CHAPTER 5

Electromagnetic Waves

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Introduction and History of EM Waves
(b) Ampere Circuital Law and Concept of Displacement Current
(c) Maxwell's Equations
(d) Sources of Electromagnetic Waves
(e) Properties of EM Waves (Energy, Intensity, Momentum, Radiation Pressure, Poynting Vector)
(f) Electromagnetic Spectrum and its Properties

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE Main are also given.

INTRODUCTION

In our daily routine, we come across a number of electromagnetic (EM) waves. The electromagnetic waves in the form of visible light enable us to view the world around us, the infrared waves warm our environment, the radio waves carry our favourite TV and radio programs and the list goes on and on.

As studied earlier that a time varying electric field produces a magnetic field and a time varying magnetic field produces an electric field. Maxwell first predicted that there exists a wave containing electric and magnetic fields, both varying with space and time and acting as sources of each other. Both electric and magnetic fields in this wave are mutually perpendicular and also perpendicular to the direction of wave-propagation.

In this chapter, we shall be studying the concept of displacement current along with various properties, uses and parts of electromagnetic waves.

HISTORICAL FACTS ABOUT ELECTROMAGNETIC WAVES

(a) In 1865 Maxwell predicted the presence of electromagnetic waves. He formulated the theory in terms of four Maxwell's equations which predict that the electromagnetic waves of all frequencies should propagate with the speed of light.

(b) Hertz in 1887 succeeded in producing and observing electromagnetic waves of wavelength of the order of 6 m in laboratory.

(c) J.C. Bose in 1894 succeeded in producing and observing electromagnetic waves of much shorter wavelength 25 mm to 5 mm.

(d) In laboratory, G. Marconi in same year succeeded in transmitting the electromagnetic waves up to a few kilometres. Marconi also discovered that if one of the spark gap terminals is connected to an antenna and the other terminal is earthed, the EM waves radiated could go up to several kilometres.

(e) The antenna and the earth wires form the two plates of a capacitor which radiate radio frequency waves. These waves could be received at a large distance by making use of an antenna earth system as detector.

(f) Using these arrangements, Marconi in 1899 first established wireless communication across the English Channel i.e. across a distance of about 50 km.

ELECTROMAGNETIC OSCILLATIONS

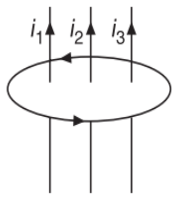
An accelerated charge produces electromagnetic field in the form of radiations. The frequency of oscillations, in LC circuit is

$$f = \frac{1}{2\pi\sqrt{LC}}$$

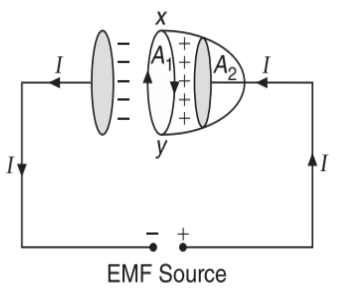
AMPERE CIRCUITAL LAW AND ITS CONTRADICTION

According to Ampere circuital law, the line integral of magnetic field along a closed loop is equal to μ_0 times the sum of steady currents threading the closed loop. So, we have

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(i_1 + i_2 + i_3\right)$$



Let us consider a parallel plate capacitor which is being charged as shown in Figure.



If at an instant, charge on the capacitor is Q then the instantaneous value of current I in the connecting wire is

$$I = \frac{dQ}{dt}$$

Now, let us consider two surfaces A_1 and A_2 which are bound by a closed loop xy. The surface A_1 lies between the two plates of capacitor and A_2 lies outside the plates of capacitor. The area A_2 is pierced by the current I but the area A_1 is not pierced by this current. So, for the surface A_2, we have

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

But for the surface A_1, we have

$$\oint \vec{B}\cdot d\vec{l} = 0$$

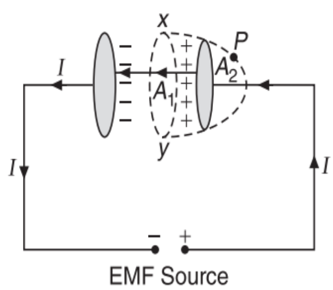
These results create an apparent contradiction in Ampere's Circuital Law (ACL) while applying it to the given situation.

CONCEPT OF DISPLACEMENT CURRENT

The displacement current I_d is a current which is produced due to the rate of change of electric flux with respect to time. Mathematically, the displacement current is given by

$$I_d = \varepsilon_0 \frac{d}{dt}(\phi_E)$$

According to Maxwell, the contradiction in Ampere's law is because of some missing term. This missing term must be such that we must get the same magnetic fields at point P, irrespective of the surface used. This missing term must be related with the changing electric field which passes through area A_1 between the plates of capacitor shown in Figure.



At any instant, if charge on the plates of capacitor is Q, area of plate is A, then electric field between the plates is uniform (but is zero outside the plates) and is given by

$$E = \frac{Q}{A\varepsilon_0} \quad \ldots(1)$$

This field is perpendicular to area A_1, so the electric flux through area A_1 is

$$\phi_E = EA_1 = \left(\frac{Q}{A\varepsilon_0}\right)A$$

where, $A = A_1$

Also, according to Gauss Law, we have

$$\phi_E = \frac{Q}{\varepsilon_0} \quad \ldots(2)$$

If charge on the capacitor plate changes with time then

$$I_d = \frac{dQ}{dt}$$

Using equation (2), we get

$$I_d = \frac{dQ}{dt} = \frac{d}{dt}(\varepsilon_0\phi_E)$$

$$\Rightarrow \quad I_d = \varepsilon_0\frac{d\phi_E}{dt}$$

This current I_d (the missing term in the Ampere's law) passes through the surface A_1 and is known as Maxwell displacement current.

The following inferences can be drawn from the above discussion.

(a) The displacement current arises due to the rate of change of electric flux or the electric field between the plates of the capacitor.

(b) The displacement current is equal to the conduction current when both are present in different parts of the circuit.

