CHAPTER 1: MAGNETIC EFFECTS OF CURRENT

iCON

Test Your Concepts-I (Based on Force and Fleming's Left Hand Rule)

1. Since $v = \frac{E}{B}$

$\Rightarrow \left[\frac{E}{B}\right] = [\text{Velocity}] = LT^{-1}$

2. Since $\vec{F} = q(\vec{v} \times \vec{B})$

By properties of cross product, we know that

$\vec{F}$ is perpendicular to $\vec{v}$ as well as $\vec{B}$. Hence

(a) $\vec{v} \cdot (\vec{v} \times \vec{B}) = \vec{v} \cdot \left(\frac{\vec{F}}{q}\right) = 0$

(b) $\vec{B} \cdot (\vec{v} \times \vec{B}) = \vec{B} \cdot \left(\frac{\vec{F}}{q}\right) = 0$

(c) $\vec{v} \times (\vec{v} \times \vec{B}) = (\vec{v} \cdot \vec{v})\vec{B} - (\vec{v} \cdot \vec{B})\vec{v}$

Since $\vec{v} \cdot \vec{B} = 0$

$\Rightarrow \quad \vec{v} \times (\vec{v} \times \vec{B}) = v^2 \vec{B}$

3. Since we have read, $\vec{F}_m \perp \vec{B}$ i.e., the acceleration $\vec{a} \perp \vec{B}$ or $\vec{a} \cdot \vec{B} = 0$

$\Rightarrow \left(x\hat{i} + 4\hat{j} + 2\hat{k}\right) \cdot \left(3\hat{i} + 2\hat{j} - 10\hat{k}\right) = 0$

$\Rightarrow 3x + 8 - 20 = 0$

$\Rightarrow x = 4$

4. $\vec{F} = q(\vec{v} \times \vec{B}) = q \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 \times 10^4 & 0 \\ 1.63 & 0.98 & 0 \end{vmatrix}$

$\Rightarrow \vec{F} = q\left[\hat{i}(0) - \hat{j}(0) + \hat{k}\left(0 - (3 \times 10^4)(1.63)\right)\right]$

$\Rightarrow \vec{F} = -(1.22 \times 10^{-8})(3 \times 10^4)(1.63)\hat{k}$

$\Rightarrow \vec{a} = \frac{\vec{F}}{m} = \frac{(1.22 \times 10^{-8})(3 \times 10^4)(1.63)}{1.81 \times 10^{-3} \text{ kg}}$

$\Rightarrow \vec{a} = -(0.33)\hat{k} \text{ ms}^{-2}$

5. (a) $\vec{F} = q(\vec{v} \times \vec{B}) = q \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ 0 & 0 & -1.25 \end{vmatrix}$

$\Rightarrow \quad \vec{F} = q\left[\hat{i}(-1.25v_y) - \hat{j}(-1.25v_x) + \hat{k}(0 - 0)\right]$

$\Rightarrow \quad \vec{F} = q\left[-1.25v_y\hat{i} + 1.25v_x\hat{j}\right] \quad \ldots(1)$

However

$\vec{F} = \left(-3.4 \times 10^{-7}\hat{i} + 7.4 \times 10^{-7}\hat{j}\right) \text{ N} \quad \ldots(2)$

From (1) and (2), by equating the components, we get

$\Rightarrow -3.4 \times 10^{-7} = -1.25qv_y$

$\Rightarrow v_y = \frac{-3.4 \times 10^{-7}}{-(1.25)(-5.6 \times 10^{-9})} = -48.6 \text{ ms}^{-1}$

Similarly, we get

$7.4 \times 10^{-7} = 1.25qv_x$

$\Rightarrow v_x = \frac{7.4 \times 10^{-7}}{(1.25)(-5.6 \times 10^{-9})} = -106 \text{ ms}^{-1}$

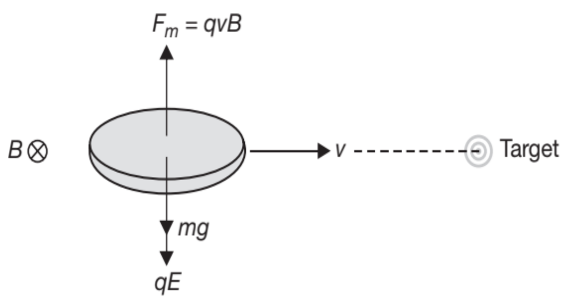
(b) Yes, $v_z = 0$

(c) Since, we know that $\vec{F}_m \perp \vec{v}$ as well as $\vec{B}$. So

$\vec{v} \cdot \vec{F} = 0$

$\Rightarrow$ Angle between $\vec{v}$ and $\vec{F}$ is $\frac{\pi}{2}$.

6. The direction of B must be perpendicular to the initial velocity of the coin as shown.



For equilibrium, we must have

$qvB = mg + qE$

$\Rightarrow B = \frac{mg + qE}{qv}$

$\Rightarrow B = \frac{(2500 \times 10^{-6})(27.5) + (5 \times 10^{-3})(9.8)}{(2500 \times 10^{-6})(12.8)}$

$\Rightarrow B = 3.7 \text{ T}$

7. Given that

$m = 2 \text{ g} = 2 \times 10^{-3} \text{ kg}$, $v = 5 \text{ ms}^{-1}$,

$B = 0.5 \text{ T}$, $q = 1 \text{ mC} = 10^{-3} \text{ C}$

Since the magnetic force is given by

$\vec{F} = q(\vec{v} \times \vec{B})$

$\Rightarrow \quad F = qvB\sin\theta$

$\Rightarrow \quad F = (10^{-3})(5)(0.5)\sin(90^\circ)$ N

$\Rightarrow \quad F = 2.5\times10^{-3}$ N

The acceleration a is given by

$$a = \frac{F}{m}$$

$$\Rightarrow \quad a = \frac{2.5\times10^{-3}}{2\times10^{-3}} \text{ ms}^{-2}$$

$\Rightarrow \quad a = 1.25 \text{ ms}^{-2}$

8. In the second part of the question, it is given that magnetic force is along x-axis when velocity is along z-axis. Hence magnetic field should be along negative y-direction. As in case of positive charge (here proton).

$$\vec{F}_m \uparrow\uparrow \vec{v}\times\vec{B}$$

So, let $\vec{B} = -B_0\hat{j}$ where B_0 = positive constant

Now applying $\vec{F}_m = q(\vec{v}\times\vec{B})$ we can find value of B_0 from the first part of the question. Substituting proper values in,

$$\vec{F}_m = q(\vec{v}\times\vec{B}) \quad \ldots(1)$$

Since $\vec{v} = (2\hat{i}+3\hat{j})\times10^6 \text{ ms}^{-1}$ and $\vec{B} = -B_0\hat{j}$

$\Rightarrow \quad \vec{v}\times\vec{B} = -(2B_0\times10^6)\hat{k}$

Also, it is given that $\vec{F}_m = -(1.28\times10^{-13}\hat{k})$ N and charge on the proton is $q = 1.6\times10^{-19}$ C, so on substituting these values in equation (1), we get

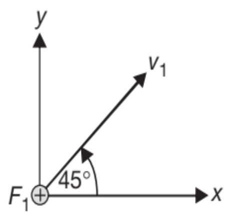
$$-(1.28\times10^{-13}\hat{k}) = (1.6\times10^{-19})(-2B_0\times10^6)\hat{k}$$

$\Rightarrow \quad 1.28 = 1.6\times2\times B_0$

$$\Rightarrow \quad B_0 = \frac{1.28}{3.2} = 0.4$$

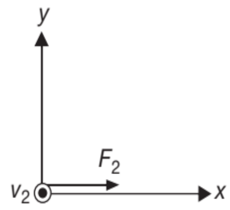
Therefore, the magnetic field is, $\vec{B} = (-0.4\hat{j})$ T

9. (a) The directions of $\vec{v}_1$ and $\vec{F}_1$ are shown in figure



Since, $\vec{F} = q\vec{v}\times\vec{B}$ so, $\vec{F}$ is perpendicular to $\vec{v}$ and $\vec{B}$. The information given here means that $\vec{B}$ can have no z-component.

The directions of $\vec{v}_2$ and $\vec{F}_2$ are shown in figure



$\vec{F}$ is perpendicular to $\vec{v}$ and $\vec{B}$, so $\vec{B}$ can have no x-component.

Both pieces of information taken together will give $\vec{B}$ is in the y-direction; $\vec{B} = B_y\hat{j}$

To calculate F_y, we have

$$\vec{F}_2 = F_2\hat{i},\ \vec{v}_2 = v_2\hat{k},\ \vec{B} = B_y\hat{j}$$

$$\vec{F}_2 = q\vec{v}_2\times\vec{B}$$

$\Rightarrow \quad F_2\hat{i} = qv_2B_y\hat{k}\times\hat{j} = qv_2B_y(-\hat{i})$ and

$$F_2 = -qv_2B_y$$

$$B_y = -\frac{F_2}{(qv_2)} = -\frac{F_2}{(qv_1)}$$

$\vec{B}$ has the magnitude $\dfrac{F_2}{(qv_1)}$ and is in the $-y$-direction

(b) $F_1 = qvB\sin\phi = \dfrac{qv_1|B_y|}{\sqrt{2}} = \dfrac{F_2}{\sqrt{2}}$

10. The acceleration of the particle is given by

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q}{m}(\vec{v}\times\vec{B})$$

Given that, $\dfrac{q}{m} = 0.2 \text{ Ckg}^{-1}$

$$\vec{v} = (2\hat{i}-3\hat{j}) \text{ ms}^{-1}$$

$$\vec{B} = (5\hat{i}+2\hat{j}) \text{ T}$$

$\Rightarrow \quad \vec{a} = 0.2(2\hat{i}-3\hat{j})\times(5\hat{i}+2\hat{j}) \text{ ms}^{-2}$

$\Rightarrow \quad \vec{a} = 0.2(4\hat{k}+15\hat{k}) \text{ ms}^{-2}$

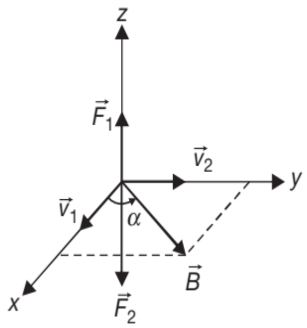
$\Rightarrow \quad \vec{a} = 3.8\hat{k} \text{ ms}^{-2}$

11. Applying Flemings Left Hand Rule to the first proton, we get $\vec{B}$ towards the $+y$-axis from $\vec{v}_1$, while when applied to the second proton requires $\vec{B}$ lie towards the $+x$-axis from $\vec{v}_2$. Thus $\vec{B}$ lies in the first quadrant of the xy-plane. The force on each proton is

$$F_1 = qv_1B\sin\alpha$$

$$F_2 = qv_2B\sin(90^\circ-\alpha) = qv_1B\cos\alpha$$

$$\Rightarrow \quad \frac{F_1}{F_2} = \frac{v_1}{v_2}\tan\alpha$$



$$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{2\times10^6\ \text{ms}^{-1}}{10^6\ \text{ms}^{-1}}\cdot\frac{1.2\times10^{-16}\ \text{N}}{4.16\times10^{-16}\ \text{N}}\right) = 30^\circ$$

The magnetic field strength is thus

$$B = \frac{F_1}{qv_1\sin\alpha} = \frac{1.2\times10^{-16}\ \text{N}}{(1.6\times10^{-19}\ \text{C})(10^6\ \text{ms}^{-1})\sin 30^\circ}$$

$$B = 1.5\times10^{-3}\ \text{T}$$

$$\Rightarrow \quad \vec{B} = (1.5\ \text{mT}, 30^\circ \text{ CCW from } +x\text{-axis})$$

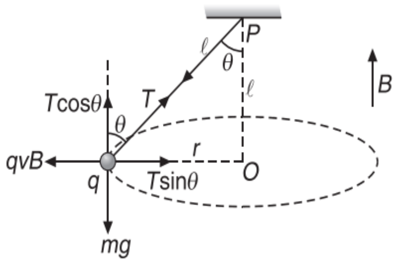
12. Do yourself

$$F_x = 0$$

$$F_y = 2.49\times10^{-3}\ \text{N}$$

$$F_z = 1.32\times10^{-3}\ \text{N}$$

13. The situation described in problem is shown in figure.



If T be the tension in the string then, we have

$$T\cos\theta = mg \qquad \ldots(1)$$

and $$T\sin\theta - qvB = \frac{mv^2}{r} \qquad \ldots(2)$$

From equation (1), we get

$$T = \frac{mg}{\cos\theta}$$

Also, if ω be the angular speed of the ball then

$v = r\omega$, where $\omega = \dfrac{2\pi}{T_0}$

So, from (2), we get

$$\left(\frac{mg}{\cos\theta}\right)\sin\theta - qB(r\omega) = mr\omega^2$$

From figure, we have

$$\sin\theta = \frac{r}{l} \text{ and } \cos\theta = \frac{\sqrt{l^2 - r^2}}{l}$$

$$\Rightarrow \quad \frac{mgr}{\sqrt{l^2 - r^2}} - qBr\omega = mr\omega^2$$

$$\Rightarrow \quad \frac{1}{\sqrt{l^2 - r^2}} = \frac{\omega^2}{g} + \frac{qB\omega}{mg}$$

$$\Rightarrow \quad \frac{1}{l^2 - r^2} = \left(\frac{\omega^2}{g} + \frac{qB\omega}{mg}\right)^2$$

$$\Rightarrow \quad \frac{1}{l^2 - r^2} = \left(\frac{4\pi^2}{gT_0^2} + \frac{2\pi qB}{mgT_0}\right)^2$$

$$\Rightarrow \quad l^2 - r^2 = \frac{1}{\left(\dfrac{4\pi^2}{gT_0^2} + \dfrac{2\pi qB}{mgT_0}\right)^2}$$

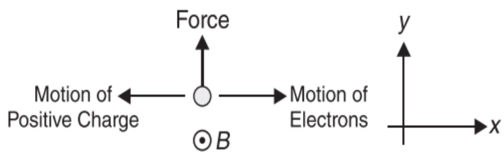
$$\Rightarrow \quad r^2 = l^2 - \frac{1}{\left(\dfrac{4\pi^2}{gT_0^2} + \dfrac{2\pi qB}{mgT_0}\right)^2}$$

$$\Rightarrow \quad r = \sqrt{l^2 - \frac{1}{\left(\dfrac{4\pi^2}{gT_0^2} + \dfrac{2\pi qB}{mgT_0}\right)^2}}$$

Test Your Concepts-II (Based on Charged Particle in a Magnetic Field)

1. Since $\vec{F} = q(\vec{v}\times\vec{B})$ and the beam consists of charged particles. So, the particles must experience some force. However, if the beam of particles enters parallel or anti parallel to the field, then either $\theta = 0^\circ$ or $\theta = 180^\circ$. Hence the charged particles will not experience any force.

2. According to Fleming's Left Hand Rule, we get the direction of $\vec{B}$ to be inwards along the $+z$ -axis.



3. Yes, No (Think Why?)

4. $r = \dfrac{mv}{qB}$ and $f = \dfrac{qB}{2\pi m}$

(a) For an electron, $r_e = \dfrac{m_e v}{eB}$

For a proton, $r_p = \dfrac{m_p v}{eB}$

Since $m_p > m_e$, so $r_p > r_e$ i.e., an electron describes a smaller circle

(b) $f_e = \dfrac{eB}{2\pi m_e}$ and $f_p = \dfrac{eB}{2\pi m_p}$

Since $m_p > m_e$, so $f_e > f_p$ i.e., frequency of electron will be more

5. (a) The kinetic energy K is given by

$$K = \frac{1}{2}mv^2 = 6 \text{ MeV}$$

$$\Rightarrow \quad K = (6\times10^6)(1.6\times10^{-19}) \text{ J}$$

$$\Rightarrow \quad K = 9.6\times10^{-13} \text{ J} = \frac{1}{2}mv^2$$

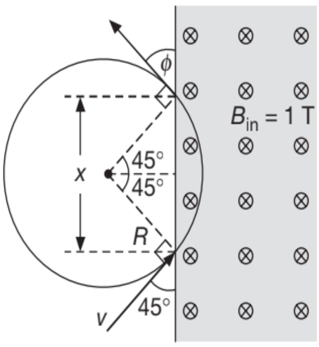
$$v = \sqrt{\frac{2(9.6\times10^{-13}\text{ J})}{1.67\times10^{-27}\text{ kg}}} = 3.39\times10^7 \text{ ms}^{-1}$$

Since, $F = qvB = \dfrac{mv^2}{R}$

$$\Rightarrow \quad R = \frac{mv}{qB}$$

$$\Rightarrow \quad R = \frac{(1.67\times10^{-27}\text{ kg})(3.39\times10^7\text{ ms}^{-1})}{(1.6\times10^{-19}\text{ C})(1\text{ T})}$$

$$\Rightarrow \quad R = 0.354 \text{ m}$$



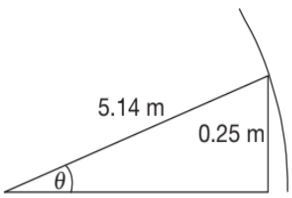
From the diagram, we get

$$x = 2R\sin(45°) = 2(0.354\text{ m})\sin(45°) = 0.501\text{ m}$$

(b) From the diagram, observe that $\phi = 45°$

6. (a) $R = \dfrac{mv}{qB} = \dfrac{(3.2\times10^{-11}\text{ kg})(1.45\times10^5\text{ ms}^{-1})}{(2.15\times10^{-6}\text{ C})(0.42\text{ T})}$

$$\Rightarrow \quad R = 5.14 \text{ m}$$



(b) The distance along the curve, d, is given by

$$d = R\theta = (5.14)\sin^{-1}\left(\frac{0.25}{5.14}\right) = 0.25\text{ m}$$

Since, $t_1 = \dfrac{d}{v} = \dfrac{0.25\text{ m}}{1.45\times10^5\text{ ms}^{-1}}$

$$\Rightarrow \quad t_1 = 1.72\times10^{-6}\text{ s}$$

(c) $\Delta x_1 = d\tan\left(\dfrac{\theta}{2}\right) = (0.25\text{ m})\tan\left(\dfrac{2.79°}{2}\right)$

$$\Rightarrow \quad \Delta x_1 = 6.08\times10^{-3}\text{ m}$$

(d) $\Delta x = \Delta x_1 + \Delta x_2$

$$\Rightarrow \quad \Delta x = 6.08\times10^{-3}\text{ m} + (0.5\text{ m})\tan(2.79°)$$

$$\Rightarrow \quad \Delta x = 0.0304\text{ m}$$

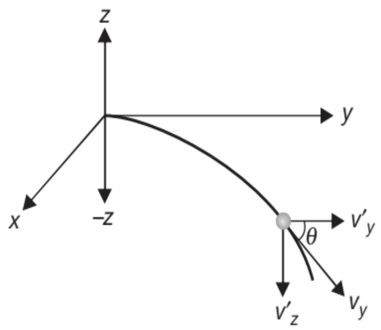
7. Since, $\vec{F} = q(\vec{v}\times\vec{B}) = q\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ v_x & v_y & 0\\ B & 0 & 0\end{vmatrix}$

$$\Rightarrow \quad \vec{F} = -(qBv_y)\hat{k}$$

The component v_x of velocity will remain unaltered, because $\vec{F}$ acting along z-direction.

Since the particle will follow a circular path in the $(y, -z)$ plane, therefore

$$\vec{v} = v_x'\hat{i} + v_y'\hat{j} - v_z'\hat{k}$$



where $v_x' = v_x$

$v_y' = v_y\cos\theta = v_y\cos(\omega t)$ and

$v_z' = v_y\sin\theta = v_y\sin(\omega t)$

where $\omega = \dfrac{eB}{m}$

$$\Rightarrow \quad \vec{v} = v_x\hat{i} + v_y\cos\left(\frac{eBt}{m}\right)\hat{j} - v_y\sin\left(\frac{eBt}{m}\right)\hat{k}$$

8. $\dfrac{1}{2}mv^2 = q(\Delta V)$

$$\Rightarrow \quad v = \sqrt{\frac{2q(\Delta V)}{m}}$$

Since, $r = \dfrac{mv}{qB}$

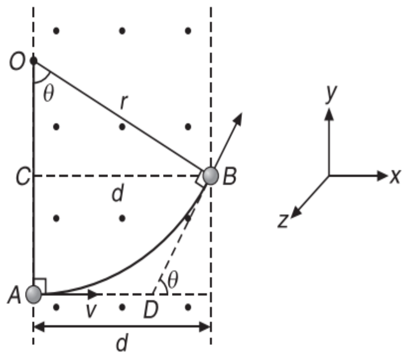
$$\Rightarrow \quad r = \frac{m\sqrt{\frac{2q(\Delta V)}{m}}}{qB}$$

$$\Rightarrow \quad r^2 = \frac{m}{q}\frac{2(\Delta V)}{B^2} \text{ and } (r')^2 = \frac{m'}{q'}\cdot\frac{2(\Delta V)}{B^2}$$

$$\Rightarrow \quad m = \frac{qB^2r^2}{2(\Delta V)} \text{ and } (m') = \frac{(q')B^2(r')^2}{2(\Delta V)}$$

$$\Rightarrow \quad \frac{m'}{m} = \left(\frac{q'}{q}\right)\left(\frac{r'}{r^2}\right)^2 = \left(\frac{2e}{e}\right)\left(\frac{2R}{R}\right)^2 = 8$$

9. The figure shows the particle entering the region at point A and leaving the region at point B.



We draw normal to the circular path followed by the particle, these normal meet at point O, centre of the circle. Applying geometry, we see that the deviation $(\angle ADB)$ of particle is same as the angular displacement of the particle about centre O i.e., θ.

$$\sin\theta = \frac{d}{r} = \frac{qBd}{mv}$$

So, the deviation is given by

$$\theta = \sin^{-1}\left(\frac{qBd}{mv}\right)$$

10. Since every electron passes through the same potential difference and the angle ϕ is small so, they all require the same time interval to travel the axial distance d. Now, all the electrons are all fired from the electron gun with the same speed v. Hence, we have

$$U_i = K_f$$

$$\Rightarrow \quad qV = \frac{1}{2}mv^2$$

$$\Rightarrow \quad (-e)(-\Delta V) = \frac{1}{2}m_e v^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2e\Delta V}{m_e}}$$

For ϕ small, $\cos\phi$ is nearly equal to 1. The time T of passage of each electron in the chamber is given by

$$d = vT$$

$$\Rightarrow \quad T = d\left(\frac{m_e}{2e\Delta V}\right)^{1/2}$$

Each electron moves in a different helix, around a different axis. If each completes just one revolution within the chamber, it will be in the right place to pass through the exit port. Its transverse velocity component $v_\perp = v\sin\phi$ swings around according to $F_\perp = ma_\perp$

$$qv_\perp B\sin 90° = \frac{mv_\perp^2}{r}$$

$$\Rightarrow \quad eB = \frac{m_e v_\perp}{r} = m_e\omega = m_e\frac{2\pi}{T}$$

$$\Rightarrow \quad T = \frac{m_e 2\pi}{eB} = d\left(\frac{m_e}{2e\Delta V}\right)^{1/2}$$

$$\Rightarrow \quad \frac{2\pi}{B}\left(\frac{m_e}{e}\right)^{1/2} = \frac{d}{(2\Delta V)^{1/2}}$$

$$\Rightarrow \quad B = \frac{2\pi}{d}\left(\frac{2m_e\Delta V}{e}\right)^{1/2}$$

11. $q(\Delta V) = \frac{1}{2}mv^2$

$$\Rightarrow \quad v = \sqrt{\frac{2q(\Delta V)}{m}}$$

Since $qvB = \frac{mv^2}{r}$

$$\Rightarrow \quad r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2q(\Delta V)}{m}} = \sqrt{\frac{2m(\Delta V)}{qB^2}}$$

$$\Rightarrow \quad r_p^2 = \frac{2m_p(\Delta V)}{eB^2}$$

$$\Rightarrow \quad r_d^2 = \frac{2m_d(\Delta V)}{q_d B^2} = \frac{2(2m_p)(\Delta V)}{eB^2}$$

$$\Rightarrow \quad r_d^2 = 2\left(\frac{2m_p(\Delta V)}{eB^2}\right) = 2r_p^2 \text{ and}$$

$$r_\alpha^2 = \frac{2m_\alpha(\Delta V)}{q_\alpha B^2} = \frac{2(4m_p)(\Delta V)}{(2e)B^2}$$

$$\Rightarrow \quad r_\alpha^2 = 2\left(\frac{2m_p(\Delta V)}{eB^2}\right) = 2r_p^2$$

$$\Rightarrow \quad r_\alpha = r_d = \sqrt{2}r_p$$

12. For each electron, $|q|vB\sin(90^\circ)=\dfrac{mv^2}{r}$

$$v=\frac{eBr}{m}$$

The electrons have no internal structure to absorb energy, so the collision must be perfectly elastic.

$$K=\frac{1}{2}mv_{1i}^2+0=\frac{1}{2}mv_{1f}^2+\frac{1}{2}mv_{2f}^2$$

$$\Rightarrow\quad K=\frac{1}{2}m\left(\frac{e^2B^2R_1^2}{m^2}\right)+\frac{1}{2}m\left(\frac{e^2B^2R_2^2}{m^2}\right)$$

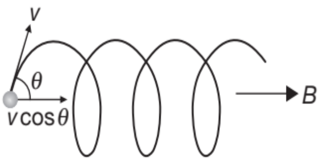
$$\Rightarrow\quad K=\frac{e^2B^2}{2m}\left(R_1^2+R_2^2\right)$$

$$\Rightarrow\quad K=\frac{(1.6\times10^{-19})^2(0.044)^2}{2(9\times10^{-31})}\left[(0.01)^2+(0.024)^2\right]$$

$$\Rightarrow\quad K=115\text{ keV}$$

13. $\dfrac{1}{2}mv^2=qV$

$$\Rightarrow\quad v=\sqrt{\frac{2qV}{m}}$$



This particle will follow a helical path of pitch p, where

$$p=\frac{2\pi mv\cos\theta}{qB}$$

If another identical particle is launched, at the same moment in the direction of B, then this particle will not experience any force due to B. Hence, if it is launched with a horizontal velocity $v\cos\theta$ then it will meet the first particle again and again during the motion. So,

$$v_2=v\cos\theta=\left(\sqrt{\frac{2qV}{m}}\right)\cos\theta$$

They will meet just at the completion of first revolution i.e.,

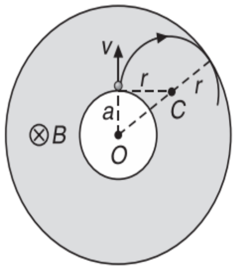
$$T_{MIN}=\frac{2\pi m}{qB}$$

Horizontal distance travelled by the second particle during this time is

$$x=v_2T_{MIN}=\left(\sqrt{\frac{2qV}{m}}\cos\theta\right)\left(\frac{2\pi m}{qB}\right)$$

$$\Rightarrow\quad x=\frac{2\pi\cos\theta}{B}\sqrt{\frac{2mV}{q}}$$

14. The electron follows a circular path with centre C such that it just escapes the outer shell tangentially and does not collide with it as shown in Figure.



The radius of the circular path followed by electron is given by

$$r=\frac{mv}{eB}\qquad\ldots(1)$$

Also, we observe that the length OC is given by

$$b-r=\sqrt{a^2+r^2}$$

$$\Rightarrow\quad b^2+r^2-2br=a^2+r^2$$

$$\Rightarrow\quad r=\frac{b^2-a^2}{2b}\qquad\ldots(2)$$

Equating (1) and (2), we get

$$\frac{mv}{eB}=\frac{b^2-a^2}{2b}$$

$$\Rightarrow\quad v=\frac{eB(b^2-a^2)}{2mb}$$

15. (a) $qvB=\dfrac{mv^2}{R}$

$$\Rightarrow\quad qRB=mv$$

But $L=mvR=qR^2B$

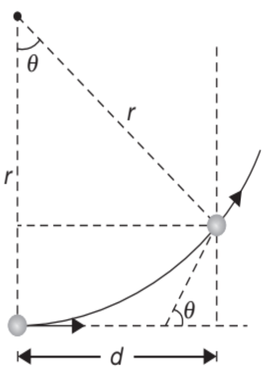
$$\Rightarrow\quad R=\sqrt{\frac{L}{qB}}=\sqrt{\frac{4\times10^{-25}\text{ J}\cdot\text{s}}{(1.6\times10^{-19}\text{ C})(1\times10^{-3}\text{ T})}}$$

$$R=0.05\text{ m}=5\text{ cm}$$

(b) Since, $v=\dfrac{L}{mR}=\dfrac{4\times10^{-25}\text{ J}\cdot\text{s}}{(9\times10^{-31}\text{ kg})(0.05\text{ m})}$

$$\Rightarrow\quad v=9\times10^{6}\text{ ms}^{-1}$$

16. $\dfrac{1}{2}mv^2=qV$



$\Rightarrow \quad v=\sqrt{\frac{2qV}{m}}$

$\sin\theta=\frac{d}{r}$, where $r=\frac{mv}{qB}=\frac{1}{B}\sqrt{\frac{2mV}{q}}$

Since $\theta=\omega t$, where $\omega=\frac{qB}{m}$

$\Rightarrow \quad \theta=\left(\frac{qB}{m}\right)t$

where $\theta=\sin^{-1}\left(\frac{d}{R}\right)=\sin^{-1}\left(Bd\sqrt{\frac{q}{2mV}}\right)$

$\Rightarrow \quad t=\left(\frac{m}{qB}\right)\theta$

$\Rightarrow \quad t=\left(\frac{m}{qB}\right)\sin^{-1}\left(Bd\sqrt{\frac{q}{2mV}}\right)$

17. The component of velocity along the field is

$$v_{\parallel}=2\times10^5\cos(60^\circ)=10^5\text{ ms}^{-1}$$

and the component of velocity perpendicular to the field is given by

$$v_{\perp}=2\times10^5\sin(60^\circ)=\sqrt{3}\times10^5\text{ ms}^{-1}$$

Proton will describe a circle in a plane perpendicular to the magnetic field. If r be the radius of the helix then

$$r=\frac{mv_{\perp}}{qB}=\frac{(1.67\times10^{-27})(\sqrt{3}\times10^5)}{(1.6\times10^{-19})(0.3)}$$

$\Rightarrow \quad r=6\times10^{-3}\text{ m}=0.6\text{ cm}$

Time taken to complete one revolution is

$$T=\frac{2\pi r}{v_{\perp}}=\frac{2\times3\cdot14\times0.006}{\sqrt{3}\times10^5}\text{ s}$$

Due to the component of velocity parallel to the magnetic field the protons will also move in the direction of magnetic field. The pitch (p) of the helical path is then given by

$$p=v_{\parallel}T=10^5\left(\frac{2\times3.14\times0.006}{\sqrt{3}\times10^5}\right)$$

$\Rightarrow \quad p=0.022\text{ m}=2.2\text{ cm}$

18. $v=\sqrt{\frac{2qV}{m}}=\sqrt{\frac{(2)(1.6\times10^{-19})(1000)}{9.1\times10^{-31}}}$

$\Rightarrow \quad v=18.8\times10^6\text{ ms}^{-1}$

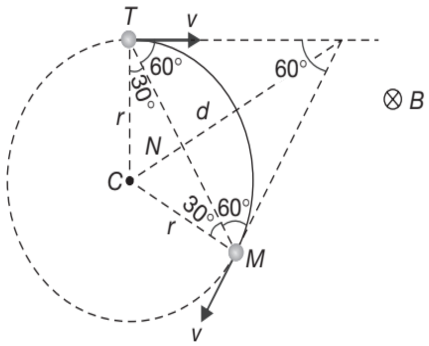
(a) In triangle CMT

$$r\cos30=\frac{d}{2}$$

$\Rightarrow \quad r\frac{\sqrt{3}}{2}=\frac{d}{2}$

$\Rightarrow \quad r=\frac{d}{\sqrt{3}}=\frac{5}{\sqrt{3}}\text{ cm}$

Since $r=\frac{mv}{qB}$



$\Rightarrow \quad \frac{0.05}{\sqrt{3}}=\frac{9.1\times10^{-31}\times18.7\times10^6}{1.6\times10^{-19}\text{ B}}$

$\Rightarrow \quad B=\frac{(9.1\times10^{-31})(18.8\times10^6)(\sqrt{3})}{(1.6\times10^{-19})(0.05)}$

$\Rightarrow \quad B=3.7\times10^{-3}\text{ T}=3.7\text{ mT}$

(b) In this case, B is parallel to TM at 60° with the velocity of electrons. So, we have the electrons following a helical path with origin at T and will hit M if it lies on the helix at $TM=n(\text{pitch})$

$\Rightarrow \quad TM=n\left(\frac{2\pi mv\cos\theta}{qB}\right)$

$\Rightarrow \quad B=\frac{n(2\pi mv\cos\theta)}{q(TM)}$

For B to be MINIMUM, $n=1$

$\Rightarrow \quad B_{MIN}=\frac{(2\pi)(mv\cos\theta)}{q(TM)}$

where $TM=\frac{5}{100}$ cm

$\Rightarrow \quad B_{MIN}=\frac{(2\pi)(9.1\times10^{-31})(18.8\times10^6)(0.5)}{(1.6\times10^{-19})(0.05)}$

$\Rightarrow \quad B_{MIN}=6.7\times10^{-3}\text{ T}=6.7\text{ mT}$

Test Your Concepts-III (Based on Charged Particle in Magnetic and Electric Field)

1. (a) The net force is the Lorentz Force given by

$$\vec{F}=q\vec{E}+q\vec{v}\times\vec{B}=q(\vec{E}+\vec{v}\times\vec{B})$$

$$\vec{F}=(3.2\times10^{-19})$$

CHAPTER 1

$$\left[\left(4\hat{i}-1\hat{j}-2\hat{k}\right)+\left(2\hat{i}+3\hat{j}-1\hat{k}\right)\times\left(2\hat{i}+4\hat{j}+1\hat{k}\right)\right]\text{ N}$$

Carrying out the indicated operations, we find

$$\vec{F}=\left(3.52\hat{i}-1.6\hat{j}\right)\times10^{-18}\text{ N}$$

(b) $\theta=\cos^{-1}\left(\frac{F_x}{F}\right)=\cos^{-1}\left(\frac{3.52}{\sqrt{(3.52)^2+(1.6)^2}}\right)=24.4^\circ$

2. $F_B=F_e$

$\Rightarrow\ qvB=qE$

where $v=\sqrt{\frac{2K}{m}}$ and K is kinetic energy of the electron

$$\Rightarrow\ E=vB=\sqrt{\frac{2K}{m}}B=\sqrt{\frac{2(750)(1.6\times10^{-19})}{9.11\times10^{-31}}}(0.015)$$

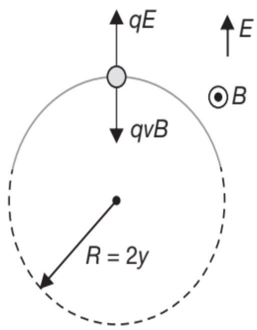
$\Rightarrow\ E=244\text{ kVm}^{-1}$

3. (a) The maximum speed occurs at the top of the cycloidal path and hence the radius of curvature is greatest there. Once the motion is beyond the top, the particle is being slowed by the electric field. As it returns to $y=0$, the speed decreases, leading to a smaller magnetic force, until the particle stops completely. Then the electric field again provides the acceleration in the y-direction of the particle, leading to the repeated motion.

(b) $W=Fd=qEd=qEy=\frac{1}{2}mv^2$

$\Rightarrow\ v=\sqrt{\frac{2qyE}{m}}$

(c) At the top, the net force on the particle $(qvB-qE)$ provides the necessary centripetal force to the particle to be in a circle of radius



$R=2y$

$$\Rightarrow\ qvB-qE=\frac{mv^2}{R}$$

$$\Rightarrow\ qvB-qE=\frac{m}{2y}\left(\sqrt{\frac{2qyE}{m}}\right)^2$$

$\Rightarrow\ qvB-qE=qE$

$\Rightarrow\ qvB=2qE$

$\Rightarrow\ v=\frac{2E}{B}$

4. Let q be the charge and m the mass of the particle. At $(x_0,\ 0,\ 0)$ speed of the particle is $\sqrt{4^2+3^2}=5$ units

Using Work-Energy Theorem, we get

$$(qE_0)x_0=\frac{1}{2}mv^2=\frac{25m}{2}$$

$$\Rightarrow\ x_0=\frac{25m}{2qE_0}$$

5. Since $\vec{E}$ and $\vec{B}$ are parallel to each other and $\vec{v}$ is perpendicular to both, so, the path of the particle is a helix of increasing pitch. The speed of particle at any time t is

$$v=\sqrt{v_x^2+v_y^2+v_z^2}\qquad\ldots(1)$$

Since the magnetic force acts along $-z$ axis, so the particle will have a circular track in yz plane, where

$$v_y^2+v_z^2=v_0^2$$

Due to the electric field along the x-axis, the x velocity of the particle keeps on increasing with time t as

$$v_x=\left(\frac{qE}{m}\right)t\qquad\left\{\because a=\frac{qE}{m}\right\}$$

According to the problem, we have

$$v=2v_0$$

Substituting the values in equation (1), we get

$$t=\sqrt{3}\left(\frac{mv_0}{qE}\right)$$

6. The net electric field

$$E=\vec{E}_1+\vec{E}_2$$

$$E=\frac{\sigma}{2\varepsilon_0}+\frac{\sigma}{2\varepsilon_0}=\frac{\sigma}{\varepsilon_0}$$

The net force acting on the electron is zero because it moves with constant velocity, due to its motion on straight line.

$\Rightarrow\ \vec{F}_{\text{net}}=\vec{F}_e+\vec{F}_m=0$

$\Rightarrow\ \left|\vec{F}_e\right|=\left|\vec{F}_m\right|$

$\Rightarrow\ eE=evB$

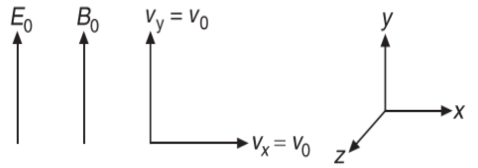
$\Rightarrow\ v=\frac{E}{B}=\frac{\sigma}{\varepsilon_0 B}$

So, the time of motion inside the capacitor is $t=\frac{\ell}{v}=\frac{\varepsilon_0\ell B}{\sigma}$

7. Since the electric and magnetic field both are parallel to the velocity of the charged particle, so $F_m = 0$ and $F_e = qE_0$. Due to the only electrostatic force acting on the charged particle, it will move along a straight line with uniform acceleration i.e. its speed keeps on increasing with time.

8. The particle will follow circular path in magnetic field due to v_x. So, its time period is

$$T = \frac{2\pi m}{qB_0}$$



However, the pitch of particle in absence of E_0 is

$$p = v_y T = v_0 T$$

But due to an electric field E_0 acting along y-direction the new pitch will be

$$p' = v_0 T + \frac{1}{2} a_y T^2 \text{, where } a_y = \frac{qE_0}{m}$$

$$\Rightarrow \quad p' = T\left(v_0 + \frac{a_y T}{2}\right)$$

$$\Rightarrow \quad p' = \frac{2\pi m}{qB_0}\left[v_0 + \frac{1}{2}\left(\frac{qE_0}{m}\right)\left(\frac{2\pi m}{qB_0}\right)\right]$$

$$\Rightarrow \quad p' = \frac{2\pi m}{qB_0}\left(v_0 + \frac{\pi E_0}{B_0}\right)$$

9. By Work-Energy Theorem, we have

Work done = Change in K.E.

$$\Rightarrow \quad (qE\hat{i})\cdot(2a\hat{i}) = \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2$$

$$\Rightarrow \quad 2aqE = \frac{3}{2}mv^2$$

$$\Rightarrow \quad E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$$

Rate of work done by $\vec{E}$ at P is $\vec{F}\cdot\vec{v}$

$$\Rightarrow \quad \frac{dW}{dt} = (qE\hat{i})\cdot(v\hat{i})$$

$$\Rightarrow \quad \frac{dW}{dt} = qEv = \frac{3}{4}\left(\frac{mv^3}{a}\right)$$

Rate of work done by $\vec{E}$ at Q is zero, because at Q, $\vec{F} \perp \vec{v}$.

Rate of work done by $\vec{B}$ at both the points P and Q is zero.

10. No change in velocity implies no acceleration i.e. no net force is acting on the charged particle under the joint influence of electric and magnetic field. This thing is possible under the following situations.

Situation 1:

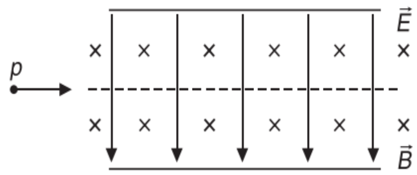
$E = 0$, $B = 0$, i.e. no field exists in the region

Situation 2:

$E = 0$ i.e. no electrostatic force. $B \neq 0$, but the charge particle enters parallel to the field, so that net force equals to zero.

Situation 3:

$E \neq 0$, $B \neq 0$ and both shown in Figure.



Because in such a situation

$F_m = qvB$ (upwards) and

$F_e = qE$ (downwards)

and if both are equal in magnitude then also the charged particle will suffer no change in its velocity and will continue to move along the dotted line as shown. In such a situation, the velocity is $v = \frac{E}{B}$.

Situation 4:

If $E \neq 0$, $B = 0$, then the charged particle must experience an electrostatic force qE and hence must accelerate. So, it will not have a constant velocity.

Hence, we conclude that for the charged particle to move with constant velocity in the region having simultaneous electric and magnetic fields, we may have,

(i) $E = 0$, $B = 0$ (as discussed in Situation 1)
(ii) $E = 0$, $B \neq 0$ (as discussed in Situation 2)
(iii) $E \neq 0$, $B \neq 0$ (as discussed in Situation 3)

Test Your Concepts-IV (Based on Biot Savart's Law and Applications)

1. (a) Since, $B_{centre} = \frac{\mu_0 I}{2R}$

and $R = 100 \text{ mm} = 100 \times 10^{-3} \text{ m} = 0.1 \text{ m}$

$$\Rightarrow \quad B_{centre} = \frac{(4\pi \times 10^{-7})(1)}{2(0.1)} = 6.3 \times 10^{-6} \text{ T} = 6.3 \text{ μT}$$

(b) Also, $B_{axis} = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$

where $R = 100\ \text{mm} = 100 \times 10^{-3}\ \text{m} = 0.1\ \text{m}$

and $x = 100\ \text{mm} = 100 \times 10^{-3}\ \text{m} = 0.1\ \text{m}$

So, we observe that $x = R = 10^{-5}\ \text{m}$

$$B_{\text{axis}} = \frac{\mu_0 I R^2}{2\left(R^2 + R^2\right)^{3/2}}$$

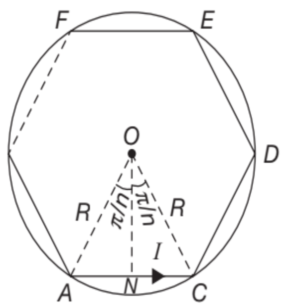
$$B_{\text{axis}} = \frac{\mu_0 I R^2}{2R^3 2^{3/2}} = \frac{\mu_0 I}{2R\left(2\sqrt{2}\right)} = \frac{\mu_0 I}{4\sqrt{2}R}$$

$$B_{\text{axis}} = \frac{\mu_0 (1)}{4\sqrt{2} \times (0.1)}$$

$$B_{\text{axis}} = \frac{4\pi \times 10^{-7}}{4(1.414)(0.1)} = 2.3 \times 10^{-6}\ \text{T}$$

$$\Rightarrow \quad B_{\text{axis}} = 2.3\ \mu\text{T}$$

2. $ON = R\cos\left(\frac{\pi}{n}\right)$



Due to the wire AC, field at the centre O, at perpendicular distance ON is

$$B_{AC} = \frac{\mu_0 I}{4\pi\left(r_\perp\right)}\left(\sin\phi_1 + \sin\phi_2\right)$$

$$\Rightarrow \quad B_{AC} = \frac{\mu_0 I}{4\pi(ON)}\left[\sin\left(\frac{\pi}{n}\right) + \sin\left(\frac{\pi}{n}\right)\right]$$

$$\Rightarrow \quad B_{AC} = \frac{\mu_0 I}{4\pi R\cos\left(\frac{\pi}{n}\right)}\left[2\sin\left(\frac{\pi}{n}\right)\right]$$

$$\Rightarrow \quad B_{AC} = \left(\frac{\mu_0 I}{2\pi R}\right)\tan\left(\frac{\pi}{n}\right), \odot$$

Similarly, all wires will give an outward equal field at O. Hence, for an n sided current carrying polygon, we get

$$B_{\text{total}} = nB_{AC} = n\left(\frac{\mu_0 I}{2\pi R}\right)\tan\left(\frac{\pi}{n}\right)$$

When the polygon has got infinite sides, then $n \to \infty$ and the polygon approaches the shape of a circle. The magnetic field due to a circular current carrying coil at the centre is $B_{\text{centre}} = \frac{\mu_0 I}{2R}$.

This can also be derived mathematically by taking the limit $n \to \infty$ on B_{total}.

$$B_{\text{total}} = \lim_{n\to\infty} n\left(\frac{\mu_0 I}{2\pi R}\right)\tan\left(\frac{\pi}{n}\right)$$

$$\Rightarrow \quad B_{total} = \lim_{n\to\infty}\left(\frac{\mu_0 I \pi}{2\pi R}\right)\frac{\tan\left(\frac{\pi}{n}\right)}{\left(\frac{\pi}{n}\right)}$$

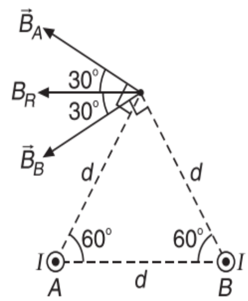
$$\Rightarrow \quad B_{\text{total}} = \frac{\mu_0 I}{2R}\lim_{n\to\infty}\left[\frac{\tan\left(\frac{\pi}{n}\right)}{\left(\frac{\pi}{n}\right)}\right]$$

The expression $\lim_{n\to\infty}\frac{\tan\left(\frac{\pi}{n}\right)}{\left(\frac{\pi}{n}\right)} = 1$

$$\Rightarrow \quad B_{\text{total}} = \frac{\mu_0 I}{2R}$$

3. CASE-1: When the wires carry current in same direction

The resultant magnetic field B_R is calculated from the given figure. So



$$B_R = 2B\cos(30°) = \sqrt{3}B$$

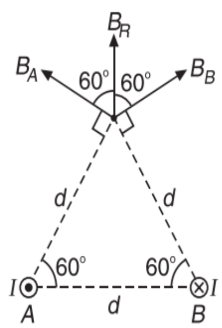
where $\left|\vec{B}_A\right| = \left|\vec{B}_B\right| = B = \frac{\mu_0 I}{2\pi d} = 10^{-7}\left(\frac{2 \times 10}{0.1}\right)$

$$\Rightarrow \quad B = 2 \times 10^{-5}\ \text{T}$$

So, $B_R = 2\sqrt{3} \times 10^{-5}\ \text{T}$

CASE-2: When the wires carry currents in the opposite direction

The resultant magnetic field B_R is calculated from the given figure. So

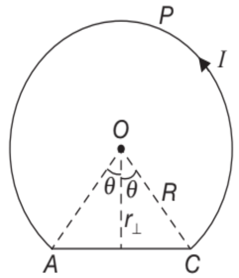


$$B_R = 2B\cos(60°) = B$$

$$\Rightarrow \quad B_R = 2\times10^{-5}\ \text{T}$$

4. $B_{arc} = \left(\dfrac{\mu_0 I}{4\pi R}\right)\theta$, where θ is in radian

So, $B_{arc} = \dfrac{\mu_0 I}{4\pi R}(2\pi - 2\theta)$, $\odot$



Due to the straight wire AC,

$$B_{AC} = \frac{\mu_0 I}{4\pi(r_\perp)}(\sin\theta + \sin\theta)$$

where $r_\perp = R\cos\theta$

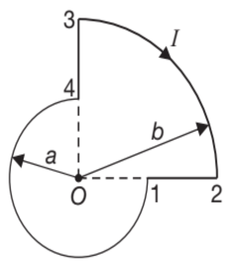
$$\Rightarrow \quad B_{AC} = \frac{\mu_0 I}{4\pi(R\cos\theta)}(2\sin\theta)$$

$$\Rightarrow \quad B_{AC} = \frac{\mu_0 I}{2\pi R}\tan\theta,\ \odot$$

So, $B_{total} = B_{arc} + B_{AC}$, $\odot$

$$\Rightarrow \quad B_{total} = \frac{\mu_0 I}{2\pi R}(\pi - \theta + \tan\theta),\ \odot$$

5. $B_{arc\ 41} = \dfrac{\mu_0 I}{4\pi a}\theta$



$$\Rightarrow \quad B_{arc\ 41} = \frac{\mu_0 I}{4\pi a}\left(\frac{3\pi}{2}\right) = \frac{3\mu_0 I}{8a},\ \otimes$$

$B_{12} = 0$ {∵ O lies at extended part of wire 12}

$B_{43} = 0$ {∵ O lies at extended part of wire 34}

$$B_{arc\ 32} = \frac{\mu_0 I}{4\pi b}\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \quad B_{arc\ 32} = \frac{\mu_0 I}{8b},\ \otimes$$

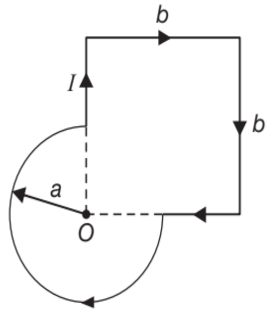
So, $B_{total} = B_{arc\ 41} + B_{arc\ 32} = \dfrac{\mu_0 I}{8}\left(\dfrac{3}{a} + \dfrac{1}{b}\right)$, $\otimes$

6. $B_{12} = B_{45} = 0$

$$B_{arc\ 51} = \frac{3\mu_0 I}{8a},\ \otimes$$

$$B_{23} = \frac{\mu_0 I}{4\pi b}(\sin 0 + \sin 45)$$

$$\Rightarrow \quad B_{23} = \frac{\mu_0 I}{(4\sqrt{2})\pi b},\ \otimes$$



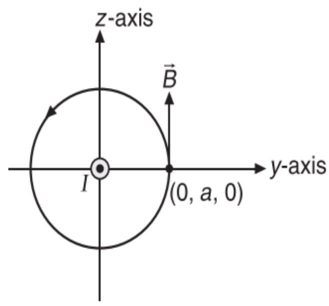
Similarly, $B_{34} = \dfrac{\mu_0 I}{(4\sqrt{2})\pi b}$, $\otimes$

$$\Rightarrow \quad B_{total} = B_{12} + B_{23} + B_{34} + B_{arc}51,\ \otimes$$

$$\Rightarrow \quad B_{total} = \frac{\mu_0 I}{4\pi}\left(\frac{3\pi}{2a} + \frac{\sqrt{2}}{b}\right),\ \otimes$$

7. CASE-1: At (0, *a*, 0)

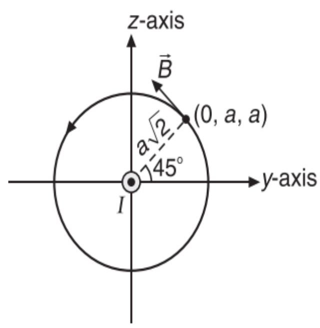
The direction of magnetic field in this case is shown in figure.



Mathematically, we have $\vec{B} = \left(\dfrac{\mu_0 I}{2\pi a}\right)\hat{k}$

CASE-2: At (0, *a*, *a*)

The direction of magnetic field in this case is shown in figure.



Since the point lies in y-z plane, so unit vector along the direction of magnetic field $\vec{B}$ is given by

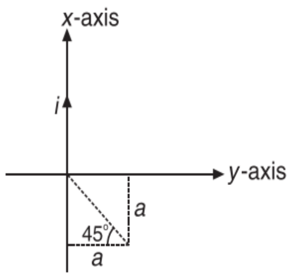
$$\hat{n} = -\frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{4\pi a\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}\right)$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{8\pi a}\left(-\hat{j} + \hat{k}\right)$$

CASE-3: At $(-a, a, 0)$

In this case we re-align the axis as shown in figure.



