CHAPTER 1: ELECTROSTATICS

Test Your Concepts-I (Based on Coulomb's Law)

1. $F_{\text{air}} = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r^2}$

Let us now replace the slab of thickness t with an equivalent amount of air such that two charges placed in air at separation x will experience the same force if the separation between them completely has a slab of dielectric constant K between them. In such a case

$$\frac{q_1 q_2}{4\pi\varepsilon_0 x^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 K t^2}$$

$\Rightarrow$ Equivalent separation in air $= x = t\sqrt{K}$

So, now the effective separation between the charges is

$$r' = r - t + x = r - t + t\sqrt{K}$$

$$\Rightarrow \quad r' = r - t + t\sqrt{K}$$

So, $F' = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r'^2}$

$$\Rightarrow \quad F' = \frac{q_1 q_2}{4\pi\varepsilon_0 \left(r - t + t\sqrt{K}\right)^2}$$

Now, as per the problem we have $t = \dfrac{r}{2}$

$$\Rightarrow \quad F' = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{\left(r - \frac{r}{2} + \frac{r}{2}\sqrt{K}\right)^2}$$

$$\Rightarrow \quad F' = \frac{1}{4\pi\varepsilon_0} \left[\frac{4 q_1 q_2}{\left(1 + \sqrt{K}\right)^2 r^2}\right]$$

$$\Rightarrow \quad \frac{F'}{F_{\text{air}}} = \frac{4}{\left(1 + \sqrt{K}\right)^2} = \frac{4}{9}$$

$$\Rightarrow \quad 1 + \sqrt{K} = 3$$

$$\Rightarrow \quad \sqrt{K} = 2$$

$$\Rightarrow \quad K = 4$$

2. 63.5 g of Cu contains 6×10^{23} Cu atoms

1 g of Cu contains $\dfrac{6 \times 10^{23}}{63.5}$ Cu atoms

10 g of Cu contains $\left[\dfrac{6 \times 10^{23}}{63.5} \times 10\right]$ Cu atoms

Since for every 1000 atoms, an electron is transferred,

So, $n = \dfrac{6 \times 10^{23} \times 10}{63.5 \times 1000}$

$$\Rightarrow \quad q = ne = \frac{6 \times 10^{24}}{63500} \times 1.6 \times 10^{-19}\ \text{C}$$

$$\Rightarrow \quad q = \frac{1920}{127}\ \text{C}$$

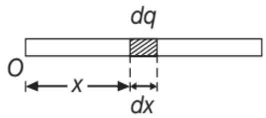
Now, one piece of Cu has positive charge and the other has a negative charge, so both will attract each other with a force of

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{r^2}$$

$$\Rightarrow \quad F = \frac{\left(9 \times 10^9\right)\left(\frac{1920}{127}\right)^2}{\left(\frac{1}{100}\right)^2} \cong 2.0 \times 10^{16}\ \text{N}$$

$$\Rightarrow \quad F = 2 \times 10^{16}\ \text{N}$$

3. Consider an element of length dx, carrying a charge dq at a distance x from O. Then, by definition



$$dq = \lambda\, dx$$

$$\Rightarrow \quad dq = \lambda_0 \left(\frac{x^2}{L}\right) dx$$

$$\Rightarrow \quad \int_O^Q dq = \frac{\lambda_0}{L} \int_O^L x^2 dx$$

$$\Rightarrow \quad Q = \frac{\lambda_0}{L}\left(\frac{L^3}{3}\right)$$

$$\Rightarrow \quad Q = \frac{\lambda_0 L^2}{3}$$

4. Each ball is in equilibrium under the following three forces:

(i) tension (T)

(ii) electric force (F)

(iii) weight (W)

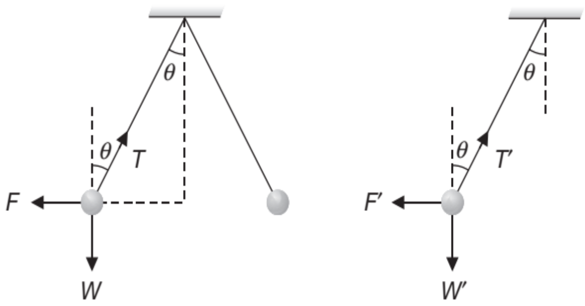
So, Lami's theorem can be applied.

$$T\cos\theta = W \qquad \ldots(1)$$

$$T\sin\theta = F \qquad \ldots(2)$$

$$T'\cos\theta = W' \qquad \ldots(3)$$

$$T'\sin\theta = F' \qquad \ldots(4)$$



In vacuum In liquid

So, from (1), (2), (3) and (4), we get

$$\frac{W}{F}=\frac{W'}{F'}$$

where $W'=W-U=V\rho g-V\sigma g$ and $F'=\frac{F}{K}$

$$\Rightarrow \quad \frac{\cancel{V}\rho \cancel{g}}{\cancel{F}}=\frac{\cancel{V}\rho\cancel{g}-\cancel{V}\sigma\cancel{g}}{\frac{\cancel{F}}{K}}$$

$$\Rightarrow \quad K=\frac{\rho}{\rho-\sigma}$$

5. Between Two Electrons:

$$F_e=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \text{ and } F_g=G\frac{m_e^2}{r^2}$$

$$\Rightarrow \quad \frac{F_e}{F_g}=\frac{e^2}{m_e^2}\frac{1}{4\pi\varepsilon_0 G}$$

$$\Rightarrow \quad \frac{F_e}{F_g}=\left(\frac{1.6\times10^{-19}}{9.1\times10^{-31}}\right)^2\frac{9\times10^9}{6.67\times10^{-11}}$$

$$\Rightarrow \quad \frac{F_e}{F_e}\cong 4\times10^{42}$$

Between Two Protons:

$$F_e=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \text{ and } F_g=\frac{Gm_p^2}{r^2}$$

$$\Rightarrow \quad \frac{F_e}{F_g}=\frac{e^2}{m_p^2}\frac{1}{4\pi\varepsilon_0 G}$$

$$\Rightarrow \quad \frac{F_e}{F_g}=\left(\frac{1.6\times10^{-19}}{1.67\times10^{-27}}\right)^2\frac{9\times10^9}{6.67\times10^{-11}}$$

$$\Rightarrow \quad \frac{F_e}{F_g}\cong 10^{36}$$

Let the specific charge be $\frac{q}{m}$ for which both the forces become equal, then

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}=\frac{Gm^2}{r^2}$$

$$\Rightarrow \quad \frac{q}{m}=\sqrt{4\pi\varepsilon_0 G}$$

$$\Rightarrow \quad \frac{q}{m}=8.6\times10^{-11}\ \text{Ckg}^{-1}$$

6. $\vec{F}=\vec{F}_{21}=\frac{q_1q_2}{4\pi\varepsilon_0\left|\vec{r}-\vec{r}_0\right|^3}\left(\vec{r}-\vec{r}_0\right)$

where $\vec{r}-\vec{r}_0=6\hat{i}-8\hat{j}$

$$\Rightarrow \quad \left|\vec{r}-\vec{r}_0\right|=10\text{ m}$$

So, $\vec{F}=\frac{50\times10^{-6}\times2\times10^{-6}\times9\times10^9}{(10)^3}\left(6\hat{i}-8\hat{j}\right)$

$$\Rightarrow \quad \vec{F}=9\times10^{-4}\left(6\hat{i}-8\hat{j}\right)\text{ N}$$

$$\Rightarrow \quad \left|\vec{F}\right|=9\times10^{-4}\sqrt{6^2+(-8)^2}$$

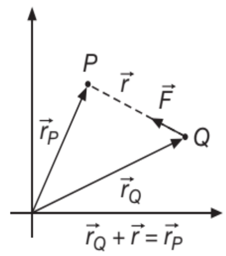
$$\Rightarrow \quad \left|\vec{F}\right|=9\times10^{-3}\text{ N}$$

7. Here, $\vec{r}_P=\hat{i}+2\hat{j}-4\hat{k}$ and $\vec{r}_Q=4\hat{i}+6\hat{j}-16\hat{k}$

$$\Rightarrow \quad \vec{r}_P-\vec{r}_Q=-3\hat{i}-4\hat{j}+12\hat{k}$$

$$\Rightarrow \quad \left|\vec{r}_P-\vec{r}_Q\right|=\sqrt{(3)^2+(-4)^2+(12)^2}=13\text{ m}$$

Since $\vec{F}=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\left|\vec{r}_P-\vec{r}_Q\right|^3}\left(\vec{r}_P-\vec{r}_Q\right)$



$$\Rightarrow \quad \vec{F}=\frac{17\times10^{-12}\times9\times10^9}{(13)^3}\left(-3\hat{i}-4\hat{j}+12\hat{k}\right)$$

$$\Rightarrow \quad \vec{F}=\frac{17\times9\times10^{-3}}{169\times13}\left(-3\hat{i}-4\hat{j}+12\hat{k}\right)$$

$$\Rightarrow \quad \vec{F}=\frac{170\times90\times10^{-5}}{169\times13}\left(-3\hat{i}-4\hat{j}+12\hat{k}\right)$$

$$\Rightarrow \quad \vec{F}\cong(1)(7)\left(10^{-5}\right)\left(-3\hat{i}-4\hat{j}+12\hat{k}\right)$$

$$\Rightarrow \quad \vec{F}\cong\left(-21\hat{i}-28\hat{j}+84\hat{k}\right)\times10^{-5}\text{ N}$$

8. Since $\lambda = \dfrac{Q}{2\pi R}$

and the tension developed in the ring when a charge is placed at its centre is given by

$$T = \frac{q_0\lambda}{4\pi\varepsilon_0 R} = \frac{q_0 Q}{8\pi^2\varepsilon_0 R^2} \text{ (we have done this already)}$$

Further, by Laws of Elasticity, we know that

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\left(\frac{T}{A}\right)}{\left(\frac{\Delta R}{R}\right)}$$

$$\Rightarrow \quad Y = \frac{TR}{A\Delta R}$$

$$\Rightarrow \quad \Delta R = \frac{TR}{AY} = \frac{q_0 Q}{8\pi^2\varepsilon_0 RAY}$$

Putting $q_0 = 10^{-8}$ C, $Q = \pi$ C, $R = 0.1$ m, $A = 10^{-6}$ m^2, $Y = 2\times 10^{-11}$ Nm^{-2}

we get $\Delta R = 2.25$ mm

9. DO YOURSELF (same kind of illustration done before) 33×10^{-9} C

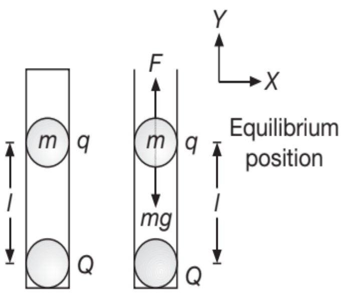
10. In equilibrium position, gravitational force is balanced by Coulomb repulsive force

$$mg = \frac{Qq}{4\pi\varepsilon_0 \ell^2}$$

If charge q is displaced in positive y – direction, such that $y \ll \ell$, from Newton's Second Law,

$$\frac{Qq}{4\pi\varepsilon_0(\ell + y)^2} - mg = ma$$

$$\frac{Qq}{4\pi\varepsilon_0\ell^2}\left[\frac{1}{\left(1+\frac{y}{\ell}\right)^2}\right] - mg = ma$$



$$\Rightarrow \quad mg\left(1 - \frac{2y}{\ell}\right) - mg = ma$$

$$\Rightarrow \quad a = -\frac{2gy}{\ell}$$

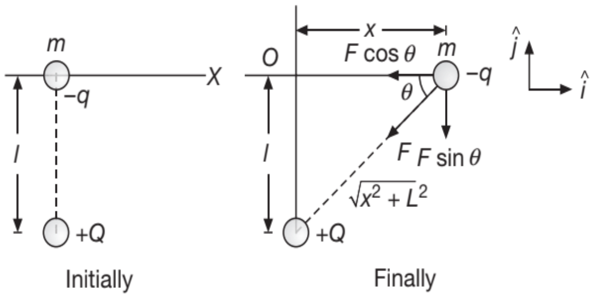
$$\Rightarrow \quad \frac{d^2y}{dt^2} + \frac{2g}{\ell}y = 0$$

Comparing with the standard equation of SHM according to which we have

$$\ddot{y} + \omega^2 y = 0$$

where we get $\omega = \sqrt{\dfrac{2g}{\ell}}$

11. When the bead is displaced through x, then the separation between the two charges becomes $\sqrt{x^2 + L^2}$ and at that instant let the Coulomb force of attraction between the two charges be F. We observe that the component of force $F\cos\theta$ restores the charge q to its mean position O.



From Newton's Second Law,

$$-F\cos\theta = ma$$

where the negative sign indicates that the component of force has a restoring nature and restores the charge $-q$ to its mean position.

$$\Rightarrow \quad -\left(\frac{Qq}{4\pi\varepsilon_0(x^2+L^2)}\right)\left(\frac{x}{\sqrt{x^2+L^2}}\right) = ma$$

$$\Rightarrow \quad a = -\left(\frac{Qq}{4\pi\varepsilon_0 m}\right)\left(\frac{x}{(x^2+L^2)^{\frac{3}{2}}}\right)$$

For a small linear displacement, $x \ll L$, we can ignore x^2 in comparison to L^2, so we get

$$a \simeq -\left(\frac{Qq}{4\pi\varepsilon_0 mL^3}\right)x$$

$$\Rightarrow \quad \ddot{x} + \left(\frac{Qq}{4\pi\varepsilon_0 mL^3}\right)x = 0$$

Comparing with the standard equation of SHM, which is given by $\ddot{x} + \omega^2 x = 0$, we get

$$\omega = \sqrt{\frac{Qq}{4\pi\varepsilon_0 mL^3}} = \frac{2\pi}{T} = 2\pi\nu$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mL^3}{Qq}}$$

12. Since the spring is metallic so the charge distributes equally on both the blocks. Hence a charge $\frac{Q}{2}$ resides on each block, due to which the blocks will repel each other with a force given by

$$F=\frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{Q}{2}\right)\left(\frac{Q}{2}\right)}{L^2}=k(L-L_0)$$

Solving for Q

$$Q=2L\sqrt{4\pi\varepsilon_0 k(L-L_0)}$$

13. Let q_1 and q_2 be the initial charges on the spheres, then according to Coulomb's Law

$$0.108=9\times10^9\frac{q_1q_2}{(0.5)^2} \Rightarrow q_1q_2=3\times10^{-12} \quad \ldots(1)$$

After connection, the charge flows from one sphere to another till both acquire the same potential (or in this case equal charge). So, final charge on both is

$$q_f=\frac{q_1-q_2}{2}$$

(as initially they have opposite nature)

Now, again according to Coulomb's Law, the repulsive force is

$$0.036=9\times10^9\frac{\left(\frac{q_1-q_2}{2}\right)^2}{(0.5)^2}$$

$$\Rightarrow q_1-q_2=\frac{0.036\times0.25\times4}{9\times10^9}$$

$$\Rightarrow q_1-q_2=2\times10^{-6} \quad \ldots(2)$$

Solving (1) and (2), we get

$$q_1-\frac{3\times10^{-12}}{q_1}=2\times10^{-6}$$

$$\Rightarrow q_1^2-2\times10^{-6}q_1-3\times10^{-12}=0$$

$$\Rightarrow q_1=\frac{2\times10^{-6}\pm\sqrt{4\times10^{-12}+12\times10^{-12}}}{2}$$

$$\Rightarrow q_1=\frac{2\times10^{-6}\pm4\times10^{-6}}{2}$$

$$\Rightarrow q_1=3\times10^{-6}\text{ C and } q_2=-1\times10^{-6}\text{ C}$$

$$\Rightarrow q_1=3\ \mu\text{C and } q_2=-1\ \mu\text{C}$$

14. $q_1+q_2=50\times10^{-6}$

$$\Rightarrow q_1+q_2=5\times10^{-5} \quad \ldots(1)$$

Also $1=9\times10^9\frac{q_1q_2}{(2)^2}$

$$\Rightarrow q_1q_2=\frac{4}{9\times10^9}$$

$$\Rightarrow q_1q_2=4.44\times10^{-10} \quad \ldots(2)$$

Put value of q_2 from (2) in (1), we get

$$q_1+\frac{4.44\times10^{-10}}{q_1}=5\times10^{-5}$$

$$\Rightarrow q_1^2-5\times10^{-5}q_1+4.44\times10^{-10}=0$$

Solving the quadratic, we get

$$q_1=3.8\times10^{-5}\text{ C}=38\ \mu\text{C}$$

OR $q_1=1.2\times10^{-5}\text{ C}=12\ \mu\text{C}$

So, we conclude that charge on one sphere is $38\ \mu\text{C}$ and that on the other is $12\ \mu\text{C}$.

15. $F_e=F=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$

$$\Rightarrow F=\frac{9\times10^9\times(1.6\times10^{-19})^2}{(0.53\times10^{-10})^2}$$

$$\Rightarrow F=8.2\times10^{-8}\text{ N}$$

This electrostatic force between the proton and electron, provides the necessary centripetal force to electron to revolve in a circle of radius 0.53 Å.

$$\Rightarrow mr\omega^2=F$$

$$\Rightarrow a_c=r\omega^2=\frac{F}{m}=\frac{8.2\times10^{-8}}{9.1\times10^{-31}}$$

$$\Rightarrow a_c=9\times10^{22}\text{ ms}^{-2}$$

Further $r\omega^2=9\times10^{22}$

$$\Rightarrow \omega^2=\frac{9\times10^{22}}{0.53\times10^{-10}}$$

$$\Rightarrow \omega=4.1\times10^{16}\text{ rads}^{-1}$$

Also, $T=\frac{2\pi}{\omega}$

$$\Rightarrow T=1.5\times10^{-16}\text{ s}$$

16. $F_g = \dfrac{Gm_e m_p}{r^2}$ ...(1)

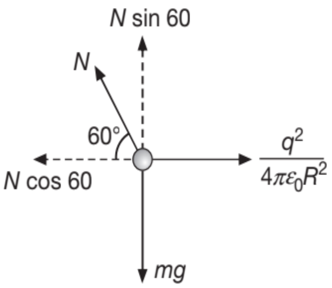
where m_e = mass of electron $= 9.1 \times 10^{-31}$ kg and m_p = mass of proton $= 1.67 \times 10^{-27}$ kg

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \quad ...(2)$$

$\Rightarrow$ $F_g \cong 3.6 \times 10^{-47}$ N and $F_e \cong 8.2 \times 10^{-8}$ N

So, we observe the gravitational force to be negligible in comparison to the electrostatic force. (But this result is not to be generalised to the solar system). So, this result is true in case of subatomic particles and we can also conclude here that gravitational forces are long range forces having significant values for massive bodies (like the sun-earth system).

17. $N\sin 60 = mg$ and $N\cos 60 = \dfrac{q^2}{4\pi\varepsilon_0 R^2}$



$$\Rightarrow \tan 60° = \frac{mg}{\left(\dfrac{q^2}{4\pi\varepsilon_0 R^2}\right)}$$

$$\Rightarrow q^2 = \frac{4\pi\varepsilon_0 mgR^2}{\sqrt{3}}$$

$$\Rightarrow q = \left(\frac{4\pi\varepsilon_0 mgR^2}{\sqrt{3}}\right)^{\frac{1}{2}}$$

18. $\dfrac{q^2}{4\pi\varepsilon_0 r^2} = \mu mg$

$$\Rightarrow \frac{9\times 10^9 \times (10^{-7})^2}{(10\times 10^{-2})^2} = \mu\left(\frac{5}{1000}\right)(10)$$

$$\Rightarrow \mu = 0.18$$

19. $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q(Q-q)}{r^2}$

For this to be minimum, we have $\dfrac{dF}{dq} = 0$

$$\Rightarrow q = \frac{Q}{2}$$

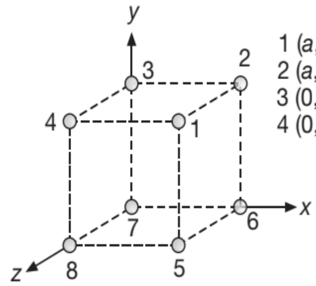
$$\Rightarrow \frac{q}{Q-q} = \frac{Q/2}{Q-Q/2} = 1$$

Test Your Concepts-II (Based on Principle of Superposition)

1. Let us redraw the diagram to locate the coordinates of all the points. Considering origin at 7, we get

Now, according to Principle of Superposition, net force on 1 is given by

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \vec{F}_{16} + \vec{F}_{17} + \vec{F}_{18}$$



1 (a, a, a)	5 (a, 0, a)
2 (a, a, 0)	6 (a, 0, 0)
3 (0, a, 0)	7 (0, 0, 0)
4 (0, a, a)	8 (0, 0, a)

From Coulomb's Law in vector form, we have

$$\vec{F}_{ij} = \frac{1}{4\pi\varepsilon_0}\frac{q_i q_j}{\left|\vec{r}_i - \vec{r}_j\right|^3}\left(\vec{r}_i - \vec{r}_j\right)$$

So, $\vec{F}_{12} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q^2}{a^3}\left(a\hat{k}\right) = \left(\dfrac{Q^2}{4\pi\varepsilon_0 a^2}\right)\hat{k}$

$$\vec{F}_{13} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{\left(\sqrt{2}a\right)^3}\left(a\hat{i} + a\hat{k}\right)$$

$$\vec{F}_{13} = \frac{Q^2}{4\pi\varepsilon_0 a^2}\left[\frac{1}{2\sqrt{2}}\left(\hat{i} + \hat{k}\right)\right]$$

$$\vec{F}_{14} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a^3}\left(a\hat{i}\right) = \left(\frac{Q^2}{4\pi\varepsilon_0 a^2}\right)\hat{i}$$

$$\vec{F}_{15} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a^3}\left(a\hat{j}\right) = \left(\frac{Q^2}{4\pi\varepsilon_0 a^2}\right)\hat{j}$$

$$\vec{F}_{16} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{\left(\sqrt{2}a\right)^3}\left(a\hat{j} + a\hat{k}\right)$$

$$\vec{F}_{16} = \frac{Q^2}{4\pi\varepsilon_0 a^2}\left[\frac{1}{2\sqrt{2}}\left(\hat{j} + \hat{k}\right)\right]$$

$$\vec{F}_{17} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{\left(\sqrt{3}a\right)^3}\left(a\hat{i} + a\hat{j} + a\hat{k}\right)$$

$$\vec{F}_{17} = \frac{Q^2}{4\pi\varepsilon_0 a^2}\left[\frac{1}{3\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right)\right]$$

$$\vec{F}_{18} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{\left(\sqrt{2}a\right)^3}\left(a\hat{i} + a\hat{j}\right)$$

$$\vec{F}_{18}=\frac{Q^2}{4\pi\varepsilon_0 a^2}\left[\frac{1}{2\sqrt{2}}\left(\hat{i}+\hat{j}\right)\right]$$

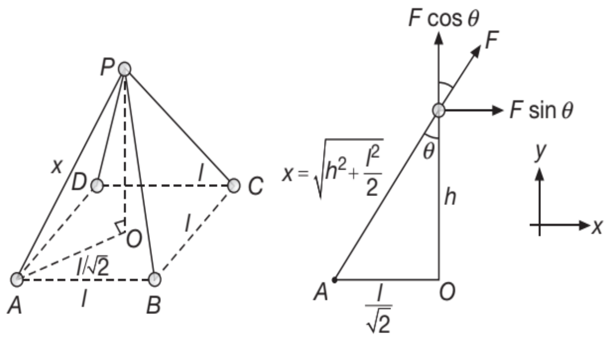
$$\Rightarrow \vec{F}_1=\frac{Q^2}{4\pi\varepsilon_0 a^2}\left[\left(\hat{i}+\hat{j}+\hat{k}\right)+\frac{1}{2\sqrt{2}}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)+\frac{1}{3\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)\right]$$

$$\Rightarrow \vec{F}_1=\frac{Q^2}{4\pi\varepsilon_0 a^2}\left[1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right]\left(\hat{i}+\hat{j}+\hat{k}\right)$$

$$\Rightarrow \left|\vec{F}_1\right|=\frac{\sqrt{3}Q^2}{4\pi\varepsilon_0 a^2}\left(1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right) \text{ MAGNITUDE}$$

The force on 1 is equally inclined to x, y and z-axis.

2. The force due to one charge on the charge at P is shown here. When all the forces are taken into account, then the sine components cancel each other. So,



$$F_{net}=F_y=4F\cos\theta$$

$$\Rightarrow F_{net}=4\frac{Q(1)}{4\pi\varepsilon_0\left(\sqrt{h^2+\frac{\ell^2}{2}}\right)^2}\frac{h}{\sqrt{h^2+\frac{\ell^2}{2}}}$$

$$\left\{\because \cos\theta=\frac{h}{\sqrt{h^2+\frac{\ell^2}{2}}}\right\}$$

$$\Rightarrow F_{net}=\frac{4Qh}{4\pi\varepsilon_0\left(h^2+\frac{\ell^2}{2}\right)^{3/2}} \quad \text{(UPWARDS)}$$

For equilibrium of m, this F_{net} must balance the weight of the particle mg (downwards).

$$\Rightarrow \frac{4Qh}{4\pi\varepsilon_0\left(h^2+\frac{\ell^2}{2}\right)^{\frac{3}{2}}}=mg$$

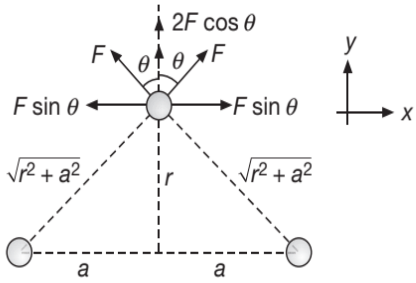
$$\Rightarrow Q=\frac{mg\pi\varepsilon_0}{h}\left(h^2+\frac{\ell^2}{2}\right)^{3/2}$$

3. From the figure drawn, we see that on resolution of the two identical forces, $F\sin\theta$ and $F\sin\theta$ cancel. So, net force equals

$$F_{net}=2F\cos\theta$$

$$\Rightarrow F_{net}=F_y=2\left[\frac{Qq_0}{4\pi\varepsilon_0\left(r^2+a^2\right)}\right]\cos\theta$$

$$\Rightarrow F_y=\frac{2Qq_0 r}{4\pi\varepsilon_0\left(r^2+a^2\right)^{3/2}}$$



This force is directed radially outwards when q_0 is positive in nature.

F_y will be MAXIMUM, when

$$\frac{d}{dr}\left(F_y\right)=0$$

$$\Rightarrow \frac{Qq_0}{2\pi\varepsilon_0}\frac{d}{dr}\left[\frac{r}{\left(r^2+a^2\right)^{3/2}}\right]=0$$

$$\Rightarrow r\left[-\frac{3}{2}\left(r^2+a^2\right)^{-5/2}2r\right]+\left(r^2+a^2\right)^{-3/2}=0$$

$$\Rightarrow -\frac{3r^2}{r^2+a^2}+1=0$$

$$\Rightarrow 3r^2=r^2+a^2$$

$$\Rightarrow 2r^2=a^2$$

$$\Rightarrow r=\pm\frac{a}{\sqrt{2}}$$

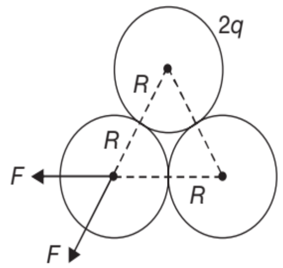
$$\Rightarrow F_{max}=\frac{Qq_0}{2\pi\varepsilon_0}\frac{\left(\frac{a}{\sqrt{2}}\right)}{\left(\frac{a^2}{2}+a^2\right)^{3/2}}$$

$$\Rightarrow F_{max}=\frac{Qq_0}{2\pi\varepsilon_0}\frac{a(2)}{3\sqrt{3}a^3}$$

$$\Rightarrow F_{max}=\frac{Qq_0}{3\sqrt{3}\pi\varepsilon_0 a^2}$$

4. $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(2q)^2}{(2R)^2} = \dfrac{q^2}{4\pi\varepsilon_0 R^2}$

$$F_{\text{net}} = \sqrt{F^2 + F^2 + 2F^2\cos 60}$$

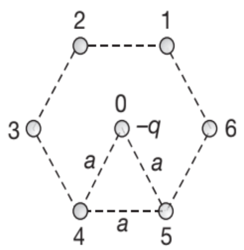


$$\Rightarrow \quad F_{\text{net}} = \sqrt{3}F = \sqrt{3}\frac{q^2}{4\pi\varepsilon_0 R^2}$$

$$\Rightarrow \quad F_{\text{net}} = \frac{\sqrt{3}q^2}{4\pi\varepsilon_0 R^2}$$

5. Assume, no charge at 6,

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + \vec{F}_{04} + \vec{F}_{05}$$



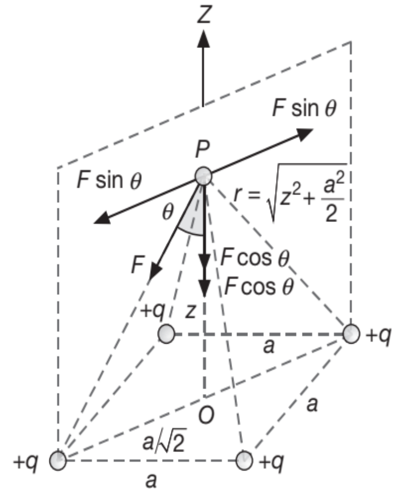
$$\Rightarrow \quad \left|\vec{F}_0\right| = \frac{q^2}{4\pi\varepsilon_0 a^2} \qquad \text{\{from 0 to 3\}}$$

6. (a) zero

(b) Assume, a to be the distance between q and Q, then

$$F = \frac{qQ}{4\pi\varepsilon_0 a^2}$$

7. Let us first visualise the situation given in the problem and draw it here. Now let us consider two diagonally opposite charges having charge $+q$ each. Each of the charge will exert an attractive force F on $-Q$ such that the horizontal component of this force F cancels, whereas vertical components add. Similarly for other two diagonally opposite charges the horizontal components cancel and the vertical components still stay so as to make the resultant force as



$$F_R = -4F\cos\theta$$

$$\Rightarrow \quad F_R = -4\left(\frac{1}{4\pi\varepsilon_0}\frac{Qq}{\left(z^2 + \frac{L^2}{2}\right)}\right)\left(\frac{z}{\sqrt{z^2 + \frac{L^2}{2}}}\right)$$

$$\Rightarrow \quad F_R = -\frac{4Qqz}{4\pi\varepsilon_0\left(z^2 + \frac{L^2}{2}\right)^{\frac{3}{2}}}$$

For $z \ll L$, we may neglect z^2 term in the denominator so as to get

$$F_R = m\ddot{z} = -4(2)^{\frac{3}{2}}\left(\frac{Qq}{4\pi\varepsilon_0 a^3}\right)z$$

$$\Rightarrow \quad \ddot{z} + \left(\frac{4(2)^{\frac{3}{2}}Qq}{4\pi\varepsilon_0 m a^3}\right)z = 0$$

Comparing with the standard equation of SHM given by $\ddot{z} + \omega^2 z = 0$, we get

$$\omega = \sqrt{\frac{4(2)^{\frac{3}{2}}Qq}{4\pi\varepsilon_0 m a^3}} = \frac{2\pi}{T} = 2\pi v$$

$$\Rightarrow \quad T = 2\pi\sqrt{2\sqrt{2}\left(\frac{Qq}{\pi\varepsilon_0 m a^3}\right)}$$

8. If we place one more charge q at the $24th$ vertex, the total force on the central charge will add up to zero. So according to the principle of superposition we have

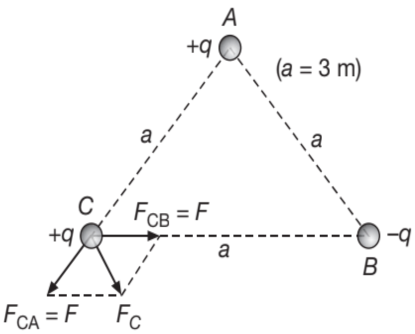
$$F = \frac{qQ}{4\pi\varepsilon_0 a^2}$$

9. $F_{CB} = F_{CA} = F = \dfrac{q^2}{4\pi\varepsilon_0 a^2}$

$$F_C^2 = F^2 + F^2 + 2F^2\cos 120$$

$$\Rightarrow F_C^2 = 2F^2 + 2F^2\left(-\frac{1}{2}\right)$$

$$\Rightarrow F_C^2 = 2F^2 - F^2 = F^2$$



$$\Rightarrow F_C = F = \frac{q^2}{4\pi\varepsilon_0 a^2} \qquad \{\text{parallel to AB}\}$$

$$\Rightarrow F_C = \frac{9\times10^9\times\left(100\times10^{-6}\right)^2}{9}$$

$$\Rightarrow F_C = 10\text{ N (parallel to AB)}$$

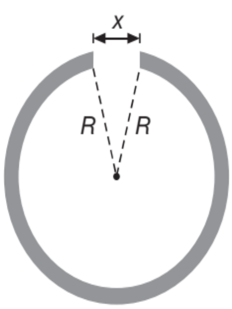
Test Your Concepts-III (Based on Electric Field)

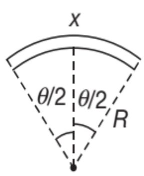
1. $E = \dfrac{q}{4\pi\varepsilon_0 a^2}$

2. Let us assume that the small length is not cut from the loop. In that case, at the centre, $\vec{E}_{\text{complete loop}} = \vec{0}$

$$\Rightarrow \vec{E}_{\substack{\text{due to removed}\\ \text{portion}}} + \vec{E}_{\substack{\text{due to the}\\ \text{remaining wire}}} = \vec{0}$$

$$\Rightarrow \left|\vec{E}_{\substack{\text{due to the remaining}\\ \text{wire}}}\right| = \left|\vec{E}_{\substack{\text{due to removed}\\ \text{portion of length } x(\ll R)}}\right|$$





Now, the removed portion is actually an arc having charge $q = \left(\dfrac{Q}{2\pi R}\right)x$ and subtending an angle θ at the centre. In that case, we know

$$E = \frac{Q}{4\pi^2\varepsilon_0 R^2}\sin\left(\frac{\theta}{2}\right) \qquad \text{(done already)}$$

$$\Rightarrow E = \frac{Q}{4\pi^2\varepsilon_0 R^2}\sin\left(\frac{x}{2R}\right)$$

Since $\dfrac{x}{R} \ll 1$, so

$$\sin\left(\frac{x}{2R}\right) \simeq \frac{x}{2R}$$

$$\Rightarrow E \simeq \frac{Qx}{8\pi^2\varepsilon_0 R^3}$$

3. Force on $+2\ \mu C = qE = \left(2\times10^{-6}\right)\left(10^5\right) = 0.2\text{ N}$ (due North)

Force on $-5\ \mu C = \left(5\times10^{-6}\right)\left(10^5\right) = 0.5\text{ N}$ (due South)

4. DO YOURSELF

$$\frac{\sqrt{6}q}{4\pi\varepsilon_0 \ell^2}$$

5. Consider the arm AB. Let us consider an element of length dx charge dq at a distance x from XY. Then torque on this element due to the field E is

$$d\tau = x(dqE)$$

$$\Rightarrow d\tau = x\left(\frac{q}{L}dx\right)E$$

$$\Rightarrow \tau = \int d\tau = \frac{qE}{L}\int_0^L x\,dx$$

$$\Rightarrow \tau = \frac{qEL}{2}$$

$$\tau_{\text{total}} = \frac{qEL}{2} + (qE)L + \frac{qEL}{2} = (2qE)L$$

$$\Rightarrow \tau = 2qEL = (mg)\frac{L}{2} + (mg)L + mg\left(\frac{L}{2}\right)$$

$$\Rightarrow \tau = 2qEL = 2mgL$$

$$\Rightarrow E = \frac{mg}{q}$$

6. Force on the particle at the centre of the ring is zero.

Let $-Q$ be now displaced through $x(\ll R)$ along axis of ring. Then a restoring force F will act on $-Q$, where

$$F = -QE_{\substack{\text{due to ring}\\ \text{at axis}}}$$

$$\Rightarrow \quad F = -\frac{Q\lambda(2\pi R)x}{4\pi\varepsilon_0\left(R^2+x^2\right)^{3/2}}$$

Since $x \ll R$

$$\Rightarrow \quad F = -\frac{Q\lambda(2\pi R)x}{4\pi\varepsilon_0 R^3} \qquad \left\{\because \left(R^2+x^2\right)^{3/2} \cong R^3\right\}$$

$$\Rightarrow \quad m\ddot{x} = -\frac{Q\lambda(2\pi R)x}{4\pi\varepsilon_0 R^3}$$

$$\Rightarrow \quad \ddot{x} + \left(\frac{Q\lambda}{2\varepsilon_0 mR^2}\right)x = 0$$

which can be compared to the standard equation of SHM i.e.,

$$\ddot{x} + \omega^2 x = 0$$

where $\omega = \sqrt{\dfrac{\lambda Q}{2\varepsilon_0 mR^2}}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{2\varepsilon_0 mR^2}{\lambda Q}}$$

7. (a) The charge Q_2 is negative and the charge Q_1 is positive.

(b) Since the electric field is zero at a distance a from the point 2, therefore

$$\frac{Q_1}{(\ell+a)^2} - \frac{Q_2}{a^2} = 0$$

$$\Rightarrow \quad \left|\frac{Q_1}{Q_2}\right| = \left(\frac{\ell+a}{a}\right)^2$$

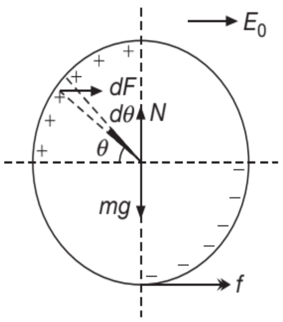
(c) For all points $x > a$,

$$E = \frac{Q_1}{(\ell+x)^2} - \frac{Q_2}{a^2}$$

For max. E, $\dfrac{dE}{dx} = 0 = -\dfrac{2Q_1}{(\ell+x)^3} + \dfrac{2Q_2}{x^3}$

Thus, $x = \dfrac{1}{\left(\dfrac{Q_1}{Q_2}\right)^{1/3} - 1} = b$

8. Consider an infinitesimal element subtending an angle $d\theta$ at the centre and at angle θ as shown in figure.



$dF = \lambda R d\theta E_0$

A force of same magnitude but in opposite direction acts on a corresponding element in the region of negative charge.

Net torque due to both types of charges is

$$d\tau = \left(\lambda R d\theta E_0\right)(2R\sin\theta)$$

$$\Rightarrow \quad \tau = 2\lambda R^2 E_0 \int_0^{\pi/2} \sin\theta\, d\theta$$

$$\Rightarrow \quad \tau = 2\lambda R^2 E_0$$

Equation for pure rolling motion is

$$\Sigma\tau = I\alpha$$

$$\Rightarrow \quad 2\lambda R^2 E_0 - fR = mR^2\alpha \qquad \ldots(1)$$

where $f = ma$ ...(2)

and $a = R\alpha$ ...(3)

Solving equations (1), (2) and (3), we get

$f = \lambda R E_0$ along positive x-axis.

9. (a) $F_{\text{net}} = F = 2\dfrac{Qq}{4\pi\varepsilon_0\left(R^2+x_0^2\right)}\dfrac{x_0}{\sqrt{R^2+x_0^2}}$

$$\Rightarrow \quad F = \frac{2Qqx_0}{4\pi\varepsilon_0\left(R^2+x_0^2\right)^{3/2}} \qquad \text{\{towards the centre\}}$$

(b) The motion of the bead is periodic between $\pm x_0$

(c) $m\dfrac{d^2x}{dt^2} = -\dfrac{2Qqx_0}{4\pi\varepsilon_0\left(R^2+x_0^2\right)^{3/2}}$ with $\dfrac{dx}{dt} = 0$ at $x = \pm x_0$

and $x = x_0$ at $t = 0$

(d) $m\dfrac{d^2x}{dt^2} = m\ddot{x} = -\left(\dfrac{2Qq}{4\pi\varepsilon_0 R^3}\right)x$

$$\Rightarrow \quad \ddot{x} + \left(\frac{2Qq}{4\pi\varepsilon_0 mR^3}\right)x = 0$$

$$\Rightarrow \quad \ddot{x} + \omega^2 x = 0$$

where $\omega = \sqrt{\dfrac{2Qq}{4\pi\varepsilon_0 mR^3}}$ and $T = 2\pi\sqrt{\dfrac{4\pi\varepsilon_0 mR^3}{2Qq}}$

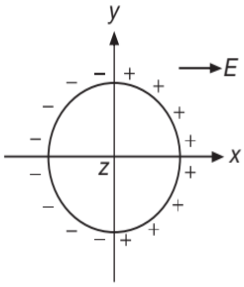
such that

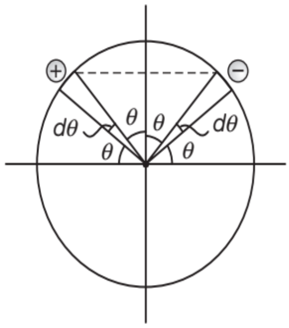
$$x = x_0\cos(\omega t) \text{ and } v = -x_0\omega\sin(\omega t)$$

(e) The velocity of bead will vanish for first time at $t = \dfrac{T}{2}$

$$\Rightarrow \quad t = \sqrt{\frac{4\pi^3\varepsilon_0 mR^3}{2Qq}}$$

10. Let us first calculate the effective dipole moment of the arrangement.

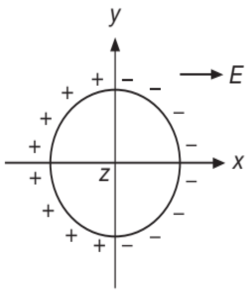




dp = (Either charge) (Separation between the charges)

$$\Rightarrow \quad dp = \left[\left(\frac{q}{\pi \cancel{R}}\right)\left(\cancel{R}\, d\theta\right)\right] 2R\sin\theta$$

$$\Rightarrow \quad dp = \frac{2qR}{\pi}\sin\theta\, d\theta$$



$$\Rightarrow \quad p = \frac{2qR}{\pi}\int_0^{\pi}\sin\theta\, d\theta = -\frac{2qR}{\pi}\left[\cos\theta\Big|_0^{\pi}\right]$$

$$\Rightarrow \quad p = -\frac{2qR}{\pi}(\cos\pi - \cos 0)$$

$$\Rightarrow \quad p = \frac{4qR}{\pi}$$

Since, $T = 2\pi\sqrt{\dfrac{I}{pE}}$

$$T = 2\pi\sqrt{\frac{mR^{\cancel{2}}}{\left(\dfrac{4q\cancel{R}}{\pi}\right)E}} = 2\pi\sqrt{\frac{mR\pi}{4qE}}$$

11. Here we can clearly see in the figure

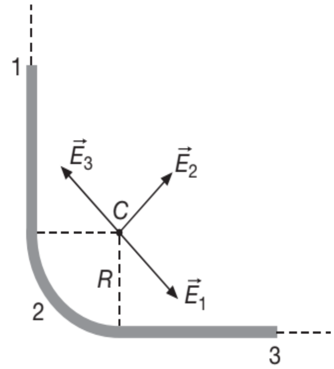
$\vec{E}_1$ (field due to wire 1) cancels out with

$\vec{E}_3$ (field due to wire 3) and

$\vec{E}_2$ (field due to wire 2)

Thus net field is $\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

$\vec{E}_{net} = \vec{E}_2$ {As $\vec{E}_1 = -\vec{E}_2$ }



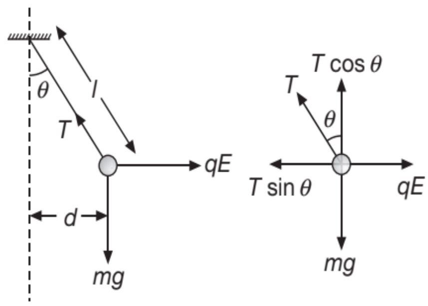
$$\vec{E}_{net} = \frac{2K\left(\lambda\dfrac{\pi R}{2}\right)\sin\left(\dfrac{\pi}{4}\right)}{\dfrac{\pi}{2}R^2}$$

$$\vec{E}_{net} = \frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 R}$$

12. From figure we can write for equilibrium of ball

$T\sin\theta = qE$

$T\cos\theta = mg$



$$\Rightarrow \quad \tan\theta = \frac{qE}{mg}$$

$$\Rightarrow \quad \frac{d}{\sqrt{\ell^2 - d^2}} = \frac{qE}{mg}$$

$$\Rightarrow \quad E = \frac{mgd}{q\sqrt{\ell^2 - d^2}}$$

13. Different forces acting on bob are shown in figure

From the figure

$T\sin\alpha = qE\sin\beta$ ...(1)

$T\cos\alpha + qE\cos\beta = mg$

$T\cos\alpha = mg - qE\cos\beta$ ...(2)

From equations (1) and (2), we have

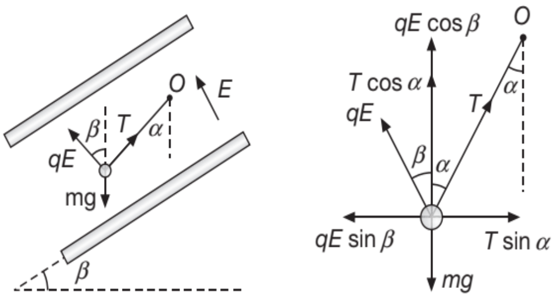
$$\tan\alpha = \frac{qE\sin\beta}{mg - qE\cos\beta}$$

$$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{qE\sin\beta}{mg - qE\cos\beta}\right) \qquad \ldots(3)$$

The net acceleration on the bob due to mg and qE is

$$a = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2 + 2g\left(\frac{qE}{m}\right)\cos(180-\beta)}$$

$$\Rightarrow \quad a = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2 - 2g\left(\frac{qE}{m}\right)\cos\beta}$$



Time period of pendulum can be given as

$$T = 2\pi\sqrt{\frac{\ell}{a}}$$

$$T = 2\pi\sqrt{\frac{\ell}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2 - 2g\left(\frac{qE}{m}\right)\cos\beta}}}$$

14. Here acceleration of block is $a = \frac{qE}{m}$

Time taken by block to reach wall

$$d = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

$$t = \sqrt{\frac{2md}{qE}}$$

Velocity at the time of impact is

$$v = \sqrt{2ad}$$

$$\Rightarrow \quad v = \sqrt{\frac{2qEd}{m}}$$

When the block rebounds, time taken by block to come to rest is given by

$$0 = \sqrt{\frac{2qEd}{m}} - \left(\frac{qE}{m}\right)t$$

$$\Rightarrow \quad t = \frac{\sqrt{\frac{2qEd}{m}}}{\frac{qE}{m}} = \sqrt{\frac{2m}{qE}}$$

Thus time period of oscillations of block is

$$T = 4t = 2\sqrt{\frac{2md}{qE}}$$

Since the restoring force is independent of x, (the displacement from mean position), hence this is not a simple harmonic motion but periodic in nature.

15. Electric Field due to 1 is

$$E_1 = \frac{\lambda}{2\pi\varepsilon_0\ell}(\cos 45° + \cos 45°)\hat{i}$$

$$\Rightarrow \quad E_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\sqrt{2}\lambda}{\ell}\right)\hat{i}$$

Electric Field due to 2 is $E_2 = \frac{1}{4\pi\varepsilon_0}\left(-\frac{4\sqrt{2}\lambda}{\ell}\right)\hat{i}$

Electric Field due to 3 is $E_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{6\sqrt{2}\lambda}{\ell}\right)\hat{j}$

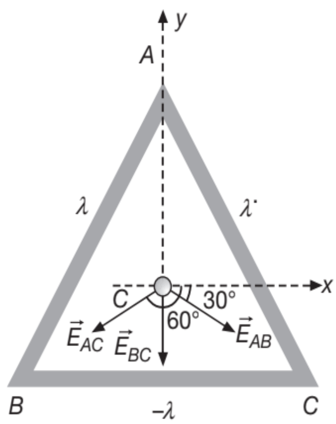
Electric Field due to 4 is $E_4 = \frac{1}{4\pi\varepsilon_0}\left(\frac{8\sqrt{2}\lambda}{\ell}\right)\hat{j}$

$$\Rightarrow \quad \vec{E}_{total} = E = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

$$\Rightarrow \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\sqrt{2}\lambda}{\ell} - \frac{4\sqrt{2}\lambda}{\ell}\right)\hat{i} + \frac{1}{4\pi\varepsilon_0}\left(\frac{6\sqrt{2}\lambda}{\ell} + \frac{8\sqrt{2}\lambda}{\ell}\right)\hat{j}$$

$$\Rightarrow \quad \vec{E} = \frac{\lambda\sqrt{2}}{4\pi\varepsilon_0\ell}\left(-2\hat{i} + 14\hat{j}\right)$$

16. The electric field strength due to the three rods AB, BC and CA are as shown in figure



$$\vec{E}_{AC}=\frac{1}{4\pi\varepsilon_0}\frac{-2\lambda}{\left(\frac{\ell}{\sqrt{3}}\right)}(2\sin 60^\circ)(\cos\theta\hat{i}+\sin\theta\hat{j})$$

$$\vec{E}_{AB}=\frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{\left(\frac{\ell}{\sqrt{3}}\right)}(2\sin 60^\circ)(\cos\theta\hat{i}-\sin\theta\hat{j})$$

$$\vec{E}_{BC}=\frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{\left(\frac{\ell}{\sqrt{3}}\right)}(2\sin 60^\circ)\hat{j}$$

$$\vec{E}_{\text{net}}=\vec{E}_{AC}+\vec{E}_{AB}+\vec{E}_{BC}$$

$$\vec{E}_{\text{net}}=-\left(\frac{3\lambda}{\pi\varepsilon_0\ell}\right)\hat{j}$$

17. Again, Newton's Second Law

$$\sum F_x=-T\sin\theta+qA=0 \qquad \ldots(1)$$

and $$\sum F_y=+T\cos\theta+qB-mg=0 \qquad \ldots(2)$$

(a) Substituting $T=\frac{qA}{\sin\theta}$, in equation (2), we get

$$\frac{qA\cos\theta}{\sin\theta}+qB=mg$$

$$\Rightarrow\quad q=\frac{mg}{(A\cot\theta+B)}$$

(b) Substituting this value in equation (1), we get

$$T=\frac{mgA}{(A\cos\theta+B\sin\theta)}$$

18. (a) Each ion moves in a quarter circle. The electric force causes the centripetal acceleration.

$$\sum F=ma$$

$$qE=\frac{mv^2}{R}$$

$$E=\frac{mv^2}{qR}$$

(b) For the x-motion, we have

$$v_x^2=u_x^2+2a_x(x_f-x_i)$$

$$\Rightarrow\quad 0=v^2+2a_xR$$

$$\Rightarrow\quad a_x=-\frac{v^2}{2R}=\frac{F_x}{m}=\frac{qE_x}{m}$$

$$\Rightarrow\quad E_x=-\frac{mv^2}{2qR}$$

Similarly for the y-motion, we have

$$v^2=0+2a_yR$$

$$\Rightarrow\quad a_y=+\frac{v^2}{2R}=\frac{qE_y}{m}$$

$$\Rightarrow\quad E_y=\frac{mv^2}{2qR}$$

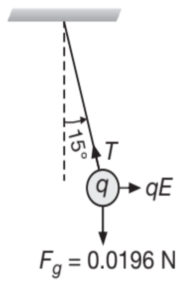
The magnitude of the field E is given by

$$E=\sqrt{E_x^2+E_y^2}$$

$$\Rightarrow\quad E=\frac{mv^2}{\sqrt{2}qR}$$

at an angle of 135° in the counterclockwise direction from the x-axis.

19. From the free-body diagram shown,



$$\sum F_y=0$$

$$\Rightarrow\quad T\cos(15^\circ)=1.96\times10^{-2}N$$

$$\Rightarrow\quad T=2.03\times10^{-2}\text{ N}.$$

Again for the equilibrium, we have

$$\sum F_x=0$$

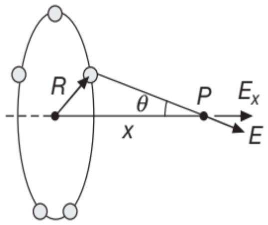
$$\Rightarrow\quad qE=T\sin(15^\circ)$$

$$\Rightarrow\quad q=\frac{T\sin(15^\circ)}{E}$$

$$\Rightarrow\quad q=\frac{(2.03\times10^{-2}\text{ N})\sin(15^\circ)}{1.00\times10^3\text{ NC}^{-1}}=5.25\times10^{-6}C$$

$$\Rightarrow\quad q=5.25\ \mu C$$

20. One of the charges creates at P a field $E=\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{n}\right)\frac{1}{R^2+x^2}$ at an angle θ to the x-axis as shown.



When all the charges produce field, for $n > 1$, the components perpendicular to the x-axis add to zero.

The total field is

$$E = \frac{1}{4\pi\varepsilon_0}\left[n\left(\frac{Q}{n}\right)\left(\frac{1}{R^2+x^2}\right)\cos\theta\right]\hat{i}$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2+x^2)^{\frac{3}{2}}}\hat{i}$$

This result is identical to the electric field due to a uniformly charged ring at a point lying at its axis.

21. $E = \int dE = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda dx}{x^2}$

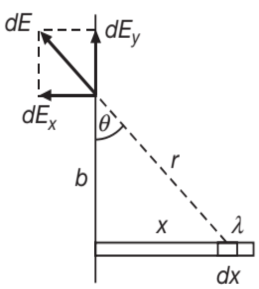
$$E = \frac{1}{4\pi\varepsilon_0}\int_{x_0}^{\infty}\frac{\lambda_0 x_0 dx}{x^3} = \frac{\lambda_0 x_0}{4\pi\varepsilon_0}\int_{x_0}^{\infty} x^{-3}dx$$

$$E = \frac{\lambda_0 x_0}{4\pi\varepsilon_0}\left(-\frac{1}{2x^2}\bigg|_{x_0}^{\infty}\right) = \frac{\lambda_0}{8\pi\varepsilon_0 x_0}$$

directed towards the negative x axis.

