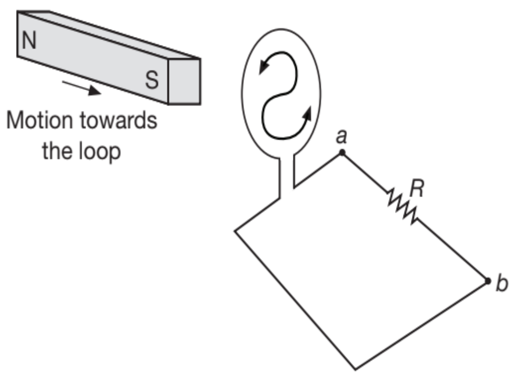
CHAPTER 3: ELECTROMAGNETIC INDUCTION

Test Your Concepts-I (Based on Magnetic Flux, Faraday's Laws and Induced EMF)

1. **METHOD I**

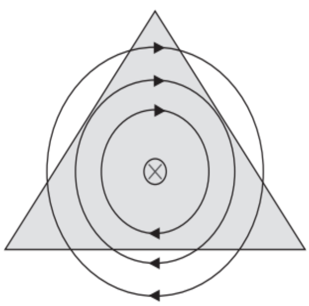
When the S pole moves towards the loop, it must behave as a S pole (as seen from the magnet's side) so, we have the current flowing from b to a and hence $V_a < V_b$ i.e., $V_a - V_b$ is negative



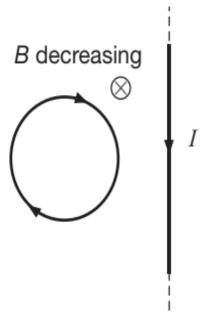
METHOD II

Look in the direction of ba. The bar magnet creates a field into the page, and the field increases. The loop will create a field out of the page by carrying a counter clockwise current. Therefore, current must flow from b to a through the resistor. Hence, $V_a - V_b$ is negative.

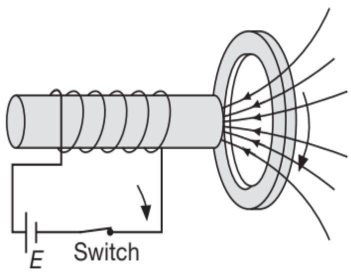
2. Magnetic field lines round the current carrying wire are as shown in figure. Since the lines are tangential to the loop $(\theta = 90°)$ the flux passing through the loop is zero, whether the current is increased or decreased. Change in flux is zero. Therefore, induced current in the loop will be zero.



3. Since I is decreasing in the wire, so the loop lies in an inward decreasing field and hence the induced current must not allow the inward field to decrease.
So, it will be in clockwise sense.

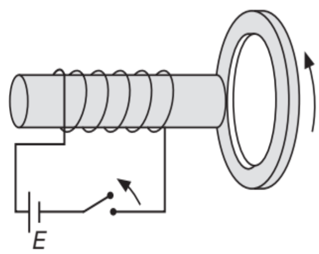


4. (a) At the instant the switch is thrown closed, the situation changes from one in which no magnetic flux exists in the ring to one in which flux exists and the magnetic field is to the left as shown in figure. The counteract this change in the flux, the current induced in the ring must set up a magnetic field directed from left to right in figure. This requires a current directed as shown.



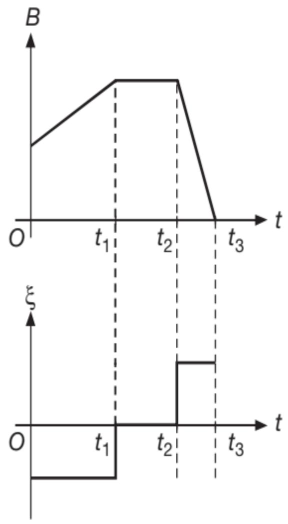
(b) After the switch has been closed for several seconds, no change in the magnetic flux through the loop occurs; hence, the induced current in the ring is zero.

(c) Opening the switch changes the situation from one in which magnetic flux exists in the ring to one in which there is no magnetic flux. The direction of the induced current is as shown in figure because current in this direction produces a magnetic field that is directed right to left and so counteracts the decrease in the flux produced by the solenoid.



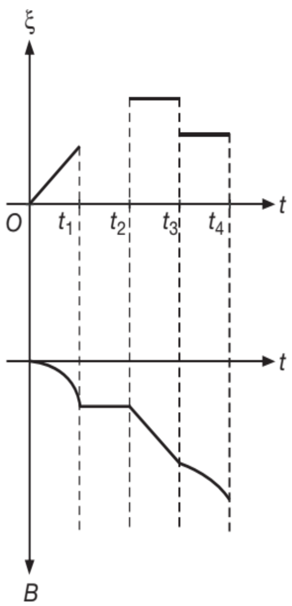
5. $\xi = -\dfrac{d\phi_B}{dt}$

$\Rightarrow \quad \xi = -A\dfrac{dB}{dt}$



6. When the wire is moved to the other side, even though the connections have not changed, bulb 1 goes out and bulb 2 glows. The bulb that is shorted depends on which side of the changing field the switch is positioned! In figure (a), because the branch containing bulb 2 is infinitely more resistant than the branch containing the resistance-free switch, we can imagine removing the branch with the bulb without altering the circuit. Then we have a simple loop containing only bulb 1, which glows.
When the wire is moved, as in figure (b), there are two possible paths for current below points a and b. We can imagine removing the branch with bulb 1, leaving only a single loop with bulb 2.

7.



8. The enclosed flux is

$$\phi_B = BA = B(\pi r^2)$$

$$qvB\sin(90^\circ) = \frac{mv^2}{r}$$

$$\Rightarrow \quad r = \frac{mv}{qB}$$

$$\Rightarrow \quad \phi_B = \frac{B\pi m^2 v^2}{q^2 B^2}$$

(a) $v = \sqrt{\dfrac{\phi_B q^2 B}{\pi m^2}} = \sqrt{\dfrac{(15\times10^{-6})(30\times10^{-9})^2(0.6)}{\pi(2\times10^{-16})^2}}$

$$\Rightarrow \quad v = 2.54\times10^5 \text{ ms}^{-1}$$

(b) Since, $q\Delta V = \dfrac{1}{2}\text{mv}^2$

$$\Rightarrow \quad \Delta V = \frac{mv^2}{2q} = \frac{(2\times10^{-16})(2.54\times10^5)^2}{2(30\times10^{-9})} = 215 \text{ V}$$

9. Due to the current in A a magnetic field is from right to left. When A is moved towards B, magnetic lines passing through B (from right to left) will increase, i.e., magnetic flux passing through B will increase. Therefore, current will be induced in B. The induced current will have such a direction that it gives a magnetic field opposite to that, was passing through B due to current in A. Therefore, induced current in B will be in opposite direction of current in A.
The current in C will decrease due to movement of coil A and this will give rise to an induced current in B in same direction as that of C, but since B is more closer to A, therefore net induced current will be opposite to current in A and C.

10. (a) The flux through the loop at time t is

$$\phi_B = BA\cos\theta = (a+bt)(\pi r^2)\cos 0^\circ = \pi(a+bt)r^2$$

At $t=0$, $\phi_B = \pi a r^2$

(b) $\xi = -\dfrac{d\phi_B}{dt} = -\pi r^2\dfrac{d(a+bt)}{dt} = -\pi b r^2$

(c) If R be the resistance of the loop, then

$$R = \lambda(2\pi r)$$

$$\Rightarrow \quad I = \frac{\xi}{R} = -\frac{\pi b r^2}{R} = -\frac{\pi b r^2}{\lambda(2\pi r)} = \frac{br}{2\lambda}$$

(d) $P = \xi I = (-\pi b r^2)\left(-\dfrac{br}{2\lambda}\right) = \dfrac{\pi b^2 r^3}{2\lambda}$

11. $\phi_i = NBA\cos(0°) = NBA$

$\phi_f = NBA\cos(180°) = -NBA$

Since $I = \dfrac{\xi}{R} = \dfrac{1}{R}\dfrac{\Delta\phi}{\Delta t}$

$\Rightarrow \quad I\Delta t = \dfrac{1}{R}\Delta\phi$

$\Rightarrow \quad |Q| = \dfrac{1}{R}(2NBA)$

$\Rightarrow \quad |Q| = \left(\dfrac{1}{50}\right)(2)(500)(0.2)(4\times10^{-4})$

$\Rightarrow \quad |Q| = 1.6 \text{ mC}$

12. The perimeter of the wire and the square loop will be the same, so

$2\pi r = 4\ell$

$\Rightarrow \quad \ell = \dfrac{\pi r}{2}$

Now, initial area of loop is $A_i = \pi r^2$ and final area of the loop is $A_f = \ell^2 = \dfrac{\pi^2 r^2}{4}$

$\xi = B\left|\dfrac{\Delta A}{\Delta t}\right| = B\left|\dfrac{A_f - A_i}{t}\right|$

$\Rightarrow \quad \xi = \left(\dfrac{100}{0.1}\right)\left|\dfrac{\pi^2 r^2}{4} - \pi r^2\right|$

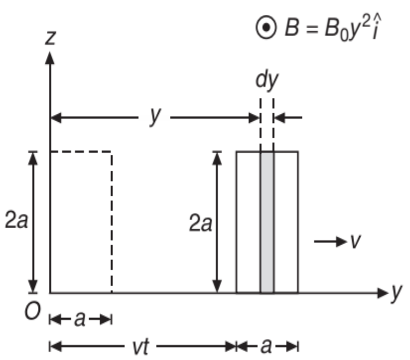
$\Rightarrow \quad \xi = \left(\dfrac{100}{0.1}\right)(0.1)^2\left|\dfrac{\pi^2}{4} - \pi\right|$

$\Rightarrow \quad \xi = (10)(0.675)$

$\Rightarrow \quad \xi = 6.75 \text{ V}$

13. Let at an instant t, the loop is at the position shown in Figure. Consider a small element of length $2a$ and width dy. If dA is the area of the element, then

$dA = 2a\,dy$



The magnetic flux associated with the loop at this instant is

$$\phi = \int \vec{B}\cdot d\vec{A} = \int_{vt}^{vt+a} B_0 y^2 (2a\,dy)$$

$\Rightarrow \quad \phi = 2aB_0\left(\dfrac{y^3}{3}\right)\Bigg|_{vt}^{vt+a}$

$\Rightarrow \quad \phi = 2aB_0\left[\dfrac{(vt+a)^3 - (vt)^3}{3}\right]$

Since $|\xi| = \dfrac{d\phi}{dt}$

$\Rightarrow \quad |\xi| = 2aB_0\left[(vt+a)^2 - (vt)^2\right]v$

$\Rightarrow \quad |\xi| = 2aB_0 v(a)(2vt + a)$

$\Rightarrow \quad |\xi| = 2a^2 B_0 v(2vt + a)$

14. $\xi = -\dfrac{d\phi}{dt}$

$\Rightarrow \quad |\xi| = \left|\dfrac{d\phi}{dt}\right| = NA\left(\dfrac{dB}{dt}\right)$

where $\dfrac{dB}{dt} = (0.01 + 0.08t)$

$\Rightarrow \quad |\xi|_{\text{at } t=5\text{ s}} = (30)\left[\pi(0.04)^2\right](0.01 + 0.4)$

$\Rightarrow \quad |\xi|_{\text{at } t=5\text{ s}} = 62 \text{ mV}$

15. The area of one turn of the coil is $(0.18 \text{ m})^2 = 0.0324 \text{ m}^2$. The magnetic flux through the coil at $t = 0$ is zero because $B = 0$ at that time. At $t = 0.8$ s, the magnetic flux through one turn is $\phi_B = BA = (0.5 \text{ T})(0.0324 \text{ m}^2) = 0.0162 \text{ Tm}^2$. Therefore, the magnitude of the induced emf is

$|\xi| = N\left(\dfrac{\Delta\phi_B}{\Delta t}\right) = 200\dfrac{(0.0162 - 0)}{0.8}$

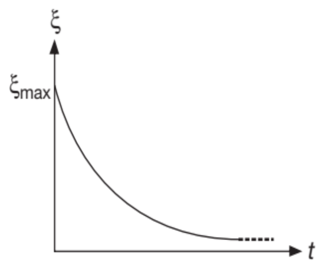
$\Rightarrow \quad |\xi| = 4.1 \text{ Tm}^2\text{s}^{-1} = 4.1 \text{ V}$

You should be able to show that

$1 \text{ Tm}^2\text{s}^{-1} = 1 \text{ V}$

16. Since $\vec{B}$ is perpendicular to the plane of the loop, the magnetic flux through the loop at time $t > 0$ is

$\phi_B = BA\cos(0°) = AB_0 e^{-at}$



Since AB_0 and a are constants, so the induced emf is given by

$$\xi = -\frac{d\phi_B}{dt} = -AB_0\frac{d}{dt}\left(e^{-at}\right)$$

$$\Rightarrow \quad \xi = aAB_0e^{-at}$$

This expression indicates that the induced emf decays exponentially in time. Note that the maximum emf occurs at $t=0$, where $\xi_{max} = aAB_0$. The plot of ξ versus t is shown in figure.

17. The upper loop has area $\pi(0.05\text{ m})^2 = 7.85\times10^{-3}\text{ m}^2$. The induced emf in it is

$$\xi = -N\frac{d}{dt}(BA\cos\theta) = -1A\cos0^\circ\frac{dB}{dt}$$

$$\Rightarrow \quad \xi = -7.85\times10^{-3}\text{ m}^2\left(2\text{ Ts}^{-1}\right) = -1.57\times10^{-2}\text{ V}$$

The minus sign indicates that it tends to produce counter clockwise current, to make its own magnetic field out of the page. Similarly, the induced emf in the lower loop is

$$\xi = -NA\cos\theta\left(\frac{dB}{dt}\right) = -\pi(0.09\text{ m})^2 2\text{ Ts}^{-1}$$

$$\Rightarrow \quad \xi = -5.09\times10^{-2}\text{ V} = +5.09\times10^{-2}\text{ V}$$

to produce counter clockwise current in the lower loop, which becomes clockwise current in the upper loop.
The net emf for current in this sense around the digit 8 is

$$\xi_{net} = 5.09\times10^{-2} - 1.57\times10^{-2} = 3.52\times10^{-2}\text{ V}$$

This net emf pushes the current in this sense through a series resistance given by

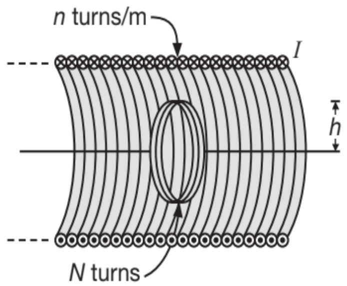
$$R = [2\pi(0.05\text{ m}) + 2\pi(0.09\text{ m})]3\ \Omega\text{m}^{-1} = 2.64\ \Omega$$

The current is $I = \dfrac{\xi_{net}}{R} = \dfrac{3.52\times10^{-2}\text{ V}}{2.64\ \Omega} = 13.3\text{ mA}$

18. $B = \mu_0 nI = \mu_0 n(30)\left(1-e^{-1.6t}\right)$

Since, $\phi_B = \int BdA = \mu_0 n(30)\left(1-e^{-1.6t}\right)\int dA$

$$\Rightarrow \quad \phi_B = \mu_0 n(30)\left(1-e^{-1.6t}\right)\pi R^2$$



From Faraday's Laws, we have

$$\xi = -N\frac{d\phi_B}{dt} = -N\mu_0 n(30)\pi R^2(1.6)e^{-1.6t}$$

$$\Rightarrow \quad \xi = -(250)\left(4\pi\times10^{-7}\right)(400)(30)\left[\pi(0.06)^2\right]\left(1.6e^{-1.6t}\right)$$

$$\Rightarrow \quad \xi = (68)e^{-1.6t}\text{ mV, counter clockwise}$$

19. From Angular Impulse - Angular Momentum Theorem, we have

$$\tau\,dt = I\omega$$

$$\Rightarrow \quad (MB\sin90^\circ)dt = \frac{2}{3}mR^2\omega$$

$$\Rightarrow \quad (NIA)Bdt = \frac{2}{3}mR^2\omega$$

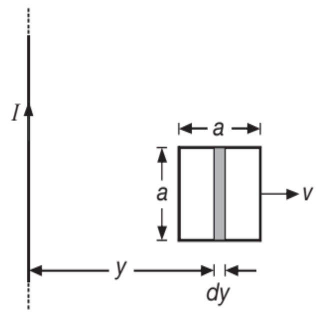
$$\Rightarrow \quad \left(N\pi R^2Bq\right) = \frac{2}{3}mR^2\omega \qquad \{I\,dt = q\}$$

$$\Rightarrow \quad \omega = \frac{1.5\pi NBq}{m} = \frac{(1.5\pi)(20)\left(0.5\times10^{-4}\right)(5\times3600)}{10^3}$$

$$\{\text{where } Q = 5\text{ Ahr} = 5\times3600\text{ C}\}$$

$$\Rightarrow \quad \omega = 27\pi\times10^{-3}\text{ rads}^{-1}$$

20. Consider a rectangular strip of the frame as shown. If dA is the area of the strip then, $dA = a\,dy$



Flux associated with this strip is

$$d\phi = BdA = \left(\frac{\mu_0 I}{2\pi y}\right)(a\,dy)$$

Total flux associated with the strip is

$$\phi = \int d\phi = \frac{\mu_0 Ia}{2\pi}\int_x^{x+a}\frac{dy}{y} = \frac{\mu_0 Ia}{2\pi}\log_e\left(\frac{x+a}{x}\right)$$

Now $\xi = -\dfrac{d\phi}{dt}$

$$\Rightarrow \quad \xi = \frac{\mu_0 Ia}{2\pi}\left(\frac{x}{x+a}\right)\frac{d}{dx}\left(1+\frac{a}{x}\right)$$

$$\Rightarrow \quad \xi = \frac{\mu_0 I a^2 x}{2\pi(x+a)}\frac{1}{x^2}\left(\frac{dx}{dt}\right)$$

$$\Rightarrow \quad \xi = \frac{\mu_0 I a^2 v}{2\pi x(x+a)} \qquad \left\{\because \frac{dx}{dt} = v\right\}$$

21. Please note that the loops are connected in a manner such that if current in one is clockwise then current in the other is counter clockwise i.e., the emf in loop b opposes the emf in loop a. If ξ_1 be the emf associated with loop a, then

$$\xi_1 = \frac{d}{dt}(a^2 B) = a^2\left(\frac{dB}{dt}\right) = a^2 B_0 \omega\cos(\omega t) \qquad \ldots(1)$$

If ξ_2 is the emf associated with loop b, then

$$\xi_2 = \frac{d}{dt}(b^2 B) = b^2\left(\frac{dB}{dt}\right) = b^2 B_0 \omega\cos(\omega t) \qquad \ldots(2)$$

So, net emf ξ is

$$\xi = B_0\omega(a^2 - b^2)\cos(\omega t)$$

Since the resistance of the circuit is $R = 4\lambda(a+b)$, hence the current amplitude is

$$I_0 = \frac{B_0\omega(a^2-b^2)}{4\lambda(a+b)}$$

$$\Rightarrow \quad I_0 = \frac{B_0\omega(a-b)}{4\lambda} = \frac{(10\times10^{-3})(100)\left(\frac{10}{100}\right)}{4(50\times10^{-3})}$$

$$\Rightarrow \quad I_0 = 0.5 \text{ A}$$

22. Since, the induced emf is given by

$$\xi = -\frac{d}{dt}(NBA) = -1\left(\frac{dB}{dt}\right)\pi a^2 = \pi a^2 K \qquad \{\because N = 1\}$$

(a) $Q = C\xi = \pi a^2 CK$

(b) Since $\vec{B}$ into the paper is decreasing, therefore the induced current will attempt to counteract this. So, the positive charge will go to upper plate.

(c) The changing magnetic field through the enclosed area induces an electric field, surrounding the magnetic field that causes the separation of the charges.

23. For a discharging RC circuit

$$q = q_0 e^{-t/RC}$$

$$\Rightarrow \quad |I| = \frac{dq}{dt} = \frac{q_0}{RC}e^{-\frac{t}{RC}} = \frac{CV_0}{RC}e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad I(t) = \frac{V_0}{R}e^{-\frac{t}{RC}}$$

where V_0 is the initial voltage across the capacitor. The resistance of the small loop is

$$(25)(0.6 \text{ m})(1\ \Omega\text{m}^{-1}) = 15\ \Omega$$

(a) The large circuit is an RC circuit with a time constant of $\tau = RC = (10\ \Omega)(20\times10^{-6}\text{ F}) = 200\ \mu s$. So, the current as a function of time is

$$I = \frac{V_0}{R}e^{-\frac{t}{RC}} = 10e^{-\frac{t}{200}}$$

At $t = 200\ \mu s$, we obtain $I = (10 \text{ A})(e^{-1}) = 3.7$ A

(b) Assuming that only the long wire nearest the small loop produces an appreciable magnetic flux through the small loop then, we get

$$\phi_B = \int_c^{c+a}\frac{\mu_0 I b}{2\pi r}dr = \frac{\mu_0 I b}{2\pi}\log_e\left(1+\frac{a}{c}\right)$$

So, the emf induced in the small loop at $t = 200\ \mu s$ is $\xi = -\frac{d\phi}{dt} = -\frac{\mu_0 b}{2\pi}\log_e\left(1+\frac{a}{c}\right)\frac{dI}{dt}$,

where $\frac{dI}{dt} = -\frac{I_0}{RC}e^{-\frac{t}{RC}}$

So, at $t = 200\ \mu s$, we get

$$\xi = -\frac{(4\pi\times10^{-7})(0.2 \text{ m})}{2\pi}\log_e(3)\left(-\frac{3.7 \text{ A}}{200\times10^{-6}\text{ s}}\right)$$

$$\xi = +0.81 \text{ mV}$$

Thus, the induced current in the small loop is

$$I' = \frac{\xi}{R} = \frac{0.81 \text{ mV}}{15\ \Omega} = 54\ \mu\text{A}$$

(c) The magnetic field from the large loop is directed out of the page within the small loop. The induced current will act to oppose the decrease in flux from the large loop. Hence, the induced current flows counter clockwise.

(d) Three of the wires in the large loop are too far away to make a significant contribution to the flux in the small loop. This can be seen by comparing the distance c to the dimensions of the large loop.

24. $$I = \frac{|\xi|}{R} = \frac{1}{R}\left(\frac{d\phi}{dt}\right) = \frac{A}{R}\left(\frac{dB}{dt}\right) \qquad \ldots(1)$$

where A is the area of the loop and R is the resistance of the loop.

Since $m = Vd = (\pi a^2)(2\pi r)d \qquad \ldots(2)$

Also, $R = \frac{\rho \ell_{wire}}{\pi a^2} = \frac{2\pi r}{\sigma(\pi a^2)} \qquad \ldots(3)$

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$$\Rightarrow \quad I=\frac{\sigma\left(\pi r^2\right)\left(\pi a^2\right)}{2\pi r}\left(\frac{dB}{dt}\right)=\frac{\sigma\left(\pi r\right)\left(\pi a^2\right)}{2\pi}\left(\frac{dB}{dt}\right)$$

Since from (2), we have

$$\left(\pi a^2\right)\left(\pi r\right)=\frac{m}{2d}$$

$$\Rightarrow \quad I=\left(\frac{\sigma m}{4\pi d}\right)\frac{dB}{dt}$$

This expression is independent of the size of the wire as well as the loop.

Test Your Concepts-II (Based on Faraday's Laws: Motional EMF)

1. $F=BI\ell$ where $I=\frac{\xi}{R}=\frac{B\ell v}{R}$

$$\Rightarrow \quad B=\frac{IR}{\ell v}$$

(a) $F=1\text{ N}=BI\ell$

$$\Rightarrow \quad 1=\left(\frac{IR}{\ell v}\right)I\ell$$

$$\Rightarrow \quad I^2R=v$$

$$\Rightarrow \quad I=\sqrt{\frac{v}{R}}=\sqrt{\frac{2}{8}}=\sqrt{\frac{1}{4}}=0.5\text{ A}$$

This could have been done directly by using the relation, $P=Fv=I^2R$

$$\Rightarrow \quad I=\sqrt{\frac{Fv}{R}}=0.5\text{ A}$$

(b) $P=I^2R=(0.5)^2 8=2\text{ W}$

(c) $P_{mech}=Fv=(1)(2)=2\text{ W}$

2. When the terminal speed is obtained, then we have

$$Mg=F_B$$

$$\Rightarrow \quad Mg=(BI)w\text{, where } I=\frac{Bwv_T}{R}$$

$$\Rightarrow \quad Mg=\frac{B^2w^2v_T}{R}$$

$$\Rightarrow \quad v_T=\frac{MgR}{B^2w^2}$$

3. $B=\frac{\mu_0 I}{2\pi r}$

$$\xi=B\ell v=\frac{\mu_0 Iv\ell}{2\pi r}$$

$$\Rightarrow \quad I=\frac{\xi}{R}=\frac{\mu_0 Iv\ell}{2\pi rR}$$

4. $I=\frac{\xi}{R_{net}}$, where $\xi=B\ell v$ and $R_{net}=R+\frac{R_1R_2}{R_1+R_2}$

$$\Rightarrow \quad I=\frac{B\ell v}{\left(R+\frac{R_1R_2}{R_1+R_2}\right)}$$

5. $mg\sin\alpha-F_m=ma$

$mg\sin\alpha-BI\ell=ma$, where $I=\frac{B\ell v}{R}$

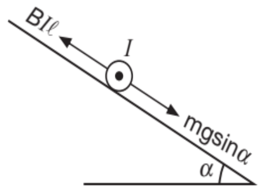
For steady state, $a=0$

$$\Rightarrow \quad mg\sin\alpha=BI\ell$$

$$\Rightarrow \quad mg\sin\alpha=B\left(\frac{B\ell v}{R}\right)\ell$$

$$\Rightarrow \quad mg\sin\alpha=\left(\frac{B^2\ell^2}{R}\right)v$$

$$\Rightarrow \quad v=\frac{mgR\sin\alpha}{B^2\ell^2}$$



Cross-sectional view from the left side

6. Potential difference between the two rails :

$$\xi=B\ell v$$

(When $\vec{B}$, $\vec{v}$ and $\vec{l}$ all are mutually perpendicular)

$$\Rightarrow \quad \xi=\left(0.2\times10^{-4}\right)\left(180\times\frac{5}{18}\right)(1)=10^{-3}\text{ V}=1\text{ mV}$$

7. Since the network given, forms a balanced Wheatstone bridge so, the net resistance of the circuit is $3\,\Omega+1\,\Omega=4\,\Omega$.

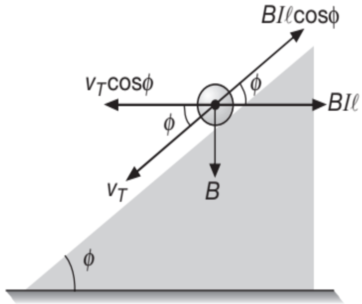
Since the induced emf is given by $B\ell v_0$. Therefore, current in the circuit is given by

$$I=\frac{B\ell v_0}{R}$$

$$\Rightarrow \quad v_0=\frac{IR}{B\ell}=\frac{\left(1\times10^{-3}\right)(4)}{2\times0.1}=0.02\text{ ms}^{-1}$$

Inward magnetic field passing through the loop is decreasing. Therefore, induced current will produced magnetic field in cross direction. So, the direction of induced current is clockwise.

8. (a)



(b) $F_{net} = 0$

$$\Rightarrow \quad mg\sin\phi = BI\ell\cos\phi = B\left(\frac{B\ell v_T\cos\phi}{R}\right)(\ell\cos\phi)$$

$$\Rightarrow \quad v_T = \frac{mgR\sin\phi}{B^2\ell^2\cos^2\phi}$$

(c) $I = \frac{B\ell v_T\cos\phi}{R} = \frac{mg\sin\phi}{B\ell\cos\phi} = \frac{mg\tan\phi}{B\ell}$

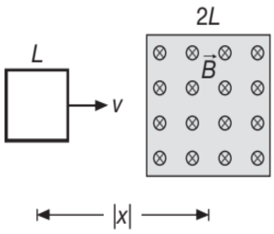
(d) $P_1 = I^2R = \frac{m^2g^2R\tan^2\phi}{B^2\ell^2}$

(e) $P_2 = Fv = (mg\sin\theta)v_T = mgv_T\sin\phi$

$$\Rightarrow \quad P_2 = \frac{m^2g^2R\tan^2\phi}{B^2\ell^2}$$

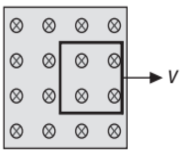
We observe that both are equal.

9. The loop before it starts to enter the magnetic field region is shown in figure.



For $x < -\frac{3L}{2}$ or $x > \frac{3L}{2}$ the loop is completely outside the field region. $\phi_B = 0$, and $\frac{d\phi_B}{dt} = 0$.

Thus $\xi = 0$ and $I = 0$, so there is no force from the magnetic field and the external force F necessary to maintain constant velocity is zero.



When the loop is completely inside the field region i.e., for $-\frac{L}{2} < x < \frac{L}{2}$, we have

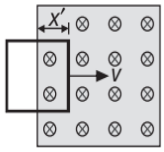
$$\phi_B = BL^2 = \text{constant}$$

$$\Rightarrow \quad \frac{d\phi_B}{dt} = 0$$

$$\Rightarrow \quad \xi = 0 \text{ and } I = 0$$

There is no force $\vec{F} = I\vec{\ell} \times \vec{B}$ from the magnetic field and the external force F necessary to maintain constant velocity is zero.

As the loop enters the magnetic field region i.e., for $-\frac{3L}{2} < x < -\frac{L}{2}$, if x' be the length of the loop that is within the field, then $|\phi_B| = BLx'$ and $\left|\frac{d\phi_B}{dt}\right| = BLv$.



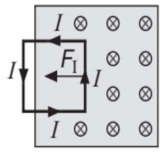
The magnitude of the induced emf is $|\xi| = \left|\frac{d\phi_B}{dt}\right| = BLv$

and the induced current is $I = \frac{|\xi|}{R} = \frac{BLv}{R}$. The current induced in the loop is counterclockwise.

The induced current and magnetic force on the loop are shown in figure for the situation when the loop is entering the field.

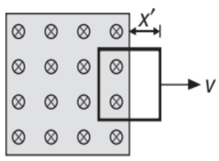
Since, $\vec{F}_1 = I\vec{\ell} \times \vec{B}$, so the force $\vec{F}_1$ exerted on the loop by the magnetic field is to the left and has magnitude

$$F_1 = ILB = \left(\frac{BLv}{R}\right)LB = \frac{B^2L^2v}{R}.$$



The external force $\vec{F}$ needed to move the loop at constant speed is equal in magnitude and opposite in direction to $\vec{F}_1$ so is to the right and has this same magnitude.

As the loop leaves the magnetic field region i.e., for $\frac{L}{2} < x < \frac{3L}{2}$ the loop is leaving the field region. Let x' be the length of the loop that lies outside the field.

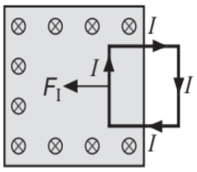


Then $|\phi_B| = BL(L - x')$ and $\left|\frac{d\phi_B}{dt}\right| = BLv$. The magnitude of the induced emf is $|\xi| = \left|\frac{d\phi_B}{dt}\right| = BLv$ and the induced current is $I = \frac{|\xi|}{R} = \frac{BLv}{R}$. The current induced in the loop is clockwise.

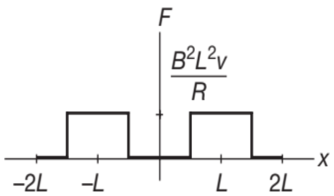
Now, the induced current and magnetic force on the loop are shown in figure for the situation when the loop is leaving the field.

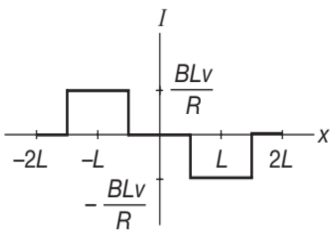
Since, $\vec{F}_I = I\vec{\ell} \times \vec{B}$, so the force $\vec{F}_I$ exerted on the loop by the magnetic field is to the left and has magnitude

$$F_I = ILB = \left(\frac{BLv}{R}\right)LB = \frac{B^2L^2v}{R}$$



The external force $\vec{F}$ needed to move the loop at constant speed is equal in magnitude and opposite in direction to $\vec{F}_I$ so is to the right and has this same magnitude. So, the graphs are shown below.



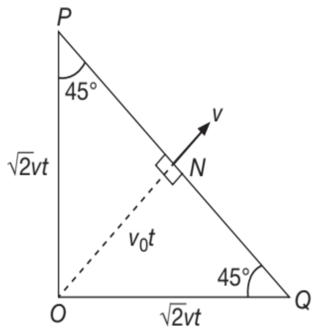


Please note that when the loop is either totally outside or totally inside the magnetic field region the flux is not changing, there is no induced current, and no external force is needed for the loop to maintain constant speed. When the loop is entering the field, the external force required is directed so as to pull the loop in and when the loop is leaving the field the external force required is directed so as to pull the loop out of the field. These directions agree with Lenz's Law according to which, the force on the induced current (opposite in direction to the required external force) is directed so as to oppose the loop entering or leaving the field.

10. $ON = vt$

$\Rightarrow \quad OQ = OP = \sqrt{2}vt$

$\Rightarrow \quad PQ = \sqrt{OP^2 + OQ^2} = 2vt$



The flux associated with this triangle at time t is

$$\phi = B(\text{Area of triangle at time } t)$$

$$\Rightarrow \quad \phi = B\left(\frac{1}{2}(2vt)(vt)\right) = Bv^2t^2$$

$$\Rightarrow \quad |\xi| = \frac{d\phi}{dt} = Bv^2(2t)$$

$$\xi = 2Bv^2t \qquad \ldots(1)$$

Since $I = \dfrac{\xi}{R}$

$$\Rightarrow \quad I = \frac{2Bv^2t}{\lambda\left(\sqrt{2}vt + \sqrt{2}vt + 2vt\right)}$$

$$\Rightarrow \quad I = \frac{Bv}{\lambda(1+\sqrt{2})} = \text{constant}$$

11. (a) $\phi = BA = B(\ell - 2vt)^2$

$$\xi = -\frac{d\phi}{dt} = -2B(\ell - 2vt)(-2v)$$

$$\Rightarrow \quad \xi = 4Bv(\ell - 2vt)$$

(b) $I = \dfrac{\xi}{R} = \dfrac{B\ell v}{\lambda\ell} = \dfrac{Bv}{\lambda}$

(c) $F = BI(\ell - 2vt)$

$$\Rightarrow \quad F = B\left(\frac{Bv}{\lambda}\right)(\ell - 2vt)$$

$$\Rightarrow \quad F = \frac{B^2v}{\lambda}(\ell - 2vt)$$

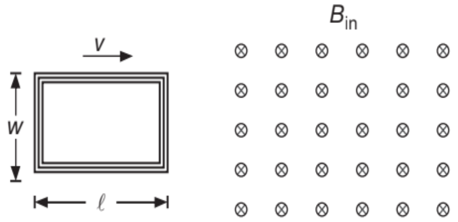
(d) $P = 4Fv = \dfrac{4B^2v^2}{\lambda}(\ell - 2vt)$

(e) $P_{\text{Thermal}} = \dfrac{\xi^2}{R_{\text{total}}} = \dfrac{16B^2v^2(\ell - 2vt)^2}{4(\ell - 2vt)\lambda}$

$$\Rightarrow \quad P_{Th} = \frac{4B^2v^2(\ell - 2vt)}{\lambda}$$

12. (a) The force on the side of the coil entering the field (consisting of N wires) is

$$F = N(ILB) = N(IwB)$$



The induced emf in the coil is

$$|\xi| = N\frac{d\phi_B}{dt} = N\frac{d(Bwx)}{dt} = NBwv$$

So, the current is $I = \frac{|\xi|}{R} = \frac{NBwv}{R}$ counter clockwise

The force on the leading side of the coil is then

$$F = N\left(\frac{NBwv}{R}\right)wB = \frac{N^2B^2w^2v}{R} \text{ to the left}$$

(b) Once the coil is entirely inside the field,

$$\phi_B = NBA = \text{constant},$$

$\Rightarrow$ $\xi = 0$, $I = 0$ and hence $F = 0$

(c) As the coil starts to leave the field, the flux decreases at the rate Bwv, so the magnitude of the current is the same as in part (a), but now the current is clockwise. Thus, the force exerted on the trailing side of the coil is

$$F = \frac{N^2B^2w^2v}{R} \text{ to the left again}$$

13. The emf induced is given by $\xi = B\left(\frac{dA}{dt}\right)$

$$\Rightarrow \xi = B\left(\frac{\text{Area Swept by the blades}}{\text{Time of one Revolution}}\right)$$

$$\Rightarrow \xi = \frac{B(\pi r^2)}{\frac{2\pi}{\omega}}$$

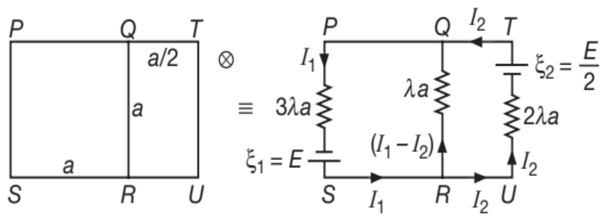
$$\Rightarrow \xi = \frac{1}{2}B\omega r^2$$

$$\Rightarrow \xi = \frac{1}{2}(50\times10^{-6})(4\pi)(9)$$

$$\Rightarrow \xi = 2.83 \text{ mV}$$

14. Let the induced emf for the loop $PQRSP$ be ξ_1 and that for $QTURQ$ be ξ_2. Then $|\xi_1| = A_1\left(\frac{dB}{dt}\right) = E(\text{say})$ and $|\xi_2| = A_2\left(\frac{dB}{dt}\right) = \frac{a^2k}{2} = \frac{E}{2}$

So, the equivalent circuit diagram for the given set up is



For $PQRSP$, we have

$$(\lambda a)(I_1 - I_2) - E + (3\lambda a)I_1 = 0$$

$$\Rightarrow (4\lambda a)I_1 - (\lambda a)I_2 = E \quad \ldots(1)$$

For $QTURQ$, we have

$$-\frac{E}{2} + (2\lambda a)I_2 - (\lambda a)(I_1 - I_2) = 0$$

$$\Rightarrow -(\lambda a)I_1 + (3\lambda a)I_2 = \frac{E}{2}$$

$$\Rightarrow -(2\lambda a)I_1 + (6\lambda a)I_2 = E \quad \ldots(2)$$

Eliminating by using 2 (2) + (1), we get

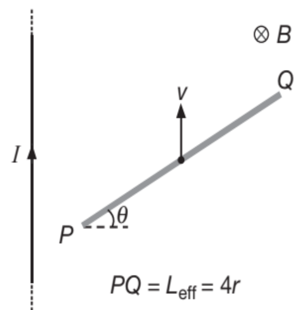
$$(11\lambda a)I_2 = 3E$$

$$\Rightarrow I_2 = \frac{3E}{11\lambda a} = \frac{3a^2k}{11\lambda a} = \frac{3ak}{11\lambda}$$

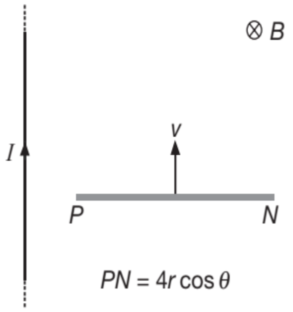
Solving for others, we get

$$I_1 = \frac{7ak}{22\lambda} \text{ and } (I_1 - I_2) = \frac{ak}{22\lambda}$$

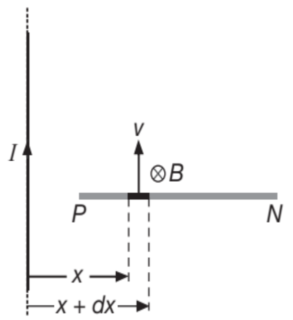
15. As discussed in theory, let us join the end point P and Q and replace the two semicircles by a straight rod of equivalent length $4r$, as shown.



The effective rod is not perpendicular to the direction of motion. Taking the projection of the rod perpendicular to its direction of motion we find the effective length of the conductors is $PN = 4r\cos\theta$.



We now have a rod of effective length $PN = 4r\cos\theta$ translating in a non-uniform magnetic field.
Consider an infinitesimal element of the wire of length dx located at a distance x from the wire. The magnetic field at this element is $B = \dfrac{\mu_0 I}{2\pi x}$.



The potential difference across this element is $d\xi = Bvdx = \dfrac{\mu_0 I}{2\pi x} v\,dx$.
The potential difference across the ends of the rod is calculated by integrating over the entire length. Therefore,

$$\xi = \int d\xi = \int_{d-2r\cos\theta}^{d+2r\cos\theta} \frac{\mu_0 I}{2\pi x} v\,dx$$

$$\Rightarrow \quad \xi = \frac{\mu_0 I v}{2\pi}\log_e\left(\frac{d+2r\cos\theta}{d-2r\cos\theta}\right)$$

16. For the suspended mass M, we have

$$Mg - T = Ma \qquad \ldots(1)$$

For the bar connected to the suspended mass M,

$$T - BI\ell = ma \qquad \ldots(2)$$

where $I = \dfrac{\xi}{R} = \dfrac{B\ell v}{R}$ ...(3)

$$\Rightarrow \quad Mg - BI\ell = (M+m)a$$

$$\Rightarrow \quad Mg - \frac{B^2\ell^2 v}{R} = (M+m)a$$

$$\Rightarrow \quad a = \frac{dv}{dt} = \frac{Mg}{M+m} - \frac{B^2\ell^2}{R(M+m)}v$$

$$\Rightarrow \quad a = \alpha - \beta v \text{ where } \alpha = \frac{Mg}{M+m},\ \beta = \frac{B^2\ell^2}{R(M+m)}$$

$$\Rightarrow \quad \frac{dv}{dt} = \alpha - \beta v$$

$$\Rightarrow \quad \int_0^v \frac{dv}{\alpha - \beta v} = \int_0^t dt$$

$$\Rightarrow \quad -\frac{1}{\beta}\log_e(\alpha - \beta v)\Big|_0^v = t$$

$$\Rightarrow \quad \log_e\left(\frac{\alpha - \beta v}{\alpha}\right) = -\beta t$$

$$\Rightarrow \quad v = \frac{\alpha}{\beta}\left(1 - e^{-\beta t}\right)$$

$$\Rightarrow \quad v = \frac{MgR}{B^2\ell^2}\left(1 - e^{-\frac{B^2\ell^2 t}{R(M+m)}}\right)$$

At the terminal velocity of the bar, say v_T we have

$$a = 0$$

$$\Rightarrow \quad \frac{Mg}{M+m} - \frac{B^2\ell^2 v_T}{R(M+m)} = 0$$

$$\Rightarrow \quad Mg = \frac{B^2\ell^2}{R}v_T$$

$$\Rightarrow \quad v_T = \frac{MgR}{B^2\ell^2}$$

Test Your Concepts-III (Based on Faraday's Laws: AC Generator)

1. $\phi = BA\cos 0$

$$\Rightarrow \quad \phi = NBA$$

$$\Rightarrow \quad \xi = N\frac{d\phi}{dt} = NA\left(\frac{dB}{dt}\right)$$

where $\dfrac{dB}{dt} = 4000\cos(1000t)\,mTs^{-1} = 4\cos(1000t)\ \text{Ts}^{-1}$

$$\Rightarrow \quad \xi = (50)(3.14)\left(7\times10^{-3}\right)^2 4\cos(1000t)$$

$$\Rightarrow \quad \xi = 30.8\cos(1000t)\ \text{mV}$$

$$\Rightarrow \quad \xi_{\text{peak}} = 30.8\ \text{mV}$$

2. The flux through the coil is $\phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta = BA\cos(\omega t)$. The induced emf is

$$\xi = -N\frac{d\phi_B}{dt} = NBA\omega\sin(\omega t)$$

(a) $\xi_{max} = \xi_0 = NBA\omega = 60(1\text{ T})(0.1\times 0.2)(30) = 36\text{ V}$

(b) $\frac{d\phi_B}{dt} = \frac{\xi}{N}$, thus

$$\left|\frac{d\phi_B}{dt}\right|_{max} = \frac{\xi_{max}}{N} = \frac{36}{60} = 0.6\text{ V} = 0.6\text{ Wbs}^{-1}$$

(c) At $t = \frac{\pi}{120}$ s, $\omega t = \frac{\pi}{4}$ radian

$$\Rightarrow \quad \xi = \xi_0\sin\left(\frac{\pi}{4}\right) = \frac{36}{\sqrt{2}}\text{V} = 18\sqrt{2}\text{ V}$$

(d) The torque on the coil at any time is

$$\vec{\tau} = |\vec{M}\times\vec{B}| = |NI\vec{A}\times\vec{B}| = (NAB)I|\sin\omega t|$$

$$\vec{\tau} = \left(\frac{\xi_{max}}{\omega}\right)\left(\frac{\xi_{max}}{R}\right)|\sin\omega t|$$

Now when $\xi = \xi_{max}$, $\sin(\omega t) = 1$

$$\Rightarrow \quad \tau = \frac{\xi_{max}^2}{\omega R} = \frac{(36)^2}{(30)(10\ \Omega)} = 4.32\text{ Nm}$$

3. (a) $\xi = -N\left(\frac{d\phi_B}{dt}\right) = -NA\left(\frac{dB}{dt}\right)$

$$\Rightarrow \quad \xi = -NA\frac{d}{dt}(\mu_0 nI) = -\mu_0 NnA\frac{dI}{dt}$$

where A = area of coil

N = number of turns in coil and

n = number of turns per unit length in solenoid.

$$\Rightarrow \quad |\xi| = N\mu_0 An\frac{d}{dt}5\sin(100\pi t)$$

$$\Rightarrow \quad |\xi| = N\mu_0 An500\pi\cos(100\pi t)$$

$$\Rightarrow \quad |\xi| = 160(4\pi\times 10^{-7})\left[\pi(0.05)^2\right](2\times 10^3)(500\pi)\cos(100\pi t)$$

$$\Rightarrow \quad |\xi| = 5\cos(100\pi t)$$

(b) $P = \frac{V^2}{R} = \frac{\xi^2}{R}$

$$\Rightarrow \quad P = \frac{(5)^2\cos^2(100\pi t)}{8}$$

Since $\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$

$$\Rightarrow \quad \langle\cos^2\theta\rangle = \frac{1}{2} + \frac{1}{2}\langle\cos(2\theta)\rangle$$

$$\Rightarrow \quad \langle\cos^2\theta\rangle = \frac{1}{2} + 0 \qquad \{\because \langle\cos(2\theta)\rangle = 0\}$$

$$\Rightarrow \quad \langle\cos^2\theta\rangle = \frac{1}{2}$$

$$\Rightarrow \quad \langle P\rangle = \frac{1}{2}\frac{(5)^2}{8} = \frac{25}{16} = 1.57\text{ W}$$

4. Outside the solenoid net magnetic field zero and inside the solenoid $B = \mu_o nI$

Induced emf

$$\xi = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA) = -\frac{d}{dt}\left(\mu_o nI\pi a^2\right)$$

$$\Rightarrow \quad \xi = -\mu_0 n\pi a^2\left(\frac{dI}{dt}\right)$$

$$\Rightarrow \quad |\xi| = \left(\mu_o n\pi a^2\right)\left(I_o\omega\cos\omega t\right)$$

Resistance of the cylindrical vessel

$$R = \frac{\rho\ell}{s} = \frac{\rho(2\pi R)}{Ld}$$

So, the induced current is given by

$$I = \frac{|\xi|}{R} = \frac{\mu_o Lnda^2 I_o\omega\cos(\omega t)}{2\rho R}$$

5. (a) $\phi = \vec{B}\cdot\vec{A} = BA\cos\theta$

$$\Rightarrow \quad \phi = B\left(\frac{\pi r^2}{2}\cos(\omega t) + ab\right)$$

(b) $\xi = -\frac{d\phi}{dt}$

$$\Rightarrow \quad \xi = \frac{\pi Br^2\omega}{2}\sin(\omega t)$$

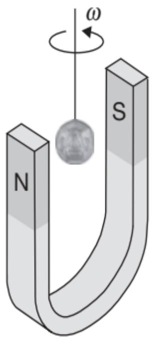
(c) $I = \frac{\xi}{R} = \frac{\pi Br^2\omega}{2R}\sin(\omega t)$

6. Since $\phi_B = BA\cos\theta = BA\cos(\omega t)$

$$\Rightarrow \quad \frac{d\phi_B}{dt} = -\omega BA\sin\theta$$

$$\Rightarrow \quad I \propto -\sin\theta$$

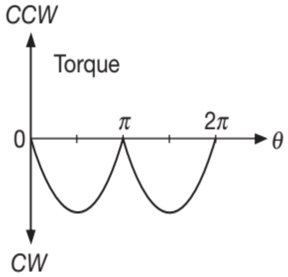
CHAPTER 3



Since $|\vec{\tau}| = MB\sin\theta = IAB\sin\theta$

$\Rightarrow \quad \tau \propto IB\sin\theta$

$\Rightarrow \quad \tau \propto -\sin^2\theta$



7. (a) $\xi = NBA\omega\sin\theta = NBA\omega\sin(\omega t)$

$$\Rightarrow \quad \xi_{MAX} = NBA\omega = BA\omega = B\left(\frac{\pi R^2}{2}\right)\omega$$

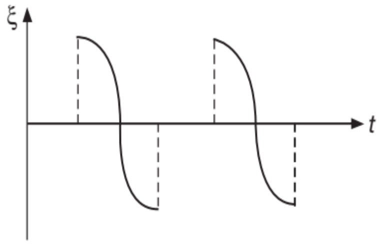
$$\Rightarrow \quad \xi_{MAX} = 1.6 \text{ V}$$

(b) $$\langle \xi \rangle = \frac{\int_0^{2\pi} \xi d\theta}{\int_0^{2\pi} d\theta}$$

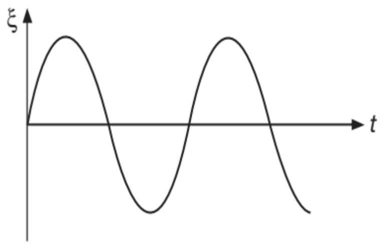
$$\Rightarrow \quad \langle \xi \rangle = \frac{BA\omega}{2\pi}\int_0^{2\pi} \sin\theta d\theta = 0$$

(c) Both remain unchanged.