The magnetic field at the point $(-a, a, 0)$ is

$$\vec{B} = \frac{\mu_0 I}{4\pi a}(\sin 90° - \sin 45°)\hat{k}$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{4\pi a}\left(1 - \frac{1}{\sqrt{2}}\right)\hat{k}$$

8. Let B_1 be the magnetic field due to circular loop and B_2 be the magnetic field due to straight current carrying wire. Then

$$B_1 = \frac{\mu_0 I}{2r}, \text{ inwards and}$$

$$B_2 = \frac{\mu_0 (4I)}{2\pi(2r)} = \frac{\mu_0 I}{\pi r}, \text{ outwards}$$

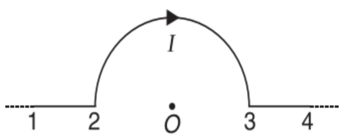
Since $B_1 > B_2$, so the net field B is

$$B = B_1 - B_2 = \frac{\mu_0 I}{2r} - \frac{\mu_0 I}{\pi r}, \text{ inwards}$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{r}\left(\frac{1}{2} - \frac{1}{\pi}\right), \text{ inwards}$$

9. The magnetic field due to the wires 12 and 34 at the point O is zero. So

$$B_{12} = B_{34} = 0$$



$$B_{\text{arc }23} = \frac{\mu_0 I}{4R}, \otimes$$

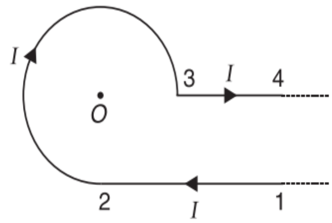
$$\Rightarrow \quad B_{\text{total}} = B_{12} + B_{\text{arc }24} + B_{34} = \frac{\mu_0 I}{4R}, \otimes$$

10. The magnetic field due to the different sections of the conductor are

$$B_{34} = 0$$

$$B_{\text{arc }23} = \frac{3\mu_0 I}{8R}, \otimes$$

$$B_{12} = \frac{\mu_0 I}{4\pi R}(\sin 0° + \sin 90°)$$

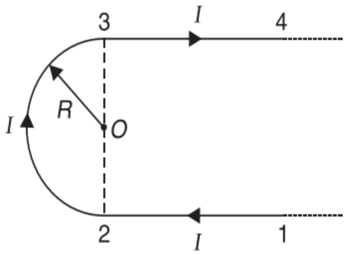


$$\Rightarrow \quad B_{12} = \frac{\mu_0 I}{4\pi R}, \otimes$$

$$\Rightarrow \quad B_{\text{total}} = \frac{\mu_0 I}{4\pi R}\left(1 + \frac{3\pi}{2}\right), \otimes$$

11. The magnetic field due to the different sections of the conductor are

$$B_{12} = \frac{\mu_0 I}{4\pi R}\left(\sin 0 + \sin\frac{\pi}{2}\right)$$



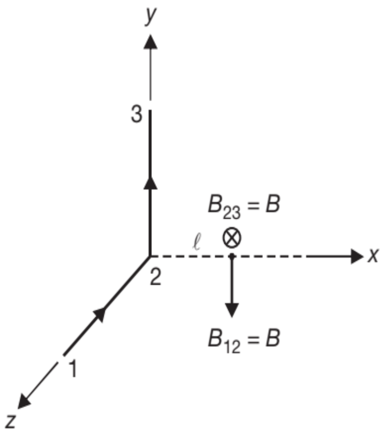
$$\Rightarrow \quad B_{12} = \frac{\mu_0 I}{4\pi R}, \otimes$$

$$B_{23} = \frac{\mu_0 I}{4R}, \otimes \text{ and } B_{34} = \frac{\mu_0 I}{4\pi R}, \otimes$$

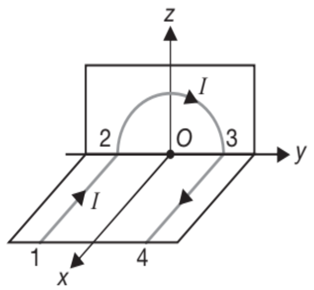
$$\Rightarrow \quad B_{\text{total}} = \frac{\mu_0 I}{4\pi R}(2 + \pi), \otimes$$

12. $B_{\text{total}} = \sqrt{B^2 + B^2}$

$$\Rightarrow \quad B_{\text{total}} = \sqrt{2}B = \sqrt{2}\left(\frac{\mu_0 I}{4\pi \ell}\right)$$



13. The magnetic field due to the different sections of the conductor are



$$\vec{B}_{12} = \frac{\mu_0 I}{4\pi R}(-\hat{k})$$

$$\vec{B}_{\text{arc }23} = \frac{\mu_0 I}{4R}(-\hat{i})$$

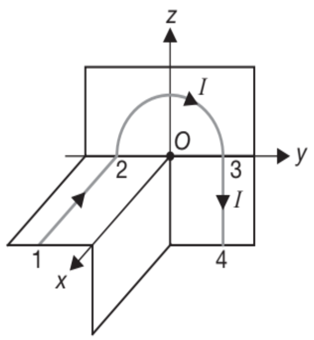
$$\vec{B}_{34} = \frac{\mu_0 I}{4\pi R}(-\hat{k})$$

$$\Rightarrow \quad \vec{B} = \vec{B}_{12} + \vec{B}_{\text{arc }23} + \vec{B}_{34}$$

$$\Rightarrow \quad \vec{B} = -\left(\frac{\mu_0 I}{4\pi R}\right)(\pi\hat{i} + 2\hat{k})$$

$$\Rightarrow \quad |\vec{B}| = \frac{\mu_0 I}{4\pi R}\sqrt{\pi^2 + 4}$$

14. The magnetic field due to the different sections of the conductor are



$$\vec{B}_{12} = \frac{\mu_0 I}{4\pi R}(-\hat{k})$$

$$\vec{B}_{\text{arc }23} = \frac{\mu_0 I}{4R}(-\hat{i})$$

$$\vec{B}_{34} = \frac{\mu_0 I}{4\pi R}(-\hat{i})$$

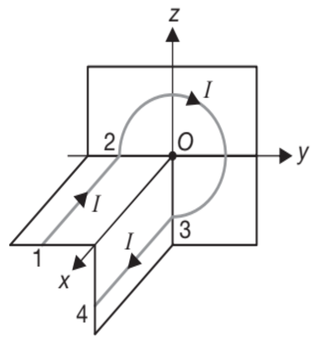
$$\Rightarrow \quad \vec{B} = B_{12} + \vec{B}_{\text{arc }23} + \vec{B}_{34}$$

$$\Rightarrow \quad \vec{B} = -\left(\frac{\mu_0 I}{4\pi R}\right)\left[(1+\pi)\hat{i} + \hat{k}\right]$$

$$\Rightarrow \quad B = |\vec{B}| = \frac{\mu_0 I}{4\pi R}\sqrt{(1+\pi)^2 + 1}$$

$$\Rightarrow \quad B = \left(\frac{\mu_0 I}{4\pi R}\right)\sqrt{2 + 2\pi + \pi^2}$$

15. The magnetic field due to the different sections of the conductor are



$$\vec{B}_{12} = \frac{\mu_0 I}{4\pi R}(-\hat{k})$$

$$\vec{B}_{\text{arc }23} = \frac{3\mu_0 I}{8R}(-\hat{i})$$

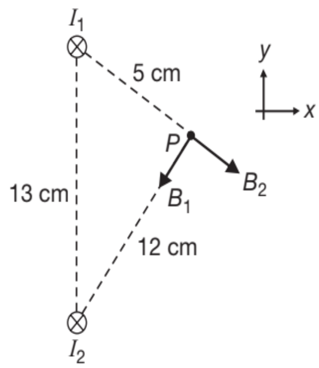
$$\vec{B}_{34} = \frac{\mu_0 I}{4\pi R}(-\hat{j})$$

$$\Rightarrow \quad \vec{B} = \vec{B}_{12} + \vec{B}_{\text{arc }23} + \vec{B}_{34}$$

$$\Rightarrow \quad \vec{B} = -\left(\frac{\mu_0 I}{4\pi R}\right)\left[\left(\frac{3\pi}{2}\right)\hat{i} + \hat{j} + \hat{k}\right]$$

$$\Rightarrow \quad B = |\vec{B}| = \frac{\mu_0 I}{4\pi R}\sqrt{\frac{9\pi^2}{4} + 2}$$

16. Take the x-direction to the right and the y-direction up in the plane of the paper.



Magnetic field at P due to current I_1 is

$$B_1 = \frac{\mu_0 I_1}{2\pi a} = \frac{(2\times 10^7 \text{ Tm})(3 \text{ A})}{A(0.05 \text{ m})}$$

$$\Rightarrow \quad B_1 = 12\ \mu\text{T}$$

Downwards and leftwards, at angle $67.4°$ below the $-x$ axis.

$\Rightarrow \quad \vec{B}_2 = (12\ \mu\text{T})\left(-\hat{i}\cos 67.4° - \hat{j}\sin 67.4°\right)$

Magnetic field at P due to current I_2 is

$$B_2 = \frac{(2\times 10^{-7}\ \text{Tm})(3\ \text{A})}{A(0.12\ \text{m})}$$

$\Rightarrow \quad B_2 = 5\ \mu\text{T}$

Clockwise perpendicular to $12\ \text{cm}$ to the right and down at angle $-22.6°$

$\Rightarrow \quad \vec{B}_2 = (5\ \mu\text{T})\left(\hat{i}\cos 22.6° - \hat{j}\sin 22.6°\right)$

The total magnetic field $\vec{B}$ is given by

$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

$$\Rightarrow \quad \vec{B} = (12)\left(-\hat{i}\cos 67.4° - \hat{j}\sin 67.4°\right) + (5)\left(\hat{i}\cos 22.6° - \hat{j}\sin 22.6°\right)$$

$\Rightarrow \quad \vec{B} = (-11.1\ \mu\text{T})\hat{j} - (1.92\ \mu\text{T})\hat{j} = (-13\ \mu\text{T})\hat{j}$

17. Since $I = \dfrac{q}{T} = \dfrac{q}{\left(\dfrac{2\pi r}{v}\right)}$

$\Rightarrow \quad B = \dfrac{\mu_0 I}{2r} = \dfrac{\mu_0}{2r}\left(\dfrac{qv}{2\pi r}\right)$

Substituting values, we get

$B = 12.5\ \text{T}$

18. Wire 1 creates at the magnetic field at the origin, given by

$\vec{B}_1 = \dfrac{\mu_0 I}{2\pi r}$, upwards along $+y$ axis.

So, $\vec{B} = \left(\dfrac{\mu_0 I}{2\pi a}\right)\hat{j}$

(a) If the total field at the origin is upwards, then

$$\frac{2\mu_0 I_1}{2\pi a}\hat{j} = \frac{\mu_0 I_1}{2\pi a}\hat{j} + \vec{B}_2$$

So, the second wire creates a field given by

$$\vec{B}_2 = \frac{\mu_0 I_1}{2\pi a}\hat{j} = \frac{\mu_0 I_2}{2\pi(2a)}, \odot$$

$\Rightarrow \quad I_2 = 2I_1$ out of the paper

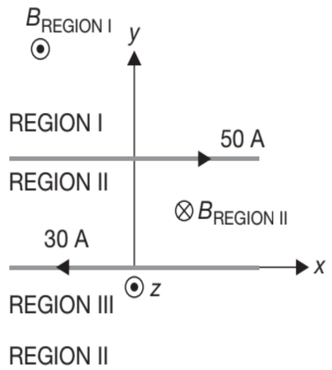
(b) The other possibility is

$$\vec{B}_1 + \vec{B}_2 = \frac{2\mu_0 I_1}{2\pi a}(-\hat{j}) = \frac{\mu_0 I_1}{2\pi a}\hat{j} + \vec{B}_2$$

$$\Rightarrow \quad \vec{B}_2 = \frac{3\mu_0 I_1}{2\pi a}(-\hat{j}) = \frac{\mu_0 I_2}{2\pi(2a)}, \otimes$$

$\Rightarrow \quad I_2 = 6I_1$ into the paper

19. (a) Above the pair of wires, in Region I, both the wires give field out of the page. Between the wires, in Region II, both produce fields into the page.



They can only add to zero below the wires, in Region III, at coordinate y. Here the total field is

$$\vec{B}_{\text{Region III}} = \frac{\mu_0 I_2}{2\pi r}\otimes + \frac{\mu_0 I_1}{2\pi r}, \odot$$

$$\Rightarrow \quad 0 = \frac{\mu_0}{2\pi}\left[\frac{50}{\left(y+\frac{2}{3}\right)} - \frac{30\ \text{A}}{y}\right]$$

$\Rightarrow \quad 50y = 30\left(y + \dfrac{2}{3}\right)$

$\Rightarrow \quad 50y = 30y + 20$

$\Rightarrow \quad 20y = 20$

$\Rightarrow \quad y = 1\ \text{m}$, below wire 1

(b) At $y = 0.1\ \text{m}$ the total field is $\vec{B} = \dfrac{\mu_0 I}{2\pi r}\otimes + \dfrac{\mu_0 I}{2\pi r}\otimes$

$$\Rightarrow \quad \vec{B} = \frac{4\pi\times 10^{-7}}{2\pi}\left(\frac{50}{\left(\frac{2}{3}-\frac{1}{3}\right)}(-\hat{k}) + \frac{30}{\frac{1}{3}}(-\hat{k})\right)$$

$\Rightarrow \quad \vec{B} = 4.8\times 10^{-5}\ T(-\hat{k})$

The force on the particle is

$$\vec{F} = q(\vec{v}\times\vec{B}) = (-2\times 10^{-6}\ \text{C})(150\times 10^{6})(i)\times (4.8\times 10^{-5})(-\hat{k}) = 14.4\times 10^{-3}\ \text{N}(-\hat{j})$$

20. On the axis of a current loop, the magnetic field is given by

$$B = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$$

where in this case $I = \dfrac{Q}{\left(\dfrac{2\pi}{\omega}\right)}$

$$\Rightarrow \quad B = \frac{\mu_0 \omega R^2 Q}{4\pi(x^2+R^2)^{3/2}}$$

when $x = \dfrac{R}{2}$

$$\Rightarrow \quad B = \frac{\mu_0\omega R^2 Q}{4\pi\left(\frac{5}{4}R^2\right)^{3/2}} = \frac{2\mu_0 Q\omega}{5\sqrt{5}\,\pi R}$$

21. Let the current I be to the right. It creates a field $B=\frac{\mu_0 I}{2\pi d}$ at the proton's location. And we have an equilibrium between the weight of the proton and the magnetic force $\vec{F}$, where

$$\vec{F}=q\left[v(-\hat{i})\times\left(\frac{\mu_0 I}{2\pi d}\right)\hat{k}\right]$$

$\Rightarrow$ $mg(-\hat{j})+qv(-\hat{i})\times\frac{\mu_0 I}{2\pi d}(\hat{k})=0$ at a distance d from the wire

$$\Rightarrow \quad d=\frac{qv\mu_0 I}{2\pi mg}$$

22. There is no magnetic field at O due to the straight portions of the current carrying loop.
Field due to an arc of radius r subtending an angle θ at the centre of the arc is given by

$$B=\frac{\mu_0 I}{4\pi R}\theta$$

Due to arc of radius $4R$, field is

$$B_1=\left(\frac{\mu_0 I}{4\pi(4R)}\right)\left(\frac{\pi}{2}\right)=\frac{\mu_0 I}{32R}$$

Due to arch of radius $2R$, field is

$$B_2=\left(\frac{\mu_0 I}{4\pi(2R)}\right)(\pi)=\frac{\mu_0 I}{8R}$$

Due to arc of radius R, field is

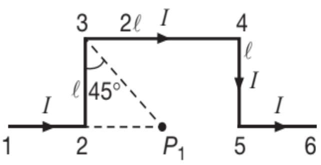
$$B_3=\left(\frac{\mu_0 I}{4\pi R}\right)\left(\frac{\pi}{2}\right)=\frac{\mu_0 I}{8R}$$

Since the current is anticlockwise in all three arcs, therefore, all the fields are outwards towards the reader. So, the net magnetic field is given by

$$B_{\text{net}}=B_1+B_2+B_3$$

$$\Rightarrow \quad B_{\text{net}}=\frac{\mu_0 I}{R}\left(\frac{1}{32}+\frac{1}{8}+\frac{1}{8}\right)=\frac{9\mu_0 I}{32R}$$

23. For the figure (a), we have



$$B_{12}=B_{56}=0$$

$$B_{23}=\frac{\mu_0 I}{4\pi\ell}\left[\sin(0)+\sin\left(\frac{\pi}{4}\right)\right]=\frac{\mu_0 I}{4\sqrt{2}\pi\ell},\ \otimes$$

$$B_{45}=B_{23}=\frac{\mu_0 I}{4\sqrt{2}\pi\ell},\ \otimes$$

$$B_{34}=\frac{\mu_0 I}{4\pi\ell}\left[\sin\left(\frac{\pi}{4}\right)+\sin\left(\frac{\pi}{4}\right)\right]=\frac{\sqrt{2}\mu_0 I}{4\pi\ell},\ \otimes$$

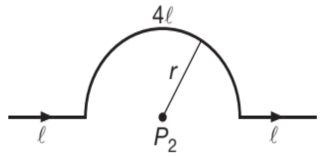
So, field at P_1 is B_1 given by

$$\Rightarrow \quad B_1=\frac{\mu_0 I}{4\pi\ell}\left(\sqrt{2}+\frac{1}{\sqrt{2}}\right),\ \otimes \qquad \ldots(1)$$

For figure (b), we observe that if r is the radius of the semicircle, then

$$\pi r=4\ell \qquad \{\because \text{ total length}=6\ell\}$$

$$\Rightarrow \quad r=\frac{4\ell}{\pi} \qquad \ldots(2)$$



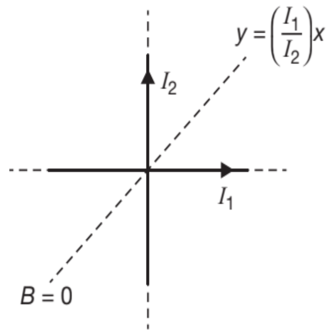
So, field at P_2 is given by,

$$B_2=\frac{\mu_0 I}{4r}=\frac{\mu_0 I}{4\left(\frac{4\ell}{\pi}\right)}=\pi\left(\frac{\mu_0 I}{16\ell}\right),\ \otimes$$

$$\frac{B_1}{B_2}=\frac{4}{\pi^2}\left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)$$

24. Let the field be zero at the point $P(x,\ y)$, then

$$\frac{\mu_0 I_1}{2\pi y}\odot+\frac{\mu_0 I_2}{2\pi x}\otimes=0$$



$$\Rightarrow \quad \frac{I_1}{y}-\frac{I_2}{x}=0$$

$$\Rightarrow \quad y=\left(\frac{I_1}{I_2}\right)x$$

25. The net field is the vector sum of the fields due to the circular loop and to the long straight wire, both at the centre of the loop.
For the long wire, $B=\frac{\mu_0 I}{2\pi D}$, and for the loop, $B=\frac{\mu_0 I_0}{2R}$
At the center of the circular loop the current I_0 generates a magnetic field that is into the page, so the current I must point to the right. For complete cancellation the two fields must have the same magnitude.

$$\vec{B}_{\text{loop}} + \vec{B}_{\text{wire}} = \vec{0}$$

$$\Rightarrow \left|\vec{B}_{\text{wire}}\right| = \left|\vec{B}_{\text{loop}}\right|$$

$$\Rightarrow \frac{\mu_0 I}{2\pi D} = \frac{\mu_0 I_0}{2R}$$

$$\Rightarrow I = \left(\frac{\pi D}{R}\right) I_0$$

Test Your Concepts-V (Based on Ampere's Circuital Law and Applications)

1. Apply Ampere's Law to a circle of radius r. The current within a radius r is $I = \int \vec{J} \cdot d\vec{A}$, where the integration is over a disk of radius r.

(a) $I_0 = \int \vec{J} \cdot d\vec{A}$

$$\Rightarrow I = \iint \left(\frac{b}{r} e^{(r-a)/\delta}\right) r dr d\theta$$

$$\Rightarrow I = 2\pi b \int_0^a e^{(r-a)/\delta} dr = 2\pi b \delta e^{(r-a)/\delta}\Big|_0^a$$

$$\Rightarrow I = 2\pi b \delta\left(1 - e^{-a/\delta}\right)$$

(b) For $r \geq a$, we have

$$\oint \vec{B} \cdot d\vec{\ell} = B(2\pi r) = \mu_0 I_{\text{encl}} = \mu_0 I_0$$

$$\Rightarrow B = \frac{\mu_0 I_0}{2\pi r}$$

(c) For $r \leq a$, we have

$$I(r) = \int \vec{J} \cdot d\vec{A} = \int_0^r \int_0^{2\pi} \left(\frac{b}{r} e^{(r-a)/\delta}\right) r dr d\theta$$

$$\Rightarrow I(r) = 2\pi b \int_0^r e^{(r-a)/\delta} dr = 2\pi b \delta e^{(r-a)/\delta}\Big|_0^r$$

$$\Rightarrow I(r) = 2\pi b \delta\left(e^{(r-a)/\delta} - e^{-a/\delta}\right)$$

$$\Rightarrow I(r) = 2\pi b \delta e^{-a/\delta}\left(e^{r/\delta} - 1\right)$$

$$\Rightarrow I(r) = I_0 \frac{\left(e^{r/\delta} - 1\right)}{\left(e^{a/\delta} - 1\right)}$$

(d) For $r \leq a$, we have

$$\oint \vec{B} \cdot d\vec{\ell} = B(r) 2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0 \frac{\left(e^{r/\delta} - 1\right)}{\left(e^{a/\delta} - 1\right)}$$

$$\Rightarrow B = \frac{\mu_0 I_0 \left(e^{r/\delta} - 1\right)}{2\pi r \left(e^{a/\delta} - 1\right)}$$

2. Apply Ampere's Law to a circular path of radius r. Let us assume the current is uniform over the cross-section of the conductor

(a) For $r < a$ we have $I_{\text{encl}} = 0$

$$\Rightarrow B = 0$$

(b) For $a < r < b$ we have

$$I_{\text{encl}} = I\left(\frac{A_{a \to r}}{A_{a \to b}}\right) = I\left(\frac{\pi\left(r^2 - a^2\right)}{\pi\left(b^2 - a^2\right)}\right)$$

$$\Rightarrow I_{\text{encl}} = I \frac{\left(r^2 - a^2\right)}{\left(b^2 - a^2\right)}$$

From Ampere's Circuital Law, we get

$$\oint \vec{B} \cdot d\vec{\ell} = B(2\pi r) = \mu_0 I \frac{\left(r^2 - a^2\right)}{\left(b^2 - a^2\right)}$$

$$\Rightarrow B = \frac{\mu_0 I \left(r^2 - a^2\right)}{2\pi r \left(b^2 - a^2\right)}$$

(c) For $r > b$ we have

$$I_{\text{encl}} = I$$

From Ampere's Circuital Law, we get

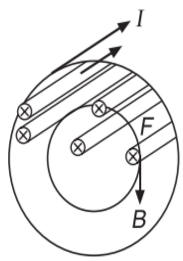
$$\oint \vec{B} \cdot d\vec{\ell} = B(2\pi r) = \mu_0 I \text{ and } B = \frac{\mu_0 I}{2\pi r}$$

3. (a) Force on a single wire due to the field of the other ninety-nine is given by

$$B = \frac{\mu_0 I_0 r}{2\pi R^2}$$

$$\Rightarrow B = \frac{\left(4\pi \times 10^{-7}\right)(99)(2)\left(0.2 \times 10^{-2}\right)}{2\pi\left(0.5 \times 10^{-2}\right)^2}$$

$$\Rightarrow B = 3.17 \times 10^{-3} \text{ T}$$

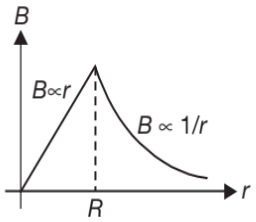


This field points tangentially to a circle of radius 0.2 cm and exerts force $\vec{F} = I\left(\vec{\ell} \times \vec{B}\right)$ towards the center of the bundle, on the single hundredth wire

$$\frac{F}{\ell} = IB\sin\theta = (2 \text{ A})\left(3.17 \times 10^{-3} \text{ T}\right)\sin 90^\circ$$

$$\Rightarrow \frac{F}{\ell} = 6.34 \text{ mNm}^{-1}$$

So, $\frac{F_B}{\ell} = 6.34 \times 10^{-3} \text{ Nm}^{-1}$, inwards



(b) $B \propto r$, so B is greatest at the outside of the bundle. Since each wire carries the same current, F is greatest at the outer surface.

4. When seen from the top, left side portion of the Amperian loop is traversed counter clockwise, whereas the other portion is traversed clockwise. So, by applying the sign convention, we see that I_1 is positive, I_2 is positive and I_3 is negative.

$$\Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 I = \mu_0 (I_1 + I_2 - I_3)$$

5. Use Ampere's Law,

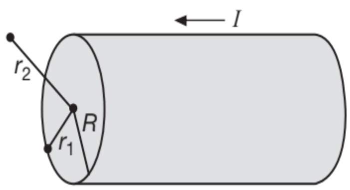
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

For current density $\vec{J}$, this becomes

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \int \vec{J} \cdot d\vec{A} \qquad \left\{ \because \mathrm{I} = \int \vec{J} \cdot d\vec{A} \right\}$$

(a) For $r_1 < R$, we get

$$B(2\pi r_1) = \mu_0 \int_0^{r_1} (br)(2\pi r dr)$$



$$\Rightarrow \quad B = \frac{\mu_0 b r_1^2}{3}$$

(for $r_1 < R$ or inside the cylinder)

(b) When $r_2 > R$, Ampere's Law gives

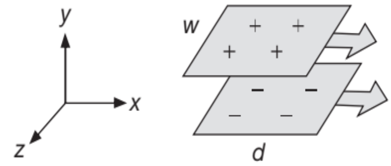
$$B(2\pi r_2)\mu_0 \int_0^{R} (br)(2\pi r dr) = \frac{2\pi \mu_0 b R^3}{3}$$

$$\Rightarrow \quad B = \frac{\mu_0 b R^3}{3 r_2}$$

(for $r_2 > R$ or outside the cylinder).

6. Let J_s be the current per unit length, then the upper sheet creates field $\vec{B} = \frac{\mu_0 J_s}{2}\hat{k}$ above it and $\frac{\mu_0 J_s}{2}(-\hat{k})$ below it.

Consider a patch of the sheet of width w parallel to the z-axis and length d parallel to the x-axis as shown in Figure.



The charge on it, $\sigma w d$ passes a point in time $\frac{d}{v'}$ so the current it constitutes is $\frac{q}{t} = \frac{\sigma w v d}{d}$ and the linear current density is $J_s = \frac{\sigma w v}{w} = \sigma v$. So, the magnitude of the magnetic field created by the upper sheet is $\frac{1}{2}\mu_0 \sigma v$. Similarly, the lower sheet in its motion toward the right constitutes current toward the left. It creates magnetic field $\frac{1}{2}\mu_0 \sigma v(-\hat{k})$ above it and $\frac{1}{2}\mu_0 \sigma v \hat{k}$ below it.

(a) Between the plates, their fields add to $\mu_0 \sigma v(-\hat{k})$ i.e. $\mu_0 \sigma v$ away from you horizontally.

(b) Above both sheets and below both the sheets, their equal magnitude fields add to zero.

(c) The upper plate exerts no force on itself. The field of the lower plate, $\frac{1}{2}\mu_0 \sigma v(-\hat{k})$ will exert a magnetic force on the current in the section of area $w \times d$ given by

$$\vec{F}_m = I(\vec{\ell} \times \vec{B}) = (\sigma w v d)\hat{i} \times \left(\frac{1}{2}\mu_0 \sigma v\right)(-\hat{k})$$

$$\Rightarrow \quad \vec{F}_m = I(\vec{\ell} \times \vec{B}) = \left(\frac{1}{2}\mu_0 \sigma^2 v^2 w d\right)\hat{j}$$

The magnetic force per area is

$$\frac{\vec{F}_m}{A} = \frac{1}{2}\left(\frac{\mu_0 \sigma^2 v^2 w d}{w d}\right)\hat{j}$$

$$\Rightarrow \quad \frac{F_m}{A} = \frac{1}{2}\mu_0 \sigma^2 v^2, \text{ upwards}$$

(d) The electrical force on the considered section of the upper plate is

$$\vec{F}_e = q\vec{E}_{\text{lower}} = (\sigma \ell w)\left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{j})$$

$$\Rightarrow \quad \vec{F}_e == \left(\frac{\ell w \sigma^2}{2\varepsilon_0}\right)(-\hat{j})$$

The electrical force per area is

$$\frac{F_e}{A} = \frac{\ell w \sigma^2}{2\varepsilon_0 \ell w} = \frac{\sigma^2}{2\varepsilon_0}, \text{ downwards}$$

The magnetic force balances the electric force acting on the plate when we have

$F_m = F_e$

$\Rightarrow \quad \frac{1}{2}\mu_0\sigma^2 v^2 = \frac{\sigma^2}{2\varepsilon_0}$

$\Rightarrow \quad v = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$\Rightarrow \quad v = \frac{1}{\sqrt{(4\pi\times10^{-7})(8.85\times10^{-12})}}$

$\Rightarrow \quad v = 3\times10^8\ \text{ms}^{-1}$

This is the speed of light which cannot be a speed that can be attained by a metal plate.

7. (a) According to Ampere's Circuital Law,

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0(I_{\text{inside}})$$

For $r_1 < R$

$I_{\text{inside}} = 0$

$\Rightarrow \quad B_{\text{inside}} = 0$

(b) According to Ampere's Circuital Law,

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0(I_{\text{inside}})$$

For $r_2 > R$

$I_{\text{inside}} = NI$

$\Rightarrow \quad B(2\pi r_2) = \mu_0 NI$

$\Rightarrow \quad B = \frac{\mu_0 NI}{2\pi r_2}$

(c) $B = \frac{(2\times10^{-7})(100)(10)}{5\times10^{-2}} = 4\times10^{-3}\ \text{T}$

$\Rightarrow \quad B = 4\ \text{mT}$

8. Apply Ampere's Law to a circle of radius r in each case.

Assume that the currents are uniform over the cross sections of the conductors.

(a) $r < a$

$\Rightarrow \quad I_{\text{encl}} = I\left(\frac{A_r}{A_a}\right) = I\left(\frac{r^2}{a^2}\right)$

From Ampere's Circuital Law, we have

$$\oint \vec{B}\cdot d\vec{\ell} = B(2\pi r) = \mu_0 I_{\text{encl.}} = \mu_0 I\left(\frac{r^2}{a^2}\right)$$

$\Rightarrow \quad B = \frac{\mu_0 I r}{2\pi a^2}$

When $r = a$ we have, $B = \frac{\mu_0 I}{2\pi a}$

(b) $b < r < c$

$\Rightarrow \quad I_{\text{encl}} = I - I\left(\frac{A_{b\to r}}{A_{b\to c}}\right) = I\left(1 - \frac{r^2-b^2}{c^2-b^2}\right)$

From Ampere's Circuital Law, we have

$$\oint \vec{B}\cdot d\vec{\ell} = B(2\pi r) = \mu_0 I_{\text{encl}}$$

$\Rightarrow \quad \oint \vec{B}\cdot d\vec{\ell} = \mu_0 I\left(1 - \frac{r^2-b^2}{c^2-b^2}\right)$

$\Rightarrow \quad \oint \vec{B}\cdot d\vec{\ell} = B(2\pi r) = \mu_0 I\left(\frac{c^2-r^2}{c^2-b^2}\right)$

$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}\left(\frac{c^2-r^2}{c^2-b^2}\right)$

When $r = b$, $B = \frac{\mu_0 I}{2\pi b}$ and

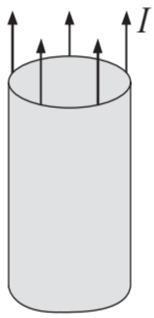
When $r = c$ then $B = 0$

9. We assume the current to be vertically upwards. Consider a circle of radius r slightly less than R. It encloses no current so from Ampere's Circuital Law, we have

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{\text{inside}}$$

$\Rightarrow \quad B(2\pi r) = 0$

We conclude that the magnetic field is zero.



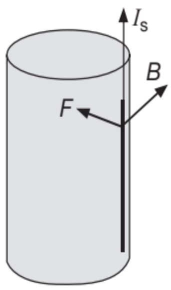
Now let the r be barely larger than R. Ampere's Law becomes

$B(2\pi R) = \mu_0 I$

$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi R}$

The field's direction is tangent to the wall of the cylinder in a counter clockwise sense.

Consider an infinitesimal strip of the wall of width dx and length ℓ. Its width is so small compared to $2\pi R$ that the field at its location would be essentially unchanged if the current in the strip were turned off.



The current carried by the strip is $I_s = \frac{Idx}{2\pi R}$ up.

The force on it is

$$\vec{F} = I_s(\vec{\ell} \times \vec{B}) = \frac{Idx}{2\pi R}\left(\ell \frac{\mu_0 I}{2\pi R}\right)$$

$$\vec{F} = \frac{\mu_0 I^2 \ell dx}{4\pi^2 R^2}, \text{ radially inwards}$$

The pressure on the strip and everywhere on the cylinder is

$$P = \frac{F}{A} = \frac{\mu_0 I^2 \ell dx}{4\pi^2 R^2 \ell dx} = \frac{\mu_0 I^2}{4\pi^2 R^2} \text{ inwards}$$

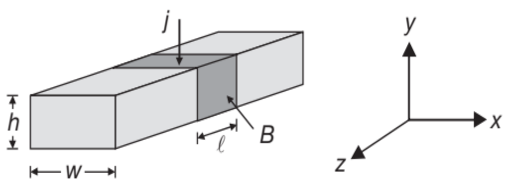
Due to the phenomenon of pinch effect, an empty aluminium can will crush itself when it carries a large current along its length.

10. In case of a long solenoid, the magnetic field is zero outside the solenoid and $B = \mu_0 nI$ inside the solenoid. So, the magnetic pressure on the surface of solenoid is given by

$$P_m = \frac{B^2}{2\mu_0} = \frac{1}{2}\mu_0 n^2 I^2$$

Test Your Concepts-VI (Based on Force on Current Carrying Conductor)

1. (a) Let us define vector $\vec{h}$ to have the downward direction of the current and vector $\vec{\ell}$ to be along the pipe into the page as shown. The electric current experiences a magnetic force $I(\vec{h} \times \vec{B})$ in the direction of $\vec{\ell}$



(b) The sodium consisting of ions and electrons, flows along the pipe transporting no net charge. But inside the section of length L, electrons drift upward to constitute downward electric current $j \times (\text{area}) = j\ell w$.

The current then feels a magnetic force

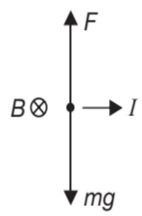
$$I|\vec{h} \times \vec{B}| = j\ell whB\sin 90°$$

This force along the pipe axis will make the fluid move, exerting pressure

$$\frac{F}{\text{area}} = \frac{j\ell whB}{hw} = j\ell B$$

2. The magnetic force $\vec{F}$ must be upward and equal to mg. The direction of $\vec{F}$ is determined by the direction of I in the circuit.

$F = I\ell B\sin\theta$ with $\theta = 90°$



From Ohm's Law, we have

$I = \frac{V}{R}$, where V is the battery voltage

(a) The forces are shown in figure. The current I in the bar must be to the right to produce $\vec{F}$ upwards. To produce current in this direction, point a must be the positive terminal of the battery.

(b) $F = mg$

$\Rightarrow I\ell B = mg$

$$\Rightarrow m = \frac{I\ell B}{g} = \frac{V\ell B}{Rg} = \frac{(175 \text{ V})(0.6 \text{ m})(1.5 \text{ T})}{(5\ \Omega)(9.8 \text{ ms}^{-2})}$$

$m = 3.21$ kg

3. $\sum F_y = 0$

$\Rightarrow N - mg = 0$

$\sum F_x = 0$

$\Rightarrow -\mu_k N + IB\sin 90° = 0$

$$\Rightarrow B = \frac{\mu_k mg}{Id} = \frac{0.1(0.2 \text{ kg})(9.8 \text{ ms}^{-2})}{(10 \text{ A})(0.5 \text{ m})} = 39.2 \text{ mT}$$

4. Call the length of the rod L and the tension in each wire alone $\frac{T}{2}$. Then, at equilibrium

$$\sum F_x = T\sin\theta - ILB\sin 90° = 0$$

$\Rightarrow T\sin\theta = ILB$

$$\sum F_y = T\cos\theta - mg = 0$$

$\Rightarrow \quad T\cos\theta = mg$

$$\Rightarrow \quad \tan\theta = \frac{ILB}{mg} = \frac{IB}{\left(\frac{m}{L}\right)g}$$

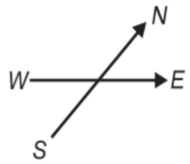
$$\Rightarrow \quad B = \frac{\left(\frac{m}{L}\right)g}{I}\tan\theta = \frac{\lambda g}{I}\tan\theta$$

5. $F = BIL\sin\theta$

For equilibrium, we have $F = mg$

$\Rightarrow \quad mg = BIL\sin\theta$

$$\Rightarrow \quad \frac{m}{L}g = IB\sin\theta$$



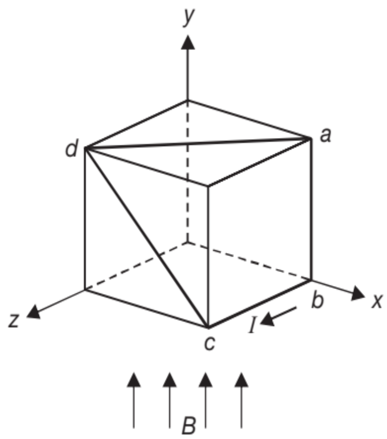
Since, $I = 2$ A and $\frac{m}{L} = 0.5\ \text{gcm}^{-1} = 5\times10^{-3}\ \text{kgm}^{-1}$

$\Rightarrow \quad (5\times10^{-2})(9.8) = (2)B\sin(90^\circ)$

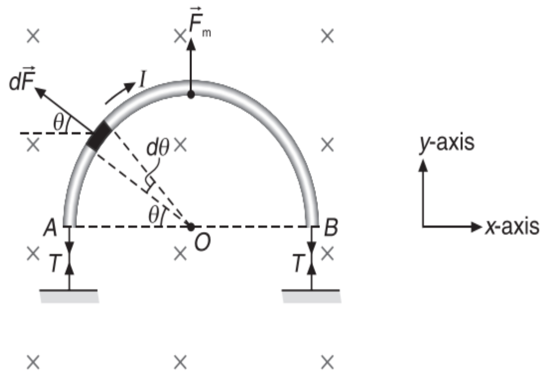
$\Rightarrow \quad B = 0.245$ T and from Fleming's Left Hand Rule the direction is eastward.

6. Since $\vec{B} = B\hat{j}$

Segment	ℓ	$\vec{F} = I(\vec{\ell}\times\vec{B})$
ab	$-\ell\hat{j}$	0
bc	$\ell\hat{k}$	$BI\ell(-\hat{i})$
cd	$-\ell\hat{i}+\ell\hat{j}$	$BI\ell(-\hat{k})$
da	$\ell\hat{i}-\ell\hat{k}$	$BI\ell(\hat{k}+\hat{i})$



7. To find the force on the conductor, consider a small arc of length $dl = Rd\theta$ as shown in Figure.



This arc can be considered as a straight conductor of length dl. The force on this arc is given by

$$dF = BIdl$$

$$\Rightarrow \quad d\vec{F} = BIdl\left(-\cos\theta\hat{i} + \sin\theta\hat{j}\right)$$

$$\Rightarrow \quad dF = BI(Rd\theta)\left(-\cos\theta\hat{i} + \sin\theta\hat{j}\right)$$

Net force on the semi circular rod is given by

$$\vec{F}_m = \int_0^{\pi} d\vec{F}$$

$$\Rightarrow \quad \vec{F}_m = BIR\left(-\int_0^{\pi}\cos\theta d\theta\hat{i} + \int_0^{\pi}\sin\theta d\theta\hat{j}\right)$$

$$\Rightarrow \quad \vec{F}_m = BIR(0+2)\hat{j}$$

$$\Rightarrow \quad \vec{F}_m = (2BIR)\hat{j}$$

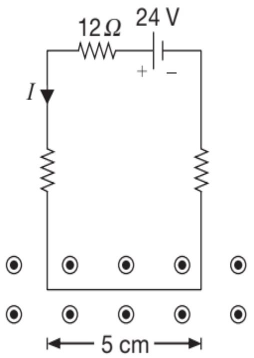
The force experienced by semi-circular wire AB is $2BiR$, perpendicular to its length AB.
For equilibrium, we must have

$$2T = F_m = 2BIR$$

$$\Rightarrow \quad T = BIR$$

8. Let x_1 be the elongation due to the weight of the wire and let x_2 be the additional elongation of the springs when the magnetic field is turned on. Then $F_{\text{magnetic}} = 2kx_2$ where k is the force constant of the spring and can be determined from $k = \frac{mg}{2x_1}$. (The factor 2 is included since there are 2 springs in parallel). Combining these two equations, we get

$$F = F_{\text{magnetic}} = 2\left(\frac{mg}{2x_1}\right)x_2 = \frac{mgx_2}{x_1}$$



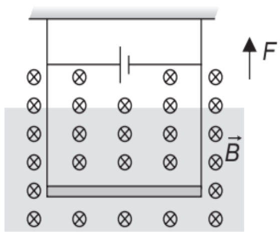
But $|\vec{F}| = I|\vec{L} \times \vec{B}| = BIL$

where $I = \dfrac{24\text{ V}}{12\ \Omega} = 2\text{ A}$

$$\Rightarrow \quad B = \frac{mgx_2}{ILx_1} = \frac{(0.1)(9.8)(3\times10^{-3})}{(2)(0.05)(5\times10^{-3})} = 0.588\text{ T}$$

9. $\dfrac{|F|}{\ell} = \dfrac{mg}{\ell} = \dfrac{I|\vec{\ell} \times \vec{B}|}{\ell}$

$$\Rightarrow \quad I = \frac{mg}{B\ell} = \frac{(0.04\text{ kgm}^{-1})(9.8\text{ ms}^{-2})}{3.6\text{ T}} = 0.109\text{ A}$$



The direction of I in the bar is to the right.

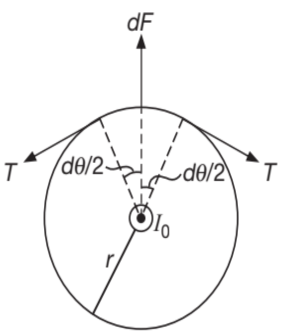
10. TRY YOURSELF

Same as Illustration 71

$$B = \frac{2\lambda g}{\sqrt{3}I}$$

11. Considering the magnetic forces between I_0 and the differential current dI we obtain

$$dF = \frac{\mu_0 I_0}{2\pi r}(dI)\left(dI = \frac{Id\theta}{2\pi}\right)$$



where dF is the force per unit length.

If T is the tension per unit length, then for the elementary portion to be in equilibrium, we have

$$2T\sin\left(\frac{d\theta}{2}\right) = dF$$

$$\Rightarrow \quad Td\theta = dF \quad \left[\text{as } d\theta \text{ is very small, } \sin\frac{d\theta}{2} \cong \frac{d\theta}{2}\right]$$

$$\Rightarrow \quad Td\theta = \frac{\mu_0 I_0}{2\pi r}\left(\frac{Id\theta}{2\pi}\right)$$

$$\Rightarrow \quad T = \frac{\mu_0 I_0 I}{4\pi^2 r}$$

12. METHOD I: Using the basic formula

The conductor $ACDEFB$ consists of 5 straight sections i.e. AC, CD, DE, EF and FB each having length a. The respective forces on these sections of conductor are

$$\vec{F}_{AC} = I\left(a\hat{i} \times B_0\hat{k}\right) = -\left(B_0 Ia\right)\hat{j}$$

$$\vec{F}_{CD} = I\left(a\hat{j} \times B_0\hat{k}\right) = \left(B_0 Ia\right)\hat{i}$$

$$\vec{F}_{DE} = I\left(a\hat{i} \times B_0\hat{k}\right) = -\left(B_0 Ia\right)\hat{j}$$

$$\vec{F}_{EF} = I\left(-a\hat{j} \times B_0\hat{k}\right) = -\left(B_0 Ia\right)\hat{i}$$

$$\vec{F}_{FB} = I\left(a\hat{i} \times B_0\hat{k}\right) = -\left(B_0 Ia\right)\hat{j}$$

So, the net force is given by

$$\vec{F} = \vec{F}_{AC} + \vec{F}_{CD} + \vec{F}_{DE} + \vec{F}_{EF} + \vec{F}_{FB}$$

$$\Rightarrow \quad \vec{F} = -3B_0 Ia\hat{j}$$

METHOD II: Using the concept of equivalent length

Instead of finding the force on individual wires, we can directly find the net force by finding effective length of the wire. The line joining A and B has length $3a$ thus net force equals.

$$\vec{F} = I\left(3a\hat{i} \times B_0\hat{k}\right)$$

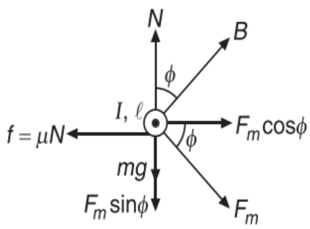
$$\Rightarrow \quad \vec{F} = 3aB_0 I\left(-\hat{j}\right)$$

$$\Rightarrow \quad \vec{F} = -3aB_0 I\hat{j}$$

13. The magnetic force is $F_m = IlB\sin\theta$, where θ is the angle between current and magnetic field. In this case $\theta = 90°$.

$$\Rightarrow \quad F_m = BIl$$

Since F_m acts at an angle ϕ with horizontal as shown in figure (according to Fleming left hand rule or right palm rule)



CHAPTER 1

Since the rod is in equilibrium, so the net force along x and y axis are given by

$$F_y = N - (mg + F_m \sin\phi) = 0$$

$$\Rightarrow N = mg + F_m \sin\phi \quad \ldots(1)$$

$$F_x = f - F_m \cos\phi = 0$$

$$\Rightarrow f = F_m \cos\phi \quad \ldots(2)$$

Also, we know that the frictional force f is given by

$$f = \mu N \quad \ldots(3)$$

Substituting (1) and (2) in (3), we get

$$\mu(mg + F_m \sin\phi) = F_m \cos\phi$$

$$\Rightarrow F_m = \frac{\mu mg}{\cos\phi - \mu \sin\phi}$$

$$\Rightarrow BIl = \frac{\mu mg}{\cos\phi - \mu \sin\phi}$$

$$\Rightarrow I = \frac{\mu mg}{Bl(\cos\phi - \mu \sin\phi)}$$

Test Your Concepts-VII
(Based on Magnetic Moment and Torque)

1. $|\tau| = MB\sin 90 = BIA$, where the effective current due to the orbiting electrons is given by

$$I = \frac{\text{Charge Circulating}}{\text{Period of one Revolution}} = \frac{q}{T}$$

and the period of the motion is

$$T = \frac{2\pi R}{v}$$

For an electron orbiting the nucleus, we have

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{R^2} = \frac{mv^2}{R}$$

$$\Rightarrow v = q\sqrt{\frac{1}{4\pi\varepsilon_0 mR}}$$

Substituting this expression for v into the equation for T, we get

$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mR^3}{q^2}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{(9\times10^{-31})(0.53\times10^{-10})^3}{(1.6\times10^{-19})^2(9\times10^9)}}$$

$$\Rightarrow T = 1.52\times10^{-16}\ \text{s}$$

Therefore, $|\tau| = BIA = B\left(\frac{q}{T}\right)A$

$$\Rightarrow |\tau| = \frac{1.6\times10^{-19}}{1.52\times10^{-16}}\pi(0.53\times10^{-10})^2(0.4)$$

$$\Rightarrow |\tau| = 3.7\times10^{-24}\ \text{Nm}$$

2. Work done in rotating the loop from $\theta_1 = 60°$ to $\theta_2 = 120°$ is

$$W = \Delta U = U_f - U_i$$

Since, we know that the change in potential energy does not depend on choice of reference zero for potential energy, so we have

$$W = -MB(\cos\theta_2 - \cos\theta_1)$$

$$\Rightarrow W = -MB(\cos 120° - \cos 60°)$$

$$\Rightarrow W = MB$$

Since we know that the result $U = -MB\cos\theta$ has been obtained for $\theta = 90°$ as the reference, because $U_{90°} = 0$. So, for this zero reference the potential energy at $\theta = 0°$ and $\theta = 45°$ respectively are

$$U_{0°} = -MB \text{ and } U_{45°} = -\frac{MB}{\sqrt{2}}$$

The potential energy difference between position $\theta_1 = 0°$ and $\theta_2 = 45°$ is

$$\Delta U = -MB\cos 45° - (-MB\cos 0°)$$

$$\Rightarrow \Delta U = MB\left(1 - \frac{1}{\sqrt{2}}\right)$$

For $\theta = 0°$ as the new reference, potential energy difference between the same positions should also be the same (because the change in potential energy is independent of the choice of reference level). So, if $U'_{45°}$ and $U'_{0°}$ be the respective potential energies with respect to the new reference, then we have

$$\Delta U = \underbrace{U'_{45°} - U'_{0°}}_{\text{reference at }0°} = \underbrace{(U_{45°} - U_{0°})}_{\text{reference at }90°}$$

Since we are given that $U'_{0°} = 0$

$$\Rightarrow \underbrace{U'_{45°} - 0}_{\text{reference at }0°} = \underbrace{(U_{45°} - U_{0°})}_{\text{reference at }90°}$$

$$\Rightarrow U'_{45°} = MB\left(1 - \frac{1}{\sqrt{2}}\right)$$

3. (a) $2\pi r = 2\ \text{m}$

$$\Rightarrow r = 0.318\ \text{m}$$

The magnetic moment M is given by

$$M = IA = (17\times10^{-3}\ \text{A})[\pi(0.318)^2\ \text{m}^2]$$

$$M = 5.41\times10^{-3}\ \text{Am}^2$$

(b) $\vec{\tau} = \vec{M}\times\vec{B}$

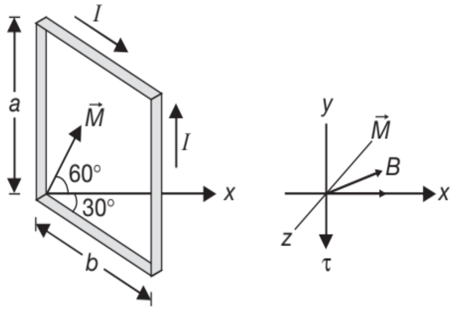
$$\Rightarrow \tau = (5.41\times10^{-3}\ \text{Am}^2)(0.8\ \text{T})$$

$$\Rightarrow \tau = 4.33\times10^{-3}\ \text{Nm}.$$

4. $\tau = NBIA\sin\theta$

$\Rightarrow \quad \tau = 100(0.8\text{ T})(0.4\times 0.3\text{ m}^2)(1.2\text{ A})\sin(60°)$

$\Rightarrow \quad \tau = 9.98\text{ Nm}$



Note that θ is the angle between the magnetic moment and the magnetic field. The loop will rotate so as to align the magnetic moment with the field. Looking down along the y-axis, the loop will rotate in a clockwise direction.

5. (a) Let θ represent the unknown angle, L, the total length of the wire and d, the length of one side of the square coil. Then, using the definition of magnetic moment and the right hand rule in figure, we find

$$\mu = (NI)A$$

$$\Rightarrow \quad \mu = \left(\frac{L}{4d}\right)d^2 I \text{ at angle } \theta \text{ with the horizontal}$$

At equilibrium, $\sum \vec{\tau} = (\vec{\mu}\times\vec{B}) - (\vec{r}\times m\vec{g}) = 0$

$$\Rightarrow \quad \left(\frac{BILd}{4}\right)\sin(90° - \theta) - \left(\frac{mgd}{2}\right)\sin\theta = 0$$

$$\text{and } \left(\frac{mgd}{2}\right)\sin\theta = \left(\frac{BILd}{4}\right)\cos\theta$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{BIL}{2mg}\right)$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{(0.01\text{ T})(3.4\text{ A})(4\text{ m})}{2(0.1\text{ kg})(9.8\text{ ms}^{-2})}\right) \simeq 4°$$

(b) $\tau_m = \left(\frac{BILd}{4}\right)\cos\theta$

$$\tau_m = \frac{1}{4}(3.4\text{ A})(4\text{ m})(0.01\text{ T})(0.1\text{ m})\cos 4°$$

$$\tau_m = 3.39\times 10^{-3}\text{ Nm}$$

6. From $\vec{\tau} = \vec{M}\times\vec{B} = (I\vec{A})\times\vec{B}$, the magnitude of the torque is $BIA\sin(90°)$

(a) Each side of the triangle is $\frac{40\text{ cm}}{3}$

Its altitude is $\sqrt{13.3^2 - 6.67^2}\text{ cm} = 11.5\text{ cm}$ and its area is

$$A = \frac{1}{2}(11.5\text{ cm})(13.3\text{ cm}) = 7.7\times 10^{-3}\text{ m}^2$$

$\Rightarrow \quad \tau = (0.52)(20)(7.7\times 10^{-3})$

$\Rightarrow \quad \tau = 80\times 10^{-3}\text{ Nm}$

(b) Each side of the square is 10 cm and its area is $100\text{ cm}^2 = 10^{-2}\text{ m}^2$

$\Rightarrow \quad \tau = (0.52\text{ T})(20\text{ A})(10^{-2}\text{ m}^2) = 0.104\text{ Nm}$

(c) $r = \frac{0.4\text{ m}}{2\pi} = 0.0637\text{ m}$

$\Rightarrow \quad A = \pi r^2 = 1.27\times 10^{-2}\text{ m}^2$

$\Rightarrow \quad \tau = (0.52)(20\text{ A})(1.27\times 10^{-2}\text{ m}^2)$

$\Rightarrow \quad \tau = 0.132\text{ Nm}$

(d) The circular loop experiences the largest torque

7. (a) $\vec{\tau} = \vec{M}\times\vec{B}$

$\Rightarrow \quad |\vec{\tau}| = |\vec{M}\times\vec{B}| = MB\sin\theta = NIAB\sin\theta$

$\Rightarrow \quad \tau_{\max} = NIAB\sin(90°)$

$\Rightarrow \quad \tau_{\max} = 1(5\text{ A})[\pi(0.05\text{ m})^2](3\times 10^{-3}\text{ T})$

$\Rightarrow \quad \tau_{\max} = 118\ \mu\text{Nm}$

(b) $U = -\vec{M}\cdot\vec{B} = -MB\cos\theta$

$U_{MIN} = -MB$, when $\theta = 0°$

$U_{MAX} = MB$, when $\theta = 180°$

$\Rightarrow \quad -MB \le U \le MB$, where

$MB = (NIA)B = 1(5)[\pi(0.05)^2](3\times 10^{-3})$

$MB = 118\ \mu\text{J}$

the range of the potential energy is

$$-118\ \mu\text{J} \le U \le +118\ \mu\text{J}$$

8. The magnetic dipole moment of the loop is initially directed along the z-axis due to which the torque exerted by the field turns it about x-axis so as to finally align its magnetic moment along the field i.e. y-axis. Since there is no external force or torque, so the total mechanical energy is of the system is conserved. Hence

$$K + U = \text{constant, where } U = -MB\cos\theta$$

$\Rightarrow \quad (K+U)_{\text{initial}} = (K+U)_{\text{final}}$

$\Rightarrow \quad 0 + 0 = \frac{1}{2}I\omega^2 - MB\cos 0° \qquad \ldots(1)$

where I is the moment of inertia of the ring about its diameter i.e. $I = \frac{1}{2}mR^2$

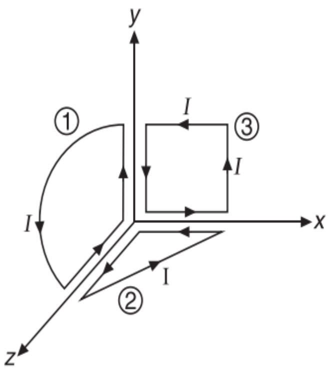
CHAPTER 1

From (1), we get

$$\omega=\sqrt{\frac{2MB}{I}}=\sqrt{\frac{2i(\pi R^2B)}{\frac{1}{2}mR^2}}$$

$$\Rightarrow \quad \omega=\sqrt{\frac{4\pi iB}{m}}$$

9. Let us split the given loop in three separate loops such that when overlapped at the three coordinate axes, the opposite currents in the wire segments in these three loops along the axes will cancel each other as shown in Figure.