22. From figure, electric field due to small length element dx at observation point at distance r.

$$dE = \frac{\lambda dx}{4\pi\varepsilon_0 r^2}$$



and $dE_y = dE\cos\theta$ and $dE_x = dE\sin\theta$

Put $x = b\tan\theta$

$$dx = b\sec^2\theta d\theta$$

Also $r = \frac{b}{\cos\theta} = b\sec\theta$

$$dE_y = \left[\frac{\lambda b\sec^2\theta d\theta}{4\pi\varepsilon_0 b^2\sec^2\theta}\right]\cos\theta$$

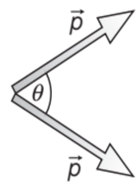
Then $E_y = \frac{\lambda}{4\pi\varepsilon_0 b}\int_0^{\frac{\pi}{2}}\cos\theta d\theta = \frac{\lambda}{4\pi\varepsilon_0 b}$ directed upward

Similarly $E_x = \frac{\lambda}{4\pi\varepsilon_0 b}\int_0^{\frac{\pi}{2}}\sin\theta d\theta = \frac{\lambda}{4\pi\varepsilon_0 b}$ to left.

If rod were infinite in both direction. The answer would become $E_x = 0$, $E_y = \frac{\lambda}{2\pi\varepsilon_0 b}$

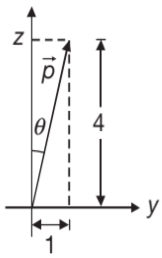
Test Your Concepts-IV
(Based on Dipoles and Dipole Moment)

1. $p_{\text{net}} = 2p\cos\frac{\theta}{2}$



2.

q_i	x_i	$q_i x_i$	y_i	$q_i y_i$	z_i	$q_i z_i$
2q	0	0	a	2qa	a	2qa
q	0	0	–a	–qa	a	qa
–q	0	0	0	0	–a	qa



$$p_x = \sum_i q_i x_i = 0$$

$$p_y = \sum_i q_i y_i = 2qa - qa$$

$$\Rightarrow \quad p_y = qa$$

$$p_z = \sum_i q_i z_i = 2qa + qa + qa$$

$$\Rightarrow \quad p_z = 4qa$$

Since, $\vec{p} = p_x\hat{i} + p_y\hat{j} + p_z\hat{k}$

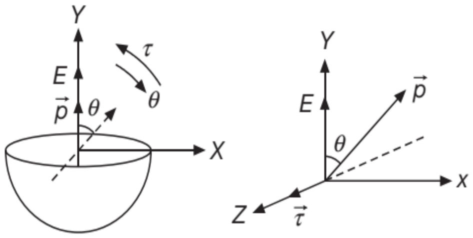
$\Rightarrow \quad \vec{p} = qa(\hat{j} + 4\hat{k})$ {in yoz plane}

$\Rightarrow \quad |\vec{p}| = \sqrt{17}qa$

at an angle $\tan^{-1}\left(\frac{1}{4}\right)$ with the z axis.

3. Thus torque experienced by dipole has magnitude

$$\tau = p\left(\frac{\sigma}{4\varepsilon_0}\right)\sin\theta$$



For small angular displacement,

$$\tau = -\left(\frac{\sigma}{4\varepsilon_0}\right)\theta = \mathrm{I}\alpha$$

$$\Rightarrow \quad \alpha = -\left(\frac{\sigma}{4\pi\varepsilon_0 \mathrm{I}}\right)\theta$$

On comparing the above expression with standard expression for SHM

$$\alpha = -\omega^2\theta$$

we obtain $\omega = \sqrt{\dfrac{\sigma}{4\pi\varepsilon_0 \mathrm{I}}} = \dfrac{2\pi}{T}$

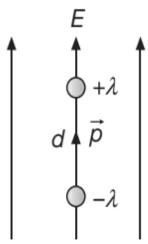
$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 \mathrm{I}}{\sigma}}$$

Potential energy of dipole is $U = -pE\cos\theta$

$$\Rightarrow \quad U = -p\left(\frac{\sigma}{4\pi\varepsilon_0}\right)\cos\theta$$

4. (a) The figure shows a side view of the system, it can be represented as an electric dipole of dipole moment

$$|\vec{p}| = (\lambda L)d$$



Slide view

The time period of oscillation of dipole is

$$T = 2\pi\sqrt{\frac{\mathrm{I}}{pE}}$$

$$T = 2\pi\sqrt{\frac{md^2/4 + md^2/4}{(\lambda Ld)E}}$$

$$T = 2\pi\sqrt{\frac{md}{2\lambda LE}}$$

(b) Work done required to turn the dipole from initial angle θ_i to θ_f is

$$W = pE(\cos\theta_i - \cos\theta_f)$$

(i) For $\theta_i = 0$, $\theta_f = \frac{\pi}{2}$,

$$W = (\lambda Ld)E[1-0]$$

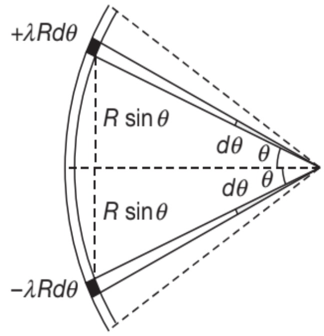
$$W = \lambda LdE$$

(ii) For $\theta_i = 0$, $\theta_f = \pi$,

$$W = \lambda LdE(\cos 0 - \cos\pi)$$

$$W = 2\lambda LdE$$

5. (a) We consider two differential elements on the rod as shown in figure



These elements constitute an electric dipole whose dipole moment is

$$|d\vec{p}| = (\lambda Rd\theta)(2R\sin\theta)$$

Net dipole moment of the system is

$$|\vec{p}| = \int_0^{2\theta_0} (\lambda Rd\theta)2R\sin\theta$$

$$|\vec{p}| = 2\lambda R^2(1 - \cos 2\theta_0)$$

$$|\vec{p}| = 4\lambda R^2\sin^2\theta_0$$

Time period of oscillation of a dipole in a uniform magnetic field is

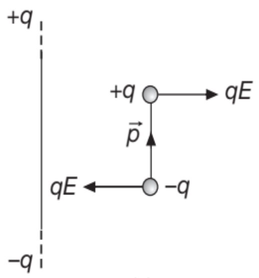
$$T = 2\pi\sqrt{\frac{\mathrm{I}}{pE}}$$

$$T = 2\pi\sqrt{\frac{mR^2}{4\lambda R^2 \sin^2\theta_0 E}} = \pi\sqrt{\frac{m}{\lambda \sin^2\theta_0 E}}$$

(b) Work done required to rotate the dipole through $180°$ is $W = pE(\cos\theta_i - \cos\theta_f) = pE(\cos 0 - \cos 180°)$

$$W = 2pE = 2\left(4\lambda R^2 \sin^2\theta_0\right)E = 8\lambda R^2 E \sin^2\theta_0$$

6. (a) In this case the resultant force on the dipole is zero as shown in figure (a)



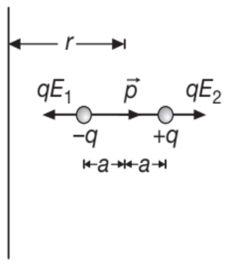
(a)

(b) The electric field at the position of negative charge $(-q)$,

$$E_1 = \frac{\lambda}{2\pi\varepsilon_0(r-a)}$$

The electric field at the position of positive charge

$$E_2 = \frac{\lambda}{2\pi\varepsilon_0(r+a)}$$



(b)

Resultant force on the dipole is

$$F = qE_1 - qE_2$$

$$F = \frac{q\lambda}{2\pi\varepsilon_0}\left[\frac{1}{r-a} + \frac{1}{r+a}\right]$$

$$F = \frac{q\lambda}{2\pi\varepsilon_0}\frac{2a}{(r^2 - a^2)}$$

for $r \gg a$, $F \simeq \frac{(2aq)\lambda}{2\pi\varepsilon_0 r^2} = \frac{p\lambda}{2\pi\varepsilon_0 r^2}$

Alternatively, the electric force on an electric dipole in a non-uniform field is given by

$$F = p\frac{dE}{dr}$$

$$F = p\frac{d}{dr}\left(\frac{\lambda}{2\pi\varepsilon_0 r}\right)$$

$$F = -\frac{p\lambda}{2\pi\varepsilon_0 r^2}$$

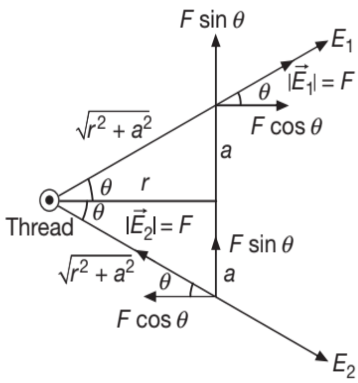
$$\Rightarrow \quad \vec{F} = -\frac{\lambda\vec{p}}{2\pi\varepsilon_0 r^2} \qquad \text{\{force is attractive\}}$$

(c) The magnitude of electric field at the position of two charges is

$$\left|\vec{E}_1\right| = \left|\vec{E}_2\right| = \frac{\lambda}{2\pi\varepsilon_0\sqrt{r^2 + a^2}}$$

and force on the charges is

$$\left|\vec{F}_1\right| = \left|\vec{F}_2\right| = F = \frac{q\lambda}{2\pi\varepsilon_0\sqrt{r^2 + a^2}}$$



(c)

As shown in figure (c), resultant force on the dipole is

$$F_R = 2F\sin\theta$$

$$F_R = \frac{2q\lambda}{2\pi\varepsilon_0\sqrt{r^2 + a^2}}\frac{a}{\sqrt{r^2 + a^2}}$$

For $r \gg a$ and $2aq = p$,

$$F_R \simeq \frac{p\lambda}{2\pi\varepsilon_0 r^2}$$

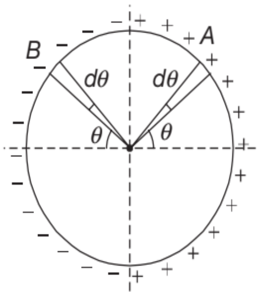
Resultant force is parallel to dipole moment

$$\vec{F}_R = \frac{\lambda\vec{p}}{2\pi\varepsilon_0 r^2}$$

7. Consider two differential charge elements at A and B as shown in figure. Dipole moment of this pair is

$$dp = \left[(\lambda_0\cos\theta)Rd\theta\right]2R\cos\theta$$

$$\Rightarrow \quad dp = 2\lambda_0 R^2 \cos^2\theta\, d\theta$$



So, the Dipole moment of the charge distribution is

$$p=2\lambda_0 R^2\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\cos^2\theta d\theta=\pi R^2\lambda_0$$

8. (a) Applying Energy Conservation Principle,

$$\begin{pmatrix}\text{Increase in}\\ \text{kinetic energy of}\\ \text{the dipole}\end{pmatrix}=\begin{pmatrix}\text{decrease in}\\ \text{electrostatic potential}\\ \text{energy of the dipole}\end{pmatrix}$$

Kinetic energy of dipole at distance d from origin $=U_i-U_f$

$$\Rightarrow\quad KE=0-\left(-\vec{p}.\vec{E}\right)=\vec{p}.\vec{E}=\left(p\hat{i}\right).\left(\frac{1}{4\pi\varepsilon_o}\frac{q}{d^2}\hat{i}\right)$$

$$\Rightarrow\quad KE=\frac{qp}{4\pi\varepsilon_o d^2}$$

(b) Electric field at origin due to the dipole,

$$\vec{E}=\frac{1}{4\pi\varepsilon_o}\frac{2p}{d^3}\hat{i}\left(\vec{E}_{axis}\uparrow\uparrow\vec{p}\right)$$

$\therefore$ Force on charge q

$$\vec{F}=q\vec{E}=\frac{pq}{2\pi\varepsilon_o d^3}\hat{i}$$

Test Your Concepts-V (Based on Flux)

1. $\phi_{\text{total}}=\left(\phi_{\text{due to }+q}\right)+\left(\phi_{\text{due to }-q}\right)$

Since, we have seen already that

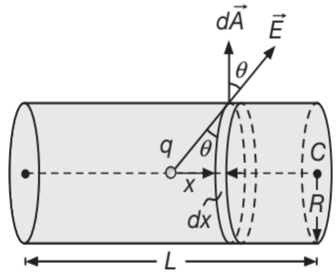
$$\phi=\frac{q}{2\varepsilon_0}(1-\cos\alpha)=\frac{q}{2\varepsilon_0}\left(1-\frac{\ell}{\sqrt{R^2+\ell^2}}\right)$$

$$\Rightarrow\quad\phi_{\text{total}}=2\phi=\frac{q}{\varepsilon_0}\left(1-\frac{\ell}{\sqrt{R^2+\ell^2}}\right)$$

(Do not expect the answer to come to be zero as one charge is $+q$ and other $-q$. If both had been $+q$ or $-q$ then it would have been zero)

2. For this we consider an infinitesimal strip of width dx on the surface of cylinder as shown. The area of this strip is

$$dA=2\pi R dx$$



The electric field due to the point charge on the strip is

$$E=\frac{q}{4\pi\varepsilon_0\left(R^2+x^2\right)}$$

If $d\phi$ is the electric flux through the strip, then

$$d\phi=E\,dA\cos\theta$$

$$d\phi=\frac{1}{4\pi\varepsilon_0}\frac{q}{\left(x^2+R^2\right)}\left(2\pi R dx\right)\left(\frac{R}{\sqrt{x^2+R^2}}\right)$$

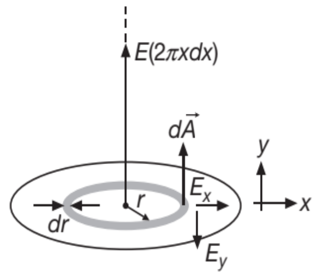
$$\Rightarrow\quad d\phi=\frac{2\pi qR^2}{4\pi\varepsilon_0}\left[\frac{dx}{\left(x^2+R^2\right)^{3/2}}\right]$$

Total flux through the lateral surface of cylinder is given by integrating the above result for the complete lateral surface. So

$$\phi=\int d\phi=\frac{qR^2}{2\varepsilon_0}\int_{-\frac{L}{2}}^{+\frac{L}{2}}\frac{dx}{\left(x^2+R^2\right)^{3/2}}$$

$$\Rightarrow\quad\phi=\frac{q}{\varepsilon_0}\left(\frac{L}{\sqrt{L^2+4R^2}}\right)$$

3. Consider an infinitesimal area element on the surface of the circle.



Let this element have radius r and thickness dr. Then

$$d\vec{A}=2\pi r dr\hat{j}$$

Since,

$$\vec{E}=E_x\hat{i}+E_y\left(-\hat{j}\right)$$

where $E_x=E_y=\dfrac{\lambda}{4\pi\varepsilon_0 r}$

Since, $d\phi=\vec{E}\cdot d\vec{A}$

$$\Rightarrow \quad d\phi = -2\pi r dr\left(\frac{\lambda}{4\pi\varepsilon_0 r}\right)$$

$$\Rightarrow \quad \phi = \frac{\lambda}{2\varepsilon_0}\int_0^R dr$$

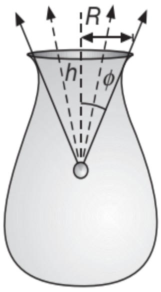
$$\Rightarrow \quad \phi = \frac{\lambda R}{2\varepsilon_0}$$

4. $\phi_{\text{lateral surface}} = \phi = \frac{q}{\varepsilon_0} -$ flux through the mouth

$$\Rightarrow \quad \phi = \frac{q}{\varepsilon_0} - \frac{q}{2\varepsilon_0}\left(1 - \frac{h}{\sqrt{h^2+R^2}}\right)$$

$$\Rightarrow \quad \phi = \frac{q}{2\varepsilon_0}\left(1 + \frac{h}{\sqrt{h^2+R^2}}\right)$$

$$\Rightarrow \quad \phi = \left(1 + \frac{\sqrt{3}}{2}\right)\frac{q}{2\varepsilon_0}$$



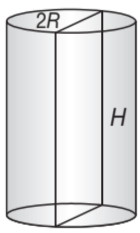
5. The cross sectional area perpendicular to the electric field is

$$A = (2R)H$$

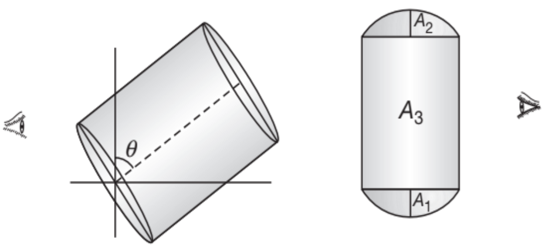
Thus electric flux crossing the cylinder is

$$\phi = EA$$

$$\Rightarrow \quad \phi = E(2RH) = 2HRE$$



6. Looking at the cylinder from $-$x-axis it appears as shown in figure.



Perpendicular component of area $A_\perp = A_1 + A_2 + A_3$

Now $A_1 = A_2$

$$A_1 = \left(\frac{\pi R^2}{2}\right)\sin\theta$$

and $A_3 = 2HR\cos\theta$

Total $\perp$ component of area is

$$A_\perp = \left(\frac{\pi R^2}{2}\right)\sin\theta \times 2 + 2HR\cos\theta$$

$$A_\perp = 2RH\cos\theta + \pi R^2\sin\theta$$

Thus flux passing through cylinder is

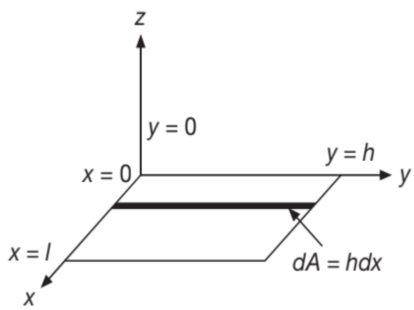
$$\phi = EA_\perp$$

$$\phi = E(2RH\cos\theta + \pi R^2\sin\theta)$$

7. In general, we have

$$E = ay\hat{i} + bz\hat{j} + cx\hat{k}$$

In the xy plane, $z = 0$ and so, $E = ay\hat{i} + cx\hat{k}$



By definition of flux, we have

$$\Phi_E = \int \vec{E}\cdot d\vec{A} = \int (ay\hat{i} + cx\hat{k})\cdot\hat{k}dA$$

$$\Rightarrow \quad \Phi_E = ch\int_{x=0}^{\ell} xdx = ch\left.\frac{x^2}{2}\right|_{x=0}^{\ell} = \frac{ch\ell^2}{2}$$

8. (a) $\Phi_E = EA\cos\theta$

$$\Phi_E = (3.5\times10^3)(0.35\times0.7)\cos0°$$

$$\Phi_E = 858 \text{ Nm}^2\text{C}^{-1}$$

(b) $\theta = 90°$

$$\Rightarrow \quad \Phi_E = 0$$

(c) $\Phi_E = (3.5\times10^3)(0.35\times0.7)\cos60°$

$$\Phi_E = 428.75 \text{ Nm}^2\text{C}^{-1}$$

9. $\Phi_E = EA\cos\theta = (2\times10^4\ \text{NC}^{-1})(18\ \text{m}^2)\cos 60° = 180\ \text{kN}\,\text{m}^2\text{C}^{-1}$

10. $\Phi_E = EA\cos\theta$

Since $A = \pi r^2 = \pi(0.4)^2 = 0.504\ \text{m}^2$

$\Rightarrow\ 5.04\times10^5 = E(0.504)\cos 0°$

$\Rightarrow\ E = 10^6\ \text{NC}^{-1} = 1\ \text{MNC}^{-1}$

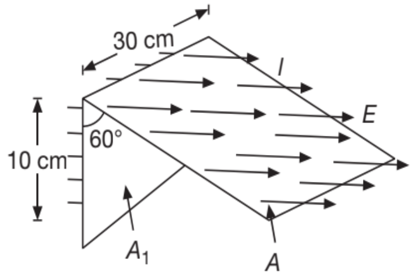
11. (a) $A_1 = (10\ \text{cm})(30\ \text{cm})$

$A_1 = 300\ \text{cm}^2 = 0.03\ \text{m}^2$

$\phi_1 = EA_1\cos\theta$

$\phi_1 = (7.8\times10^4)(0.03)\cos 180°$

$\phi_1 = -2.34\ \text{kNm}^2\text{C}^{-1}$



(b) $\phi_2 = EA\cos\theta = (7.8\times10^4)(A)\cos 60°$

Now, $A = (30\ \text{cm})(\ell) = (30\ \text{cm})\left(\dfrac{10\ \text{cm}}{\cos 60°}\right)$

$\Rightarrow\ A\cos 60 = 30\times10$

$\Rightarrow\ \phi_2 = 7.8\times10^4\times30\times10 = +2.34\ \text{kN}\,\text{m}^2\text{C}^{-1}$

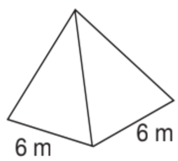
(c) The bottom and the two triangular sides all lie parallel to E, so $\Phi_E = 0$ for each of these. Thus,

$$\Phi_{E,\text{ total}} = -2.34\ \text{kN m}^2\text{C}^{-1} + 2.34\ \text{kN m}^2\ \text{C}^{-1} + 0 + 0 + 0 = 0$$

12. $\Phi_E = EA\cos\theta$ through the base

$\Phi_E = (52)(36)\cos 180° = -1.87\ \text{kNm}^2\text{C}^{-1}$

Note the same number of electric field lines pass through the base as are passing through the surface of the pyramid excluding the base.



For the inclined surfaces, $\Phi_E = +1.87\ \text{kNm}^2\text{C}^{-1}$

13. The flux entering the closed surface equals the flux exiting the surface. The flux entering the left side of the cone is given by

$$\Phi_E = \int \vec{E}\cdot d\vec{A} = ERh$$

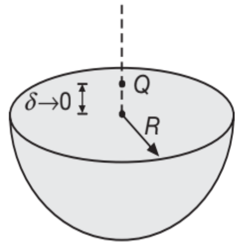
This is the same flux that exits the right side of the cone. Here we must observe that for a uniform field only the cross sectional area matters, note the shape.

14. (a) With δ being very small, all points on the hemisphere are nearly at a distance R from the charge, so the field everywhere on the curved surface is $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R^2}$ radially outward (normal to the surface). Therefore, the flux is this field strength times the area of half a sphere or hemisphere

$$\Rightarrow\ \Phi_{\text{curved}} = \int E\cdot dA = E_{\text{local}}A_{\text{hemisphere}}$$

$$\Rightarrow\ \Phi_{\text{curved}} = \left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\right)\left(\frac{1}{2}4\pi R^2\right)$$

$$\Rightarrow\ \Phi_{\text{curved}} = \frac{1}{4\pi\varepsilon_0}Q(2\pi) = +\frac{Q}{2\varepsilon_0}$$



(b) The closed surface encloses zero charge, so according to Gauss's Law we have

$$\Phi_{\text{curved}} + \Phi_{\text{flat}} = 0$$

$$\Rightarrow\ \Phi_{\text{flat}} = -\Phi_{\text{curved}} = -\frac{Q}{2\varepsilon_0}$$

Test Your Concepts-VI (Based on Gauss's Law)

1. Net charge enclosed by the sphere is $q_{\text{enc}} = 0$. Therefore, according to Gauss's Law net flux passing through the sphere is zero.

2. (a) According to Gauss's Law,

$$\phi_{\text{total}} = \frac{q_{\text{in}}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

(b) The cube is a symmetrical body with 6 faces and the point charge is at its centre, so electric flux linked with each face will be,

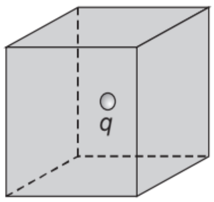
$$\phi_{\text{each face}} = \frac{\phi_{\text{total}}}{6} = \frac{q}{6\varepsilon_0}$$

We can also do this by another method discussed below :

Here the total solid angle subtended by cube surface at the point charge q is 4π. As q is at centre of cube, we can say the each face of cube subtend equal solid angle at the centre, thus solid angle subtended by each face at point charge is

$$\Omega_{\text{face}} = \frac{4\pi}{6} \text{ steradian}$$

Thus electric flux through each face is



$$\phi = \left(\frac{q}{4\pi\varepsilon_0}\right)\Omega$$

$$\Rightarrow \quad \phi_{\text{face}} = \frac{q}{4\pi\varepsilon_0}\left(\frac{4\pi}{6}\right)$$

$$\Rightarrow \quad \phi_{\text{face}} = \frac{q}{6\varepsilon_0}$$

If charge is not at the centre, the answer of part (a) will remain same while that of part (b) will change.

3. At any point $P(x, y, z)$ on the sphere a unit vector perpendicular to the sphere directed radially outwards is,

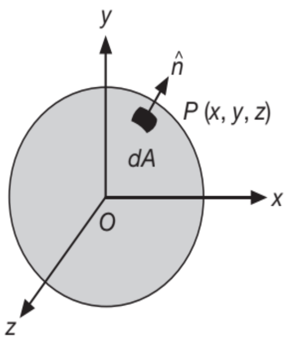
$$\hat{n} = \frac{x}{\sqrt{x^2+y^2+z^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{j} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{k}$$

$$\Rightarrow \quad \hat{n} = \frac{x}{R}\hat{i} + \frac{y}{R}\hat{j} + \frac{z}{R}\hat{k} \qquad \{\because x^2+y^2+z^2 = R^2\}$$

Let us find the electric flux passing through a small area dA at point P on the sphere, then

$$d\phi = \vec{E}\cdot\hat{n}\,dA = \left\{\frac{\alpha x^2}{R(x^2+y^2)} + \frac{\alpha y^2}{R(x^2+y^2)}\right\}dA$$

$$\Rightarrow \quad d\phi = \left(\frac{\alpha}{R}\right)dA$$



Here, we note that $d\phi$ is independent of the co-ordinates x, y and z. Therefore, total flux passing through the sphere

$$\phi = \int d\phi = \frac{\alpha}{R}\int dA = \left(\frac{\alpha}{R}\right)(4\pi R^2)$$

$$\Rightarrow \quad \phi = 4\pi\alpha R$$

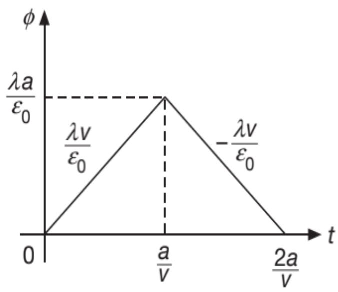
From Gauss's Law, $\phi = \dfrac{q_{\text{enc}}}{\varepsilon_0}$

$$\Rightarrow \quad (4\pi\alpha R) = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$\Rightarrow \quad q_{\text{enc}} = 4\pi\varepsilon_0\alpha R$$

4. At time t, length of rod inside the cube is $\ell = vt$

So, charge enclosed $q_{\text{enc}}(t) = \lambda\ell = \lambda vt$



$$\Rightarrow \text{Flux } = \phi_1(t) = \frac{1}{\varepsilon_0}\left[q_{\text{enc}}(t)\right] = \frac{\lambda vt}{\varepsilon_0} \text{ \{By Gauss's Law\}}$$

So, flux increases as a function of time

When the rod enters the cube completely, flux is

$$\phi_{\max} = \frac{1}{\varepsilon_0}(\lambda a)$$

CHAPTER 1

and again when the rod starts leaving the cube it starts decreasing as

$$\phi_2(t)=\frac{\lambda}{\varepsilon_0}(a-vt)$$

and ultimately falls to zero as soon as rod is out of the cube

5. Draw a Gaussian cylinder of length ℓ and radius s. For this surface, Gauss's Law states

$$\oint_S E\cdot d\vec{A}=\frac{1}{\varepsilon_0}Q_{\text{enc}}$$

The enclosed charge is

$$Q_{enc}=\int\rho dV=\frac{2}{3}\pi k\ell x^3$$

Now, symmetry dictates that $\vec{E}$ must point radially outward, so for the curved portion of the Gaussian cylinder we have

$$\int\vec{E}\cdot d\vec{A}=\int|\vec{E}|dA=|\vec{E}|\int dA=|\vec{E}|2\pi x\ell$$

while the two ends contribute nothing (here $\vec{E}$ is perpendicular to $d\vec{A}$). Thus,

$$|\vec{E}|2\pi s\ell=\frac{1}{\varepsilon_0}\frac{2}{3}\pi k\ell x^3$$

$$\Rightarrow\quad |\vec{E}|=E=\frac{1}{3\varepsilon_0}kx^2$$

6. (a) One half of the total flux created by the charge q goes through the plane. Thus,

$$\phi_{E,\text{ plane}}=\frac{1}{2}\phi_{E,\text{ total}}=\frac{1}{2}\left(\frac{q}{\varepsilon_0}\right)=\frac{q}{2\varepsilon_0}$$

(b) The square looks like an infinite plane to a charge very close to the surface. Hence,

$$\phi_{E,\text{ square}}\approx\phi_{E,\text{ plane}}=\frac{q}{2\varepsilon_0}$$

(c) The plane and the square behave in the same manner for the charge.

7. The total charge enclosed is $Q_{\text{enc}}=Q+(-6q)=Q-6q$.

The total outward flux from the cube is $\phi_E=\dfrac{Q-6q}{\varepsilon_0}$,

of which one-sixth goes through each face so , flux through each face is given by

$$(\phi_E)_{\text{one face}}=\frac{Q-6q}{6\varepsilon_0}$$

For, the cube not to be associated with any flux, we must have $q=\dfrac{Q}{6}$

8. The charge density is determined by $Q=\dfrac{4}{3}\pi a^3\rho$

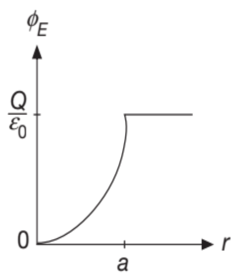
$$\rho=\frac{3Q}{4\pi a^3}$$

(a) The flux is that created by the enclosed charge within radius r.

$$\phi_E=\frac{q_{\text{enc}}}{\varepsilon_0}=\frac{4\pi r^3\rho}{3\varepsilon_0}=\left(\frac{4\pi r^3}{3\varepsilon_0}\right)\left(\frac{3Q}{4\pi a^3}\right)=\frac{Qr^3}{\varepsilon_0 a^3}$$

(b) $\phi_E=\dfrac{Q}{\varepsilon_0}$. Note that the answers to parts (a) and (b) agree at $r=a$.

(c)



9. **CASE-1**

When $R\le d$, the sphere encloses no charge and $\phi_E=\dfrac{q_{\text{enc}}}{\varepsilon_0}=0$

CASE-2

When $R>d$, the length of line falling within the sphere is $2\sqrt{R^2-d^2}$. So, according to Gauss's Law

$$\phi_E=\frac{q_{\text{enc}}}{\varepsilon_0}=\frac{2\lambda\sqrt{R^2-d^2}}{\varepsilon_0}$$

10. $\oint\vec{E}\cdot d\vec{A}=E(4\pi r^2)=\dfrac{q_{\text{enc}}}{\varepsilon_0}$

(a) For $r>R$, we have

$$q_{\text{enc}}=\int_0^R\rho_0 r^2(4\pi r^2)dr=\frac{4\pi\rho_0R^5}{5}$$

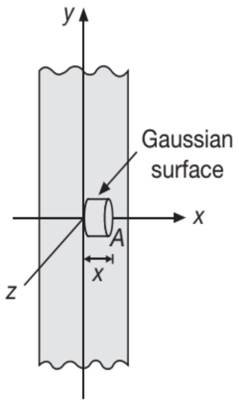
$$\Rightarrow\quad E_{\text{outside}}=\frac{\rho_0R^5}{5\varepsilon_0r^2}$$

(b) For $r<R$, we have

$$q_{\text{enc}}=\int_0^r\rho_0 r^2(4\pi r^2)dr=\frac{4\pi\rho_0r^5}{5}$$

$$\Rightarrow\quad E_{\text{inside}}=\frac{\rho_0r^3}{5\varepsilon_0}$$

11. Consider the Gaussian surface, a Gaussian pill box as shown



(a) For $x > \frac{d}{2}$

$$dq = \rho dV = \rho A dx = A\rho_0 x^2 dx$$

Since $\int \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}\int dq$

$$\Rightarrow EA = \frac{\rho_0 A}{\varepsilon_0}\int_0^{\frac{d}{2}} x^2 dx = \frac{1}{3}\left(\frac{\rho_0 A}{\varepsilon_0}\right)\left(\frac{d^3}{8}\right)$$

$$\Rightarrow E = \frac{\rho_0 d^3}{24\varepsilon_0}$$

$$\Rightarrow E = \begin{cases} \frac{\rho_0 d^3}{24\varepsilon_0}\hat{i} & \text{for } x > \frac{d}{2} \\ -\frac{\rho_0 d^3}{24\varepsilon_0}\hat{i} & \text{for } x < -\frac{d}{2} \end{cases}$$

(b) For $-\frac{d}{2} < x < \frac{d}{2}$

$$\int \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}\int dq = \frac{\rho_0 A}{\varepsilon_0}\int_0^x x^2 dx = \frac{\rho_0 A x^3}{3\varepsilon_0}$$

$$\Rightarrow E = \begin{cases} \frac{\rho_0 x^3}{3\varepsilon_0}\hat{i} & \text{for } x > 0 \\ -\frac{\rho_0 x^3}{3\varepsilon_0}\hat{i} & \text{for } x < 0 \end{cases}$$

12. $\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_0^r \frac{\rho_0}{r} 4\pi r^2 dr$

$$\Rightarrow E\left(4\pi r^2\right) = \frac{4\pi\rho_0}{\varepsilon_0}\int_0^r r dr = \frac{4\pi\rho_0 r^2}{2\varepsilon_0}$$

$$\Rightarrow E = \frac{\rho_0}{2\,\varepsilon_0} = \text{constant in magnitude}$$

(The direction is radially outward from the center for positive ρ_0 and radially inward for negative ρ_0)

13. (a) A point mass m creates a gravitational acceleration $\vec{g} = -\frac{Gm}{r^2}\hat{r}$ at a distance r.

The flux of this field through a sphere is

$$\oint \vec{g}\cdot d\vec{A} = -\frac{Gm}{r^2}\left(4\pi r^2\right) = -4\pi Gm$$

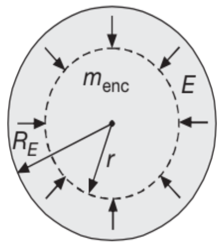
Since the flux is independent of r, so we can visualize the field as unbroken field lines. The same flux would go through any other closed surface around the mass. If there are several or no masses inside a closed surface, each creates field to make its own contribution to the net flux according to

$$\oint \vec{g}\cdot d\vec{A} = -4\pi G\sum m_{enc}$$

(b) Take a spherical Gaussian surface of radius r. The field is inward so

$$\phi = \oint \vec{g}\cdot d\vec{A} = g\left(4\pi r^2\right)\cos 180^\circ = -g\left(4\pi r^2\right)$$

$$\Rightarrow \phi = -g\left(4\pi r^2\right)$$



Now, we have

$$m_{enc} = \frac{4}{3}\pi r^3 \rho$$

So, according to Gauss's Law, we have

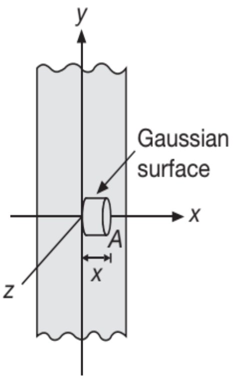
$$\oint \vec{g}\cdot d\vec{A} = -4\pi G\sum m_{enc}$$

$$\Rightarrow -g\left(4\pi r^2\right) = -4\pi G\left(\frac{4}{3}\pi r^3 \rho\right)$$

$$\Rightarrow g = \frac{4}{3}\pi G r \rho$$

But $\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3} \Rightarrow g = \left(\frac{GM_E}{R_E^3}\right)r$, inwards

14. (a) Consider a cylindrical shaped Gaussian surface, also called Gaussian Pill box, perpendicular to the yz plane with one end in the yz plane and the other end containing the point x.



According to Gauss's Law, we have

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

By symmetry, the electric field is zero in the yz plane and is perpendicular to dA over the wall of the gaussian cylinder. Therefore, the only contribution to the integral is over the end cap containing the point x:

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \quad \Rightarrow \quad EA = \frac{\rho(Ax)}{\varepsilon_0}$$

so that at distance x from the mid-line of the slab,

$$E = \frac{\rho x}{\varepsilon_0}$$

(b) $a = \frac{F}{m} = \frac{(-e)E}{m} = -\left(\frac{\rho e}{m\varepsilon_0}\right)x$

The acceleration of the electron is of the form

$$a = -\omega^2 x \text{ with } \omega = \sqrt{\frac{\rho e}{m\varepsilon_0}} = \frac{2\pi}{T}$$

Thus, the motion is simple harmonic with period

$$T = 2\pi\sqrt{\frac{m\varepsilon_0}{\rho e}}$$

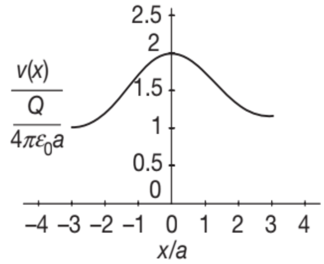
15. DO YOURSELF (Already Discussed)

$$\vec{E} = \frac{\rho\vec{a}}{2\varepsilon_0}$$

Test Your Concepts-VII (Based on Electrostatic Potential and Energy)

1. (a) $V(x) = \frac{2Q}{4\pi\varepsilon_0\sqrt{x^2+a^2}} = \frac{Q}{4\pi\varepsilon_0 a}\left(\frac{2}{\sqrt{\left(\frac{x}{a}\right)^2+1}}\right)$

$$\Rightarrow \frac{V(x)}{\left(\frac{Q}{4\pi\varepsilon_0 a}\right)} = \frac{2}{\sqrt{\left(\frac{x}{a}\right)^2+1}}$$

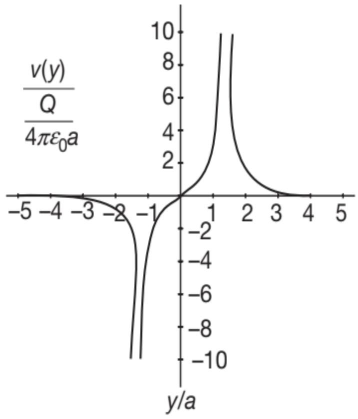


(b) $V(y) = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_2}$

$$\Rightarrow V(y) = \frac{1}{4\pi\varepsilon_0}\left(\frac{+Q}{|y-a|} + \frac{-Q}{|y+a|}\right)$$

$$\Rightarrow V(y) = \frac{Q}{4\pi\varepsilon_0 a}\left(\frac{1}{\left|\frac{y}{a}-1\right|} - \frac{1}{\left|\frac{y}{a}+1\right|}\right)$$

$$\Rightarrow \frac{V(y)}{\left(\frac{Q}{4\pi\varepsilon_0 a}\right)} = \left(\frac{1}{\left|\frac{y}{a}-1\right|} - \frac{1}{\left|\frac{y}{a}+1\right|}\right)$$



2. In this problem,

$$r_{41} = r_{43} = r_{32} = r_{21} = 1 \text{ m}$$

and $r_{42} = r_{31} = \sqrt{(1)^2+(1)^2} = \sqrt{2}$ m

$$\Rightarrow U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_4}{r_{34}} + \frac{q_4q_1}{r_{14}} + \frac{q_2q_4}{r_{24}} + \frac{q_1q_3}{r_{13}}\right]$$

Substituting the proper values with sign in above equation, we get

$$U = (9\times10^9)(10^{-6})(10^{-6})\left[\frac{(1)(2)}{1} + \frac{(2)(-3)}{1} + \frac{(-3)(4)}{1} + \frac{(1)(4)}{1} + \frac{(2)(4)}{\sqrt{2}} + \frac{(1)(-3)}{\sqrt{2}}\right]$$

$$U = (9\times10^{-3})\left[-12 + \frac{5}{\sqrt{2}}\right]$$

$$U = -7.62\times10^{-2} \text{ J}$$

The negative sign of U implies that positive work has to be done by electrostatic forces in assembling these charges at respective positions from infinity.

3. $\Delta V = -14\text{ V}$ and

$$Q = -N_A e = -(6.02\times10^{23})(1.6\times10^{-19}) = -9.63\times10^{4}\text{ C}$$

Since $\Delta V = \dfrac{W}{Q}$

$$\Rightarrow\quad W = Q\Delta V = (-9.63\times10^{4}\text{ C})(-14\text{ JC}^{-1})$$

$$\Rightarrow\quad W = 1.35\times10^{6}\text{ J} = 1.35\text{ MJ}$$

4. The net electric potential at origin is,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1}+\frac{q_2}{r_2}+\frac{q_3}{r_3}\right]$$

Substituting the values, we have

$$V = (9\times10^{9})\left(\frac{1}{1}-\frac{2}{2}+\frac{3}{3}\right)\times10^{-6}$$

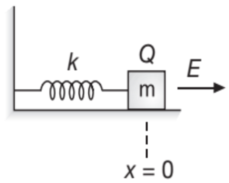
$$\Rightarrow\quad V = 9\times10^{3}\text{ V}$$

5. (a) Let us consider $V = 0$ at 0. Then at other points

$$V = -Ex \text{ and } U_e = QV = -Q(Ex)$$

Between the extreme points of the motion, we have

$$(K+U_s+U_e)_i = (K+U_s+U_e)_f$$



$$\Rightarrow\quad 0+0+0 = 0+\frac{1}{2}kx_{\text{max}}^2 - QEx_{\text{max}}$$

$$\Rightarrow\quad x_{\text{max}} = \frac{2QE}{k}$$

(b) At equilibrium,

$$\sum F_x = -F_s + F_e = 0$$

$$\Rightarrow\quad kx = QE$$

So the equilibrium position is at $x = \dfrac{QE}{k}$

(c) The block's equation of motion is

$$\sum F_x = -kx + QE = m\frac{d^2x}{dt^2}$$

$$\Rightarrow\quad m\frac{d^2x}{dt^2} = -k\left(x-\frac{QE}{k}\right)$$

Let $X = x - \dfrac{QE}{k}$,

$$\Rightarrow\quad \frac{d^2X}{dt^2} = \frac{d^2X}{dt^2}$$

$$\Rightarrow\quad m\frac{d^2X}{dt^2} = -kX$$

$$\Rightarrow\quad \frac{d^2X}{dt^2} + \frac{k}{m}X = 0$$

$$\Rightarrow\quad \ddot{X} + \omega^2 X = 0$$

This is the equation for simple harmonic motion with $\omega = \sqrt{\dfrac{k}{m}}$

The period of the motion is then

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

(d) $(K+U_s+U_e)_i + \Delta E_{\text{mech}} = (K+U_s+U_e)_f$

$$\Rightarrow\quad 0+0+0-\mu_k mgx_{\text{max}} = 0+\frac{1}{2}kx_{\text{max}}^2 - QEx_{\text{max}}$$

$$x_{\text{max}} = \frac{2(QE-\mu_k mg)}{k}$$

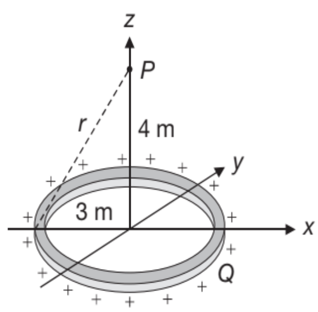
6. The electric potential at point P, at distance r from circumference is

$$V = \frac{Q}{2\pi\varepsilon_0 r}$$

where $r = \sqrt{(3)^2+(4)^2} = 5\text{ m}$ and

$$Q = 10\ \mu\text{C} = 10^{-5}\text{ C}$$

$$\Rightarrow\quad V = \frac{(9.0\times10^{9})(10^{-5})}{(5.0)} = 1.8\times10^{4}\text{ V}$$



7. For the entire motion, $y_f - y_i = u_y t + \frac{1}{2}a_y t^2$

$$\Rightarrow \quad 0 - 0 = u_y t + \frac{1}{2}a_y t^2$$

$$\Rightarrow \quad a_y = -\frac{2u_y}{t}$$

Since, $\sum F_y = ma_y$

$$\Rightarrow \quad -mg - qE = -\frac{2mu_y}{t}$$

$$\Rightarrow \quad E = \frac{m}{q}\left(\frac{2u_y}{t} - g\right) \text{ OR } \vec{E} = -\frac{m}{q}\left(\frac{2u_y}{t} - g\right)\hat{j}$$

For the upward flight, we have

$$v_y^2 = u_y^2 + 2a_y\left(y_f - y_i\right)$$

$$\Rightarrow \quad 0 = u_y^2 + 2\left(-\frac{2u_y}{t}\right)\left(y_{\max} - 0\right)$$

$$\Rightarrow \quad y_{\max} = \frac{1}{4}u_y t$$

Since $\Delta V = -\int_0^{y_{\max}} E\,dy = +\frac{m}{q}\left(\frac{2u_y}{t} - g\right)y\Big|_0^{y_{\max}}$

$$\Rightarrow \quad \Delta V = \frac{m}{q}\left(\frac{2u_y}{t} - g\right)\left(\frac{1}{4}u_y t\right)$$

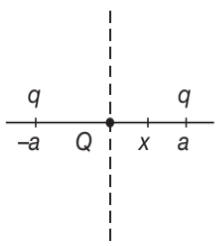
$$\Rightarrow \Delta V = \frac{2}{5\times10^{-6}}\left(\frac{2(20.1)}{4.1} - 9.8\right)\left[\frac{1}{4}(20.1)(4.1)\right] \cong 40\,\text{kV}$$

8. Initially, the potential energy of Q is

$$U_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qq}{a} + \frac{Qq}{a}\right]$$

$$\Rightarrow \quad U_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{a}$$

When Q is displaced by a small amount x to the right, the potential energy of Q becomes



$$U_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qq}{(a+x)} + \frac{Qq}{(a-x)}\right] = \frac{1}{4\pi\varepsilon_0}\frac{2Qqa}{\left(a^2 - x^2\right)}$$

$$\Rightarrow \quad U_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{a}\left(1 - \frac{x^2}{a^2}\right)^{-1}$$

Expanding binomially and neglecting higher powers of $\frac{x^2}{a^2}$ as $x \ll a$

$$U_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{a}\left(1 + \frac{x^2}{a^2}\right)$$

$$\left\{\because \left(1 - \frac{x^2}{a^2}\right)^{-1} \cong 1 + \frac{x^2}{a^2} \text{ for } x^2 \ll a^2\right\}$$

So, the change in electrostatic potential energy of the system,

$$dU = U_2 - U_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{a}\frac{x^2}{a^2}$$

$$\Rightarrow \quad dU = \frac{1}{4\pi\varepsilon_0}\frac{2Qqx^2}{a^3}$$

9. Arbitrarily take $V = 0$ at the initial point. Then at distance d downfield, where L is the rod length,

$$V = -Ed \text{ and } U_e = -\lambda LEd$$

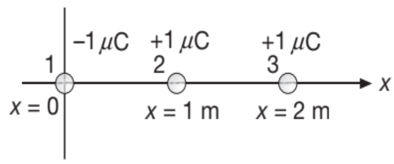
$$(K+U)_i = (K+U)_f$$

$$\Rightarrow \quad 0 + 0 = \frac{1}{2}\mu L v^2 - \lambda LEd$$

$$\Rightarrow \quad v = \sqrt{\frac{2\lambda Ed}{\mu}}$$

$$\Rightarrow \quad v = \sqrt{\frac{2(40\times10^{-6}\,\text{Cm}^{-1})(100\,\text{NC}^{-1})(2\,\text{m})}{\left(0.1\,\text{kgm}^{-1}\right)}} = 0.4\,\text{ms}^{-1}$$

10. (a) The work that must be done on q_3 by an external force is equal to the difference of potential energy U when the charge is at $x = 2$ m and the potential energy when it is at infinity.



$$\therefore \quad W_{\text{ext}} = U_f - U_i = U_f - U_\infty$$

$$V_3 = V_{13} + V_{23} = \frac{q_1}{4\pi\varepsilon_0 r_{13}} + \frac{q_2}{4\pi\varepsilon_0 r_{23}}$$

Now $W_{\infty\to 3} = q_3V_3$

$$\Rightarrow \quad W = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3q_2}{r_{23}} + \frac{q_3q_1}{r_{13}}\right]$$

Substituting the values, we have

$$\Rightarrow \quad W = (9\times 10^9)(10^{-12})\left[\frac{(1)(1)}{(1)} + \frac{(1)(-1)}{(2)}\right]$$

$$\Rightarrow \quad W = 4.5\times 10^{-3}\ \text{J}$$

(b) The total potential energy of the three charges is given by,

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_3q_2}{r_{23}} + \frac{q_3q_1}{r_{13}} + \frac{q_2q_1}{r_{12}}\right)$$

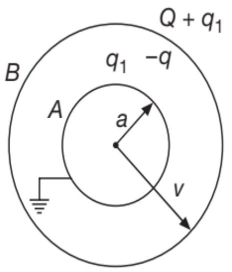
$$\Rightarrow \quad U = (9\times 10^9)\left[\frac{(1)(1)}{(1)} + \frac{(1)(-1)}{(2)} + \frac{(1)(-1)}{(1)}\right](10^{-12})$$

$$\Rightarrow \quad U = -4.5\times 10^{-3}\ \text{J}$$

Since $U < 0$, the system of three charges has lower potential energy then it would if the three charges were infinitely far apart. An external force would have to do negative work to bring the three charges from infinity to assemble this entire arrangement and would have to do positive work to move the three charges back to infinity.

11. When an object is connected to earth (grounded), its potential is reduced to zero. Let q' be the charge on A after it is earthed.

The charge q' on A induces $-q'$ on inner surface of B and $+q'$ on outer surface of B, in equilibrium the charge distribution is as shown in figure.



$$\begin{pmatrix}\text{Potential}\\ \text{due to}\\ \text{inner sphere}\end{pmatrix} = \begin{pmatrix}\text{potential}\\ \text{due to}\\ \text{charge on A}\end{pmatrix} + \begin{pmatrix}\text{potential}\\ \text{due to}\\ \text{charge on B}\end{pmatrix}$$

$$V_A = \frac{q'}{4\pi\varepsilon_0 a} - \frac{q'}{4\pi\varepsilon_0 b} + \frac{Q+q'}{4\pi\varepsilon_0 b} = V_{\text{Earth}} = 0$$

$$\Rightarrow \quad q' = -Q\left(\frac{a}{b}\right)$$

This implies that a charge $+Q\left(\frac{a}{b}\right)$ has been transferred to earth leaving negative charge on A.

12. (a) $E = \dfrac{|Q|}{4\pi\varepsilon_0 r^2}$

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad r = \frac{|V|}{|E|} = \frac{3000\ \text{V}}{500\ \text{Vm}^{-1}} = 6\ \text{m}$$

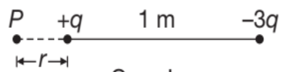
(b) $V = -3000$ V

$$\Rightarrow \quad -3000 = \frac{Q}{4\pi\varepsilon_0(6)}$$

$$\Rightarrow \quad Q = -\frac{3000\times 6}{9\times 10^9} = -2\ \mu\text{C}$$

13. Let P be the point on the axis either to the left or to the right of charge $+q$ at a distance r where potential is zero. Hence,

CASE-1:

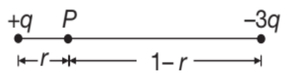


Case I

$$V_P = \frac{q}{4\pi\varepsilon_0 r} - \frac{3q}{4\pi\varepsilon_0(1+r)} = 0$$

Solving this, we get $r = 0.5$ m

CASE-2:



Case II

$$V_P = \frac{q}{4\pi\varepsilon_0 r} - \frac{3q}{4\pi\varepsilon_0(1-r)} = 0,$$

which gives $r = 0.25$ m

Thus, the potential will be zero at point P on the axis which is either 0.5 m to the left or 0.25 m to the right of charge $+q$.

14. The original electrical potential energy is

$$U_e = qV = q\left(\frac{q}{4\pi\varepsilon_0 d}\right) = \frac{q^2}{4\pi\varepsilon_0 d}$$

In the final configuration we have mechanical equilibrium. The spring and electrostatic forces on each charge are

$$-k(2d)+\frac{q^2}{4\pi\varepsilon_0(3d)^2}=0$$

$$\Rightarrow \quad k=\frac{q^2}{4\pi\varepsilon_0(18d^3)}$$

In the final configuration the total potential energy is

$$\frac{1}{2}kx^2+qV=\frac{1}{2}\frac{q^2}{4\pi\varepsilon_0(18d^3)}(2d)^2+q\frac{q}{4\pi\varepsilon_0(3d)}=$$

$$\frac{4}{9}\frac{q^2}{4\pi\varepsilon_0 d}$$

The missing energy must have become internal energy, as the system is isolated

So, $\frac{q^2}{4\pi\varepsilon_0 d}=\frac{4}{9}\left(\frac{q^2}{4\pi\varepsilon_0 d}\right)+\Delta E_{\text{int}}$

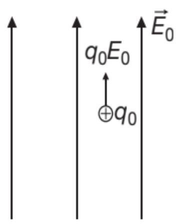
$$\Rightarrow \quad \Delta E_{\text{int}}=\frac{5}{9}\left(\frac{q^2}{4\pi\varepsilon_0 d}\right)$$

15. $\Delta V=V_{2R}-V_0=\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+(2R)^2}}-\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

$$\Delta V=\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\left(\frac{1}{\sqrt{5}}-1\right)=-0.553\frac{Q}{4\pi\varepsilon_0 R}$$

16. Electrostatic force F on the test charge,

$F=q_0E_0$ {along positive y-direction}



Since, $W_{i\to f}=-\Delta U$

$$\Rightarrow \quad \Delta U=-W_{i\to f}=-\left[q_0E_0(2a-a)\right]$$

$$\Rightarrow \quad \Delta U=-q_0E_0a$$

Here work done by electrostatic force is positive. Hence the potential energy must be decreasing.

