CHAPTER 3: ELECTRIC CURRENT AND CIRCUITS

Test Your Concepts-I (Based on Current Definition)

1. (a) Using $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}=\frac{mv^2}{r}$, we get

$$v=\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{mr}}=2.19\times10^6\ \text{ms}^{-1}$$

(b) The time for the electron to revolve around the proton once is:

$$t=\frac{2\pi r}{v}=\frac{2\pi\left(5.29\times10^{-11}\ \text{m}\right)}{2.19\times10^6\ \text{ms}^{-1}}=1.52\times10^{-16}\ \text{s}.$$

The total charge flow in this time is 1.60×10^{-19} C, so the current is

$$I=\frac{1.60\times10^{-19}\ \text{C}}{1.52\times10^{-16}\ \text{s}}=1.05\times10^{-3}\ \text{A}=1.05\ \text{mA}$$

2. $q=4t^3+5t+6$

$$A=\left(2\ \text{cm}^2\right)\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2=2\times10^{-4}\ \text{m}^2$$

(a) $I(1\ \text{s})=\left.\frac{dq}{dt}\right|_{t=1\ \text{s}}=\left.\left(12t^2+5\right)\right|_{t=1\ \text{s}}=17\ \text{A}$

(b) $J=\frac{I}{A}=\frac{17\ \text{A}}{2\times10^{-4}\ \text{m}^2}=85\ \text{kAm}^{-2}$

3. $I=\frac{dq}{dt}$

$$q=\int dq=\int Idt=\int_0^{\frac{1}{240}\text{s}}(100\ \text{A})\sin\left(\frac{120\pi t}{s}\right)dt$$

$$q=\frac{-100\ \text{C}}{120\pi}\left[\cos\left(\frac{\pi}{2}\right)-\cos\ 0\right]=\frac{+100\ \text{C}}{120\pi}=0.265\ \text{C}$$

4. Let us write the condition of the zero potential of the sphere, and hence, of any point inside it (in particular, its centre), at any instant of time t. We shall find out three times intervals:

(1) $t<\frac{a}{v}$, (2) $\frac{a}{v}\le t<\frac{b}{v}$, (3) $t\ge\frac{b}{v}$

Denoting the charge of the sphere by $q(t)$, we obtain the following expression for an instant t from the first time interval:

$$\frac{q_1}{a}+\frac{q_2}{b}+\frac{q(t)}{vt}=0,$$

Hence, $q(t)=-v\left(\frac{q_1}{a}+\frac{q_2}{b}\right)t$,

$$I_1(t)=-v\left(\frac{q_1}{a}+\frac{q_2}{b}\right)$$

For an instant t from the second time interval, we find that the fields inside and outside the sphere are independent, and hence

$$\frac{q(t)+q_1}{vt}=-\frac{q_2}{b},$$

$$I_2(t)=-v\frac{q_2}{b}$$

Finally, as soon as the sphere absorbs the two point charges q_1 and q_2, the current will stop flowing through the earthing conductor, and we can write $I_3(t)=0$.

Thus,

$$I(t)=\begin{cases}-v\left(\frac{q_1}{a}+\frac{q_2}{b}\right), & t<\frac{a}{v},\\ -v\frac{q_2}{b}, & \frac{a}{v}\le t<\frac{b}{v},\\ 0, & t\ge\frac{b}{v}.\end{cases}$$

5. (a) $I=\frac{dq}{dt}$

$$\Rightarrow\ \int_0^q dq=\int_0^5 Idt$$

$$\Rightarrow\ q=\int_0^5(3+2t)dt$$

$$\Rightarrow\ q=\left[3t+t^2\right]_0^5=[15+25]$$

$$\Rightarrow\ q=40\ \text{C}$$

(b) $I=\frac{q}{t}=\frac{40}{4}$

$$\Rightarrow\ I=10\ \text{A}$$

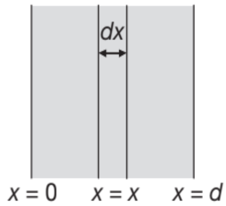
6. Let n be the number of electrons at distance x. Now,

$$\frac{dn}{dx}=\text{increase in } n \text{ per unit length}$$

$$\Rightarrow\ \frac{dn}{dx}=\alpha n$$

$$\Rightarrow \frac{dn}{n} = \alpha dx$$

$$\Rightarrow \int_{n_0}^{n} \frac{dn}{n} = \alpha \int_0^d dx$$



$$\Rightarrow \log_e\left(\frac{n}{n_0}\right) = \alpha d$$

$$\Rightarrow \frac{n}{n_0} = e^{\alpha d}$$

$$\Rightarrow n = n_0 e^{\alpha d}$$

where, n is number of electrons reaching at other plate per unit time.

$$\left(\begin{array}{c}\text{The electronic current}\\ \text{at opposite plate}\end{array}\right) = \left(\begin{array}{c}\text{charge reaching}\\ \text{per unit time}\end{array}\right)$$

$$\Rightarrow I = en$$

$$\Rightarrow I = en_0 e^{\alpha d}$$

7. Convectional current is the current which is developed due to the transportation of charge

$$I = \frac{q_{\text{transported}}}{t} = 2\lambda v$$

Test Your Concepts-II (Based on Resistance, Resistivity and Ohm's Law)

1. $\Delta V = -E\ell$

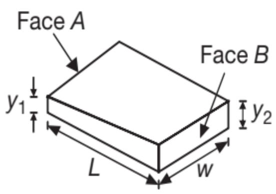
$$\Rightarrow dV = -Edx$$

$$\Delta V = -IR = -E\ell$$

$$\Rightarrow I = \frac{dq}{dt} = \frac{E\ell}{R} = \frac{A}{\rho\ell}E\ell = \frac{A}{\rho}E = -\sigma A\frac{dV}{dx} = \sigma A\left|\frac{dV}{dx}\right|$$

Current flows in the direction of decreasing voltage. Energy flows as heat in the direction of decreasing temperature.

2. $R = \int \frac{\rho dx}{A} = \int \frac{\rho dx}{wy}$ where $y = y_1 + \frac{y_2 - y_1}{L}x$



$$\Rightarrow R = \frac{\rho}{w}\int_0^L \frac{dx}{y_1 + \left[\frac{(y_2 - y_1)}{L}\right]x}$$

$$\Rightarrow R = \frac{\rho L}{w(y_2 - y_1)}\log_e\left(y_1 + \frac{y_2 - y_1}{L}x\right)_0^L$$

$$\Rightarrow R = \frac{\rho L}{w(y_2 - y_1)}\log_e\left(\frac{y_2}{y_1}\right)$$

3. Resistance of original wire,

$$R_1 = \frac{\rho\ell_1}{A_1} = 8\ \Omega c$$

Resistance of modified wire,

$$R_2 = \frac{\rho\ell_2}{A_2} = \frac{\rho(2\ell_1)}{4A_1} = \frac{1}{2}\frac{\rho\ell_1}{A} = \frac{1}{2}(8\ \Omega) = 4\ \Omega$$

4. (a) The resistance of a wire is given by

$$R = \rho\frac{\ell}{A}$$

As volume is constant, $V = A\ell =$ constant

$$R = \rho\left(\frac{\ell^2}{V}\right)$$

The fractional charge, $\frac{dR}{R} = 2\frac{d\ell}{\ell} = 2(0.1\%) = 0.2\%$

The percentage change in resistance is 0.2%

(b) Similarly $R = \rho\frac{V}{A^2} \Rightarrow \frac{dR}{R} = -2\frac{dA}{A}$

Since cross-sectional area $A = \pi r^2$

$$\Rightarrow \frac{dA}{A} = 2\frac{dr}{r} \Rightarrow \frac{dR}{R} = -2\frac{dA}{A} = -4\frac{dr}{r}$$

If radius is increased by 1%, the resistance is decreased by 4%. If area is increased by 1%, the resistance is decreased by 2%. The significance of negative sign is that change in resistance is opposite to that of radius and area.

Here please note that calculus can be employed for small percentage change only.

5. In electrostatic condition, electric field inside a conductor is zero. But when a current flows through the conductor electric field is non-zero.

6. (a) $R = \int_a^b \frac{\rho dr}{4\pi r^2}$ $\left\{\text{Resistance} = \frac{\rho\ell}{A}\right\}$

$$\Rightarrow R = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$

(b) $I = \frac{V_{ab}}{R}$ and $j = \frac{I}{A} = \frac{I}{4\pi r^2}$

$$\Rightarrow j = \frac{V_{ab}}{4\pi Rr^2} = \frac{V_{ab}}{\rho\left(\frac{1}{a} - \frac{1}{b}\right)r^2}$$

$$\Rightarrow j = \frac{abV_{ab}}{\rho(b-a)r^2}$$

Test Your Concepts-III (Based on Variation of Resistance with Temperature)

1. Resistance at temperature T is given by

$$R_T = R_0(1+\alpha_0 T) = R(t)$$

From the given data,

$$R_0 = R_1[1+\alpha_1(0-20^\circ)]$$

$$\Rightarrow \quad R_0 = 2(1-20\times 0.00575)$$

$$\Rightarrow \quad R_0 = 1.77\ \Omega$$

Since, $T = 20+10t$

$$\Rightarrow \quad R(t) = 1.77[1+0.0065(20+10t)]$$

$$\Rightarrow \quad R(t) = (2+0.115t)\ \Omega$$

2. Since $R_T = R_0(1+\alpha_0 T)$ ...(1)

and $R_0 = R_T(1-\alpha_T T)$ ...(2)

From equation (2), we have

$$\alpha_T = \frac{R_T - R_0}{TR_T} \quad \text{...(3)}$$

Substituting for R_T in equation (3), we have

$$\alpha_T = \frac{R_0(1+\alpha_0 T)-R_0}{TR_0(1+\alpha_0 T)} = \frac{\alpha_0}{1+\alpha_0 T}$$

3. Since, we have

$$\frac{1}{\alpha_T} = \frac{1+\alpha_0 T}{\alpha_0} = \frac{1}{\alpha_0}+T$$

So we have $\dfrac{1}{\alpha_1} = \dfrac{1}{\alpha_0}+T_1$

and $\dfrac{1}{\alpha_2} = \dfrac{1}{\alpha_0}+T_2$

Subtracting, we get

$$\frac{1}{\alpha_1}-\frac{1}{\alpha_2} = T_1 - T_2$$

$$\Rightarrow \quad \alpha_2 = \frac{1}{\left(\dfrac{1}{\alpha_1}\right)+(T_2-T_1)} = \frac{\alpha_1}{1+\alpha_1(T_2-T_1)}$$

4. Let R_0 be the resistance at $0\ ^\circ\text{C}$. Then at the temperatures T_1 and T_2, we have

$$R_1 = R_0(1+\alpha_0 T_1) \quad \text{...(1)}$$

and $R_2 = R_0(1+\alpha_0 T_2)$ ...(2)

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{1+\alpha_0 T_1}{1+\alpha_0 T_2} \quad \text{...(3)}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = (1+\alpha_0 T_1)(1+\alpha_0 T_2)^{-1}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = (1+\alpha_0 T_1)\left[1-\alpha_0 T_2+(\alpha_0 T_2)^2-\ldots\right]$$

$$\Rightarrow \quad \frac{R_1}{R_2} \simeq 1+\alpha_0(T_1-T_2)$$

{We have neglected the higher powers of $\alpha_0 T_2$}

5. From figure, the slope of the straight line is

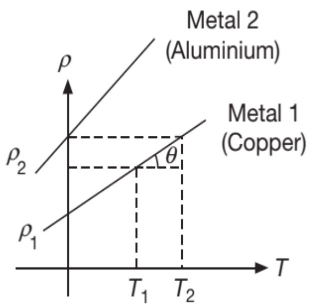
$$\tan\theta = m = \frac{\rho_2-\rho_1}{T_2-T_1}$$

which on rearranging yields

$$\rho_2 = \rho_1 + m(T_2-T_1)$$

$$\Rightarrow \quad \rho_2 = \rho_1\left[1+\frac{m}{\rho_1}(T_2-T_1)\right]$$

where m is the slope from the $\rho-T$ graph.



6. If coefficient of linear expansion is α, then the percentage change in length is

$$\left(\frac{d\ell}{\ell}\right)(100) = (\alpha dT)(100)$$

Coefficient of area ℓ expansion is 2α, so percentage change in area is

$$\left(\frac{dA}{A}\right)(100) = (2\alpha dT)(100)$$

percentage change in resistivity is

$$\left(\frac{d\rho}{\rho}\right)(100) = (\alpha_R dT)(100)$$

where α_R denotes thermal coefficient of resistance.
All the variables ℓ, A and ρ are functions of T. Consequently, R is also a function of T. We find, therefore,

$$\frac{dR}{dT} = \frac{d}{dT}\left(\frac{\rho\ell}{A}\right)$$

$$\Rightarrow \quad \frac{dR}{dt} = \frac{\ell}{A}\frac{d\rho}{dT}+\frac{\rho}{A}\frac{d\ell}{dT}+\rho\ell\frac{d}{dT}\left(\frac{\ell}{A}\right)$$

$$\Rightarrow \quad \frac{dR}{dt} = \alpha_R\rho\frac{\ell}{A}+\alpha\ell\frac{\rho}{A}-\frac{1}{A^2}2\,\alpha A\,\rho\ell$$

$$\Rightarrow \quad \frac{dR}{dt} = (\alpha_R+\alpha-2\alpha)\frac{\rho\ell}{A}$$

$$\Rightarrow \quad \frac{dR}{dt} = (\alpha_R-\alpha)\frac{\rho\ell}{A}$$

where we have written $\frac{d\rho}{dT}=\alpha_R\rho$ from equation $d\rho=\alpha_R\rho dT$. Similarly $\frac{d\ell}{dT}=\alpha\ell$ and $\frac{dA}{dT}=2\alpha A$

Thus change in resistance due to temperature change is given by

$$dR=(\alpha_R-\alpha)RdT$$

We find that since α is quite small as compared to α_R, we have approximately

$$dR=\alpha_R RdT$$

and, therefore, percentage change in R is the same as that in resistivity.

So, percentage change in Length $=0.0017\%$

percentage change in Area $=0.0034\%$

percentage change in Resistance $=0.39\%$

7. $R_{50\,°C}=(R_1)_{50\,°C}+(R_2)_{50\,°C}$

$$R_{50\,°C}=(R_1)_{20\,°C}(1+\alpha_1\Delta\theta)+(R_2)_{20\,°C}(1+\alpha_2\Delta\theta)$$

$$R_{50\,°C}=600(1+0.001\times 30)+300(1+0.004\times 30)$$

$$R_{50\,°C}=954\ \Omega$$

Further, $R_{50\,°C}=R_{20\,°C}(1+\alpha\Delta\theta)$

$$\Rightarrow\quad \alpha=\frac{\left(\frac{R_{50\,°C}}{R_{20\,°C}}-1\right)}{\Delta\theta}=\frac{\left(\frac{954}{900}-1\right)}{30}$$

$$\Rightarrow\quad \alpha=0.002\ (°C)^{-1}$$

Test Your Concepts-IV (Based on Series and Parallel Combination of Resistances)

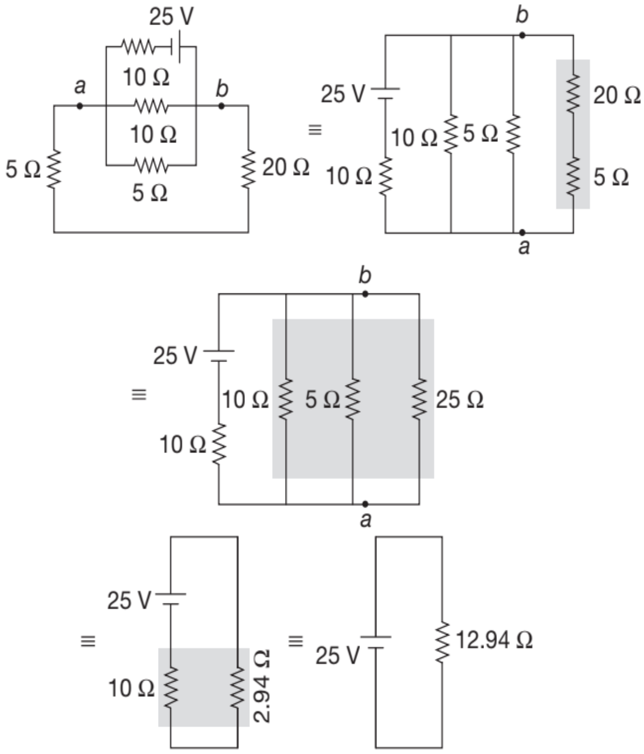
1. $120\text{ V}=IR_{eq}=I\left(\frac{\rho\ell}{A_1}+\frac{\rho\ell}{A_2}+\frac{\rho\ell}{A_3}+\frac{\rho\ell}{A_4}\right)$

$$\Rightarrow\quad I\rho\ell=\frac{(120\text{ V})}{\left(\frac{1}{A_1}+\frac{1}{A_2}+\frac{1}{A_3}+\frac{1}{A_4}\right)}$$

$$\Rightarrow\quad \Delta V_2=\frac{I\rho\ell}{A_2}=\frac{(120\text{ V})}{A_2\left(\frac{1}{A_1}+\frac{1}{A_2}+\frac{1}{A_3}+\frac{1}{A_4}\right)}=29.5\text{ V}$$

2. The equivalent circuit is shown for the convenience of evaluating.

$$R_{eq}=\frac{1}{\left(\frac{1}{10\ \Omega}+\frac{1}{5\ \Omega}+\frac{1}{25\ \Omega}\right)}=2.94\ \Omega$$



Applying $I=\frac{\Delta V}{R}$ and $\Delta V=IR$ alternately to every resistor, real and equivalent. The $12.94\ \Omega$ resistor is connected across 25 V, so the current through the battery in every diagram is

$$I=\frac{\Delta V}{R}=\frac{25\text{ V}}{12.94\ \Omega}=1.93\text{ A}$$

This 1.93 A goes through the $2.94\ \Omega$ equivalent resistor to give a potential difference of

$$\Delta V=IR=(1.93\text{ A})(2.94\ \Omega)=5.68\text{ V}$$

We see that this potential difference is the same across ab, the $10\ \Omega$ resistor, and the $5\ \Omega$ resistor.

(a) Therefore, $\Delta V_{ab}=5.68\text{ V}$

(b) Since the current through the $20\ \Omega$ resistor is also the current through the $25\ \Omega$ line ab,

$$I=\frac{\Delta V_{ab}}{R_{ab}}=\frac{5.68\text{ V}}{25\ \Omega}=0.227\ A=227\text{ mA}$$

3. When S is open, R_1, R_2 and R_3 are in series with the battery. So,

$$R_1+R_2+R_3=\frac{6\ V}{10^{-3}\ \text{A}}=6\text{ k}\Omega \qquad \ldots(1)$$

When S is closed in position 1, the parallel combination of the two R_2's is in series with R_1, R_3, and the battery. So,

$$R_1 + \frac{1}{2}R_2 + R_3 = \frac{6\text{ V}}{1.2\times10^{-3}\text{ A}} = 5\text{ k}\Omega \quad \ldots(2)$$

When S is closed in position 2, R_1 and R_2 are in series with the battery. R_3 is shorted. Hence,

$$R_1 + R_2 = \frac{6\text{ V}}{2\times10^{-3} A} = 3\text{ k}\Omega \quad \ldots(3)$$

From (1) and (3), we get $R_3 = 3\text{ k}\Omega$

Subtract (2) from (1), we get $R_2 = 2\text{ k}\Omega$ and from (3), we get $R_1 = 1\text{ k}\Omega$

$\Rightarrow \quad R_1 = 1\text{ k}\Omega, R_2 = 2\text{ k}\Omega, R_3 = 3\text{ k}\Omega$

4. Denoting the two resistors as x and y, we get

$$x + y = 690 \text{ and } \frac{1}{150} = \frac{1}{x} + \frac{1}{y}$$

$$\Rightarrow \quad \frac{1}{150} = \frac{1}{x} + \frac{1}{690-x} = \frac{(690-x)+x}{x(690-x)}$$

$$\Rightarrow \quad x^2 - 690x + 103\ 500 = 0$$

$$\Rightarrow \quad x = \frac{690 \pm \sqrt{(690)^2 - 414000}}{2}$$

$$\Rightarrow \quad x = 470\ \Omega \text{ and } y = 220\ \Omega$$

5. $I = \dfrac{200}{5+10+25} = 5\text{ A}$

(i) $0 - (10+5)\times 5 = V_1$

$\Rightarrow \quad V_1 = -75\text{ V}$

(ii) $0 - 10\times 5 = V_2$

$\Rightarrow \quad V_2 = -50\text{ V}$

(iii) $0 + 25\times 5 = V_3$

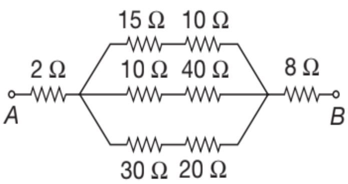
$\Rightarrow \quad V_3 = 125\text{ V}$

(iv) $V_{32} = V_3 - V_2 = 175\text{ V}$

(v) $V_{12} = V_1 - V_2 = -25\text{ V}$ and

(vi) $V_{13} = V_1 - V_3 = -200\text{ V}$

6. The simple circuit is as shown in figure which gives $R_{AB} = 22.5\ \Omega$



7. Two resistances are in parallel. Hence, net resistance across the battery,

$$R = \frac{12\times 8}{12+8} = 4.8\ \Omega$$

$$\Rightarrow \quad I = \frac{V}{R} = \frac{24}{4.8}\text{ A} = 5\text{ A}$$

8. All the four resistances are in parallel. Hence the equivalent resistance is,

$$\frac{1}{R} = \frac{1}{8} + \frac{1}{4} + \frac{1}{6} + \frac{1}{12} = \frac{15}{24}\ \Omega$$

$$\Rightarrow \quad R = \frac{8}{5}\ \Omega$$

$$\Rightarrow \quad I = \frac{V}{R} = \frac{24}{8/5} = 15\text{ A}$$

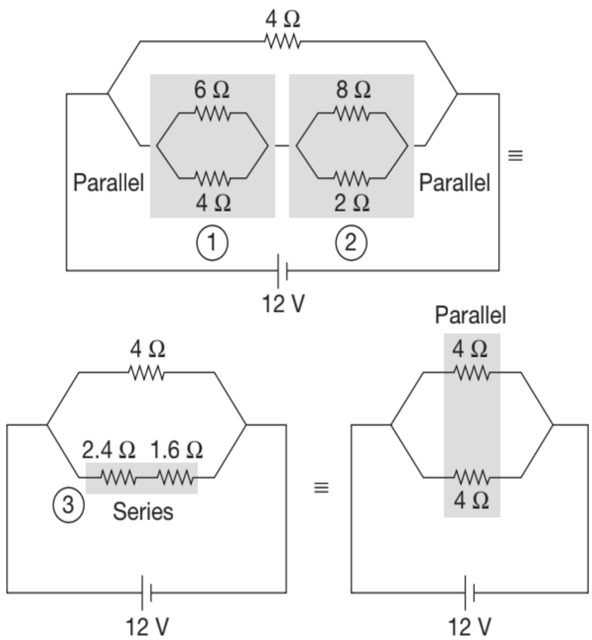
9. All the three resistances are in parallel. Hence, the equivalent resistance will be given by,

$$\frac{1}{R} = \frac{1}{8} + \frac{1}{6} + \frac{1}{12} = \frac{9}{24}$$

$$\Rightarrow \quad R = \frac{8}{3}\ \Omega$$

$$\Rightarrow \quad \text{I} = \frac{V}{R} = \frac{24}{8/3} = 9\text{ A}$$

10. The equivalent circuit is as shown in figure.



For ①: $\dfrac{6\times 4}{6+4} = 2.4\ \Omega$

For ②: $\dfrac{8\times 2}{8+2} = 1.6\ \Omega$

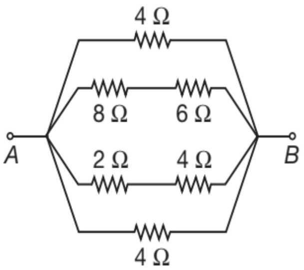
$$\Rightarrow \quad R = \frac{4(2.4+1.6)}{4+2.4+1.6} = 2\ \Omega$$

$$\Rightarrow \quad I = \frac{V}{R} = \frac{12}{2} = 6\text{ A}$$

11. (a) The simplified circuit is as shown in figure,

$$\frac{1}{R_{AB}} = \frac{1}{4} + \frac{1}{8+6} + \frac{1}{2+4} + \frac{1}{4}$$

$$\Rightarrow \quad R_{AB} = \frac{42}{31}\ \Omega$$

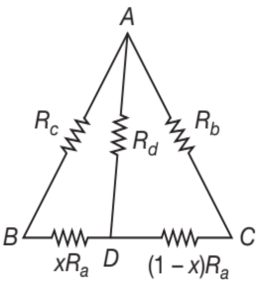


(b) The $6\ \Omega$ and $4\ \Omega$ resistances are in parallel. Similarly $8\ \Omega$ and $2\ \Omega$ resistances are also in parallel. These two are then in series. Hence,

$$R_{AB} = \left(\frac{6\times 4}{6+4}\right) + \left(\frac{8\times 2}{8+2}\right) = 4\ \Omega$$

12. First find the value of R_{AD} as a function of x and then for obtaining its maximum value we perform $\frac{dR_{AD}}{dx} = 0$, we get

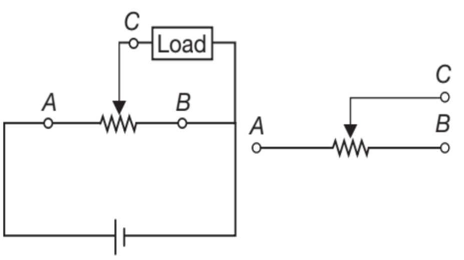
$$(R_{AD})_{MAX} = \frac{R_d(R_d + 2R_b)(R_a + 2R_c)}{4R_d(R_a + R_b + R_c) + (R_a + 2R_b)(R_a + 2R_c)}$$



13. $R_{CD} = \frac{R_2 r}{R_2 + r} = \frac{60\times 120}{60+120} = 40\ \Omega$

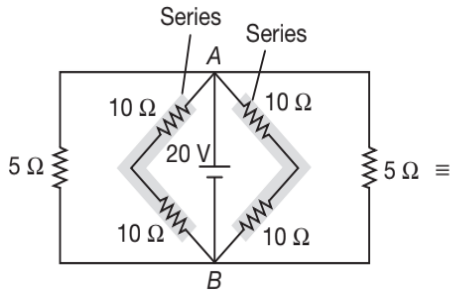
$$\Rightarrow \quad V_{CD} = \left(\frac{R_{CD}}{R_{CD} + R_1}\right)V_{AB} = \left(\frac{40}{40+60}\right)120 = 48\ \text{V}$$

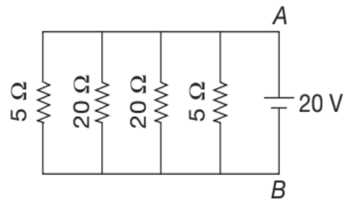
14. The rheostat is as shown in figure. Battery should be connected between A and B and the load between C and B.



15. Note that both the $5\ \Omega$ resistors are connected to same points A and B, across which battery is connected. Therefore each of $5\ \Omega$ resistors is in parallel arrangement with battery, therefore the current through it,

$$I = \left(\frac{20}{5}\right)\text{A} = 4\ \text{A}$$





The equivalent resistance between A and B is

$$\frac{1}{R_{eq}} = \frac{1}{20} + \frac{1}{20} + \frac{1}{5} + \frac{1}{5} = \frac{10}{20}$$

$$R_{eq} = 2\ \Omega$$

Thus the current supplied by battery $= \left(\frac{20}{2}\right)\text{A} = 10\ \text{A}$

16. It is zero, because the current will follow the path of least resistance. In fact a path of zero resistance is not available. Hence $I = 0$.

Test Your Concepts-V (Based on Series and Parallel Combination of Resistances)

1. (a) Let us split the medium into infinitesimal differential cylindrical shell elements, of width dr, in series. The current flow is cylindrically symmetric $(L \gg b)$. The area through which the current flows across a shell of radius r is $A(r) = 2\pi rL$. The length the current flows, passing through a shell of radius r is dr. So, the resistance of the shell of radius r is

$$dR = \frac{1}{\sigma}\frac{dr}{2\pi rL} \qquad \ldots(1)$$

Since the shells are connected in a series, we get

$$R_{ab} = \int_a^b dR = \frac{\log_e\left(\frac{b}{a}\right)}{2\pi\sigma L} = \frac{1}{2\pi\sigma L}\log_e\left(\frac{b}{a}\right) \qquad \ldots(2)$$

(b) Using $\sigma(r)=\frac{\sigma_0}{r}$, we have from equation (1),

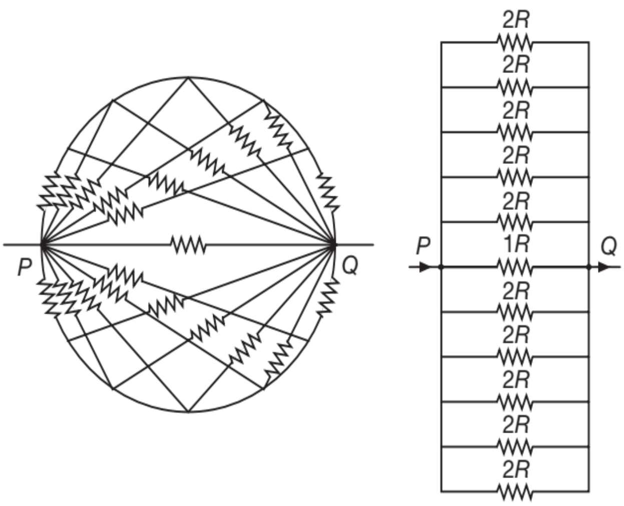
$$dR=\frac{dr}{2\pi rL\sigma(r)}=\frac{dr}{2\pi rL\left(\frac{\sigma_0}{r}\right)}=\frac{dr}{2\pi L\sigma_0} \quad \ldots(3)$$

$$\Rightarrow \quad R_{ab}=\int_a^b dR=\frac{b-a}{2\pi L\sigma_0} \quad \ldots(4)$$

2. (a) Since each of the terminals is connected by an insulated wire to each of the remaining 11 terminals by a resistor R, the symmetry shows all the twelve terminals to be symmetrically equivalent before any voltage is applied.
However, asymmetry is introduced just at the point where current enters and also at the point where the current leaves the circuit. All other ten points are symmetry points, all at the same potential. Hence the given network reduces to the following network
Thus if X be the equivalent resistance between P and Q

$$\frac{1}{X}=\frac{1}{2R}+\frac{1}{2R}+\ldots \text{ to 10 terms }+\frac{1}{R}$$

$$\frac{1}{X}=\frac{10}{2R}+\frac{1}{R}=\frac{6}{R}$$



Hence $X=\frac{R}{6}$

The equivalent resistance $=\frac{R}{6}\ \Omega=1\ \Omega$

(b) Proceeding in the same manner as in (a) we find that the equivalent resistor is given by X where

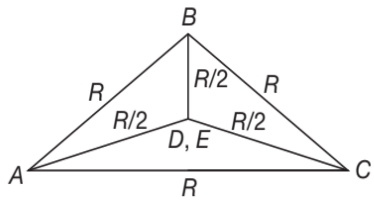
$$\frac{1}{X}=\frac{1}{2R}+\frac{1}{2R}+\ldots \text{ to } (n-2) \text{ terms }+\frac{1}{R}$$

$$\frac{1}{X}=\frac{n-2}{2R}+\frac{1}{R}=\frac{n}{2R}$$

Hence equivalent resistance $=\frac{2R}{n}$

The above symmetry simplification can be done only if n is even.

3. (a) Points D and E are symmetrically located with respect to points A and C. The circuit can be redrawn as shown in figure.

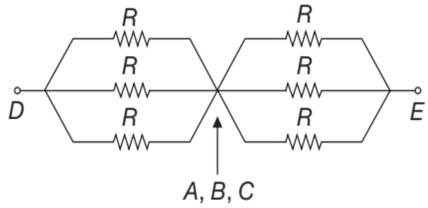


This is a combination of a balanced Wheatstone bridge which is in parallel with a resistance R. So, the resistance between B and D (or E) can be removed to get

$$\frac{1}{R_{AC}}=\frac{1}{R}+\frac{1}{\frac{R}{2}+\frac{R}{2}}+\frac{1}{R}$$

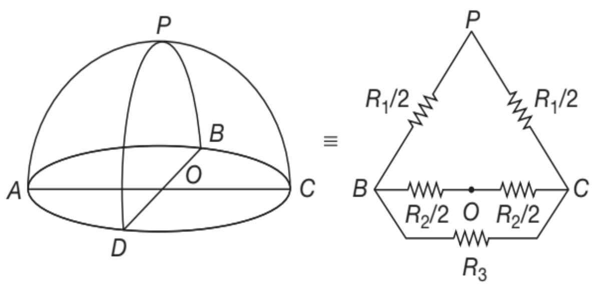
$$\Rightarrow \quad R_{AC}=\frac{2R}{5}$$

(b) With respect to D and E the points A, B and C all are symmetrically located. Hence, the simplified equivalent circuit is shown here.



$$\Rightarrow \quad R_{DE}=\frac{R}{3}+\frac{R}{3}=\frac{2R}{3}$$

4. Points $(A \text{ and } C)$ and $(D \text{ and } B)$ are symmetrically located with respect to points O and P. Hence, the circuit can be drawn as shown in figure.



This is a balanced Wheatstone bridge between P and O

Hence, R_3 can be removed. And,

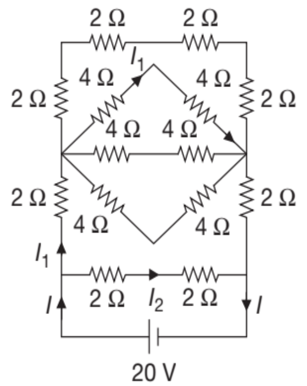
$$R_{PO} = \frac{R_1 + R_2}{4}$$

where $R_1 = R_{PB} = R_{PD} = \dfrac{(\pi a)\lambda}{2}$

and $R_2 = R_{OB} = (a)\lambda$

$$\Rightarrow \quad R_{PO} = \frac{(2+\pi)a\lambda}{8}$$

5. The simplified circuit is as shown in figure



Total resistance of circuit

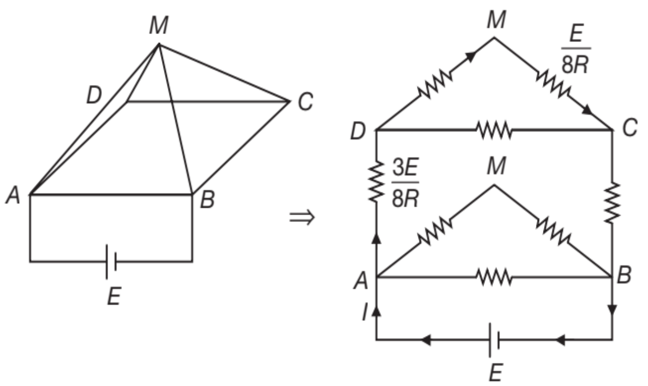
$$R = 2.4\ \Omega$$

$$\Rightarrow \quad I = \frac{20}{2.4} = 8.33\ \text{A}$$

So, $I_1 = \left(\dfrac{4}{10}\right)(I) = 3.33$ A

The desired current is $I' = \dfrac{I_1}{4} = 0.833$ A

6. (a) $R_{\text{net}} = \dfrac{8R}{15}$

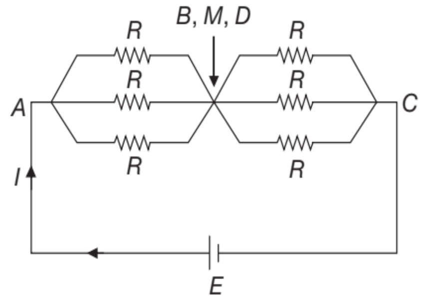


Connection removal method

$$\Rightarrow \quad I = \frac{E}{8R/15} = \frac{15E}{8R}$$

Hence current through MC is $I = \dfrac{E}{8R}$

(b) Points B, D and M are symmetrical with respect to points A and C both. Hence, the simple equivalent circuit can be drawn as shown

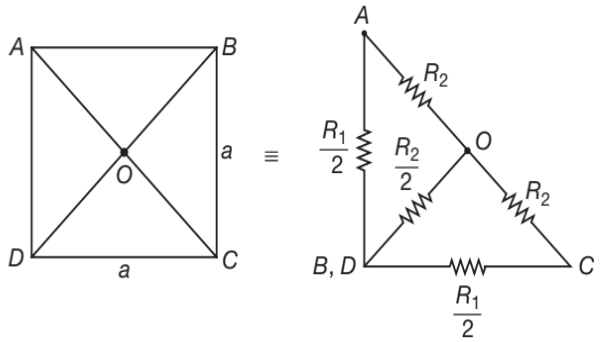


$$R_{\text{net}} = \frac{2R}{3}$$

$$\Rightarrow \quad I = \frac{E}{R_{\text{net}}} = \frac{3E}{2R}$$

Hence, current through MA (or AM) $= \dfrac{I}{3} = \dfrac{E}{2R}$

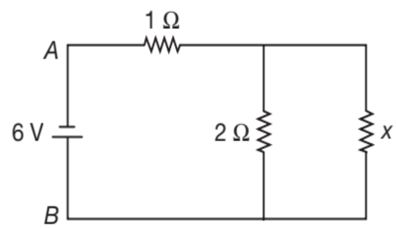
7. Since points B and D are symmetrically located with respect to points A and O. So they are at same potential. The simple circuit can be drawn as shown.



Here, $R_1 = \rho a$ and $R_2 = \dfrac{\rho a}{\sqrt{2}}$

$$\Rightarrow \quad R_{AO} = \left(\frac{\sqrt{2}}{2\sqrt{2}+1}\right)\lambda a$$

8. (a) Let $R_{AB} = x$. Then, we can break one chain and connect a resistance of magnitude x that replaces the remaining circuit.



Thus, the equivalent circuit remains as shown in the figure.

Now, 2 Ω and x are in parallel. So, their combined is $\frac{2x}{2+x}$

$$\Rightarrow \quad R_{AB} = 1 + \frac{2x}{2+x}$$

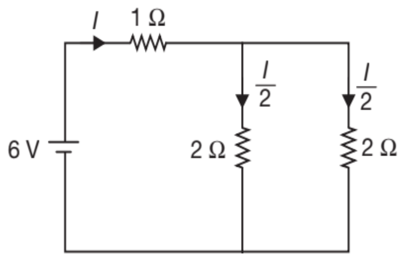
But R_{AB} is assumed to x. Therefore,

$$x = 1 + \frac{2x}{2+x}$$

Solving this equation, we get

$x = 2\Omega$ Hence proved.

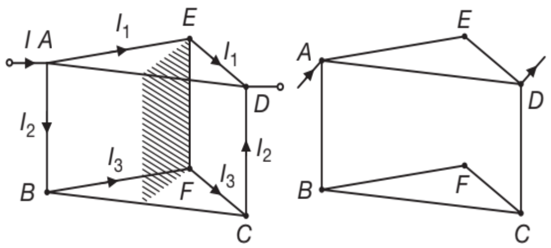
(b) Net resistance of circuit $R = 1 + \frac{2\times 2}{2+2} = 2\ \Omega$



$\therefore$ Current through battery $I = \frac{6}{2} = 3$ A

This current is equally distributed in $2\,\Omega$ and $2\,\Omega$ resistances. Therefore, the desired current is $\frac{I}{2}$ or 1.5 A.

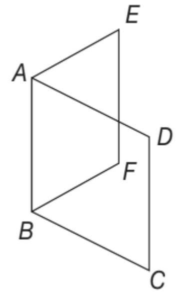
9. (a) Due to symmetry about the shaded plane, current distribution on either side of the plane will be identical and points E and F will be at same potential and no current will flow through it,



$$\Rightarrow \quad R_{AD} = \frac{\frac{2}{3}r \times \frac{8}{3}r}{\frac{2}{3}r + \frac{8}{3}r} = \frac{8}{15}r$$

(b) Redrawing the given arrangement for resistance across AB. Potentials $V_D = V_E$

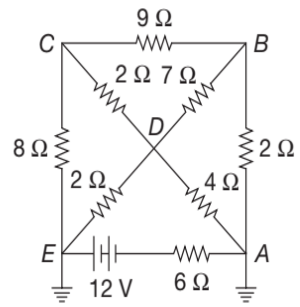
$V_C = V_F$



Therefore no current flows through DE and CF.

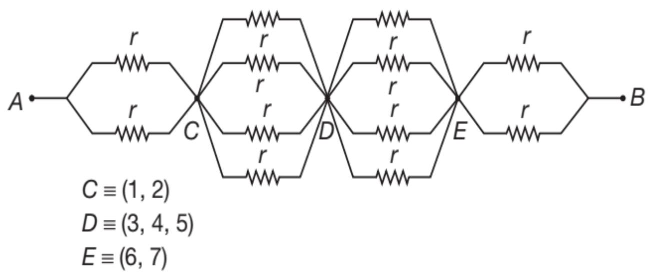
$$\Rightarrow \quad R_{AB} = \frac{\frac{3}{2}r \times r}{\frac{3}{2}r + r} = \frac{3}{5}r$$

10. Both the ends of the bottom branch are grounded, so the net potential difference between E and A is zero. If we traverse from E to A there is a potential drop of 12 V across battery, so there must be a potential gain of 12 V in the resistor. Therefore



the current in 6 Ω resistor is $I = \left(\frac{12}{6}\right)\text{A} = 2$ A and to the left. The current in the 9 Ω resistor is zero because there is no potential difference across E and A. The entire current of the battery goes from E into the zero resistance path back to A via ground. Please remember that when ground connections are shown, it is assumed that all such points are wired to a common line even if not shown.

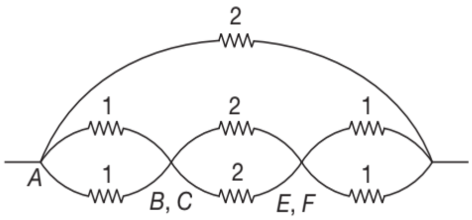
11. In figure points $C(1, 2)$, $D(3, 4, 5)$ and $E(6, 7)$ are at same potential. Equivalent circuit can be redrawn as in figure.



The equivalent resistance of this series combination is

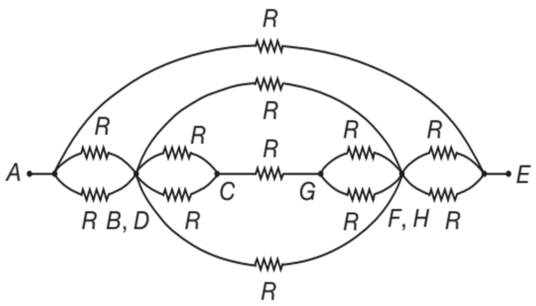
$$R_{eq} = \frac{r}{2} + \frac{r}{4} + \frac{r}{4} + \frac{r}{2} = \frac{3r}{2}$$

12. Points B and C, E and F are at the same potential, so the circuit can be redrawn as in figure.



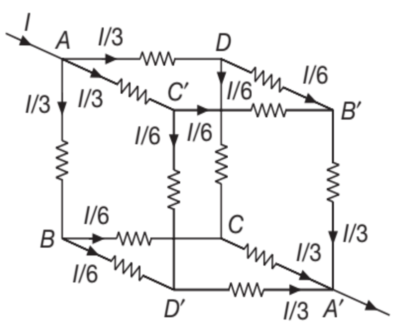
Thus the equivalent resistance is $1\ \Omega$.

13. Points B and D have same potential, similarly F and H have same potential. The equivalent circuit is shown in figure.



The equivalent resistance of network is $\frac{7R}{2}$.

14. (a) The potential difference between any two points is same no matter what path we take to arrive at the second beginning from first.
The circuit is symmetrical as shown in figure, the entry point and exit point are identical. Therefore at A, a current $\frac{I}{3}$ flows in each branch, similarly at exit point A' each branch must have current $\frac{I}{3}$.
At point B the current divides equally to $\frac{I}{6}$ and $\frac{I}{6}$ in branches BC and BD'.
Similarly current divides equally to $\frac{I}{6}$ and $\frac{I}{6}$ in branches C' and D' respectively.



Let equivalent resistance between A and A' be R_{eq} and the potential drop across it IR_{eq}

From given circuit the potential difference between A and A' can be determined as follows

$$V_A - \frac{I}{3}R - \frac{I}{6}R - \frac{I}{3}R = V_B$$

$$\Rightarrow \quad V_A - V_B = \frac{I}{3}R + \frac{I}{3}R + \frac{I}{3}R$$

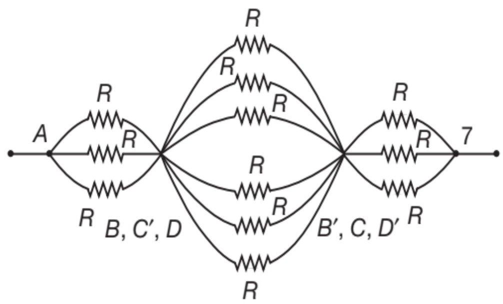
From equivalent circuit $V_A - V_B = IR_{eq}$

Thus we have, $IR_{eq} = \frac{5}{6}IR$

$$R_{eq} = \frac{5}{6}R$$

Method 2. In between A and A', symmetry of the circuit indicates that B, C', D are at equal potential and similarly B', C, D'. So the cube may be redrawn as

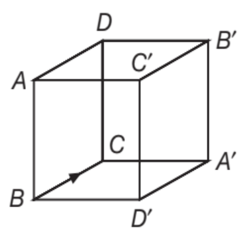
$$R_{eq} = \frac{R}{3}\ '+'\ \frac{R}{6}\ '+'\ \frac{R}{3}$$



where '+' stands for the series

$$= \frac{R}{3} + \frac{R}{6} + \frac{R}{3} = \frac{5}{6}R$$

(b) Once again the circuit is symmetrical. Figure shows the current distribution at junctions A and B. The incoming current at D must be equal to outgoing current, similar situation exists at C'. Therefore the current in branches CD and $C'D'$ are zero or we can say that the points C and D are equipotential, similarly C' and D' have same potential.
The equivalent circuit is reduced to figure.



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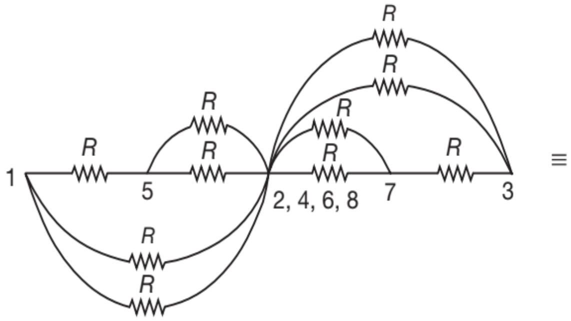
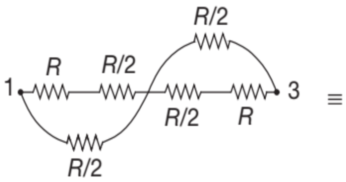
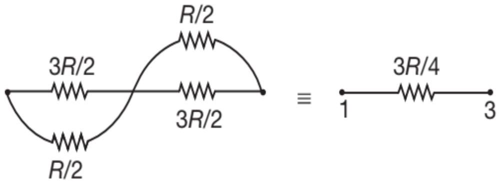
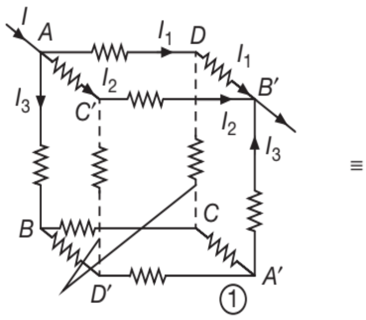
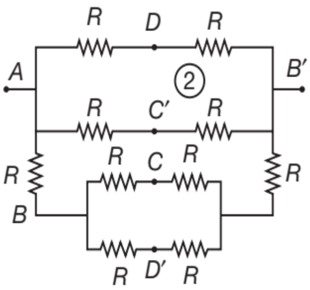


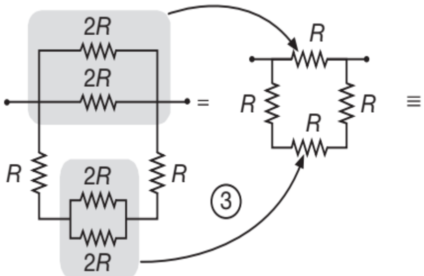


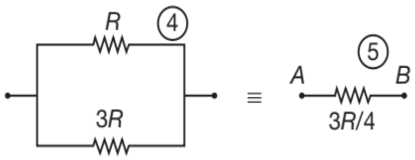




Resistors irrelevant from point of view of current







$$\Rightarrow \quad R_{AB'} = \frac{3R}{4}$$

Method 2. In between points 1 and 3, 2 and 4 are at the same potential. Current in the wires $2-6$ and $4-8$ is zero. Potential difference between $2-6$ and $4-8$ is zero.

(c) Let a current I enter the point A and leave the point B. Figure shows distribution of current in the circuit. the circuit is symmetrical, note the currents at entry and exit points. The wires AC' and AD have equal resistance and located symmetrically, so they have same current; similar situation exists with DC and $C'D'$. From KCL we have

$$I = I_1 + 2I_2 \quad \ldots(1)$$

For the loop $AC'D'BA$, applying KVL, we have

$$-I_2R - I_3R - I_2R + I_1R = 0$$

$$\Rightarrow \quad I_1 = 2I_2 + I_3 \quad \ldots(2)$$

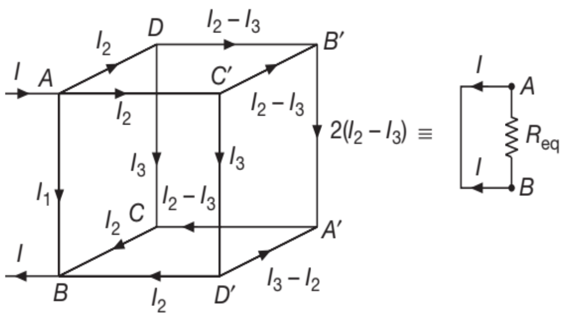
For the loop $C'D'A'B'$, applying KVL we have

$$-I_3R + (I_2 - I_3)R + 2(I_2 - I_3)R + (I_2 - I_3)R = 0$$

$$\Rightarrow \quad 4I_2 - 5I_3 = 0 \quad \ldots(3)$$

On solving equations (1), (2) and (3), we get

$$I_1 = \frac{7}{12}I,\ I_2 = \frac{5}{24}I \text{ and } I_3 = \frac{1}{6}I$$



The potential difference across $AB = I_1R = \frac{7}{12}IR$

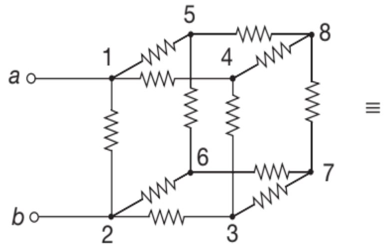
The potential difference across equivalent resistance is $V = IR_{eq}$

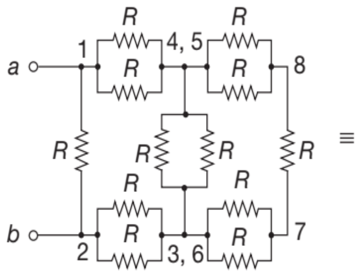
$$\Rightarrow \quad IR_{eq} = \frac{7}{12}IR$$

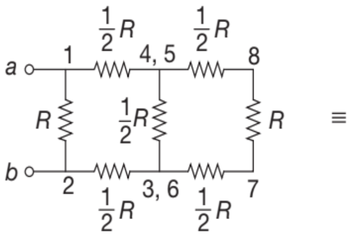
$$\Rightarrow \quad R_{eq} = \frac{7}{12}R$$

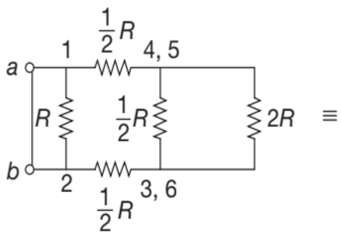
Method 2: From considerations of symmetry alone, points 3 and 6 must be at the same potential, and so must points 4 and 5.

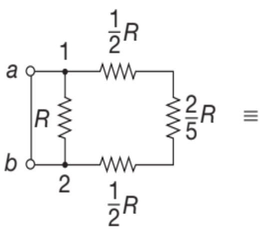
If two points in a circuit have the same potential, the currents in the circuit do not change if they are connected at these points by a wire. There is no current in the wire because there is no potential difference between its ends. Points 3 and 6 may therefore be connected by a wire, and similarly points 4 and 5 may be connected.

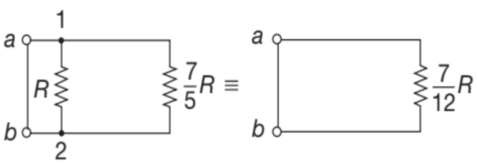












Thus the equivalent resistance between a and b is $\frac{7}{12}R$

15. Before connecting B with D and C with E,

$$R_{AF} = 5R$$

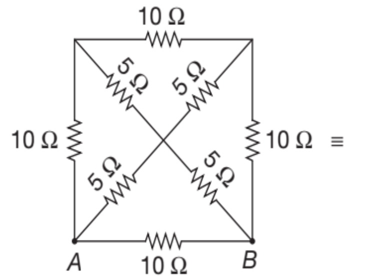
After connecting B with D and C with E, a balanced Wheatstone bridge is formed between B and E.

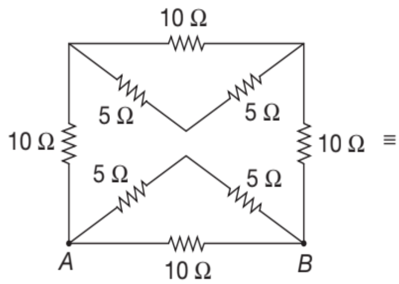
So, $R_{BE} = R$

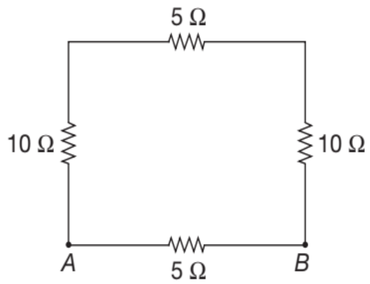
and $R'_{AF} = 3R = 0.6R_{AF}$

i.e., the new equivalent resistance in the circuit has become 0.6 times.

16. The simplified circuit is as shown in figure.

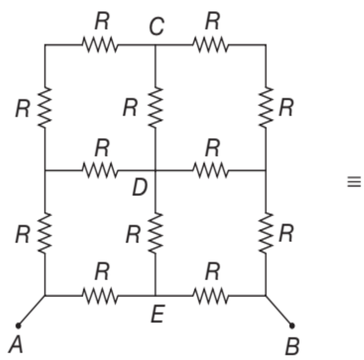


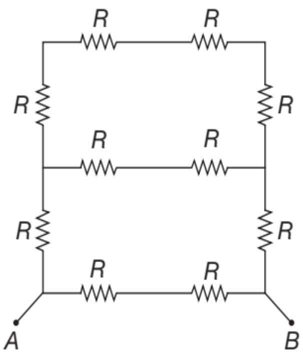




$$R_{AB} = \frac{(25)(5)}{(25)+(5)} = \frac{25}{6}\ \Omega$$

17.

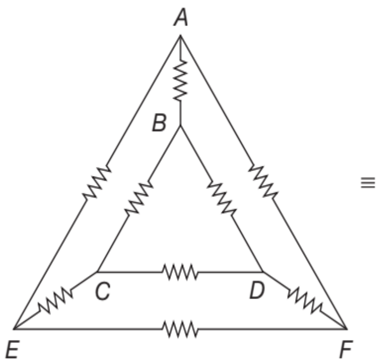


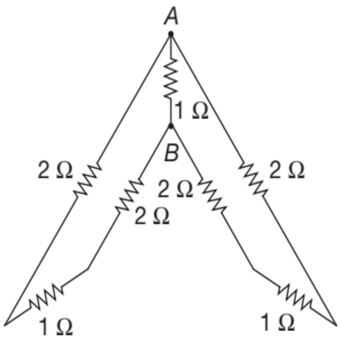


Points C, D and E are at same potential. The circuit can now be redrawn as,

The equivalent resistance between A and B can now be easily determined as $\frac{5R}{4}$.

