CHAPTER 2

Magnetism and Matter

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Bar Magnet, Magnetic Poles and Properties
(b) Magnetic Moment of an Orbital Electron
(c) Magnetic Dipole and Properties
(d) Earth's Magnetism
(e) Tangent Law and Tangent Galvanometer
(f) Vibration Magnetometer
(g) Properties of Magnetic Materials

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE Main are also given.

INTRODUCTION TO BAR MAGNET AND MAGNETIC POLES

Moving charges or current loops cause magnetism. This phenomenon was known long before the magnetic effect of current was discovered. Bar magnets are the simplest source of magnetic field (apart from electric current). A bar magnet possesses the following properties.

(a) A freely suspended bar magnet always orients itself (approximately) along the North-South direction. It is important to note here that the end which points towards geographical north is called the **North pole** (not the South pole) and the end towards geographical South is called the **South pole** (and not the north pole).

(b) Like poles repel each other and unlike poles attract each other with a force which obeys the inverse square law.

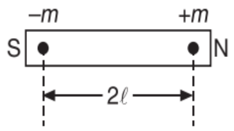
(c) A magnet attracts certain substances e.g., small pieces of iron, iron filings sprinkled on a sheet of paper held over a bar magnet form characteristic patterns similar to the lines of force of an electric dipole.

Conceptual Note(s)

It was thought that, like two types of electric charges, there are two types of magnetic charges (or poles). However, every effort to find such magnetic charges, or to isolate the poles of a magnet have failed. If we break a magnet into two parts, the two pieces become two new magnets, each having both N-pole and S-pole. Even if we break up a magnet into the electrons and nuclei, it will be found that even these are magnetic dipoles. Thus, **a magnetic monopole does not exist.**

A bar magnet consists of two equal and opposite magnetic poles separated by a distance, hence the magnet is also called the **magnetic dipole**.

If m is pole strength and $2l$ the separation between poles, then



Magnetic moment of bar magnet,

$$M = m(2l)$$

M is a vector directed from South Pole to the North Pole of magnet.
If I is the current flowing in a coil of area A, then magnetic moment of current loop

$$M = IA \left(\text{in Am}^2\right)$$

If the coil contains N turns, then

$$M = NIA$$

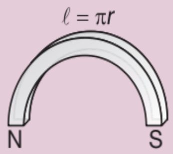
The unit of pole strength m is ampere-metre (Am).

Conceptual Note(s)

(a) On bending a magnet its pole strength remains unchanged whereas its magnetic moment changes.

(b) The unit of magnetic moment is Am^2 and its dimensional formula is $M^0L^2T^0A$

(c) If a magnet of length ℓ and magnetic moment M is bent in the form of a semi-circular arc then its new magnetic moment will be $M' = \dfrac{2M}{\pi}$



$$M' = m_p(2r) = \frac{m_p 2\ell}{\pi} = \frac{2M}{\pi}$$

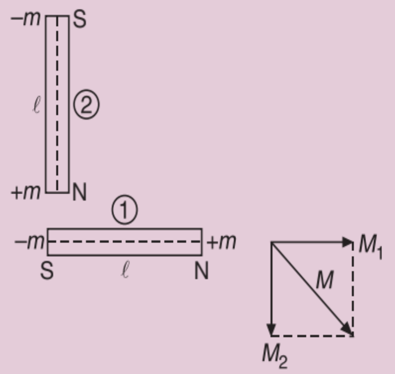
(d) The property of magnetism in materials is on account of magnetic moment in that material.

(e) In para magnetic and ferro magnetic materials the direction of M is in the direction of H whereas in diamagnetic materials it is opposite to that of H.

(f) If a magnet is cut along its length then the magnetic moment decreases i.e.

$$m_p \propto A$$

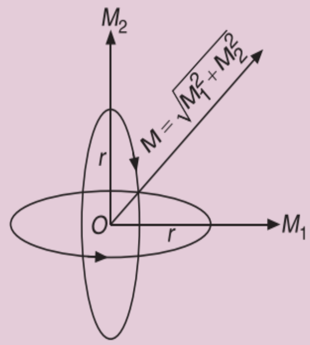
(g) When two bar magnets are lying mutually perpendicular to each other, then the resultant magnetic moment is



$$M = \sqrt{M_1^2 + M_2^2} = \sqrt{2}m\ell$$

(h) When two coils, each of radius r and carrying current I, are lying concentrically with their planes at right angles to each other, then the resultant magnetic moment is

$$M = \sqrt{M_1^2 + M_2^2} = \sqrt{2}I\pi r^2 \text{ for } M_1 = M_2$$



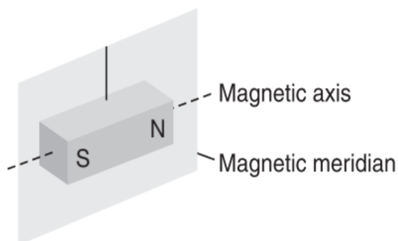
(i) The magnetic moment of a magnet decreases on increasing temperature.

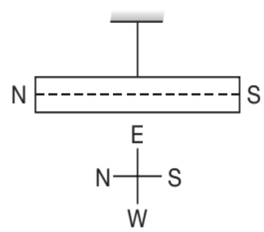
Properties of a Magnet

A magnet has following properties.

(a) It attracts ferromagnetic materials towards it.

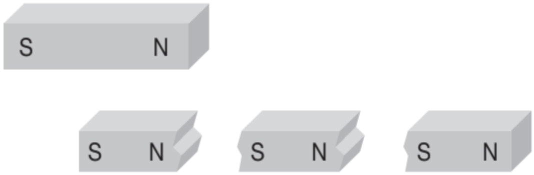
(b) When suspended freely it rests in a particular direction i.e. North-South direction.





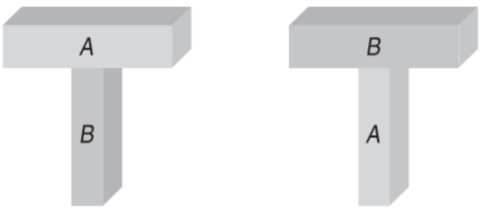
(c) The pole strength of its two poles is the same.

(d) The two poles of a magnet cannot be isolated i.e. separated out *i.e. magnetic monopoles do not exist*. If a bar magnet is broken into a number of pieces, then each piece behaves a s an individual magnet rather than behaving like an isolated pole. So, magnetic monopoles do not exist



(e) Like poles repel and unlike poles attract each other.

(f) For two rods A and B as shown, if both the rods attract in case (i) and do not attract in case (ii) then,
B is a magnet and A is a simple iron rod showing that repulsion is the sure test of magnetism.



(i) (ii)

(f) It can be magnetically saturated.

(g) It can be demagnetized by beating, mechanical jerks, heating and with lapse of time.

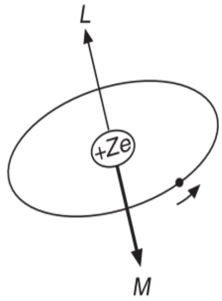
(h) It produces magnetism in other materials by induction.

MAGNETIC MOMENT OF AN ORBITAL ELECTRON

The magnetic moment of an electron due to its orbital motion is $l\mu_B$ whereas that due to its spin motion it is $\frac{\mu_B}{2}$.

So, $M_{\text{orbital}} = l\left(\frac{eh}{4\pi m}\right) = l\left(\frac{e\hbar}{2m}\right)$

and $M_{\text{spin}} = s\left(\frac{eh}{4\pi m}\right) = s\left(\frac{e\hbar}{2m}\right) = s\mu_B = \frac{\mu_B}{2}$



where μ_B is the Bohr magneton. The value of Bohr magneton is

$$\mu_B = \frac{eh}{4\pi m} = \frac{e\hbar}{2m} \qquad \left\{\text{where } \hbar = \frac{h}{2\pi}\right\}$$

$$\Rightarrow \quad \mu_B = 0.93 \times 10^{-23}\ \text{Am}^2$$

The magnetic moment associated with the electron revolving in the first Bohr orbit is known as Bohr magneton.

Problem Solving Technique(s)

(a) Other formulae for magnetic moment M

(i) $M = ni\pi r^2$

(ii) $M = \frac{evr}{2} = \frac{er^2\omega}{2} = \frac{er^2 2\pi f}{2} = \frac{er^2\pi}{T}$

(iii) $M = \frac{eL}{2m}$, where L = Angular Momentum of the Electron

(iv) $M = n\mu_B$

(b) About 90% of magnetic moment is due to spin motion of electrons whereas 10% part is due to their orbital motion.

(c) The net magnetic moment of an atom is equal to the vector sum of magnetic moments of all its electrons.

ILLUSTRATION 1

The electron in hydrogen atom moves with a speed of 2.2×10^6 ms^{-1} in an orbit of radius 5.3×10^{-11} cm. Calculate the magnetic moment of the orbiting electron.

SOLUTION

Frequency of revolution of electron is

$$f = \frac{v}{2\pi r}$$

Since the moving charge is equivalent to a current loop, so

$$I = qf$$

$$\Rightarrow \quad I = \frac{ev}{2\pi r}$$

If A be the area of the orbit, then the magnetic moment of the orbiting electron is,

$$M = IA = \left(\frac{ev}{2\pi r}\right)(\pi r^2) = \frac{evr}{2}$$

Substituting the values, we get

$$M = \frac{(1.6\times10^{-19})(2.2\times10^{6})(5.3\times10^{-11}\times10^{-2})}{2}$$

$$\Rightarrow \quad M = 9.3\times10^{-26}\ \text{Am}^2$$

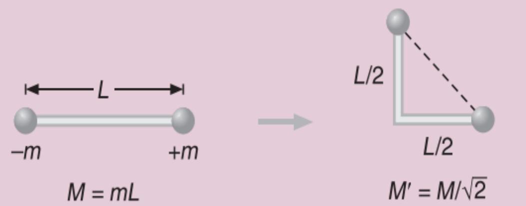
Conceptual Note(s)

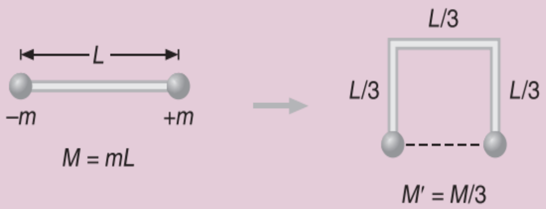
(a) Bohr magneton $\mu_B = \dfrac{eh}{4\pi m} = 9.27\times10^{-24}\ \text{Am}^{-2}$. It serves as natural unit of magnetic moment. Bohr magneton can be defined as the orbital magnetic moment of an electron circulating in inner most orbit.

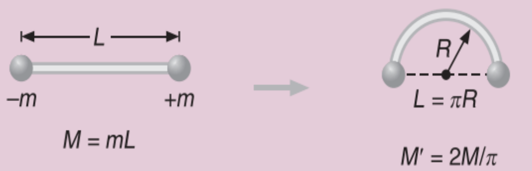
(b) Magnetic moment of straight current carrying wire is zero.

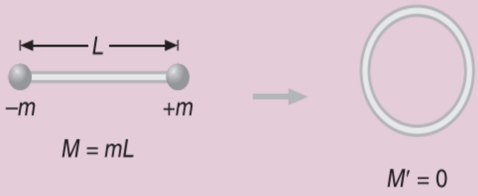
(c) Magnetic moment of toroid is zero.

(d) If a magnetic wire of magnetic moment (M) is bent into any shape then it's magnetic moment M decreases as it's length (L) always decreases but pole strength remains constant.

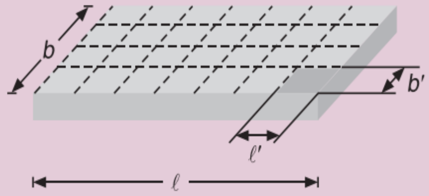








(e) Cutting of a bar magnet: Consider a rectangular bar magnet having length ℓ, breadth b and mass w, then if the magnet is cut in n equal parts along its length as well as perpendicular to its length simultaneously as shown in the figure, then



(i) length of each part is $\ell' = \dfrac{\ell}{\sqrt{n}}$

(ii) breadth of each part $b' = \dfrac{b}{\sqrt{n}}$ and

(iii) Mass of each part $w' = \dfrac{w}{n}$,

(iv) The pole strength of each part is $m' = \dfrac{m}{\sqrt{n}}$

(v) Magnetic moment of each part is $M' = m'L' = \dfrac{m}{\sqrt{n}} \times \dfrac{\ell}{\sqrt{n}} = \dfrac{M}{n}$

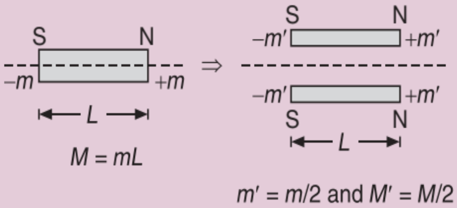
(vi) If initially moment of inertia of bar magnet about the axes passing from centre and perpendicular to its length is $I = w\left(\dfrac{L^2+b^2}{12}\right)$ then moment of inertia of each part $I' = \dfrac{I}{n^2}$

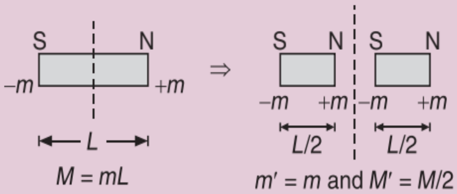
(f) For short bar magnet i.e. for a magnet of small width, we have

$b = 0$, so $\ell' = \dfrac{\ell}{n}$, $w' = \dfrac{w}{n}$, $m' = m$, $M' = \dfrac{M}{n}$ and

$$I' = \frac{I}{n^3}$$

(g) If a magnet of monopole strength m and magnetic moment M is cut parallel to its length in two equal halves or is cut parallel to width in two equal haves then the divided pieces of magnet will have monopole strength m' and magnetic moment M' as shown.





MAGNETISM

(a) That property of a magnet, by virtue of which it attracts ferromagnetic materials towards it and rests itself in North-South direction, is known as magnetism.

(b) The magnetism produced by induction depends upon the distance between inducing pole and induced pole.

(c) The magnetism of materials is mainly due to the spin motion of its electrons.

MAGNETIC POLES

The two points at the ends of a magnet at which magnetism is maximum, are defined as magnetic poles. The magnetic poles are of two types.

(a) North pole

(b) South pole

North Pole

(a) The pole of a magnet which when freely suspended always points towards geographical north, is known as north pole.

(b) The pole, from which the magnetic lines of force emerge out, is known as north pole.

(c) This is also called as the **positive pole**.

(d) Here north direction means geographical north.

South Pole

(a) The pole of the magnet, which when freely suspended always points towards south, is known as south pole.

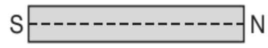
(b) The magnetic pole into which the lines of force merge into, is known as south pole.

(c) This is also called as **negative pole**.

(d) It is not necessary that magnetic poles are exactly in the middle of its ends rather these can be situated away from the middle.

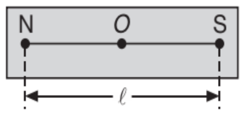
MAGNETIC AXIS

The imaginary line joining the two poles of a magnet is defined as magnetic axis.



EFFECTIVE LENGTH OF A MAGNET

The distance between the two poles of a magnet is defined as its **effective length**. This length is measured along the axis of the magnet.



The distance of magnetic pole from the centre of the magnet is known as **half-length of magnet**. Effective length of magnet is $\frac{5}{6}$ times its geometric length.

MAGNETIC MERIDIAN

An imaginary plane passing through the axis of a freely suspended stationary magnet is defined as **magnetic meridian**. All imaginary planes drawn parallel to this plane are also called **magnetic meridian**. It is not a particular plane rather it is the direction of earth's magnetic field. This plane cuts the surface of earth in a line, hence it is represented by line on the piece of paper.

POLE STRENGTH (*M*)

The strength of a magnetic pole to attract magnetic materials towards it, is known as pole strength. Greater the number of unit poles in a magnetic pole, greater will be its strength.

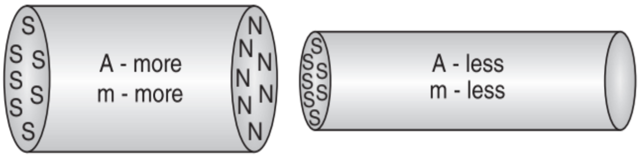
$$m = \frac{F}{B} = \frac{\text{Magnetic force}}{\text{Magnetic induction}}$$

Its unit is ampere-metre (Am)

(a) It is a scalar quantity (just like charge)

(b) Pole strength of N and S pole of a magnet is conventionally represented by $+m$ and $-m$ respectively.

(c) Pole strength of the magnet depends on the nature of material of magnet and area of cross section. It doesn't depend upon length

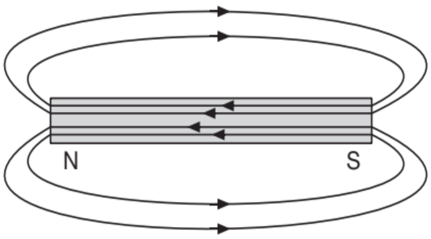


MAGNETIC LINES OF FORCE

The imaginary lines straight or curved which represent the direction of magnetic field, are known as **magnetic lines of force**. The imaginary path traced by an isolated (imaginary) unit north pole is defined as a **line of force**.

Properties

(a) Magnetic lines of force are closed curves. Outside the magnet their direction is from north pole to south pole and inside the magnet these are from south to north pole.



(b) They neither have an origin nor an end.

(c) These lines do not intersect, because if they do so then it would mean two directions of magnetic field at a single point, which is not possible.

(d) The tangent drawn at any point to the line of force indicates the direction of magnetic field at that point.

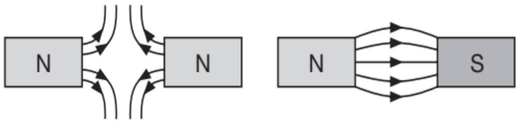
(e) The number of magnetic lines of force passing through unit normal area is defined as **magnetic induction** (B) whereas the number of lines of force passing through any area is known as **magnetic flux**.

(f) The larger the number of field lines crossing a unit area, the stronger is the magnetic field.

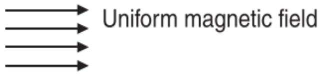
(g) At the poles of the magnet the magnetic field is stronger because the lines of force there are crowded together and away from the poles the magnetic field is week. So, we conclude that Magnetic Field Intensity $\propto$ Number of Lines of Force.

(h) The lines of force can emerge out of the north pole of magnet at any angle and these can merge into the south pole at any angle.

(i) Magnetic field lines have tendency to contract longitudinally indicating attraction between unlike magnetic poles. The lines also have tendency to dilate laterally, indicating repulsion between like magnetic poles as shown below.



(j) Uniform magnetic field lines are shown below.



(k) Non-uniform magnetic field lines are shown below.

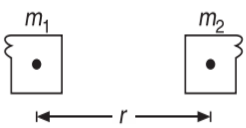
Conceptual Note(s)

The Electrostatic Analog: We can write most of the results for magnetism using analogy with electrostatics by making the following replacements:

$$q \to m,\ \vec{E} \to \vec{B},\ \vec{p} \to \vec{M},\ \frac{1}{4\pi\varepsilon_0} \to \frac{\mu_0}{4\pi}$$

COULOMB'S LAW IN MAGNETISM

The force between two magnetic poles of strength m_1 and m_2 lying at a distance r in vacuum is given by



$$F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2}$$

ILLUSTRATION 2

Two magnetic poles, one of which is four times stronger than the other, exert a force of $10\ \text{gf}$ on each other when placed at a distance of $20\ \text{cm}$. Find the strength of each pole.

SOLUTION

Let the pole strength of the two dipoles be m and $4m$

Here, $F = 10\ \text{gf} = 10\times10^{-3}\ \text{Kgf} = 10\times10^{-3}\times9.8\ \text{N}$

and $r = 20\ \text{cm} = 0.2\ \text{m}$

Using Coulomb's law of magnetism

$$F = \frac{\mu_0}{4\pi}\left(\frac{m_1 m_2}{r^2}\right)$$

Substituting the values,

$$10\times10^{-3}\times9.8 = \frac{10^{-7}\times m\times4m}{(0.2)^2}$$

$$\Rightarrow \quad m^2 = \frac{10\times9.8\times(0.2)^2\times10^4}{4} = 9800$$

$$\Rightarrow \quad m_1 = m = 98.9\ \text{Am}$$

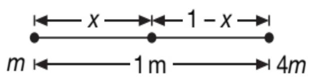
$$\Rightarrow \quad m_2 = 4m = 4\times98.9 = 396\ \text{Am}$$

ILLUSTRATION 3

Two similar magnetic poles, having pole strengths in the ratio $1:3$ and placed $1\ \text{m}$ apart. Find the point where a unit pole experiences no net force due to these two poles.

