ICON

# CHAPTER 2: CAPACITANCE AND APPLICATIONS

## Test Your Concepts-I (Based on General Capacitance)

**1.** The common potential,

$$V=\frac{C_1V_1+C_2V_2}{C_1+C_2}$$

Here $C_1=20\ \mu F=20\times10^{-6}$ F, $V_1=500$ V

$C_2=10\ \mu F=10\times10^{-6}$ F, $V_2=200$ V

$$\Rightarrow\quad V=\frac{20\times10^{-6}\times500+10\times10^{6}\times200}{20\times10^{-6}+10\times10^{-6}}$$

$$\Rightarrow\quad V=400\text{ V}$$

**2.** The energy stored in the condenser is given by

$$U=\frac{1}{2}CV^2$$

where $C=100\ \mu F=100\times10^{-6}$ F, $V=200$ V

$$U=\frac{1}{2}\times100\times10^{-6}\times(200)^2=2\text{ J}$$

When discharged through a resistor $(2\ \Omega)$, the whole energy is dissipated as heat. So, heat produced is given by

$$Q=U=2\text{ J}$$

**3.** When the dial is at $0°$, the capacitance of the capacitor is given by

$$C_1=50\text{ pF}=50\times10^{-12}\text{ F}$$

When dial is at $180°$, the capacitance is given by

$$C_2=950\text{ pF}=950\times10^{-12}\text{ F}$$

The potential difference across capacitor $C_2$, is given by

$$V_2=400\text{ V}$$

Charge on capacitor $C_2$ is

$$q=C_2V_2=950\times10^{-12}\times400$$

$$\Rightarrow\quad q=380\times10^{-9}\text{ C}$$

(a) When battery is disconnected the charge remains the same and so, $q=$ constant. Let $V_1$ be the potential difference across capacitor when dial reads $0°$. Then

$$q=C_1V_1$$

$$\Rightarrow\quad 380\times10^{-9}=50\times10^{-12}\times V_1$$

$$\Rightarrow\quad V_1=\frac{380\times10^{-9}}{50\times10^{-12}}=7600\text{ V}$$

(b) Work required to turn the dial from $180°$ to $0°$ is
$W=$ Gain in energy of capacitor

$$\Rightarrow\quad W=\frac{q^2}{2C_1}-\frac{q^2}{2C_2}=\frac{q^2}{2}\left(\frac{1}{C_1}-\frac{1}{C_2}\right)=\frac{q^2(C_2-C_1)}{2C_1C_2}$$

$$\Rightarrow\quad W=\frac{\left(380\times10^{-9}\right)^2(950-50)\times10^{-12}}{2\times50\times10^{-12}\times950\times10^{-12}}$$

$$\Rightarrow\quad W=1.368\times10^{-3}\text{ J}$$

**4.** Initial energy stored in the capacitors

$$U_1=\frac{1}{2}C_1V_1^2+\frac{1}{2}C_2V_2^2$$

$$\Rightarrow\quad U_1=\frac{1}{2}\times0.1\times10^2+\frac{1}{2}C_2\times0=5\text{ J}$$

When charging battery is removed, the charge remains constant. This charge is collected by first capacitor $(q_1=C_1V_1)$ then redistributed equally in such a way that their potentials are equal i.e.,

$$V_1'=V_2'$$

$$\Rightarrow\quad \frac{q_1'}{C_1}=\frac{q_2'}{C_2}$$

Now since, $q_1'=q_2'$ {given}

$$\Rightarrow\quad C_1=C_2=C(\text{say})$$

$\therefore$ Common potential

$$V=\frac{q_1+q_2}{C_1+C_2}=\frac{CV_1+0}{C+C}=\frac{V_1}{2}=5\text{ V}$$

So, final energy stored is

$$U_2=\frac{1}{2}(C_1+C_2)V^2$$

$$\Rightarrow\quad U_2=\frac{1}{2}(2C)V^2=CV^2=0.1\times(5)^2=2.5\text{ J}$$

$$\Rightarrow\quad \frac{U_2}{U_1}=\frac{2.5}{5}=\frac{1}{2}$$

**5.** (a) Let the charge on sphere of radius $R_1$ be $q_1(=q)$ and that on the sphere of radius $R_2$ be $q_2(=Q-q)$, then the total energy is

$$U = U_1 + U_2 = \frac{1}{2}\frac{q_1^2}{C_1} + \frac{1}{2}\frac{q_2^2}{C_2}$$

$$\Rightarrow \quad U = \frac{q^2}{8\pi\varepsilon_0 R_1} + \frac{(Q-q)^2}{8\pi\varepsilon_0 R_2}$$

For $U$ to be minimum we must have

$$\frac{dU}{dq} = 0$$

$$\Rightarrow \quad \frac{2q}{8\pi\varepsilon_0 R_1} + \frac{2(Q-q)}{8\pi\varepsilon_0 R_2}(-1) = 0$$

$$\Rightarrow \quad \frac{q}{R_1} = \frac{Q-q}{R_2}$$

$$\Rightarrow \quad q = \left(\frac{R_1}{R_1+R_2}\right)Q = q_1$$

and $Q - q = \left(\frac{R_2}{R_1+R_2}\right)Q = q_2$

(b) $V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{R_1} = \frac{1}{4\pi\varepsilon_0}\frac{R_1 Q}{R_1(R_1+R_2)}$

$$\Rightarrow \quad V_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_1+R_2}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{R_2} = \frac{1}{4\pi\varepsilon_0}\frac{R_2 Q}{R_2(R_1+R_2)}$$

$$\Rightarrow \quad V_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_1+R_2}$$

$$\Rightarrow \quad V_1 - V_2 = 0$$

**6.** Let $V_1$ be the potential of $C_0$ after first charging, then

$$(C+C_0)V_1 = C_0 V_0$$

$$\Rightarrow \quad V_1 = \frac{C_0 V_0}{C+C_0}$$

Let $V_2$ be the potential of $C_0$ after second charging then

$$C_0 V_1 = (C+C_0)V_2$$

$$\Rightarrow \quad V_2 = \left(\frac{C_0}{C+C_0}\right)\left(\frac{C_0}{C+C_0}\right)V_0 = \left(\frac{C_0}{C+C_0}\right)^2 V_0$$

and so on

$$\Rightarrow \quad V_{10} = V = \left(\frac{C_0}{C+C_0}\right)^{10} V_0$$

$$\Rightarrow \quad C = \left[\left(\frac{V_0}{V}\right)^{\frac{1}{10}} - 1\right]C_0$$

## Test Your Concepts-II (Based on Series and Parallel Combination of Capacitors)

**1.** (a) $U = \frac{1}{2}C(\Delta V)^2 + \frac{1}{2}C(\Delta V)^2 = C(\Delta V)^2$

(b) The altered capacitor has capacitance $C' = \frac{C}{2}$. The total charge is the same as before.

So, $q_{\text{initial}} = q_{\text{final}}$

$$\Rightarrow \quad C(\Delta V) + C(\Delta V) = C(\Delta V') + \frac{C}{2}(\Delta V')$$

$$\Rightarrow \quad \Delta V' = \frac{4\Delta V}{3}$$

(c) $U' = \frac{1}{2}C\left(\frac{4\Delta V}{3}\right)^2 + \frac{1}{2}\left(\frac{C}{2}\right)\left(\frac{4\Delta V}{3}\right)^2$

$$\Rightarrow \quad U' = 4C\frac{(\Delta V)^2}{3}$$

(d) The extra energy comes from work put into the system by the agent pulling the capacitor plates apart.

**2.** (a) Capacitors connected in series.

Initially, capacitance $C$ is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{\varepsilon_0 A} + \frac{d}{\varepsilon_0 A}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{2d}$$

When the plates are moved in the given manner, net capacity $C'$ of the system is given by

$$\frac{1}{C'} = \frac{d-\Delta d}{\varepsilon_0 A} + \frac{d+\Delta d}{\varepsilon_0 A}$$

$$\left\{\because C_1' = \frac{\varepsilon_0 A}{d-\Delta d} \text{ and } C_2' = \frac{\varepsilon_0 A}{d+\Delta d}\right\}$$

$$\Rightarrow \quad C' = \frac{\varepsilon_0 A}{2d}$$

Net capacitance remains same as before.

(b) Two identical capacitors $C_1$ and $C_2$ connected in parallel. Initially the net capacity of the system is

$$C = C_1 + C_2 = 2\frac{\varepsilon_0 A}{d}$$

where, $A$ is the area of the plates, and $d$ the distance between the plates, for both the capacitors. When the plates are brought closer by $\Delta d$ in one capacitor, its capacity increases to $\frac{\varepsilon_0 A}{d-\Delta d}$; whereas the capacity of the second capacitor, whose plates are moved away by $\Delta d$, decreases to $\dfrac{\varepsilon_0 A}{d+\Delta d}$. Net capacity of the system becomes

$$C' = \varepsilon_0 A\left(\frac{1}{d-\Delta d}+\frac{1}{d+\Delta d}\right) = \frac{2\varepsilon_0 A d}{d^2-(\Delta d)^2}$$

$$\Rightarrow \quad C' = \frac{2\varepsilon_0 A}{d-\left(\dfrac{\Delta d^2}{d}\right)}$$

We find that $C' > C$ and hence net capacitance increases.

**3.** The initial charge on the larger capacitor is

$$Q = C\Delta V$$

$$\Rightarrow \quad Q = (10\ \mu\text{F})(15\ \text{V}) = 150\ \mu\text{C}$$

An additional charge $q$ is pushed through the 50 V battery, giving the smaller capacitor charge $q$ and the larger charge $150+q$ (in $\mu$C).

$$\Rightarrow \quad 50\ \text{V} = \frac{q}{5\ \mu\text{F}} + \frac{150\ \mu\text{C}+q}{10\ \mu\text{F}}$$

$$\Rightarrow \quad 500\ \mu\text{C} = 2q + 150\ \mu\text{C} + q$$

$$\Rightarrow \quad q = 117\ \mu\text{C}$$

Across the 5 $\mu$F capacitor $\Delta V = \dfrac{q}{C} = \dfrac{117\ \mu\text{C}}{5\ \mu\text{F}} = 23.3\ \text{V}$

Across the 10 $\mu$F capacitor

$$\Delta V = \frac{150\ \mu\text{C} + 117\ \mu\text{C}}{10\ \mu\text{F}} = 26.7\ \text{V}$$

**4.** (a) $U_i = \dfrac{q^2}{2C_i} = \left(\dfrac{q^2}{2\varepsilon_0 A}\right)x_1$ and

$$U_f = \frac{q^2}{2C_f} = \left(\frac{q^2}{2\varepsilon_0 A}\right)x_2$$

$$\Rightarrow \quad W_{\text{ext}} = W = U_f - U_i = \frac{q^2}{2\varepsilon_0 A}(x_2 - x_1)$$

(b) $U_i = \dfrac{1}{2}C_i V^2 = \dfrac{1}{2}\left(\dfrac{\varepsilon_0 A}{x_1}\right)V^2$

$$U_f = \frac{1}{2}C_f V^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{x_2}\right)V^2$$

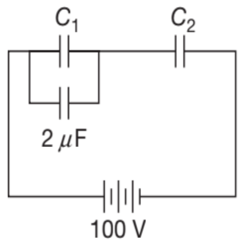
$$\Rightarrow \quad W_{\text{external}} = U_f - U_i = \frac{1}{2}\varepsilon_0 A V^2\left(\frac{1}{x_2}-\frac{1}{x_1}\right)$$

**5.** Initially $C_1(60) = C_2(40)$

$$3C_1 = 2C_2$$

Now, $q_2 = q_1 + q_{\text{across } 2\ \mu\text{F}}$

$$\Rightarrow \quad C_2(90) = C_1(10) + (2)(10)$$




$$\Rightarrow \quad 9C_2 = C_1 + 2$$

$$\Rightarrow \quad 9C_2 = \frac{2C_2}{3} + 2$$

$$\Rightarrow \quad 25C_2 = 6$$

$$\Rightarrow \quad C_2 = \frac{6}{25} = 0.24\ \mu\text{F}$$

and $C_1 = 0.16\ \mu$F

**6.** $U = \dfrac{1}{2}C_P V^2 = \dfrac{1}{2}(C_1 + C_2)V^2$

$$\Rightarrow \quad \frac{1}{2}(C_1+C_2)(2)^2 = 0.1$$

$$\Rightarrow \quad (C_1 + C_2) = 5\times 10^{-2} \qquad \ldots(1)$$

Similarly $\dfrac{1}{2}C_S V^2 = 1.6\times 10^{-2}$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)(2)^2 = 1.6\times 10^{-2}$$

$$\Rightarrow \quad C_1C_2 = 4\times 10^{-4} \qquad \ldots(2)$$

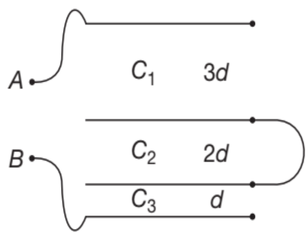
Solving equations (1) and (2), we get

$$C_1 = 40\ \text{mF} \ \text{ and } \ C_2 = 10\ \text{mF}$$

**7.** In the figure, $C_1 = \dfrac{\varepsilon_0 A}{3d}$

$$C_2 = \frac{\varepsilon_0 A}{2d}$$

and $C_3 = \dfrac{\varepsilon_0 A}{d}$




Now, $C_2$ is short circuited so that $C_1$ and $C_3$ are in series.

$$\Rightarrow \quad C_{AB} = \frac{C_1C_3}{C_1+C_3}$$

$$\Rightarrow \quad C_{AB} = \frac{\varepsilon_0 A}{4d}$$

**8.** In series the charge on each capacitor is same. Therefore, potential difference given by $V = \dfrac{q}{C}$ across the capacitors are in inverse ratio of their capacitors. Hence

$$V_1 = \frac{q}{C_1} = 130 \text{ V} \quad \text{and}$$

$$V_2 = \frac{q}{C_2} = 100 \text{ V}$$

$$\Rightarrow \frac{C_2}{C_1} = \frac{V_1}{V_2} = \frac{130}{100}$$

$$\Rightarrow \frac{C_1}{C_2} = \frac{10}{13} \qquad \ldots(1)$$

and so the smaller capacitance is $C_1$.
As capacitance of a capacitor is proportional to the dielectric constant $(K)$, so
New capacitance $C_1'$ of the first capacitor is

$$C_1' = \frac{5}{2}C_1 \qquad \left\{\because \frac{C_1'}{C_1} = \frac{K_1'}{K_1} = \frac{5}{2}\right\}$$

New capacitance of second capacitor $C_2'$ does not change, so

$$C_2' = C_2$$

$$\Rightarrow \frac{C_1'}{C_2'} = \frac{\frac{5}{2}C_1}{C_2} = \frac{5}{2} \times \frac{10}{13} = \frac{25}{13} \qquad \{\text{using (1)}\}$$

If $V_1'$ and $V_2'$ are the respective potential differences, then

$$\frac{V_1'}{V_2'} = \frac{C_2'}{C_1'} = \frac{13}{25} \qquad \ldots(2)$$

Also, $V_1' + V_2' = 230$ V ...(3)
Solving (2) and (3), we get

$$V_1' = 78.7 \text{ V} \quad \text{and} \quad V_2' = 151.3 \text{ V}.$$

**9.** Let $d$ be the separation between the plates of first capacitor, the separation between the plates of second capacitor will be $[a - (b + d)]$.
The capacitances of these capacitors are

$$C_1 = \frac{\varepsilon_0 A}{d} \quad \text{and} \quad C_2 = \frac{\varepsilon_0 A}{a - (b + d)}$$

Since the capacitors are in series, so the equivalent capacitance of the combination $C$ is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\left(\frac{\varepsilon_0 A}{d}\right)} + \frac{1}{\left(\frac{\varepsilon_0 A}{[a - (b + d)]}\right)}$$

$$\Rightarrow \frac{1}{C} = \frac{1}{(\varepsilon_0 A)}[d + a - b - d] = \frac{a - b}{\varepsilon_0 A}$$

$$\Rightarrow C = \frac{\varepsilon_0 A}{a - b}$$

This is the required expression for the net capacitance and is independent of $d$ and hence of the position of the central H-shaped part.

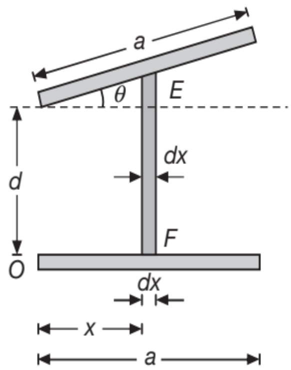
**10.** The given capacitor may be supposed to be formed of a large number of differential capacitors each connected in parallel. Consider one such capacitor of width $dx$ at a distance $x$ from $O$. The area of each plate of this small capacitor, $dA (= a dx)$.
Separation between these plates, $EF = d + x \tan\theta$
If $dC$ be the capacitance of this small capacitor, then

$$dC = \frac{\varepsilon_0 dA}{(EF)} = \frac{\varepsilon_0 (a dx)}{d + x\tan\theta} = \frac{\varepsilon_0 (a dx)}{d + x\theta}$$

{since for small $\theta$, $\tan\theta = \theta$ }

$$\Rightarrow dC = \frac{\varepsilon_0 (a dx)}{d\left(1 + \frac{x\theta}{d}\right)} = \frac{\varepsilon_0 (a dx)}{d}\left(1 + \frac{x\theta}{d}\right)^{-1}$$

$$\Rightarrow dC = \frac{\varepsilon_0 (a dx)}{d}\left(1 - \frac{\theta}{d}x\right) \quad \{\text{using Binomial Theorem}\}$$

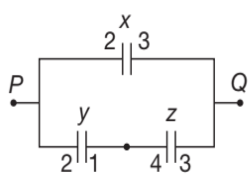



The net capacitance of capacitor is obtained by integrating the expression with respect to $x$ between 0 to $a$ i.e.,

$$C = \frac{\varepsilon_0 a}{d}\int_0^a \left(dx - \frac{\theta}{d}x dx\right) = \frac{\varepsilon_0 a}{d}\left[x - \frac{\theta}{d}\frac{x^2}{2}\right]_0^a$$

$$\Rightarrow C = \frac{\varepsilon_0 a}{d}\left[a - \frac{a\theta^2}{2d}\right] = \frac{\varepsilon_0 a^2}{d}\left[1 - \frac{a\theta}{2d}\right]$$

**11.** (a) The equivalent of Arrangement 1 is shown. Let $C$ be capacitance of each capacitor.



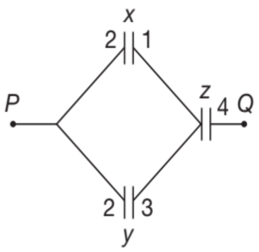

The net capacitance of $y$ and $z$ connected in series is

$$C_{yz} = \frac{C}{2}$$

Now $C_{yz}$ and capacitor $x$ are connected in parallel. Therefore net capacitance between $P$ and $Q$,

$$C_{PQ} = C + \frac{C}{2} = \frac{3}{2}C = \frac{3}{2}\left(\frac{\varepsilon_0 A}{d}\right)$$

(b) The equivalent of Arrangement 2 is shown.




The net capacitance of $x$ and $y$ connected in parallel is

$$C_{xy} = 2C$$

Now $C_{xy}$ and $z$ are connected in series. Therefore net capacitance of arrangement between $P$ and $Q$ is

$$C_{PQ} = \frac{CC_{xy}}{C + C_{xy}} = \frac{C(2C)}{C + 2C} = \frac{2}{3}C = \frac{2}{3}\left(\frac{\varepsilon_0 A}{d}\right)$$

**12.** The capacitance is maximum when the plates of one group are parallel to the plates of other group, so that the effective area of each plate is $A$.
Since the alternate plates are connected together the potential difference across any two consecutive plates is same. The $n$ plates of given arrangement form $(n-1)$ capacitors connected in parallel. Therefore the maximum capacitance of radio capacitor is

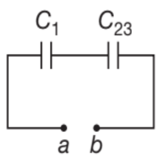
$$C = \frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{d} + \ldots (n-1) \text{ times}$$

$$\Rightarrow \quad C = (n-1)\frac{\varepsilon_0 A}{d}$$

**13.** (a) Capacitors $C_2$ and $C_3$ are in parallel, so their equivalent capacitance is

$$C_{23} = C_2 + C_3 = 4 + 8$$

$$\Rightarrow \quad C_{23} = 12\ \mu\text{F}$$




Capacitors $C_1$ and $C_{23}$ are in series; so the equivalent capacitance $C$ of the entire combination is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{23}} = \frac{1}{6} + \frac{1}{12} = \frac{3}{12}$$

$$C_{eq} = 4\ \mu\text{F}$$

(b) The total charge that flows from the battery is

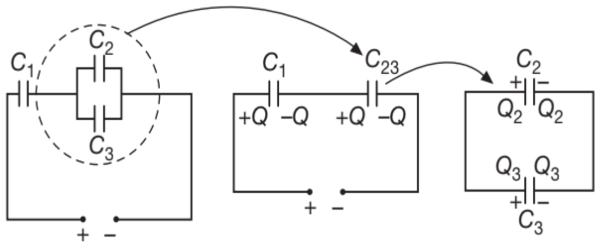
$$Q = C_{eq}V = (4\times10^{-6})(12) = 4.8\times10^{-5}\ \text{C}$$

Capacitors $C_1$ and $C_{23}$ are in series, hence they carry the same charge $Q = 4.8\times10^{-5}$ C.
The voltage across $C_1$,

$$V_1 = \frac{Q}{C_1} = \frac{4.8\times10^{-5}}{6\times10^{-6}} = 8\ \text{V}$$

The voltage across the combination $C_{23}$ is

$$V_{23} = \frac{Q}{C_{23}} = \frac{4.8\times10^{-5}}{12\times10^{-6}} = 4\ \text{V}$$




$C_{23}$ is parallel combination of $C_2$ and $C_3$ and hence they have the same potential

$$V_2 = V_3 = 4\ \text{V}$$

The charges on $C_2$ and $C_3$ are

$$Q_2 = C_2V_2 = (4\times10^{-6})(4) = 1.6\times10^{-5}\ \text{C}$$

$$Q_3 = C_3V_3 = (8\times10^{-6})(4) = 3.2\times10^{-5}\ \text{C}$$

So we have $V_1 = 8$ V, $Q_1 = 48\ \mu$C

$V_2 = 4$ V, $Q_2 = 16\ \mu$C

$V_3 = 4$ V, $Q_3 = 32\ \mu$C

Also, we note that $Q_2 + Q_3 = Q_1 = Q$

**14.** (a) The two condensers in the middle are connected together and hence act as a single conductor. Thus effectively there are three plates which form two capacitors in parallel.
Capacitance of each capacitor,

$$C = \frac{\varepsilon_0 A}{d}$$

So, net capacitance of combination is

$$C_{PQ} = 2C = \frac{2\varepsilon_0 A}{d}$$

(b) The four plates are alternatively connected and form three capacitors in parallel. So, the net capacitance of combination is given by

$$C_{PQ} = 3C = \frac{3\varepsilon_0 A}{d}$$

**15.** (a) The capacitance of capacitor $(A,B)$ is

$$C_1 = \frac{\varepsilon_0 A_1}{d_1} = \frac{\varepsilon_0 \pi R_1^2}{d_1}$$

$$\Rightarrow \quad C_1 = \frac{\left(\frac{1}{36\pi\times10^9}\right)\pi\left(\frac{0.1}{2}\right)^2}{2\times10^{-3}} = \frac{1}{288\times10^8}\text{ F}$$

The capacitance of capacitor $(C,D)$ is

$$C_2 = \frac{\varepsilon_0 A}{d_2} = \frac{\varepsilon_0 \pi R_2^2}{d_2}$$

$$\Rightarrow \quad C_2 = \frac{\left(\frac{1}{36\pi\times10^9}\right)\pi\left(\frac{0.12}{2}\right)^2}{3\times10^{-3}} = \frac{1}{300\times10^8}\text{ F}$$

The capacitors $C_1$ and $C_2$ are in series. Therefore, equivalent capacitance $C$ is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = 288\times10^8 + 300\times10^8$$

$$\Rightarrow \quad \frac{1}{C} = 588\times10^8$$

$$\Rightarrow \quad C = \frac{1}{588\times10^8} = 17\times10^{-12}\text{ F} = 17\text{ pF}$$

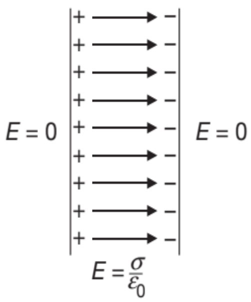
(b) Energy stored by capacitors

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\times(17\times10^{-12})\times(120)^2$$

$$\Rightarrow \quad U = 1.224\times10^{-7}\text{ J}$$

**16.** (a) Electric field is uniform between the plates of the capacitor. The magnitude of this field is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$$




Therefore, the energy density $(u)$ should also be constant.

$$U = \frac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{2A^2\varepsilon_0}$$

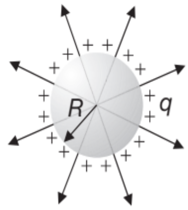
So, total stored energy, $U = (u)$ (total volume)

$$U = \left(\frac{q^2}{2A^2\varepsilon_0}\right)(A.d) = \frac{q^2}{2\left(\frac{A\varepsilon_0}{d}\right)}$$

$$\Rightarrow \quad U = \frac{q^2}{2C} \qquad \left\{\because C = \frac{A\varepsilon_0}{d}\right\}$$

(b) In case of a spherical conductor (of radius $R$) the excess charge resides on the outer surface of the conductor. The field inside the conductor is zero. It extends from surface to infinity. And since the potential energy is stored in the field only, it will be stored in the region extending from surface to infinity. Also the field is non-uniform, the energy density $u$ is also non-uniform. Hence, the total energy can be calculated using the concept of integration. Electric field at a distance $r$ from the centre is,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

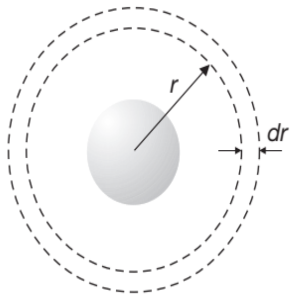



$$u(r) = \frac{1}{2}\varepsilon_0 E^2$$

$$\Rightarrow \quad u(r) = \frac{1}{2}\varepsilon_0\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)^2$$

Energy stored in a volume $dV = (4\pi r^2)dr$ is

$$dU = u\,dV$$




$$\Rightarrow \quad \text{Total energy stored is, } U = \int_{r=R}^{r=\infty} dU$$

Substituting the values and integrating, we get

$$U = \frac{q^2}{2(4\pi\varepsilon_0 R)}$$

$$\Rightarrow \quad U = \frac{q^2}{2C} \qquad \{\because C = 4\pi\varepsilon_0 R\}$$

CHAPTER 2

## Test Your Concepts-III (Based on Dielectrics and Breakdown)

**1.** Arrangement A is equivalent to a combination of two capacitors each of area $A$, separation $\frac{d}{2}$ and connected in series. So,

$$\frac{1}{C_S}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{(d/2)}{\varepsilon_0 K_1 A}+\frac{d/2}{\varepsilon_0 K_2 A}$$

$$\frac{1}{C_S}=\frac{\varepsilon_0 A}{d}\left[\frac{K_1K_2}{K_1+K_2}\right]$$

where $K_S=\frac{K_1K_2}{K_1+K_2}$

Arrangement $B$ is equivalent to combination of two capacitors each of area $\frac{A}{2}$, separation $d$ and connected in parallel. So,

$$C_p=C_3+C_4=\frac{\varepsilon_0K_1\left(\frac{A}{2}\right)}{d}+\frac{\varepsilon_0K_2\left(\frac{A}{2}\right)}{d}$$

$$C_p=\frac{\varepsilon_0 A}{d}\left[\frac{K_1+K_2}{2}\right]$$

where $K_P=\frac{K_1+K_2}{2}$

Now $\frac{C_S}{C_P}=\frac{2K_1K_2}{K_1+K_2}\times\frac{2}{(K_1+K_2)}=\frac{4K_1K_2}{(K_1+K_2)^2}$

$$\Rightarrow \frac{C_S}{C_P}=\frac{4\times2\times3}{(2+3)^2}=\frac{24}{25}$$

**2.** (a) $C_0=\frac{\varepsilon_0 A}{d}=\frac{Q_0}{\Delta V_0}$

When the dielectric is inserted at constant voltage, then

$$C=\kappa C_0=\frac{Q}{\Delta V_0}$$

Since, $U_0=\frac{1}{2}C_0(\Delta V_0)^2$

and $U=\frac{1}{2}C(\Delta V)^2=\frac{\kappa C_0\left(\Delta V_0^2\right)}{2}$

$$\Rightarrow \frac{U}{U_0}=\kappa$$

The extra energy comes from (part of the) electrical work done by the battery in separating the extra charge.

(b) $Q_0=C_0\Delta V_0$

and $Q=C\Delta V_0=\kappa C_0\Delta V_0$

$$\Rightarrow \frac{Q}{Q_0}=\kappa$$

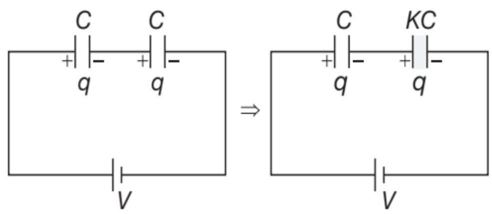
Please note that this situation is not the same as the situation where the battery is removed from the circuit before inserting the dielectric.

**3.** (a) Net capacitance without inserting the slab is, $\frac{C}{2}$

$$\Rightarrow q=\left(\frac{C}{2}\right)V$$

Since, electric field

$$E=\frac{\text{Potential Difference}}{\text{separation between the plates}}=\frac{\left(\frac{V}{2}\right)}{d}=\frac{V}{2d}$$




Net capacitance after inserting the slab is

$$C'=\left(\frac{K}{K+1}\right)C$$

$$\Rightarrow q'=CV\left(\frac{K}{K+1}\right)$$

Electric field

$$E'=\frac{\text{Potential Difference}}{\text{Separation between the plates}}$$

$$E'=\frac{V\left(\frac{1}{K+1}\right)}{d}\frac{V}{(K+1)d}$$

Electric field decreases by a factor

$$\frac{E}{E'}=\frac{\left(\frac{V}{2d}\right)}{\left(\frac{V}{(K+1)d}\right)}=\frac{K+1}{2}$$

(b) Charge that flows through the battery is $q'-q$

$$\text{Charge Flowing}=\frac{CV(K-1)}{2(K+1)}$$

**4.** $C=\frac{\varepsilon_0 A}{d-t+\frac{t}{k}}=\frac{C_0}{1-\frac{t}{d}+\frac{t}{kd}}$ $\left\{\because C_0=\frac{\varepsilon_0 A}{d}\right\}$

where $t=2(0.1\text{ mm})=0.2\text{ mm}$, $d=4\text{ mm}$ and $K=3$

Putting values, we get $C_0=44.25\text{ pF}$

Now, $C=\dfrac{44.25}{1-\dfrac{0.2}{4}+\dfrac{0.2}{(3)(4)}}$

$\Rightarrow\ C=\dfrac{44.25}{1-\dfrac{1}{20}+\dfrac{1}{60}}pF$

$\Rightarrow\ C=\dfrac{44.25\times 60}{60-3+1}=\dfrac{2655}{58}$

$\Rightarrow\ C=45.77\%$

So, %age change is

$$\frac{\Delta C}{C_0}\times 100=\left(\frac{45.77-44.25}{44.25}\right)\times 100=3.4\%$$

$\Rightarrow$ Percentage increase in the value of capacitance is 3.4%

**5.** (a) $C=\dfrac{\varepsilon_0 A}{d}$

(b) $C=\dfrac{\varepsilon_0 A}{d-t}$

Since, $t\to 0$

(c) $C=\dfrac{\varepsilon_0 A}{d/2}=\dfrac{2\varepsilon_0 A}{d}$

**6.** (a) Since both are connected to the same potential, hence they are in parallel. So,

$$C_{\text{net}}=K_1\left(\frac{\varepsilon_0 A}{d_1}\right)+K_2\left(\frac{\varepsilon_0 A}{d_2}\right)$$

$$\Rightarrow\ C_{\text{net}}=\varepsilon_0 A\left(\frac{K_1}{d_1}+\frac{K_2}{d_2}\right)$$

(b) $\sigma=\dfrac{Q}{A}$

$$\Rightarrow\ \sigma=\frac{CV}{A}$$

$$\Rightarrow\ \sigma=\frac{\varepsilon_0 AV}{A}\left(\frac{K_1}{d_1}+\frac{K_2}{d_2}\right)$$

$$\Rightarrow\ \sigma=\varepsilon_0 V\left(\frac{K_1}{d_1}+\frac{K_2}{d_2}\right)$$

(c) $U_1=\dfrac{1}{2}\varepsilon E_1^2$

Since $\varepsilon=K_1\varepsilon_0$ and $E_1=\dfrac{V_1}{d}$, so

$$U_1=\frac{1}{2}(K_1\varepsilon_0)\left(\frac{V}{d_1}\right)^2$$

$$\Rightarrow\ U_1=\frac{1}{2}(K_1\varepsilon_0)\left(\frac{V^2}{d_1^2}\right)$$

$$\Rightarrow\ U_1=\frac{1}{2}\left(\frac{\varepsilon_0 K_1 V^2}{d_1^2}\right)$$

**7.** The system of three parallel plates form two parallel plate capacitors connected in series.
The capacitance of first capacitor

$$C_1=\frac{K_1\varepsilon_0 A}{d_1}$$

and capacitance of second capacitor

$$C_2=\frac{K_2\varepsilon_0 A}{d_2}$$

The net capacitance $C$ is given by

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{\left(\dfrac{K_1\varepsilon_0 A}{d_1}\right)}+\frac{1}{\left(\dfrac{K_2\varepsilon_0 A}{d_2}\right)}$$

$$\Rightarrow\ \frac{1}{C}=\frac{1}{\varepsilon_0 A}\left[\frac{d_1}{K_1}+\frac{d_2}{K_2}\right]=\frac{1}{\varepsilon_0 A}\frac{d_1K_2+d_2K_1}{K_1K_2}$$

$$\Rightarrow\ C=\frac{\varepsilon_0 K_1K_2A}{d_1K_2+d_2K_1}$$

**8.** The capacitances $C_2$ and $C_3$ are in series, each having area $\dfrac{A}{2}$ and separation $\dfrac{d}{2}$.

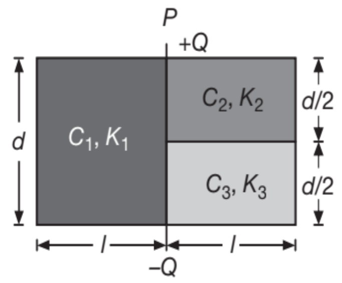
$$C_2=K_2\left(\frac{\varepsilon_0\dfrac{A}{2}}{\dfrac{d}{2}}\right)=K_2\left(\frac{\varepsilon_0 A}{d}\right)$$

$$\Rightarrow\ C_3=K_3\left(\frac{\varepsilon_0\left(\dfrac{A}{2}\right)}{\dfrac{d}{2}}\right)=K_3\left(\frac{\varepsilon_0 A}{d}\right)$$

Equivalent capacitance of $C_2$ and $C_3$ is

$$C'=\frac{C_2C_3}{C_2+C_3}=\frac{K_2K_3\left(\dfrac{\varepsilon_0 A}{d}\right)^2}{(K_2+K_3)\left(\dfrac{\varepsilon_0 A}{d}\right)}=\left(\frac{K_2K_3}{K_2+K_3}\right)\frac{\varepsilon_0 A}{d}$$

Also $C_1=K_1\left(\dfrac{\varepsilon_0\left(\dfrac{A}{2}\right)}{d}\right)=K_1\left(\dfrac{\varepsilon_0 A}{2d}\right)$




From figure it is clear that $C_1$ is in parallel with $C'$ (combination of $C_2$ and $C_3$). Hence net capacitance between $P$ and $Q$.

$$C = C_1 + C' = K_1\left(\frac{\varepsilon_0 A}{2d}\right) + \left(\frac{K_2K_3}{K_2+K_3}\right)\frac{\varepsilon_0 A}{d}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{d}\left(\frac{K_1}{2} + \frac{K_2K_3}{(K_2+K_3)}\right)$$

Substituting given values

$$C = \frac{8.8\times10^{-12}\times(1\times10^{-4})}{2\times10^{-3}}\left(\frac{4}{2}+\frac{6\times2}{6+2}\right)$$

$$\Rightarrow \quad C = 0.44\times10^{-12}\left(\frac{7}{2}\right)$$

$$\Rightarrow \quad C = 1.54\times10^{-12}\text{ F} = 1.54\text{ pF}$$

**9.** With the switch $S$ closed, the potential difference across capacitors $A$ and $B$ is same. So,

$$V = \frac{Q_A}{C} = \frac{Q_B}{C}$$

The initial charges on the capacitors are given by

$$Q_A = Q_B = CV$$

When dielectric is introduced, the new capacitance of either capacitor.

$$C' = KC = 3C$$

Now, when the switch $S$ is opened, let the potential difference across capacitor $A$ be $V$ volt and that across the capacitor $B$ be $V'$ volt.
When dielectric is introduced with the switch open (i.e., battery disconnected) then, the charge on capacitor $B$ remains unchanged, so

$$Q_B = CV = C'V'$$

$$\Rightarrow \quad V' = \frac{C}{C'}V = \frac{V}{3}\text{ volt}$$

Initial energy of both capacitors

$$U_i = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$$

Final energy of both capacitors

$$U_f = \frac{1}{2}C'V^2 + \frac{1}{2}C'V'^2$$

$$\Rightarrow \quad U_f = \frac{1}{2}(3C)V^2 + \frac{1}{2}(3C)\left(\frac{V}{3}\right)^2 = \frac{5}{3}CV^2$$

$$\Rightarrow \quad \frac{U_i}{U_f} = \frac{CV^2}{\frac{5}{3}CV^2} = \frac{3}{5}$$

