CHAPTER 3

Electric Current and Circuits

Learning Objectives

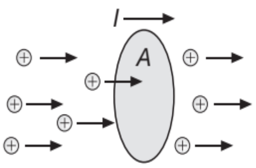
After reading this chapter, you will be able to understand concepts and problems based on:

(a) Current
(b) Ohm's Law
(c) Resistance
(d) Resistance Combinations
(e) Kirchhoff's Laws
(f) Nodal Analysis
(g) Cells
(h) Cell Combinations
(i) Ammeter
(j) Voltmeter
(k) Heating Effects of Current
(l) Wheatstone Bridge
(m) Post Office Box
(n) Potentiometer
(o) RC Circuit and Applications

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

ELECTRIC CURRENT

Whenever a charge moves (i.e. a charge moving with respect to an observer) we get current. So, moving charges (positive and negative) constitute electric current. Flow of electric charge is a direct measure of electric current. Suppose a collection of charges is moving perpendicular to a surface of area A, as shown in figure.



Charges moving through a cross section

The electric current is defined as the rate at which charges flow across any cross-sectional area. If an amount of charge ΔQ passes through a surface in a time interval Δt, then the average current I_{avg} is given by

$$I_{avg} = \frac{\Delta Q}{\Delta t} \qquad \ldots(1)$$

The SI unit of current is the ampere (A), where $1\text{ A} = 1\text{ coulomb sec}^{-1}$.

Common currents range from mega-amperes (in lightning) to nano-amperes (in your nerves).

In the limit $\Delta t \to 0$, the instantaneous current I may be defined as

$$I = \lim_{t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \qquad \ldots(2)$$

Since flow has a direction, so we have implicitly introduced a convention that the **direction of current corresponds to the direction in which positive charges are flowing**. This current is called the *Conventional Current*.

The flowing charges inside wires are negatively charged electrons that move in the opposite direction of the current and is called 'Electronic Current'. Electric currents flow in conductors : solids (metals, semiconductors), liquids (electrolytes, ionized) and gases (ionized), but the flow is impeded in non-conductors or insulators.

According to its magnitude and direction we categorise the current in two types.

(a) **Direct Current (DC):** If the magnitude and direction of current does not vary with time, it is said to be direct current (DC). This type of current is provided by a Cell, a battery or DC dynamo.

(b) **Alternating Current (AC):** If a current is periodic (with constant amplitude) and has half cycle positive and half negative, it is said to be alternating current (AC). This type of current is generally sinusoidal in nature. An AC dynamo provides this type of current.

Remark(s)

(a) In all electric circuits, the current shown (or the arrows drawn) must represent (or indicate) the "Conventional Current".

(b) A negative charge moving to the right is equivalent to a positive charge moving to the left. So current due to this negative charge is towards the right.

(c) If a charge Q revolves in a circle of radius R with angular frequency ω or frequency f, then equivalent current,

$$I = \frac{\text{Charge Circulating}}{\text{Time to Complete one Revolution}}$$

$$I = \frac{Q}{T} = \frac{Q\omega}{2\pi} = \frac{Qv}{2\pi R}$$

DIFFERENT SITUATIONS PRODUCING CURRENT

Due to Translatory Motion of Charges

(a) If N particles, each having a charge q, pass through a given area in time t, the current is given by

$$I = \frac{\Delta Q}{\Delta t} = \frac{Nq}{t}$$

(b) If n particles (each having a charge q) pass per second per unit area, then

$$n = \frac{N}{A\Delta t}$$

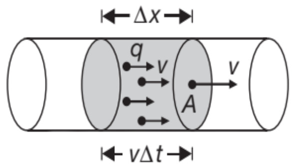
So, the current associated with cross-sectional area A is

$$I = \frac{\Delta Q}{\Delta t} = \frac{Nq}{\Delta t} = nqA$$

(c) If there are n particles per unit volume in the conductor (each having a charge q) moving with velocity v, then

$$n = \frac{N}{A\Delta x}$$

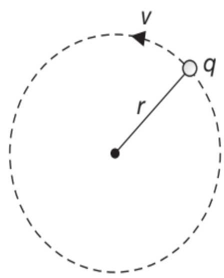
So, the current through cross-sectional area A is



$$I = \frac{\Delta Q}{\Delta t} = \frac{Nq}{\Delta t} = nqA\frac{\Delta x}{\Delta t} = nqvA$$

Due to Circular Motion of Charge

If a point charge q is moving in a circle of radius r with speed v, then its time period $T = \frac{2\pi}{\omega} = \left(\frac{2\pi r}{v}\right) = \frac{1}{f}$.



So through a given hypothetical cross-section (perpendicular to motion), the current is

$$I = \frac{\text{Charge Circulating}}{\text{Period of One Revolution}}$$

$$\Rightarrow \quad I = \frac{q}{T} = qf = \frac{qv}{2\pi r} = \frac{q\omega}{2\pi}$$

where ω is the angular velocity of the charge.

Due to Convectional Motion of Charge

It is possible that a charged body is transported from one place to another. The **convectional current** is the current which is developed due to the transportation of charge or the mechanical transfer of a charge.

Conceptual Note(s)

Instantaneous current through a cross-section is

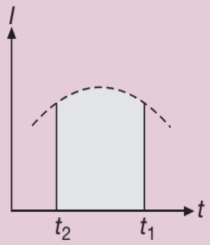
$$I = \frac{dq}{dt}$$

Charge passed through the cross-section in the interval t to $t + dt$

$$dq = Idt$$

Total charge in the interval t_1 to t_2 is given by

$\Delta q = \int_{t_1}^{t_2} Idt$ = Area under I versus t graph in the interval t_1 to t_2 as shown in figure.



Average current in the interval t_1 to t_2 is

$$I_{av} = \frac{\Delta q}{\Delta t} = \frac{\Delta q}{t_2 - t_1}$$

$$\Rightarrow I_{av} = \frac{\Delta q}{t_2 - t_1} = \frac{\int_{t_1}^{t_2} Idt}{t_2 - t_1} = \frac{\text{Area under } I \text{ versus } t \text{ graph}}{\text{Time interval}}$$

ILLUSTRATION 1

The current in a wire varies with time according to the relation $I = a + bt^2$, where current I is in ampere and time t is in second and $a = 4$ A, $b = 2$ As^{-2}.

(a) How many coulomb pass a cross-section of the wire in the time interval between $t = 5$ s and $t = 10$ s ?

(b) What constant current could transport the same charge in same time interval ?

SOLUTION

(a) $\Delta q = \int_5^{10} I\, dt = \int_5^{10} \left(4 + 2t^2\right) dt$

$$\Rightarrow \quad \Delta q = \left| 4t + \frac{2}{3}t^3 \right|_5^{10} = 4(10-5) + \frac{2}{3}(1000-125)$$

$$\Delta q = 603.33 \text{ C}$$

(b) $I_e = \frac{\Delta q}{\Delta t} = \frac{603.33}{10-5} = 120.67$ A

ILLUSTRATION 2

In the Bohr model of hydrogen atom, the electron is pictured to rotate in a circular orbit of radius 5×10^{-11} m, at a speed of 2.2×10^6 ms^{-1}. What is the current associated with electron motion ?

SOLUTION

The time taken to complete one rotation is

$$T = \frac{2\pi r}{v}$$

Therefore, the current is

$$I = \frac{q}{t} = \frac{e}{T} = \frac{ev}{2\pi r} = \frac{1.6 \times 10^{-19} \times 2.2 \times 10^6}{2 \times 3.14 \times 5 \times 10^{-11}} = 1.12 \text{ mA}$$

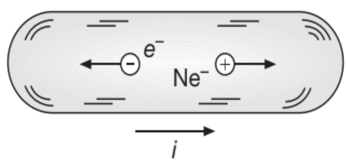
ILLUSTRATION 3

In a neon discharge tube 2.9×10^{18} Ne^+ ions move to the right each second while 1.2×10^{18} electrons move to the left per second. Electron charge is 1.6×10^{-19} C. Calculate the current in the discharge tube.

SOLUTION

Since current is actually due to the flow of positive charges. Also we know that negative charge moving to the left is equivalent to positive charge moving to the right, so

$$\text{Net current } i = i_+ + i_- = \frac{(n_+)(q_+)}{t} + \frac{(n_-)(q_-)}{t}$$



$$\Rightarrow \quad i = \frac{(n_+)e}{t} + \frac{(n_-)e}{t}$$

$$\Rightarrow \quad i = \left(2.9 \times 10^{18} + 1.2 \times 10^{18}\right) \times 1.6 \times 10^{-19}$$

$$\Rightarrow \quad i = 0.66 \text{ A}$$

ILLUSTRATION 4

A long cylinder with uniformly charged surface and cross-sectional radius $a = 1\text{ cm}$ moves with a constant velocity $v = 10\text{ ms}^{-1}$ along its axis. An electric field strength at the surface of the cylinder is equal to $E = 0.9\text{ kVcm}^{-1}$. Find the resulting convection current, that is, the current caused by mechanical transfer of a charge.

SOLUTION

The convection current is

$$I = \frac{dq}{dt} \qquad \ldots(1)$$

where, $dq = \lambda dx$, λ is the linear charge density

But, from the Gauss's Law, electric field at the surface of the cylinder,

$$E = \frac{\lambda}{2\pi\varepsilon_0 a}$$

Substituting the value of λ and subsequently of dq in equation (1), we get

$$I = \frac{dq}{dt} = \frac{\lambda dx}{dt}$$

$$\Rightarrow \quad I = \frac{2E\pi\varepsilon_0 a dx}{dt}$$

$$\Rightarrow \quad I = 2\pi\varepsilon_0 Eav \qquad \left\{\because \frac{dx}{dt} = v\right\}$$

Substituting the given values, we get $I = 0.5\ \mu\text{A}$

Test Your Concepts-I

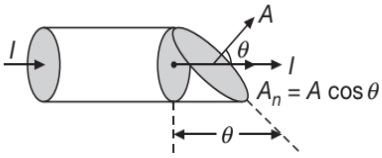
Based on Current Definition

(Solutions on page H.198)

1. In the Bohr model of the hydrogen atom, an electron in the lowest energy state follows a circular path 5.29×10^{-11} m from the proton.
   (a) Show that the speed of the electron is $2.19 \times 10^{6}\ \text{ms}^{-1}$.
   (b) What is the effective current associated with this orbiting electron?
2. The quantity of charge q (in coulombs) that has passed through a surface of area $2\ \text{cm}^2$ varies with time according to the equation $q = 4t^3 + 5t + 6$, where t is in seconds.
   (a) What is the instantaneous current through the surface at $t = 1$ s?
   (b) What is the value of the current density?
3. An electric current is given by the expression $I(t) = 100\sin(120\pi t)$, where I is in ampere and t is in second. What is the total charge carried by the current from $t = 0$ to $t = \left(\frac{1}{240}\right)$s?
4. A very small earthed conducting sphere is at a distance a from a point charge q_1 and at a distance b from a point charge q_2 $(a < b)$. At a certain instant, the sphere starts expanding so that its radius grows according to the law $R = vt$. Determine the time dependence $I(t)$ of the current in the earthing conductor, assuming that the point charges and the centre of the sphere are at rest, and in due time the initial point charges get into the expanding sphere without touching it (through small holes).
5. The current in a wire varies with time according to the relation
   $$I = (3A) + \left(2\frac{A}{s}\right)t$$
   (a) How many coulomb of charge passes a cross-section of the wire in the time interval between $t = 0$ and $t = 5$ s?
   (b) What constant current would transport the same charge in the same time interval?
6. The gap between two plane plates of a capacitor equal to d is filled with a gas. One of the plates emits n_0 electrons per second, which while moving in an electric field, ionize the gas molecules. This way each electron produces α new electrons (and ions) along a unit length of its path. Find the electronic current at the opposite plate, neglecting the ionization of gas molecules by the ions so formed. Take charge on an electron as e.
7. A long conductor of charge q, with charge density λ is moving with a velocity 2v parallel to its own axis. Find the convectional current due to motion of conductor.

CURRENT DENSITY $(\vec{J})$

The current density at any point inside a conductor is defined as a vector quantity whose magnitude is equal to current per unit infinitesimal area at that point, the area held normal to the direction of flow of current.



Current density $\vec{J}$ points along the direction of current flow.
If A_n is small area normal to current I, then

$$\text{Current density } J = \frac{I}{A_n}$$

If the plane of the small area A is not normal to current, but makes an angle θ with the to current, then

$$J = \frac{I}{A_n} = \frac{I}{A\cos\theta}$$

The unit of current density is Am^{-2}.
From above, we have

$$I = JA\cos\theta = \vec{J}\cdot\vec{A}$$

OHM'S LAW AND ELECTRICAL RESISTANCE

On applying a potential difference across a conductor, a current I is set up in the conductor. According to Ohm's Law "Under given physical conditions the potential difference (V) applied across a conductor produces a proportionate amount of current (I) in the conductor" i.e.,

$$V \propto I$$

$$\Rightarrow \quad V = IR$$

where the constant R is called the electrical resistance of the given conductor.
The unit of resistance R is volt/ampere.

$$1\ \text{VA}^{-1} = 1\ \text{ohm}(\Omega)$$

RESISTIVITY (ρ), CONDUCTIVITY (σ) AND CONDUCTANCE (G)

For a given conductor of uniform cross-section A and length l, it has been observed that the electrical resistance R is

(a) directly proportional to length l and
(b) inversely proportional to cross-sectional area A

$$\Rightarrow \quad R \propto \frac{l}{A}$$

$$\Rightarrow \quad R = \frac{\rho l}{A} \quad \ldots(1)$$

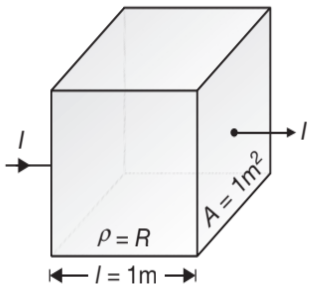
where ρ is a constant of proportionality called the **specific resistance** or **resistivity of the metal of the conductor** at a given temperature. Since

$$\rho = \frac{RA}{l}$$

If $l = 1$ m, $A = 1\ \text{m}^2$, then $\rho = R$ (numerically).

So, the specific resistance of the material of the conductor is defined as the resistance offered by the conductor of length 1 m and cross sectional area $1\ \text{m}^2$, when current flows normal to the area.

Alternatively, the specific resistance of a material is defined as the resistance offered by the opposite faces of a cube of that material of side 1 m. The unit of resistivity is ohm metre (Ωm).



The reciprocal of resistivity (ρ) is called the **conductivity** (σ)

$$\Rightarrow \quad \sigma = \frac{1}{\rho}$$

The unit of conductivity is mho metre^{-1} $(\Omega^{-1}\ \text{m}^{-1})$
Ohm's Law in alternative form may be expressed as

$$J = \sigma E$$

where J = Current Density and E = Electric Field Strength.

Conductance (G) is the reciprocal of resistance

$$\Rightarrow \quad G=\frac{1}{R}$$

Unit of conductance is siemen or mho $\left(=\Omega^{-1}\right)$.

ILLUSTRATION 5

A 3000 km long cable consists of seven copper wires, each of diameter 0.73 mm, bundled together and surrounded by an insulating sheath. Calculate the resistance of the cable. Use $3\times10^{-6}\ \Omega$ cm for the resistivity of the copper.

SOLUTION

The resistance R of a conductor is related to the resistivity ρ by $R=\dfrac{\rho l}{A}$, where l and A are the length of the conductor and the cross-sectional area, respectively. Since the cable consists of $N=7$ copper wires, the total cross sectional area is

$$A=N\pi r^2=N\frac{\pi d^2}{4}=7\frac{\pi(0.073\text{ cm})^2}{4}$$

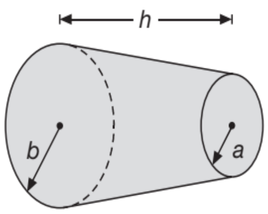
The resistance then becomes

$$R=\frac{\rho l}{A}=\frac{\left(3\times10^{-6}\ \Omega\text{cm}\right)\left(3\times10^{8}\text{ cm}\right)}{\dfrac{7\pi(0.073\text{ cm})^2}{4}}=3.1\times10^{4}\ \Omega$$

$$\Rightarrow \quad R=31\text{ k}\Omega$$

ILLUSTRATION 6

Consider a material of resistivity ρ in a shape of a truncated cone of altitude h, and radii a and b, for the right and the left ends, respectively, as shown in the figure.



A truncated cone

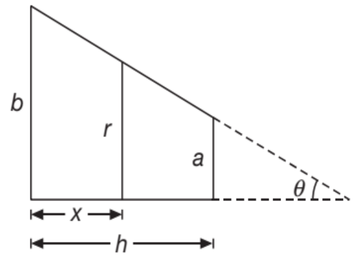
Assuming that the current is distributed uniformly throughout the cross-section of the cone, what is the resistance between the two ends?

SOLUTION

Consider a thin disk of radius r at a distance x from the left end. From the figure shown, we have

$$\frac{b-r}{x}=\frac{b-a}{h}$$

$$\Rightarrow \quad r=(a-b)\frac{x}{h}+b$$



Since resistance R is related to resistivity ρ by $R=\dfrac{\rho l}{A}$, where l is the length of the conductor and A is the cross section, the contribution to the resistance from the disk having a thickness dy is

$$dR=\frac{\rho\,dx}{\pi r^2}=\frac{\rho\,dx}{\pi\left[b+\dfrac{(a-b)x}{h}\right]^2}$$

Straight forward integration then yields

$$R=\int_0^h\frac{\rho\,dx}{\pi\left[b+\dfrac{(a-b)x}{h}\right]^2}=\frac{\rho h}{\pi ab}$$

$$\Rightarrow \quad R=\frac{\rho h}{\pi ab}$$

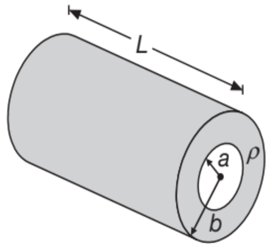
where we have used

$$\int\frac{du}{(\alpha u+\beta)^2}=-\frac{1}{\alpha(\alpha u+\beta)}$$

Note that if $b=a$, we get $R=\dfrac{\rho h}{\pi a^2}=\dfrac{\rho l}{A}$

ILLUSTRATION 7

Consider a hollow cylinder of length L and inner radius a and outer radius b, as shown in figure. The material has resistivity ρ.



A hollow cylinder

(a) Suppose a potential difference is applied between the ends of the cylinder and produces a current flowing parallel to the axis. What is the resistance measured?

(b) If instead the potential difference is applied between the inner and outer surfaces so that current flows radially outward, what is the resistance measured?

SOLUTION

(a) When a potential difference is applied between the ends of the cylinder, current flows parallel to the axis. In this case, the cross-sectional area is $A=\pi\left(b^2-a^2\right)$, and the resistance is given by

$$R=\frac{\rho L}{A}=\frac{\rho L}{\pi\left(b^2-a^2\right)}$$

(b) Consider a differential element which is made up of a thin cylinder of inner radius r and outer radius $r+dr$ and length L. Its contribution to the resistance of the system is given by

$$dR=\frac{\rho\, dl}{A}=\frac{\rho\, dr}{2\pi rL}$$

where $A=2\pi rL$ is the area normal to the direction of current flow. The total resistance of the system becomes

$$R=\int_a^b \frac{\rho dr}{2\pi rL}=\frac{\rho}{2\pi L}\log_e\left(\frac{b}{a}\right)$$

RECASTING A WIRE OF GIVEN MASS

Consider a conductor of mass m, density d, resistivity ρ to be recast to a wire of length l, cross-sectional area A (or radius r), then we have

$$R=\frac{\rho l}{A} \quad \ldots(1)$$

Since, $m=\text{volume}\times\text{density}=Ald$ ...(2)

$$\Rightarrow \quad A=\frac{m}{ld}$$

Then from (1), we get

$$R=\frac{\rho l}{(m/ld)}=\left(\frac{\rho d}{m}\right)l^2 \quad \ldots(3)$$

As ρ, d, m are constants, so we have

$$R\propto l^2$$

$$\Rightarrow \quad \frac{R_2}{R_1}=\left(\frac{l_2}{l_1}\right)^2 \quad \ldots(4)$$

So, we conclude that if a given mass of wire is stretched (or drawn) to n times its original length, then the new resistance is n^2 times the original resistance of wire. Hence

$$R_{\text{new}}=n^2R_{\text{old}}$$

Further if we put value of l from (2) i.e. $l=\frac{m}{Ad}$ in (1), then we get

$$R=\frac{\rho m}{AdA}=\left(\frac{m\rho}{d}\right)\left(\frac{1}{A^2}\right)$$

$$\Rightarrow \quad R\propto\frac{1}{A^2} \quad \ldots(5)$$

Since $A=\pi r^2$

$$\Rightarrow \quad R\propto\frac{1}{r^4}$$

Remark(s)

Thus if given mass of material is stretched to make its length ℓ, then $R\propto\ell^2$ and if the given mass of material is stretched to make its radius r, then

$$R\propto\frac{1}{r^4}$$

or $$\frac{R_2}{R_1}=\left(\frac{r_1}{r_2}\right)^4$$

So, we conclude that if a given mass of wire is stretched (or draw) such that new radius of wire is n times its old radius, then

$$R_{\text{new}}=\frac{1}{n^4}R_{\text{old}}$$

FRACTIONAL CHANGE IN RESISTANCE

The resistance of a wire depends on its length L, radius r and cross-sectional area A (where $A=\pi r^2$). If the wire is stretched by a small amount, its resistance R changes.

CASE-1: Suppose the length L increases by a small amount $\Delta L(\ll L)$. Then, we can write

$$R=\frac{\rho L}{A}=\rho\left(\frac{L^2}{V}\right)$$

{as $AL=$ volume, $V=$ constant}

Hence fractional change in resistance with a small change in length is

$$\frac{\Delta R}{R}=2\frac{\Delta L}{L}$$

CASE-2: Suppose the wire is stretched so that the cross-sectional area A reduces by small amount $\Delta A(\ll A)$. Then we can write

$$R=\frac{\rho L}{A}=\rho\left(\frac{V}{A^2}\right)$$

$$\Rightarrow \quad \frac{\Delta R}{R}=-2\frac{\Delta A}{A}$$

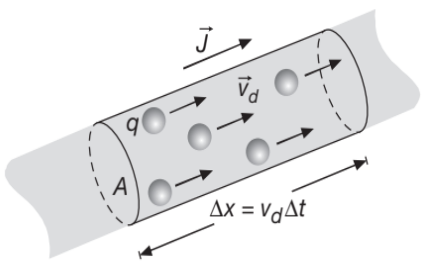
CASE-3: Suppose the wire is stretched so that the radius reduces by $\Delta r(\ll r)$. Then, we can write

$$R=\frac{\rho L}{A}=\rho\left(\frac{LA}{A^2}\right)=\rho\frac{V}{\left(\pi r^2\right)^2}=\left(\frac{\rho V}{\pi^2}\right)\frac{1}{r^4}$$

$$\Rightarrow \quad \frac{\Delta R}{R}=-4\frac{\Delta r}{r}$$

CURRENT DENSITY AND DRIFT VELOCITY

To relate current, a macroscopic quantity, to the microscopic motion of the charges, let's examine a conductor of cross-sectional area A, as shown in figure.



A microscopic picture of current flowing in a conductor

Let the total current through a surface be written as

$$I=\int \vec{J}\cdot d\vec{A} \qquad \ldots(1)$$

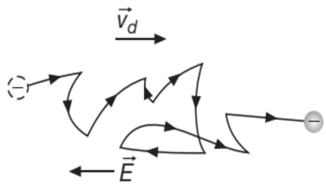
where $\vec{J}$ is the current density (the SI unit of current density are Am^{-2}). If q is the charge of each carrier, and n is the number of charge carriers per unit volume (also called "Charge Carrier Density"), the total amount of charge in this section is then

$$\Delta Q=q(nA\Delta x).$$

Suppose that the charge carriers move with a speed v_d, then the displacement in a time interval Δt will be $\Delta x=v_d\Delta t$. So, by definition of average current, we get

$$I_{\text{avg}}=\frac{\Delta Q}{\Delta t}=nqAv_d \qquad \ldots(2)$$

The speed v_d (along with the random speed) at which the charge carriers are moving is known as the **drift speed**. Physically, v_d is the average speed of the charge carriers inside a conductor when an external electric field is applied. Actually an electron inside the conductor does not travel in a straight line. Instead, its path is rather erratic, as shown in figure.



Motion of an electron in a conductor

From the above equations (1) and (2), the current density $\vec{J}$ can be written as

$$\vec{J}=nq\vec{v}_d \qquad \ldots(3)$$

Thus, we see that $\vec{J}$ and $\vec{v}_d$ point in the same direction for positive charge carriers, in opposite directions for negative charge carriers.

To find the drift velocity of the electrons, we first note that an electron in the conductor experiences an electric force $\vec{F}_e=-e\vec{E}$ which gives an acceleration

$$\vec{a}=\frac{\vec{F}_e}{m_e}=-\frac{e\vec{E}}{m_e}$$

Let the velocity of a given electron immediate after a collision be $\vec{u}$. The velocity of the electron immediately before the next consecutive collision is then given by

$$\vec{v} = \vec{u} + \vec{a}t = \vec{u} + \left(-\frac{e\vec{E}}{m_e}\right)t$$

where t is the time between two consecutive collisions. The average of $\vec{v}$ over all time intervals is

$$\langle \vec{v} \rangle = \langle \vec{u} \rangle - \frac{e\vec{E}}{m_e}\langle t \rangle$$

This average, $\langle v \rangle$ is equal to the drift velocity $\vec{v}_d$. Since in the absence of electric field, the velocity of the electron is completely random, it follows that $\langle \vec{u} \rangle = \vec{0}$. If $\tau = \langle t \rangle$ is the average characteristic time between successive collisions (the average relaxation time or the mean free time), we have

$$\vec{v}_d = \langle \vec{v} \rangle = -\left(\frac{e\vec{E}}{m_e}\right)\tau$$

The current density, from equation (3) becomes

$$\vec{J} = -ne\vec{v}_d = -ne\left(-\frac{e\vec{E}}{m_e}\tau\right) = \left(\frac{ne^2\tau}{m_e}\right)\vec{E} \qquad \ldots(4)$$

Note that $\vec{J}$ and $\vec{E}$ will be in the same direction for either negative or positive charge carriers.

Also, we compare (4) with $\vec{J} = \sigma\vec{E}$, then we get

$$\sigma = \frac{ne^2\tau}{m_e}$$

$$\Rightarrow \quad \rho = \frac{m_e}{ne^2\tau}$$

Remark(s)

(a) $\vec{J}$ and $\vec{v}_d$ point in the same direction for positive charge carriers, in opposite directions for negative charge carriers.

(b) $\vec{J}$ and $\vec{E}$ will be in the same direction for either negative or positive charge carriers.

OHM'S LAW: REVISITED

In many materials, the current density is linearly dependent on the external electric field $\vec{E}$ with their relation expressed as

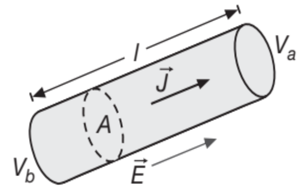
$$\vec{J} = \sigma\vec{E}$$

where σ is called the conductivity of the material. The above equation is known as the **Microscopic Ohm's Law**. A material that obeys this relation is said to be **ohmic**, otherwise, the material is **non-ohmic**.

Also, we have seen that the conductivity σ is given by

$$\sigma = \frac{ne^2\tau}{m_e}$$

To obtain a more useful form of Ohm's Law for practical applications let us consider a segment of straight wire of length l and cross-sectional area A, as shown in figure.



A uniform conuctor of lenght l
and potential difference $\Delta V = V_b - V_a$

Suppose a potential difference $\Delta V = V_b - V_a$ is applied across the ends of the wire, creating an electric field $\vec{E}$ and hence make a current I to flow through it. Assuming $\vec{E}$ to be uniform, then

$$\Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} = El$$

The current density can then be written as

$$J = \sigma E = \sigma\left(\frac{\Delta V}{l}\right)$$

where $J = \dfrac{I}{A}$. The potential difference becomes

$$\Delta V = \frac{l}{\sigma}J = \left(\frac{l}{\sigma A}\right)I = RI$$

where $R = \dfrac{\Delta V}{I} = \dfrac{l}{\sigma A}$, is the resistance of the conductor.

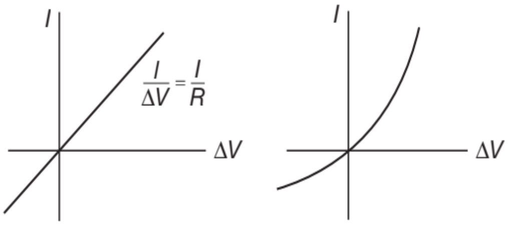
The equation $\Delta V = IR$ is the **"macroscopic"** version of the **Ohm's Law**.

The SI unit of R is the ohm (Ω, the Greek letter Omega), where

$$1\ \Omega \equiv \frac{1\ \text{V}}{1\ \text{A}}$$

Once again, a material that obeys the above relation is **ohmic**, and **non-ohmic** if the relation is not obeyed.

Most metals, with good conductivity and low resistivity, are ohmic. We shall focus mainly on **ohmic materials**.



Ohmic vs. Non-ohmic behaviour

The resistivity ρ of a material is defined as the reciprocal of conductivity

$$\rho = \frac{1}{\sigma} = \frac{m_e}{ne^2\tau}$$

From the above equations, we see that ρ can be related to the resistance R of an object by

$$\rho = \frac{E}{J} = \frac{\frac{\Delta V}{l}}{\frac{I}{A}} = \frac{RA}{l}$$

$$\Rightarrow \quad R = \frac{\rho l}{A} \text{ where } \rho = \frac{m_e}{ne^2\tau}$$

MOBILITY

Conductivity arises from mobile charge carriers. These mobile charge carriers are electrons in case of metal, electrons and positive ions in case of an ionized gas and both positive and negative ions in case of an electrolyte. In semiconductor's material such as germanium and silicon, conduction is partly due to electrons and partly due to electron vacancies called holes. Holes are sites of missing electrons which acts like positive charges.
The mobility μ of a charge carrier is defined as the drift velocity per unit electric field.

$$\mu = \frac{v_d}{E} = \frac{|v_d|}{E}$$

Mobility is positive for both electrons and holes, although their drift velocities are opposite to each other. The electric conductivity for a semiconductor containing electrons and holes as charge carriers can be expressed as

$$\sigma = e(n_e\mu_e + n_h\mu_h)$$

where μ_e and μ_h are electron and hole mobilities and n_e and n_h are electron and hole concentrations (in charge carriers per unit volume) also called as electron and hole density.

Also, $v_d = \frac{q\tau E}{m}$ $\quad \left\{ \because v_d = a\tau = \left(\frac{qE}{m}\right)\tau \right\}$

$$\Rightarrow \quad \mu = \frac{v_d}{E} = \frac{q\tau}{m}$$

Hence, $\mu_e = \frac{e\tau_e}{m_e}$ and $\mu_h = \frac{e\tau_h}{m_h}$

τ_e and τ_h are relaxation time for electrons and holes, respectively. m_e and m_h refer to mass of electron and holes respectively. Charge on either of the carriers is e.

ILLUSTRATION 8

The resistivity of sea water is about $25\ \Omega\text{cm}$. The charge carriers are chiefly Na^+ and Cl^- ions, and of each there are about $3\times10^{20}\ \text{cm}^{-3}$. If we fill a plastic tube 2 m long with sea water and connect a 12 V battery to the electrodes at each end, what is the resulting average drift velocity of the ions, in cms^{-1}?

SOLUTION

The current in a conductor of cross sectional area A is related to the drift speed v_d of the charge carriers by

$$I = neAv_d$$

where n is the number of charges per unit volume. We can then rewrite the Ohm's Law as

$$V = IR = (neAv_d)\left(\frac{\rho l}{A}\right) = nev_d(\rho l)$$

$$\Rightarrow \quad v_d = \frac{V}{ne\rho l}$$

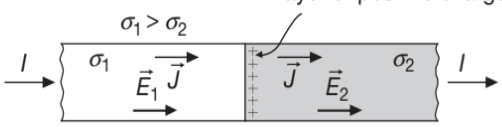
Substituting the values, we get

$$v_d = \frac{12\text{ V}}{\begin{matrix}(6\times10^{20}\text{ cm}^{-3})(1.6\times10^{-19}\text{ C}) \\ (25\ \Omega\text{ cm})(200\text{ cm})\end{matrix}}$$

$$\Rightarrow \quad v_d = 2.5\times10^{-5}\frac{\text{Vcm}}{\text{C}\Omega} = 2.5\times10^{-5}\ \text{cms}^{-1}$$

ILLUSTRATION 9

Show that the total amount of charge at the junction of the two materials in figure is $\varepsilon_0 I\left(\frac{1}{\sigma_2}-\frac{1}{\sigma_1}\right)$, where I is the current flowing through the junction, and σ_1 and σ_2 are the conductivities for the two materials.



Charge at a junction

SOLUTION

In a steady state of current flow, the normal component of the current density $\vec{J}$ must be the same on both sides of the junction. Since $J=\sigma E$, we have $\sigma_1 E_1 = \sigma_2 E_2$

$$\Rightarrow \quad E_2 = \left(\frac{\sigma_1}{\sigma_2}\right)E_1$$

Let the charge on the interface be q_{enc}, then from the Gauss's Law, we have

$$\oint \vec{E}\cdot d\vec{A} = (E_2 - E_1)A = \frac{q_{enc}}{\varepsilon_0}$$

$$\Rightarrow \quad E_2 - E_1 = \frac{q_{enc}}{A\varepsilon_0}$$

Substituting the expression for E_2 from above then

$$q_{enc} = \varepsilon_0 A E_1\left(\frac{\sigma_1}{\sigma_2}-1\right) = \varepsilon_0 A \sigma_1 E_1\left(\frac{1}{\sigma_2}-\frac{1}{\sigma_1}\right)$$

Since the current is $I = JA = (\sigma_1 E_1)A$. So, the amount of charge on the interface becomes

$$q_{enc} = \varepsilon_0 I\left(\frac{1}{\sigma_2}-\frac{1}{\sigma_1}\right)$$

Test Your Concepts-II

Based on Resistance, Resistivity and Ohm's Law

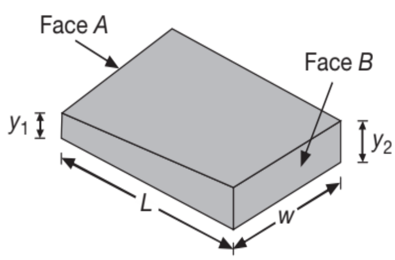
(Solutions on page H.199)

1. A close analogy exists between the flow of energy by heat because of a temperature difference and the flow of electric charge because of a potential difference. The energy dQ and the electric charge dq can both be transported by free electrons in the conducting material. Consequently, a good electrical conductor is usually a good thermal conductor as well. Consider a thin conducting slab of thickness dx, area A, and electrical conductivity σ, with a potential difference dV between opposite faces. Show that the current $I=\frac{dq}{dt}$ is given by the equation on the left below:

Charge Conduction	Thermal Conduction
$\frac{dq}{dt} = \sigma A\left\lvert\frac{dV}{dx}\right\rvert$	$\frac{dQ}{dt} = kA\left\lvert\frac{dT}{dx}\right\rvert$

In the analogous thermal conduction equation on the right, the rate of energy flow $\frac{dQ}{dt}$ (in SI units of joules per second) is due to a temperature gradient $\frac{dT}{dx}$, in a material of thermal conductivity k. State analogous rules relating the direction of the electric current to the change in potential, and relating the direction of energy flow to the change in temperature.

2. Material with uniform resistivity ρ is formed into a wedge as shown in figure. Show that the resistance between face A and face B of this wedge is



$$R = \rho\frac{L}{w(y_2 - y_1)}\log_e\left(\frac{y_2}{y_1}\right)$$

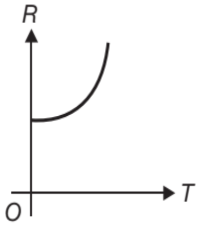
3. A piece of wire of uniform cross section has a resistance 8 Ω. If the length of the wire is doubled and its area of cross section is increased four times,

calculate the new resistance. Neglect the temperature variation of resistance.

4. **(a)** If a copper wire is stretched to make it 0.1% longer, keeping volume constant, what is the percentage change in its resistance?
   (b) What is percentage change in its resistance if radius is increased by 1% and area is increased by 1%?
5. A steady current passes through a cylindrical conductor. Is there an electric field inside the conductor?
6. The region between two concentric conducting spheres with radii a and b is filled with a material with resistivity ρ.
   (a) Show that the resistance between the spheres is given by
   $$R=\frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$$
   (b) Find the current density as a function of radius, in terms of the potential difference V_{ab} between the spheres.

VARIATION OF RESISTANCE WITH TEMPERATURE

The resistance of a conductor varies with temperature. The graph of variation of resistance of pure metal with temperature is shown.



Mathematically the dependence of (R) on temperature (T) is expressed as

$$R_T=R_0\left(1+\alpha T+\beta T^2\right) \quad ...(1)$$

where α and β are **Temperature Coefficients of Resistance (TCR)**. The values of α and β vary from metal to metal.

If temperature T is not sufficiently large (as in most practical cases), then equation (1) may be expressed as

$$R_T=R_0\left(1+\alpha T\right) \quad ...(2)$$

The constant α is called the **Temperature Coefficient of Resistance (TCR)** of the material. α is positive for metals and negative for semiconductors and electrolytes. If R_1 and R_2 are the resistances of the same specimen at temperatures T_1 °C and T_2 °C, then

$$R_1=R_0\left(1+\alpha T_1\right) \quad ...(3)$$

$$R_2=R_0\left(1+\alpha T_2\right) \quad ...(4)$$

So, the temperature coefficient of resistance is given by

$$\alpha=\left|\frac{R_2-R_1}{R_1T_2-R_2T_1}\right|$$

The resistivity of a material actually varies with temperature T. For metals, the variation is linear over a large range of T

$$\rho_T=\rho_0\left(1+\alpha T\right)$$

where α is the temperature coefficient of resistivity. Typical values of ρ, σ and α (at 20 °C) for different types of materials are given in the table below.

Material	Resistivity ρ (Ωm)	Conductivity σ (Ωm)$^{-1}$	Temperature coefficient α (°C)$^{-1}$
Elements			
Silver	1.59×10^{-8}	6.29×10^{7}	0.0038
Copper	1.72×10^{-8}	5.81×10^{7}	0.0039
Aluminium	2.82×10^{-8}	3.55×10^{7}	0.0039
Tungsten	5.6×10^{-8}	1.8×10^{7}	0.0045
Iron	10×10^{-8}	1×10^{7}	0.0050
Platinum	10.6×10^{-8}	1×10^{7}	0.0039
Alloys			
Brass	7×10^{-8}	1.4×10^{7}	0.002
Manganin	44×10^{-8}	0.23×10^{7}	1×10^{-5}
Nichrome	100×10^{-8}	0.1×10^{7}	0.0004

(Continued)

Material	Resistivity ρ (Ωm)	Conductivity σ $(\Omega m)^{-1}$	Temperature coefficient α $(°C)^{-1}$
Semiconductors			
Carbon (graphite)	3.5×10^{-5}	2.9×10^{4}	−0.0005
Germanium (pure)	0.46	2.2	−0.048
Silicon (pure)	640	1.6×10^{-3}	−0.075
Insulators			
Glass	$10^{10}-10^{14}$	$10^{-14}-10^{-10}$	
Sulfur	10^{15}	10^{-15}	
Quartz (fused)	75×10^{16}	1.33×10^{-18}	

Variation of Resistivity of Metals with Temperature

Since, $R=\dfrac{m\ell}{ne^2\tau A}$

and $\rho=\dfrac{m}{ne^2\tau}$

The number density of free electrons, n, is practically independent of temperature for most metals. However, an increase of temperature increases the amplitude of vibration of atoms and also the average speed of the free electrons. Therefore the average relaxation time τ decreases and hence resistivity increases with temperature.

If ρ_0 and ρ_T are the values of resistivity of a material at 0°C and T°C respectively then over a temperature range that is not too large, we have approximately,

$$\rho_T=\rho_0(1+\alpha T)$$

ILLUSTRATION 10

The temperature coefficient of resistivity α is given by $\alpha=\left(\dfrac{1}{\rho}\right)\dfrac{d\rho}{dT}$, where ρ is the resistivity at temperature T.

(a) Assume that α is not constant and is given by $\alpha=-\dfrac{a}{T}$, where T is the absolute temperature where a is a constant, show that the resistivity ρ is given by $\rho=\dfrac{b}{T^n}$, where b is another constant.

(b) Using the values $\rho=3.5\times10^{-5}\ \Omega$m and $\alpha=-5\times10^{-4}\ (C°)^{-1}$ for graphite at 293 K, determine a and b.

(c) Using your result from part (b), determine the resistivity of graphite at −196 °C and 300 °C. (Remember to express T on absolute scale).

SOLUTION

(a) $\alpha=\dfrac{1}{\rho}\dfrac{d\rho}{dT}$

$$\Rightarrow \quad \frac{d\rho}{\rho}=\alpha\, dT=-a\frac{dT}{T}$$

Let $\rho=\rho_0$ at $T=T_0$, then

$$\int_{\rho_0}^{\rho}\frac{d\rho}{\rho}=-a\int_{T_0}^{T}\frac{dT}{T}$$

$$\Rightarrow \quad \log_e\left(\frac{\rho}{\rho_0}\right)=-a\log_e\left(\frac{T}{T_0}\right)=\log_e\left(\frac{T_0}{T}\right)^a$$

$$\Rightarrow \quad \rho=\left(\rho_0T_0^a\right)\frac{1}{T^a}=\frac{b}{T^a}$$

Here, $b=\rho_0T_0^a$

(b) Given that $\rho_0=3.5\times10^{-5}\ \Omega$m and

$$\alpha_0=-5\times10^{-4}\,(°C)^{-1} \text{ at } T_0=293\text{ K}$$

$$a=-\alpha_0T_0 \qquad \left\{\text{as } \alpha=-\frac{a}{T}\right\}$$

$$\Rightarrow \quad a\approx0.15$$

and $b=\rho_0T_0^a=\left(3.5\times10^{-5}\right)(293)^{0.15}$

$\Rightarrow \quad b=8\times10^{-5}\ \Omega\text{mK}^{0.15}$

{be careful while taking units of a and b}

(c) Now at $T=-196+273=77$ K,

$$\rho=\frac{b}{T^a}=\frac{8\times10^{-5}}{(77)^{0.15}}\approx4.3\times10^{-5}\ \Omega\text{m}$$

Similarly at $T=300+273=573$ K

$$\rho=\frac{8\times10^{-5}}{(573)^{0.15}}\approx3\times10^{-5}\ \Omega\text{m}$$

COLOUR CODE FOR CARBON RESISTANCES

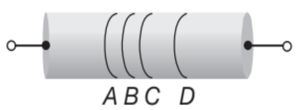
Carbon resistances find their wide use in electronic circuits at low voltages due to following reasons.

(a) They may have values ranging from few ohms to $100\ M\Omega$.
(b) They are made up of small handy sizes.
(c) They are quite cheap.

A Colour Code is used to indicate the resistance and its percentage reliability. The carbon resistor has a set of concentric rings of various colours on it with their significance in the table.

Colour	Number	Multiplier	% Tolerance
Black (B)	0	$10^0 = 1$	Gold 5%
Brown (B)	1	10^1	Silver 10%
Red (R)	2	10^2	No Colour 20%
Orange (O)	3	10^3	
Yellow (Y)	4	10^4	
Green (G)	5	10^5	
Blue (B)	6	10^6	
Violet (V)	7	10^7	
Grey (G)	8	10^8	
White (W)	9	10^9	

The colour bands are formed from left to right. The first three bands $(A, B \text{ and } C)$ give the value of resistance. The colours of first and second bands indicate the first and second significant digits while the colour of third band gives decimal multiplier (i.e., the number of zeros which follow the first two digits). The colour of fourth band represents its tolerance. Absence of any colour means a tolerance of 20%.



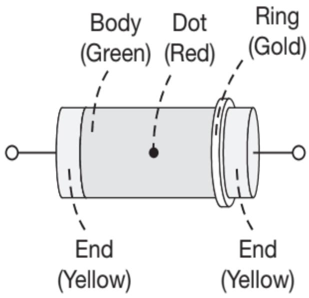
$R = AB \times 10^C \pm$ Tolerance (in %)

e.g. if A is red, B is green, C is grey and there is no colour at D, then

$$R = 25 \times 10^8\ \Omega \pm 20\%$$

The table may be memorised by the mnemonic
BB ROY of **G**reat **B**ritain has a **V**ery **G**ood **W**ife.
Capital letters represent the first letter of colour (*from Darkest to the lightest colour*).
Also you may try to remember the following mnemonic
Black **B**ears **R**oar, **O**rangutans **Y**ell, **G**oats **B**leat **V**iolently, **G**o **W**eep.

Another way of representation is the Dot-Body convention in which the body of the resistance is given one colour, the ends are given the other second same colour and a dot is marked over the body. A ring (silver or gold) on one side indicates the tolerance. The colour code table remains the same.

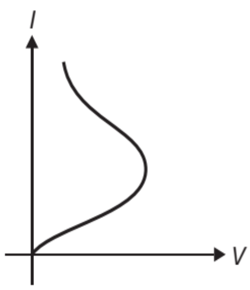


Colour of body describes the first significant digit.
Identical end colours give the second significant figure.
Colour of dot gives the decimal multiplier.
The colour of the ring gives the tolerance.
So, for the colours specified in the figure the resistance has a value

$$R = 54 \times 10^2\ \Omega \pm 5\%$$

THERMISTOR

A thermistor is a heat sensitive resistor usually made of semiconductor material. The temperature coefficient of a thermistor is negative but is unusually large, of the order of $-0.04(°C)^{-1}$. The V-I curve of a thermistor is unusual and is shown in Figure.



Uses of Thermistor

(a) They are used for resistance thermometers in very low temperature measurement of the order of 10 K.

(b) They may be used to safeguard electronic circuits against current jumps because initially (when cold) thermistor has high resistance and its resistance drops appreciably when it warms up.

Test Your Concepts-III

Based on Variation of Resistance with Temperature

(Solutions on page H.200)

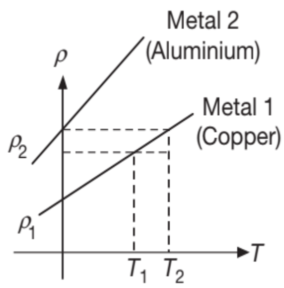
1. When the temperature of a heating element changes, its resistance also changes and so does the temperature coefficient. Suppose the temperature varies linearly with time and is given by $T(°C) = (20 + 10t)$ where t is time in second. The temperature coefficient of the material is $0.0065\ (°C)^{-1}$ at 0°C. If the initial resistance of the heating element is $2r$, then express the resistance as a function of time.

2. Obtain a general relationship between α_0 and α_T, the respective temperature coefficients at 0°C and T°C.

3. Obtain a general relationship between α_1 and α_2, the respective temperature coefficients at T_1°C and T_2°C.

4. Obtain the expression for the ratio of resistances of a coil at two temperatures T_1 and T_2 assuming that the only other given quantity is the temperature coefficient α_0 at 0°C.

5. It has been experimentally found that the resistivity of conducting materials such as copper and aluminium varies linearly with temperature. Obtain a relation between resistivities of a metal ρ_1 and ρ_2 at two temperatures T_1 and T_2 respectively.

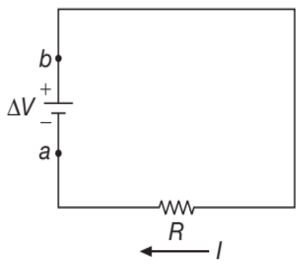


6. When a metal rod is heated, not only its resistance but also its length and area of cross-section changes. Find the percent change in R, ℓ and A of a copper wire for a temperature rise of 1°C. Coefficient of linear expansion for copper is $1.7 \times 10^{-5}\ (°C)^{-1}$ and its thermal coefficient of resistance is $3.9 \times 10^{-3}\ (°C)^{-1}$.

7. Two coils connected in series have resistance of 600 Ω and 300 Ω and temperature co-efficient of $0.001\ (°C)^{-1}$ and $0.004\ (°C)^{-1}$ respectively at 20°C. Find resistance of the combination at a temperature of 50°C. What is the effective temperature co-efficient of combination?

ELECTRICAL ENERGY AND POWER

Consider a circuit consisting of a battery and a resistor with resistance R.



Let the potential difference between two points a and b be $\Delta V = V_b - V_a > 0$. If a charge Δq is moved from a through the battery, its electric potential energy is increased by $\Delta U = \Delta q \Delta V$. On the other hand, as the charge moves across the resistor, the potential energy is decreased due to collisions with atoms in the resistor. If we neglect the internal resistance of the battery and the connecting wires, upon returning to a the potential energy of Δq remains unchanged.

Thus, the rate of energy loss through the resistor is given by

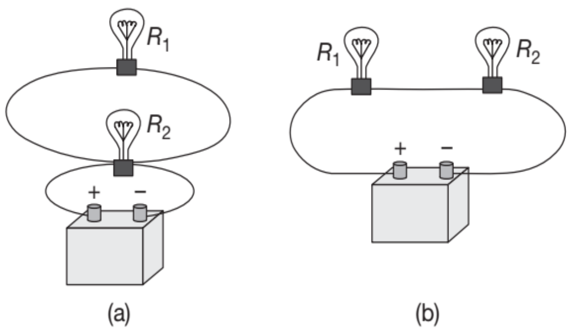
$$P = \frac{\Delta U}{\Delta t} = \left(\frac{\Delta q}{\Delta t}\right)\Delta V = I\Delta V$$

This is precisely the power supplied by the battery. Using $\Delta V = IR$, the above equation can be rewritten as

$$P = I^2R = \frac{(\Delta V)^2}{R}$$

Introduction to Electrical Circuits

Electrical circuits connect power supplies to loads such as resistors, motors, heaters, or lamps.



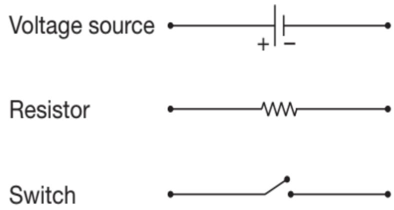
Elements connected (a) in parallel and (b) in series

The connection between the supply and the load is made with the help of wires (that are often called **leads**), or with many kinds of **connectors** and **terminals**. Energy is delivered from the source to the user on demand at the flick of a switch. Sometimes many circuit elements are connected to the same lead, which is called a **common lead** for those elements. Various parts of the circuits are called **circuit elements**, which can be in series or in parallel, as we have already seen in the case of capacitors.

Elements are said to be in parallel when they are connected across the same potential difference. Generally, loads are connected in parallel across the power supply.

On the other hand, when the elements are connected one after another, so that the current passes through each element without any branches, the elements are in series.

There are pictorial diagrams that show wires and components roughly as they appear, and schematic diagrams that use conventional symbols, somewhat like road maps. Some frequently used symbols are shown here for the purpose of convenience.



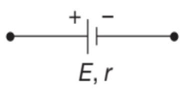
Often, in circuits we observe a switch to be connected in series. When the switch is open the load is disconnected and when the switch is closed, the load is connected.