(d)



(e)



8. (a) $\phi = BA\cos\theta = BA\cos(\omega t) = BA\cos(2\pi ft)$

$$\Rightarrow \quad \phi = (0.8)(0.1)^2\cos(2\pi(60)t)$$

$$\Rightarrow \quad \phi = (8\times10^{-3})\cos(377t) \ Tm^2$$

(b) $\xi = -\frac{d\phi}{dt} = (8\times10^{-3})(377)\sin(377t)$

$$\Rightarrow \quad \xi = 3\sin(377t) \text{ V}$$

(c) $I = \frac{\xi}{R} = 3\sin(377t) \text{ A}$

(d) $P = I^2R = 9\sin^2(377t) \text{ W}$

(e) $P = Fv = \tau\omega$

$$\Rightarrow \quad \tau = \frac{P}{\omega} = \frac{9\sin^2(377t)}{120\pi}$$

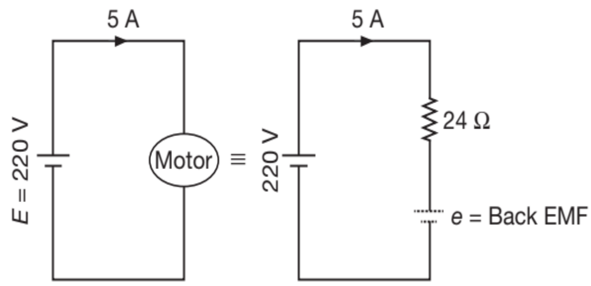
$$\Rightarrow \quad \tau \cong 24\times10^{-3}\sin^2(377t) \text{ Nm}$$

9. (a) According to Kirchhoff's Law, we have

$$220 - (5)(24) - e = 0$$

$$\Rightarrow \quad e = 220 - 120$$

$$\Rightarrow \quad e = 100 \text{ V}$$



(b) $P = I^2R = (5)^2(24) = 600 \text{ W}$

(c) With no motion, the motor does not function and so we have back emf $e = 0$

$$\Rightarrow \quad 220 - I(24) = 0$$

$$\Rightarrow \quad I' = 9.2 \text{ A}$$

Since $P' = I'^2R$

$$\Rightarrow \quad P' = (9.2)^2(24) \cong 2 \text{ kW}$$

Test Your Concepts-IV (Based on Faraday's Laws: Induced Electric Field)

1. Since $\oint \vec{E}\cdot d\vec{\ell} = \left|\frac{d\phi_B}{dt}\right|$

$$\Rightarrow \quad (2\pi r)E = (\pi r^2)\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E = \frac{r}{2}\left(\frac{dB}{dt}\right)$$

For a solenoid, we have $B = \mu_0 n I$

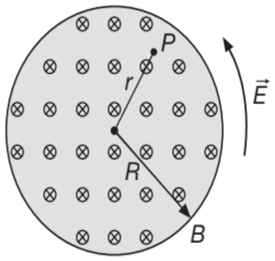
$$\Rightarrow \quad \frac{dB}{dt} = \mu_0 n\left(\frac{dI}{dt}\right), \text{ where } \frac{dI}{dt} = I_0 \omega \cos(\omega t)$$

$$\Rightarrow \quad E = \left(\frac{\mu_0 n r \omega I_0}{2}\right)\cos(\omega t)$$

The electric field is always opposite to the increasing B, so if must be clockwise.

2. Since we know that

$$\oint \vec{E}\cdot d\vec{\ell} = \left|\frac{d\phi_B}{dt}\right|$$



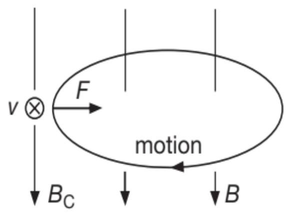
$$\Rightarrow \quad E(2\pi r) = (\pi r^2)\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E = \frac{r}{2}\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E = \left(\frac{0.02}{2}\right)(0.06 \times 3)$$

$$\Rightarrow \quad E = 1.8 \times 10^{-3}\ \text{NC}^{-1}$$

3. Let us assume the field to be acting vertically downwards. Consider an electron that is moving away from us as shown. The force on it is in the radially inward direction, so as to make them circulate clockwise.



(a) Now, when the downward field increases, an emf is induced to produce some current that in turn produces an upward field. This current is directed counter clockwise direction (using Right Hand Thumb Rule) carried by negative electrons moving clockwise. Therefore, the original electron motion speeds up.

(b) For the electrons moving in a circle, we have

$$|q| v B_c \sin(90°) = \frac{mv^2}{r}$$

$$\Rightarrow \quad mv = |q| r B_c$$

The increasing magnetic field $\vec{B}_{av}$ in the area enclosed by the orbit produces a tangential electric field given by

$$\left|\oint \vec{E}\cdot d\vec{\ell}\right| = \left|\frac{-d\phi_B}{dt}\right|\left|\frac{d}{dt}(\vec{B}_{av}\cdot\vec{A})\right|$$

$$\Rightarrow \quad E(2\pi r) = \pi r^2 \frac{dB_{av}}{dt}$$

$$\Rightarrow \quad E = \frac{r}{2}\frac{dB_{av}}{dt}$$

An electron feels a tangential force when $\sum F_t = ma_t$

$$\Rightarrow \quad |q|E = m\frac{dv}{dt}$$

$$\Rightarrow \quad |q|\frac{r}{2}\frac{dB_{av}}{dt} = m\frac{dv}{dt}$$

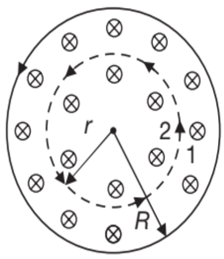
$$\Rightarrow \quad |q|\frac{r}{2}B_{av} = mv = |q| r B_c$$

$$\Rightarrow \quad B_{av} = 2B_c$$

4. Since $\xi = -\dfrac{d\phi}{dt}$

$$\Rightarrow \quad \oint \vec{E}\cdot d\vec{\ell} = -A\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad \oint \vec{E}\cdot d\vec{\ell} = \left|A\frac{d\vec{B}}{dt}\right|$$



For $r \le R$:

Using $E\ell = A\left|\dfrac{dB}{dt}\right|$, we get

$$E(2\pi r) = (\pi r^2)\alpha$$

$$\Rightarrow \quad E = \frac{r\alpha}{2}$$

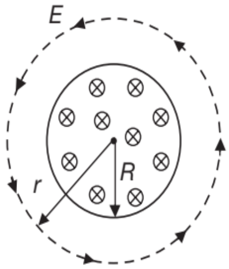
$$\Rightarrow \quad E \propto r$$

i.e., E-r graph is a straight line passing through origin.

At $r = R$, $E = \dfrac{R\alpha}{2}$

For $r \ge R$:

Using $E\ell = A\left|\frac{dB}{dt}\right|$



$\Rightarrow \quad E(2\pi r) = (\pi R^2)(\alpha)$

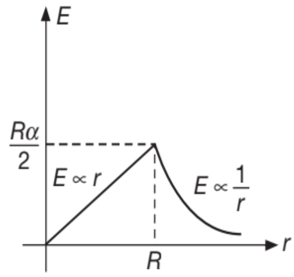
$\Rightarrow \quad E = \frac{\alpha R^2}{2r}$

$\Rightarrow \quad E \propto \frac{1}{r}$

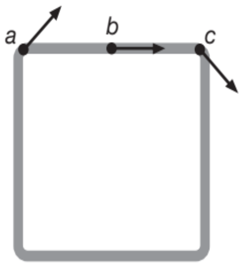
i.e. E-r graph is a rectangular hyperbola.

The E-r graph is as shown in figure.

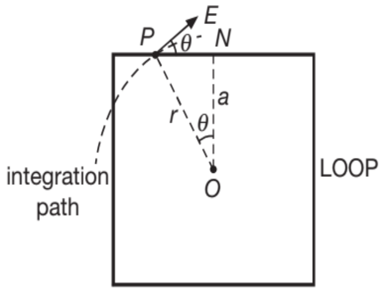
Direction of electric field is shown in figure.



5. (a)



(b) Consider an integration path which is a circle of radius r. By symmetry the induced field is tangent to this path and constant in magnitude at all the points on the path.



For the loop, we have $E_{\text{Loop}} = E\cos\theta$

Since $E(2\pi r) = \xi \qquad \left\{\because |\xi| = \int \vec{E}\cdot d\vec{\ell}\right\}$

$\Rightarrow \quad E = \frac{\xi}{2\pi r}$

From triangle OPN

$r\cos\theta = a$

$\Rightarrow \quad E = \frac{\xi\cos\theta}{2\pi a}$

$\Rightarrow \quad E_{loop} = E\cos\theta = \frac{\xi\cos^2\theta}{2\pi a} \qquad \ldots(1)$

But $\xi = \frac{d\phi_B}{dt} = A\left(\frac{dB}{dt}\right) = \pi r^2\left(\frac{dB}{dt}\right)$

$\Rightarrow \quad \xi = \frac{\pi a^2}{\cos^2\theta}\left(\frac{dB}{dt}\right)$

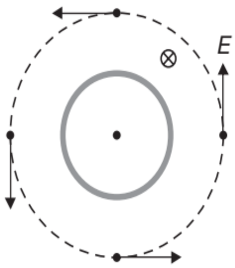
$\Rightarrow \quad \xi\cos^2\theta = \pi a^2\left(\frac{dB}{dt}\right)$

But from (1) $\xi\cos^2\theta = E_{\text{loop}}(2\pi a)$

$\Rightarrow \quad E_{\text{loop}}(2\pi a) = \pi a^2\left(\frac{dB}{dt}\right)$

$\Rightarrow \quad E_{\text{loop}} = \frac{a}{2}\left(\frac{dB}{dt}\right)$

6. (a) The ring will rotate in the direction of induced electric field $\vec{E}$ as shown in Figure.



$$E(2\pi R) = (\pi R^2)\frac{dB}{dt}$$

$\Rightarrow \quad E = \left(\frac{R}{2}\right)(B_0)$

Since, $F = qE = \frac{qB_0R}{2}$ and $\tau = FR = \frac{qB_0R^2}{2}$

$\Rightarrow \quad \alpha = \frac{\tau}{I} = \frac{\tau}{mR^2} = \frac{qB_0}{2m}$

Since, $\alpha = $ constant, so we have

$\omega = \omega_0 + \alpha t$

$$\Rightarrow \quad \omega = 0 + \left(\frac{qB_0}{2m}\right)t = \left(\frac{qB_0}{2m}\right)t$$

(b) $P = \frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) = I\cdot\omega\cdot\frac{d\omega}{dt} = I\omega\alpha$

$$\Rightarrow \quad P = I(\alpha t)\alpha = I\alpha^2 t = (mR^2)\left(\frac{qB_0}{2m}\right)^2 t$$

$$\Rightarrow \quad P = \frac{q^2B_0^2R^2t}{4m}$$

7. The induced electric field is set up as a result of the changing current through the solenoid.
Inside the solenoid i.e. for $r < a$, the induced electric field is

$$E_{nc} = \frac{r}{2}\left(\frac{dB}{dt}\right)$$

where, $B = \mu_0 ni$ for $r < a$ i.e. inside the solenoid

$$\Rightarrow \quad E_{nc} = \frac{r}{2}\frac{d}{dt}(\mu_0 ni) = \frac{\mu_0 nr}{2}\left(\frac{di}{dt}\right)$$

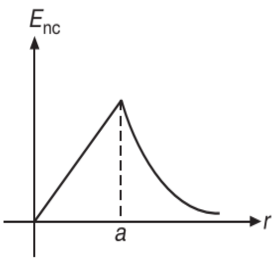
According to the problem, we have

$$\frac{di}{dt} = I$$

$$\Rightarrow \quad E_{nc} = \left(\frac{\mu_0 nI}{2}\right)r$$

Outside the solenoid i.e. for $r > a$, the induced electric field is

$$E_{nc} = \frac{a^2}{2r}\left(\frac{dB}{dt}\right)E = \left(\frac{\mu_0 nIa^2}{2}\right)\frac{1}{r}$$



8. $\xi = \frac{d\phi}{dt} = -A\left(\frac{dB}{dt}\right)$

$$\Rightarrow \quad \xi = -\frac{1}{2}(L)\sqrt{R^2 - \frac{L^2}{4}}\frac{dB}{dt}$$

$$\Rightarrow \quad |\xi| = \left(\frac{L}{2}\sqrt{R^2 - \frac{L^2}{4}}\right)\frac{dB}{dt}$$

9. Induced emf in the loop is

$$|\xi| = A\left(\frac{dB}{dt}\right) = \left(\frac{\pi r^2}{2}\right)(6t^2 + 6t)$$

Net resistance of the loop is

$$R_{net} = R\left(\frac{r}{2} + \frac{r}{2} + 2r + \pi r\right) = rR(3+\pi)$$

Since the induced emf has opposing nature, so the current in the loop is

$$i = \frac{\text{Net Voltage}}{\text{Net Resistance}} = \frac{E - \xi}{R_{net}}$$

$$\Rightarrow \quad i = \frac{E - \frac{\pi r^2}{2}(6t^2 + 6t)}{rR(3+\pi)} = \frac{E - 3\pi r^2(t^2 + t)}{rR(3+\pi)}$$

$$\Rightarrow \quad i = \frac{E - 18\pi r^2}{rR(3+\pi)}$$

Test Your Concepts-V (Based on Faraday's Laws: Self Induction)

1. $|\xi| = L\frac{dI}{dt} = L(2t - 6)$

(a) $|\xi| = |(90\times10^{-3})(2-6)| = 360$ mV

(b) $|\xi| = |(90\times10^{-3})(2)| = 180$ mV

(c) $|\xi| = 0$

$$\Rightarrow \quad t = 3 \text{ s}$$

2. Since $\xi = \xi_0 e^{-kt}$ and $\xi = -L\frac{dI}{dt}$

$$\Rightarrow \quad -L\frac{dI}{dt} = \xi_0 e^{-kt}$$

Since we are to find the current as a function of t, so let the current be I at time t. Then as $t \to \infty$, $I \to 0$ $(\because \xi \to 0)$

$$\Rightarrow \quad \int_I^0 dI = -\frac{\xi_0}{L}\int_t^\infty e^{-kt}dt$$

$$\Rightarrow \quad -I = \frac{\xi_0}{kL}e^{-kt}\Big|_t^\infty$$

$$\Rightarrow \quad -I = \frac{\xi_0}{kL}(e^{-\infty} - e^{-kt})$$

$$\Rightarrow \quad I = \frac{\xi_0}{kL}e^{-kt}$$

Now, we know that $|Q| = \left|\int_0^{\infty} Idt\right|$

$$\Rightarrow \quad |Q| = \left|\frac{\xi_0}{kL}\int_0^{\infty} e^{-kt}dt\right|$$

$$\Rightarrow \quad |Q| = \frac{\xi_0}{k^2L}$$

3. (a) Self induced emf,

$$\xi = -L\frac{dI}{dt} = (-0.54)(-0.03) \text{ V}$$

$$\Rightarrow \quad \xi = 1.62 \times 10^{-2} \text{ V}$$

(b) $V_{ba} = L\frac{dI}{dt} = -1.62 \times 10^{-2}$ V

Since, $V_{ba}(= V_b - V_a)$ is negative. It implies that $V_a > V_b$ or a is at higher potential.

4. P.D. across inductor,

$$V_L = L\frac{dI}{dt} = (5)(-1) = -5 \text{ V}$$

Now, $V_a - IR - V_L - E = V_b$

$$\Rightarrow \quad V_{ab} = V_a - V_b = E + IR + V_L$$

$$\Rightarrow \quad V_{ab} = 20 + (2)(10) - 5 = 35 \text{ V}$$

5. (a) The inductance of a solenoid is given by,

$$L = \mu_0 n^2 A\ell = \frac{\mu_0 N^2 A}{\ell} \qquad \left\{\because n = \frac{N}{\ell}\right\}$$

Substituting the values, we get

$$L = \frac{(4\pi \times 10^{-7})(300)^2(4 \times 10^{-4})}{(25 \times 10^{-2})} \text{ H}$$

$$\Rightarrow \quad L = 1.81 \times 10^{-4} \text{ H}$$

(b) $\xi = -L\frac{dI}{dt}$

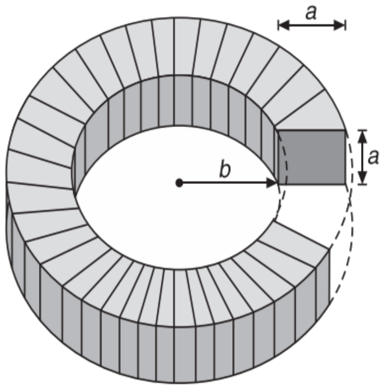
where $\frac{dI}{dt} = -50 \text{ As}^{-1}$

$$\Rightarrow \quad \xi = -(1.81 \times 10^{-4})(-50)$$

$$\Rightarrow \quad \xi = 9.05 \times 10^{-3} \text{ V}$$

$$\Rightarrow \quad \xi = 9.05 \text{ mV}$$

6. If we consider an elemental ring of radius r and width dr as shown in Figure.



The number of turns per unit length in this ring-shaped element is

$$n = \frac{N}{2\pi r}$$

The magnetic induction inside the elemental toroid is

$$B = \mu_r \mu_0 nI = \frac{\mu_r \mu_0 NI}{2\pi r}$$

The magnetic flux through a cross-section of the elemental ring-shaped toroid is

$$d\phi = \left(\frac{\mu_r \mu_0 NI}{2\pi r}\right)(adr)$$

Magnetic flux linked with all N turns of the toroid in the elemental ring-shaped element is

$$Nd\phi = \left(\frac{\mu_r \mu_0 N^2 Ia}{2\pi}\right)\frac{dr}{r}$$

Total magnetic flux linked with the given toroid is given as

$$\phi_{\text{total}} = \int d\phi = \frac{\mu_r \mu_0 N^2 Ia}{2\pi}\int_b^{b+a}\frac{dr}{r}$$

$$\Rightarrow \quad \phi = \frac{\mu_r \mu_0 N^2 Ia}{2\pi}\ln\left(\frac{b+a}{a}\right)$$

Self-inductance of the toroid is

$$L = \frac{\phi}{I}$$

$$\Rightarrow \quad L = \frac{\mu_r \mu_0}{2\pi}N^2 a \ln\left(\frac{b+a}{a}\right)$$

7. (a) $\Delta V_R = IR = (2)(8) = 16\text{ V}$

(b) $\Delta V_L + \Delta V_R = E$

$\Rightarrow \quad \Delta V_R = 36 - 16 = 20\text{ V}$

(c) $\Delta V_L = E - \Delta V_R = E - IR$

$\Rightarrow \quad \Delta V_L = 36 - (4.5)(8)$

$\Rightarrow \quad \Delta V_L = 36 - 36 = 0\text{ V}$

So, at $I = 4.5\text{ A}$, voltage across the inductor is zero volt.

8. Magnetic induction due to the solenoid at its centre is given by

$B = \mu_0 n i$

Total flux linked with the coil is

$\phi = NBA = N(\mu_0 n i)A$

EMF induced in the coil is

$$|\xi| = \frac{d\phi}{dt} = \mu_0 nNA\frac{di}{dt}$$

According to the problem, we have

$$\frac{di}{dt} = \frac{4}{0.05} = 80$$

$$\Rightarrow \quad \xi = (4\pi \times 10^{-7})(2 \times 10^4)(100)(\pi \times 10^{-4})(80)$$

$$\Rightarrow \quad \xi = 6.4\pi^2 \times 10^{-3}\text{ V}$$

$$\Rightarrow \quad \xi \approx 64 \times 10^{-3}\text{ V} = 64\text{ mV}$$

Charge flown through the coil is

$$q = i\Delta t = \frac{\xi}{R}\Delta t$$

$$\Rightarrow \quad q = \frac{(6.4\pi^2 10^{-3})(0.05)}{10\pi^2}$$

$$\Rightarrow \quad q = 3.2 \times 10^{-5}\text{ C}$$

$$\Rightarrow \quad q = 32\ \mu\text{C}$$

9. Let a be the radius of each wire b be the separation between the wires of double line in which current in the wires is in the opposite direction. If each wire carries a current I but in opposite direction, then the magnetic field between the wires at a distance x from one of the wires is

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{b-x}\right)$$

If we consider an elemental strip of length l, width dx at a distance x from one wire, then magnetic flux associated with this strip is

$$d\phi = BdA = B(ldx)$$

$$\Rightarrow \quad \phi = \int d\phi = \frac{\mu_0 Il}{2\pi}\left(\int_a^{b-a}\frac{dx}{x} + \int_a^{b-a}\frac{dx}{b-x}\right)$$

$$\Rightarrow \quad \phi = \frac{\mu_0 Il}{\pi}\ln\left(\frac{b-a}{a}\right)$$

The self-inductance of this double line is

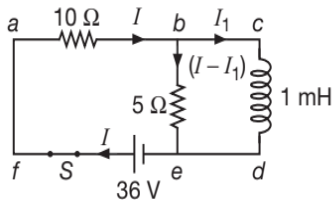
$$L = \frac{\phi}{I} = \frac{\mu_0 l}{\pi}\ln\left(\frac{b-a}{a}\right)$$

Hence the self-inductance per unit length of this double line is

$$\Rightarrow \quad \frac{L}{l} = \frac{\mu_0}{\pi}\ln\left(\frac{b}{a}\right) = \frac{\mu_0}{\pi}\ln(\eta)$$

Test Your Concepts-VI (Based on Growth and Decay of Current in LR Circuits)

1.



For loop $abefa$, we get

$$-10I - 5(I - I_1) + 36 = 0$$

$$-5I_1 + 15I = 36 \quad \ldots(1)$$

For loop $bcdeb$, we get

$$-L\frac{dI_1}{dt} + 5(I - I_1) = 0$$

$$-5I_1 + 5I = L\frac{dI_1}{dt} \quad \ldots(2)$$

Multiply (2) by (3) and subtracting from (1)

$$-5I_1 + 15I_1 = 36 - 3L\frac{dI_1}{dt}$$

$$\Rightarrow \quad 10I_1 = 36 - 3L\frac{dI_1}{dt}$$

$$\Rightarrow \quad 3L\frac{dI_1}{dt} = 36 - 10\text{I}_1$$

$$\Rightarrow \int_0^{I} \frac{dI_1}{36-10I_1} = \frac{1}{3L}\int_0^{t} dt$$

$$\Rightarrow -\frac{1}{10}\log_e\left(36-10I_1\right)\Big|_0^{I_1} = \frac{t}{3L}$$

$$\Rightarrow \frac{36-10I_1}{36} = 10e^{-\frac{t}{3L}}$$

$$\Rightarrow 36-10I_1 = 36e^{-\frac{t}{3L}}$$

$$\Rightarrow 10I_1 = 36\left(1-e^{-\frac{t}{3L}}\right)$$

$$\Rightarrow I_1 = 3.6\left(1-e^{-\frac{t}{3L}}\right)A$$

Since $L = 1$ mH

$$\Rightarrow I_1 = 3.6\left(1-e^{-\frac{t}{3\times10^{-4}}}\right)$$

The current through $10\ \Omega$ resistor is I which is not a constant and varies with t.

2. (a) This is the case of growth of current in an LR circuit. Hence, current at time t is given by,

$$I = I_0\left(1-e^{-t/\tau_L}\right),\ \text{where } \tau_L = \frac{L}{R}$$

$$\Rightarrow I = I_0\left(1-e^{-\frac{Rt}{L}}\right)$$

Rate of increase of current is given by

$$\frac{dI}{dt} = \frac{I_0}{\tau_L}e^{-t/\tau_L}$$

At $t=0$, $\frac{dI}{dt} = \frac{I_0R}{L} = \frac{R}{L}\left(\frac{E}{R}\right) = \frac{E}{L}$

Substituting the value, we have

$$\frac{dI}{dt} = \frac{200}{0.5} = 400\ \text{As}^{-1}$$

(b) At $t = \frac{L}{R}$, $\frac{dI}{dt} = (400)e^{-1} = (0.37)(400) = 148\ \text{As}^{-1}$

(c) The steady state current in the circuit,

$$I_0 = \frac{E}{R} = \frac{200}{20} = 10\ \text{A}$$

3. (a) $\tau_L = \frac{L}{R} = \frac{2}{10} = 0.2$ s

(b) $I_{max} = I_0 = \frac{E}{R} = \frac{100}{10} = 10$ A

(c) $I = I_0\left(1-e^{-\frac{t}{\tau_L}}\right) = 10\left(1-e^{-\frac{1}{0.2}}\right)$

$$\Rightarrow I = 10\left(1-e^{-5}\right) = 9.93\ \text{A}$$

(d) At steady state $I = I_{max} = I_0$

$$\Rightarrow U = \frac{1}{2}LI_{max}^2 = \frac{1}{2}(2)(10)^2 = 100\ \text{J}$$

4. (a) $I = I_0e^{-\left(\frac{R_{total}}{L}\right)t}$

$$\Rightarrow I = I_0e^{-\left(\frac{R+r}{L}\right)t}$$

At $t=0$, when the switch is opened, just before that the current in the circuit must be maximum and must have been passing through L only. So, we have

$$I_0 = \frac{E}{r}$$

$$\Rightarrow I = \frac{E}{r}e^{-\left(\frac{R+r}{L}\right)t}$$

(b) $dH = I^2r\,dt$

$$\Rightarrow dH = \frac{E^2}{r}e^{-2\left(\frac{R+r}{L}\right)t}dt$$

$$\Rightarrow H = \int dH = \frac{E^2}{r}\int_0^{\infty} e^{-2\left(\frac{R+r}{L}\right)t}dt$$

$$\Rightarrow H = -\frac{E^2L}{2r(R+r)}e^{-2\left(\frac{R+r}{L}\right)t}\Big|_0^{\infty}$$

$$\Rightarrow H = \frac{E^2L}{2r(R+r)}$$

5. At $t=0$, when the key is closed, the inductor does not allow current to pass through it. So, the Bulb-1 does not glow but Bulb-2 starts glowing at its full intensity instantly.

After a long time, the current through branch of Bulb-1 is more, as the branch has lesser resistance, so finally, Bulb-1 glows brighter.

When the key is opened, then both bulbs are in series and due to the inductor connected in this series combination of bulbs, the current in both the bulbs decays slowly and hence both bulbs die slowly. However, both bulbs glow with equal intensities at any time after the key is opened.

6. $20-(I_1+I_2)R = 5\frac{dI_1}{dt} = 10\frac{dI_2}{dt_2}$ ...(1)

At $t=0$, $I_1 = I_2 = 0$

$\Rightarrow\ 5I_1 = 10I_2$ ...(2)

In steady state, we have

$$\frac{dI_1}{dt} = \frac{dI_2}{dt} = 0$$

$$\Rightarrow\ I_1 + I_2 = \frac{20}{R} = \frac{20}{5} = 4\text{ A}$$

$\Rightarrow\ I_1 + \frac{I_1}{2} = 4\text{ A}$ $\{\because I_1 = 2I_2\}$

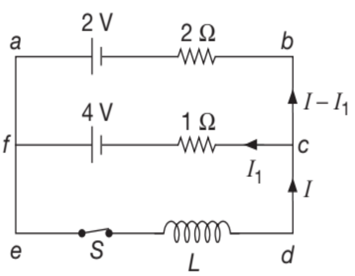
$$\Rightarrow\ 3I_1 = 8$$

$$\Rightarrow\ I_1 = \frac{8}{3}\text{ A}$$

7. For loop $abcfa$, we get

$$-2+2(I-I_1)-I_1+4=0$$

$\Rightarrow\ -3I_1+2I=-2$ ...(1)



For loop $fcdef$, we get

$$-4+I_1+L\frac{dI}{dt}=0$$

$I_1 + L\frac{dI}{dt} = 4$ ...(2)

Multiply (2) by (3) and adding to (1), we get

$2I + 3L\frac{dI}{dt} = 10$, where $L = 10^{-3}$ H

$$\Rightarrow\ 3L\frac{dI}{dt} = 10-2I$$

$$\Rightarrow\ \int_0^I \frac{dI}{10-2I} = \int_0^t \frac{dt}{3L}$$

$$\Rightarrow\ -\frac{1}{2}\log_e\left(\frac{10-2I}{10}\right) = \frac{t}{3L}$$

$$\Rightarrow\ \left(\frac{10-2I}{10}\right) = e^{-\frac{2t}{3L}}$$

$$\Rightarrow\ 10-2I = 10e^{-\frac{2t}{3L}}$$

$$\Rightarrow\ I = 5\left(1-e^{-\frac{2000t}{3}}\right)\text{ A}$$

8. $R^2 = \frac{L}{C}$

$$\Rightarrow\ RC = \frac{L}{R}$$

$\Rightarrow\ \tau_C = \tau_L$ ...(1)

Now, $I = I_L + I_C$ ...(2)

where $I_L = I_0\left(1-e^{-\frac{t}{\tau_L}}\right) = \frac{V}{R}\left(1-e^{-\frac{t}{\tau_L}}\right)$ ...(3)

$$I_C = \frac{dq}{dt} = \frac{d}{dt}\left[q_0\left(1-e^{-\frac{t}{\tau_C}}\right)\right] = \frac{q_0}{\tau_C}e^{-\frac{t}{\tau_C}}$$

Since $\tau_C = RC$ and $q_0 = CV$

$\Rightarrow\ I_C = \frac{V}{R}e^{-\frac{t}{\tau_C}}$ ...(4)

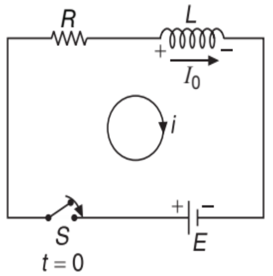
So, $I = \frac{V}{R}\left(1-e^{-\frac{t}{\tau_L}}+e^{-\frac{t}{\tau_C}}\right)$

Since $\tau_C = \tau_L$, so, we get

$$I = \frac{V}{R}$$

9. In the given circuit, applying KLL, we get

$$E - iR - L\frac{di}{dt} = 0$$



$$\Rightarrow \int_{I_0}^{i} \frac{di}{E - iR} = \int_{0}^{t} \frac{dt}{L}$$

$$\Rightarrow -\frac{1}{R}\ln\left(\frac{E - iR}{E - I_0 R}\right) = \frac{t}{L}$$

$$\Rightarrow \frac{E - iR}{E - I_0 R} = e^{-\frac{Rt}{L}}$$

$$\Rightarrow i = \frac{1}{R}\left[E - (E - I_0 R)e^{-\frac{Rt}{L}}\right]$$

10. After a long time, resistance across an inductor becomes zero while resistance across capacitor becomes infinite. Hence, net external resistance,

$$R_{net} = \frac{\frac{R}{2} + R}{2} = \frac{3R}{4}$$

Current through the batteries,

$$I = \frac{2E}{\frac{3R}{4} + r_1 + r_2}$$

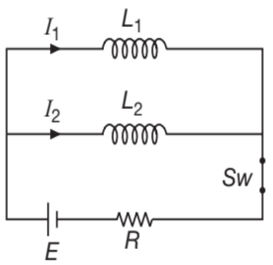
Given that potential across the terminals of cell A is zero, so

$$E - Ir_1 = 0$$

$$\Rightarrow E - \left(\frac{2E}{\frac{3R}{4} + r_1 + r_2}\right) r_1 = 0$$

$$\Rightarrow R = \frac{4}{3}(r_1 - r_2)$$

11. In steady state, the inductors behave like short circuits so the current through battery is



$$I = I_1 + I_2 = \frac{E}{R}$$

Since the current in inductors connected in parallel combination is divided in inverse ratio of their inductances, so we have currents through inductors given by

$$I_1 = \frac{EL_2}{R(L_1 + L_2)} \text{ and } I_2 = \frac{EL_1}{R(L_1 + L_2)}$$

12. Current before closing the switch is

$$I_0 = \frac{V}{R + R_0} \qquad \ldots(1)$$

Let current in the circuit be I at any time t. Applying KVL in the circuit, we get

$$V = L\frac{dI}{dt} + RI$$

$$\Rightarrow \frac{dI}{dt} + \frac{RI}{L} = \frac{V}{L}$$

$$\frac{dI}{dt} = \frac{V - IR}{L}$$

$$\Rightarrow \int_{I_0}^{I} \frac{dI}{V - IR} = \int_{0}^{t} \frac{dt}{L}$$

$$\Rightarrow -\frac{1}{R}\log_e\left(\frac{V - IR}{V - I_0 R}\right) = \frac{t}{L}$$

$$\Rightarrow \frac{V - IR}{V - I_0 R} = e^{-\frac{Rt}{L}}$$

$$\Rightarrow V - IR = (V - I_0 R)e^{-\frac{Rt}{L}}$$

$$\Rightarrow IR = V - (V - I_0 R)e^{-\frac{Rt}{L}}$$

where $I_0 = \frac{V}{R + R_0}$

$$\Rightarrow I = \frac{V}{R} - \left(V - \frac{VR}{R + R_0}\right)e^{-\frac{Rt}{L}}$$

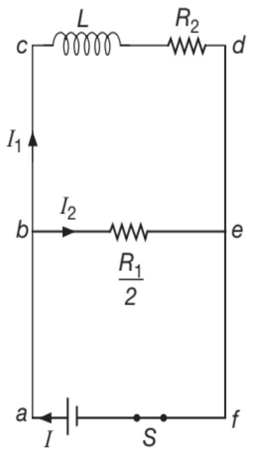
$$\Rightarrow I = \frac{V}{R} - \left(\frac{VR_0}{R + R_0}\right)e^{-\frac{Rt}{L}}$$

13. Applying Kirchhoff's Law in loop *abcdefa*, we get

$$E - L\left(\frac{dI_1}{dt}\right) - I_1R_2 = 0$$

$$\Rightarrow \int_0^{I_1} \frac{L dI_1}{E - I_1R_2} = \int_0^t dt$$

$$\Rightarrow I_1 = \frac{E}{R_2}\left(1 - e^{-\frac{R_2 t}{L}}\right)$$



Potential drop across inductor is

$$V_L = L\frac{dI_1}{dt} = Ee^{-\frac{R_2 t}{L}} = 10e^{-5t} \text{ V}$$

in steady state $I_1 = \dfrac{E}{R_2}$

When the switch S is opened, we have

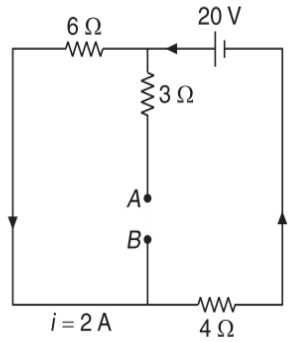
$$I = \left(\frac{E}{R_2}\right)e^{-\left(\frac{R_1}{2}+R\right)\frac{t}{L}}$$

Substituting the values, we get, $I = 5\,e^{-10t}$ A (direction of current is from c to d)

14. Remove the inductor, short the battery and then calculate the equivalent resistance across the open terminals (lets call then A and B) after removing the inductor is

$$R_{AB} = \frac{4 \times 6}{4+6} + 3 = 5.4\ \Omega$$

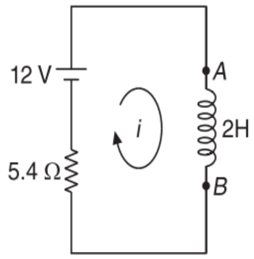
Now calculate the current in the battery circuit at the moment key is closed i.e. when inductor offers infinite resistance path to the flow of current, then from the circuit shown in Figure, the current is



$$I = \frac{20}{6+4} = 2\text{A}$$

So, open circuit potential difference across A and B is

$$V_{AB} = (2)(6) = 12 \text{ V}$$



Hence, current through the inductor is given by

$$I_L = \frac{V_{AB}}{R_{AB}}\left(1 - e^{-\frac{R_{AB}t}{L}}\right)$$

$$\Rightarrow I_L = \frac{12}{5.4}\left(1 - e^{-\frac{5.4t}{2}}\right) = 2.22\left(1 - e^{-2.7t}\right) \text{ A}$$

$$\Rightarrow i = 2.22\left(1 - e^{-2.7t}\right) \text{ A}$$

15. The set up discussed is a series LR circuit with time constant $\tau_L = \dfrac{L}{R}$. So, we just need to verify that the power delivered by the battery equals the sum of power dissipated as heat in the resistor and the power stored in the inductor i.e.,

$$EI = I^2R + LI\frac{dI}{dt} \qquad \ldots(1)$$

Since at any instant t, current in LR circuit is given by

$$I = I_0\left(1-e^{-\frac{Rt}{L}}\right) = \frac{E}{R}\left(1-e^{-\frac{Rt}{L}}\right)$$

At $t = \tau_C = \frac{L}{R}$, we have

$$I = \frac{E}{R}\left(1-\frac{1}{e}\right) = \frac{5}{10}\left(1-\frac{1}{2.718}\right) = 0.316 \text{ A}$$

The rate at which energy is delivered by the battery is

$$P_1 = EI = (5)(0.316) \text{ W} = 1.58 \text{ W} \quad \ldots(2)$$

Energy dissipated as heat in the resistance, given by

$$P_2 = I^2R = (0.316)^2(10) = 0.998 \text{ W}$$

$$\Rightarrow \quad P_2 = 0.998 \text{ W} \quad \ldots(3)$$

The energy stored in the magnetic field, linked with the inductor is given by the relation

$$P_3 = LI\left(\frac{dI}{dt}\right), \text{ where}$$

$$\frac{dI}{dt} = \frac{E}{L}e^{-\frac{Rt}{L}} = \frac{E}{L}\left(\frac{1}{e}\right) \quad \left\{\text{at } \left(t=\frac{L}{R}\right)\right\}$$

$$\Rightarrow \quad P_3 = (LI)\left(\frac{E}{L}\right)\left(\frac{1}{e}\right)$$

$$\Rightarrow \quad P_3 = \frac{EI}{e} = \frac{(5)(0.316)}{2.718} = 0.582 \text{ W} \quad \ldots(4)$$

From equations (1), (2), (3) and (4) we observe that

$$P_1 = P_2 + P_3$$

It is same as required by the Principle of Conservation of Energy.

Test Your Concepts-VII (Based on Magnetic Energy and Magnetic Energy Density)

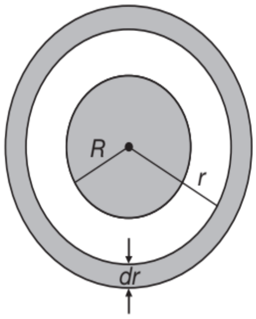
1. We have, $I = 200$ A and $U = 1 \text{ kWh} = 3.6\times10^6$ J

Since, $L = \frac{2U}{I^2}$ $\quad \left\{\because U = \frac{1}{2}LI^2\right\}$

$$\Rightarrow \quad L = \frac{2(3.6\times10^6)}{(200)^2} = 180 \text{ H}$$

2. Since $u = u_m = \frac{B^2}{2\mu_0}$

$$\Rightarrow \quad u = \left(\frac{B_0^2}{2\mu_0}\right)\left(\frac{R}{r}\right)^4$$



So, the magnetic energy in a spherical shell of radius r thickness dr is given by

$$dU = udV, \text{ where } dV = 4\pi r^2 dr$$

$$\Rightarrow \quad dU = \left(\frac{B_0^2R^4}{2\mu_0}\right)\frac{4\pi r^2 dr}{r^4}$$

$$U = \int dU = \frac{2\pi B_0^2R^4}{\mu_0}\int_R^\infty r^{-2}dr$$

$$\Rightarrow \quad U = \frac{2\pi B_0^2R^3}{\mu_0} \quad \left\{\because \int_R^\infty r^{-2}dr = \frac{1}{R}\right\}$$

3. From Ampere's Circuital Law, we have

$$B(2\pi r) = \mu_0(I_{\text{enc}})$$

$$\Rightarrow \quad B(2\pi r) = \mu_0 I$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$

If dU is the energy associated with the cylindrical element, then

$$dU = u_m dV$$

$$\Rightarrow \quad dU = u_m(2\pi r\ell dr)$$

where $u_m = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2 = \frac{\mu_0 I^2}{8\pi^2 r^2}$

$$\Rightarrow \quad dU = u_m(2\pi r\ell dr) = \frac{\mu_0 I^2}{8\pi^2 r^2}(2\pi r\ell dr) = \left(\frac{\mu_0 I^2 l}{4\pi}\right)\frac{dr}{r}$$

$$\Rightarrow \quad U = \int dU = \frac{\mu_0 I^2 l}{4\pi}\int_R^{2R}\frac{dr}{r} = \frac{\mu_0 I^2 l}{4\pi}\ln r\Big|_R^{2R} = \left(\frac{\mu_0 I^2 l}{4\pi}\right)\ln 2$$

$$\Rightarrow \quad \frac{U}{\ell} = \frac{\mu_0 I^2}{4\pi}\ln 2$$

So, we observe that the magnetic energy per unit length within the wire is independent of its radius.

4. $u_e = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}(8.85\times10^{-12})(1000)^2$

$$\Rightarrow \quad u_e = 44.25 \text{ nJm}^{-3}$$

$$u_m = \frac{B^2}{2\mu_0} = \frac{(50\times10^{-6})^2}{8\pi\times10^{-7}} = 995\ \mu\text{Jm}^{-3}$$

For $u_e = u_m$

$$\frac{1}{2}\varepsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

$$\Rightarrow \quad \frac{E}{B} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c = 3\times10^8 \text{ ms}^{-1}$$

Yes, this ratio happens to be the velocity of light.

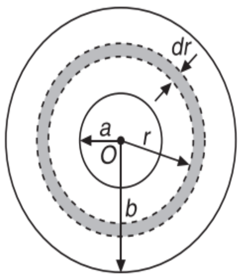
5. The magnetic field B in the space between the two conductors is given by

$$B = \frac{\mu_0 i}{2\pi r}$$

The magnetic energy density in the space between the conductors is

$$u_m = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 i}{2\pi r}\right)^2 = \frac{\mu_0 i^2}{8\pi^2 r^2}$$

The cross-sectional view of the coaxial cable described in the problem is shown in Figure.



Consider a volume element dV in the form of a cylindrical shell of radii r and $(r+dr)$. Energy stored in this elemental volume is

$$dU = u_m dV = \left(\frac{\mu_0 i^2}{8\pi^2 r^2}\right)2\pi r l dr$$

$$\Rightarrow \quad dU = \frac{\mu_0 i^2 l}{4\pi}\left(\frac{dr}{r}\right)$$

$$\Rightarrow \quad U = \int dU = \frac{\mu_0 i^2 l}{4\pi}\int_a^b \frac{dr}{r}$$

$$\Rightarrow \quad U = \frac{\mu_0 i^2 l}{4\pi}\ln\left(\frac{b}{a}\right)$$

6. (a) $|\xi| = L\frac{dI}{dt}$

$$\Rightarrow \quad 10 = L(16) \qquad \left\{\because \frac{dI}{dt} = 16\right\}$$

$$\Rightarrow \quad L = 6.25\times10^{-4}\text{ H}$$

(b) $E = \frac{1}{2}LI^2$

$$\Rightarrow \quad E = \frac{1}{2}(6.25\times10^{-4})(5+16t)^2$$

$$\Rightarrow \quad E|_{\text{at } t=1\text{ s}} = \frac{1}{2}(6.25\times10^{-4})(21)^2 \cong 138 \text{ mJ}$$

Since $P = VI = (10\times10^{-3})(5+16t)$

$$\Rightarrow \quad P|_{\text{at } t=1\text{ s}} = (10\times10^{-3})(21)$$

$$\Rightarrow \quad P|_{\text{at } t=1\text{ s}} = 210 \text{ mW}$$

7. (a) Since $\xi = -L\frac{dI}{dt} = -Lk$

(b) $I = \frac{dQ}{dt}$

$$\Rightarrow \quad Q = \int_0^t I dt = \int_0^t k t dt = \frac{kt^2}{2}$$

Since $V = V_C = \frac{Q}{C}$

$$\Rightarrow \quad |V| = \frac{kt^2}{2C}$$

(c) $\frac{1}{2}CV^2 \geq \frac{1}{2}LI^2$

$$\Rightarrow \quad \frac{1}{2}C\left(\frac{k^2t^4}{4C^2}\right) \geq \frac{1}{2}L(k^2t^2)$$

$\Rightarrow \quad t \geq 2\sqrt{LC}$

$\Rightarrow \quad t_{MIN} = 2\sqrt{LC}$

8. $U = \frac{1}{2}LI^2 \Rightarrow U \propto I^2$

U will reach $\frac{1}{4}th$ of its maximum value when current reaches half of its maximum value. In series LR circuit, equation of current growth is

$$I = I_o\left(1 - e^{-t/\tau_L}\right)$$

where, $I_o =$ Maximum value of current $= \frac{E}{R}$ and $= \frac{10\text{ H}}{2\ \Omega} = 5\text{ s}^{-1}$

$\Rightarrow \quad I = \frac{I_0}{2} = I_0\left(1 - e^{-t/5}\right)$

$\Rightarrow \quad \frac{1}{2} = 1 - e^{-t/5}$

$\Rightarrow \quad e^{-t/5} = \frac{1}{2}$

$\Rightarrow \quad -\frac{t}{5} = \ln\left(\frac{1}{2}\right)$

$\Rightarrow \quad \frac{t}{5} = \ln(2) = 0.693$

$\Rightarrow \quad t = (5)(0.693)\ s$

$\Rightarrow \quad t = 3.465\ s$

Test Your Concepts-VIII (Based on Faraday's Laws: Mutual Induction)

1. (a) $\phi_2 = MI_1$

$\Rightarrow \quad (6 \times 10^{-3})(1000) = M(3)$

$\Rightarrow \quad M = 2\text{ H}$

(b) $\xi_2 = -M\frac{dI_1}{dt}$

$\Rightarrow \quad \xi_2 = -(2)\left(\frac{0\text{-}3}{0.2}\right)$

$\Rightarrow \quad \xi_2 = 30\text{ V}$

(c) $\phi_1 = LI_1$

$\Rightarrow \quad (600)(5 \times 10^{-3}) = L(3)$

$\Rightarrow \quad L = 1\text{ H}$

2. Since $\xi = M\left(\frac{\Delta I}{\Delta t}\right)$

$\Rightarrow \quad 50 \times 10^{-3} = M\left(\frac{12-4}{0.5}\right)$

$\Rightarrow \quad M = \frac{25 \times 10^{-3}}{8} = 3.125\text{ mH}$

$$\xi_2 = (3.125 \times 10^{-3})\left(\frac{9-3}{0.02}\right) = 937.5\text{ mV}$$

3. The magnetic induction B at the location of coil P due to coil Q is

$$B = \frac{\mu_0 N_Q i}{2R}$$

Flux linked with the coil P is

$$\phi = N_P BA = N_P B(\pi r^2)$$

$\Rightarrow \quad \phi = \left(\frac{\mu_0 \pi r^2 N_Q N_P}{2R}\right)i$

Mutual inductance between the two coils is

$$M = \frac{\phi}{i} = \frac{\mu_0 \pi r^2 N_Q N_P}{2R}$$

$\Rightarrow \quad M = \frac{(4\pi \times 10^{-7})\pi(2 \times 10^{-2})^2(100)(1000)}{2(0.2)}$

$\Rightarrow \quad M \approx 4 \times 10^{-4}\text{ H}$

The emf induced in the coil P is

$$\xi = M\frac{di}{dt}$$

$\Rightarrow \quad \xi = (4 \times 10^{-4})\left(\frac{5-3}{0.04}\right)$

$\Rightarrow \quad \xi = 0.02\text{ V} = 20\text{ mV}$

Rate of change of flux through coil P is given by

$$\frac{d\phi}{dt} = \xi = 20 \times 10^{-3}\text{ Wbs}^{-1}$$

Charge passing through coil P is given by

$$q = \frac{\Delta\phi}{R}$$

The total change in flux in the coil in this time interval is

$$\Delta\phi = \xi\Delta t = (0.02)(0.04) = 8 \times 10^{-4}\text{ Wb}$$

$\Rightarrow \quad q = \frac{\Delta\phi}{R} = \frac{8 \times 10^{-4}}{8} = 10^{-4}\text{ C}$

$\Rightarrow \quad q = 10^{-4}\text{ C} = 100\ \mu\text{C}$

4. Field produced by the larger coil at the centre of the smaller coil is

$$B_1 = \frac{\mu_0 N_1 I_1 R_1^2}{2\left(x^2 + R_1^2\right)^{3/2}}$$

This field B_1 is normal to area of coil 2 and is nearly uniform over this area. So, it produces a flux,

$$\phi_{12} = N_2 B_1 A_2 = \frac{\mu_0 N_1 N_2 I_1 R_1^2\left(\pi R_2^2\right)}{2\left(x^2 + R_1^2\right)^{3/2}}$$

When I_1 varies, then

$$\xi_2 = -\frac{d\phi_{12}}{dt} = -\frac{\mu_0 \pi (N_1 N_2)(R_1 R_2)^2}{2\left(x^2 + R_1^2\right)^{3/2}}\left(\frac{dI_1}{dt}\right) = -M\frac{dI_1}{dt}$$

$$\Rightarrow \quad M = \frac{\mu_0 \pi (N_1 N_2)(R_1 R_2)^2}{2\left(x^2 + R_1^2\right)^{3/2}}$$

5. The magnetic field at any point inside the straight solenoid of primary with n_1 turns per unit length carrying a current I_1 is given by the relation,

$$B = \mu_0 n_1 I_1$$

The magnetic flux through the secondary of N_2 turns each of area A is given by,

$$N_2 \phi_2 = N_2 (BA) = \mu_0 n_1 N_2 I_1 A$$

$$\Rightarrow \quad M = \frac{N_2 \phi_2}{I_1} = \mu_0 n_1 N_2 A$$

Substituting the values, we get

$$M = \left(4\pi \times 10^{-7}\right)\left(\frac{50}{10^{-2}}\right)(200)\left(4 \times 10^{-4}\right)$$

$$\Rightarrow \quad M = 5 \times 10^{-4}\ \text{H}$$

6. The magnetic field at the location of one loop due to the other loop carrying a current i in it, considering it as a magnetic dipole is

$$B = \frac{\mu_0 i a^2}{2\left(a^2 + l^2\right)^{3/2}}$$

Since, $l \gg a$ so we have $\left(a^2 + l^2\right)^{3/2} \approx l^3$

$$\Rightarrow \quad B = \frac{\mu_0 i a^2}{2l^3}$$

Flux associated with the first loop is

$$\phi = B\left(\pi a^2\right)$$

$$\Rightarrow \quad \phi = \left(\frac{\mu_0 i a^2}{2l^3}\right)\left(\pi a^2\right)$$

$$\Rightarrow \quad \phi = \left(\frac{\mu_0 \pi a^4}{2l^3}\right) i$$

If M is the mutual inductance between the two loops, then we have

$$M = \frac{\phi}{i} = \frac{\mu_0 \pi a^4}{2l^3}$$

7. (a) $L_1 = \mu_0 n^2 A \ell_1 = \dfrac{\mu_0 N^2 A}{\ell_1}$

$$\Rightarrow \quad L_1 = \frac{\left(4\pi \times 10^{-7}\right)(1000)^2\left(10^{-4}\right)}{0.5} = 251\ \mu\text{H}$$

(b) $M = \dfrac{N_2 \phi_2}{I_1} = \dfrac{N_2 \phi_1}{I_1} = \dfrac{N_2 BA}{I_1} = \dfrac{\mu_0 N_1 N_2 A}{\ell_1}$

$$\Rightarrow \quad M = \frac{\left(4\pi \times 10^{-7}\right)(1000)(100)\left(10^{-4}\right)}{0.5}$$

$$\Rightarrow \quad M = 25.1\ \mu\text{H}$$

(c) $\xi_1 = -M\dfrac{dI_2}{dt}$

$$\Rightarrow \quad I_1 R_1 = -M\frac{dI_2}{dt}$$

$$\Rightarrow \quad \frac{dq_1}{dt} = -\frac{M}{R_1}\frac{dI_2}{dt}$$

$$\Rightarrow \quad q_1 = -\frac{M}{R_1}\int_1^0 dI_2$$

$$\Rightarrow \quad q_1 = \left(\frac{M}{R_1}\right)(1) = \frac{\left(2.51 \times 10^{-5}\right)}{1000}$$

$$\Rightarrow \quad q_1 = 25.1\ \text{nC}$$

8. The magnetic field B inside the solenoid is

$$B = \mu_0 n_s i_s$$

The magnetic flux linked with the coaxial coil placed inside the solenoid is

$$\phi_c = N_c B A_c = N_c(\mu_0 n_s i_s)A_c$$

where A_c is the cross sectional area of coil.

