CHAPTER 4: ALTERNATING CURRENTS

Test Your Concepts-I (Based on AC)

1. For an ac, $I = I_0 \sin(\omega t)$

Therefore, instantaneous value of heat produced in time dt across a resistance R is,

$$dH = I^2 R dt = I_0^2 R \sin^2(\omega t) dt$$

Average value of heat produced during a cycle

$$H_{av} = \frac{\int_0^T dH}{\int_0^T dt} = \frac{\int_0^{2\pi/\omega} \left(I_0^2 R \sin^2(\omega t)\right) dt}{\int_0^{2\pi/\omega} dt}$$

$$\Rightarrow \quad H_{av} = \frac{I_0^2}{2} R \left(\frac{2\pi}{\omega}\right)$$

$$\Rightarrow \quad H_{av} = I_{\text{rms}}^2 RT$$

i.e., ac produces same heating effects as produced by dc of value $I = I_{\text{rms}}$

2. Since hot wire voltmeter reads only rms value we will have to find rms value of the given voltage. Considering one complete cycle,

$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V^2 d\theta} \text{ where } \theta = \omega t$$

$$V_{\text{rms}}^2 = \frac{2}{2\pi}\int_0^{2\pi} \left(200\sin\theta + 100\sin(3\theta) + 50\sin(5\theta)\right)^2 d\theta$$

$$V_{\text{rms}}^2 = \frac{1}{2\pi}\int_0^{2\pi} \left(200^2\sin^2\theta + 100^2\sin^2 3\theta + 50^2\sin^2 5\theta\right) d\theta$$

The integral of all cross terms comes to be zero over one complete cycle. So,

$$V_{\text{rms}}^2 = \frac{1}{2\pi}\left(\frac{200^2}{2} + \frac{100^2}{2} + \frac{50^2}{2}\right) 2\pi = 26,250$$

$$\Rightarrow \quad V_{\text{rms}} = \sqrt{26250} = 162 \text{ V}$$

3. By definition, $I_{\text{rms}}^2 = \dfrac{\int_0^T I^2 dt}{T}$

$$\Rightarrow \quad I_{\text{rms}}^2 = \frac{\int_0^2 I^2 dt}{2} = \frac{1}{2}\int_0^2 (kt)^2 dt = \frac{k^2}{2}\left(\frac{t^3}{3}\right)_0^2 = \frac{4k^2}{3}$$

$$\Rightarrow \quad I_{\text{rms}} = \frac{2k}{\sqrt{3}}$$

4. For the given time function as shown in problem, the RMS value of the voltage is given by

$$E_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^{\frac{T}{2}} E_0^2 dt}$$

$$\Rightarrow \quad V_{\text{rms}} = E_0\sqrt{\frac{1}{T}\left(\frac{T}{2}\right)}$$

$$\Rightarrow \quad E_{\text{rms}} = \frac{E_0}{\sqrt{2}}$$

5. $$V_{\text{rms}} = \sqrt{\frac{\int_0^T V^2 dt}{T}} = \sqrt{\langle V^2 \rangle}$$

$$\Rightarrow \quad V_{\text{rms}} = \sqrt{\left\langle V_0^2 + V_1^2\cos^2\omega t + 2V_0V_1\cos\omega t\right\rangle}$$

$$\Rightarrow \quad V_{\text{rms}} = \sqrt{\left\langle V_0^2\right\rangle + V_1^2\left\langle\cos^2\omega t\right\rangle + 2V_0V_1\left\langle\cos\omega t\right\rangle}$$

Since, $\langle\cos\omega t\rangle = 0$ and $\langle\cos^2\omega t\rangle = \dfrac{1}{2}$ for one cycle

$$\Rightarrow \quad V_{\text{rms}} = \sqrt{V_0^2 + \frac{V_1^2}{2}}$$

6. (a) $I = \dfrac{V_R}{R} = \dfrac{(2\text{ V})\sin\left((10^3 \text{ rads}^{-1})t\right)}{100}$

$$I = (2\times10^{-2}\text{ A})\sin\left((10^3\text{ rads}^{-1})t\right)$$

(b) $X_L = L\omega = (4\text{ H})(10^3\text{ rads}^{-1})$

$$\Rightarrow \quad X_L = 4\times10^3\ \Omega$$

(c) The amplitude of voltage across inductor is

$$V_0 = I_0 X_L = (2\times10^{-2}\text{ A})(4\times10^3\ \Omega)$$

$$\Rightarrow \quad V_0 = 80\text{ V}$$

For an ac input, voltage across the inductor leads the current by $90°$ or $\frac{\pi}{2}$ rad. Hence

$$V_L = V_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$V_L = 80\sin\left\{\left(10^3 \text{ rads}^{-1}\right)t + \frac{\pi}{2}\right\} \text{ V}$$

7. Please remember that the designated or assigned values are the rms values. So, $V_{rms} = 200$ V.

Since, in case of an ac, the voltage leads the current in phase by an angle,

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{L\omega}{R}\right)$$

Here, $X_L = \omega L = (2\pi f L)$

$$X_L = (2\pi)(50)(0.01) = \pi\Omega$$

and $R = 1\,\Omega$

$$\Rightarrow \phi = \tan^{-1}(\pi) = \tan^{-1}(3.14) \approx 72.3°$$

Further, $I_{rms} = \frac{V_{rms}}{|Z|}$

$$\Rightarrow I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + X_L^2}}$$

Substituting the values, we get

$$I_{rms} = \frac{200}{\sqrt{(1)^2 + (\pi)^2}}$$

$$\Rightarrow I_{rms} = 60.67 \text{ A}$$

8. (a) $T = \frac{1}{f} = \frac{1}{60}$ s

$$\omega = 2\pi f = 377 \text{ rads}^{-1}$$

$$X_L = L\omega = (377)(0.04) = 15.08\ \Omega$$

$$Z = \sqrt{X_L^2 + R^2} = 25.05\ \Omega$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}(0.754)$$

$$\Rightarrow \phi = 37°$$

(b) Amplitudes (maximum value) are,

$$I_0 = \frac{V_0}{Z} = \frac{150}{25.05} \approx 6 \text{ A}$$

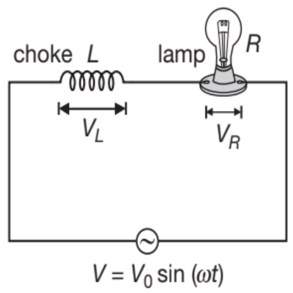
$$(V_0)_R = I_0 R = 120 \text{ V}$$

$$(V_0)_L = I_0 X_L = 90.5 \text{ V}$$

Also, we observe that $V_0 = \sqrt{(V_0)_R^2 + (V_0)_L^2}$

9. For lamp, $V_R = IR = 10 \times 5 = 50$ V

In series, $V^2 = V_R^2 + V_L^2$



$$\Rightarrow V_L = \sqrt{V^2 - V_R^2}$$

$$\Rightarrow V_L = \sqrt{(160)^2 - (50)^2}$$

$$\Rightarrow V_L = 152 \text{ V}$$

Since $V_L = IX_L = 2\pi I f L$

$$\Rightarrow L = \frac{V_L}{2\pi f I}$$

Substituting the values, we get

$$L = \frac{152}{(2\pi)(50)(10)} = 4.84 \times 10^{-2} \text{ H} = 48.4 \text{ mH}$$

Now, when the lamp is operated at 160 V dc and instead of choke, an additional resistance R' is put in series with it then, we have

$$V = I(R + R')$$

$$\Rightarrow 160 = 10(5 + R')$$

$$\Rightarrow R' = 11\ \Omega$$

In case of ac, as the choke has no resistance, power loss in choke is zero.

In case of dc, the loss in additional resistance R' is,

$$P = I^2 R' = (10)^2(11) = 1100 \text{ W}$$

10. $I_0 = \frac{V_0}{R} = \frac{2.5}{300} = 8.33 \times 10^{-3}$ A

(a) $I = 8.33 \times 10^{-3} \cos\left[\left(950 \text{ rads}^{-1}\right)t\right]$

(b) $X_L = L\omega = 0.8 \times 950 = 760\ \Omega$

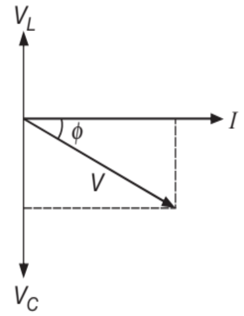
(c) $V_L = (V_L)_0 \cos\left(\left(950 \text{ rads}^{-1}\right)t + \frac{\pi}{2}\right)$

$$\Rightarrow V_L = \left(8.33 \times 10^{-3} \times 760\right)\cos\left(950t + \frac{\pi}{2}\right)$$

$$\Rightarrow V_L = (6.33 \text{ V})\cos\left[\left(950 \text{ rads}^{-1}\right)t + \frac{\pi}{2}\right]$$

$$\Rightarrow V_L = -(6.33 \text{ V})\sin\left(\left(950 \text{ rads}^{-1}\right)t\right)$$

11. $X_L = L\omega = 100\ \Omega$, $X_C = \dfrac{1}{C\omega} = 312.5\ \Omega$ and $R = 300\ \Omega$



(a) $I = \dfrac{V}{Z} = \dfrac{120}{\sqrt{(312.5-100)^2+(300)^2}}$

$\Rightarrow \quad I = \dfrac{120}{368} = 326\ \text{mA}$

(b) Since $X_C > X_L$, so

$V_C > V_L$

$\Rightarrow \quad \tan\phi = \dfrac{X_C - X_L}{R}$

$\Rightarrow \quad V$ lags behind I by ϕ, where

$\phi = \tan^{-1}\left(\dfrac{312.5-100}{300}\right) \cong 35.3°$

So, we have voltage lagging behind current by 35.3°

(c) $V_R = I_R = (0.326)(300) = 97.8\ \text{V}$

$V_L = IX_L = (0.326)(100) = 32.6\ \text{V}$

$V_C = IX_C = (0.326)(312.5) = 102\ \text{V}$

12. (a) $X_L = L\omega = (2)(2\pi(50)) = 200\pi = 628\ \Omega$

(b) $X_L = L\omega$

$\Rightarrow \quad 2 = L(2\pi(50))$

$\Rightarrow \quad L = 6.37\ \text{mH}$

(c) $X_C = \dfrac{1}{C\omega} = \dfrac{1}{2\pi fC} = \dfrac{1}{(314)(2\times10^{-6})} = 1.6\ \text{k}\Omega$

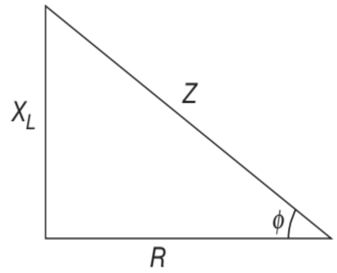
(d) $C = \dfrac{1}{2\pi fX_C} = \dfrac{1}{(314)(2)} = 1.6\ \text{mF}$

13. (a) In series, the impedance of the circuit is,

$$Z = \sqrt{R^2+\omega^2L^2} = \sqrt{R^2+(2\pi fL)^2}$$

$\Rightarrow \quad Z = \sqrt{(220)^2+(2\times3.14\times50\times0.7)^2}$

$\Rightarrow \quad Z = 311\ \Omega$



Since $I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{220}{311} = 0.707\ \text{A}$

and $\cos\phi = \dfrac{R}{Z} = \dfrac{220}{311} = 0.707$

So, the power absorbed in the circuit is

$P = V_{rms}I_{rms}\cos\phi$

$\Rightarrow \quad P = (220)(0.707)(0.707)\ \text{W}$

$\Rightarrow \quad P = 110.08\ \text{W}$

(b) When the resistance and choke are in parallel, the entire power is absorbed in resistance, because the choke (having zero resistance) absorbs no power.

$\Rightarrow \quad P = \dfrac{V_{rms}^2}{R} = \dfrac{(220)^2}{220}$

$\Rightarrow \quad P = 220\ \text{W}$

14. Inductive reactance

$$X_L = \omega L = (50)(2\pi)(35\times10^{-3}) \approx 11\ \Omega$$

Impedance $Z = \sqrt{R^2+X_L^2} = \sqrt{(11)^2+(11)^2} = 11\sqrt{2}\ \Omega$

Given $V_{rms} = 220\ \text{V}$

Hence, amplitude of voltage

$$V_0 = \sqrt{2}\ V_{rms} = 220\sqrt{2}\ \text{V}$$

The amplitude of current is

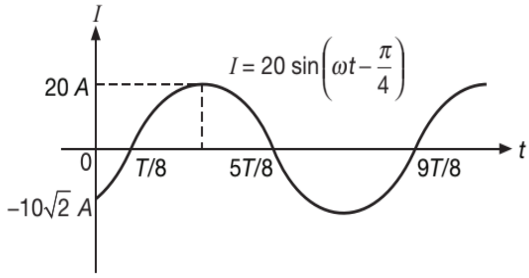
$$I_0 = \frac{V_0}{Z} = \frac{220\sqrt{2}}{11\sqrt{2}} = 20\ \text{A}$$

Phase difference $\phi = \tan^{-1}\left(\dfrac{X_L}{R}\right) = \tan^{-1}\left(\dfrac{11}{11}\right) = \dfrac{\pi}{4}$

In LR circuit voltage leads the current. Hence, instantaneous current in the circuit is,

$$I = (20\,A)\sin\left(\omega t - \frac{\pi}{4}\right)$$

Corresponding I-t graph is shown in figure.



15. Given $R = X_L$

When the plate separation of the capacitor is halved, then C is also halved. So, $X_C = \dfrac{1}{C\omega}$ is also halved.

