mirror, the second image will be formed at a distance of 60 cm to the left of lens system.

Refraction from lens

$$\frac{1}{v_3} - \frac{1}{60} = \frac{1}{15}$$

$$\Rightarrow \quad v_3 = 12 \text{ cm}$$

Therefore, the final image is formed at 12 cm to the left of the lens system.

Hence, the correct answer is (C).

16. From the lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, we have

$$\frac{1}{f} = \frac{1}{10} - \frac{1}{-10}$$

$$\Rightarrow \quad f = +5 \text{ cm}$$

Further from the graph, $\Delta u = 0.1$ cm and $\Delta v = 0.1$ cm

Now, differentiating the lens formula we get

$$\frac{\Delta f}{f^2} = \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}$$

$$\Rightarrow \quad \Delta f = \left(\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}\right)f^2$$

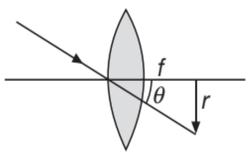
Substituting the values, we get

$$\Delta f = \left(\frac{0.1}{10^2} + \frac{0.1}{10^2}\right)(5)^2 = 0.05 \text{ cm}$$

$$\Rightarrow \quad f \pm \Delta f = (5 \pm 0.05) \text{ cm}$$

Hence, the correct answer is (B).

17.



$$r = f\tan\theta$$

$$\Rightarrow \quad r \propto f$$

$$\Rightarrow \quad \pi r^2 \propto f^2$$

Hence, the correct answer is (B).

18. Let focal length of convex lens is $+f$, then length of concave lens would be $-\frac{3}{2}f$.

Since $\frac{1}{f_{comb}} = \frac{1}{f_1} + \frac{1}{f_2}$

$\Rightarrow \quad \frac{1}{30} = \frac{1}{f} - \frac{2}{3f} = \frac{1}{3f}$

$\Rightarrow \quad f = 10 \text{ cm}$

Therefore, focal length of convex lens is $+10$ cm and that of concave lens is -15 cm

Hence, the correct answer is (D).

19. Distance of object from mirror is $15 + \frac{33.25}{1.33} = 40$ cm

Distance of image from mirror is $15 + \frac{25}{1.33} = 33.8$ cm

Applying mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$\Rightarrow \quad \frac{1}{-33.8} + \frac{1}{-40} = \frac{1}{f}$

$\Rightarrow \quad f = -18.3$ cm

Hence, the correct answer is (C).

20. Critical angle $C = \sin^{-1}\left(\frac{1}{\mu}\right)$

Wavelength increases in the sequence of VIBGYOR. According to Cauchy's formula refractive index (μ) decreases as the wavelength increases. Hence the refractive index will increase in the sequence of ROYGBIV. The critical angle C will thus increase in the same order VIBGYOR.

For green light the incidence angle is just equal to the critical angle. For yellow, orange and red the critical angle will be greater than the incidence angle. So these colours will emerge from the glass air interface.

Hence, the correct answer is (A).

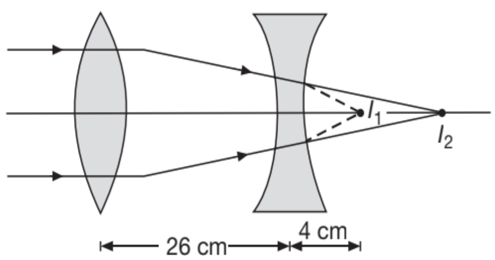
21. For minimum deviation the ray inside the prism is parallel to the base of the prism in case of an equilateral prism.

Hence, the correct answer is (B).

22. When the object is placed at the centre of the glass sphere, the rays from the object fall normally at the surface of the sphere and emerge undeviated.

Hence, the correct answer is (C).

23. Image formed by convex lens at I_1 will act as a virtual object for concave lens.



For concave lens applying $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$\frac{1}{v} - \frac{1}{4} = \frac{1}{-20}$

$\Rightarrow \quad v = 5$ cm

Magnification for concave lens

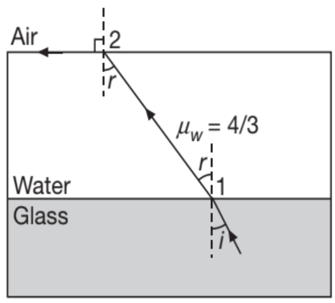
$m = \frac{v}{u} = \frac{5}{4} = 1.25$

Since, size of the image at I_1 is 2 cm, therefore, size of image at I_2 will be $2 \times 1.25 = 2.5$ cm

Hence, the correct answer is (B).

24. Applying Snell's Law $(\mu \sin i = \text{constant})$ at 1 and 2, we get

$\mu_1 \sin i_1 = \mu_2 \sin i_2$



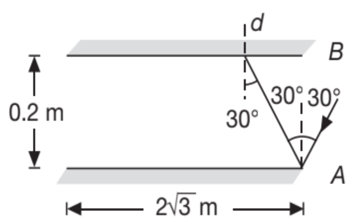
$\Rightarrow \quad \mu_g \sin i = \mu_w \sin r = 1 \sin(90^\circ)$

$\Rightarrow \quad \mu_g \sin i = (1)(\sin 90^\circ)$

$\Rightarrow \quad \mu_g = \frac{1}{\sin i}$

Hence, the correct answer is (B).

25. $\frac{d}{0.2} = \tan(30^\circ)$



$\Rightarrow \quad d = \frac{0.2}{\sqrt{3}} = \frac{2}{10\sqrt{3}}$

Total number of reflections $= n = \frac{\ell}{d} = \frac{2\sqrt{3}}{\left(\frac{2}{10\sqrt{3}}\right)}$

$\Rightarrow \quad n = 30$

Hence, the correct answer is (B).

26. $\dfrac{1}{f}=(\mu-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)$

For no dispersion, we have $d\left(\dfrac{1}{f}\right)=0$

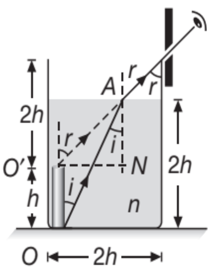
$\Rightarrow \quad d\mu\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)=0$

$\Rightarrow \quad R_1=R_2$

Hence, the correct answer is (C).