(c) The conduction current and the displacement current are individually discontinuous, however the currents together possess the property of continuity through any closed electric circuit.

(d) Just like the conduction current, the displacement current acts as the source of varying magnetic field.

Conceptual Note(s)

(a) For consistency of Ampere circuital law, there must be a current between the plates of capacitor. It is called displacement (I_d) current because it is produced by changing electric field or electric displacement.

(b) The current in conductor is due to the flow of charge carrier and hence is called conduction current I_c.

(c) In **Modified Ampere Circuital Law**, the line integral of magnetic field along a closed loop in free space is equal to μ_0 times the total current (sum of conduction and displacement) threading the loop.

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0(I_c + I_d)$$

ILLUSTRATION 1

A parallel plate capacitor consists of two circular plates each of radius 0.1 m separated by a distance of 0.5 mm. If electric field between the capacitor plates changes at rate of $5\times10^{13}\ \text{Vm}^{-1}\text{s}^{-1}$, find the displacement current between the plates.

SOLUTION

Area of each capacitor plate is

$$A = \pi r^2 = 3.14\times(0.1)^2\ \text{m}^2$$

The rate of change of electric field is

$$\frac{dE}{dt} = 5\times10^{13}\ \text{Vm}^{-1}\text{s}^{-1}$$

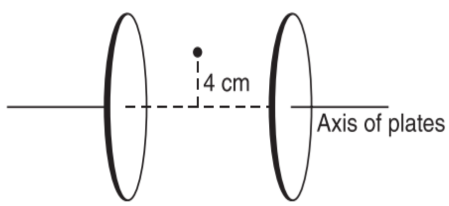
Since, $I_d = \frac{d\phi_E}{dt} = \varepsilon_0 A\left(\frac{dE}{dt}\right)$ $\quad \{\because\ \phi_E = EA\}$

$$\Rightarrow \quad I_d = (8.85\times10^{-12})(3.14)(0.1)^2(5\times10^{13})\ \text{A}$$

$$\Rightarrow \quad I_d = 13.9\ \text{A}$$

ILLUSTRATION 2

Two circular plates each of radius 0.1 m are used to form a parallel plate capacitor. If displacement current between the plates is 2π ampere, then calculate the magnetic field produced by displacement current 4 cm from the axis of the plates.



SOLUTION

Current density of the displacement current is

$$J_d = \frac{2\pi}{\pi(0.1)^2} = \frac{2}{0.01} = 200\ \text{Am}^{-2}$$

Since, $J_d = \frac{I_d}{A}$

$\Rightarrow \quad i_d = J_d \times \pi(0.04)^2$

$\Rightarrow \quad i_d = 200 \times \pi \times 16 \times 10^{-4}$

$\Rightarrow \quad i_d = 32 \times \pi \times 10^{-2}\ \text{A}$

$\Rightarrow \quad i_d \approx 1\ \text{A}$

Since, $\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{\text{enclosed}}$

$\Rightarrow \quad B \times 2\pi(0.04) = 4\pi \times 10^{-7} \times 1$

$\Rightarrow \quad B = \frac{2 \times 10^{-7}}{4 \times 10^{-2}} = 0.5 \times 10^{-5}$

$\Rightarrow \quad B = 5 \times 10^{-6}\ \text{T}$

MAXWELL'S EQUATIONS

Maxwell's equations govern the basics laws of electricity and magnetism by giving the complete idea of all electromagnetic interactions. These equations relate the electric field E and magnetic field B to their sources which are electric charges and current respectively. Maxwell predicted the existence of electromagnetic waves on the basis of these equations. *Just like Newton's Laws being fundamental laws for Mechanics, these equations are same way fundamental equations for electromagnetics.*

In free space, the Maxwell's equations are

(a) Gauss Law For Electrostatics (Maxwell's First Equation)

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

According to Gauss's Law for electrostatics, "the net electric flux associated with any closed surface is $\frac{1}{\varepsilon_0}$ times the total charge enclosed by the surface". This law relates electric field with the charge distribution. The Gauss law is in accordance with the Coulomb's inverse square law which has experimentally been confirmed.

(b) Gauss Law for Magnetism (Maxwell's Second Equation)

$$\oint \vec{B} \cdot d\vec{A} = 0$$

According to Gauss Law for magnetism, "the net magnetic flux associated with a closed surface is zero". This simply implies that the number of magnetic field lines entering the closed surface is equal to number of magnetic field lines leaving the closed surface due to which we donot have isolated magnetic monopoles.

(c) Faraday's Laws of EMI (Maxwell's Third Equation)

$$\oint \vec{E} \cdot d\vec{l} = \frac{d\phi_B}{dt}$$

According to Faraday's Laws of EMI, "the line integral of electric field along a closed path is equal to the rate of change of magnetic flux through the surface". This law relates electric field with a changing magnetic flux and vice versa. The induced current in a conducting loop placed in a time varying magnetic field confirms this equation.

(d) Ampere-Maxwell's Circuital Law (Maxwell's Fourth Equation)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I_C + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

This is also called as Modified Ampere's Circuital Law, according to which "the line integral of magnetic field along a closed loop is equal to μ_0 times the total current threading the surface bound by the closed loop". This law describes how a magnetic field can be produced by both changing electric flux and a conduction current.

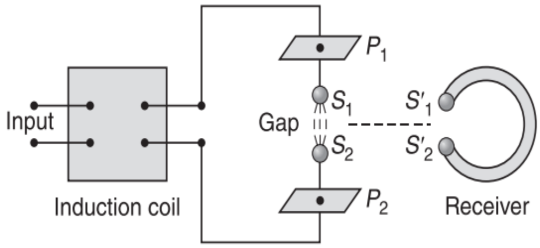
Problem Solving Technique(s)

Important consequence of the Maxwell's equations is that these can be used to derive the Law of Conservation of Charge.

EXPERIMENTAL SETUP FOR PRODUCING ELECTROMAGNETIC WAVES

Hertz experiment was based on the fact that when an oscillating charge is accelerating continuously, then it will radiate electromagnetic waves continuously.