**10.** (a) $C = \frac{\varepsilon_0}{d}[(\ell - x)\ell + \kappa\ell x] = \frac{\varepsilon_0}{d}[\ell^2 + \ell x(\kappa - 1)]$

(b) $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\left(\frac{\varepsilon_0(\Delta V)^2}{d}\right)[\ell^2 + \ell x(\kappa-1)]$

(c) $F = -\left(\frac{dU}{dx}\right)\hat{i} = \frac{\varepsilon_0(\Delta V)^2}{2d}\ell(\kappa-1)$ to the left (out of the capacitor)

(d) $F = \frac{(2000)^2(8.85\times10^{-12})(0.0500)(4.50-1)}{2(2.00\times10^{-3})}$

$F = 1.55\times10^{-3}$ N

**11.** The vertical orientation sets up two capacitors in parallel, with equivalent capacitance

$$C_P = \frac{\varepsilon_0\left(\frac{A}{2}\right)}{d} + \frac{\kappa\varepsilon_0\left(\frac{A}{2}\right)}{d} = \left(\frac{\kappa+1}{2}\right)\frac{\varepsilon_0 A}{d}$$

where $A$ is the area of either plate and $d$ is the separation of the plates. The horizontal orientation produces two capacitors in series. If $f$ is the fraction of the horizontal capacitor filled with dielectric, the equivalent capacitance is

$$\frac{1}{C_s} = \frac{fd}{\kappa\varepsilon_0 A} + \frac{(1-f)d}{\varepsilon_0 A} = \left[\frac{f+\kappa(1-f)}{\kappa}\right]\frac{d}{\varepsilon_0 A}$$

$$\Rightarrow \quad C_s = \left[\frac{\kappa}{f+\kappa(1-f)}\right]\frac{\varepsilon_0 A}{d}$$

For having $C_p = C_s$ we get $\frac{\kappa+1}{2} = \frac{\kappa}{f+\kappa(1-f)}$

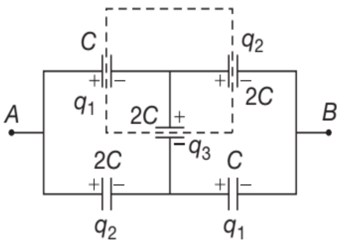
$$\Rightarrow \quad (\kappa+1)[f+\kappa(1-f)] = 2\kappa$$

For $\kappa = 2$, we get $3[2-(1)f] = 4$

$$\Rightarrow \quad f = \frac{2}{3}$$

## Test Your Concepts-IV (Based on Capacitor Circuits, Kirchhoff's Laws, Charge Flown and Generation of Heat)

**1.** The given circuit forms an unbalanced Wheatstone bridge. Let us suppose that the point $A$ is connected to the positive terminal of a hypothetical battery and $B$ to the negative terminal of the same battery. Hence, a total charge $q$ is stored in the capacitors. Seeing the symmetry about the input and output points, we can say that charges will be distributed as shown.




$$q_1 + q_2 = q \qquad \ldots(1)$$

Applying Second Law, we have

$$-\frac{q_1}{C} - \frac{q_3}{2C} + \frac{q_2}{2C} = 0$$

$$\Rightarrow \quad q_2 - q_3 - 2q_1 = 0 \qquad \ldots(2)$$

Plates inside the dotted line form an isolated system. Hence,

$$q_2 + q_3 - q_1 = 0 \qquad \ldots(3)$$

Solving these three equations, we have

$$q_1 = \frac{2}{5}q,\ q_2 = \frac{3}{5}q \text{ and } q_3 = -\frac{q}{5}$$

Now, let $C_{eq}$ be the equivalent capacitance between $A$ and $B$. Then,

$$V_A - V_B = \frac{q}{C_{eq}} = \frac{q_1}{C} + \frac{q_2}{2C}$$

$$\Rightarrow \quad \frac{q}{C_{eq}} = \frac{2q}{5C} + \frac{3q}{10C} = \frac{7q}{10C}$$

$$\Rightarrow \quad C_{eq} = \frac{10}{7}C$$

**2.** $V_A - \frac{q_1}{C_1} + E - \frac{q_2}{C_2} - V_B = 0$

$$5 - \frac{q_1}{1} + 10 - \frac{q_2}{2} = 0$$

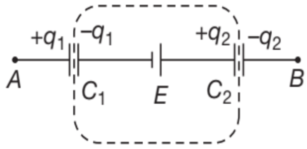
$$\Rightarrow \quad q_1 + \frac{q_2}{2} = 15 \qquad \ldots(1)$$

Also, we observe that

$$-q_1 + q_2 = 0$$

$$\Rightarrow \quad q_1 = q_2 \qquad \ldots(2)$$

From (1) and (2), we get




$$q_1 + \frac{q_1}{2} = 15$$

$$\Rightarrow \quad q_1 = 10\ \mu C$$

$$q_2 = 10\ \mu C$$

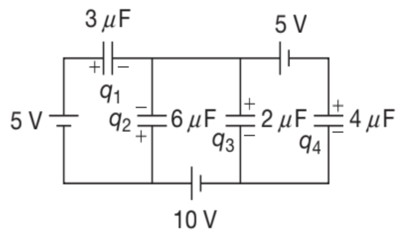
So, voltage across the first capacitor is

$$V_1 = \frac{q_1}{C_1} = 10 \text{ V}$$

and across the second is

$$V_2 = \frac{q_2}{C_2} = 5 \text{ V}$$

**3.** Here, we observe that $q_1$, $q_2$, $q_3$ and $q_4$ are in $\mu$C (microcoulomb) and any battery supplies same magnitude of charge from its both terminals.




Select the sign of the charges on the capacitors arbitrarily, we get

$$-q_1 - q_2 + q_3 + q_4 = 0 \qquad \ldots(1)$$

Applying Loop Law in three loops, one by one, we get

$$5 - \frac{q_1}{3} + \frac{q_2}{6} = 0 \qquad \ldots(2)$$

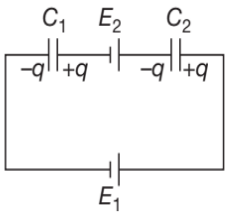
$$10 - \frac{q_2}{6} - \frac{q_3}{2} = 0 \qquad \ldots(3)$$

$$\text{and } 5 - \frac{q_3}{2} + \frac{q_4}{4} = 0 \qquad \ldots(4)$$

Solving these four equations, we get

$$q_2 = q_3 = 10\ \mu C \text{ and } q_1 = q_4 = \frac{40}{3}\ \mu C$$

**4.** The charge distribution is shown in figure. The circuit is equivalent to two capacitors connected in series with a battery in between them. Let the charge on $C_1$ is $q$. The same charge will also appear on $C_2$. The reason is as follows : Whatever charge appears on plate of $C_1$ (say-$q$) then equal but opposite charge goes to $C_2$ through $E_1$. The potential drops across $C_1$ and $C_2$ will be $\frac{q}{C_1}$ and $\frac{q}{C_2}$ respectively. Which plate would be at higher potential will be decided by relative strength of $E_1$ and $E_2$. If we move along the complete loop, the total potential difference must be zero. Let $E_2 > E_1$.



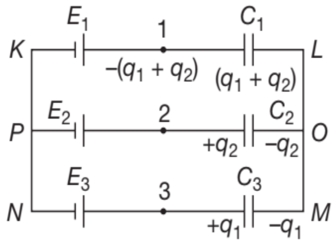

Thus, $\frac{q}{C_1}+E_1+\frac{q}{C_2}-E_2=0$

$$\Rightarrow \quad q=\frac{(E_2-E_1)C_1C_2}{C_1+C_2}$$

$$|q|=\frac{|E_2-E_1|C_1C_2}{C_1+C_2}$$

It is obvious from the figure that the addition of a battery in between two capacitors is equivalent to charging the potential applied to the two capacitors in series.

**5.** Charge distribution is shown in figure. Consider the loop $NMLKN$. Applying $-\Delta V=0$, we have




$$-E_3+\frac{q_1}{C_3}+\left(\frac{q_1+q_2}{C_1}\right)+E_1=0 \quad \ldots(1)$$

Similarly, for loop $OLKPO$, we get

$$\left(\frac{q_1+q_2}{C_1}\right)+E_1-E_2+\frac{q_2}{C_2}=0 \quad \ldots(2)$$

Solving these equations, we get

$$q_1+q_2=\frac{E_2C_2-E_1C_2-E_1C_3+E_3C_3}{\frac{C_3}{C_1}+\frac{C_2}{C_1}+1} \quad \ldots(3)$$

Now $V_1-V_0=V_1=-\frac{(q_1+q_2)}{C_1}$ $\quad \{\because V_0=0\}$

$$\Rightarrow \quad V_1=\frac{E_1(C_2+C_3)-E_2C_2-E_3C_3}{C_1+C_2+C_3}$$

Similarly, $V_2=\frac{E_2(C_1+C_3)-E_1C_1-E_3C_3}{C_1+C_2+C_3}$

$$V_3=\frac{E_3(C_1+C_2)-E_1C_1-E_2C_2}{C_1+C_2+C_3}$$

**6.** Capacitors $C_1$ and $C_2$ are in series; their effective capacitance is

$$C_{12}=\frac{C_1C_2}{C_1+C_2}$$

Effective capacitance of $C_3$ and $C_4$ is

$$C_{34}=\frac{C_3C_4}{C_3+C_4}$$

$C_{12}$ and $C_{34}$ are in parallel; equivalent capacitance of the circuit is

$$C_{eq}=C_{12}+C_{34}$$

total charge on the capacitors is

$$Q=C_{eq}V$$

Charge $Q$ divides into charge $Q_{12}$ in $C_1$ or $C_2$ and $Q_{34}$ in $C_3$ or $C_4$. We have

$$Q_{12}=C_{12}V \text{ and } Q_{34}=C_{34}V$$

Potential across capacitor $C_1$ is

$$V_1=\frac{Q_{12}}{C_1}=\frac{C_2}{C_1+C_2}V$$

Potential drop at capacitor $C_3$

$$V_3=\frac{Q_{34}}{C_3}=\frac{C_4}{C_3+C_4}V$$

Potential of $A$ and $B$ are

$$V_A=V-V_1=\left(1-\frac{C_2}{C_1+C_2}\right)V$$

and $V_B=V-V_3=\left(1-\frac{C_4}{C_3+C_4}\right)V$

Thus, potential differences between $A$ and $B$ are equal to

$$V_A-V_B=\left(\frac{C_4}{C_3+C_4}-\frac{C_2}{C_1+C_2}\right)V=\frac{C_1C_4-C_2C_3}{(C_1+C_2)(C_3+C_4)}V$$

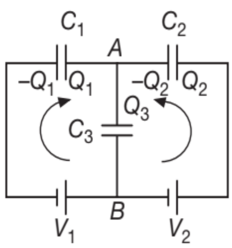
This will be zero, if $C_1C_4-C_2C_3=0$

$$\Rightarrow \quad \frac{C_1}{C_2}=\frac{C_3}{C_4}$$

This is the condition for a balanced Wheatstone bridge.

**7.** Let the charges on capacitors $C_1$, $C_2$, and $C_3$ be $Q_1$, $Q_2$ and $Q_3$. Potential differences are therefore $\frac{Q_1}{C_1},\frac{Q_2}{C_2}$ and $\frac{Q_3}{C_3}$ respectively. Look at the inner plates of the three capacitors connected to point $A$. From conservation of charge, we have

$$Q_1-Q_2+(\pm Q_3)=0 \quad \ldots(1)$$



Because all the inner three plates were neutral initially, therefore, whatever charge is induced on one is obviously drawn from others. $Q_3$, the charge on the upper plate of $C_3$, may be positive or negative depending upon the value of $(Q_1 - Q_2)$.

As we move along the closed loops in the sense shown, we have

$$-V_1 + \frac{Q_1}{C_1} \mp \frac{Q_3}{C_3} = 0 \quad \ldots(2)$$

and $$V_2 - \frac{Q_2}{C_2} \mp \frac{Q_3}{C_3} = 0 \quad \ldots(3)$$

If $Q_3$ is positive, then potential falls as we go from $A$ to $B$. Hence $\mp Q_3$ in (1) implies $\mp\left(\frac{Q_3}{C_3}\right)$ in (2) and (3).

From (2) and (3), we get

$$Q_1 = \left(V_1 \mp \frac{Q_3}{C_3}\right)C_1 \text{ and } Q_2 = \left(V_2 \mp \frac{Q_3}{C_3}\right)C_2$$

Substituting for $Q_1$ and $Q_2$ in (1),we get

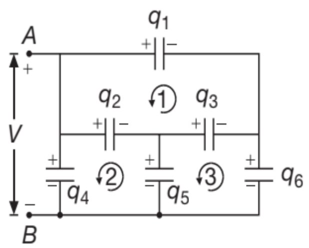
$$\frac{V_1C_1 - V_2C_2 \pm Q_3(C_1 + C_2 + C_3)}{C_3} = 0$$

$$\Rightarrow \frac{Q_3}{C_3} = (\pm)\frac{V_2C_2 - V_1C_1}{C_1 + C_2 + C_3}$$

Thus potential difference between $A$ and $B$

$$V_{AB} = \frac{V_2C_2 - V_1C_1}{C_1 + C_2 + C_3}$$

8. Let the potential difference across the battery terminals be $V$ and the charge of the battery $Q$. To find the capacitance of the battery means to find the capacitance of a capacitor which would have the same charge $Q$ on its plates as the battery at voltage $V$. Hence,




$$C_{AB} = \frac{Q}{V} \quad \ldots(1)$$

where from conservation of charge we must have

$$Q = q_1 + q_2 + q_3 = q_4 + q_5 + q_6 \quad \ldots(2)$$

and $V = V_4 = \frac{q_4}{C}$

In a closed loop the net potential drop must be zero, from $KVL$. Therefore, for loop 1, loop 2 and loop 3, we have

**For Loop 1**

$$\frac{q_1}{C} - \frac{q_2}{C} - \frac{q_3}{C} = 0$$

**For Loop 2**

$$\frac{q_2}{C} - \frac{q_4}{C} + \frac{q_5}{C} = 0$$

**For Loop 3**

$$\frac{q_3}{C} - \frac{q_5}{C} + \frac{q_6}{C} = 0 \quad \ldots(3)$$

The conductor that connects the second, third and fifth capacitors is electrically neutral. Hence,

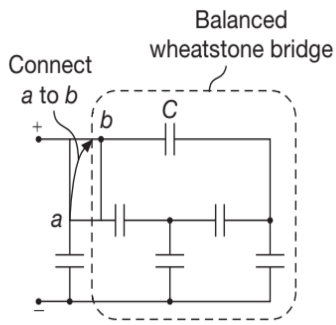
$$q_3 + q_5 - q_2 = 0 \quad \ldots(4)$$

On solving, we get

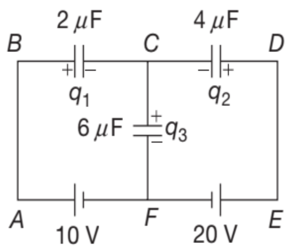
$$q_1 = q_2 = q_5 = q_6 = \frac{q_4}{2} \text{ and } q_3 = 0$$

Therefore, $C_{AB} = 2C$

**Method 2.** Connect the connection of point $a$ to $b$ and note the indicated Wheatstone bridge. Now simplify the circuit to obtain the desired result.




9. Let the charges in three capacitors be as shown in figure




Charge supplied by 10 V battery is $q_1$ and that from 20 V battery is $q_2$. Then,

$$q_1 + q_2 = q_3$$

This relation can also be obtained by using the Junction Law. The charges on the three plates which are in contact add to zero. Because these plates taken together form an isolated system which can't receive charges from the batteries. Thus,

$$q_3 - q_1 - q_2 = 0$$

$$\Rightarrow \quad q_3 = q_1 + q_2 \qquad \ldots(1)$$

Applying Kirchhoff's Second Law to loops $BCFAB$ and $CDEFC$, we get

$$-\frac{q_1}{2} - \frac{q_3}{6} + 10 = 0$$

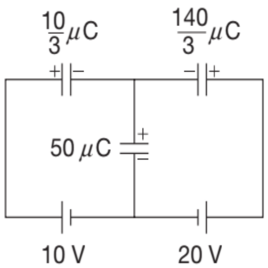
$$\Rightarrow \quad q_3 + 3q_1 = 60 \qquad \ldots(2)$$

and $\dfrac{q_2}{4} - 20 + \dfrac{q_3}{6} = 0$

$$\Rightarrow \quad 3q_2 + 2q_3 = 240 \qquad \ldots(3)$$

Solving equations (1), (2) and(3), we have

$$q_1 = \frac{10}{3}\ \mu\text{C},\ q_2 = \frac{140}{3}\ \mu\text{C and } q_3 = 50\ \mu\text{C}$$




**10.** Initially, when the switch is closed on position 1, the capacitor $C$ is connected in series with batteries $V_1$ and $V_2$.
From KVL we have

$$\frac{Q_i}{C} - V_2 + V_1 = 0$$

$$\Rightarrow \quad Q_i = (V_2 - V_1)C \qquad \ldots(1)$$

depending upon the sign of $(V_2 - V_1)$, charge $Q_i$ on the left plate may be positive (if $V_2 > V_1$), or negative (if $V_2 < V_1$). However, charge on the right plate would be equal and opposite.
When the switch is moved to position 2, the left plate earlier having charge $+Q_i$, will have charge

$$Q_f = -V_1C \qquad \ldots(2)$$

The net charge flow through the circuit is

$$\Delta Q = Q_f - Q_i = [-V_1 - (V_2 - V_1)]C = -V_2C$$

We can say that a net positive charge equal to $V_2C$ is pulled by the battery of emf $V_1$ from the left plate of the capacitor, which flows through battery $V_1$ and is transferred to the right plate of the capacitor. Work done by battery $V_1$ in the process of charge transfer is given by

$$W_{\text{battery}} = V_1|\Delta Q| = V_1V_2C \qquad \ldots(3)$$

A part of this work changes the energy of the capacitor

$$\Delta U_C = \frac{Q_f^2}{2C} - \frac{Q_i^2}{2C}$$

$$\Delta U_C = \frac{1}{2}V_1^2C - \frac{1}{2}(V_2 - V_1)^2C$$

$$\Delta U_C = \frac{1}{2}(2V_1V_2 - V_2^2)C$$

and the remaining part is lost as Joule heating. Hence the heat generated $H$ is given by

$$\Delta H = W_{\text{battery}} - \Delta U_C = \frac{1}{2}V_2^2C$$

**11.** When the switch is open, then $C_{\text{eq}} = 4\ \mu$F

So, $q = C_{eq}V$

$$\Rightarrow \quad q = 800\ \mu\text{C}$$

This charge divides equally amongst the upper branch and lower branch. So,

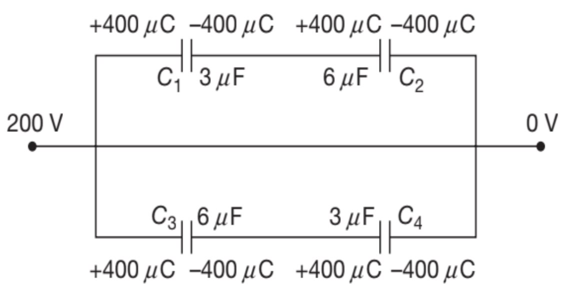
$$V_1 - V_b = \frac{400}{3}\text{ V}$$

$$V_1 - V_a = \frac{400}{6}\text{ V} = \frac{200}{3}\text{ V}$$

$$\Rightarrow \quad V_a - V_b = \frac{200}{3}\text{ V}$$

When the switch is closed, then $V_a - V_b = 0$

The charges on the capacitors before the switch $S$ is closed are shown.



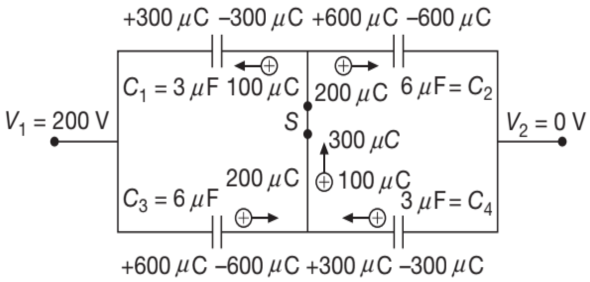

Now, when the switch is closed, then $C'_{eq} = \frac{9}{2}\ \mu$F

$$\Rightarrow \quad q' = C'_{eq}V = 900\ \mu\text{C}$$

This $900\ \mu$C is shared amongst $3\ \mu$F and $6\ \mu$F in direct ratio of their capacitances. So

$$q_{3\,\mu\text{F}} = \left(\frac{3}{3+6}\right)900 = 300\ \mu\text{C and}$$

$$q_{6\,\mu F} = \left(\frac{6}{3+6}\right)900 = 600\ \mu C$$

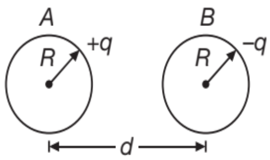



| | Initial Charge | Final Charge | Charge Flown $\|\Delta q\|$ |
|---|---|---|---|
| $C_1$ | 400 μC | 300 μC | $\|\Delta q_1\| = 100$ μC |
| $C_2$ | 400 μC | 600 μC | $\|\Delta q_2\| = 200$ μC |
| $C_3$ | 400 μC | 600 μC | $\|\Delta q_3\| = 200$ μC |
| $C_4$ | 400 μC | 300 μC | $\|\Delta q_4\| = 100$ μC |

## Test Your Concepts-V (Based on Spherical and Cylindrical Capacitors)

**1.** Let us assume a positive charge $+q$ on $A$ and a negative charge $-q$ on $B$. Then their potentials are

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} - \frac{1}{4\pi\varepsilon_0}\frac{q}{d} \text{ and } V_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{d} - \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$




The potential difference is

$$V = V_A - V_B = \frac{1}{4\pi\varepsilon_0}\frac{2q}{R} - \frac{1}{4\pi\varepsilon_0}\frac{2q}{d}$$

$$\Rightarrow \quad V = 2\frac{1}{4\pi\varepsilon_0}q\left[\frac{1}{R} - \frac{1}{d}\right] = \frac{q}{2\pi\varepsilon_0}\left(\frac{d-R}{Rd}\right)$$

Since $d \gg R$

$$\Rightarrow \quad V = \frac{q}{2\pi\varepsilon_0 R}$$

Since, $C = \dfrac{q}{V}$

$$\Rightarrow \quad C = 2\pi\varepsilon_0 R$$

$$\Rightarrow \quad C = \frac{2}{9\times 10^9}\times\frac{9}{1000}$$

$$\Rightarrow \quad C = 2\times 10^{-12}\ \text{F}$$

$$\Rightarrow \quad C = 2\ \text{pF}$$

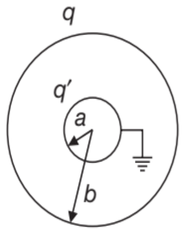
**2.** (a) Given, $a = 2\times 10^{-2}$ m, $b = 4\times 10^{-2}$ m and $q = 2\times 10^{-6}$ C

Let $q'$ charge comes on the inner sphere. Then,

$$V_{\text{inner}} = 0$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\left[\frac{q'}{a} + \frac{q}{b}\right] = 0$$

$$\Rightarrow \quad q' = -\frac{a}{b}\cdot q = -10^{-6}\ \text{C}$$

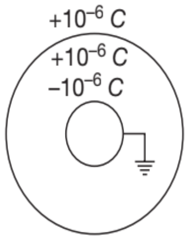



Now, $V_{\text{inner}} - V_{\text{outer}} = \dfrac{q'}{4\pi\varepsilon_0}\left(\dfrac{1}{a} - \dfrac{1}{b}\right)$

$$\Rightarrow \quad 0 - V_{\text{outer}} = -10^{-6}\times 10^9 \times 10^2 \times \frac{1}{4}$$

$$\Rightarrow \quad V_{\text{outer}} = 2.5\times 10^4\ \text{V}$$

(b) Charges appearing on different surfaces are as shown in figure. Hence the charge retained on the outer surface of the outer sphere is $+10^{-6}$ C or $+1\ \mu$C.




**3.** Let $q$, $v$ and $c$ be the charge, potential and capacitance of individual (small) drop. Also let $Q$, $V$, $C$ be the corresponding quantities for bigger drop.

As total charge is conserved, therefore the charge of bigger drop

$$Q = N \times \text{charge on small drop}$$

$$\Rightarrow \quad Q = Nq$$

$$\Rightarrow \quad \frac{Q}{q} = N \qquad \ldots(1)$$

Since the capacitance of a spherical drop is proportional to its radius $(C = 4\pi\varepsilon_0 r)$, therefore if $R$ and $r$ are radii of big drop and small drop respectively, then

$$\frac{C}{c} = \frac{R}{r} \qquad \ldots(2)$$

Now, since total mass of the drops is conserved, so, we have

Mass of bigger drop $= N\times$ Mass of small drop

$$\Rightarrow \frac{4}{3}\pi R^3 \rho = N\left(\frac{4}{3}\pi r^3 \rho\right) \quad \{\rho \text{ is density of mercury}\}$$

$$\Rightarrow R = N^{1/3} r$$

$$\Rightarrow \frac{C}{c} = \frac{R}{r} = N^{1/3} \quad \ldots(3)$$

$$\Rightarrow \frac{C}{c} = N^{\frac{1}{3}}$$

Again, by definition

$$V = \frac{Q}{C} \text{ and } v = \frac{q}{c}$$

$$\Rightarrow \frac{V}{v} = \frac{Q}{C}\cdot\frac{c}{q} = \left(\frac{Q}{q}\right)\left(\frac{c}{C}\right) = \frac{N}{N^{1/3}} = N^{2/3} \quad \ldots(4)$$

$$\Rightarrow \frac{V}{v} = N^{\frac{2}{3}}$$

Also, $U = \dfrac{Q^2}{2C}$ and $u = \dfrac{q^2}{2c}$

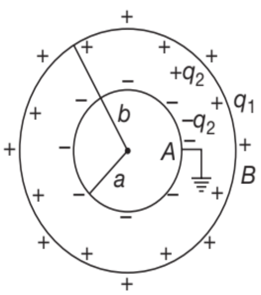
$$\Rightarrow \frac{U}{u} = \frac{\frac{Q^2}{2C}}{\frac{q^2}{2c}} = \left(\frac{Q}{q}\right)^2\left(\frac{c}{C}\right)$$

$$\Rightarrow \frac{U}{u} = N^2 \times N^{-\frac{1}{3}} = N^{\frac{5}{3}}$$

$$\Rightarrow \frac{U}{u} = N^{\frac{5}{3}}$$

**4.** Let $Q$ be the charge given to outer sphere $B$. This charge will be partly on outer surface and partly on inner surface. Let $q_1$ be charge on outer surface and $q_2$ on inner surface. Then

$$q_1 + q_2 = Q \quad \ldots(1)$$




The charge induced on inner sphere = $-q_2$

The potential of $B$,

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1 + q_2 - q_2}{b} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{b} \quad \ldots(2)$$

The potential of $A$,

$$V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{-q_2}{a} + \frac{q_1 + q_2}{b}\right)$$

As sphere $A$ is earthed $V_A = 0$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\left[\frac{-q_2}{a} + \frac{q_1 + q_2}{b}\right] = 0 \quad \ldots(3)$$

Equation (3) gives

$$\frac{-q_2}{a} + \frac{q_1 + q_2}{b} = 0 \text{ i.e., } \frac{-q_2}{a} + \frac{Q}{b} = 0$$

$$\Rightarrow \frac{q_2}{Q} = +\frac{a}{b} \quad \ldots(4)$$

Using this equation (1) gives

$$q_1 + \frac{a}{b}Q = Q$$

$$\Rightarrow q_1 = Q\left(1 - \frac{a}{b}\right) = \frac{b-a}{b}Q$$

i.e., $$\frac{Q}{q_1} = \frac{b}{b-a} \quad \ldots(5)$$

Potential difference between $A$ and $B$

$$V_{ab} = V_B - V_A = V_B - 0 = V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{b}$$

$\therefore$ Capacitance

$$C = \frac{Q}{V_{AB}} = \frac{Q}{\frac{1}{4\pi\varepsilon_0}\frac{q_1}{b}} = 4\pi\varepsilon_0 b\frac{Q}{q_1}$$

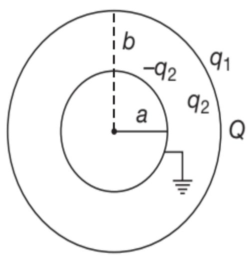
Using (5), we get

$$C = 4\pi\varepsilon_0 b\left(\frac{b}{b-a}\right) = \frac{4\pi\varepsilon_0 b^2}{b-a} \quad \{\text{done already}\}$$

**5.** Charge on inner sphere is given by

$$-q_2 = -\frac{a}{b}Q$$

$$\Rightarrow -q_2 = -\frac{8 \text{ cm}}{10 \text{ cm}} \times 30 \text{ nC} = -24 \text{ nC}$$



Charge on inner surface of outer sphere is

$$q_2 = +24 \text{ nC}$$

The charge $Q$ is distributed into $q_1$ and $q_2$ so that the electrostatic potential of inner sphere is zero.

6. The given arrangement is a parallel combination of two capacitors.
The capacity of a cylindrical capacitor is given by

$$C = \frac{2\pi\varepsilon_0 L}{\log_e\left(\frac{b}{a}\right)}$$

$$\Rightarrow \quad C_1 = \frac{2\pi\varepsilon_0 K_1\left(\frac{L}{2}\right)}{\log_e\left(\frac{b}{a}\right)}$$

$$\Rightarrow \quad C_2 = \frac{2\pi\varepsilon_0 K_2\left(\frac{L}{2}\right)}{\log_e\left(\frac{b}{a}\right)}$$

$$\Rightarrow \quad C_{eq} = C_1 + C_2 = \frac{\pi L}{\log_e\left(\frac{b}{a}\right)}(K_1 + K_2)$$

## Single Correct Choice Type Questions

1. When the switch is closed, the inner plates of the two capacitors get connected whereas the outer plates still are not connected and hence the circuit is not complete.
**Hence, the correct answer is (A).**

3. $C = k\varepsilon_0 \dfrac{A}{d}$

$$\Rightarrow \quad 1.77 \times 10^{-6} = 200(8.85 \times 10^{-12})\frac{A}{d}$$

$$\Rightarrow \quad \frac{A}{d} = 10^3$$

This ratio is satisfied by both **OPTIONS (A)** and **(B)**. So to arrive at a conclusion we take help of the break through strength.

FOR OPTION (A)

$$d = 10^{-6} m$$

$$\Rightarrow \quad E = \frac{V}{d}$$

$$\Rightarrow \quad E = \frac{20}{10^{-6}}$$

$$\Rightarrow \quad E = 2 \times 10^7 \ Vm^{-1} > 3 \times 10^6 \ Vm^{-1}$$

which is impossible

FOR OPTION (B)

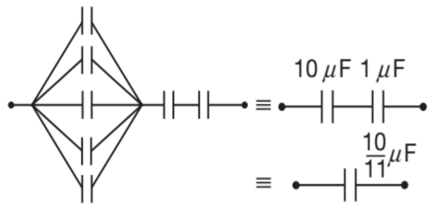
$$d = 10^{-5} \text{m}$$

$$\Rightarrow \quad E = \frac{V}{d}$$

$$\Rightarrow \quad E = \frac{20}{10^{-5}}$$

$$\Rightarrow \quad E = 2 \times 10^6 \ \text{Vm}^{-1} < 3 \times 10^6 \ \text{Vm}^{-1}$$

Hence **OPTION (B)** is right to give as an answer.
**Hence, the correct answer is (B).**

4. 


**Hence, the correct answer is (A).**

5. The circuit show is a balanced Wheat Stone Bridge.
**Hence, the correct answer is (D).**

6. The given system is equivalent to a spherical capacitor of inner radius $b$ and outer radius $c$. So, the capacitance of the system will be $4\pi\varepsilon_0\left(\dfrac{bc}{c-b}\right)$.
**Hence, the correct answer is (D).**

7. $C = 4\pi\varepsilon_0 R$

$$\Rightarrow \quad C = 711 \ \mu\text{F}$$

$$\Rightarrow \quad C \cong 1000 \ \mu\text{F}$$

$$\Rightarrow \quad C \cong 10^{-3} \text{ F} = 1 \text{ mF}$$

**Hence, the correct answer is (B).**

8. $Q_2 = -Q_3$ {By induction}

Further,

$$\Delta V = \frac{Q_2}{C} = \frac{2Q_2}{2C} = \frac{Q_2 - (-Q_2)}{2C}$$

$$\Rightarrow \quad \Delta V = \frac{Q_2 - Q_3}{2C}$$

**Hence, the correct answer is (C).**

10. On connecting both with a thin wire, the common potential V is given by

$$(C_1 + C_2)V = C_1V_1 + C_2V_2$$

$$\Rightarrow \quad V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

Total initial energy $= U_i = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$

Total final energy $= U_f = \frac{1}{2}(C_1 + C_2)V^2$

Loss = Total Initial Energy – Total Final Energy

$\Rightarrow$ $\text{Loss} = U_i - U_f$

$\Rightarrow$ $\text{Loss} = \frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)(V_1 - V_2)^2$

**Hence, the correct answer is (C).**

**12.** $C' = \frac{C}{1 - \frac{1}{2}}$ $\quad \left\{\because C' = \frac{C}{1 - \frac{t}{d}}\right\}$

$C' = 2C$

**Hence, the correct answer is (B).**

**14.** The capacitance of the capacitor increases initially and then decreases and hence the positive charge on plate $A$ first increases and then decreases. Due to this the current in the outer circuit first flows from $B$ to $A$ and then from $A$ to $B$.

**Hence, the correct answer is (C).**

**15.** $C_S = 2\ pF$

$q = C_SV$

$\Rightarrow$ $q = 10\ \mu C$

when connected in parallel

$C_P = C_1 + C_2$

$\Rightarrow$ $C_P = 9\ pF$

$\Rightarrow$ $V = \frac{q}{C_P}$

$\Rightarrow$ $V = \frac{10}{9}\ kV$

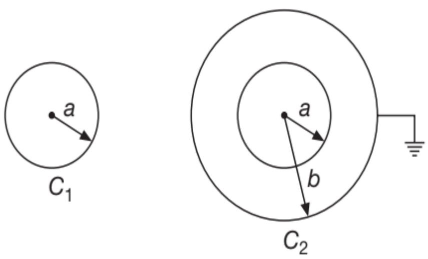
$\Rightarrow$ $V = 1111\ V$

**Hence, the correct answer is (D).**

**16.** $C_1 = 4\pi\varepsilon_0 a$

and $C_2 = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$

Given that $C_2 = nC_1$




$\Rightarrow$ $\frac{ab}{b-a} = na$

$\Rightarrow$ $\frac{b}{b-a} = n$

$\Rightarrow$ $b = nb - na$

$\Rightarrow$ $na = (n-1)b$

$\Rightarrow$ $\frac{b}{a} = \frac{n}{n-1}$

**Hence, the correct answer is (B).**

**17.** $\frac{1}{C} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots\ldots$

$\Rightarrow$ $\frac{1}{C} = \frac{1/2}{1 - 1/2}$

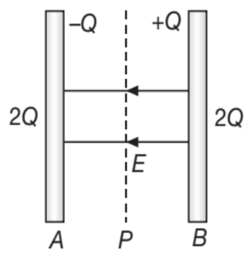
$\Rightarrow$ $C = 1$

**Hence, the correct answer is (D).**

**18.** Since, the charges on the facing surfaces are $-Q$ and $Q$ (used Gauss's Law to find them)

So, $E = \frac{Q}{A\varepsilon_0}$

$\Rightarrow$ $V_P - V_A = +\frac{Ed}{2}$




$\Rightarrow$ $V_P - V_A = \frac{Qd}{2A\varepsilon_0} = \frac{Q}{2\left(\frac{\varepsilon_0 A}{d}\right)} = \frac{Q}{2C_0}$

**Hence, the correct answer is (B).**

**19.** For series combination

$C_S = \frac{C_1C_2}{C_1 + C_2}$

$\Rightarrow$ $C_S = \frac{2}{3}\mu F$

When connected in series the maximum charge that can flow through the combination equals the lower value of charge accommodated by the first capacitor i.e., $6000\ \mu C$

$\therefore$ $Q_1 = 6000\ \mu C$ and $Q_2 = 8000\ \mu C$

$\Rightarrow \quad V_S = \dfrac{Q_1}{C_S}$

$\Rightarrow \quad V_S = \dfrac{6000\ \mu C}{\dfrac{2}{3}\ \mu C}$

$\Rightarrow \quad V_S = 9\ kV$

**Hence, the correct answer is (C).**

**20.** Let $C$ be the capacitance of the capacitor without slab. Then after removing the slab i.e., finally the net capacitance is $\dfrac{C}{2}$

$$q_1 = q_2 = \frac{CE}{2}$$

Before the slab is removed

$$C_1 = C \text{ and } C_2 = KC$$

$\Rightarrow \quad C_{\text{net}} = \left(\dfrac{K}{K+1}\right)C$

$\Rightarrow \quad Q_1 = Q_2 = \left(\dfrac{KCE}{K+1}\right)$

$\Rightarrow \quad \dfrac{q_2}{Q_2} = \dfrac{K+1}{2K}$

**Hence, the correct answer is (A).**

**21.** $U_i = \dfrac{1}{2}C_0E^2 \quad \ldots(1)$

On inserting a dielectric new capacitance is

$$C = kC_0 \quad \ldots(2)$$

and new potential difference is $\{V = Ed\}$

$E' = f_e'd$ where $f_e$ is the electrostatic field. But on inserting the dielectric, the new field $f_e'$ becomes $\dfrac{1}{k}$ times the original field.