SOLUTION

Let the pole strengths of the two magnetic poles be m and $3m$. Suppose the required point is located at distance x from the first pole. Then at this point,



Force on unit pole due to first pole

= Force on unit pole due to second pole

$$\Rightarrow \quad \frac{\mu_0}{4\pi}\left(\frac{m\times1}{x^2}\right) = \frac{\mu_0}{4\pi}\left(\frac{3m\times1}{(1-x)^2}\right)$$

$$\Rightarrow \quad 3x^2 = (1-x)^2 \text{ or } \sqrt{3}x = 1-x$$

$$\Rightarrow \quad x = \frac{1}{1+\sqrt{3}} = 0.366\ \text{m}$$

STRENGTH OF MAGNETIC FIELD

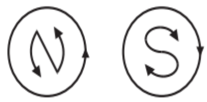
It is defined as the force experienced by a unit north pole or unit text monopole (m_0) placed at the given point in the magnetic field.

Hence, magnetic field due to an imaginary magnetic pole with pole strength m is

$$B = \frac{F}{m_0} = \frac{\mu_0}{4\pi}\frac{m}{r^2}$$

CURRENT LOOP AS A MAGNETIC DIPOLE

If we compare the field produced by a bar magnet and a current carrying solenoid, we find that they are similar. Thus, a solenoid behaves as a dipole. Since, a solenoid is a collection of current loops, therefore we can assert that a single current loop is the most elementary magnetic dipole with its one face behaving as a north pole and the other as a south pole.



The face in which the current is anticlockwise acts as N pole because the lines of force emerge out of this face, and the face in which the current is clockwise acts as S pole.

For a loop of n turns or for a solenoid,

$$m = nIA$$

The S.I. unit of m is Am^2 or JT^{-1}

ILLUSTRATION 4

Each atom of an iron bar $(5\ \text{cm}\times1\ \text{cm}\times1\ \text{cm})$ has a magnetic moment $1.8\times10^{-23}\ \text{Am}^2$. Knowing that the density of iron is $7.78\times10^3\ \text{kgm}^{-3}$, atomic weight is 56 and Avogadro's number is 6.02×10^{23}. Calculate the magnetic moment of bar in the state of magnetic saturation

SOLUTION

The number of atoms per unit volume in a specimen,

$$n = \frac{\rho N_A}{A}$$

For iron,

$$\rho = 7.8\times10^3\ \text{kgm}^{-3},$$

$$N_A = 6.02\times10^{26}/\text{kg mol},\ A = 56$$

$$\Rightarrow \quad n=\frac{7.8\times10^{3}\times6.02\times10^{26}}{56}$$

$$n=8.38\times10^{28}\ \text{m}^{-3}$$

Total number of atoms in the bar is

$$N_0=nV=8.38\times10^{28}\times\left(5\times10^{-2}\times1\times10^{-2}\times1\times10^{-2}\right)$$

$$N_0=4.19\times10^{23}$$

The saturated magnetic moment of bar is

$$M=4.19\times10^{23}\times1.8\times10^{-23}=7.54\ \text{Am}^2$$

ILLUSTRATION 5

The moment of a magnet is $0.1\ \text{Am}^2$ and the force acting on each pole in a uniform magnetic field of strength 0.36 oersted is 1.224×10^{-4} N. Find the distance between the poles of the magnet.

SOLUTION

$$F=mB$$

$$\Rightarrow \quad m=\frac{F}{B}=\frac{1.224\times10^{-4}}{0.36\times10^{-4}}$$

$$\Rightarrow \quad m=4\ \text{Am}$$

Since, $M=m(2l)$

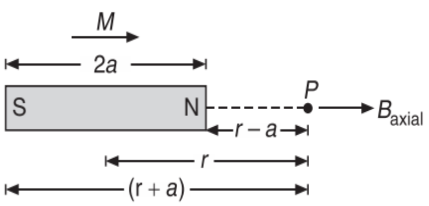
$$\Rightarrow \quad 2l=\frac{M}{m}=\frac{0.1}{4}\ \text{m}=\frac{10}{4}\ \text{cm}$$

$$\Rightarrow \quad 2l=2.50\ \text{cm}$$

MAGNETIC FIELD STRENGTH

At Point *P* Lying on Axial Line (End on Position)

The intensity of magnetic field due to a magnetic dipole of length $2a$ at a point P distant r on its axis from the centre of the magnet is given by



$$B_{\text{axial}}=B_a=\frac{\mu_0}{4\pi}\frac{2Mr}{\left(r^2-a^2\right)^2}\quad(\text{along } \vec{M})$$

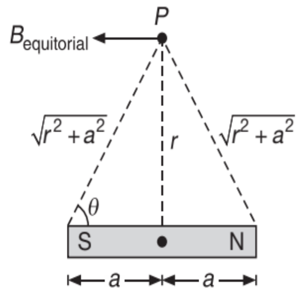
In case of a magnetic dipole if the two poles are situated very close to each other, then

$$a\ll r$$

$$\Rightarrow \quad B_{\text{axial}}\cong\frac{\mu_o}{4\pi}\frac{2M}{r^3}$$

At Point *P* Lying on Equitorial Line (Broad-side on Position)

The intensity of magnetic field due to a magnetic dipole at a point P distant r on its equatorial line from the centre of dipole is given by



$$B_{\text{equitorial}}=B_e=\frac{\mu_o}{4\pi}\frac{M}{\left(r^2+a^2\right)^{3/2}}\quad(\text{opposite to } \vec{M})$$

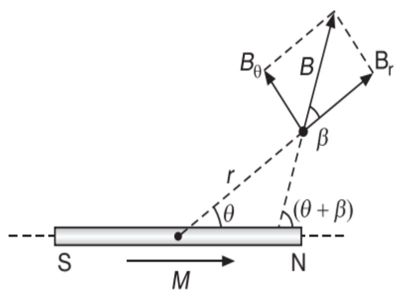
If $a\ll r$, then

$$B_{\text{equitorial}}\cong\frac{\mu_0}{4\pi}\frac{M}{r^3}$$

At any Point *P*

The intensity of magnetic field at any point P with polar co-ordinates (r,θ) i.e. point P is and inclined at angle θ with dipole moment M of the magnet distant r from the centre of magnetic dipole (magnet) is given by

$$B=\sqrt{B_r^2+B_\theta^2}$$



where $B_r=\begin{pmatrix}\text{Field in the direction}\\ \text{of increasing } r\end{pmatrix}=\frac{\mu_0}{4\pi}\left(\frac{2M\cos\theta}{r^3}\right)$

and $B_\theta=\begin{pmatrix}\text{Field in the direction}\\ \text{of increasing } \theta\end{pmatrix}=\frac{\mu_0}{4\pi}\left(\frac{M\sin\theta}{r^3}\right)$

$$\Rightarrow \quad B = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{1+3\cos^2\theta}$$

The direction of this intensity is given by

$$\tan\beta = \frac{1}{2}\tan\theta$$

$$\Rightarrow \quad \beta = \tan^{-1}\left(\frac{1}{2}\tan\theta\right)$$

and net field B makes an angle $(\theta+\beta)$ with the magnetic moment (M) of the magnetic dipole.

ILLUSTRATION 6

A short bar magnet has a magnetic moment of $0.48\ \text{JT}^{-1}$. Find the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of magnet on

(a) the axis
(b) the equatorial lines (normal bisector) of the magnet.

SOLUTION

(a) When the point lies on the axis, then let B_1 be the magnetic field at P on the axial line.
So, for $r = 10\ \text{cm} = 0.1\ \text{m}$, we have

$$B_1 = \frac{\mu_0}{4\pi}\left(\frac{2M}{r^3}\right) = 10^{-7}\times\frac{2\times 0.48}{(0.1)^3}$$

$$\Rightarrow B_1 = 0.96\times10^{-4}\ \text{T from } S \text{ pole to } N \text{ pole}$$

(b) Let B_2 be the magnetic field at point P on the equatorial line, then

$$B_2 = \frac{\mu_0}{4\pi}\frac{M}{r^3} = 10^{-7}\times\frac{0.48}{(0.1)^3}$$

$\Rightarrow B_2 = 0.48\times10^{-4}\ \text{T} = 0.48\ \text{G}$ along from N pole to S pole.

ILLUSTRATION 7

What is the magnitude of the equatorial and axial fields due to a bar magnet of length 5.0 cm at a distance of 50 cm from its mid-point? The magnetic moment of the bar magnet is $0.40\ \text{Am}^2$.

SOLUTION

$$B_{\text{eq}} = \frac{\mu_0}{4\pi}\frac{M}{r^3} = \frac{10^{-7}\times 0.4}{(0.5)^3} = 3.2\times10^{-7}\ \text{T}$$

$$B_{\text{axial}} = \frac{\mu_0}{4\pi}\frac{2M}{r^3} = \frac{10^{-7}\times2\times0.4}{(0.5)^3} = 6.4\times10^{-7}\ \text{T}$$

ILLUSTRATION 8

Calculate the magnetic induction at a point $1\ \text{Å}$ away from a proton, measured along its axis of spin. The magnetic moment of the proton is $1.4\times10^{-26}\ \text{Am}^2$.

SOLUTION

On the axis of a magnetic dipole, magnetic induction is given by

$$B = \frac{\mu_0}{4\pi}\left(\frac{2M}{r^3}\right)$$

Substituting the values, we get

$$B = \frac{(10^{-7})(2)(1.4\times10^{-26})}{(10^{-10})^3} = 2.8\times10^{-3}\ \text{T} = 2.8\ \text{mT}$$

ILLUSTRATION 9

A bar magnet of length 0.1 m has pole strength of 50 Am. Calculate the magnetic field at distance of 0.2 m from its centre on

(a) its axial line and
(b) its equatorial line.

SOLUTION

Here, $m = 50\ \text{Am}$, $r = 0.2\ \text{m}$, $2l = 0.1\ \text{m}$

or $\quad l = 0.05\ \text{m}$

So, Magnetic dipole moment is

$$M = m(2l) = 50\times0.1 = 5\ \text{Am}^2$$

Since r and l are comparable, so

(a) $$B_{\text{axial}} = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2-l^2)^2} = \frac{10^{-7}\times2\times5\times0.2}{(0.2^2-0.05^2)^2}$$

$$\Rightarrow B_{\text{axial}} = 1.42\times10^{-4}\ \text{T}$$

(b) $$B_{\text{eq}} = \frac{\mu_0}{4\pi}\frac{M}{(r^2+l^2)^{3/2}} = \frac{10^{-7}\times5}{(0.2^2+0.05^2)^{3/2}}$$

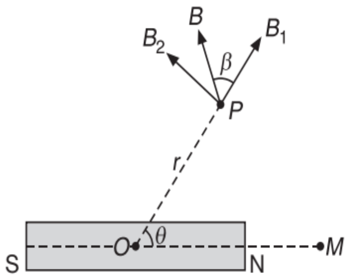
$$\Rightarrow B_{\text{eq}} = 5.71\times10^{-5}\ \text{T}$$

ILLUSTRATION 10

Find the magnetic field due to a dipole of magnetic moment $3\ \text{Am}^2$ at a point 5 m away from it in the direction making angle of $45°$ with the dipole axis.

SOLUTION

The condition given in the figure can be drawn as



So, the magnetic field at point P is,

$$\Rightarrow \quad B = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{1+3\cos^2\theta}$$

Here, $M = 3\ \text{Am}^2$, $r = 5\ \text{m}$, $\theta = 45°$

$$\Rightarrow \quad B = 10^{-7} \times \frac{3}{(5)^3}\sqrt{1+3\cos^2(45°)}$$

$$\Rightarrow \quad B = 10^{-7} \times \frac{3}{125} \times \sqrt{1+1.5}$$

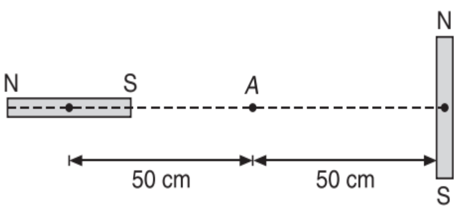
$$\Rightarrow \quad B = 10^{-7} \times \frac{3}{125} \times 1.58 = 3.79 \times 10^{-9}\ \text{T}$$

Also, $\tan\beta = \frac{1}{2}\tan\theta = \frac{1}{2}\tan 45° = \frac{1}{2}$

$$\Rightarrow \quad \beta = \tan^{-1}(0.5)$$

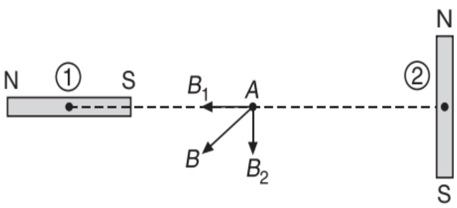
ILLUSTRATION 11

Calculate the magnetic field at point A due to the arrangement of two small dipoles each having a dipole moment of $1\ \text{Am}^2$ as shown in Figure.



SOLUTION

Point A lies on the axial line of first magnet and the field produced by the first magnet is



$$B_1 = \frac{\mu_0}{4\pi}\frac{2M}{r^3} = \frac{10^{-7} \times 2 \times 1}{(0.5)^3} = 16 \times 10^{-7}\ \text{T}$$

Point A lies on the equatorial line of second magnet and the field produced by the second magnet is

$$B_2 = \frac{\mu_0}{4\pi}\frac{M}{r^3} = \frac{10^{-7} \times 1}{(0.5)^3} = 8 \times 10^{-7}\ \text{T}$$

Therefore, net magnetic field at A is

$$B = \sqrt{B_1^2 + B_2^2} = 8\sqrt{5} \times 10^{-7}\ \text{T}$$

MAGNETIC FORCE BETWEEN TWO SHORT MAGNETS

(a) When they are coaxial, the force between the dipoles is

$$F = \frac{\mu_0}{4\pi}\left(\frac{6M_1M_2}{r^4}\right)$$

(b) When their axes are mutually perpendicular, then the force between the dipoles is

$$F = \frac{\mu_0}{4\pi}\left(\frac{3M_1M_2}{r^4}\right)$$

ILLUSTRATION 12

Two small magnets each of magnetic moment $10\ \text{Am}^2$ are placed in end on position 0.1 m apart from their centres. Calculate the force acting between them.

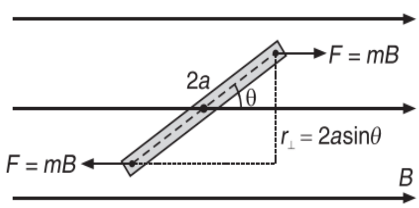
SOLUTION

$$F = \frac{\mu_0}{4\pi}\frac{6M_1M_2}{r^4} = (10^{-7})\frac{6(10 \times 10)}{(0.1)^4}$$

$$\Rightarrow \quad F = 0.06\ \text{N}$$

TORQUE ON A DIPOLE IN A UNIFORM MAGNETIC FIELD

Consider a bar magnet or a small compass needle of known magnetic moment $\vec{M}$, moment of inertia I is placed in a uniform magnetic field $\vec{B}$ at an angle θ as shown in Figure



We observe that, both of its poles experience equal and opposite forces due to which net force on it is always zero. Hence the magnetic dipole is always in translational equilibrium in a uniform magnetic field.

However, since these forces do not have the same line of action so it experiences a non-zero torque on it. This torque τ is given by

$$\tau = Fr_{\perp} = (mB)(2a\sin\theta)$$

$$\Rightarrow \quad \tau = m(2a)(B\sin\theta)$$

$$\Rightarrow \quad \tau = MB\sin\theta \qquad \{\because m(2a) = M\}$$

In vector form, $\vec{\tau} = \vec{M}\times\vec{B}$

Maximum torque is obtained when

$$\sin\theta = 1 \text{ or } \theta = \frac{\pi}{2}$$

$$\Rightarrow \quad \tau_{max} = MB$$

$$\Rightarrow \quad M = \frac{\tau_{max}}{B}$$

If $B = 1$ tesla, then $M = \tau_{max}$

So, the magnetic moment of a magnetic dipole is numerically equal to the torque acting on the dipole placed perpendicular to a magnetic field of strength 1 tesla.

SPECIAL CASES

(a) When $\theta = 0°$, $\vec{\tau} = 0$

The dipole is in rotational equilibrium also.

(b) When $\theta = 180°$, $\vec{\tau} = 0$ again.

(c) When $\theta = 90°$, $|\vec{\tau}| = \tau_{max} = MB$

ILLUSTRATION 13

A magnetic dipole is placed at an angle of $30°$ with a magnetic field of intensity 10^4 T. It experiences a torque equal to 5 Nm. Calculate the pole strength of the dipole, if dipole length is 1 cm.

SOLUTION

As $\tau = MB\sin\theta$, $M = \dfrac{\tau}{B\sin\theta}$

$$\Rightarrow \quad M = \frac{5}{((10^4)(0.5))} = 10^{-3} \text{ Am}^2$$

Since $M = md$

$$\Rightarrow \quad m = \frac{M}{d} = \frac{10^{-3}}{10^{-2}} = 10^{-1} \text{ Am}$$

ILLUSTRATION 14

A bar magnet when placed at an angle of $30°$ to the direction of magnetic field of 5×10^{-2} T, experiences a moment of couple 2.5×10^{-6} Nm. If the length of the magnet is 5 cm, then calculate its pole strength.

SOLUTION

Since torque acting on the magnet is

$$\tau = MB\sin\theta = m(2l)B\sin\theta, \text{ where}$$

$$\theta = 30°,\ B = 5\times10^{-2} \text{ T},\ \tau = 2.5\times10^{-6} \text{ Nm}$$

$$2l = 5 \text{ cm} = 0.05 \text{ m and } m = ?$$

$$\Rightarrow \quad m = \frac{\tau}{B(2l)\sin\theta} = \frac{2.5\times10^{-6}}{5\times10^{-2}(0.05)\sin 30°}$$

$$\Rightarrow \quad m = 2\times10^{-3} \text{ Am}$$

ILLUSTRATION 15

A bar magnet of length 10 cm and having the pole strength equal to 10^{-3} Wb is kept in a magnetic field having magnetic induction (B) equal to $4\pi\times10^{-3}$ T. It makes an angle of $30°$ with the direction of magnetic induction (B). Calculate the value of the torque acting on the magnet. Given that $(\mu_0 = 4\pi\times10^{-7} \text{ WbA}^{-1}\text{m}^1)$

SOLUTION

The unit of pole strength in S.I. system is Am, but here it is given in weber and since the unit of $\mu_0 m$ is weber, so we have

$$\mu_0 m = 10^{-3} \text{ weber}$$

$$\Rightarrow \quad m = \frac{10^{-3}}{\mu_0}$$

Magnetic moment $M = m(2\ell)$

$$\Rightarrow \quad M = \frac{10^{-3}}{\mu_0}\times0.10 = \frac{10^{-4}}{\mu_0}$$

Further $\tau = MB\sin\theta$

$$\Rightarrow \quad \tau = \frac{10^{-4}}{\mu_0}\times(4\pi\times10^{-3})\sin 30°$$

$$\Rightarrow \quad \tau = \frac{4\pi}{\mu_0}\times10^{-7}\times0.5 = 0.5 \text{ Nm}$$

ILLUSTRATION 16

A magnet of magnetic moment $50\hat{i}\ \text{Am}^2$ is placed along the x-axis in a magnetic field $\vec{B}=\left(0.5\hat{i}+3\hat{j}\right)$ T. Calculate the torque acting on the magnet.