The mutual inductance between solenoid and coil is

$$M = \frac{\phi_c}{i_s} = \mu_0 n_s N_c A_c$$

Substituting the given values, we get

$$M = (4\pi \times 10^{-7})(800)(100)(\pi \times 10^{-4})\text{ H}$$

$$\Rightarrow \quad M = 3.2 \times 10^{-5}\text{ H}$$

9. (a) $\phi(t) = BA\cos\omega t = \left(\frac{\mu_0 I}{2b}\right)(\pi a^2)\cos(\omega t)$

$$\Rightarrow \quad \phi(t) = \frac{\mu_0 \pi a^2 I}{2b}\cos(\omega t)$$

$$\xi = \left|-\frac{d\phi}{dt}\right| = \frac{\mu_0 \omega \pi a^2 I}{2b}\sin(\omega t)$$

$$\Rightarrow \quad i = \frac{\xi}{R} = \frac{\mu_0 \omega \pi a^2 I}{2bR}\sin(\omega t)$$

(b) $\tau = MB\sin(\omega t) = (\pi a^2 i)\left(\frac{\mu_0 I}{2b}\right)\sin(\omega t)$

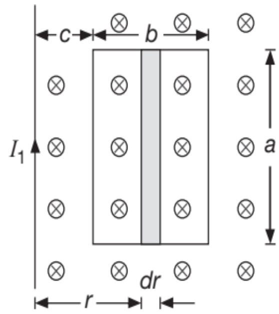
$$\Rightarrow \quad \tau = \frac{\mu_0^2 \omega \pi^2 a^4 I^2}{4b^2 R}\sin^2(\omega t)$$

(c) Induced emf in larger loop,

$$|\xi| = \left|-M\frac{di}{dt}\right| = \left(\frac{\mu_0 \pi a^2}{2b}\right)\left(\frac{\mu_0 \omega^2 \pi a^2 I}{2b}\right)\cos(\omega t)$$

$$\Rightarrow \quad |\xi| = \frac{\mu_0^2 \pi^2 a^4 \omega^2 I}{4b^2}\cos(\omega t)$$

10. The magnetic field due to the straight wire has magnitude $B_1 = \frac{\mu_0 I_1}{2\pi r}$ at a distance r. In accordance with right hand rule B_1 points inward to the plane of page. We consider a differential strip of thickness dr, area $dA_2 = a\,dr$. Magnetic flux through area dA, $d\phi_B = B_1(a\,dr)$.



Total flux through the loop,

$$\phi_{B_{12}} = \int B_1 dA_2 = \int_c^{c+b} \frac{\mu_0 I_1}{2\pi r}a\,dr$$

$$\phi_{B_{12}} = \frac{\mu_0 I_1 a}{2\pi}\int_c^{c+b}\frac{dr}{r} = \frac{\mu_0 I_1 a}{2\pi}\log_e\left(\frac{c+b}{c}\right)$$

Therefore, the mutual inductance,

$$M = M_{12} = \frac{\phi_{12}}{I_1} = \frac{\mu_0 a}{2\pi}\log_e\left(1+\frac{b}{c}\right)$$

11. When the given inductor is cut in identical parts then each part will have an inductance $\frac{L}{3}$. When two such parts are connected in series then equivalent self-inductance of the combination is given as

$$L_s = L_1 + L_2$$

$$\Rightarrow \quad L_s = \frac{L}{3} + \frac{L}{3} = \frac{2L}{3}$$

When connected in parallel, then

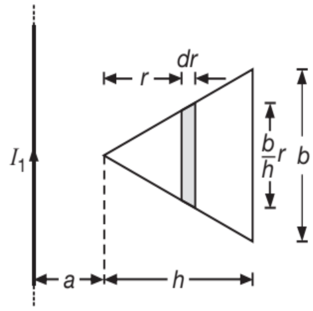
$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$\Rightarrow \quad \frac{1}{L_p} = \frac{1}{\frac{L}{3}} + \frac{1}{\frac{L}{3}}$$

$$\Rightarrow \quad L_p = \frac{L}{6}$$

12. Consider a differential strip of area dA, then from property of similar triangles length of the strip is $\frac{br}{h}$.

$$\Rightarrow \quad dA = \left(\frac{b}{h}r\right)dr$$



Let I_1 be the current in wire then, its magnetic field on the strip is

$$B = \frac{\mu_0 I}{2\pi(a+r)}$$

If $d\phi_{B_{12}}$ is the flux associated with the strip, then

$$d\phi_{B_{12}} = B_1 dA = \frac{\mu_0 I_1}{2\pi h}\frac{br\,dr}{a+r}$$

$$\Rightarrow \quad \phi_{B_{12}} = \frac{\mu_0 Ib}{2\pi h}\int_0^h \left[\frac{(a+r)-a}{(a+r)}\right]dr$$

$$\Rightarrow \quad \phi_{B_{12}} = \frac{\mu_0 Ib}{2\pi h}\int_0^h \left[1-\frac{a}{a+r}\right]dr$$

$$\Rightarrow \quad \phi_{B_{12}} = \frac{\mu_0 Ib}{2\pi h}\left[h - a\log_e\left(\frac{a+h}{a}\right)\right]$$

Therefore, mutual inductance M_{12} is

$$M_{12} = \frac{\phi_{B_{12}}}{I_1} = \frac{\mu_0 b}{2\pi h}\left[h - a\log_e\left(\frac{a+h}{a}\right)\right]$$

Test Your Concepts-IX (Based on LC Oscillations)

1. (a) $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.1)(10^{-6})}} \cong 503$ Hz

(b) $Q = CE = (10^{-6})(12) = 12\ \mu$C

(c) $\frac{1}{2}CE^2 = \frac{1}{2}LI_{max}^2$

$$\Rightarrow \quad I_{max} = E\sqrt{\frac{C}{L}} = 12\sqrt{\frac{10^{-6}}{0.1}} = 38 \text{ mA}$$

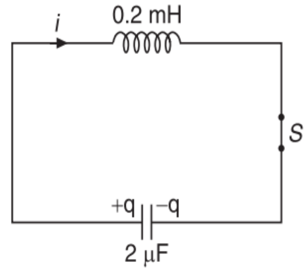
(d) $U = \frac{1}{2}CE^2 = \frac{1}{2}(10^{-6})(144) = 72\ \mu$J

2. $\frac{1}{2}CV^2 = \frac{1}{2}LI_0^2$

$$\Rightarrow \quad V = \sqrt{\frac{L}{C}}I_0 = \left(\sqrt{\frac{20\times10^{-3}}{0.5\times10^{-6}}}\right)(0.1)$$

$$\Rightarrow \quad V = 20 \text{ V}$$

3. When switch is closed, current flows in the circuit as shown in Figure.



As per the condition given in the problem, we have

$$U_L = \frac{1}{3}U_C$$

If Q_0 be the initial charge on the capacitor, then, the total energy of system is given by

$$U_L + U_C = \frac{Q_0^2}{2C} \quad \ldots(1)$$

At any instant, let q be the charge on the capacitor, then we have

$$U_C = \frac{q^2}{2C}$$

So, equation (1) becomes

$$\frac{1}{3}U_C + U_C = \frac{Q_0^2}{2C}$$

$$\Rightarrow \quad \frac{4}{3}\frac{q^2}{2C} = \frac{Q_0^2}{2C}$$

$$\Rightarrow \quad q = \frac{\sqrt{3}}{2}Q_0$$

For LC oscillations, the angular frequency is given by

$$\omega = \frac{1}{\sqrt{LC}}$$

Charge on the capacitor as a function of time is given by

$$q = Q_0 \cos\omega t$$

$$\Rightarrow \frac{\sqrt{3}}{2}Q_0 = Q_0 \cos\left(\frac{t}{\sqrt{LC}}\right)$$

$$\Rightarrow \frac{t}{\sqrt{LC}} = \frac{\pi}{6}$$

$$\Rightarrow t = \frac{\pi}{6}\sqrt{LC} = \frac{\pi}{6}\sqrt{0.2\times10^{-3}\times2\times10^{-6}}$$

$$\Rightarrow t = \frac{\pi}{6}\times2\times10^{-5} = 10.5\ \mu s$$

4. Initially, the current in the circuit is $I_0 = \frac{E}{R}$. When the switch is shifted to B, then by Law of Conservation of Energy, the magnetic energy stored in the conductor gets converted to the electrostatic energy in the capacitor. So, we have

$$\frac{1}{2}LI_0^2 = \frac{Q_0^2}{2C}$$

$$\Rightarrow \frac{1}{2}L\left(\frac{E}{R}\right)^2 = \frac{Q_0^2}{2C}$$

$$\Rightarrow Q_0 = \frac{E\sqrt{LC}}{R}$$

$$\Rightarrow I_{max} = V\sqrt{\frac{C}{2L}}$$

5. (a) When current is zero, charge and hence potential across capacitor is maximum. So,

$$V_0 = \frac{q_0}{C} = \frac{5\times10^{-6}}{400\times10^{-6}}$$

$$\Rightarrow V_0 = 12.5\text{ mV}$$

(b) $\frac{1}{2}LI_0^2 = \frac{1}{2}CV_0^2$

$$\Rightarrow I_0 = V_0\sqrt{\frac{C}{L}} = 1.25\times10^{-2}\sqrt{\frac{400\times10^{-6}}{90\times10^{-3}}}$$

$$\Rightarrow I_0 = 8.33\times10^{-4}\text{ A}$$

(c) $U_{max} = \frac{1}{2}LI_0^2 = \frac{1}{2}CV_0^2$

$$\Rightarrow U_{max} = \frac{1}{2}\times(400\times10^{-6})\times(12.5\times10^{-3})^2$$

$$\Rightarrow U_{max} = 3.125\times10^{-8}\text{ J}$$

(d) $U_{max} = \frac{1}{2}L\left(\frac{I_0}{2}\right)^2 + \frac{Q^2}{2C}$

$$3.125\times10^{-8} = \left(\frac{0.09}{2}\right)\left(\frac{8.33\times10^{-4}}{2}\right)^2 + \frac{Q^2}{2(4\times10^{-4})}$$

$$\Rightarrow Q = 4.33\times10^{-6}\text{ C}$$

$$\Rightarrow U = \frac{1}{2}LI^2 = 7.8\times10^{-7}\text{ J}$$

6. (a) Since both capacitors are in parallel, so

$$f = \frac{1}{2\pi\sqrt{L(C_1+C_2)}}$$

$$\Rightarrow f = \frac{1}{2\pi\sqrt{(0.5\times10^{-3})(5\times10^{-6})}}$$

$$\Rightarrow f = \frac{1}{(2\pi)(5\times10^{-5})} = \frac{10^4}{\pi}\text{ Hz}$$

(b) $\frac{1}{2}LI_0^2 = \frac{1}{2}(C_1+C_2)V^2$

$$\Rightarrow \frac{1}{2}(0.5\times10^{-3})I_0^2 = \frac{1}{2}(5\times10^{-6})(200)^2$$

$$\Rightarrow I_0^2 = \frac{(5\times10^{-6})(4\times10^4)}{(5\times10^{-4})}$$

$$\Rightarrow I_0^2 = 4\times10^2$$

$$\Rightarrow I_0 = 20\text{ A}$$

7. Since, $E = \frac{Q'^2}{2C} + \frac{1}{2}LI^2$

$$\Rightarrow \frac{Q^2}{2C} = \frac{\left(\frac{Q}{2}\right)^2}{2C} + \frac{1}{2}LI^2$$

$$\Rightarrow I = \frac{Q}{2}\sqrt{\frac{3}{LC}}$$

The flux through each turn of the coil is

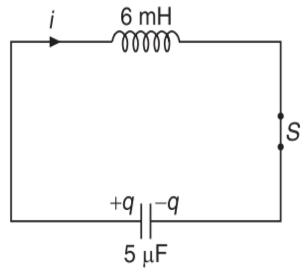
$$\phi = \frac{LI}{N}$$

$$\Rightarrow \phi = \frac{QL}{2N}\sqrt{\frac{3}{LC}}$$

$$\Rightarrow \phi = \frac{Q}{2N}\sqrt{\frac{3L}{C}}$$

8. At $t=0$ when switch is closed capacitor starts discharging through the inductor and current in circuit increases. When charge on capacitor is q and current in circuit is i by conservation of energy we have

$$\frac{q^2}{2C}+\frac{1}{2}Li^2=\frac{Q^2}{2C}$$



Current in circuit in terms of charge on capacitor is given as

$$i=\frac{1}{\sqrt{LC}}\sqrt{Q^2-q^2}=\frac{\sqrt{\left[(100)^2-(50)^2\right]\times10^{-12}}}{\sqrt{3\times10^{-8}}}$$

$$\Rightarrow \quad i=\frac{10^{-6}}{\sqrt{3}\times10^{-4}}\sqrt{7500}$$

$$\Rightarrow \quad i=0.5\text{ A}$$

Maximum energy is stored in inductor when

$$\frac{1}{2}LI_{\max}^2=\frac{Q^2}{2C}$$

$$\Rightarrow \quad I_{\max}=\frac{Q}{\sqrt{LC}}=\frac{100\times10^{-6}}{\sqrt{3\times10^{-8}}}=\frac{1}{\sqrt{3}}\text{ A}$$

9. At $t=0$, we have $Q=Q_{\max}=Q_0$. Since the charge on the capacitor at any time t is

$$Q=Q_0\cos(\omega t)\text{, where } \omega=\frac{1}{\sqrt{LC}}$$

Energy stored in the capacitor at time t is,

$$U=\frac{1}{2}\left(\frac{Q^2}{C}\right)=\frac{Q_0^2}{2C}\cos^2(\omega t)=U_0\cos^2(\omega t)$$

Since, $U=\dfrac{U_0}{4}$

$$\Rightarrow \quad \frac{1}{4}=\cos^2(\omega t)$$

$$\Rightarrow \quad \omega t=\frac{\pi}{3}$$

$$\Rightarrow \quad \frac{t}{\sqrt{LC}}=\frac{\pi}{3}$$

$$\Rightarrow \quad \frac{t^2}{LC}=\frac{\pi^2}{9}$$

$$\Rightarrow \quad L=\frac{9t^2}{\pi^2 C}$$

10. (a) $\omega=2\pi f$

Since $f=10^3$ cps $=10^3$ Hz, so

$$\omega=6.28\times10^3\text{ rads}^{-1}$$

$$T=\frac{1}{f}=10^{-3}\text{ s}$$

(b) $q=q_0\cos(\omega t)$

$$\Rightarrow \quad q=CV_0\cos(\omega t)$$

$$\Rightarrow \quad q=10^{-4}\cos(6.28\times10^3\text{t})$$

(c) $\omega^2=\dfrac{1}{LC}$

$$\Rightarrow \quad 4\pi^2f^2=\frac{1}{LC}$$

$$\Rightarrow \quad (40)(10^6)=\frac{1}{L(10^{-6})}$$

$$\Rightarrow \quad L=\frac{1}{40}=25\text{ mH}$$

(d) Since

$$I=\frac{dq}{dt}=-(10^{-4})(6.28\times10^3)\sin(6.28\times10^3\text{ t})$$

$$\Rightarrow \quad \langle I\rangle=(10^{-4})(6.28\times10^3)\frac{2}{\pi}$$

$$\left\{\because\ \langle\sin(\omega t)\rangle=\frac{2}{\pi}\right\}$$

$$\Rightarrow \quad \langle I\rangle=(10^{-4})(6.28\times10^3)\left(\frac{2}{3.14}\right)$$

$$\Rightarrow \quad \langle I\rangle=0.4\text{ A}$$

CHAPTER 3

11. If q_0 is the maximum charge on capacitor then by Law of Conservation of Energy, we have

$$\frac{1}{2}LI_0^2 = \frac{q_0^2}{2C}$$

$$\Rightarrow \quad q_0 = \sqrt{LC}I_0$$

Charge on capacitor as a function of time is given by

$$q = q_0 \sin(\omega t + \alpha)$$

where $\omega = \frac{1}{\sqrt{LC}}$

Since at $t = 0$ we have $q = 0$, so we get $\alpha = 0$.

Thus, charge on capacitor is given by

$$q = I_0\sqrt{LC}\sin\left(\frac{t}{\sqrt{LC}}\right)$$

12. The time period of oscillation is

$$T = 2\pi\sqrt{LC_{eq}}$$

where $C_{eq} = \frac{(C)(C)}{C+C} = \frac{C}{2}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{LC}{2}}$$

The initial charge on the first capacitor is $q_1 = CV$ and that on the second capacitor is $q_2 = 2CV$.

When these capacitors are connected along with an inductor, then due to the charge sharing between the capacitors there will be a loss in energy of the capacitor combination. This loss in energy of the capacitor combination is given by

$$\text{Loss} = -\Delta U = \frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)(V_1 - V_2)^2$$

$$\Rightarrow \quad \text{Loss} = -\Delta U = \frac{1}{2}\left(\frac{(C)(C)}{C+C}\right)(2V - V)^2$$

$$\Rightarrow \quad \text{Loss} = -\Delta U = \frac{1}{2}\left(\frac{C}{2}\right)(V)^2 = \frac{CV^2}{4}$$

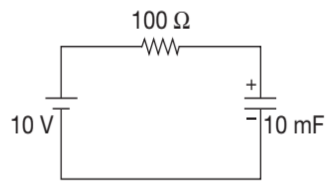
This loss in energy of the capacitor combination will be equal to the gain in magnetic energy in the inductor. So, we have

$$\frac{CV^2}{4} = \frac{1}{2}LI_{max}^2$$

$$\Rightarrow \quad I_{max} = V\sqrt{\frac{C}{2L}}$$

13. (a) Initially, charge on the capacitor is $Q_0 = CV_0$

$$\Rightarrow \quad Q_0 = (10 \times 10^{-3})(5\text{ V}) = 50\text{ mC}$$



When the capacitor is connected to position 1, then we get from Kirchhoff's Loop Law,

$$E - IR - \frac{q}{C} = 0$$

$$\Rightarrow \quad \int_0^t \frac{1}{RC}dt = \int_{Q_0}^{q} \frac{dq}{CE - q}$$

$$\frac{t}{RC} = -\log_e\left(\frac{CE - q}{CE - Q_0}\right)$$

$$\Rightarrow \quad CE - q = (CE - Q_0)e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad q = CE - (CE - Q_0)e^{-\frac{t}{RC}}$$

Since $CE = 100\text{ mC}$ and $Q_0 = 50\text{ mC}$

$$\Rightarrow \quad q = 100 - (100 - 50)e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad q = 100 - 50e^{-t/RC}$$

$$\Rightarrow \quad q = 50\left(2 - e^{-\frac{t}{RC}}\right)$$

At $t = 1$ s, we have

$$q = 50(2 - e^{-1})\text{ mC}$$

$$\Rightarrow \quad q = 81.5\text{ mC}$$

Voltage across capacitor at that time is

$$V = \frac{q}{C} = \frac{81.5\times10^{-3}}{10\times10^{-3}} = 8.15\text{ V}$$

(b) $\frac{1}{2}LI_{\max}^2 = \frac{1}{2}CV_{\max}^2$

$$\Rightarrow \quad I_{\max} = \left(\sqrt{\frac{C}{L}}\right)V_{\max}$$

$$\Rightarrow \quad I_{\max} = \left(\sqrt{\frac{10}{2.5}}\right)8.15 = 16.3\text{ A}$$

Frequency, $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1000}{2\pi\times25}$

$$\Rightarrow \quad f = \frac{10^3}{50\pi}\text{ Hz} = \frac{20}{\pi}\text{ Hz}$$

14. (a) By Law of Conservation of energy, we have

$$\frac{1}{2}Li_0^2 = \frac{1}{2}CV_0^2$$

$$\Rightarrow \quad L = \frac{CV_0^2}{i_0^2}$$

$$\Rightarrow \quad L = \frac{(4\times10^{-6})(1.5)}{(50\times10^{-3})^2}$$

$$\Rightarrow \quad L = 3.6\times10^{-3}\text{ H}$$

(b) Oscillation frequency of the LC circuit

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow \quad f = \frac{1}{2\pi\sqrt{(3.6\times10^{-3})(4\times10^{-6})}}$$

$$\Rightarrow \quad f = 0.133\times10^4\text{ Hz}$$

$$\Rightarrow \quad f = 1.33\text{ kHz}$$

(c) During oscillations, the time taken for charge to rise from zero to maximum is one fourth of the oscillation period which is given by

$$t = \frac{T}{4} = \frac{1}{4f}$$

$$\Rightarrow \quad t = \frac{1}{4\times1.33\times10^3}\text{ s}$$

$$\Rightarrow \quad t = 0.188\times10^{-3}\text{ s}$$

$$\Rightarrow \quad t = 0.188\text{ ms}$$

15. $q_1 = 8CV_0$ and $q_2 = CV_0$

$$I = -\frac{dq_1'}{dt} = \frac{dq_2'}{dt} = \frac{dq}{dt}$$

$$\Rightarrow \quad \frac{dI}{dt} = \frac{d^2q}{dt^2}$$

Applying loop law, we get

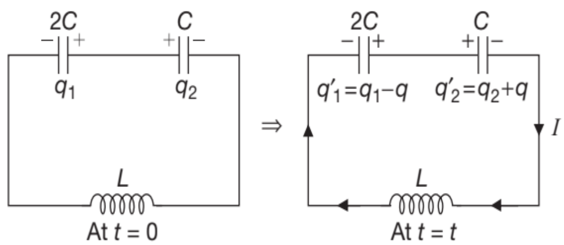
$$\frac{q_1'}{2C} - \frac{q_2'}{C} - L\frac{dI}{dt} = 0$$

$$\Rightarrow \quad \left(\frac{q_1 - q}{2C}\right) - \left(\frac{q_2 + q}{C}\right) - L\frac{d^2q}{dt} = 0$$

$$\Rightarrow \quad L\frac{d^2q}{dt^2} = \left(\frac{q_1 - 2q_2}{2C}\right) - \frac{3q}{2C}$$

The solution of this equation with the condition $q = 0$ at $t = 0$ is, $q = q_0(1 - \cos\omega t)$

where $\omega = \sqrt{\frac{3}{2LC}}$ and $q_0 = \frac{q_1 - 2q_2}{3} = 2CV_0$



(a) $I = \frac{dq}{dt} = q_0\omega\sin\omega t$

$$\Rightarrow \quad I_{\max} = q_0\omega \text{ when } \omega t = \frac{\pi}{2}$$

(b) At this instant $q = q_0$, so we get

$$V_{2C} = \frac{q_1'}{2C} = \frac{q_1 - q_0}{2C} = \frac{8CV_0 - 2CV_0}{2C} = 3V_0 \text{ and}$$

$$V_C = \frac{q_2'}{C} = \frac{q_2 + q_0}{C} = \frac{CV_0 + 2CV_0}{C} = 3V_0$$

(c) $I = q_0\omega\sin(\omega t)$

Single Correct Choice Type Questions

1. At the terminal speed, the net force on the loop equals zero. So, we have

$$F_B = mg$$

$$\Rightarrow \quad BI\ell = mg$$

$$\Rightarrow \quad \frac{B^2\ell^2 v_T}{R} = mg \qquad \left\{\because I = \frac{\xi}{R} = \frac{B\ell v}{R}\right\}$$

$$\Rightarrow \quad v_T = \frac{mgR}{B^2\ell^2}$$

Now, $m = \rho V = \rho(4\ell)\pi\left(\frac{d}{2}\right)^2 = \pi\rho\ell d^2$ and

$$R = \frac{\ell_{net}}{\sigma A} = \frac{4\ell}{\sigma\left(\frac{\pi d^2}{4}\right)} = \frac{16\ell}{\pi\sigma d^2}$$

$$\Rightarrow \quad v_T = \frac{(\pi\rho\ell d^2)g\left(\frac{16\ell}{\pi\sigma d^2}\right)}{B^2\ell^2}$$

$$\Rightarrow \quad v_T = \frac{16\rho g}{\sigma B^2}$$

Hence, the correct answer is (C).

2. $\phi = 6t^2 - 5t + 1$

$$\Rightarrow \quad \xi = -\frac{d\phi}{dt} = -(12t - 5)$$

At $t = 0.25$ sec, $\xi = 2$ V

$$\Rightarrow \quad I = \frac{\xi}{R} = \frac{2}{10} = 0.2 \text{ A}$$

Hence, the correct answer is (D).

3. $U = \frac{1}{2}Li^2$

$$\Rightarrow \quad [L] = \left[\frac{U}{i^2}\right] = \left[\frac{ML^2T^{-2}}{A^2}\right]$$

$$\Rightarrow \quad [L] = [ML^2T^{-2}A^{-2}]$$

Hence, the correct answer is (C).

4. The net flux associated with the loop is

$$\phi = k\pi(a^2 - b^2)$$

The induced current is

$$I = \frac{\xi}{R} = \frac{1}{R}\left|\frac{d\phi}{dt}\right| = \frac{k\pi(a^2 - b^2)}{2\pi(a+b)\lambda} = \frac{k(a-b)}{2\lambda}$$

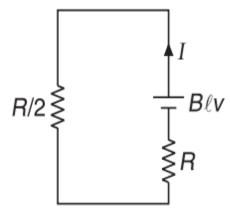
Hence, the correct answer is (C).

5. The voltage across inductance coil is

$$V_L = L\frac{di}{dt}$$

Hence, the correct answer is (C).

6. The wire PQ can be replaced with a battery of emf $\xi = Blv$ and internal resistance R. As the other two resistances are in parallel, their equivalent resistance is $\frac{R}{2}$.



$$\Rightarrow \quad I = \frac{Blv}{R + R/2} = \frac{2Blv}{3R}$$

Hence, the correct answer is (D).

7. $\frac{\text{Electric Flux}}{\text{Magnetic Flux}} = \frac{EA}{BA} = \frac{E}{B}$

$$\left[\frac{E}{B}\right] = [\text{Velocity}] = \left[\frac{1}{\sqrt{\mu_0\varepsilon_0}}\right]$$

Hence, the correct answer is (C).

8. At the instant the rod crosses point $(a, 0)$, its effective length is $l = 2\sqrt{4a \times a} = 4a$

The induced emf in the rod at this instant is

$$\xi = Blv = 4Bav$$

Hence, the correct answer is (D).

9. Value remains $\frac{1}{4}$th in 20 ms time. Hence, two half-lives are equal to 20 ms. So, one half-life is 10 ms

$$\Rightarrow \quad t_{1/2} = (\ln 2)\tau_C = (\ln 2)\frac{L}{R}$$

$$\Rightarrow \quad R = \frac{(\ln 2)L}{t_{1/2}}$$

$$\Rightarrow \quad R = \frac{(\ln 2)(2)}{10 \times 10^{-3}} = (100\ln 4)\ \Omega$$

Hence, the correct answer is (C).

10. The peak value of induced emf is

$$\xi_0 = NBA\omega = 30 \times 1 \times (400 \times 10^{-4}) \times \left(\frac{1800 \times 2\pi}{60}\right)$$

$$\Rightarrow \quad \xi_0 = 226 \text{ V}$$

Hence, the correct answer is (B).

11. Induced electric field at a distance R from centre is

$$E = \frac{R}{2}\left(\frac{dB}{dt}\right)$$

Since $B(t) = B_0 + \alpha t$

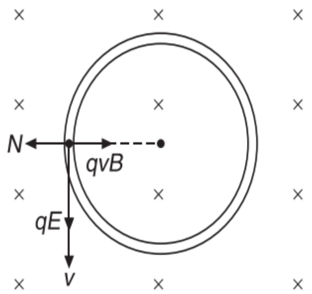
$$\Rightarrow \quad \frac{dB}{dt} = \alpha$$

$$\Rightarrow \quad E = \frac{R}{2}\alpha$$

Speed attained by the bead is

$$v = at \text{ , where } a = \frac{qE}{m}$$

$$\Rightarrow \quad v = \left(\frac{qE}{m}\right)t$$



For circular motion of bead, we have

$$qvB - N = \frac{mv^2}{R}$$

$$\Rightarrow \quad N = \frac{\alpha q^2 Rt}{4m}(2B_0 + \alpha t)$$

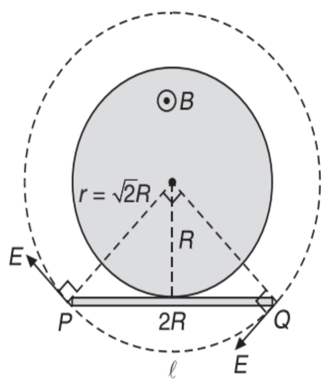
Hence, the correct answer is (B).

12. Consider a circle of radius r passing through points P and Q. The electric field at P and Q is given by

$$E(2\pi r) = A\frac{dB}{dt}$$

$$\Rightarrow \quad E\left[2\pi\left(\sqrt{2}R\right)\right] = \pi R^2 \frac{dB}{dt} = \pi\alpha R^2$$

$$\Rightarrow \quad E = \frac{\alpha R}{2\sqrt{2}}$$



Therefore, emf across rod is

$$\xi = El$$

where $l = r\theta$

$$\Rightarrow \quad l = \sqrt{2}R\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \quad \xi = \frac{\alpha R}{2\sqrt{2}} \times \frac{\pi\left(\sqrt{2}R\right)}{2} = \frac{\alpha\pi R^2}{4}$$

Hence, the correct answer is (D).

13. Since, $I = (10t + 5)$ A

$$\Rightarrow \quad \frac{dI}{dt} = 10 \text{ As}^{-1} = \text{constant}$$

At, $t = 0$, $I = 5$ A

Applying KLL, we get

$$V_A - 3 \times 5 - 1 \times 10 + 10 = V_B$$

$$\Rightarrow \quad V_A - V_B = 15 \text{ V}$$

Hence, the correct answer is (A).

15. The mutual inductance due to smaller ring carrying current I is same as the mutual inductance due to larger ring. If the larger ring carries current I, then the field at the location of smaller ring is

$$B = \frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{\frac{3}{2}}}$$

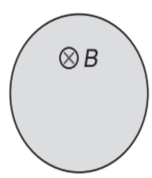
The flux associated with the smaller ring is

$$\phi = B\left(\pi r^2\right) = \frac{\mu_0 \pi R^2 r^2}{2\left(R^2 + x^2\right)^{\frac{3}{2}}} I$$

$$\Rightarrow \quad M = \frac{\phi}{I} = \frac{\mu_0 \pi R^2 r^2}{2\left(R^2 + x^2\right)^{\frac{3}{2}}}$$

Hence, the correct answer is (D).

16. If magnetic field in the cylindrical region shown in Figure is changing, then induced electric field exists both inside and outside the cylindrical regions i.e. induced electric field also exists in the region where magnetic field does not exist.



Hence, the correct answer is (C).

CHAPTER 3

17. $q = q_0 \cos(\omega_0 t)$

$$\Rightarrow \quad I = \frac{dq}{dt} = -q_0\omega_0 \sin(\omega_0 t)$$

$$\Rightarrow \quad \frac{dI}{dt} = -q_0\omega_0^2 \cos(\omega_0 t)$$

$$\Rightarrow \quad \left(\frac{dI}{dt}\right)_{max} = q_0\omega_0^2 = \frac{q_0}{LC}$$

We can also think that since,

$$(V_C)_{max} = (V_L)_{max}$$

$$\Rightarrow \quad \frac{q_0}{C} = L\left(\frac{dI}{dt}\right)_{max}$$

$$\Rightarrow \quad \left(\frac{dI}{dt}\right)_{max} = \frac{q_0}{LC}$$

Hence, the correct answer is (A).

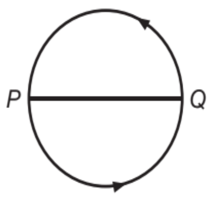
18. For wire ab, velocity vector $\vec{v}$ is parallel to length $\vec{l}$, hence $\xi = 0$

Hence, the correct answer is (C).

19. If the coil is rotated about an axis perpendicular to the plane of coil passing through O, there will be no change in flux and hence no emf will be induced.

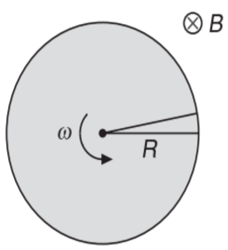
Hence, the correct answer is (D).

20. Since the inward field is decreasing at a constant rate, so the induced current must be set up in the loop so that if does not allow the inward field to decrease. Hence the induced current must set up an outward field. So, the induced current must flow in the loop in the Counter-Clockwise sense. Due to this fact no current flows through the branch PQ.



Hence, the correct answer is (D).

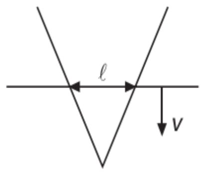
21. Consider a thin section of the disc shown in Figure.



The emf induced across it is $\xi = \frac{BR^2\omega}{2}$. Same emf is induced across all such sections. So, $\xi = \frac{BR^2\omega}{2}$ is the induced emf between the rim and the axis.

Hence, the correct answer is (D).

22. $i = \frac{Bvl}{R}$



Let λ be the resistance per unit length of conducting rod, then

$$i = \frac{Bvl}{\lambda l} = \frac{Bv}{\lambda} = \text{constant}$$

Hence, the correct answer is (C).

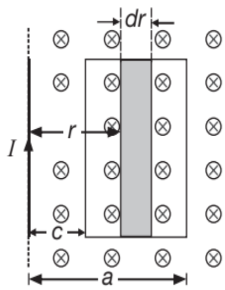
23. The effective length of a straight rod joining points P and Q is $l = 2L\sin\left(\frac{\theta}{2}\right)$

The emf induced across it is

$$\xi = BlV = 2BLV\sin\left(\frac{\theta}{2}\right)$$

Hence, the correct answer is (C).

24. $B = \frac{\mu_0 I}{2\pi r}$



Since $\phi_m = \int B dA$

$$\Rightarrow \quad \phi_m = \frac{\mu_0 I}{2\pi}\int\frac{dA}{r}$$

$$\Rightarrow \quad \phi_m = \frac{\mu_0 I}{2\pi} b \int_c^{a+c}\frac{dr}{r}$$

$$\Rightarrow \quad \phi_m = \frac{\mu_0 Ib}{2\pi}\ell n\left(\frac{a+c}{c}\right)$$

Hence, the correct answer is (C).

25. The force acting on the rod is $F = BIL$ towards left. If d is the distance moved by the rod, then by Work-Energy Theorem, we have

$$W_F = \Delta KE$$

$$\Rightarrow \quad -Fd = 0 - \frac{1}{2}mV_0^2$$

$$\Rightarrow \quad d = \frac{mV_0^2}{2F} = \frac{mV_0^2}{2BIL}$$

Hence, the correct answer is (B).

26. Imagine the rod OQ of length $2l$ with P as its centre.

Then, $V_Q - V_O = \frac{1}{2}B(2l)^2\omega$

and $V_P - V_O = \frac{1}{2}Bl^2\omega$

$$\Rightarrow \quad V_Q - V_P = \frac{1}{2}B(2l)^2\omega - \frac{1}{2}Bl^2\omega = \frac{3}{2}Bl^2\omega$$

Hence, the correct answer is (C).

27. The current through inductor is

$$I_1 = \frac{20}{4+6}\left(1 - e^{-(4+6)t/0.005}\right) = 2\left(1 - e^{-2000t}\right)$$

The current through capacitor is

$$I_2 = \frac{20}{5+5}e^{-t/(5+5)(10^{-4})} = 2e^{-1000t}$$

The current through the key is

$$I = I_1 + I_2 = 2\left(1 - e^{-2000t}\right) + 2e^{-1000t}$$

At $t = 10^{-3}\ln 2$,

$$I = \frac{3}{2} + 1 = 2.5 \text{ A}$$

Hence, the correct answer is (C).

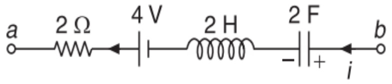
28. $i = \frac{dq}{dt} = (8t)$ A

$$\Rightarrow \quad \frac{di}{dt} = 8 \text{ As}^{-1}$$

At $t = 1$ s, $q = 4$ C, $i = 8$ A

and $\frac{di}{dt} = 8 \text{ As}^{-1}$

Since charge on capacitor is increasing, hence charge on positive plate is also increasing. So, the direction of current is towards left.



Applying KLL from a to b, we get

$$V_a + 2\times 8 - 4 + 2\times 8 + \frac{4}{2} = V_b$$

$$\Rightarrow \quad V_a - V_b = -30 \text{ V}$$

Hence, the correct answer is (B).

29. The change in flux between the initial and final position is $|\Delta\phi| = |0 - BA| = BA$

Time taken to turn through 90° is

$$\Delta t = \frac{T}{4} = \frac{1}{4}\times\frac{2\pi}{\omega} = \frac{\pi}{2\omega}$$

$$\Rightarrow \quad \xi_{av} = \left|-\frac{\Delta\phi}{\Delta t}\right| = \frac{BA}{\pi/2\omega} = \frac{2\omega BA}{\pi}$$

Hence, the correct answer is (D).

30. By short-circuiting the battery, net resistance across inductor is $\frac{R}{2}$, because R and R become parallel.

$$\Rightarrow \quad \tau_{\text{net}} = \frac{L}{R_{\text{net}}} = \frac{2L}{R}$$

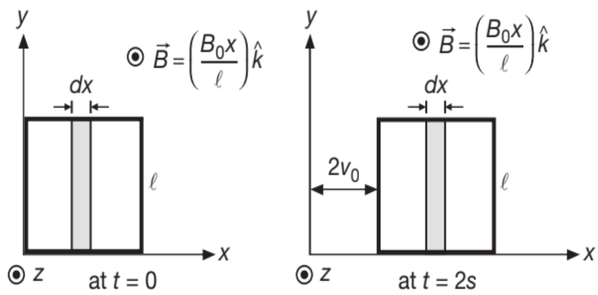
Hence, the correct answer is (B).

31. The motion of charged rod is equivalent to the current flowing along the rod. The field produced due to this current is in the plane of the loop. Hence, the induced emf is zero.

Hence, the correct answer is (D).

32. Total flux linked with the coil at time $t = 0$ is

$$\phi_i = \int_0^{\phi} d\phi = \int_0^{\ell}\left(\frac{B_0 x}{\ell}\right)(\ell\, dx) = \frac{B_0\ell^2}{2}$$



and flux linked with the coil at time $t = 2$ s is

$$\phi_f = \int_{2v_0}^{2v_0+\ell} d\phi$$

$$\Rightarrow \quad \phi_f = \int_{2v_0}^{2v_0+\ell} \frac{B_0 x}{\ell}\cdot(\ell dx)$$

$$\Rightarrow \quad \phi_f = \frac{B_0}{2}\left[(2v_0+\ell)^2 - (2v_0)^2\right]$$

$$\phi_f = \frac{B_0}{2}\left(4v_0^2 + \ell^2 + 4v_0\ell - 4v_0^2\right) = \frac{B_0\ell}{2}(\ell + 4v_0)$$

Since $\xi = \frac{\Delta\phi}{\Delta t} = \frac{1}{2}\left(\frac{B_0\ell^2}{2} + \frac{4B_0\ell v_0}{2} - \frac{B_0\ell^2}{2}\right)$

$$\Rightarrow \quad \xi = B_0\ell v_0$$

Hence, the correct answer is (A).

33. The flux associated with the semicircle is

$$\phi = BA\cos\theta = B\frac{\pi r^2}{2}\cos(\omega t)$$

$$\Rightarrow \quad \xi = -\frac{d\phi}{dt} = \frac{B\omega\pi r^2}{2}\sin(\omega t)$$

Since, $P = \frac{\xi^2}{R} = \frac{B^2\omega^2\pi^2 r^4}{4R}\sin^2\omega t$

$$\Rightarrow \quad P_{av} = \langle P \rangle = \frac{1}{T}\int_0^T \frac{B^2\omega^2\pi^2 r^4}{4R}\sin^2(\omega t)dt$$

$$\Rightarrow \quad P_{av} = \langle P \rangle = \frac{(B\pi r^2\omega)^2}{8R}$$

Hence, the correct answer is (B).

34. Since all i.e. $\vec{B}$, $\vec{l}$ and $\vec{v}$ are coplanar, so $\xi = 0$.

Hence, the correct answer is (A).

35. The magnitude of electric field at $(r, 0, 0)$ is

$$E_{\text{nc}} = \left|\frac{\pi a^2 \frac{dB}{dt}}{2\pi r}\right| = \frac{a^2 B_0}{2r}$$

The force on charge q is $F = qE_{\text{nc}} = \frac{qB_0a^2}{2r}$

Hence, the correct answer is (B).

36. $\Delta q = \frac{\Delta\phi}{R}$

$$\Rightarrow \quad i\Delta t = \frac{\Delta\phi}{R}$$

$$\Rightarrow \quad \Delta\phi = i(\Delta t)R = (10\times10^{-3})(5)(0.5)$$

$$\Rightarrow \quad \Delta\phi = 25\times10^{-3}\text{ Wb}$$

Hence, the correct answer is (B).

37. Let E be the emf of batteries and R be the resistances,

then, $i_1 = 0$, $i_2 = \frac{E}{R}$, $i_3 = \frac{E}{2R}$

$$\Rightarrow \quad i_2 > i_3 > i_1$$

Hence, the correct answer is (A).

38. Since, $E_{nc}(2\pi r) = A\left(\frac{dB}{dt}\right)$

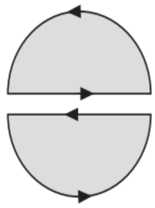
$$\Rightarrow \quad E_{nc}(2\pi r) = \pi r^2\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E_{nc} = \frac{r}{2}\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E \propto r$$

Hence, the correct answer is (B).

39. On breaking the loop in two parts as shown in Figure, we observe that equal and opposite currents flow through the diameter.



They cancel each other. So, no current flows through diameter.

Hence, the correct answer is (D).

40. $\oint \vec{E}\cdot d\vec{\ell} = \left|\frac{d\phi}{dt}\right|$

$$\Rightarrow \quad E(2\pi r) = \pi r^2\left(\frac{dB}{dt}\right)$$

$\Rightarrow \quad E = \frac{r}{2}\left(\frac{dB}{dt}\right)$, tangential to the ring

Now

$$\tau = Fr = qEr$$

$$\Rightarrow \quad \tau = q\left(\frac{r}{2}\frac{dB}{dt}\right)r$$

$$\Rightarrow \quad \tau = \frac{1}{2}qr^2\left(\frac{dB}{dt}\right)$$

Hence, the correct answer is (C).

41. There is no change in flux through the ring. Hence, emf induced is zero.

Hence, the correct answer is (A).

42. Since $\xi = \left|\frac{d\phi}{dt}\right| = a\tau - 2at$

$$\Rightarrow \quad i = \frac{\xi}{R} = \frac{a\tau - 2at}{R}$$

Heat generated in the loop is

$$H = \int_0^\tau i^2Rdt = \frac{a^2\tau^3}{3R}$$

Hence, the correct answer is (B).

43. Let the magnetic field be into the paper. Then, if the wire cd moves with velocity v, the current through it is $I = \frac{\xi}{R} = \frac{Blv}{R}$ towards right.

The magnetic force on it is

$$F = IlB = \frac{Blv}{R}lB \text{ upwards.}$$

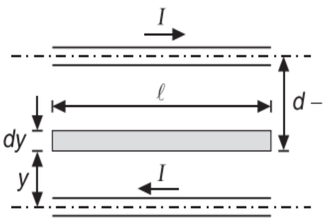
This magnetic force is balanced by the weight mg of rod acting downwards.

$$\Rightarrow \quad \frac{B^2l^2v}{R} = mg$$

$$\Rightarrow \quad v = \frac{mgR}{B^2l^2}$$

Hence, the correct answer is (B).

44. Let us calculate the coefficient of self-induction for this arrangement. For doing this, we need to find the flux between the wires. Let us consider a strip of width dy and length l as shown in Figure.



Magnetic field at the strip due to current in two wires is

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{y} + \frac{1}{d-y}\right) \otimes$$

Area of the strip, $dA = ldy$

Flux associated with the strip is

$$d\phi = \frac{\mu_0 I}{2\pi}\left(\frac{1}{y} + \frac{1}{d-y}\right) ldy$$

Total flux associated with the area between two wires is

$$\phi = \int d\phi = \frac{\mu_0 Il}{2\pi}\left(\int_a^{d-a}\frac{dy}{y} + \int_a^{d-a}\frac{dy}{d-y}\right)$$

$$\Rightarrow \quad \phi = \frac{\mu_0 Il}{2\pi}\left[\ln\left(\frac{d-a}{a}\right) - \ln\left(\frac{d-d+a}{d-a}\right)\right]$$

$$\Rightarrow \quad \phi = \frac{\mu_0 Il}{\pi}\ln\left(\frac{d-a}{a}\right)$$

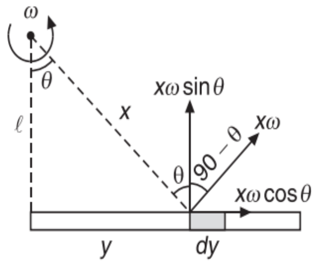
$$\Rightarrow \quad L = \frac{\phi}{I} = \frac{\mu_0 l}{\pi}\ln\left(\frac{d-a}{a}\right)$$

Hence the inductance (L) per unit length of this arrangement,

$$\frac{L}{l} = \frac{\mu_0}{\pi}\ln\left(\frac{d-a}{a}\right)$$

Hence, the correct answer is (A).

45. METHOD-I



$$d\xi = B(dy)(x\omega\sin\theta)$$

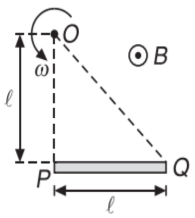
Now $\sin\theta = \dfrac{y}{x}$

$$\Rightarrow \quad d\xi = B(dy)\left(x\omega\left(\frac{y}{x}\right)\right)$$

$$\Rightarrow \quad \int d\xi = \int_0^l B\omega y dy$$

$$\Rightarrow \quad \xi = \frac{1}{2}B\omega l^2$$

METHOD-II



Consider two imaginary straight rods OP of length l and OQ of length $\sqrt{2}l$.

Then, $V_P - V_O = \dfrac{1}{2}B\omega l^2$

and $V_Q - V_O = \dfrac{1}{2}B\omega(\sqrt{2}l)^2$

$$\Rightarrow \quad V_Q - V_P = \frac{1}{2}B\omega(\sqrt{2}l)^2 - \frac{1}{2}B\omega l^2 = \frac{1}{2}B\omega l^2$$

Hence, the correct answer is (A).

46. $P = i_0^2 R$

$$\Rightarrow \quad i_0^2 = \frac{P}{R}$$

Since $\tau = \dfrac{L}{R}$

$$\Rightarrow \quad L = \tau R$$

Heat dissipated $= \dfrac{1}{2}Li_0^2 = \dfrac{1}{2}(\tau R)\left(\dfrac{P}{R}\right) = \dfrac{1}{2}P\tau$

Hence, the correct answer is (B).

47. The magnetic flux associated with the loop is

$$\phi = \int_c^{a+c} \frac{\mu_0 I}{2\pi x} b dx = \frac{\mu_0 I b}{2\pi} \ln\left(\frac{a+c}{c}\right)$$

$$\Delta I = 0 - I_0 = -I_0$$

$$\Rightarrow \quad \Delta\phi = -\frac{\mu_0 I_0 b}{2\pi} \ln\left(\frac{a+c}{c}\right)$$

$$\Rightarrow \quad Q = -\frac{\Delta\phi}{R} = \frac{\mu_0 I_0 b}{2\pi R} \ln\left(\frac{a+c}{c}\right)$$

Hence, the correct answer is (C).

48. $\tau = R_{eq} C_{eq}$

$$\tau = (5)(1)$$

$$\tau = 5 \text{ s}$$

Hence, the correct answer is (A).

49. According to Lenz's law, the induced current in the ring opposes the motion of magnet. The magnetic field produced in the ring will exert an upward force on the magnet. The acceleration of magnet will be thus, less than acceleration due to gravity.

Hence, the correct answer is (B).

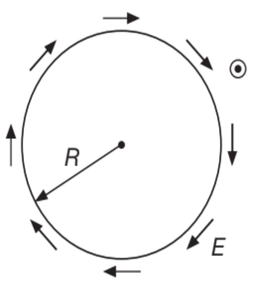
50. Due to the time varying magnetic field, an induced electric field E is set up, so the force on the ring is

$$F = qE$$

If f be the frictional force between the ring and the table, then the ring starts rotating when

$$F = f$$

$$\Rightarrow \quad qE = \mu mg \qquad \ldots(1)$$



Also $\oint \vec{E} \cdot d\vec{\ell} = \left|\frac{d\phi}{dt}\right|$

$$\Rightarrow \quad E(2\pi R) = A\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E(2\pi R) = (\pi R^2)(16t) \qquad \left\{\because \frac{dB}{dt} = 16t\right\}$$

$$\Rightarrow \quad E = 8Rt \qquad \ldots(2)$$

Substituting (2) in (1), we get

$$q(8Rt) = \mu mg$$

$$\Rightarrow \quad \mu = \left(\frac{8qR}{mg}\right)t$$

$$\Rightarrow \quad \mu|_{t=3} = \frac{24qR}{mg}$$

Hence, the correct answer is (D).