One can have closed circuits, through which current flows, or open circuits in which there are no currents. Usually by accident, wires may touch, causing a short circuit. Most of the current flows through the short, very little will flow through the load. This may burn out a piece of electrical equipment such as a transformer. To prevent damage, a fuse or circuit breaker is put in series. When there is a short the fuse blows, or the breaker opens.

In electrical circuits, a point (or some common lead) is chosen as the ground. This point is assigned an arbitrary voltage, usually zero, and the voltage V at any point in the circuit is defined as the voltage difference between that point and ground.

Active and Passive Elements in Electric Circuits

An **electric circuit** is an arrangement of active and passive elements, having one or more closed paths for electric current to flow. It is due to the active elements (such as cells, batteries, dynamos, generators, etc.) that the current is forced to flow in a closed path. These supply energy to the circuit. The active element in a dc circuit is a **cell** or a **battery**. Its circuit symbol is



On the contrary, the passive elements either consume or store electric energy. There are three basic passive elements.

	1	2	3
Passive element (property)	Resistor (Resistance)	Capacitor (Capacitance)	Inductor (Inductance)
Symbol	R	C	L
Units	Ω	F	H
Circuit Symbol	 or 		
V-I Relationship	$V = IR$	$I = C\frac{dV}{dt}$	$V = L\frac{dI}{dt}$
Energy consumed/ Stored	I^2Rt	$\frac{1}{2}CV^2$	$\frac{1}{2}LI^2$
Characteristic property	Converts electric energy into heat	Opposes variations in voltage	Opposes variations in current

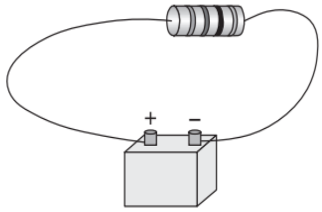
ELECTROMOTIVE FORCE (EMF)

We are aware of the fact that energy has to be supplied to maintain a constant current in a closed circuit. The source of energy is commonly referred to as the **electromotive force**, or the **emf** (symbol E). The ability of a cell or a battery to make the current flow in the circuit is measured in terms of its **electromotive** force or emf. Batteries, solar cells and thermocouples are some examples of emf source. They can be thought of as a "charge pump" that moves charges from lower potential to the higher potential. Mathematically emf is defined as

$$E = \frac{W}{q} \qquad \text{...(1)}$$

So, emf is the work done to move a unit positive charge in the direction of the applied external filed or from higher potential to the lower potential. SI unit for E is volt (V).

Consider a simple circuit consisting of a battery as the emf source and a resistor of resistance R, as shown in figure.

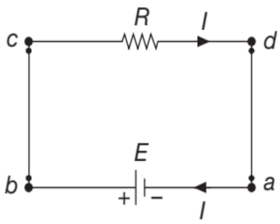


A simple circuit consisting of a battery and a resistor

Assuming that the battery has no internal resistance, the potential difference ΔV (or terminal voltage) between the positive and the negative terminals of the battery is equal to the emf E. To drive the current around the circuit, the battery undergoes a discharging process which converts chemical energy to emf. The current I is found by noting that no work is done in moving a charge q around a closed loop due to the conservative nature of the electrostatic force. So,

$$W = q\oint \vec{E}\cdot d\vec{l} = 0 \qquad \text{...(2)}$$

Consider the loop *abcda* and the point *a* in figure be the point of start.

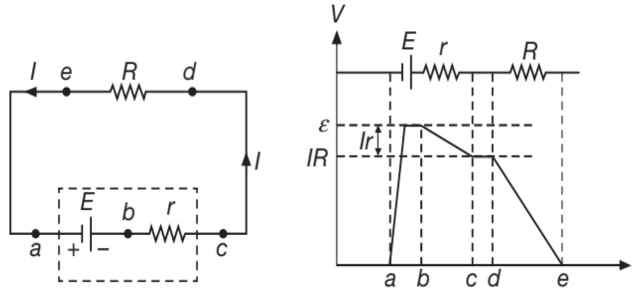


When crossing from the negative to the positive terminal, the potential increases by E. On the other hand, as we cross the resistor, the potential decreases by an amount IR, and the potential energy is converted into thermal energy in the resistor. *Assuming that the connecting wire carries no resistance* so, upon completing the loop the net change in potential difference must be zero. Hence, we must have

$$E - IR = 0 \qquad \text{...(3)}$$

$$\Rightarrow \quad I = \frac{E}{R} \qquad \text{...(4)}$$

However in reality a battery always carries an internal resistance r and the potential difference across the



(a) (b)

(a) Circuit with an emf source having an internal resistance r and a resistor of resistance R.
(b) Change in electric potential around the circuit.

battery terminals, also called as the **Terminal Potential Difference (TPD)**, becomes

$$\Delta V = E - Ir \quad \ldots(5)$$

Since there is no net change in potential difference around a closed loop $abcdea$, we have

$$E - Ir - IR = 0 \quad \ldots(6)$$

$$\Rightarrow \quad I = \frac{E}{R+r} \quad \ldots(7)$$

Remark(s)

(a) Figure (b) depicts the change in electric potential as we traverse the circuit clockwise. From the figure, we see that the highest voltage is immediately after the battery. The voltage drops as each resistor is crossed. Note that the voltage is essentially constant along the wires. This is because the wires have a negligibly small resistance compared to the resistors.

(b) For a source with emf E, the power or the rate at which energy is delivered is

$$P = IE = I(IR + Ir) = I^2R + I^2r$$

That the power of the source emf is equal to the sum of the power dissipated in both the internal and load resistance is required by energy conservation.

(c) V (potential difference across the terminals of the cell) may be less than E, equal to E or greater than E.

(i) $V < E$, when the circuit draws current I from the cell and in that case $V = E - Ir$.

(ii) $V = E$, when the circuit draws no current from the cell i.e. when $I = 0$.

(iii) $V > E$, when the cell is being charged and in that case $V = E + Ir$.

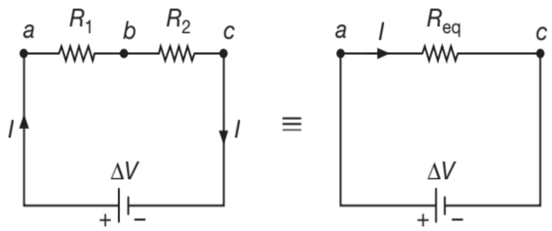
RESISTORS IN SERIES AND IN PARALLEL

Series Combination

The two resistors R_1 and R_2 in figure are connected in series to a voltage source ΔV. By current conservation, the same current I is flowing through each resistor.

The total voltage drop from a to c across both elements is the sum of the voltage drops across the individual resistors

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$



(a) Resistors in series

(b) Equivalent circuit

The two resistors in series can be replaced by one equivalent resistor R_{eq} (figure (b)) with the identical voltage drop $\Delta V = IR_{eq}$ which implies that

$$R_{eq} = R_1 + R_2$$

The above argument can be extended to N resistors placed in series. The equivalent resistance is just the sum of the original resistances,

$$R_{eq} = R_1 + R_2 + \ldots = \sum_{i=1}^{N} R_i$$

(a) Notice that if one resistance R_1 is much larger than the other resistances R_i, then the equivalent resistance R_{eq} is approximately equal to the largest resistor R_1.

(b) The current flowing in each resistor is same.

(c) The total potential difference V across the combination is equal to the sum of the potential difference across individual resistances i.e.,

$$V = V_1 + V_2 + V_3$$

such that $V_1 : V_2 : V_3 \equiv R_1 : R_2 : R_3$

(d) The equivalent or effective resistance (R_S) of the combination is equal to the sum of individual resistances i.e.,

$$R_S = R_1 + R_2 + R_3$$

(e) Net Conductance (G) is

$$\frac{1}{G_S}=\frac{1}{G_1}+\frac{1}{G_2}+\frac{1}{G_3}$$

(f) Net Resistance R_s has a value greater than the largest resistance of the combination.

(g) $V_1=\left(\frac{R_1}{R_S}\right)V$

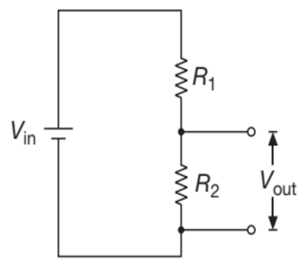
$$V_2=\left(\frac{R_2}{R_S}\right)V$$

$$V_3=\left(\frac{R_3}{R_S}\right)V$$

where $R_S=R_1+R_2+R_3$

Voltage Divider Circuit

Consider a voltage source V_{in} that is connected in series to two resistors, R_1 and R_2.



Voltage divider

The voltage difference, V_{out}, across resistor R_2 will be less than V_{in}. This circuit is called a voltage divider. From the loop rule,

$$V_{in}-IR_1-IR_2=0$$

So the current in the circuit is given by

$$I=\frac{V_{in}}{R_1+R_2}$$

Thus the voltage difference, V_{out}, across resistor R_2 is given by

$$V_{out}=IR_2=\frac{R_2}{R_1+R_2}V_{in}$$

Note that the ratio of the voltages characterizes the voltage divider and is determined by the resistors

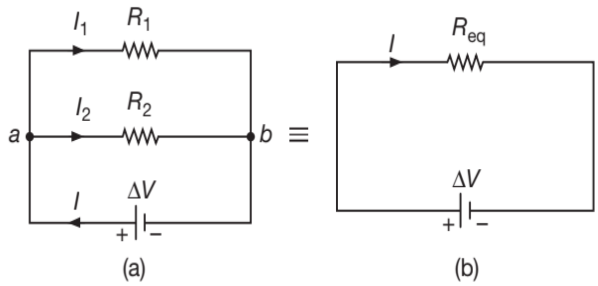
$$\frac{V_{out}}{V_{in}}=\frac{R_2}{R_1+R_2}$$

Parallel Combination

Next let's consider two resistors R_1 and R_2 that are connected in parallel across a voltage source ΔV.

By current conservation, the current I that passes through the voltage source must divide into a current I_1 that passes through resistor R_1 and a current I_2 that passes through resistor R_2. Each resistor individually satisfies Ohm's Law, $\Delta V_1=I_1R_1$ and $\Delta V_2=I_2R_2$. However, the potential across the resistors are the same, $\Delta V_1=\Delta V_2=\Delta V$. Current conservation then implies

$$I=I_1+I_2=\frac{\Delta V}{R_1}+\frac{\Delta V}{R_2}=\Delta V\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$



(a) Two resistors in parallel (b) Equivalent resistance

The two resistors in parallel can be replaced by one equivalent resistor R_{eq} with $\Delta V=IR_{eq}$. Comparing these results, the equivalent resistance for two resistors that are connected in parallel is given by

$$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}$$

This result easily generalizes to N resistors connected in parallel

$$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+....=\sum_{i=1}^{N}\frac{1}{R_i}$$

(a) The potential difference (V) across each resistor is the same.

(b) The current is different in different resistances such that the total current flowing in the combination is shared by the individual resistances in the inverse ratio of their resistances i.e.,

$$I=I_1+I_2+I_3$$

such that $I_1:I_2:I_3\equiv\frac{1}{R_1}:\frac{1}{R_2}:\frac{1}{R_3}$

(c) The equivalent or effective resistance (R_P) of the combination is given by

$$\frac{1}{R_P}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

(d) Effective conductance G_P is the sum of conductance of individual resistors i.e.,

$$G_P=G_1+G_2+G_3$$

(e) Net Resistance R_P has a value smaller than the least resistance of the combination.

(f) $I_1=\left(\frac{R_P}{R_1}\right)I$, $I_2=\left(\frac{R_P}{R_2}\right)I$ and $I_3=\left(\frac{R_P}{R_3}\right)I$ where

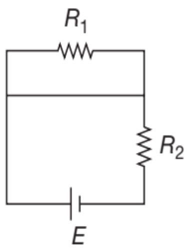
$$\frac{1}{R_P}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

(g) When one resistance R_1 is much smaller than the other resistances, then the equivalent resistance R_{eq} is approximately equal to the smallest resistor R_1. In the case of two resistors,

$$R_{eq}=\frac{R_1R_2}{R_1+R_2}\approx\frac{R_1R_2}{R_2}=R_1$$

This means that almost all of the current that enters the node point will pass through the branch containing the smallest resistance. So, when a short develops across a circuit, all of the current passes through this path of nearly zero resistance. **Hence SHORT CIRCUITING a branch implies taking the resistance of that branch to be zero or close to zero.**

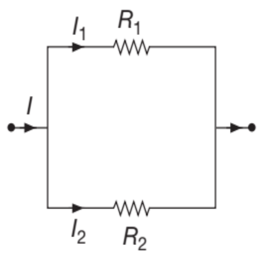
Shown here is an electric circuit whose two points are directly connected by a conducting wire.



Here we say that the resistance R_1 in the circuit is short circuited. Under such condition, both points are at same potential and hence the potential difference across R_1 is zero. Hence, no current will flow through R_1 and the current through R_2 is $I=\frac{E}{R_2}$.

Current Divider

Consider two resistors connected in parallel as shown. From figure, we have



$$I=I_1+I_2 \qquad \ldots(1)$$

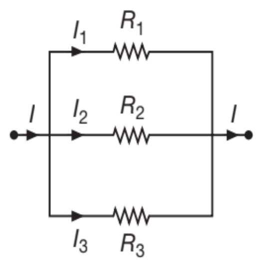
$$I_1R_1=I_2R_2 \qquad \ldots(2)$$

On solving equations (1) and (2), we get

$$I_1=\left(\frac{R_2}{R_1+R_2}\right)I \text{ and } I_2=\left(\frac{R_1}{R_1+R_2}\right)I$$

So, we observe that the division of current in the branches of a parallel circuit is inversely proportional to their resistances.

Similarly, consider three resistors connected in parallel as shown.



From figure, we have

$$I=I_1+I_2+I_3$$

$$I_1R_1=I_2R_2=I_3R_3$$

and $$R_{eq}=\frac{R_1R_2R_3}{R_1R_2+R_2R_3+R_3R_1}$$

$$I_1=I\left[\frac{R_2R_3}{R_1R_2+R_2R_3+R_3R_1}\right]$$

$$\Rightarrow \quad I_1=\left(\frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}\right)I=\left(\frac{\frac{1}{R_1}}{\frac{1}{R_P}}\right)I$$

$$I_2 = I\left[\frac{R_1R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

$$\Rightarrow \quad I_2 = \left(\frac{\dfrac{1}{R_2}}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}}\right)I = \left(\frac{\dfrac{1}{R_2}}{\dfrac{1}{R_P}}\right)I$$

$$I_3 = I\left[\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

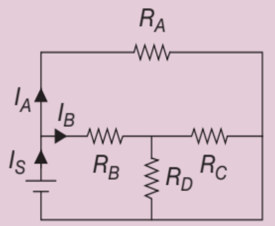
$$\Rightarrow \quad I_3 = \left(\frac{\dfrac{1}{R_3}}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}}\right)I = \left(\frac{\dfrac{1}{R_3}}{\dfrac{1}{R_P}}\right)I$$

It is easy to remember the expressions for I_1, I_2 and I_3 just by noticing which resistance is missing in the term at the numerator.

Word of Advice

Be **careful** in applying current division. Without parallel resistors, there is no way that the current division can be applied. To say

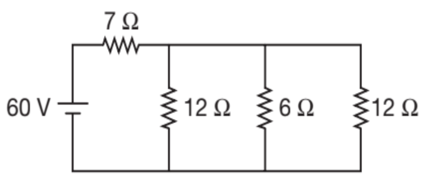
$$I_A = I_S \frac{R_B}{R_A + R_B} \text{ is } \textbf{wrong}.$$



Remember, parallel resistor must be the resistors connected between the same pair of terminals.

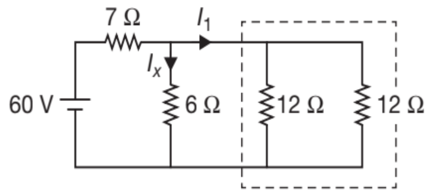
ILLUSTRATION 11

In the given network, calculate the current through 6 Ω resistor.

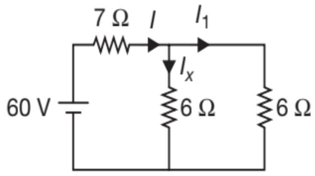


SOLUTION

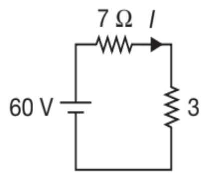
In this problem, our aim is to calculate only the current through 6 Ω resistor, we can do it **easily** by using current divider concept.



(a)



(b)



(c)

We first calculate the current I supplied by the battery. For this we rearrange the network and simplify it step by step as follows:

(a) We interchange the position of 6 Ω and 12 Ω resistors. This will not make any difference, as the two resistors are in parallel [Fig. (a)].

(b) We then combine the two 12 Ω resistors to give a single resistor of 6 Ω [Fig. (b)]

(c) We again combine the two 6 Ω resistors to give a single resistor of 3 Ω [Fig. (c)].

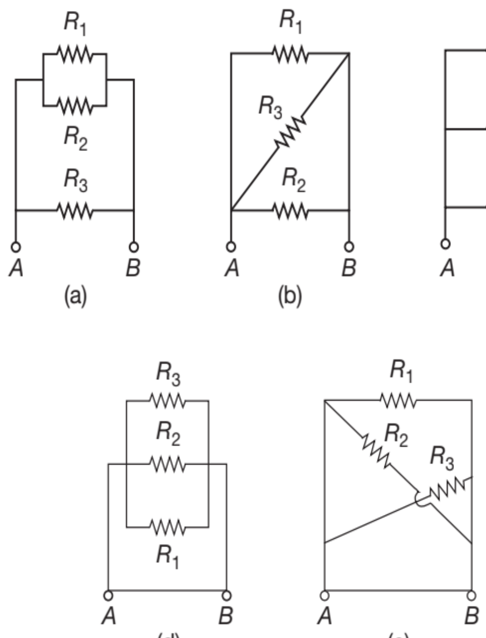
(d) The current I is clearly given as $I = \dfrac{60}{7+3} = 6$ A

(e) Using Fig. (b), and the current divider concept, we get the current through 6 Ω resistor as

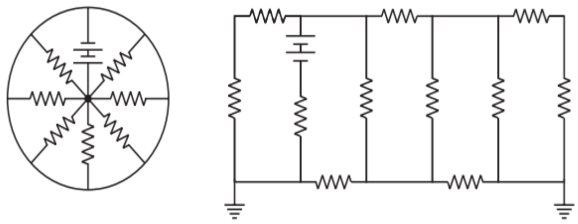
$$I_x = I\left(\frac{6}{6+6}\right) = 3 \text{ A}$$

NETWORK ANALYSIS

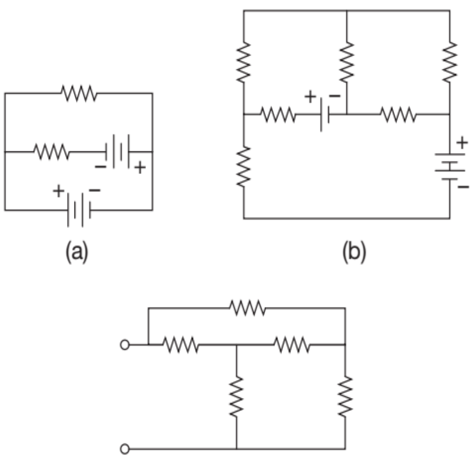
(a) The first step in analyzing a circuit is to simplify it to simplest equivalent configuration. In figure, each of the circuits has the same equivalent resistance between A and B, in each version, R_1, R_2 and R_3 are in parallel.

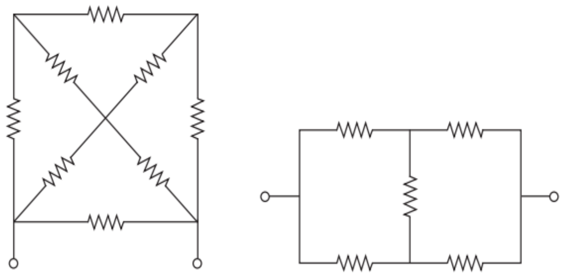


(b) All the resistors in figure (a) are in parallel arrangement. All the resistors in figure (b) are in series arrangement.

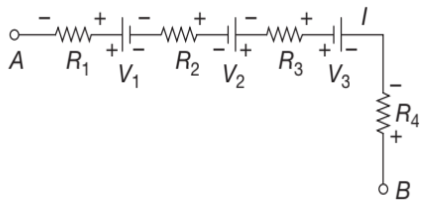

(a) (b)

(c) Figure shows several circuits in which the circuit elements are neither in series nor in parallel.

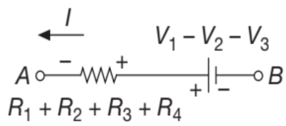

(a) (b) (c)


(d) (e)

(d) If the same current passes through every resistor in a given branch, respective of the presence of sources in that branch, the resistors are in series even though they are not directly connected to each other. Same is true about capacitors.



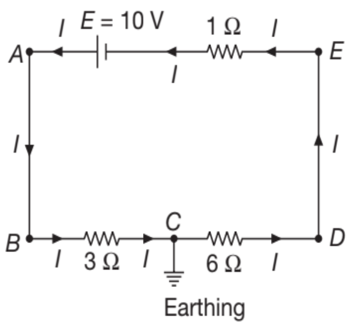
(a)



(b)

EARTHING OR GROUNDING IN AN ELECTRICAL CIRCUIT

If a certain point in an electrical circuit is earthed, its potential is taken as zero. Consider the following circuit. When only one point is earthed, then there is no effect on current flowing in the circuit.



Using *KVL* in *ABCDEA*,

$$-3I-6I-I+10=0$$

$\Rightarrow \quad I=1\text{ A}$

Now, $V_C = 0\text{ V}$ (because C is grounded)

Using KVL, $V_B - 3I = V_C$

$\Rightarrow \quad V_B = V_C + 3I$

$\Rightarrow \quad V_B = 3\text{ V}$

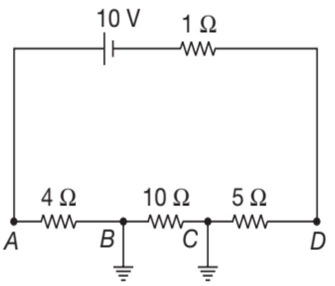
Similarly $V_C - 6I = V_D$

$\Rightarrow \quad V_D = -6\text{ V}$

Hence, earthing point gives us the reference potential i.e., zero potential and potentials of different points can then be calculated with respect to this new reference.

ILLUSTRATION 12

In the electrical circuit shown, points B and C are earthed. Find the potentials of points A and D.



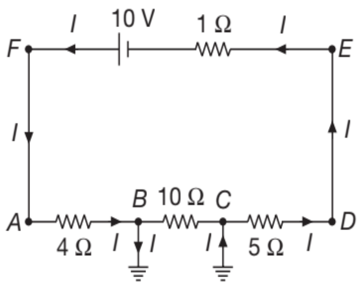
SOLUTION

Since B and C both are earthed, so

$$V_B = V_C = 0$$

$\Rightarrow \quad V_B - V_C = 0$

$\Rightarrow \quad$ Current through 10 Ω is zero



Applying *KJL* to closed loop *ABCDEFA*, we get

$$-4I+0-5I-(1)I+10=0$$

$\Rightarrow \quad I=1\text{ A}$

Now, $V_A - V_B = 4I = 4\text{ V}$

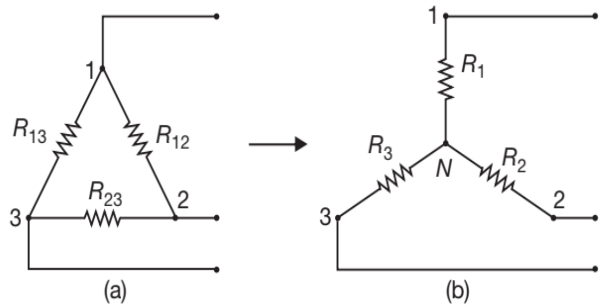
$\Rightarrow \quad V_A = 4\text{ V}$

Also, $V_C - V_D = 5I$

$\Rightarrow \quad V_D = -5\text{ V}$

DELTA TO STAR OR DELTA-STAR TRANSFORMATION

Suppose we are given three resistances R_{12}, R_{23} and R_{13} connected in delta fashion between terminals 1, 2 and 3 as in figure (a). So far as the respective terminals are concerned, these three given resistances can be replaced by the three resistances R_1, R_2 and R_3 connected in star as shown in figure (b).



These two arrangements will be electrically equivalent because resistance as measured between any pair of terminals is the same in both the arrangements.

$$R_1 = \frac{R_{12}R_{13}}{R_{12}+R_{23}+R_{13}}$$

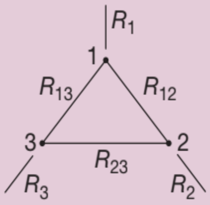
$$R_2 = \frac{R_{23}R_{12}}{R_{12}+R_{23}+R_{13}}$$

and $$R_3 = \frac{R_{13}R_{23}}{R_{12}+R_{23}+R_{13}}$$

Problem Solving Technique(s)

How to Remember (Delta to Star)?

Resistance of each arm of the star is given by the product of the resistances of the two delta sides that meet at its end divided by the sum of the three delta resistances.



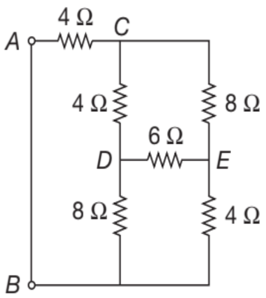
$$R_1 = \frac{R_{12}R_{13}}{R_{12}+R_{23}+R_{13}}$$

$$R_2 = \frac{R_{23}R_{12}}{R_{12}+R_{23}+R_{13}} \text{ and}$$

$$R_3 = \frac{R_{13}R_{23}}{R_{12}+R_{23}+R_{13}}$$

ILLUSTRATION 13

Find the input resistance of the circuit between the points A and B of figure.



SOLUTION

For finding R_{AB}, we will convert the delta CDE of figure into its equivalent star as shown in figure.

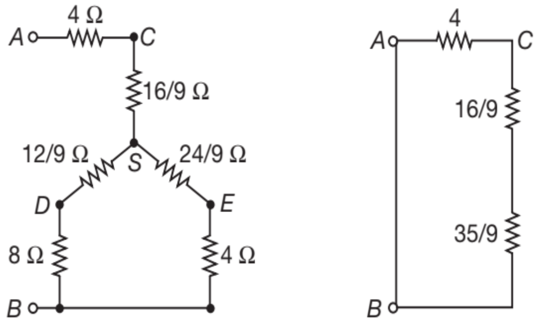
$$R_{CS} = 8\times\frac{4}{18} = \frac{16}{9}\ \Omega;\ R_{ES} = 8\times\frac{6}{18} = \frac{24}{9}\ \Omega$$

$$R_{DS} = 6\times\frac{4}{18} = \frac{12}{9}\ \Omega$$

The two parallel resistances between S and B can be reduced to a single resistance of $\frac{35}{9}\ \Omega$

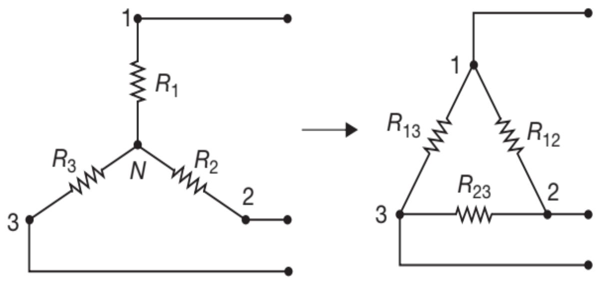
As seen from figure

$$R_{AB} = 4+\left(\frac{16}{9}\right)+\left(\frac{35}{9}\right) = \frac{87}{9}\ \Omega$$



STAR TO DELTA OR STAR-DELTA TRANSFORMATION

Suppose we are given three resistances R_1, R_2 and R_3 connected in star fashion between terminals 1, 2 and 3 as shown in figure (a). So far as the respective terminals are concerned, these three resistances can now be replaced by the three resistances R_{12}, R_{23} and R_{13} connected in the delta network as shown in figure (b).



(a) (b)

These two arrangements will be electrically equivalent because resistance as measured between any pair of terminals is the same in both the arrangements.

$$R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3}$$

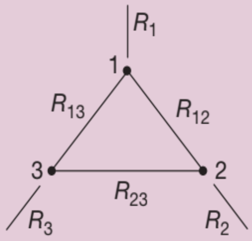
$$R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1}$$

and $$R_{13} = R_1 + R_3 + \frac{R_1R_3}{R_2}$$

Problem Solving Technique(s)

How to Remember (Star to Delta)?

The equivalent delta resistance between any two terminals is given by the sum of star resistances between those terminals plus the product of these two star resistances divided by the third star resistance.



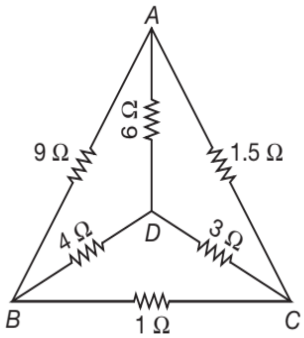
$$R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3},$$

$$R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1} \text{ and}$$

$$R_{13} = R_1 + R_3 + \frac{R_1R_3}{R_2}$$

ILLUSTRATION 14

A network of resistances is formed as follows:



(a)

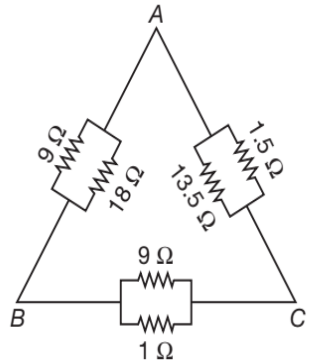
$AB = 9\ \Omega$; $BC = 1\ \Omega$; $CA = 1.5\ \Omega$ forming a delta and $AD = 6\ \Omega$, $BD = 4\ \Omega$ and $CD = 3\ \Omega$ forming a star. Compute the network resistance measured between

(a) A and B,
(b) B and C, and
(c) C and A.

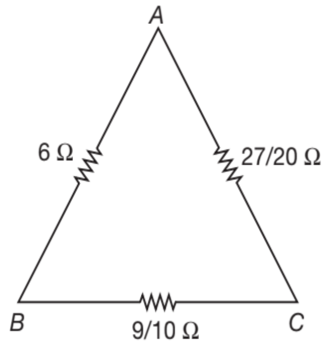
SOLUTION

The star of figure (a) may be converted into the equivalent delta and combined in parallel with the given delta ABC. The three equivalent delta resistances of the given star become as shown in figure (b).

When combined together, the final circuit is as shown in figure (c).



(b)



(c)

(a) As seen, there are two parallel paths across points A and B.

(i) First, directly from A to B having a resistance of $6\ \Omega$, and

(ii) the other via C having a total resistance of

$$\left(\frac{27}{20} + \frac{9}{10}\right) = 2.25\ \Omega$$

$$\Rightarrow \quad R_{AB} = \frac{6 \times 2.25}{(6 + 2.25)} = \frac{18}{11}\ \Omega$$

(b) $$R_{BC} = \frac{\frac{9}{10} \times \left(6 + \frac{27}{20}\right)}{\left(\frac{9}{10} + 6 + \frac{27}{10}\right)} = \frac{441}{550}\ \Omega$$

(c) $$R_{CA} = \frac{\frac{27}{20} \times \left(6 + \frac{9}{10}\right)}{\left(\frac{9}{10} + 6 + \frac{27}{20}\right)} = \frac{621}{550}\ \Omega$$

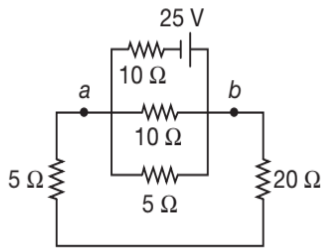
Test Your Concepts-IV

Based on Series and Parallel Combination of Resistances

(Solutions on page H.201)

1. Four copper wires of equal length are connected in series. Their cross-sectional areas are 1 cm^2, 2 cm^2, 3 cm^2, and 5 cm^2. A potential difference of 120 V is applied across the combination. Determine the voltage across the wire having the area 2 cm^2.

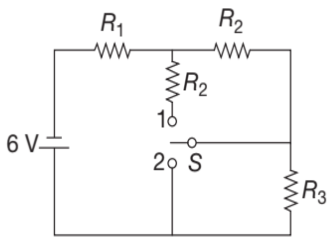
2. Consider the circuit shown in figure. Find



(a) the current in the 20 Ω resistor and

(b) the potential difference between points *a* and *b*.

3. A 6 V battery supplies current to the circuit shown in figure. When the double-throw switch *S* is open, as shown in the figure, the current in the battery is 1 mA. When the switch is closed in position 1, it is observed that the current in the battery is 1.2 mA and when the switch is closed in position 2, it is observed that the current in the battery is 2 mA. Find the resistances R_1, R_2 and R_3.

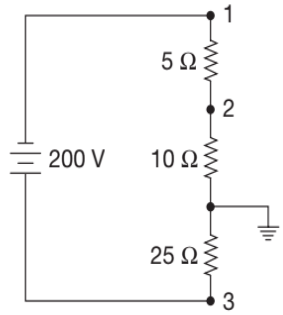


4. Two resistors connected in series have an equivalent resistance of 690 Ω. When they are connected in parallel, their equivalent resistance is 150 Ω. Find the resistance of each resistor.

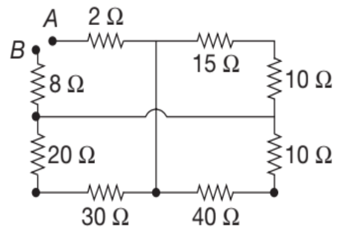
5. In the circuit shown, find the magnitude and polarity of the voltages mentioned below:

(i) V_1, (ii) V_2, (iii) V_3,

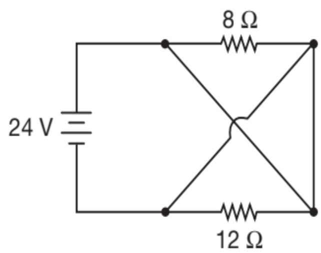
(iv) V_{3-2}, (v) V_{1-2}, (vi) V_{1-3}.



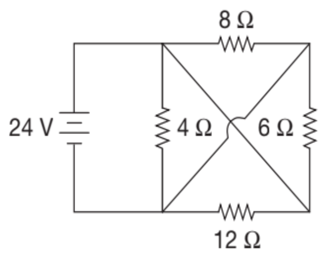
6. In the circuit shown, calculate the net resistance between the points *A* and *B*.



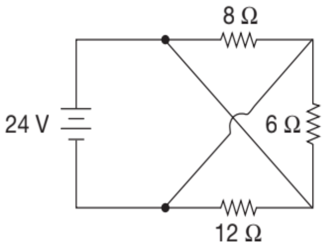
7. In the circuit shown, find the current supplied by the battery.



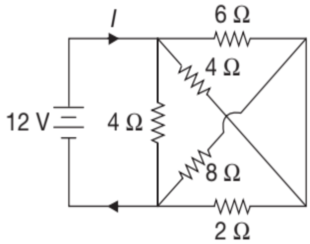
8. Find the current through the battery and the equivalent resistance of the network shown.



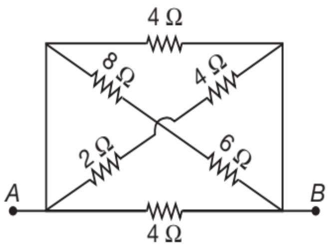
9. Find the total equivalent circuit resistance and current through the battery in the circuit shown.



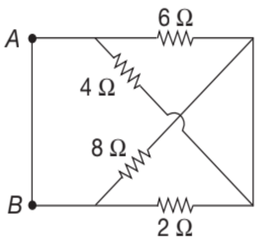
10. Compute the value of battery current I shown in figure.



11. Find the equivalent resistance between A and B in the following circuits:

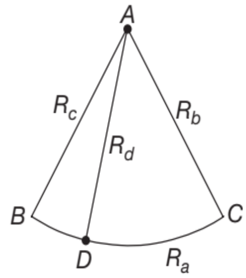


(a)

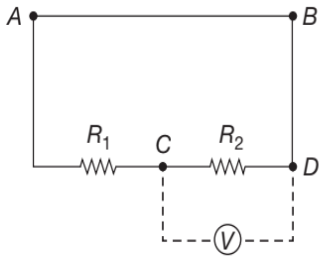


(b)

12. Three wires having resistance R_a, R_b and R_c are joined in the form of a triangle as shown. Another wire is joined at A and it makes a free sliding contact with BC. The resistance of this wire is R_d. Find the maximum resistance of the circuit (assume all wires to be uniform). The arc BC is a part of a circle whose centre is at A.

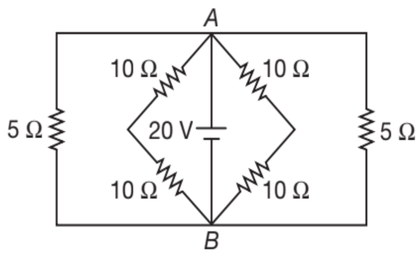


13. Resistances R_1 and R_2, each 60 Ω, are connected in series. The potential difference between points A and B is 120 V. Find the reading of a voltmeter connected to points C and D if its internal resistance is $r = 120$ Ω.

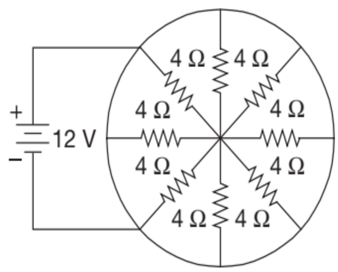


14. Show by diagram, how can we use a rheostat as the potential divider?

15. In the figure shown, determine the current through each 5 Ω resistor and the battery.

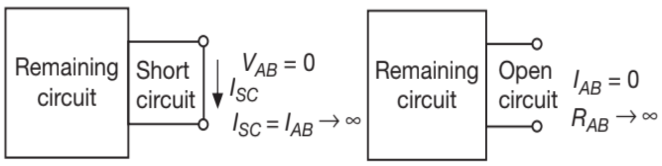


16. In figure shown we wish to determine current in one of the 4 Ω resistors in the circuit.



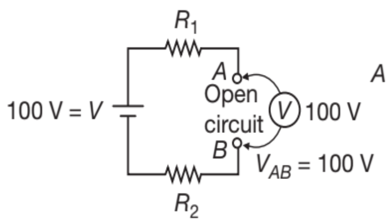
SHORT AND OPEN CIRCUITS

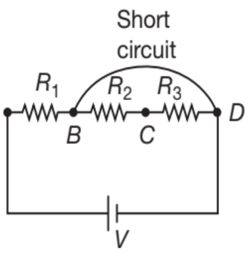
When two points of a circuit are connected together by a conducting wire, they are said to be **short circuited**. The connecting wire is assumed to have zero resistance. No voltage can exist across a short, secondly current through it is very large (theoretically infinity).



Two points are said to be open circuited when there is not direct connection between them, a break in the continuity of circuit exists. Due to this break the resistance between the two points is infinite and there is no flow of current between the two points.

Please note that the entire applied voltage is felt across the open, i.e., across terminals A and B, so, $V_{AB} = 100\text{ V}$.



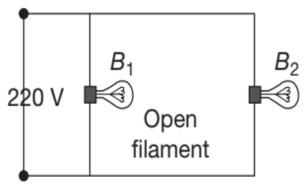


In the circuit shown if B and D are shorted, then

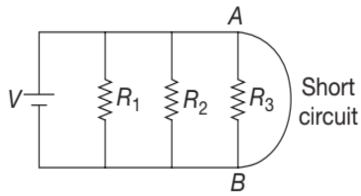
$$R_{eq} = R_1$$

$$\Rightarrow \quad I = \frac{V}{R_{eq}} = \frac{V}{R_1} \text{ and } V_B = V_D$$

Bulb B_1 acts as open circuit, bulb B_1 will not glow. However, other bulb B_2 remains connected across the voltage supply, it will operate normally.



Short circuit across R_3 shorts R_1 and R_2 as well, short across one branch in parallel means short across all branches. There is no current in shorted resistors. The shorted components are not damaged, they will function normally when short circuit is removed.



In figure shown, short circuit across R_3 may short out R_2 but not R_1, because it is protected by R_4.

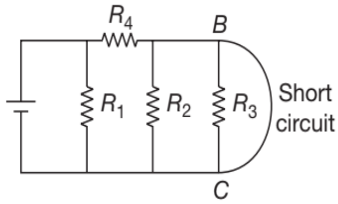
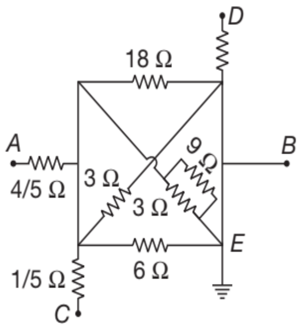


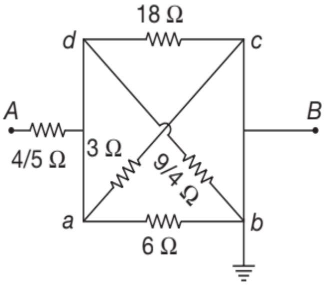
ILLUSTRATION 15

In figure shown, if a battery is connected between points A and B, emf $E = 18\text{ V}$, what is current through it?

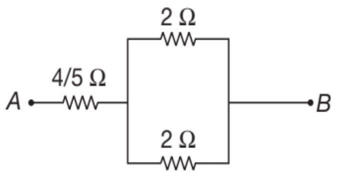


SOLUTION

The resistors at C and D are redundant, they are open branches, therefore no current through them. The point E is grounded, but no current will leak to ground, no return path for the current. Points a and d are at the same potential, similarly points b and c are at the same potential, because these points are connected by conducting wires without any circuit element.



Notice the $6\,\Omega$ and $3\,\Omega$ resistors, similarly $18\,\Omega$, $\frac{9}{4}\,\Omega$ resistors, these pairs are a parallel arrangement.



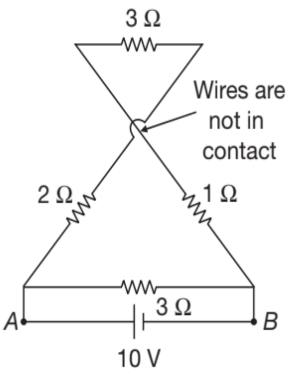
(Parallel of 3 Ω and 6 Ω)

Thus the equivalent resistance of the circuit between A and B is $1.8\,\Omega$. The current supplied by battery.

$$I = \left(\frac{18}{1.8}\right)\,\text{A} = 10\,\text{A}$$

ILLUSTRATION 16

In figure shown, determine the current provided by the battery.



SOLUTION

Just flip the circuit about the dotted line shown in the figure. Now the circuit is simple with equivalent resistance $2\,\Omega$ and current $I = \left(\frac{10}{2}\right)\,\text{A} = 5\,\text{A}$.

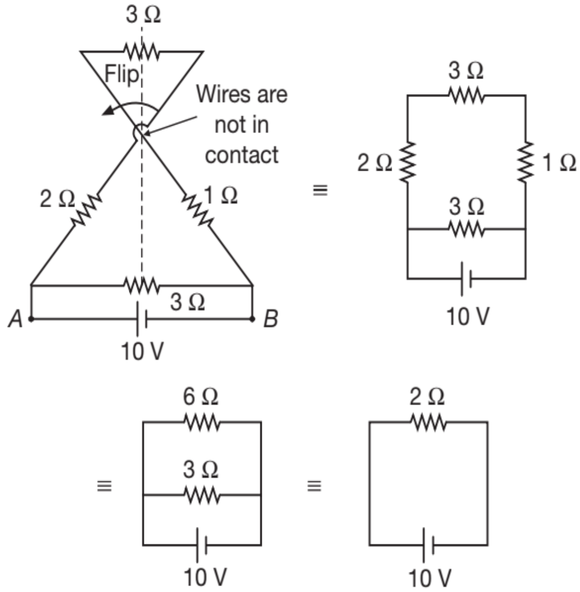
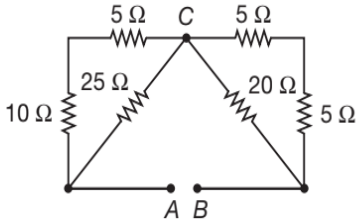


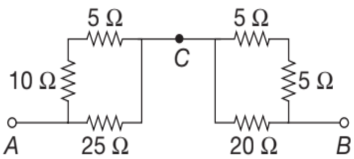
ILLUSTRATION 17

Find the equivalent resistance across AB of the circuit given.



SOLUTION

The circuit is redrawn to make it easily evaluable.



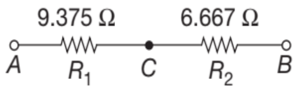
The left block is equivalent to $15\,\Omega$ and $25\,\Omega$ in parallel, and its resistance is given as

$$R_1 = \frac{25 \times 15}{25 + 15} = 9.375\,\Omega$$

The right block is equivalent to $10\,\Omega$ and $20\,\Omega$ in parallel, and its resistance is given as

$$R_2 = \frac{10 \times 20}{10 + 20} = \frac{200}{30} = 6.667\,\Omega$$

The circuit now reduces to two resistors in series, as shown.



Hence, the total resistance,

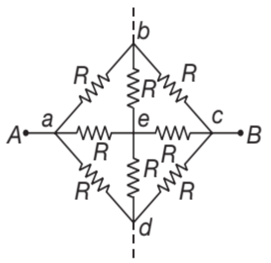
$$R = R_1 + R_2 = 9.375 + 6.667 = 16.042\ \Omega$$

PRINCIPLE OF SYMMETRY

If a network is symmetrical on both sides of a line, all the points lying on the line will have same potential. Hence, no current will flow in a resistance connected between two such points. Such resistances can be ignored while calculating the equivalent resistance. This simplifies the network. Or, else all the points on this line may be treated as shorted.

SYMMETRICAL CIRCUITS

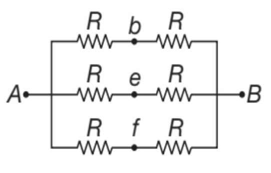
When a circuit is symmetrical about a line (By symmetry we mean that two parts are mirror images of each others), then the potential and current must also be symmetrical.



(a)

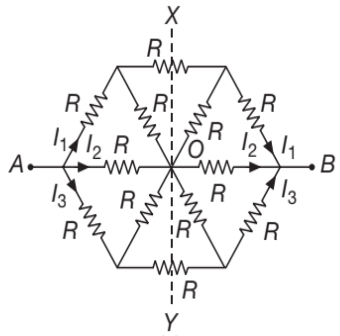
Therefore, currents in ab and ad are same. Currents in dc and bc are same. Potentials of the points b, e and d are same. The equivalent circuit is redrawn, the equivalent resistance is $\frac{2}{3}R$. Note that there is no current in branches be and ef.

Another symmetry is visible along line bd.



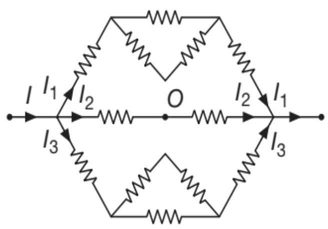
(b)

The current flow is not a mirror image in branches ab and bc because the flow is in same direction. This is called asymmetric condition. The special thing about this asymmetry is that current incoming at b is equal to outgoing current, similar situation exists at b and d also. Thus resistors in branches be and de are ineffective. In the following figure there is a symmetry along the xy.



(a)

The current I_2 reaching O is equal to outgoing current, which simply means there is no mingling of current from upper branch and lower branch into middle branch and hence we can remove the connection of the respective upper and lower branches as shown.

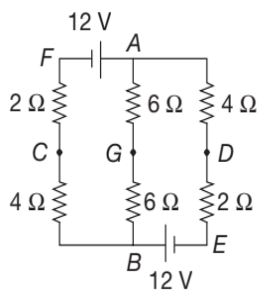


(b)

The resulting circuit is simple enough and the equivalent resistance is $\frac{4R}{5}$.

ILLUSTRATION 18

Find the current in the branch AB of the circuit. Also find the current in the branches BF and EA.

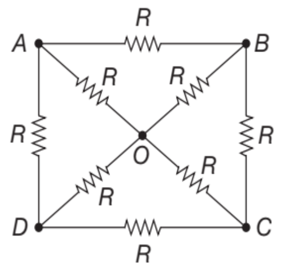


SOLUTION

Referring to figure we see that symmetry demands that current only circulates in outer branch. Points A and B are at the same potential because the circuit is symmetrical. Therefore, no current can go across the resistors in that branch. The current through BF and EA is 2 A.

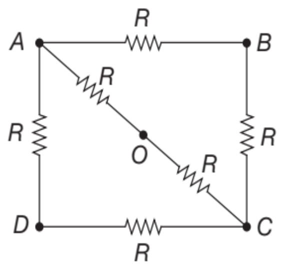
ILLUSTRATION 19

Calculate effective resistance between points A and C for the networks shown.



SOLUTION

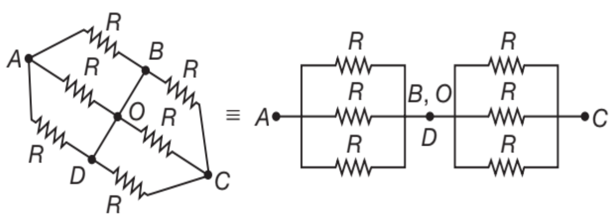
The two sides of the line BOD, are symmetrical. Thus, if the points A and C are connected to a source of emf, points B, O and D of the network will have same potential. Therefore, no current flows through R_{BO} and R_{DO}. These resistances can be ignored. The network then reduces to that shown in figure.



Thus, between points A and C, we now have three branches in parallel, each having resistance of $2R$.

$$\Rightarrow \quad R_{eq} = \frac{2R}{3}$$

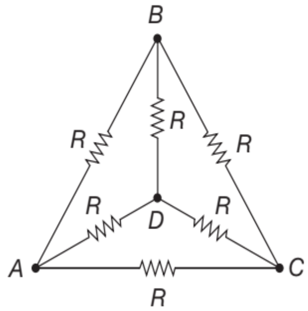
Alternately, since the points B, O and D are at same potential, we can treat them shorted.



$$\Rightarrow \quad R_{eq} = \frac{R}{3} + \frac{R}{3} = \frac{2R}{3}$$

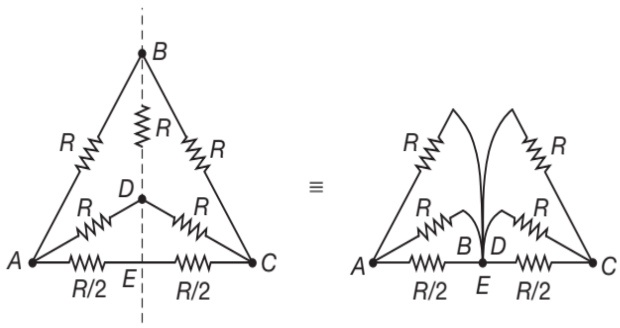
ILLUSTRATION 20

Calculate the effective resistance between points A and C, by applying symmetry principle.



SOLUTION

Break the branch AC into two resistors in series, each $\frac{R}{2}$, and consider the dotted line passing through B, D and E. The network on the two sides of this line is symmetrical. Hence, one can short-circuit the points B, D and E and calculate $(R_{eq})_{AE}$.