18. The extended line AB is a line of symmetry. Hence, points C and D and also points E and F are at the same potentials. So, the resistance between them can be removed from the circuit.





The equivalent resistance between A and B can now be worked out to be $\frac{5}{7}\,\Omega$.

19. (a) A thin cylindrical shell of radius r, thickness dr, and length L contributes resistance

$$dR = \frac{\rho d\ell}{A} = \frac{\rho dr}{(2\pi r)L} = \left(\frac{\rho}{2\pi L}\right)\frac{dr}{r}$$

The resistance of the whole annulus is the series summation of the contributions of the thin shells:

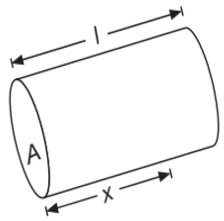
$$R = \frac{\rho}{2\pi L}\int_{r_a}^{r_b}\frac{dr}{r} = \frac{\rho}{2\pi L}\log_e\left(\frac{r_b}{r_a}\right)$$

(b) In this equation $\frac{\Delta V}{I} = \frac{\rho}{2\pi L}\log_e\left(\frac{r_b}{r_a}\right)$

$$\Rightarrow \quad \rho = \frac{2\pi L\Delta V}{I\log_e\left(\frac{r_b}{r_a}\right)}$$

20. (a) Consider the cylinder as composed of thin discs of width dx connected in series. The resistance of a disc at a distance x away from the cylinder end is given by

$$dR = \frac{1}{\sigma(x)}\frac{dx}{A} = \frac{\sqrt{x}\,dx}{A\ell\sigma_0} \quad \text{...(1)}$$



where A is the cross-section area of the disc and dx is its width. Since the discs are connected in series, the total resistance is

$$R = \int_0^\ell dR = \frac{1}{A\ell\sigma_0}\int_0^\ell \sqrt{x}dx = \frac{2\sqrt{\ell}}{3A\sigma_0} \quad \text{...(2)}$$

(b) From Ohm's Law, we deduce that the current flowing across the cylinder is given by

$$I = \frac{V_0}{R} = \frac{3A\sigma_0 V_0}{2\sqrt{\ell}} \quad \text{...(3)}$$

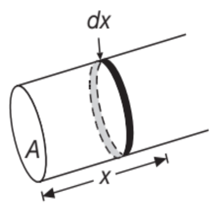
The current density is, therefore

$$J = \frac{I}{A} = \frac{3\sigma_0 V_0}{2\sqrt{\ell}} \quad \text{...(4)}$$

The electric field in the cylinder may be found by using Ohm's Law, according to which

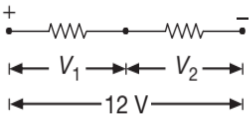
$$J = \sigma(x)E(x)$$

$$\Rightarrow \quad E(x) = \frac{J}{\sigma(x)} = \frac{J}{\sigma_0\left(\frac{1}{\sqrt{x}}\right)} = \frac{3V_0\sqrt{x}}{2\ell^{\frac{3}{2}}} \quad \text{...(5)}$$



21. Refer to solution of ILLUSTRATION 24, we get on solving $R_{AB} = \left(\frac{\sqrt{7}-1}{3}\right)R_0$

22. Note that points a, h, g and f have same potential, they are connected by conducting wires without any circuit elements between them. Similarly points b, c, d and e have same potential. Hence the potential drop across branch e and f and a and b is same. The two resistors ($6\ \Omega$ and $4\ \Omega$ in series) are directly connected across the terminals of 12 V battery.
The complex circuitry in the middle has no effect on the potential drop across the upper $10\ \Omega$ branch. If the current through it is I



Potential drop across R_1, $V_1 = IR_1$

Potential drop across R_2, $V_2 = IR_2$

Potential drop across branch,

$$V = V_1 + V_2 = I(R_1 + R_2)$$

The current $I = \frac{V}{R_1 + R_2} = \frac{12}{10} = 1.2\ \text{A}$

Hence $V_1 = (1.2)(6) = 7.2\ \text{V}$

$$V_2 = (1.2)(4) = 4.8\ \text{V}$$

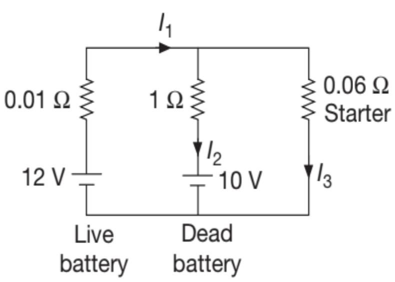
23. Since the polarity of batteries in the lowest horizontal branch and in the right vertical branch oppose each other, so the net emf in these branches is zero. So there is no current anywhere in the circuit.

Test Your Concepts-VI (Based on Kirchhoff's Laws and Nodal Analysis)

1. Using Kirchhoff's Rules,

$$12 - (0.01)I_1 - (0.06)I_3 = 0$$

$$10 + (1)I_2 - (0.06)I_3 = 0$$



and $I_1 = I_2 + I_3$

$$12 - (0.01)I_2 - (0.07)I_3 = 0$$

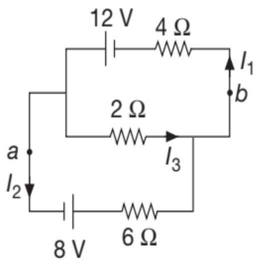
$$10 + (1)I_2 - (0.06)I_3 = 0$$

Solving simultaneously,

$I_2 = 0.283\ \text{A}$, downwards in the dead battery

and $I_3 = 171\ \text{A}$, downwards in the starter.
The currents are forward in the live battery and in the starter, relative to normal starting operation. The current is backward in the dead battery, tending to charge it up.

2. Let the currents be I_1, I_2, and I_3 as shown.



(a) $I_1 = I_2 + I_3$ ...(1)

Traversing the top loop counter clockwise, we get

$$12\ \text{V} - (2\ \Omega)I_3 - (4\ \Omega)I_1 = 0$$

$\Rightarrow \quad I_1 = 3 - \frac{1}{2}I_3$ ...(2)

Traversing the bottom loop, we get

$$8\ \text{V} - (6\ \Omega)I_2 + (2\ \Omega)I_3 = 0$$

$\Rightarrow \quad I_2 = \frac{4}{3} + \frac{1}{3}I_3$ ...(3)

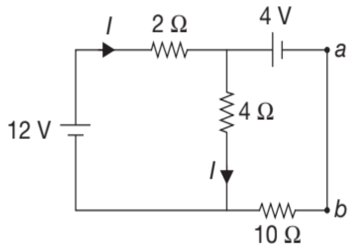
From (1), (2) and (3), we get

$$I_3 = 909\ \text{mA}$$

(b) $V_a - (0.909\ \text{A})(2\ \Omega) = V_b \Rightarrow V_b - V_a = -1.82\ \text{V}$

3. Using Kirchhoff's Loop Rule for the closed loop, $+12 - 2I - 4I = 0$, so $I = 2\ \text{A}$

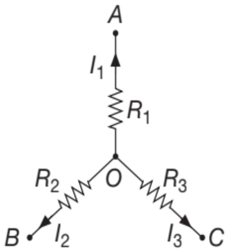
$$V_b - V_a = +4\ \text{V} - (2\ \text{A})(4\ \Omega) - (0)(10\ \Omega) = -4\ \text{V}$$



Thus, $|\Delta V_{ab}| = 4\ \text{V}$ and point a is at the higher potential.

4. Let the current through each branch be I_1, I_2, I_3 as shown in figure. Applying KCL to O, we get

$I_1 + I_2 + I_3 = 0$ ...(1)



Now if V_0 is the potential at junction point O, then

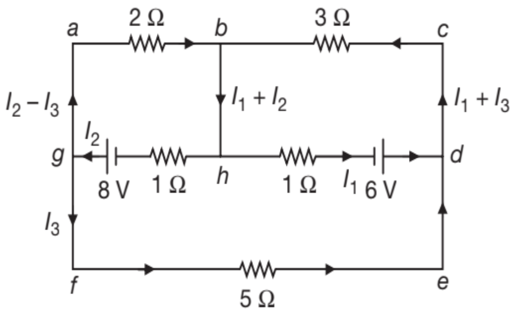
$$I_1=\frac{(V_0-V_1)}{R_1},\ I_2=\frac{(V_0-V_2)}{R_2}\ \text{and}\ I_3=\frac{(V_0-V_3)}{R_3}$$

So substituting these values of I_1, I_2 and I_3 in equation (1), we have

$$V_0\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)-\left(\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right)=0$$

$$\Rightarrow\quad V_0=\frac{\left(\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right)}{\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)}$$

5. Applying Kirchhoff's Loop Law in three loops, we have,



For loop *abhga* $8-2(I_2-I_3)-1(I_2)=0$ …(1)

For loop *dhbcd* $-6+1(I_1)+3(I_1+I_3)=0$ …(2)

For loop hgfedh $-8+1(I_2)-1(I_1)+6+5I_3=0$ …(3)

Solving these three equations, we have

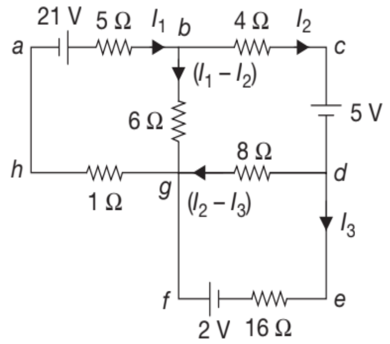
$$I_1=\frac{108}{77}\ \text{A},\ I_2=\frac{212}{77}\ \text{A and } I_3=\frac{10}{77}\ \text{A}$$

$V_A-V_B=(I_2-I_3)(2\ \Omega)$ and $V_B-V_C=-(I_1+I_3)(3\ \Omega)$

$$\Rightarrow\quad V_A-V_B=\left(\frac{212-10}{77}\right)(2)=\frac{404}{77}\ \text{V}$$

and $V_B-V_C=-\dfrac{354}{77}$ V

6. Using the loop current method.
Three equations are as under



For loop abgha $21-5I_1-6(I_1-I_2)-1I_1=0$ …(1)

For loop cdgbc $-5-8(I_2-I_3)+6(I_1-I_2)-4I_2=0$ …(2)

For loop efgde $2+8(I_2-I_3)-16I_3=0$ …(3)

On solving, we get

$$I_1=2\ \text{A},\ I_2=0.5\ \text{A and } I_3=0.25\ \text{A}$$

So, current through $1\ \Omega$ and $5\ \Omega$ is $I_1=2$ A

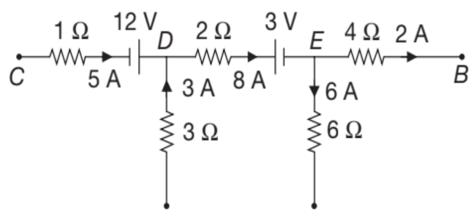
current through $6\ \Omega$ is $I_1-I_2=1.5$ A

current through $8\ \Omega$ is $I_2-I_3=0.25$ A

current through $4\ \Omega$ is $I_2=0.5$ A and

current through $16\ \Omega$ is $I_3=0.25$ A

7. Applying Kirchhoff's Junction Law at E current in wire DE is 8 A from D to E. Now further applying Junction Law at D. The current in $3\ \Omega$ resistance will be 3 A towards D.



Power dissipated in $3\ \Omega$ resistance

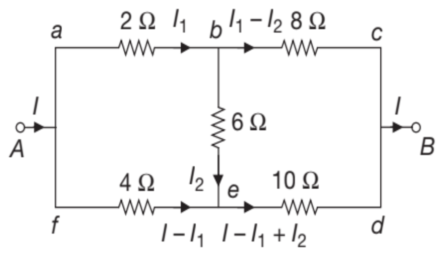
$$=I^2R=(3)^2(3)=27\ \text{W}$$

$$V_C-V_B:\ V_C-5\times1+12-8\times2-3-4\times2=V_B$$

$$\Rightarrow\quad V_C-V_B=5-12+16+3+8$$

$$\Rightarrow\quad V_C-V_B=20\ \text{V}$$

8. This is an unbalanced Wheatstone bridge.



Applying Kirchhoff's Second Law in loops *abefa* and *bcdeb* we have,

$$-2I_1 - 6I_2 + 4(I - I_1) = 0 \quad \ldots(1)$$

$$-8(I_1 - I_2) + 10(I - I_1 + I_2) + 6I_2 = 0 \quad \ldots(2)$$

Solving these two equations we have,

$$I_1 = \frac{13}{21}I \text{ and } I_2 = \frac{I}{21}$$

For $AabcB$, we have

$$V_A - 2I_1 - 8(I_1 - I_2) - V_B = 0$$

$$\Rightarrow \quad V_{AB} = 2I_1 + 8(I_1 - I_2)$$

Substituting values of I_1 and I_2,

$$V_{AB} = \frac{122}{21}I \quad \ldots(3)$$

According to Thevenin's Theorem, "any complicated circuit can be reduced to an equivalent resistance" and hence the potential difference (across the terminals for which equivalent resistance has been calculated) equals the total current (entering the terminal) times the equivalent resistance. So,

$$V_{AB} = IR_{AB} \quad \ldots(4)$$

Equating equations (3) and (4), we get

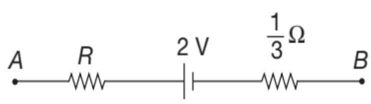
$$R_{AB} = \frac{122}{21}\ \Omega$$

9. (a) Equivalent emf of three batteries would be :

$$E_{eq} = \frac{\Sigma(E/r)}{\Sigma(1/r)} = \frac{(3/1+2/1+1/1)}{(1/1+1/1+1/1)} = 2\text{ V}$$

Further r_1, r_2 and r_3 each are of $1\ \Omega$. Therefore, internal resistance of the equivalent battery will be $\frac{1}{3}\ \Omega$ as all three are in parallel.

The equivalent circuit is therefore, shown in the figure.



Since, no current is taken from the battery.

$$V_{AB} = 2\ V \ (\text{From } V = E - Ir)$$

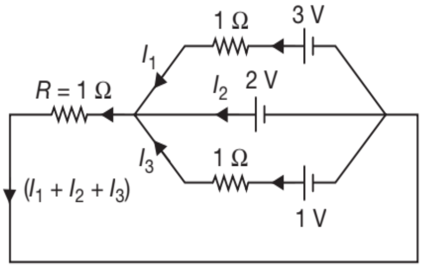
Further, $V_A + I_1r_1 - E_1 = V_B$

$$\Rightarrow \quad I_1 = \frac{V_B - V_A + E_1}{r_1} = \frac{-2+3}{1} = 1\text{ A}$$

Similarly, $I_2 = \frac{V_B - V_A + E_2}{r_2} = \frac{-2+2}{1} = 0$

and $I_3 = \frac{V_B - V_A + E_3}{r_3} = \frac{-2+1}{1} = -1\text{ A}$

(b) r_2 is short circuited means resistance of this branch becomes zero. Making a closed circuit with a battery and resistance R, applying Kirchhoff's Second Law in three loops so formed.



$$3 - I_1 - (I_1 + I_2 + I_3) = 0 \quad \ldots(1)$$

$$2 - (I_1 + I_2 + I_3) = 0 \quad \ldots(2)$$

$$1 - I_3 - (I_1 + I_2 + I_3) = 0 \quad \ldots(3)$$

From equation (2),

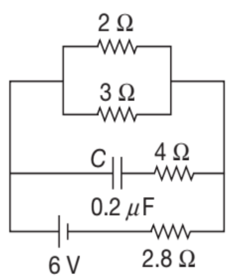
$$I_1 + I_2 + I_3 = 2\text{ A}$$

$\Rightarrow$ Substituting in (1) we get, $I_1 = 1\text{ A}$

Substituting in (3), we get $I_3 = -1\text{ A}$

$\Rightarrow \quad I_2 = 2\text{ A}$

10. In steady state situation no current will flow through the capacitor, $2\ \Omega$ and $3\ \Omega$ are in parallel.



Therefore, their combined resistance will be

$$R = \frac{2 \times 3}{2+3} = 1.2\ \Omega$$

Net current through the battery

$$I = \frac{6}{1.2 + 2.8} = 1.5\text{ A}$$

This current will distribute in inverse ratio of their resistances $2\ \Omega$ and $3\ \Omega$.

CHAPTER 3

$$\Rightarrow \quad \frac{I_2}{I_3} = \frac{3}{2}$$

$$\Rightarrow \quad I_2 = \left(\frac{3}{3+2}\right)(1.5) = 0.9 \text{ A}$$

11. Applying Kirchhoff's Second Law in Loop $ADBA$:

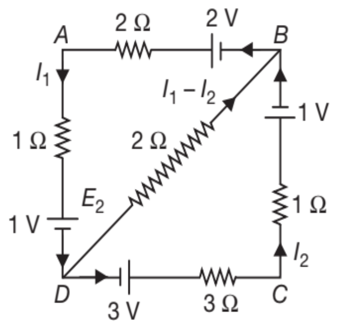
$$2 - 2I_1 - I_1 - 1 - 2(I_1 - I_2) = 0 \qquad \ldots(1)$$

Similarly applying Kirchhoff's Second Law in Loop $BDCB$

$$2(I_1 - I_2) + 3 - 3I_2 - I_2 - 1 = 0 \qquad \ldots(2)$$

Solving in equations (1) and (2) we get,

$$I_1 = \frac{5}{13}, I_2 = \frac{6}{13} \text{ and } I_1 - I_2 = -\frac{1}{13}$$



(a) Potential difference between B and D.

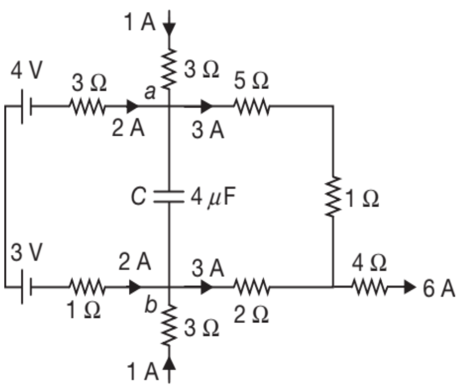
$$V_B + 2(I_1 - I_2) = V_D$$

$$\Rightarrow \quad V_B - V_D = -2(I_1 - I_2) = \frac{2}{13} \text{ V}$$

(b) $V_G = E_G - I_2 r_G = 3 - \frac{6}{13} \times 3 = \frac{21}{13}$ V

$$V_H = E_H + I_2 r_H = 1 + \frac{6}{13} \times 1 = \frac{19}{13} \text{ V}$$

12. Using Kirchhoff's First Law at junctions a and b, we have found the current in other wires of the circuit on which currents were not shown.



Now, to calculate the energy stored in the capacitor we will have to first find the potential difference V_{ab} across it.

$$V_a - 3 \times 5 - 3 \times 1 + 3 \times 2 = V_b$$

$$\Rightarrow \quad V_a - V_b = V_{ab} = 12 \text{ V}$$

$$\Rightarrow \quad U = \frac{1}{2}CV_{ab}^2 = \frac{1}{2}(4 \times 10^{-6})(12)^2 \text{ J} = 0.288 \text{ mJ}$$

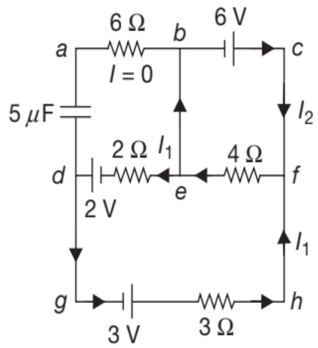
13. In steady state no current will flow through $R_1 = 6\ \Omega$.

Potential difference across R_3 or $4\ \Omega$ is E_1 or 6 V.

$\therefore$ Current through it will be $\frac{6}{4} = 1.5$ A from right to left.

Because left hand side of this resistance is at higher potential.

Now, suppose this 1.5 A distributes in I_1 and I_2 as shown.



Applying Kirchhoff's Second Law in Loop $dghfed$:

$$3 - 3I_1 - 4 \times 1.5 - 2I_1 + 2 = 0$$

$$\Rightarrow \quad I_1 = -\frac{1}{5} \text{ A} = -0.2 \text{ A}$$

To find energy stored in capacitor we will have to find potential difference across it. Or V_{ad}.

Now, $V_a - 2I_1 + 2 = V_d$

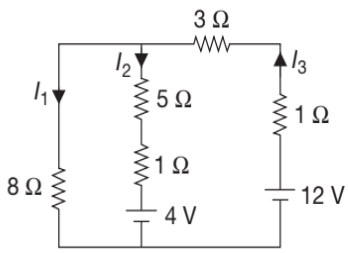
$$\Rightarrow \quad V_a - V_d = 2I_1 - 2 = -2.4 \text{ V}$$

$$\Rightarrow \quad V_d - V_a = 2.4 \text{ V} = V_{da}$$

Energy stored in capacitor:

$$U = \frac{1}{2}CV_{da}^2 = \frac{1}{2}(5 \times 10^{-6})(2.4)^2 = 1.44 \times 10^{-5} \text{ J}$$

14. We name currents I_1, I_2 and I_3 as shown.



From Kirchhoff's Current Rule, $I_3 = I_1 + I_2$.

Applying Kirchhoff's Voltage Rule to the loop containing I_2 and I_3

$$12\text{ V}-(4)I_3-(6)I_2-4\text{ V}=0$$

$$8=(4)I_3+(6)I_2$$

Applying Kirchhoff's Voltage Rule to the loop containing I_1 and I_2,

$$-(6)I_2-4\text{ V}+(8)I_1=0$$

$$(8)I_1=4+(6)I_2.$$

Solving the above linear system, we proceed to the pair of simultaneous equations:

$$\begin{cases}8=4I_1+4I_2+6I_2\\ 8I_1=4+6I_2\end{cases}\text{ or }\begin{cases}8=4I_1+10I_2\\ I_2=1.33I_1-0.667\end{cases}$$

and to the single equation $8=4I_1+13.3I_1-6.67$

$$I_1=\frac{14.7\text{ V}}{17.3\ \Omega}=0.846\text{ A}$$

Then $I_2=1.33(0.846\text{ A})-0.667$

and $I_3=I_1+I_2$ give $I_1=846$ mA, $I_2=462$ mA, $I_3=1.31$ A

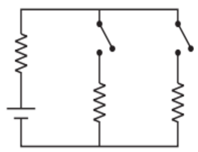
All currents are in the directions indicated by the arrows in the circuit diagram.

Test Your Concepts-VII (Based on emf, Internal Resistance and Combination of Cells)

1. (a) Here $E=I(R+r)$,

so $I=\dfrac{E}{R+r}=\dfrac{12.6\text{ V}}{(5\ \Omega+0.08\ \Omega)}=2.48\text{ A}$

Then, $\Delta V=IR=(2.48\text{ A})(5\ \Omega)=12.4\text{ V}$



(b) Let I_1 and I_2 be the currents flowing through the battery and the headlights, respectively.

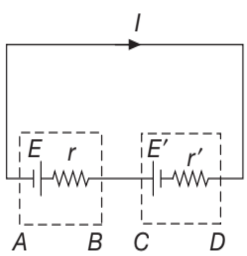
Then, $I_1=I_2+35$ A, and $E-I_1r-I_2r=0$

$\Rightarrow\quad E=(I_2+35\text{ A})(0.08\ \Omega)+I_2(5\ \Omega)=12.6\text{ V}$

giving $I_2=1.93$ A

Thus, $\Delta V_2=(1.93\text{ A})(5\ \Omega)=9.65\text{ V}$

2. Two batteries AB and CD emf's E and $E'(E'>E)$ and internal resistances r and r' respectively are connected in series as shown in the figure.



If I is the current in the circuit, the total potential drop in the circuit must be equal to net emf:

$$I(r+r')=E'-E$$

$$\Rightarrow\quad I=\frac{E'-E}{r+r'}$$

The potential difference across terminals of battery AB is

$$V_B-V_A=Ir+E$$

Obviously $(V_B-V_A)>E$ by Ir, i.e.,

$$(V_B-V_A)-E=\frac{(E'-E)r}{r+r'}$$

when a current is flowing in any battery opposite to its emf, then terminal voltage is given by

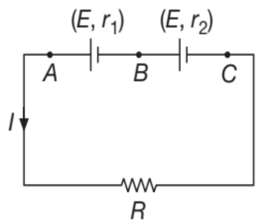
$$V=E+Ir$$

3. Two batteries are connected in series. The effective emf in the circuit is therefore $2E$ because both push the charge in the same direction. Hence emf's are added.

Net resistance in the circuit is (r_1+r_2+R)

Therefore, current in the circuit

$$I=\frac{2E}{(r_1+r_2+R)}$$



The potential difference between the terminals of first battery is (V_A-V_B), terminal potential difference is given by

$$(V_A-V_B)=E-Ir_1$$

where E is the emf of the battery and r_1 is its internal resistance. Substituting the value of I, we get

$$V_A-V_B=E-\frac{2Er_1}{r_1+r_2+R}=E\frac{(R+r_2-r_1)}{(R+r_2+r_1)}$$

For (V_A-V_B) to be zero, we must have

$$R=(r_1-r_2)$$

This gives meaningful result only if $r_1>r_2$. Otherwise, if $r_2>r_1$, then $R=r_2-r_1$ will produce terminal voltage across second cell to be zero $(V_{BC}=0)$.

4. The voltage supplied by the charging plant is here constant which is equal to,

$$V=E_I+I_Ir=(90)+(10)(2)$$

$$V=110\text{ V}$$

Let I_f be the current at the end of charging

Then, $V = E_f + I_f r$

$$\Rightarrow \quad I_f = \frac{V - E_f}{r} = \frac{110 - 100}{2}$$

$$\Rightarrow \quad I_f = 5 \text{ A}$$

5. In the first case current will be maximum when, Total external resistance = total internal resistance

$$\Rightarrow \quad R = \frac{nr}{m} \quad \Rightarrow \quad r = \frac{mR}{n} \qquad \ldots(1)$$

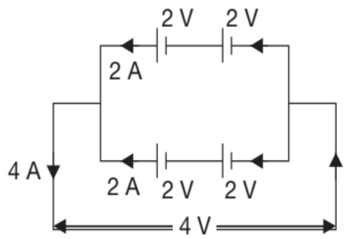
$$\Rightarrow \quad I = \frac{nE}{2R} \qquad \ldots(2)$$

In the second case,

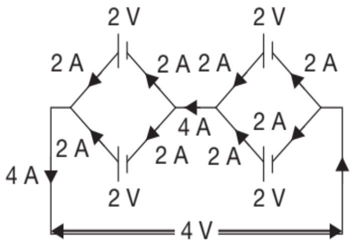
$$I' = \frac{mE}{R + \frac{mr}{n}} = \frac{mE}{R + \frac{m^2}{n^2}R}$$

$$\Rightarrow \quad I' = \frac{(mE)n^2}{R(n^2 + m^2)} = \frac{nE}{2R}\left(\frac{2mn}{n^2 + m^2}\right) = \left(\frac{2mn}{m^2 + n^2}\right)I$$

6. To get 4 V, two cells must be connected in series, and to get 4 A two cells must be connected in parallel. Following two arrangements of cells are possible, which can supply 4 A at 4 V.

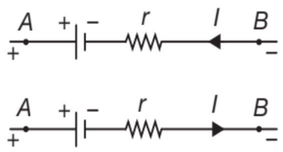


(A)



(B)

7. Let A, B represent the terminals of the cell.



$$V_{AB} = E - IR$$

$$\Rightarrow \quad 8.5 = E - 3R \qquad \ldots(1)$$

$$V'_{AB} = E + IR$$

$$\Rightarrow \quad 11 = E + 2R \qquad \ldots(2)$$

Solving (1) and (2), we get,

$$E = 10 \text{ V} \text{ and } r = 0.5\ \Omega$$

8. The last cell is a voltage divider that reduces the potential of the nth point k times as compared with the $(n-1)$ th point. Hence,

$$U_n = \frac{U_{n-1}}{R_1 + R_3} R_3 = \frac{U_{n-1}}{k}$$

$$\Rightarrow \quad \frac{R_1}{R_3} = k - 1$$

The relation $U_i = \frac{U_{i-1}}{k}$ should be true for any cell. For this reason the resistance of the last cell, of the last two, of the last three, etc., cells should also be R_3. Therefore,

$$\frac{1}{R_3} = \frac{1}{R_2} + \frac{1}{R_1 + R_3}$$

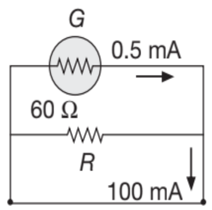
$$R_2 = \frac{R_3(R_1 + R_3)}{R_1} = R_3 \frac{k}{k-1}$$

$$\Rightarrow \quad R_1 : R_2 : R_3 = (k-1)^2 : k : (k-1)$$

Test Your Concepts-VIII (Based on Galvanometer, Voltmeter and Ammeter)

1. $\Delta V = I_g r_g = (I - I_g) R_p$

$$\Rightarrow \quad R_p = \frac{I_g r_g}{(I - I_g)} = \frac{I_g (60\ \Omega)}{(I - I_g)}$$

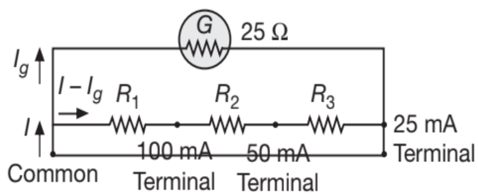


Therefore, to have $I = 0.1 \text{ A} = 100 \text{ mA}$ when $I_g = 0.5 \text{ mA}$:

$$R_p = \frac{(0.5 \text{ mA})(60\ \Omega)}{99.5 \text{ mA}} = 0.302\ \Omega$$

2. Consider the circuit diagram, realizing that $I_g = 1 \text{ mA}$.

For the 25 mA scale, we have $(I - I_g)S = I_g G$



$$\Rightarrow \quad (24\text{ mA})(R_1+R_2+R_3)=(1\text{ mA})(25\ \Omega)$$

$$\Rightarrow \quad R_1+R_2+R_3=\left(\frac{25}{24}\right)\Omega \qquad \ldots(1)$$

For the 50 mA scale:

$$(49\text{ mA})(R_1+R_2)=(1\text{ mA})(25\ \Omega+R_3)$$

$$\Rightarrow \quad 49(R_1+R_2)=25\ \Omega+R_3 \qquad \ldots(2)$$

For the 100 mA scale:

$$(99\text{ mA})R_1=(1\text{ mA})(25\ \Omega+R_2+R_3)$$

$$\Rightarrow \quad 99R_1=25\ \Omega+R_2+R_3 \qquad \ldots(3)$$

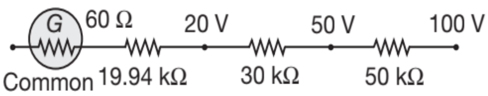
Solving (1), (2) and (3) simultaneously yields

$$R_1=0.26\ \Omega,\ R_2=0.261\ \Omega,\ R_3=0.521\ \Omega$$

3. $\Delta V=IR$ and for a voltmeter we have $V=I_g(G+R)$

(a) $20\text{ V}=(1\times10^{-3}\text{ A})(R_1+60\ \Omega)$

$R_1=1.994\times10^4\ \Omega=19.94\text{ k}\Omega$



(b) $50\text{ V}=(1\times10^{-3}\text{ A})(R_2+R_1+60\ \Omega)$

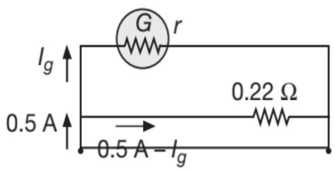
$R_2=30\text{ k}\Omega$

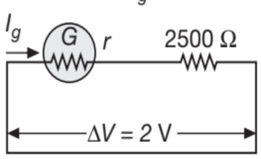
(c) $100\text{ V}=(1\times10^{-3}\text{ A})(R_3+R_1+60\ \Omega)$

$R_3=50\text{ k}\Omega$

4. For ammeter, we have $I_g r=(0.5\text{ A}-I_g)(0.22\ \Omega)$ …(1)

$$\Rightarrow \quad I_g(r+0.22\ \Omega)=0.11\text{ V}$$





For voltmeter, we have $2\text{ V}=I_g(r+2500\ \Omega)$ …(2)

Solve (1) and (2) simultaneously, we get

$$I_g=0.756\text{ mA and } r=145\ \Omega$$

5. (a) Potential difference across $R=(I_{meas}-I_1)R$; potential difference across voltmeter $=I_1R_v=V_{meas}$ and this is measured potential difference. The two branches are in parallel.

$$(I_{meas}-I_1)R=V_{meas}$$

$$\Rightarrow \quad \left(I_{meas}-\frac{V_{meas}}{R_v}\right)R=V_{meas}$$

$$\Rightarrow \quad R=\frac{V_{meas}}{\left(I_{meas}-\dfrac{V_{meas}}{R_v}\right)}$$

(b) From the expression for R, we get

$$\frac{1}{R}=\frac{I_{meas}}{V_{meas}}-\frac{1}{R_v}$$

If $$R_v\gg\frac{V_{meas}}{I_{meas}}$$

$$\frac{I}{R_v}\ll\frac{I_{meas}}{V_{meas}}$$

$$\Rightarrow \quad \frac{1}{R}\simeq\frac{I_{meas}}{V_{meas}}$$

$$\Rightarrow \quad R=\frac{V_{meas}}{I_{meas}}$$

6. (a) The voltmeter and ammeter branches are in parallel arrangement. Hence,

$$V_{meas}=(I-I_{meas})R_V=I_{meas}(R_A+R)$$

Hence, $$R_A+R=\frac{V_{meas}}{I_{meas}}$$

$$R=\left(\frac{V_{meas}}{I_{meas}}-R_A\right)$$

(b) For $\dfrac{V_{meas}}{I_{meas}}\gg R_A$, $R\simeq\dfrac{V_{meas}}{I_{meas}}$

7. The given parameters are

$$I_S=1\text{ mA}=1\times10^{-3}\text{ A}$$

$$G=20\ \Omega$$

$$I=I_{max}=50\times10^{-3}\text{ A}$$

Since, $$S=\frac{I_GG}{(I-I_G)}$$

$$\Rightarrow \quad S=\frac{(1\times10^{-3})(20)}{50\times10^{-3}-1\times10^{-3}}$$

$$\Rightarrow \quad S=0.408\ \Omega$$

The equivalent resistance of the instrument is

$$\frac{1}{R_{eq}}=\frac{1}{G}+\frac{1}{S}=\frac{1}{20}+\frac{1}{0.408}$$

$$R_{eq}=0.4\ \Omega$$

Note that shunt resistance is so small in comparison to the galvanometer resistance that the equivalent resistance is very nearly equal to the shunt resistance.

8. The maximum current through the galvanometer is

$$I_G=\frac{V}{G}$$

With a multiplier resistor in series with galvanometer the potential difference across the entire branch is

$$nV = I_G G + I_G R$$

$$\Rightarrow \quad nV = \left(\frac{V}{G}\right)G + \left(\frac{V}{G}\right)R$$

$$\Rightarrow \quad nV = V + \left(\frac{V}{G}\right)R$$

$$\Rightarrow \quad R = (n-1)G$$

Usually n is referred to as multiplication factor, e.g., a galvanometer with a range 0 to 1 mV is to be converted to a voltmeter of range 0 to 1 V.

We have $n = \dfrac{1\text{ V}}{1\text{ mV}} = 10^3$

The multiplier resistor $R = (10^3 - 1)G = 999G$.

So you can guess how large the multiplier resistor is in comparison to the galvanometer resistance G.

9. The currents in the ammeters are proportional to deflections produced, let α_1 and α_2 be the proportionality constants.

$$I_1 = \alpha_1 n_1,$$

$$I_2 = \alpha_2 n_2$$

The ammeters are in series, hence

$$I_1 = I_2$$

$$\Rightarrow \quad \alpha_1 n_1 = \alpha_2 n_2 \qquad \ldots(1)$$

In the second arrangement the potential drops across resistors are equal as they are in parallel arrangement. Resistances of ammeters have been ignored assuming them to be ideal.

Thus we have $I_1' R_1 = I_2' R_x$

Also $I_1' = \alpha_1 N_1$ and $I_2' = \alpha_2 N_2$

$$\Rightarrow \quad R_1 \alpha_1 N_1 = R_x \alpha_2 N_2 \qquad \ldots(2)$$

From equations (1) and (2), eliminating α_1 and α_2, we get

$$\frac{R_1 N_1}{n_1} = \frac{R_x N_2}{n_2}$$

$$\Rightarrow \quad R_x = \frac{R_1 n_2 N_1}{n_1 N_2}$$

10. Before the ammeter is connected, $I_0 = \dfrac{V}{R}$, and after it is connected, $I = \dfrac{V}{R + R_0}$, where R_0 is the resistance of the ammeter. The error is given by

$$\varepsilon = \frac{I_0 - I}{I_0} = \frac{1}{1 + \dfrac{R}{R_0}} = \frac{R_0}{R + R_0}$$

When $R_0 \ll R$, then the error may be neglected.

11. The sought error is $\varepsilon = \dfrac{V_0 - V}{V_0}$, where V_0 is the voltage across the resistance R before the voltmeter is switched on and V the voltage after it is switched on.

According to Ohm's Law, $V_0 = IR$ and $V = I\left(\dfrac{RR_0}{R + R_0}\right)$, where R_0 is the resistance of the voltmeter. Hence, the error is

$$\varepsilon = \frac{\dfrac{R}{R_0}}{1 + \dfrac{R}{R_0}} = \frac{R}{R + R_0}$$

This is determined only by the ratio between the resistances of the section of the circuit and the voltmeter. When $R_0 \gg R$, the error may be neglected.

12. A reduction in the sensitivity n times means that the galvanometer carries a current I_1 which is n times smaller than the current in the remaining circuit before the branching off, Hence the current I_2 through the shunt is $\left(\dfrac{n-1}{n}\right)I$ in the remaining circuit. Hence,

$$\frac{r}{R} = \frac{I_1}{I_2} = \frac{1}{n-1}$$

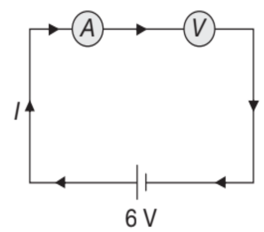
$$\Rightarrow \quad r = \frac{R}{n-1} \cong 204\ \Omega$$

13. Let R_1 = resistance of ammeter,

R_2 = combined resistance of ammeter and voltmeter

V = Voltage across voltmeter and

V_A = Voltage across ammeter



In the first case current in the circuit,

$$I = \frac{6}{R_2} \qquad \ldots(1)$$

and voltage across voltmeter $V = 6 -$ voltage across ammeter

$$\Rightarrow \quad V = 6 - V_A$$

$$\Rightarrow \quad V = 6 - IR_1$$

$$\Rightarrow \quad V = 6 - \frac{6R_1}{R_2} \qquad \ldots(2)$$

In the second case reading of ammeter becomes two times, i.e., the total resistance becomes half while the resistance of ammeter remains unchanged. Hence,

$$I' = \frac{6}{R_2/2} = \frac{12}{R_2} \quad \ldots(3)$$

and $V' = 6 - (I')R_1$

$$\Rightarrow \quad V' = 6 - \frac{12R_1}{R_2} \quad \ldots(4)$$

Further, it is given that $V' = \dfrac{V}{2}$

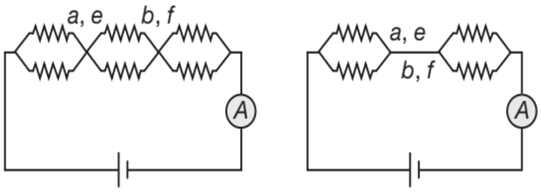
$$\Rightarrow \quad 6 - \frac{12R_1}{R_2} = 3 - \frac{3R_1}{R_2} \quad \Rightarrow \quad \frac{R_1}{R_2} = \frac{1}{3}$$

Substituting this value in equation (4), we have

$$V' = 6 - (12)\left(\frac{1}{3}\right)$$

$$\Rightarrow \quad V' = 2 \text{ V}$$

14. The two conductors ace and bdf have different potentials, with the result that when the switch S is closed a current will flow from c to d, while the currents will flow from a to c, from e to c, from d to f and from d to b.



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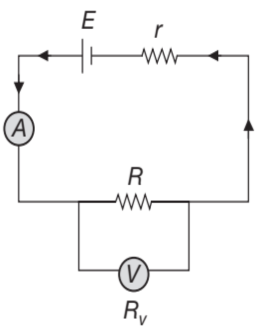
So, the closing of switch leads to an increase in the current flowing through the ammeter. Please note that the sections ab and ef of the resistors are shorted.

15. Net resistance $= \dfrac{E}{I} = \dfrac{3.4}{0.04}\ \Omega = 85\ \Omega$



$$\Rightarrow \quad R_A + r + \frac{RR_V}{R + R_V} = 85$$

$$\Rightarrow \quad 2 + 3 + \frac{100R_V}{100 + R_V} = 85$$

$$\Rightarrow \quad R_V = 400\ \Omega$$

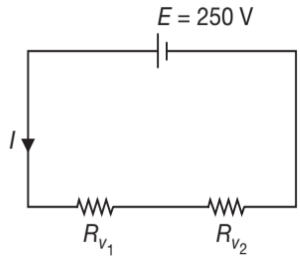
Further, $\dfrac{100R_V}{100 + R_V} = 80\ \Omega$

Hence, reading of voltmeter $= (0.04)(80) = 3.2$ V

Had it been an ideal voltmeter $(R_V \to \alpha)$, its reading would had been,

$$\left(\frac{3.4}{100 + 5}\right)(100) = 3.238 \text{ V}$$

16. (a) When the switch is open, no current flows through the branch that contains R_1 and R_2. Hence



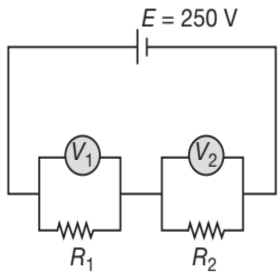
$$I = \frac{250}{10000} = \frac{1}{40} \text{ A}$$

According to Ohm's Law

$$V_1 = \frac{1}{40}(6000) = 150 \text{ V} \quad \text{and}$$

$$V_2 = (250 - 150) \text{ V} = 100 \text{ V}$$

(b) When the switch is closed the equivalent circuit is shown.



Since we observe that V_1 and R_1 are in parallel and V_2 and R_2 are in parallel and both in series and also simultaneously we have

$$\frac{R_1(R_{V_1})}{R_1 + R_{V_1}} = \frac{R_2(R_{V_2})}{R_2 + R_{V_2}} = 2.4 \text{ k}\Omega$$

Hence the potential of 250 V must be equally divided. So, reading of both the voltmeters is

$$V_1 = V_2 = \frac{E}{2} = 125 \text{ V}$$

17. (a) $\dfrac{RR_V}{R+R_V}=\dfrac{50\times 200}{50+200}=40\ \Omega$

Current in the circuit, $I=\dfrac{4.3}{1+40+2}=0.1\text{ A}$

and voltage across voltmeter $=40I=4\text{ V}$

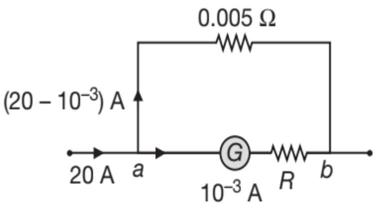
(b) $\dfrac{(R+R_A)R_V}{R+R_A+R_V}=\dfrac{52\times 200}{52+200}=41.27\ \Omega$

$I=\dfrac{4.3}{1+41.27}=0.102\text{ A}$

Reading of voltmeter $=41.27I=4.2\text{ V}$

Reading of ammeter $=\left(\dfrac{R_V}{R+R_A+R_V}\right)(I)=0.08\text{ A}$

18. $V_{ab}=(0.005)(20-10^{-3})=10^{-3}(20+R)$



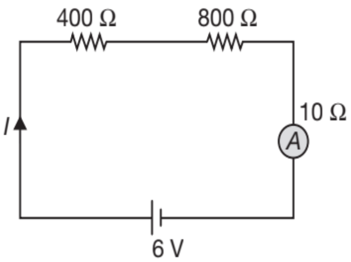
$\Rightarrow\quad R=79.995\ \Omega$

19. $\Delta R=G-\dfrac{GS}{G+S}=\dfrac{G^2}{G+S}$

20. $100=\left(\dfrac{2500\times r}{2500+r}\right)\times 5$

$\Rightarrow\quad r=20.16\ \Omega$

21. Refer figure (a): Current through ammeter,



(a)

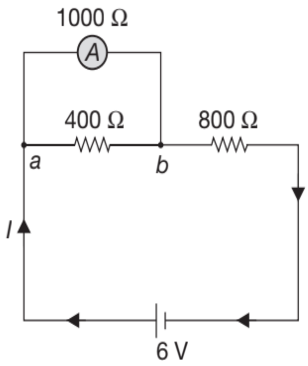
$I=\dfrac{\text{net emf}}{\text{net resistance}}=\dfrac{6}{400+800+10}=$

$4.96\times 10^{-3}\text{ A}=4.96\text{ mA}$

Refer figure (b): Combined resistance of $1000\ \Omega$ voltmeter and $400\ \Omega$ resistance is,

$R=\dfrac{1000\times 400}{1000+400}=285.71\ \Omega$

$\Rightarrow\quad I=\dfrac{6}{(285.71+800)}=5.53\times 10^{-3}\text{ A}$

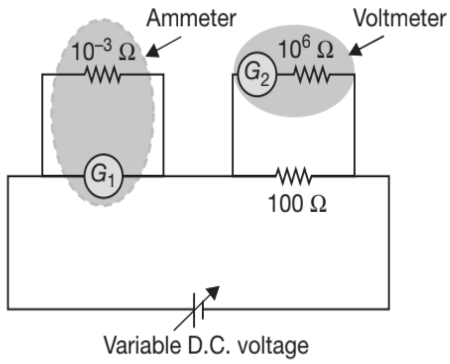


(b)

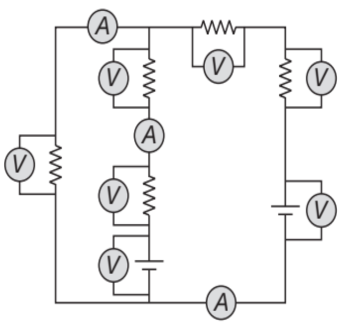
Reading of voltmeter

$=V_{ab}=I'R=(5.53\times 10^{-3})(285.71)=1.58\text{ V}$

22.



23. The usage of voltmeters and ammeters is shown in the figure.



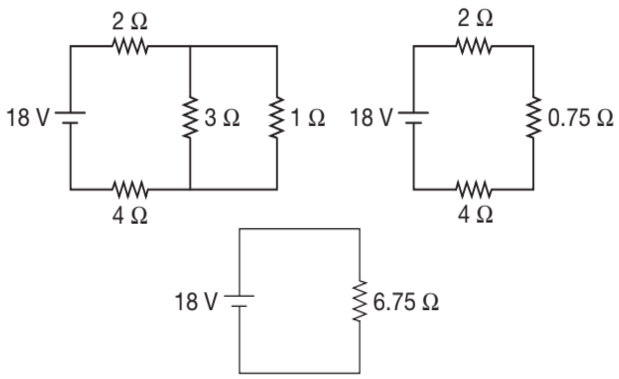
Test Your Concepts-IX (Based on Heating Effects and Power Consumption)

1. $\dfrac{1}{R_p}=\left(\dfrac{1}{3}+\dfrac{1}{1}\right)$

$\Rightarrow\quad R_p=0.75\ \Omega$

$R_s=(2+0.75+4)\Omega=6.75\ \Omega$

$I_{\text{battery}}=\dfrac{\Delta V}{R_S}=\dfrac{18\text{ V}}{6.75\ \Omega}=2.67\text{ A}$



Since, $P = I^2R \Rightarrow P_2 = (2.67\ \text{A})^2(2\ \Omega)$

$\Rightarrow\ P_2 = 14.2$ W in 2 Ω

$$P_4 = (2.67\ \text{A})^2(4\ \text{A}) = 28.4 \text{ W in } 4\ \Omega$$
$$\Delta V_2 = (2.67\ \text{A})(2\ \Omega) = 5.33\ \text{V}$$
$$\Delta V_4 = (2.67\ \text{A})(4\ \Omega) = 10.67\ \text{V}$$
$$\Delta V_p = 18\ \text{V} - \Delta V_2 - \Delta V_4 = 2\ \text{V}(= \Delta V_3 = \Delta V_1)$$

So, $P_3 = \dfrac{(\Delta V_3)^2}{R_3} = \dfrac{(2\ \text{V})^2}{3\ \Omega} = 1.33$ W in 3 Ω

$$P_1 = \frac{(\Delta V_1)}{R_1} = \frac{(2\ \text{V})^2}{1\ \Omega} = 4 \text{ W in } 1\ \Omega$$

2. A certain quantity of energy $\Delta E_{\text{int}} = P(\text{time})$ is required to raise the temperature of the water to 100°C. For the power delivered to the heaters we have $P = I\Delta V = \dfrac{(\Delta V)^2}{R}$ where ΔV is a constant. Thus comparing coils 1 and 2, we have for the energy

$$H = \frac{(\Delta V)^2 \Delta t}{R_1} = \frac{(\Delta V)^2 2\Delta t}{R_2}$$

$\Rightarrow\ R_2 = 2R_1$.

(a) When connected in parallel, the coils present equivalent resistance

$$R_p = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{2R_1}} = \frac{2R_1}{3}$$

and $H = \dfrac{(\Delta V)^2 \Delta t}{R_1} = \dfrac{(\Delta V)^2 \Delta t_p}{\dfrac{2R_1}{3}}$

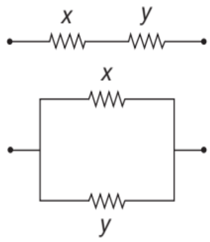
$$\Rightarrow\ \Delta t_p = \frac{2\Delta t}{3}$$

(b) For the series connection, $R_s = R_1 + R_2 = R_1 + 2R_1 = 3R_1$ and $\dfrac{(\Delta V)^2 \Delta t}{R_1} = \dfrac{(\Delta V)^2 \Delta t_s}{3R_1}$

$$\Rightarrow\ \Delta t_s = 3\Delta t$$

3. Let the two resistances be x and y.

Then, $R_s = x + y = \dfrac{P_s}{I^2}$ and $R_p = \dfrac{xy}{x+y} = \dfrac{P_p}{I^2}$



From the first equation, $y = \dfrac{P_s}{I^2} - x$ and

the second becomes $\dfrac{x\left(\dfrac{P_s}{I^2 - x}\right)}{x + \left(\dfrac{P_s}{I^2 - x}\right)} = \dfrac{P_p}{I^2}$

$$\Rightarrow\ x^2 - \left(\frac{P_s}{I^2}\right)x + \frac{P_sP_p}{I^4} = 0$$

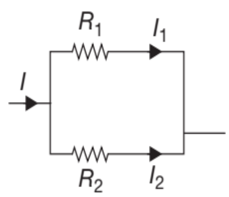
Using the quadratic formula, $x = \dfrac{P_s \pm \sqrt{P_s^2 - 4P_sP_p}}{2I^2}$

Then, $y = \dfrac{P_s}{I^2} - x$ gives $y = \dfrac{P_s \mp \sqrt{P_s^2 - 4P_sP_p}}{2I^2}$

The two resistances are $\dfrac{P_s + \sqrt{P_s^2 - 4P_sP_p}}{2I^2}$ and $\dfrac{P_s - \sqrt{P_s^2 - 4P_sP_p}}{2I^2}$

4. (a) $\Delta V_1 = \Delta V_2$

$I_1R_1 = I_2R_2$



$$\Rightarrow\ I = I_1 + I_2 = I_1 + \frac{I_1R_1}{R_2} = I_1\frac{R_2 + R_1}{R_2}$$

$$\Rightarrow\ I_1 = \frac{IR_2}{R_1 + R_2} \text{ and } I_2 = \frac{I_1R_1}{R_2} = \frac{IR_1}{R_1 + R_2} = I_2$$

(b) The power delivered to the pair is $P = I_1^2R_1 + I_2^2R_2 = I_1^2R_1 + (I - I_1)^2 R_2$. For minimum power let us find I_1 such that $\dfrac{dP}{dI_1} = 0$

$$\Rightarrow\ \frac{dP}{dI_1} = 2I_1R_1 + 2(I - I_1)(-1)R_2 = 0$$

$\Rightarrow \quad I_1R_1 - IR_2 + I_1R_2 = 0$

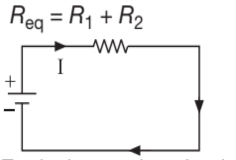
$\Rightarrow \quad I_1 = \dfrac{IR_2}{R_1 + R_2}$

This is the same condition as that derived in part (a).

5. For the resistors in series, from KVL we have

$$V_0 - I(R_1 + R_2) = 0$$

$$I = \frac{V_0}{(R_1 + R_2)}$$



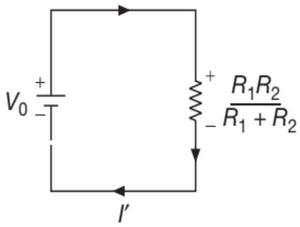
Equivalent series circuit

The power absorbed by equivalent resistor,

$$R_s = R_1 + R_2$$

$$\Rightarrow \quad I = \frac{V_0}{R_s} = \frac{V_0}{R_1 + R_2}$$

$$P = IV = \left(\frac{V_0}{R_1 + R_2}\right)V_0 = \frac{V_0^2}{R_1 + R_2}$$



Equivalent parallel circuit

For a parallel connection the equivalent resistance is

$$R_p = \frac{R_1R_2}{R_1 + R_2}$$

If I' be the current in the circuit , then

$$I' = \frac{V_0}{R_p} = \frac{V_0(R_1 + R_2)}{R_1R_2}$$

The power absorbed by equivalent resistor is,

$$P' = I'V = \left(V_0\frac{R_1 + R_2}{R_1R_2}\right)V_0 = V_0^2\left(\frac{R_1 + R_2}{R_1R_2}\right)$$

The ratio of the power absorbed in parallel and series circuit is

$$\frac{P'}{P} = \frac{V_0^2\left(\frac{(R_1 + R_2)}{R_1R_2}\right)}{\frac{V_0^2}{(R_1 + R_2)}} = \frac{(R_1 + R_2)^2}{R_1R_2}$$

$$\Rightarrow \quad \frac{P'}{P} = \frac{(R_1 + R_2)^2}{R_1R_2} = 2 + \frac{R_1}{R_2} + \frac{R_2}{R_1} > 1$$

The ratio is greater than unity. Therefore, power absorbed in a parallel connection of two resistors is greater than in a series connection of the same resistors for a given voltage.