Now, the current doubles only when Z becomes half of its initial value.

$$\Rightarrow \quad Z_f = \frac{1}{2}Z_i$$

$$\Rightarrow \quad 2Z_f = Z_i$$

$$\Rightarrow \quad 4\left[R^2 + \left(R - \frac{X_C}{2}\right)^2\right] = R^2 + (R - X_C)^2$$

$$\Rightarrow \quad 4R^2 + 4\left(R - \frac{X_C}{2}\right)^2 = R^2 + (R - X_C)^2$$

$$\Rightarrow \quad 8R^2 - 4RX_C + X_C^2 = 2R^2 - 2RX_C + X_C^2$$

$$\Rightarrow \quad X_C = 3R$$

16. $L = 2\,\text{H}$, $C = 10\times10^{-6}\,\text{F}$, $R = 10\,\Omega$ and $V = 100\sin(\omega t)$ volt

(a) $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2)(10\times10^{-6})}} = 224\ \text{rads}^{-1}$$

(b) $$P = \frac{V_0^2}{2R} = \frac{(100)^2}{2(10)} = 500\ \text{W}$$

(c) Since $P = E_v I_v \cos\phi = E_{\text{rms}} I_{\text{rms}} \cos\phi$

$$\Rightarrow \quad P = \frac{E_0}{\sqrt{2}}\frac{I_0}{\sqrt{2}}\cos\phi$$

Since, $\cos\phi = \dfrac{R}{Z}$ and $I_0 = \dfrac{E_0}{Z}$

$$\Rightarrow \quad P = \frac{E_0^2 R}{2Z^2} \qquad \ldots(1)$$

Further P is maximum, when Z is minimum i.e., at resonance. So, maximum value of P is

$$P_0 = \frac{E_0^2}{2R} \qquad \ldots(2)$$

As per the question we must have

$$P = \frac{P_0}{2}$$

$$\Rightarrow \quad \frac{E_0^2 R}{2Z^2} = \frac{E_0^2}{4R}$$

$$\Rightarrow \quad Z^2 = 2R^2$$

$$\Rightarrow \quad \left(L\omega - \frac{1}{C\omega}\right)^2 + R^2 = 2R^2$$

$$\Rightarrow \quad \left(L\omega - \frac{1}{C\omega}\right)^2 = R^2$$

$$\Rightarrow \quad (L^2C^2)\omega^4 - (2LC + R^2C^2)\omega^2 + 1 = 0$$

$$\omega^2 = \frac{(2LC + R^2C^2) \pm \sqrt{(2LC + R^2C^2)^2 - 4L^2C^2}}{2L^2C^2}$$

Substituting values, we get

$$\omega^2 = 51130 \text{ OR } \omega^2 = 48894$$

$$\Rightarrow \quad \omega_1 = 226\ \text{rads}^{-1} \text{ OR } \omega_2 = 221\ \text{rads}^{-1}$$

17. (a) $\omega = 6280$

$$\Rightarrow \quad 2\pi f = 6280$$

$$\Rightarrow \quad 6.28 f = 6280$$

$$\Rightarrow \quad f = 1\ \text{kHz}$$

(b) $$\cos\phi = \cos\left(\frac{\pi}{2} - \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

(c) $$\cos\phi = \frac{R}{Z}$$

where $Z = \dfrac{170}{8.5} = 20\ \Omega$

$$\Rightarrow \quad R = 20\frac{\sqrt{3}}{2} = 10\sqrt{3} = 17.32\ \Omega$$

Since this circuit has voltage lagging behind the current, so this must contain a resistance and a capacitance. Hence

$$Z^2 = \frac{1}{C^2\omega^2} + R^2$$

$$\Rightarrow \quad 400 = \frac{1}{4\pi^2 f^2 C^2} + 300$$

$$\Rightarrow \quad 100 = \frac{1}{(2\pi f C)^2}$$

$$\Rightarrow \quad 2\pi f C = 0.1$$

$$\Rightarrow \quad C = \frac{0.1}{(2)(3.14)(1000)} \cong 16\ \mu\text{F}$$

18. (a) The resonance frequency of a rejector LCR circuit or the parallel circuit is given by,

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f=\frac{1}{2\pi}\sqrt{\frac{1}{(1.6\times10^{-2})(250\times10^{-12})}-\frac{(20)^2}{(1.6\times10^{-2})^2}}$$

$$\Rightarrow \quad f=7.96\times10^4\ \text{Hz}$$

(b) The circuit impedance at resonance is given by,

$$Z=\frac{L}{CR}=\frac{1.6\times10^{-2}}{(250\times10^{-12})(20)}$$

$$\Rightarrow \quad Z=3.2\times10^6\ \Omega$$

19. (a) $f=\frac{1}{2\pi\sqrt{LC}}=1000\times10^6$ Hz

$$\Rightarrow \quad C=\frac{1}{4\pi^2f^2L}=\frac{1}{(4)(10)(10^{18})(400\times10^{-12})}$$

$$\Rightarrow \quad C=62.5\times10^{-12}\ \text{F}=62.5\ \text{pF}$$

(b) $C=\frac{\varepsilon_0 A}{d}$

$$\Rightarrow \quad 62.5\times10^{-12}=\frac{8.85\times10^{-12}\,\ell^2}{10^{-3}}$$

$$\Rightarrow \quad \ell^2=\frac{62.5\times10^{-3}}{8.85}$$

$$\Rightarrow \quad \ell=84\ \text{mm}$$

(c) $X_L=L\omega=2\pi fL=(2)(3.14)(10^9)(400\times10^{-12})$

$$\Rightarrow \quad X_L=2.51\ \Omega$$

20. $f_0=\frac{1}{2\pi\sqrt{L_1C_1}}=\frac{1}{2\pi\sqrt{L_2C_2}}$

$$\Rightarrow \quad L_1C_1=L_2C_2 \qquad \ldots(1)$$

When connected in series, the new resonant frequency be say f, then

$$f=\frac{1}{2\pi\sqrt{L_SC_S}}$$

where $L_S=L_1+L_2$ and $C_S=\frac{C_1C_2}{C_1+C_2}$

$$\Rightarrow \quad f=\frac{1}{2\pi\sqrt{(L_1+L_2)\left(\frac{C_1C_2}{C_1+C_2}\right)}}$$

$$\Rightarrow \quad f=\frac{1}{2\pi\sqrt{\frac{L_1C_1C_2+L_2C_1C_2}{C_1+C_2}}}=\frac{1}{2\pi\sqrt{L_1C_1\left(\frac{C_1+C_2}{C_1+C_2}\right)}}$$

$$\Rightarrow \quad f=\frac{1}{2\pi\sqrt{L_1C_1}}=\frac{1}{2\pi\sqrt{L_2C_2}}=f_0$$

Test Your Concepts-II (Based on Transformer)

1. By applying a dc voltage, no flux variation takes place in the core of transformer. Hence no voltage is induced in the secondary coil.

2. (a) $\frac{N_2}{N_1}=\frac{V_2}{V_1}=\frac{10\times10^3}{120}=83.3$

(b) $\eta=\frac{V_2I_2}{V_1I_1}$

$$\Rightarrow \quad I_2=\frac{\eta(V_1I_1)}{V_2}$$

But $I_1=\frac{V_1}{R_1}=\frac{120}{24}=5$ A

$$\Rightarrow \quad I_2=\frac{(0.9)(120)(5)}{10\times10^3}$$

$$\Rightarrow \quad I_2=54\times10^{-3}\ \text{A}=54\ \text{mA}$$

(c) $Z_2=\frac{V_2}{I_2}=\frac{10\times10^3}{54\times10^{-3}}\cong185\ \text{k}\Omega$

3. Given that, $N_S=25000$ and $N_P=10000$

$$\Rightarrow \quad \frac{N_S}{N_P}=\frac{25}{10}=2.5$$

Hence it is a step-up transformer. Since, we have

$$\frac{\xi_1}{\xi_2}=\frac{N_1}{N_2}$$

$$\Rightarrow \quad \xi_2=\left(\frac{N_2}{N_1}\right)\xi_1=\left(\frac{25000}{10000}\right)(50)\ \text{V}$$

$$\Rightarrow \quad \xi_2=125\ \text{V}$$

4. (a) $R=(5\times10^{-4})(600\times10^3\ \text{m})=300\ \Omega$

Since $I_{\text{rms}}=\frac{P}{V_{rms}}=\frac{5\times10^6}{500\times10^3}=10$ A

So, power loss is

$$P_{\text{loss}}=I_{rms}^2R=(100)(300)=30\ \text{kW}$$

(b) Fraction $f=\frac{P_{\text{loss}}}{P_{\text{input}}}=\frac{30\times10^3\ \text{W}}{5\times10^6\ \text{W}}=6\times10^{-3}$

(c) It is impossible to transmit so much power at such a low voltage. Since we know that from Maximum Power Transfer Theorem, the power transferred is maximum for load resistance to be equal to the line resistance of $300\ \Omega$ and

CHAPTER 4

$$P_{max} = \frac{V^2}{4R} = \frac{\left(4.5\times10^3\right)^2}{4(300)} = 16.9 \text{ kW}$$

This value happens to be far below 5 MW.

Single Correct Choice Type Questions

1. For series RC circuit impedance,

$$Z = \sqrt{R^2 + \frac{1}{C^2\omega^2}}$$

$$\Rightarrow \quad Z = \sqrt{10^8 + \frac{1}{\left(10^{-12}\right)\left(10^4\right)}}$$

$$\Rightarrow \quad Z = 10^4\sqrt{2}\ \Omega$$

Ammeter will give a reading which is the virtual value or the value that is the labelled value. So

$$I = \frac{E_{rms}}{Z} = \frac{E_0}{\sqrt{2}Z}$$

$$\Rightarrow \quad I = \frac{200\sqrt{2}}{\sqrt{2}\left(10^4\sqrt{2}\right)}$$

$$\Rightarrow \quad I = 10\sqrt{2} \text{ mA}$$

Hence, the correct answer is (B).

2. At resonance, voltage across resistance is 60 V

$$\Rightarrow \quad 60 = I_0 R$$

$$\Rightarrow \quad I_0 = \frac{60}{120} = 0.5 \text{ A}$$

Also, voltage across inductance is 40 V

$$\Rightarrow \quad 40 = I_0 X_L$$

$$\Rightarrow \quad 40 = 0.5(L\omega)$$

$$\Rightarrow \quad 80 = L\left(4\times10^5\right)$$

$$\Rightarrow \quad L = 0.2 \text{ mH}$$

Since $\omega_0 = \frac{1}{\sqrt{LC}}$

$$4\times10^5 = \frac{1}{\sqrt{0.2\times10^{-3}\ C}}$$

$$\Rightarrow \quad 16\times10^{10} = \frac{1}{C\left(0.2\times10^{-3}\right)}$$

$$\Rightarrow \quad C = \frac{1}{3.2\times10^7}$$

$$\Rightarrow \quad C = \frac{1}{32}\times10^{-6} \text{ F}$$

$$\Rightarrow \quad C = \frac{1}{32}\ \mu\text{F}$$

Hence, the correct answer is (A).

3. Output current $= \frac{140}{24}$ A

$$\Rightarrow \quad I_S = \frac{140}{24} \text{ A}$$

$$\eta = \frac{\xi_S I_S}{\xi_P I_P} = \frac{24}{240}\frac{\left(\frac{140}{24}\right)}{0.7}$$

$$\Rightarrow \quad \eta = 0.833$$

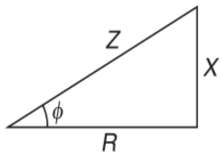
$$\Rightarrow \quad \eta = 83.3\%$$

Hence, the correct answer is (B).

4. According to the problem, we have

$$2Z = \sqrt{3}X$$

From the figure, we get



$$\sin\phi = \frac{X}{Z} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \phi = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$$

Hence, the correct answer is (A).

5. $I_{rms} = \frac{V_{rms}}{X_C}$

$$\Rightarrow \quad I_{rms} = \left(\frac{300\sqrt{2}}{\sqrt{2}}\right)(C\omega)$$

$$\Rightarrow \quad I_{rms} = (300)\left(10^{-6}\right)(50)$$

$$\Rightarrow \quad I_{rms} = 15 \text{ mA}$$

Hence, the correct answer is (B).

6. Power factor is $\cos\phi = \frac{R}{Z}$, where

$$Z^2 = \left(X_L - X_C\right)^2 + R^2$$

$$\Rightarrow \quad Z = (100-20)^2 + (40+20)^2$$

$$\Rightarrow \quad Z^2 = (80)^2 + (60)^2$$

$$\Rightarrow \quad Z = 100\ \Omega$$

So, $\cos\phi = \frac{(40+20)}{100} = 0.6$

Hence, the correct answer is (C).

7. An ideal choke coil should have almost zero internal resistance. Otherwise, it will consume some power.
Hence, the correct answer is (A).

8. $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\Rightarrow \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow \quad 10^6 = \frac{1}{2\pi\sqrt{10^{-2}C}}$$

$$\Rightarrow \quad 10^{12} = \frac{1}{4(10)(10^{-2}C)}$$

$$\Rightarrow \quad C = 2.5 \times 10^{-12}\ F$$

$$\Rightarrow \quad C = 2.5\ \text{pF}$$

Hence, the correct answer is (A).

9. At resonance

$$I = \frac{E_0}{R} = \frac{0.1 \times 10^{-3}}{50}$$

$$\Rightarrow \quad I = 2 \times 10^{-6}\ \text{A}$$

$$\Rightarrow \quad I = 2\ \mu\text{A}$$

Hence, the correct answer is (C).

10. The inductive reactance is

$$X_L = \omega L = (5 \times 10^{-3})(2000) = 10\ \Omega$$

The inductive capacitance is

$$X_C = \frac{1}{\omega C} = \frac{1}{(2000)(50 \times 10^{-6})} = 10\ \Omega$$

Since, $X_L = X_C$, so the circuit is in resonance and hence we have

$$Z = Z_{\text{min}} = R = 6 + 4 = 10\ \Omega$$

$$\Rightarrow \quad I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{\left(\frac{20}{\sqrt{2}}\right)}{10} = 1.414\ \text{A}$$

This is also the reading of ammeter.
The reading of the voltmeter is

$$V = I_{\text{rms}} Z_{AB}, \text{ where } Z_{AB} = 4\ \Omega$$

$$\Rightarrow \quad V = I_{\text{rms}} Z_{AB} \approx 5.6 \text{ volt}$$

Hence, the correct answer is (C).

11. When a dc is applied across the coil (an inductor) only resistance is offered to flow of dc
Hence

$$I = \frac{E_0}{R}$$

$$\Rightarrow \quad I = \frac{100}{R}$$

$$\Rightarrow \quad R = 100\ \Omega$$

On application of 100 V ac of frequency 50 Hz we have $I' = 0.5$ A

$$\Rightarrow \quad I' = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + L^2\omega^2}}$$

$$\Rightarrow \quad I' = \frac{E}{\sqrt{R^2 + 4\pi^2 f^2 L^2}}$$

$$\Rightarrow \quad 0.5 = \frac{100}{\sqrt{10^4 + 4(10)(2500)L^2}}$$

$$\Rightarrow \quad 10^4 + 10^5 L^2 = 4 \times 10^4$$

$$\Rightarrow \quad L^2 = \frac{3 \times 10^4}{10^5} = 0.3$$

$$\Rightarrow \quad L = \sqrt{0.3}\ \text{H}$$

So, $R = 100\ \Omega$, $L = \sqrt{0.3}$ H

Hence, the correct answer is (D).

12. Given that $V_0 = 240$ V

$$\Rightarrow \quad V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{240}{\sqrt{2}} = 170\ \text{V}$$

$$\omega = 120\ \text{rads}^{-1}$$

$$\Rightarrow \quad f = \frac{\omega}{2\pi} = \frac{120}{2 \times 3.14} = 19\ \text{Hz}$$

Hence, the correct answer is (A).

13. $P = \frac{E_v^2}{Z}\cos\phi$

$$\Rightarrow \quad 3000 = \frac{(240)^2}{Z}(0.75)$$

$$\Rightarrow \quad Z = \frac{(240)^2}{3000} \times \frac{3}{4}$$

$$\Rightarrow \quad Z = 14.4\ \Omega$$

Hence, the correct answer is (C).