Now, we find that resistances R, R and $\frac{R}{2}$ are in parallel across A and E. The parallel combination of R and R gives $\frac{R}{2}$. This $\frac{R}{2}$ in parallel with $\frac{R}{2}$ gives $\frac{R}{4}$. Hence,

$$(R_{eq})_{AE} = \frac{R}{4}$$

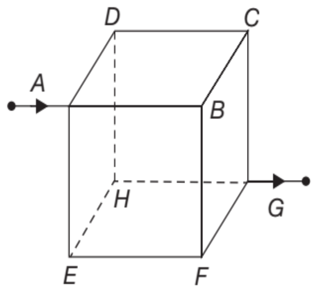
$$\Rightarrow \quad (R_{eq})_{AC} = 2(R_{eq})_{AE} = 2\left(\frac{R}{4}\right) = \frac{R}{2}$$

IDENTICAL POTENTIAL POINTS

In some networks, you may not find symmetry on the two sides of a line. But, you may find that the network contains some set of points having identical potential. Such a set of points can be joined together to make the network simple.

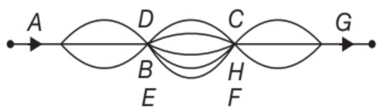
ILLUSTRATION 21

Twelve equal wires of resistances R each are joined up to form the edges of a cube, as shown in figure. The cube is connected into a circuit across the diagonal AG. Find the equivalent resistance of the network.



SOLUTION

Let us search the points of identical potential. Since the three edges of the cube from A, viz., AB, AD and AE are identical in all respects in the circuit, the points B, D and E are at the same potential. Similarly, for the point G, the sides GC, GH and GF are symmetrical and the points C, H and F are at the same potential.



Next, to simplify the circuit, we bring together the points, B, D and E and also C, H and F.

Now, it becomes obvious that

the resistance between A and D = R/3

the resistance between D and C = R/6

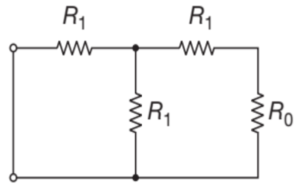
the resistance between C and G = R/3

Thus, the circuit is equivalent to three resistances of value. R/3, R/6 and R/3, in series, and hence the net resistance between A and G is

$$R_{AG} = \frac{R}{3} + \frac{R}{6} + \frac{R}{3} = \frac{5R}{6}$$

ILLUSTRATION 22

Consider the circuit shown in figure. For a given resistance R_0, what must be the value of R_1 so that the equivalent resistance between the terminals is equal to R_0?



SOLUTION

The equivalent resistance, R', due to the three resistors on the right is

$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_0 + R_1} = \frac{R_0 + 2R_1}{R_1(R_0 + R_1)}$$

$$\Rightarrow \quad R' = \frac{R_1(R_0 + R_1)}{R_0 + 2R_1}$$

Since R' is in series with the fourth resistor R_1, the equivalent resistance of the entire configuration becomes

$$R_{eq} = R_1 + \frac{R_1(R_0 + R_1)}{R_0 + 2R_1} = \frac{3R_1^2 + 2R_1R_0}{R_0 + 2R_1}$$

If $R_{eq} = R_0$, then

$$R_0(R_0 + 2R_1) = 3R_1^2 + 2R_1R_0$$

$$\Rightarrow \quad R_0^2 = 3R_1^2$$

$$\Rightarrow \quad R_1 = \frac{R_0}{\sqrt{3}}$$

INFINITE LADDER AND GRID

Some networks make a ladder (or a grid) and extend to infinity. To reduce such networks we use the following steps.

STEP-1: Let us assume the total resistance of the infinite network to be X (say).

STEP-2: Now just retain one basic repetitive unit and we observe the remaining circuit to be the same as the original circuit. So, resistance of this left out circuit must be X.

STEP-3: Now the equivalent circuit, is the combination of basic unit and original repetitive circuit of resistance X, such that the net resistance of the entire circuit is X.

The following illustrations are done on the basis of these three steps.

ILLUSTRATION 23

(a) Find effective resistance between points A and B of an infinite chain of resistors joined as shown in Figure 1.

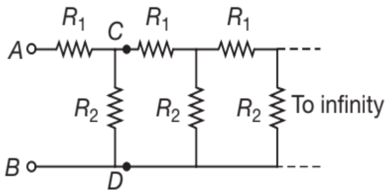


Figure 1

(b) For what value of R_0 in the circuit shown in Figure 2 will the net effective resistance is independent of the number of cells in the chain?

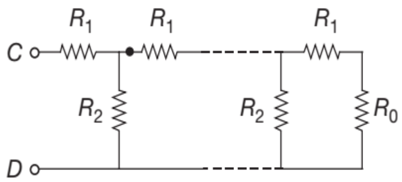


Figure 2

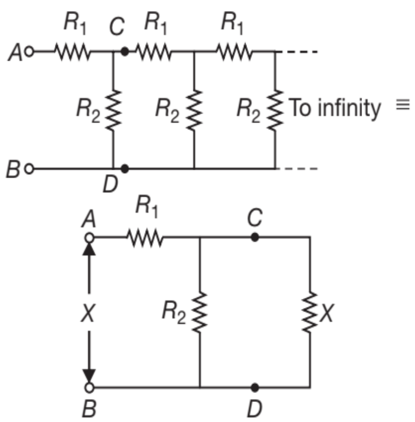
SOLUTION

(a) Suppose the effective resistance between A and B is X. Applying the steps discussed, we get

$$\Rightarrow \quad X = R_1 + [R_2 \parallel X] = R_1 + \frac{R_2 X}{(R_2 + X)}$$

$$\Rightarrow \quad X^2 - R_1 X - R_1 R_2 = 0$$

$$\Rightarrow \quad X = \frac{1}{2}\left(R_1 \pm \sqrt{(R_1^2 + 4R_1R_2)}\right)$$



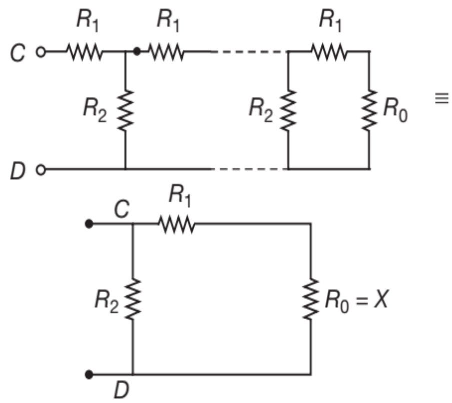
And as resistance can not be negative, we have

$$X = \frac{R_1}{2}\left(1 + \sqrt{1 + \frac{4R_2}{R_1}}\right)$$

However, if $R_1 = R_2 = R$ we get

$$X = \frac{R}{2}(1 + \sqrt{5}) = 1.6\ \mathrm{R}$$

(b) Suppose there are n sections between points A and B and the network is terminated by R_0 with equivalent resistance X. Now, if we add one more to the network between the C and D, the equivalent resistance of the network X will be independent of number of cells if the resistance between points C and D still remains R_0 (or X). So, the circuit reduces to an equivalent circuit as shown.



$$\Rightarrow \quad R_1 + \frac{R_2 R_0}{(R_2 + R_0)} = R_0$$

$$\Rightarrow \quad X = R_0 = \frac{1}{2}\left(1 + \sqrt{1 + \frac{4R_2}{R_1}}\right)$$

If $R_1 = R_2 = R$, then we get

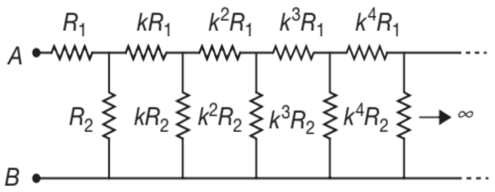
$$X = R_0 = \frac{R}{2}(1 + \sqrt{5}) = 1.6R$$

Problem Solving Technique(s)

Whenever each element of the circuit is multiplied by a factor, say k, then the equivalent resistance is also multiplied by the same factor.

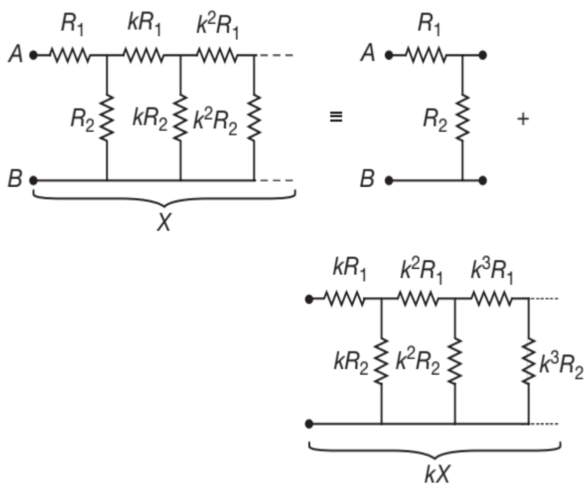
ILLUSTRATION 24

The circuit diagram shown consists of a large number of elements (each element has two resistors R_1 and R_2). The resistances of the resistors in each subsequent element differs by a factor of $k=\frac{1}{2}$ from the resistances of the resistors in the previous elements. Find the equivalent resistance between A and B shown in figure.

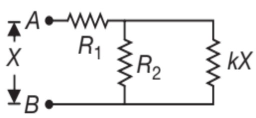


SOLUTION

When each element of circuit is multiplied by a factor k then equivalent resistance also becomes k times. Let the equivalent resistance between A and B be X.



So, equivalent circuit becomes



For $k=\frac{1}{2}$

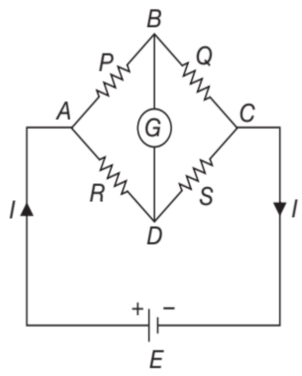
$$\Rightarrow \quad X=\frac{(R_1-R_2)+\sqrt{R_1^2+R_2^2+6\,R_1R_2}}{2}$$

BALANCED WHEATSTONE BRIDGE LIKE SITUATIONS

Condition of balance is

$$\frac{P}{Q}=\frac{R}{S}$$

So, to conclude

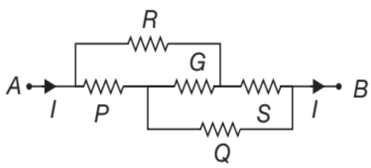


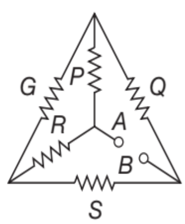
(a) When battery and galvanometer arms of a Wheatstone's Bridge are interchanged, the balance position remains undisturbed while sensitivity of bridge changes.

(b) When Wheatstone's Bridge is balanced, the resistance in arm BD may be ignored while calculating the equivalent resistance of bridge between A and C. So, P and Q are in series, R and S are in series and both in parallel to each other.

$$\Rightarrow \quad \frac{1}{R_{net}}=\frac{1}{P+Q}+\frac{1}{R+S}$$

Three other common forms of balanced Wheatstone's Bridge are shown here.





and

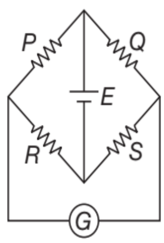
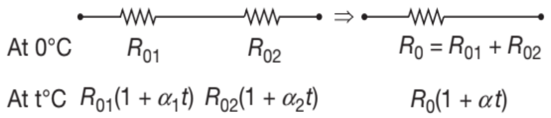


ILLUSTRATION 25

Two resistors with temperature coefficients of resistance α_1 and α_2 have resistances R_{01} and R_{02} at 0 °C. Find the temperature coefficient of the compound resistor consisting of the two resistors connected, (a) in series (b) in parallel

SOLUTION

In Series:



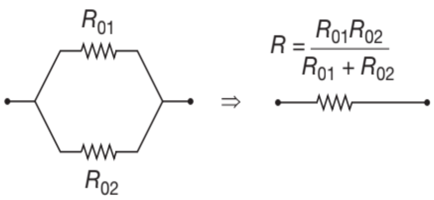
$$R_{01}(1+\alpha_1 t)+R_{02}(1+\alpha_2 t)=R_0(1+\alpha t)$$

$$\Rightarrow R_{01}(1+\alpha_1 t)+R_{02}(1+\alpha_2 t)=(R_{01}+R_{02})(1+\alpha t)$$

$$\Rightarrow R_{01}+R_{01}\alpha_1 t+R_{02}+R_{02}\alpha_2 t=R_{01}+R_{02}+(R_{01}+R_{02})\alpha t$$

$$\Rightarrow \alpha=\frac{R_{01}\alpha_1+R_{02}\alpha_2}{R_{01}+R_{02}}$$

In Parallel:



At t °C, $$\frac{1}{R_0(1+\alpha t)}=\frac{1}{R_{01}(1+\alpha_1 t)}+\frac{1}{R_{02}(1+\alpha_2 t)}$$

$$\Rightarrow \frac{R_{01}+R_{02}}{R_{01}R_{02}(1+\alpha t)}=\frac{1}{R_{01}(1+\alpha_1 t)}+\frac{1}{R_{02}(1+\alpha_2 t)}$$

Since $\alpha t \ll 1$, so by using the Binomial expansion, we get

$$\frac{1}{R_{02}}(1-\alpha t)+\frac{1}{R_{01}}(1-\alpha t)=\frac{1}{R_{01}}(1-\alpha_1 t)+\frac{1}{R_{02}}(1-\alpha_2 t)$$

$$\Rightarrow \alpha t\left(\frac{1}{R_{01}}+\frac{1}{R_{02}}\right)=\frac{\alpha_1}{R_{01}}t+\frac{\alpha_2}{R_{02}}t$$

$$\Rightarrow \alpha=\frac{\alpha_1 R_{02}+\alpha_2 R_{01}}{R_{01}+R_{02}}$$

ILLUSTRATION 26

A rod of length L and cross-section area A lies along the x-axis between $x=0$ and $x=L$. The material obeys Ohm's Law and its resistivity varies along the rod according to,

$$\rho(x)=\rho_0 e^{-x/L}$$

The end of the rod at $x=0$ is at a potential V_0 and it is zero at $x=L$

(a) Find the total resistance of the rod and the current in the wire.

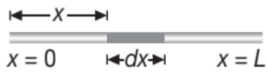
(b) Find the electric potential $V(x)$ in the rod as a function of x.

SOLUTION

(a) Resistance of elementary section dx at $x=x$ is,

$$dR=\frac{\rho(x)dx}{A} \qquad \left\{\because\ R=\rho\frac{l}{A}\right\}$$

$$\Rightarrow dR=\frac{\rho_0 e^{-\frac{x}{L}}dx}{A}$$



Since all such elements are in series, hence

$$R=\int_0^L dR=\frac{\rho_0}{A}\int_0^L e^{-\frac{x}{L}}dx=\frac{\rho_0 L}{A}(1-e^{-1})$$

$$\Rightarrow R=\frac{\rho_0 L}{A}\left(1-\frac{1}{e}\right)$$

Current in the wire is given by

$$I=\frac{V_0}{R}=\frac{V_0 A}{\rho_0 L}\left(\frac{e}{e-1}\right)$$

(b) $dV=IdR$

$$dV=\frac{V_0 A}{\rho_0 L}\left(\frac{e}{e-1}\right)\frac{\rho_0 e^{-x/L}}{A}\cdot dx$$

$$dV=\frac{V_0}{L}\left(\frac{e}{e-1}\right)e^{-x/L}dx$$

$$\Rightarrow \int_{V_0}^{V(x)}dV=\frac{V_0}{L}\left(\frac{e}{e-1}\right)\int_0^x e^{-x/L}dx$$

$$\Rightarrow V(x)-V_0=V_0\left(\frac{e}{1-e}\right)(1-e^{-x/L})$$

$$\Rightarrow V(x)=\frac{V_0(e^{-x/L}-e^{-1})}{1-e^{-1}}$$

Test Your Concepts-V

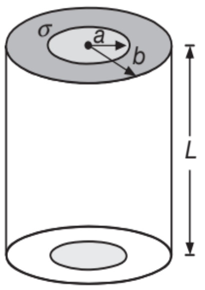
Based on Series and Parallel Combination of Resistances

(Solutions on page H.203)

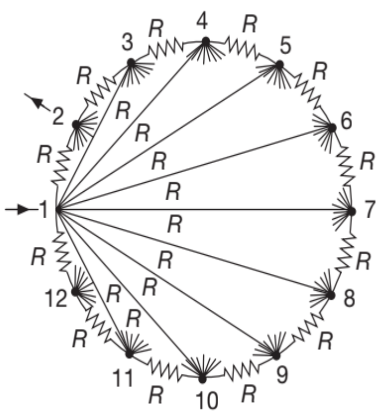
1. The space between two coaxial cylinders, whose radii are a and b (where $a < b$) is filled with a conducting medium. The specific conductivity of the medium is σ.

(a) Compute the resistance between the cylinders in the radial direction. Assume that the cylinders are very long compared to their radii, i.e., $L \gg b$, where L is the length of the cylinders.

(b) Calculate the resistance, assuming σ varies as $\sigma(r) = \dfrac{\sigma_0}{r}$

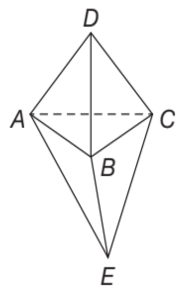


2. (a) A network of 12 resistors each of value $R = 6\ \Omega$ are interconnected as shown in figure, being placed along the sides of a regular dodecagon. Each of the terminals 1, 2, 3, …, 12 has been connected to each of the 9 terminals (other than nearest) directly by insulated wires each of resistance R, there being 9 wires from each terminal making 108 wire connections totally. [Only one set of 9 wires, from terminal 1 have been shown]. Find the equivalent resistance of the network when the current enters at the terminal 1 and leaves at terminal 2.



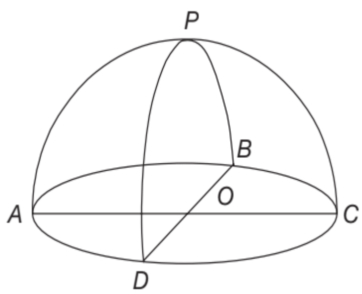
(b) If the above network were generalised so that there are n $(n = even)$ resistors each of resistance R placed along the sides of a regular n sided polygon and if each terminal point of a resistor were connected by $(n - 2)$ insulated wires each of resistance R, directly to the $(n - 2)$ terminals, other than its, nearest terminals, find the equivalent resistance across any two terminals of the network (i.e., current entering at one of the two terminals and leaving by the other.

3. Calculate the equivalent resistance of the triangular bipyramid between the points specified. Assume the resistance of each branch to be R.

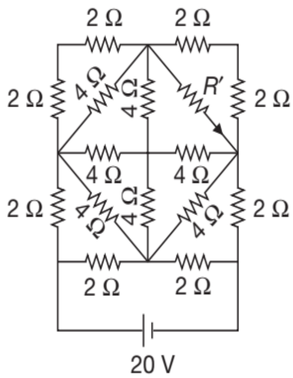


(a) A and C
(b) D and E

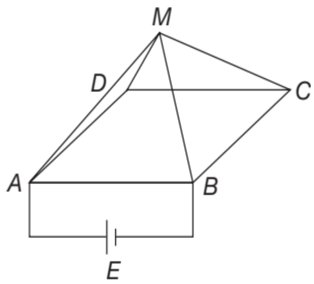
4. A hemispherical network of radius a is made by using a conducting wire of resistance per unit length λ. Calculate the equivalent resistance across OP.



5. In the circuit shown, all the side resistances are of value $2\ \Omega$ and all inner resistances are of value $4\ \Omega$. Find the current via branch AB.



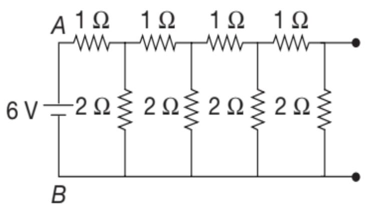
6. A square pyramid, having vertex at M is formed by joining 8 equal resistances R across the edges. The square base of the pyramid has the corners at A, B, C and D. Calculate the current in the edge



(a) MC if an ideal cell of emf E is connected across the adjacent corners A and B.
(b) MA if an ideal cell of emf E is connected across the opposite corners A and C.

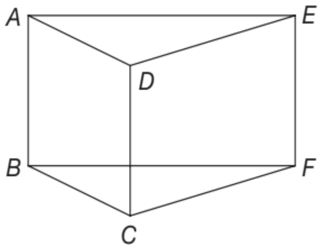
7. A length $4a$ of uniform wire having resistance per unit length λ, is bent in the form of a square and the opposite angular points are formed with straight pieces of the same wire which are in contact at the intersection. A given current enters at the intersection of diagonals and leaves at an angular point. Find the resistance of the whole network.

8. An infinite ladder network of resistances is constructed with 1 Ω and 2 Ω resistances, as shown in figure. (1987; 7M)
The 6 volt battery between A and B has negligible internal resistance.



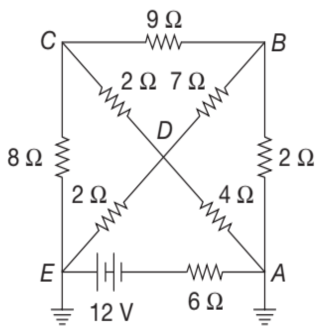
(a) Show that the effective resistance between A and B is 2 Ω
(b) What is the current that passes through the 2 Ω resistance nearest to the battery?

9. Nine wires each of resistance r are connected to make a prism as shown in figure. Find the equivalent resistance of the arrangement across.

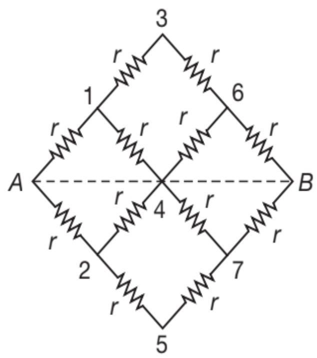


(a) AD
(b) AB

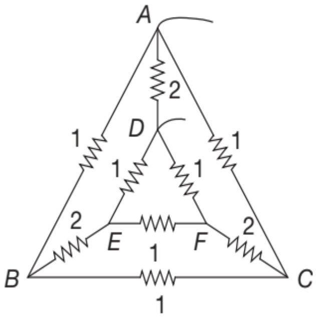
10. In figure, determine the current through 6 Ω and 9 Ω resistor.



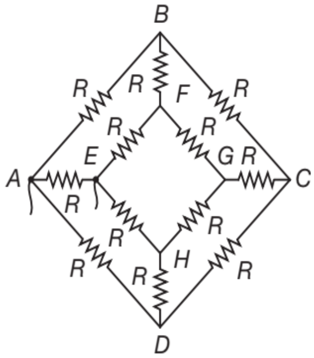
11. Determine equivalent resistance between A and B.



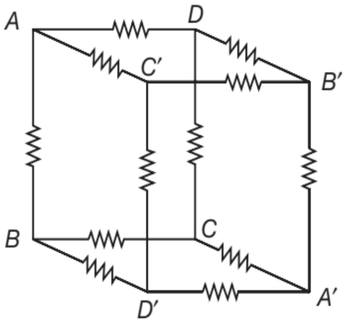
12. In the figure shown, the resistances specified are in ohms. Determine the equivalent resistance between points A and D.



13. In the network shown in figure all the resistances are equal. Determine equivalent resistance between A and E.

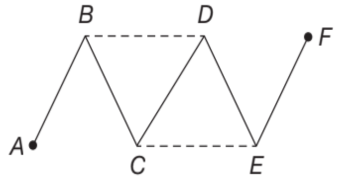


14. Twelve equal resistors each $R\ \Omega$ are connected to form the edges of a cube. Find the equivalent resistance of the network:

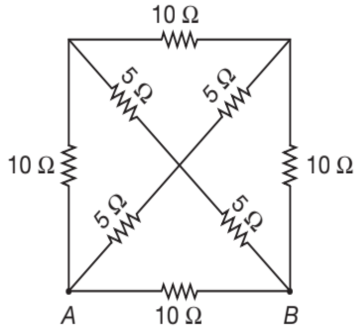


(a) when current enters at A and leaves at A'.
(b) when current enters at A and leaves at B'.
(c) when current enters at A and leaves at B.

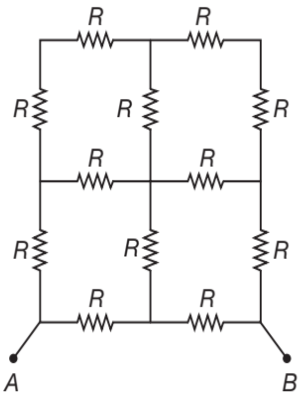
15. What will be the change in the resistance of a circuit between A and F consisting of five identical conductors if two similar conductors added as shown by the dashed line in figure.



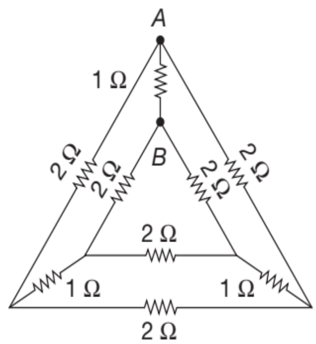
16. Find the equivalent resistance between A and B in the following circuit:



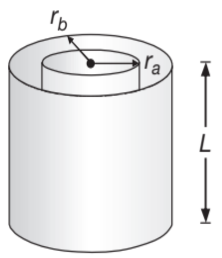
17. Find the equivalent resistance between A and B in the following circuit:



18. Find the equivalent resistance between A and B in the following circuit:



19. An oceanographer is studying the variation of the ion concentration in sea water with depth. This is done by lowering into the water a pair of concentric metallic cylinders at the end of a cable and taking data to determine the resistance between these electrodes as a function of depth. The water between the two cylinders forms a cylindrical shell of inner radius r_a, outer radius r_b and length L much larger than r_b. A potential difference ΔV is applied between the inner and outer surfaces, producing an outward radial current I. Let ρ represent the resistivity of the water. Find the



(a) resistance of the water between the cylinders in terms of L, ρ, r_a and r_b.

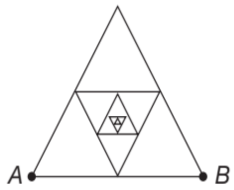
(b) resistivity of the water, ρ in terms of the measured quantities L, r_a, r_b, ΔV and I.

20. The cross section area and length of *a* cylindrical conductor are A and ℓ, respectively. The specific conductivity varies as $\sigma(x) = \sigma_0 \frac{1}{\sqrt{x}}$, where x is the distance along the axis of the cylinder from one of its ends.

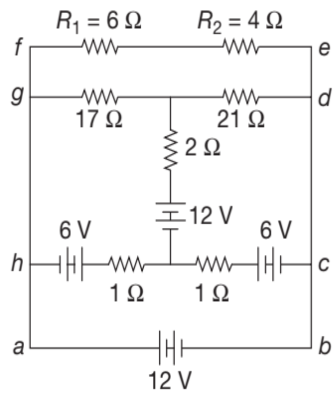
(a) Compute the resistance of the system along the cylindrical axis.

(b) Compute the current density if the potential drop along the cylinder is V_0. What is the electric field at each point in the cylinder in the case described?

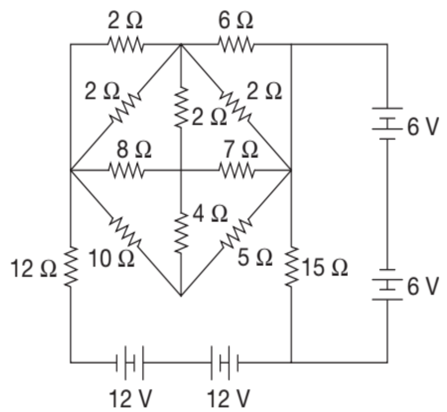
21. A frame made of thin homogeneous wire is shown in figure. Assume that the number of successively embedded equilateral triangle with sides decreasing by half tends to infinity. The side AB has a resistance R_0. Find the equivalent resistance between A and B.



22. In the circuit shown in figure, determine the current and potential drop across resistor R_1.



23. Determine the current through the 8 Ω resistor in figure.



KIRCHHOFF'S CIRCUIT RULES

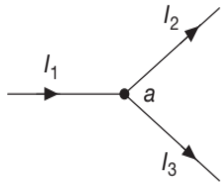
In analyzing circuits, there are two fundamental (Kirchhoff's) rules

Junction Rule

At any point where there is a junction between various current carrying branches, by current conservation the sum of the currents into the node must equal the sum of the currents out of the node (otherwise charge would build up at the junction)

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

In other words this law states that current or charge cannot accumulate at a junction. This law is in accordance with the Law of Conservation of Charge.



Kirchhoff's junction rule

According to the junction rule, the three currents shown here are related by

$$I_1 = I_2 + I_3$$

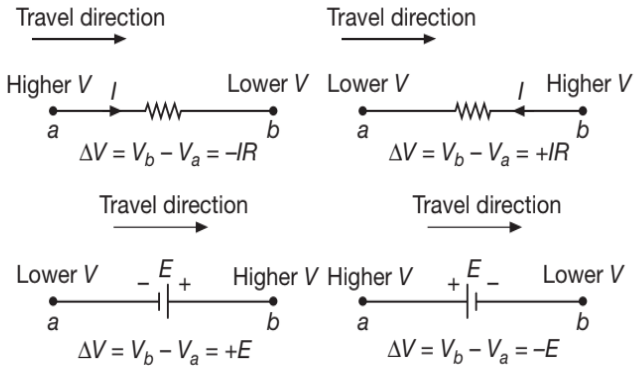
Loop Rule

The sum of the voltage drops ΔV, across any circuit elements that form a closed circuit is zero.

$$\sum_{\text{closed loop}} \Delta V = 0$$

This rule is simply saying that there cannot be a potential difference between a point and itself and is also in accordance with the Law of Conservation of Energy.

The rules for determining ΔV across a resistor and a battery with a designated travel direction are shown below:



Convention for determining ΔV

Note that the choice of travel direction is arbitrary. The same equation is obtained whether the closed loop is traversed clockwise or counter clockwise. Also, remember that a closed loop must start and end at the same point.

Problem Solving Technique(s)

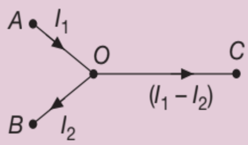
Applying Kirchhoff's Rules

In this chapter we have seen how Kirchhoff's rules can be used to analyze multiloop circuits. The steps are summarized below:

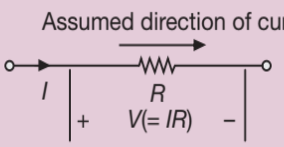
(a) Draw a circuit diagram, and label all the quantities, both known and unknown. The number of unknown quantities is equal to the number of linearly independent equations we must look for.

(b) Assign a direction to the current in each branch of the circuit. (If the actual direction is opposite to what you have assumed, your result at the end will be a negative number).

(c) Apply *KCL* to each junction. Often, it is found more convenient to apply *KCL* while marking the currents in the branches. This reduces the number of unknowns to be solved. For example, if we have assumed currents in the branches *AO* and *OB* as I_1 and I_2, we need not mark the current in branch *OC* as I_3. Instead we apply *KCL* and say that the current in branch *OC* is $(I_1 - I_2)$.



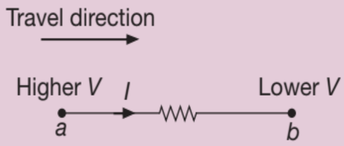
(d) Mark the polarity of the voltage across each element. In a resistor, the polarity depends upon the assumed direction of current. The end into which the current enters is marked positive.



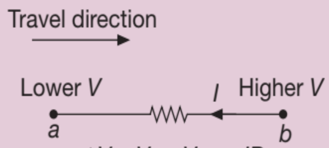
(e) Apply the loop rule to the loops until the number of independent equations obtained is the same as the number of unknowns. For example, if there are three unknowns, then we must write down three linearly independent equations in order to have a unique solution.

Traverse the loops using the convention below for ΔV

Resistor

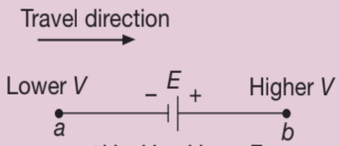


$\Delta V = V_b - V_a = -IR$

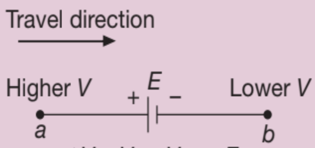


$\Delta V = V_b - V_a = +IR$

EMF Source

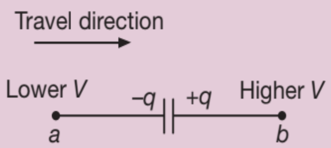


$\Delta V = V_b - V_a = +E$

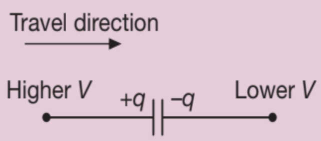


$\Delta V = V_b - V_a = -E$

Capacitor



$\Delta V = V_b - V_a = +q/C$

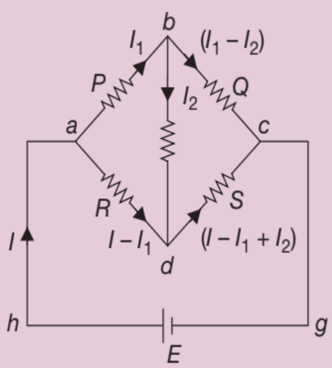


$\Delta V = V_b - V_a = -q/C$

The same equation is obtained whether the closed loop is traversed clockwise or counterclockwise. Try to take as many equations as the number of variables.

Start from any point and mentally go around the loop in the designated direction. Write down directly the voltage of each element whose positive (+) terminal is entered, and write down the negative of every voltage whose negative (–) terminal is entered.

(f) If necessary, choose other loops and repeat above steps until you get as many independent equations as the unknowns. The set of equations obtained will be independent provided each new loop equation contains a voltage change not included in a previous equation. The easiest way to ensure the independency of equations is to select all meshes (just like the panes of a window) as the loops for writing *KVL*. You may have to select loops as per the number of variables to be calculated. Another thing we must keep in mind while selecting loops is that we must select the loops independently. Suppose we require three loops to be selected and we select a bigger loop along with two loops (these two loops collectively form the bigger loop) and write three equations. We will notice that actually we may not be getting three equations but instead we shall be getting only two equations. So avoid taking such type of loops and try to identify and take independent loops. Suppose, in this circuit, we take the currents as shown. Hence we observe that at least three equations are required to get I, I_1 and I_2.



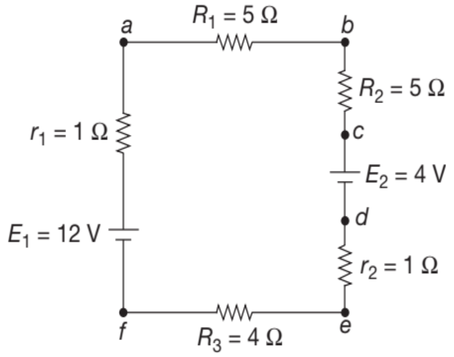
Please be careful and take loops as *abda*, *bcdb* (or *abcda*) because *abcda* = *abda* + *bcdb*. So these are just two loops. The third loop must be *abcgha* (or *adcgha* or *adbcgha* or *abdcgha*).

So, while taking the loops, keep in mind to take unique loops. Avoid taking big loops made from smaller loops (for which equations have already been taken) because the big loop made from the combination of small loops will again fetch you the identical equations which cannot be used as such.

(g) Solve the simultaneous equations to obtain the solutions for the unknowns.

ILLUSTRATION 27

In the arrangement shown, find the



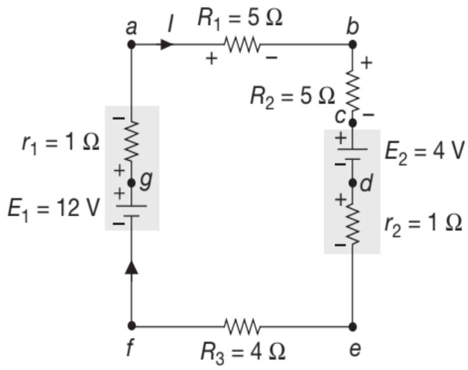
(a) current I in the circuit,

(b) potential at each of the labelled points a, b, c, d, e, assuming potential at f to be zero,

(c) power input and output in the circuit.

SOLUTION

(a) First we assume that current is clockwise. Now we apply KVL in the assumed direction of current.



Circuit element	Changes in potential	Sign
$R_1\ (a \to b)$	Drop IR_1	minus
$R_2\ (b \to c)$	Drop IR_2	minus
$E_2\ (c \to d)$	Drop E_2	minus
$r_2\ (d \to e)$	Drop Ir_2	minus
$R_3\ (e \to f)$	Drop IR_3	minus
$E_1\ (f \to g)$	Gain E_1	plus
$r_1\ (g \to a)$	Drop Ir_1	minus

Always remember the signs of emf are independent of the current.

Hence, we have

$$-IR_1 - IR_2 - E_2 - Ir_2 - IR_3 + E_1 - Ir_1 = 0$$

$$I = \frac{E_1 - E_2}{R_1 + R_2 + R_3 + R_4 + R_5}$$

Note that if E_2 is greater than E_1, we get a negative value of I, which shows that assumed direction of current is wrong.

$$I = \frac{12-4}{5+5+4+1+1} = 0.5 \text{ A}$$

(b) Now we determine the potential at each labelled point in the circuit.

$$V_g = V_f + E_1 = 0 + 12 = 12 \text{ V}$$

$$V_a = V_g - Ir_1 = 12 - (0.5)(1) = 11.5 \text{ V}$$

$$V_b = V_a - IR_1 = 11.5 - (0.5)(5) = 9 \text{ V}$$

$$V_c = V_b - IR_2 = 9 - (0.5)(5) = 6.5 \text{ V}$$

$$V_d = V_c - E_2 = 6.5 - 4 = 2.5 \text{ V}$$

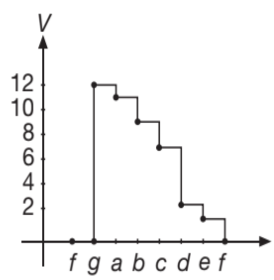
$$V_e = V_d - Ir_2 = 2.5 - (0.5)(1) = 2 \text{ V}$$

$$V_f = V_e - IR_3 = 2 - (0.5)(4) = 0$$

Note that, here we have assigned f to be at zero potential however, we can choose any point of the circuit to be at zero potential and then determine the potentials of the other points relative to it. The zero potential point is indicated by the ground symbol ⏚ at point f.

(c) Power delivered by source of emf E_1,

$$P_{E_1} = E_1 I = (12)(0.5) = 6 \text{ W}$$



Power dissipated in resistors,

$$P_R = I^2 (R_1 + R_2 + R_3 + r_1 + r_2)$$

$$P_R = (0.5)^2 (5+5+4+1+1) = 4 \text{ W}$$

The power consumed by battery 2 in getting charged,

$$P_{E_2} = E_2 I = 4(0.5) = 2 \text{ W}$$

Note that battery E_1 is discharging, the current comes out of its positive terminal. The terminal voltage across its terminals is

$$V_1 = E_1 - Ir_1$$

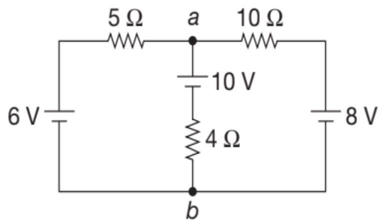
$$\Rightarrow \quad V_1 = 12 - (0.5 \times 1)$$

$$\Rightarrow \quad V_1 = 11.5 \text{ V}$$

The battery E_2 is charging, the current goes in from its positive terminal. The terminal voltage across it $V_2 = E_2 + Ir_2 = 4 + (0.5)(1) = 4.5$ V. The terminal voltage is greater than the emf of the battery.

ILLUSTRATION 28

Calculate the currents through each resistance in the given circuit. Also calculate the potential difference between points a and b.

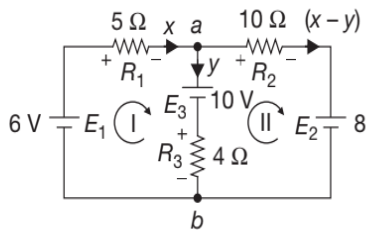


SOLUTION

Let the current through R_1 be x and through R_3 be y, as shown.

Applying KCL at junction a, the current through R_2 will be $(x - y)$.

According to the assumed direction of currents, we mark the polarity of voltage drops across different resistances.



Writing KVL for loop I, we get

$$xR_1 + E_3 + yR_3 - E_1 = 0$$

$$\Rightarrow \quad 5x + 10 + 4y - 6 = 0$$

$$\Rightarrow \quad 5x + 4y = -4 \qquad \ldots(1)$$

Writing KVL for loop II, we get

$$(x - y)R_2 + E_2 - yR_3 - E_3 = 0$$

$$\Rightarrow \quad (x - y)10 + 8 - 4y - 10 = 0$$

$$\Rightarrow \quad 10x - 14y = 2 \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$x = -\frac{24}{25} \text{ A}, \; y = -\frac{5}{11} \text{ A and } x - y = \frac{1}{55} \text{ A}$$

The signs indicate that the directions of x and y were assumed incorrectly, whereas the assumed direction of current x-y through R_2 was correct.

Potential difference between a and b,

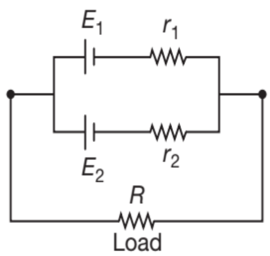
V_{ab} = Potential of a with respect to b is $V_a - V_b$. So,

$V_a - V_b$ = Potential rise when we go from b to a

$$\Rightarrow \quad V_a - V_b = yR_3 + E_3 = -\frac{5}{11} \times 4 + 10 = \frac{90}{11} \text{ V}$$

ILLUSTRATION 29

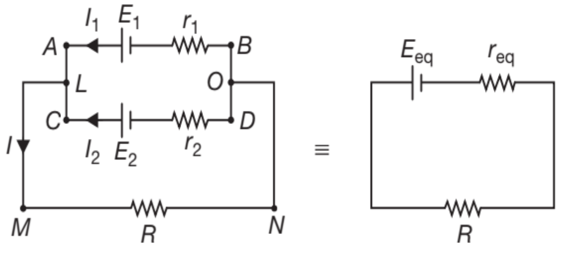
Two dissimilar cells of emfs E_1 and E_2 and internal resistances r_1 and r_2 respectively are connected in parallel across a load resistance R, as shown. Find the emf and internal resistance of the equivalent cell of this combination.



SOLUTION

Let I_1 and I_2 be the currents supplied by the two cells. Let the current through load resistance R be I. According to KCL at junction L,

$$I = I_1 + I_2 \qquad \ldots(1)$$



(a) (b)

For loop $ALMNOBA$, writing KVL equation,

$$IR + I_1 r_1 - E_1 = 0$$

$$\Rightarrow \quad I_1 = \frac{E_1 - IR}{r_1} \qquad \ldots(2)$$

Similarly, for loop $CLMNODC$, the KVL equation,

$$IR + I_2 r_2 - E_2 = 0$$

$$\Rightarrow \quad I_2 = \frac{E_2 - IR}{r_2} \qquad \ldots(3)$$

Substituting the value of I_1 and I_2 from equation (2) and (3) into (1), we get

$$I = \left[\frac{E_1}{r_1} + \frac{E_2}{r_2}\right] - IR\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$$

$$\Rightarrow \quad I\left[1 + R\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\right] = \left[\frac{E_1}{r_1} + \frac{E_2}{r_2}\right]$$

$$\Rightarrow \quad I\left[R + \left(\frac{1}{r_1} + \frac{1}{r_2}\right)^{-1}\right] = \left[\frac{E_1}{r_1} + \frac{E_2}{r_2}\right]\left[\frac{1}{r_1} + \frac{1}{r_2}\right]^{-1} \qquad \ldots(4)$$

Now if E_{eq} and r_{eq} are the equivalent emf and internal resistance of a single cell as shown in Fig. (3.139(b)), then according to loop rule, we have

$$-IR - Ir_{eq} + E_{eq} = 0$$

$$\Rightarrow \quad I\left(R + r_{eq}\right) = E_{eq} \qquad \ldots(5)$$

So, comparing equations (4) and (5), we get

$$r_{eq} = \left[\frac{1}{r_1} + \frac{1}{r_2}\right]^{-1} \text{ and } E_{eq} = \left[\frac{E_1}{r_1} + \frac{E_2}{r_2}\right]\left[\frac{1}{r_1} + \frac{1}{r_2}\right]^{-1} \qquad \ldots(6)$$

$$\Rightarrow \quad r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \text{ and } E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} \qquad \ldots(7)$$

Problem Solving Technique(s)

(a) If in the circuit battery E_2 is reversed $E_2 \Rightarrow -E_2$, equation (7) reduces to

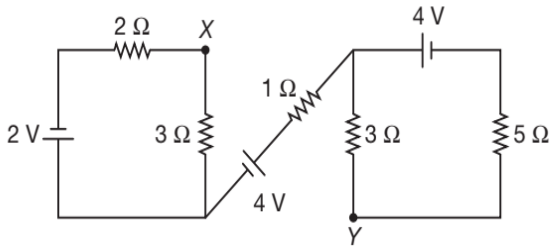
$$r_{eq} = \left[\frac{r_1 r_2}{r_1 + r_2}\right] \text{ and } E_{eq} = \frac{E_1 r_2 - E_2 r_1}{\left(r_1 + r_2\right)}$$

(b) If large number of dissimilar cells are joined in parallel, in the light of equation (6), we have

$$r_{eq} = \left[\sum \frac{1}{r_i}\right]^{-1} \text{ and } E_{eq} = \left[\sum \frac{E_i}{r_i}\right]\left[\sum \frac{1}{r_i}\right]^{-1}$$

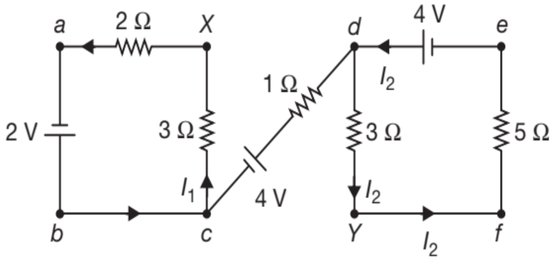
ILLUSTRATION 30

In the given network, determine the potential of point X with respect to the point Y.



SOLUTION

There are two closed circuits or loops. Let the current in the two loops be I_1 and I_2 as shown.



Note that no current can flow through branch cd, since it is not a part of any closed loop. The whole current I_1 goes round the left-hand loop, and whole current I_2 goes round the right-hand loop.

$$\therefore \quad I_1 = \frac{2\text{ V}}{2\ \Omega + 3\ \Omega} = 0.4\text{ A}$$

$$\text{and} \quad I_2 = \frac{4\text{ V}}{3\ \Omega + 5\ \Omega} = 0.5\text{ A}$$

The potential of point X with respect to point Y is written as V_{XY}. If we traverse any path starting from point Y and end at point X, the net potential rise will be V_{XY}. Now, let us find V_{XY}, by traversing the path $YdcX$,

$$V_{XY} = 3I_2 + 1(0) - 4 - 3I_1$$

$$V_{XY} = 3(0.5) + 0 - 4 - 3(0.4) = 1.5 - 4 - 1.2 = -3.7\text{ V}$$

It means that point X is 3.7 V lower in potential than the point Y. We can also write

$$V_{YX} = -V_{XY} = -(-3.7)\text{ V} = 3.7\text{ V}$$

It means point Y is 3.7 V higher in potential than the point X.

Note that whichever path we traverse from Y to X, the result should be the same, that is, -3.7 V. Let us check if by traversing the path $YfedcbaX$,

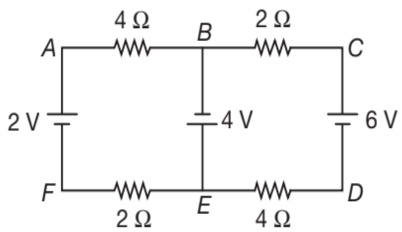
$$V_{XY} = 0 - 5I_2 + 4 + 1(0) - 4 + 0 - 2 + 2I_1$$

$$V_{XY} = 0 - 5(0.5) + 4 + 1(0) - 4 + 0 - 2 + 2(0.4)$$

$$V_{XY} = -2.5 + 4 - 4 - 2 + 0.8 = 4.8 - 8.5 = -3.7\text{ V}$$

ILLUSTRATION 31

Find currents in different branches of the electric circuit shown in figure. Also find the potential difference between points F and C.



SOLUTION

In this Illustration there are three wire segments $EFAB$, BE and $BCDE$ and two junctions at B and E. Therefore we have three unknowns I, I_1 and I_2 and hence we require three equations. One equation will be obtained by applying Kirchhoff's Junction Law (either at B or at E) and the remaining two equations, we get from the Second Law (Loop Law). Consider three loops $ABEFA$, $ACDFA$ and $BCDEB$. But we have to choose any two of them. Further, we can choose any arbitrary directions of I, I_1 and I_2.

Applying Kirchhoff's First Law (Junction Law) at junction B,

$$I = I_1 + I_2 \qquad \ldots(1)$$

Applying Kirchhoff's Second Law in Loop $ABEFA$,

$$-4I + 4 - 2I + 2 = 0 \qquad \ldots(2)$$

$$\Rightarrow \quad I_1 = 6\text{ A}$$

Applying Kirchhoff's Second Law in Loop $BCDEB$,

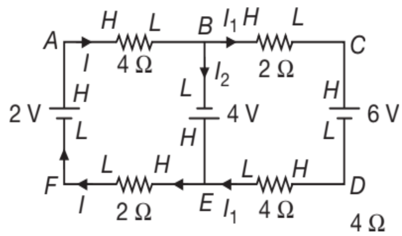
$$-2I_1 - 6 - 4I_1 - 4 = 0 \qquad \ldots(3)$$

$$\Rightarrow \quad I_1 = -\frac{10}{6} = -\frac{5}{3}\text{ A}$$

Using (1), we get

$$I_1 = -\frac{5}{3}\text{ A}$$

Here, negative sign of I_1 implies that current I_1 is in the direction opposite to that assumed by us in the figure i.e., it is from C to B (and not from B to C).



To find the potential difference between any two points of a circuit let us reach from one point to the other via any path of the circuit. It is advisable to choose a path in which we come across the least number of resistors preferably a path which has no resistance.

Let us reach from F to C via A and B,

$$V_F + 2 - 4I - 2I_1 = V_C$$

$$\Rightarrow \quad V_F - V_C = 4I + 2I_1 - 2$$

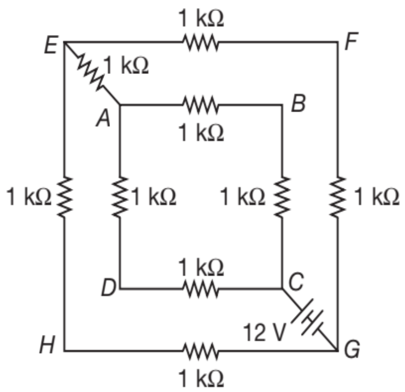
Substituting, $I = 1$ A and $I_1 = -\frac{5}{3}$ A, we get

$$V_F - V_C = -\frac{4}{3}\text{ V}$$

Here negative sign implies that $V_F < V_C$ or F is at a lower potential than C.

ILLUSTRATION 32

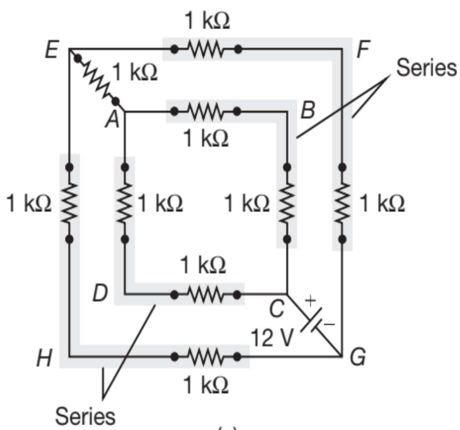
In the arrangement shown, find the



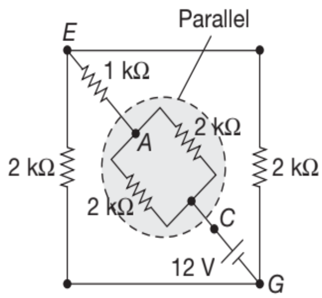
(a) equivalent resistance between C and G.
(b) current provided by the source.
(c) voltage across points G and E.