6. (a) Power consumed in resistor 1 is

$$P_1 = \frac{V^2}{R_1} = \frac{V^2}{\left(\frac{\rho\ell}{A_1}\right)} = \frac{V^2A_1}{\rho\ell} = \frac{\pi}{4}\left(\frac{V^2D_1^2}{\rho\ell}\right)$$

Similarly power consumed in resistor 2 is

$$P_2 = \frac{V^2}{R_2} = \frac{\pi}{4}\frac{V^2D_2^2}{\rho\ell}$$

Since $D_1 = 2D_2$

$$\Rightarrow \quad \frac{P_1}{P_2} = \frac{D_1^2}{D_2^2} = \frac{4D_2^2}{D_2^2} = 4$$

(b) Power consumed in resistor 1 is

$$P_1 = I^2R_1 = I^2\frac{\rho\ell}{A_1} = \frac{4}{\pi}\frac{I^2\rho\ell}{D_1^2}$$

Power consumed in resistor 2 is

$$P_2 = I^2R_2 = \frac{4}{\pi}\frac{I^2\rho\ell}{D_2^2}$$

If $D_1 = 2D_2$, then $\dfrac{P_1}{P_2} = \dfrac{\frac{1}{D_1^2}}{\frac{1}{D_2^2}} = \dfrac{D_2^2}{4D_2^2} = \dfrac{1}{4}$

7. The ratio of resistances at the two temperatures is given by

$$\frac{R_{20}}{R_{2000}} = \frac{1 + 20\,\alpha_0}{1 + 2000\,\alpha_0} = \frac{1 + 20 \times 0.0055}{1 + 2000 \times 0.0055}$$

$$\Rightarrow \quad \frac{R_{20}}{R_{2000}} = 9.25 \times 10^{-2}$$

At 2000 °C: $R_{2000} = \dfrac{V^2}{P} = \dfrac{(110)^2}{100} = 121\ \Omega$

At 20 °C: $R_{20} = 121 \times 9.25 \times 10^{-2} = 11.2\ \Omega$

Current at 20 °C, $I_{20} = \dfrac{P}{V} = \dfrac{110}{11.2} = 9.82\ \text{A}$

Current at 2000 °C, $I_{2000} = \dfrac{P}{V} = \dfrac{100}{110} = 0.91\ \text{A}$

Thus we can see that at the instant the bulb is turned on and when temperature of filament is 20 °C, the current is very large (9.82 A) as compared to current at 2000 °C ($I_{2000} = 0.91$ A)

8. (a) When a current is passed through a wire, its temperature rises with time due to heat produced. Since the fuse wire is exposed to the surroundings it also loses heat mainly due to radiation, heat loss depends upon temperature and it increases as temperature rises. The temperature attains a steady value when the rate at which heat is produced in the wire is equal to the rate at which heat is lost due to radiation. Let the steady temperature be equal to the melting point of the wire and I be the current at the melting point. The rate at which heat is produced in the wire is given by

$$P = I^2R$$

where R is the resistance of the wire given by

$$R = \frac{\rho\ell}{\pi r^2}$$

where ρ = resistivity, ℓ = length and r = radius of the wire. Thus

$$P = \frac{I^2\rho\ell}{\pi r^2} \quad \ldots(1)$$

Now let h be the rate of loss of heat per unit surface area of the wire. The rate of loss of heat from the wire is

$$P' = 2\pi r\ell \times h \quad \ldots(2)$$

In the steady state, $P = P'$. Therefore equating equations (1) and (2) we have

$$\frac{I^2\rho\ell}{\pi r^2} = 2\pi r\ell h$$

$$\Rightarrow \quad I = \sqrt{\frac{2\pi^2 r^3 h}{\rho}} \quad \ldots(3)$$

Thus the current at which the melting point is reached is independent of the length, ℓ of the wire, it depends upon the values of r, ρ and h. If the two wires have the same value of r, they are made of the same material (ρ is the same for both wires), the rate of loss of heat per unit surface area (h) depends upon the material of the wire and the melting point which is same for both the wires, both fuses will melt at the same value of current, inspite of different lengths.

(b) From equation (3), we have

$$I = \sqrt{\frac{2\pi^2 r^3 h}{\rho}}$$

$$\Rightarrow \quad \frac{I^2}{r^3} = \frac{2\pi^2 h}{\rho} = \text{constant}$$

$$\Rightarrow \quad \frac{I_1^2}{r_1^3} = \frac{I_2^2}{r_2^3}$$

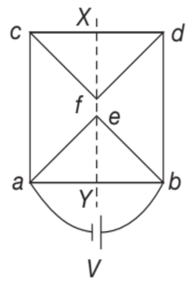
$$\Rightarrow \quad r_2 = r_1\left(\frac{I_2}{I_1}\right)^{\frac{2}{3}}$$

Substituting numerical values $r_1 = 0.1$ mm, $I_1 = 10$ A and $I_2 = 20$ A, we have

$$r_2 = (0.1)\left(\frac{20}{10}\right)^{\frac{2}{3}} = (0.1)(2)^{\frac{2}{3}}$$

$$\Rightarrow \quad r_2 = 0.159 \text{ mm} \approx 0.16 \text{ mm}$$

9. The circuit has asymmetry about line XY, i.e., the current in left and right are not mirror images. Just imagine the central junction of wires in the form of two junctions connected by wire ef as shown. Then it follows from symmetry consideration that there is no current in the wire ef, thus we can remove it from the circuit. The resistors of the wires will be proportional to their lengths.



$$R_{ab} = R_{ac} = R_{cd} = R_{bd} = r$$

$$R_{ae} = R_{be} = R_{cf} = R_{df} = \frac{r}{\sqrt{2}}$$

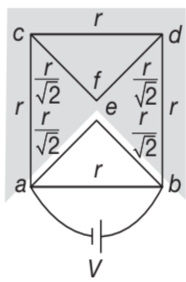
The power dissipated in resistor ab is

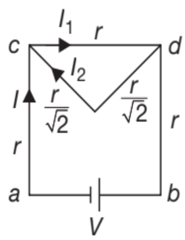
$$P_{ab} = \frac{V^2}{r}$$

By Applying Ohm's Law to the upper part, we can determine current through cd,

$$I_{cd} = \frac{V}{(\sqrt{2}+3)r} \qquad \{\because V_{ab} = V_{aeb} = V_{\text{shaded}} = V\}$$

The power dissipated in the conductor cd,





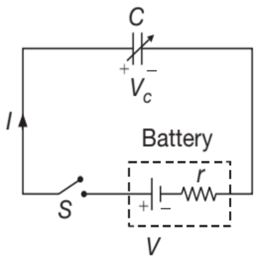
$$P_2 = I_{cd}^2 r = \frac{V^2}{(\sqrt{2}+3)^2 r}$$

Thus the required ratio is

$$\frac{P_1}{P_2}=\left(\sqrt{2}+3\right)^2=11+6\sqrt{2}$$

10. (a) The current in the circuit is related to the voltage across the capacitor as

$$I=C\frac{dV_c}{dt}$$



As the capacitor charges up, V_C rises toward its limiting value and its rate of change goes to zero. Therefore, in order to keep I constant, capacitance, C, should compensate for this drop in $\frac{dV_C}{dt}$ (observe that, as time $t\to\infty$, $\frac{dV_c}{dt}\to 0$ and we should have $C_\infty\to\infty$). Ignoring the practical difficulties of maintaining a constant I, asymptotically we have a continuous charge flow in the circuit

$$I=\frac{dq}{dt}$$

and $V_C=V$

The battery supplies a constant power since its emf is constant,

$$P_B=VI$$

(b) The capacitor is being continuously charged. Once the voltage across the capacitor stabilizes at $t\to\infty$, we have

$$V=Ir+V_{C,\infty}$$

$$\Rightarrow \quad V_{C,\infty}=V-Ir$$

The energy stored in the capacitor becomes

$$W=\frac{1}{2}q_\infty V_{C,\infty}$$

$$\Rightarrow \quad W=\frac{1}{2}q_\infty(V-Ir)$$

The rate of change of W with time is

$$P_c=\frac{dE}{dt}=\frac{1}{2}V_{c,\infty}\frac{dq_\infty}{dt}=\frac{1}{2}I(V-Ir)$$

The rate of dissipation of energy in the resistor r is $P_r=I^2r$,

Therefore the power supplied directly to the capacitor is

$$P=P_B-P_r=IV-I^2r=I(V-Ir),$$

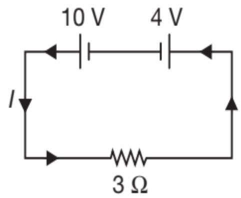
which is twice as large as that used in charging the capacitor. So,

$$P=2P_c$$

The battery supplies twice as much energy to the capacitor as is stored in the capacitor. The difference is the work done by the capacitor against the external agent which is changing the capacitance.

11. Net emf of the circuit $=(10-4)$ V $=6$ V

Total resistance of the circuit $=3\ \Omega$



$\Rightarrow$ Current in the circuit $I=\frac{\text{net emf}}{\text{total resistance}}=\frac{6}{3}=2$ A

(a) Power supplied by 10 V battery

$=EI=(10)(2)=20$ W

(b) Power consumed by 4 V battery

$=EI=(4)(2)=8$ W

(c) Power consumed by $3\ \Omega$ resistance

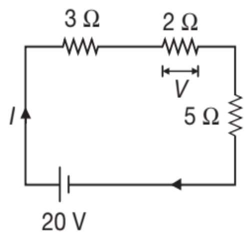
$=I^2R=(2)^2(3)=12$ W

Here we can see that total power supplied by 10 V battery (i.e., 20 W) = power consumed by 4 V battery and $3\ \Omega$ resistance. Which proves that conservation of energy holds good in electric circuits also.

12. The $6\ \Omega$ and $3\ \Omega$ resistances are in parallel. So their combined resistance is,

$$\frac{1}{R}=\frac{1}{6}+\frac{1}{3}=\frac{1}{2} \text{ or } R=2\ \Omega$$

The equivalent simple circuit can be drawn as shown. Current in the circuit,



$$I=\frac{\text{net emf}}{\text{total resistance}}=\frac{20}{3+2+5}=2\text{ A}$$

$$V=IR=(2)(2)=4\text{ V}$$

i.e., potential difference across $6\ \Omega$ and $3\ \Omega$ resistances are 4 V. Now,

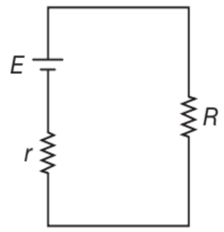
$$H_{3\,\Omega} = I^2Rt = (2)^2(3)(2) = 24 \text{ J}$$

$$H_{6\,\Omega} = \frac{V^2}{R}t = \frac{(4)^2}{6}(2) = \frac{16}{3} \text{ J}$$

$$H_{3\,\Omega} = \frac{V^2}{R}t = \frac{(4)^2(2)}{3} = \frac{32}{3} \text{ J}$$

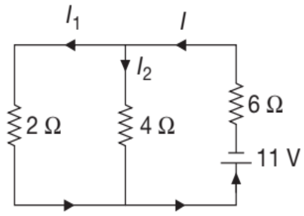
and $H_{5\,\Omega} = I^2Rt = (2)^2(5)(2) = 40$ J

13. Suppose, we insert the battery with $2\,\Omega$ resistance. Then we can take $2\,\Omega$ as the internal resistance (r) of the battery and combined resistance of the other two as the external resistance (R). The circuit in that case shown in figure.



Now power, $P = \dfrac{E^2}{R+r}$

This power will be minimum where $R+r$ is maximum and we can see that $(R+r)$ will be maximum when the battery is inserted with $6\,\Omega$ resistance as shown in figure.



Net resistance in this case is

$$6 + \frac{2\times 4}{2+4} = \frac{22}{3}\ \Omega$$

$$\Rightarrow \quad I = \frac{11}{22/3} = 1.5 \text{ A}$$

This current will be distributed in $2\,\Omega$ and $4\,\Omega$ in the inverse ratio of their resistances.

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{4}{2} = 2$$

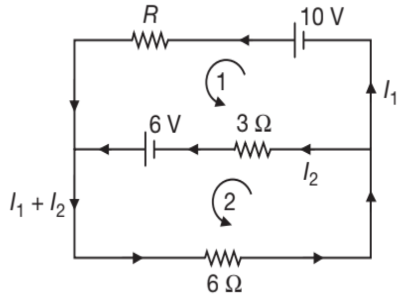
$$\Rightarrow \quad I_1 = \left(\frac{2}{2+1}\right)(1.5) = 1 \text{ A}$$

So, $P_6 = (1.5)^2 6 = 13.5$ W

$P_4 = (0.5)^2 4 = 1$ W

$P_2 = (1)^2(2) = 2$ W

14. Applying Kirchhoff's Laws to the two loops we have,



$$10 - I_1R - 6 + 3I_2 = 0 \qquad \ldots(1)$$

$$6 - 6(I_1 + I_2) - 3I_2 = 0 \qquad \ldots(2)$$

Solving these two equations we get,

$$I_1 = \frac{6}{R+2}$$

Power developed in R,

$$P = I_1^2R = \frac{36}{(R+2)^2}R \qquad \ldots(3)$$

For power to be maximum,

$$\frac{dP}{dR} = 0$$

$\Rightarrow \quad (R+2)^2(36) - (36R)(2)(R+2) = 0$

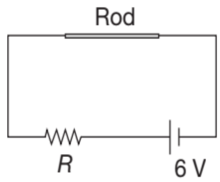
$\Rightarrow \quad R + 2 - 2R = 0$

$\Rightarrow \quad R = 2\,\Omega$

For maximum power from equation (3), we have

$P_{max} = 4.5$ W

15. (a) At $I = 0.4$ A, voltage across the rod,



$$V = \left(\frac{0.4}{0.2}\right)^2 = 4 \text{ V}$$

$\Rightarrow \quad V_R = 6 - 4 = 2$ V

Now, $IR = V_R = 2$ V

$$\Rightarrow \quad R = \frac{2}{0.4} = 5\,\Omega$$

(b) $P_{supplied} = EI = 6I$

$P_R = I^2R$

$\Rightarrow \quad P_{rod} = 6I - I^2R$

Given that, $6I - I^2R = 2I^2R$

$\Rightarrow \quad 6 - IR = 2IR$

CHAPTER 3

$\Rightarrow \quad IR = 2 \qquad \ldots(1)$

Further, $6 = V_R + V_{rod} = IR + \dfrac{I^2}{0.04} \qquad \ldots(2)$

$$\left\{\because I = 0.2\,V^{\frac{1}{2}} \Rightarrow I^2 = 0.04V_{rod}\right\}$$

Solving equations (1) and (2), we get

$R = 5\,\Omega$

16. $V = E - Ir$

$\Rightarrow \quad 2 = 2.6 - (1)(r)$

$\Rightarrow \quad r = 0.6\,\Omega$

Power generated by battery $= VI = 2$ W

and power developed in it by electric forces $= I^2 r = 0.6$ W

17. (a) By the 4 V battery

$$\Delta U = (\Delta V)I\Delta t = (4\text{ V})(-0.462\text{ A})120\text{ s} = -222\text{ J}$$

By the 12 V battery

$$(12\text{ V})(1.31\text{ A})120\text{ s} = 1.88\text{ kJ}$$

(b) By the $8\,\Omega$ resistor

$$I^2R\Delta t = (0.846\text{ A})^2(8\,\Omega)120\text{ s} = 687\text{ J}$$

By the $5\,\Omega$ resistor

$$I^2R\Delta t = (0.462\text{ A})^2(5\,\Omega)120\text{ s} = 128\text{ J}$$

By the $1\,\Omega$ resistor

$$I^2R\Delta t = (0.462\text{ A})^2(1\,\Omega)120\text{ s} = 25.6\text{ J}$$

By the $3\,\Omega$ resistor

$$I^2R\Delta t = (1.31\text{ A})^2(3\,\Omega)120\text{ s} = 616\text{ J}$$

By the $1\,\Omega$ resistor

$$I^2R\Delta t = (1.31\text{ A})^2(1\,\Omega)120\text{ s} = 205\text{ J}$$

(c) $-222\text{ J} + 1.88\text{ kJ} = 1.66\text{ kJ}$ from chemical to electrical

$687\text{ J} + 128\text{ J} + 25.6\text{ J} + 616\text{ J} + 205\text{ J} = 1.66\text{ kJ}$ from electrical to internal

Test Your Concepts-X (Based on Wheatstone Bridge and Potentiometer)

1. (a) The value of E is given by ($E_1 = 1.02$ V is the emf of the standard cell C_1)

$$E = \frac{1.02 \times 82.3}{67.3} = 1.25\text{ V}$$

(b) The high resistance R keeps the current drawn from the standard cell within permissible limit and prevents a large current to flow through the galvanometer when far away from the balance point.

(c) At null point no current flows through R, hence no effect on null point.

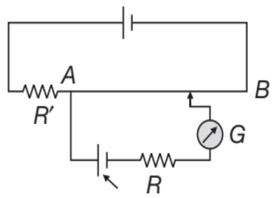
(d) The null point depends on the terminal voltage of C and the emf of C_1 only, hence no effect.

(e) The method would not work if the potential difference across AB due to the driver cell C became less than the emf of cell C_1, because there would be no null point on the wire.

(i) In this case the potential difference across AB due to cell C of emf 2 V will be less than 1 V. Since the emf of the standard cell is greater than this value, there will be no null point on the wire. So, it will not work.

(ii) Similarly, the method would not work if the emf of C were 1 V instead of 2 V.

(f) Suppose the emf E to be measured is, say, $1\text{ mV} = 1 \times 10^{-3}$ V, the potential difference across AB due to C is 2 V, the length AB of the wire is 100 cm and its resistance per centimetre is λ (say). Then the balance length will be



1×10^{-3} V = P.D. across AJ due to C = current in the main circuit $\times$ resistance of AJ

$$\Rightarrow \quad 1 \times 10^{-3} = \frac{2}{100\lambda}\lambda AJ = \frac{2\,AJ}{100}\text{ V}$$

$\Rightarrow \quad AJ = 0.05$ cm

Thus the null point will be very close to A and there is an extremely large percentage error in its measurement. To have a large balance length, the circuit shown in figure is modified by putting a suitable resistor R'. The balance length will then be measurable and the percentage error will be much smaller.

2. Let E be the emf of the cell C' and r its internal resistance. Let $\ell = AJ$ be the balance length when switch S is open. When a resistance R is introduced by closing the switch, a current begins to flow through the cell C' and resistance R. The potential difference between the terminals of the cell falls and the balance length decreases to $\ell' = AJ'$. The internal resistance of the cell is given by

$$r = \frac{E - V}{I}$$

where V is the terminal voltage of C' and I is the current in the circuit involving C' and R. Also

$$I = \frac{V}{R}$$

$$\Rightarrow \quad r = \left(\frac{E}{V} - 1\right)R$$

But $\frac{E}{V} = \frac{\ell}{\ell'}$. Hence

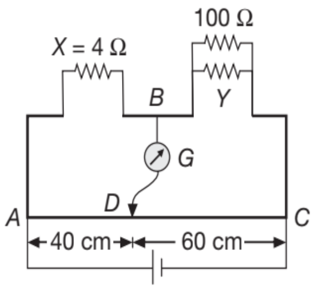
$$\Rightarrow \quad r = R\left(\frac{\ell - \ell'}{\ell'}\right) = 4\left(\frac{76.3 - 60}{60}\right) = 1.1\ \Omega$$

3. The resistances of the two segments of the wire AD and DC are in the ratio of their lengths. If R_0 is the resistance of Y in melting ice $(0\ °C)$, the balance condition of Wheatstone bridge gives

$$\frac{X}{R_0} = \frac{\lambda\ell}{\lambda(100-\ell)} = \frac{\ell}{100-\ell}$$

where λ is the resistance per centimetre of wire AC. Now, $\ell = 40$ cm and $X = 4\ \Omega$. Substituting these values, we get $R_0 = 6\ \Omega$. Let R_t be the resistance of Y when heated to a temperature $t = 100\ °C$. When it is connected in parallel with $100\ \Omega$ resistor as shown in figure, the net resistance becomes

$$R' = \frac{100R_t}{R_t + 100}$$



Since the null point remains unchanged, we have

$$\frac{X}{R'} = \frac{40}{60}$$

$$\Rightarrow \quad R' = 6\ \Omega$$

$$\Rightarrow \quad 6 = \frac{100R_t}{R_t + 100}$$

$$\Rightarrow \quad R_t = 6.38\ \Omega$$

Temperature coefficient of resistance of the coil Y is

$$\alpha = \frac{R_t - R_0}{R_0 t} = \frac{6.38 - 6}{6 \times 100} = 6.3 \times 10^{-4}\ \text{K}^{-1}$$

4. $$\frac{E}{V} = \frac{0.52}{0.4} = \frac{13}{10} = 1.3$$

$$\Rightarrow \quad \frac{E}{E - Ir} = 1.3$$

$$\Rightarrow \quad 1.3Ir = 0.3E$$

$$\Rightarrow \quad 1.3\left(\frac{Er}{R+r}\right) = 0.3E \qquad \left\{\because I = \frac{E}{R+r}\right\}$$

$$\Rightarrow \quad r = 0.3R = (0.3)(5) = 1.5\ \Omega$$

5. Emf of cell = (potential gradient) × (balancing length)

$$\Rightarrow \quad E_1 = \frac{V_{AB}}{L} \times x$$

$$\Rightarrow \quad 2.4 = \frac{V_{AB}}{10} \times 8$$

$$\Rightarrow \quad V_{AB} = 3\ \text{V}$$

Consider the loop containing E and applying potential divider concept, we get

$$V_{AB} = E\frac{R_{AB}}{R_{AB} + r}$$

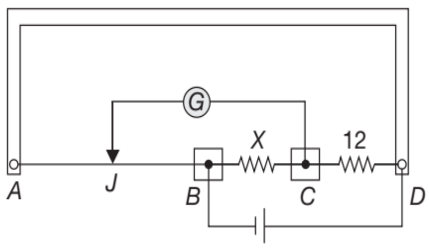
$$\Rightarrow \quad 3 = 4\frac{(1.6)(10)}{(1.6)(10) + r} \qquad \left\{\because R_{AB} = (1.6\ \Omega\text{m}^{-1})(10\ \text{m})\right\}$$

$$\Rightarrow \quad r = \frac{16}{3}\ \Omega$$

Note that as there is no current through the cell and galvanometer, the battery E, the internal resistance r and the potentiometer wire AB are in series.

6. (a) There are no positive and negative terminals on the galvanometer because only zero deflection is needed.

(b)



(c) $AJ = 60$ cm

$$\Rightarrow \quad BJ = 40\ \text{cm}$$

If no deflection is taking place. Then, the Wheatstone bridge is said to be balanced.

Hence, $\frac{X}{12} = \frac{R_{BJ}}{R_{AJ}}$

$$\Rightarrow \quad \frac{X}{12} = \frac{40}{60} = \frac{2}{3}$$

$$\Rightarrow \quad x = 8\ \Omega$$

7. Slide wire bridge is most sensitive when the resistance of all the four arms of bridge is same.
Hence, for the resistor B the value of X is the most accurate answer.

CHAPTER 3

8. Resistance of x metre length of wire,

$$R_1 = \frac{R_0}{L}x$$

Resistance R and R_1 are in parallel, so their effective resistance,

$$R' = \frac{RR_1}{R+R_1}$$

This is in series with length $(L-x)$ of wire. Resistance of length $(L-x)$ of wire is

$$R_2 = \frac{R_0}{L}(L-x)$$

Hence, total resistance is given by

$$R_{eff} = R' + R_2 = \frac{RR_1}{R+R_1} + R_2$$

So, current in the main circuit is given by

$$I = \frac{V_0}{R_{eff}} = \frac{V_0}{\left(\frac{RR_1}{R+R_1} + R_2\right)}$$

Potential difference across the length x is

$$V = IR' = I\frac{RR_1}{R+R_1}$$

$$\Rightarrow \quad V = \frac{V_0}{\left(\frac{RR_1}{R+R_1} + R_2\right)}\left(\frac{RR_1}{R+R_1}\right)$$

$$\Rightarrow \quad V = \frac{V_0RR_1}{RR_1 + RR_2 + R_1R_2}$$

$$\Rightarrow \quad V = \frac{V_0RR_1}{R(R_1+R_2)+R_1R_2} = \frac{V_0RR_1}{R_0R + R_1R_2}$$

$$\Rightarrow \quad V = \frac{V_0R\left(\frac{R_0}{L}x\right)}{R_0R + \left(\frac{R_0}{L}x\right)\left(\frac{R_0}{L}(L-x)\right)}$$

$$\Rightarrow \quad V = \frac{V_0Rx}{RL + R_0x\left(1-\frac{x}{L}\right)}$$

For $R \gg R_0$, we get $V \cong \frac{V_0x}{L}$

9. Since $r = \left(\frac{\ell_{\text{key open}}}{\ell_{\text{key closed}}} - 1\right)R$

$$\Rightarrow \quad r = \left(\frac{60}{40} - 1\right)4$$

$$\Rightarrow \quad r = 2\ \Omega$$

10. Since $r = \left(\frac{\ell_{\text{key open}}}{\ell_{\text{key closed}}} - 1\right)R$

$$\Rightarrow \quad r = \left(\frac{75}{60} - 1\right)6$$

$$\Rightarrow \quad r = \left(\frac{5}{4} - 1\right)6$$

$$\Rightarrow \quad r = 1.5\ \Omega$$

Test Your Concepts-XI (Based on RC Circuit)

1. $Q(t) = \int_0^t I dt = I_0\tau\left(1 - e^{-\frac{t}{\tau}}\right)$

(a) $Q(\tau) = I_0\tau(1-e^{-1}) = (0.632)I_0\tau$

(b) $Q(10\tau) = I_0\tau(1-e^{-10}) = (0.99995)I_0\tau$

(c) $Q(\infty) = I_0\tau(1-e^{-\infty}) = I_0\tau$

2. (a) The resistance of the dielectric block is $R = \frac{\rho\ell}{A} = \frac{d}{\sigma A}$

The capacitance of the capacitor is $C = \frac{\kappa\varepsilon_0 A}{d}$

Then $RC = \frac{d}{\sigma A}\frac{\kappa\varepsilon_0 A}{d} = \frac{\kappa\varepsilon_0}{\sigma}$ is a characteristic of the material only.

(b) $R = \frac{\kappa\varepsilon_0}{\sigma C} = \frac{\rho\kappa\varepsilon_0}{C} = \frac{75\times10^{16}(3.78)8.85\times10^{-12}}{14\times10^{-9}}$

$R = 1.79\times10^{15}\ \Omega$

3. (a) $I(t) = \frac{dq}{dt} = \frac{d}{dt}\left(Qe^{-\frac{t}{RC}}\right) = -I_0e^{-\frac{t}{RC}}$

where, $I_0 = \frac{Q}{RC} = \frac{5.1\times10^{-6}}{(1300)(2\times10^{-9})} = 1.96\text{ A}$

$$\Rightarrow \quad I(t) = -(1.96\text{ A})\exp\left[\frac{-9\times10^{-6}}{(1\,300)(2\times10^{-9})}\right]$$

$$I(t) = -61.6\text{ mA}$$

(b) $q(t) = Qe^{-\frac{t}{RC}} = (5.1\ \mu\text{C})\exp\left[\frac{-8\times10^{-6}}{(1\,300)(2\times10^{-9})}\right]$

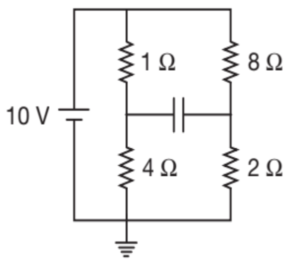
$= 0.235\ \mu\text{C}$

(c) The magnitude of the maximum current is $I_0 = 1.96\text{ A}$

4. $U=\frac{1}{2}C(\Delta V)^2$ and $\Delta V=\frac{Q}{C}$

Therefore, $U=\frac{Q^2}{2C}$ and when the charge decreases to half its original value, the stored energy is one-quarter its original value and so, $U_f=\frac{1}{4}U_0$.

5. (a) Let us denote the potential at the left junction V_L and at the right V_R. After a long time, the capacitor is fully charged.



$V_L=8$ V because of voltage divider

$$I_L=\frac{10\text{ V}}{5\ \Omega}=2\text{ A}$$

$$V_L=10\text{ V}-(2\text{ A})(1\ \Omega)=8\text{ V}$$

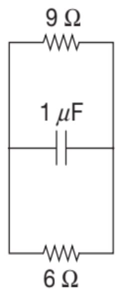
Similarly, $V_R=\left(\frac{2\ \Omega}{2\ \Omega+8\ \Omega}\right)(10\text{ V})=2\text{ V}$

$$\Rightarrow\quad I_R=\frac{10\text{ V}}{10\ \Omega}=1\text{ A}$$

$$V_R=(10\text{ V})-(8\ \Omega)(1\text{ A})=2\text{ V}$$

Therefore, $\Delta V=V_L-V_R=8-2=6$ V

(b) Redraw the circuit



$$R=\frac{1}{\left(\frac{1}{9}\ \Omega\right)+\left(\frac{1}{6}\ \Omega\right)}=3.6\ \Omega$$

$$RC=3.6\times10^{-6}\text{ s}$$

and $e^{-\frac{t}{RC}}=\frac{1}{10}$

$$\Rightarrow\quad t=RC\log_e 10=8.29\ \mu\text{s}$$

6. The total resistance between points b and c is

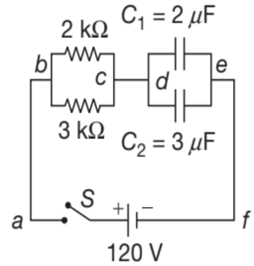
$$R=\frac{(2)(3)}{2+3}=1.2\text{ k}\Omega$$

The total capacitance between points d and e is

$$C=2\ \mu\text{F}+3\ \mu\text{F}=5\ \mu\text{F}$$

The potential difference between point d and e in this series RC circuit at any time is

$$\Delta V=E\left[1-e^{-\frac{t}{RC}}\right]=(120\text{ V})\left[1-e^{-\frac{1000t}{6}}\right]$$



Therefore, the charge on each capacitor between points d and e is

$$q_1=C_1\Delta V=(2\ \mu\text{F})(120\text{ V})\left[1-e^{-\frac{1000t}{6}}\right]$$

$$\Rightarrow\quad q_1=(240\ \mu\text{C})\left[1-e^{-\frac{1000t}{6}}\right]$$

$$q_2=C_2(\Delta V)=(3\ \mu\text{F})(120\text{ V})\left[1-e^{-\frac{1000t}{6}}\right]$$

$$\Rightarrow\quad q_2=(360\ \mu\text{C})\left[1-e^{-\frac{1000t}{6}}\right]$$

7. The current in a charging series RC circuit is given by

$$I=I_0e^{-\frac{t}{RC}}$$

when the current is halved,

$$\frac{I_0}{2}=I_0e^{-\frac{t}{RC}}$$

$$\Rightarrow\quad \frac{1}{2}=e^{-\frac{t}{RC}}$$

Taking the natural logarithms on both sides,

$$\log_e 1-\log_e 2=-\frac{t}{RC}$$

On solving for t, we get

$$t=RC\log_e 2=\tau\log_e 2=0.693\tau$$

8. The charge at time t in a discharging circuit is given by

$$q=Q_0e^{-\frac{t}{RC}}=Q_0e^{-\frac{t}{\tau}}$$

When the charge is halved,

$$\frac{Q_0}{2} = Q_0 e^{-\frac{t}{\tau}}$$

$$\Rightarrow \frac{1}{2} = e^{-\frac{t}{\tau}}$$

Taking the natural logarithms on both sides, we get

$$-0.693 = -\frac{t}{\tau}$$

$$\Rightarrow t = 0.693\tau$$

9. The electric potential energy stored in the capacitor is

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2C}Q_0^2 e^{-\frac{2t}{\tau}}$$

Since $U_0 = \frac{Q_0^2}{2C}$ is initial potential energy, we have

$$U = U_0 e^{-\frac{2t}{\tau}}$$

When the potential energy is half its initial value, we have

$$\frac{1}{2} = e^{-\frac{2t}{\tau}}$$

Taking natural logarithms of both sides, we get

$$-0.693 = -\frac{2t}{\tau}$$

$$\Rightarrow t = 0.34\tau$$

The potential energy decreases to half its value in half the time it takes the charge to decrease to half its value.

10. (a) Rate at which energy is dissipated in the resistor is

$$P(t) = I^2 R$$

$$\Rightarrow P(t) = \left(\frac{V_0}{R}e^{-\frac{t}{RC}}\right)^2 R$$

$$\Rightarrow P(t) = \frac{V^2}{R}e^{-\frac{2t}{RC}}$$

(b) We find the total joule heat by integrating from $t = 0$ to $t \to \infty$

$$W = \int P(t)\,dt = \int_0^\infty \frac{V_0^2}{R}e^{-\frac{2t}{RC}}\,dt$$

$$\Rightarrow W = \frac{V_0^2 C}{2}\left(e^{-\frac{2t}{RC}}\right)_0^\infty$$

$$\Rightarrow W = \frac{V_0^2 C}{2}(-0+1)$$

$$\Rightarrow W = \frac{1}{2}CV_0^2$$

Observation: The result is independent of R. Therefore, when a capacitor is charged by a battery with a constant emf, half the energy provided by the battery is stored in the capacitor and half goes into thermal energy, independent of resistance; the thermal energy includes the energy that goes into the internal resistance of the battery.

While charging, the charge flowing through battery = CV_0

Work done by battery $= (CV_0)V_0 = CV_0^2$

Power dissipated in the resistor $= \frac{1}{2}CV_0^2$

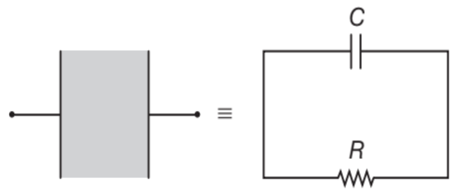
Energy stored in the capacitor $= \frac{1}{2}CV_0^2$

11. The problem is basically of discharging of CR circuit, because between the plates of the capacitor, there is capacitor as well as resistance.

$$R = \frac{d}{\sigma A} \qquad \left\{\because R = \frac{\ell}{\sigma A}\right\}$$

and $C = \frac{K\varepsilon_0 A}{d}$

$\therefore$ Time constant, $\tau_C = CR = \frac{K\varepsilon_0}{\sigma}$



Substituting the values, we have

$$\tau_1 = \frac{5 \times 8.86 \times 10^{-12}}{7.4 \times 10^{-12}} = 5.98\ s$$

Charge at any time decrease exponentially as

$$q = q_0 e^{-t/\tau_C}$$

Here $q_o = 8.85 \times 10^{-6}\,C$ (Charge at time $t = 0$)

Therefore, discharging (leakage) current at time t will be given by

$$I = \left(-\frac{dq}{dt}\right) = \frac{q_o}{\tau_C}e^{-t/\tau_C}$$

or current at $t = 12\ s$ is

$$I = \frac{\left(8.85 \times 10^{-6}\right)}{5.98}e^{-12/5.98} = 0.198 \times 10^{-6}\,A = 0.198\ \mu A$$

12. In SITUATION 1, the capacitor and resistor form a series RC circuit. Charge at time t is

$$q = q_0\left(1 - e^{-\frac{t}{CR}}\right)$$

$$\Rightarrow \quad CV = CV_0\left(1 - e^{-\frac{t}{CR}}\right)$$

$$\Rightarrow \quad \frac{V}{V_0} = \left(1 - e^{-\frac{t}{CR}}\right)$$

$$\Rightarrow \quad \frac{0.75}{1.5} = \left(1 - e^{-\frac{t}{CR}}\right)$$

$$\Rightarrow \quad e^{-\frac{t}{CR}} = \frac{1}{2}$$

Taking natural logarithm on both sides, we have

$$t = CR\log_e 2$$

$$\Rightarrow \quad t = \left(10\times10^{-6}\right)\left(0.1\times10^{6}\right)\log_e 2$$

$$\Rightarrow \quad t = \log_e 2$$

$$\Rightarrow \quad t = 0.693 \text{ s}$$

In SITUATION 2, the capacitor is connected in parallel with the battery. Initially it acts like a short circuit and will be charged instantaneously. Now,

$$RC = \frac{8.846\times10^{-12}\times5}{7.4\times10^{-12}}$$

$$\Rightarrow \quad RC = 6$$

$$\Rightarrow \quad I = \frac{8.85\times10^{-6}}{6}e^{-\frac{12}{6}}$$

$$\Rightarrow \quad I = 0.24\ \mu\text{A} \qquad \{\text{where } e = 2.718 \text{ and } e^2 = 7.39\}$$

13. The charge on a capacitor at time t is

$$q = q_0\left(1 - e^{-\frac{t}{RC}}\right)$$

$$\Rightarrow \quad CV = CV_0\left(1 - e^{-\frac{t}{RC}}\right)$$

So, the potential difference at time t is given by

$$V = V_0\left(1 - e^{-\frac{t}{RC}}\right) \qquad \ldots(1)$$

Let the potential difference V_e be attained at time t and V_f at time $t+T$, we have

$$V_e = V_0\left(1 - e^{-\frac{t}{RC}}\right) \qquad \ldots(2)$$

$$V_f = V_0\left(1 - e^{-\frac{(t+T)}{RC}}\right) \qquad \ldots(3)$$

From equation (2),

$$e^{-\frac{t}{RC}} = V_0 - V_e \qquad \ldots(4)$$

From equation (3),

$$e^{-\frac{(t+T)}{RC}} = V_0 - V_f \qquad \ldots(5)$$

Dividing equation (5) by (4), we get

$$\frac{V_0 - V_f}{V_0 - V_e} = e^{-\frac{(t+T)}{RC}+\frac{t}{RC}} = e^{-\frac{T}{RC}} \qquad \ldots(6)$$

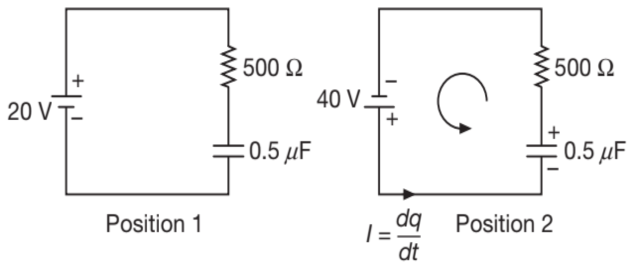
Now taking logarithm of both sides of equation (6), we get

$$T = RC\log_e\left(\frac{V_0 - V_e}{V_0 - V_f}\right)$$

14. When the switch is shifted to position 1, the capacitor gets charged and the instantaneous charge is

$$q = \left(0.5\times10^{-6}\right)(20)\left(1 - e^{-4000t}\right)$$

$$\Rightarrow \quad q = 10\left(1 - e^{-4000t}\right)\ \mu\text{C} \qquad \ldots(1)$$

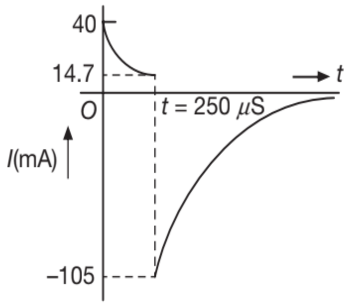


Since time constant, $\tau = RC$

$$\Rightarrow \quad \tau = (500)\left(0.5\times10^{-6}\right) = 250\ \mu\text{s}$$

$$\Rightarrow \quad \frac{1}{\tau} = \frac{10^6}{250} = 4000\ \text{s}^{-1}$$

At $t = 250\ \mu$s, we have $q = 10\left(1 - e^{-1}\right) = 6.3\ \mu$C



When the switch is shifted to position 2 at $t = 250\ \mu$s, applying KVL, we get

$$-R\frac{dq}{dt} - \frac{q}{C} - 40 = 0$$

Putting $R = 500\ \Omega, C = 0.5\ \mu\text{F}$ and $\frac{1}{\tau} = 4000\ \text{s}^{-1}$, we get

$$\int_{6.3\ \mu C}^{q}\frac{dq}{q+20} = -4000\int_{250\ \mu s}^{t} dt$$

$$\Rightarrow \quad \log_e\left|\frac{q+20}{26.3}\right| = -(4000t-1)$$

$$\Rightarrow \quad q = 26.3e^{-(4000t-1)} - 20 = \left(26.3e^{-4000t}\right)(2.718) - 20$$

$$\Rightarrow \quad q = 71.5e^{-4000t} - 20 \qquad \ldots(2)$$

Differentiating equations (1) and (2), we get

$$I(in\ \text{mA}) = \frac{dq}{dt} = \begin{bmatrix} 40e^{-4000t} & 0 < t < 250\ \mu s \\ -286e^{-4000t} & t > 250\ \mu s \end{bmatrix}$$

15. Time constant of the circuit,

$$\tau = CR = (10^{-4})\left(\frac{20\times 40}{20+40}\right)\ \text{s} = \frac{40}{3}\times 10^{-4}\ \text{s}$$

The potential across capacitor will reach to 20 V (or half its steady state value) after a time,

$$t = \tau \log_e 2 = (0.693)\times\frac{40}{3}\times 10^{-4}\ \text{s} = 9.24\times 10^{-4}\ \text{s}$$

Current through R_2 at time t would be,

$$I = \frac{2}{3}I_0 e^{-t/\tau}$$

Here $I_0 = \dfrac{E}{R} = \dfrac{40}{\left(\dfrac{40}{3}\right)} = 3\ \text{A}$

$$\Rightarrow \quad I = 2e^{-\frac{3\times 10^4 t}{40}} = 2e^{-\frac{3\times 10^3 t}{4}}$$

$$\Rightarrow \quad H = \int_0^t I^2 R_2 dt = \int_0^{9.24\times 10^{-4}} \left(4e^{-1.5\times 10^3 t}\right)(20)dt$$

$$\Rightarrow \quad H = \frac{80}{1.5\times 10^3}\left(1 - e^{-1.5\times 9.24\times 10^{-1}}\right)\ \text{J}$$

$$\Rightarrow \quad H = 40\times 10^{-3}\ \text{J} = 40\ \text{mJ}$$

16. (a) Half of the energy supplied by the battery is stored in the capacitor and rest half is lost as a heat. So,

$$(\Delta H) = \frac{1}{2}(2E)^2 C = 2CE^2$$

Alternate Method:

$$(\Delta H) = \int_0^\infty I^2 R\,dt = \int_0^\infty \left(I_0 e^{-t/\tau_c}\right)^2 R\,dt = 2CE^2$$

Here $I_0 = \dfrac{2E}{R}$ and $\tau_c = CR$

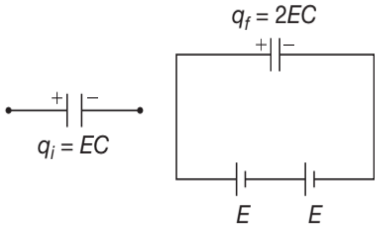
(b) $\Delta H = (\Delta H)_1 + (\Delta H)_2$

Here $(\Delta H)_1 = \dfrac{1}{2}CE^2$

Calculation of $(\Delta H)_2$:

Charge transferred from the battery =

$$\Delta q = q_f - q_i = EC$$



So, energy supplied by the battery $= (\Delta q)(2E) = 2CE^2$

$$\Rightarrow \quad \Delta U = \frac{1}{2}C\left[4E^2 - E^2\right] = \frac{3}{2}CE^2$$

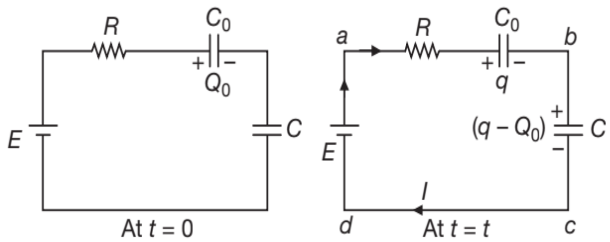
$$\Rightarrow \quad (\Delta H)_2 = 2CE^2 - \frac{3}{2}CE^2 = \frac{1}{2}CE^2$$

$$\Rightarrow \quad \Delta H = CE^2$$

17. Applying Loop Law to abcda, we get

$$-IR + E - \frac{q}{C_0} - \left(\frac{q-Q_0}{C}\right) = 0$$

Substituting $C_0 = C$, we get



$$E + \frac{Q_0}{C} - \frac{2q}{C} - IR = 0$$

$$\Rightarrow \quad \left(\frac{dq}{dt}\right)R = E + \frac{Q_0}{C} - \frac{2q}{C} \qquad \left\{\because I = \frac{dq}{dt}\right\}$$

$$\Rightarrow \quad \int_{Q_0}^{q} \frac{dq}{E + \dfrac{Q_0}{C} - \dfrac{2q}{C}} = \frac{1}{R}\int_0^t dt$$

Solving this, we get

$$q = \frac{1}{2}\left[(EC + Q_0) - (EC - Q_0)e^{-\frac{2t}{CR}}\right]$$

and $q - Q_0 = \dfrac{1}{2}\left[(EC - Q_0) - (EC - Q_0)e^{-\frac{2t}{CR}}\right]$

So, $I = \dfrac{dq}{dt} = \dfrac{1}{CR}(EC - Q_0)e^{-\frac{2t}{CR}}$

$$\Rightarrow \quad I = \left(\frac{E}{R} - \frac{Q_0}{CR}\right)e^{-\frac{2t}{CR}}$$

Substituting the given values, we have

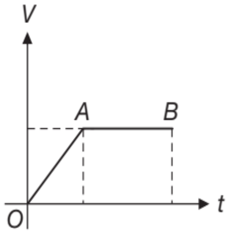
$$I = \left(\frac{4}{100} - \frac{2 \times 10^{-6}}{2 \times 10^{-6} \times 100} \right) e^{-\frac{2t}{2 \times 10^{-6} \times 100}}$$

or $\quad I = \left(0.03 e^{-10^4 t} \right)$ A

In this case, though the capacitors are in series yet the charges on the capacitors are not equal (even in steady state).

18. **From O to A:** $V = at$, where a is a positive constant

$$\Rightarrow \quad V = at = \frac{q}{C} + IR$$



Differentiating w.r.t. time, we get

$$a = \frac{1}{C}\left(\frac{dq}{dt}\right) + \left(\frac{dI}{dt}\right)R$$

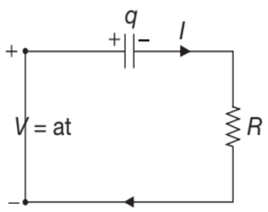
$$\Rightarrow \quad \left(\frac{dI}{dt}\right)R = a - \frac{I}{C} \qquad \left\{ \because I = \frac{dq}{dt} \right\}$$

$$\Rightarrow \quad \int_0^I \frac{dI}{a - \frac{I}{C}} = \int_0^t \frac{dt}{R}$$

$$\Rightarrow \quad I = aC\left(1 - e^{-\frac{t}{CR}}\right)$$

i.e., current in the circuit increases exponentially and hence

$$V_{CD} = IR = aCR\left(1 - e^{-\frac{t}{CR}}\right)$$



i.e., V_{CD} also increases exponentially.

From A onwards: When V = constant (say V_0), then

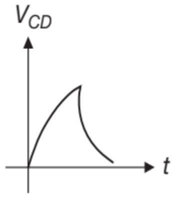
$$V_0 = at$$

$$\Rightarrow \quad t = \frac{V_0}{a}$$

$$\Rightarrow \quad V_{CD} = aCR\left(1 - e^{\frac{-V_0}{aCR}}\right)$$

After this V_{CD} will decrease exponentially.

Hence, a rough graph is as shown in figure.

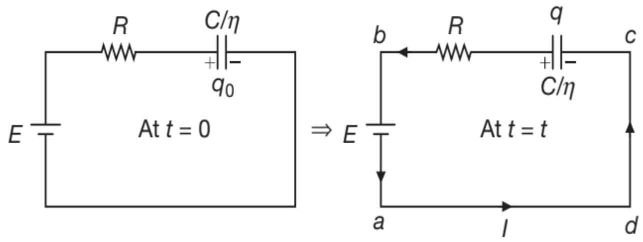


19. At $t = 0$, charge stored in the capacitor is,

$$q_0 = EC$$

Since, the capacity of the capacitor has been decreased η fold, the charge stored in the capacitor will decrease. Hence, the current in the circuit will flow anticlockwise, with

$$I = -\frac{dq}{dt}$$



Applying Loop Law to $abcda$, we get, $E + IR - \frac{q}{C/\eta} = 0$

$$\Rightarrow \quad \left(\frac{dq}{dt}\right)R = E - \left(\frac{\eta}{C}\right)q$$

$$\Rightarrow \quad \int_{q_0}^{q} \frac{dq}{E - \frac{\eta}{C}q} = \frac{1}{R}\int_0^t dt$$

$$\Rightarrow \quad q = \frac{EC}{\eta}\left[1 + (\eta - 1)e^{-\frac{\eta t}{CR}}\right]$$

$$\Rightarrow \quad I = -\frac{dq}{dt} = \frac{E}{R}(\eta - 1)e^{-\frac{\eta t}{CR}}$$

20. Charge on capacitor

$$q = CV \qquad \ldots(1)$$

But from Gauss Theorem

$$\Rightarrow \quad q = KE_0 \int_S \vec{E} \cdot d\vec{A} \qquad \ldots(2)$$

Also, $I = \int \vec{J} \cdot d\vec{A}$

$$\Rightarrow \quad I = \int \sigma \vec{E} \cdot d\vec{A} = \sigma \int \vec{E} \cdot d\vec{A} \qquad \ldots(3)$$

$\{\because \ \vec{J} = \sigma \vec{E}$ where $\sigma =$ conductivity$\}$

Now using (2) and (3), we get

$$I = \frac{\sigma q}{KE_0} = \frac{1}{\rho}\left(\frac{CV}{KE_0}\right) \qquad \left\{\because \sigma = \frac{1}{\rho}\right\}$$

$$\Rightarrow \quad I = \frac{(4\times10^{-9})(2\times10^{3})}{(10^{11})(6)(8.85\times10^{-12})}$$

$$\Rightarrow \quad I = 1.5\times10^{-6}\ \text{A} = 1.5\ \mu\text{A}$$

Single Correct Choice Type Questions

1. $j = \frac{I}{A} = nqv$

where v is the drift velocity of charge carriers each with charge q.

Hence, the correct answer is (D).

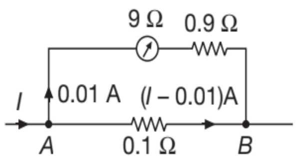
2. $\hbar = \frac{h}{2\pi}$ (Just for the sake of knowledge)

$$I = \frac{\text{Charge Circulating}}{\text{Time for one revolution}}$$

$$\Rightarrow \quad I = \frac{e}{\left(\frac{2\pi r}{v}\right)} = \frac{e\left(\frac{e^2}{\hbar}\right)}{2\pi\frac{\hbar^2}{me^2}} = \frac{me^5}{2\pi\hbar^3}$$

Hence, the correct answer is (A).

3. In parallel, current distributes in inverse ratio of resistance.



$$\Rightarrow \quad \frac{0.01}{I-0.01} = \frac{0.1}{9+0.9}$$

Solving we get, $I = 1$ A

Hence, the correct answer is (C).

4. $Q = Q_0 e^{-t/\tau} = \frac{Q_0}{\eta}$

$$\Rightarrow \quad e^{-t/\tau} = \frac{1}{\eta}$$

$$\Rightarrow \quad e^{t/\tau} = \eta$$

$$\Rightarrow \quad \frac{t}{\tau} = \log_e \eta$$

Hence, the correct answer is (B).

5. Let us assume that charge flow between the rod and cylinder be radial. Then at point P at distance r from centre of rod the current density J is

$$J = \frac{I}{2\pi r\ell}$$

Further

$$E = \rho J = \frac{\rho I}{2\pi r\ell} \qquad \{\because J = \sigma E\}$$

with both J and E directed radially outwards.

Since

$$dV = -E\,dr$$

$$\Rightarrow \quad dV = -\frac{\rho I}{2\pi\ell}\frac{dr}{r}$$

$$\Rightarrow \quad \int_a^b dV = -\frac{\rho I}{2\pi\ell}\int_a^b \frac{dr}{r}$$

$$\Rightarrow \quad -V = -\frac{\rho I}{2\pi\ell}\log_e\left(\frac{b}{a}\right)$$

$$\Rightarrow \quad I = \frac{2\pi\ell V}{\rho\log_e\left(\frac{b}{a}\right)}$$

Hence, the correct answer is (B).

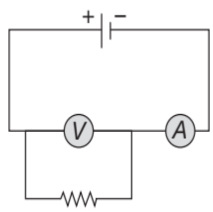
6. Let V be the voltage of the source and R be the resistance of each bulb. Then, we have

$R = \frac{V^2}{P}$ when N bulbs are joined in series across V, current in each bulb is given by $I = \frac{V}{NR}$. Power drawn by each bulb is $I^2R = \left(\frac{V^2}{N^2R^2}\right)R = \frac{V^2}{N^2R} = \frac{P}{N^2}$

Hence, total power drawn is $Np = N\left(\frac{P}{N^2}\right) = \frac{P}{N}$

Hence, the correct answer is (C).

7. Whenever a resistance is joined in parallel with the voltmeter, the total resistance of the circuit decreases so that the current increases and hence ammeter reading also increases. The equivalent resistance across the voltmeter decreases and hence its reading will decrease.



Hence, the correct answer is (A).

8. Since the resistivity of a current carrying conductor carrying charge carriers each of charge q, mass m is given by

$$\rho = \frac{m}{nq^2\tau}$$

where n is number density of charge carriers and τ is the average relaxation time.

Hence, the correct answer is (B).

9. Convectional current is the current which is developed due to the transportation of charge.

$$\Rightarrow \quad I = \frac{q_{\text{transported}}}{t} = 2\lambda v$$

Hence, the correct answer is (B).

10. This is basically a RC circuit, being charged from a battery. The resistance (R) of the voltmeter is the resistance in the circuit. The voltage across R is

$$V = (\text{Circuit Current})(\text{Resistance}) = \text{Reading of the voltmeter } (V)$$

Hence the nature of the V-t curve is the same as that of the I-t curve.

Hence, the correct answer is (C).

13. $R_1 + R_2 = \text{Constant}$

$$\Rightarrow \quad \Delta(R_1 + R_2) = 0$$

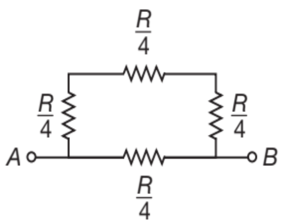
$$\Rightarrow \quad |\Delta R_1| = |\Delta R_2|$$

$$\Rightarrow \quad R_1\alpha\Delta T = R_2\beta\Delta T$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{\beta}{\alpha}$$

Hence, the correct answer is (D).

14.



$$\Rightarrow \quad R_{AB} = \frac{3R}{16} = \frac{3(16)}{16} = 3\ \Omega$$

Hence, the correct answer is (C).

15. $I = \dfrac{2(1.5)}{R + 2r}$

$$\Rightarrow \quad R + 2r = 3 \qquad \ldots(1)$$

$$0.6 = \frac{1.5}{R + (r/2)}$$

$$\Rightarrow \quad 2R + r = 5 \qquad \ldots(2)$$

From (1) and (2), we get

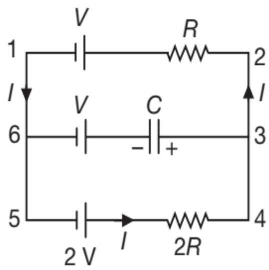
$$R = \frac{7}{3}\ \Omega,\ r = \frac{1}{3}\ \Omega$$

Hence, the correct answer is (C).

16. In steady state the branch containing the capacitor can be omitted and hence current in the circuit is

$$I = \frac{2V - V}{R + 2R}$$

$$\Rightarrow \quad I = \frac{V}{3R}$$



For loop 36543

$$-V_C - V + 2V - I(2R) = 0$$

$$\Rightarrow \quad V_C = -V + 2V - \frac{V}{3R}(2R)$$

$$\Rightarrow \quad V_C = V - \frac{2V}{3} = \frac{V}{3}$$

Hence, the correct answer is (C).

17. $G = 25\ \Omega$

$$I_g = (2 \times 10^{-4})(50)$$

$$\Rightarrow \quad I_g = 0.01\ \text{A}$$

Since

$$R = \frac{V}{I_g} - G$$

$$\Rightarrow \quad R = \frac{25}{0.01} - 25$$

$$\Rightarrow \quad R = 2500 - 25$$

$$\Rightarrow \quad R = 2475\ \Omega$$

Hence, the correct answer is (C).

18. On short circuiting

$$I = \frac{E}{R}$$

$$\Rightarrow \quad 3 = \frac{1.5}{R}$$

$$\Rightarrow \quad R = 0.5\ \Omega$$

Hence, the correct answer is (A).

19. When K is open

$$R_{\text{net}} = \frac{3R}{2}$$

$$\Rightarrow \quad i_1 = \frac{E}{\left(\frac{3R}{2}\right)} = \frac{2E}{3R}$$

When K is closed

$$R_{\text{net}} = 2\left[\frac{R \times 2R}{R + 2R}\right] = \frac{4}{3}R$$

$$\Rightarrow \quad i_2 = \frac{E}{\left(\frac{4R}{3}\right)} = \frac{3E}{4R}$$

$$\Rightarrow \quad \frac{i_1}{i_2} = \frac{8}{9}$$

Hence, the correct answer is (C).

20. 7 Ω and 3 Ω are in parallel; 6 Ω and 4 Ω are in parallel and both in series.
So

$$\Rightarrow \quad R_{\text{eq}} = \frac{7 \times 3}{7+3} + \frac{4 \times 6}{4+6}$$

$$\Rightarrow \quad R_{\text{eq}} = 2.1 + 2.4$$

$$\Rightarrow \quad R_{\text{eq}} = 4.5\,\Omega$$

Hence, the correct answer is (A).