14. When coils are connected in series, we have $R_{eq} = R_1 + R_2$ and $L_{eq} = L_1 + L_2$. Further, we have

$$\text{Power Factor } = \cos\phi = \frac{R}{Z} = \frac{R_1 + R_2}{Z}$$

$$\Rightarrow \quad \frac{3}{4} = \frac{5 + R_2}{14.4}$$

$$\Rightarrow \quad R_2 = 5.8\ \Omega$$

Hence, the correct answer is (B).

15. Since $Z = \sqrt{R_{eq}^2 + L_{eq}^2\omega^2} = \sqrt{R_{eq}^2 + 4\pi^2 f^2 L_{eq}^2}$

$$\Rightarrow \quad (14.4)^2 = (5.8)^2 + L_{eq}^2(4)(10)(2500)$$

$$\Rightarrow \quad 207.36 = 33.64 + 100000\ L_{eq}^2$$

$$\Rightarrow \quad L_{eq} = 0.04\ H$$

$$\Rightarrow \quad L_1 + L_2 = 0.04$$

$\Rightarrow \quad L_2 = 0.04 - 0.02$

$\Rightarrow \quad L_2 = 0.02\ H$

Hence, the correct answer is (B).

16. $\xi = M\dfrac{dI}{dt}$ (in magnitude)

$$\xi = MI_0 \frac{d}{dt}\left[\sin(\omega t)\right]$$

$$\Rightarrow \quad \xi = MI_0\omega\cos(\omega t)$$

$$\Rightarrow \quad \xi_{max} = MI_0\omega$$

$$\Rightarrow \quad \xi_{max} = (0.005)(10)(100\pi)$$

$$\Rightarrow \quad \xi_{max} = 5\pi$$

Hence, the correct answer is (B).

18. $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

$$\Rightarrow \quad f_0 = \frac{1}{2\pi\sqrt{8\left(0.5\times 10^{-6}\right)}}$$

$$\Rightarrow \quad f_0 = \frac{1000}{4\pi} = \frac{250}{\pi}\ \text{Hz}$$

Hence, the correct answer is (C).

19. The current in the circuit is

$$I = \frac{V}{Z}$$

$$\Rightarrow \quad I = \frac{V}{\sqrt{R^2 + \left(\omega L \sim \dfrac{1}{\omega C}\right)^2}}$$

If R is increased, then the current will definitely decrease. However, by changing L or C, the current may increase or decrease.

Hence, the correct answer is (A).

20. $\tan\phi = \dfrac{X_L}{R}$

$$\Rightarrow \quad \tan 45 = \frac{L\omega}{R}$$

$$\Rightarrow \quad L = \frac{R}{\omega} = \frac{100}{2\pi(1000)}$$

$$\Rightarrow \quad L = \frac{1}{20\pi}$$

Hence, the correct answer is (B).

21. For the given circuit, the current flowing through the capacitor is

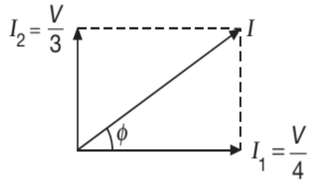
$$I_2 = \frac{V}{X_C} = \frac{V}{3}$$

where, V is the rms value of the ac input.

Similarly, the current I_1 flowing through the resistor is

$$I_1 = \frac{V}{R} = \frac{V}{4}$$

The current I_2 leads the applied voltage by a phase angle of $90°$ whereas the current I_1 is in phase with the applied voltage.



$$\tan\phi = \frac{V/3}{V/4} = \frac{4}{3}$$

$$\Rightarrow \quad \phi = 53°$$

Hence, the correct answer is (C).

23. Since, we have

$$i = -\sqrt{2}\cos\left(\omega t + \frac{\pi}{4}\right)$$

$$\Rightarrow \quad i = \sqrt{2}\sin\left(\omega t + \frac{\pi}{4} + \frac{\pi}{2}\right)$$

Hence, phase difference between V and i is $\dfrac{\pi}{2}$. So, power consumed is zero.

Hence, the correct answer is (D).

24. RMS value $= \sqrt{\langle E^2 \rangle}$

$$\langle E^2 \rangle = \frac{\int_0^T E^2 dt}{\int_0^T dt}$$

$$\Rightarrow \quad \langle E^2 \rangle = \frac{1}{T}\int_0^T \left[64\sin^2(\omega t) + 36\sin^2(2\omega t) + 96\sin(\omega t)\sin(2\omega t)\right]dt$$

$$\Rightarrow \quad \langle E^2 \rangle = \frac{1}{T}\left[64\left(\frac{T}{2}\right) + 36\left(\frac{T}{2}\right) + 0\right]$$

$$\Rightarrow \quad \langle E^2 \rangle = \frac{64+36}{2} = 50$$

$$\Rightarrow \quad \text{RMS value } = \sqrt{\langle E^2 \rangle} = \sqrt{50} = 5\sqrt{2}\ \text{V}$$

Hence, the correct answer is (A).

Problem Solving Technique(s)

The RMS value can also be calculated using the formula

$$E_{rms} = \sqrt{\frac{8^2+6^2}{2}} = 5\sqrt{2}\text{ V}$$

25. Since we know that, for an ac passing through the series combination of inductor and resistor, we have

$$V = \sqrt{V_R^2 + V_L^2}$$

$$\Rightarrow \quad V = \sqrt{(20)^2+(15)^2} = 25\text{ V}$$

Since this is the rms value, so the peak value is given by

$$V_0 = \sqrt{2}V_{rms} = 25\sqrt{2}\text{ V}$$

Hence, the correct answer is (C).

26. $M = \sqrt{L_1L_2}$

When number of turns in the coil are interchanged then M remains same.

Hence, the correct answer is (C).

27. $I_P = \frac{P}{\xi_P} = \frac{6.6\times10^3}{220} = 30\text{ A}$

Further

$$N_S = \frac{\xi_S}{\xi_P}N_P = \frac{4.4\times1000\times1000}{220} = 20{,}000$$

Also $I_S = \frac{N_P}{N_S}I_P = 1.5\text{ A}$

Hence, the correct answer is (C).

28. $X_L = L\omega = 2\pi\nu L$

$X_L \propto \nu$ which indicates a graph for a straight line.

Hence, the correct answer is (A).

29. Since the resistance R equals the capacitive reactance, so we have

$$R = X_C = \frac{1}{\omega C}$$

The impedance of the circuit is

$$Z = \sqrt{R^2+X_C^2} = \sqrt{2}R \qquad \{\because X_C = R\}$$

$$\Rightarrow \quad I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{2}R} \qquad \ldots(1)$$

When ω becomes $\frac{1}{\sqrt{3}}$ times, X_C will become $\sqrt{3}$ times or $\sqrt{3}R$, so the new impedance is

$$Z' = \sqrt{(R^2)+(\sqrt{3}R)^2} = 2R$$

$$\Rightarrow \quad I_0' = \frac{V_0}{Z'} = \frac{V_0}{2R} = \frac{I_0}{\sqrt{2}}$$

Hence, the correct answer is (B).

30. The power factor is given by

$$\cos\phi = \frac{R}{Z}$$

Where the impedance can be

$$Z = \sqrt{R^2+L^2\omega^2}$$

OR $Z = \sqrt{R^2+\frac{1}{C^2\omega^2}}$

OR $Z = \sqrt{(X_L - X_C)^2+R^2}$

Hence, the correct answer is (B).

31. For an ideal capacitor or an ideal inductor, the phase difference between current and voltage is $90°$.
Hence, the correct answer is (D).

32. These ammeters use the heating property of current to calculate its value and hence can be used to measure both ac and dc.
Hence, the correct answer is (C).

33. Since we know that

$$V = \sqrt{V_C^2+V_R^2}$$

$$\Rightarrow \quad V_R = \sqrt{V^2-V_C^2} = \sqrt{(10)^2-(8)^2} = 6\text{ V}$$

$$\Rightarrow \quad \tan\phi = \frac{X_C}{X_R} = \frac{V_C}{V_R} = \frac{8}{6} = \frac{4}{3}$$

Hence, the correct answer is (A).

34. At resonance, no phase difference exists between applied voltage and current.
Hence, the correct answer is (B).

35. Applied voltage V is a sine function and we see that the current is leading the voltage by $45°$. So, the circuit should be capacitive in nature.
Since $\phi = 45°$

$$\Rightarrow \quad \tan 45° = \frac{X_C}{R} = \frac{1}{RC\omega}$$

$$\Rightarrow \quad X_C = R$$

$$\Rightarrow \quad \omega C = R$$

$$\Rightarrow \quad C = \frac{R}{\omega} = \frac{R}{100} = 0.01\text{ R}$$

The best suitable combination for satisfying the given condition is in OPTION (B).

Hence, the correct answer is (B).

36. The average emf during the positive half cycle of an ac is

$$E_{av} = \frac{2E_0}{\pi}$$

Hence, the correct answer is (D).

37. In first case, $X_C = \frac{V}{I} = \frac{220}{0.25} = 880\ \Omega$

In the second case, $R = \frac{V}{I} = \frac{220}{0.25} = 880\ \Omega$

In the combination of P and Q,

$$\tan\phi = \frac{X_C}{R} = 1$$

$$\Rightarrow \quad \phi = 45^\circ$$

Since the circuit is capacitive in nature, so current leads the voltage by a phase angle of 45°.

The impedance of the circuit is

$$Z = \sqrt{R^2 + X_C^2} = 880\sqrt{2}\ \Omega$$

$$\Rightarrow \quad I = \frac{V}{Z} = \frac{220}{880\sqrt{2}} = \frac{1}{4\sqrt{2}}\ \text{A}$$

Hence, the correct answer is (B).

38. $I = \sqrt{(I_C - I_L)^2 + I_R^2}$

$$\Rightarrow \quad I = V\sqrt{\left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2 + \frac{1}{R^2}}$$

$$\Rightarrow \quad I = 100\sqrt{\left(\frac{1}{20} - \frac{1}{10}\right)^2 + \frac{1}{400}}$$

$$\Rightarrow \quad I = 100\sqrt{\frac{1}{400} + \frac{1}{400}}$$

$$\Rightarrow \quad I = \frac{100}{20}\sqrt{2}$$

$$\Rightarrow \quad I = 5\sqrt{2}\ \text{A}$$

Hence, the correct answer is (D).

39. $\frac{N_S}{N_P} = \frac{550}{22000} = \frac{1}{40}$

Hence, the correct answer is (C).

40. $\omega_0 = \frac{1}{\sqrt{LC}}$

C goes to $4C$, so to keep ω_0 the same L must go to $\frac{L}{4}$

Hence, the correct answer is (C).

41. The capacitive reactance is given by

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 10^{-6}} = 10^4\ \Omega$$

The ac ammeter measures the rms current, so we have

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \frac{\left(\frac{200\sqrt{2}}{\sqrt{2}}\right)}{10^4} = 0.02\ \text{A}$$

$$\Rightarrow \quad I_{\text{rms}} = 20\ \text{mA}$$

Hence, the correct answer is (C).

42. $I_S = \frac{E_S}{Z} = \frac{22}{220} = 0.1\ \text{A}$

Further

$$I_P = \frac{E_S}{E_P} I_S = \frac{22}{220}(0.1) = 0.01\ \text{A}$$

Hence, the correct answer is (A).

43. According to the problem, we have

$$T = \frac{1}{f} = \frac{1}{50}\ \text{s}$$

$$\Rightarrow \quad t = \frac{T}{4} = \frac{1}{200}\ \text{s}$$

$$\Rightarrow \quad t = 5 \times 10^{-3}\ \text{s}$$

$$\Rightarrow \quad t = 5\ \text{ms}$$

Hence, the correct answer is (A).

44. $I_S = \frac{N_P}{N_S} I_P$

$$I_S = \left(\frac{100}{20}\right)(2)$$

$$I_S = 10\ A$$

Hence, the correct answer is (D).

45. For the series combination of resistor and a capacitor, we have

$$V_C = \sqrt{V^2 - V_R^2} = \sqrt{(20)^2 - (12)^2}$$

$$\Rightarrow \quad V_C = 16\ \text{V}$$

Hence, the correct answer is (B).

46. $I_v \sin\phi$ component of current gives no power consumption and hence is called **wattless current**.

Hence, the correct answer is (B).

47. Since we observe that

$$i = \frac{V}{R}$$

Hence the given circuit is in resonance.

$\Rightarrow \quad V_C = V_L = 200 \text{ V}$

Hence, the correct answer is (B).

48. For steady voltage, we have

$$x = \frac{(10)^2}{R}$$

For AC voltage, we have

$$\frac{x}{2} = \frac{E^2}{2R}$$

$\Rightarrow \quad E = 10V$

Hence, the correct answer is (C).

49. Current in the circuit is maximum at resonance and

$$I_{max} = \frac{V}{R}$$

$\Rightarrow \quad 6 = \frac{24}{R}$

$\Rightarrow \quad R = 4\ \Omega$

$\Rightarrow \quad I_{DC} = \frac{V}{R+r} = \frac{12}{4+4} = 1.5 \text{ A}$

Hence, the correct answer is (C).

50. $V = V_0 \cos(\omega t)$

$$-L\frac{dI}{dt} = V_0 \cos(\omega t)$$

$\Rightarrow \quad dI = -\frac{V_0}{L}\cos(\omega t)dt$

$\Rightarrow \quad I = -\left(\frac{V_0}{L\omega}\right)\sin(\omega t)$

$\Rightarrow \quad I = -I_0 \sin(\omega t)$

Hence, the correct answer is (B).

51. The power consumed in the circuit is

$$P = V_{rms} I_{rms} \cos\phi$$

$\Rightarrow \quad P = \left(\frac{V_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\cos 60°$

$\Rightarrow \quad P = \left(\frac{220}{\sqrt{2}}\right)\left(\frac{4}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$\Rightarrow \quad P = 220 \text{ W}$

Hence, the correct answer is (C).

52. For current to lag behind the voltage, circuit should be inductive in nature i.e. $X_L > X_C$

$\Rightarrow \quad L\omega > \frac{1}{C\omega}$

$\Rightarrow \quad \omega^2 > \frac{1}{LC}$

$\Rightarrow \quad \omega^2 > \omega_0^2$

$\Rightarrow \quad \omega > \omega_0$

Hence, the correct answer is (B).

53. $\tan\phi = \frac{X}{R} = \frac{4}{3}$

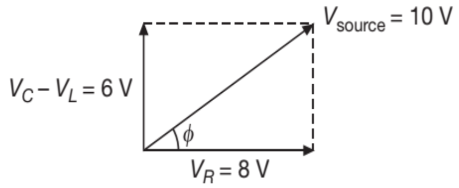
Power Factor $= \cos\phi = \frac{3}{5} = 0.6$

Hence, the correct answer is (B).

54. The voltage of the source is given by

$$V_{source} = \sqrt{V_R^2 + (V_C - V_L)^2} = 10 \text{ V}$$

Since $V_C > V_L$, hence current leads the voltage.