SOLUTION

Since torque, $\vec{\tau}=\vec{M}\times\vec{B}$

where, $\vec{M}=50\hat{i}\ \text{Am}^2$, $\vec{B}=\left(0.5\hat{i}+3\hat{j}\right)$ T

$\Rightarrow\quad \vec{\tau}=50\hat{i}\times\left(0.5\hat{i}+3\hat{j}\right)$

$\Rightarrow\quad \vec{\tau}=150\left(\hat{i}\times\hat{j}\right)=150\hat{k}$ Nm

WORK DONE IN ROTATING A DIPOLE IN A UNIFORM MAGNETIC FIELD

Since the torque acting on a dipole placed in a uniform magnetic field is given by

$$\tau=MB\sin\theta$$

If dipole is rotated by a small angle $d\theta$, then work is done against torque which is given by

$$dW=\tau d\theta=MB\sin\theta\, d\theta$$

Total work done is given by

$$W=\int dW=\int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta$$

$$\Rightarrow\quad W=-MB\left(\cos\theta_2-\cos\theta_1\right)$$

The work done in rotating the dipole from equilibrium position $\theta_1=0^\circ$ through an angle θ is

$$W=-MB\left(\cos\theta-1\right)=MB\left(1-\cos\theta\right)$$

The work done in rotating the magnetic dipole from $\theta_1=90^\circ$ to $\theta_2=\theta$ is

$$W=-MB\left(\cos\theta-\cos 90^\circ\right)=-MB\cos\theta$$

This work done is stored in the form of potential energy U of system, so the potential energy of magnetic dipole is given by

$$U=-\vec{M}\cdot\vec{B}=-MB\cos\theta$$

POTENTIAL ENERGY OF A DIPOLE IN A UNIFORM MAGNETIC FIELD

The work done by external torque in rotating the dipole from an angle θ_1 to an angle θ_2 is

$$W=-MB\left(\cos\theta_2-\cos\theta_1\right) \qquad \ldots(1)$$

Since we know that work done by an external force or torque equals the change in potential energy, so from equation (1), we have

$$W=U_f-U_i=-MB\cos\theta_2-\left(-MB\cos\theta_1\right)$$

This makes us conclude that

$$U_f=-MB\cos\theta_2 \text{ and } U_i=-MB\cos\theta_1$$

and while drawing this conclusion, we have simply assumed that the potential energy of the dipole is zero at $\theta=90^\circ$. So, we have

$$U_{\text{initial}}=0,\text{ when } \theta_1=90^\circ$$

and $U_{\text{final}}=U=-MB\cos\theta$, when $\theta_2=\theta$

$$\Rightarrow\quad U=-MB\cos\theta=-\vec{M}.\vec{B}$$

SPECIAL CASES

(a) When $\theta=0^\circ$, $\vec{\tau}=0$, $U=U_{\min}=-MB$

The dipole is in total equilibrium and its potential energy is minimum, so its equilibrium is stable.

(b) When $\theta=180^\circ$, $\vec{\tau}=0$, $U=U_{\max}=MB$

(c) When $\theta=90^\circ$, $\left|\vec{\tau}\right|=\tau_{\max}=MB$, $U=0$

ILLUSTRATION 17

A magnetic dipole of length 5 cm having pole strength $\pm 2\times 10^{-3}$ Am, placed at 53° with the uniform magnetic field, experiences a torque of 8 Nm. Calculate the

(a) magnitude of magnetic field.
(b) potential energy of the dipole.

Given that $\tan\left(53^\circ\right)=\dfrac{4}{3}$

SOLUTION

(a) Since $\tau=MB\sin\theta$

$$\Rightarrow\quad B=\frac{\tau}{M\sin\theta}$$

$$\Rightarrow\quad B=\frac{8}{\left(2\times 10^{-3}\right)(0.05)(0.8)}=10^5\ \text{T}$$

(b) Since $U=-MB\cos\theta$

$$\Rightarrow\quad U=-\left(2\times 10^{-3}\right)(0.05)\left(10^5\right)(0.6)=-6\ \text{J}$$

TIME PERIOD OF SMALL OSCILLATIONS OF MAGNETIC DIPOLE/BAR MAGNET IN UNIFORM MAGNETIC FIELD

Let a bar magnet of dipole moment M and moment of inertia I is slightly disturbed by a small angle θ from its stable equilibrium position in a uniform magnetic field B. Restoring torque acting on it is

$$\tau = -MB\sin\theta$$

For small angle $\sin\theta \approx \theta$

$$\Rightarrow \quad \tau = -MB\theta \quad \text{...(1)}$$

Since, $\tau = I_{CM}\alpha = I_{CM}\left(\frac{d^2\theta}{dt^2}\right) = I_{CM}\ddot{\theta}$ ...(2)

From (1) and (2), we get

$$I_{CM}\left(\frac{d^2\theta}{dt^2}\right) = -MB\theta$$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} + \left(\frac{MB}{I_{CM}}\right)\theta = 0 \quad \text{...(3)}$$

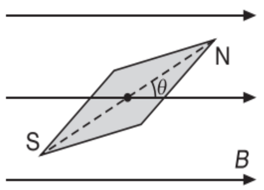
Comparing equation (3), with the standard equation of angular SHM i.e. $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$, we get

$$\omega = \sqrt{\frac{MB}{I_{CM}}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{I_{CM}}{MB}}$$

ILLUSTRATION 18

A magnetic needle is free to oscillate in a uniform magnetic field as shown in Figure.



The magnetic moment of magnetic needle is 7.2 Am^2 and moment of inertia $I = 6.5\times10^{-6}$ kgm^2. The number of oscillations performed in $5\,s$ is 10. Calculate the magnitude of magnetic field.

SOLUTION

Given that

$$T = \frac{\text{Number of revolutions}}{\text{Time taken}} = \frac{5}{10} = 0.5 \text{ s}$$

$$M = 7.2 \text{ Am}^2,\ I = 6.5\times10^{-6} \text{ kgm}^2$$

Since $T = 2\pi\sqrt{\frac{I}{MB}}$

$$\Rightarrow \quad T^2 = 4\pi^2\left(\frac{I}{MB}\right)$$

So, the magnitude of the magnetic field is

$$B = \frac{4\pi^2 I}{MT^2} = \frac{4\times(3.14)^2\times6.5\times10^{-6}}{7.2\times(0.5)^2} = 1.42\times10^{-4} \text{ T}$$

PROPERTIES OF ELECTRIC AND MAGNETIC DIPOLES

Property	Electric dipole	Magnetic dipole
Torque in an external field	$\vec{\tau} = \vec{p}\times\vec{E}$	$\vec{\tau} = \vec{M}\times\vec{B}$
Field at distant point along axis (end-on position)	$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{x^3}$	$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{x^3}$
Field at distant point along perpendicular bisector (broad-side-on position)	$\vec{E} = -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{x^3}$	$\vec{B} = -\frac{\mu_0}{4\pi}\frac{\vec{M}}{x^3}$
Work done in rotating the dipole in an external field from the equilibrium position	$W = pE(1-\cos\theta)$ $\Rightarrow W = pE - \vec{p}\cdot\vec{E}$	$W = MB(1-\cos\theta)$ $\Rightarrow W = MB - \vec{M}\cdot\vec{B}$
Potential energy in an external field	$U = -\vec{p}\cdot\vec{E}$	$U = -\vec{M}\cdot\vec{B}$

ILLUSTRATION 19

The work done in turning a magnet of magnetic moment M through an angle of $90°$ from the meridian, is n times the corresponding work done to turn it through an angle of $60°$. Calculate n.

SOLUTION

Since, we know that

$$W = MB(1-\cos\theta)$$

According to the problem, we have

$$W_{0°\to 90°} = n\left(W_{0°\to 60°}\right)$$

$$\Rightarrow \quad MB\left(1-\cos\left(90°\right)\right) = nMB\left(1-\cos\left(60°\right)\right)$$

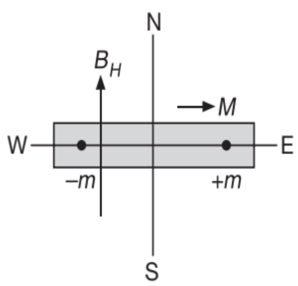
$$\Rightarrow \quad n = 2$$

ILLUSTRATION 20

A bar magnet of magnetic moment $2\ \text{Am}^2$ is free to rotate about a vertical axis passing through its centre. The magnet is released from rest from east-west position. If horizontal component of earth's field is $25\ \mu\text{T}$ and it is directed from south to north, then calculate the kinetic energy of the magnet as it takes north-south position.

SOLUTION

Initially the magnet is along east-west direction, so the angle between M and B_H is $90°$ i.e. $\theta_1 = 90°$ as shown in Figure.



So, initial potential energy of the magnet is

$$U_1 = -MB_H \cos 90° = 0$$

Finally, when the magnet takes the north-south position, then it aligns along the earths magnetic field, so the angle between M and B_H is $0°$ i.e. $\theta_2 = 0°$.
So, the final potential energy of the magnet is

$$U_2 = -MB_H \cos 0° = -MB_H$$

According to Law of Conservation of Energy, we know that the loss in potential energy of the magnet equals the gain in its kinetic energy, so we have

$$\Delta K = U_i - U_f = MB_H$$

$$\Rightarrow \quad \Delta K = 2\left(25\times 10^{-6}\right) = 50\times 10^{-6}\ \text{J} = 50\ \mu\text{J}$$

GAUSS'S LAW FOR MAGNETISM

The law states that

$$\oint \vec{B}\cdot d\vec{A} = 0$$

for all closed surfaces. This is a precise expression of the fact that magnetic monopoles do not exist.

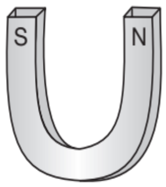
The flux emanating out of a surface is taken positive and the flux entering the surface is to be taken as negative. So, we conclude that total magnetic flux entering a closed surface equals the total magnetic flux leaving. This can also be attributed to the fact that magnetic field lines are continuous and form closed loops unlike electrostatic field lines which begin at a positive charge and end at a negative charge.

Test Your Concepts-I

Based on Bar Magnet and Properties

(Solutions on page H.107)

1. The work done in turning a magnet of magnetic moment M by an angle 90° from the meridian is two times the corresponding work done to turn it through an angle of θ. Calculate θ.
2. Calculate the geometric length of a bar magnet that has a magnetic length of 10 cm.
3. The distance between the poles of a horse shoe magnet is 0.1 m and its pole strength is 0.01 Am. Calculate the induction of magnetic field at a point midway between the poles.



4. A current I is flowing in a conducting wire of length L. If the wire is bent to form a circular loop, then calculate its magnetic dipole moment.
5. The length of a magnetised steel wire is l and its magnetic moment is M. It is bent into an L shaped wire with two sides equal. Calculate the new magnetic moment of the wire.
6. A magnetised wire of moment M is bent into an are of a circle subtending an angle of 60° at the centre. Calculate the new magnetic moment.
7. A short bar magnet of magnetic moment $m = 0.32\ \text{JT}^{-1}$ is placed in a uniform magnetic field 0.15 T. If the bar magnet is free to rotate in the plane of the field, which orientation would correspond to its stable and unstable equilibrium. Also calculate the potential energy of the magnet in each case.
8. A thin bar magnet of length $2L$ is bent at the mid-point, so that the angle between them is 60°. Calculate the new length of the magnetic dipole.
9. A wire of length ℓ is bent in the form a circular coil of some turns. A current i flows through the coil. The coil is placed in a uniform magnetic field B. Calculate the maximum torque acting on the coil.
10. A bar magnet of magnetic moment 4 Am^2 is free to rotate about a vertical axis passing through its centre. The magnet is released from rest from east-west position. If horizontal component of earth's field is 25 μT and it is directed from south to north, then calculate the kinetic energy of the magnet as it takes north-south position.

EARTH'S MAGNETISM

The earth behaves like a magnet. When a bar magnet is suspended freely in the earth's magnetic field, it stays along north-south direction. The north and south poles of magnet stay along South and North poles of earth respectively. It is observed that the magnetic poles are at some distance from geographical poles. The latest theories providing explanation to earth's magnetism are given below.

(a) The earth rotates about its axis and has a surrounding ionised region due to interaction of cosmic rays. Due to rotation of earth, the surrounding ionised region gives rise to strong electric currents which cause magnetisation.

(b) There exists molten iron and nickel within the core of earth and when the earth rotates, then due to the convective motion of these metallic fluids, a magnetic field is produced which causes magnetisation and this effect is called as "*the dynamo effect*".

Both theories given above, show that earth's magnetism will be destroyed if the earth stops rotating.

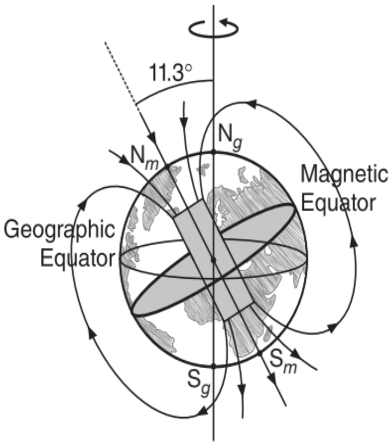
PROPERTIES OF EARTH'S MAGNETIC FIELD

Originally, it was assumed that the magnetic field of earth is similar to one which would be obtained if a huge magnet is assumed to be buried deep inside the earth at its centre. The strength of the earth's magnetic field is not constant and changes irregularly from place to place at the earth's surface and even at a given place, it varies with time too. The order of magnitude of the earth's magnetic field is 10^{-5} T. The following things must also be kept in mind while understanding the earth's magnetic field.

(a) The axis of rotation of earth is called geographic axis and the points where it cuts the surface of earth are called geographical poles i.e. geographical north N_g and geographical south S_g. The circle on the earth's surface perpendicular to the geographical axis is called equator.

(b) A vertical plane passing through the geographical axis is called Geographical Meridian (GM).

(c) The axis of the huge magnet assumed to be lying inside the earth is called magnetic axis of the earth. The points where the magnetic axis cuts the surface of earth are called magnetic poles. The circle on the earth's surface perpendicular to the magnetic axis is called magnetic equator.



(d) Magnetic axis and Geographical axis do not coincide but they make an angle of $11.3°$ with each other.

(e) A vertical plane passing through the magnetic axis is called Magnetic Meridian (MM).

(f) Direction of earth's magnetic field is from S (geographical south) to N (Geographical north).

COMPONENTS OF EARTH'S MAGNETIC FIELD

It has been observed that the magnitude and direction of the earth's magnetic field at a place can be completely calculated by the three quantities known as magnetic elements. These magnetic elements also called as components of earth's magnetic field. These essential components of earth's magnetic field are

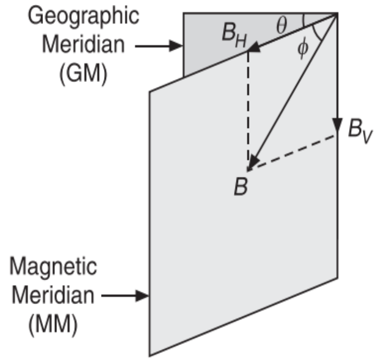
(a) Angle of Declination or Magnetic Declination (θ)

(b) Angle of Dip or Magnetic Dip (ϕ)

(c) Horizontal component of earth's magnetic field (B_H).

ANGLE OF DECLINATION (θ)

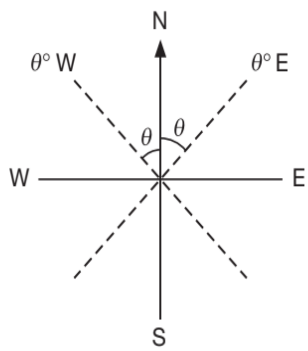
The vertical plane passing through the line joining magnetic north and south poles is called the **magnetic meridian** while that passing through the line joining the geographical north and south poles is called the **geographical meridian** figure.



The angle between magnetic and geographical meridian is called the **angle of declination** and is denoted by θ.

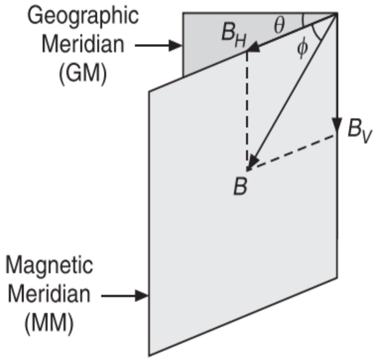
If, at a place, the north pole of the compass needle lies $\theta°$ to the east of geographical axis, then declination at that place is expressed at $\theta° E$.

If, at a place, the north pole of the compass needle lies $\theta°$ to the west of geographical axis, then declination at that place is expressed at $\theta° W$.



ANGLE OF DIP OR ANGLE OF INCLINATION (ϕ)

The magnetic field of earth varies in magnitude and direction. The angle of dip is the angle made by resultant earth's magnetic field (B) and the horizontal line in the magnetic meridian as shown in Figure.



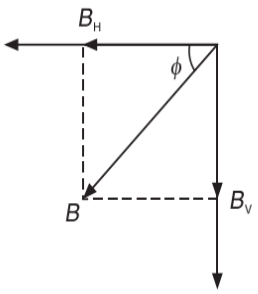
It is also the angle which the axis of a freely suspended magnet makes with the horizontal. The angle of dip is measured by dip circle.

At poles $\phi = 90°$ and at equator $\phi = 0°$.

In the northern hemisphere the north pole of the magnet points below the horizontal and hence the angle is called as angle of dip, whereas in the southern hemisphere the North Pole points above the horizontal and hence the angle is called as the Angle of Inclination.

HORIZONTAL COMPONENT OF EARTH'S MAGNETIC FIELD (B_H)

The earth's magnetic field strength (B) makes an angle ϕ with the horizontal as shown in Figure.



It can be resolved into two components, the

(a) horizontal component B_H, which is from south to north and

(b) vertical component B_V

From figure, we observe that

$$B_H = B\cos\phi \text{ and } B_V = B\sin\phi$$

$$\Rightarrow \quad B = \sqrt{B_H^2 + B_V^2} \text{ and } \tan\phi = \frac{B_V}{B_H}$$

Conceptual Note(s)

At equator $\phi = 0$

$\Rightarrow$ $B_H = B$ and $B_V = 0$

while at poles $\phi = 90°$

$\Rightarrow$ $B_H = 0$ and $B_V = B$

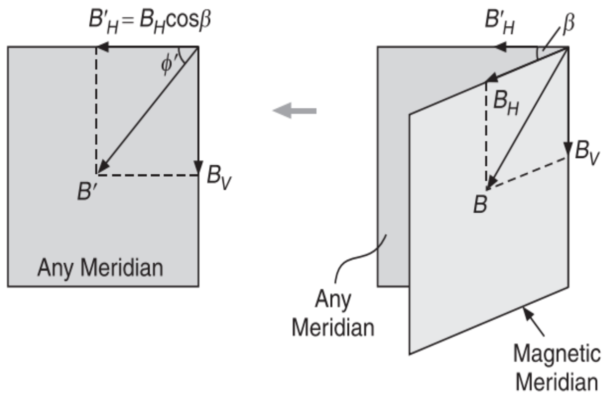
APPARENT DIP (ϕ')

The value of dip at a place is determined with the help of an instrument known as a **dip circle**. It consists of a magnetised needle capable of rotating in the vertical plane about a horizontal axis. The ends of the needle move over a vertical scale graduated in degrees.

When the plane of the scale of the dip circle is in the magnetic meridian, the needle rests in the direction of the earth's magnetic field. The angle made by the needle with the horizontal is called **true dip**.

If the plane of the scale of the dip circle is not in the magnetic meridian, then the needle will not indicate the correct direction of the earth's magnetic field. The angle made by the needle with the horizontal is called the **apparent dip**.

So **apparent dip** is the angle of dip shown by the dip circle when its plane is placed in any general meridian at some angle with the magnetic meridian as shown in Figure.



Suppose the dip circle is set at an angle β to the magnetic meridian. In this new vertical plane inclined at an angle β to the magnetic meridian, the vertical component of the earth's magnetic field remains the same whereas in this new plane the horizontal component of the earth's magnetic field will have a value

$$B'_H = B_H \cos\beta$$

Now, the apparent dip ϕ' is given by

$$\tan\phi' = \frac{B_V}{B'_H} = \frac{B_V}{B_H \cos\beta}$$

But $\tan\phi = \frac{B_V}{B_H}$, where ϕ is the true dip, so we get

$$\tan\phi' = \frac{\tan\phi}{\cos\beta}$$

Problem Solving Technique(s)

(a) When the magnetic needle oscillates in the vertical east-west plane at right angle to magnetic meridian, then only vertical component of earth's magnetic (B_V) field acts on it.

(b) When the dip needle oscillates at right angles to the magnetic meridian in a horizontal plane, then only horizontal component of earth's magnetic field (B_H) acts on it.

(c) When the dip needle oscillates in the vertical plane in magnetic meridian then both the components B_V and B_H act on it.

(d) The horizontal component of earth's magnetic field is from S to N

(e) If at any place the angle of dip is θ and magnetic latitude is λ then,

$$\tan\theta = 2\tan\lambda$$

(f) The total intensity of earth's magnetic field

$$I = I_0\sqrt{1+3\sin^2\lambda}$$

here $I_0 = \frac{M}{R^3}$

Here M and R are the magnetic moment of bar magnet of earth and radius of earth respectively.

(g) At magnetic equator of earth $\lambda = 0^\circ$ and at poles $\lambda = 90^\circ$

$$I_{pole} = 2I_{equator}$$

(h) At the poles and equator of earth, the values of total intensity are 0.66 oersted and 0.33 oersted respectively.

ILLUSTRATION 21

The magnetic field of the earth at the equator is approximately 0.4 G. Taking the radius of earth to be 6400 km, estimate the dipole moment of the earth.