21. Let E be the e.m.f. and r be the internal resistance of battery. Then

$$0.5 = \frac{E}{20 + r}$$

$$\Rightarrow \quad E = 10 + 0.5\,r \qquad \ldots(1)$$

and $0.8 = \dfrac{E}{10 + r}$

$$\Rightarrow \quad E = 8 + 0.8\,r \qquad \ldots(2)$$

From (1) and (2)

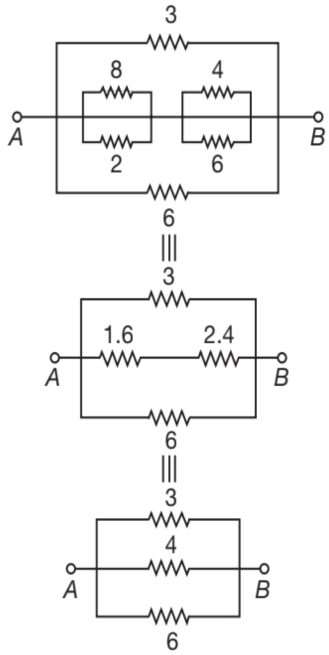
$$2 = 0.3\,r$$

$$\Rightarrow \quad r = \frac{20}{3}\,\Omega \text{ and}$$

$$E = \frac{40}{3}\,\text{V}$$

Hence, the correct answer is (D).

22.



3, 4 and 6 Ω all in parallel. Hence

$$\frac{1}{R_P} = \frac{1}{3} + \frac{1}{4} + \frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{R_P} = \frac{4+3+2}{12}$$

$$\Rightarrow \quad R_P = \frac{12}{9}\,\Omega = \frac{4}{3}\,\Omega$$

Hence, the correct answer is (A).

24. $0.2 = \dfrac{1.5}{5 + r}$

$$\Rightarrow \quad 1 + 0.2\,\text{r} = 1.5$$

$$\Rightarrow \quad r = \frac{5}{2} = 2.5\,\Omega$$

Hence, the correct answer is (C).

25. In parallel, the potential across all the resistances is the same.
Hence, the correct answer is (D).

26. Since $I = \dfrac{E}{R_{\text{eq}}} = \dfrac{E}{R + 10}$

$$\Rightarrow \quad I = \frac{2}{10 + R}$$

100 cm has a resistance of 10 Ω

$\Rightarrow$ 40 cm has a resistance of $\dfrac{10}{100} \times 40$

$\Rightarrow$ $R' = 4\,\Omega$

So, potential difference across $4\ \Omega$ is

$$V' = IR' = 10\ \text{mV}$$

$$\Rightarrow \quad V' = \left(\frac{2}{10+R}\right)4 = 10^{-2}$$

$$\Rightarrow \quad 800 = 10 + R$$

$$\Rightarrow \quad R = 790\ \Omega$$

Hence, the correct answer is (B).

27. If A is fused, then complete circuit is broken.
Hence, the correct answer is (C).

28. $R_1 + R_2 = 18$ and $\dfrac{R_1R_2}{R_1+R_2} = 4$

Hence, the correct answer is (C).

29. If a wire is stretched to n times its original length, then new resistance is n^2 times original resistance. Stretching to make the wire 0.1% longer implies its new length is

$$\frac{0.1}{100}\ell_0 + \ell_0 = \frac{1001}{1000}\ell_0$$

So, $R_{new} = \left(\dfrac{1001}{1000}\right)^2 R_0$

$\Rightarrow$ %age increment is

$$\left[\left(\frac{1001}{1000}\right)^2 - 1\right]\times 100\%$$

$$= \left[\left(1+\frac{1}{1000}\right)^2 - 1\right]\times 100\%$$

$$= \left[1+\frac{2}{1000} - 1\right]\times 100 = 0.2\%$$

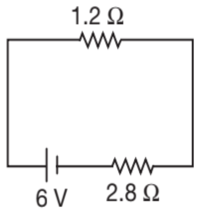
(Since according to Binomial Theorem

$$(1+x)^n \simeq 1 + nx$$

For $|x| << 1$ or $-1 < x < 1$)

Hence, the correct answer is (C).

30. Since capacitor is a dc blocking element, so no current flows through the branch containing the capacitor. However, a voltage drop will exist across this branch. $R_{net} = 4\ \Omega$



$$\Rightarrow \quad I_{total} = \frac{V}{R_{net}} = 1.5\ \text{A}$$

Voltage drop across $2.8\ \Omega$ is $V_1 = (2.8)(1.5)$

$$\Rightarrow \quad V_1 = 4.2\ \text{V}$$

So, potential across parallel combination of $2\ \Omega$ and $3\ \Omega$ is

$$V_2 = 6 - 4.2 = 1.8\ \text{V}$$

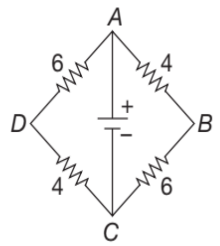
If I_2 is the current in $2\ \Omega$ resistor, then

$$I_2 = \frac{1.8}{2} = 0.9\ \text{A}$$

Hence, the correct answer is (B).

31. Let $E =$ emf of the cell

$$V_A - V_B = \frac{E}{4\ \Omega + 6\ \Omega}\times 4\ \Omega = \frac{4E}{10}$$



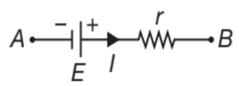
$$V_A - V_D = \frac{E}{6\ \Omega + 4\ \Omega}\times 6\ \Omega = \frac{6E}{10}$$

$$V_B - V_D = \frac{E}{5}$$

Hence, the correct answer is (C).

32. Current in the circuit is $I = \dfrac{NE}{Nr} = \dfrac{E}{r}$

The equivalent circuit of one cell is shown in the figure.



Potential difference across the cell equals

$$V_A - V_B = -E + Ir = -E + \left|\frac{E}{r}\right| r = 0$$

Hence, the correct answer is (D).

33. $I = \dfrac{2Q}{2\pi/\omega}$

$$\Rightarrow \quad I = \frac{Q\omega}{\pi}$$

Hence, the correct answer is (C).

34. No current will flow through voltmeter. As it is ideal (infinite resistance). Current through two batteries

$$i = \frac{1.5 - 1.3}{r_1 + r_2} = \frac{0.2}{r_1 + r_2}$$

Now, $V = E_2 - ir_2$

$$\Rightarrow \quad 1.45 = 1.5 - \left(\frac{0.2}{r_1 + r_2}\right)(r_2)$$

Solving this equation, we get

$$r_1 = 3r_2$$

Hence, the correct answer is (B).

35. $F = \frac{dp}{dt} = \dot{p}$

Momentum of each charge carrier moving with a drift velocity v is mv.

Total number of charge carriers in the sample is $N = n(A\ell)$, where n is number of charge carriers per unit volume, A is area of cross-section of the conductor.

Total momentum = $p = N(mv) = nA\ell mv$

Further we have $v = \frac{I}{neA}$

$$\Rightarrow \quad p = nA\ell m\left(\frac{I}{neA}\right)$$

$$\Rightarrow \quad p = \ell\left(\frac{m}{e}\right)I$$

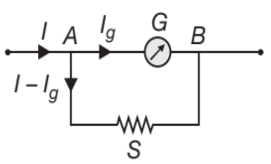
Since $F = \frac{dp}{dt}$

$$\Rightarrow \quad F = \frac{\ell}{s}\dot{I} \qquad \left\{\because \ s = \text{specific charge} = \frac{e}{m}\right\}$$

$$\Rightarrow \quad \frac{F}{\ell} = \frac{\dot{I}}{s}$$

Hence, the correct answer is (C).

36. $I_g = 10 \text{ mA} = 0.01 \text{ A},\ G = 1\,\Omega,\ I = 1 \text{ A}$



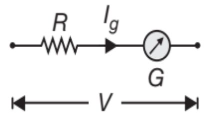
Since, $V_A - V_B = I_g G = (I - I_g)S$

$$S = \frac{I_g G}{I - I_g} = \frac{(0.01 \text{ A} \times 1\ \Omega)}{1 \text{ A} - 0.01 \text{ A}} = \frac{1}{99}\ \Omega$$

Hence, the correct answer is (C).

37. $I_g = 0.01 \text{ A},\ G = 1\,\Omega,\ V = 10 \text{ V}$

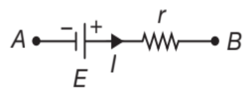
Since, $V = (G + R)I_g$



$$\Rightarrow \quad R = \frac{V}{I_g} - G = \frac{10 \text{ V}}{0.01 \text{ A}} - 1\ \Omega = 999\ \Omega$$

Hence, the correct answer is (C).

38. $I = \frac{(N-2)E}{Nr}$



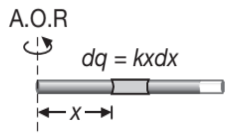
$$V_B - V_A = -Ir + E = E - \frac{(N-2)E}{Nr}r$$

$$\Rightarrow \quad V_B - V_A = E\left(1 - \frac{N-2}{N}\right) = \frac{2E}{N}$$

Hence, the correct answer is (A).

39. Consider an element of length dx at a distance x from the axis of rotation (A.O.R.)

$$\Rightarrow \quad q = \int_0^L dq = k\int_0^L x\,dx$$



$$\Rightarrow \quad q = \frac{1}{2}kL^2 = \frac{\lambda L}{2}$$

$$\Rightarrow \quad I = \frac{q}{T} = \frac{\left(\frac{\lambda L}{2}\right)}{\left(\frac{2\pi}{\omega}\right)}$$

$$\Rightarrow \quad I = \frac{\lambda L\omega}{4\pi} = \frac{kL^2\omega}{4\pi}$$

Hence, the correct answer is (C).

40. For every cell that is wrongly connected, the emf decreases by $2E_0$. However, internal resistance does not depend on direction and therefore remains the same for all cells.

Hence, the correct answer is (D).

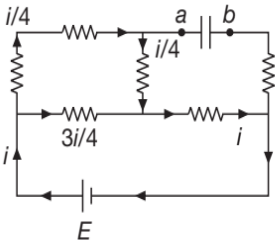
41. Let x, y and $\frac{x}{3}$ be the dimensions of the block.

$$\Rightarrow \quad R_{max} = \frac{\rho x}{y\left(\frac{x}{3}\right)}$$

$$\Rightarrow \quad R_{\min} = \frac{\rho\left(\frac{x}{3}\right)}{xy}$$

$$\Rightarrow \quad \frac{R_{\max}}{R_{\min}} = 9$$

Hence, the correct answer is (B).

42. 

$$V_a - \frac{i}{4}R - iR = V_b$$

$$\Rightarrow \quad V_a - V_b = \frac{5}{4}iR = 10 \qquad \ldots(1)$$

$$R_{\text{net}} = R + \frac{(3R)(R)}{(3R+R)}$$

$$\Rightarrow \quad R_{\text{net}} = \frac{7}{4}R$$

$$\Rightarrow \quad i = \frac{E}{R_{\text{net}}}$$

$$\Rightarrow \quad i = \frac{E}{\left(\frac{7}{4}\right)R} = \left(\frac{4E}{7R}\right)$$

Substituting in Equation (1), we get

$$\left(\frac{5}{4}\right)\left(\frac{4E}{7R}\right) \times R = 10$$

$$\Rightarrow \quad E = 14 \text{ V}$$

Hence, the correct answer is (A).

43. Since Mass = (Density)(Volume)

$$\Rightarrow \quad m = (d)(A\ell)$$

$$\Rightarrow \quad A = \frac{m}{\ell d}$$

Further since

$$\Rightarrow \quad R = \frac{\rho\ell}{A}$$

$$\Rightarrow \quad R = \frac{\rho\ell}{\left(\frac{m}{\ell d}\right)}$$

$$\Rightarrow \quad R = \frac{\rho\ell^2 d}{m}$$

As all the three wires are made of Cu, so all have same d and ρ.

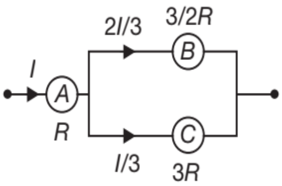
$$\Rightarrow \quad R \propto \frac{\ell^2}{m}$$

$$\Rightarrow \quad R_1 : R_2 : R_3 :: 25 : 3 : \frac{1}{5}$$

$$\Rightarrow \quad R_1 : R_2 : R_3 :: 125 : 15 : 1$$

Hence, the correct answer is (C).

46. $V_A = IR$, $V_B = \frac{2I}{3}\left(\frac{3}{2}R\right) = IR$



and $V_C = \left(\frac{I}{3}\right)(3R) = IR$

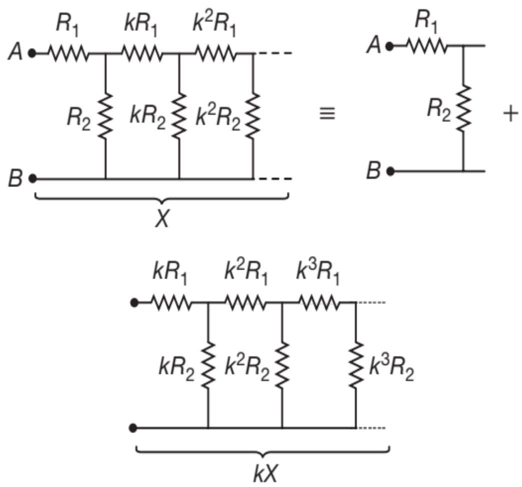
Hence, the correct answer is (D).

47. $r = R\left(\frac{E}{V} - 1\right) = 5\left(\frac{2.2}{1.8} - 1\right) = \frac{10}{9}\ \Omega$

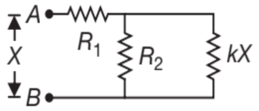
Hence, the correct answer is (A).

48. The filament of the heater attains steady resistance when the heater reaches its steady temperature, which is much higher than the room temperature. The resistance at room temperature is thus much lower than the resistance at its steady state. When the heater is switched on, it draws a larger current than its steady state current. As the filament heats up, its resistance increases and the current falls to its steady state value.
Hence, the correct answer is (D).

49. When each element of circuit is multiplied by a factor k then equivalent resistance also becomes k times.
Let the equivalent resistance between A and B be X.



So, equivalent circuit becomes



For $k = \frac{1}{2}$

$$\Rightarrow \quad X = \frac{(R_1 - R_2) + \sqrt{R_1^2 + R_2^2 + 6R_1R_2}}{2}$$

Hence, the correct answer is (C).

50. Since the capacitors are identical so they will finally attain a charge $\frac{Q}{2}$ each. Now,

Initial energy of the system $= E_i = \frac{Q^2}{2C}$

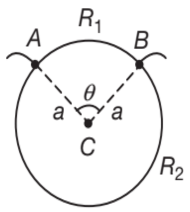
Final energy of the system $= E_f = 2\left[\frac{\left(\frac{Q}{2}\right)^2}{2C}\right] = \frac{Q^2}{4C}$

So, heat produced = Loss in energy $= E_i - E_f = \frac{Q^2}{4C}$

Hence, the correct answer is (B).

51. Since $R \propto \ell$

$$\Rightarrow \quad R_1 = \frac{R}{2\pi a}(a\theta)$$



$$\Rightarrow \quad R_1 = \frac{R\theta}{2\pi}$$

Similarly $R_2 = \frac{R}{2\pi a}(2\pi a - a\theta)$

$$\Rightarrow \quad R_2 = \frac{R}{2\pi}(2\pi - \theta)$$

Since, $R_{AB} = \frac{R_1R_2}{R_1 + R_2}$

$$\Rightarrow \quad R_{AB} = \frac{R\theta(2\pi - \theta)}{4\pi^2}$$

Hence, the correct answer is (A).

52. Both the resistances are in parallel.

So, $\frac{1}{R_p} = \frac{1}{R_{Cu}} + \frac{1}{R_{Ni}}$

where $R_{Cu} = \frac{\rho_C \ell}{\pi r^2}$

$$R_{Ni} = \frac{\rho_N \ell}{\pi(2R)^2 - \pi R^2} = \frac{\rho_N \ell}{3\pi R^2}$$

$$\Rightarrow \quad \frac{1}{R_P} = \frac{\pi r^2}{P_C \ell} + \frac{3\pi R^2}{\rho N \ell}$$

$$\Rightarrow \quad R_P = \frac{\ell}{\pi r^2\left(\frac{1}{\rho_C} + \frac{3}{\rho_N}\right)}$$

Hence, the correct answer is (C).

55. Since the materials are same, the resistivity is same. If R_1 and R_2 are their resistances, we have $R_1\pi r_1^2 / \ell_1 = R_2\pi r_2^2 / \ell_2$, where ℓ_1 and ℓ_2 are their respective lengths. Here

$$\ell_1 = \ell_2, \; r_1^2/r_2^2 = R_2/R_1 .$$

If ℓ is the balancing length measured from the left of metre bridge, we have

$$R_1/R_2 = \ell/(100 - \ell) = 25/75 .$$

This gives $r_1/r_2 = \sqrt{75/25} = \sqrt{3}$

Hence, the correct answer is (A).

56. $I = \frac{E}{R_2 + r} = \frac{5}{4+1}$

$$\Rightarrow \quad I = 1\,A$$

Voltage across R_2 is

$$V = IR_2 = 4\text{ V}$$

So, voltage across each capacitor is 2 V.

$$\Rightarrow \quad Q = CV = 3(2)$$

$$\Rightarrow \quad Q = 6\,\mu C$$

Hence, the correct answer is (B).

57. For metre bridge to be balanced

$$\frac{P}{Q} = \frac{40}{60} = \frac{2}{3}$$

$$\Rightarrow \quad P = \frac{2}{3}Q$$

When Q is shunted i.e. a resistance of $10\,\Omega$ is connected in parallel across Q. So net resistance is $\frac{10Q}{10+Q}$.

Now the balance point shifts to 50 cm i.e.

$$\frac{P}{\left(\frac{10Q}{10+Q}\right)} = 1$$

$$\Rightarrow \quad \frac{2}{3} = \frac{10}{10+Q}$$

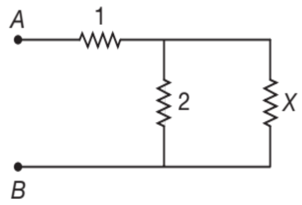
$$\Rightarrow \quad 20 + 2Q = 30$$

$$\Rightarrow \quad Q = 5\ \Omega \text{ and } P = \frac{10}{3}\ \Omega$$

Hence, the correct answer is (A).

58. Let the equivalent resistance between A and B be X. If we retain the basic repetitive unit then the remaining circuit is identical to the total circuit and hence will also possess a resistance X.

$$R_{AB} = X = \frac{2X}{2+X} + 1$$



$$\Rightarrow \quad 2X = (2+X)(X-1)$$

$$\Rightarrow \quad 2X = 2X + X^2 - X - 2$$

$$\Rightarrow \quad X^2 - X - 2 = 0$$

$$\Rightarrow \quad X^2 - 2X + X - 2 = 0$$

$$\Rightarrow \quad (X+1)(X-2) = 0$$

$$\Rightarrow \quad X = 2\ \Omega$$

Hence, the correct answer is (B).

59. $I_{\text{total}} = \frac{V}{R_{AB}} = \frac{6}{2}$

$$\Rightarrow \quad I_{\text{total}} = 3\text{ A}$$

Voltage drop across $2\ \Omega$ resistor is 3 V.

So, $I = \frac{3}{2} = 1.5\text{ A}$

Hence, the correct answer is (A).

60. Let the potential of the battery be V

$$R_1 = \frac{V}{1}$$

$$R_2 = \frac{V}{2}$$

$$R_3 = \frac{V}{4}$$

On connecting in series

$$R_S = R_1 + R_2 + R_3$$

$$\Rightarrow \quad R_S = V\left(1 + \frac{1}{2} + \frac{1}{4}\right)$$

$$\Rightarrow \quad R_S = \frac{7}{4}\text{V}$$

Since, by definition

$$I = \frac{V}{R_S}$$

$$\Rightarrow \quad I = \frac{4}{7}A$$

Hence, the correct answer is (C).

62. Since A and B will remain connected in parallel across the capacitor so, at all the instants they will produce the same readings.
Hence, the correct answer is (D).

63. $2 = 1 + (0.00125)(T - 300)$

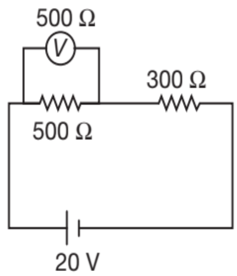
$$\Rightarrow \quad T - 300 = \frac{100000}{125}$$

$$\Rightarrow \quad T - 300 = 800$$

$$\Rightarrow \quad T = 1100\ K$$

Hence, the correct answer is (B).

64.



When we do not connect the voltmeter then potential across $500\ \Omega$ resistor is

$$V_1 = \frac{500}{800} \times 20 = 12.5\text{ V}$$

The measure this value of potential difference we connect a voltmeter in parallel across $500\ \Omega$ resistor. Net resistance of parallel combination is $250\ \Omega$ and also voltage drop across both $500\ \Omega$ and voltmeter is the same (as both are in parallel). So, voltage drop across combination is

$$V_2 = \frac{250}{250+300} \times 20 = 9.1\text{ V}$$

So, error in the reading is $12.5 - 9.1 = 3.4$ V
Hence, the correct answer is (C).

65. For potential difference between B and D to be zero

$$\frac{10}{5+x} = \frac{0.5}{1} \quad \Rightarrow \quad 20 = 5 + x$$

$$\Rightarrow \quad x = 15\ \Omega$$

Hence, the correct answer is (C).

66. Since net resistance is to be found between A and B. So let a current I enter at A and then exit at B.
When I enters at A then by symmetry a current $\frac{I}{6}$ must flow in the branch AB from A to B.
For current I to exit from B, a current $\frac{I}{6}$ must flow in the branch AB from A to B. Super-imposing the two, we conclude that a current $\left(\frac{I}{6}+\frac{I}{6}\right)$ must flow in the branch AB from A to B.
According to Thevenin's Theorem we have

$$I_{\text{total}} R_{\text{eq}} = V_{AB} = \left(\frac{I}{6}+\frac{I}{6}\right)R_0 = \frac{IR_0}{3}$$

$$\Rightarrow \quad IR_{\text{eq}} = \frac{IR_0}{3}$$

$$\Rightarrow \quad R_{\text{eq}} = \frac{R_0}{3}$$

Hence, the correct answer is (C).

67. Equivalent emf of two batteries E_1 and E_2 is

$$E = \frac{\frac{E_1}{r_1}+\frac{E_2}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}} = \frac{\left(\frac{2}{2}\right)+\left(\frac{4}{6}\right)}{\left(\frac{1}{2}\right)+\left(\frac{1}{6}\right)}$$

$$\Rightarrow \quad E = 2.5 \text{ V}$$

Now, $V_{AN} = E$

$$\Rightarrow \quad (I_{AN})(R_{AN}) = E$$

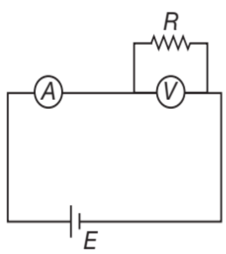
$$\Rightarrow \quad \left(\frac{12}{4+4\times 4}\right)(4)(\ell) = 2.5$$

Solving this equation, we get

$$\ell = \frac{25}{24} \text{ m}$$

Hence, the correct answer is (D).

68. When a resistance is joined in parallel to a voltmeter the net resistance of parallel combination decreases as a result of which total resistance of circuit decreases, hence net current increases. So reading of ammeter increases. Since net resistance of parallel combination decreases so reading of voltmeter decreases



Hence, the correct answer is (D).

69. $$I = \frac{\text{Charge Circulating}}{\text{Time to Complete One Revolution}}$$

$$\Rightarrow \quad I = \frac{Q}{\left(\frac{2\pi}{\omega}\right)}$$

$$\Rightarrow \quad I = \frac{Q\omega}{2\pi}$$

Hence, the correct answer is (C).

70. $R \propto \frac{1}{A}$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{A_2}{A_1}$$

$$\Rightarrow \quad \frac{10}{R_2} = \frac{1}{3}$$

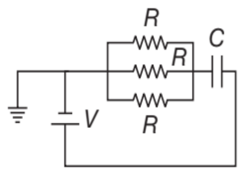
$$\Rightarrow \quad R_2 = 30\ \Omega$$

$$\Rightarrow \quad R_s = 30 + 10 = 40\ \Omega$$

Hence, the correct answer is (A).

71. The simple circuit is as shown below.

$$R_{net} = \frac{R}{3}$$



$$\Rightarrow \quad \tau_C = CR_{\text{net}} = \frac{CR}{3}$$

$$\Rightarrow \quad q = q_0\left(1 - e^{-\frac{t}{\tau_C}}\right),$$

where, $q_0 = CV$

Hence, the correct answer is (C).

72. Since, $P = \frac{V^2}{R}$, so the bulb with higher power rating has lower resistance, and vice versa. When the bulbs are joined in series, they draw the same current. Since $P = I^2R$, the bulb with the lower power rating draws more power.
Hence, the correct answer is (C).

73. Just after the switch is closed C_1 is short-circuited and current passes through R_1 and C_1 only.

Hence, the correct answer is (B).

74. Current can flow in a circuit or part of a circuit only when it has a return path, i.e., it can return to its starting point. Here, current flowing from PQ cannot return to P.
Hence, the correct answer is (C).

76. For series connection, $I_{\max} = \frac{NE}{Nr} = \frac{E}{r}$

For parallel connection, $I_{\max} = \frac{E}{r/N} = \frac{NE}{r}$

Hence, the correct answer is (D).

77. In the steady state no current will flow in the branch containing the capacitor. $\{\because R_2$ and C are in parallel$\}$

Hence $I = \dfrac{V}{R_1 + R_2}$

However potential across the capacitor will be there and equals the potential across the resistor R_2.

$$\Rightarrow \quad V_C = \left(\frac{R_2}{R_1 + R_2}\right)V$$

Hence, the correct answer is (D).

78. In a Wheatstone bridge, the deflection in the galvanometer does not change whenever the battery and galvanometer are interchanged.
Hence, the correct answer is (D).

79. $$\frac{E^2 r_1}{(r + r_1)^2} = \frac{E^2 r_2}{(r + r_2)^2}$$

$$\Rightarrow \quad r_1\left(r^2 + r_2^2 + 2\,rr_2\right) = r_2\left(r^2 + r_1^2 + 2\,rr_1\right)$$

$$\Rightarrow \quad r_1 r^2 + r_1 r_2^2 + 2rr_1 r_2 = r_2 r^2 + r_2 r_1^2 + 2rr_1 r_2$$

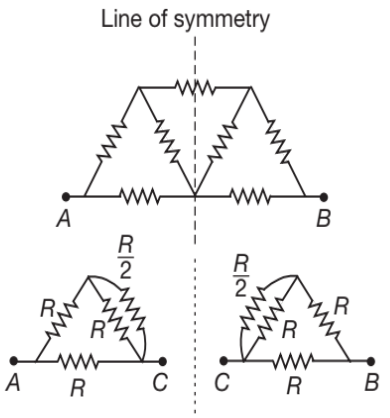
$$\Rightarrow \quad r^2(r_1 - r_2) = r_1 r_2 (r_1 - r_2)$$

$$\Rightarrow \quad r = \sqrt{r_1 r_2}$$

Hence, the correct answer is (B).

80. When we move in the direction of the current in a uniform conductor, the potential decreases linearly. When we pass through the cell, from its negative to its positive terminal, the potential increase by an amount equal to its potential difference.
This is less than its emf, as there is some potential drop across its internal resistance when the cell is driving current.
Hence, the correct answer is (D).

81. $R_{AB} = R_{AC} + R_{CB}$.



Hence, the correct answer is (D).

82. Current in the circuit $= I = \dfrac{E}{R + r}$

Potential difference across cell = potential difference across R is $IR = \dfrac{ER}{R + r}$

Hence, the correct answer is (C).

84. Since $q_g = q_0\left(1 - e^{-\frac{t}{RC}}\right)$

$$\Rightarrow \quad I_g = \frac{dq_g}{dt} = \frac{q_0}{RC}e^{-\frac{t}{RC}}$$

Also $q_d = q_0 e^{-\frac{t}{RC}}$

$$\Rightarrow \quad I_d = \frac{dq_d}{dt} = \frac{-q_0}{RC}e^{-\frac{t}{RC}}$$

$$\Rightarrow \quad I_g = -I_d$$

Hence, the correct answer is (C).

85. By Kirchhoff's Laws

$$E - \frac{q_0}{C_0} - I_C R = 0$$

$$\Rightarrow \quad I_C = \frac{\left(E - \dfrac{q_0}{C_0}\right)}{R}$$

Hence, the correct answer is (C).

86. $I = \dfrac{E}{R_{AB} + R} = \dfrac{4}{2 + 2.4}$

$$\Rightarrow \quad I = \frac{40}{44} = \frac{10}{11}\text{ A}$$

Now, potential difference across wire AB is

$$V_{AB} = IR_{AB} = \frac{20}{11}\text{ V}$$

$$\Rightarrow \quad \frac{20}{11}\text{ V} \xrightarrow[\text{length of}]{\text{across a}} 100\text{ cm}$$

$$\Rightarrow \quad 1\text{ V} \xrightarrow[\text{length of}]{\text{across a}} \frac{100}{\left(\frac{20}{11}\right)}$$

$$\Rightarrow \quad 1.5\text{ V} \xrightarrow[\text{length of}]{\text{across a}} \frac{1100}{20} \times 1.5\text{ cm} = 82.5\text{ cm}$$

Hence, the correct answer is (A).

87. The equivalent resistance of the circuit is $\dfrac{7}{5}R$. Hence the time constant $\tau = \dfrac{7}{5}RC$.

Practically, the steady state value is attained after five time constants
i.e., $t_0 \simeq 5\tau = 7\,RC$
Hence, the correct answer is (B).

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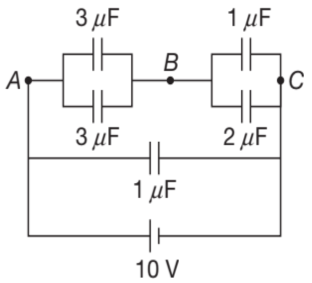
88.



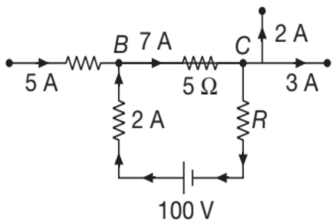
$$\frac{V_{AB}}{V_{BC}}=\frac{(C)_{BC}}{(C)_{AB}}=\frac{3}{6}=\frac{1}{2}$$

$\Rightarrow$ $V_{AB}=\left(\frac{1}{1+2}\right)(10)\text{ V}$

$\Rightarrow$ $V_{AB}=\frac{10}{3}\text{ V}$

Hence, the correct answer is (D).

89.



Applying loop equation in closed loop we have,

$+100-30-35-2R=0$

$\Rightarrow$ $2R=35\text{ V}=V_R$

$V_{5\,\Omega}=7\times 5=35\text{ V}$

$\Rightarrow$ $\frac{V_{5\,\Omega}}{V_R}=1$

Hence, the correct answer is (D).

90. At $t=0$ when capacitors are initially uncharged, their equivalent resistance is zero. Hence, whole current passes through these capacitors.
Hence, the correct answer is (D).

91. Since the balancing length is at the mid point, each wire has a resistance equal to the known resistance value R. When they are in series, if ℓ is the balancing length measured from the left, we have resistance R in the left gap and $2R$ in the right gap.

Thus $\frac{R}{2R}=\frac{\ell}{100-\ell}$

$\Rightarrow$ $\ell=33.3\text{ cm}$

Hence, the correct answer is (B).

93. $\frac{1}{R_{AB}}=\frac{1}{2R}+\frac{1}{2R}+\frac{1}{R}$

Hence, the correct answer is (B).

94. Slope $=\frac{V}{I}=$ Resistance

So, $R_1>R_2$

$\Rightarrow$ $T_1>T_2$

Hence, the correct answer is (A).

95. The circuit is a balanced Wheatstone Bridge with net resistance

$$R_{\text{total}}=\frac{(3R)(6R)}{3R+6R}=2R$$

For maximum power

$R_{\text{internal}}=R_{\text{total external}}$

$\Rightarrow$ $4=2R$

$\Rightarrow$ $R=2\,\Omega$

Hence, the correct answer is (B).

96. $\frac{E_1+E_2}{E_1-E_2}=\frac{204}{36}=\frac{51}{9}$

$\Rightarrow$ $9E_1+9E_2=51E_1-51E_2$

$\Rightarrow$ $42E_1=60E_2$

$\Rightarrow$ $E_1=\frac{60}{42}(1.4)$

$\Rightarrow$ $E_1=2\,V$

Hence, the correct answer is (D).

97. According to Kirchhoff's Junction Law

$I_{CO}+I_{BO}+I_{AO}=I_{OD}$

$\Rightarrow$ $\frac{V_C-V_O}{R}+\frac{V_B-V_O}{R}+\frac{V_A-V_O}{R}=\frac{V_O-V_D}{R}$

$\Rightarrow$ $3\left(\frac{V_A-V_O}{R}\right)=\frac{V_O-V_A+V_A-V_D}{R}$

$\Rightarrow$ $3\left(\frac{V_A-V_O}{R}\right)=-\left(\frac{V_A-V_O}{R}\right)+\left(\frac{V_A-V_D}{R}\right)$

$\Rightarrow$ $4\left(\frac{V_A-V_O}{R}\right)=\frac{V_A-V_D}{R}$

$\Rightarrow$ $4(V_A-V_O)=40$

$\Rightarrow$ $V_A-V_O=10\text{ V}$

Hence, the correct answer is (A).

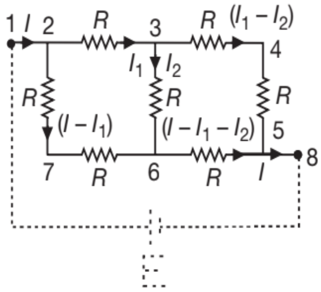
98. e.m.f. is the value when no current is withdrawn from the circuit. So

$E=2\text{ V}$

Also $r=\text{slope}=\frac{2}{5}=0.4\,\Omega$

Hence, the correct answer is (B).

99. Let a fictitious battery E be connected across the circuit.



Loop 23672

$$-I_1R - I_2R + 2(I - I_1)R = 0$$

$\Rightarrow \quad 3I_1 + I_2 = 2I$ ...(1)

Loop 34563

$$-2(I_1 - I_2)R + (I - I_1 + I_2)R + I_2R = 0$$

$\Rightarrow \quad -3I_1 + 4I_2 = -I$ ...(2)

Add (1) and (2), we get

$$5I_2 = I$$

$$\Rightarrow \quad I_2 = \frac{I}{5}$$

and $I_1 = \dfrac{3I}{5}$

For $127658EI$

$$-2R(I - I_1) - R(I - I_1 + I_2) + E = 0$$

$$\Rightarrow \quad E = 2R\left(I - \frac{3I}{5}\right) + R\left(I - \frac{3I}{5} + \frac{I}{5}\right)$$

$$\Rightarrow \quad E = \frac{4IR}{5} + \frac{3IR}{5}$$

$$\Rightarrow \quad E = \frac{7}{5}IR$$

Finally, we have

$$E = I_{total} R_{net}$$

$$\Rightarrow \quad IR_{net} = \frac{7}{5}IR$$

$$\Rightarrow \quad R_{net} = \frac{7}{5}R = 7\ \Omega$$

Hence, the correct answer is (B).

101. $R_{max} = \dfrac{\rho \ell_{max}}{A_{min}}$

$$\Rightarrow \quad R_{max} = \frac{\rho(3\ell_0)}{\ell_0^2}$$

$\Rightarrow \quad \ell_0 =$ Minimum Length

$$R_{min} = \frac{\rho \ell_{min}}{A_{max}}$$

$$\Rightarrow \quad R_{min} = \frac{\rho(\ell_0)}{3\ell_0^2}$$

$$\Rightarrow \quad \frac{R_{max}}{R_{min}} = 9$$

Hence, the correct answer is (C).

102. Since

$$r = \left(\frac{\ell}{\ell'} - 1\right) \times R$$

$$\Rightarrow \quad r = \left(\frac{75}{60} - 1\right) \times 10$$

$$\Rightarrow \quad R = \frac{15}{60} \times 10$$

$$\Rightarrow \quad R = \frac{5}{2} = 2.5\ \Omega$$

Hence, the correct answer is (B).

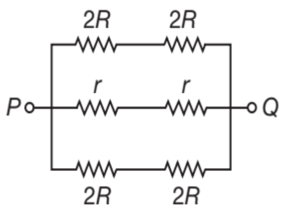
103. Current decreases $\dfrac{20}{30}$ times or $\dfrac{2}{3}$ times. Therefore, net resistance should become $\dfrac{3}{2}$ times.

$$\Rightarrow \quad R + 50 = \frac{3}{2}(2950 + 50)$$

Solving we get, $R = 4450\ \Omega$

Hence, the correct answer is (A).

104.



Hence, the correct answer is (A).

105. Out of 5 one is wrongly connected i.e. 4 are rightly connected

Net $EMF = 4E - E = 3E$

Net resistance $= 5r$

Hence, the correct answer is (B).

106. $I = I_1 + I_2$ ...(1)

$r = \dfrac{r_1 r_2}{r_1 + r_2}$ ...(2)

Further

$$I = \frac{E_1}{r_1} + \frac{E_2}{r_2}$$

$$\Rightarrow \quad \frac{E}{r} = \frac{E_1}{r_1} + \frac{E_2}{r_2}$$

$$\Rightarrow \quad E = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

Hence, the correct answer is (C).

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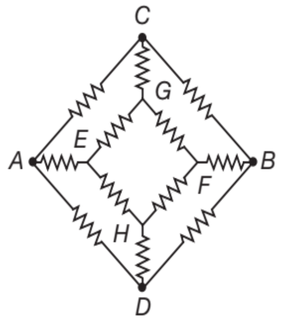
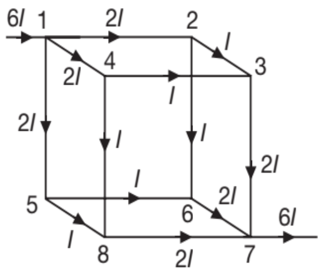
107. The circuit reduces to the following



Hence, the correct answer is (D).

108. Resistance between diagonal corners of cube



$\Rightarrow \quad V_{17} = (V_1 - V_5) + (V_5 - V_8) + (V_8 - V_7)$

$\Rightarrow \quad V_{17} = 2IR_0 + IR_0 + 2IR_0$

$\Rightarrow \quad V_{17} = 5IR_0$

Also,

$V_{17} = x(6I)$

$\Rightarrow \quad x(6I) = 5IR_0$

$\Rightarrow \quad x = \frac{5R_0}{6}$

Hence, the correct answer is (D).

109. $R_{\text{total}} = \frac{5R}{11} + \frac{6R}{11} = R$

Since $I = \frac{V}{R_{\text{total}}}$

$\Rightarrow \quad 2 = \frac{12}{R}$

$\Rightarrow \quad R = 6\Omega$

Hence, the correct answer is (C).

110. In a galvanometer deflection is directly proportional to the current. So 20 divisions correspond to current through galvanometer and 30 divisions for current through shunt.

So, voltage drop a cross galvanometer $= 20\,G$

where G is resistance of galvanometer. Since a fall of 30 divisions is registered when a $12\,\Omega$ shunt (a resistance) is connected in parallel across a galvanometer. Hence voltage drop across shunt = (30)(12)

$\Rightarrow \quad 20\,G = (30)(12)$

$\Rightarrow \quad G = 18\,\Omega$

Hence, the correct answer is (A).

111. Since

$I = neAv$

If N is total number of electrons in the wire then

$N = nA\ell$

Momentum possessed by a single electron is mv.

Total Momentum $= Nmv$

$\Rightarrow \quad p = (nA\ell)m\frac{I}{neA}$

$\Rightarrow \quad p = \frac{m\ell I}{e}$

$\Rightarrow \quad \frac{p}{\ell} = \frac{I}{e/m} = \frac{I}{s}$

Hence, the correct answer is (B).

112. Since $E \propto \ell$. So, for $E_1 > E_2$ we have $\ell_1 > \ell_2$ and hence null point will be obtained at shorter length i.e. to left of C.

Hence, the correct answer is (A).

113. Voltage drop across $50\,\Omega$ and $30\,\Omega$ (n series) is

$V' = (80)(0.1) = 8\text{ V}$

Voltage across $20\,\Omega$ branch is also 8 V.

So, current in $20\,\Omega$ branch is

$I_2 = \frac{8}{20} = 0.4\text{ A}$

Hence total current in the circuit is 0.5 A.

Also voltage drop across R is $12 - 8 = 4$ V.

$\Rightarrow \quad 4 = I_{\text{total}}R$

$\Rightarrow \quad 4 = (0.5)R$

$\Rightarrow \quad R = 8\,\Omega$

Hence, the correct answer is (B).

114. Net resistance of the balanced Wheatstone Bridge is R which is connected in parallel with another resistance of same value.

Hence

$R_{AB} = R/2$

Hence, the correct answer is (A).

115. $R_{\text{eq}} = 220\,\Omega$

$\Rightarrow \quad I_{\text{total}} = 1\text{ A}$

Reading of ammeter is the current which flows through 3 branches.

Hence reading $= \frac{3}{5}$ A

Hence, the correct answer is (C).

116. Shunt $= 0.1\ \Omega = S$

$G_{net} = 9.9\ \Omega$

$I_g = 10$ mA

$I = ?$

Since $S = \frac{I_g G}{I - I_g}$

$\Rightarrow\ 0.1 = \frac{(10\times10^{-3})(9\cdot9)}{I - 10\times10^{-3}}$

$\Rightarrow\ I - 10^{-2} = 99\times10^{-2}$

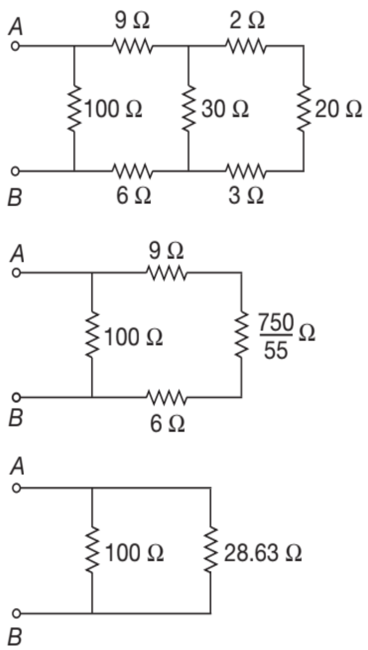
$\Rightarrow\ I = 100\times10^{-2}$

$\Rightarrow\ I = 1$ A

Hence, the correct answer is (C).

117. $\Rightarrow\ R_{AB} = \frac{(28.63)(100)}{128.63}$

$\Rightarrow\ R_{AB} \simeq 22\cdot26\ \Omega$



Hence, the correct answer is (C).

118. Taking the total current as $6I$, current through $4R$ is $2I$ and current through $2R$ is $4I$. Applying Kirchhoff's Second Law, we get current through $2R$ is $2E/7R$.

Hence, the correct answer is (C).

119. $R_{total} = R_1 + r$

(Since capacitor is a dc blocking element)

$V_C = V_{R_1}$

$V_C = \frac{ER_1}{R_1 + r}$ $\left\{\because\ I = \frac{E}{R_1 + r}\right\}$

Since

$Q = CV_C$

$\Rightarrow\ Q = \frac{CER_1}{R_1 + r}$

Hence, the correct answer is (B).

120. $N = 24 = mn$ ...(1)

For current to be MAXIMUM

$R_{internal} = R_{external}$

$\Rightarrow\ \frac{mr}{n} = 3$

$\Rightarrow\ \frac{m}{n}(0.5) = 3$

$\Rightarrow\ \frac{m}{n} = 6$

$\Rightarrow\ m = 6n$ ...(2)

Substituting (2) in (1), we get

$24 = 6n^2$

$\Rightarrow\ n = 2$

$\Rightarrow\ m = 12$

Hence, the correct answer is (A).

121. Initial current, $i_1 = \frac{E_1 + E_2}{R + r_1 + r_2}$

Final current, when second battery is short circuited is

$i_2 = \frac{E_1}{R + r_1}$

$i_2 > i_1$ if $\frac{E_1}{R + r_1} > \frac{E_1 + E_2}{R + r_1 + r_2}$

$\Rightarrow\ E_1R + E_1r_1 + E_1r_2 > E_1R + E_1r_1 + E_2R + E_2r_1$

$\Rightarrow\ E_1r_2 > E_2(R + r_1)$

Hence, the correct answer is (B).

122. $V_0 = i_0R = (10)(10) = 100$ V

After 2 s, current becomes $\frac{1}{4}$th. Therefore, after 1 s, current will remain half also called half-life.

$t_{\frac{1}{2}} = (\ln 2)\tau_C = (\ln 2)CR$

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$$\Rightarrow \quad C = \frac{\left(t_{\frac{1}{2}}\right)}{(\ln 2)R} = \frac{1}{10\ln 2}F$$

Total heat $\Delta H = \frac{1}{2}CV_0^2$

$$\Rightarrow \quad \Delta H = \frac{1}{2} \times \frac{1}{10\ln 2}(100)^2$$

$$\Rightarrow \quad \Delta H = \frac{500}{\ln 2}J$$

Hence, the correct answer is (D).

123. $\frac{E_1 + E_2}{E_1 - E_2} = \frac{400}{200}$

$$\Rightarrow \quad E_1 + E_2 = 2(E_1 - E_2)$$

$$\Rightarrow \quad E_1 = 3E_2$$

$$\Rightarrow \quad \frac{E_1}{E_2} = \frac{3}{1}$$

Hence, the correct answer is (B).

124. $I = \frac{dQ}{dt}$

$$\Rightarrow \quad I = \alpha - 2\beta t$$

Since at $t_0 = t = \frac{\alpha}{2\beta}$, I becomes zero. Hence we shall calculate the heat produced from

$$t = 0 \text{ to } t_0 = \frac{\alpha}{2\beta}$$

$$\Rightarrow \quad H = \int_0^{t_0} I^2 R\, dt$$

$$\Rightarrow \quad H = R\int_0^{\frac{\alpha}{2\beta}} (\alpha - 2\beta t)^2\, dt$$

$$\Rightarrow \quad H = R\left.\frac{(\alpha - 2\beta t)^{2+1}}{-6\beta}\right|_0^{\frac{\alpha}{2\beta}}$$

$$\Rightarrow \quad H = R\left(0 - \frac{\alpha^3}{-6\beta}\right)$$

$$\Rightarrow \quad R = \frac{\alpha^3 R}{6\beta}$$

Hence, the correct answer is (D).

125. In position-1, initial maximum current is

$$i_0 = \frac{V}{R} = \frac{10}{5} = 2\text{ A}$$

At the given time, given current is 1 A of half of the above value. Hence, at this is instant capacitor is also charged to half of the final value of 5 V.
Now, it is shifted to position-2 wherein steady state it is again charged to 5 V but with opposite polarity.

$$U_i = U_f = \frac{1}{2}CV^2 \qquad \{\because\ V = 5\text{ V}\}$$

So, total energy supplied by the lower battery is converted into heat. But double charge transfer (from the normal) takes place from this battery.

$\Rightarrow$ Heat produced = Energy supplied by the battery

$$\Rightarrow \quad \Delta H = (\Delta q)V = (2CV)(V) = 2CV^2$$

$$\Rightarrow \quad \Delta H = 2 \times 2 \times 10^{-6} \times (5)^2$$

$$\Rightarrow \quad \Delta H = 100 \times 10^{-6}\text{ J}$$

$$\Rightarrow \quad \Delta H = 100\ \mu\text{J}$$

Hence, the correct answer is (C).

126. $\frac{R_1 + 10}{R_2} = \frac{50}{50}$

$$\Rightarrow \quad R_1 + 10 = R_2 \qquad \ldots(1)$$

Now, when $10\ \Omega$ is removed, then

$$\frac{R_1}{R_2} = \frac{40}{60}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{2}{3}$$

$$\Rightarrow \quad R_2 = \frac{3}{2}R_1 \qquad \ldots(2)$$

Put (2) in (1), we get

$$R_1 + 10 = \frac{3}{2}R_1$$

$$\Rightarrow \quad 2R_1 + 20 = 3R_1$$

$$\Rightarrow \quad R_1 = 20\ \Omega$$

Hence, the correct answer is (A).

127. Resistance between A and B can be removed due to balanced Wheatstone bridge concept. Now, R_{DE} and R_{GH} are in series and they are connected in parallel with 10 V battery.

$$\Rightarrow \quad I_{DE} = \frac{10}{R_{DE} + R_{HG}} = \frac{10}{2+2}$$

$$\Rightarrow \quad I_{DE} = 2.5\text{ A}$$

Hence, the correct answer is (B).

128. $R_{AB} = 2$ [Net resistance of infinite series] +1
In parallel net resistance is always less than the smallest one. Hence, net resistance of infinite series is less than $1\,\Omega$.

$\Rightarrow \quad 1\,\Omega < R_{AB} < 3\,\Omega$

Hence, the correct answer is (C).

129. $V = iR$

$\Rightarrow \quad V \propto R \qquad (\text{as } i = \text{constant})$

$$\Rightarrow \quad \frac{V_A}{V_B} = \left(\frac{\rho \ell_A}{\pi r_A^2}\right)\left(\frac{\pi r_B^2}{\rho \ell_B}\right)$$

$$\Rightarrow \quad \frac{r_B}{r_A} = \sqrt{\frac{V_A}{V_B} \times \frac{\ell_B}{\ell_A}}$$

$$= \sqrt{\frac{3}{2} \times \frac{1}{6}} = \frac{1}{2}$$

Hence, the correct answer is (B).

130. $\tau_C = CR$

$$\Rightarrow \quad \tau_C = \left(\frac{K\varepsilon_0 A}{d}\right)\left(\frac{d}{A\sigma}\right) \qquad \left\{\because R = \frac{\ell}{\sigma A}\right\}$$

$$\Rightarrow \quad \tau_C = \frac{K\varepsilon_0}{\sigma}$$

$$\Rightarrow \quad \tau_C = \frac{5 \times 8.86 \times 10^{-12}}{7.4 \times 10^{-12}}$$

$\Rightarrow \quad \tau_C = 6$ s

Hence, the correct answer is (D).

131. $r = R\left(\frac{\ell_1}{\ell_2} - 1\right) = 132.4\left(\frac{70}{60} - 1\right) \approx 22.1\,\Omega$

Hence, the correct answer is (A).

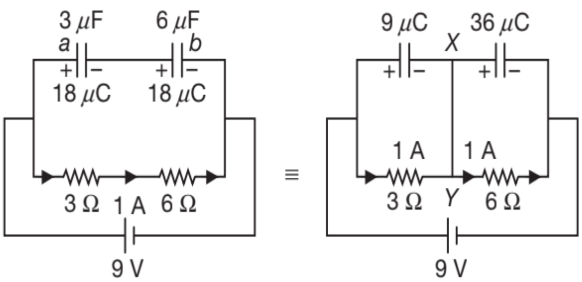
132. Since $\frac{X}{20} = \frac{1}{80}$

$\Rightarrow \quad X = 0.25\,\Omega$

Hence, the correct answer is (A).

133. From Y to X charge flows to plates a and b

$$(q_a + q_b)_i = 0, \ (q_a + q_b)_f = 27\ \mu\text{C}$$



Initial figure (When switch was open) Final figure (When switch is closed)

$\therefore \quad 27\ \mu$C charge flows from Y to X

Hence, the correct answer is (C).

134. Initially, the rate of charging is fast.

Hence, the correct answer is (B).

135. $\alpha(T) = \frac{1}{R_0}\frac{dR}{dT}$

$$\Rightarrow \quad (3T^2 + 2T) = \frac{1}{R_0}\frac{dR}{dT}$$

$$\Rightarrow \quad dR = R_0(3T^2 + 2T)dT$$

$$\Rightarrow \quad \int_{R_0}^{R} dR = R_0\left[3\int_0^T T^2\,dT + 2\int_0^T T\,dT\right]$$

$$\Rightarrow \quad R = R_0[1 + T^2 + T^3]$$

Hence, the correct answer is (C).

136. Let $R = at + b$

At $t = 10$ s, $R = 20\,\Omega$

$\Rightarrow \quad 20 = 10a + b \qquad \ldots(1)$

At $t = 30a + b \qquad \ldots(2)$

Solving these two equations, we get

$a = 1.0\ \Omega\text{s}^{-1}$

and $b = 10\,\Omega$

$\Rightarrow \quad R = (t + 10)$

Since, $i = \frac{E}{R} = \frac{10}{t + 10}$

and $\Delta q = \int_{10}^{30} i\,dt$

$$\Rightarrow \quad \Delta q = \int_{10}^{30}\left(\frac{10}{t + 10}\right)dt$$

$\Rightarrow \quad \Delta q = 10\log_e(2)$

Hence, the correct answer is (D).

137. During charging capacitor and resistance of its wire are independently connected with the battery. Hence,

$\tau_C = CR$

During discharging capacitor is discharged through both resistors (in series). Hence,

$\tau_C = C(2R) = 2CR$

Hence, the correct answer is (C).

138. Mean free path is the average distance travelled by the charge carriers between two successive collisions

$$\Rightarrow \quad \ell = v_d\tau = \frac{j\tau}{nq}$$

Hence, the correct answer is (B).

139. The given time is the half-life time of the exponentially decreasing equation. So,

$$t = t_{\frac{1}{2}} = (\ln 2)\tau_C = (\ln 2)CR_{net}$$

$$\Rightarrow \quad R_{net} = \frac{t}{(\ln 2)C}$$

$$\Rightarrow \quad R_{net} = \frac{2(\ln 2)\mu s}{(\ln 2)(0.5\ \mu F)} = 4\ \Omega$$

So, resistance of ammeter is $2\ \Omega$

Hence, the correct answer is (C).

140. 9, 9 and 9 are in parallel to give $3\ \Omega$.
This $3\ \Omega$ is further in series with the other $3\ \Omega$ to give $6\ \Omega$

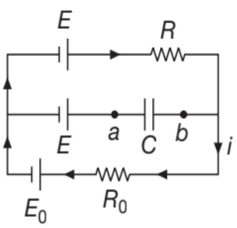
Hence, the correct answer is (B).

141. $V_A - 6 - 3 \times 2 + \frac{9}{1} - 3 \times 3 = V_B$

$$\Rightarrow \quad V_A - V_B = 12 \text{ V}$$

Hence, the correct answer is (A).

142.



$$i = \frac{[E - E_0]}{R + R_0}$$

Now, $V_a - E + E_0 + iR_0 = V_b$

$$\Rightarrow \quad V_a - V_b = (E - E_0) - iR_0$$

$$\Rightarrow \quad V_a - V_b = (E - E_0)\left[1 - \frac{R_0}{R + R_0}\right]$$

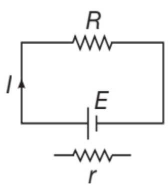
$$\Rightarrow \quad V_a - V_b = \frac{R(E - E)}{R + R_0}$$

$$\Rightarrow \quad q = C(V_a - V_b) = \frac{CR(E - E_0)}{R + R_0}$$

Hence, the correct answer is (D).

143. $I = \frac{E}{R + r}$

$\Rightarrow \quad P = I^2 R$ = Power consumed across external resistance



$$\Rightarrow \quad P = \frac{E^2 R}{(R + r)^2}$$

For P to be Maximum

$$\Rightarrow \quad \frac{dP}{dR} = 0$$

$$\Rightarrow \quad R = r$$

Hence, the correct answer is (A).

144. $i_1 = \left(\frac{V}{2R}\right)e^{-\frac{t}{6CR}}$

$$\Rightarrow \quad i_2 = \left(\frac{V}{R}\right)e^{-\frac{t}{CR}}$$

$$\Rightarrow \quad \frac{i_1}{i_2} = \frac{e^{\frac{5t}{6CR}}}{2}$$

We can see that this ratio is increasing with time.

Hence, the correct answer is (B).