SOLUTION

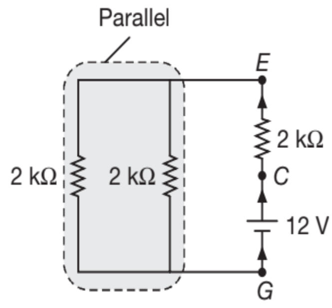
(a) Imagine the wires to be flexible, lift up the inside square with the resistors along with the source attached and then following figures (a), (b), (c) and (d) to arrive at equivalent circuit. The equivalent resistance is $3\ \text{k}\Omega$.



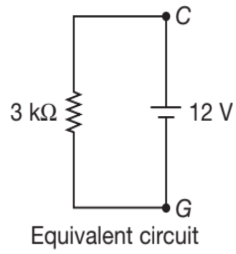
(a)



(b)



(c)



(d)

(b) Since $V = IR_{\text{eq}}$, $I = \dfrac{V}{R_{\text{eq}}} = 4\ \text{mA}$

(c) Start at point G, assign it a potential V_G, proceed toward E along any path. When you reach point E after adding potential drops and gains you get potential of E

$$V_G + 12 - IR = V_E$$

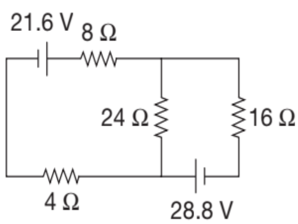
$$\Rightarrow \quad V_E - V_G = 12 - IR$$

$$\Rightarrow \quad V_E - V_G = 12 - \left(4 \times 10^{-3} \times 2 \times 10^{3}\right) = 4\ \text{V}$$

THE PRINCIPLE OF SUPERPOSITION OR SUPERPOSITION THEOREM

Complex network problems can sometimes be solved easily by using the principle of superposition. This principle essentially states that when a number of emf's act in a network, the solution is the same as the superposition of the solutions for one emf acting at a time, the others being shorted.

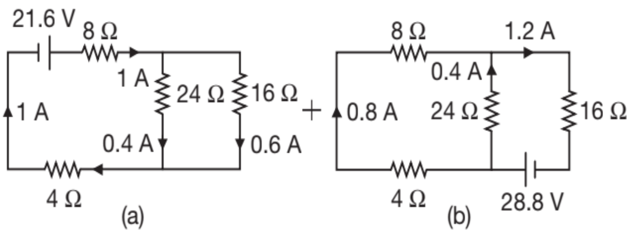
Figure shows a network with two loops.



The currents in various branches can be calculated using Kirchhoff's Laws. We can get the same solution by considering only one battery at a time and then superposing the two solutions. If a battery has an internal resistance, it must be left in place when the emf of the battery is removed. Figure shows how the superposition principle can be applied to the present problem.

The current values in figure (a) and (b) are easily verified. For example when the 21.6 V battery alone is acting, the total resistance in the circuit is,

$$8+\frac{24\times16}{24+16}+4=21.6\ \Omega$$



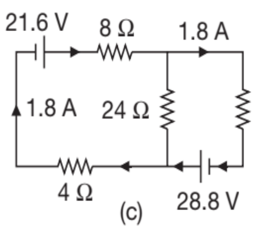
This makes the total current $\frac{21.6\text{ V}}{21.6\ \Omega}=1\text{ A}$. This current splits between $16\ \Omega$ and $24\ \Omega$ in the ratio $3:2$.

Similarly, the total resistance when only the 28.8 V battery is acting is,

$$16+\frac{24\times12}{24+12}=24\ \Omega$$

Therefore, the total current is $\frac{28.8\text{ V}}{24\ \Omega}=1.2\text{ A}$

The superposition principle shows that there is no current in the $24\ \Omega$ resistance.



Only a current of 1.8 A flows through the outer loop. All these conclusions can also be verified by analyzing the circuit using Kirchhoff's Laws.

Problem Solving Technique(s)

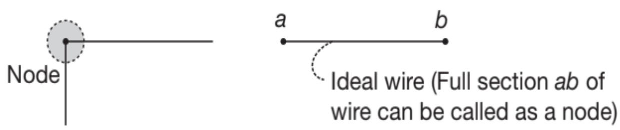
(a) If there are a number of emfs acting simultaneously in any linear network, then each emf acts independent of the others, i.e., as if the other emfs did not exist. The value of current in any conductor is the algebraic sum of the currents due to each emf. Similarly, voltage across any conductor is the algebraic sum of the voltages which each emf would have produced while acting singly.

(b) This theorem is applicable only to linear networks where current is linearly related to voltage as per Ohm's Law.

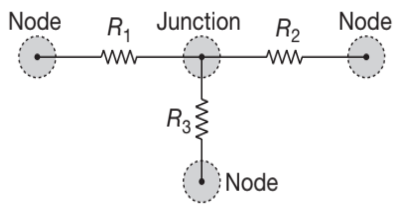
(c) Please note that while applying the superposition theorem only the current sources (or batteries) and the currents have to be superimposed. **Do not superimpose the resistances, because then you will be getting inaccurate results.**

NODAL ANALYSIS IN CIRCUITS

Node is a point at which two or more elements are joined together. For two nodes to be different, their voltages must be different. A conductor with a substantially zero resistance is considered to be a node for the purpose of analysis. So, in circuit diagrams where connections are done using ideal wires (i.e. wires with zero resistance), a node may consist of the entire section of wire between elements, not just a single point.



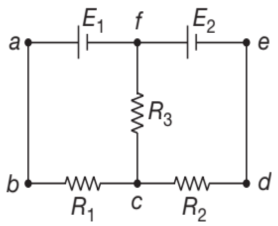
Junction is a point where three or more branches meet together as shown in figure.



Conceptual Note(s)

A junction can also be called a node but a node may or may not be a junction.

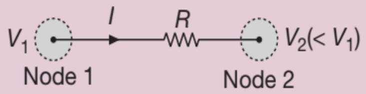
Think about two wires/branches meeting at a point, which makes it a Node, whereas think about three wires meeting at a point which may be called a Node (two or more branches meeting at a point) as well as a Junction (three or more branches meeting at a point).



In the above figure we can say that points a, b, c, d, e, f are **Nodes** and point c and f are called **Junctions**. (or can also be called as Nodes)

Conceptual Note(s)

A current through a resistor can be calculated by using node-voltages as follows:



$$I=\frac{V_1-V_2}{R} \text{ (when } V_1>V_2\text{)}$$

$$I=\frac{V_2-V_1}{R} \text{ (when } V_2>V_1\text{)}$$

Problem Solving Technique(s)

The nodal analysis is a systematic way of applying Kirchhoff's Circuit Laws (*KCL*) at each essential node of a circuit and represents the branch current in terms of the node voltages. This will give us a set of equations that we solve together to find the node voltages. Once we find the node voltages, we can use them to calculate any other currents or voltages of interest.

STEP-1: Identify all the essential nodes and select one of them as a reference node i.e. the node at zero potential i.e. 0 V. This reference node (can generally be taken as the ground) has usually most elements tied to it.

STEP-2: Now assign voltages (with respect to the reference node) to all the nodes except the reference node.

STEP-3: Apply the *KCL* at each node except the reference node and write the equations. In this step, for each node, we assume that all branch currents are either leaving from the node (or entering the node), and then we describe the branch current(s) in term of node voltages.

Always keep in mind that a Junction may be called a node but a node may or may not be a junction.

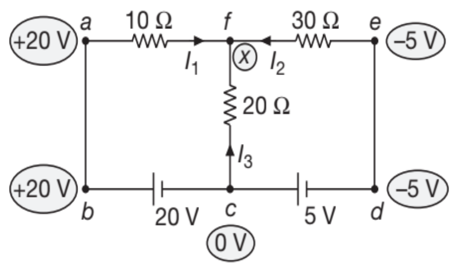
STEP-4: Solve the system of linear equations obtained to find the unknown node voltage(s) and hence the current through different branches of the circuit.

NODAL ANALYSIS METHOD

To solve problems on the basis of nodal analysis, we use the following steps.

STEP-1: Assign potential to every node of circuit. For doing this, we assign zero potential to any one of the node of the circuit. This Node (which is assigned zero potential) is called as **Reference Node** OR **Reference Junction**.

For example, in the circuit shown, we assign zero potential to the point c.



Then, $V_c=0\text{ V}$, $V_b=+20\text{ V}$, $V_d=-5\text{ V}$. Since ab and de are perfectly conducting sections of the circuit. So, $V_a=V_b=+20\text{ V}$, $V_e=V_d=-5\text{ V}$.

Let the potential of node/junction f be x.

STEP-2: Now at junction f, apply KJL i.e., $\Sigma I=0$ where $I=\frac{\Delta V}{R}$. Assuming all currents I_1, I_2 and I_3 to enter the junction, we get

$$I_1+I_2+I_3=0$$

$$\Rightarrow \frac{20-x}{10}+\frac{-5-x}{30}+\frac{0-x}{20}=0$$

$$\Rightarrow 6(20-x)+2(-5-x)+3(-x)=0$$

$$\Rightarrow 120-6x-10-2x-3x=0$$

$$\Rightarrow 11x=110$$

$$\Rightarrow x=10\text{ V}$$

So, potential of junction f is $V_f = x = 10\ \text{V}$

$$\Rightarrow \quad I_1 = \frac{20-x}{10} = \frac{20-10}{10} = 1\ \text{A}$$

$$\Rightarrow \quad I_2 = \frac{-5-x}{30} = \frac{-5-10}{30} = -\frac{1}{2}\ \text{A}$$

(Negative sign with I_2 implies that its direction is opposite to the direction as assigned in the circuit i.e., actually from f to e)

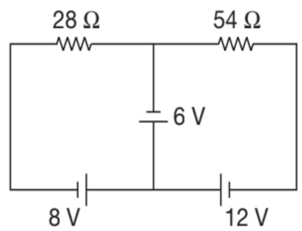
$$\Rightarrow \quad I_3 = -\frac{x}{20} = -\frac{10}{20} = -\frac{1}{2}\ \text{A}$$

$$\Rightarrow \quad I_3 = \frac{1}{2} A\,(\text{from}\, f \text{ to } c)$$

Also, we observe that $I_1 = I_2 + I_3$

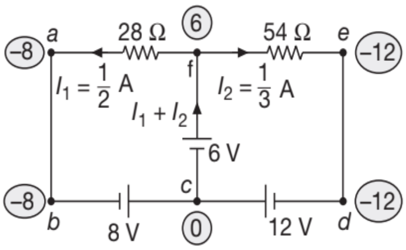
ILLUSTRATION 33

Using Nodal Analysis method, calculate the current through the resistors $28\ \Omega$, $54\ \Omega$ and $6\ \text{V}$ battery.



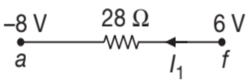
SOLUTION

Let us redraw the circuit and take a zero reference point in the circuit. So let potential of the point c be taken as zero. Then with respect to this point potential of point b is $-8\ \text{V}$, point d is $-12\ \text{V}$, point f is $6\ \text{V}$.

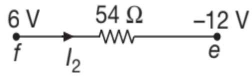


Also sections ab and de are perfectly conducting, so, $V_a = V_b = -8\ \text{V}$, $V_e = V_d = -12\ \text{V}$.

A careful observation of this circuit reveals that



$$I_1 = \frac{6-(-8)}{28} = \frac{1}{2}\ \text{A}$$



$$I_2 = \frac{6-(-12)}{54} = \frac{1}{3}\ \text{A}$$

So, current through $6\ \text{V}$ battery is

$$(I_1 - I_2) = \frac{5}{6}\ \text{A (from } e \text{ to } f\text{)}$$

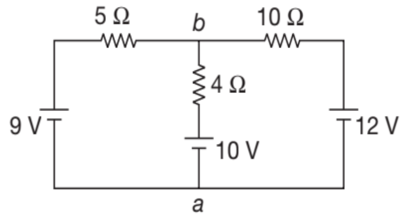
$$I_1 = I_{28\ \Omega} = \frac{1}{2}\ \text{A (from } f \text{ to } a\text{)}$$

$$I_2 = I_{54\ \Omega} = \frac{1}{3}\ \text{A (from } f \text{ to } e\text{) and}$$

$$I_{6\ \text{V battery}} = I_1 + I_2 = \frac{5}{6}\ \text{A (from } c \text{ to } f\text{)}$$

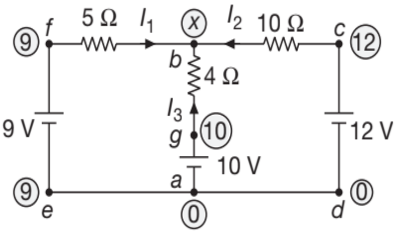
ILLUSTRATION 34

Using Nodal Analysis method, calculate the current through $5\ \Omega$, $4\ \Omega$, $10\ \Omega$ resistors. Also calculate potential difference between the points a and b.

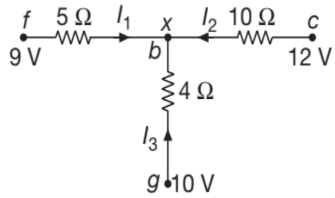


SOLUTION

Let $V_a = 0$ and the circuit is redrawn as shown.



Let x be the potential at junction b.



Applying *KJL* at *b*, we have

$$\Sigma I_b = 0$$

$$\Rightarrow \quad I_1 + I_2 + I_3 = 0$$

$$\Rightarrow \quad \frac{9-x}{5} + \frac{12-x}{10} + \frac{10-x}{4} = 0$$

$$\Rightarrow \quad 4(9-x) + 2(12-x) + 5(10-x) = 0$$

$$\Rightarrow \quad 36 - 4x + 24 - 2x + 50 - 5x = 0$$

$$\Rightarrow \quad 11x = 110$$

$$\Rightarrow \quad x = 10 \text{ V}$$

$$\Rightarrow \quad I_1 = \frac{9-x}{5} = \frac{9-10}{5} = -\frac{1}{5} \text{ A}$$

$$\Rightarrow \quad I_1 = \frac{1}{5} \text{ A (from } b \text{ to } f\text{)}$$

$$\Rightarrow \quad I_2 = \frac{12-x}{10} = \frac{12-10}{10} = \frac{1}{5} \text{ A}$$

$$\Rightarrow \quad I_2 = \frac{1}{5} \text{ A (from } c \text{ to } b\text{)}$$

$$\Rightarrow \quad I_3 = \frac{10-x}{4} = \frac{10-10}{4} = 0 \text{ A}$$

Since no current flows through the branch ab, so

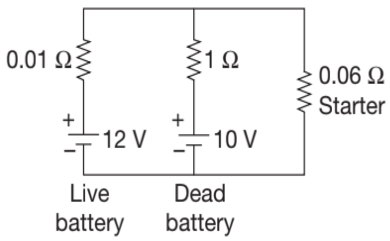
$$V_a - V_b = V_{ab} = 10 \text{ V}$$

Test Your Concepts-VI

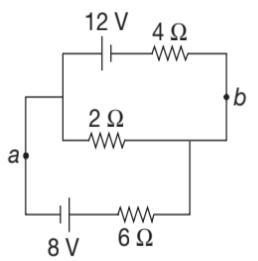
Based on Kirchhoff's Laws and Nodal Analysis

(Solutions on page H.211)

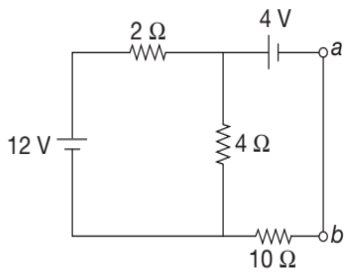
1. A dead battery is charged by connecting it to the live battery of another car with jumper cables. Determine the current in the starter and in the dead battery.



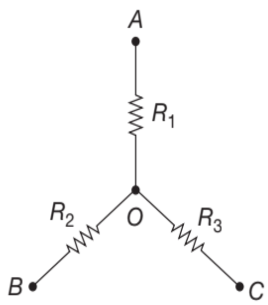
2. For the circuit shown in figure, calculate
(a) The current in the 2 Ω resistor and
(b) The potential difference between points *a* and *b*.



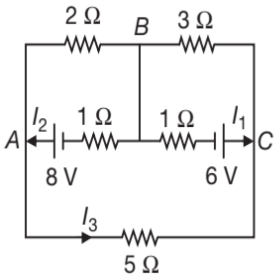
3. Calculate the potential difference between points *a* and *b* in figure and identify which point is at the higher potential.



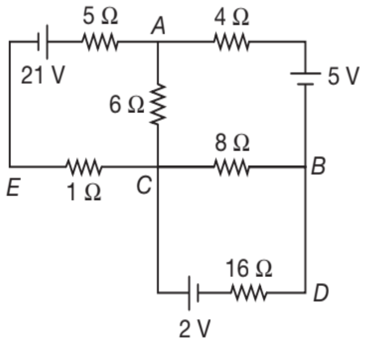
4. A circuit has a section *ABC* as shown in figure. If the potentials at points *A*, *B* and *C* are V_1, V_2 and V_3 respectively, calculate the potential at point *O*.



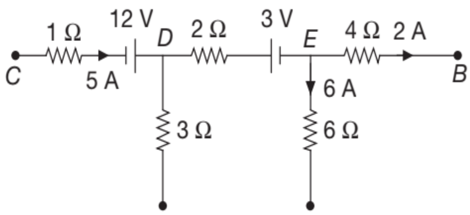
5. In the circuit shown in figure, calculate the currents I_1, I_2 and I_3. Also find out the potential differences between the points *A* and *B* and *B* and *C*.



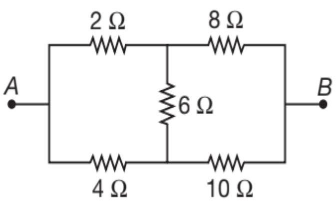
6. Find the current in each branch of the circuit shown in figure.



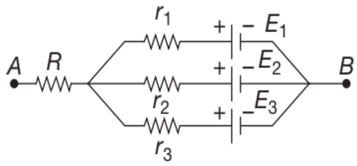
7. Figure shows part of a circuit. Calculate the power dissipated in 3 Ω resistance. What is the potential difference $V_C - V_B$?



8. Find the equivalent resistance between A and B in the following circuit:



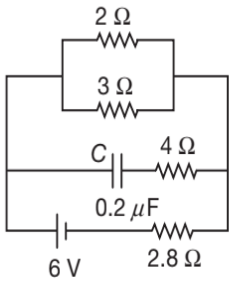
9. In the circuit shown in figure $E_1 = 3$ V, $E_2 = 2$ V, $E_3 = 1$V and $R = r_1 = r_2 = r_3 = 1\,\Omega$



(a) Find the potential difference between the points A and B and the currents through each branch.

(b) If r_2 is short circuited and the point A is connected to point B, find the currents through E_1, E_2, E_3 and the resistor R.

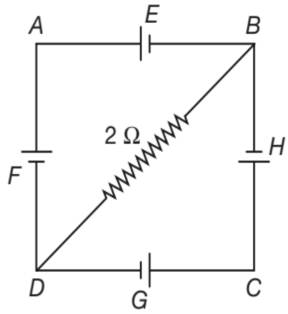
10. Calculate the steady state current in the 2 Ω resistor shown in the circuit (shown in figure). The internal resistance of the battery is negligible and the capacitance of the condenser C is 0.2 μF.



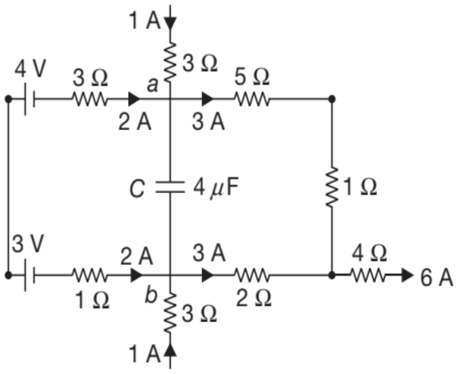
11. In the circuit shown in figure E, F, G, H are cells of emf 2 V, 1 V, 3 V and 1 V respectively, and their internal resistances are 2 Ω, 1 Ω, 3 Ω and 1 Ω respectively. Calculate

(a) the potential difference between B and D and

(b) the potential difference across the terminals of each cells G and H.



12. A part of circuit in a steady state along with the currents flowing in the branches, the values of resistances etc., is shown in the figure. Calculate the energy stored in the capacitor C(4 μF).

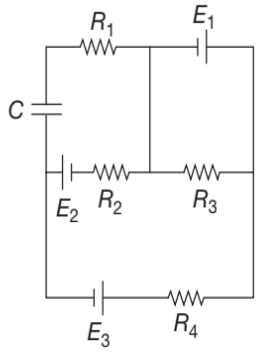


13. In the given circuit:

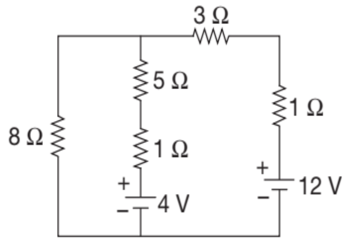
$E_1 = 3E_2 = 2E_3 = 6$ V, $R_1 = 2R_4 = 6\ \Omega$

$R_3 = 2R_2 = 4\ \Omega,\ C = 5\ \mu F$

Find the current in R_3 and the energy stored in the capacitor.



14. Determine the current in each branch of the circuit shown in Figure.



PRIMARY AND SECONDARY CELLS/BATTERIES

A battery consists of a series or parallel combination of two or more similar cells. Cells or batteries can be divided into **primary** and **secondary** types. The secondary cell is rechargeable, whereas the primary is not. The battery used in a car is of secondary type, since it can be recharged. But the cells used in a torch are primary type, as they cannot be recharged.

The resistance offered by the electrolyte to the flow of current through the cell is called the internal resistance of the cell. The internal resistance of a cell depends on

(a) the distance between the electrodes $(r \propto d)$

(b) the area of the electrodes $\left[r \propto \left(\frac{1}{A}\right)\right]$

(c) the concentration of the electrolyte $(r \propto C)$

(d) the absolute temperature of the electrolyte $\left[r \propto \left(\frac{1}{T}\right)\right]$

Internal resistance is different for different types of cells and even for a given type of cell it varies from cell to cell.

IDEAL VOLTAGE SOURCE

An ideal voltage source is one that can maintain constant voltage across its terminals whatever be the current drawn from it. The internal resistance of an ideal cell is zero. In practice, no cell can be ideal.

PRACTICAL VOLTAGE SOURCE

It is due to the internal resistance that the source does not behave as an ideal voltage source. It is usual practice to show a practical voltage source as consisting of an ideal voltage source of voltage E in series with a resistance r. E is the emf of the cell and r is its internal resistance. If such a cell is connected to load of resistance R, the current supplied by the cell is

$$I = \frac{\text{total emf in the circuit}}{\text{total resistance}} = \frac{E}{R+r}$$

Therefore, the potential difference that is now existing between the terminals A and B of the cell is given as

$$V = E - Ir$$

This quantity V is called the **terminal voltage** of the cell.

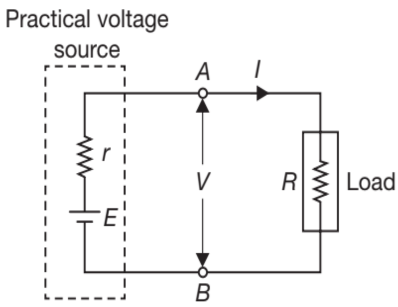
DISCHARGING OF CELL

When current is drawn from a cell, the potential difference across it is less than the emf of the cell and is given by

$$\boxed{V = E - Ir}$$

If the load resistance i.e. $R \to \infty$ (i.e., when the terminals A and B are open-circuited), then $V \to E$. Thus, the emf of a cell is nothing but its terminals voltage when its terminals are open-circuited.

Suppose a battery of emf E and internal resistance r is connected to a box which contains sources with greater emfs.



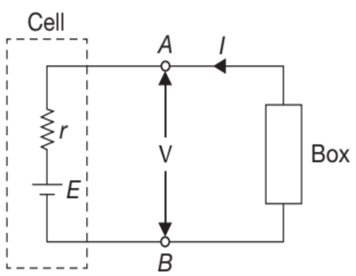
Then the current I may flow into the positive terminal of the battery (instead of its flowing out of the battery). The battery does not supply energy to the box; on the other hand the box supplies energy to the battery. Now, the terminal voltage is given as

$$V = E + Ir$$

CHARGING OF CELL

When a cell is being charged, the voltage across its terminals is greater than its emf and is given by

$$\boxed{V = E + Ir}$$



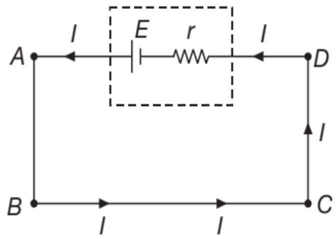
SHORT-CIRCUITING OF A CELL

The terminals A and D of a cell are connected by a conducting wire of zero resistance.

Using KVL in $ABCDA$, $-Ir + E = 0$

$$\Rightarrow \quad E = Ir$$

$$\Rightarrow \quad I = \frac{E}{r}$$



Terminal potential difference across the cell is

$$V_{AD} = E - Ir = E - \left(\frac{E}{r}\right)r$$

$$\Rightarrow \quad V_{AD} = 0$$

INTERNAL RESISTANCE OF CELL (r)

The internal resistance of a cell is the resistance offered by the electrolyte of the cell to the flow of current between its electrodes. It is denoted by r. The internal resistance of a cell

(a) varies directly as concentration of the electrolyte solution of the cell.

(b) varies directly as the separation between electrodes i.e., length of electrolyte solution between electrodes.

(c) varies inversely as the area of immersed electrodes.

(d) is independent of the material of electrodes.

Remark(s)

The internal resistance of a circuit is always in series with external resistance of the circuit.

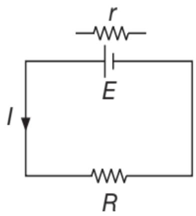
TERMINAL POTENTIAL DIFFERENCE ACROSS A CELL

If a cell of e.m.f. E, internal resistance r sends a current I, through an external resistance, then terminal potential difference

$$V = IR = E - Ir$$

In general the terminal potential difference (T.P.D.) is the potential difference across the external resistance of the circuit.

where $I = \frac{E}{R+r}$

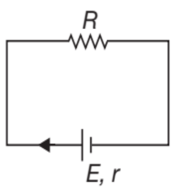


During the process of charging of a battery current is driven into a battery in the reverse direction. In such a case positive charge enters the battery at the positive terminal and leaves the battery from the negative terminal. So, when a cell is being charged by an external battery, then terminal potential difference

$$V = E + Ir$$

MAXIMUM POWER TRANSFER THEOREM

The power consumption across the external resistance of the circuit is maximum when the net external resistance of the circuit equals the net internal resistance. Consider the circuit shown. The power consumed across R is I^2R, where $I = \frac{E}{R+r}$.



$$\Rightarrow \quad P = \frac{E^2R}{(R+r)^2}$$

$$\Rightarrow \quad P = \frac{E^2R}{(R-r)^2 + 4Rr}$$

For P to be MAXIMUM

$$R - r = 0$$

$$\Rightarrow \quad R = r$$

So, we observe that for maximum power consumption across the external resistance, we must have External Resistance = Internal Resistance of the battery.

$$\Rightarrow \quad P_{max} = \frac{E^2}{4R} = \frac{E^2}{4r}$$

This is an example of "impedance matching", in which the variable resistance R is adjusted so that the power delivered to it is maximized. The behaviour of P as a function of R is depicted in figure below.

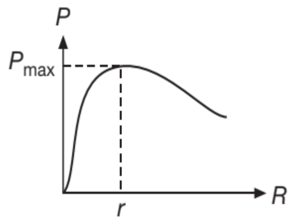
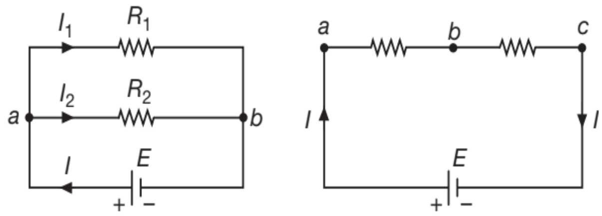


ILLUSTRATION 35

Shown here are two resistors with resistances R_1 and R_2 connected in parallel and in series. The battery has a terminal voltage of E.



Suppose R_1 and R_2 are connected in parallel, then

(a) find the power delivered to each resistor.

(b) show that the sum of the power used by each resistor is equal to the power supplied by the battery.

Suppose R_1 and R_2 are now connected in series, then

(c) find the power delivered to each resistor.

(d) show that the sum of the power used by each resistor is equal to the power supplied by the battery.

Which configuration, parallel or series, uses more power?

SOLUTION

(a) When two resistors are connected in parallel, the current through each resistor is

$$I_1 = \frac{E}{R_1} \text{ and } I_2 = \frac{E}{R_2}$$

and the power delivered to each resistor is given by

$$P_1 = I_1^2 R_1 = \frac{E^2}{R_1} \text{ and } P_2 = I_2^2 R_2 = \frac{E^2}{R_2}$$

The results indicate that the smaller the resistance, the greater the amount of power delivered. If the loads are the light bulbs, then the one with smaller resistance will be brighter since more power is delivered to it.

(b) The total power delivered to the two resistors is

$$P_R = P_1 + P_2 = \frac{E^2}{R_1} + \frac{E^2}{R_2} = \frac{E^2}{R_{eq}}$$

where $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

$$\Rightarrow \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

is the equivalent resistance of the circuit. On the other hand, the total power P supplied by the battery is $P = IE$, where $I = I_1 + I_2$, as seen from the figure. Thus,

$$P = I_1 E + I_2 E = \left(\frac{E}{R_1}\right)E + \left(\frac{E}{R_2}\right)E$$

$$P = \frac{E^2}{R_1} + \frac{E^2}{R_2} = \frac{E^2}{R_{eq}} = P_R$$

The above results are in accordance with Law of Conservation of Energy.

(c) When the two resistors are connected in series, the equivalent resistance becomes

$$R'_{eq} = R_1 + R_2$$

and the currents through the resistors are

$$I_1 = I_2 = I = \frac{E}{R_1 + R_2}$$

Therefore, the power delivered to each resistor is

$$P_1 = I_1^2 R_1 = \left(\frac{E}{R_1 + R_2}\right)^2 R_1$$

and $P_2 = I_2^2 R_2 = \left(\frac{E}{R_1 + R_2}\right)^2 R_2$

Contrary to what we have seen in the parallel case, when connected in series, the greater the resistance, the greater the fraction of the power delivered. Once again, if the loads are light bulbs, the one with greater resistance will be brighter.

(d) The total power delivered to the resistors is

$$P'_R = P_1 + P_2 = \left(\frac{E}{R_1 + R_2}\right)^2 R_1 + \left(\frac{E}{R_1 + R_2}\right)^2 R_2$$

$$P'_R = \frac{E^2}{R_1 + R_2} = \frac{E^2}{R'_{eq}}$$

On the other hand, the power supplied by the battery is

$$P' = IE = \left(\frac{E}{R_1 + R_2}\right)E = \frac{E^2}{R_1 + R_2} = \frac{E^2}{R'_{eq}}$$

Again, we see that $P' = P'_R$, as required by energy conservation

Comparing the results obtained in (b) and (d), we see that

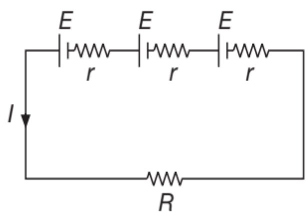
$$P = \frac{E^2}{R_1} + \frac{E^2}{R_2} > \frac{E^2}{R_1 + R_2} = P'$$

$$\Rightarrow \quad P > P'$$

which means that the parallel connection uses more power because of the fact that the equivalent resistance of two resistors connected in parallel is always smaller than that connected in series.

IDENTICAL CELLS IN SERIES

In this arrangement the positive terminal of one cell is connected to negative terminal of the other in succession. Figure represents n cells, each of e.m.f. E and internal resistance r connected in series and an external resistance R is connected across the combination.



Since all the cells are in series, so
Net e.m.f. $= nE$
Similarly all resistances are in series, so
Net internal resistance $= nr$

$\Rightarrow$ Total resistance of circuit $= R + nr$

$\Rightarrow$ Current $I = \dfrac{\text{Net e.m.f.}}{\text{Net resistance}}$

$$I = \frac{nE}{R + nr}$$

(a) If $R >> nr$, then

$$I = n\frac{E}{R} = n \text{ (current due one cell)}$$

(b) If $R << nr$, then

$$I = \frac{E}{r} \approx \text{ current due to one cell}$$

So, when net external resistance >> net internal resistance, then to get the maximum current, the cells must be connected in series for maximum current, the cells should be connected in series, when net external resistance >> net internal resistance.

Conceptual Note(s)

Note that if in series grouping of m cells, x cells are reversed i.e. x cells are being charged, then

$$E_{eq} = (m - x)E - xE = (m - 2x)E \text{ and } r_{eq} = mr$$

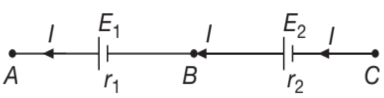
$$\Rightarrow I = \frac{(m - 2x)E}{R + mr}$$

NON-IDENTICAL CELLS IN SERIES

E_1 and E_2 are emf of two cells used in series as shown in figure. If r_1 and r_2 are their respective internal resistances and I is the current flowing from C to A, potentials at points A, B and C respectively be V_A, V_B and V_C, then

$$V_{AB} = V_A - V_B = E_1 - Ir_1 \text{ and}$$

$$V_{BC} = V_B - V_C = E_2 - Ir_2$$

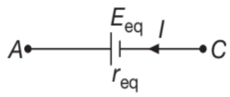


Potential difference between terminals A and C is

$$V_{AC} = V_A - V_C = V_A - V_B + V_B - V_C$$

$$\Rightarrow V_{AC} = (E_1 + E_2) - I(r_1 + r_2) \quad \ldots(1)$$

We can replace the above combination by a single cell of emf E_{eq} and internal resistance r_{eq} as shown below.



Since, $V_{AC} = E_{eq} - Ir_{eq}$ ...(2)

Comparing (1) and (2), we get

$$E_{eq} = E_1 + E_2 \text{ and}$$

$$r_{eq} = r_1 + r_2$$

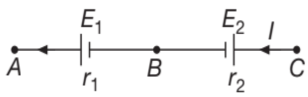
(a) If n cells are connected correctly in series, then

$$E_{eq} = E_1 + E_2 + \ldots.. + E_n \text{ and}$$

$$r_{eq} = r_1 + r_2 + \ldots\ldots + r_n$$

If cells are identical then $E_{eq} = nE$ and $r_{eq} = nr$.

(b) If cells of emf E_1 and E_2 are connected wrongly as shown below



then, $V_{AC} = E_1 - Ir_1 - E_2 - Ir_2$

$$\Rightarrow V_{AC} = (E_1 - E_2) - I(r_1 + r_2)$$

$$\Rightarrow V_{AC} = E_{eq} - Ir_{eq}$$

Thus, equivalent emf $E_{eq} = E_1 - E_2$ and internal resistance $r_{eq} = r_1 + r_2$

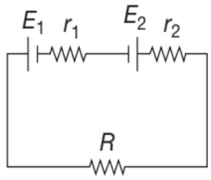
(c) If out of N identical cells, n are wrongly connected then

$$E_{eq} = (N - n)E - nE = (N - 2n)E$$

and $r_{eq} = Nr$

ILLUSTRATION 36

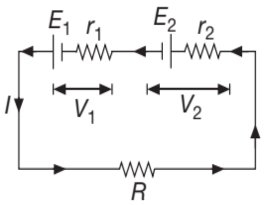
In the circuit shown in figure, $E_1 = 10\text{ V}$, $E_2 = 4\text{ V}$, $r_1 = r_2 = 1\,\Omega$ and $R = 2\,\Omega$. Find the potential difference across battery 1 and battery 2.



SOLUTION

Net emf of the circuit $= E_1 - E_2 = 6$ V

Total resistance of the circuit $= R + r_1 + r_2 = 4\ \Omega$



$\therefore$ Current in the circuit

$$I = \frac{\text{net emf}}{\text{total resistance}} = \frac{6}{4} = 1.5 \text{ A}$$

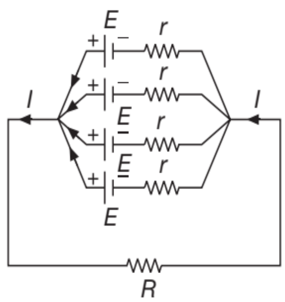
Now, $V_1 = E_1 - Ir_1 = 10 - (1.5)(1) = 8.5$ V

and $V_2 = E_2 + Ir_2 = 4 + (1.5)(1) = 5.5$ V

IDENTICAL CELLS IN PARALLEL

In this arrangement the positive terminals of all cells are connected to one point and negative terminals to the other point. Figure represents m cells, each of e.m.f. E and internal resistance r, connected in parallel and an external resistance R is connected across the combination.

Since all the cells are in parallel, so net e.m.f. equals to the e.m.f. due to a single cell.



$\Rightarrow$ Net e.m.f. $= E$

Similarly all internal resistances are in parallel, so

$$\text{Net internal resistance} = R_{\text{int}} = \frac{r}{m}$$

$$\Rightarrow \quad \text{Total resistance of circuit} = R + \frac{r}{m}$$

$$\Rightarrow \quad \text{Current } I = \frac{E}{R + \left(\frac{r}{m}\right)}$$

(a) If $R << \frac{r}{m}$, then

$$I = \frac{mE}{r} = m(\text{current due to one cell})$$

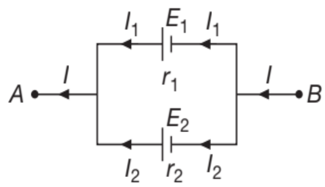
(b) If $R >> \frac{r}{m}$, then

$$I = \frac{E}{R} \approx \text{current due to one cell}.$$

So, when Net Internal Resistance >> Net External Resistance, then to get maximum current, the cells must be connected in parallel.

NON-IDENTICAL CELLS IN PARALLEL

Consider two cells of emf E_1 and E_2 connected in parallel across AB.



If I_1 and I_2 are the currents leaving positive electrodes of both cells, then total current from B to A.

$$I = I_1 + I_2 \qquad \ldots(1)$$

V_A and V_B are potentials at A and B respectively, then potential difference between A and B, can be written as

$$V = V_A - V_B = E_1 - I_1 r_1 \qquad \ldots(2)$$

and $V = V_A - V_B = E_2 - I_2 r_2 \qquad \ldots(3)$

Using equation (1), (2) and (3), we get

$$I = I_1 + I_2$$

$$\Rightarrow \quad I = \frac{E_1 - V}{r_1} + \frac{E_2 - V}{r_2} = \left(\frac{E_1}{r_1} + \frac{E_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\Rightarrow \quad V = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} - I\left(\frac{r_1 r_2}{r_1 + r_2}\right) \qquad \ldots(4)$$

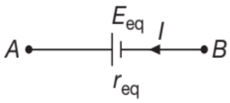
We can replace the given combination by a single cell of emf equation and internal resistance r_{eq} across AB as shown in the figure.

We have,

$$V_{AB} = V_A - V_B = E_{eq} - I r_{eq} \qquad \ldots(5)$$

Comparing (4) and (5), we have

$$E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$



and $r_{eq} = \dfrac{r_1 r_2}{r_1 + r_2}$

$$\Rightarrow \quad \frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2}, \text{ where } \frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

(a) If n cells of emf E_1, E_2,……, E_n and internal resistances r_1, r_2,……, r_n are in parallel, then

$$\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \ldots\ldots + \frac{E_n}{r_n}$$

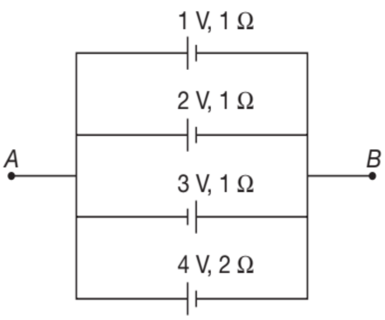
where, $\dfrac{1}{r_{eq}} = \dfrac{1}{r_1} + \dfrac{1}{r_2} + \ldots\ldots + \dfrac{1}{r_n}$

(b) For identical cells, $r_1 = r_2 = \ldots\ldots = r_n = r$ (say) and $E_1 = E_2 = \ldots\ldots = E_n = E$ (say), then

$$r_{eq} = \frac{r}{n} \text{ and } E_{eq} = E$$

ILLUSTRATION 37

Calculate equivalent emf of four different cells connected in parallel as shown.



SOLUTION

$$\text{Since } E_{eq} = \frac{\dfrac{E_1}{r_1} + \dfrac{E_2}{r_2} + \dfrac{E_3}{r_3} + \dfrac{E_4}{r_4}}{\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3} + \dfrac{1}{r_4}}$$

$$\Rightarrow \quad E_{eq} = \frac{\dfrac{1}{1} + \dfrac{2}{1} - \dfrac{3}{1} + \dfrac{4}{2}}{\dfrac{1}{1} + \dfrac{1}{1} + \dfrac{1}{1} + \dfrac{1}{2}} = \frac{2}{\left(\dfrac{7}{2}\right)} = \frac{4}{7} \text{ V}$$

The equivalent cell arrangement between A and B is shown for reference.

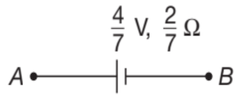
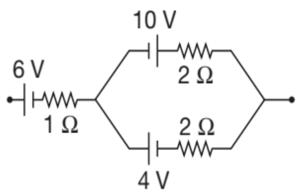


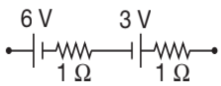
ILLUSTRATION 38

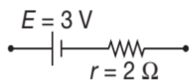
Find the emf and internal resistance of a single battery which is equivalent to a combination of three batteries as shown in figure.



SOLUTION

The given combination consists of two batteries in parallel and resultant of these two in series with the third one.





For parallel combination we can apply,

$$E_{eq} = \frac{\dfrac{E_1}{r_1} - \dfrac{E_2}{r_2}}{\dfrac{1}{r_1} + \dfrac{1}{r_2}} = \frac{\dfrac{10}{2} - \dfrac{4}{2}}{\dfrac{1}{2} + \dfrac{1}{2}} = 3 \text{ V}$$

$$\text{Further, } \frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{2} + \frac{1}{2} = 1$$

$$\Rightarrow \quad r_{eq} = 1\ \Omega$$

Now this resistance is in series with the third resistance so the equivalent emf of these two is $(6-3)\ \text{V} = 3\ \text{V}$ and the internal resistance will be $(1+1) = 2\ \Omega$.

Conceptual Note(s)

The equivalent emf can also be found by another method. Suppose we wish to find the equivalent emf of the circuit. Here we shall be using the fact that when no current is drawn by the cell then, we must have

$$E = V$$

However we may note that the current in the internal circuit may be non zero. This current is,

$$I = \frac{10+4}{2+1} = \frac{14}{3}\ \text{A}$$

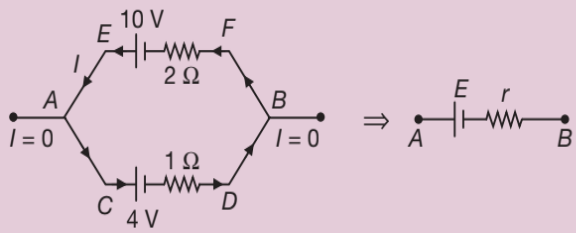
For path ACDB, we have

$$\text{Now, } V_A + 4 - 1\left(\frac{14}{3}\right) - V_B = 0$$

$$\Rightarrow\ V_A - V_B = \frac{14}{3} - 4 = \frac{2}{3}\ \text{V}$$

$$\Rightarrow\ E = V_A - V_B = V = \frac{2}{3}\ \text{V}$$

$$\Rightarrow\ E = \frac{2}{3}\ \text{V}$$



The identical result can also be obtained for the path AEFB for which we will get

$$V_A - 10 + 2\left(\frac{14}{3}\right) + V_B = 0$$

$$\Rightarrow\ E = V_A - V_B = \frac{2}{3}\ \text{V}$$

Further, since, $V_A - V_B$ is positive, i.e., $V_A > V_B$. So, we conclude that A is connected to the positive terminal of the battery and B to the negative.

Internal resistance of the equivalent battery is found by the same routine procedure. For example, here 2 Ω and 1 Ω resistances are in parallel. Hence, their combined resistance is

$$\frac{1}{r} = \frac{1}{1} + \frac{1}{2} = \frac{3}{2}$$

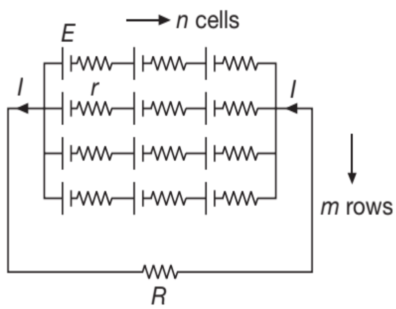
$$\Rightarrow\ r = \frac{2}{3}\ \Omega$$

MIXED GROUPING OF IDENTICAL CELLS

In this arrangement a total of N cells are used. This arrangement comprises of m rows of cells in parallel, each row containing n cells in series. So, $N = mn$. The e.m.f. of each cell is E and internal resistance of each cell is r. The combination is connected to external resistance R.

Net e.m.f. $= nE$

Net internal resistance $R_{\text{int}} = \dfrac{nr}{m}$



Net resistance of circuit $= R + \dfrac{nr}{m}$

$$\Rightarrow\quad \text{Current } I = \frac{nE}{R + \frac{nr}{m}} = \frac{NE}{mR + nr}$$

$$\Rightarrow\quad I = \frac{NE}{\left(\sqrt{mR} - \sqrt{nr}\right)^2 + 2\sqrt{mnRr}}$$

For maximum current

$$R = \frac{nr}{m}$$

$$\Rightarrow\quad R_{\text{ext}} = R_{\text{int}}$$

Thus for maximum current, the cells should be connected in mixed grouping when external resistance

R = net internal resistance i.e.,

$$R_{ext} = R_{int} = \frac{nr}{m}$$

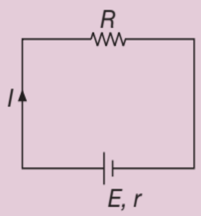
and $$I_{max} = \frac{1}{2}\frac{mE}{r} = \frac{1}{2}\frac{nE}{R}$$

Conceptual Note(s)

Please note that, for the current in the circuit to be maximum or for maximum power to be consumed by the external resistance R, following cases exist.

(a) In the first case, we arrange the cells in such a manner that current and power in the circuit should be maximum. And this happens when we arrange the cells in such a manner that total internal resistance comes out to be equal to total external resistance i.e., load resistance.

(b) In the second case, when a battery of emf E, internal resistance r is connected in series with a load of resistance R, then



$$I = \frac{E}{R+r}$$

$$P_R = I^2R = \left(\frac{E}{R+r}\right)^2 R$$

(i) Now if r can be varied keeping E and R are fixed, then both I and P_R are maximum when $r = 0$.

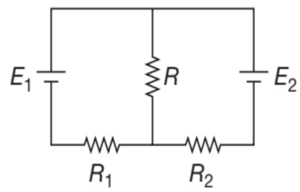
(ii) However, if R can be varied keeping E and r are fixed. Then, current in the circuit is maximum when $R = 0$ (i.e., the battery is shorted), but P_R will be maximum, when

$R = r$

$\Rightarrow$ External Resistance = Internal Resistance

ILLUSTRATION 39

In the circuit shown in figure, the emfs of batteries are E_1 and E_2 which have internal resistances R_1 and R_2. At what value of the resistance R will the thermal power generated in it be the highest? What it is?



SOLUTION

The two batteries are in parallel. Thermal power generated in R will be maximum when,

Total internal resistance = total external resistance

$$\Rightarrow \quad R = \frac{R_1R_2}{R_1+R_2}$$

$$\Rightarrow \quad E_{eq} = \frac{\left(\frac{E_1}{R_1}+\frac{E_2}{R_2}\right)}{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)} = \left(\frac{E_1R_2+E_2R_1}{R_1+R_2}\right)$$

Since, $$R_{net} = \frac{2R_1R_2}{R_1+R_2}$$

$$\Rightarrow \quad I = \frac{E_{eq}}{R_{net}} = \frac{E_1R_2+E_2R_1}{2R_1R_2}$$

Maximum power through R.

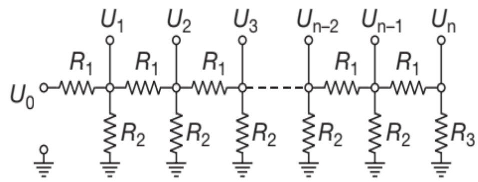
$$P_{max} = I^2R = \frac{(E_1R_2+E_2R_1)^2}{4R_1R_2(R_1+R_2)}$$

Test Your Concepts-VII

Based on EMF, Internal Resistance and Combination of Cells

(Solutions on page H.215)

1. An automobile battery has an emf of 12.6 V and an internal resistance of 0.08 Ω. The headlights together present equivalent resistance 5 Ω (assumed constant). What is the potential difference across the headlight bulbs when
   (a) they are the only load on the battery and
   (b) the starter motor is operated, taking an additional 35 A from the battery?
2. Under what circumstances can the terminal potential difference of a battery exceeds its emf?
3. Two batteries having the same emf E but different internal resistances r_1 and r_2 are connected in series with an external resistor R. For what value of R does the potential difference between the terminals of the first battery become zero?
4. The emf of a storage battery is 90 V before charging and 100 V after charging. When charging began the current was 10 A. What is the current at the end of charging if the internal resistance of the storage battery during the whole process of charging may be taken as constant and equal to 2 Ω?
5. A battery is made by joining m rows of identical cells in parallel each row consists n cells joined in series. This battery sends a maximum current I in a given external circuit. Now the cells are so arranged that instead of m rows, n rows are joined in parallel and each row consists of m cells joined in series. Show that current through the same external circuit is now given by $I' = \frac{2mn}{m^2+n^2}I$.
6. We are made available with 4 batteries each of emf 2 V and supplying a current of 2 A. Show by a circuit diagram how these can be arranged to supply 4 A of current at 4 V.
7. The potential difference across a battery is 8.5 V when there is a current of 3 A in the battery from the negative to the positive terminals. When the current is 2 A in the reverse direction, the potential difference becomes 11 V. What is the emf and the internal resistance of the battery ?
8. The output voltage can be reduced in the output circuits of generators as desired by means of an attenuator designed as the voltage divider shown in figure.



A special selector switch makes it possible to connect the output terminal either to the point with a potential U_0 produced by the generator, or to any of the points U_1, U_2 ..., U_n, each having a potential k times smaller $(k>1)$ than the previous one. The second output terminal and the lower ends of the resistances are earthed.
Find the ratio between the resistances $R_1:R_2:R_3$ with any number of cells in the attenuator.