This arrangement of three loops is equivalent to the single loop arrangement asked in the question. The magnetic moment of the loop is the vector sum of the three individual loops. So

$$\vec{M}=\vec{M}_1+\vec{M}_2+\vec{M}_3$$

$$\Rightarrow \quad \vec{M}=I\left(\frac{\pi R^2}{4}\right)\hat{i}+I\left(\frac{R^2}{2}\right)\hat{j}+I(R^2)\hat{k}$$

10. For a moving coil galvanometer, we have

$$i=\left(\frac{C}{NAB}\right)\theta$$

The torsional constant of the spring is given as

$$C=\frac{NABi}{\theta}$$

Substituting the values in SI units we get

$$C=\frac{(60)(5.0\times10^{-4})(9\times10^{-3})(0.2\times10^{-3})}{18\times\frac{\pi}{180}}$$

$$\Rightarrow \quad C=1.71\times10^{-7}\ \text{Nmrad}^{-1}$$

11. $M=IA$

where $I=\dfrac{\text{Charge Circulating}}{\text{Period of one Revolution}}$

$$I=\frac{e}{\left(\frac{2\pi r_n}{v_n}\right)}$$

where $r_n=$ radius of the nth orbit

$v_n=$ velocity of the electron in the nth orbit

$$r_n=\frac{n^2h^2\varepsilon_0}{\pi me^2} \text{ and } v_n=\frac{e^2}{2h\varepsilon_0 n}$$

$$M=(\pi r_n^2)\left(\frac{ev_n}{2\pi r_n}\right)$$

$$\Rightarrow \quad M=\frac{ev_nr_n}{2}$$

$$\Rightarrow \quad M=\frac{e}{2}\left(\frac{n^2h^2\varepsilon_0}{\pi me^2}\right)\left(\frac{e^2}{2h\varepsilon_0 n}\right)$$

$$\Rightarrow \quad M=\frac{neh}{4\pi m}\text{, where } m=\text{mass of electron}$$

$$B=\frac{\mu_0 I_n}{2r_n}$$

$$\Rightarrow \quad B=\left(\frac{\mu_0}{2r_n}\right)\left(\frac{ev_n}{2\pi r_n}\right)$$

$$\Rightarrow \quad B=\left(\frac{\mu_0 e}{4\pi}\right)\left(\frac{v_n}{r_n^2}\right)$$

$$\Rightarrow \quad B=\frac{\mu_0 e}{4\pi}\left(\frac{e^2}{2h\varepsilon_0 n}\right)\left(\frac{\pi^2m^2e^4}{n^4h^4\varepsilon_0^2}\right)$$

$$\Rightarrow \quad B=\frac{\mu_0\pi m^2e^7}{8\varepsilon_0h^5n^5}$$

12. DO YOURSELF

$$\vec{F}_{AB}=-\vec{F}_{CD}=-N(BI\ell\sin\theta)\hat{k}$$

$$\vec{F}_{BC}=-\vec{F}_{DA}=N(BIb\sin\theta)\hat{j}$$

$$\vec{\tau}=\sqrt{2}(NBI\ell b\sin\theta\cos\theta)\hat{k}$$

or $\vec{\tau}=(MB\cos\theta)\hat{k}$ where $M=\sqrt{2}(NI\ell b)\sin\theta$

13. Magnitude of torque is given by

$$|\vec{\tau}|=MB\sin\theta$$

Here, $M=NIA=(1)(1)(\pi)(0.2)^2=(0.04\,\pi)\ \text{Am}^2$

$B=2\ T$ and $\theta=$ angle between $\vec{M}$ and $\vec{B}$ is $90°$

$$\Rightarrow \quad |\vec{\tau}|=(0.04\,\pi)(2)\sin 90°=0.25\ \text{Nm}$$

Vectorially, we have

$$\vec{B}=2\cos(45°)\hat{i}+2\sin(45°)\hat{j}=\sqrt{2}(\hat{i}+\hat{j})\ \text{T}$$

and $\vec{M}=-(0.04\pi)\hat{k}\ \text{Am}^2$

Since, we know that $\vec{\tau}=\vec{M}\times\vec{B}=\left(0.04\sqrt{2}\pi\right)\left(-\hat{j}+\hat{i}\right)$

$\Rightarrow \quad \vec{\tau}=0.18\left(\hat{i}-\hat{j}\right)$ Nm

14. (a) The magnetic field intensity inside the solenoid is $B=\mu_0 nI$

The magnetic moment of the coil is $\mu=NIA$

$$\Rightarrow \quad \mu=NI_0\left(\pi r^2\right)=\pi I_0 r^2 \qquad \{\because N=1\}$$

The maximum torque $=\tau_{\max}=\left|\vec{\mu}\times\vec{B}\right|_{\max}=\mu B$

$$\Rightarrow \quad \tau=\left(I_0\pi r^2\right)\left(\mu_0 nI\right)=\mu_0\pi nII_0 r^2$$

(b) For small displacement from mean position, $\tau=\mu B\theta$

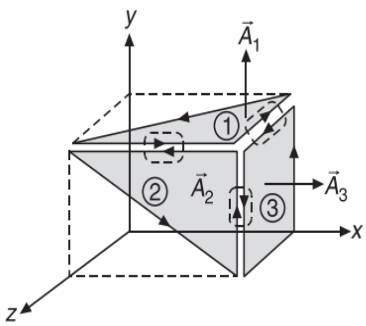
$\therefore$ Motion will be angular SHM

Period of oscillation, $T=2\pi\sqrt{\dfrac{I}{\mu B}}$

$$T=2\pi\sqrt{\frac{\left(\frac{2}{5}\right)mr^2}{\pi r^2 I_0\mu_0 nI}}=2\pi\sqrt{\frac{2m}{5\pi\mu_0 nII_0}}$$

15. The net force on a current carrying loop of any arbitrary shape placed in a uniform magnetic field is zero. Hence, $\vec{F}_{\text{net}}=0$.

The given loop can be considered to be a superposition of three loops (1, 2 and 3) as shown in Figure.



The respective area vectors (in square metre) of the three loops 1, 2 and 3 are

$$\vec{A}_1=\frac{1}{2}\left(\frac{10}{100}\right)\left(\frac{10}{100}\right)\hat{j}=(0.005)\hat{j}$$

$$\vec{A}_2=\frac{1}{2}\left(\frac{10}{100}\right)\left(\frac{10}{100}\right)\hat{k}=(0.005)\hat{k} \text{ and}$$

$$\vec{A}_3=\left(\frac{10}{100}\right)\left(\frac{10}{100}\right)\hat{i}=(0.01)\hat{i}$$

Magnetic moment vector, $\vec{M}=i\vec{A}$

$$\Rightarrow \quad \vec{M}=10\left(0.01\hat{i}+0.005\hat{j}+0.005\hat{k}\right)\text{ Am}^2$$

$$\Rightarrow \quad \vec{M}=\left(0.1\hat{i}+0.05\hat{j}+0.05\hat{k}\right)\text{ Am}^2$$

The torque $\vec{\tau}$ acting on the loop is given by

$$\vec{\tau}=\vec{M}\times\vec{B}$$

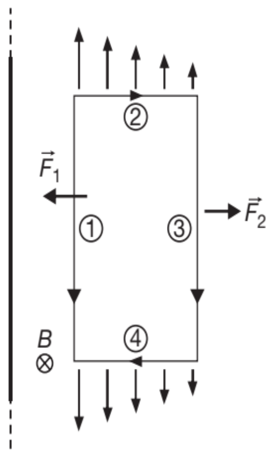
$$\vec{\tau}=\left(0.1\hat{i}+0.05\hat{j}+0.05\hat{k}\right)\times\left(2\hat{i}-3\hat{j}+\hat{k}\right)$$

$$\Rightarrow \quad \vec{\tau}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0.1 & 0.05 & 0.05\\ 2 & -3 & 1\end{vmatrix}=\left(-0.1\hat{i}-0.4\hat{k}\right)\text{ Nm}$$

Test Your Concepts-VIII (Based on Force between Current Carrying Conductors)

1. By symmetry, we note that the magnetic forces on the top and bottom segments of the rectangle cancel. The net force on the vertical segments of the rectangle is

$$\vec{F}=\vec{F}_1+\vec{F}_2=\frac{\mu_0 I_1 I_2\ell}{2\pi}\left(\frac{1}{c+a}-\frac{1}{c}\right)\hat{i}$$



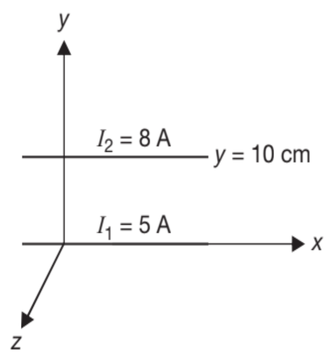
$$\Rightarrow \quad \vec{F}=\frac{\mu_0 I_1 I_2\ell}{2\pi}\left(\frac{-a}{c(c+a)}\right)\hat{i}$$

$$\Rightarrow \quad \vec{F}=-\frac{\left(4\pi\times10^{-7}\right)(5)(10)(0.45)}{2\pi}\left(\frac{0.15}{(0.1)(0.25)}\right)\hat{i}$$

$$\Rightarrow \quad \vec{F}=-\left(2.7\times10^{-5}\hat{i}\right)\text{ N}$$

$$\Rightarrow \quad \vec{F}=2.7\times10^{-5}\text{ N toward the left}$$

2. Consider both wires to carry current in the x direction, the first at $y=0$ and the second at $y=10$ cm.



(a) $\vec{B}=\left(\dfrac{\mu_0 I}{2\pi r}\right)\hat{k}=\dfrac{(4\pi\times10^{-7}\ \text{T}\cdot\text{mA}^{-1})(5\ \text{A})}{2\pi(0.1\ \text{m})}\hat{k}$

$\vec{B}=1\times10^{-5}\ \text{T}$ out of the page

(b) $\vec{F}=I_2(\vec{\ell}\times\vec{B})=(8\ \text{A})\left[(1\ \text{m})\hat{i}\times(1\times10^{-5}\ \text{T})\hat{k}\right]$

$\vec{F}=(8\times10^{-5}\ \text{N})(-\hat{j})$

$\vec{F}=8\times10^{-5}\ \text{N}$ toward the first wire

(c) $\vec{B}=\left(\dfrac{\mu_0 I}{2\pi r}\right)(-\hat{k})=\dfrac{(4\pi\times10^{-7})(8)}{2\pi(0.1)}(-\hat{k})$

$\vec{B}=(1.6\times10^{-5}\ \text{T})(-\hat{k})$

$\vec{B}=1.6\times10^{-5}\ \text{T}$ into the page

(d) $\vec{F}=I_1(\vec{\ell}\times\vec{B})=(5)\left[(1)\hat{i}\times(1.6\times10^{-5})(-\hat{k})\right]$

$\vec{F}=(8\times10^{-5}\ \text{N})(+\hat{j})$

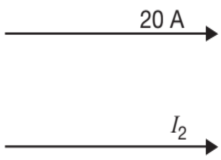
$\vec{F}=8\times10^{-5}\ \text{N}$ towards the second wire

3. To attract, both currents must be to the right. The attraction is described by

$$F=BI_2\ell\sin90^\circ=I_2\ell\frac{\mu_0 I}{2\pi r}$$

$$\Rightarrow\ I_2=\frac{F}{\ell}\frac{2\pi r}{\mu_0 I_1}=(320\times10^{-6})\left(\frac{2\pi(0.5)}{4\pi\times10^{-7}(20)}\right)$$

$$\Rightarrow\ I_2=40\ \text{A}$$



Let y represent the distance of the zero field point below the upper wire.

Then $\vec{B}=\dfrac{\mu_0 I}{2\pi r}\otimes+\dfrac{\mu_0 I}{2\pi r}\odot$

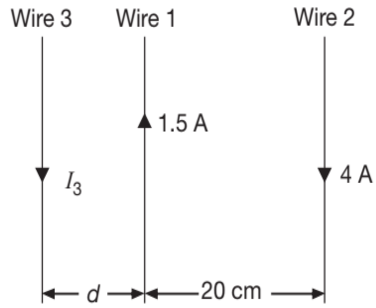
$$\Rightarrow\ 0=\frac{\mu_0}{2\pi}\left(\frac{20\ \text{A}}{y}-\frac{40\ \text{A}}{(0.5-y)}\right)$$

$$\Rightarrow\ 20(0.5-y)=40y$$

$$\Rightarrow\ 20(0.5\ \text{m})=60y$$

$$\Rightarrow\ y=0.167\ \text{m}\ \text{below the upper wire}$$

4. Since wire 1 and 2 carry currents in opposite directions, so they repel each other.
If wire 3 were between them, it would have to repel either 1 or 2, so the force on that wire could not be zero.



If wire 3 were to the right of wire 2, it would feel a larger force exerted by 2 than that exerted by 1, so the total force on 3 could not be zero.
Therefore wire 3 must be to the left of both other wires as shown. It must carry downward current so that it can attract wire 2.
For the equilibrium of wire 3 we have

$$F_{31}=F_{32}$$

$$\Rightarrow\ \frac{\mu_0(1.5\ \text{A})I_3}{2\pi d}=\frac{\mu_0(4\ \text{A})I_3}{2\pi(20\ \text{cm}+d)}$$

$$\Rightarrow\ 1.5(20\ \text{cm}+d)=4d$$

$$\Rightarrow\ d=\frac{30\ \text{cm}}{2.5}=12\ \text{cm}\ \text{to the left of wire 1}$$

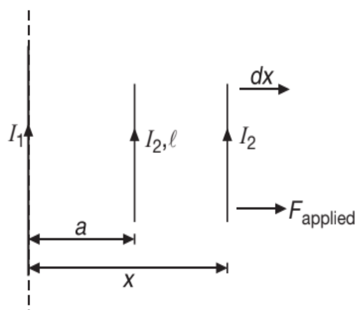
For the equilibrium of wire 1,

$$\frac{\mu_0 I_3(1.5\ \text{A})}{2\pi(12\ \text{cm})}=\frac{\mu_0(4\ \text{A})(1.5\ \text{A})}{2\pi(20\ \text{cm})}$$

$$\Rightarrow\ I_3=\frac{12}{20}4\ \text{A}=2.4\ \text{A}\ \text{down}$$

We know that wire 2 must be in equilibrium because the forces on it are equal in magnitude to the forces that it exerts on wires 1 and 3, which are equal because they both balance the equal magnitude forces that 1 exerts on 3 and that 3 exerts on 1.

5. Let the separation between the conductors be increased through a distance b as shown in Figure.



The force acting on the conductor of length l when it is at a distance x from the infinite conductor is given by

$$F=\left(\frac{\mu_0 I_1 I_2}{2\pi x}\right)l$$

The minimum work required to move it away by small displacement dx is by applying an external force F_{applied} such that the conductor moves very slowly and hence the applied force must be equal to the magnetic force i.e.

$$F_{\text{applied}} = F = \left(\frac{\mu_0 I_1 I_2}{2\pi x}\right) l$$

If dW be the small work done in displacing the conductor through dx, then

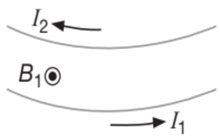
$$dW = F dx$$

$$\Rightarrow \quad dW = \left(\frac{\mu_0 I_1 I_2 l}{2\pi}\right)\frac{dx}{x}$$

$$\Rightarrow \quad W = \frac{\mu_0 I_1 I_2 l}{2\pi}\int_a^{a+b}\frac{dx}{x} = \left(\frac{\mu_0 I_1 I_2 l}{2\pi}\right)\ln x\Big|_a^{a+b}$$

$$\Rightarrow \quad W = \frac{\mu_0 I_1 I_2 l}{2\pi}\ln\left(\frac{a+b}{a}\right)$$

6. Let us arrange two wires as straight parallel wires as shown.



(a) $F = \frac{\mu_0 I^2 \ell}{2\pi r}$

$$\Rightarrow \quad F = \frac{(4\pi\times10^{-7})(140)^2(2\pi)(0.1)}{2\pi(1\times10^{-3})}$$

$$\Rightarrow \quad F = 2.46\text{ N upward}$$

(b) Since $F - mg = ma$, where $m =$ mass of the loop $a =$ acceleration of the loop.

$$\Rightarrow \quad a = \frac{F - mg}{m}$$

$$\Rightarrow \quad a = \frac{2.46-(0.021)(10)}{0.021}$$

$$\Rightarrow \quad a \cong 107\text{ ms}^{-2}\text{, upwards}$$

7. At equilibrium, $\frac{F}{\ell} = \frac{\mu_0 I_A I_B}{2\pi r} = \frac{mg}{\ell}$

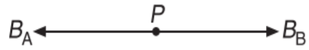
$$\Rightarrow \quad I_B = \frac{2\pi r\left(\frac{m}{\ell}\right)g}{\mu_0 I_A}$$

$$\Rightarrow \quad I_B = \frac{2\pi(0.025\text{ m})(0.01\text{ kgm}^{-1})(9.8\text{ ms}^{-2})}{(4\pi\times10^{-7}\text{ TmA}^{-1})(150\text{ A})}$$

$$\Rightarrow \quad I_B = 81.7\text{ A}$$

$$\Rightarrow \quad I_B \cong 82\text{ A}$$

8. At the point P, the field due to the wires A and B is shown in Figure.



For the net magnetic field at P to be zero, the direction of current at B should be perpendicular to paper outwards. Let current in the wire B be I_B. Then,

$$\frac{\mu_0}{2\pi}\frac{I_A}{\left(2+\frac{10}{11}\right)} = \frac{\mu_0}{2\pi}\frac{I_B}{\left(\frac{10}{11}\right)}$$

$$\Rightarrow \quad \frac{I_B}{I_A} = \frac{10}{32}$$

$$\Rightarrow \quad I_B = \frac{10}{32}\times I_A = \frac{10}{32}\times 9.6 = 3\text{ A}$$

Force per unit length on wire B is given by

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_A I_B}{r} \qquad \{\because r = AB = 2\text{ m}\}$$

$$\Rightarrow \quad \frac{F}{l} = \frac{(2\times10^{-7})(9.6\times3)}{2}$$

$$\Rightarrow \quad \frac{F}{l} = 2.88\times10^{-6}\text{ Nm}^{-1}$$

Single Correct Choice Type Questions

1. $B = \frac{\mu_0 q}{4\pi}\left(\frac{\vec{v}\times\vec{r}}{r^3}\right)$

$\vec{v}$ is the velocity of charge with respect to observer. Let the observers A and B have velocities $\vec{v}_A$ and $\vec{v}_B$ and since both see that a point charge produces same magnetic field at the same point, so

$$\vec{v}_{CA}\times\vec{r} = \vec{v}_{CB}\times\vec{r}$$

where $\vec{v}_{CA}$ is velocity of charge w.r.t. A and $\vec{v}_{CB}$ is velocity of charge w.r.t. B.

$$\Rightarrow \quad (\vec{v}_C - \vec{v}_A)\times\vec{r} = (\vec{v}_C - \vec{v}_B)\times\vec{r}$$

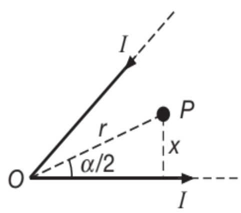
$$\Rightarrow \quad (\vec{v}_A - \vec{v}_B)\times\vec{r} = 0$$

$$\Rightarrow \quad (\vec{v}_A - \vec{v}_B)\parallel\vec{r}$$

So, the relative velocity of observers is parallel to $\vec{r}$

Hence, the correct answer is (A).

2.



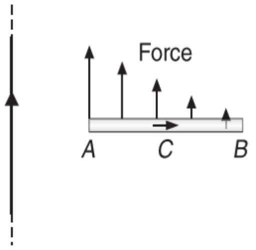
$$\Rightarrow \quad B_P = 2\left(\frac{\mu_0 I}{4\pi x}\right)\left[\sin\left(90^\circ - \frac{\alpha}{2}\right) + \sin 90^\circ\right]$$

Since $x = r\sin\left(\frac{\alpha}{2}\right)$

$$\Rightarrow \quad B_P = \frac{\mu_0 I}{2\pi r}\left(\frac{1+\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}\right) = \frac{\mu_0 I}{2\pi r}\left[\operatorname{cosec}\left(\frac{\alpha}{2}\right) + \cot\left(\frac{\alpha}{2}\right)\right]$$

Hence, the correct answer is (C).

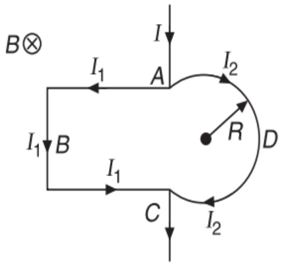
3. Variation of magnetic force on wire ACB is as shown in Figure.



Point of application of net force lies somewhere between A and C.

Hence, the correct answer is (D).

4.



Since $F = BIl_{\text{eff}}$

$\Rightarrow \quad F_{ABC} = BI_1 l$ (towards right)

Also, $F_{ADC} = BI_2 l$ (towards right)

$\Rightarrow \quad F_{\text{loop}} = F_{ABC} + F_{ADC}$

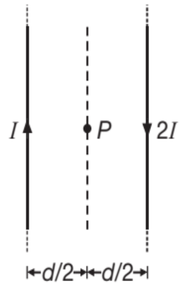
$\Rightarrow \quad F_{\text{loop}} = B(I_1 + I_2)l$

$\Rightarrow \quad F_{\text{loop}} = BIl \qquad \{\because I_1 + I_2 = I\}$

Hence, the correct answer is (B).

5. Consider the gas as a thick conductor carrying a uniform current, use Ampere's Law to find the magnetic field and then apply the left-hand rule to find the direction of the Ampere force.
Hence, the correct answer is (B).

6. The magnetic field at point P is perpendicular to paper inwards due to both the wires. Since, the charged particle is also projected in the same direction. So, force on charged particle is zero.



$\Rightarrow \quad \vec{F}_m = q(\vec{v} \times \vec{B}) = \vec{0} \qquad \{\because \vec{v} \parallel \vec{B}\}$

Hence, the correct answer is (D).

7. Since $\frac{F_{\text{magnetic}}}{F_{\text{electrostatic}}} = \frac{v^2}{c^2}$

So, the ratio of electric field to magnetic field is independent of r.

Hence, the correct answer is (D).

8. $\vec{F} = I(\vec{l}_{\text{eff}} \times \vec{B})$

where $\vec{l}_{\text{eff}} = 2[\cos(45^\circ)\hat{i} + \sin(45^\circ)\hat{j}]$

$\Rightarrow \quad \vec{l}_{\text{eff}} = \sqrt{2}\hat{i} + \sqrt{2}\hat{j}$

$$\vec{F} = \begin{vmatrix} \overset{\oplus}{\hat{i}} & \overset{\ominus}{\hat{j}} & \overset{\oplus}{\hat{k}} \\ \sqrt{2} & \sqrt{2} & 0 \\ 3 & 4 & 1 \end{vmatrix}$$

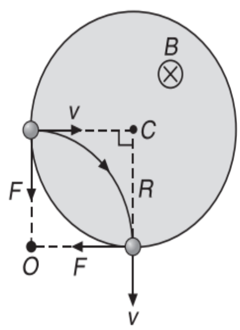
$\Rightarrow \quad \vec{F} = \sqrt{2}\hat{i} - \sqrt{2}\hat{j} + \sqrt{2}\hat{k}$

$\Rightarrow \quad \vec{F} = \sqrt{2}(\hat{i} - \hat{j} + \hat{k})$

Hence, the correct answer is (B).

9. Net force on a current carrying loop in a uniform magnetic field is zero. So magnetic force cannot balance its weight.
Hence, the correct answer is (D).

10. The particle will move along a circle of radius that equals the radius of circular region in which magnetic field is present (as shown in Figure).



Radius of the circular path followed by the particle is

$$R=\frac{mv}{qB}$$

$$\Rightarrow \quad v=\frac{qBR}{m}$$

$$\Rightarrow \quad v=\frac{5\times10^{-6}\times4\times(0.1)}{2\times10^{-3}}=10^{-3}\ \text{ms}^{-1}=1\ \text{mms}^{-1}$$

Hence, the correct answer is (D).

11. The pitch p of the helical path is given by

$$p=\frac{2\pi mv\cos\theta}{qB}$$

$$\Rightarrow \quad p=\frac{2\pi m}{qB}(v\cos45^\circ)=\frac{2\pi m}{qB}(v\sin45^\circ)$$

Since $r=\dfrac{mv\sin\theta}{qB}$

$$\Rightarrow \quad r=\frac{mv\sin45^\circ}{qB}=\frac{p}{2\pi}$$

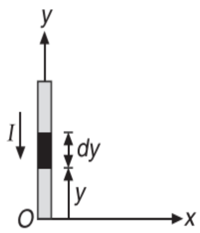
Hence, the correct answer is (D).

12. Since $d\vec{F}=I(d\vec{l}\times\vec{B})$

where $I=2\times10^{-3}$ A

$$Id\vec{l}=-(Idy)\hat{j}$$

$$\vec{B}=0.3y\hat{i}+0.4y\hat{j}$$



$$\Rightarrow \quad d\vec{F}=I(d\vec{l}\times\vec{B})=0.3Iydy\hat{k}$$

$$\Rightarrow \quad \vec{F}=\int d\vec{F}=0.3I\left(\frac{y^2}{2}\bigg|_0^1\right)\hat{k}=\left(\frac{0.3I}{2}\right)\hat{k}$$

$$\Rightarrow \quad \vec{F}=(3\times10^{-4})\hat{k}$$

Hence, the correct answer is (D).

13. $B_{\text{centre}}=0$

$$\Rightarrow \quad \frac{\mu_0I}{4\pi b}\odot+\frac{\mu_0I}{4\pi b}\odot+\frac{\mu_0I}{2b}\odot+\frac{\mu_0I}{2a}\otimes=0$$

$$\Rightarrow \quad \frac{\mu_0I}{2\pi b}+\frac{\mu_0I}{2b}-\frac{\mu_0I}{2a}=0$$

$$\Rightarrow \quad \frac{1}{2\pi b}+\frac{1}{2b}=\frac{1}{2a}$$

$$\Rightarrow \quad \frac{a}{b}=\frac{\pi}{\pi+1}$$

Hence, the correct answer is (B).

14. Let the capacitor discharge in time Δt, then a current will flow in the conducting bar due to which magnetic force will act on bar for small time

Impulse due to force is $J=F\Delta t$

$$\Rightarrow \quad J=BIl\Delta t$$

$$\Rightarrow \quad J=B\left(\frac{\Delta q}{\Delta t}\right)l\Delta t$$

$$\Rightarrow \quad J=B(\Delta q)l$$

where, Δq is the total charge flowing through capacitor in time Δt.

Change in momentum equals impulse, so

$$\Delta p=mv=\text{Impulse}$$

$$\Rightarrow \quad mv=B(\Delta q)l$$

Since $\Delta q=CV_0$

$$\Rightarrow \quad mv=B(CV_0)l \qquad \ldots(1)$$

Also, two springs are in parallel, so the equivalent stiffness constant is $k_{eq}=2k$

By Law of Conservation of Energy, we have

$$\frac{1}{2}mv^2=\frac{1}{2}(2k)A^2,\ \text{where } A \text{ is amplitude}$$

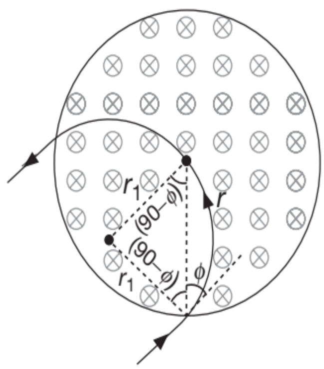
$$\Rightarrow \quad A=\frac{BlCV_0}{\sqrt{2km}}$$

Hence, the correct answer is (A).

15. $r_1=\dfrac{mV}{qB}=\dfrac{5\times10^{-5}\times\left(\dfrac{1}{\sqrt{3}}\right)}{5\times10^{-5}\times1}=\dfrac{1}{10\sqrt{3}}$

By sine rule, we have

$$\frac{r_1}{\sin(90^\circ-\phi)}=\frac{r}{\sin2\phi}$$



$\Rightarrow \quad \dfrac{1}{10\sqrt{3}\cos\phi} = \dfrac{0.1}{2\sin\phi\cos\phi}$

$\Rightarrow \quad \sin\phi = \dfrac{0.1\times10\sqrt{3}}{2} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad \phi = 60°$

Hence, the correct answer is (C).

16. The field at A is directed into the page.
Hence, the correct answer is (A).

17. Since $\vec{E}$ and $\vec{B}$ are parallel to each other and $\vec{v}$ is perpendicular to both, so, the path of the particle is a helix of increasing pitch. The speed of particle at any time t is

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \quad \ldots(1)$$

Since the magnetic force acts along $-z$ axis, so the particle will have a circular track in yz plane, where

$$v_y^2 + v_z^2 = v_0^2$$

Due to the electric field along the x-axis, the x velocity of the particle keeps on increasing with time t as

$$v_x = \left(\frac{qE}{m}\right)t \qquad \left\{\because a = \frac{qE}{m}\right\}$$

According to the problem, we have

$$v = 2v_0$$

Substituting the values in equation (1), we get

$$t = \sqrt{3}\left(\frac{mv_0}{qE}\right)$$

Hence, the correct answer is (D).

18. To enter region II, radius of particle in region I should be greater than d, so we have

$$R > d$$

$\Rightarrow \quad \dfrac{mv}{qB} > d$

$\Rightarrow \quad v > \dfrac{qBd}{m}$

$\Rightarrow \quad v > \dfrac{1.6\times10^{-19}\times0.001\times5\times10^{-2}}{9\times10^{-31}}$

$\Rightarrow \quad v > \dfrac{8}{9}\times10^{7}\ \text{ms}^{-1}$

To come out of region II, we must have

$$2R > d$$

$\Rightarrow \quad \dfrac{2mv}{qB} > d$

$\Rightarrow \quad v > \dfrac{qBd}{2m}$

$\Rightarrow \quad v > \dfrac{1.6\times10^{-19}\times0.002\times5\times10^{-12}}{2\times9\times10^{-31}}$

$\Rightarrow \quad v > \dfrac{8}{9}\times10^{7}\ \text{ms}^{-1}$

Hence, the correct answer is (A).

19. $W_E \neq 0$, because a charged particle placed in a field experiences a force parallel (if positive) or antiparallel (if negative) to the field.
When a charged particle moves in a magnetic field, the force experienced by it is $\vec{F} = q(\vec{v}\times\vec{B})$. The $\vec{F}$ is perpendicular to $\vec{v}$ as well as $\vec{B}$. So $W_M = 0$.
Hence, the correct answer is (C).

20. Along the wire $d\vec{l}\times\vec{r} = 0$

$\Rightarrow \quad B = 0$

Hence, the correct answer is (D).

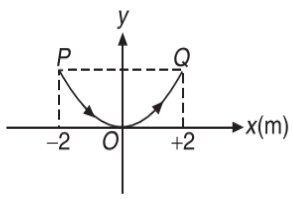
21. Since we know that

$$\vec{F} = q(\vec{v}\times\vec{B}) = q\left[2\hat{i}\times(\hat{i}+2\hat{j}+3\hat{k})\right]$$

$\Rightarrow \quad \vec{F} = 2q(-3\hat{j}+2\hat{k})$

So, this force lies in the yz plane.
Hence, the correct answer is (D).

22. In uniform field,



Magnetic force on POQ is equal to magnetic force on straight wire PQ having the same current. Hence,

$$\vec{F} = i(\vec{l}_{\text{eff}}\times\vec{B}) = i(\overrightarrow{PQ}\times\vec{B})$$

$\Rightarrow \quad \vec{F} = 2\left[(4\hat{i})\times(-0.02\hat{k})\right]$

$\Rightarrow \quad \vec{F} = (0.16\hat{j})$

$\Rightarrow \quad \vec{a} = \dfrac{\vec{F}}{m} = \dfrac{(0.16\hat{j})}{0.1} = 1.6\hat{j}$

Hence, the correct answer is (C).

23. Since $\vec{B} = \dfrac{\mu_0}{4\pi}\left(\dfrac{q\vec{v}\times\vec{r}}{r^3}\right)$ and $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q\vec{r}}{r^3}$

$\Rightarrow \quad \vec{B} = \mu_0\varepsilon_0(\vec{v}\times\vec{E}) = \dfrac{\vec{v}\times\vec{E}}{c^2}$

Now $\vec{v}\times\vec{E}=\begin{vmatrix} \overset{\oplus}{\hat{i}} & \overset{\ominus}{\hat{j}} & \overset{\oplus}{\hat{k}} \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{vmatrix}=6\hat{i}-2\hat{j}$

$\Rightarrow \quad \vec{B}=\dfrac{6\hat{i}-2\hat{j}}{c^2}$

Hence, the correct answer is (A).

24. Let $\vec{B}=\left(x\hat{i}+y\hat{j}+z\hat{k}\right)T$. Since $\vec{F}=q\left(\vec{v}\times\vec{B}\right)$, so we get

$$e\left(-\hat{j}+\hat{k}\right)=e\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ x & y & z \end{vmatrix}=e\left(-z\hat{j}+y\hat{k}\right)$$

$\Rightarrow \quad z=1,\ y=1$

Also, we have

$$e\left(\hat{i}-\hat{k}\right)=e\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 0 \\ x & y & z \end{vmatrix}=e\left(z\hat{i}-x\hat{k}\right)$$

$\Rightarrow \quad z=1,\ x=1$

$\Rightarrow \quad \vec{B}=\hat{i}+\hat{j}+\hat{k}$

Hence, the correct answer is (D).

25. $\omega=10^8\times 2\pi \text{ rads}^{-1}$

Also, $\omega=\dfrac{qB}{m}=\dfrac{eB}{m}$

$\Rightarrow \quad 2\pi\times 10^8=\dfrac{1.6\times 10^{-19}B}{9.1\times 10^{-31}}$

$\Rightarrow \quad B=\left(\dfrac{2\pi\times 9.1\times 10^{-23}}{1.6\times 10^{-19}}\right)\text{T}$

For a solenoid $B=\mu_0 ni$

$\Rightarrow \quad \dfrac{2\pi\times 9.1\times 10^{-23}}{1.6\times 10^{-19}}=\left(4\pi\times 10^{-7}\right)n(2)$

$\Rightarrow \quad n\approx 1420 \text{ turns/m}$

Hence, the correct answer is (B).

26. Since $F=BI\ell$

$\Rightarrow \quad \left[\dfrac{F}{I\ell}\right]=\dfrac{MLT^{-2}}{AL}=MT^{-2}A^{-1}$

Since $[\phi]=[BA]=MT^{-2}A^{-1}L^2$

$\Rightarrow \quad [\phi]=ML^2T^{-2}A^{-1}$

Hence, the correct answer is (C).

27. $F_m=qvB$, and directed radially outward. Since,

$$N-mg\sin\theta-qvB=\frac{mv^2}{R}$$

$\Rightarrow \quad N=\dfrac{mv^2}{R}+mg\sin\theta+qvB$

Hence at $\theta=\dfrac{\pi}{2}$, we get the maximum value of N as

$\Rightarrow \quad N_{\max}=\dfrac{2mgR}{R}+mg+qB\sqrt{2gR}$

$\Rightarrow \quad N_{\max}=3mg+qB\sqrt{2gR}$

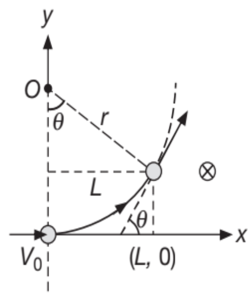
Hence, the correct answer is (C).

28. Since net force on the current carrying closed loop placed in uniform magnetic field is zero.

So, $F_{PQ}=\sqrt{\left(F_3-F_1\right)^2+F_2^2}$

Hence, the correct answer is (D).

29. $\sin\theta=\dfrac{L}{r}=\dfrac{L}{\dfrac{mv}{qB}}$



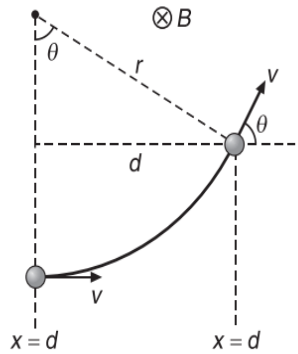
$\Rightarrow \quad v\propto\dfrac{1}{\sin\theta}$

$\Rightarrow \quad \dfrac{v}{v_0}=\dfrac{\sin 60^\circ}{\sin 30^\circ}$

$\Rightarrow \quad v=v_0\sqrt{3}$

Hence, the correct answer is (D).

30. Figure below shows the path of motion of the particle.



Deviation (θ) angle of particle is given by

$$\sin\theta = \frac{d}{r} = \frac{d}{\frac{mv}{Bq}}$$

$$\Rightarrow \frac{q}{m} = \frac{v\sin\theta}{Bd}$$

Hence, the correct answer is (D).

31. The electric field at the axis is

$$E = E_{\text{axis}} = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(R^2+x^2)^{3/2}}$$

At $x=R$, $E = \frac{qR}{4\pi\varepsilon_0(2R^2)^{3/2}}$

$$\Rightarrow E = \frac{q}{4\pi\varepsilon_0 R^2(2)^{3/2}}$$

The magnetic field at the axis is

$$B = B_{\text{axis}} = \frac{\mu_0 iR^2}{2(R^2+x^2)^{3/2}}$$

where, $i = qf = q\left(\frac{v}{2\pi R}\right)$

At $x=R$, $B = \frac{\mu_0 iR^2}{2R^3(2)^{3/2}} = \frac{\mu_0 i}{2R(2)^{3/2}}$

$$\Rightarrow B = \frac{\mu_0\left(\frac{qv}{2\pi R}\right)}{2R(2)^{3/2}} = \frac{\mu_0 qv}{4\pi R^2(2)^{3/2}}$$

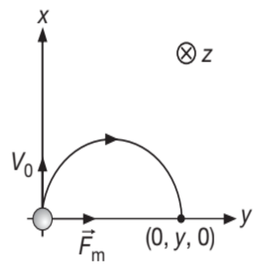
$$\Rightarrow \frac{E}{B} = \frac{1}{\mu_0\varepsilon_0 v}$$

Since $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$$\Rightarrow \frac{E}{B} = \frac{c^2}{v}$$

Hence, the correct answer is (A).

32.



$$y = 2r = \frac{2mv_0}{qB_0} = \frac{2v_0}{B_0\alpha}$$

where the specific charge α equals $\frac{q}{m}$

Hence, the correct answer is (D).

33. For the charge to be prevented from colliding the plate, we have

$$r = d$$

$$\Rightarrow \frac{\sqrt{2mK}}{qB} = d$$

$$\Rightarrow B = \sqrt{\frac{2mK}{q^2d^2}}$$

$$\Rightarrow B = \sqrt{\frac{2\times1.6\times10^{-26}\times2\times10^3\times e}{e^2\times10^{-4}}}$$

$$\Rightarrow B = \sqrt{\frac{2\times1.6\times10^{-26}\times2\times10^3}{1.6\times10^{-19}\times10^{-4}}}$$

$$\Rightarrow B = 2\text{ T}$$

Hence, the correct answer is (A).

34. Since the magnetic field is uniform and the particle is projected in a direction perpendicular to the field hence, it will describe a circular path. The particle will not hit the y-z plane only if the radius of the circle happens to be smaller than d. For the maximum value of v the radius is just equal to d.

So, $r = d = \frac{mv_0}{qB}$

$$\Rightarrow v = \frac{qBd}{m}$$

Hence, the correct answer is (C).

35. Pitch p of helical path is,

$$p = (v\cos\theta)\times\frac{2\pi m}{qB} = \frac{2\pi mv\cos(45^\circ)}{qB}$$

$$\Rightarrow p = \frac{2\pi mv}{\sqrt{2}qB}$$

Also, radius of helical path is,

$$r = \frac{mv\sin\theta}{qB} = \frac{mv}{\sqrt{2}qB}$$

$$\Rightarrow \frac{r}{p} = \left(\frac{mv}{\sqrt{2}qB}\right)\left(\frac{\sqrt{2}qB}{2\pi mv}\right) = \frac{1}{2\pi}$$

$$\Rightarrow r = \frac{p}{2\pi}$$

Hence, the correct answer is (C).

36. $B_{\text{centre}} = B_c = \frac{\mu_0 I}{2a}$, $B_{\text{axis}} = B_a = \frac{\mu_0 I a^2}{2(a^2+9a^2)^{3/2}}$

So, the desired ratio is

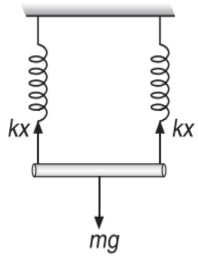
$$\frac{B_c}{B_a} = \frac{\frac{\mu_0 I}{2a}}{\frac{\mu I a^2}{2(10a^2)^{3/2}}} = (10)^{3/2}$$

$$\Rightarrow \quad \frac{B_c}{B_a} = 10\sqrt{10}$$

Hence, the correct answer is (C).

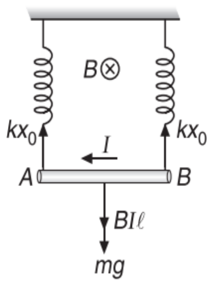
37. Initially, when B does not exist, then we have

$$mg = (2k)x \qquad \ldots(1)$$



When B is switched on, then

$$mg + BIl = 2kx_0 \qquad \ldots(2)$$



From equations (1) and (2), we get

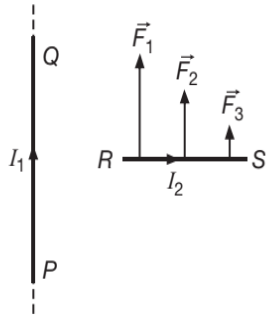
$$\frac{mg+BIl}{mg} = \frac{x_0}{x}$$

$$\Rightarrow \quad BIl = mg\left(\frac{x_0}{x}-1\right)$$

$$\Rightarrow \quad B = \frac{mg}{Il}\left(\frac{x_0-x}{x}\right)$$

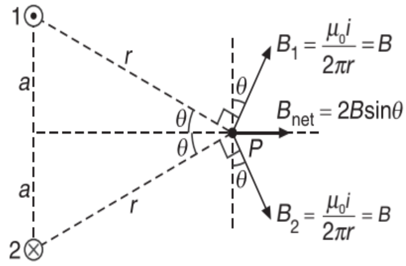
Hence, the correct answer is (D).

38. Since, the magnetic field due to wire PQ is non uniform. Neither the force nor the torque on wire RS is zero. Therefore, it will have both translational as well as rotational motion.



Hence, the correct answer is (C).

39. Net magnetic field at P due to wires 1 and 2 at point P is



$$B_{\text{net}} = 2B\sin\theta = 2\left(\frac{\mu_0 i}{2\pi r}\right)\left(\frac{a}{r}\right)$$

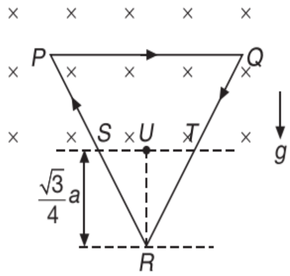
$$\Rightarrow \quad B_{\text{net}} = \frac{\mu_0 i a}{\pi r^2}$$

Hence, the correct answer is (D).

40. The force on section $MPQN$ is

$$\vec{F}_{MPQN} = \vec{F}_{\text{MN}}$$

and this force should be upwards to balance the weight.



$$\Rightarrow \quad IlB = mg\text{, where } l = ST = 2(UT)$$

$$\Rightarrow \quad l = 2\left[\frac{\sqrt{3}a}{4}\tan(30°)\right] = \frac{a}{2}$$

$$\Rightarrow \quad I\left(\frac{a}{2}\right)B = Mg$$

$$\Rightarrow \quad I = \frac{2Mg}{aB}$$

Force is upwards if current in loop is clockwise.

Hence, the correct answer is (B).

CHAPTER 1

41. Since the point $P(0, 0, -a)$ lies on z-axis. Therefore, magnetic field due to current along z-axis is zero and due to other two wires is $\frac{\mu_0}{2\pi}\frac{I}{a}$ in mutually perpendicular directions along positive y-direction and negative x-direction. So,

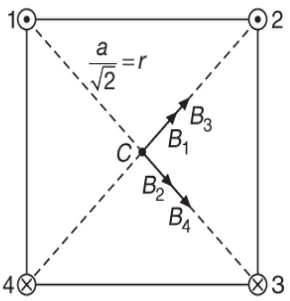
$$\vec{B} = \frac{\mu_0}{2\pi}\frac{I}{a}\left(-\hat{i}+\hat{j}\right)$$

Hence, the correct answer is (C).

42. The given point lies on the wire, so $B = 0$.

Hence, the correct answer is (D).

43.



Since $B_1 = B_2 = B_3 = B_4 = B = \frac{\mu_0 I}{2\pi r}$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi\left(\frac{a}{\sqrt{2}}\right)} = \frac{\sqrt{2}\mu_0 I}{2\pi a}$$

Resultant magnetic field abc is

$$B_{\text{net}} = \sqrt{(2B)^2 + (2B)^2}$$

$$\Rightarrow \quad B_{\text{net}} = 2B\sqrt{2}$$

$$\Rightarrow \quad B_{\text{net}} = 2\sqrt{2}\frac{\mu_0 I}{2\pi\left(\frac{a}{\sqrt{2}}\right)}$$

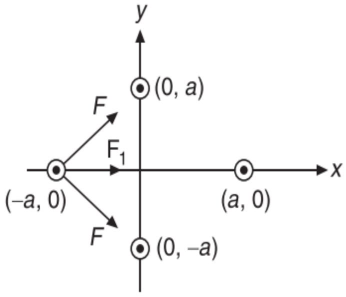
$$\Rightarrow \quad B_{\text{net}} = \frac{2\mu_0 I}{\pi a}$$

Hence, the correct answer is (A).

44. Forces acting on the loop, due to I_1 act in the plane of the loop and hence give zero torque.

Hence, the correct answer is (A).

45.

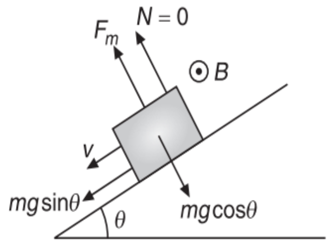


Net force is along $+x$-axis

Hence, the correct answer is (C).

46. At break-off from the surface, we have $N = 0$

So, the magnetic force on block is given by



$$F_m = mg\cos\theta$$

$$\Rightarrow \quad qvB = mg\cos\theta$$

Since $v = at$

$$\Rightarrow \quad v = (g\sin\theta)t$$

$$\Rightarrow \quad q(gt\sin\theta)B = mg\cos\theta$$

$$\Rightarrow \quad t = \frac{m\cot\theta}{qB}$$

Hence, the correct answer is (C).

47. $B = \frac{\mu_0 I}{4\pi(r_\perp)}(\sin\theta_2 - \sin\theta_1)$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi(r\sin\alpha)}\left[\sin\frac{\pi}{2} - \sin\left(\frac{\pi}{2}-\alpha\right)\right]$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi(r\sin\alpha)}(1-\cos\alpha)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi r\left(2\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)\right)}\left(2\sin^2\left(\frac{\alpha}{2}\right)\right)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi r}\tan\left(\frac{\alpha}{2}\right)$$

Hence, the correct answer is (D).

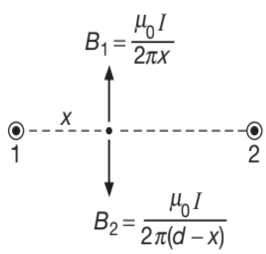
48. $F_{AB} = F_{BC}$

$$\Rightarrow \quad \frac{\mu_0}{2\pi}\frac{(i)(2i)}{d_1} = \frac{\mu_0}{2\pi}\frac{(2i)(3i)}{d_2}$$

$$\Rightarrow \quad \frac{d_1}{d_2} = \frac{1}{3}$$

Hence, the correct answer is (A).

49.



$$B_{net} = B = \frac{\mu_0 I}{2\pi x} - \frac{\mu_0 I}{2\pi(d-x)} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} - \frac{1}{d-x}\right)$$

The above equation is best represented by (D).

Hence, the correct answer is (D).

50. At point A, the magnetic field due to both the cylinders is along $+y$ direction. The magnitude of field due to one cylinder is given by

$$B_1 = \frac{\mu_0 J d}{4}$$

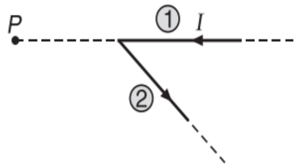
So, net magnetic field at point A is given by

$$B = B_1 + B_2$$

$$\Rightarrow \quad B = \frac{\mu_0 J d}{2}, \text{ along the } +y \text{ direction}$$

Hence, the correct answer is (A).

51. Magnetic field at P due to 1 is zero. So, magnetic field is only due to the straight current carrying wire 2, which is given by

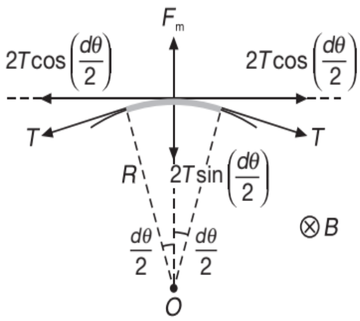


$$B = \frac{\mu_0 I}{4\pi\left(\frac{R}{\sqrt{2}}\right)}(\sin 90° + \sin 135°)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi R}(\sqrt{2} - 1)$$

Hence, the correct answer is (A).

52.



Magnetic force on the circular arc is $F_m = BId\ell$

$$F_m = BI(Rd\theta)$$

For the arc to be in equilibrium, $F = 2T\sin\left(\frac{d\theta}{2}\right)$

For small angle $d\theta$, $\sin\left(\frac{d\theta}{2}\right) \approx \frac{d\theta}{2}$

$$\Rightarrow \quad 2Td\theta = BIRd\theta$$

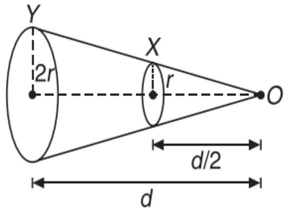
$$\Rightarrow \quad T = BIR$$

Since $L = 2\pi R$

$$\Rightarrow \quad T = BIR = \frac{BIL}{2\pi}$$

Hence, the correct answer is (C).

53.



The two coils subtend the same solid angle at O and since

$$\Rightarrow \quad \text{Solid angle} = \frac{\text{Area}}{(\perp \text{distance})^2}$$

Hence area of coil Y is 4 times the area of coil X

$\Rightarrow$ Radius of coil Y is 2 times the radius of coil X

Since the magnetic field at the axis of the coil is

$$B = \frac{\mu_0 N I R^2}{2(R^2 + x^2)^{\frac{3}{2}}}$$

$$\Rightarrow \quad B_Y = \frac{\mu_0}{4\pi} \times \frac{2\pi I(2r)^2}{\left[(2r)^2 + (d^2)\right]^{\frac{3}{2}}}$$

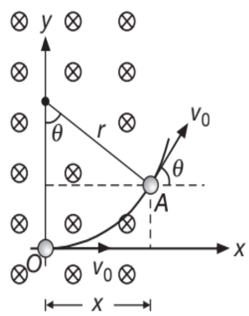
$$\Rightarrow \quad B_X = \frac{\mu_0}{4\pi} \times \frac{2\pi I(r)^2}{\left[r^2 + \left(\frac{d}{2}\right)^2\right]^{\frac{3}{2}}}$$

$$\Rightarrow \quad \frac{B_Y}{B_X} = \frac{4}{(4r^2 + d^2)^{\frac{3}{2}}} \times \left[\frac{4r^2 + d^2}{4}\right]^{\frac{3}{2}}$$

$$\Rightarrow \quad \frac{B_Y}{B_X} = \frac{4}{(4)^{\frac{3}{2}}} = \frac{4}{8} = \frac{1}{2}$$

Hence, the correct answer is (C).

54. $r = \dfrac{mv_0}{qB_0}$



Since, $\frac{x}{r}=\frac{\sqrt{3}}{2}=\sin\theta$

$\Rightarrow \quad \theta=60^\circ$

$\Rightarrow \quad t_{OA}=\frac{T}{6}=\frac{m\pi}{3qB_0}$

Therefore, x co-ordinate of particle at any time

$t>\frac{m\pi}{3qB_0}$ is

$$x=\frac{\sqrt{3}}{2}\frac{mv_0}{qB_0}+v_0\left(t-\frac{m\pi}{3qB_0}\right)\cos(60^\circ)$$

$$\Rightarrow \quad x=\frac{\sqrt{3}}{2}\frac{mv_0}{qB_0}+\frac{v_0}{2}\left(t-\frac{m\pi}{3qB_0}\right)$$

Hence, the correct answer is (D).

55. $KE=\frac{1}{2}mv^2=eV$

$\Rightarrow \quad mv=\sqrt{2emV}$

The electron will be refocused after travelling a distance equal to pitch of helix. So

$$p=\frac{2\pi mv\cos\theta}{qB}$$

Since θ is small, so $\cos\theta\approx 1$

$$\Rightarrow \quad p=\frac{2\pi mv}{eB}=\frac{2\pi\sqrt{2emV}}{eB}$$

$$\Rightarrow \quad p=\sqrt{\frac{8\pi^2 mV}{eB^2}}$$

Hence, the correct answer is (A).

56. Using Ampere's Circuital Law

For $r\le a$, $B=0$

For $a\le r\le b$, $B=\frac{\mu_0 I}{2\pi(b^2-a^2)}\left(\frac{r^2-a^2}{r}\right)$

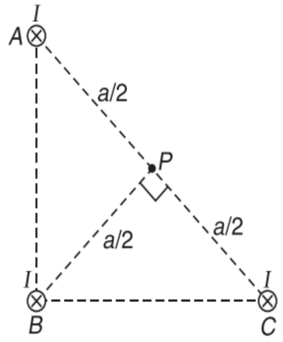
and for $r\ge b$, $B=\frac{\mu_0}{2\pi}\frac{I}{r}$

The corresponding B-r graph will be as shown in OPTION (C).

Hence, the correct answer is (C).

57. At P, net magnetic field is given by

$$B_{net}=B_A+B_B+B_C$$

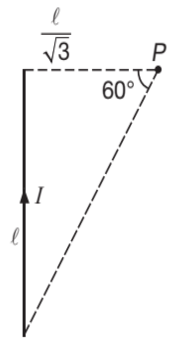


Since B_A and B_C cancel so, net field at P is only due to wire B given by

$$B_{net}=B=\frac{\mu_0 I}{2\pi r_\perp}=\frac{\mu_0 I}{2\pi\left(\frac{a}{2}\right)}=\frac{\mu_0 I}{\pi a}$$

Hence, the correct answer is (A).

58. Since, $B=\frac{\mu_0}{4\pi}\frac{I}{r_\perp}(\sin\alpha+\sin\beta)$ where $r_\perp=\frac{\ell}{\sqrt{3}}$



In this case, $\alpha=0^\circ$ and $\beta=60^\circ$

$$\Rightarrow \quad B=\frac{\mu_0 I}{4\pi\frac{\ell}{\sqrt{3}}}(\sin 0^\circ+\sin 60^\circ)=\frac{3\mu_0 I}{8\pi\ell}$$

Hence, the correct answer is (D).

59. $B=\frac{\mu_0 I}{2r}\left[1-\frac{1}{2}+\frac{1}{2^2}-\frac{1}{2^3}+\frac{1}{2^4}+\ldots\right]$

$$\Rightarrow \quad B=\frac{\mu_0 I}{2r}\left[\frac{1}{1-\left(-\frac{1}{2}\right)}\right]$$

$$\Rightarrow \quad B=\frac{\mu_0 I}{2r}\left(\frac{2}{3}\right)$$

$$\Rightarrow \quad B=\frac{\mu_0 I}{3r}$$

Hence, the correct answer is (D).

60. $\vec{F}_{CAD} = \vec{F}_{CD} = \vec{F}_{CED} = BI\ell_{\text{eff}} = BI(2a)$

Net force on the frame is $\vec{F} = 3\vec{F}_{CD}$

$\Rightarrow \quad |\vec{F}| = 6BIa \qquad \{F = I\ell B\}$

Hence, the correct answer is (A).