51. The emf induced is

$$\xi = \frac{Bl^2\omega}{2}$$

$$\Rightarrow \quad \omega = \frac{2\xi}{Bl^2}$$

$$\Rightarrow \quad f = \frac{\omega}{2\pi} = \frac{\xi}{\pi B l^2} = \frac{0.01}{\pi \times 5 \times 10^{-4} \times 1^2}$$

$$\Rightarrow \quad f = \frac{20}{\pi} \text{ rev/sec}$$

Hence, the correct answer is (B).

52. Inductive time constant is

$$\tau_L = \frac{L}{R} = 2 \text{ s}$$

Since, $i_0 = \frac{E}{R} = 3$ A

So, at $t = 2$ s, we get

$$i = i_0\left(1 - e^{-\frac{t}{\tau_L}}\right) = 3(1 - e^{-1}) \text{ A}$$

Hence, the correct answer is (A).

53. Since $\xi = E - IR$

$$\Rightarrow \quad \xi = E - E\left(1 - e^{-\frac{t}{\tau}}\right) = Ee^{-\frac{t}{\tau}}$$

Therefore, ξ vs t graph is best represented by OPTION (C).

Hence, the correct answer is (C).

54. Already derived in Theory.

The correct answer is (D).

55. The electric field at point P is given by

$$E(2\pi R) = \pi R^2 \frac{dB}{dt}$$

$$\Rightarrow \quad E = \frac{R}{2}\frac{dB}{dt}$$

Force on charge q is

$$F = qE = \frac{qR}{2}\frac{dB}{dt}$$

$$\Rightarrow \int_{B_0}^{0} dB = \frac{2}{qR}\int_0^t F dt = \frac{2}{qR}\times(\text{impulse})$$

Since $\int_0^t F dt = mv$

$$\Rightarrow |0 - B_0| = \frac{2}{qR}mv$$

$$\Rightarrow v = \frac{qRB_0}{2m}$$

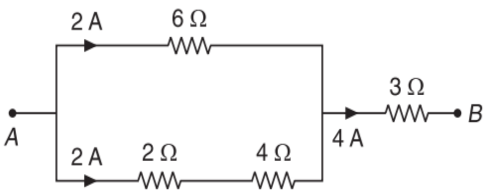
Hence, the correct answer is (C).

56. For the second position, $\Delta\phi = 0$

$$\Rightarrow |Q_2| = \frac{\Delta\phi}{R} = 0$$

Hence, the correct answer is (D).

57. In steady state, inductor acts as a pure conductor. Steady state circuit is shown in Figure.



$$V_A - (2)(6) - (4)(3) - V_B = 0$$

$$\Rightarrow V_A - V_B = 24 \text{ V}$$

Hence, the correct answer is (C).

58. According to Lenz's law, the motion of magnet will always be opposed. So, M will repel R when moving towards it and it will attract R when moving away from it.

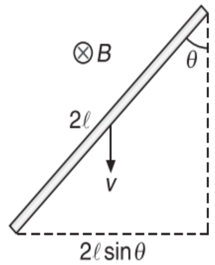
Hence, the correct answer is (A).

59. $\frac{1}{2}mv_0^2 = \frac{1}{2}Li_{max}^2$

$$\Rightarrow i_{max} = \sqrt{\frac{m}{L}}v_0$$

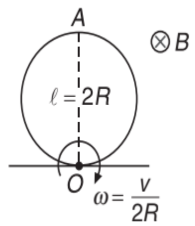
Hence, the correct answer is (B).

60. The projected length of the rod in the direction perpendicular to both $\vec{v}$ and $\vec{B}$ is $2l\sin\theta$. Therefore, potential difference between the two ends of the rod equals $Bv(2l\sin\theta)$



Hence, the correct answer is (B).

61. Let a hypothetical conducting rod of length $l = 2R$ joins O to A, then emf induced across ends of rod is



$$\xi_{OA} = \frac{B\omega l^2}{2} = \frac{(B)\left(\frac{v}{2R}\right)(4R)^2}{2} = 4BvR$$

Hence, the correct answer is (D).

62. Applying Kirchhoff's rule to the path A to B, we get

$$V_A - V_B = L\,di/dt + iR$$

$$\Rightarrow 0.5 = L\times 8 + 0.5\times 0.2$$

$$\Rightarrow L = 0.05 \text{ H}$$

Hence, the correct answer is (C).

63. Applying Kirchhoff's Loop Law (KLL), we get

$$V_A - IR + E - L\frac{dI}{dt} - V_B = 0$$

According to the problem, we have

$$\frac{dI}{dt} = -10^3 \text{ As}^{-1}$$

$$\Rightarrow V_A - (5)(1) + 15 - (5\times10^{-3})(-10^3) - V_B = 0$$

$$\Rightarrow V_A - V_B = 15\ V$$

Hence, the correct answer is (B).

64. Applying Kirchhoff's Loop Law (KLL), we get

$$V_A + IR + E + L\frac{dI}{dt} - V_B = 0$$

According to the problem, we have

$$\frac{dI}{dt} = -10^3 \text{ As}^{-1}$$

CHAPTER 3

$\Rightarrow \quad V_A + (5)(1) + 15 + (5\times10^{-3})(-10^3) - V_B = 0$

$\Rightarrow \quad V_A - V_B = -15\text{ V}$

$\Rightarrow \quad V_B - V_A = 15\text{ V}$

Hence, the correct answer is (C).

65. At mean position, velocity is maximum. Hence motional EMF Bvl is also maximum. Velocity v oscillates simple harmonically so motional emf will also vary simple harmonically. Further, polarity of induced emf will keep on changing.

Hence, the correct answer is (B).

66. The current as a function of time in LR circuit after closing the switch is

$$i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right) = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$$

When energy stored in inductor is half the maximum value, then

$$\frac{1}{2}Li^2 = \frac{1}{2}\left(\frac{1}{2}Li_0^2\right)$$

$$\Rightarrow \quad i = \frac{i_0}{\sqrt{2}} = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$\Rightarrow \quad e^{-\frac{Rt}{L}} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{Rt}{L} = \ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)$$

$$\Rightarrow \quad t = \frac{L}{R}\ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)$$

Hence, the correct answer is (A).

67. Since $E_{nc}(2\pi r) = A\left(\frac{dB}{dt}\right)$

$$\Rightarrow \quad E_{nc}(2\pi r) = (\pi r^2)\frac{dB}{dt}$$

$$\Rightarrow \quad E_{nc} = \frac{r}{2}\frac{dB}{dt}$$

Force on the charge q is

$$F = qE = \frac{qr}{2}\left(\frac{dB}{dt}\right)$$

Work done is

$$W = Fd = (2\pi r)$$

$$\Rightarrow \quad W = \pi r^2 q\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad W = \left(\frac{22}{7}\right)(1)^2(10^{-6})(2\times10^{-3})$$

$$\Rightarrow \quad W = 2\pi\times10^{-9}\text{ J}$$

Hence, the correct answer is (B).

68. Maximum speed is attained when the entire energy stored in inductor is converted into kinetic energy of rod.

$$\Rightarrow \quad \frac{1}{2}L\left(\frac{E}{R}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \left(\frac{E}{R}\right)\sqrt{\frac{L}{m}}$$

Hence, the correct answer is (A).

69. Since both L_1 and L_2 are in parallel, so

$$V_1 = V_2$$

$$\Rightarrow \quad L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt}$$

$$\Rightarrow \quad L_1 dI_1 = L_2 dI_2$$

$$\Rightarrow \quad L_1\int_0^{I_1} dI_1 = L_2\int_0^{I_2} dI_2$$

$$\Rightarrow \quad L_1 I_1 = L_2 I_2$$

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{L_2}{L_1}$$

Hence, the correct answer is (A).

70. At $t = 0$, potential difference between A and B is V and resistance is $R + R = 2R$.

$$\Rightarrow \quad i_1 = \left(\frac{V}{2R}\right)$$

As $t \to \infty$, potential difference between A and B is V and resistance is R

$$\Rightarrow \quad i_2 = \frac{V}{R}$$

$$\Rightarrow \quad \frac{i_1}{i_2} = \frac{1}{2}$$

Hence, the correct answer is (D).

71. Since, $\frac{1}{2}Li^2 = \frac{1}{4}\left(\frac{1}{4}Li_0^2\right)$

$\Rightarrow \quad i = \frac{i_0}{2}$, half value

$$\Rightarrow \quad t = t_{1/2} = (\ln 2)\tau_L = (\ln 2)\left(\frac{L}{R}\right)$$

Hence, the correct answer is (A).

72. The magnitude of emf induced is $\xi = \frac{d\phi}{dt}$

The current through the circuit is

$$i = \frac{dQ}{dt} = \frac{\xi}{R} = \frac{1}{R}\frac{d\phi}{dt}$$

$$\Rightarrow \quad dQ = \frac{d\phi}{R}$$

$$\Rightarrow \quad Q = \frac{\Delta\phi}{R}$$

Hence, the correct answer is (D).

73. $\xi = \frac{d\phi}{dt}$ {in magnitude}

$$\Rightarrow \quad i = \frac{\xi}{R} = \frac{1}{R}\frac{d}{dt}(BA) = \frac{A}{R}\frac{dB}{dt}$$

where $A = \pi r^2$ is the area of loop of radius r and R is the resistance of the loop of length $(2\pi r)$ and area of cross-section πa^2.

$$\Rightarrow \quad R = \frac{\rho\ell}{\pi a^2} = \frac{\rho(2\pi r)}{\pi a^2}$$

Further mass of wire is $m = (\pi a^2)(2\pi r)(d)$

$$\Rightarrow \quad i = \frac{(\pi a^2)(\pi r^2)}{\rho(2\pi r)}\frac{dB}{dt}$$

$$\Rightarrow \quad i = \frac{(\pi a^2)(2\pi r)}{4\pi\rho}\frac{dB}{dt}$$

$$\Rightarrow \quad i = \frac{m}{4\pi\rho d}\frac{dB}{dt}$$

Hence, the correct answer is (A).

74. When the sides shrink at the rate α, we have

$$\frac{da}{dt} = -\alpha$$

Since, $\xi = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA)$

$$\Rightarrow \quad \xi = -B\frac{d}{dt}(a^2) = -2Ba\frac{da}{dt} = 2a\alpha B$$

Hence, the correct answer is (A).

75. The induced emf is given by

$$\xi = A\frac{dB}{dt} = \pi r^2 B_0\left[\frac{d}{dt}(e^{-t})\right] = \pi r^2 B_0 e^{-t}$$

$$\Rightarrow \quad P = \frac{V^2}{R} = \left(\frac{\pi^2 r^4 B_0^2}{R}\right)e^{-2t}$$

At $t = 0$, $P = \frac{B_0^2\pi^2 r^4}{R}$

Hence, the correct answer is (D).

76. $I = I_0\left(1 - e^{-\frac{Rt}{L}}\right) = I_0(1 - e^{-2t})$

$$\Rightarrow \quad \frac{I(t \to \infty)}{I(t = 1)} = \frac{I_0}{I_0(1 - e^{-2})} = \frac{e^2}{e^2 - 1}$$

Hence, the correct answer is (B).

77. $\frac{dI}{dt} = I_0\omega\cos\omega t$

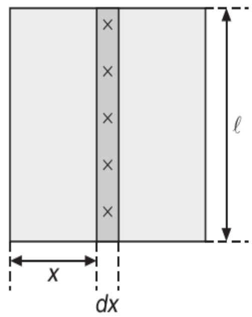
Since $\xi = M\frac{dI}{dt} = MI_0\omega\cos\omega t$

$$\Rightarrow \quad \xi_{max} = MI_0\omega$$

$$\Rightarrow \quad \xi_{max} = 0.005 \times 10 \times 100\pi = 5\pi \text{ V}$$

Hence, the correct answer is (B).

78. The flux associated with the loop in initial position is



$$\phi_i = \int_0^l Bl\,dx = B_0 l\int_0^l x\,dx = \frac{B_0 l^3}{2}$$

The flux associated with the loop in final position is

$$\phi_f = -\int_l^{2l} Bl\,dx = -B_0 l\int_l^{2l} x\,dx = -\frac{3B_0 l^3}{2}$$

$$\Rightarrow \quad |\Delta\phi| = |\phi_i - \phi_f| = 2B_0 l^3$$

Hence, the correct answer is (B).

79. At $t = 0$ i.e. when the key is just pressed, no current exists inside the inductor. So $10\ \Omega$ and $20\ \Omega$ resistors are in series and a net resistance of $(10 + 20) = 30\ \Omega$ exists across the circuit.

Hence $I_1 = \frac{2}{30} = \frac{1}{15}A$

As $t \to \infty$, the current in the inductor grows to attain a maximum value i.e. the entire current passes through the inductor and no current passes through $10\ \Omega$ resistor

Hence $I_2 = \frac{2}{20} = \frac{1}{10}A$

Hence, the correct answer is (A).

80. Energy stored in the inductor is

$$U=\frac{1}{2}LI^2=\frac{1}{2}L\left(\frac{E}{R}\right)^2\left(1-e^{-\frac{Rt}{L}}\right)^2$$

The rate at which energy is stored is

$$\frac{dU}{dt}=\frac{E^2}{R}\left(1-e^{-\frac{Rt}{L}}\right)\left(e^{-\frac{Rt}{L}}\right)$$

Its value is maximum, when

$$1-e^{-\frac{Rt}{L}}=e^{-\frac{Rt}{L}}$$

$$\Rightarrow \quad e^{-\frac{Rt}{L}}=\frac{1}{2}$$

$$\Rightarrow \quad \left(\frac{dU}{dt}\right)_{\max}=\frac{E^2}{4R}$$

Hence, the correct answer is (A).

81. At time $t=0$, resistance offered by a capacitor is zero and resistance offered by an inductor is ∞.

$$\Rightarrow \quad R_{\text{net}}=\frac{R}{2}+\frac{R}{3}=\frac{5R}{6}=5\ \Omega$$

So, current from the battery is

$$i=\frac{E}{R_{\text{net}}}=\frac{5}{5}=1\text{ A}$$

Hence, the correct answer is (A).

82. The charge flowing through galvanometer is

$$Q=\frac{|\Delta\phi|}{R}=\frac{AB}{R}$$

Hence, the correct answer is (C).

83. The emf induced in the triangular loop is

$$\xi=\frac{d\phi}{dt}$$

$$\Rightarrow \quad \xi=A\frac{dB}{dt}$$

Since area of an equilateral triangle is

$$A=\frac{\sqrt{3}}{4}\times(\text{side})^2$$

$$\Rightarrow \quad A=\frac{\sqrt{3}}{4}(2)^2=\sqrt{3}\text{ m}^2$$

$$\Rightarrow \quad \xi=\sqrt{3}\times\sqrt{3}=3\text{ V}$$

Due to symmetry, emf developed across each wire of frame will be 1 V.

The current in the triangular loop is

$$i=\frac{\xi}{R}=\frac{3}{2+2+1}=0.6\text{ A}$$

Voltage across AB is

$$\Rightarrow \quad V_{AB}=\xi_{AB}-iR_{AB}=1-(0.6)(1)$$

$$\Rightarrow \quad V_{AB}=1-0.6=0.4\text{ V}$$

Hence, the correct answer is (A).

84. Flux linked with the loop, when it is at a distance r from the wire is

$$\phi=BA\cos 0=\left(\frac{\mu_0 I}{2\pi r}\right)(\pi a^2)$$

$$\Rightarrow \quad \phi=\frac{\mu_0 I a^2}{2r}$$

Since $\xi=-\frac{d\phi}{dt}$

$$\Rightarrow \quad \xi=\frac{\mu_0 I a^2}{2}\frac{d}{dt}(r^{-1})$$

$$\Rightarrow \quad \xi=\frac{\mu_0 I a^2}{2r^2}\left(\frac{dr}{dt}\right)$$

$$\Rightarrow \quad \xi=\left(\frac{\mu_0 I a^2}{2r^2}\right)v \qquad \left\{\because \frac{dr}{dt}=v\right\}$$

Hence, the correct answer is (C).

85. The current through the wire PQ is $I=\frac{\xi}{R}$

When the wire just starts sliding, the force IlB towards right is equal to the force μmg towards left.

$$\Rightarrow \quad \mu mg=IlB=\left(\frac{\xi}{R}\right)lB$$

$$\Rightarrow \quad \mu=\frac{\xi lB}{mgR}=\frac{6\times 4.9\times 10^{-2}\times 0.8}{10^{-2}\times 9.8\times 20}=0.12$$

Hence, the correct answer is (D).

86. Relative velocity $v_r=0$

So, change in flux $\Delta\phi=0$

Hence, the correct answer is (D).

87. At $t=0$, current through L is zero.

So, $i_1=\frac{10}{6+4}=1\text{ A}$

As $t\to\infty$ effective resistance of circuit is

$$R_{\text{eff}}=6+\frac{4}{2}=8\ \Omega$$

$$\Rightarrow \quad i_2=\frac{10}{8}=\frac{5}{4}\text{ A}$$

$$\Rightarrow \quad \frac{i_1}{i_2}=0.8$$

Hence, the correct answer is (B).

88. $\frac{L}{R} = 2 \times 10^{-3}$ ...(1)

$\frac{L}{R+90} = 0.5 \times 10^{-3}$ ...(2)

From (1) and (2), on solving we get

$L = 60$ mH and $R = 30\ \Omega$

Hence, the correct answer is (C).

89. The distance of points P and Q from centre is $2R$. Electric field at these points is given by

$$E(2\pi \times 2R) = \pi R^2 \left(\frac{dB}{dt}\right) = \pi R^2 b$$

$$\Rightarrow \quad E = \frac{Rb}{4}$$

So, potential difference between P and Q is

$$\xi = El = E\left[\frac{2\pi(2R)}{3}\right] = \frac{\pi b R^2}{3}$$

The current induced in loop is

$$I = \frac{\xi}{R_{\text{net}}} = \frac{\xi}{(2R + 2R + 2\sqrt{3}R)\lambda}$$

$$\Rightarrow \quad I = \frac{\pi b R}{6\lambda(2+\sqrt{3})} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\Rightarrow \quad I = \frac{\pi R b(2-\sqrt{3})}{6\lambda}$$

Hence, the correct answer is (A).

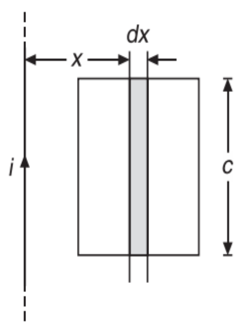
90. At $t = 0$, $V_L = -E$

Hence, the correct answer is (C).

91. The emf induced across wire HE is $\xi = Blv$ and is a constant. Once capacitor attains this potential difference, no current flows in $HKDE$.

Hence, the correct answer is (D).

92. Consider an element of length c, thickness dx at a distance x from the wire as shown in Figure.



Area (dA) of element is

$$dA = c\,dx$$

Magnetic field at the element is

$$B = \frac{\mu_0 i}{2\pi x}$$

Since $d\phi = B\,dA$

$$\Rightarrow \quad d\phi = \frac{\mu_0 i c}{2\pi x} dx$$

$$\Rightarrow \quad \phi = \int_a^b d\phi = \frac{\mu_0 i c}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$\Rightarrow \quad M = \frac{\phi}{i} = \frac{\mu_0 c}{2\pi} \ln\left(\frac{b}{a}\right)$$

Hence, the correct answer is (B).

93. The field produced due to induced current is directed into the paper. So, current in outer loop is clockwise and in inner loop is anticlockwise. Hence, current will flow from A to B and D to C.

Hence, the correct answer is (C).

94. For PQ, $\vec{l}$ is parallel to $\vec{v}$. Hence, $\xi = 0$

Hence, the correct answer is (A).

95. The horizontal distance between points P and Q is $l = a$. The potential difference between these points is

$$\xi = Blv = Bva$$

Hence, the correct answer is (D).

96. The equivalent resistance of five resistors is $3\ \Omega$

{It is a balanced Wheat Stone Bridge}

So $R_{\text{total}} = 3 + 1 = 4\ \Omega$

$$\Rightarrow \quad (10^{-3})(4) = (2)(0.1)v \qquad \{\because \xi = IR = B\ell v\}$$

$$\Rightarrow \quad v = 2\ \text{cms}^{-1}$$

Hence, the correct answer is (C).

97. The current through L and C are respectively

$$I = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right) \text{ and } I = \frac{V}{R} e^{-\frac{t}{RC}}$$

Both are equal when

$$1 - e^{-\frac{Rt}{L}} = e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad R = \sqrt{\frac{L}{C}}$$

$$\Rightarrow \quad \frac{R}{L} = \frac{1}{RC}$$

CHAPTER 3

$\Rightarrow \quad 1-e^{-\frac{Rt}{L}}=e^{-\frac{t}{RC}}$

$\Rightarrow \quad e^{-\frac{Rt}{L}}=\frac{1}{2}$

$\Rightarrow \quad t=CR(\ln 2)$

Hence, the correct answer is (B).

98. By Law of Conservation of Energy, we get

$$\frac{1}{2}Li^2=\frac{1}{2}CV^2$$

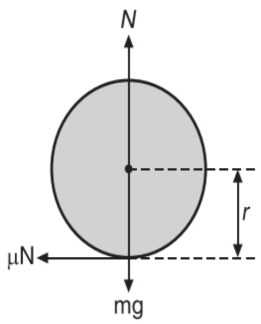
$\Rightarrow \quad V=\sqrt{\frac{L}{C}}$

$\Rightarrow \quad i=\sqrt{\frac{2}{4\times 10^{-6}}}(2)=\sqrt{2}\times 10^3\text{ V}$

Hence, the correct answer is (A).

99. The electric field induced at the ring is

$$E=\frac{\pi r^2\left(\frac{dB}{dt}\right)}{2\pi r}=\frac{r}{2}kt=\frac{rt}{2}$$



Since the magnetic field is increasing with time, so an anticlockwise torque acts on the ring and is given by

$$\tau=Fr_{\perp}=QEr=\frac{Qr^2t}{2}$$

The ring starts motion when

$$\tau=\mu Nr=\mu mgr$$

$\Rightarrow \quad \frac{Qr^2t}{2}=\mu mgr$

$\Rightarrow \quad t=\frac{2\mu mg}{rQ}$

Hence, the correct answer is (B).

100. $i_0=\frac{E}{R}=\frac{12}{0.3}=40\text{ A}$

Energy stored in the inductor is

$$U=\frac{1}{2}Li_0^2=\frac{1}{2}\times 50\times 10^{-3}(40)^2$$

$\Rightarrow \quad U=40\text{ J}$

Hence, the correct answer is (A).

101. Induced EMF across the rod is

$$\xi=\vec{B}\cdot(\vec{l}\times\vec{v})=\begin{vmatrix} B_x & B_y & B_z \\ l_x & l_y & l_z \\ v_x & v_y & v_z \end{vmatrix}$$

The magnetic field in the region is

$$\vec{B}=3\hat{j}+4\hat{k}$$

The length vector of the rod is

$$\vec{l}=(10\cos 53^\circ)\hat{i}+(10\sin 53^\circ)\hat{j}=6\hat{i}+8\hat{j}$$

The velocity of the rod is

$$\vec{v}=1\hat{i}$$

So, EMF induced is

$$\xi=\begin{vmatrix} 3 & 0 & 4 \\ 6 & 8 & 0 \\ 1 & 0 & 0 \end{vmatrix}$$

$\Rightarrow \quad \xi=4(0-8)=-32\text{ V}$

Hence, the correct answer is (A).

102. $\Delta\phi=0-BA$

$\Rightarrow \quad \Delta\phi=-BA \text{ and } \Delta t=\frac{T}{4} \text{ for } \theta=\frac{\pi}{2}$

$\Rightarrow \quad \xi_{av}=\left|\frac{\Delta\phi}{\Delta t}\right|=\frac{BA}{\left(\frac{T}{4}\right)}$

$\Rightarrow \quad \xi_{av}=\frac{BA}{\frac{2\pi}{4\omega}}=\frac{2\omega BA}{\pi}$

Hence, the correct answer is (D).

103. At any time t, the current in the circuit grows in accordance with formula

$$I=I_0\left(1-e^{-Rt/L}\right)=I_0\left(1-e^{-t/\tau}\right)$$

$\Rightarrow \quad I=\frac{E}{R}\left(1-e^{-t/\tau}\right)$

$\Rightarrow \quad \frac{dq}{dt}=\frac{E}{R}\left(1-e^{-t/\tau}\right)$

$\Rightarrow \quad dq=\frac{E}{R}dt-\frac{E}{R}e^{-t/\tau}dt$

$\Rightarrow \quad q=\frac{E}{R}\int_0^{\tau}dt-\frac{E}{R}\int_0^{\tau}e^{-t/\tau}dt$

$$\Rightarrow \quad q = \frac{E\tau}{R} - \left[-\frac{E\tau}{\text{Re}} + \frac{E\tau}{R} \right] = \frac{E\tau}{\text{Re}}$$

Hence, the correct answer is (C).

104. While moving towards the loop, the flux through the loop will increase and while moving away, the flux will decrease. So, the current will change direction as the electron passes by.

Hence, the correct answer is (D).

105. $M \propto N_1 N_2$

$\Rightarrow \quad M' = 4M$

Hence, the correct answer is (A).

106. Efficiency of a d.c. motor is the ratio of the back emf to the applied emf.

Hence, the correct answer is (B).

107. $\xi = L\left|\frac{di}{dt}\right| = L$ (Slope of i-t graph)

Initially, slope $= 0$

$\Rightarrow \quad \xi = 0$

In the remaining two regions, slopes are constants but of opposite signs. Hence, induced emfs are constants but of opposite signs.

Hence, the correct answer is (C).

108. Due to increasing field, the induced current in both loops individually must be anticlockwise which actually oppose each other. However, as the emf induced in larger loop is more, the net current in larger loop is anticlockwise and in smaller loop is clockwise. Hence, its direction is B to A and D to C

Hence, the correct answer is (D).

109. Since we know that

$$dq = \frac{d\phi}{R}$$

$$\Rightarrow \quad Q = \frac{1}{R}(\Delta\phi) = \frac{1}{R}(\phi_f - \phi_i)$$

where $\phi_i = 0$ and $\phi_f = \left(\frac{\mu_0 I}{2b}\right)(\pi a^2)$

$$\Rightarrow \quad \Delta\phi = \frac{\mu_0 \pi I a^2}{2b}$$

So, $Q = \dfrac{\mu_0 \pi I a^2}{2bR}$

Hence, the correct answer is (D).

110. $\vec{v}$ is parallel to $\vec{l}$, so $\xi = 0$

Hence, the correct answer is (D).

111. On increasing current, the flux through Q increases. An opposing magnetic field is developed due to induced current. The loops will therefore repel each other.

Hence, the correct answer is (B).

112. The time in which the charge decays to 0.368 of its initial value is equal to the time constant of the RC circuit i.e. $\tau = RC$

$\Rightarrow \quad \tau = 5\times10^4 \times 2\times10^{-6}\,s = 10\times10^{-2}$ s

$\Rightarrow \quad \tau = 0.1$ s

Hence, the correct answer is (D).

113. For the case of free fall, we have

$$s = \frac{1}{2}gt^2$$

$$\Rightarrow \quad s = \frac{1}{2}(10)(1)^2 = 5 \text{ m}$$

Here due to repulsion from induced effects, we get

$a < g$

$\Rightarrow \quad s < 5$ m

Hence, the correct answer is (C).

114. Applying Kirchhoff's rule to the path A to B, we get

$$V_A - V_B = IR + L\,dI/dt - 10$$

where, $dI/dt = 10$ As^{-1}

At $t = 0$, $I = 5$ A

$\Rightarrow \quad V_A - V_B = 5\times3 + 1\times10 - 10 = 15$ V

Hence, the correct answer is (A).

115. $V_C = BLv$

$\Rightarrow \quad q = CV_C = BvLC = \text{constant}$

$$\Rightarrow \quad I_C = \frac{dq}{dt} = 0$$

$$\Rightarrow \quad U_C = \frac{1}{2}CV^2 = \frac{1}{2}CB^2L^2v^2$$

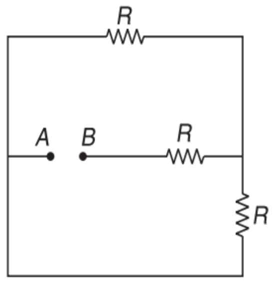
Hence, the correct answer is (C).

116. As the ring and magnetic field remain in same plane, no emf will be induced in all the motions given in different options.

Hence, the correct answer is (D).

117. By short circuiting the battery, the resistance across the open ends of inductor is

CHAPTER 3



$$R_{AB}=R+\frac{R}{2}=\frac{3R}{2}$$

$$\Rightarrow \quad \tau=\frac{L}{R_{AB}}=\frac{2L}{3R}$$

Hence, the correct answer is (D).

118. Magnetic field due to toroid is $B=\mu_0 nI$, where $n=\dfrac{N}{2\pi r}$ (n is number of turns per unit length)

$$\Rightarrow \quad B=\frac{\mu_0 NI}{2\pi r}$$

Further, flux ϕ is given by NBA

$$\Rightarrow \quad \phi=NBA=\frac{\mu_0 N^2 AI}{2\pi r}$$

Now, by definition of L we have

$$L=\frac{\phi}{I}=\frac{\mu_0 N^2 A}{2\pi r}$$

$$\Rightarrow \quad L=\frac{4\pi\times10^{-7}\times(1200)^2\times\left(\frac{12}{1000}\right)}{2\pi\left(\frac{15}{100}\right)}$$

$$\Rightarrow \quad L=2\cdot3\times10^{-3}\text{ H}$$

$$\Rightarrow \quad L=2.3\text{ mH}$$

Hence, the correct answer is (C).

119. Flux across the secondary ϕ_s is

$$\phi_s=N_sBA$$

$$\Rightarrow \quad \phi_s=\frac{\mu_0 N_s NAI}{2\pi r}$$

Since, $\xi=-\dfrac{d\phi_s}{dt}$

$$\Rightarrow \quad |\xi|=\frac{\mu_0 N_s NA}{2\pi r}\frac{dI}{dt}$$

$$\Rightarrow \quad |\xi|=\frac{(4\pi\times10^{-7})(300)(1200)\left(\frac{12}{10000}\right)}{2\pi\left(\frac{15}{100}\right)}\left(\frac{2}{0.05}\right)$$

$$\Rightarrow \quad |\xi|=2.3\times10^{-2}\text{ V}$$

$$\Rightarrow \quad |\xi|=0.023\text{ V}$$

Hence, the correct answer is (C).

120. The flux associated with the loop is $\phi=B\pi\left(a^2+b^2\right)$

The induced current is

$$I=\frac{\xi}{R}=\frac{1}{R}\left|\frac{d\phi}{dt}\right|=\frac{k\pi\left(a^2+b^2\right)}{2\pi(a+b)\lambda}=\frac{k\left(a^2+b^2\right)}{2(a+b)\lambda}$$

Hence, the correct answer is (D).

121. Since, $i=i_0\left(1-e^{-\frac{Rt}{L}}\right)=\dfrac{E}{R}\left(1-e^{-\frac{Rt}{L}}\right)$

$$\Rightarrow \quad i=\frac{E}{R}-\frac{Ee^{-\frac{Rt}{L}}}{R}=i_0-\left(\frac{V_L}{R}\right) \qquad \left\{\because V_L=Ee^{-\frac{Rt}{L}}\right\}$$

$$\Rightarrow \quad V_L=(i_0R)-(R)i$$

Hence, V_L versus i graph is a straight line with positive intercept and negative slope.

Hence, the correct answer is (D).

122. Since, $N_P\phi_P=L_PI_P$

$$\Rightarrow \quad 2000\times0.8\times10^{-3}=L_P\times1$$

Similarly, $N_S\phi_S=MI_P$

$$\Rightarrow \quad 5000\times0.4\times10^{-3}=M\times1$$

$$\Rightarrow \quad \frac{L_P}{M}=\frac{4}{5}$$

Hence, the correct answer is (C).

123. $i=\dfrac{\xi}{R}=\dfrac{M}{R}\dfrac{dI}{dt}$

Since $I=I_0t$

$$\Rightarrow \quad \frac{dI}{dt}=I_0$$

$$\Rightarrow \quad i=\frac{M}{R}I_0$$

Hence, the correct answer is (B).

124. In small time dt, rod PQ moves by a distance vdt. The increase in area is $lvdt$ and the increase in flux, $d\phi=Blvdt$

The magnitude of induced emf is

$$\xi=\left|\frac{d\phi}{dt}\right|=Blv$$

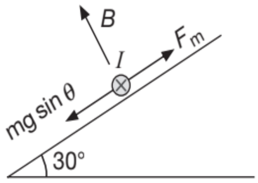
Hence, the correct answer is (A).

125. When current is passed through the straight wire, magnetic lines are circular and tangential to the loop. So, no flux is linked with the loop.

Hence, the correct answer is (B).

126. In the rod, an induced current I is set up in the inward direction $\otimes$, as shown. Due to this I, the rod experiences a magnetic force $F_m = BI\ell$, up the incline. At $v = v_T$, we have

$$mg\sin\theta - F_m = 0$$



Cross Sectional View

$$\Rightarrow \quad mg\sin\theta = BI\ell$$

$$\Rightarrow \quad mg\sin\theta = B\left(\frac{B\ell v_T}{R}\right)\ell \qquad \left\{\because I = \frac{\xi}{R} = \frac{B\ell v_T}{R}\right\}$$

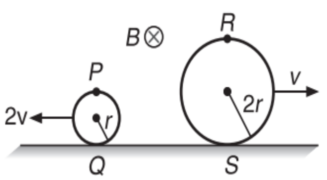
$$\Rightarrow \quad v_T = \frac{mgR\sin\theta}{B^2\ell^2}$$

Since $\theta = 30°$

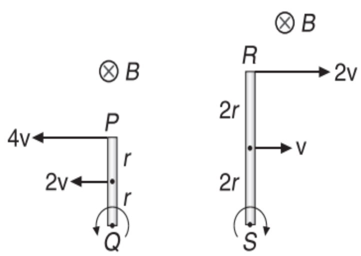
$$\Rightarrow \quad v_T = \frac{mgR}{2B^2\ell^2}$$

Hence, the correct answer is (C).

127. Let the magnetic field be directed into the paper as shown in Figure.



We can solve this problem by taking two conducting rods PQ and RS as shown in Figure.



The emf induced across the rod PQ is

$$\xi_1 = \frac{1}{2}B\omega_1 l_1^2 = \frac{1}{2}B\left(\frac{2v}{r}\right)(2r)^2 = 4Bvr$$

The emf induced across the rod RS is

$$\xi_2 = \frac{1}{2}B\omega_2 l_2^2 = \frac{1}{2}B\left(\frac{v}{2r}\right)(4r)^2 = 4Bvr$$

Using Fleming's Right Hand Rule, we see that points Q and R are at high potential whereas points P and S are at low potential. So

$$V_Q - V_P = \xi_1 = 4Bvr$$

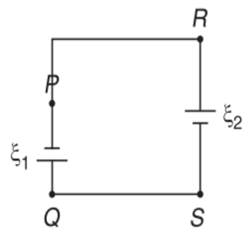
$$V_R - V_S = \xi_2 = 4Bvr$$

Since the horizontal surface is conducting, so we have

$$V_Q = V_S$$

$$\Rightarrow \quad V_R - V_P = 8Bvr$$

A simple circuit diagram for the equivalent batteries in shown in Figure.



Hence, the correct answer is (D).

128. Clearly, $V_P = V_Q$, $V_S = V_R$ and $V_P > V_S$

Hence, the correct answer is (D).

129. $I = I_0\left(1 - e^{-\frac{Rt}{L}}\right)$

At $I = \frac{I_0}{2}$, we have

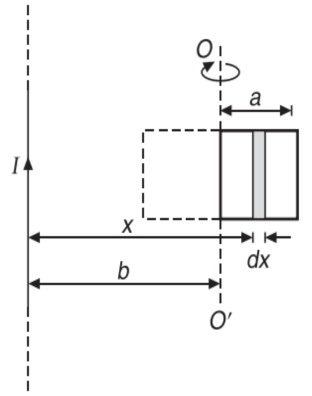
$$e^{-\frac{Rt}{L}} = \frac{1}{2}$$

$$\Rightarrow \quad t = \frac{L}{R}\ln 2 = \frac{300 \times 10^{-3}}{2} \times 0.693 \approx 0.1 \text{ s}$$

Hence, the correct answer is (D).

130. Initial magnetic flux associated with the square loop is calculated by integrating the flux through an elemental strip considered in the square loop as shown in Figure.

$$\phi_i = \frac{\mu_0 ia}{2\pi}\int_b^{b+a}\frac{dx}{x}$$



$$\phi_i = \frac{\mu_0 ia}{2\pi}\ln\left(\frac{b+a}{b}\right)$$

Similarly, after 180° rotation, final flux associated with the loop is

$$\phi_f = \frac{\mu_0 ia}{2\pi}\ln\left(\frac{b-a}{b}\right)$$

$$\Rightarrow \quad \Delta\phi = \left|\phi_i - \phi_f\right| = \frac{\mu_0 ia}{2\pi}\ln\left(\frac{b+a}{b-a}\right)$$

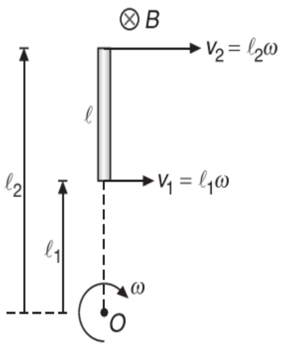
So, charge flown through the loop in process of rotation is

$$\Delta q = \frac{\Delta\phi}{R} = \frac{\mu_0 ia}{2\pi R}\ln\left(\frac{b+a}{b-a}\right)$$

Hence, the correct answer is (D).

131. Using our knowledge of rotational dynamics, we know that

$$\omega = \frac{v_2 - v_1}{l}$$



When this rod rotates with angular velocity ω in the magnetic field B, then

$$\xi = \frac{1}{2}B\omega\left(l_2^2 - l_1^2\right)$$

$$\Rightarrow \quad \xi = \frac{1}{2}B\omega\left[\left(\frac{v_2}{\omega}\right)^2 - \left(\frac{v_1}{\omega}\right)^2\right]$$

$$\Rightarrow \quad \xi = \frac{1}{2}B\omega\left(\frac{v_2^2 - v_1^2}{\omega^2}\right)$$

$$\Rightarrow \quad \xi = \frac{1}{2}B\left(\frac{v_2^2 - v_1^2}{\omega}\right)$$

$$\Rightarrow \quad \xi = \frac{1}{2}B\left[\frac{v_2^2 - v_1^2}{\left(\frac{v_2 - v_1}{l}\right)}\right]$$

$$\Rightarrow \quad \xi = \frac{1}{2}Bl\left(v_2 + v_1\right)$$

Hence, the correct answer is (C).

132. The wire ab can be replaced with a battery of emf $\xi = Blv$ and internal resistance R. The current flowing through the battery is I. Therefore, potential difference between a and b is

$$V = \xi - IR = Blv - IR$$

Hence, the correct answer is (C).

133. When the solenoid is cut into two equal parts, then each divided part has a self-inductance $\frac{L}{2}$ and resistance $\frac{R}{2}$. So, time constant τ which is the ratio of self-inductance to the resistance of the circuit remains the same.

$$\Rightarrow \quad \tau_{\text{initial}} = \tau_{\text{final}} = \frac{L_{eq}}{R_{eq}} = \frac{L/4}{R/4}$$

Hence, the correct answer is (D).

134. The steady state current is

$$\Rightarrow \quad I_0 = \frac{E}{R_{\text{equivalent}}} = \frac{E}{R/4}$$

$$\Rightarrow \quad I_0 = \frac{4E}{R}$$

Hence, the correct answer is (B).

135. When X is joined to Y, the time constant is $\tau = \frac{L}{R}$ and the rate of heat produced in steady state is $P = I_0^2 R$

$$\Rightarrow \quad I_0^2 = \frac{P}{R}$$

When X is joined to Z, the total heat produced is

$$H = \frac{1}{2}LI_0^2 = \frac{1}{2}L\frac{P}{R} = \frac{1}{2}\left(\frac{L}{R}\right)P = \frac{1}{2}\tau P$$

Hence, the correct answer is (B).

136. Since we know that

$$q = \frac{1}{R}\left(\Delta\phi\right) \qquad \ldots(1)$$

where q is the area under the I-t graph

So, $q = \left(\frac{1}{2}\right)(0.2)(8)$

$\Rightarrow \quad q = 0.8$ C

Now, from (1), we get

$$\Delta\phi = q(R) = (0.8)(10) = 8 \text{ Wb}$$

Hence, the correct answer is (D).

137. $\phi_i = BA\cos 0° = 2$ Wb and

$\phi_f = BA\cos 180° = -2$ Wb

$\Rightarrow \quad |\Delta\phi| = 4$ Wb

Since, $|\Delta q| = \frac{|\Delta\phi|}{R}$

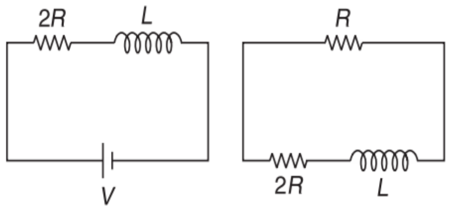
$\Rightarrow \quad |\Delta q| = \frac{4}{10} = 0.4$ C

Hence, the correct answer is (A).

138. During charging, $\tau_1 = \frac{L}{2R}$

During discharging, $\tau_2 = \frac{L}{3R}$

$\Rightarrow \quad \tau_1 : \tau_2 = 3:2$



Charging Discharging

Hence, the correct answer is (B).

139. Even if radius is doubled, flux is not going to change, because B being constant we get ϕ as constant and hence

$\xi = 0$ and $i = 0$

Hence, the correct answer is (C).

140. The emf induced is $\xi = BLv$.

Since, AC is in parallel to $(AB + BC)$, so their combined resistance is

$$R = \frac{\lambda L \times \lambda\sqrt{2}L}{\lambda L + \lambda\sqrt{2}L} = \frac{\sqrt{2}\lambda L}{\sqrt{2}+1}$$

$$\Rightarrow \quad I = \frac{\xi}{R} = \frac{Bv(\sqrt{2}+1)}{\sqrt{2}\lambda}$$

The force required to pull the triangle is

$$F = BIL = \frac{B^2Lv(\sqrt{2}+1)}{\sqrt{2}\lambda}$$

Hence, the correct answer is (A).

141. $i = i_0 e^{-\frac{Rt}{L}}$

$\Rightarrow \quad \frac{i_0}{\beta} = i_0 e^{-\frac{Rt}{L}}$

$\Rightarrow \quad \frac{L}{R} = \frac{T}{\ln\beta}$

Hence, the correct answer is (C).

142. $I = I_0\left(1 - e^{-\frac{t}{\tau_L}}\right)$ where $I_0 = \frac{E}{R}$ and $\tau_L = \frac{L}{R}$

$$Q = \int_0^{\tau_L} I dt = I_0 \int_0^{\tau_L}\left(1 - e^{-\frac{t}{\tau_L}}\right)dt$$

$$\Rightarrow \quad Q = I_0 t\Big|_0^{\tau_L} - \left(\frac{I_0 e^{-\frac{t}{\tau_L}}}{-\frac{1}{\tau_L}}\right)\Bigg|_0^{\tau_L}$$

$$\Rightarrow \quad Q = I_0\tau_L + I_0\tau_L\left[e^{-1} - e^0\right]$$

$$\Rightarrow \quad Q = I_0\tau_L + \frac{I_0\tau_L}{e} - I_0\tau_L$$

$$\Rightarrow \quad Q = \frac{I_0\tau_L}{e} = \frac{1}{e}\left(\frac{E}{R}\right)\left(\frac{L}{R}\right) = \frac{EL}{eR^2}$$

Hence, the correct answer is (C).

143. At $t = 0$, capacitor acts as a pure conductor. The effective resistance of the circuit is R and current $i_1 = \frac{\xi}{R}$

As $t \to \infty$ capacitor acts as a pure insulator and inductor acts as a pure conductor. The effective resistance of the circuit is $\frac{3R \times 6R}{3R + 6R} + R = 3R$

and the current $i_2 = \frac{\xi}{3}R$

$\Rightarrow \quad i_1 : i_2 = 3:1$

Hence, the correct answer is (A).

144. Since $V_{PQ} = L\dfrac{dI}{dt} + IR$

For the first case, we have

$$I = 4\text{ A},\ \frac{dI}{dt} = 4\text{ As}^{-1}\text{ and } V_{PQ} = 16\text{ V}$$

$\Rightarrow\quad 16 = L(4) + 4R$

$\Rightarrow\quad 4 = L + R \qquad \ldots(1)$

For the second case, we have

$$I = 2\text{ A},\ \frac{dI}{dt} = -1\text{ As}^{-1}\text{ and } V_{PQ} = 5\text{ V}$$

$\Rightarrow\quad 5 = -L + 2R \qquad \ldots(2)$

From (1) and (2), we get

$3R = 9$

$\Rightarrow\quad R = 3\ \Omega$ and $L = 1$ H

Hence, the correct answer is (A).

145. The relative velocity of approach becomes $2v$ (i.e. doubled), so induced emf is also doubled i.e., becomes 2ξ.

Hence, the correct answer is (C).

146. Since inward magnetic field is increasing, so induced electric lines are circular and anti-clockwise. Hence the negative charge experiences a force opposite to electric field.

Hence, the correct answer is (A).

147. At time $t = 0$, resistance capacitor behaves like short circuit and inductor behaves like open circuit so at this instant circuit resistance across the battery will be

$$R_{net} = \frac{R}{2} + \frac{R}{3} = \frac{5R}{6} = 5\ \Omega$$

Current through the battery at $t = 0$ is given as

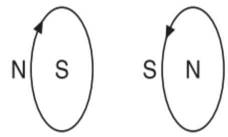
$$i = \frac{E}{R_{net}} = \frac{5}{5} = 1\text{ A}$$

Hence, the correct answer is (D).

148. $\xi = \dfrac{B\omega l^2}{2} = \text{constant}$

Hence, the correct answer is (C).

149. When brought closer, the induced effects should produced repulsion. So, currents should increase, because of which that pole strength increases. Hence, repulsion increases.



Hence, the correct answer is (B).

150. $I = \dfrac{\xi}{R} = -\dfrac{B}{R}\left(\dfrac{dA}{dt}\right)$

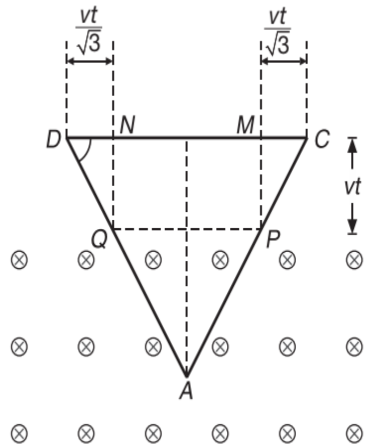
$$DN = CM = \frac{vt}{\sqrt{3}}$$

$$PQ = 2a - (DN + CM) = 2a - \frac{2vt}{\sqrt{3}}$$

At time t, area of the triangle APQ, as a function of t is

$$Ar(\Delta APQ) = Ar(\Delta ACD) - Ar\text{ of Trap }(PQDC)$$

$$\Rightarrow\quad A = \sqrt{3}a^2 - \frac{1}{2}\left(2a + 2a - \frac{2vt}{\sqrt{3}}\right)(vt)$$



$$\Rightarrow\quad A = \sqrt{3}a^2 - 2avt + \frac{v^2t^2}{\sqrt{3}}$$

$$\frac{dA}{dt} = -2av + \frac{2v^2t}{\sqrt{3}}$$

$$\Rightarrow\quad I = \frac{B}{R}\left(2av - \frac{2v^2t}{\sqrt{3}}\right)$$

$$\Rightarrow\quad I = \frac{2vB}{R}\left(a - \frac{vt}{\sqrt{3}}\right)$$

$$\Rightarrow\quad I = \frac{2Bva}{R} - \left(\frac{2Bv^2}{\sqrt{3}R}\right)t$$

Which happens to be a straight line with negative slope, So, the I-t graph is best represented by (C).

Hence, the correct answer is (C).

151. Magnetic field due to infinite wire at a distance x is

$$B = \frac{\mu_0 I}{2\pi x}$$

Consider an infinitesimal element of length dx of conductor EF. EMF induced across this element is

$$d\xi = Bvdx = \frac{\mu_0 I}{2\pi x} vdx$$

$$\Rightarrow \quad \xi = \int_a^b d\xi = \frac{\mu_0 I v}{2\pi} \ln\left(\frac{b}{a}\right)$$

So, induced current in the conductor EF is

$$i = \frac{\xi}{R} = \frac{\mu_0 I v}{2\pi R} \ln\left(\frac{b}{a}\right)$$

Force on conductor EF due to field of PQ is

$$dF = Bidx$$

$$\Rightarrow \quad dF = \left(\frac{\mu_0 I}{2\pi x}\right)\left[\frac{\mu_0 I v}{2\pi R} \ln\left(\frac{b}{a}\right)\right]dx$$

$$\Rightarrow \quad F = \int_a^b dF = \frac{1}{vR}\left[\frac{\mu_0 I v}{2\pi} \log_e\left(\frac{b}{a}\right)\right]^2$$

Hence, the correct answer is (A).

152. The magnetic field produced at the site of wire CD is into the plane of paper. The force on positive charges on the wire CD is in the direction of $\vec{v} \times \vec{B}$ which is towards point C. So, C will be at higher potential.

Hence, the correct answer is (A).

154. The induced emf between points O and Q is $2Brv$. However, as there is no change in magnetic flux through the ring, the net emf induced in the ring is zero and hence, the induced current is zero.

Hence, the correct answer is (A).

155. The back emf equals the applied voltage potential drop across armature coil

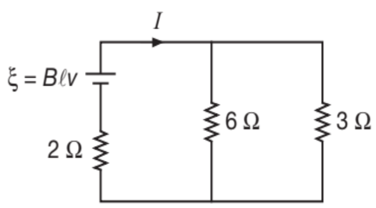
$$\Rightarrow \quad \xi = 200 - iR$$

$$\Rightarrow \quad \xi = 200 - 1.5 \times 20$$

$$\Rightarrow \quad \xi = 170 \text{ V}$$

Hence, the correct answer is (B).

156. The equivalent circuit for this set up is



$$F = BI\ell = B\left(\frac{\xi}{R}\right)\ell$$

$$\Rightarrow \quad F = B\left(\frac{B\ell v}{R}\right)\ell = \frac{B^2\ell^2 v}{R}, \text{(opposite to } v\text{)}$$

where $R = \frac{(6)(3)}{6+3} + 2 = 4\ \Omega$

$$\Rightarrow \quad F = \frac{(2)^2(2)^2(4)}{2} = 32 \text{ N}$$

Hence, the correct answer is (D).

157. $L_{eq} = \frac{(5.5)(5)}{5.5+5} = 2.619\ H$

Hence, the correct answer is (A).

158. In steady state, the entire current passes through the inductor.

Hence, the correct answer is (D).