17. $V=\frac{1}{4\pi\varepsilon_0}\int_{\text{all charge}}\frac{dq}{r}$

$$\Rightarrow \quad V=\frac{1}{4\pi\varepsilon_0}\left[\int_{3R}^{R}\frac{\lambda dx}{x}+\int_{\text{semicircle}}\frac{\lambda ds}{R}+\int_{R}^{3R}\frac{\lambda dx}{x}\right]$$

$$\Rightarrow \quad V=\frac{1}{4\pi\varepsilon_0}\lambda\log_e(x)\Big|_{3R}^{R}+\frac{1}{4\pi\varepsilon_0}\left[\frac{\lambda}{R}\pi R+\lambda\log_e x\Big|_{R}^{3R}\right]$$

$$\Rightarrow \quad V=\frac{1}{4\pi\varepsilon_0}\left[\log_e 3+\lambda\pi+\log_e 3\right]$$

$$\Rightarrow \quad V=\frac{\lambda}{4\pi\varepsilon_0}(\pi+2\log_e 3)$$

18. For an element of area which is a ring of radius r and width dr, $dV=\frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{r^2+x^2}}$

$dq=\sigma dA=\sigma_0 r(2\pi r dr)$ and

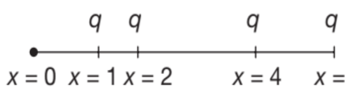
$$V=\sigma_0\left(\frac{2\pi}{4\pi\varepsilon_0}\right)\int_0^R\frac{r^2dr}{\sqrt{r^2+x^2}}$$

$$\Rightarrow \quad V=\left(\frac{\pi\sigma_0}{4\pi\varepsilon_0}\right)\left[R\sqrt{R^2+x^2}+x^2\log_e\left(\frac{x}{R+\sqrt{R^2+x^2}}\right)\right]$$

19. (i) Potential at $x=0$ will be

$$V=\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{1}+\frac{q}{2}+\frac{q}{4}+\frac{q}{8}+\ldots\infty\right)$$

$$\Rightarrow \quad V=\frac{q}{4\pi\varepsilon_0}\left[1+\frac{1}{2}+\frac{1}{4}+\ldots\infty\right]$$



This is an infinite G.P. sum of ∞ terms of G.P. is

$$S_n=\frac{a}{1-r}$$

$$\Rightarrow \quad V=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1-\frac{1}{2}}\right]=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1/2}\right]=\frac{2q}{4\pi\varepsilon_0}$$

Now electric field

$$E=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(1)^2}+\frac{q}{(2)^2}+\frac{q}{(4)^2}+\ldots\infty\right]$$

$$\Rightarrow \quad E=\frac{q}{4\pi\varepsilon_0}\left[1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots\infty\right]$$

{This is also an infinite G.P.}

$$\Rightarrow \quad E=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1-1/4}\right]=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{3/4}\right]$$

$$\Rightarrow \quad E=\frac{4q}{4\pi\varepsilon_0\times 3}=\frac{q}{3\pi\varepsilon_0}$$

(ii) When consecutive charges are negative then

$$V=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1}-\frac{q}{2}+\frac{q}{4}-\frac{q}{8}+\frac{q}{16}-\frac{q}{32}+\ldots\infty\right]$$

$$\Rightarrow \quad V=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1}+\frac{q}{4}+\frac{q}{16}+\ldots\infty\right]-\left[\frac{q}{2}+\frac{q}{8}+\frac{q}{32}+\ldots\infty\right]$$

$$\Rightarrow\ V=\frac{q}{4\pi\varepsilon_0}\left\{\left[\frac{1}{\left[1-\left(\frac{1}{4}\right)\right]}\right]-\frac{1}{2}\left[\frac{1}{\left[1-\left(\frac{1}{4}\right)\right]}\right]\right\}$$

$$\Rightarrow\ V=\frac{q}{4\pi\varepsilon_0}\left[\frac{4}{3}-\frac{2}{3}\right]=\frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{3}\right)$$

Electric field at $x=0$ is

$$E=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(1)^2}-\frac{q}{(2)^2}+\frac{q}{(4)^2}-\frac{q}{(8)^2}+\frac{q}{(16)^2}+\ldots\infty\right]$$

$$\Rightarrow\ E=\frac{q}{4\pi\varepsilon_0}\left[1+\frac{1}{16}+\frac{1}{256}+\ldots\right]-\left[\frac{1}{4}+\frac{1}{64}+\frac{1}{1024}+\ldots\right]$$

$$\Rightarrow\ E=\frac{q}{4\pi\varepsilon_0}\left\{\left[\frac{1}{\left[1-\frac{1}{16}\right]}\right]\right\}-\frac{1}{4}\left[\frac{1}{1-\frac{1}{16}}\right]$$

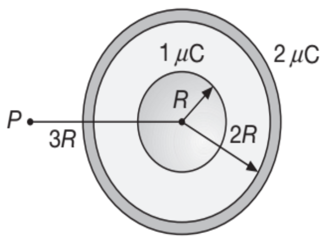
$$\Rightarrow\ E=\frac{q}{4\pi\varepsilon_0}\left[\frac{16}{15}-\frac{1}{4}\frac{16}{15}\right]=\frac{q}{4\pi\varepsilon_0}\left[\frac{4q}{5}\right]$$

20. Potential at point P

$$9000=\frac{K\left(1\times10^{-6}\right)}{3R}+\frac{K\left(2\times10^{-6}\right)}{3R}$$

$$3R=3$$

$$R=1\text{ m}$$



21. We can here use Law of Conservation of Energy, according to which

$$(U+K)_{\text{at }\frac{\ell}{4}}+(U+K)_{\text{at }A}$$

$$\Rightarrow\ \frac{-qQ}{4\pi\varepsilon_0\left(\frac{\ell}{4}\right)}+\frac{1}{2}mv^2=\frac{-qQ}{4\pi\varepsilon_0(2\ell)}+\frac{1}{2}m(0)^2$$

$$\Rightarrow\ \frac{1}{2}mv^2=\frac{qQ}{4\pi\varepsilon_0}\left[\frac{4}{\ell}-\frac{1}{2\ell}\right]$$

$$\Rightarrow\ \frac{1}{2}mv^2=\frac{qQ}{4\pi\varepsilon_0}\left[\frac{7}{2\ell}\right]$$

$$\Rightarrow\ v=\sqrt{\frac{7Qq}{4\pi\varepsilon_0 m\ell}}$$

22. For potential energy of the system of charges, total number of charge pairs will be ${}^8C_2$ or 28 of these 28 pairs, 12 unlike charges are at a separation 'a', 12 like charges are at separation $\sqrt{2}a$ and 4 unlike charges are at separation $\sqrt{3}a$. Therefore, the potential energy of the system

$$U=\frac{1}{4\pi\varepsilon_o}\left[\frac{(12)(q)(-q)}{a}+\frac{(12)(q)(q)}{\sqrt{2}a}+\frac{(4)(q)(-q)}{\sqrt{3}a}\right]$$

$$\Rightarrow\ U=-5.824\left(\frac{1}{4\pi\varepsilon_o}\frac{q^2}{a}\right)$$

The binding energy of this system is therefore,

$$|U|=5.824\left(\frac{1}{4\pi\varepsilon_o}\frac{q^2}{a}\right)$$

So, work done by external forces in disassembling, this system of charges is

$$W=5.824\left(\frac{1}{4\pi\varepsilon_o}\frac{q^2}{a}\right)$$

23. Let q be the charge on the bubble, then

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{a}$$

$$\Rightarrow\ q=4\pi\varepsilon_0 aV$$

After collapsing, let the radius of droplet becomes R. Then equating the volume, we get

$$\left(4\pi a^2\right)t=\frac{4}{3}\pi R^3$$

$$\Rightarrow\ R=\left(3a^2t\right)^{1/3}$$

Now, potential of new droplet will be $V'=\frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

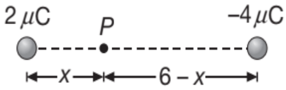
Substituting the values, we have

$$V'=\frac{1}{4\pi\varepsilon_0}\frac{4\pi\varepsilon_0 aV}{\left(3a^2t\right)^{1/3}}$$

$$\Rightarrow\ V'=V\left(\frac{a}{3t}\right)^{1/3}$$

24. There will be two points where electric potential will be zero. One point lies between the charges and other point lies in region outside the charges.

For inside point,



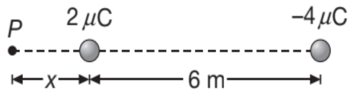
$$V_P = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\times10^{-6}}{x}+\frac{-4\times10^{-6}}{6-x}\right)=0$$

$$\Rightarrow \quad \frac{2}{x}=\frac{4}{6-x}$$

$$\Rightarrow \quad 12-2x=4x$$

$$\Rightarrow \quad x=2\text{ m}$$

For outside point



$$V_P = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\times10^{-6}}{x}+\frac{-4\times10^{-6}}{6+x}\right)=0$$

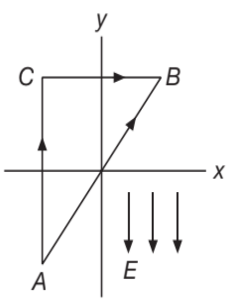
$$\Rightarrow \quad \frac{2}{x}=\frac{4}{6+x}$$

$$\Rightarrow \quad 12+2x=4x$$

$$\Rightarrow \quad x=6\text{ m}$$

Test Your Concepts-VIII (Based on Equipotential Surfaces)

1. $V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{\ell} = -\int_A^C \vec{E}\cdot d\vec{\ell} - \int_C^B \vec{E}\cdot d\vec{\ell}$



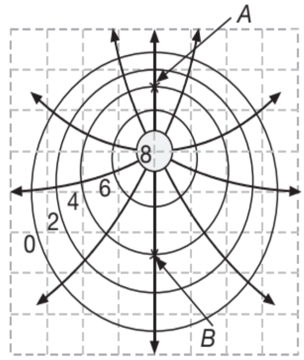
$$\Rightarrow \quad V_B - V_A = \left(-E\cos180^o\right)\int_{-0.3}^{0.5} dy - \left(E\cos90^o\right)\int_{-0.2}^{0.4} dx$$

$$\Rightarrow \quad V_B - V_A = (325)(0.8) = +260V$$

2. (a) $E_A > E_B$ since $E = \frac{\Delta V}{\Delta \ell}$

(b) $E_B = -\frac{\Delta V}{\Delta \ell} = -\frac{(6-2)\text{ V}}{0.02\text{ m}} = 200\text{ NC}^{-1}$ downwards

(c) The figure is shown here with the field lines sketched.



3. For the given charge distribution,

$$V(x,\ y,\ z) = \frac{1}{4\pi\varepsilon_0}\frac{(q)}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{(-2q)}{r_2} \quad \ldots(1)$$

where $r_1 = \sqrt{(x+R)^2+y^2+z^2}$ and $r_2 = \sqrt{x^2+y^2+z^2}$

The surface on which $V(x,\ y,\ z)=0$

is given by $\frac{1}{4\pi\varepsilon_0}q\left(\frac{1}{r_1}-\frac{2}{r_2}\right)=0 \quad \Rightarrow \quad 2r_1 = r_2$

From (1), we get $4(x+R)^2+4y^2+4z^2 = x^2+y^2+z^2$

$$\Rightarrow \quad x^2+y^2+z^2+\left(\frac{8}{3}R\right)x+\left(\frac{4}{3}R^2\right)=0 \quad \ldots(2)$$

The general equation for a sphere of radius r centered at $(x_0,\ y_0,\ z_0)$ is

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2-r^2=0$$

$$\Rightarrow \quad x^2+y^2+z^2+(-2x_0)x+(-2y_0)y+ (-2z_0)z+\left(x_0^2+y_0^2+z_0^2-r^2\right)=0 \ \ldots(3)$$

Comparing equations (2) and (3), it is seen that the equipotential surface for which $V=0$ is indeed a sphere and that

$$-2x_0=\frac{8}{3}R, -2y_0=0, -2z_0=0, x_0^2+y_0^2+z_0^2-r^2=\frac{4}{3}R^2$$

$$\Rightarrow \quad x_0=-\frac{4}{3}R,\ y_0=z_0=0, \text{ and } r^2=\left(\frac{16}{9}-\frac{4}{3}\right)R^2=\frac{4}{9}R^2$$

The equipotential surface is therefore a sphere centered at $\left(-\frac{4}{3}R,\ 0,\ 0\right)$, having a radius $\frac{2}{3}R$

4. Let us consider that the equipotential surface corresponding to this situation mentioned lies at a perpendicular distance r from all the charge densities. Then, at the equipotential surface, we have

$$\Sigma V = \text{constant}$$

$$\Rightarrow \frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{r_1}{r}\right)+\frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{r_2}{r}\right)-\frac{2\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{r_3}{r}\right)=\text{constant}$$

$$\Rightarrow \log_e\left(\frac{r_1r_2}{r^2}\right)-2\log_e\left(\frac{r_3}{r}\right)=\text{constant}$$

$$\Rightarrow \log_e\left(\frac{r_1r_2}{r^2}\right)+\log_e\left(\frac{r^2}{r_3^2}\right)=\text{constant} \quad \left\{\because \log_e x=-\log_e\left(\frac{1}{x}\right)\right\}$$

$$\Rightarrow \log_e\left(\frac{r_1r_2}{r_3^2}\right)=\text{constant} \quad \{\because \log_e m+\log_e m=\log_e(mn)\}$$

$$\Rightarrow \frac{r_1r_2}{r_3^2}=\text{constant}$$

Test Your Concepts-IX (Based on Relation Between Electrostatic Field and Potential)

1. $V=5x-3x^2y+2yz^2$

Since $\vec{E}=-\vec{\nabla}V=-\left(\frac{\partial V}{\partial x}\hat{i}+\frac{\partial V}{\partial y}\hat{j}+\frac{\partial V}{\partial z}\hat{k}\right)$

$$\Rightarrow \vec{E}=(-5+6xy)\hat{i}+(3x^2-2z^2)\hat{j}-4yz\hat{k}$$

$$\Rightarrow \vec{E}\Big|_{(1,\,0,\,-2)}=-5\hat{i}-5\hat{j}$$

$$\Rightarrow E=\sqrt{E_x^2+E_y^2+E_z^2}$$

$$E=\sqrt{(-5)^2+(-5)^2+0^2}=5\sqrt{2}\ \text{NC}^{-1}$$

2. Inside the sphere, $E_x=E_y=E_z=0$.

Outside, $E_x=-\frac{\partial V}{\partial x}$

$$\Rightarrow E_x=-\frac{\partial}{\partial x}\left[V_0-E_oz+E_oa^3z\left(x^2+y^2+z^2\right)^{-\frac{3}{2}}\right]$$

$$\Rightarrow E_x=-\left[E_0a^3z\left(-\frac{3}{2}\right)\left(x^2+y^2+z^2\right)^{-\frac{5}{2}}(2x)\right]$$

$$\Rightarrow E_x=3E_0a^3xz\left(x^2+y^2+z^2\right)^{-\frac{5}{2}}$$

$$E_y=-\frac{\partial V}{\partial y}$$

$$\Rightarrow E_y=-\frac{\partial}{\partial y}\left(V_0-E_0z+E_0a^3z\left(x^2+y^2+z^2\right)^{-\frac{3}{2}}\right)$$

$$\Rightarrow E_y=-E_0a^3z\left(-\frac{3}{2}\right)\left(x^2+y^2+z^2\right)^{-\frac{5}{2}}2y$$

$$\Rightarrow E_y=3E_0a^3yz\left(x^2+y^2+z^2\right)^{-\frac{5}{2}}$$

$$E_z=-\frac{\partial V}{\partial z}$$

$$\Rightarrow E_z=E_0+E_0a^3\left(2z^2-x^2-y^2\right)\left(x^2+y^2+z^2\right)^{-\frac{5}{2}}$$

3. $\vec{E}=-\left(\frac{\partial V}{\partial x}\hat{i}+\frac{\partial V}{\partial y}\hat{j}+\frac{\partial V}{\partial z}\hat{k}\right)$

where, $\frac{\partial V}{\partial x}=\frac{\partial}{\partial x}(2x+3y-z)=2$

$$\frac{\partial V}{\partial y}=\frac{\partial}{\partial y}(2x+3y-z)=3$$

$$\frac{\partial V}{\partial z}=\frac{\partial}{\partial z}(2x+3y-z)=-1$$

$$\Rightarrow \vec{E}=-2\hat{i}-3\hat{j}+\hat{k}$$

4. $dV=-\vec{E}\cdot d\vec{\ell}$

$$\int_B^A dV=-\int_{(0,2,4)}^{(2,1,0)}\left(x\hat{i}-2y\hat{j}+z\hat{k}\right)\cdot\left(dx\,\hat{i}+dy\,\hat{j}+dz\,\hat{k}\right)$$

$$\Rightarrow V_A-V_B=-\int_{(0,2,4)}^{(2,1,0)}(xdx-2ydy+zdz)$$

$$\Rightarrow V_{AB}=-\left[\left(\frac{x^2}{2}-y^2+\frac{z^2}{2}\right)\Bigg|_{(0,2,4)}^{(2,1,0)}\right]$$

$$\Rightarrow V_{AB}=3\ \text{V}$$

5. (a) $dV=-\vec{E}\cdot d\vec{\ell}$

$$\int_B^A dV=-\int_{(1,1,1)}^{(0,0,0)}\left(y\hat{i}+x\hat{j}\right)\cdot\left(dx\,\hat{i}+dy\,\hat{j}+dz\,\hat{k}\right)$$

$$\Rightarrow \quad V_A - V_B = -\int_{(1,1,1)}^{(0,0,0)} (y\,dx + x\,dy)$$

$$\Rightarrow \quad V_{AB} = -\int_{(1,1,1)}^{(0,0,0)} d(xy) \quad \{\text{as } y\,dx + x\,dy = d(xy)\}$$

$$\Rightarrow \quad V_{AB} = -\left[(xy)\Big|_{(1,1,1)}^{(0,0,0)}\right] = 1\text{ V}$$

(b) $dV = -\vec{E}\cdot d\vec{\ell}$

$$\Rightarrow \quad \int_B^A dV = -\int_{(1,1,1)}^{(0,0,0)} (3x^2y\hat{i} + x^3\hat{j})\cdot(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

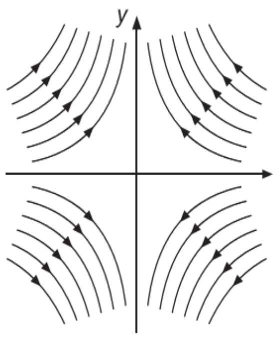
$$\Rightarrow \quad V_A - V_B = -\int_{(1,1,1)}^{(0,0,0)} (3x^2y\,dx + x^3\,dy)$$

$$\Rightarrow \quad V_A - V_B = -\int_{(1,1,1)}^{(0,0,0)} d(x^3y)$$

$$\Rightarrow \quad V_{AB} = -\left[(x^3y)\Big|_{(1,1,1)}^{(0,0,0)}\right] = 1\text{ V}$$

Since in both the cases, the line integral of the field is an exact differential and hence both the fields are conservative in nature.

6. (a) Given, $\varphi = a(x^2 - y^2)$



(a)

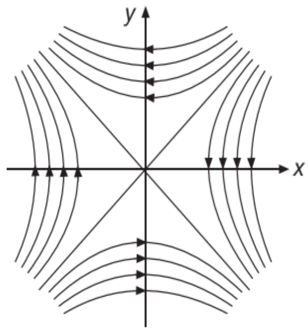
So, $\vec{E} = -\vec{\nabla}\varphi = -2a(x\hat{i} - y\hat{j})$

The sought shape of field lines is as shown in the figure.

(b) Since $\varphi = axy$

So, $\vec{E} = -\vec{\nabla}\varphi = -ay\hat{i} - ax\hat{j}$

The sought shape of field lines is as shown in the figure.



(b)

7. Given, $V = a(x^2 + y^2) + bz^2$

Since, $\vec{E} = -\vec{\nabla}V$

$$\Rightarrow \quad \vec{E} = -(2ax\hat{i} + 2ay\hat{j} + 2bz\hat{k})$$

Hence $|\vec{E}| = 2\sqrt{a^2(x^2 + y^2) + b^2z^2}$

Shape of the equipotential surface i.e. for V = constant

Since, $V = a(x^2 + y^2) + bz^2$

$$\Rightarrow \quad \frac{x^2}{\left(\frac{V}{a}\right)} + \frac{y^2}{\left(\frac{V}{a}\right)} + \frac{z^2}{\left(\frac{V}{b}\right)} = 1$$

$$\Rightarrow \quad \frac{x^2}{\left(\sqrt{\frac{V}{a}}\right)^2} + \frac{y^2}{\left(\sqrt{\frac{V}{a}}\right)^2} + \frac{z^2}{\left(\sqrt{\frac{V}{b}}\right)^2} = 1$$

which is an ellipsoid of revolution with semi-axis $\sqrt{\frac{V}{a}}, \sqrt{\frac{V}{a}}, \sqrt{\frac{V}{b}}$.

8. Given, again

$$dV = -\vec{E}\cdot d\vec{\ell}$$

$$\Rightarrow \quad dV = -\left(ay\hat{i} + (ax + bz)\hat{j} + by\hat{k}\right)\left(dx\hat{i} + dy\hat{j} + dx\hat{k}\right)$$

$$\Rightarrow \quad dV = -a(ydx + axdy) + b(zdy + ydz)$$

$$\Rightarrow \quad dV = -\left[ad(xy) + bd(yz)\right]$$

Integrating we get,

$$V = -(axy + byz) + C$$

Test Your Concepts-X (Based on Motion of Charged Particles in Electric Field)

1. (a) $|\Delta V| = Ed = (6\times10^3\ \text{Vm}^{-1})(0.01\text{ m}) = 60\text{ V}$

(b) $\frac{1}{2}mv_f^2 = |q\Delta V|$

$$\Rightarrow \frac{1}{2}(9.1\times10^{-31})v_f^2 = (1.6\times10^{-19})(60)$$

$$\Rightarrow v_f = 4.6\times10^6 \text{ ms}^{-1}$$

2. (a) Energy of the proton-field system is conserved as the proton moves from higher to lower potential, which can be defined for this problem as moving from 120 V down to 0 V.

$$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$$

$$\Rightarrow 0 + qV + 0 = \frac{1}{2}mv_p^2 + 0$$

$$\Rightarrow (1.6\times10^{-19})(120) = \frac{1}{2}(1.67\times10^{-27})v_p^2$$

$$\Rightarrow v_p = 1.52\times10^5 \text{ ms}^{-1}$$

(b) The electron will gain speed in moving the other way, from $V_i = 0$ to $V_f = 120$ V. So,

$$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$$

$$\Rightarrow 0 + 0 + 0 = \frac{1}{2}mv_e^2 + qV$$

$$\Rightarrow 0 = \frac{1}{2}mv_e^2 + (-e)V$$

$$\Rightarrow \frac{1}{2}(9.1\times10^{-31})v_e^2 = (1.6\times10^{-19})(120)$$

$$\Rightarrow v_e = 6.5\times10^6 \text{ ms}^{-1}$$

3. (a) $$U = \frac{10^{-18}}{4\pi\varepsilon_0}\left[\frac{(20)(10)}{0.04} + \frac{(10)(-20)}{0.04} + \frac{(20)(-20)}{0.08}\right]$$

$$\Rightarrow U = 9\times10^{-9}\times100\left[\frac{200}{4} - \frac{200}{4} - \frac{400}{8}\right]$$

$$\Rightarrow U = 9\times10^{-9}\times100\times\left(-\frac{400}{8}\right)$$

$$\Rightarrow U = -4.5\times10^{-5} \text{ J}$$

(b) The three fixed charges create this potential at the location where the fourth is released:

$$V = V_1 + V_2 + V_3$$

$$\Rightarrow V = (9\times10^9)\left(\frac{20\times10^{-9}}{0.05} + \frac{10\times10^{-9}}{0.03} - \frac{20\times10^{-9}}{0.05}\right)$$

$$\Rightarrow V = 3\times10^3 \text{ V}$$

Energy of the system of four charged objects is conserved as the fourth charge flies away

$$\left(\frac{1}{2}mv^2 + qV\right)_i = \left(\frac{1}{2}mv^2 + qV\right)_f$$

$$\Rightarrow 0 + (40\times10^{-9}\text{C})(3\times10^3\text{ V}) = \frac{1}{2}(2\times10^{-13}\text{ kg})v^2 + 0$$

$$\Rightarrow v = \sqrt{\frac{2(1.2\times10^{-4}\text{ J})}{2\times10^{-13}\text{ kg}}} = 3.46\times10^4 \text{ ms}^{-1}$$

4. Using conservation of energy for the alpha particle-nucleus system,

we have $K_f + U_f = K_i + U_i$.

where $U_i = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_\alpha q_{\text{gold}}}{r_i}$

Since $r_i \to \infty$

$$\Rightarrow U_i = 0$$

also $K_f = 0$ because, at turning point, we have $v_f = 0$

so we get, $U_f = K_i$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\text{gold}}}{r_{\min}} = \frac{1}{2}m_\alpha v_\alpha^2$$

$$\Rightarrow r_{\min} = \frac{1}{4\pi\varepsilon_0}\frac{2q_\alpha q_{\text{gold}}}{m_\alpha v_\alpha^2}$$

$$\Rightarrow r_{\min} = \frac{2(9\times10^9)(2)(79)(1.6\times10^{-19})^2}{(6.67\times10^{-27})(2\times10^7)^2}$$

$$\Rightarrow r_{\min} = 2.73\times10^{-14} \text{ m} = 27.3 \text{ fm}$$

5. Each charge moves off on its diagonal line. All charges have equal speeds.

$$\sum(K+U)_i = \sum(K+U)_f$$

$$\Rightarrow 0 + \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L} + \frac{2q^2}{\sqrt{2}L}\right) = 4\left(\frac{1}{2}mv^2\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{2L} + \frac{2q^2}{2\sqrt{2}L}\right)$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\left(2 + \frac{1}{\sqrt{2}}\right)\frac{q^2}{L} = 2mv^2$$

$$\Rightarrow v = \sqrt{\left(1 + \frac{1}{\sqrt{8}}\right)\frac{1}{4\pi\varepsilon_0}\frac{q^2}{mL}}$$

6. The potential created by the ring at the electron's starting point is

$$V_i = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x_i^2 + a^2}} = \frac{1}{4\pi\varepsilon_0}\frac{(2\pi\lambda a)}{\sqrt{x_i^2 + a^2}}$$

CHAPTER 1

while at the center, it is $V_f=\frac{1}{4\pi\varepsilon_0}(2\pi\lambda)$

From Law of Conservation of Energy, we have

$$(U+K)_i=(U+K)_f$$

$$\Rightarrow\quad -eV_i+0=-eV_f+\frac{1}{2}m_eV_f^2$$

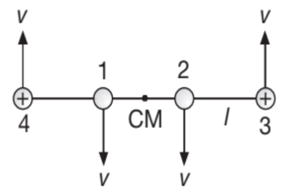
$$\Rightarrow\quad v_f^2=\frac{2e}{m_e}(V_f-V_i)=\frac{1}{4\pi\varepsilon_0}\frac{4\pi e\lambda}{m_e}\left(1-\frac{a}{\sqrt{x_i^2+a^2}}\right)$$

$$\Rightarrow\quad v_f^2=\frac{4\pi(1.6\times10^{-19})(9\times10^9)(1\times10^{-7})}{9.1\times10^{-31}}\times\left(1-\frac{0.2}{\sqrt{(0.1)^2+(0.2)^2}}\right)$$

$$\Rightarrow\quad v_f=1.45\times10^7\ \text{ms}^{-1}$$

7. Initially, since no external horizontal forces act on the set of four balls, so the center of mass of the system stays fixed at the initial location of the center of the square. As the charged balls 4 and 3 swing out and away from each other, balls 1 and 2 move down with equal y-components of velocity. The maximum kinetic energy point is illustrated. For system energy to be conserved,

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{\ell}=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{3\ell}+\frac{1}{2}mv^2+\frac{1}{2}mv^2+\frac{1}{2}mv^2+\frac{1}{2}mv^2$$



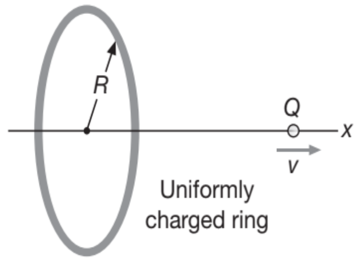
$$\Rightarrow\quad \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{3\ell}=2mv^2$$

$$\Rightarrow\quad v=\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{q^2}{3m\ell}}$$

8. From Law of Conservation of Energy,

$$K_f+U_f=K_i+U_i$$

$$\Rightarrow\quad \frac{1}{2}Mv_f^2+0=0+\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{R}$$



$$\Rightarrow\quad v_f=\sqrt{\frac{Q^2}{2\pi\varepsilon_0MR}}$$

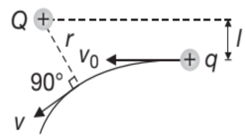
9. By Law of Conservation of Energy,

$$(U+K)_{\substack{\text{initial for}\\(A+B)\text{ system}}}=(U+K)_{\substack{\text{final for}\\(A+B)\text{ system}}}$$

$$\Rightarrow\quad mg\ell+\frac{q^2}{4\pi\varepsilon_0\sqrt{2}\ell}+0+0=\frac{q^2}{4\pi\varepsilon_0(2\ell-\ell)}+\frac{1}{2}mu^2+K_A$$

$$\Rightarrow\quad K_A=mg\ell-\frac{1}{2}mu^2+\frac{q^2}{4\pi\varepsilon_0}\frac{1}{\sqrt{2}\ell}(1-\sqrt{2})$$

10. Let $r_{\min}$ be the distance of closest approach between the two charges. Then, from Conservation of Angular Momentum, we get



$$mv_0\ell=mvr$$

$$\Rightarrow\quad v_0\ell=vr \qquad \ldots(1)$$

From Law of Conservation of Energy,

$$\frac{1}{2}mv_0^2=\frac{1}{2}mv^2+\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{r} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$r_{\min}=\frac{Qq}{4\pi\varepsilon_0mv_0^2}+\sqrt{\left(\frac{Qq}{4\pi\varepsilon_0mv_0^2}\right)^2+\ell^2}$$

11. Since $|E|=\frac{V}{\ell}=\frac{at}{\ell}$

$$\Rightarrow\quad m\frac{dv}{dt}=\left(\frac{at}{\ell}\right)e$$

$$\Rightarrow\quad \int_0^v dv=\frac{ae}{m\ell}\int_0^t t\,dt$$

$$\Rightarrow\quad v=\left(\frac{ae}{2m\ell}\right)t^2 \qquad \ldots(1)$$

$$\Rightarrow\quad \frac{dx}{dt}=\left(\frac{ae}{2m\ell}\right)t$$

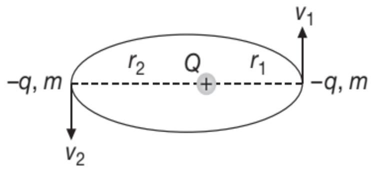
$$\Rightarrow \quad x\Big|_0^{\ell} = \frac{ae}{2m\ell}\left(\frac{t^3}{3}\right)\Bigg|_0^{t}$$

$$\Rightarrow \quad \ell = \left(\frac{ae}{6m\ell}\right)t^3 \qquad \ldots(2)$$

Substituting value of t from (2) in (1), we get

$$v = \left(\frac{ae}{2m\ell}\right)\left(\frac{6m\ell^2}{ae}\right)^{\frac{2}{3}}$$

12. Using Law of Conservation of Angular Momentum about $+Q$, we get



$$mv_1r_1 = mv_2r_2$$

$$\Rightarrow \quad v_1r_1 = v_2r_2 \qquad \ldots(1)$$

From Law of Conservation of Energy,

$$(U+K)_{\text{initial}} = (U+K)_{\text{final}}$$

$$\Rightarrow \quad \frac{1}{2}mv_1^2 - \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_1} = \frac{1}{2}mv_2^2 - \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_2} \qquad \ldots(2)$$

On solving equations (1) and (2), we get

$$v_1 = \sqrt{\frac{Qqr_2}{2\pi\varepsilon_0 mr_1(r_1+r_2)}} \text{ and } v_2 = \sqrt{\frac{Qqr_1}{2\pi\varepsilon_0 mr_2(r_1+r_2)}}$$

13. Since, no external force acts on the system of protons, so by Law of Conservation of Linear Momentum, we get

$$mv_0 = 2mv$$

$$\Rightarrow \quad v = \frac{v_0}{2} \qquad \ldots(1)$$

At the closest distance of approach, both the protons have same velocity, so

By Law of Conservation of Energy, we get

$$\frac{1}{2}mv_0^2 = 2\left[\frac{1}{2}mv^2\right] + \frac{e^2}{4\pi\varepsilon_0 r_{\min}} \qquad \ldots(2)$$

On solving equations (1) and (2) for $r_{\min}$, we get

$$r_{\min} = \frac{e^2}{\pi\varepsilon_0 mv_0^2}$$

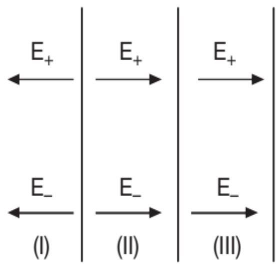
Test Your Concepts-XI (Based on Conductors)

1. The charge divides equally between the identical spheres, with charge $\frac{Q}{2}$ on each. Then they repel like point charges at their centers so, we have

$$F = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{Q}{2}\right)\left(\frac{Q}{2}\right)}{(L+R+R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{4(L+2R)^2}$$

$$F = \frac{9\times10^9\times(60\times10^{-6})^2}{4\times(2.01)^2} \cong 2\text{ N}$$

2. Since $E = \frac{\sigma}{2\varepsilon_0}$



Let us designate left to right as positive direction. Then

$$E_I = -\frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B)$$

$$E_{II} = \frac{1}{2\varepsilon_0}(\sigma_A - \sigma_B)$$

$$E_{III} = \frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B)$$

For $\sigma_A = +\sigma$ and $\sigma_B = -\sigma$, we get

$$E_I = E_{III} = 0 \text{ and } E_{II} = \frac{\sigma}{\varepsilon_0}$$

3. Consider as a Gaussian surface a box with horizontal area A, lying between 500 m and 600 m elevation.

$$\oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad (+120\text{ NC}^{-1})A + (-100\text{ NC}^{-1})A = \frac{\rho A(100\text{ m})}{\varepsilon_0}$$

$$\Rightarrow \quad \rho = \frac{(20\text{ NC}^{-1})(8.85\times10^{-12}\text{ C}^2\text{Nm}^{-2})}{100\text{ m}}$$

$$\rho = 1.77\times10^{-12}\text{ Cm}^{-3} = 1.77\text{ pCm}^{-3}$$

The charge has to be positive so that it produces the net outward flux of electric field.

4. The electric field on the surface of a conductor varies inversely with the radius of curvature of the surface. Thus, the field is most intense where the radius of curvature is smallest and vice-versa. The local charge density and the electric field intensity are related by

$$E=\frac{\sigma}{\varepsilon_0}$$

$\Rightarrow \quad \sigma = E\varepsilon_0$

(a) At the point on the surface where the radius of curvature is the greatest, we have

$\sigma=\varepsilon_0 E_{min}=(8.85\times10^{-12}\,C^2Nm^{-2})(5.6\times10^4\,NC^{-1})$

$\Rightarrow \quad \sigma = 496\ nCm^{-2}$

(b) At the point on the surface where the radius of curvature is the smallest, we have

$\sigma=\varepsilon_0 E_{max}=(8.85\times10^{-12}\,C^2Nm^{-2})(11.2\times10^4\,NC^{-1})$

$\Rightarrow \quad \sigma = 992\ nCm^{-2}$

5. (a) The charge $+q$ at the center induces charge $-q$ on the inner surface of the conductor, where its surface density is given by

$$\sigma_a=\frac{-q}{4\pi a^2}$$

(b) The outer surface carries charge $Q+q$ with density

$$\sigma_b=\frac{Q+q}{4\pi b^2}$$

6. Charge on the outer surface of the sphere is $-Q$

Charge on the inner surface of the shell is $+Q$

Charge on the outer surface of the shell is $+2Q$

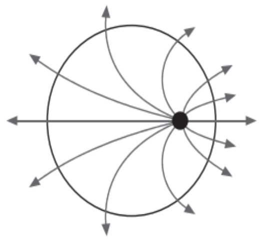
Using Gauss's Law we evaluate the electric field in each region, keeping in mind that the electric field is zero everywhere within conducting materials. Then

Inside the sphere and within the material of the shell, we have $E=0$

Between the sphere and shell the field is directed radially inwards given by $E=\dfrac{Q}{4\pi\varepsilon_0 r^2}$

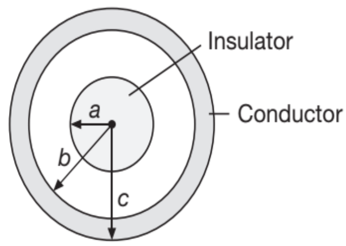
Outside the shell the field is directed radially outwards given by $E=\dfrac{2Q}{4\pi\varepsilon_0 r^2}$

7. An approximate sketch is given at the right. Note that the electric field lines should be perpendicular to the conductor both inside and outside.



8. (a) $\oint \vec{E}\cdot d\vec{A}=E(4\pi r^2)=\dfrac{q_{enc}}{\varepsilon_0}$

For $r<a$, $q_{enc}=\rho\left(\dfrac{4}{3}\pi r^3\right)$



$\Rightarrow \quad E=\dfrac{\rho r}{3\varepsilon_0}$

For $a<r<b$ and $c<r$, $q_{enc}=Q$

$\Rightarrow \quad E=\dfrac{Q}{4\pi r^2\varepsilon_0}$

For $b\le r\le c$, $E=0$ (since $E=0$ inside a conductor).

(b) Let q_1 = induced charge on the inner surface of the hollow sphere. Since $E=0$ inside the conductor, the total charge enclosed by a spherical surface of radius $b\le r\le c$ must be zero.

Therefore, $q_1+Q=0$

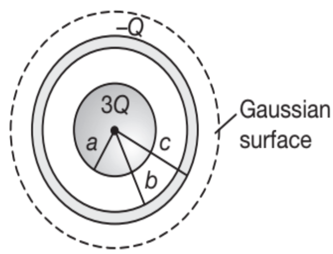
$\Rightarrow \quad \sigma_1=\dfrac{q_1}{4\pi b^2}=\dfrac{-Q}{4\pi b^2}$

Let q_2 = induced charge on the outside surface of the hollow sphere. since the hollow sphere is uncharged, we require

$q_1+q_2=0$

$\Rightarrow \quad \sigma_2=\dfrac{q_1}{4\pi c^2}=\dfrac{Q}{4\pi c^2}$

9. (a) $q_{enc}=+3Q-Q=+2Q$



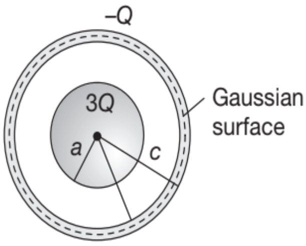
(b) The charge distribution is spherically symmetric and $q_{enc} > 0$. Thus, the field is directed radially outward.

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_{enc}}{r^2}\right) = \frac{2Q}{4\pi\varepsilon_0 r^2} \text{ for } r \geq c.$$

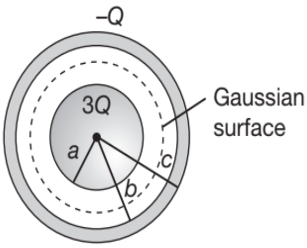
(c) Since all points within this region are located inside conducting material, $E = 0$ for $b < r < c$.

(d) $\phi_E = \int E \cdot dA = 0$

$\Rightarrow \quad q_{enc} = \varepsilon_0 \phi_E = 0$

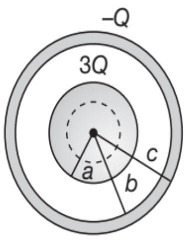


(e) $q_{enc} = +3Q$



(f) $E = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_{enc}}{r^2}\right) = \frac{3Q}{4\pi\varepsilon_0 r^2}$ (radially outward) for $a \leq r < b$

(g) $q_{enc} = \rho V = \left(\frac{+3Q}{\frac{4}{3}\pi a^3}\right)\left(\frac{4}{3}\pi r^3\right) = +3Q\left(\frac{r^3}{a^3}\right)$



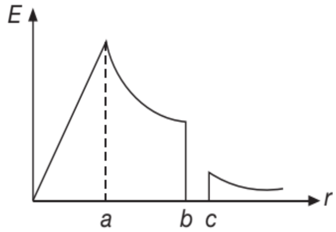
(h) $E = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_{enc}}{r^2}\right) = \frac{1}{4\pi\varepsilon_0 r^2}\left(+3Q\frac{r^3}{a^3}\right) = \left(\frac{3Q}{4\pi\varepsilon_0 a^3}\right) r$

(radially outwards) for $0 \leq r \leq a$

(i) From part (c), $E = 0$ for $b < r < c$. Thus, for a spherical Gaussian surface with $b < r < c$, we have $q_{enc} = +3Q + q_{inner} = 0$ where q_{inner} is the charge on the inner surface of the conducting shell. This yields $q_{inner} = -3Q$

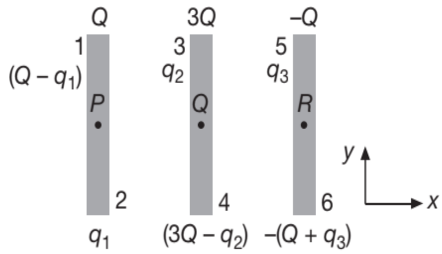
(j) Since the total charge on the conducting shell is $q_{net} = q_{outer} + q_{inner} = -Q$, we have $q_{outer} = -Q - q_{inner} = -Q - (-3Q) = +2Q$

(k) This is shown in the figure.



Test Your Concepts-XII (Based on Charge Distribution)

1. We assume that the charges on surfaces 2, 3 and 5 are q_1, q_2 and q_3 in equilibrium. Following conservation of charge, we see that surfaces 1, 4 and 6 have charges $-Q - q_1$, $3Q - q_2$ and $Q - q_3$ respectively. The electric field inside a metallic plate is zero and hence the fields at points P, Q and R are zero.



Resultant field at P: Considering field to the right as positive, we get

$E_P = 0$

$$\frac{1}{2\varepsilon_0 A}[(Q - q_1) - q_1 - q_2 - (3Q - q_2) - q_3 + (Q + q_3)] = 0$$

$\Rightarrow \quad -Q - 2q_1 = 0$

$\Rightarrow \quad q_1 = -\frac{Q}{2}$

Resultant field at Q is $E_Q = 0$

$$\frac{1}{2\varepsilon_0 A}[(Q - q_1) + q_1 + q_2 - (3Q - q_2) - q_3 + (Q + q_3)] = 0$$

$\Rightarrow \quad -Q + 2q_2 = 0$

$\Rightarrow \quad q_2 = +\frac{Q}{2}$

Resultant field at R:

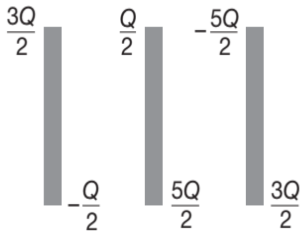
$E_R = 0$

$$\Rightarrow \quad \frac{1}{2\varepsilon_0 A}[(Q - q_1) + q_1 + q_2 + (3Q - q_2) + q_3 + (Q + q_3)] = 0$$

$\Rightarrow \quad 5Q + 2q_3 = 0$

$\Rightarrow \quad q_3 = -\dfrac{5Q}{2}$

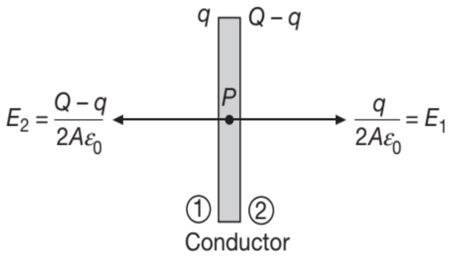
Thus the final charge distribution on all the faces is:



Also note that the faces opposite to each other have opposite charges equal in magnitude.

2. Let a charge q appears on left side of sheet, so a charge $(Q-q)$ will appear on the right side of sheet. Since field at a point P (say) inside the sheet is zero, so

$E_P = 0$



$$\Rightarrow \quad \frac{q}{2A\varepsilon_o} - \frac{Q-q}{2A\varepsilon_o} = 0$$

$$\Rightarrow \quad \frac{2q}{2A\varepsilon_o} = \frac{Q}{2A\varepsilon_o}$$

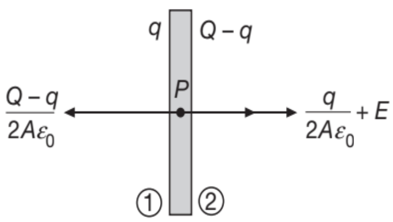
$$\Rightarrow \quad q = \frac{Q}{2} \text{ and } (Q-q) = \frac{Q}{2}$$

So charge is equally distributed on both sides.

3. Let charge on left side of plate be q, then $(Q-q)$ is charge on right side of plate.

Since inside the plate, at point P, we have

$E_P = 0$



$$\Rightarrow \quad \frac{q}{2A\varepsilon_0} + E = \frac{Q-q}{2A\varepsilon_0}$$

$$\Rightarrow \quad \frac{q}{A\varepsilon_0} = \frac{Q}{2A\varepsilon_0} - E$$

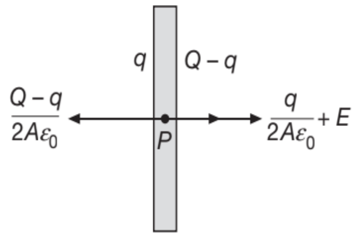
$$\Rightarrow \quad q = \frac{Q}{2} - \varepsilon_o AE \text{ and } (Q-q) = \frac{Q}{2} + \varepsilon_o AE$$

So charge on one side is $\left(\dfrac{Q}{2} - \varepsilon_0 AE\right)$ and other side $\left(\dfrac{Q}{2} + \varepsilon_0 AE\right)$

If sheet is neutral, then $Q = 0$, so charges $-\varepsilon_0 AE$ and $+\varepsilon_0 AE$ appear on left and right side of sheet.

4. Let a charge q appear on left side of plate, so a charge $(Q-q)$ appears on right side of plate.

$E_P = 0$



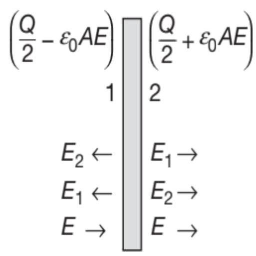
$$\Rightarrow \quad \frac{q}{2A\varepsilon_0} + E = \frac{Q-q}{2A\varepsilon_0}$$

$$\Rightarrow \quad \frac{q}{A\varepsilon_0} = \frac{Q}{2A\varepsilon_0} - E$$

$$\Rightarrow \quad q = \frac{Q}{2} - \varepsilon_0 AE \text{ and } Q - q = \frac{Q}{2} + \varepsilon_0 AE$$

So charge on one side is $\dfrac{Q}{2} - \varepsilon_0 AE$ and other side $\dfrac{Q}{2} + \varepsilon_0 AE$

Electric field due to conducting sheet at a point outside the sheet $= \dfrac{Q_{net}}{2A\varepsilon_0} = \dfrac{Q}{2A\varepsilon_0}$ and having direction away from plate



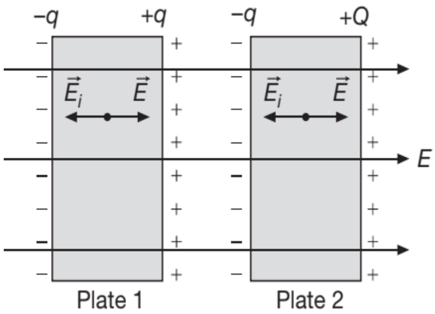
If E_1 and E_2 be electric field due to charges on left and right side of the plate respectively, then

$$E_{\text{left}} = E - E_1 - E_2 = E - \frac{\frac{Q}{2} - \varepsilon_0 AE + \frac{Q}{2} + \varepsilon_0 AE}{2A\varepsilon_0}$$

$$\Rightarrow \quad E_{\text{left}} = E - \frac{Q}{2A\varepsilon_0} \qquad \text{(towards right)}$$

and $E_{\text{right}} = E + E_1 + E_2 = E + \frac{Q}{2A\varepsilon_0}$ (towards right)

5. Since plates are conducting so net electric field inside these plates should be zero. So, electric field due to induced charges (on the surface of the plate) must balance the outside electric field. The magnified view of plates 1 and 2 is shown below:



Let $\vec{E}_i$ be the induced electric field, then

for both plates, inside than $\vec{E}_i + \vec{E} = 0$

$$\Rightarrow \quad \vec{E}_i = -\vec{E} \qquad \text{...(1)}$$

Let charge induced on surfaces be $+q$ and $-q$, then

$$\left|\vec{E}_i\right| = \frac{Q}{A\varepsilon_0}$$

Using equation (1), we get

$$\frac{q}{A\varepsilon_0} = E$$

$$\Rightarrow \quad q = AE\varepsilon_0$$

6. Electric field near a large metallic plate is given by $E = \frac{\sigma}{\varepsilon_o}$. In between the plates the two fields will be in opposite direction. Hence

$$E_{\text{net}} = \frac{\sigma_1 - \sigma_2}{\varepsilon_o} = E_o \text{ (say)}$$

Now, $W = (q)(\text{potential difference}) = q(E_o a \cos 45^\circ)$

$$\Rightarrow \quad W = q\left(\frac{\sigma_1 - \sigma_2}{\varepsilon_o}\right)\left(\frac{a}{\sqrt{2}}\right) = \frac{(\sigma_1 - \sigma_2)qa}{\sqrt{2}\,\varepsilon_o}$$

Test Your Concepts-XIII (Based on Concept of Self and Interaction Energy)

1. The initial self potential energy of the spherical shell is

$$U_i = \frac{q^2}{8\pi\varepsilon_0 R_1}$$

The final self potential energy of the spherical shell is

$$U_f = \frac{q^2}{8\pi\varepsilon_0 R_2}$$

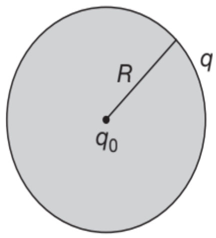
$$\Rightarrow \quad \Delta U = U_f - U_i = \frac{q^2}{8\pi\varepsilon_0}\left[\frac{1}{R_2} - \frac{1}{R_1}\right]$$

Work done by electrical forces (conservative forces) is given by

$$W = -\Delta U = \frac{q^2}{8\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

2. The electrical potential energy of the system is :

$$U = \begin{pmatrix}\text{Self potential}\\ \text{energy of shell}\end{pmatrix} + \begin{pmatrix}\text{Interaction energy of}\\ \text{shell and point charge}\end{pmatrix}$$



$$\Rightarrow \quad U = \frac{q^2}{8\pi\varepsilon_0 R} + \frac{qq_0}{4\pi\varepsilon_0 R}$$

Initial potential energy of system is

$$U_i = \frac{q^2}{8\pi\varepsilon_0 R_1} + \frac{qq_0}{4\pi\varepsilon_0 R_1}$$

Final potential energy of system is

$$U_f = \frac{q^2}{8\pi\varepsilon_0 R_2} + \frac{qq_0}{4\pi\varepsilon_0 R_2}$$

Since, work done by a conservative force (electrostatic force) is actually equal to the decrease in electrostatic potential energy. So,

$$W = -\Delta U = -(U_f - U_i) = U_i - U_f \qquad \text{...(1)}$$

On substituting the values of U_i and U_f in equation (1), we get

$$W = \frac{q\left(q_0 + \frac{q}{2}\right)}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

3. Electrical field is conservative in nature. For a conservative force,

$$F = \frac{-\partial U}{\partial r}$$

Since, $U = \frac{q^2}{8\pi\varepsilon_0 r}$

$$\Rightarrow \quad F = -\frac{\partial U}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{q^2}{8\pi\varepsilon_0 r}\right) \qquad \left\{\because F = -\frac{\partial U}{\partial r}\right\}$$

$$\Rightarrow \quad F = -\frac{q}{8\pi\varepsilon_0}\frac{\partial}{\partial r}\left(r^{-1}\right) = -\frac{q^2}{8\pi\varepsilon_0}\left(-\frac{1}{r^2}\right) = \frac{q^2}{8\pi\varepsilon_0 r^2}$$

Force per unit area is

$$P = \frac{F}{4\pi r^2} = \frac{q^2}{8\pi\varepsilon_0 r^2 \times 4\pi r^2}$$

Since $q = \sigma A = \sigma\left(4\pi r^2\right)$

$$\Rightarrow \quad P = \frac{\sigma^2\left(4\pi r^2\right)^2}{8\pi\varepsilon_0 r^2 \times 4\pi r^2} = \frac{\sigma^2}{2\varepsilon_0}$$

4. $W = \int_0^Q V dq$, where $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

$$\Rightarrow \quad W = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2R}$$

Single Correct Choice Type Questions

1. By Law of Conservation of Moments about O,

$$\left(\begin{array}{c}\text{Total Clockwise}\\ \text{(CW) Moments}\end{array}\right) = \left(\begin{array}{c}\text{Total Counter}\\ \text{Clockwise (CCW)}\\ \text{Moments}\end{array}\right)$$

$$\Rightarrow \quad \frac{qQ}{4\pi\varepsilon_0 h^2}\left(\frac{L}{2}\right) + W\left(x - \frac{L}{2}\right) = \frac{2qQ}{4\pi\varepsilon_0 h^2}\left(\frac{L}{2}\right)$$

$$\Rightarrow \quad W\left(x - \frac{L}{2}\right) = \frac{QqL}{8\pi\varepsilon_0 h^2} \quad \Rightarrow \quad x = \frac{QqL + 4\pi\varepsilon_0 h^2 LW}{8\pi\varepsilon_0 h^2 W}$$

Hence, the correct answer is (D).