61. When electron is projected in an electric field, then we have

$$\frac{mv_0^2}{r_1} = eE \qquad \ldots(1)$$

When electron is projected in magnetic field, then we have

$$\frac{mv_0^2}{r_2} = ev_0B \qquad \ldots(2)$$

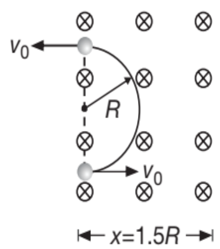
Dividing equation (1) and (2), we get

$$\frac{r_2}{r_1} = \frac{eE}{ev_0B}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{Bv_0}{E}$$

Hence, the correct answer is (D).

62. From the figure it is clear that deviation is $180°$ i.e., π radian (π^c), for $x > R$.



Hence, the correct answer is (D).

63. Magnetic induction at point P due current I_1 is given by

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi(AP)}\hat{k} = \frac{4\pi\times10^{-7}\times2}{2\pi\times1\times10^{-2}}\hat{k}$$

$$\Rightarrow \quad \vec{B}_1 = (4\times10^{-5}\text{ T})\hat{k}$$

Magnetic induction at point P due current I_2 is given by

$$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi(BP)}\hat{j} = \frac{4\pi\times10^{-7}\times3}{2\pi\times2\times10^{-2}}\hat{j}$$

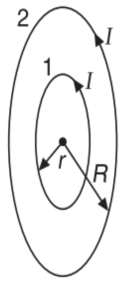
$$\Rightarrow \quad \vec{B}_2 = (3\times10^{-5}\text{ T})\hat{j}$$

So net magnetic induction at point P is given as

$$\vec{B} = (3\times10^{-5}\text{ T})\hat{j} + (4\times10^{-5}\text{ T})\hat{k}$$

Hence, the correct answer is (B).

64.



Work done in rotating the smaller I coil about any of its diameter through an angle π is given by

$$W = -M_1B_2(\cos\pi - \cos 0) = 2M_1B_2$$

where $M_1 = n(\pi r^2)I$ and $B_2 = N\left(\frac{\mu_0 I}{2R}\right)$

$$\Rightarrow \quad W = 2M_1B_2 = \frac{\mu_0\pi nNI^2r^2}{R}$$

Hence, the correct answer is (C).

65. $\vec{B} = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+\ldots\right)\hat{k} - \left(\frac{1}{2}+\frac{1}{8}+\ldots\right)\hat{k}\right]$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{1-\frac{1}{4}}\right)\hat{k} - \frac{1}{2}\left(\frac{1}{1-\frac{1}{4}}\right)\hat{k}\right]$$

$$\Rightarrow \quad \vec{B} = \frac{2}{3}\frac{\mu_0}{2\pi}\hat{k} = \left(\frac{2}{3}\times2\times10^{-7}\right)\hat{k}$$

$$\Rightarrow \quad \vec{B} = 1.33\times10^{-7}\hat{k}$$

Hence, the correct answer is (C).

66. Force on the wires parallel to x-axis will be obtained by integration (as $B \propto x$ and x coordinates vary along these wires). But on a loop, there are two such wires. Force on them will be equal and opposite. However, forces on two wires parallel to y-axis can be obtained directly (without integration) as value of B is same along these wires. But their values will be different as x-coordinate and therefore B is different.

$$F_{\text{net}} = \Delta F \text{ (on two wires)}$$

$$\Rightarrow \quad F_{\text{net}} = Ia(\Delta B) = Ia(B_0)(\Delta x)$$

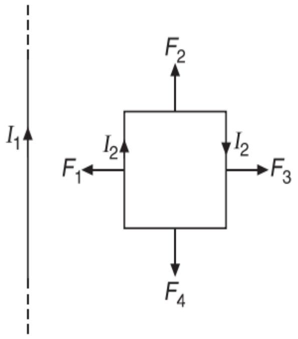
$$\Rightarrow \quad F_{\text{net}} = IaB_0(a) = IB_0a^2$$

This is independent of x

$$\Rightarrow \quad F_1 = F_2 = IBla^2 \neq 0$$

Hence, the correct answer is (D).

67.



F_2 and F_4 cancel each other and $F_1 > F_3$

$$\Rightarrow \quad F_{net} = F_1 - F_3 = \left(\frac{\mu_0 I_1 I_2}{2\pi a}\right)a - \left(\frac{\mu_0 I_1 I_2}{4\pi a}\right)a$$

$$\Rightarrow \quad F_{net} = \frac{\mu_0 I_1 I_2}{4\pi}$$

Hence, the correct answer is (B).

68. Electrostatic force on electron should be equal and opposite to magnetic force, i.e.,

$$\vec{F}_e = -\vec{F}_m$$

$$\Rightarrow \quad q\vec{E} = -q(\vec{v} \times \vec{B})$$

$$\Rightarrow \quad -e\vec{E} = -(-e)(\vec{v} \times \vec{B})$$

$$\Rightarrow \quad \vec{E} = -(\vec{v} \times \vec{B})$$

So, $\vec{B}$ should be along negative z-axis.

Hence, the correct answer is (C).

69. Work done is

$W = U_f - U_i$, where

$U_f = -M_f B$ and $U_i = -M_i B$

Since $M_i = Ia^2$

Keeping the length same, the shape of loop is changed from square to an equilateral triangle of side l (say), so we have

$$4a = 3l$$

$$\Rightarrow \quad l = \frac{4a}{3}$$

Since area of an equilateral triangle of side l is

$$A = \frac{\sqrt{3}}{4} l^2$$

$$\Rightarrow \quad A = \frac{\sqrt{3}}{4}\left(\frac{16a^2}{9}\right) = \frac{4\sqrt{3}a^2}{9}$$

$$\Rightarrow \quad M_f = IA = \frac{4\sqrt{3}Ia^2}{9}$$

$$\Rightarrow \quad W = U_f - U_i = -M_f B - (-M_i B)$$

$$\Rightarrow \quad W = B(M_i - M_f)$$

$$\Rightarrow \quad W = BIa^2\left(1 - \frac{4\sqrt{3}}{9}\right)$$

Hence, the correct answer is (A).

70. Since we know that $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$. Also, we are aware of the fact that the ratio of electric field to magnetic field equals velocity. So, we have

$$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{E}{B}$$

$$\Rightarrow \quad \left[\mu_0 \varepsilon_0 \left(\frac{B}{E}\right)^2\right] = M^0 L^0 T^0$$

Hence, the correct answer is (C).

71. Magnetic field due to wire

$$B = \frac{\mu_0 I}{2\pi r} = \left(\frac{4\pi \times 10^{-7}}{2\pi}\right)\left(\frac{30}{2 \times 10^{-2}}\right)$$

$$\Rightarrow \quad B = 3 \times 10^{-4} \text{ T}$$

This magnetic field will be perpendicular to external magnetic field. So, net magnetic field is

$$B_{net} = \sqrt{B^2 + B_0^2}$$

$$\Rightarrow \quad B_{net} = \sqrt{(3 \times 10^{-4})^2 + (4 \times 10^{-4})^2}$$

$$\Rightarrow \quad B_{net} = 5 \times 10^{-4} \text{ T}$$

Hence, the correct answer is (C).

72. Conceptual

Hence, the correct answer is (A).

73. If I_1 is current in $ACDB$ then

$$\frac{\mu_0}{4}\frac{I_1}{r} = \frac{\mu_0}{4}\frac{I_2}{4r}$$

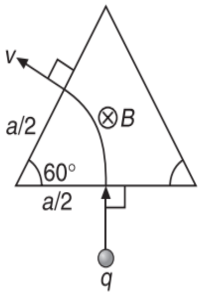
$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{1}{4}$$

$$\Rightarrow \quad I_1 = \frac{\text{I}}{5}$$

$$\Rightarrow \quad \frac{I_1}{I} = \frac{1}{5}$$

Hence, the correct answer is (D).

74. The charged particle moves in a circle of radius $\frac{a}{2}$



$$\Rightarrow \quad qvB = \frac{mv^2}{\frac{a}{2}}$$

$$\Rightarrow \quad B = \frac{2mv}{qa}$$

Hence, the correct answer is (B).

75. Let N be the number of turns and R be the radius of the coil. Then

$$\ell = 2\pi RN$$

$$\Rightarrow \quad R = \frac{\ell}{2\pi N} \qquad \ldots(1)$$

The magnetic moment of the coil is

$$M = NIA = NI(\pi R^2)$$

$$\Rightarrow \quad M = (NI\pi)\left(\frac{\ell^2}{4\pi^2 N^2}\right) = \frac{I\ell^2}{4\pi N}$$

Maximum value of M can be only when $N = 1$, then

$$M_{\max} = \frac{I\ell^2}{4\pi}$$

So, $\tau_{\max} = \tau_0 = M_{\max} B \sin 90° = \frac{BI\ell^2}{4\pi}$

Hence, the correct answer is (C).

76. $\tau = MB\sin(90-\theta) = C\theta$

$$\Rightarrow \quad (10)(0.025)\cos\theta = (1)\theta$$

$$\Rightarrow \quad \frac{1}{4}\sqrt{1-\sin^2\theta} = \theta$$

Since θ is small, so $\sin\theta \approx \theta$

$$\Rightarrow \quad \frac{1}{4}\sqrt{1-\theta^2} = 4\theta$$

$$\Rightarrow \quad 1-\theta^2 = 16\theta^2$$

$$\Rightarrow \quad 17\theta^2 = 1$$

$$\Rightarrow \quad \theta = \frac{1}{\sqrt{17}} \text{ radian} = 0.24 \text{ radian}$$

Hence, the correct answer is (A).

77. The particles will not collide, when

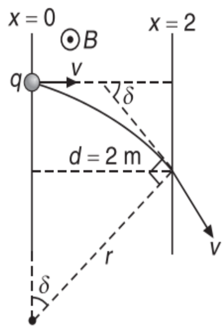
$$\ell > 2(r_1 + r_2)$$

$$\Rightarrow \quad \ell > 2\left(\frac{mv_1}{qB} + \frac{mv_2}{qB}\right)$$

$$\Rightarrow \quad \ell > \frac{2m}{qB}(v_1 + v_2)$$

Hence, the correct answer is (D).

78.



$$\sin\delta = \frac{d}{r}$$

where $r = \frac{mv}{qB} = \frac{p}{qB}$

$$\Rightarrow \quad \sin(60°) = \frac{qBd}{p}$$

$$\Rightarrow \quad p = \frac{qBd}{\sin(60°)} = \frac{2\times 2\times 10^{-6}\times 2\sqrt{3}\times 10^{-3}\times 2}{\sqrt{3}}$$

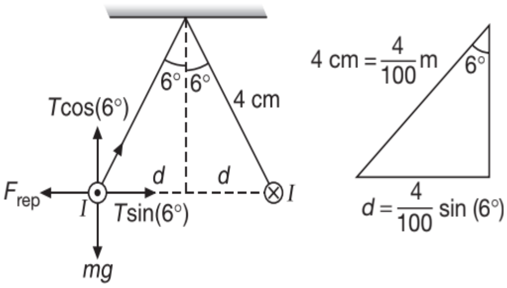
$$\Rightarrow \quad p = 10\times 10^{-9} = 1.6\times 10^{-8} \text{ kgms}^{-1}$$

Hence, the correct answer is (B).

79. Since, every current carrying loop behaves like a magnetic dipole. If it lies in the plane of paper and current in it is in clockwise direction magnetic field at all points lying within the loop is perpendicular to paper inwards and at points outside the loop magnetic field is perpendicular to paper in outward direction.

For $\theta < 180°$, the centre O lies outside the loop and current is clockwise. Therefore, magnetic field is perpendicular to paper in outward direction.
Hence, the correct answer is (B).

80. The magnified view of the arrangement is shown in Figure.



$$T\cos(6°) = mg \qquad ...(1)$$

$$T\sin(6°) = \frac{\mu_0 I^2 l}{2\pi(2d)} \qquad ...(2)$$

Dividing (2) by (1), we get

$$\tan(6°) = \frac{\mu_0 I^2 l}{4\pi mgd} = \frac{\mu_0 I^2}{4\pi\lambda gd}$$

where $\lambda = \frac{m}{l} = 0.0125\ \text{kgm}^{-1}$ and

$$\tan(6°) \approx \sin(6°) = \frac{6\times\pi}{180} \approx 0.1$$

$$\Rightarrow \quad 0.1 = \frac{10^{-7} I^2}{(0.0125)(10)\left(\frac{4}{100}\times 0.1\right)}$$

$$\Rightarrow \quad I^2 = 500$$

$$\Rightarrow \quad I = 22.3\ \text{A}$$

Hence, the correct answer is (B).

81. $B = \frac{\mu_0 I_{in}}{2\pi x}$ where

$$I_{in} = I - \left[\frac{I}{\pi(c^2 - b^2)}\right]\left[\pi(x^2 - b^2)\right]$$

$$\Rightarrow \quad I_{in} = I\left(\frac{c^2 - x^2}{c^2 - b^2}\right)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi x}\left(\frac{c^2 - x^2}{c^2 - b^2}\right)$$

Hence, the correct answer is (C).

82. The path of the particle will be a helix of time period

$$T = \frac{2\pi m}{qB_0} = \frac{2\pi}{B_0\alpha} \qquad \left\{\because \alpha = \frac{q}{m}\right\}$$

The given time $t = \frac{\pi}{B_0\alpha} = \frac{T}{2}$

So, the co-ordinates of particle at time $t = \frac{T}{2}$ are $\left(\frac{v_x T}{2}, 0, -2r\right)$

where $r = \frac{mv_y}{qB_0} = \frac{v_0}{B_0\alpha}$

Hence the required co-ordinates are $\left(\frac{v_0\pi}{B_0\alpha}, 0, \frac{-2v_0}{B_0\alpha}\right)$

Hence, the correct answer is (C).

83. On a horizontal rough surface, minimum force required to just slide the wire of mass m is

$$F_{\min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

$$\Rightarrow \quad B_{\min} iL = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

$$\Rightarrow \quad B_{\min} = \frac{\mu mg}{iL\sqrt{1+\mu^2}}$$

Hence, the correct answer is (A).

84. $B = 2\left(\frac{\mu_0}{4\pi}\frac{I}{\frac{d}{\sqrt{2}}}\right)(\sin 90° - \sin 45°)$

$$\Rightarrow \quad B = \frac{\mu_0 I}{\sqrt{2}\pi d}\left(1 - \frac{1}{\sqrt{2}}\right), \otimes$$

Hence, the correct answer is (B).

85. According to Ampere's law, we have

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

For loop A, $\oint \vec{B}\cdot d\vec{l} = \mu_0(0) = 0$

For loop B, $\oint \vec{B}\cdot d\vec{l} = \mu_0(I + I - I) = \mu_0 I$

For loop C, $\oint \vec{B}\cdot d\vec{l} = -\mu_0 I$

For loop D, $\oint \vec{B}\cdot d\vec{l} = -\mu_0 I$

$$\Rightarrow \quad B > A > C = D$$

Hence, the correct answer is (C).

86. Just like electric field is negative gradient of electric potential, we can also say that magnetic field is negative gradient of magnetic scalar potential. So, we have

$$B = -\frac{\Delta V_B}{\Delta x_\perp}$$

where $\Delta x_\perp$ is the perpendicular separation between the equipotential surfaces drawn for magnetic scalar potential.

$$\Rightarrow \quad |B| = \frac{2\times10^{-5} - 1\times10^{5}}{(0.1)\sin 30^\circ} = 2\times10^{-4}\ \text{T}$$

Hence, the correct answer is (D).

87. $x = \frac{B}{M} = \left(\frac{\mu_0 I}{2r}\right)\left(\frac{1}{I(\pi r^2)}\right)$

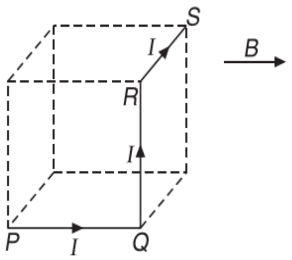
$$\Rightarrow \quad x \propto \frac{1}{r^3}$$

So, x becomes $\frac{x}{8}$ when radius and current both are doubled.

$\Rightarrow$ Decrease in value of x is $\left(x - \frac{x}{8}\right) = \frac{7x}{8}$

Hence, the correct answer is (C).

88.



Let the field be directed along the side PQ, then

$$F_{PQ} = 0$$

$$F_{QR} = BIl$$

$$F_{RS} = BIl$$

Since, F_{QR} and F_{RS} are perpendicular to each other, so

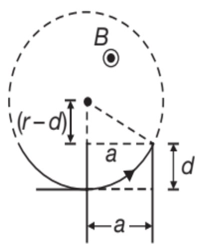
$$F_{\text{net}} = \sqrt{(BIl)^2 + (BIl)^2} = \sqrt{2}BIl$$

Hence, the correct answer is (C).

89. $qvB\sin 90 = \frac{mv^2}{r}$

$$\Rightarrow \quad mv = qBr \qquad \ldots(1)$$

Since $a^2 + (r-d)^2 = r^2$



$$\Rightarrow \quad a^2 = r^2 - (r-d)^2$$

$$\Rightarrow \quad a^2 = (r - r + d)(r + r - d)$$

$$\Rightarrow \quad a^2 = (d)(2r - d)$$

$$\Rightarrow \quad 2r - d = \frac{a^2}{d}$$

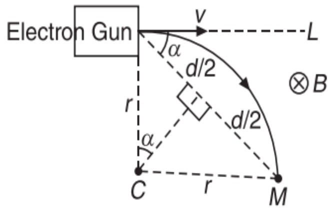
$$\Rightarrow \quad r = \frac{a^2}{2d} + \frac{d}{2} \qquad \ldots(2)$$

Put (2) in (1), we get

$$p = mv = qBr = \frac{qB}{2}\left(\frac{a^2}{d} + d\right)$$

Hence, the correct answer is (A).

90. For electron (negatively charged) to hit the target M, magnetic field B must be directed inwards



Since $r = \frac{mv}{eB}$

$$\Rightarrow \quad r = \frac{\sqrt{2mK}}{qB} = \frac{\sqrt{2mqV}}{qB}$$

$$\Rightarrow \quad r = \frac{1}{B}\sqrt{\frac{2mV}{q}} = \frac{1}{B}\sqrt{\frac{2mV}{e}}$$

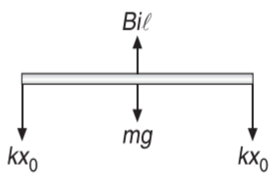
Also, $r = \frac{d}{2\sin\alpha}$

$$\Rightarrow \quad \frac{d}{2\sin\alpha} = \frac{1}{B}\sqrt{\frac{2mV}{e}}$$

$$\Rightarrow \quad B = 2\sqrt{\frac{2mV}{e}}\frac{\sin\alpha}{d}$$

Hence, the correct answer is (A).

91. If the springs are extended through x_0, then for equilibrium of rod, we have



$$2kx_0 + mg = ilB$$

$$\Rightarrow \quad x_0 = \frac{ilB - mg}{2K}$$

Length of spring now becomes

$$l = l_0 + x_0$$

$$\Rightarrow \quad l = l_0 + \frac{ilB - mg}{2k}$$

Hence, the correct answer is (B).

92. Using Ampere's Circuital Law, we get the desired results.

Hence, the correct answer is (B).

93. At the centre of loop, magnetic field is

$$B_{\text{centre}} = \frac{\mu_0 I}{2a}$$

At axial point, the magnetic field is

$$B_{\text{axis}} = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{\frac{3}{2}}}$$

$$\frac{B_{\text{axis}}}{B_{\text{centre}}} = \frac{a^3}{(a^2 \times x^2)^{\frac{3}{2}}}$$

$$\Rightarrow \quad B_{\text{axis}} = B_{\text{centre}} \frac{a^3}{(a^2 + x^2)^{\frac{3}{2}}}$$

$$\Rightarrow \quad B_{\text{axis}} = 0.5 \times 10^{-4} \times \frac{(12)^3}{(144 + 25)^{\frac{3}{2}}}$$

$$\Rightarrow \quad B_{\text{axis}} = 0.5 \times 10^{-4} \times \left(\frac{12}{13}\right)^3$$

$$\Rightarrow \quad B_{\text{axis}} = 3.9 \times 10^{-5} \text{ T}$$

Hence, the correct answer is (A).

94. Since $B = \mu_0 n I$

Also, we know that the magnetic energy density equals pressure. So,

$$P = \frac{B^2}{2\mu_0} = \frac{(\mu_0 n I)^2}{2\mu_0} = \frac{\mu_0 n^2 I^2}{2}$$

Hence, the correct answer is (B).

95. $r = \dfrac{mv}{qB} = \dfrac{\sqrt{2mK}}{qB}$

$$\Rightarrow \quad r_p = \frac{\sqrt{2m_p K}}{eB}$$

$$\Rightarrow \quad r_d = \frac{\sqrt{2(2m_p)K}}{eB}$$

$$\Rightarrow \quad r_\alpha = \frac{\sqrt{2(4m_p)K}}{(2e)B}$$

$$\Rightarrow \quad r_p : r_d : r_\alpha = 1 : \sqrt{2} : 1$$

Hence, the correct answer is (A).

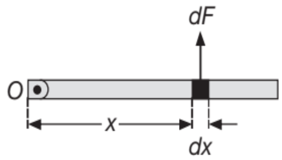
96. The force acting on the elementary portion of the current carrying conductor is given as,

$$dF = I(dx)B\sin 90°$$

$$\Rightarrow \quad dF = BIdx$$

The torque applied by dF about $O = d\tau = xdF$

$\Rightarrow$ The total torque about O is τ given by



$$\tau = \int d\tau = \int x(BIdx)$$

$$\Rightarrow \quad \tau = IB\int_0^L x\,dx = \frac{IBL^2}{2}$$

The angular acceleration

$$\alpha = \frac{\tau}{\left(\begin{array}{c}\text{Moment of Inertia} \\ \text{of Rod about O}\end{array}\right)} = \frac{\tau}{\left(\frac{1}{3}mL^2\right)}$$

$$\Rightarrow \quad \alpha = \frac{\left(\frac{BIL^2}{2}\right)}{\left(\frac{mL^2}{3}\right)}$$

$$\Rightarrow \quad \alpha = \frac{3IB}{2m}$$

Hence, the correct answer is (C).

97. Current divides in the inverse ratio of resistances, so we get

$$i_1 = \frac{20}{47}\text{ A}, \ i_2 = \frac{15}{47}\text{ A}, \ i_3 = \frac{12}{47}\text{ A}$$

For force on wire 2 to be zero, we have

$$\frac{i_1 i_2}{d_1} = \frac{i_2 i_3}{d_2}$$

$$\Rightarrow \frac{20\times 15}{d_1}=\frac{15\times 12}{d_2}$$

$$\Rightarrow \frac{d_1}{d_2}=\frac{5}{3}$$

Hence, the correct answer is (B).

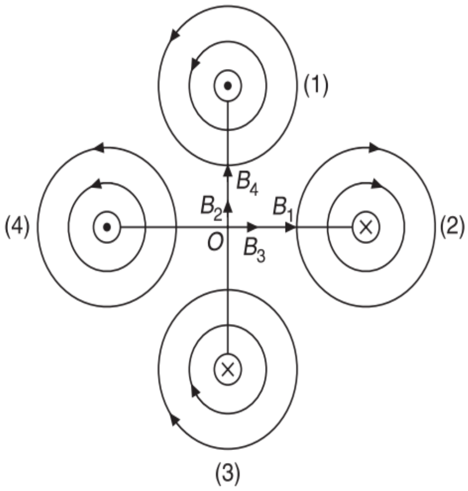
98. Diameter, $D=2r=\frac{2mv}{qB}$

$$\Rightarrow D\propto\frac{m}{q}$$

Here $\frac{m}{q}$ is maximum for C^+. So, it will hit the farthest.

Hence, the correct answer is (B).

99. The field at origin O due to the wires is shown in Figure.



$$\Rightarrow \vec{B}_{origin}=\vec{B}_0=2B\hat{i}+2B\hat{j}$$

$$\Rightarrow \left|\vec{B}_0\right|=2\sqrt{2}B$$

Hence, the correct answer is (C).

100. The central wire creates field $B=\frac{\mu_0 I_1}{2\pi R}$ counter clockwise. The curved portions of the loop experience no force since $\vec{\ell}\times\vec{B}=\vec{0}$. The straight portions both experience a force $I(\vec{\ell}\times\vec{B})$ to the right, amounting to

$$\vec{F}=I_2(2L)\left(\frac{\mu_0 I_1}{2\pi R}\right)=\frac{\mu_0 I_1 I_2 L}{\pi R} \text{ to the right.}$$

Hence, the correct answer is (C).

101. $mg=kx_0$ and $mg+BiL=2(kx_0)$

$$\Rightarrow mg+B\left(\frac{E}{R}\right)L=2mg$$

$$\Rightarrow B=\frac{mgR}{EL}$$

Hence, the correct answer is (B).

102. $\frac{F_{AB}}{\ell}=\frac{\mu_0 I I_0}{2\pi\left(\frac{\ell}{2}\right)}$ {towards XY}

$$\Rightarrow F_{AB}=\frac{\mu_0 I I_0}{\pi} \quad \{\text{towards } XY\}$$

Similarly, $\frac{F_{CD}}{\ell}=\frac{\mu_0 I I_0}{2\pi\left(\frac{3\ell}{2}\right)}$ {away from XY}

$$\Rightarrow F_{CD}=\frac{\mu_0 I I_0}{3\pi} \quad \{\text{away from } XY\}$$

$$\Rightarrow F_{net}=\frac{\mu_0 I I_0}{\pi}-\frac{\mu_0 I I_0}{3\pi}=\frac{2}{3}\left(\frac{\mu_0 I I_0}{\pi}\right) \quad \{\text{towards } XY\}$$

Hence, the correct answer is (D).

103. $\vec{F}=I(\vec{l}\times\vec{B})$, where

$\vec{l}=10^{-2}\hat{i}$ and $\vec{B}=(0.74\hat{j}-0.3\hat{k})$ T

$$\Rightarrow \vec{F}=3.5\left[10^{-2}\hat{i}\times(0.74\hat{j}-0.36\hat{k})\right]$$

$$\Rightarrow \vec{F}=(2.59\hat{k}+1.26\hat{j})\times 10^{-2}\text{ N}$$

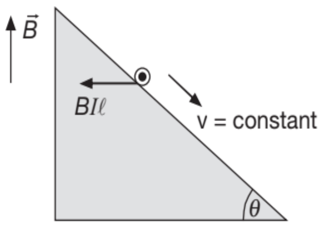
Hence, the correct answer is (A).

104. $\left|\vec{F}_{ABC}\right|=\left|\vec{F}_{AC}\right|=BIl_{eff}$

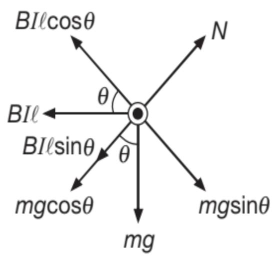
$$\Rightarrow F_{ABC}=(2)(5)(2)=20\text{ N}$$

Hence, the correct answer is (B).

105. Magnetic force $\left|\vec{F}_m\right|=BI\ell$ acts in the direction shown in Figure.



Since the rod moves downwards with constant velocity, hence the net force on it is zero.



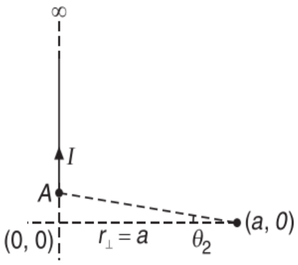
$\Rightarrow \quad F_m \cos\theta = mg\sin\theta$

$\Rightarrow \quad BI\ell\cos\theta = mg\sin\theta$

$\Rightarrow \quad B = \left(\dfrac{mg}{I\ell}\right)\tan\theta$

Hence, the correct answer is (B).

106.



Since $B = \dfrac{\mu_0 I}{4\pi r_\perp}(\sin\theta_1 - \sin\theta_2)$

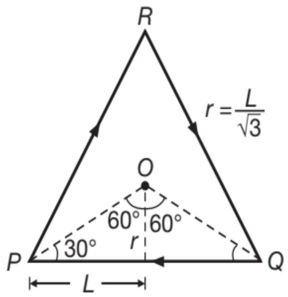
where $\theta_1 = 90°$ and $\sin\theta_2 = \dfrac{b}{\sqrt{a^2+b^2}}$

$\Rightarrow \quad B = \dfrac{\mu_0 I}{4\pi a}\left(1 - \dfrac{b}{\sqrt{a^2+b^2}}\right)$

Hence, the correct answer is (B).

107. Magnetic field at O is

$B = 3B_{PQ} = 3\left[\dfrac{\mu_0}{4\pi}\dfrac{I}{r}(\sin 60° + \sin 60°)\right]$



$\Rightarrow \quad B = 3\left(\dfrac{\mu_0 I}{4\pi}\right)\left(\dfrac{\sqrt{3}}{L}\right)(\sqrt{3})$

$\Rightarrow \quad B = \dfrac{9\mu_0 I}{4\pi L}$

Hence, the correct answer is (D).

108. Velocity magnitude will not change as magnetic field does not work on charge. So tangential acceleration (i.e., rate of change of speed) is zero.
Hence, the correct answer is (D).

109. $\overrightarrow{DA} = -2\cos(30°)\hat{i} - 2\sin(30°)\hat{k} = (-\sqrt{3}\hat{i} - \hat{k})$ and

$\overrightarrow{AB} = 2\hat{j}$

Since, $\vec{M} = I(\overrightarrow{DA} \times \overrightarrow{AB})$

$\Rightarrow \quad \vec{M} = \dfrac{1}{2}\left[(-\sqrt{3}\hat{i} - \hat{k}) \times (2\hat{j})\right]$

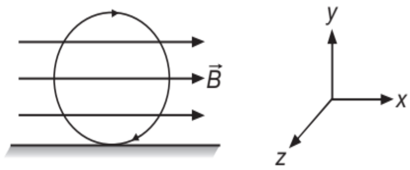
$\Rightarrow \quad \vec{M} = -\sqrt{3}\hat{k} + \hat{i}$

$\Rightarrow \quad \vec{M} = (\hat{i} - \sqrt{3}\hat{k})\ \text{Am}^2$

Hence, the correct answer is (C).

110. $\vec{M} = IA(-\hat{k}) = -(4)(\pi)(0.5)^2\hat{k}$

$\Rightarrow \quad \vec{M} = -\pi\hat{k}(Am^2)$



Since, $\vec{\tau} = \vec{M} \times \vec{B} = (-\pi\hat{k}) \times (10\hat{i}) = -(10\pi)\hat{j}$

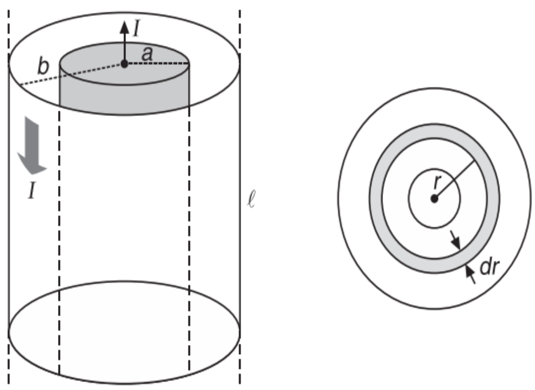
The axis of rotation is along $\vec{\tau}$ i.e., axis of rotation is the y-axis, moment of inertia about which is

$$I = \frac{1}{2}mR^2 = \frac{1}{2}(2)(0.5)^2 = \frac{1}{4}\ \text{kgm}^2$$

$\Rightarrow \quad \alpha = \dfrac{|\vec{\tau}|}{I} = \dfrac{10\pi}{\left(\frac{1}{4}\right)} = 40\pi\ \text{rads}^{-2}$

Hence, the correct answer is (D).

111. Consider an infinitesimal element of radius r, thickness dr, length l such that volume of element is $dV = (2\pi r dr)l$



The magnetic field at this element is $B = \dfrac{\mu_0 I}{2\pi r}$

The magnetic energy density u_m is given by

$$u_m = \frac{B^2}{2\mu_0} = \frac{\mu_0 I^2}{8\pi^2 r^2}$$

If dU be the energy associated with this infinitesimal element, then

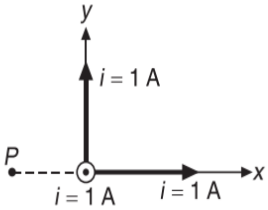
$$dU = u_m dV = \left(\frac{\mu_0 I^2}{8\pi^2 r^2}\right)(2\pi r dr)l$$

$$\Rightarrow \quad U = \int dU = \frac{\mu_0 I^2 l}{4\pi}\int_a^b \frac{dr}{r}$$

$$\Rightarrow \quad U = \frac{\mu_0 I^2 l}{4\pi}\log_e\left(\frac{b}{a}\right)$$

Hence, the correct answer is (A).

112.



No magnetic field at P due to wire along x-axis

Magnetic field at P due to wire along y-axis is

$$\vec{B}_1 = \left(\frac{\mu_0 I}{4\pi a}\right)\hat{k}$$

Magnetic field at P due to wire along z-axis is

$$\vec{B}_2 = \left(\frac{\mu_0 I}{4\pi a}\right)(-\hat{j})$$

$$\Rightarrow \quad \vec{B}_P = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{4\pi a}(-\hat{j}+\hat{k})$$

Since $a = 2$ m

$$\Rightarrow \quad \vec{B}_P = \frac{\mu_0 I}{8\pi}(-\hat{j}+\hat{k})$$

Hence, the correct answer is (C).

113. Let q be the charge and m the mass of the particle. At $(x_0, 0, 0)$ speed of the particle is $\sqrt{4^2+3^2} = 5$ units

Using Work-Energy Theorem, we get

$$(qE_0)x_0 = \frac{1}{2}mv^2 = \frac{25m}{2}$$

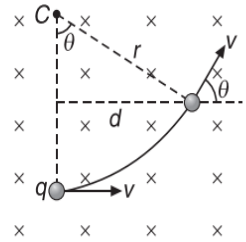
$$\Rightarrow \quad x_0 = \frac{25m}{2qE_0}$$

Hence, the correct answer is (C).

114. For $qBd \ll mv$, we have

$$d \ll \frac{mv}{qB}$$

$$\Rightarrow \quad d \ll r$$



Since impulse is equal to change in momentum. The initial and final momentum are

$$p_i = mv\hat{i}$$

$$\vec{p}_f = mv\cos\theta\hat{i} + mv\sin\theta\hat{j}$$

Since $\sin\theta = \frac{d}{r}$ and $d \ll r$, so θ is very small and hence

$$\sin\theta \approx \theta \text{ and } \cos\theta \approx 1$$

$$\Rightarrow \quad \vec{p}_f \approx mv\hat{i} + mv\theta\hat{j}$$

$$\Rightarrow \quad \Delta\vec{p} = mv\hat{i} + mv\theta\hat{j} - mv\hat{i}$$

$$\Rightarrow \quad \Delta\vec{p} = (mv\theta)\hat{j}$$

Since, $\theta = \frac{d}{r}$

$$\Rightarrow \quad |\Delta\vec{p}| = (mv)\frac{d}{r} \text{ where } r = \frac{mv}{qB}$$

$$\Rightarrow \quad |\Delta\vec{p}| = qBd$$

Hence, Impulse $J = qBd$

Hence, the correct answer is (C).

115. The following forces act on the particle.

- Force T acting radially inwards.
- Centrifugal force $\frac{mv^2}{r}$ acting radially outwards.
- Magnetic force qvB acting radially inwards.

$$\Rightarrow \quad T + qvB = \frac{mv^2}{r}$$

$$\Rightarrow \quad \frac{mv^2}{r} - qvB - T = 0$$

$$\Rightarrow \quad v^2 - \left(\frac{qBr}{m}\right)v - \frac{Tr}{m} = 0$$

$$\Rightarrow \quad v = \frac{1}{2}\left[\frac{qBr}{m} + \sqrt{\frac{q^2B^2r^2}{m^2} + \frac{4Tr}{m}}\right]$$

$$\Rightarrow \quad v=\frac{r}{2}\left[\frac{qB}{m}+\sqrt{\frac{q^2B^2}{m^2}+\frac{4T}{mr}}\right]$$

Hence, the correct answer is (D).

116. Energy gained in one movement across the gap is 100 KeV. So, the energy gained in one turn is 200 KeV

$$\Rightarrow \quad N=\frac{20\times10^6}{200\times10^3}=100$$

Hence, the correct answer is (D).

117. $B_c=\dfrac{\mu_0 NI}{2r}$ and

$$B_a=\frac{\mu_0 NIr^2}{2\left(r^2+h^2\right)^{\frac{3}{2}}}$$

$$\Rightarrow \quad B_a=\frac{\mu_0 NI}{2r\left(1+\frac{h^2}{r^2}\right)^{\frac{3}{2}}}$$

$$\Rightarrow \quad B_a=B_c\left(1+\frac{h^2}{r^2}\right)^{-\frac{3}{2}}$$

$$\Rightarrow \quad B_a\simeq B_c\left(1-\frac{3h^2}{2r^2}\right) \qquad \{\because h^2\ll r^2\}$$

$$\Rightarrow \quad \frac{B_c-B_a}{B_c}=\frac{3h^2}{2r^2}$$

Hence, the correct answer is (D).

119. $\left[\dfrac{B^2}{2\mu_0}\right]=\left[\dfrac{\text{Energy}}{\text{Volume}}\right]=ML^{-1}T^{-2}$

$$\left[R^2C^2\right]=[\text{Time}]^2=T^2$$

$$\left[\frac{B^2R^2C^2}{2\mu_0}\right]=ML^{-1}$$

Since the velocity of transverse wave in a string is given by $v=\sqrt{\dfrac{T}{\lambda}}$ where T is the tension in the string of mass per unit length λ. So, we have

$$[v]=\left[\sqrt{\frac{T}{\lambda}}\right]=\left[\sqrt{\frac{F}{\lambda}}\right]$$

$$\Rightarrow \quad \left[v^2\right]=\left[\frac{F}{\lambda}\right]$$

$$\Rightarrow \quad [\lambda]=\left[\frac{F}{v^2}\right]=\left[\frac{qvB}{v^2}\right]=\left[\frac{qB}{v}\right]$$

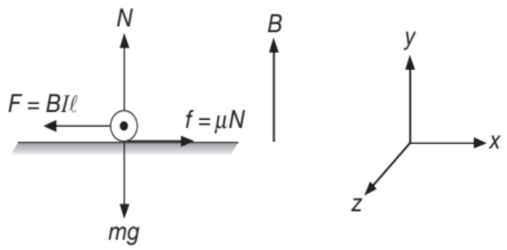
Hence, the correct answer is (B).

120. $B_P=\dfrac{\mu_0 I}{2\pi}\left(\dfrac{1}{2}-\dfrac{1}{3}\right)=\dfrac{\mu_0 I}{12\pi}$

Hence, the correct answer is (C).

121. $F_m=(2)(1)(1)=2\text{ N}$

$$f=\mu N=(0.2)(2)(10)=4\text{ N}$$



Since, the applied force i.e., the magnetic force is less than the force of friction, so the wire will not move at all.

Hence, the correct answer is (D).

122. $B=\dfrac{\mu_0 Ni}{2r}$

Since $l=(2\pi r)N$

$$\Rightarrow \quad r=\frac{l}{2\pi N}$$

$$\Rightarrow \quad B=\frac{\mu_0 Ni}{2\left(\frac{l}{2\pi N}\right)}$$

$$\Rightarrow \quad B\propto N^2$$

$$\Rightarrow \quad B=9B_0$$

Hence, the correct answer is (B).

123. The torque on the loop must be equal to the gravitational torque exerted about an axis that is tangential the loop.

The gravitational torque is given by

$$\tau_1=mgr \qquad \ldots(1)$$

Since $\vec{M}=(IA)\hat{k}$ and only B_x causes a torque. Therefore, torque due to the magnetic field on the current loop is

$$\tau_2 = |\vec{M}\times\vec{B}| = MB\sin 90° = \pi r^2 IB_x \quad ...(2)$$

Equations (1) and (2), we get

$$I = \frac{mg}{\pi r B_x}$$

Hence, the correct answer is (B).

124. Since we know that $B = \frac{\mu_0 I}{4\pi r_\perp}(\sin\theta_1 + \sin\theta_2)$

So, magnetic field due to wire 1 at P is given by

$$B_1 = \frac{\mu_0 i}{4\pi(a\cos 30°)}(\sin 30° + \sin 30°)$$

$$\Rightarrow B_1 = \frac{\mu_0 i}{2\pi\sqrt{3}a}, \text{ outwards}$$

Due to wire 2, magnetic field at point P is given by

$$B_2 = \frac{\mu_0 i}{2\pi\sqrt{3}(2a)}, \text{ inwards}$$

Total magnetic field at point P is given by

$$B = \frac{\mu_0 i}{2\pi\sqrt{3}a}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}...\right]$$

$$\Rightarrow B = \frac{\mu_0 i 2\ln 2}{4\pi\sqrt{3}a} = \frac{\mu_0 I \ln 4}{4\pi\sqrt{3}a}\hat{k}$$

Hence, the correct answer is (B).

125. Charge on a ring of radius x and width dx

$$dq = (2\pi x dx)\sigma$$

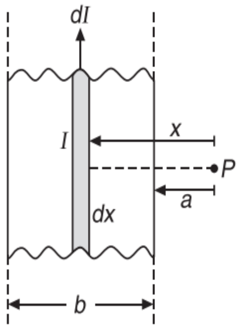
Current, $dI = \frac{dq}{dt} = \frac{2\pi x\sigma dx}{dt} = \omega\sigma x\, dx$

$$dB = \frac{\mu_0 dI r^2}{2(x^2 + y^2)^{3/2}}$$

$$B = \frac{\mu_0\sigma\omega}{2}\left(\frac{r^2 + 2y^2}{\sqrt{r^2 + y^2}} - 2y\right)$$

Hence, the correct answer is (C).

126. Consider an infinitesimal thin very long element carrying a current dI, at a distance x from it as shown in Figure. Then



$$dI = \frac{I}{b}dx$$

If dB be the magnetic field due to this strip at point P, then

$$dB = \frac{\mu_0(dI)}{2\pi x} = \frac{\mu_0 I}{2\pi b}\frac{dx}{x}$$

$$\Rightarrow B = \int dB = \frac{\mu_0 I}{2\pi b}\int_a^{a+b}\frac{dx}{x}$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi b}\log_e\left(\frac{a+b}{a}\right)$$

$$\Rightarrow B = \frac{2\mu_0 I}{4\pi b}\log_e\left(1 + \frac{b}{a}\right)$$

Hence, the correct answer is (B).

127. Since, we have studied that

$$\frac{\text{Magnetic moment}}{\text{Angular momentum}} = \frac{q}{2m}$$

$$\Rightarrow \text{Magnetic moment} = (\text{Angular Momentum})\frac{q}{2m}$$

$$\Rightarrow \text{Magnetic moment} = (I\omega)\frac{q}{2m}$$

$$\Rightarrow \text{Magnetic moment} = \left(\frac{1}{2}mR^2\right)(\omega)\left(\frac{q}{2m}\right) = \frac{1}{4}q\omega R^2$$

Hence, the correct answer is (D).

128. In time $t = \frac{\pi m}{qB} = \frac{T}{2}$, which is half the revolution time, the velocity of the particle will get reversed in yz plane, so we have

$$\vec{v} = 2\hat{i} - 3\hat{j} - 4\hat{k}$$

Since, net force on the particle is zero, so we have

$$q(\vec{E} + \vec{v}\times\vec{B}) = 0$$

$$\Rightarrow \vec{E} = -(\vec{v}\times\vec{B}) = -(2\hat{i} - 3\hat{j} - 4\hat{k})\times 2\hat{i}$$

$$\Rightarrow \vec{E} = -6\hat{k} + 8\hat{j}$$

Hence, the correct answer is (C).

129. $$\frac{\mu_0 I_1}{4\pi(a+x)} = \frac{\mu_0 I_2}{4\pi(a-x)}$$

$$\Rightarrow \frac{a-x}{a+x} = \frac{I_2}{I_1}$$

$$\Rightarrow I_1 a - I_1 x = I_2 a + I_2 x$$

$$\Rightarrow \quad x = \left(\frac{I_1 - I_2}{I_1 + I_2}\right)a$$

Hence, the correct answer is (C).

130. For a closed current carrying loop placed in a uniform magnetic field,

$$F_{\text{net}} = \text{zero}.$$

Hence, the correct answer is (D).

131. Since we know (have already calculated) the magnetic field due to the cross sectional (half current carrying) cylinder at its centre is

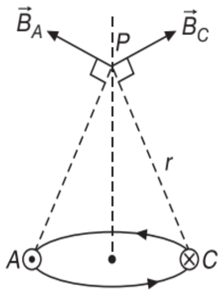
$$B = \frac{\mu_0 I}{\pi^2 R}$$

Further, $F = BI\ell$, so for 1 metre of length, we have

$$F = \frac{\mu_0 I^2}{\pi^2 R}$$

Hence, the correct answer is (C).

132. The magnetic field vectors at a point P on axis of circle are $\vec{B}_A$ and $\vec{B}_C$ at the instants the point charge is at A and C respectively as shown in Figure.



Since the particles rotates in circle, only magnitude of magnetic field remains constant at the point on axis P but it's direction changes. We can also think that, the magnetic field at point on the axis due to charged particle moving along a circular path is given by

$$\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{q\vec{v} \times \vec{r}}{r^3}\right)$$

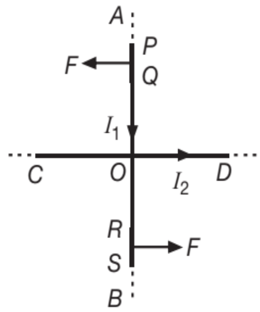
So, it can be seen that the magnitude of the magnetic field at a point on the axis remains constant. But the direction of the field keeps on changing, because at any instant $\vec{B}$ must be perpendicular to $\vec{r}$ (as shown)

Hence, the correct answer is (A).

133. Force on BC and DE are equal and opposite and hence cancel out. So, the force on CD is BIL

Hence, the correct answer is (A).

135. Consider two elements PQ and RS at equal distance point O. Magnetic field at PQ due to I_2 is perpendicular to paper outwards $(\odot)$ and at RS perpendicular to paper inwards $(\otimes)$. Therefore, magnetic force on these elements will be in the directions shown in Figure.



Hence, net force on wire AB will be zero but it will have an anticlockwise torque.

So, $\vec{F} = \vec{0}$ and $\vec{\tau} \neq \vec{0}$ (CCW)

Hence, the correct answer is (B).

137. Magnetic field due to ADB is

$$B_1 = \left(\frac{\mu_0 I}{4\pi a}\right)\theta, \odot$$

and magnetic field due to ACB is

$$B_2 = \frac{\mu_0 I}{4\pi a}(2\pi - \theta), \otimes$$

$$\Rightarrow \quad B_{\text{net}} = B_2 - B_1 = \frac{\mu_0 I}{4\pi a}(2\pi - 2\theta) = \frac{\mu_0 I}{2\pi a}(\pi - \theta), \otimes$$

Hence, the correct answer is (C).

138. Since, $R = \frac{\sqrt{2mK}}{qB}$ and $R_d = R_p$

$$\Rightarrow \quad \frac{\sqrt{2m_d K_d}}{q_d B} = \frac{\sqrt{2m_p K_p}}{q_p B}$$

$$\Rightarrow \quad \frac{\sqrt{2(2m_p)K_d}}{eB} = \frac{\sqrt{2m_p K_p}}{eB}$$

$$\Rightarrow \quad K_p = 2K_d = 2 \times 40 \text{ MeV} = 80 \text{ MeV}$$

Hence, the correct answer is (B).

139. Using Ampere's Circuital Law, we get the desired results.

Hence, the correct answer is (D).

140. Electron velocity v is directed along magnetic field lines, so

$$F = 0$$

Hence, the correct answer is (D).

141. $\vec{B} = \frac{\mu_0 I}{2R}(\pm\hat{i}) + \frac{\mu_0 I}{2R}(\pm\hat{j}) + \frac{\mu_0 I}{2R}(\pm\hat{k})$

$\Rightarrow \quad |\vec{B}| = \frac{\mu_0 I}{2R}\sqrt{3}$

Hence, the correct answer is (B).

142. Since $|\vec{\tau}| = MB\sin\theta$

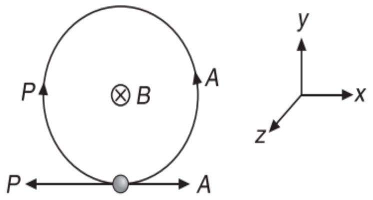
$$\Rightarrow \quad W = \int dW = \int \tau d\theta = \int_0^\theta MB\sin\theta d\theta = MB(-\cos\theta)\Big|_0^\theta$$

$\Rightarrow \quad W = MB(1-\cos\theta)$

Hence, the correct answer is (D).

143. When A emits P, then both have charge of equal magnitude. Also, A and P will have the momentum same in magnitude, but they will move in opposite directions.

Since $r = \frac{mv}{qB} = \frac{p}{qB}$, so A and P will move in the circle of same radius and the same centre but in opposite sense as shown.



If they meet after time t, then we have

$\omega_A t + \omega_P t = 2\pi$

$$\Rightarrow \quad t = \frac{2\pi}{\omega_A + \omega_P} = \frac{2\pi}{\frac{2eB}{4m} + \frac{2eB}{(A-4)m}}$$

$$\Rightarrow \quad t = \frac{4(A-4)m\pi}{eBA}$$

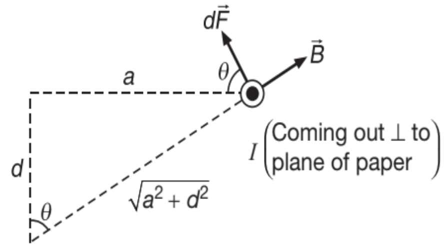
Since $\theta = \omega t$

$$\Rightarrow \quad \theta_A = \omega_A t = \frac{2eB}{4m} \times \frac{4m(A-4)}{eBA}$$

$$\Rightarrow \quad \theta_A = \frac{2(A-4)\pi}{A} = \frac{48\pi}{25} \text{ radian}$$

Hence, the correct answer is (D).

144. The situation, for the cross-sectional view of the wire is shown in figure in which the current is coming out perpendicular to the plane of paper.



Force on the current element ds is $dF = BIds$.

By symmetry, the forces in the plane of the current loop gives zero when contributions around the loop are added. But vertically net force F is calculated as

$F = I(B\sin\theta)$ (circumference of loop)

$\Rightarrow \quad F = (IB\sin\theta)(2\pi a)$

$\Rightarrow \quad F = 2\pi aIB\sin\theta$

Hence, the correct answer is (C).

145. Since $\vec{F} = I(\vec{l} \times \vec{B})$

where, $I\vec{l} = (Il)\hat{i}$ and $\vec{B} = B_0(\hat{i}+\hat{j}+\hat{k})$

$$\Rightarrow \quad \vec{F} = B_0 Il \begin{vmatrix} \overset{\oplus}{\hat{i}} & \overset{\ominus}{\hat{j}} & \overset{\oplus}{\hat{k}} \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$\Rightarrow \quad \vec{F} = B_0 Il(-\hat{j}+\hat{k})$

$\Rightarrow \quad |\vec{F}| = B_0 Il\sqrt{2} = \sqrt{2}B_0 Il$

Hence, the correct answer is (D).

146. The work done is given by

$W = U_f - U_i$

where, $U =$ potential energy

$\Rightarrow \quad W = (-M_f B\cos 0^\circ) - (-M_i B\cos 0^\circ)$

$\Rightarrow \quad W = IB(A_i - A_f)$

where $A_i = a^2$ and $A_f = \pi R^2$

Since, $2\pi R = 4a$

$\Rightarrow \quad R = \frac{2a}{\pi}$

$\Rightarrow \quad A_f = \frac{4a^2}{\pi}$

$\Rightarrow \quad W = BIa^2\left(1 - \frac{4}{\pi}\right)$

Hence, the correct answer is (C).

147. At the point A, field due to both the wire and the loop is inwards, so

$$B_{net} = B_{wire} + B_{loop}$$

So, net field is more than the field due to loop/coil.
Hence, the correct answer is (B).

148. Maximum magnetic field is at surface of the wire and is given by

$$B = \frac{\mu_0}{2\pi}\frac{I}{R} \qquad \text{\{using Ampere's Circuital Law\}}$$

$$\Rightarrow \quad I = \frac{BR}{\frac{\mu_0}{2\pi}}$$

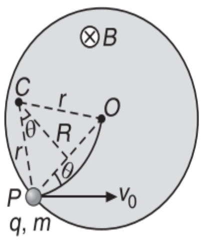
$$\Rightarrow \quad I = \frac{(5\times10^{-3})\left(\frac{3.2}{2}\times10^{-3}\right)}{2\times10^{-7}}$$

$$\Rightarrow \quad I = 40 \text{ A}$$

Hence, the correct answer is (A).

149. Since the particle enters perpendicular to the field, so it follows a circular path of radius r given by

$$r = \frac{mv_0}{qB}$$



For particle to pass through O, P and O must lie on the perimeter of this circle of radius $r = \frac{mv_0}{qB}$.

Since, $R = 2r\sin\theta$

$$\Rightarrow \quad R = 2\frac{mv_0}{qB}\sin\theta$$

$$\Rightarrow \quad v_0 = \frac{qBR}{2m\sin\theta}$$

Hence, the correct answer is (B).

150. If R be the radius of the loop, then

$$A = \pi R^2$$

$$\Rightarrow \quad R = \sqrt{\frac{A}{\pi}}$$

Since, $B = \frac{\mu_0 I}{2R}$

$$\Rightarrow \quad I = \frac{2BR}{\mu_0}$$

By definition $M = IA = \left(\frac{2BR}{\mu_0}\right)A = \left(\frac{2BA}{\mu_0}\right)\sqrt{\frac{A}{\pi}}$

Hence, the correct answer is (C).

151. B at the centre of smallest square is given by

$$B_1 = 4\left[\frac{\mu_0 i}{4\pi\left(\frac{a}{2}\right)}2\sin(45^\circ)\right]$$

$$\Rightarrow \quad B_1 = \frac{2\sqrt{2}\mu_0 i}{\pi a}, \text{ inside}$$

Similarly, we get

$$B_2 = \frac{2\sqrt{2}\mu_0 i}{\pi(2a)}, \text{ outside}$$

$$B_3 = \frac{2\sqrt{2}\mu_0 i}{\pi(3a)}, \text{ inside}$$

and so on.
Hence, net magnetic field at the centre of arrangement is

$$B_{net} = B_1 + B_2 + B_3 + B_4 + B_5 + \ldots\ldots\ldots\ldots$$

$$\Rightarrow \quad B_{net} = \frac{2\sqrt{2}\mu_0 i}{\pi a}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}\ldots\ldots\ldots\right)$$

$$\Rightarrow \quad B_{net} = \frac{2\sqrt{2}\mu_0 i}{\pi a}\log_e(2)$$

Hence, the correct answer is (B).

152. For $\vec{B}$ to be zero at the centre, sum of field due to the ring and that due to the semi-infinite straight section should be equal and opposite

$$\Rightarrow \quad \frac{\mu_0 I}{2\pi R} - \frac{\mu_0 I}{2\pi R}\left(\frac{\theta}{2}\right) = 0$$

$$\Rightarrow \quad \frac{\mu_0 I}{2\pi R}\left(1 - \frac{\theta}{2}\right) = 0$$

$$\Rightarrow \quad \theta = 2 \text{ rad}$$

Hence, the correct answer is (C).