27. $\dfrac{\sin i}{\sin r}=\dfrac{1}{n}$

Since



$\tan r=\dfrac{2h}{2h}=1$

$\Rightarrow \quad r=45°$

$\Rightarrow \quad \sin i=\dfrac{h}{h\sqrt{5}}$

$\Rightarrow \quad \sin i=\dfrac{1}{\sqrt{5}}$

$\Rightarrow \quad \dfrac{1}{n}=\dfrac{\frac{1}{\sqrt{5}}}{\frac{1}{\sqrt{2}}}$

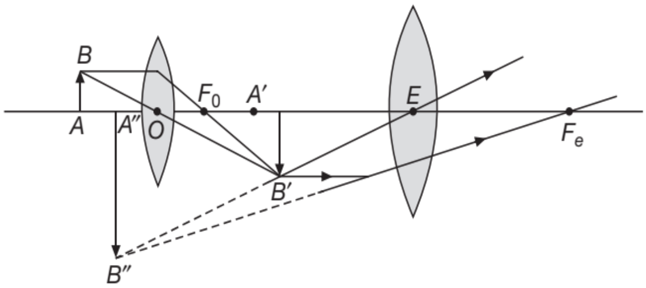
$\Rightarrow \quad n=\sqrt{\dfrac{5}{2}}$

Hence, the correct answer is (B).

28. *CD* is parallel to *AB*, so both media must have equal refractive indices.
Hence, the correct answer is (D).

29. Since all the prisms *P*, *Q* and *R* are made of same material, so the deviation suffered by the combination remains the same as the combination forms a part of the bigger sphere.
Hence, the correct answer is (C).

30. The ray diagram is shown in Figure.



From the figure it is clear that image formed by objective (or the intermediate image) is real, inverted and magnified.
Hence, the correct answer is (C).

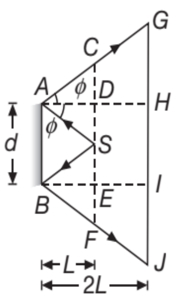
31. The lens makers' formula is

$$\frac{1}{f}=\left(\frac{n_L}{n_m}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

where $n_L=$ Refractive index of lens and $n_m=$ Refractive index of medium.
In case of double concave lens, R_1 is negative and R_2 is positive. Therefore $\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)$ will be negative.
For the lens to be diverging in nature, focal length f should be negative or $\left(\dfrac{n_L}{n_m}-1\right)$ should be positive or $n_L>n_m$ but since $n_2>n_1$ (given), therefore the lens should be filled with L_2 and immersed in L_1.
Hence, the correct answer is (D).

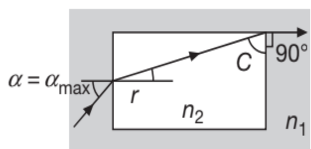
32. Since rays after passing through the glass slab just suffer lateral displacement hence we have angle between the emergent rays as α.
Hence, the correct answer is (B).

33.



Hence, the correct answer is (D).

34.



$$\frac{\sin\alpha}{\sin r}=\frac{n_1}{n_2}$$

$$\Rightarrow \quad \sin\alpha=\frac{n_1}{n_2}\sin r \qquad \ldots(1)$$

For TIR at other end

$$\Rightarrow \quad \sin C=\frac{n_2}{n_1}$$

Also $C=90-r$

$$\Rightarrow \quad \sin\alpha=\frac{n_1}{n_2}\sin(90-C) \qquad \{\text{Put } r=90-C \text{ in (1)}\}$$

$$\Rightarrow \quad \sin\alpha=\frac{n_1}{n_2}\cos C$$

$$\Rightarrow \quad \sin\alpha=\frac{n_1}{n_2}\left[\cos\left[\sin^{-1}\left(\frac{n_2}{n_1}\right)\right]\right]$$

Hence, the correct answer is (A).

35. $\frac{1}{f'}=\left(\frac{1.5}{1.75}-1\right)\left(-\frac{2}{R}\right)$

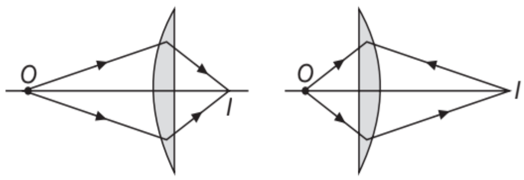
$$\Rightarrow \quad \frac{1}{f'}=\frac{0.25}{1.75}\times\frac{2}{R}$$

$$\Rightarrow \quad f'=\frac{R}{2}\left(\frac{175}{25}\right)$$

$$\Rightarrow \quad f'=3.5R$$

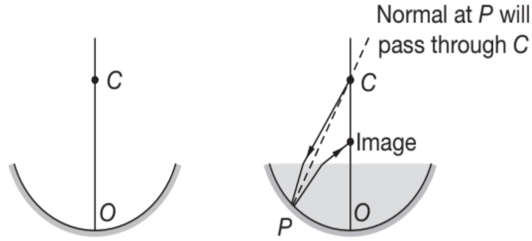
Hence, the correct answer is (A).

36. In general spherical aberration is minimum when the total deviation produced by the system is equally divided on all refracting surfaces. A planoconvex lens is used for this purpose. In order that the total deviation be equally divided on two surfaces, it is essential that more parallel beam (or the incident and refracted) be incident on the convex side. Thus, when the object is far away from the lens, incident rays will be more parallel than the refracted rays, therefore, the object should face the convex side, but if the object is near the lens, the object should face the plane side. This has been shown in figure.



Hence, the correct answer is (B).

37. The ray diagram is shown in figure. Therefore, the image will be real and between C and O.



Hence, the correct answer is (D).

38. $\frac{1}{-(-x)}+\frac{1.5}{x}=\frac{1.5-1}{R}$

$$\Rightarrow \quad \frac{2.5}{x}=\frac{0.5}{R}$$

$$\Rightarrow \quad x=5R$$

Hence, the correct answer is (A).

39. The focal length of combination is given by

$$\frac{1}{f_{\text{comb}}}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$\Rightarrow \quad \frac{1}{f_{\text{comb}}}=\frac{1}{40}-\frac{1}{25}$$

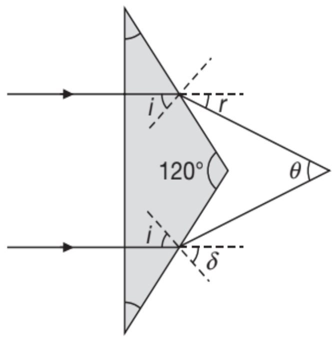
$$\Rightarrow \quad f_{\text{comb}}=-\frac{200}{3}\text{ cm}=-\frac{2}{3}\text{ m}$$

Power of the combination (in dioptre) is $P=\frac{1}{f}$

$$\Rightarrow \quad P=-\frac{3}{2}=-1.5 \qquad \left\{P=\frac{1}{f(m)}\right\}$$

Hence, the correct answer is (B).