So $\quad f_e' = \dfrac{f_e}{k}$

$\Rightarrow \quad E' = \dfrac{E}{k} \quad \ldots(3)$

So $\quad U_f = \dfrac{1}{2}CE'^2$

$\Rightarrow \quad U_f = \dfrac{1}{2}(kC_0)\left(\dfrac{E}{k}\right)^2$

$\Rightarrow \quad U_f = \left(\dfrac{1}{2}C_0E^2\right)\dfrac{1}{k}$

$\Rightarrow \quad \Delta U = \dfrac{1}{2}C_0E^2\left(\dfrac{1}{k} - 1\right)$

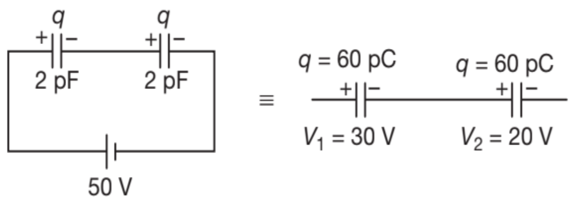
**Hence, the correct answer is (D).**

**22.** $k_1$ in series with half of $k_3$ and hence equivalent dielectric constant is $\dfrac{k_1k_3}{k_1+k_3}$

$k_2$ in series with half of $k_3$ and hence equivalent dielectric constant is $\dfrac{k_2k_3}{k_2+k_3}$ and then both in parallel to give $k = \dfrac{k_1k_3}{k_1+k_3} + \dfrac{k_2k_3}{k_2+k_3}$

**Hence, the correct answer is (D).**

**24.** Charges on the capacitors are




$q_1 = (30)(2) = 60$ pC

$\Rightarrow \quad q_2 = (20)(3) = 60$ pC

$\Rightarrow \quad q_1 = q_2 = q \quad$ (say)

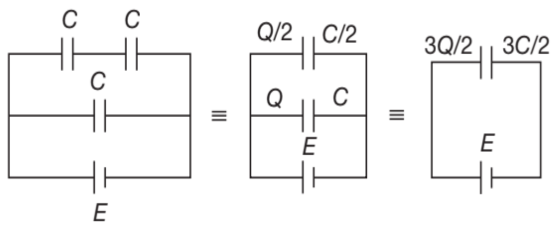
The situation is similar to the two capacitors in series which are first charged with a battery of emf 50 V and then disconnected.

So, when $S_3$ is closed, $V_1 = 30$ V and $V_2 = 20$ V

Other way of looking at the thing is, when $S_1$ and $S_3$ both are closed, due to attraction with opposite charge, no flow of charge takes place through $S_3$. Therefore, potential difference across capacitor plates remains unchanged or $V_1 = 30$ V and $V_2 = 20$ V.

**Hence, the correct answer is (D).**

**26.**




$\{\because Q = CE\}$

**Hence, the correct answer is (B).**

**27.** $W = \dfrac{1}{2}C(100 - 25) \qquad \left\{\because W = \dfrac{1}{2}C\left(V_2^2 - V_1^2\right)\right\}$

$\Rightarrow \quad W = \dfrac{75}{2}C \quad \ldots(1)$

$\Rightarrow \quad W' = \dfrac{1}{2}C(225 - 100)$

$\Rightarrow \quad W' = \dfrac{125}{2}C \quad \ldots(2)$

CHAPTER 2

From (1) and (2)

$$W' = \frac{5}{3}W$$

**Hence, the correct answer is (C).**

**28.** $V = 4\pi\sigma\left(\frac{t}{k_1} + \frac{d-t}{k_2}\right)$

$$\Rightarrow \quad V = \frac{4\pi Q}{A}\left(\frac{t}{k_1} + \frac{d-t}{k_2}\right)$$

**Hence, the correct answer is (B).**

**29.** $V_{\text{combined}} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$$\Rightarrow \quad V_{\text{combined}} = \frac{(1)(200) + 2(100)}{1+2}$$

$$V_{\text{combined}} = \frac{400}{3} \simeq 133\ V$$

**Hence, the correct answer is (C).**

**30.** $\text{Loss} = \frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)(V_1 - V_2)^2$

$$\Rightarrow \quad \text{Loss} = \frac{1}{2}\left(\frac{C}{2}\right)(V_1 - V_2)^2 = \frac{1}{4}C(V_1 - V_2)^2$$

**Hence, the correct answer is (C).**

**31.** During electrostatic equilibrium.
Electrostatic attraction between the plates = Spring force

$$\Rightarrow \quad \frac{q^2}{2\varepsilon_0 A} = kx$$

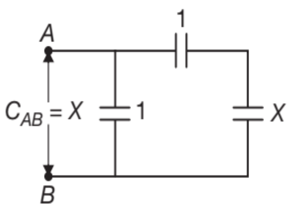
$$\Rightarrow \quad \frac{(CV)^2}{2\varepsilon_0 A} = k\left(d - \frac{4d}{5}\right) \qquad \{\because q = CV\}$$

$$\Rightarrow \quad \frac{\left(\frac{\varepsilon_0 A}{\left(\frac{4d}{5}\right)}\right)^2 V^2}{2\varepsilon_0 A} = 0.2kd$$

$$\Rightarrow \quad k = \frac{\varepsilon_0 AV^2}{0.256d^3} \approx \frac{4\varepsilon_0 AV^2}{d^3}$$

**Hence, the correct answer is (C).**

**32.** $\frac{X(1)}{X+1} + 1 = X$




$$\Rightarrow \quad X = X^2 - 1$$

$$\Rightarrow \quad X^2 - X - 1 = 0$$

$$\Rightarrow \quad X = \frac{1+\sqrt{5}}{2}$$

$$\Rightarrow \quad X = \frac{1+2.236}{2}$$

$$\Rightarrow \quad X \simeq \frac{3.2}{2}$$

$$\Rightarrow \quad X \simeq 1.6\ \mu F$$

**Hence, the correct answer is (C).**

**33.** Let $d$ be the thickness of the slab, then

$$t = \frac{d}{2}$$

$$\Rightarrow \quad \frac{\varepsilon_0 A}{d - \frac{d}{2} + \frac{d}{2K}} = \frac{4}{3}\left(\frac{\varepsilon_0 A}{d}\right)$$

$$\Rightarrow \quad \frac{1}{\frac{d}{2}\left(1 + \frac{1}{K}\right)} = \frac{4}{3d}$$

$$\Rightarrow \quad 1 + \frac{1}{K} = \frac{3}{2}$$

$$\Rightarrow \quad K = 2$$

**Hence, the correct answer is (A).**

**34.** $C_{AB} = \frac{3C}{2}$

Total potential across the series combination of 2C, C and 2C is 90 V. If Q is the total charge then

$$Q = C_{AB}(90)$$

$$\Rightarrow \quad Q = 135C$$

Charge in SR branch $= Q_1 = 90C$

Charge in SPQR branch $= Q_2 = 45C$

$\Rightarrow \quad V_{MN} =$ Potential across C

$$\Rightarrow \quad V_{MN} = \frac{Q_2}{C} = 45\ V$$

**Hence, the correct answer is (C).**

**35.** The spring force $\vec{F}_S$ acting on plate $a$ is given by

$$\vec{F}_S = -kx\hat{i}$$

Similarly, the electrostatic force $\vec{F}_e$ due to the electric field created by plate $b$ is

$$\vec{F}_e = QE\hat{i} = Q\left(\frac{\sigma}{2\varepsilon_0}\right)\hat{i} = \frac{Q^2}{2A\varepsilon_0}\hat{i}$$

where $A$ is the area of the plate. Notice that charges on plate $a$ cannot exert a force on itself, as required by Newton's Third Law. Thus, only the electric field due to plate $b$ is considered. At equilibrium the two forces cancel and we have

$$kx = Q\left(\frac{Q}{2A\varepsilon_0}\right)$$

which gives

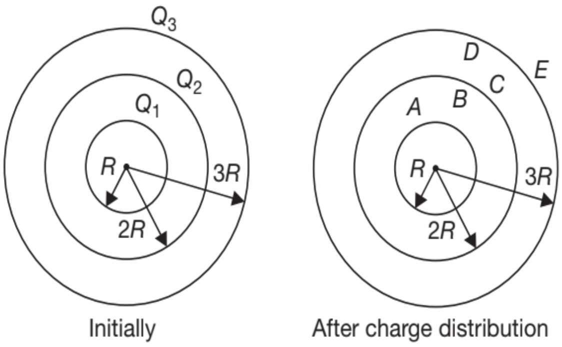
$$x = \frac{Q^2}{2kA\varepsilon_0}$$

**Hence, the correct answer is (C).**

**36.** The charge distribution on the surfaces of the shells are given. As per the given condition, the surface charge densities of the outer surfaces are equal. So,

$$\frac{Q_1}{4\pi R^2} = \frac{Q_1 + Q_2}{4\pi(2R)^2} = \frac{Q_1 + Q_2 + Q_3}{4\pi(3R)^2}$$

$$\Rightarrow \quad \frac{Q_1}{1} = \frac{Q_2}{3} = \frac{Q_3}{5}$$



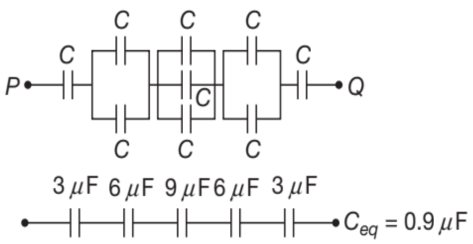

$q_A = Q_1$ $\quad$ $q_D = -(Q_2 + Q_1)$

$q_B = -Q_1$ $\quad$ $q_E = Q_3 + Q_1 + Q_2$

$q_C = Q_2 + Q_1$

**Hence, the correct answer is (B).**

**37.**




**Hence, the correct answer is (C).**

**38.** $V = \dfrac{q}{C}$

Since $C \propto \dfrac{1}{d}$

So, as $d$ is halved, C becomes double

$$\Rightarrow \quad V' = \frac{q}{C'}$$

$$\Rightarrow \quad \frac{V}{V'} = \frac{C'}{C} = 2$$

$$\Rightarrow \quad V' = \frac{V}{2} = 250\ V$$

$$U_i = \frac{1}{2}CV^2$$

$$U_f = \frac{1}{2}C'V'^2 = \frac{1}{2}(2C)\left(\frac{V}{2}\right)^2$$

$$U_f = \frac{1}{4}CV^2$$

$$\Rightarrow \quad Change = \frac{1}{4}CV^2$$

$$\Rightarrow \quad \text{Change} = \frac{1}{4}(100\times10^{-6})(500)^2$$

$$\Rightarrow \quad \Delta U = 6.25\ J$$

**Hence, the correct answer is (B).**

**39.** $C = \dfrac{\varepsilon_0 A}{d}$

$$\Rightarrow \quad C = \frac{1}{4\pi\times9\times10^9}\ \frac{\pi(0.08)^2}{10^{-3}}$$

$$\Rightarrow \quad C = 1.8\times10^{-10}\ \text{F}$$

Since $Q = CV$

$$\Rightarrow \quad Q = 1.8\times10^{-8}\ \text{C}$$

**Hence, the correct answer is (B).**

**40.** Total Initial Charge

$$Q_i = (2C)(2V) - CV = 3CV$$

Total Final Charge

$$Q_f = 2CV' + CV'$$

where $V'$ is common potential.

By Law of Conservation of Charge

$$Q_i = Q_f$$

$$\Rightarrow \quad 3CV = 3CV'$$

$$\Rightarrow \quad V' = V$$

So, final energy of combination is

$$U_f = \frac{1}{2}(C + 2C)V^2 = \frac{3}{2}CV^2$$

**Hence, the correct answer is (B).**

**41.** $C = 4\pi\varepsilon_0\left(\dfrac{ab}{b-a}\right)$

$$C = \frac{1}{9\times10^9}\ \frac{(0.5)(0.6)}{0.1}$$

$$\Rightarrow \quad C = \frac{0.3}{9\times10^8}$$

$\Rightarrow \quad C' = kC$

$\Rightarrow \quad C' = 2\times10^{-9}$ F

**Hence, the correct answer is (B).**

**42.** The $3\ \mu$F capacitor has energy $600\ \mu$J

$$\Rightarrow \quad 600\times10^{-6} = \frac{1}{2}(3\times10^{-6})V_0^2$$

$\Rightarrow \quad V_0 = 20$ V

$\Rightarrow \quad V_{AB} = 3V_0 = 60$ V

This 60 V has to divide in between $2\ \mu$F and $6\ \mu$F in the inverse ratio of their capacitances i.e., $3:1$.

$$\text{So, } V_{2\ \mu\text{F}} = \left(\frac{3}{3+1}\right)60\text{ V} = 45\text{ V}$$

**Hence, the correct answer is (C).**

**45.** The arrangement is a balanced Wheatstone Bridge. So, no charge will exist across the branch PQ. Hence

$C_{net} = 10\ \mu F$

**Hence, the correct answer is (D).**

**47.** On connecting by a wire, let V be the common potential. Then, by Law of Conservation of Charge

$Q_{initial} = Q_{final}$

$\Rightarrow \quad C_1V_1 + C_2V_2 = C_1V + C_2V$

$$\Rightarrow \quad V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

If $E_i$ be total initial energy and $E_f$ be the total final energy, then

$\text{Loss} = E_i - E_f$

$$\Rightarrow \quad \text{Loss} = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}(C_1 + C_2)V^2$$

$$\Rightarrow \quad \text{Loss} = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

We get above result by putting value of V and then solving.

**Hence, the correct answer is (C).**

**48.** $C_P = C + \frac{C}{2} + \frac{C}{4} + \frac{C}{8} + \ldots$

$$\Rightarrow \quad C_P = C\left(\frac{1}{1-\frac{1}{2}}\right)$$

$\Rightarrow \quad C_P = 2C = 2\ \mu F$

**Hence, the correct answer is (C).**

**49.** Total initial energy is

$$E_i = 2\left(\frac{1}{2}CV^2\right) = CV^2$$

Initial charge across $A$ is $CV$

Initial charge across $B$ is $CV$

On, opening the switch when a dielectric with $k = 3$ is introduced in both $A$ and $B$, then

$C_A = 3C$ and $C_B = 3C$

The new potential across $A$ is still $V$. Let $V'$ be new potential across $B$. So, new charge across $B$ is $(3C)V'$.

By Law of Conservation of Charge

$CV = (3C)V'$

$$\Rightarrow \quad V' = \frac{V}{3}$$

So, total final energy is

$$E_f = \frac{1}{2}(3C)V^2 + \frac{1}{2}(3C)\left(\frac{V}{3}\right)^2$$

$$\Rightarrow \quad E_f = \frac{3}{2}CV^2\left(1 + \frac{1}{9}\right)$$

$$\Rightarrow \quad E_f = \frac{5}{3}CV^2$$

$$\Rightarrow \quad \frac{E_i}{E_f} = \frac{3}{5}$$

**Hence, the correct answer is (B).**

**50.** $C_s = \frac{4\times6}{4+6}\ \mu F$

$\Rightarrow \quad C_s = 2.4\ \mu F$

$q = C_sV = 2.4\times500\times10^{-6}$

$q = 1200\ \mu C$

**Hence, the correct answer is (C).**

**52.** $Q_1 = C_0V_0$

$Q_2 = (kC_0)V_0$

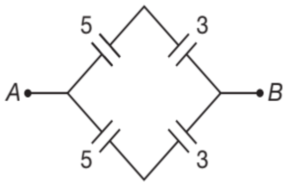
$$\Rightarrow \quad \frac{Q_1}{Q_2} = \frac{1}{k}$$

$$\Rightarrow \quad \frac{40}{100} = \frac{1}{k}$$

$\Rightarrow \quad k = 2.5$

**Hence, the correct answer is (B).**

**53.** It is a balanced Wheatstone Bridge.




$$\Rightarrow \quad C_{AB} = \frac{5\times 3}{5+3} + \frac{5\times 3}{5+3}$$

$$\Rightarrow \quad C_{AB} = 2\times\frac{15}{8}$$

$$\Rightarrow \quad C_{AB} = 3.75\ \mu F$$

**Hence, the correct answer is (A).**

**54.** Initial charge on first capacitor is $CV = Q_1$

Initial charge on second capacitor is $2CV = Q_2$

Final capacitance of first capacitor is $KC$

If $V'$ is the common potential then

$$V' = \frac{Q_1+Q_2}{C_1'+C_2}$$

$$\Rightarrow \quad V' = \frac{CV+2CV}{KC+2C}$$

$$\Rightarrow \quad V' = \frac{3V}{2+K}$$

**Hence, the correct answer is (A).**

**55.** $U_i = \dfrac{q^2}{2C_i} = \dfrac{q^2}{2(3C_0)}$

$$U_f = \frac{q^2}{2C_f} = \frac{q^2}{2(2C_0)}$$

$$\Rightarrow \quad \frac{U_f}{U_i} = \frac{3}{2}$$

**Hence, the correct answer is (B).**

**56.** By Law of Conservation of Charge

$$C_1V = C_1V' + C_2V'$$

$$\Rightarrow \quad V' = \left(\frac{C_1}{C_1+C_2}\right)V$$

**Hence, the correct answer is (A).**

**57.** $U_i = U = \dfrac{1}{2}C_1V^2$

$$U_f = \frac{1}{2}C_1V'^2 + \frac{1}{2}C_2V'^2$$

$$\Rightarrow \quad \frac{U_f}{U} = \left(\frac{C_1+C_2}{C_1}\right)\frac{V'^2}{V^2}$$

$$\Rightarrow \quad U_f = \left[\frac{C_1+C_2}{C_1}\left(\frac{C_1}{C_1+C_2}\right)^2\right]U$$

$$\Rightarrow \quad U_f = \left(\frac{C_1}{C_1+C_2}\right)U$$

**Hence, the correct answer is (A).**

**58.** While drawing the dielectric plate outside, the capacitance decreases till the entire plate comes out and then becomes constant.

So, V increases and then becomes constant.

**Hence, the correct answer is (B).**

**59.** $C_2$ and $C_3$ in parallel

$\Rightarrow \quad V_2 = V_3$. Also,

$$V = V_1 + V_2 = V_1 + V_3$$

$$Q_2 + Q_3 = Q_1$$

**Hence, the correct answer is (C).**

**60.** When we move from left to right the voltage increases and this increase is just within the capacitors and remains constant in the conducting wires. Also for series combination $V \propto \dfrac{1}{C}$ and according to graph $V_2 > V_1$

$$\Rightarrow \quad C_1 > C_2$$

**Hence, the correct answer is (C).**

**61.** By Law of Conservation of Charge

$$Q_{\text{initial}} = Q_{\text{final}}$$

$$Q_{\text{initial}} = 5\times 12 = 60\ \mu C$$

$$Q_{\text{final}} = (C+5)3\ \mu C$$

$$\Rightarrow \quad (C+5)3 = 60$$

$$\Rightarrow \quad C = 15\ \mu F$$

**Hence, the correct answer is (B).**

**62.** Potential difference across the branch de is 6 V.

Net capacitance of de branch is $2.1\ \mu F$.

So, $q = CV$

$$\Rightarrow \quad q = 2.1\times 6\ \mu C$$

$$\Rightarrow \quad q = 12.6\ \mu C$$

Potential across $3\ \mu F$ capacitance is

$$V = \frac{12.6}{3} = 4.2\ volt$$

Potential across 2 and 5 combination in parallel is $6 - 4.2 = 1.8\ V$

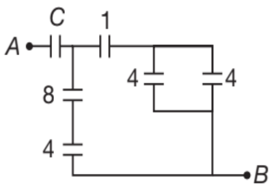
So,

$$q' = (1.8)(5) = 9\ \mu C$$

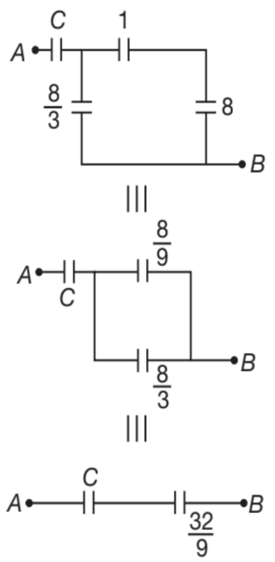
**Hence, the correct answer is (B).**

**63.** Since all are in series.
**Hence, the correct answer is (D).**

**64.**




Next equivalent circuit diagram is




$$\Rightarrow \frac{\frac{32}{9}C}{\frac{32}{9}+C} = 1$$

$$\Rightarrow C = \frac{32}{23}\ \mu F$$

**Hence, the correct answer is (A).**

**65.** $\frac{1}{C_s} = 1 + \frac{1}{2} + \frac{1}{4} + \ldots$

$$\frac{1}{C_s} = \frac{1}{1-\frac{1}{2}} = 2$$

$$\Rightarrow C_s = 0.5\ \mu F$$

**Hence, the correct answer is (C).**

**66.** $C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$

$$\Rightarrow C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

**Hence, the correct answer is (C).**

**68.** $V_0 = \frac{q}{C_0}$

$$V = \frac{q}{C}$$

$$\Rightarrow \frac{V}{V_0} = \frac{C_0}{C}$$

$$\Rightarrow \frac{C}{C_0} = \frac{500}{75} = \frac{20}{3}$$

By definition

$$C = kC_0$$

$$\Rightarrow k = \frac{20}{3}$$

**Hence, the correct answer is (C).**

**69.** $\frac{\sigma}{\varepsilon_0} d = \frac{\sigma}{\varepsilon_0}\left(d + 2.4 - 3 + \frac{3}{k}\right)$

$$\Rightarrow d = d - 3 + 2.4 + \frac{3}{k}$$

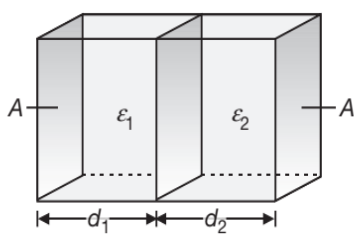
$$\Rightarrow 3 - 2.4 = \frac{3}{k}$$

$$\Rightarrow 0.6 = \frac{3}{k}$$

$$\Rightarrow k = 5$$

**Hence, the correct answer is (B).**

**70.**



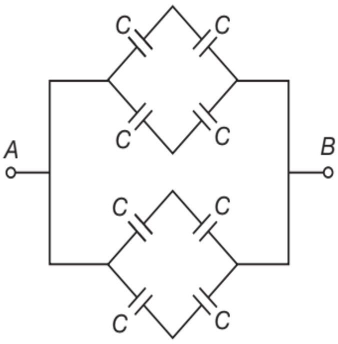

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \frac{1}{C_s} = \frac{1}{\frac{\varepsilon_1 A}{d_1} + \frac{\varepsilon_2 A}{d_2}}$$

$$\Rightarrow \frac{1}{C_s} = \frac{1}{A}\left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right)$$

$$\Rightarrow \quad C_s = \frac{A\varepsilon_1\varepsilon_2}{\varepsilon_2 d_1 + \varepsilon_1 d_2}$$

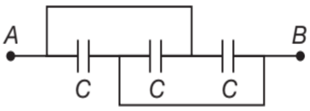
**Hence, the correct answer is (A).**

**71.** 


The figure shows two independent balanced Wheatstone Bridges connected in parallel each having a capacitance C. So,

$$C_{net} = C_{AB} = 2C$$

**Hence, the correct answer is (B).**

**72.** 


$$C_{AB} = 3C$$

The circuit contains such two combinations in series. Hence

$$C_{\text{total}} = \frac{(3C)(3C)}{3C+3C}$$

$$\Rightarrow \quad C_{\text{total}} = \frac{3C}{2}$$

**Hence, the correct answer is (B).**

**73.** (A), (B) and (C) are different forms of wheatstone bridge. **Hence, the correct answer is (D).**

**74.** When disconnected from the source, $q = \text{constant}$.

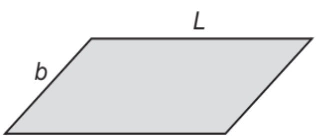
When immersed in dielectric, we have $C' = KC$. So, we get

$$V' = \frac{q}{C'} = \frac{q}{KC} = \frac{V}{K}$$

Hence $E' = Vd = \dfrac{Vd}{K} = \dfrac{Vd}{K} = \dfrac{E}{K}$

**Hence, the correct answer is (A).**

**75.** Let area of plate be $A$, then $A = bL$, where $b$ is width of plate




On partial insertion of dielectric, the net capacitance of capacitor is

$$C = \frac{\varepsilon_0 b(L-x)}{d} + \frac{K\varepsilon_0 bx}{d}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 b}{d}[L + (K-1)x]$$

Now $U = \dfrac{q^2}{2C}$ $\qquad \{\because q = \text{constant}\}$

Since, $F = -\dfrac{dU}{dx}$, so we get

$$dU = \frac{q^2}{2}(-1)C^{-2}\frac{dC}{dx}$$

$$\Rightarrow \quad dU = -\frac{q^2}{2C^2}\frac{d}{dx}\left\{\frac{\varepsilon_0 b}{d}(L+(K-1)x)\right\}$$

$$\Rightarrow \quad dU = -\frac{q^2}{2\left(\frac{\varepsilon_0 b}{d}\right)^2}\frac{\frac{\varepsilon_0 b}{d}(K-1)dx}{[L+(K-1)x]^2}$$

$$\Rightarrow \quad F = -\frac{dU}{dx} = \frac{q^2(K-1)d}{2\varepsilon_0 b[L+(K-1)x]^2}$$

$$\Rightarrow \quad F = \frac{q^2(K-1)}{2\left(\frac{\varepsilon_0(bL)}{d}\right)L\left(1+\left(\frac{K-1}{L}\right)x\right)^2}$$

But $\dfrac{\varepsilon_0(bL)}{d} = C_0$

$$\Rightarrow \quad F = \frac{q^2(K-1)}{2C_0 L\left(1+\left(\frac{K-1}{L}\right)x\right)^2}, \text{ attractive}$$

**Hence, the correct answer is (C).**

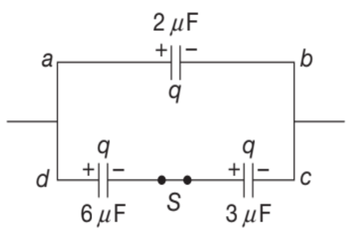
**76.** Potential across the first capacitor is V, equal to emf of the battery.

Thus no charge will further flow through the circuit.

**Hence, the correct answer is (D).**

**77.** As soon as the switch is closed, for the charge to flow in the circuit (through $S$), there must exist a potential difference in the circuit. Here, we observe, that for the closed loop $abcd$, we have

$$\Delta V = -\frac{q}{2} + \frac{q}{3} + \frac{q}{6} = 0$$

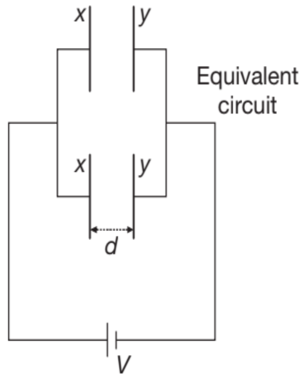


Hence, no charge will flow through $S$, when closed.

**Hence, the correct answer is (A).**

**78.** Equivalent capacitance of the circuit

$$C = 2\left(\frac{\varepsilon_0 A}{d}\right)$$




Energy stored, $U = \frac{1}{2}CV^2$

$$\Rightarrow \quad U = \frac{1}{2}\left(\frac{2(\varepsilon_0 A)}{d}\right)V^2 = \frac{\varepsilon_0 A V^2}{d}$$

**Hence, the correct answer is (B).**

**79.** Starting from positive plate, at higher potential (say $V_0$) and going to negative plate at lower potential, we have potential after covering a

(a) distance $d$ in air, is $V_1 = V_0 - E_0 d$ where $E_0 = \frac{\sigma}{\varepsilon_0}$.

(b) further $d$ in conductor, is still $V_1$, because field inside conductor is zero and hence potential stays constant.

(c) further $d$ in air is $V_2 = V_1 - E_0 d$ where $E_0 = \frac{\sigma}{\varepsilon_0}$.

(d) further $d$ in dielectric is $V_3 = V_2 - Ed$ where $E = \frac{\sigma}{K\varepsilon_0} (< E_0)$

(e) last $d$ in air is $V_4 = V_3 - E_0 d$ where $E_0 = \frac{\sigma}{\varepsilon_0}$.

In all the cases except (b), potential decreases linearly with field and in (d) the field has lesser value than cases (a), (c) and (e). So OPTION (C) is correct.

**Hence, the correct answer is (C).**

**80.** $\frac{4}{3}\left(\frac{\varepsilon_0 A}{d}\right) = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}}$, where $t = \frac{d}{2}$

$$\Rightarrow \quad \frac{4}{3}\left(\frac{\varepsilon_0 A}{d}\right) = \frac{\varepsilon_0 A}{d - \frac{d}{2} + \frac{d}{2k}}$$

$$\Rightarrow \quad \frac{4}{3d} = \frac{1}{\frac{d}{2}\left(1 + \frac{1}{K}\right)}$$

$$\Rightarrow \quad 1 + \frac{1}{K} = \frac{3}{2}$$

$$\Rightarrow \quad K = 2$$

**Hence, the correct answer is (A).**

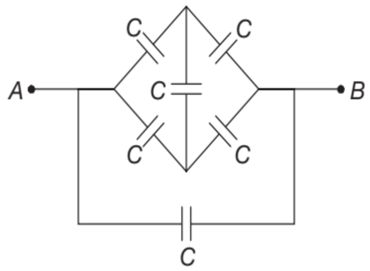
**81.** $F = \frac{q^2}{2\varepsilon_0 A}$

$$\Rightarrow \quad F \propto q^2$$

Since capacitor stays connected to the battery, so $V =$ constant. Hence when separation is halved, $C$ becomes twice. So, $q(= CV)$ becomes twice. Hence $F$ becomes four times.

**Hence, the correct answer is (C).**

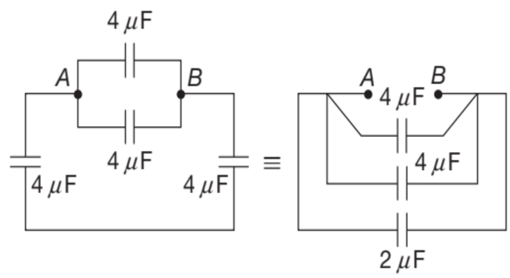
**82.** This circuit is a balanced Wheatstone Bridge with one capacitor connected in parallel to the bridge. So, the equivalent circuit is shown here. The net capacitance of the circuit between the points A and B is $2C$.




**Hence, the correct answer is (D).**

**83.** Since both free terminals are earthed, hence they being at same potential can be connected to give the following equivalent circuit.

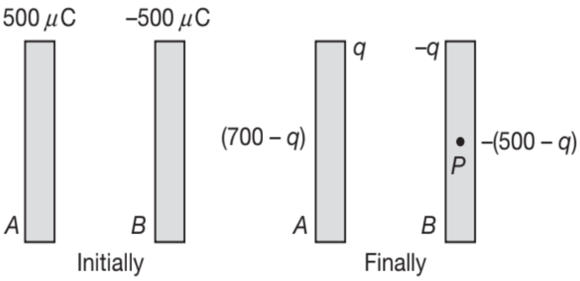
Now, all three being in parallel between A and B, will given net capacitance as $10\ \mu\text{F}$.




**Hence, the correct answer is (D).**

**84.** Charge attained by the plates of capacitor is

$$q_0 = CV_0 = (10\ \mu\text{F})(50\ \text{V}) = 500\ \mu\text{C}$$




Now, when a charge $200\ \mu C$ is given to the positive plate of capacitor, then the net positive charge becomes $700\ \mu C$, while charge on negative plate is stil $-500\ \mu C$.

Let a charge $q$ appear on inner surface of $A$, then surface of $B$ facing $A$ gets a charge $-q$. Such that a charge $(700-q)$ and $-(500-q)$ appears on outer surfaces of A and B respectively. Consider a point $P$ inside plate $B$. Then net field at $P$, inside the conductor is zero. So

$$E_P = 0$$

$$\Rightarrow \quad \frac{700-q}{2\varepsilon_0 A} + \frac{q}{2\varepsilon_0 A} - \frac{q}{2\varepsilon_0 A} + \frac{500-q}{2\varepsilon_0 A} = 0$$

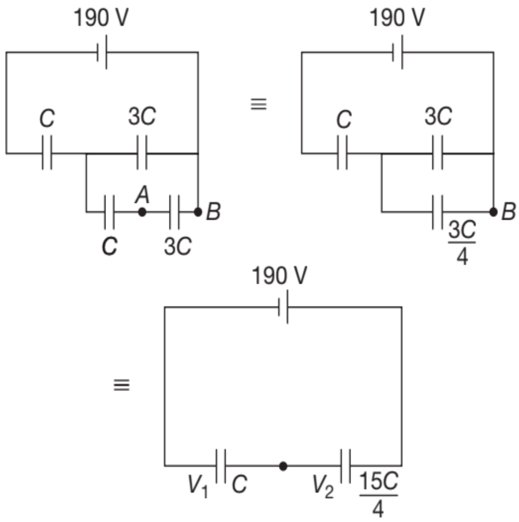
$$\Rightarrow \quad q = 600\ \mu C$$

So, potential difference between the plates is

$$\Delta V = \frac{q}{C} = \frac{600\ \mu C}{10\ \mu F} = 60\ V$$

**Hence, the correct answer is (B).**

**85.** $C_{net} = \dfrac{15C}{19}$



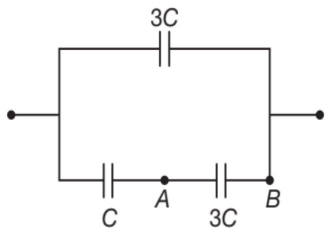

$$\Rightarrow \quad q = C_{net} V$$

$$\Rightarrow \quad q = \left(\frac{15C}{19}\right)(190) = 150C$$

Now, potential across $\dfrac{15C}{4}$ is

$$V_2 = \frac{q}{\left(\frac{15C}{4}\right)} = \frac{150C}{\frac{15C}{4}} = 40\ V$$

So, potential across $3C$ and combination of $3C$ and $C$ is also $40\ V$.

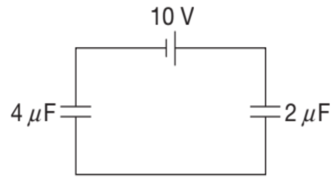



This $40\ V$ is distributed between $C$ and $3C$ (in series), in the inverse ratio of their capacitance i.e., $3:1$. Hence

$$V_{AB} = 10\ V$$

**Hence, the correct answer is (A).**

**86.** The equivalent circuit is shown here for convenience




Potential across each capacitor is $5\ V$ (equally divided)

$$\Rightarrow \quad q = CV = (4\ \mu F)(5\ V) = 20\ \mu C$$

**Hence, the correct answer is (B).**

**87.** Using Kirchhoffs' Law, we get

$$8 - \frac{q_1}{C_1} - 2 - \frac{q_3}{C_3} - 3 = 0$$

Since, $C_1 = C_3 = 2\ \mu F$

$$\Rightarrow \quad q_1 + q_2 = 6 \qquad \ldots(1)$$

Also

$$8 - \frac{q_1}{C_1} + \frac{q_2}{C_2} - 2 = 0$$

Since $C_1 = C_2 = 2\ \mu F$

$$\Rightarrow \quad q_1 - q_2 = 12 \qquad \ldots(2)$$

Also, we observe that

$$-q_1 - q_2 + q_3 = 0$$

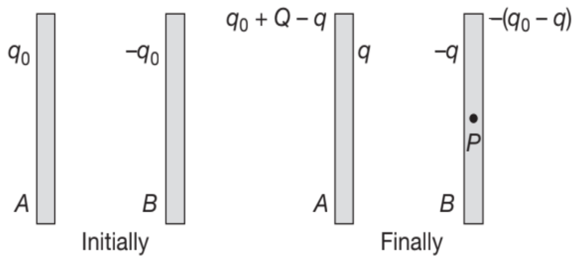
$$\Rightarrow \quad q_1 + q_2 = q_3 \qquad \ldots(3)$$

Solving equations (1), (2) and (3) we get

$$q_1 = 6\ \mu C,\ q_2 = -6\ \mu C \text{ and } q_3 = 0$$

**Hence, the correct answer is (C).**

**88.** Initial charge on plates of capacitor will be $q_0(say)$. Then $q_0 = CV$




When charge $Q$ is given to the positive plate of capacitor, then the net positive charge becomes $q_0 + Q$ while charge on negative plate is still $-q_0$. Finally the charge distribution on the plates is also shown such that both inner faces have charge $q$ and $-q$ respectively. When a point $P$ is considered inside plate $B$, then net field at $P$, inside the conductor, is always zero. So

$$E_P = 0$$

$$\Rightarrow \frac{1}{2\varepsilon_0 A}\left[(q_0 + Q - q) + q - q + (q_0 - q)\right] = 0$$

$$\Rightarrow q_0 + Q + q_0 - 2q = 0$$

$$\Rightarrow 2q = 2q_0 + Q$$

Since $q_0 = CV$, so we get

$$q = CV + \frac{Q}{2}$$

So, potential difference between the plates is

$$\Delta V = \frac{q}{C} = \frac{CV + \frac{Q}{2}}{C} = V + \frac{Q}{2C}$$

**Hence, the correct answer is (C).**

## Problem Solving Technique(s)

We can also arrive at the result by using the concept that

**(a)** Charges on the surfaces facing each other are equal and opposite.
**(b)** Charges on the outer surfaces are equal.
**(c)** Capacitance of a parallel plate capacitor depends upon its shape i.e., A and d, but not on charge distribution.