153. Magnetic field due to circle is

$$B_1 = \frac{\mu_0 I}{2R}$$

Since, $L = 2\pi R$

$$\Rightarrow \quad 2R = \frac{L}{\pi}$$

$$\Rightarrow \quad B_1 = \frac{\mu_0 \pi I}{L} \approx 3.14\left(\frac{\mu_0 I}{L}\right)$$

Magnetic field due to square coil is

$$B_2 = 2\sqrt{2}\left(\frac{\mu_0 I}{\pi x}\right)$$

Since $L = 4x$

$$\Rightarrow \quad x = \frac{L}{4}$$

$$\Rightarrow \quad B_2 = 8\sqrt{2}\frac{\mu_0 I}{\pi L} \approx 3.60\left(\frac{\mu_0 I}{L}\right)$$

$$\Rightarrow \quad B_1 < B_2$$

Hence, the correct answer is (B).

154. Consider an element of length dx at a distance x from the wire.

Field at the element is $B = \dfrac{\mu_0 i}{2\pi x}$

If dF is the force experienced by the element then

$$dF = BI\,dx$$

$$\Rightarrow \quad dF = \frac{\mu_0 iI}{2\pi}\frac{dx}{x}$$

$$\Rightarrow \quad F = \frac{\mu_0 iI}{2\pi}\int_{L/2}^{3L/2}\frac{dx}{x}$$

$$\Rightarrow \quad F = \frac{\mu_0 iI}{2\pi}\ell n x\Big|_{L/2}^{3L/2}$$

$$\Rightarrow \quad F = \frac{\mu_0 iI}{2\pi}\left[\ell n\frac{3L}{2} - \ell n\frac{L}{2}\right]$$

$$\Rightarrow \quad F = \frac{\mu_0 iI}{2\pi}\ell n 3$$

Hence, the correct answer is (C).

155. Since Impulse = change in linear momentum, so we have

$$\int F\,dt = mv$$

$$\Rightarrow \quad \int BI\ell\,dt = mv$$

$$\Rightarrow \quad B\ell\int I dt = mv$$

$$\Rightarrow \quad B\ell q = mv \qquad \left\{\because \int I dt = q\right\}$$

$$\Rightarrow \quad q = \frac{mv}{B\ell}$$

Hence, the correct answer is (B).

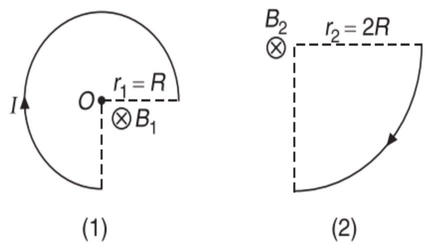
156. Because current in both the loops is zero, so $B = 0$

Hence, the correct answer is (A).

157. Field at centre of an arc subtending an angle θ is

$$B = \left(\frac{\mu_0 I}{4\pi r}\right)\theta$$

Net magnetic field B is



$$B = B_1 + B_2$$

where $B_1 = \dfrac{3\mu_0 I}{8R}$ and $B_2 = \dfrac{\mu_0 I}{8(2R)}$

$$\Rightarrow \quad B = \frac{3}{8}\frac{\mu_0 I}{R} + \frac{\mu_0 I}{2(2R)} \times \frac{1}{4}$$

$$\Rightarrow \quad B = \frac{7}{16}\left(\frac{\mu_0 I}{R}\right)\otimes$$

Hence, the correct answer is (A).

158. Applying Ampere's Law to the Rectangular Loop 12341 shown, we get

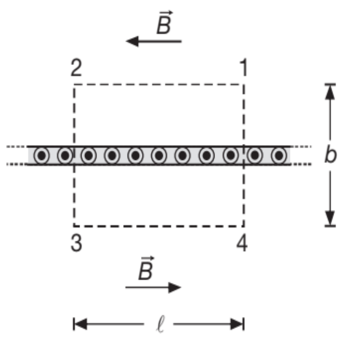
$$\oint \vec{B}\cdot d\vec{\ell} = \int_1^2 \vec{B}\cdot d\vec{\ell} + \int_2^3 \vec{B}\cdot d\vec{\ell} + \int_3^4 \vec{B}\cdot d\vec{\ell} + \int_4^1 \vec{B}\cdot d\vec{\ell}$$

For 14 and 32 (or 41 and 23),

$$\vec{B} \perp d\vec{\ell} \text{ and hence } \int \vec{B}\cdot d\vec{\ell} = 0$$

For 1 to 2 and 3 to 4 $\vec{B} \parallel d\vec{\ell}$

$$\Rightarrow \quad \oint \vec{B}\cdot d\vec{\ell} = 2B\ell$$



According to Ampere's Circuital Law,

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$\Rightarrow \quad 2B\ell = \mu_0(\lambda\ell)$$

$$\Rightarrow \quad B = \frac{\mu_0\lambda}{2}$$

Hence, the correct answer is (B).

159. Due to symmetry of the circuit, it is clear that for each current carrying wire there is another wire that cancels the field of the first.
For example, the field at P due to wire AB is cancelled by field of wire GF. Field of wire CD is cancelled by wire HE. Similarly, AH and CF, DG and BE, BC and HG, AD and EF fields are cancelled and hence net field is zero at P.
Hence, the correct answer is (C).

160. Since, kinetic energy $K = q\Delta V$

$$\Rightarrow \quad K = QU$$

For the particle to move in a circle, we have

$$R = \frac{mv}{QB} = \frac{\sqrt{2mK}}{QB} = \frac{\sqrt{2mQU}}{QB}$$

Since the magnetic moment M is

$$M = iA = i(\pi R^2)$$

$$\Rightarrow \quad M = \frac{Q}{T}(\pi R^2) = \frac{Q}{\left(\frac{2\pi m}{QB}\right)}\pi\left(\frac{2mQU}{Q^2B^2}\right)$$

$$\Rightarrow \quad M = \frac{Q^2B}{2\pi m}\frac{2\pi mQU}{Q^2B^2} = \frac{QU}{B}$$

If U is doubled and B is also doubled, then M remains same
Hence, the correct answer is (D).

161. Since magnetic field at the centre of an arc is equal to

$$B = \frac{\mu_0 I}{4\pi r}\theta$$

Hence net

$$B = \frac{\mu_0 I}{4\pi}\left[\frac{1}{r} - \frac{1}{2r} + \frac{1}{3r} - \frac{1}{4r} + \frac{1}{5r} - \frac{1}{6r}\right]\theta$$

$$\Rightarrow \quad B = \frac{\mu_0 I\theta}{4\pi r}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}\right)$$

$$\Rightarrow \quad B = \frac{\mu_0 I\theta}{4\pi r}\left(\frac{60-30+20-15+12-10}{60}\right)$$

$$\Rightarrow \quad B = \frac{\mu_0 I\theta}{4\pi r}\left(\frac{37}{60}\right) = \frac{37\mu_0 I\theta}{240\pi r}$$

Hence, the correct answer is (B).

162. When the current in the loop flows clockwise, then only the loop will be attracted towards the conductor because AB is attracted strongly towards conductor and CD is repelled weakly away from it (Think Why?). So, net force will attract the loop towards the conductor.
Hence, the correct answer is (D).

163. $F = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{r}$

$$F' = \frac{\mu_0}{2\pi}\frac{\left(\frac{i_1}{3}\right)\left(\frac{i_2}{3}\right)}{3r} = \frac{F}{27}$$

Hence, the correct answer is (D).

164. When the plane of the loop aligns itself normally to the field, then torque on it becomes zero, because angle between $\vec{M}$ and $\vec{B}$ is $0°$. For this to happen the loop must turn through an angle $\left(\frac{\pi}{2}+\theta\right)$.
Hence, the correct answer is (B).

165. τ due to B_0 is $\tau_B = MB_0 = I(\pi R^2)B_0$

τ due to mg is $\tau_{mg} = mgR$

For equilibrium, $\tau_B = \tau_{mg}$

$$\Rightarrow \quad I(\pi R^2)B_0 = mgR$$

$$\Rightarrow \quad I = \frac{mg}{\pi B_0 R}$$

Hence, the correct answer is (A).

166. From Ampere's Circuital Law, we get

$$B_1(2\pi x) = \mu_0 I$$

$$\Rightarrow \quad B_1 = \frac{\mu_0 I}{2\pi x}$$

Again, using Ampere's Circuital Law, we get

$$B_2(2\pi)(2x) = \mu_0(I+I)$$

$$\Rightarrow \quad B_2 = \frac{\mu_0 I}{2\pi x} = B_1$$

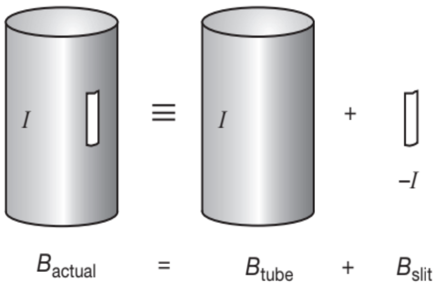
Hence, the correct answer is (C).

167. The magnetic field due to a lengthy thin walled tube at any internal point is zero. When a longitudinal slit of width w is removed, then a net field will exist inside. The tube with a slit can be visualised as equivalent to a full tube carrying a current I and a slit carrying current $-I$ (i.e. current equal in magnitude and opposite in direction). So, the net field inside the tube is due to the slit carrying current I in opposite direction.

Since

$$B_{\text{tube}} = 0$$

$$\Rightarrow \quad B_{\text{actual}} = B_{\text{slit}} = \frac{\mu_0 I_{\text{slit}}}{2\pi R}$$



(Since width of slit is very small, so field due to slit is equal to that of a thin wire).

Further

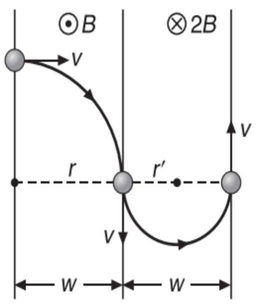
$$I_{\text{slit}} = \left(\frac{I}{2\pi R}\right)w$$

$$\Rightarrow \quad B_{\text{actual}} = \frac{\mu_0\left(\frac{I}{2\pi R}\right)w}{2\pi R} = \frac{\mu_0 I w}{4\pi^2 R^2}$$

Hence, the correct answer is (C).

168. For the first, $r = \frac{mv}{qB}$

For the second region, $r' = \frac{mv}{q(2B)} = \frac{r}{2}$



This makes us conclude that the particle will cover a quarter circle in the first region and a semicircle in the second region, so that it can come out of the magnetic field. Hence, we have

$$t = \frac{T}{4} + \frac{T'}{2}$$

$$\Rightarrow \quad t = \frac{1}{4}\left(\frac{2\pi m}{qB}\right) + \frac{1}{2}\left[\frac{2\pi m}{q(2B)}\right]$$

$$\Rightarrow \quad t = \frac{\pi m}{2qB} + \frac{\pi m}{2qB} = \frac{\pi m}{qB}$$

Hence, the correct answer is (B).

169. Since we know that the GYROMAGNETIC RATIO equals,

$$\frac{\text{Magnetic moment}}{\text{Angular momentum}} = \frac{q}{2m} = \frac{e}{2m}$$

So, magnetic moment is

$$M = \left(\frac{e}{2m}\right)$$

(angular momentum of the electron in n^{th} orbit)

From Bohr's Quantisation Rule, we have

$$L = \frac{nh}{2\pi}$$

$$\Rightarrow \quad M = \left(\frac{e}{2m}\right)\left(\frac{nh}{2\pi}\right) = \frac{neh}{4\pi m}$$

Hence, the correct answer is (B).

170. Magnetic field is non zero only in the region between the two solenoids, where $B = \mu_0 n_2 i_2$

So, energy stored per unit volume is

$$u_2 = \frac{B^2}{2\mu_0} = \frac{\mu_0 n_2^2 i_2^2}{2}$$

The energy stored per unit length is the energy per unit volume times the area of cross section in which $B \neq 0$

$$\Rightarrow \quad e = \frac{\mu_0 n_2^2 i_1^2}{2}\left[\pi\left(r_2^2 - r_1^2\right)\right]$$

Since $n_1 i_1 = n_2 i_2$

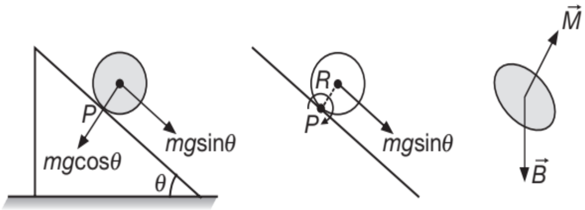
$$\Rightarrow \quad e = \frac{\mu_0 n_1^2 i_1^2}{2}\left[\pi\left(r_2^2 - r_1^2\right)\right]$$

$$\Rightarrow \quad e = \frac{\mu_0 \pi}{2} n_1^2 i_1^2\left(r_2^2 - r_1^2\right)$$

Hence, the correct answer is (A).

172. For rotational equilibrium of sphere about point P, we have

$$(mg\sin\theta)R = M(B\sin(180-\theta))$$

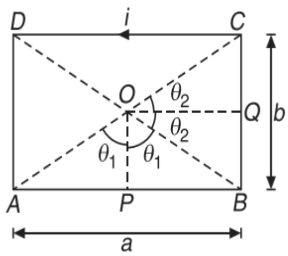


$$\Rightarrow \quad (mgR\sin\theta) = I(\pi R^2)B\sin\theta$$

$$\Rightarrow \quad B = \frac{mg}{\pi IR}$$

Hence, the correct answer is (D).

173. Due to symmetry, $B_{AB} = B_{CD}$ and $B_{BC} = B_{DA}$



$$B_{AB} = \frac{\mu_0 i}{4\pi\left(\frac{b}{2}\right)}(\sin\theta_1 + \sin\theta_1)$$

$$\Rightarrow \quad B_{AB} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{4i}{b}\right)\left(\frac{a}{\sqrt{a^2+b^2}}\right)$$

Similarly, $B_{BC} = B_{DA} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{4i}{a}\right)\left(\frac{b}{\sqrt{a^2+b^2}}\right)$

$$\Rightarrow \quad B = B_{AB} + B_{BC} + B_{CD} + B_{DA} = 2(B_{AB} + B_{CD})$$

$$\Rightarrow \quad B = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{4i}{\sqrt{a^2+b^2}}\right)\left(\frac{a}{b} + \frac{b}{a} + \frac{a}{b} + \frac{b}{a}\right)$$

$$\Rightarrow \quad B = \frac{\mu_0 i}{\pi\sqrt{a^2+b^2}}\left(2\frac{a}{b} + 2\frac{b}{a}\right)$$

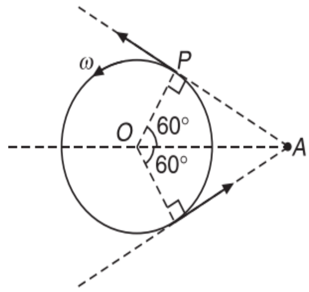
$$\Rightarrow \quad B = \frac{2\mu_0 i}{\pi\sqrt{a^2+b^2}}\left(\frac{a^2+b^2}{ab}\right)$$

$$\Rightarrow \quad B = \frac{2\mu_0 i}{\pi ab}\sqrt{a^2+b^2}$$

$$\Rightarrow \quad B = \frac{\mu_0 i}{4\pi}\left(\frac{8\sqrt{a^2+b^2}}{ab}\right)$$

Hence, the correct answer is (A).

174. The point A shall record zero magnetic field (due to α particle) when the α particle is at position P and Q as shown in Figure.



The time taken by α-particle to go from P to Q is

$$t = \frac{1}{3}\left(\frac{2\pi}{\omega}\right)$$

$$\Rightarrow \quad \omega = \frac{2\pi}{3t}$$

Hence, the correct answer is (B).

175. The net electric field

$$E = \vec{E}_1 + \vec{E}_2$$

$$\Rightarrow \quad E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

The net force acting on the electron is zero because it moves with constant velocity, due to its motion on straight line.

$$\Rightarrow \quad \vec{F}_{net} = \vec{F}_e + \vec{F}_m = 0$$

$$\Rightarrow \quad |\vec{F}_e| = |\vec{F}_m|$$

$$\Rightarrow \quad eE = evB$$

$$\Rightarrow \quad v = \frac{E}{B} = \frac{\sigma}{\varepsilon_0 B}$$

The time of motion inside the capacitor is

$$t = \frac{\ell}{v} = \frac{\varepsilon_0 \ell B}{\sigma}$$

Hence, the correct answer is (B).

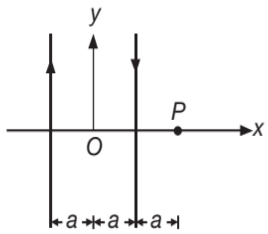
176. $F = BIl\sin\theta$

$$\Rightarrow \quad F_{PQ} = BIl\sin 0^\circ = 0$$

$$\Rightarrow \quad F_{QR} = BIl\sin 90^\circ = BIl$$

Hence, the correct answer is (C).

177. If the currents are in the directions shown in Figure.



Then $\vec{B}_0 = -2\left(\frac{\mu_0}{2\pi}\frac{I}{a}\right)\hat{k} = -\left(\frac{\mu_0 I}{\pi a}\right)\hat{k}$

$$\vec{B} = \left(\frac{\mu_0}{2\pi}\frac{I}{a}\right)\hat{k} - \left(\frac{\mu_0}{2\pi}\frac{I}{3a}\right)\hat{k}$$

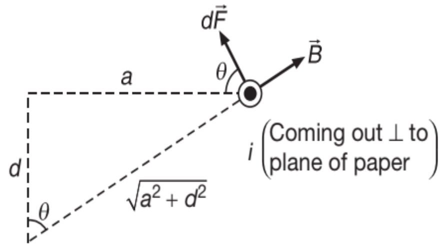
$$\Rightarrow \quad \vec{B} = \left(\frac{\mu_0}{3\pi}\frac{I}{a}\right)\hat{k} = -\frac{B_0}{3}\hat{k}$$

$$\Rightarrow \quad B = \frac{B_0}{3} \text{ along } -z \text{ axis}$$

$$\Rightarrow \quad B = \frac{B_0}{3}, \otimes$$

Hence, the correct answer is (C).

178. The situation, for the cross-sectional view of the wire is shown in Figure in which the current is coming out perpendicular to the plane of paper.



Force on the current element ds is $dF = Bids$.
By symmetry, the forces in the plane of the current loop gives zero when contributions around the loop are added. But vertically net force F is calculated as

$$F = i(B\sin\theta) \text{ (circumference of loop)}$$

$$\Rightarrow \quad F = (iB\sin\theta)(2\pi a)$$

$$\Rightarrow \quad F = 2\pi aiB\sin\theta$$

Since $\sin\theta = \dfrac{a}{\sqrt{a^2+d^2}}$

$$\Rightarrow \quad F = \frac{2\pi a^2 iB_0}{\sqrt{a^2+d^2}}$$

Hence, the correct answer is (B).

179. $\vec{F}_{aob} = \vec{F}_{ab} = I\left(\vec{\ell}_{ab} \times \vec{B}\right)$

where $\ell_{ab} = 2\sqrt{(2)(2)} = 4$ m

$$\Rightarrow \quad \vec{F}_{ab} = 2\left[\left(-4\hat{j}\right)\times\left(-4\hat{k}\right)\right] = 32\hat{i}$$

Hence, the correct answer is (B).

180. According to Modified Work-Energy Theorem, we get,

$$W = qE\Delta x = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 - 0$$

$$\Rightarrow \quad \left(4\times10^{-6}\right)(4)(x) = \frac{1}{2}m\left(4^2+3^2\right)$$

$$\Rightarrow \quad \left(16\times10^{-6}\right)x = \frac{1}{2}\times10\times10^{-6}\times25$$

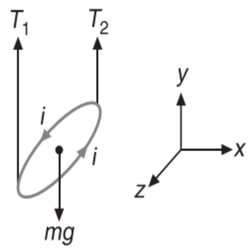
$$\Rightarrow \quad x = \frac{250}{32}\text{ m} = \frac{125}{16}\text{ m}$$

Hence, the correct answer is (B).

181. For equilibrium, we have

$$\sum F = 0 \text{ and } \sum\tau = 0$$

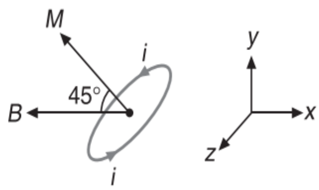
$$\Rightarrow \quad T_1 + T_2 = mg \qquad \ldots(1)$$



Torque on ring due to current is

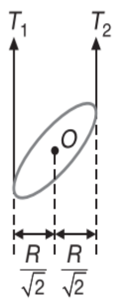
$$\tau_i = MB\sin\theta = i\pi R^2 B\sin(45°)$$

$$\Rightarrow \quad \tau_i = \frac{i\pi R^2 B}{\sqrt{2}}, \text{ counter clockwise in } xy \text{ plane}$$



Since $\sum\tau = 0$

$$\Rightarrow \quad \tau_i + \tau_{T_1} + \tau_{T_2} = 0 \qquad \{\text{about } 0\}$$



$$\Rightarrow \underbrace{\frac{i\pi R^2 B}{\sqrt{2}}}_{CCW} + \underbrace{T_1\left(\frac{R}{\sqrt{2}}\right)}_{CW} + \underbrace{T_2\left(\frac{R}{\sqrt{2}}\right)}_{CCW} = 0$$

Taking CW as positive, we get

$$\frac{T_1 R}{\sqrt{2}} = \frac{T_2 R}{\sqrt{2}} + \frac{i\pi R^2 B}{\sqrt{2}}$$

$$\Rightarrow T_1 = T_2 + \pi i R B$$

$$\Rightarrow T_1 - T_2 = \frac{mg}{4} \quad \text{...(2)}$$

Using equations (1) and (2), we get

$$T_1 = \frac{5mg}{8},\ T_2 = \frac{3mg}{8}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{5}{3}$$

Hence, the correct answer is (B).

182. Since, $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$ ...(1)

$$\Rightarrow \Delta V = V = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{a}{b}\right) \quad \text{...(2)}$$

$$\Rightarrow \frac{\lambda}{2\pi\varepsilon_0} = \frac{V}{\ln\left(\frac{a}{b}\right)}$$

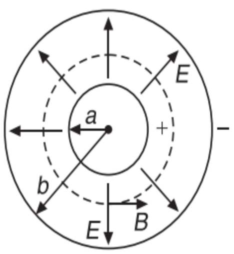
From (1) and (2), we get

$$E = \frac{V}{r\ln\left(\frac{a}{b}\right)}$$

B varies with r as

$$B = \frac{\mu_0 I}{2\pi r}$$

Since, B and E are perpendicular to each other, so for the electron to travel undeviated parallel to axis in the evacuated region, we have



$$F_m = F_e$$

$$\Rightarrow qv_0 B = qE$$

$$\Rightarrow v_0 = \frac{E}{B} = \frac{2\pi V}{\mu_0 I \ln\left(\frac{a}{b}\right)}$$

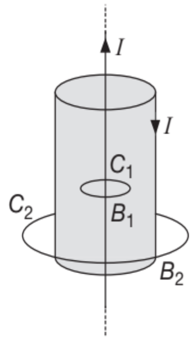
Hence, the correct answer is (B).

183. By Right hand thumb rule, the field due to both the segments is out of the plane that is along +ve z axis.

Hence, the correct answer is (B).

Multiple Correct Choice Type Questions

1. We apply Ampere's Law to the co-axial circular loops C_1 and C_2 with corresponding magnetic fields B_1 and B_2 respectively. For C_1 and C_2 we have the OPTION (B) correct. On the other hand, when the currents are in same direction, $B_1 \neq 0$ and $B_2 \neq 0$. Hence, the OPTIONS (C) and (D) are also correct.



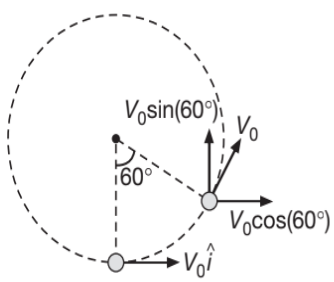
Hence, (B), (C) and (D) are correct.

2. $B_P = 0$, because current in upper part and lower part is the same due to which the magnetic field for upper part is into the page and for the lower part the field is out of the page. $B_Q \neq 0$, because current in upper and lower section is not same. Hence net field at Q will not be zero. $B_R = 0$, because current in upper and lower section is same.

Hence, (A) and (C) are correct.

3. $\vec{v} \perp \vec{B}$

Therefore, path of the particle is a circle. In magnetic field speed of particle remains constant. Therefore, distance moved by the particle in time $t = \dfrac{\pi}{\alpha B_0}$ is $v_0 t$ or $\dfrac{\pi v_0}{\alpha B_0}$



Since $T = \frac{2\pi m}{qB_0} = \frac{2\pi}{\alpha B_0}$

$\Rightarrow t = \frac{\pi}{3\alpha B_0} = \frac{T}{6}$

$\Rightarrow \vec{v} = v_0\left(\cos(60°)\hat{i} + \sin(60°)\hat{j}\right)$

$\Rightarrow \vec{v} = \frac{v_0}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$

Hence, (B) and (C) are correct.

Please note that since the magnitude of velocity remains same in the magnetic field. This is true but will not help us to conclude whether (D) is correct or false.

4. $T = \frac{2\pi m}{qB}$

$\Rightarrow a = \frac{T_1}{T_2} = 1$

$r = \frac{mv\sin\theta}{qB}$

$\Rightarrow b = \frac{r_1}{r_2} = \frac{\sin 30°}{\sin 60°} = \frac{1}{\sqrt{3}}$

$p = (T)(v\cos\theta) = \frac{2\pi mv\cos\theta}{qB}$

$\Rightarrow c = \frac{p_1}{p_2} = \frac{\cos 30°}{\cos 60°} = \sqrt{3}$

From above, we get

$abc = 1$, $a = bc$ and $c = 3ab$

Hence, (B), (C) and (D) are correct.

5. At the axis of the coil, we have

$B_x = \frac{\mu_0 iR^2}{2(R^2+x^2)^{\frac{3}{2}}}$ **{OPTION (A)}**

Let us now calculate $\int_0^\infty B_x dx$

$\Rightarrow \int_0^\infty B_x dx = \frac{\mu_0 iR^2}{2}\int_0^\infty \frac{dx}{(R^2+x^2)^{\frac{3}{2}}}$

Substituting $x = R\tan\theta$, we get

$dx = R\sec^2\theta d\theta$

When $x = 0$, $\theta = 0$ and when $x \to \infty$, $\theta \to \frac{\pi}{2}$

$\Rightarrow \int_0^\infty B_x dx = \frac{\mu_0 iR^2}{2}\int_0^{\frac{\pi}{2}} \frac{R\sec^2\theta d\theta}{R^3\sec^3\theta}$

$\Rightarrow \int_0^\infty B_x dx = \frac{\mu_0 i}{2}\int_0^{\frac{\pi}{2}} \cos\theta d\theta$

$\Rightarrow \int_0^\infty B_x dx = \frac{\mu_0 i}{2}\left(\sin\frac{\pi}{2} - \sin 0°\right)$

$\Rightarrow \int_0^\infty B_x dx = \frac{\mu_0 i}{2}$ **{OPTION (B)}**

Similarly, $\int_{-\infty}^{+\infty} B_x dx = \frac{\mu_0 i}{2}\left(\sin\frac{\pi}{2} + \sin\frac{\pi}{2}\right)$

$\Rightarrow \int_{-\infty}^{\infty} B_x dx = \left(\frac{\mu_0 i}{2}\right)(2) = \mu_0 i$ **{OPTION (C)}**

Hence, (A), (B) and (C) are correct.

6. $F = Bqv$

$\Rightarrow B = \frac{F}{qv} = \frac{N}{Cms^{-1}} = NA^{-1}m^{-1}$ $\{\because 1\,Cs^{-1} = 1\,A\}$

Further $B = \frac{\mu_0 I}{2R}$

$\Rightarrow \mu_0 = \frac{2BR}{I} = \frac{(NA^{-1}m^{-1})m}{A} = NA^{-2}$

Since $\phi_B = BA = (NA^{-1}m^{-1})(m^2) = NmA^{-1}$

Since we know that

$\left[\frac{L}{R}\right] = [CR]$

$\Rightarrow [R] = \left[\sqrt{\frac{L}{C}}\right]$

So, $\sqrt{\frac{L}{C}}$ will have units same as that of resistance VA^{-1}.

Hence, (A), (B) and (D) are correct.

7. $B = \frac{\mu_0 NIr^2}{2(r^2+x^2)^{3/2}}$ along x-axis

$\Rightarrow B = \frac{\mu_0 NI}{2r}\frac{r^3}{(r^2+x^2)^{\frac{3}{2}}}$

$$\Rightarrow \quad B = \frac{\mu_0 NI}{2r}\sin^3\theta$$

According to which, the coil with lesser radius is to make more contribution.

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{r_2}{r_1} = \frac{1}{2}$$

Hence, (B) and (D) are correct.

8. $\vec{B} \perp \vec{v}$, so, it may along y-axis, so, **OPTION (A)** is correct

$\vec{F} \perp \vec{v}$,

$\Rightarrow \quad \vec{a} \perp \vec{v} = 0$

$\Rightarrow \quad \vec{a} \cdot \vec{v} = 0$

$\Rightarrow \quad a_1 b_1 + a_2 b_2 = 0$, so, **OPTION (B)** is correct

Since $\vec{B} \perp \vec{v}$, so for B along x-axis this cannot be possible. Hence, **OPTION (C)** is incorrect.
Magnetic force cannot change the kinetic energy of a particle. Hence, **OPTION (D)** is also correct.
Hence, (A), (B) and (D) are correct.

9. Under the state of equilibrium,

$$\begin{pmatrix}\text{Electrostatic force}\\ \text{acting downward}\end{pmatrix} = \begin{pmatrix}\text{Magnetic force}\\ \text{acting upward}\end{pmatrix}$$

$$\Rightarrow \quad qE = qvB\sin 90^\circ$$

$$\Rightarrow \quad v = \frac{E}{B}$$

and under this state, the particle must hit at O

Hence, (C) and (D) are correct.

10. $B_a = \dfrac{\mu_0 i}{4\pi R} + \dfrac{3\mu_0 i}{8R} = \dfrac{\mu_0 i}{4R}\left(\dfrac{1}{\pi} + \dfrac{3}{2}\right)$

$$\Rightarrow \quad B_b = \frac{3\mu_0 i}{8R} - \frac{\mu_0 i}{2\pi R} = \frac{\mu_0 i}{2R}\left(\frac{3}{4} - \frac{1}{\pi}\right)$$

$$\Rightarrow \quad B_c = \frac{\mu_0 i}{4\pi R} + \frac{3\mu_0 i}{8R} - \frac{\mu_0 i}{4\pi R} = \frac{3\mu_0 i}{8R}$$

Hence, (A), (B) and (D) are correct.

12. Areal velocity $= \dfrac{\text{Area Swept}}{\text{Period of one Revolution}} = \dfrac{\pi r^2}{T}$

$$\Rightarrow \quad \text{Areal velocity} = \frac{\pi\left(\dfrac{mv}{qB}\right)^2}{\left(\dfrac{2\pi m}{qB}\right)} = \frac{mv^2}{2qB} = \frac{K}{qB}$$

$$\Rightarrow \quad \frac{dA}{dt} = \frac{K}{qB} = \frac{p^2}{2mqB}$$

Hence, (A), (B), (C) and (D) are correct.

13. Since, $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$

So, for the proton to suffer deflection along $+x$ axis, the force acting on it must be along $+x$ axis. For that to happen, we must/may have

CASE-1: $\vec{E}$ along x-axis ($\vec{B}$ is switched off)

CASE-2: $\vec{v}$ along $+y$ axis and $\vec{B}$ along $+z$ axis, so that $\vec{v} \times \vec{B}$ is along $+x$ axis ($\vec{E}$ switched off)

CASE-3: $\vec{E}$ along x-axis, $\vec{B}$ along $+y$ axis

$$\vec{v} = \pm v_y \hat{j} - v_z \hat{k}$$

$\vec{v} \times \vec{B}$ is along $+x$ axis

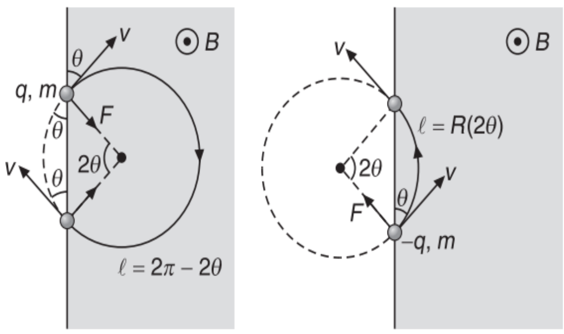
CASE-4: $\vec{E}$ along $+x$ axis $\vec{B}$ along $-y$ axis

$$\vec{v} = \pm v_y \hat{j} + v_z \hat{k}$$

$\vec{v} \times \vec{B}$ is along $+x$ axis

Hence, (B), (C) and (D) are correct.

14. Distance described by the positively charged particle in the magnetic field is



(For +q) (For −q)

$$l = 2(\pi - \theta)R$$

$$\Rightarrow \quad l = 2(\pi - \theta)\frac{mv}{qB}$$

Time taken by the positive charge particle in the magnetic field is

$$t = \frac{l}{v} = \frac{2(\pi - \theta)}{v}\frac{mv}{qB}$$

$$\Rightarrow \quad t = \frac{2m(\pi - \theta)}{qB} \qquad \ldots(1)$$

For the case of negative charge particle entering the field, we have

$l = R(2\theta)$, where $R = \dfrac{mv}{qB}$

$$\Rightarrow \quad t = \frac{l}{v} = \frac{2R\theta}{v} = \frac{2mv\theta}{vqB}$$

$$\Rightarrow \quad t = \frac{2m\theta}{qB} \qquad \ldots(2)$$

Hence, for $q_1 = q_2$, $t_1 = t_2$, **OPTION (B)** is correct.

for $q_1 > 0$, $q_2 < 0$, $t_1 > t_2$, **OPTION (C)** is incorrect.

for $q_1 < 0$, $q_2 > 0$, $t_1 < t_2$, **OPTION (D)** is correct.

Hence, (B) and (D) are correct.

16. Restoring torque $\tau = -MB\sin\theta$

$$\Rightarrow \quad \tau = -\left(\pi r^2 I_2\right)\left(\frac{\mu_0 I_1}{2R}\right)\theta \quad \{\because \text{ for small } \theta, \sin\theta \cong \theta\}$$

$$\Rightarrow \quad \left(\frac{1}{2}mr^2\right)\alpha = -\left(\frac{\mu_0 \pi I_1 I_2 r^2}{2R}\right)\theta$$

$$\Rightarrow \quad \alpha = -\left(\frac{\mu_0 \pi I_1 I_2}{mR}\right)\theta$$

$$\Rightarrow \quad \ddot{\theta} + \left(\frac{\mu_0 \pi I_1 I_2}{mR}\right)\theta = 0$$

Comparing with standard equation of SHM, i.e., $\ddot{\theta} + \omega^2\theta = 0$, we get

$$\omega = \sqrt{\frac{\mu_0 \pi I_1 I_2}{mR}}$$

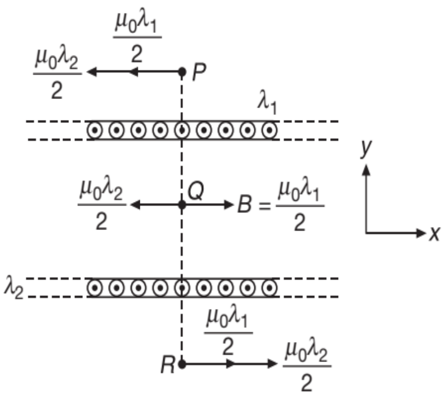
$$\Rightarrow \quad T = 2\pi\sqrt{\frac{mR}{\mu_0 \pi I_1 I_2}}$$

$$\Rightarrow \quad T \propto \frac{1}{\sqrt{I_1 I_2}}, \; T \propto \sqrt{m}, \; T \propto \sqrt{R}$$

and is independent of r

Hence, (A), (B) and (D) are correct.

17. If λ is the current per unit length, then



$$B = \frac{\mu_0 \lambda}{2}$$

$$B_Q = \frac{\mu_0 \lambda_1}{2} - \frac{\mu_0 \lambda_2}{2}$$

$$B_R = \frac{\mu_0 \lambda_1}{2} + \frac{\mu_0 \lambda_2}{2} = B_P$$

Hence, (B) and (C) are correct.

18. In REGION 1, the charged particle passes undeviated (See Figure given with PROBLEM), so we conclude that the electrostatic force must be balanced by magnetic force. Hence

$$qE = qvB$$

$$\Rightarrow \quad v = \frac{E}{B} \qquad \text{\{OPTION (A)\}}$$

Since in both regions, the charged particle has $\vec{F}$ and $\vec{s}$ perpendicular to each other, so work done in both regions is zero. **{OPTION (B)}**

Further, REGION 2, has got only the magnetic field, so the charged particle will move in a circle of radius

$$r = \frac{mv}{qB_0} \qquad \{\because B_0 \text{ is the field in REGION 2}\}$$

$$\Rightarrow \quad r = \frac{m}{qB_0}\frac{E}{B}$$

$$\Rightarrow \quad r = \frac{E}{(q/m)BB_0} = \frac{E}{sBB_0} \qquad \text{\{OPTION (C)\}}$$

For $\ell_2 > r$ i.e. $\ell_2 > \frac{E}{sBB_0}$ the charged particle will complete the semicircle to emerge out from region with a velocity directed opposite to the initial velocity. Hence $\vec{v}' = -\vec{v}$ **{OPTION (D)}**

Hence, (A), (B), (C) and (D) are correct.

19. Since $\vec{v} \perp \vec{B}$, so the particle is performing circular motion in xy plane with radius of circle given by

$$r = \frac{mv}{qB} = \frac{1 \times 10}{2} = 5 \text{ m}$$

The radius of the circle given by the equation

$x^2 + y^2 - 4x - 21 = 0$ is 5 m

Also, the radius of circle given by the equation

$x^2 + y^2 = 25$ is 5 m

The time period of revolution of particle is given by

$$T = \frac{2\pi m}{qB} = \frac{2\pi \times 1}{1 \times 2} = 3.14 \text{ s}$$

Hence, (A), (B) and (D) are correct.

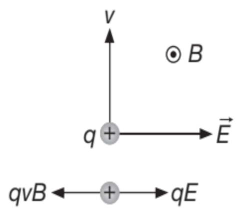
21. Torque acting on loop is given by

$$\tau = ilbB_0 \sin\theta$$

The torque direction can be given by right hand thumb rule according to which τ acts along $-y$ direction and hence it has a tendency to decrease θ.

Hence, (A), (B) and (D) are correct.

22. A uniform magnetic field B and a uniform electric field E exist perpendicular to each other and the particle moves along a direction perpendicular to both of these fields, then forces exerted by these two fields may be opposite to each other as shown in Figure. If magnitudes of these forces are equal, then resultant force on the particle will become equal to zero.



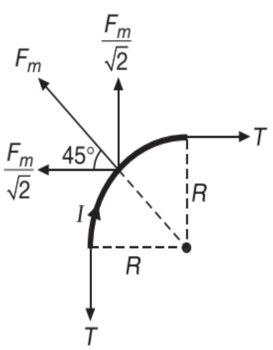
Hence, particle will move with constant velocity. Hence, the OPTION (A) is correct, obviously, the OPTION (B) is wrong.
If E is equal to zero, then the particle will experience a force due to magnetic field alone. But the force exerted by the magnetic field is always perpendicular to the direction of its motion, hence, no power is associated with this force. In other words, no work is done by the magnetic field on the particle. Therefore, KE of the particle will remain constant. Hence, the OPTION (C) is also correct.

Hence, (A) and (C) are correct.

23. To find the Ampere force on a conductor of any shape just replace the conductor by an imaginary straight conductor joining the two ends of the given conductor having effective length $\ell_{eff} = \lambda$.

Hence, (A) and (C) are correct.

25. Since the wire is held in equilibrium, so net force on the wire is zero.



Net magnetic force on the wire is $F = BIl_{eff} = BI(\sqrt{2}R)$

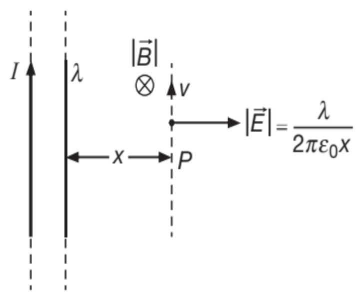
$\Rightarrow \quad F_m = \sqrt{2}BIR$

From the figure it is clear that

$$T = \frac{F_m}{\sqrt{2}} = BIR$$

Hence, (A), (B) and (C) are correct.

26. At P, electric field $E = \frac{\lambda}{2\pi\varepsilon_0 x}$ (to the right), and magnetic field $B = \frac{\mu_0 I}{2\pi x}$ (into the paper).



For no deflection, $qE = qvB\sin(90°)$

$$\Rightarrow \quad v = \frac{E}{B}$$

$$\Rightarrow \quad v = \left(\frac{\lambda}{2\pi\varepsilon_0 x}\right)\left(\frac{2\pi x}{\mu_0 I}\right) = \frac{\lambda}{I}\frac{1}{\varepsilon_0\mu_0} = \frac{\lambda c^2}{I}$$

Hence, (A) and (D) are correct.

27. The pairs $\vec{F}$ and $\vec{v}$ and $\vec{F}$ and $\vec{B}$ are always at right angles to each other, because $\vec{F}$ is always perpendicular to the plane containing $\vec{B}$ and $\vec{v}$. Vectors $\vec{B}$ and $\vec{v}$ have any angle between them.

Hence, (A) and (B) are correct.

28. Magnetic field at the centre of first coil is

$$B_1 = \frac{\mu_0 N_1 i_1}{2R_1} = \frac{(4\pi\times10^{-7})(50)(2)}{2(5\times10^{-2})}$$

$$\Rightarrow \quad B_1 = 4\pi\times10^{-4}\text{ T}$$

Magnetic field at the centre of second coil is

$$B_2 = \frac{\mu_0 N_2 i_2}{2R_2} = \frac{(4\pi\times10^{-7})(100)(2)}{(2)(10\times10^{-2})}$$

$$\Rightarrow \quad B_2 = 4\pi\times10^{-4}\text{ T}$$

When currents are in the same sense, then we have

$$B_{net} = B_1 + B_2 = 8\pi\times10^{-4}\text{ T}$$

When currents are in the opposite sense, then we have

$$B_{net} = B_1 - B_2 = 0$$

Hence, (A) and (C) are correct.

29. Pitch $= \left(\frac{2\pi m}{QB}\right)v\cos\theta$

Also, $r = \frac{mv\sin\theta}{QB}$

Maximum distance of the particle from the x-axis is

$$d_{max} = 2r$$

$$\Rightarrow \left(\frac{2\pi m}{QB}\right)v\cos\theta = 2\frac{mv}{QB}\sin\theta$$

$$\Rightarrow \tan\theta = \pi$$

Hence, (A), (B) and (C) are correct.

31. Magnetic field at the centre of a circular coil is given by

$$B = \frac{\mu_0 I}{2R}$$

Since we know that $c^2 = \frac{1}{\mu_0\varepsilon_0}$

$$\Rightarrow \mu_0 = \frac{1}{c^2\varepsilon_0}$$

$$\Rightarrow B = \frac{I}{2c^2\varepsilon_0 R}$$

Hence, (A) and (C) are correct.

32. When electric force qE and magnetic force qvB balance each other, then the particle will move in a straight line.

$$\Rightarrow qE = qv_2B$$

$$\Rightarrow v_2 = \frac{E}{B}$$

If electric field force is not balanced by the magnetic field force, then either $qE > qvB$ OR $qE < qvB$

When $qE > qv_3B$, then $v_3 < \frac{E}{B}$ and when $qE < v_1Bq$, then $v_1 > \frac{E}{B}$.

Hence, (A), (B) and (C) are correct.

33. Magnetic force is always perpendicular to the velocity of the particle i.e., $\vec{v} \perp \vec{a}$

$$\Rightarrow \vec{v}\cdot\vec{a} = 0$$

$$\Rightarrow 12 + 4x = 0$$

$$\Rightarrow x = -3$$

Further since $\vec{F} = q(\vec{v}\times\vec{B})$, so the magnetic field is perpendicular to the plane of velocity i.e., xy plane. Hence the magnetic field is along z-direction.
Also, we know that work done by a magnetic force is zero so from Work Energy Theorem, i.e., $W = \Delta K$, we get

$$K_{\text{final}} = K_{\text{initial}}$$

Hence, (A), (C) and (D) are correct.

34. Apply Ampere's Circuital Law we get

$$B(r) = 0 \quad \{\text{inside the tube}\}$$

$$\Rightarrow B(2\pi r) = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

$$\Rightarrow B \propto \frac{1}{r}$$

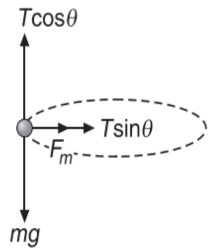
Hence, (A) and (C) are correct.

35. For the conical pendulum, force equations on the charge can be written as

$$T\sin\theta + F_m = mr\omega^2, \text{ where } r = L\sin\theta$$

$$\Rightarrow F_m = qvB = q(L\sin\theta)\omega B$$

$$\Rightarrow T\cos\theta = mg$$



$$\Rightarrow \frac{mg}{\cos\theta}\sin\theta + q(\omega L\sin\theta)B = m\omega^2 L\sin\theta$$

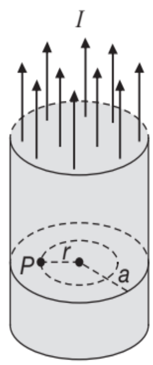
$$\Rightarrow B = \frac{m\omega^2 L\sin\theta}{(\omega L\sin\theta)q} - \frac{\frac{mg\sin\theta}{\cos\theta}}{(\omega L\sin\theta)q}$$

$$\Rightarrow B = \frac{m\omega}{q} - \frac{mg}{q\omega L\cos\theta} = \frac{1}{\beta}\left(\omega - \frac{g}{\omega L\cos\theta}\right)$$

Rate of change of angular momentum i.e., torque of the bob about point of suspension is not constant.

Hence, (A) and (D) are correct.

37. $dI = JdA$



$\Rightarrow \quad dI = kr^2(2\pi r\, dr)$

$\Rightarrow \quad I = 2\pi k \int_0^a r^3 dr$

$\Rightarrow \quad I = \frac{1}{2}\pi k a^4$

Further, field for $r > a$ is

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 k a^4}{4r}$$

and field for $r < a$ is

$$B = \frac{\mu_0\left(\frac{\pi k r^4}{2}\right)}{2\pi r} = \frac{1}{4}\mu_0 k r^3$$

Hence, (A), (C) and (D) are correct.

38. Time period of motion is given as

$$T = \frac{2\pi m}{qB} = \frac{2\pi}{\alpha B_0}$$

At $t = \frac{\pi}{\alpha B_0} = \frac{T}{2}$, the velocity of particle is

$$\vec{v}' = -v_0\hat{i} + v_0\hat{k}$$

Speed of charge in magnetic field always remains constant, so we have

$$v' = v = v_0\sqrt{2}$$

At $t = \frac{2\pi}{\alpha B_0} = T$, displacement is equal to pitch. So, displacement is given by

$$\Delta x = v_0 T = \frac{2\pi v_0}{\alpha B_0}$$

At $t = \frac{2\pi}{\alpha B_0} = T$, the distance (l) is given as product of speed and time, so we have

$$l = vT = \frac{2\sqrt{2}v_0\pi}{\alpha B_0}$$

Hence, (A) and (C) are correct.

40. Velocity vector is perpendicular to magnetic field. Therefore, path of the particle is a circle or radius

$$r = \frac{mv_0}{qB}$$

$v = v_0$ as the speed of the particle does not change in magnetic field.

Centre of the circle is C.

$CP = CQ$

$\Rightarrow \quad \angle CPQ = \angle CQP$

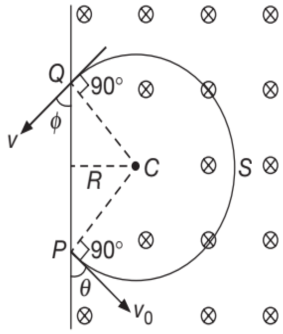
$\Rightarrow \quad 90° - \theta = 90° - \phi$

$\Rightarrow \quad \theta = \phi$

Further $PQ = 2PR = 2r\cos(90° - \theta)$

$\Rightarrow \quad PQ = 2r\sin\theta$

$\Rightarrow \quad PQ = \frac{2mv_0 \sin\theta}{qB}$



Also, $\angle PCR = \angle RCQ = \theta$

$\Rightarrow \quad \angle PCQ = 2\theta$

$\Rightarrow \quad \widehat{PSQ} = (2\pi - 2\theta)r = \frac{2mv_0(\pi - \theta)}{qB}$

$\Rightarrow \quad t_{PSQ} = \frac{\widehat{PSQ}}{v_0} = \frac{2m(\pi - \theta)}{qB}$

Hence, (A), (B), (C) and (D) are correct.

42. Let d = distance of the target T from the point of projection. P will strike T if d is an integral multiple of the pitch.

$$\text{Pitch} = \left(\frac{2\pi m}{QB}\right)v\cos\theta$$

Here, m, Q and θ are constant

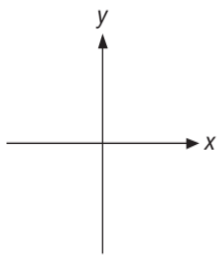
$\therefore \quad \text{Pitch} = k\left(\frac{v}{B}\right)$

where k = constant

Initially, $d = k\left(\frac{v_0}{B_0}\right)$

Hence, (A) and (B) are correct.

43. The velocity of proton makes an angle of 45° with the direction of magnetic field. Therefore, path of the proton is a helix. The plane of the circle of this helix is the plane formed by negative x and positive z-axis. Therefore, x-coordinate can never be positive. Further, x and z co-ordinates will become zero simultaneously after every pitch and y-coordinate of proton at any time t is



$$y = v_0 t$$

$$\Rightarrow \quad y \propto t$$

Hence, (B) and (D) are correct.

45. When $\alpha = \beta$ then $\vec{v} \parallel \vec{B}$

$$\Rightarrow \quad \vec{F} = \vec{0}$$

So, **OPTION (A)** is correct

Since, $\vec{F} = q(\vec{v} \times \vec{B}) = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & \beta & 0 \\ \beta & \alpha & 0 \end{vmatrix}$

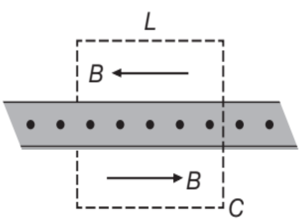
$$\Rightarrow \quad \vec{F} = q(\alpha^2 - \beta^2)\hat{k} \qquad \ldots(1)$$

$$\Rightarrow \quad |\vec{F}| \propto (\alpha^2 - \beta^2)$$

So, **OPTION (B)** is also correct.
Also, from (1), we get **OPTION (C)** to be correct.

Hence, (A), (B) and (C) are correct.

47. Consider only one plate as shown in Figure.



The field on both sides of plate is shown in Figure.
Applying Ampere's Circuital Law to the contour C, we get

$$2BL = \mu_0(jL)$$

$$B = \frac{\mu_0 j}{2}$$

Superimposing, the field due to two plates we get at P both fields cancel each other and at Q, they add to give $B_Q = \mu_0 j$

Hence, (A) and (D) are correct.

48. $B(2\pi r) = \mu_0 I_{\text{enclosed}}$

$$B(2\pi r) = \mu_0\left(\frac{I}{\pi a^2}\pi r^2\right)$$

$B = \dfrac{\mu_0 I r}{2\pi a^2}$ for $0 < r \le a$,

$B(r) = 0$ at $r = 0$,

$B(r) \propto \dfrac{1}{r}$ for $r > a$ and

$B(r)$ is maximum at the surface i.e., at $r = a$

Hence, (A), (B), (C) and (D) are correct.

50. $a_e = \dfrac{F}{m_e}$, $a_p = \dfrac{F}{m_p}$

Since, $m_e \ll m_p$

So, $a_e \gg a_p$

$$\Rightarrow \quad d = \frac{1}{2}a_e t_1^2,\ d = \frac{1}{2}a_p t_2^2$$

$$\Rightarrow \quad \frac{t_1}{t_2} = \left(\frac{m_e}{m_p}\right)^{1/2}$$

Hence, (A) and (C) are correct.

Reasoning Based Questions

2. $\dfrac{mv^2}{r} = qvB$

$\Rightarrow \quad a = \dfrac{v^2}{r} = \dfrac{qvB}{m}$, is called a centripetal acceleration

If the particle moves parallel or antiparallel to the field then acceleration is zero.
Hence, the correct answer is (A).

3. Work done by magnetic force on moving charge is zero.
Hence, the correct answer is (A).

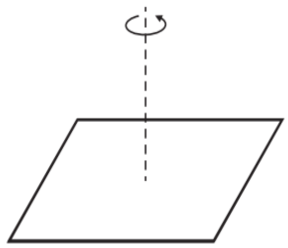
4. $v = \dfrac{E}{B}$

Hence, the correct answer is (A).

6. $|\vec{B}|$ is proportional to number of magnetic field lines per unit area (area should be normal to field).
Hence, the correct answer is (D).

CHAPTER 1

7. $U = -\vec{M} \cdot \vec{B} = -MB\cos\theta$



Since, M, B as well as θ remains constant, U does not change.
Hence, the correct answer is (A).

8. $r = \dfrac{mv}{qB}$

Since $T = \dfrac{2\pi r}{v} = \dfrac{2\pi m}{qB}$

So, we observe T is independent of r. Hence Statement-1 is incorrect.
Hence, the correct answer is (D).

9. The winding of helix carry currents in same direction. Hence, they experience an attractive force pulling the lower and out of mercury. As a result of this the circuit breaks and so the force of attraction vanishes and the helix comes back to its initial position, completing the circuit again.
Hence, the correct answer is (B).

11. See Theory.
The correct answer is (C).

12. Initially moment $M = I\pi r^2$

And afterwards $M' = I\pi(2r)^2 = 4I(\pi r^2) = 4M$
So magnetic moment becomes four times when radius is doubled.
Hence, the correct answer is (A).

13. $\vec{F} = q\vec{v} \times \vec{B}$

$\Rightarrow \quad \vec{F} \perp \vec{v}$
So, power produced by magnetic force is zero
$\Rightarrow$ Kinetic energy of particle will remain conserved
Hence, the correct answer is (A).

14. Use $\vec{F} = \oint Id\vec{\ell} \times B = 0$

$\vec{\tau} = MB\sin\theta = 0$
Hence, the correct answer is (B).

15. The magnetic lines of force due to current carrying straight solenoid is same as that of bar magnet.
Hence, the correct answer is (A).