SOLUTION

The equatorial magnetic field for the earth can be approximated as

$$B_e = \frac{\mu_0 M}{4\pi R_e^3}$$

where, $B_e \approx 0.4\text{ G} = 4\times10^{-5}\text{ T}$, $R_e = 6.4\times10^6\text{ m}$

$$\Rightarrow \quad M = \frac{4\pi B_e R_e^3}{\mu_0} = \frac{(4\times10^{-5})(6.4\times10^6)^3}{\mu_0/4\pi}$$

$$\Rightarrow \quad M = \frac{(4\times10^{-5})(6.4\times10^6)^3}{10^{-7}} = 1.05\times10^{23}\text{ Am}^2$$

This value is close to the value $8\times10^{22}\text{ Am}^2$ quoted in texts related to geomagnetism.

ILLUSTRATION 22

The plane of the dip circle is set in the geographical meridian and the apparent dip is θ_1. It is then set in a vertical plane perpendicular to the geographical meridian, the apparent dip becomes θ_2. Find the angle of declination α in terms of θ_1 and θ_2 at that place.

SOLUTION

Since, $\tan\theta_1 = \frac{\tan\theta}{\cos\alpha}$

and $\tan\theta_2 = \frac{\tan\theta}{\cos(90^\circ-\alpha)} = \frac{\tan\theta}{\sin\alpha}$

$$\Rightarrow \quad \cos\alpha = \frac{\tan\theta}{\tan\theta_1} \quad \ldots(1)$$

and $\sin\alpha = \frac{\tan\theta}{\tan\theta_2} \quad \ldots(2)$

Dividing (2) by (1), we get

$$\tan\alpha = \frac{\tan\theta_1}{\tan\theta_2}$$

ILLUSTRATION 23

A magnet 10 cm long and having a pole strength 2 Am is deflected through $30°$ from the magnetic meridian. The horizontal component of earth's induction is $0.32\times10^{-4}\text{ T}$. Find the value of deflecting couple.

SOLUTION

Magnetic moment $M = m(2l)$

$$\Rightarrow \quad M = 2\times(10\times10^{-2}) = 0\cdot 20\text{ Am}^2$$

Since, $\tau = MB_H \sin\theta$

$$\Rightarrow \quad \tau = (0.20)(0.32\times10^{-4})\sin 30°$$

$$\Rightarrow \quad \tau = 32\times10^{-7}\text{ Nm}$$

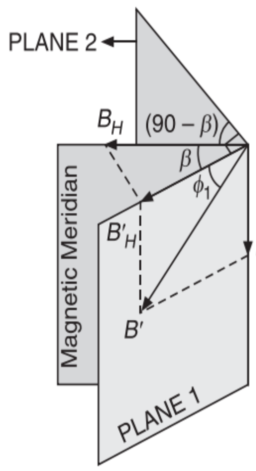
ILLUSTRATION 24

If ϕ_1 and ϕ_2 be the angles of dip observed in two vertical planes at right angles to each other and ϕ be the true angle of dip, then prove that

$$\cot^2\phi = \cot^2\phi_1 + \cot^2\phi_2$$

SOLUTION

Let α be the angle which one of the planes make with the magnetic meridian the other plane makes an angle $(90° - \beta)$ with it. The components of B_H in these planes will be $B_H\cos\alpha$ and $B_H\sin\alpha$ respectively. If ϕ_1 and ϕ_2 are the apparent dips in these two planes, then



$$\tan\phi_1 = \frac{B_V}{B_H\cos\beta}$$

$$\Rightarrow \quad \cos\beta = \frac{B_V}{B_H\tan\phi_1} \qquad \ldots(1)$$

$$\tan\phi_2 = \frac{B_V}{B_H\sin\beta}$$

$$\Rightarrow \quad \sin\beta = \frac{B_V}{B_H\tan\phi_2} \qquad \ldots(2)$$

Squaring and adding (1) and (2), we get

$$\cos^2\beta + \sin^2\beta = \left(\frac{B_V}{B_H}\right)^2\left(\frac{1}{\tan^2\phi_1} + \frac{1}{\tan^2\phi_2}\right)$$

$$\Rightarrow \quad 1 = \frac{B_V^2}{B_H^2}\left(\cot^2\phi_1 + \cot^2\phi_2\right)$$

$$\Rightarrow \quad \frac{B_H^2}{B_V^2} = \cot^2\phi_1 + \cot^2\phi_2$$

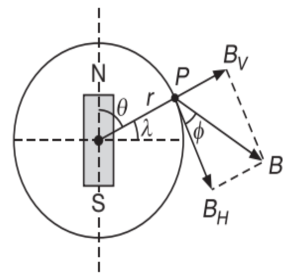
$$\Rightarrow \quad \cot^2\phi = \cot^2\phi_1 + \cot^2\phi_2$$

ILLUSTRATION 25

Consider the earth as a short magnetic with its centre coinciding with the centre of earth and dipole moment M. Calculate the angle of dip ϕ at a place where the magnetic latitude is λ.

SOLUTION

For a short magnet/dipole, the radial (or vertical) component of magnetic field is found by taking the field due to the small dipole at any general point $P(r, \theta)$



The vertical and horizontal components of earth's magnetic field are

$$B_V = \frac{\mu_0}{4\pi}\frac{2M\cos\theta}{r^3} \text{ and } B_H = \frac{\mu_0}{4\pi}\frac{M\sin\theta}{r^3}$$

where $\theta = (90° - \lambda)$

$$\Rightarrow \quad B_V = \frac{\mu_0}{4\pi}\frac{2M\cos(90° - \lambda)}{r^3} = \frac{\mu_0}{4\pi}\frac{2M\sin\lambda}{r^3}$$

$$\Rightarrow \quad B_H = \frac{\mu_0}{4\pi}\frac{2M\sin(90° - \lambda)}{r^3} = \frac{\mu_0}{4\pi}\frac{M\cos\lambda}{r^3}$$

Angle of dip ϕ is the angle made by resultant magnetic field intensity with the horizontal. So,

$$\tan\phi = \frac{B_V}{B_H}$$

$$\Rightarrow \quad \tan\phi = \frac{\frac{\mu_0}{4\pi}\left(\frac{2M\sin\lambda}{r^3}\right)}{\frac{\mu_0}{4\pi}\left(\frac{M\cos\lambda}{r^3}\right)} = 2\tan\lambda$$

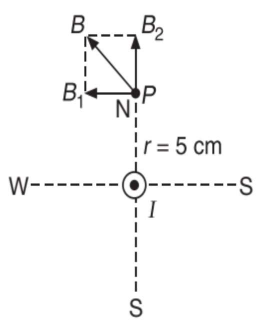
$$\Rightarrow \quad \phi = \tan^{-1}(2\tan\lambda)$$

ILLUSTRATION 26

A long vertical wire carries a steady current of 10 A flowing upwards through it at a place where horizontal component of earth's magnetic field is 0.3 G. Calculate the total magnetic induction at a point 5 cm from the wire due magnetic north of wire.

SOLUTION

The magnetic field B_1 due to current carrying wire at distance $r = 5\text{ cm} = 5\times10^{-2}\text{ m}$ is shown in Figure and is given by



$$B_1 = \frac{\mu_0 I}{2\pi r} = \frac{4\pi\times10^{-7}\times10}{2\pi\times5\times10^{-2}}$$

$$\Rightarrow \quad B_1 = 0.4\times10^{-4}\text{ T}$$

$$\Rightarrow \quad B_1 = 0.4\text{ G, due west}$$

The earth's field is

$$B_2 = B_H = 0.4\text{ G, towards north}$$

So, net magnetic field is given by

$$B = \sqrt{B_1^2 + B_2^2}$$

$$\Rightarrow \quad B = \sqrt{(0.4)^2 + (0.3)^2} = 0.5\text{ G}$$

ILLUSTRATION 27

A compass needle whose magnetic moment is 60 Am^2 pointing geographical north at a certain place where the horizontal component of earth's magnetic field is 40 μT, experiences a torque of 1.2×10^{-3} Nm. Calculate the declination of the place.

SOLUTION

Torque acting on compass needle is

$$\tau = MB_H\sin\theta$$

$$\Rightarrow \quad 1.2\times10^{-3} = 60\times(40\times10^{-6})\sin\theta$$

$$\Rightarrow \quad \sin\theta = \frac{1.2\times10^{-3}}{50\times40\times10^{-6}} = \frac{1}{2}$$

So, declination at that place is

$$\theta = 30^\circ$$

ILLUSTRATION 28

A magnet is suspended in the magnetic meridian using an untwisted wire. The upper end of wire is rotated through 180° to deflect the magnet by 30° from magnetic meridian. When this magnet is replaced by another magnet, the upper end of wire is rotated through 270° to deflect the magnet 30° from magnetic meridian. Calculate the ratio of magnetic moments of magnets.

SOLUTION

Let M_1 and M_2 be the magnetic moments of magnets and H the horizontal component of earth's field. Since

$$\tau = MB_H\sin\theta$$

If ϕ is the twist of wire, then $\tau = C\phi$, C being restoring couple per unit twist of wire also called as torsional constant. At equilibrium, we have

$$C\phi = MB_H\sin\theta \quad \ldots(1)$$

$$\Rightarrow \quad \frac{\phi_1}{\phi_2} = \frac{M_1}{M_2}$$

where $\phi_1 = (180^\circ - 30^\circ) = 150^\circ$

$$\Rightarrow \quad \phi_1 = 150\times\frac{\pi}{180}\text{ radian}$$

and $\phi_2 = (270^\circ - 30^\circ) = 240^\circ$

$$\Rightarrow \quad \phi_2 = 240\times\frac{\pi}{180}\text{ radian}$$

$$\Rightarrow \quad \frac{M_1}{M_2} = \frac{\phi_1}{\phi_2} = \frac{150\times\left(\frac{\pi}{180}\right)}{240\times\left(\frac{\pi}{180}\right)} = \frac{15}{24} = \frac{5}{8}$$

ABOUT MAGNETIC MAPS

It is observed that dip, declination and horizontal component of earth's magnetic field over the earth's surface varies from place to place. However, there are many places on the earth which possess the same value of magnetic elements. Lines are drawn joining all the places on the earth having the same value of magnetic elements and these lines form magnetic maps.

Isogonic Lines

The lines joining the equal angle of declination are called **isogonic lines**.

Agonic Lines

The lines joining the zero angle of declination are called **agonic lines**.

Isoclinic Lines

The lines joining the equal angle of dip are called **isoclinic lines**.

Aclinic Lines (or Magnetic Equator)

The lines joining the zero angle of dip are called **aclinic lines**.

Isodynamic Lines

The lines joining equal horizontal component of earth's magnetic field are called **isodynamic lines**.

NEUTRAL POINT

A neutral point is obtained when horizontal component of earth's field is balanced by the field produced by the magnet. i.e. at neutral point the intensity of magnetic field is equal to the horizontal component of earth's magnetic field but its direction is opposite to that of B_H.

NEUTRAL POINT WHEN BAR MAGNET IS PLACED HORIZONTALLY ON A HORIZONTAL PLANE/TABLE

Consider a bar magnet that is placed horizontally on a horizontal plane/table such that

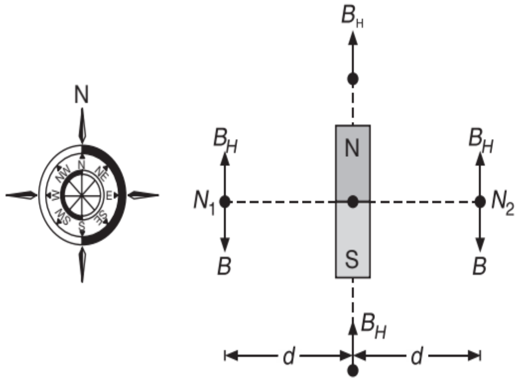
(a) the north pole of magnet is facing geographical north of earth

(b) the north pole of magnet is facing geographical south of earth

In both these cases two neutral points are obtained as discussed below.

CASE-1:

When the bar magnet is placed horizontally in the horizontal plane such that the north pole of magnet is facing geographical north of earth, then two neutral points N_1 and N_2 are obtained in the plane of the magnet on the equatorial line of bar magnet at a distance d from the centre of the magnet as shown.



This is because the field of the earth B_H is from geographical south to geographical north and the field of the magnet B is from north pole to south pole of magnet, outside it.

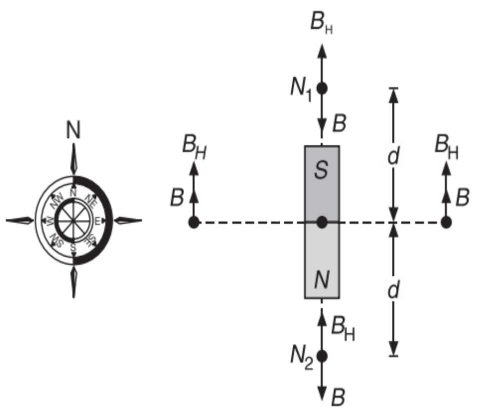
At Neutral points, we have

$$B_H = B_{\text{equitorial}}$$

$$\Rightarrow \quad B_H = \frac{\mu_0}{4\pi}\frac{M}{d^3}$$

CASE-2:

When the bar magnet is placed horizontally in the horizontal plane such that the south pole of magnet is facing geographical north of earth, then two neutral points N_1 and N_2 are obtained in the plane of the magnet on the equatorial line of bar magnet at a distance d from the centre of the magnet as shown.



This is because the field of the earth B_H is from geographical south to geographical north and the field of the magnet B is from north pole to south pole of magnet, outside it.

At neural point, in this case, we have

$$B_H = B_{axial}$$

$$\Rightarrow \quad B_H = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$$

NEUTRAL POINT WHEN BAR MAGNET IS PLACED VERTICALLY ON A HORIZONTAL PLANE/TABLE

Consider a magnet placed vertically in a horizontal plane just like a magnet placed vertically on a horizontal table surface, such that

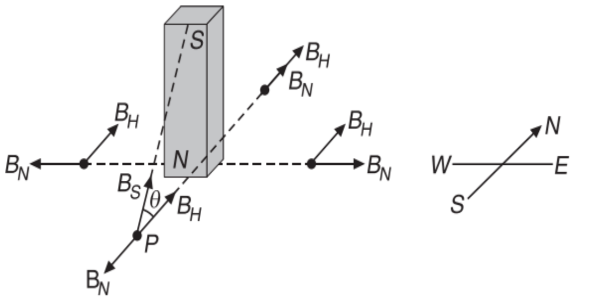
(a) the north pole of the magnet lies in the plane of the table

(b) the south pole of the magnet lies in the plane of the table.

In both the said cases, only one neutral point is obtained as discussed below.

CASE-1:

North pole of the magnet lies in the horizontal plane i.e. the plane of the table.



B_H = Magnetic field due to N-pole

B_H = Magnetic field due to S-pole

M = Pole strength of each pole of the magnet

At neutral point P we have

$$B_N - B_S\cos\theta = B_H \qquad \{B_S < B_N\}$$

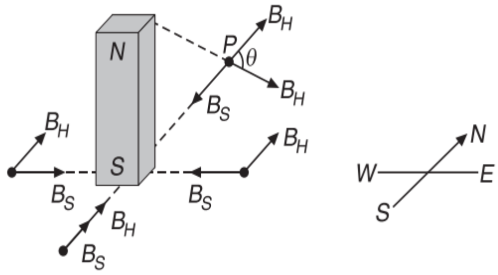
If we neglect the effect of South pole, then too only one neutral point is obtained (as seen from above) as shown and then at the neutral point, we have

$$B_N = B_H$$

$$\Rightarrow \quad \frac{\mu_0}{4\pi}\frac{m}{r^2} = B_H$$

CASE-2:

South pole of the magnet lies in the horizontal plane i.e. the plane of the table.



At neutral point P, we have

$$B_S - B_N\cos\theta = B_H \qquad \{B_S > B_N\}$$

Here too, if we neglect the effect of North pole, then also only one neutral point is obtained (as seen from above) as shown and then at the neutral point, we have

$$B_S = B_H$$

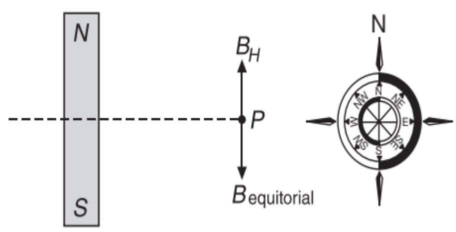
$$\Rightarrow \quad \frac{\mu_0}{4\pi}\frac{m}{r^2} = B_H$$

ILLUSTRATION 29

A short bar magnet with its north pole facing north forms a neutral point at P in the horizontal plane. If the magnet is rotated by $90°$ in the horizontal plane, calculate the net magnetic induction at P, assuming that the horizontal component of earth's magnetic field is B_H.

SOLUTION

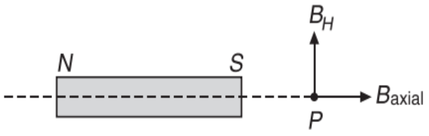
Initially the magnet is placed as shown in Figure.



So, neutral point obtained equitorial line and at neutral point, we have

$$|B_H| = |B_{equitorial}|$$

where B_H is the horizontal component of earth's magnetic field, $B_{\text{equitorial}}$ is the magnetic field due to bar magnet on its equitorial line.
Finally, we have



Now the point P lies on axial line of the magnet and at P, net magnetic field is given by

$$B = \sqrt{B_{\text{axial}}^2 + B_H^2}$$

$$\Rightarrow \quad B = \sqrt{(2B_e)^2 + (B_H)^2}$$

Since $B_e = B_{\text{equitorial}} = B_H$

$$\Rightarrow \quad B = \sqrt{(2B_H)^2 + B_H^2} = \sqrt{5}B_H$$

Test Your Concepts-II

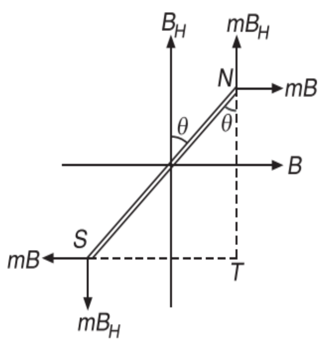
Based on Earth's Magnetism

(Solutions on page H.108)

1. A magnetic needle suspended in a vertical plane at 30° from the magnetic meridian makes an angle of 45° with the horizontal. Find the true angle of dip.
2. A short magnet of moment 6.75 Am^2 produces a neutral point on its axis. If horizontal component of earth's magnetic field is $5 \times 10^{-5} \text{Wbm}^{-2}$, then calculate the distance of the neutral point.
3. A dip circle shows an apparent dip of 45° at a place where the true dip is 30°. If the dip circle is rotated through 90°, what apparent dip will it show?
4. A compass needle of magnetic moment 60 Am^2 is pointing geographical north at a certain place. It experiences a torque of 1.2×10^{-3} Nm. The horizontal component of earth's magnetic field at that place is 40 μWbm^{-2}. Calculate the angle of declination at that place.
5. At a certain location on earth, a compass point 12° west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points 60° above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. Specify the direction and magnitude of the earth's field at the location.
6. A very small magnet is placed in the magnetic meridian with its south pole pointing north. The null point is obtained 20 cm away from the centre of the magnet. If the earth's magnetic field (horizontal component) at this point be 0.3 G, then calculate the magnetic moment of the magnet.
7. A ship is to reach a place 10° south of west. In which direction should it be steered if the declination at the place is 18° west of north?
8. The horizontal and vertical component of earth's field at a place are 0.22 G and 0.38 G respectively. Calculate the angle of dip and resultant intensity of earth's field.
9. A dip circle is adjusted so that its needle moves freely in the magnetic meridian. In this position, the angle of dip is 60°. Now the dip circle is rotated so that the plane in which the needle moves makes and angle of 30° with the magnetic meridian. In this new position, calculate the angle of dip.
10. In the magnetic meridian of a certain place, the horizontal component of earth's magnetic field is 0.26 G and the dip angle is 60°. Calculate the vertical component of earth's magnetic field and the net magnetic field of earth at this place.

TANGENT LAW

It is a relation which states the condition of equilibrium of a magnet subjected to two uniform magnetic fields at right angle to each other.



Consider a magnet NS of length $2l$ suspended in two uniform fields, B and B_H, at right angles to each other. Ordinarily field B_H is the horizontal component of earth's magnetic field and B is due to some magnet. Let it come to rest making an angle θ with the lines of force of one of the fields, say B_H. Two couples act on the magnet.