So, we have

$$q_0 + Q - q = -(q_0 - q), \text{ where } q_0 = CV$$

$$CV + Q - q = -CV + q$$

$$\Rightarrow 2q = 2CV + Q$$

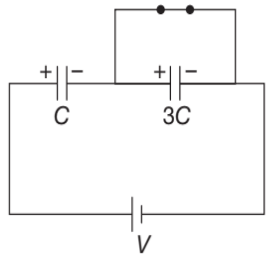
$$\Rightarrow q = CV + \frac{Q}{2}$$

$$\Rightarrow \Delta V = \frac{q}{C} = \frac{CV + \frac{Q}{2}}{C} = V + \frac{Q}{2C}$$

**89.** In the circuit $C_{net} = \frac{3C}{4}$

When key $K$ was open, charge stored in the capacitors was $q_{\text{initial}} = \frac{3CV}{4}$

When key $K$ is closed, capacitor $3C$ becomes short circuited and hence the new charge on $C$ is $q_{\text{final}} = CV$




So, charge flowing is

$$\Delta q = q_{\text{final}} - q_{\text{initial}}$$

$$\Rightarrow \Delta q = CV - \frac{3CV}{4} = \frac{CV}{4}$$

**Hence, the correct answer is (C).**

**90.** Energy stored in capacitor is, $\frac{1}{2}CV^2 = 3\text{ J}$

On connecting this capacitor to an uncharged capacitor, since charge distributes equally, hence both capacitors must be same capacitance.

Now common potential, $V' = \frac{CV + C(0)}{C + C} = \frac{V}{2}$

Total energy stored in two capacitors is

$$U' = \frac{1}{2}CV'^2 + \frac{1}{2}CV'^2 = C\left(\frac{V}{2}\right)^2 = \frac{1}{4}CV^2$$

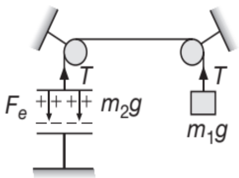
$$\Rightarrow U' = \frac{3}{2} = 1.5\text{ J}$$

**Hence, the correct answer is (A).**

**91.** $C_1 + C_2 = \frac{25}{6}\left(\frac{C_1C_2}{C_1 + C_2}\right)$

**Hence, the correct answer is (A).**

**92.**




For upper plate, we have

$$T = F_e + m_2g \qquad \ldots(1)$$

For $m_1$, we have

$$T = m_1g \qquad \ldots(2)$$

Equating (1) and (2), we get

$$m_1g = \frac{q^2}{2A\varepsilon_0} + m_2g$$

$$\Rightarrow \quad m_1 = \frac{q^2}{2A\varepsilon_0 g} + m_2$$

**Hence, the correct answer is (C).**

**93.** $V_{C_1} = 10 - 4 = 6\text{ V}$

$$V_{C_2} = 4 - 0 = 4\text{ V}$$

Since, in series, we have

$$\frac{V_{C_1}}{V_{C_2}} = \frac{C_2}{C_1}$$

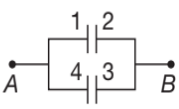
$$\Rightarrow \quad \frac{C_1}{C_2} = \frac{V_{C_2}}{V_{C_1}} = \frac{4}{6} = \frac{2}{3}$$

**Hence, the correct answer is (B).**

**94.** It is a Wheatstone bridge

**Hence, the correct answer is (A).**

**95.**




$$C_{eq} = \frac{2\varepsilon_0 A}{d}$$

**Hence, the correct answer is (B).**

**96.** The capacitor is charged by a battery of $25\text{ V}$. Let the magnitude of surface charge density on each plate be $\sigma$. Before inserting the dielectric slab, electric field strength between the plates,

$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{V}{d}$$

$$\Rightarrow \quad E_0 = \frac{\sigma}{\varepsilon_0} = \frac{25}{5\times10^{-3}} = 5000\text{ NC}^{-1}$$

The capacitor is disconnected from the battery but charge on it will not change so that $\sigma$ has the same value. When a dielectric slab of thickness $3\text{ mm}$ is placed between the plates, the thickness of air between the plates will be

$$t = 5 - 3 = 2\text{ mm}$$

Electric field strength in air will have the same value $(5000\text{ NC}^{-1})$ but inside the dielectric, it will be

$$E = \frac{5000}{K} = \frac{5000}{10} = 500\text{ NC}^{-1}$$

So potential difference is $V = E_{\text{air}}d_{\text{air}} + E_{\text{med}}d_{\text{med}}$

$$\Rightarrow \quad V = 5000\times(2\times10^{-3}) + 500\times(3\times10^{-3})$$

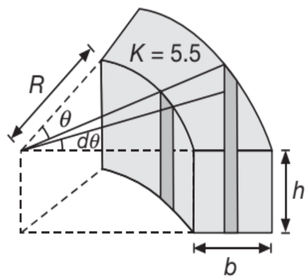
$$\Rightarrow \quad V = 11.5\text{ V}$$

**Hence, the correct answer is (C).**

**97.** $dC = \frac{\varepsilon_0 KRd\theta\times h}{b}$

Since all small $dC$ are in parallel, so

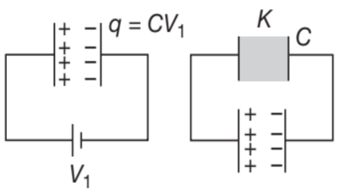
$$C_{eq} = \Sigma dC = \int_{\theta=0}^{\frac{\pi}{6}} dC$$




$$\Rightarrow \quad C_{eq} = \frac{6KhR}{b}\int_{\theta=0}^{\frac{\pi}{6}} d\theta = \frac{\pi}{6}\left(\frac{\varepsilon_0 KhR}{b}\right)$$

**Hence, the correct answer is (A).**

**99.**




$$V = \frac{q_1 + q_2}{C_1 + C_2}$$

$$\Rightarrow \quad V = \frac{CV_1}{C + KC}$$

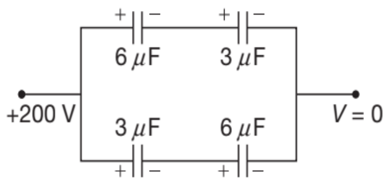
$$\Rightarrow \quad CV + KCV = CV_1$$

$$\Rightarrow \quad K = \frac{CV_1 - CV}{CV}$$

$$\Rightarrow \quad K = \frac{V_1 - V}{V}$$

**Hence, the correct answer is (D).**

**100.** **When switch is open**

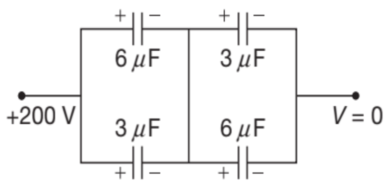


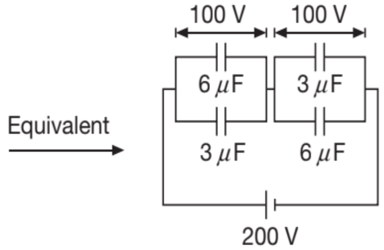

$q = C_{eq}V$ (for both branches)

$$\Rightarrow \quad q = 2 \times 200 = 400$$

Hence charge on each capacitor is $400\ \mu C$

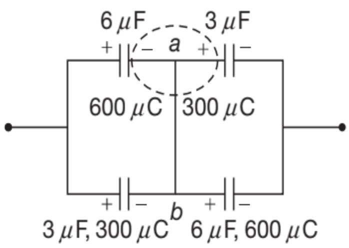
**When switch is closed**



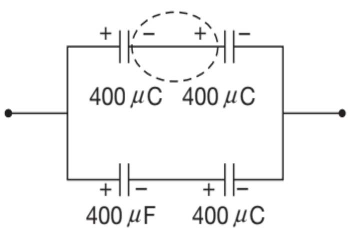




$q_6 = 600\ \mu C$ and $q_3 = 300\ \mu C$

Final charge on both plates is $300\ \mu C$




Initial charge on both plates is zero




Hence charge flown is 300 μC.

**Hence, the correct answer is (A).**

**101.** Since $F = kx$

$$\frac{q^2}{2A\varepsilon_0} = kx$$

$$\Rightarrow \quad x = \frac{q^2}{2A\varepsilon_0 k}$$

**Hence, the correct answer is (A).**

**102.** $C_{eq} = C_1 + C_2 = 4\pi\varepsilon_0 (r_1 + r_2)$

**Hence, the correct answer is (B).**

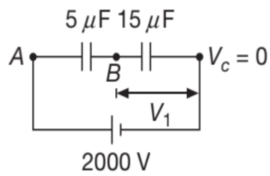
**104.** $\text{Loss} = -\Delta U = \dfrac{C_1C_2}{2(C_1+C_2)}(V_1 - V_2)^2$

Since $V_1 = V$ and $V_2 = -V$

$$\Rightarrow \quad -\Delta U = \frac{C^2}{4C}(2V)^2 = CV^2$$

**Hence, the correct answer is (B).**

**105.**




$$V_1 = \frac{2000 \times 5}{20} = 500$$

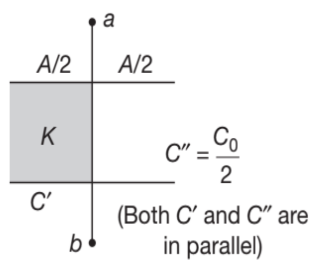
$$\Rightarrow \quad V_B - V_C = 500$$

Since $V_C = 0$

$$\Rightarrow \quad V_B = 500$$

**Hence, the correct answer is (D).**

**106.** Equivalent capacitance of Arrangement-1



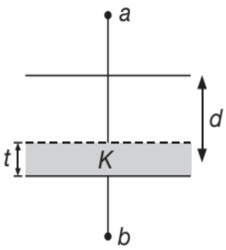

$$C' = \frac{\varepsilon_0 A}{2d}K = \frac{C_0 K}{2}$$

$$\Rightarrow \quad C_1 = C' + C''$$

$$\Rightarrow \quad C_1 = \frac{C_0}{2} + \frac{C_0 K}{2} = \frac{C_0}{2}(1 + K)$$

Equivalent capacitance of Arrangement-2

$$C_2 = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$




Both are same, so

$$\frac{C_0}{2}(K+1) = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

$$\Rightarrow \quad \frac{\varepsilon_0 A}{2d}(2+1) = \frac{\varepsilon_0 A}{d - t + \frac{t}{2}}$$

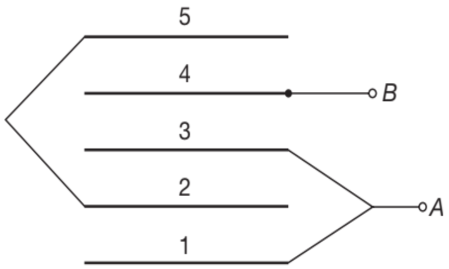
$$\Rightarrow \quad 3d - \frac{3t}{2} = 2d$$

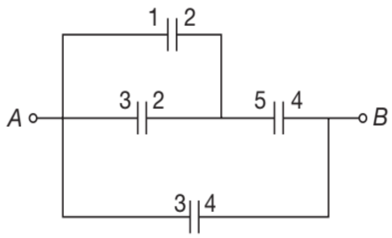
$$\Rightarrow \quad d = \frac{3t}{2}$$

$$\Rightarrow \quad t = \frac{2d}{3}$$

**Hence, the correct answer is (A).**

**107.**




The capacity of each capacitor is, $C_0 = \frac{\varepsilon_0 A}{d}$

From figure it is clear that $C_{eq} = \frac{5}{3}C_0 = \frac{5}{3}\left(\frac{\varepsilon_0 A}{d}\right)$

**Hence, the correct answer is (C).**

**108.** Before the switch $S$ is closed, then

$$q_i = C_{eq}V$$

$$\Rightarrow \quad q_i = 5 \times 4 = 20\ \mu C$$

When the switch $S$ is closed, then

$$q_f = CV$$

$$\Rightarrow \quad q_f = 10 \times 4 = 40\ \mu C$$

$$\Rightarrow \quad W_{battery} = (\Delta q)V_{battery} = (q_f - q_i)V_{battery}$$

$$\Rightarrow \quad W_{battery} = 20 \times 4 = 80\ \mu J$$

**Hence, the correct answer is (C).**

**109.** Since between the plates, we have

$$E = \frac{2Q}{2A\varepsilon_0} + \frac{Q}{2A\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{3Q}{2A\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{3}{2}\frac{Q}{Cd} \qquad \left\{\because C = \frac{\varepsilon_0 A}{d}\right\}$$

$$\Rightarrow \quad Ed = \frac{3Q}{2C} = V$$

Also, force on one plate due to other is

$$F = Q_2 E_1$$

$$\Rightarrow \quad F = (-Q)\left(\frac{2Q}{2\varepsilon_0 A}\right)$$

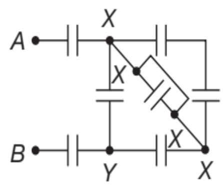
$$\Rightarrow \quad F = -\frac{Q^2}{A\varepsilon_0} = -\frac{Q^2}{Cd}$$

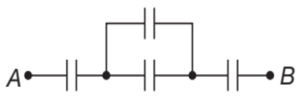
$$\text{Energy} = \frac{1}{2}\varepsilon_0 E^2 Ad$$

$$\Rightarrow \quad U = \frac{1}{2}\varepsilon_0\left(\frac{3Q}{2Cd}\right)^2 Ad = \frac{9}{8}\frac{Q^2}{C}$$

**Hence, the correct answer is (D).**

**110.** When no capacitor, resistor or battery exist between any two points in a branch, then those two points are taken at same potential. Using this concept to solve the circuit, we get



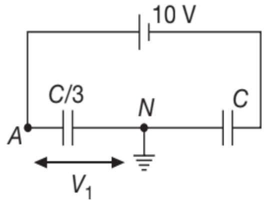




$$\Rightarrow \quad \frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{C}$$

$$\Rightarrow \quad C_{eq} = \frac{2C}{5}$$

**Hence, the correct answer is (B).**

**112.** Earthing a single point in a circuit does not change the charge or current flowing in the circuit, So,




$$V_A - V_N = \frac{10 \times C}{C + \frac{C}{3}} = \frac{30C}{4C} = 7.5$$

$\Rightarrow \quad V_A - 0 = 7.5$

$\Rightarrow \quad V_A = 7.5 \text{ V}$

**Hence, the correct answer is (B).**

**113.** $W = \Delta U = U_f - U_i = \frac{Q^2}{2C_f} - \frac{Q^2}{2C_i} = \frac{Q^2}{2}\left(\frac{2d}{\varepsilon_0 A} - \frac{d}{\varepsilon_0 A}\right)$

$\Rightarrow \quad W = \frac{Q^2}{2}\frac{d}{\varepsilon_0 A} = \frac{C^2V^2}{2C} = \frac{CV^2}{2} = \frac{\varepsilon_0 AV^2}{2d}$

**Hence, the correct answer is (A).**

**114.** Energy density $= \frac{1}{2}\varepsilon_0 E^2$ and $E = \frac{V}{d}$

**Hence, the correct answer is (C).**

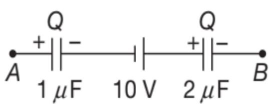
**115.** $Q = CV = 120\ \mu\text{C}$ and

$Q' = C'V' = 50 \times 12 = 600\ \mu\text{C}$

$\Rightarrow$ Charge Flown $= \Delta Q = 600 - 120 = 480\ \mu\text{C}$

**Hence, the correct answer is (C).**

**116.**




Applying Kirchhoff's Voltage Law $(KVL)$ between $A$ and $B$, we get

$$V_A - \frac{Q}{1 \times 10^{-6}} + 10 - \frac{Q}{2 \times 10^{-6}} - V_B = 0$$

$\Rightarrow \quad (V_A - V_B) + 10 = \frac{3Q}{2 \times 10^{-6}}$

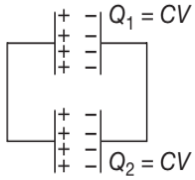
$\Rightarrow \quad Q = 10\ \mu\text{C}$

$\Rightarrow \quad V_{1\,\mu\text{C}} = \frac{Q}{C} = \frac{10\ \mu\text{C}}{1\ \mu\text{F}} = 10 \text{ V}$ and

$V_{2\,\mu\text{C}} = \frac{Q}{C} = \frac{10\ \mu\text{C}}{2\ \mu\text{F}} = 5 \text{ V}$

**Hence, the correct answer is (C).**

**117.**




Since loss in energy is given by

$$-\Delta U = \frac{C_1C_2}{2(1 + C_2)}(V_1 - V_2)^2$$

$\Rightarrow \quad -\Delta U = \frac{C^2}{2 \times 2C}[V - (-V)]^2$

$\Rightarrow \quad -\Delta U = \frac{C}{4} \times 4V^2 = CV^2$

**Hence, the correct answer is (B).**

**119.** $i = \frac{dq}{dt}$

$\Rightarrow \quad q = it + a$, where $a =$ constant

Since $V = \frac{q}{C}$

$\Rightarrow \quad V = \frac{it + a}{C}$

So, $V$ is proportional to time

**Hence, the correct answer is (D).**

**120.** $C_{eff} = \frac{\varepsilon_0 A}{d}$

Since effective capacitance between plates $A$ and $E$ is zero, so

$$U = \frac{1}{2}CV^2 = \frac{\varepsilon_0 A}{2d}V^2$$

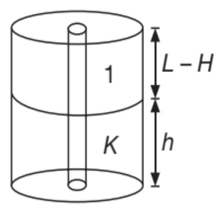
**Hence, the correct answer is (C).**

**121.** Gravitational force $= mg = \rho\pi(b^2 - a^2)gh$

Net upward force $F = \frac{dU}{dh} = \frac{1}{2}V^2\frac{dC}{dh}$

where $h$ is the height of liquid.

Now let us calculate $C$ as a function of $h$.




Since, $C_{eq} = C_{Air} + C_K = \frac{2\pi\varepsilon_0(L - h)}{\log_e\left(\frac{b}{a}\right)} + \frac{2\pi\varepsilon_0 Kh}{\log_e\left(\frac{b}{a}\right)}$

$$\Rightarrow \quad C = \frac{2\pi\varepsilon_0 (L + (K-1)h)}{\log_e\left(\frac{b}{a}\right)}$$

$$\Rightarrow \quad F = \frac{1}{2}V^2 \times \frac{dC}{dh} = \frac{1}{2}V^2 \times \frac{2\pi\varepsilon_0 (K-1)}{\log_e\left(\frac{b}{a}\right)}$$

Since $F = mg$, so we get

$$\frac{\frac{1}{2}V^2 \times 2\pi\varepsilon_0 (K-1)}{\log_e\left(\frac{b}{a}\right)} = \rho\pi(b^2 - a^2)gh$$

$$\Rightarrow \quad h = \frac{\pi\varepsilon_0 V^2 (K-1)}{\rho\pi(b^2 - a^2) g \log_e\left(\frac{b}{a}\right)}$$

$$\Rightarrow \quad h = \frac{\varepsilon_0 V^2 (K-1)}{\rho(b^2 - a^2) g \log_e\left(\frac{b}{a}\right)}$$

**Hence, the correct answer is (B).**

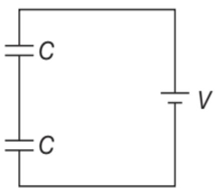
**122.** $U = \frac{1}{2}qV$

Work done, $W = qV$

So, Loss of energy $= qV - \frac{1}{2}qV = \frac{qV}{2}$

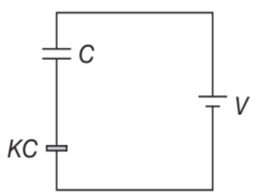
**Hence, the correct answer is (D).**

**123.**




Initially, energy stored is

$$U_i = \frac{1}{2}\left(\frac{C}{2}\right)V^2 = \frac{CV^2}{4}$$




Finally, energy stored is

$$U_f = \frac{1}{2}\left(\frac{C \times KC}{C + KC}\right)V^2 = \frac{KCV^2}{2(1+K)}$$

Since, $W = \Delta U = U_f - U_i$

$$\Rightarrow \quad W = \frac{KCV^2}{2(1+K)} - \frac{CV^2}{4} = \frac{CV^2}{2}\left(\frac{K}{1+K} - \frac{1}{2}\right)$$

$$\Rightarrow \quad W = \frac{CV^2}{2}\left(\frac{2K - 1 - K}{2(1+K)}\right) = \frac{CV^2}{4}\left(\frac{K-1}{K+1}\right)$$

**Hence, the correct answer is (B).**

**124.** Total charge $= 1.25\ \mu C + 0.75\ \mu C = 2\ \mu C$

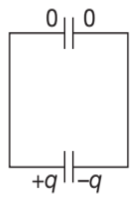
Since, $q_1' : q_2' = R_1 : R_2 = 1:2$

$$\Rightarrow \quad q_1' = \frac{1}{3} \times 2 = \frac{2}{3}\ \mu C$$

and $q_2' = \frac{2}{3} \times 2 = \frac{4}{3}\ \mu C$

**Hence, the correct answer is (A).**

**125.**




Energy loss $= -\Delta U = \frac{1}{2}\left(\frac{C^2}{C + C}\right)V^2 = \frac{1}{2}\frac{C}{2} \times \frac{q^2}{C^2}$

$$\Rightarrow \quad -\Delta U = \frac{q^2}{4C} = \frac{q^2 d}{4\varepsilon_0 A}$$

**Hence, the correct answer is (A).**

**126.** $U_i = \frac{Q^2}{2C_i}$

$$U_f = \frac{Q^2}{2C_f}$$

As surface was deformed in such a way that charge on original surface are coming closer or moving perpendicular to electric force acting on them, so total energy of foil increases.

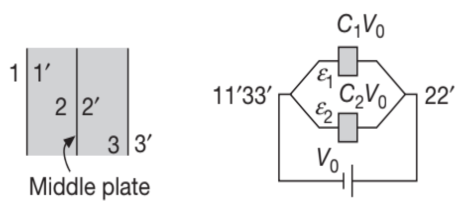
$$\Rightarrow \quad U_f > U_i$$

$$\Rightarrow \quad \frac{Q^2}{2C_f} > \frac{Q^2}{2C_i}$$

$$\Rightarrow \quad C_i > C_f$$

**Hence, the correct answer is (B).**

**127.**




CHAPTER 2

Total charge on 2 and 2′ plate is

$$q = C_{eq}V = \left(\frac{\varepsilon_1\varepsilon_0 A}{d_1} + \frac{\varepsilon_2\varepsilon_0 A}{d_2}\right)V$$

$$\Rightarrow \quad \sigma = \frac{q}{A} = \varepsilon_0 V\left(\frac{\varepsilon_1}{d_1} + \frac{\varepsilon_2}{d_2}\right)$$

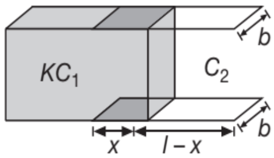
**Hence, the correct answer is (A).**

**128.** $V_{\text{common}} = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2} = \dfrac{C_1 \times V + C_2 \times 0}{C_1 + C_2}$

$$\Rightarrow \quad V_{\text{common}} = \frac{C_1V}{C_1 + C_2}$$

**Hence, the correct answer is (B).**

**129.**




$$C_{eq} = C_1 + C_2 = \frac{\varepsilon_0 bxK}{d} + \frac{\varepsilon_0(\ell - x)b}{d}$$

$$\Rightarrow \quad C_{eq} = \frac{\varepsilon_0 b}{d}(\ell b + x(K-1))$$

$$\Rightarrow \quad \frac{dC}{dt} = \frac{\varepsilon_0 b}{d}(K-1)\frac{dx}{dt} = \frac{\varepsilon_0 b}{d}(K-1)v$$

**Hence, the correct answer is (B).**

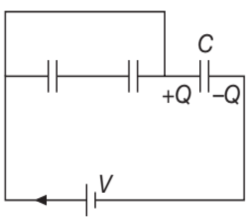
**130.** Charge in the circuit flows only when potential difference across $C_1$ is either greater or less than that across $C_2$

$$\Rightarrow \quad \frac{q_1}{C_1} \neq \frac{q_2}{C_2}$$

$$\Rightarrow \quad q_1C_2 \neq q_2C_1$$

**Hence, the correct answer is (D).**

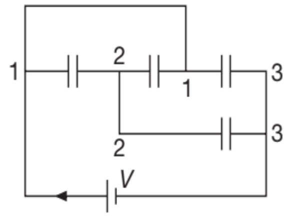
**131.** When $K_1$ is closed



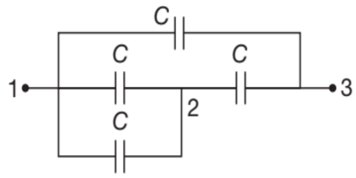

$$Q = CV$$

So, initial Energy is $U_i = \dfrac{1}{2}CV^2$

When $K_2$ is also closed




Equivalent circuit is shown here.




$$C_{eq} = C + \frac{2C}{3} = \frac{5C}{3}$$

So, final Energy is $U_f = \dfrac{1}{2} \times \dfrac{5C}{3}V^2 = \dfrac{5}{6}CV^2$

Charge supplied by battery after closing $K_2$ is

$$\Delta q = \frac{5}{3}CV - CV = \frac{2}{3}CV$$

Energy supplied by battery is $(\Delta q)V_{battery}$

Also energy supplied by battery $= U_f - U_i + \Delta H$

$$\Rightarrow \quad \frac{2}{3}CV^2 = \frac{5}{6}CV^2 - \frac{1}{2}CV^2 + \Delta H$$

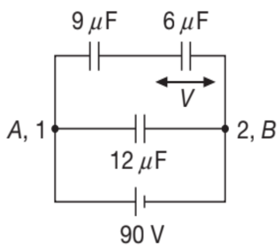
$$\Rightarrow \quad \Delta H = \frac{1}{3}CV^2$$

**Hence, the correct answer is (C).**

**132.** $E = \dfrac{V}{d}$

**Hence, the correct answer is (A).**

**133.**




$$V' = \frac{90 \times 9}{15} = 54$$

$$\Rightarrow \quad q = CV' = 6 \times 54 = 324\ \mu\text{C}$$

**Hence, the correct answer is (C).**

**134.** $C = \dfrac{q}{V}$. If potential difference between plates is zero then capacitance will be infinite.
**Hence, the correct answer is (D).**

**135.** $C_K = \dfrac{\varepsilon_0 A}{d - b + \dfrac{b}{K}}$

If we set $b = 0$, we get

$$C_K = \frac{\varepsilon_0 A}{d} = C$$

If $C_K = 2C$

$$\Rightarrow \quad \frac{\varepsilon_0 A}{d - b + \dfrac{b}{K}} = \frac{2\varepsilon_0 A}{d}$$

$$\Rightarrow \quad K = \frac{2b}{2b - d}$$

Since, we know that $K > 0$ and $b \le d$

$$\Rightarrow \quad K = \frac{2b}{2b - d} \text{ and}$$

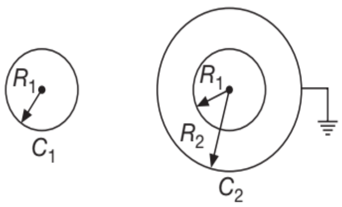
$$2b - d > 0$$

$$\Rightarrow \quad b > \frac{d}{2}$$

$$\Rightarrow \quad \frac{d}{2} < b \le d$$

**Hence, the correct answer is (B).**

**136.**




$$C_1 = 4\pi\varepsilon_0 R_1$$

and $C_2 = 4\pi\varepsilon_0 \left( \dfrac{R_1 R_2}{R_2 - R_1} \right)$

Given that $C_2 = nC_1$

$$\Rightarrow \quad \frac{R_2 R_1}{R_2 - R_1} = nR_1$$

$$\Rightarrow \quad \frac{\dfrac{R_2}{R_1}}{\dfrac{R_2}{R_1} - 1} = n$$

$$\Rightarrow \quad \frac{R_2}{R_1} = \frac{n}{n-1}$$

**Hence, the correct answer is (B).**

**137.** For Capacitor 1, we have $\begin{cases} C = 1\ \mu\text{F} \\ V_{max} = 16\text{ KV} \\ Q_{max} = 16\text{ mC} \end{cases}$

For Capacitor 2, we have $\begin{cases} C = 2\ \mu\text{F} \\ V_{max} = 4\text{ KV} \\ Q_{max} = 8\text{ mC} \end{cases}$

For series combination of capacitors, we have

$$\begin{cases} C_{eq} = \dfrac{1 \times 2}{1 + 2} = \dfrac{2}{3}\ \mu\text{F} \\ Q_{max} = 8\text{ mC} \\ \text{and } V_{max} = \dfrac{Q_{max}}{C_{eq}} = \dfrac{8\text{ mC}}{\dfrac{2}{3}\ \mu\text{F}} = 12\text{ KV} \end{cases}$$

**Hence, the correct answer is (B).**

**138.** All the $3C$ capacitors are in series, so

$$\frac{1}{3C} + \frac{1}{3C} + \frac{1}{3C} = \frac{1}{C_{eq}}$$

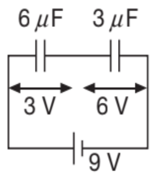
$$\Rightarrow \quad C_{eq} = C$$

Since $C$ and $C$ are in parallel

$$\Rightarrow \quad C_{eq} = 2C$$

**Hence, the correct answer is (B).**

**139.** Initially

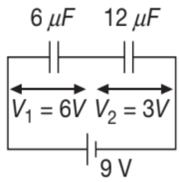



$$q_{6\ \mu F} = 18\ \mu C$$

$$q_{1\ \mu F} = 6\ \mu C$$

$$q_{2\ \mu F} = 12\ \mu C$$

Finally




$$q_{6\ \mu F} = 36\ \mu C$$

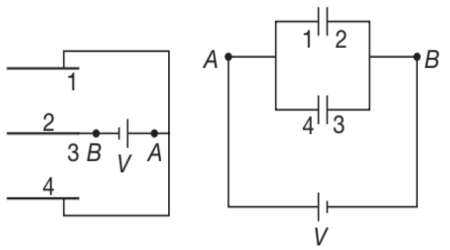
$$q_{2\ \mu F} = 6\ \mu C$$

$$q_{1\ \mu F} = 3\ \mu C$$

Hence charge on $C_1$ increases but on $C_2$ decreases.

**Hence, the correct answer is (C).**

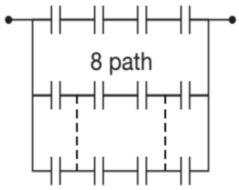
**140.**




$$U = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2 = \frac{\varepsilon_0 A V^2}{d}$$

**Hence, the correct answer is (B).**

CHAPTER 2

**141.** 

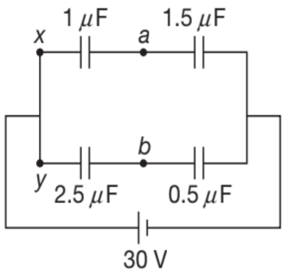

So, $N = 4 \times 8 = 32$

**Hence, the correct answer is (D).**

**142.** Since, $Q = CV$

So, for $Q = 5C$, we get $V = 2.5$ V

Since the capacitor is charged at a steady rate, hence it should be a straight line graph.
**Hence, the correct answer is (A).**

**143.** 

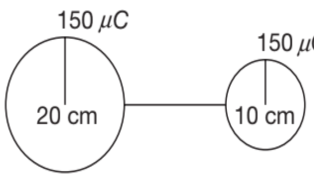

$$V_x - V_a = \frac{30 \times 1.5}{2.5} = 18 \quad \ldots(1)$$

$$V_y - V_b = \frac{30 \times 0.5}{3} = 5 \quad \ldots(2)$$

Subtracting (1) from (2), we get

$$V_a - V_b = -13 \text{ V}$$

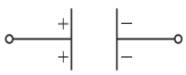
**Hence, the correct answer is (C).**

**144.** 

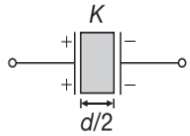

Since $V = \dfrac{Q_1 + Q_2}{C_1 + C_2} = \dfrac{150 \times 2 \times 10^{-6}}{4\pi\varepsilon_0 (r_1 + r_2)}$

$$\Rightarrow \quad V = \frac{300 \times 10^{-6} \times 9 \times 10^{9}}{30 \times 10^{-2}} = 9 \times 10^{6} \text{ V}$$

**Hence, the correct answer is (A).**

**146.** 

$$V = \frac{q}{C} = \frac{qd}{\varepsilon_0 A}$$




$$V' = \frac{q}{C} = \frac{q\left(\dfrac{d - t + t}{K}\right)}{\varepsilon_0 A}$$

$$\Rightarrow \quad V' = \frac{q}{\varepsilon_0 A}\left(d - \frac{d}{2} + \frac{d}{2 \times 2}\right)$$

$$\Rightarrow \quad V' = \frac{qd}{\varepsilon_0 A}\left(\frac{1}{2} + \frac{1}{4}\right)$$

$$\Rightarrow \quad V' = \frac{qd}{\varepsilon_0 A}\frac{3}{4}$$

$$\Rightarrow \quad V' = \frac{3}{4}V$$

**Hence, the correct answer is (D).**

**147.** Let, $Q_0 = C_0 V_0 =$ Initial charge on capacitor

Battery connected means $V$ remains unchanged

As separation is doubled, so $C = \dfrac{C_0}{2}$

$$\Rightarrow \quad U_1 = \frac{1}{2}\left(\frac{C_0}{2}\right)V_0^2 = \frac{C_0 V_0^2}{4}$$

When battery disconnected, charge remains unchanged

As separation is doubled, so $C = \dfrac{C_0}{2}$

$$\Rightarrow \quad U_2 = \frac{Q_0^2}{2C} = \frac{C_0^2 V_0^2}{2\left(\dfrac{C_0}{2}\right)} = C_0 V_0^2$$

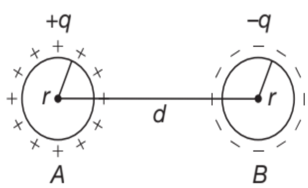
$$\Rightarrow \quad \frac{U_1}{U_2} = \frac{1}{4}$$

**Hence, the correct answer is (A).**

**148.** $C_{eq} = C_1 + C_2 + C_3$

$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{3d} + \frac{\varepsilon_0 A}{3d \times 2} = \frac{\varepsilon_0 A}{3 \times 3d} = \frac{11\varepsilon_0 A}{18d}$$

**Hence, the correct answer is (C).**

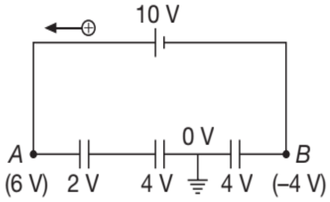
**150.** 


Since $V_A - V_B = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{r} - \left(-\dfrac{Kq}{r}\right)\right) = \dfrac{2q}{4\pi\varepsilon_0 r}$

$\Rightarrow \quad C = \dfrac{q}{V_A - V_B} = 2\pi\varepsilon_0 r$

**Hence, the correct answer is (B).**

**151.**




**Hence, the correct answer is (A).**

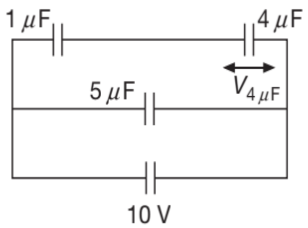
**153.** $Q_1 = C_1 V$ and $Q_2 = C_2 V$

$\Rightarrow \quad \dfrac{50}{150} = \dfrac{\varepsilon_0}{K\varepsilon_0}$

$\Rightarrow \quad K = 3$

**Hence, the correct answer is (D).**

**156.**




$2\ \mu$F and $2\ \mu$F are reduced to 1 capacitor of $1\ \mu$F and $3\ \mu$F and $1\ \mu$F are reduced to $4\ \mu$F .