AMMETER

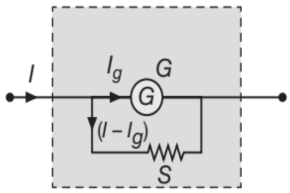
Measurement of Current: The instrument used for measuring current in a circuit is called **ammeter**. It is basically a moving coil galvanometer in which a low resistance wire (shunt) is connected in parallel with the coil. The shunt is connected for two reasons.

(a) To increase the range of current measurement: The moving coil galvanometer is a very sensitive device and its coil gets a full scale deflection for a very small current $(\sim 1\text{ mA})$. By connecting a shunt of suitable resistance, the excess current passes through it, without damaging the galvanometer.

(b) To reduce the error of measurement: Generally the galvanometer coil has a fairly high resistance (10 to 1000 Ω) and therefore it will give error in the measurement of current-the measured current will be less than the actual current. By connecting a low resistance wire in parallel with the coil, the effective resistance of the ammeter can be made small and hence the error is reduced.

SHUNT RESISTANCE TO CONVERT A GALVANOMETER INTO AN AMMETER OF DESIRED RANGE

If I is the range of the ammeter, I_g is the galvanometer current for full scale deflection, G is the resistance of the coil and S is the shunt resistance then, clearly,



Ammeter

$$\left(I-I_g\right)S=I_gG$$

$$\Rightarrow \quad S=\frac{I_gG}{I-I_g}$$

The resistance of the ammeter so obtained is

$$R_A=\frac{GS}{G+S}$$

Generally $S \ll G$, so

$$G+S\simeq G$$

$$R_A\simeq S$$

i.e., ammeter is a low resistance device and hence it is always connected in series in a circuit.

Remark(s)

(a) The reading of an ammeter is always lesser than the actual current in the circuit. If V is the potential difference across a resistance R, the true current is $I=\left(\frac{V}{R}\right)$. However, when an ammeter of resistance r is used to measure it, the reading will be

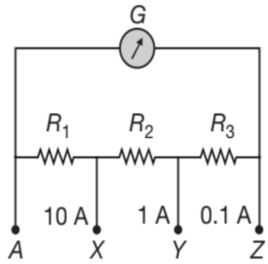
$$I'=\frac{V}{(R+r)}$$

which is less than the true current I

(b) Smaller the resistance of an ammeter, the more accurate will be its reading. An ammeter is said to be ideal if its resistance (r) is zero. However, ideal ammeter cannot be realized in practice.

ILLUSTRATION 40

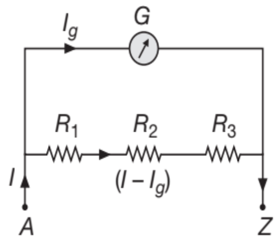
The galvanometer shown in figure has resistance $50\ \Omega$ and current required for full scale



deflection is 1 mA. Find the resistances R_1, R_2 and R_3 required to convert it into ammeter having ranges as indicated.

SOLUTION

For the range 0.1 A, R_1, R_2 and R_3 are in series combination.

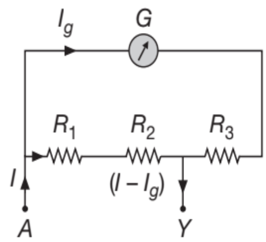


So, equating potential difference across galvanometer and series combination of R_1, R_2 and R_3, we get

$$I_gG=\left(I-I_g\right)\left(R_1+R_2+R_3\right)$$

Hence $\quad R_1+R_2+R_3=\frac{I_gG}{\left(I-I_g\right)}=\frac{10^{-3}\times 50}{\left(0.1-10^{-3}\right)}=\frac{50}{99}\ \Omega \quad \ldots(1)$

For range 1 A, $\left(R_1+R_2\right)$ is in parallel to $\left(G+R_3\right)$



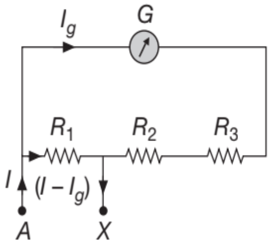
$$\Rightarrow \quad I_g\left(G+R_3\right)=\left(I-I_g\right)\left(R_1+R_2\right)$$

$$\Rightarrow \quad I_g\left(G+R_1+R_2+R_3\right)=I\left(R_1+R_2\right)$$

Substituting for I_g, G, I and $(R_1+R_2+R_3)$, we have

$$R_1+R_2=10^{-3}\left(\frac{50+\frac{50}{99}}{1}\right)=\frac{5}{99}\ \Omega \qquad \ldots(2)$$

For range 10 A, R_1 is in parallel to $(G+R_2+R_3)$



So, $I_g(G+R_2+R_3)=(I-I_g)R_1$

$$\Rightarrow \quad R_1=\frac{I_g(G+R_1+R_2+R_3)}{I}$$

$$\Rightarrow \quad R_1=\frac{10^{-3}\left(50+\frac{50}{99}\right)}{10}=\frac{5}{990}\ \Omega=\frac{1}{198}\ \Omega \qquad \ldots(3)$$

So, $R_2=\frac{5}{99}-\frac{5}{990}=\frac{1}{22}\ \Omega$

$$\Rightarrow \quad R_3=\frac{50}{99}-(R_1+R_2)$$

$$\Rightarrow \quad R_3=\frac{50}{99}-\frac{5}{99}=\frac{45}{99}$$

$$\Rightarrow \quad R_3=\frac{15}{33}\ \Omega$$

Hence, $R_1=\frac{1}{198}\ \Omega$, $R_2=\frac{1}{22}\ \Omega$ and $R_3=\frac{15}{33}\ \Omega$

VOLTMETER

Measurement of Potential Difference: The most commonly used instrument for measurement of potential difference is voltmeter. It is a moving coil galvanometer to which a high resistance is connected in series. The high resistance is connected for two reasons.

(a) To increase the range of measurement: Since the galvanometer has a full scale deflection for very small currents and hence it can measure only a small potential difference. However, if a large resistance is connected in series with the coil, any desired potential difference can be measured.

(b) To reduce the error of measurement: The voltmeter is connected in parallel with that part of the circuit across which the potential difference is to be measured. Therefore its own resistance should be very high, otherwise it will change the current in (and hence the potential difference across) that part of the circuit. The measured potential difference will be less than the actual value. By connecting a high resistance in series with the coil, we can minimize this error.

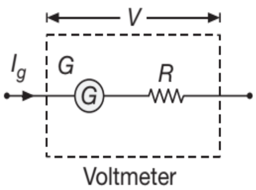
SERIES RESISTANCE TO CONVERT A GALVANOMETER INTO A VOLTMETER OF DESIRED RANGE

If V is the desired range and R is the additional series resistance, then

$$V=I_g(G+R)$$

$$\Rightarrow \quad R=\frac{V}{I_g}-G$$

Voltmeter is a high resistance device and hence it is always connected in parallel across a circuit whose voltage is to be measured.



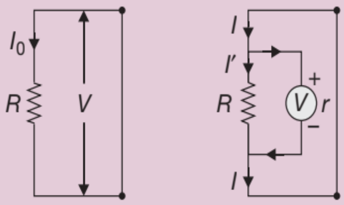
Remark(s)

(a) The reading of a voltmeter is always lesser than the true value. If a current I_0 is passing through a resistance R, the true value $V=I_oR$. However, when a voltmeter having resistance r is connected across R, the current through R will become

$$I'=\frac{r}{(R+r)}I_o$$

and so $V'=I'R=\frac{V}{[1+(R/r)]}$

When the voltmeter is connected across R, its reading will also be V' which is less than V.



(b) Greater the resistance of voltmeter, the more accurate will be its reading. A voltmeter is said to be **ideal** if its resistance r is infinite. An ideal voltmeter draws no current from the circuit element for its operation.

ILLUSTRATION 41

How can we make a galvanometer with $G = 20\ \Omega$ and $I_g = 1\text{ mA}$ into a voltmeter with a maximum range of 10 V?

SOLUTION

Using $R = \dfrac{V}{I_g} - G$

We have, $R = \dfrac{10}{10^{-3}} - 20 = 9980\ \Omega$

Thus, a resistance of $9980\ \Omega$ is to be connected in series with the galvanometer to convert it into the voltmeter of desired range.

Please note that at full scale deflection current through the galvanometer, the voltage drop across the galvanometer

$$V_g = I_g G = 20 \times 10^{-3}\text{ V} = 0.02\text{ V}$$

and the voltage drop across the series resistance R is,

$$V = I_g R = 9980 \times 10^{-3}\text{ V} = 9.98\text{ V}$$

Conceptual Note(s)

(a) If an ammeter of range I and resistance G is to be converted into a voltmeter of range V, then the resistance R to be connected in series is

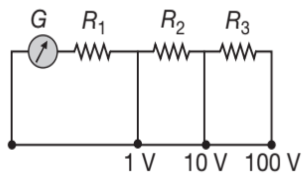
$$R = \frac{V}{I} - G$$

(b) If the voltmeter of range V and resistance G is to be converted into an ammeter of range I, then shunt S to be connected in parallel is

$$S = \frac{VG}{IG - V}$$

ILLUSTRATION 42

A galvanometer has an internal resistance of $50\ \Omega$ and current required for full scale deflection is 1 mA. Find the series resistances required (as shown in figure) to use it as a voltmeter with different ranges, as indicated in figure.



SOLUTION

For range 1 V, galvanometer and R, are in series

$$I_g = \frac{V}{(G + R_1)}$$

$$\Rightarrow \quad 10^{-3} = \frac{1}{(50 + R_1)}$$

$$\Rightarrow \quad 50 + R_1 = 1000$$

$$\Rightarrow \quad R_1 = 1000 - 50 = 950\ \Omega$$

For range 10 V, galvanometer and R_2, R_3 are in series.

$$10^{-3} = \frac{10}{(G + R_1 + R_2)}$$

$$\Rightarrow \quad G + R_1 + R_2 = \frac{10}{10^{-3}} = 10 \times 10^3$$

$$\Rightarrow \quad R_2 = 10000 - (50 + 950) = 9000$$

$$R_2 = 9\text{ k}\Omega$$

and for range 100 V, galvanometer, R_1, R_2 and R_3 are in series.

$$10^{-3} = \frac{100}{(G + R_1 + R_2 + R_3)}$$

$$\Rightarrow \quad G + R_1 + R_2 + R_3 = \frac{100}{10^{-3}} = 100 \times 10^3$$

$\Rightarrow \quad R_3 = 100 \times 10^3 - (G + R_1 + R_2)$

$\Rightarrow \quad R_3 = 100 \times 10^3 - 10 \times 10^3$

$\Rightarrow \quad R_3 = 90 \times 10^3 = 90\ \text{k}\Omega$

ILLUSTRATION 43

The scale of galvanometer is divided into 150 equal divisions. The galvanometer has the current sensitivity of 10 divisions per mA and the voltage sensitivity of 2 divisions per mV. How can you convert this galvanometer into a/an

(a) ammeter of 6 A per division
(b) voltmeter of 1 V per division

Calculate the galvanometer resistance G, value of shunt S to be connected to convert into ammeter and value of series resistance R to be connected to convert into voltmeter.

SOLUTION

Since galvanometer resistance is

$$G = \frac{\text{Full scale voltage}}{\text{Full scale current}}$$

Also, full scale voltage =

$$\frac{\text{No. of divisions}}{\text{Voltage sensitivity}} = \frac{150}{2} = 75\ \text{V}$$

and full scale current =

$$\frac{\text{No. of divisions}}{\text{Current sensitivity}} = \frac{150}{10} = 15\ \text{mA}$$

$$= 15 \times 10^{-3}\ \text{A}$$

$$\Rightarrow \quad G = \frac{75}{15 \times 10^{-3}} = 5000\ \Omega$$

$$\Rightarrow \quad S = \frac{I_g G}{I - I_g} = \frac{15 \times 10^{-3} \times 5000}{150 \times 6 - 15 \times 10^{-3}} = 83\ \text{m}\Omega$$

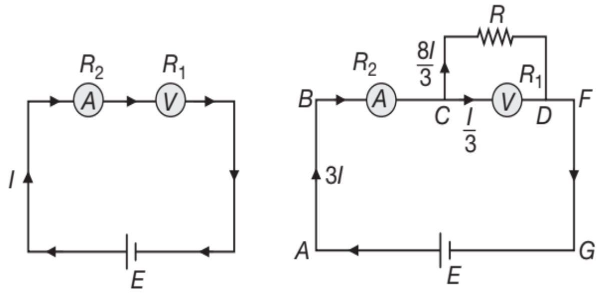
$$\Rightarrow \quad R = \frac{V}{I_g} - G = \frac{150}{15 \times 10^{-3}} - 5 = 5000\ \Omega$$

ILLUSTRATION 44

A voltmeter of resistance R_1 and an ammeter of resistance R_2 are connected in series across a battery of negligible internal resistance. When a resistance R is connected in parallel to voltmeter, reading of ammeter increases three times while that of voltmeter reduces to one third. Find R_1 and R_2 in terms of R.

SOLUTION

Let E be the emf of the battery



In the first case let I be the current in the circuit, then

$$E = I(R_1 + R_2) \qquad \ldots(1)$$

In the second case main current increases three times while current through voltmeter will reduce to $\frac{I}{3}$. Hence, the remaining $3I - \frac{I}{3} = \frac{8I}{3}$ passes through R as shown in figure.

$$V_C - V_D = \left(\frac{I}{3}\right) R_1 = \left(\frac{8I}{3}\right) R$$

$$\Rightarrow \quad R_1 = 8R$$

Applying Kirchhoff's Second Law in Loop $ABFGA$,

$$E = 3I(R_2) + \left(\frac{I}{3}\right)(R_1) = I\left(3R_2 + \frac{R_1}{3}\right) \qquad \ldots(2)$$

From equations (1) and (2), we get

$$R_1 + R_2 = 3R_2 + \frac{R_1}{3}$$

$$\Rightarrow \quad 2R_2 = \frac{2R_1}{3}$$

$$\Rightarrow \quad R_2 = \frac{R_1}{3} = \frac{8R}{3}$$

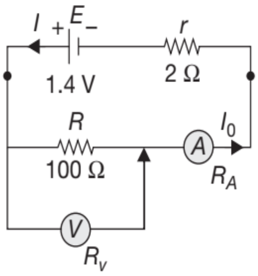
ILLUSTRATION 45

A battery of emf 1.4 V and internal resistance $2\ \Omega$ is connected to a resistor of $100\ \Omega$. In order to measure the current through the resistance and the potential difference across its ends, an ammeter is connected in series with it and a voltmeter is connected across its ends. The resistance of the ammeter is $\frac{4}{3}\ \Omega$ and that of the voltmeter is $200\ \Omega$. What are the readings of the two instruments? What would be their reading if they were ideal instruments?

SOLUTION

Let R_A and R_V be the resistances of the ammeter and voltmeter respectively. Then the total resistance across the emf E is

$$R_{\text{eq}} = \frac{RR_V}{R+R_V} + R_A + r = \frac{100\times200}{100+200} + \frac{4}{3} + 2 = 70\ \Omega$$



Therefore, the current

$$I_o = \frac{E}{R_{\text{eq}}} = \frac{1.4}{70} = 0.02\ \text{A}$$

This is the current through ammeter. Hence, the reading of ammeter is 0.02 A

Reading of voltmeter is the potential difference across its terminals. So reading of voltmeter is

$$V = I_o\left(\frac{RR_V}{R+R_V}\right) = 0.02\left(\frac{100\times200}{100+200}\right) = 1.33\ \text{V}$$

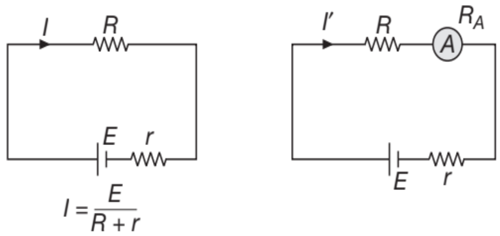
If the ammeter and the voltmeter were **ideal**, $R_A = 0$ and $R_V \to \infty$. Then,

$$\text{The reading of ammeter} = \frac{E}{r+R} = \frac{1.4}{2+100} = 0.0137\ \text{A}$$

$$\text{The reading of voltmeter} = I_o R = \frac{1.4}{102}\times100 = 1.37\ \text{V}$$

ERROR IN THE MEASUREMENT BY AMMETER

To measure current flowing in a certain branch, we connect an ammeter in that branch. Consider the following circuit. To measure I, we connect an ammeter in the wire.



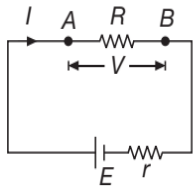
Connecting the ammeter changes the current. Let R_A is the resistance of the ammeter, then

$$I' = \frac{E}{R+R_A+r}$$

Clearly $I' < I$. So an ammeter reads less than actual value. Error in the reading $= I' - I$

ERROR IN THE MEASUREMENT BY VOLTMETER

Consider the same circuit as before. Now, we wish to find out the potential drop across the external resistance R. To measure the potential difference, a voltmeter of resistance R_V is connected between A and B.



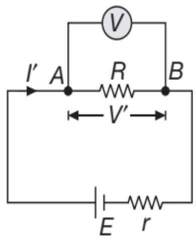
Current through the cell is

$$I = \frac{E}{R+r}$$

Now, $V = V_{AB} = IR$

$$\Rightarrow \quad V = V_{AB} = \frac{ER}{(R+r)} = \frac{E}{1+\dfrac{r}{R}}$$

$$\text{Now, } R' = \frac{R\times R_V}{R+R_V} = \frac{R}{1+\dfrac{R}{R_V}} < R$$



Current through the cell is

$$I' = \frac{E}{R'+r},\ R' = \frac{R\times R_V}{R+R_V}$$

$$\Rightarrow \quad V' = I'R' = \frac{ER'}{(R'+r)} = \frac{E}{1+\dfrac{r}{R'}}$$

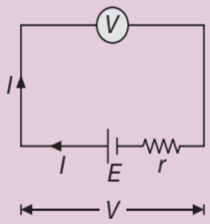
As $R' < R$, $V' < V$ (error in the reading is $V' - V$). So a voltmeter reads less than the actual value.

Conceptual Note(s)

(a) An ideal ammeter should have zero resistance, but a non-ideal ammeter has small but non-zero resistance.

(b) An ideal voltmeter should have infinite resistance, but a non-ideal voltmeter has large but not infinite resistance.

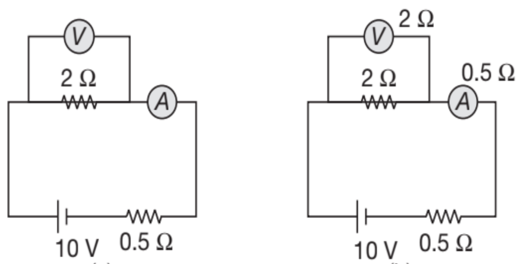
(c) A non-ideal voltmeter (i.e., a practical voltmeter) always draws some current from the circuit and there is always a potential drop across a non-ideal ammeter (i.e., a practical ammeter).



Therefore, when it is used to measure the emf of a cell, it will read $V = E - Ir$. As $I \neq 0$ therefore, reading of a voltmeter will not be equal to emf of the cell.

ILLUSTRATION 46

In the electrical circuits shown in figure (a) and figure (b), the instruments are ideal in figure (a) while their resistances are marked in figure (b). Determine their readings.



(a) (b)

SOLUTION

For figure (a), we have

$$I = \frac{10}{2+0.5} = 4 \text{ A}$$

So, reading of ammeter is 4 A and reading of voltmeter is $IR = 8$ V

For figure (b), we have

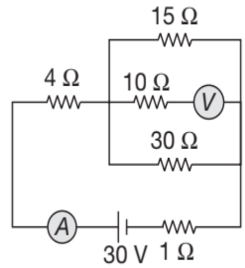
$$I' = \frac{10}{R'+0.5+0.5}, \text{ where } R' = \frac{2\times 2}{2+2} = 1\ \Omega$$

$$\Rightarrow \quad I' = \frac{10}{2} = 5 \text{ A}$$

So, reading of ammeter is 5 A and reading of voltmeter is $I'R' = 5\times(1) = 5$ V

ILLUSTRATION 47

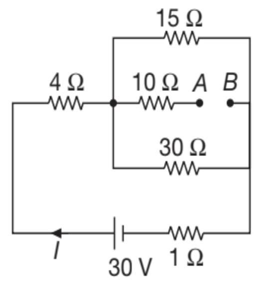
In the electrical circuit shown, the instruments are ideal. Determine their readings.



SOLUTION

Since the ideal ammeter has zero resistance and an ideal voltmeter has infinite resistance.

So, no current flows through the voltmeter. The given circuit can be redrawn as shown in figure.



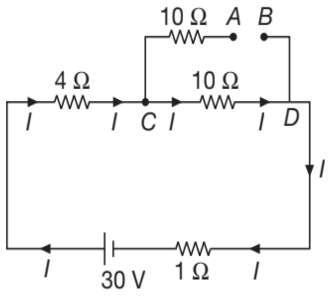
The effective external resistance of the circuit is

$$R_{\text{net}} = 4 + \frac{15\times 30}{15+30} = 14\ \Omega$$

$$\Rightarrow \quad I = \frac{30}{14+1} = 2 \text{ A}$$

Hence, reading of ammeter is 2 A

Since 15 Ω and 30 Ω are in parallel, so their effective resistance is 10 Ω as shown in figure.



The reading of voltmeter is the potential difference between A and B. Since $V_{AB} = I(10) = (2)(10) = 20$ V.

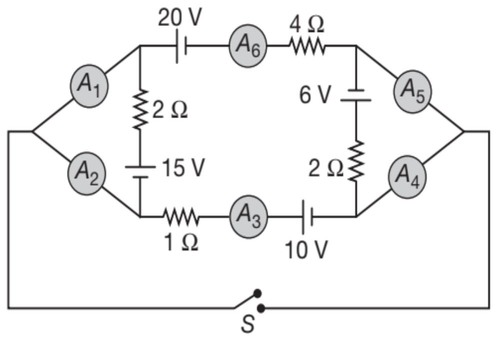
So, reading of voltmeter is 20 V

ILLUSTRATION 48

In the circuit shown, all the ammeters are ideal.

(a) If the switch S is open, find the reading of all ammeters and the potential difference across the switch.

(b) If the switch S is closed, find the current through all ammeters and the switch also.



SOLUTION

Using the loop current method.

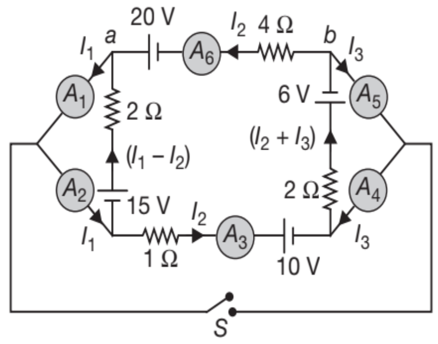
(a) $-2(I_1 - I_2) + 15 = 0$ ...(1)

$-4I_2 + 20 - 2(I_2 - I_1) - 15 - I_2 - 10 - 2(I_2 + I_3) - 6 = 0$

$\Rightarrow \quad 2I_1 - 9I_2 - 2I_3 - 11 = 0$ ...(2)

$-2(I_2 + I_3) - 6 = 0$

$\Rightarrow \quad I_2 + I_3 + 3 = 0$ ...(3)



Solving these three equation, we get

$I_1 = 9.5$ A

$I_2 = 2$ A and $I_3 = -5$ A

Ammeter	A_1	A_2	A_3	A_4	A_5	A_6
Reading (Amp)	9.5	9.5	2	5	5	2

P.D. across switch $= 10 + (1)I_2 = 10 + 2 = 12$ V

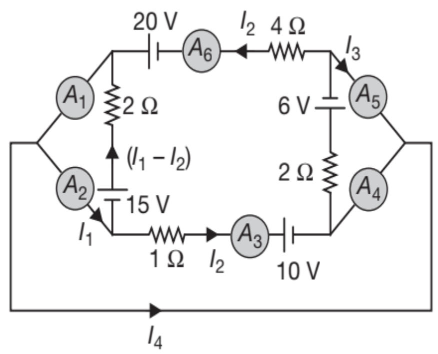
(b) When switch is closed

$-2(I_1 - I_2) + 15 = 0$ ...(1)

$2I_1 - 9I_2 - 2I_3 - 11 + I_4 = 0$ ...(2)

$I_2 + I_3 + 3 = 0$ ...(3)

$10 - (I_4 - I_2) = 0$ ...(4)



Solving these four equation, we get

$I_1 = 12.5$ A

$I_2 = 5$ A

$I_3 = -8$ A

$I_4 = 15$ A

Ammeter	A_1	A_2	A_3	A_4	A_5	A_6
Reading (Amp)	12.5	2.5	10	7	8	5

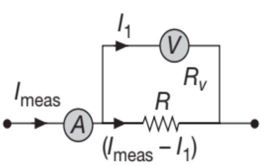
And the current through switch is 15 A.

Test Your Concepts-VIII

Based on Galvanometer, Voltmeter and Ammeter

(Solutions on page H.216)

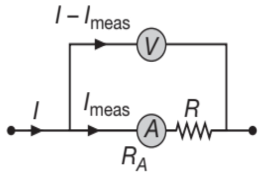
1. Assume that a galvanometer has an internal resistance of 60 Ω and requires a current of 0.5 mA to produce full scale deflection. What resistance must be connected in parallel with the galvanometer if the combination is to serve as an ammeter that has a full scale deflection for a current of 0.1 A?
2. Design a multirange ammeter capable of full scale deflection for 25 mA, 50 mA, and 100 mA. Assume the meter movement is a galvanometer that has a resistance of 25 Ω and gives a full scale deflection for 1 mA.
3. Design a multirange voltmeter capable of full scale deflection for 20 V, 50 V and 100 V. Assume the meter movement is a galvanometer that has a resistance of 60 Ω and gives a full scale deflection for a current of 1 mA.
4. A particular galvanometer serves as a 2 V full scale voltmeter when a 2500 Ω resistor is connected in series with it. It serves as a 0.5 A full scale ammeter when a 0.22 Ω resistor is connected in parallel with it. Determine the internal resistance of the galvanometer and the current required to produce full scale deflection.
5. To measure the resistance R of a resistor, a voltmeter of resistance R_v is placed across a resistor and an ammeter is placed in series with the combination as shown in figure



   (a) Find the resistance R in terms of the measured readings on the ammeter I_{meas} and voltmeter V_{meas}

   (b) Discuss the result for $R_v \gg \dfrac{V_{meas}}{I_{meas}}$

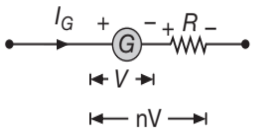
6. To measure the resistance R of a resistor, an ammeter of resistance R_A is placed in series with the resistor and the voltmeter is placed across the series combination as shown in figure.



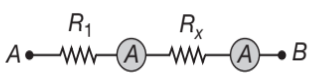
   (a) Find the resistance R in terms of the measured readings on the ammeter I_{meas} and voltmeter V_{meas}

   (b) Discuss the result for $\dfrac{V_{meas}}{I_{meas}} \gg R_A$.

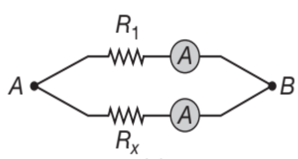
7. What shunt resistance is required to make the 1 mA, 20 Ω galvanometer into an ammeter with a range of 0 A to 50 mA?
8. A voltmeter has a resistance G ohm and range V volt. Calculate the resistance to be used in series with it to extend its range to nV volt.



9. Consider two different ammeters in which the currents are proportional to the respective deflections of the needle. The first ammeter is connected to a resistor of resistance R_1 and the second to a resistor of unknown resistance R_x. Firstly, the ammeters are connected in series between points A and B (as shown in figure). In this case the readings of the ammeters are n_1 and n_2. Then the ammeters are connected in parallel between A and B (as shown in figure) and indicate N_1 and N_2. Determine the unknown resistance R_x of the second resistor.



(a)



(b)

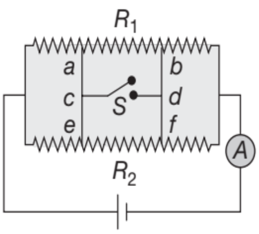
10. An ammeter is connected to measure the current intensity in a circuit with a resistance R. What relative error ε will be made if connection of the ammeter does not change the current intensity in the circuit? The voltage across the ends of the circuit is kept constant.

11. Determine the voltage across a resistance R using a voltmeter connected to its ends. What relative error, ε will be made if the readings of the voltmeter are taken as the voltage applied before it was switched on? The current intensity in the circuit is constant.

12. What resistance r should be used to shunt a galvanometer with an internal resistance of $R = 10$ kΩ to reduce its sensitivity $n = 50$ times?

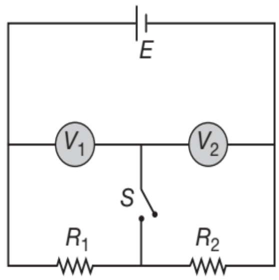
13. An ammeter and a voltmeter are connected in series to a battery of emf $E = 6$ V. When a certain resistance is connected in parallel with the voltmeter, the reading of the latter decreases two times, whereas the readings of the ammeter increase the same number of times. Find the voltmeter readings after the connection of the resistance.

14. Two conductors ace and bdf are connected at equal potentials on resistors R_1 and R_2. Will there any currents flowing through them and through the cd section if the switch S is closed. Will this lead to a change in the reading of an ammeter?



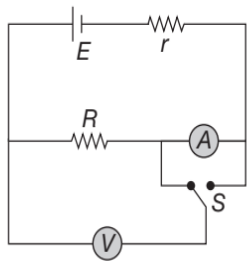
15. A cell of emf 3.4 V and internal resistance 3 Ω is connected to an ammeter having resistance 2 Ω and to an external resistance of 100 Ω. When a voltmeter is connected across the 100 Ω resistance the ammeter reading is 0.04 A. Find the voltage read by the voltmeter and its resistance. Had the voltmeter been an ideal one what would have been its reading?

16. In the circuit shown in figure, V_1 and V_2 are two voltmeters having resistances 6 kΩ and 4 kΩ respectively. The emf of the battery is 250 V, having negligible internal resistance. Two resistances $R_1 = 4$ kΩ and $R_2 = 6$ kΩ are also connected in the circuit as shown. Find the reading of the voltmeters V_1 and V_2 when



(a) the switch S is open and
(b) the switch S is closed.

17. The emf E and the internal resistance r of the battery shown in figure are 4.3 V and 1 Ω respectively. The external resistance R is 50 Ω. The resistances of the ammeter and voltmeter are 2 Ω and 200 Ω respectively.

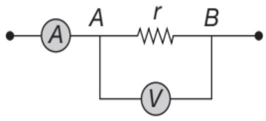


(a) Find the readings of the two meters.
(b) The switch is thrown to the other side. What will be the readings of the two meters now?

18. A moving coil galvanometer of resistance 20 Ω gives a full scale deflection when a current of 1 mA is passed through it. It is to be converted into an ammeter reading 20 A on full scale. But the shunt of 0.005 Ω only is available. What resistance should be connected in series with the galvanometer coil?

19. A resistance box, a battery and a galvanometer of resistance G Ω are connected in series. If the galvanometer is shunted by resistance of S Ω, find the change in resistance in the box required to maintain the current from the battery unchanged.

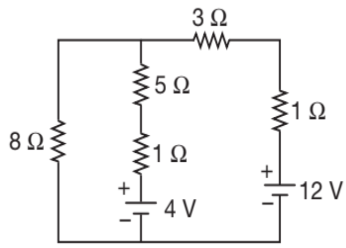
20. Determine the resistance r if an ammeter shows a current of $I = 5$ A and a voltmeter 100 V. The internal resistance of the voltmeter is $R = 2500$ Ω.



21. Two resistors, 400 Ω, and 800 Ω are connected in series with a 6 V battery. It is desired to measure the current in the circuit. An ammeter of 10 Ω resistance is used for this purpose. What will be the reading in the ammeter? Similarly, if a voltmeter of 1000 Ω resistance is used to measure the potential difference across the 400 Ω resistor, what will be the reading in the voltmeter?

22. Draw the circuit for experimental verification of Ohm's Law using a source of variable d.c. voltage, a main resistance of 100 Ω, two galvanometers and two resistances of values 10^6 Ω and 10^{-3} Ω respectively. Clearly show the positions of the voltmeter and the ammeter.

23. In figure, show how to add just enough ammeters to measure every different current. Show how to add just enough voltmeters to measure the potential difference across each resistor and across each battery.



HEATING EFFECT OF CURRENT

When a charge dq passes across a potential difference V, the work done dW is given by,

$$dW = Vdq$$

This work represents the loss of potential energy of charges. The flow of charge dq in time dt is equivalent to current I, where

$$I = \frac{dq}{dt}$$

$\Rightarrow \quad dq = Idt$

$\Rightarrow \quad dW = VIdt$

If constant current I passes through conductor for time t under a potential difference V, then

$$W = VIt \qquad \ldots(1)$$

According to Ohm's Law, $V = IR$

$\Rightarrow$ Work done $= I^2Rt = \frac{V^2}{R}t = VIt \qquad \ldots(2)$

This work is converted into the energy of random thermal motion of molecules of the conductor. That is the electric current through a conductor produces thermal energy in the conductor and the conductor gets heated. This phenomenon is called **Joule's heating effect of current**.

If V is in volt, I in amp, R in ohm, then Joule's heat is equivalent to

$$W = VIt = I^2Rt = \frac{V^2}{R}t \text{ joule}$$

OR

Heat produced Q is given by

$$Q = \frac{W}{J} = \frac{VIt}{J} = \frac{I^2Rt}{J} = \frac{\left(\frac{V^2}{R}\right)t}{J} \text{ cal}$$

POWER

The rate of work done in an electrical circuit is called the **power** and is dissipated in the form of heat.

Power dissipated,

$$P = \frac{dW}{dt} = VI = I^2R = \frac{V^2}{R}$$

The SI unit of power is watt (W)

$$1 \text{ W} = 1 \text{ Js}^{-1}$$

ELECTRIC ENERGY

The usual unit of energy is joule but for convenience electrical energy (measured as a large unit) is measured in kilowatthour (kWh).

$$1\ \text{kWh} = P(\text{in kilowatt}) \times (t \text{ in hour})$$

$$1\ \text{kWh} = 1000 \times 3600 \text{ joule}$$

$$1\ \text{kWh} = 3.6 \times 10^6 \text{ joule}$$

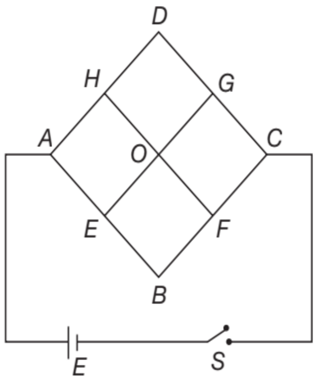
In houses the electric appliances (e.g., bulbs, refrigerator, T.V., cooler, heater etc.) are connected in parallel and the electric energy consumed is measured in kilowatthour (kWh).

$$\text{Number of units consumed } (N) = \frac{\text{watt} \times \text{hours}}{1000}$$

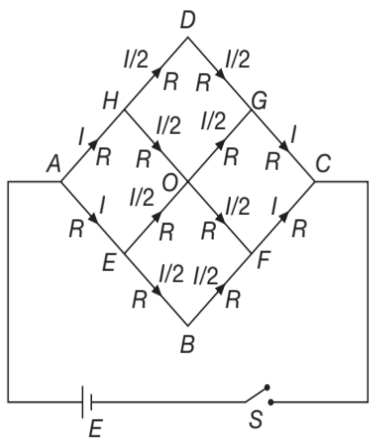
$$\Rightarrow N = \frac{(\text{Power Consumed})(\text{Time of Consumption})}{1000}$$

ILLUSTRATION 49

In the given figure each of the segments has resistance R. EMF of battery is E and internal resistance is negligible. Find ratio of power generated in AE to that in HO.



SOLUTION



$$\frac{P_{AE}}{P_{HO}} = \frac{I^2 R}{\left(\frac{I}{2}\right)^2 R} = 4$$

SPECIFICATION OF A BULB OR OTHER ELECTRIC APPLIANCES

Every electrical appliance like Bulb, Heater, Geyser etc. has wattage and voltage printed on it. These are called rated values or the specified values. When a bulb has specified voltage V and power P, then resistance R and maximum allowed current I may be determined.

$$\text{Power}(P) = \text{Voltage}(V) \times \text{Current}(I)$$

$\Rightarrow$ Maximum allowed current in bulb is

$$I = \frac{P}{V}$$

Resistance of its filament is

$$R = \frac{V}{I} = \frac{V}{P/V} = \frac{V^2}{P}$$

The more the current flowing through a bulb the more is its glow.

The bulbs and other electric appliances are manufactured for parallel combination. If they are connected in series, the effect is reversed. For example if two bulbs of $50\ \text{W}$, $200\ \text{W}$ are given, then for parallel combination of the bulbs, the $200\ \text{W}$ bulb would glow more brightly. However, for the series combination of the bulbs, the $50\ \text{W}$ bulb would glow more brightly as more current flows through it.

Conceptual Note(s)

(a) S.I. unit of power is Js^{-1} and $1\,Js^{-1} = 1$ watt

(b) **Rated values of Voltage and Power:** Every electrical appliance like Bulb, Heater, Geyser etc. has wattage and voltage printed on it. These are called rated values.

FOR EXAMPLE: A 40 W, 220 V bulb has a rated power $P_R = 40$ W and the rated voltage $V_R = 220$ V. This simply means that when this bulb is connected across the rated voltage i.e. 220 V, then the power consumed across it will also be the rated power that is 40 W. So,

when $V_{\text{applied}} < V_{\text{rated}}$, then $P_{\text{consumed}} < P_{\text{rated}}$ and

when $V_{\text{applied}} > V_{\text{rated}}$, then $P_{\text{consumed}} > P_{\text{rated}}$

(c) **Resistance of electrical appliance:** If variation of resistance with temperature is neglected then resistance of any electrical appliance can be calculated by rated power and rated voltage i.e. by using $R=\frac{V_R^2}{P_R}$.

(d) It is observed that for a bulb

Brightness $\propto P_{consumed}$

(e) Power consumed and illumination: An electrical appliance (Bulb, heater, ... etc.) consume rated power P_R only when the applied voltage V_A is equal to the rated voltage V_R. So

(i) If $V_A = V_R$, then $P_{consumed} = P_R$

(ii) If $V_A < V_R$, then $P_{consumed} = \frac{V_A^2}{R}$

Since, $R=\frac{V_R^2}{P_R}$ and hence

$$\Rightarrow \quad P_{consumed} = \left(\frac{V_A^2}{V_R^2}\right)P_R \propto (\text{Brightness})$$

POWER TRANSFORMATION RULE

CASE-I: Bulbs Connected in Series

When bulbs/appliances having rated voltage V, and rated powers P_1, P_2, P_3, ... are connected in series across a voltage supply V, then the power consumption across the series combination of the appliances will be

$$P_s = \frac{V^2}{R_s},$$

where $R_s = R_1 + R_2 + R_3$ and

$$R_1 = \frac{V^2}{P_1},\ R_2 = \frac{V^2}{P_2} \text{ and } R_3 = \frac{V^2}{P_3}$$

$$\Rightarrow \quad P_s = \frac{V^2}{\left(\frac{V^2}{P_1}+\frac{V^2}{P_2}+\frac{V^2}{P_3}\right)}$$

$$\Rightarrow \quad \frac{1}{P_s} = \frac{1}{P_1}+\frac{1}{P_2}+\frac{1}{P_3}$$

CASE-2: Bulbs Connected in Parallel

When bulbs/appliances having rated voltage V, and rated powers P_1, P_2, P_3, ... are connected in parallel across a voltage supply V, then the power consumption across the parallel combination of the appliances will be

$$P_p = \frac{V^2}{R_p},$$

where $\frac{1}{R_p} = \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$ and

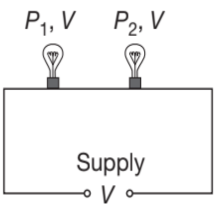
$$R_1 = \frac{V^2}{P_1},\ R_2 = \frac{V^2}{P_2} \text{ and } R_3 = \frac{V^2}{P_3}$$

$$\Rightarrow \quad P_s = V^2\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)$$

$$\Rightarrow \quad P_s = P_1 + P_2 + P_3$$

COMBINATION OF BULBS

Series Combination



(a) Total power consumed $\frac{1}{P_{total}} = \frac{1}{P_1}+\frac{1}{P_2}+......$

(b) If n bulbs are identical, $P_{total} = \frac{P}{n}$

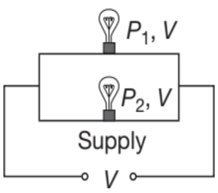
(c) $P_{consumed}$ (Brightness) $\propto V \propto R \propto \frac{1}{P_{rated}}$

i.e. in series combination, the bulb of lesser wattage will give more bright light and potential difference appeared across it will be more.

Parallel Combination

(a) Total power consumed

$$P_{total} = P_1 + P_2 + P_3 ...... + P_n$$



(b) If n identical bulbs are in parallel, then $P_{total} = nP$

(c) $P_{consumed}$ (Brightness) $\propto P_R \propto i \propto \dfrac{1}{R}$

i.e. in parallel combination, bulb of greater wattage will give more bright light and more current will pass through it.

Conceptual Note(s)

(a) If $V_{Applied} < V_{Rated}$ then percentage drop in output power of electrical device is

$$\%\text{age drop} = \frac{(P_R - P_{consumed})}{P_R} \times 100$$

(b) Different bulbs

25 W	100 W	1000 W
220 V	220 V	220 V

$\Rightarrow$ Resistance $R_{25} > R_{100} > R_{1000}$

$\Rightarrow$ Thickness of filament $t_{1000} > t_{100} > t_{40}$

$\Rightarrow$ Brightness $B_{1000} > B_{100} > B_{25}$

(c) Necessary series resistance to glow a bulb of rated power P_R, if $V_{Applied} > V_{Rated}$ is

$$R = \left(\frac{V_{Applied} - V_{Rated}}{P_R}\right) \times V_R$$

(d) When some potential difference applied across the conductor then collision of free electrons with ions of the lattice result's in conversion of electrical energy into heat energy

(e) If a heating coil of resistance R, (length ℓ) consumed power P, when voltage V is applied to it then by keeping V constant if it is cut in n equal parts then resistance of each part will be $\frac{R}{n}$ and from $P_{consumed} \propto \frac{1}{R}$, power consumed by each part $P' = nP$.

(f) In series a device of higher power rating consumes less power.

(g) Consider that n bulbs are connected in series across V volt supply. If one bulb gets fused and $(n - 1)$ bulbs are again connected in series across same supply, the illumination will be more with $(n - 1)$ bulbs than with n bulbs, but risk of fusing of bulbs will increases.

(h) When a heavy current appliance such us motor, heater or geyser is switched on, it will draw a heavy current from the source so that terminal voltage of source decreases. Hence power consumed by the bulb decreases, so the light of bulb becomes less.

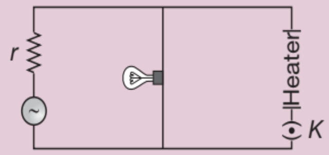


ILLUSTRATION 50

A wire of length L and 3 identical cells of negligible internal resistances are connected in series. Due to this current, the temperature of the wire is raised by ΔT in time t. A number N of similar cells is now connected in series with a wire of the same material and cross-section but of length $2L$. The temperature of wire is raised by same amount ΔT in the same time t. Find the value of N.

SOLUTION

Since we have

$$\text{Heat} = mc\Delta T = i^2 Rt$$

CASE-1: Length (L), Resistance R and mass m

CASE-2: Length $(2L)$, Resistance $2R$ and mass $2m$

$$\Rightarrow \frac{m_1 S_1 \Delta T_1}{m_2 S_2 \Delta T_2} = \frac{i_1^2 R_1 t_1}{i_2^2 r_2 t_2}$$

$$\Rightarrow \frac{mS\Delta T}{2mS\Delta T} = \frac{i_1^2 Rt}{i_2^2 2Rt}$$

$$\Rightarrow i_1 = i_2$$

$$\Rightarrow \frac{(3E)^2}{12} = \frac{(NE)^2}{2R}$$

$$\Rightarrow N = 6$$

TIME RELATION FOR SERIES AND PARALLEL COMBINATION

Let an electrical appliance having rated power P_1 can generate energy E in time t_1, then $E = P_1 t_1$.

Similarly another electrical appliance having rated power P_2 can generate same energy E in time t_2, then $E = P_2 t_2$.

Now when these appliances are connected across the same voltage source

CASE-1: In Series

Then power generated is

$$\frac{1}{P_S} = \frac{1}{P_1} + \frac{1}{P_2}$$

So time required to generate the same energy E is

$$t_S = \frac{E}{P_S} = \frac{E}{P_1} + \frac{E}{P_2} = \frac{E}{\left(\frac{E}{t_1}\right)} + \frac{E}{\left(\frac{E}{t_2}\right)} = t_1 + t_2$$

$$\Rightarrow \boxed{t_S = t_1 + t_2}$$

CASE-2: In Parallel

Then power generated is

$$P_P = P_1 + P_2$$

So, time required to generate the same energy E is

$$t_P = \frac{E}{P_P} = \frac{E}{P_1 + P_2} = \frac{E}{\frac{E}{t_1} + \frac{E}{t_2}} = \frac{t_1 t_2}{t_1 + t_2}$$

$$\Rightarrow \boxed{\frac{1}{t_P} = \frac{1}{t_1} + \frac{1}{t_2}}$$

ILLUSTRATION 51

An electric tea kettle has a multi-position switch and two heating coils. When only one of the coils is switched on, the well-insulated kettle makes a full pot of water to a boil in a time interval Δt. When only the other coil is switched on, it requires a time interval of $2\Delta t$ to boil the same amount of water. Find the time interval required to boil the same amount of water if both coils are switched while being used

(a) in a parallel connection and
(b) in a series connection.

SOLUTION

A certain quantity of energy $\Delta E_{int} = P(\text{time})$ is required to raise the temperature of the water to 100°C. For the power delivered to the heaters we have $P = I\Delta V = \frac{(\Delta V)^2}{R}$ where ΔV is a constant. Thus comparing coils 1 and 2, we have for the energy

$$H = \frac{(\Delta V)^2 \Delta t}{R_1} = \frac{(\Delta V)^2 2\Delta t}{R_2}$$

$$\Rightarrow \quad R_2 = 2R_1$$

(a) When connected in parallel, the coils present equivalent resistance

$$R_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{1}{R_1} + \frac{1}{2R_1}} = \frac{2R_1}{3}$$

and $H = \frac{(\Delta V)^2 \Delta t}{R_1} = \frac{(\Delta V)^2 \Delta t_p}{\frac{2R_1}{3}}$

$$\Rightarrow \quad \Delta t_p = \frac{2\Delta t}{3}$$

(b) For the series connection, $R_s = R_1 + R_2 = R_1 + 2R_1$ $= 3R_1$ and $\frac{(\Delta V)^2 \Delta t}{R_1} = \frac{(\Delta V)^2 \Delta t_s}{3R_1}$

$$\Rightarrow \quad \Delta t_s = 3\Delta t$$

CHARACTERISTICS OF A FUSE

Fuse is used with the main electrical circuit for the safety of electrical appliances.

A fuse wire must have high resistance and low melting point. Hence, generally it is made of tin-lead alloy.

Let R be the resistance, ρ resistivity, l length, a cross-sectional area and I ampere be its current carrying capacity.

When the fuse is safe, then for its steady state temperature, heat produced per second must be equal to heat radiated by it per second. Heat produced in fuse wire per second

$$H = \frac{Q}{t} = I^2 R = I^2\left(\frac{\rho l}{A}\right)$$

$$H = \frac{I^2 \rho l}{\pi r^2} \text{ joule sec}^{-1} \quad \ldots(1)$$

If e is emissivity of fuse material of radius r and T is the excess safe temperature of wire above surroundings then according to **Newton's Law of Cooling**, the energy radiated per second

$$H = e(2\pi rl)T \quad \ldots(2)$$

For steady state

$$e(2\pi rl)T = I^2 \frac{\rho l}{\pi r^2}$$

$$\Rightarrow \quad T = \frac{I^2 \rho}{2\pi^2 e r^3} \quad \ldots(3)$$

So, we observe that the steady state temperature of a fuse is independent of its length.
Hence length is immaterial for an electric fuse.
For a given material of a fuse wire

$$I^2 \propto r^3$$

ILLUSTRATION 52

What amount of heat will be generated in a coil of resistance R due to a charge q passing through it if the current in the coil decreases down to zero

(a) uniformly during a time interval t_0?

(b) halving its value every t_0 seconds?

SOLUTION

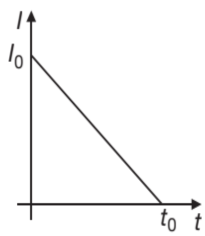
Heat generated in a resistance is given by,

$$H = I^2 Rt$$

We can directly use this formula provided I is constant. Here, I is varying. So, first we will calculate I at any time t, then find a small heat dH in a short interval of time dt. Then by integrating it with proper limits we can obtain the total heat produced.

(a) The corresponding $I-t$ graph will be a straight line with I decreasing from a peak value (say I_0) to zero in time t_0. $I-t$ equation will be as

$$\Rightarrow \quad I = I_0 - \left(\frac{I_0}{t_0}\right)t \qquad (y = -mx + c) \quad \ldots(1)$$



Here, I_0 is unknown, which can be obtained by using the fact that area under I-t graph gives the flow of charge. Hence,

$$q = \frac{1}{2}(t_0)(I_0)$$

$$\Rightarrow \quad I_0 = \frac{2q}{t_0}$$

Substituting in (1), we get,

$$I = \frac{2q}{t_0}\left(1 - \frac{t}{t_0}\right)$$

$$\Rightarrow \quad I = \left(\frac{2q}{t_0} - \frac{2qt}{t_0^2}\right)$$

Now at time t, heat produced in a short interval dt is,

$$dH = I^2 R\, dt$$

$$\Rightarrow \quad dH = \left(\frac{2q}{t_0} - \frac{2qt}{t_0^2}\right)^2 R dt$$

So, total heat produced is $H = \int_0^{t_0} dH$

$$\Rightarrow \quad H = \int_0^{t_0} \left(\frac{2q}{t_0} - \frac{2qt}{t_0^2}\right)^2 R\, dt$$

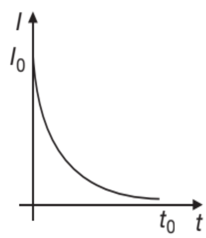
$$\Rightarrow \quad H = \frac{4}{3}\frac{q^2 R}{t_0}$$

(b) Here, current decreases from some peak value (say I_0) to zero exponentially with half life t_0. I-t equation in this case will be

$$I = I_0 e^{-\lambda t}$$

Here, $\lambda = \dfrac{\log_e(2)}{t_0}$

Now, $q = \int_0^{\infty} I\, dt = \int_0^{\infty} I_0 e^{-\lambda t} dt = \left(\frac{I_0}{\lambda}\right)$



$$\Rightarrow \quad I_0 = \lambda q$$

$$\Rightarrow \quad I = (\lambda q)e^{-\lambda t}$$

$$\Rightarrow \quad dH = I^2 R dt = \lambda^2 q^2 e^{-2\lambda t} R dt$$

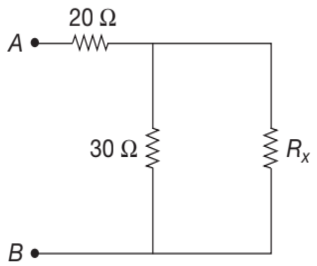
$$\Rightarrow \quad H = \int_0^\infty dH = \lambda^2 q^2 R \int_0^\infty e^{-2\lambda t} dt = \frac{q^2 \lambda R}{2}$$

Substituting $\lambda = \dfrac{\log_e(2)}{t_0}$, we have

$$H = \frac{q^2 R \log_e(2)}{2t_0}$$

ILLUSTRATION 53

A circuit shown in the figure has resistances 20 Ω and 30 Ω. At what value of resistance R_x will the thermal power generated in it be practically independent of small variations of that resistance? The voltage between points A and B is supposed to be constant in this case.