The metallic plates P_1 and P_2 act as a capacitor and the wires connecting spheres S_1 and S_2 to the plates provide a low inductance as shown in Figure.



When a high voltage is applied across the metallic plates, then these plates get discharged by sparking across the narrow gap. This spark gives rise to oscillations which in turn send out electromagnetic waves having frequency given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

The succession of sparks sends out a train of such waves which are received by the receiver.

SOURCES OF ELECTROMAGNETIC WAVES

The waves that are produced by accelerated charged particles and composed of electric and magnetic field vibrating transversely and sinusoidally perpendicular to each other and to the direction of propagation are called electromagnetic waves or electromagnetic radiations. These waves are produced in the following physical phenomena.

(a) An accelerating charge produces both electric field and magnetic field which varies with space and time which forms electromagnetic wave.

(b) An accelerating charge (in case of LC oscillation) emits electromagnetic wave of same frequency as frequency of accelerating charge (i.e., frequency of oscillating LC circuit)

(c) An electron orbiting in a stationary orbit around the nucleus does not emit an electromagnetic wave. However, during a transition of an electron from a higher orbit to a lower orbit, it emits electromagnetic waves of some definite frequency governed by Bohr's Transition Rule.

(d) Electromagnetic wave (*X*-ray) is produced when high speed electron enters into a target of high atomic weight.

(e) Electromagnetic wave (γ-rays) is produced during de-excitation of nucleus in radioactivity.

However, please note that electromagnetic waves will not be produced when

(a) an electric charge is at rest, because it only produces electrostatic field around it.

(b) a charge is moving with uniform velocity (i.e. steady current), because then it produces both electric and magnetic field. However, this magnetic field does not change with time and hence it does not produce time varying electric field.

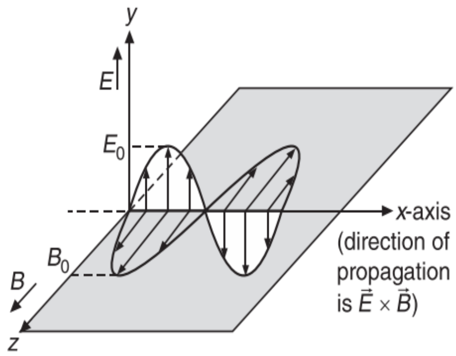
PLANE PROGRESSIVE EM WAVE

In an EM wave, at any point in space the electric field E and the magnetic field B are sinusoidal functions of time t and at any instant, the spatial variation (variation with x) of these fields is also sinusoidal.

If at any instant the fields are uniform over any plane perpendicular to the direction of wave propagation, then the wave is called a plane wave.

Also, it is observed that $\vec{E}$ and $\vec{B}$ are at right angles to the direction of propagation of EM wave. So, electromagnetic waves are transverse in nature.

A sinusoidal electromagnetic wave travelling in the $+x$ direction as shown in Figure.



The $\vec{E}$ and $\vec{B}$ vectors are shown for few points on the x-axis. The point where $\vec{E}$ is in the $+y$ direction has B is in the $+z$ direction and the point where E is in $-y$ direction has B is in $-z$ direction.

Let E and B represent the instantaneous values, E_0 and B_0 represent the peak values of corresponding fields. The equation of the travelling electromagnetic wave is then given by

$$\vec{E} = E_0 \sin(\omega t - kx)\hat{j}$$

$$\vec{B} = B_0 \sin(\omega t - kx)\hat{k}$$

where, $\omega = 2\pi f$ is the angular frequency, $k = \frac{2\pi}{\lambda}$ is the propagation constant of the EM wave.

The speed of the EM wave in vacuum is

$$c = \frac{\omega}{k} = f\lambda$$

The speed of electromagnetic wave in vacuum is

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

The speed of electromagnetic wave in medium is

$$c_{\text{medium}} = v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \mu_r \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$$

where, $\sqrt{\mu_r \varepsilon_r} = \frac{c}{v} = n$ is called the refractive index of medium. E_0 and B_0 are related to each other according to equation

$$B_0 = \frac{E_0}{c}$$

The speed of electromagnetic wave is given by

$$c = \frac{E_0}{B_0}$$

ILLUSTRATION 3

An electromagnetic wave is propagating in vacuum along x-axis, which is produced by oscillating charge of frequency 3×10^{10} Hz. The amplitude of magnetic field (B_0) is 1×10^{-7} T along z-axis. Calculate the wavelength of the EM wave, its propagation constant and the equation for oscillating electric field and magnetic field.

SOLUTION

Since we know that

$$\lambda = \frac{c}{f} = \frac{3\times10^{8}}{3\times10^{10}} = 1\times10^{-2} \text{ m}$$

$$\Rightarrow \quad k = \frac{2\pi}{\lambda} = \frac{2(3.14)}{10^{-2}} = 6.28\times10^{2} = 628 \text{ m}^{-1}$$

The equation for oscillating electric field is

$$E = cB_0 \sin\left(2\pi\times3\times10^{10}t - 628x\right)$$

$$\Rightarrow \quad \vec{E} = 30\sin\left(6\pi\times10^{10}t - 628x\right)\hat{j} \text{ NC}^{-1} \text{ and}$$

$$\vec{B} = 1\times10^{-7}\sin\left(6\pi\times10^{10}t - 628x\right)\hat{k} \text{ T}$$

ENERGY OF AN EM WAVE

The energy possessed by an EM wave is equally divided between the electric and magnetic fields. Since we are already aware that the energy density associated with an electric field is given by

$$u_e = \frac{1}{2}\varepsilon_0 E^2$$

and the energy density associated with a magnetic field is given by

$$u_m = \frac{B^2}{2\mu_0}$$

The total energy per unit volume possessed by an EM wave is

$$u = u_e + u_m = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}$$

Since we know that $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{E}{B}$

$$\Rightarrow \quad u_e = u_m = \frac{1}{2}\varepsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

$$\Rightarrow \quad u = u_e + u_m = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$$