Power factor of the circuit is

$$\cos\phi = \frac{8}{10} = 0.8$$

Hence, the correct answer is (D).

55. $\tan\phi = \frac{X_L}{R} = \frac{L\omega}{R}$

$\Rightarrow \quad \tan\phi = \frac{\left(\frac{1}{\pi}\right)(2\pi)(200)}{300}$

$\Rightarrow \quad \tan\phi = \frac{4}{3}$

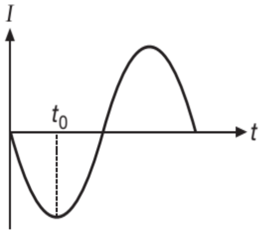
$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{4}{3}\right)$

Hence, the correct answer is (D).

56. Current will lead the voltage by $90°$, so we have

$$i = i_0 \cos\left(\frac{\pi t}{2} + 90°\right) = -i_0 \sin\left(\frac{\pi t}{2}\right)$$

So, the current function plot versus time is shown in Figure.



$$t_0 = \frac{T}{4} = \frac{\left(\frac{2\pi}{\omega}\right)}{4} = \frac{\pi}{2\omega}$$

$$\Rightarrow \quad t_0 = \frac{\pi}{2\left(\frac{\pi}{2}\right)} = 1\text{ s}$$

Hence, the correct answer is (A).

57. $\frac{\xi_S}{\xi_P} = \frac{I_P}{I_S}$

$$\Rightarrow \quad \frac{4.6}{230} = \frac{I_P}{5}$$

$$\Rightarrow \quad I_p = 0.1\text{ A}$$

Hence, the correct answer is (A).

58. For a resistor, the voltage and current are in the same phase, whereas for an inductor the voltage leads the current by $90°$ or current lags behind the voltage by $90°$. Hence, we have

$$I_R = I_0 \sin(\omega t) \text{ and}$$

$$I_L = I_0 \sin(\omega t - 90°) = -I_0 \cos(\omega t)$$

Hence, the correct answer is (C).

59. For ac of rms value 10 A

$$P_{ac} = I_v^2 R \quad \ldots(1)$$

For dc of value I

$$P_{dc} = I^2 R \quad \ldots(2)$$

Equating (1) and (2), we get

$$I = I_v = 10\ A$$

Hence, the correct answer is (A).

60. Resistance does not depend on the frequency of AC.

Hence, the correct answer is (A).

61. Effective current is the rms value. Here 220 V is the labelled value of ac which is also the rms value. Hence

$$I_{\text{rms}} = \frac{E_{\text{rms}}}{R} = \frac{220}{100 \times 10^3}$$

$$\Rightarrow \quad I_{\text{rms}} = 2.2\text{ mA}$$

Hence, the correct answer is (A).

62. The power consumed in the circuit is

$$P = I_{\text{rms}}^2 R = \left(\frac{V_{\text{rms}}}{Z}\right)^2 R$$

$$\Rightarrow \quad P = \left[\frac{\left(\frac{V_0}{\sqrt{2}}\right)^2}{R^2 + \omega^2 L^2}\right] R$$

$$\Rightarrow \quad P = \frac{V_0^2 R}{2\left(R^2 + \omega^2 L^2\right)}$$

Hence, the correct answer is (C).

63. $\xi_S = \left(\frac{N_S}{N_P}\right)\xi_P$

$$\Rightarrow \quad \xi_S = \left(\frac{3000}{1000}\right)(80)$$

$$\Rightarrow \quad \xi_s = 240\ V$$

So, potential difference across each turn of the secondary is

$$\frac{\xi_S}{N_S} = \frac{240}{3000} = 0.08\ V/\text{turn}$$

Hence, the correct answer is (D).

65. $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5 \times 8 \times 10^{-6}}}$

$$\Rightarrow \quad \omega_0 = 500\text{ rads}^{-1}$$

Hence, the correct answer is (A).

66. $I^2 = 16t^2$

$$\langle I^2 \rangle = \frac{\int_0^2 16t^2 dt}{\int_0^2 dt} = \frac{16\left.\frac{t^3}{3}\right|_0^2}{2} = 8\left.\frac{t^3}{3}\right|_0^2 = \frac{8(8)}{3}$$

$$\Rightarrow \quad \langle I^2 \rangle = \frac{64}{3}$$

$$\Rightarrow \quad I_{RMS} = \sqrt{\langle I^2 \rangle} = \frac{8}{\sqrt{3}}\text{ A}$$

Hence, the correct answer is (C).

67. Since current in the capacitor I_C is $90°$ ahead of the applied voltage and current in the inductor I_L lags behind the applied voltage by $90°$. So, there is a phase difference of $180°$ between I_L and I_C. Hence

$$I = I_C - I_L = 0.2\text{ A}$$

Hence, the correct answer is (C).

68. $V_{rms} = \sqrt{\frac{0^2 + V_0^2}{2}} = \frac{V_0}{\sqrt{2}}$

Hence, the correct answer is (B).

69. The inductive reactance is

$X_L = \omega L$

Since ω is very low, so $X_L \approx 0$

$\Rightarrow V_L \approx 0$

$\Rightarrow V = V_C = V_0$

Hence, the correct answer is (B).

70. A Leclanche cell is a dc voltage source and transformer does not work for dc sources.

Hence, the correct answer is (A).

71. The impedance of this circuit is

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

and $I_0 = \frac{V_0}{Z} = V_0\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$

Hence, the correct answer is (B).

72. $I_0 = \frac{E_0}{X_L}$

$\Rightarrow I_0 = \frac{E_0}{L\omega}$

$\Rightarrow I_0 = \frac{220\sqrt{2}}{1(2\pi(50))}$

$\Rightarrow I_0 = \frac{220\sqrt{2}}{100\pi}$

$\Rightarrow I_0 \simeq 1$ A

Hence, the correct answer is (C).

73. Power consumed is

$P = E_V I_V \cos\phi$

Also $\phi = \frac{\pi}{2}$

$\Rightarrow P = 0$

Hence, the correct answer is (D).

74. For DC current passing through the resistor, we have

$H_{DC} = I^2 Rt$

For AC current passing through the resistor, we have

$\Rightarrow H_{AC} = I_{\text{rms}}^2 Rt = \left(\frac{I}{\sqrt{2}}\right)^2 Rt = \frac{I^2 Rt}{2}$

$\Rightarrow \frac{H_{DC}}{H_{AC}} = \frac{2}{1}$

Hence, the correct answer is (A).

75. $V = V_0 \sin(\omega t) + V_0 \sin(2\omega t)$

$\Rightarrow V = V_1 + V_2$

where $V_1 = V_0 \sin(\omega t)$ and $V_2 = V_0 \sin(2\omega t)$

Now, $I_1 = \frac{V_0}{Z_1}$ and $I_2 = \frac{V_0}{Z_2}$ where

$$Z_1 = \sqrt{R^2 + \frac{1}{C^2\omega^2}} \text{ and } Z_2 = \sqrt{R^2 + \frac{1}{4C^2\omega^2}}$$

Since $Z_2 < Z_1$

$\Rightarrow I_1 < I_2$

Hence, the correct answer is (C).

76. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

$\Rightarrow X_C \propto \frac{1}{f}$

So, X_C versus f graph is a rectangular hyperbola

Hence, the correct answer is (B).

77. $\omega_0 = 2\pi(100) = 600$ rad s^{-1} $\{\text{Given } \pi = 3\}$

Further

$\omega_0 = \frac{1}{\sqrt{LC}}$ ...(1)

Also $X_C = \frac{1}{C\omega_0} = 60$

$\Rightarrow C = \frac{1}{\omega_0(60)}$

$\Rightarrow C = \frac{1}{600 \times 60}$

$\Rightarrow C = \frac{1}{36 \times 10^{-3}}$ F

So, put values in (1), we get

$$600 = \frac{1}{\sqrt{L\left(\frac{1}{36 \times 10^{-3}}\right)}}$$

$\Rightarrow 36 \times 10^4 = \frac{36 \times 10^{-3}}{L}$

$\Rightarrow L = 0.1$ H

Hence, the correct answer is (A).

CHAPTER 4

78. $I_0 = \frac{E_0}{R}$ (at resonance)

$\Rightarrow \quad I_0 = \frac{300}{10} = 30\text{ A}$

Hence, the correct answer is (C).

79. The net source voltage is

$V_S = \sqrt{V_R^2 + V_L^2}$

$\Rightarrow \quad V_S = \sqrt{(70)^2 + (20)^2} = 72.8\text{ V}$

$\Rightarrow \quad \tan\phi = \frac{X_L}{R} = \frac{V_L}{V_R} = \frac{20}{70} = \frac{2}{7}$

Hence, the correct answer is (A).

80. $I = \frac{V}{Z}$ where

$Z^2 = R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2$

$L\omega = 2\pi f L = 120 \times 0.3\pi$

$\Rightarrow \quad L\omega = 36\pi = 113.04$

$\frac{1}{C\omega} = \frac{1}{2\pi f C} = \frac{10^6}{7200\pi}$

$\Rightarrow \quad \frac{1}{C\omega} = 44.23$

$\Rightarrow \quad Z = 85.05\ \Omega$

$\Rightarrow \quad I = \frac{120}{85.05} \cong 1.5\text{ A}$

Hence, the correct answer is (A).

81. $P = E_V I_V \cos\phi$

$\Rightarrow \quad P = (I_V Z)(I_V)\left(\frac{R}{Z}\right)$

$\Rightarrow \quad P = I_V^2 R$

$\Rightarrow \quad P = (1.5)^2 (50)$

$\Rightarrow \quad P = 112.5\text{ W}$

Hence, the correct answer is (D).

83. $V = V_0 \sin(2\pi f t)$

$\Rightarrow \quad V = 10\sin\left(\frac{100\pi}{600}\right)$

$\Rightarrow \quad V = \frac{10}{2} = 5\text{ V}$

Hence, the correct answer is (C).

84. $\frac{N_S}{N_P} = \frac{22000}{220} = 100$

Hence, the correct answer is (B).

85. Average value of $5\sin 100\omega t$ is zero. But average value of 6 A (which is a constant current) is 6 A. Hence, average value of current is 6 A.

Hence, the correct answer is (B).

86. $P_{av} = E_v I_v \cos\phi = \frac{E_0}{\sqrt{2}}\frac{I_0}{\sqrt{2}}\cos\phi$

$\Rightarrow \quad P_{av} = \frac{E_0 I_0}{2}\cos\phi$

$\Rightarrow \quad P_{av} = \frac{100 \times 100 \times 10^{-3}}{2} \times \frac{1}{2}$

$\Rightarrow \quad P_{av} = 2.5\text{ W}$

Hence, the correct answer is (C).

Multiple Correct Choice Type Questions

1. The average power consumed in the circuit is

$$\langle P \rangle = \frac{V_0^2 R}{2\left[R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2\right]}$$

$\langle P \rangle = 0$, when $\omega \to 0$ or when $\omega \to \infty$ i.e. $\langle P \rangle$ is zero for extremely small and extremely large frequencies. So, (A) is correct.

The average power consumed in the circuit varies with the frequency, because the impedance Z depends upon the frequency. So, (B) is incorrect.

Also, $\langle P \rangle = \frac{V_0^2}{2R} = P_{max}$ at resonance i.e. when

$\omega = \frac{1}{\sqrt{LC}}$

$\Rightarrow \quad \omega^2 LC = 1$

So, (C) is correct.

Also, $\langle P \rangle = \frac{V_0^2}{4R}$ i.e. $\langle P \rangle = \frac{V_0^2}{4R} = \frac{P_{max}}{2}$ at two frequencies that are also called half power frequencies, such that $\Delta\omega = \frac{R}{L}$. So, (D) is also correct.

Hence, (A), (C) and (D) are correct.

2. Power factor, $\cos\phi = \frac{R}{Z}$

When circuit is purely resistive, then

$Z = R$

$\Rightarrow \quad \cos\phi = 1$

When circuit is purely capacitive or inductive, then

$$R = 0$$

$\Rightarrow \quad \cos\phi = 0$

When the difference of inductive reactance and capacitive reactance i.e. $X_L - X_C$ is 1.732 i.e. $\sqrt{3}$ times the resistance R in the circuit, then, we have

$$X_L - X_C = 1.732R = \sqrt{3}R$$

Since, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$\Rightarrow \quad Z = \sqrt{R^2 + (\sqrt{3}R)^2} = 2R$$

$$\Rightarrow \quad \cos\phi = \frac{R}{Z} = \frac{R}{2R} = 0.5$$

Hence, (A), (C) and (D) are correct.

3. Mean current $\langle i \rangle = \frac{\text{Area}}{\text{Time}}$

$$\langle i \rangle = i_m = \frac{\frac{1}{2} \times i_0 \times \frac{T}{2}}{\frac{T}{2}}$$

$$\Rightarrow \quad i_m = \frac{i_0}{2}$$

As $i = \frac{2i_0}{T}t$

$$\Rightarrow \quad i_2 = \frac{\frac{4i_0^2}{T^2}\int_0^{T/2} t^2 dt}{\frac{T}{2}} = \frac{2 \times 4i_0^2}{T^3}\frac{T^3}{3 \times 8} = \frac{i_0^2}{3}$$

$$\Rightarrow \quad i_{\text{rms}} = \sqrt{i^2} = \frac{i_0}{\sqrt{3}}$$

Hence, (B) and (C) are correct.

4. Since we know that

$$P_R = V_R i$$

$$\Rightarrow \quad i = \frac{P_R}{V_R} = \frac{60}{60} = 1 \text{ A}$$

Also, for the given circuit

$$V_L = \sqrt{V^2 - V_R^2}$$

$$\Rightarrow \quad V_L = \sqrt{(100)^2 - (60)}^2$$

$$\Rightarrow \quad V_L = 80 \text{ V} = iX_L = i(2\pi fL)$$

$$\Rightarrow \quad L = \frac{80}{2\pi fi}$$

$$\Rightarrow \quad L = \frac{80}{(2\pi)(50)(1)} = \frac{4}{5\pi} \text{ H}$$

When we connect another resistance R in series, then it should consume 40 V, so that remaining 60 V is used by the tube light.

$$R = \frac{V}{i} = \frac{40}{1} = 40 \ \Omega$$

Hence, (A) and (D) are correct.

5. Since $V_L = V_C$

So, $V = \sqrt{(V_L - V_C)^2 + V_R^2} = V_R$

$$\Rightarrow \quad I = \frac{V}{R} = \frac{V_R}{R} = \frac{200}{100} = 2 \text{ A}$$

Hence, (C) and (D) are correct.