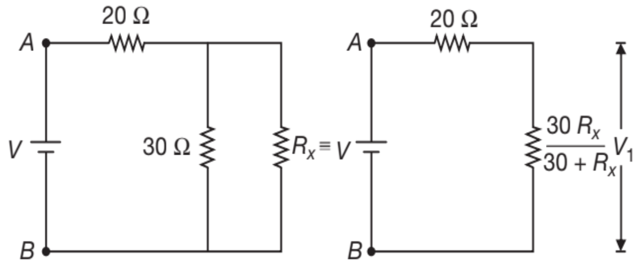


SOLUTION

$$V = \text{Constant}$$

Now, voltage across R_x is equal to the voltage across the 30 Ω resistor, because both are in parallel. Hence, if V_1 is the voltage across R_x, then

$$V_1 = V_{30\,\Omega} = \left(\frac{\dfrac{30R_x}{30+R_x}}{\dfrac{30R_x}{30+R_x} + 20} \right) V = \left(\frac{30R_x}{50R_x + 600} \right) V$$



Original circuit Equivalent circuit

Now, power generated in R_x is

$$P = \frac{V_1^2}{R_x} = \frac{900 R_x V^2}{(50R_x + 600)^2}$$

For P to be constant, we have

$$\frac{dP}{dR_x} = 0$$

$$\Rightarrow \quad \frac{(50R_x + 600)^2 (900V^2) - (1800)(50)(R_x V^2)(50R_x + 600)}{(50R_x + 600)^4} = 0$$

$$\Rightarrow \quad 50R_x + 600 - 100R_x = 0$$

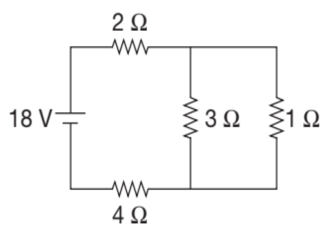
$$\Rightarrow \quad R_x = 12\ \Omega$$

Test Your Concepts-IX

Based on Heating Effects and Power Consumption

(Solutions on page H.220)

1. Calculate the power delivered to each resistor in the circuit shown.



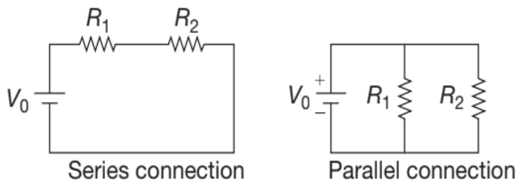
2. An electric teakettle has a multiposition switch and two heating coils. When only one of the coils is switched on, the well-insulated kettle makes a full pot of water to a boil in a time interval Δ*t*. When only the other coil is switched on, it requires a time interval of 2Δ*t* to boil the same amount of water. Find the time interval required to boil the same amount of water if both coils are switched while being used

(a) in a parallel connection and
(b) in a series connection.

3. When two unknown resistors are connected in series with a battery, the battery delivers total power P_s and carries a total current of I. For the same total current, a total power P_p is delivered when the resistors are connected in parallel. Determine the values of the two resistors.

4. Two resistors R_1 and R_2 are in parallel with each other. Together they carry total current I.
(a) Determine the current in each resistor.
(b) Prove that this division of the total current I between the two resistors results in less power delivered to the combination than any other division. In other words prove the general principle that current in a direct current circuit distributes itself so that the total power delivered to the circuit is a minimum.

5. Consider an ideal battery with emf V_0 and two resistors with resistances R_1 and R_2. For the power absorbed by resistors to be maximum, should they be connected in series or in parallel, as in figure.

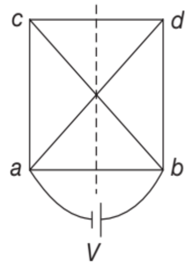


6. Two resistors are made of identical material, have the same length but one has the diameter twice that of the other. Determine the ratio of powers dissipated in the two resistors when
(a) each resistor is being connected across the same voltage source i.e., in parallel
(b) they are connected in series

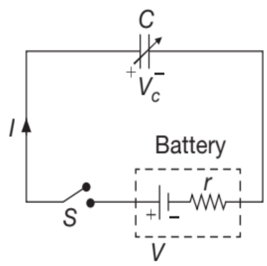
7. A 100 W, 110 V light bulb has a filament made of an alloy having a temperature coefficient of 0.0055 $°C^{-1}$ at 0°C. The normal operating temperature of the bulb is 2000°C. How much current will the bulb draw at the instant it is turned on when the room temperature is 20°C and 2000°C?

8. **(a)** Two wires made of same tinned copper alloy having equal cross-sectional area but of different lengths are to be used as fuses. Show that the fuses will melt at the same value of current which is independent of the length of the wires.
(b) A fuse wire of radius 0.1 mm blows when a current of 10 A passes through it. What should be the radius of a fuse wire made of the same material which will blow at a current of 20 A?

9. In the circuit shown in figure *abcd* is a square. All the wires forming the square and its diagonals are homogeneous and have same cross-section. Find the ratio of power dissipated in resistors *ab* and *cd*.

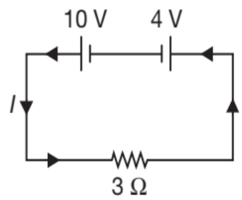


10. The variable capacitor in figure is connected to a battery of emf V and internal resistance r. The current in the circuit is kept constant by changing the capacitance. (Assume that we are able to increase the capacitance indefinitely in order to accomplish this).



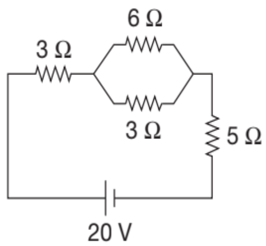
(a) Calculate the power supplied by the battery.
(b) Compare the rate at which energy is supplied by the battery with the rate of change of the energy stored in the capacitor.

11. In the circuit shown in figure, find the power

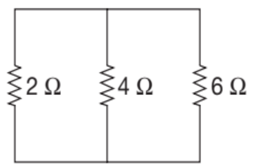


(a) supplied by 10 V battery
(b) consumed by 4 V battery and
(c) dissipated in 3 Ω resistance.

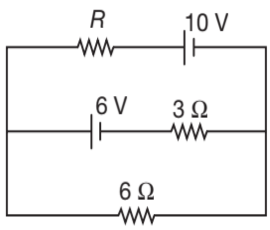
12. In the circuit shown in figure, find the heat developed across each resistance in $t = 2$ s.



13. In which branch of the circuit shown, a 11 V battery be inserted so that it dissipates minimum power. What will be the current through the 2 Ω resistance for this position of the battery?



14. In a circuit shown in figure if the internal resistances of the sources are negligible then at what value of resistance R will the thermal power generated in it be the maximum. What is its value?



15. The current I through a rod of a certain metallic oxide is given by $I = 0.2\ V^{5/2}$, where V is the potential difference across the rod. The rod is connected in series with a resistance to a 6 V battery of negligible internal resistance. What value should the series resistance have so that:

(a) the current in the circuit is 0.4 A.

(b) the power dissipated in the rod is twice that dissipated in the resistance.

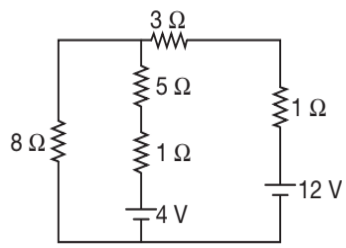
16. A storage battery with emf 2.6 V loaded with external resistance produces a current 1 A. In this case the potential difference between the terminals of the storage battery equals 2 V. Find the thermal power generated in the battery and the power developed in it by electric forces.

17. The circuit considered shown in figure is connected for a duration of 2 minute.

(a) Find the energy delivered by each battery.

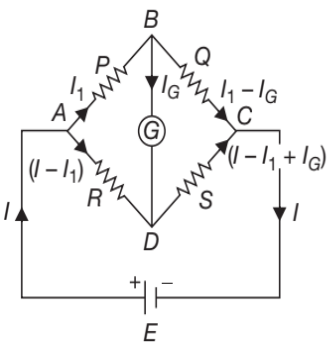
(b) Find the energy delivered to each resistor.

(c) Identify the types of energy transformations that occur in the operation of the circuit and the total amount of energy involved in each type of transformation.



WHEATSTONE BRIDGE: CONDITION OF BALANCE

The Wheatstone's Bridge is shown in figure.



P, Q, R and S are four resistances, G is galvanometer and E is a battery. The Wheatstone's Bridge is said to be balanced when no current flows in galvanometer. So,

Potential of $B =$ Potential of D.

The condition of balance is best achieved by applying Kirchhoff's Laws

Applying Kirchhoff's 2nd Law to loop ABDA

$$-I_1P - I_GG + (I - I_1)R = 0 \qquad \ldots(1)$$

Applying Kirchhoff's 2nd Law to loop BCDB

$$-(I_1 - I_G)Q + S(I - I_1 + I_G) + I_GG = 0 \qquad \ldots(2)$$

For balance $I_G = 0$, so that (1) and (2) give

$$I_1 P = (I - I_1) R \text{ and} \quad \ldots(3)$$

$$I_1 Q = (I - I_1) S \quad \ldots(4)$$

Dividing (3) by (4), we get
Condition of balance is

$$\frac{P}{Q} = \frac{R}{S}$$

Remark(s)

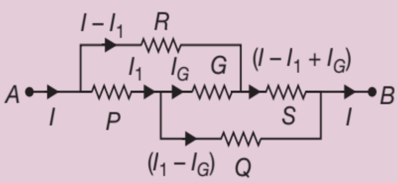
So, to conclude

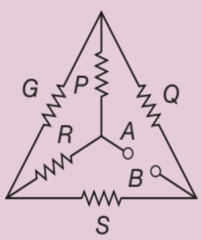
(a) When battery and galvanometer arms of a Wheatstone's Bridge are interchanged, the balance position remains undisturbed while sensitivity of bridge changes.

(b) When Wheatstone's Bridge is balanced, the resistance in arm *BD* may be ignored while calculating the equivalent resistance of bridge between *A* and *C*. So, *P* and *Q* are in series, *R* and *S* are in series and both in parallel to each other.

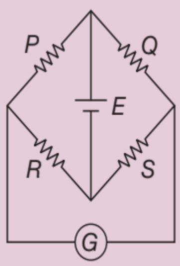
$$\Rightarrow \quad \frac{1}{R_{net}} = \frac{1}{P+Q} + \frac{1}{R+S}$$

Three other common forms of balanced Wheatstone's Bridge are shown here.





and



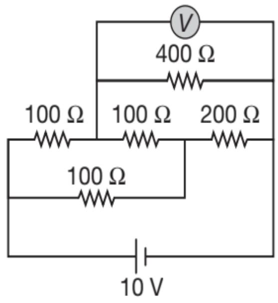
(c) If the bridge is not balanced, then the current will flow

(i) from *D* to *B* if $\frac{P}{Q} > \frac{R}{S}$

(ii) from *B* to *D* if $\frac{P}{Q} < \frac{R}{S}$

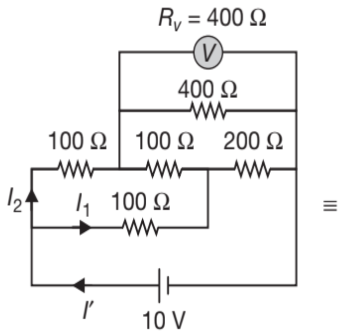
ILLUSTRATION 54

An electrical circuit is shown in figure. Calculate the potential difference across the resistor of $400\ \Omega$ as will be measured by the voltmeter V of resistance $400\ \Omega$ either by applying Kirchhoff's rules or otherwise.

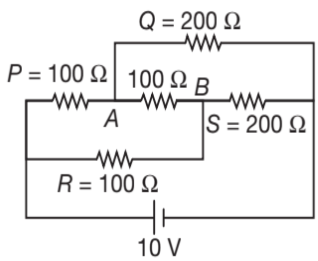


SOLUTION

The given circuit actually forms a balanced Wheatstone bridge (including the voltmeter) as shown in figure.

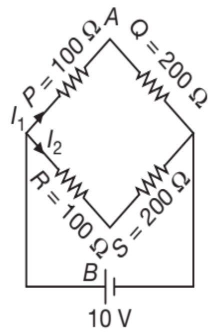


(a)



(b)

≡



(c)

Here, we see that $\frac{P}{Q} = \frac{R}{S}$ {Bridge is balanced}

Therefore, resistance between A and B can be ignored and equivalent simple circuit can be drawn as shown in figure (c).

The voltmeter will read the potential difference across resistance Q. So, currents I_1 and I_2 are given by

$$I_1 = I_2 = \frac{10}{100+200} = \frac{1}{30}\ \text{A}$$

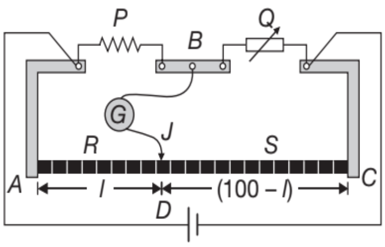
$\Rightarrow$ Potential difference across voltmeter is

$$\Delta V = I_1 Q = (200)\left(\frac{1}{30}\right)\ \text{V} = \frac{20}{3}\ \text{V}$$

So, reading of voltmeter is $\frac{20}{3}$ V

THE METRE BRIDGE

The metre bridge is the practical application of the Wheatstone network principle in which the ratio of two of the resistances, say R and S, is deduced from the ratio of their balancing lengths. AC is a 1 m long uniform wire. If $AD = l$ cm, then $DC = (100 - l)$ cm
Since Resistance $\propto$ Length



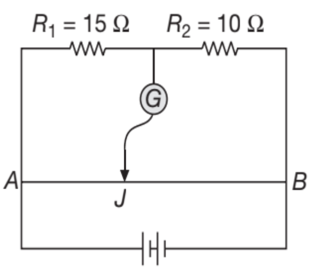
$$\Rightarrow \quad \frac{P}{Q} = \frac{l}{100-l}$$

If P is known then Q can be determined.

At the balancing point galvanometer G gives no deflection at all. At A and C, the galvanometer must have deflections in opposite direction because, then only zero deflection can be expected when the jockey (J) attached to the galvanometer is moved from A to C.

ILLUSTRATION 55

In the meter bridge circuit shown in figure. Calculate the length AJ for null deflection in galvanometer.



SOLUTION

Let $AJ = l$ cm, then $JB = (100 - l)$ cm

At zero deflection of Galvanometer

$$\frac{R_1}{R_2} = \frac{R_{AJ}}{R_{JB}} = \frac{l}{100-l}$$

$$\Rightarrow \quad \frac{15}{10} = \frac{l}{100-l}$$

$$\Rightarrow \quad \frac{3}{2} = \frac{l}{100-l}$$

$$\Rightarrow \quad 300 - 3l = 2l$$

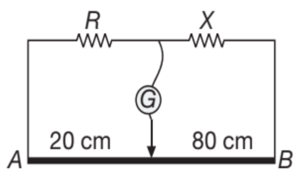
$$\Rightarrow \quad 5l = 300$$

$$\Rightarrow \quad l = 60\ \text{cm}$$

ILLUSTRATION 56

In a meter bridge, null point is found to be at 20 cm. When the known resistance R is shunted by 10 Ω resistance, null point is found to be shifted by 10 cm. Find the unknown resistance X.

SOLUTION



$$\frac{R}{20} = \frac{X}{80}$$

$$\Rightarrow \quad \frac{R}{X} = \frac{1}{4} \qquad \ldots(1)$$

Now shunting R by 10 Ω, i.e., 10 Ω connected in parallel to R, then new resistance becomes $\frac{10R}{10+R}$ (which is less than R) so new balance length is $20 - 10 = 10$ cm. Hence

$$\frac{\left(\frac{10R}{10+R}\right)}{10} = \frac{X}{90}$$

$$\Rightarrow \quad \frac{10}{10+R} = \frac{4}{9}$$

$$\Rightarrow \quad 90 = 40 + 4R$$

$$\Rightarrow \quad R = 12.5\ \Omega$$

$$\Rightarrow \quad X = 4R = 50\ \Omega$$

ILLUSTRATION 57

If resistance R_1 in resistance box is $300\ \Omega$, then the balanced length is found to be 75 cm from end A. The diameter of unknown wire is 1 mm and length of the unknown wire is 31.4 cm. Find the specific resistance of the unknown wire.

SOLUTION

Since, $\dfrac{R}{X}=\dfrac{l}{100-l}$

$\Rightarrow \quad X=\left(\dfrac{100-l}{l}\right)R$

$\Rightarrow \quad X=\left(\dfrac{100-75}{75}\right)(300)=100\ \Omega$

where $X=\dfrac{\rho l}{A}=\dfrac{\rho l}{\left(\dfrac{\pi d^2}{4}\right)}$

$\Rightarrow \quad \rho=\dfrac{\pi d^2 X}{4l}$

$\Rightarrow \quad \rho=\dfrac{\left(\dfrac{22}{7}\right)(10^{-3})^2(100)}{(4)(0.314)}$

$\Rightarrow \quad \rho=2.5\times10^{-4}\ \Omega\text{m}$

END CORRECTIONS IN METRE BRIDGE

In meter bridge, some extra length (under the metallic strips) comes at points A and C. Therefore, some additional length x_1 and x_2 should be included at the ends. So, x_1 and x_2 are called the end corrections. Hence, in place of l we use $l+x_1$ and in place of $(100-l)$ we use $100-l+x_2$. To find x_1 and x_2, we use known resistors R_1 and R_2 in place of R and X and suppose we get null point length equal to l_1 Then,

$$\frac{R_1}{R_2}=\frac{l_1+x_1}{100-l_1+x_2} \quad \ldots(1)$$

Now, we interchange the positions of R_1 and R_2 and suppose the new null point length is l_2. Then,

$$\frac{R_2}{R_1}=\frac{l_2+x_1}{100-l_2+x_2} \quad \ldots(2)$$

Solving equations (1) and (2), we get

$$x_1=\frac{R_2l_1-R_1l_2}{R_1-R_2}$$

and $\quad x_2=\dfrac{R_1l_1-R_2l_2}{R_1-R_2}-100$

ILLUSTRATION 58

When we use $100\ \Omega$ and $200\ \Omega$ in place of R and X we get null point deflection at $l=33\text{ cm}$. On interchanging the resistors, the null point length is found to be 67 cm. Calculate the end corrections x_1 and x_2.

SOLUTION

$$x_1=\frac{R_2l_1-R_1l_2}{R_1-R_2}$$

$\Rightarrow \quad x_1=\dfrac{(200)(33)-(100)(67)}{100-200}$

$\Rightarrow \quad x_1=1\text{ cm}$

$$x_2=\frac{R_1l_1-R_2l_2}{R_1-R_2}-100$$

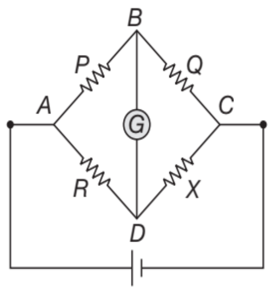
$\Rightarrow \quad x_2=\dfrac{(100)(33)-(200)(67)}{100-200}-100$

$\Rightarrow \quad x_2=1\text{ cm}$

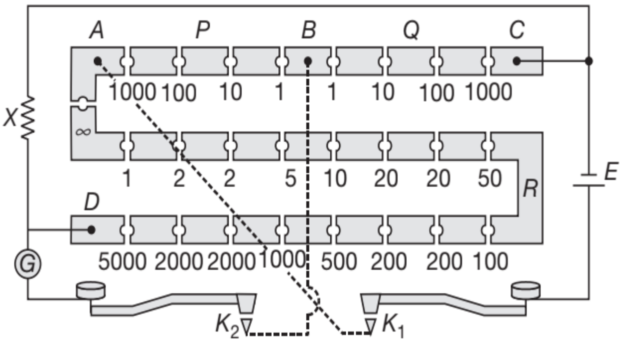
POST OFFICE BOX

Post office box also works on the principle of Wheatstone's bridge. In a Wheatstone's bridge circuit, when $\dfrac{P}{Q}=\dfrac{R}{X}$, then the bridge is said to be balanced. So, unknown resistance X is given by

$$X=\frac{Q}{P}R$$



It is observed that P and Q are set in arms AB and BC, where we can have the ratio $\dfrac{Q}{P}$ can be set by using $10\ \Omega$, $100\ \Omega$ or $1000\ \Omega$ resistances.



These arms are called ratio arm, initially we take $Q = 10\ \Omega$ and $P = 10\ \Omega$ to set $\frac{Q}{P} = 1$. The unknown resistance (X) is connected between C and D, the battery is connected across A and C.

Now, adjust resistance in part A to D such that the bridge gets balanced. For this, keep on increasing the resistance with $1\ \Omega$ interval, check the deflection in galvanometer by first pressing key K_1 then galvanometer key K_2.

Suppose at $R = 4\ \Omega$, we get deflection towards left and at $R = 5\ \Omega$, we get deflection towards right. Then, we can say that for balanced condition, R should lie between $4\ \Omega$ to $5\ \Omega$.

Now, $X = \frac{Q}{P}R = \frac{10}{10}R = R = 4\ \Omega$ to $5\ \Omega$.

To get closer value of X, in the second observation, let us choose $\frac{Q}{P} = \frac{1}{10}$ i.e., $\left(\frac{P = 100}{Q = 10}\right)$

Suppose, now at $R = 42$ we get deflection towards left and at $R = 43\ \Omega$ deflection is towards right.

So R lies between $42\ \Omega$ and $43\ \Omega$

Now, $X = \frac{Q}{P}R = \frac{10}{100}R = \frac{1}{10}R$

$\Rightarrow$ R lies between $4.2\ \Omega$ and $4.3\ \Omega$

Now, to get further closer value take $\frac{Q}{P} = \frac{1}{100}$ and so on.

The observation table is shown below

S. No.	Resistance in ratio arm		Resistance in arm *AD* (is *R* ohm)	Direction of deflection	Unknown resistance $X = \frac{Q}{P} \times R$ (ohm)
	AB (is *P* ohm)	*BC* (is *Q* ohm)			
1.	10	10	4	Left	4 to 5
			5	Right	
2.	100	10	40	Left (large)	4.2 to 4.3
			50	Right (large)	
			42	Left	
			43	Right	
3.	1000	10	420	Left	4.25
			424	Left	
			425	No deflection	
			426	Right	

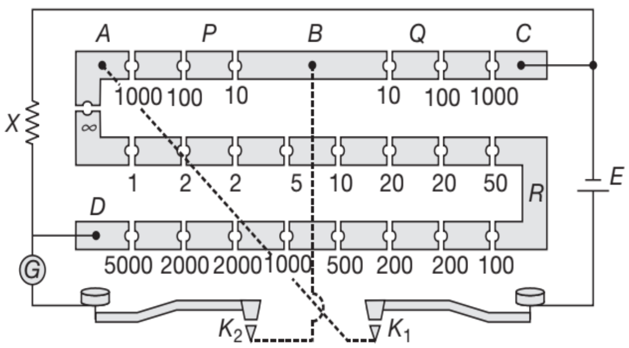
So, the correct value of X is $4.25\ \Omega$

Conceptual Note(s)

To locate the null point, deflection battery key (K_1) is pressed slightly earlier than the galvanometer key (K_2) because if galvanometer key K_2 is pressed first, then just after closing the battery key K_1, the current suddenly increases and due to self-induction, a large back emf is generated in the galvanometer, which may damage the galvanometer.

ILLUSTRATION 59

Calculate the maximum and minimum values of unknown resistance X, which can be determined using the post office box shown in the figure?



SOLUTION

$$X=\frac{QR}{P}$$

So, maximum value of x is given by

$$X_{\max}=\frac{Q_{\max}R_{\max}}{P_{\min}}$$

$$\Rightarrow \quad X_{\max}=\frac{1000}{10}(11110)$$

$$\Rightarrow \quad X_{\max}=1111\text{ k}\Omega$$

$$X_{\min}=\frac{Q_{\min}R_{\min}}{P_{\max}}$$

$$\Rightarrow \quad X_{\min}=\frac{(10)(1)}{1000}$$

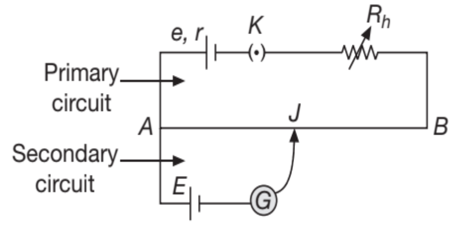
$$\Rightarrow \quad X_{\min}=0.01\ \Omega$$

POTENTIOMETER

Potentiometer is a device mainly used to measure emf of a given cell and to compare emf's of cells. It is also used to measure internal resistance of a given cell.

Circuit Diagram

Potentiometer consists of a long resistive wire AB of length L (about 6 m to 10 m long) made up of manganin or constantan and a battery of known voltage e and internal resistance r called supplier battery or driver cell. Connection of these two forms primary circuit.



J = Jockey
K = Key
$R_{AB}=R$ = Resistance of potentiometer wire
ρ = Specific resistance of potentiometer wire
R_h = Variable resistance which controls the current through the wire AB

One terminal of another cell (whose emf E is to be measured) is connected at one end of the main circuit and the other terminal at any point on the resistive wire through a galvanometer G. This forms the secondary circuit. Other details are as follows :

(a) The specific resistance (ρ) of potentiometer wire must be high but its temperature coefficient of resistance (α) must be low.

(b) All higher potential points (terminals) of primary and secondary circuits must be connected together at point A and all lower potential points must be connected to point B or jockey.

(c) The value of known potential difference must be greater than the value of unknown potential difference to be measured.

(d) The potential gradient must remain constant. For this the current in the primary circuit must remain constant and the jockey must not slide while in contact with the wire.

(e) The diameter of potentiometer wire must be uniform everywhere.

Potential Gradient (x)

Potential difference (or fall in potential) per unit length of wire is called potential gradient i.e.,

$$x=\frac{V}{L}\frac{\text{volt}}{\text{m}} \text{ where}$$

$$V = IR_{AB} = \left(\frac{e}{R + R_h + r}\right)R$$

So, potential gradient x is given by

$$x = \frac{V}{L} = \frac{IR}{L} = \frac{I\rho}{A} = \frac{e}{(R + R_h + r)}\frac{R}{L}$$

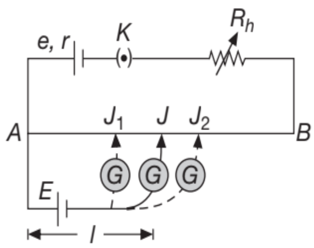
(a) The potential gradient directly depends upon the
- (i) resistance per unit length $\left(\frac{R}{L}\right)$ of potentiometer wire.
- (ii) radius of potentiometer wire (i.e., Area of cross-section)
- (iii) specific resistance of the material of potentiometer wire (i.e., ρ)
- (iv) current flowing through potentiometer wire (I)

(b) Potential gradient indirectly depends upon the
- (i) emf of battery in the primary circuit (i.e., e)
- (ii) resistance of rheostat in primary circuit (i.e., R_h)

Working

Suppose the jockey is made to touch a point J on wire. Then potential difference between A and J will be $V = xl$. At this length (l), two potential difference values are obtained.

(a) V due to battery e and
(b) E due to unknown cell



If $V > E$, then current will flow in galvanometer circuit in one direction, shown as 

If $V < E$, then current will flow in galvanometer circuit in opposite direction, shown as 

If $V = E$, then no current will flow in galvanometer circuit this condition to known as null deflection position, length l is known as balancing length, shown as 

In balanced condition $E = xl$

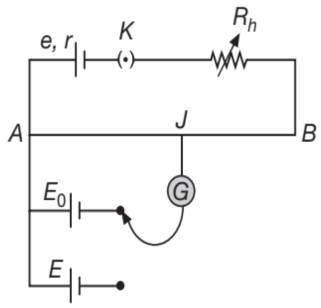
$$\Rightarrow \quad E = xl = \frac{V}{L}l = \frac{IR}{L}l = \left(\frac{e}{R + R_h + r}\right)\frac{R}{L} \times l$$

If V is constant then $L \propto l$

$$\Rightarrow \quad \frac{x_1}{x_2} = \frac{L_1}{L_2} = \frac{l_1}{l_2}$$

Standardization of Potentiometer

The process of determining potential gradient experimentally is known as **standardization of potentiometer**.



Let the balancing length for the standard emf E_0 is l_0, then by the principle of potentiometer we have $E_0 = xl_0$

$$\Rightarrow \quad x = \frac{E_0}{l_0}$$

Sensitivity of Potentiometer

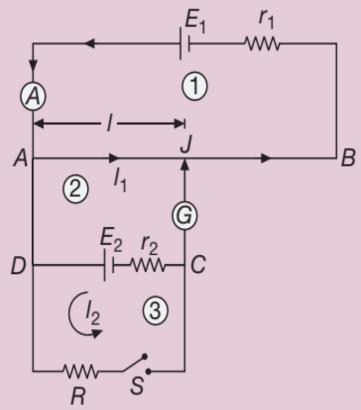
A potentiometer is said to be more sensitive, if it measures a small potential difference more accurately.

(a) The sensitivity of potentiometer is assessed by its potential gradient. The sensitivity is inversely proportional to the potential gradient.

(b) In order to increase the sensitivity of potentiometer, the
- (i) resistance in primary circuit will have to be decreased.
- (ii) length of potentiometer wire will have to be increased so that the length may be measured more accuracy.

Conceptual Note(s)

(a) In the potentiometer arrangement shown,



under balanced condition (when $I_G = 0$) Loop ① and Loop ③ are independent of each other. All problems in this condition can be solved by a single equation. i.e.,

$$V_{AJ} = V_{DC}$$

$$\Rightarrow \quad I_1 R_{AJ} = E_2 - I_2 r_2$$

$$\Rightarrow \quad I_1 \lambda \ell = E_2 - I_2 r_2 \qquad \text{...(1)}$$

where, λ is the resistance per unit length of potentiometer wire AB. The length ℓ is called the balance point length. Currents I_1 and I_2 are independent with each other. Current $I_2 = 0$, if switch is open.

(b) From equation (1), the null point length is given by

$$\ell = \frac{E_2 - I_2 r_2}{I_1 \lambda}$$

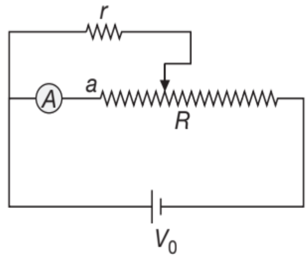
Now, suppose E_1 is increased then I_1 will also increase and null point length ℓ will decrease.

(c) Under balanced condition, a part of potential difference of E_1 is balanced by the lower circuit. So, normally $E_2 < E_1$ for taking balance point length. Similarly, we observe that $V_{AJ} = V_{DC}$. Also, $V_A = V_D$ and $V_J = V_C$, so we conclude that positive terminals of both batteries should be on same side and negative terminals on the other side.

(d) If we do not get any balanced condition $(I_G \neq 0)$, then the given circuit is simply a three loop problem, which can be solved by applying Kirchhoff's Laws.

ILLUSTRATION 60

A constant voltage V_0 is applied to a potentiometer of resistance R connected to an ammeter. A constant resistor r is connected to the sliding contact of the potentiometer and the fixed end of the potentiometer. How will the reading of ammeter vary as the sliding contact is moved from one end of the potentiometer to the other. The resistance of ammeter is assumed to be negligible.



SOLUTION

If x is the resistance of the potentiometer between point a and the sliding contact, the total resistance between a and the sliding contact is $\frac{rx}{r+x}$, while the resistance of the entire circuit is $(R-x)+\frac{rx}{r+x}$. The current supplied by the source is

$$I = \frac{V_0}{R - x + \frac{rx}{r+x}}$$

The potential difference between the sliding contact and point a is

$$V = \frac{V_0 rx}{(R-x)(r+x)+rx} = \frac{V_0 rx}{Rx - x^2 + Rr}$$

The current passing through ammeter is,

$$I_a = \frac{V_0 r}{Rx - x^2 + Rr} \qquad \text{...(1)}$$

To find the extremum (maximum, minimum or constant)

$$\frac{dI_a}{dx} = 0$$

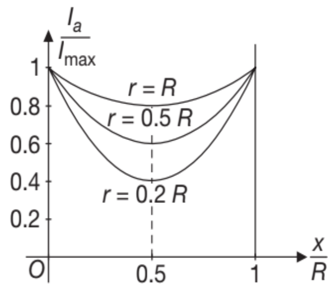
$$\Rightarrow \quad V_0 r\left[\frac{-R+2x}{(Rx - x^2 + Rr)^2}\right] = 0$$

$$\Rightarrow \quad x = \frac{R}{2}$$

Substituting x in equation (1) we find the minimum current

$$I_{min} = \frac{V_0 r}{R\left(r + \frac{R}{4}\right)}$$

Thus, as the sliding contact is moved the current through the ammeter passes through a minimum and the smaller the r the deeper the minimum. At $x = 0$ and $x = R$, a current of $I_{max} = \frac{V_0}{R}$ passes through the ammeter. The $\frac{I_a}{I_{max}}$ versus $\frac{x}{R}$ curves for several values of $\frac{r}{R}$ are shown in figure.



APPLICATIONS OF POTENTIOMETER

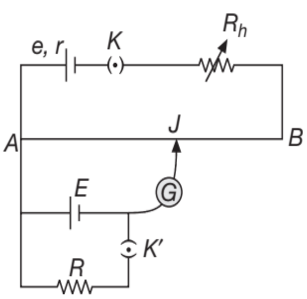
To Determine the Internal Resistance of a Primary Cell

(a) Initially, in the secondary circuit the key K' remains open and balancing length $l_1 = l_{\text{key open}}$ is obtained. Since cell E is in open circuit so it's emf balances for a length l_1 i.e.,

$$E = xl_1 \quad \ldots(1)$$

(b) Now key K' is closed so cell E comes in closed circuit. If the process of balancing repeated again then potential difference V balances on length $l_2 = l_{\text{key closed}}$ i.e.,

$$V = xl_2 \quad \ldots(2)$$



(c) Since we know that the internal resistance is given by

$$r = \left(\frac{E}{V} - 1\right)R$$

$$\Rightarrow \quad r = \left(\frac{l_1 - l_2}{l_2}\right)R = \left(\frac{l_{\text{key open}}}{l_{\text{key closed}}} - 1\right)R$$

ILLUSTRATION 61

In a potentiometer circuit to find the internal resistance r of the cell of emf E, when K is open, the balance point is obtained at 60 cm. When K is closed, the balance point is obtained at 50 cm. What is the value of r? If the cell is shorted by a $10\ \Omega$ resistor, then calculate the internal resistance of the cell.

SOLUTION

$l_1 = 60$ cm, when K is open and

$l_2 = 50$ cm, when K is closed

Since $r = \left(\frac{l_{\text{key open}}}{l_{\text{key closed}}} - 1\right)R$

$$\Rightarrow \quad r = \left(\frac{l_1}{l_2} - 1\right)R = \left(\frac{60}{50} - 1\right)10\ \Omega = \frac{1}{5} \times 10\ \Omega = 2\ \Omega$$

ILLUSTRATION 62

When a resistor of $5\ \Omega$ is connected across a cell its terminal potential difference is balanced by 150 cm of potentiometer wire and when a resistor of $10\ \Omega$ is connected across the cell, then the terminal potential difference is balanced by 175 cm of the potentiometer wire. Calculate the internal resistance of the cell.

SOLUTION

Since the same cell is used in both cases, so $l_{\text{key open}} = l$ will remain the same for both.

For the first case, we have $l_{\text{key closed}} = l_1$ and $R = R_1$. So

$$r = \left(\frac{l_{\text{key open}}}{l_{\text{key closed}}} - 1\right)R = \left(\frac{l}{l_1} - 1\right)R_1 \quad \ldots(1)$$

For the second case, we have $l_{\text{key closed}} = l_2$ and $R = R_2$. So

$$r = \left(\frac{l_{\text{key open}}}{l_{\text{key closed}}} - 1\right)R = \left(\frac{l}{l_2} - 1\right)R_2 \quad \ldots(2)$$

From (1), we get

$$r\left(\frac{l_1}{R_1}\right)=l-l_1 \quad \ldots(3)$$

From (2), we get

$$r\left(\frac{l_2}{R_2}\right)=l-l_2 \quad \ldots(4)$$

Subtracting (4) from (3), we get

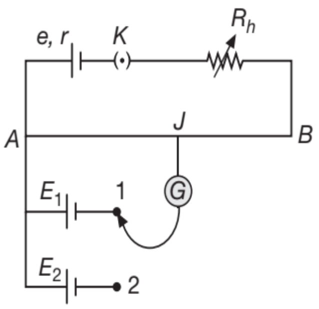
$$r\left(\frac{l_1}{R_1}-\frac{l_2}{R_2}\right)=l_2-l_1$$

$$\Rightarrow \quad r=\frac{l_2-l_1}{\left(\frac{l_1}{R_1}-\frac{l_2}{R_2}\right)}=\frac{175-150}{\frac{150}{5}-\frac{175}{10}}=\frac{25}{12.5}=2$$

$$\Rightarrow \quad r=2\ \Omega$$

Comparison of EMF's of Two Cell

Let l_1 and l_2 be the balancing lengths with the cells E_1 and E_2 respectively then $E_1=xl_1$ and $E_2=xl_2$



$$\Rightarrow \quad \frac{E_1}{E_2}=\frac{l_1}{l_2}$$

Let $E_1>E_2$ and both are connected in series. If balancing length is l_1, when cells assist each other and is l_2, when they oppose each other (as shown), then

$$(E_1+E_2)=xl_1$$

$$(E_1-E_2)=xl_2$$

$$\Rightarrow \quad \frac{E_1+E_2}{E_1-E_2}=\frac{l_1}{l_2}$$

$$\Rightarrow \quad \frac{E_1}{E_2}=\frac{l_1+l_2}{l_1-l_2}$$

ILLUSTRATION 63

In a potentiometer arrangement a cell of emf 1.5 V gives a balance point at 30 cm length of wire. Now, when the cell is replaced by another cell, the balance point shifts to 50 cm. Calculate the emf of second cell?

SOLUTION

$E_1=1.5\text{ V}$, $l_1=30\text{ cm}$

$E_2=?$, $l_2=50\text{ cm}$

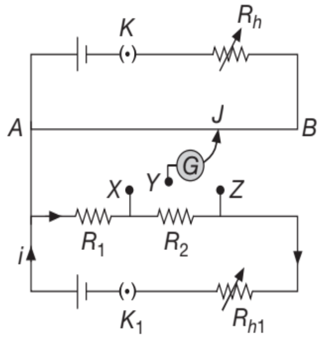
Using the formula for comparison of emf of cells by potentiometer, we have

$$\frac{E_2}{E_1}=\frac{l_2}{l_1}$$

$$\Rightarrow \quad E_2=\frac{l_2}{l_1}\times E_1=\frac{50}{30}\times 1.5\text{ V}=2.5\text{ V}$$

Comparison of Resistances

Let the balancing length for resistance R_1 (when XY is connected) is l_1 and let balancing length for resistance R_1+R_2 (when YZ is connected) is l_2.



Then $IR_1=xl_1$ and $I(R_1+R_2)=xl_2$

$$\Rightarrow \quad \frac{R_2}{R_1}=\frac{l_2-l_1}{l_1}$$

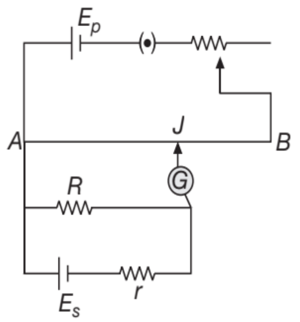
ILLUSTRATION 64

One of the circuits for the measurement of resistance using a potentiometer is shown. The galvanometer is connected and zero deflection is observed at length $AJ=30\text{ cm}$. In second case the secondary cell is changed.

Take $E_S=10\text{ V}$ and $r=1\ \Omega$ in 1^{st} reading

and $E_S=5\text{ V}$ and $r=2\ \Omega$ in 2^{nd} reading.

In second case, the zero deflection is observed at length $AJ = 10$ cm. Calculate the resistance R (in ohm).



SOLUTION

At zero deflection, we have

$$\frac{V}{l} = \frac{I_1 R}{30} = \frac{I_2 R}{10} \quad \ldots(1)$$

Now $I_1 = \dfrac{E_{S_1}}{r_1 + R}$

$\Rightarrow \quad I_1 = \dfrac{10}{1+R}$

$I_2 = \dfrac{E_{S_2}}{r_2 + R}$

$\Rightarrow \quad I_2 = \dfrac{5}{2+R}$

Substituting in (1), we get

$$\frac{10R}{(1+R)\times 30} = \frac{5R}{(2+R)\times 10}$$

$\Rightarrow \quad 4 + 2R = 3 + 3R$

$\Rightarrow \quad R = 1\,\Omega$

Calibration of Ammeter

Checking the correctness of ammeter readings with the help of potentiometer is called **calibration of ammeter.**

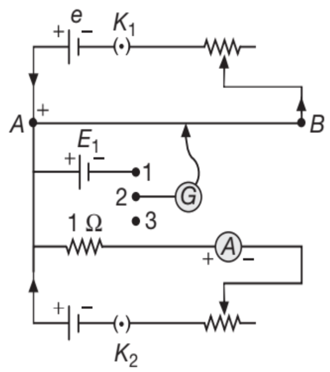
(a) In the process of calibration of an ammeter the current flowing in a circuit is measured by an ammeter and the same current is also measured with the help of potentiometer. By comparing both the values, the error(s) in the ammeter readings can be determined.

(b) For the calibration of an ammeter, $1\,\Omega$ standard resistance coil is specifically used in the secondary circuit of the potentiometer, because the potential difference across $1\,\Omega$ is equal to the current flowing through it. So, from Ohm's Law we get $V = I$

(c) If the balancing length for the emf E_0 is l_0 then

$$E_0 = x l_0$$

$$\Rightarrow \quad x = \frac{E_0}{l_0} \text{ (Process of standardisation)}$$



(d) Let I' be the current that flows through $1\,\Omega$ resistance, thus giving the potential difference as $V' = I'(1) = x l_1$ where l_1 is the balancing length. So error can be found as

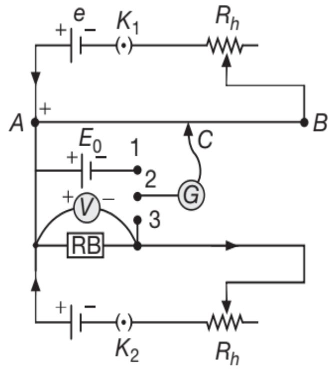
$$\Delta I = I - I' = I - x l_1 = I - \left(\frac{E_0}{l_0}\right) l_1$$

Calibration of Voltmeter

(a) Practical voltmeters are not ideal, because these do not have infinite resistance. The error of such practical voltmeter can be found by comparing the voltmeter reading with calculated value of potential difference by potentiometer.

(b) If l_0 is balancing length for E_0, the emf of standard cell (by connecting 1 and 2 of bi-directional key), then $x = \dfrac{E_0}{l_0}$.

(c) The balancing length l_1 for unknown potential difference V' is given by (by closing 2 and 3), then $V' = x l_1 = \left(\dfrac{E_0}{l_0}\right) l_1$



If the voltmeter reading is V then the error will be $(V - V')$ which may be positive, negative or zero.

DIFFERENCE BETWEEN VOLTMETER AND POTENTIOMETER

Voltmeter	Potentiometer
It's resistance is high but finite.	It's resistance is infinite.
It draws some current from source of emf.	It does not draw any current from the source of unknown emf.
The potential difference measured by it is lesser than the actual potential difference.	The potential difference measured by it is equal to actual potential difference.
Its sensitivity is low.	Its sensitivity is high.
It is a versatile instrument.	It measures only emf or potential difference.
It is based on deflection method.	It is based on zero deflection method.

ILLUSTRATION 65

A cell having a steady emf of 2 V is connected across the potentiometer wire of length 10 m. The potentiometer wire is of manganin and having resistance of $11.5\ \Omega m^{-1}$. An another cell gives a null point at 6.9 m. If a resistance of $5\ \Omega$ is put in series with the potentiometer wire, find the new position of the null point.

SOLUTION

Length of potentiometer wire $= 10$ m

So, fall in potential per meter (also known as potential gradient) $= \dfrac{2}{10} = 0.2\ \text{Vm}^{-1}$

Since the cell gives a null point at 6.9 m, hence its emf is,

$$E = (0.2)(6.9) = 1.38\ \text{V}$$

Resistance of potentiometer wire $= 11.5 \times 10 = 115\ \Omega$. When a resistance of $5\ \Omega$ is connected, total resistance becomes $R_{\text{net}} = 115 + 5 = 120\ \Omega$ and

Current, $I = \dfrac{2}{120}$ A

So, new potential gradient = (current) (resistance per unit length)

$$\Rightarrow \quad \text{Potential Gradient} = \left(\frac{2}{120}\right)(11.5) = \frac{23}{120}\ \text{Vm}^{-1}$$

Let l' be the position of new null point, then

$$\left(\frac{23}{120}\right)l' = E = 1.38$$

$$\Rightarrow \quad l' = \frac{1.38 \times 120}{23} = 7.2\ \text{m}$$

ILLUSTRATION 66

In a potentiometer circuit, the emf of driver cell is 2 V and internal resistance is $0.5\ \Omega$. The potentiometer wire is 1 m long. It is found that a cell of emf 1 V and internal resistance $0.5\ \Omega$ is balanced against 60 cm length of the wire. Calculate the resistance of potentiometer wire.

SOLUTION

Let resistance of potentiometer wire be $R_{AB} = R$. At balance, we have

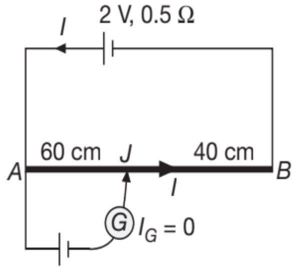
$$I = \frac{2}{R + 0.5}$$

$$\Rightarrow \quad V_{AB} = IR_{AB} = \frac{2R}{R + 0.5} \qquad \ldots(1)$$

Since, $V_{AJ} = 1$ V $\qquad \{\because$ There is no deflection$\}$

Across 60 cm length of potentiometer wire, potential difference is 1 V

So, across 1 cm of potentiometer wire, potential difference is $\frac{1}{60}$ V



Hence, across 100 cm of potentiometer wire, potential difference is

$$V_{AB} = \left(\frac{1}{60}\right)100 = \frac{5}{3}\text{ V} \quad \ldots(2)$$

Equating (1) and (2), we get

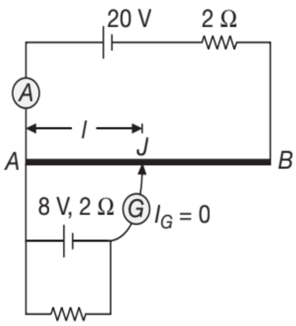
$$\frac{5}{3} = \frac{2R}{R+0.5}$$

$$\Rightarrow \quad 5R + 2.5 = 6R$$

$$\Rightarrow \quad R = 2.5\ \Omega$$

ILLUSTRATION 67

In the figure shown, wire AB has a length of 100 cm and resistance 8 Ω. Find the balance point length l.

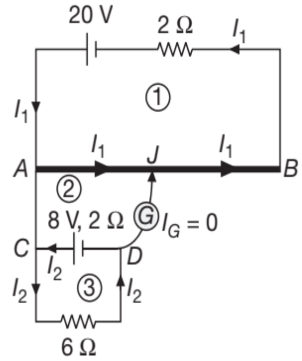


SOLUTION

For condition of balance, we have $I_G = 0$. So loops ① and ③ are independent of each other. Hence we observe that

$$V_{AJ} = V_{CD}$$

$$\Rightarrow \quad I_1 R_{AJ} = E - I_2 r \quad \ldots(1)$$



where $E = 8$ V, $r = 2\ \Omega$

$$I_1 = \left(\frac{20}{2+8}\right) = 2\text{ A}$$

$$R_{AJ} = \left(\frac{R_{AB}}{l_{AB}}\right)l_{AJ} = \left(\frac{8}{100}\right)l = \frac{2l}{25}$$

$$I_2 = \frac{8}{2+6} = 1\text{ A}$$

Substituting these values in equation (1), we get

$$2\left(\frac{2l}{25}\right) = 8 - (1)(2)$$

$$\Rightarrow \quad \frac{4l}{25} = 6$$

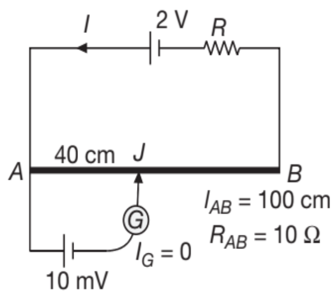
$$\Rightarrow \quad 4l = 150$$

$$\Rightarrow \quad l = \frac{150}{4} = 37.5\text{ cm}$$

ILLUSTRATION 68

A potentiometer wire of length 100 cm has a resistance of 10 Ω. It is connected in series with a resistance R and a cell of emf 2 V and of negligible internal resistance. A source of emf 10 mV is balanced against a length of 40 cm of the potentiometer wire. What is the value of R?

SOLUTION



For the condition of balance i.e., when $I_G = 0$, we have

$$I = \frac{2}{R + R_{AB}} = \frac{2}{R + 10}$$

$$\Rightarrow \quad V_{AB} = IR_{AB} = \left(\frac{20}{R+10}\right)V$$

Since $V_{AJ} = 10 \text{ mV} = 10 \times 10^{-3} \text{ V} = 10^{-2} \text{ V}$ ...(1)

For a length of 100 cm, potential difference across AB is

$$V_{AB} = \frac{20}{R+10}$$

For a length of 1 cm, potential difference is

$$= \left(\frac{20}{R+10}\right)\frac{1}{100}$$

For a length of 40 cm, potential difference is

$$V_{AJ} = \frac{20}{100(R+10)} \times 40$$

$$\Rightarrow \quad \left[\frac{20}{10(R+10)}\right]40 = 10^{-2} \qquad \{\because \text{ of } (1)\}$$

$$\Rightarrow \quad 800 = R + 10$$

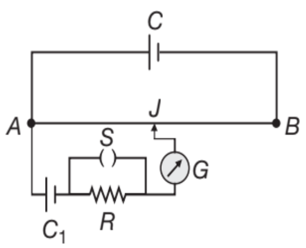
$$\Rightarrow \quad R = 790\ \Omega$$

Test Your Concepts-X

Based on Wheatstone Bridge and Potentiometer

(Solutions on page H.226)

1. Figure shows a potentiometer using a cell C of emf 2 V and internal resistance 0.4 Ω connected to a resistor wire AB. A standard cell of constant emf of 1.02 V gives a balance point at 67.3 cm length of the wire. A very high resistance R = 600 kΩ is put in series with the standard cell. This resistance is shorted by inserting switch S when close to the balance point. The standard cell is then replaced by a cell of unknown emf E and the null point turns out to be 82.3 cm length of the wire.



(a) What is the value of E?

(b) What is the purpose of using the high resistance R?

(c) Is the null point affected by this high resistance?

(d) Is the null point affected by the internal resistance of the cell C?