Deflecting Couple (mB - mB)

This couple acts on the magnet due to the magnetic field B. It tries to increase the angle θ and hence the name deflecting couple. Its torque τ_d (clockwise) is given by

$$\tau_d = mB \times NT = mB \times 2l\cos\theta$$

$$\Rightarrow \quad \tau_d = MB\cos\theta \text{ (clockwise)}$$

Restoring Couple (mB_H - mB_H)

This couple acts on the magnet due to the field B_H. It tries to decrease the value of θ and hence the name restoring couple. Its torque τ_r (anti-clockwise) is given by

$$\tau_r = mB_H \times ST = 2mlB_H \sin\theta$$

$$\Rightarrow \quad \tau_r = MB_H \sin\theta \text{ (anti-clockwise)}$$

At equilibrium, we have

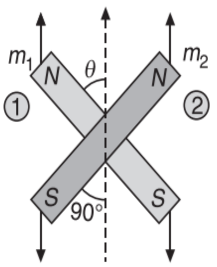
$$\tau_r = \tau_d$$

$$\Rightarrow \quad MB\cos\theta = MB_H \sin\theta$$

$$\Rightarrow \quad B = B_H \tan\theta$$

ILLUSTRATION 30

Two magnets of equal mass are joined at right angles to each other as shown the magnet 1 has a magnetic moment 3 times that of magnet 2. This arrangement is pivoted so that it is free to rotate in the horizontal plane. Calculate the angle which the magnet 1 makes with the magnetic meridian in equilibrium.



SOLUTION

For equilibrium of the system, torques on M_1 and M_2 due to B_H must counter balance each other i.e.,

$$\vec{M}_1 \times \vec{B}_H = \vec{M}_2 \times \vec{B}_H$$

If θ is the angle between M_1 and B_H, then angle between $\vec{M}_2$ and $\vec{B}_H$ is $(90-\theta)$

$$\Rightarrow \quad M_1 B_H \sin\theta = M_2 B_H \sin(90-\theta)$$

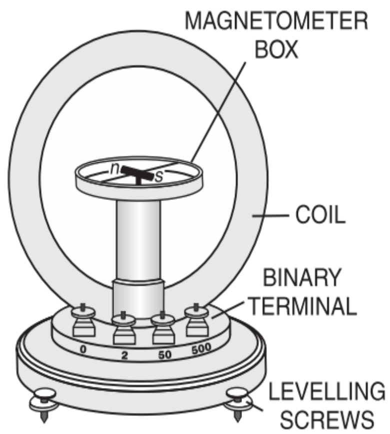
$$\Rightarrow \quad \tan\theta = \frac{M_2}{M_1} = \frac{M}{3M} = \frac{1}{3}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{3}\right)$$

TANGENT GALVANOMETER

It is an instrument based on Tangent Law, and used for detection of electric current in a circuit.

The vertical coil of the galvanometer is placed in the magnetic meridian.



When a current is passed through the coil having N turns, radius R, then a magnetic field (B) is produced at right angles to the plane of the coil i.e. at right angles to the horizontal component of earth's magnetic field (B_H).

The magnetic needle of the galvanometer undergoes a deflection θ under the influence of two crossed magnetic fields B and B_H, such that at equilibrium, according to Tangent Law, we have

$$B = B_H \tan\theta$$

Since, $B = \dfrac{\mu_0 NI}{2r}$

$$\Rightarrow \quad \frac{\mu_0 NI}{2r} = B_H \tan\theta$$

$$\Rightarrow \quad I = \frac{2rB_H}{\mu_0 N}\tan\theta$$

$$\Rightarrow \quad I = \frac{B_H}{G}\tan\theta$$

where $G = \frac{\mu_0 N}{2r}$, which is known as the **Galvanometer Constant**.

$$\Rightarrow \quad I \propto \tan\theta$$

Reduction Factor (K)

The quantity $\frac{B_H}{G}$ in the above equation is known as the reduction factor of tangent galvanometer.

$$\Rightarrow \quad K = \frac{B_H}{G} = \frac{2rB_H}{\mu_0 n}$$

$$\Rightarrow \quad I = K\tan\theta$$

If $\theta = 45°$, then $K = I$

Thus, the reduction factor of tangent galvanometer is defined as the amount of current required to produce a deflection of $45°$ in it.

Sensitivity and Accuracy of Tangent Galvanometer

Sensitivity is the measure of change in deflection produced by a unit current. Mathematically it is given by $\frac{d\theta}{dI}$. Since we have

$$I = K\tan\theta$$

Differentiating, $dI = K\sec^2\theta\, d\theta$

$$\Rightarrow \quad \frac{d\theta}{dI} = \frac{1}{K\sec^2\theta} = \frac{1}{K(1+\tan^2\theta)}$$

$$\Rightarrow \quad \frac{d\theta}{dI} = \frac{1}{K\left(1+\frac{I^2}{K^2}\right)}$$

A tangent galvanometer is both sensitive and accurate if the change in its deflection is large for a given fractional change in current.

Since, $I = K\tan\theta$

Differentiating, $dI = K\sec^2\theta\, d\theta$

$$\Rightarrow \quad \frac{dI}{I} = \frac{\sec^2\theta\, d\theta}{\tan\theta} = \frac{\cos\theta\, d\theta}{\cos^2\theta\sin\theta} = \frac{d\theta}{\sin\theta\cos\theta}$$

Since $2\sin\theta\cos\theta = \sin(2\theta)$

$$\Rightarrow \quad \frac{dI}{I} = \frac{2d\theta}{\sin 2\theta}$$

$$\Rightarrow \quad d\theta = \frac{\sin 2\theta}{2}\frac{dI}{I}$$

Clearly, $d\theta$ would be maximum if $\sin 2\theta$ is maximum i.e., 1. This is possible if $2\theta = 90°$ or $\theta = 45°$. So, tangent galvanometer has maximum sensitivity when the deflection is $45°$.

ILLUSTRATION 31

The coil of a tangent galvanometer of radius 12 cm is having 200 turns. If the horizontal component of earth's magnetic field is $25\,\mu T$. Find the current which gives a deflection of $60°$.

SOLUTION

Since, $I = K\tan\theta$

where, $K = \frac{2rB_H}{\mu_0 N}$

$$\Rightarrow \quad I = \frac{2rB_H}{\mu_0 N}\tan\theta$$

It is given that

$r = 12\text{ cm} = 0.12\text{ m}$, $N = 200$,

$B_H = 25\,\mu T = 25\times10^{-6}$ T and $\phi = 60°$

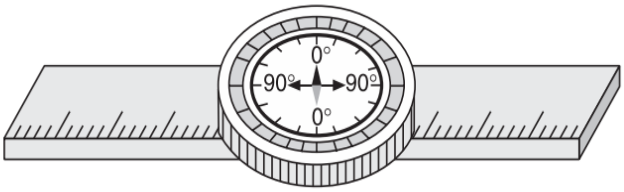
$$\Rightarrow \quad I = \frac{2\times0.12\times25\times10^{-6}\times\tan 60°}{4\pi\times10^{-7}\times200}$$

$$\Rightarrow \quad I = 0.042\text{ A}$$

DEFLECTION MAGNETOMETER

It's working is based on the principle of tangent law.

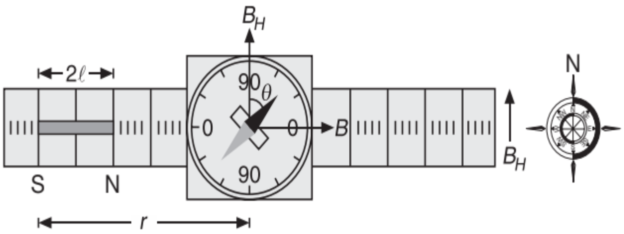
It consists of a small compass needle, pivoted at the centre of a circular box. The box is kept in a wooden frame having two-meter scale fitted on it's two arms. Reading of a scale at any point directly gives the distance of that point from the centre of compass needle.



Different position of deflection magnetometer: Deflection magnetometer can be used according to two following positions.

tan *A* Position

For this position, the arms of magnetometer are placed along E-W direction such that magnetic needle is acted upon by only horizontal component of earth's magnetic field (B_H) as shown.



If a bar magnet of magnetic moment M is placed on one arm with its length parallel to arm, then the magnetic needle comes under the influence of two mutual perpendicular magnetic fields

(a) Horizontal component of the earth's magnetic field, B_H and

(b) The axial magnetic field of experimental bar magnet.

At equilibrium, we have $B = B_H \tan\theta$ (Based on Tangent Law)

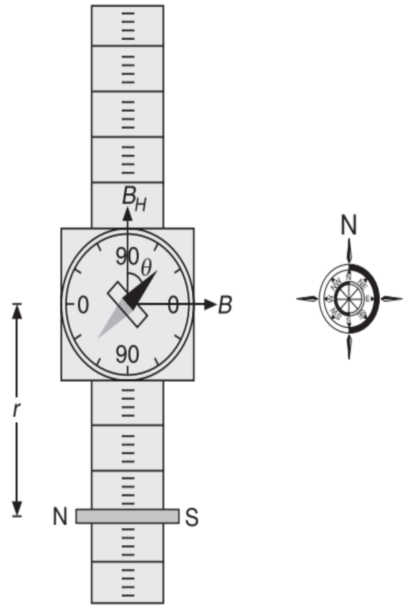
$$B_H \tan\theta = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2 - l^2)^2}$$

where r is the distance of needle from centre of magnet and $2l$ is the length of magnet.
If the magnet is small, then we have

$$\Rightarrow \quad B_H \tan\theta = \frac{\mu_0}{4\pi}\frac{2M}{r^3}$$

tan *B* Position

Arms of magnetometer are placed along N-S direction such that magnetic needle align itself in the direction of earth's magnetic field (*i.e.* B_H) as shown.



If a bar magnet of magnetic moment M is placed on one arm with its length perpendicular to arm, so that the magnetic needle comes under the influence of two mutual perpendicular magnetic fields

(a) Horizontal component of the earth's magnetic field, B_H and

(b) The equatorial magnetic field of experimental bar magnet.

At equilibrium, we have $B = B_H \tan\theta$ (Based on Tangent Law)

$$\Rightarrow \quad B_H \tan\theta = \frac{\mu_0}{4\pi}\frac{M}{(r^2 + l^2)^{3/2}}$$

where r is the distance of needle from centre of magnet and $2l$ is the length of magnet.
If the magnet is small, then we have

$$B_H \tan\theta = \frac{\mu_0}{4\pi}\frac{M}{r^3}$$

Problem Solving Technique(s)

Deflection magnetometer also used to compare the magnetic moments either by deflection method or by null deflection method.

(a) By Deflection method, we have $\dfrac{M_1}{M_2} = \dfrac{\tan\theta_1}{\tan\theta_2}$

(b) By Null deflection method, we have $\dfrac{M_1}{M_2} = \left(\dfrac{d_1}{d_2}\right)^3$

where d_1 and d_2 are the position of two bar magnets placed simultaneously on each arm.

ILLUSTRATION 32

The needle of a deflection galvanometer shows a deflection of $60°$ due to a short bar magnet at a certain distance in $\tan A$ position. Find the deflection when the distance is doubled.

SOLUTION

For short bar magnet in $\tan A$-position

$$\frac{\mu_0}{4\pi}\frac{2M}{d^3} = B_H \tan\theta \quad \ldots(1)$$

When distance is doubled, then new deflection θ' is given by

$$\frac{\mu_0}{4\pi}\frac{2M}{(2d)^3} = B_H \tan\theta' \quad \ldots(2)$$

Dividing (2) by (1), we get

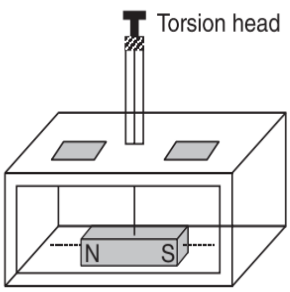
$$\frac{\tan\theta'}{\tan\theta} = \frac{1}{8}$$

$$\Rightarrow \quad \tan\theta' = \frac{\tan\theta}{8} = \frac{\tan 60°}{8} = \frac{\sqrt{3}}{8}$$

$$\Rightarrow \quad \theta' = \tan^{-1}\left(\frac{\sqrt{3}}{8}\right)$$

VIBRATION MAGNETOMETER

Vibration magnetometer is used for comparison of magnetic moments and magnetic fields. This device works on the principle, that whenever a freely suspended magnet in a uniform magnetic field, is disturbed from its equilibrium position, it starts vibrating about the mean position.



Time period of oscillation of experimental bar magnet (magnetic moment M) in earth's magnetic field (B_H) is given by the formula

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

where, I is the moment of inertia of short bar magnet of mass w and is given by $I = \frac{wL^2}{12}$.

TIME PERIOD OF A MAGNETIC NEEDLE IN EARTH'S MAGNETIC FIELD

Let a small magnetic needle of moment M be in earth field. When it is given a displacement such that it makes an angle θ with the magnetic field B_H, then restoring couple on needle is

$$\tau = -mB_H(2l\sin\theta)$$

$$\Rightarrow \quad \tau = -[m(2l)]B_H\sin\theta = -MB_H\sin\theta$$

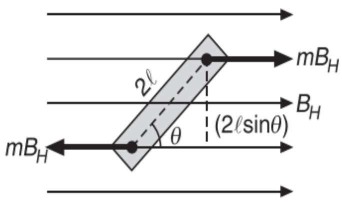
If I is moment of inertia of needle and $\alpha\left(=\ddot{\theta}\right)$ its angular acceleration, then

$$I\alpha = -MB_H\sin\theta$$

For small θ, $\sin\theta \simeq \theta$

$$\alpha = -\left(\frac{MB_H}{I}\right)\theta$$

$$\Rightarrow \quad \ddot{\theta} = -\left(\frac{MB_H}{I}\right)\theta \qquad \left\{\because\ \alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}\right\}$$



This is condition of angular SHM.

Since, $T = 2\pi\sqrt{\frac{\theta}{|\ddot{\theta}|}}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{I}{MB_H}}$$

Conceptual Note(s)

When a dip needle oscillates in a vertical plane in the magnetic meridian, it oscillates under the action of total intensity B_e, of earth's field, but if the needle oscillates in a vertical plane at right angles to the magnetic meridian (i.e., vertical *E-W* plane) it oscillates under the action of vertical component *V* only, when it oscillates in horizontal plane it oscillates under horizontal component *H*.

USES OF VIBRATION MAGNETOMETER

Determination of Magnetic Moment of a Magnet

The given magnet is put into vibration magnetometer and its time period T is determined.

Now $T = 2\pi\sqrt{\dfrac{I}{MB_H}}$

$$\Rightarrow \quad M = \frac{4\pi^2 I}{B_H T^2}$$

The moment of inertia I can be determined by the geometry of magnet.

Comparison of Horizontal Components of Earth's Magnetic Field at Two Places

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

Since, I and M of the magnet are constant, so

$$T^2 \propto \frac{1}{B_H}$$

$$\Rightarrow \quad \frac{B_{H_1}}{B_{H_2}} = \frac{T_2^2}{T_1^2}$$

Comparison of Magnetic Moments of Two Magnets of Same Size and Same Mass

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

where I and B_H are constants

$$\Rightarrow \quad M \propto \frac{1}{T^2}$$

$$\Rightarrow \quad \frac{M_1}{M_2} = \frac{T_2^2}{T_1^2}$$

If two magnets have same magnetic length then

$$2ml \propto \frac{1}{T^2}$$

$$\Rightarrow \quad m \propto \frac{1}{T^2} \qquad \{\because M = m(2l)\}$$

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{T_2^2}{T_1^2}$$

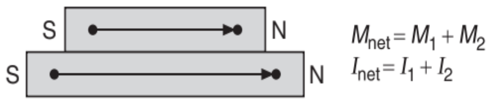
Comparison of Magnetic Moments of Two Magnets of Unequal Sizes and Masses

(a) When magnetic moments of two magnets align. Then

$$I = I_1 + I_2$$

$$M = M_1 + M_2$$

$$\Rightarrow \quad T_1 = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 + M_2)B_H}}$$

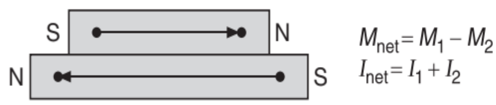


(b) When magnetic moments of two magnets do not align. Then

$$I = I_1 + I_2$$

$$M = M_1 - M_2$$

$$\Rightarrow \quad T_2 = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 - M_2)H}}$$



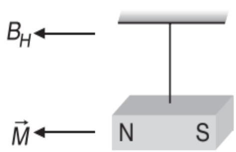
$$\Rightarrow \quad \frac{T_1}{T_2} = \sqrt{\frac{M_1 - M_2}{M_1 + M_2}}$$

$$\Rightarrow \quad \frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2}$$

So, by calculating the time periods in the two cases M_1 and M_2 can be compared.

To Find the Ratio of Magnetic Field with the Horizontal Component of Earth's Field

Suppose it is required to find the ratio $\dfrac{B}{B_H}$ where B is the field created by magnet and B_H is the horizontal component of earth's magnetic field.



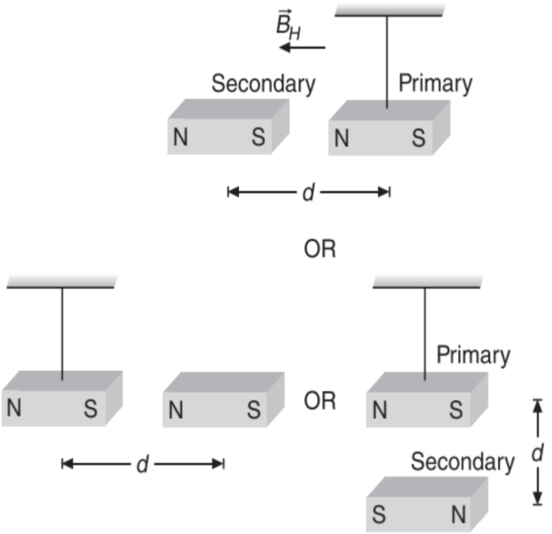
To determine $\dfrac{B}{B_H}$ a primary (main) magnet is made to first oscillate in earth's magnetic field (B_H) alone and it's time period of oscillation (T) is noted.

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

and frequency $\nu = \frac{1}{2\pi}\sqrt{\frac{MB_H}{I}}$

Now a secondary magnet placed near the primary magnet so primary magnet oscillate in a new field with is the resultant of B and B_H and now time period, is noted again.
There are two important possibilities for placing secondary magnet.

Possibility 1:
New field increases so time period of oscillation of primary magnet decreases.



Now time period $T = 2\pi\sqrt{\frac{I}{M(B+B_H)}}$

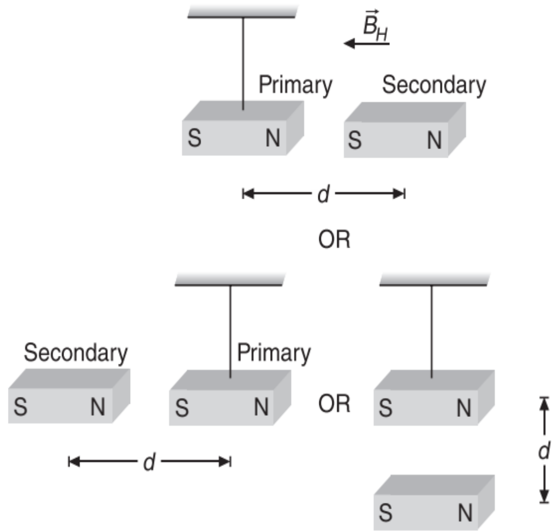
or new frequency $\nu = \frac{1}{2\pi}\sqrt{\frac{M(M+B_H)}{I}}$

Also $\frac{\nu'}{\nu} = \sqrt{\frac{B+B_H}{B_H}}$

$$\Rightarrow \left(\frac{\nu'}{\nu}\right)^2 = \frac{B}{B_1}+1$$

$$\Rightarrow \frac{B}{B_H} = \left(\frac{\nu'}{\nu}\right)^2 - 1$$

Possibility 2:
Net field decreases so time period of oscillation of primary magnet increases.



$$T = 2\pi\sqrt{\frac{I}{M(B_H - B)}}\,(B_H > B)$$

and $\nu' = \frac{1}{2\pi}\sqrt{\frac{M(B_H - B)}{I}}$

Also $\frac{\nu'}{\nu} = \sqrt{\frac{B_H - B}{B_H}}$

$$\Rightarrow \left(\frac{\nu'}{\nu}\right)^2 = 1 - \left(\frac{B}{B_1}\right)$$

$$\Rightarrow \frac{B}{B_H} = 1 - \left(\frac{\nu'}{\nu}\right)^2$$

Conceptual Note(s)

If a rectangular bar magnet is cut in n equal parts then time period of each part will be $\frac{1}{\sqrt{n}}$ times that of complete magnet (i.e., $T = \frac{T}{\sqrt{n}}$) while for short magnet $T = \frac{T}{n}$. If nothing is said then bar magnet is treated as short magnet.

ILLUSTRATION 33

A freely suspended magnet oscillates with period T in earth's horizontal magnetic field. When a bar magnet is brought near it, the period decreases to $\frac{T}{2}$. Calculate the ratio of the field of the magnet B to the earth's magnetic field (B_H).