3. The transfer of electrons from one body to the other produces charge on the body

Hence,

$$\left(\begin{array}{c}\text{number of electrons}\\ \text{given by one body}\end{array}\right) = \left(\begin{array}{c}\text{number of electrons}\\ \text{obtained by the other}\end{array}\right)$$

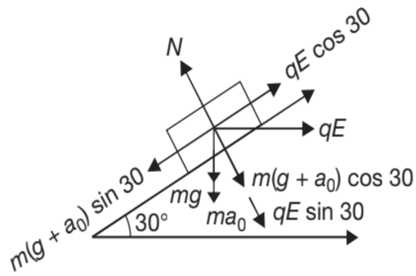
So, mass of negatively charged body slightly increases while mass of positively charged body slightly increases However, the total mass of the system remains the same.

Hence, the correct answer is (D).

4. Net acc of block down the incline is

$$a = \frac{\left(g + a_0\right)}{2} - \frac{\sqrt{3}qE}{2m}$$

$$\Rightarrow \quad a = \frac{1}{2}\left(g + a_0 - \frac{\sqrt{3}qE}{m}\right)$$



Now $\frac{h}{\sin 30} = \frac{1}{2}at^2$

$$\Rightarrow \quad 2h = \frac{1}{4}\left[g + a_0 - \sqrt{3}\left(\frac{qE}{m}\right)\right]t^2$$

$$\Rightarrow \quad t = \sqrt{\frac{8h}{g + a_0 - \sqrt{3}\left(\frac{qE}{m}\right)}}$$

$$\Rightarrow \quad t = 2\sqrt{\frac{2h}{g + a_0 - \sqrt{3}\left(\frac{qE}{m}\right)}}$$

Hence, the correct answer is (C).

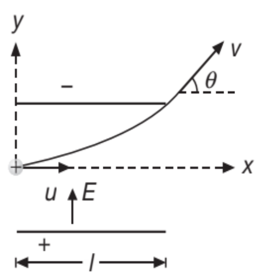
5. $a = \frac{eE}{m}$

Since $\ell = \frac{1}{2}at^2 = \frac{1}{2}a'T^2$

$$\Rightarrow \quad \frac{T}{t} = \sqrt{\frac{a}{a'}} = \sqrt{\frac{qE/m}{qE/M}} = \sqrt{\frac{M}{m}}$$

Hence, the correct answer is (C).

6. $\frac{1}{2}mu^2 = eV$



$$\Rightarrow \quad u = u_x = \sqrt{\frac{2eV}{m}}$$

$$t = \frac{\ell}{u_x} = \ell\sqrt{\frac{m}{2eV}}$$

Since, $a_y = \frac{eE}{m}$

$$\Rightarrow \frac{dv_y}{dt} = \frac{eat}{m} \qquad \{\because E = at\}$$

$$\Rightarrow \int_0^{v_y} dv_y = \frac{ea}{m}\int_0^t t\,dt$$

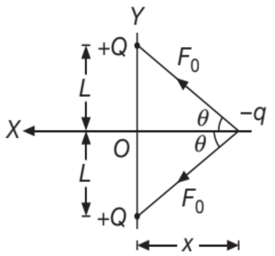
$$\Rightarrow v_y = \frac{ea}{2m}t^2 = \left(\frac{ea}{2m}\right)\left(\frac{\ell^2 m}{2eV}\right) = \frac{a\ell^2}{4V}$$

Since $\tan\theta = \frac{v_y}{v_x}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left\{\frac{a\ell^2}{4V}\sqrt{\frac{m}{2eV}}\right\}$$

Hence, the correct answer is (C).

7. The net force F on the particle towards O is the cause of acceleration a.



$$F = 2F_0\cos\theta$$

$$F = \frac{2}{4\pi\varepsilon_0}\frac{qQ}{(x^2+\ell^2)}\frac{x}{\sqrt{x^2+\ell^2}} = \frac{Qq}{2\pi\varepsilon_0}\frac{x}{(x^2+\ell^2)^{3/2}}$$

We observe that F is zero for large values of x and also for $x = 0$.

So, a must increase to a maximum and fall to zero at O

To the right of O, the net force is to the left while motion is to the right. Thus, the direction of a is opposite to the particle's direction of motion and is taken as negative as shown.

Hence, the correct answer is (D).

8. $-\int_{\ell=\infty}^{\ell=0}\vec{E}\cdot d\vec{\ell}$ is the line integral of electric field which is also called electrostatic potential. Since, charge is distributed non-uniformly, so $E \neq 0$ at the centre.

$$\Rightarrow -\int_{\ell=\infty}^{\ell=0}\vec{E}\cdot d\vec{\ell} = \frac{q}{4\pi\varepsilon_0 R} = 2$$

Hence, the correct answer is (A).

9. If charges were placed at all the corners, the field at the centre would be zero. Hence, the field at the centre due to any one charge is equal (and opposite) to the field due to all the other $(n-1)$ charges.

Hence, the correct answer is (A).

10. Since the potential function is not defined for an infinite conducting sheet, hence to solve this problem we either calculate potential difference or use force equations Electric field due to an infinite dielectric sheet

$$E_1 = \frac{\sigma}{2\varepsilon_0}$$

Electric field at the axis of a disc of radius R is

$$E_2 = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]$$

The resultant electric field is given by

$$E = E_1 - E_2 = \frac{\sigma}{2\varepsilon_0}\frac{x}{\sqrt{x^2+R^2}}$$

Force on an electron is

$$F = -eE \quad \Rightarrow \quad F = -\frac{\sigma ex}{2\varepsilon_0\sqrt{x^2+R^2}}$$

$$\Rightarrow mv\frac{dv}{dx} = -\frac{\sigma ex}{2\varepsilon_0\sqrt{x^2+R^2}}$$

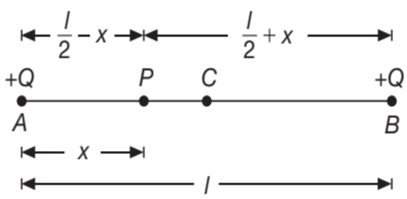
$$\Rightarrow m\int_0^v v\,dv = -\frac{\sigma e}{2\varepsilon_0}\int_{\sqrt{3}R}^0 \frac{x}{\sqrt{x^2+R^2}}dx$$

$$\Rightarrow \frac{mv^2}{2} = -\frac{\sigma e}{2\varepsilon_0}\left(\sqrt{x^2+R^2}\right)\Big|_{\sqrt{3}R}^0$$

$$\Rightarrow v = \sqrt{\frac{\sigma eR}{m\varepsilon_0}}$$

Hence, the correct answer is (B).

11. $E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\left(\frac{\ell}{2}-x\right)^2} - \frac{1}{\left(\frac{\ell}{2}+x\right)^2}\right]$



The magnitude of E will increase sharply for $x \to 0$ and $x \to \ell$.

Hence, the correct answer is (B).

12. $V = \frac{q}{4\pi\varepsilon_0 x_0}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} ....\right)$

$\Rightarrow \quad V = \frac{q}{4\pi\varepsilon_0 x_0}\log_e 2$

Hence, the correct answer is (D).

13. Charge density $\rho = \frac{Q}{\frac{4}{3}\pi\left(R^3 - \left(\frac{R}{2}\right)^3\right)} = \frac{6Q}{7\pi R^3}$

Using the Superposition Principle,

$$V = \frac{\rho\left(\frac{4}{3}\pi R^3\right)}{4\pi\varepsilon_0 x} - \frac{\rho\left(\frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right)}{4\pi\varepsilon_0\left(x - \frac{R}{2}\right)} = \frac{\rho R^3}{3\varepsilon_0}\frac{(7x - 4R)}{4x(2x - R)}$$

At $x = 2R$, we get $V = \frac{5}{36}\left(\frac{\rho R^2}{\varepsilon_0}\right)$

Similarly, $E = \frac{\rho\left(\frac{4}{3}\pi R^3\right)}{4\pi\varepsilon_0 x^2} - \frac{\rho\left(\frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right)}{4\pi\varepsilon_0\left(x - \frac{R}{2}\right)^2}$

$$E = \frac{\rho R^3}{3\varepsilon_0}\left(\frac{1}{x^2} - \frac{1}{2(2x - R)^2}\right)$$

So, at $x = 2R$, we get $E = \frac{2\rho R}{27\varepsilon_0} = \frac{2}{27}\left(\frac{\rho R}{\varepsilon_0}\right)$

Hence, the correct answer is (B).

14. $d = \frac{u^2\sin^2\theta}{2a}$ and $\ell = \frac{2u^2\sin\theta\cos\theta}{a}$

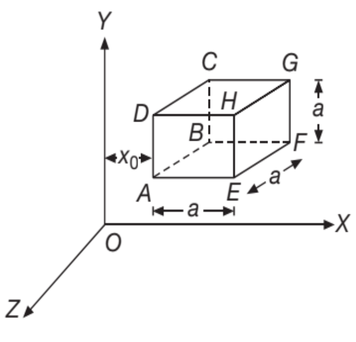
$\Rightarrow \quad \frac{d}{\ell} = \frac{\tan\theta}{4}$

$\Rightarrow \quad \tan\theta = \frac{4d}{\ell}$

Hence, the correct answer is (B).

15. The field at the face $ABCD = \alpha x_0\hat{i}$

So, flux over the face $ABCD = -(\alpha x_0)a^2$



The negative sign arises as the field is directed into the cube

The field at the face $EFGH = \alpha(x_0 + a)\hat{i}$

So, flux over the face $EFGH = \alpha(x_0 + a)a^2$

The flux over the other four faces is zero as the field is parallel to the surfaces

So, total flux over the cube $= \alpha a^3 = \frac{q}{\varepsilon_0}$, where q is the total charge inside the cube

$\Rightarrow \quad q = \varepsilon_0 \alpha a^3$

Hence, the correct answer is (A).

16. Since, $\vec{E} = -\vec{\nabla}V \quad \Rightarrow \quad \vec{E} = -k(y\hat{i} + x\hat{j})$

Further,

$$dW = q_0\vec{E}\cdot d\vec{r}$$

$\Rightarrow \quad dW = q_0\left[-k(y\hat{i} + x\hat{j})\cdot(\hat{i}dx + \hat{j}dy)\right]$

$\Rightarrow \quad dW = -kq_0(ydx + xdy) = -kq_0 d(xy)$

$\Rightarrow \quad W = \int dW = -kq_0\int_{(0,0)}^{(a,a)} d(xy)$

$\Rightarrow \quad W = -kq_0 a^2$

Hence, the correct answer is (A).

17. Since, $V = \frac{Q}{8\pi\varepsilon_0 R^3}(3R^2 - r^2)$

So, at $r = 0$, potential at the centre is

$$V_C = \frac{3Q}{8\pi\varepsilon_0 R}$$

As per the question we require a point where

$$V = \frac{V_C}{2} = \frac{3Q}{16\pi\varepsilon_0 R}$$

This point cannot lie inside the sphere where $V \geq \frac{Q}{4\pi\varepsilon_0 R}$

Let the point lie outside the sphere, at a distance $r + R$ from the centre. Then,

$$V = \frac{Q}{4\pi\varepsilon_0(R + r)} = \frac{3Q}{16\pi\varepsilon_0 R}$$

$\Rightarrow \quad R + r = \frac{4}{3}R$

$\Rightarrow \quad r = \frac{4R}{3} - R = \frac{R}{3}$

Distance from the surface is $\frac{R}{3}$

Hence, the correct answer is (D).

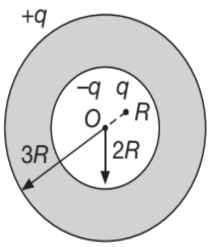
18. $V_f - V_i = \frac{\lambda}{2\pi\varepsilon_0}\log\left(\frac{r_f}{r_i}\right)$

$\Rightarrow \quad V = \frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{\sqrt{5^2+12^2}}{\sqrt{1^2+2^2+4^2}}\right)$

$\Rightarrow \quad V = \frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{13}{5}\right)$

Hence, the correct answer is (A).

19. Induced charges will be $-q$ and $+q$ as shown in figure



Potential at centre O is

$$V_O = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R} - \frac{q}{2R} + \frac{q}{3R}\right]$$

$$V_O = \frac{1}{4\pi\varepsilon_0}\left(\frac{5q}{6R}\right) = \frac{5q}{24\pi\varepsilon_0 R}$$

Hence, the correct answer is (D).

20. A negative charge, when free to move will always move from lower to higher potential.

Hence, the correct answer is (D).

21. $V_1 = \frac{Q_1}{4\pi\varepsilon_0 a} + \frac{Q_2}{4\pi\varepsilon_0\sqrt{2}a}$

$$V_2 = \frac{Q_1}{4\pi\varepsilon_0\sqrt{2}a} + \frac{Q_2}{4\pi\varepsilon_0 a}$$

$\Rightarrow \quad W_{1\to2} = q(V_2 - V_1)$

$\Rightarrow \quad W_{1\to2} = \frac{q(Q_1 - Q_2)}{4\pi\varepsilon_0 a}\left(1 - \frac{1}{\sqrt{2}}\right)$

$\Rightarrow \quad W = \frac{q(Q_1 - Q_2)(\sqrt{2}-1)}{4\sqrt{2}\pi\varepsilon_0 a}$

Hence, the correct answer is (B).

22. Charges will be induced as shown

$$V_0 = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r} + \frac{1}{2R} - \frac{1}{R}\right]$$

$$V_0 = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{2R}\right]$$

Hence, the correct answer is (D).

23. For the motion of the electron along Y-axis

$$d = \frac{1}{2}\frac{eE}{m}t^2 \qquad \ldots(1)$$

where E is the electric field due to the charged plate given by

$$E = \frac{\sigma}{\varepsilon_0} \qquad \ldots(2)$$

where σ is the surface density of charge of the plate.

The time taken by the electron is

$$t = \frac{\ell}{u} \qquad \ldots(3)$$

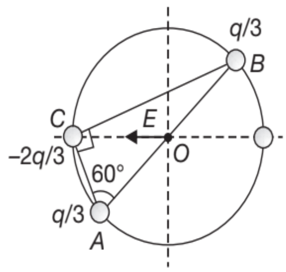
From equations (1), (2) and (3), we obtain

$$d = \frac{1}{2}\frac{e\sigma}{\varepsilon_0 m}\left(\frac{\ell}{u}\right)^2$$

$\Rightarrow \quad \sigma = \frac{2d\varepsilon_0 mu^2}{e\ell^2}$

Hence, the correct answer is (C).

24. Net electric field due to both charges $\frac{q}{3}$, will get cancelled. Electric field due to $\left(-\frac{2q}{3}\right)$ will be directed in –ve axis



$$E = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{2q}{3}\right)}{R^2}$$

$\Rightarrow \quad E = \frac{q}{6\pi\varepsilon_0 R^2}$

P.E. of system $= \frac{1}{4\pi\varepsilon_0}\left(\frac{\left(\frac{q}{3}\right)^2}{2R} + \frac{\frac{q}{3}\left(-\frac{2q}{3}\right)}{2R\sin 60^\circ} + \frac{\frac{q}{3}\left(-\frac{2q}{3}\right)}{2R\cos 60^\circ}\right)$

P.E. of system $\neq 0$

Force between B and C

$$F = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{2q}{3}\right)\left(\frac{q}{3}\right)}{(2R\sin 60^\circ)^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\times 2q^2}{9\times 4\times 3R^2}$$

$$\Rightarrow \quad F=\frac{2q^2}{9\times 3\times 4\pi\varepsilon_0 R^2} \qquad \text{(attractive)}$$

$$\Rightarrow \quad F=\frac{1}{54}\frac{q^2}{\pi\varepsilon_0 R^2}$$

Potential at O is

$$V=\frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q}{3}+\frac{q}{3}-\frac{2q}{3}\right)}{R}=0$$

Hence, the correct answer is (C).

25. Since both the particles exert equal force on each other, so the angle of deflection for both must be the same i.e. $\alpha=\beta$.

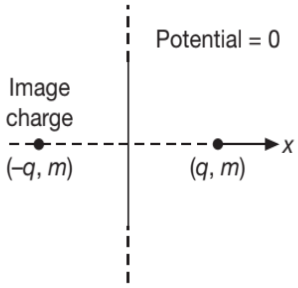
Hence, the correct answer is (C).

27. According to the METHOD OF IMAGES, we place a mirror charge $-q$ of mass m at $x=-d$.

The two charges will move towards each other till they collide i.e., until the charge q reaches the plane. Let at any instant the charge q be at a distance $x(t)$ from the plane. By Law of Conservation of Energy we have

$$\frac{-q^2}{(d+d)} = \frac{-q^2}{2x(t)}+2\left[\frac{1}{2}m\{\dot{x}(t)\}^2\right]$$

$$\left(\begin{array}{c}\text{Energy at}\\ t=0\text{ in}\\ \text{c.g.s. system}\end{array}\right) = \left(\begin{array}{c}\text{Energy at any}\\ \text{instant } t \text{ in}\\ \text{c.g.s system}\end{array}\right)$$



$$\Rightarrow \quad \frac{dx}{dt}=\frac{q}{\sqrt{2md}}\sqrt{\frac{d-x}{x}}$$

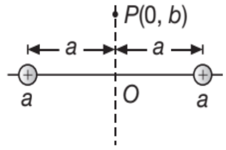
$$\Rightarrow \quad dt=\frac{\sqrt{2md}}{q}\sqrt{\frac{x}{d-x}}dx$$

$$\Rightarrow \quad t=\int dt=\frac{\sqrt{2md}}{q}\int_d^0\sqrt{\frac{x}{d-x}}dx$$

$$\Rightarrow \quad t=\frac{2}{3q}\sqrt{2md^3}$$

Hence, the correct answer is (A).

28.



$$OP=\sqrt{a^2+\left(\sqrt{3}a\right)^2}=2a$$

By Law of Conservation of Energy

$$(U+k)_P=(U+K)_O$$

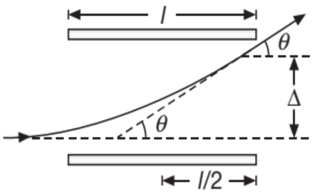
$$\Rightarrow \quad -\frac{2Qq}{4\pi\varepsilon_0(2a)}+0=-\frac{2Qq}{4\pi\varepsilon_0 a}+\frac{1}{2}mv_0^2$$

$$\Rightarrow \quad \frac{1}{2}mv_0^2=\frac{Qq}{4\pi\varepsilon_0 a}$$

$$\Rightarrow \quad v_0=\sqrt{\frac{Qq}{2\pi\varepsilon_0 ma}}$$

Hence, the correct answer is (B).

29. $\tan\theta=\dfrac{\Delta}{\left(\frac{\ell}{2}\right)}=\dfrac{2\Delta}{\ell}$



Hence, the correct answer is (B).

30. In ΔABN $\sin(90-\alpha)=\dfrac{h}{x}$

$$\cos\alpha=\frac{h}{x} \qquad \text{...(1)}$$

In ΔAOB $x=2\ell\sin\left(\dfrac{\theta}{2}\right)$

$$\Rightarrow \quad \sin\left(\frac{\theta}{2}\right)=\frac{x}{2\ell} \qquad \text{...(2)}$$

From (1), we get

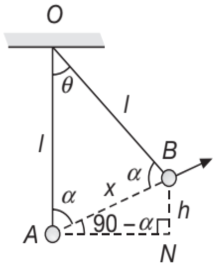
$$\cos\left(90-\frac{\theta}{2}\right)=\frac{h}{x} \qquad \left\{\because \alpha=90-\frac{\theta}{2}\right\}$$

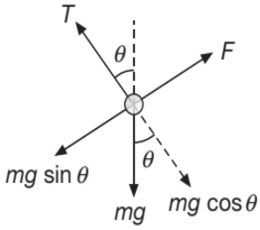
$$\Rightarrow \quad \sin\left(\frac{\theta}{2}\right)=\frac{h}{x} \qquad \text{...(3)}$$

From (2) and (3), we get

$$\Rightarrow \quad \frac{x}{2\ell}=\frac{h}{x}$$

$$\Rightarrow \quad x^2=2h\ell$$





Also, for equilibrium of q

$$T = mg\cos\theta$$

$$F = mg\sin\theta$$

Since θ is small, so

$$F = mg\left(\frac{x}{\ell}\right)$$

$$\Rightarrow \quad \frac{qQ}{4\pi\varepsilon_0 x^2} = mg\left(\frac{x}{\ell}\right)$$

$$\Rightarrow \quad \frac{qQ}{4\pi\varepsilon_0 (2h\ell)} = mg\left(\frac{x}{\ell}\right)$$

$$\Rightarrow \quad \frac{qQ}{4\pi\varepsilon_0 x} = 2mgh$$

$$\Rightarrow \quad W_{\text{ext}} = mgh + 2mgh = 3mgh$$

Hence, the correct answer is (B).

31. According to option (D), the electric field due to P and S and due to Q and T add to zero. While due to U and R will be added up.

Hence, the correct answer is (D).

32. Each of the charged sphere will have potential energy due to its own charge or self energy

$$U_1 = \frac{1}{8\pi\varepsilon_0}\frac{q^2}{r}$$

The mutual potential energy stored in one sphere in the electric field of the other is

$$\frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} = U_2 \quad \Rightarrow \quad U_2 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{R}$$

Therefore the P.E. stored in the system of spheres is

$$U = 2U_1 + U_2 = \frac{q^2}{4\pi\varepsilon_0}\left[\frac{1}{R} + \frac{1}{r}\right]$$

Hence, the correct answer is (A).

33. At the surface of the charged sphere, whether it consists of a single piece or two pieces close together, the electric field strength from Gauss's Law is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

The electric charge per unit surface area is

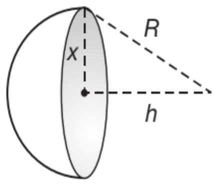
$$\sigma = \frac{Q}{4\pi R^2}$$

This electric field exerts a force $\Delta F = \frac{1}{2}E\Delta Q$ on the charge $\Delta Q = \sigma\Delta A$ which resides on a surface of area ΔA, as illustrated in figure. The reason for the factor of $\frac{1}{2}$ is that the electric field strength is E at the outer surface of the sphere and zero inside and hence its average value comes out to be $\frac{E}{2}$.

The force per unit area exerted by the charges on the pieces of the sphere is therefore

$$\frac{\Delta F}{\Delta A} = \frac{Q^2}{32\pi^2\varepsilon_0 R^4} = p$$

The required force can be considered to be analogous to the force with which a liquid having a pressure p would push apart the two pieces of the sphere and this force is also the product of p and the cross-sectional area of the intersection of the plane and sphere.



So, $F = pa = p\left(\pi x^2\right)$

$$\Rightarrow \quad F = p\pi\left(R^2 - h^2\right)$$

Hence it follows that the two parts of the sphere can be held together by a force

$$F = \frac{Q^2}{32\pi\varepsilon_0 R^4}\left(R^2 - h^2\right)$$

Hence, the correct answer is (D).

34. At any point over the spherical Gaussian surface, net electric field is the vector sum of electric fields due to $+q_1, -q_1$ and q_2.

Don't confuse with the electric flux which is zero (net) passing over the Gaussian surface as the net charge enclosing the surface is zero.

Hence, the correct answer is (C).

35. All the three plates will produce electric field at P along negative z-axis. Hence ,

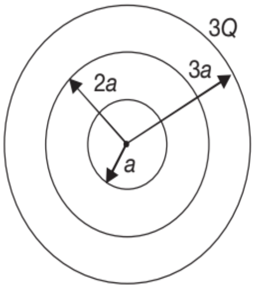
$$\vec{E}_P=\left[\frac{\sigma}{2\varepsilon_o}+\frac{2\sigma}{2\varepsilon_o}+\frac{\sigma}{2\varepsilon_o}\right](-\hat{k})=-\frac{2\sigma}{\varepsilon_o}\hat{k}$$

Hence, the correct answer is (B).

36. Since $V=\sum V_i=\frac{1}{4\pi\varepsilon_0}\sum\left(\frac{q_i}{r_i}\right)$

$$\Rightarrow\quad V_{N^{th}}=\underset{\text{due to 1}}{V_{\text{at }N^{th}}}+\underset{\text{due to 2}}{V_{\text{at }N^{th}}}+\ldots\ldots+\underset{\text{due to itself}}{V_{\text{at }N^{th}}}$$

$$\Rightarrow\quad V_{N^{th}}=\frac{Q}{4\pi\varepsilon_0(Na)}+\frac{2Q}{4\pi\varepsilon_0(Na)}+\ldots\ldots+\frac{NQ}{4\pi\varepsilon_0(Na)}$$



$$\Rightarrow\quad V_{N^{th}}=\frac{Q}{4\pi\varepsilon_0(Na)}(1+2+3+\ldots\ldots+N)$$

$$\Rightarrow\quad V_{N^{th}}=\frac{Q}{4\pi\varepsilon_0(Na)}\frac{N(N+1)}{2}$$

$$\left\{\because\ 1+2+3+\ldots..+N=\frac{N(N+1)}{2}\right\}$$

$$\Rightarrow\quad V_{N^{th}}=\frac{Q(N+1)}{8\pi\varepsilon_0 a}$$

Hence, the correct answer is (A).

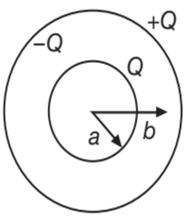
37. Charge on smaller sphere

$$q=C\left(\frac{V}{2}\right)=\frac{4\pi\varepsilon_0 RV}{2}$$

$$\text{Potential difference}=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{2R}\right)=\left(\frac{VR}{2}\right)\left(\frac{1}{2R}\right)=\frac{V}{4}$$

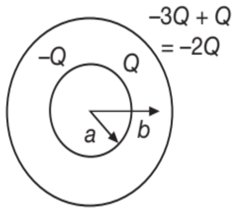
Hence, the correct answer is (B).

38.



$$V=\underset{\text{sphere}}{V_{\text{surface}}}-\underset{\text{shell}}{V_{\text{outer}}}$$

$$\Rightarrow\quad V=\frac{Q}{4\pi\varepsilon_0 a}-\frac{Q}{4\pi\varepsilon_0 b}$$



$$\text{Potential difference}=V'=\underset{\text{sphere}}{V_{\text{surface}}}-\underset{\text{shell}}{V_{\text{outer}}}$$

$$\Rightarrow\quad V'=\left(\frac{Q}{4\pi\varepsilon_0 a}+\frac{-2Q}{4\pi\varepsilon_0 b}\right)-\left(\frac{Q}{4\pi\varepsilon_0 b}+\frac{-2Q}{4\pi\varepsilon_0 b}\right)$$

$$\Rightarrow\quad V'=\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)=V$$

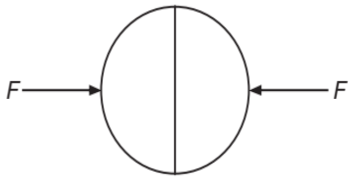
Hence, the correct answer is (A).

39. From Gauss's Law, we have flux equal to

$$\phi=\frac{\Sigma Q_{\text{enc}}}{\varepsilon_0}=\frac{\left(\frac{8C}{4}\right)-7C+\left(\frac{6C}{2}\right)}{\varepsilon_0}=-\frac{2C}{\varepsilon_0}$$

Hence, the correct answer is (A).

40.



Electrostatics repulsive force :

$$F_{ele}=\left(\frac{\sigma^2}{2\varepsilon_0}\right)\pi R^2$$

$$F=F_{ele}=\frac{\sigma^2\pi R^2}{2\varepsilon_0}$$

Hence, the correct answer is (A).

41. In equilibrium, $mg=qE$

In absence of electric field, $mg=6\pi\eta rv$

$$\Rightarrow\quad qE=6\pi\eta rv$$

$$m=\frac{4}{3}\pi Rr^3 d=\frac{qE}{g}$$

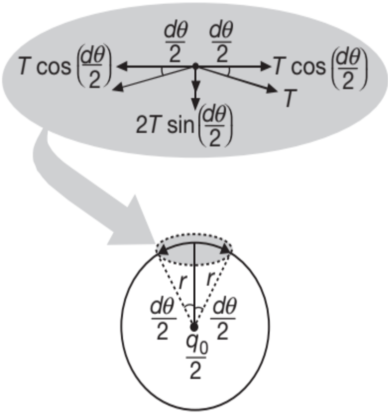
$$\frac{4}{3}\pi\left(\frac{qE}{6\pi\eta v}\right)^3 d=\frac{qE}{g}$$

After substituting value we get,

$$q=8\times10^{-19}\text{ C}$$

Hence, the correct answer is (D).

42. Let us consider an element of arc length dl having a charge dq. Then $dq = \left(\frac{q_0}{2\pi r}\right)dl$.



If dF is the force of repulsion between the element and the charge $\frac{q_0}{2}$ at the centre, then

$$dF = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_0}{2}\right)\frac{dq}{r^2} \qquad \text{(radially outwards)}$$

For equilibrium to be there

$$dF = 2T\sin\left(\frac{d\theta}{2}\right)$$

$$\Rightarrow \frac{1}{8\pi\varepsilon_0}\frac{q_0 dq}{r^2} \simeq 2T\left(\frac{d\theta}{2}\right)$$

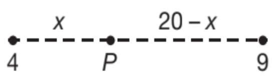
$$\left\{\because \text{ for small angle, } \sin\left(\frac{d\theta}{2}\right) \simeq \frac{d\theta}{2}\right\}$$

$$\Rightarrow \frac{1}{8\pi\varepsilon_0}\frac{q_0\left(\frac{q_0}{2\pi r}\right)dl}{r^2} = T\left(\frac{dl}{r}\right)$$

$$\Rightarrow T = \frac{q_0^2}{16\pi^2\varepsilon_0 r^2}$$

Hence, the correct answer is (B).

43.



$$E_P = 0$$

$$\Rightarrow \frac{4}{x^2} = \frac{9}{(20-x)^2}$$

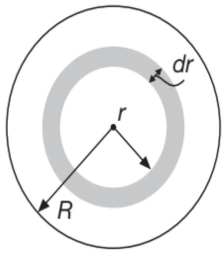
$$\Rightarrow \frac{20-x}{x} = \frac{3}{2}$$

$$\Rightarrow 40 - 2x = 3x$$

$$\Rightarrow x = 8 \text{ cm}$$

Hence, the correct answer is (A).

44. Consider an infinitesimal concentric spherical element of thickness dr, radius r. If dq is the charge on this element, then



$$dq = \rho dV \qquad \{\text{where } dV = 4\pi r^2 dr\}$$

$$\Rightarrow dq = \left(\rho_0 r^3\right)\left(4\pi r^2 dr\right)$$

$$\Rightarrow dq = 4\pi\rho_0 r^5 dr$$

$$\Rightarrow Q = \int_0^Q dq = 4\pi\rho_0\int_0^R r^5 dr$$

$$\Rightarrow Q = 4\pi\rho_0\left[\frac{r^6}{6}\Big|_0^R\right]$$

$$\Rightarrow Q = \frac{2}{3}\pi\rho_0 R^6$$

Hence, the correct answer is (D).

45. $qE = Mg$

$$q\frac{\sigma}{\varepsilon_0} = Mg$$

$$q\frac{Q}{A\varepsilon_0} = Mg$$

$$\Rightarrow q = \frac{MgA\varepsilon_0}{Q}$$

Hence, the correct answer is (B).

46. $F_{\text{air}} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$

Let us now replace the slab of thickness t with an equivalent amount of air such that two charges placed in air at separation x will experience the same force if the separation between them completely has a slab of dielectric constant K between them. In such a case

$$\frac{q_1 q_2}{4\pi\varepsilon_0 x^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 K t^2}$$

$\Rightarrow$ Equivalent separation in air $= x = t\sqrt{K}$

So, now the effective separation between the charges is

$$r' = r - t + x = r - t + t\sqrt{K}$$

$\Rightarrow \quad r' = r - t + t\sqrt{K}$

So, $F' = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r'^2}$

$\Rightarrow \quad F' = \dfrac{q_1 q_2}{4\pi\varepsilon_0 \left(r - t + t\sqrt{K}\right)^2}$

Now, as per the problem we have $t = \dfrac{r}{3}$

$\Rightarrow \quad F' = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{\left(r - \dfrac{r}{3} + \dfrac{r}{3}\sqrt{K}\right)^2}$

$\Rightarrow \quad F' = \dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{9 q_1 q_2}{\left(2+\sqrt{K}\right)^2 r^2}\right]$

$\Rightarrow \quad \dfrac{F'}{F_{\text{air}}} = \dfrac{9}{\left(1+\sqrt{K}\right)^2} = \dfrac{9}{25}$

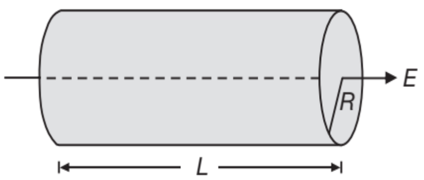
$\Rightarrow \quad 2 + \sqrt{K} = 5$

$\Rightarrow \quad \sqrt{K} = 3$

$\Rightarrow \quad K = 9$

Hence, the correct answer is (C).

47.

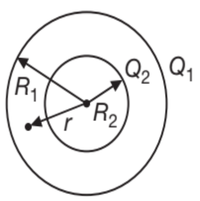


Flux due to curved surface = 0

Since field lines enter at one end and exit at other, so total flux equals zero.

Hence, the correct answer is (D).

48.



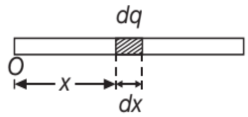
$$V_r = \frac{Q_2}{4\pi\varepsilon_0 r} + \frac{Q_1}{4\pi\varepsilon_0 R_1}$$

$$V_r = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_2}{r} + \frac{Q_1}{R_1}\right)$$

Hence, the correct answer is (C).

49. Consider an element of length dx, carrying a charge dq at a distance x from O. Then, by definition

$$dq = \lambda dx$$



$\Rightarrow \quad dq = \lambda_0\left(\dfrac{x^3}{L}\right)dx$

$\Rightarrow \quad \displaystyle\int_O^Q dq = \frac{\lambda_0}{L}\int_O^L x^3 dx$

$\Rightarrow \quad Q = \dfrac{\lambda_0}{L}\left(\dfrac{L^4}{4}\right)$

$\Rightarrow \quad Q = \dfrac{\lambda_0 L^3}{4}$

Hence, the correct answer is (C).

51. $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

$\Rightarrow \quad E \propto \dfrac{1}{r}$

Hence, the correct answer is (C).

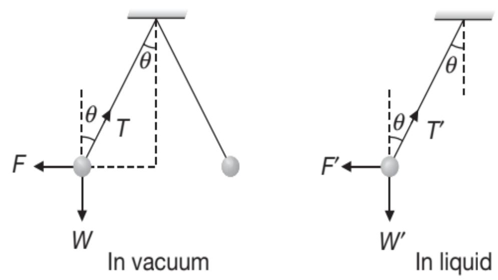
52. Each ball is in equilibrium under the following three forces:

(i) tension (T)

(ii) electric force (F)

(iii) weight (W)

So, Lami's theorem can be applied.



$T\cos\theta = W$ ...(1)

$T\sin\theta = F$ ...(2)

$T'\cos\theta = W'$ ...(3)

$T'\sin\theta = F'$ ...(4)

So, from (1), (2), (3) and (4), we get

$$\frac{W}{F} = \frac{W'}{F'}$$

where $W' = W - U = V\rho g - V\sigma g$ and $F' = \dfrac{F}{K}$

$\Rightarrow \quad \dfrac{\cancel{V}\rho \cancel{g}}{\cancel{F}} = \dfrac{\cancel{V}\rho\cancel{g} - \cancel{V}\sigma\cancel{g}}{\dfrac{\cancel{F}}{K}}$

$\Rightarrow \quad K = \dfrac{\rho}{\rho - \sigma}$

Hence, the correct answer is (D).

53. $V = \dfrac{Q}{4\pi\varepsilon_0 R} - \dfrac{Q}{4\pi\varepsilon_0(3R)}$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{3R}\right)$$

$$\Rightarrow \quad Q = (4\pi\varepsilon_0)\left(\frac{3VR}{2}\right)$$

$$\Rightarrow \quad E = \frac{Q}{4\pi\varepsilon_0(3R)^2}$$

$$\Rightarrow \quad E = \frac{4\pi\varepsilon_0\left(\frac{3VR}{2}\right)}{4\pi\varepsilon_0(3R)^2}$$

$$\Rightarrow \quad E = \frac{V}{6R}$$

Hence, the correct answer is (D).

55. $\vec{F} = \vec{F}_{21} = \dfrac{q_1 q_2}{4\pi\varepsilon_0|\vec{r}-\vec{r}_0|^3}(\vec{r}-\vec{r}_0)$

where $\vec{r}-\vec{r}_0 = 6\hat{i} - 8\hat{j}$

$$\Rightarrow \quad |\vec{r}-\vec{r}_0| = 10 \text{ m}$$

So, $\vec{F} = \dfrac{50\times10^{-6}\times2\times10^{-6}\times9\times10^{9}}{(10)^3}(6\hat{i}-8\hat{j})$

$$\Rightarrow \quad \vec{F} = 9\times10^{-4}(6\hat{i}-8\hat{j}) \text{ N}$$

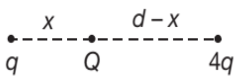
$$\Rightarrow \quad |\vec{F}| = 9\times10^{-4}\sqrt{6^2+(-8)^2}$$

$$\Rightarrow \quad |\vec{F}| = 9\times10^{-3} \text{ N} = 9 \text{ mN}$$

Hence, the correct answer is (B).

56. For equilibrium of Q

$$k\frac{qQ}{x^2} = k\frac{4qQ}{(d-x)^2}$$



$$\Rightarrow \quad \frac{d-x}{x} = 2$$

$$\Rightarrow \quad d - x = 2x$$

$$\Rightarrow \quad x = \frac{d}{3}$$

For equilibrium of q

$$\frac{kQq}{x^2} + \frac{k4q^2}{d^2} = 0$$

$$\Rightarrow \quad \frac{Qq}{(d^2/9)} + \frac{4q^2}{d^2} = 0$$

$$\Rightarrow \quad 9Q + 4q = 0$$

$$\Rightarrow \quad Q = -\frac{4q}{9}$$

Hence, the correct answer is (C).

57. Since $\lambda = \dfrac{Q}{2\pi R}$

and the tension developed in the ring when a charge is placed at its centre is given by

$$T = \frac{q_0\lambda}{4\pi\varepsilon_0 R} = \frac{q_0 Q}{8\pi^2\varepsilon_0 R^2} \text{ (we have done this already)}$$

Further, by Laws of Elasticity, we know that

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\left(\frac{T}{A}\right)}{\left(\frac{\Delta R}{R}\right)}$$

$$\Rightarrow \quad Y = \frac{TR}{A\Delta R}$$

$$\Rightarrow \quad \Delta R = \frac{TR}{AY} = \frac{q_0 Q}{8\pi^2\varepsilon_0 RAY}$$

Substituting $q_0 = 10^{-8}$ C, $Q = \pi$ C, $R = 0.1$ m, $A = 10^{-6}$ m^2, $Y = 2\times10^{-11}$ Nm^{-2}, we get

$$\Delta R = 2.25 \text{ mm}$$

Hence, the correct answer is (B).

58. Same electric field near surfaces gives

$$\frac{Q_1}{4\pi\varepsilon_0 R_1^2} = \frac{Q_2}{4\pi\varepsilon_0 R_2^2}$$

$$\Rightarrow \quad \frac{Q_1}{Q_2} = \frac{R_1^2}{R_2^2}$$

Further

$$\frac{V_1}{V_2} = \frac{Q_1}{Q_2}\frac{R_2}{R_1} = \frac{R_1}{R_2} \qquad \left\{\because V = \frac{Q}{4\pi\varepsilon_0 R}\right\}$$

Hence, the correct answer is (A).

59. Since the spring is metallic so the charge distributes equally on both the blocks. Hence a charge $\dfrac{Q}{2}$ resides on each block, due to which the blocks will repel each other with a force given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{Q}{2}\right)\left(\frac{Q}{2}\right)}{L^2} = k(L - L_0)$$

Solving for Q

$$Q = 2L\sqrt{4\pi\varepsilon_0 k(L-L_0)}$$

Substituting $L = \frac{3L_0}{2}$, we get

$$Q = 3L_0\sqrt{2\pi\varepsilon_0 L_0}$$

So, OPTION (A) is correct.

Hence, the correct answer is (A).

60. Velocity of particle at time t is $v = at$ where $a = \frac{F}{m} = \frac{qE}{m}$.

$$\Rightarrow \quad v = \frac{qEt}{m}$$

So, $K.E. = \frac{1}{2}mv^2 = \frac{q^2E^2t^2}{2m}$

Hence, the correct answer is (C).

61. Let q_1 and q_2 be the initial charges on the spheres, then according to Coulomb's Law

$$0.108 = 9\times10^9\frac{q_1q_2}{(0.5)^2}$$

$$\Rightarrow \quad q_1q_2 = 3\times10^{-12} \qquad \ldots(1)$$

After connection, the charge flows from one sphere to another till both acquire the same potential (or in this case equal charge). So, final charge on both is

$$q_f = \frac{q_1-q_2}{2}$$

(as initially they have opposite nature)

Now, again according to Coulomb's Law, the repulsive force is

$$0.036 = 9\times10^9\frac{\left(\frac{q_1-q_2}{2}\right)^2}{(0.5)^2}$$

$$\Rightarrow \quad q_1 - q_2 = \frac{0.036\times0.25\times4}{9\times10^9}$$

$$\Rightarrow \quad q_1 - q_2 = 2\times10^{-6} \qquad \ldots(2)$$

Solving (1) and (2), we get

$$q_1 - \frac{3\times10^{-12}}{q_1} = 2\times10^{-6}$$

$$\Rightarrow \quad q_1^2 - 2\times10^{-6}q_1 - 3\times10^{-12} = 0$$

$$\Rightarrow \quad q_1 = \frac{2\times10^{-6}\pm\sqrt{4\times10^{-12}+12\times10^{-12}}}{2}$$

$$\Rightarrow \quad q_1 = \frac{2\times10^{-6}\pm4\times10^{-6}}{2}$$

$$\Rightarrow \quad q_1 = 3\times10^{-6}\text{ C and } q_2 = -1\times10^{-6}\text{ C}$$

$$\Rightarrow \quad q_1 = 3\ \mu\text{C and } q_2 = -1\ \mu\text{C}$$

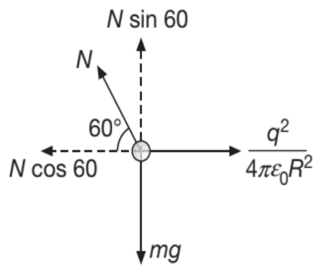
Hence, the correct answer is (B).

62. Since, all A, B, C, D and E lie on an equipotential surface so,

$$W = 0$$

Hence, the correct answer is (C).

63. $N\sin 60 = mg$ and $N\cos 60 = \frac{q^2}{4\pi\varepsilon_0R^2}$



$$\Rightarrow \quad \tan 60° = \frac{mg}{\left(\frac{q^2}{4\pi\varepsilon_0R^2}\right)}$$

$$\Rightarrow \quad q^2 = \frac{4\pi\varepsilon_0 mgR^2}{\sqrt{3}}$$

$$\Rightarrow \quad q = \left(\frac{4\pi\varepsilon_0 mgR^2}{\sqrt{3}}\right)^{\frac{1}{2}}$$

Hence, the correct answer is (C).

64. $U = 4\left(\frac{-q^2}{4\pi\varepsilon_0 a}\right) + 2\left(\frac{q^2}{4\pi\varepsilon_0\sqrt{2}a}\right)$

$$U = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q^2}{a}\left(-4+\sqrt{2}\right)$$

Hence, the correct answer is (A).

65. $\frac{q^2}{4\pi\varepsilon_0 r^2} = \mu mg$

$$\Rightarrow \quad \frac{9\times10^9\times(10^{-7})^2}{(10\times10^{-2})^2} = \mu\left(\frac{5}{1000}\right)(10)$$

$$\Rightarrow \quad \mu = 0.18$$

Hence, the correct answer is (C).

66. Here we shall use a logical trick. If a charge of $4\ \mu$C had also been placed at A then

$$\vec{E}_A + \vec{E}_B + \vec{E}_C + \vec{E}_D = \vec{0}$$

$$\Rightarrow \quad \vec{E}_B + \vec{E}_C + \vec{E}_D = -\vec{E}_A$$

$$\Rightarrow \quad \left|\vec{E}_B + \vec{E}_C + \vec{E}_D\right| = \frac{k(4\times10^{-6})}{(1/\sqrt{2})^2}$$

$$\Rightarrow \quad \left|\vec{E}_B + \vec{E}_C + \vec{E}_D\right| = (9\times10^9)(4\times10^{-6})(2)$$

$$\Rightarrow \quad \left|\vec{E}_B + \vec{E}_C + \vec{E}_D\right| = 7.2\times10^4\ NC^{-1}$$

Hence, the correct answer is (A).

67. $\int \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}q$

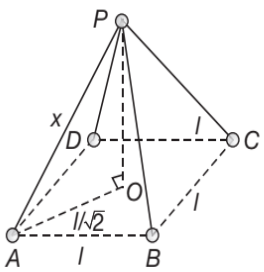
$$\Rightarrow \quad E(2\pi r\ell) = \frac{1}{\varepsilon_0}(\lambda\ell)$$

$$\Rightarrow \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Hence, the correct answer is (C).

68. The force due to one charge on the charge at P is shown here. When all the forces are taken into account, then the sine components cancel each other. So,

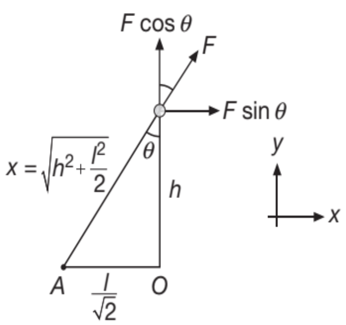
$$F_{\text{net}} = F_y = 4F\cos\theta$$



Since, we see that $\cos\theta = \dfrac{h}{\sqrt{h^2+\dfrac{\ell^2}{2}}}$

$$\Rightarrow \quad F_{\text{net}} = 4\frac{Q(1)}{4\pi\varepsilon_0\left(\sqrt{h^2+\dfrac{\ell^2}{2}}\right)^2}\frac{h}{\sqrt{h^2+\dfrac{\ell^2}{2}}}$$

$$\Rightarrow \quad F_{\text{net}} = \frac{4Qh}{4\pi\varepsilon_0\left(h^2+\dfrac{\ell^2}{2}\right)^{3/2}} \qquad \text{(UPWARDS)}$$



For equilibrium of m, this F_{net} must balance the weight of the particle mg (downwards).

$$\Rightarrow \quad \frac{4Qh}{4\pi\varepsilon_0\left(h^2+\dfrac{\ell^2}{2}\right)^{\frac{3}{2}}} = mg$$

$$\Rightarrow \quad Q = \frac{mg\pi\varepsilon_0}{h}\left(h^2+\frac{\ell^2}{2}\right)^{3/2}$$

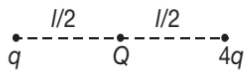
Hence, the correct answer is (B).

69. Field lines always enter or leave a surface normal to it.

Hence, the correct answer is (D).

70. $(F_{net})_q = 0$

$$\Rightarrow \quad k\frac{Qq}{\left(\dfrac{\ell}{2}\right)^2} + k\frac{4q^2}{\ell^2} = 0$$



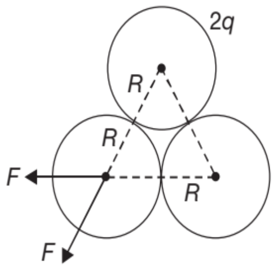
where $k = \dfrac{1}{4\pi\varepsilon_0}$

$$\Rightarrow \quad 4Qq + 4q^2 = 0$$

$$\Rightarrow \quad Q = -q$$

Hence, the correct answer is (A).

71.



$$F = \frac{1}{4\pi\varepsilon_0}\frac{(2q)^2}{(2R)^2} = \frac{q^2}{4\pi\varepsilon_0 R^2}$$

$$F_{\text{net}} = \sqrt{F^2+F^2+2F^2\cos 60}$$

$$\Rightarrow \quad F_{\text{net}} = \sqrt{3}F = \sqrt{3}\frac{q^2}{4\pi\varepsilon_0 R^2}$$

$$\Rightarrow \quad F_{\text{net}} = \frac{\sqrt{3}q^2}{4\pi\varepsilon_0 R^2}$$

Hence, the correct answer is (D).

72. According to Gauss Theorem

$$\text{Flux} = \phi = \frac{q}{\varepsilon_0}$$

Hence, the correct answer is (A).

73. Since charge resides only on the outer surface of conductor, So (D) is the correct option.

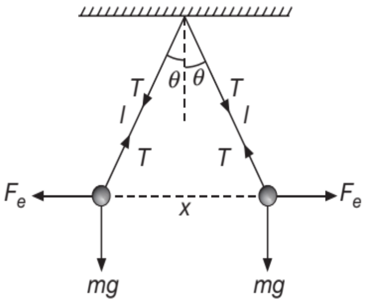
Hence, the correct answer is (D).

74. In this problem, let us first calculate the value of x in terms of other known parameters. Let us consider the particles to be in equilibrium, then

$$T\cos\theta = mg \quad \ldots(1)$$

$$T\sin\theta = \frac{q^2}{4\pi\varepsilon_0 x^2} \quad \ldots(2)$$

$$\Rightarrow \tan\theta = \frac{q^2}{4\pi\varepsilon_0 mgx^2}$$



Since $x \ll \ell$

$\therefore$ $\tan\theta$ is very small and hence $\tan\theta \simeq \theta = \frac{x}{2\ell}$

$$\Rightarrow \frac{x}{2\ell} = \frac{q^2}{4\pi\varepsilon_0 mgx^2}$$

$$\Rightarrow x^3 = \frac{q^2\ell}{2\pi\varepsilon_0 mg} \quad \ldots(3)$$

$$\Rightarrow x = \left(\frac{q^2\ell}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$$

Take derivative of (3) w.r.t. time, we get

$$3x^2\frac{dx}{dt} = \left(\frac{\ell}{2\pi\varepsilon_0 mg}\right)2q\left(\frac{dq}{dt}\right)$$

But according to the problem, $v = \frac{dx}{dt} = \frac{6}{\sqrt{x}}$

$$\Rightarrow 3x^2\frac{6}{\sqrt{x}} = \left(\frac{x^3}{q^2}\right)2q\left(\frac{dq}{dt}\right)$$

$$\Rightarrow \frac{dq}{dt} = 9\left(\frac{q}{x^{3/2}}\right)$$

But $\frac{x^3}{q^2} = \frac{\ell}{2\pi\varepsilon_0 mg}$

$$\Rightarrow \frac{dq}{dt} = 9\sqrt{\frac{2\pi\varepsilon_0 mg}{\ell}}$$

Hence, the correct answer is (D).

75. Since, $\vec{E} = 5\hat{i} + 2\hat{j}$

$$\vec{A} = 2\hat{i}$$

(as area vector is always directed along the outward normal)

So, $\phi = \vec{E}\cdot\vec{A} = 10$ SI units

Hence, the correct answer is (A).

76. Consider an infinitesimal ring element of inner radius r and outer radius $r + dr$ having a charge dq on it. Then

$$dq = \sigma \text{ (Area of infinitesimal ring element)}$$

$$\Rightarrow dq = \sigma(2\pi r dr)$$

But $\sigma = \frac{\sigma_0}{r^2}$

$$\Rightarrow dq = \frac{\sigma_0}{r^2}(2\pi r dr) = 2\pi\sigma_0\frac{dr}{r}$$

$$\Rightarrow q = \int dq = 2\pi\sigma_0\int_a^b \frac{dr}{r} = 2\pi\sigma_0 \log_e\left(\frac{b}{a}\right)$$

Hence, the correct answer is (D).

78. Since the spheres attract with a force F, so let q_1 and $-q_2$ be the charges on the spheres, then

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{d^2} \quad \ldots(1)$$

When the spheres are brought in contact, then final charge on both is q (say), given by

$$q = \frac{q_1 + (-q_2)}{2} = \frac{q_1 - q_2}{2}$$

These spheres with new charge q, placed at separation d, again repel with force F. So

$$F = \frac{q^2}{4\pi\varepsilon_0 d^2} = \frac{\left(\frac{q_1 - q_2}{2}\right)^2}{4\pi\varepsilon_0 d^2} \quad \ldots(2)$$

So, from (1) & (2), we get

$$\frac{\left(\frac{q_1 - q_2}{2}\right)^2}{4\pi\varepsilon_0 d^2} = \frac{q_1q_2}{4\pi\varepsilon_0 d^2}$$

$$\Rightarrow q_1^2 + q_2^2 - 2q_1q_2 = 4q_1q_2$$

$$\Rightarrow q_1^2 + q_2^2 - 6q_1q_2 = 0$$

$$\Rightarrow \left(\frac{q_1}{q_2}\right)^2 - 6\left(\frac{q_1}{q_2}\right) + 1 = 0$$

$\Rightarrow \quad \dfrac{q_1}{q_2}=\dfrac{6\pm\sqrt{32}}{2}=\dfrac{6\pm2\sqrt{8}}{2}$

$\Rightarrow \quad \dfrac{q_1}{q_2}=3\pm\sqrt{8}$

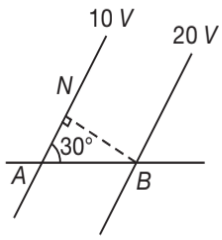
So, ratio between the initial charges is

$$\frac{q_1}{-q_2}=-\left(3\pm\sqrt{8}\right)$$

$\Rightarrow$ Either $\dfrac{q_1}{-q_2}=-3-\sqrt{8}$ OR $\dfrac{q_1}{-q_2}=-3+\sqrt{8}$

Hence, the correct answer is (C).