6. The voltage across the resistor is

$$V_R = IR = 80 \text{ V}$$

The capacitive reactance is

$$X_C = \frac{V_C}{I} = \frac{100}{2} = 50 \ \Omega$$

Voltage across the inductor is

$$V_L = IX_L = 40 \text{ V}$$

The net voltage across the combination is

$$V = V_{\text{rms}} = \sqrt{V_R^2 + (V_C - V_L)^2}$$

$$\Rightarrow \quad V = \sqrt{(80)^2 + (100 - 40)^2} = 60 \text{ V}$$

So, the peak value of voltage is

$$V_0 = \sqrt{2}V_{\text{rms}} = 60\sqrt{2} \text{ V}$$

Hence, (A), (B) and (C) are correct.

7. $P = \frac{E_0 I_0}{2}\cos\phi$

At resonance, $\phi = 0°$

$\Rightarrow \quad \cos\phi = \text{Power Factor} = 1$

Also, $Z = Z_{\text{MIN}} = R$ and

$$P = \frac{I_0^2 R}{2}$$

No power is consumed across the capacitor and the inductor at resonance.

Hence, (A), (C) and (D) are correct.

8. Current through the resistor is

$$I_R = \frac{V_{\text{rms}}}{R} = \frac{200}{100} = 2 \text{ A}$$

Capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{(2\pi)(5\times10^3)\left(\frac{1}{\pi}\times10^{-6}\right)}$$

$$\Rightarrow \quad X_C = 100\ \Omega$$

$$\Rightarrow \quad I_C = \frac{V_{rms}}{X_C} = \frac{200}{100} = 2\text{ A}$$

Since I_C is 90° ahead of the applied voltage and I_R is in phase with the applied voltage, so there is a phase difference of 90° between I_R and I_C too. Hence

$$I = \sqrt{I_R^2 + I_C^2}$$

$$\Rightarrow \quad I = \sqrt{(2)^2 + (2)^2} = 2\sqrt{2} \approx 2.83\text{ A}$$

Hence, (A), (C) and (D) are correct.

9. For voltmeter V_1, we have

$$\sqrt{V_R^2 + V_L^2} = 100\text{ V} \quad \ldots(1)$$

For voltmeter V_2, we have

$$|V_L - V_C| = 120\text{ V} \quad \ldots(2)$$

Since the applied voltage is 130 V, so we have

$$\sqrt{V_R^2 + (V_L - V_C)^2} = 130\text{ V} \quad \ldots(3)$$

Solving these three equations, we get

$V_R = 50$ V, $V_L = 86.6$ V and $V_C = 206.6$ V

Power factor $\cos\phi = \frac{R}{Z} = \frac{V_R}{V} = \frac{50}{130} = \frac{5}{13}$

Since $V_C > V_L$, circuit is capacitive in nature.

Hence, (A), (C) and (D) are correct.

10. In series LCR circuit

$$V_{rms}^2 = (V_R)^2 + (V_L - V_C)^2$$

$$(170)^2 = V_R^2 + (V_L - V_C)^2$$

Obviously $V_R \le (170)^2$

Here V_L and V_C may be greater than 170 volt or less than 170 volt

But $|V_L - V_C| \le (170)$ volt

Hence, (A), (C) and (D) are correct.

11. The resonance frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Since $X_L < X_C$

$$\Rightarrow \quad L\omega < \frac{1}{C\omega}$$

$$\Rightarrow \quad \omega^2 < \frac{1}{LC}$$

$$\Rightarrow \quad \omega^2 < \omega_0^2$$

$$\Rightarrow \quad \omega < \omega_0$$

When the frequency is increased from the given value, X_C will further increase, whereas X_L will increase, so $X_C - X_L$ will decrease and hence the net impedance will decrease.

When frequency is increased from the given value, then till the resonant frequency ω_0, the current increase, becomes maximum at resonant frequency and after that it again starts decreasing.

Hence, (A), (C) and (D) are correct.

12. $f = \frac{1}{2\pi\sqrt{LC}}$

$$\Rightarrow \quad f_1 = f_2 = f_3 = \frac{1}{2\pi\sqrt{LC}}$$

Also, we have, from Law of Conservation of Energy,

$$\frac{1}{2}LI_{max}^2 = \frac{1}{2}CV^2$$

$$\Rightarrow \quad I_{max} = \sqrt{\frac{C}{L}}V$$

$$\sqrt{\frac{C_1}{L_1}} = \sqrt{\frac{C}{L}}$$

$$\sqrt{\frac{C_2}{L_2}} = \frac{1}{2}\sqrt{\frac{C}{L}} \text{ and}$$

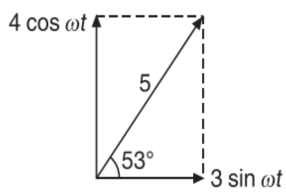
$$\sqrt{\frac{C_3}{L_3}} = 2\sqrt{\frac{C}{L}}$$

$$\Rightarrow \quad I_2 < I_1 < I_3$$

Hence, (A) and (D) are correct.

13. The given function $i = 3\sin\omega t + 4\cos\omega t$ can be re-written as

$$i = 5\sin(\omega t + 53°)$$



So, we have $i_0 = 5$ A

$$\Rightarrow \quad i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} = 2.5\sqrt{2}\text{ A}$$

Mean value of current in positive half cycle is

$$\langle i \rangle = \frac{2i_0}{\pi} = \left(\frac{2}{\pi}\right)(5) = \left(\frac{10}{\pi}\right) = 3.2 \text{ A}$$

For $V = V_0 \sin\omega t$, the current $i = 5\sin(\omega t + 53°)$ leads the voltage. Hence, circuit is capacitive in nature. However, for $V = V_0 \cos\omega t = V_0 \sin(\omega t + 90°)$, the current $i = 5\sin(\omega t + 53°)$ lags behind the voltage and hence the circuit is inductive in nature.

Hence, (A), (B), (C) and (D) are correct.

14. Since, $X_L > X_C$, hence voltage function will lead the current function.

Also, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\Rightarrow \quad Z = \sqrt{(10)^2 + (20-10)^2}$

$\Rightarrow \quad Z = 10\sqrt{2}\ \Omega$

$\Rightarrow \quad \cos\phi = \frac{R}{Z} = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \phi = 45°$

So, power factor is given by

$$\cos\phi = \frac{R}{Z} = \frac{1}{\sqrt{2}}$$

Hence, (A), (B), (C) and (D) are correct.

Reasoning Based Questions

1. At resonant frequency

$X_L = X_C$

$\therefore \quad Z = R$ (minimum)

Therefore, current in the circuit is maximum.

Hence, the correct answer is (A).

4. For half cycle $I_{\text{mean}} = 0.636 I_0$

or $\quad E_{\text{mean}} = 0.636 E_0$

Average value is always defined over a half cycle cause in next half cycle it will be opposite in direction. Hence for one complete cycle, average value will be zero.

Hence, the correct answer is (B).

5. The capacitive reactance of capacitor is given by

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

So, this is infinite for d.c. $(f = 0)$ and has a very small value for a.c. therefore a capacitor block d.c.

Hence, the correct answer is (A).

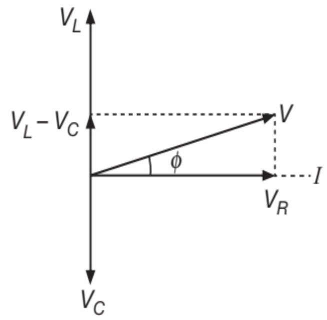
6. $\omega > \omega_0$

Since, $X_L - X_C = L\omega - \frac{1}{C\omega} = \frac{CL\omega^2 - 1}{C\omega} = X_C\left[\left(\frac{\omega}{\omega_0}\right)^2 - 1\right]$

$\Rightarrow \quad X_L - X_C > 0 \qquad \{\because \omega > \omega_0\}$

$\Rightarrow \quad \tan\phi = \frac{X_L - X_C}{R}$

$\Rightarrow$ V leads I by ϕ and the circuit will be inductive in nature.



Hence, the correct answer is (A).

8. The maximum value of r.m.s. current is $I_{\text{rms}} = \frac{E_{\text{r.m.s.}}}{Z} = \frac{E_{\text{r.m.s.}}}{R}$. It does not depend upon ω.

Hence, the correct answer is (D).

Linked Comprehension Type Questions

1. The given equation $E = 310\sin(314t)$ is compared with the equation $E = E_0 \sin 2\pi f t$, we get

$2\pi f = 314$

$\Rightarrow \quad 2 \times \frac{22}{7} \times f = 314$

$\Rightarrow \quad f = \frac{7 \times 314}{2 \times 22}$ cps ≈ 50 cps

Hence, the correct answer is (B).

2. Inductive reactance is

$$X_L = L\omega = 2\pi f L = 2 \times \frac{22}{7} \times 50 \times 0.1 \text{ ohm} = 31.4\ \Omega$$

Capacitive reactance,

$$X_C = \frac{1}{C\omega} = \frac{1}{2\pi f C} = \frac{1 \times 7}{2 \times 22 \times 50 \times 25 \times 10^{-6}}\ \Omega$$

$\Rightarrow \quad X_C = 127.3\ \Omega$

Net reactance

$$X = X_C - X_L = (127.3 - 31.4)\ \Omega = 95.9\ \Omega$$

$\{\because X_C$ and X_L differ in phase by $180°\}$

Hence, the correct answer is (A).

3. Impedance, $Z=\sqrt{R^2+\left(X_L-X_C\right)^2}$

$\Rightarrow \quad Z=\sqrt{25^2+95.9^2}\ \Omega=99.1\ \Omega$

Hence, the correct answer is (C).

4. Current, $I_v=\dfrac{E_v}{Z}=\dfrac{E_0}{\sqrt{2}Z}=\dfrac{310}{\sqrt{2}\times 99.1}A=2.21\ A$

Hence, the correct answer is (B).

5. If ϕ be the phase angle, then

$$\tan\phi=\frac{X_C-X_L}{R}=\frac{95.9}{25}=3.84$$

$\Rightarrow \quad \phi=75.4°$

Thus, the current leads the voltage by $75.4°$. This is because the capacitive reactance is greater than inductive reactance and, therefore, the circuit is capacitive in nature.

Hence, the correct answer is (A).

6. The current at any time t is given by

$$I=I_0\sin\left(\omega t+\phi\right)$$

$\Rightarrow \quad I=I_0\sin\left(2\pi ft+\phi\right)$

where $I_0=\sqrt{2}\times I_v=\sqrt{2}\times 2.21\ A=3.13\ A$

$\Rightarrow \quad I=3.13\sin\left(314t+1.32\right) \quad \left\{\because\ \phi=75.4°=1.32\text{ rad}\right\}$

Hence, the correct answer is (A).

7. Effective voltage across inductor is

$$V_L=I_v\times X_L=2.21\ A\times 31.4\ \Omega=69.4\text{ V}$$

Effective voltage across capacitor is

$$V_C=I_v\times X_C=2.21\text{ A}\times 127.3\ \Omega=281.3\text{ V}$$

Effective voltage across resistor is

$$V_R=I_vR=2.21\text{ A}\times 25\ \Omega=55.3\text{ V}$$

Hence, the correct answer is (A).

8. Impedance will be minimum, if $X_L=X_C$

$\Rightarrow \quad 2\pi fL=\dfrac{1}{2\pi fC}$

$\Rightarrow \quad L=\dfrac{1}{4\pi^2f^2C}$

$\Rightarrow \quad L=\dfrac{1\times 7\times 7}{4\times 22\times 22\times 50\times 50\times 25\times 10^{-6}}\text{ H}$

$\Rightarrow \quad L=0.41\text{ H}$

Hence, the correct answer is (D).

9. $I_0=\dfrac{E_0}{\sqrt{R^2\left(\dfrac{1}{\omega C}-\omega L\right)^2}}$

For maximum current

$$I_0=\frac{E_0}{R} \qquad \left\{\because\ \frac{1}{C\omega}-L\omega=0\right\}$$

Hence, the correct answer is (B).

10. $f_0=\dfrac{1}{2\pi\sqrt{LC}}$

$\Rightarrow \quad f_0=\dfrac{250}{3}\text{ Hz}$

Hence, the correct answer is (D).

12. $\dfrac{1}{2}CV^2=\dfrac{1}{2}LI^2$

$\Rightarrow \quad I=V\sqrt{\dfrac{C}{L}}$

Hence, the correct answer is (A).

13. $f=\dfrac{1}{2\pi\sqrt{LC}}$

Hence, the correct answer is (C).

14. $V_{\text{rms}}=\dfrac{V_m}{\sqrt{2}}=\dfrac{283}{\sqrt{2}}\text{ V}=\dfrac{283}{1.414}\text{ V}\approx 200\text{ V}$

Hence, the correct answer is (C).

15. $X_L=\omega L=2\pi vL=2\times 3.14\times 50\times 25.48\times 10^{-3}\ \Omega$

$\Rightarrow \quad X_L=8\ \Omega$

Hence, the correct answer is (D).

16. $X_C=\dfrac{1}{\omega C}=\dfrac{1}{2\pi vC}$

$\Rightarrow \quad X_C=\dfrac{1}{2\times 3.14\times 50\times 7.96\times 10^{-4}}\approx\dfrac{1}{0.25}=4\ \Omega$

Hence, the correct answer is (B).

17. $Z=\sqrt{R^2+\left(\omega L-\dfrac{1}{\omega C}\right)^2}=\sqrt{3^2+(8-4)^2}$

$\Rightarrow \quad Z=\sqrt{9+16}\ \Omega=\sqrt{25}\ \Omega=5\ \Omega$

Hence, the correct answer is (D).

18. $I_m=\dfrac{V_m}{Z}=\dfrac{283}{5}\text{A}=56.6\text{ A}$

Hence, the correct answer is (A).

19. $\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\frac{4}{3} = 53.13°$

Hence, the correct answer is (B).

20. $I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{56.6}{1.414}\text{ A} = 40\text{ A}$

$X_L > X_C$

So, the current lags the voltage

Hence, the correct answer is (D).

21. $(V_C)_{rms} = 4 \times 40\text{ V} = 160\text{ V}$

Hence, the correct answer is (B).

22. $(V_L)_{rms} = 8 \times 40\text{ V} = 320\text{ V}$

Hence, the correct answer is (C).

23. $(V_R)_{rms} = 3 \times 40\text{ V} = 120\text{ V}$

Hence, the correct answer is (A).

24. $P = I_{rms}^2 R = 40^2 \times 3\text{ W} = 4800\text{ W}$

Hence, the correct answer is (A).

25. Power factor $= \cos\phi = \cos 53.13° = 0.6$

Hence, the correct answer is (D).

26. Power input $= V_{rms} I_{rms} \cos\theta$

$\Rightarrow P_{in} = 200 \times 40 \times 0.6\text{ W} = 4800\text{ W}$

It may be noted that the power input is the same as the power dissipated in the resistor. This is because the capacitor and the inductor do not absorb or produce any net power.

Hence, the correct answer is (A).

27. For resonance $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\omega_0 = \frac{1}{\sqrt{25.48 \times 10^{-3} \times 7.96 \times 10^{-4}}}$$

$$\omega_0 = \frac{10^4}{\sqrt{2028}} = 222.1\text{ rads}^{-1}$$

$$\omega_0 = 2\pi v_0,\ v_0 = \frac{\omega_0}{2\pi} = \frac{222.1}{2 \times 3.14}\text{ Hz} = 35.4\text{ Hz}$$

Hence, the correct answer is (A).