The average energy density associated by the electric field is

$$\langle u_e \rangle = \left\langle \frac{1}{2}\varepsilon_0 E^2 \right\rangle = \frac{1}{2}\varepsilon_0 \langle E^2 \rangle$$

Since, we know that $E = E_0 \sin(\omega t - kx)$

$$\Rightarrow \quad \langle u_e \rangle = \frac{1}{2}\varepsilon_0 \langle E^2 \rangle = \frac{1}{2}\varepsilon_0 E_0^2 \langle \sin^2(\omega t - kx) \rangle$$

Also, we are aware that

$$\langle \sin^2(\omega t - kx) \rangle = \langle \cos^2(\omega t - kx) \rangle = \frac{1}{2}$$

$$\Rightarrow \quad \langle u_e \rangle = \frac{1}{2}\varepsilon_0 \left(\frac{E_0^2}{2}\right) = \frac{1}{4}\varepsilon_0 E_0^2 \qquad \ldots(1)$$

Similar arguments when applied to the magnetic part of the EM wave also give us

$$\langle u_m \rangle = \frac{1}{2\mu_0}\langle B^2 \rangle = \frac{B_0^2}{4\mu_0} \qquad \ldots(2)$$

Since, we know that $c = \dfrac{E_0}{B_0} = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

So, from (1) and (2), we get

$$\langle u_e \rangle = \langle u_m \rangle = \frac{1}{4}\varepsilon_0 E_0^2 = \frac{B_0^2}{4\mu_0}$$

$$\Rightarrow \quad \langle u \rangle = \langle u_e \rangle + \langle u_m \rangle = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$$

Conceptual Note(s)

Average energy density of electromagnetic wave is equal to average value of total instantaneous energy density in one time period.

$$\langle u \rangle = \frac{1}{T}\int_0^T u\,dt$$

$$\Rightarrow \langle u \rangle = \frac{1}{T}\int_0^T \varepsilon_0 E^2 dt$$

$$\Rightarrow \langle u \rangle = \frac{\varepsilon_0 E_0^2}{T}\int_0^T \sin^2(kx - \omega t)\,dt$$

Since, $\displaystyle\int_0^T \sin^2(kx - \omega t)\,dt = \frac{T}{2}$

$$\Rightarrow \langle u \rangle = \left(\frac{\varepsilon_0 E_0^2}{T}\right)\left(\frac{T}{2}\right)$$

$$\Rightarrow \langle u \rangle = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$$

ILLUSTRATION 4

In an electromagnetic wave, the amplitude of electric field is $1\ \text{Vm}^{-1}$. What is

(a) the amplitude of magnetic field?
(b) average energy density of electric field?
(c) average energy density of magnetic field?
(d) average energy density of wave?

SOLUTION

(a) As $\dfrac{E_0}{B_0} = c$

$$\Rightarrow B_0 = \frac{E_0}{c}$$

$$\Rightarrow B_0 = \frac{1}{3\times 10^8} = 3.33\ \text{nT}$$

(b) $U_E = \dfrac{1}{4}\varepsilon_0 E^2 = \dfrac{1}{4}\times 8.85\times 10^{-12}\times 1^2$

$$\Rightarrow U_E = 2.2125\times 10^{-12}\ \text{Jm}^{-3}$$

(c) $U_B = U_E = 2.2125\times 10^{-12}\ \text{Jm}^{-3}$

(d) $U = U_E + U_B = 4.425\times 10^{-12}\ \text{Jm}^{-3}$

INTENSITY OF AN EM WAVE

The intensity of an EM wave is defined as the amount of energy crossing per unit time per unit area of a surface held normally to the direction of propagation of electromagnetic wave. Mathematically, we have

$$I = \frac{U}{A\Delta t} \qquad \ldots(1)$$

where U is the total energy of the EM wave.
Let us consider a cylindrical region of length L, area of cross section A through which EM wave is passing.

Volume of cylinder is

$$V = AL$$

Since, the intensity of the EM wave is

$$I = \frac{U}{A\Delta t}$$

$$\Rightarrow \quad I = \left(\frac{U}{AL}\right)\left(\frac{L}{\Delta t}\right) = uc$$

where, $u = \dfrac{U}{V}$ is the total energy density of the EM wave and $c = \dfrac{L}{\Delta t}$ is the speed of the EM wave.

Total energy of EM wave in the cylinder is

$$E = \langle u \rangle (AL)$$

$$\Rightarrow \quad I_{av} = \langle u \rangle c$$

Since, $\langle u \rangle = \frac{1}{2}\varepsilon_0 E_0^2$

$$\Rightarrow \quad I_{av} = \left(\frac{1}{2}\varepsilon_0 E_0^2\right)c$$

Also, we know that

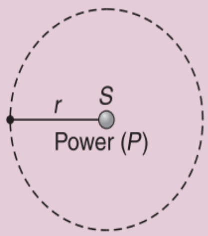
$$E_{rms} = \frac{E_0}{\sqrt{2}}$$

$$\Rightarrow \quad I_{av} = \left(\frac{1}{2}\varepsilon_0 E_0^2\right)c = \varepsilon_0\left(\frac{E_0}{\sqrt{2}}\right)^2 c = \varepsilon_0 E_{rms}^2 c$$

Conceptual Note(s)

INTENSITY DUE TO A POINT SOURCE

For a point source S having power P, the intensity of EM wave at a distance r from the source is



$$I = \frac{U}{A\Delta t} = \left(\frac{U}{\Delta t}\right)\frac{1}{A} = \frac{P}{A} = \frac{P}{4\pi r^2}\ \text{Wm}^{-2}$$

Since, $I_{av} = \langle u \rangle c$

$$\Rightarrow \quad \frac{P}{4\pi r^2} = \langle u \rangle c\text{, where } \langle u \rangle = \frac{1}{2}\varepsilon_0 E_0^2$$

$$\Rightarrow \quad \left(\frac{1}{2}\varepsilon_0 E_0^2\right)c = \frac{P}{4\pi r^2}$$

$$\Rightarrow \quad E_0 = \sqrt{\frac{P}{2\pi r^2 c\varepsilon_0}} \text{ and } B_0 = \frac{E_0}{c} = \sqrt{\frac{P\mu_0}{2\pi r^2 c}}$$

ILLUSTRATION 5

A point source of electromagnetic radiation has average power output of 800 W, then calculate the maximum value of electric field at a distance 3.5 m from the source, maximum value of magnetic field and the average energy density at 3.5 m from the source.