79.



Since $|E|=\dfrac{\Delta V}{\Delta r_{\perp}}$

In ΔABN

$$\sin(30^\circ)=\frac{BN}{AB}$$

$\Rightarrow \quad BN=\left(\dfrac{10}{100}\right)\sin(30^\circ)$

$\Rightarrow \quad E=\dfrac{10}{\left(\dfrac{10}{100}\right)\sin(30^\circ)}$

$\Rightarrow \quad E=200\ Vm^{-1}$

E must be perpendicular to the line indicating equipotential surface. Hence it must be directed at 120° with X-axis.

Hence, the correct answer is (C).

80. For Solid Sphere

$$q_1=\int\rho dV=\int_0^{R_1}\frac{\rho_0}{r}\left(4\pi r^2dr\right)$$

$\Rightarrow \quad q_1=4\pi\rho_0\displaystyle\int_0^{R_1}r\,dr=4\pi\rho_0\left(\frac{R_1^2}{2}\right)$

$\Rightarrow \quad q_1=2\pi R_1^2\rho_0 \qquad$ ...(1)

For Hollow Sphere

$$q_2=\left(4\pi R_2^2\right)(-\sigma)$$

$\Rightarrow \quad q_2=-4\pi R_2^2\sigma \qquad$ ...(2)

Since, total charge on the system is zero, so

$$q_1+q_2=0$$

$\Rightarrow \quad 2\pi R_1^2\rho_0-4\pi R_2^2\sigma=0$

$\Rightarrow \quad R_1^2\rho_0=2R_2^2\sigma$

$\Rightarrow \quad \dfrac{R_2}{R_1}=\sqrt{\dfrac{\rho_0}{2\sigma}}$

Hence, the correct answer is (C).

81. $\phi_{total}=\dfrac{q}{\varepsilon_0}$

$$\phi_{face}=\frac{1}{6}\phi_{total}=\frac{q}{6\varepsilon_0}$$

Hence, the correct answer is (A).

82. Since electrostatic force is conservative in nature, so work done by the force in a closed path must be zero.

$$W_{P\to Q}+W_{Q\to R}+W_{R\to S}+W_{S\to P}=0$$

$\Rightarrow \quad W_{P\to S}=-W_{S\to P}$

$\Rightarrow \quad \left(\begin{array}{c}\text{Work done in}\\ \text{the process}\end{array}\right)=W_{P\to Q\to R\to S}=W_{P\to S}=-W_{S\to P}$

Further $W_{S\to P}=\vec{F}\cdot\overrightarrow{SP}$

$\Rightarrow \quad W_{S\to P}=\left(qE\hat{i}\right)\cdot\left(a\hat{i}+b\hat{j}\right)$

$\Rightarrow \quad W_{S\to P}=qEa$

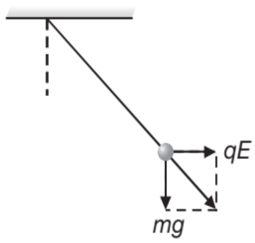
$\Rightarrow \quad$ Work done in the process $=-qEa$

Hence, the correct answer is (B).

83. The velocity of the particle will first increase and then decrease. Thus total potential energy will first decrease and then increase.

Hence, the correct answer is (C).

84. $a_{net}=\dfrac{\sqrt{m^2g^2+q^2E^2}}{m}$



$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{\ell}{a_{net}}}$

$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{\ell}{\sqrt{g^2 + \left(\dfrac{qE}{m}\right)^2}}}$

Hence, the correct answer is (B).

85. $q_1 + q_2 = 20\ \mu C$ ...(1)

According to Coulomb's Law

$$0.075 = 9 \times 10^9 \frac{q_1 q_2}{(3)^2}$$

$\Rightarrow \quad q_1 q_2 = 75 \times 10^{-12}$ ...(2)

Since

$$(q_1 - q_2)^2 = (q_1 + q_2)^2 - 4q_1 q_2$$

$\Rightarrow \quad q_1 - q_2 = \sqrt{400 - 300}\ \mu C$

$\Rightarrow \quad q_1 - q_2 = 10\ \mu C$ ...(3)

Solving (1) and (3), we get

$$q_1 = 15\ \mu C,\ q_2 = 5\ \mu C$$

Hence, the correct answer is (B).

86. Electric potential along the x-axis will be zero as $\Sigma q = 0$ and distances of the point from the charges is zero. So,

$$x = y = 0$$

Hence, the correct answer is (C).

88. Whatever may be the charge on the surface of a shell field inside it is always zero.

Hence, the correct answer is (C).

89. Charge on the inner surface will be –q and that on the outer surface will be zero so to make electric field inside the conductor zero and potential of the surface to be zero.

Hence, the correct answer is (D).

90. The potential at the centre of a hollow sphere is equal to the potential at its surface.

Hence, the correct answer is (B).

91. Radius of the semicircle, $R = \dfrac{L}{\pi}$

Electric potential, $V = \dfrac{q}{4\pi\varepsilon_o R} = \dfrac{q}{4\varepsilon_o L}$

Hence, the correct answer is (B).

92. When connected by a long wire then

$$\frac{Q_1}{Q_2} = \frac{C_1 V}{C_2 V} = \frac{4\pi\varepsilon_0 R_1 V}{4\pi\varepsilon_0 R_2 V}$$

$\Rightarrow \quad \dfrac{Q_1}{Q_2} = \dfrac{R_1}{R_2}$

Hence, the correct answer is (A).

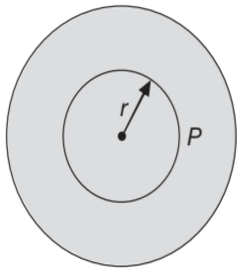
93. $E_1 = \dfrac{kQ_1}{R_1^2}$ and $E_2 = \dfrac{kQ_2}{R_2^2}$

$\Rightarrow \quad \dfrac{E_1}{E_2} = \dfrac{Q_1}{Q_2}\dfrac{R_2^2}{R_1^2}$

$\Rightarrow \quad \dfrac{E_1}{E_2} = \dfrac{R_2}{R_1}$

Hence, the correct answer is (B).

94. Field at the point P is only due to inner core, so



$$E = \frac{1}{4\pi\varepsilon_o}\frac{\left(\rho\left(\frac{4}{3}\right)\pi r^3\right)}{r^2} = \frac{\rho r}{3\varepsilon_o}$$

Hence, the correct answer is (A).

96. $V_0 = 0$

$$\vec{E}_0 = \vec{E}_A + \vec{E}_B + \vec{E}_C + \vec{E}_D$$

$$\vec{E}_A + \vec{E}_B = \frac{\sqrt{2}q}{4\pi\varepsilon_0\left(\frac{a}{\sqrt{2}}\right)^2}(-\hat{j})$$

$\Rightarrow \quad \vec{E}_A + \vec{E}_B = \dfrac{2\sqrt{2}q}{4\pi\varepsilon_0 a^2}(-\hat{j})$

Similarly

$$\vec{E}_C + \vec{E}_D = \frac{2\sqrt{2}q}{4\pi\varepsilon_0 a^2}(-\hat{j})$$

$\Rightarrow \quad \vec{E}_0 = \dfrac{4\sqrt{2}q}{4\pi\varepsilon_0 a^2}(-\hat{j})$

$$\Rightarrow \quad |\vec{E}_0| = \left(\frac{4\sqrt{2}q}{a^2}\right)\frac{1}{4\pi\varepsilon_0}$$

Hence, the correct answer is (B).

97. Let $P(x, y)$ be the point where potential is zero.

$$\frac{-2Q}{4\pi\varepsilon_o\sqrt{(x+3a)^2+y^2}} + \frac{Q}{4\pi\varepsilon_o\sqrt{(x-3a)^2+y^2}} = 0$$

$$\Rightarrow \quad (x-5a)^2 + y^2 = 16a^2$$

So, OPTIONS (A) and (B) are correct.
Since potential at the center is positive, OPTION (C) is also correct.

Hence, the correct answer is (D).

98. $W_{A\to B} = Q(V_B - V_A)$

$$W_{A\to C} = Q(V_C - V_A)$$

Since $V_C = V_B$ as both B and C lie on an equipotential surface.

Hence, the correct answer is (B).

99. Kinetic energy of the particle $= \frac{1}{2}mv^2 + qEy$

Hence, the correct answer is (A).

101. The ball will be at equilibrium at an angle θ_0, where

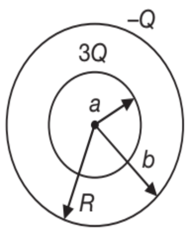
$$\tan\theta_0 = \frac{qE}{mg}$$

The tension will be minimum for

$$\theta = \pi + \theta_0 = \pi + \tan^{-1}\left(\frac{qE}{mg}\right)$$

Hence, the correct answer is (D).

102.



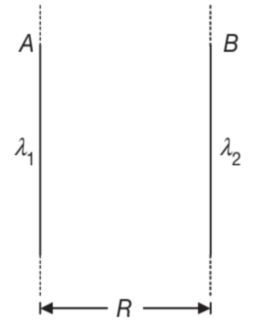
Since field inside the shell is zero. So,

$$E = \frac{3Q}{4\pi\varepsilon_0 R^2}$$

Hence, the correct answer is (C).

103. Electric field due to wire A at a distance R from it is

$$E_1 = \frac{\lambda_1}{2\pi\varepsilon_0 R}$$



Wire B is now in the field of wire A. Let us consider a length ℓ of wire B. Charge on this portion will be $q_2 = \lambda_2\ell$. So, force experienced by this portion of B (of charge q_2) due to field of wire A is

$$F_{21} = q_2E_1 = (\lambda_2\ell)\frac{\lambda_1}{2\pi\varepsilon_0 R}$$

$$\Rightarrow \quad \frac{F_{21}}{\ell} = \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0 R} = \frac{2\lambda_1\lambda_2}{4\pi\varepsilon_0 R}$$

$$\Rightarrow \quad \frac{F_{21}}{\ell} = K\left(\frac{2\lambda_1\lambda_2}{R}\right)$$

Hence, the correct answer is (B).

104. The motion of the charge will be like projectile with range

$$AB = 2\left(\frac{R}{2}\right) = \frac{v_0^2\sin(2\theta)}{a}$$

So, $\quad \frac{R}{\sin(2\theta)} = \frac{v_0^2}{a}$

For minimum velocity, $\sin(2\theta) = 1$

$$\Rightarrow \quad \frac{v_0^2}{a} = R$$

where $a = \frac{qE}{m} = \frac{q\left(\frac{\rho}{3\varepsilon_0}\frac{R}{2}\right)}{m}$

$$\Rightarrow \quad a = \frac{2\times10^{-4}\times3\varepsilon_0\times10^{-3}}{1\times10^{-3}\times3\varepsilon_0} = 2\times10^{-4}\ \text{ms}^{-2}$$

$$\Rightarrow \quad v_0 = 0.02\ \text{ms}^{-1}$$

Hence, the correct answer is (A).

105. Since, $U = -\vec{p}\cdot\vec{E}$ and

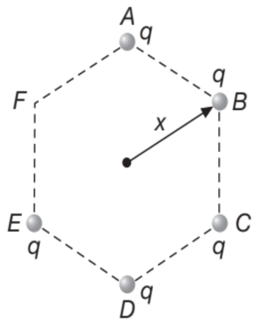
$$\vec{N} = \vec{p}\times\vec{E}$$

U is MINIMUM when $\theta = 0°$ and for $\theta = 0°$

$$\vec{N} = \vec{0}$$

Hence, the correct answer is (A).

106. Electric fields at the center due to the charges on opposite corners will cancel each other. Therefore at the center net electric field will be due to only that charge which does not has charge on the opposite corner.



$$E_{net} = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(2a)^2} = \frac{1}{16\pi\varepsilon_0}\frac{q}{a^2}$$

Hence, the correct answer is (B).

107. Let Q be divided into q and $Q-q$. Then

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q(Q-q)}{r^2}$$

For F to be MAXIMUM $\frac{dF}{dq} = 0$

$$\Rightarrow \frac{d}{dq}(qQ - q^2) = 0$$

$$Q - 2q = 0$$

$$\Rightarrow q = \frac{Q}{2}$$

Hence, the correct answer is (A).

108. Since the point where electric field is zero is nearer to q_1 therefore

$$q_2 > q_1$$

Again electric field is zero between q_1 and q_2, therefore, both charges must be of same nature.

Since electric field due to q_1 is towards right and that of q_2 is towards left therefore q_1 and q_2 are positive charges.

Hence, the correct answer is (A).

109. Both A and B have same radius and same potential. Hence

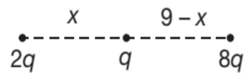
$$Q_A = Q_B \qquad \left\{\because V = \frac{Q}{4\pi\varepsilon_0 r} \text{ for both}\right\}$$

Hence, the correct answer is (C).

110. For potential energy to be MINIMUM, the charges with maximum value must be placed the farthest.

Also MINIMUM potential energy implies equilibrium i.e.

$$\sum F = 0$$



For equilibrium of q

$$\frac{1}{4\pi\varepsilon_0}\frac{2q^2}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{8q^2}{(9-x)^2}$$

$$\Rightarrow \left(\frac{9-x}{x}\right)^2 = 4$$

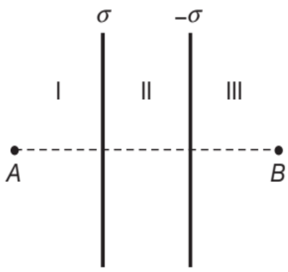
$$\Rightarrow 9 - x = 2x$$

$$\Rightarrow x = 3 \text{ cm}$$

Hence, the correct answer is (B).

111. In the region I and III net electric field is zero.

In the region II net electric field is



$$E_{II} = \frac{\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o} = \frac{\sigma}{\varepsilon_o} \text{ and is uniform.}$$

Hence, the correct answer is (B).

112. $V = \frac{q}{4\pi\varepsilon_0}\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\right)$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{1-\frac{1}{2}}\right)$$

$$\Rightarrow V = \frac{1}{4\pi\varepsilon_0}(2q)$$

$$E = \frac{q}{4\pi\varepsilon_0}\left(1 + \frac{1}{4} + \frac{1}{16} + \ldots\right)$$

$$\Rightarrow E = \frac{q}{4\pi\varepsilon_0}\frac{1}{\left(1-\frac{1}{4}\right)}$$

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q}{3}\right)$$

Hence, the correct answer is (D).

113. $V = \frac{q}{4\pi\varepsilon_0}\left(1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + ...\right)$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{1+\frac{1}{2}}\right)$$

$$\Rightarrow V = \frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{3}\right)$$

$$E = E_+ - E_-$$

$$\Rightarrow E = \frac{q}{4\pi\varepsilon_0}\left(1 + \frac{1}{16} + ...\right) - \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{4} + \frac{1}{64} + ...\right)$$

$$\Rightarrow E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1-\frac{1}{16}} - \frac{\frac{1}{4}}{1-\frac{1}{16}}\right]$$

$$\Rightarrow E = \frac{q}{4\pi\varepsilon_0}\left(\frac{16}{15} - \frac{4}{15}\right)$$

$$\Rightarrow E = \frac{q}{4\pi\varepsilon_0}\left(\frac{12}{15}\right)$$

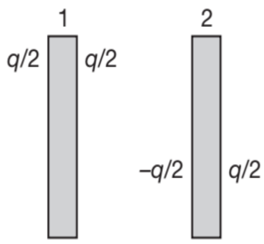
$$\Rightarrow E = \frac{q}{4\pi\varepsilon_0}\left(\frac{4}{5}\right)$$

Hence, the correct answer is (C).

114. Net electric field between the plates is $E = \frac{\sigma}{\varepsilon_0}$

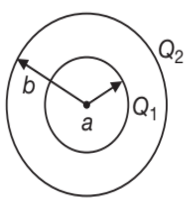
Since $\sigma = \frac{\frac{q}{2}}{A} = \frac{q}{2A}$

$$\Rightarrow E = \frac{q}{2\varepsilon_0 A}$$



Hence, the correct answer is (A).

115.



$$V_{\text{inner}} = \frac{Q_1}{4\pi\varepsilon_0 a} + \frac{Q_2}{4\pi\varepsilon_0 b}$$

$$V_{\text{outer}} = \frac{Q_2}{4\pi\varepsilon_0 b} + \frac{Q_1}{4\pi\varepsilon_0 b}$$

$$\Rightarrow V_{\text{inner}} > V_{\text{outer}} \text{ as } b > a$$

Hence, the correct answer is (B).

117. Electric field at O depends only on the induced charge on the outer surface of the spherical conductor. Shifting of q within the cavity does not change distribution of induced charge over the outer surface.

Hence, the correct answer is (D).

118. $\vec{E} = -\vec{\nabla}V$

$$\Rightarrow \vec{E} = -\left(\hat{i}\frac{\partial U}{\partial x} + \hat{j}\frac{\partial U}{\partial y} + \hat{k}\frac{\partial U}{\partial z}\right)$$

$$\frac{\partial U}{\partial x} = 6 - 8y^2 - 8x$$

$$\frac{\partial U}{\partial y} = -16xy - 8 + 6z$$

$$\frac{\partial U}{\partial z} = 6y$$

At (0, 0, 0)

$$\frac{\partial U}{\partial x} = 6$$

$$\frac{\partial U}{\partial y} = -8$$

$$\frac{\partial U}{\partial z} = 0$$

$$\Rightarrow \vec{E}\Big|_{(0,\,0,\,0)} = -6\hat{i} + 8\hat{j}$$

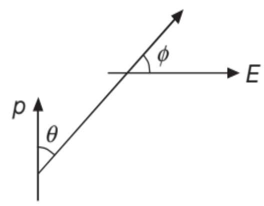
$$\Rightarrow |\vec{E}| = 10 \text{ NC}^{-1}$$

$$\Rightarrow |\vec{F}| = q|\vec{E}|$$

$$\Rightarrow |\vec{F}| = 20\ N$$

Hence, the correct answer is (D).

119. If angle made by the electric field due to the dipole with the radius vector is ϕ then



$$\tan\phi = \frac{1}{2}\tan\theta$$

In this equation

$$\phi = \frac{\pi}{2} - \theta$$

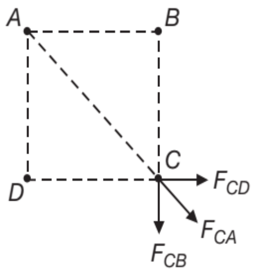
$$\Rightarrow \quad \tan\left(\frac{\pi}{2} - \theta\right) = \frac{1}{2}\tan\theta$$

$$\Rightarrow \quad \cot\theta = \frac{1}{2}\tan\theta$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\sqrt{2}\right)$$

Hence, the correct answer is (D).

120.



$$F_{net} = \frac{\sqrt{2}q^2}{4\pi\varepsilon_0 a^2} + \frac{q^2}{4\pi\varepsilon_0\left(\sqrt{2}a\right)^2}$$

$$F_{net} = \left(\frac{1+2\sqrt{2}}{2}\right)\frac{q^2}{4\pi\varepsilon_0 a^2}$$

Hence, the correct answer is (C).

121. $V_B - V_0 = -\int E_x dx = -\frac{400}{\sqrt{2}}(0.03)$

$$V_A - V_B = -\int E_y dy = -\frac{400}{\sqrt{2}}(0.02)$$

where V_0 is potential at origin. So,

$$V_A - V_B = \frac{400}{\sqrt{2}}(0.03 - 0.02)$$

$$\Rightarrow \quad V_A - V_B = \frac{4}{\sqrt{2}} = 2.8 \text{ V}$$

Hence, the correct answer is (D).

122. $q_r + q_R = Q$ ...(1)

$$\frac{q_r}{q_R} = \frac{4\pi r^2\sigma}{4\pi R^2\sigma} = \frac{r^2}{R^2}$$

$$\Rightarrow \quad \frac{q_r}{q_R} = \frac{r^2}{R^2} \quad \text{...(2)}$$

From (1) & (2)

$$q_r = \frac{Qr^2}{R^2 + r^2} \text{ and } q_R = \frac{QR^2}{R^2 + r^2}$$

So, $V = \frac{q_r}{4\pi\varepsilon_0 r} + \frac{q_R}{4\pi\varepsilon_0 R}$

$$\Rightarrow \quad V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{r+R}{r^2+R^2}\right)$$

Hence, the correct answer is (A).

124. Initially potential difference between the spheres is $V_A - V_B$

Since earthing of B will make potential of B zero but potential difference will not change because potential difference between two concentric spheres depends only on the charge on the inner sphere. Hence

Final Potential Difference = Initial Potential Difference

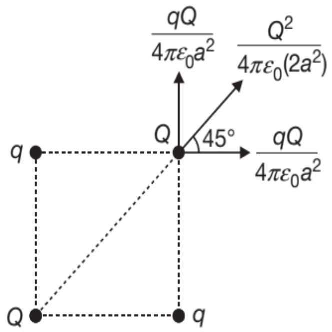
$$\Rightarrow \quad V'_A - V'_B = V_A - V_B$$

$$\Rightarrow \quad V'_A - 0 = V_A - V_B$$

$$\Rightarrow \quad V'_A = V_A - V_B$$

Hence, the correct answer is (C).

125.



For net force on Q to be zero

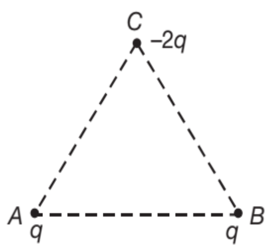
$$\frac{qQ}{4\pi\varepsilon_0 a^2} + \frac{Q^2}{4\pi\varepsilon_0\left(2a^2\right)}\cos 45 + \frac{qQ}{4\pi\varepsilon_0 a^2}\cos 90 = 0$$

$$\Rightarrow \quad Q = -2\sqrt{2}q$$

Hence, the correct answer is (D).

126. Method 1

Let origin be at A, then co-ordinates of points A, B and C are $(0, 0)$, $(\ell, 0)$ and $\left(\frac{\ell}{2}, \frac{\sqrt{3}\ell}{2}\right)$ respectively.



$$p_x = q(0) + q(\ell) + (-2q)\frac{\ell}{2}$$

$$\Rightarrow \quad p_x = 0$$

$$p_y = q(0) + q(0) + (-2q)\frac{\sqrt{3}\ell}{2}$$

$$\Rightarrow \quad p_y = -\sqrt{3}q\ell$$

$$\Rightarrow \quad p = \sqrt{p_x^2 + p_y^2}$$

$$\Rightarrow \quad p = \sqrt{3}q\ell$$

Method 2 (Shortcut)

$$p = (2q)\left(\frac{\sqrt{3}\ell}{2}\right)$$

because

$$p = \left(\begin{matrix}\text{Either}\\\text{charge}\end{matrix}\right)\left(\begin{matrix}\text{Distance of}\\\text{separation}\end{matrix}\right)$$

Hence, the correct answer is (C).

128. Angular momentum is given by

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\Rightarrow \quad L = rp\sin\theta$$

$$\Rightarrow \quad L = x_0(mv)\sin(90°) = x_0 mv$$

Now, at time t, the velocity is given by

$$v = at = \left(\frac{qE_0}{m}\right)t \qquad \left\{\because a = \frac{qE_0}{m}\right\}$$

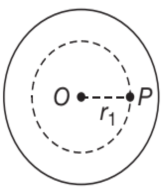
$$\Rightarrow \quad L = x_0 m\left(\frac{qE_0}{m}\right)t = (qE_0 x_0)t$$

Since the speed of the particle is increasing with time, therefore angular momentum is increasing with time.

Hence, the correct answer is (C).

129. P is any inside point at distance r_1 from O. Consider a spherical surface of radius r_1 as Gaussian surface. So,

$$\oint_S \vec{E}\cdot\vec{d}A = \frac{q_{enc}}{\varepsilon_0}$$



By symmetry, E at all points on the surface is same and angle between $\vec{E}$ and $\vec{d}A$ is zero everywhere. So,

$$\oint_S \vec{E}.d\vec{A} = EA = \frac{q_{enc}}{\varepsilon_0}$$

$$\Rightarrow \quad E(4\pi r_1^2) = \frac{q_{enc}}{\varepsilon_0} \qquad \ldots(1)$$

Now, let us calculate q_{enc}. The sphere can be regarded as consisting of a large number of spherical shells. Consider a shell of inner and outer radii r and $r + dr$. Its volume will be $dV = 4\pi r^2 dr$. Charge dq on this shell will be

$$dq = \rho dV = \frac{A}{r}(4\pi r^2 dr) = 4\pi A(rdr)$$

So, total charge enclosed by Gaussian-surface is

$$q_{enc} = \int dq = 4\pi A\int_0^{r_1} rdr = 4\pi A\left(\frac{r_1^2}{2}\right)$$

$$\Rightarrow \quad q_{enc} = 4\pi A\int_0^{r_1} rdr = 4\pi A\left(\frac{r_1^2}{2}\right) = 2\pi A r_1^2$$

From Equation (1), we get

$$E(4\pi r_1^2) = \frac{2\pi A r_1^2}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{A}{2\varepsilon_0}$$

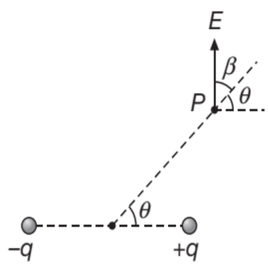
Hence, the correct answer is (D).

130. Direction of electric field with the axis of dipole is given by

$$\theta + \tan^{-1}\left(\frac{\tan\theta}{2}\right)$$

Here given that E is along y-axis, so

$$\theta + \tan^{-1}\left(\frac{\tan\theta}{2}\right) = \frac{\pi}{2}$$



$$\Rightarrow \quad \frac{\tan\theta}{2} = \cot\theta$$

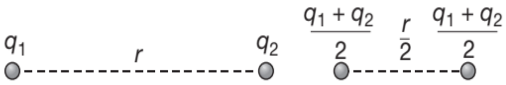
$$\Rightarrow \quad \tan\theta = 2\cot\theta$$

$$\Rightarrow \quad \tan^2\theta = 2$$

$$\Rightarrow \quad \tan\theta = \sqrt{2}$$

Hence, the correct answer is (B).

131.



Initially Finally

Initially, $F=\dfrac{q_1q_2}{4\pi\varepsilon_0r^2}$

After contact i.e., finally, we have

$$F'=\frac{(q_1+q_2)^2}{4\pi\varepsilon_0r^2}$$

Since, $F'=4.5F$

$$\Rightarrow \frac{(q_1+q_2)^2}{r^2}=\frac{4.5q_1q_2}{r^2}$$

$$\Rightarrow 2q_1^2+2q_2^2+4q_1q_2=9q_1q_2$$

$$\Rightarrow 2q_1^2+2q_2^2-5q_1q_2=0$$

$$\Rightarrow \frac{q_1^2}{q_2^2}+2-5\left(\frac{q_1}{q_2}\right)=0$$

$$\Rightarrow 2x^2-5x+2=0\text{, where } x=\frac{q_1}{q_2}$$

$$\Rightarrow 2x^2-4x-x+2=0$$

$$\Rightarrow 2x(x-2)-1(x-2)=0$$

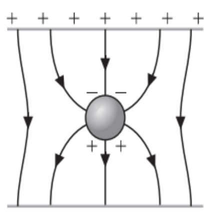
$$\Rightarrow (2x-1)(x-2)=0$$

$$\Rightarrow x=\frac{1}{2}\text{ or } x=2$$

$$\Rightarrow q_1=2q_2$$

Hence, the correct answer is (A).

132. Due to induction, charges will appear on the surface of sphere as shown in figure. Electric field lines never enter any conducting surface and are perpendicular at the surface.



Hence, the correct answer is (C).

133. $F=\dfrac{k(10)(90)}{r^2}$

Charge on each, when contact is made is given by

$$q_1'=q_2'=\frac{10+(-90)}{2}=-40\ \mu\text{C}$$

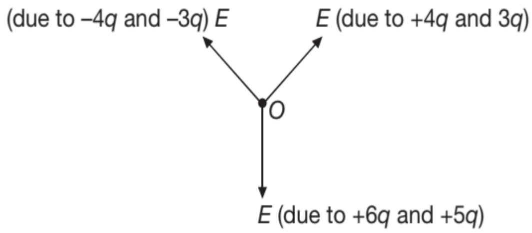
So, new force $F'=\dfrac{k(40)(40)}{r^2}$

$$\Rightarrow \frac{F'}{F}=\frac{40\times 40}{10\times 90}=\frac{16}{9}$$

$$\Rightarrow F'=\frac{16}{9}F$$

Hence, the correct answer is (C).

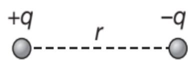
134.



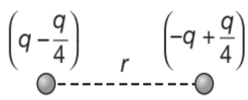
Since all fields are equal in magnitude and are inclined to each other at 120°, so $E_{\text{net}}=0$

Hence, the correct answer is (D).

135.



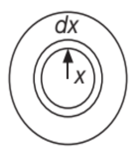
$F=\dfrac{q^2}{4\pi\varepsilon_0r^2}$ and when 25% of charge of one is transferred to the other, then we have,



$$F'=\frac{\left(\frac{3q}{4}\right)\left(\frac{3q}{4}\right)}{4\pi\varepsilon_0r^2}=\frac{9}{16}F$$

Hence, the correct answer is (B).

136. We can consider all the charge inside the sphere to be concentrated at the centre of sphere, consider an elementary shell of radius x and thickness dx, then



$$E=\frac{1}{4\pi\varepsilon_0}\frac{\int dq}{r^2}=\frac{\int_0^r 4\pi x^2dx(\alpha x)}{4\pi\varepsilon_0r^2}=\frac{\alpha r^2}{4\varepsilon_0}$$

Hence, the correct answer is (B).

137. If 20th charge is also placed, then

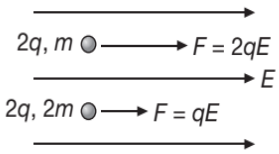
$$\vec{F}_{net}=\vec{0}\text{ (by symmetry)}$$

$$\Rightarrow \vec{F}_{0,1}+\vec{F}_{0,2}+\vec{F}_{0,3}+\ldots+\vec{F}_{0,20}=\vec{0}$$

$$\Rightarrow \left|\vec{F}_{0,1}+\vec{F}_{0,2}+\ldots+\vec{F}_{0,19}\right|=\left|-\vec{F}_{0,20}\right|=F_{0,20}=\frac{Qq}{4\pi\varepsilon_0 a^2}$$

Hence, the correct answer is (B).

138.



$$\frac{KE_A}{KE_B}=\frac{\frac{1}{2}m_A v_A^2}{\frac{1}{2}m_B v_B^2}$$

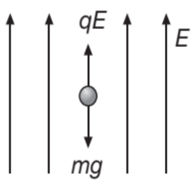
Since, $V_A = u_A + a_A t$

$$\Rightarrow V_A=\frac{2qE}{m}t \text{ and}$$

$$\Rightarrow \frac{KE_A}{KE_B}=\frac{\frac{1}{2}m\left(\frac{2qEt}{m}\right)^2}{\frac{1}{2}(2m)\left(\frac{2qEt}{2m}\right)^2}$$

Hence, the correct answer is (A).

140.



$$\Rightarrow qE = mg$$

$$\Rightarrow neE = mg$$

$$\Rightarrow n=\frac{mg}{eE}=\frac{1.6\times10^{-3}\times9.8}{1.6\times10^{-19}\times10^{9}}$$

$$\Rightarrow n = 9.8\times10^{7}$$

Hence, the correct answer is (A).

141. $F=\dfrac{K(Q)(2Q)}{r^2}$

When allowed to touch, we have

$$F'=\frac{K\left(\frac{Q}{2}\right)\left(\frac{Q}{2}\right)}{r^2}$$

$$\Rightarrow F'=\frac{F}{8}$$

Hence, the correct answer is (C).

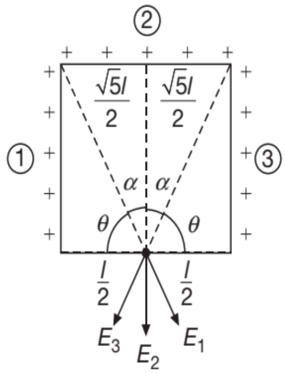
142. Since work done by electric force is equal to the gain in elastic potential energy of the spring, so

$$\frac{1}{2}kx^2 = qEx$$

$$\Rightarrow x=\frac{2qE}{k}$$

Hence, the correct answer is (A).

143.



By Symmetry, $\left|\vec{E}_1\right|=\left|\vec{E}_3\right|$

So, on resolution, their x components will cancel. However their y - components will add up to give

$$(E_{1+3})y=\frac{2\lambda}{4\pi\varepsilon_0\frac{\ell}{2}}(\cos0^\circ-\cos\theta)=\frac{4\lambda}{4\pi\varepsilon_0\ell}\left(1-\frac{1}{\sqrt5}\right)$$

Similarly, we have

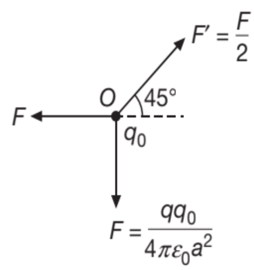
$$E_2=(E_2)_y=\frac{\lambda}{4\pi\varepsilon_0\ell}2\sin\alpha=\frac{2\lambda}{4\pi\varepsilon_0\ell}\left(\frac{1}{\sqrt5}\right)$$

$$\Rightarrow E_{\text{net}}=(E_{1+3})_y+(E_2)_y=\frac{2\lambda}{4\pi\varepsilon_0\ell}\left(2-\frac{2}{\sqrt5}+\frac{1}{\sqrt5}\right)$$

$$\Rightarrow E_{\text{net}}=\frac{\lambda}{2\sqrt5\pi\varepsilon_0\ell}\left(2\sqrt5-1\right)$$

Hence, the correct answer is (A).

144.



So, $F_{net}=\sqrt2F-\dfrac{F}{2}$

$$\Rightarrow F_{net}=\left(\sqrt2-\frac{1}{2}\right)\frac{qq_0}{4\pi\varepsilon_0 a^2}$$

Hence, the correct answer is (C).

145. $mg = qE$

$$\Rightarrow mg=(Ne)E \qquad \{\because q = \text{Ne}\}$$

$$\Rightarrow \quad \frac{4}{3}\pi r^3 \times \rho \times g = NeE$$

$$\Rightarrow \quad N = \frac{4\pi r^3 \rho g}{3eE}$$

Number of moles of the metal in sphere is $n = \frac{m}{A}$

$$\Rightarrow \quad n = \frac{4}{3}\frac{\pi r^3 \rho}{A}$$

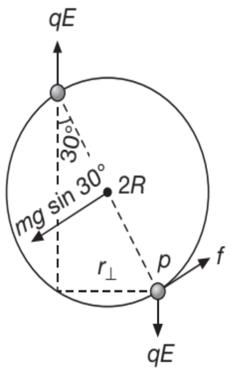
So, number of metal atoms $= \frac{4}{3}\frac{\pi r^3 \rho \times N_A}{A}$

Total number of electrons $= \frac{4}{3}\frac{\pi r^3 \rho N_A \times Z}{A} = N_0$

So, required fraction $= \frac{N}{N_0} = \frac{gA}{eEZN_A}$

Hence, the correct answer is (C).

146.



For system to be in equilibrium, $\tau = 0$

Let us take the torque about the point P, so

$$qE[2R\sin(30°)] = [mg\sin(30°)]R$$

$$\Rightarrow \quad E = \frac{mg}{2q}$$

Hence, the correct answer is (B).

147. Assume a charge q to be placed at the eight corner too. Then by symmetry, we have

$$\vec{E} = \vec{0}$$

$$\Rightarrow \quad \vec{E}_{01} + \vec{E}_{02} + \ldots + \vec{E}_{08} = \vec{0}$$

$$\Rightarrow \quad \left|\vec{E}_{01} + \vec{E}_{02} + \ldots + \vec{E}_{07}\right| = \left|\vec{E}_{08}\right| = \frac{q}{4\pi\varepsilon_0\left(\frac{\sqrt{3}\ell}{2}\right)^2}$$

$$\Rightarrow \quad E_{\text{centre}} = E_0 = \frac{q}{3\pi\varepsilon_0 \ell^2}$$

Hence, the correct answer is (B).

148.

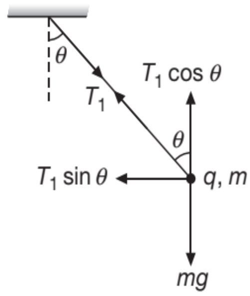


Figure 1

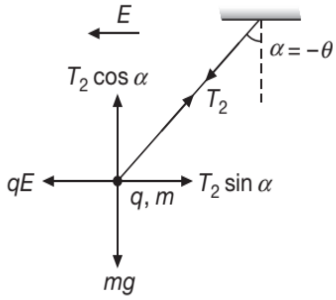


Figure 2

When no field is applied as in Figure 1. Then, we have,

$$T_1 \cos\theta = mg \qquad \ldots(1)$$

$$T_1 \sin\theta = \frac{mv^2}{r} \qquad \ldots(2)$$

$$\Rightarrow \quad \tan\theta = \frac{v^2}{rg} \qquad \ldots(3)$$

When an external field is applied such that the pendulum makes an angle θ to the opposite side, then $\alpha = -\theta$. From Figure 2, we get

$$T_2 \cos\alpha = mg \qquad \ldots(4)$$

$$T_2 \sin\alpha - qE = \frac{mv^2}{r}$$

$$\Rightarrow \quad T_2 \sin\alpha = \frac{mv^2}{r} + qE \qquad \ldots(5)$$

$$\Rightarrow \quad \tan\alpha = -\tan\theta = \frac{\frac{mv^2}{r} + qE}{mg}$$

$$\Rightarrow \quad -\frac{v^2}{rg} = \frac{\frac{mv^2}{r} + qE}{mg}$$

$$\Rightarrow \quad -\frac{mv^2}{r} = \frac{mv^2}{r} + qE$$

$$\Rightarrow \quad qE = -\frac{2mv^2}{r}$$

$$\Rightarrow \quad E = -\frac{2mv^2}{qr}$$

(negative sign implies opposite direction)

$$\Rightarrow \quad |E| = E = \frac{2mv^2}{qr}$$

Hence, the correct answer is (C).

149. $F = 10\times10^{-3}\times10\text{ N} = 10^{-1}$

Since $F = \dfrac{q^2}{4\pi\varepsilon_0 r^2}$

$\Rightarrow \quad q = \sqrt{4\pi\varepsilon_0 F r^2}$

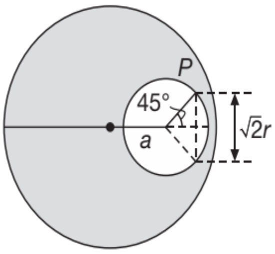
$\Rightarrow \quad q = \sqrt{\dfrac{0.1\times0.36}{9\times10^9}} = 2\times10^{-6}\text{ C}$

Hence, the correct answer is (D).

150. $E = \dfrac{\rho a}{3\varepsilon_0}$

$\Rightarrow \quad F = eE = \dfrac{e\rho a}{3m\varepsilon_0}$

Acceleration $= \dfrac{\rho e a}{3m\varepsilon_0}$



Since $s = \dfrac{1}{2}\left(\dfrac{\rho e a}{3m\varepsilon_0}\right)t^2 = \sqrt{2}r$

$\Rightarrow \quad t = \sqrt{\dfrac{6\sqrt{2}r\varepsilon_0 m}{\rho e a}}$

Hence, the correct answer is (A).

151. $F = \left(\dfrac{\sigma^2}{2\varepsilon_0}\right)A$

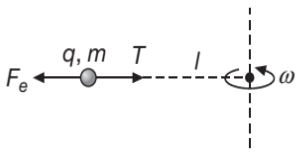
Since a soap bubble has 2 free surfaces, so $F = 2T(2\pi R)$

$\Rightarrow \quad 2[T(2\pi R)] = \dfrac{\sigma^2}{2\varepsilon_0}(\pi R^2)$

$\Rightarrow \quad T = \dfrac{\sigma^2 R}{8\varepsilon_0}$

Hence, the correct answer is (A).

152.



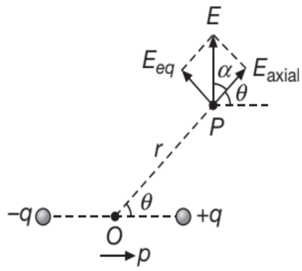
$T - F_e = m\ell\omega^2$

$\Rightarrow \quad T = \dfrac{q^2}{4\pi\varepsilon_0(2\ell)^2} + m\ell\omega^2$

$\Rightarrow \quad T = \dfrac{q^2}{16\pi\varepsilon_0\ell^2} + m\ell\omega^2$

Hence, the correct answer is (D).

153. Angle of resultant with x-axis will be $\alpha+\theta$, where $\tan\alpha = \dfrac{1}{2}\tan\theta$



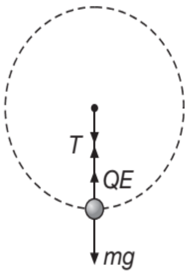
Angle $\phi = \tan^{-1}\left(\dfrac{\tan\theta}{2}\right)+\theta$

$\Rightarrow \quad \phi = \tan^{-1}\left(\dfrac{\tan 45}{2}\right)+\theta$

$\Rightarrow \quad \phi = \tan^{-1}(0.5)+\dfrac{\pi}{4}$

Hence, the correct answer is (B).

154.



Since $(T+QE) - mg = \dfrac{mv^2}{L}$

$\Rightarrow \quad 10mg + QE - mg = \dfrac{mv^2}{L}$

$\Rightarrow \quad 9mg + \dfrac{QE}{m} = \dfrac{mv^2}{L}$

$\Rightarrow \quad v = \sqrt{L\left(9g+\dfrac{QE}{m}\right)}$

Hence, the correct answer is (C).

155. $F = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r^2}$

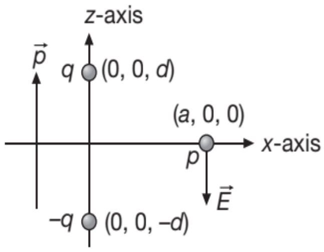
$\Rightarrow \quad \log_e F = \log_e \dfrac{q_1 q_2}{4\pi\varepsilon_0} - 2\log_e r$

$\Rightarrow \quad y = c + mx$

$\Rightarrow \quad m = -2$

Hence, the correct answer is (A).

157.



Since the point $p\ (a, 0\ 0)$ lies at the equitorial line of dipole, so we have $E = \dfrac{p}{4\pi\varepsilon_0 (r^2 + \ell^2)^{\frac{3}{2}}}$

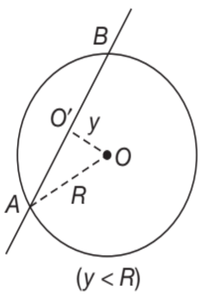
where $p = q(2d)$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{q \times 2d}{(a^2 + d^2)^{\frac{3}{2}}}$$

$$\Rightarrow \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2qd}{(a^2 + d^2)^{\frac{3}{2}}}(-\hat{k})$$

Hence, the correct answer is (C).

158. Electric flux $\oint_S \vec{E}\cdot\vec{dS} = \dfrac{q_{enc}}{\varepsilon_0}$, where q_{enc} is the charge enclosed by the Gaussian-surface which, in the present case, is the surface of given sphere. As shown, length AB of the line lies inside the sphere.



In $\Delta OO'A \quad R^2 = y^2 + (O'A)^2$

$$\Rightarrow \quad O'A = \sqrt{R^2 - y^2}$$

and $AB = 2\sqrt{R^2 - y^2}$

Charge on length AB is $q_{enc} = 2\sqrt{R^2 - y^2} \times \lambda$

So, electric flux is $\phi = \oint_S \vec{E}\cdot d\vec{A} = \dfrac{2\lambda\sqrt{R^2 - y^2}}{\varepsilon_0}$

Hence, the correct answer is (C).

159. $\vec{\tau} = \vec{p} \times \vec{E} = q(2\vec{a}) \times \vec{E}$

Here, $2\vec{a} = (2-1)\hat{i} + (-1-0)\hat{j} + (5-4)\hat{k} = \hat{i} - \hat{j} + \hat{k}$

$$\vec{E} = 0.20\hat{i}\ \text{Vcm}^{-1} = 20\hat{i}\ \text{Vm}^{-1}$$

$$\Rightarrow \quad \vec{\tau} = (4 \times 10^{-6})\left[(\hat{i} - \hat{j} + \hat{k}) \times 20\hat{i}\right] = 8 \times 10^{-5}(\hat{k} + \hat{j})$$

Magnitude of torque is

$$\tau = 8\sqrt{2} \times 10^{-5}\ \text{Nm}$$

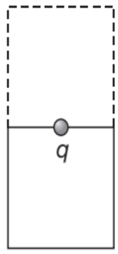
Hence, the correct answer is (C).

160. $\dfrac{\Sigma q}{\varepsilon_0} = \phi_{net} = -8 \times 10^3 + 4 \times 10^3 = -4 \times 10^3$

$$\Rightarrow \quad \Sigma q_{enc} = -4000\varepsilon_0 \text{ coulomb}$$

Hence, the correct answer is (D).

161.

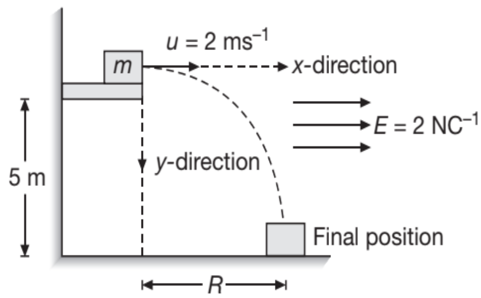


$$\phi_{Gaussian} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad \phi_{vessel} = \frac{1}{2}\phi_{Gaussian} = \frac{q}{2\varepsilon_0}$$

Hence, the correct answer is (A).

162.



Along y-direction

$$S_y = u_y t + \frac{1}{2}a_y t^2 \qquad \ldots(1)$$

$$\Rightarrow \quad 5 = 0 + \frac{1}{2} \times g \times t^2$$

$$t = \sqrt{\frac{10}{g}} = 1\ s$$

Along x-direction

$$S_x = u_x t + \frac{1}{2}a_x t^2 \qquad \ldots(2)$$

$$\Rightarrow \quad R = 2\times t + \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

$$\Rightarrow \quad R = 2\times 1 + \frac{1}{2}\times\frac{2\times 10^{-9}\times 2}{2\times 10^{-9}}\times 1$$

$$\Rightarrow \quad R = 2 + 1 = 3 \text{ m}$$

Hence, the correct answer is (D).

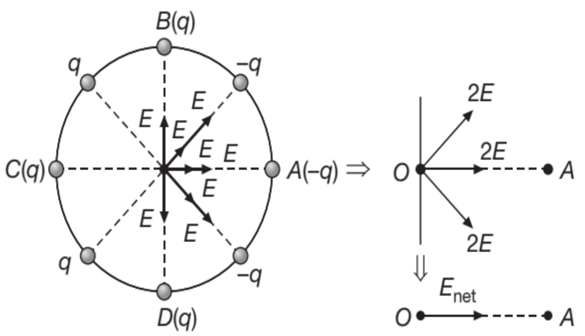
163. $E_{\text{arc}} = \dfrac{2\lambda}{4\pi\varepsilon_0 R}\sin\left(\dfrac{\theta}{2}\right)$

Field due to a single arc subtending an angle of $60°$ at centre is $E_1 = \dfrac{2\lambda}{4\pi\varepsilon_0 R}\left(\dfrac{1}{2}\right) = \dfrac{\lambda}{4\pi\varepsilon_0 R}$, where $\lambda = \dfrac{6Q}{2\pi R}$

$$\Rightarrow \quad E_{\text{total}} = \frac{6\lambda}{4\pi\varepsilon_0 R}, \text{ where } \lambda = \frac{3Q}{\pi R}$$

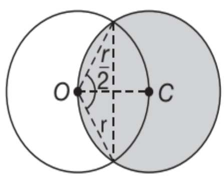
Hence, the correct answer is (D).

164. Since magnitude of each charges are same and situated at equal distance from centre O, so all charges will produce same magnitude of electric field at centre.



Hence, the correct answer is (A).

165.



$$\cos\theta = \frac{\left(\frac{r}{2}\right)}{r} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 60°$$

Length of ring in sphere is

$$\ell = r\times 2\theta = \frac{2\pi r}{3}$$

So, charge enclosed in the sphere is

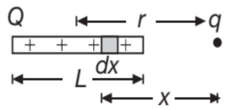
$$q_{\text{enc}} = \frac{Q}{2\pi r}\times\frac{2\pi r}{3} = \frac{Q}{3}$$

Hence, flux by Gauss Law is

$$\phi = \frac{Q}{3\varepsilon_0}$$

Hence, the correct answer is (C).

166.



$$dF = \frac{q\left(\frac{Q}{L}\right)dx}{4\pi\varepsilon_0 x^2}$$

$$\Rightarrow \quad F = \frac{Qq}{4\pi\varepsilon_0 L}\int_{r-\frac{L}{2}}^{r+\frac{L}{2}}\frac{dx}{x^2} = -\frac{Qq}{4\pi\varepsilon_0 L}\left(\frac{1}{r+\left(\frac{L}{2}\right)} - \frac{1}{r-\left(\frac{L}{2}\right)}\right)$$

$$\Rightarrow \quad F = \frac{Qq}{4\pi\varepsilon_0\left(r^2 - \frac{L^2}{4}\right)} = \frac{Qq}{\pi\varepsilon_0\left(4r^2 - L^2\right)}$$

Hence, the correct answer is (B).

167. Effective distance in vacuum is given by

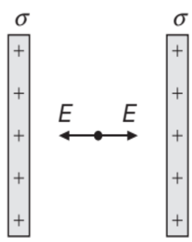
$$r = \sqrt{k_1}t_1 + \sqrt{k_2}t_2 - (t_1 + t_2)$$

So, $F = \dfrac{Q_1Q_2}{4\pi\varepsilon_0\left(t_1\sqrt{k_1} + t_2\sqrt{k_2} - t_1 - t_2\right)^2}$

$$\Rightarrow \quad F = \frac{Q_1Q_2}{4\pi\varepsilon_0\left[\left(\sqrt{k_1}-1\right)t_1 + \left(\sqrt{k_2}-1\right)t_2\right]^2}$$

Hence, the correct answer is (D).

168.



Electric field due to infinite sheet is independent of distance and hence electric field due to both sheets will be equal and opposite.

Hence, the correct answer is (D).

170. $f_\ell \geq F_e$, where $F_e = \dfrac{q^2}{4\pi\varepsilon_0 r^2}$

Since $f = \mu N = \mu mg$

$$\Rightarrow \quad \mu mg \geq \frac{q^2}{4\pi\varepsilon_0 r^2}$$

$\Rightarrow \quad \mu \geq \dfrac{q^2}{4\pi\varepsilon_0 r^2 mg}$

Hence, the correct answer is (B).

171. Using Gauss Theorem, we have

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \text{ for } r \leq R \text{ and}$$

$$E = 0 \text{ for } r \geq R$$

So, correct graph is (A)

Hence, the correct answer is (A).

172. $(-10\ °\text{C ice})$ $(0\ °\text{C water})$

$q_1 \underset{r_1 = 25\text{ cm}}{\overset{k_1}{\text{———}}} q_2 \qquad q_1 \underset{r_2 = 5\text{ cm}}{\overset{k_2 = 80}{\text{———}}} q_2$

Given $F_{\text{ice}} = F_{\text{water}}$

$\Rightarrow \quad \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{k_1r_1^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{k_2r_2^2}$

$\Rightarrow \quad k_1r_1^2 = k_2r_2^2$

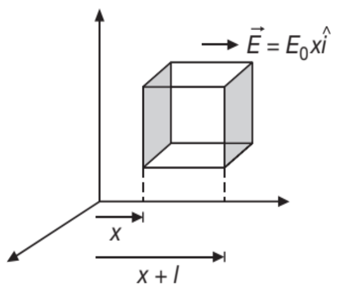
$\Rightarrow \quad k_1 = k_2\left(\dfrac{r_2}{r_1}\right)^2 = 80 \times \left(\dfrac{5}{25}\right)^2 = 3.2$

Hence, the correct answer is (B).

173. $\phi_{\text{net}} = E_0(x+\ell)\ell^2 - E_0x\ell^2 = E_0\ell^3$

Since, $\Sigma q = (\phi_{\text{net}})\varepsilon_0$

$\Rightarrow \quad q_{\text{enc}} = \varepsilon_0 E_0 \ell^3$



Hence, the correct answer is (B).

175. Since $\phi_A + \phi_C + \phi_B = \dfrac{q}{\varepsilon_0}$

and $\phi_A = \phi_C$

$\Rightarrow \quad \phi_A + \phi_A + \phi = \dfrac{q}{\varepsilon_0}$

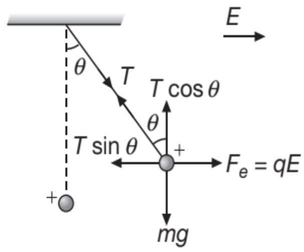
$\Rightarrow \quad \phi_A = \dfrac{1}{2}\left(\dfrac{q}{\varepsilon_0} - \phi\right)$

Hence, the correct answer is (D).

176. θ is inversely proportional to L

Hence, the correct answer is (A).

177.



For equilibrium

$$T\cos\theta = mg \text{ and}$$

$$T\sin\theta = qE$$

$\Rightarrow \quad T = \sqrt{(mg)^2 + (qE)^2}$

$\Rightarrow \quad T = \sqrt{(80\times10\times10^{-6})^2 + (2\times10^{-8}\times2\times10^4)^2}$

$\Rightarrow \quad T = \sqrt{64\times10^{-8} + 16\times10^{-8}}$

$\Rightarrow \quad T = \sqrt{80\times10^{-8}} = 8.8\times10^{-4}\text{ N}$

Hence, the correct answer is (C).