18. The period of a charged particle in a magnetic field is given by

$$T = \frac{2\pi m}{qB}$$

Hence, the correct answer is (A).

21. Force on the loop is not zero; because magnetic field is not constant.
Hence, the correct answer is (D).

22. Due to both positive and negative charges the wire is electrically neutral and hence no electric field is present and only magnetic field is created.
Hence, the correct answer is (D).

23. Magnetic force is always perpendicular to magnetic field and small element. Also, the force depends on the current direction.
Hence, the correct answer is (B).

25. $F = 0$ in both case.
Hence, the correct answer is (C).

26. As we know every atom of a magnet act as a dipole. So, poles cannot be separated. When magnet is broken into two equal pieces. Magnetic moment of each part will be half of the original magnet.
Hence, the correct answer is (B).

27. For a solenoid $B_{\text{ends}} = \dfrac{1}{2}B_{\text{inside}}$. Also, for a long solenoid magnetic field is uniform within it but this reason is not explaining the statement (1).
Hence, the correct answer is (B).

28. Velocity is a vector quantity even if direction changes, velocity is said to be changing, no matter speed remains same or different.
Hence, the correct answer is (C).

29. $C\phi = BINA$

$\Rightarrow \quad \phi = \left(\dfrac{BNA}{C}\right)I$

Using iron core, value of magnetic field increases. So, deflection increases for same current. Hence sensitivity increases. So, Statement-1 is true.
Statement-2 is false as we know soft iron can be easily magnetized or demagnetized.
Hence, the correct answer is (C).

Linked Comprehension Type Questions

1. When the ends of the wire are in contact with the mercury and current flows in the wire, the magnetic field exerts an upward force and the wire has an upward

acceleration. After the ends leave the mercury the electrical connection is broken and the wire is in free fall.

After the wire leaves the mercury its acceleration is g, downward. The wire travels upward a total distance of 0.35 m from its initial position. Its ends lose contact with the mercury after the wire has travelled 0.025 m, so the wire travels upward 0.325 m after it leaves the mercury. Consider the motion of the wire after it leaves the mercury. Take $+y$ to be upward and take the origin at the position of the wire as it leaves the mercury

$$a_y = -9.8 \text{ ms}^{-2},\ y - y_0 = +0.325 \text{ m},\ v_y = 0$$

(at maximum height), $v_{0y} = ?$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.8 \text{ ms}^{-2})(0.325 \text{ m})}$$

$$v_{0y} = 2.52 \text{ ms}^{-1} \simeq 2.5 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

2. Now consider the motion of the wire while it is in contact with the mercury. Take $+y$ to be upward and the origin at the initial position of the wire. Calculate the acceleration

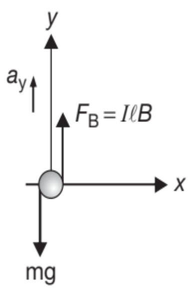
$$y - y_0 = +0.025 \text{ m},\ v_{0y} = 0 \text{ (starts from rest)},$$

$$v_y = +2.52 \text{ ms}^{-1} \text{ (from part (a))},\ a_y = ?$$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$a_y = \frac{v_y^2}{2(y - y_0)} = \frac{(2.52 \text{ ms}^{-1})^2}{2(0.025 \text{ m})} = 127 \text{ ms}^{-2}$$

The free body diagram for the wire is given in figure



$$\sum F_y = ma_y$$

$$\Rightarrow F_B - mg = ma_y$$

$$\Rightarrow I\ell B = m(g + a_y)$$

$$\Rightarrow I = \frac{m(g + a_y)}{\ell B}$$

ℓ is the length of the horizontal section of the wire,

$$\ell = 0.15 \text{ m}$$

$$I = \frac{(5.4 \times 10^{-5} \text{ kg})(9.8 \text{ ms}^{-2} + 127 \text{ ms}^{-2})}{(0.15 \text{ m})(0.0065 \text{ T})} = 7.58 \text{ A}$$

$$\Rightarrow I \cong 7.6 \text{ A}$$

Hence, the correct answer is (A).

3. Use Ohm's Law

$$V = IR \text{ so } R = \frac{V}{I} = \frac{1.5 \text{ V}}{7.58 \text{ A}} = 0.198\ \Omega$$

The current is large and the magnetic force provides a large upward acceleration. During this upward acceleration the wire moves a much shorter distance as it gains speed than the distance is moves while in free-fall with a much smaller acceleration, as it loses the speed it gained. The large current means the resistance of the wire must be small.

Hence, the correct answer is (C).

4. Since $\vec{F} = q(\vec{v} \times \vec{B})$

$$\Rightarrow \vec{F} = 10^6 \times q(8\hat{i} - 6\hat{j} + 4\hat{k}) \times (-0.4)\hat{k}$$

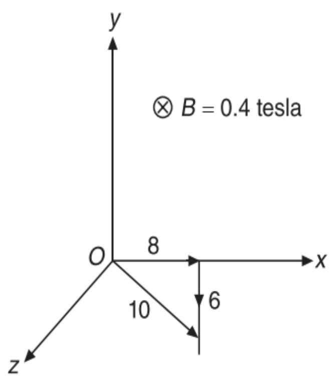
$$\Rightarrow \vec{F} = 10^6 \times q(+3.2\hat{j} + 2.4\hat{i})$$

$$\Rightarrow |\vec{F}| = q \times 10^6 \sqrt{(3.2)^2 + (2.4)^2}$$

$$\Rightarrow 1.6 = q \times 4 \times 10^6$$

$$\Rightarrow q = 0.4 \times 10^{-6} = 0.4\ \mu\text{C}$$

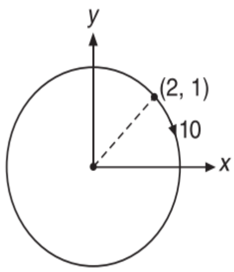
Hence, the correct answer is (C).

5. 

Resultant velocity is 10×10^6 in x-y plane. The circular motion of the particle is in x-y plane and translational component along z-axis.

Hence, the correct answer is (C).

6.



After time 3 T, its x and y coordinate will remain same, whereas z coordinate is

$$z = 4 \times 3\text{ T} = 12\text{ T}$$

Since $T = \dfrac{2\pi m}{qB}$

$$\Rightarrow \quad T = \frac{2 \times \pi\left(4 \times 10^{-15}\right)}{0.4 \times 10^{-6} \times 0.4} = 0.157 \times 10^{-6}\text{ s}$$

So, z-coordinate of particle is

$$z = v_z t = \left(4 \times 10^6\right)(3\text{ T})$$

$$\Rightarrow \quad z = \left(4 \times 10^6\right)\left(3 \times 0.157 \times 10^{-6}\right)$$

$$\Rightarrow \quad z = 1.884\text{ m}$$

Hence, the correct answer is (C).

7. $\vec{\tau} = \vec{\mu} \times \vec{B}_0$ where $\vec{\mu}$ is the magnetic moment of the loop

$$\Rightarrow \quad \vec{\tau} = I_0 \pi R^2 \hat{k} \times \left(\frac{B_0}{\sqrt{2}}\hat{i} + \frac{B_0}{\sqrt{2}}\hat{j}\right)$$

$$\Rightarrow \quad \vec{\tau} = \frac{I_0 \pi R^2 B_0}{\sqrt{2}}\left(\hat{j} - \hat{i}\right)$$

$$\Rightarrow \quad |\vec{\tau}| = I_0 \pi R^2 B_0$$

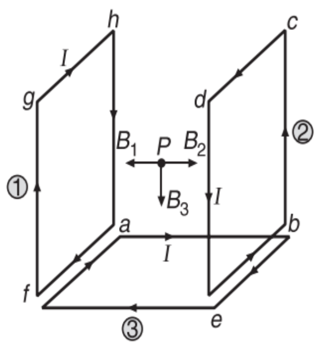
Hence, the correct answer is (A).

8. $\theta = \dfrac{1}{2}\alpha(\Delta t)^2$

$$\Rightarrow \quad \theta = \frac{\pi I_0 B_0}{M}(\Delta t)^2$$

Hence, the correct answer is (D).

9. The current path $abcdefgha$ can be treated as superposition of three loops $fghaf$, $fabef$ and $ebcde$ each carrying a current I.



Hence, the correct answer is (A).

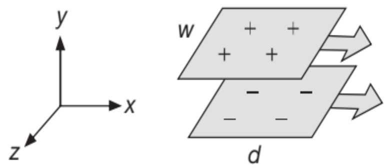
10. The magnetic field at centre P of cube cancels due to two square loops (1) and (2) each carrying current I as shown earlier. So, the magnetic field at centre P of cube is only due to loop 3. Hence the magnetic field at centre of cube is in the negative y-direction.
Hence, the correct answer is (B).

11. The dipole moment of the loop is also along negative y direction.

Since $\vec{B}_{\text{ext}} = B_0\hat{j}$, so $\vec{\tau} = \vec{M} \times \vec{B} = \vec{0}$

Hence, the correct answer is (D).

12. If J_S be the current per unit length, then the upper sheet creates field $\vec{B} = \dfrac{\mu_0 J_s}{2}\hat{k}$ above it and $\dfrac{\mu_0 J_s}{2}(-\hat{k})$ below it.
Consider a patch of the sheet of width w parallel to the z-axis and length d parallel to the x-axis as shown in Figure.



The charge on it, σwd passes a point in time $\dfrac{d}{v'}$ so the current it constitutes is $\dfrac{q}{t} = \dfrac{\sigma wvd}{d}$ and the linear current density is $J_s = \dfrac{\sigma wv}{w} = \sigma v$. So, the magnitude of the magnetic field created by the upper sheet is $\dfrac{1}{2}\mu_0 \sigma v$. Similarly, the lower sheet in its motion toward the right constitutes current toward the left. It creates magnetic field $\dfrac{1}{2}\mu_0 \sigma v\left(-\hat{k}\right)$ above it and $\dfrac{1}{2}\mu_0 \sigma v\hat{k}$ below it.
Between the plates, their fields add to $\mu_0 \sigma v\left(-\hat{k}\right)$ i.e. $\mu_0 \sigma v$ away from you horizontally.

Hence, the correct answer is (A).

13. Above both sheets and below both the sheets, their equal magnitude fields add to zero.
Hence, the correct answer is (D).

14. The upper plate exerts no force on itself. The field of the lower plate, $\dfrac{1}{2}\mu_0 \sigma v\left(-\hat{k}\right)$ will exert a magnetic force on the current in the $w \times d$ section, given by

$$\vec{F} = I\left(\vec{l} \times \vec{B}\right) = (\sigma wvd)\hat{i} \times \left(\frac{1}{2}\mu_0 \sigma v\right)\left(-\hat{k}\right)$$

$$\Rightarrow \quad \vec{F} = I\left(\vec{\ell} \times \vec{B}\right) = \left(\frac{1}{2}\mu_0 \sigma^2 v^2 wd\right)\hat{j}$$

The magnetic force per area is

$$\frac{F}{A}=\frac{1}{2}\left(\frac{\mu_0\sigma^2 v^2 wd}{wd}\right)\hat{j}=\frac{1}{2}\mu_0\sigma^2 v^2\text{, upwards}$$

Hence, the correct answer is (D).

15. The electrical force on the considered section of the upper plate is

$$\vec{F}_e = q\vec{E}_{\text{lower}} = (\sigma\ell w)\left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{j})$$

$$\Rightarrow\quad \vec{F}_e=\left(\frac{\ell w\sigma^2}{2\varepsilon_0}\right)(-\hat{j})$$

The electrical force per area is

$$\frac{F_e}{A}=\frac{\ell w\sigma^2}{2\varepsilon_0\ell w}=\frac{\sigma^2}{2\varepsilon_0}\text{, downwards}$$

Hence, the correct answer is (B).

16. The magnetic force balances the electric force, when we have

$$\frac{1}{2}\mu_0\sigma^2 v^2=\frac{\sigma^2}{2\varepsilon_0}$$

$$\Rightarrow\quad v=\frac{1}{\sqrt{\mu_0\varepsilon_0}}=c$$

This is the speed of light which cannot be a speed that can be attained by a metal plate.

Hence, the correct answer is (C).

17. The correct answer is (C).

18. The correct answer is (A).

Combined Solution to 17 and 18

Since we know that $\vec{F}=q(\vec{v}\times\vec{B})$

For the first case, we get

$$-(5\sqrt{2}\times10^{-3})\hat{k}=(10^{-5})\left(\frac{10^6}{\sqrt{2}}\right)(\hat{i}+\hat{j})\times\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)$$

$$\Rightarrow\quad -(5\sqrt{2}\times10^{-3})\hat{k}=\frac{10}{\sqrt{2}}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 0\\ B_x & B_y & B_z\end{vmatrix}$$

$$\Rightarrow\quad -(5\sqrt{2}\times10^{-3})\hat{k}=\frac{10}{\sqrt{2}}\left[B_z\hat{i}-B_z\hat{j}+(B_y-B_x)\hat{k}\right]$$

$$\Rightarrow\quad B_z=0 \text{ and } B_y-B_x=-10^{-3}\text{ T} \qquad \ldots(1)$$

For the second case, we have

$$F_2\hat{j}=(10^{-5})(10^6)\hat{k}\times\left(B_x\hat{i}+B_y\hat{j}\right)$$

$$\Rightarrow\quad F_2\hat{j}=10\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & 1\\ B_x & B_y & 0\end{vmatrix}$$

$$\Rightarrow\quad F_2\hat{j}=10B_x\hat{j}-10B_y\hat{i} \qquad \ldots(2)$$

From (2), we get

$$B_y=0$$

$$\Rightarrow\quad B_x=10^{-3}\text{ T and } F_2=10B_x=10^{-2}\text{ N}$$

Hence $\vec{B}=(10^{-3}\text{ T})\hat{i}$ and $F_2=10^{-2}$ N

19. $B=\dfrac{\mu_0 I}{2\pi r}=\dfrac{(4\pi\times10^{-7}\text{ TmA}^{-1})(25\text{ A})}{2\pi(0.0125\text{ m})}$

$$\Rightarrow\quad B=4\times10^{-4}\text{ T}=400\ \mu\text{T}$$

Hence, the correct answer is (D).

20. At point C, conductor AB produces a field

$$B=\frac{B_0}{2}=\frac{1}{2}(4\times10^{-4}\text{ T})(-\hat{j}),$$

conductor DE produces a field of $B=\dfrac{B_0}{2}=\dfrac{1}{2}(4\times10^{-4}\text{ T})(-\hat{j})$, BD produces no field, and AE produces negligible field. The total field at C is $B_C=\dfrac{B_0}{2}+\dfrac{B_0}{2}=B_0=400\ \mu$T

Hence, the correct answer is (A).

21. $\vec{F}_B=I(\vec{\ell}\times\vec{B})=(25\text{ A})(0.025\text{ m}\hat{k})\times\left[5(4\times10^{-4}\text{ T})(-\hat{j})\right]$

$$\vec{F}_B=(1.25\times10^{-3}\text{ N})\hat{i}$$

Hence, the correct answer is (B).

22. $a=\dfrac{\sum F}{m}=\dfrac{(1.25\times10^{-3}\text{ N})\hat{i}}{5\times10^{-3}\text{ kg}}$

$$\Rightarrow\quad a=(0.25\text{ ms}^{-2})\hat{i}=(25\text{ cms}^{-2})\hat{i}$$

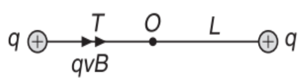
Hence, the correct answer is (C).

23. $v_f^2=v_i^2+2ax=0+2(0.25\text{ ms}^{-2})(1.2\text{ m})$, so

$$v_f=0.6\text{ ms}^{-1}=60\text{ cms}^{-1}\ \hat{i}$$

Hence, the correct answer is (D).

24. The free body diagram of the arrangement is shown in Figure.



For particle to move in a circle of radius L, we have

$$T + qvB = \frac{mv^2}{L}$$

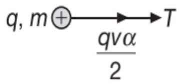
For $T = 0$, we get

$$q\left(\frac{q\alpha L}{m}\right)B = \frac{mv^2}{L}$$

$\Rightarrow \quad B = \alpha$

Hence, the correct answer is (B).

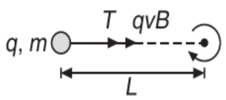
25. Since, $T + \frac{qv\alpha}{2} = \frac{mv^2}{L}$



$$\Rightarrow \quad T = \frac{m(q\alpha L)^2}{m^2 R} - \left[\frac{q(q\alpha L)}{m}\right]\frac{\alpha}{2} = \frac{q^2\alpha^2 L}{2m}$$

Hence, the correct answer is (C).

26. $T + qvB = \frac{mv^2}{L}$



$$\Rightarrow \quad qvB = \frac{mv^2}{L} - T$$

$$\Rightarrow \quad q\left(\frac{q\alpha L}{m}\right)B = \frac{m}{L}\left(\frac{q^2\alpha^2 L^2}{m^2}\right) - \frac{3}{4}\left(\frac{q^2\alpha^2 L}{m}\right)$$

$$\Rightarrow \quad B = \frac{\alpha}{4}$$

When the string breaks due to this magnetic field, then the radius of the path followed by the particle is

$$R = \frac{mv}{qB} = \frac{mv}{q\left(\frac{\alpha}{4}\right)} = \left(\frac{4m}{q\alpha}\right)v$$

$$\Rightarrow \quad R = \left(\frac{4m}{q\alpha}\right)\left(\frac{q\alpha L}{m}\right) = 4L$$

So, maximum separation between the particles is

$$r_{max} = (2R + 2R) - 2L$$

$$\Rightarrow \quad r_{max} = 4R - 2L = 16L - 2L$$

$$\Rightarrow \quad r_{max} = 14L$$

Hence, the correct answer is (C).

27. $\vec{\tau} = \vec{M} \times \vec{B} = (3\hat{i} - 4\hat{j}) \times (4\hat{i} + 3\hat{j}) = (25\hat{k})$ Nm

Moment of inertia about an axis, parallel to $\vec{\tau}$ and passing through centre of mass is

$$I = \frac{2 \times 10^{-2}}{I} = 10^{-2}\ \text{kgm}^2$$

$$\Rightarrow \quad \alpha = \frac{\vec{\tau}}{I} = \frac{25}{10^{-2}} = 2500\ \text{rads}^{-2}$$

Hence, the correct answer is (C).

28. At time $t = 0$, $\vec{M} \cdot \vec{B}$ i.e., $\vec{M} \perp \vec{B}$. Angular velocity will have a maximum value when $\vec{M} \parallel \vec{B}$, as a result of which magnetic potential energy will decrease and rotational kinetic energy will increase. Hence,

$$\frac{1}{2}I\omega^2 = U_i - U_f$$

$$\Rightarrow \quad \frac{1}{2}I\omega^2 = -MB\cos 90^\circ - (-MB\cos 0^\circ)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2MB}{I}} = \sqrt{\frac{(2)(5)(5)}{10^{-2}}} = 50\sqrt{2}\ \text{rads}^{-1}$$

Hence, the correct answer is (B).

29. For neutrons charge, $q = 0$

Hence, the correct answer is (D).

31. Due to extremely small mass of an electron, the frequency of revolution of electrons is very high and oscillators of such high frequencies are not easily obtainable.

Hence, the correct answer is (A).

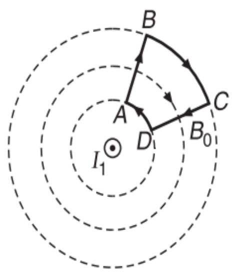
32. $B_{AD} = \frac{\mu_0 I\alpha}{4\pi R} = \frac{\mu_0 I\pi}{4\pi a \times 6} = \frac{\mu_0 I}{24a}$, $\odot$

$$B_{BC} = \frac{\mu_0 I\alpha}{4\pi R} = \frac{\mu_0 I\pi}{4\pi b \times 6} = \frac{\mu_0 I}{24b}, \otimes$$

$$\Rightarrow \quad B_{net} = \frac{\mu_0 I}{24}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\mu_0 I(b-a)}{24ab}, \odot$$

Hence, the correct answer is (B).

33. Since magnetic field lines and current carrying wires are parallel hence no force is exerted on wire DA and BC, because $\vec{I}$ and $\vec{B}$ are collinear for BC and DA.

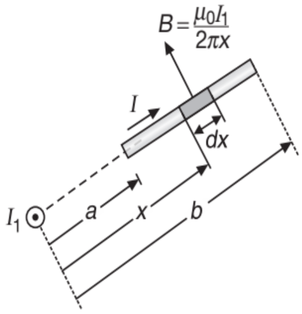


Similarly, F_{AB} and F_{CD} are equal in magnitude but opposite in direction, so

$$F_{net} = \text{zero}$$

Hence, the correct answer is (B).

34.



Force on small element dx is

$$dF = B_1 I dx = \frac{\mu_0 I I_1}{2\pi}\frac{dx}{x}$$

$$\Rightarrow \quad F = \int dF = \frac{\mu_0 I I_1}{2\pi}\int_a^b \frac{dx}{x} = \frac{\mu_0 I I_1}{2\pi}\log_e\left(\frac{b}{a}\right)$$

Hence, the correct answer is (B).

35. Since the electric field is in y-direction, therefore, electrostatic force will provide an acceleration to the particle in y-direction. y component of its velocity will go on increasing. Since, magnetic field is also in y-direction, so magnetic force $\vec{F}_m$ will act in x-z plane i.e. the particle will rotate in circle with its plane in x-z plane. Hence, path of particle will be helix with increasing pitch.
Hence, the correct answer is (C).

36. $a_y = \frac{F_y}{m} = \frac{qE_0}{m} = E_0\alpha$

$$\Rightarrow \quad y = \frac{1}{2}a_y t^2 = \frac{1}{2}E_0\alpha t^2$$

Particle will touch the y-axis for nth time, when the time (t) equals integral multiple of T i.e.

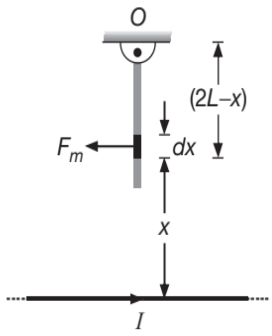
$$t = nT = n\left(\frac{2\pi m}{qB_0}\right) = \frac{2\pi n}{B_0\alpha}$$

$$\Rightarrow \quad y_n = \frac{1}{2}\alpha E_0\left(\frac{2n\pi}{B_0\alpha}\right)^2$$

$$\Rightarrow \quad y_n = \frac{2\pi^2 n^2 E_0}{B_0^2\alpha}$$

Hence, the correct answer is (B).

37. Consider an element of length dx (of the suspended wire) at a distance x from the horizontal wire as shown in Figure.



If $d\tau$ is the torque on this element due to the magnetic force F_m, then

$$\tau_0 = \int_{x=L}^{x=2L} F_m(2L-x) = \int_L^{2L}\left(\frac{\mu_0}{2\pi}\frac{I}{x}\right)(I)dx(2L-x)$$

$$\Rightarrow \quad \tau_0 = \frac{\mu_0 I^2}{2\pi}\left(2L\log_e x - x\right)\Big|_L^{2L}$$

$$\Rightarrow \quad \tau_0 = \frac{\mu_0 I^2}{2\pi}\left(2L\log_e(2) - L\right) = \frac{0.4\mu_0 I^2 L}{2\pi} \qquad \ldots(1)$$

$$\Rightarrow \quad \tau_0 = (0.4)(2\times 10^{-7})(2) = 3.2\times 10^{-7}\ \text{Nm}$$

Since the moment of inertia of the wire is I_0 given by

$$I_0 = \frac{mL^2}{3} = \frac{(0.1)(1)^2}{3} = \frac{1}{30}\ \text{kgm}^2$$

$$\Rightarrow \quad \alpha = \frac{\tau_0}{I_0} = 9.6\times 10^{-6}\ \text{rads}^{-2}$$

Hence, the correct answer is (D).

38. Net torque about O should be zero. Magnetic field on the suspended wire due to already placed wire is perpendicular to paper outwards. Hence, magnetic field on suspended wire due to new wire should be perpendicular to paper inwards.
Hence, the correct answer is (B).

39. Torque about O due to the new wire will be,

$$\tau = \left(\frac{\mu_0}{2\pi}\frac{I}{r}\right)(L)(I)\left(\frac{L}{2}\right) = \left(\frac{\mu_0}{2\pi}\right)\left(\frac{I^2L^2}{2r}\right)$$

Since $\tau = \tau_0$

$$\Rightarrow \quad \frac{L}{2r} = 0.386$$

$$\Rightarrow \quad r = \frac{L}{2\times 0.4} = \frac{1}{2\times 0.4} = 1.25\ \text{m}$$

Hence, the correct answer is (C).

40. When the currents in two parallel wires are in opposite direction, then they repel each other.
Hence, the correct answer is (B).

41. $\left(\dfrac{\mu_0 i_1 i_2}{2\pi h}\right) l = mg$

$$\Rightarrow \frac{\mu_0 i_1 i_2}{2\pi h} = \left(\frac{m}{l}\right) g$$

$$\Rightarrow \frac{\mu_0 i_1 i_2}{2\pi h} = \lambda g$$

$$\Rightarrow h = \frac{\mu_0 i_1 i_2}{2\pi \lambda g}$$

Hence, the correct answer is (A).

42. At equilibrium, we have

$$\frac{\mu_0 i_1 i_2}{2\pi h} = \lambda g$$

After displacing it dh upwards net force per unit length now becomes

$$F' = \lambda g - \frac{\mu_0 i_1 i_2}{2\pi(h + dh)}$$

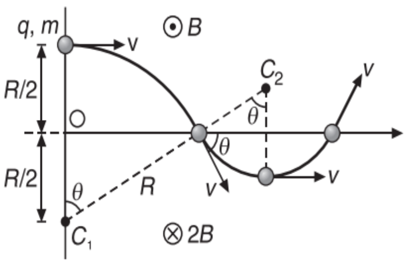
$$\Rightarrow F' = \lambda g - \frac{\lambda g}{\left(1 + \dfrac{dh}{h}\right)} = \frac{\lambda g dh}{h}$$

Hence, the correct answer is (D).

43. **The correct answer is (B).**

44. **The correct answer is (C).**

Combined Solution to 43 and 44



From Figure, we have

$$\cos\theta = \frac{R/2}{R} = \frac{1}{2}$$

$$\Rightarrow \theta = 60° = \frac{\pi}{3} \text{ radian}$$

So, time to hit x-axis is

$$t_1 = \frac{\theta}{\omega}, \text{ where } \omega = \frac{qB}{m},\ \theta = \frac{\pi}{3}$$

$$\Rightarrow t_1 = \frac{\pi/3}{qB/m} = \frac{\pi m}{3qB}$$

Let t_2 be time when its velocity becomes parallel to x-axis first time after entering the magnetic field, then

$$t_2 = t_1 + \frac{\theta}{\omega'}, \text{ where } \omega' = \frac{q(2B)}{m}$$

$$\Rightarrow t_2 = \frac{\pi m}{3qB} + \frac{\pi/3}{\dfrac{q(2B)}{m}}$$

$$\Rightarrow t_2 = \frac{\pi m}{3qB} + \frac{\pi m}{6qB}$$

$$\Rightarrow t_2 = \frac{\pi m}{2qB}$$

45. $\vec{\tau} = \vec{M} \times \vec{B}$, where $\vec{M}$ is the dipole moment

$$\Rightarrow \vec{\tau} = nI(\pi r^2)\hat{j} \times B\hat{i}$$

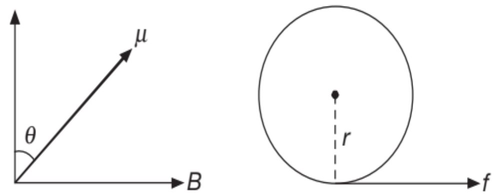
$$\Rightarrow \vec{\tau} = -(\pi n I r^2 B)\hat{k}$$

Hence, the correct answer is (A).

46. According to Newton's Second Law for rotational motion, we have

$$\Sigma\tau = I_{cm}\alpha$$

$$\Rightarrow \mu B\cos\theta - fr = I_{cm}\alpha \quad \ldots(1)$$



where I_{cm} is the moment of inertia of the ball about the centre of mass. (Do not confuse I_{cm} with the current I) and

$$f = ma_{cm} = mr\alpha \quad \ldots(2)$$

$$\Rightarrow \alpha = \frac{5}{7}\left(\frac{\pi n I B}{m}\right)\cos\theta$$

Hence, the correct answer is (A).

47. $\omega\dfrac{d\omega}{d\theta} = \dfrac{5}{7}\left(\dfrac{\pi n I B}{m}\right)\cos\theta$

$$\Rightarrow \frac{\omega^2}{2} = \frac{5}{7}\left(\frac{\pi n I B}{m}\right)\sin\theta$$

$$\Rightarrow \omega = \sqrt{\frac{10}{7}\left(\frac{\pi n I B}{m}\right)\sin\theta}$$

Hence, the correct answer is (A).

48. Since, $f = mr\alpha$

$$\Rightarrow \mu mg = mr\left(\frac{5}{7}\frac{\pi n I B}{m}\right)$$

$$\mu = \frac{5\pi}{7g}\left(\frac{nIB}{m}\right)r$$

Hence, the correct answer is (B).

49. Since there is no friction, there is no force acting on the sphere, which is responsible for the translation motion of the ball.

Hence, the correct answer is (A).

50. The component of velocity of charged particle along the magnetic field does not change. The component of velocity of charged particle normal to magnetic field only changes in direction but always remains normal to magnetic field. So, the angle between velocity and magnetic field remains the same.

Hence, the correct answer is (A).

51. $f = \frac{qB}{2\pi m} = \frac{10^3}{13} \times \frac{26}{2\pi}$

$\Rightarrow \quad f = \frac{1000}{\pi} = 318 \text{ Hz}$

Hence, the correct answer is (B).

52. Pitch $p = v_{\parallel}T$

where $v_{\parallel}$ is the component of velocity along the field i.e.

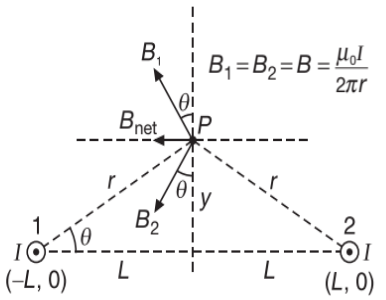
$$v_{\parallel} = \frac{\vec{v}\cdot\vec{B}}{|\vec{B}|} = \frac{36}{26} = \frac{16}{13} \text{ ms}^{-1}$$

$\Rightarrow \quad p = \left(\frac{16}{13}\right)\left(\frac{\pi}{1000}\right) \approx \frac{50}{13\times 1000}$

$\Rightarrow \quad p \approx \frac{1}{260} \text{ m}$

Hence, the correct answer is (C).

53. At point $P(0, y, 0)$, net magnetic field is



$B_{net} = 2B\sin\theta$, where

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi\sqrt{L^2+y^2}} \text{ and } \sin\theta = \frac{y}{\sqrt{L^2+y^2}}$$

$\Rightarrow \quad B_{net} = \frac{2\mu_0 I y}{2\pi(L^2+y^2)} = \frac{\mu_0 I}{\pi}\left(\frac{y}{L^2+y^2}\right)$

Hence, the correct answer is (B).

54. For B to be maximum, we have

$$\frac{dB}{dy} = 0$$

$\Rightarrow \quad \frac{2\mu_0 I}{2\pi}\left[\frac{(L^2+y^2)-2y^2}{L^2+y^2}\right] = 0$

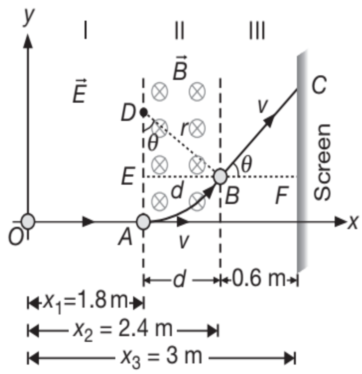
$\Rightarrow \quad y = \pm L$

Hence, the correct answer is (D).

55. $B_{maximum} = B\big|_{y=\pm L} = \frac{\mu_0 I}{2\pi L}$

Hence, the correct answer is (B).

56.



In REGION (I), we have only the electric field. So, velocity with which the particle enters REGION (II) is given by

$$v^2 = 2\left(\frac{qE}{m}\right)x_1$$

$\Rightarrow \quad v = \sqrt{2\left(\frac{qE}{m}\right)x_1}$

$\Rightarrow \quad v = \sqrt{\frac{2\times 1\times 10\times 1.8}{1}} \qquad \{\because q = 1\text{ C}, m = 1\text{ kg}\}$

$\Rightarrow \quad v = 6 \text{ ms}^{-1}$

In magnetic field speed does not change. Hence, particle will collide with 6 ms^{-1}.

Hence, the correct answer is (C).

57. In magnetic field path of the particle is circle. Radius of circular particle is

$$r = \frac{mv}{Bq} = \frac{(1)(6)}{(5)(1)} = 1.2 \text{ m}$$

$$d = (2.4 - 1.8) \text{ m} = 0.6 \text{ m}$$

Since, $d < r$

$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{d}{r}\right) = \sin^{-1}\left(\frac{0.6}{1.2}\right) = 30°$

Now, $AE = r(1-\cos\theta) = 1.2\left(1 - \frac{\sqrt{3}}{2}\right)$

CHAPTER 1

$\Rightarrow \quad AE = 0.6(2-\sqrt{3})$ and $FC = BF\tan\theta = \dfrac{0.6}{\sqrt{3}}$

So, y-coordinate is

$$y = AE + FC = 0.6\left(2-\sqrt{3}+\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \quad y = \frac{1.2(\sqrt{3}-1)}{\sqrt{3}}\ \text{m}$$

Hence, the correct answer is (B).

58. Total time is given by $t = t_{OA} + t_{AB} + t_{BC}$

Where $t_{OA} = \sqrt{\dfrac{2x_1}{a}} = \sqrt{\dfrac{2x_1}{\left(\dfrac{qE}{m}\right)}} = \sqrt{\dfrac{2mx_1}{qE}}$

$$\Rightarrow \quad t_{OA} = \sqrt{\frac{2\times1\times1.8}{(1)(10)}}$$

$$\Rightarrow \quad t_{OA} = \frac{3}{5}\ \text{s}$$

$$t_{AB} = \left(\frac{30^\circ}{360^\circ}\right)T = \left(\frac{1}{12}\right)\left(\frac{2\pi m}{Bq}\right)$$

$$\Rightarrow \quad t_{AB} = \frac{(2\pi)(1)}{(12)(5)(1)} = \frac{\pi}{30}\ \text{s}$$

$$t_{BC} = \frac{BC}{v} = \frac{0.6\sec\theta}{v} = \frac{(0.6)\left(\dfrac{2}{\sqrt{3}}\right)}{6}$$

$$\Rightarrow \quad t_{BC} = \frac{1}{5\sqrt{3}}\text{s}$$

$$\Rightarrow \quad t = \frac{1}{5}\left(3+\frac{\pi}{6}+\frac{1}{\sqrt{3}}\right)\text{s}$$

Hence, the correct answer is (A).

59. Let us consider a length (dx) of the wire at a distance x from the long straight conductor. Magnetic field at this point is

$$B = \frac{\mu_0 i}{2\pi x}$$

Force acting on the length (dx) of the wire is

$$dF = \left(\frac{\mu_0 i i_0}{2\pi x}\right)dx$$

This force $d\vec{F}$ will be in the upward direction parallel to the current i.
So, the disc will start rotating in anticlockwise direction about its centre in its plane.

Torque due to this force $d\vec{F}$ is

$$d\tau = (dF)r_\perp$$

$$\Rightarrow \quad d\tau = \left(\frac{\mu_0 i i_0}{2\pi x}\right)dx(x-r) \qquad \{\because r_\perp = x-r\}$$

Total initial torque is

$$\tau = \frac{\mu_0 i i_0}{2\pi}\int_r^{r+R}\frac{(x-r)}{x}dx = \frac{\mu_0 i i_0}{2\pi}\int_r^{r+R}\left(1-\frac{r}{x}\right)dx$$

$$\Rightarrow \quad \tau = \frac{\mu_0 i i_0}{2\pi}\left[x\Big|_r^{R+r} - r\log_e x\Big|_r^{R+r}\right]$$

$$\Rightarrow \quad \tau = \frac{\mu_0 i i_0}{2\pi}\left[R - r\log_e\left(\frac{r+R}{r}\right)\right]$$

Since $\tau = I\alpha$

$$\Rightarrow \quad \alpha = \frac{\tau}{I} = \frac{\mu_0 i i_0}{2\pi\left(\frac{1}{2}mR^2\right)}\left[R - r\log_e\left(\frac{r+R}{r}\right)\right]$$

$$\Rightarrow \quad \alpha = \frac{\mu_0 i i_0}{\pi m R^2}\left[R - r\log_e\left(1+\frac{R}{r}\right)\right]$$

Hence, the correct answer is (B).

60. Since the C.M. of the (disc + wire) is at rest, so net force on $(\text{wire}+\text{disc}) = 0$

$$\Rightarrow \quad N - F = 0$$

$$\Rightarrow \quad N = F = \int dF$$

$$\Rightarrow \quad N = \frac{\mu_0 i i_0}{2\pi}\int_r^{r+R}\frac{dx}{x}$$

$$\Rightarrow \quad N = \frac{\mu_0 i i_0}{2\pi}\log_e x\Big|_r^{R+r}$$

$$\Rightarrow \quad N = \frac{\mu_0 i i_0}{2\pi}\log_e\left(\frac{R+r}{r}\right) = \frac{\mu_0 i i_0}{2\pi}\log_e\left(1+\frac{R}{r}\right)$$

Hence, the correct answer is (D).

61. $\vec{\tau} = \vec{\mu}\times\vec{B} = 0.75(-\hat{k})\times0.2\hat{i}$

$\vec{\tau} = -0.15\,\hat{j}$

Hence, the correct answer is (B).

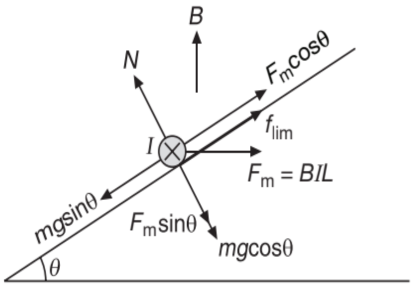
63. $W = MB\cos0^\circ - MB\cos\pi$

$W = 2\,\mu B$

$W = 0.3$ J

Hence, the correct answer is (C).

64. When magnetic field is minimum, then the rod tends to slide down the incline and hence f_{lim} acts up the incline as shown in Figure.



For equilibrium, $mg\sin\theta = BIL\cos\theta + f_{lim}$

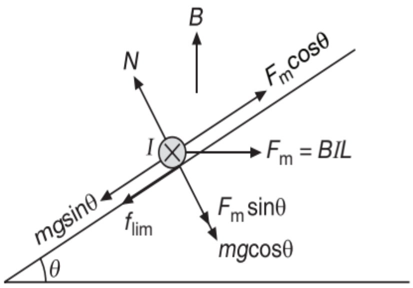
$$\Rightarrow \quad mg\sin\theta = BIL\cos\theta + \mu N$$

$$\Rightarrow \quad mg\sin\theta = BIL\cos\theta + \mu(mg\cos\theta + BIL\sin\theta)$$

$$\Rightarrow \quad B_{min} = \frac{mg}{IL}\left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right)$$

Hence, the correct answer is (B).

65. When magnetic field is maximum, then the rod tends to slide up the inclined plane and hence f_{lim} acts down the incline as shown in Figure.



For equilibrium of the rod, we have

$$mg\sin\theta + f_{lim} = BIL\cos\theta$$

$$\Rightarrow \quad mg\sin\theta + \mu N = BIL\cos\theta$$

$$\Rightarrow \quad mg\sin\theta + \mu(BIL\sin\theta + mg\cos\theta) = BIL\cos\theta$$

$$\Rightarrow \quad B_{max} = \frac{mg}{IL}\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)$$

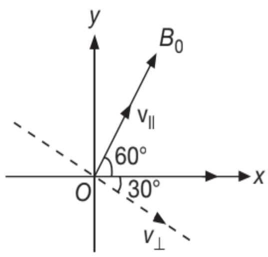
This is the maximum value of the required magnetic field.

Hence, the correct answer is (C).

66. Since the particle enters the field and has a velocity component parallel and perpendicular to the field, so it will follow a helical path.

Hence, the correct answer is (B).

67. Let us show the situation discussed diagrammatically. We observe that $\vec{B}$ makes an angle of $60°$ with x-axis. Let us now resolve $\vec{v}$ along $\vec{B}$ and perpendicular to $\vec{B}$ and label them as $v_{\parallel}$ and $v_{\perp}$ respectively. Then



$$v_{\parallel} = v_0\cos 60° = \frac{v_0}{2} \text{ and}$$

$$v_{\perp} = v_0\sin 60° = \frac{\sqrt{3}v_0}{2}$$

Since pitch of the helix is

$$p = v_{\parallel} \times T = \frac{v_0}{2}\left(\frac{2\pi m}{qB_0}\right)$$

$$\Rightarrow \quad p = \frac{\pi m v_0}{qB_0}$$

Hence, the correct answer is (B).

68. $v_z = v_{\perp} = \frac{\sqrt{3}v_0}{2}$ after $t = \frac{T}{4}$

and $v_z = -v_{\perp} = -\frac{\sqrt{3}v_0}{2}$ after $t = \frac{3T}{4}$

Hence, the correct answer is (B).

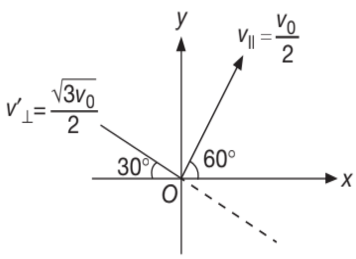
69. $Z_{max} = 2r = \frac{2mv_{\perp}}{qB_0} = \frac{\sqrt{3}mv_0}{qB_0}$

Hence, the correct answer is (C).

70. When z-coordinate has its maximum value, $v_{\parallel}$ will remain as it is whereas $v_{\perp}$ will change its direction by $180°$.

$$\Rightarrow \quad v_x = \frac{v_0}{2}\cos 60° - \frac{\sqrt{3}v_0}{2}\cos 30° = -\frac{v_0}{2} \text{ and}$$

$$v_y = \frac{v_0}{2}\sin 60° + \frac{\sqrt{3}v_0}{2}\sin 30° = \frac{\sqrt{3}}{2}v_0$$



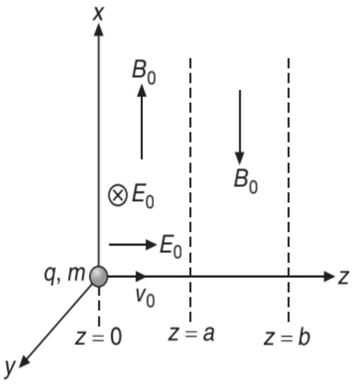
Hence, the correct answer is (C).

71. At origin, $F_m = qv_0B_0$, along $+y$ direction and $\vec{F}_e = q\vec{E} = qE_0(-\hat{j} + \hat{k})$.

For particle to move undeviated, $(F_m)_y = (F_e)_y$

$\Rightarrow \quad qv_0B_0 = qE_0$

$\Rightarrow \quad E_0 = B_0v_0$



Hence, the correct answer is (B).

72. For particle to reverse its direction of motion, it should move in the magnetic field (from $z = a$ to $z = b$) in a circle of radius $r = b - a$.
Let v be the speed of the particle as it enters the magnetic field at $z = a$. Then by Work-Energy Theorem we have

$$W = \Delta K$$

$$\Rightarrow \quad (qE_0)a = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$\Rightarrow \quad v = \sqrt{v_0^2 + \frac{2qE_0a}{m}}$$

$$\Rightarrow \quad v = \sqrt{v_0^2 + \frac{2qE_0a}{\frac{2}{3}\left(\frac{qE_0a}{v_0^2}\right)}}$$

$$\Rightarrow \quad v = 2v_0$$

Since we have

$$r = \frac{mv}{qB} = \left(\frac{2}{3}\frac{qE_0a}{v_0^2}\right)\left(\frac{2v_0}{qB_0}\right)$$

$$\Rightarrow \quad b - a = \frac{4}{3}\left(\frac{E_0a}{B_0v_0}\right)$$

$$\Rightarrow \quad b = a\left(1 + \frac{4E_0}{3B_0v_0}\right)$$

Hence, the correct answer is (B).

73. After all the fields are switched off, the velocity remains constant, so

$$t = \frac{a}{v} = \frac{a}{2v_0}$$

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. A → (q)
B → (r)
C → (s)
D → (s)

At (a, a), B due to both conductors is along $+z$ direction. At $(-a, -a)$, B due to both conductors is along $-z$ direction

At $(a, -a)$ and $(-a, a)$ B due to both conductors is zero.

2. A → (p, q)
B → (q)
C → (p, q, r)
D → (q, s)

A charge always has an electric field associated with it, whether moving or stationary.
So, (A) → (p, q)
However magnetic field is associated with current. Which can be due to a moving charge only.
So, (B) → (q)
A charged particle, whether moving or stationary will experience an electric force which will accelerate it or decelerate it depending upon the nature of charge and the direction of electric field. So, the velocity of a charge and hence the kinetic energy of a charge changes when it is in the influence of electrostatic force.
So, (C) → (p, q, r)

Since $\vec{F} = q(\vec{v} \times \vec{B})$, so the magnetic force acts on a moving charged particle (for $0° < \theta < 180°$). This force is perpendicular to the velocity of the particle (at any instant) as well as to $\vec{B}$. Also work done by a magnetic force is zero (as studied) so the kinetic energy of a charged particle does not change when it is under the influence of a magnetic force.

Hence, (D) → (q, s)

3. A → (q)
B → (p, t)
C → (s)
D → (r)

According to Amperes Circuital Law, we have

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

where $I_{enc} = I = \int \vec{J} \cdot d\vec{A}$

$$\Rightarrow \quad I = \int_0^R br(2\pi r dr) = \frac{2}{3}\pi bR^3$$

So, for $r < R$ we have $I_{\text{inside}} = \dfrac{2}{3}\pi b r^3$

Hence, (D) → (r)

Now, for $r < R$, we have

$$\mu_0 I_{\text{enc}} = \oint \vec{B} \cdot d\vec{\ell}$$

$$\mu_0\left(\frac{2}{3}\pi b r^3\right) = B(2\pi r)$$

$$\Rightarrow \quad B_{\text{inside}} = \frac{\mu_0 b r^2}{3}$$

Hence, (A) → (q)

Also, at the surface we shall observe the B to be maximum and continuous.

So, (B) → (p, t)

Now, for $r > R$ i.e., outside we have

$$\mu_0\left(\frac{2\pi b R^3}{3}\right) = B(2\pi r)$$

$$\Rightarrow \quad B_{\text{outside}} = \frac{\mu_0 b R^3}{3r}$$

Hence, (C) → (s)

4. A → (r)
B → (q)
C → (p)
D → (r)

For (A): $\vec{F} = q(\vec{v} \times \vec{B})$

Since, q is negative, $\vec{v}$ is along $+\hat{i}$ and $\vec{B}$ along $+\hat{j}$. Therefore, $\vec{F}$ is along negative z.

For (B): $\vec{F} = q(\vec{v} \times \vec{B})$ Since q is negative, v along $+\hat{i}$ and $\vec{B}$ along $+\hat{k}$, so $\vec{F}$ is along $+y$ axis.

For (C): $\vec{B}$ is parallel to $\vec{v}$. So, magnetic force is zero. Charge is negative so electric force is opposite to $\vec{E}$.

For (D): Charge is negative. So, electrostatic force is in opposite direction of $\vec{E}$.

5. Done already, see theory
A → (s)
B → (q)
C → (p)
D → (r)

6. Done already, see theory
A → (r)
B → (p)
C → (q, r)
D → (s)

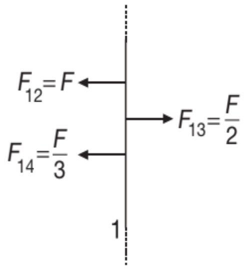
7. A → (q)
B → (r)
C → (q)
D → (r)

The force (F) per unit length is

$$\frac{F}{l} = \frac{\mu_0 I^2}{2\pi r}$$

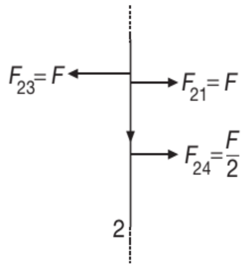
Current in same direction means attraction and current in opposite direction means repulsion.

Force on wire 1 due to other wires 2, 3 and 4 is shown in Figure.



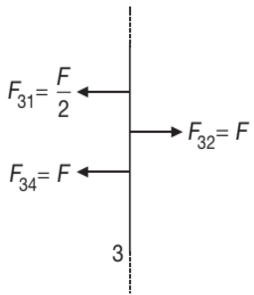
So, force on 1 is leftwards

Force on wire 2 due to other three wires 1, 3 and 4 is shown in Figure.

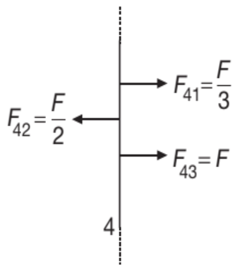


So, net force on 2, i.e. F_2 is rightwards.

Force on wire 3 due to others is also leftwards



Force on wire 4 due to others is rightwards



8. Done already, see theory
A → (p, t)
B → (p, q, r)
C → (p)
D → (s)

9. A → (p, s)
B → (p, q)
C → (p, r)
D → (p, s)
Force on a current carrying loop is zero for all angles. τ is maximum when angle between $\vec{M}$ and $\vec{B}$ is 90°. Potential energy is minimum when $\theta = 0°$. Positive Potential energy is positive for obtuse angle. The direction of $\vec{M}$ is obtained by right hand thumb rule.

10. Done already, see theory
A → (q, r)
B → (p)
C → (q, r)
D → (q)

11. Done already, see theory
A → (s)
B → (t)
C → (q)
D → (p)

12. A → (r)
B → (s)
C → (q)
D → (p)
For direction of magnetic force applying Fleming's left hand rule, according to which w and x are positively charged particles and y and z negatively charged particle.
Also, we know that

$$r = \frac{\sqrt{2mK}}{Bq}$$

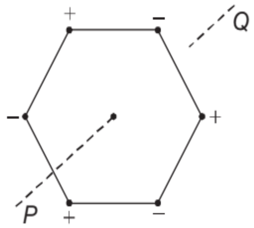
Since K i.e. the kinetic energy is same for all particles, so

$$r \propto \frac{\sqrt{m}}{q}$$

13. Done already, see theory
A → (q)
B → (p)
C → (s, t)
D → (q)

14. A → (p, r, s)
B → (r, s)
C → (p, q, t)
D → (r, s)

For (p)

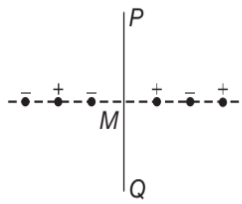


By symmetry $E = 0$, $V = 0$, $B = 0$

and $\mu = NIA$ but $I_{\text{effective}} = 0$

So $\mu = 0$

For (q)

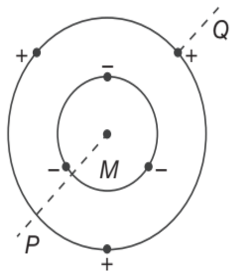


$E \neq 0$, $V = 0$

Since, $I_{\text{effective}} = 0$

$\Rightarrow \quad B = 0$ and $\mu = 0$

For (r)



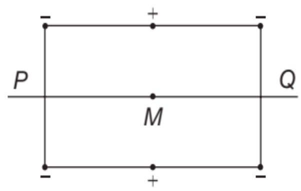
$E = 0$ (by symmetry)

$V \neq 0$ (since distances are different)

$B \neq 0$ (since radius is different)

$\mu \neq 0$

For (s)



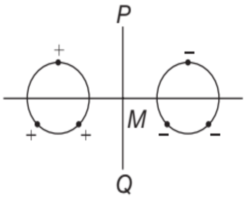
$E = 0$ (by symmetry)

$V \neq 0$ (since distances are different)

$B \neq 0$

$\mu \neq 0$

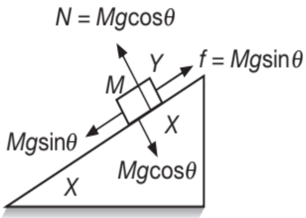
For (t)



$E \neq 0$, $V = 0$, $\mu = 0$, $B = 0$ and given each rotating charge to be equivalent to a steady current so $B = 0$ so $(t \to C)$ and if it was not given that each rotating charge to be equivalent to steady current then $B \neq 0$

15. A → (p, t)
B → (q, s, t)
C → (p, r, t)
D → (q)

For (p)



Net force on Y due to

$$X = \sqrt{(Mg\cos\theta)^2 + (Mg\sin\theta)^2} = Mg$$

(B) As the inclined is fixed. So, gravitational P.E. of X is constant.

(C) As K.E. is constant and P.E. of Y is decreasing. So mechanical energy of $(X+Y)$ is decreasing.

For (q)

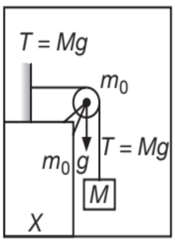
(A) Force on Y due to X will be greater than Mg which is equal to $(Mg + \text{repulsion force})$

(B) As the system is moving up, P.E. of X is increasing

(C) Mechanical energy of $(X+Y)$ is increasing

(D) Torque of the weight of Y about point $P = 0$

For (r)



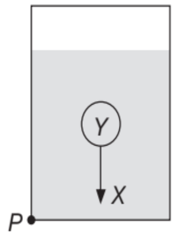
(A) Force on Y due to $X = \sqrt{[(M+m_0)g]^2 + (Mg)^2}$

(B) As the system moves down, gravitational P.E. of X decreases

(C) As the system moves down, total mechanical energy of $(X+Y)$ also decreases

(D) $\tau_P \neq 0$

For (s)



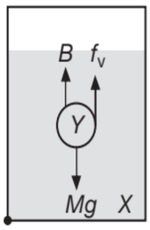
(A) Force on Y due to X = Buoyancy force which is less than mg

(B) As the sphere moves down, that volume of water comes up, so gravitational P.E. of X increases.

(C) As there is no non-conservative force, so total mechanical energy of $X+Y$ remains conserved.

(D) $\tau_P \neq 0$

For (t)



(A) As the sphere is moving with constant velocity

$$B + f_v = Mg$$

so force on Y due to X is $B + f_v = Mg$

(B) As the sphere moves down, that volume of water comes up, so gravitational P.E. of X will increase

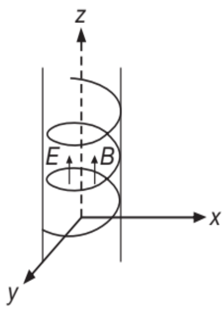
(C) Increase in mechanical energy $= w_{fr} = -ve$

(D) $\tau_P = 0$

Integer/Numerical Answer Type Questions

1. The Z component of the force $\vec{F}$ is a constant electrical force. It produces a constant acceleration a along Z-axis given as

$$Z = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{eE}{m}\right)t^2$$



The other component of the force F is a magnetic force provides the necessary centripetal force to keep the proton in a circular path of radius, r say. The period of revolution of the proton

$$T = \frac{2\pi r}{v_0} \text{ where}$$

$$F_{c.p.} = \frac{mv_0^2}{r} = ev_0 B$$

$$\Rightarrow \quad v_0 = \frac{eBr}{m}$$

$$\Rightarrow \quad T = \frac{2\pi m}{eB}$$

Since the particle (proton) moves in circular path having a period of revolution T in x, y plane and moves along Z-axis with a constant acceleration $a = \frac{eE}{m}$, the path of the proton is an helix.