SOLUTION

The intensity of EM wave is

$$I = \frac{P}{4\pi r^2}$$

$$\Rightarrow \quad I = \frac{800}{4 \times 3.14 \times (3.5)^2}$$

$$\Rightarrow \quad I = 5.2\ \text{Wm}^{-2}$$

Since, we know that

$$I = \left(\frac{1}{2}\varepsilon_0 E_0^2\right)c$$

$$\Rightarrow \quad E_0 = \sqrt{\frac{2 \times I}{\varepsilon_0 \times c}}$$

$$\Rightarrow \quad E_0 = \sqrt{\frac{2 \times 5.2}{8.85 \times 10^{-12} \times 3 \times 10^8}}$$

$$\Rightarrow \quad E_0 = 0.626 \times 10^2 = 62.6\ \text{NC}^{-1}$$

$$\Rightarrow \quad B_0 = \frac{E_0}{c} = \frac{62.6}{3 \times 10^8} = 20.87 \times 10^{-8}$$

$$\Rightarrow \quad B_0 = 2.087 \times 10^{-7}\ \text{T}$$

Since, we know that

$$\langle u \rangle = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{8.85 \times 10^{-12}}{2} \times (62.6)^2$$

$$\Rightarrow \quad \langle u \rangle = 1.74 \times 10^{-8}\ \text{Jm}^{-3}$$

ILLUSTRATION 6

Calculate the electric and magnetic field amplitude in an electromagnetic wave radiated by a 200 W bulb at a distance 2 m from it. Assume that the efficiency of bulb is 5% and it behaves like a point source.

SOLUTION

Effective power output of the bulb is

$$P = \frac{5}{100} \times 200 = 10\ \text{watt}$$

Intensity at distance r is given by

$$I = \frac{P}{4\pi r^2}$$

$$\Rightarrow \quad I = \frac{10}{4 \times 3.14 \times 2^2}\ \text{Wm}^{-2}$$

$$\Rightarrow \quad I=\frac{10}{16\times 3.14}\text{ Wm}^{-2}=0.199\text{ Wm}^{-2}$$

$$\Rightarrow \quad I\approx 0.2\text{ watt m}^{-2}$$

Since, $I=\frac{1}{2}\varepsilon_0 E_0^2 c$

$$\Rightarrow \quad E_0^2=\frac{2I}{c\times\varepsilon_0}$$

$$\Rightarrow \quad E_0=\sqrt{\frac{2I}{c\times\varepsilon_0}}$$

$$\Rightarrow \quad E_0=\sqrt{\frac{2\times 0.2}{3\times 10^8\times 8.85\times 10^{-12}}}$$

$$\Rightarrow \quad E_0=12.27\text{ NC}^{-1}$$

$$\Rightarrow \quad B_0=\frac{E_0}{c}=\frac{12.27}{3\times 10^8}=\frac{12.27}{3}\times 10^{-8}\text{ T}$$

$$\Rightarrow \quad B_0=4.09\times 10^{-8}\text{ T}$$

MOMENTUM POSSESSED BY AN EM WAVE

It is interesting to note that, though EM waves do not possess mass, yet they have finite momentum p given by

$$p=\frac{U}{c}$$

If an EM wave is incident on a perfectly absorbing surface, then the momentum delivered by the wave to the surface is equal to the change in momentum of the EM wave. So, we have

$$p_{\substack{\text{imparted to}\\\text{the surface}}}=\Delta p_{\text{wave}}=\frac{U}{c}$$

Similarly, if an EM wave is incident on a perfectly reflecting surface, then the momentum delivered by the wave to the surface is also equal to the change in momentum of the EM wave. So, we have

$$p_{\substack{\text{imparted to}\\\text{the surface}}}=\Delta p_{\text{wave}}=\frac{2U}{c}$$

RADIATION PRESSURE OF AN EM WAVE

When the EM wave is incident on a surface, perfectly absorbing or perfectly reflecting, then the momentum delivered by the wave to the surface is equal to the change in momentum of the EM wave. Due to this change in momentum, the surface experiences a force and hence pressure called as radiation pressure given by

$$P=\frac{F}{A}=\frac{1}{A}\left(\frac{\Delta p}{\Delta t}\right)$$

For a perfectly absorbing surface, we have

$$\Delta p=\frac{U}{c}$$

$$P=\frac{1}{A}\left(\frac{\Delta p}{\Delta t}\right)=\frac{1}{A}\left(\frac{U}{c\Delta t}\right)=\frac{1}{c}\left(\frac{U}{A\Delta t}\right)=\frac{I}{c}$$

Similarly, for a perfectly reflecting surface, we have

$$\Delta p=\frac{2U}{c}$$

$$P=\frac{1}{A}\left(\frac{\Delta p}{\Delta t}\right)=\frac{2}{A}\left(\frac{U}{c\Delta t}\right)=\frac{2}{c}\left(\frac{U}{A\Delta t}\right)=\frac{2I}{c}$$

ILLUSTRATION 7

An electromagnetic wave of intensity 10 Wm^{-2} strikes a small mirror of area $20\times 10^{-4}\text{ m}^2$, held perpendicular to the approaching wave. What is the radiation force on the mirror?

SOLUTION

Radiation force is the momentum transferred per second by electromagnetic wave to the mirror.