178. $\phi = \dfrac{q_{\text{enclosed}}}{\varepsilon_0} = \dfrac{\lambda\ell}{\varepsilon_0} = \dfrac{1\text{ C/cm}\times100\text{ cm}}{\varepsilon_0} = \dfrac{100\text{ C}}{\varepsilon_0}$

Hence, the correct answer is (D).

179. Since, positive charge diverts in the direction of field, so field must be upwards

So, particle (2) deflects down, (3) up and particle (4) deflects down

Hence, the correct answer is (A).

180. $\vec{A} = 100\hat{k}$

$\Rightarrow \quad \phi = \vec{E}\cdot\vec{A} = 300$ units

Hence, the correct answer is (B).

181. $E_P = \dfrac{q}{4\pi\varepsilon_0 r^2}$

$$E_Q = \frac{q}{4\pi\varepsilon_0(2r)^2} + \frac{3q}{4\pi\varepsilon_0(2r)^2} = \frac{q}{4\pi\varepsilon_0 r^2}$$

$\Rightarrow \quad E_P : E_Q = 1:1$

Hence, the correct answer is (C).

182. For downward motion with constant velocity, we have

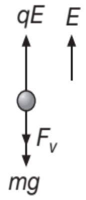
$$q = 2e \text{ and } m = 3.2\times10^{-17}\text{ kg}$$

Since motion is downwards, so viscous force is upwards.

$$6\pi\eta rv = mg \quad \text{...(1)}$$

For upwards motion with same constant speed, we have

$$6\pi\eta rv + mg = qE \quad \text{...(2)}$$



Substitute (1) in (2), we get

$$qE = 2mg$$

$$\Rightarrow \quad E = \frac{2mg}{q} = \frac{2 \times 3.2 \times 10^{-17} \times 10}{2 \times 1.6 \times 10^{-19}} = 2 \times 10^3 \text{ Vm}^{-1}$$

Hence, the correct answer is (A).

183. $\phi_{\text{cube}} = \dfrac{q}{\varepsilon_0}$

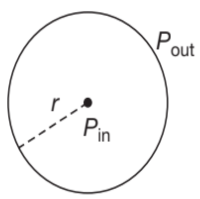
$$\Rightarrow \quad \phi_{\text{face}} = \frac{1}{6}\phi_{\text{cube}} = \frac{q}{6\varepsilon_0} = \frac{4\pi q}{24\pi\varepsilon_0}$$

Hence, the correct answer is (A).

184. The electric field due to Q at any point of square plate will be along the plane of square, therefore angle between electric field and area vector of plate is 90° and hence $\phi = 0$

Hence, the correct answer is (D).

185. Inside pressure must be $\dfrac{4T}{r}$ greater than outside pressure in bubble. This excess pressure is provided by charge on bubble. So, we have



$$\frac{4T}{r} = \frac{\sigma^2}{2\varepsilon_0}$$

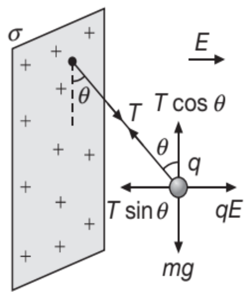
Since $\sigma = \dfrac{Q}{4\pi r^2}$

$$\Rightarrow \quad \frac{4T}{r} = \frac{Q^2}{16\pi^2 r^4 \times 2\varepsilon_0}$$

$$\Rightarrow \quad Q = 8\pi r\sqrt{2rT\varepsilon_0}$$

Hence, the correct answer is (B).

186.



$$T\sin\theta = qE \quad \text{...(1)}$$

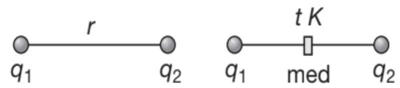
$$T\cos\theta = mg \quad \text{...(2)}$$

$$\Rightarrow \quad \tan\theta = \frac{qE}{mg} = \frac{q\sigma}{2\varepsilon_0 mg}$$

$$\Rightarrow \quad \sigma = \frac{2\varepsilon_0 mg\tan\theta}{q}$$

Hence, the correct answer is (B).

187.



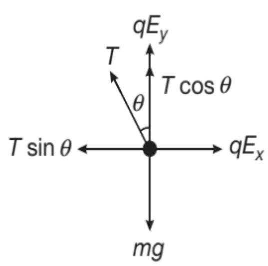
$$F_1 = \frac{Kq_1q_2}{r^2} = \frac{q_1q_2}{4\pi\varepsilon_0 r^2} \text{ and } F_2 = \frac{q_1q_2}{4\pi\varepsilon_0(r - t + t\sqrt{k})^2}$$

Hence, the correct answer is (C).

Multiple Correct Choice Type Questions

1. $T\cos\theta + qE_y = mg$ ...(1)

$T\sin\theta = qE_x$ ...(2)



$$\Rightarrow \quad T(0.6) = 3 \times 10^5 q$$

$$\Rightarrow \quad q = \frac{6}{30} \times 10^{-5}\, T$$

$$\Rightarrow \quad q = 2 \times 10^{-6} T$$

$\Rightarrow \quad T(0.8)+(2\times10^{-6}T)(5\times10^{5})=\dfrac{1}{1000}(10)$

$\Rightarrow \quad T(0.8)+T=\dfrac{10}{1000}$

$\Rightarrow \quad T=\dfrac{1}{(100)(1.8)}$

$\Rightarrow \quad T=5.55\times10^{-3}\text{ N}$

$\Rightarrow \quad q=11.1\times10^{-9}\text{ C}$

$\Rightarrow \quad q\simeq 11\text{ nC}$

Hence, (A) and (C) are correct.

2. $\vec{E}_A$ is along $\overrightarrow{OA}$ and $\vec{E}_B$ is along $\overrightarrow{OB}$, where

$$\overrightarrow{OA}=\hat{i}+2\hat{j}+3\hat{k} \text{ and } \overrightarrow{OB}=\hat{i}+\hat{j}-\hat{k}$$

Since, we observe that $\overrightarrow{OA}\cdot\overrightarrow{OB}=0$

$\Rightarrow \quad \vec{E}_A\perp\vec{E}_B$ (dot product of perpendicular vectors is zero)

Also, $|\vec{E}|\propto\dfrac{1}{r^2}$

and $|\overrightarrow{OC}|=2|\overrightarrow{OB}|$

$\Rightarrow \quad |\vec{E}_B|=4|\vec{E}_C|$

Hence, (A) and (C) are correct.

5. Charge density at A < charge density at B. Since field inside cavity is zero, hence
Potential at A = potential at B = a constant value.
By Gauss Theorem,

$$\begin{pmatrix}\text{Total electric}\\ \text{flux}\end{pmatrix}=\frac{1}{\varepsilon_0}\begin{pmatrix}\text{charge}\\ \text{enclosed}\end{pmatrix}=\left(\frac{q}{\varepsilon_0}\right)$$

Hence, (C) and (D) are correct.

7. Field lines must enter a surface or leave a surface normally.

Hence, option (D) is correct.

8. $t=\sqrt{\dfrac{2d}{a}}$ when $a=\dfrac{qE}{m}=\dfrac{qV}{md}$

$\Rightarrow \quad t=\sqrt{\dfrac{2d}{\frac{qV}{md}}}=\sqrt{\dfrac{2md^2}{qV}}$

$\Rightarrow \quad \dfrac{t}{T}=\sqrt{\dfrac{(2d)^2}{d^2}}=2$

$\Rightarrow \quad t=2T$

Hence, option (D) is correct.

10. $\dfrac{mv^2}{r}=qE=q\left(\dfrac{\lambda}{2\pi\varepsilon_0 r}\right)$

$\Rightarrow \quad v=\sqrt{\dfrac{q\lambda}{2\pi\varepsilon_0 m}}$

Hence, (A), (B) and (C) are correct.

14. $E=E_0t$

Since $a=\dfrac{qE}{m}=\dfrac{qE_0t}{m}=E_0st$

$\Rightarrow \quad \dfrac{dv}{dt}=E_0st \quad\Rightarrow\quad dv=E_0st\,dt$

$\Rightarrow \quad v=E_0s\displaystyle\int_0^t t\,dt=\frac{1}{2}E_0st^2$

$\Rightarrow \quad \dfrac{dx}{dt}=\dfrac{1}{2}E_0st^2$

$\Rightarrow \quad dx=\dfrac{1}{2}E_0st^2dt$

$\Rightarrow \quad \displaystyle\int_0^x dx=\frac{1}{2}E_0s\int_0^t t^2dt$

$\Rightarrow \quad x=\dfrac{1}{6}E_0st^3$

Hence, (B) and (D) are correct.

16. $\dfrac{1}{2}mv^2=QV$

$\Rightarrow \quad \dfrac{p^2}{2m}=QV$

$\Rightarrow \quad p=\sqrt{2mQV}$

$\Rightarrow \quad t=\dfrac{v}{a}=\dfrac{\sqrt{\frac{2QV}{m}}}{\frac{Q}{m}\left(\frac{V}{d}\right)} \qquad \{\because v=0+at\}$

$\Rightarrow \quad t=\sqrt{\dfrac{2md^2}{QV}}$

Hence, (B) and (D) are correct.

18. $a=\dfrac{qE}{m}=sE$

Since, $L=ut$

$\Rightarrow \quad t=\dfrac{L}{u}$

Also, $\Delta = \frac{1}{2}at^2$

$$\Rightarrow \quad \Delta = \frac{1}{2}(sE)\frac{L^2}{u^2}$$

$$\Rightarrow \quad \Delta = \frac{EsL^2}{2u^2}$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{at}{u}$$

$$\Rightarrow \quad \tan\theta = \frac{Est}{u} = \frac{EsL}{u^2}$$

Hence, (A), (B), (C) and (D) are correct.

20. $\frac{1}{4\pi\varepsilon_0}\left(\frac{q_A}{R} + \frac{q_B}{2R}\right) = 2V \quad \ldots(1)$

and, $\frac{1}{4\pi\varepsilon_0}\left(\frac{q_A}{2R} + \frac{q_B}{2R}\right) = \frac{3}{2}V \quad \ldots(2)$

Solving equations (1) and (2), we get

$$\frac{q_A}{q_B} = \frac{1}{2}$$

When B is earthed, potential of B becomes zero. So,

$$q'_B = -q'_A = -q_A \qquad \{\text{charge on } A \text{ remains same}\}$$

$$\Rightarrow \quad \frac{q'_A}{q'_B} = -1$$

Also after earthing,

$$V_A - V_B = \frac{q_A}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{2R}\right) = \frac{q_A}{8\pi\varepsilon_0 R}$$

Substituting $q_B = 2q_A$ in equation (1), we get

$$\frac{1}{4\pi\varepsilon_0}\frac{q_A}{2R} = \frac{V}{2}$$

$$\Rightarrow \quad V_A - V_B = \frac{V}{2}$$

Since B is earthed, so $V_B = 0$ and hence $V_A = \frac{V}{2}$

Hence, (A) and (D) are correct.

22. Potential of innermost shell is zero, so

$$\frac{Q_1}{r} + \frac{Q_2}{2r} + \frac{Q_3}{3r} = 0$$

$$\Rightarrow \quad 6Q_1 + 3Q_2 + 2Q_3 = 0 \quad \ldots(1)$$

Similarly, potential of the outermost shell is also zero. So,

$$\Rightarrow \quad \frac{Q_1}{3r} + \frac{Q_2}{3r} + \frac{Q_3}{3r} = 0$$

$$\Rightarrow \quad Q_1 + Q_3 = -Q_2 \quad \ldots(2)$$

Solving equations (1) and (2), we get

$$Q_1 = -\frac{Q_2}{4}, \ \frac{Q_3}{Q_1} = 3 \text{ and } \frac{Q_3}{Q_2} = -\frac{3}{4}$$

Hence, (A), (B) and (C) are correct.

24. $a = \frac{F}{m} = \frac{qE}{m} = \frac{q}{m}(\alpha - \beta x) \quad \ldots(1)$

$$\Rightarrow \quad a = \frac{q\alpha}{m} - \frac{q\beta}{m}x$$

$$\Rightarrow \quad \ddot{x} = \frac{q\alpha}{m} - \frac{q\beta}{m}x$$

Let $\frac{q\alpha}{m} - \frac{q\beta}{m}x = X$

$$\Rightarrow \quad -\frac{q\beta}{m}\ddot{x} = \ddot{X}$$

$$\Rightarrow \quad \ddot{x} = -\frac{m}{q\beta}\ddot{X}$$

$$\Rightarrow \quad -\frac{m}{q\beta}\ddot{X} = X$$

$$\Rightarrow \quad \ddot{X} + \frac{q\beta}{m}X = 0$$

$$\Rightarrow \quad \ddot{X} + \omega^2 X = 0$$

So, the motion of the particle is oscillatory and we observe that $a = 0$ at $x = \frac{\alpha}{\beta}$

At the mean position, no force acts on the particle

$$\Rightarrow \quad F = 0 \text{ at } x = x_0 = \frac{\alpha}{\beta}$$

Equation (1) can be written as,

$$v \cdot \frac{dv}{dx} = \frac{q}{m}(\alpha - \beta x)$$

$$\Rightarrow \quad \int_0^v v\,dv = \frac{q}{m}\int_0^x (\alpha - \beta x)\,dx$$

$$\Rightarrow \quad v = \sqrt{\frac{2qx}{m}\left(\alpha - \frac{\beta}{2}x\right)}$$

$$v = 0 \text{ at } x = 0 \text{ and } x = \frac{2\alpha}{\beta}$$

So, the particle reverses its direction of motion at $x = \frac{2\alpha}{\beta}$ (extreme position) and hence the particle will oscillate between $x = 0$ to $x = \frac{2\alpha}{\beta}$ with mean position at $x=\dfrac{\alpha}{\beta}$. So, amplitude of particle is $\dfrac{\alpha}{\beta}$ with maximum acceleration of particle at extreme positions (at $x=0$ and $x=\dfrac{2\alpha}{\beta}$ and $a_{max}=\dfrac{q\alpha}{m}$ {from equation (1)}.

Hence, (A), (B), (C) and (D) are correct.

26. The electron will experience a downward acceleration a, given by

$$a=\frac{eE}{m}=\frac{1.6\times10^{-19}\times2000}{9\times10^{-31}}$$

$$\Rightarrow\quad a=\frac{eE}{m}=3.6\times10^{14}\ \text{ms}^{-2}$$

So, we can consider the electron to be a projectile launched with some initial velocity between the plates under influence of downward acceleration a.

From the point of launch let us consider the rightward horizontal as the x-axis and upward vertical as y-axis, let us calculate the height to which the electron will rise and its horizontal range. Considering upward vertical motion, we have

$$0=6\times10^{6}\sin45^\circ t+\frac{1}{2}\times\left(-3.5\times10^{14}\right)t^2$$

$$\Rightarrow\quad t=2.4\times10^{-8}\ \text{s}$$

Considering motion along x-axis, we get

$$x(\text{range})=6\times10^{6}\cos45^\circ\times2.4\times10^{-8}$$

$$\Rightarrow\quad x=0.102\ \text{m}=10.2\ \text{cm}$$

The vertical displacement in half the total time i.e., at maximum height, we have

$$t=\frac{T}{2}$$

The vertical height is given by

$$y=\left(6\times10^{6}\sin45^\circ\right)\left(1.2\times10^{-8}\right)-\frac{1}{2}\left(3.5\times10^{14}\right)\left(1.2\times10^{-8}\right)^2$$

$$\Rightarrow\quad y=0.0509-0.0252$$

$$\Rightarrow\quad y=0.0257=2.57\ \text{cm}$$

So, the electron will not touch the upper plate and will strike the lower plate at the other edge.

Hence, (B) and (C) are correct.

27. $\dfrac{1}{2}mv^2=\dfrac{Q^2}{4\pi\varepsilon_0 d}$

$$\Rightarrow\quad d=\frac{Q^2}{2\pi\varepsilon_0 mv^2}$$

Hence, (A), (B) and (D) are correct.

28. Retardation $a=\dfrac{\text{force}}{\text{mass}}=\dfrac{eE}{m}$

$$\Rightarrow\quad a=\frac{1.6\times10^{-19}\times1000}{9\times10^{-31}}=1.78\times10^{14}\ \text{ms}^{-2}$$

Since, $v^2-v_0^2=2as$

$$\Rightarrow\quad 0^2-\left(5\times10^{6}\right)^2=2\times\left(-1.78\times10^{14}\right)s$$

$$\Rightarrow\quad s=0.07\ \text{m}$$

Now, $v=v_0+at$

$$\Rightarrow\quad 0=5\times10^{6}-\left(1.78\times10^{14}\right)t$$

$$\Rightarrow\quad t=2.8\times10^{-8}\ \text{s}=0.03\ \mu\text{s}$$

Loss of energy = work done $W=Fd=(eE)d$

$$\Rightarrow\quad \text{Fraction } f=\frac{eEd}{\frac{1}{2}mv^2}=\frac{2eEd}{mv^2}$$

$\Rightarrow$ %age Fraction is

$$f=\frac{2\times1.6\times10^{-19}\times1000\times0.8\times10^{-2}}{9\times10^{-31}\times\left(5\times10^{6}\right)^2}=11\%$$

Hence, (B), (C) and (D) are correct.

30. The potential due to a pair of rings each of radius R and carrying charges $+q$ and $-q$ at a distance x from the mid point along the axis is given by

$$V=\frac{q\ell x}{4\pi\varepsilon_0\left(R^2+x^2\right)^{3/2}}$$

Now, consider a pair of rings of radius z and thickness dz. If dq be the infinitesimal charge on each ring, then

$$dq=2\pi z\sigma dz$$

and if dV be the potential due to this infinitesimal pair of elements at P

$$\Rightarrow\quad dV=\frac{(2\pi z\sigma dz)\ell x}{4\pi\varepsilon_0\left(z^2+x^2\right)^{3/2}}$$

$$\Rightarrow\quad dV=\frac{\sigma\ell x}{2\varepsilon_0}\frac{zdz}{\left(z^2+x^2\right)^{3/2}}$$

$$\Rightarrow\quad V=\frac{\sigma\ell x}{2\varepsilon_0}\int_R^\infty\frac{zdz}{\left(z^2+x^2\right)^{3/2}}$$

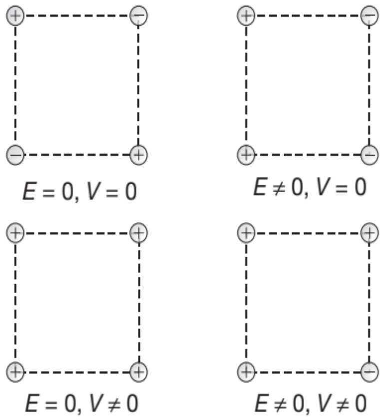
$$\Rightarrow\quad V=\frac{\sigma\ell x}{2\varepsilon_0\left(R^2+x^2\right)^{1/2}}$$

Since, $E=-\dfrac{\partial V}{\partial x}$

$$\Rightarrow \quad E = -\frac{\sigma \ell R^2}{2\varepsilon_0 (R^2 + x^2)^{3/2}}$$

Hence, (A) and (B) are correct.

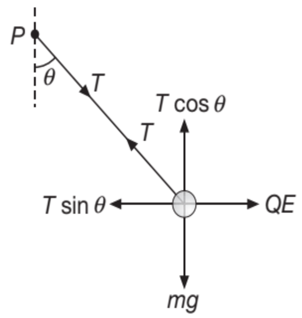
31.



Hence, (A), (B), (C) and (D) are correct.

32. Equating the forces in the vertical and horizontal directions

$$T\cos\theta = mg \text{ and } T\sin\theta = F = QE$$



Dividing, $\tan\theta = \frac{QE}{mg} = \frac{2\times10^{-8}\times2\times10^{4}}{71\times10^{-6}\times9.8}$

$$\Rightarrow \quad \tan\theta = 0.57 \cong \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = 30^\circ$$

Now, $T = \frac{71\times10^{-6}\times9.8}{\cos(30^\circ)} = 803\ \mu\text{N}$

Hence, (C) and (D) are correct.

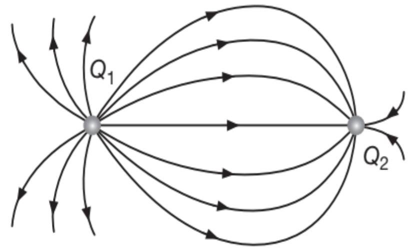
34. The given graph is of charged conducting sphere of radius. The entire charge distributes on the surface of the sphere.

Hence, (A), (B), (C) and (D) are correct.

36. Torque about Q of charge $-q$ is zero, so angular momentum charge $-q$ is constant, but distance between charges is changing, so force is charging, so speed and velocity are changing.

Hence, option (A) is correct.

38.



From the diagram, it can be observed that Q_1 is positive, Q_2 is negative.

Number of lines on Q_1 is greater and number of lines is directly proportional to magnitude of charge.

So, $|Q_1| > |Q_2|$

Electric field will be zero to the right of Q_2 as it has small magnitude and opposite sign to that of Q_1.

Hence, (A) and (D) are correct.

40. Using Work Energy Theorem, we get

$$\frac{1}{2}mv^2 = qE(\ell - \ell\cos(60^\circ))$$

$$\Rightarrow \quad v = \sqrt{\frac{qE\ell}{m}}$$

So, OPTION (B) is correct.

At point B, we have

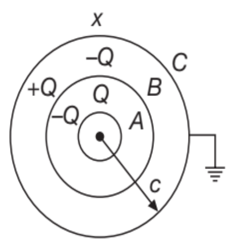
$$T - qE = \frac{mv^2}{\ell}$$

$$\Rightarrow \quad T = qE + \frac{m}{\ell}\left(\sqrt{\frac{qE\ell}{m}}\right)^2 = 2qE$$

So, OPTION (D) is also correct.

Hence, (B) and (D) are correct.

41. The distribution of charges is shown in figure. We see that the inner surface of B and C possess the same charge $-Q$. Also if x is charge on outer surface of C, then



$$V_C = 0$$

$$\Rightarrow \quad \frac{Q}{4\pi\varepsilon_0 c} + \left(\frac{-Q+Q}{4\pi\varepsilon_0 c}\right) + \frac{-Q+x}{4\pi\varepsilon_0 c} = 0$$

$$\Rightarrow \quad x = 0$$

Hence, (A) and (D) are correct.

CHAPTER 1

42. $\vec{E} = \dfrac{Q_1}{4\pi\varepsilon_0\left(2\dfrac{\sqrt{5}}{2}\right)^3}\left(\dfrac{a}{2}\hat{i}+a\hat{j}\right)+\dfrac{Q_2}{4\pi\varepsilon_0\left(a\dfrac{\sqrt{5}}{2}\right)^3}\left(-\dfrac{a}{2}\hat{i}+a\hat{j}\right)$

Since, x component is zero, so

$Q_1 = Q_2$

Hence, (A) and (C) are correct.

43. Now electric field outside the sphere is due to charge on outer surface of sphere as E due Q and $-Q$ (induced charge) gets cancelled. Also E due to conducting sphere for from it will be zero

Hence, (B) and (D) are correct.

44. If $q_1 = q_2 = Q$, then

$F_i \propto q_1 q_2$

$\Rightarrow \quad F_i \propto Q^2$

After touching, new charges on each ball is

$\dfrac{Q_1+Q_2}{2} = \dfrac{2Q}{2} = Q$

$F_{\text{new}} \propto Q^2$

$\Rightarrow \quad F_i = F_{\text{new}}$

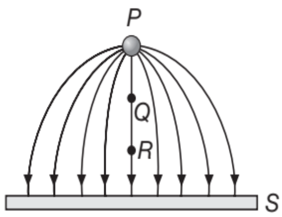
If $q_1 \neq q_2$, then

$F_{\text{new}} = \left(\dfrac{q_1+q_2}{2}\right)^2 = \dfrac{q_1^2}{4}+\dfrac{q_2^2}{4}+\dfrac{2q_1q_2}{4}$

$\Rightarrow \quad F_{\text{new}} > F_i$

Hence, (A) and (B) are correct.

46. As we are moving away from P towards sheet S spacing between electric lines of force is increasing.



$\Rightarrow \quad E_R < E_Q$

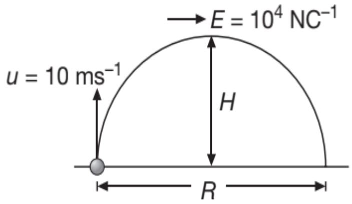
Also, in the direction of electric field potential decreases.

$\Rightarrow \quad V_R < V_Q$

Hence, (A) and (C) are correct.

47. Time of flight $(t) = \dfrac{2u}{g} = \dfrac{2\times 10}{10} = 2$ sec.

$H = \dfrac{u^2}{2g} = \dfrac{10\times 10}{2\times 10} = 5$ m



$R = 0 + \dfrac{1}{2}\left(\dfrac{qE}{m}\right)t^2$

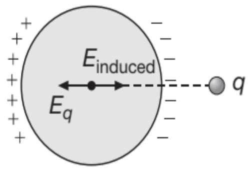
$\Rightarrow \quad R = \dfrac{1}{2}\left(\dfrac{10^{-3}\times 10^4\times 2\times 2}{2}\right) = 10$ m

Hence, (A), (B) and (C) are correct.

50. Charge is distributed over the surface of conductor in such a way that net field due to the charges inside and outside the conductor is zero inside. Field only due to the charge q inside the conductor is non-zero

Hence, (A), (B) and (C) are correct.

51.



Inside the conducting sphere, we have $E_{\text{net}} = 0$

Since, $E_{\text{net}} = E_{\text{induced}} - E_{\text{due to q}}$

$\Rightarrow \quad E_{\text{due to } q} = \dfrac{kq}{d^2} = E_{\text{induced}}$

Hence, (A) and (D) are correct.

52. At a distance h above the sheet

$E = E_{\text{sheet}} + E_{\text{slab}}$

$\Rightarrow \quad E = \dfrac{-\sigma}{2\varepsilon_0}+\dfrac{\rho D}{2\varepsilon_0} = \dfrac{\rho D-\sigma}{2\varepsilon_0}$

At a distance h below the top surface of slab, we have

$E_{\text{slab}} = \dfrac{\rho(D-2h)}{2\varepsilon_0}$

$\Rightarrow \quad E = E_{\text{sheet}} + E_{\text{slab}} = \dfrac{\sigma}{2\varepsilon_0}+\dfrac{\rho(D-2h)}{2\varepsilon_0}$

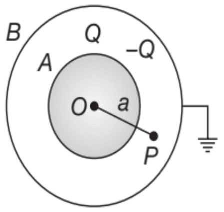
$\Rightarrow \quad E = \dfrac{\sigma+\rho(D-2h)}{2\varepsilon_0}$

At a distance h below the bottom surface of the slab

$E = \dfrac{-\sigma}{2\varepsilon_0}+\dfrac{\rho D}{2\varepsilon_0} = \dfrac{\rho D-\sigma}{2\varepsilon_0}$

Hence, (A), (B) and (D) are correct.

53.



$$E = \frac{Q}{4\pi\varepsilon_0 r^2}, \text{ when } a \le r \le b$$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r} - \frac{Q}{b}\right), \text{ where } a \le r \le b$$

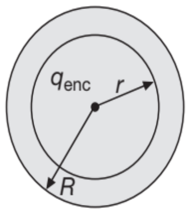
Potential of B is $V_B = 0$

Potential of A is $V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{a} - \frac{Q}{b}\right)$

So, potential difference is $V_A - V_B = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$

Hence, (A), (C) and (D) are correct.

54.

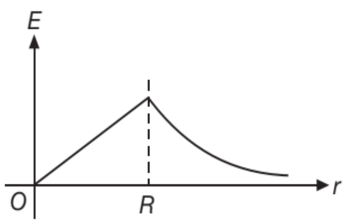


$$E(4\pi r^2) = \frac{q_{enc}}{\varepsilon_0} = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho r}{3\varepsilon_0}$$

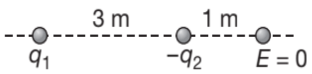
Slope of E-r graph (inside) i.e. for $r < R$ is $\frac{\rho}{3\varepsilon_0}$.

$\Rightarrow$ Slope $\propto \rho$



Hence, (B) and (C) are correct.

56.

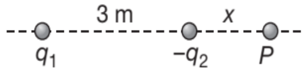


Since $\frac{kq_2}{1^2} = \frac{kq_1}{16}$

$\Rightarrow \quad q_1 = 16q_2$

Now, potential can be zero both inside and outside the charges on the line joining then

For a point P lying outside the two charges



$$V_{\text{at } P} = 0$$

$$\Rightarrow \quad -\frac{kq_2}{x} + \frac{kq_1}{3+x} = 0$$

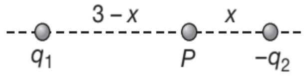
$$\Rightarrow \quad +\frac{q_2}{x} = \frac{16q_2}{3+x}$$

$$\Rightarrow \quad 3 + x = 16x$$

$$\Rightarrow \quad 3 = 15x$$

$$\Rightarrow \quad x = \frac{1}{5} = 0.2 \text{ m}$$

For a point P lying between the charges $V_{\text{at } P} = 0$



$$\Rightarrow \quad \frac{kq_1}{3-x} = \frac{kq_2}{x}$$

$$\Rightarrow \quad 16x = 3 - x$$

$$\Rightarrow \quad x = \frac{3}{17} \text{ meter}$$

Hence, (B) and (C) are correct.

58. $qEx_{\max} = \frac{1}{2}kx_{\max}^2$

$$\Rightarrow \quad x_{\max} = \frac{2qE}{k}$$

At Equilibrium, we have

$$qE = kx_{eq}$$

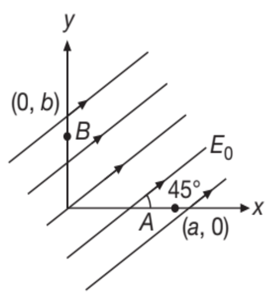
$$\Rightarrow \quad x_{eq} = \frac{qE}{k}$$

Amplitude of oscillation is equal to the displacement of block from the mean position, So, amplitude is

$$A = \frac{2qE}{k} - \frac{qE}{k} = \frac{qE}{k}$$

Hence, (A), (B) and (C) are correct.

59. When E_0 makes an angle of $45°$ with x-axis, then



$$\vec{E} = \frac{E_0\hat{i}}{\sqrt{2}} + \frac{E_0\hat{j}}{\sqrt{2}}$$

Now, $V_A - V_B = -\vec{E} \cdot \overrightarrow{BA}$

$$\Rightarrow \quad V_A - V_B = -\left(\frac{E_0}{\sqrt{2}}\hat{i} + \frac{E_0}{\sqrt{2}}\hat{j}\right) \cdot \left(+a\hat{i} - b\hat{j}\right)$$

$$\Rightarrow \quad V_A - V_B = -\frac{E_0 a}{\sqrt{2}} + \frac{E_0 b}{\sqrt{2}} = \frac{E_0}{\sqrt{2}}(b - a)$$

If $a = b$

$\Rightarrow \quad V_A - V_B = 0$

If $a < b$

$\Rightarrow \quad V_A - V_B > 0$

$\Rightarrow \quad V_A > V_B$

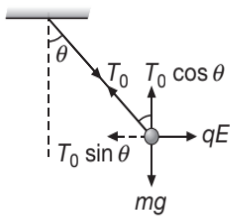
If $a > b$

$\Rightarrow \quad V_A - V_B < 0$

$\Rightarrow \quad V_A < V_B$

Hence, (B), (C) and (D) are correct.

60.



$$E = \frac{\sigma}{2\varepsilon_0}$$

Since, $T_0 \cos\theta = mg$

$T_0 \sin\theta = qE$

$$\Rightarrow \quad \tan\theta = \frac{q}{mg}\left(\frac{\sigma}{2\varepsilon_0}\right)$$

$$\Rightarrow \quad T_0 = \sqrt{(mg)^2 + (qE)^2}$$

$\Rightarrow \quad T_0 > mg$

i.e. the effective value of g is increased, hence time period T of oscillation decreases.

Hence, (A) and (D) are correct.

61. Near positive charge net potential is positive and near negative charge potential is negative. Q_1 is positive & Q_2 is negative, at 1 potential is zero and this point is near to Q_2. So, magnitude of Q_1 is more than magnitude of Q_2. Also, slope of $V - r$ graph gives E, which is zero at 3.

Hence, (A), (C) and (D) are correct.

Reasoning Based Questions

1. Coulombic attraction exists even when one body is charged and the other is uncharged.

Hence, the correct answer is (C).

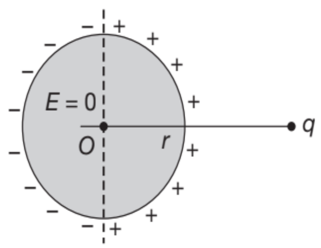
2. Force of interaction between two charge is independent of presence of other charge.

Hence, the correct answer is (B).

3. The rate of decrease of electric field is different in the two cases. In case of a point charge, it decreases as $\frac{1}{r^2}$ but in the case of electric dipole it decreases more rapidly, as $E = \frac{1}{r^2}$.

Hence, the correct answer is (D).

4. $$V_0 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{\sum \text{Induced charge}}{r}\right)$$



$$\Rightarrow \quad V_0 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} + 0$$

Hence, potential is constant

Hence, the correct answer is (C).

6. Electric field due to line charge $E = \frac{\lambda}{4\pi\varepsilon_0 r}(\sin\alpha + \sin\beta)$

Hence, the correct answer is (D).

7. Electron being a negative charge, will move from lower to higher potential.

Hence, the correct answer is (D).

10. Since field is zero for a charged conductor on the surface and inside it also, so the surface of a charged conductor is always equipotential. Also, $\Delta V = 0$ for equipotential surface. Hence $\int \vec{E} \cdot d\vec{\ell} = 0$

$\Rightarrow \quad \vec{E} \perp d\vec{\ell}$. Hence both are true and Statement-2 is the correct explanation to Statement-1.

Hence, the correct answer is (A).

12. Path of charged particle is parabolic.

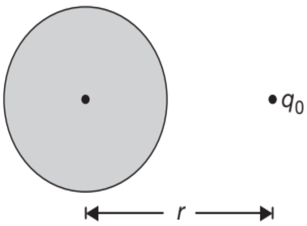
Hence, the correct answer is (C).

13. Inside the conductor, charged atoms are also present.
Hence, the correct answer is (B).

14. See theory part in detail to get the answer.
Hence, the correct answer is (B).

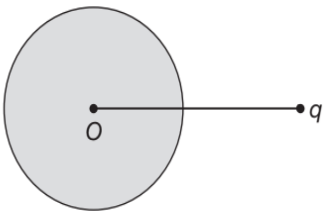
15. Statement-1 is also practical experience based; so it is true.
Statement-2 is also true but is not the correct explanation of Statement-1. Correct explanation is "there is increase in normal reaction when the object is pushed and there is decrease in normal reaction when object is pulled".
Hence, the correct answer is (B).

16. Electric field due to q_0 is towards left and is $\frac{q_0}{4\pi\varepsilon_0 r^2}$ but electric field due to induced charge is towards right and will have same magnitude $\frac{q_0}{4\pi\varepsilon_0 r^2}$ so that electric field inside the sphere is zero.



Hence, the correct answer is (D).

17.



At point O

$$\vec{E}_{\text{net}} = \vec{E}_{\text{in}} + \vec{E}_q$$

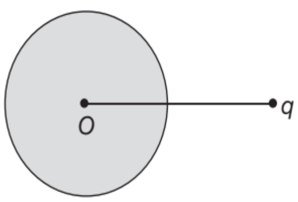
$$\vec{E}_{\text{net}} = 0$$

$$\Rightarrow \quad E_{in} + E_q$$

$$\Rightarrow \quad E_q = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \neq 0$$

Hence, the correct answer is (D).

18.



$$V_0 = \frac{q}{4\pi\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0}\left(\frac{\int dq_{\text{induced}}}{R}\right)$$

$$\Rightarrow \quad V_0 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad \left\{\because \int dq_{\text{inside}} = 0\right\}$$

Hence, the correct answer is (A).

19. In a hollow spherical shield, the charge is present only on its surface but charge is zero at every point inside the hollow sphere. Hence, the metallic shield in form of hollow shell may be built to block an electric field.
Hence, the correct answer is (A).

20. Apply the concept of electric image.
Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. The magnitude of the force is given by

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

Now we can substitute our numerical values and find that the magnitude of the force between the proton and the electron in the hydrogen atom is

$$F_e = \frac{(9.0\times10^9\ \text{Nm}^2\text{C}^{-2})(1.6\times10^{-19}\ \text{C})^2}{(5.3\times10^{-11}\ \text{m})^2}$$

$$F_e = 8.2\times10^{-8}\ \text{N}$$

Hence, the correct answer is (C).

2. The magnitude of the electric field due to the proton is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{(90\times10^9\ \text{Nm}^2\text{C}^{-2})(1.6\times10^{-19}\ \text{C})}{(0.5\times10^{-10}\ \text{m})^2}$$

$$E = 5.76\times10^{11}\ \text{NC}^{-1}$$

Hence, the correct answer is (B).

3. The mass of the electron is $m_e = 9.1\times10^{-31}$ kg and the mass of the proton is $m_p = 1.7\times10^{-27}$ kg. Thus, the ratio of the magnitudes of the electric and gravitational force is given by

$$\text{Ratio} = \frac{\left(\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\right)}{\left(G\frac{m_p m_e}{r^2}\right)} = \frac{\frac{1}{4\pi\varepsilon_0}e^2}{Gm_p m_e}$$

$$\text{Ratio} = \frac{(9\times 10^{9})(1.6\times 10^{-19})^2}{(6.67\times 10^{-11})(1.7\times 10^{-27})(9.1\times 10^{-31})}$$

$$\text{Ratio} = 2.2\times 10^{39}$$

which is independent of r, the distance between the proton and the electron.

Hence, the correct answer is (C).

4. $M = \rho_{\text{oil}}V = \rho_{\text{oil}}\left(\frac{4}{3}\pi r^3\right)$

where the oil drop is assumed to be a sphere of radius r with volume $V = \frac{4\pi r^3}{3}$

$$\Rightarrow \quad M = \rho_{\text{oil}}\left(\frac{4}{3}\pi r^3\right)$$

$$\Rightarrow \quad M = (8.51\times 10^{2}\ \text{kgm}^{-3})\left(\frac{4\pi}{3}\right)(1.64\times 10^{-6}\ \text{m})^3$$

$$\Rightarrow \quad M = 1.57\times 10^{-14}\ \text{kg}$$

Hence, the correct answer is (A).

5. The oil drop will be in static equilibrium when the gravitational force exactly balances the electrical force

$$\Rightarrow \quad \vec{F}_g + \vec{F}_e = 0$$

Since the gravitational force points downward so, the electric force on the oil must be acting upwards. So, we get

$$0 = m\vec{g} + q\vec{E}$$

$$\Rightarrow \quad mg = -qE$$

With the electrical field pointing downwards, we conclude that the charge on the oil drop must be negative.

$$q = -\frac{mg}{E_y}$$

$$q = -\frac{(1.57\times 10^{-14}\ \text{kg})(9.80\ \text{ms}^{-2})}{1.92\times 10^{5}\ \text{NC}^{-1}}$$

$$q = -8.03\times 10^{-19}\ \text{C}$$

Since the electron has charge $e = 1.6\times 10^{-19}\ \text{C}$, the charge of the oil drop in units of e is

$$N = \frac{q}{e} = \frac{8.02\times 10^{-19}\ \text{C}}{1.6\times 10^{-19}\ \text{C}} = 5$$

Hence, the correct answer is (D).

6. Since the electron has a negative charge, $q = -e$, the force on the electron is

$$\vec{F}_e = q\vec{E} = -e\vec{E} = (-e)(-E)\hat{j} = eE\hat{j}$$

where the electric field is written as $\vec{E} = -E\hat{j}$, with $F_y > 0$. The force on the electron is upward. Note that the motion of the electron is analogous to the motion of a mass that is thrown horizontally in a constant gravitational field. The mass follows a parabolic trajectory downward. Since the electron is negatively charged, the constant force on the electron is upward and the electron will be deflected upwards on a parabolic path.

Hence, the correct answer is (D).

7. The acceleration of the electron is

$$\vec{a} = \frac{q\vec{E}}{m} = -\frac{qE}{m}\hat{j} = \frac{eE}{m}\hat{j}$$

and its direction is upward.

The time of passage for the electron is given by $t_1 = \frac{\ell}{v_0}$.

The time t_1 is not affected by the acceleration because v_0, the horizontal component of the velocity which determines the time is not affected by the field.

Hence, the correct answer is (B).

8. The electron has an initial horizontal velocity, $\vec{v}_0 = v_0\hat{i}$. Since the acceleration of the electron is in the $+y$ direction, only the y-component of the velocity changes. The velocity at a later time t_1 is given by

Since $\vec{v} = v_x\hat{i} + v_y\hat{j} = v_0\hat{i} + a_y t_1\hat{j}$

$$\Rightarrow \quad \vec{v} = v_0\hat{i} + \left(\frac{eE}{m}\right)t_1\hat{j}$$

$$\Rightarrow \quad \vec{v} = v_0\hat{i} + \left(\frac{eE}{m}\right)\hat{j}$$

Hence, the correct answer is (C).

9. From the figure (in the question), we see that the electron travels a horizontal distance ℓ in the time $t_1 = \frac{\ell}{v_0}$ and then emerges from the plates with a vertical displacement y_1. Then

$$y_1 = \frac{1}{2}a_y t_1^2 = \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{\ell}{v_0}\right)^2 = \frac{eE\ell^2}{2mv_0^2}$$

Hence, the correct answer is (C).

10. When the electron leaves the plates at time t_1, the electron makes an angle θ with the horizontal given by

$$\tan\theta = \frac{v_y}{v_x} = \frac{\left(\frac{eE}{m}\right)\left(\frac{\ell}{v_0}\right)}{v_0} = \frac{eE\ell}{mv_0^2}$$

Hence, the correct answer is (C).

11. After the electron leaves the plate, there is no longer any force on the electron so it travels in a straight path. The deflection y_2 is

$$y_2 = L\tan\theta = \frac{eE\ell L}{mv_0^2}$$

and the total deflection becomes

$$y = y_1 + y_2 = \frac{eE\ell^2}{2mv_0^2} + \frac{eE\ell L}{mv_0^2} = \frac{eE\ell}{mv_0^2}\left(\frac{\ell}{2} + L\right)$$

Hence, the correct answer is (B).

12. Since, $E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{36\times10^{-9}\ \text{Cm}^{-2}}{8.85\times10^{-12}\ \text{C}^2\text{Nm}^{-2}} = 4.07\ \text{kNC}^{-1}$

and $V = Ed$

So, the potential difference between the plates is

$$V = Ed = \left(4.07\times10^3\ \text{NC}^{-1}\right)(0.12\ \text{m})$$

$$V = 488\ \text{V}$$

Hence, the correct answer is (D).

13. $\dfrac{1}{2}mv^2 = qV$

$$\Rightarrow \quad KE = \left(1.6\times10^{-19}\right)(488) = 7.81\times10^{-17}\ \text{J}$$

Hence, the correct answer is (B).

14. Since $\dfrac{1}{2}mv^2 = qV$

$$\Rightarrow \quad v = \sqrt{\frac{2qV}{m}}$$

$$\Rightarrow \quad v = 306\ \text{ms}^{-1}$$

Hence, the correct answer is (C).

15. Since $v^2 - u^2 = 2a(\Delta x)$

$$\Rightarrow \quad \left(3.06\times10^5\right)^2 = 2a(0.12)$$

$$\Rightarrow \quad a = 3.9\times10^{11}\ \text{ms}^{-2}$$

Hence, the correct answer is (A).

16. Since, we know that $F = ma$

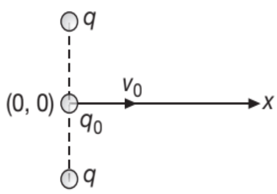
$$\Rightarrow \quad F = \left(1.67\times10^{-27}\ \text{kg}\right)\left(3.90\times10^{11}\ \text{ms}^{-2}\right)$$

$$F = 6.51\times10^{-16}\ \text{N}$$

Hence, the correct answer is (A).

17. For q_0, we have $(U+K)_{\text{at }0} = (U+K)_{\text{at }\infty}$

$$\Rightarrow \quad \frac{2qq_0}{4\pi\varepsilon_0 a} + 0 = \frac{1}{2}mv_0^2 + 0$$



$$\Rightarrow \quad v_0 = \sqrt{\frac{qq_0}{\pi\varepsilon_0 ma}}$$

Hence, the correct answer is (B).

18. $v = \dfrac{v_0}{2}$

$$(U+K)_{\text{initial}} = (U+K)_{\text{final}}$$

Here $U_{\text{initial}} \to 0$ and $K_{\text{initial}} = \dfrac{1}{2}mv^2$

where as $U_{\text{final}} = \dfrac{qq_0}{4\pi\varepsilon_0\sqrt{x^2+a^2}}$ and $K_{\text{final}} = 0$

$$\Rightarrow \quad 0 + \frac{1}{2}m\left(\frac{v_0^2}{4}\right) = \frac{0 + 2qq_0}{4\pi\varepsilon_0\sqrt{x^2+a^2}}$$

$$\frac{1}{2}m\left(\frac{qq_0}{4\pi\varepsilon_0 ma}\right) = \frac{qq_0}{2\pi\varepsilon_0\sqrt{x^2+a^2}}$$

$$\Rightarrow \quad \sqrt{x^2+a^2} = 4a$$

$$\Rightarrow \quad x = \sqrt{15}\,a$$

Hence, the correct answer is (D).

19. $0 + 0 = -\dfrac{2qq_0}{4\pi\varepsilon_0 a} + \dfrac{1}{2}mv^2$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{qq_0}{2\pi\varepsilon_0 a}$$

$$\Rightarrow \quad v = v_0 = \sqrt{\frac{qq_0}{\pi\varepsilon_0 ma}}$$

Hence, the correct answer is (C).

20. The forces of interaction, the force of gravity and Coulomb force are conservative, so we can apply the Law of Conservation of Energy. We have

$$GPE_i + EPE_i = GPE_f + KE_f + EPE_f$$

$$m_1gh + \frac{Q_1Q_2}{4\pi\varepsilon_0 h} = m_1gh_1 + \frac{m_1v^2}{2} + \frac{Q_1Q_2}{4\pi\varepsilon_0 h_1}$$

On solving for v, we get

$$v = \sqrt{2g\left(h - h_1\right) - \frac{2Q_1Q_2\left(h - h_1\right)}{4\pi\varepsilon_0 mhh_1}} \qquad \ldots(1)$$

Assuming that $h_1 \ll h$, equation (1) reduces to

$$v = \sqrt{2gh - \frac{2Q_1Q_2}{4\pi\varepsilon_0 m_1h_1}} = \sqrt{2gh - \frac{Q_1Q_2}{2\pi\varepsilon_0 m_1h_1}} \qquad \ldots(2)$$

Hence, the correct answer is (A).

CHAPTER 1

21. Substituting $v = 0$ in (2), and replacing h_1 by h_2, we get

$$Q_2 = \frac{4\pi\varepsilon_0 g m_1 h h_2}{Q_1}$$

Hence, the correct answer is (B).

22. The object m_1 will be in equilibrium where gravitational and Coulombic force balance,

i.e., $\frac{Q_1 Q_2}{4\pi\varepsilon_0 h_3^2} = m_1 g$

$$\Rightarrow \quad h_3 = \sqrt{\frac{Q_1 Q_2}{4\pi\varepsilon_0 m_1 g}}$$

If the object is displaced upward, the force of gravity will restore it to equilibrium position and if displacement is downward. Coulombic force will be restoring. Thus the motion of the object will be periodic and oscillatory. Motion will be simple harmonic for very small displacement from equilibrium position.

Hence, the correct answer is (C).

23. Let the charges on A and C be q_A and q_C respectively . From conservation of charge, we have

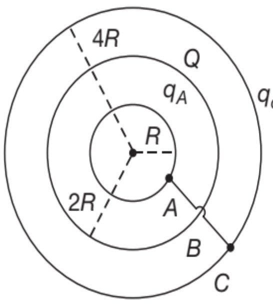
$$q_A + q_C = 0$$

Hence $q_A = -q_C$

Since A and C are connected by a conducting wire, so they have same potential. So, potential of A and C is given by

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q_A}{R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{q_C}{4R}$$

$$V_C = \frac{1}{4\pi\varepsilon_0}\frac{q_A}{4R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{4R} + \frac{1}{4\pi\varepsilon_0}\frac{q_C}{4R}$$



Equalising the potentials of A and C, we get

$$V_A = V_C$$

$$\frac{q_A}{R} + \frac{Q}{2R} + \frac{q_C}{4R} = \frac{q_A}{4R} + \frac{Q}{4R} + \frac{q_C}{4R}$$

$$\Rightarrow \quad 4q_A + 2Q = q_A + Q$$

$$q_A = -\frac{Q}{3}$$

Hence, $q_C = \frac{Q}{3}$

Hence, the correct answer is (D).

24. $V_A = \frac{1}{4\pi\varepsilon_0 R}\left[-\frac{Q}{3} + \frac{Q}{2} + \frac{Q}{12}\right] = \frac{Q}{16\pi\varepsilon_0 R}$

Hence, the correct answer is (C).

25. $V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_A}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{q_C}{4R}$

$$V_B = \frac{1}{8\pi\varepsilon_0 R}\left[-\frac{Q}{3} + Q + \frac{Q}{6}\right] = \frac{5Q}{48\pi\varepsilon_0 R}$$

Hence, the correct answer is (D).

26. $[\alpha] = \left[\frac{\lambda}{x}\right] = \left[\frac{C}{m}\left(\frac{1}{m}\right)\right] = \left[\frac{C}{m^2}\right] = M^0 L^{-2} T A$

Hence, the correct answer is (B).

27. $V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda dx}{r} = \frac{1}{4\pi\varepsilon_0}\alpha \int_0^L \frac{x\,dx}{d+x}$

$$V = \frac{1}{4\pi\varepsilon_0}\alpha\left[L - d\log_e\left(1 + \frac{L}{d}\right)\right] = \frac{\alpha}{4\pi\varepsilon_0}[L - \log_e 5]$$

Hence, the correct answer is (A).

28. $V = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int \frac{\alpha x\,dx}{\sqrt{b^2 + \left(\frac{L}{2} - x\right)^2}}$

Consider $z = \frac{L}{2} - x$

Then $x = \frac{L}{2} - z$, and $dx = -dz$

$$V = \frac{\alpha}{4\pi\varepsilon_0}\int \frac{\left(\frac{L}{2} - z\right)(-dz)}{\sqrt{b^2 + z^2}}$$

$$V = -\frac{\alpha L}{8\pi\varepsilon_0}\int \frac{dz}{\sqrt{b^2 + z^2}} + \frac{\alpha}{4\pi\varepsilon_0}\int \frac{z dz}{\sqrt{b^2 + z^2}}$$

$$\Rightarrow \quad V = -\frac{\alpha L}{8\pi\varepsilon_0}\log_e\left(z + \sqrt{z^2 + b^2}\right) + \frac{\alpha}{4\pi\varepsilon_0}\sqrt{z^2 + b^2}$$

$$\Rightarrow \quad V = -\frac{\alpha L}{8\pi\varepsilon_0}\log_e\left[\left(\frac{L}{2}-x\right)+\sqrt{\left(\frac{L}{2}-x\right)^2+b^2}\right]\Bigg|_0^L +$$

$$\frac{\alpha}{4\pi\varepsilon_0}\sqrt{\left(\frac{L}{2}-x\right)^2+b^2}\Bigg|_0^L$$

$$\Rightarrow \quad V = -\frac{\alpha L}{8\pi\varepsilon_0}\log_e\left[\frac{\frac{L}{2}-L+\sqrt{\left(\frac{L}{2}\right)^2+b^2}}{\frac{L}{2}+\sqrt{\left(\frac{L}{2}\right)^2+b^2}}\right]+$$

$$\frac{\alpha}{4\pi\varepsilon_0}\left[\sqrt{\left(\frac{L}{2}-L\right)^2+b^2}-\sqrt{\left(\frac{L}{2}\right)^2+b^2}\right]$$

$$\Rightarrow \quad V = -\frac{\alpha L}{8\pi\varepsilon_0}\log_e\left[\frac{\sqrt{b^2+\left(\frac{L^2}{4}\right)}-\frac{L}{2}}{\sqrt{b^2+\left(\frac{L^2}{4}\right)}+\frac{L}{2}}\right]$$

But at $b=\frac{\sqrt{3}L}{2}$, we get

$$V = -\frac{\alpha L}{8\pi\varepsilon_0}\log_e\left[\frac{\left(\frac{L}{2}\right)}{\left(\frac{3L}{2}\right)}\right] = -\frac{\alpha L}{8\pi\varepsilon_0}\log_e\left(\frac{1}{3}\right)$$

$$\Rightarrow \quad V = \frac{\alpha L}{8\pi\varepsilon_0}\log_e(3)$$

Hence, the correct answer is (C).

29. **The correct answer is (B).**

30. **The correct answer is (A).**

31. **The correct answer is (D).**

32. **The correct answer is (C).**

Combined solution to 29, 30, 31, 32

In *ab*

$$V+10 = 25(x+3)$$

$$\Rightarrow \quad V = 25x+65 \qquad \ldots(1)$$

$$\Rightarrow \quad E_x = -\frac{dV}{dx} = -25 \text{ Vm}^{-1}$$

$$\Rightarrow \quad |E_x| = 25$$

In *bc*

$$V-15 = -\frac{10}{3}(x+2)$$

$$\Rightarrow \quad V = -\frac{10}{3}x+\frac{25}{3}$$

$$\Rightarrow \quad E_x = -\frac{dV}{dx} = \frac{10}{3}\text{ Vm}^{-1}$$

In *cd*

$$V = \text{constant } = 5\text{ V}$$

$$\Rightarrow \quad E = 0$$

In *de*

$$V-10 = 5(x-3)$$

$$\Rightarrow \quad V = 5x-5$$

$$\Rightarrow \quad E_x = -\frac{dV}{dx} = -5\text{ V}$$

Hence, the correct answer is (D).