After three revolutions, putting $t = 3T$ we obtain

$$z = \frac{1}{2}\left(\frac{eE}{m}\right)(3T)^2 = \frac{9}{2}\frac{eET^2}{m}$$

Putting $T = \frac{2\pi m}{eB}$ we get

$$z = \frac{18\pi^2 Em}{eB^2} = 37 \text{ m}$$

2. Let at time t, particle bet at point $P(x, y)$ and its velocity be

$$\vec{v} = \left(v_x\hat{i} + v_y\hat{j}\right)$$

Since, $|v| = |v_0|$

$$\Rightarrow \quad v_0^2 = v_x^2 + v_y^2$$

Since work done by magnetic field is always zero, so there is no change in magnitude of velocity, hence

$$|\vec{v}| = |\vec{v}_0|$$

$$\Rightarrow \quad v_0^2 = v_x^2 + v_y^2$$

Then, magnetic force on the particle at point $P(x, y)$ is

$$\vec{F} = q\left(v_x\hat{i} + v_y\hat{j}\right) \times B_0\left(1 + \frac{y}{d}\right)\left(-\hat{k}\right)$$

$$\Rightarrow \quad \vec{F} = qv_x B_0\left(1 + \frac{y}{d}\right)\hat{j} - qv_y B_0\left(1 + \frac{y}{d}\right)\hat{i}$$

$$\Rightarrow \quad F_x = \frac{mdv_x}{dt} = -qv_y B_0\left(1 + \frac{y}{d}\right), \text{ where } v_d = \frac{dy}{dt}$$

$$\Rightarrow \quad mdv_x = -qB_0\left(1 + \frac{y}{d}\right)dy \qquad \ldots(1)$$

When the particle leaves the field at $y = d$, then let the velocity in x-direction be v_x. Integrating equation (1), we get

$$\int_{v_0}^{v_x} dv = -\frac{qB_0}{m}\int_0^d \left(1 + \frac{y}{d}\right)dy$$

$$\Rightarrow \quad v_x - v_0 = -\frac{qB_0}{m}\left(d + \frac{d^2}{2d}\right) = -\frac{3qB_0 d}{2m}$$

$$\Rightarrow \quad v_x = v_0 - \frac{3qB_0 d}{2m}$$

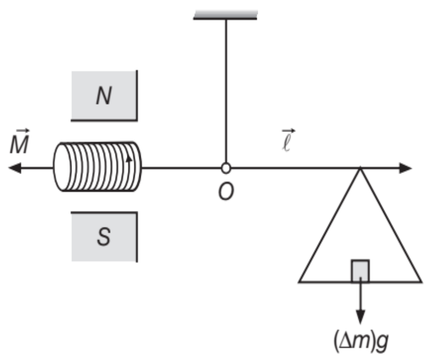
$$\Rightarrow \quad k = 1.5$$

3. We know that the torque acting on a magnetic dipole is

$$\vec{\tau} = \vec{M} \times \vec{B}$$

where $\vec{M} = (IA)\hat{n}$, where $\hat{n}$ is the normal on the plane of the loop and the direction of which is given by Right Hand Thumb Rule.

On passing a current through the coil, this torque acting on the magnetic dipole, is counterbalanced by the moment of additional weight, about O. Hence, the direction of current in the loop must be in the direction, shown in the figure.



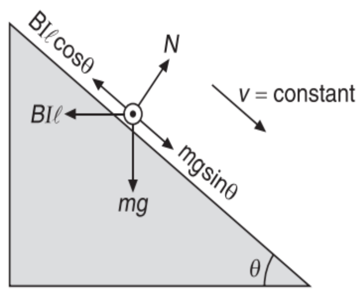
$$\vec{M} \times \vec{B} + \vec{\ell} \times (\Delta m)\vec{g} = \vec{0}$$

$$\Rightarrow \quad NBIA = (\Delta m)g\ell$$

$$\Rightarrow \quad B = \frac{(\Delta m)g\ell}{NIA} = 0.4 \text{ T}$$

$$\Rightarrow \quad B = 400 \text{ mT}$$

4. The forces acting on conductor are shown in Figure.



For conductor to move down the incline with constant velocity, we have

$$mg\sin\theta = BIl\cos\theta$$

$$\Rightarrow \quad B = \frac{mg}{IL}\tan\theta$$

$$\Rightarrow \quad B = \frac{\left(\frac{50}{1000}\right)(10)}{(2.5)\left(\frac{50}{100}\right)}\frac{3}{4}$$

$$\Rightarrow \quad B = 0.3\ \text{T} = 300\times10^{-3}\ \text{T}$$

$$\Rightarrow \quad B = 300\ \text{mT}$$

5. Let the wire connected to the $25\ \Omega$ resistor be designated as WIRE 2 and the wire connected to the $10\ \Omega$ resistor be designated as WIRE 1. Both I_1 and I_2 are directed toward the right in the figure, so at the location of the proton B_2 is $\otimes$ and B_1 is $\odot$, where

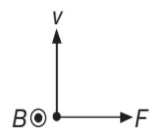
$$B_1 = \frac{\mu_0 I_1}{2\pi r} \text{ and } B_2 = \frac{\mu_0 I_2}{2\pi r}, \text{ with } r = 0.025\ \text{m}$$

$$I_1 = (100\ \text{V})(10\ \Omega) = 10\ \text{A} \text{ and}$$

$$I_2 = (100\ \text{V})(25\ \Omega) = 4\ \text{A}$$

$$B_1 = \frac{\mu_0 I_1}{2\pi r} = 8\times10^{-5}\ \text{T},\ B_2 = \frac{\mu_0 I_2}{2\pi r} = 3.2\times10^{-5}\ \text{T}$$

$$\Rightarrow \quad B = B_1 - B_2 = 4.8\times10^{-5}\ \text{T} \text{ and in the direction } \odot$$



Force is to the right

Since $F = qvB = (1.6\times10^{-19})(650\times10^3)(4.8\times10^{-5})$

$$\Rightarrow \quad F = 4.99\times10^{-18}\ \text{N} \cong 5\times10^{-18}\ \text{N}$$

$$\Rightarrow \quad x = 5$$

6. Since $R = \frac{mv}{qB}$

Also, $v^2 = \frac{2q\Delta V}{m}$

$$\Rightarrow \quad \frac{m}{q} = \frac{2\Delta V}{v^2}$$

$$\Rightarrow \quad \frac{m}{q} = \frac{2(12\times1000)}{10^{12}}$$

$$\Rightarrow \quad \frac{m}{q} = \frac{24\times10^3}{10^{12}}$$

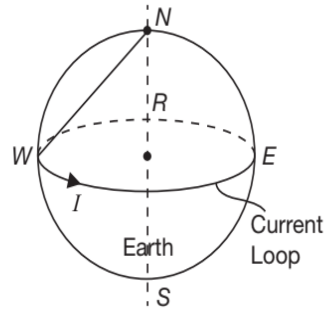
$$\Rightarrow \quad R = \frac{mv}{qB} = \frac{24\times10^3\times10^6}{10^{12}\times0.2}$$

$$\Rightarrow \quad R = 12\times10^{-2}\ \text{m}$$

$$\Rightarrow \quad R = 12\ \text{cm}$$

7. Since $B_{axis} = \frac{\mu_0 IR^2}{2(R^2+x^2)^{3/2}}$

Here $x = R =$ Radius of Earth



$$B_{\text{axis}} = 7\times10^{-5}\ \text{T}$$

Substituting values, we get

$$I = 2\times10^9\ \text{A}$$

$$\Rightarrow \quad I = 2000\ \text{MA}$$

8. Maximum tension that the ring can bear is

$$T = BIR$$

$$\Rightarrow \quad 1.5 = B(10)\left(\frac{15}{100}\right)$$

$$\Rightarrow \quad B = 1\ \text{T}$$

9. We observe here that, reducing the normal force will reduce the friction force $F = BIL$

$$\Rightarrow \quad B = \frac{F}{IL}$$

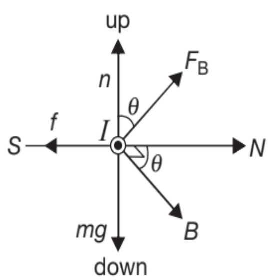
When the wire is just able to move,

$$\sum F_y = N + F\cos\theta - mg = 0$$

$$\Rightarrow \quad N = mg - F\cos\theta \text{ and since}$$

$$f = \mu N$$

$$\Rightarrow \quad f = \mu(mg - F\cos\theta)$$



Also, $\sum F_x = F\sin\theta - f = 0$

$$\Rightarrow \quad F_B\sin\theta = f$$

$$\Rightarrow \quad F\sin\theta = \mu(mg - F\cos\theta)$$

$$\Rightarrow \quad F = \frac{\mu mg}{\sin\theta + \mu\cos\theta}$$

We minimize B by minimizing F

$$\Rightarrow \quad \frac{dF}{d\theta} = 0$$

$$\Rightarrow \quad (\mu mg)\frac{\cos\theta - \mu\sin\theta}{(\sin\theta + \mu\cos\theta)^2} = 0$$

CHAPTER 1

$\Rightarrow \quad \mu \sin\theta = \cos\theta$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{\mu}\right) = \tan^{-1}\left(\frac{1}{0.75}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 53°$

for the smallest field, and

$$\Rightarrow \quad B = \frac{F}{IL} = \left(\frac{\mu g}{I}\right)\left(\frac{\left(\frac{m}{L}\right)}{\sin\theta + \mu\cos\theta}\right)$$

$$\Rightarrow \quad B_{min} = \left[\frac{(0.75)(10)}{2\ A}\right]\frac{0.1}{\sin(53°)+(0.75)\cos(53°)}$$

$\Rightarrow \quad B_{min} = 0.128\ \text{T}$

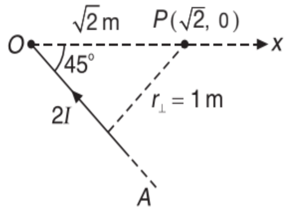
Pointing north at an angle of 78.7° below the horizontal

$$\Rightarrow \quad B_{min} = \frac{(0.75)(10)(0.1)}{(2)\left(\frac{4}{5}+\left(\frac{3}{4}\right)\left(\frac{3}{5}\right)\right)}$$

$$\Rightarrow \quad B_{min} = \frac{0.75}{(2)\left(\frac{5}{4}\right)}$$

$$\Rightarrow \quad B_{min} = \frac{3}{10} = 0.3\ \text{T} = 300\ \text{mT}$$

10. Magnetic field at P due to section OA is



$$B_1 = \frac{\mu_0(2I)}{4\pi(1)}(\sin 45° + \sin 90°)$$

$$\Rightarrow \quad B_1 = \frac{\mu_0 I}{4\pi}(\sqrt{2}+2)$$

Magnetic field at P due to section OB is

$$B_2 = \frac{\mu_0 I}{4\pi(1)}(\sin 45° + \sin 90°)$$

$$\Rightarrow \quad B_2 = \frac{\mu_0 I}{4\pi}\left(\frac{1}{\sqrt{2}}+1\right)$$

Magnetic field at P due to section OC is

$$B_3 = \frac{\mu_0 I}{4\pi\sqrt{2}}(\sin 0° + \sin 90°)$$

$$\Rightarrow \quad B_3 = \frac{\mu_0 I}{4\pi\sqrt{2}}$$

Net magnetic field B is given by

$$B = B_1 + B_2 + B_3$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi}\left(\sqrt{2}+2+\frac{1}{\sqrt{2}}+1+\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi}(2\sqrt{2}+2+1)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi}(\sqrt{2}+1)^2$$

$$\Rightarrow \quad B = \frac{2\mu_0 I}{8\pi}(\sqrt{2}+1)^2$$

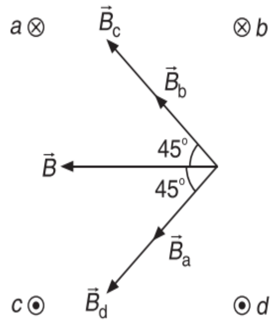
$\Rightarrow \quad x = 2$

11. The net magnetic field at the center of the square is the vector sum of the fields due to each wire.

For each wire, $B = \frac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is given by the Right Hand Rule.

For situations (a) and (b), $B = 0$ since the magnetic fields due to currents at opposite corners of the square cancel.

For (c) the fields due to each wire are shown in figure.



$$\Rightarrow \quad B = B_a\cos 45° + B_b\cos 45° + B_c\cos 45° + B_d\cos 45°$$

$$\Rightarrow \quad B = 4B_a\cos 45° = 4\left(\frac{\mu_0 I}{2\pi r}\right)\cos 45°$$

where $r = \sqrt{(10\ \text{cm})^2 + (10\ \text{cm})^2}$

$r = 10\sqrt{2}\ \text{cm} = (0.1)\sqrt{2}\ \text{m}$

$$\Rightarrow \quad B = 4\frac{(4\pi\times10^{-7}\ \text{TmA}^{-1})(100\ \text{A})}{2\pi(0.1\sqrt{2}\ \text{m})}\cos 45°$$

$B = 4\times10^{-4}\ \text{T}$, to the left

$\Rightarrow \quad B = 400\ \mu\text{T}$

12. Since $E_y = -E$, $E_x = 0$

$$\Rightarrow \quad a_y = -\frac{qE}{M},\ a_x = 0$$

Also, $v_x = u_x + a_x t$ and $v_y = u_y + a_y t$

$$\Rightarrow \quad v_x = \frac{v}{\sqrt{2}} \text{ and } v_y = \frac{v}{\sqrt{2}} - \left(\frac{qE}{M}\right)t$$

At $x = 0.5$ m, we have

$$0.5 = v_x t$$

$$\Rightarrow \quad t = \frac{0.5}{v/\sqrt{2}} = \frac{0.5\sqrt{2}}{v}$$

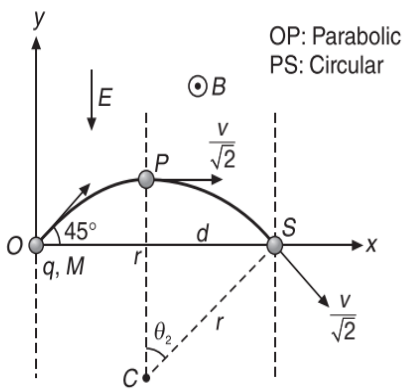
$$\Rightarrow \quad v_y = \frac{v}{\sqrt{2}} - v^2\left(\frac{0.5\sqrt{2}}{v}\right) = \frac{v}{\sqrt{2}} - \frac{v}{\sqrt{2}}$$

$$\Rightarrow \quad v_y = 0$$

i.e., deviation suffered by particle is $\theta_1 = 45°(\text{CW})$

So, at $x = 0.5$ m, the charged particle enters the magnetic field with a velocity $\frac{v}{\sqrt{2}}$, perpendicular to the field so, it will follow a circular path of radius

$$r = \frac{M(v/\sqrt{2})}{qB} = \frac{Mv}{\sqrt{2}qB} \text{ as shown in Figure.}$$



So, deviation suffered by particle is

$$\sin\theta_2 = \frac{d}{r} = \frac{1}{2}$$

$$\Rightarrow \quad \theta_2 = 30°(\text{CW})$$

So, total deviation is $\theta = \theta_1 + \theta_2 = 75°(\text{CW})$

$$\Rightarrow \quad \theta = 75 \times \frac{\pi}{180} \text{ radian}$$

$$\Rightarrow \quad \theta = \frac{5\pi}{12} \text{ radian}$$

$$\Rightarrow \quad x = 5,\ y = 12$$

13. (a) $BI\ell = mg$

$$\Rightarrow \quad I = \frac{mg}{\ell B} = \frac{(0.75\text{ kg})(10\text{ ms}^{-2})}{(0.5\text{ m})(0.5\text{ T})} = 30\text{ A}$$

Since $\varepsilon = IR = (30\text{ A})(25\ \Omega) = 750$ V

(b) $R = 2\ \Omega$

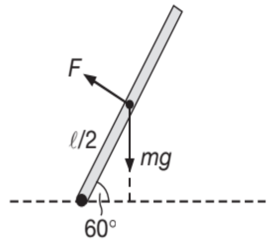
From Ohm's Law, we have

$$I = \frac{\varepsilon}{R} = \frac{(750\text{ V})}{(2\ \Omega)} = 375\text{ A}$$

$$F_I = BI\ell = 93.75\text{ N}$$

$$\Rightarrow \quad a = \frac{(F_I - mg)}{m} = \frac{93.75 - 7.5}{0.75} = 115\text{ ms}^{-2}$$

14. The contact at a will break if the bar rotates about b. The magnetic field is directed out of the page, so the magnetic torque is counter clockwise, whereas the gravity torque is clockwise in the figure in the problem. The maximum current corresponds to zero net torque, in which case the torque due to gravity is just equal to the torque due to the magnetic field.



The magnetic force F is perpendicular to the bar and has moment arm $\frac{\ell}{2}$, where $\ell = 1$ m is the length of the bar. The gravity torque is $mg\left(\frac{\ell}{2}\cos(60°)\right)$ and the torque to the magnetic field is

$$\tau_B = (BI\ell\sin 90)\frac{\ell}{2}$$

Since, $\tau_{\text{gravity}} = \tau_B$

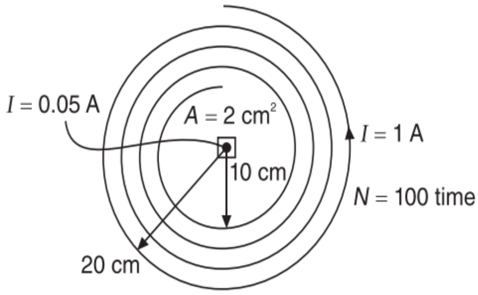
$$\Rightarrow \quad mg\left(\frac{\ell}{2}\right)\cos(60°) = BI\ell\sin(90°)\frac{\ell}{2}. \text{ This gives}$$

$$I = \frac{mg\cos(60°)}{\ell B\sin 90°}$$

$$\Rightarrow \quad I = \frac{(0.5)(10)(0.5)}{(1)(0.5)}$$

$$\Rightarrow \quad I = 5\text{ A}$$

15. Magnetic field at centre of spiral loop is



$$B = \frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$$

$$\Rightarrow \quad B = \frac{\mu_0(100)(1)}{2\times 10\times 10^{-2}}\ln(2)$$

$\Rightarrow \quad B=\frac{\mu_0}{2}(10^3)\ln(2)$

$\Rightarrow \quad B=500\mu_0 \ln 2$

Magnetic dipole moment of the loop is

$$M=IA=0.05\times 2\times 10^{-4}=1\times 10^{-5}\ \text{Am}^{-2}$$

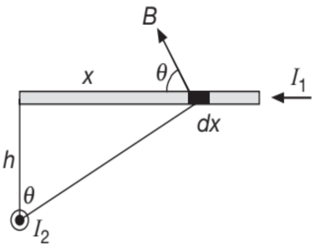
Since $U=-\vec{M}\cdot\vec{B}$

$\Rightarrow \quad (U)=MB\cos\theta$

$\Rightarrow \quad U=\frac{\mu_0}{2}(10^3)(\ln 2)(10^{-5})=\frac{\mu_0}{200}\ln(2)$

$\Rightarrow \quad x=200$

16. At a point at distance x from the left end of the bar, current I_2 creates magnetic field $\vec{B}=\frac{\mu_0 I_2}{2\pi\sqrt{h^2+x^2}}$ to the left and above the horizontal at angle θ where $\tan\theta=\frac{x}{h}$. This field exerts force on an element of the rod of length dx



$$d\vec{F}=I_1(d\vec{\ell}\times\vec{B})=I_1\frac{\mu_0 I_2 dx}{2\pi\sqrt{h^2+x^2}}\sin\theta$$

$\Rightarrow \quad d\vec{F}=\frac{\mu_0 I_1 I_2 dx}{2\pi\sqrt{h^2+x^2}}\frac{x}{\sqrt{h^2+x^2}}$, into the page

$\Rightarrow \quad d\vec{F}=\frac{\mu_0 I_1 I_2 x dx}{2\pi(h^2+x^2)}(-\hat{k})$

The net force is the sum of the forces on all of the elements of the bar. So,

$$\vec{F}=\int_{x=0}^{\ell}\frac{\mu_0 I_1 I_2 x dx}{2\pi(h^2+x^2)}(-\hat{k})$$

$\Rightarrow \quad \vec{F}=\frac{\mu_0 I_1 I_2(-\hat{k})}{4\pi}\int_0^{\ell}\frac{2x\,dx}{h^2+x^2}$

$\Rightarrow \quad \vec{F}=\frac{\mu_0 I_1 I_2(-\hat{k})}{4\pi}\log_e(h^2+x^2)\Big|_0^{\ell}$

$\Rightarrow \quad \vec{F}=\frac{\mu_0 I_1 I_2(-\hat{k})}{4\pi}\left[\log_e(h^2+\ell^2)-\log_e h^2\right]$

$\Rightarrow \quad \vec{F}=10^{-7}(100)(200)(-\hat{k})\log_e\left[\frac{(0.5)^2+(10)^2}{(0.5)^2}\right]$

$\Rightarrow \quad \vec{F}=2\times 10^{-3}\ \text{N}(-\hat{k})\log_e(401)=1.2\times 10^{-2}\ \text{N}(-\hat{k})$

$\Rightarrow \quad |\vec{F}|=12\ \text{mN}$

17. Magnetic induction at O is given by

$$\vec{B}=\frac{\mu_0 i}{4\pi a}\left[\left(1-\frac{1}{\sqrt{2}}\right)(-\hat{j})+\left(\frac{1}{\sqrt{2}}\right)\hat{j}+\hat{k}\right]$$

Comparing with equation given in problem, we get

$$x=2$$

18. Force of magnetic interaction is

$$\vec{F}_{\text{mag}}=e(\vec{v}\times\vec{B})$$

where $\vec{B}=\frac{\mu_0}{4\pi}\frac{e(\vec{v}\times\vec{r})}{r^3}$

$\Rightarrow \quad \vec{F}_{\text{mag}}=\frac{\mu_0}{4\pi}\frac{e^2}{r^3}[\vec{v}\times(\vec{v}\times\vec{r})]$

$\Rightarrow \quad \vec{F}_m=\frac{\mu_0}{4\pi}\frac{e^2}{r^3}[(\vec{v}\cdot\vec{r})\times\vec{v}-(\vec{v}\cdot\vec{v})\times\vec{r}]$

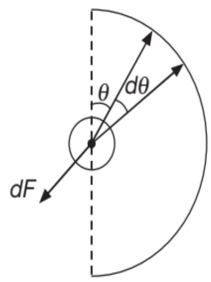
$\Rightarrow \quad \vec{F}_m=\frac{\mu_0}{4\pi}\frac{e^2}{r^3}(-v^2\vec{r})$

and $\vec{F}_{\text{ele}}=\frac{e^2\vec{r}}{4\pi\varepsilon_0|\vec{r}|^3}$

$\Rightarrow \quad \frac{|\vec{F}_{\text{mag}}|}{|\vec{F}_{\text{electric}}|}=-v^2\mu_0\varepsilon_0=\left(\frac{v}{c}\right)^2=1\times 10^{-6}$

$\Rightarrow \quad x=1$

19. Magnetic field due to small ring at distance R from the centre



$$B=\frac{\mu_0}{4\pi}\frac{M}{R^3}\quad \text{where } M=I\pi a^2$$

$\Rightarrow \quad B=\frac{\mu_0}{4\pi}\frac{I\pi a^2}{R^3}=\frac{\mu_0 I a^2}{4R^3}$

$\Rightarrow \quad dF=BI_0 d\ell=BI_0(Rd\theta)=I_0 Rd\theta\frac{\mu_0 I a^2}{4R^3}$

$$\Rightarrow \quad dF_x = dF\sin\theta$$

$$\Rightarrow \quad dF_x = \frac{\mu_0 I I_0 a^2 \sin\theta d\theta}{4R^2}$$

$$\Rightarrow \quad F_x = \frac{\mu_0 I I_0 a^2}{4R^2}\int_0^{\pi}\sin\theta d\theta$$

$$\Rightarrow \quad F_x = \left(\frac{\mu_0 I I_0 a^2}{4R^2}\right)2$$

$$\Rightarrow \quad F_x = \frac{\mu_0 I I_0 a^2}{2R^2}$$

and $F_y = 0$

$$\Rightarrow \quad F_{\text{net}} = F_x = \frac{\mu_0 I I_0 a^2}{2R^2} = 8 \text{ newton}$$

20. Force on a small element of length dx of wire is given by

$$d\vec{F} = Id\vec{x}\times\vec{B}$$

$$\Rightarrow \quad d\vec{F} = \left[IB_0\left(1+\frac{x}{a}\right)dx\right](\hat{i}\times\hat{k})$$

$$\Rightarrow \quad d\vec{F} = IB_0\left(1+\frac{x}{a}\right)dx(-\hat{j})$$

So, magnitude of total force $\vec{F}$ on wire will be

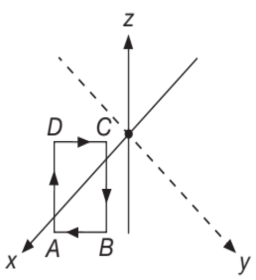
$$F = \int_a^{2a} dF = \int_a^{2a} IB_0\left(1+\frac{x}{a}\right)dx$$

$$\Rightarrow \quad F = IB_0\left(x^2+\frac{x^2}{2a}\right)\Bigg|_a^{2a}$$

$$\Rightarrow \quad F = IB_0\left(\frac{5a}{2}\right), \text{ along negative } y\text{-axis}$$

$$\Rightarrow \quad k = 5$$

21. $\vec{m} = I\vec{S}$, where $\vec{S}$ is the area of the loop.



$$\Rightarrow \quad \vec{S} = \overrightarrow{BA}\times\overrightarrow{AD}$$

$$\Rightarrow \quad \overrightarrow{BA} = 2d\hat{i} - 2a\hat{j}$$

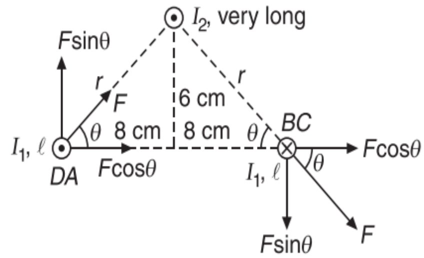
$$\Rightarrow \quad \overrightarrow{AD} = 2b\hat{k}$$

$$\Rightarrow \quad \vec{S} = 2\left(d\hat{i} - a\hat{j}\right)\times 2b\hat{k}$$

$$\Rightarrow \quad \vec{M} = -4bI\left(d\hat{j} + a\hat{i}\right)$$

$$\Rightarrow \quad |\vec{M}| = 4bI\sqrt{d^2+a^2} = 2 \text{ Am}^2$$

22. For wires AB and CD, the force due to field of very long wire P on the wires $(AB \text{ and } CD)$ is zero. However force on wire DA will be attractive (towards P) and that on wire BC will be repulsive (away from P), as shown in Figure.



Let F_0 be force on wire DA due to P, then

$$F_{DA} = F_{BC} = F_0 = \left(\frac{\mu_0 I_1 I_2}{2\pi r}\right)l$$

The components of force i.e. $F\sin\theta$ and $F\sin\theta$ cancel, so net force on the loop is

$$F_{\text{net}} = 2F_0\cos\theta = \left[2\left(\frac{\mu_0 I_1 I_2}{2\pi r}\right)l\right]\cos\theta$$

where $\cos\theta = \dfrac{8}{\sqrt{8^2+b^2}} = \dfrac{4}{5}$

$$\Rightarrow \quad F_{\text{net}} = \frac{(4\times10^{-7})(10)(5)\left(\frac{16}{100}\right)}{10}\times\frac{4}{5}$$

$$\Rightarrow \quad F_{\text{net}} = 256\times10^{-7} \text{ N}$$

$$\Rightarrow \quad F = 256$$

23. $\left(\vec{B}_1+\vec{B}_2+\vec{B}_3+\vec{B}_4\right)_{\text{at the centre}} = \vec{0}$

$$\frac{\mu_0 I_1}{2\pi a}\otimes + \vec{B}_2 + \frac{\mu_0 I_3}{2\pi a}\odot + \frac{\mu_0 I_4}{2\pi a}\otimes = \vec{0} \qquad \ldots(1)$$

Assume inward direction to be positive, then on substituting $a = \frac{20}{100}m = 0.2$ m in (1), we get

$$\vec{B}_2 = \frac{\mu_0}{2\pi a}(I_3 - I_1 - I_4)$$

$$\Rightarrow \quad \vec{B}_2 = \frac{\mu_0}{2\pi a}(20-10-8) = \frac{\mu_0}{2\pi a}(2\text{ A}) \qquad \ldots(2)$$

Since $\vec{B}_2$ is $\oplus$, so we conclude that the field due to wire 2 must be directed inwards $\otimes$. For the field due to wire 2 to be directed inwards the current in the wire 2 must be in the downward direction. Hence, we get, from (1),

$$\frac{\mu_0 I}{2\pi a}=\frac{\mu_0}{2\pi a}(2\text{ A})$$

$\Rightarrow \quad I = 2\text{ A}$ (downwards)

24. Magnetic induction at origin is due to one semi infinite wire and two quarter circle of radii R and $2R$.

$$\vec{B}_0=\frac{\mu_0 I}{8R}\hat{k}+\frac{\mu_0 I}{8(2R)}\hat{k}+\frac{\mu_0 I}{4\pi R}\hat{j}$$

$$\Rightarrow \quad \vec{B}_0=\frac{3\mu_0 I}{16R}\hat{k}+\frac{\mu_0 I}{4\pi R}\hat{j}$$

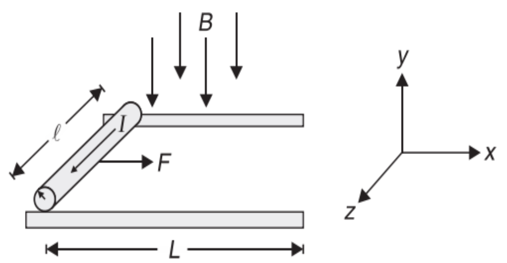
$$\Rightarrow \quad \vec{B}_0=\frac{\mu_0 I}{4R}\left(\frac{3}{4}\hat{k}+\frac{1}{\pi}\hat{j}\right)$$

$\Rightarrow \quad a=3,\ b=1$

$\Rightarrow \quad ab=3$

25. The magnetic force acting on the rod is given by

$$\vec{F}=I(\vec{\ell}\times\vec{B})=I\ell(\hat{k})\times B(-\hat{j})=I\ell B(\hat{i})$$



From Work Energy Theorem we have

$$(K_{\text{trans}}+K_{\text{rot}})_{\text{initial}}+\Delta E=(K_{\text{trans}}+K_{\text{rot}})_{\text{final}}$$

$$0+0+Fs\cos\theta=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2,\text{ where } I=\frac{1}{2}mR^2$$

$$\Rightarrow \quad I\ell BL\cos 0^\circ=\frac{1}{2}mv^2+\frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2$$

$$\Rightarrow \quad I\ell BL=\frac{3}{4}mv^2$$

$$\Rightarrow \quad v=\sqrt{\frac{4I\ell BL}{3m}}$$

$$\Rightarrow \quad v=\sqrt{\frac{4(48\text{ A})(0.12\text{ m})(0.24\text{ T})(0.45\text{ m})}{3(0.72\text{ kg})}}$$

$$\Rightarrow \quad v=1.07\text{ ms}^{-1}$$

$$\Rightarrow \quad v=107\text{ cms}^{-1}$$

26. Here torque on the magnetic dipole because of magnetic force must be balanced by torque by weight

So, $\tau_{mg}=mg\sin\theta\times R$ (about point of contact)

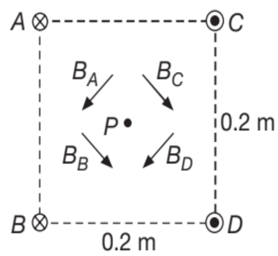
$$|\vec{M}\times\vec{B}|=mg\sin\theta\times R$$

$$\Rightarrow \quad i\times 2R\times L\times B\times\sin\theta=mg\sin\theta\times R$$

$$\Rightarrow \quad i=\frac{mg}{2BL}=\frac{1\times10\times2\times10}{4\times2\times1\times1}=\frac{100}{4}=25\text{ A}$$

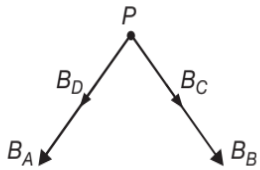
27. Distance of the point P from each of the wires is

$$r=\ell\cos(45^\circ)=\frac{\ell}{\sqrt{2}}$$



Each wire produces a field at P of magnitude say B, then

$$B=\frac{\mu_0 I}{2\pi r}=\frac{\mu_0 I\sqrt{2}}{2\pi\ell}$$

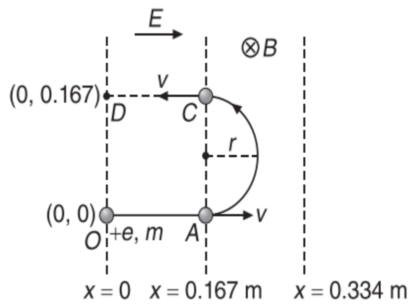


The field directions due to wires A, B, C and D are shown.

$$|\vec{B}_A|=|\vec{B}_B|=|\vec{B}_C|=|\vec{B}_D|=\frac{\sqrt{2}\mu_0 I}{2\pi\ell}$$

$B_P=4B\sin(45^\circ)=\dfrac{4\sqrt{2}\mu_0 I}{2\pi\ell}=20\ \mu\text{T}$ toward the bottom of the page.

28. The situation described in the problem is shown in Figure.



Since the electric field is along x-axis, so proton will be accelerated by the electric field and will enter the magnetic field at A (i.e., $x=0.167$, $y=0$) with velocity v along x-axis such that

$$\frac{1}{2}mv^2=W=q\Delta V=q(Ed)$$

$$\Rightarrow \quad v=\sqrt{\frac{2qEd}{m}}$$

$$\Rightarrow \quad v=\left[\frac{2\times1.6\times10^{-19}\times100\times0.167}{1.67\times10^{-27}}\right]^{\frac{1}{2}}$$

$$\Rightarrow \quad v=4\sqrt{2}\times10^4\text{ ms}^{-1}$$

Now as proton is moving perpendicular to magnetic field, so it will describe a circular path in the magnetic field with radius r such that

$$r = \frac{mv}{qB}$$

For proton to strike the point $(0,\ 0.167)$ m, we have $2r = 0.167$

$$\Rightarrow\ 2\left(\frac{mv}{qB}\right) = 0.167$$

$$\Rightarrow\ B = \frac{2\times 1.67\times 10^{-27}\times 4\sqrt{2}\times 10^{4}}{1.6\times 10^{-19}\times 0.167}$$

$$\Rightarrow\ B = \frac{1}{\sqrt{2}}\times 10^{-2}$$

$$\Rightarrow\ B = 7.07\text{ mT}$$

$$\Rightarrow\ B \approx 7\text{ millitesla}$$

29. From Ampere's Law, the magnetic field at point a is given by $B_a = \frac{\mu_0 I_a}{2\pi r_a}$, where I_a is the net current through the area of the circle of radius r_a. In this case, $I_a = 1$ A out of the page (the current in the inner conductor), so

$$B_a = \frac{(4\pi\times 10^{-7})(1\text{ A})}{2\pi(1\times 10^{-3}\text{ m})}$$

$\Rightarrow\ B_a = 200\ \mu\text{T}$, towards top of page

Similarly at point $b: B_b = \frac{\mu_0 I_b}{2\pi r_b}$, where I_b is the net current through the area of the circle having radius r_b.

Taking out of the page as positive, $I_b = 1\text{ A} - 3\text{ A} = -2\text{ A}$

$\Rightarrow\ I_b = 2$ A into the page

$$\Rightarrow\ B_b = \frac{(4\pi\times 10^{-7})(2\text{ A})}{2\pi(3\times 10^{-3}\text{ m})}$$

$\Rightarrow\ B_b = 133\ \mu\text{T}$, towards bottom of page

30. Using Ampere's law, we have

$\int \vec{B}\cdot d\vec{l} = \mu_0 I = \mu_0 \int J dA$, where $dA = 2\pi r dr$

For $r > R$, we have

$$B\int dl = \mu_0 \int_0^R (br)(2\pi r dr)$$

$$B(2\pi r) = \frac{2\pi\mu_0 b R^3}{3}$$

$$\Rightarrow\ B = \frac{\mu_0 b R^3}{3r}$$

$$\Rightarrow\ x = 3 \text{ and } y = 3$$

$$\Rightarrow\ \frac{x}{y} = 1$$

31. At equilibrium, we have

$$\frac{F}{\ell} = \frac{\mu_0 I_A I_B}{2\pi r} = \frac{mg}{\ell}$$

$$\Rightarrow\ \frac{\mu_0 I_A I_B}{2\pi r} = \left(\frac{m}{\ell}\right)g$$

where $\frac{m}{\ell} = 0.15\text{ gcm}^{-1} = \left(0.15\times\frac{100}{1000}\right)\text{kgm}^{-1}$

$$\Rightarrow\ \frac{(4\pi\times 10^{-7})(100)(I)}{(2\pi)\left(\frac{2}{100}\right)} = \left(0.15\times\frac{100}{1000}\right)(9.8)$$

$$\Rightarrow\ 10^{-3} I = 0.147$$

$$\Rightarrow\ I = 0.147\times 10^{3}$$

$$\Rightarrow\ I = 147\text{ A}$$

32. (a) $|\vec{\tau}| = |\vec{M}\times\vec{B}| = NBIA$

$$\Rightarrow\ \tau_{\max} = 1000(10^{-2})(0.025\times 0.04)(0.8)\sin 90^\circ$$

$$\Rightarrow\ \tau_{\max} = 8\times 10^{-3}\text{ Nm} = 8\text{ mNm}$$

(b) $\mathbf{P}_{\max} = \tau_{\max}\omega = (8\times 10^{-3})\left(3600\times\frac{2\pi}{60}\right) = 3\text{ W}$

(c) In one half revolution the work is

$$W = U_{\max} - U_{\min} = -MB\cos 180^\circ - (-MB\cos 0^\circ)$$

$$\Rightarrow\ W = 2MB$$

$$\Rightarrow\ W = 2NBIA = 2(8\times 10^{-3}) = 16\times 10^{-3}\text{ J}$$

In one full revolution

$$W = 2(16\times 10^{-3}\text{ J}) = 32\times 10^{-3}\text{ J}$$

$$\Rightarrow\ W = 32\text{ mJ}$$

(d) $\mathbf{P}_{\text{avg}} = \frac{W}{t} = \frac{32\times 10^{-3}\text{ J}}{\left(\frac{1}{60}\right)\text{s}}$

$$\Rightarrow\ \mathbf{P}_{\text{avg}} = 1920\times 10^{-3}\text{ W} = 1920\text{ mW}$$

(e) $\frac{\text{Peak Power}}{\text{Average Power}} = \frac{3}{1.92} = \frac{300}{192} = \frac{100}{64} = \frac{25}{16} = \frac{(5)^2}{16}$

So, $* = 5$

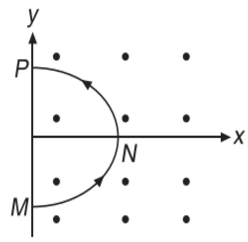
33. At $x = 0$,

$$y = \pm 2m$$

Since, $\vec{F}_{\text{MNP}} = \vec{F}_{\text{MP}} = i[\overrightarrow{MP}\times\vec{B}]$

$$\Rightarrow\ \vec{F}_{\text{MNP}} = 3[(4\hat{j})\times(5\hat{k})]$$

$$\Rightarrow\ \vec{F}_{\text{MNP}} = 60\hat{i}$$



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1. $F = -kx - IlB = -kx - \left(\frac{Blv}{R}\right)(lB)$

$\Rightarrow \quad F = -kx - \left(\frac{B^2l^2}{R}\right)v$

So, it is case of damped oscillation, hence

$$A = A_0 e^{-\frac{bt}{2m}}$$

Given that $A = \frac{A_0}{e}$

$\Rightarrow \quad \frac{A_0}{e} = A_0 e^{-\frac{bt}{2m}}$

$\Rightarrow \quad t = \frac{2m}{\left(\frac{B^2l^2}{R}\right)} = \frac{2 \times 50 \times 10^{-3} \times 10}{0.01 \times 0.01} = 10000 \text{ s}$

Time period, $T = 2\pi\sqrt{\frac{m}{k}} \approx 2 \text{ s}$

So, the number N is

$$N = \frac{10000}{2} = 5000$$

Hence, the correct answer is (B).

2. $|\vec{\tau}| = |\vec{\mu} \times \vec{B}|$

$\Rightarrow \quad \tau = (NIA)(B)$

Since, $A = \pi r^2$

$\Rightarrow \quad \tau = NI(\pi r^2)B$

Hence, the correct answer is (C).

3. $\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{2\pi d}(\hat{k} - \hat{k}) = 0$

Hence, the correct answer is (A).

4. The force on loop is given by

$$F = I_2 a(B_1 - B_2)$$

where

$$B_1 = \frac{\mu_0 I_1}{2\pi a} \text{ and } B_2 = \frac{\mu_0 I_1}{4\pi a}$$

$\Rightarrow \quad F = \frac{\mu_0 I_1 I_2}{4\pi}$

Hence, the correct answer is (A).

5. $NBIA = C\theta$

$\Rightarrow \quad (175)B(1 \times 10^{-3})(1 \times 10^{-4}) = \frac{10^{-6} \times \pi}{180}$

$\Rightarrow \quad B = \frac{\pi}{180} \times \frac{10}{175} \approx 9.97 \times 10^{-4} \text{ T}$

$\Rightarrow \quad B = 10^{-3} \text{ T}$

Hence, the correct answer is (D).

6. $\sum F = 0$

$\Rightarrow \quad \frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi(x - d)}$

As both wires A and B carry currents in opposite directions, so $x > d$

$\Rightarrow \quad I_1 x - I_1 d = I_2 x$

$\Rightarrow \quad x = \frac{I_1 d}{I_1 - I_2}$

Hence, the correct answer is (A).

7. Radius of circular path (r) in a perpendicular uniform magnetic field is $r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$

For proton, electron and helium nucleus i.e. α-particle, we have

$m_\alpha = 4m_p$ and $m_p \gg m_e$

Also, $q_\alpha = 2q_p$ and $q_p = q_e$

Since kinetic energy (KE) of all the particles is same, so

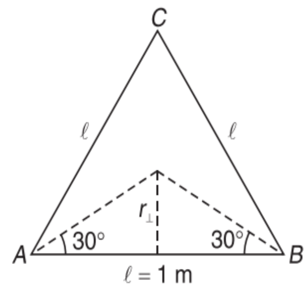
$$r \propto \frac{\sqrt{m}}{q}$$

$\Rightarrow \quad r_\alpha = r_p > r_e$

Hence, the correct answer is (C).

8. The magnetic field due to wire AB is

$$B_{AB} = \frac{\mu_0 I}{4\pi r_\perp}(\sin\theta_1 + \sin\theta_2)$$



where

$$\tan 30° = \frac{r_\perp}{l/2}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{r_\perp}{l/2}$$

$$\Rightarrow r_\perp = \frac{l}{2\sqrt{3}}$$

$$\Rightarrow r_\perp = \frac{1}{2\sqrt{3}} \text{ m}$$

Also, $\theta_1 = \theta_2 = 60°$

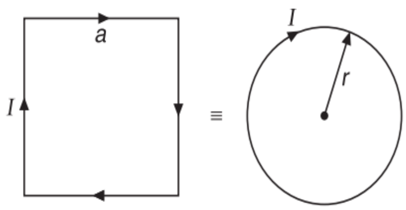
$$\Rightarrow B_{AB} = \frac{\mu_0 I}{4\pi r_\perp} 2\sin(60°) = \frac{\mu_0 I}{4\pi r_\perp}\sqrt{3}$$

$$\Rightarrow B_{AB} = \frac{6\mu_0 I}{4\pi}$$

$$\Rightarrow B_{\text{total}} = 3B_{AB} = \frac{18\mu_0 I}{4\pi} = 18\ \mu\text{T}$$

Hence, the correct answer is (A).

9. The square loop is changed to a circular loop. So, we have



$$2\pi r = 4a$$

$$\Rightarrow r = \left(\frac{2a}{\pi}\right)$$

For Square loop,

$$m = Ia^2$$

For Circular loop,

$$m' = I(\pi r^2)$$

$$\Rightarrow m' = I\pi\left(\frac{4a^2}{\pi^2}\right)$$

$$\Rightarrow m' = \frac{4Ia^2}{\pi}$$

$$\Rightarrow m' = \frac{4m}{\pi}$$

Hence, the correct answer is (B).

10. $B = \dfrac{\mu_0 I}{2a}$ and $I = \dfrac{q\omega}{2\pi}$

$$\Rightarrow B = \left(\frac{\mu_0}{2a}\right)\left(\frac{q\omega}{2\pi}\right)$$

$$\Rightarrow B = \frac{(10^{-7})(40)}{0.1} q\pi$$

$$\Rightarrow q = 3 \times 10^{-5} \text{ C}$$

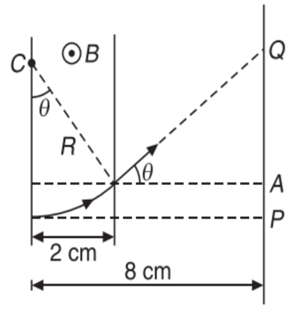
Hence, the correct answer is (B).

11. Radius of path R is given by

$$R = \frac{mv}{eB} = \frac{\sqrt{2m(KE)}}{eB}$$

$$\Rightarrow R = \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times 100e}}{e \times 1.5 \times 10^{-3}}$$

$$\Rightarrow R = \frac{\sqrt{2 \times 9.1 \times 10^{-29}}}{\sqrt{1.6 \times 10^{-19}} \times 1.5 \times 10^{-3}} \text{ m}$$



$$\Rightarrow R = \frac{3.37 \times 10^{-5}}{1.5 \times 10^{-3}} \times 100 \text{ cm} = 2.25 \text{ cm}$$

Since, $\sin\theta = \dfrac{2}{2.25} = \dfrac{8}{9}$

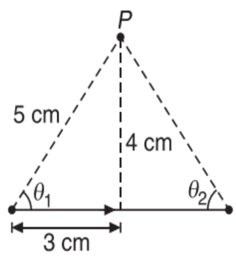
$$\Rightarrow PQ = PA + AQ$$

$$\Rightarrow PQ = 2.25(1 - \cos\theta) + 11.64$$

$$\Rightarrow PQ = 1.22 + 11.64 = 12.86 \text{ cm}$$

Hence, the correct answer is (B).

12.



$$B_P = \frac{\mu_0 I}{4\pi d}(\cos\theta_1 + \cos\theta_2)$$

$$\Rightarrow B_P = \frac{(10^{-7})(5)}{0.04}\left(2 \times \frac{3}{5}\right) = 1.5 \times 10^{-5} \text{ T}$$

Hence, the correct answer is (B).

13. For shifting of loop along x-direction, the potential energy is

$$U(x) = -\vec{M} \cdot \vec{B}$$

$$\Rightarrow U(x) = -\left[(\pi a^2)i\right]\left(\frac{\mu_0 I_0}{2\pi x}\right)$$

$$\Rightarrow \quad U(x) = -\frac{\mu_0 i I_0 a^2}{2x}$$

PE decreases as it comes closer to wire.

So, attractive force $F(x)$ is given by

$$F(x) = -\frac{dU}{dx} = \frac{\mu_0 i I_0 a^2}{2}\left(-\frac{1}{x^2}\right)$$

$$\Rightarrow \quad F(x) = \frac{\mu_0 i I_0 a^2}{2d^2} \text{ (Attractive)}$$

$$\Rightarrow \quad F \propto \frac{a^2}{d^2}$$

Hence, the correct answer is (B).

14. $B = \frac{\mu_0 I}{4\pi R}\theta$ where $\theta = \frac{\pi}{4}$ radian

$$\Rightarrow \quad B = \frac{\mu_0 I}{16R}$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{16}\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{\mu_0 I}{120}$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{120} = 1.047 \times 10^{-7} \approx 1.0 \times 10^{-7} \text{ T}$$

Hence, the correct answer is (D).

15. Since $2\pi r_1 = L$

$$\Rightarrow \quad r_1 = \frac{L}{2\pi}$$

Also, magnetic field at centre of loop is

$$B_L = \frac{\mu_0 I}{2r_1}$$

Now, $L = (2\pi r_2)N$

$$\Rightarrow \quad r_2 = \frac{L}{2N\pi}$$

Magnetic field at centre of coil is

$$B_C = \frac{N\mu_0 I}{2r_2}$$

$$\Rightarrow \quad \frac{B_L}{B_C} = \frac{1}{N^2}$$

Hence, the correct answer is (D).

16. For particle to move in a straight line, we have

$$eE = evB$$

Since $R = \frac{mv}{eB}$

$$\Rightarrow \quad v = \frac{eBR}{m}$$

$$\Rightarrow \quad E = \left(\frac{eBR}{m}\right)B$$

$$\Rightarrow \quad m = \frac{eB^2R}{E}$$

$$\Rightarrow \quad m = \frac{1.6 \times 10^{-19} \times (0.5)^2 \times 0.5 \times 10^{-2}}{100}$$

$$\Rightarrow \quad m = 2.0 \times 10^{-24} \text{ kg}$$

Hence, the correct answer is (D).

17. Since, $E = vB$

$$\Rightarrow \quad E = 0.6 \text{ Vm}^{-1}$$

Since, $V = Ed$

$$\Rightarrow \quad V = 0.6 \times 2 \times 10^{-2}$$

$$\Rightarrow \quad V = 12 \text{ mV}$$

Hence, the correct answer is (A).

18. Since, $r = \frac{mv}{Bq} = \frac{\sqrt{2mqV}}{Bq}$

$$\Rightarrow \quad r = \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times V}}{B\sqrt{q}}$$

$$\Rightarrow \quad r = \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times 500}}{100 \times 10^{-3}\sqrt{1.6 \times 10^{-19}}}$$

$$\Rightarrow \quad r = \frac{1}{100 \times 10^{-3}}\frac{\sqrt{2 \times 9.1 \times 500 \times 10^{-12}}}{1.6}$$

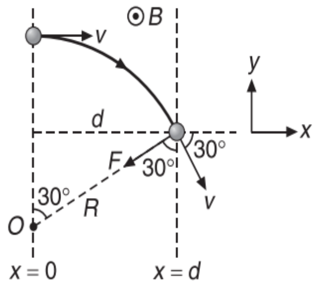
$$\Rightarrow \quad r = \frac{75.4 \times 10^{-6}}{100 \times 10^{-3}} = 7.5 \times 10^{-4} \text{ m}$$

Hence, the correct answer is (D).

19. *The data given in the question is incorrect.*

However, if the magnetic field $\vec{B} = B\hat{k}$ given in the question would have been between $x = 0$ and $x = d$ instead of $y = 0$ and $y = d$, then we solve the question as given below but we still do not get the matching answer.

So, let us solve the problem assuming that the particle enters the magnetic field that exists in the region $x = 0$ to $x = d$. On entering the magnetic field, the charge particle follows a circular path of radius $R = \frac{mv}{qB}$ as shown in Figure.



Deviation suffered by the charged particle when it leaves the field at $x = d$ is

$$\sin\theta = \frac{d}{R} = \frac{mv/2qB}{mv/qB} = \frac{1}{2}$$

$\Rightarrow \quad \theta = 30°$

At the point of leaving the field, force acting on the particle is towards the centre of the circle i.e. O and has a magnitude given by

$$F = qvB$$

This force is making an angle of $30°$ with the vertical, so

$$\vec{F} = -qvB\left(\sin 30°\hat{i} + \cos 30°\hat{j}\right)$$

$$\Rightarrow \quad \vec{F} = -\frac{qvB}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$$

$$\Rightarrow \quad \vec{a} = \frac{\vec{F}}{m} = -\frac{qvB}{2m}\left(\hat{i} + \sqrt{3}\hat{j}\right)$$

*So, none of the OPTIONS given is correct.

20. The straight path from origin to $P(x = 1, y = 1)$ is $y = x$

Since work is only done by electric force, so we have

$$W = q\int \vec{E}\cdot d\vec{r} = q\int_0^1 2dx + q3\int_0^1 dy$$

$\Rightarrow \quad W = 2q + 3q = 5q$

Hence, the correct answer is (B).

21. Since, $r = \dfrac{mv}{qB} = \dfrac{\sqrt{2m \times (qV)}}{qB}$

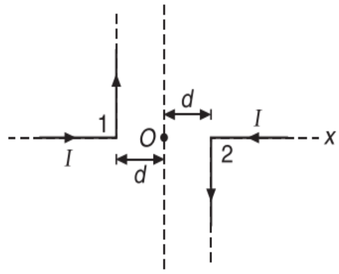
$$\Rightarrow \quad r \propto \sqrt{\frac{m}{q}}$$

$$\Rightarrow \quad \frac{r_p}{r_\alpha} = \sqrt{\frac{m_p}{m_\alpha} \times \frac{q_\alpha}{q_p}}$$

$$\Rightarrow \quad \frac{r_p}{r_\alpha} = \sqrt{\frac{1}{4} \times \frac{2}{1}} = \frac{1}{\sqrt{2}}$$

Hence, the correct answer is (D).

22. At O, $\vec{B} = \vec{B}_1 + \vec{B}_2$



where, $\vec{B}_1 = \vec{B}_2 = \dfrac{\mu_0 I}{4\pi d}$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi d} = 10^{-4}$$

$$\Rightarrow \quad \frac{2\times 10^{-7} \times I}{4 \times 10^{-2}} = 10^{-4}$$

$$\Rightarrow \quad I = \frac{2}{10^{-1}} = 20 \text{ A}$$

Hence, the correct answer is (B).

23. Radius of circular path followed by a charged particle in a uniform magnetic field (B) is given by

$$r = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2mK}}{qB}$$

For electron, $r_e = \dfrac{\sqrt{2m_e K}}{eB}$

For proton, $r_p = \dfrac{\sqrt{2m_p K}}{eB}$

For α particle, $r_\alpha = \dfrac{\sqrt{2m_\alpha K}}{2eB} = \dfrac{\sqrt{2(4m_p K)}}{2eB} = \dfrac{\sqrt{2m_p K}}{eB}$

Since, $m_p > m_e$, hence $r_\alpha = r_p > r_e$

Hence, the correct answer is (B).