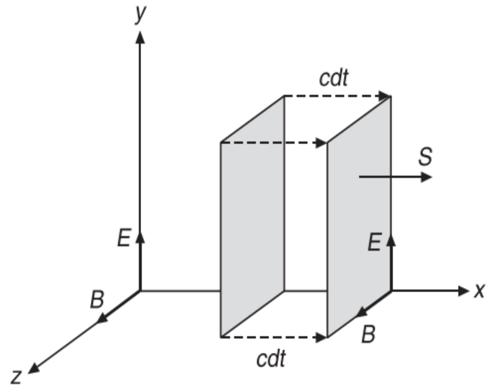
$$F=PA=\frac{2IA}{c}=\frac{2\times(10)\times 20\times 10^{-4}}{3\times 10^8}$$

$$\Rightarrow \quad F=1.33\times 10^{-10}\text{ N}$$

POYNTING VECTOR FOR AN EM WAVE

The concept of Poynting vector $\vec{S}$ was introduced by the British physicist John Poynting (1852-1914). It is defined as the flow of energy in the direction of the propagation of a wave per unit time through a unit cross-sectional area perpendicular to the propagation direction.

Consider a plane EM wave front of area A travelling with speed c along the $+x$ direction as shown in Figure.



In time dt, this wave front moves through a distance $dx = cdt$. The energy in the space between these two positions of the wave front is the product of energy density (u) of the EM wave and volume (dV) of the space between these two positions. So, we have

$$dU = udV, \text{ where } dV = Adx = Acdt$$

$$\Rightarrow \quad dU = udV = (\varepsilon_0 E^2)(Acdt) \quad \ldots(1)$$

The energy flowing per unit time per unit area is called the Poynting Vector given by

$$S = \frac{1}{A}\frac{dU}{dt} = \varepsilon_0 cE^2 = uc$$

$$\Rightarrow \quad S = \varepsilon_0 cE^2 = \varepsilon_0\left(\frac{1}{\sqrt{\varepsilon_0\mu_0}}\right)E^2 = \sqrt{\frac{\varepsilon_0}{\mu_0}}E^2$$

Since, we know that $\frac{E}{B} = c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$$\Rightarrow \quad S = \sqrt{\frac{\varepsilon_0}{\mu_0}}E^2 = \sqrt{\frac{\varepsilon_0}{\mu_0}}E(Bc) = \sqrt{\frac{\varepsilon_0}{\mu_0}}\left(\frac{EB}{\sqrt{\mu_0\varepsilon_0}}\right) = \frac{EB}{\mu_0}$$

Vectorially, $\vec{S}$ is parallel to x-axis, cdt is also parallel to x-axis, $\vec{E}$ is along y axis and $\vec{B}$ is along z axis. So, we can also say that the Poynting vector $\vec{S}$ describes both the magnitude and direction of energy flow rate. For vacuum, the Poynting vector is given by

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$$

Its SI unit is Jsm^{-2} or Wm^{-2}

The magnitude $\frac{EB}{\mu_0}$ gives the flow of energy through a cross section perpendicular to the propagation direction, per unit area per unit time at an instant. The electric and magnetic fields at any point vary with time, so the Poynting vector at any point is also function of time.

The average value of Poynting vector at any point is energy flowing on an average in unit time from unit area. This is also called intensity of the radiation at that point. We can also write

$$|\vec{S}| = S = \frac{E_0B_0}{\mu_0}\sin^2(\omega t - kx)$$

$$\Rightarrow \quad \langle S\rangle = I = \frac{E_0B_0}{\mu_0}\langle\sin^2(\omega t - kx)\rangle = \frac{E_0B_0}{2\mu_0}$$

$$\Rightarrow \quad I = S_{av} = \frac{E_0B_0}{2\mu_0} = \frac{E_0H_0}{2} \qquad \left\{\because H_0 = \frac{B_0}{\mu_0}\right\}$$

$$\Rightarrow \quad I = \frac{E_0B_0}{2\mu_0} = \left(\frac{E_0}{\sqrt{2}}\right)\left(\frac{B_0}{\sqrt{2}}\right)\frac{1}{\mu_0} = \frac{E_{rms}B_{rms}}{\mu_0}$$

Problem Solving Technique(s)

(a) The average value of Poynting vector represents the intensity (I) of the EM wave.

(b) Poynting vector $\vec{S}$ gives the direction of EM wave propagation.

ILLUSTRATION 8

The oscillating magnetic field in a plane electromagnetic wave is given as

$$B_y = 8\times10^{-6}\sin(5000\pi x - 3\times10^{11}\pi t)\ \text{T}$$

Calculate the frequency, wavelength, speed of EM wave, electric field amplitude and the expression for oscillating electric field.

SOLUTION

Comparing given equation with standard equation i.e. $B_y = B_0(kx - \omega t)$, we get

$$\omega = 3\times10^{11}\pi$$

$$\Rightarrow \quad 2\pi f = 3\times10^{11}\pi$$

$$\Rightarrow \quad f = 1.5\times10^{11}\ \text{Hz}$$

Since, $k = \frac{2\pi}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{2\pi}{5000\pi} = 0.4\times10^{-3}\ \text{m} = 0.4\ \text{mm}$$

$$\Rightarrow \quad v = \frac{\omega}{k} = \frac{3\times10^{11}\times\pi}{5000\pi} = 0.6\times10^{8} = 6\times10^{7}\ \text{ms}^{-1}$$

Since, $v = \dfrac{E_0}{B_0}$

$\Rightarrow \quad E_0 = vB_0 = (6\times 10^7)(8\times 10^{-6}) = 480\ \text{Vm}^{-1}$

Since the wave is travelling along the x-direction, so $\vec{E}\times\vec{B}$ should be along x-axis. Since $-(\hat{k}\times\hat{j}) = \hat{i}$, hence we have

$$E_z = 480\sin(500\pi x - 3\times 10^{11}\ \pi t)\ \text{Vm}^{-1}$$

SUMMARY OF IMPORTANT CHARACTERISTICS AND NATURE OF ELECTROMAGNETIC WAVES

Electromagnetic waves are transverse waves made up of oscillating electric and magnetic fields, which oscillate perpendicular to each other as well as to the direction of propagation of the wave. They possess the following properties.