159. The combined inductance of L and $2L$ in parallel is $2\frac{L}{3}$. The steady state current is independent of inductance and is equal to $\frac{V}{R}$.

Hence, the correct answer is (A).

160. $|V| = L\frac{dI}{dt}$

$$|V| = (1)\frac{d}{dt}(3t\sin t)$$

$$|V| = 3\sin t + 3t\cos t$$

Hence, the correct answer is (B).

161. In horizontal position, the motion of rod in the plane of magnetic field and hence, induced emf is zero. In vertical position,

$$\xi = Blv = 4 \times 10^{-5} \times 3 \times \left(30 \times \frac{5}{18}\right) = 10^{-3} \text{ V}$$

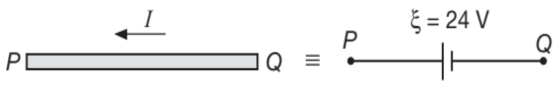
Hence, the correct answer is (B).

162. $\xi = B\ell v_{\perp}$

$$\Rightarrow \quad \xi = (3)(2)(8\sin 30)$$

$$\Rightarrow \quad \xi = 24 \text{ V}$$

From Fleming's Right Hand Rule, the induced current in the rod PQ is directed from Q to B, thus giving an equivalent emf replacement of the motional emf as



So, P is at a higher potential.

Hence, the correct answer is (B).

Please note that when the moving rod is replaced by its motional emf ξ, then actually the induced current going from Q to P creates a confusion that Q must be at a higher potential. However, do not forget that current goes from higher potential to lower potential in the external circuit but inside a single battery connected in a circuit it always goes from lower potential to higher potential. The most important thing is that the rod is showing the internal part of the battery and not the external circuit. So, it is P that is at a higher potential and not Q.

163. Since $V_A - V_B = L\dfrac{di}{dt}$

$\Rightarrow \quad V_A - V_B = L(-\alpha) = -\alpha L$

Hence, the correct answer is (B).

164. $\vec{A} = (ab)\hat{k}$ (perpendicular to xy-plane)

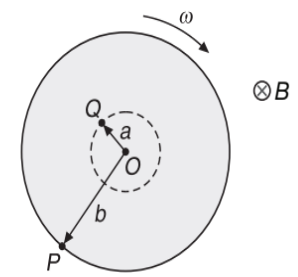
$\phi = \vec{B}\cdot\vec{A} = (50)(ab) = \text{constant}$

$\Rightarrow \quad \dfrac{d\phi}{dt} = 0$

$\Rightarrow \quad \xi = 0$

Hence, the correct answer is (D).

165. If the disc were without the hole then

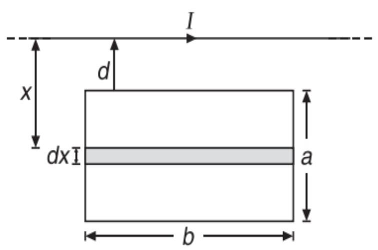


$$V_P - V_O = \frac{1}{2}Bb^2\omega \text{ and } V_Q - V_O = \frac{1}{2}Ba^2\omega$$

$$\Rightarrow \quad V_P - V_Q = \frac{B\omega}{2}(b^2 - a^2)$$

Hence, the correct answer is (B).

166. To calculate the magnetic flux through the rectangular loop, we consider an elemental strip of width dx at a distance x from the wire carrying current I as shown in Figure.



The magnetic flux associated with the strip is

$$d\phi = \frac{\mu_0 I}{2\pi x} b\,dx$$

Total magnetic flux linked with the loop is

$$\phi = \int d\phi$$

$$\Rightarrow \quad \phi = \frac{\mu_0 I b}{2\pi}\int_a^{a+b}\frac{dx}{x}$$

$$\Rightarrow \quad \phi = \left(\frac{\mu_0 I b}{2\pi}\right)\ln x\Big|_a^{a+b} = \frac{\mu_0 I b}{2\pi}\ln\left(\frac{a+b}{a}\right)$$

The induced emf in the loop is

$$\xi = -\frac{d\phi}{dt}$$

$$\Rightarrow \quad \xi = -\frac{\mu_0 b}{2\pi}\ln\left(\frac{a+b}{a}\right)\frac{dI}{dt}$$

$$\Rightarrow \quad \xi = -\frac{\mu_0 b}{2\pi}\ln\left(\frac{a+b}{a}\right)\frac{d}{dt}\left(I_0 e^{-\frac{t}{\tau}}\right)$$

$$\Rightarrow \quad \xi = \frac{\mu_0 b I_0}{2\pi\tau}\ln\left(\frac{a+b}{a}\right)e^{-\frac{t}{\tau}}$$

At $t = \tau$, we have

$$\xi = \frac{\mu_0 b I_0}{2\pi\tau}\ln\left(\frac{a+b}{a}\right)e^{-1}$$

$$\Rightarrow \quad \xi = \frac{\mu_0 b I_0}{2\pi e\tau}\ln\left(\frac{a+b}{a}\right)$$

Hence, the correct answer is (B).

167. Since $\xi = B\ell v$

So, if Q is the charge across the capacitor, then current in the branch containing the capacitor is

$$I = \frac{dQ}{dt}, \text{ where } Q = C\xi = B\ell Cv$$

$$\Rightarrow \quad I = B\ell C\left(\frac{dv}{dt}\right)$$

Since $\dfrac{dv}{dt} = 0$ $\quad \{\because v = \text{constant}\}$

So, $I = 0$

Hence, the correct answer is (D).

168. Current increases with time. So, flux passing through B will increase with time. From Lenz's law, it should have a tendency to move away from the coil to decrease flux.

Hence, the correct answer is (B).

169. Magnetic field through Q (by I_2) is downwards. By decreasing I_1, downward magnetic field through Q will decrease. Hence, induced current in Q should produce magnetic field in same direction.

Hence, the correct answer is (A).

170. $E = L\dfrac{dI}{dt} + IR$

$\Rightarrow \quad E = \xi + IR$

$\Rightarrow \quad \xi = E - IR$

So, the graph of ξ vs I is a straight line with negative slope.

Hence, the correct answer is (B).

171. For $E \neq 0$, ϕ must change

$$\Rightarrow \quad \frac{d\phi}{dt} \neq 0$$

Hence, the correct answer is (C).

172. For $r < R$, i.e. inside

$$E_{\text{in}}(2\pi r) = \left|\frac{d\phi}{dt}\right| = \pi r^2 \frac{dB}{dt}$$

$$\Rightarrow \quad E_{\text{in}} = \frac{r\,dB/dt}{2}$$

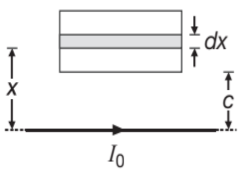
For $r > R$, i.e., outside

$$E_{\text{out}}(2\pi r) = \left|\frac{d\phi}{dt}\right| = \pi R^2 \frac{dB}{dt}$$

$$\Rightarrow \quad E_{\text{out}} = \frac{R^2}{2r}\left(\frac{dB}{dt}\right)$$

Hence, the correct answer is (D).

173. Consider an infinitesimal element of length b, thickness dx at a distance x from the wire.



Since $d\phi = BdA$

$$\Rightarrow \quad d\phi = \frac{\mu_0 I}{2\pi x}(bdx)$$

$$\Rightarrow \quad \phi = \int d\phi = \frac{\mu_0 I b}{2\pi}\int_c^{c+a}\frac{dx}{x}$$

$$\Rightarrow \quad \phi = \frac{\mu_0 b I}{2\pi}\log_e\left(\frac{c+a}{c}\right)$$

Given that $I = I_0\left(1 - \dfrac{t}{\tau}\right)$

$$\Rightarrow \quad \phi = \frac{\mu_0 b}{2\pi}\log_e\left(\frac{c+a}{c}\right)\left[I_0\left(1 - \frac{t}{\tau}\right)\right]$$

Since, $\xi = -\dfrac{d\phi}{dt}$

$$\Rightarrow \quad \xi = -\frac{\mu_0 b I_0}{2\pi}\log_e\left(\frac{c+a}{c}\right)\left(-\frac{1}{\tau}\right)$$

$$\Rightarrow \quad \xi = \frac{\mu_0 b I_0}{2\pi\tau}\log_e\left(\frac{c+a}{c}\right)$$

$$\Rightarrow \quad I = \frac{\xi}{R} = \frac{\mu_0 b I_0}{2\pi\tau R}\log_e\left(\frac{c+a}{c}\right)$$

$$\Rightarrow \quad \frac{dq}{dt} = \frac{\mu_0 b I_0}{2\pi\tau R}\log_e\left(\frac{c+a}{c}\right)$$

$$\Rightarrow \quad dq = \frac{\mu_0 b I_0}{2\pi\tau R}\log_e\left(\frac{c+a}{c}\right)dt$$

$$\Rightarrow \quad q = \frac{\mu_0 b I_0}{2\pi\tau R}\log_e\left(\frac{c+a}{c}\right)\int_0^{\tau} dt$$

$$\Rightarrow \quad q = \frac{\mu_0 b I_0}{2\pi R}\log_e\left(\frac{c+a}{c}\right)$$

Hence, the correct answer is (C).

174. At time t side of square is

$$l = l_0 - \alpha t$$

Area of square is

$$A = l^2 = (l_0 - \alpha t)^2$$

At given time, we have

$$\frac{dl}{dt} = -\alpha$$

Since, $\phi = BA = B(l_0 - \alpha t)^2$

$$\Rightarrow \quad \xi = \left|\frac{d\phi}{dt}\right| = 2B\alpha(l_0 - \alpha t)$$

But, $(l_0 - \alpha t) = a$

$$\Rightarrow \quad \xi = 2a\alpha B$$

Hence, the correct answer is (A).

175. Consider a point at distance r from centre.

For $r < R$, emf developed is

$$\xi = \frac{d\phi}{dt} = \pi r^2 \frac{dB}{dt} = \pi r^2 \alpha$$

$$\Rightarrow \quad E_{\text{nc}}(2\pi r) = \pi r^2 \alpha$$

$$\Rightarrow \quad E_{\text{nc}} \propto r$$

For $r > R$, $E_{\text{nc}}(2\pi r) = \pi R^2 \alpha$

$$\Rightarrow \quad E_{\text{nc}} \propto \frac{1}{r}$$

Hence, the correct answer is (D).

176. If I be the current flowing in the larger coil, then field at the centre of coil is $B=\frac{\mu_0 I}{2R}$

Flux linked with smaller coil is

$\phi=(\text{Area of smaller coil})B$

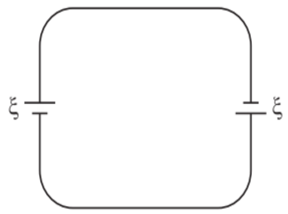
$\Rightarrow \quad \phi=(\pi r^2)B$

$\Rightarrow \quad \phi=\frac{\mu_0 \pi r^2}{2R}I$

$\Rightarrow \quad M=\frac{\phi}{I}=\frac{\mu_0 \pi r^2}{2R}$

Hence, the correct answer is (B).

177. The left and right arms of the tubes can be replaced by a battery of emf $\xi=Blv$ each. The emf induced in the circuit is $2\xi=2Blv$



Hence, the correct answer is (B).

178. $I=\frac{\xi}{R_{eq}}=\frac{B\ell v}{\left(\frac{R_{\text{Half}}}{2}\right)}$ where

$R_{\text{Half}}=\pi(2)\left(\frac{2}{\pi}\right)=4\ \Omega$

$\Rightarrow \quad I=\frac{(2)(4)(2)}{2}=8\text{ A}$

So, if F_m is the magnetic force, then

$F_m=BI\ell$

$\Rightarrow \quad F_m=(2)(8)(4)=64\text{ N}$

Hence, the correct answer is (D).

179. Steady state current through inductor in $\frac{E}{R}$.

So, at $t=0$, current in closed loop will remain same.

Hence, the correct answer is (C).

180. At $t=0$, $i=\frac{E}{R}$

Now, this current will decay in closed loop in anti-clockwise direction. So, $|i_2|=i_2=\frac{E}{R}$ in upward or opposite direction.

$\Rightarrow \quad i_2=-\frac{E}{R}$

Hence, the correct answer is (B).

181. $\frac{I}{I_0}=10\%=\frac{1}{10}$

Since, $I=I_0\left(1-e^{-t/\tau}\right)$

$\Rightarrow \quad \frac{1}{10}=1-e^{-t/\tau}$

$\Rightarrow \quad \frac{9}{10}=e^{-t/\tau}$

$\Rightarrow \quad \frac{t}{\tau}=\log_e\left(\frac{10}{9}\right)$

$\Rightarrow \quad t=\tau\log_e(1.1)$

Hence, the correct answer is (A).

182. 10% less than the steady state value implies a growth of 90%

$\Rightarrow \quad \frac{I}{I_0}=90\%=\frac{9}{10}$

Since

$I=I_0\left(1-e^{-t/\tau}\right)$

$\Rightarrow \quad \frac{1}{10}=e^{-t/\tau}$

$\Rightarrow \quad t=\tau\log_e(10)$

Hence, the correct answer is (B).

183. From right hand rule, we can see that points P and Q are at higher potential than the point O.

Hence, the correct answer is (A).

184. $U=\frac{1}{2}LI^2$ and $P=I^2R$

$\Rightarrow \quad L=\frac{2U}{I^2}$ and $R=\frac{P}{I^2}$

So, $\tau_L=\frac{L}{R}=\frac{2U}{P}$

Hence, the correct answer is (C).

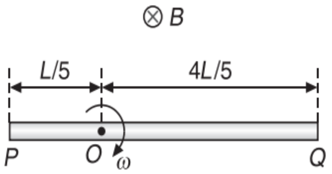
185. When the loops approach, the field between them becomes strong and according to Lenz's law the field must not become strong, so the current decreases in both the loops.

Hence, the correct answer is (A).

186. Magnetic field of ring is also along its axis, or in the direction of velocity of charged particle.
Hence, no magnetic force will act on charged particle. However, due to g velocity of charged particle will increase.
Hence, the correct answer is (C).

187. According to Lenz's Law, induced effects always oppose the cause due to which they are produced. So, when the first loop is moved towards the smaller loop, it will face repulsion.
Hence, the correct answer is (B).

188. From the figure, we have



$$V_Q - V_O = \frac{1}{2}B\omega\left(\frac{4L}{5}\right)^2 = \frac{16}{50}B\omega L^2 \text{ and}$$

$$V_P - V_O = \frac{1}{2}B\omega\left(\frac{L}{5}\right)^2 = \frac{1}{50}B\omega L^2$$

$$\Rightarrow \quad V_Q - V_P = \frac{15}{50}B\omega L^2 = \frac{3}{10}B\omega L^2$$

Hence, the correct answer is (C).

189. Time constant (τ) is the time during which the current grows from zero to 0.632 (or 63.2%) times the maximum value. Since, this is being done in a duration of 1 second, so $\tau = 1\ s$
Hence, the correct answer is (C).

190. Since, the motional emf, ξ, is

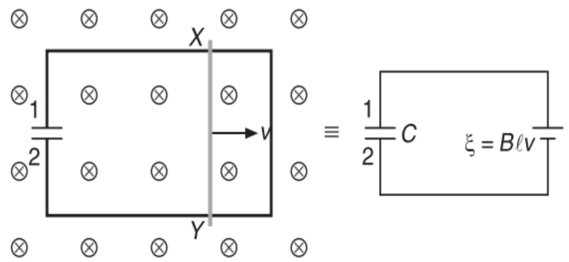
$$\xi = B\ell v$$

$$\Rightarrow \quad \xi = (2)(0.5)(4) = 4 \text{ V}$$

Since $Q = C\xi$

$$\Rightarrow \quad Q = (4)(5) = 20\ \mu\text{C}$$

The equivalent circuit for the arrangement is



So, $q_1 = +20\ \mu\text{C}$ and $q_2 = -20\ \mu\text{C}$

Hence, the correct answer is (D).

191. $|\xi| = \frac{d\phi}{dt}$

$$\Rightarrow \quad |\xi| = A\frac{dB}{dt}$$

$$\Rightarrow \quad |\xi| = (d^2)\frac{d}{dt}\left[B_0\left(1+\frac{x}{a}\right)\right]$$

$$\Rightarrow \quad |\xi| = (d^2)B_0\left(0+\frac{1}{a}\frac{dx}{dt}\right)$$

$$\Rightarrow \quad |\xi| = \frac{B_0 d^2}{a}\left(\frac{dx}{dt}\right)$$

$$\Rightarrow \quad |\xi| = \frac{B_0 d^2 v_0}{a}$$

Hence, the correct answer is (D).

192. Initial current is

$$I_{\text{initial}} = \frac{10}{10} = 1 \text{ A}$$

$$\Rightarrow \quad \phi_{\text{initial}} = L(I_{\text{initial}}) = 500 \text{ mWb} = 0.5 \text{ Wb}$$

Final current is

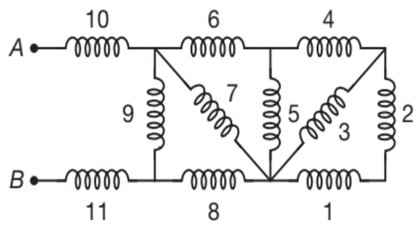
$$I_{\text{final}} = \frac{20}{5} = 4 \text{ A}$$

$$\Rightarrow \quad \phi_{\text{final}} = L(I_{\text{final}}) = (0.5)\times 4 = 2 \text{ Wb}$$

$$\Rightarrow \quad \Delta\phi = 1.5 \text{ Wb}$$

Hence, the correct answer is (B).

193. 1 and 2 in series in parallel with 3. The result of this is in series with 4, which is in parallel with 5, again in series with 6, which again is in parallel with 7, in series with 8 and in parallel with 9. Finally result of above steps, 10 and 11 are in series to get 2.618 H.



Hence, the correct answer is (D).

194. In decay of current through L-R circuit, current cannot remain constant.
Hence, the correct answer is (D).

195. $\frac{1}{2} = e^{-t/RC}$

$$\Rightarrow \quad \frac{1}{2} = e^{-t/2}$$

$\Rightarrow \quad e^{t/2} = 2$

$\Rightarrow \quad \frac{t}{2} = 0.693$

$\Rightarrow \quad t = 1.38$ s

Hence, the correct answer is (A).

196. Applying Kirchhoff's law to the circuit starting from E_1, we get

$$E_1 + iR + L\frac{di}{dt} - E_2 = 0$$

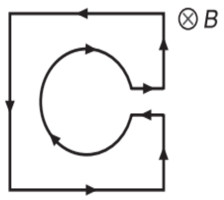
$$\Rightarrow \quad E_2 - E_1 = iR + L\frac{di}{dt}$$

Hence, the correct answer is (B).

197. $|\xi| = \frac{d\phi}{dt} = A\frac{dB}{dt} = (4b^2 - \pi a^2)B_0$

$$i = \frac{|\xi|}{R} = \frac{(4b^2 - \pi a^2)B_0}{R}$$

Inward ⊗ magnetic field is increasing. So, an outward ⊙ magnetic field is produced due to induction.



Hence, the correct answer is (D).

198. Since $U = \frac{1}{2}LI^2$

$$\Rightarrow \quad \frac{dU}{dt} = LI\left(\frac{dI}{dt}\right)$$

For the growth of current, we have

$$I = I_0\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$\Rightarrow \quad \frac{dI}{dt} = \frac{I_0R}{L}e^{-\frac{Rt}{L}}$$

$$\Rightarrow \quad \frac{dU}{dt} = L\left(\frac{I_0R}{L}\right)I_0\left(1 - e^{-\frac{Rt}{L}}\right)e^{-\frac{Rt}{L}}$$

$$\Rightarrow \quad \frac{dU}{dt} = I_0^2R\left(e^{-\frac{Rt}{L}} - e^{-\frac{2Rt}{L}}\right)$$

This is best represented by the curve shown in OPTION (C).

Hence, the correct answer is (C).

199. At time t side of square is

$$l = (a + 2v_0t)$$

Area of square is

$$A = l^2 = (a + 2v_0t)^2$$

Since $\phi = BA = B(a + 2v_0t)^2$

$$\Rightarrow \quad \xi = \frac{d\phi}{dt} = 4Bv_0(a + 2v_0t)$$

The resistance of the square frame at this instant is

$$R = \lambda(4l) = 4\lambda(a + 2v_0t)$$

$$\Rightarrow \quad i = \frac{\xi}{R} = \frac{Bv_0}{\lambda}$$

Hence, the correct answer is (C).

200. The induced emf is

$$\xi = Bvl = 0.5 \times 4 \times 0.25 = 0.5 \text{ V}$$

Since, $12\ \Omega$ and $4\ \Omega$ are parallel. Hence, their net resistance is $R = 3\ \Omega$.

$$\Rightarrow \quad i = \frac{\xi}{R + r} = \frac{0.5}{3 + 2} = 0.1 \text{ A}$$

Hence, the correct answer is (A).

201. At any instant

$$I = I_0(1 - e^{-t/\tau})$$

$$\Rightarrow \quad \frac{dI}{dt} = \frac{I_0}{\tau}e^{-t/\tau}$$

$$\Rightarrow \quad \frac{dI}{dt} = \frac{E}{R}\frac{R}{L}e^{-t/\tau} \qquad \left\{\because \tau = \frac{L}{R}\right\}$$

$$\Rightarrow \quad V_L = L\frac{dI}{dt} = Ee^{-t/\tau}$$

So, potential difference across the coil is $Ee^{-t/\tau}$

Hence, the correct answer is (D).

202. At any instant

$$I = I_0(1 - e^{-t/\tau})$$

$$\Rightarrow \quad \frac{dI}{dt} = \frac{I_0}{\tau}e^{-t/\tau}$$

$$\Rightarrow \quad \frac{dI}{dt} = \frac{E}{R}\frac{R}{L}e^{-t/\tau} \qquad \left\{\because \tau = \frac{L}{R}\right\}$$

$$\Rightarrow \quad L\frac{dI}{dt} = Ee^{-t/\tau}$$

Since, potential difference across the inductor coil is

$$V_L = L\frac{dI}{dt}$$

$$\Rightarrow \quad V_L = L\frac{dI}{dt} = Ee^{-t/\tau}$$

Let t be the time when potential difference across coil equals potential difference across the resistor R.

$$\Rightarrow \quad Ee^{-t/\tau} = IR$$

$$\Rightarrow \quad Ee^{-t/\tau} = (I_0R)\left(1-e^{-t/\tau}\right) \qquad \left\{\because I = I_0\left(1-e^{-t/\tau}\right)\right\}$$

$$\Rightarrow \quad Ee^{-t/\tau} = E\left(1-e^{-t/\tau}\right)$$

$$\Rightarrow \quad 2e^{-t/\tau} = 1$$

$$\Rightarrow \quad e^{t/\tau} = 2$$

$$\Rightarrow \quad t = \tau\log_e(2)$$

Hence, the correct answer is (C).

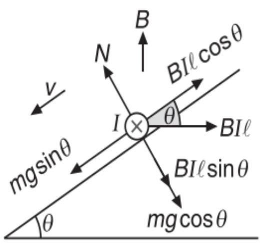
203. In steady state, the impedance of an ideal inductor is zero. In steady state, the entire current will pass throughout the inductor and therefore, the final value of current in 10 Ω resistor is zero.

Hence, the correct answer is (D).

204. For conductor to move with constant speed v, we have

$$mg\sin\theta = IlB\cos\theta$$

where $I = \dfrac{Blv\cos\theta}{R}$



$$\Rightarrow \quad mg\sin\theta = \left(\frac{Blv}{R}\right)lB\cos^2\theta$$

$$\Rightarrow \quad B^2 = \frac{mgR\sin\theta}{vl^2\cos^2\theta}$$

$$\Rightarrow \quad B = \sqrt{\frac{mgR\sin\theta}{vl^2\cos^2\theta}}$$

Hence, the correct answer is (D).

205. The induced emf is

$$\xi = \frac{1}{2}B\omega l^2$$

Since $v = r\omega = \dfrac{l}{2}\omega$

$$\Rightarrow \quad \xi = Blv$$

Hence, the correct answer is (A).

206. At time t, angle rotated by loop is $\theta = \omega t$. This is also the angle between $\vec{B}$ and $\vec{A}$. So, flux associated is

$$\phi = BA\cos\theta$$

$$\Rightarrow \quad \phi = Bb^2\cos\omega t$$

$$\Rightarrow \quad \xi = \left|\frac{d\phi}{dt}\right| = b^2B\omega\sin\omega t$$

Hence, the correct answer is (A).

207. When solenoid is cut in two parts, then

$$L_1 = \left(\frac{\eta}{\eta+1}\right)L,\ R_1 = \left(\frac{\eta}{\eta+1}\right)R$$

$$L_2 = \left(\frac{1}{\eta+1}\right)L,\ R_2 = \left(\frac{1}{\eta+1}\right)R$$

When these parts are connected in parallel, then

$$L_{net} = \frac{L_1L_2}{L_1+L_2}$$

$$R_{net} = \frac{R_1R_2}{R_1+R_2}$$

$$\Rightarrow \quad \tau_L = \frac{L_{net}}{R_{net}} = \frac{L}{R}$$

Hence, the correct answer is (A).

208. In steady state, the inductor has a lower resistance, so the majority of current passes through it. When the switch is suddenly opened, this current now passes through the bulb until it decays to zero.
So, the bulb will glow very brightly for a moment.

Hence, the correct answer is (C).

209. $\tau_L = \dfrac{L}{R} = \dfrac{0.01}{10} = 10^{-3}$ s

$$\tau_C = CR = \left(0.1\times10^{-3}\right)(10) = 10^{-3}\ \text{s}$$

$$\Rightarrow \quad (i_0)_L = \frac{20}{10} = 2\ \text{A}$$

$$\Rightarrow \quad (i_0)_C = \frac{20}{10} = 2\ \text{A}$$

The given time is the half-life time of both the circuits, so

$$i_L = i_C = \frac{2}{2} = 1\ \text{A}$$

Hence, total current is 2 A

Hence, the correct answer is (A).

210. I_0 = Peak value $= \dfrac{E}{2R}$

Total heat produced across R is $H = \dfrac{1}{2}LI_0^2$

CHAPTER 3

$\Rightarrow \quad H = \frac{1}{2}(2L)\frac{E^2}{4R^2}$

$\Rightarrow \quad H = \frac{LE^2}{4R^2}$

Hence, the correct answer is (C).

211. $I = \frac{I_0}{\alpha}$ at $t = t_0$

Since, $I = I_0 e^{-t_0/\tau}$

$\Rightarrow \quad \frac{1}{\alpha} = e^{-t_0/\tau}$

$\Rightarrow \quad \alpha = e^{t_0/\tau}$

$\Rightarrow \quad t_0 = \tau \log_e \alpha$

$\Rightarrow \quad \tau = \frac{t_0}{\log_e \alpha}$

Hence, the correct answer is (C).

212. Since the charge moves along the magnetic field of the solenoid, the force acting on it is zero. So, its acceleration is equal to g.

Hence, the correct answer is (C).

213. $V_L = L\frac{dI}{dt}$

$4t = 2\frac{dI}{dt}$

$\Rightarrow \quad dI = 2t\,dt$

$\Rightarrow \quad I = \int dI = 2\int_0^4 t\,dt = 2\left(\frac{16}{2}\right) = 16 \text{ A}$

If U be the energy stored in the coil, then

$$U = \frac{1}{2}LI^2 = \left(\frac{1}{2}\right)(2)(16)^2 = 256 \text{ J}$$

Hence, the correct answer is (B).

214. Since, $\xi = \frac{d\phi}{dt} = \frac{d}{dt}(NBA)$

$\Rightarrow \quad \xi = \frac{d}{dt}\left[NA(\mu_0 n I)\right]$

$\Rightarrow \quad \xi = NA\mu_0 n\left(\frac{dI}{dt}\right)$

where N is total number of turns in the coil and n is the number of turns per unit length in the solenoid.

$$\xi = (300)(1.2\times10^{-3})(4\pi\times10^{-7})\times\frac{2000}{0.3}\times\frac{4}{0.25}$$

$\Rightarrow \quad \xi = 4.8\times10^{-2} \text{ V} = 48 \text{ mV}$

Hence, the correct answer is (D).

215. Let M be the mutual inductance between X and Y. By definition.

$$\xi_y = M\frac{dI_x}{dt}$$

$\Rightarrow \quad E = M\frac{dI}{dt} = M\dot{I}$

$\Rightarrow \quad M = \frac{E}{\dot{I}}$

The flux linked with X is

$$\phi_X = MI_Y = \left(\frac{E}{\dot{I}}\right)I_0$$

Hence, the correct answer is (B).

216. Let the field be E at a distance x from the centre of the disc. Then

$$eE = mx\omega^2$$

$\Rightarrow \quad E = \left(\frac{m\omega^2}{e}\right)x$

Since $\Delta V = \left|\int_0^a E\,dx\right| = \frac{m\omega^2}{e}\int_0^a x\,dx = \frac{m\omega^2 a^2}{2e}$

Hence, the correct answer is (A).

Multiple Correct Choice Type Questions

1. When the magnet is above R, it is repelled by the ring and when it is below R, it is attracted by the ring. In both cases, its acceleration is less than g.

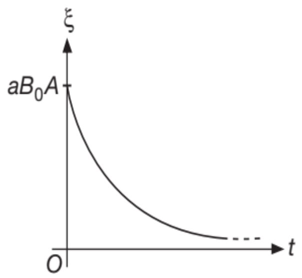
Hence, (A) and (C) are correct.

3. $V = |\xi| = Blv$, where $l = \frac{L}{2}$

The polarity of this motional emf is obtained by using Flemings Right Hand Rule.

Hence, (A) and (C) are correct.

4.



$$\xi = -A\frac{dB}{dt}$$

$$\Rightarrow \quad \xi = AB_0 a e^{-at}$$

Hence, (B) and (D) are correct.

5. $$Q = \frac{1}{R}|\Delta\phi|$$

$$\phi_i = NBA\cos 0 = NBA = 2NB\ell^2$$

$$\phi_f = NBA\cos\theta = 2NB\ell^2\cos\theta$$

$$R = 2\lambda(2\ell + \ell) = 6\lambda\ell$$

$$\Rightarrow \quad Q = \frac{2NB\ell^2}{6\lambda\ell}|1-\cos\theta|$$

For $\theta = 90°$, $Q = \frac{NB\ell}{3\lambda}$

For $\theta = 180°$, $Q = \frac{2NB\ell}{3\lambda}$

For $\theta = 270°$, $Q = \frac{NB\ell}{3\lambda}$

For $\theta = 360°$, $Q = \frac{NB\ell}{3\lambda}$

Hence, (A) and (B) are correct.

6. In the case of LC oscillations, when the capacitor if fully charged, then we have

$$q = q_0\cos\omega t \text{ and } i = -q_0\omega\sin\omega t$$

where, $\omega = \frac{1}{\sqrt{LC}}$

According to the problem, the electric and magnetic fields store equal energy, so

$$\frac{q^2}{2C} = \frac{1}{2}Li^2$$

$$\Rightarrow \quad \frac{q_0\cos^2\omega t}{2C} = \frac{1}{2}Lq_0^2\omega^2\sin^2\omega t$$

$$\Rightarrow \quad \cot^2\omega t = 1$$

$$\Rightarrow \quad \omega t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \ldots.$$

$$\Rightarrow \quad t = \frac{\pi\sqrt{LC}}{4}, \frac{3\pi\sqrt{LC}}{4}, \frac{5\pi\sqrt{LC}}{4}, \frac{7\pi\sqrt{LC}}{4} \ldots.$$

Hence, (A) and (C) are correct.

8. Since $\tau_L = \frac{L}{R} = \frac{2}{2} = 1$ s

$$\Rightarrow \quad t_{\frac{1}{2}} = (\ln 2)\tau_L = (\ln 2) \text{ s}$$

Hence, the given time is half-life time.

$$\Rightarrow \quad i = \frac{i_0}{2} = \frac{8/2}{2} = 2 \text{ A}$$

Rate of energy supplied by battery is

$$P_{\text{supplied}} = Ei = 8\times 2 = 16 \text{ Js}^{-1}$$

Rate of heat dissipated across the resistor is

$$P_R = i^2R = (2)^2(2) = 8 \text{ Js}^{-1}$$

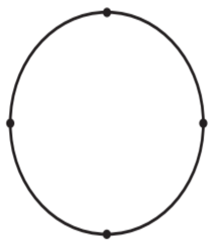
Also, $V_a - V_b = E - iR = 8 - 2\times 2 = 4$ V

Hence, (A), (B) and (D) are correct.

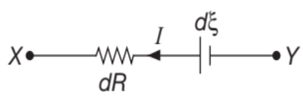
9. $$\xi = \left|\frac{d\phi}{dt}\right| = A\left(\frac{dB}{dt}\right) = \pi r^2 B \qquad \left\{\because \frac{dB}{dt} = B\right\}$$

Consider an infinitesimal element of length $d\ell$ on the ring. If $d\xi$ be emf induced across this element and dR be the resistance of this element, then

$$d\xi = \left(\frac{\xi}{2\pi r}\right)d\ell \text{ and } dR = \left(\frac{R}{2\pi r}\right)d\ell$$



For the arrangement of infinitesimal element and infinitesimal emf source, we have



$$V_X + I(dR) - d\xi - V_Y = 0$$

$$\Rightarrow \quad V_X - V_Y = -Idr + d\xi$$

$$\Rightarrow \quad V_X - V_Y = -\left(\frac{\xi}{R}\right)\left(\frac{R}{2\pi r}\right)d\ell + \left(\frac{\xi}{2\pi r}\right)d\ell = 0$$

So, all the points on the ring are at the same potential.

Hence, (B) and (D) are correct.

10. $$e = -\frac{d\phi}{dt}$$

$$e = -a(n^2-1)t^{n^2-2}$$

$\Rightarrow \quad |e| = a(n^2-1)t^{n^2-2}$

For $n=\pm 1$, $e=0$

For $n^2-1=1$ we get $\phi = at$ i.e. $|e|=a$

$\Rightarrow \quad n=\pm\sqrt{2}$ for $|e|=a$

Hence, (A) and (B) are correct.

11. By Law of Conservation of Energy,

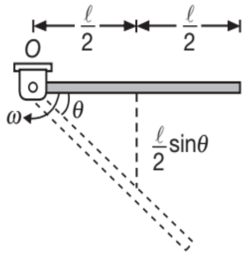
$$\begin{pmatrix}\text{Loss in Gravitational}\\ \text{Potential Energy}\end{pmatrix} = \begin{pmatrix}\text{Gain in Rotational}\\ \text{Kinetic Energy}\end{pmatrix}$$

$\Rightarrow \quad mg\left(\frac{\ell}{2}\sin\theta\right) = \frac{1}{2}I\omega^2$

$\Rightarrow \quad mg\left(\frac{\ell}{2}\sin\theta\right) = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$

$\Rightarrow \quad \omega = \sqrt{\frac{3g\sin\theta}{\ell}}$

Further



$$\xi = \frac{1}{2}B\omega\ell^2$$

$\Rightarrow \quad \xi = \frac{B}{2}\sqrt{\frac{3g\sin\theta}{\ell}}\,\ell^2$

$\Rightarrow \quad \xi = \frac{1}{2}B\sqrt{3g}(\sin\theta)^{\frac{1}{2}}\ell^{\frac{3}{2}}$

$\Rightarrow \quad \xi \propto B$

$\Rightarrow \quad \xi \propto \sqrt{\sin\theta}$

$\Rightarrow \quad \xi \propto \ell^{3/2}$

Hence, (A), (C) and (D) are correct.

12. As ba and bc are equal the potential difference of a and c will be same so we have

$$V_a - V_c = 0$$

and $V_a - V_b = V_c - V_b = \frac{B\omega l^2}{2}$

Hence, (A) and (C) are correct.

13. When an iron core is inserted inside a coil, its inductance increases. The steady state current, $I_0 = \frac{E}{R}$ remains the same.

The time constant, $\tau = \frac{L}{R}$ increases.

Hence, (A), (B) and (D) are correct.

14. Due to Lenz's Law

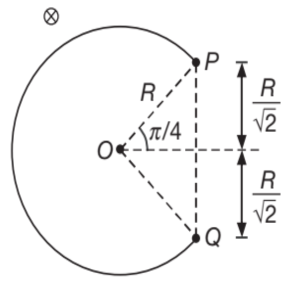
$$s < \frac{1}{2}g(2)^2$$

$\Rightarrow \quad s < 20\ m$

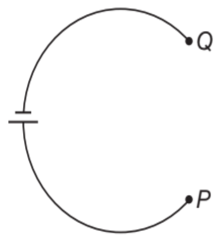
Hence, (A) and (C) are correct.

15. Length $PQ = 2\left(\frac{R}{\sqrt{2}}\right) = \sqrt{2}R$

So, $\xi = B(\ell_{PQ})v = \sqrt{2}BRv$



From Fleming's Right Hand Rule, the induced current in the equivalent rod PQ is from Q to P, so that the equivalent circuit diagram is



So, P is positive w.r.t. Q.

Hence, (B) and (C) are correct.

16. The flux initially and finally is

$$\phi_i = BA\cos 0^\circ = (4)(2) = 8\text{ Wb}$$

$$\phi_f = BA\cos 90^\circ = 0$$

$\Rightarrow \quad \Delta\phi = 8\text{ Wb}$

Since, $|\xi| = \frac{\Delta\phi}{\Delta t} = \frac{8}{0.1} = 80\text{ V}$

$\Rightarrow \quad i = \frac{|\xi|}{R} = 20\text{ A}$

Also, $\Delta q = \frac{|\Delta\phi|}{R} = 2\text{ C}$

We cannot find the heat generated without knowing the variation of current with time.

Hence, (B), (C) and (D) are correct.

17. $v < g(t)$

$\Rightarrow \quad v < 2g$ {due to Lenz's Law}

Also, $s < \frac{1}{2}gt^2$

$\Rightarrow \quad s < 2g$ {due to Lenz's Law}

Hence, (B) and (D) are correct.

18. At $t=0$, $I_2 = 0$ and $I_3 = \frac{E}{3R}$

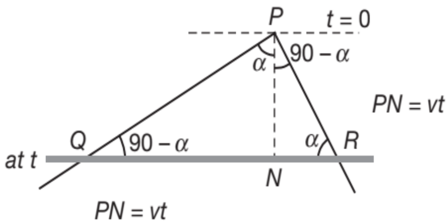
$\Rightarrow \quad I_1 = I_2 + I_3 = \frac{E}{3R}$

As $t \to \infty$, $I_3 = 0$, $I_2 = \dfrac{E}{2R}$

$$\Rightarrow \quad I_1 = I_2 + I_3 = \frac{E}{2R}$$

Hence, (B) and (D) are correct.

19.



In ΔPQN

$$\tan(90-\alpha) = \frac{PN}{QN} = \frac{vt}{QN}$$

$$\Rightarrow \quad QN = vt\tan\alpha \qquad \ldots(1)$$

In ΔPNR

$$\tan\alpha = \frac{PN}{RN} = \frac{vt}{RN}$$

$$\Rightarrow \quad RN = vt\cot\alpha \qquad \ldots(2)$$

Induced emf in the circuit at time t is

$$\xi = B(QN + NR)v$$

$$\Rightarrow \quad \xi = B(vt)(\tan\alpha + \cot\alpha)v$$

$$\Rightarrow \quad \xi = Bv^2 t(\tan\alpha + \cot\alpha)$$

$$\Rightarrow \quad \xi \propto v^2 \text{ and } \xi \propto t$$

Hence, (B) and (C) are correct.

20. $\xi(t) = -\dfrac{d\phi_m}{dt}$

$$\Rightarrow \quad \xi(t) = -\frac{d}{dt}[Ba]$$

$$\Rightarrow \quad \xi(t) = -a\frac{dB}{dt}$$

$$\Rightarrow \quad \xi(t) = -a\beta < 0 \qquad \{\because \beta > 0\}$$

The emf is constant and negative, so that induced electric field points around the ring F_2 towards F_1. So, face F_1 will develop an excess positive charge.

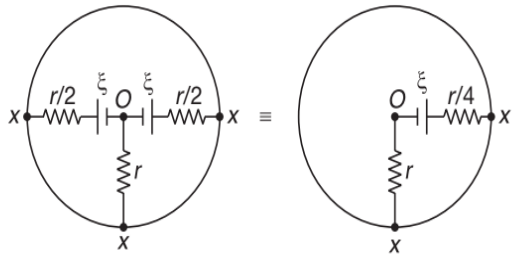
Hence, the correct answer is (A).

21. $E = \dfrac{|\xi|}{\delta} = \dfrac{a\beta}{\delta}$

This expression is independent of R as long as the radius of the ring exceeds the radius $\sqrt{\dfrac{a}{\pi}}$ of the solenoid.

Hence, (A) and (D) are correct.

22. The two halves of the rotating rod produce the motional emf, so the equivalent circuit of the given arrangement is shown in Figure.



Induced emf in each half of rod is

$$\xi = \frac{1}{2}B\omega a^2$$

Let x be the potential of ring and lets take the potential of the point O to be zero, then by making use of Kirchhoff's Loop Law (KLL), we get

$$\left(\frac{x-\xi}{r/4}\right) + \left(\frac{x-0}{r}\right) = 0$$

$$\Rightarrow \quad 5x = 4\xi$$

$$\Rightarrow \quad x = \frac{4\xi}{5} = \frac{2}{5}B\omega a^2$$

$$\Rightarrow \quad i = \frac{x}{r} = \frac{2B\omega a^2}{5r}$$

The direction of current in r will be from higher potential to lower potential i.e. from the circumference to the centre O.

Hence, (B) and (D) are correct.

23. The magnitude of emf developed in the ring is

$$\xi = \left|\frac{d\phi}{dt}\right| = \pi a^2\left(\frac{dB}{dt}\right) = \pi a^2\alpha$$

So, OPTION (B) is correct.

If R is the resistance of ring, then

$$I = \frac{\xi}{R} = \frac{\pi a^2\alpha}{R}$$

Consider a small length l of the ring, then

$$\xi' = \frac{\xi}{2\pi a}l = \frac{a\alpha dl}{2}$$

Its resistance is $r = \left(\dfrac{R}{2\pi a}\right)l$

The potential difference across it is

$$V = \xi' - Ir = \frac{a\alpha l}{2} - \left(\frac{\pi a^2\alpha}{R}\right)\left(\frac{R}{2\pi a}\right)l = 0$$

So, all points on the ring are at the same potential.

So, OPTION (A) is correct

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Since, $E(2\pi a) = \xi = \pi a^2 \alpha$

$$\Rightarrow \quad E = \frac{\alpha a}{2}$$

So, OPTION (D) is correct.

Hence, (A), (B) and (D) are correct.

24. $\xi = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\phi}{dt}$

Inside $(r < R)$

$$\Rightarrow \quad E(2\pi r) = -\frac{d}{dt}(B\pi r^2)$$

$$\Rightarrow \quad E = -\frac{1}{2} r \frac{dB}{dt}$$

Outside $(r > R)$

$$E(2\pi r) = -\pi R^2 \frac{dB}{dt}$$

$$\Rightarrow \quad E = -\frac{1}{2} \frac{R^2}{r} \frac{dB}{dt}$$

Hence, (A) and (C) are correct.

25. Using Fleming's Right Hand Rule, we get

$$\xi_1 = V_0 - V_P = \frac{1}{2} B\omega (3\ell)^2 = \frac{9}{2} B\omega \ell^2$$

$$\xi_2 = V_0 - V_Q = \frac{1}{2} B\omega (5\ell)^2 = \frac{25}{2} B\omega \ell^2$$

$$\Rightarrow \quad \xi_2 - \xi_1 = V_0 - V_Q - V_0 + V_P$$

$$\Rightarrow \quad \xi_2 - \xi_1 = V_P - V_Q = 8B\omega \ell^2$$

Hence, (B), (C) and (D) are correct.

26. $L = \frac{N\phi}{i}$

$$\Rightarrow \quad \phi = \frac{Li}{N}$$

So, SI unit of flux is henry-ampere.

$$L = \frac{-\xi}{\Delta i / \Delta t} = -\frac{\xi \Delta t}{\Delta i}$$

Hence, SI unit of L is $\frac{\text{V-s}}{\text{ampere}}$

Hence, (A) and (C) are correct.

27. $I = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right) = 5\left(1 - e^{-2t}\right)$

When potential difference across R is $4\ \text{V}$, the current is

$$I = \frac{4}{4} = 1\ \text{A}$$

$$\Rightarrow \quad 1 = 5\left(1 - e^{-2t}\right)$$

$$\Rightarrow \quad e^{-2t} = 0.8$$

At this instant, we have

$$\frac{dI}{dt} = 10e^{-2t} = 100 \times 0.8 = 8\ \text{As}^{-1}$$

Power supplied by battery is

$$P = EI = 20 \times 1 = 20\ \text{W}$$

Power consumed by resistor is

$$P_R = I^2 R = 1^2 \times 4 = 4\ \text{W}$$

So, power stored in magnetic field is

$$P_m = 20\ \text{W} - 4\ \text{W} = 16\ \text{W}$$

Hence, (A), (B), (C) and (D) are correct.

28. $L_1 \frac{dI_1}{dt} = L_2 \frac{dI_2}{dt}$

$$\Rightarrow \quad L_1 I_1 = L_2 I_2$$

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{L_2}{L_1} \qquad \ldots(1)$$

Steady state current passing through the resistor is

$$I = I_1 + I_2 = \frac{E}{R}$$

$$\Rightarrow \quad I_1 + \left(\frac{L_1}{L_2}\right) I_1 = \frac{E}{R}$$

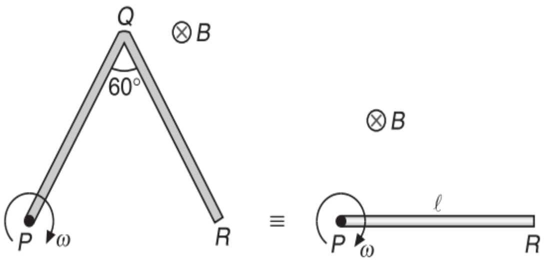
$$\Rightarrow \quad I_1 = \frac{EL_2}{R(L_1 + L_2)} \text{ and } I_2 = \frac{EL_1}{R(L_1 + L_2)}$$

Hence, (A) and (D) are correct.

29. The emf induced across the rod PQ is

$$\xi_{PQ} = V_Q - V_P = \frac{1}{2} B\omega l^2 \qquad \ldots(1)$$

Also, we see that the equivalent conducting rod that fits between the points P and R also has an equivalent length l (because the triangle is equilateral) as shown in Figure.



So, the emf induced across the ends of this rod is

$$\xi_{PR} = V_R - V_P = \frac{1}{2}B\omega l^2 \qquad \ldots(2)$$

From equations (1) and (2), we get

$$V_Q - V_R = 0$$

$$\Rightarrow \quad V_Q = V_R$$

Hence, (B), (C) and (D) are correct.

30. $\phi = BA = B(\ell + 2vt)^2$

$$\Rightarrow \quad \xi = -\frac{d\phi}{dt} = -4Bv(\ell + 2vt)$$

$$\Rightarrow \quad |\xi| = 4Bv(\ell + 2vt)$$

Further $I = \frac{|\xi|}{R} = \frac{4Bv(\ell + 2vt)}{4\lambda(\ell + 2vt)} = \frac{Bv}{\lambda}$

So, the correct graphs are the ones depicted in (A) and (D).

Hence, (A) and (D) are correct.

31. $\xi = A\frac{dB}{dt} = (\pi a^2)(2)$

$$\Rightarrow \quad \xi = 2\pi a^2$$

Since $I = \frac{dq}{dt} = \frac{\xi}{R} = \frac{2\pi a^2}{R} = \frac{2\pi a^2}{\lambda(2\pi a)} = \frac{a}{\lambda}$

$$Q = It = \left(\frac{a}{\lambda}\right)(3) = \frac{6\pi a^2}{R}$$

$$P = I^2R = \left(\frac{a^2}{\lambda^2}\right)R = \left(\frac{2\pi a^2}{R}\right)^2 R = \frac{4\pi^2 a^4}{R}$$

Hence, (A), (B), (C) and (D) are correct.

32. Just after closing switch, capacitor offers no impedance. So, current through R_3 is zero. Long after closing switch, inductor offers no impedance. So, current through R_3 is zero.

Hence, (A) and (B) are correct.

33. Since the peak value (steady state) current for both 1 and 2 is the same. So, we have

$$\frac{V}{R_1} = \frac{V}{R_2}$$

$$\Rightarrow \quad R_1 = R_2$$

Further 1 grows faster than 2, so we conclude that

$$\tau_{L_2} < \tau_{L_1}$$

$$\Rightarrow \quad L_2 < L_1 \qquad \left\{\because \tau_L = \frac{L}{R}\right\}$$

Hence, (B) and (C) are correct.

34. Given that $q = 2t^2$

$$\Rightarrow \quad i = \frac{dq}{dt} = 4t$$

$$\Rightarrow \quad \frac{di}{dt} = 4 \text{ As}^{-1}$$

At $t = 1$ s, $q = 2$ C, $i = 4$ A and $\frac{di}{dt} = 4 \text{ As}^{-1}$

$$\Rightarrow \quad V_a - V_b = V_{ab} = L\frac{di}{dt} = 1 \times 4 = 4 \text{ V}$$

$$\Rightarrow \quad V_b - V_c = V_{bc} = \frac{q}{C} = \frac{2}{2} = 1 \text{ V}$$

$$\Rightarrow \quad V_c - V_d = V_{cd} = iR = 4 \times 4 = 16 \text{ V}$$

$$\Rightarrow \quad V_{ad} = V_a - V_d = V_{ab} + V_{bc} + V_{cd} = 21 \text{ V}$$

Hence, (A), (B) and (C) are correct.

35. $R = \sqrt{\frac{L}{C}}$

$$\Rightarrow \quad R^2 = \frac{L}{C}$$

$$\Rightarrow \quad RC = \frac{L}{R}$$

$$\Rightarrow \quad \tau_C = \tau_L = \tau \text{ (say)}$$

Since $I_L = I_0\left(1 - e^{-\frac{t}{\tau_L}}\right) = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$ and

$$I_C = \frac{dq}{dt} = \frac{V}{R}e^{-\frac{t}{\tau}}$$

For $I_L = I_C$, we have

$$1 - e^{-\frac{t}{\tau}} = e^{-\frac{t}{\tau}}$$

$$\Rightarrow \quad 1 = 2e^{-\frac{t}{\tau}}$$

$$\Rightarrow \quad e^{\frac{t}{\tau}} = 2$$

$$\Rightarrow \quad \frac{t}{\tau} = \log_e(2) = \ln(2)$$

$$\Rightarrow \quad t = \tau\ln(2) = RC\ln(2) = \frac{L}{R}\ln(2)$$

Hence, (B) and (C) are correct.