SOLUTION

For freely suspended magnet of magnetic moment M oscillating in earth's magnetic field

$$T = 2\pi\sqrt{\frac{I}{MH}} \quad \ldots(1)$$

Let bar magnet produce magnetic field F in the vicinity of the oscillations of magnet.
As time period decreases, the net magnetic field must increase, so that magnetic field is $(B_H = B)$

$$\Rightarrow \quad \frac{T}{2} = 2\pi\sqrt{\frac{I}{M(B_H + B)}} \quad \ldots(2)$$

Dividing (1) by (2)

$$2 = \sqrt{\frac{B_H + B}{H}}$$

$$\Rightarrow \quad 4 = 1 + \frac{B}{B_H}$$

$$\Rightarrow \quad \frac{B}{B_H} = 3$$

ILLUSTRATION 34

A dip needle vibrates in the vertical plane perpendicular to the magnetic meridian. The time period of vibration is found to be 2 s. The same needle is then allowed to vibrate in the horizontal plane and the time period is again found to be 2 s. Calculate the angle of dip.

SOLUTION

In vertical plane perpendicular to magnetic meridian, we have

$$T = 2\pi\sqrt{\frac{I}{MB_V}} \quad \ldots(1)$$

In horizontal plane, we have

$$T' = 2\pi\sqrt{\frac{I}{MB_H}} \quad \ldots(2)$$

Since $T' = T$, so from (1) and (2), we get

$$B_V = B_H$$

$$\Rightarrow \quad \tan\phi = \frac{B_V}{B_H}$$

$$\Rightarrow \quad \tan\phi = 1$$

$$\Rightarrow \quad \phi = 45^\circ$$

ILLUSTRATION 35

A small bar magnet having a magnetic moment of $9\times10^{-3}\ \text{Am}^2$ is suspended at its centre of gravity by a light torsion less string at a distance of 10^{-2} m vertically above a long, straight horizontal wire carrying a current of $1.0\ \text{A}$ from east to west. Find the frequency of oscillation of the magnet about its equilibrium position. The moment of inertia of the magnet is $6\times10^{-9}\ \text{kgm}^2$ and horizontal component of earth's magnetic field is $3\times10^{-5}\ \text{T}$.

SOLUTION

The magnetic moment of the bar magnet is

$$M = 9\times10^{-9}\ \text{Am}^2$$

The magnitude of the magnetic field at the location of the magnet due to current carrying wire is

$$B = \frac{\mu_0 i}{2\pi r} = \frac{(2\times10^{-7})(1.0)}{10^{-2}}$$

$$\Rightarrow \quad B = 2\times10^{-5}\ \text{Wbm}^{-2}, \text{ from } S \text{ to } N$$

The earth's horizontal magnetic field is,

$$B_H = 3\times10^{-5}\ \text{T} = 3\times10^{-5}\ \text{Wbm}^{-2}, \text{ from } S \text{ to } N$$

$$\Rightarrow \quad B_{\text{net}} = B + B_H = 5\times10^{-5}\ \text{Wbm}^{-2}$$

The frequency of oscillation will be,

$$v = \frac{1}{2\pi}\sqrt{\frac{M(B + B_H)}{I}}$$

where I is moment of inertia of the magnet

$$\Rightarrow \quad v = \frac{1}{2\pi}\sqrt{\frac{(9\times10^{-9})\times(5\times10^{-5})}{6\times10^{-9}}}$$

$$\Rightarrow \quad v = 1.38\times10^{-3}\ \text{Hz}$$

Test Your Concepts-III

Based on Tangent Law, Tangent Galvanometer and Vibration Magnetometer

(Solutions on page H.110)

1. A magnet performs 15 oscillations per minute in a horizontal plane, where angle of dip is 60° and earth's total field is 0.5 G. At another place, where total field is 0.6 G, the magnet performs 20 oscillations per minute. What is the angle of dip at this place?
2. For what deflection, error in measuring the current using a tangent galvanometer is minimum?
3. A bar magnet of length 5 cm, width 3 cm and height 2 cm takes 5 s to complete an oscillation in vibration magnetometer placed in a horizontal magnetic field of 20 μT. The mass of this bar magnet is 250 g.
   (a) Find the magnetic moment of the magnet.
   (b) If the magnet is put in the magnetometer with its 0.5 cm edge horizontal, what would be the new time period?
4. A magnet is suspended in such a way that it oscillates in the horizontal plane. If it makes 20 oscillations per minute at a place where dip angle is 30° and 15 oscillations per minute at a place where dip angle is 60°. Calculate the ratio of total earth's magnetic field at the two places.
5. Two tangent galvanometers having coils of the same radius are connected in series. A current flowing in them produced deflections of 60° and 45° respectively. Calculate the ratio of the number of turns in the coils.
6. In a tangent galvanometer, when a current of 10 mA is passed, the deflection is 31°. By what percentage, the current has to be increased, so as to produce a deflection of 42°?
7. A compass needle placed at a distance r from a short magnet in $\tan A$ position shows a deflection of 60°. If the distance is increased to $r(3)^{\frac{1}{3}}$, then calculate the deflection of the compass needle.
8. The time period of vibration of two magnets in sum position (magnets placed with similar poles on one side one above the other) is 3 s. When polarity of weaker magnet is reversed the combination makes 12 oscillations per minute. What is the ratio of magnetic moments of two magnets?
9. The ratio of magnetic moments of two bar magnet is 13 : 5. These magnets are held together in a vibration magnetometer are allowed to oscillate in earth's magnetic field with like poles together 15 scillations per minutes are made. Calculate the frequency of oscillations of system if unlike poles are together.

MAGNETIC PROPERTIES OF MATERIALS

Magnetic Induction (B)

Magnetic induction inside a magnetic substance is the number of magnetic lines of force crossing a unit area normal to their direction. It is also called as magnetic flux density (MFD). Its SI unit is tesla or weber/m^2 and cgs unit is gauss.

$$1\ \text{T} = 1\ \text{Wbm}^{-2} = 10^4\ \text{G}$$

INTENSITY OF MAGNETISATION ($\vec{I}$)

When a material is placed in a magnetising field, it acquires magnetic moment M. The intensity of magnetisation is defined as the magnetic moment per unit volume (V).

$$\Rightarrow \quad I = \frac{M}{V}$$

If the material is in the form of a bar magnet of cross-sectional area A, length $2l$ and pole strength m, then $M = m(2l)$ and $V = A(2l)$

$$\Rightarrow \quad I = \frac{M}{V} = \frac{m(2l)}{A(2l)} = \frac{m}{A}$$

So, the intensity of magnetisation may also be defined as the pole strength per unit cross-sectional area.
Its SI unit is amperemetre^{-1} $(= \text{Am}^{-1})$.

It is a vector quantity. The magnetic field due to the magnetisation of the material (B_m) is proportional to the intensity of magnetisation (I) i.e. $B_m \propto I$

$$\Rightarrow \quad B_m = \mu_0 I$$

MAGNETISING FIELD ($\vec{H}$)

It is the degree or extent to which a magnetic field can magnetise a substance. In order to magnetise a material, it has to be kept in an external field B. So, it is defined as

$$H = \frac{B}{\mu_0} - I$$

In vacuum or in the absence of any material, $I = 0$, so we have

$$H = \frac{B}{\mu_0} \text{ (in vacuum)}$$

It's SI unit is amperemetre^{-1} $(= \text{Am}^{-1})$

It's CGS unit is oersted and 1 oersted = 80 Am^{-1}

For a solenoid having n turns per unit length, carrying a current i, we have $B = \mu_0 ni$

$$\Rightarrow \quad H = \frac{B}{\mu_0} = ni$$

So, the magnetising field is independent of the material of the core of the solenoid.
It is a vector quantity.

Conceptual Note(s)

(a) The Unit of intensity of magnetisation is ampere/metre (Am^{-1}) and its dimensional formula is $M^0L^{-1}T^0A^1$.

(b) $\vec{I}$ is a vector quantity whose direction is along the magnetic field.

(c) In paramagnetic and ferromagnetic materials its direction is in the direction of H and in diamagnetic materials it is opposite to that of H.

(d) I is produced in materials due to spin motion of electrons.

(e) The value of I and its direction in a material depend on the nature of that material.

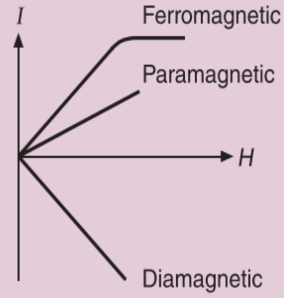
(f) The value of $\vec{I}$ depends on temperature.

(g) $\vec{I}$ is produced on account of induction in a material. For low magnetising field $I \propto H$.

$$\Rightarrow \quad I \propto H$$

$$\Rightarrow \quad I = \chi_m H$$

(i) I-H curve



MAGNETIC SUSCEPTIBILITY (χ_m)

It is the property of substance which tells how easily a substance can be magnetized.

The magnetic susceptibility is defined as the intensity of magnetisation per unit magnetising field or it is defined as the ratio of I to H, so mathematically,

$$\chi_m = \frac{I}{H}$$

When $H = 1$ oersted then $\chi_m = I$.

So, the intensity of magnetisation induced in a material by unit magnetising field is defined as magnetic susceptibility.
It is a scalar quantity and has no dimensional formula.

Problem Solving Technique(s)

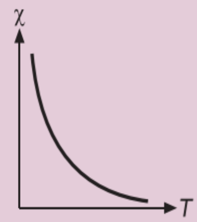
(a) χ_m has no unit and is dimensionless.

(b) χ_m is a measure of ease with which a material can be magnetised by a magnetising field (H).

(c) Magnetic susceptibility of various materials

- **(i)** For diamagnetic materials-χ_m is low and negative.
- **(ii)** For paramagnetic materials-χ_m is low but positive.
- **(iii)** For ferromagnetic materials-χ_m is high and positive.

(d) For paramagnetic substances it is inversely proportional to temperature i.e. $\chi_m \propto \frac{1}{T}$



(e) For low magnetising field the value of χ_m is constant.

MAGNETIC PERMEABILITY

The magnetic permeability of a material is the measure of degree to which the material can be permeated by a magnetic field and is defined as the ratio of magnetic induction (B) in the material to the magnetising field (H).

$$\Rightarrow \quad \mu = \frac{B}{H}$$

In other words, the extent to which magnetic lines of force can enter a medium is known as **magnetic permeability** of that medium. It is the characteristic property of a magnetic material because it represents the amplification of magnetising field in that material. It is always positive and is different for different materials.

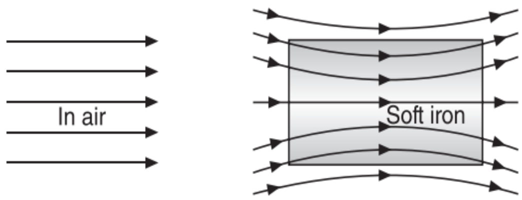
RELATIVE PERMEABILITY (μ_r)

The ratio of magnetic permeability of medium (μ) to the magnetic permeability of free space (μ_0) is defined as **relative permeability** (μ_r). So,

$$\mu_r = \frac{\mu}{\mu_0} = \frac{B}{B_0} = \frac{\text{Magnetic flux density in material}}{\text{Magnetic flux density in vacuum}}$$

$\mu = \mu_0\mu_r$, where μ_0 is the absolute permeability of air or free space is $4\pi \times 10^{-7}\ \text{TmA}^{-1}$

Permeability is the characteristic of a medium which allows magnetic flux to pass through the medium e.g. permeability of soft iron is 1000 times greater than that of air.



So, μ_r for soft iron is 1000

Conceptual Note(s)

(a) $\mu_r = \dfrac{\left(\text{Number of magnetic lines of force passing through unit area in medium}\right)}{\left(\text{Number of magnetic lines of force passing through unit area in vacuum}\right)}$

(b) $\mu_r = \dfrac{\text{Magnetic flux density in material}}{\text{Magnetic flux density in vacuum}}$

(c) The limit upto which a magnetic field penetrates matter, is known as relative permeability of that material.

(d) It has no units and is dimensionless.

(e) For Diamagnetic substances, $\mu_r < 1$

For Paramagnetic substances, $\mu_r > 1$

For Ferromagnetic substances, $\mu_r \gg 1$

RELATION BETWEEN MAGNETIC SUSCEPTIBILITY (χ_m) AND RELATIVE PERMEABILITY (μ_r)

When a magnetic material is placed in a magnetic field B_0, then the material gets magnetised and produces a magnetic field B_m of its own inside it. Due to this the total magnetic field in the material is

$$B = B_0 + B_m$$

where

$$B = \mu H,\ B_0 = \mu_0 H \text{ and } B_m = \mu_0 I$$

$$\Rightarrow \quad B = \mu_0 H + \mu_0 I$$

$$\Rightarrow \quad B = \mu_0 H\left(1 + \frac{I}{H}\right) = \mu_0 H(1 + \chi_m)$$

$$\Rightarrow \quad \frac{B}{H} = \mu_0(1 + \chi_m)$$

Since $\dfrac{B}{H} = \mu$

$$\Rightarrow \quad \mu = \mu_0(1 + \chi_m) \qquad \ldots(1)$$

Also, we know that $\dfrac{\mu}{\mu_0} = \mu_r$

So, from equation (1), the relative magnetic permeability is given by

$$\mu_r = \frac{\mu}{\mu_0} = 1 + \chi_m$$

Conceptual Note(s)

Comparison Table of Relative permeability and susceptibility of various substances

MATERIAL	μ_r	χ_m
Diamagnetic	slightly less than unity	constant, small and negative
Paramagnetic	slightly more than unity	constant, small and positive
Ferromagnetic	much greater than unity ($\sim 10^3$)	variable, large and positive

ILLUSTRATION 36

The magnetising field of 20 CGS units produces a flux of 2400 CGS units in a bar of iron of cross-section 0.2 cm^2. Calculate the

(a) permeability and
(b) susceptibility of the bar.

SOLUTION

Since Magnetic field,

$$B = \frac{\phi}{A} = \frac{2400\times10^{-8}}{0.2\times10^{-4}} = 1.20\text{ Wbm}^{-2}$$

(a) The permeability of the bar material is

$$\mu = \frac{B}{H} = \frac{1.2}{\frac{20}{4\pi\times10^{-3}}} = 7.54\times10^{-4}\text{ Hm}^{-1}$$

(b) The magnetic susceptibility and permeability of a material are related with each other as,

$$\mu = \mu_0(1+\chi_m)$$

$$\Rightarrow \chi_m = \frac{\mu}{\mu_0} - 1 = \frac{7.54\times10^{-4}}{4\pi\times10^{-7}} - 1 = 599$$

MAGNETIC SHIELDING PROCESS

The process of protecting any apparatus from the effect of earth's magnetic field is known as **magnetic shielding**.

When an iron box is placed in a magnetic field then the value of magnetic field inside the box is zero, whereas if a magnet is enclosed in an iron box then magnetic field outside the box is zero. So, to protect any instrument from external magnetic field, it is put inside an iron box. This is why, passengers travelling by aeroplane are allowed to carry magnets inside iron box only.

MAGNETOMOTIVE FORCE (F_M)

The line integral of magnetising field around a closed path in a magnetic field is defined as magnetomotive force. So

$$F_m = \oint \vec{H}\cdot d\vec{l} = \frac{1}{\mu}\oint \vec{B}\cdot d\vec{l} = \Sigma I$$

Its unit is ampere turns. Its value around the closed path is equal to the algebraic sum of electric currents enclosed by the closed path.

ILLUSTRATION 37

An iron rod of volume 10^{-4} m^3 and relative permeability 1000 is placed inside a long solenoid wound with 5 turns/cm. If a current of 0.5 A is passed through the solenoid, then find the magnetic moment of the rod.

SOLUTION

Since, we have, $\mu_r = \dfrac{\mu}{\mu_0} = 1 + \chi_m = 1 + \dfrac{I}{H}$

$$\Rightarrow I = (\mu_r - 1)H$$

For a solenoid having n turns per unit length and current i, we have

$$H = ni$$

$$\Rightarrow I = (\mu_r - 1)ni$$

$$\Rightarrow I = (1000-1)\times500\times0.5$$

$$\Rightarrow I = 2.5\times10^5\text{ Am}^{-1}$$

Since, magnetic moment is $M = IV$

$$\Rightarrow M = 2.5\times10^5\times10^{-4}$$

$$\Rightarrow M = 25\text{ Am}^2$$

ILLUSTRATION 38

The space within a current carrying solenoid is filled with magnesium having magnetic susceptibility $\chi = 1.2\times10^{-5}$. Calculate the percentage increase in magnetic field.

SOLUTION

Magnetic field without magnesium is

$$B_0 = \mu_0 H$$

Magnetic field with magnesium is

$$B = \mu H = \mu_0 (1+\chi) H$$

Fractional increase in magnetic field is given by

$$\frac{\Delta B}{B_0} = \frac{B - B_0}{B_0}$$

$$\Rightarrow \quad \frac{\Delta B}{B_0} = \frac{\mu_0 (1+\chi) H - \mu_0 H}{\mu_0 H} = \chi$$

Percentage increase is

$$\frac{\Delta B}{B_0} \times 100\% = \chi_{Mg} \times 100\% = 1.2\times10^{-5} \times 100$$

$$\Rightarrow \quad \frac{\Delta B}{B_0} \times 100\% = 1.2\times10^{-3}$$

ILLUSTRATION 39

A solenoid having 2000 turns/m has a core of a material with relative permeability 220. The area of core is 4 cm^2 and carries a current of 5 A. Calculate

(a) magnetic intensity
(b) magnetic field
(c) magnetisation (I) of the core. Also calculate the pole strength developed.

SOLUTION

Given that $n = 2000$ turns, $I = 5$ A, $\mu_r = 220$ and $A = 4\ \text{cm}^2$

(a) Magnetic intensity

$$H = nI = 2000 \times 5 = 10000\ \text{Am}^{-1}$$

(b) Magnetic field, $B = \mu H = \mu_0 \mu_r H$

$$\Rightarrow \quad B = 4\pi\times10^{-7} \times 220 \times 10{,}000 = 88\pi\times10^{-2}\ \text{T}$$

(c) As we know, $B = B_0 (H + I)$

$$\Rightarrow \quad 88\pi\times10^{-2} = 4\pi\times10^{-7}(10000 + I)$$

$$\Rightarrow \quad 2.20\times10^{6} = 10^{4} + I$$

$$\Rightarrow \quad I = \left(2.20\times10^{6} - 10^{4}\right) = 2.19\times10^{6}\ \text{Am}^{-1}$$

(d) Pole strength,

$$m = IA = \left(2.19\times10^{6}\right)\left(4\times10^{-4}\right) = 876\ \text{Am}$$

ILLUSTRATION 40

Consider a bar magnet having pole strength 2 Am, magnetic length 4 cm and area of cross-section 1 cm^2. Find

(a) the magnetisation I
(b) the magnetic intensity H and
(c) the magnetic field at the centre of magnet.

SOLUTION

Given that $m = 2$ Am, $2l = 4$ cm and $A = 1\ \text{cm}^2$

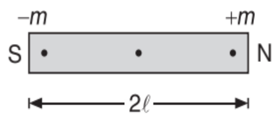
(a) Magnetisation

$$I = \frac{M}{V} = \frac{m(2l)}{A(2l)} = \frac{m}{A} = \frac{2}{1\times10^{-4}}$$

$$\Rightarrow \quad I = 2\times10^{4}\ \text{Am}^{-1}$$

The direction will be from S to N-pole

(b)



At centre, $H_N = \dfrac{\dfrac{\mu_0}{4\pi}\dfrac{m}{l^2}}{\mu_0} = \dfrac{m}{4\pi l^2}$, along north pole

$$H_S = \frac{m}{4\pi l^2}, \text{ along south pole}$$

$$H = H_N + H_S = \frac{m}{2\pi l^2} = \frac{2}{2\pi\left(2\times10^{-2}\right)^2}$$

$$\Rightarrow \quad H = \frac{1}{4\pi}\times10^{4}\ \text{Am}^{-1}, \text{ along south pole}$$

(c) Magnetic field at the centre of magnet, $B = \mu_0 (H + I)$

$$\Rightarrow \quad B = 4\pi\times10^{-7}\left(-\frac{1}{4\pi}\times10^{4} + 2\times10^{4}\right)$$

$$\Rightarrow \quad B = -10^{-3} + 8\pi\times10^{-3} = (8\pi - 1)\times10^{-3}$$

$$\Rightarrow \quad B = 2.4\times10^{-2}\ \text{T}, \text{ along north pole}$$

MAGNETISATION

The phenomenon of magnetising an unmagnetised substance by the process of magnetic induction is defined as **magnetisation** or it is the phenomenon of increasing the pole strength of a magnet.