24. Initially, the dipole moment of circular loop is

$$M = IA = I\left(\pi R^2\right)$$

and magnetic field is

$$B_1 = \frac{\mu_0 I}{2R}$$

Finally, the dipole moment becomes double, keeping the current constant, so radius of the loop must have become $\sqrt{2}R$. Hence the magnetic field is given is

$$B_2 = \frac{\mu_0 I}{2\left(\sqrt{2}R\right)} = \frac{B_1}{\sqrt{2}}$$

CHAPTER 1

$$\Rightarrow \frac{B_1}{B_2} = \sqrt{2}$$

Hence, the correct answer is (C).

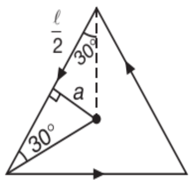
25. Required magnetic field is given by

$$B = 2\left(\frac{\mu_0 NIR^2}{2\left(R^2 + \frac{R^2}{4}\right)^{\frac{3}{2}}}\right) = \frac{\mu_0 NIR^2}{\frac{5^{\frac{3}{2}}}{8}} = \frac{8\mu_0 NI}{5^{\frac{3}{2}} R}$$

Hence, the correct answer is (C).

26. The magnetic field at the centre of the triangle is

$$B_{net} = 3\left[\frac{\mu_0 I}{4\pi a}(\cos 30° + \cos 30°)\right]$$



Since, from the figure, we see that

$$\tan 30° = \frac{a}{\left(\frac{l}{2}\right)}$$

$$\Rightarrow a = \frac{l}{2}\tan 30° = \frac{l}{2\sqrt{3}}$$

$$\Rightarrow B_{net} = \frac{3 \times (10^{-7}) \times 1}{\frac{(4.5 \times 10^{-2})}{(2\sqrt{3})}}\left(2 \times \frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow B_{net} = \frac{2 \times 9 \times 10^{-5}}{4.5} = 4 \times 10^{-5} \text{ Wbm}^{-2}$$

Hence, the correct answer is (A).

27. Magnetic moment is given by $M = iA = \frac{q}{T}(\pi r^2)$

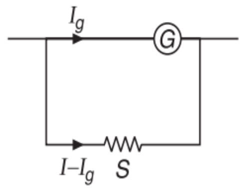
Also, $T = \frac{2\pi}{\omega}$

$$\Rightarrow M = \frac{q}{\left(\frac{2\pi}{\omega}\right)}(\pi r^2)$$

$$\Rightarrow M = \frac{1}{2} q\omega r^2$$

Hence, the correct answer is (A).

28. Since we know that



$$I_g G = (I - I_g)S$$

As the current through galvanometer is very small, so we have

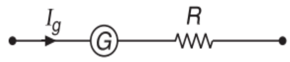
$$S \approx \frac{10^{-3} \times 25}{2}$$

$$\Rightarrow S \approx 12.5 \times 10^{-3} = 1.25 \times 10^{-2}\ \Omega$$

Hence, the correct answer is (B).

29. Given that $I_g = 5$ mA, $G = 15\ \Omega$

Let R be the resistance to be connected in series with the galvanometer as shown in Figure.



Since $V = I_g(R + G)$

$$\Rightarrow 10 = 5 \times 10^{-3}(R + 15)$$

$$\Rightarrow 2000 = R + 15$$

$$\Rightarrow R = 1985\ \Omega = 1.985 \times 10^3\ \Omega$$

Hence, the correct answer is (A).

30. Here, $\vec{B} = B_0(\hat{i} + 2\hat{j} - 4\hat{k})$, $\vec{v} = v_0(3\hat{i} - \hat{j} + 2\hat{k})$

Since, $\vec{F} = \vec{F}_e + \vec{F}_m = \vec{0}$

$$\Rightarrow \vec{F}_e = -\vec{F}_m$$

$$\Rightarrow \vec{F}_e = -q(\vec{v} \times \vec{B})$$

$$\Rightarrow \vec{F}_e = -qv_0B_0\left[(3\hat{i} - \hat{j} + 2\hat{k}) \times (\hat{i} + 2\hat{j} - 4\hat{k})\right]$$

$$\Rightarrow \vec{F}_e = -qv_0B_0(14\hat{j} + 7\hat{k})$$

The electric field produced by the charge q, will be given by

$$\vec{E} = \frac{\vec{F}_e}{q} = -\frac{qv_0B_0(14\hat{j} + 7\hat{k})}{q}$$

$$\Rightarrow \vec{E} = -v_0B_0(14\hat{j} + 7\hat{k})$$

Hence, the correct answer is (A).

31. When a magnetic dipole of dipole moment M is placed in a uniform magnetic field, it will experience a torque,

$$\tau = MB\sin\theta$$

Torque is maximum when $\theta = 90°$

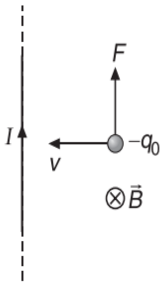
$$\tau_{max} = MB\sin 90° = MB$$

Potential Energy of a magnetic dipole in a uniform magnetic field is,

$$U = -MB\cos\theta = -MB\cos 90° = 0$$

Hence, the correct answer is (A).

32. Given situation is shown in Figure.



Since we know that

$$\vec{F} = q(\vec{v}\times\vec{B})$$

$$\Rightarrow \quad \vec{F} = -q_0(\vec{v}\times\vec{B})$$

According to the problem, force acting on the electron is parallel to the direction of current. Hence the motion of test charge is towards the wire.

Hence, the correct answer is (D).

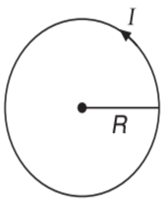
33. Magnetic potential energy of the dipole in a magnetic field is

$$U = -\vec{M}\cdot\vec{B}$$

For stable equilibrium, M and B must be parallel to each other and in this case the potential energy should be minimum.

Hence, the correct answer is (D).

34. Wire A is bent into a circle of radius R as shown, then

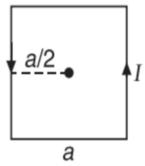


$$l = 2\pi R$$

$$\Rightarrow \quad R = \frac{l}{2\pi}$$

$$\Rightarrow \quad B_A = \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{2\times\left(\frac{l}{2\pi}\right)} = \frac{\mu_0\pi I}{l}$$

Wire B is bent into a square of side a as shown, then



$$l = 4a$$

$$\Rightarrow \quad a = \frac{l}{4}$$

$$\Rightarrow \quad B_B = 4\times\left[\frac{\mu_0 I}{4\pi\left(\frac{a}{2}\right)}(\sin 45° + \sin 45°)\right]$$

$$\Rightarrow \quad B_B = \frac{2\mu_0 I}{\pi a}\times\frac{2}{\sqrt{2}} = \frac{16\mu_0 I}{\sqrt{2}\pi l}$$

$$\Rightarrow \quad \frac{B_A}{B_B} = \frac{\mu_0\pi\left(\frac{I}{l}\right)}{16\mu_0\left(\frac{I}{\sqrt{2}\pi l}\right)} = \frac{\pi^2}{8\sqrt{2}}$$

Hence, the correct answer is (D).

35. Given that $i_g = 1\text{ mA}$, $G = 100\ \Omega$, $i = 10\text{ A}$, $S = ?$

Since, $(i - i_g)S = i_g G$

$$\Rightarrow \quad S = \frac{i_g G}{i - i_g} = \frac{1\times 10^{-3}\times 100}{(10 - 10^{-3})}$$

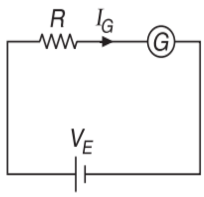
$$\Rightarrow \quad S \approx 10^{-2}\ \Omega$$

$$\Rightarrow \quad S = 0.01\ \Omega$$

Hence, the correct answer is (A).

36. **CASE-1:**

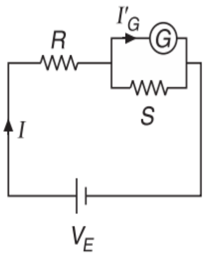
This situation is shown in Figure.



$$I_G = \frac{V_E}{R+G} \qquad \ldots(1)$$

CASE-2:

In this case a shunt is connected across the galvanometer as shown in Figure.



The current in the circuit is given by

$$I = \frac{V_E}{R + \frac{GS}{G+S}} \quad \text{...(2)}$$

According to the problem,

$$I'_G = \frac{I_G}{2} = \frac{IS}{G+S} \quad \text{...(3)}$$

From (1), (2) and (3), we get

$$\frac{V_E}{2(R+G)} = \frac{V_E}{R + \frac{GS}{G+S}} \times \frac{S}{G+S}$$

$$\Rightarrow \frac{1}{2(R+G)} = \frac{(G+S)}{(RG+RS+GS)} \times \frac{S}{(G+S)}$$

$$\Rightarrow RG + RS + GS = 2RS + 2GS$$

$$\Rightarrow RS + GS = RG$$

$$\Rightarrow S(R+G) = RG$$

Hence, the correct answer is (A).

37. Current in the circuit without ammeter is

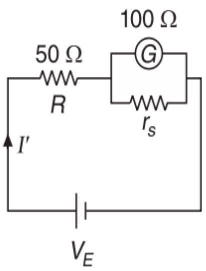
$$I = \frac{V}{R} = \frac{5\text{ V}}{50\ \Omega} = 0.1\text{ A}$$

So, the allowed current with ammeter is

$$I' = 0.099\text{ A}$$

Also, $I' = \frac{V}{R_{eq}}$ where $R_{eq} = 50 + \frac{100r_s}{100+r_s}$

$$\Rightarrow 0.099 = \frac{5}{50 + \frac{100r_s}{100+r_s}}$$



$$\Rightarrow 50 + \frac{100r_s}{100+r_s} = \frac{5}{0.099}$$

$$\Rightarrow \frac{100r_s}{100+r_s} = 0.5$$

$$\Rightarrow 100r_s = 50 + 0.5r_s$$

$$\Rightarrow r_s = \frac{50}{99.5} = 0.5\ \Omega$$

Hence, the correct answer is (D).

38. Magnetic force on electron in the metal sheet,

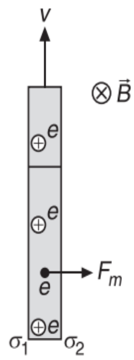
$$\vec{F}_m = -e(\vec{v} \times \vec{B})$$

At equilibrium,

$$F_m = F_e = eE \text{ (induced)}$$

Since $\sigma_2 = -\sigma_1$ and assuming $\sigma_1 = \sigma$, we get

$$evB = e\left(\frac{\sigma}{\varepsilon_0}\right)$$

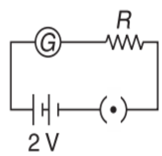


$$\Rightarrow \sigma = \sigma_1 = \varepsilon_0 vB$$

$$\Rightarrow \sigma_2 = -\sigma_1 = -\varepsilon_0 vB$$

Hence, the correct answer is (B).

39. The circuit given in the question is incorrect. The given figure shows the correct circuit.



Let the current which produces full scale deflection in the galvanometer be I_g.

Then according to question

$$\frac{4}{5}I_g = \frac{V}{G+R} = \frac{2}{G+2400} \quad \text{...(1)}$$

$$\frac{2}{5}I_g = \frac{2}{G+4900} \quad \text{...(2)}$$

From equations (1) and (2)

$$\frac{4}{2}=\frac{G+4900}{G+2400}$$

$\Rightarrow \quad G=100\ \Omega$

Putting G in equation (1)

$$\frac{4}{5}I_g=\frac{2}{100+2400}$$

$$\Rightarrow \quad I_g=\frac{2\times 5}{4\times 2500}=1\text{ mA}$$

For a deflection of 10 divisions

$$\frac{1}{5}I_g=\frac{V}{G+R}$$

$$\Rightarrow \quad \frac{1}{5}\times 10^{-3}=\frac{2}{100+R}$$

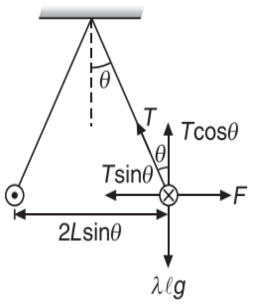
$\Rightarrow \quad R=9900\ \Omega$

Now, current sensitivity is

$$\frac{I_g}{n}=\frac{1\text{ mA}}{50\text{ div}}=20\ \mu\text{A/division}$$

Hence, the correct answer is (A).

40. Let the length of right wire be ℓ, then its mass is $\lambda\ell$.



Forces acting on this wire are tension (T), weight $(\lambda\ell g)$ and force of repulsion due to other wire (F).

From figure, we get

$$T\cos\theta=\lambda\ell g \qquad \ldots(1)$$

$$T\sin\theta=F \qquad \ldots(2)$$

where, $F=\dfrac{\mu_0}{2\pi}\dfrac{I^2\ell}{(2L\sin\theta)}$

From (2), we get

$$T\sin\theta=\frac{\mu_0}{2\pi}\frac{I^2\ell}{(2L\sin\theta)}$$

From (1), we get

$$\Rightarrow \quad \frac{\lambda\ell g}{\cos\theta}\sin\theta=\frac{\mu_0}{2\pi}\frac{I^2\ell}{(2L\sin\theta)}$$

$$\Rightarrow \quad I=\sqrt{\frac{4\pi L\lambda g\sin^2\theta}{\mu_0\cos\theta}}=2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$$

Hence, the correct answer is (D).

41. Since, $I=12\text{ A}$, $\vec{B}=0.3\hat{k}\text{ T}$

Also, $A=10\times 5\text{ cm}^2=50\times 10^{-4}\text{ m}^2$

$\Rightarrow \quad \vec{M}=(IA)\hat{n}=12\times 50\times 10^{-4}\hat{n}\text{ Am}^2$

$\Rightarrow \quad M=6\times 10^{-2}\text{ Am}^2$

Here, $\vec{M}_1=6\times 10^{-2}\hat{i}\text{ Am}^2$, $\vec{M}_2=6\times 10^{-2}\hat{k}\text{ Am}^2$

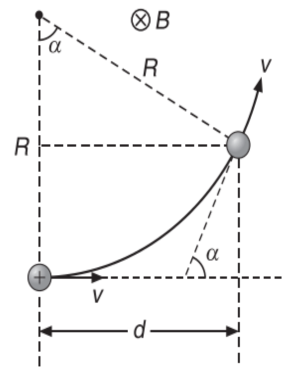
$\vec{M}_3=-6\times 10^{-2}\hat{j}\text{ Am}^2$, $\vec{M}_4=-6\times 10^{-2}\hat{k}\text{ Am}^2$

Since $\vec{M}_2$ is parallel to $\vec{B}$, it means potential energy is minimum, therefore in orientation (ii) the loop is in stable equilibrium. $\vec{M}_4$ is antiparallel to $\vec{B}$, it means potential energy is maximum, therefore in orientation (iv) the loop is in unstable equilibrium.

Hence, the correct answer is (A).

43. Energy of proton is $\frac{1}{2}mv^2=qV$

$$\Rightarrow \quad v=\sqrt{\frac{2qV}{m}}$$



Magnetic force, $qvB\sin 90^\circ=\dfrac{mv^2}{R}$

$$\Rightarrow \quad R=\frac{mv}{qB}$$

Since, $\sin\alpha=\dfrac{d}{R}=\dfrac{qBd}{mv}=\dfrac{qBd}{m}\sqrt{\dfrac{m}{2qV}}$

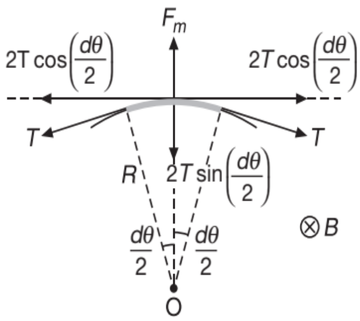
$$\Rightarrow \quad \sin\alpha=Bd\sqrt{\frac{q}{2mV}}$$

Hence, the correct answer is (C).

44. When the currents are parallel, $I_1 I_2$ is positive and the force between them is attractive (i.e. negative). Similarly, when currents are antiparallel, $I_1 I_2$ is negative and the force between them is repulsive (i.e. positive). So, option (D) satisfies the condition.

Hence, the correct answer is (D).

45.



Magnetic force on the circular arc is $F_m = BId\ell$

$$F_m = BI(Rd\theta)$$

For the arc to be in equilibrium, $F = 2T\sin\left(\frac{d\theta}{2}\right)$

For small angle $d\theta$, $\sin\left(\frac{d\theta}{2}\right) \approx \frac{d\theta}{2}$

$$\Rightarrow \quad 2Td\theta = BIRd\theta$$

$$\Rightarrow \quad T = BIR$$

Hence, the correct answer is (A).

46. Force on conductor, 0.325 m

$+y \quad a_y = -9.8\ \text{ms}^{-2}$

$y - \quad v_y = 0$ along $v_{0y} = ?$

Work done $v_{''}^2$: on the conductor in moving along

$v_{''}$ – is

$$v_{0y} = 2.52\ \text{ms}^{-1} \simeq 2.5\ \text{ms}^{-1}$$

$+y$

$v_{0y} = 0$

$y -$

So, average power is $v_y = +2.52$ m

$a_{''}$: $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$$\sum F_{''} = ma_{''}$$

Hence, the correct answer is (C).

48. The radius of the circular path of a charged particle in the magnetic field is given by $r = \frac{mv}{Bq}$

Kinetic energy of a charged particle is

$$K = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2K}{m}}$$

$$\Rightarrow \quad r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2Km}}{qB}$$

Since K and B are constants, so we have

$$r \propto \frac{\sqrt{m}}{q}$$

$$r_p : r_d : r_\alpha = \frac{\sqrt{m_p}}{q_p} : \frac{\sqrt{m_d}}{q_d} : \frac{\sqrt{m_\alpha}}{q_\alpha}$$

$$\Rightarrow \quad r_p : r_d : r_\alpha = \frac{\sqrt{m}}{e} : \frac{\sqrt{2m}}{e} : \frac{\sqrt{4m}}{2e} = 1 : \sqrt{2} : 1$$

$$\Rightarrow \quad r_\alpha = r_p < r_d$$

Hence, the correct answer is (A).

49. $dB = \frac{\mu_0(dq)}{2r}\left(\frac{\omega}{2\pi}\right)$

$$\Rightarrow \quad B = \int dB = \frac{\mu_0\omega}{4\pi}\left(\frac{Q}{\pi R^2}\right)2\pi\int_0^R \frac{rdr}{r}$$

$$\Rightarrow \quad B = \left(\frac{\mu_0\omega Q}{2\pi R^2}\right)R = \frac{\mu_0\omega Q}{2\pi R}$$

$$\Rightarrow \quad B \propto \frac{1}{R}$$

Hence, the correct answer is (D).

50. Let the current I in the wire be inwards then

$$\lambda = \frac{I}{\pi R}$$

If dI be an infinitesimal current due to an element subtending angle $d\theta$ at centre, then

$$dI = \lambda d\ell = \lambda(Rd\theta)$$

Magnetic field dB due to this infinitesimal element is

$$dB = \frac{\mu_0 dI}{2\pi R}$$

$$B_x = \int dB_x = \int dB\sin\theta$$

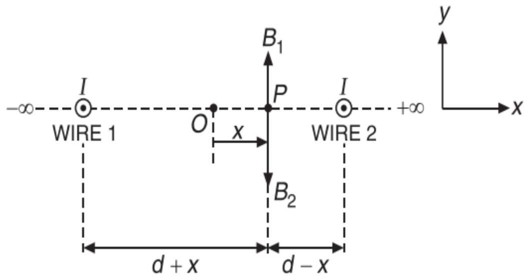
$$\Rightarrow \quad B_x = \frac{\mu_0 \lambda}{2\pi} \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \sin\theta d\theta = 0$$

$$\text{and } B_y = \frac{\mu_0 \pi}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta d\theta = \frac{\mu_0 \lambda}{\pi} = \frac{\mu_0 I}{\pi^2 R}$$

Hence, the correct answer is (A).

51. Taking upward direction as positive, the magnetic field at a point P between the wires at point P to the right of origin at a distance x from O is

$$B_P = \frac{\mu_0 I}{2\pi}\left(\frac{1}{d-x} - \frac{1}{d+x}\right)$$



The field will remain negative as we move from $-\infty$ to Wire 1, has a zero value at ∞ and becomes very large near the Wire 1.
As we move from Wire 1 to Wire 2 the field changes sign from positive to negative, becoming zero at O and then again increasing as we approach Wire 2.
As we move from Wire 2 to $+\infty$, field is positive everywhere, very large near Wire 2 and then decreases to zero at $+\infty$.
Hence, the correct answer is (B).

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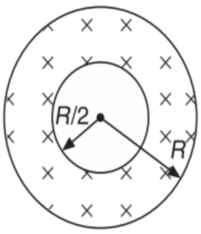
Single Correct Choice Type Problems

1.



Let r be the distance of a point from centre, then

For $r \le \frac{R}{2}$, using Ampere's circuital law,

$$\oint \vec{B} \cdot dl$$

$$\Rightarrow \quad Bl = \mu_0 (I_{\text{in}})$$

$$\Rightarrow \quad B(2\pi r) = \mu_0 (I_{\text{in}})$$

$$\Rightarrow \quad B = \frac{\mu_0}{2\pi} \frac{I_{\text{in}}}{r} \qquad \ldots(1)$$

Since, $I_{\text{in}} = 0$

$$\Rightarrow \quad B = 0$$

If J be the current per unit area, then

for $\frac{R}{2} \le r \le R$, we have

$$I_{\text{in}} = \left[\pi r^2 - \pi\left(\frac{R}{2}\right)^2\right] J$$

Substituting in equation (1), we have

$$B = \frac{\mu_0}{2\pi} \frac{\left(\pi r^2 - \pi \frac{R^2}{4}\right) J}{r}$$

$$\Rightarrow \quad B = \frac{\mu_0 J}{2r}\left(r^2 - \frac{R^2}{4}\right)$$

At $r = \frac{R}{2}$, $B = 0$

At $r = R$, $B = \frac{3\mu_0 JR}{8}$

For $r \ge R$, we have

$$I_{\text{in}} = I_{\text{Total}} = I(\text{say})$$

Therefore, substituting in equation (1), we have

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\Rightarrow \quad B \propto \frac{1}{r}$$

Hence, the correct answer is (D).

2. Area A of the given loop is equal to area of two circles of radius $\frac{a}{2}$ and area of a square of side a

$$\Rightarrow \quad A = 2\pi\left(\frac{a}{2}\right)^2 + a^2 = \left(\frac{\pi}{2} + 1\right) a^2$$

Since, $|\vec{M}| = IA = \left(\frac{\pi}{2}+1\right)a^2 I$

From Right Hand Thumb Rule, direction of $\vec{M}$ is outwards or in positive z-direction.

$$\Rightarrow \quad \vec{M} = \left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$$

Hence, the correct answer is (B).

3. The radial width $(b-a)$ is having N turns. So, number of turns per unit radial width is $n = \frac{N}{b-a}$.

Consider a circular coil of radius x, radial thickness dx. If dN is the number of turns in it, then we have

$$dN = \frac{N dx}{b-a}$$

If dB is the field due to this element at the centre, then

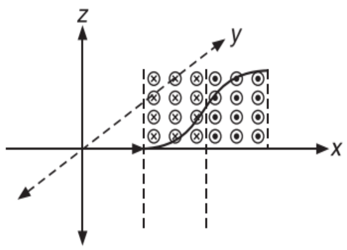
$$dB = \frac{\mu_0 NI\, dx}{2(b-a)x}$$

$$\Rightarrow \quad B = \int_a^b dB = \frac{\mu_0 NI}{2(b-a)}\log_e\left(\frac{b}{a}\right)$$

Hence, the correct answer is (A).

4. When $a < x < 2a$

$\vec{F}_m$ acts along positive z-direction



When $2a < x < 3a$

$\vec{F}_m$ acts along negative z-direction.

Hence, the correct answer is (A).

5. Magnetic force does not change the speed of charged particle. Hence $v = u$. Further magnetic field in the given condition is along negative z-axis in the starting. Or it describes a circular path in clockwise direction. The direction is found by using Fleming's Left Hand Rule. Hence when it exits from the field, $y < 0$.
Hence, the correct answer is (D).

6. $U = -\vec{M}\cdot\vec{B}$

Hence, the correct answer is (C).

7. Because of magnetic force acting radially outwards on the loop, it has a tendency to expand.
Hence, the correct answer is (B).

8. Plane of motion must be perpendicular to at least one of the component of the magnetic field.
Hence, the correct answer is (B).

9. $r > (b-a)$

$$\Rightarrow \quad \frac{mv}{qB} > (b-a)$$

$$\Rightarrow \quad v > \frac{q(b-a)B}{m}$$

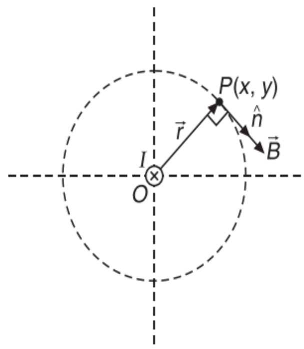
$$\Rightarrow \quad v_{\min} = \frac{q(b-a)B}{m}$$

Hence, the correct answer is (B).

10. Outside a bar magnet, field lines go from N to S, whereas inside a magnet they go from S to N. Also, we note that, magnetic field lines can form closed loops.
Hence, the correct answer is (D).

11. In magnitude, field at point $P(x, y)$ is

$$B = \frac{\mu_0 I}{2\pi r}$$



However, vectorially the field is given by

$$\vec{B} = \left(\frac{\mu_0 I}{2\pi r}\right)\hat{n}$$

where $\hat{n}$ is a unit vector perpendicular to $Id\vec{l}$ as well as $\vec{r}$.

In this case, $\vec{r} = x\hat{i} + y\hat{j}$

A unit vector perpendicular to $Id\vec{l}$ as well as $\vec{r}$ is given by

$$\hat{n} = \frac{y\hat{i} - x\hat{j}}{\sqrt{x^2+y^2}}$$

Because only then, we have $\vec{r}\cdot\hat{n} = 0$

$$\Rightarrow \quad \vec{B} = \left(\frac{\mu_0 I}{2\pi\sqrt{x^2+y^2}}\right)\left(\frac{y\hat{i} - x\hat{j}}{\sqrt{x^2+y^2}}\right)$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{2\pi}\left(\frac{y\hat{i} - x\hat{j}}{x^2+y^2}\right)$$

Hence, the correct answer is (A).

12. Since $r_A > r_B$

$$\Rightarrow \quad m_A v_A > m_B v_B \qquad \left\{\because r = \frac{mv}{qB}\right\}$$

Hence, the correct answer is (B).

13. The radial width $(b-a)$ of the coil is having N turns.

So, number of turns per unit radial width is $n = \dfrac{N}{b-a}$.

Consider a circular coil of radius x, radial thickness dx. If dN is the number of turns in it, then we have

$$dN = \frac{Ndx}{b-a}$$

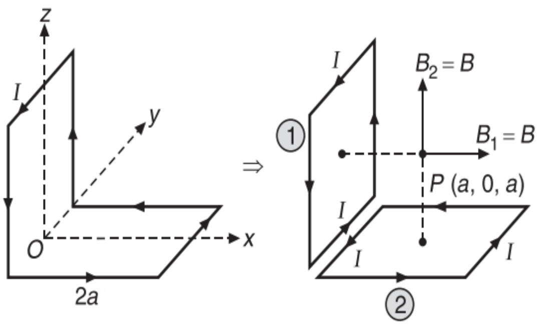
If dB is the field due to this element at the centre, then

$$dB = \frac{\mu_0 NI\,dx}{2(b-a)x}$$

$$\Rightarrow \quad B = \int_a^b dB = \frac{\mu_0 NI}{2(b-a)}\log_e\left(\frac{b}{a}\right)$$

Hence, the correct answer is (C).

14. Consider the bigger loop to be made up of two loops 1 and 2 as shown in Figure.



Magnetic field due to loop 1 and 2 at point P has same value say B. So,

$$\vec{B}_P = B\hat{i} + B\hat{k} = B(\hat{i} + \hat{k})$$

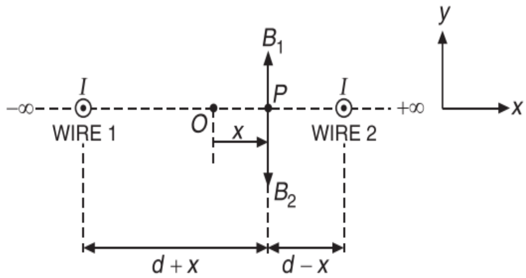
So, magnetic field points along the direction $\hat{n}$ given by

$$\hat{n} = \frac{\vec{B}_P}{|\vec{B}_P|} = \frac{B(\hat{i}+\hat{k})}{\sqrt{2}B} = \frac{\hat{i}+\hat{k}}{\sqrt{2}}$$

Hence, the correct answer is (D).

15. Taking upward direction as positive, the magnetic field at a point P between the wires at point P to the right of origin at a distance x from O is

$$B_P = \frac{\mu_0 I}{2\pi}\left(\frac{1}{d-x} - \frac{1}{d+x}\right)$$



The field will remain negative as we move from $-\infty$ to Wire 1, has a zero value at ∞ and becomes very large near the Wire 1.

As we move from Wire 1 to Wire 2 the field changes sign from positive to negative, becoming zero at O and then again increasing as we approach Wire 2.

As we move from Wire 2 to $+\infty$, field is positive everywhere, very large near Wire 2 and then decreases to zero at $+\infty$.

Hence, the correct answer is (B).

16. Magnetic moment $= iA = \dfrac{q}{(2\pi/\omega)}$

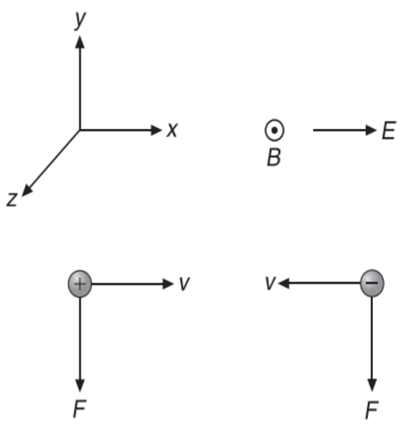
$$\Rightarrow \quad M = \frac{1}{2}q\omega r^2$$

Further angular momentum $L = I\omega = mr^2\omega$

$$\Rightarrow \quad \frac{M}{L} = \frac{q}{2m}$$

Hence, the correct answer is (C).

17. According to Fleming's left-hand rule, both positive and negative experience force along $-y$ axis.



Hence, the correct answer is (C).

18. $H_1 = \dfrac{\mu_o I}{4\pi(QM)}$ and

$$H_2 = \frac{\mu_o I}{4\pi(QM)} + \frac{\mu_o(I/2)}{4\pi(QM)}$$

$\Rightarrow \quad H_2 = H_1 + \frac{1}{2}H_1$

$\Rightarrow \quad H_1 = \left(1+\frac{1}{2}\right)H_1 = \frac{3}{2}H_1$

$\Rightarrow \quad \frac{H_2}{H_1} = \frac{2}{3}$

Hence, the correct answer is (C).

19. Total magnetic flux passing through whole of the x-y plane will be zero, because magnetic lines form a closed loop. So as many lines will move in $-z$ direction same will return to $+z$ direction from the x-y plane.
Hence, the correct answer is (D).

20.

Since the charged particle is at rest, so initially, the magnetic field will not make it move. But initially, the charged particle will experience an electrostatic force in the direction of electric field as a result of which it gains a velocity parallel to E or parallel to B and hence even after the motion of the particle it will not experience a magnetic force i.e. its trajectory is a straight line.
Hence, the correct answer is (A).

21. Since both wires carry currents in opposite directions, so B at midpoint of the two wires is normal to the plane containing the wires and hence no net magnetic force is exerted on the charge q.

Hence, the correct answer is (D).

22. Magnetic Moment $M = iA$

$\Rightarrow \quad M = \frac{2q}{\left(\frac{2\pi}{\omega}\right)}(\pi R^2)$

$\Rightarrow \quad M = q\omega R^2$

Further, angular momentum L is

$L = I\omega$, where

I = Moment of Inertia of system

$\Rightarrow \quad L = (mR^2 + mR^2)\omega$

$\Rightarrow \quad L = 2mR^2\omega$

$\Rightarrow \quad \frac{M}{L} = \frac{q\omega R^2}{2mR^2\omega}$

$\Rightarrow \quad \frac{M}{L} = \frac{q}{2m}$

Hence, the correct answer is (A).

23. Radius of the circular path is given by

$$r = \frac{mv}{Bq} = \frac{\sqrt{2Km}}{Bq}$$

Here, K is the kinetic energy to the particle.

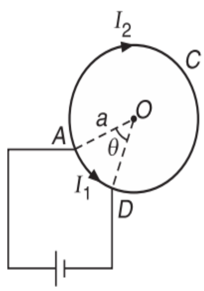
Therefore, $r \propto \frac{\sqrt{m}}{q}$ if K and B are same.

$\Rightarrow \quad r_p : r_d : r_\alpha = \frac{\sqrt{1}}{1} : \frac{\sqrt{2}}{1} : \frac{\sqrt{4}}{2} = 1 : \sqrt{2} : 1$

Hence, $r_\alpha = r_p < r_d$

Hence, the correct answer is (A).

24. For a current flowing in the circular arc of radius a shown in Figure the magnetic induction at the centre is



$$B = \left(\frac{\mu_o I_1}{4\pi a}\right)\theta - \frac{\mu_0 I_2}{4\pi a}(2\pi - \theta) = \frac{\mu_0}{4\pi a}\left(I_1\theta - I_2(2\pi - \theta)\right)$$

$\Rightarrow \quad B \propto I\theta$ ...(1)

In the given problem, the total current is divided into two arcs such that it divides in the inverse ratio of resistances of arcs i.e.

$$\frac{I_1}{I_2} = \frac{\lambda a(2\pi - \theta)}{\lambda a\theta}$$

$\Rightarrow \quad I_1\theta = I_2(2\pi - \theta)$

Substituting in equation (1), we get

$B = 0$

Hence, the correct answer is (D).

25. Using Ampere's Circuital Law over a circular loop of any radius less than the radius of the pipe, we can see that net current inside the loop is zero. Hence, magnetic field at every point inside the loop will be zero.
Hence, the correct answer is (B).

26. $R = \dfrac{\sqrt{2qVm}}{Bq}$

$\Rightarrow \quad R \propto \sqrt{m}$

$\Rightarrow \quad \dfrac{R_1}{R_2} = \sqrt{\dfrac{m_X}{m_Y}}$

$\Rightarrow \quad \dfrac{m_X}{m_Y} = \left(\dfrac{R_1}{R_2}\right)^2$

Hence, the correct answer is (C).

27. Force per unit length between two wires carrying currents I_1 and I_2 at distance r is given by

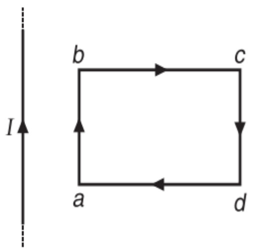
$$\frac{F}{\ell} = \frac{\mu_o}{2\pi}\frac{I_1 I_2}{r}$$

Here, $I_1 = I_2 = I$ and $r = b$

$\Rightarrow \quad \dfrac{F}{\ell} = \dfrac{\mu_o I^2}{2\pi b}$

Hence, the correct answer is (B).

28. Straight wire will produce a non-uniform field to the right of it. $\vec{F}_{bc}$ and $\vec{F}_{dc}$ will be calculated by integration but these two forces will cancel each other. Further force on wire ab will be towards the long wire and on wire cd will be away from the long wire. But since the wire ab is nearer to the long wire, force of attraction towards the long wire will be more. Hence, the loop will move towards the wire.



Hence, the correct answer is (C).

29. Magnetic force on a current carrying loop in uniform magnetic field is zero.
Hence, the correct answer is (D).

30. In non-uniform magnetic field, the needle will experience both a force and a torque.
Hence, the correct answer is (A).

Multiple Correct Choice Type Problems

1. Magnetic field due to loop at origin is

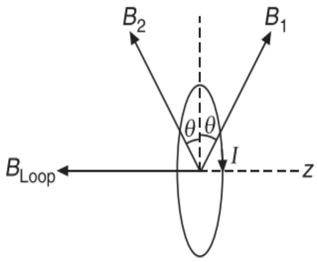
$$\frac{\mu_0 I R^2}{2.8R^3}(-\hat{k}) = \frac{\mu_0 I}{16R}(-\hat{k})$$

Magnetic field at origin due to wires

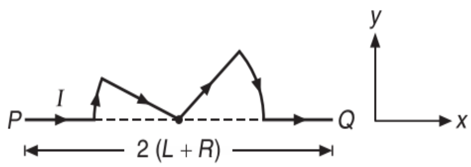
$$= \left(\frac{\mu_0 I_1}{2\pi R} - \frac{\mu_0 I_2}{2\pi R}\right)\hat{k}$$

(A) If $I_1 = I_2$ then $\vec{B}_0 = \dfrac{\mu_0 I}{16R}(-\hat{k})$

(B) It can be zero if $I_1 > 0$, $I_2 < 0$

(D) 

Hence, (A), (B) and (D) are correct.

2. 

Force on the complete wire equals the force on straight wire PQ carrying a current I.

$$\vec{F} = I\left(\overrightarrow{PQ} \times \vec{B}\right) = I\left[\left\{2(L+R)\hat{i}\right\} \times \vec{B}\right]$$

This force is zero, if $\vec{B}$ is along $\hat{i}$ direction or x-direction. If magnetic field is along $\hat{j}$ direction or $\hat{k}$ direction, then

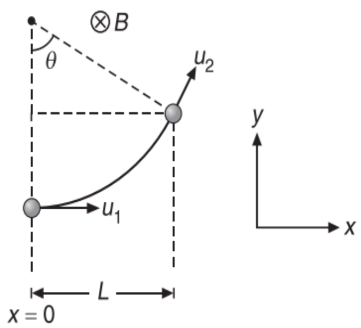
$$|\vec{F}| = F = (I)(2)(L+R)B\sin(90°)$$

$\Rightarrow \quad F = 2I(L+R)B$

$\Rightarrow \quad F \propto (L+R)$

Hence, (A), (B) and (C) are correct.

3. $\vec{u} = 4\hat{i}$ and $\vec{v} = 2\left(\sqrt{3}\hat{i} + \hat{j}\right)$



According to the figure, magnetic field should be in ⊗ direction, or along $-z$ direction.

Further, $\tan\theta = \dfrac{v_y}{v_x} = \dfrac{2}{2\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad \theta = 30^\circ$

$\Rightarrow \quad \dfrac{\pi}{6} =$ Angle of $\vec{v}$ with x-axis is $\theta = \dfrac{\pi}{6}$

Since $\theta = \omega t$, where $\omega = \dfrac{QB}{M}$ and $t = 10^{-2}$ s

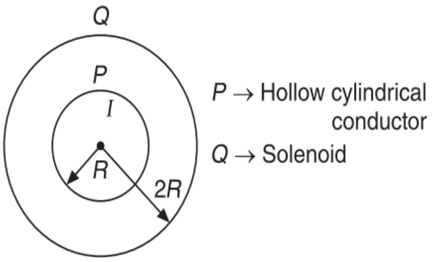
$\Rightarrow \quad \dfrac{\pi}{6} = \left(\dfrac{QB}{M}\right)t$

$\Rightarrow \quad B = \dfrac{\pi M}{6Qt} = \dfrac{\pi M}{6Q(10^{-2})}$

$\Rightarrow \quad B = \dfrac{50\pi M}{3Q}$

Hence, (A) and (C) are correct.

4.



In the region, $0 < r < R$

$B_P = 0$,

$B_Q \neq 0$, along the axis

So, $B_{\text{net}} \neq 0$

In the region, $R < r < 2R$

$B_P \neq 0$, tangential to the circle of radius r, centred on the axis.

$B_Q \neq 0$, along the axis.

So, $B_{\text{net}} \neq 0$ and is neither in the directions mentioned in options (B) or (C).

In region, $r > 2R$

$B_P \neq 0$

$B_Q \neq 0$

So, $B_{\text{net}} \neq 0$

Hence, (A) and (D) are correct.

5. For (A) and (B)

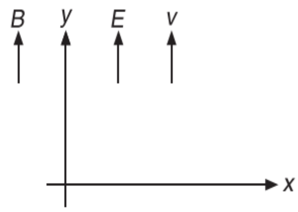
Magnetic field will rotate the particle in a circular path (in x-z plane or perpendicular to B). Electric field will exert a constant force on the particle in positive y-direction. Therefore, resultant path is neither purely circular nor helical, hence the options (A) and (B) both are incorrect.

For (C)

$v_\perp$ and $\vec{B}$ will rotate the particle in a circular path in x-z plane (or perpendicular to $\vec{B}$). Further $v_\parallel$ and $\vec{E}$ will move the particle (with increasing speed) along positive y-axis (or along the axis of above circular path). Therefore, the resultant path is helical with increasing pitch, along the y-axis (or along $\vec{B}$ and $\vec{E}$). Therefore, option (C) is correct.

For (D)

Magnetic force is zero, as θ between $\vec{B}$ and $\vec{v}$ is zero.



But electric force will act in y-direction. Therefore, motion is 1-D and uniformly accelerated (towards positive y-direction). So, option (D) is correct.

Hence, (C) and (D) are correct.

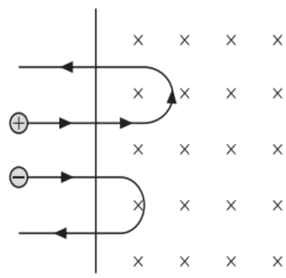
6. $r = \dfrac{mv}{Bq}$

$\Rightarrow \quad r \propto m$

$\Rightarrow \quad r_e < r_p$ because, $m_e < m_p$

Further, $T = \dfrac{2\pi m}{Bq}$

$\Rightarrow \quad T \propto m$



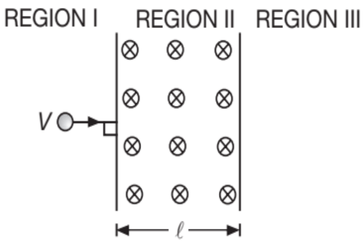
$\Rightarrow \quad T_e < T_p$

Since, $t_e = \dfrac{T_e}{2}$ and $t_p = \dfrac{T_p}{2}$

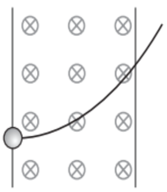
$\Rightarrow \quad t_e < t_p$

Hence, (B) and (D) are correct.

7. The radius of circle of path of charged particle is $R = \frac{mv}{qB}$



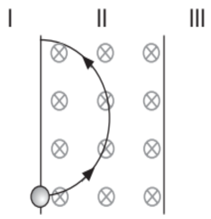
For particle to enter REGION III $R > \ell$ or $\frac{mv}{qB} > \ell$



For path length of particle in REGION II to be maximum

$\ell = R$

$\Rightarrow \quad V = \frac{q\ell B}{m}$



The period of revolution of charged particle is $\omega = \frac{qB}{m}$

The time spent in REGION II is $t = \frac{\pi}{\omega} = \frac{\pi m}{qB}$, which is same for all the cases when it returns to REGION II.

Hence, (A), (C) and (D) are correct.

8. Since, $\vec{F}_{BA} = 0$, because magnetic lines are parallel to this wire.

Also, $\vec{F}_{CD} = 0$, because magnetic lines are antiparallel to this wire.

Now, we see that $\vec{F}_{CB}$ is perpendicular to paper inwards and $\vec{F}_{AD}$ is perpendicular to paper outwards. These two forces (although calculated by integration) will cancel each other but produce a torque which tend to rotate the loop in clockwise direction about an axis OO'.

Hence, (A) and (C) are correct.

9. $r = \frac{mv}{qB} = \frac{\sqrt{2mE}}{qB}$

$\Rightarrow \quad r \propto \frac{\sqrt{m}}{q}$

$\Rightarrow \quad \text{Deflection} \propto \frac{q}{\sqrt{m}}$

Hence, (A) and (C) are correct.

10. By Work-Energy Theorem, we have

Work done = Change in K.E.

$\Rightarrow \quad (qE\hat{i})\cdot(2a\hat{i}) = \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2$

$\Rightarrow \quad 2aqE = \frac{3}{2}mv^2$

$\Rightarrow \quad E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$

Rate of work done by $\vec{E}$ at P is $\vec{F}\cdot\vec{v}$

$\Rightarrow \quad \frac{dW}{dt} = (qE\hat{i})\cdot(v\hat{i})$

$\Rightarrow \quad \frac{dW}{dt} = qEv = \frac{3}{4}\left(\frac{mv^3}{a}\right)$

Rate of work done by $\vec{E}$ at Q is zero, because at Q, $\vec{F} \perp \vec{v}$.

Hence, (A), (B) and (D) are correct.

11. No change in velocity implies no acceleration i.e. no net force is acting on the proton, even under the joint influence of electric and magnetic field. This thing is possible under the following situations.

OPTION (A):

$E = 0$, $B = 0$, i.e. no field exists in the region

OPTION (B):

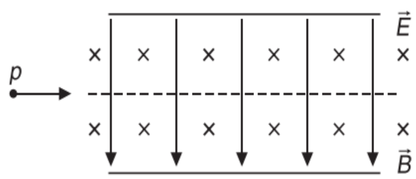
$E = 0$ i.e. no electrostatic force. $B \neq 0$, but the charge particle enters parallel to the field, so that net force equals to zero.

OPTION (C):

$E \neq 0$, i.e. the charged particle proton must experience an electrostatic force eE and hence must accelerate.

OPTION (D):

$E \neq 0$, $B \neq 0$ and both shown in Figure



Because in such a situation

$F_m = evB$ (upwards) and

$F_e = eE$ (downwards)

and if both are equal in magnitude then even the proton will suffer no change in its velocity and will continue to move along the dotted line as shown in Figure. In such a situation, the velocity is $v = \frac{E}{B}$.

Hence, (A), (B) and (D) are correct.

Reasoning Based Questions

1. $C\phi = BINA$

$\Rightarrow \quad \phi = \left(\frac{BNA}{C}\right)I$

Using iron core, value of magnetic field increases. So, deflection increases for same current. Hence sensitivity increases. So, Statement-1 is true.
Statement-2 is false as we know soft iron can be easily magnetized or demagnetized.
Hence, the correct answer is (C).

Linked Comprehension Type Questions

1. $F_M = Bev = Be\left(\frac{I}{nAe}\right) = \frac{BI}{nA}$ and $F_e = eE$

Since, $F_e = F_m$

$\Rightarrow \quad eE = \frac{BI}{nA}$

$\Rightarrow \quad E = \frac{B}{nAe}$

Also, $V = Ed$

$\Rightarrow \quad V = \left(\frac{BI}{nAe}\right)w = \frac{BIw}{n(wd)e} = \frac{BI}{ned}$

$\Rightarrow \quad \frac{V_1}{V_2} = \frac{d_2}{d_1}$

So, if $w_1 = 2w_2$ and $d_1 = d_2$ then

$V_1 = V_2$

Hence, (A) and (D) are correct.

2. Since, $V = \frac{BI}{ned}$

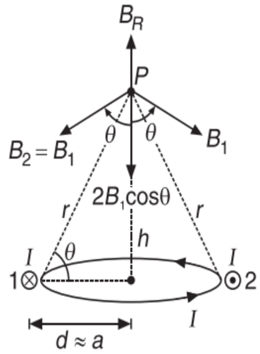
$\Rightarrow \quad \frac{V_1}{V_2} = \left(\frac{B_1}{B_2}\right)\left(\frac{n_2}{n_1}\right)$

If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$

If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$

Hence, (A) and (C) are correct.

3. Let the magnetic field due to the ring, wire 1 and wire 2 be B_R, B_1 and B_2 respectively.



In magnitudes, we have

$$B_1 = B_2 = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi\sqrt{a^2+h^2}}$$

Resultant of B_1 and B_2 is

$$2B_1\cos\theta = 2\left(\frac{\mu_0 I}{2\pi r}\right)\left(\frac{a}{r}\right) = \frac{\mu_0 I a}{\pi r^2}$$

Also, $B_R = \frac{\mu_0 I R^2}{2(R^2+x^2)^{\frac{3}{2}}} \approx \frac{2\mu_0 I\pi a^2}{4\pi r^3}$

Since $d \approx a$, so $a^2 + h^2 = r^2$

For zero magnetic field at P, we have

$$2B_1\cos\theta = B_R$$

$\Rightarrow \quad \frac{\mu_0 I a}{\pi r^2} = \frac{2\mu_0 I\pi a^2}{4\pi r^3}$

$\Rightarrow \quad \pi a = 2r = 2\sqrt{a^2+h^2}$

$\Rightarrow \quad \pi^2 a^2 = 4a^2 + 4h^2$

$\Rightarrow \quad \frac{(\pi^2-4)a^2}{4} = h^2$

$\Rightarrow \quad h^2 = 1.46a^2$

$\Rightarrow \quad h \approx 1.2a$

Hence, the correct answer is (C).

4. Magnetic field at mid-point of two wires is

$B = 2$ (magnetic field due to one wire)

$\Rightarrow \quad B = 2\left(\frac{\mu_0}{2\pi}\frac{I}{d}\right) = \frac{\mu_0 I}{\pi d} \otimes$

Magnetic moment of loop is

$$M = IA = I(\pi a^2)$$

Torque on loop is

$$\tau = MB\sin 150°$$

$$\Rightarrow \quad \tau = \frac{\mu_0 I^2 a^2}{2d}$$

Hence, the correct answer is (B).

5. If $B_2 > B_1$, critical temperature, (at which resistance of semiconductors abruptly becomes zero) in case-2 will be less than compared to case-1.
Hence, the correct answer is (A).

6. With increase in temperature, T_C is decreasing.

$$T_C(0) = 100\text{ K}$$

$$T_C = 75\text{ K at } B = 7.5\text{ T}$$

Hence, at $B = 5$ T, T_C should lie between 75 K and 100 K.

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. For particle to move in negative y-direction, either its velocity must be in negative y-direction (if initial velocity $\neq 0$) and force should be parallel to velocity or it must experience a net force in negative y-direction only (if initial velocity $= 0$)
Hence, the correct answer is (C).

2. $\vec{F}_{net} = \vec{F}_e + \vec{F}_m$

$$\Rightarrow \quad \vec{F}_{net} = q\vec{E} + q\vec{v} \times \vec{B}$$

For particle to move in straight line with constant velocity, we have

$$\vec{F}_{net} = \vec{0}$$

$$\Rightarrow \quad q\vec{E} + q\vec{v} \times \vec{B} = 0$$

Hence, the correct answer is (A).

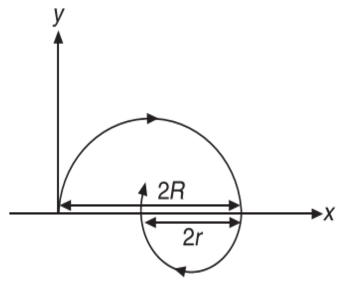
3. For path to be helix with axis along positive z-direction, particle should experience a centripetal acceleration in xy-plane.
For the given set of options, only option (C) satisfy the condition. Path is helical with increasing pitch.
Hence, the correct answer is (C).

4. Done already, see theory
A → (q, r)
B → (p)
C → (q, r)
D → (q)

5. A → (q)
B → (r, s)
C → (s)
D → (p, q, r)

Integer/Numerical Answer Type Questions

1. The particle will follow the path as shown



$$\text{Average speed} = \frac{\frac{2mv}{qB} + \frac{2mv}{4qB}}{\frac{\pi m}{qB} + \frac{\pi m}{4qB}} = 2.00\text{ ms}^{-1}$$

2. $\tau = BANi_m = k\theta$

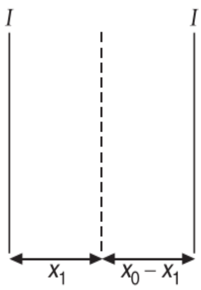
$$i_m = \frac{K\theta}{BAN} = \frac{10^{-4} \times 0.2}{0.02 \times 2 \times 10^{-4} \times 50}$$

$$\Rightarrow \quad i_m = \frac{0.2}{2} = 0.1\text{ A}$$

$$0.1 \times 50 = 0.9\text{ S}$$

$$\Rightarrow \quad S = \frac{50}{9}\ \Omega = 5.56\ \Omega$$

3.



When currents are in same direction, then

$$B_1 = \frac{\mu_0 I}{2\pi x_1} - \frac{\mu_0 I}{2\pi(x_0 - x_1)}$$

When currents are in opposite direction, then

$$B_2 = \frac{\mu_0 I}{2\pi x_1} + \frac{\mu_0 I}{2\pi(x_0 - x_1)}$$

Substituting $x_1 = \frac{x_0}{3}$, we get

$$B_1 = \frac{3\mu_0 I}{2\pi x_0} - \frac{3\mu_0 I}{4\pi x_0} = \frac{3\mu_0 I}{4\pi x_0}$$

$$\Rightarrow \quad R_1 = \frac{mv}{qB_1}$$

and $B_2 = \frac{9\mu_0 I}{4\pi x_0}$

$$\Rightarrow \quad R_2 = \frac{mv}{qB_2}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{B_2}{B_1} = \frac{9}{3} = 3$$

4. If B be the magnetic field due to the given cylinder, B_F be the magnetic field due to full cylinder and B_C be the magnetic field due to removed cylinder i.e. cavity, then

$$B = B_F - B_C$$

$$\Rightarrow \quad B = \frac{\mu_0 I_F}{2a\pi} - \frac{\mu_0 I_C}{2\left(\frac{3a}{2}\right)\pi} \qquad \ldots(1)$$

where, $I_F = J(\pi a^2)$

$$I_C = J\left(\frac{\pi a^2}{4}\right)$$

Substituting the values in equation (1), we get

$$B = \frac{\mu_0}{a\pi}\left(\frac{I_F}{2} - \frac{I_C}{3}\right)$$

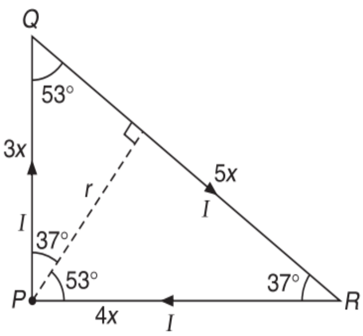
$$\Rightarrow \quad B = \frac{\mu_0}{a\pi}\left(\frac{\pi a^2 J}{2} - \frac{\pi a^2 J}{12}\right) = \frac{5\mu_0 aJ}{12}$$

$$\Rightarrow \quad N = 5$$

5. Magnetic field at point P due to wires RP and RQ is zero. Only wire QR will produce magnetic field at P. Since

$$r_\perp = r = 3x\cos 37^\circ$$

$$\Rightarrow \quad r_\perp = (3x)\left(\frac{4}{5}\right) = \frac{12x}{5}$$



Also, $B = \frac{\mu_0 I}{4\pi r_\perp}(\sin\theta_1 + \sin\theta_2)$

$$\Rightarrow \quad B = \frac{\mu_0}{4\pi}\frac{I}{\frac{12x}{5}}[\sin 37^\circ + \sin 53^\circ]$$

$$\Rightarrow \quad B = 7\left(\frac{\mu_0 I}{48\pi x}\right)$$