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36. Due to current i in the straight conductor, the flux associated with the loop is

$$\phi = \int_{r}^{r+a} \frac{\mu_0 i}{2\pi x} a dx = \frac{\mu_0 i a}{2\pi} \ln\left(\frac{r+a}{r}\right)$$

$$\Rightarrow \quad M = \frac{\phi}{i} = \frac{\mu_0 a}{2\pi} \ln\left(1+\frac{a}{r}\right)$$

If the loop is pulled with speed v, then

$$\xi = -\frac{d\phi}{dt} = -\left(\frac{d\phi}{dr}\right)\left(\frac{dr}{dt}\right) = \left(\frac{a}{r^2}\frac{\mu_0 i a}{2\pi}\right)\left(\frac{r}{r+a}\right)(v)$$

$$\Rightarrow \quad \xi = \frac{\mu_0 i a^2 v}{2\pi r(r+a)}$$

If i increases, current in loop is induced in anticlockwise direction resulting in repulsion.

Hence, (A), (B) and (D) are correct.

37. The induced emf will first increase and then decrease. It is maximum when the loop is half outside the field. In this case,

$$\xi = B(\sqrt{2}l)v = Blv\sqrt{2}$$

Hence, (A) and (C) are correct.

39. Since there is no change in flux so, no current flows in the ring.

$$V_C = V_E \text{ and } V_D = V_A = B(2R)v$$

Hence, (A), (C) and (D) are correct.

40. By Law of Conservation of Energy, we have

$$P_B = P_R + P_L$$

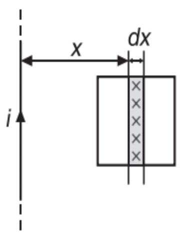
When the circuit is just closed, then we have

$$P_R = i^2R \text{ and } P_L = Li\frac{di}{dt}$$

Since $\frac{di}{dt}$ has greater value at the instant when the circuit is just closed, so we have $P_L > P_R$ and in steady state $P_R > P_L$ because P_L becomes zero as $\frac{dI}{dt} = 0$.

Hence, (A) and (C) are correct.

41. Consider an element of length a, width dx at a distance x from wire as shown in Figure.



Then magnetic field at the element due to wire is

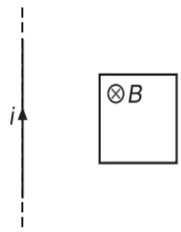
$$B = \frac{\mu_0 i}{2\pi x}$$

Flux associated with this element is

$$d\phi = (B_x)dA = \left(\frac{\mu_0 i}{2\pi x}\right)(adx)$$

$$\Rightarrow \quad \phi = \int_{a}^{2a} d\phi = \frac{\mu_0 i a}{2\pi}\ln 2$$

$$\Rightarrow \quad M = \frac{\phi}{i} = \frac{\mu_0 a}{2\pi}\ln 2$$



The wire produces an inward ⊗ magnetic field over the loop. If the loop is brought closer to the wire, ⊗ magnetic field passing through the loop increases. Hence, induced current produces an outward ⊙ magnetic field so, induced current is anti-clockwise.

Hence, (A) and (C) are correct.

42. When inward magnetic field ⊗ increases, then induced electric lines are anti-clockwise. When inward magnetic field ⊗ decreases, then induced electric lines are clockwise (both inside and outside the cylindrical region).
On a positive charge, force is in the direction of $\vec{E}$ and on a negative charge, force is in the direction opposite to $\vec{E}$.

Hence, (B), (C) and (D) are correct.

43. When I increases, field associated with P increases. So Q lies in a field that increases or we can say that apparently Q moves closer to P. Hence P and Q must repel each other. Similarly I decreases, then P attracts Q.

Hence, (A) and (D) are correct.

44. According to Lenz's law, induced effects always oppose the change. Since i_1 and i_2 both are in same direction, so magnetic lines from B due to both currents are from right to left. By bringing A closer to B or increasing i_1 right to left, the magnetic field from B will increase. So, i_2 should decrease.

Hence, (A) and (C) are correct.

46. After time t, the velocity of wire is $v = at$, the emf induced is $\xi = Blv$ and charge stored on capacitor is

$$Q = C\xi = BlCv = (BLC)at$$

The current in the circuit is $I = \frac{dQ}{dt} = BlCa$

The opposing force on wire is

$$F' = BIl = B^2l^2Ca$$

According to Newton's Second Law, we have

$$F - F' = ma$$

$$\Rightarrow \quad ma = F - B^2l^2Ca$$

$$\Rightarrow \quad a = \frac{F}{m + B^2l^2C}$$

Hence, (A) and (C) are correct.

48. L is the analogue of m

Q is analogue of x

$$\frac{dQ}{dt} = I \text{ is analogue of } \frac{dx}{dt} = v$$

$$\frac{dI}{dt} = I \text{ is analogue of } \frac{dv}{dt} = \dot{v}$$

Hence, (A), (C) and (D) are correct.

49. $\frac{1}{2}LI_{\max}^2 = \frac{1}{2}CV_0^2$

$$\Rightarrow \quad I_{\max} = V_0\sqrt{\frac{C}{L}}$$

Also, we know that for this series LC circuit,

$$q = q_0\cos(\omega t) \text{ and } V = V_0\cos(\omega t) \quad \{\because q = CV\}$$

where $\omega = \frac{2\pi}{T} = \frac{1}{\sqrt{LC}}$

So, potential across the capacitor becomes zero when

$$\omega t = \frac{\pi}{2}$$

$$\Rightarrow \quad \left(\frac{2\pi}{T}\right)t = \frac{\pi}{2}$$

$$\Rightarrow \quad t = \frac{T}{4} = \frac{2\pi\sqrt{LC}}{4} = \frac{\pi}{2}\sqrt{LC}$$

Since at this moment $t = \frac{\pi}{2}\sqrt{LC}$, energy across the capacitor is zero, so energy across the inductor is maximum and has a value $\frac{1}{2}LI_{\max}^2 = \frac{1}{2}CV_0^2$

Hence, (A), (C) and (D) are correct.

50. Since $\vec{F} = q(\vec{v}\times\vec{B})$, the force on positive charge is towards D and on negative charge is towards A. Accordingly, all options are correct.

Hence, (A), (B), (C) and (D) are correct.

51. The voltage across the inductor is given as

$$V_L = L\frac{di}{dt}$$

$$\Rightarrow \quad V_L = (2)\frac{d}{dt}(10e^{-4t})$$

$$\Rightarrow \quad V_L = -80e^{-4t}$$

Writing Equation of potential drop from point A to B, we get

$$V_A - iR - L\frac{di}{dt} - V_B = 0$$

$$\Rightarrow \quad V_A - V_B = iR + L\frac{di}{dt}$$

$$\Rightarrow \quad V_{AB} = (10e^{-4t})(4) - 80e^{-4t} = -40e^{-4t}$$

Hence, (B) and (D) are correct.

Reasoning Based Questions

1. As the coil rotates, the magnetic flux linked with the coil (being $\vec{B}\cdot\vec{A}$) will change and e.m.f. may be induced in the loop.

Hence, the correct answer is (D).

3. $\Delta W = q(\Delta V)$

Here ΔV = non-zero in a closed loop.

Hence, the correct answer is (A).

4. Lenz's Law is based on conservation of energy and induced e.m.f. always opposes the cause of it i.e., change in magnetic flux.

Hence, the correct answer is (D).

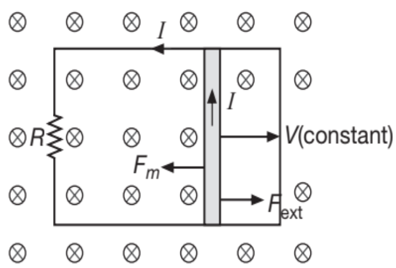
5. $\dot{L}$ is dependent only upon geometrical parameter.

Hence, the correct answer is (C).

6. $\xi = B\ell V$

$$I = \frac{B\ell V}{R}$$

$$F_B = I\ell B = \frac{B^2\ell^2 V}{R}$$



Hence, the correct answer is (A).

7. Even though flux through individual lines changes, it remains unchanged for the solenoid as a whole. Therefore no e.m.f. is induced in the long solenoid.
Hence, the correct answer is (D).

8. Electric field generated from time dependent magnetic field obeys Lenz's Law.
Hence, the correct answer is (C).

9. $\phi = BA\cos(\omega t) = 0$

$$\Rightarrow \quad \omega t = \frac{\pi}{2}$$

Since $|\xi| = NBA\omega\sin(\omega t)$

$$\Rightarrow \quad |\xi|_{max} = NBA\omega \qquad \left\{\because \ \sin(\omega t) = \sin\left(\frac{\pi}{2}\right) = 1\right\}$$

Hence, the correct answer is (B).

12. Charged particle(s) in motion produce both electric and magnetic field.
So, Statement-2 is false.
Hence, the correct answer is (C).

13. Coefficient of coupling between them

$$M = K^2(L_1L_2)$$

When 2 coils are wound on each other, the coefficient of coupling is maximum and hence mutual inductance between the coils is maximum.
Hence, the correct answer is (C).

15. The current in LR circuit grows exponentially $I = I_0\left(1 - e^{-Rt/L}\right)$
Hence, the correct answer is (D).

16. Lenz's Law gives the nature or polarity of induced emf or the direction of current.
Hence, the correct answer is (B).

Linked Comprehension Type Questions

1. When mass is moving with constant velocity, then ring is rotating at constant angular velocity.
So, net torque $\tau = 0$
Hence, torque by mg equals the torque by magnetic force

$$\Rightarrow \quad \tau_{\text{magnetic force}} = mgr = \left(\frac{20}{1000}\right)(10)(0.2) = 0.04 \text{ Nm}$$

Hence, the correct answer is (B).

2. Induced emf in spoke is

$$\xi = \frac{Bvr}{2}$$

$$\Rightarrow \quad i = \frac{\xi}{R} = \frac{Bvr}{2R}$$

$$\Rightarrow \quad i = 1 \text{ A}$$

Hence, the correct answer is (A).

3. Current through the resistor is

$$i = \frac{B\omega r^2}{2R}$$

$$\omega = \frac{v_T}{r}$$

$$i = \frac{Bv_Tr}{2R}$$

$$\Rightarrow \quad i^2R = mgv_T \text{ (Energy conservation)}$$

$$\Rightarrow \quad \frac{Bv_T^2r^2}{4R} = mgv_T$$

$$\Rightarrow \quad v_T = \frac{4mgR}{B^2r^2}$$

$$\Rightarrow \quad v_T = \frac{(4)\left(\frac{20}{1000}\right)(10)(0.15)}{(0.5)^2(0.2)^2} = 12 \text{ ms}^{-1}$$

Hence, the correct answer is (D).

4. The magnetic flux at time t is given by

$$\phi_B = BA = (B_0 + bt)(\pi a^2) = \pi(B_0 + bt)a^2$$

where we have chosen the area vector to point into the page, so that $\phi_B > 0$ g. At $t = 0$, we have

$$\phi_B = \pi B_0 a^2$$

Hence, the correct answer is (B).

5. Using Faraday's Law, the induced emf is

$$\xi = -\frac{d\phi_B}{dt} = -A\frac{dB}{dt} = (\pi a^2)\frac{d(B_0 + bt)}{dt} = -\pi ba^2$$

Hence, the correct answer is (A).

6. The induced current is

$$I = \frac{|\xi|}{R} = \frac{\pi ba^2}{R} = \frac{\pi ba^2}{\lambda(2\pi a)} = \frac{ba}{2\lambda}$$

and its direction is counterclockwise by Lenz's Law.
Hence, the correct answer is (D).

7. The power dissipated due to the resistance R is

$$P = I^2R = \left(\frac{\pi ba^2}{R}\right)^2 R = \frac{(\pi ba^2)^2}{R} = \frac{\pi^2 b^2 a^4}{\lambda(2\pi a)} = \frac{\pi b^2 a^3}{2\lambda}$$

Hence, the correct answer is (C).

8. $|\xi| = \frac{d\phi}{dt} = A\frac{dB}{dt}$

$\Rightarrow \quad \xi = (0.2\times0.4)(2) = 0.16 \text{ V}$

$\Rightarrow \quad i = \frac{|\xi|}{R} = \frac{0.16}{(1)(40+40+20)\times10^{-2}}$

$\Rightarrow \quad i = 0.16 \text{ A}$

Outward magnetic field passing through the loop is increasing. So, induced current should produce an inward magnetic field and hence, induced current is clockwise.

Hence, the correct answer is (A).

9. At $t = 2$ s, rod will move 10 cm towards left, so 40 cm side will become 30 cm

$\Rightarrow \quad |\xi| = \xi_1(\text{say}) = A\left(\frac{dB}{dt}\right)$

$\Rightarrow \quad \xi_1 = (0.2\times0.3)(2) = 0.12 \text{ V}$

At $t = 2$ s, $B = 4$ T

$\Rightarrow \quad \xi_2 = Bvl$

$\Rightarrow \quad \xi_2 = (4)(5\times10^{-2})(0.2)$

$\Rightarrow \quad \xi_2 = 0.04 \text{ V}$

So, $\xi_{net} = \xi_1 - \xi_2 = 0.08 \text{ V}$

Hence, the correct answer is (B).

10. $i = \frac{\xi_{net}}{R} = \frac{0.08}{(1)(30+30+20)\times10^{-2}}$

$\Rightarrow \quad i = 0.1 \text{ A}$

Since $F = Bil$

$\Rightarrow \quad F = (0.1)(0.2)(4) = 0.08 \text{ N}$

Hence, the correct answer is (C).

11. Consider an annulus of radius r, width dr, height b and resistivity ρ. Around its circumference, a voltage is induced according to

$$\xi = -N\frac{d}{dt}(\vec{B}\cdot\vec{A}) = -1\frac{d}{dt}\left[B_0(\cos\omega t)\pi r^2\right]$$

$\Rightarrow \quad \xi = B_0\pi r^2\omega\sin(\omega t)$

The resistance around the loop is

$$\frac{\rho\ell}{A_x} = \frac{\rho(2\pi r)}{bdr}$$

The eddy current in the ring is

$$dI = \frac{\xi}{\text{resistance}} = \frac{B_0\pi r^2\omega\sin(\omega t)bdr}{\rho(2\pi r)}$$

$$dI = \frac{B_0 rb\omega dr\sin(\omega t)}{2\rho}$$

The infinitesimal instantaneous power is

$$dP_i = \xi dI = \frac{B_0^2\pi r^3 b\omega^2 dr\sin^2(\omega t)}{2\rho}$$

$\Rightarrow \quad$ Since, $\langle\sin^2(\omega t)\rangle = \frac{1}{2}$

So, the infinitesimal time-averaged power delivered to the annulus is

$$dP = \frac{B_0^2\pi r^3 b\omega^2 dr}{4\rho}$$

The power delivered to the disk is

$$P = \int dP = \int_0^R \frac{B_0^2\pi b\omega^2}{4\rho}r^3 dr$$

$$\Rightarrow \quad P = \frac{B_0^2\pi b\omega^2}{4\rho}\left(\frac{R^4}{4} - 0\right) = \frac{\pi B_0^2 R^4 b\omega^2}{16\rho}$$

Hence, the correct answer is (D).

12. When B_0 gets two times larger, P gets four times larger.

Hence, the correct answer is (B).

13. When f and $\omega = 2\pi f$ is doubled, P gets four times larger.

Hence, the correct answer is (B).

14. When R is doubled, P becomes 2^4 i.e. 16 times larger.

Hence, the correct answer is (D).

15. Since, $\frac{dB}{dt} = (6t^2 + 24)\text{ Ts}^{-1}$

At $t = 2$ s, $\frac{dB}{dt} = 48 \text{ Ts}^{-1}$

Also, $E_{nc}l = \frac{d\phi}{dt} = A\left(\frac{dB}{dt}\right)$

$\Rightarrow \quad E_{nc}(2\pi r) = \pi r^2\left(\frac{dB}{dt}\right)$

$\Rightarrow \quad E_{nc} = \frac{r}{2}\left(\frac{dB}{dt}\right)$

$\Rightarrow \quad F = qE_{nc} = \frac{qr}{2}\left(\frac{dB}{dt}\right)$

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$\Rightarrow \quad F = \dfrac{(1.6\times10^{-19})(1.25\times10^{-2})}{2}(48)$

$\Rightarrow \quad F = 48\times10^{-21}\text{ N}$

Hence, the correct answer is (B).

16. Since $E_{nc} = E = \dfrac{r}{2}\left(\dfrac{dB}{dt}\right)$

$\Rightarrow \quad E \propto r$

So, E-r graph is a straight line passing through origin.

Hence, the correct answer is (C).

17. The inward magnetic field is increasing so, an outward magnetic field is produced by a conducting circular loop placed there. For producing an outward magnetic field, the induced current should be anti-clockwise. So, direction of induced circular electric lines are also anti-clockwise.

Hence, the correct answer is (B).

18. Since $B_{\text{centre}} = \dfrac{\mu_0 I}{2a}$

$\Rightarrow \quad \phi = BA = \left(\dfrac{\mu_0 I}{2a}\right)(\pi a^2) = \dfrac{\mu_0 \pi a I}{2}$

Hence, the correct answer is (A).

19. Since $\phi = \dfrac{\mu_0 \pi a I}{2}$

$\Rightarrow \quad \xi = -\dfrac{d\phi}{dt}$

$\Rightarrow \quad IR = -\dfrac{d}{dt}\left(\dfrac{\mu_0 \pi a I}{2}\right)$

$\Rightarrow \quad IR = -\dfrac{\mu_0 \pi a}{2}\left(\dfrac{dI}{dt}\right)$

$\Rightarrow \quad -\dfrac{dI}{dt} = \left(\dfrac{2R}{\mu_0 \pi a}\right)I \qquad \ldots(1)$

So, the rate of fall of current is $\left(\dfrac{2R}{\mu_0 \pi a}\right)I$

Hence, the correct answer is (C).

20. From (1), we have

$\dfrac{dI}{I} = -\left(\dfrac{2R}{\mu_0 \pi a}\right)dt$

$\Rightarrow \quad \displaystyle\int_{I_0}^{I}\frac{dI}{I} = -\frac{2R}{\mu_0 \pi a}\int_0^t dt$

$\Rightarrow \quad \log_e\left(\dfrac{I}{I_0}\right) = -\left(\dfrac{2R}{\mu_0 \pi a}\right)t$

$\Rightarrow \quad I = I_0 e^{-\left(\frac{2R}{\mu_0 \pi a}\right)t}$

Hence, the correct answer is (D).

21. Torque due to frictional force F is

$\tau = FR$

Since $\tau = NBIA$

$\Rightarrow \quad NIAB = FR$

$\Rightarrow \quad I = \dfrac{FR}{NAB}$

$\Rightarrow \quad I = 23.15\text{ A}$

Hence, the correct answer is (B).

22. Back emf is $\xi = NBA\omega\sin(\omega t)$

Since the magnetic field is always parallel to the plane of coil, so

$\omega t = 90^\circ$ i.e., $\sin(\omega t) = 1$

$\Rightarrow \quad \xi = NBA\omega$

Since $\omega = \dfrac{v}{R}$

$\Rightarrow \quad \xi = \dfrac{NBAv}{R}$

$\Rightarrow \quad \xi = 90\text{ V}$

Hence, the correct answer is (C).

23. Power supplied by battery is $P_{\text{input}} = VI$

$\Rightarrow \quad P_{\text{in}} = 10\times12\times23$

$\Rightarrow \quad P_{\text{in}} = 2760$

Power generated in the coil is $P_{\text{output}} = \xi I$

$\Rightarrow \quad P_{\text{out}} = 90\times23.15$

$\Rightarrow \quad P_{\text{out}} = 2083$

Efficiency $\eta = \dfrac{P_{\text{out}}}{P_{\text{in}}} = \dfrac{\text{Power generated}}{\text{Power supplied}}$

$\Rightarrow \quad \eta = \dfrac{2083}{2760} = 75\%$

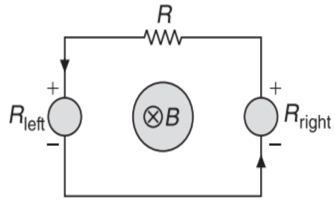
Hence, the correct answer is (A).

26. **The correct answer is (B).**

Combined Solution to 24, 25 and 26

Given that $V_{\text{left}} = +0.1$ volt

So, current enters through positive terminal and is anticlockwise.



Also, $R_{left} = R_{right}$

$\Rightarrow \quad V_{left} = V_{right} = 0.1$ Volt

$$i = \frac{\Delta V}{R} = \frac{V_{left}}{R_{left}} = \frac{0.1}{10^4} = 10^{-5} \text{ ampere}$$

Induced emf $\xi = i\left(R + R_{left} + R_{right}\right)$

$\Rightarrow \quad \xi = 10^{-5}\left(2.5 \times 10^4\right)$

$\Rightarrow \quad \xi = 0.25$ V

27. $|\xi| = \left|\frac{d\phi}{dt}\right| = A\frac{dB}{dt} = \left(\pi a^2\right)B_0$

Hence, the correct answer is (A).

28. Since, $El = \frac{d\phi}{dt}$

$\Rightarrow \quad E(2\pi a) = \left(\pi a^2\right)B_0$

$\Rightarrow \quad E = \frac{1}{2}aB_0$

Hence, the correct answer is (C).

29. Since, $F = qE = \frac{1}{2}qaB_0$

$\Rightarrow \quad \tau = Fa = \frac{1}{2}qa^2B_0$

$$\Rightarrow \quad \alpha = \frac{\tau}{I} = \frac{\left(\frac{1}{2}qa^2B_0\right)}{ma^2} = \frac{qB_0}{2m}$$

Hence, the correct answer is (A).

30. Angular velocity ω is

$$\omega = \alpha t = \left(\frac{qB_0}{2m}\right)t$$

$$\Rightarrow \quad P = \tau\omega = \left(\frac{1}{2}qa^2B_0\right)\left(\frac{qB_0}{2m}\right)$$

$$\Rightarrow \quad P = \frac{q^2B_0^2a^2}{4m}$$

Hence, the correct answer is (D).

31. At $v = v_T$, i.e., equilibrium condition, we have

$$mg - BI\ell = 0$$

$\Rightarrow \quad mg = BI\ell$ where $I = \frac{\xi}{R} = \frac{B\ell v_T}{R}$

$$\Rightarrow \quad mg = \frac{B^2\ell^2 v_T}{R}$$

$$\Rightarrow \quad v_T = \frac{mgR}{B^2\ell^2}$$

Hence, the correct answer is (C).

32. Energy dissipated as heat $= mgh - \frac{1}{2}mv^2$

$$\Rightarrow \quad H = (1)(10)(1) - \left(\frac{1}{2}\right)(1)(4)^2 = 2 \text{ J}$$

Hence, the correct answer is (A).

33. Once the terminal speed is attained, then the energy dissipated in the resistor per unit time is

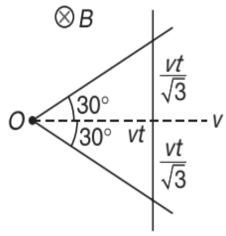
$$H = I^2R = \left(\frac{B^2\ell^2v_T^2}{R^2}\right)R = \frac{B^2\ell^2v_T^2}{R}$$

However, $\frac{B^2\ell^2v_T}{R} = mg$

$$\Rightarrow \quad H = mgv_T = \frac{B^2\ell^2v_T^2}{R}$$

Hence, (B) and (C) are correct.

34. $A = \frac{1}{2}\left(\frac{2vt}{\sqrt{3}}\right)(vt)$



$$\Rightarrow \quad A = \frac{v^2t^2}{\sqrt{3}}$$

Since, $\phi = BA = \frac{Bv^2t^2}{\sqrt{3}}$

$$\Rightarrow \quad |\xi| = \frac{d\phi}{dt} = \frac{2Bv^2t}{\sqrt{3}}$$

So, induced emf increases with time.

Hence, the correct answer is (A).

35. Since, $i = \frac{\xi}{R}$

where, $R = \frac{2vt}{\sqrt{3}} + \frac{2vt}{\sqrt{3}} + \frac{2vt}{\sqrt{3}}$

Since resistance per unit length is $1\,\Omega$

$$\Rightarrow \quad R = \frac{6vt}{\sqrt{3}}$$

$$\Rightarrow \quad i = \frac{2Bv}{6} = \frac{Bv}{3} = \text{constant}$$

Hence, the correct answer is (C).

36. Work done

$$W=\int_0^{10} i^2 R dt=\int_0^{10}\left(\frac{B^2v^2}{9}\right)\left(\frac{6vt}{\sqrt{3}}\right)dt$$

$$\Rightarrow \quad W=\left(\frac{6B^2v^3}{9\sqrt{3}}\right)\left(\frac{10^2}{2}\right)=\frac{6\times 0.16\times 0.001\times 100}{9\sqrt{3}\times 2}$$

$$\Rightarrow \quad W=3\times 10^{-3}\text{ J}$$

Hence, the correct answer is (A).

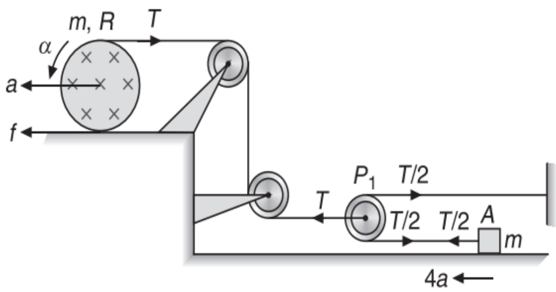
39. The correct answer is (A).

Combined Solution to 37, 38 and 39

$$f-T=ma \qquad \ldots(1)$$

$$\frac{T}{2}=m\times 4a$$

$$T=8ma \qquad \ldots(2)$$



Torque at disc due to time varying magnetic field is

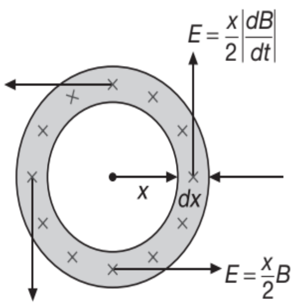
$$\tau=\int_0^R (2\pi x dx)\frac{Q}{\pi R^2}\frac{x}{2}B_0 x$$

$$\Rightarrow \quad \tau=\frac{QB_0}{R^2}\frac{R^4}{4}=\frac{QB_0R^2}{4}$$

Torque about centre of disc is

$$\frac{QB_0R^2}{4}-TR-fR=\frac{MR^2}{2}\frac{a}{R}$$

$$\Rightarrow \quad \frac{QB_0R}{4}-T-f=\frac{ma}{2} \qquad \ldots(3)$$



Solving these three equations, we get

$$a=\frac{QB_0R}{70m}\text{ ms}^{-2}$$

$$T=\frac{4QB_0R}{35}\text{ N}$$

$$\alpha=\frac{a}{R}=\frac{QB_0}{70m}$$

Acceleration of block is

$$a_{\text{block}}=4a=\frac{2QB_0R}{35m}$$

41. Since, $\phi=BA\cos 0°$

As area is changing, so we have

$$\xi=-\frac{d\phi}{dt}$$

$$\Rightarrow \quad \xi=-NB\left(\frac{dA}{dt}\right)$$

$$\Rightarrow \quad \xi=\frac{(NB)(\pi r^2)}{\Delta t}$$

$$\Rightarrow \quad i=\frac{(NB)(\pi r^2)}{\Delta t R}=\frac{100\times 4\times \pi\times (0.1)^2}{3\times 2}=2.09\text{ A}$$

Hence, the correct answer is (A).

42. Power supplied which is converted into heat is

$$P_{\text{supplied}}=I^2R$$

So, energy supplied is $\Delta U=I^2R\times\Delta t=(2.09)^2\times 2\times 3$

$$\Rightarrow \quad \Delta U=26\text{ joule}$$

Loop was initially $2r$ wide and it become πr long after it is flattened.

Since, $\Delta U=F\Delta r$

$$\Rightarrow \quad 26=F(\pi r-2r)$$

$$\Rightarrow \quad F=231\text{ N}$$

Hence, the correct answer is (B).

45. The correct answer is (B).

Combined Solution to 43, 44 and 45

$$y=2A\sin(kx)\cos(\omega t)$$

$$\Rightarrow \quad v=\frac{dy}{dt}=-2A\omega\sin(kx)\sin(\omega t)$$

$$\Rightarrow \quad v_{\max}=-2A\omega\sin(kx)$$

Induced emf is

$$\xi = \int_0^l B v_{max} dx$$

For third harmonic, $l = \frac{3\lambda}{2}$, so we get

$$\xi = \int_0^l B(-2A\omega \sin(kx))dx$$

$$\Rightarrow \quad \xi = -2AB\omega\left(-\frac{\cos kx}{k}\right) = \frac{2AB\omega}{k}\left(\cos(kx)\big|_0^l\right)$$

$$\Rightarrow \quad \xi = \frac{2AB\omega}{k}(\cos 3\pi - \cos 0) = -\frac{4BA\omega}{k}$$

$\Rightarrow \quad \xi$ is maximum

$$\Rightarrow \quad \omega t = \frac{\pi}{2}$$

$$\Rightarrow \quad t = \frac{\pi}{2\omega}$$

For second harmonic $k = \frac{2\pi}{AB}$

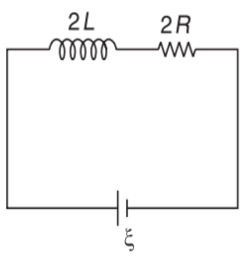
47. $\phi = BA = B\left(\ell^2 + b^2\right)$

$$\Rightarrow \quad |\xi| = \left|\frac{d\phi}{dt}\right| = \left(\ell^2 + b^2\right)\left(\frac{dB}{dt}\right) = \left[\left(\frac{20}{100}\right)^2 + \left(\frac{10}{100}\right)^2\right]10$$

$$\Rightarrow \quad \xi = 0.5 \text{ V}$$

Hence, the correct answer is (D).

48. $I = \frac{\xi}{2R}\left(1 - e^{-\left(\frac{2R}{2L}\right)t}\right)$



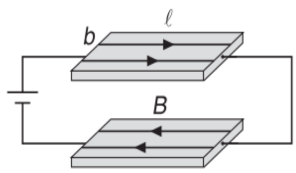
$$\Rightarrow \quad I = \frac{0.5}{20}\left(1 - e^{-t}\right)$$

$$\Rightarrow \quad I = \frac{1}{40}\left(1 - e^{-t}\right)$$

Hence, the correct answer is (B).

49. Magnetic field in the region between the plates is

$$B = \frac{\mu_0 \lambda}{2} + \frac{\mu_0 \lambda}{2}$$



$$\Rightarrow \quad B = \mu_0 \lambda$$

where $\lambda = \frac{i}{b}$

Flux $\phi = BA$

$$\Rightarrow \quad \phi = (\mu_0 \lambda)(al)$$

$$\Rightarrow \quad \phi = \frac{\mu_0 ila}{b}$$

Since $\phi = Li$

$$\Rightarrow \quad L = \frac{\phi}{i} = \frac{\mu_0 la}{b}$$

Hence, the correct answer is (A).

50. Voltage at a distance x from shorted end is

$$V_x = L_x \frac{di}{dt} \quad \ldots(1)$$

Using Kirchhoff's Law, we get

$$V_0 - L\frac{di}{dt} = 0$$

$$\Rightarrow \quad di = \frac{V_0}{L}dt$$

$$\Rightarrow \quad i = \int di = \frac{V_0}{L}\int_0^t dt = \frac{V_0 t}{L}$$

Since, $L_x = \frac{\mu_0 xa}{b}$

So, from (1), we get

$$V_x = \left(\frac{\mu_0 xa}{b}\right)\frac{di}{dt} = \left(\frac{\mu_0 xa}{b}\right)\left(\frac{V_0}{L}\right)$$

Hence, the correct answer is (A).

51. Energy flow rate i.e. $P = V_x \times i$

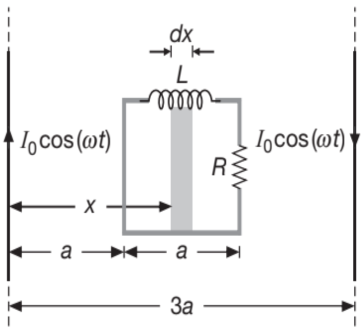
$$\Rightarrow \quad P = \left(\frac{\mu_0 xaV_0}{bL}\right)\frac{V_0 t}{L}$$

$$\Rightarrow \quad P = \frac{\mu_0 xaV_0^2 t}{bL^2}$$

Hence, the correct answer is (B).

52. $d\phi = BdA$, where $B = \frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(3a - x)}$

$$\Rightarrow \quad d\phi = \left[\frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(3a - x)}\right]adx$$



CHAPTER 3

$$\Rightarrow \quad \phi = \frac{\mu_0 I}{2\pi}\left[\int_a^{2a}\frac{dx}{x} + \int_a^{2a}\frac{dx}{(3a-x)}\right]a$$

$$\Rightarrow \quad \phi = \frac{\mu_0 I a}{\pi}\log_e(2)$$

Hence, the correct answer is (A).

53. Magnitude of emf in this circuit

$$\xi = \left|\frac{d\phi}{dt}\right| = \frac{\mu_0 a \log_e(2)}{\pi}\left|\frac{dI}{dt}\right|$$

$$\Rightarrow \quad \xi = \frac{\mu_0 a \log_e(2)}{\pi} I_0 \omega \sin(\omega t)$$

$$\Rightarrow \quad \xi = \frac{\mu_0 I_0 a \omega \log_e(2)}{\pi}\sin(\omega t)$$

Hence, the correct answer is (A).

54. Since in series LR circuit, current lags behind the voltage by $\phi = \tan^{-1}\left(\frac{L\omega}{R}\right)$, so we have

$$I = \frac{\xi_0}{Z}\sin(\omega t - \phi)$$

$$\Rightarrow \quad I = \frac{\mu_0 I_0 \omega a \log_e(2)}{\pi\sqrt{R^2 + L^2\omega^2}}\sin(\omega t - \phi)$$

Hence, the correct answer is (D).

56. **The correct answer is (B).**

Combined Solution to 55 and 56

$$Q = C\xi = B\ell C v$$

$$I = \frac{dQ}{dt} = B\ell C\left(\frac{dv}{dt}\right) \qquad \dots(1)$$

$$F - BI\ell = ma$$

$$\Rightarrow \quad F - (B^2\ell^2 C)a = ma$$

$$\Rightarrow \quad F = (m + B^2\ell^2 C)a$$

$$\Rightarrow \quad a = \frac{F}{m + B^2\ell^2 C} \qquad \dots(2)$$

From (2), we get

$$a = \frac{dv}{dt} = \frac{F}{m + B^2\ell^2 C}$$

Substituting in (1), we get

$$I = \frac{BC\ell F}{m + B^2\ell^2 C}$$

59. **The correct answer is (D).**

Combined Solution to 57, 58 and 59

At terminal velocity, we have

$$BiL = mg$$

$$\Rightarrow \quad i = \frac{mb}{LB} = \frac{0.2\times 98}{1\times 0.6}$$

$$\Rightarrow \quad i = 3.27\ \text{A} \qquad \dots(1)$$

If v is the terminal velocity, then

$$\xi = BLv$$

$$\Rightarrow \quad \xi = (0.6)(v)(1)$$

$$\Rightarrow \quad \xi = 0.6v$$

Also, $P_{R_1} = \frac{\xi^2}{R_1}$

$$\Rightarrow \quad 0.76 = \frac{0.36v^2}{R_1} \qquad \dots(2)$$

Also, $P_{R_2} = \frac{\xi^2}{R_2}$

$$\Rightarrow \quad 1.2 = \frac{0.36v^2}{R_2} \qquad \dots(3)$$

Since R_1 and R_2 are in parallel, so we have

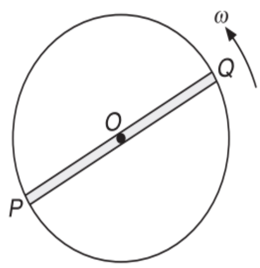
$$R_{\text{net}} = \frac{R_1 R_2}{R_1 + R_2} \qquad \dots(4)$$

$$\Rightarrow \quad i = \frac{\xi}{R_{\text{net}}} = 3.27\ \text{A} \qquad \dots(5)$$

Solving these five equations, we can get the desired results.

60. Induced emf across OP is $\xi = \frac{1}{2}B\omega\left(\frac{l}{2}\right)^2$

$$\Rightarrow \quad \xi = \frac{B\omega l^2}{8}$$



The current is

$$i = \frac{B\omega l^2}{8R} \qquad \dots(1)$$

Torque on the rod is

$$\tau = 2\int_0^{\frac{l}{2}} Bixdx = \frac{Bil^2}{4} \quad \ldots(2)$$

Since $\tau = I\alpha$, where $\alpha = \frac{d\omega}{dt}$

$$\Rightarrow \left(\frac{Ml^2}{12}\right)\frac{d\omega}{dt} = -\frac{B^2\omega l^4}{32R}$$

$$\Rightarrow -\int_{\omega_0}^{\omega}\frac{d\omega}{\omega} = \frac{3B^2l^2}{8RM}\int_0^t dt$$

$$\Rightarrow -\log\left(\frac{\omega}{\omega_0}\right) = \left(\frac{3B^2l^2}{8RM}\right)t$$

$$\Rightarrow \omega = \omega_0 e^{-\left(\frac{3B^2l^2}{8RM}\right)t} = \omega_0 e^{-\alpha t}$$

where $\alpha = \frac{3B^2l^2}{8RM}$

So, from equation (1), we get

$$i = \left(\frac{B\omega_0 l^2}{8R}\right)e^{-\alpha t}$$

Hence, the correct answer is (C).

61. $$Q = \int_0^{\infty} idt = \frac{B\omega_0 l^2}{8R}\int_0^{\infty} e^{-\alpha t}dt$$

where $\alpha = \frac{3B^2l^2}{8RM}$

$$\Rightarrow Q = \frac{B\omega_0 l^2}{8R}\left(\left.\frac{e^{-\alpha t}}{-\alpha}\right|_0^{\infty}\right)$$

$$\Rightarrow Q = -\frac{B\omega_0 l^2}{8\alpha R}\left(e^{-\infty} - e^0\right)$$

$$\Rightarrow Q = \frac{B\omega_0 l^2}{8R\left(\frac{3B^2l^2}{8RM}\right)}$$

$$\Rightarrow Q = \frac{\omega_0 M}{3B}$$

Hence, the correct answer is (B).

62. Heat generated equals the loss in kinetic energy of rod.

So, $\Delta H = |\Delta K| = \frac{1}{2}I\omega_0^2 = \frac{1}{2}\left(\frac{Ml^2}{12}\right)\omega_0^2$

$$\Rightarrow \Delta H = \frac{Ml^2\omega_0^2}{24}$$

Hence, the correct answer is (A).

63. Using Fleming's Right Hand Rule, we see that the induced current in the wire PQ is from Q to P. So, we have

$$V_P - V_Q = \xi = Blv = (4)(1)(2) = 8 \text{ V}$$

Hence, the correct answer is (D).

64. Inductor being connected in parallel to the emf source PQ has same potential difference developed across the emf source. So

$$V_L = \xi = Blv = 8 \text{ V}$$

Hence, the correct answer is (D).

65. Capacitor being connected in parallel to the emf source PQ has same potential difference developed across the emf source. So

$$V_L = \xi = Blv = 8 \text{ V}$$

Hence, the correct answer is (D).

66. Resistor being connected in parallel to the emf source PQ has same potential difference developed across the emf source. So

$$V_L = \xi = Blv = 8 \text{ V}$$

Hence, the correct answer is (D).

67. Since $V_L = 8$ V

$$\Rightarrow L\frac{di_1}{dt} = 8$$

$$\Rightarrow 4\frac{di_1}{dt} = 8$$

$$\Rightarrow \frac{di_1}{dt} = 2 \text{ As}^{-1}$$

Hence, the correct answer is (C).

68. Charge on the capacitor is

$$Q = C\xi = (1)(8) = 8 \text{ coulomb}$$

$$\Rightarrow i_2 = \frac{dQ}{dt} = \frac{d}{dt}(8) = 0$$

Hence, the correct answer is (A).

69. Current through resistor is

$$i_3 = \frac{\xi}{R} = \frac{8}{2} = 4 \text{ A}$$

Hence, the correct answer is (C).

70. Since rate of change of current in inductor is

$$\frac{di_1}{dt} = 2 \text{ As}^{-1}$$

So, current in the inductor at $t = 2$ s is

$$i_1 = \left(\frac{di_1}{dt}\right)\Delta t = (2)(2) = 4 \text{ A}$$

Hence, the correct answer is (C).

71. Since wire PQ is acting as source of emf and supplies current i_1 to inductor, i_2 to capacitor and i_3 to resistor, so current through wire PQ is

$$i = i_1 + i_2 + i_3 = 4 + 0 + 4 = 8 \text{ A}$$

Hence, the correct answer is (D).

72. To keep the wire PQ moving with constant velocity, an external force that equals the magnetic force on the current carrying conductor should be applied on it.

$$\Rightarrow \quad F_{\text{external}} = F_{\text{magnetic}} = Bil = (4)(8)(1) = 32 \text{ N}$$

Hence, the correct answer is (D).

73. Energy supplied per second by the source is

$$P_{\text{supplied}} = \xi i = (8)(8) = 64 \text{ W}$$

Hence, the correct answer is (C).

74. Power generated by the applied external force is

$$P_{\text{external force}} = F_{\text{ext}} v = (32)(2) = 64 \text{ W}$$

Hence, the correct answer is (C).

75. Magnetic energy stored per second in inductor is

$$\frac{dU_m}{dt} = \frac{d}{dt}\left(\frac{1}{2}Li_1^2\right) = \frac{L}{2}\left(2i_1\frac{di_1}{dt}\right) = Li_1\frac{di_1}{dt}$$

Since $\frac{di_1}{dt} = 2 \text{ As}^{-1}$ and $i_1 = 4$ A at $t = 2$ s

$$\Rightarrow \quad \frac{dU_m}{dt} = Li_1\frac{di_1}{dt} = (4)(4)(2) = 32 \text{ W}$$

Hence, the correct answer is (B).

76. Energy dissipated per second in the resistor at $t = 2$ s is

$$P_{\text{resistor}} = i_3^2 R = (4)^2(2) = 32 \text{ W}$$

Hence, the correct answer is (B).

77. Electrostatic stored per second in capacitor at $t = 2$ s is

$$\frac{dU_e}{dt} = \frac{d}{dt}\left(\frac{Q^2}{2C}\right) = \frac{1}{2C}\frac{d}{dt}(Q^2)$$

$$\Rightarrow \quad \frac{dU_e}{dt} = \frac{1}{2C}\left(2Q\frac{dQ}{dt}\right) = \frac{Q}{C}\frac{dQ}{dt} = \frac{Q}{C}i_2$$

Since $i_2 = 0$

$$\Rightarrow \quad \frac{dU_e}{dt} = 0$$

Hence, the correct answer is (A).

80. The correct answer is (B).

Combined Solution to 78, 79 and 80

The fan is operating at 200 V, consuming 1000 W

then $I = \frac{1000}{200} = 5$ A

Since the coil resistance is $1\,\Omega$, so power dissipated by internal resistance as heat is

$$P_1 = I^2 R = 25 \text{ W}$$

If V is the net emf across coil, then

$$\frac{V^2}{R} = 25$$

$$\Rightarrow \quad V = 5 \text{ volt}$$

Net Emf = Source Emf – Back Emf

$$\Rightarrow \quad V = V_S - \xi$$

$$\Rightarrow \quad \xi = 195 \text{ V}$$

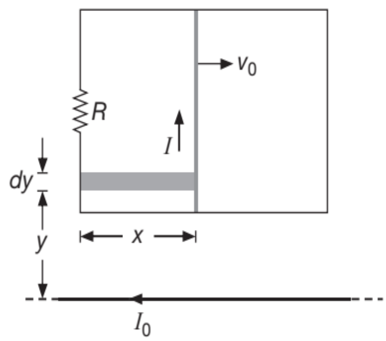
The work done is $P_2 = 1000 - 25 = 975$ W

81. $$\phi = \int_a^b x \frac{\mu_0 I_0}{2\pi y} dy = \frac{\mu_0 I_0 x}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$\xi = \frac{d\phi}{dt} = \frac{\mu_0 I_0 v_0}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$\Rightarrow \quad I = \frac{\mu_0 I_0 v_0}{2\pi R}\ln\left(\frac{b}{a}\right)$$

Current induced in the circuit will be anticlockwise to reduce the change in flux.

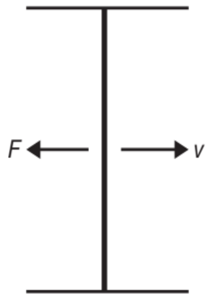


Hence, the correct answer is (C).

82. At any instant when the velocity of the sliding wire is v, current through the wire is

$$I = \frac{\mu_0 I_0 v}{2\pi R}\ln\left(\frac{b}{a}\right)$$

Force on the wire $F = I(dy)B$



$$\Rightarrow \quad F = \int_a^b \frac{\mu_0 I_0 v}{2\pi R} \ln\left(\frac{b}{a}\right)\left(\frac{\mu_0 I_0}{2\pi y}\right) dy$$

$$\Rightarrow \quad F = \frac{\mu_0^2 I_0^2 v}{4\pi^2 R}\left(\ln\frac{b}{a}\right)\ln\left(\frac{b}{a}\right)\frac{\mu_0^2 I_0^2}{4\pi^2 R}\left[\ln\left(\frac{b}{a}\right)\right]^2$$

$$\Rightarrow \quad dv = -\frac{\mu_0^2 I_0^2 v}{4\pi^2 mR}\left(\ln\frac{b}{a}\right)^2 \int_0^t dt$$

$$\Rightarrow \quad \int_{v_0}^{\frac{v_0}{2}} \frac{dv}{v} = -\frac{\mu_0^2 I_0^2}{4\pi^2 mR}\left(\log\left(\frac{b}{a}\right)\right)^2 \int_0^t dt$$

$$\Rightarrow \quad t = \frac{4\pi^2 mR \ln(2)}{\mu_0^2 I_0^2 \left(\ln\left(\frac{b}{a}\right)\right)^2}$$

Hence, the correct answer is (A).

83. $$-\frac{dv}{dt} = -v\frac{dv}{dx} = \frac{\mu_0^2 I_0^2 v}{4\pi^2 mR}\left(\ln\frac{b}{a}\right)^2$$

$$\Rightarrow \quad \frac{dv}{dx} = -\frac{\mu_0^2 I_0^2}{4\pi^2 mR}\left(\ln\frac{b}{a}\right)^2$$

$$\Rightarrow \quad \int_{v_0}^{0} dv = -\frac{\mu_0^2 I_0^2}{4\pi^2 mR}\left(\log_e\left(\frac{b}{a}\right)\right)^2 \int_0^x dx$$

$$\Rightarrow \quad v_0 = \frac{\mu_0^2 I_0^2}{4\pi^2 mR}\left(\log_e\left(\frac{b}{a}\right)\right)^2 x$$

$$\Rightarrow \quad x = \left[\frac{4\pi^2 mR}{\left(\mu_0 I_0 \ln\left(\frac{b}{a}\right)\right)^2}\right] v_0$$

$$\Rightarrow \quad x \propto v_0$$

Hence, the correct answer is (A).

84. For CASE-1, the total area inside the coils is $L^2 + l^2$ and for CASE-2, it is $L^2 - l^2$.

The corresponding flux are

$(L^2 + l^2)B$ and $(L^2 - l^2)B$.

Hence, the correct answer is (D).

85. Since the field is decreasing, so the field produced due to induced current is inwards. Hence the current is from b to a and from d to c.

Hence, the correct answer is (C).

86. Since the field is decreasing, so field produced due to induced current is inwards. Hence, the current is from b to a and f to e.

Hence, the correct answer is (B).

87. The induced Current is

$$I = \left|\frac{1}{R}\frac{d\phi}{dt}\right| = \left|\frac{A}{R}\frac{dB}{dt}\right|$$

Since R is same for both and area A inside the coil in CASE-1 is more, so $I_1 > I_2$.

Hence, the correct answer is (B).

88. Total initial energy

$$U_{\text{initial}} = \frac{Q_0^2}{2C} = \frac{10^{-2} \times 10^{-2}}{2 \times 50 \times 10^{-6}}\ \text{J} = 1\ \text{J}$$

This energy shall remain conserved in the absence of resistance.

Hence, the correct answer is (C).

89. Natural frequency, $f = \dfrac{1}{2\pi\sqrt{LC}}$

$$f = \frac{1}{2 \times 3.14 (20 \times 10^{-3} \times 50 \times 10^{-6})^{1/2}}\ \text{Hz}$$

$$\Rightarrow \quad f = 159\ \text{Hz}$$

Hence, the correct answer is (B).

90. $Q = Q_0 \cos \omega t$

$$\Rightarrow \quad Q = Q_0 \cos\left(\frac{2\pi}{T}t\right), \text{ where } T = \frac{1}{f}$$

Energy stored is completely electrical at $t = 0$, $\frac{T}{2}$, T, $\frac{3T}{2}$

Hence, the correct answer is (A).

91. Electrical energy is zero i.e., energy stored is completely magnetic at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}$

Hence, the correct answer is (D).

92. At times $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \ldots$ we have

$$Q = Q_0 \cos\left(\frac{\omega T}{8}\right) = Q_0 \cos\left(\frac{\pi}{4}\right) = \frac{Q_0}{\sqrt{2}}$$

Electrical energy at this instant is

$$U_e = \frac{Q^2}{2C} = \frac{1}{2}\frac{Q_0^2}{2C}$$

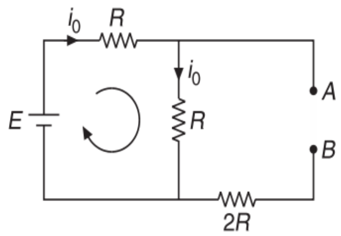
which is half of the total energy

Hence, the correct answer is (C).

93. R damps out the LC oscillations eventually. The entire initial energy 1 J is eventually dissipated as heat.

Hence, the correct answer is (C).

94. Since we know that at $t = 0$, the inductor offers infinite resistance path to the flow of current, so the given circuit can be redrawn as shown in Figure.



Current in the inductor just when the switch is closed is

$$i_0 = \frac{E}{R+R} = \frac{E}{2R}$$

Hence, the correct answer is (D).

95. The internal resistance across terminals A and B can be calculated by shorting the battery which gives

$$R_{AB} = 2R + \frac{R}{2} = \frac{5R}{2}$$

So, the time constant of the circuit is

$$\tau = \frac{L}{R_{AB}} = \frac{2L}{5R}$$

Hence, the correct answer is (C).