DEMAGNETISATION

The phenomenon of decreasing or spoiling magnetic strength of a material is known as **demagnetisation**. It occurs due to mechanical jerks, thermal changes and temporal variations. When a magnet is beaten with a hammer then its magnetism gets spoiled. When a magnet is heated and then cooled then too its magnetism is spoiled. Keeping a magnet at a place for a long time also spoils its magnetism.

MAGNETIC SATURATION

The state of a material after which the increase in its magnetic strength stops is known as **magnetic saturation**.

CLASSIFICATION OF MAGNETIC MATERIALS

The root cause of magnetism in matter is the motion of electric charges. The motion of electrons and protons in atoms is responsible for their magnetic properties. The variation in the number of fundamental charged particles and variation in their arrangement in different materials are responsible for differences in their magnetic properties.

Curie and Faraday observed that almost all substances have certain magnetic properties. On the basis of mutual interactions or magnetic behaviour of various materials in an external magnetic field, the materials are divided in three main categories.

(a) Diamagnetic Substances
(b) Paramagnetic Substances
(c) Ferromagnetic Substances

MAGNETIC PROPERTIES OF MATERIALS

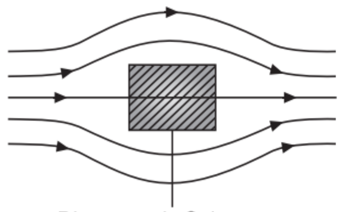
Diamagnetic Materials

Diamagnetic materials are repelled by magnetic field. In other words, as we know that a bar magnet attracts metals like iron, however, it is observed that a bar magnet will repel a diamagnetic material. Following are the properties of diamagnetic materials.

(a) Diamagnetic materials have a tendency to move from the stronger to weaker part of external magnetic field.

(b) They develop this tendency because they are feebly magnetized in a direction opposite to that of external magnetising field.

(c) Some of diamagnetic materials are copper, lead, bismuth, silicon, nitrogen (at STP), water and sodium chloride.



(d) The magnetic field lines are expelled by these materials.

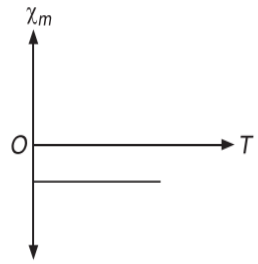
(e) Magnetic field inside diamagnetic material (B) is less than in free space (B_0), therefore for a diamagnetic material, we have

$$\frac{B}{B_0}<1,\ \frac{\mu}{\mu_0}<1,\ \mu_r<1.$$

(f) Relative permeability of diamagnetic material is less than one.

(g) Since $\mu_r=(1+\chi_m)$ and $\mu_r<1$, for diamagnetic material, so χ_m is negative and small for a diamagnetic material.

(h) Magnetic susceptibility χ_m of diamagnetic material is independent of temperature.



(i) Diamagnetism is a universal property i.e. it is present in all substances. However, the effect is so weak in most cases that it gets dominated by other effects like paramagnetism, ferromagnetism etc.

EXPLANATION OF DIAMAGNETISM

Diamagnetism is the intrinsic property of every material and it is generated due to mutual interaction between the applied magnetic field and orbital motion of electrons.

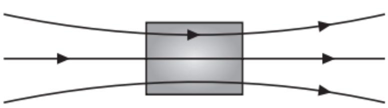
Electrons in an atom orbiting around nucleus possess orbital angular momentum. These orbiting electrons are equivalent to current carrying loop and thus possess orbital magnetic moment. Diamagnetic substances are those in which net magnetic dipole moment of an atom is zero. When magnetic field is applied, those electrons having orbital magnetic moment in the same direction slow down and those in opposite direction speed up. This is due to induced current in accordance with Lenz's law. Thus, the substance develops a net magnetic moment opposite to applied field due to which it is repelled from stronger field to weaker field.

Conceptual Note(s)

(a) In these materials the electron number is even and every two electrons get coupled.
(b) In these materials all the orbitals or atoms are completely filled.
(c) The resultant magnetic moment is zero for diamagnetic substances.
(d) Although diamagnetism is an inherent property of all materials, even then due to other properties like paramagnetism and ferromagnetism being much stronger, the property of diamagnetism is suppressed.
(e) Diamagnetism is the result of small variations in the velocity of electrons moving in atomic orbits.

Paramagnetic Materials

(a) These materials have tendency to move from region of weak magnetic field to strong magnetic field i.e. they get weakly attracted to a magnet.
(b) This is because these substances get feebly magnetized in the direction of applied external magnetic field.
(c) Examples of some paramagnetic substances are aluminum, sodium, calcium, oxygen (at STP) and copper chloride.
(d) Magnetic field lines tend to pass through these substances therefore magnetic field inside substance is more than the outside.

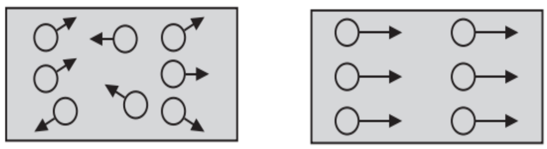


(e) $B > B_0$, $\frac{B}{B_0} > 1$, $\mu > \mu_0$, $\frac{\mu}{\mu_0} > 1$, $\mu_r > 1$
(f) The relative permeability of paramagnetic substances is greater than one.
(g) Since $\mu_r = 1 + \chi_m$, so χ_m is positive. (The magnetic susceptibility of paramagnetic substance is small and positive).

Explanation of Paramagnetism

In paramagnetic substances, vector sum of orbital magnetic moment of electrons is not zero, therefore, each atom behaves like tiny magnetic dipole and has some finite dipole moment. Due to thermal agitations the atomic dipoles in substance are randomly oriented hence total net dipole moment becomes zero. In paramagnetic substances, the inner orbits of atoms are incomplete.

Since the electron spins are uncoupled, consequently on applying a magnetic field the magnetic moment generated due to spin motion align in the direction of magnetic field and induces magnetic moment in its direction due to which the material gets feebly magnetised. In these materials the electron number is odd.



(a) When no field is applied (b) On application of field (B)

NOTE: The magnetic susceptibility of paramagnetic substances is around hundred times higher than that of diamagnetic substances.

CURIE LAW AND CURIE TEMPERATURE

This law states that the intensity of magnetisation of a paramagnetic material is inversely proportional to the absolute temperature T.

$$I \propto \frac{B_0}{T}$$

$$\Rightarrow \quad I = \frac{CB_0}{T}$$

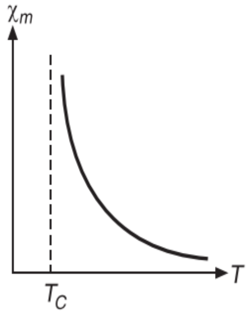
Dividing both sides of this equation by H, we get

$$\Rightarrow \quad \frac{I}{H} = \frac{C(B_0/H)}{T}$$

Since $\chi_m = \frac{I}{H}$ and $\frac{B_0}{H} = \mu_0$

$$\Rightarrow \quad \chi_m = \frac{C\mu_0}{T}$$

where C is called the Curie's constant.



Thus, for a paramagnetic substance both χ_m and μ_r depend not only on the material but also on the sample temperature. As the field is increased or the temperature is lowered, then the magnetisation increases until it reaches the saturation value (m_s) and at this instant all atomic dipoles are aligned with the applied field. Beyond this Curie's Law is not applicable.

Ferromagnetic Substances

(a) These are the substances which get strongly magnetized when placed in an external magnetic field.
(b) They have strong tendency to move from a region of weak magnetic field to strong magnetic field.
(c) They get strongly attracted to the magnet.
(d) Some of the ferromagnetic substances are as follows: iron, cobalt, nickel, alloys like alnico etc.
(e) Magnetic field lines tend to crowd into ferromagnetic material.
(f) Magnetic susceptibility χ_m of ferromagnetic substance is very high, therefore, they can be magnetized easily and strongly.
(g) With rise in temperature, susceptibility of ferromagnetic materials decreases. At a certain temperature ferromagnetic substance is converted into paramagnetic substance. This transition temperature is called Curie temperature of Curie point T_C.

ILLUSTRATION 41

A solenoid having 5000 turns/m carries a current of 2 A. An aluminium ring at temperature 300 K inside the solenoid provides the core.

(a) If the magnetisation I is $2\times10^{-2}\ \text{Am}^{-1}$, find the susceptibility of aluminium at 300 K.
(b) If temperature of the aluminium ring is 320 K, what will be the magnetisation?

SOLUTION

(a) $H = I = 5000\times2 = 10^4\ \text{Am}^{-1}$

Since, $I = \chi H$

$$\Rightarrow \chi = \frac{I}{H} = \frac{2\times10^{-2}}{10^4} = 2\times10^{-6}$$

(b) According to Curie Law, we have

$$\chi = \frac{C}{T} \Rightarrow \frac{\chi_2}{\chi_1} = \frac{T_2}{T_1}$$

$$\Rightarrow \chi_2 = \frac{T_2}{T_1}\chi_1 = \frac{320}{300}\times2\times10^{-6} = 2.13\times10^{-6}$$

Magnetisation at 320 K

$$I = \chi_2 H = 2.13\times10^{-6}\times10^4 = 2.13\times10^{-2}\ \text{Am}^{-1}$$

CURIE-WEISS LAW

At temperature above the Curie temperature, a ferromagnetic substance becomes an ordinary paramagnetic substance whose magnetic susceptibility obeys the **Curie-Weiss law.** At temperatures above Curie Temperature the magnetic susceptibility of ferromagnetic materials is inversely proportional to $(T - T_c)$ i.e.

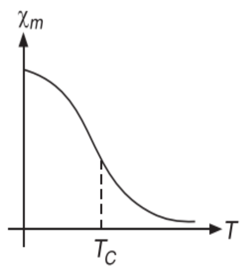
$$\chi \propto \frac{1}{T - T_C}$$

$$\Rightarrow \quad \chi = \frac{C}{(T - T_C)}$$

where T_c = Curie Temperature

according to which

$$\chi_m = \frac{C}{T - T_C}$$

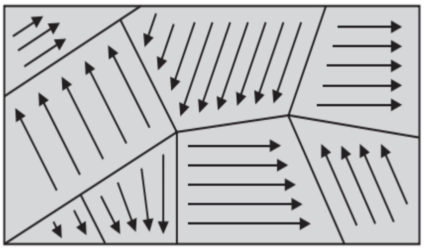


EXPLANATION OF FERROMAGNETISM

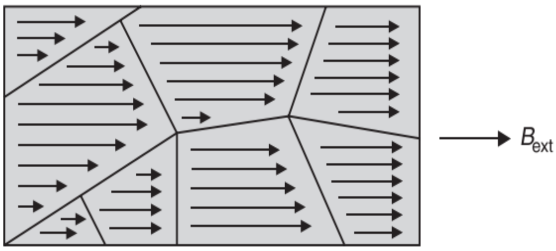
The individual atom in a ferromagnetic substance has net dipole moment. However, they interact with one another in such a way that they spontaneously align themselves in a common direction over a macroscopic volume called domain. Each domain has net magnetisation but magnetisation of the whole sample is zero (in absence of an external field).

At ordinary temperature, every piece of iron is not a magnet because at this temperature the distribution of domains is random as a result of which the resultant magnetic moment is zero.

So, every ferromagnetic material is made of a very large number of miniature regions called as **domains**. The linear dimension of domains ranges from 10^{-2} m to 10^{-5} m. Each domain contains 10^{17} to 10^{21} atoms whose axes are aligned in the same direction. All spin magnetic moments are in the same direction in a particular domain but it is different than that in any other domain.
At ordinary temperatures these domains do not align, rather these are scattered randomly as shown.



When external field B_{ext} is applied, the domains the domains orient themselves in the direction of B_{ext} and simultaneously the domains oriented in direction of B_{ext} grow in size as shown.



Curie Temperature (T_C)

The temperature above which a ferromagnetic material behaves like a paramagnetic material is defined as **Curie Temperature** (T_c).

OR

The minimum temperature at which a ferromagnetic substance is converted into paramagnetic substance is defined as **Curie Temperature**.
For various ferromagnetic materials its values are different. e.g. for Ni, $T_{C_{Ni}} = 358°C$

for Fe, $T_{C_{Fe}} = 770°C$

for CO, $T_{C_{CO}} = 1120°C$

At this temperature the ferromagnetism of the substances suddenly vanishes.

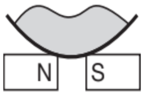
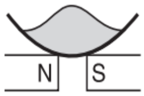
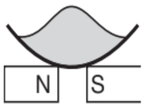
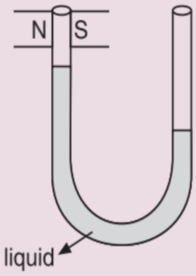
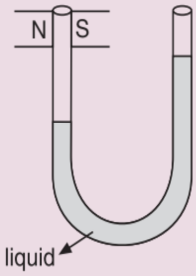
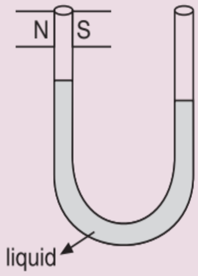
Materials and their Curie Temperature

Material	$T_C(K)$
Cobalt	1394
Iron	1043
Fe_2O_3	893
Nickel	631
Godolinium	317

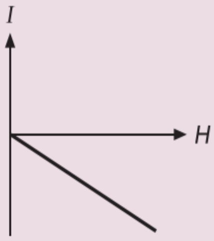
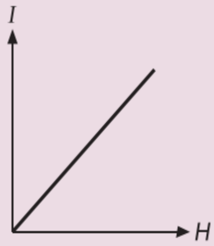
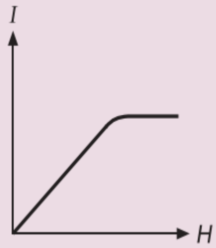
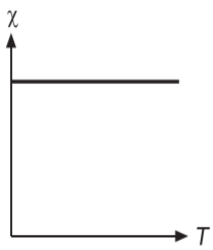
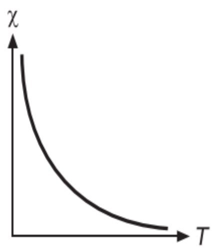
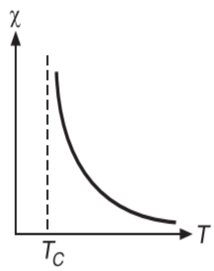
COMPARATIVE STUDY OF THESE MATERIALS

Property	Diamagnetic Substances	Paramagnetic Substances	Ferromagnetic Substances
Cause of magnetism	Orbital motion of electrons.	Spin motion of electrons.	Formation of domains.
Explanation of magnetism	On the basis of orbital motion of electrons.	On the basis of spin and orbital motion of electrons.	On the basis of domains formed.

(Continued)

Property	Diamagnetic Substances	Paramagnetic Substances	Ferromagnetic Substances
Behaviour In a non-uniform magnetic field	These are repelled in an external magnetic field i.e. have a tendency to move from high to low field region.	These are feebly attracted in an external magnetic field i.e. have a tendency to move from low to high field region	These are strongly attracted in an external magnetic field. i.e. they easily move from low to high field region.
State of magnetisation	These are weekly magnetised in a direction opposite to that of applied magnetic field	These get weekly magnetised in the direction of applied magnetic field	These get strongly magnetised in the direction of applied magnetic field
Liquid or powder in a watch glass when placed between the pole pieces (a) when poles are far apart (b) when poles are close to each other	(a) The liquid gets bulged at the middle  (a) (b) The liquid gets depressed at the middle  (b)	(a) The liquid gets depressed at the middle  (a) (b) The liquid gets bulged at the middle  (b)	(a) The liquid is very much depressed at the middle  (a) (b) The liquid gets very much bulged at the middle  (b)
When the material in the form of liquid is filled in the U-tube and placed between pole pieces.	Liquid level in that limb gets depressed 	Liquid level in that limb rises up 	Liquid level in that limb rises up very much 
On placing the gaseous materials between pole pieces	The gas expands at right angles to the magnetic field.	The gas expands in the direction of magnetic field	The gas rapidly expands in the direction of magnetic field
The value of magnetic induction B	$B < B_0$	$B > B_0$	$B >> B_0$
	where B_0 is the magnetic induction in vacuum		
Magnetic susceptibility χ	Low and negative $\lvert\chi\rvert \approx 1$	Low but positive $\chi \approx 1$	Positive and high $\chi \approx 10^2$
Dependence of χ on temperature	Does not depend on temperature (except *Bi* at low temperature)	Inversely proportional to temperature $\chi \propto \frac{I}{T}$ or $\chi = \frac{C}{T}$. This is called Curie Law, where C = Curie constant.	$\chi \propto \frac{1}{T - T_C}$ or $\chi = \frac{C}{T - T_C}$ This is called Curie-Weiss Law. T_C = Curie Temperature.

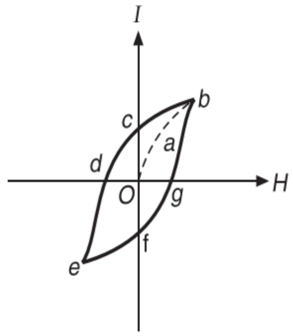
(*Continued*)

Property	Diamagnetic Substances	Paramagnetic Substances	Ferromagnetic Substances
Dependence of χ on H	Does not depend Independent	Does not depend Independent	Does not depend Independent
Relative Permeability (μ_r)	$\mu_r < 1$	$\mu_r > 1$	$\mu_r >> 1$ $\mu_r \approx 10^2$
Intensity of magnetisation (I)	I is in a direction opposite to that of H and its value is very low.	I is in the direction of H but value is low.	I is in the direction of H and value is very high.
I-H Curves			
Magnetic moment (M)	The value of M is very low (≈ 0 and is in a direction opposite to H.)	The value of M is very low and is in the direction of H	The value of M is very high and is in the direction of H.
Transition of materials (at Curie temperature)	These do not change.	On cooling, these get converted to ferro-magnetic materials at Curie temperature.	These get converted into paramagnetic materials above Curie temperature.
χ-T Curve			
The property of magnetism	Diamagnetism is found in those materials the atoms of which have even number electrons.	Paramagnetism is found in those materials the atoms of which have majority of electron spins in the same direction.	Ferro-magnetism is found in those materials which when placed in an external magnetic field are strongly magnetised.
Examples	*Cu, Ag, Au, Zn, Bi, Sb, NaCl,* H_2O air and diamond etc.	*Al, Mn, Pt, Na,* $CuCl_2$, O_2, and crown glass.	*Fe, Co, Ni, Cd,* Fe_3O_4 etc.
Nature of effect	Distortion effect	Orientation effect	Hysteresis effect

HYSTERESIS

When a bar of ferromagnetic material is magnetised by a varying magnetic field and the intensity of magnetisation I induced is measured for different values of magnetising field H, the graph of I versus H is shown in figure. From graph, it is observed that

(a) when magnetising field is increased from O, the intensity of magnetisation I increases and becomes maximum. This maximum value is called the **saturation value**.
The state of magnetic material in which the value of I becomes maximum and does not increase further on increasing the value of H is called the state of **Magnetic saturation**.



(b) when H is reduced, I reduces but is not zero when $H=0$. The remainder value Oc of magnetisation when $H=0$ is called the residual magnetism or retentivity.
The property by virtue of which the magnetism (I) remains in a material even on the removal of magnetising field is called **Retentivity** or **Residual Magnetism** or **Remnant Magnetism**.

(c) when magnetic field H is reversed, the magnetisation decreases and for a particular value of H, denoted by H_c, it becomes zero *i.e.*, $H_c = Od$ when $I = 0$. This value of H is called the **coercivity**.
So, the process of demagnetising a material completely by applying magnetising field in a negative direction is defined as **Coercivity**.
Coercivity assesses the magnetic softness or hardness of a magnetic material. If the ***coercivity*** of a magnetic material is ***low*** then it is ***magnetically soft*** and if the ***coercivity*** is ***high*** then material is ***magnetically hard***. So,
Magnetic hard substance (steel) $\rightarrow$ High coercivity
Magnetic soft substance (soft iron) $\rightarrow$ Low coercivity

(d) when field H is further increased in reverse direction, the intensity of magnetisation attains saturation value in reverse direction (i.e., point e).

(e) when H is decreased to zero and changed direction in steps, we get the part $efgb$.
Thus, complete cycle of magnetisation and demagnetisation is represented by $bcdefgb$.

In the complete cycle the intensity of magnetisation I is lagging behind the applied magnetising field. This is called **hysteresis** and the closed loop $bcdefgb$ is called **hysteresis cycle**.

The energy loss in magnetising and demagnetising a specimen is proportional to the area of hysteresis loop.