40. The diagramatic representation of the given problem is shown in figure.



From figure it follows that $\angle i=\angle A=30°$

From Snell's Law, we get

$$n_1\sin i=n_2\sin r$$

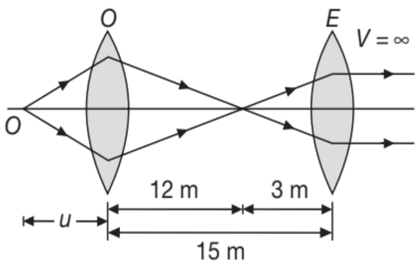
$$\Rightarrow \quad \sin r=\frac{1.44\sin(30°)}{1}=0.72$$

Now, $\angle\delta = \angle r - \angle i = \sin^{-1}(0.72) - 30°$

$\Rightarrow \quad \theta = 2(\angle\delta) = 2\left(\sin^{-1}(0.72) - 30°\right)$

Hence, the correct answer is (C).

41. Since, the final image is formed at infinity, the image formed by the objective will be at the focal point of the eye-piece, which is 3 cm. The image formed by the objective will be at a distance of 12 cm $(= 15\text{ cm} - 3\text{ cm})$ from the objective.



If u is the distance of the object from the objective, then by applying lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we get

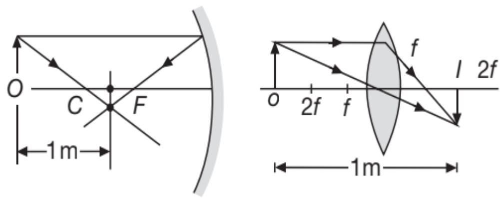
$$\frac{1}{12} - \frac{1}{u} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{1}{u} = \frac{1}{12} - \frac{1}{2} = \frac{2-12}{24} = -\frac{10}{24}$$

$\Rightarrow \quad u = -2.4$ cm

Hence, the correct answer is (A).

42. Image can be formed on the screen if it is real. Real image of reduced size can be formed by a concave mirror or a convex lens as shown in figure.



A diminished real image is formed by a convex lens when the object is placed beyond $2f$ and the image of such object is formed beyond $2f$ on other side.

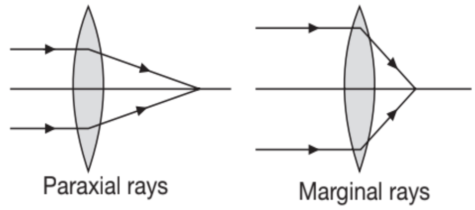
Thus, $d > (2f + 2f)$

$\Rightarrow \quad 4f < 0.1$ m

$\Rightarrow \quad f < 0.25$ m

Hence, the correct answer is (C).

43. Spherical aberration is caused due to spherical nature of lens. Paraxial and marginal rays are focussed at different places on the axis of the lens. Therefore, image so formed is blurred. This aberration can be reduced by either stopping paraxial rays or marginal rays, which can be done by using a circular annular mask over the lens.



Hence, the correct answer is (C).

44. For the refraction through the first lens, we have $u \to \infty$, so

$$v_1 = f_1$$

Since, $d < f_2$, the first image (formed by L_1) lies to the right of second lens L_2, so

$$u_2 = +(f_1 - d)$$

Applying Lens Formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v_2} - \frac{1}{(f_1 - d)} = \frac{1}{f_2}$$

$$\Rightarrow \quad v_2 = \frac{f_2(f_1 - d)}{f_1 + f_2 - d}$$

$$\Rightarrow \quad x = v_2 + d = \frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$$

Magnification for second lens is given by

$$m = \frac{v_2}{u_2} = \frac{f_2}{f_1 + f_2 - d}$$

The image due to the second lens is formed below its principal axis and is of the size mb. So, the y coordinate of the focal point system is given by

$$y = b - mb$$

$$\Rightarrow \quad y = b - \frac{f_2 b}{f_1 + f_2 - d}$$

$$\Rightarrow \quad y = \frac{(f_1 - d)b}{f_1 + f_2 - d}$$

So, the coordinates of the focal point of this system are

$$(x, y) = \left[\frac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}, \frac{(f_1 - d)b}{f_1 + f_2 - d}\right]$$

Hence, the correct answer is (C).

CHAPTER 1

45. $A_2 = \left(\dfrac{n_1 - 1}{n_2 - 1}\right) A_1$

$\Rightarrow \quad A_2 = \left(\dfrac{1.54 - 1}{1.72 - 1}\right) 4$

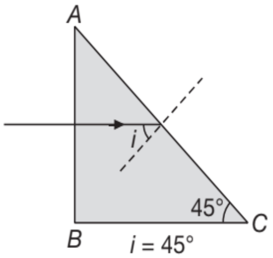
$\Rightarrow \quad A_2 = 3°$

Hence, the correct answer is (C).

46. The colours for which $i > C$, will get total internal reflection :

$i > C$

$\Rightarrow \quad \sin i > \sin C$



$\Rightarrow \quad \sin 45° > \dfrac{1}{\mu}$

$\Rightarrow \quad \dfrac{1}{\sqrt{2}} > \dfrac{1}{\mu}$

or for which $\mu > \sqrt{2}$ or $\mu > 1.414$.

Hence, the rays for which $\mu > 1.414$ will get TIR.

For green and blue $\mu > 1.414$, so they will suffer TIR on face AC only red comes out from this face.

Hence, the correct answer is (A).

47. $f_o + f_e = 36$ cm ...(1)

$\dfrac{f_o}{f_e} = 5$ ...(2)

$\Rightarrow \quad f_o = 30$ cm and $f_e = 6$ cm.

Hence, the correct answer is (D).

48. From the mirror formula

$\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f} \qquad (f = \text{constant})$ ...(1)

$-v^{-2}dv - u^{-2}du = 0$

$\Rightarrow \quad |dv| = \left|\dfrac{v^2}{u^2}\right| |du|$ ...(2)

Here, $|dv|$ = size of image.

$|du|$ = size of object (short) lying along the axis $= b$

Further, from equation (1), we can find

$\dfrac{v^2}{u^2} = \left(\dfrac{f}{u - f}\right)^2$

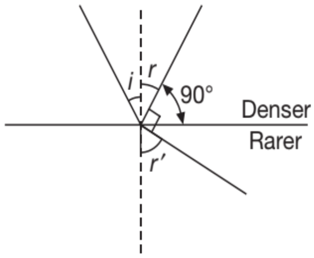
Substituting these values in equation (2), we get

size of image $= b\left(\dfrac{f}{u - f}\right)^2$

Hence, the correct answer is (D).