96. Due to removal of inductor right loop of circuit is now open and a current flow in the left loop which is given as

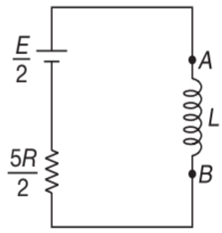
$$i = \frac{E}{2R}$$

The open circuit potential difference across terminals A and B is given as

$$V_{AB} = iR = \frac{E}{2}$$

Hence, the correct answer is (B).

97. The simpler form of the circuit given in the problem is shown in Figure.



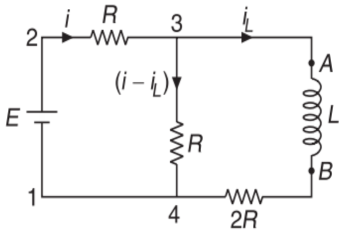
This circuit is now reduced to simple RL circuit and in this case the current as a function of time can be given as

$$i_L = \frac{V_{AB}}{R_{AB}} = \frac{\left(\frac{E}{2}\right)}{\left(\frac{5R}{2}\right)}\left(1 - e^{-\frac{5Rt}{2L}}\right)$$

$$\Rightarrow \quad i_L = \frac{E}{5R}\left(1 - e^{-\frac{5Rt}{2L}}\right)$$

Hence, the correct answer is (B).

98. To calculate the current supplied by the battery as a function of time, we redraw the circuit as shown in Figure.



Applying KLL, to loop 12341, we get

$$E - iR - (i - i_L)R = 0$$

$$\Rightarrow \quad 2iR = E + i_L R$$

$$\Rightarrow \quad i = \frac{E}{2R} + \frac{i_L}{2}$$

$$\Rightarrow \quad i = \frac{E}{2R} + \frac{E}{10R}\left(1 - e^{-\frac{5Rt}{2L}}\right)$$

Hence, the correct answer is (C).

99. On shorting the battery, the resistances across the inductor are $3\ \Omega$ and $6\ \Omega$ in parallel. Their combined resistance is $R_{\text{net}} = 2\ \Omega$. So,

$$\tau = \frac{L}{R_{\text{net}}} = \frac{1}{2}\text{ sec}$$

The current through inductor is

$$i_1 = i_0\left(1 - e^{-\frac{t}{\tau}}\right) = 3\left(1 - e^{-2t}\right)$$

Potential difference across $3\,\Omega$ resistance is equal to Potential difference across inductor, so

$$|V_R| = L\frac{di}{dt} = 6e^{-2t}$$

Hence, the correct answer is (B).

100. Current through $3\,\Omega$ resistor is

$$i_R = \left|\frac{V_R}{3}\right| = 2e^{-2t}$$

Current through inductor is $i_L = 3(1-e^{-2t})$

So, current from battery is

$$i = i_L + i_R = 3 - e^{-2t}$$

Hence, the correct answer is (D).

101. $i_R = i_L$

$$\Rightarrow \quad 2e^{-2t} = 3 - e^{-2t}$$

$$\Rightarrow \quad e^{-2t} = \frac{3}{5}$$

$$\Rightarrow \quad t = \ln\sqrt{\frac{5}{3}}$$

Hence, the correct answer is (A).

102. Current through inductor is towards right and hence positive throughout. Since

$$i_L = 3(1-e^{-2t})$$

At $t = 0$, $i_L = 0$

and at $t \to \infty$,

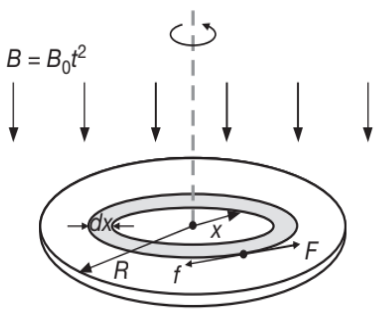
$$i_L = 3\text{ A}$$

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. A → (s)
B → (p)
C → (p, r, t)
D → (q)

Assume the disc to be made of a number of infinitesimal concentric rings. Consider one such ring of radius x, thickness



dx having mass dm. Then the frictional force on this infinitesimal element is

$$df = \mu(dN) = \mu(dm)g \qquad \{\because dN = (dm)g\}$$

where $dm = (2\pi x dx)\dfrac{M}{\pi R^2} = \left(\dfrac{2M}{R^2}\right)(x\,dx)$

Infinitesimal torque due to this frictional force is

$$d\tau = (df)x$$

$$\Rightarrow \quad d\tau = \mu\left(\frac{2M}{R^2}x\,dx\right)g\,x$$

$$\Rightarrow \quad d\tau = \frac{2\mu Mg}{R^2}x^2\,dx$$

$$\Rightarrow \quad \tau = \int d\tau = \frac{2\mu Mg}{R^2}\int_0^R x^2dx$$

$$\Rightarrow \quad \tau = \frac{2\mu Mg}{R^2}\left(\frac{R^3}{3}\right) = \frac{2}{3}(\mu MgR)$$

This expression is independent of t. So

(B) → (p)

Now, let us calculate the torque due to the varying magnetic field.

The varying magnetic field produces an electric field which will be tangential to the disc (or the infinitesimal element). If dq be the charge on the infinitesimal element, then

$$dq = \left(\frac{Q}{\pi R^2}\right)(2\pi x\,dx) = \left(\frac{2Q}{R^2}\right)x\,dx$$

Electrostatic force on this element in the presence of tangential electric field E_t is

$$dF = (dq)E_t$$

So, if $d\tau_m$ is the torque due to the magnetic field, then

$$\tau_m = \int d\tau_m = \int x\,dF = \int\left(\frac{2Q}{R^2}x^2dx\right)E_t \qquad \ldots(1)$$

From Faraday's Laws, we know that

$$\left|\oint \vec{E}_t \cdot d\vec{\ell}\right| = \left|\frac{d\phi_B}{dt}\right|$$

$$\Rightarrow \quad (2\pi x)E_t = (\pi x^2)\frac{dB}{dt}$$

$$\Rightarrow \quad E_t = \frac{x}{2}(2B_0t) = (B_0t)x$$

$$\Rightarrow \quad \tau_m = \frac{(2QB_0)t}{R^2}\int_0^R x^3dx$$

$$\Rightarrow \quad \tau_m = \frac{(2QB_0)t}{R^2}\left(\frac{R^4}{4}\right) = \left(\frac{QB_0R^2}{2}\right)t$$

$$\Rightarrow \quad \tau_m = \left(\frac{QB_0R^2}{2}\right)t \qquad \ldots(2)$$

The disc will start rotating at $t = t_0$, when

$$\tau_m = \tau_f$$

$$\Rightarrow \quad \left(\frac{QB_0R^2}{2}\right)t_0 = \frac{2}{3}\mu MgR$$

$$\Rightarrow \quad t_0 = \frac{4}{3}\left(\frac{\mu Mg}{QB_0R}\right)$$

So, we get (A) → (s)

Also, from (2), we observe that torque due to magnetic field

at $t = 0$ is zero

at $t = t_0$ is $\left(\frac{QB_0R^2}{2}\right)t_0 = \frac{2}{3}\mu MgR$

at $t = 3t_0$ is $\frac{3}{2}\left(QB_0R^2\right)t_0 = 2\mu MgR$

So, (C) → (p, r, t)

Let us calculate the net torque τ on the disc at time $t(>t_0)$

$$\tau = \tau_m - \tau_f = \left(\frac{QB_0R^2}{2}\right)t - \frac{2}{3}(\mu MgR)$$

$$\Rightarrow \quad I\alpha = \left(\frac{QB_0R^2}{2}\right)t - \frac{2}{3}(\mu MgR)$$

$$\Rightarrow \quad I\frac{d\omega}{dt} = \left(\frac{QB_0R^2}{2}\right)t - \frac{2}{3}(\mu MgR)$$

$$\Rightarrow \quad \frac{1}{2}MR^2\frac{d\omega}{dt} = \left(\frac{QB_0R^2}{2}\right)t - \frac{2}{3}(\mu MgR)$$

$$\Rightarrow \quad d\omega = \frac{QB_0}{M}t\,dt - \frac{4\mu g}{3R}dt$$

$$\Rightarrow \quad \omega = \frac{QB_0}{M}\int_{t_0}^{2t_0} t\,dt - \frac{4\mu g}{3R}\int_{t_0}^{2t_0} dt$$

$$\Rightarrow \quad \omega = \left(\frac{QB_0}{2M}\right)\left(3t_0^2\right) - \left(\frac{4\mu g}{3R}\right)t_0 \qquad \ldots(3)$$

Substituting $t_0 = \frac{4}{3}\left(\frac{\mu Mg}{QB_0R}\right)$ in (3), we get

$$\omega = \frac{8}{9}\left(\frac{M\mu^2g^2}{QB_0R^2}\right)$$

So, (D) → (q)

2. A → (q)
B → (p)
C → (r)
D → (s)

$$V_L = \frac{LdI}{dt} = Ee^{-\frac{t}{\tau_L}} = 10e^{-\frac{t}{\tau_L}}, \text{ where } \tau_L = \frac{L}{R}$$

$$\Rightarrow \quad \tau_L = \frac{L}{R} = 1\text{ s}$$

$$\Rightarrow \quad V_L = 10e^{-t}$$

At $t = 0$, $V_L = 10$ V

At $t = 1$ s, $V_L = \frac{10}{e}$ V

$$\Rightarrow \quad V_R = E - V_L = 10\left(1 - e^{-t}\right)$$

At $t = 0$, $V_R = 0$

At $t = 1$ s, $V_R = 10\left(1 - \frac{1}{e}\right)$ V

3. A → (q)
B → (p)
C → (s)
D → (r)

farad is the unit of capacitance, so

$$[C] = \left[\frac{Q}{V}\right] = \left[\frac{Q^2}{W}\right] = \frac{A^2T^2}{ML^2T^{-2}}$$

$$\Rightarrow \quad [C] = M^{-1}L^{-2}T^4A^2$$

weber is the unit of magnetic flux, so

$$[\phi] = [BA] = \left[\frac{F}{qv}A\right] = \frac{\left(MLT^{-2}\right)L^2}{(AT)\left(LT^{-1}\right)} = ML^2T^{-2}A^{-1}$$

ohm is the unit of resistance, so

$$[R] = \left[\frac{V}{I}\right] = \left[\frac{W}{Iq}\right] = \left[\frac{W}{I^2t}\right] = \frac{ML^2T^{-2}}{A^2T} = ML^2T^{-3}A^{-2}$$

henry is the unit of inductance, since

$$\left[\frac{L}{R}\right] = T$$

$$\Rightarrow \quad [L] = \left(ML^2T^{-3}A^{-2}\right)T = ML^2T^{-2}A^{-2}$$

4. A → (s)
B → (q)
C → (p)
D → (p)

Steady state current through inductor is

$$i_0 = \frac{9}{3} = 3\text{ A}$$

Now, this current decay exponentially across inductor and two resistors. The inductive time constant of the circuit is

$$\tau_L = \frac{L}{R} = \frac{9}{6+3} = 1 \text{ s}$$

$$\Rightarrow \quad t_{1/2} = (\ln 2)\tau_L = (\ln 2) \text{ s}$$

So, given time is half-life time

Hence, current will be $\frac{i_0}{2} = 1.5$ A

Since $i = i_0 e^{-\frac{t}{\tau_L}}$

$$\Rightarrow \quad i = 3e^{-t}$$

$$\Rightarrow \quad \left(-\frac{di}{dt}\right) = 3e^{-t}$$

In the beginning, $\left(-\frac{di}{dt}\right) = 3 \text{ As}^{-1}$ $\{\because t = 0\}$

After one half-life time $\left(-\frac{di}{dt}\right) = 1.5 \text{ As}^{-1}$ $\{\because t = \ln 2\}$

So, $V_L = L\left|-\frac{di}{dt}\right| = 9 \times 1.5 = 13.5 \text{ V}$

$$V_{3\,\Omega} = iR = 1.5 \times 3 = 4.5 \text{ V}$$

$$V_{6\,\Omega} = iR = 1.5 \times 6 = 9 \text{ V and}$$

$$V_{bc} = V_L - V_{3\,\Omega} = 9 \text{ V}$$

5. A → (q)
B → (s)
C → (p)
D → (r)

For (A) $\tau_C = \frac{R}{2}(C_1 + C_2)$

For (B) $\tau_L = \frac{L_1 L_2}{(L_1 + L_2)(R_1 + R_2)}$

For (C) $\tau_L = \frac{L_1 + L_2}{R_1 + R_2}$

For (D) $\tau_C = R(C_1 + C_2)$

6. A → (s)
B → (q)
C → (s)
D → (p)

Since, $\phi = 2t$

$$\Rightarrow \quad |\xi| = \frac{d\phi}{dt} = 2 \text{ V}$$

$$\Rightarrow \quad i = \frac{\xi}{R} = 1 \text{ A} = \text{constant}$$

$$\Rightarrow \quad \Delta q = i\Delta t = 1 \times 2 = 2 \text{ C}$$

$$\Rightarrow \quad H = i^2 R\Delta t = (1)^2(2)(2) = 4 \text{ J}$$

7. A → (p, r)
B → (p, r)
C → (q, s)
D → (p, r, t)

For case (A)

$$I = \frac{V}{R} \text{ and } U = \frac{1}{2}LI^2 = \frac{LV^2}{2R^2}$$

For case (B)

$$I = \frac{V}{R} \text{ and } U = \frac{1}{2}LI^2 = \frac{LV^2}{2R^2}$$

For case (C)

$$I = \frac{V}{3R} \text{ and } U = \frac{1}{2}\left(\frac{L}{2}\right)I^2 = \frac{LV^2}{36R^2}$$

For case (D)

$$I = \frac{V}{R} \text{ and } U_1 = \frac{1}{2}LI^2 = \frac{LV^2}{2R^2}$$

and $U_2 = \frac{1}{2}\left(\frac{L}{9}\right)I^2 = \frac{LV^2}{18R^2}$

8. A → (q, s)
B → (p, r)
C → (p, r)
D → (q, s)

9. A → (q)
B → (r, s)
C → (s)
D → (p, q, r)

10. A → (r)
B → (s)
C → (p)
D → (t)

In steady state, all the currents and voltages reach their final maximum value. At steady state both the inductors can be shorted so we get current through L_1 is

$$I_1 = \frac{V}{R_1} = \frac{16}{4 \times 10^3} = 4 \times 10^{-3} \text{ A} = 4 \text{ mA}$$

Voltage across L_2 is

$$V_2 = L_2 \frac{dI_2}{dt}$$

At steady state $I_2 = \text{constant}$

$\Rightarrow \quad V_2 = 0$

Energy stored in inductor L_1 is

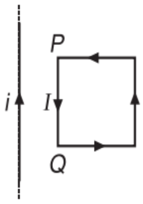
$$E_1 = \frac{1}{2}L_1 I_1^2 = 16\ \mu\text{J}$$

Energy stored in inductor L_2 is

$$E_2 = \frac{1}{2}L_2 I_1^2$$

$\Rightarrow \quad E_2 = 24\ \mu\text{J}$

11. A → (q, s)
B → (p, r)
C → (p, r)
D → (q, s)
When current is increased, the inward magnetic field passing through loop will increase. So, induced current will produce an outward magnetic field. Hence, induced current is anti-clockwise.



When i and I currents in PQ are in opposite directions, then they will repel each other.

12. A → (q)
B → (p)
C → (t)
D → (r)
Also for 1, the induced current will set up an inward field, so direction of I is from h → g → f → e → d → c → b → a → h and for 2, the induced current will be from h → g → f → e → d → c → b → a → h

13. A → (q)
B → (s)
C → (p)
D → (r)

LC Oscillations (Electrical Oscillations)	Spring Block Oscillations (Mechanical Oscillations)
Q	x
I	v
V	F
C	$\frac{1}{k}$
L	m
$I = \frac{dQ}{dt}$	$v = \frac{dx}{dt}$
$\frac{dI}{dt}$	$\frac{dv}{dt} = \text{acceleration}$
$U_L = \frac{1}{2}LI^2$	$K = \frac{1}{2}mv^2$
$U_C = \frac{Q^2}{2C}$	$U = \frac{1}{2}kx^2$

(Continued)

14. A → (r)
B → (p)
C → (q)
D → (q)
At $t = 0$, current through R_1 is zero and current through R_2 is $i_{R_2} = \frac{E}{R_2} = \frac{18}{6} = 3$ A

At $t \to \infty$, current through R_1 is $i_{R_1} = \frac{E}{R_1} = \frac{18}{3} = 6$ A

and current through R_2 is $i_{R_2} = \frac{E}{R_2} = \frac{18}{6} = 3$ A

Integer/Numerical Answer Type Questions

1. (a) Since, according to Faraday's Laws of Electromagnetic Induction, we have

$$\xi = -N\frac{\Delta\phi_B}{\Delta t}, \text{ with } N = 1$$

Given $a = 5\times10^{-3}$ m and $h = 0.5$ m,

$$\Delta\phi_B = B_2A - B_1A = A(B_2 - B_1)$$

$$\Rightarrow \quad \Delta\phi_B = a^2\left(\frac{\mu_0 I}{2\pi(h+a)} - \frac{\mu_0 I}{2\pi a}\right)$$

$$\Rightarrow \quad \Delta\phi_B = \frac{a^2\mu_0 I}{2\pi}\left(\frac{1}{h+a} - \frac{1}{a}\right) = -\frac{\mu_0 a h I}{2\pi(h+a)}$$

The time for the washer to drop a distance h (from rest) is given by

$$\Delta t = \sqrt{\frac{2h}{g}}$$

From Faraday's Laws, we have

$$\xi = -\frac{\Delta\phi_B}{\Delta t} = \frac{\mu_0 ahI}{2\pi(h+a)\Delta t}$$

$$\Rightarrow \quad \xi = \frac{\mu_0 ahI}{2\pi(h+a)}\sqrt{\frac{g}{2h}} = \frac{\mu_0 aI}{2\pi(h+a)}\sqrt{\frac{gh}{2}}$$

$$\Rightarrow \quad \xi = \frac{(4\pi\times10^{-7})(5\times10^{-3})(10)}{2\pi(0.5+0.005)}\sqrt{\frac{(9.8)(0.5\text{ m})}{2}}$$

$$\Rightarrow \quad \xi = 30.9\text{ nV} \cong 31\text{ nV}$$

(b) Since the magnetic flux going through the washer (into the plane of the paper) is decreasing in time, a current will form in the washer so as to oppose that decrease. Therefore, the current will flow in a clockwise direction.

2. Power dissipated in the loop is

$$P = \frac{\xi^2}{R} \qquad \ldots(1)$$

where R is the resistance of the loop given by

$$R = \lambda(2\pi a) = 2\pi\lambda a$$

where ξ = induced emf in the loop given by

$$\xi = \left|\frac{d\phi}{dt}\right| = A\left|\frac{dB}{dt}\right| = (\pi a^2)\left|\frac{dB}{dt}\right|$$

where $B(x) = \frac{\mu_0}{2\pi}\frac{I_0}{x} = B$

$$\Rightarrow \quad \left(\frac{dB}{dt}\right) = -\left(\frac{\mu_0 I_0}{2\pi x^2}\right)\left(\frac{dx}{dt}\right) = -\frac{\mu_0 I_0 v}{2\pi x^2} \qquad \left\{\because \frac{dx}{dt} = v\right\}$$

$$\Rightarrow \quad \xi = \frac{\pi a^2 \mu_0 I_0 v}{2\pi x^2} \qquad \ldots(2)$$

From equations (1) and (2)

$$P = \frac{\pi^2 a^4 \mu_0^2 I_0^2 v^2}{4\pi^2 x^4 \cdot R}$$

$$\Rightarrow \quad v = \sqrt{\frac{4Px^4 R}{\mu_0^2 I_0^2 a^4}}$$

$$\Rightarrow \quad v = \frac{2x^2}{\mu_0 I_0 a^2}\sqrt{PR} = \frac{2x^2}{\mu_0 I_0 a^2}\sqrt{2\pi\lambda Pa}$$

$$\Rightarrow \quad v = \frac{x^2}{\mu_0 I_0 a^2}\sqrt{8a\pi\lambda P}$$

$$\Rightarrow \quad k = 8$$

3. Instantaneous flux through loop is

$$\phi = \pi a^2 B\cos 0^\circ + \pi b^2 B\cos 180^\circ$$

$$\Rightarrow \quad \phi = \pi(a^2 - b^2)B$$

$$\Rightarrow \quad \phi = \pi(a^2 - b^2)B_0\sin(\omega t)$$

Differentiating w.r.t. time

$$\left|\frac{d\phi}{dt}\right| = \pi(a^2 - b^2)B_0\omega\cos(\omega t)$$

$$\Rightarrow \quad I = \frac{\pi(a^2 - b^2)B_0\omega\cos(\omega t)}{R}$$

$I_{max} = I_0 = \frac{\pi(a^2 - b^2)B_0\omega}{2\pi(a+b)\lambda}$, λ = resistance per unit length

$$I_{max} = I_0 = \frac{1}{2\lambda}(a-b)B_0\omega$$

$$I_{max} = I_0 = \frac{1}{2}(20-10)\times10^{-2}\times10\times10^{-3}\times\frac{100}{50\times10^{-3}}$$

$$I_{max} = I_0 = 1\text{ A}$$

4. Let l_0 be the length of the solenoid, then according to the problem, we have

$$l_0 = 100\text{ cm}$$

Self-inductance of the solenoid is

$$L = \mu_0 n^2 A l_0 = \frac{\mu_0 N^2 \pi r^2}{l_0}$$

If l be the total length of wire used to make the solenoid, then

$$l = N(2\pi r)$$

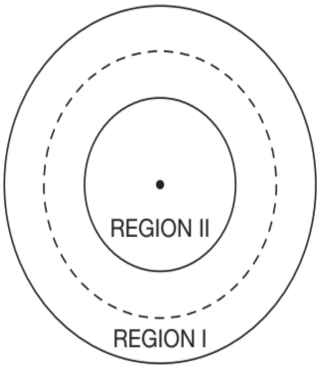
$$\Rightarrow \quad r = \frac{l}{2\pi N}$$

Substitute this value in equation (1), we get

$$L = \frac{\mu_0 l^2}{4\pi l_0}$$

$$\Rightarrow \quad l = \sqrt{\frac{4\pi l_0 L}{\mu_0}} = \sqrt{(10^7)(1)(10^{-3})} = 100\text{ m}$$

5. Let the current in the outer coil at any instant of time, say t, be $I = \alpha t$ and in the inner coil is $2I = 2\alpha t$, where α is a constant. Because of these currents the magnetic field in the outer coil is $B = \mu_0 n(\alpha t)$, where n is the number of turns per unit length. Because in the annular part (region between two coils) no field will be there due to the inner coil, because the annular region is the region outside the inner coil, so $(B_{annular})_{inner\ coil} = 0$ and $(B_{annular})_{outer\ coil} = \mu_0 nI$



Similarly, the field in the Region II inside the inner coil will be

$$B_{\text{REGION II}} = B_{\text{outer coil}} + B_{\text{inner coil}}$$

$$\Rightarrow B_{\text{REGION II}} = \mu_0 nI + \mu_0 n(2I) = 3\mu_0 nI = 3B$$

The magnetic flux enclosed by the particle's trajectory of radius r is

$$\phi = (\pi R^2)(2B) + (\pi r^2)(B)$$

$$\Rightarrow \phi = (2R^2 + r^2)\pi\mu_0 n\alpha t$$

Since we have

$$\left|\oint \vec{E}\cdot d\vec{\ell}\right| = \frac{d\phi}{dt}$$

$$\Rightarrow E(2\pi r) = \frac{d\phi}{dt}$$

$$\Rightarrow E(2\pi r) = (2R^2 + r^2)\pi\mu_0 n\alpha$$

$$\Rightarrow E = \frac{(2R^2 + r^2)}{r}\frac{\mu_0 n\alpha}{2} \qquad \ldots(1)$$

The charged particle is held in its circular orbit by the magnetic field, and so we have

$$\frac{mv^2}{r} = qvB \qquad \ldots(2)$$

Further, due to the induced electric field the particle gets accelerated along its circular orbit by the tangential component of the net force.

$$\Rightarrow ma_t = qE$$

where m is the mass and q the electric charge of the particle. Further, since the magnitude of the electric field is constant, the speed of the particle increases uniformly with time. So,

$$v = a_t t = \left(\frac{qE}{m}\right)t$$

Substituting the value of E from (1), we get

$$v = \frac{q}{m}\left(\frac{2R^2 + r^2}{r}\frac{\mu_0 n\alpha}{2}\right)t \qquad \ldots(3)$$

Again, substituting this value of v and the value of $B(=\mu_0 n\alpha t)$, into equation (2), we get

$$\frac{m}{r}\frac{(2R^2 + r^2)}{r}\frac{\mu_0 n\alpha}{2}\frac{q}{m}t = q\mu_0 n\alpha t,$$

$$\Rightarrow \frac{(2R^2 + r^2)}{2r^2} = 1$$

$$\Rightarrow r = \sqrt{2}R$$

$$\Rightarrow k = 2$$

6. Induced electric field

$$E(2\pi r) = \frac{d\phi}{dt}$$

$$\Rightarrow E = \frac{\pi a^2 2B_0 t}{2\pi r}$$

Torque due to field about centre of ring

$$\tau_1 = (qE)r = (\lambda(2\pi r))\left(\frac{2\pi a^2 B_0 t}{2\pi r}\right)r$$

ring starts rotating when

$$\tau_{\text{due to electric field}} = \tau_{\text{due to friction}}$$

$$\Rightarrow \tau_1 = (\mu mg)r$$

Solving, we get $t = \dfrac{\mu mg}{2\pi a^2 B_0 \lambda} = 4\text{ s}$

7. When the switch is closed for a long time, then $I_{max} = I_0 = \dfrac{E}{R}$. Now when the switch is opened, the current in the battery and the resistor drops to zero. However, the coil carries this same current because the oscillations begin in the LC loop. So, we have

$$\frac{1}{2}CV^2 = \frac{1}{2}LI_0^2$$

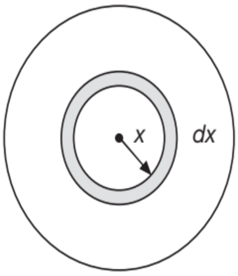
$$\Rightarrow L = \frac{CV^2}{I_0^2} = \frac{(0.5\times10^{-6})(150)^2(250)^2}{(50)^2} \quad \left\{\because I_0 = \frac{E}{R}\right\}$$

$$\Rightarrow L = 281\text{ mH}$$

8. $E = \dfrac{x}{2}\dfrac{dB}{dt}$

$$\Rightarrow E = \frac{3Kxt^2}{2}$$

$$\Rightarrow d\tau = (dq)Ex = \left(\frac{3Kxt^2}{2}\right)\left(\frac{2\pi x dx}{\pi r^2}\right)qx$$



$$\Rightarrow \quad \tau = \frac{3Kt^2q}{r^2}\int_0^r x^3 dx$$

$$\Rightarrow \quad \tau = \frac{3Kqt^2}{4}r^2 \qquad \ldots(1)$$

Torque due to friction force

$$d\tau = \mu gxdm = \mu gx(2\pi xdx)\frac{m}{\pi r^2}$$

$$\Rightarrow \quad \tau = 2\mu g\frac{m}{r^2}\int_0^r x^2dx = \frac{2}{3}\mu mgr \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{3Kqt^2r^2}{4} = \frac{2}{3}\mu mgr$$

$$\Rightarrow \quad t = \sqrt{\frac{8\mu mg}{9Kqr}}$$

Substituting values, we get

$$t = 2 \text{ s}$$

9. $\xi = \frac{d\phi}{dt} = \frac{d}{dt}(NB\ell^2\cos\theta) = N\ell^2\cos\theta\frac{dB}{dt}$

$$\ell = \sqrt{\frac{\xi}{N\left(\frac{dB}{dt}\right)\cos\theta}}$$

where $\xi = 80\times10^{-3}$ V,

$N = 50$ and $\frac{dB}{dt} = \left(\frac{600-200}{0.4}\right)\times10^{-6}$ Ts^{-1}

$$\Rightarrow \quad \ell = \sqrt{\frac{80\times10^{-3}}{(50)\left(\frac{400\times10^{-6}}{0.4}\right)\cos(30^\circ)}}$$

$$\Rightarrow \quad \ell = 1.36 \text{ m}$$

$\Rightarrow$ Total length $= N(4\ell) = (50)(4)(1.36) = 272$ m

10. Since $\phi = BA = \mu_0 nI(A_{\text{solenoid}})$

$\Rightarrow \quad \phi = \mu_0 n(\pi r^2_{\text{solenoid}})I$, where $n = 1000$

According to Faraday's Laws, we have

$$\xi = -\frac{Nd\phi}{dt} = -N\mu_0 n(\pi r^2_{\text{solenoid}})\left(\frac{dI}{dt}\right), \text{ where } N = 15$$

Since $I = 4\cos(250t)$

$$\Rightarrow \quad \frac{dI}{dt} = -1000\sin(250t)$$

$$\xi = (15)(4\pi\times10^{-7})(1000)\pi(0.02)^2(1000)\sin(250t)$$

$$\Rightarrow \quad \xi = (60\pi^2)(4\times10^{-4})(10^{-1})\sin(250t)$$

$$\Rightarrow \quad \xi = 600\times4\times10^{-5}\sin(250t)$$

$$\Rightarrow \quad \xi = 2400\times10^{-5}\sin(250)t$$

$$\Rightarrow \quad \xi = (24\times10^{-3})\sin(250t)\text{volt}$$

$$\Rightarrow \quad \xi_{\text{peak}} = 24 \text{ mV}$$

11. We know the magnetic field at P a distance $x = a$ from the centre of the coil lying on the axis is

$$B = \frac{\mu_0 Ia^2}{2\pi(a^2+a^2)^{3/2}}$$

The current due to revolving charge is

$$I = \frac{q\omega}{2\pi}$$

$$\Rightarrow \quad B = \frac{\mu_0}{4\pi}\frac{q\omega a^2}{(2a^2)^{3/2}} = \frac{\mu_0 q\omega}{8\sqrt{2}\pi a}$$

The energy density due to the magnetic field is

$$u_M = \frac{B^2}{2\mu_0}$$

The electric field strength at an axial point is given as

$$E = \frac{1}{4\pi\varepsilon_0}\frac{qa}{(a^2+a^2)^{3/2}} = \frac{q}{8\sqrt{2}\pi\varepsilon_0 a^2}$$

The energy density of the electric field is given as

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

Ratio of energy densities of magnetic and electric field is

$$\frac{u_M}{u_E} = \frac{\left(\frac{B}{E}\right)^2}{\mu_0\varepsilon_0} = (\omega a)^2\mu_0\varepsilon_0$$

$$\Rightarrow \quad \frac{u_M}{u_E} = \frac{a^2\omega^2}{c^2} = \frac{(81\times10^{-4})(10^4)}{9\times10^{16}} = 9\times10^{-16}$$

$$\Rightarrow \quad * = 9$$

CHAPTER 3

12. After reading the statement carefully, we note that the flux inside the tube is constant. So,

$$\phi = B_{\text{initial}}A_{\text{initial}} = B_{\text{final}}A_{\text{final}}$$

$$\Rightarrow \quad B_iA_i = B_fA_f$$

$$\Rightarrow \quad B_f = B_i\left(\frac{A_i}{A_f}\right)$$

$$\Rightarrow \quad B_f = (2.5)\left(\frac{R}{r}\right)^2 \qquad \{\because A_i = \pi R^2 \text{ and } A_f = \pi r^2\}$$

$$\Rightarrow \quad B_f = (2.5)(12)^2$$

$$\Rightarrow \quad B_f = (2.5)(144) = 360 \text{ T}$$

13. (a) $|\vec{F}| = I|\vec{\ell}\times\vec{B}| = BI\ell$, where $I = \frac{\xi}{R} = \frac{B\ell v}{R}$

$$\Rightarrow \quad F = \frac{B^2\ell^2 v}{R} = \frac{(2.5)^2(1.2)^2(2)}{6} = 3 \text{ N}$$

(b) $P = I^2R = Fv = \frac{B^2\ell^2v^2}{R} = 6 \text{ W}$

14. The magnetic field at the axis of a solenoid is

$$B = \mu ni = (\mu_r\mu_0)ni \qquad \ldots(1)$$

where, μ_r is the relative permeability of the medium.

The magnetic flux associated with the cross-section of the solenoid is 1.6×10^{-3} Wb. This simply means that the flux associated with each turn of the solenoid is 1.6×10^{-3} Wb i.e. Since the magnetic flux is

$$\phi = (1)BA = 1.6\times10^{-3}$$

$$\Rightarrow \quad B = \frac{\phi}{A} = \frac{1.6\times10^{-3}}{10^{-3}} = 1.6 \text{ T}$$

Substituting the values in Equation (1), we get

$$1.6 = \mu_r(4\pi\times10^{-7})\times400\times5$$

$$\Rightarrow \quad \mu_r \approx 637$$

The total number of turns in the solenoid are

$$N = n\times l = 400\times0.5 = 200$$

Hence, the total magnetic flux linked with the solenoid is

$$\phi_T = N\phi = 200\times1.6\times10^{-3} = 0.32 \text{ Wb}$$

$$\Rightarrow \quad L = \frac{N\phi}{i} = \frac{0.35}{5} = 0.064 \text{ H} = 64 \text{ mH}$$

15. (a) $\xi = B\ell v = (2.5)\left(\frac{50}{100}\right)(8) = 10 \text{ V}$

$$\Rightarrow \quad I_1 = \frac{10}{5/3} = 6 \text{ A and } I_2 = \frac{10}{5} = 2 \text{ A}$$

(b) $P = \xi(I_1 + I_2) = (10)(8) = 80 \text{ W}$

(c) $F = \frac{P}{v} = \frac{80}{8} = 10 \text{ N}$

16. Since, $\phi_B = 3(at^3 - bt^2) = (6t^3 - 18t^2)$

$$\Rightarrow \quad \xi = -\frac{d\phi_B}{dt} = -18t^2 + 36t$$

ξ is maximum when, $\frac{d\xi}{dt} = -36t + 36 = 0$ which gives

$$t = 1 \text{ s}$$

Therefore, the maximum current (at $t = 1$ s) is

$$I = \frac{\xi}{R} = \frac{(-18+36) \text{ V}}{3\ \Omega} = 6 \text{ A}$$

17. Magnetic energy stored in inductor is

$$U = \frac{1}{2}Li^2$$

This energy is completely utilised in melting the ice. If L_f be the latent heat of fusion of ice, then we have

$$\frac{1}{2}Li^2 = mL_f$$

$$\Rightarrow \quad m = \frac{Li^2}{2L_f}$$

Substituting the values, we get

$$m = \frac{(42)(20)^2}{2(80\times4200)} = 0.025 \text{ kg}$$

$$\Rightarrow \quad m = 25 \text{ g}$$

18. $\frac{1}{2}CV^2 = \frac{1}{2}LI_0^2$

$$\Rightarrow \quad I_0 = \sqrt{\frac{CV^2}{L}}$$

$$\Rightarrow \quad I_0 = \sqrt{\frac{(10^{-6})(1600)}{10\times10^{-3}}}$$

$$\Rightarrow \quad I_0 = 0.4 \text{ A} = 400 \text{ mA}$$

19. (a) $I_0 = I_{\max} = \frac{E}{R} = \frac{12 \text{ V}}{12\ \Omega} = 1 \text{ A}$

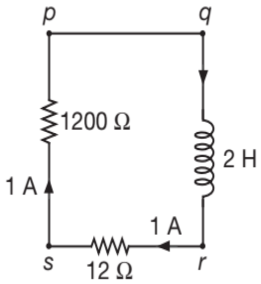
(b) At the instant the switch is thrown quickly from *a* to *b*, the current through the circuit is 1 A i.e., current through $12\ \Omega$ and now $1200\ \Omega$ both is 1 A.

$$\Rightarrow \quad \Delta V_{12\ \Omega} = (1 \text{ A})(12\ \Omega) = 12 \text{ V and}$$

$$\Delta V_{1200\ \Omega} = (1 \text{ A})(1200\ \Omega) = 1200 \text{ V}$$

Consider the circuit at the instant immediately after the switch is thrown quickly from a to b. For the loop $pqrs$, we get

$$-(12)(1)-(1200)(1)-\Delta V_L=0$$



$$\Rightarrow \quad \Delta V_L=-1212\text{ V}$$

$$\Rightarrow \quad |\Delta V_L|=1212\text{ V}$$

(c) Since $I=I_{max}e^{-\frac{Rt}{L}}=I_0e^{-\frac{Rt}{L}}$

$$\Rightarrow \quad \frac{dI}{dt}=-\frac{I_0R}{L}e^{-\frac{Rt}{L}}$$

$$\Rightarrow \quad -L\frac{dI}{dt}=\Delta V_L=(I_0R)e^{-\frac{Rt}{L}}$$

$$\Rightarrow \quad 12=(1)(1200+12)e-\frac{(1200+12)t}{2}$$

$$\Rightarrow \quad \frac{12}{1212}=e^{-606t}$$

$$\Rightarrow \quad \log_e\left(\frac{1212}{12}\right)=606t$$

$$\Rightarrow \quad 606t=\log_e(101)$$

$$\Rightarrow \quad 606t=4.6$$

$$\Rightarrow \quad t=76\times10^{-4}\text{ s}$$

$$\Rightarrow \quad x=76$$

20. Let the magnitude of the magnetic field generated by the wire carrying a current I, at a distance x from it be B. Then

$$B=\frac{\mu_0 I}{2\pi x}$$

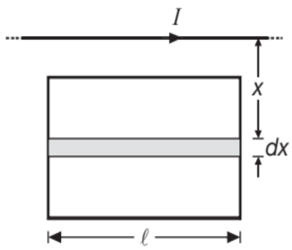
Consider a rectangular strip of width dx, area $dA=\ell dx$

$$d\phi=\vec{B}\cdot d\vec{A}=BdA\cos 0$$

$$\Rightarrow \quad d\phi=\frac{\mu_0 I}{2\pi x}\ell\,dx$$

$$\Rightarrow \quad \phi=\int d\phi=\frac{\mu_0 I\ell}{2\pi}\int_{0.4\text{ mm}}^{1.7\text{ mm}}\frac{dx}{x}$$

$$\Rightarrow \quad \phi=\frac{\mu_0 I\ell}{2\pi}\log_e\left(\frac{1.7}{0.4}\right)=\frac{\mu_0 I\ell}{2\pi}\log_e\left(\frac{17}{4}\right)$$



The mutual inductance between the wire and the loop is

$$M=\frac{N_2\phi_{12}}{I_1}=\frac{N_2\mu_0 I\ell}{2\pi I}\log_e\left(\frac{17}{4}\right)=\frac{N_2\mu_0\ell}{2\pi}(1.45)$$

$$\Rightarrow \quad M=\frac{(1)(4\pi\times10^{-7})(2.7\times10^{-3})}{2\pi}(1.45)$$

$$\Rightarrow \quad M=783\times10^{-12}\text{ H}$$

$$\Rightarrow \quad M=783\text{ pH}$$

21. $\tau_C=\tau_L$

$$\Rightarrow \quad CR=\frac{L}{R}$$

$$\Rightarrow \quad R=\sqrt{\frac{L}{C}}=\sqrt{\frac{3}{3\times10^{-6}}}=1000\ \Omega=1\text{ k}\Omega$$

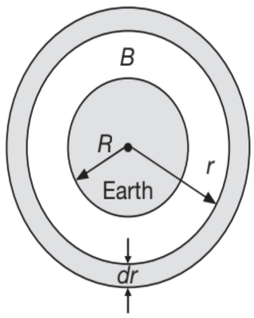
Also, $\tau=RC=(1000)(3\times10^{-6})=3$ ms

22. Since, the magnetic energy density is

$$u_m=\frac{B^2}{2\mu_0}=\frac{B_0^2R^4}{2\mu_0 r^4}$$

Consider an infinitesimal spherical shell (concentric with earth) of radius r, thickness dr and volume dV. Then

$$dV=4\pi r^2dr$$



If dU is the energy associated with the shell, then

$$dU=u_m dV$$

$$\Rightarrow \quad dU=\left(\frac{B_0^2R^4}{2\mu_0}\right)\left(\frac{4\pi r^2dr}{r^4}\right)$$

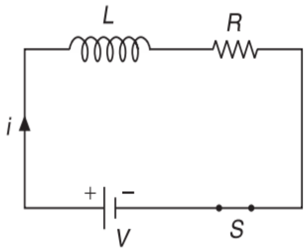
$\Rightarrow \quad U = \int dU = \dfrac{2\pi B_0^2 R^4}{\mu_0}\int_R^{\infty} r^{-2}dr$

$\Rightarrow \quad U = \dfrac{2\pi B_0^2 R^3}{\mu_0} = \dfrac{(2\pi)(50\times10^{-6})^2(6\times10^6)^3}{4\pi\times10^{-7}}$

$\Rightarrow \quad U = 2700\times10^{15}$ J

$\Rightarrow \quad U = 2700$ peta joule $\quad \{\because 1 \text{ peta} = 10^{15}\}$

23. Let at a time instant t, after shorting of switch, current through the circuit be i as shown in Figure.



For the given loop, the current in the circuit as a function of time is

$$i = \frac{V}{R}\left(1-e^{-Rt/L}\right) = i_0\left(1-e^{-Rt/L}\right) \quad \ldots(1)$$

where, $i_0 = \dfrac{V}{R}$ is the maximum current in circuit.

The magnetic energy stored in inductor is

$$U = \frac{1}{2}Li^2$$

Given that at an instant the magnetic energy attains one fourth of the maximum value, so we have

$$\frac{1}{2}Li^2 = \frac{1}{4}\left(\frac{1}{2}Li_0^2\right)$$

$\Rightarrow \quad i = \dfrac{i_0}{2} = \dfrac{V}{2R}$

Substituting the values in Equation (1), we get

$$\frac{V}{2R} = \frac{V}{R}\left(1-e^{-2t/20}\right)$$

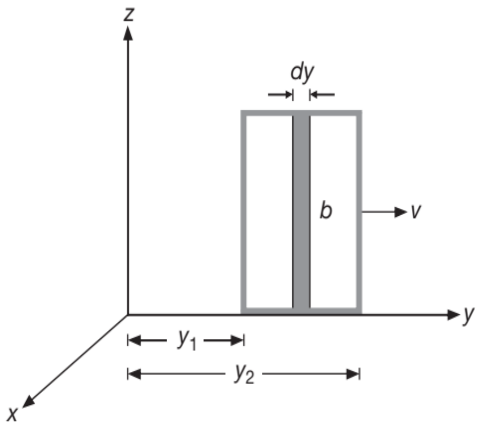
$\Rightarrow \quad \dfrac{1}{2} = 1 - e^{-t/10}$

$\Rightarrow \quad e^{-t/10} = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{t}{10} = 0.7$

$\Rightarrow \quad t = 7$ s

24. (a) Magnetic flux linked with the loop



Consider a strip of length b, width dy, then

$d\phi = BdA$

$\Rightarrow \quad d\phi = (6-y)(b\,dy)$

$\Rightarrow \quad \phi = \int_{y_1}^{y_2}(6-y)b\,dy$, where $y_1 = vt$ and $y_2 = vt+\ell$

$\Rightarrow \quad \phi = 6b(y_2-y_1) - \dfrac{b}{2}(y_2-y_1)(y_2+y_1) \quad \ldots(1)$

$\Rightarrow \quad \phi = 6\ell b - \dfrac{\ell b}{2}(\ell+2vt)$

Induced e.m.f. $\xi = -\dfrac{d\phi}{dt} = b\ell v$

$\Rightarrow \quad \xi = (5)(2)(3)$ V

$\Rightarrow \quad \xi = 30$ V

(b) From (1), we get

$$\phi = 6b(y_2-y_1) - \frac{b}{2}(y_2-y_1)(y_2+y_1)$$

Now, in this case we have, if a to be the acceleration, then

$y_1 = \dfrac{1}{2}at^2$ and $y_2 = \dfrac{1}{2}at^2 + \ell$

$\Rightarrow \quad y_2 - y_1 = \ell$ and $y_2 + y_1 = at^2 + \ell$

$\Rightarrow \quad \phi = 6\ell b - \dfrac{\ell b}{2}(at^2+\ell)$

Since $\xi = -\dfrac{d\phi}{dt}$

$\Rightarrow \quad \xi = (\ell b a)t$

$\Rightarrow \quad \xi = (2)(5)(2) = 20$ V

25. Volume of the balloon at any instant, when radius is r

$$V = \frac{4}{3}\pi r^3$$

Time rate of change of volume

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$$

Flux through the band at the given instant is

$$\phi = B(\pi r^2)$$

Induced emf $\xi = -\frac{d\phi}{dt} = -\frac{d}{dt}(B\pi r^2)$

$$\Rightarrow \xi = -2\pi r B\frac{dr}{dt}$$

$$\Rightarrow \xi = -2\pi r B\left(\frac{1}{4\pi r^2}\frac{dV}{dt}\right)$$

$$\Rightarrow \xi = -\frac{B}{2r}\frac{dV}{dt}$$

Since, the volume of the balloon is decreasing, so $\frac{dV}{dt}$ is negative

$$\Rightarrow \xi = -\frac{(0.04)}{2\times 6\times 10^{-2}}\times(-100\times 10^{-6})$$

$$\Rightarrow \xi = 333\ \mu V$$

26. Flux linked with the loop at any instant

$$\phi = \int_x^{x+\ell}\left(\frac{\mu_0 I}{2\pi y}\right)\ell dy$$

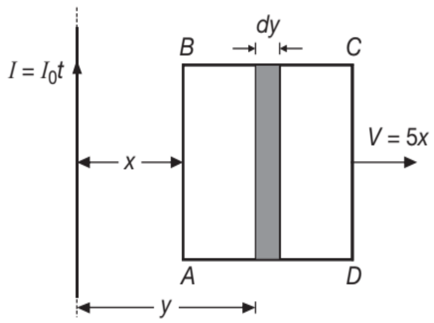
$$\Rightarrow \phi = \frac{\mu_0 I_0 \ell t}{2\pi}\log_e\left(\frac{x+\ell}{x}\right) \quad \ldots(1)$$

$$\Rightarrow \xi = \frac{-d\phi}{dt}$$

$$\xi = -\left[\frac{\mu_0 I_0 \ell}{2\pi}\log_e\frac{(x+\ell)}{x} + \frac{\mu_0 I_0 \ell t}{2\pi}\left(\frac{x}{x+\ell}\right)\left(-\frac{\ell}{x^2}\right)\left(\frac{dx}{dt}\right)\right]$$

$$\left\{\because \frac{d}{dt}\log_e(u) = \frac{1}{u}\frac{du}{dt}\right\}$$

$$\Rightarrow I = \frac{\xi}{R} = -\frac{\mu_0 I_0 \ell}{2\pi R}\log_e\left(\frac{x+\ell}{x}\right) + \frac{\mu_0 I_0 \ell^2 t}{2\pi R x(x+\ell)}\frac{dx}{dt} \quad \ldots(2)$$



Since, we have

$$\frac{dx}{dt} = 5x$$

$$\Rightarrow \int_{\ell/10}^{10\ell}\frac{dx}{x} = 5\int_0^t dt$$

$$\Rightarrow t = \frac{\log_e 100}{5}$$

Putting the value of t and $R = \frac{\mu_0 I_0 \ell}{2\pi}$ in equation (2), we get

$$I = -\log_e(1.1) + \frac{1}{11}\log_e(100)$$

$$\Rightarrow I = -0.095 + \frac{4.605}{11}$$

$$\Rightarrow I = 324\ \text{mA}$$

27. (a) Voltage across the inductor is given by

$$V_L = L\frac{dI}{dt}$$

$$\Rightarrow V_L = 10^{-3}(20)\ \text{V}$$

$$\Rightarrow V_L = 20\ \text{mV}$$

(b) Voltage across the capacitor is given by

$$V_C = \frac{q}{C} = \frac{1}{C}\int_0^t Idt$$

$$\Rightarrow V_C = \left(\frac{1}{10^{-6}}\right)\int_0^t (20t)dt$$

$$\Rightarrow V_C = (10^6)\frac{20t^2}{2}\ V = (10^7 t^2)\ V$$

So, at $t = 1$ ms, we have

$$V_C = 10^7(10^{-6}) = 10\ \text{V}$$

(c) $\frac{1}{2}CV_C^2 > \frac{1}{2}LI^2$

$$\Rightarrow CV_C^2 > LI^2$$

$$\Rightarrow (10^{-6})(10^7 t^2)^2 > (10^{-3})(20t)^2$$

$$\Rightarrow t > 63.2\times 10^{-6}\ \text{s}$$

$$\Rightarrow t > 63.2\ \mu s$$

$$\Rightarrow t = 63\ \mu s$$

28. $I=\frac{dq}{dt}=\frac{\xi}{R}$ where $\xi=-N\frac{d\phi_B}{dt}$

$\Rightarrow \int_0^Q dq=\frac{N}{R}\int_{\phi_1}^{\phi_2} d\phi_B$

$\Rightarrow |Q|=\frac{N}{R}(\phi_2-\phi_1)$

$\Rightarrow Q=\frac{N}{R}\left[BA\cos(0^\circ)-BA\cos\left(\frac{\pi}{2}\right)\right]=\frac{BAN}{R}$

$\Rightarrow B=\frac{RQ}{NA}=\frac{(200)(5\times10^{-4})}{(100)(40\times10^{-4})}=0.25\text{ T}=250\text{ mT}$

29. $I=\frac{\xi}{R}=\frac{B}{R}\left(\frac{\Delta A}{\Delta t}\right)$

$\Rightarrow I\Delta t=Q=\left|\frac{B}{R}(\Delta A)\right|$

$\Rightarrow Q=\left|\frac{B}{R}(A_f-A_i)\right|=\left|\frac{B}{R}(0-\ell^2)\right|=\frac{B\ell^2}{R}$

$\Rightarrow Q=\frac{(25\times10^{-6})(0.04)}{0.5}$

$\Rightarrow Q=2\times10^{-6}C=2\,\mu\text{C}$

$\Rightarrow Q=2\,\mu\text{C}$

30. Since we know that

$I=-\frac{N}{R}\frac{d\phi}{dt}$

$\Rightarrow Idt=-\frac{N}{R}d\phi$

$\Rightarrow dq=-\frac{N}{R}d\phi$

$\Rightarrow \int dq=-\frac{N}{R}\int d\phi$

$\Rightarrow \Delta q=-\frac{N}{R}\Delta\phi=-\frac{N}{R}(\phi_f-\phi_i)$

$\Rightarrow \Delta q=-\frac{N}{R}(-BA-BA)$

$\{\because \phi_f=BA\cos(180^\circ)$ and $\phi_i=BA\cos(0^\circ)\}$

$\Rightarrow \Delta q=\frac{2NBA}{R}$

$\Rightarrow B=\frac{R\Delta q}{2NA}$

$\Rightarrow B=\frac{(40)(4.5\times10^{-6})}{(2)(30)(3\times10^{-6})}$

$\Rightarrow B=1\text{ T}$

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1. Energy dissipated per second by resistor is

$P_R=i^2\times R$

Energy supplied per second by battery is

$P_B=E\times i$

Since $P_B=P_R+P_L$

$\Rightarrow P_L=Ei-i^2R$

$\Rightarrow Ei-i^2R=i^2R$

$\Rightarrow i=\frac{E}{2R}$

Since $i=\frac{E}{R}\left(1-e^{-\frac{t}{\tau}}\right)$, where $\tau=\frac{L}{R}=2$

$\Rightarrow \frac{E}{2R}=\frac{E}{R}\left(1-e^{-\frac{t}{\tau}}\right)$

$\Rightarrow t=\tau\ln(2)=\frac{20}{10}\ln(2)=2\ln(2)$

Hence, the correct answer is (B).