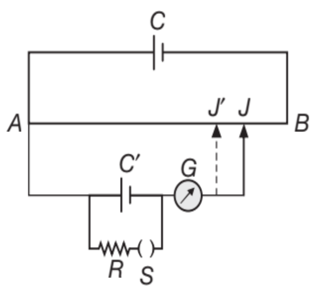
(e) Would this method work if:

(i) the internal resistance of cell C were higher than the resistance of wire AB and

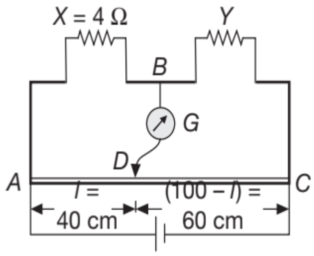
(ii) the emf of cell C were 1 V instead of 2 V?

(f) Would the circuit work well for determining extremely small emfs of the order of a few millivolts? What modification do you suggest in the circuit.

2. Figure shows a potentiometer circuit for determining the internal resistance of a cell. When switch S is open, the balance point is found to be at 76.3 cm of the wire. When switch S is closed and the value of R is 4 Ω, the balance point shifts to 60 cm. Find the internal resistance of cell C'.

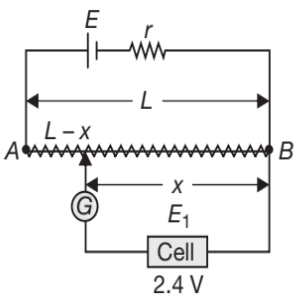


3. Figure shows a metre bridge consisting of two resistances X and Y together in parallel with a metre-long constantan wire AC of uniform cross-section. D is a movable contact that can slide along the wire AC. The resistors X, Y and resistances of segments AD and DC of the wire constitute the four arms of the bridge. The length of wire AC is 100 cm. X is a standard 4 Ω resistor and Y is a coil of wire. With Y immersed in melting ice the null point is found to be at a distance of 40 m from point A. When the coil Y is heated to 100°C, a 100 Ω resistor has to be connected in parallel with Y in order to keep the bridge balanced at the same point. Calculate the temperature coefficient of resistance of the coil.

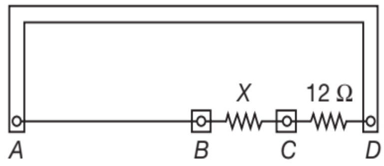


4. In a potentiometer experiment it is found that no current passes through the galvanometer when the terminals of the cell are connected across 0.52 m of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω a balance is obtained when the cell is connected across 0.4 m of the wire. Find the internal resistance of the cell.

5. A battery of emf 4 V is connected across a 10 m long potentiometer wire having a resistance per unit length 1.6 Ωm^{-1}. A cell of emf 2.4 V is connected so that its negative terminal is connected to the low potential end of the potentiometer wire and the other end is connected through a galvanometer to a sliding contact along the wire. It is found that the no-deflection point occurs against the balancing length of 8 m. Calculate the internal resistance of the 4 V battery.



6. A thin uniform wire AB of length 1 m, an unknown resistance X and a resistance of 12 Ω are connected by thick conducting strips, as shown in the figure. A battery and a galvanometer (with a sliding jockey connected to it) are also available. Connections are to be made to measure the unknown resistance X using the principle of Wheatstone bridge. Answer the following questions.

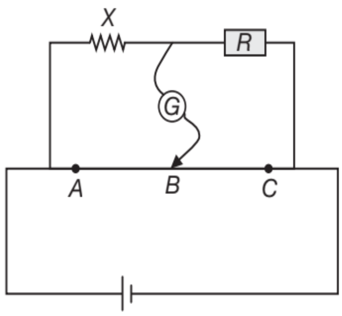


(a) Are there positive and negative terminals on the galvanometer?

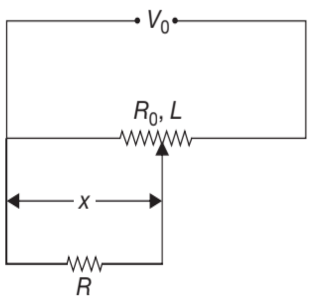
(b) Copy the figure in your answer book and show the battery and the galvanometer (with jockey) connected at appropriate points.

(c) After appropriate connections are made, it is found that no deflection takes place in the galvanometer when the sliding jockey touches the wire at a distance of 60 cm from A. Obtain the value of the resistance X.

7. R_1, R_2, R_3 are different values of R. A, B, C are the null points obtained corresponding to R_1, R_2 and R_3 respectively. For which resistor, the value of X will be the most accurate and why?

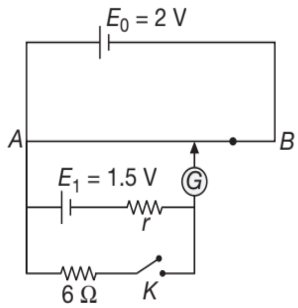


8. Figure illustrates potentiometer circuit by means of which we can vary a voltage V applied to a certain device possessing a resistance R. The potentiometer wire has a length L and a resistance R_0. A voltage V_0 is applied to the terminals of wire. Find the voltage V fed to the device as a function of distance x. Analyse the case $R \gg R_0$.



9. For the potentiometer arrangement to measure the internal resistance of a cell, when the switch is open in lowermost loop the balance point length is 60 cm. When the switch is closed with a known resistance of $R = 4\ \Omega$, the balance point length decreases to 40 cm. Calculate the internal resistance of the unknown battery.

10. For the potentiometer arrangement shown in the figure, the balance point is obtained at a distance 75 cm from A when the key K is open.



The second balance point is obtained at 60 cm from A when the key K is closed. Find the internal resistance (in Ω) of the battery E_1.

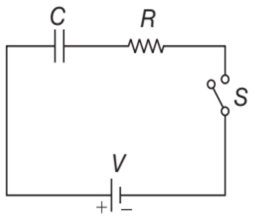
RC CIRCUIT AND APPLICATIONS

RC CIRCUIT

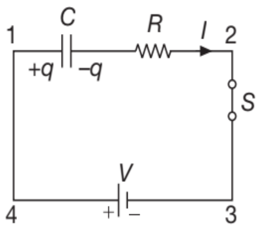
Charging a Capacitor

Consider the circuit shown below. The capacitor is connected to a DC voltage source of voltage V. At time $t = 0$, the switch S is closed. The capacitor initially is uncharged,

$$q(\text{initial}) = q(\text{at } t = 0) = 0$$



(a)
RC circuit diagram for $t < 0$



(b)
Circuit diagram for $t < 0$

In particular, for $t < 0$, there is no voltage across the capacitor so the capacitor acts like a short circuit. At $t = 0$, the switch is closed and current begins to flow and has a value

$$I_0 = \frac{V}{R}$$

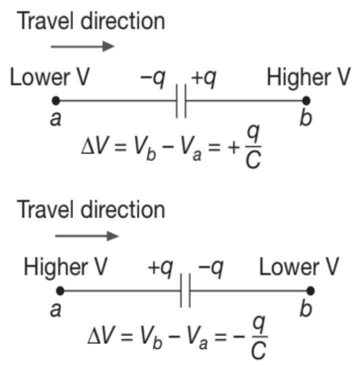
At this instant, the potential difference from the battery terminals is the same as that across the resistor. This initiates the charging of the capacitor. As the capacitor starts to charge, the voltage across the capacitor increases in time. At any instant, the voltage across the capacitor is

$$V_C(t) = \frac{q(t)}{C}$$

Applying Kirchhoff's loop rule to 12341 shown in figure (b), we obtain

$$V - IR - \frac{q}{C} = 0$$

$$\Rightarrow \quad V - R\frac{dq}{dt} - \frac{q}{C} = 0 \qquad \left\{\because I = \frac{dq}{dt}\right\}$$



Kirchhoff's rule for capacitors

Since I must be the same in all parts of the series circuit, the current across the resistance R is equal to the rate of increase of charge on the capacitor plates. The current flow in the circuit will continue to decrease because the charge already present on the capacitor makes it harder to put more charge on the capacitor. Once the charge on the capacitor plates reaches its maximum value q_0 the current in the circuit will drop to zero.

Thus, the charging capacitor satisfies a first order differential equation that relates the rate of change of charge to the charge on the capacitor

$$\frac{dq}{dt} = \frac{1}{R}\left(V - \frac{q}{C}\right)$$

This equation can be solved by the method of separation of variables. The first step is to separate terms involving charge and time, (this means putting terms involving dq and q on one side of the equality sign and terms involving dt on the other side),

$$\frac{dq}{\left(V - \frac{q}{C}\right)} = \frac{1}{R}dt$$

$$\Rightarrow \quad \frac{dq}{CV - q} = \frac{1}{RC}dt$$

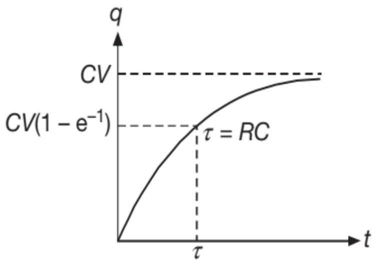
Integrating both sides of the above equation, we get

$$\int_0^q \frac{dq}{CV - q} = \frac{1}{RC}\int_0^t dt$$

$$\Rightarrow \quad \log_e\left(\frac{CV-q}{CV}\right) = -\frac{t}{RC}$$

$$\Rightarrow \quad q(t) = CV\left(1-e^{-t/RC}\right) = q_0\left(1-e^{-t/RC}\right)$$

where $q_0 = CV$ is the maximum amount of charge stored on the plates. The time dependence of $q(t)$ is plotted in figure below:



Once we know the charge on the capacitor we also can determine the voltage across the capacitor,

$$V_C(t) = \frac{q(t)}{C} = V\left(1-e^{-t/RC}\right)$$

The graph of voltage as a function of time has the same form as shown in figure. From the figure, we see that after a sufficiently long time the charge on the capacitor approaches the value

$$q(t\to\infty) = CV = q_0$$

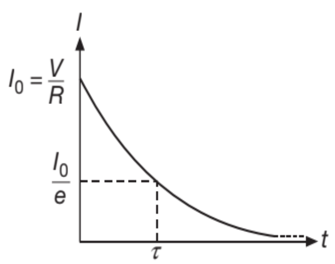
At that time, the voltage across the capacitor is equal to the applied voltage source and the charging process effectively ends,

$$V_C = \frac{q(t\to\infty)}{C} = \frac{q_0}{C} = V \text{ (Battery Voltage)}$$

The current that flows in the circuit is given by

$$I(t) = \frac{dq}{dt} = \left(\frac{V}{R}\right)e^{-t/RC} = I_0 e^{-t/RC}$$

The coefficient in front of the exponential (I_0) is equal to the initial current that flows in the circuit when the switch was closed at $t=0$. The graph of current as a function of time is shown in figure below:



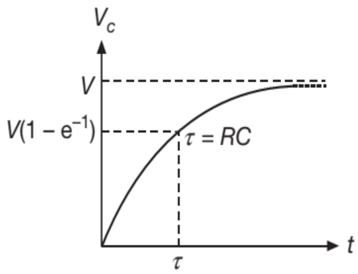
The current in the charging circuit decreases exponentially in time, $I(t) = I_0 e^{-t/RC}$. This function is often written as $I(t) = I_0 e^{-t/\tau}$ where $\tau = RC$ is called the time constant. The SI unit of τ is second, as can be seen from the dimensional analysis

$$[\tau] = [RC] = [\Omega][F] = \left(\frac{[V]}{[A]}\right)\left(\frac{[C]}{[V]}\right)$$

$$[\tau] = \frac{[C]}{[A]} = \frac{[C]}{[C]/[s]} = [s]$$

The time constant τ is a measure of the decay time for the exponential function. This decay rate satisfies the following property

$$I(t+\tau) = I(t)e^{-1}$$



which shows that after one time constant τ has elapsed, the current falls off by a factor of $e^{-1} = 0.368$, as indicated in figure above. Similarly, the voltage across the capacitor (shown in figure) can also be expressed in terms of the time constant τ

$$V_C(t) = V\left(1-e^{-t/\tau}\right)$$

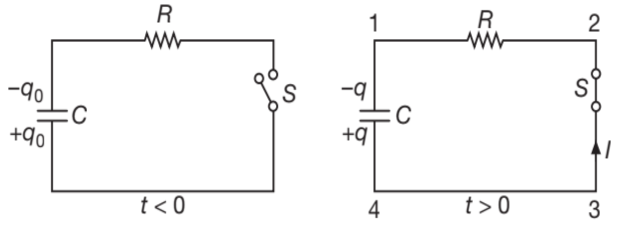
Notice that initially at time $t=0$, $V_C(t=0)=0$.

After one time constant τ has elapsed, the potential difference across the capacitor plates has increased by a factor $\left(1-e^{-1}\right) = 0.632$ of its final value

$$V_C(\tau) = V\left(1-e^{-1}\right) = 0.632V$$

Discharging a Capacitor

Suppose initially the capacitor has been charged to some value q_0. For $t<0$, the switch is open and the potential difference across the capacitor is given by $V_C = \frac{q_0}{C}$. On the other hand, the potential difference across the resistor is zero because there is no current flow, that is, $I=0$. Now suppose at $t=0$ the switch is closed (shown in figure). The capacitor will begin to discharge.



Discharging the RC circuit

The charged capacitor is now acting like a voltage source to drive current around the circuit. When the capacitor discharges (electrons flow from the negative plate through the wire to the positive plate), the voltage across the capacitor decreases. The capacitor is losing strength as a voltage source. Applying the Kirchhoff's loop rule to 12341, the equation that describes the discharging process is given by

$$IR - \frac{q}{C} = 0$$

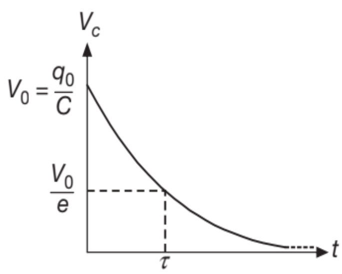
The current that flows away from the positive plate is proportional to the charge on the plate,

$$I = -\frac{dq}{dt}$$

The negative sign in the equation is an indication that the rate of change of the charge is proportional to the negative of the charge on the capacitor. This is due to the fact that the charge on the positive plate is decreasing as more positive charges leave the positive plate. Thus, charge satisfies a first order differential equation

$$\frac{q}{C} + R\frac{dq}{dt} = 0$$

This equation when integrated by using the method of separation of variables



$$\frac{dq}{q} = -\frac{1}{RC}dt$$

$$\int_{q_0}^{q}\frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

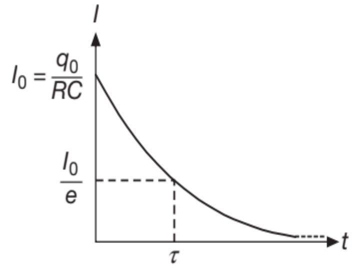
$$\Rightarrow \quad \log_e\left(\frac{q}{q_0}\right) = -\frac{t}{RC}$$

$$\Rightarrow \quad q(t) = q_0 e^{-t/RC}$$

The voltage across the capacitor is then

$$V_C(t) = \frac{q(t)}{C} = \left(\frac{q_0}{C}\right)e^{-t/RC}$$

A graph of voltage across the capacitor vs. time for the discharging capacitor is shown in figure.



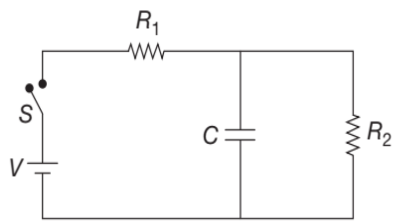
The current also exponentially decays in the circuit as can be seen by differentiating the charge on the capacitor

$$I = -\frac{dq}{dt} = \left(\frac{q_0}{RC}\right)e^{-t/RC}$$

A graph of the current flowing in the circuit as a function of time also has the same form as the voltage graph depicted in figure.

ILLUSTRATION 69

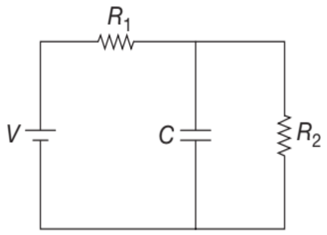
At $t = 0$, switch S is closed. The charge on the capacitor is varying with time as $Q = Q_0\left(1 - e^{-\alpha t}\right)$. Obtain the value of Q_0 and α in the given circuit parameters.



SOLUTION

Q_0 is the steady state charge stored in the capacitor.

$Q_0 = C$ (Potential difference across capacitor in steady state)



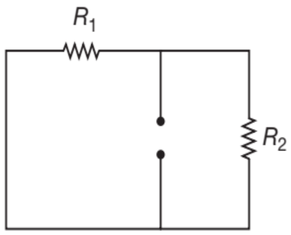
$\Rightarrow \quad Q_0 = C\,(\text{Steady state current through } R_2)\,(R_2)$

$$\Rightarrow \quad Q_0 = C\left(\frac{V}{R_1 + R_2}\right)R_2$$

$$\Rightarrow \quad Q_0 = \frac{CVR_2}{R_1 + R_2}$$

Since, $\alpha = \dfrac{1}{\tau_C} = \dfrac{1}{CR_{net}}$

where, R_{net} is the equivalent resistance across capacitor after short circuiting the battery. So,

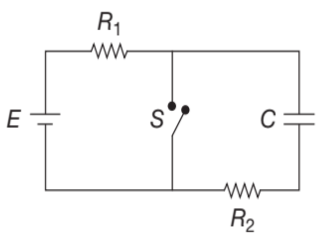


$$R_{net} = \frac{R_1 R_2}{R_1 + R_2} \qquad \{\because R_1 \text{ and } R_2 \text{ are in parallel}\}$$

$$\Rightarrow \quad \alpha = \frac{1}{C\left(\dfrac{R_1 R_2}{R_1 + R_2}\right)} = \frac{R_1 + R_2}{CR_1R_2}$$

ILLUSTRATION 70

In the circuit in figure, suppose the switch has been open for a very long time. At time $t = 0$, it is suddenly closed.



(a) What is the time constant before the switch is closed?

(b) What is the time constant after the switch is closed?

(c) Find the current through the switch as a function of time after the switch is closed.

SOLUTION

(a) Before the switch is closed, the two resistors R_1 and R_2 are in series with the capacitor.
Since the equivalent resistance is $R_{eq} = R_1 + R_2$, the time constant is given by

$$\tau = R_{eq}C = (R_1 + R_2)C$$

The amount of charge stored in the capacitor is

$$q(t) = CE\left(1 - e^{-t/\tau}\right)$$

(b) After the switch is closed, the closed loop on the right becomes a decaying RC circuit with time constant $\tau' = R_2C$. Charge begins to decay according to

$$q'(t) = CEe^{-t/\tau'} = CEe^{-t/R_2C}$$

(c) The current passing through the switch consists of two sources :

(i) the steady current I_1 from the left circuit given by $I_1 = \dfrac{E}{R_1}$ and

(ii) the decaying current I_2 from the RC circuit, given by

$$I'(t) = \frac{dq'}{dt} = -\left(\frac{CE}{\tau'}\right)e^{-t/\tau'} = -\left(\frac{E}{R_2}\right)e^{-t/R_2C}$$

The negative sign in $I'(t)$ indicates that the direction of flow is opposite of the charging process. Since both I_1 and I' move downward across the switch, the total current is given by

$$I(t) = I_1 + I'(t) = \frac{E}{R_1} + \left(\frac{E}{R_2}\right)e^{-t/R_2C}$$

Problem Solving Technique(s)

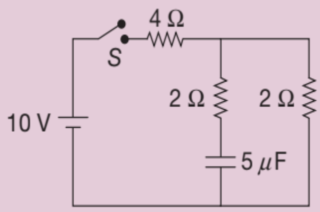
(a) An uncharged capacitor offers zero resistance to the current in a circuit, i.e., the branch containing the uncharged capacitor can be assumed to be short circuited in terms of the capacitance. That is the capacitor can just be thought of being absent initially.

(b) A fully charged capacitor offers an infinite resistance to the current and hence no current will pass through the branch that contains a fully charged capacitor.

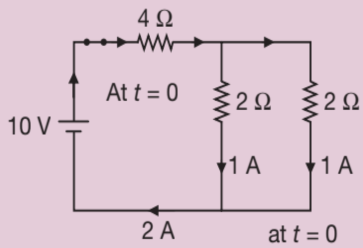
(c) While solving problems, whenever we come across the branch that contains a fully charged capacitor then we can omit that branch to calculate the net resistance of the circuit.

(d) Please be alert that no current will pass through the branch of the circuit that contains a fully charged capacitor However, this does not imply that potential across the capacitor is zero. The potential across the capacitor will be $\Delta V = \pm\frac{Q}{C}$.

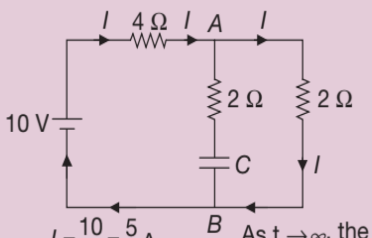
For instance, consider the circuit shown.



The circuit at time $t = 0$, when switch is closed, becomes as shown in figure.



When the capacitor becomes fully charged i.e., when $t \to \infty$, the circuit in the figure becomes as if no current passes through the branch containing the capacitor.



$I = \frac{10}{6} = \frac{5}{3}$ A

As $t \to \infty$, the capacitor C blocks the current flow in the branch AB

Equivalent Time Constant

To find the equivalent time constant of a circuit, follow the steps mentioned here :

STEP-1: Short-circuit the battery i.e., just remove the battery and join the ends across which it was connected.

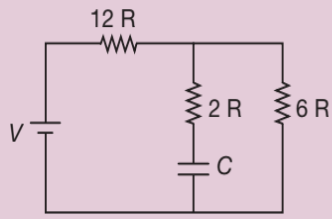
STEP-2: Find net resistance across the capacitor (say it R_{net}), Just think the capacitor is not there and the two plates of the capacitor had been converted to the ends across which we wish to find the net equivalent resistance R_{net} of the circuit.

OR

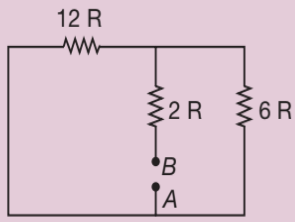
For doing this just remove the capacitor and make the points across which the capacitor was connected as the current inlet and outlet points.

STEP-3: Then, the capacitive time constant is $\tau = (R_{net})C$

For example, in the circuit shown in figure, after short circuiting the battery $6R$ and $12R$



are in parallel, so their combined resistance is $\frac{(12R)(6R)}{12R+6R} = 4R$. Now this $4R$ is in series with the remaining $2R$.



STEP-1: Battery Shorted

STEP-2: Capacitor removed to find equivalent resistance across A and B.

Hence, $R_{net} = 4R + 2R = 6R$

$\Rightarrow \tau = (R_{net})C = 6RC$

Alternate Method of Finding Current in the Circuit and Charge on the Capacitor at Any Time *t*:

In a complicated *RC* circuit it is easy to find current in the circuit and charge stored in the capacitor at time $t = 0$ and $t \to \infty$ by conditions which we have already discussed. But to find the current and charge as function of time *t* following steps may be followed.

(a) Find equivalent time constant (τ) of the circuit.

(b) Find steady state charge q_0 (at time $t \to \infty$) on the capacitor.

(c) Charge on the capacitor at any time *t* is,

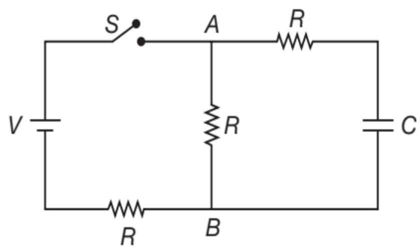
$$q = q_0\left(1 - e^{-t/\tau}\right)$$

By differentiating it w.r.t. time we can find current through the capacitor at time *t*. Then by using Kirchhoff's Laws we can calculate currents in other parts of the circuit also.

Otherwise we can also find the current in the circuit as shown and calculated in the following ILLUSTRATION because, it may not always be easy and convenient for us to find the current through the capacitor as a function of time.

ILLUSTRATION 71

In the circuit shown in figure, the battery is an ideal one, with emf V. The capacitor is initially uncharged. The switch S is closed at time $t = 0$.



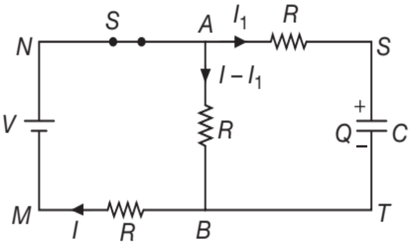
(a) Find the charge Q on the capacitor at time t.

(b) Find the current in AB at time t. What is its limiting value as $t \to \infty$?

SOLUTION

Let at any time t charge on capacitor C be Q and currents are as shown. Since, charge Q will increase with time t. Therefore,

$$I_1 = \frac{dQ}{dt}$$



(a) Applying Kirchhoff's Second Law in Loop $MNABM$

$$V = (I - I_1)R + IR$$

$$\Rightarrow \quad V = 2IR - I_1R \qquad \ldots(1)$$

Similarly, applying Kirchhoff's Second Law in Loop $MNSTM$, we have

$$V = I_1R + \frac{Q}{C} + IR \qquad \ldots(2)$$

Eliminating I from equations (1) and (2), we get

$$V = 3I_1R + \frac{2Q}{C}$$

$$\Rightarrow \quad 3I_1R = V - \frac{2Q}{C}$$

$$\Rightarrow \quad I_1 = \frac{1}{3R}\left(V - \frac{2Q}{C}\right)$$

$$\Rightarrow \quad \frac{dQ}{dt} = \frac{1}{3R}\left(V - \frac{2Q}{C}\right)$$

$$\Rightarrow \quad \frac{dQ}{V - \frac{2Q}{C}} = \frac{dt}{3R}$$

$$\Rightarrow \quad \int_0^Q \frac{dQ}{V - \frac{2Q}{C}} = \int_0^t \frac{dt}{3R}$$

This equation gives

$$Q = \frac{CV}{2}\left(1 - e^{-2t/3RC}\right)$$

(b) $I_1 = \dfrac{dQ}{dt} = \dfrac{V}{3R}e^{-2t/3RC}$

From equation (1), we get

$$I = \frac{V + I_1R}{2R} = \frac{V + \frac{V}{3}e^{-2t/3RC}}{2R}$$

So, current through AB is given by

$$I_2 = I - I_1 = \frac{V + \frac{V}{3}e^{-2t/3RC}}{2R} - \frac{V}{3R}e^{-2t/3RC}$$

$$I_2 = \frac{V}{2R} - \frac{V}{6R}e^{-2t/3RC}$$

$$I_2 = \frac{V}{2R} \text{ as } t \to \infty$$

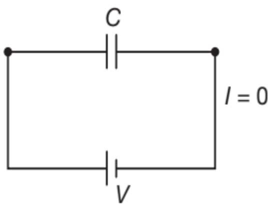
Problem Solving Technique(s)

In the above ILLUSTRATION, please note that it is advisable to take the current in the branch containing the capacitor as I_1 because then, directly we get $I_1 = \frac{dQ}{dt}$ otherwise we had to take

$$(I - I_1) = \frac{dQ}{dt}$$

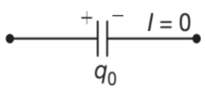
LEAKAGE CURRENT THROUGH A CAPACITOR

In ideal situation, when the space between the capacitor is filled with a dielectric (insulator) then no current flows through it when it is connected to a battery as shown in figure below



(a) Ideal Situation

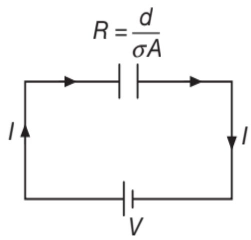
Another ideal case where no current flows through the capacitor is when a capacitor is charged and the charge on the capacitor is left over for a longer duration of time, as shown.



(b) Ideal Situation

However, in both the cases discussed, we get some small amount of current flowing through the capacitor. This non zero current (of the order of microampere) is called the leakage current. This is due to the fact that every insulator has some conductivity (or very lightly conducting), on account of which some current flows through the capacitor which is connected to battery as in Figure (a). Similarly when the capacitor is charged and left over, then the charge does not sustain (at its value) over a longer duration of time and starts discharging. This situation can be thought of as being equivalent to the case of discharging of a capacitor in an RC series circuit. In both the cases discussed above, we calculate the resistance of the dielectric using the Ohm's Law, according to which, we get

$$R = \frac{l}{\sigma A} \qquad \{\sigma = \text{specific conductance}\}$$



(c) Non-Ideal Practical Situation

Here $l = d$ (the separation between the plates of the capacitor)

$$\Rightarrow \quad R = \frac{d}{\sigma A}$$

Thus, the leakage current in the circuit shown in figure is

$$I = \frac{V}{R}$$

(d) Non-Ideal Practical Situation

Similarly, if the capacitor is given a charge q_0 at time $t = 0$, and left over, then after time t the charge that will remain on it is q, given by

$$q = q_0 e^{-t/\tau} \qquad \{\text{discharging of a capacitor}\}$$

Here, $\tau = RC = \left(\frac{K\varepsilon_0 A}{d}\right)\left(\frac{d}{\sigma A}\right)$

$$\Rightarrow \quad \tau = \frac{K\varepsilon_0}{\sigma}$$

Remark(s)

(a) Dielectric leakage occurs in a capacitor as the result of LEAKAGE CURRENT through the dielectric. Normally it is assumed that the dielectric will effectively prevent the flow of current through the capacitor. Although the resistance of the dielectric is extremely high, a minute amount of current does flow. Ordinarily this current is so small that for all practical purposes it is ignored. However, if the leakage through the dielectric is abnormally high, there will be a rapid loss of charge and an overheating of the capacitor.

(b) The power loss of a capacitor is determined by loss in the dielectric. If the loss is negligible and the capacitor returns the total charge to the circuit, it is considered to be a perfect capacitor with a power loss of zero.

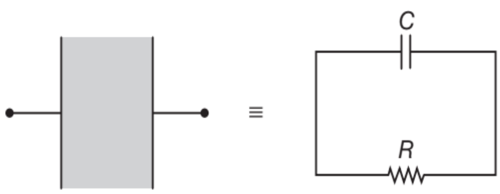
(c) The dielectric used inside the capacitor is not a perfect insulator resulting in a very small current flowing or "leaking" through the dielectric when applied to a constant supply voltage. This small current flow (in the region of micro amperes) is called the **Leakage Current**. This **leakage current** is a result of electrons physically making their way through the dielectric medium, around its edges or across the leads. The leakage current of a capacitor is sometimes called the **insulation resistance**.

ILLUSTRATION 72

A leaky parallel plane capacitor is filled completely with a material having dielectric constant $k=5$ and electrical conductivity $\sigma=7.4\times10^{-12}\ \Omega^{-1}\ \text{m}^{-1}$. If the charge on the plane at instant $t=0$ is $q=8.85$ mC, then calculate the leakage current at the instant $t=12$ s.

SOLUTION

The problem deals with discharging of CR circuit, because between the plates of the capacitor, there is capacitor as well as resistance.



$$R=\frac{\rho d}{A}=\frac{d}{\sigma A}$$

$$\{\text{where } \rho=\text{resistivity } \& \ \sigma=\text{conductivity}\}$$

and $C=\dfrac{K\varepsilon_o A}{d}$

The capacitive time constant of the circuit is

$$\tau=CR=\frac{K\varepsilon_o}{\sigma}$$

Substituting the values, we have

$$\tau=\frac{5\times8.86\times10^{-12}}{7.4\times10^{-12}}=5.98\ \text{s}$$

Charge at any time decrease exponentially as

$$q=q_o e^{-t/\tau}$$

where $q_o=8.85\times10^{-6}$ C is the charge at time $t=0$

Therefore, discharging (leakage) current at time t is given by

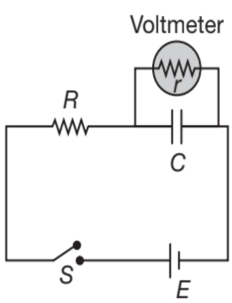
$$I=\left(-\frac{dq}{dt}\right)=\frac{q_o}{\tau}e^{-t/\tau}$$

So, the current at $t=12$ s is

$$I=\frac{\left(8.85\times10^{-6}\right)}{5.98}e^{-12/5.98}=0.198\times10^{-6}\ \text{A}=0.198\ \mu\text{A}$$

ILLUSTRATION 73

A digital voltmeter of internal resistance r is used to measure the voltage across a capacitor after the switch in Figure is closed. Because the meter has finite resistance, part of the current supplied by the battery passes through the meter.



(a) Apply Kirchhoff's rules to this circuit, and use the fact that $I_C=\dfrac{dq}{dt}$ to show that this leads to the differential equation

$$R_{eq}\frac{dq}{dt}+\frac{q}{C}=\frac{r}{R+r}E$$

where $R_{eq}=\dfrac{rR}{R+r}$

(b) Show that the solution to this differential equation is

$$q=\frac{CEr}{R+r}\left(1-e^{-\frac{t}{R_{eq}C}}\right)$$

and that the voltage across the capacitor as a function of time is

$$V_C=\frac{Er}{R+r}\left(1-e^{-\frac{t}{R_{eq}C}}\right)$$

(c) If the capacitor is fully charged, and the switch is then opened, how does the voltage across the capacitor behave in this case?

SOLUTION

(a) Let I represent the current in the battery and I_C the current charging the capacitor. Then $I-I_C$ is the current in the voltmeter. The loop rule applied to the inner loop is $+E-IR-\dfrac{q}{C}=0$. The loop rule for the outer perimeter is $E-IR-(I-I_C)r=0$. With $I_C=\dfrac{dq}{dt}$, this becomes $E-IR-Ir+\dfrac{dq}{dt}r=0$. Between the two loop equations we eliminate $I=\dfrac{E}{R}-\dfrac{q}{RC}$ by substitution to obtain

$$E-(R+r)\left(\frac{E}{R}-\frac{q}{RC}\right)+\frac{dq}{dt}r=0$$

$$\Rightarrow\quad E-\left(\frac{R+r}{R}\right)E+\left(\frac{R+r}{RC}\right)q+r\frac{dq}{dt}=0$$

$$\Rightarrow\quad -\left(\frac{r}{R+r}\right)E+\frac{q}{C}+\left(\frac{Rr}{R+r}\right)\frac{dq}{dt}=0$$

This is the required differential equation

(b) From above, we get

$$\frac{dq}{dt}=\frac{E}{R}-\frac{R+r}{RrC}q=-\frac{R+r}{RrC}\left(q-\frac{ErC}{R+r}\right)$$

$$\Rightarrow\quad \int_0^q\frac{dq}{q-\frac{ErC}{(R+r)}}=-\frac{R+r}{RrC}\int_0^t dt$$

$$\Rightarrow\quad \log_e\left(q-\frac{ErC}{R+r}\right)\Bigg|_0^q=-\frac{R+r}{RrC}t\Bigg|_0^t$$

$$\Rightarrow\quad \log_e\left(\frac{q-\frac{ErC}{(R+r)}}{\frac{-ErC}{(R+r)}}\right)=-\left(\frac{R+r}{RrC}\right)t$$

$$\Rightarrow\quad q-\frac{ErC}{R+r}=-\frac{ErC}{R+r}\exp\left[\left(-\frac{R+r}{RrC}\right)t\right]$$

$$\Rightarrow\quad q=\frac{r}{R+r}CE\left(1-e^{-\frac{t}{R_{eq}C}}\right)\text{ where }R_{eq}=\frac{Rr}{R+r}$$

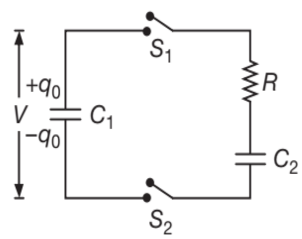
The voltage across the capacitor is

$$V_C=\frac{q}{C}=\frac{Er}{R+r}\left(1-e^{-t/R_{eq}C}\right)$$

(c) As $t\to\infty$ the capacitor voltage approaches $\dfrac{Er}{R+r}(1-0)=\dfrac{Er}{R+r}$. If the switch is then opened, the capacitor discharges through the voltmeter. Its voltage decays exponentially according to $\dfrac{Er}{R+r}e^{-\frac{t}{rC}}$.

ILLUSTRATION 74

The capacitor C_1 in figure initially carries a charge q_0. When the switches S_1 and S_2 are shut, capacitor C_1 is connected in series to a resistor R and a second capacitor C_2, which initially does not carry any charge.



(a) Find the charges deposited on the capacitors and the current through R as a function of time.

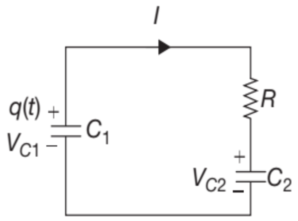
(b) What is the heat lost in the resistor after a long time of closing the switch?

SOLUTION

(a) Suppose at a moment t the charge deposited on C_1 is $q(t)$. Then,

$$V_{C_1}=\frac{q(t)}{C_1} \text{ and } V_{C_2}=\frac{q_0-q(t)}{C_2}$$

$$V_R=IR \text{ where } I=-\frac{dq}{dt}$$



Applying KVL, we get

$$\frac{q}{C_1}-\frac{(q_0-q)}{C_2}=IR=-R\frac{dq}{dt}$$

$$\Rightarrow \quad R\frac{dq}{dt}+\left(\frac{1}{C_1}+\frac{1}{C_2}\right)q=\frac{q_0}{C_2}$$

Put $\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$, we get

$$R\frac{dq}{dt}+\frac{q}{C}=\frac{q_0}{C_2}$$

$$\Rightarrow \quad R\frac{dq}{dt}=\frac{q_0}{C_2}-\frac{q}{C}$$

$$\Rightarrow \quad R\frac{dq}{dt}=\frac{1}{C}\left(\frac{C}{C_2}q_0-q\right)$$

$$\Rightarrow \quad \int_{q_0}^{q}\frac{dq}{q-\frac{C}{C_2}q_0}=-\int_0^t\frac{dt}{RC}$$

$$\Rightarrow \quad \log_e\left(\frac{q-\frac{C}{C_2}q_0}{q_0-\frac{C}{C_2}q_0}\right)=-\frac{t}{RC}$$

$$\Rightarrow \quad q-\frac{C}{C_2}q_0=q_0\left(1-\frac{C}{C_2}\right)e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad q=q(t)=q_0\left[\left(1-\frac{C}{C_2}\right)e^{-\frac{t}{RC}}+\frac{C}{C_2}\right] \text{ where}$$

$$C=\frac{C_1C_2}{C_1+C_2}$$

$$\Rightarrow \quad I(t)=-\frac{dq}{dt}=\frac{q_0}{RC_1}e^{-\frac{t}{RC}}$$

Charge on C_2,

$$q_2=q_0-q(t)=q_0\frac{C}{C_1}\left(1-e^{-\frac{t}{RC}}\right)$$

So, we observe that the charge on q_1 decays and that on q_2 grows exponentially.

(b) Electrostatic energy at $t=0$ is

$$U(0)=\frac{q_0^2}{2C_1}$$

Electrostatic energy at $t\to\infty$ is final energy given by

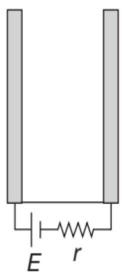
$$U_f=U(\infty)=\frac{q_0^2}{2(C_1+C_2)}$$

Since, Heat Loss $=-\Delta U=U_i-U_f$

$$\Rightarrow \quad -\Delta U=U(0)-U(\infty)=\frac{q_0^2C_2}{2C_1(C_1+C_2)}$$

ILLUSTRATION 75

Two large conducting plates are arranged to form a parallel-plate capacitor of capacitance C. The plates are given charges Q_1 and Q_2 respectively. The plates are then connected to the terminals of a battery of emf E and internal resistance r at $t=0$. Find the charges on the four surfaces as function of time.



SOLUTION

When the battery is connected, the charges on the inner surfaces will always be equal and opposite. Since the electric field inside the plates will be zero, the cancellation of electric field must be accomplished by the charges on the outer surfaces. For this, they have to be of same magnitude and nature. It thus follows that the charges on the outer surfaces will always be what they initially were, i.e., $\frac{Q_1+Q_2}{2}$.

Let q be the charges at time t on the inner surfaces. Applying KVL,

$$-\frac{q}{C}-Ir+E=0$$

$$\Rightarrow \quad \frac{dq}{E-\frac{q}{C}}=\frac{dt}{r}$$

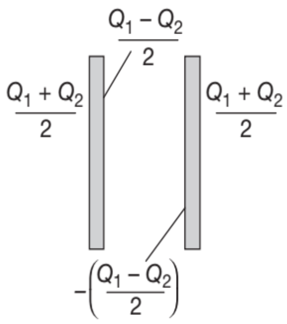
$$\int_{\frac{Q_1-Q_2}{2}}^{q}\frac{dq}{CE-q}=\int_0^t\frac{dt}{rC}$$

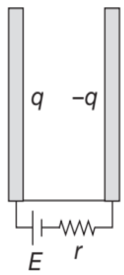
$$-\log_e(CE-q)\Big|_{\frac{Q_1-Q_2}{2}}^{q}=\frac{t}{rC}$$

$$\Rightarrow \quad \log_e\left[\frac{CE-q}{CE-\left(\frac{Q_1-Q_2}{2}\right)}\right]=-\frac{t}{rC}$$

$$\Rightarrow \quad \frac{CE-q}{CE-\left(\frac{Q_1-Q_2}{2}\right)}=e^{-\frac{t}{rC}}$$

$$\Rightarrow \quad CE-q=\left(CE-\frac{Q_1-Q_2}{2}\right)e^{-\frac{t}{rC}}$$

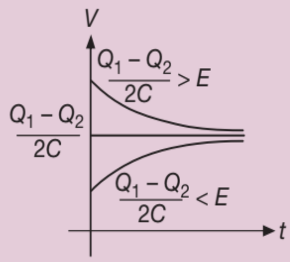




$$\Rightarrow \quad q=CE\left(1-e^{-\frac{t}{rC}}\right)+\left(\frac{Q_1-Q_2}{2}\right)e^{-\frac{t}{rC}}$$

Observation

We can interpret this result as a superposition of discharging and charging of a capacitor. Also note that in steady state, the charge on the inner surfaces will be CE and $-CE$ respectively, regardless of the initial value. In particular if $\frac{Q_1-Q_2}{2C}$ (initial potential difference between the plates) is equal to E, there is no charge flow through the battery. If $\frac{Q_1-Q_2}{2C}>E$, an extra charge is given to the battery and if $\frac{Q_1-Q_2}{2C}<E$, the battery supplies the charge to the capacitor. This all is shown in the graph shown in figure, wherein potential difference across the plates has been plotted as function of time.



Test Your Concepts-XI

Based on RC Circuit

(Solutions on page H.228)

1. Suppose that the current through a conductor decreases exponentially with time according to the equation $I(t)=I_0e^{-\frac{t}{\tau}}$ where I_0 is the initial current (at t = 0), and τ is a constant having dimensions of time. Consider a fixed observation point within the conductor. Calculate the charge passes through this point between

(a) $t=0$ and $t=\tau$. **(b)** $t=0$ and $t=10\tau$.

(c) $t=0$ and $t\to\infty$.

2. The dielectric material between the plates of a parallel plate capacitor always has some nonzero conductivity σ. Let A represent the area of each plate and d the distance between them. Let κ represent the dielectric constant of the material

(a) Show that the resistance R and the capacitance C of the capacitor are related by $RC=\frac{\kappa\varepsilon_0}{\sigma}$.

(b) Find the resistance between the plates of a 14 nF capacitor with a fused quartz dielectric having $\kappa=3.78$

3. A 2 nF capacitor with an initial charge of 5.1 μC is discharged through a 1.3 kΩ resistor.

(a) Calculate the current in the resistor 9 μs after the resistor is connected across the terminals of the capacitor.

(b) What charge remains on the capacitor after 8 μs?

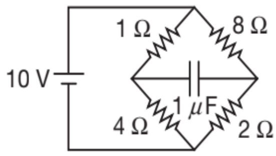
(c) What is the maximum current in the resistor?

4. A fully charged capacitor stores energy U_0. How much energy remains when its charge has decreased to half its original value?

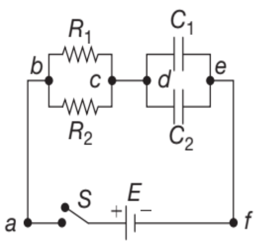
5. The circuit in figure has been connected for a long time.

(a) What is the voltage across the capacitor?

(b) If the battery is disconnected, how long does it take the capacitor to discharge to one tenth of its initial voltage?



6. The circuit in figure contains two resistors, $R_1 = 2$ kΩ and $R_2 = 3$ kΩ, and two capacitors, $C_1 = 2$ μF and $C_2 = 3$ μF, connected to a battery with emf $E = 120$ V. Charge is on either capacitor before switch S is closed is zero. Determine the charges q_1 and q_2 on capacitors C_1 and C_2, respectively, after the switch is closed.



7. During the process of charging of a capacitor, how long does it take for the current to decay to half its value?

8. At what time the charge on the capacitor in a discharging circuit equal to one half its initial value?

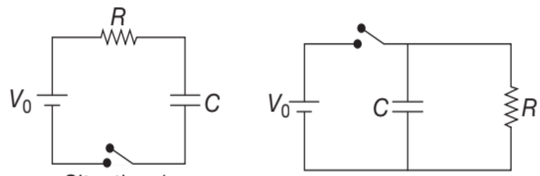
9. In a discharging *RC* circuit, at what time is the electrical potential energy becomes half of its initial value?

10. (a) Calculate the power absorbed by a charging capacitor in the series *RC*circuit as a function of time.

(b) Integrate $P(t)$ from $t = 0$ to $t \to \infty$ to show that the energy stored in the capacitor is $\left(\frac{1}{2}\right)CV_0^2$, when it is fully charged.

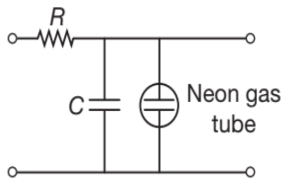
11. A leaky parallel plate capacitor is filled completely with a material having dielectric constant $K = 5$ and electrical conductivity $\sigma = 7.4 \times 10^{-12}\ \Omega^{-1}\ \text{m}^{-1}$. If the charge on the plate at the instant $t = 0$ is $q = 8.85$ μC, then calculate the leakage current at $t = 12$ s.

12. A 10 μF capacitor is charged through a resistance of 0.1 MΩ from a battery of 1.5 V. Calculate the time required for the capacitor to get charged upto 0.75 V for situations shown.

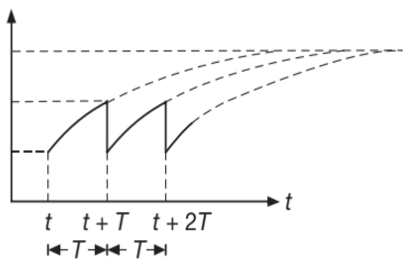


Situation 1 Situation 2

13. Figure (a) shows a circuit in which time interval of flashes of a neon gas tube is controlled by a capacitor. When cool, the gas is a good insulator. The tube "fires" (ionizes and emits light) when the potential difference across it reaches the firing value V_f. Its resistance becomes very small and so the capacitor rapidly discharges through it. As the potential difference drops, the gas cools down and becomes an insulator. The extension potential difference is V_e. At this stage the capacitor again starts to charge. The variation of potential difference is shown in Figure (b). Determine the time period T of flashing of the neon gas tube.

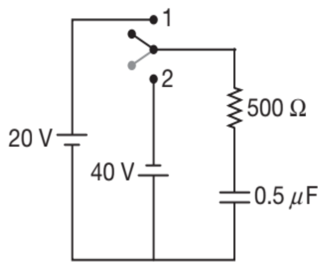


(a)

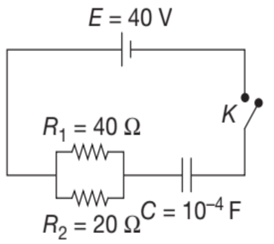


(b)

14. In the given circuit, the switch is closed in the position 1 at $t = 0$ and then moved to 2 after 250 μs. Derive an expression for current as a function of time for $t > 0$. Also, plot the variation of current with time.



15. The circuit shown in figure is closed at time $t = 0$. Calculate the total amount of heat generated in R_2 during the time in which the capacitor is charged to a voltage of 20 V.



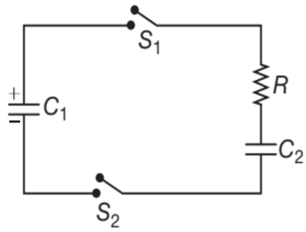
16. A battery consists of 2 identical cells, each of e.m.f. E, connected in series. The battery is used to charge a capacitor, with capacitance C, through a resistor R.

(a) by connecting it across the terminals of the entire battery and

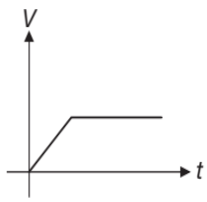
(b) by connecting it first across a single cell, then across two cells until it is again fully charged.

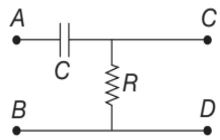
Determine, in each case, the amount of energy transferred from the battery, which is not stored in the capacitor. What has happened to this energy?

17. The switch S is closed at $t = 0$. The capacitor C is uncharged but C_0 has a charge $Q_0 = 2$ μC at $t = 0$. If $R = 100\ \Omega$, $C = 2$ μF, $C_0 = 2$ μF, $E = 4$ V. Calculate $I(t)$ in the circuit.



18. A time varying voltage is applied to the clamps A and B such that voltage across the capacitor plates is as shown in the figure. Plot the time dependence of voltage across the terminals of the resistance C and D.





19. A circuit consists of a source of a constant emf E a resistance R and a capacitor with capacitance C connected series. The internal resistance of the source is negligible. At a moment $t = 0$, the capacitance of the capacitor is abruptly decreased η-fold. Find the current flowing through the circuit as a function of time t.

20. The gap between the plates of a parallel plate capacitor is filled with glass of resistivity $\rho = 10^{11}\ \Omega$m. The capacitance of the capacitor $C = 4$ nF. Find the leakage current of the capacitor when a voltage $V = 2$ kV is applied across it. (Dielectric constant of glass $K = 6$).

SOLVED PROBLEMS

PROBLEM 1

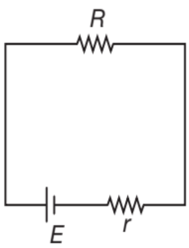
A battery has an open circuit potential difference of 6 V between its terminals. When a load resistance of 60 Ω is connected across the battery, the total power dissipated by the battery is 0.4 W. What should be the load resistance R, so that maximum power will be dissipated in R. Calculate this power. What is the total power supplied by the battery when such a load is connected?

SOLUTION

When the circuit is open, $V = E$

$$\Rightarrow \quad E = 6\text{ V}$$

Let r be the internal resistance of the battery. Then the power supplied by the battery in this case is,



$$P = \frac{E^2}{R+r}$$

Substituting the values, we have

$$0.4 = \frac{(6)^2}{60+r}$$

$$\Rightarrow \quad r = 30\ \Omega$$

According to Maximum Power Transfer Theorem, maximum power is dissipated in the circuit when, net external resistance is equal to net internal resistance. So, we have

$$R = r$$

$$\Rightarrow \quad R = 30\ \Omega$$

Further, total power supplied by the battery under this condition is,

$$P_{\text{Total}} = \frac{E^2}{R+r} = \frac{(6)^2}{30+30} = 0.6\text{ W}$$

Out of this 0.6 W, half of the power is dissipated in R and half in r. Therefore, maximum power dissipated in R would be

$$P_{\text{MAX}} = \frac{0.6}{2} = 0.3\text{ W}$$

PROBLEM 2

A conductor has a temperature independent resistance R and a total heat capacity C. At the moment $t = 0$ it is connected to a dc voltage V. Find the time dependence of the conductor's temperature T assuming the thermal power dissipated into surrounding space to vary as $q = k(T - T_0)$ where k is a constant, T_0 is the environmental temperature (equal to conductor's temperature at the initial moment).