$\Rightarrow \quad V_{4\ \mu F} = \dfrac{10 \times 1}{5} = 2$

(potential difference across $4\ \mu$F capacitor is calculated by voltage division Rule)

Since, $V_{5\ \mu F} = 10$ V

(potential difference across $5\ \mu$F is same as Battery)

$\Rightarrow \quad \dfrac{V_{1\ \mu F}}{V_{5\ \mu F}} = \dfrac{2}{10} = \dfrac{1}{5}$

**Hence, the correct answer is (C).**

**157.** When switch is open then

$Q_1 = \dfrac{8 \times 4}{12} \times 40 = \dfrac{320}{3}\ \mu$C

When switch is closed, then

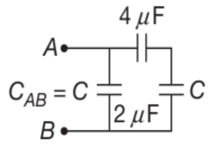
$Q_2 = 8 \times 40 = 320\ \mu$C

Charge flown through $AB$ is

$\Delta q = q_2 - q_1 = 320 - \dfrac{320}{3} = \dfrac{640}{3}\ \mu$C

**Hence, the correct answer is (B).**

**158.**




$\dfrac{4C}{4 + C} + 2 = C$

$\Rightarrow \quad C = 4\ \mu$F

**Hence, the correct answer is (A).**

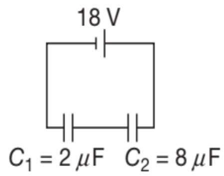
**160.** $U = \dfrac{1}{2} C V_0^2 = \dfrac{1}{2} \dfrac{\varepsilon_0 A}{x} V_0^2$

$\dfrac{dU}{dt} = -\dfrac{V_0^2 \varepsilon_0 A}{2} \dfrac{1}{x^2} \left( \dfrac{dx}{dt} \right) = -\dfrac{V_0^2 \varepsilon_0 A v}{2x^2}$

**Hence, the correct answer is (A).**

**161.** $V_1 = \dfrac{18 \times C_2}{C_1 + C_2} = \dfrac{18 \times 8}{10} = 14.6$ V

and $V_2 = \dfrac{18 \times 1}{10} = 1.8$ V




**Hence, the correct answer is (D).**

**162.** $C_{eq} = \left( \dfrac{\varepsilon r_1 + \varepsilon r_2}{2} \right) C$

$\Rightarrow \quad C_{eq} = \left( \dfrac{4 + 6}{2} \right) (1\ \mu\text{F}) = 5\ \mu$F

**Hence, the correct answer is (A).**

**164.** $C_1 = 10\ \mu$F , $C_2 = ?$

$V_1 = 100$ V , $V_2 = 0$

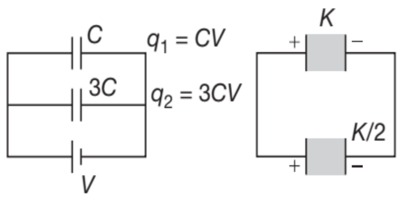
and $V_{\text{common}} = 40$ V

Since $V_{\text{common}} = \dfrac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

$\Rightarrow \quad C_2 = 15\ \mu$F

**Hence, the correct answer is (A).**

**165.**



Common potential is given by

$$V_{\text{common}} = \frac{q_1+q_2}{C_1+C_2} = \frac{CV+3CV}{KC+\frac{K}{2}3C}$$

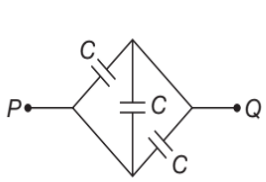
$$\Rightarrow \quad V_{\text{common}} = \frac{4CV}{\frac{5KC}{2}} = \frac{8V}{5K}$$

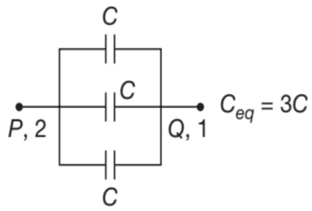
**Hence, the correct answer is (C).**

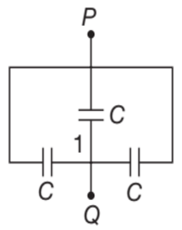
**166.** $\text{Loss} = -\Delta U = \dfrac{C_1C_2}{2(C_1+C_2)}(V_1-V_2)^2$

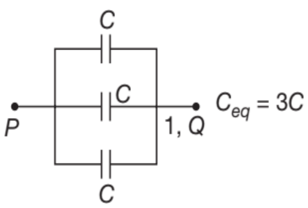
**Hence, the correct answer is (C).**

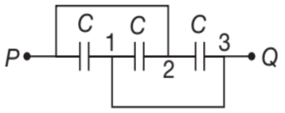
**167.**

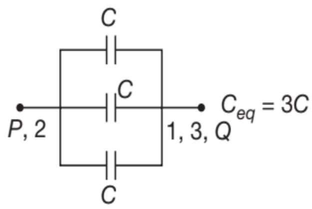


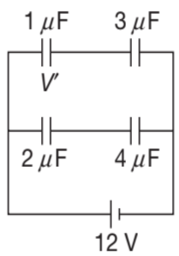




**Hence, the correct answer is (C).**

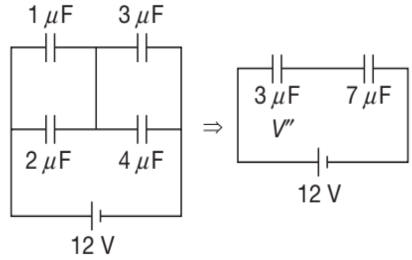
**168. When only $S_1$ is closed**




$$V' = \frac{12\times 3}{4} = 9V$$

$$\Rightarrow \quad q_{1\,\mu F} = 1\times 9 = 9\ \mu C$$

**When both $S_1$ and $S_2$ are closed**




$$V'' = \frac{12\times 7}{10} = \frac{42}{5}\ \text{V}$$

$$\Rightarrow \quad q'_{1\,\mu F} = 1\times\frac{42}{5} = \frac{42}{5}\ \mu C$$

$$\Rightarrow \quad \frac{q_1}{q'_1} = \frac{9}{\frac{42}{5}} = \frac{45}{42} = \frac{15}{14}$$

**Hence, the correct answer is (C).**

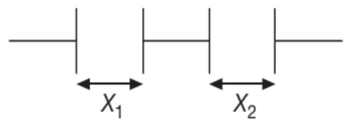
**169.** $I_0 = \dfrac{E_0}{R}$ Since both capacitors are charged to same potential difference hence current at $t=0$ will be same in both circuits.

Time taken in 50% discharging is $t = 0.693RC$

So, 1 $\mu$F will take less time compared to 2 $\mu$F

**Hence, the correct answer is (C).**

**171.** When two capacitors are in series




$$\Rightarrow \quad \frac{1}{C_{eq}} = \frac{x_1}{\varepsilon_0 A} + \frac{x_2}{\varepsilon_0 A} = \frac{x_1+x_2}{\varepsilon_0 A}$$

$$\Rightarrow \quad C_{eq} = \frac{\varepsilon_0 A}{x_1+x_2}$$

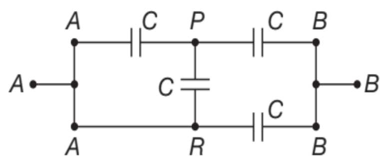
$$\Rightarrow \quad C_{eq} = \frac{\varepsilon_0 A}{\text{Sum of separations between plates}}$$

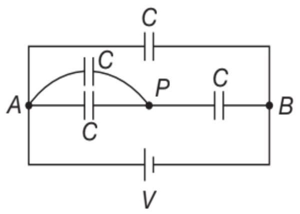
Now in given arrangement, the capacitors are in series and sum of separations $= a-b$

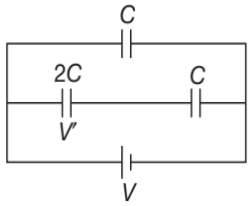
$$\Rightarrow \quad C_{eq} = \frac{\varepsilon_0 A}{a-b}$$

**Hence, the correct answer is (B).**

**172.**



⇒ 


⇒ 


Potential across $2C$ be $V'$, then

$$V' = \frac{V \times C}{3C} = \frac{V}{3}$$

$$\Rightarrow \quad q = \frac{CV}{3}$$

**Hence, the correct answer is (B).**

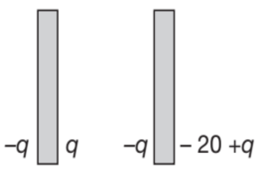
**173.** Battery disconnected

⇒ $Q$ = constant

⇒ As $d$ increases, $C$ decreases. So, $U$ increases, $V$ decreases

**Hence, the correct answer is (A).**

**174.** Since outer surfaces both have equal charge, so

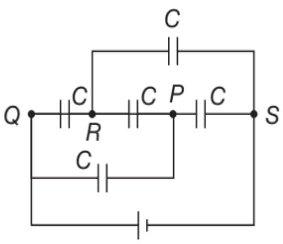



$$-q = -20 + q$$

$$\Rightarrow \quad q = \frac{20}{2} = 10\ \mu\text{C}$$

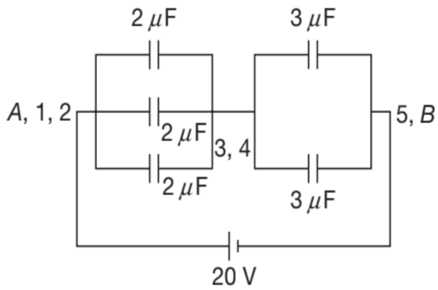
**Hence, the correct answer is (A).**

**175.** Capacitor between $P$ and $R$ will not acquire any charge.

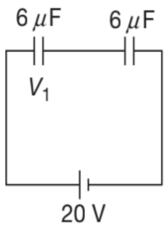



**Hence, the correct answer is (A).**

**178.**




⇓




$$V_1 = 10\ \text{V}$$

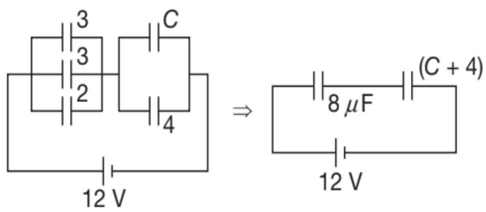
$$\Rightarrow \quad q_{C_0} = 10 \times 2 = 20\ \mu\text{C}$$

**Hence, the correct answer is (B).**

**179.** $U_i = \frac{1}{2}C_1V_0^2$ and $U_f = \frac{C_1^2V_0^2}{2(C_1 + C_2)}$

**Hence, the correct answer is (A).**

**180.** Using point potential method, the circuit can be redrawn as




Since, $q = C_{eq}V$

$$\Rightarrow \quad 48 = \frac{8 \times (C+4)}{8 + 4 + C} \times 12$$

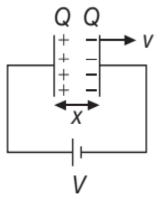
$$\Rightarrow \quad 48 + 4C = 8C + 32$$

$$\Rightarrow \quad 16 = 4C$$

$$\Rightarrow \quad C = 4$$

**Hence, the correct answer is (A).**

**181.**




CHAPTER 2

Since $U=\frac{1}{2}\frac{\varepsilon_0 A}{x}V^2$

$$\Rightarrow \frac{dU}{dt}=\frac{1}{2}\varepsilon_0 AV^2\frac{d^{(x^{-1})}}{dt}$$

$$\Rightarrow \frac{dU}{dt}=\frac{1}{2}\left(\frac{\varepsilon_0 A}{x^2}\right)\left(\frac{dx}{dt}\right)V^2$$

$$\Rightarrow \frac{dU}{dt}\propto\frac{1}{x^2}\propto x^{-2}$$

**Hence, the correct answer is (A).**

**182.** $C_{eq}=C_1+C_2=2+2=4\ \mu F$

$$\Rightarrow q=C_{eq}V=4\ \mu F\times 8=32\ \mu C$$

**Hence, the correct answer is (D).**

**183.** $q_1=C_{eq}V$

$$\Rightarrow q_1=\frac{10\times 20\times 10^{-6}}{30}\times 3\times 10^3=2\times 10^{-2}$$

$$\Rightarrow q_1=20000\times 10^{-6}=20000\ \mu C$$

**Hence, the correct answer is (B).**

## Multiple Correct Choice Type Questions

**1.** The capacitances

$$C_1=\frac{K_1\varepsilon_0 A}{\left(\frac{d}{2}\right)}\text{ and } C_2=\frac{K_2\varepsilon_0 A}{\left(\frac{d}{2}\right)}$$

The capacitors are in series. So, the effective capacitance is

$$C=\frac{C_1C_2}{C_1+C_2}=\frac{2K_1K_2}{K_1+K_2}\left(\frac{\varepsilon_0 A}{d}\right)$$

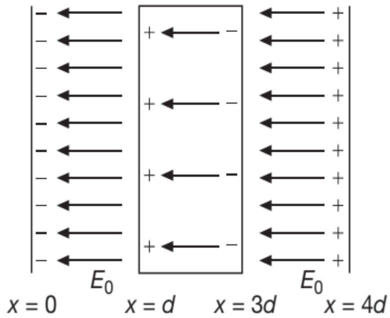
So, the **OPTIONS (A), (B), (C)** and **(D)** all are correct

**Hence, (A), (B), (C) and (D) are correct.**

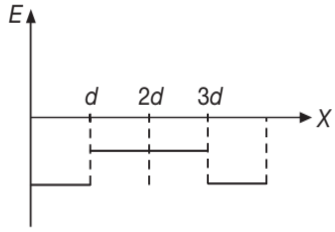
**3.** Since $P$ and $R$ are earthed and so they can be assumed to be connected to each other. Hence, $'P+Q'$ and $'Q+R'$ are two capacitors with the same potential difference. If $Q$ is closer to $P$ than to $R$ then the capacitance $C_{PQ}$ is $>C_{QR}$. The upper surface of $Q$ will have greater charge than the lower surface. As the force of attraction between the plates of a capacitor is proportional to $Q^2$ so, there will be a net upward force on $Q$ which can balance its weight.

**Hence, (B) and (D) are correct.**

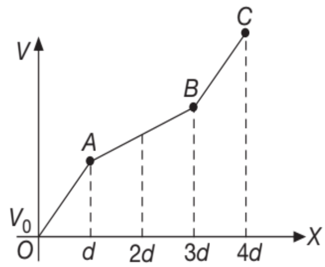
**4.** The magnitude and direction of electric field in different regions are shown in figure. The direction of the electric field remains the same. Hence, **OPTION (B)** is correct.




Similarly, electric lines always flow from higher to lower potential, therefore, electric potential increases continuously as we move from $x=0$ to $x=4d$. Therefore, **OPTION (C)** is also correct.



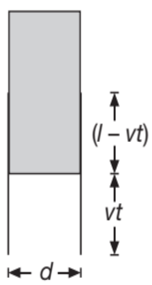

The variation of electric field $(E)$ and potential $(V)$ with $x$ will be as follows :




**8.** Let the length, width, thickness and dielectric constant of the slab be $\ell$, $b$, $d$ and $K$ respectively. The initial capacitance of the capacitor is given by

$$C_0=\frac{\varepsilon_0 K\ell b}{d}$$

When the dielectric is pulled out of the capacitor with a speed $v$ then, at time $t$, the length of the slab that has been pulled out is $vt$. At this instant, the capacitor may be considered as a parallel combination of two capacitors, an air capacitor having area $b(vt)$ and a dielectric capacitor having area $b(\ell-vt)$.



Hence, the net capacitance as a function of time will be equal to

$$C = \frac{\varepsilon_0 (vt) b}{d} + \frac{\varepsilon_0 K b (\ell - vt)}{d}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 b}{d}\left(vt + K(\ell - vt)\right)$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 b}{d}\left(K\ell - (K-1)vt\right)$$

It is observed that the net capacitance $C$ varies with time $t$ linearly and slope of this line is negative and has positive intercept on C-axis. Hence, the **OPTION (D)** is correct.

When the battery is disconnected, the charge on the capacitor remains constant and at time $t$, the potential difference across the capacitor will be $V = \frac{q}{C}$ and hence the curve between $C$ and $V$ is a rectangular hyperbola as shown in **OPTION (C)**. Now since we observe that $C$ decreases with time as a result of which $V$ also increases from initial value $V_0 = \frac{q}{C_0}$. So, **OPTION (A)** is also correct.

Let the potential difference between the plates of the capacitor be $V$, then the energy stored in it is equal to $U = \frac{1}{2}qV$, where $q$ remains constant. Hence, energy $U \propto V$. So, the curve between $U$ and $V$ will be a straight line passing through origin but since the minimum possible non-zero value of $C$ is $C_{MIN} = \frac{\varepsilon_0 \ell b}{d}$, therefore, the curve must start from this minimum value of $C$. Hence, the **OPTION (B)** is also correct.

**Hence, (A), (B), (C) and (D) are correct.**

**9.** Since the given circuit is a series combination of two identical dielectric capacitors, therefore, charges on these two capacitors are equal. So, when the dielectric slab of the capacitor $B$ is pulled out, its capacitance decreases, as a result of which the equivalent capacitance of the series combination also decreases. Hence the charge on series combination begins to decrease as a result of which an anticlockwise current starts flowing through the circuit or a charge flows from $a$ to $b$. Hence, the **OPTION (A)** is correct.

Also the capacitors are in series with each other, so the charge on two capacitors remains same or both the capacitors possess a common charge at all instants and hence, the **OPTION (B)** is incorrect.

As studied and discussed earlier, we know that a charged capacitor always attracts the dielectric in the space between the plates. So, capacitor $B$ exerts a force of attraction on the dielectric slab. On pulling the dielectric slab out of the capacitor, a work has to be done by the external force $F$. But if the slab is pulled out slowly then a very small current will flow through the circuit. In that case, heat generated in the circuit will be negligible. Hence, it is not necessary that significant amount of heat is produced in the circuit. Therefore, the **OPTION (C)** is incorrect.

It is also observed that an anticlockwise current flows in the circuit during the process i.e., the battery is being charged which further means that the internal energy of the battery increases. Therefore, the **OPTION (D)** is also correct.

**Hence, (A) and (D) are correct.**

**10.** Let $C_0$ be the capacitance initially and $C$ be the capacitance finally . Then $C_0 = \frac{\varepsilon_0 A}{d}$

Since, $Q = C_0 V$

$$\Rightarrow \quad Q = \frac{\varepsilon_0 A V}{d}$$

Further $E_0 = \frac{V}{d}$ and $E = \frac{E_0}{K}$

$$\Rightarrow \quad E = \frac{V}{Kd}$$

Also, if $U_i$ is the initial energy, then $U_i = \frac{1}{2}C_0 V^2$

After the introduction of slab if $U_f$ be the final energy, then

$$U_f = \frac{1}{2}CV_{slab}^2 = \frac{1}{2}(KC_0)\left(\frac{V}{K}\right)^2$$

$$\Rightarrow \quad U_f = \frac{1}{2}\frac{C_0 V^2}{K}$$

$$\Rightarrow \quad \Delta U = U_2 - U_1$$

$$\Rightarrow \quad \Delta U = \frac{1}{2}C_0 V^2\left(\frac{1}{K} - 1\right)$$

Since work done = Decrease in Potential Energy

$$\Rightarrow \quad W = -\Delta U$$

$$\Rightarrow \quad W = \frac{1}{2}\frac{\varepsilon_0 A V^2}{d}\left(1 - \frac{1}{K}\right)$$

**Hence, (A), (C) and (D) are correct.**

**11.** Capacitance without water is

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \times 0.01}{0.01} = \varepsilon_0$$

$$\Rightarrow \quad C_0 = 8.85 \times 10^{-12}\ \text{F}$$

CHAPTER 2

When water rises by 1 cm

$$C=\frac{\varepsilon_0\times 81\times 0.001}{0.01}+\frac{\varepsilon_0(0.01-0.001)}{0.01}$$

$\Rightarrow \quad C=8.1\varepsilon_0+0.9\varepsilon_0=9\varepsilon_0$

$\Rightarrow \quad C=9\left(8.85\times 10^{-12}\text{ F}\right)=80\text{ F}$

Let $m$ be the mass of water that rises to a height of 100 cm, then

$$m=(0.001\times 0.01)\times 1000=0.01\text{ kg}$$

By Law of Conservation of Energy, we have

$$\frac{q^2}{2C_0}=\frac{q^2}{2C}+\frac{mgh}{2}$$

$\Rightarrow \quad \frac{q^2}{C_0}=\frac{q^2}{C}+mgh$

Now, $q=C_0V$

$\Rightarrow \quad \frac{V^2C_0^2}{C_0}=\frac{V^2C_0^2}{C}+mgh$

$\Rightarrow \quad V^2C_0\left(1-\frac{C_0}{C}\right)=mgh$

$\Rightarrow \quad V^2\varepsilon_0\left(1-\frac{\varepsilon_0}{9\varepsilon_0}\right)=0.01\times 9.8\times 0.01$

$\Rightarrow \quad V=\sqrt{\frac{9\times 0.01\times 9.8\times 0.01}{8.85\times 10^{-12}\times 8}}$

$\Rightarrow \quad V=1110\text{ V}$

**Hence, (A), (C) and (D) are correct.**

12. On connecting the conducting sphere having charge $Q$ with a conducting shell having no charge, the entire charge must flow out to the uncharged shell.
**{OPTION (C)}**
The capacitance of the outer shell of radius $2R$ is $C=4\pi\varepsilon_0(2R)$.

So, energy liberated is $U=\frac{Q^2}{2C}=\frac{Q^2}{16\pi\varepsilon_0R}$.

**Hence, (C) and (D) are correct.**

13. The spring force $\vec{F}_S$ acting on plate $a$ is given by

$$\vec{F}_S=-kx\hat{i}$$

Similarly, the electrostatic force $\vec{F}_e$ due to the electric field created by plate $b$ is

$$\vec{F}_e=QE\hat{i}=Q\left(\frac{\sigma}{2\varepsilon_0}\right)\hat{i}=\frac{Q^2}{2A\varepsilon_0}\hat{i}$$

where $A$ is the area of the plate. Notice that charges on plate $a$ cannot exert a force on itself, as required by Newton's Third Law. Thus, only the electric field due to plate $b$ is considered. At equilibrium the two forces cancel and we have

$$kx=Q\left(\frac{Q}{2A\varepsilon_0}\right)=QE$$

which gives

$$x=\frac{Q^2}{2kA\varepsilon_0}=\frac{QE}{k}$$

**Hence, (A), (B) and (C) are correct.**

16. The effective capacitance

$$C=\frac{4\times 5}{9}=\frac{20}{9}\ \mu\text{F}$$

The charge on $C_1$ is given by

$$Q_1=CV=\frac{20}{9}\times 9=20\ \mu\text{C}$$

The charge on $C_2$ is given by

$$Q_2=\left(\frac{C_2}{C_2+C_3}\right)Q_1=\frac{2}{5}\times 20=8\ \mu\text{C}$$

The charge on $C_3$ is given by

$$Q_3=\left(\frac{C_3}{C_2+C_3}\right)Q_1=\frac{3}{5}\times 20=12\ \mu\text{C}$$

Since, $\frac{Q_2}{C_2}=\frac{Q_3}{C_3}$

$\Rightarrow \quad Q_1=Q_2+Q_3=Q_2+\frac{C_3}{C_2}Q_2=Q_2\left(\frac{C_2+C_3}{C_2}\right)$

$\Rightarrow \quad Q_2=\left(\frac{C_2}{C_2+C_3}\right)Q_1=\left(\frac{2}{2+3}\right)20=8\ \mu\text{C}$

and $Q_3=\left(\frac{C_3}{C_2+C_3}\right)Q_1=\left(\frac{3}{2+3}\right)20=12\ \mu\text{C}$

**Hence, (A), (C) and (D) are correct.**

17. When either $A$ or $C$ is earthed (both not earthed together), a parallel plate capacitor is formed with $B$, with $\pm Q$ charges on the inner surfaces. The other plate, which is not earthed, plays no role, hence charge of amount $+Q$ flows to the earth.
When both are earthed together, $A$ and $C$ effectively become connected. The plates now form two capacitors in parallel, with capacitances in the ratio $1:2$, and hence share charge $Q$ in the same ratio.

**Hence, (A), (B) and (C) are correct.**

**18.** Condition for a balanced Wheat Stone Bridge.

**Hence, (A) and (C) are correct.**

**19.** Since, $F = \dfrac{q^2}{2\varepsilon_0 A}$

**Hence, (A) and (D) are correct.**

**21.** The distribution of charges is shown in figure. Applying Kirchhoff's loop rules for loops ①, ②, and ③, we get

**LOOP ①**

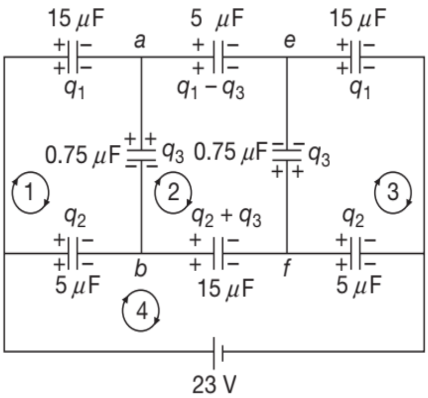
$$-\frac{q_2}{5} + \frac{q_3}{0.75} + \frac{q_1}{15} = 0$$

$$\Rightarrow \quad q_1 - 3q_2 + 20q_3 = 0 \qquad \ldots(1)$$

**LOOP ②**

$$-\frac{q_2 + q_3}{15} - \frac{q_3}{0.75} + \frac{q_1 - q_3}{5} - \frac{q_3}{0.75} = 0$$

$$\Rightarrow \quad 3q_1 - q_2 - 44q_3 = 0 \qquad \ldots(2)$$




**LOOP ③**

$$23 - \frac{q_2}{5} - \frac{q_2 + q_3}{15} - \frac{q_2}{5} = 0$$

$$\Rightarrow \quad 345 = 7q_2 + q_3 \qquad \ldots(3)$$

Solving for $q_1$, $q_2$, $q_3$, we get

$$q_1 = \frac{19 \times 345}{92}, \; q_2 = \frac{13 \times 345}{92}, \; q_3 = \frac{345}{92}$$

Potential Difference between $a$ and $b$ is

$$V_{ab} = \frac{q_3}{0.75} = \frac{345}{92} \times \frac{4}{3} = 5 \text{ V}$$

Potential Difference between $e$ and $f$ is

$$V_{ef} = -5 \text{ V}$$

**Hence, (A) and (B) are correct.**

**27.** The equivalent capacitance of the arrangement here is $C$ where

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \Rightarrow C = \frac{20}{3} \mu\text{F}$$

The potential difference across the combination is 90 V and hence the total charge is

$$Q = CV = \frac{20}{3} \times 10^{-6} \times 90 = 6 \times 10^{-4} \text{ C} = 600 \,\mu\text{C}$$

Since all the capacitors are in series and hence all capacitors will have the same charge given by

$$Q = CV$$

The potential difference across each capacitor will be in the inverse ratio of their capacitances.
So, the potential difference across $C_1$ is 30 V, the potential difference across $C_2$ is 20 V and the potential difference across $C_3$ is 40 V and hence all the **OPTIONS (A), (B), (C)** and **(D)** are correct.

**Hence, all the options are correct.**

**30.** $q = CV = C_0 V_0$

where $C_0 = \dfrac{\varepsilon_0 A}{d}$ and $q = \dfrac{\varepsilon_0 A V_0}{d} = \dfrac{\varepsilon_0 A V}{(d+\ell)}$

$$\Rightarrow \quad \frac{\varepsilon_0 A}{d} V_0 = \frac{\varepsilon_0 A}{(d+\ell)} V \;\Rightarrow\; V = \frac{(d+\ell)}{d} V_0 = \left(1 + \frac{\ell}{d}\right) V_0$$

$$\text{and } C = \frac{\varepsilon_0 A}{d+\ell} = \frac{\varepsilon_0 A}{d\left(1 + \frac{\ell}{d}\right)} = \frac{C_0}{\left(1 + \frac{\ell}{d}\right)}$$

**Hence, (B) and (C) are correct.**

**31.** $C_s = \dfrac{2 \times 8}{2 + 8} = 1.6 \,\mu F$

Since $Q = C_s V = 1.6 \times 10^{-6} \times 300$

$$Q = 4.8 \times 10^{-4} \, C$$

$$V_1 = \frac{4.8 \times 10^{-4}}{2 \times 10^{-6}} = 240 \, V$$

$$V_2 = \frac{4.8 \times 10^{-4}}{8 \times 10^{-6}} = 60 \, V$$

$$U = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2}$$

$$\Rightarrow \quad U = \frac{(4.8 \times 10^{-4})^2}{2} \left(\frac{1}{1.6 \times 10^{-6}}\right)$$

$$\Rightarrow \quad U = 3 \times 2.4 \times 10^{-2} \, J$$

$$\Rightarrow \quad U = 2.4 \times 10^{-2} \, J$$

**Hence, (A) and (D) are correct.**

**32.** Since battery is still in connection. So,

$$V = V_0$$

$\Rightarrow \quad Q_0 = C_0V_0$ and

$Q = kC_0V_0$

$\Rightarrow \quad Q = kQ_0$

Since $k > 1$

$\Rightarrow \quad Q > Q_0$

Also $U_0 = \frac{1}{2}Q_0V_0$ and

$$U = \frac{1}{2}QV = kU_0 \qquad \{\because Q = kQ_0 \text{ and } V = V_0\}$$

Hence $U > U_0$

**Hence, (A) and (D) are correct.**

**33.** Since same charge flows through $7\ \mu F$ and $3\ \mu F$ (as both are in series), so

$Q_7 = Q_3 = 42\ \mu C$

$\Rightarrow \quad 7 \times 6 = 3V_3$

$\Rightarrow \quad V_3 = 14\ V$

So, $\quad V_{3.9} = 20\ V$

Charge on $12\ \mu F$ capacitor is

$Q_{12} = Q_{3.9} + Q_7$

$$\Rightarrow \quad Q_{12} = 3.9 \times 20 + \frac{7 \times 3}{7 + 30} \times 20$$

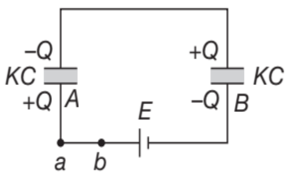
$\Rightarrow \quad Q_{12} = 120\ \mu C =$ Total charge in the circuit.

$$\Rightarrow \quad V_{12} = \frac{120}{12} = 10\ V$$

Voltage across battery is $14 + 6 + 10 = 30\ V$

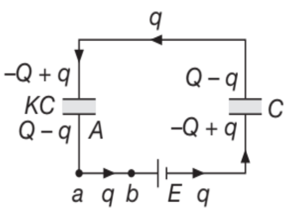
**Hence, (A), (C) and (D) are correct.**

**34.**




$$Q = \frac{KCE}{2}$$

On removal of the dielectric from $B$, let a charge $q$ flow from $a$ to $b$. Then by applying Kirchhoff's Loop Law, we have




$$\frac{Q-q}{KC} + \frac{Q-q}{C} = E$$

$$\Rightarrow \quad Q - x = \left(\frac{K}{K+1}\right)CE$$

$$\Rightarrow \quad \frac{KCE}{2} - q = \left(\frac{K}{K+1}\right)CE$$

$$\Rightarrow \quad q = KCE\left(\frac{1}{2} - \frac{1}{K+1}\right) = \frac{KCE}{2}\left(\frac{K-1}{K+1}\right)$$

$$\Rightarrow \quad q = Q\left(\frac{K-1}{K+1}\right)$$

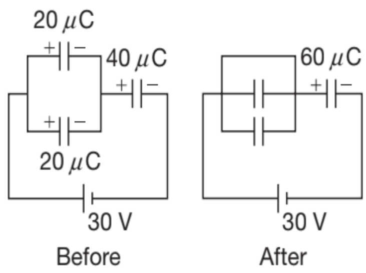
Since $K$ is always greater than one, so $q$ is positive and hence we have the charge flowing from $a$ to $b$. So, option (A) is correct.

Final charge on $B$ equals the final charge on $A$, so option (B) is incorrect.

During this process, energy received by the is battery is $(\Delta q)V_{\text{battery}} = qE$. So, option (D) is correct.

**Hence, (A) and (D) are correct.**

**35.** The charges stored in different capacitors before and after the switch $S$ is closed are shown in figure(s).




The amount of charge flown through the battery is

$$\Delta q = q_f - q_i = 20\ \mu C$$

So, energy supplied by the battery is

$$W_{\text{battery}} = (\Delta q)V_{\text{battery}}$$

$$\Rightarrow \quad W_{\text{battery}} = (20 \times 10^{-6})(30) = 0.6 \text{ mJ}$$

Energy stored in all the capacitors before closing the switch $S$ is

$$U_i = \frac{1}{2}C_{\text{net}}V^2 = \frac{1}{2}\left(\frac{4}{3} \times 10^{-6}\right)(30)^2 = 0.6 \text{ mJ}$$

and after closing the switch is

$$U_f = \frac{1}{2}C_{\text{net}}V^2 = \frac{1}{2}(2 \times 10^{-6})(30)^2 = 0.9 \text{ mJ}$$

Since we know that $W_{\text{battery}} = \Delta U_C + \Delta H$

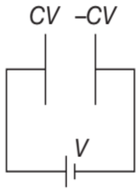
So, heat generated $\Delta H$ is given by

$$\Delta H = W_{\text{battery}} - \Delta U_C = 0.3 \text{ mJ}$$

and charge flown through the switch is $60\ \mu C$.

**Hence, (A), (C) and (D) are correct.**

**37.**

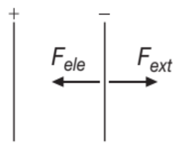



When the battery is disconnected, then the charge will remain constant. Now when the plates are pulled apart, then we have

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

So, $E$ remains constant and hence option (A) is correct

Work is done against attractive force by the external force $F_{ext}$




So, option (B) is also correct.

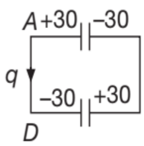
Since, $U = \frac{1}{2}CV^2$

When the battery stays connected to the capacitor, then $V =$ constant. When the separation between the plates is increased, then capacitance given by $C = \frac{\varepsilon_0 A}{d}$ decreases and hence $U$ also decreases.

So option (C) is also correct.

**Hence, (A), (B) and (C) are correct.**

**38.**




The common potential is given by

$$V = \frac{C_1V_1 - C_2V_2}{C_1 + C_2} = \frac{30 - 30}{C_1 + C_2} = 0$$

Final charge on the first capacitor is

$$Q_1' = C_1V = 0$$

Final charge on the second capacitor is

$$Q_2' = C_2V = 0$$

So, final energy stored in the arrangement is zero.

Let a charge $q$ flow from $A$ to $D$, then

$$30 - q = 0$$

$$\Rightarrow \quad q = 30\ \mu C$$

**Hence, (A) and (C) are correct.**

**39.** $\tau_{C_1} = \tau_{C_2} = 2RC = \tau_C$ (say)

$$q_1 = (CE)\left(1 - e^{-\frac{t}{\tau_C}}\right)$$

and $q_2 = (2CE)\left(1 - e^{-\frac{t}{\tau_C}}\right)$

$$\Rightarrow \quad \frac{q_1}{q_2} = \frac{1}{2} \text{ (at any time)}$$

So, the ratio of charges in steady state is also $1:2$

Now, $\frac{dq_1}{dt} = \frac{CE}{\tau_C}e^{-\frac{t}{\tau_C}}$

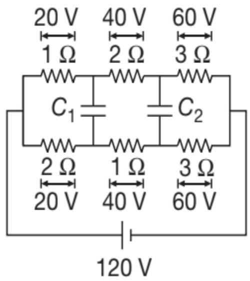
and $\frac{dq_2}{dt} = \frac{2CE}{\tau_C}e^{-\frac{t}{\tau_C}}$

$$\Rightarrow \quad \left(\frac{dq_1}{dt}\right) \neq \left(\frac{dq_2}{dt}\right)$$

The option (A) is wrong.

**Hence, (B), (C) and (D) are correct.**

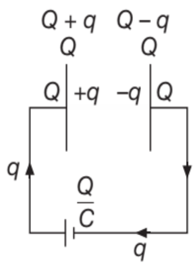
**40.** In steady state, the potential difference across $C_1$ is $20\ V$ and potential difference across $C_2$ is $0\ V$. Then, charge stored on $C_1$ is $40\ \mu C$ and charge stored on $C_2$ is $0\ \mu C$.




**Hence, (B) and (D) are correct.**

**41.** Let the charge flowing from the positive terminal to the negative terminal of the battery be $q$, then

$$q = CV = C\left(\frac{Q}{C}\right) = Q$$




Total charge on plate $x$ is

$$Q + q = Q + Q = 2Q$$

Total charge on plate y is

$$Q - q = Q - Q = 0$$

The energy supplied by the cell is

$$W_{\text{battery}} = \left(\frac{Q}{C}\right)q = \frac{Q^2}{C} = CV^2$$

**Hence, all the options are correct.**

42. (A) $U = \frac{1}{2}CV^2$

As source between the plates is connected, so potential difference remains constant. But capacitance $C$ becomes $KC$ hence energy stored is increased by factor $K$.

(B) Electric field $\frac{V}{d}$ is not changed.

(C) Charge on each plate is increased by factor $K$ hence force between them increases by a factor of $K^2$.

(D) Since $Q = CV$

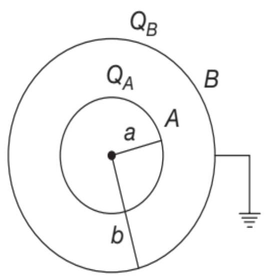
Hence charge becomes $KQ$ as $C$ becomes $KC$ and $V$ remains unchanged.

**Hence, (A), (C) and (D) are correct.**

## Reasoning Based Questions

6. $V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_A}{a} + \frac{Q_B}{b}\right)$

$$V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_A}{b} + \frac{Q_B}{b}\right)$$




$$\Rightarrow \quad V_A - V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_A}{a} - \frac{Q_A}{b}\right) = \frac{Q_A}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

**Hence, the correct answer is (A).**

7. Equivalent capacitance of parallel combination is

$$C_{\text{parallel}} = C_1 + C_2 + C_3$$

**Hence, the correct answer is (C).**

8. When the battery is disconnected from the capacitor, then $Q =$ constant

$$\text{Energy} = \frac{Q^2}{2C} = \frac{Q^2 d}{2\varepsilon_0 A}$$

$\Rightarrow$ Energy $\propto d$

**Hence, the correct answer is (D).**

9. Since $V = \frac{Q}{C}$

If sizes of two capacitors are different then potentials will also be different. Thus potential difference may exist between them although they carry same amount of positive charge.

**Hence, the correct answer is (D).**

10. The sum of charges on both the plates should be zero.

**Hence, the correct answer is (A).**

11. The capacitance depends upon the geometrical parameters only. And if $Q$ is increased then $V$ increase.

**Hence, the correct answer is (C).**

12. In the given case $V = V_0$ (constant)

Energy stored in the capacitor $= \frac{1}{2}CV^2$

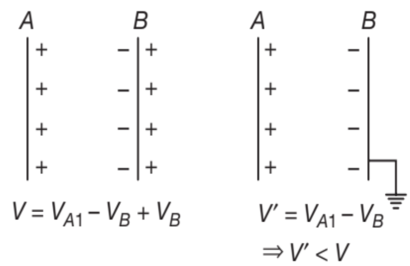
$C' = CK$, so energy stored will become $A$ times

$Q = CV$, so $Q$ will become $K$ times

$\therefore$ surface charge density $\sigma' = \frac{Kq}{A} = K\sigma_0$

**Hence, the correct answer is (C).**

13.