145. $mn = 24$ …(1)

For power consumption across the load to be maximum

$$\begin{pmatrix}\text{External} \\ \text{Resistance}\end{pmatrix} = \begin{pmatrix}\text{Total Internal} \\ \text{Resistance}\end{pmatrix}$$

$$\Rightarrow \quad R = \frac{nr}{m}$$

$$\Rightarrow \quad \frac{n}{m} = 10 \qquad \text{…(2)}$$

$$\Rightarrow \quad n = 10\ m$$

$$\Rightarrow \quad 10\ m^2 = 24$$

$$\Rightarrow \quad m = \sqrt{2.4}$$

$$\Rightarrow \quad m \simeq 1.55$$

Option-1

$m = 1;\ n = 24$

Since

$$\Rightarrow \quad I_1 = \frac{nE}{R + \frac{nr}{m}}$$

$$\Rightarrow \quad I_1 = \frac{24}{34}E$$

Option-2

$m = 2;\ n = 12$

Since

$$\Rightarrow \quad I_2 = \frac{nE}{R + \frac{nr}{m}} = \frac{12}{16}E$$

$$\Rightarrow \quad I_2 = \frac{24}{32}E$$

From above we observe that $I_2 > I_1$

Hence $n = 12$, $m = 2$ for the power consumption across the load to be maximum.
Hence, the correct answer is (B).

146. In steady state condition, current flows from outermost loop.

$$i = \frac{12}{6+2} = 1.5 \text{ A}$$

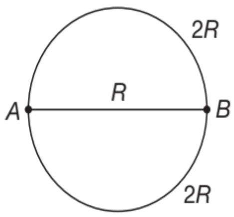
Now, $V_C = V_{6\,\Omega} = iR$

$\Rightarrow \quad V_C = 1.5 \times 6 = 9 \text{ V}$

$\Rightarrow \quad q = CV_C = 18\ \mu C$

Hence, the correct answer is (D).

147.



$$R = \left(\frac{4}{2\pi r}\right)(2r) = \frac{4}{\pi}$$

Now $2\,\Omega$, $2\,\Omega$ and R are in parallel.

Hence, the correct answer is (A).

149. $E - ir = 0$

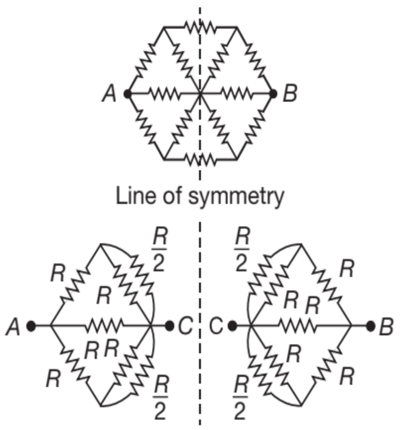
$$\Rightarrow \quad 3 - \left(\frac{3+15}{1+2+R}\right)(1) = 0$$

Solving this equation, we get

$R = 3\,\Omega$

Hence, the correct answer is (C).

150.



Let each resistance be having a value R

$\Rightarrow \quad R_{AB} = R_{AC} + R_{CB}$

Hence, the correct answer is (B).

151. At $t = 0$, when capacitor is under charged equivalent resistance of capacitor $= 0$

In this case, $6\,\Omega$ and $3\,\Omega$ are parallel $(\text{equivalent} = 2\,\Omega)$

$\Rightarrow \quad R_{\text{net}}(1+2)\Omega = 3\,\Omega$

$\Rightarrow \quad \text{Current from battery} = \frac{12}{3} = 4 \text{ A}$

$= \text{Current through } 1\,\Omega \text{ resistor}$

Hence, the correct answer is (B).

152. $\tau_C = CR = 6 \text{ s}$

$q_0 = CV = 10\ \mu C$

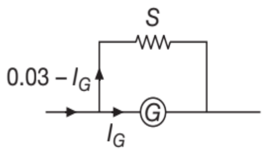
Now, $q = q_0 e^{-\frac{t}{\tau C}} = (10\ \mu\text{C})e^{-\frac{12}{6}}$

$$\Rightarrow \quad q = \left(\frac{1}{e}\right)^2 (10\ \mu\text{C})$$

$\Rightarrow \quad q = (0.37)^2 (10\ \mu\text{C})$

Hence, the correct answer is (B).

154.



In parallel, current distributers in inverse ratio of resistance.

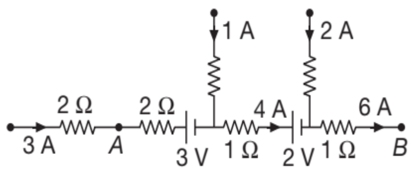
$$\frac{0.03 - I_G}{I_G} = \frac{G}{S} = \frac{r}{\left(\frac{r}{4}\right)} = 4$$

Solving this equation, we get

$I_G = 0.006 \text{ A}$

Hence, the correct answer is (C).

156.

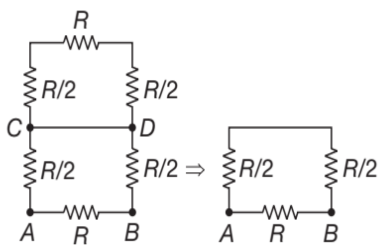


$V_A - 3 \times 2 - 3 - 1 \times 4 + 2 - 1 \times 6 = V_B$

$\Rightarrow \quad V_A - V_B = 17 \text{ V}$

Hence, the correct answer is (D).

157.



Hence, the correct answer is (D).

159. $V_{1\,\Omega} = 5 + 2 = 7$ V

$\Rightarrow \quad i_{1\,\Omega} = \frac{V}{R} = 7$ A

$V_{2\,\mu F} = 6$ V

$\Rightarrow \quad q_{2\,\mu F} = CV = 12\ \mu C$

Hence, the correct answer is (B).

161. $H_1 = H_2 = U_i - U_f$

The only change is by increasing the resistance τ_C increase. Hence, process of redistribution of charge slows down.

Hence, the correct answer is (A).

162. When K_1 and K_2 both are closed R_1 is short-circuited,

$R_{net} = (50 + r)\ \Omega$

When K_1 is open and K_2 is closed, current remains half.

Therefore, net resistance of the circuit becomes two times.

$\Rightarrow \quad (50 + r) + R_1 = 2(50 + r)$

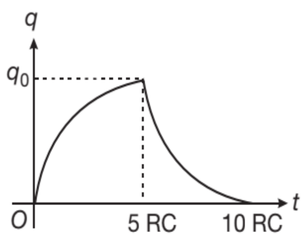
Of the given options, the above equation is satisfied if

$r = 0$ and $R_1 = 50\ \Omega$

Hence, the correct answer is (D).

163. Practically, the steady state is attained when five time constants have elapsed. As soon as the key is opened the peak value of charge also decays in five time constants. Hence pulse period is

$5RC + 5RC = 10RC$



Hence, the correct answer is (C).

164. Potential drop across potentiometer wire

$= (0.2 \times 10^{-3})(100) = 0.02$ V

Now given resistance and potentiometer wire are in series with given battery. So, potential will drop in direct ratio of resistance.

$\Rightarrow \quad \frac{0.02}{2 - 0.02} = \frac{R}{490}$

$\Rightarrow \quad R = 4.9\ \Omega$

Hence, the correct answer is (A).

165. $q_g + q_d = q_0 = 10\ \mu C$

$\Rightarrow \quad q_d = 7\ \mu C$

Hence, the correct answer is (B).

166. Total heat produced $\Delta H = H = \frac{1}{2}CV^2$

$\Rightarrow \quad \Delta H = \frac{1}{2}(2\ \mu F)(5)^2$

$\Rightarrow \quad \Delta H = 25\ \mu J$

Now, this should distribute in inverse ratio of resistors, as they are in parallel.

$\Rightarrow \quad \frac{H_{5\,\Omega}}{H_R} = \frac{R}{5}$

$\Rightarrow \quad H_{5\,\Omega} = \left(\frac{R}{R+5}\right)$ (Total heat)

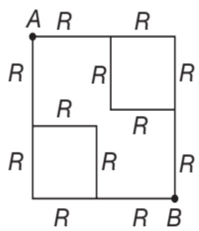
$\Rightarrow \quad 10 = \left(\frac{R}{R+5}\right)(25)$

Solving this equation, we get

$R = \left(\frac{10}{3}\right)\Omega$

Hence, the correct answer is (C).

167.



Connection can be removed from centre. $3R$ and $3R$ from two sides of AB are in parallel.

Hence, the correct answer is (B).

168. $\frac{1}{10} = 1 - e^{-t/\tau}$

$\Rightarrow \quad e^{-t/\tau} = \frac{9}{10}$

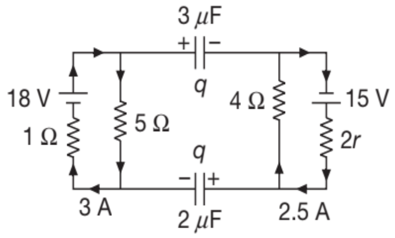
$\Rightarrow \quad e^{t/\tau} = \frac{10}{9}$

$\Rightarrow \quad \frac{t}{\tau} = \ell n\left(\frac{10}{9}\right)$

$\Rightarrow \quad t = \tau \ell n\left(\frac{10}{9}\right)$

Hence, the correct answer is (B).

169. Applying Kirchhoff's loop law in outermost loop, we have



$$-\frac{q}{3}+15-2\times 2.5-\frac{q}{2}-3\times 1+18=0$$

Solving this equation, we get

$$q=30\ \mu\text{C}$$

Hence, the correct answer is (A).

171. Suppose $n(<1)$ fraction of length is stretched to m times.

Then, $(1-n)\ell+(n\ell)m=1.5\ell$

$$\Rightarrow\quad nm-n=0.5 \qquad \ldots(1)$$

$$R=\frac{\rho\ell}{A}=\frac{\rho\ell}{\left(\frac{V}{\ell}\right)} \qquad (V=\text{volume})$$

$$\Rightarrow\quad R=\frac{\rho\ell^2}{V}$$

$$\Rightarrow\quad R\propto\ell^2 \qquad (\text{if } V=\text{constant})$$

Now, the second condition is

$$(1-n)R+(nR)m^3=4R$$

$$\Rightarrow\quad nm^2-n=3 \qquad \ldots(2)$$

Solving these two equations, we get

$$n=\frac{1}{8}$$

Hence, the correct answer is (B).

172. Total potential of 10 V equally distributes between 50 Ω and other parallel combination of 100 Ω and voltmeter. Hence, their net resistance should be same.

$$\Rightarrow\quad \frac{100\times R}{100+R}=50$$

$$\Rightarrow\quad R=100\ \Omega=\text{resistance of voltmeter}$$

Hence, the correct answer is (B).

174. All these resistors are in parallel.

$$\Rightarrow\quad R_{\text{net}}=\frac{R}{3}+r=4\ \Omega$$

Hence. the main current

$$i=\frac{E}{R_{\text{net}}}=1\text{ A}$$

Current through either of the resistance is $\frac{i}{3}$

$$\Rightarrow\quad \frac{1}{3}\text{ A}$$

$$\Rightarrow\quad V=iR=\left(\frac{1}{3}\right)(9)=3\text{ V}$$

Hence, the correct answer is (A).

175. Since capacitors are identical, hence they share equal amount of charge $\frac{Q_0}{2}$.

$$E_i=\frac{Q_0^2}{2C}$$

$$\Rightarrow\quad E_f=2\left[\frac{\left(\frac{Q_0}{2}\right)^2}{2C}\right]=\frac{Q_0^2}{4C}$$

{Due to sharing the new capacitance is 2C}}

Heat produced $=E_i-E_f=\frac{Q_0^2}{4C}$

Hence, the correct answer is (C).

Multiple Correct Choice Type Questions

1. Let us assume that charge flow between the rod and cylinder be radial. Then at point P at distance r from centre of rod the current density J is

$$J=\frac{I}{2\pi r\ell}$$

Further

$$E=\rho J=\frac{\rho I}{2\pi r\ell} \qquad \{\because J=\sigma E\}$$

with both J and E directed radially outwards.

Since

$$dV=-E\,dr$$

$$\Rightarrow\quad dV=-\frac{\rho I}{2\pi\ell}\frac{dr}{r}$$

$$\Rightarrow\quad \int_a^b dV=-\frac{\rho I}{2\pi\ell}\int_a^b\frac{dr}{r}$$

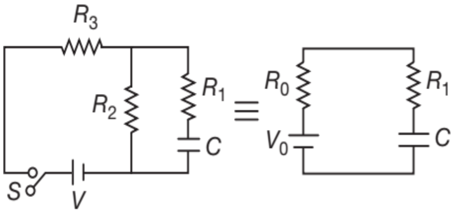
$$\Rightarrow\quad -V=-\frac{\rho I}{2\pi\ell}\log_e\left(\frac{b}{a}\right)$$

$$\Rightarrow\quad I=\frac{2\pi\ell V}{\rho\log_e\left(\frac{b}{a}\right)}$$

Hence, (A) and (B) are correct.

2. $R_0=\frac{R_2R_3}{R_2+R_3}$ and $V_0=\frac{V\,R_2}{R_2+R_3}$

CHAPTER 3



$$\Rightarrow \quad \tau = C(R_1 + R_0) = C\left(R_1 + \frac{R_2R_3}{R_2+R_3}\right)$$

$$\Rightarrow \quad q = q_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\Rightarrow \quad q = CV_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

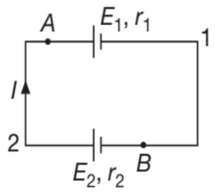
$$\Rightarrow \quad q = \frac{CVR_2}{R_2+R_3}\left(1 - e^{-\frac{t}{\tau}}\right)$$

Hence, (B) and (C) are correct.

3. Let $E_2 > E_1$, then

$$I = \frac{E_2 - E_1}{r_1 + r_2}$$

For potential difference across battery 1, apply KVL to $A1B$, we get



$$V_A - E_1 - Ir_1 - V_B = 0$$

$$\Rightarrow \quad V_A - V_B = E_1 + Ir_1 \quad \ldots(1)$$

$$\Rightarrow \quad V_A - V_B = E_1 + \left(\frac{E_2 - E_1}{r_1 + r_2}\right)r_1$$

$$\Rightarrow \quad V_A - V_B = \frac{E_1r_1 + E_1r_2 + E_2r_1 - E_1r_1}{r_1 + r_2}$$

$$\Rightarrow \quad V_A - V_B = \frac{E_1r_2 + E_2r_1}{r_1 + r_2} \quad \ldots(2)$$

For potential difference across battery 2, apply KVL to $A2B$, we get

$$V_A + Ir_2 - E_2 - V_B = 0$$

$$\Rightarrow \quad V_A - V_B = E_2 - Ir_2 \quad \ldots(3)$$

$$\Rightarrow \quad V_A - V_B = E_2 - \left(\frac{E_2 - E_1}{r_1 + r_2}\right)r_2$$

$$\Rightarrow \quad V_A - V_B = \frac{E_2r_1 + E_2r_2 - E_2r_2 + E_1r_2}{r_1 + r_2}$$

$$\Rightarrow \quad V_A - V_B = \frac{E_1r_2 + E_2r_1}{r_1 + r_2} \quad \ldots(4)$$

Also, we could have thought of equating equation (1) and (2), to get

$I = \dfrac{E_2 - E_1}{r_1 + r_2}$, which also satisfies the same result obtained.

Also, from equation (1), we get that potential difference across 1 is greater than its emf E_1.

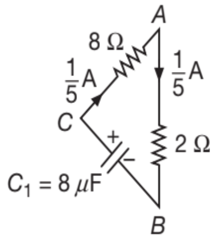
Now, since the current I flows from the positive plate to the negative plate inside the battery 2, hence energy is absorbed by it or in other words the battery will continuously get the energy supplied by the other battery.

Hence all the statements A, B, C and D are correct.

Hence, (A), (B), (C) and (D) are correct.

4. In steady state, no current passes through the branches containing the capacitors. So,

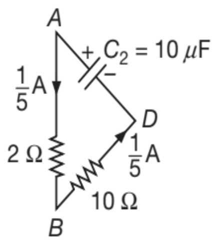
$$I = \frac{4}{8+2+10} = \frac{4}{20} = \frac{1}{5}\text{ A}$$



Loop ABCA

$$-\left(\frac{1}{5}\right)8 - \left(\frac{1}{5}\right)2 + \Delta V_1 = 0$$

$$\Rightarrow \quad \Delta V_1 = \frac{10}{5} = 2\text{ V}$$



Loop ABDA

$$-\left(\frac{1}{5}\right)2 - \left(\frac{1}{5}\right)(10) + \Delta V_2 = 0$$

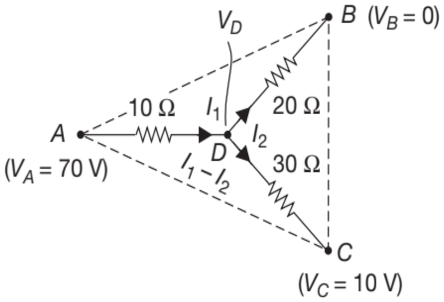
$$\Rightarrow \quad \Delta V_2 = 2 + 0.4$$

$$\Rightarrow \quad \Delta V_2 = 2.4\text{ V}$$

Hence, (B) and (D) are correct.

6. Let V_D be the potential at the point D, then

$$70 - V_D = I_1 \times 10\ \Omega$$
$$V_D - 0 = I_2 \times 20\ \Omega$$
$$V_D - 10 = (I_1 - I_2) \times 30\ \Omega$$



Solve for I_1, I_2 and V_D, we get

$$V_D = 40\text{ V}$$
$$I_1 = 3\text{ A},\ I_2 = 2\text{ A and } (I_1 - I_2) = 1\text{ A}$$
$$P_{\text{total}} = (3)^2(10) + (2)^2(20) + (1)^2(30)$$
$$\Rightarrow P_{\text{total}} = 90 + 80 + 30 = 200\text{ W}$$
$$\Rightarrow I_1 : I_2 : (I_1 - I_2) :: 3:2:1 \text{ and } P_{\text{total}} = 200\text{ W}$$

Hence, (A), (B) and (D) are correct.

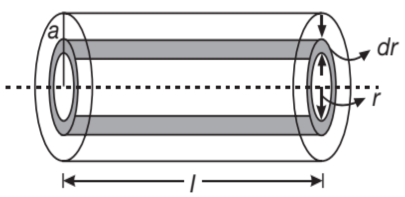
7. Consider a cylindrical element of radius r, thickness dr. If dR is the resistance of this element then

$$dR = \frac{\rho(r)\ell}{2\pi r\, dr}$$

Total resistance of the cylinder is given by

$$\frac{1}{R_{\text{total}}} = \int \frac{1}{dR} = \frac{2\pi}{\alpha\ell}\int_0^a r^3 dr$$

$$\Rightarrow \frac{1}{R_{\text{total}}} = \frac{2\pi}{\alpha\ell}\left(\frac{a^4}{4}\right)$$



$$\Rightarrow R_{\text{total}} = \frac{2\alpha\ell}{\pi a^4}$$

$$\Rightarrow \frac{R_{\text{total}}}{\ell} = \frac{2\pi\alpha}{(\pi a^2)^2}$$

$$\Rightarrow R = \frac{2\pi\alpha}{A^2}$$

Since $E = \frac{V}{\ell}$ (in magnitude)

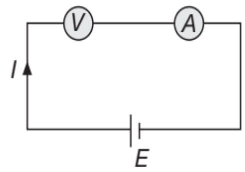
$$\Rightarrow E = I\left(\frac{R_{\text{total}}}{\ell}\right) \qquad \text{\{By Ohm's Law\}}$$

$$\Rightarrow E = \frac{2\pi\alpha I}{A^2}$$

Hence, (A) and (C) are correct.

8. $I = \frac{E}{R_A + R_V}$

$$\Rightarrow R_A + R_V = \frac{E}{I} \qquad \ldots(1)$$



Since $E = V + IR_A$

$$\Rightarrow V = E - IR_A < E \qquad \ldots(2)$$

Also, $V = IR_V$

$$\Rightarrow R_V = \frac{V}{I} \qquad \ldots(3)$$

Potential difference across the ammeter is $\Delta V_A = IR_A$

$$\Rightarrow \Delta V_A = IR_A = E - V \qquad \ldots(4)$$

So, from (1), (2), (3) and (4), we observe all the options to be correct.

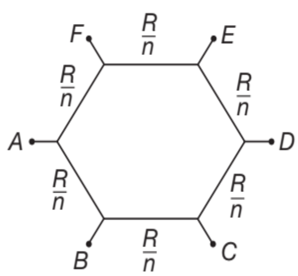
Hence, (A), (B), (C) and (D) are correct.

10. Consider the 'n' sided polygon, shown below

For the points that face each other like (A, D), (B, E) and (C, F), we have

$$R_{\text{MAX}} = R_{AD} = R_{BE} = R_{CF}$$

Resistance for the upper half between A and D is $\left(\frac{R}{n}\right)\left(\frac{n}{2}\right) = \frac{R}{2}$ and that for the lower half between A and D is also $\frac{R}{2}$.



These both are in parallel across A and D, hence

CHAPTER 3

$$R_{MAX} = R_{AD} = R_{BE} = R_{CF} = \frac{\left(\frac{R}{2}\right)\left(\frac{R}{2}\right)}{\frac{R}{2}+\frac{R}{2}} = \frac{R}{4}$$

We shall get the minimum value of resistance between the adjacent points of the polygon i.e., AB, BC, CD, DE, EF or FA. Between two adjacent points we have two resistors connected in parallel one having resistance $\frac{R}{n}$ and other having resistance $(n-1)\frac{R}{n}$

So, $$R_{MIN} = R_{\substack{\text{ADJACENT}\\ \text{POINTS}}} = \frac{\left(\frac{R}{n}\right)(n-1)\left(\frac{R}{n}\right)}{\frac{R}{n}+(n-1)\frac{R}{n}} = \left(\frac{n-1}{n^2}\right)R$$

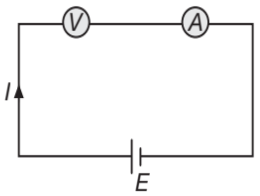
Hence, (A) and (C) are correct.

11. Let the respective resistance of the voltmeter and ammeter be R_V and R_A. Then, initially

$$I_{\text{initial}} = \frac{E}{R_V + R_A} \qquad \ldots(1)$$

Finally, when another resistance of value R_A is connected in parallel with the ammeter, then we have

$$I_{\text{final}} = \frac{E}{R_V + \frac{R_A}{2}}$$



Initially

New ammeter reading is

$$\frac{I_{\text{final}}}{2} = \frac{E}{2R_V + R_A}$$

Initial reading of ammeter was

$$I_{\text{initial}} = \frac{E}{R_V + R_A}$$

So, we observe that

$$\frac{E}{2R_V + R_A} > \frac{1}{2}\left(\frac{E}{R_V + R_A}\right)$$

$$\Rightarrow \quad I_{\text{final}} > \frac{1}{2}(I_{\text{initial}})$$

Initial reading of the voltmeter is

$$V_{\text{initial}} = \left(\frac{E}{R_V + R_A}\right)R_V$$

Final reading of the voltmeter is

$$V_{\text{final}} = \left(\frac{E}{R_V + \frac{R_A}{2}}\right)R_V > V_{\text{initial}}$$

Hence, (B) and (D) are correct.

12. In steady state,

$$q_C = EC \text{ and } q_{2C} = 2EC$$

$$\tau_C = 2CR \text{ of both circuits}$$

At time t,

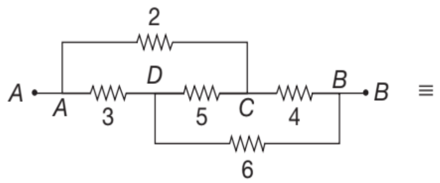
$$q_C = EC\left(1 - e^{\frac{t}{\tau_C}}\right)$$

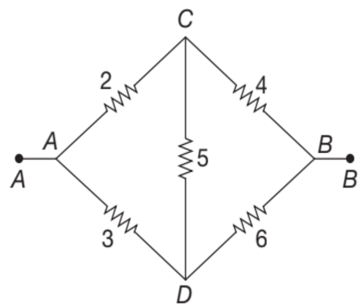
$$q_{2C} = 2EC\left(1 - e^{-\frac{t}{\tau_C}}\right)$$

$$\Rightarrow \quad \frac{q_C}{q_{2C}} = \frac{1}{2}$$

Hence, (B), (C) and (D) are correct.

14.

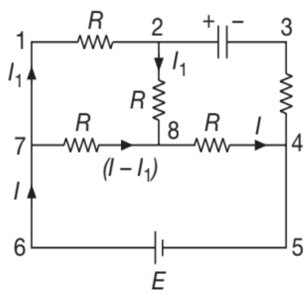




Rearrangement of the circuit as shown gives a balanced Wheatstone bridge, and no current flows through the $5\,\Omega$ resistor. It can thus be removed from the circuit to find the net resistance. When the current in the $5\,\Omega$ branch is zero, then $\Delta V = 0$.

Hence, (B) and (D) are correct.

15. In steady state, no current passes through the branch that contains a fully charged capacitor, because a fully charged capacitor is a dc blocking element. Hence the circuit becomes



For Loop 1284561

$$-2I_1R - IR + E = 0$$

$$\Rightarrow \quad 6I_1 + 3I = 15 \qquad \ldots(1)$$

For Loop 784567

$$-(I - I_1)R - IR + E = 0$$

$$-2IR + I_1R + E = 0$$

$$\Rightarrow \quad 3I_1 - 6I = -15$$

$$\Rightarrow \quad \frac{3I_1}{2} - 3I = -\frac{15}{2} \qquad \ldots(2)$$

Add (1) and (2), we get

$$\left(\frac{3}{2} + 6\right)I_1 = 15 - \frac{15}{2}$$

$$\Rightarrow \quad \left(\frac{3+12}{2}\right)I_1 = \frac{30-15}{2}$$

$$\Rightarrow \quad 15I_1 = 15$$

$$\Rightarrow \quad I_1 = 1 \text{ A}$$

$$\Rightarrow \quad 6 + 3I = 15$$

$$\Rightarrow \quad 3I = 9$$

$$\Rightarrow \quad I = 3 \text{ A}$$

For Loop 23482

$$-V_C + I(3) + I_1(3) = 0$$

$$\Rightarrow \quad V_C = 9 + 3 = 12 \text{ V}$$

Hence, (C) and (D) are correct.

Please note that though no current flows through the branch containing the fully charged capacitor, yet potential difference across the capacitor will be finite non zero value given by $\Delta V = \pm \frac{q}{C}$.

16. $V_1 = \left(\frac{R_1}{R_1 + R_2 + R_3 + \ldots\ldots}\right)V = \left(\frac{R_1}{R_s}\right)V$

$$\Rightarrow \quad V_2 = \left(\frac{R_2}{R_1 + R_2 + R_3 + \ldots\ldots}\right)V = \frac{R_2}{R_s}V$$

Hence, (A), (B) and (C) are correct.

17. Acceleration of each electron is

$$a = \frac{eE}{m} = \frac{eV}{md}$$

$$\Rightarrow \quad v^2 - 0^2 = 2ax$$

$$\Rightarrow \quad v = \sqrt{\frac{2eVx}{md}}$$

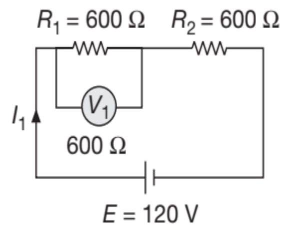
Since $I = neAv$

and $\rho = en = e\frac{I}{veA}$

$$\Rightarrow \quad \rho = I\sqrt{\frac{md}{2eVxA^2}}$$

Hence, (A) and (B) are correct.

18. **CASE-1:** Net resistance of the circuit is



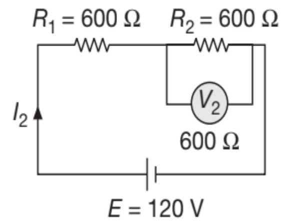
$$R = \frac{(600)(600)}{600 + 600} + 300$$

$$R = 600\ \Omega$$

$$\Rightarrow \quad I_1 = \frac{120}{600} = \frac{1}{5} \text{ A}$$

$$V_1 = \frac{1}{5}(300) = 60 \text{ V}$$

CASE-2: Net resistance of circuit is



$$R' = 600 + \frac{(300)(600)}{300 + 600}$$

$$\Rightarrow \quad R' = 600 + 200 = 800\ \Omega$$

$$\Rightarrow \quad I_2 = \frac{E}{R'} = \frac{120}{800} = \frac{3}{20} \text{ A}$$

Now $V_2 = \left(\frac{3}{20}\right)\left(\frac{300 \times 600}{300 + 600}\right) = 30 \text{ V}$

Hence, (C) and (D) are correct.

20. For conductors, the resistance decreases with the increase in temperature as random motion of the charge carriers decreases, so $\alpha_2 > 0$
For semiconductors, the resistance increases with the decrease in temperature and hence $\alpha_1 < 0$.
Hence, (B) and (C) are correct.

21. $\Rightarrow \quad I = \frac{9}{9} = 1 \text{ A}$

At A a current of 1 A divides into 0.5 A and 0.5 A.
At B the current of 0.5 A divides into 0.25 A and 0.25 A
Hence, (A) and (D) are correct.

22. See Theory
Hence, (A) and (C) are correct.

23. The electrical paths AC_1D_2B and AD_1C_2B are in parallel. The resistance of each of them is $10\ \Omega$, and hence the current is 1 A.
When C_1C_2 and D_1D_2 are joined, the ammeters are in parallel combination. As they are of a same resistance, their readings will be equal.
Hence, (B), (C) and (D) are correct.

24. Since $R = \frac{\rho\ell}{A} = \frac{\ell}{\sigma A}$

and $C = \frac{K\varepsilon_0 A}{\ell}$

$\Rightarrow \quad \tau = RC = \frac{\varepsilon_0 K}{\sigma}$

Since $q = q_0 e^{-t/\tau}$

$\Rightarrow \quad q = q_0 e^{-\frac{\sigma t}{K\varepsilon_0}}$

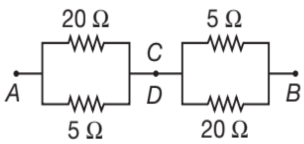
Leakage current $= I = \frac{dq}{dt}$

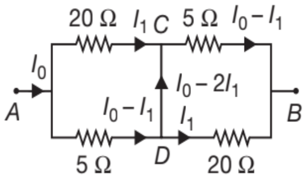
$\Rightarrow \quad I = -\frac{q_0\sigma}{\varepsilon_0 K} e^{-\frac{t\sigma}{\varepsilon_0 K}}$

$\Rightarrow \quad I = I_0 e^{-\frac{t\sigma}{\varepsilon_0 K}}$ where $I_0 = -\frac{q_0\sigma}{\varepsilon_0 K}$

Hence, (A), (C) and (D) are correct.

25. As C and D are joined, they must be at the same potential, and may be treated as the same point. This gives the equivalent resistance as $8\ \Omega$. When we distribute current in the network using symmetry,





$$V_A - V_D = V_A - V_C$$

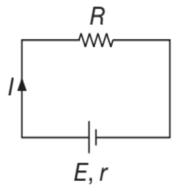
$\Rightarrow \quad 20I_1 = 5(I_0 - I_1)$

$\Rightarrow \quad I_1 = \frac{I_0}{5}$

$\Rightarrow \quad I_0 - 2I_1 = I_0 - \frac{2I_0}{5} = \frac{3I_0}{5}$ which is the current flowing from D to C.

Hence, (A), (B) and (C) are correct.

28. $I = \frac{E}{R+r}$



$$P = EI$$

$\Rightarrow \quad P = \frac{E^2}{R+r}$

So, heat produced per second across the external circuit is

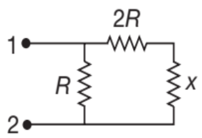
$$H = I^2R = I(IR) = \frac{E(IR)}{R+r}$$

and that across the internal circuit is

$$h = I^2 r = \frac{E^2 r}{(R+r)^2}$$

Hence, (A), (B), (C) and (D) are correct.

29. CIRCUIT A



$$x = \frac{R(2R+x)}{3R+x}$$

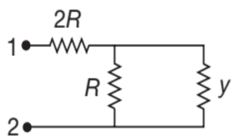
$\Rightarrow \quad 3Rx + x^2 = 2R^2 + Rx$

$\Rightarrow \quad x^2 + 2Rx - 2R^2 = 0$

$\Rightarrow \quad x = -\frac{2R \pm \sqrt{4R^2 + 8R^2}}{2}$

$\Rightarrow \quad x = \frac{-2R + 2\sqrt{3}R}{2}$

$\Rightarrow \quad x = \left(\sqrt{3} - 1\right)R$



CIRCUIT B

$$y = \frac{yR}{y+R} + 2R$$

$\Rightarrow \quad y^2 + Ry = yR + 2Ry + 2R^2$

$\Rightarrow \quad y^2 - 2yR - 2R^2 = 0$

$\Rightarrow \quad y = (\sqrt{3}+1)R$

Hence, (A), (B), (C) and (D) are correct.

31. Since voltmeter is a device connected in parallel across the circuit, hence

$$R_{\text{equivalent}} = \frac{R_V R_0}{R_V + R_0}$$

For $R_0 \ll R_V$

$\Rightarrow \quad R_{\text{equivalent}} \simeq R_0$

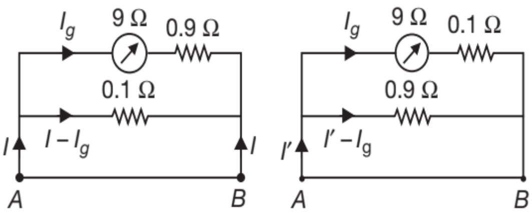
{i.e., resistance of the circuit remains unaltered when a voltmeter of extremely high resistance is applied across the circuit}

Hence, (A) and (B) are correct.

33. $I_g = 10 \text{ mA} = 0.01 \text{ A}$

$V_A - V_B = (I - I_g)(0.1 \text{ A}) = I_g(9.9)$

$\Rightarrow \quad I \times 0.1 = I_g \times 10$



$\Rightarrow \quad I = \frac{0.01 \text{ A} \times 10}{0.1} = 1 \text{ A}$

Similarly $(I' - I_g)(0.9) = I_g(9.1)$

$\Rightarrow \quad I'(0.9) = I_g(10)$

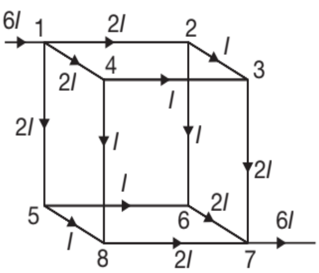
$\Rightarrow \quad I' = \frac{1}{9} \text{ A} = 111 \text{ mA}$

Hence, (A) and (C) are correct.

34. Resistance between diagonal corners of cube

$\Rightarrow \quad V_{17} = (V_1 - V_5) + (V_5 - V_8) + (V_8 - V_7)$

$\Rightarrow \quad V_{17} = 2IR_0 + IR_0 + 2IR_0$



$\Rightarrow \quad V_{17} = 5IR_0$

Also,

$V_{17} = x(6I)$

$\Rightarrow \quad x(6I) = 5IR_0$

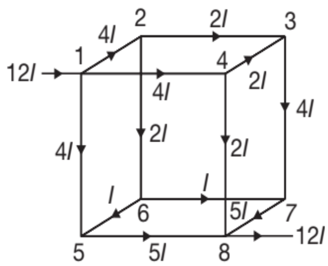
$\Rightarrow \quad x = \frac{5R_0}{6}$

$\Rightarrow \quad V_{18} = (V_1 - V_5) + (V_5 - V_8)$

$\Rightarrow \quad V_{18} = 4IR_0 + 5IR_0$

$\Rightarrow \quad V_{18} = 9IR_0$

Also $V_{18} = y(12I)$



$\Rightarrow \quad (12I)y = 9IR_0$

$\Rightarrow \quad y = \frac{3}{4}R_0$

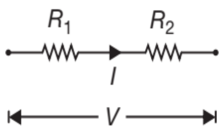
$\Rightarrow \quad \frac{x}{y} = \frac{\left(\frac{5}{6}\right)}{\left(\frac{3}{4}\right)} = \frac{20}{18} = \frac{10}{9}$

$\Rightarrow \quad x - y = \frac{5}{6}R_0 - \frac{3}{4}R_0$

$\Rightarrow \quad x - y = \frac{R_0}{12}$

Hence, (A) and (B) are correct.

35. Let $V = 220$ V and R_1 and R_2 be the resistances of the 25 W and 100 W bulbs.



Since, $P_A = 25 \text{ W} = \frac{V^2}{R_1}$ and $P_B = 100 \text{ W} = \frac{V^2}{R_2}$

$\Rightarrow \quad R_1 = \frac{V^2}{25 \text{ W}}$ and $R_2 = \frac{V^2}{100 \text{ W}}$

When the bulbs are joined in series then the current is

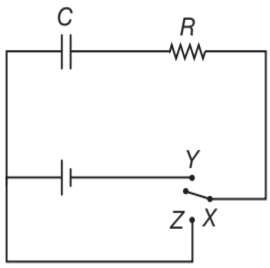
$I = \frac{V}{R_1 + R_2}$

Power in the 25 W bulb is R_1I^2 and that in the 100 W bulb is R_2I^2.

Hence, (A) and (C) are correct.

36. When X is joined to Y for a long time (charging), the energy stored in the capacitor is equal to the heat produced in R i.e., H_1.

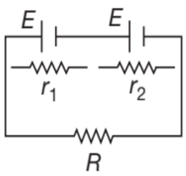
When X is joined to Z (discharging), the energy stored in $C(=H_1)$ reappears as heat (H_2) in R. So, $H_1 = H_2$. Also, we observe that energy supplied by the battery is $H = H_1 + H_2$.



Hence, (A), (C) and (D) are correct.

37. $I = \dfrac{2E}{R + r_1 + r_2}$

$V_1 = 0$, for terminal potential difference to be zero across source 1.



$\Rightarrow \quad E - Ir_1 = 0$

$\Rightarrow \quad E - \dfrac{2Er_1}{R + r_1 + r_2} = 0$

$\Rightarrow \quad 2r_1 = R + r_1 + r_2$

$\Rightarrow \quad R = r_1 - r_2$

$V_2 = 0$, for terminal potential difference to be zero across source 2.

$\Rightarrow \quad E - Ir_2 = 0$

$\Rightarrow \quad E - \dfrac{2\,Er_2}{R + r_1 + r_2} = 0$

$\Rightarrow \quad 2r_2 = R + r_1 + r_2$

$\Rightarrow \quad R = r_2 - r_1$

Hence, (A) and (C) are correct.

38. Let R_1 and R_2 be the resistances of the two heaters. Let, initially, the heat produced be H, then

$$H = \left(\frac{V^2}{R_1}\right)t_A = \left(\frac{V^2}{R_2}\right)t_B \quad \ldots(1)$$

When used in series,

$$H = \left(\frac{V^2}{R_1 + R_2}\right)T \quad \ldots(2)$$

When used in parallel

$$H = \left(\frac{V^2}{R_1} + \frac{V^2}{R_2}\right)t \quad \ldots(3)$$

Using (1), (2) and (3), we get the desired results.

Hence, (B) and (D) are correct.

39. When steady state is reached no current will flow in the branch having the capacitor. So

$$R_{total} = 2.8 + \frac{(2)(3)}{2+3}$$

$\Rightarrow \quad R_{total} = 4\ \Omega$

$\Rightarrow \quad I = \dfrac{6}{4} = 1.5\text{ A}$

Current in 2 Ω resistor $= I_2 = \left(\dfrac{1/2}{1/2 + 1/3}\right)1.5$

$\Rightarrow \quad I_2 = \left(\dfrac{3}{5}\right)(1.5)$

$\Rightarrow \quad I_2 = 0.9\text{ A}$

Potential drop across 2.8 Ω is

$$V = (2.8)(1.5)$$

$\Rightarrow \quad V = 4.2\text{ V}$

Hence, (A), (B) and (D) are correct.

40. $V_A - V_B = 9\text{ V}$

15 V battery and 24 V battery form the dominating combination and hence I must flow from D to A through R.

$\Rightarrow \quad I = \dfrac{24 + 15 - 6}{R + 1 + 2 + 1}$

$\Rightarrow \quad I = \dfrac{33}{R + 4}$

Since inside the 6 V battery the current is going from positive terminal to the negative terminal so

$$V_A - V_B = 6 + Ir$$

$\Rightarrow \quad 9 = 6 + \dfrac{33}{R+4}$

$\Rightarrow \quad R + 4 = 11$

$\Rightarrow \quad R = 7\ \Omega$

$\Rightarrow \quad V_{BC} = 15 - 3(2) = 9\text{ V} \qquad \left\{\because I = \dfrac{33}{11} = 3\text{ A}\right\}$

$\Rightarrow \quad V_{BD} = 39 - 3(3)$

$\Rightarrow \quad V_{BD} = 30\ V$

Hence, (B) and (C) are correct.

41. $V = V_0\left(1 - e^{-t/\tau}\right)$

$$\tau = \frac{RC}{2}$$

$V_0 = \frac{E}{2}$ (as both R and R are in series)

$$\Rightarrow \quad V = \frac{E}{2}\left(1 - e^{-\frac{2t}{RC}}\right)$$

Hence, (C) and (D) are correct.

42. $H = \frac{V^2}{R_1}t_1$

$$\Rightarrow \quad R_1 = \frac{V^2 t_1}{H}$$

Similarly, $R_2 = \frac{V^2 t_2}{H}$

In series, $H = \left(\frac{V^2}{R_1 + R_2}\right)t$

$$t = \frac{H(R_1 + R_2)}{V^2}$$

Substituting the values of R_1 and R_2, we get

$$t = t_1 + t_2$$

In parallel, $H = \frac{V^2}{R_{\text{net}}}t = V^2 t\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$$= V^2 t\left(\frac{H}{V^2 t_1} + \frac{H}{V^2 t_2}\right)$$

Solving we get, $t = \frac{t_1 t_2}{t_1 + t_2}$

Hence, (A) and (C) are correct.

43. $I = \frac{dQ}{dt} = a - 2bt$, which decreases linearly with t.

$I = 0$ for $t = \frac{a}{2b}$ and rate of charging is $\frac{dI}{dt}$. So,

$$\frac{dI}{dt} = -2b$$

Hence the correct options are **(A), (C)** and **(D)**

Hence, (A), (C) and (D) are correct.

45. The resistance in the middle plays no part in the charging process of C, as it does not alter either the potential difference across the RC combination or the current through it.

C discharges through $R + R$ in series.

Hence, (A) and (B) are correct.

46. $Q = Q_0 e^{-t_1/\pi}$ and potential difference across C is proportional to Q.

For the potential difference to fall by 10%, Q must fall by 10%. So,

$$Q = 0.9\,Q_0 = Q_0 e^{-t_1/\tau}$$

$$\Rightarrow \quad e^{t_1/\tau} = \frac{10}{9}$$

$$\Rightarrow \quad \frac{t_1}{\tau} = \log_e\left(\frac{10}{9}\right)$$

$$\Rightarrow \quad t_1 = \tau \log_e\left(\frac{10}{9}\right)$$

$$Q = 0.1\,Q_0 = Q_0 e^{-t_2/\tau}$$

$$\Rightarrow \quad e^{t_2/\tau} = 10$$

$$\Rightarrow \quad \frac{t_2}{\tau} = \log_e 10 = 2.303$$

$$\Rightarrow \quad t_2 = 2.303\,\tau$$

Hence, (B) and (C) are correct.

47. $Q = Q_0\left(1 - e^{-t_1/\tau}\right) = 0.1Q_0$

$$\Rightarrow \quad e^{-t_1/\tau} = 0.9$$

$$\Rightarrow \quad e^{t_1/\tau} = \frac{10}{9}$$

$$\Rightarrow \quad t_1 = \tau \log_e\left(\frac{10}{9}\right)$$

$$Q = Q_0\left(1 - e^{-t_2/\tau}\right) = 0.9Q_0$$

$$\Rightarrow \quad e^{-t_2/\tau} = 0.1$$

$$\Rightarrow \quad e^{t_2/\tau} = 10$$

$$\Rightarrow \quad t_2 = \tau \log_e(10)$$

$$\Rightarrow \quad (t_1 + t_2) = \tau \log_e\left(\frac{100}{9}\right) = 2\tau \log_e\left(\frac{10}{3}\right)$$

and $(t_2 - t_1) = 2\tau \log_e(3)$

Hence, (B) and (D) are correct.

49. By closing S_1, net external resistance will decrease. So, main current will increase.

By closing S_2, net emf will remain unchanged but net internal resistance will decrease. Hence, main current will increase.

Hence, (A) and (C) are correct.

50. $11.4 = I_1(9.5)$

$$\Rightarrow \quad I_1 = 1.2\ \text{A}$$

Since, $E-11.4=I_1r$

$\Rightarrow$ $E-11.4=1.2r$ ...(1)

Similarly $I_2=1$ A

$\Rightarrow$ $E-11.5=1r$ ...(2)

$0.1=0.2r$

$\Rightarrow$ $r=0.5\ \Omega$ and $E=12$ V

Hence, (A) and (B) are correct.

51. $C_s=\dfrac{10}{7}\ \mu F$

$$Q=C_sV=\frac{10}{7}\times 14$$

$\Rightarrow$ $Q=20\ \mu C$

Further $V_{C_1}=\dfrac{Q}{C_1}=\dfrac{20}{5}=4$ V

Similarly $V_{C_2}=\dfrac{Q}{C_2}=\dfrac{20}{2}=10$ V

Hence, (B) and (C) are correct.

52. At $t=0$, emf of the circuit $=PD$ across the capacitor $=6$ V.

$\Rightarrow$ $i=\dfrac{6}{1+2}=2$ A

Half-life of the circuit

$$=(\ln 2)\tau_C(\ln 2)CR=(6\ln 2)s.$$

In half-life time, all values get halved. For example

$$V_C=\frac{6}{2}=3\text{ V}$$

$$i=\frac{2}{1}=1\text{ A}$$

$\Rightarrow$ $V_{1\,\Omega}=iR=1$ V and

$V_{2\,\Omega}=iR=2$ V

Hence, (A), (B), (C) and (D) are correct.

54. If switch S is open,

$$i_1\lambda\ell=E_2$$

where, $i_1=$ current in upper circuit and λ is resistance per unit length of potentiometer wire.

$\Rightarrow$ Null point length, $\ell=\dfrac{E_2}{i_1\lambda}$

(a) If jockey is shifted towards right, resistance in upper circuit will increase. So, current i_1 will decrease. Hence, ℓ will increase.

(b) If E_1 is increased, i_1 will also increase, So, ℓ will decrease.

(c) $\ell\propto E_2$

(d) If switch is closed, then null point will be obtained corresponding to

$$V_2=E_2-i_2r_2$$

which is less than E_2. Hence, null point length will decrease.

Hence, (A), (B), (C) and (D) are correct.

55. Current through A is the main current passing through the battery. So, this current is more than the current passing through B. Hence, during charging more heat is produced in A.

In steady state,

$$i_C=0$$

and $i_A=i_B$

Hence, heat is produced at the same rate in A and B.

Further, in steady state

$$V_C=V_B=\frac{\varepsilon}{2}$$

$\Rightarrow$ $U=\dfrac{1}{2}CV_C^2=\dfrac{1}{8}C\varepsilon^2$

Hence, (A), (B) and (D) are correct.

Reasoning Based Questions

2. The electrons are in motion which constitute electric current in a conductor but no. of positive and negative charges are same.

Hence, the correct answer is (D).

3. When current flows through a conductor, it always remains uncharged. Hence no electric field is produced outside it.

Hence, the correct answer is (A).

4. Direction of flow of current is from higher potential to lower potential.

Hence, the correct answer is (D).

5. Since current arises due to continuous flow of charged particles. There is no free charge in insulator. Hence no flow of charges is possible. Therefore current does not flow through insulators.

Hence, the correct answer is (A).

6. $V=IR$

Hence, the correct answer is (D).

8. Since, in the case of stretching the length n times, resistance becomes n^2 times the original value. So, Statement 1 is false, however Statement 2 is true.

Because $R=\rho\dfrac{\ell}{A}$

Hence, the correct answer is (D).

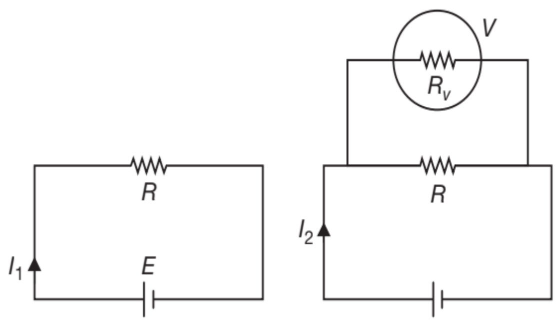
9. On increasing temperature of wire, the kinetic energy of the electrons increases and so they collide more rapidly with each other as a result of which their drift velocity decreases and resistivity increases which is inversely proportional to the conductivity of material.
Hence, the correct answer is (B).

10. Voltmeter gives terminal potential (V) thought it can give emf if internal resistance of the cell is zero.
Hence, the correct answer is (D).

12. $I_1 = \frac{E}{R}$

$$R_{eq} = \frac{RR_V}{R+R_V}$$

$$I_2 = \frac{E}{\left(\frac{RR_V}{R+R_V}\right)}$$



Hence, the correct answer is (A).

13. In a simple battery circuit the point at lowest potential is the negative terminal of battery. The current flows in the circuit from positive terminal to negative terminal.
Hence, the correct answer is (D).

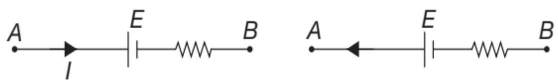
15. Internal resistance is always in series with the battery.
Hence, the correct answer is (D).

16. $P = \frac{V^2}{R}$

Hence, the correct answer is (D).

17. $V_A - E - Ir = V_B$

$\Rightarrow \quad V_A - V_B = E + Ir$



Similarly $V_A - V_B = E - Ir$

Hence, the correct answer is (A).

18. Power used $= I^2R$
Hence, power is consumed and not the current
Hence, the correct answer is (D).

19. $P = I^2R$

$$\Rightarrow \quad \left(\frac{dP}{P}\right)(100\%) = 2\left(\frac{dI}{I}\right)100\%$$

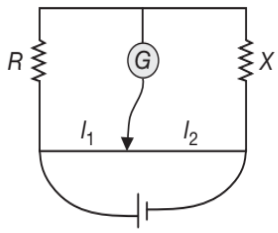
Hence, the correct answer is (B).

20. If either the e.m.f. of the driver cell or potential difference across whole potentiometer wire is lesser than the e.m.f. of the experimental cell, then balance point will not obtained.
Hence, the correct answer is (A).

21. $V = E + Ir$ when charging of cell take place, i.e., $V > E$
Hence, the correct answer is (D).

22. The resistance of galvanometer is fixed. In metre bridge experiments, to protect the galvanometer from a high current, high resistance is connected to the galvanometer in order to protect it from damage (because then less current can pass through the galvanometer).
Hence, the correct answer is (C).

23. $R\ell_2 = \ell_1 x$



To get null point at the same position, means ℓ_1 and ℓ_2 are still the same. As temperature increases, value of unknown resistance increases. To get the same null point, R must be increased. So Statement-1 is wrong. Statement-2 is True.
Hence, the correct answer is (D).

25. Charge on capacitor $q = CE\left(1 - e^{-t/CR_{eq}}\right)$

$$\Rightarrow \quad I = \frac{dq}{dt} = \frac{E}{R_{eq}} e^{-t/CR}$$

At $t = 0$

$$I = \frac{E}{R_{eq}} = \frac{E}{R+r}$$

$\Rightarrow$ Resistance offered by capacitor is zero.
Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. To find the charge per pulse let us integrate the expression $dQ = Idt$. Since the current to be constant for the entire duration of the pulse, so

$$Q_{pulse} = I\int dt = I\Delta t = \left(250\times10^{-3}\,\text{A}\right)\left(200\times10^{-9}\,\text{s}\right)$$

CHAPTER 3

$\Rightarrow \quad Q_{\text{pulse}} = 5\times10^{-8}$ C

Dividing this quantity of charge per pulse by the electronic charge gives the number of electrons per pulse. So, we have

$$\text{Electrons per pulse} = \frac{5\times10^{-8}\ \text{C/Pulse}}{1.6\times10^{-19}\ \text{C/electron}} = 3.13\times10^{11}\ \text{electrons/pulse}$$

Hence, the correct answer is (C).

2. Average current is given by Equation, $I_{av} = \frac{\Delta Q}{\Delta t}$. Because the time interval between pulses is 4 ms, and also we have already calculated the charge per pulse, so we obtain

$$I_{av} = \frac{Q_{\text{pulse}}}{\Delta t} = \frac{5\times10^{-8}\ \text{C}}{4\times10^{-3}\ \text{s}} = 12.5\ \mu\text{A}$$

This represents only 0.005% of the peak current, which is 250 mA.

Hence, the correct answer is (A).

3. Since, power is defined as the energy delivered per unit time interval. Thus, the peak power is equal to the energy delivered by a pulse divided by the pulse duration.

$$P_{\text{peak}} = \frac{\text{pulse energy}}{\text{pulse duration}} \quad \ldots(1)$$

$$P_{\text{peak}} = \frac{(3.13\times10^{11}\ \text{electrons/pulse})(40\ \text{MeV/electron})}{2\times10^{-7}\ \text{s/pulse}}\left(\frac{1.6\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right)$$

$\Rightarrow \quad P_{\text{peak}} = 1\times10^{7}\ \text{W} = 10\ \text{MW}$

Also we could have calculated the power directly by assuming that each electron has zero energy before being accelerated. Thus, by definition, each electron must go through a potential difference of 40 MV to acquire a final energy of 40 MeV. Hence, we have

$$P_{\text{peak}} = I_{\text{peak}}\Delta V \quad \ldots(2)$$

$\Rightarrow \quad P_{\text{peak}} = (250\times10^{-3}\ \text{A})(40\times10^{6}\ \text{V})$

$\Rightarrow \quad P_{\text{peak}} = 10\ \text{MW}$

Hence, the correct answer is (B).

4. To calculate the average power, we shall make use of the time interval between pulses rather than the duration of a pulse. So, the average power is given by

$$P_{av} = \frac{\text{pulse energy}}{\text{time interval between pulses}}$$

$$P_{av} = \frac{(3.13\times10^{11}\ \text{electrons/pulse})(40\ \text{MeV/electron})}{4\times10^{-3}\ \text{s/pulse}}\times\left(\frac{1.6\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right) = 500\ \text{W}$$

Otherwise we could have calculated the average power using the similar formula where we have the average power given by the relation

$$P_{av} = I_{av}\Delta V = (12.5\times10^{-6}\ \text{A})(40\times10^{6}\ \text{V})$$

$\Rightarrow \quad P_{av} = 500\ \text{W}$

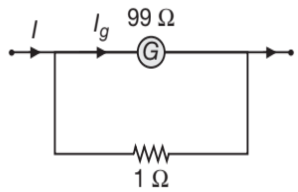
Notice that these two calculations agree with each other and that the average power is much lower than the peak power.

Hence, the correct answer is (D).

5. For ammeter $99I_g = (I - I_g)1$

$\Rightarrow \quad I = 100I_g \quad \ldots(1)$

I_g is the full scale deflection current of the galvanometer I and the range of ammeter



For the circuit in figure, given in the question

$$\frac{12V}{2 + r + \frac{99\times1}{99+1}} = 3\ \text{A}$$

$\Rightarrow \quad r = 1.01\ \Omega$

Hence, the correct answer is (C).

6. **The correct answer is (C).**

7. **Hence, the correct answer is (B).**

Combined Solution to 6 & 7

For voltmeter, range

$$V = I_g(99 + 101)$$

$$V = 200I_g \quad \ldots(2)$$

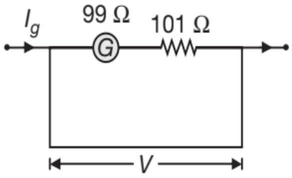
Also resistance of the voltmeter is R_V = 99 + 101 = 200 Ω. In figure resistance across the terminals of the battery is given by

$$R_1 = r + \frac{200\times2}{200} = 2.99\ \Omega$$

$\Rightarrow$ Current drawn from the battery,

$$I_1 = \frac{12}{2.99} = 4.01\ \text{A}$$

$\Rightarrow$ Voltmeter reading



$$\frac{4}{5}V = 12 - I_1 r$$

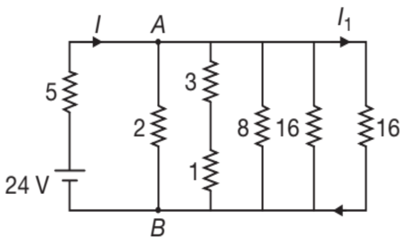
$$\Rightarrow \quad \frac{4}{5}V = 12 - 4.01 \times 1.01$$

$$\Rightarrow \quad V = 7.96 \times \frac{5}{4} = 9.95 \text{ V}$$

Using (2), $I_g = \frac{9.95}{200} = 0.05$ A

Using (1) range of the ammeter $I = 100 I_g = 5$ A

8. The equivalent circuit is as shown in figure.



Net resistance of circuit $R = 6\ \Omega$. Hence,

$$I = \frac{V}{R} = \frac{24}{6} = 4 \text{ A}$$

Hence, the correct answer is (B).