49. $r + r' + 90° = 180°$

$\Rightarrow \quad r' = 90 - r$



Further, $i = r$

Applying Snell's Law,

$\mu_D \sin i = \mu_R \sin r'$

$\Rightarrow \quad \mu_D \sin r = \mu_R \sin(90 - r) = \mu_R \cos r$

$\Rightarrow \quad \dfrac{\mu_R}{\mu_D} = \tan r$

$\theta_C = \sin^{-1}\left(\dfrac{\mu_R}{\mu_D}\right) = \sin^{-1}(\tan r)$

Hence, the correct answer is (A).

50. $\dfrac{1}{f} = \dfrac{1}{40} + \dfrac{1}{-25}$

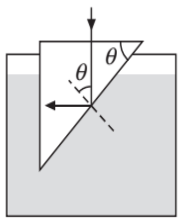
$\Rightarrow \quad f = \dfrac{-40 \times 25}{40 - 25}$

$\Rightarrow \quad f = -66.67$ cm

$\Rightarrow \quad P = \dfrac{10}{f(\text{in cm})} = -1.5$ D

Hence, the correct answer is (A).

51. $\sin C = \dfrac{\mu_r}{\mu_d} = \dfrac{4/3}{3/2} = \dfrac{8}{9}$



$\Rightarrow \quad C = \sin^{-1}\left(\frac{8}{9}\right)$

For TIR

$i > C$

$\Rightarrow \quad \theta > C$

$\Rightarrow \quad \sin\theta > \sin C$

$\Rightarrow \quad \sin\theta > \frac{8}{9}$

Hence, the correct answer is (A).

52. According to Snell's Law

$\mu\lambda = \text{constant}$

$\Rightarrow \quad \mu_a\lambda_a = \mu_g\lambda_g$

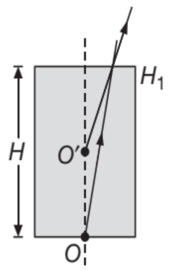
$\Rightarrow \quad \lambda_g = \frac{\mu_a\lambda_a}{\mu_g}$

$\Rightarrow \quad \lambda_g < \lambda_a$

Hence, the correct answer is (A).

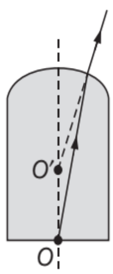
Multiple Correct Choice Type Problems

1. CASE-1:



$$H_1 = \frac{H}{\mu} = \frac{30 \times 2}{3} = 20 \text{ cm (below)}$$

CASE-2:



$$\frac{1}{V} + \frac{3}{2H} = \frac{1-1.5}{-R}$$

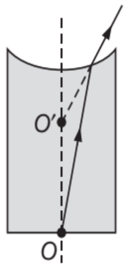
$$\Rightarrow \quad \frac{1}{V} + \frac{1}{20} = \frac{1}{2 \times 300}$$

$$\Rightarrow \quad \frac{1}{V} = \frac{1}{600} - \frac{1}{20}$$

$$\Rightarrow \quad \frac{1}{V} = \frac{-29}{600}$$

$$\Rightarrow \quad H_2 = \frac{600}{29} = 20.68 \text{ cm (below)}$$

CASE-3:



$$\frac{1}{V} + \frac{3}{2H} = \frac{-1}{2 \times 300}$$

$$\Rightarrow \quad \frac{1}{V} = \frac{-1}{600} - \frac{1}{20}$$

$$\Rightarrow \quad \frac{1}{V} = \frac{-31}{600}$$

$$\Rightarrow \quad H_3 = \frac{600}{31} = 19.35 \text{ cm}$$

$$H_2 - H_1 \simeq 20.68 - 20 \simeq 0.68$$

Since $H_2 > H_1$ and $H_2 > H_3$

$\Rightarrow \quad H_1 > H_3$

Hence, (B) and (D) are correct.

2. For the combination of lens, we have

$$\frac{1}{f} = (n_1 - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) + (n_2 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$$

$$\Rightarrow \quad \frac{1}{f} = \frac{(n_1-1)}{R} + \frac{(n_2-1)}{R} = \frac{(n_1+n_2-2)}{R} \quad \ldots(1)$$

For $n_1 = n_2 = n$, we get

$$\frac{1}{f} = (n-1)\frac{2}{R} \quad \ldots(2)$$

$$\Rightarrow \quad -\frac{\Delta f}{f^2} = \frac{\Delta n}{R} \quad \ldots(3)$$

Since, from (1), we get

$\frac{f}{R} = \frac{1}{n_1 + n_2 - 2}$, so, equation (3) becomes

$$-\frac{\Delta f}{f} = \frac{\Delta n}{(n_1 + n_2 - 2)}$$

CHAPTER 1

Since $n_1 = n$ and $n_2 = n + \Delta n$, so $\frac{\Delta f}{f} = \frac{\Delta n}{(2n + \Delta n - 2)}$

When $n_1 = n_2 = 1.5$, $\Delta n = 10^{-3}$, $f = 20$ cm we get

$R = 20$ cm

$$|\Delta f| = \frac{10^{-3} \times 20}{(2 \times 1.5 + 10^{-3} - 2)} \approx 0.02 \text{ cm}$$

When $\frac{\Delta n}{n} < 0$ then, $\frac{\Delta f}{f} > 0$

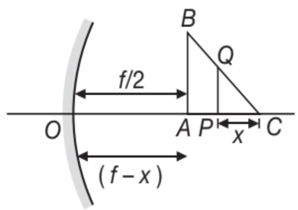
When the surfaces are replaced by concave surfaces of same radius of curvature, then but has same magnitude such that

$$-\frac{\Delta f}{f} \approx \frac{\Delta n}{(2n-2)} = \frac{\Delta n}{2(n-1)}$$

i.e., both $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remain unchanged.

Hence, (B), (C) and (D) are correct.