28. At resonance condition

$$Z = R = 3\ \Omega$$

Hence, the correct answer is (C).

29. At resonance, $I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} = \frac{200}{3} A$

$\Rightarrow I_{rms} = 66.67\ A$

Hence, the correct answer is (A).

30. Power consumed at resonance is

$$P = I_{rms}^2 R = 66.67 \times 66.67 \times 3\text{ W} = 13.33\text{ kW}$$

Hence, the correct answer is (A).

31. $z = \frac{V_0}{I_0} = \frac{170}{8.5} = 20,\ \phi = \frac{\pi}{6}$ and

$\omega = 6280$

$\tan\phi = \frac{X_C}{R} = \frac{1}{R(C\omega)}$

$\Rightarrow R = \sqrt{3}\frac{1}{C\omega}$

$\Rightarrow C\omega = \frac{\sqrt{3}}{R}$

Since, $Z = \sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2}$

$\Rightarrow Z = \sqrt{R^2 + \frac{R^2}{3}}$

$\Rightarrow Z = \frac{2R}{\sqrt{3}}$

$\Rightarrow R = 17.32\ \Omega$

Hence, the correct answer is (B).

32. $\frac{1}{C\omega} = 10$

$\Rightarrow C = 15.92\ \mu\Omega$

Hence, the correct answer is (A).

33. $\cos\phi = \cos\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

Hence, the correct answer is (C).

34. I_0 is maximum at $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$

$\Rightarrow \omega_0 = \frac{1}{\sqrt{0.12 \times 480 \times 10^{-9}}}\text{ rad s}^{-1}$

$\Rightarrow \omega_0 = 4167\text{ rad s}^{-1}$

If f_0 is the frequency corresponding to resonant angular frequency ω_0, then $2\pi f_0 = \omega_0$

$$\Rightarrow \quad f_0 = \frac{\omega_0}{2\pi} = \frac{4167}{2\pi}\text{ Hz} = 663\text{ Hz}$$

Hence, the correct answer is (D).

35. Maximum current is

$$I = \frac{E_0}{R} = \frac{\sqrt{2}\times 230}{23}\text{ ampere}$$

Hence, the correct answer is (D).

36. Average power $= \frac{1}{2}I_0^2 R$

It is maximum at the same frequency i.e., resonant frequency of 663 Hz for which I_0 is maximum.

Hence, the correct answer is (D).

37. Maximum average power

$$P = \frac{1}{2}(I_0)_{\max}^2 R = \frac{1}{2}(14.1)^2 \times 23\text{ watt}$$

$$P = 2286\text{ watt}$$

Hence, the correct answer is (C).

38. At $\omega \simeq \omega_0 \pm \frac{R}{2L}$ $\quad \left\{\text{iff } \frac{R}{2L} \ll \omega_0\right\}$

$$\Rightarrow \quad \Delta\omega = \frac{R}{2L} = \frac{23}{0.24}\text{ rad s}^{-1} = 95.8\text{ rad s}^{-1}$$

$$\Rightarrow \quad \Delta f = \frac{\Delta\omega}{2\pi} = \frac{95.8}{2\times 3.14}\text{ Hz} = 15.3\text{ Hz}$$

Power absorbed is half the peak power at

$$f = 648\text{ Hz and } f_2 = 678\text{ Hz}$$

Hence, the correct answer is (A).

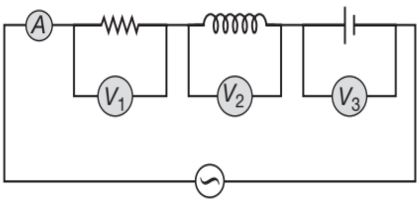
39. At these frequencies, current amplitude is $\frac{1}{\sqrt{2}}$ times $(I_0)_{\max}$ i.e., current amplitude (at half the peak power points) is 10 A.

Hence, the correct answer is (D).

40. $Q = \frac{\omega_0 L}{R} = \frac{4167 \times 0.12}{23} = 21.7$

Hence, the correct answer is (A).

41.



rms values of voltages are

$$V_R = 100\text{ V},\ V_L = 125\text{ V and } V_C = 150\text{ V}$$

Since, $Z = \frac{V_{rms}}{I_{rms}}$

$$\Rightarrow \quad Z = \frac{\sqrt{(V_L - V_C)^2 + V_R^2}}{I_{rms}} = 5\sqrt{17}\ \Omega$$

Hence, the correct answer is (C).

42. Power factor $\cos\phi = \frac{R}{Z} = \frac{100}{26\sqrt{17}} = \frac{4}{\sqrt{17}}$

Hence, the correct answer is (A).

43. $V_{\text{rms}} = 25\sqrt{17} = \frac{E_0}{\sqrt{2}}$

$$\Rightarrow \quad E_0 = 25\sqrt{34}\text{ V}$$

Hence, the correct answer is (D).

44. If $E = E_0\cos\omega t$, then

$$I = \frac{E_0}{\left|\omega L - \frac{1}{\omega C}\right|}\cos\left(\omega t \pm \frac{\pi}{2}\right),\text{ if } R \text{ is zero.}$$

If $\omega L > \frac{1}{\omega C}$, then –ve sign appears. However, if $\omega L < \frac{1}{\omega C}$, then +ve sign appears.

$$L\omega - \frac{1}{C\omega} = 2\times\pi\times 50\times 80\times 10^{-3} - \frac{1}{2\pi\times 50\times 60\times 10^{-6}}$$

$$\Rightarrow \quad X_L - X_C = 25.12 - 53.08 = -27.96\ \Omega \simeq -28\ \Omega$$

The negative sign indicates that the reactance is capacitive in nature.
Current amplitude,

$$I_0 = \frac{E_0}{28} = \frac{1.414\times 230}{28}\text{ A} = 11.6\text{ A}$$

Hence, the correct answer is (D).

45. $I_v = \frac{230}{28}\text{ ampere} = 8.2\text{ A}$

Hence, the correct answer is (D).

46. $(E_L)_v = I_v(L\omega) = 25.12\times 8.2\text{ volt} \simeq 206\text{ V}$

Hence, the correct answer is (B).

47. $(E_C)_v = I_v\left(\frac{1}{C\omega}\right) = 53.08\times 8.2\text{ volt} = 435.3\text{ V}$

Hence, the correct answer is (D).

48. Applied rms voltage is

$$E_V = (435.3 - 206)\text{ volt} = 229.3\text{ V}$$

(The voltages across L and C get subtracted because they are $180°$ out of phase).

Hence, the correct answer is (A).

49. Whatever the value of current in the inductor, the actual voltage leads the current by $\frac{\pi}{2}$.

So, the average power consumed by L is zero.

Hence, the correct answer is (A).

50. For C, voltage lags by $\frac{\pi}{2}$. So, average power consumed by C is zero.

Hence, the correct answer is (A).

51. Total average power absorbed by the circuit is zero.

Hence, the correct answer is (A).

Matrix Match/Column Match Type Questions

1. A → (r)
B → (s)
C → (t)
D → (q)

For a series LCR circuit, current lags behind the voltage when $X_L > X_C$, so we have

$$\tan\phi = \frac{X_L - X_C}{R}$$

Also, $V_L = 2\,V_C$

$$\Rightarrow \quad IX_L = 2IX_C$$

$$\Rightarrow \quad X_L = 2X_C$$

So, for $\phi = 45°$, we get

$$\tan(45°) = 1 = \frac{2X_C - X_C}{R}$$

$$\Rightarrow \quad X_C = R = 20\ \Omega \text{ and } X_L = 40\ \Omega$$

So, (A) → (r) and (B) → (s)
Further

$$Z = \sqrt{(X_L - X_C)^2 + R^2} = \sqrt{(20)^2 + (20)^2}$$

$$\Rightarrow \quad Z = 20\sqrt{2}\ \Omega$$

So, (C) → (t)

Further, when $\phi = 60°$, then

$$\tan(60°) = \frac{X_L - X_C}{R}$$

$$\Rightarrow \quad X_L - X_C = \sqrt{3}R$$

$$\Rightarrow \quad X_L - X_C = 20\sqrt{3}\ \Omega$$

$$\Rightarrow \quad \text{(D)} \rightarrow \text{(q)}$$

2. A → (q)
B → (p)
C → (r)
D → (s)

The resistance is given by

$$R = \frac{V_R}{I} = \frac{40}{2} = 20\ \Omega$$

Since, we know that

$$V_C = IX_C = 2 \times 30 = 60\text{ V}$$

$$V_L = IX_L = 2 \times 15 = 30\text{ V}$$

$$V = \sqrt{V_R^2 + (V_C - V_L)^2} = 50\text{ V}$$

3. A → (r)
B → (q)
C → (s)
D → (p)

For a non-ideal inductor, the voltage leads the current.
For both ideal inductor and capacitor, phase angle between voltage and current is $90°$.
For series LCR circuit, impedance is minimum at resonance, so there is no phase lag between the voltage and current.
For a non-ideal capacitor, the current leads the voltage.

4. A → (q, s)
B → (r, s)
C → (r, s)
D → (r, s)

Since, $P = I_{\text{rms}}^2 R$, where $I_{\text{rms}} = \frac{E_{\text{rms}}}{Z}$ and the impedance Z is given by $Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$

By increasing R, current I_{rms} will decrease but the power, $P = I_{\text{rms}}^2 R$ may increase or decrease. Similarly when C (or L) is increased, then Z may increase or decrease and hence P may increase or decrease.

5. A → (p, t)
B → (q)
C → (r, t)
D → (s, t)

For (A), the phase difference between voltage and current is $\phi = 0°$. This is possible when the circuit is only resistive in nature.

For (B), $I = I_0 \sin(\omega t - 90^\circ)$ i.e. voltage leads the current by $\phi = 90^\circ$, which is possible only for an ideal inductor.

For (C), $I = I_0 \sin\left(\omega t + \frac{\pi}{6}\right)$ i.e. current leads the voltage by $\phi = 30^\circ$. This is possible only when circuit is capacitive in nature

For (D), $I = I_0 \sin\left(\omega t - \frac{\pi}{3}\right)$ i.e. voltage leads the current by $\phi = 60^\circ$. This is possible only when the circuit is inductive in nature.

Integer/Numerical Answer Type Questions

1. (a) At resonance frequency $X_L = X_C$, so we have

$$Z = R$$

Since $\cos\phi = \frac{R}{Z}$

$$\Rightarrow \quad \cos\phi = 1$$

(b) $$\langle P \rangle = V_{\text{rms}} I_{\text{rms}} \cos\phi = \left(\frac{300}{\sqrt{2}}\right) \times \frac{\left(\frac{300}{\sqrt{2}}\right)}{300} \times 1$$

$$\Rightarrow \quad \langle P \rangle = 150 \text{ W}$$

(c) Average power delivered is still 150 W, because still we have $Z = R$ and $\cos\phi = 1$.

2. Inductive reactance of the inductor is

$$X_L = 2\pi f L = 100\ \Omega$$

Impedance of the circuit is

$$Z = \sqrt{R^2 + X_L^2}$$

$$\Rightarrow \quad Z = \sqrt{(100)^2 + (100)^2} = 100\sqrt{2}\ \Omega$$

$$\Rightarrow \quad I_{\text{rms}} = \frac{E_{\text{rms}}}{Z} = \frac{220}{100\sqrt{2}} = 1.1\sqrt{2} = 1.56 \text{ A}$$

3. The coil consists of an inductance (L) and a resistance (R) and for the dc input only resistance is effective. So,

$$R = \frac{V}{I} = \frac{12}{4} = 3\ \Omega$$

For AC, $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \omega^2 L^2}}$

$$\Rightarrow \quad L^2 = \frac{1}{\omega^2}\left[\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2\right]$$

$$\Rightarrow \quad L = \frac{1}{\omega}\sqrt{\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - R^2}$$

Substituting the values, we get

$$L = \frac{1}{50}\sqrt{\left(\frac{12}{2.4}\right)^2 - (3)^2} = 0.08 \text{ H}$$

$$\Rightarrow \quad L = 80 \text{ mH}$$

When capacitor is connected to the circuit, the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Here $R = 3\ \Omega$, $X_L = \omega L = (50)(0.08) = 4\ \Omega$ and

$$X_C = \frac{1}{\omega C} = \frac{1}{(50)(2500 \times 10^{-6})} = 8\ \Omega$$

$$\Rightarrow \quad Z = \sqrt{(3)^2 + (4-8)^2} = 5\ \Omega$$

Now, $\langle P \rangle = V_{\text{rms}} I_{\text{rms}} \cos\phi = (V_{\text{rms}})\left(\frac{V_{\text{rms}}}{Z}\right)\left(\frac{R}{Z}\right)$

$$\langle P \rangle = \left(\frac{V_{\text{rms}}}{Z}\right)^2 R$$

Substituting the values, we get

$$\langle P \rangle = \left(\frac{12}{5}\right)^2 \times 3$$

$$\Rightarrow \quad \langle P \rangle = 17.28 \text{ W}$$

$$\Rightarrow \quad \langle P \rangle = 17 \text{ W}$$

4. Since an AC voltmeter reads the rms value of voltage. The applied rms source voltage is

$$V = V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{50\sqrt{2}}{\sqrt{2}} = 50 \text{ V}$$

Let the reading of voltmeter V_2 be x, then the combined reading of voltmeters V_1 and V_2 equals the applied rms source voltage, so we have

$$\sqrt{x^2 + 40^2} = 50 \text{ V}$$

$$\Rightarrow \quad x = 30 \text{ V}$$

5. $$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow \quad C = \frac{1}{4\pi^2 f_0^2 L}$$

$$\Rightarrow \quad C = \frac{1}{(40)(25 \times 10^{12})(2 \times 10^{-6})}$$

$$\Rightarrow \quad C = 500 \times 10^{-12} \text{ F} = 500 \text{ pF}$$

6. When capacitor is removed, then the circuit is simply an LR circuit, so we have

$$\tan\phi = \frac{X_L}{R}$$

$$\Rightarrow \quad \tan\phi = \tan 45° = \frac{X_L}{R}$$

$$\Rightarrow \quad X_L = R$$

When the inductor is removed, then the circuit simply becomes a CR circuit and then

$$\tan\phi' = \frac{X_C}{R} = \frac{1}{2}$$

$$\Rightarrow \quad X_C = \frac{R}{2}$$

So, the impedance of the complete LCR circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\Rightarrow \quad Z = \sqrt{R^2 + \left(R - \frac{R}{2}\right)^2} = \frac{\sqrt{5}}{2}R$$

Since, $I_{rms} = \frac{E_{rms}}{Z} = \frac{200}{250} = 0.8 \text{ A}$ and $\cos\phi = \frac{R}{Z} = \frac{2}{\sqrt{5}}$

$$\Rightarrow \quad P_{av} = V_{rms} I_{rms} \cos\phi = 64\sqrt{5} \text{ W} = 143 \text{ W}$$