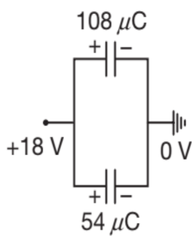
9. $I_1 = \frac{V_{AB}}{16} = \frac{4}{16} = \frac{1}{4}$ A

Hence, the correct answer is (C).

10. $V_{AB} = E - Ir = 24 - (4)(5) = 4$ V

Hence, the correct answer is (A).

11. When switch S is open circuit is as shown in figure.



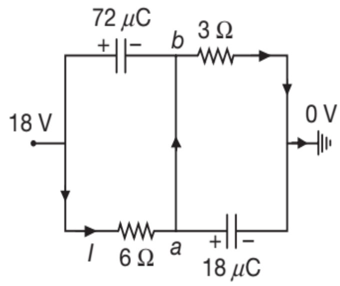
$$V_a = +18 \text{ V}$$

and $V_b = 0$

$$\Rightarrow \quad V_{ab} = 18 \text{ V}$$

Hence, the correct answer is (B).

12.



When switch S is closed, a current,

$$I = \frac{18}{6+3} = 2 \text{ A, flows in the circuit}$$

$$V_b - 0 = 3 \times 2 = 6 \text{ V}$$

$$\Rightarrow \quad V_b = 6 \text{ V}$$

Hence, the correct answer is (C).

13. $\Delta q = (\Delta q)_{3\ \mu\text{F}} = (18 - 54)\ \mu\text{C} = -36\ \mu\text{C}$

and $\Delta q' = (\Delta q)_{6\ \mu\text{F}} = (72 - 108)\ \mu\text{C} = -36\ \mu\text{C}$

Hence, the correct answer is (A).

14. **The correct answer is (C).**

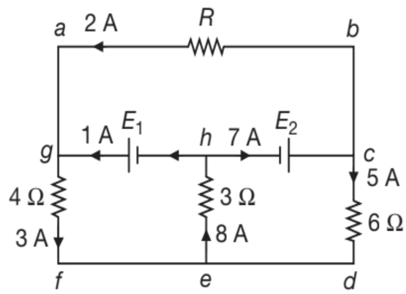
15. **The correct answer is (B).**

16. **Hence, the correct answer is (D).**

Combined Solution to 14, 15 & 16

Applying Junction Law, we get

$$I_{3\ \Omega} = 5 + 3 = 8 \text{ A}$$



Applying Kirchhoff's Loop Law, we have for

Loop $ghefg$, $E_1 - 12 - 24 = 0$

$$\Rightarrow \quad E_1 = 36 \text{ V}$$

Loop $chedc$, $-E_2 + 24 + 30 = 0$

$$\Rightarrow \quad E_2 = 54 \text{ V}$$

Loop $cbagc$, $-2R - E_1 + E_2 = 0$

$$\Rightarrow \quad R = \frac{E_2 - E_1}{2} = 9\ \Omega$$

17. Time constant of circuit is

$$\tau = CR = \left(\frac{K\varepsilon_0 A}{d}\right)\left(\frac{\rho d}{A}\right) = K\varepsilon_0 \rho$$

$\Rightarrow \quad \tau = 18\left(\dfrac{1}{4\pi \times 9 \times 10^9}\right)(4\pi \times 10^3)$

$\Rightarrow \quad \tau = 2 \times 10^{-6}\ \text{s} = 2\ \mu\text{s}$

Hence, the correct answer is (B).

18. Since, $q = q_0 e^{-\frac{t}{CR}}$

$\Rightarrow \quad q = 2 \times 10^{-6} \times e^{-\frac{2}{2}} = \dfrac{2 \times 10^{-6}}{e}$

$\Rightarrow \quad q = 0.37 \times 2 \times 10^{-6} = 0.74\ \mu\text{C}$

Hence, the correct answer is (D).

19. $q = q_0 e^{-\frac{t}{CR}}$

$\Rightarrow \quad I = -\dfrac{dq}{dt} = -q_0 e^{-\frac{t}{CR}}\left(-\dfrac{1}{CR}\right)$

$\Rightarrow \quad I = \dfrac{q_0}{CR} e^{-\frac{t}{CR}}$

$\Rightarrow \quad I = \dfrac{2}{2} e^{-\frac{4}{2}} = \dfrac{1}{e^2} = \dfrac{1}{7.39}$ (As, $e = 2.718$ and $e^2 = 7.39$)

$\Rightarrow \quad I = 0.13\ \mu\text{A}$

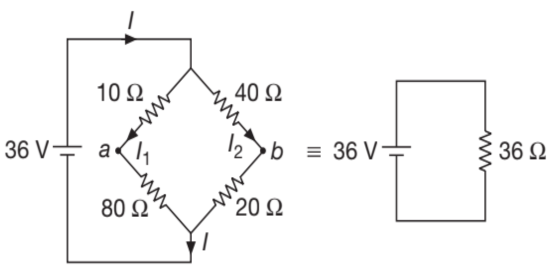
Hence, the correct answer is (D).

20. In steady state there is no current in the capacitor, the equivalent circuit is shown.

$R_{eq} = 36\ \Omega$

The current I is given by

$I = \left(\dfrac{36}{36}\right)\text{A} = 1\ \text{A}$



The current divides into two parallel branches, as I_1 and I_2. In parallel branches current divides in inverse ratio of resistance, thus

$I_1 = \left(\dfrac{60}{90+60}\right)I = 0.4\ \text{A} \qquad \{\because\ I = 1\ \text{A}\}$

$I_2 = \left(\dfrac{90}{90+60}\right)I = 0.6\ \text{A} \qquad \{\because\ I = 1\ \text{A}\}$

The capacitor is connected across points a and b. From KVL we have

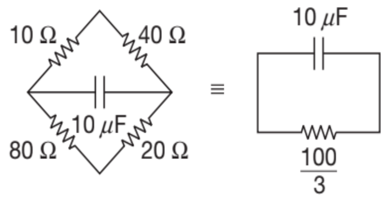
$V_a + 10I_1 - 40I_2 = V_b$

$V_a - V_b = 40I_2 - 10I_1 = 20\ \text{V}$

Thus, the voltage across capacitor is $20\ \text{V}$

Hence, the correct answer is (C).

21. When battery is disconnected, the equivalent circuit is shown here.



The initial charge on capacitor is

$Q_0 = (10 \times 20)\ \mu\text{C}$

$\Rightarrow \quad Q_0 = 200\ \mu\text{C}$

The time constant of circuit is

$\tau = RC = \left(\dfrac{100}{3} \times 10\right)\mu\text{s} = \left(\dfrac{1000}{3}\right)\mu\text{s}$

The charge on a capacitor, Q as a function of time in a discharging circuit is given by the relation

$Q = Q_0 e^{-\frac{t}{RC}}$

$\Rightarrow \quad Q = 200 e^{-\frac{t}{\left(\frac{1000}{3}\right)}}$

$\Rightarrow \quad Q = 200 e^{-\frac{3t}{1000}} = 200\ \exp\left(-\dfrac{3t}{1000}\right)\mu\text{C}$

Hence, the correct answer is (C).

22. Voltage, V across capacitor as a function of time is

$CV = CV_0 e^{-\frac{t}{RC}}$

$\Rightarrow \quad V = V_0 e^{-\frac{t}{RC}}$

$\Rightarrow \quad V = 200 e^{-\frac{t}{RC}}$

$I = 200 e^{-\frac{t}{RC}}$

Taking log both sides, we get

$t = RC \log_e 200 = \dfrac{1000}{3} \log_e 200$

$\Rightarrow \quad t = 1766.1\ \mu\text{s}$

$\Rightarrow \quad t = 1.766\ \text{ms}$

Hence, the correct answer is (B).

23. $\tau_1 = RC = (1.5\times10^5\ \Omega)(10\times10^{-6}\ \text{F}) = 1.5\ \text{s}$

Hence, the correct answer is (C).

24. $\tau_2 = (1\times10^5\ \Omega)(10\times10^{-6}\ \text{F}) = 1\ \text{s}$

Hence, the correct answer is (B).

25. The battery carries current

$$\frac{10\ \text{V}}{50\times10^3\ \Omega} = 200\ \mu\text{A}$$

The $100\ \text{k}\Omega$ carries current of magnitude

$$I = I_o e^{-\frac{t}{RC}} = \left(\frac{10\ \text{V}}{100\times10^3\ \Omega}\right)e^{-\frac{t}{1\,\text{s}}} = 100e^{-t}\ \mu\text{A}$$

So the switch carries downward current, I' given by

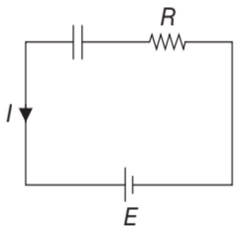
$$I' = 200\ \mu\text{A} + (100\ \mu\text{A})e^{-\frac{t}{1\,\text{s}}}$$

$$\Rightarrow\quad I' = 100(2+e^{-t})\ \mu\text{A}$$

Hence, the correct answer is (C).

26. Taking loop equation, we get

$$E - \frac{q}{C_f} + RI = 0 \quad [C_f \text{ is the capacitance at } t>0] \quad \ldots(1)$$



As charge on the capacitor cannot change abruptly, the current I_0, just after the capacitance has been changed, will be given by

$$E - \frac{q_0}{C_f} + RI_0 = 0$$

where q_0 is charge on capacitor at $t=0$,

$$\text{Hence, } I_0 = \frac{q_0 - EC_f}{RC_f}$$

As $q_0 = C_i E$ $\qquad \{C_i = \text{initial capacitance}\}$

putting the values, we get

$$I_0 = \frac{5\times10}{5\times50} = \frac{1}{5}\ \text{A}$$

Hence, the correct answer is (A).

27. Differentiating equation (1), we get

$$R\frac{dI}{dt} = \frac{I}{C_f}\frac{dq}{dt}$$

as $I = \left|\frac{dq}{dt}\right|$

$$\Rightarrow\quad R\frac{dI}{dt} = -\frac{I}{C_f}$$

$$\Rightarrow\quad R\int_{I_0}^{I}\frac{dI}{I} = -\int_0^t \frac{dt}{C_f}$$

$$\Rightarrow\quad I = I_0 e^{-\frac{t}{RC_f}} \qquad \ldots(2)$$

Hence, $I = (0.2\ \text{A})e^{-\frac{t}{(2.5\times10^{-4})}} = 0.2\exp(-4000t)$ A where t is in seconds.

Hence, the correct answer is (B).

28. Substituting I in equation (1) we get

$$E - \frac{q}{C_f} + RI_0 e^{-\frac{t}{RC_f}} = 0$$

$$\Rightarrow\quad q = C_f\left(E + RI_0 e^{-\frac{t}{RC_f}}\right)$$

$$\Rightarrow\quad q = (5)\left(10 + 10e^{-\frac{t}{2.5\times10^{-4}}}\right)$$

$$\Rightarrow\quad q = (5\times10^{-5})\left(1 + e^{-\frac{t}{2.5\times10^{-4}}}\right)\text{C}$$

$$\Rightarrow\quad q = 5\times10^{-5}(1+e^{-4000t})$$

$$\Rightarrow\quad q = 50(1+e^{-4000t})\ \mu\text{C}$$

Hence, the correct answer is (C).

29. Power in resistor $= I^2R$

Hence, heat produced in the wire as function of time is H, given by

$$H = \int_0^t (I^2R)dt = I_0^2R\int_0^t e^{-\frac{2t}{RC_f}}dt = \frac{-I_0^2R^2C_f}{2}\left(e^{-\frac{2t}{RC_f}}\right)\Bigg|_0^t$$

$$\Rightarrow\quad H = +\frac{I_0^2R^2C_f}{2}\left(1 - e^{-\frac{2t}{RC_f}}\right)$$

$$\Rightarrow\quad H = \frac{(0.2\times50)^2\times(5\times10^{-6})}{2}[1-e^{-8000t}]\ \text{J}$$

$$\Rightarrow\quad H = (2.5\times10^{-4})[1-e^{-8000t}]\ \text{J}$$

$$\Rightarrow\quad H = 250(1-e^{-8000t})\ \mu\text{J}$$

Hence, the correct answer is (C).

30. Finally, the capacitors are in parallel and total charge $(=q_0)$ distributes between them in direct ratio of capacity.

$$\Rightarrow \quad q_{C_2} = \left(\frac{C_2}{C_1+C_2}\right)q_0 \rightarrow \text{ in steady state.}$$

But this charge increases exponentially.

Hence, charge on C_2 at any time t is

$$q_{C_2} = \left(\frac{C_2 q_0}{C_1+C_2}\right)\left(1-e^{-\frac{t}{\tau_C}}\right)$$

Initially, C_2 is uncharged so, whatever is the charge on C_2, it is charge flown through switches.

Hence, the correct answer is (B).

31. Common potential in steady state when they finally come in parallel is

$$V = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{q_0}{C_1+C_2}$$

Total heat dissipated is $\Delta H = U_i - u_f$

$$\Rightarrow \quad \Delta H = \frac{q_0^2}{2C_1} - \frac{1}{2}(C_1+C_2)\left(\frac{q_0}{C_1+C_2}\right)^2$$

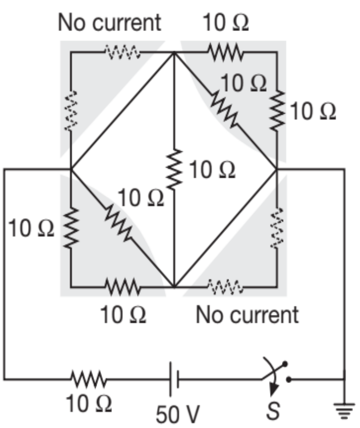
$$\Rightarrow \quad \Delta H = \left(\frac{q_0^2}{2C_1}\right)\left(\frac{C_1C_2}{C_1+C_2}\right)$$

Hence, the correct answer is (A).

32. Immediately after closing the switch a capacitor behaves as a short circuit. Figure shows the effective circuit from the point of view of current.

Figure shows the equivalent circuit, current I from battery is

$$I = \frac{50}{\frac{5}{2}+10} = 4 \text{ A}$$



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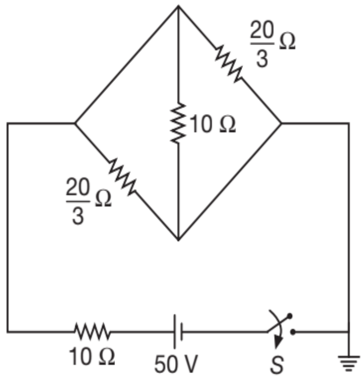
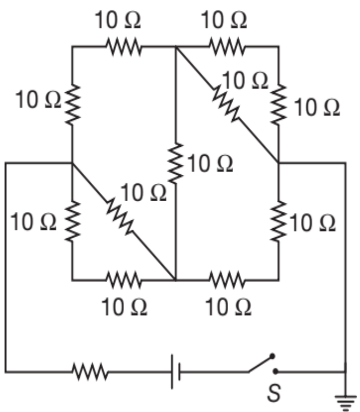


Hence, the correct answer is (B).

33. A long time after closing the switch a capacitor behaves as an open circuit. The equivalent circuits are shown. Using Kirchhoff's Laws, we get

$$I = \frac{70}{33}\text{ A} \quad \text{and} \quad I_1 = \frac{25}{33}\text{ A}$$



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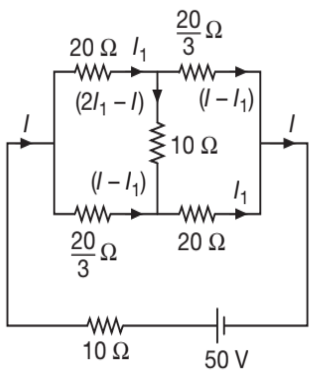


Hence, the correct answer is (C).

34. The $10\ \mu$F capacitor is connected across the $20\ \Omega$ resistor of figure (see question)

$$\text{P.D. across } 20\ \Omega \text{ resistor } = 20 \times I_1 = \frac{500}{33}\text{ V}$$

So, charge on $10\ \mu$F capacitor is given by

$$q_{10} = \left(10 \times \frac{500}{33}\right) \mu\text{C} = \frac{5000}{33}\ \mu\text{C}$$

The $5\ \mu\text{F}$ capacitor is connected in parallel to the $20\ \Omega$ resistor hence charge on the $5\ \mu\text{F}$ capacitor, is

$$q_5 = \left(\frac{5 \times 500}{33}\right) \mu\text{C} = \frac{2500}{33}\ \mu\text{C}$$

Hence, the correct answer is (C).

35. Immediately after closing switch S_1 the capacitor C_1 short-circuits the part of circuit to the right of it. So, the current will flow through the loop $ABGHA$ only, with a value

$$I_0 = I(t = 0) = \frac{V}{R} = \frac{12}{100} = 0.12\ \text{A}$$

Hence, the correct answer is (C).

36. Long time after closing the switches the capacitors do not allow the current to pass through them. So, the current will now flow in the loop $ABCFGHA$ (with no capacitor in the branch BG). So, at $t \to \infty$, the net resistance offered to the flow of current is given by

$$R_{eq} = 100 + 50 + 150 = 300\ \Omega$$

$$\Rightarrow \quad I = I(t \to \infty) = \frac{V}{R_{eq}} = \frac{12}{300} = 0.04\ \text{A}$$

Hence, the correct answer is (B).

37. C_1 is connected across points B and G. We traverse the circuit along $BAHGB$ and apply KVL, we have

$$V_B + 100I - 12 - V_G = 0$$

$$V_B - V_G = 12 - 100I = 12 - (100)(0.04) = 8\ \text{V}$$

Final voltage across C_1 is 8 V

Hence, the correct answer is (D).

38. Capacitor C_2 is connected across the $150\ \Omega$ resistor, hence voltage across C_2 is

$$V_2 = (150)(0.04) = 6\ \text{V}$$

Hence, the correct answer is (C).

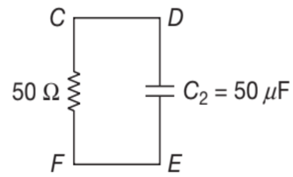
39. When switch S_2 is opened, the loop $CDEF$ is a discharging CR circuit. The charge as function of time is

$$Q = Q_0 e^{-\left(\frac{t}{RC_2}\right)}$$

$$\Rightarrow \quad Q = (C_2 \times 6) e^{-\left(\frac{t}{RC_2}\right)} \qquad \{\because Q_0 = C_2V_2 = 6C_2\}$$

Current $I = \dfrac{dQ}{dt}$

$$\Rightarrow \quad I = -\frac{6C_2}{RC_2} e^{-\left(\frac{t}{RC_2}\right)}$$



$$\Rightarrow \quad I = -\frac{6}{R} e^{-\left(\frac{t}{RC_2}\right)}$$

$$\Rightarrow \quad I = -\frac{6}{150} e^{-\frac{t}{(50\times10^{-6})(150)}}$$

$$\Rightarrow \quad I = -0.04 e^{-\frac{400t}{3}}$$

$$\Rightarrow \quad I = -0.04 \exp\left(-\frac{400t}{3}\right)$$

Hence, the correct answer is (C).

Matrix Match/Column Match Type Questions

1. A → (q)
B → (s)
C → (p)
D → (r)

2. A → (s)
B → (q)
C → (q)
D → (p)

Since $R = \dfrac{\rho \ell}{A}$ or $R \propto \dfrac{\ell}{A}$

$$\Rightarrow \quad R_1 : R_2 : R_3 = \left(\frac{\ell}{A}\right)_1 : \left(\frac{\ell}{A}\right)_2 : \left(\frac{\ell}{A}\right)_3$$

$$\Rightarrow \quad R_1 : R_2 : R_3 = \frac{1}{2} : \frac{1}{2} : \frac{1}{3} = 1 : 1 : 6$$

Hence (A) → (s)
As V is constant so

$$I \propto \frac{1}{R} \text{ and } P \propto \frac{1}{R}, \text{ so } P \propto I$$

Hence, $I_1 : I_2 : I_3 = P_1 : P_2 : P_3 = \left(\frac{2}{1}\right) : \left(\frac{2}{1}\right) : \left(\frac{1}{3}\right)$

$$\Rightarrow \quad I_1 : I_2 : I_3 = 6 : 6 : 1$$

So (B) → (q), (C) → (q)
Since wires are of same material, so

$$\rho_1 : \rho_2 : \rho_3 = 1 : 1 : 1$$

Hence (D) → (p)

3. A → (q, r, s)
B → (q, s)
C → (p, s)
D → (p, s)

Since,

$$q = q_0\left(1 - e^{-t/RC}\right)$$

So, the charging current is

$$I = \frac{dq}{dt} = \frac{V}{R}e^{-t/RC}$$

$$\Rightarrow \quad I = \frac{V}{R}e^{-t/RC} = I_0 e^{-t/RC}$$

At $t = 0$, we get

$$I = I_0 = \frac{V}{R}$$

Hence current is maximum and capacitor acts as a short circuit.

So, (A) $\rightarrow$ (q, r, s)

The decay charge is given by

$$q = q_0 e^{-t/RC}$$

Hence, the discharging current is

$$I = -\frac{dq}{dt} = I_0 e^{-t/RC}$$

$$\Rightarrow \quad I = I_0 e^{-t/RC}$$

at $t = 0$, we have

$$I = I_0 = \frac{V}{R} \text{ (maximum)}$$

Hence, (B) $\rightarrow$ (q, s)

Now, when the capacitor is fully charged, energy stored is

$$U = \frac{1}{2}CV^2$$

$$\Rightarrow \quad U = \frac{1}{2}C\left[V_0\left(1 - e^{-t/RC}\right)\right]^2$$

$$\Rightarrow \quad U = \frac{1}{2}CV_0^2\left(1 - e^{-t/RC}\right)^2$$

Hence, U is an exponential function of t

So, (C) $\rightarrow$ (p, s)

Energy dissipated as heat is

$$dH = I^2 R\,dt$$

$$\Rightarrow \quad \int dH = \int_0^\infty \left(I_0 e^{-t/RC}\right)^2 R\,dt$$

$$\Rightarrow \quad H = \frac{1}{2}CV^2$$

Hence, (D) $\rightarrow$ (p, s)

4. A $\rightarrow$ (q)
B $\rightarrow$ (s)
C $\rightarrow$ (q)
D $\rightarrow$ (s)

Let potential of point e is V volts. Then,

$$I_{ae} + I_{be} + I_{ce} + I_{de} = 0$$

$$\Rightarrow \quad \left(\frac{2-V}{1}\right) + \left(\frac{4-V}{2}\right) + \left(\frac{6-V}{1}\right) + \left(\frac{4-V}{2}\right) = 0$$

$$\Rightarrow \quad V = 4 \text{ V}$$

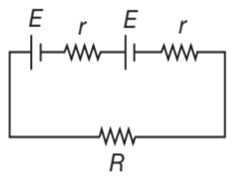
Now current through any wire can be obtained by the equation,

$$I = \frac{\Delta V}{R}$$

5. A $\rightarrow$ (q)
B $\rightarrow$ (q)
C $\rightarrow$ (s)
D $\rightarrow$ (r)

For series combination

$$R_{ext} = R,\ R_{int} = 2r \text{ and } E_{net} = 2E$$



Acc to Maximum Power Transfer Theorem

$$R = 2r$$

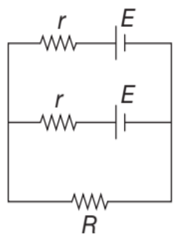
$$\Rightarrow \quad P = \left[\frac{4E^2}{(R+2r)^2}\right]R = \frac{4E^2}{4R^2}R = \frac{E^2}{R} = \frac{E^2}{2r}$$

For parallel combination

$$E_{net} = E,\ R_{ext} = R,\ R_{int} = \frac{r}{2}$$

Acc to Maximum Power Transfer Theorem,

$$R_{ext} = R_{int}$$



$$\Rightarrow \quad R = \frac{r}{2}$$

$$\Rightarrow \quad r = 2R$$

Now $P = \dfrac{E^2}{\left(R + \frac{r}{2}\right)^2}R = \left(\dfrac{E^2}{4R^2}\right)R$

$$\Rightarrow \quad P = \frac{E^2}{4R} = \frac{E^2}{4\left(\frac{r}{2}\right)} = \frac{E^2}{2r}$$

6. A → (p)
B → (p)
C → (p)
D → (p)

$i_1 = i_2$ or i is same at both sections.

$A_1 < A_2$

(A) Current density $= \frac{i}{A} \propto \frac{1}{A}$

(C) $\frac{\text{Resistance}}{\text{length}} = \frac{\rho}{A} \propto \frac{1}{A}$

(D) and (B) E or potential difference per unit length $= (i)$ (Resistance per unit length)

$$\Rightarrow \frac{P.D.}{\ell} = (i)\left(\frac{\rho}{A}\right) \propto \frac{1}{A}$$

7. A → (q)
B → (r)
C → (t)
D → (p)

As 3 Ω and 7 Ω are in series across 6 V and in series potential divides in proportion to resistance

$$\text{So, } V_A - V_D = \frac{R(V)}{(R+S)} = \frac{3\times 6}{3+7} = \frac{18}{10} = 1.8 \text{ V}$$

Hence (A) → (q)

$$\text{Similarly, } V_A - V_B = \frac{X(V)}{(X+Y)} = \frac{2\times 6}{2+4} = 2 \text{ V}$$

$$(V_A - V_B) - (V_A - V_D) = V_D - V_B = 2 - 1.8 = 0.2 \text{ V}$$

Hence (B) → (r)
Energy stored in the capacitor will be zero if $V_B = V_D$. So for condition of a balanced wheatstone bridge, we have

$$\frac{2}{Y} = \frac{3}{7}$$

$$\Rightarrow Y = \frac{14}{3} \Omega$$

So (C) → (t)
Also, in steady state no current flows through the branch containing the capacitor. Hence (D) → (p)

8. A → (r)
B → (s)
C → (q)
D → (p)

9. A → (p, q)
B → (p, q)
C → (r, s)
D → (r, s)

Resistance of bulbs A and B are

$$R_A = \frac{V_A^2}{P}$$

and $R_B = \frac{V_B^2}{P}$

$$\Rightarrow \frac{R_A}{R_B} = \frac{V_A^2}{V_B^2}$$

When the bulbs are connected in series, current through them will be same. Hence, potential difference across them will be proportional to their resistances. Therefore, ratio of potential difference across them will be equal to ratio of their resistances, $R_A : R_B$ which is $V_A^2 : V_B^2$

So, (A) → (p, q)
Power consumed will be

$$P_A = I^2 R_A$$

and $P_B = I^2 R_B$

$$\Rightarrow \frac{P_A}{P_B} = \frac{R_A}{R_B} = \frac{V_A^2}{V_B^2}$$

Hence, (B) → (p, q)
If bulbs are connected in parallel, then

$$I_1 R_A = I_2 R_B$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{R_B}{R_A} = \frac{V_B^2}{V_A^2}$$

So, (C) → (r, s)
and power consumed is given by

$$\frac{P_A}{P_B} = \frac{\frac{V^2}{R_A}}{\frac{V^2}{R_B}} = \frac{R_B}{R_A} = \frac{V_B^2}{V_A^2}$$

Hence, (D) → (r, s)

10. A → (s)
B → (r)
C → (p)
D → (q)

11. A → (s)
B → (r)
C → (s)
D → (r)

$$i = \frac{4-1}{1+1+1} = 1 \text{ A} \qquad \text{(anti-clockwise)}$$

(A) $V_A = E - ir = 4 - 1\times 1 = 3$ V

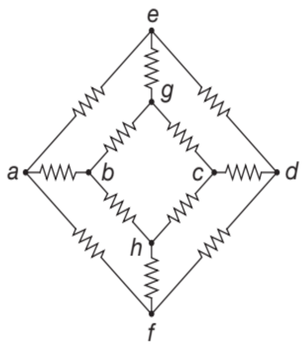
(B) $V_B = E + ir = 1 + 1\times 1 = 2$ V

CHAPTER 3

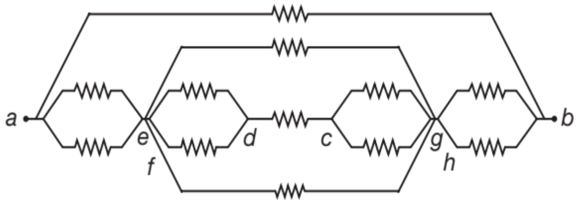
(C) $|P_A| = Ei - i^2r = (4\times1) - (1)^2(1) = 3\text{ W}$

(D) $|P_B| = Ei + i^2r = (1)(1) + (1)^2(1) = 2\text{ W}$

12. A → (s)
B → (q)
C → (r)
D → (p)



Symmetry of circuit shows that e and f are at same potential and g and h are another same potential. So, this circuit can be redrawn as shown.



Equivalent resistance in the middle line between e and h is

$$\frac{R}{2} + R + \frac{R}{2} = 2R$$

The total equivalent resistance between e and h is R' such that

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R} + \frac{1}{2R}$$

$$\Rightarrow \quad R' = \frac{2R}{5}$$

The equivalent resistance between a and b along path aeb

$$\frac{R}{2} + \frac{2R}{5} + \frac{R}{2} = \frac{7R}{5}$$

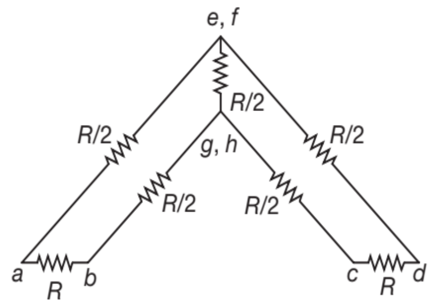
The total equivalent resistance between a and b is the resistance of R and $\frac{7R}{5}$ in parallel.

$$\Rightarrow \quad R_{ab} = \frac{R\times\frac{7R}{5}}{R+\frac{7R}{5}} = \frac{\frac{7R^2}{5}}{\frac{12R}{5}} = \frac{7R}{12}$$

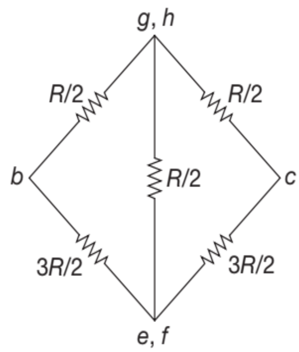
$$\Rightarrow \quad R_{ab} = \frac{7R}{12}$$

So, (A) → (s)

Resistance between b and c can be calculated as shown. Due to symmetry condition, network can be folded along the line bc. The circuit looks like



Finally it becomes balanced wheatsone bridge, so resistance of $\frac{R}{2}$ between g and e can be neglected. Equivalent resistance between b and c is the parallel combination of R and $3R$



So, $R_{bc} = \frac{R\times3R}{4R} = \frac{3R}{4}$

So (B) → (q)

Similarly, we can get $R_{ac} = \frac{5R}{6}$

So, (C) → (r)

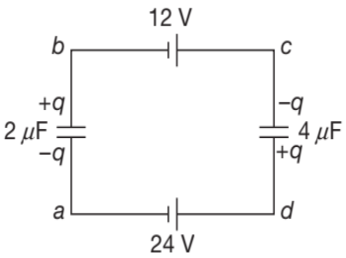
when c and h an shorted, $R_{ch} = 0$

So, (D) → (p)

13. A → (q)
B → (p)
C → (r)
D → (s)

Apply Kirchhoff's Law

$$V_a = \frac{q}{2} + 12 + \frac{q}{4} - 24 = V_a$$



$$\frac{3}{4}q = 12$$

$$\Rightarrow \quad q = 16\ \mu C$$

$$V_a + \frac{16}{2} = V_b$$

$$\Rightarrow \quad V_a - V_b = -8V$$

Similarly $V_d - V_c = -4$ V

Work done by cell 12 V is $-12 \times 16 = 192\ \mu J$

Work done by cell 24 V is $24 \times 16 = 384\ \mu J$

Integer/Numerical Answer Type Questions

1. Since, $I = nqAv_d$, where n is the number of charge carriers per unit volume, and is identical to the number of atoms per unit volume. We assume a contribution of 1 free electron per atom in the relationship above. For aluminium, which has a molar mass of 27, we know that Avogadro's number of atoms, N_A, has a mass of 27 g. Thus, the mass per atom is

$$\frac{27\text{ g}}{N_A} = \frac{27\text{ g}}{6.02 \times 10^{23}} = 4.49 \times 10^{-23}\ \text{gatom}^{-1}$$

$$\text{Thus, } n = \frac{\text{density of aluminium}}{\text{mass per atom}} = \frac{2.7\text{ gcm}^{-3}}{4.49 \times 10^{-23}\text{ gatom}^{-1}}$$

$$\Rightarrow \quad n = 6.02 \times 10^{22}\ \text{atomscm}^{-3} = 6.02 \times 10^{28}\ \text{atomsm}^{-3}$$

Therefore,

$$v_d = \frac{I}{nqA} = \frac{5\text{ A}}{(6.02 \times 10^{28}\text{ m}^{-3})(1.6 \times 10^{-19}\text{ C})(4 \times 10^{-6}\text{ m}^2)}$$

$$\Rightarrow \quad v_d = 1.3 \times 10^{-4}\ \text{ms}^{-1}$$

$$\Rightarrow \quad v_d = 130 \times 10^{-6}\ \text{ms}^{-1} = 130\ \mu\text{ms}^{-1}$$

2. $R = R_c + R_n = R_c[1 + \alpha_c(T - T_0)] + R_n[1 + \alpha_n(T - T_0)]$

$$0 = R_c\alpha_c(T - T_0) + R_n\alpha_n(T - T_0)$$

$$\Rightarrow \quad R_c = -R_n\frac{\alpha_n}{\alpha_c}$$

So, $R = R_c + R_n = -R_n\dfrac{\alpha_n}{\alpha_c} + R_n$

$$\Rightarrow \quad R_n = R\left(1 - \frac{\alpha_n}{\alpha_c}\right)^{-1}$$

$$\Rightarrow \quad R_c = R\left(1 - \frac{\alpha_c}{\alpha_n}\right)^{-1}$$

$$\Rightarrow \quad R_n = 10\text{ k}\Omega\left[1 - \frac{(0.4 \times 10^{-3}\ {}^\circ\text{C}^{-1})}{(-0.5 \times 10^{-3}\ {}^\circ\text{C}^{-1})}\right]^{-1}$$

$$\Rightarrow \quad R_n \cong 5560\ \Omega \text{ and } R_c = 4440\ \Omega$$

3. The drift velocity is $I = nqv_dA = nqv_d(\pi r^2)$

$$\Rightarrow v_d = \frac{I}{nq\pi r^2} = \frac{1000\text{ A}}{8.49 \times 10^{28}\text{ m}^{-3}(1.6 \times 10^{-19}\text{ C})\pi(10^{-2}\text{ m})^2}$$

$$\Rightarrow v_d = 2.34 \times 10^{-4}\ \text{ms}^{-1}$$

Since $v = \dfrac{x}{t}$

$$\Rightarrow \quad t = \frac{x}{v} = \frac{200 \times 10^3\text{ m}}{2.34 \times 10^{-4}\text{ ms}^{-1}} = 8.54 \times 10^8\text{ s} = 27\text{ yr}$$

4. $R = R_0[1 + \alpha(T - T_0)]$

$$\Rightarrow \quad T = T_0 + \frac{1}{\alpha}\left[\frac{R}{R_0} - 1\right] = T_0 + \frac{1}{\alpha}\left[\frac{I_0}{I} - 1\right]$$

In this case, $I = \dfrac{I_0}{10}$

$$\Rightarrow \quad T = T_0 + \frac{1}{\alpha}(9) = 20^\circ + \frac{9}{0.00450\ {}^\circ\text{C}^{-1}} = 2020\ {}^\circ\text{C}$$

5. Label the currents in the branches as shown in the first figure. Reduce the circuit by combining the two parallel resistors as shown in the second figure, we get the resistance in the middle branch as (1.71) R.

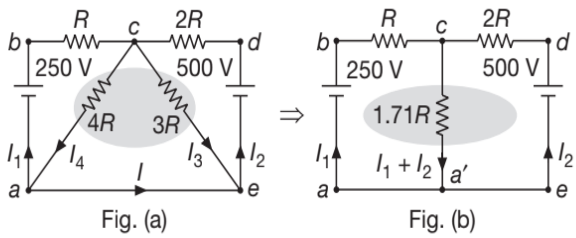


Fig. (a) Fig. (b)

Apply Kirchhoff's Loop Rule to both loops in figure (b) to obtain

$$(2.71R)I_1 + (1.71R)I_2 = 250$$

and $(1.71R)I_1 + (3.71R)I_2 = 500$

With $R = 1000\ \Omega$, simultaneous solution of these equations yields

$$I_1 = 10\text{ mA}$$

and $I_2 = 130$ mA

From figure (b), $V_c - V_a = (I_1 + I_2)(1.71R) = 240$ V

Thus, from figure (a), $I_4 = \frac{V_c - V_a}{4R} = \frac{240\text{ V}}{4000\ \Omega} = 60\text{ mA}$

Finally, Applying Kirchhoff's Point Rule at point a in figure (a) gives :

$$I = I_4 - I_1 = 60\text{ mA} - 10\text{ mA} = +50\text{ mA},$$

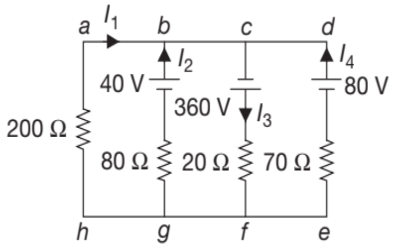
$\Rightarrow$ $I = 50\text{ mA}$ from point a to point e.

6. Name the currents as shown in the figure. Then $I_1 + I_2 + I_4 = I_3$. Loop equations are

Loop *abgha* $\quad -200I_1 - 40 + 80I_2 = 0$

Loop *bcfgb* $\quad -80I_2 + 40 + 360 - 20I_3 = 0$

Loop *cfedc* $\quad +360 - 20I_3 - 70I_4 + 80 = 0$



Eliminate I_3 by substitution

$$\begin{cases} I_2 = 2.5I_1 + 0.5 \\ 400 - 100I_2 - 20I_1 - 200I_4 = 0 \\ 440 - 20I_1 - 20I_2 - 90I_4 = 0 \end{cases}$$

Eliminate I_2 $\cdot$ $\begin{cases} 350 - 270I_1 - 20I_4 = 0 \\ 430 - 70I_1 - 90I_4 = 0 \end{cases}$

Eliminate $I_4 = 17.5 - 13.5I_1$ to obtain

$$430 - 70I_1 - 1575 + 1215I_1 = 0$$

$$I_1 = \frac{70}{70} = 1\text{ A upward in }200\ \Omega$$

Now $I_4 = 4$ A upward in 70 Ω

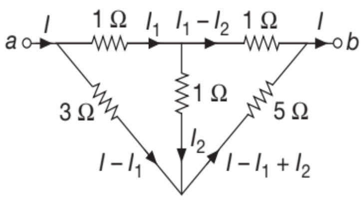
$I_2 = 3$ A upward in 80 Ω

$I_3 = 8$ A downward in 20 Ω

and for the 200 Ω, $\Delta V = IR = (1\text{ A})(200\ \Omega) = 200\text{ V}$

7. $\Delta V_{ab} = (1)I_1 + (1)(I_1 - I_2)$

$$\Delta V_{ab} = (1)I_1 + (1)I_2 + (5)(I_1 - I_1 + I_2)$$



$$\Delta V_{ab} = (3)(I - I_1) + (5)(I - I_1 + I_2)$$

Let $I = 1$ A, $I_1 = x$, and $I_2 = y$

Then, the three equations become

$\Delta V_{ab} = 2x - y$,

$\Rightarrow$ $y = 2x - \Delta V_{ab}$

$\Delta V_{ab} = -4x + 6y + 5$

$\Rightarrow$ $\Delta V_{ab} = 8 - 8x + 5y$

Substituting the first into the last two gives

$7\ \Delta V_{ab} = 8x + 5$

$\Rightarrow$ $6\ \Delta V_{ab} = 2x + 8$

Solving these simultaneously yields $\Delta V_{ab} = \frac{27}{17}$ V

Then, $R_{ab} = \frac{\Delta V_{ab}}{I} = \frac{\frac{27}{17}\text{ V}}{1\text{ A}}$

$\Rightarrow$ $R_{ab} = \frac{27}{17}\ \Omega = 1 + \frac{x}{17}$

$\Rightarrow$ $x = 10$

8. (a) $RC = (1\times10^6\ \Omega)(5\times10^{-6}\text{ F}) = 5\text{ s}$

(b) $Q = CE = (5\times10^{-6}\text{ C})(30\text{ V}) = 150\ \mu\text{C}$

(c) $I(t) = \frac{E}{R}e^{-\frac{t}{RC}} = \left(\frac{30}{1\times10^6}\right)\exp\left[\frac{-10}{(1\times10^6)(5\times10^{-6})}\right]$

$= 4.06\ \mu\text{A} \simeq 4\ \mu\text{A}$

9. $q(t) = Q\left[1 - e^{-\frac{t}{RC}}\right]$

$\Rightarrow$ $\frac{q(t)}{Q} = 1 - e^{-\frac{t}{RC}}$

$\Rightarrow$ $0.6 = 1 - e^{-\frac{0.9}{RC}}$

$\Rightarrow$ $e^{-\frac{0.9}{RC}} = 1 - 0.6 = 0.4$

$\Rightarrow$ $-\frac{0.9}{RC} = \log_e(0.4)$

$\Rightarrow$ $RC = \frac{-0.9}{\log_e(0.4)} = 0.982\text{ s} \simeq 1\text{ s}$

10. The potential difference across the capacitor

$$\Delta V(t) = \Delta V_{\max}\left(1 - e^{-\frac{t}{RC}}\right)$$

$\Rightarrow$ $4\text{ V} = (10\text{ V})\left[1 - e^{\left[\frac{-(3\text{ s})}{R(10\times10^{-6}\text{ s}\Omega^{-1})}\right]}\right]$

Therefore, $0.4 = 1 - e^{-\frac{(3\times10^5\ \Omega)}{R}}$

$\Rightarrow$ $e^{-\frac{(3\times10^5\ \Omega)}{R}} = 0.6$

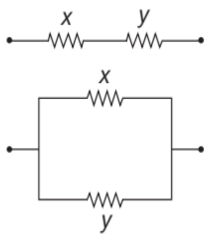
Taking the natural logarithm of both sides,

$$-\frac{3\times10^5\ \Omega}{R}=\log_e(0.6)$$

$$\Rightarrow\quad R=-\frac{3\times10^5\ \Omega}{\log_e(0.6)}=+\,5.87\times10^5\ \Omega=587\ \text{k}\Omega$$

11. Let the two resistances be x and y.

Then, $R_s=x+y=\dfrac{P_s}{I^2}=\dfrac{225\ \text{W}}{(5\ \text{A})^2}=9\ \Omega$



$$y=9\ \Omega-x$$

$$\Rightarrow\quad R_P=\frac{xy}{x+y}=\frac{P_p}{I^2}=\frac{50\ \text{W}}{(5\ \text{A})^2}=2\ \Omega$$

$$\Rightarrow\quad \frac{x\ (9\ \Omega-x)}{x+(9\ \Omega-x)}=2\ \Omega$$

$$x^2-9x+18=0$$

Factoring the second equation, $(x-6)(x-3)=0$

$\Rightarrow\quad x=6\ \Omega$

$\Rightarrow\quad x=3\ \Omega$

Then, $y=9\ \Omega-x$ gives $y=3\ \Omega$

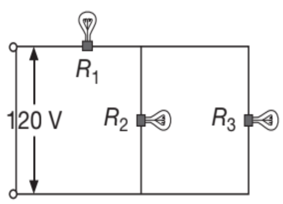
$\Rightarrow\quad y=6\ \Omega$

The two resistances are found to be $6\ \Omega$ and $3\ \Omega$

12. (a) First determine the resistance of each light bulb:

$$P=\frac{(\Delta V)^2}{R}$$

$$R=\frac{(\Delta V)^2}{P}=\frac{(120\ \text{V})^2}{60\ \text{W}}=240\ \Omega$$



We obtain the equivalent resistance R_{eq} of the network of light bulbs by identifying series and parallel equivalent resistances

$$R_{eq}=R_1+\frac{1}{\left(\frac{1}{R_2}\right)+\left(\frac{1}{R_3}\right)}=240\ \Omega+120\ \Omega=360\ \Omega$$

The total power dissipated in the $360\ \Omega$ is

$$P=\frac{(\Delta V)^2}{R_{eq}}=\frac{(120\ \text{V})^2}{360\ \Omega}=40\ \text{W}$$

(b) The current through the network is given by $P=I^2R_{eq}$

$$\Rightarrow\quad I=\sqrt{\frac{P}{R_{eq}}}=\sqrt{\frac{40\ \text{W}}{360\ \Omega}}=\frac{1}{3}\ \text{A}$$

The potential difference across R_1 is

$$\Delta V_1=IR_1=\left(\frac{1}{3}\ \text{A}\right)(240\ \Omega)=80\ \text{V}$$

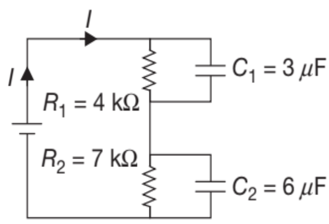
The potential difference ΔV_{23} across the parallel combination of R_2 and R_3 is

$$\Delta V_{23}=IR_{23}=\left(\frac{1}{3}\ \text{A}\right)\left(\frac{1}{\frac{1}{240\ \Omega}+\frac{1}{240\ \Omega}}\right)=40\ \text{V}$$

13. (a) With the switch closed, current exists in a simple series circuit as shown. The capacitors carry no current. For R_2 we have

$$P=I^2R_2$$

$$\Rightarrow\quad I=\sqrt{\frac{P}{R_2}}=\sqrt{\frac{2.4\ \text{VA}}{7000\ \frac{V}{A}}}=18.5\ \text{mA}$$



The potential difference across R_1 and C_1 is

$$\Delta V=IR_1=(1.85\times10^{-2}\ \text{A})\left(4000\ \frac{V}{A}\right)=74.1\ \text{V}$$

The charge on C_1 is

$$Q_1=C_1\ \Delta V=(3\times10^{-6}\ \text{CV}^{-1})(74.1\ \text{V})=222\ \mu\text{C}$$

The potential difference across R_2 and C_2 is

$$\Delta V=IR_2=(1.85\times10^{-2}\ \text{A})(7000\ \Omega)=130\ \text{V}$$

The charge on C_2 is

$$Q_2=C_2\ \Delta V=\left(6\times10^{-6}\ \frac{C}{V}\right)(130\ \text{V})=778\ \mu\text{C}$$

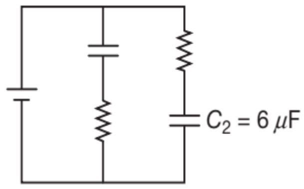
The battery emf is given by

$$IR_{eq}=I(R_1+R_2)$$

$$\Rightarrow\quad IR_{eq}=1.85\times10^{-2}(4000+7000)=204\ \text{V}$$

(b) In equilibrium, after the switch has been opened for a long time, no current can flow in the circuit. The potential difference across each resistor is zero. The full 204 V appears across both capacitors. The new charge C_2 is

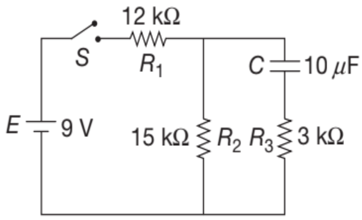
$$Q_2' = C_2\,\Delta V = (6\times10^{-6}\ \text{CV}^{-1})(204\ \text{V}) = 1222\ \mu\text{C}$$



Change in the value of charge on Q_2 is given by

$$\Delta Q_2 = Q_2' - Q_2 = 1222\ \mu\text{C} - 778\ \mu\text{C} = 444\ \mu\text{C}$$

14. (a) After steady state conditions have been reached, there is no DC current through the capacitor Thus, for R_3, we have $I_{R_3} = 0$ (steady state)



For the other two resistors, the steady state current is simply determined by the 9 V emf across the 12 kΩ and 15 kΩ resistors in series

For R_1 and R_2, we have

$$I_{(R_1+R_2)} = \frac{E}{R_1+R_2} = \frac{9\ \text{V}}{(12\ \text{k}\Omega + 15\ \text{k}\Omega)} = 333\ \mu\text{A}$$

(steady state)

(b) After the transient currents have ceased, the potential difference across C is the same as the potential difference across $R_2\,(=IR_2)$ because there is no voltage drop across R_3. Therefore, the charge Q on C is

$$Q = C(\Delta V)_{R_2} = C(IR_2) = (10\ \mu\text{F})(333\ \mu\text{A})(15\ \text{k}\Omega)$$

$$\Rightarrow \quad Q = 50\ \mu\text{C}$$

(c) When the switch is opened, the branch containing R_1 is no longer a part of the circuit. The capacitor discharges through (R_2+R_3) with a time constant of

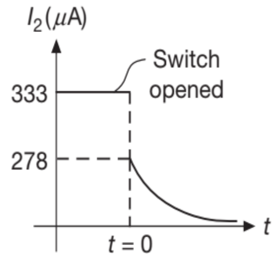
$$(R_2+R_3)C = (15\ \text{k}\Omega + 3\ \text{k}\Omega)(10\ \mu\text{F}) = 0.18\ \text{s}$$

The initial current I_0 in this discharge circuit is determined by the initial potential difference across the capacitor applied to (R_2+R_3) in series. So,

$$I_0 = \frac{(\Delta V)_C}{(R_2+R_3)} = \frac{IR_2}{(R_2+R_3)}$$

$$\Rightarrow \quad I_0 = \frac{(333\ \mu\text{A})(15\ \text{k}\Omega)}{(15\ \text{k}\Omega + 3\ \text{k}\Omega)} = 278\ \mu\text{A}$$

Thus, when the switch is opened, the current through R_2 changes instantaneously from $333\ \mu$A (downward) to $278\ \mu$A (downward) as shown in the graph.



Afterwards, it decays according to

$$I = I_0 e^{-t/(R_2+R_3)C} = (278\ \mu\text{A})e^{-t/(0.18\ \text{s})}\ (\text{for } t > 0)$$

(d) The charge q on the capacitor decays from Q_0 to $\frac{Q_0}{5}$ according to

$$q = Q_0 e^{-t/(R_2+R_3)C}$$

$$\Rightarrow \quad \frac{Q_0}{5} = Q_0 e^{(-t/0.18\ \text{s})}$$

$$\Rightarrow \quad 5 = e^{t/0.18\ \text{s}}$$

$$\Rightarrow \quad \log_e 5 = \frac{t}{180\ \text{ms}}$$

$$\Rightarrow \quad t = (0.18\ \text{s})(\log_e 5) = 290\ \text{ms}$$

15. Since α can vary at different temperatures, so it is advisable to make a rough estimate

$$R = R_{20}(1+\alpha\Delta T)$$

$$\Rightarrow \quad \Delta T = \frac{R - R_{20}}{\alpha R_{20}}$$

$$\Rightarrow \quad T - 20\ ^\circ\text{C} = \frac{(190-15)}{(4.5\times10^{-3})(15)} = 2592\ ^\circ\text{C}$$

$$\Rightarrow \quad T = 2612\ ^\circ\text{C}$$

$$\Rightarrow \quad T = 2612 + 273 = 2885\ \text{K}$$

16. Let R_x and R_y be the respective resistances at a given temperature. The total resistance after a charge ΔT becomes

$$R = R_x(1+0.0025\ \Delta T) + R_y(1+0.00075\ \Delta T) \quad \ldots(1)$$

As $0.001(°C)^{-1}$ is the temperature coefficient of the combination, we also have

$$R = (R_x + R_y)(1 + 0.001\,\Delta T) \quad \ldots(2)$$

From equations (1) and (2), we get

$$R_x(1 + 0.0025\,\Delta T) + R_y(1 + 0.00075\,\Delta T)$$
$$= (R_x + R_y)(1 + 0.001\,\Delta T)$$
$$\Rightarrow \quad R_x(0.0015\,\Delta T) = R_y(0.00025\,\Delta T)$$

Thus we get

$$R_x = \frac{5}{3}R_y \quad \ldots(3)$$

Also, $R_x + R_y = 1000\ \Omega \quad \ldots(4)$

On solving equations (3) and (4), we get

$$R_x = 625\ \Omega \text{ and } R_y = 375\ \Omega$$

The respective lengths are

$$L_x = \frac{1}{100} \times 625 = 6.25 \text{ km} = 6250 \text{ m}$$

and $L_y = \frac{1}{50} \times 375 = 7.5 \text{ km} = 7500 \text{ m}$

17. The power consumed (P) in time t is given by

$$P = \frac{dU}{dt} = \frac{V^2}{R(t)}$$
$$\Rightarrow \quad dU = \frac{V^2}{R(t)}dt$$
$$\Rightarrow \quad U = \int_0^t \frac{V^2}{R(t)}dt$$
$$\Rightarrow \quad U = \frac{(110)^2}{0.5}\int_0^t e^{-2t}dt$$
$$\Rightarrow \quad U = \frac{(110)^2}{2 \times 0.5}(e^{-2t})_0^t$$
$$\Rightarrow \quad U = (110)^2(1 - e^{-2t}) \text{ J}$$

According to problem,

$$U = 7644 \text{ J}$$
$$\Rightarrow \quad 1 - e^{-2t} = \frac{7644}{(110)^2} = 0.632$$
$$\Rightarrow \quad e^{-2t} = 0.367$$
$$\Rightarrow \quad -2t\log_e e = \log_e 0.367$$
$$\Rightarrow \quad -2t = -1$$
$$\Rightarrow \quad t = 0.5 \text{ s}$$
$$\Rightarrow \quad t = 500 \text{ ms}$$

18. Heat produced in a resistance R in time t is

$$H = Pt = \frac{V^2}{R}t$$

For coil 1,

$$H_1 = \frac{V^2}{R_1}(15 \times 60) \quad \ldots(1)$$

For coil 2,

$$H_2 = \frac{V^2}{R_2}(30 \times 60) \quad \ldots(2)$$

But according to given problem $H_1 = H_2 = H$ i.e.,

$$\frac{15}{R_1} = \frac{30}{R_2}$$
$$\Rightarrow \quad R_2 = 2R_1 \quad \ldots(3)$$

(a) When both the coils are used in series, then

$$H_s = \frac{V^2}{(R_1 + R_2)}t_s = \frac{V^2}{3R_1} \times t_s \quad \{\because\ R_2 = 2R_1\}$$

But as here, $H_s = H_1 = H_2 = H$

$$\Rightarrow \quad \frac{V^2}{R_1}(15 \times 60) = \frac{V^2}{3R_1}t_s$$
$$\Rightarrow \quad t_s = (45 \times 60)\text{s} = 45 \text{ min}$$

(b) When both the coils are used in parallel, then

$$H_P = H = \left(\frac{V^2}{R_1} + \frac{V^2}{R_2}\right)t_p = \left(\frac{3V^2}{2R_1}\right)t_p \quad \{\because\ R_2 = 2R_1\}$$
$$\Rightarrow \quad \frac{3V^2}{2R_1} \times t_p = \frac{V^2}{R_1} \times (15 \times 30)$$
$$\Rightarrow \quad t_p = (10 \times 60)\text{s} = 10 \text{ min}$$

19. Resistance of the wire at 300 K is

$$R_0 = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = \frac{\pi^2 \times 10^{-8} \times 1}{\pi \times (10^{-3})^2} = \pi \times 10^{-2}\ \Omega$$

Resistance at 900 K is

$$R_t = R_0(1 + \alpha t)$$
$$\Rightarrow \quad R_t = \pi \times 10^{-2}[1 + 7.8 \times 10^{-3} \times (900 - 300)]$$
$$\Rightarrow \quad R_t = 5.68\ \pi \times 10^{-2}\ \Omega$$

The rate at which the wire radiates energy is

$$W = P = I^2R_t = 5.68\pi \times 10^{-2}I^2 \text{ watt} \quad \ldots(1)$$

The rate at which a black body radiates energy is given by Stefan's Law

$$W = \sigma A_s\ (T^4 - T_0^4)$$

where A_s = surface area of the wire = $(2\pi r)L$, $T = 900$ K, $T_0 = 300$ K and $\sigma = 5.68 \times 10^{-8}$ Wm^{-2}K^{-4}

Thus $W=5.68\times10^{-8}\times(2\pi\times10^{-3})\times1\left[(900)^4-(300)^4\right]$

$\Rightarrow \quad W=5.68\times12.96\ \pi$ watt ...(2)

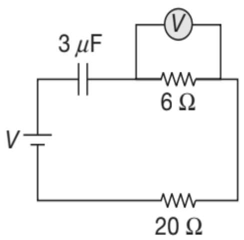
Equating (1) and (2), we get

$$5.68\ \pi\times10^{-2}\times I^2=5.68\times12.96\ \pi$$

$\Rightarrow \quad I^2=1296$

$\Rightarrow \quad I=36$ A

20. (a) Since, a capacitor is an open circuit to DC in steady state. The effective circuit through which current will pass initially after closing switch S_1 is shown. When steady state is reached the capacitor will not allow current. Thus there is no current from battery and hence the reading of voltmeter is zero.