3. For image of point A, we have $u = -\frac{f}{2}$, $v = ?$, $f = -f$



Since $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \frac{1}{v} + \frac{1}{-\left(\frac{f}{2}\right)} = \frac{1}{-f}$$

$$\Rightarrow v = f$$

$$\Rightarrow \frac{I_{AB}}{AB} = -\frac{f}{-\frac{f}{2}}$$

$$\Rightarrow I_{AB} = 2AB$$

For height of PQ, we have $u = -(f - x)$, $v = ?$, $f = -f$

Since $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \frac{1}{v} + \frac{1}{-(f-x)} = \frac{1}{-f}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{(f-x)} - \frac{1}{f}$$

$$\Rightarrow v = \frac{f(f-x)}{x}$$

$$\Rightarrow \frac{I_{PQ}}{PQ} = -\frac{f(f-x)}{x[-(f-x)]} = \left(\frac{f}{x}\right)$$

$\Rightarrow I_{PQ} = \frac{f}{x} PQ$ Since from triangle ABC and PQC, we have

$$\frac{AB}{\left(\frac{f}{2}\right)} = \frac{PQ}{x}$$

$$\Rightarrow PQ = \frac{2(AB)x}{f}$$

$$\Rightarrow I_{PQ} = \frac{f}{x}\left[\frac{2(AB)x}{f}\right] = 2AB$$

$$\Rightarrow I_{PQ} = 2AB$$

So, Size of image is independent of x and hence the final image will be of same height terminating at infinity.

Hence, the correct answer is (D).

4. The minimum deviation produced by a prism

$$\delta_m = 2i - A = A$$

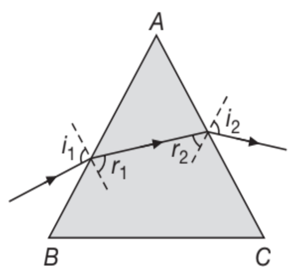
$$\Rightarrow i_1 = i_2 = A \text{ and } r_1 = r_2 = \frac{A}{2}$$

$$\Rightarrow r_1 = \frac{i_1}{2}$$

Now, using Snell's law

$$\sin A = \mu \sin\left(\frac{A}{2}\right)$$

$$\Rightarrow \mu = 2\cos\left(\frac{A}{2}\right)$$



For this prism when the emergent ray at the second surface is tangential to the surface

$$i_2 = \frac{\pi}{2}$$

$$\Rightarrow r_2 = \theta_c$$

$$\Rightarrow r_1 = A - \theta_c$$

$\Rightarrow \quad \sin i_1 = \mu \sin(A - \theta_c)$

$\Rightarrow \quad i_i = \sin^{-1}\left[\sin A\sqrt{4\cos^2\left(\frac{A}{2}\right) - 1} - \cos A\right]$

For minimum deviation through isosceles prism, the ray inside the prism is parallel to the base of the prism if $\angle B = \angle C$.

But it is not necessarily parallel to the base if,

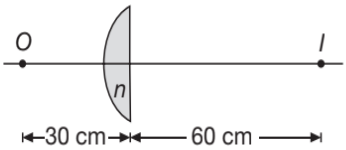
$\angle A = \angle B$

$\Rightarrow \quad \angle A = \angle C$

Hence, (A), (B) and (C) are correct.

5. Since $m = \frac{v}{u} = 2$

$\Rightarrow \quad v = 60$ cm



Using lens formula, we get

$$\frac{1}{60} - \frac{1}{(-30)} = \frac{1}{f_1}$$

$\Rightarrow \quad \frac{1}{f_1} = \frac{1}{60} + \frac{2}{60}$

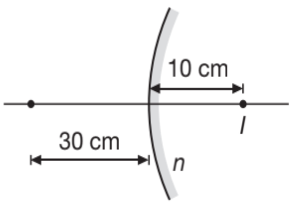
$\Rightarrow \quad f_1 = 20$ cm

Further, $\frac{1}{f_1} = (n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$

$\Rightarrow \quad f_1 = \frac{R}{n-1}$

Since $f_1 = 20$ cm

$\Rightarrow \quad \frac{R}{n-1} = +20$ cm



Now using mirror formula, we get

$$\frac{1}{10} + \frac{1}{-30} = \frac{1}{f_2}$$

$\Rightarrow \quad \frac{3}{30} - \frac{1}{30} = \frac{1}{f_2} = \frac{2}{30}$

$\Rightarrow \quad f_2 = 15 = \frac{R}{2}$

$\Rightarrow \quad R = 30$ cm

$\Rightarrow \quad \frac{R}{n-1} = 20 \text{ cm} = \frac{30}{n-1}$

$\Rightarrow \quad \frac{30}{n-1} = 20$

$\Rightarrow \quad 2n - 2 = 3$

$\Rightarrow \quad n = 2.5$

$\Rightarrow \quad f_1 = +20$ cm

So, refractive index of lens is 2.5, radius of curvature of convex surface is 30 cm and the faint image is erect and virtual focal length of lens is 20 cm.

Hence, (A) and (D) are correct.

6. For, film, $\frac{1}{f_{\text{film}}} = (n_1 - 1)\left(\frac{1}{R} - \frac{1}{R}\right)$

$\Rightarrow \quad f_{\text{film}} \to \infty$

$\Rightarrow \quad P = 0$

$\Rightarrow$ There is no effect of presence of film.

From Air to Glass

Using the equation $\frac{n_2}{v} - \frac{1}{u} = \frac{n_2 - 1}{R}$, we get

$$\frac{1.5}{v} - \frac{1}{\infty} = \frac{1.5 - 1}{R}$$

$\Rightarrow \quad v = 3R$

Since parallel rays will meet at focus, so

$f_1 = 3R$

From Glass to Air

Again using the same equation, $\frac{1}{v} - \frac{n_2}{u} = \frac{1 - n_2}{-R}$, we get

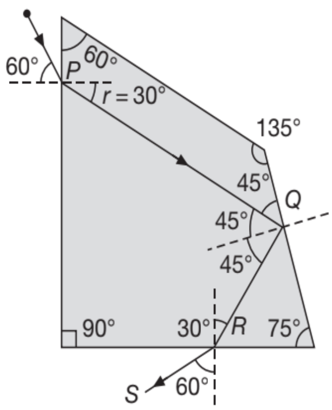
$$\frac{1}{v} - \frac{1.5}{\infty} = \frac{1 - 1.5}{-R}$$

$\Rightarrow \quad v = 2R$

$\Rightarrow \quad f_2 = 2R$

Hence, (A) and (C) are correct.

7. $\sqrt{3} = \frac{\sin(60°)}{\sin r}$



$\Rightarrow \quad r = 30°$

Since, $\theta_C = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) = 0.577$

At point Q, angle of incidence inside the prism is $i = 45°$.

Since $\sin i = \frac{1}{\sqrt{2}}$ is greater than $\sin\theta_C = \frac{1}{\sqrt{3}}$, ray gets totally internally reflected at face CD. Path of ray of light after point Q is shown in figure.