7. Since $V_0 = 100\sqrt{2}$ volt, so the rms value of voltage across the source is

$$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{100\sqrt{2}}{\sqrt{2}} = 100 \text{ V}$$

Since $\omega = 1000 \text{ rads}^{-1}$ and the impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where, $X_L = L\omega = (2)(1000) = 2000\ \Omega$ and

$$X_C = \frac{1}{C\omega} = \frac{1}{(10^{-6})(1000)} = 1000\ \Omega$$

$$\Rightarrow \quad Z = \sqrt{(1000)^2 + (2000 - 1000)^2} = 1000\sqrt{2}\ \Omega$$

$$\Rightarrow \quad I_{rms} = \frac{V_{rms}}{|Z|} = \frac{100}{1000\sqrt{2}} = \frac{1}{10\sqrt{2}} \text{ A}$$

$$\Rightarrow \quad I_{rms} = 0.0707 \text{ A}$$

The current will be same everywhere in the circuit, therefore, if potential difference across the inductor, capacitor and resistor is denoted by V_L, V_C and V_R respectively, then

$$V_L = (I_{rms})X_L = 141 \text{ V}$$

$$V_C = (I_{rms})X_C = 71 \text{ V}$$

$$V_R = (I_{rms})R = 71 \text{ V}$$

8. The resonant frequency is given by

$$f = \frac{1}{2\pi\sqrt{LC}}, \text{ where}$$

$$L = 0.4 \text{ mH} = 0.4 \times 10^{-3} \text{ H and}$$

$$C = 400 \text{ pF} = 4 \times 10^{-10} \text{ F}$$

$$\Rightarrow \quad f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2(3.14)\sqrt{0.4 \times 10^{-3} \times 4 \times 10^{-10}}}$$

$$\Rightarrow \quad f = \frac{10^7}{8\pi} \text{ Hz}$$

The speed of electromagnetic wave produced is

$$c = 3 \times 10^8 \text{ ms}^{-1}$$

$$\Rightarrow \quad \lambda = \frac{c}{f} = \frac{3 \times 10^8}{\left(\frac{10^7}{8\pi}\right)} = 240\pi$$

$$\Rightarrow \quad * = 240$$

9. Since, at resonance, the angular frequency ω_0 is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Now, when $\omega = 2\omega_0 = \frac{2}{\sqrt{LC}}$, then

$$X_L = L\omega = \frac{2L}{\sqrt{LC}} = 2\sqrt{\frac{L}{C}} \text{ and}$$

$$X_C = \frac{1}{C\omega} = \frac{\sqrt{LC}}{2C} = \frac{1}{2}\sqrt{\frac{L}{C}}$$

Since by definition, we have

$$Z^2 = (X_L - X_C)^2 + R^2$$

$$\Rightarrow \quad Z^2 = R^2 + 2.25\left(\frac{L}{C}\right)$$

Further, we know that

$$P = \left(\frac{V^2}{Z}\right)\cos\phi = \left(\frac{V^2}{Z}\right)\left(\frac{R}{Z}\right) = \frac{V^2 R}{Z^2}$$

$$\Rightarrow \quad P = \frac{V^2 R}{R^2 + 2.25\left(\frac{L}{C}\right)}$$

So, energy delivered to the circuit in one cycle is

$$E = PT = P\left(\frac{2\pi}{\omega}\right) = \left(\frac{2\pi RCV^2}{R^2C + 2.25L}\right)\frac{\sqrt{LC}}{2}$$

CHAPTER 4

$$\Rightarrow \quad E = \frac{4\pi RCV^2\sqrt{LC}}{4R^2C + 9L}$$

Substituting values, we get

$$E = 242 \text{ mJ}$$

10. For DC supply, we have

$$I = \frac{V}{R}$$

$$\Rightarrow \quad R = \frac{V}{I} = 50\ \Omega$$

For AC supply, we have

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$$

$$\Rightarrow \quad 1 = \frac{100}{Z}$$

$$\Rightarrow \quad Z = 100\ \Omega$$

Since, we know that for a series LR circuit, the impedance Z is

$$Z = \sqrt{R^2 + X_L^2}$$

$$\Rightarrow \quad X_L = 50\sqrt{3}\ \Omega \approx 87\ \Omega$$

11. The power factor $\cos\phi$ is given by

$$\cos\phi = \frac{R}{Z}$$

where, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$\Rightarrow \quad \cos\phi = \frac{60}{\sqrt{60^2 + (100 - 20)^2}}$$

$$\Rightarrow \quad \cos\phi = \frac{60}{100} = 0.6$$

12. At resonance, the current in the circuit is

$$I = \frac{E}{Z_{\text{min}}} = \frac{E_R}{R} = \frac{60}{120} = 0.5 \text{ A}$$

Voltage across inductor is

$$V_L = IX_L = (0.5)(L\omega) = 40 \text{ V}$$

$$\Rightarrow \quad L = \frac{40}{(0.5)(4000)} = 0.02 \text{ H}$$

$$\Rightarrow \quad L = 20 \text{ mH}$$

At resonance, we have

$$X_L = X_C$$

$$\Rightarrow \quad L\omega_0 = \frac{1}{C\omega_0}, \text{ where } \omega_0 = 4000 \text{ rads}^{-1}$$

$$\Rightarrow \quad C = \frac{1}{L\omega_0^2} = \frac{1}{(20\times10^{-3})(4000)^2}$$

$$\Rightarrow \quad C = \frac{1}{32}\times10^{-4} = \frac{25}{8}\ \mu\text{F} = 3.125\ \mu\text{F}$$

13. The impedance of the circuit is given by

$$Z = \sqrt{R^2 + \omega^2L^2}$$

$$\Rightarrow \quad Z = \sqrt{R^2 + (2\pi fL)^2} = \sqrt{R^2 + 4\pi^2f^2L^2}$$

$$\Rightarrow \quad Z = \sqrt{(12)^2 + 4\pi^2(50)^2\left(\frac{0.05}{\pi}\right)^2}$$

$$\Rightarrow \quad Z = \sqrt{144 + 25} = 13\ \Omega$$

Current in the circuit is given by

$$I_{\text{rms}} = \frac{E_{\text{rms}}}{Z} = \frac{130}{13} = 10 \text{ A}$$

Potential difference across resistance is

$$V_R = I_{\text{rms}}R = 10\times12 = 120 \text{ V}$$

Inductive reactance of coil is

$$X_L = L\omega = 2\pi fL$$

$$\Rightarrow \quad X_L = 2\pi(50)\left(\frac{0.05}{\pi}\right) = 5\ \Omega$$

Potential difference across inductance is

$$V_L = I_{\text{rms}}X_L = 10\times5 = 50 \text{ V}$$

14. From the given voltage and current equations, we conclude that current leads the voltage by a phase angle ϕ given by

$$\phi = \frac{\pi}{12} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{4} = 45^\circ$$

So, the power factor is given by

$$\cos\phi = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

Average power $\langle P\rangle$ dissipated in the circuit is

$$\langle P\rangle = \frac{E_0I_0}{2}\cos\phi = \frac{(10)(2)}{2}\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \langle P\rangle = 5\sqrt{2} \text{ W} \approx 7 \text{ W}$$

15. Required current for bulb is

$$I_{\text{rms}} = \frac{P}{E_{\text{rms}}} = \frac{10 \text{ W}}{30 \text{ V}} = \frac{1}{3} \text{ A}$$

$$\Rightarrow \quad I_{\text{rms}} = \frac{E_{\text{rms}}}{X_C} = \frac{1}{3} \text{ A}$$

$$\Rightarrow \quad I_{\text{rms}} = \frac{E_{\text{rms}}}{\left(\frac{1}{2\pi fC}\right)} = E_{\text{rms}}(2\pi fC)$$

$$\Rightarrow \quad C = \frac{I_{\text{rms}}}{2\pi fE_{\text{rms}}} = \frac{1}{3(220)2\pi(50)}$$

$$\Rightarrow \quad C = \frac{1}{660\times\pi\times100}$$

$$\Rightarrow \quad C = \frac{1}{66\times10^3\times\pi}\text{ F} = 4.82\ \mu\text{F}$$

$$\Rightarrow \quad C \approx 4.8\ \mu\text{F}$$

16. Since the power consumed across a resistor is

$$P = E_{\text{rms}}I_{\text{rms}} = \frac{E_{\text{rms}}^2}{R}$$

$$\Rightarrow \quad R = \frac{E_{\text{rms}}^2}{P} = \frac{220\times220}{200} = \frac{22\times22}{2} = \frac{484}{2}$$

$$\Rightarrow \quad R = 242\ \Omega$$

The rms value of current is

$$I_{\text{rms}} = \frac{P}{E_{\text{rms}}} = \frac{200}{220} = \frac{10}{11}\text{ A} = 0.9\text{ A}$$

17. The circuit can be thought of as a parallel combination of $3L$, C and R.
Since we have been given that each capacitor has a reactance of $4R$, so we have

$$4R = \frac{1}{(C/4)\omega}$$

$$\Rightarrow \quad R = \frac{1}{C\omega}$$

So, for four capacitors in parallel the equivalent capacitive reactance is

$$X_C = \frac{1}{C_{\text{eq}}\omega} = \frac{1}{C\omega} = R$$

Since we have been given that each inductor has a reactance of $\frac{R}{3}$, so we have

$$\frac{R}{3} = L\omega$$

$$\Rightarrow \quad R = (3L)\omega$$

So, for three inductors in series the equivalent inductive reactance is

$$\Rightarrow \quad X_L = L_{\text{eq}}\omega = (3L)\omega = R$$

Reading of the ammeter is

$$I_{\text{rms}} = E_{\text{rms}}\sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

Since here we see that $X_L = X_C = R$, so we have

$$I_{\text{rms}} = \frac{E_{\text{rms}}}{R} = \frac{10}{5} = 2\text{ A}$$

So, the reading of the ammeter is 2 A

18. When current is in phase with the voltage, then we get the condition of resonance. So, we have

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$4\pi^2 f_0^2 LC = 1$$

$$\Rightarrow \quad C = \frac{1}{4\pi^2 f_0^2 L}$$

$$\Rightarrow \quad C = \frac{1}{4(3.14)^2(50)^2(40\times10^{-3})} = 250\ \mu\text{F}$$

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1. Since $I = I_m\sin(100\pi t)$

$$\Rightarrow \quad \frac{I_m}{2} = I_m\sin(100\pi t_1)$$

$$\Rightarrow \quad \frac{\pi}{6} = 100\pi t_1$$

$$\Rightarrow \quad t_1 = \frac{1}{600}\text{ s}$$

Since $w = 100\pi$

$$\Rightarrow \quad T = \frac{2\pi}{100\pi} = \frac{1}{50}\text{ s}$$

So, $t_{\text{req}} = \frac{T}{4} - t_1$

$$\Rightarrow \quad t_{\text{req}} = \frac{1}{200} - \frac{1}{600} = \frac{2}{600} = \frac{1}{300}\text{ s} = 3.3\text{ ms}$$

Hence, the correct answer is (D).

2. As $\phi = \frac{\pi}{4}$, $x_c = R$

Hence, the correct answer is (C).

3. $Q = 1\times4200\times80 + 2260\times10^3$ J

$$\Rightarrow \quad Q = (336+2260)\times10^3\text{ J} = 2596\times10^3\text{ J}$$

CHAPTER 4

Since $Q = I_{rms}V_{rms}t = 200 \times \frac{200}{20}t = 2000t$

$\Rightarrow \quad t = \frac{2596}{2}\text{ s} \approx 21.6 \text{ minute} \approx 22 \text{ minute}$

Hence, the correct answer is (C).

4. Inductive Reactance

$$X_L = L\omega = (20\times10^{-3})(2\pi\times50) = 2\pi\ \Omega = 6.28\ \Omega$$

Capacitive Reactance

$$X_C = \frac{1}{C\omega} = \frac{1\times10^6}{(120)(2\pi\times50)} = \frac{1000}{2\pi\times6}\ \Omega = 26.53\ \Omega$$

$\Rightarrow \quad Z = \sqrt{R^2 + (X_C - X_L)^2}$

$\Rightarrow \quad |Z| = \sqrt{(60)^2 + (20.25)^2} = 63.32\ \Omega$

$\Rightarrow \quad I_{rms} = \frac{V_{rms}}{Z} = \frac{24}{63.32}$

Since, $\cos\phi = \frac{R}{Z} = \frac{60}{63.32}$

$\Rightarrow \quad \Delta E = \frac{24}{63.32}\times24\times\frac{60}{63.32}\times60 = 5.17\times10^2\text{ J}$

Hence, the correct answer is (B).

5. Power input is $P_{input} = (V_p)(I_p) = 2300\times5\text{ W}$

Since $\eta = \frac{P_{out}}{P_{in}} = 0.9$

$\Rightarrow \quad P_{output} = 0.9\ P_{input} = V_S I_S$

$\Rightarrow \quad I_S = \frac{0.9\times2300\times5}{230} = 45\text{ A}$

Hence, the correct answer is (C).

6. Capacitive reactance is

$$X_C = \frac{1}{C\omega} = \frac{1\times2}{100\times\sqrt{3}}\times10^6 = \frac{20}{\sqrt{3}}\text{ k}\Omega$$

Inductive reactance is

$$X_L = L\omega = 10\sqrt{3}\ \Omega$$

For the C-R_2 path, since, $X_C \gg R_2$, so we have $\phi_2 \to 90^\circ$ and hence I_2 leads V by 90°

For the L-R_1 path, we have $\tan\phi_1 = \frac{L\omega}{R_1} = \sqrt{3}$ i.e. $\phi_1 = 60^\circ$, hence I_1 lags V by 60°

So, phase difference between I_1 and I_2 is 150°.

*No given option is correct.

Please note that, if we had been given $R_2 = 20\text{ k}\Omega$ instead of $R_2 = 20\ \Omega$, then we would have got $\phi_2 = \tan^{-1}\left(\frac{X_C}{R_2}\right) = 30^\circ$ and then our result would have been $30^\circ + 60^\circ = 90^\circ$

7. Average power, $P_{av} = e_{rms}i_{rms}\cos\phi = \frac{e_0}{\sqrt{2}}\frac{i_0}{\sqrt{2}}\cos\phi$

where, $e_0 = 100$, $i_0 = 20$, $\phi = \frac{\pi}{4}$

$\Rightarrow \quad P_{av} = \frac{100}{\sqrt{2}}\times\frac{20}{\sqrt{2}}\cos45^\circ = \frac{1000}{\sqrt{2}}$ units

Wattless current is given by

$$i_w = i_{rms}\sin\phi = \frac{i_0}{\sqrt{2}}\sin45^\circ = 10\text{ units}$$

Hence, the correct answer is (B).

8. Quality factor is given by

$$Q = \frac{L\omega_0}{R}$$

Hence, the correct answer is (A).