(b) Without current the resistors act as conducting wires. Hence potential difference across capacitor is 12 V.

Charge on capacitor, 3 μF, is

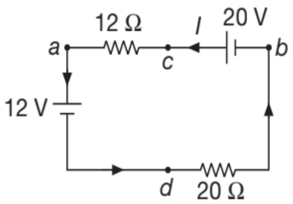
$$Q=(3\times12)\mu\text{C}=36\ \mu\text{C}$$

(c) When all the switches are closed, the branch containing capacitor will not allow current after steady state is reached. The equivalent circuit from point of view of current is redrawn. Let the current in the single loop circuit be I. We begin at left lower corner and traverse the circuit anticlockwise while applying KVL. We get

$$-20I+20-12I-12=0$$

$$\Rightarrow \quad I=\frac{8}{32}$$

$$\Rightarrow \quad I=\frac{1}{4}\ \text{A}$$



In the steady state power is consumed only in 12 Ω and 20 Ω resistors. All the resistors in a single loop circuit are in series, thus power consumed is

$$P=I^2R_{eq}=\left(\frac{1}{4}\right)^2(32)=2\ \text{W}$$

(d) The capacitor is connected between points c and d. In order to determine the potential difference across it we begin at point c and traverse along any path to d, to get

$$V_c-12I-12=V_d$$

$$V_c-V_d=12I+12=3+12=15\ \text{V}$$

Charge on 2 μF capacitor = $(15)\times2=30\ \mu$C

21. (a) If there are N resistors of resistance R each, connected in parallel, the net effective R_e of such a combination is given by

$$\frac{1}{R_e}=\frac{1}{R}+\frac{1}{R}+\ldots N \text{ terms}$$

$$\Rightarrow \quad R_e=\frac{R}{N}$$

Now, this combination is connected to a source of emf E and internal resistance r. Hence current in the main circuit would be

$$I=\frac{E}{r+R_e}$$

This current will be equally divided between all the N resistors (so that potential drop across each resistor is same). Hence power generated in any single resistor will be

$$P=\left(\frac{I}{N}\right)^2R=\frac{I^2R}{N^2}=\frac{E^2R}{N^2\left(r+\dfrac{R}{N}\right)^2}$$

$$\Rightarrow \quad P=\frac{(200)^2(300)}{(200)^2\left(0.5+\dfrac{300}{200}\right)^2}=\frac{(200)^2(300)}{(200)^2(2)^2}$$

$$\Rightarrow \quad P=\frac{300}{4}=75\ \text{W}$$

(b) In the next stage, one of the bulb burns out, i.e., now only $(N-1)$ resistors are left out. Power generated by any single resistor can be obtained by just replacing N by $(N-1)$ in the previous equation. So, we get

$$P'=\frac{E^2}{(N-1)^2}\frac{R}{\left(r+\dfrac{R}{N-1}\right)^2}=\frac{E^2R}{(N-1)^2\left(r+\dfrac{R}{N-1}\right)^2}$$

The relative change f, by definition, is given by

$$f=\frac{P'-P}{P}$$

$$\Rightarrow \quad f=\left[\frac{(rN+R)^2}{(r(N-1)+R)^2}-1\right]$$

$$\Rightarrow \quad f = \frac{1}{\left(1 - \frac{r}{rN+R}\right)^2} - 1$$

Now, $(Nr+R) \gg r$, since there are many bulbs $(N = 200)$. Hence, we can approximate,

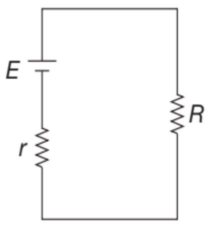
$$\left(1 - \frac{r}{rN+R}\right)^{-2} \approx \left(1 + \frac{2r}{rN+R}\right)$$

Substituting $r = 0.5\ \Omega$, $R = 300\ \Omega$ and $N = 200$, we get

$$f = \frac{2(0.5)}{(0.5)(200)+300} = \frac{1}{400} = 0.0025$$

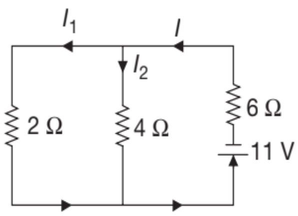
$$\Rightarrow \quad 10000 f = 25$$

22. Suppose, we insert the battery with $2\ \Omega$ resistance. Then we can take $2\ \Omega$ as the internal resistance (r) of the battery and combined resistance of the other two as the external resistance (R). The circuit in that case shown in figure.



Now power, $P = \frac{E^2}{R+r}$

This power will be minimum where $(R+r)$ is maximum and we observe that $(R+r)$ will be maximum only when the battery is inserted with $6\ \Omega$ resistance as shown in figure.



Net resistance in this case is

$$R_{eq} = 6 + \frac{2 \times 4}{2+4} = \frac{22}{3}\ \Omega$$

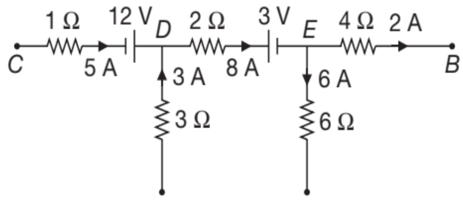
$$\Rightarrow \quad I = \frac{11}{22/3} = 1.5\ \text{A}$$

This current will be distributed in $2\ \Omega$ and $4\ \Omega$ in the inverse ratio of their resistances.

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{4}{2} = 2$$

$$\Rightarrow \quad I_1 = \left(\frac{2}{2+1}\right)(1.5) = 1\ \text{A}$$

23. Applying Kirchhoff's Junction Law at E current in wire DE is 8 A from D to E. Now further applying Junction Law at D. The current in $3\ \Omega$ resistance will be 3 A towards D.



Power dissipated in $3\ \Omega$ resistance

$$= I^2 R = (3)^2(3) = 27\ \text{W}$$

For path $CDEB$, we have

$$V_C - (5)(1) + 12 - (8)(2) - 3 - (4)(2) = V_B$$

$$\Rightarrow \quad V_C - V_B = 5 - 12 + 16 + 3 + 8$$

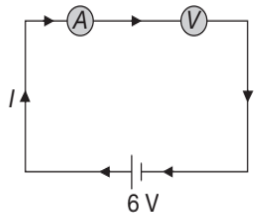
$$\Rightarrow \quad V_C - V_B = 20\ \text{V}$$

24. Let R_1 = resistance of ammeter and R_2 = combined resistance of ammeter and voltmeter

In the first case, current in the circuit is given by

$$I = \frac{6}{R_2} \quad \ldots(1)$$

and voltage across voltmeter is given by



$$V = 6 - (\text{voltage across ammeter})$$

$$\Rightarrow \quad V = 6 - IR_1$$

$$\Rightarrow \quad V = 6 - \frac{6R_1}{R_2} \quad \ldots(2)$$

In the second case reading of ammeter becomes two times, i.e., the total resistance becomes half whereas the resistance of ammeter remains unchanged. Hence,

$$I' = \frac{6}{R_2/2} = \frac{12}{R_2} \quad \ldots(3)$$

and $V' = 6 - (I')R_1$

$$\Rightarrow \quad V' = 6 - \frac{12R_1}{R_2} \quad \ldots(4)$$

Further, it is given that $V' = \frac{V}{2}$

$$\Rightarrow \quad 6-\frac{12R_1}{R_2}=3-\frac{3R_1}{R_2}$$

$$\Rightarrow \quad \frac{R_1}{R_2}=\frac{1}{3}$$

Substituting this value in equation (4), we have

$$V'=6-(12)\left(\frac{1}{3}\right)$$

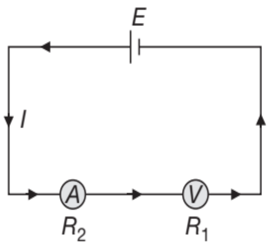
$$\Rightarrow \quad V'=2\text{ V}$$

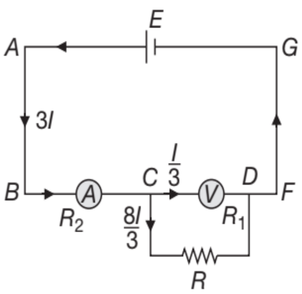
25. Let E be the emf of the battery

In the first case let I be the current in the circuit, then

$$E=I(R_1+R_2) \qquad \ldots(1)$$

In the second case main current increases three times while current through voltmeter will reduce to $\frac{I}{3}$. Hence, the remaining $3I-\frac{I}{3}=\frac{8I}{3}$ passes through R as shown in figure.





$$V_C-V_D=\left(\frac{I}{3}\right)R_1=\left(\frac{8I}{3}\right)R$$

$$\Rightarrow \quad R_1=8R=8(3\ \Omega)=24\ \Omega$$

Applying Kirchhoff's Second Law in loop $ABFGA$,

$$E=3I(R_2)+\left(\frac{I}{3}\right)(R_1)=I\left(3R_2+\frac{R_1}{3}\right) \qquad \ldots(2)$$

From equations (1) and (2),

$$R_1+R_2=3R_2+\frac{R_1}{3}$$

$$\Rightarrow \quad 2R_2=\frac{2R_1}{3}$$

$$\Rightarrow \quad R_2=\frac{R_1}{3}$$

$$\Rightarrow \quad R_2=\frac{8R}{3}=\frac{8}{3}(3\ \Omega)=8\ \Omega$$

26. The voltage supplied by the charging plant is here constant which is equal to,

$$V=E_i+I_ir=(90)+(10)(2)$$

$$V=110\text{ V}$$

Let I_f be the current at the end of charging process. Then,

$$V=E_f+I_fr$$

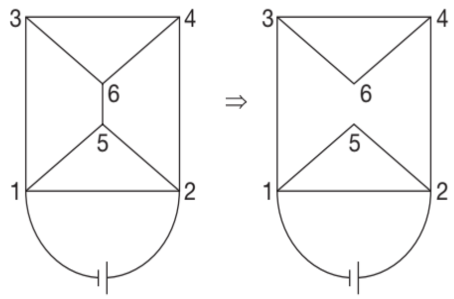
$$\Rightarrow \quad I_f=\frac{V-E_f}{r}=\frac{110-100}{2}$$

$$\Rightarrow \quad I_f=5\text{ A}$$

27. Let us represent the central junction of wires in the form of two junctions connected by the wire 5-6 as shown in figure. Then it follows from symmetry that there is no current through this wire. Therefore, the central junction can be removed from the initial circuit. Further,

$$R_{12}=R_{13}=R_{34}=R_{24}=r$$

and $R_{15}=R_{25}=R_{36}=R_{46}=\frac{r}{\sqrt{2}}$



Let V be the voltage between 1 and 2. Then the amount of heat liberated in conductor 1-2.

$$H_{12}=\frac{V^2}{r} \qquad \ldots(1)$$

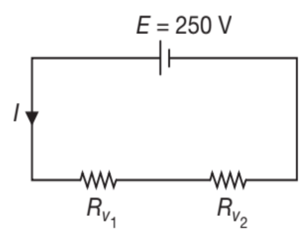
Current through 3-4, $I_{3-4}=\frac{V}{r(\sqrt{2}+3)}$

$$\Rightarrow \quad H_{3-4}=I_{3-4}^2r=\frac{V^2}{r(\sqrt{2}+3)^2}$$

$$\Rightarrow \quad \frac{H_{1-2}}{H_{3-4}}=(\sqrt{2}+3)^2=11+6\sqrt{2}$$

Comparing with $11x+y\sqrt{2x}$, we get $x=1,\ y=6$

28. (a) When the switch is open, no current flows through the branch that contains R_1 and R_2. Hence



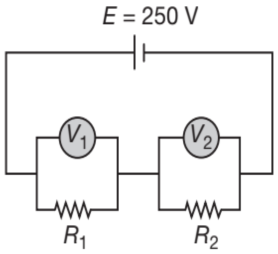
$$I = \frac{250}{10000} = \frac{1}{40}\ \text{A}$$

According to Ohm's Law

$$V_1 = \frac{1}{40}(6000) = 150\ \text{V} \ \text{and}$$

$$V_2 = (250 - 150)\ \text{V} = 100\ \text{V}$$

(b) When the switch is closed the equivalent circuit is shown.



Since we observe that V_1 and R_1 are in parallel and V_2 and R_2 are in parallel and both in series and also simultaneously we have

$$\frac{R_1(R_{V_1})}{R_1 + R_{V_1}} = \frac{R_2(R_{V_2})}{R_2 + R_{V_2}} = 2.4\ \text{k}\Omega$$

Hence the potential of 250 V must be equally divided. So, reading of both the voltmeters is

$$V_1 = V_2 = \frac{E}{2} = 125\ \text{V}$$

29. Let N be the total number of cells required to be grouped in m rows, each row carrying n cells, then $N = mn$

We have, $I = \dfrac{nE}{R + \dfrac{nr}{m}} = \dfrac{mnE}{mR + nr} = \dfrac{NE}{mR + nr}$

$$\Rightarrow \quad 8 = \frac{2N}{5m + \dfrac{0.5N}{m}}$$

$$\Rightarrow \quad 20m^2 + 2N = Nm \qquad \ldots(1)$$

For least value of N, $\dfrac{dN}{dm} = 0$,

Differentiating equation (1), we have

$$40m + 2\frac{dN}{dm} = m\frac{dN}{dm} + N$$

$$\Rightarrow \quad N = 40\ \text{m} \qquad \left\{\because \frac{dN}{dm} = 0\right\} \qquad \ldots(2)$$

But $N = mn$ $\qquad \ldots(3)$

Solving equations (1), (2) and (3), we get

$$n = 40 \ \text{and} \ m = 4$$

$$\Rightarrow \quad N = mn = 160$$

30. Before connecting B with D and C with E,

$$R_{AF} = 5R = 5(2\ \Omega) = 10\ \Omega$$

After connecting B with D and C with E, a balanced Wheatstone bridge is formed between B and E.

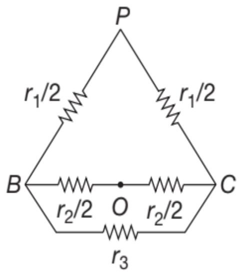
So, $R_{BE} = R$

and $R'_{AF} = 3R = 3(2\ \Omega) = 6\ \Omega$

$$\Rightarrow \quad \Delta R = 10 - 6 = 4\ \Omega$$

i.e., the new resistance decreases by $4\ \Omega$

31. Points $(A \text{ and } C)$ and $(D \text{ and } B)$ are symmetrically located with respect to points O and P. Hence, the circuit can be drawn as shown in figure.



This is a balanced Wheatstone bridge between P and O Hence, r_3 can be removed. And,

$$R_{PO} = \frac{r_1 + r_2}{4}$$

Here $r_1 = R_{PB} = R_{PD} = \dfrac{(\pi a)\lambda}{2}$

and $r_2 = R_{OB} = (a)\lambda$

$$\Rightarrow \quad R_{PO} = \frac{(2+\pi)a\lambda}{8} = \left(\frac{2+\pi}{8}\right)a\left(\frac{64}{2+\pi}\right)\frac{1}{a} = 8\ \Omega$$

32. $v_d = \dfrac{I}{neA} = \dfrac{1}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-4}}$

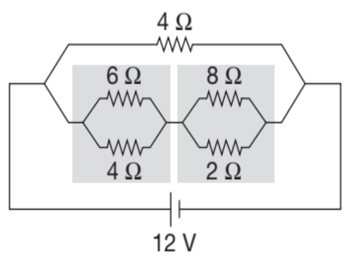
$$\Rightarrow \quad v_d = 0.735 \times 10^{-6}\ \text{ms}^{-1}$$

$$\Rightarrow \quad v_d = 0.735\ \mu\text{ms}^{-1}$$

Since, $t = \dfrac{\ell}{v_d} = \dfrac{10^4}{0.735 \times 10^{-6} \times 365 \times 24 \times 3600}$ year

$$\Rightarrow \quad t = 431\ \text{year}$$

33. The equivalent circuit is as shown in figure.



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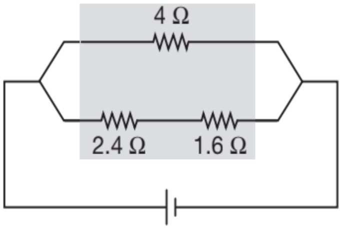

$$\Rightarrow \quad R_{\text{series}} = \frac{6\times 4}{6+2} + \frac{8\times 2}{8+2} = 4\ \Omega$$

$$R_{\text{total}} = \frac{4(2.4+1.6)}{4+2.4+1.6} = \frac{(4)(4)}{4+4} = 2\ \Omega$$

$$\Rightarrow \quad I = \frac{V}{R} = \frac{12}{2} = 6\ \text{A}$$

34. (a) Current in the circuit,

$$I = \frac{\text{net emf}}{\text{net resistance}} = \frac{6+12}{1+2+3} = 3\ \text{A}$$

Now, $V_G = 0$ V

$$V_A = 12\ \text{V},\ V_B = 12 - 1\times 3 = 9\ \text{V},$$
$$V_C = 12 - 3\times 3 = 3\ \text{V}$$

and $V_D = 12 - 3\times 6 = -6$ V

(b) $I = \dfrac{12-6}{1+2+3} = 1$ A

$V_G = 0$

$$\Rightarrow \quad V_A = 12\ \text{V}, V_B = 12 - 1\times 1 = 11\ \text{V},$$
$$V_C = 12 - 3\times 1 = 9\ \text{V and } V_D = 12 - 6\times 1 = 6\ \text{V}$$

35. Let n cells be wrongly connected in the battery. Let e.m.f. of each cell be E and R be the total resistance of the circuit

$$(12-n)E - nE = (12-2n)E$$

When the cells aid the battery, the net e.m.f. is

$$E_1 = (12-2n)E + 2E$$

So, current in circuit is

$$I_1 = 3 = \frac{(12-2n)E + 2E}{R} \quad \ldots(1)$$

When the cells oppose the battery, the net e.m.f. is

$$E_2 = (12-2n)E - 2E$$

So, current in circuit is

$$I_2 = 2 = \frac{(12-2n)E - 2E}{R} \quad \ldots(2)$$

Dividing (1) by (2), we get

$$\frac{3}{2} = \frac{(12-2n)+2}{(12-2n)-2}$$

Solving, we get $n = 1$

Hence one cell of the battery is wrongly connected.

36. Leakage current I is given by

$$I = \frac{CV}{K\rho\varepsilon_0} = \frac{\left(\dfrac{K\varepsilon_0 A}{d}\right)V}{K\varepsilon_0\rho}$$

$$\Rightarrow \quad I = \frac{AV}{\rho d} = \frac{(10\times 10^{-4})(6)}{(10^{13})(1\times 10^{-3})}$$

$$\Rightarrow \quad I = 6\times 10^{-13}\ \text{A}$$

So, $* = 6$

37. The resistance of a layer of medium of thickness dx at a distance x from the first plate of capacitor is

$$dR = \frac{\rho(x)dx}{A} = \frac{1}{\sigma(x)}\frac{dx}{A}$$

Since σ varies linearly with distance from the first plate, so we have

$$\sigma = \sigma_1 + \left(\frac{\sigma_2-\sigma_1}{d}\right)x \ \Rightarrow\ dR = \left[\frac{1}{\sigma_1 + \left(\dfrac{\sigma_2-\sigma_1}{d}\right)x}\right]\frac{dx}{A}$$

Hence, total resistance R is given by

$$R = \frac{1}{A}\int_0^d \frac{dx}{\sigma_1 + \left(\dfrac{\sigma_2-\sigma_1}{d}\right)x}$$

$$\Rightarrow \quad R = \frac{d}{A(\sigma_2-\sigma_1)}\log_e\left(\frac{\sigma_2}{\sigma_1}\right)$$

The current I is given by

$$I = \frac{V}{R} = \frac{AV(\sigma_2-\sigma_1)}{d\log_e\left(\dfrac{\sigma_2}{\sigma_1}\right)} = \frac{AV}{d}\left[\frac{\sigma_2-\sigma_1}{\log_e\left(\dfrac{\sigma_2}{\sigma_1}\right)}\right]$$

$$\Rightarrow \quad I = \frac{(230\times 10^{-4})(300)(2-1)\times 10^{-12}}{(2\times 10^{-3})(2.3\log_{10}2)}$$

$$\Rightarrow \quad I = 5\times 10^{-9}\ \text{A} = 5\ \text{nA}$$

38. Resistance of bulb is given by

$$R_0 = \frac{V_0^2}{P} = \frac{100\times 100}{500} = 20\ \Omega$$

Current in bulb is $I = \dfrac{V_0}{R_0} = \dfrac{100}{20} = 5$ A

For same power dissipation in bulb, the current in circuit must be 5 A

When bulb is in a circuit having supply voltage 220 V, the safe resistance of the circuit

$$R' = \frac{V'}{I} = \frac{200}{5} = 40\ \Omega$$

Hence, required resistance in series is

$$R = R' - R_0 = 40 - 20 = 20\ \Omega$$

39. The resistance R of fuse wire of length ℓ, cross sectional area A and resistivity ρ is

$$R = \frac{\rho\ell}{A}$$

Here $\rho = 22\times10^{-6}\ \Omega\text{cm} = 22\times10^{-8}\ \Omega\text{m}$

$$A = 0.2\ \text{mm}^2 = 0.2\times10^{-6}\ \text{m}^2 = 2\times10^{-7}\ \text{m}^2$$

Since, $Q = I^2Rt$ (in joule)

$$Q = I^2\left(\frac{\rho\ell}{A}\right)t \quad \ldots(1)$$

Again if m is mass of wire, d is its density, c its specific heat and ΔT, the rise of temperature, then

$$Q = mc\Delta T$$

$$\Rightarrow \quad Q = (\text{Volume}\times\text{density})\times c\times(327-20)$$

$$\Rightarrow \quad Q = (A\ell dc)(307) \quad \ldots(2)$$

From (1) and (2), we get

$$(A\ell d)c\times307 = I^2\left(\frac{\rho\ell}{AJ}\right)t$$

$$\Rightarrow \quad t = \frac{(A^2cd)(307)J}{I^2\rho} \quad \ldots(3)$$

Here density $d = 11.34\ \text{gcm}^{-3} = 11.34\times10^3\ \text{kgm}^{-3}$, current $I = 20\ \text{A}$, $\rho = 22\times10^{-6}\ \Omega\text{cm} = 22\times10^{-8}\ \Omega\text{m}$ and specific heat $c = 0.032\ \text{calg}^{-1}(°\text{C})^{-1}$

$$\Rightarrow \quad c = (0.032)(4200)\ \text{Jkg}^{-1}(°\text{C})^{-1},$$

$$A = 0.2\ \text{mm}^2 = 2\times10^{-7}\ \text{m}^2$$

Substituting given values in (3), we get

$$t = \frac{(2\times10^{-7})^2\times4.2\times10^3\times11.34\times10^3\times0.032\times307}{(20)^2\times22\times10^{-8}}$$

$$\Rightarrow \quad t = 0.095\ \text{s} = 95\times10^{-3}\ \text{s} = 95\ \text{ms}$$

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1. From Colour Code of Carbon Resistors table, we get

$$200\ \Omega = \text{Red} + \text{Black} + \text{Brown}$$

Since, Green $\equiv 5$

$$\Rightarrow \quad \text{Green} + \text{Black} + \text{Brown} \equiv 500\ \Omega$$

Hence, the correct answer is (B).

2. $$E_{eq} = \frac{\dfrac{E_1}{2R_1} + \dfrac{E_2}{R_2} + \dfrac{E_3}{2R_1}}{\dfrac{1}{2R_1} + \dfrac{1}{R_2} + \dfrac{1}{2R_1}}$$

$$\Rightarrow \quad E_{eq} = \frac{\dfrac{2}{2} + \dfrac{4}{2} + \dfrac{4}{2}}{\dfrac{1}{2} + \dfrac{1}{2} + \dfrac{1}{2}}$$

$$\Rightarrow \quad E_{eq} = \frac{5}{\dfrac{3}{2}} = \frac{10}{3} = 3.3\ \text{V}$$

Hence, the correct answer is (D).

3. For maximum power consumption in external resistance

$$\text{Internal resistance} = \text{External resistance}$$

$$\Rightarrow \quad R = r$$

Hence, the correct answer is (B).

4. Resistance of potentiometer wire is

$$R_p = 400\times0.01$$

$$\Rightarrow \quad R_p = 4\ \Omega$$

$$\Rightarrow \quad I = \frac{3}{6} = 0.5\ \text{A}$$

Reading of voltmeter is

$$V_{AJ} = IR_{AJ}$$

$$\Rightarrow \quad V_{AJ} = 0.5\times50\times0.01$$

$$\Rightarrow \quad V_{AJ} = 0.25\ \text{V}$$

Hence, the correct answer is (D).

5. Equivalent resistance of the given circuit is

$$R_{eq} = 45 + \frac{10\times50}{10+50} = \left(45 + \frac{50}{6}\right)\Omega = \frac{160}{3}\ \Omega$$

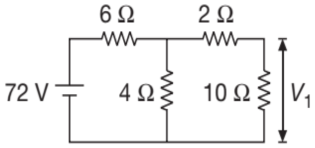
$$\Rightarrow \quad I = \frac{15}{\dfrac{160}{3}} = \frac{9}{32}\ \text{A}$$

Hence, the correct answer is (B).

6. At steady state, current through capacitor is zero.

$\Rightarrow \quad V_C = V_1$

where, $V_1 = \frac{5}{6} \times V_0 \times \frac{3}{9}$



$\Rightarrow \quad V_1 = \frac{5 \times 72 \times 3}{6 \times 9} = 20 \text{ V}$

Since $Q_1 = CV_1$

$\Rightarrow \quad Q_1 = 200\ \mu\text{C}$

Hence, the correct answer is (A).

7. Between D and C, the resistance is

$R_1 = \frac{R}{8}$

and between E and C, the resistance is

$R_2 = \frac{7R}{8}$

$\Rightarrow \quad \frac{1}{R_{eq}} = \frac{8}{R} + \frac{8}{7R}$

$\Rightarrow \quad R_{eq} = \frac{7R}{64}$

Hence, the correct answer is (D).

8. $V = I_g(R_s + R_g)$

$\Rightarrow \quad V = 4 \times 10^{-3}[5050]$

$\Rightarrow \quad V \approx 20 \text{ V}$

Hence, the correct answer is (B).

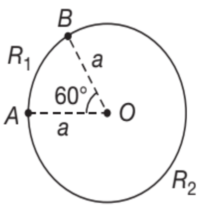
9. $R_0 = \rho \frac{\ell}{A} = 3\ \Omega$

When wire is stretched to twice its original length, then

$R' = n^2 R_0$

$\Rightarrow \quad R' = (2)^2(3) = 12\ \Omega$

We have to find resistance between A and B.



$\angle AOB = 60° = \frac{\pi}{3}$

$\Rightarrow \quad R_1 = \left(\frac{12}{2\pi a}\right) a \left(\frac{\pi}{3}\right)$

$\Rightarrow \quad R_1 = 2\ \Omega$

and $R_2 = \left(\frac{12}{2\pi a}\right) a \left(\frac{5\pi}{3}\right)$

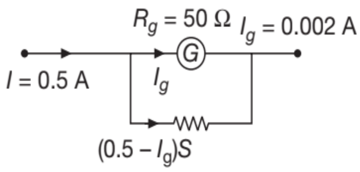
$\Rightarrow \quad R_2 = 10\ \Omega$

$\Rightarrow \quad \frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{10} = \frac{6}{10}$

$\Rightarrow \quad R_{eq} = \frac{5}{3}\ \Omega$

Hence, the correct answer is (A).

10.



Since, $I_g R_g = (0.5 - I_g)\ S$

$\Rightarrow \quad (0.002)(50) = (0.5 - I_g)\ S$

$\Rightarrow \quad S \approx \frac{0.002 \times 50}{0.5} = 0.2\ \Omega$

Hence, the correct answer is (A).

11. Since $\rho = \frac{m}{ne^2\tau}$

$\Rightarrow \quad \rho = \frac{9.1 \times 10^{-31}}{8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 25 \times 10^{-15}}\ \Omega \text{ m}$

$\Rightarrow \quad \rho = \frac{9.1}{8.5 \times 2.56 \times 25} \times 10^{(-59+53)}$

$\Rightarrow \quad \rho = 1.67 \times 10^{-8}\ \Omega \text{ m}$

Hence, the correct answer is (D).

12. $\ell n R(T) = a - \frac{a}{b} - \frac{1}{T^2}$

$a,\ b$ are constant

$\Rightarrow \quad R(T) = R_0 e^{\frac{-T_0^2}{T^2}}$

Hence, the correct answer is (B).

13. $I_g = 10^{-4}$ A, $R = 2 \times 10^6\ \Omega$

and $V = 5$ volt

Since $V = I_g(R + G)$

$\Rightarrow \quad 5 = 10^{-4}(2\times10^{6}+G)$

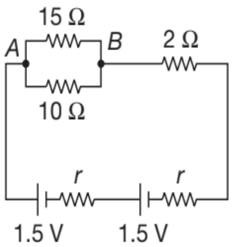
$\Rightarrow \quad G =$ NEGATIVE

Which is NOT POSSIBLE.

Data Contradictory.

*No given option is correct.

14. For the given circuit, it is given that



$V_{AB} = 2\ V$

$\Rightarrow \quad I = \frac{2}{15}+\frac{2}{10}=\frac{1}{3}$ A

Also $I(2r+2) = 1.5+1.5-V_{AB}$

$\Rightarrow \quad 2r+2=(3-2)3$

$\Rightarrow \quad r=\frac{1}{2}\ \Omega$

Hence, the correct answer is (A).

15. $\frac{R}{X}=\frac{\ell}{100-\ell}$

$\Rightarrow \quad X=\frac{R(100-\ell)}{\ell}$

For 1, $X=\frac{100\times40}{60}=\frac{2000}{3}\ \Omega=667\ \Omega$

For 2, $X=\frac{100\times87}{13}=\frac{8700}{13}\ \Omega=669\ \Omega$

For 3, $X=\frac{10\times98.5}{1.5}=\frac{1970}{3}\ \Omega=656\ \Omega$

For 4, $X=\frac{1\times99}{1}=99\ \Omega$

Clearly, we can see that the value of X calculated in CASE-4 is inconsistent with the other values.

Hence, the correct answer is (D).

16. Mobility is $\mu=\frac{v_d}{E}$

and resistivity $\rho=\frac{E}{j}=\frac{EA}{i}$

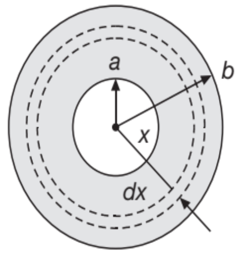
$\Rightarrow \quad \mu=\frac{v_dA}{i\rho}$

$\Rightarrow \quad \mu=\frac{1.1\times10^{-3}\times\pi\times(5\times10^{-3})^2}{5\times1.7\times10^{-8}}$

$\Rightarrow \quad \mu=1.0\ m^2V^{-1}s^{-1}$

Hence, the correct answer is (D).

17.



Resistance of infinitesimal element is given by

$dR=\frac{\rho dx}{4\pi x^2}$

$\Rightarrow \quad R=\int dR$

$\Rightarrow \quad R=\left(\frac{\rho}{4\pi}\right)\int_a^b\frac{dx}{x^2}$

$\Rightarrow \quad R=\left(\frac{\rho}{4\pi}\right)\left(\frac{1}{a}-\frac{1}{b}\right)$

Hence, the correct answer is (A).

18. $R_{eq}=2R+R+4R+R=8R$

Since, $P=\frac{E^2}{R_{eq}}$

$\Rightarrow \quad \frac{16\times16}{8R}=4$ watt

$\Rightarrow \quad R=8\ \Omega$

Hence, the correct answer is (A).

19. **For R_1,**

Since $I_g=10^{-3}$ Amp

$\Rightarrow \quad 10^{-3}(R_1+100)=2$ V

$\Rightarrow \quad R_1=1900\ \Omega$

For R_2,

$10^{-3}(R_1+R_2+100)=10$ V

$\Rightarrow \quad R_1+R_2+100=10000$

$\Rightarrow \quad R_2=8000\ \Omega$

For R_3,

$10^{-3}(R_1+R_2+R_3+100)=20$ V

$\Rightarrow \quad R_1+R_2+R_3+100=20\times1000$

$\Rightarrow \quad R_3=10000\ \Omega$

Hence, the correct answer is (A).

20. $V = \dfrac{ER}{R+r}$

For $R \to \infty$, $V = E = 1.5$ V

For $R = 0$, $I = \dfrac{E}{r} = 1$

$\Rightarrow \quad r = 1.5\ \Omega$

Hence, the correct answer is (A).

21. $V = I_g(R_V + G) = GI_0$ ...(1)

Also, $(I_0 - I_g)R_A = I_g G$ ...(2)

From (1), we get

$$R_V = \frac{G(I_0 - I_g)}{I_g}$$

From (2), we get

$$R_A = \frac{I_g G}{I_0 - I_g}$$

$\Rightarrow \quad R_A R_V = G^2$

$\Rightarrow \quad \dfrac{R_A}{R_V} = \left(\dfrac{I_g}{I_0 - I_g}\right)^2$

Hence, the correct answer is (A).

22. Since $A\ell = \text{Constant}$

and $R = \rho\dfrac{\ell}{A}$

$\Rightarrow \quad \dfrac{\Delta R}{R} = \dfrac{\Delta \ell}{\ell} + \dfrac{\Delta A}{A}$

$\Rightarrow \quad \dfrac{\Delta R}{R} = \dfrac{2\Delta \ell}{\ell} = 2 \times 0.5\%$

$\Rightarrow \quad \dfrac{\Delta R}{R}\% = 1\%$

Hence, the correct answer is (D).

23.

$$\begin{matrix} R & V & O & S \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 7 & 3 & \pm 10\% \end{matrix}$$

$\Rightarrow \quad R = 27 \times 10^3 \pm 10\%$

Hence, the correct answer is (C).

24. Applying Kirchhoff's Loop Law, we get

$$20 - 2i_1 - 2(i_1 + i_2) = 0$$

$\Rightarrow \quad 20 - 2i_1 = 10 - 4i_2$ ...(1)

Also, $10 - 4i_1 - 2(i_1 + i_2) = 0$

$\Rightarrow \quad \dfrac{10 - 2i_1}{4} = i_2$ ...(2)

From (1) and (2), we get

$$20 = 2i_1 + 2i_1 + 2i_2$$

$\Rightarrow \quad 20 = 4i_1 + 5 - i_1$

$\Rightarrow \quad 3i_1 = 15$

$\Rightarrow \quad i_1 = 5$ and $i_2 = 0$

$\Rightarrow \quad i = i_1 + i_2 = 5$ A

Hence, the correct answer is (B).

25. $v_d = \dfrac{I}{neA}$

$\Rightarrow \quad v_d = \dfrac{1.5}{(9 \times 10^{28})(1.6 \times 10^{-19})(5 \times 10^{-6})}$

$\Rightarrow \quad v_d = 2.08 \times 10^{-5}\ \text{ms}^{-1}$

$\Rightarrow \quad v_d = 0.02\ \text{mms}^{-1}$

Hence, the correct answer is (A).

26. G O Y Golden

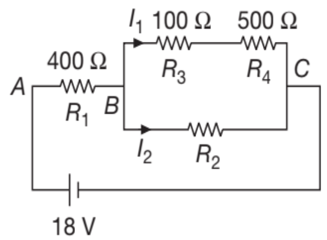
Since $G = 5$, $O = 3$, $Y = 4$ and Golden $= 5\%$

$\Rightarrow \quad R = 53 \times 10^4 \pm 5\%$

$\Rightarrow \quad R = (530\ \text{k}\Omega \pm 5\%)$

Hence, the correct answer is (D).

27.



Since voltmeter is ideal, so

$$I_1 = \frac{V}{R_4} = \frac{5}{500} = 0.01\ \text{A}$$

$\Rightarrow \quad V_B - V_C = 600 I_1 = 6$ V

Since $V_A - V_C = 18$ V

$\Rightarrow \quad (V_A - V_B) + (V_B - V_C) = 18$ V

$\Rightarrow \quad V_A - V_B = 12$ V

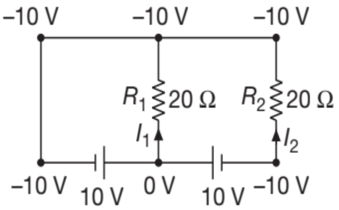
So, $I_1 + I_2 = \dfrac{12}{400} = 0.03$ A

$\Rightarrow \quad I_2 = 0.03 - 0.01 = 0.02$ A

$\Rightarrow \quad R_2 = \dfrac{6}{0.02} = 300\ \Omega$

Hence, the correct answer is (D).

28.



The potentials of various points is shown in figure after assigning 0 V potential to the junction. So,

$$I_1 = \frac{0-(-10)}{20} = \frac{1}{2}\ A = 0.5\ \text{A} \text{ and}$$

$$I_2 = \frac{-10-(-10)}{20} = 0\ \text{A}$$

Hence, the correct answer is (C).

29. G B R Br

Resistance $= 50 \times 10^2 \pm 1\%$

$P = I^2 R$

$\Rightarrow\ 2 = (5000) I^2$

$\Rightarrow\ \frac{4}{10000} = I^2$

$\Rightarrow\ I = \frac{2}{100}$

$\Rightarrow\ I = 20$ mA

Hence, the correct answer is (A).

30. $R_{AB} = \frac{12 \times 6}{12+6} = 4\ \Omega$

Hence, the correct answer is (A).

31. For no deflection, $I = \frac{\varepsilon}{r+12r} = \frac{\varepsilon}{13r}$

$\Rightarrow\ V_{AB} = IR_{AB} = \left(\frac{\varepsilon}{13r}\right)12r = \frac{12\varepsilon}{13r}$

So, $\frac{12\varepsilon}{13}$ potential is developed across L

1 unit potential is developed across $\frac{L}{\left(\frac{12\varepsilon}{13}\right)} = \frac{13L}{12\varepsilon}$

So, potential of $\frac{\varepsilon}{2}$ is developed

across a length given by

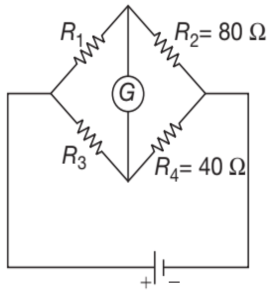
$\ell_{AJ} = \left(\frac{13L}{12\varepsilon}\right)\frac{\varepsilon}{2}$

$\Rightarrow\ \ell_{AJ} = \frac{13L}{24}$

Hence, the correct answer is (D).

32. From colour code table, we get

$R_1(\text{O, R, B}) = 320\ \Omega$



Since $R_2 = 80\ \Omega$

$R_4 = 40\ \Omega$

For a Balanced Wheatstone Bridge, we have

$\frac{R_1}{R_2} = \frac{R_3}{R_4}$

$\Rightarrow\ R_3 = \frac{320 \times 40}{80}$

$\Rightarrow\ R_3 = 160\ \Omega = 16 \times 10^1\ \Omega$

So, colour for R_3 is Brown, Blue, Brown

Hence, the correct answer is (D).

33. $R' = \frac{RR_V}{R+R_V}$

Since, $R' = \frac{95}{100} \times 30 = 28.5\ \Omega$

$\Rightarrow\ 28.5 = \frac{30R_V}{30+R_V}$

$\Rightarrow\ 855 + 28.5R_V = 30R_V$

$\Rightarrow\ R_V = \frac{855}{1.5}$

$\Rightarrow\ R_V = 570\ \Omega$

Hence, the correct answer is (A).

34. $I^2R = 4.4$ watt

$\Rightarrow\ R = \frac{4.4}{4 \times 10^{-6}}\ \Omega$

Since $P_{(\text{dissipated})}$ at voltage V is 11 volt

$\Rightarrow\ P = \frac{V^2}{R} = \frac{11 \times 11 \times 4 \times 10^{-6}}{4.4}$

$\Rightarrow\ P = \frac{11 \times 11 \times 4 \times 10^{-6} \times 10}{44}$

$\Rightarrow\ P = 1.1 \times 10^{-4}$ watt

$\Rightarrow\ P = 11 \times 10^{-5}$ watt

Hence, the correct answer is (A).

35. When in series, $P_S = \dfrac{P_1 P_2}{P_1 + P_2} = 60\text{ W}$

Since $P_1 = P_2 = P$

$\Rightarrow \quad P_S = \dfrac{P}{2}$

$\Rightarrow \quad P = 120\text{ watt}$

When in parallel,

$P_P = 2P = 2\times 120 = 240\text{ W}$

Hence, the correct answer is (D).

36. $\dfrac{P}{R} = \dfrac{Q}{X}$

$\Rightarrow \quad \dfrac{P}{400} = \dfrac{Q}{X}$ ...(1)

On interchanging P and Q, we have

$\dfrac{Q}{405} = \dfrac{P}{X}$

$\Rightarrow \quad P = \dfrac{QX}{405}$

Substitute in (1), we get

$\dfrac{QX}{400\times 405} = \dfrac{Q}{X}$

$\Rightarrow \quad X = \sqrt{400\times 405}$

$\Rightarrow \quad X = 402.5\ \Omega$

Hence, the correct answer is (C).

37. At null point, $E = V_{AJ} = IR_{AJ}$

where $I = \left(\dfrac{6}{R_{AB} + R_H}\right) R_{AJ}$

Since AB is uniform wire, so if λ is resistance per unit length of AB, then $R_{AJ} = 4\lambda$

$\Rightarrow \quad E_1 = 0.5 = \left(\dfrac{6}{4+2}\right)(4\lambda)$

Also, $E_2 = \left(\dfrac{6}{4+6}\right)(4\lambda)$

$\Rightarrow \quad \dfrac{E_2}{0.5} = \dfrac{6}{10}$

$\Rightarrow \quad E_2 = 0.3\text{ V}$

Hence, the correct answer is (C).

38. Full scale deflection current is

$I_g = 30\times 0.005 = 0.15\text{ A}$

Since $\gamma = I_g(G+R)$

$\Rightarrow \quad 15 = 0.15(20+R)$

$\Rightarrow \quad R = 80\ \Omega$

Hence, the correct answer is (C).

39. Since $\dfrac{1}{r_{eq}} = 3$

$\Rightarrow \quad r_{eq} = \dfrac{1}{3}\ \Omega$

Also, $\dfrac{E_{eq}}{r_{eq}} = \dfrac{E_1}{r_1} + \dfrac{E_2}{r_2} + \dfrac{E_3}{r_3}$

$\Rightarrow \quad \dfrac{E_{eq}\times 3}{1} = \dfrac{1}{1} + \dfrac{2}{1} + \dfrac{3}{1} = 6$

$\Rightarrow \quad E_{eq} = 2\text{ V}$

Hence, the correct answer is (C).

40. For Condition of Balance, we have

$\dfrac{R_1}{R_2} = \dfrac{40}{60} = \dfrac{2}{3}$ ...(1)

Now, $\dfrac{R_1 + 10}{R_2} = 1$ ...(2)

From (1) and (2), we get

$\dfrac{R_1}{R_1 + 10} = \dfrac{2}{3}$

$\Rightarrow \quad R_1 = 20\ \Omega$ and $R_2 = 30\ \Omega$

Let resistance to be connected in parallel to

$R_1 + 10$ i.e., $30\ \Omega$ be R,

then for balance point to shift to the initial situation, we have

$\dfrac{\dfrac{30R}{30+R}}{30} = \dfrac{40}{60}$

$\Rightarrow \quad \dfrac{30\times R}{30+R} = 20$

$\Rightarrow \quad 30R = 600 + 20R$

$\Rightarrow \quad R = 60\ \Omega$

Hence, the correct answer is (A).

41. Since deflection θ is proportional to i, so

$\theta \propto i$

Let $R_G = R$

$\Rightarrow \quad i_1 = \dfrac{V}{220+R} = k\theta_0$ ...(1)

$i_2 = \dfrac{V}{220 + \dfrac{5\times R}{5+R}} \times \dfrac{5}{R+5} = k\times\dfrac{\theta_0}{5}$ ...(2)

From (1) and (2), we get

$$\frac{25}{220\times(5+R)+5R}=\frac{1}{(220+R)}$$

$$\Rightarrow \quad \frac{1}{45R+220}=\frac{1}{5\times(220+R)}$$

$$\Rightarrow \quad R = 22\ \Omega$$

Hence, the correct answer is (A).

42. Since $\frac{dR}{d\ell}=\frac{K}{\sqrt{\ell}}$

$$\Rightarrow \quad \int_0^R dR = K\int_0^\ell \frac{d\ell}{\sqrt{\ell}}$$

$$\Rightarrow \quad R = 2K\sqrt{\ell}$$

$$\Rightarrow \quad \frac{R}{R'}=\frac{2K\sqrt{\ell}}{2K\left(1-\sqrt{\ell}\right)}=\frac{\sqrt{\ell}}{1-\sqrt{\ell}}$$

Since $R = R'$

$$\Rightarrow \quad \frac{\sqrt{\ell}}{1-\sqrt{\ell}}=1$$

$$\Rightarrow \quad 2\sqrt{\ell}=1$$

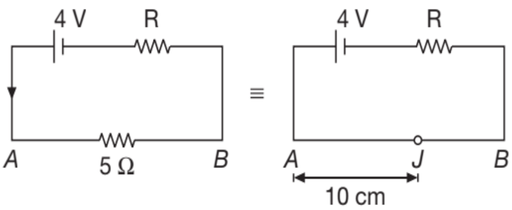
$$\Rightarrow \quad \ell=\frac{1}{4}=0.25\ \text{m}$$

Hence, the correct answer is (C).

43. $V_{AJ}=5\times10^{-3}$

$$R_{AJ}=\frac{5}{1}\times\frac{10}{100}=0.5\ \Omega$$

$$\Rightarrow \quad i=\frac{V_{AJ}}{R_{AJ}}=10^{-2}\ \text{A}$$



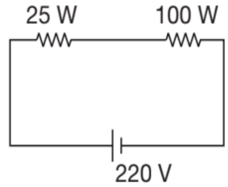
Since, $i=\frac{4}{R+5}=10^{-2}$

$$\Rightarrow \quad R+5=400\ \Omega$$

$$\Rightarrow \quad R=395\ \Omega$$

Hence, the correct answer is (D).

44. $\frac{1}{P}=\frac{1}{25}+\frac{1}{100}$



$$\Rightarrow \quad P = 20\ \text{W}$$

Since Power $\propto R$

$$\Rightarrow \quad P_1=\frac{PR_1}{R_1+R_2}=16\ \text{W}$$

$$\Rightarrow \quad P_2 = 4\ \text{W}$$

Hence, the correct answer is (D).

45. $I_g = 25\times4\times10^{-4}$

$$\Rightarrow \quad I_g=10^{-2}\ \text{A}$$

Since $V = I_g(R+50)$

$$\Rightarrow \quad R = 200\ \Omega$$

Hence, the correct answer is (C).

46. At Junction S, we have

$$I_4 = I_3 + I_5$$

$$\Rightarrow \quad I_4 = I_3 + 0.4$$

$$\Rightarrow \quad 0.8 - 0.4 = I_3$$

$$\Rightarrow \quad I_3 = 0.4\ \text{A}$$

At Junction R, we have

$$I_1 + I_2 = I_4$$

$$\Rightarrow \quad -0.3 + I_2 = 0.8$$

$$\Rightarrow \quad I_2 = 1.1\ \text{A}$$

At Junction Q, we have

$$I_3 + I_6 = I_1 + I_2$$

$$\Rightarrow \quad 0.4 + I_6 = -0.3 + 1.1$$

$$\Rightarrow \quad I_6 = 0.4\ \text{A}$$

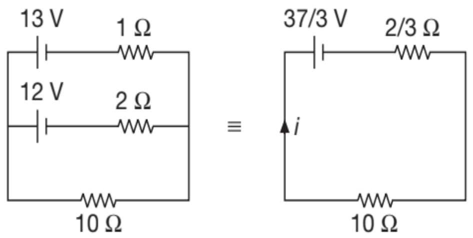
Hence, the correct answer is (A).

47. Equivalent e.m.f. of parallel batteries

$$E=\frac{\frac{E_1}{r_1}+\frac{E_2}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}}=\frac{\frac{12}{1}+\frac{13}{2}}{\frac{1}{1}+\frac{1}{2}}=\frac{37}{3}\ \text{V}$$

CHAPTER 3

Equivalent resistance of parallel batteries,



$$r_{eq} = \frac{2 \times 1}{2+1} = \frac{2}{3}\ \Omega$$

Now, its equivalent circuit is as drawn.

Current in the circuit, $i = \dfrac{\frac{37}{3}}{10 + \left(\frac{2}{3}\right)} = \dfrac{37}{32}$

Voltage across the load,

$$V_{10\,\Omega} = i \times 10 = \frac{37}{32} \times 10 = \frac{370}{32} = 11.56\ \text{V}$$

Hence, the correct answer is (B).

48. Let R_1 (left slot) and R_2 (right slot) be two resistances in two slots of a meter bridge.
Initially ℓ be the balancing length

Then, $\dfrac{R_1}{R_2} = \dfrac{\ell}{(100-\ell)}$ ...(1)

Also, $R_1 + R_2 = 1000\ \Omega$ ...(2)

On interchanging the resistances, balancing length becomes $(\ell - 10)$, so

$$\frac{R_2}{R_1} = \frac{\ell - 10}{100 - (\ell - 10)} = \frac{\ell - 10}{110 - \ell}$$

$\Rightarrow \dfrac{100-\ell}{\ell} = \dfrac{\ell - 10}{110 - \ell}$ (From equation (1))

$\Rightarrow 11000 + \ell^2 - 210\ell = \ell^2 - 10\ell$

$\Rightarrow 200\ell = 11000$

$\Rightarrow \ell = 55$ cm

From equation (1), we get

$$\frac{R_1}{R_2} = \frac{55}{45}$$

$\Rightarrow R_1 = \dfrac{55}{45} R_2$

$\Rightarrow R_1 = \dfrac{55}{45}(1000 - R_1)$ (From equation (2))

$\Rightarrow R_1 + \dfrac{55}{45} R_1 = 1000 \times \dfrac{55}{45}$

$\Rightarrow 100 R_1 = 1000 \times 55$

$\Rightarrow R_1 = 550\ \Omega$

Hence, the correct answer is (C).

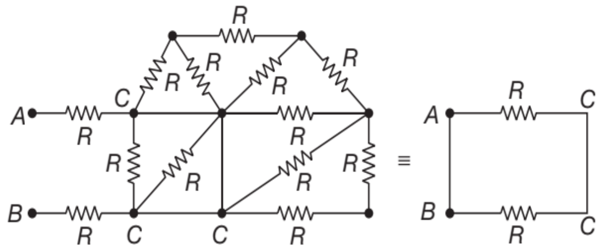
49. For a balanced meter bridge

$$Y(39.5) = X(100 - 39.5)$$

$\Rightarrow X = \dfrac{12.5 \times 39.5}{60.5} = 8.16\ \Omega$

When X and Y are interchanged so ℓ_1 and $(100 - \ell_1)$ will also interchange and hence $\ell_2 = 60.5$ cm.
Hence, the correct answer is (B).

50.



$$R_{AB} = R + R = 2R$$

Hence, the correct answer is (A).

51. Rate of heat developed, $P = \dfrac{V^2}{R}$

For a given V, we have

$$P \propto \frac{1}{R} = \frac{A}{\rho\ell} = \frac{\pi r^2}{\rho\ell}$$

$\Rightarrow \dfrac{P_1}{P_2} = \left(\dfrac{r_1^2}{r_2^2}\right)\left(\dfrac{\ell_2}{\ell_1}\right)$

Since it is given that, $\ell_2 = \dfrac{\ell_1}{2}$ and $r_2 = 2r_1$

$\Rightarrow \dfrac{P_1}{P_2} = \dfrac{1}{4} \times \dfrac{1}{2} = \dfrac{1}{8}$

$\Rightarrow P_2 = 8P_1$

Hence, the correct answer is (A).

52. Current flowing through copper rod is given by

$I = neAv_d = \rho A v_d$ $\{\because \rho = ne\}$

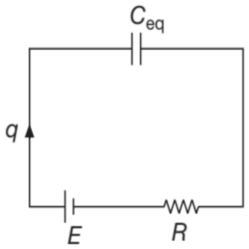
$\Rightarrow v_d = \dfrac{I}{\rho A}$

Time taken by charges to travel distanced d is

$$t = \frac{d}{v_d} = \frac{d}{\left(\frac{I}{\rho A}\right)} = \frac{\rho A d}{I}$$

Hence, the correct answer is (A).

53. Equivalent circuit is shown in figure. Charging of capacitor is given by



$$q = q_{max}\left(1 - e^{-\frac{t}{RC_{eq}}}\right), \text{ where } q_{max} = C_{eq}E$$

$$\Rightarrow \quad q = C_{eq}E\left(1 - e^{-\frac{t}{RC_{eq}}}\right)$$

Both capacitors will have same charge as they are connected in series.

Hence, the correct answer is (A).

54. Resistance of element at $500\ °C$ is

$$R_T = \frac{\text{Voltage applied}}{\text{Current}} = \frac{220}{2} = 110\ \Omega$$

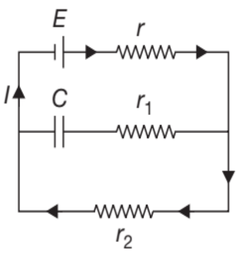
Since $R_T = R_0(1 + \alpha\Delta T)$

$$\Rightarrow \quad 110 = 100(1 + \alpha \times 500)$$

$$\Rightarrow \quad \alpha = \frac{10}{100 \times 500} = 2 \times 10^{-4}\ °C^{-1}$$

Hence, the correct answer is (B).

55. In steady state, flow of current through capacitor will be zero.



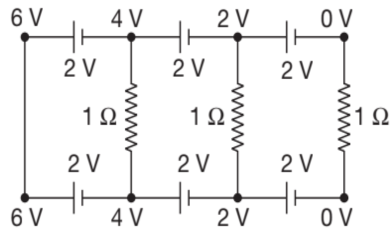
$$I = \frac{E}{r + r_2}$$

$$V_C = Ir_2 = \frac{Er_2}{r + r_2}$$

$$\Rightarrow \quad q_C = CV_C = CE\frac{r_2}{r + r_2}$$

Hence, the correct answer is (C).

56. Using Nodal Analysis method, we observe that the potential difference across each resistor is zero.



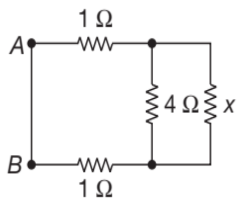
So, no current will flow through each resistor.

Hence, the correct answer is (D).

57. In a balanced Wheatstone bridge, the null point remains unchanged even if cell and galvanometer are interchanged.

Hence, the correct answer is (B).