From the figure, we can see that angle between incident ray OP and emergent ray RS is $90°$.

Hence, (A), (B) and (C) are correct.

8. Values of options (C) and (D) don't match with the mirror formula.

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Hence, (C) and (D) are correct.

9. When upper half of the lens is covered, image is formed by the rays coming from lower half of the lens i.e., the image will be formed by lesser number of rays. Therefore, intensity of image will decrease. However, complete image will be formed.

Hence, (B) and (D) are correct.

10. Objective and eye piece are separated by a distance $(f_o + f_e) = 16.02$ m OPTION **(A)**}

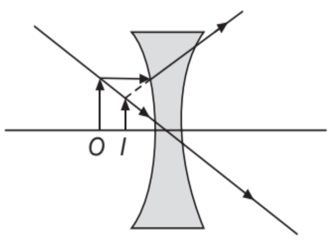
Angular Magnification $= -\frac{f_o}{f_e} = -\frac{16}{0.02} = -800$ {OPTION **(B)**}

A telescope produces an image which is always inverted. {OPTION **(C)**}

In a telescope an objective is larger than the eye piece. OPTION **(D)**}

Hence, (A), (B), (C) and (D) are correct.

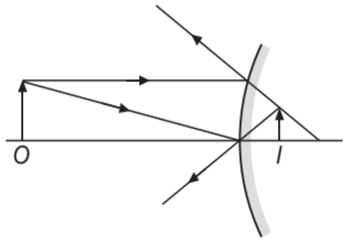
11. For a lens $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$



i.e., $\frac{1}{v}=\frac{1}{f}+\frac{1}{u}$

For a concave lens, f and u are negative, i.e., v will always be negative and image will always be virtual.

For a mirror



$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

I.e., $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$

Here, f is positive and u is negative for a convex mirror. Therefore, v is always positive and image is always virtual.

Hence, (C) and (D) are correct.

12. For TIR, we have

$i > C$

$\Rightarrow \quad \sin i > \sin C$

$\Rightarrow \quad \sin(45°) > \sin C$

Since, $\sin C = \frac{1}{n}$

$\Rightarrow \quad \frac{1}{\sqrt{2}} > \frac{1}{n}$

$\Rightarrow \quad n > \sqrt{2}$

$\Rightarrow \quad n > 1.414$

Hence, (C) and (D) are correct.

Comprehension Type Questions

1. For S_1 immersed in water, we have

$$\frac{4}{3}\sin i = \frac{\sqrt{45}}{4}\sin(90-\theta_c) = \frac{\sqrt{45}}{4}\cos\theta_c$$

Since, $\sin\theta_c = \frac{n_2}{n_1}$

$\Rightarrow \quad \cos\theta_c = \sqrt{1-\left(\frac{n_2}{n_1}\right)^2}$

$\Rightarrow \quad \frac{4}{3}\sin i = \frac{\sqrt{45}}{4}\frac{3}{\sqrt{45}}$

$\Rightarrow \quad \sin i = \frac{9}{16}$

In second case, i.e. for S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$, we have

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{7}{8}$$

$$\Rightarrow \quad \cos\theta_c = \frac{\sqrt{15}}{8}$$

Since $\frac{16}{3\sqrt{15}}\sin i = \frac{8}{5}\sin(90-\theta_c)$

Simplifying we get,

$$\sin i = \frac{9}{16}$$

Same approach can be adopted for other options. Correct answers are (A) and (C).

Hence, the correct answer is (A).

2. Since, $1\sin i_m = n_1 \sin(90-\theta_c)$

$$\Rightarrow \quad \sin i_m = n_1 \cos\theta_c$$

$$\Rightarrow \quad NA = n_1\sqrt{1-\sin^2\theta_c}$$

$$\Rightarrow \quad NA = n_1\sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$\Rightarrow \quad NA = \sqrt{n_1^2 - n_2^2}$$

Substituting the values we get,

$$NA_1 = \frac{3}{4} \text{ and } NA_2 = \frac{\sqrt{15}}{5} = \sqrt{\frac{3}{4}}$$

$$\Rightarrow \quad NA_2 < NA_1$$

Therefore, the numerical aperture of combined structure is equal to the lesser of the two numerical aperture, which is NA_2.

Hence, the correct answer is (D).

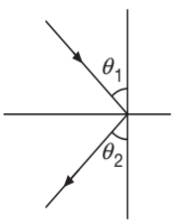
3. Since value of n in meta-material is negative.

$$\therefore \quad v = \frac{c}{|n|}$$

Hence, the correct answer is (B).

4. $1\sin\theta_1 = -n\sin\theta_2$

$$\sin\theta_2 = -\frac{1}{n}\sin\theta_1$$

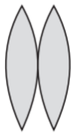


Hence, the correct answer is (C).

Matrix Match/Column Match Type Questions

1. A → (p)
B → (s)
C → (r)
D → (p)

(P)  $\frac{1}{f} = \left(\frac{3}{2}-1\right)\left(\frac{1}{r}+\frac{1}{r}\right) = \frac{1}{r}$

 $\Rightarrow f = r$

$$\Rightarrow \quad \frac{1}{f_{eq}} = \frac{1}{f}+\frac{1}{f} = \frac{2}{r}$$

$$\Rightarrow \quad f_{eq} = \frac{r}{2}$$

(Q)  $\frac{1}{f} = \left(\frac{3}{2}-1\right)\left(\frac{1}{r}\right)$

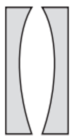
$$\Rightarrow \quad f = 2r$$

 $\Rightarrow \frac{1}{\phi}+\frac{1}{\phi} = \frac{2}{\phi} = \frac{1}{\rho}$

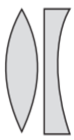
$$\Rightarrow \quad f_{eq} = r$$

(R)  $\frac{1}{f} = \left(\frac{3}{2}-1\right)\left(-\frac{1}{r}\right) = -\frac{1}{2r}$

$$\Rightarrow \quad f = -2r$$

 $\Rightarrow \frac{1}{f_{eq}} = \frac{1}{f}+\frac{1}{f} = -\frac{2}{2r}$

$$\Rightarrow \quad f_{eq} = -r$$

(S)  $\Rightarrow \frac{1}{f_{eq}} = \frac{1}{r}+\frac{1}{-2r} = \frac{1}{2r}$

$$\Rightarrow \quad f_{eq} = 2r$$

2. A → (q)
B → (r)
C → (s)
D → (p)

For $e \to i$

$\Rightarrow \quad \sin 45° > \theta_c$

$\Rightarrow \quad \sin 45° > \sin\theta_c$

$\Rightarrow \quad \dfrac{1}{\sqrt{2}} > \dfrac{\mu_2}{\mu_1}$

$\Rightarrow \quad \mu_1 > \sqrt{2}\mu_2$

For $e \to f$

angle of refraction is lesser than angle of incidence, so $\mu_2 > \mu_1$ and then $\mu_2 > \mu_3$

For $e \to g$, $\mu_1 = \mu_2$

for $e \to h$, $\mu_2 < \mu_1 < \sqrt{2}\mu_2$ and $\mu_2 > \mu_3$

3. A → (p, r)
B → (q, s, t)
C → (p, r, t)
D → (q, s)

(A)→ since $\mu_1 < \mu_2$, the ray of light will bend towards normal after first refraction.