9. Using Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in}\\ \text{Electrostatic Energy}\\ \text{of capacitor}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{Magnetic Energy}\\ \text{of Inductor}\end{pmatrix}$$

$\Rightarrow \quad \frac{1}{2}(0.2\times10^{-6})[(10)^2-(5)^2] = \frac{1}{2}(0.5\times10^{-3})I^2$

$\Rightarrow \quad I = \sqrt{3}\times10^{-1}\text{ A} = 0.17\text{ A}$

Hence, the correct answer is (B).

10. Efficiency of the transformer is

$$\eta = 0.9 = \frac{P_s}{P_p}$$

$\Rightarrow \quad V_s I_s = 0.9\times V_p I_p$

$\Rightarrow \quad I_s = \frac{0.9\times2300\times5}{230} = 45\text{ A}$

Hence, the correct answer is (C).

11. According to the problem, we have

$E_0 = 283\text{ V}$, $\omega = 320\text{ s}^{-1}$, $R = 5\ \Omega$,

$L = 25\text{ mH} = 25\times10^{-3}\text{ H}$, $C = 1000\ \mu\text{F} = 10^{-3}\text{ F}$

$\Rightarrow \quad X_L = \omega L = 320 \times 25 \times 10^{-3} = 8\ \Omega$

$\Rightarrow \quad X_C = \dfrac{1}{\omega C} = \dfrac{1}{320 \times 10^{-3}} = \dfrac{1000}{320} = 3.125\ \Omega$

Impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

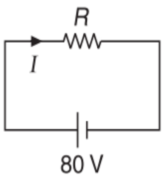
$$\Rightarrow \quad Z = \sqrt{5^2 + (8 - 3.125)^2} \approx \sqrt{49} = 7\ \Omega$$

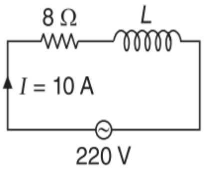
Since $X_L > X_C$, so the voltage leads the current by a phase angle

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{4.875}{5}\right) \approx 45^\circ$$

Hence, the correct answer is (D).

12. For a dc source $I = 10$ A, $V = 80$ V





Resistance of the arc lamp is

$$R = \frac{V}{I} = \frac{80}{10} = 8\ \Omega$$

For an ac source,

$e_{\text{rms}} = 220$ V

$\nu = 50$ Hz

$\omega = 2\pi \times 50 = 100\pi\ \text{rads}^{-1}$

Arc lamp will glow if $I_{\text{rms}} = 10$ A,

$$\Rightarrow \quad I_{\text{rms}} = \frac{E_{\text{rms}}}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\Rightarrow \quad R^2 + \omega^2 L^2 = \left(\frac{E_{\text{rms}}}{I_{\text{rms}}}\right)^2$$

$$\Rightarrow \quad 8^2 + (100\pi)^2 L^2 = \left(\frac{220}{10}\right)^2$$

$$\Rightarrow \quad L^2 = \frac{22^2 - 8^2}{(100\pi)^2}$$

$$\Rightarrow \quad L = \frac{\sqrt{30 \times 14}}{100\pi} = 0.065\ \text{H}$$

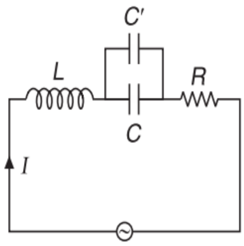
Hence, the correct answer is (D).

13. Current in LR circuit is $I = \dfrac{V_0}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t - \dfrac{\pi}{2}\right)$,

i.e., it is sinusoidal in nature.

Hence, the correct answer is (D).

14. Since the current is observed to lead the applied voltage, so the circuit is capacitive in nature. Now to make the power factor 1, effective capacitance should be increased thus the capacitor should be connected in parallel.



$\cos\phi = 1$

$\Rightarrow \quad \phi = 0$

$\Rightarrow \quad L\omega = \dfrac{1}{(C + C')\omega}$

$\Rightarrow \quad C + C' = \dfrac{1}{\omega^2 L}$

$\Rightarrow \quad C' = \dfrac{1}{\omega^2 L} - C$

$\Rightarrow \quad C' = \dfrac{1 - \omega^2 LC}{\omega^2 L}$ in parallel

Hence, the correct answer is (D).

15. Given that $R = 200\ \Omega$, $V_{\text{rms}} = 220$ V, $\nu = 50$ Hz

When only the capacitor is removed, the phase difference between the current and voltage is $\tan\phi = \dfrac{X_L}{R}$

$\Rightarrow \quad \tan 30^\circ = \dfrac{X_L}{R}$

$\Rightarrow \quad X_L = \dfrac{1}{\sqrt{3}} R$

When only the inductor is removed, the phase differ

$\tan 30^\circ = \dfrac{X_C}{R}$

$\Rightarrow \quad X_C = \dfrac{1}{\sqrt{3}} R$

Since we see that $X_L = X_C$, therefore the given series LCR circuit is in resonance. Hence the impedance of the circuit is minimum and is given by

$Z = R = 200\ \Omega$

The power dissipated in the circuit is

$P = V_{\text{rms}} I_{\text{rms}} \cos\phi$, where $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z}$

$\Rightarrow \quad P = \dfrac{V_{\text{rms}}^2}{Z} \cos\phi$

At resonance, power factor $\cos\phi = 1$

$\Rightarrow \quad P = \dfrac{V_{\text{rms}}^2}{Z} = \dfrac{(220\ \text{V})^2}{(200\ \Omega)} = 242$ W

Hence, the correct answer is (A).

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Single Correct Choice Type Problems

1. $I_{rms} = \dfrac{V_{rms}}{\sqrt{R^2 + \dfrac{1}{C^2\omega^2}}}$

when ω increases, I_{rms} increases so the bulb glows brighter.

Hence, the correct answer is (B).

2. As the current i leads the emf e by $\dfrac{\pi}{4}$, it is an R-C circuit.

$$\tan\phi = \frac{X_C}{R}$$

$$\Rightarrow \tan\frac{\pi}{4} = \frac{\frac{1}{\omega C}}{R}$$

$$\Rightarrow \omega CR = 1$$

As $\omega = 100 \text{ rads}^{-1}$

The product of RC should be $\dfrac{1}{100} \text{ s}^{-1}$

Hence, the correct answer is (A).

Multiple Correct Choice Type Problems

1. At $\omega \approx 0$, $X_C = \dfrac{1}{\omega C} \to \infty$. So, current is nearly zero.

Further at resonance, current and voltage are in phase. This resonance frequency is given by,

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-6} \times 10^{-6}}} = 10^6 \text{ rads}^{-1}$$

So, we observe that this frequency is independent of R.

Further, $X_L = \omega L$, $X_C = \dfrac{1}{\omega C}$

At, $\omega = \omega_0 = 10^6 \text{ rads}^{-1}$, $X_L = X_C$

For $\omega > \omega_0$, $X_L > X_C$ and hence circuit is inductive in nature.

Hence, (A) and (B) are correct.

2. The potential difference between the points X and Y is

$$V_{XY} = V_X - V_Y$$

If ϕ be the phase difference between the two voltages, then the peak value of this potential difference is

$$(V_{XY})_0 = \sqrt{(V_X)_0^2 + (V_Y)_0^2 - 2(V_X)_0(V_Y)_0\cos\phi}$$

From the problem, we see that the phase difference between V_X and V_Y is $\dfrac{2\pi}{3}$, so we have

$$(V_{XY})_0 = \sqrt{V_0^2 + V_0^2 - 2V_0^2\cos\left(\frac{2\pi}{3}\right)} = \sqrt{3}V_0$$

Similarly, we have phase difference between V_Y and V_Z to be $\dfrac{2\pi}{3}$ and hence

$$(V_{XY})_0 = (V_{YZ})_0 = (V_{ZX})_0 = \sqrt{3}V_0$$

$$\Rightarrow (V_{XY})_{rms} = (V_{YZ})_{rms} = \frac{\sqrt{3}V_0}{\sqrt{2}} = \sqrt{\frac{3}{2}}V_0$$

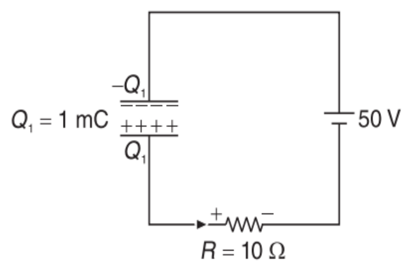
Hence, (B) and (C) are correct.

3. Since $I = \dfrac{dQ}{dt}$

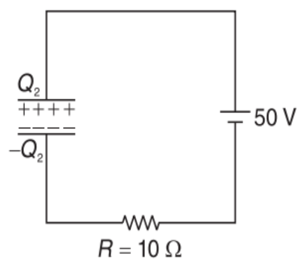
$$\Rightarrow Q = \int I dt = \int (I_0 \cos\omega t) dt$$

$$\Rightarrow Q_{max} = \frac{I_0}{\omega} = \frac{1}{500} = 2 \times 10^{-3} \text{ C} = 2 \text{ mC}$$

JUST AFTER SWITCHING



IN STEADY STATE



At $t = \dfrac{7\pi}{6\omega}$, $I(t) = \text{NEGATIVE}$

So, current is anti-clockwise sense. Charge supplied by source from $t = 0$ to $t = \dfrac{7\pi}{6\omega}$ is

$$Q=\int_0^{\frac{7\pi}{6\omega}}\cos(500t)\,dt=\left(\frac{\sin 500t}{500}\right)\Bigg|_0^{\frac{7\pi}{6\omega}}$$

$$\Rightarrow\quad Q=\frac{\sin\left(\frac{7\pi}{6}\right)}{500}=-1\text{ mC}$$

Apply Kirchhoff's loop law just after changing the switch to position D, we get

$$50+\frac{Q_1}{C}-IR=0$$

Substituting the values of Q_1, C and R, we get

$$I=10\text{ A}$$

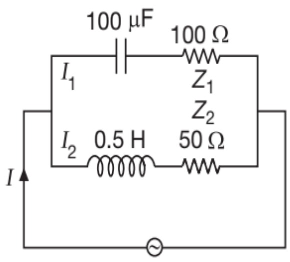
In steady state, we have

$$Q_2=CV=1\text{ mC}$$

So, net charge flown from battery is $\Delta q=2$ mC

Hence, (C) and (D) are correct.

4.



CIRCUIT 1:

The capacitive reactance is

$$X_C=\frac{1}{\omega C}=100\ \Omega$$

$$\Rightarrow\quad Z_1=\sqrt{(100)^2+(100)^2}$$

$$\Rightarrow\quad Z_1=100\sqrt{2}\ \Omega$$

$$\Rightarrow\quad \phi_1=\cos^{-1}\left(\frac{R_1}{Z_1}\right)=45^\circ$$

In this circuit, current leads the voltage by 45°.
So, we have

$$I_1=\frac{V}{Z_1}=\frac{20}{100\sqrt{2}}=\frac{1}{5\sqrt{2}}\text{ A}$$

$$\Rightarrow\quad V_{100\ \Omega}=(100)I_1=(100)\frac{1}{5\sqrt{2}}\text{ V}=10\sqrt{2}\text{ V}$$

CIRCUIT 2:

The inductive reactance is

$$X_L=\omega L=(100)(0.5)=50\ \Omega$$

$$\Rightarrow\quad Z_2=\sqrt{(50)^2+(50)^2}=50\sqrt{2}\ \Omega$$

$$\Rightarrow\quad \phi_2=\cos^{-1}\left(\frac{R_2}{Z_2}\right)=45^\circ$$

In this circuit, voltage leads the current by 45°.
So, we have

$$I_2=\frac{V}{Z_2}=\frac{20}{50\sqrt{2}}=\frac{\sqrt{2}}{5}\text{ A}$$

$$\Rightarrow\quad V_{50\ \Omega}=(50)I_2=50\left(\frac{\sqrt{2}}{5}\right)=10\sqrt{2}\text{ V}$$

Since phase difference between the currents I_1 and I_2 is 90°, so we have

$$I=\sqrt{I_1^2+I_2^2}=0.34$$

Hence, (A) and (C) are correct.

5. $Z=\sqrt{R^2+X_C^2}=\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}$

In case (B), capacitance C will be more. Therefore, impedance Z will be less. Hence, current will be more.

Further, $V_C=\sqrt{V^2-V_R^2}$

$$\Rightarrow\quad V_C=\sqrt{V^2-(IR)^2}$$

In case (B), since current I is more. Therefore, V_C will be less.

Hence, (B) and (C) are correct.

Linked Comprehension Type Questions

1. The power consumed is

$$P=Vi$$

$$\Rightarrow\quad i=\frac{P}{V}=\frac{600\times10^3}{4000}=150\text{ A}$$

Total resistance of cables is

$$R=0.4\times20=8\ \Omega$$

SO, power loss in cables is

$$P=i^2R=(150)^2(8)=180000\text{ W}=180\text{ kW}$$

This loss is 30% of 600 kW

Hence, the correct answer is (B).

2. For a step-up transformer, we have

$$\frac{N_p}{N_s}=\frac{V_p}{V_s}$$

$$\Rightarrow\quad \frac{1}{10}=\frac{4000}{V_s}$$

$\Rightarrow \quad V_s = 40000 \text{ V}$

For a step-down transformer, we have

$$\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{40000}{200} = \frac{200}{1}$$

Hence, the correct answer is (A).

Matrix Match/Column Match Type Questions

1. A → (r, s, t)
   B → (q, r, s, t)
   C → (q, p)
   D → (q, r, s, t)

For Circuit (p):

I cannot be non-zero in steady state

For Circuit (q):

$V_1 = 0$ and $V_2 = 2I = V$ (also)

For Circuit (r):

$$V_1 = IX_L = I(2\pi fL)$$

$$\Rightarrow \quad V_1 = (2\pi \times 50 \times 6 \times 10^{-3})I = 1.88I$$

$$\Rightarrow \quad V_2 = 2I$$

For Circuit (s):

$V_1 = IX_L = 1.88I$ and

$$V_2 = IX_C = I\left(\frac{1}{2\pi fC}\right)$$

$$\Rightarrow \quad V_2 = \left(\frac{1}{2\pi \times 50 \times 3 \times 10^{-6}}\right)I = (1061)I$$

For Circuit (t):

$V_1 = IR = (1000)I$ and $V_2 = X_C I = (1061)I$

Integer/Numerical Answer Type Questions

1. For the circuit, impedance is

$$Z = \sqrt{R^2 + X_C^2} = R\sqrt{1.25}$$

$$\Rightarrow \quad R^2 + X_C^2 = 1.25R^2$$

$$\Rightarrow \quad X_C = \frac{R}{2}$$

$$\Rightarrow \quad \frac{1}{\omega C} = \frac{R}{2}$$

$$\Rightarrow \quad \text{Time constant } = CR = \frac{2}{\omega} = \frac{2}{500}\text{ s} = 4 \text{ ms}$$