33. In the region ab,

$$V+10 = 25(x+3)$$

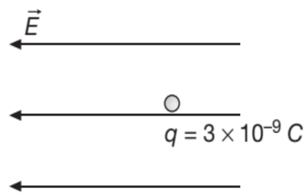
$$\Rightarrow \quad V = 25x+65$$

So, for $V=0$, we get

$$x = -\frac{65}{25} = -\frac{13}{5}\text{ m}$$

Hence, the correct answer is (C).

34. Since, work done by electric forces on a charge particle is the gain in K.E.



Work done $=\Delta$K.E.

Thus work done $=4.5\times10^{-5}$ J

Hence, the correct answer is (B).

35. Work done is also equal to qE

$$\Rightarrow \quad 4.5\times10^{-5} = 3\times10^{-9}\times5\times10^{-2}\times E$$

$$\Rightarrow \quad E = 3\times10^{5}\text{ NC}^{-1}$$

Hence, the correct answer is (A).

36. By energy conservation $q(V_1-V_2)=\frac{1}{2}mv^2$

$$(V_1-V_2) = \frac{\frac{1}{2}mv^2}{q}$$

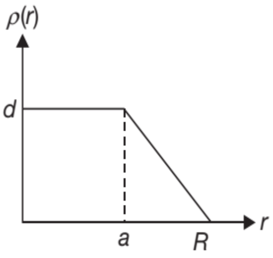
$$(V_1-V_2) = \frac{4.5\times10^{-5}}{3\times10^{-9}}$$

$$(V_1 - V_2) = 1.5 \times 10^4 \text{ V}$$

Hence, the correct answer is (D).

37. Electric field at $r = R$ is

$$E = \frac{Q}{4\pi\varepsilon_0 R^2}$$



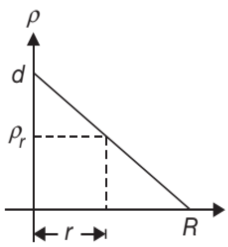
Q = Total charge within the nucleus $= Ze$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Ze}{R^2}$$

Hence the electric field is independent of a.

Hence, the correct answer is (A).

38. $Q = \int \rho_r 4\pi r^2 dr$ for $a = 0$, and the figure for the distribution looks like as shown



$$\text{Slope } = \frac{d}{R} = \frac{\rho_r}{R - r}$$

$$\Rightarrow \quad \rho_r = \frac{d}{R}(R - r)$$

$$\Rightarrow \quad Q = \int_0^R \frac{d}{R}(R - r) 4\pi r^2 dr$$

$$\Rightarrow \quad Q = \frac{4\pi d}{R}\left(R\int_0^R r^2 dr - \int_0^R r^3 dr\right) = \frac{4\pi d}{R}\left(\frac{R^4}{3} - \frac{R^4}{4}\right) =$$

$$\Rightarrow \quad Q = \frac{\pi d R^3}{3}$$

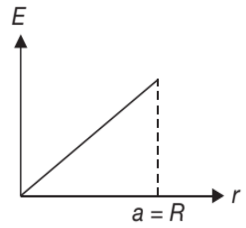
$$\Rightarrow \quad Q = Ze = \frac{\pi d R^3}{3} \qquad \{\because Q = Ze\}$$

$$\Rightarrow \quad d = \frac{3Ze}{\pi R^3}$$

Hence, the correct answer is (B).

39. Let us review the formula of uniformly (volume) charged solid sphere according to which we have

$$E = \frac{\rho r}{3\varepsilon_0}$$



$\Rightarrow$ $E \propto r$, ρ should be constant throughout the volume of nucleus.

This will be possible only when $a = R$.

Hence, the correct answer is (C).

40. $$Q_1 = \int_0^{R/2} \rho(r) 4\pi r^2 dr = \alpha \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{\pi\alpha R^3}{6}$$

$$Q_2 = \int_{R/2}^{R} \rho(r) 4\pi r^2 dr$$

$$\Rightarrow \quad Q_2 = \int_{R/2}^{R} 2\alpha\left(1 - \frac{r}{R}\right) 4\pi r^2 dr$$

$$\Rightarrow \quad Q_2 = 2\alpha\frac{4}{3}\pi\left(R^3 - \frac{R^3}{8}\right)$$

$$\Rightarrow \quad Q_2 = \frac{7\alpha\pi R^3}{3} - \frac{15\alpha\pi R^3}{8} = \frac{11}{24}\pi\alpha R^3$$

So, total charge is

$$Q = Q_1 + Q_2$$

$$\Rightarrow \quad Q = \frac{1}{6}\pi\alpha R^3 + \frac{11}{24}\pi\alpha R^3$$

$$\Rightarrow \quad Q = \frac{15}{24}\pi\alpha R^3$$

$$\Rightarrow \quad \frac{Q_1}{Q} = \frac{4}{15}$$

Hence, the correct answer is (A).

41. Since $F = ma = qE$ where $q = -e$ and $E = \frac{\rho r}{3\varepsilon_0} = \frac{\alpha r}{3\varepsilon_0}$

$$\Rightarrow \quad a = \left(\frac{-e\alpha}{3\varepsilon_0 m}\right) r$$

$$\Rightarrow \quad \ddot{r} + \left(\frac{e\alpha}{3\varepsilon_0 m}\right) r = 0$$

$$\Rightarrow \quad \omega = \sqrt{\frac{e\alpha}{3\varepsilon_0 m}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{3\varepsilon_0 m}{e\alpha}}, \text{ where } Q = \frac{15}{24}\pi\alpha R^3$$

$$\Rightarrow \quad \alpha = \frac{8}{5Q\pi R^3}$$

Substituting the value of α, we get

$$T = 2\pi\sqrt{\frac{15\pi\varepsilon_0 m R^3}{8Qe}}$$

Hence, the correct answer is (B).

42. Since, $\oint \vec{E}\cdot d\vec{A} = \frac{Q_1}{\varepsilon_0} + \int_{\frac{R}{2}}^{r} \frac{2\alpha}{\varepsilon_0}\left(1-\frac{r}{R}\right)4\pi r^2 dr$

$$\Rightarrow \quad E(4\pi r^2) = \frac{\pi\alpha R^3}{6\varepsilon_0} + \int_{\frac{R}{2}}^{r} \frac{2\alpha}{\varepsilon_0}\left(1-\frac{r}{R}\right)4\pi r^2 dr$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{\pi\alpha R^3}{6\varepsilon_0} + \frac{8\pi\alpha}{\varepsilon_0}\int_{\frac{R}{2}}^{r}\left(r^2 - \frac{r^3}{R}\right)dr$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{\pi\alpha R^3}{6\varepsilon_0} + \frac{8\pi\alpha}{\varepsilon_0}\left[\frac{r^3}{3}\Bigg|_{\frac{R}{2}}^{r} - \frac{r^4}{4R}\Bigg|_{\frac{R}{2}}^{r}\right]$$

$$\Rightarrow E(4\pi r^2) = \frac{\pi\alpha R^3}{6\varepsilon_0} + \frac{8\pi\alpha}{\varepsilon_0}\left[\frac{1}{3}\left(r^3 - \frac{R^3}{8}\right) - \frac{1}{4R}\left(r^4 - \frac{R^4}{16}\right)\right]$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{\pi\alpha R^3}{6\varepsilon_0} + \frac{8\pi\alpha}{\varepsilon_0}\left[\frac{r^3}{3} - \frac{r^4}{4R} - \frac{R^3}{24} + \frac{R^3}{64}\right]$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{\pi\alpha R^3}{6\varepsilon_0} + \frac{8\pi\alpha}{\varepsilon_0}\left[r^3\left(\frac{1}{3} - \frac{r}{4R}\right) + \frac{R^3}{8}\left(-\frac{1}{3} + \frac{1}{8}\right)\right]$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{\pi\alpha R^3}{6\varepsilon_0} + \frac{8\pi\alpha}{\varepsilon_0}\left[r^3\left(\frac{1}{3} - \frac{r}{4R}\right) + \frac{-5R^3}{8(24)}\right]$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{\pi\alpha R^3}{6\varepsilon_0} + \frac{8\pi\alpha}{\varepsilon_0}\left(\frac{1}{3} - \frac{r}{4R}\right)r^3 - \frac{5\pi\alpha R^3}{24\varepsilon_0}$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{\pi\alpha R^3}{6\varepsilon_0}\left(1 - \frac{5}{4}\right) + \frac{8\pi\alpha}{\varepsilon_0}\left(\frac{1}{3} - \frac{r}{4R}\right)r^3$$

$$\Rightarrow \quad E(4\pi r^2) = -\frac{\pi\alpha R^3}{24\varepsilon_0} + \frac{8\pi\alpha}{\varepsilon_0}\left(\frac{1}{3} - \frac{r}{4R}\right)r^3$$

$$\Rightarrow \quad E(4\pi r^2) = -\frac{\pi\alpha R^3}{24\varepsilon_0} + \frac{8\pi\alpha}{3\varepsilon_0}\left(1 - \frac{3r}{4R}\right)r^3$$

$$\Rightarrow \quad E = -\frac{\alpha R^3}{96\varepsilon_0 r^2} + \frac{2\alpha}{3\varepsilon_0}\left(1 - \frac{3r}{4R}\right)r$$

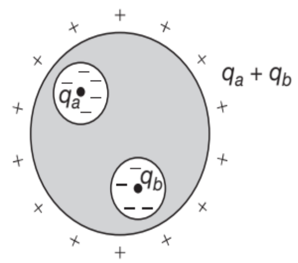
$$\Rightarrow \quad E = -\frac{\alpha R^3}{96\varepsilon_0 r^2} + \frac{2\alpha r}{3\varepsilon_0}\left(1 - \frac{3r}{4R}\right)$$

Hence, the correct answer is (D).

43. In cavity there is no electric field, so no force acts on charge q_a.

Hence, the correct answer is (D).

44.



Now E outside the sphere is due to $(q_a + q_b)$ because E due to q_a, q_b and $-q_a$ and $-q_b$ on the inner surface of cavity gets cancelled. So, we have

$$E = \frac{q_a + q_b}{4\pi\varepsilon_0 r_0^2}$$

Hence, the correct answer is (C).

45. Due to presence of charge on outside of sphere nothing will happen to the field inside cavity. So, E will remain same.

Hence, the correct answer is (C).

46. Correct answer is (B). Since ultraviolet light ejects electrons from zinc so the ball becomes positively charged. This is known as photoelectric effect.

Hence, the correct answer is (B).

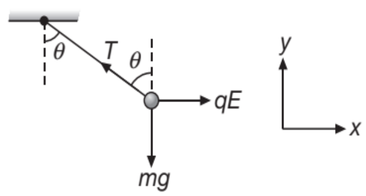
47. Let n be the number of electrons lost, so that the ball acquires a positive charge given by $q = ne$.

So, ball experiences a force qE along horizontal direction.

The situation is shown in following figure.

For equilibrium,

$$\Sigma F_x = 0$$



$$\Rightarrow \quad qE - T\sin\theta = 0 \qquad \ldots(1)$$

$$\Sigma F_y = 0$$

CHAPTER 1

$\Rightarrow \quad T\cos\theta - mg = 0 \quad \ldots(2)$

From (1) and (2), we get

$$\frac{qE}{mg} = \tan\theta$$

$$\Rightarrow \quad q = \frac{mg\tan\theta}{E}$$

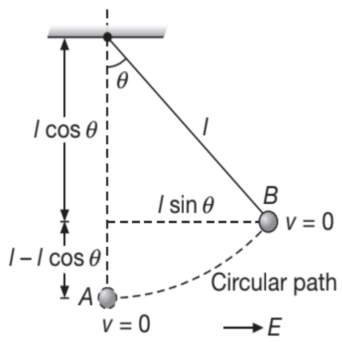
Now by Charge Quantisation, are have $q = ne$. So

$$ne = \frac{mg\tan\theta}{E}$$

$$\Rightarrow \quad n = \frac{mg\tan\theta}{eE}$$

Hence, the correct answer is (C).

48. As soon as a charge (q) appears on the ball it starts experiencing a horizontal force (qE) due to presence of electric field. Due to the constraint of string,it will start moving on a circular path in a vertical plane. At maximum deflection of the string, velocity of the ball reduces to zero as shown in figure.



By Modified Work-Energy Theorem, we have

$$W_{ext} = \Delta U + \Delta K$$

Between the two positions A and B, total change in KE is zero i.e. $\Delta K = 0$

$$\Rightarrow \quad W_{ext} = \Delta U$$

Also T acts along radial direction, so $W_{Tension} = 0$

$$\Rightarrow \quad qE(\ell\sin\theta) = mg\ell(1-\cos\theta)$$

Since $1-\cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$

$$\Rightarrow \quad mg\ell \times 2\sin^2\left(\frac{\theta}{2}\right) = qE\ell\sin\theta$$

Since $\sin\theta = 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$

$$\Rightarrow \quad mg \times 2 \times \sin^2\left(\frac{\theta}{2}\right) = qE\left[2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right]$$

$$\Rightarrow \quad mg\sin\left(\frac{\theta}{2}\right) = qE\cos\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \quad \tan\left(\frac{\theta}{2}\right) = \frac{qE}{mg}$$

$$\Rightarrow \quad \theta = 2\tan^{-1}\left(\frac{qE}{mg}\right)$$

Hence, the correct answer is (C).

49. For calculating the electric field outside the ball, we should calculate first the total charge present in the ball.

Since $Q = \int \rho dV = \int \rho(4\pi r^2 dr)$

$$\Rightarrow \quad Q = 4\pi\rho_0 \int_0^R \left(1 - \frac{r}{R}\right) r^2 dr$$

$$\Rightarrow \quad Q = 4\pi\rho_0\left(\frac{R^3}{3} - \frac{R^4}{4R}\right) = 4\pi\rho_0\left(\frac{R^3}{12}\right)$$

To find the electric field outside the ball, the total charge Q is considered to be concentrated at the centre of ball. Hence, electric field at a distance r from the centre of ball is

$$E = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2} = \frac{1}{4\pi\varepsilon}4\pi\rho_0\left(\frac{R^3}{12}\right) \times \frac{1}{r^2}$$

$$\Rightarrow \quad E = \frac{\rho_0 R^3}{12\varepsilon r^2}$$

Hence, the correct answer is (B).

50. For E to be maximum (which is possible inside), we have

$$\frac{dE}{dr} = 0$$

where $E = E_{\text{inside}} = \frac{Q_{\text{inside}}}{4\pi\varepsilon r^2} = \frac{\rho_0}{\varepsilon}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$

$$\Rightarrow \quad \frac{\rho_0}{\varepsilon}\left(\frac{1}{3} - \frac{2r}{4R}\right) = 0$$

$$\Rightarrow \quad r_m = \frac{2R}{3}$$

Hence, the correct answer is (C).

51. $E_m = \frac{\rho_0}{\varepsilon}\left(\frac{r_m}{3} - \frac{r_m^2}{4R}\right) = \frac{\rho_0}{\varepsilon}\left(\frac{2R}{9} - \frac{4R^2}{36R}\right)$

$$\Rightarrow \quad E_m = \frac{\rho_0 R}{9\varepsilon}$$

Hence, the correct answer is (A).

52. Let the speed of charges A and B be v_A and v_A when the separation between them is ℓ. Then by Conservation of Linear Momentum, we have

$$-mv_A + (2m)v_B = 0$$

$$\Rightarrow \quad v_A = 2v_B$$

By Law of Conservation of Energy, we have

$$U_i + K_i = U_f + K_f$$

$$\Rightarrow \quad \frac{2q^2}{4\pi\varepsilon_0 \ell} + 0 + 0 = \frac{2q^2}{4\pi\varepsilon_0 (2\ell)} + \frac{1}{2}mv_A^2 + \frac{1}{2}(2m)v_B^2$$

$$\Rightarrow \quad \frac{1}{2}mv_A^2 + \frac{1}{2}2mv_B^2 = \frac{2q^2}{4\pi\varepsilon_0 \ell}\left(1 - \frac{1}{2}\right)$$

$$\Rightarrow \quad \frac{1}{2}mv_A^2 + \frac{1}{2}(2m)\left(\frac{v_A}{2}\right)^2 = \frac{q^2}{4\pi\varepsilon_0 \ell}$$

Solving, we get the speed of charge A as

$$v_A = \sqrt{\frac{q^2}{3\pi\varepsilon_0 m\ell}}$$

Hence, the correct answer is (D).

53. The work done by electrostatic force on charge A, from work energy theorem, in the given duration equals the change in kinetic energy of A. So,

$$W = \Delta K_A = \frac{1}{2}mv_A^2 = \frac{q^2}{6\pi\varepsilon_0 \ell}$$

Hence, the correct answer is (B).

54. The net work done by electrostatic force on system of two charged particle is equal to decrease in electrostatic potential energy of the system. So,

$$W_{e\ell} = U_i - U_f$$

$$\Rightarrow \quad W_{e\ell} = \frac{2q^2}{4\pi\varepsilon_0 \ell}\left(1 - \frac{1}{2}\right)$$

$$\Rightarrow \quad W_{e\ell} = \frac{q^2}{4\pi\varepsilon_0 \ell}$$

Hence, the correct answer is (C).

55. By symmetry, potential at any point at y-axis is zero.

Hence potential at $P\left(0, \frac{R}{2}\right)$ is zero.

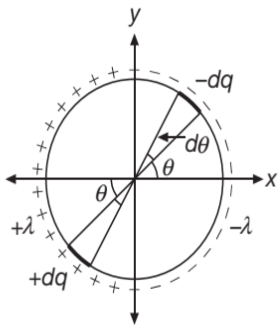
Hence, the correct answer is (B).

56. Since all charge lies in x-y plane, hence direction of electric field at point P should be in x-y plane. Also y-axis is an equipotential (zero potential) line. Hence direction of electric field at all points on y-axis should be normal to y-axis.

So, the direction of electric field at P should be in x-y plane and normal to y-axis. Hence direction of electric field is along positive x direction.

Hence, the correct answer is (A).

57. Consider two small elements of ring having charge $+dq$ and $-dq$ as shown in figure. The pair constitutes a dipole of dipole moment dp given by



$$dp = (dq)(2R)$$

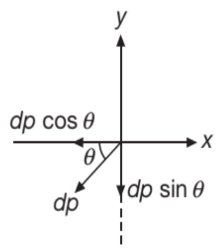
Since $dq = \lambda d\ell = \lambda(Rd\theta)$

$$\Rightarrow \quad dp = \lambda(Rd\theta)2R$$

By symmetry the resultant dipole moment is along negative x-direction.

So, $p_x = \int dp_x = \int dp\cos\theta$

$$\Rightarrow \quad p_x = 2\lambda R^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta d\theta$$



$$\Rightarrow \quad p_x = 2\lambda R^2 \sin\theta \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

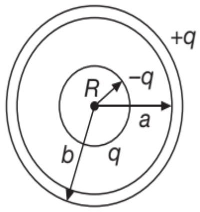
$$\Rightarrow \quad p_x = 4\lambda R^2$$

Similarly $p_y = \int dp_y = \int dp\sin\theta$

$$\Rightarrow \quad p_y = 2\lambda R^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin\theta d\theta = 0$$

Hence, the correct answer is (C).

58.



$$\sigma_R = \frac{q}{4\pi R^2}$$

$$\sigma_a = \frac{-q}{4\pi a^2}$$

$$\sigma_b = \frac{q}{4\pi b^2}$$

Hence, the correct answer is (B).

59. $V_{\text{centre}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{b}+\frac{q}{R}-\frac{q}{a}\right)$

Hence, the correct answer is (D).

60. On earthing the outer surface, we have

$$V_{\text{outer surface}} = 0$$

If x be the charge on outer surface, then

$$v = \frac{1}{4\pi\varepsilon_0 b}(q-q+x)=0$$

$$\Rightarrow \quad x = 0$$

$$\Rightarrow \quad V_{\text{centre}} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{a}\right)$$

Hence, the correct answer is (A).

Matrix Match/Column Match Type Questions

1. A → (p, q, r, s)
B → (p, q, s)
C → (p, q, s)
D → (p, q, r, s, t)
Electrostatic force is conservative in nature, obeys Newton's Third Law (i.e. action-reaction force), depends upon the nature of medium between the interacting particles, obeys Principle of Superposition, attractive for unlike charges and repulsive for like charges. So, A → (p, q, r, s)
Similarly, gravitational force is conservative in nature, obeys Newton's Third Law (i.e. action-reaction force), does not depend upon the nature of medium between the interacting particles, obeys Principle of Superposition and is always attractive in nature. So, B → (p, q, s), C → (p , q, s) and D → (p, q, r, s, t)

2. A → (p, r, s, t)
B → (p, q)
C → (p, q)
D → (p, r, s, t)
When $Q = +q$ is displaced along $+x$ axis, then before being displaced, force on Q is zero. However after being displaced along x-axis (say towards B), then the repulsive force due to B is stronger on Q than the repulsive force due to A. So, Q has a tendency to go back to O (i.e. the mean position) and hence equilibrium is stable i.e. U is minimum. On the similar approach, the remaining answers can be given.

4. A → (q, r, s)
B → (p)
C → (p, q, r, s)
D → (p, q, r, s)
Electric field is zero inside a charged conducting sphere or a charged conducting shell or a charged non-conducting shell. However for all of these, electric field outside them is inversely proportional to r^2 i.e, $E_{\text{outside}} \propto \frac{1}{r^2}$. For the charged non-conducting sphere $E_{\text{inside}} = \frac{\rho r}{3\varepsilon_0}$, $E_{centre} = 0$ and E is maximum at the surface.

5. A → (q, r)
B → (s)
C → (t)
D → (p)

$$E_1 = \frac{\lambda R x}{2\varepsilon_0\left(R^2+x^2\right)^{\frac{3}{2}}}$$

$$E_2 = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{x}{\sqrt{R^2+x^2}}\right)$$

$$E_3 = \frac{\lambda}{2\pi\varepsilon_0 x}$$

$$E_4 = \frac{\sigma}{2\varepsilon_0}$$

6. A → (s, t)
B → (p, q, r)
C → (q, r)
D → (p)
$E = 0$ inside the body of the conductor. So $E = 0$ in the region between inner and outer surface of shell (both small as well as big). So, A → (s, t).
For the space between the shells and outside the shells, $E \neq 0$. So, B → (p, q, r).
Charge is $3Q$ on outersurface of inner shell and $-3Q$ on inner surface of outershell. So, magnitude of charge on these surfaces is $3Q$. So, C → (q, r).

Charge on outer surface of bigger (or outer) shell is $8Q$. So, D → (p)

7. A → (r, s)
B → (p, r, s)
C → (q)
D → (t)

For dipole, $V_{\text{centre}} = 0$ and $V_{\text{axial}} = 0$. So, A → (r, s)

E_{axial} is parallel to $\vec{p}$ (i.e. dipole moment) and $E_{\text{equitorial}}$ is anti-parallel to $\vec{p}$. So, at C, $\vec{E}$ is along $\vec{p}$ and at D, $\vec{E}$ is opposite to $\vec{p}$. So, B → (p, r, s) and C → (q). For $r \gg a$, $E_{\text{axial}} = 2\left(E_{\text{equitorial}}\right)$. Hence $E_C > E_D$. So D → (t)

8. A → (q, s)
B → (p)
C → (t)
D → (r)

Since E and V are field and potential at centre of square having charges placed at the corners, so

For (p), $E = 0$, $V \neq 0$

For (q), $E \neq 0$, $V \neq 0$

For (r), $E \neq 0$, $V = 0$

For (s), $E \neq 0$, $V \neq 0$

For (t) $E = 0$, $V = 0$

9. A → (s, t)
B → (q)
C → (p)
D → (r, s)

For a small dipole, $E \propto \dfrac{1}{r^3}$

For a uniformly charged ring of radius R, field at a point P on axis (at a distance $r \gg R$) is $E \approx \dfrac{Q}{4\pi\varepsilon_0 r^2}$

$\Rightarrow \quad E \propto \dfrac{1}{r^2}$

For line charge, charged cylinder or charged conducting cylinder $E \propto \dfrac{1}{r}$ (outside them)

For an infinite charged conducting plate, $E = \dfrac{\sigma}{2\varepsilon_0}$, independent of r

So, $E \propto \dfrac{1}{r^0}$

10. A → (q, t)
B → (s)
C → (p)
D → (r)

Jm^{-3} is the unit of energy density $\left(= \dfrac{1}{2}\varepsilon_0 E^2\right)$ and hence electrostatic pressure $(= P_e)$. So, A → (q, t)

$\dfrac{V}{E}$ will have SI unit of metre. So, B → (s)

Since $U = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r}$, so $\varepsilon_0 = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 U r}$. Hence $C^2 J^{-1} m^{-1}$ is the unit of ε_0. So, C → (p)

The unit of capacitance is farad. Since energy stored (U) in the capacitor is $U = \dfrac{1}{2}CV^2$, so farad represents $\dfrac{U}{V^2}$. So, D → (r)

11. A → (r)
B → (s)
C → (p)
D → (p)

Between the plates, $E = \text{constant}$. So, D → (p)

When a particle of charge $-q$, mass m moves between the plates from $-V$ to V, then K increases, U decreases, however $T(= U + K)$ remains constant.

So, A → (r), B → (s), (C) → (p)

12. A → (s)
B → (r)
C → (q, t)
D → (p)

$$[\varepsilon_0] = M^{-1}L^{-3}T^4A^2,$$

So, A → (s)

$$[\phi_E] = ML^3T^{-3}A^{-1}$$

So, B → (r)

$$[E] = \left[\frac{\sigma}{2\varepsilon_0}\right] = MLT^{-3}A^{-1}$$

So, C → (q, t)

$$[V] = ML^2T^{-3}A^{-1}$$

So, D → (p)

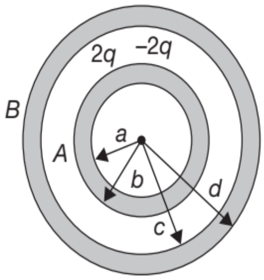
13. A → (p)
B → (q, s)
C → (p)
D → (q, r)

There will be no field inside the body of the conductor, so $|E| = 0$ for $a < r < b$ and $c < r < d$. So, C → (p). Also, inside shell A, $|E| = 0$. So, A → (p)

In the region $b < r < c$, $|\vec{E}|$ is directed radially outwards and $|E| = \dfrac{2q}{4\pi\varepsilon_0 r^2} = \dfrac{q}{2\pi\varepsilon_0 r^2}$

So, B → (q, s).

CHAPTER 1



Finally, in the region outside B i.e. for $r > d$, $|\vec{E}|$ is directed radially inwards (because charge on outer surface of B is $-2q$). Hence $|E| = \frac{2q}{4\pi\varepsilon_0 r^2} = \frac{q}{2\pi\varepsilon_0 r^2}$

So, D → (q, r)

14. A → (r, s)
B → (r, s)
C → (p, s)
D → (p, s)

When S is open then in Region II, $E = \frac{q}{4\pi\varepsilon_0 r^2}$

$\Rightarrow \quad E \neq 0$

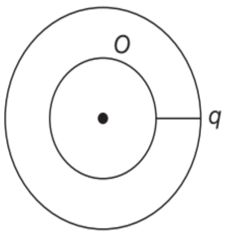
In Region III, $E = \frac{q}{4\pi\varepsilon_0 r^2}$

$\Rightarrow \quad E \neq 0$

Potential is $\frac{q}{4\pi\varepsilon_0 r}$ in both regions.

$\Rightarrow \quad V \neq 0$

When S is closed, then all charge goes from inner sphere to outer sphere.



So, in Region I and II

$E = 0$

But in Region I and II, $V \neq 0$

15. A → (p, s)
B → (q, s)
C → (q, s)
D → (s)

(A) Electrostatic potential energy

$$U = \frac{1}{4\pi\varepsilon_0}\frac{(-Q)^2}{2a} = \frac{Q^2}{8\pi\varepsilon_0 a}$$

(B) Electrostatic potential energy

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{(-Q)\times(-Q)}{\frac{5a}{2}} + \frac{(-Q)^2}{2\left(\frac{5a}{2}\right)}\right) = \frac{3}{20}\frac{Q^2}{\pi\varepsilon_0 a}$$

(C) Electrostatic potential energy

$$U = \frac{1}{4\pi\varepsilon_0}\frac{3Q^2}{5a} = \frac{3}{20}\frac{Q^2}{\pi\varepsilon_0 a}$$

(D) Electrostatic potential energy

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{3Q^2}{5a} + \frac{(-Q^2)}{2(2a)} + \frac{(-Q)\times(-Q)}{2a}\right) = \frac{27Q^2}{80\pi\varepsilon_0 a}$$

16. A → (p, r, s, t)
B → (p, r, s, t)
C → (p, q, r, s)
D → (q)

For arrangement (A), potential at any point on z-axis is zero, y-component of electric field at any point on z-axis is non-zero, U_{system} is negative, z-component of electric field at any point on z-axis is zero. So A → (p, r, s, t).

Similarly B → (p, r, s, t), C → (p , q, r, s) and D → (q)

17. A → (q)
B → (r)
C → (s)
D → (p)

For hemispherical shell, E at centre is $E = \frac{Q}{8\pi\varepsilon_0 R^2}$

So, A → (q)

For hemispherical sphere, $E = \frac{3Q}{8\pi\varepsilon_0 R^2}$

So, B → (r)

For ring at a point near its axis $(x \ll R)$, we get

$$E = \frac{\lambda x}{2\varepsilon_0 R^2}\left(1 - \frac{3x^2}{2R^2}\right)$$

So, C → (s)

For semi-infinite wire, E at the given point is

$$E = \frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 x}$$

So, D → (p)

18. A → (s)
B → (r)
C → (p)
D → (q)

Since $E_x = -\frac{\partial V}{\partial x}$ and $E_y = -\frac{\partial V}{\partial y}$. Also, tangent to electric lines of force will give direction of electric field. So,

$$\frac{dy}{dx} = \frac{E_y}{E_x}$$

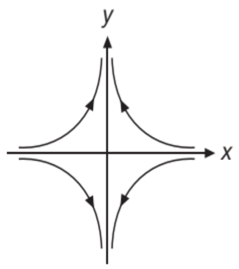
(A) $V = x^2 - y^2$

$$\Rightarrow \quad \vec{E} = -2x\hat{i} + 2y\hat{j}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{E_y}{E_x} = \frac{-y}{x}$$

$$\Rightarrow \quad \log y = -\log x + c$$

$$\Rightarrow \quad xy = c \text{ (Rectangular Hyperbola)}$$



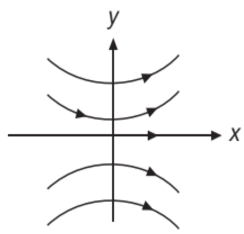
Corresponding curve is drawn for electric lines of force.

(B) $V = xy$

$$\vec{E} = -y\hat{i} - x\hat{j}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x}{y}$$

$$\Rightarrow \quad y^2 = x^2 + c$$



Corresponding curve for field lines is shown

(C) $V = x^2 + y^2$

$$\Rightarrow \quad \vec{E} = -2x\hat{i} - 2y\hat{j}$$

Tangent to electric line of force will give direction of electric field. So

$$\frac{dy}{dx} = \frac{E_y}{E_x} = \frac{y}{x}$$

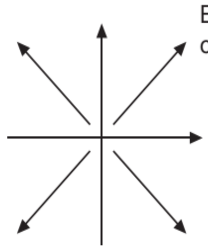
$$\Rightarrow \quad \frac{dy}{y} = \frac{dx}{x}$$

$$\Rightarrow \quad \log y = \log x + c$$

$$\Rightarrow \quad \frac{y}{x} = c$$

$$\Rightarrow \quad y = cx$$

Straight line equation passing through origin



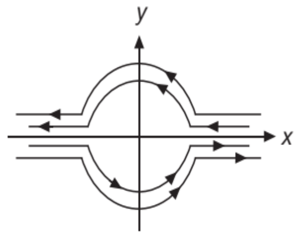
(D) $V = \frac{x}{y}$

$$\vec{E} = -\frac{1}{y}\hat{i} + \frac{x}{y^2}\hat{j}$$

$$\frac{dy}{dx} = \frac{\frac{x}{y^2}}{-\frac{1}{y}} = -\frac{x}{y}$$

$$-ydy = xdx$$

$$\Rightarrow \quad x^2 + y^2 = \text{constant (circle)}$$



Corresponding curve is given.

19. A → (r)
B → (q)
C → (p)
D → (s)

For S_1, imagine the charge q to be enclosed inside a cube with S_1 as one of its face, then $\phi_{cube} = \frac{q}{\varepsilon_0}$. Since S_1 is only one sixth of the total surface area of cube

$$\Rightarrow \quad \phi_{S_1} = \frac{1}{6}\left(\frac{q}{\varepsilon_0}\right) = \frac{q}{6\varepsilon_0}.$$

So, A → (r)

For an sheet having charge q placed near it, we have

$$\phi = \frac{q}{2\varepsilon_0}(1 - \cos\theta).$$

Since sheet is infinite, so $\theta \to \frac{\pi}{2}$

$\Rightarrow \quad \phi_{S_2} = \frac{q}{2\varepsilon_0}$.

So, B → (q)

Since $\phi_{S_3} = \frac{q}{2\varepsilon_0}(1-\cos\theta)$ where $\cos\theta = \frac{a}{\sqrt{a^2+15a^2}} = \frac{1}{4}$

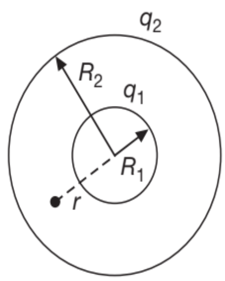
$\Rightarrow \quad \phi_{S_3} = \frac{q}{2\varepsilon_0}\left(1-\frac{1}{4}\right) = \frac{3q}{8\varepsilon_0}$.

So, C → (p)

Similarly $\phi_{S_4} = \frac{1}{4}\left(\frac{q}{\varepsilon_0}\right) = \frac{q}{4\varepsilon_0}$.

So, D → (s)

20. A → (s)
B → (r)
C → (p)
D → (q)



For, $r < R_1$

$$E = 0 + 0 = 0$$

and $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1}+\frac{q_2}{R_2}\right) = \text{constant}$

$R_1 < r < R_2$

$$E = \frac{q_1}{4\pi\varepsilon_0 r^2} + 0$$

and $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r}+\frac{q_2}{R_2}\right)$

21. A → (p)
B → (r)
C → (p)
D → (p)
n drops coalesce, then radius (R) of bigger drop having charge nq is

$$n\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$

$\Rightarrow \quad R^3 = nr^3$

$\Rightarrow \quad R = n^{\frac{1}{3}} r$

Now $E_{big} = \frac{nq}{4\pi\varepsilon_0 R^2}$

$\Rightarrow \quad E_{big} = \frac{nq}{4\pi\varepsilon_0 n^{\frac{2}{3}} r^2} = n^{\frac{1}{3}} E_{\text{small}}$

So, A → (p)

Similarly $V_{big} = \frac{nq}{4\pi\varepsilon_0 R} = n^{\frac{2}{3}} V_{small}$

So, B → (r)

Since $C_{big} = 4\pi\varepsilon_0 R = n^{\frac{1}{3}} C_{\text{small}}$.

So, C → (p)

Since $\sigma_{big} = \frac{nq}{4\pi R^2} = n^{\frac{1}{3}}\sigma_{small}$.

So, D → (p)

22. A → (q)
B → (r)
C → (p)
D → (s)
For (A), we have

$$\underset{(r<a)}{E_{inside}} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$E_{a<r<b} = 0$$

$$\underset{(r>b)}{E_{outside}} = zero$$

So A → (q)
For (B), we have

$$\underset{(r<a)}{E_{inside}} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$E_{(a<r<b)} = 0$$

$$\underset{(r>b)}{E_{outside}} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

So, B → (r)
For (C), we have

$$\underset{(r<a)}{E_{inside}} = 0$$

$$E_{(a<r<b)} = 0$$

$$\underset{(r>b)}{E_{outside}} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

So, C → (p)

For (D), we have

$$E_{\underset{(r<a)}{inside}} = \frac{-Q}{4\pi\varepsilon_0 r^2}$$

$$E_{(a<r<b)} = 0$$

$$E_{\underset{(r>b)}{outside}} = \frac{2Q-Q}{4\pi\varepsilon_0 r^2} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

So, D $\rightarrow$ (s).

Please note that, while answering all these we must keep in mind that the field inside the body of the conductor is zero.

Integer/Numerical Answer Type Questions

1. 33

2. Since $\lambda = \dfrac{Q}{2\pi R}$

and the tension developed in the ring when a charge is placed at its centre is given by

$$T = \frac{q_0\lambda}{4\pi\varepsilon_0 R} = \frac{q_0 Q}{8\pi^2\varepsilon_0 R^2} \text{ (we have done this already)}$$

Further, by Laws of Elasticity, we know that

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\left(\dfrac{T}{A}\right)}{\left(\dfrac{\Delta R}{R}\right)}$$

$$\Rightarrow \quad Y = \frac{TR}{A\Delta R}$$

$$\Rightarrow \quad \Delta R = \frac{TR}{AY} = \frac{q_0 Q}{8\pi^2\varepsilon_0 RAY}$$

Substituting, $q_0 = 10^{-8}$ C, $Q = \pi$C, $R = 0.1$ m, $A = 10^{-6}$ m^2 and $Y = 2\times 10^{-11}$ Nm^{-2}, we get

$$\Delta R = 225\times 10^{-5} \text{ m}$$

$$\Rightarrow \quad x = 225$$

3. $t = \dfrac{2t_1t_2}{t_1 - t_2} = 12$ ms

4. 2

5. $y = u_y t + \dfrac{1}{2}a_y t^2$, where

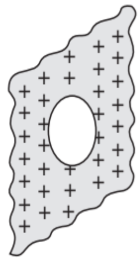
$$u_y = 0,\ a_y = \frac{eE}{m} \text{ and } t = \frac{\ell}{u}$$

$$\Rightarrow \quad y = \frac{1}{2}a_y t^2$$

$$\Rightarrow \quad \frac{1}{200} = \frac{1}{2}\frac{(1.6\times 10^{-19})(E)}{9.1\times 10^{-31}}\left(\frac{2}{4\times 10^8}\right)^2$$

$$\Rightarrow \quad E = \frac{2\times 9.1\times 10^{-31}\times 16\times 10^{16}}{1.6\times 10^{-19}\times 4\times 200} = 2275 \text{ NC}^{-1}$$

6. $E_{\text{net}} = \dfrac{\sigma}{2\varepsilon_0} - E_{\text{disc}}$



$$\Rightarrow \quad E_{\text{net}} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+a^2}}\right)$$

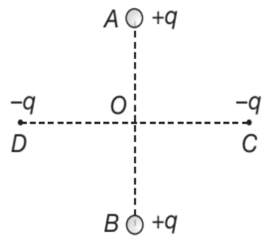
$$\Rightarrow \quad E_{\text{net}} = \frac{\sigma z}{2\varepsilon_0\sqrt{z^2+a^2}}$$

$$\Rightarrow \quad K = 9\times 10^9\times 1.6\times 10^{-19}\times\frac{4}{9}\times 10^{-8}$$

$$\Rightarrow \quad x = 12$$

7. From figure $AC = \sqrt{(AO)^2 + (OC)^2}$

$$AC = \sqrt{3^2+4^2} = 5 \text{ m}$$



Similarly, $BC = 5$ m

P.E. at C is $U_C = 2\times 9\times 10^9\times\dfrac{(q)\times(-q)}{AC}$

$$\Rightarrow \quad U_C = -2\times 9\times 10^9\times\frac{(5\times 10^{-5})^2}{5}$$

$$\Rightarrow \quad U_C = -9 \text{ J}$$

K.E. at C is 4 J

Total energy at C is E_C = P.E. + K.E.

$$\Rightarrow \quad E_C = -9+4 = -5 \text{ J}$$

Potential energy at D is U_D, given by

$$U_D = \frac{-2\times 9\times 10^9(5\times 10^{-5})^2}{r}$$

where $AD = BD = r$ (say)

$$\Rightarrow \quad U_D = \frac{-45}{r} \text{ J}$$

K.E. at $D = 0$

So, total energy at D is $E_D = 0 + \dfrac{-45}{r} = \dfrac{-45}{r}$ J

By Law of Conservation of Energy,

$$U_C + K_C = U_D + K_D$$

$$\Rightarrow \quad \frac{-45}{r} = -5$$

$$\Rightarrow \quad r = 9 \text{ m}$$

Now $OD=\sqrt{(AD)^2-(AO)^2}$

$\Rightarrow \quad OD=\sqrt{9^2-3^2}=\sqrt{72}$ m

$\Rightarrow \quad x=72$

8. Done already (Illustration 61)

$$v_{\min}=\sqrt{\frac{39}{40}\left(\frac{Qq}{2\pi\varepsilon_0 m}\right)}$$

$$\Rightarrow \quad x=\frac{39}{40}\cong 1$$

9. Since, $\theta=2\sin^{-1}\left[\left(\frac{Qq}{32\pi\varepsilon_0 mg\ell^2}\right)^{\frac{1}{3}}\right]$

(As already done in Illustration 6)

$$\frac{Qq}{mg\ell^2}=4\pi\varepsilon_0$$

$$\Rightarrow \quad \theta=2\sin^{-1}\left[\left(\frac{4\pi\varepsilon_0}{32\pi\varepsilon_0}\right)^{\frac{1}{3}}\right]$$

$$\Rightarrow \quad \theta=2\sin^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad \theta=2\left(\frac{\pi}{6}\right)$$

$$\Rightarrow \quad \theta=\frac{\pi}{3} \Rightarrow \theta=60^\circ$$

10. $mg=\dfrac{q^2}{4\pi\varepsilon_0 r^2}$

$$\Rightarrow \quad 1.7\times10^{-27}\times10=\frac{9\times10^9\times(1.6\times10^{-19})^2}{r^2}$$

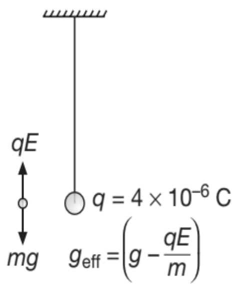
$$\Rightarrow \quad r^2=\frac{9\times10^9\times(1.6\times10^{-19})^2}{1.7\times10^{-26}}$$

$\Rightarrow \quad r=0.116$ m

$\Rightarrow \quad r=11.6$ cm $\cong 12$ cm

11. Initially $9\times10^9\cdot q^2\left(\frac{1}{0.40}-\frac{1}{0.50}\right)$ ...(1)

Finally $T'=2\pi\sqrt{\dfrac{L}{\left(g-\frac{qE}{m}\right)}}$ ...(2)



Dividing equation (1) and (2), we get

$$\frac{T}{T'}=\sqrt{\frac{g-\frac{qE}{m}}{g}}$$

$$\Rightarrow \quad \frac{T}{T'}=\sqrt{1-\frac{qE}{mg}}=\sqrt{1-\frac{4\times10^{-6}\times2.5\times10^4}{40\times10^{-3}\times10}}$$

$\Rightarrow \quad T'=2.6$ s

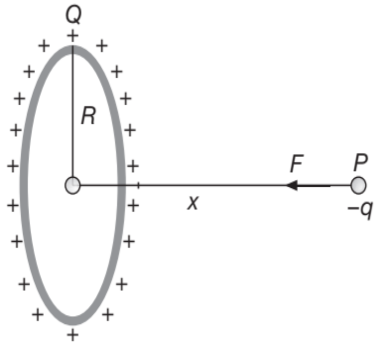
For 20 oscillation, time taken is $t=20\times2.6$

$\Rightarrow \quad t=52$ s

12. Let us first the force on a-q charge placed at a distance x from centre of ring along its axis.

Figure shows the respective situation. In this case force on particle P is

$$F_P=-qE$$



$$F_P=-\frac{q}{4\pi\varepsilon_0}\frac{Qx}{(x^2+R^2)^{3/2}}$$

For small x, $x\ll R$, we can neglect x, compared to $v^2=2\times(1.76\times10^{11})\times9\times10^9\times$. So

$$F=-\frac{1}{4\pi\varepsilon_0}\frac{qQx}{R^3}$$

Now, acceleration of particle is

$$a=\frac{F}{m}=-\frac{1}{4\pi\varepsilon_0}\frac{qQ}{mR^3}x$$

$$\Rightarrow \quad \frac{6\times10^{-8}\times0.15\times0.30}{(0.30-0.15)}\ddot{x}+\left(\frac{qQ}{4\pi\varepsilon_0 mR^3}\right)x=0$$

which is a standard SHM equation. So

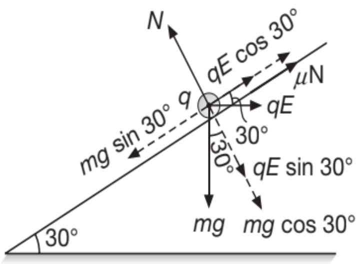
$$\omega=\sqrt{\frac{Qq}{4\pi\varepsilon_0 mR^3}}$$

$$\Rightarrow \quad T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{4\pi\varepsilon_0 mR^3}{Qq}}$$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{0.9\times10^{-3}\times(1)^3}{9\times10^9\times10^{-5}\times10^{-6}}}$$

$$\Rightarrow \quad T=\frac{\pi}{5}\text{ s} \quad \Rightarrow \quad k=5$$

13. The different forces on the particle are shown in figure



From figure,

$$N = mg\cos 30° + qE\cos 60°$$

By definition, function is

$$f = \mu N = \mu mg\cos 30° + \mu qE\cos 60°$$

If a is the acceleration of the particle down the incline, then

$$mg\sin 30° - \mu N - qE\cos 30° = ma$$

$$\Rightarrow \quad ma = mg\sin 30° - \mu mg\cos 30° - \mu qE\cos 60° - qE\cos 30°$$

Thus acceleration of the particle down the incline is

$$a = g\sin 30° - \mu g\cos 30° - \frac{\mu qE}{m}\cos 60° - \frac{qE}{m}\cos 30°$$

$$\Rightarrow a = 10\left(\frac{1}{2}\right) - (0.2)(10)\left(\frac{\sqrt{3}}{2}\right) - \frac{(0.2)(0.01)(100)}{1}\left(\frac{1}{2}\right) - \frac{(0.01)(100)}{1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad a = 5 - \sqrt{3} - 0.1 - \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad a = 2.3 \text{ ms}^{-2}$$

Now, distance travelled in time t is

$$s = 0 + \frac{1}{2}at^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2\times 2}{a}} \qquad \left\{\because s = \frac{1}{\sin 30} = 2\right\}$$

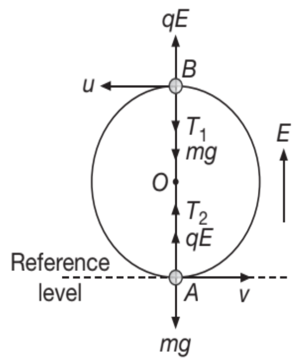
$$\Rightarrow \quad t = \sqrt{\frac{4}{2.3}}$$

$$\Rightarrow \quad t = 1.32 \text{ s}$$

$$\Rightarrow \quad t = 1320\times 10^{-3} \text{ s}$$

$$\Rightarrow \quad t = 1320 \text{ ms}$$

14. The corresponding situation is shown in figure



Here by using work energy theorem between position B and A, we have

$$\frac{1}{2}mu^2 + 2mg\ell - 2qE\ell = \frac{1}{2}mv^2 \qquad \ldots(1)$$

at position A we have

$$T + qE = \frac{mv^2}{\ell} + mg$$

Since $T = 15$mg

$$\Rightarrow \quad qE\ell = mv^2 - 14mg\ell$$

{$\because$ at the lowest point, $T = 15$ mg}

From equation (1), we have

$$qE\ell + 14mg\ell = mu^2 + 4mg\ell - 4qE\ell$$

$$\Rightarrow \quad u^2 = \frac{5qE}{m} + \frac{10mg}{m}$$

$$\Rightarrow \quad u = \sqrt{\frac{5qE\ell}{m} + 10g\ell}$$

$$\Rightarrow \quad u = \sqrt{\frac{5\ell}{m}(qE + 2mg)}$$

$$\Rightarrow \quad u = \sqrt{\frac{5\ell}{m}\left(\frac{6mg}{5} + 2mg\right)} = \sqrt{\frac{5\ell}{m}\left(\frac{16mg}{5}\right)}$$

$$\Rightarrow \quad u = 4\sqrt{g\ell} = 4\sqrt{(10)(3.6)} = (4)(6) = 24 \text{ ms}^{-1}$$

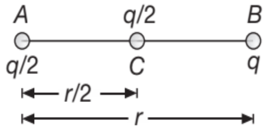
15. Let q be the initial charge on each of spheres A and B, then given

$$F_{AB} = 2.0\times 10^{-5} \text{ N}$$

So from Coulomb's Law

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = 2.0\times 10^{-5} \qquad \ldots(1)$$

When the uncharged sphere C is touched with A, the charge on A and C is distributed equally i.e.,



$$q_A = q_C = \frac{0+q}{2} = \frac{q}{2}$$

The arrangement is shown, where C is mid point of A and B

$$\Rightarrow \quad AC = BC = \frac{AB}{2} = \frac{r}{2}$$

The force on C due to A

$$\left|\vec{F}_{CA}\right| = \frac{1}{4\pi\varepsilon_0}\frac{q_B q_C}{r_{BC}^2} = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q}{2}\right)\left(\frac{q}{2}\right)}{\left(\frac{r}{2}\right)^2}$$

$$\Rightarrow \quad F_{CA} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \text{ from } A \to C$$

The force on C due to B is

$$\left|\vec{F}_{CB}\right| = \frac{1}{4\pi\varepsilon_0}\frac{q_A q_C}{r_{BC}^2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{q}{2}(q)}{\left(\frac{r}{2}\right)^2}$$

$$\Rightarrow \quad F_{CB} = \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{r^2} \text{ from } B \to C$$

By Principle of Superposition, net force on C is

$$\vec{F}_C = \vec{F}_{CA} + \vec{F}_{CB}$$

$$\Rightarrow \quad F_C = \left(\frac{1}{4\pi\varepsilon_0}\frac{2q^2}{r^2} - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}\right) \text{ from } B \text{ to } C$$

$$\Rightarrow \quad F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \text{ from } B \text{ to } C$$

Using (1), we get

$$\left|\vec{F}_C\right| = 2.0 \times 10^{-5} \text{ N from } B \text{ to } C$$

$$\Rightarrow \quad F = 20\ \mu\text{N}$$

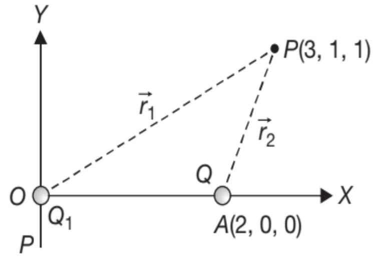
16. 2

17. $T = 7.72 \times 10^{-2}$ N $\simeq 77$ milli newton

$$m = 7.96 \text{ g} \simeq 8 \text{ g}$$

18. The electric field strength at a point having position vector $\vec{r}$ relative to charge Q is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3} \text{ (vector form)}$$



The position vector of P relative to origin is

$$\vec{r}_1 = \left(3\hat{i} + \hat{j} + \hat{k}\right) \text{ m}$$

modulus of $\vec{r}_1$ is

$$r_1 = \sqrt{(3)^2 + (1)^2 + (1)^2} = \sqrt{11} \text{ m}$$

The position vector of P relative to point $A(2,\ 0,\ 0)$ is given by

$$\vec{r}_1 = \left(3\hat{i} + \hat{j} + \hat{k}\right) - \left(2\hat{i}\right) = \hat{i} + \hat{j} + \hat{k}$$

modulus of $\vec{r}_2$

$$r_2 = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3} \text{ m}$$

The electric field at P due to charge Q, at O is

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1\left(3\hat{i} + \hat{j} + \hat{k}\right)}{(11)^{3/2}} \qquad \ldots(1)$$

The electric field at P due to charge Q, at A is

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q\left(\hat{i} + \hat{j} + \hat{k}\right)}{(3)^{3/2}} \qquad \ldots(2)$$

$\therefore$ Net field at P,

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1\left(3\hat{i} + \hat{j} + \hat{k}\right)}{(11)^{3/2}} + \frac{Q\left(\hat{i} + \hat{j} + \hat{k}\right)}{(3)^{3/2}}\right] \qquad \ldots(3)$$

X-component of electric field at P is

$$E_x = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1(3)}{(11)^{3/2}} + \frac{Q(1)}{(3)^{3/2}}\right] = 0$$

This gives

$$Q = -\left(\frac{3}{11}\right)^{3/2}(3Q_1) = -\frac{3^{5/2} \times 10^{-9}}{11^{3/2}} \text{ C}$$

$$\Rightarrow \quad Q = -\frac{3^{5/2}}{\cancel{11^{3/2}}} \times 11 \times \frac{\cancel{11^{3/2}}}{3^{3/2}} \times 10^{-9}$$

$$\Rightarrow \quad Q = -3 \times 11 \times 10^{-9} \text{ C}$$

$$\Rightarrow \quad Q = -33 \text{ nC}$$

$$\Rightarrow \quad |Q| = 33 \text{ nC}$$

19. (a) The forces acting on each balloon are shown in figure and are as under

(i) The effective weight of balloon W acting vertically upward

$$W = V\rho_{\text{air}}g - V\rho_{He}g$$

$$W = V(\rho_{\text{air}} - \rho_{He})g \qquad \ldots(1)$$

where V = volume of balloon, ρ_{air} and ρ_{He} are densities of air and helium respectively