**Hence, the correct answer is (C).**

14. $C_1 = \frac{\varepsilon_0 KA}{d}$

$$\frac{C_1}{C_2} = \left(\frac{K_1}{d_1}\right)\left(\frac{d_2}{K_2}\right) = \left(\frac{K_1}{K_2}\right)\left(\frac{d_2}{d_1}\right) = \left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = \frac{1}{6}$$

**Hence, the correct answer is (B).**

## Linked Comprehension Type Questions

1. Since,

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{d - \frac{d}{3} + \frac{d}{6}}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 A}{d - \frac{d}{6}}$$

$$\Rightarrow \quad C = \frac{6}{5}\left(\frac{\varepsilon_0 A}{d}\right)$$

$$\Rightarrow \quad C = \frac{6}{5} C_0$$

**Hence, the correct answer is (D).**

**2.** According to question

$$\frac{\varepsilon_0 A}{d - \frac{t}{2}} = \frac{3}{2}\frac{\varepsilon_0 A}{d}$$

Solving we get $\frac{t}{d} = \frac{2}{3}$

**Hence, the correct answer is (A).**

**3.** For dielectric $U_1 = \frac{q^2}{2C_1} = \frac{q^2}{2\left(\frac{\varepsilon_0 A}{d - \frac{t}{2}}\right)} = \frac{q^2}{2\left(\frac{3\varepsilon_0 A}{2d}\right)}$

For conductor with air filling the complete space, we have

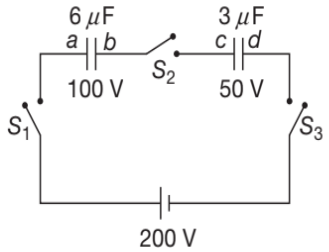
$$U_2 = \frac{q^2}{2C_2} = \frac{q^2}{2\left(\frac{\varepsilon_0 A}{d}\right)} \quad \Rightarrow \quad \frac{U_2}{U_1} = \frac{3}{2}$$

**Hence, the correct answer is (B).**

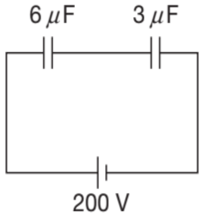
**4.** Plate $b$ and $c$ are joined together and they are neither connected to any of the terminals of the battery nor to any other source of charge. So, they together form an isolated system.

**Hence, the correct answer is (D).**

**5.** The charges, before the switches are closed are shown here




In steady state, after closing the switch, let $q$ charge goes from battery.




Then charge on each will increase by $q$

Applying Kirchhoff's Voltage Law, we get

$$200 - \left(\frac{600+q}{6}\right) - \left(\frac{150+q}{3}\right) = 0 \quad \Rightarrow \quad q = 100\ \mu\text{C}$$

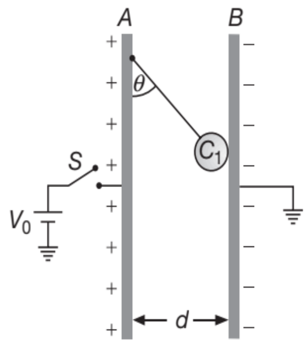
Hence, charge on $6\ \mu$F capacitor will be $700\ \mu$C and that on the $3\ \mu$F capacitor will be $250\ \mu$C.

**Hence, the correct answer is (B).**

**6.** From all the switches $100\ \mu$C of charge will flow.

**Hence, the correct answer is (D).**

**7.** Let $d$ be the separation between the plates and $\ell$ be the length of the silk thread, then in the final position, we have




$$\sin\theta = \frac{d}{\ell} = \frac{50}{100} = \frac{1}{2}$$

$\Rightarrow \quad \theta = 30°$ {with plate $A$}

**Hence, the correct answer is (A).**

**8.** When the ball again touches the plate $A$, the charge on plate $A$, i.e., $Q_0 = C_0 V_0$, redistributes on $A$ and ball. Let the charge on plate be $Q_0'$ and on ball be $Q$ after redistribution such that the common potential is $V$. Then

$$Q_0' + Q = C_0 V_0$$

$$\frac{Q_0'}{Q} = \frac{C_0}{C}$$

$$\Rightarrow \quad 1 + \frac{Q_0'}{Q} = 1 + \frac{C_0}{C}$$

$$\Rightarrow \quad Q=\left(\frac{C}{C+C_0}\right)(Q+Q_0')$$

$$\Rightarrow \quad Q=\left(\frac{C}{C+C_0}\right)C_0V_0$$

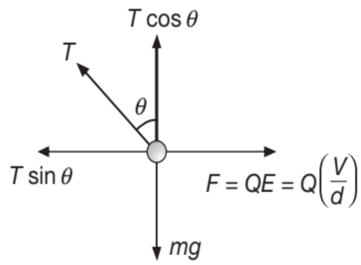
$$\Rightarrow \quad V=\frac{Q}{C}=\frac{C_0V_0}{C+C_0}$$

**Hence, the correct answer is (B).**

9. In the final position of the ball, the free body diagram shows that

$$T\sin\theta=QE=Q\left(\frac{V}{d}\right)$$

and $T\cos\theta=mg$




$$\Rightarrow \quad \tan\theta=\frac{QV}{mgd}$$

$$\Rightarrow \quad mg\tan\theta=\frac{QV}{d}=\frac{(CV)V}{d}=\frac{CV^2}{d}$$

Since $V=\dfrac{C_0V_0}{C+C_0}$

$$\Rightarrow \quad mg\tan\theta=\frac{C}{d}\left(\frac{C_0V_0}{C_0+C}\right)^2$$

$$\Rightarrow \quad \frac{C_0^2V_0^2}{(C_0+C)^2}=\frac{mgd\tan\theta}{C}$$

$$\Rightarrow \quad V_0=\frac{C_0+C}{C_0}\sqrt{\frac{mgd\tan\theta}{C}}$$

$$\Rightarrow \quad V_0=\left(1+\frac{C}{C_0}\right)\sqrt{m(10)\left(\frac{50}{1000}\right)\frac{\tan 30}{C}}$$

$$\Rightarrow \quad V_0=\left(1+\frac{C}{C_0}\right)\sqrt{\frac{m}{2\sqrt{3}C}}$$

**Hence, the correct answer is (C).**

10. $\dfrac{C_A}{C_B}=\dfrac{1}{K}$

**Hence, the correct answer is (B).**

15. $C\propto r$

$$\Rightarrow \quad \frac{C_A}{C_B}=\frac{r_A}{r_B}=\frac{2}{3}$$

**Hence, the correct answer is (A).**

16. The charge will be shared till both attain a common potential $V$ (say).
Then

$$\frac{Q_2}{Q_1}=\frac{C_2V}{C_1V}=\frac{C_2}{C_1}=\frac{3}{2}$$

$$\Rightarrow \quad \frac{Q_2}{Q_1}+1=\frac{3}{2}+1$$

$$\Rightarrow \quad \frac{Q_2+Q_1}{Q_1}=\frac{3+2}{2}$$

$$\Rightarrow \quad \frac{Q}{Q_1}=\frac{5}{2}$$

$$\Rightarrow \quad Q_1=\frac{2Q}{5}$$

**Hence, the correct answer is (C).**

17. $\dfrac{(U_A)_i}{(U_A)_f}=\dfrac{q_i^2}{q_f^2}=\left(\dfrac{2}{5}\right)^2=\dfrac{4}{25}$

**Hence, the correct answer is (C).**

18. $\dfrac{U_i}{U_f}=\dfrac{\frac{1}{2}C_1V^2}{\frac{1}{2}(C_1+C_2)V^2}=\dfrac{C_1}{C_1+C_2}=\dfrac{2}{2+3}=\dfrac{2}{5}$

**Hence, the correct answer is (B).**

22. Capacitor $A$ is a combination of two capacitors $C_K$ and $C_O$ in parallel. Hence,

$$C_A=C_K+C_O=\frac{K\varepsilon_oA}{d}+\frac{\varepsilon_oA}{d}=(K+1)\frac{\varepsilon_oA}{d}$$

Here, $A=0.02\text{ m}^2$ Substituting the values, we have

$$C_A=(9+1)\frac{8.85\times10^{-12}(0.02)}{\left(8.85\times10^{-4}\right)}\text{ F}$$

$$\Rightarrow \quad C_A=2\times10^{-9}\text{ F}=2\text{ nF}$$

**Hence, the correct answer is (B).**

23. Energy stored in capacitor $A$, when connected with a 110 V battery is

$$U_A=\frac{1}{2}C_AV^2=\frac{1}{2}\left(2\times10^{-9}\right)(110)^2$$

$$\Rightarrow \quad U_A=1.21\times10^{-5}\text{ J}\cong 12\ \mu\text{J}$$

**Hence, the correct answer is (C).**

**24.** Charge stored in the capacitor

$$q_A = C_A V = (2\times10^{-9})(110)$$

$$\Rightarrow\quad q_A = 2.2\times10^{-7}\,\text{C}$$

Now, this charge remains constant even after battery is disconnected. But when the slab is removed, capacitance of $A$ will get reduced. Let the new capacitance be $C'_A$. So

$$C'_A = \frac{\varepsilon_o(2A)}{d} = \frac{(8.85\times10^{-12})(0.04)}{8.85\times10^{-4}}\,\text{F}$$

$$\Rightarrow\quad C'_A = 0.4\times10^{-9}\,\text{F}$$

Energy stored in this case would be

$$U'_A = \frac{1}{2}\frac{(q_A)^2}{C'_A} = \frac{1}{2}\frac{(2.2\times10^{-7})^2}{(0.4\times10^{-9})}\,\text{J}$$

$$\Rightarrow\quad U'_A = 6.05\times10^{-5}\,\text{J} > U_A$$

So, work done to remove the slab would be

$$W = U'_A - U_A = (6.05-1.21)\times10^{-5}\,\text{J}$$

$$\Rightarrow\quad W = 4.84\times10^{-5}\,\text{J} = 48.4\,\mu\text{J}$$

**Hence, the correct answer is (A).**

**25.** Capacitance of $B$ when filled with dielectric is

$$C_B = \frac{K\varepsilon_0 A}{d} = \frac{(9)(8.85\times10^{-12})(0.02)}{(8.85\times10^{-4})}\,\text{F}$$

$$\Rightarrow\quad C_B = 1.8\times10^{-9}\,\text{F}$$

These two capacitors are in parallel. Therefore, net capacitance of the system is

$$C = C'_A + C_B = (0.4+1.8)\times10^{-9}\,\text{F}$$

$$\Rightarrow\quad C = 2.2\times10^{-9}\,\text{F}$$

Charge stored in the system is $q = q_A = 2.2\times10^{-7}\,\text{C}$

So, energy stored, $U = \frac{1}{2}\frac{q^2}{C} = \frac{q_A^2}{2C}$

$$\Rightarrow\quad U = \frac{1}{2}\frac{(2.2\times10^{-7})^2}{(2.2\times10^{-9})}$$

$$\Rightarrow\quad U = 1.1\times10^{-5}\,\text{J} = 11\,\mu\text{J}$$

**Hence, the correct answer is (B).**

**26.** Capacitors 2 and 3 are in parallel and give equivalent capacitance 6C, Which further is in series with capacitor 1, so the battery sees capacitance $\frac{1}{C_{\text{net}}} = \frac{1}{3C} + \frac{1}{6C}$

$$\Rightarrow\quad C_{\text{net}} = 2C$$

**Hence, the correct answer is (C).**

**27.** If they were initially uncharged, $C_1$ stores the same charge as $C_2$ and $C_3$ together. With greater capacitance, $C_3$ stores more charge than $C_2$. Then $Q_1 > Q_2 > Q_3$

Hence $C_1 > C_2 > C_3$ (in terms of charges)

**Hence, the correct answer is (A).**

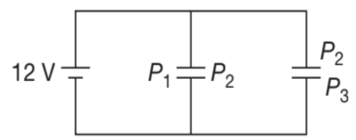
**28.** The $(C_2 \parallel C_3)$ equivalent capacitor stores the same charge as $C_1$. Since it has greater capacitance, so $\Delta V = \frac{Q}{C}$ implies that it has smaller potential difference across it than $C_1$. In parallel with each other, $C_2$ and $C_3$ have equal voltages. $\Delta V_1 > \Delta V_2 = \Delta V_3$. Hence $C_1 > C_2 = C_3$ (in terms of potentials)

**Hence, the correct answer is (C).**

**29.** If $C_3$ is increased, the overall equivalent capacitance increases. More charge moves through the battery and $Q$ increases. As $\Delta V_1$ increases, $\Delta V_2$ must decrease so $Q_2$ decreases. Then $Q_3$ must increase even more: $Q_3$ and $Q_1$ increase; $Q_2$ decreases. Hence charge on $C_1$ and $C_3$ increase but that on $C_2$ decreases.

**Hence, the correct answer is (B).**

**30.** Each face of $P_2$ carries charge, so the three-plate system is equivalent to




Each capacitor by itself has capacitance

$$C = \frac{K\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12})(7.5\times10^{-4})}{(1.19\times10^{-3})} = 5.58\,\text{pF}$$

So, equivalent capacitance = $5.58 + 5.58 = 11.2$ pF

**Hence, the correct answer is (B).**

**31.** $Q = C\Delta V + C\Delta V$

$$\Rightarrow\quad Q = (11.2\times10^{-12})(12\,\text{V}) = 134\,\text{pC}$$

**Hence, the correct answer is (C).**

**32.** Now $P_3$ has charge on two surfaces and in effect three capacitors are in parallel

$$C = 3(5.58\,\text{pF}) = 16.7\,\text{pF}$$

**Hence, the correct answer is (D).**

**33.** Only one face of $P_4$ carries charge

$$Q = C\Delta V = 5.58\times10^{-12}\,\text{F}(12\,\text{V}) = 67\,\text{pC}$$

**Hence, the correct answer is (B).**

CHAPTER 2

**34.** Let $V_1$ and $V_2$ be the voltage across 1 and 2 $\mu$F capacitors respectively. Then

$$V_1 + V_2 = 300 \text{ V}$$

In series the charge $Q$ on each capacitor is same, therefore we have

$$\frac{Q}{C_1} + \frac{Q}{C_2} = 300$$

$$\Rightarrow \left(\frac{1}{C_1} + \frac{1}{C_2}\right)Q = 300$$

Here $C_1 = 1\ \mu\text{F}$, $C_2 = 2\ \mu\text{F}$

$$\Rightarrow Q = \frac{300C_1C_2}{C_1 + C_2} = \frac{300(1\times10^{-6})(2\times10^{-6})}{(1\times10^{-6} + 2\times10^{-6})}$$

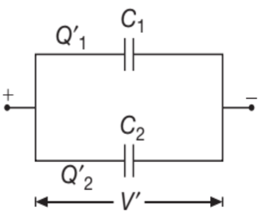
$$\Rightarrow Q = 200\times10^{-6} \text{ C} = 200\ \mu\text{C}$$

$$\Rightarrow V_1 = \frac{Q}{C_1} = \frac{200\ \mu\text{C}}{1\ \mu\text{F}} = 200 \text{ V}$$

$$V_2 = \frac{Q}{C_2} = \frac{200\ \mu\text{C}}{2\ \mu\text{F}} = 100 \text{ V}$$

**Hence, the correct answer is (D).**

**35.** When the plates of same polarity are connected together, the capacitors are in usual parallel combination. In parallel the potential difference across each capacitor is the same.




$\therefore$ The common potential difference

$$V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{2Q}{C_1 + C_2}$$

$$\Rightarrow V = \frac{2\times(200\ \mu\text{C})}{(1\ \mu\text{F} + 2\ \mu\text{F})} = \frac{400\ \mu\text{C}}{3\ \mu\text{F}} = \frac{400}{3} \text{ V}$$

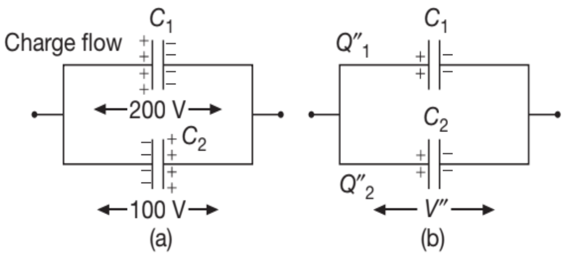
If $Q_1'$ and $Q_2'$ are new charges on the respective capacitors, then

$$Q_1' = C_1V = (1\ \mu\text{F})\times\left(\frac{400}{3} \text{ V}\right) = \frac{400}{3}\ \mu\text{C}$$

$$Q_2' = C_2V = (2\ \mu\text{F})\times\left(\frac{400}{3} \text{ V}\right) = \frac{800}{3}\ \mu\text{C}$$

**Hence, the correct answer is (C).**

**36.** When the opposite plates of **SITUATION (I)** are connected together, the charge will flow from the capacitor of higher potential to that of lower potential until their potentials are same. The net charge is then $(Q_1 - Q_2)$.




If $V''$ is common potential difference across the capacitors, then

$$V'' = \frac{Q_1 - Q_2}{C_1 + C_2} = \frac{(200\ \mu\text{F} - 200\ \mu\text{C})}{1\ \mu\text{F} + 2\ \mu\text{F}} = 0$$

The charges after sharing

$$Q_1'' = C_1V'' = C_1(0) = 0$$

$$Q_2'' = C_2V'' = C_2(0) = 0$$

**Hence, the correct answer is (A).**

**37.** When the opposite plates of **SITUATION (II)** are connected together, the charge will flow from one having larger quantity of charge of that of smaller quantity of charge i.e., from $C_2$ to $C_1$, to make common potential $V'''$ (say) given by

$$V''' = \frac{(Q_2' - Q_1')}{C_1 + C_2} = \frac{\left(\frac{800}{3} - \frac{400}{3}\right)\mu\text{C}}{(1+2)\ \mu\text{F}} = \frac{400}{9} \text{ V}$$

$\Rightarrow$ Charges after sharing are

$$Q_1''' = C_1V''' = \left(1\ \mu\text{F}\times\frac{400}{9} \text{ V}\right) = \frac{400}{9}\ \mu\text{C}$$

and $Q_2''' = C_2V''' = \dfrac{800}{9}\ \mu\text{C}$

**Hence, the correct answer is (B).**

**38.** Initial charges on first and second capacitors are $q_1 = C_1V_0 = 100\ \mu\text{C}$ and $q_2 = -C_2V_0 = -300\ \mu\text{C}$ finally, let the common potential be $V$, then $q_{\text{final}} = (C_1 + C_2)V$. So, by Law of Conservation of Charge, we get

$$(C_1 + C_2)V = C_1V_0 + C_2(-V_0)$$

$$\Rightarrow V = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)V_0$$

$$\Rightarrow V = \left(\frac{3-1}{3+1}\right)100 = 50 \text{ V}$$

**Hence, the correct answer is (B).**

**39.** $(q_1)_f = C_1V = (1\ \mu F)(50\ V) = 50\ \mu C$

**Hence, the correct answer is (A).**

**40.** $(q_2)_f = C_2V = (3\ \mu F)(50\ V) = 150\ \mu C$

**Hence, the correct answer is (C).**

**41.** $U_f = \frac{1}{2}(C_1 + C_2)V^2$

$\Rightarrow\ U_f = \frac{1}{2}(4\times10^{-6})(2500)$

$\Rightarrow\ U_f = 2\times10^{-6}\times2500$

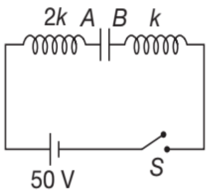
$\Rightarrow\ U_f = 5\times10^{-3}\ J = 5\ mJ$

and $U_i = \frac{1}{2}(C_1 + C_2)V_0^2 = 200\ mJ$

$\Rightarrow$ Loss $= U_i - U_f = 15\ mJ$

**Hence, the correct answer is (C).**

**42.**




Final energy of capacitor is $10\times10^{-3}$ J

$\Rightarrow\ 10\times10^{-3} = \frac{1}{2}C_{final}(50)^2$

$\Rightarrow\ C_{final} = C_2 = 8\ \mu F$

Since, $C = \frac{\varepsilon_0 A}{d}$

$\Rightarrow\ \frac{C_1}{C_2} = \frac{d_2}{d_1}$

$\Rightarrow\ C_1 = \left(\frac{d_2}{d_1}\right)C_2 = \left(\frac{2}{8}\right)(8) = 2\ \mu F$

So, the capacitance of the capacitor when the switch $S$ is not closed is $2\ \mu F$.

**Hence, the correct answer is (C).**

**43.** If $x_1$ and $x_2$ be the extensions in springs connected to plates A and B respectively, then we observe that

$F = (2k)x_1 = kx_2$

$\Rightarrow\ 2x_1 = x_2$

Also, $x_1 + x_2 = d_1 - d_2 = 8 - 2 = 6$ mm

$\Rightarrow\ \frac{x_2}{2} + x_2 = 6$

$\Rightarrow\ \frac{3x_2}{2} = 6$

$\Rightarrow\ x_2 = 4$ mm

So, $x_1 = \frac{x_2}{2} = 2$ mm

**Hence, the correct answer is (B).**

**44.** Since force between plates of the capacitor connected to the battery is given by

$$F = \frac{1}{2}\frac{C_2V^2}{d_2} = \frac{10\times10^{-3}}{2\times10^{-3}} = 5\ N$$

$\Rightarrow\ (2k)x_1 = 5$

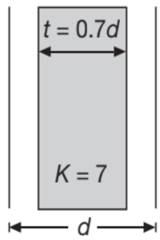
$\Rightarrow\ k_A = 2k = \frac{5}{x_1} = \frac{5}{2\times10^{-3}} = 2500\ Nm^{-1}$

**Hence, the correct answer is (C).**

**45.** Since, we know that

$$C_{eq} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)} = \frac{\varepsilon_0 A}{\frac{0.3d}{1} + \frac{0.7d}{7}} = \frac{\varepsilon_0 A}{0.4d} \quad \text{...(1)}$$

When plate was absent, then




$$C_0 = \frac{\varepsilon_0 A}{d} = 10\ \mu F$$

So from equation (1), we get

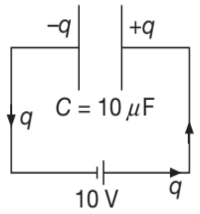
$$C_{eq} = \frac{10}{0.4} = 25\ \mu F$$

Charge stored $= C_{eq}V = 25\times10 = 250\ \mu C$

Energy stored $= \frac{1}{2}(25)(10)^2 = 1250\ \mu J$

**Hence, the correct answer is (B).**

**46.** After disconnecting the battery, when the slab is removed, then let new charge on the capacitor plate be $q_f$. So, we have




$$-10+\frac{q_f}{10}=0$$

$$\Rightarrow \quad q_f = 100\ \mu C$$

$$\text{New energy } = \frac{(100)^2}{2\times 10} = 500\ \mu J$$

**Hence, the correct answer is (A).**

**47.** Since $W_{battery} = \Delta U_C + \Delta H$

where, $W_{battery} = (\Delta q)V_{battery}$

Since the positive plate of the capacitor is connected to the negative of the battery, so $q_{final} = -100\ \mu C$, whereas we observe that $q_{initial} = 250\ \mu C$. Hence the charge flown through the battery is

$$|\Delta q| = |q_{final} - q_{initial}| = 350\ \mu C$$

Also, $\Delta U_C = U_{final} - U_{initial} = 500 - 1250 = -750\ \mu J$

$$\Rightarrow \quad 3500 = -750 + \Delta H$$

$$\Rightarrow \quad \Delta H = 4250\ \mu J$$

**Hence, the correct answer is (D).**

**48. Method-1**

$$E_1 = \frac{Q}{2A\varepsilon_0} \text{ and } E_2 = \frac{2Q}{2A\varepsilon_0}$$

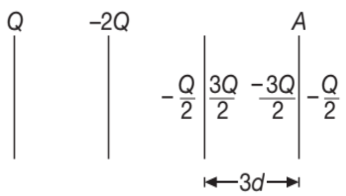
$$\Rightarrow \quad E = E_1 + E_2$$

$$\Rightarrow \quad E = \frac{3Q}{2A\varepsilon_0}$$

Potential difference between the plates is

$$\Delta V = E(3d) = \left(\frac{3Q}{2A\varepsilon_0}\right)3d = \frac{9Q}{2C}$$

**Method-2**




$$C_{eq} = \frac{\varepsilon_0 A}{3d}$$

$$V = \frac{Q_{inside}}{C_{eq}} = \frac{\left(\frac{3Q}{2}\right)}{\left(\frac{\varepsilon_0 A}{3d}\right)} = \frac{9Q}{2C}$$

where, $C = \frac{\varepsilon_0 A}{d}$ as said in the question.

$$\Rightarrow \quad V = \frac{9Q}{2C}$$

**Hence, the correct answer is (B).**

**49. Method-1**

Since field between the plates is

$$E = \frac{3Q}{2A\varepsilon_0}$$

Since electric energy density is given by

$$u_{electric} = \frac{1}{2}\varepsilon_0 E^2$$

$$\Rightarrow \quad u_{electric} = \frac{1}{2}\varepsilon_0\left(\frac{3Q}{2A\varepsilon_0}\right)^2 = \frac{9Q^2}{8\varepsilon_0 A^2}$$

So, the electric energy $U$ is obtained by taking the product of the volume of the capacitor with the energy density. Hence

$$U = A(3d)\left(\frac{1}{2}\varepsilon_0 E^2\right)$$

$$U = A(3d)\left(\frac{1}{2}\varepsilon_0 E^2\right) = 3Ad\left(\frac{9Q^2}{8\varepsilon_0 A^2}\right) = \frac{27Q^2}{8C}$$

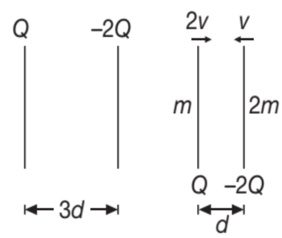
**Method-2**

$$U = \frac{(Q_{inside})^2}{2C_{eq}}$$

$$\Rightarrow \quad U = \frac{\left(\frac{3Q}{2}\right)^2}{2\left(\frac{\varepsilon_0 A}{3d}\right)} = \frac{27Q^2}{8C}$$

**Hence, the correct answer is (C).**

**50.**




By Law of conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in Electrostatic}\\ \text{Energy of system}\end{pmatrix} = \begin{pmatrix}\text{Gain in Kinetic}\\ \text{Energy of system}\end{pmatrix}$$

$$\Rightarrow \quad U_{\text{initial}} - U_{\text{final}} = \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)v^2$$

$$\Rightarrow \quad \frac{27Q^2}{8C} - \frac{9Q^2}{8C} = 3mv^2$$

$$\Rightarrow \quad \frac{9Q^2}{4C} = 3mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{3Q^2}{4mC}} = \frac{3Q}{2}\sqrt{\frac{1}{3mC}}$$

So the relative velocity of approach of the plates is

$$v_{\text{rel}} = 3v = \frac{9Q}{2}\sqrt{\frac{1}{3mC}}$$

**Hence, the correct answer is (D).**

**52.** Since both the capacitors are in parallel, so we have

$$C = C_1 + C_2$$

$$\Rightarrow \quad C = \frac{K\varepsilon_0 Wh}{d} + \frac{\varepsilon_0 W(L-h)}{d}$$

$$\Rightarrow \quad C = \frac{\varepsilon_0 W}{d}[(K-1)h + L]$$

$$\Rightarrow \quad K_{\text{eff}} = \left[\frac{(K-1)h}{L} + 1\right]$$

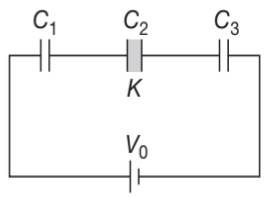
**Hence, the correct answer is (A).**

**53.** When the tank is one-fourth empty then it is three-fourth full, so $h = \frac{3L}{4}$ and hence

$$K_{\text{eff}} = \frac{K+1}{4}$$

**Hence, the correct answer is (D).**

**54.** It is observed that potential difference across $C_3$ is $12\text{ V}$, so we have

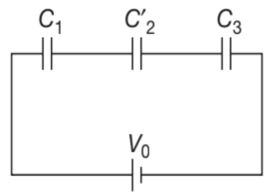



$$12 = \left(\frac{\frac{1}{C_3}}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}\right)V_0 \qquad \ldots(1)$$

Now, when the dielectric material $(K = 3)$ between the plates of $C_2$ is removed then the new capacitance becomes $C_2'$, so that

$$C_2' = \frac{C_2}{3}$$

In this situation, the new potential difference across $C_3$ is found to be $9\text{ V}$, so




$$9 = \left(\frac{\frac{1}{C_3}}{\frac{1}{C_1} + \frac{1}{C_2'} + \frac{1}{C_3}}\right)V_0 \qquad \ldots(2)$$

It is also observed that the potential differences across $C_1$ and $C_2'$ are equal, so we have

$$\Rightarrow \quad C_1V_1 = C_2'V_1$$

$$\Rightarrow \quad C_1 = C_2' = \frac{C_2}{3}$$

$$\Rightarrow \quad C_2 = 3C_1 \qquad \ldots(3)$$

Substituting (3) in (1) and (2), we get

$$12 = \left(\frac{\frac{1}{C_3}}{\frac{4}{3C_1} + \frac{1}{C_3}}\right)V_0 = \frac{V_0}{\frac{4}{3}\frac{C_3}{C_1} + 1} \qquad \ldots(4)$$

$$9 = \left(\frac{\frac{1}{C_3}}{\frac{2}{C_2'} + \frac{1}{C_3}}\right)V_0 = \frac{V_0}{\frac{2C_3}{C_1} + 1} \qquad \ldots(5)$$

Now assuming $\frac{C_3}{C_1} = x$ and dividing equation (4) by equation (5), we get

$$\frac{12}{9} = \frac{2x+1}{4x+3}$$

$$\Rightarrow \quad 16x + 12 = 18x + 9$$

$$\Rightarrow \quad x = \frac{3}{2}$$

$$\Rightarrow \quad x = \frac{C_3}{C_1} = \frac{3}{2}$$

Substituting the value of $x$ in equation (4), we get

$$12 = \frac{V_0 \times 3}{4x + 3}$$

$\Rightarrow \quad 4 = \frac{V_0}{6+3}$

$\Rightarrow \quad V_0 = 36 \text{ V}$

**Hence, the correct answer is (D).**

**55.** If $C_2$ is $6\ \mu F$, then $C_2 = 3C_1$

$\Rightarrow \quad C_1 = \frac{C_2}{3} = \frac{6}{3} = 2\ \mu\text{F}$

**Hence, the correct answer is (D).**

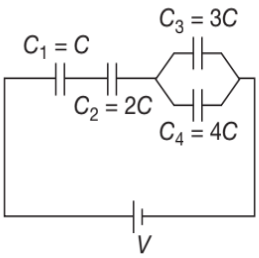
**56.** $C_3 = xC_1 = \frac{3}{2} \times 2 = 3\ \mu\text{F}$

**Hence, the correct answer is (C).**

## Matrix Match/Column Match Type Questions

**1.** A → (q, s)
B → (p)
C → (q, s)
D → (r)

**2.** A → (q)
B → (p)
C → (s)
D → (r)




Effective capacitance is given by

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{7C}$$

$$\Rightarrow \quad \frac{1}{C_{eq}} = \frac{14+7+2}{14C} = \frac{23}{14C}$$

$$\Rightarrow \quad C_{eq} = \frac{14C}{23}$$

So, charge across $C_1$ and $C_2$ is $q = \frac{14CV}{23}$

Charge will get distributed across $C_3$ and $C_4$ in direct ratio of their capacitances and hence charge across $C_3$ is

$$q_{C_3} = \frac{3q}{7} = \frac{3}{7} \times \frac{14CV}{23} = \frac{6CV}{23}$$

Hence, (A) → (q)

Potential differences across $C_3$ and $C_4$ would be the same, because they are in parallel so,

$$V_{C_3} = V_{C_4} = \frac{q}{7C}$$

$$\Rightarrow \quad V_{C_3} = V_{C_4} = \frac{1}{7C}\left(\frac{14CV}{23}\right) = \frac{2V}{23}$$

Hence (B) → (p)

Energy stored across $C_3$ is

$$U_3 = \frac{1}{2} C_3 V_{C_3}{}^2 = \frac{1}{2}(3C)\left(\frac{q}{7C}\right)^2$$

$$\Rightarrow \quad U_3 = \frac{1}{2}(3C)\frac{q^2}{49C^2} = \frac{3q^2}{98C}$$

$$\Rightarrow \quad U_3 = \frac{3q^2}{98C} = \frac{3}{98C}\left(\frac{14CV}{23}\right)^2$$

$$\Rightarrow \quad U_3 = \frac{3 \times 196CV^2}{98 \times 23 \times 23} \simeq \frac{CV^2}{88}$$

Hence (C) → (s)

Energy stored across $C_4$ is

$$U_4 = \frac{1}{2} C_4 V_{C_4}^2 = \frac{1}{2} 4C\left(\frac{q}{7C}\right)^2$$

$$\Rightarrow \quad U_4 = \frac{1}{2}(4C)\left(\frac{q^2}{49C^2}\right)$$

$$\Rightarrow \quad U_4 = \frac{4q^2}{98C} = \frac{4 \times 196CV^2}{98 \times 23 \times 23} \simeq \frac{CV^2}{66}$$

Hence (D) → (r)

**3.** A → (p, r, s)
B → (p, r)
C → (p, r, s)
D → (p, r)

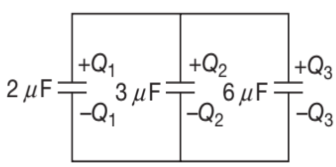
**4.** A → (s)
B → (q)
C → (r)
D → (p)

After reconnection, charges are redistributed as shown

$$Q_1 = (2\ \mu\text{F})V$$

$$Q_2 = (3\ \mu\text{F})V$$

$$Q_3 = (6\ \mu\text{F})V$$



Since, initially the capacitors were in series, so, 12 $\mu$C charge appears on each of the positive plates and $-12\ \mu$C charge appears on each of negative plates. So a total of 36 $\mu$C charge on the three positive plates now redistribute as $Q_1$, $Q_2$ and $Q_3$.

$$Q_1 + Q_2 + Q_3 = 36\ \mu C$$

$$(11\ \mu F)V = 36\ \mu C$$

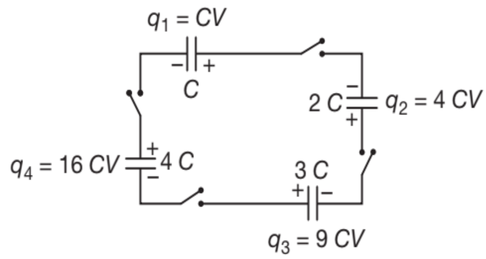
$$\Rightarrow \quad V = \frac{36}{11}\ V$$

$$\Rightarrow \quad Q_1 = \frac{72}{11}\ \mu C$$

$$\Rightarrow \quad Q_2 = \frac{108}{11}\ \mu C$$

$$\Rightarrow \quad Q_3 = \frac{216}{11}\ \mu C$$

**5.** A → (r)
B → (p)
C → (q)
D → (s)

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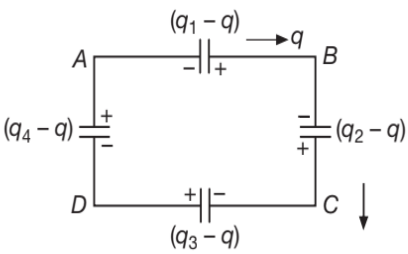



When circuit is closed, let charge $q$ flows in the circuit; then applying Loop Law in ABCDA

$$\frac{q_1 - q}{C} + \frac{q_2 - q}{2C} + \frac{q_3 - q}{3C} + \frac{q_4 - q}{4C} = 0$$

Substituting the value of $q_1$, $q_2$, $q_3$ and $q_4$ we get

$$q = \left(\frac{24}{5}\right)(CV)$$

Hence $V_1 = \frac{q_1 - q}{C} = \frac{19V}{5}$

$$V_2 = \frac{q_2 - q}{2C} = \frac{2V}{5}$$

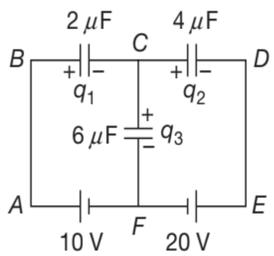
$$V_3 = \frac{q_3 - q}{3C} = \frac{7V}{5}$$

$$V_4 = \frac{q_4 - q}{4C} = \frac{14V}{5}$$

**6.** A → (q)
B → (r)
C → (p)
D → (s)

The charges on three plates which are in contact add to zero, because these plates taken together form an isolated system which cannot receive charges from the batteries

Thus, $q_3 - q_1 - q_2 = 0$ ...(1)




Applying Kirchhoff's Law in loop ABCFA and CDEFC

$$-\frac{q_1}{2} - \frac{q_3}{6} + 10 = 0$$

$$\Rightarrow \quad q_3 + 3q_1 = 60 \quad \text{...(2)}$$

and $20 - \frac{q_2}{4} - \frac{q_3}{6} = 0$

$$\Rightarrow \quad 3q_2 + 2q_3 = 240 \quad \text{...(3)}$$

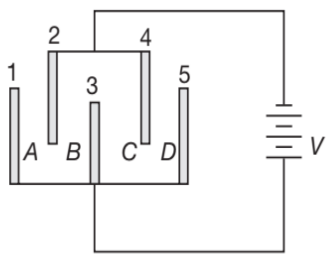
Solving the above three equations, we have

$$q_1 = \frac{10}{3}\ \mu C,\ q_2 = \frac{140}{3}\ \mu C,\ q_3 = 50\ \mu C$$

Potential difference across 6 $\mu$C is

$$V_{6\ \mu C} = \frac{q_3}{6} = \frac{50\ \mu C}{6\ \mu F} = \frac{25}{3}\ V$$

**7.** A → (q)
B → (p)
C → (s)
D → (r)



These five plates constitute four identical capacitors in parallel, each of capacity $\frac{\varepsilon_0 A}{d}$. Now as plate 1 is connected to positive terminal of battery and is a part of one capacitor only, so charge on it

$$q_1 = +\left(\frac{\varepsilon_0 AV}{d}\right) = 1\left(\frac{\varepsilon_0 AV}{d}\right)$$

So (A) $\rightarrow$ (q)

However the plate 4 is connected to negative terminal of battery and in common to two identical capacitors in parallel.