(a) The fundamental sources of electromagnetic waves are accelerating electric charges. For examples radio waves emitted by an antenna arise from the continuous oscillations (and hence acceleration) of charges within the antenna structure.

(b) Electromagnetic waves obey the principle of superposition.

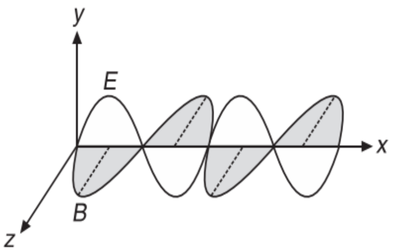
(c) The electric and magnetic fields of a sinusoidal plane electromagnetic wave propagating in the positive x-direction can also be written as

$$E = E_0\sin(kx - \omega t)$$
$$B = B_0\sin(kx - \omega t)$$

where ω is the angular frequency of the wave and k is angular wave number or propagation constant which are given by

$$\omega = 2\pi f \text{ and } k = \frac{2\pi}{\lambda}$$

(d) In an EM wave, $\vec{E}$ and $\vec{B}$ vary sinusoidally. $\vec{E}$ and $\vec{B}$ become maximum at same place and at the same time, are perpendicular to each other as well as to the direction of propagation. So, the phase difference between the two fields is zero.



(e) EM Waves travel in free space with speed equal to $3\times 10^8\ \text{ms}^{-1}$ which is given by $c = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$.

(f) Electromagnetic waves travel with speed of light. In vacuum their speed is

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

In isotropic medium, their speed is

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_r\mu_0\varepsilon_r\varepsilon_0}}$$

$$\Rightarrow \quad v = \frac{1}{\sqrt{\mu_r\varepsilon_r}}\cdot\frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{c}{n}$$

where $n = \sqrt{\mu_r\varepsilon_r}$
is refractive index of medium.
μ_r = relative permeability of medium and
ε_r = relative permittivity of medium or electric dielectric constant.

(g) The velocity of electromagnetic wave in a medium is decided by electric and magnetic properties of medium and not by the amplitude of electric and magnetic field vector. The speed of electromagnetic wave in a medium is

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

where μ is the permeability of the medium and ε is the permittivity of the medium.

(h) EM Waves do not require material medium for their propagation.

(i) The electric and magnetic fields satisfy the following wave equations, which can be obtained from Maxwell's third and fourth equations.

$$\frac{\partial^2 E}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 B}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2}$$

(j) The amplitude of electric field and the magnetic field in EM wave are related to each other as

$$c = \frac{E_0}{B_0}.$$

(k) The direction of propagation of an EM wave is determined by $\vec{E}\times\vec{B}$.

(l) The electric field vector of an electromagnetic wave produces optical effects and hence it is also called as Light vector.

(m) Electromagnetic waves are not deflected by electric and magnetic field because EM waves are made up of uncharged particles called photons.

(n) The energy carried by an electromagnetic wave is equally divided between the electric field and magnetic field. Total average energy density is

$$u = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\frac{B_0^2}{\mu_0}$$

(o) The intensity of an electromagnetic wave is defined as energy crossing per second per unit area of a surface normal to the direction of propagation of electromagnetic wave.
Average intensity is given by

$$I_{av} = \langle I \rangle = \frac{1}{2}\varepsilon_0 E_0^2 c = \varepsilon_0 E_{rms}^2 c$$

(in terms of electric field)

$$I_{av} = \left(\frac{B_0^2}{2\mu_0}\right)c = \left(\frac{B_{rms}^2}{\mu_0}\right)c$$

(in terms of magnetic field)

(p) The electromagnetic waves carry both energy as well as momentum. The momentum of an EM wave is

$$p = \frac{U}{c}$$

where, U is the energy carried by an EM wave in free space and c is the speed of an EM wave in free space.

(q) The radiation pressure is defined as force exerted by an EM Wave on a unit area of a surface. Nichols and Hull measured radiation pressure of visible light and found it to be of the order of 7×10^{-6} Nm^{-2}.

(r) For an EM wave of intensity I falling normally on a perfectly absorbing surface, the radiation pressure is

$$P = \frac{I}{c}$$

(s) For an EM wave of intensity I falling normally on a perfectly reflecting surface, the radiation pressure is

$$P = \frac{2I}{c}$$

For all other surfaces, the radiation pressure lies between

$$\frac{I}{c} < P < \frac{2I}{c}$$

(t) The **Poynting vector** $\vec{S} = \vec{E} \times \vec{H}$ represents the direction of energy flow per unit area per sec along the direction of wave propagation. Its unit is Wm^{-2}.

(u) The intensity of a sinusoidal plane electromagnetic wave is defined as the **average value of Poynting vector** taken over one cycle.

$$S_{av} = \frac{E_0 B_0}{2\mu_0} = \frac{E_0^2}{2\mu_0 c} = \frac{c}{2\mu_0}B_0^2 \qquad \left\{\because c = \frac{E_0}{B_0}\right\}$$

(v) The medium in which EM wave is travelling, offers hindrance to the propagation of wave. Such a hindrance is called **Wave Impedance** (Z) and its value in a medium is given by

$$Z = \frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r}{\varepsilon_r}}\sqrt{\frac{\mu_0}{\varepsilon_0}}$$

because, $\mu = \mu_r \mu_0$ and $\varepsilon = \varepsilon_r \varepsilon_0$

where μ_r and ε_r are relative permeability and relative permittivity of medium.
For vacuum or free space

$$Z = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.6\ \Omega$$

ELECTROMAGNETIC SPECTRUM

The electromagnetic spectrum has a very wide range-from very large to very small wavelengths. Different methods are used for producing them and they have different properties. However, the boundaries of different types of electromagnetic waves are not sharp and they overlap.