SOLUTION

Here energy is being generated in the resistance at a rate of $\dfrac{V^2}{R}$. Of which part of energy is being lost in the environment and the rest is utilized in raising the temperature of conductor. So, applying the Law of Conservation of Energy, we get

$$\begin{pmatrix}\text{Energy}\\\text{supplied}\\\text{by the dc}\\\text{source per}\\\text{unit time}\end{pmatrix} = \begin{pmatrix}\text{Energy lost}\\\text{in the}\\\text{environment}\\\text{per}\\\text{unit time}\end{pmatrix} + \begin{pmatrix}\text{Energy used}\\\text{in raising the}\\\text{temperature of}\\\text{the conductor}\\\text{per unit time}\end{pmatrix}$$

Hence, $\dfrac{V^2}{R} = k(T - T_0) + C\left(\dfrac{dT}{dt}\right)$

$$\Rightarrow \quad C\left(\frac{dT}{dt}\right) = \frac{V^2}{R} - k(T - T_0)$$

$$\Rightarrow \quad \frac{dT}{\dfrac{V^2}{R} - k(T - T_0)} = \frac{dt}{C}$$

Integrating the above expression, we get

$$\int_{T_0}^{T} \frac{dT}{\dfrac{V^2}{R} - k(T - T_0)} = \int_0^t \frac{dt}{C}$$

Solving this equation, we get

$$T = T_0 + \frac{V^2}{kR}\left(1 - e^{-\frac{kt}{C}}\right)$$

PROBLEM 3

A battery of emf 2 V and negligible internal resistance is connected across a uniform wire of length 10 m and resistance $30\ \Omega$. The appropriate terminals of a cell of emf 1.5 V and internal resistance $1\ \Omega$ is connected to one end of the wire, and the other terminal of the cell is connected through a sensitive galvanometer to a slider on the wire.

(a) What length of the wire will be required to produce zero deflection of the galvanometer?

How will the balancing change,

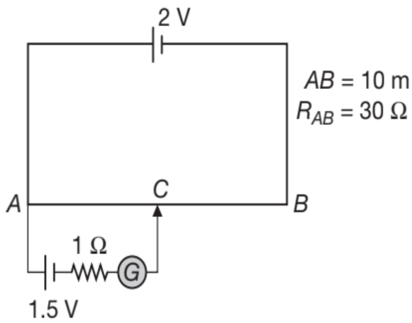
(b) when a coil of resistance $5\ \Omega$ is placed in series with the battery.

(c) When the cell of 1.5 V is shunted with $5\ \Omega$ resistor?

SOLUTION

(a) Potential gradient across wire AB is

$$\frac{\Delta V}{\Delta l} = \frac{2}{10} = 0.2\ \text{Vm}^{-1}$$



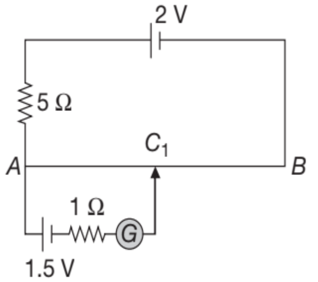
Now, $V_{AC} = 1.5$ V

$$\Rightarrow \left(\frac{\Delta V}{\Delta l}\right)(AC) = 1.5$$

$$\Rightarrow (0.2)(AC) = 1.5 = E$$

$$\Rightarrow AC = 7.5\ \text{m}$$

(b) $V_{AB} = \left(\frac{R_{AB}}{R_{AB}+5}\right)\times 2 = \left(\frac{30}{30+5}\right)\times 2 = \frac{12}{7}$ V



Now, the potential gradient across AB is

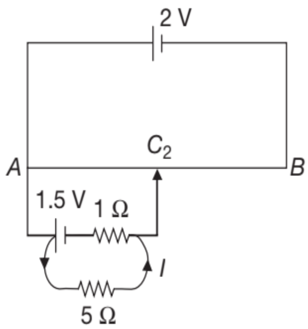
$$\left(\frac{\Delta V}{\Delta l}\right)_{AB} = \frac{12}{70}\ \text{voltm}^{-1}$$

Since, $V_{AC_1} = 1.5\ \text{V} = E$

$$\Rightarrow \left(\frac{12}{70}\right)(AC_1) = 1.5$$

$$\Rightarrow AC_1 = 8.75\ \text{m}$$

(c) V_{AC_2} = Voltage drop across 1.5 V battery



$$\Rightarrow (0.2)(AC_2) = \left(\frac{5}{5+1}\right)(1.5)$$

$$\Rightarrow AC_2 = 6.25\ \text{m}$$

PROBLEM 4

A 10 m long potentiometer wire has resistance $10\ \Omega$ and is connected to an accumulator of 2 V and internal resistance zero. Two resistance boxes B_1 and B_2 are connected in series with the accumulator. A cell of emf 1.018 V and a shunted galvanometer are connected in parallel across the box B_1. Reading of galvanometer is zero.

(a) For what values of resistances R_1 and R_2 both having integral values will be required from the boxes B_1 and B_2 to maintain a potential gradient of $10^{-3}\ \text{Vm}^{-1}$ in the potentiometer wire.

(b) Determine the length of potentiometer wire that balances the thermo emf of iron-copper couple at 300 °C which develops 1.7×10^{-5} volt °C^{-1}.

SOLUTION

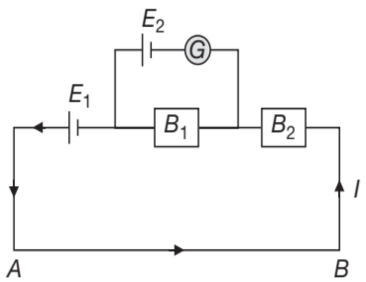
According to the statement, we are provided with the following data

$$E_1 = 2\text{ V}, E_2 = 1.018\text{ V}, AB = 10\text{ m and } R_{AB} = 10\,\Omega$$

(a) Potential gradient across potentiometer wire $AB = 10^{-3}\text{ Vm}^{-1}$

$$\therefore \quad V_{AB} = (10^{-3})(10) = 10^{-2}\text{ V}$$

$$I = \frac{V_{AB}}{R_{AB}} = \frac{10^{-2}}{10} = 10^{-3}\text{ A} \qquad \ldots(1)$$



Further, potential difference across $B_1 = E_2 = 1.018$ V

$$\Rightarrow \quad IR_1 = 1.018\text{ V}$$

$$\Rightarrow \quad R_1 = \frac{1.018}{I} = 1018\,\Omega$$

Also, $I = \dfrac{E_1}{R_1 + R_2 + R_{AB}}$

$$\Rightarrow \quad 10^{-3} = \frac{2}{1018 + R_2 + 10}$$

$$\Rightarrow \quad R_2 = 972\,\Omega$$

(b) $V = (1.7\times10^{-5})(300) = 5.1\times10^{-3}$ V

$$\Rightarrow \quad l = \frac{V}{\text{Potential gradient}} = \frac{5.1\times10^{-3}}{10^{-3}} = 5.1\text{ m}$$

PROBLEM 5

It is desired to send a current of 8 A through a circuit whose resistance is $5\,\Omega$. Find the least number of cells which must be used for this purpose and how should they be connected. The emf of each cell is 2 V and the internal resistance is $0.5\,\Omega$.

SOLUTION

Let N = total number of cells required to be grouped in m rows, each row carrying n cells $= mn$

Since, we know that for n cells in series connected in m rows in parallel, the current in the external circuit is given by

$$I = \frac{nE}{R + \frac{nr}{m}} = \frac{mnE}{mR + nr} = \frac{NE}{mR + nr}$$

$$\Rightarrow \quad 8 = \frac{2N}{5m + \frac{0.5N}{m}}$$

$$\Rightarrow \quad 20m^2 + 2N = Nm \qquad \ldots(1)$$

For least value of N, we have

$$\frac{dN}{dm} = 0$$

Differentiating equation (1), we have

$$40m + 2\frac{dN}{dm} = m\frac{dN}{dm} + N$$

$$\Rightarrow \quad \frac{dN}{dm} = \frac{N - 40m}{2 - m} = 0$$

$$\Rightarrow \quad N = 40m \qquad \ldots(2)$$

Since $N = mn$ $\qquad \ldots(3)$

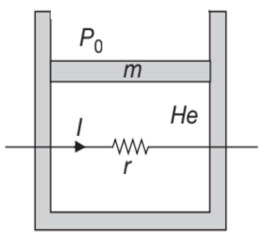
Solving equations (1), (2) and (3), we get

$$n = 40 \text{ and } m = 4$$

$$\Rightarrow \quad N = mn = 160$$

PROBLEM 6

A resistance coil of resistance r connected to an external battery, is placed inside an adiabatic cylinder fitted with a frictionless piston of mass m and same area A. Initially cylinder contains one mole of ideal gas He. A current I flows through the coil such that temperature of gas varies as $T = T_0 + at + bt^2$, keeping pressure constant with time t. Atmosphere pressure above piston is P_0. Find



(a) Current I flowing through the coil as function of time and

(b) Speed of piston as function of time.

SOLUTION

Heat produced by coil inside the cylinder in time dt is

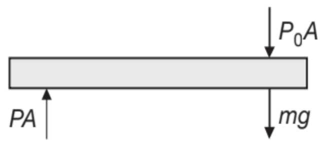
$$dQ = I^2 r\, dt \qquad \ldots(1)$$

(a) From First Law of Thermodynamics

$$\Delta Q = \Delta U + \Delta W \qquad \ldots(2)$$

$$\Rightarrow \quad I^2 r dt = C_v dT + PdV$$

$$\Rightarrow \quad I^2 r = C_v \frac{dT}{dt} + R\frac{dT}{dt} = C_p \frac{dT}{dt}$$



As $T = T_0 + at + bt^2$

$$\Rightarrow \quad \frac{dT}{dt} = (a + 2bt)$$

$$\Rightarrow \quad I = \sqrt{\frac{5R}{2r}(2bt + a)} \qquad \left(C_p = \frac{5R}{2}\right)$$

(b) $PV = RT$

$$\Rightarrow \quad PdV = RdT \qquad \text{(pressure is constant)}$$

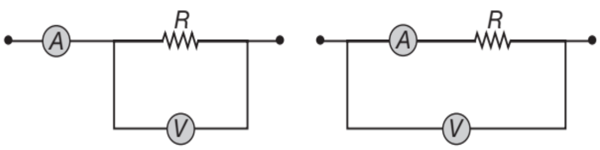
$$\Rightarrow \quad PAdx = RdT$$

$$\Rightarrow \quad \text{Velocity } v = \frac{dx}{dt} = \frac{R}{PA}\left(\frac{dT}{dt}\right)$$

$$\Rightarrow \quad v = \frac{R}{PA}(2bt + a) \qquad (\text{where } P_0A + mg = PA)$$

PROBLEM 7

The value of a resistor R is to be determined using the ammeter-voltmeter setup shown in figure. The ammeter has a resistance of $0.5\ \Omega$, and the voltmeter has a resistance of $20000\ \Omega$.



Circuit (a) Circuit (b)

Within what range of actual values of R will the measured values be correct to within 5% if the measurement is made using the circuit shown in figure.

SOLUTION

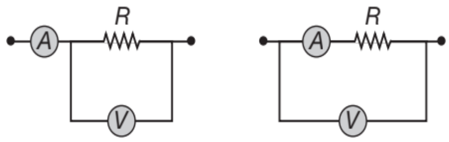
Let R_m = measured value, R = actual value,

I_R = current through the resistor R

I = current measured by the ammeter.

(a) When using circuit (a), $I_R R = \Delta V = 20000(I - I_R)$

$$\Rightarrow \quad R = 20000\left[\frac{I}{I_R} - 1\right]$$



Circuit (a) Circuit (b)

But since $I = \dfrac{\Delta V}{R_m}$ and $I_R = \dfrac{\Delta V}{R}$, we have $\dfrac{I}{I_R} = \dfrac{R}{R_m}$

$$\Rightarrow \quad R = 20000\frac{(R - R_m)}{R_m} \qquad \ldots(1)$$

When $R > R_m$, we require $\left(\dfrac{R - R_m}{R}\right) \le 0.05$

$\Rightarrow \quad R_m \ge R(1 - 0.05)$ and from (1) we get

$R \le 1050\ \Omega$.

(b) When using circuit (b), we have

$I_R R = \Delta V - I_R(0.5\ \Omega)$

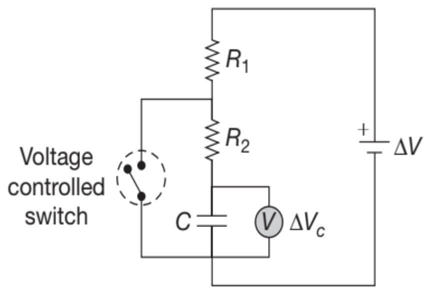
But since $I_R = \dfrac{\Delta V}{R_m}$ and $R_m = (0.5 + R)$ $\qquad \ldots(2)$

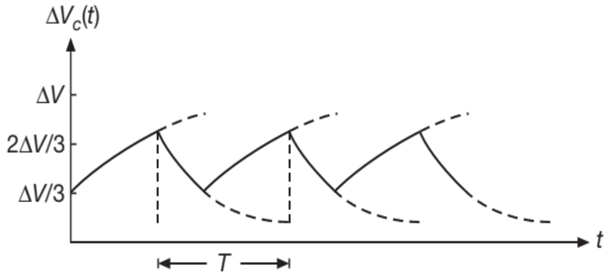
When $R_m > R$, we require $\dfrac{(R_m - R)}{R} \le 0.05$

From (2) we find $R \ge 10\ \Omega$

PROBLEM 8

The switch in figure closes when $\Delta V_c > \dfrac{2\Delta V}{3}$ and opens when $\Delta V_c < \dfrac{\Delta V}{3}$ The voltmeter reads a voltage as plotted in figure. What is the period T of the waveform in terms of R_1, R_2 and C?

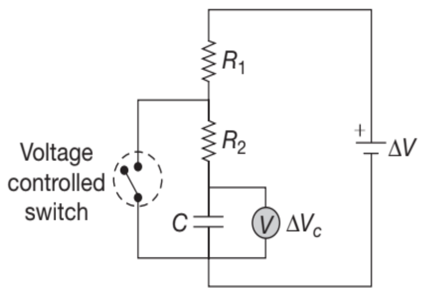




SOLUTION

Start at the point when the voltage has just reached $\frac{2}{3}\Delta V$ and the switch has just closed. The voltage is $\frac{2}{3}\Delta V$ and is decaying towards 0 V with a time constant R_2C

$$\Delta V_C(t)=\left[\frac{2}{3}\Delta V\right]e^{-\frac{t}{R_2C}}$$



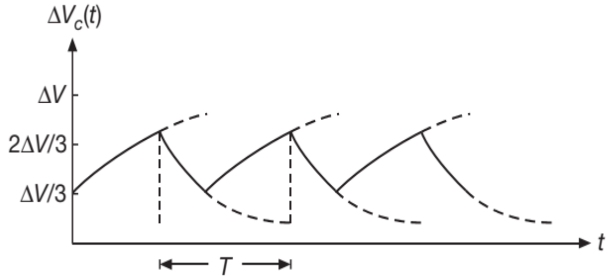
Let, $\Delta V_C(t)=\frac{1}{3}\Delta V$, at time t_1, then

$$\frac{1}{3}\Delta V=\left[\frac{2}{3}\Delta V\right]e^{-\frac{t_1}{R_2C}}$$

$$\Rightarrow \quad e^{-\frac{t_1}{R_2C}}=\frac{1}{2} \quad \Rightarrow \quad t_1=R_2C\log_e 2$$

After the switch opens, the voltage is $\frac{1}{3}\Delta V$, increasing toward ΔV with time constant $(R_1+R_2)C$, so

$$\Delta V_C(t)=\Delta V-\left[\frac{2}{3}\Delta V\right]e^{-\frac{t}{(R_1+R_2)C}}$$



Let $\Delta V_C(t)=\frac{2}{3}\Delta V$, in further time t_2, then

$$\Rightarrow \quad \frac{2}{3}\Delta V=\Delta V-\frac{2}{3}\Delta V e^{-\frac{t_2}{(R_1+R_2)C}}$$

$$\Rightarrow \quad e^{-\frac{t_2}{(R_1+R_2)C}}=\frac{1}{2}$$

$$\Rightarrow \quad t_2=(R_1+R_2)C\log_e 2$$

$$\Rightarrow \quad T=t_1+t_2=(R_1+2R_2)C\log_e 2$$

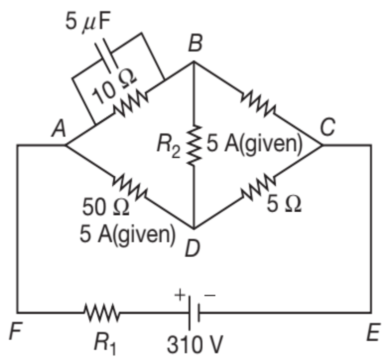
PROBLEM 9

Figure shows a circuit in steady state. If charge on the capacitor is 1000 μC, find

(a) The battery current

(b) The resistance R_1, R_2, R_3.

The current in 50 Ω resistor and R_2 is given to be 5 A.



SOLUTION

(a) The potential difference across 5 μC capacitor, is

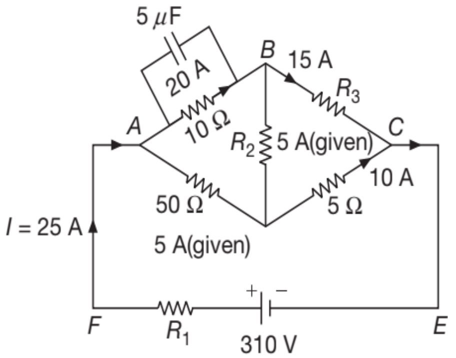
$$V=\frac{Q}{C}=\left(\frac{1000}{5}\right)\text{V}=200\text{ V}$$

The 10 Ω resistor and capacitor are in parallel arrangement and therefore current I through 10 Ω resistor is given by

$$I = \frac{V}{R} = \frac{200}{10} = 20 \text{ A}$$

(b) From KCL at junction A we get

$I = 25$ A



Now we apply KVL in the loop $ABDA$,

$$-(10)(20) - 5R_2 + (50)(5) = 0$$

$$\Rightarrow \quad R_2 = 10\ \Omega$$

Apply KVL in loop $ADCEFA$, we get

$$-(50)(5) - (5)(10) + 310 - 25R_1 = 0$$

$$\Rightarrow \quad R_1 = \frac{10}{25}\ \Omega = 0.4\ \Omega$$

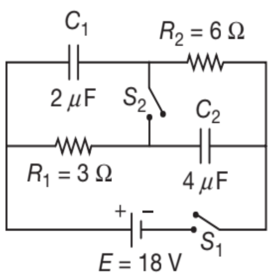
Apply KVL in loop $ABCDA$,

$$-(20)(10) - 15R_3 + (10)(5) + (50)(5) = 0$$

$$\Rightarrow \quad R_3 = \frac{100}{15} = 6.66\ \Omega$$

PROBLEM 10

In the circuit shown, initially the switches are open and the capacitors are uncharged. Switches S_1 and S_2 are closed simultaneously at $t = 0$.



(a) Obtain the expression for current through switch S_2 as function of time.

(b) The switch S_2 is opened after a long time interval. Find the heat dissipated in resistors and the charge flowing through S_1.

SOLUTION

(a) The distribution of current in the circuit is shown in the figure. Apply KVL to loop $abefa$, we get

$$-\frac{q_1}{C_1} + (I_2 + I_3 - I_1)R_1 = 0 \qquad \ldots(1)$$

Apply KVL to loop $bcdeb$, we get

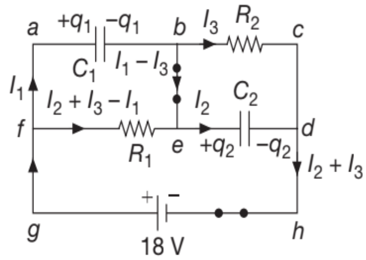
$$-I_3R_2 + \frac{q_2}{C_2} = 0 \qquad \ldots(2)$$

Apply KVL to loop $achga$, we get

$$I_3R_2 + \frac{q_1}{C_1} = E \qquad \ldots(3)$$

The current through capacitor C_1 is given by

$$I_1 = \frac{dq_1}{dt}$$



The current through capacitor C_2 is given by

$$I_2 = \frac{dq_2}{dt}$$

From equations (2) and (3), we get, at any instant,

$$\frac{q_1}{C_1} + \frac{q_2}{C_2} = E \qquad \ldots(4)$$

From equation (1), we get

$$I_2 - I_1 = \frac{q_1}{R_1C_1} - I_3$$

$$\Rightarrow \quad \frac{d}{dt}(q_2 - q_1) = \frac{q_1}{C_1R_1} - \frac{q_2}{C_2R_2} \qquad \ldots(5)$$

$$\Rightarrow \quad \frac{d}{dt}\left[q_2 - C_1\left(E - \frac{q_2}{C_2}\right)\right] = \frac{1}{C_1R_1}\left[C_1\left(E - \frac{q_2}{C_2}\right)\right] - \frac{q_2}{C_2R_2}$$

$$\Rightarrow \quad \left(1 + \frac{C_1}{C_2}\right)\frac{dq_2}{dt} = \frac{E}{R_1} - \frac{q_2}{C_2}\left(\frac{1}{R_2} + \frac{1}{R_1}\right)$$

$$\Rightarrow \left(\frac{C_1+C_2}{\cancel{C_2}}\right)\left(\frac{dq_2}{dt}\right)=\frac{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}{\cancel{C_2}}\left[\frac{EC_2}{R_1\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}-q_2\right]$$

$$\Rightarrow \frac{dq_2}{dt}=\frac{1}{(C_1+C_2)\left(\frac{R_1R_2}{R_1+R_2}\right)}\left[\frac{EC_2R_2}{R_1+R_2}-q_2\right]$$

$$\Rightarrow \int_0^{q_2}\frac{dq_2}{\left(\frac{EC_2R_2}{R_1+R_2}-q_2\right)}=\int_0^t\frac{dt}{(C_1+C_2)\left(\frac{R_1R_2}{R_1+R_2}\right)}$$

$$\Rightarrow -\log_e\left(\frac{EC_2R_2}{R_1+R_2}-q_2\right)\Bigg|_0^{q_2}=\frac{t}{(C_1+C_2)\left(\frac{R_1R_2}{R_1+R_2}\right)}$$

$$\Rightarrow \log_e\left(\frac{\frac{EC_2R_2}{R_1+R_2}-q_2}{\frac{EC_2R_2}{R_1+R_2}}\right)=-\frac{t}{(C_1+C_2)\left(\frac{R_1R_2}{R_1+R_2}\right)}$$

$$\Rightarrow \frac{EC_2R_2}{R_1+R_2}-q_2=\left(\frac{EC_2R_2}{R_1+R_2}\right)\exp\left[-\frac{t}{(C_1+C_2)\left(\frac{R_1R_2}{R_1+R_2}\right)}\right]$$

$$\Rightarrow q_2=\left(\frac{EC_2R_2}{R_1+R_2}\right)\left(1-e^{-\frac{t}{RC}}\right) \qquad \ldots(6)$$

where $R=\frac{R_1R_2}{R_1+R_2}$ and $C=C_1+C_2$ ...(7)

Substituting the value of q_2 from above in equation (4), we get

$$q_1=C_1E-\frac{C_1}{C_2}\left(\frac{EC_2R_2}{R_1+R_2}\right)\left(1-e^{-\frac{t}{RC}}\right)$$

$$\Rightarrow q_1=C_1E-\frac{C_1ER_2}{R_1+R_2}\left(1-e^{-\frac{t}{RC}}\right)$$

$$\Rightarrow q_1=C_1E\left[1-\frac{R_2}{R_1+R_2}(1-e)^{-\frac{t}{RC}}\right] \qquad \ldots(8)$$

where $R=\frac{R_1R_2}{R_1+R_2}$ and $C=C_1+C_2$

Now, $I_1=\frac{dq_1}{dt}=-\left(\frac{C_1E}{RC}\right)\left(\frac{R_2}{R_1+R_2}\right)e^{-\frac{t}{RC}}$

$$\Rightarrow I_1=-\frac{C_1E}{\left(\frac{R_1\cancel{R_2}}{\cancel{R_1+R_2}}\right)(C_1+C_2)}\left(\frac{\cancel{R_2}}{\cancel{R_1+R_2}}\right)e^{-\frac{t}{RC}}$$

$$\Rightarrow I_1=-\frac{C_1E}{R_1(C_1+C_2)}e^{-\frac{t}{RC}}$$

Similarly,

$$I_2=\frac{dq_2}{dt}=\frac{EC_2\cancel{R_2}}{(\cancel{R_1+R_2})}\frac{1}{(C_1+C_2)\left(\frac{R_1\cancel{R_2}}{\cancel{R_1+R_2}}\right)}e^{-\frac{t}{RC}}$$

$$\Rightarrow I_2=\frac{EC_2}{R_1(C_1+C_2)}e^{-\frac{t}{RC}}$$

Let us first now substitute the values of all given in equations (6), (7), (8) to get

$$q_2=\frac{(18)(4)(6)}{(3+6)}\left(1-e^{-\frac{t}{(2)(6)}}\right)$$

$$\Rightarrow q_2=48\left(1-e^{-\frac{t}{12}}\right) \quad (\text{in } \mu\text{C}) \qquad \ldots(9)$$

Similarly

$$q_1=(2)(18)-\frac{(2)(18)(6)}{(3+6)}\left(1-e^{-\frac{t}{12}}\right)$$

$$\Rightarrow q_1=36-24\left(1-e^{-\frac{t}{12}}\right)$$

$$\Rightarrow q_1=12+24e^{-\frac{t}{12}}$$

$$\Rightarrow q_1=12\left(1+2e^{-\frac{t}{12}}\right) \quad (\text{in } \mu\text{C}) \qquad \ldots(10)$$

Please note that as per equation (4), we must have $\frac{q_1}{C_1}+\frac{q_2}{C_2}=E$ (at any instant) and as per the relation the above two equations satisfy it at all the instants of time ranging from $t=0$ to $t\to\infty$. Finally, from (9) and (10), we get

$$I_1=\frac{dq_1}{dt}=-2e^{-\frac{t}{12}} \quad \text{(in A)}$$

$$I_2=\frac{dq_2}{dt}=4e^{-\frac{t}{12}} \quad \text{(in A)}$$

The current through the switch S_2 is given by

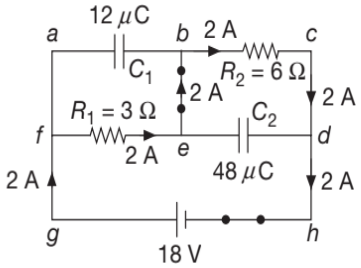
$$I=I_1-I_3=-\frac{q_1}{R_1C_1}+I_2$$

$$\Rightarrow \quad I=-\frac{12\left(1+2e^{-\frac{t}{12}}\right)}{(3)(2)}+48\left(1-e^{-\frac{t}{12}}\right)$$

$$\Rightarrow \quad I=-2\left(1+2e^{-\frac{t}{12}}\right)+4e^{-\frac{t}{12}}$$

$$\Rightarrow \quad I=-2\text{ A}$$

(b) In the steady state, no current will pass through the capacitors. The current and charges on capacitor will be as shown.



Total resistance of circuit is

$$R=9\ \Omega$$

So, current is given by

$$I'=\frac{18}{9}=2\text{ A}$$

Potential difference across C_1 is

$$V_f-V_e=I'R_1=6\text{ V}$$

Charge on C_1 is

$$Q_1=(6)(2)=12\ \mu\text{C}$$

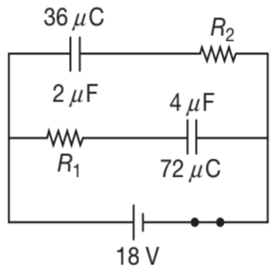
Potential difference across C_2 is

$$V_b-V_c=(6)(2)=12\text{ V}$$

Charge on C_2 is

$$Q_2=(12)(4)=48\ \mu\text{C}$$

After a long time, when switch S_2 is open and switch S_1 still remains closed then no current is drawn from battery in steady state. Both capacitors are in parallel arrangement with battery, so as to given equivalent capacitance of $(2+4)=6\ \mu$F



So, $Q_{\text{total}}=(C_{\text{eq}})(V)=(6)(18)$

$$\Rightarrow \quad Q_{\text{total}}=108\ \mu\text{C}$$

Hence $Q_{2\ \mu\text{F}}=(2\ \mu\text{F})(18\text{ V})=36\ \mu\text{C}$ and

$Q_{4\ \mu\text{F}}=(4\ \mu\text{F})(18\text{ V})=72\ \mu\text{C}$

Hence, the charge flowing through S_1 is

$$\Delta Q=Q_f-Q_i$$

$$\Rightarrow \quad \Delta Q=(36+72)-(12+48)$$

$$\Rightarrow \quad \Delta Q=48\ \mu\text{C}$$

Total heat dissipated in the resistors is

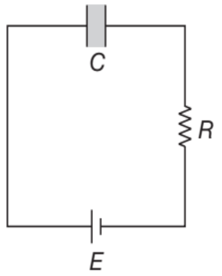
$$\Delta H=\begin{pmatrix}\text{Initial}\\ \text{Energy}\end{pmatrix}+\begin{pmatrix}\text{Work done by}\\ \text{battery when}\\ \text{charge } 48\ \mu\text{C}\\ \text{flows through}\\ \text{battery after}\\ \text{switch } S_2\\ \text{is opened}\end{pmatrix}-\begin{pmatrix}\text{Final}\\ \text{Energy}\end{pmatrix}$$

$$\Rightarrow \quad \Delta H=\frac{1}{2}\left(\frac{Q_1^2}{C_1}\right)+\frac{1}{2}\left(\frac{Q_2^2}{C_2}\right)+E(\Delta Q)-\frac{1}{2}(C_1+C_2)V^2$$

$$\Rightarrow \quad \Delta H=136\ \mu\text{J}$$

PROBLEM 11

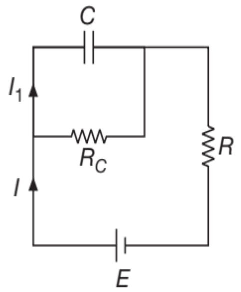
Consider a parallel plate capacitor of capacitance C with partially conducting medium between its plates having a resistance R_C. If this capacitor is connected to a battery of emf E and a resistor R as shown. Find the



(a) charge on the capacitor as a function of time.
(b) current through 'R' as a function of time.

SOLUTION

The equivalent circuit can be drawn as shown here.



Applying Kirchhoff's Law for the two loops,

$$(I-I_1)R_C+IR=E \qquad \ldots(1)$$

$$\frac{q}{C}=R_C(I-I_1) \qquad \ldots(2)$$

(a) Eliminating I between (1) and (2), we get

$$\frac{q}{C}=\left(1+\frac{R}{R_C}\right)+I_1R=E$$

For capacitor C, $\dfrac{dq}{dt}=I_1$

$$\Rightarrow \quad \frac{dq}{dt}=\frac{CE-q(1+\alpha)}{RC} \qquad \left\{\because \ \alpha=\frac{R}{R_C}\right\}$$

(b) Solving, we get $q=\dfrac{EC}{1+\alpha}\left(1-e^{-\frac{t(1+\alpha)}{RC}}\right)$

Adding (1) and (2), we get

$$\frac{Q}{C}+IR=E$$

$$\Rightarrow \quad I=\frac{CE-q}{RC}$$

Substituting for q, we get

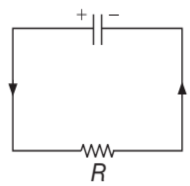
$$I=\frac{E}{R}\left(1-\frac{1}{1+\alpha}\left(1-e^{-\frac{t(1+\alpha)}{RC}}\right)\right)$$

PROBLEM 12

A capacitor initially given a charge Q_0 is connected across a resistor R at $t=0$. The separation d between the plates changes according to the relation

$$d=\frac{d_0}{(1+t)} \qquad (0\le t<1)$$

A small bulb is connected across the plates of the capacitor which lights when potential difference across the plates of the capacitor reaches V_0. Find the



(a) variation of charge with time
(b) time when the bulb will light?

SOLUTION

(a) Capacitance at $t=0$ is given by

$$C_0=\frac{\varepsilon_0A}{d_0}$$

and at time t is $C=\dfrac{\varepsilon_0A}{d}=\dfrac{\varepsilon_0A}{d_0}(1+t)=C_0(1+t)$

Using Kirchhoff's Law, we get

$$\frac{q}{C}-IR=0$$

$$\Rightarrow \quad \frac{q}{C_0(1+t)}+R\frac{dq}{dt}=0$$

$$\Rightarrow \quad \frac{dq}{q}=-\frac{1}{RC_0}\frac{dt}{(1+t)}$$

$$\Rightarrow \quad \log_e q\Big|_{Q_0}^{q}=-\frac{1}{RC_0}\log_e(1+t)\Big|_0^t$$

$$\Rightarrow \quad \log_e\left(\frac{q}{Q_0}\right)=-\frac{1}{RC_0}\log_e(1+t)$$

$$\Rightarrow \quad \log_e\left(\frac{q}{Q_0}\right) = \log_e(1+t)^{-\frac{1}{RC_0}}$$

$$\Rightarrow \quad q = Q_0(1+t)^{-\frac{1}{RC_0}}$$

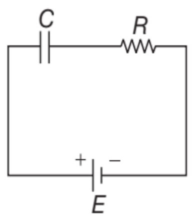
(b) $$V = \frac{q}{C} = \frac{Q_0(1+t)^{-\frac{1}{RC_0}}}{C_0(1+t)}$$

$$\Rightarrow \quad V = \frac{Q_0}{C_0}(1+t)^{-\frac{1}{RC_0}+1}$$

$$\Rightarrow \quad t = 1 - \left(\frac{V_0 C_0}{Q_0}\right)^{-\left(\frac{RC_0}{RC_0+1}\right)}$$

PROBLEM 13

In the following RC circuit, the capacitor is in the steady state. The initial separation of the capacitor plates is x_0. If at $t=0$, the separation between the plates starts changing so that a constant current flows through R, find the velocity of the moving plates as a function of time. The plate area is A.



SOLUTION

Let q be the instantaneous charge on the capacitor when a steady current I flows through the circuit. Applying KVL on the circuit, we have

$$E = \frac{q}{C} + IR$$

$$\Rightarrow \quad E = \frac{qx}{\varepsilon_0 A} + IR \qquad \ldots(1)$$

At the instant separation between the plates of the capacitor is x, its capacity $C = \frac{\varepsilon_0 A}{x}$

Differentiating equation (1) with respect to time, we get

$$0 = \frac{q}{\varepsilon_0 A}\left(\frac{dx}{dt}\right) + \frac{Ix}{\varepsilon_0 A} + 0 \qquad \left\{\because I = \frac{dq}{dt}\right\}$$

$$\Rightarrow \quad q = -\frac{Ix}{\left(\frac{dx}{dt}\right)} \quad \left(\text{where } \frac{dx}{dt} = \text{velocity}\right) \qquad \ldots(2)$$

From equation (2), substituting the value of q in equation (1), we have

$$E = -I\frac{x^2}{\varepsilon_0 A\left(\frac{dx}{dt}\right)} + IR$$

$$\Rightarrow \quad \frac{dx}{dt} = v = -\left(\frac{I}{\varepsilon_0 A}\right)\left(\frac{x^2}{E-IR}\right) \qquad \ldots(3)$$

$$\Rightarrow \quad -\frac{dx}{x^2} = \left(\frac{I}{\varepsilon_0 A}\right)\left(\frac{dt}{E-IR}\right) \qquad \ldots(4)$$

Integrating the above expression w.r.t. time, we get

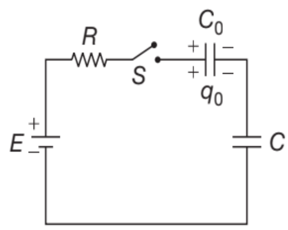
$$\frac{1}{x} - \frac{1}{x_0} = \left(\frac{I}{\varepsilon_0 A(E-IR)}\right)t \qquad \ldots(5)$$

From equations (3) and (5), we get

$$v = \frac{I}{\varepsilon_0 A(IR-E)} \frac{1}{\left[\left(\frac{\frac{I}{\varepsilon_0 A}}{E-IR}\right)t + \frac{1}{x_0}\right]^2}$$

PROBLEM 14

The switch S is closed at $t=0$. The capacitor C is uncharged but C_0 has a charge q_0 at $t=0$. Calculate the current $I(t)$ in the circuit.



SOLUTION

Let q_0 and q be the instantaneous charges on C_0 and C respectively. Applying KVL to the circuit, we have

$$\frac{q_0}{C_0} + \frac{q}{C} + IR = E \qquad \ldots(1)$$

Differentiating this equation, we get

$$\frac{1}{C_0}\frac{dq_0}{dt} + \frac{1}{C}\frac{dq}{dt} + R\frac{dI}{dt} = 0$$

$$\Rightarrow \quad I\left(\frac{1}{C_0}+\frac{1}{C}\right)=-R\frac{dI}{dt} \qquad \left\{\text{where } I=\frac{dq_0}{dt}=\frac{dq}{dt}\right\}$$

$$\Rightarrow \quad \frac{dI}{I}=-\frac{dt}{RC_{eq}} \qquad \left\{\text{where } C_{eq}=\frac{C_0C}{C+C_0}\right\}$$

Integrating this expression, we have

$$\int_{I_0}^{I(t)}\frac{dI}{I}=-\int_0^t\frac{dt}{RC_{eq}}$$

$$\Rightarrow \quad \log_e I\Big|_{I_0}^{I(t)}=-\frac{t}{RC_{eq}}$$

$$\Rightarrow \quad I(t)=I_0e^{-\frac{t}{RC_{eq}}} \qquad \ldots(2)$$

where I_0 is the initial current.

Further, $I_0R+\frac{q_0}{C_0}=E$

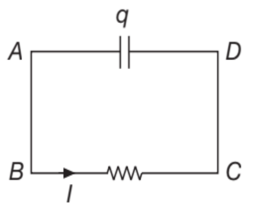
$$\Rightarrow \quad I_0=\frac{\left(E-\frac{q_0}{C_0}\right)}{R} \qquad \ldots(3)$$

Substituting I_0 from equation (3) into equation (2), we get

$$I(t)=\frac{1}{R}\left[E-\frac{q_0}{C_0}\right]e^{-\frac{t}{RC_{eq}}}$$

PROBLEM 15

Plates of a parallel plate capacitor initially charged to Q_0 are connected to a variable resistance R whose resistance is given by $R=R_0+\alpha t$. At time $t=0$, $R=R_0$. Find the current through the capacitor as function of time.



SOLUTION

The resistance varies according to the relation given by

$$R=R_0+\alpha t$$

Applying Kirchhoff's Law to the loop $ABCDA$, at time t, when charge on capacitor is q, we have

$$\frac{q}{C}-IR=0 \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{q}{C}-R\frac{dq}{dt}=0 \qquad \left\{\because I=\frac{dq}{dt}\right\}$$

$$\Rightarrow \quad \int_{Q_0}^{q}\frac{dq}{q}=\int_0^t\frac{dt}{RC}$$

$$\Rightarrow \quad \log_e\left(\frac{q}{Q_0}\right)=\int_0^t\frac{dt}{(R_0+\alpha t)C}$$

$$\Rightarrow \quad \log_e\left(\frac{q}{Q_0}\right)=\frac{1}{\alpha C}\log_e(R_0+\alpha t)\Big|_0^t$$

$$\Rightarrow \quad \log_e\left(\frac{q}{Q_0}\right)=\frac{1}{\alpha C}\log_e\left(\frac{R_0+\alpha t}{R_0}\right)$$

$$\Rightarrow \quad \frac{q}{Q_0}=\left(\frac{R_0+\alpha t}{R_0}\right)^{\frac{1}{\alpha C}}$$

$$\Rightarrow \quad q=Q_0\left(\frac{R_0+\alpha t}{R_0}\right)^{\frac{1}{\alpha C}}$$

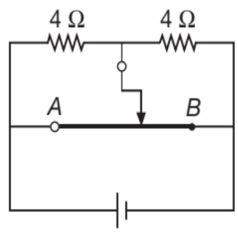
Since $I=\frac{q}{CR}$

$$\Rightarrow \quad I=\frac{Q_0}{RC}\left(\frac{R_0+\alpha t}{R_0}\right)^{\frac{1}{\alpha C}}$$

$$\Rightarrow \quad I=\frac{Q_0}{C(R_0+\alpha t)}\left(\frac{R_0+\alpha t}{R_0}\right)^{\frac{1}{\alpha C}}$$

PROBLEM 16

The wire AB of a meter bridge changes linearly from radius r to $2r$ from left end to right end. Where should the free end of the galvanometer be connected on AB so that the deflection in the galvanometer is zero?



SOLUTION

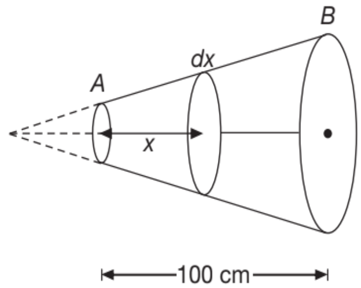
Let the galvanometer be connected at a point $x = x_1$ from end A where $x = 0$.

Let $R_1 =$ resistance of left part, i.e., AX_1 and

$R_2 =$ resistance of right part, i.e., X_1B

Length = 100 cm = 1 m

Consider an element of thickness dx at a distance x from end A and of radius r_x.



Thus, $r_x = \left(r + \frac{r}{1}x\right) = r(1+x)$

Resistance of this element will be, $dR_x = \frac{\rho\, dx}{\pi r_x^2}$

$$R_1 = \int_0^1 \frac{\rho\, dx}{\pi(1+x)^2 r^2} = \frac{\rho}{\pi r^2}\left[1 - \frac{1}{1+x_1}\right]$$

$$R_2 = \int_1 \frac{\rho\, dx}{(1+x)^2 r^2} = \frac{\rho}{\pi r^2}\left[\frac{1}{1+x_1} - \frac{1}{1+1}\right]$$

For null point of zero deflection,

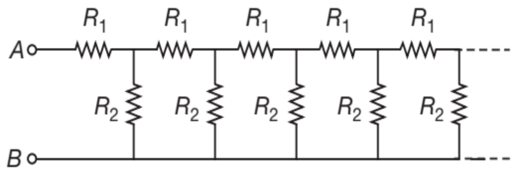
$$\frac{R_1}{R_2} = \frac{4}{4}$$

$$\Rightarrow \quad 1 - \frac{1}{1+x_1} = \frac{1}{1+x_1} - \frac{1}{1+1}$$

$$\Rightarrow \quad x_1 = \frac{1}{3}\text{ m} = 33.33\text{ cm}$$

PROBLEM 17

Consider an infinite ladder of network shown. A voltage is applied between points A and B. If the voltage is halved after each section, find the ratio $\frac{R_1}{R_2}$. Suggest a method to terminate it after a few sections without introducing much error in its attenuation.

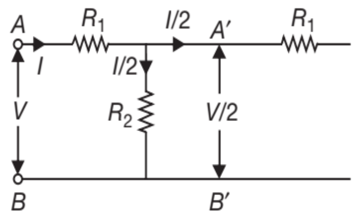


SOLUTION

Voltage across $AB = V$

Voltage across $A'B' = \frac{V}{2}$

Voltage across $R_2 = \frac{V}{2}$



Now from Kirchhoff's Law it is obvious that voltage across must be $\left(V - \frac{V}{2}\right) = \frac{V}{2}$

Now when the voltage is halved then the current is also halved. So, current in R_2 is half of that in R_1

$$\Rightarrow \quad IR_1 = \left(\frac{I}{2}\right)R_2$$

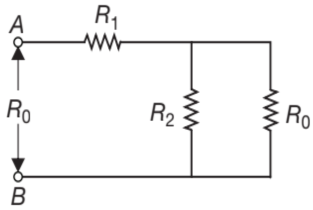
$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{1}{2}$$

Now the attenuation produced by circuit on termination by a resistance will not be affected if equivalent resistance R becomes independent of number of sections in the circuit. This is only possible if the terminating resistance R_0 is itself equal to equivalent resistance as shown. The equivalent resistance of R_0 and R_2 is

$$R' = \frac{R_0R_2}{R_0 + R_2}$$

R_1 is in series with it, so equivalent resistance between A and B is

$$R_{eq} = R_1 + R' = R_1 + \frac{R_0R_2}{R_0 + R_2}$$



According to proposition we have

$$R_{eq} = R_0$$

$$R_0 = R_1 + \frac{R_0 R_2}{R_0 + R_2}$$

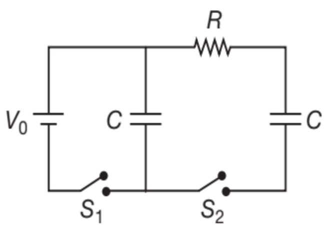
Solving for R_0, we get

$$R_0 = \frac{R_1}{2}\left(1 + \sqrt{1 + \frac{4R_2}{R_1}}\right)$$

Hence, the circuit may be terminated after a few sections if resistance R_0 is connected in parallel as shown in figure.

PROBLEM 18

In a circuit shown in figure the capacitance of each capacitor is equal to C and the resistance equal to R. One of the capacitance was connected to a voltage V_0 by closing switch S_1. Now at $t = 0$, the switch S_1 is opened and S_2 is closed. Calculate



(a) the current I in the circuit as a function of time t

(b) the amount of heat generated provided the dependence of current $I(t)$ on time t is known.

SOLUTION

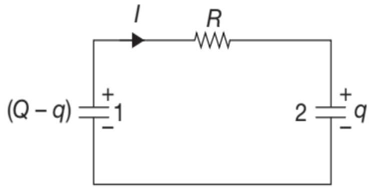
(a) Initial charge on capacitor '1' is $Q = CV_0$

Let charge q flow in the current in time t.

Then current $I = \frac{dq}{dt}$

After time t, charge on capacitor '1' $= (Q - q)$

Charge on capacitor 2 is q



According to Kirchhoff's Loop Law, we have

$$\frac{Q-q}{C} - IR - \frac{q}{C} = 0$$

$$\Rightarrow \quad IR = \frac{Q-2q}{C}$$

$$\Rightarrow \quad R\frac{dq}{dt} = \frac{Q-2q}{C}$$

$$\Rightarrow \quad \frac{dq}{Q-2q} = \frac{dt}{RC}$$

$$\Rightarrow \quad \int_0^q \frac{dq}{Q-2q} = \int_0^t \frac{dt}{RC}$$

$$\Rightarrow \quad \left(\frac{\log_e(Q-2q)}{-2}\right)\Bigg|_0^q = \frac{t}{RC}$$

$$\Rightarrow \quad \log_e(Q-2q) - \log_e Q = -\frac{2t}{RC}$$

$$\Rightarrow \quad \log_e\left(\frac{(Q-2q)}{Q}\right) = -\frac{2t}{RC}$$

$$\Rightarrow \quad 1 - \frac{2q}{Q} = e^{-2t/RC}$$

$$\Rightarrow \quad \frac{2q}{Q} = 1 - e^{-2t/RC}$$

$$\Rightarrow \quad q = \frac{Q}{2}\left[1 - e^{-2t/RC}\right]$$

Since, $Q = CV_0$

$$\Rightarrow \quad q = \frac{CV_0}{2}\left[1 - e^{-2t/RC}\right]$$

Current $I = \frac{dq}{dt} = \frac{CV_0}{2}\left[0 + \left(\frac{2}{RC}\right)e^{-2t/RC}\right]$

$$\Rightarrow \quad I = \frac{V_0}{R}e^{-2t/RC}$$

(b) Heat produced is given by

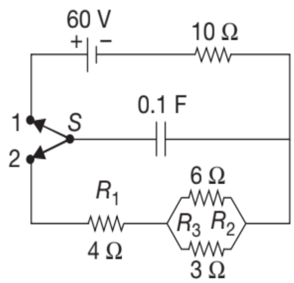
$$H=\int_0^{\infty} I^2 R dt=\int_0^{\infty}\left(\frac{V_0}{R} e^{-2t/RC}\right)^2 R dt$$

$$\Rightarrow \quad H=\frac{V_0^2}{R}\int_0^{\infty} e^{-4t/RC} dt=\frac{V_0^2}{R}\left(\frac{e^{-4t/RC}}{-\frac{4}{RC}}\right)\Bigg|_0^{\infty}$$

$$\Rightarrow \quad H=-\frac{CV_0^2}{4}\left(e^{-\infty}-e^{-0}\right)=\frac{1}{4}CV_0^2$$

PROBLEM 19

A two way switch S is used in the circuit as shown in figure. First the capacitor is charged by putting the switch in position 1. Now in switch is thrown to position 2. Calculate the heat generated in each resistor.



SOLUTION

Initially the switch S is in position 1. Therefore battery of 60 V is in the circuit. The capacitor begins to collect charge. When capacitor is fully charged, the steady state is reached. In steady state there is no current in $R = 10\ \Omega$ resistance.
Energy stored by capacitor,

$$U=\frac{1}{2}CV^2=\frac{1}{2}0.1\times(60)^2=180 \text{ J}$$

When switch S is thrown to position 2, the capacitor starts discharging through three resistances $4\ \Omega$ and $6\ \Omega$ and $3\ \Omega$ in parallel.

Let I_1 be the instantaneous current in $4\ \Omega$, I_2 current in $6\ \Omega$ and I_3 current in $3\ \Omega$, then

$$I_2+I_3=I_1 \text{ and } 6I_2=3I_3$$

$$\Rightarrow \quad I_3=2I_2$$

Also, $I_2+2I_2=I_1$

$$\Rightarrow \quad I_2=\frac{I_1}{3} \text{ and so } I_3=\frac{2I_1}{3}$$

The ratio of currents

$$I_1:I_2:I_3 \equiv I_1:\frac{I_1}{3}:\frac{2}{3}I_1=1:\frac{1}{3}:\frac{2}{3}=3:1:2$$

This ratio remains constant throughout.
Therefore ratio of heat energy dissipated in $R_1:R_2$ and R_3 is

$$H_1:H_2:H_3=(3)^2\times 4:(1)^2\times 6:(2)^2\times 3$$

$$\Rightarrow \quad H_1:H_2:H_3=6:1:2$$

If K is ratio constant, then

$$H_1=6K,\ H_2=K \text{ and } H_3=2K$$

Also, $H_1+H_2+H_3=180$ J

$$\Rightarrow \quad 6K+K+2K=180 \text{ J}$$

$$\Rightarrow \quad K=\frac{180}{9}=20$$

Heat generated in $R_1=4\ \Omega$ is

$$H_1=6K=6\times 20=120 \text{ J}$$

Heat generated in $R_2=6\ \Omega$ is

$$H_2=K=20 \text{ J}$$

Heat generated in $R_3=3\ \Omega$ is

$$H_3=2K=2\times 20=40 \text{ J}$$

During discharging of capacitor, no current flows in $R=10\ \Omega$, so heat generated in $10\ \Omega$ is zero.