58. Let equivalent resistance of the infinite network be x. Then, equivalent resistance between points A and B is also x.



$$\Rightarrow \quad x = \frac{4x}{4 + x} + 2$$

$$\Rightarrow \quad x^2 - 2x - 8 = 0$$

$$x = \frac{2 \pm \sqrt{4 - 4(1)(-8)}}{2} = \frac{2 \pm \sqrt{36}}{2}$$

$$\Rightarrow \quad x = \frac{2 \pm 6}{2} = 4\ \Omega$$

(Since negative value of resistance is not accepted)

$$\Rightarrow \quad I_1 = \frac{9}{4 + 0.5} = 2\ \text{A}$$

So, reading of A_1 is $2\ \text{A}$.

Hence, the correct answer is (B).

59. When key is plugged between 2 and 1, then

$$V_1 = iR_1 = k\ell_1 \quad \ldots(1)$$

where k is the potential gradient of wire PQ

When key is plugged between 3 and 1, then

$$V_2 = i(R_1 + R_2) = k\ell_2 \quad \ldots(2)$$

On dividing equation (2) by equation (1), we get

$$\frac{R_1}{R_1 + R_2} = \frac{\ell_1}{\ell_2}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{\ell_1}{\ell_2 - \ell_1}$$

Hence, the correct answer is (B).

60. For $P = 4\ \Omega$, $\ell_1 = 60$ cm

$$\Rightarrow \frac{P}{Q} = \frac{\ell_1}{100 - \ell_1} = \frac{60}{40} = \frac{3}{2}$$

$$\Rightarrow Q = \frac{2}{3}P = \frac{8}{3}\ \Omega$$

Now, $P' = P + R$ and $\ell_1' = 80$ cm

$$\Rightarrow \frac{P'}{Q} = \frac{\ell_1'}{100 - \ell_1'} = \frac{80}{20} = 4$$

$$\Rightarrow \frac{P + R}{Q} = 4$$

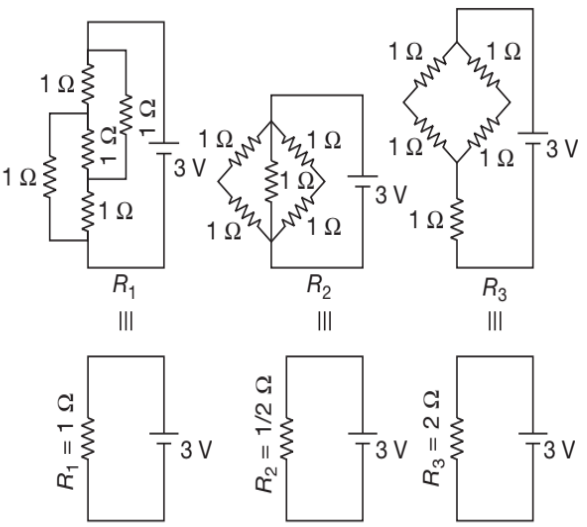
$$\Rightarrow \frac{4 + R}{\frac{8}{3}} = 4$$

$$\Rightarrow 4 + R = \frac{32}{3}$$

$$\Rightarrow R = \frac{32}{3} - 4 = \frac{20}{3}\ \Omega$$

Hence, the correct answer is (A).

61. The given three circuits are equivalent to the following three simpler circuits.



So, $P_1 = \frac{3^2}{1} = 9$ W

$$P_2 = \frac{3^2}{\frac{1}{2}} = 18 \text{ W and}$$

$$P_3 = \frac{3^2}{2} = 4.5 \text{ W}$$

$$\Rightarrow P_2 > P_1 > P_3$$

Hence, the correct answer is (B).

62. Resistance of a wire of length ℓ and radius r is given by

$$R = \frac{\rho\ell}{A} = \frac{\rho\ell}{A} \times \frac{A}{A} = \frac{\rho V}{A^2} = \frac{\rho V}{\pi^2 r^4} \qquad \{\because V = A\ell\}$$

$$\Rightarrow R \propto \frac{1}{r^4}$$

$$\Rightarrow \frac{R_1}{R_2} = \left(\frac{r_2}{r_1}\right)^4$$

According to the problem, we have

$$R_1 = 100\ \Omega,\ r_1 = r,\ r_2 = \frac{r}{2},\ R_2 = ?$$

$$\Rightarrow R_2 = R_1\left(\frac{r_1}{r_2}\right)^4 = 16R_1 = 1600\ \Omega$$

Hence, the correct answer is (D).

63. Cu is conductor, so with increase in temperature, resistance will increase.
Si is semiconductor, so with increase in temperature resistance will decrease.
Hence, the correct answer is (D).

64. Let the source voltage be V. Then, equivalent resistance of the circuit when $r = fR$ is

$$R_{eq} = R + \frac{r \times R}{r + R} = R + \frac{fR}{f + 1} = \frac{(2f + 1)R}{(f + 1)}$$

$\Rightarrow$ Current in the circuit is

$$I = \frac{V}{R_{eq}} = \frac{V(f + 1)}{R(2f + 1)}$$

Current through the resistance $r(= fR)$ is

$$I_2 = \frac{I}{f + 1} = \frac{V}{R(2f + 1)}$$

So, heat generated per unit time in the resistor r is

$$H = I_2^2 r = \frac{V^2 f}{R(2f + 1)^2}$$

For maximum H, we have $\frac{dH}{df} = 0$

$$\Rightarrow \frac{V^2}{R}\left(\frac{1}{(2f + 1)^2} - \frac{4f}{(2f + 1)^3}\right) = 0$$

$$\Rightarrow 2f + 1 - 4f = 0$$

$$\Rightarrow f = \frac{1}{2}$$

Hence, the correct answer is (A).

65. Here, $R(T) = R_0\left[1 + \alpha(T - T_0)\right]$

At $T_0 = 300\text{ K},\ R_0 = 100\ \Omega$ and

at $T = 500\text{ K},\ R = 120\ \Omega$

$\Rightarrow\ 120 = 100(1 + \alpha(200))$

$\Rightarrow\ 200\alpha = \frac{6}{5} - 1 = \frac{1}{5}$

$\Rightarrow\ \alpha = 10^{-3}\ ^\circ\text{C}^{-1}$

Temperature of the toaster is raised at constant rate from 300 K to 500 K is 30 s.

So, increment in the temperature in time t is

$$\Delta T = \frac{(500 - 300)}{30}t$$

$$\Delta T = \frac{20}{3}t$$

Total work done during this time in raising the temperature is

$$W = \int_0^t \frac{V^2 dt}{R(t)} = \int_0^t \frac{V^2 dt}{R_0(1 + \alpha\Delta T)}$$

$$\Rightarrow\ W = \int_0^{30} \frac{(200)^2 dt}{100\left(1 + 10^{-3} \times \frac{20}{3}t\right)} = 400\int_0^{30} \frac{dt}{\left(1 + \frac{t}{150}\right)}$$

$$\Rightarrow\ W = 400 \times 150\left[\ell n\left(1 + \frac{t}{150}\right)\right]_0^{30}$$

$$\Rightarrow\ W = 60000\left[\ell n\left(1 + \frac{30}{150}\right) - \ell n 1\right] = 60000\,\ell n\left(\frac{6}{5}\right)\text{ J}$$

$$\Rightarrow\ W = 60\,\ell n\left(\frac{6}{5}\right)\text{ kJ}$$

Hence, the correct answer is (C).

66. $v_d = 2.5 \times 10^{-4}\ \text{ms}^{-1},\ n = 8 \times 10^{28}\ \text{m}^{-3}$

Since, $I = neAv_d$

$$\Rightarrow\ \frac{V}{R} = neAv_d$$

$$\Rightarrow\ \frac{V}{\left(\frac{\rho\ell}{A}\right)} = neAv_d$$

$$\Rightarrow\ \rho = \frac{V}{nev_d\ell}$$

Substituting values, we get

$$\rho = 1.6 \times 10^{-5}\ \Omega\text{m}$$

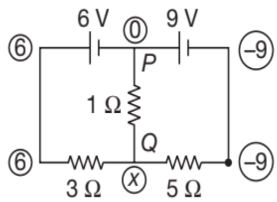
Hence, the correct answer is (D).

67. Let us redraw the circuit and assign potential at P to be zero and that at Q to be x. Then at the junction Q, $\Sigma I_Q = 0$

$$\Rightarrow\ \frac{x - 6}{3} + \frac{x - 0}{1} + \frac{x + 9}{5} = 0$$

$$\Rightarrow\ x\left(\frac{5 + 15 + 3}{15}\right) = 2 - \frac{9}{5}$$

$$\Rightarrow\ x = \frac{3}{23}\text{ V}$$



$$\Rightarrow\ I = \frac{x - 0}{1} = \frac{3}{23}\text{ A}$$

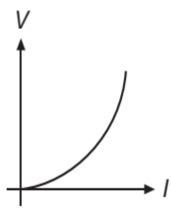
$\Rightarrow\ I = 0.13$ A from Q to P

Hence, the correct answer is (C).

68. Given, $v_d \propto \sqrt{E}$

Since, $I = neAv_d$

$\Rightarrow\ I \propto \sqrt{E}$ ...(1)



Since $E = \frac{V}{\ell}$

$\Rightarrow\ E \propto V$

So from (1), we get

$$I \propto \sqrt{V}$$

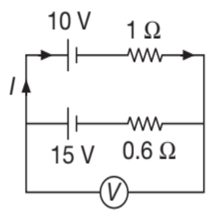
$\Rightarrow\ I^2 \propto V$

Hence, the correct answer is (C).

69. Current in the circuit, $I = \frac{5}{1.6}$

$$\Rightarrow\ I = \frac{50}{16} = \frac{25}{8}\text{ A}$$

Reading of the voltmeter is

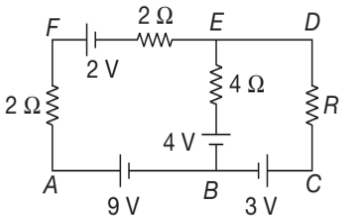


$$V = 15 - \frac{25}{8} \times 0.6$$

$$\Rightarrow \quad V = 15 - \frac{15}{8} = 13.1 \text{ V}$$

Hence, the correct answer is (C).

70.



Current in $4\ \Omega$ is zero.

Start from C and go to D (via B and E), we get

$$V_C - 3 + 4 - V_D = 0$$

$$\Rightarrow \quad V_C - V_D = -1 \text{ V} \qquad \ldots(1)$$

Starting from A and going to C (via B), we get

$$V_A - 9 + 3 - V_C = 0$$

$$\Rightarrow \quad V_A - V_C = 6 \text{ V} \qquad \ldots(2)$$

Add (1) and (2), we get

$$V_A - V_D = 5 \text{ V}$$

Hence, the correct answer is (C).

71. Total power P is given by

$$P = (15)(40) + (5)(100) + (5)(80) + (1)(1000)$$

$$\Rightarrow \quad P = 600 + 500 + 400 + 1000 = 2500\ W$$

Since, $P = VI$

$$\Rightarrow \quad I = \frac{2500}{220} A$$

$$\Rightarrow \quad I = \frac{125}{11} A = 11.3 \text{ A}$$

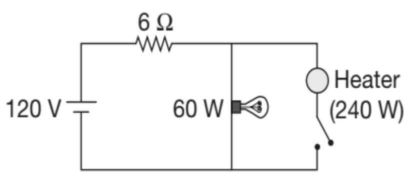
So, minimum capacity of main fuse of building must be 12 A.

Hence, the correct answer is (C).

72. Assuming that both bulb and the heater have a rating of 120 V, then

$$R_{\text{bulb}} = \frac{120 \times 120}{60} = 240\ \Omega$$

and $R_{\text{heater}} = 60\ \Omega$



Initial current is $i = \dfrac{120}{240 + 6} = \dfrac{120}{246}$

New current is $i' = \dfrac{120}{48 + 6} \times \dfrac{60}{60 + 240} = \dfrac{120}{54} \times \dfrac{1}{5} = \dfrac{24}{54}$

Decrease in voltage $= 240 \times \left(\dfrac{120}{246} - \dfrac{24}{54} \right) = 10.4$ V

So the nearest approximate answer is (D)

Hence, the correct answer is (D).

73. During discharging of a capacitor, we have $V = V_0 e^{-t/\tau}$ where $\tau (= RC)$ is the time constant of RC circuit. At $t = \tau$, we have $V = \dfrac{V_0}{e} = 0.37\ \text{V}_0$

From the graph, $t = 0$, $V_0 = 25$ V

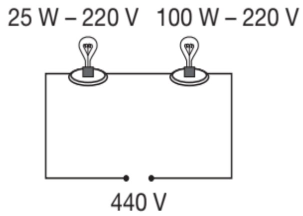
$$\Rightarrow \quad V = 0.37 \times 25$$

$$\Rightarrow \quad \text{V} = 9.25 \text{ V}$$

This voltage occurs at time that lies between 100 sec and 500 sec. Hence, time constant τ of this circuit lies between 100 sec and 150 sec.

Hence, the correct answer is (C).

74.



Since $R = \dfrac{(\text{Rated voltage})^2}{\text{Rated power}}$

So, the resistance of 25 W-220 V bulb is

$$R_1 = \frac{(220)^2}{25}\ \Omega$$

and resistance of 100 W-200 V bulb is

$$R_2 = \frac{(220)^2}{100}\ \Omega$$

When these two bulbs are connected in series, the total resistance is

$$R_S = R_1 + R_2 = (220)^2 \left(\frac{1}{25} + \frac{1}{100} \right) = \frac{(220)^2}{20}\ \Omega$$

So, the current, $I = \dfrac{440}{\dfrac{(220)^2}{20}} = \dfrac{2}{11}$ A

Potential difference across 25 W bulb is

$$V_1 = IR_1 = \frac{2}{11} \times \frac{(220)^2}{25} = 352 \text{ V}$$

Potential difference across 100 W bulb is

$$V_2 = IR_2 = \frac{2}{11} \times \frac{(220)^2}{100} = 88 \text{ V}$$

Thus the bulb 25 W will be fused, because it can tolerate only 220 V while the voltage across it is 352 V.

Hence, the correct answer is (B).

75. Resistance of wire $R = \frac{\rho \ell}{A}$ ...(1)

On stretching, volume (V) remains constant.

So $V = A\ell$

$$\Rightarrow \quad A = \frac{V}{\ell}$$

$$\Rightarrow \quad R = \frac{\rho \ell^2}{V} \qquad \text{\{From (1)\}}$$

$$\Rightarrow \quad R \propto \ell^2$$

$$\Rightarrow \quad \frac{\Delta R}{R} = \frac{2\Delta \ell}{\ell} \qquad \{\because V \text{ and } \rho \text{ are constants}\}$$

Hence, when wire is stretched by 0.1% its resistance will increase by 0.2%.

Hence, the correct answer is (B).

76. Since, by definition $\Delta R = R\alpha \Delta T$

For series combination, we have

$$R_s = R_1 + R_2$$

$$\Rightarrow \quad \Delta R_s = \Delta R_1 + \Delta R_2$$

$$\Rightarrow \quad 2R_0 \alpha_s \Delta T = R_0 \alpha_1 \Delta T + R_0 \alpha_2 \Delta T$$

$$\Rightarrow \quad \alpha_s = \frac{\alpha_1 + \alpha_2}{2}$$

For parallel combination, we have

$$R_P = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1 R_2}{R_s}$$

$$\frac{\Delta R_P}{R_p} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} - \frac{\Delta R_s}{R_s}$$

$$\Rightarrow \quad \alpha_P \Delta T = \alpha_1 \Delta T + \alpha_2 \Delta T - \alpha_s \Delta T$$

$$\Rightarrow \quad \alpha_P = \alpha_1 + \alpha_2 - \alpha_s$$

$$\Rightarrow \quad \alpha_P = \alpha_1 + \alpha_2 - \left(\frac{\alpha_1 + \alpha_2}{2}\right)$$

$$\Rightarrow \quad \alpha_P = \frac{\alpha_1 + \alpha_2}{2}$$

Hence, the correct answer is (A).

77. From the statement given, $\alpha = 2.5 \times 10^{-3} {}^\circ C^{-1}$.

The resistance of a wire change from 100 Ω to 150 Ω when the temperature is increased from 27 °C to 227 °C.

Hence, the correct answer is (A).

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Single Correct Choice Type Problems

1. For infinite line charge, we have

$$E = \frac{\lambda}{2\pi \varepsilon r}$$

Also, we know that

$$j = \sigma E$$

$$\Rightarrow \quad j = \sigma \left(\frac{\lambda}{2\pi \varepsilon r}\right)$$

But $i = jA = j(2\pi r \ell)$

$$\Rightarrow \quad \frac{dq}{dt} = j(2\pi r \ell) = \frac{\sigma \lambda}{2\pi \varepsilon r}(2\pi r \ell)$$

$$\Rightarrow \quad i = -\frac{\sigma \lambda \ell}{\varepsilon}$$

(Negative sign because i is flowing radially outwards).

$$\Rightarrow \quad \frac{dq}{dt} = -\frac{\sigma \lambda \ell}{\varepsilon}$$

$$\Rightarrow \quad \frac{d(\lambda \ell)}{dt} = -\frac{\sigma \lambda \ell}{\varepsilon}$$

$$\Rightarrow \quad \int_{\lambda_0}^{\lambda} \frac{d\lambda}{\lambda} = -\frac{\sigma}{\varepsilon} \int_0^t dt$$

$$\log_e \lambda \Big|_{\lambda_0}^{\lambda} = -\left(\frac{\sigma}{\varepsilon}\right) t$$

$$\Rightarrow \quad \lambda = \lambda_0 e^{-\left(\frac{\sigma}{\varepsilon} t\right)}$$

Since, $j = \frac{\sigma \lambda}{2\pi \varepsilon r}$

$$\Rightarrow \quad j = \left(\frac{\lambda_0 \sigma}{2\pi \varepsilon r}\right) e^{-\left(\frac{\sigma}{\varepsilon} t\right)}$$

Hence, the correct answer is (C).

2. $$\frac{1}{R} = \frac{1}{R_{Al}} + \frac{1}{R_{Fe}} = \left(\frac{A_{Al}}{\rho_{Al}} + \frac{A_{Fe}}{\rho_{Fe}}\right)\frac{1}{\ell}$$

$$= \left[\frac{(7^2 - 2^2)}{27} + \frac{2^2}{10}\right] \frac{10^{-6}}{10^{-8}} \times \frac{1}{50 \times 10^{-3}}$$

Solving we get,

$$R = \frac{1875}{64} \times 10^{-6}\ \Omega$$

$$\Rightarrow \quad R = \frac{1875}{64}\ \mu\Omega$$

Hence, the correct answer is (B).

3. For balanced meter bridge

$$\frac{X}{R} = \frac{\ell}{(100-\ell)} \qquad \{\text{where, } R = 90\ \Omega\}$$

$$\Rightarrow \quad \frac{X}{90} = \frac{40}{100-40}$$

$$\Rightarrow \quad X = 60\ \Omega$$

$$\Rightarrow \quad X = R\frac{\ell}{(100-\ell)}$$

$$\Rightarrow \quad \frac{\Delta X}{X} = \frac{\Delta \ell}{\ell} + \frac{\Delta \ell}{100-\ell}$$

$$\Rightarrow \quad \frac{\Delta X}{X} = \frac{0.1}{40} + \frac{0.1}{60}$$

$$\Rightarrow \quad \Delta X = 0.25$$

So, $X = (60 \pm 0.25)\ \Omega$

Hence, the correct answer is (C).

4. Using the concept of balanced Wheat stone bridge, we have

$$\frac{P}{Q} = \frac{R}{S}$$

$$\Rightarrow \quad \frac{X}{(52+1)} = \frac{10}{(48+2)}$$

$$\Rightarrow \quad X = \frac{10 \times 53}{50} = 10.6\ \Omega$$

Hence, the correct answer is (B).

5. We will require a voltmeter, an ammeter, a test resistor and a variable battery to verify Ohm's law.
Voltmeter which is made by connecting a high resistance with a galvanometer is connected in parallel with the test resistor.
Further, an ammeter which is formed by connecting a low resistance in parallel with galvanometer is required to measure the current through test resistor.
Hence, the correct answer is (C).

6. $R = \frac{\rho \ell}{A}$

$$\Rightarrow \quad R = \frac{\rho L}{tL} = \frac{\rho}{t}$$

Independent of L
Hence, the correct answer is (C).

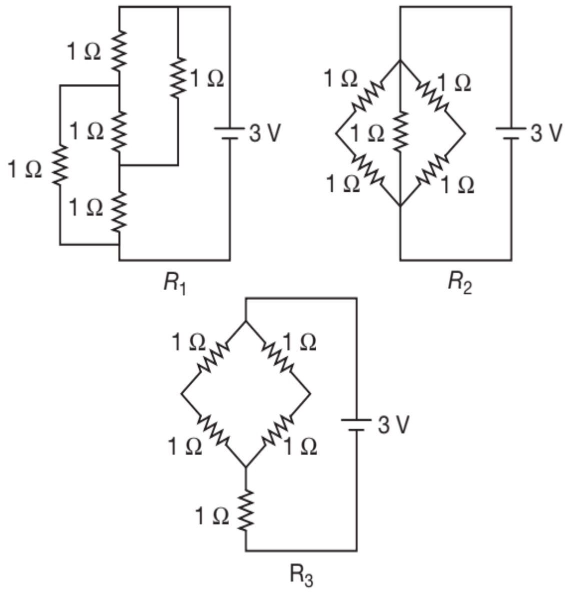
7. $R = \frac{V^2}{P}$

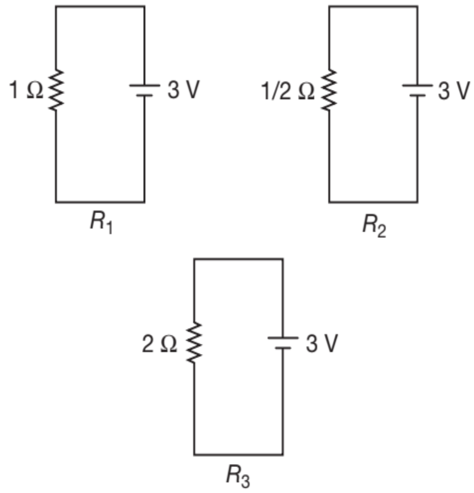
$$\Rightarrow \quad R \propto \frac{1}{P}$$

$$\Rightarrow \quad \frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$$

Hence, the correct answer is (D).

8. The given three circuits R_1, R_2 and R_3 are equivalent to the following three circuits.





$$P_1 = \frac{3^2}{1} = 9\ \text{W}$$

$$P_2 = \frac{3^2}{\frac{1}{2}} = 18\ \text{W}$$

$$P_3 = \frac{3^2}{2} = 4.5\ \text{W}$$

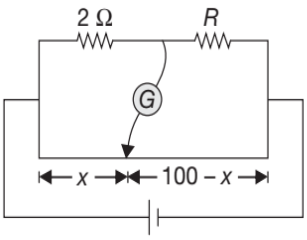
$P_2 > P_1 > P_3$

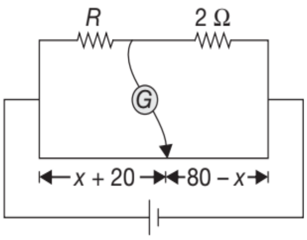
Hence, the correct answer is (C).

9. $R > 2\,\Omega$

$\Rightarrow \quad 100 - x > x$

Applying $\frac{P}{Q} = \frac{R}{S}$





We have $\frac{2}{R} = \frac{x}{100 - x}$ …(1)

$\frac{R}{2} = \frac{x + 20}{80 - x}$ …(2)

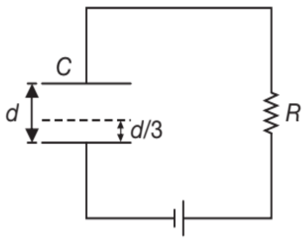
Solving equations (1) and (2), we get

$R = 3\,\Omega$

Hence, the correct answer is (A).

10. Time constant $= \tau = RC$, where

$$C = \frac{C_1C_2}{C_1 + C_2} = \frac{\left(\frac{A\varepsilon_0}{d - x}\right)\left(\frac{KA\varepsilon_0}{x}\right)}{\frac{A\varepsilon_0}{d - x} + \frac{A\varepsilon_0}{x}}$$



$\Rightarrow \quad C = \frac{KA\varepsilon_0}{x + K(d - x)}$ where $x = \frac{d}{3} - Vt$

$\Rightarrow \quad \tau = \frac{RKA\varepsilon_0}{\frac{d}{3} - Vt + K\left(d - \frac{d}{3} + Vt\right)}$ where $A = 1\text{ m}^2$ and

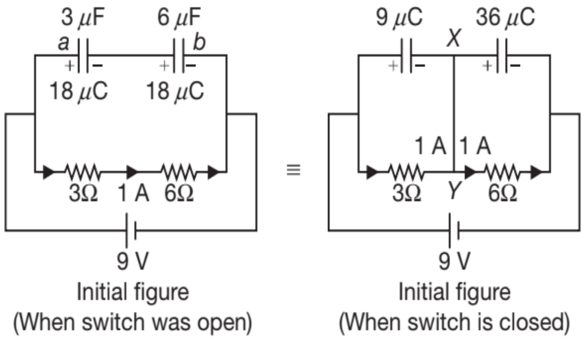
$K = 2$

$\Rightarrow \quad \tau = \frac{3(2R\varepsilon_0)}{d - 3Vt + 6d - 2d + 6Vt} = \frac{6R\varepsilon_0}{5d + 3Vt}$

Hence, the correct answer is (A).

11. From Y to X charge flows to plates a and b

$(q_a + q_b)_i = 0$, $(q_a + q_b)_f = 27\ \mu C$



$\therefore$ $27\ \mu C$ charge flows from Y to X

Hence, the correct answer is (C).

12. Current flowing through both the bars is equal. Now, the heat produced is given by

$H = I^2Rt$

$\Rightarrow \quad H \propto R$

$\Rightarrow \quad \frac{H_{AB}}{H_{BC}} = \frac{R_{AB}}{R_{BC}} = \frac{(1/2r)^2}{(1/r)^2} = \frac{1}{4} \quad \left\{\text{as } R \propto \frac{1}{A} \propto \frac{1}{r^2}\right\}$

$\Rightarrow \quad H_{BC} = 4H_{AB}$

Hence, the correct answer is (A).

13. $\tau = CR$

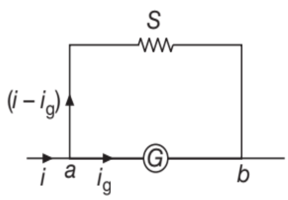
$\tau_1 = (C_1 + C_2)(R_1 + R_2) = 18\ \mu s$

$\tau_2 = \left(\frac{C_1C_2}{C_1 + C_2}\right)\left(\frac{R_1R_2}{R_1 + R_2}\right) = \frac{8}{6} \times \frac{2}{3} = \frac{8}{9}\ \mu s$

$\tau_3 = (C_1 + C_2)\left(\frac{R_1R_2}{R_1 + R_2}\right) = (6)\left(\frac{2}{3}\right) = 4\ \mu s$

Hence, the correct answer is (B).

14.



$V_{ab} = i_g.G = (i - i_g)S$

$\therefore \quad i = \left(1 + \frac{G}{S}\right)i_g$

CHAPTER 3

Substituting the values, we get

$$i = 100.1 \text{ mA}$$

Hence, the correct answer is (A).

15. $W = 0$. Therefore, from First Law of Thermodynamics,

$$\Delta U = \Delta Q = I^2Rt = (1)^2(100)(5\times 60)\text{ J} = 30 \text{ kJ}$$

Hence, the correct answer is (D).

16. Current in the respective loop will remain confined in the loop itself.
Therefore, current through $2\ \Omega$ resistance $= 0$

Hence, the correct answer is (C).

17. Given: $V_C = 3V_R = 3(V - V_C)$

Here, V is the applied potential.

$$\Rightarrow \quad V_C = \frac{3}{4}V$$

$$\Rightarrow \quad V\left(1 - e^{-t/\tau_C}\right) = \frac{3}{4}V$$

$$\Rightarrow \quad e^{-t/\tau_C} = \frac{1}{4} \qquad \ldots(1)$$

Here, $\tau_C = cR = 10$ sec

Substituting this value of τ_C in equation (1) and solving for t we get

$$t = 13.86 \text{ sec}$$

Hence, the correct answer is (A).

18. $R_{PQ} = \frac{5}{11}r$, $R_{QR} = \frac{4}{11}r$ and $R_{PR} = \frac{3}{11}r$

$\Rightarrow$ R_{PQ} is maximum.

Hence, the correct answer is (A).

19. BC, CD and BA are known resistance.
The unknown resistance is connected between A and D.
Hence, the correct answer is (C).

20. Charging current, $I = \frac{E}{R}e^{-\frac{t}{RC}}$

Taking log both sides, $\log I = \log\left(\frac{E}{R}\right) - \frac{t}{RC}$

When R is doubled, slope of curve increases. Also at $t = 0$, the current will be less. Graph Q represents the best.
Hence, the correct answer is (B).

21. Ammeter is always connected in series and voltmeter in parallel.
Hence, the correct answer is (A).

22. The ratio $\frac{AC}{CB}$ will remain unchanged.

Hence, the correct answer is (A).

23. $P = I^2R$

Current is same, so $P \propto R$.

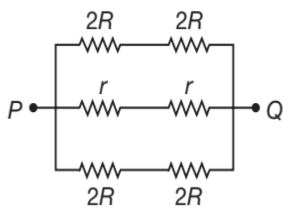
In the first case it is $3r$, in second case it is $\frac{2}{3}r$, in third case it is $\frac{r}{3}$ and in fourth case the net resistance is $\frac{3r}{2}$.

$$R_{III} < R_{II} < R_{IV} < R_I$$

$$\Rightarrow \quad P_{III} < P_{II} < P_{IV} < P_I$$

Hence, the correct answer is (A).

24.



Hence, the correct answer is (A).

25. $P = \frac{V^2}{R}$ so, $R = \frac{V^2}{P}$

$$\Rightarrow \quad R_1 = \frac{V^2}{100} \text{ and } R_2 = R_3 = \frac{V^2}{60}$$

Now, $W_1 = \frac{(250)^2}{(R_1 + R_2)^2}R_1$, $W_2 = \frac{(250)^2}{(R_1 + R_2)^2}R_2$

and $W_3 = \frac{(250)^2}{R_3}$

$$W_1 : W_2 : W_3 = 15 : 25 : 64$$

$$\Rightarrow \quad W_1 < W_2 < W_3$$

Hence, the correct answer is (D).

26. Current I can be independent of R_6 only when R_1, R_2, R_3, R_4 and R_6 from a balanced Wheatstone bridge.

Therefore, $\frac{R_1}{R_2} = \frac{R_3}{R_4}$ or $R_1R_4 = R_2R_3$.

Hence, the correct answer is (C).

27. $H = \frac{V^2}{R}t$

In the first case (wire of length ℓ)

$$mc\,\Delta T = \frac{(3E)^2}{R}t \qquad \ldots(1)$$

In the second case (wire of length 2ℓ)

$$(2m)c\Delta T = \frac{(nE)^2}{2R}t \qquad \ldots(2)$$

{Because doubling the length will simply double the mass and the resistance of the wire} so, from (1) and (2) we get,

$$n^2 = 36$$

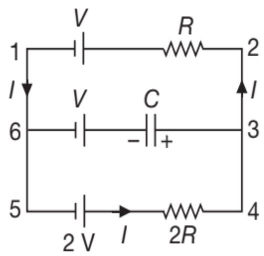
$$\Rightarrow \quad n = 6$$

Hence, the correct answer is (B).

28. In steady state the branch containing the capacitor can be omitted and hence current in the circuit is

$$I = \frac{2V - V}{R + 2R}$$

$$\Rightarrow \quad I = \frac{V}{3R}$$



For loop 36543

$$-V_C - V + 2V - I(2R) = 0$$

$$\Rightarrow \quad V_C = -V + 2V - \frac{V}{3R}(2R)$$

$$\Rightarrow \quad V_C = V - \frac{2V}{3} = \frac{V}{3}$$

Hence, the correct answer is (C).

29. Reading of Galvanometer remains same whether S is opened or closed i.e. no current must be flowing through this branch. Hence P and Q are in series and R and G are in series too.

So, $I_R = I_G$

Hence, the correct answer is (A).

30. $\Rightarrow \quad I = \frac{9}{9} = 1$ A

At A a current of 1 A divides into 0.5 A and 0.5 A.

At B the current of 0.5 A divides into 0.25 A and 0.25 A

Hence, the correct answer is (D).

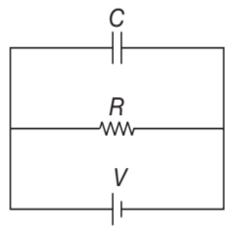
31. $i = neAv_d$

Drift speed, $v_d = \frac{1}{neA} \propto \frac{1}{A}$

Therefore, for non-uniform cross-section (different values of A) drift speed will be different at different sections. Only current (or rate of flow of charge) will be same.

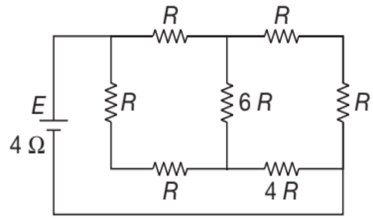
Hence, the correct answer is (D).

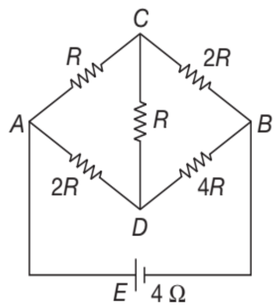
32. Since, the capacitor plates are directly connected to the battery, it will take no time in charging.



Hence, the correct answer is (D).

33. The given circuit is that of a Wheatstone bridge.

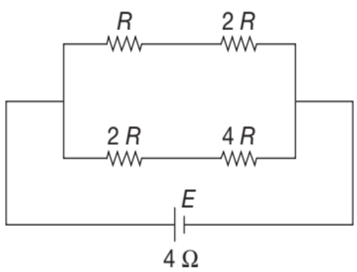


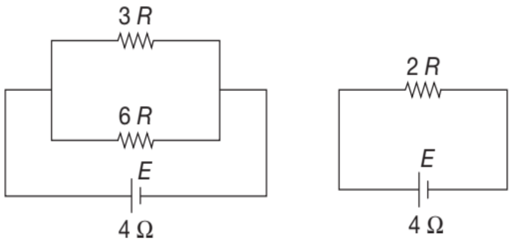


The circuit is a balanced one since,

$$\frac{\text{Resistance across } AC}{\text{resistance across } AD} = \frac{\text{resistance across } CB}{\text{resistance across } BD}$$

Thus, no current will flow across $6R$ of the side CD. The given circuit will now be equivalent to





For maximum power, net external resistance = Total internal resistance.

$\Rightarrow \quad 2R = 4$

$\Rightarrow \quad R = 2\,\Omega$

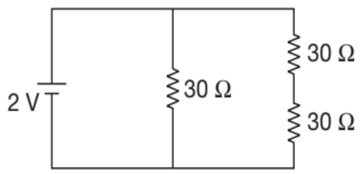
Hence, the correct answer is (B).

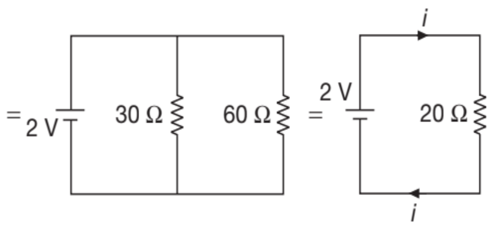
34. Resistivity of conductors increases with increase in temperature because rate of collisions between free electrons and ions increase with increase of temperature. However, the resistivity of semiconductors decreases with increase in temperature, because more and more covalent bonds are broken at higher temperatures.

Hence, the correct answer is (C).

35. Cooling increases the resistance of a semiconductor and decreases the resistance of a conductor.
Hence, the correct answer is (D).

36. The simplified circuit is shown in the figure.

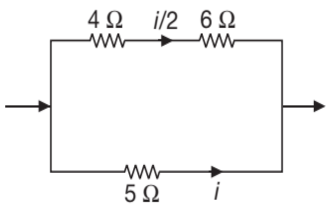




Therefore, current $i = \frac{2}{20} = \frac{1}{10}\,A$

Hence, the correct answer is (C).

37. Since, resistance in upper branch of the circuit is twice the resistance in lower branch. Hence, current there will be half.



Now, $P_4 = \left(\frac{i}{2}\right)^2 (4) \qquad \left(P = i^2 R\right)$

$P_5 = (i)^2 (5)$

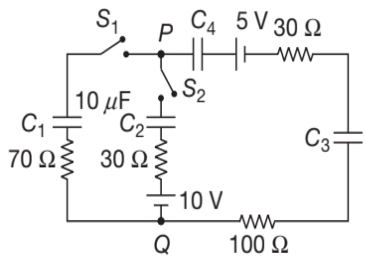
$\Rightarrow \quad \frac{P_4}{P_5} = \frac{1}{5}$

$\Rightarrow \quad P_4 = \frac{P_5}{5} = \frac{10}{5} = 2\text{ cals}^{-1}$

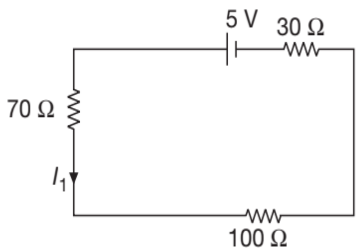
Hence, the correct answer is (B).

Multiple Correct Choice Type Problems

1.



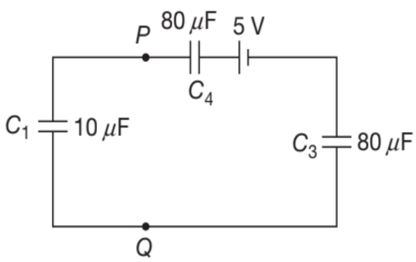
When switch S_1 is closed, so the equivalent circuit at $t = 0$ is shown in figure.



$\Rightarrow \quad I_1 = \frac{5}{200} = 25\text{ mA}$

At steady state, for S_1 closed, we have

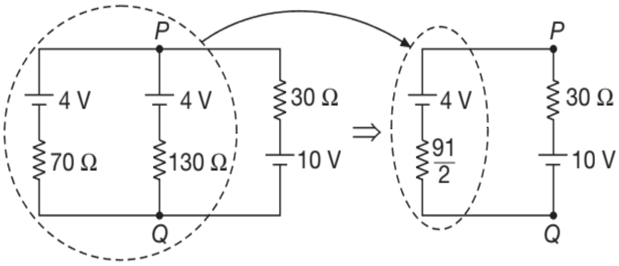
$C_{eq} = 8\,\mu\text{F}$ and $Q = 40\,\mu\text{C}$



So, $V(C_1) = 4$ volt and $V(C_3) = V(C_4) = 0.5$ volt

At steady state potential difference between P and Q is 4 volt.

Now when switch S_2 is closed, then equivalent circuit is shown in figure.



$$\Rightarrow \quad I_{PQ} = \frac{6 \times 2}{151} = 0.08 \text{ A}$$

Hence, (A) and (C) are correct.

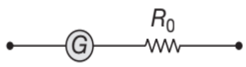
2. Maximum coil current is

$I_c = 2 \times 10^{-6}$ A and coil resistance is

$R_c = 10\ \Omega$

So, maximum potential difference across coil is

$$V_{\max} = 2 \times 10^{-6} \times 10 = 2 \times 10^{-5} \text{ volt}$$

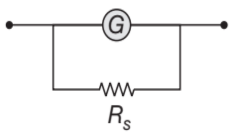


When converted into voltmeter of range 100 mV, then we have

$$(10 + R_0) \times 2 \times 10^{-6} = 100 \times 10^{-3}$$

$$\Rightarrow \quad R_0 = 49990\ \Omega$$

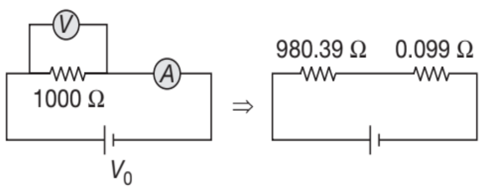
When converted into an ammeter of range 1 mA, then we have



$$R_s \times 9.98 \times 10^{-4} = 2 \times 10^{-5}\ \Omega$$

$$\Rightarrow \quad R_s = 0.02\ \Omega$$

Now, Ohm's Law setup is shown in the figure.



If R_{eq} is the equivalent resistance of the circuit, then

$$R_{eq} = 980.48\ \Omega$$

$$\Rightarrow \quad I = \frac{V_0}{980.48}$$

If V' be the reading of the voltmeter, then

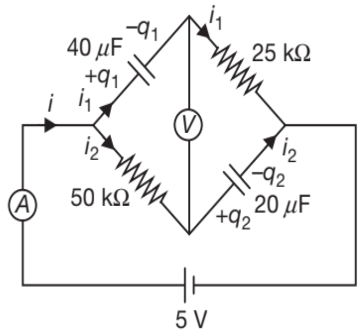
$$V' = \frac{V_0 \times 50000}{980.48 \times 51}$$

So, the measured value of resistance is

$$R_{(\text{measured})} = \frac{V'}{I} = \frac{50000}{51} = 980.4\ \Omega$$

Hence, (A) and (B) are correct.

3.



Just after pressing key,

$$5 - 25000 i_1 = 0$$

$$5 - 50000 i_2 = 0$$

{As charge on both capacitors is zero}

$$\Rightarrow \quad i_1 = 0.2 \text{ mA and } i_2 = 0.1 \text{ mA}$$

Also, $V_B + 25000 i_1 = V_A$

$$\Rightarrow \quad V_B - V_A = -5V$$

After a long time, steady state is achieved, so i_1 and $i_2 = 0$

$$\Rightarrow \quad 5 - \frac{q_1}{40} = 0$$

$$\Rightarrow \quad q_1 = 200\ \mu\text{C}$$

and $5 - \frac{q_2}{20} = 0$

$$\Rightarrow \quad q_2 = 100\ \mu\text{C}$$

So, $V_B - \frac{q_2}{20} = V_A$

$$\Rightarrow \quad V_B - V_A = +5 \text{ V}$$

Hence, (A) is correct

For capacitor 1, $q_1 = 200\left(1 - e^{-t/1}\right)\ \mu\text{C}$

$$\Rightarrow \quad i_1 = \frac{dq_1}{dt} = \frac{1}{5} e^{-t/1} \text{ mA}$$

For capacitor 2, $q_2 = 100\left[1 - e^{-t/1}\right]\ \mu\text{C}$

$$\Rightarrow \quad i_2 = \frac{dq_2}{dt} = \frac{1}{10} e^{-t/1} \text{ mA}$$

$$V_B - \frac{q_2}{20} + i_1 \times 25 - V_A = 0$$

CHAPTER 3

$\Rightarrow \quad V_B - V_A = 5(1-e^{-t}) - 5e^{-t} = 5(1-2e^{-t})$

At $t = \ln 2$, $V_B - V_A = 5(1-1) = 0 \qquad \left\{\because e^{-\ln 2} = -\frac{1}{2}\right\}$

So, (B) is correct

At $t = 1$, $i = i_1 + i_2 = \frac{1}{5}e^{-1} + \frac{1}{10}e^{-1} = \left(\frac{3}{10}\right)\frac{1}{e}$

At $t = 0$, $i = i_1 + i_2 = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$

So, (C) is correct

After a long time, $q_1 = q_2 =$ constant

So, $i_1 = i_2 = 0$

Hence, (D) is correct.

Hence, (A),(B), (C) and (D) are correct.

4. When the filament breaks, the temperature of the filament will be high. Since according to Wein's Law, $\lambda_m \propto \frac{1}{T}$, so

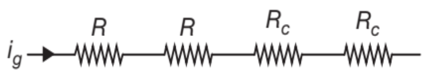
$$v_m \propto T \qquad \left\{\because v_m \propto \frac{1}{\lambda_m}\right\}$$

Hence the filament emits more light at higher band of frequencies.

Towards the end of life of the bulb, R (resistance) increases as temperature increases. Since $V =$ constant, so $P = \frac{V^2}{R}$. So, less power is consumed towards the end of life of bulb.

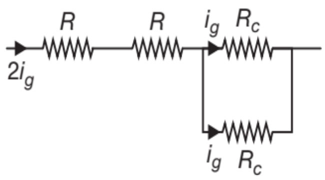
Hence, (C) and (D) are correct.

5. **For (A),**



$V_1 = 2i_g R + 2i_g R_C$

For (B),



$$V_2 = 4i_g R + i_g R_C = 2i_g R + 2i_g R_C + 2i_g R - i_g R_C$$

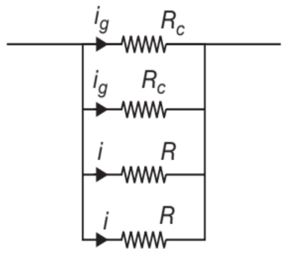
$$V_2 = V_1 + i_g(2R - R_C)$$

$$\Rightarrow \quad V_2 > V_1 \text{ as } \frac{R}{2} > R_C$$

Hence, $2R > R_C$

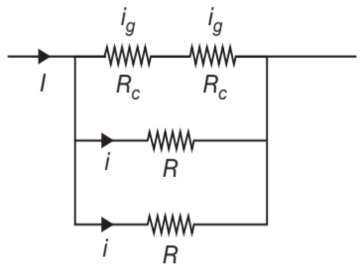
So, (B) is correct

For (C)



$I_1 = 2i_g + 2i = 2i_g + 2i_g \frac{R_C}{R}$

For (D)



$$I_2 = i_g + 2i = i_g + 2 \times \frac{2i_g R_C}{R}$$

$$\Rightarrow \quad I_2 = 2i_g + 2i_g \frac{R_C}{R} + 2i_g \frac{R_C}{R} - i_g$$

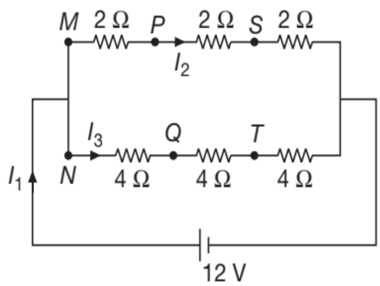
$$\Rightarrow \quad I_2 = I_1 + \frac{2i_g}{R}\left(R_C - \frac{R}{2}\right)$$

$$\Rightarrow \quad I_2 < I_1 \text{ as } R_C < \frac{R}{2}$$

So, (C) is correct

Hence, (B) and (C) are correct.

6. Due to symmetry on upper side and lower side, points P and Q are at same potentials. Similarly, points S and T are at same potentials. Therefore, the simple circuit can be drawn as shown below.



$$I_2 = \frac{12}{2+2+2} = 2 \text{ A}$$

$$\Rightarrow \quad I_3 = \frac{12}{4+4+4} = 1 \text{ A}$$

$$\Rightarrow \quad I_1 = I_2 + I_3 = 3 \text{ A}$$

$\Rightarrow \quad I_{PQ} = 0$

$\Rightarrow \quad V_P = V_Q$

Potential drop (from left to right) across each resistance is

$$\frac{12}{3} = 4 \text{ V}$$

$\Rightarrow \quad V_{MS} = 2 \times 4 = 8 \text{ V}$

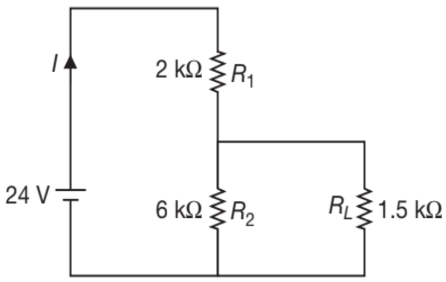
$\Rightarrow \quad V_{NQ} = 1 \times 4 = 4 \text{ V}$

$\Rightarrow \quad V_S < V_Q$

Hence, (A), (B), (C) and (D) are correct.

7. $R_{eq} = \left(\frac{6 \times 1.5}{7.5} + 2\right) = \frac{6}{5} + 2 = \frac{16}{5} \text{ k}\Omega$

$$I = \frac{V}{R_{eq}} = \frac{24}{16} \times 5 \text{ mA} = \frac{3}{2} \times 5 = 7.5 \text{ mA}$$



for potential difference across R_1

$\Rightarrow \quad V_1 = 7.5 \times 2 = 15 \text{ V}$

for potential difference across R_2

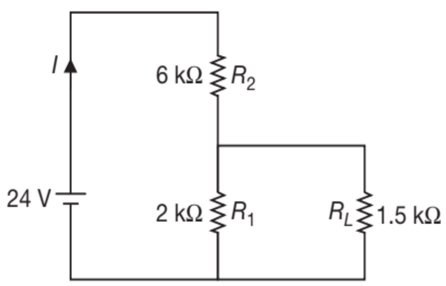
$\Rightarrow \quad V_2 = 24 - 15 = 9 \text{ V}$

for power

$$P_{R_1} : P_{R_2} = \frac{V_1^2}{R_1} : \frac{V_2^2}{R_2} = \frac{(15)^2}{2} : \frac{9^2}{6} = \frac{225}{2} \times \frac{6}{81} = \frac{25}{3}$$

$$P_{R_L} = \frac{V_2^2}{R_L} = \frac{9^2}{1.5} = 54 \text{ mW}$$

If R_1 and R_2 are interchanged



$$R' = R_1 \parallel R_2 = \left[\frac{(2)(1.5)}{2 + 1.5}\right] = \frac{3}{3.5}$$

$$V_L' = \frac{R'}{R_2 + R'} \times 24 \text{ V} = 3 \text{ V}$$

Now power dissipated in R_L is

$$P_L' = \frac{V_L'^2}{R_L} = \frac{3^2}{1.5} = 6 \text{ mW}$$

Hence, (A) and (D) are correct.

8. At 0 K, a semiconductor becomes a perfect insulator. Therefore, at 0 K, if some potential difference is applied across an insulator or semiconductor, current is zero. But a conductor will become a super conductors at 0 K. Therefore, current will be infinite. In reverse biasing at 300 *K* through a *p-n* junction diode, a small finite current flows due to minority charge carriers.
Hence, (A), (B) and (D) are correct.

9. To increase the range of ammeter a parallel resistance (called shunt) is required which is given by

$$S = \left(\frac{i_g}{i - i_g}\right) G$$

For option (C): $S = \left(\frac{50 \times 10^{-6}}{5 \times 10^{-3} - 50 \times 10^{-6}}\right)(100) \approx 1 \ \Omega$

To change it in voltmeter, a high resistance R is put in series, where R is given by

$$R = \frac{V}{i_g} - G$$

For option (B):

$$R = \frac{10}{50 \times 10^{-6}} - 100 \approx 200 \text{ k}\Omega$$

Hence, (B) and (C) are correct.

10. $q = q_0 e^{-\frac{t}{RC}}$

$$\Rightarrow \quad q = (CV_0) e^{-\frac{t}{RC}}$$

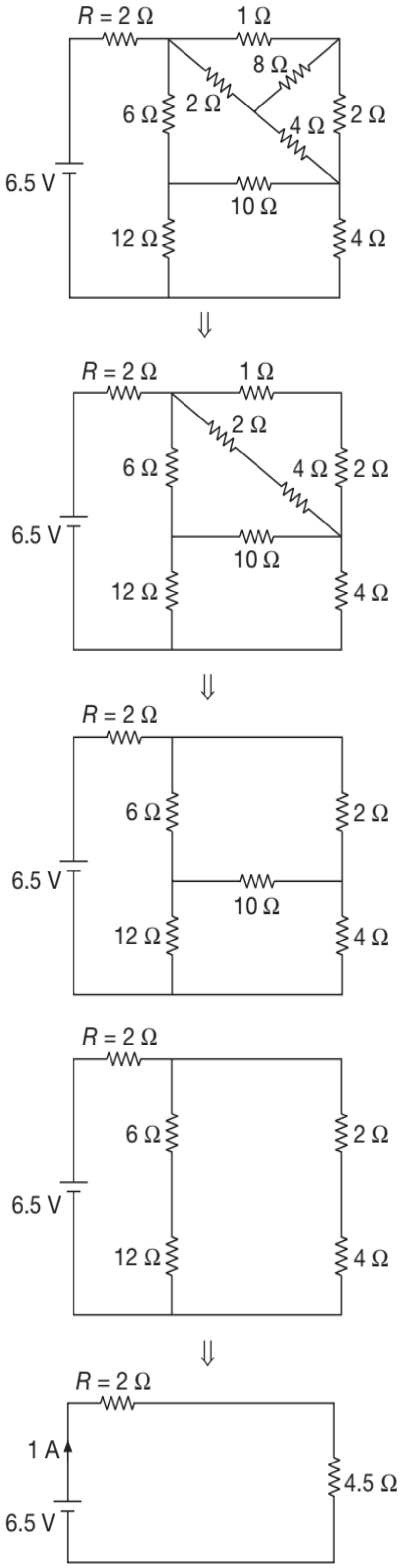
$$I = \frac{dq}{dt} = -\frac{CV_0}{RC} e^{-\frac{t}{RC}}$$

$$I(t = 0) = -\frac{V_0}{R}$$

which we observe is independent of the capacitance and only depends upon resistance.
Hence, the correct answer is (B).

Integer/Numerical Answer Type Questions

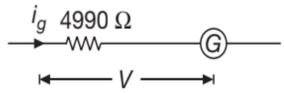
1.



2. V_{AB} = Equivalent emf of two batteries in parallel

$$= \frac{\frac{E_1}{r_1}+\frac{E_2}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}} = \frac{\frac{6}{1}+\frac{3}{2}}{\frac{1}{1}+\frac{1}{2}} = 5 \text{ V}$$

3.



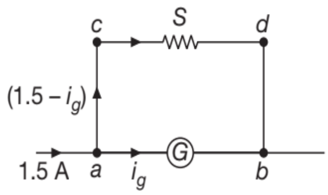
$$i_g(G+4990) = V$$

$$\Rightarrow \quad \frac{6}{1000}(G+4990) = 30$$

$$\Rightarrow \quad G+4990 = \frac{30000}{6} = 5000$$

$$\Rightarrow \quad G = 10\ \Omega$$

$$V_{ab} = V_{cd}$$



$$\Rightarrow \quad i_g G = (1.5 - i_g)S$$

$$\Rightarrow \quad \frac{6}{1000}\times 10 = \left(1.5 - \frac{6}{1000}\right)S$$

$$\Rightarrow \quad S = \frac{60}{1494} = \frac{2n}{249}$$

$$\Rightarrow \quad n = \frac{249\times 30}{1494}$$

$$\Rightarrow \quad n = \frac{2490}{498} = 5$$

4. In series, $i = \dfrac{2E}{2+R}$

$$\Rightarrow \quad J_1 = i^2 R = \left(\frac{2E}{2+R}\right)^2 \cdot R$$

In parallel, $i = \dfrac{E}{0.5+R}$

$$\Rightarrow \quad J_2 = i^2 R = \left(\frac{E}{0.5+R}\right)^2 \cdot R$$

Since, $\dfrac{J_1}{J_2} = 2.25$

$$\Rightarrow \quad \frac{J_1}{J_2} = \frac{4(0.5+R)^2}{(2+R)^2}$$

$$\Rightarrow \quad 1.5 = \frac{2(0.5+R)}{(2+R)}$$

Solving we get, $R = 4\ \Omega$

5. Voltage across the capacitors will increase from 0 to 10 V exponentially. The voltage at time t will be given by

$$V = 10\left(1 - e^{\frac{-t}{\tau}C}\right)$$

Here $\tau_c = C_{net}R_{net}$

$$= \left(1\times 10^{6}\right)\left(4\times 10^{-6}\right) = 4s$$

$$\therefore \quad V = 10\left(1 - e^{\frac{-t}{4}}\right)$$

Substituting $V = 4$ volt, we have

$$4 = 10\left(1 - e^{\frac{-t}{4}}\right) \text{ or } e^{\frac{-t}{4}} = 0.6 = \frac{3}{5}$$

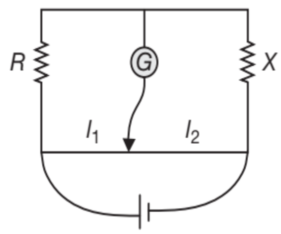
Taking log both sides we have,

$$-\frac{t}{4} = \text{In } 3 - \text{In } 5$$

or $t = 4(\text{In } 5 - \text{In } 3) = 2\,s$

Assertion and Reasoning Type Problems

1. $R\ell_2 = \ell_1 x$



To get null point at the same position, means ℓ_1 and ℓ_2 are still the same. As temperature increases, value of unknown resistance increases. To get the same null point, R must be increased. So Statement-1 is wrong. Statement-2 is True.

Hence, the correct answer is (D).