So $q_4 = -\frac{2\varepsilon_0 AV}{d} = -2\left(\frac{\varepsilon_0 AV}{d}\right)$

Hence (B) $\rightarrow$ (p)

From circuit, it is obvious that between the plates 2 and 3, battery is connected so potential difference will be $V\left(=\frac{1}{2}(2V)\right)$. Between 1 and 5. Plates 1 and 5 gets connected through connecting wire, so potential difference is zero $(=0(2\text{ V}))$.

Hence (C) $\rightarrow$ (s), (D) $\rightarrow$ (r)

**8.** A $\rightarrow$ (q)
B $\rightarrow$ (r)
C $\rightarrow$ (s)
D $\rightarrow$ (p)
Done Already

**9.** A $\rightarrow$ (p, s)
B $\rightarrow$ (q, r, s)
C $\rightarrow$ (q, r, s)
D $\rightarrow$ (p)

**10.** A $\rightarrow$ (p, q)
B $\rightarrow$ (r, q)
C $\rightarrow$ (p, q, s)
D $\rightarrow$ (r, q)

(A) For an isolated capacitor

$Q = \text{constant}$

So, if distance between plate decreases, then capacitance will increase.

Since $Q = CV$

$\Rightarrow \quad V \propto \frac{1}{C}$

So as $C$ increases $V$ decreases.

Since Energy $U = \frac{Q^2}{2C}$

So as $C$ increases energy decreases.

Since Force $F = \frac{Q^2}{2A\varepsilon_0}$

So, when $Q = \text{constant}$, then $F = \text{constant}$.

(B) As battery is connected to capacitor, so $V$ is constant

when slab is inserted, $C$ increases

So, energy $U = \frac{1}{2}CV^2$ also increases

Since $F = \frac{CV^2}{2d}$

So, as $U$ increases, $F$ also increases.

(C) If area of plates of capacitor increases, then $C$ increases.

As capacitor is isolated, so

$Q = \text{constant}$

Since, $V = \frac{Q}{C}$

As $C$ increases, $V$ decreases.

Since, $U = \frac{Q^2}{2C}$

As $C$ increases, $U$ decreases.

Since, $F = \frac{q^2}{2\varepsilon_0 A} = \frac{q^2}{2Cd}$

As $U$ decreases i.e. $C$ increases, $F$ decreases.

(D) Capacitor is connected with battery

$V = \text{constant}$

As $d$ decreases, $C$ increases

Since, $U = \frac{1}{2}CV^2$

As $C$ increases, $U$ increases.

Since, $F = \frac{Q^2}{2A\varepsilon_0} = \frac{C^2V^2}{2A\varepsilon_0}$

As $C$ increases, so $F$ increases.

**11.** A $\rightarrow$ (q, s)
B $\rightarrow$ (r, s)
C $\rightarrow$ (p, r, s)
D $\rightarrow$ (p, r, s)

**For A:** If $V = E$, i.e. potential difference across the capacitor is same as that of e.m.f. of battery, no charge will be supplied by battery and hence work done by battery $= 0$.

$U_i = U_f$ so $\Delta H = 0$

In all cases, zero charge appears on outer surfaces of the plates of capacitor.

**For B:** If $V > E$, so capacitor charges the battery and hence thermal energy will be developed in circuit.

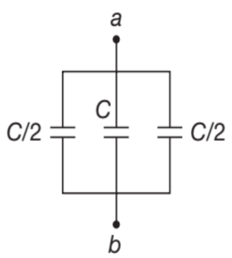
**For C:** If $V < E$ so battery performs some work on the capacitor.

**For D:** Combination of $B$ and $C$.

**12.** A $\rightarrow$ (q)
B $\rightarrow$ (p)
C $\rightarrow$ (r)
D $\rightarrow$ (s)

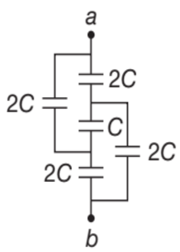
**13.** A $\rightarrow$ (p)
B $\rightarrow$ (p)
C $\rightarrow$ (r)
D $\rightarrow$ (q)

(A)



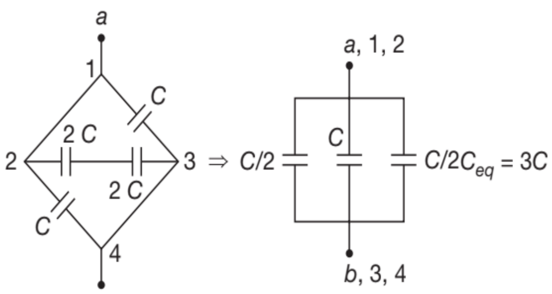

$C_{eq} = 2C$

(B)



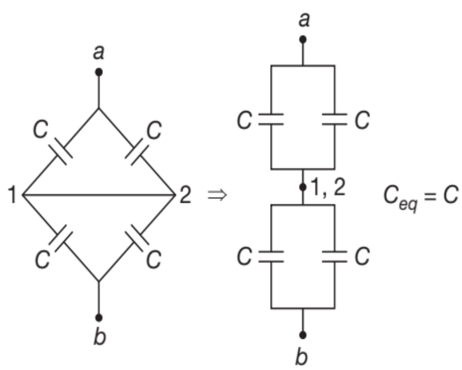

$C_{eq} = 2C$

(C)




$C_{eq} = 3C$

(D)




$C_{eq} = C$

**14.** A $\rightarrow$ (p, s)
B $\rightarrow$ (r, s)
C $\rightarrow$ (p, q)
D $\rightarrow$ (q)

**For A:** Charge is constant, $C$ increases, so energy stored decreases. Since system is isolated, so

$$\Delta U < 0 \text{ and } W_{ext} = \Delta U < 0$$

**For B:** Since potential is constant, so $C$ increases, so energy stored increases

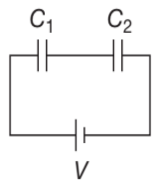
$$\Rightarrow \quad \Delta U > 0 \text{ and } W_{e\ell} = -\Delta U < 0$$

Since charge of capacitor increases, which means that work done by battery is greater then zero.

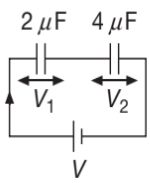
**For C:** Potential is constant, $C$ decreases, $Q$ decreases and $U$ also decreases. So, $W_{battery} < 0$, $\Delta U < 0$, $W_{ext} > 0$

**For D:** Charge is constant, $C$ decreases, $U$ increases So, $\Delta U > 0$, $W_{ext} > 0$

**15.** A $\rightarrow$ (p, r)
B $\rightarrow$ (q)
C $\rightarrow$ (s, q)
D $\rightarrow$ (q)



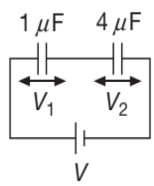

$$C = \frac{\varepsilon_0 A}{d}$$




So, $V_1 = \frac{4}{6} \times V = \frac{4}{6} V = \frac{2}{3} V = 0.6 \text{ V}$ and

$$V_2 = \frac{2}{6} \times V = \frac{V}{3} = 0.3 \text{ V}$$

When $d$ is double $d$, then $C$ becomes $C' = \frac{2}{2} = 1\,\mu\text{F}$




$$V_1' = \frac{4V}{5} = 0.8 \text{ V}$$

$$V_2' = \frac{1V}{5} = 0.2 \text{ V}$$

So, potential difference across $C_1$ increases and that across $C_2$ decreases.

Initial energy stored in

$$C_1 \text{ is } (U_1)_i = \frac{1}{2}C_1V_1^2 = \frac{1}{2}\times 2\times\frac{16}{36}V^2 = \frac{16}{36}V^2$$

$$C_2 \text{ is } (U_2)_i = \frac{1}{2}C_2V_2^2 = \frac{1}{2}\times 4\times\frac{V^2}{9} = \frac{8}{9}V^2$$

Final energy stored in

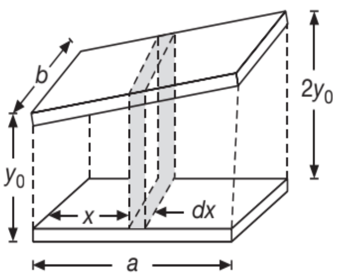
$$C_1 \text{ is } (U_1)_f = \frac{1}{2}\times 1\times\frac{16}{25}V^2 = \frac{8}{25}V^2$$

$$C_2 \text{ is } (U_2)_f = \frac{1}{2}\times 4\times\frac{V^2}{25} = \frac{2}{25}V^2$$

So, potential energy of $C_1$ decreases by a factor of $\frac{18}{25}$ and potential energy of $C_2$ decreases.

## Integer/Numerical Answer Type Questions

1. Let us consider an infinitesimal strip of width $dx$ and length $b$ to approximate a differential capacitor of area $bdx$ and separation $d = y_0 + \left(\frac{y_0}{a}\right)x$. All such differential capacitors are in parallel arrangement.




$$dC = \frac{\varepsilon_0(bdx)}{y_0 + \left(\frac{y_0}{a}\right)x}$$

$$C = \varepsilon_0 b\int_0^a \frac{dx}{\left(y_0 + \frac{y_0}{a}x\right)}$$

$$C = \frac{\varepsilon_0 b}{\left(\frac{y_0}{a}\right)}\left[\log_e\left(\frac{y_0 + \frac{y_0}{a}\times a}{y_0}\right)\right]$$

$$C = \frac{\varepsilon_0 ab}{y_0}\log_e 2$$

$\Rightarrow \quad \alpha = 2$

2. The original kinetic energy of the particle is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(2\times10^{-16}\text{ kg}\right)\left(2\times10^{6}\text{ ms}^{-1}\right)^2$$

$\Rightarrow \quad K = 4\times10^{-4}$ J

The potential difference across the capacitor is

$$\Delta V = \frac{Q}{C} = \frac{1000\ \mu\text{C}}{10\ \mu\text{F}} = 100\text{ V}$$

For the particle to reach the negative plate, the particle-capacitor system would need energy

$$U = q\Delta V = \left(-3\times10^{6}\text{ C}\right)(-100\text{ V})$$

$\Rightarrow \quad U = 3\times10^{-4}$ J

Since its original kinetic energy is greater than this, the particle will reach the negative plate
As the particle moves, the system keeps constant total energy

$$(K+U)_{\text{at + plate}} = (K+U)_{\text{at-plate}}$$

$$4\times10^{-4}\text{ J} + \left(-3\times10^{-6}\text{ C}\right)(+100\text{ V}) = \frac{1}{2}\left(2\times10^{-16}\right)v_f^2 + 0$$

$$\Rightarrow \quad v_f = \sqrt{\frac{2\left(1\times10^{-4}\text{ J}\right)}{2\times10^{-16}\text{ kg}}} = 10^6\text{ ms}^{-1} = 1000\text{ kms}^{-1}$$

3. Initially (capacitors charged in parallel),

$$q_1 = C_1(\Delta V) = (6\ \mu\text{F})(250\text{ V}) = 1500\ \mu\text{C}$$

$$q_2 = C_2(\Delta V) = (2\ \mu\text{F})(250\text{ V}) = 500\ \mu\text{C}$$

After reconnection (positive plate to negative plate),

$$q'_{\text{total}} = q_1 - q_2 = 1000\ \mu\text{C}$$

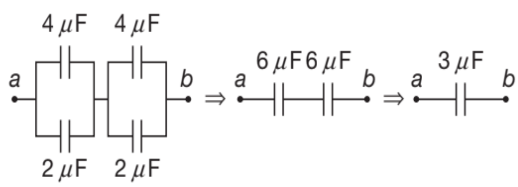
and $\Delta V' = \dfrac{q'_{\text{total}}}{C_{\text{total}}} = \dfrac{1000\ \mu\text{C}}{8\ \mu\text{F}} = 125\text{ V}$

Therefore,

$$q'_1 = C_1(\Delta V') = (6\ \mu\text{F})(125\text{ V}) = 750\ \mu\text{C}$$

$$q'_2 = C_2(\Delta V') = (2\ \mu\text{F})(125\text{ V}) = 250\ \mu\text{C}$$

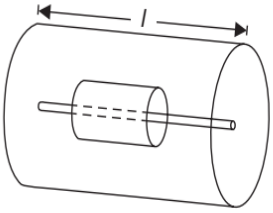
4. Assume a potential difference across $a$ and $b$, and notice that the potential difference across the $8\ \mu$F capacitor must be zero by symmetry. Then the equivalent capacitance can be determined from the following circuit.




$$C_{ab} = 3\ \mu\text{F}$$

5. $2\pi r \ell E = \frac{q_{in}}{\varepsilon_0}$

So $E = \frac{\lambda}{2\pi r \varepsilon_0}$




$$\Delta V = -\int_{r_1}^{r_2} E \cdot dr = \int_{r_1}^{r_2} \frac{\lambda}{2\pi r \varepsilon_0} dr = \frac{\lambda}{2\pi\varepsilon_0} \log_e\left(\frac{r_1}{r_2}\right)$$

$$\frac{\lambda_{\max}}{2\pi\varepsilon_0} = E_{\max} r_{\text{inner}}$$

$$\Delta V = \left(1.2\times10^6 \text{ Vm}^{-1}\right)\left(0.1\times10^{-3} \text{ m}\right)\log_e\left(\frac{25}{0.2}\right)$$

$$\Delta V_{\max} = 579 \text{ V}$$

6. Capacitance of metal surface of radius

$r = 0.5\times10^{-3}$ m

$$C = 4\pi\varepsilon_0 r = \frac{0.5\times10^{-3}}{9\times10^9} = \frac{1}{18}\times10^{-12} \text{ farad}$$

Rate of escape of charge from surface is $R$ is

$$R = \frac{80}{100}\times ne$$

$$\Rightarrow \quad R = \frac{80}{100}\times 6.25\times10^{10}\times1.6\times10^{-19} \text{ Cs}^{-1}$$

$$\Rightarrow \quad R = 8\times10^{-9} \text{ Cs}^{-1}$$

$\therefore$ If $t$ is the required time, then charge escaped

$$q = Rt = \left(8\times10^{-9}\right)t$$

From relation $q = CV$, we have

$$8\times10^{-9} t = \frac{1}{18}\times10^{-12}\times1 \qquad \{\because V = 1 \text{ V}\}$$

$$\Rightarrow \quad t = \frac{10^{-12}}{8\times10^{-9}\times18} = \frac{10^{-3}}{144}$$

$$\Rightarrow \quad t = \frac{1000\times10^{-6}}{144} s = 6.95\ \mu s = 6950 \text{ ns}$$

7. If $d$ is the separation between the plates (each of area $A_0$) of a parallel plate condenser in air, then its capacitance $C = \frac{\varepsilon_0 A}{d}$ ...(1)

If a slab of thickness $t$ is introduced between the plates with new separation $d'$, then its new capacitance,

$$C' = \frac{\varepsilon_0 A}{d' - t\left(1 - \left(\frac{1}{K}\right)\right)} \qquad \text{...(2)}$$

As $Q = CV$, the charge on the capacitor is same in both cases, therefore to maintain the same potential difference the capacitance $C$ and $C'$ must be same i.e., from (1) and (2).

$$\frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d' - t\left(1 - \left(\frac{1}{K}\right)\right)}$$

$$\Rightarrow \quad d = d' - t\left(1 - \left(\frac{1}{K}\right)\right)$$

Here, $d' = d + 2.4 \text{ mm} = d + 2.4\times10^{-3}\text{m}$

$t = 3 \text{ mm} = 3\times10^{-3}\text{m}$

$$\Rightarrow \quad d = d + 2.4\times10^{-3} - 3\times10^{-3}\left\{1 - \left(\frac{1}{K}\right)\right\}$$

$$\Rightarrow \quad 3\left\{1 - \left(\frac{1}{K}\right)\right\} = 2.4$$

$$\Rightarrow \quad K = 5$$

8. (a) The capacitors of 3, 5 and 4 $\mu$F are in parallel, therefore their equivalent capacitance

$$C_1 = 3 + 5 + 4 = 12\ \mu\text{F}$$

Again the capacitor of 4 and 2 $\mu$F are equivalent to a single capacitor of capacitance

$$C_3 = 4 + 2 = 6\ \mu\text{F}$$

For equivalent capacitance between $x$ and $y$, $C_1$ and $C_2$ are in series. If $C'$ is the equivalent capacitance of $C_1$ and $C_2$, then

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{12\times4}{12+4} = \frac{48}{16} = 3\ \mu\text{F}$$

Similarly, $C_3$ and $C_4$ are in series and their equivalent capacitance C" is given by

$$C'' = \frac{C_3 C_4}{C_3 + C_4} = \frac{6\times3}{6+3} = \frac{18}{9} = 2\ \mu\text{F}$$

For equivalent capacitance between $x$ and $y$, $C'$ and $C''$ are in parallel, therefore the equivalent capacitance between $x$ and $y$

$$C = C' + C'' = 3 + 2 = 5\ \mu\text{F}$$

(b) The charge of 5 $\mu$F capacitor

$$= 120\ \mu\text{C} = 120\times10^{-6} \text{ C}$$

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The p.d. across $5\ \mu\text{F}$ capacitor

$$= \frac{120 \times 10^{-6}}{5 \times 10^{-6}} = 24\ \text{V}$$

As the capacitor 3, 5 and $4\ \mu\text{F}$ are in parallel, the p.d. across each of these capacitors is 24 V.

$\therefore$ The charge on $3\ \mu\text{F}$ capacitor

$$= 3 \times 10^{-6} \times 24\ \text{C}$$

$$= 72 \times 10^{-6}\ \text{C}$$

and the charge on $4\ \mu\text{F}$ capacitor

$$= 4 \times 10^{-6} \times 24 = 96 \times 10^{-6}\ \text{C}$$

Therefore, the total charge flowing through $C_1$ and $C_2$ is

$$= (72 + 120 + 96) \times 10^{-6}\ \text{C}$$

$$= 288 \times 10^{-6}\ \text{C}$$

The potential difference across $C_2$

$$= \frac{288 \times 10^{-6}}{4 \times 10^{-6}} = 72\ \text{V}$$

$\therefore$ Net potential difference (p.d.) between $x$ and $y$

$$= 24 + 72 = 96\ \text{V}$$

As equivalent capacitance of $C_3$ and $C_4$ is $2\ \mu\text{F}$, the charge flowing through $C_3$ and $C_4$

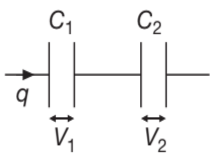
$$= 96 \times 2 \times 10^{-6}\ \text{C}$$

$$= 192 \times 10^{-6}\ \text{C}$$

The potential difference between $x$ and $z$ is

$$V_{xz} = \frac{192 \times 10^{-6}}{3 \times 10^{-6}} = 64\ \text{V}$$

**9.** Given $V_1 = 6\ \text{kV}$, $V_2 = 4\ \text{kV}$




Charge on first capacitor

$$q_1 = C_1 V_1 = 1\ \mu\text{F} \times (6 \times 10^3\ \text{V})$$

$\Rightarrow$ $q_1 = 6000\ \mu\text{C}$

Charge on second capacitor

$$q_2 = C_2 V_2$$

$\Rightarrow$ $q_2 = (2\ \mu\text{F}) \times (4 \times 10^3\ \text{V}) = 8000\ \mu\text{C}$

In series the charge on each capacitor remains same, but maximum charge on first capacitor will be $6000\ \mu\text{C}$.

Therefore charge on second capacitor must also be $6000\ \mu\text{C}$

The potential across second capacitor

$$V_2 = \frac{q}{C_2} = \frac{6000\ \mu\text{C}}{2\ \mu\text{F}} = 3000\ \text{V} = 3\ \text{kV}$$

So, maximum voltage across system will be

$$= V_1 + V_2 = 6\ \text{kV} + 3\ \text{kV} = 9\ \text{kV}$$

**10.** Capacitance of parallel plate capacitor.

$$C = \frac{\varepsilon_0 A}{d}$$

Charge on capacitor $q = CV = \frac{\varepsilon_0 A}{d} V$

If the separation is doubled, the capacitance becomes half, i.e., here capacitance $C' = \frac{C}{2}$. If battery is disconnected the charge remains same i.e., $q$ = constant.

$\therefore$ Net potential $V'$ is given by $CV = C'V'$

$$V' = \frac{CV}{C'}$$

As $C' = \frac{C}{2}$

$$V' = \frac{CV}{\left(\frac{C}{2}\right)} = 2V = 2 \times 10^3\ \text{V}$$

Work required, $W = U_2 - U_1$

$$= \frac{q^2}{2C'} - \frac{q^2}{2C} = \frac{q^2}{2}\left[\frac{1}{\frac{C}{2}} - \frac{1}{C}\right] = \frac{q^2}{2C}$$

$$= \frac{(CV)^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} \frac{\varepsilon_0 A}{d} V^2$$

Here $A = 0.2\ \text{m}^2$, $d = 10^{-2}\ \text{m}$

$$\Rightarrow \quad W = \frac{1}{2}\left(\frac{1}{36\pi \times 10^7}\right) \times \frac{0.2}{10^{-2}} \times (10^3)^2$$

$$\Rightarrow \quad W = \frac{1}{36\pi} \times 10^{-2}\ \text{J} = 8.8 \times 10^{-5}\ \text{J}$$

Final voltage $(V')$ across capacitor is given by

$$q = CV = C'V'$$

i.e., $V' = \frac{CV}{C'} = \frac{C}{\frac{C}{2}} V = 2V = 2\times 10^3$ V

**11.** First of all we shall calculate the equivalent capacitance of this arrangement. The condensers (1) and (2) are in series. Their equivalent capacitance is give by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}$$

$$\Rightarrow \quad C' = \frac{C_1 C_2}{C_1 + C_2}$$

$$\Rightarrow \quad C' = \frac{C_1 \times \eta C_1}{C_1 + \eta C_1} = \frac{\eta C_1}{1+\eta} \qquad \ldots(1)$$

Now $C'$ and capacitor (4) are in parallel. Hence

$$C'' = C' + C_2 = \frac{\eta C_1}{1+\eta} + \eta C_1$$

$$\Rightarrow \quad C'' = \frac{\left(\eta^2 + 2\eta\right)C_1}{1+\eta} \qquad \ldots(2)$$

Further $C''$ and capacitor (3) are in series. So the equivalent capacitance $C$ is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{(1+\eta)}{\left(\eta^2 + 2\eta\right)C_1} = \frac{\left(\eta^2+2\eta\right)+(1+\eta)}{\left(\eta^2+2\eta\right)C_1}$$

$$\Rightarrow \quad C = \frac{\left(\eta^2+2\eta\right)C_1}{\eta^2 + 3\eta + 1} \qquad \ldots(3)$$

$$\Rightarrow \quad q = CE = \frac{\left(\eta^2+2\eta\right)C_1 E}{\eta^2+3\eta+1} \qquad \ldots(4)$$

The voltage $V$ across equivalent capacitor is given by

$$V = \frac{q}{C''} = \frac{\left(\eta^2+2\eta\right)C_1 E}{\eta^2+3\eta+1} \times \frac{(1+\eta)}{\left(\eta^2+2\eta\right)C_1}$$

$$\Rightarrow \quad V = \frac{(1+\eta)E}{\eta^2+3\eta+1} \qquad \ldots(5)$$

($\because$ The charge $q$ will be across $q_1$ and equivalent capacitor $C''$)

The voltage $V$ will be across $C_2$ and equivalent capacitor $C'$

$$q' = C'V = \left(\frac{\eta C_1}{1+\eta}\right)\left[\frac{(1+\eta)E}{\eta^2+3\eta+1}\right]$$

$$\Rightarrow \quad q' = \frac{\eta C_1 E}{\eta^2+3\eta+1} \qquad \ldots(6)$$

The charge will be distributed across $C_1$ and $C_2$.

Voltage across $C_2$ is given by

$$V = \frac{q'}{C_2} = \frac{q'}{\eta C_1} = \frac{1}{\eta C_1}\left[\frac{\eta C_1 E}{\eta^2+3\eta+1}\right]$$

$$\Rightarrow \quad V = \frac{E}{\eta^2+3\eta+1} \qquad \ldots(7)$$

Substituting the values, we get

$$V = \frac{110}{4+6+1} = \frac{110}{11} = 10 \text{ V}.$$

**12.** Charge on $10\ \mu$F condenser

$$q_1 = 10\times 10^{-6} \times 150 \text{ C} = 2.5\times 10^{-3} \text{ C}$$

Charge on $20\ \mu$F condenser

$$q_2 = 20\times 10^{-6}\times 300 \text{ C} = 6\times 10^{-3} \text{ C}$$

When the two condensers are connected in parallel, the total charge

$$q = \left(1.5\times 10^{-3} + 6\times 10^{-3}\right) = 7.5\times 10^{-3} \text{ C} = 7500\ \mu\text{C}$$

The equivalent capacitance $= 30\ \mu$F

If $V$ be the potential, then

$$30\times 10^{-6} V = 7.5\times 10^{-3}$$

$$\Rightarrow \quad V = \frac{7.5\times 10^{-3}}{30\times 10^{-6}} = 250 \text{ V}$$

Energy of first condenser before connection

$$U_1 = \frac{1}{2}\times\left(1.5\times 10^{-3}\right)150$$

Energy of second condenser before connection

$$U_2 = \frac{1}{2}\times\left(6\times 10^{-3}\right)300$$

Total energy of condensers before connection

$$U_{\text{initial}} = \frac{1}{2}\times\left(1.5\times 10^{-3}\right)150 + \frac{1}{2}\left(6\times 10^{-3}\right)300$$

$$\Rightarrow \quad U_i = 1.0125 \text{ J}$$

Energy after connection

$$U_f = \frac{1}{2}\times\left(7.5\times 10^{-3}\right)250 = 0.9375 \text{ J}$$

Energy lost

$$-\Delta U = 1.0125 - 0.9375 = 0.075 \text{ J} = 75 \text{ mJ}$$

**13.** The capacitance $C_0$ before the slab is introduced

$$V_0 = \frac{\varepsilon_0 A}{d} = \frac{\left(8.9\times 10^{-12}\right)\left(10^{-2}\right)}{10^{-2}} = 8.9\times 10^{-12} \text{ farad}$$

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$\therefore$ Free charge

$$q = C_0V_0 = (8.9\times10^{-12})\times100 = 890\times10^{-12}\ \text{C} = 890\ \text{pC}$$

Now, Electric field intensity

$$E_0 = \frac{V_0}{d} = \frac{100}{10^{-2}} = 1\times10^4\ \text{Vm}^{-1} = 10\ \text{kVm}^{-1}$$

Electric field intensity in dielectric

$$E = \frac{E_0}{K} = \frac{1\times10^4}{7} = 1.43\times10^3\ \text{Vm}^{-1} = 1430\ \text{Vm}^{-1}$$

Potential difference between the plates with dielectric present is given by

$$V = E_0(d-b) + Eb$$

$$\Rightarrow\quad V = (1\times10^4)(10^{-2} - 0.6\times10^{-2}) + (1.43\times10^3)(0.5\times10^{-2})$$

$$\Rightarrow\quad V = 57\ \text{V}$$

The free charge on the plate is the same as before. The capacitance with dielectric present is

$$C = \frac{q}{V} = \frac{8.9\times10^{-10}\ \text{C}}{57\ \text{V}} = 16\times10^{-12}\ \text{farad} = 16\ p\text{F}$$

**14.** Potential difference without introducing the slab is given by

$$V_0 = E\times d = 300\times5 = 1500\ \text{V}$$

The capacitance $C_0$ is given by

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{0.05}\ \text{farad}$$

$$\Rightarrow\quad \varepsilon_0 A = 0.05C_0 \qquad \ldots(1)$$

When the slab is inserted, the capacitance $C$ is given by

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}} \quad \text{where } t = \text{thickness of slab}$$

$$\Rightarrow\quad C = \frac{0.05C_0}{0.05 + \frac{0.01}{5}} = \frac{25C_0}{21} \qquad \ldots(2)$$

Now, $C_0V_0 = CV$

$$C_0\times1500 = \frac{25C_0}{21}\times V$$

$$\Rightarrow\quad V = \frac{1500\times21}{25} = 1260\ \text{V}$$

Let $x$ be the thickness of the metal plate. Now the effective distance in air between the plates is $(5-x)$ cm.

Here it should be remembered that the metal sheet splits the single capacitor into two in series.

$$\therefore\quad C_f = \frac{\varepsilon_0 A}{\frac{(5-x)}{100}}\ \text{farad}$$

Again $CV = C_0V_0 = C_fV_f$

$$\Rightarrow\quad V_f = V_0\frac{C_0}{C_f} = 1500\times\frac{\varepsilon_0 A}{0.05}\times\frac{(5-x)}{\varepsilon_0 A\times100}$$

According to the problem $V_f = 1260$ V

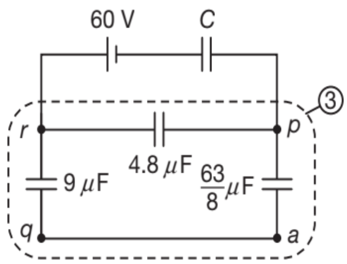
$$\Rightarrow\quad 1500\times\frac{\varepsilon_0 A}{0.05}\times\frac{(5-x)}{\varepsilon_0 A\times100} = 1260$$

$$\Rightarrow\quad (5-x) = 4.2$$

$$\Rightarrow\quad x = 0.8\ \text{cm} = 8\ \text{mm}$$

**15.** The $3\ \mu$F and $5\ \mu$F capacitors are in series, the potential difference across them will be in inverse ratio of capacitance.

Thus P.D. across $5\ \mu$F capacitor $= 10\times\frac{3}{5} = 6$ V

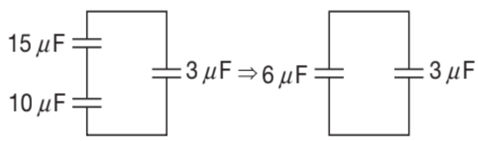



P.D. across $p$ and $b$ $= (10+6)V = 16$ V

Charge on $6\ \mu$F capacitor $= (16\times6)\mu\text{C} = 96\ \mu C$

Total charge in enclosed portion 1 of the circuit

$$= (96+30)\ \mu\text{C} = 126\ \mu\text{C}$$




The two enclosed portions (1) and (2) are in series combination as shown in figure. Charge on $9\ \mu$F capacitor $= 126\ \mu C$.

If $Q_1$ and $Q_2$ are charges on $3\ \mu$F and $6\ \mu$F branches respectively.

$$\frac{Q_1}{3} = \frac{Q_2}{6} \quad \text{...(1)}$$

and $Q_1 + Q_2 = 126\ \mu C$ ...(2)

Now solving equations (1) and (2) simultaneously, we get

$Q_1 = 42\ \mu C$,

$Q_2 = 84\ \mu C$

The P.D. across $3\ \mu F$ capacitor (that is between points $q$ and $r$)

$$= \frac{Q_1}{C} = \frac{42}{3} = 14\text{ V}$$

Thus the P.D. between $p$ and $r = (16+14)$ V=30 V

The total charge on $4.8\ \mu F$ capacitor

$= (4.8 \times 30)\mu C = 144\ \mu C$

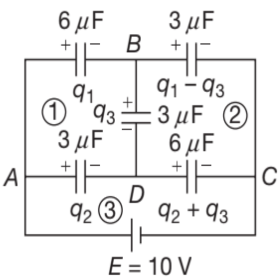
Total charge in the enclosed region 3

$= 144\ \mu C + 126\ \mu C = 270\ \mu C$

The capacitor C is in series with enclosed portion 3, so charge on it is $270\ \mu C$ and potential $(60-30)$ V

Hence $C = \left(\frac{270}{60-30}\right)\mu F = 9\ \mu F$

**16.** Let the charge on each capacitor be as shown in figure. Applying $KVL$ in the loop (1), traversing clock-wise,




$$-\frac{q_1}{6} - \frac{q_3}{3} + \frac{q_2}{3} = 0$$

$\Rightarrow -q_1 - 2q_3 + 2q_2 = 0$ ...(1)

For loop (2), traversing clockwise, we have

$$-\left(\frac{q_1 - q_3}{3}\right) + \frac{q_2 + q_3}{6} + \frac{q_3}{3} = 0$$

$-2q_1 + 5q_3 + q_2 = 0$ ...(2)

For loop $ABCEA$, traversing clockwise, we have

$$-\frac{q_1}{6} - \frac{(q_1 - q_3)}{3} + 10 = 0$$

$3q_1 - 2q_3 = 60$ ...(3)

Solving equations (1), (2) and (3), we get

$q_1 = 24\ \mu C$;

$q_2 = 18\ \mu C$;

$q_3 = 6\ \mu C$

Charge on $6\ \mu F$ capacitor is $24\ \mu C$

Charge on $3\ \mu F$ capacitor is $18\ \mu C$

**17.** (a) $C = \frac{Q}{\Delta V}$

$\Rightarrow 6 \times 10^{-6} = \frac{Q}{20}$ and $Q = 120\ \mu C$

(b) Since $Q = 120\ \mu C$

$\Rightarrow Q_1 + Q_2 = 120\ \mu C$

$\Rightarrow Q_1 = 120 - Q_2$ ...(1)

Also, $\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$

$$\Rightarrow \frac{120 - Q_2}{6} = \frac{Q_2}{3}$$

$\Rightarrow Q_2 = 40\ \mu C$ and $Q_1 = 80\ \mu C$

**18.** With switch closed, distance $d' = \frac{d}{2}$ and capacitance

$$C' = \frac{\varepsilon_0 A}{d'} = \frac{2\varepsilon_0 A}{d} = 2C$$

(a) $Q = C'(\Delta V) = 2C(\Delta V) = 2(2 \times 10^{-6}\text{ F})(100\text{ V})$

$\Rightarrow Q = 400\ \mu C$

(b) The force stretching out one spring is

$$F = \frac{Q^2}{2\varepsilon_0 A} = \frac{4C^2(\Delta V)^2}{2\varepsilon_0 A} = \frac{2C^2(\Delta V)^2}{\left(\frac{\varepsilon_0 A}{d}\right)d}$$

$$F = \frac{2C(\Delta V)^2}{d}$$

One spring stretches by distance $x = \frac{d}{4}$

CHAPTER 2

$$\Rightarrow \quad k=\frac{F}{x}=\frac{2C(\Delta V)^2}{d}\left(\frac{4}{d}\right)=\frac{8C(\Delta V)^2}{d^2}$$

$$\Rightarrow \quad k=\frac{8\left(2\times10^{-6}\ \text{F}\right)(100\ \text{V})^2}{\left(8\times10^{-3}\ \text{m}\right)^2}$$

$$\Rightarrow \quad k=2500\ \text{Nm}^{-1}$$

**19.** The energy transferred is

$$H_{ET}=\frac{1}{2}Q\Delta V=\frac{1}{2}(50\ \text{C})\left(1\times10^{8}\ V\right)$$

$$H_{ET}=2.5\times10^{9}\ \text{J}$$

and 1% of this is absorbed by the tree
So, energy absorbed is given by

$$\Delta E_{\text{absorbed}}=2.5\times10^{7}\ \text{J}$$

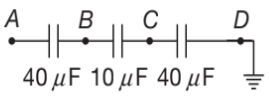
If $m$ is the amount of water boiled away, then

$$\Delta E_{\text{int}}=m\left(4186\ \text{Jkg}^{-1}\ {}^\circ\text{C}\right)(100\ {}^\circ\text{C}-30\ {}^\circ\text{C})+ m\left(2.26\times10^{6}\ \text{Jkg}^{-1}\right)=2.5\times10^{7}\ \text{J}$$

$$\Rightarrow \quad m=9.79\ \text{kg}$$

So, $m\cong 10$ kg

**20.** The given combination of capacitors is equivalent to the circuit diagram shown in the figure




Assume the charge on point $A$ to be $Q$. Then,

$$Q=(40\ \mu\text{F})\Delta V_{AB}=(10\ \mu\text{F})\Delta V_{BC}=(40\ \mu\text{F})\Delta V_{CD}$$

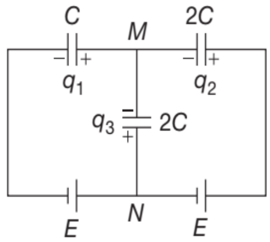
$$\Rightarrow \quad \Delta V_{BC}=4\Delta V_{AB}=4\Delta V_{CD}$$

So, the centre capacitor will break down first, at $\Delta V_{BC}=15$ V and when this occurs, we have

$$\Delta V_{AB}=\Delta V_{CD}=\frac{1}{4}(\Delta V_{BC})=4\ \text{V}$$

$$\Rightarrow \quad V_{AD}=V_{AB}+V_{BC}+V_{CD}=4\ \text{V}+16\ \text{V}+4\ \text{V}=24\ \text{V}$$

**21.** Let $C_1=C$ and $C_2=2C$ and the charges on the different capacitors be as shown in figure.



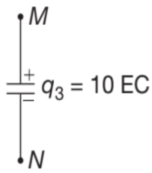

Net charge on isolated system should be zero.