(B) → $\mu_1 > \mu_2$, the ray of light will bend away from the normal after first refraction.

(C) → since $\mu_2 = \mu_3$ means in second refraction there will be no change in the path of ray of light.

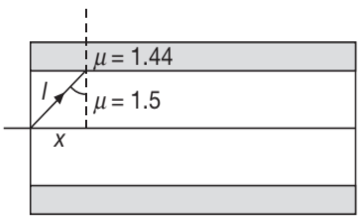
(D) → Since $\mu_2 > \mu_3$, ray of light will bend away from the normal after second refraction.

Therefore, the correct options are as under.
(A) → p, r; (B) → q, s, t; (C) → p, r, t; (D) → q, s

Integer/Numerical Answer Type Questions

1. $1.5\sin\theta_C = 1.44\sin(90°)$

$\Rightarrow \quad \sin\theta_C = \dfrac{24}{25}$



Since $\ell = \dfrac{x}{\sin\theta_C} = \dfrac{25}{4}x$

So, total length for light to travel is given by

$$\ell' = \frac{25}{4} \times 9.6 = 10 \text{ m}$$

Hence total time is given by

$$T = \frac{\ell'}{c/1.5} = 5 \times 10^{-8} \text{ s}$$

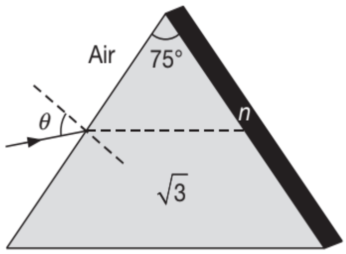
$\Rightarrow \quad T = 50 \times 10^{-9} \text{ s} = t \times 10^{-9} \text{ s}$

$\Rightarrow \quad t = 50.00$

2. For TIR at other face, we have

$$\sin C = \frac{n}{n_0}$$

$\Rightarrow \quad \sin C = \dfrac{n}{\sqrt{3}}$ ...(1)



Applying Snell's Law at first surface, we get

$$\sin\theta = \sqrt{3}\sin(75 - C)$$

For $\theta = 60°$, we have

$$\frac{\sqrt{3}}{2} = \sqrt{3}\sin(75 - C)$$

$\Rightarrow \quad \sin(75 - C) = \dfrac{1}{2}$

$\Rightarrow \quad C = 45°$

From (1), we get

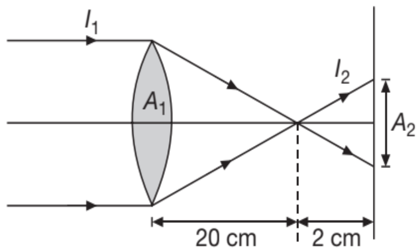
$$\frac{1}{\sqrt{2}} = \frac{n}{\sqrt{3}}$$

$\Rightarrow \quad n = \sqrt{\dfrac{3}{2}}$

$\Rightarrow \quad n^2 = \dfrac{3}{2} = 1.5$

3. $I_1 = \dfrac{P}{A_1}$

$\Rightarrow \quad P = I_1 A_1$ ...(1)



Also, $I_2 = \dfrac{P}{A_2}$

$$\Rightarrow \quad P = I_2 A_2 \qquad \ldots(2)$$

From (1) and (2), we get

$$I_2 = \frac{I_1 A_1}{A_2} = (1.3)\left(\frac{20}{2}\right)^2 = 130 \text{ kWm}^{-2}$$

4. $1.6\sin\theta = (n - m\Delta n)\sin(90°)$

$$\Rightarrow \quad 1.6\sin\theta = n - m\Delta n$$

$$\Rightarrow \quad 1.6 \times \frac{1}{2} = 1.6 - m(0.1)$$

$$\Rightarrow \quad 0.8 = 1.6 - m(0.1)$$

$$\Rightarrow \quad m \times 0.1 = 0.8$$

$$\Rightarrow \quad m = 8$$

Theoretically $m = 8$ satisfies $m > 1$ but such a large value of refractive index may not be physically possible to obtain.

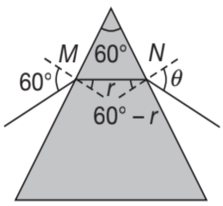
5. Applying Snell's law at M and N,

$$\sin(60°) = n\sin r \qquad \ldots(1)$$

$$\sin\theta = n\sin(60 - r) \qquad \ldots(2)$$

Differentiating we get

$$\cos\theta \frac{d\theta}{dn} = -n\cos(60 - r)\frac{dr}{dn} + \sin(60 - r)$$



Differentiating Equation (1), we get

$$n\cos r \frac{dr}{dn} + \sin r = 0$$

$$\Rightarrow \quad \frac{dr}{dn} = -\frac{\sin r}{n\cos r} = \frac{-\tan r}{n}$$

$$\Rightarrow \quad \cos\theta \frac{d\theta}{dn} = -n\cos(60 - r)\left(\frac{-\tan r}{n}\right) + \sin(60 - r)$$

$$\frac{d\theta}{dn} = \frac{1}{\cos\theta}[\cos(60 - r)\tan r + \sin(60 - r)]$$

From Equation (1), for $n = \sqrt{3}$, we get $r = 30°$

$$\Rightarrow \quad \frac{d\theta}{dn} = \frac{1}{\cos(60°)}[\cos(30°) \times \tan(30°) + \sin(30°)]$$

$$\Rightarrow \quad \frac{d\theta}{dn} = 2\left(\frac{1}{2} + \frac{1}{2}\right) = 2$$

$$\Rightarrow \quad m = 2$$

6. **CASE-1: Arrangement kept in air**

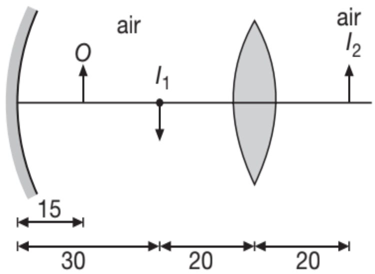
Applying mirror formula

$$\frac{1}{v_1} + \frac{1}{u_1} = \frac{1}{f_1}, \text{ we get}$$

$$\frac{1}{v_1} + \frac{1}{(-15)} = \frac{1}{(-10)}$$

$$\Rightarrow \quad v_1 = -30$$

$$\Rightarrow \quad \left|\frac{v_1}{u_1}\right| = \frac{30}{15} = 2$$



For lens

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\Rightarrow \quad \frac{1}{10} = \frac{1}{v_2} - \frac{1}{-20}$$