Problem Solving Technique(s)

$$\begin{pmatrix}\text{Hysteresis}\\\text{Energy}\\\text{Loss}\end{pmatrix} = \begin{pmatrix}\text{Area bound by}\\\text{the hysteresis}\\\text{loop}\end{pmatrix} = VAnt \text{ joule}$$

where,
V is the volume of Ferromagnetic sample,
A is the area of B-H
n is the frequency of alternating magnetic field and
t is the time for which the material is being magnetised

ILLUSTRATION 42

A ferromagnetic substance of volume 10^{-3} m^3 is placed in an alternating field of frequency 50 Hz. If the area of the hysteresis curve obtained is 0.1 m^2, then calculate the heat produced due to energy loss per second in the substance.

SOLUTION

Since we know that

$$(\text{Heat Loss}) = VAnt$$

where $V = 10^{-3}\text{ m}^3$, $A = 0.1\text{ m}^2$, $n = 50\text{ Hz}$ and $t = 1\text{ s}$

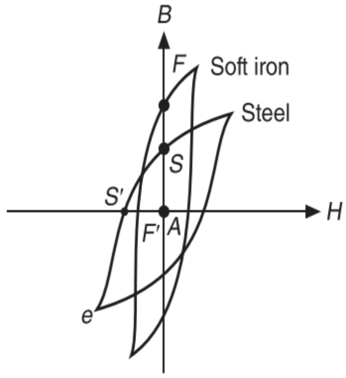
$$\Rightarrow \begin{pmatrix}\text{Heat}\\\text{Loss}\end{pmatrix} = (10^{-3})(0.1)(50)(1) = 5\times10^{-3}\text{ J} = 5\text{ mJ}$$

PROPERTIES OF SOFT IRON AND STEEL

For soft iron the susceptibility, permeability and retentivity are greater while coercivity and hysteresis loss per cycle are smaller than those of steel.

For permanent magnet, the substance must have high retentivity and high coercivity while for electromagnet the substance must have low retentivity and high permeability.

Permanent magnets are made of steel and cobalt while electromagnets are made of soft iron. Hysteresis curves for soft iron and steel are shown.



Comparison Chart

Soft Iron	Steel
The hysteresis loss for this is low.	The hysteresis loss for this is high.
The area of hysteresis curve for this is less.	The area of hysteresis curve for this is more.
For this the value of remnant magnetism is high (AF in the diagram).	For this value of remnant magnetism is low (AS in the diagram).
For this the coercivity is low (AF′ in the diagram).	For this the coercivity is high (AS′ in the diagram).
For this magnetic permeability (μ) is high.	For this magnetic permeability (μ) is low.
For this the magnetic susceptibility (χ) is high.	For this the magnetic susceptibility (χ) is low.
For this intensity of magnetisation (I) is high.	For this intensity of magnetisation (I) is low.

(Continued)

Soft Iron	Steel
For this the crystal arrangement is simple.	For this the crystal arrangement is complicated.
Its magnetisation and demagnetisation is easy.	Its magnetisation and demagnetisation is complicated and takes place with difficulty.

Selection of Materials

(a) **For permanent magnets: Permanent magnets should have high retentivity, so that the magnet is strong and high coercivity so that the magnetisation is not easily erased by stray magnetic fields, temperature fluctuations or minor mechanical damages.** An alloy Alnico (Al + Ni + Co) is used for permanent magnets. Also, it is observed that steel has slightly smaller retentivity than soft iron but this fact is dominated by the fact that steel has much smaller coercivity than soft iron and due to this reason steel is preferred over soft iron to make permanent magnets.

(b) **For electromagnets: Since an electromagnet must magnetise and demagnetise easily so it should have high permeability and low retentivity.** Soft iron is used for electromagnets. If a soft iron rod is placed in a solenoid and current is passed through the solenoid, then the magnetic field in the solenoid is increased 1000 times as $\mu_r = 1000$ for soft iron. As soon as the current is switched off, the magnetic field quickly becomes zero as the soft iron core has low retentivity.

(c) **For transformer, dynamo and tape recorder tapes:** Soft iron is used for these because its permeability is high, hysteresis loss is less and coercivity is low. For transformer core a special alloy permalloy is used. For soft materials μ, K and I are high and hysteresis loss or the area of I-H or B-H curve is low.

Hard and Soft Magnets

(a) The ferromagnetic material which retain magnetization for a long period of time are called hard magnetic material or hard ferromagnets. Some hard-magnetic materials are Alnico (an alloy of iron, aluminum, nickel, cobalt and copper) and naturally occurring lodestone. They are used for permanent magnets. For permanent magnet material should have high retentivity and high coercivity.

(b) The ferromagnetic material which retain magnetization as long as the external field persists are called soft magnetic materials or soft ferromagnets. Soft ferromagnets is soft iron. Such material is used for making electromagnets. For electromagnets material should have low retentivity and low coercivity. Electromagnets are used in electric bells, loudspeakers and telephone diaphragms.

Conceptual Note(s)

(a) Area under B-H loop is equal to the energy loss per cycle per unit volume, because unit of $B\times H$ is Jm^{-3}

(b) Area of I-H loop is $I\times H$

So, unit of Area of I-H loop is A^2m^{-2}

$$\because [IH]=\left(\frac{\text{A}}{\text{m}}\right)\left(\frac{\text{A}}{\text{m}}\right)=\text{A}^2\text{m}^{-2}$$

(c) Unit of area of B-H loop is μ_0 times the unit of area of I-H loop.

Test Your Concepts-IV

Based on Magnetic Properties of Materials

(Solutions on page H.112)

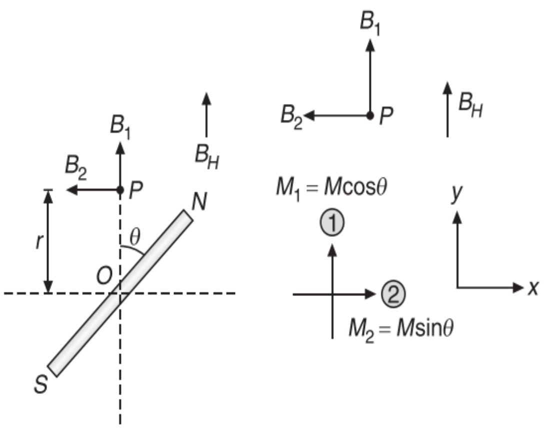
1. A bar magnet has coercivity $4\times10^3\ \text{Am}^{-1}$. It is desired to demagnetise it by inserting it inside a solenoid 12 cm long and having 60 turns. Calculate the current that should be sent through the solenoid.
2. The magnetic moment of a magnet $(15\ \text{cm}\times2\ \text{cm}\times1\ \text{cm})$ is $1.2\ \text{Am}^2$. Calculate its intensity of magnetisation.
3. The dipole moment of each molecule of a paramagnetic gas is $1.5\times10^{-23}\ \text{Am}^2$. The temperature of gas is 27°C and the number of molecules per unit volume in it is $2\times10^{26}\ \text{m}^{-3}$. Calculate the maximum possible intensity of magnetisation in the gas.
4. The coercivity of a certain permanent magnet is $4.0\times10^4\ \text{Am}^{-1}$. The magnet is placed inside a solenoid 20 cm long and having 700 turns and a current is passed in the solenoid to demagnetise it completely. Find the current.
5. An iron rod of $0.2\ \text{cm}^2$ cross-sectional area is subjected to a magnetizing field of $1200\ \text{Am}^{-1}$. The susceptibility of iron is 599. Calculate the magnetic flux produced in the rod.
6. The magnetic susceptibility of a paramagnetic material at -73 °C is 0.0075. Find its value at -173 °C.
7. The hysteresis loss for a specimen of iron weighing 15 kg is equivalent to $300\ \text{Jm}^{-3}\ \text{cycle}^{-1}$. Calculate the loss of energy per hour at $25\ \text{cycles}^{-1}$. Density of iron is $7500\ \text{kgm}^{-3}$.
8. A cylindrical rod magnet has a length of 5 cm and a diameter of 1 cm. It has a uniform magnetisation of $5.30\times10^3\ \text{Am}^{-3}$. Find the magnetic dipole moment of the rod.
9. Relative permeability of iron is 5500, calculate its magnetic susceptibility.

SOLVED PROBLEMS

PROBLEM 1

A short magnet $(M = 4\times10^{-2}\ \text{Am}^2)$ lying in a horizontal plane with its north-pole points 37° east of north. Find the net horizontal field at a point of the magnet 0.1 m away from it $(B_H = 11\ \mu\text{T})$ $\left(\sin 37° = \frac{3}{5},\ \cos 37° = \frac{4}{5}\right)$

SOLUTION



Due to magnet, magnetic field at P is

$$B_1 = \frac{\mu_0}{4\pi}\left(\frac{2M_1}{r^3}\right) = \frac{\mu_0}{4\pi}\left(\frac{2M\cos\theta}{r^3}\right)$$

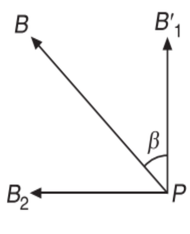
$$\Rightarrow \quad B_1 = \frac{10^{-7}\times2\times4\times10^{-2}\times\frac{4}{5}}{(0.1)^3} = 6.4\times10^{-6}\ \text{T}$$

and $B_2 = \frac{\mu_0}{4\pi}\left(\frac{M_2}{r^3}\right) = \frac{\mu_0}{4\pi}\left(\frac{M\sin\theta}{r^3}\right)$

$$\Rightarrow \quad B_2 = \frac{10^{-7}\times4\times10^{-2}\times\frac{3}{5}}{(0.1)^3} = 2.4\times10^{-6}\ \text{T}$$

Since, B_1 and B_H are in same direction, so we get

$$B_1' = B_1 + B_H = 6.4\times10^{-6} + 11\times10^{-6} = 17.4\times10^{-6}\ \text{T}$$



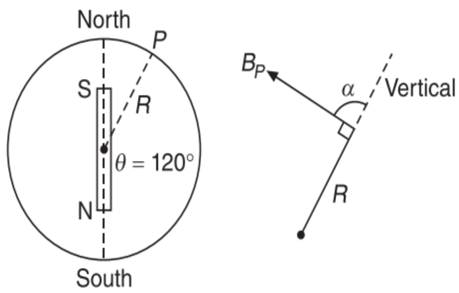
$$\Rightarrow \quad B_P = \sqrt{B_1'^2 + B_2^2} = \sqrt{(17.4)^2 + (2.4)^2}\times10^{-6}$$

$$\Rightarrow \quad B_P = \sqrt{308.52}\times10^{-6}\ \text{T}$$

$$\tan\beta = \frac{B_2}{B_1'} = \frac{2.4\times10^{-6}}{17.4\times10^{-6}} = 0.14°$$

PROBLEM 2

The earth's magnetic field at geomagnetic poles has a magnitude 8×10^{-5} T. Find the magnitude and the direction of the field at a point on the earth's surface where the radius makes an angle of 120° with the axis of the earth's assumed magnetic dipole. What is the inclination (dip) at this point?



SOLUTION

The geomagnetic poles are lying in end on position. The magnetic field at geomagnetic poles is

$$B' = \frac{\mu_0}{4\pi}\left(\frac{2M}{R^3}\right) = 8\times10^{-5}\ \text{T}$$

The magnetic field at point P is

$$B_P = \frac{\mu_0 M}{4\pi R^3}\sqrt{1+3\cos^2\theta}$$

$$\Rightarrow \quad B_P = \frac{B'}{2}\sqrt{1+3\cos^2\theta}$$

$$\Rightarrow \quad B_P = \frac{8\times10^{-5}}{2}\sqrt{1+3\cos^2 120°}$$

$$\Rightarrow \quad B_P = 4\times10^{-5}\sqrt{1+\frac{3}{4}} = 2\sqrt{7}\times10^{-5}\ \text{T}$$

Also, $\tan\alpha = \frac{1}{2}\tan\theta = \frac{1}{2}\tan 120°$

$$\Rightarrow \quad \tan\alpha = \frac{1}{2}\times\left(-\sqrt{3}\right) = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \alpha = \tan^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

Dip ϕ is an angle made by the earth's magnetic field with the horizontal plane, so we have

$$\phi = \alpha - 90^\circ = \tan^{-1}\left(-\frac{\sqrt{3}}{2}\right) - 90^\circ$$

PROBLEM 3

A bar magnet 30 cm long is placed in the magnetic meridian with its north pole pointing south. The neutral point is observed at a distance of 30 cm from its one end. Calculate the pole strength of the magnet. Given horizontal component of earth's field $= 0.34\text{ G}$.

SOLUTION

When magnet is placed with its north pole pointing south, then neutral point is obtained on its axial line.

$$\Rightarrow \quad B_{\text{axial}} = B_H$$

$$\Rightarrow \quad \frac{\mu_0}{4\pi}\times\frac{2Mr}{\left(r^2 - l^2\right)^2} = B_H$$

$$\Rightarrow \quad M = \frac{4\pi B_H\left(r^2 - l^2\right)^2}{2\mu_0 r}$$

Since $2l = 30\text{ cm}$

$\Rightarrow \quad l = 15\text{ cm} = 0.15\text{ m}$, $r = 30\text{ cm} = 0.30\text{ m}$

and $B_H = 0.34\text{ G} = 0.34\times10^{-4}\text{ T}$

$$\Rightarrow \quad M = \frac{1}{10^{-7}}\left[\frac{0.34\times10^{-4}\times\left(0.30^2 - 0.15^2\right)^2}{2\times0.30}\right]$$

$$\Rightarrow \quad M = \frac{0.34\times10^{-4}\times(0.0675)^2}{10^{-7}\times2\times0.30} = 2.582\text{ Am}^2$$

The pole strength of the magnet is

$$m = \frac{M}{2l} = \frac{2.582}{0.30} = 8.606\text{ Am}$$

PROBLEM 4

A short bar magnet is placed with its north pole pointing north. The neutral point is 10 cm away from the centre of the magnet. If $H = 0.4\text{ G}$. Calculate the magnetic moment of the magnet.

SOLUTION

When north pole of the magnet points towards magnetic north, null point is obtained on perpendicular bisector of the magnet. Simultaneously, magnetic field due to the bar magnet should be equal to the horizontal component of earth's magnetic field B_H. So, we have

$$B_H = \left(\frac{\mu_0}{4\pi}\right)\frac{M}{r^3}$$

$$\Rightarrow \quad M = \frac{B_H r^3}{\left(\frac{\mu_0}{4\pi}\right)}$$

Substituting the values, we get

$$M = \frac{\left(0.4\times10^{-4}\right)\left(10\times10^{-2}\right)^3}{10^{-7}} = 0.4\text{ Am}^2$$

PROBLEM 5

A magnetic needle performs 20 oscillations per minute in a horizontal plane. if the angle of dip be 30°, then how many oscillations per minute will this needle perform in vertical north-south plane and in vertical east-west plane?

SOLUTION

In horizontal plane, the magnetic needle oscillates in horizontal component of earths magnetic field. So,

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

In the vertical north-south plane (magnetic meridian), the needle oscillates in the total earth's magnetic field B_e and in vertical east-west plane (plane perpendicular to the magnetic meridian) it oscillates only in earth's vertical component B_V. If its time period be T_1 and T_2, then

$$T_1 = 2\pi\sqrt{\frac{I}{MB_e}} \text{ and } T_2 = 2\pi\sqrt{\frac{I}{MB_V}}$$

From above equations, we get

$$\frac{T_1^2}{T^2} = \frac{B_H}{B_e}$$

$$\Rightarrow \quad \frac{n_1^2}{n^2} = \frac{B_e}{B_H}$$

Similarly, $\frac{n_2^2}{n^2} = \frac{B_V}{B_H}$

Further, $\frac{B_e}{B_H} = \sec\phi = \sec(30°) = \frac{2}{\sqrt{3}}$

and $\frac{B_V}{B_H} = \tan\phi = \tan 30° = \frac{1}{\sqrt{3}}$

$$\Rightarrow \quad n_1^2 = n^2\left(\frac{B_e}{B_H}\right) = (20)^2\left(\frac{2}{\sqrt{3}}\right)$$

$$\Rightarrow \quad n_1 = 21.5 \text{ oscillations/min}$$

and $n_2^2 = n^2\left(\frac{B_V}{B_H}\right) = (20)^2\left(\frac{1}{\sqrt{3}}\right)$

$$\Rightarrow \quad n_2 = 15.2 \text{ oscillations/min}$$

PROBLEM 6

A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now, it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T'. Then, find the ratio T'/T.

SOLUTION

When magnet is divided into two equal parts the pole strength remains same, so, the magnetic dipole moment is

$$M' = (\text{Pole Strength})\frac{l}{2} = \frac{M}{2}$$

Also, the mass of magnet becomes half i.e., $m' = \frac{m}{2}$

Moment of inertia of magnet is

$$I = \frac{ml^2}{12}$$

New moment of inertia is

$$I' = \frac{1}{12}\left(\frac{m}{2}\right)\left(\frac{l}{2}\right)^2 = \frac{ml^2}{12\times 8}$$

$$\Rightarrow \quad I' = \frac{I}{8}$$

Since, $T = 2\pi\sqrt{\frac{I}{MB_H}}$

and $T' = 2\pi\sqrt{\frac{I'}{M'B_H}} = 2\pi\sqrt{\frac{I/8}{(M/2)B_H}}$

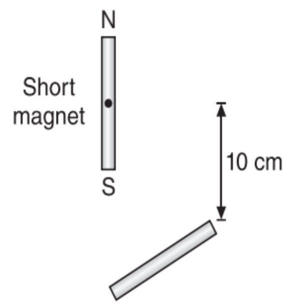
$$\Rightarrow \quad T' = \frac{T}{2}$$

$$\Rightarrow \quad \frac{T'}{T} = \frac{1}{2}$$

PROBLEM 7

The time period of the magnetic in an oscillation magnetometer in the earth magnetic field is 2 s. A short bar magnet is placed to the north of the magnetometer, at a separation 10 cm from the oscillating magnet, with its north pole pointing towards north. The time period becomes half. Calculate the magnetic moment of this short magnet.

$B_H = 12\ \mu\text{T}$.



SOLUTION

Since time period $T = 2\pi\sqrt{\frac{I}{MB}}$

$$\Rightarrow \quad T \propto \frac{1}{\sqrt{H}}$$

$$\Rightarrow \quad B \propto \frac{1}{T^2}$$

Let M is magnet moment due to short magnet and B' be the magnetic field due to short magnet, along south to north, then

$$\frac{(B+B')}{B} = \frac{T_1^2}{T_2^2}$$

where $T_1 = 2\text{ s}$, $B_H = 12\ \mu\text{T}$

$T_2 = 1\text{ s}$, $B_H = B + B' = 12 + B'$

$$\frac{12+B'}{12} = \left(\frac{2}{1}\right)^2 = 4$$

$\Rightarrow \quad B' = 36\ \mu\text{T}$

Since, $B' = \dfrac{\mu_0}{4\pi}\dfrac{2M}{r^3}$

$\Rightarrow \quad 36\times10^{-6} = 10^{-7}\times\dfrac{2M}{(0.10)^3}$

$\Rightarrow \quad M = 0.18\ \text{Am}^2$

PROBLEM 8

Each atom of an iron bar $(5\ \text{cm}\times1\ \text{cm}\times1\ \text{cm})$ has a magnetic moment $1.8\times10^{-23}\ \text{Am}^2$. Knowing that the density of iron is $7.78\times10^3\ \text{kg}^{-3}\text{m}$, atomic weight is 56 and Avogadro's number is 6.02×10^{23}, calculate the magnetic moment of bar in the state of magnetic saturation.

SOLUTION

The number of atoms per unit volume in a specimen is

$$n = \frac{N_A}{V} = \frac{N_A}{\left(\dfrac{m_{Fe}}{\rho_{Fe}}\right)} = \frac{N_A\rho_{Fe}}{m_{Fe}}$$

For iron, $\rho = 7.8\times10^{-3}\ \text{kgm}^{-3}$

$$N_A = 6.02\times10^{23},\ m_{Fe} = 56\ \text{g} = \frac{56}{1000}\ \text{kg}$$

$$\Rightarrow \quad n = \frac{7.8\times10^3\times6.02\times10^{23}}{(56/1000)} = 8.38\times10^{28}\ \text{m}^{-3}$$

The volume of the iron bar is

$$V = (5\times10^{-2})(1\times10^{-2})(1\times10^{-2}) = 5\times10^{-6}\ \text{m}^3$$

Total number of atoms in the bar is

$$N_0 = nV = (8.38\times10^{28})(5\times10^{-6})$$

$$\Rightarrow \quad N_0 = 4.19\times10^{23}$$

The saturated magnetic moment of bar is

$$M_{\text{saturated}} = N_0 M_{\text{each atom}}$$

$$\Rightarrow \quad M_{\text{saturated}} = (4.19\times10^{23})(1.8\times10^{-23})$$

$$\Rightarrow \quad M_{\text{saturated}} = 7.54\ \text{Am}^2$$