2. Since $B=\mu_0 ni$

$\Rightarrow \phi=(\mu_0 ni)(nL)A$

So, inductance $L'=\frac{\phi}{i}\mu_0 n^2AL$

Since $n=\frac{N}{L}$

$\Rightarrow L'=\mu_0\left(\frac{N^2A}{L}\right)$

$\Rightarrow L'\propto\frac{1}{L}$

Hence, the correct answer is (A).

3. $\phi_Q=Mi$

$\Rightarrow 10^{-3}=M(3)$

$\Rightarrow M=\frac{1}{3}\times10^{-3}\text{ H}$

Since $\phi_P=M(2)$

$$\Rightarrow \quad \frac{10^{-3}}{3} = \frac{\phi_P}{2}$$

$$\Rightarrow \quad \phi_P = \frac{20}{3} \times 10^{-4} = 6.67 \times 10^{-4} \text{ Wb}$$

Hence, the correct answer is (B).

4. $i = kte^{-\alpha t}$

Since $\phi = BA = (\mu_0 ni)A = \mu_0 n(kte^{-\alpha t})\pi(2R)^2$

$$\Rightarrow \quad \phi = (4\pi R^2 \mu_0 nK)te^{-\alpha t} = k'(te^{-\alpha t})$$

Since $\xi = -\left(\frac{d\phi}{dt}\right) = -k'e^{-\alpha t} + k'\alpha te^{-\alpha t}$

$$\Rightarrow \quad \xi = -k'e^{-\alpha t}(1 - \alpha t)$$

So, induced current I is given by

$$I = \frac{E}{r} = -k'e^{-\alpha t}(1 - \alpha t)$$

At $t = 0$, $I = -k'$

Hence, the correct answer is (B).

5. $I = I_{sat}\left(1 - e^{\frac{Rt}{L}}\right)$, where $R = R_L + r = 1\,\Omega$

Since $I = 0.8 I_{sat}$

$$\Rightarrow \quad 0.8 I_{sat} = I_{sat}\left(1 - e^{-\frac{t}{0.01}}\right)$$

$$\Rightarrow \quad \frac{4}{5} = 1 - e^{-100t}$$

$$\Rightarrow \quad e^{-100t} = \left(\frac{1}{5}\right)$$

$$\Rightarrow \quad 100t = \ln 5$$

$$\Rightarrow \quad t = \frac{1}{100}\ln 5$$

$$\Rightarrow \quad t = 0.016 \text{ s}$$

Hence, the correct answer is (D).

6. $BLv = IR_{eq}$

where $R_{eq} = \frac{4}{3}\Omega + 1.7 = 3\,\Omega$

$$\Rightarrow \quad I = \frac{BLv}{R_{eq}} = \frac{(1)(5 \times 10^{-2}) \times 10^{-2}}{3}$$

$$\Rightarrow \quad I = \frac{5}{3} \times 10^{-4} \text{ A} \approx 1.7 \times 10^{-4} \text{ A}$$

$$\Rightarrow \quad I = 170\ \mu\text{A}$$

Hence, the correct answer is (A).

7. $i = \frac{E}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$ where $\tau = \frac{L}{R}$

$$\Rightarrow \quad q = \int_0^q dq = \frac{E}{R}\int\left(1 - e^{-\frac{t}{\tau}}\right)dt$$

$$\Rightarrow \quad q = \frac{E}{R}\left(t + e^{-\frac{t}{\tau}} \times \tau\right)\Bigg|_0^t$$

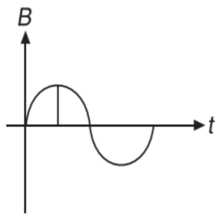
$$\Rightarrow \quad q = \frac{E}{R}\left(\frac{L}{R} + \frac{L}{R}e^{-1} - \frac{L}{R}\right)$$

$$\Rightarrow \quad q = \frac{EL}{eR^2} = \frac{EL}{2.7R^2} \qquad \{\because e \approx 2.7\}$$

Hence, the correct answer is (B).

8. Since, $\Delta Q = \frac{\Delta\phi}{R}$

Given that,



$$B(t) = 0.4\sin(50\pi t)$$

$$\Rightarrow \quad \frac{2\pi}{T} = 50\pi$$

$$\Rightarrow \quad T = \frac{1000}{25} \text{ ms}$$

$$\Rightarrow \quad T = 40 \text{ ms}$$

$$\Rightarrow \quad \Delta Q = \frac{0.4 \times 3.5 \times 10^{-3}}{10}$$

$$\Rightarrow \quad \Delta Q = 1.4 \times 10^{-4} \text{ C}$$

*No given option is correct.

9. Field inside solenoid is

$$B = \mu_0 ni, \text{ where } n = \frac{100 \times 10}{2} = 500 \text{ m}$$

$$\Rightarrow \quad B = \mu_0 \times 500i$$

So, corresponding $H = \frac{B}{\mu_0} = ni$

$$\Rightarrow \quad H = 500 \times i$$

$$\Rightarrow \quad H = 500 \times 5.2 \text{ A}$$

$$\Rightarrow \quad H = 2600 \text{ Am}^{-1}$$

Hence, the correct answer is (B).

10. Change in energy is

$$\Delta E = \frac{1}{2}LI_f^2 - \frac{1}{2}LI_{in}^2$$

Since, $L\frac{dI}{dt} = 25$

$$\Rightarrow \quad L\left(\frac{25 - 10}{1}\right) = 25$$

$\Rightarrow \quad L = \frac{25}{15} = \frac{5}{3}$ H

$\Rightarrow \quad \Delta E = \frac{1}{2}\left(\frac{5}{3}\right)(625 - 100)$

$\Rightarrow \quad \Delta E = 437.5$ J

Hence, the correct answer is (A).

11. $i(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right),\ t \le t_0$

$\Rightarrow \quad i(t) = \frac{V}{R} e^{-\frac{R}{L}(t - t_0)},\ t > t_0$

* No given option is correct

12. The mutual inductance M and self-inductance L are given by

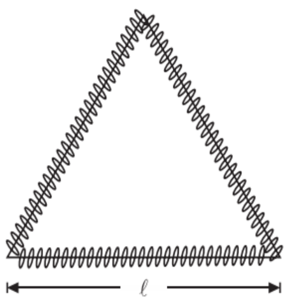
$M = \mu_0 n_1 n_2 \pi r_1^2 l$

$L = \mu_0 n_1^2 \pi r_1^2 l$

$\Rightarrow \quad \frac{M}{L} = \frac{n_2}{n_1}$

Hence, the correct answer is (B).

13.



$\phi = Ll = (\mu_0 n l) A (n)(3l)$

where n is the number of turns/length

$\Rightarrow \quad L \propto l$

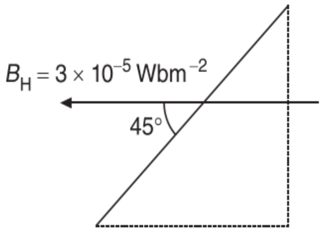
Hence, the correct answer is (A).

14. After long time, the inductor will behave like a wire.

$I = \frac{15}{\left(\frac{R}{2}\right)} = \frac{30}{5} = 6$ A

Hence, the correct answer is (C).

15. $B_H = 3 \times 10^{-5}$ Wbm^{-2}



$\xi = Blv = 5 \times 3 \times 10^{-5} \times 10 \times \frac{1}{\sqrt{2}} = 1.06 \times 10^{-3}$ V

$\Rightarrow \quad \xi \approx 1.1 \times 10^{-3}$ V

Hence, the correct answer is (A).

16. Since coil is small, so B is assumed to be constant in this region.

Emf induced in smaller coil is given by

$\xi = -\frac{d\phi}{dt} = -BA\frac{d}{dt}[\cos(\omega t)]$

$\Rightarrow \quad \xi = \frac{\mu_0 I}{2R}\omega \pi r^2 \sin\omega t$

Hence, the correct answer is (D).

17. Copper rod will acquire terminal velocity when the magnetic force equals the gravitation force.

$\Rightarrow \quad I\ell B = mg\sin\theta$ ...(1)

Also, $I = \frac{\text{induced emf}}{R} = \frac{B\ell v}{R}$ ...(2)

From equations (1) and (2), we get

$\frac{B^2\ell^2 v}{R} = mg\sin\theta$

$\Rightarrow \quad v = \frac{mgR\sin\theta}{B^2\ell^2}$

Hence, the correct answer is (A).

18. EMF induced in the coil is

$\xi = N_n BA\omega\sin(\omega t)$

$\Rightarrow \quad \xi_{\max} = N_n BA\omega$

Hence, the correct answer is (B).

19. Since $\Delta q = \frac{\Delta\phi}{R}$

$\Rightarrow \quad \Delta\phi = R\Delta q = R(\text{Area under } I\text{-}t \text{ graph})$

$\Rightarrow \quad \Delta\phi = R\left(\frac{1}{2} \times 10 \times 0.5\right) = 100\left(\frac{1}{2}(10)(0.5)\right)$

$\Rightarrow \quad \Delta\phi = 250$ Wb

Hence, the correct answer is (C).

20. The induced emf, $\xi = -M\dfrac{dI}{dt}$ ...(1)

where mutual inductance M is given by

$$M = \frac{\mu_0 \pi a^2}{2b}$$

Since, $I = I_0 \cos(\omega t)$

$$\Rightarrow \quad \xi = \frac{-\mu_0 \pi a^2}{2b}\frac{d}{dt}\left[I_0 \cos(\omega t)\right] = \frac{\mu_0 \pi a^2}{2b} I_0 \omega \sin(\omega t)$$

$$\Rightarrow \quad \xi = \frac{\pi \mu_0 I_0}{2}\frac{a^2}{b}\omega \sin(\omega t)$$

Hence, the correct answer is (C).

21. Given that, $B = B_0 e^{-\frac{t}{\tau}}$

Area of the circular loop is $A = \pi r^2$

Flux linked with the loop at any time t is

$$\phi = BA = \pi r^2 B_0 e^{-\frac{t}{\tau}}$$

Emf induced in the loop is

$$\xi = -\frac{d\phi}{dt} = \left(\pi r^2 B_0\right)\left(\frac{1}{\tau}e^{-\frac{t}{\tau}}\right)$$

Net heat generated in the loop is

$$H = \int_0^\infty \frac{\xi^2}{R}dt = \frac{\pi^2 r^4 B_0^2}{\tau^2 R}\int_0^\infty e^{-\frac{2t}{\tau}}\, dt$$

$$\Rightarrow \quad H = \frac{\pi^2 r^4 B_0^2}{\tau^2 R} \times \frac{1}{\left(-\frac{2}{\tau}\right)}\left(e^{-\frac{2t}{\tau}}\right)\Bigg|_0^\infty$$

$$\Rightarrow \quad H = -\frac{\pi^2 r^4 B_0^2}{2\tau^2 R}\left[\tau(0-1)\right] = \frac{\pi^2 r^4 B_0^2}{2\tau R}$$

Hence, the correct answer is (B).

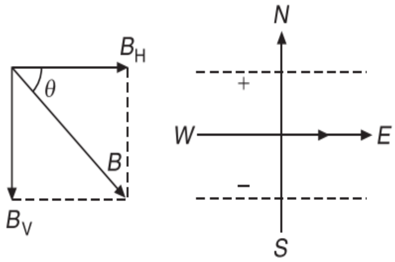
22. Given that, Length of the plane is $\ell = 20$ m

Wing span is $\ell' = 15$ m

Height of plane is $h = 5$ m

Velocity of plane is $v = 240\ \text{ms}^{-1}$, towards east

$$\sin\theta = \frac{2}{3},\ B = 5\times10^{-5}\ \text{T},\ V_B = ?,\ V_W = ?$$



If V_B be the voltage developed between the lower and upper side of the plane, then

$$V_B = vhB\cos\theta$$

$$\Rightarrow \quad V_B = 240\times5\times5\times10^{-5}\times\frac{\sqrt{5}}{3}$$

$$\Rightarrow \quad V_B = 44.72\times10^{-3}\ \text{V} \approx 45\ \text{mV}$$

The vertical component of earth's magnetic field is

$$B_V = B\sin\theta$$

$$\Rightarrow \quad B_V = 5\times10^{-5}\times\frac{2}{3} = \frac{1}{3}\times10^{-4}\ \text{T}$$

If V_W is the voltage developed between tips of the wings, then

$$V_W = B_V \ell' V = \frac{1}{3}\times10^{-4}\times15\times240 = 1200\times10^{-4}$$

$$\Rightarrow \quad V_W = 120\ \text{mV}$$

Hence, the correct answer is (D).

23. When key K_1 is kept closed, a steady current $I_0\left(=\frac{E}{R}\right)$ flows through the circuit.

When K_1 is opened and K_2 is closed, current at any time t in the circuit is

$$I = I_0 e^{-\frac{t}{\tau}} = \frac{E}{R}e^{-\frac{tR}{L}} \qquad \left(\because\ \tau = \frac{L}{R}\right)$$

Given that, $\xi = 15$ V, $R = 0.15\ \text{k}\Omega = 150\ \Omega$

$$L = 0.03\ \text{H},\ t = 1\ \text{ms} = 10^{-3}\ \text{s}$$

$$\Rightarrow \quad I = \frac{15}{150}e^{-\left(\frac{10^{-3}\times150}{0.03}\right)} = \frac{e^{-5}}{10} = \frac{1}{10e^5} = \frac{1}{10\times150}$$

$$\Rightarrow \quad I = 6.67\times10^{-4}\ \text{A} = 0.67\ \text{mA}$$

Hence, the correct answer is (B).

24. At any time t, the equation of the given circuit is

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0 \qquad ...(1)$$

which is equivalent to that of a damped pendulum.

The solution to this equation (1) is

$$q = Q_0 e^{-\frac{Rt}{2L}}\cos(\omega' t + \phi)$$

where, $\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

The square of maximum charge on capacitor at any time t is

$$Q_{\max}^2 = Q_0^2 e^{-\frac{Rt}{L}}\cos^2(\omega' t + \phi)$$

It decays exponentially with time

For $L_2 < L_1$, the curve is more steep

Hence, the correct answer is (C).

25. $I_1 = 5\text{ A}$, $I_2 = 2\text{ A}$

$\Rightarrow \quad \Delta I = 2 - 5 = -3\text{ A}$

Also, $\Delta t = 0.1\text{ s}$ and $\xi = 50\text{ V}$

Since, $\xi = -L\dfrac{\Delta I}{\Delta t}$

$$\Rightarrow \quad 50 = -L\left(\frac{-3}{0.1}\right)$$

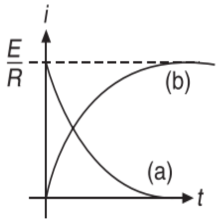
$$\Rightarrow \quad 50 = 30L$$

$$\Rightarrow \quad L = \frac{5}{3} = 1.67\text{ H}$$

Hence, the correct answer is (B).

26. For RC circuit, we have

$$i = \frac{E}{R}e^{-\frac{t}{RC}}$$



For RL circuit, we have

$$i = \frac{E}{R}\left(1 - e^{-t\left(\frac{L}{R}\right)}\right)$$

Hence, the correct answer is (A).

27. Initially current in the circuit is I_0

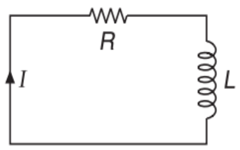
After time t, current falls to new value given by

$$I = I_0 e^{\left(-\frac{t}{\tau}\right)}$$

So, voltage drop across the resistance is

$$V_R = IR = V_0 e^{-\frac{t}{\tau}} \qquad \ldots(1)$$

Voltage across the inductor is



$$V_L = L\frac{dI}{dt} = L\left[-\frac{I_0}{\tau}e^{\left(-\frac{t}{\tau}\right)}\right]$$

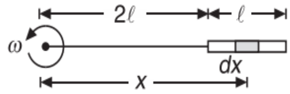
$$\Rightarrow \quad V_L = -I_0\text{R}e^{-\frac{t}{\tau}} = -V_0 e^{-\frac{t}{\tau}} \qquad \ldots(2)$$

From equation (1) and (2), we get

$$\frac{V_R}{V_L} = -1$$

Hence, the correct answer is (D).

28. Consider an element of length dx at a distance x from the fixed end of the string as shown in figure.



Induced emf in the element is

$$d\xi = B(\omega x)dx$$

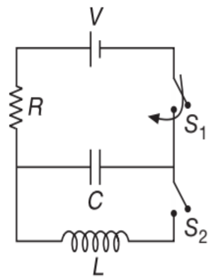
Hence, the emf induced across the ends of the rod is

$$\xi = \int_{2\ell}^{3\ell} B\omega x dx = B\omega\left(\frac{x^2}{2}\right)\Bigg|_{2\ell}^{3\ell} = \frac{B\omega}{2}\left[(3\ell)^2 - (2\ell)^2\right]$$

$$\Rightarrow \quad \xi = \frac{5B\omega\ell^2}{2}$$

Hence, the correct answer is (A).

29. As switch S_1 is closed and switch S_2 is kept open. Now, capacitor is charging through a resistor R.



Charge on a capacitor at any time t is

$$q = q_0\left(1 - e^{-\frac{t}{\tau}}\right), \text{ where } q_0 = CV \text{ and } \tau = RC$$

$$\Rightarrow \quad q = CV\left(1 - e^{-\frac{t}{\tau}}\right) \qquad [\text{As } q_0 = CV]$$

At $t = \dfrac{\tau}{2}$, we have

$$q = CV\left(1 - e^{-\frac{\tau}{2\tau}}\right) = CV\left(1 - e^{-\frac{1}{2}}\right)$$

At $t = \tau$, we have

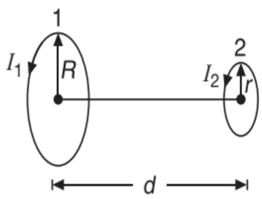
$$q = CV\left(1 - e^{-\frac{\tau}{\tau}}\right) = CV\left(1 - e^{-1}\right)$$

At $t = 2\tau$, we have

$$q = CV\left(1 - e^{-\frac{2\tau}{\tau}}\right) = CV\left(1 - e^{-2}\right)$$

Hence, the correct answer is (D).

30.



As field due to current loop 1 at an axial point

$$\Rightarrow \quad B_1 = \frac{\mu_0 I_1 R^2}{2\left(d^2 + R^2\right)^{3/2}}$$

Flux linked with smaller loop 2 due to B_1 is

$$\phi_2 = B_1A_2 = \frac{\mu_0 I_1 R^2}{2\left(d^2+R^2\right)^{3/2}}\pi r^2$$

The coefficient of mutual inductance between the loops is

$$M = \frac{\phi_2}{I_1} = \frac{\mu_0 R^2 \pi r^2}{2\left(d^2+R^2\right)^{3/2}}$$

Flux linked with bigger loop 1 is

$$\phi_1 = MI_2 = \frac{\mu_0 R^2 \pi r^2 I_2}{2\left(d^2+R^2\right)^{3/2}}$$

Substituting the given values, we get

$$\phi_1 = \frac{4\pi\times10^{-7}\left(20\times10^{-2}\right)^2\times\pi\times\left(0.3\times10^{-2}\right)^2\times2}{2\left[\left(15\times10^{-2}\right)^2+\left(20\times10^{-2}\right)^2\right]^{3/2}}$$

$\Rightarrow \quad \phi_1 = 9.1\times10^{-11}$ weber

Hence, the correct answer is (B).

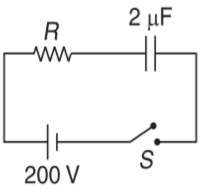
31. Given that $B_H = 5.0\times10^{-5}\ \text{NA}^{-1}\text{m}^{-1}$, $\ell = 2$ m and $v = 1.5\ \text{ms}^{-1}$

Induced emf, $\xi = B_H \ell v = \left(5\times10^{-5}\right)(2)(1.50)$

$\Rightarrow \quad \xi = 15\times10^{-5}\ \text{V} = 0.15$ mV

Hence, the correct answer is (D).

32.



In the case charging of capacitor through the resistance, we have

$$V = V_0\left(I - e^{-\frac{t}{RC}}\right)$$

where, $V = 120$ V, $V_0 = 200$ V, $R = ?$, $C = 2\ \mu$F and $t = 5$ s.

$$\Rightarrow \quad 120 = 200\left(1 - e^{-\frac{5}{R}\times2\times10^{-6}}\right)$$

$$\Rightarrow \quad e^{-\frac{5}{R}\times2\times10^{-6}} = \frac{80}{200}$$

Taking the natural logarithm on both sides, we get

$$\frac{-5}{R\times2\times10^{-6}} = \ln(0.4) = -0.916$$

$\Rightarrow \quad R = 2.7\times10^6\ \Omega$

Hence, the correct answer is (C).

33. Charge on the capacitor at any instant t is

$$q = q_0\cos\omega_0 t \qquad \ldots(1)$$

Equal sharing of energy means that energy stored in the capacitor is half the total energy

$$\frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\left(\frac{1}{2}\frac{q_0^2}{C}\right)$$

$$\Rightarrow \quad q = \frac{q_0}{\sqrt{2}}$$

From equation (1)

$$\frac{q_0}{\sqrt{2}} = q_0\cos\omega_0 t, \text{ where } \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \quad \cos\omega_0 t = \frac{1}{\sqrt{2}}$$

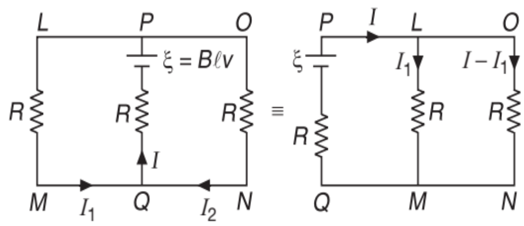
$$\Rightarrow \quad \omega_0 t = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$\Rightarrow \quad t = \frac{\pi}{4\omega_0} = \frac{\pi}{4}\sqrt{LC}$$

Hence, the correct answer is (B).

34. Emf induced across PQ is $\xi = B\ell v$.

The equivalent circuit diagram is as shown in Figure.



So $I - I_1 = I_1$

$$\Rightarrow \quad I_1 = \frac{I}{2}$$

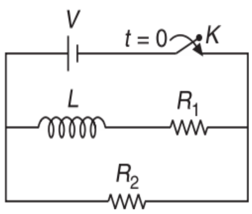
$$\text{Now } I = \frac{\xi}{R_{net}} = \frac{Blv}{R + \frac{(R)(R)}{R+R}} = \frac{Blv}{3R/2}$$

$$\Rightarrow \quad I = \frac{2Blv}{3R}$$

$$\Rightarrow \quad I_1 = \frac{Blv}{3R} \text{ and } I - I_1 = \frac{Blv}{3R}$$

Hence, the correct answer is (C).

35.

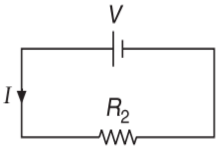


CHAPTER 3

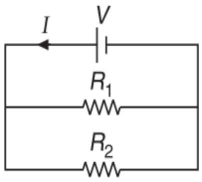
A time $t = 0$, the inductor acts as an open circuit. The corresponding equivalent circuit diagram is as shown in Figure.

The current through battery is

$$I = \frac{V}{R_2}$$



At time $t = \infty$, the inductor acts as a short circuit. The corresponding equivalent circuit diagram is as shown in Figure.



So, the current through the battery is

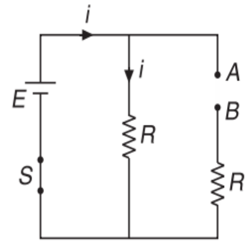
$$I = \frac{V}{R_{eq}} = \frac{V}{\frac{R_1 R_2}{R_1 + R_2}}$$

$$\Rightarrow \quad I = \frac{V(R_1 + R_2)}{R_1 R_2}$$

Hence, the correct answer is (C).

36. Equivalent time constant of circuit is

$$\tau = \frac{L}{R_{AB}} = \frac{L}{R} \qquad \{\because R_{AB} = R\}$$



(at $t = 0$)

Current in the circuit at $t = 0$ is $i = \frac{E}{R}$

So, open circuit voltage across AB is

$$V_{AB} = iR = E$$

Hence current through inductor is

$$i_L = \frac{V_{AB}}{R_{AB}}\left(1 - e^{-\frac{R_{AB}t}{L}}\right)$$

$$\Rightarrow \quad i_L = \frac{12}{2}\left(1 - e^{-\frac{2t}{0.4}}\right) = 6\left(1 - e^{-5t}\right)$$

$$V_L = L\frac{di_L}{dt}$$

$$\Rightarrow \quad V_L = 0.4 \times 6 \times 5e^{-5t}$$

$$\Rightarrow \quad V_L = 12e^{-5t}$$

Hence, the correct answer is (D).

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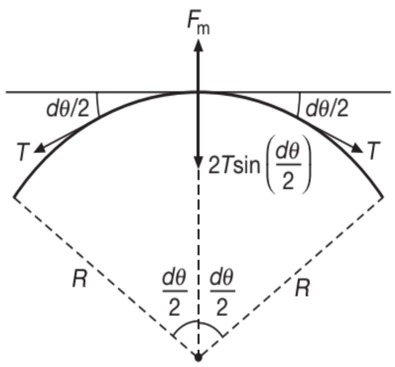
Single Correct Choice Type Problems

1. The induced electric field lines (produced by change in magnetic field) and magnetic field lines form closed loops.

Hence, the correct answer is (C).

2.



$$L = 2\pi R$$

$$\Rightarrow \quad R = \frac{L}{2\pi}$$

$$2T\sin\left(\frac{d\theta}{2}\right) = F_m = BI(dl)$$

For small angles, $\sin\left(\frac{d\theta}{2}\right) \approx \frac{d\theta}{2}$

$$2T\left(\frac{d\theta}{2}\right) = BIdl$$

Since $dl = Rd\theta$

$$\Rightarrow \quad Td\theta = BI(Rd\theta)$$

$$\Rightarrow \quad T = BIR$$

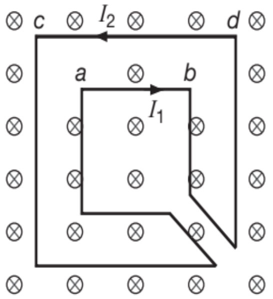
$$\Rightarrow \quad T = BI\left(\frac{L}{2\pi}\right) = \frac{IBL}{2\pi}$$

Hence, the correct answer is (C).

3. Current $I_1 = I_2$

Since magnetic field increases with time

So induced net flux should be outward i.e., current will flow from a to b.



Hence, the correct answer is (D).

4. In uniform magnetic field, change in magnetic flux is zero. Therefore, induced current will be zero.

Hence, the correct answer is (C).

5. Polarity of emf will be opposite in the two cases while entering and while leaving the coil. Only in option (B) polarity is changing.

Hence, the correct answer is (B).

6. Power $P = \dfrac{\xi^2}{R}$

where, ξ is the induced emf given by

$$\xi = -\left(\frac{d\phi}{dt}\right) \text{ and } \phi = NBA$$

$$\Rightarrow \quad \xi = -NA\left(\frac{dB}{dt}\right)$$

The resistance of wire R is

$$R = \frac{\rho l}{A}$$

$$\Rightarrow \quad R \propto \frac{l}{r^2}$$

where, r is radius, ℓ is length of wire.

$$\Rightarrow \quad P \propto \frac{N^2 r^2}{l}$$

$$\Rightarrow \quad \frac{P_1}{P_2} = 1$$

Hence, the correct answer is (B).

8. Electric field will be induced in both AD and BC.

Hence, the correct answer is (D).

9. When current flows in any of the coils, the flux linked with the other coil will be maximum in the first case. Therefore, mutual inductance will be maximum in case (a).

Hence, the correct answer is (A).

10. Let L be the inductance of coil and R be its resistance. Then current in the coil $I_1(t)$ at time t is given by

$$I_1(t) = I_0\left(1 - e^{-\frac{Rt}{L}}\right) = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$\Rightarrow \quad I_1(t) = k_1\left(1 - e^{-k_2 t}\right) \text{ where } k_1 = \frac{E}{R} \text{ and } k_2 = \frac{R}{L}$$

The magnetic field $B(t)$ at the axis of coil is

$$B(t) = \mu_0 n I_1(t)$$

$$\Rightarrow \quad B(t) = \mu_0 n\left[\frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)\right]$$

$$\Rightarrow \quad B(t) = \frac{\mu_0 n E}{R}\left[1 - e^{-\frac{Rt}{L}}\right]$$

Assuming $k_3 = \dfrac{\mu_0 n E}{R}$, we get

$$B(t) = k_3\left(1 - e^{-k_2 t}\right) \qquad \ldots(1)$$

If $I_2(t)$ be the current induced in the ring, then

$$I_2(t) = \left|-M\left(\frac{dI_1}{dt}\right)\right| = M\frac{dI_1}{dt}$$

$$\Rightarrow \quad I_2(t) = M k_1 k_2 e^{-k_2 t}$$

$$\Rightarrow \quad I_2(t) = k_4 e^{-k_2 t} \qquad \ldots(2)$$

$$\Rightarrow \quad I_2(t)B(t) = k_3 k_4\left(1 - e^{-k_2 t}\right)e^{-k_2 t}$$

$$\Rightarrow \quad I_2(t)B(t) \propto \left(1 - e^{-k_2 t}\right)\left(e^{-k_2 t}\right) \qquad \ldots(3)$$

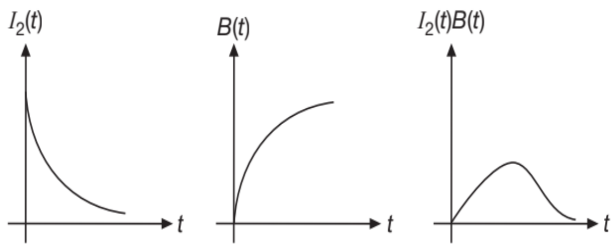
The product, $I_2(t)\ B(t)$ is zero at $t = 0$ as well as at $t \to \infty$

So, this product should pass through a maximum value.

Interestingly this product is maximum when

$$e^{-k_2 t} = \frac{1}{2} \qquad \text{\{try yourself\}}$$

The corresponding graphs will be as shown.



Hence, the correct answer is (D).

11. $\oint \vec{E}\cdot d\vec{r} = (2\pi r)E = \xi = -\dfrac{d\phi}{dt}$

$\Rightarrow \quad 2\pi rE = \pi a^2 - \dfrac{dB}{dt}$

$\Rightarrow \quad E = -\dfrac{1}{2}\dfrac{a^2}{r}\dfrac{dB}{dt}$

Hence, the correct answer is (B).

12. Total magnetic flux passing through whole of the x-y plane will be zero, because magnetic lines form a closed loop. So, as many lines will move in $-z$ direction same will return to $+z$ direction from the x-y plane.

Hence, the correct answer is (D).

13. $I = I_0\left(1 - e^{-\frac{Rt}{L}}\right)$, where

$I_0 = \dfrac{12}{6}\text{ A} = 2\text{ A}$

$\Rightarrow \quad 1 = 2\left(1 - e^{-\frac{6t}{8.4\times10^{-3}}}\right)$

$\Rightarrow \quad e^{-\frac{6t}{8.4\times10^{-3}}} = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{6t}{8.4\times10^{-3}} = \ln 2$

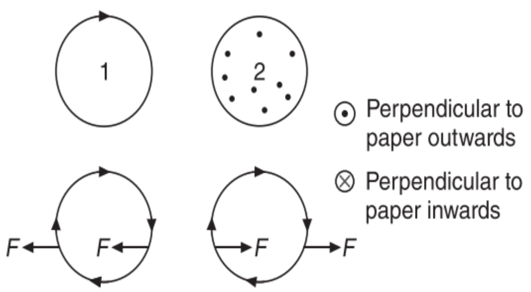
$\Rightarrow \quad t = (1.4\times0.693)\text{ ms}$

$\Rightarrow \quad t = 0.97\text{ ms}$

Hence, the correct answer is (D).

14. For understanding, let us assume that the two loops are lying in the plane of paper as shown. The current in loop 1 will produce • magnetic field in loop 2. Therefore, increase in current in loop 1 will produce an induced current in loop 2 which produces ⊗ magnetic field passing through it i.e. induced current in loop 2 will also be clockwise as shown in Figure.

The loops will now repel each other as the currents at the nearest and farthest points of the two loops flow in the opposite directions.



Hence, the correct answer is (C).

15. $B = \dfrac{2\sqrt{2}\mu_0 I}{\pi L}$

The magnetic flux ϕ_{12} linking big loop with the small square loop of side $l(<<L)$ is

$$\phi_{12} = Bl^2 = \frac{2\sqrt{2}\mu_0 I}{\mu}\left(\frac{l^2}{L}\right)$$

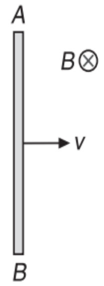
Mutual Inductance is $M_{12} = \dfrac{\phi_{12}}{\text{I}}$

$\Rightarrow \quad M_{12} = \dfrac{2\sqrt{2}\mu_0}{\pi}\left(\dfrac{l^2}{L}\right)$

$\Rightarrow \quad M \propto \dfrac{l^2}{L}$

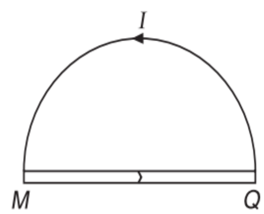
Hence, the correct answer is (B).

16. A motional emf, $\xi = B/v$ is induced in the rod. Or we can say a potential difference is induced between the two ends of the rod AB, with A at higher potential and B at lower potential. Due to this potential difference, there is an electric field in the rod.



Hence, the correct answer is (B).

17. The induced emf across a conductor does not depend upon its shape but only at the extreme (end) points. As a simple case just replace the actual conductor by an imaginary straight conductor placed in between the end points. Then induced emf is



$\xi = Blv = B(2R)V$

$\Rightarrow \quad \xi = 2BVR$

The direction of induced current in this imaginary conductor MQ is from M to Q. So, in the loop the current flows from Q to N to M thus indicating that Q is at a higher potential.

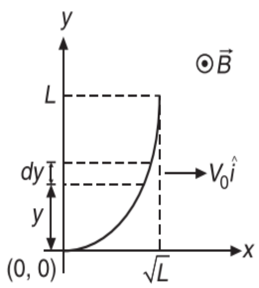
Hence, the correct answer is (D).

18. Since the flux linked with the loop does not change. Hence the induced current is zero.
Hence, the correct answer is (D).

Multiple Correct Choice Type Problems

1. Given that $y = x^2$, $V = V_0\hat{i}$ and $B = B_0\left(1+\left(\frac{y}{2}\right)^{\beta}\right)\hat{k}$

Also, the end points of the conducting wire are $(0, 0)$ and $(\sqrt{L}, L)$. Let us consider an infinitesimal element of length dy at a distance y from the x-axis as shown.



If $d\phi$ be the emf induced across this element, then

$$d\phi = V_0 B dy$$

$$\Rightarrow \quad d\phi = V_0 B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)dy$$

$$\Delta\phi = V_0 B_0\left[\left(y+\frac{y^{\beta+1}}{(\beta+1)L^{\beta}}\right)\Bigg|_0^L\right]$$

$$\Rightarrow \quad \Delta\phi = V_0 B_0\left[L+\frac{L^{\beta+1}}{(\beta+1)L^{\beta}}\right]$$

$$\Rightarrow \quad \Delta\phi = B_0 L V_0\left(\frac{\beta+2}{\beta+1}\right)$$

$$\Rightarrow \quad \Delta\phi \propto L$$

When $\beta = 2$, then $\Delta\phi = \frac{4}{3}B_0 L V_0$

When $\beta = 0$, then $\Delta\phi = 2B_0 L V_0$

Hence, (B), (C) and (D) are correct.

2. The net magnetic flux through the loops at time t is

$$\phi = B(2A - A)\cos\omega t = BA\cos\omega t$$

$$\Rightarrow \quad \left|\frac{d\phi}{dt}\right| = B\omega A\sin\omega t$$

So, $\left|\frac{d\phi}{dt}\right|$ is maximum when $\phi = \omega t = \frac{\pi}{2}$

The emf induced in the smaller loop,

$$\xi_{\text{smaller}} = -\frac{d}{dt}(BA\cos\omega t) = B\omega A\sin\omega t$$

So, amplitude of maximum net emf induced in both the loops is equal to amplitude of maximum emf induced in the smaller loop alone.
Hence, (B) and (D) are correct.

3. For right edge of loop from $x = 0$ to $x = L$

$$i = +\frac{BLv}{R}$$

Since, $F = iLB = \frac{vB^2L^2}{R}$ (leftwards)

$$\Rightarrow \quad -mv\frac{dv}{dx} = \frac{vB^2L^2}{R}$$

$$\Rightarrow \quad v(x) = v_0 - \frac{B^2L^2}{mR}x$$

$$\Rightarrow \quad i(x) = \frac{v_0 BL}{R} - \frac{B^3L^3}{mR^2}x \text{ and}$$

$$F(x) = \frac{v_0 B^2L^2}{R} - \frac{B^4L^4}{mR^2}x \text{ (leftwards)}$$

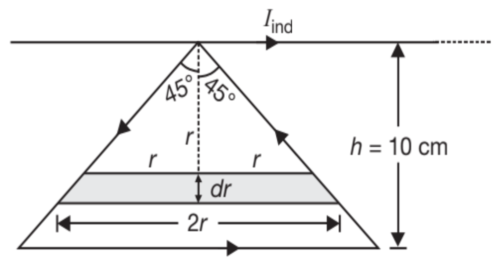
Hence, (C) and (D) are correct.

4. $$\phi = \int_0^h \frac{\mu_0 I}{2\pi r}2rdr = \frac{\mu_0 Ih}{\pi}$$

$$\Rightarrow \quad M = \frac{\phi}{I} = \frac{\mu_0 h}{\pi}$$

Now, $\xi = \left|M\frac{dI}{dt}\right| = \frac{\mu_0 h}{\pi}(10)$

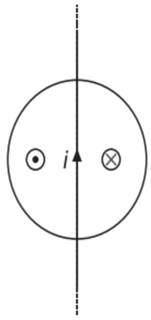
$$\Rightarrow \quad \xi = \frac{\mu_0}{\pi}\left(\frac{10}{100}\right)(10) = \frac{\mu_0}{\pi} \text{ volt}$$



Since, current in triangular loop is increasing at rate of $10\ As^{-1}$, due to which the outward field also increases. According to Lenz's Law, induced current in conductor should produce an inward field at the triangular loop, so I_{ind} in the conductor must be rightwards due to which a repulsive force acts between the wire and the loop. Further, due to rotation of the loop there will not be any change in flux through the loop, so there is no extra emf induced in the wire.

Hence, (A) and (D) are correct.

5. Due to the current in the straight wire, net magnetic flux from the circular loop is zero. Because in half of the circle, magnetic field is inwards and in other half, magnetic field is outwards.

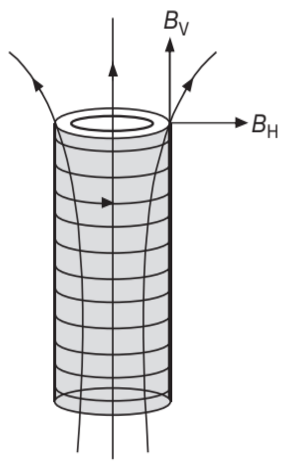


Therefore, change in current will not cause any change in magnetic flux from the loop. Therefore, induced emf under all conditions through the circular loop is zero.

Hence, (A) and (C) are correct.

6. The horizontal component of magnetic field due to solenoid will exert force on ring in vertical direction

$F = B_H I(2\pi r)$



Since $F\Delta t = MV$

$$\Rightarrow \quad I = \frac{\left(\frac{\Delta\phi}{\Delta t}\right)}{\left(\rho\frac{(2\pi r)}{A}\right)}$$

$$B_H I(2\pi r)\Delta t = MV$$

$$V = \frac{B_H \Delta\phi A}{\rho M} = \frac{K}{\rho M}$$

$$h = \frac{V^2}{2g} = \frac{K^2}{\rho^2 M^2}$$

$$h_A > h_B$$

$$\frac{K^2}{\rho_A^2 M_A^2} > \frac{K^2}{\rho_B^2 M_B^2}$$

$$\Rightarrow \quad \rho_B M_B > \rho_A M_A$$

So, we get (B) and (D) as the correct options.

Hence, (B) and (D) are correct.

7. Electrostatic and gravitational field do not make closed loops.

Hence, (B) and (D) are correct.

8. Since $v = L\frac{di}{dt}$

$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{L_1\left(\frac{di_1}{dt}\right)}{L_2\left(\frac{di_2}{dt}\right)} = \frac{L_1}{L_2} = \frac{8}{2} = 4$$

$$\Rightarrow \quad \frac{v_2}{v_1} = \frac{1}{4}$$

Also, $L_1 i_1 = L_2 i_2$

$$\Rightarrow \quad \frac{i_1}{i_2} = \frac{L_2}{L_1} = \frac{2}{8} = \frac{1}{4} \text{ and}$$

$$\frac{w_2}{w_1} = \frac{L_2 i_2^2}{L_1 i_1^2} = \left(\frac{2}{8}\right)(16) = 4$$

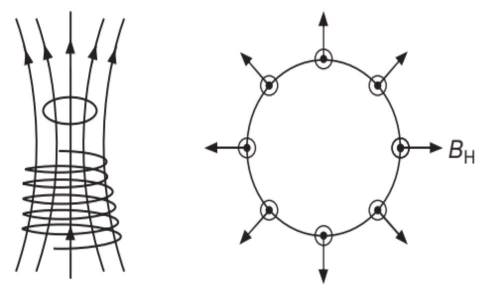
Hence, (A), (C) and (D) are correct.

Reasoning Based Questions

1. The arrow $\rightarrow$ represents horizontal component of magnetic field

⊙ Represents vertical component of magnetic field

The top view of ring is



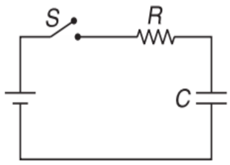
From diagram it is clear that horizontal component of magnetic field will interact with induced current. The resulting magnetic force will act opposite to weight of ring.

Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. When switch is closed for a very long-time capacitor will get fully charged and charge on capacitor will be $q = CV$

Energy stored in capacitor, $E_C = \frac{1}{2}CV^2$ ...(1)



Work done by a battery, $W = qV = (CV)V = CV^2$

Energy dissipated across resistance is

$$E_D = \left(\begin{matrix}\text{Work done}\\ \text{by Battery}\end{matrix}\right) - \left(\begin{matrix}\text{Energy}\\ \text{Stored}\end{matrix}\right)$$

$$\Rightarrow \quad E_D = CV^2 - \frac{1}{2}CV^2 = \frac{1}{2}CV^2 \qquad ...(2)$$

From equations (1) and (2)

$$E_D = E_C$$

Hence, the correct answer is (B).

2. When a capacitor is charged from v_i to v_f, heat dissipated is given by

$$\Delta H = W_{\text{battery}} - \Delta U_C = (\Delta q)V_f - \Delta U_C$$

$$\Rightarrow \quad \Delta H = C(V_f - V_i)V_f - \frac{1}{2}C(V_f^2 - V_i^2) = \frac{1}{2}C(V_f - V_i)^2$$

So, total heat dissipated is

$$\Delta H = \frac{1}{2}C\left[\left(\frac{V_0}{3} - 0\right)^2 + \left(\frac{2V_0}{3} - \frac{V_0}{3}\right)^2 + \left(V_0 - \frac{2V_0}{3}\right)^2\right]$$

$$\Rightarrow \quad \Delta H = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$$

Hence, the correct answer is (A).

3. The induced electric field is given by,

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi}{dt}$$

$$\Rightarrow \quad El = -A\left(\frac{dB}{dt}\right)$$

$$\Rightarrow \quad E(2\pi R) = -(\pi R^2)(B)$$

$$\Rightarrow \quad E = -\frac{BR}{2}$$

Hence, the correct answer is (B).

4. Since the Gyro-Magnetic Ratio is

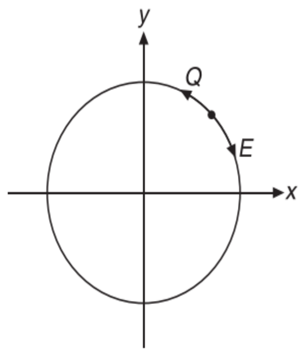
$$\frac{|\vec{M}|}{|\vec{L}|} = \frac{Q}{2m}$$

$$\Rightarrow \quad |\vec{M}| = \left(\frac{Q}{2m}\right)|\vec{L}|$$

$$\Rightarrow \quad |\vec{M}| \propto |\vec{L}|, \text{ where } \gamma = \frac{Q}{2m}$$

$$\Rightarrow \quad |\vec{M}| = \left(\frac{Q}{2m}\right)(I\omega) = \left(\frac{Q}{2m}\right)(mR^2\omega)$$

$$\Rightarrow \quad |\vec{M}| = \frac{Q\omega R^2}{2}$$



Since induced electric field is opposing in nature, so

$$\omega' = \omega - \alpha t$$

where, $\alpha = \frac{\tau}{I} = \frac{(QE)R}{mR^2}$

$$\Rightarrow \quad \alpha = \frac{(Q)\left(\frac{BR}{2}\right)R}{mR^2} = \frac{QB}{2m}$$

Since we have $t = 1$ s (see paragraph)

$$\Rightarrow \quad \omega' = \omega - \left(\frac{QB}{2m}\right)1 = \omega - \frac{QB}{2m}$$

$$\Rightarrow \quad M_f = \frac{Q\omega' R^2}{2} = Q\left(\omega - \frac{QB}{2m}\right)\frac{R^2}{2}$$

$$\Rightarrow \quad \Delta M = M_f - M_i = -\frac{Q^2BR^2}{4m}$$

$$\Rightarrow \quad \Delta M = -\gamma\frac{QBR^2}{2} \qquad \left\{\because \gamma = \frac{Q}{2m}\right\}$$

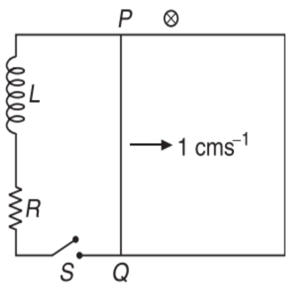
Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. A → (p)
B → (p, q, s)
C → (q, s)
D → (q, r, s)

CHAPTER 3

Integer/Numerical Answer Type Questions

1. 

Since $B=1\text{ T}$, $l=10\text{ cm}$, $v=1\text{ cms}^{-1}$

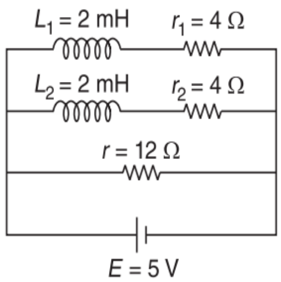
$$\Rightarrow \quad \xi = Blv = (1)\left(\frac{1}{10}\right)\left(\frac{1}{100}\right) = 1\times 10^{-3}\text{ V}$$

Since, $\tau = \frac{L}{R} = \frac{1\times 10^{-3}}{1} = 10^{-3}$ sec

$$\Rightarrow \quad I = I_0\left(1-e^{-\frac{t}{\tau}}\right)$$

$$\Rightarrow \quad I = \frac{10^{-3}}{1}\left(1-e^{-1}\right)$$

$$\Rightarrow \quad I = 10^{-3}(1-0.37) = 0.63\text{ mA}$$

2. 

Initially i.e. at $t=0$ current in the circuit is maximum, so

$$I_{\max} = \frac{E}{R} = \frac{5}{12}\text{ A}$$

Long time after i.e. in steady state, current in the circuit is minimum, so

$$I_{\min} = \frac{E}{R_{eq}} = E\left(\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{R}\right)$$

$$\Rightarrow \quad I_{\min} = 5\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{12}\right) = \frac{10}{3}\text{ A}$$

$$\Rightarrow \quad \frac{I_{\max}}{I_{\min}} = 8$$

3. If I current flows through the circular loop, then magnetic flux at the location of square loop is

$$B = \frac{\mu_0 I R^2}{2\left(R^2+z^2\right)^{\frac{3}{2}}}$$

Substituting the value of $z=\sqrt{3}R$, we get

$$B = \frac{\mu_0 I}{16R}$$

Now, total flux through the square loop is

$$\phi = NBA\cos\theta = (2)\left(\frac{\mu_0 I}{16R}\right)a^2\cos 45^\circ$$

Mutual inductance is given by

$$M = \frac{\phi}{I} = \frac{\mu_0 a^2}{2^{\frac{7}{2}}R}$$

$$\Rightarrow \quad p = 7$$

4. Flux through the ring is

$$\phi = BA = \left(\mu_0 nI\right)\left(\pi r^2\right)$$

Since $n = \frac{1}{l}$ and $I = I_0\cos(300t)$

$$\Rightarrow \quad \phi = \frac{\mu_0}{L}\left(\pi r^2\right)I_0\cos(300t)$$

EMF induced in the loop is

$$\xi = -\frac{d\phi}{dt}$$

$$\Rightarrow \quad \xi = \frac{300\mu_0 I_0\left(\pi r^2\right)}{l}\sin(300t)$$

Induced current in the loop is

$$i = \frac{\xi}{R} = \frac{300\mu_0 I_0\left(\pi r^2\right)}{lR}\sin(300t)$$

Magnetic moment of the loop is

$$M = iA = i\left(\pi r^2\right)$$

$$\Rightarrow \quad M = \frac{300\mu_0 I_0 \pi^2 r^4}{lR}\sin(300t)$$

Comparing with $M = N\mu_0 I_0\sin(300t)$, we get

$$N = \frac{300\pi^2 r^4}{lR}$$

Taking $\pi^2 \approx 10$, we get

$$N = \frac{(300)(10)(0.1)^4}{(0.005)(10)}$$

$$\Rightarrow \quad N = \frac{(300)(10)\left(10^{-4}\right)}{5\times 10^{-2}}$$

$$\Rightarrow \quad N = \frac{3\times 10^{-1}}{5\times 10^{-2}} = \frac{30}{5} = 6$$