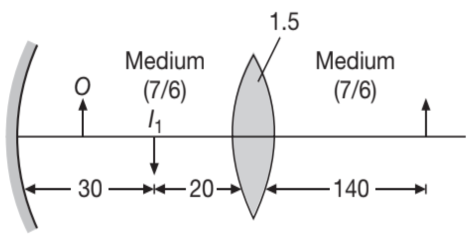
$$\Rightarrow \quad v_2 = 20$$

$$\Rightarrow \quad \left|\frac{v_2}{u_2}\right| = \frac{20}{20} = 1$$

$$\Rightarrow \quad |M_1| = \left|\frac{v_1}{u_1}\right|\left|\frac{v_2}{u_2}\right| = \left(\frac{30}{15}\right)\left(\frac{20}{20}\right) = 2 \times 1 = 2 \qquad \text{(in air)}$$

CASE-2: Arrangement kept in a medium with refractive index $\frac{7}{6}$.

For mirror, there is no change.



For lens, $\dfrac{1}{f_{air}} = \dfrac{1}{10} = \left(\dfrac{\frac{3}{2}}{1} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$, because

$f_{air} = 10$ cm

$$\Rightarrow \quad \frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{5}$$

Since $\dfrac{1}{f_{\text{medium}}} = \left(\dfrac{\frac{3}{2}}{\frac{7}{6}} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

$\Rightarrow \quad \dfrac{1}{f_{\text{medium}}} = \dfrac{2}{35}\ \text{cm}^{-1}$

Applying lens formula, we get

$$\frac{1}{v_2} - \frac{1}{(-20)} = \frac{2}{35}$$

$\Rightarrow \quad \dfrac{1}{v_2} + \dfrac{7}{140} = \dfrac{8}{140}$

$\Rightarrow \quad \dfrac{1}{v_2} = \dfrac{8}{140} - \dfrac{7}{140} = \dfrac{1}{140}$

$\Rightarrow \quad v_2 = 140\ \text{cm}$

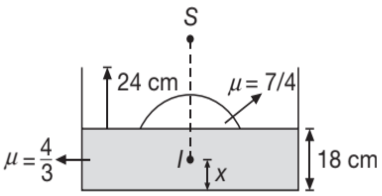
$\Rightarrow \quad \left|\dfrac{v_2}{u_2}\right| = \dfrac{140}{20} = 7$

$\Rightarrow \quad |M_2| = \left|\dfrac{v_1}{u_1}\right|\left|\dfrac{v_2}{u_2}\right| = \left(\dfrac{30}{15}\right)\left(\dfrac{140}{20}\right),$

$\Rightarrow \quad M_2 = 2 \times 7 = 14$

$\Rightarrow \quad \left|\dfrac{M_2}{M_1}\right| = \dfrac{14}{2} = 7$

7.



Two refractions will take place, first, from spherical surface and the other from the plane surface.
So, applying

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

two times with proper sign convention
Ray of light is travelling downwards. Therefore, downward direction is taken as positive direction.

$$\frac{\frac{7}{4}}{v} - \frac{1.0}{(-24)} = \frac{\frac{7}{4} - 1.0}{+6} \qquad \ldots(1)$$

$$\frac{\frac{4}{3}}{(18 - x)} - \frac{\frac{7}{4}}{v} = \frac{\frac{4}{3} - \frac{7}{4}}{\infty} \qquad \ldots(2)$$

Solving these equations, we get, $x = 2$ cm

8. Applying mirror formula when image is at $\dfrac{25}{3}$ m, we get

$$\frac{1}{+\frac{25}{3}} + \frac{1}{-u_1} = \frac{1}{+10}$$

$\Rightarrow \quad \dfrac{1}{u_1} = \dfrac{3}{25} - \dfrac{1}{10}$

$\Rightarrow \quad u_1 = 50$ m

Again applying mirror formula when image is at $\dfrac{50}{7}$ m, we get

$$\frac{1}{\left(+\frac{50}{7}\right)} + \frac{1}{-u_2} = \frac{1}{+10}$$

$\Rightarrow \quad \dfrac{1}{u_2} = \dfrac{7}{50} - \dfrac{1}{10}$

$\Rightarrow \quad u_2 = 25$ m

Speed of object $= \dfrac{u_1 - u_2}{\text{time}} = \dfrac{25}{30}\text{ms}^{-1} = 3\ \text{kmh}^{-1}$

9. $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$\Rightarrow \quad \dfrac{u}{v} - 1 = \dfrac{u}{f}$

$\Rightarrow \quad \dfrac{u}{v} = \left(\dfrac{u+f}{f}\right)$

$\Rightarrow \quad m = \dfrac{v}{u} = \left(\dfrac{f}{u+f}\right)$

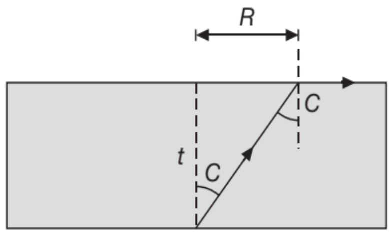
$\Rightarrow \quad \dfrac{m_{25}}{m_{50}} = \dfrac{\left(\frac{20}{-25+20}\right)}{\left(\frac{20}{-50+20}\right)} = 6$

10. Since $\dfrac{R}{t} = \tan C$

$\Rightarrow \quad R = t(\tan C)$

But, $\sin C = \dfrac{1}{\mu} = \dfrac{3}{5}$

$\Rightarrow \quad \tan C = \dfrac{3}{4}$



$\Rightarrow \quad R = \dfrac{3}{4}t = \dfrac{3}{4}(8\ \text{cm}) = 6\ \text{cm}$