Name	Discoverer	Approximate Wavelength Range	Generated by	Properties	Applications
Radio waves	Marconi	0.1 m – 10^4 m	Oscillating circuits	Exhibit waves like properties more than particle like properties	Radio Communication
Microwaves	Hertz (Heinrich Rudolph Hertz)	10^3 m – 0.1 m	Special electronic devices such as klystron tube	Phenomena of reflection, refraction and diffraction.	(a) Radar and Telecommunication. (b) Analysis of fine details of molecular structure.
Infra-Red	William Herschell	8000 Å to 10^7 Å	(a) Rearrangement of outer orbital electrons in atoms and molecules. (b) Change of molecular vibrational and rotational energies. (c) By bodies at high temperature.	(a) Thermal effect (b) All properties similar to those of light except λ	(a) Used in industry, medicine and astronomy. (b) Useful for fog or haze photography. (c) Elucidating molecular structure.
Visible light and Sub parts of visible spectrum	Newton		Rearrangement of outer orbital electrons in atoms and molecules, e.g., gas discharge tube, incandescent solids and liquids.	Sensitive to human eye	(a) To see objects. (b) To study molecular structure.
(a) Violet		3990 Å to 4550 Å			
(b) Blue		4550 Å to 4920 Å			
(c) Green		4920 Å to 5770 Å			
(d) Yellow		5770 Å to 5970 Å			
(e) Orange		5970 Å to 6220 Å			
(f) Red		6220 Å to 7880 Å			
Ultraviolet	Ritter	10 Å to 1000 Å	Rearrangement of orbital electrons of atoms, as in high voltage gas discharge tube, arc, the sun and the mercury vapour lamp.	(a) All properties of light. (b) Photoelectric effect.	Sterilization of water due to its destructive action on bacteria. To detect adulteration, writing and signature.

(Continued)

Name	Discoverer	Approximate Wavelength Range	Generated by	Properties	Applications
X-rays	Roentgen	0.1 Å to 10 Å	(a) Bombardment of high energy electrons on a high atomic number target. (b) Energy changes of innermost orbital electrons.	(a) Low penetrating power. (b) Other properties similar to γ-rays except wavelength.	(a) Medical diagnosis and treatment. (b) Study of crystal structure. (c) Industrial radiography.
Gamma Rays	Henry Becquerel and Madam Curie	0.006 Å to 1 Å	(a) Change of nuclear energy levels. (b) Radioactive substances.	(a) High penetrating power. (b) Uncharged. (c) Low ionising power.	(a) Gives information on nuclear structure. (b) Medical treatment.

SUB PARTS OF RADIO SPECTRUM

Name	Approximate Wavelength Range	Applications
(a) SHF (Super High Frequency)	0.001 m to 0.1 m	Radar, Radio and satellite communication (Microwaves) Radar and television broadcast, short distance communication, Television communication.
(b) UHF (Ultra High Frequency)	0.1 m to 1 m	
(c) VHF (Very High Frequency)	1 m to 10 m	
(d) HF (High Frequency)	10 m to 100 m	Medium distance communication, Telephone communication, Marine and navigation use, Long range communication, Long distance communication.
(e) MF (Medium Frequency)	100 m to 1000 m	
(f) LF (Low Frequency)	1000 m to 10000 m	
(g) VLF (Very Low Frequency)	10000 m to 30000 m	

ATMOSPHERE AND VARIOUS PARTS OF ELECTROMAGNETIC SPECTRUM

(a) Atmosphere is transparent to visible radiations.

(b) Atmosphere absorbs most of infrared radiations.

(c) The ozone layer which is somewhere 30 km – 50 km above the ground (mesosphere) absorbs ultraviolet radiations, thus protects us from harmful radiations of sun. Practically all radiations of wavelength less than 3000 Å $\left(3\times10^{-7}\text{ m}\right)$ are absorbed by ozone layer.

(d) Earth is heated by sun's infrared radiations. The earth also emits radiations most of which are in infrared region. These radiations are reflected back by atmosphere. These reflected radiations keep the earth's surface warm at night. This phenomenon is called **Green House Effect**.

(e) Waves of wavelength 1 mm or higher are broadly called **Radiowaves**. They include radio, television and microwaves. Waves having wavelength 10 cm or more (frequencies less than 30 MHz) are termed as **Amplitude Modulated Waves**. These waves are transmitted in lower atmosphere but are reflected by top-most region of atmosphere called **Ionosphere**. These waves further are of two types.

(i) Ground waves: These follow the surface of earth the ground waves are attenuated depending on frequency. The communication upto 1500 kHz may be via **ground waves**.

(ii) Sky waves: They are reflected back by Ionosphere. For frequencies above 1500 kHz the communication is via **Sky Waves**.

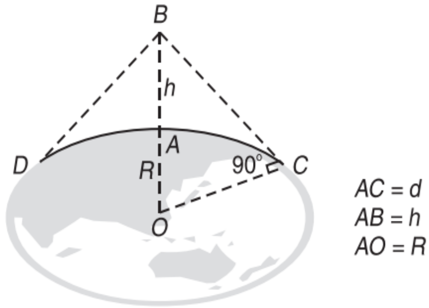
These two regions of waves of Amplitude modulated waves are termed as **Medium Waves (MW)** and **Short Waves (SW)** respectively.

For waves above 40 MHz, the Ionosphere deviates incident waves but does not reflect back to earth.

T.V. Signal waves of frequency 100 MHz – 200 MHz called **Frequency Modulated Wave** neither follow the curvature of earth nor get reflected by Ionosphere, their reception is possible either by large antenna or by geostationary satellites.

COVERING RANGE OF A TRANSMITTER OR T.V. ANTENNA

Consider a TV tower of height h. If $d = AC$ is the covering range of transmitter, then from triangle OBC.



$$(OB)^2 = (OC)^2 + (CB)^2$$

We have

$$OB = R + h$$

$$OC = R$$

and $BC = AC = d$

$$\Rightarrow \quad (R+h)^2 = R^2 + d^2$$

$$\Rightarrow \quad d^2 = (R+h)^2 - R^2$$

$$\Rightarrow \quad d^2 = 2hR + h^2$$

$$\Rightarrow \quad d = \sqrt{2hR + h^2}$$

For $h \ll R$ covering range is given by

$$d = \sqrt{2hR}$$