Hence, $q_1-q_2-q_3=0$ …(1)

Applying Loop's Law in two loops we have,

$$E-\frac{q_3}{2C}-\frac{q_1}{C}=0 \quad \ldots(2)$$

$$E-\frac{q_2}{2C}+\frac{q_3}{2C}=0 \quad \ldots(3)$$




Solving these three equations, we get

$$q_3=-10EC$$

$$\Rightarrow \quad V_{MN}=\frac{q_3}{2C}=5E$$

$$\Rightarrow \quad V_{MN}=550\ \text{V}$$

**22.** $W_{\text{ext}}=\dfrac{q^2}{2\varepsilon_0 A}(x_f-x_i)=\dfrac{q^2}{2\varepsilon_0 A}(2d-d)=\dfrac{q^2}{2\varepsilon_0 A}d$

$$\Rightarrow \quad W_{\text{ext}}=\frac{q^2}{2\left(\dfrac{\varepsilon_0 A}{d}\right)}=\frac{q^2}{2C}=\frac{\left(20\times10^{-12}\times200\right)^2}{2\times\left(20\times10^{-12}\right)}$$

$$\Rightarrow \quad W_{\text{ext}}=0.4\ \mu\text{J}=400\ \text{nJ}$$

**23.** $W=U_f-U_i=\dfrac{\varepsilon_0 AV^2}{2}\left(\dfrac{1}{x_f}-\dfrac{1}{x_i}\right)=\dfrac{V^2}{2}\left(C_f-C_i\right)$

$$\Rightarrow \quad W=160\ \text{mJ}$$

## ARCHIVE: JEE MAIN

**1.** $C = \frac{k\varepsilon_0 A}{d}$

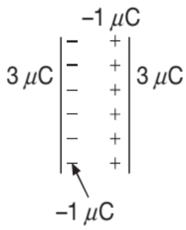
Since, $E = \frac{V}{d}$

$\Rightarrow \quad 15 \times 10^{-12} = \frac{k \times 8.86 \times 10^{-12} \times 10^{-4} \times 10^{6}}{500}$

$k = 8.5$

**Hence, the correct answer is (C).**

**2.**




Since $V = \frac{q}{C} = \frac{1\ \mu C}{1\ \mu F}$

$\Rightarrow \quad V = 1\ V$

**Hence, the correct answer is (B).**

**3.** $U_i = \frac{Q^2}{2C_1}$ and $U_f = \frac{Q^2}{2C_2}$

Since $W = U_f - U_i$

$\Rightarrow \quad W = \frac{Q^2}{2}\left(\frac{1}{C_2} - \frac{1}{C_1}\right)$

$\Rightarrow \quad W = \frac{(5)^2}{2}\left[\frac{1}{2} - \frac{1}{5}\right]$

$\Rightarrow \quad W = 3.75\ \mu J$

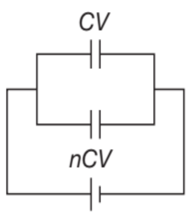
**Hence, the correct answer is (C).**

**4.** Initially, we have $C_{eq} = C + nC = (1+n)C$

$\Rightarrow \quad Q = CV = CV(1+n)$ ...(1)

Finally, $C'_{eq} = KC + nC = (K+n)C$

$\Rightarrow \quad Q' = C'V' = (K+n)CV'$ ...(2)




Since $Q =$ constant because system is disconnected from battery, so

$Q = Q'$

$\Rightarrow \quad CV(1+n) = (K+n)CV'$

$\Rightarrow \quad V' = \frac{V(n+1)}{K+n}$

$\Rightarrow \quad V = \frac{CV(1+n)}{(K+n)C} = \frac{V(1+n)}{(K+n)}$

**Hence, the correct answer is (A).**

**5.** Equivalent capacitance for series combination is

$C' = \frac{C_1 C_2}{C_1 + C_2}$

For parallel combination, $C'' = C_1 + C_2$

Since, $C'' > C'$

$\Rightarrow \quad C_1 + C_2 = \frac{500}{10} = 50\ \mu F$ ...(1)

and $\frac{C_1 C_2}{C_1 + C_2} = \frac{80}{10} = 8\ \mu F$

$\Rightarrow \quad C_1 C_2 = 400\ \mu F$ ...(2)

Solving equations (1) and (2), we get

$C_1 = 40\ \mu F \quad C_2 = 10\ \mu F$

**Hence, the correct answer is (A).**

**6.** $\frac{1}{C_1} = \frac{d}{3A\varepsilon_0}\left(\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}\right)$

$\Rightarrow \quad C_1 = \frac{3A\varepsilon_0 (K_1 K_2 K_3)}{d(K_1K_2 + K_2K_3 + K_3K_1)}$

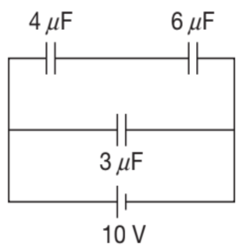
$\Rightarrow \quad C_2 = \frac{A\varepsilon_0}{3d}(K_1 + K_2 + K_3)$

$\Rightarrow \quad \frac{E_1}{E_2} = \frac{C_1}{C_2} = \frac{3K_1K_2K_3}{(K_1K_2 + K_2K_3 + K_3K_1)} \times \frac{3}{(K_1 + K_2 + K_3)}$

$\Rightarrow \quad \frac{E_1}{E_2} = \frac{9K_1K_2K_3}{(K_1 + K_2 + K_3)(K_1K_2 + K_2K_3 + K_3K_1)}$

**Hence, the correct answer is (C).**

**7.**

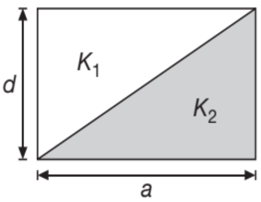


$$Q = Q_{(4\,\mu F)} = \left(\frac{4\times 6}{4+6}\right)\times 10\ \mu C$$

$\Rightarrow$ $Q = 24\ \mu C$

**Hence, the correct answer is (D).**

**8.** Since $C_{eq} = \dfrac{\varepsilon_0 K_1 K_2 a^2 \ln\dfrac{K_1}{K_2}}{(K_1 - K_2)d}$ where $K_2 = K$ and $K_1 = 1$




$$\Rightarrow\ C_{eq} = \frac{\varepsilon_0 K a^2 \ln\left(\dfrac{1}{K}\right)}{d(1-K)}$$

$$\Rightarrow\ C_{eq} = \frac{\varepsilon_0 K a^2}{d(K-1)}\ln K$$

**Hence, the correct answer is (A).**

**9.** $C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2} + \dfrac{C_3 C_4}{C_3 + C_4}$

where $C_1 = K_1 C_0$ and $C_0 = \dfrac{\varepsilon_0 \dfrac{A}{2}}{\dfrac{d}{2}}$

Similarly

$C_2 = K_2 C_0$

$C_3 = K_3 C_0$

$C_4 = K_4 C_0$

$$\Rightarrow\ K_{eq}\left(\frac{\varepsilon_0 A}{d}\right) = \left(\frac{K_1 K_2}{K_1 + K_2} + \frac{K_3 K_4}{K_3 + K_4}\right)\frac{\varepsilon_0 \dfrac{A}{2}}{\dfrac{d}{2}}$$

$$\Rightarrow\ K_{eq} = \frac{K_1 K_2}{K_1 + K_2} + \frac{K_3 K_4}{K_3 + K_4}$$

*No given option is correct.

**10.** $C = C_1 + C_2 + C_3$

$$\frac{\varepsilon_0 K A}{d} = \frac{\varepsilon_0 K_1 A}{3d} + \frac{\varepsilon_0 K_2 A}{3d} + \frac{\varepsilon_0 K_3 A}{3d}$$

$$\Rightarrow\ K = \frac{K_1 + K_2 + K_3}{3} = \frac{10+12+14}{3} = 12$$

**Hence, the correct answer is (C).**

**11.** $U_{\text{in}} = \dfrac{1}{2}\times 12\times 10^{-12}\times 100$

$\Rightarrow$ $U_{in} = 600$ pJ

Since $Q = CV$

$\Rightarrow$ $Q = 120$ pC

$$U_{\text{final}} = \frac{1}{2}\times\frac{120\times 120\times 10^{-24}\times 2}{12\times 10^{-12}\times 13}$$

$\Rightarrow$ $U_{\text{final}} = 92.3$ pJ

$\Rightarrow$ $W_{cap} = U_{\text{initial}} - U_{final} \approx 508$ pJ

**Hence, the correct answer is (C).**

**12.** Let the charges be $Q_1$ and $Q_2$, then

$$\frac{Q_1}{6} = \frac{Q_2}{4}$$

Also, $Q_1 + Q_2 = 30$

$\Rightarrow$ $Q_1 = 18\ \mu C$, $Q_2 = 12\ \mu C$

**Hence, the correct answer is (A).**

**13.** Since for combination in OPTION (A), we have

$$C_{eq} = \frac{6\times\left(\dfrac{2}{4}\right)}{6+\dfrac{2}{4}}$$

$$\Rightarrow\ C_{eq} = \frac{3}{6+\dfrac{1}{2}} = \frac{6}{13}\ \mu F$$

**Hence, the correct answer is (A).**

**14.** Heat produced is given by

$\Delta H = \Delta U$

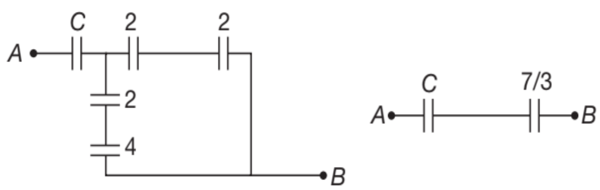
$$\Rightarrow\ \Delta H = \frac{1}{2}\times\frac{C\times 3C}{C+3C}(V^2)$$

$$\Rightarrow\ \Delta H = \frac{1}{2}\times\frac{3}{4}CV^2 = \frac{3}{8}CV^2$$

$$\Rightarrow\ \Delta H = \frac{3}{8}\frac{Q^2}{C}$$

**Hence, the correct answer is (A).**

**15.**



$$\Rightarrow \frac{\frac{7}{3}C}{C+\frac{7}{3}} = \frac{1}{2}$$

$$\Rightarrow 14C = 3C + 7$$

$$\Rightarrow C = \frac{7}{11}\ \mu F$$

**Hence, the correct answer is (A).**

**16.** $I = \frac{dq}{dt} =$ Slope of $q - t$ graph.

$$\Rightarrow I = 0$$

**Hence, the correct answer is (B).**

**17.** Induced charge on dielectric, $Q_{ind} = Q\left(1 - \frac{1}{K}\right)$

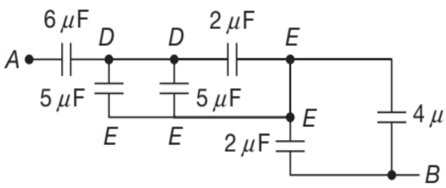
Final charge on capacitor, $Q = K(C_0 V)$

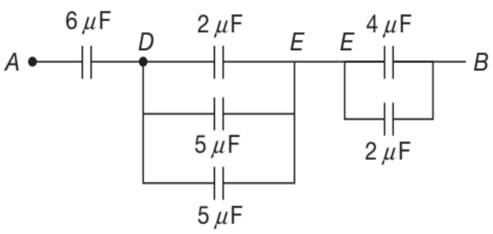
$$\Rightarrow Q = \frac{5}{3} \times 90 \times 10^{-12} \times 20 = 3 \times 10^{-9} C = 3\text{ nC}$$

$$\Rightarrow Q_{ind} = 3\left(1 - \frac{3}{5}\right) = 3 \times \frac{2}{5} = 1.2\text{ nC}$$

**Hence, the correct answer is (A).**

**18.**




$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{12} + \frac{1}{6} = \frac{5}{12}$$

$$\Rightarrow C_{eq} = \frac{12}{5} = 2.4\ \mu F$$

**Hence, the correct answer is (A).**

**19.** Given $A = 200\text{ cm}^2 = 2 \times 10^{-2}\text{ m}^2$

$d = 1.5\text{ cm} = 1.5 \times 10^{-2}\text{ m}$, $F = 25 \times 10^{-6}\text{ N}$, $V = ?$

Since, $F = \frac{1}{2}(\varepsilon_0 A)\frac{V^2}{d^2}$

$$\Rightarrow V = d\sqrt{\frac{2F}{\varepsilon_0 A}}$$

$$\Rightarrow V = 1.5 \times 10^{-2}\sqrt{\frac{2 \times 25 \times 10^{-6}}{8.85 \times 10^{-12} \times 2 \times 10^{-2}}}$$

$$\Rightarrow V = 1.5 \times 10^{2}\sqrt{\frac{25}{8.85}}$$

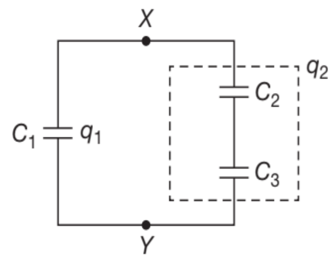
$$\Rightarrow V \approx 250\text{ V}$$

**Hence, the correct answer is (C).**

**20.** Initially, potential on $C_1$ is $V_0 = 60$ V

$$\Rightarrow q_0 = C_1 V_0 = 1(\mu F)(60\text{ V}) = 60\ \mu C$$

Finally, circuit can be redrawn as shown.




Charge starts flowing from $C_1$ till the potential difference across $C_1$ is equal to potential difference across series combination of $C_2$ and $C_3$.

$$\Rightarrow \frac{q_1}{C_1} = \frac{q_2}{\frac{C_2 C_3}{C_2 + C_3}}$$

Since, $C_1 = 1\ \mu F$, $C_2 = 3\ \mu F$, $C_3 = 6\ \mu F$

$$\Rightarrow q_1 = \frac{q_2}{2}$$

$$\Rightarrow q_2 = 2q_1 \quad \text{...(1)}$$

Also, $q_1 + q_2 = 60$ ...(2)

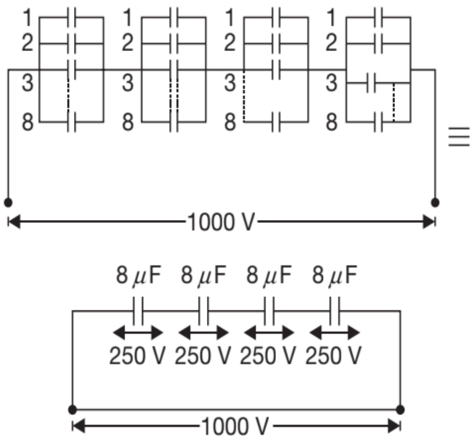
From (1) and (2), we get

$$q_1 = 20\ \mu C \text{ and } q_2 = 40\ \mu C$$

**Hence, the correct answer is (C).**

**21.** Following arrangement will meet the required condition.

Eight capacitors of $1\ \mu F$ in parallel with four such branches in series.




**Hence, the correct answer is (D).**

**22.** The energy stored in the electric field produced by a metal sphere is $4.5\ \text{J}$

$$\Rightarrow \quad \frac{Q^2}{2C} = 4.5$$

$$\Rightarrow \quad C = \frac{Q^2}{2\times 4.5} \qquad \ldots(1)$$

Capacitance of spherical conductor is $C = 4\pi\varepsilon_0 R$

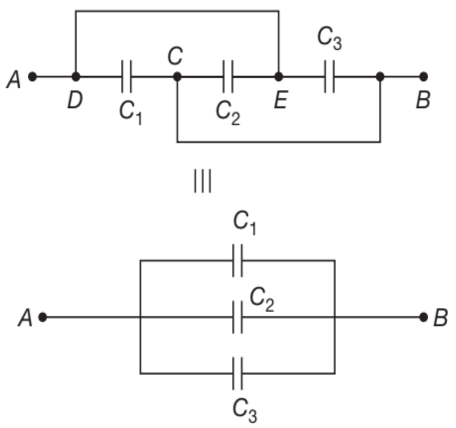
$$\Rightarrow \quad C = 4\pi\varepsilon_0 R = \frac{Q^2}{2\times 4.5} \qquad \text{\{From (1)\}}$$

$$\Rightarrow \quad R = \frac{1}{4\pi\varepsilon_0}\times\frac{\left(4\times10^{-6}\right)^2}{2\times 4.5} = 9\times10^9\times\frac{16}{9}\times10^{-12}$$

$$\Rightarrow \quad R = 16\times10^{-3}\ \text{m} = 16\ \text{mm}$$

**Hence, the correct answer is (C).**

**23.**




As the capacitors are in parallel combination so they have equal potential differences.

$$C_{\text{before}} = \frac{\varepsilon_0 A}{3} \qquad \ldots(1)$$

$$C_{\text{after}} = \frac{\left(\frac{k\varepsilon_0 A}{3}\right)\left(\frac{\varepsilon_0 A}{2.4}\right)}{\frac{k\varepsilon_0 A}{3}+\frac{\varepsilon_0 A}{2.4}} \qquad \ldots(2)$$

From (1) and (2), $\dfrac{\varepsilon_0 A}{3} = \dfrac{\left(\frac{k\varepsilon_0 A}{3}\right)\left(\frac{\varepsilon_0 A}{2.4}\right)}{k\frac{\varepsilon_0 A}{3}+\frac{\varepsilon_0 A}{2.4}}$

$$\Rightarrow \quad 3k = 2.4\,k + 3$$

$$\Rightarrow \quad 0.6k = 3$$

$$\Rightarrow \quad k = \frac{3}{0.6}$$

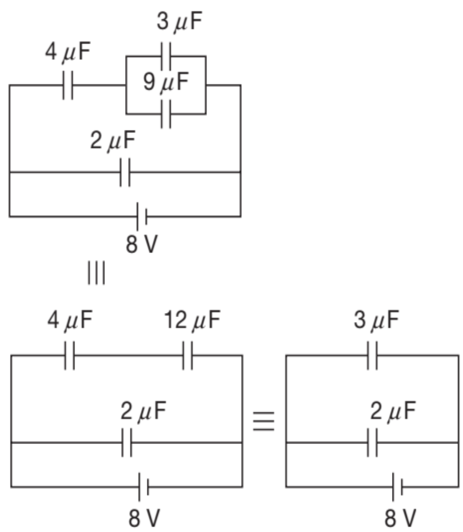
$$\Rightarrow \quad k = 5$$

**Hence, the correct answer is (B).**

**24.** $3\ \mu F$ and $9\ \mu\text{F}$ are in parallel combination, so the equivalent capacitance is $C_{eq} = (3+9) = 12\ \mu\text{F}$

Now, $4\ \mu\text{F}$ and $12\ \mu\text{F}$ are in series, so their equivalent capacitance is $C'_{eq} = \dfrac{4\times12}{16} = 3\ \mu\text{F}$

Charge on $3\ \mu\text{F}$ is $q_1 = (3\ \mu\text{F})\times(8\ \text{V}) = 24\ \mu\text{C}$




So charge on $4\ \mu\text{F}$ and $12\ \mu\text{F}$ are same (i.e. $24\ \mu C$) as they are in series.

Charge on $9\ \mu F$ is $q_2 = \left(\dfrac{9}{9+3}\right)\times 24\ \mu C = 18\ \mu\text{C}$

So, required charge $Q =$ Charge on $4\ \mu F +$ Charge on $9\ \mu\text{F}$

$\Rightarrow \quad Q=(24+18)\ \mu C=42\ \mu C$ and hence the required electric field is

$$E=\frac{1}{4\pi\varepsilon_0}\times\frac{Q}{r^2}$$

$$\Rightarrow \quad E=(9\times10^9)\frac{(42\times10^{-6})}{(30)^2}=420\ NC^{-1}$$

**Hence, the correct answer is (C).**

**25.** For (A), $\frac{1}{C_a}=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{3}{4} \Rightarrow C_a=\frac{4}{3}\mu F$

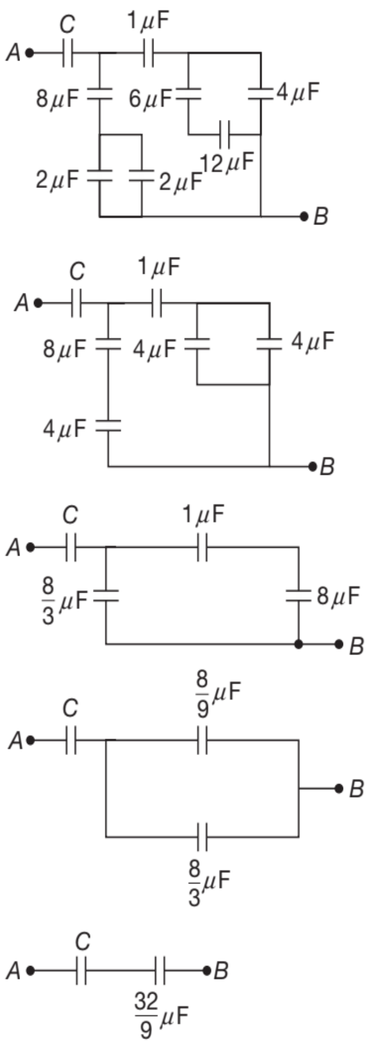
For (B), $C_b=4+4+4=12\ \mu F$

For (C), $C_c=\frac{(4+4)\times4}{(4+4)+4}=\frac{8}{3}\mu F$

For (D), $C_d=\frac{4\times4}{4+4}+4=6\ \mu F$

**Hence, the correct answer is (D).**

**26.**




Here $C_{AB}=1\ \mu F=\frac{C\times\frac{32}{9}}{C+\frac{32}{9}}$

$$\Rightarrow \quad C+\frac{32}{9}=\frac{32C}{9}$$

$$\Rightarrow \quad \frac{23}{9}C=\frac{32}{9}$$

$$\Rightarrow \quad C=\frac{32}{23}\mu F$$

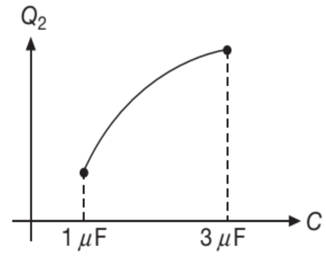
**Hence, the correct answer is (A).**

**27.** $Q_2=\frac{2Q}{2+1}=\frac{2Q}{3}$

Now, $Q=C_{net}E$

$$\Rightarrow \quad Q=\left(\frac{3C}{3+C}\right)E$$

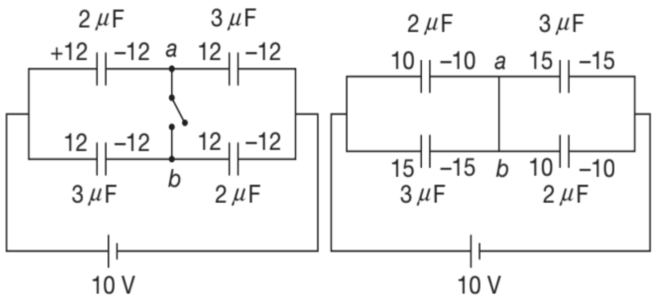
$$\Rightarrow \quad Q_2=\frac{2}{3}\left(\frac{3CE}{3+C}\right)=\frac{2CE}{3+C}$$




**Hence, the correct answer is (B).**

**28.** For upper and lower links, $C_{eq}=\frac{6}{5}\mu F$

$$\Rightarrow \quad Q_{upper}=Q_{lower}=12\ \mu C$$




On closing the switch, charge on $2\ \mu F$ is $10\ \mu C$ and that on $3\ \mu F$ is $15\ \mu C$

At $a$, we have $q_i=-12+12=0$ and

$$q_f = 15 - 10 = 5\ \mu\text{C}$$

So, charge flowing from $b$ to a is $5\ \mu\text{C}$.

**Hence, the correct answer is (A).**

**29.** Inside the dielectric,

$$E = \frac{E_0}{K} = \frac{\sigma}{K\varepsilon_0} = 3\times10^4$$

$$\Rightarrow \quad \sigma = K\varepsilon_0\left(3\times10^4\right) = 6\times10^{-7}\ \text{Cm}^{-2}$$

**Hence, the correct answer is (A).**

**30.** For this, charge must be same $Q = C_1V_1 = C_2V_2$

$$\Rightarrow \quad 120C_1 = 200C_2$$

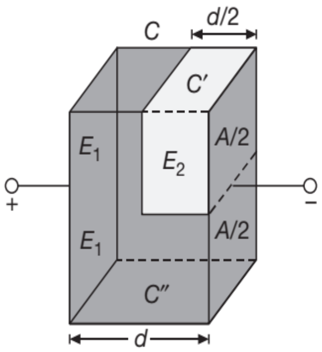
$$\Rightarrow \quad 3C_1 = 5C_2$$

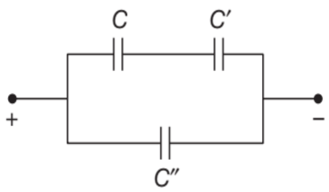
**Hence, the correct answer is (B).**

## ARCHIVE: JEE ADVANCED

### Single Correct Choice Type problems

**1.**




$$C_1 = \frac{\varepsilon_0 A}{d},\ C = \frac{2\varepsilon_0\left(\frac{A}{2}\right)}{\frac{d}{2}} = \frac{2\varepsilon_0 A}{d}$$

$$C' = \frac{4\varepsilon_0\left(\frac{A}{2}\right)}{\frac{d}{2}} = \frac{4\varepsilon_0 A}{d}$$

and $$C'' = \frac{2\varepsilon_0\left(\frac{A}{2}\right)}{d} = \frac{\varepsilon_0 A}{d}$$

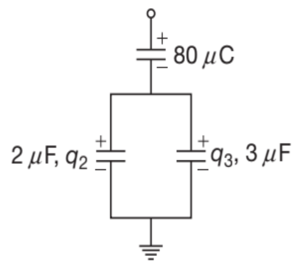
Since, $$C_2 = \frac{CC'}{C+C'} + C'' = \frac{4}{3}\frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{d}$$

$$\Rightarrow \quad C_2 = \frac{7}{3}\frac{\varepsilon_0 A}{d}\ \frac{C_2}{C_1} = \frac{7}{3}$$

**Hence, the correct answer is (D).**

**2.** Total charge of $80\ \mu\text{C}$ will distribute between $2\ \mu\text{F}$ and $2\ \mu\text{F}$ capacitors (in parallel) in direct ratio of capacitances. So

$$\frac{q_3}{q_2} = \frac{3}{2}$$




$$\Rightarrow \quad q_3 = \left(\frac{3}{3+2}\right)(80) = 48\ \mu\text{C}$$

**Hence, the correct answer is (C).**

**3.** $q_i = C_1V = 2V = q$

This charge remains constant after the switch is shifted from position 1 to position 2. So,

$$U_i = \frac{1}{2}\frac{q^2}{C_i} = \frac{q^2}{2\times2} = \frac{q^2}{4} \quad \text{and}$$

$$U_f = \frac{1}{2}\frac{q^2}{C_f} = \frac{q^2}{2\times10} = \frac{q^2}{20}$$

So, energy dissipated is

$$-\Delta U = U_i - U_f = \frac{q^2}{5}$$

This energy dissipated $\left(=\frac{q^2}{5}\right)$ is 80% of the initial stored energy $\left(=\frac{q^2}{4}\right)$

**Hence, the correct answer is (D).**

**4.** $\text{Loss} = \frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)(V_1-V_2)^2$

$\Rightarrow \quad \text{Loss} = \frac{1}{2}\left(\frac{C}{2}\right)(V_1-V_2)^2 = \frac{1}{4}C(V_1-V_2)^2$

**Hence, the correct answer is (C).**

**5.** When the switch is closed, the inner plates of the two capacitors get connected whereas the outer plates still are not connected and hence the circuit is not complete.

**Hence, the correct answer is (A).**

**6.** $k_1$ in series with half of $k_3$ and hence equivalent dielectric constant is $\frac{k_1k_3}{k_1+k_3}$

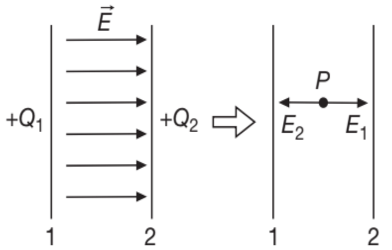
$k_2$ in series with half of $k_3$ and hence equivalent dielectric constant is $\frac{k_2k_3}{k_2+k_3}$ and then both in parallel to give $k = \frac{k_1k_3}{k_1+k_3} + \frac{k_2k_3}{k_2+k_3}$

**Hence, the correct answer is (D).**

**7.** Electric field at point $P$ within the plates is $\vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2}$

$$E = E_1 - E_2 = \frac{Q_1}{2A\varepsilon_o} - \frac{Q_2}{2A\varepsilon_o}$$

$$\Rightarrow \quad E = \frac{Q_1 - Q_2}{2A\varepsilon_o}$$

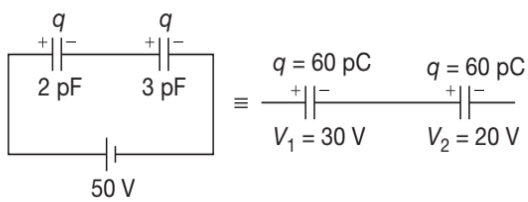



So, potential difference between the plates is

$$V_A - V_B = Ed = \left(\frac{Q_1-Q_2}{2A\varepsilon_o}\right)d = \frac{Q_1-Q_2}{2\left(\frac{A\varepsilon_o}{d}\right)} = \frac{Q_1-Q_2}{2C}$$

**Hence, the correct answer is (D).**

**8.** Charges on the capacitors are




$q_1 = (30)(2) = 60 \text{ pC}$

$\Rightarrow \quad q_2 = (20)(3) = 60 \text{ pC}$

$\Rightarrow \quad q_1 = q_2 = q \text{ (say)}$

The situation is similar to the two capacitors in series which are first charged with a battery of emf 50 V and then disconnected.

So, when $S_3$ is closed, $V_1 = 30$ V and $V_2 = 20$ V

Other way of looking at the thing is, when $S_1$ and $S_3$ both are closed, due to attraction with opposite charge, no flow of charge takes place through $S_3$. Therefore, potential difference across capacitor plates remains unchanged or $V_1 = 30$ V and $V_2 = 20$ V.

**Hence, the correct answer is (D).**

**9.** $\oint \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}q$

$\Rightarrow \quad E(2\pi r\ell) = \frac{1}{\varepsilon_0}(\lambda\ell)$

$\Rightarrow \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$

**Hence, the correct answer is (C).**

**10.** Total Initial Charge

$$Q_i = (2C)(2V) - CV = 3CV$$

Total Final Charge

$$Q_f = 2CV' + CV'$$

where $V'$ is common potential.

By Law of Conservation of Charge

$$Q_i = Q_f$$

$\Rightarrow \quad 3CV = 3CV'$

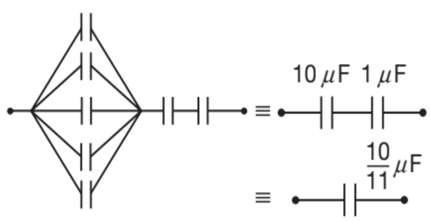
$\Rightarrow \quad V' = V$

So, final energy of combination is

$$U_f = \frac{1}{2}(C+2C)V^2 = \frac{3}{2}CV^2$$

**Hence, the correct answer is (B).**

**11.**




**Hence, the correct answer is (A).**

## Multiple Correct Choice Type Problems

**1.** $C = C_1 + C_2$

$$C_1 = \frac{k\varepsilon_0 \frac{A}{3}}{d}$$

$$C_2 = \frac{\frac{\varepsilon_0 2A}{3}}{d}$$

$$\Rightarrow \quad C = \frac{(K+2)\varepsilon_0 A}{3d}$$

$$\Rightarrow \quad \frac{C}{C_1} = \frac{K+2}{K}$$

Also, $E_1 = E_2 = \frac{V}{d}$, where $V$ is potential difference between the plates.

**Hence, (A) and (D) are correct.**

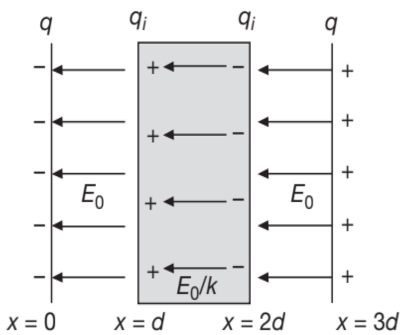
**2.** After pressing $S_1$ charge on upper plate of $C_1$ is $+2CV_0$. After pressing $S_2$ this charge equally distributes in two capacitors. Therefore charge an upper plates of both capacitors will be $+CV_0$.

When $S_2$ is released and $S_3$ is pressed, charge on upper plate of $C_1$ remains unchanged $(=+CV_0)$ but charge on upper plate of $C_2$ is according to new battery $(=-CV_0)$.

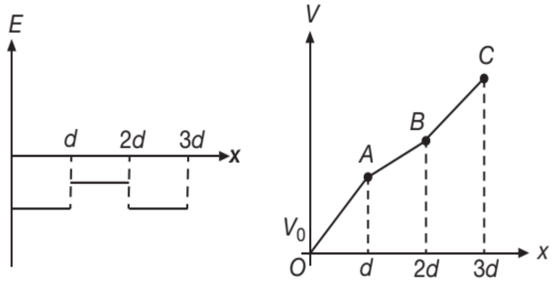
**Hence, (B) and (D) are correct.**

**3.** The magnitude and direction of electric field at different points are shown in figure. The direction of the electric field remains the same. Hence, answer (B) is correct.

Similarly, electric lines always flow from higher to lower potential, therefore, electric potential increases continuously as we move from $x = 0$ to $x = 3d$.




Therefore, answer (C) is also correct. The variation of electric field $(E)$ and potential $(V)$ with $x$ is shown.




$$OA \parallel BC \text{ and } (\text{Slope})_{OA} > (\text{Slope})_{AB}$$

So, $E_{0\to d} = E_{2d\to 3d}$ and $E_{0\to d} > E_{d\to 2d}$

**Hence, (B) and (C) are correct.**

**4.** Let $C_0$ be the capacitance initially and $C$ be the capacitance finally. Then $C_0 = \frac{\varepsilon_0 A}{d}$

Since, $Q = C_0 V$

$$\Rightarrow \quad Q = \frac{\varepsilon_0 AV}{d}$$

Further $E_0 = \frac{V}{d}$ and $E = \frac{E_0}{K}$

$$\Rightarrow \quad E = \frac{V}{Kd}$$

Also, if $U_i$ is the initial energy, then $U_i = \frac{1}{2}C_0V^2$

After the introduction of slab if $U_f$ be the final energy, then

$$U_f = \frac{1}{2}CV_{slab}^2 = \frac{1}{2}(KC_0)\left(\frac{V}{K}\right)^2$$

$$\Rightarrow \quad U_f = \frac{1}{2}\frac{C_0V^2}{K}$$

$$\Rightarrow \quad \Delta U = U_2 - U_1$$

$$\Rightarrow \quad \Delta U = \frac{1}{2}C_0V^2\left(\frac{1}{K} - 1\right)$$

Since work done = Decrease in Potential Energy

$$\Rightarrow \quad W = -\Delta U$$

$$\Rightarrow \quad W = \frac{1}{2}\frac{\varepsilon_0 AV^2}{d}\left(1 - \frac{1}{K}\right)$$

**Hence, (A), (C) and (D) are correct.**

**5.** Charging battery is removed. Therefore, $q =$ constant.

Distance between the plates is increased. Therefore, $C$ decreases.

Now, $V = \frac{q}{C}$, $q$ is constant and $C$ is decreasing.

Therefore, $V$ should increase.

$$U = \frac{1}{2}\frac{q^2}{C} \text{ again } q \text{ is constant and } C \text{ is decreasing.}$$

Therefore $U$ should increase.

**Hence, (B) and (D) are correct.**

**6.** Since battery is still in connection. So,

$$V = V_0$$

$$\Rightarrow Q_0 = C_0V_0 \text{ and}$$

$$Q = kC_0V_0$$

$$\Rightarrow Q = kQ_0$$

Since $k > 1$

$$\Rightarrow Q > Q_0$$

Also $U_0 = \frac{1}{2}Q_0V_0$ and

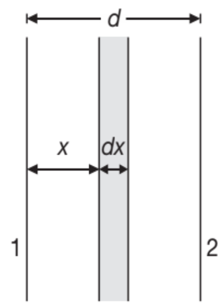
$$U = \frac{1}{2}QV = kU_0 \qquad \{\because Q = kQ_0 \text{ and } V = V_0\}$$

Hence $U > U_0$

**Hence, (A) and (D) are correct.**

## Integer/Numerical Answer Type Questions

**1.** $dC = \frac{K\varepsilon_0 A}{dx}$




$x = m\delta = m\left(\frac{d}{N}\right)$

Since all capacitors connected in series, so

$$\frac{1}{C} = \int \frac{1}{dC}$$

$$\Rightarrow \frac{1}{C} = \int \frac{dx}{K\varepsilon_0 A\left(1+\frac{m}{N}\right)}$$

Distance of $m^{\text{th}}$ layer from plate 1 is

$$x = m\delta = m\left(\frac{d}{N}\right)$$

$$\Rightarrow \frac{1}{C} = \int \frac{dx}{K\varepsilon_0 A\left(1+\frac{x}{d}\right)}$$

$$\Rightarrow \frac{1}{C} = \frac{d}{K\varepsilon_0 A}\int_0^x \frac{dx}{d+x}$$

$$\Rightarrow \frac{1}{C} = \frac{d\ln(2)}{K\varepsilon_0 A}$$

$$\Rightarrow C = \frac{K\varepsilon_0 A}{d\ln(2)}$$

$$\Rightarrow \alpha = 1$$