(ii) Electric repulsive force acting horizontally given by

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = 9\times10^9 \times \frac{q^2}{(0.6)^2} \qquad \ldots(2)$$

(iii) Tension in string T along $\vec{AC}$ for balloon A

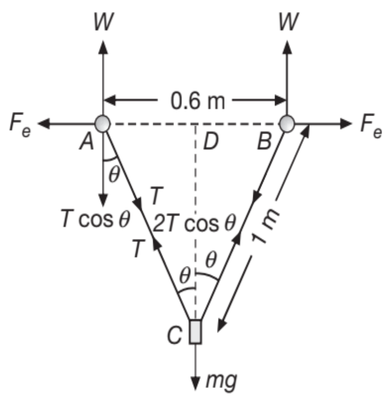
For equilibrium of balloon A say

$$W = T\cos\theta$$

$$F_e = T\sin\theta$$

Dividing (2) by (1)

$$\tan\theta = \frac{F_e}{W} \qquad \ldots(3)$$



For equilibrium of weight C

$$2T\cos\theta = mg$$

$$\therefore \quad T\cos\theta = \frac{mg}{2} \qquad \ldots(4)$$

From (3) and (4)

$$W = T\cos\theta = \frac{mg}{2}$$

$$W = \frac{1}{2}\times5\times10^{-3}\times9.8$$

$$W = 24.5\times10^{-3}\ \text{N} \qquad \ldots(5)$$

From figure, $\tan\theta = \dfrac{AD}{CD}$

$$\Rightarrow \quad \tan\theta = \frac{0.3}{\sqrt{(1)^2-(0.3)^2}} = \frac{0.3}{0.95} \qquad \ldots(6)$$

Using (2), (5) and (6), we get from (3)

$$\frac{0.3}{0.95} = \frac{9\times10^9\times\dfrac{q^2}{(0.6)^2}}{24.5\times10^{-3}}$$

$$\Rightarrow \quad q^2 = \frac{(0.6)^2\times(24.5\times10^{-3})}{9\times10^9}\times\frac{0.3}{0.95}$$

$$\Rightarrow \quad q = 55\times10^{-6}\ \text{C}$$

$$\Rightarrow \quad q = 55\ \mu\text{C}$$

(b) From equations (1) and (5), we get

$$\frac{mg}{2} = V(\rho_{\text{air}} - \rho_{He})g$$

So, volume of balloon, is given by

$$V = \frac{m}{2(\rho_{\text{air}} - \rho_{He})}$$

$$\Rightarrow \quad V = \frac{5\times10^{-3}}{2(1.29-0.2)}$$

$$\Rightarrow \quad V = \frac{5\times10^{-3}}{2\times1.09} = 2.293\times10^{-3}\ \text{m}^3$$

$$\Rightarrow \quad V = 2293\ \text{cc}$$

20. Let q_1 and q_2 be the respective charges distributed over two concentric spheres of radii r and R such that

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{(d-R)}\right] \qquad \ldots(1)$$

As surface densities are given to be equal, therefore

$$\sigma_1 = \sigma_2$$

$$\Rightarrow \quad \frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2} \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{q_1}{q_2} = \frac{r^2}{R^2}$$

$$\Rightarrow \quad \frac{q_1}{q_2} + 1 = \frac{r^2}{R^2} + 1$$

$$\Rightarrow \quad \frac{q_1+q_2}{q_2} = \frac{r^2+R^2}{R^2}$$

Using (1), we get

$$\frac{Q}{q_2} = \frac{r^2+R^2}{R^2}$$

This gives $q_2 = \left[\dfrac{R^2}{r^2+R^2}\right]Q$

Therefore $q_1 = Q - q_2$

$$\Rightarrow \quad q_1 = Q - \left(\frac{R^2}{r^2+R^2}\right)Q = \left(\frac{r^2}{r^2+R^2}\right)Q$$

The potential V_1 at common centre due to charge q_1 is given by

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

$$\Rightarrow \quad V_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{r^2}{r^2+R^2}\right)\frac{Q}{r}$$

$$\Rightarrow \quad V_1 = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{r^2+R^2}$$

The potential V_2 at common centre due to charge q_2 is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{R} = \frac{1}{4\pi\varepsilon_0}\left(\frac{R^2}{r^2+R^2}\right)\frac{Q}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{2e(\Delta V')}{m}} = \sqrt{\frac{2\times 1.602\times 10^{-19}\times 30\times 10^2}{1.672\times 10^{-27}}}$$

$\therefore$ Net potential at common centre,

$$V = V_1 + V_2$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(r^2+R^2)}[r+R]$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q(R+r)}{R^2+r^2}$$

$$\Rightarrow \quad V = 9\times 10^9 \times \frac{5}{100}\times 10^{-9}\times \frac{9}{45}\times 100$$

$$\Rightarrow \quad V = 9\text{ V}$$

21. The horizontal force on dust particle is $F = qE$

If v is speed of particle in air, then by Stoke's Law,

Viscous force $= 6\pi\eta r v$

For uniform speed v, we have

$$qE = 6\pi\eta r v$$

If n is the number of excess electrons, then

$$q = ne$$

$$\Rightarrow \quad neE = 6\pi\eta r v$$

$$\Rightarrow \quad n = \frac{6\pi\eta r v}{eE}$$

Substituting given values

$$n = \frac{6\times 3.14\times 1.6\times 10^{-5}\times 5\times 10^{-7}\times 0.02}{1.6\times 10^{-19}\times 6.28\times 10^5}$$

$$\Rightarrow \quad 0 + \frac{1}{4\pi\varepsilon_0}\frac{(-q_1)q_2}{d} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\Rightarrow \quad n = 30$$

22. Potential at the centre of a loop is the sum of potentials due to charge of its own and that due to other loop.

Potential at the centre C_1 of loop A

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R_1} + \frac{q}{r_1}\right)$$

$$\Rightarrow \quad V_1 = \frac{1}{4\pi\varepsilon_0}q\left[\frac{1}{R_1} + \frac{1}{\sqrt{R_2^2+d^2}}\right]$$

$$\Rightarrow \quad V_1 = 9\times 10^9\times 10^{-6}\left[\frac{1}{R_1} + \frac{1}{\sqrt{(0.09)^2+(0.12)^2}}\right]$$

$$\Rightarrow \quad V_1 = 9\times 10^3\left[\frac{1}{0.05} + \frac{1}{0.15}\right]$$

$$\Rightarrow \quad V_1 = 9\times 10^3 \times \frac{80}{3} = 2.4\times 10^5\text{ V}$$

The potential at the centre C_2 of loop B

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R_2} + \frac{q}{r_2}\right]$$

$$\Rightarrow \quad V_2 = \frac{1}{4\pi\varepsilon_0}q\left[\frac{1}{R_1} + \frac{1}{\sqrt{R_2^2+d^2}}\right]$$

$$\Rightarrow \quad V_2 = 9\times 10^9\times 10^{-6}\left[\frac{1}{R_1} + \frac{1}{\sqrt{(0.05)^2+(0.12)^2}}\right]$$

$$\Rightarrow \quad V_2 = 9\times 10^3\left[\frac{1}{0.09} + \frac{1}{0.13}\right]$$

$$\Rightarrow \quad V_2 = 9\times 10^3\times 100\left[\frac{1}{9} + \frac{1}{13}\right]$$

$$\Rightarrow \quad V_2 = 9\times 10^5\left[\frac{1}{9} + \frac{1}{13}\right]$$

$$\Rightarrow \quad V_2 = 9\times 10^5\left[\frac{13+9}{117}\right]$$

$$\Rightarrow \quad V_2 = 9\times 10^5\times \frac{22}{117}$$

$$\Rightarrow \quad V_2 = 1.692\times 10^5\text{ V}$$

Potential difference, ΔV is given by

$$\Delta V = V_1 - V_2$$

$$\Rightarrow \quad \Delta V = 2.4\times 10^5 - 1.692\times 10^5$$

$\Rightarrow \quad \Delta V = 7.08 \times 10^4$ volt

$\Rightarrow \quad \Delta V = 7080$ V

23. The potential differences between spherical conductors

$$V_{ab} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{a} - \frac{q}{b}\right)$$

Here $a = 1 \text{ cm} = 10^{-2}$ m,

$b = 4 \text{ cm} = 4 \times 10^{-2}$ m

$$\Rightarrow \quad 3000 = 9 \times 10^9 q\left(\frac{1}{10^{-2}} - \frac{1}{4 \times 10^{-2}}\right)$$

$$\Rightarrow \quad 3000 = 9 \times 10^9 q\left[\frac{(4-1)}{4 \times 10^{-2}}\right]$$

$$\Rightarrow \quad q = \frac{3000 \times 4 \times 10^{-2}}{9 \times 10^9 \times 3} = \frac{4}{9} \times 10^{-8} \text{ C}$$

The potential at point distant $r_1 = 3$ cm from centre

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{b}\right)$$

The potential at point distant $r_2 = 2$ cm from centre,

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_2} - \frac{q}{b}\right)$$

The gain in kinetic energy of electron

$$K = e(V_2 - V_1)$$

$$\Rightarrow \quad K = e\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_2} - \frac{q}{b} - \frac{q}{r_1} + \frac{q}{b}\right]$$

$$\Rightarrow \quad K = \frac{1}{4\pi\varepsilon_0} eq\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$\Rightarrow \quad K = 9 \times 10^9 \times 1.6 \times 10^{-19} \times \frac{4}{9} \times 10^{-8} \times \left[\frac{1}{2 \times 10^{-2}} - \frac{1}{3 \times 10^{-2}}\right]$$

$$\Rightarrow \quad K = \frac{32}{3} \times 10^{-17} \text{ J}$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{32}{3} \times 10^{-17} \text{ J}$$

$$\Rightarrow \quad v^2 = \frac{2}{m} \times \frac{32}{3} \times 10^{-17} \text{ J}$$

$$\Rightarrow \quad v^2 = \left(\frac{2}{9 \times 10^{-31}}\right) \times \frac{32}{3} \times 10^{-17}$$

$$\Rightarrow \quad v^2 = \frac{64}{27} \times 10^{14}$$

$\Rightarrow \quad v = 1.54 \times 10^7 \text{ ms}^{-1}$

$\Rightarrow \quad v = 154 \times 10^5 \text{ ms}^{-1}$

$\Rightarrow \quad x = 154$

24. Initial potential energy of system

$$U_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_1}{r_{13}}\right]$$

$$\Rightarrow \quad U_1 = q \times 10^9\left[\frac{2 \times 3}{1} + \frac{3 \times 1}{1} + \frac{2 \times 1}{1}\right]$$

$$\Rightarrow \quad U_1 = 9 \times 10^9 \times 11$$

$$\Rightarrow \quad U_1 = 99 \times 10^9 \text{ J}$$

Final potential energy of system

$$U_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r'_{12}} + \frac{q_2q_3}{r'_{23}} + \frac{q_3q_1}{r'_{13}}\right]$$

$$\Rightarrow \quad U_2 = 9 \times 10^9\left[\frac{2 \times 3}{0.5} + \frac{3 \times 1}{0.5} + \frac{2 \times 1}{0.5}\right]$$

$$\Rightarrow \quad U_2 = 9 \times 10^9 \times 22 = 198 \times 10^9 \text{ J}$$

So, work done $W = U_2 - U_1$

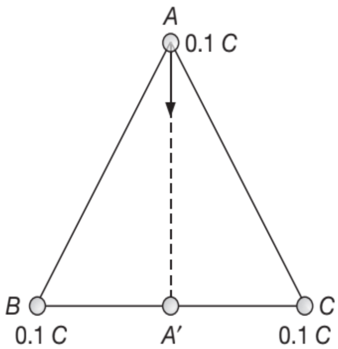
$\Rightarrow \quad W = 198 \times 10^9 - 99 \times 10^9$

$\Rightarrow \quad W = 99 \times 10^9$ J

$\Rightarrow \quad x = 99$

25. Initial electrostatic energy of system of three charges,

$$U_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_1}{r_{13}}\right)$$



Here $q_1 = q_2 = q_3 = q$ (say) $= 0.1$ C

$r_{12} = r_{23} = r_{13} = r$ (say) $= 1$ m

$$U_i = 3\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}\right)$$

$\Rightarrow \quad U_i = 3\times 9\times 10^9 \times \dfrac{(0.1)^2}{1}$

$\Rightarrow \quad U_i = 2.7\times 10^8$ J

When charge A is moved to new position A' which is mid point of side BC, then final electrostatic energy,

$$U_f = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}}+\frac{q_2q_3}{r_{23}}+\frac{q_3q_1}{r_{13}}\right)$$

$$\Rightarrow \quad U_f = \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{(A'B)}+\frac{q^2}{[BC]}+\frac{q^2}{(A'C)}\right]$$

But $A'B = A'C = \dfrac{r}{2} = 0.5$ m, so we get

$$U_f = 9\times 10^9\left[\frac{(0.1)^2}{0.5}+\frac{(0.1)^2}{1}+\frac{(0.1)^2}{0.5}\right]$$

$\Rightarrow \quad U_f = 4.5\times 10^8$ J

Work done

$W = U_f - U_i$

$\Rightarrow \quad W = 4.5\times 10^8 - 2.7\times 10^8$

$\Rightarrow \quad W = 1.8\times 10^8$ J

Since, by definition, Power $= \dfrac{\text{Work done}}{\text{Time}}$

$\Rightarrow \quad P = \dfrac{W}{t}$

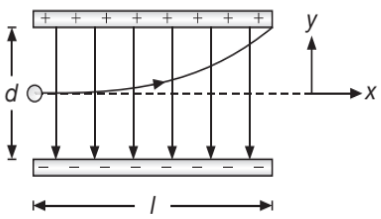
$\Rightarrow \quad t = \dfrac{W}{P} = \dfrac{1.8\times 10^8}{10^3}$ second

$\Rightarrow \quad t = 1.8\times 10^5$ s

$\Rightarrow \quad t = \dfrac{1.8\times 10^5}{3600}$ h

$\Rightarrow \quad t = 50$ h

26. The electron on entering the electric field, will experience a force $F = eE$ opposite to direction of electric field. On account of this force, it will be deflected in transverse direction. The path of electron is shown in figure.



Assuming initial velocity along X-axis, the transverse deflection will be along Y-direction,

$$a_y = \frac{F}{m} = \frac{eE}{m} = \text{constant}$$

$\therefore$ Initial velocity along Y-direction is zero and initial velocity along X-direction is u (say) therefore transverse deflection

$$y = \frac{1}{2}a_y t^2$$

$$\Rightarrow \quad y = \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{\ell}{u}\right)^2 \qquad \left\{\because t = \frac{\ell}{u}\right\}$$

So, $y_{\text{MIN}} = \dfrac{1}{2}\dfrac{eE}{m}\left(\dfrac{\ell^2}{u^2_{\text{MAX}}}\right)$

Here $y_{\text{MIN}} = \dfrac{d}{2}$

and $E = \dfrac{V}{d}$

$$\Rightarrow \quad \frac{d}{2} = \frac{\ell}{2}\frac{e}{m}\left(\frac{V}{d}\right)\frac{\ell^2}{u^2_{\text{MAX}}}$$

$$\Rightarrow \quad u^2_{\text{MAX}} = \frac{e}{m}\frac{V}{d}\left(\frac{\ell^2}{d}\right)$$

$$\Rightarrow \quad u_{\text{MAX}} = \frac{\ell}{d}\sqrt{\frac{eV}{m}}$$

Substituting given values

$$u_{MAX} = \frac{10^{-1}}{2\times 10^{-2}}\sqrt{\left(\frac{1.6\times 10^{-19}\times 300}{9\times 10^{-31}}\right)}$$

$\Rightarrow \quad u_{\text{MAX}} = 3.65\times 10^7$ ms^{-1}

$\Rightarrow \quad u_{\text{MAX}} = 365\times 10^5$ ms^{-1}

$\Rightarrow \quad x = 365$

27. (a) For equilibrium of oil drop between the plates the weight (mg) of drop must be balanced by upward electric force qE where E is electric field strength and q the charge on drop.

$$qE = mg$$

$$\Rightarrow \quad q = \frac{mg}{E}$$

where $E = \dfrac{V}{d} = \dfrac{1.5\text{ kV}}{1.5\times 10^{-2}\text{ m}}$

$$\Rightarrow \quad E = \frac{1.5\times 10^3}{1.5\times 10^{-2}}\text{ Vm}^{-1} = 10^5\text{ Vm}^{-1}$$

and $m = 4.9\times 10^{-15}$ kg

$$\Rightarrow \quad q = \frac{4.9\times10^{-15}\times9.8}{10^5}$$

$$\Rightarrow \quad q = 4.8\times10^{-19}\text{ C}$$

If n is the number of electrons attached to oil drop and e is elementary charge, then by charge quantisation

$$q = ne$$

$$\Rightarrow \quad n = \frac{q}{e} = \frac{4.8\times10^{-19}}{1.6\times10^{-19}} = 3$$

(b) If the polarity of plates is reversed, then both weight of drop and electric force act downward therefore net downward force,

$$F = mg + qE = 2mg \qquad \{\text{since } mg = qE\}$$

Initial Acceleration of drop,

$$a = \frac{F}{m} = \frac{2mg}{q} = 2g \text{ downward}$$

$$\Rightarrow \quad a = 2\times10 = 20\text{ ms}^{-2} \text{ downward}$$

(c) If the downward force F is balanced by upward viscous force, then drop attains terminal velocity v_T given by Stoke's Law

$$F = 6\pi\eta r v_T$$

$$\Rightarrow \quad 2mg = 6\pi\eta r v_T$$

$$\Rightarrow \quad v_T = \frac{2mg}{6\pi\eta r} = \frac{mg}{3\pi\eta r}$$

$$\Rightarrow \quad v_T = \frac{4.9\times10^{-15}\times10}{3\times3.14\times1.8\times10^{-5}\times5\times10^{-6}}$$

$$\Rightarrow \quad v_T = 5.8\times10^{-5}\text{ ms}^{-1}$$

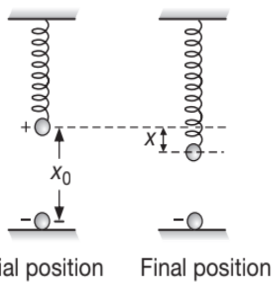
$$\Rightarrow \quad v_T = 58\times10^{-6}\text{ ms}^{-1}$$

$$\Rightarrow \quad v_T = 58\ \mu\text{ms}^{-1}$$

28. When the thread is burnt, the positively charged sphere moves downward due to two forces

(i) Weight mg, acting vertically downward

(ii) Coulomb force of attraction $\frac{1}{4\pi\varepsilon_0}\frac{qq}{x^2}$, acting downward



At the position of maximum elongation of spring, the sphere moving downward comes to rest.

Let maximum elongation of spring be x, then by Law of Conservation of Energy

$$(U+K)_{\text{initial}} = (U+K)_{\text{final}}$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{(q)(-q)}{x_0} + mgx_0 + 0 = \frac{1}{4\pi\varepsilon_0}\frac{q(-q)}{(x_0-x)} + mg(x_0-x) + \frac{1}{2}K(x^2)$$

Total energy in initial position = Total energy in final position

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}q^2\left(\frac{1}{x_0-x} - \frac{1}{x_0}\right) = \frac{1}{2}Kx^2 - mgx \qquad \ldots(1)$$

Given $x = 0.1$ m, $x_0 = 0.5$ m,

$$K = 10^4\text{ Nm}^{-1},\ g = 10\text{ ms}^{-2}$$

$$\Rightarrow \quad \frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{0.4} - \frac{1}{0.5}\right) = \frac{1}{2}(10^4)(0.1)^2 - (5)(10)(0.1)$$

$$\Rightarrow \quad 4.5\times10^9 q^2 = 45$$

$$\Rightarrow \quad q^2 = \frac{45}{4.5\times10^9} = 10^8$$

$$\Rightarrow \quad q = 10^4\text{ C} = 100\ \mu\text{C}$$

29. Electric field due to a straight conductor,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}$$

$$\therefore \quad \text{Force, } F = qE = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda q}{r}$$

$$\Rightarrow \quad F = 9\times10^9\cdot\frac{2\times6\times10^4\times1.6\times10^{-19}}{0.5}$$

$$\Rightarrow \quad F = 3.45\times10^{-4}\text{ N}$$

$$\Rightarrow \quad F = 345\times10^{-6}\text{ N} = 345\ \mu\text{N}$$

Since, by definition, we have

$$W = q(V_2 - V_1)$$

$$\Rightarrow \quad W = q\cdot\frac{1}{4\pi\varepsilon_0}2\lambda\log_e\left(\frac{r_2}{r_1}\right)$$

$$\Rightarrow \quad W = \frac{1}{4\pi\varepsilon_0}(2\lambda q)\log_e\frac{r_2}{r_1}$$

$$\Rightarrow \quad W = 9\times10^9\times(2\times6\times10^4\times1.6\times10^{-19})\times\log_e\left(\frac{0.5}{0.2}\right)$$

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$\Rightarrow \quad W = 1.58 \times 10^{-4}$ J

$\Rightarrow \quad W \cong 16$ mJ

30. Gain in KE $= e(V_2 - V_1)$

$$\Rightarrow \quad \frac{1}{2}mv^2 = e\frac{1}{4\pi\varepsilon_0}q\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$\Rightarrow \quad v^2 = \frac{2e}{m}\left[\frac{1}{4\pi\varepsilon_0}\left(q\frac{ab}{b-a}\right)\right]$$

$$\Rightarrow \quad v = 8 \times 10^6 \text{ ms}^{-1}$$

So, the part not readable is 8.

31. The electric potential at P is due to two spheres

(i) A large solid sphere of charge density (ρ)

(ii) A small solid sphere of charge density $(-\rho)$

The electric potential due to whole solid sphere of radius R at a distance r from centre

$$V_1 = \frac{1}{4\pi\varepsilon_0}q\frac{(3R^2 - r^2)}{2R^3}$$

$$\Rightarrow \quad V_1 = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{4}{3}\pi R^3\rho\right)(3R^2 - r^2)}{2R^3}$$

The electric potential at external point at a distance

$$r' = \sqrt{x^2 + y^2} = 5 \text{ cm}$$

$$\Rightarrow \quad V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r'} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\frac{4}{3}\pi r'^3(-\rho)}{r'}$$

So, net potential

$$V = V_1 + V_2$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\left[\frac{2}{3}\pi R\rho(3R^2 - r^2) - \frac{4}{3}\pi r^2\rho\right]$$

where $R = 5 \text{ cm} = 5 \times 10^{-2}$ m,

$r = 4 \text{ cm} = 4 \times 10^{-2}$ m,

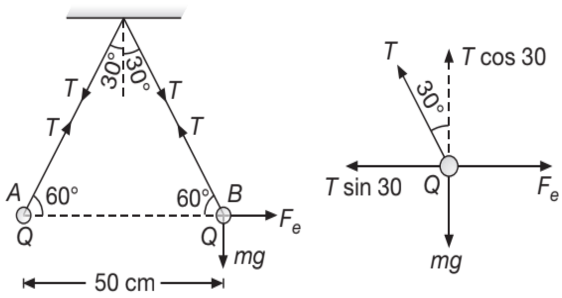
$r' = 5 \text{ cm} = 5 \times 10^{-2}$ m,

$$\rho = \frac{10^{-6}}{\pi}$$

Substituting given values, $V = 35.16$ V

So, $V \cong 35$ V

32. Considering the equilibrium of ball P.



$$T\sin(30°) = F = F_e \quad \ldots(1)$$

$$T\cos(30°) = mg \quad \ldots(2)$$

$$\Rightarrow \quad T = \frac{mg}{\cos 30} = \frac{(0.866 \times 10^{-3}) \times 10}{0.866}$$

$$\Rightarrow \quad T = 10^{-2} \text{ N}$$

From equation (1)

$$T\sin 30° = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$$

$$\Rightarrow \quad 10^{-2} \times \frac{1}{2} = \frac{9 \times 10^9 \times q^2}{(0.5)^2}$$

Solving we get,

$$q = 3.726 \times 10^{-7} \text{ C}$$

$$q = 373 \text{ nC}$$

33. $\phi_E = \dfrac{q}{2\varepsilon_0}\left(1 - \dfrac{a}{\sqrt{R^2 + a^2}}\right)$ {as derived earlier}

$$\Rightarrow \quad \phi_E = \frac{8.85 \times 10^{-6}}{2(8.85 \times 10^{-12})}\left(1 - \frac{40}{50}\right)$$

$$\Rightarrow \quad \phi_E = \frac{10^6}{2}\left(1 - \frac{4}{5}\right)$$

$$\Rightarrow \quad \phi_E = \frac{10^6}{2} \times \frac{1}{5} = 10^5 \text{ NC}^{-1}$$

$$\Rightarrow \quad \phi_E = 100 \text{ kNC}^{-1}$$

34. By Law of Conservation of Energy

$$(U + K)_{\text{initial}} = (U + K)_{\text{final}}$$

$$\Rightarrow \quad \frac{1}{2}mv^2 + \frac{Qq}{4\pi\varepsilon_0 r_1} = \frac{1}{2}m(0)^2 + \frac{Qq}{4\pi\varepsilon_0 r_2}$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$\Rightarrow \frac{1}{2}(2\times10^{-6})(1)^2=(10^{-9})(10^{-8})(9\times10^{9})\left(\frac{1}{r_2}-\frac{1}{\left(\frac{10}{100}\right)}\right)$$

$$\Rightarrow \frac{100}{9}=\frac{1}{r_2}-10$$

$$\Rightarrow \frac{1}{r_2}=\frac{190}{9}\text{ m}$$

$$\Rightarrow r_2=\frac{9}{190}\text{ m}$$

$$\Rightarrow r_2=4.7\times10^{-2}\text{ m}$$

$$\Rightarrow r_2=47\times10^{-3}\text{ m}$$

$$\Rightarrow r_2=47\text{ mm}$$

35. 63.5 g copper contains $N_A=6\times10^{23}$ copper atoms.

Therefore number of copper atoms in 10 g copper is

$$N=\frac{6\times10^{23}}{63.5}\times10$$

Since only one electron is transferred for every 1000 atoms, therefore the number of electrons transferred,

$$n=\frac{6\times10^{23}\times10}{63.5\times1000}$$

Magnitude of charge q is given by

$$q=ne=\frac{6\times10^{23}\times10}{63.5\times1000}\times1.6\times10^{-19}\text{ C}$$

$$\Rightarrow q=\frac{1920}{127}\text{ C}$$

Separation between pieces is $r=1\text{ cm}=10^{-2}\text{ m}$

One piece of copper has positive charge and the other negative charge, so force of attraction between the pieces is

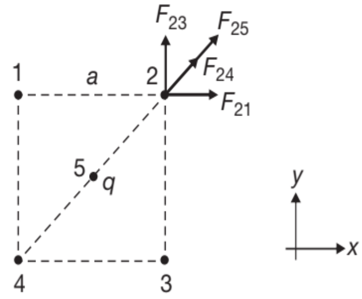
$$F=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

$$\Rightarrow F=\frac{9\times10^{9}\left(\frac{1920}{127}\right)\times\left(\frac{1920}{127}\right)}{(10^{-2})^2}\text{ N}$$

$$\Rightarrow F=2.057\times10^{6}\text{ N}$$

So, $x\cong2$

36. Let us consider the equilibrium of charge Q placed at 2. Then for equilibrium



$$\vec{F}_2=\vec{0}$$

$$\Rightarrow \vec{F}_{2x}\hat{i}+F_{2y}\hat{j}=\vec{0}$$

$$\Rightarrow F_{2x}=0$$

$$\Rightarrow F_{21}+F_{24}\cos45+F_{25}\cos45+F_{23}\cos90=0$$

$$\Rightarrow \frac{Q^2}{4\pi\varepsilon_0a^2}+\frac{Q^2}{4\pi\varepsilon_0(\sqrt{2}a)^2}\frac{1}{\sqrt{2}}+\frac{Qq}{4\pi\varepsilon_0\left(\frac{a}{\sqrt{2}}\right)^2}\frac{1}{\sqrt{2}}+0=0$$

$$\Rightarrow Q+\frac{Q}{2\sqrt{2}}+\sqrt{2}q=0$$

$$\Rightarrow Q(2\sqrt{2}+1)+4q=0$$

$$\Rightarrow q=-\frac{Q}{4}(2\sqrt{2}+1)$$

Since $Q=\frac{8}{7}(1-2\sqrt{2})$

$$\Rightarrow q=-\frac{1}{4}\left[\frac{8}{7}(1-2\sqrt{2})(1+2\sqrt{2})\right]$$

$$\Rightarrow q=-\frac{1}{4}\left(\frac{8}{7}\right)(-7)=2\text{ C}$$

37. Beta particles (i.e., electrons) emitted in t seconds

$$N=5\times10^{10}\text{ t}$$

∴ The deficiency of electrons from a conductor results in positive charge

As 40% beta-particles escape from the source, positive charge gained by source (metal sphere),

$$q=\frac{40}{100}Ne=-\frac{40}{100}\times5\times10^{10}et$$

$$\Rightarrow q=2\times10^{10}t\times1.6\times10^{-19}=3.2\times10^{-9}t\text{ C}$$

Due to this charge the potential acquired by sphere

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

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Substituting given values

$$2 = 9\times10^9 \times \frac{3.2\times10^{-9}t}{10^{-2}}$$

$$\Rightarrow \quad t = \frac{2\times10^{-2}}{9\times10^9\times3.2\times10^{-9}}$$

$$\Rightarrow \quad t = 7\times10^{-4}\ \text{s} = 700\ \mu\text{s}$$

38. For the ball just to complete the circle, the tension must vanish at the topmost point, i.e., $T_2 = 0$

From Newton's Second Law,

$$T_2 + mg - \frac{q^2}{4\pi\varepsilon_0\ell^2} = \frac{mv^2}{\ell} \qquad \ldots(1)$$

At the topmost point $T_2 = 0$

$$mg - \frac{q^2}{4\pi\varepsilon_0\ell^2} = \frac{mv^2}{\ell} \qquad \ldots(2)$$

By Law of Conservation of Energy

$$\begin{pmatrix}\text{Energy at the}\\ \text{lowest point L}\end{pmatrix} = \begin{pmatrix}\text{Energy at the}\\ \text{highest point H}\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg(2\ell) \qquad \ldots(3)$$

$$\Rightarrow \quad v^2 = u^2 - 4g\ell \qquad \ldots(4)$$

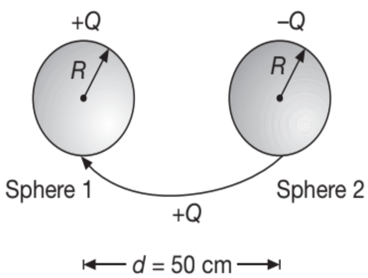
From equation (2), $v^2 = g\ell - \dfrac{q^2}{4\pi\varepsilon_0 m\ell}$ $\qquad \ldots(5)$

Equating (4) and (5), $u = \sqrt{5g\ell - \dfrac{q^2}{4\pi\varepsilon_0 m\ell}}$

$$\Rightarrow \quad u = \left(\frac{275}{8}\right)^{1/2} = 5.86\ \text{ms}^{-1}$$

$$\Rightarrow \quad u = 586\ \text{cms}^{-1}$$

39. Since the spheres are initially neutral, so when a charge Q is transferred from one sphere to the other, the charges on the sphere become $+Q$ and $-Q$ respectively. The potential difference between them is



$$\Delta V = \frac{Q}{4\pi\varepsilon_0 R} - \left(-\frac{Q}{4\pi\varepsilon_0 R}\right)$$

$$\Rightarrow \quad Q = 2\pi\varepsilon_0 R\Delta V$$

$$\Rightarrow \quad Q = \frac{2}{9\times10^9}\times100$$

Potential at the surface of first sphere,

V_1 = Potential due to charge on 1 + Potential due to charge on 2

$$\Rightarrow \quad V_1 = \frac{Q}{4\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0(d-R)} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{(d-R)}\right]$$

Potential at the surface of second sphere,

V_2 = Potential due to charge on 1 + Potential due to charge on 2

$$\Rightarrow \quad V_2 = \frac{+Q}{4\pi\varepsilon_0(d-R)} - \frac{Q}{4\pi\varepsilon_0 R} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{(d-R)} - \frac{1}{R}\right]$$

Potential difference between the surfaces of the two spheres is $\Delta V'$ given by

$$\Delta V' = V_1 - V_2$$

$$\Rightarrow \quad \Delta V' = \frac{200}{10^{-2}}\left[\frac{1}{10} - \frac{1}{40}\right] - \frac{200}{10^{-2}}\left[\frac{1}{40} - \frac{1}{10}\right]$$

$$\Rightarrow \quad \Delta V' = 30\times10^2\ \text{V}$$

Kinetic energy gained by proton $= +e\Delta V' = \dfrac{1}{2}mv^2$

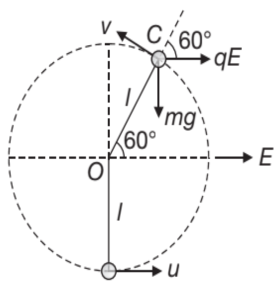
$$\Rightarrow \quad v = \sqrt{\frac{2e(\Delta V')}{m}}$$

$$\Rightarrow \quad v = \sqrt{\frac{2\times1.602\times10^{-19}\times30\times10^2}{1.672\times10^{-27}}}$$

$$\Rightarrow \quad v = 7.58\times10^5\ \text{ms}^{-1}$$

$$\Rightarrow \quad v = 758\times10^3\ \text{ms}^{-1} = 758\ \text{kms}^{-1}$$

40. Using the concept of Work-Energy Theorem,



we get

$$\frac{1}{2}m(v^2 - u^2) = -mg\ell(1+\sin60^\circ) + qE\ell\cos60^\circ$$

Substituting the values, we get

$$u^2 - v^2 = 32.32 \qquad \ldots(1)$$

Since, tension in the string at C is zero.

$$\Rightarrow \quad \frac{mv^2}{\ell} = mg\sin 60^\circ - qE\cos 60^\circ$$

$$\Rightarrow \quad v^2 = 3.66 \qquad \ldots(2)$$

From equations (1) and (2), we get

$$u = 6 \text{ ms}^{-1}$$

41. (a) Consider the two spheres as a system

By Law of Conservation of Momentum

$$0 = m_1 v_1 \hat{i} + m_2 v_2 (-\hat{i})$$

$$\Rightarrow \quad v_2 = \frac{m_1 v_1}{m_2}$$

By Law of Conservation of Energy

$$0 + \frac{1}{4\pi\varepsilon_0}\frac{(-q_1)q_2}{d} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{4\pi\varepsilon_0}\frac{(-q_1)q_2}{r_1 + r_2}$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_1 + r_2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}\frac{m_1^2 v_1^2}{m_2}$$

$$\Rightarrow \quad v_1 = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{2m_2 q_1 q_2}{m_1(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}$$

Substituting values, we get

$$v_1 = \sqrt{\frac{2(0.7)(9\times 10^9)(2\times 10^{-6})(3\times 10^{-6})}{(0.1)(0.8)}\left(\frac{1}{8\times 10^{-3}} - \frac{1}{1}\right)}$$

$$\Rightarrow \quad v_1 = 10.8 \text{ ms}^{-1}$$

$$\Rightarrow \quad v_2 = \frac{m_1 v_1}{m_2} = \frac{0.1 \text{ kg}(10.8 \text{ ms}^{-1})}{0.7 \text{ kg}}$$

$$\Rightarrow \quad v_2 = 1.55 \text{ ms}^{-1}$$

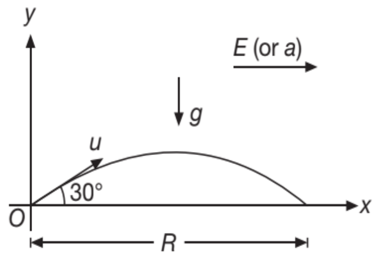
So, $v_1 = 1080 \text{ cms}^{-1}$ and $v_2 = 155 \text{ cms}^{-1}$

(b) If the spheres are conductors/metal, then electrons will move around them such that the centres of the excess charges can be placed inside the spheres without any loss in energy. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving faster than that calculated in (a).

42. $u = 20 \text{ ms}^{-1}$, $\alpha = 45^\circ$

$$a_x = \frac{qE}{m} = 40 \text{ ms}^{-2}$$

$$a_y = g = -10 \text{ ms}^{-2}$$



Since at maximum height $v_y = 0$, so

$$v_y = u_y + a_y t$$

$$\Rightarrow \quad 0 = u\sin\alpha + (-g)T$$

$$\Rightarrow \quad T = \frac{2u_y}{|a_y|} = \frac{2u\sin\alpha}{g} = \frac{2\times 20}{10}\times\frac{1}{\sqrt{2}} = 2\sqrt{2} \text{ s}$$

In time $t = T$, the projectile touches the ground at a distance R from the launch point. So,

Range is given by

$$R = u_x T + \frac{1}{2}a_x T^2$$

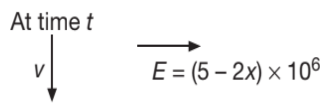
$$\Rightarrow \quad R = \left(\frac{20}{\sqrt{2}}\right)(2\sqrt{2}) + \left(\frac{1}{2}\right)(40)(2\sqrt{2})^2$$

$$\Rightarrow \quad R = 40 + 160$$

$$\Rightarrow \quad R = 200 \text{ m}$$

43. $a_x = \dfrac{qE}{m}$

$$\Rightarrow \quad \frac{dv_x}{dt} = \frac{q}{m}(5 - 2x)$$



$$\Rightarrow \quad v\frac{dv}{dx} = \frac{q}{m}(5 - 2x)$$

$$\Rightarrow \quad \frac{v_x^2}{2} = \frac{q}{m}(5x - x^2)$$

$$\Rightarrow \quad v_x^2 = \frac{2q}{m}(5x - x^2)$$

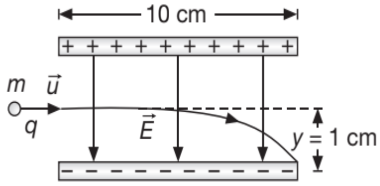
$$\Rightarrow \quad \frac{dx}{dt} = \sqrt{\frac{2q}{m}(5x - x^2)}$$

For x to be maximum $\dfrac{dx}{dt} = 0$

$$\Rightarrow \quad 5x - x^2 = 0$$

$$\Rightarrow \quad x = 5 \text{ m}$$

44. The situation is visualised in the figure



Since, $E=\dfrac{V}{d}$

$$\Rightarrow \quad E=\frac{300}{2/100}=15000\ \text{Vm}^{-1}$$

Since the particle does not come out hence its maximum deflection in vertical direction has to be

$$y=1\ \text{cm}=10^{-2}\ \text{m}$$

$$y=\frac{1}{2}at^2=\frac{1}{2}\frac{qE}{m}\left(\frac{\ell}{u}\right)^2 \qquad \left\{\text{As } a=\frac{qE}{m} \text{ and } t=\frac{\ell}{u}\right\}$$

$$\Rightarrow \quad u^2=\frac{1}{2}\left(\frac{qE}{my}\right)x^2$$

$$\Rightarrow \quad u^2=\frac{1}{2}\frac{(1.6\times10^{-19})(15000)}{(12\times10^{-24})(10^{-2})}\left(\frac{1}{10}\right)^2=10^8$$

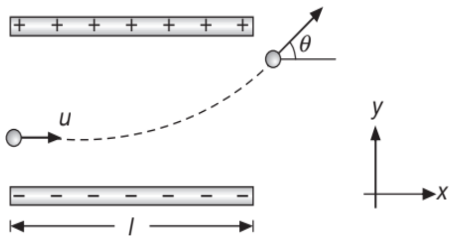
$$\Rightarrow \quad u=10^4\ \text{ms}^{-1}$$

$$\Rightarrow \quad u=10\ \text{kms}^{-1}$$

45. Along the x-direction speed of electron remains uniform; because of no component of field along horizontal. So,

$$v_x=u \text{ and } x=ut$$

$$\Rightarrow \quad \ell=ut \qquad \ldots(1)$$



Along y direction

$$u_y=0$$

Acceleration along y-direction of motion of electron is given by

$$a_y=a=\frac{eE}{m}$$

Since, $v_y=u_y+a_yt$

$$\Rightarrow \quad v_y=0+\left(\frac{eE}{m}\right)t$$

From (1), we get $t=\dfrac{\ell}{u}$

$$v_{y_{\text{final}}}=v_{y_{\text{initial}}}+at$$

$$\Rightarrow \quad v_y=\left(\frac{eE}{m}\right)\left(\frac{\ell}{u}\right)$$

If θ be the angle of deviation, then

$$\tan\theta=\frac{v_y}{v_x}=\frac{\left(\frac{eE\ell}{mu}\right)}{u}$$

$$\Rightarrow \quad \theta=\tan^{-1}\left(\frac{eE\ell}{mu^2}\right)$$

$$\Rightarrow \quad \theta=\tan^{-1}\left[\frac{eE\ell}{m\left(\sqrt{3}\frac{eE\ell}{m}\right)}\right]=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=30^\circ$$

46. Lets consider $V=0$ at point P. Then the potential at the original position of the charge is

$$V_i=-Ex=-EL\cos\theta$$

At the final point A, $V_f=-EL$. Since the table is frictionless, by Law of Conservation of Energy

$$(K+U)_i=(K+U)_f$$

$$\Rightarrow \quad 0-qEL\cos\theta=\frac{1}{2}mv^2-qEL$$

$$\Rightarrow \quad v=\sqrt{\frac{2qEL(1-\cos\theta)}{m}}$$

$$\Rightarrow \quad v=\sqrt{\frac{2(2\times10^{-6}\ \text{C})(300\ \text{NC}^{-1})(1.5\ \text{m})(1-\cos60^\circ)}{0.01\ \text{kg}}}$$

$$\Rightarrow \quad v=0.3\ \text{ms}^{-1}$$

$$\Rightarrow \quad v=30\ \text{cms}^{-1}$$

47. Total charge

$$Q=\int\rho dV=\int_{r=0}^{r=R}(Kr^a)(4\pi r^2dr)=\frac{4\pi k}{a+3}(R^{a+3})$$

$$Q'=\int\rho dV=\int_{r=0}^{r=\frac{R}{2}}(Kr^a)(4\pi r^2dr)=\frac{4\pi k}{a+3}\left(\frac{R}{2}\right)^{a+3}$$

According to question

$$\frac{1}{4\pi\varepsilon_0}\frac{Q'}{\left(\frac{R}{2}\right)^2}=\frac{1}{8}\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\right)$$

Putting the value of Q and Q' get

$$a=2$$

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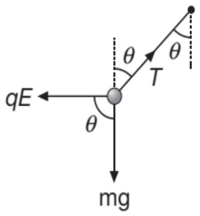
1. $T\cos\theta = mg$

$T\sin\theta = qE$



$$\Rightarrow \tan\theta = \frac{qE}{mg}$$

$$\Rightarrow \tan\theta = \frac{5\times10^{-6}\times2000}{2\times10^{-3}\times10} = \frac{1}{2}$$

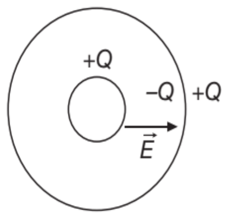
$$\Rightarrow \tan^{-1}\left(\frac{1}{2}\right)$$

Hence, the correct answer is (D).

2. For CASE-1

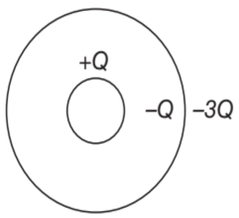
Electric field between spherical surface

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$$



For CASE-2

Electric field between the surfaces remains unchanged.



So, potential difference between them also remains unchanged.

Hence, the correct answer is (B).

3. Since $dV = -\vec{E}d\vec{r} = -(Ax+B)dx$

$$\Rightarrow \int_{v_2}^{v_1} dV = \int_{-5}^{1} -(Ax+B)dx$$

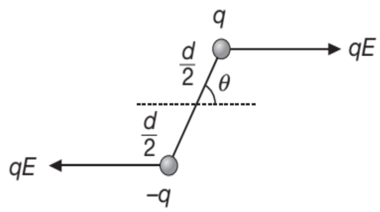
$$\Rightarrow V_1 - V_2 = \left(-A\frac{x^2}{2} - Bx\right)_{-5}^{1}$$

$$\Rightarrow V_1 - V_2 = \left(-\frac{A}{2} - B\right) + \left(\frac{A}{2}25 + B(-5)\right)$$

$$\Rightarrow V_1 - V_2 = 12A - 6B = 240 - 60 = 180 \text{ V}$$

Hence, the correct answer is (A).

4. Since $\tau = I\alpha$



$$\Rightarrow -(qE)d\theta = I\frac{d^2\theta}{dt^2} = 2\left(\frac{md^2}{4}\right)\left(\frac{d^2\theta}{dt^2}\right)$$

$$\Rightarrow \frac{d^2\theta}{dt^2} - \frac{2qE\theta}{md}$$

$$\Rightarrow \omega = \sqrt{\frac{2qE}{md}}$$

Hence, the correct answer is (A).

5. $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

$$\int_{V_G}^{V_P} dV = \int -\frac{\lambda}{2\pi\varepsilon_0 r}dr$$

$$\Rightarrow V_P - V_G = \frac{\lambda}{2\pi\varepsilon_0}\ell n\left(\frac{r}{r_0}\right)$$

Since, $\frac{1}{2}mv^2 = q(V_P - V_G)$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{q\lambda}{2\pi\varepsilon_0}\ell n\left(\frac{r}{r_0}\right)$$

$$\Rightarrow v \propto \left[\ell n\left(\frac{r}{r_0}\right)\right]^{\frac{1}{2}}$$

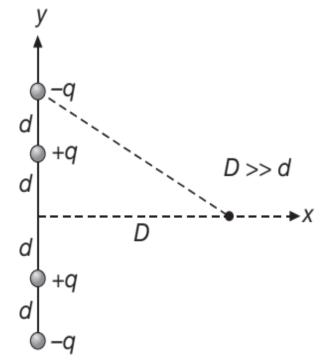
Hence, the correct answer is (A).

6. $$U=\frac{1}{4\pi\varepsilon_0}\left(-\frac{q^2}{d}+\frac{Qq}{\left(\frac{d}{2}+D\right)}-\frac{Qq}{\left(D-\frac{d}{2}\right)}\right)$$

$$\Rightarrow \quad U=\frac{1}{4\pi\varepsilon_0}\left(-\frac{q^2}{d}-\frac{qQd}{D^2}\right)$$

Hence, the correct answer is (A).

7.



$$|\vec{E}|=\frac{2qD}{4\pi\varepsilon_0\left(D^2+4d^2\right)^{\frac{3}{2}}}-\frac{2qD}{4\pi\varepsilon_0\left(D^2+d^2\right)^{\frac{3}{2}}}$$

(along -ive x direction)

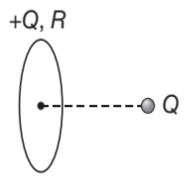
$$\Rightarrow \quad |\vec{E}|=\frac{2qD}{4\pi\varepsilon_0 D^3}\left[\frac{1}{\left(1+\left(\frac{2d}{D}\right)^2\right)^{\frac{3}{2}}}-\frac{1}{\left(1+\left(\frac{d}{D}\right)^2\right)^{\frac{3}{2}}}\right]$$

$$\Rightarrow \quad |\vec{E}|=\frac{2q}{4\pi\varepsilon_0 D^2}\left(1-\frac{3}{2}\frac{4d^2}{D^2}-1+\frac{3}{2}\frac{d^2}{D^2}\right)$$

$$\Rightarrow \quad |\vec{E}|=\frac{9qd^2}{4\pi\varepsilon_0 D^4}$$

Hence, the correct answer is (C).

8.



Potential at any point of the charged ring

$$V_P=\frac{Q}{4\pi\varepsilon_0\sqrt{R^2+x^2}}$$

If v_0 is the minimum velocity for the point charge to reach the centre, then

$$\frac{1}{2}mv_0^2=Q\left(V_C-V_P\right)$$

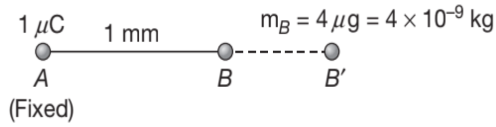
$$\Rightarrow \quad \frac{1}{2}mv_0^2=\frac{Q}{4\pi\varepsilon_0}\left(\frac{Q}{3a}-\frac{Q}{5a}\right)$$

$$\Rightarrow \quad v_0^2=\frac{4}{15}\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q^2}{ma}\right)$$

$$\Rightarrow \quad v_0=\sqrt{\frac{2}{m}}\left[\frac{2}{15}\left(\frac{q^2}{4\pi\varepsilon_0 a}\right)\right]^{\frac{1}{2}}$$

Hence, the correct answer is (D).

9.



Since, $U_1=\frac{q_1q_2}{4\pi\varepsilon_0 r_i}$ $\quad U_f=\frac{q_1q_2}{4\pi\varepsilon_0 r_2}$

By Law of Conservation of Energy, we have

$$(U+K)_i=(U+K)_f$$

$$\frac{q_1q_2}{4\pi\varepsilon_0 r_1}=\frac{q_1q_2}{4\pi\varepsilon_0 r_2}+\frac{1}{2}mv^2$$

$$v^2=\frac{2q_1q_2}{4\pi\varepsilon_0 m}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$\Rightarrow \quad v^2=\frac{2\times 9\times 10^9\times 10^{-12}}{4\times 10^{-9}\times 10^{-3}}\left(1-\frac{1}{9}\right)=4\times 10^9$$

$$\Rightarrow \quad v=\sqrt{40}\times 10^4\ \text{ms}^{-1}=6.32\times 10^4\ \text{ms}^{-1}$$

Hence, the correct answer is (D).

10. $$T=2\pi\sqrt{\frac{L}{g_{\text{eff}}}}$$

where, $g_{\text{eff}}=\sqrt{g^2+\left(\frac{gE}{m}\right)^2}$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{L}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2}}}$$

Hence, the correct answer is (C).

11. $V=0$ and

$$\vec{E}=\frac{\vec{p}}{4\pi\varepsilon_0 r^3}\sqrt{3\cos^2\theta+1}$$

For $\theta=\frac{\pi}{2}$ and $r=d$, we get

$$\vec{E}=\frac{-\vec{p}}{4\pi\varepsilon_0 d^3}$$

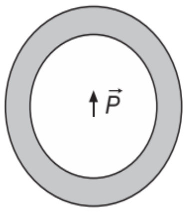
Hence, the correct answer is (C).

12. Since net charge on the dipole is zero, so the inside surface shall have non-zero and non-uniform charge distribution. Net field outside the region would be same as that would have been for point charge at surface.

Hence, the correct answer is (C).

13. According to Gauss Law, we have

$$E(4\pi a^2) = \frac{q_{enclosed}}{\varepsilon_0} \quad \ldots(1)$$



$$\Rightarrow \quad q_{enc} = \int_0^a (kr) \times 4\pi r^2 dr$$

$$\Rightarrow \quad q_{enc} = 4\pi k\left(\frac{a^4}{4}\right) = 2Q \quad \ldots(2)$$

From (1), we get

$$E(4\pi a^2) = \frac{4\pi k a^4}{4\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{ka^2}{4\varepsilon_0}$$

Since both charges do not experience any force, so

$$(F_{\text{net}})_A = 0$$

$$\Rightarrow$$

$$\Rightarrow \quad \frac{ka^2}{4\varepsilon_0} \times Q = \frac{Q^2}{4\pi\varepsilon_0 \times 4a^2}$$

$$\text{Since, } 2Q = \frac{4\pi k R^2}{4}$$

$$\Rightarrow \quad k = \frac{2Q}{\pi R^4}$$

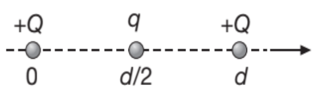
$$\left(\frac{2Q}{\pi R^4}\right)\left(\frac{a^2 Q}{4\varepsilon_0}\right) = \frac{Q^2}{4\pi\varepsilon_0 \times 4a^2}$$

$$\Rightarrow \quad 8a^4 = R^4$$

$$\Rightarrow \quad a = \frac{R}{8^{\frac{1}{4}}} = 8^{-\frac{1}{4}}R$$

Hence, the correct answer is (D).

14.



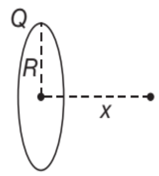
$$F_Q = \frac{Qq}{4\pi\varepsilon_0\left(\frac{d}{2}\right)^2} + \frac{Q^2}{4\pi\varepsilon_0 d^2} = 0$$

$$\Rightarrow \quad 4Qq + Q^2 = 0$$

$$\Rightarrow \quad q = -\frac{Q}{4}$$

Hence, the correct answer is (C).

15. $E(x) = \dfrac{Qx}{4\pi\varepsilon_0 (R^2 + x^2)^{\frac{3}{2}}}$



For maximum, $\dfrac{dE}{dx} = 0$

$$\Rightarrow \quad \frac{Q}{4\pi\varepsilon_0}\left[\frac{(R^2+x^2)^{\frac{3}{2}} - (x)\frac{3}{2}(R^2+x^2)^{\frac{1}{2}}(2x)}{(R^2+x^2)^3}\right] = 0$$

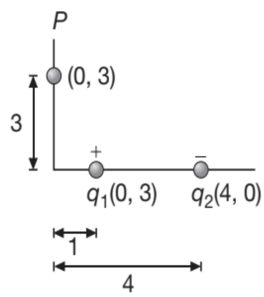
$$\Rightarrow \quad \frac{(R^2+x^2)^{\frac{1}{2}}Q}{4\pi\varepsilon_0 (R^2+x^2)^3}(x^2 + R^2 - 3x^2) = 0$$

$$\Rightarrow \quad x = \frac{R}{\sqrt{2}}$$

$$\Rightarrow \quad h = \frac{R}{\sqrt{2}}$$

Hence, the correct answer is (A).

16.



By Principle of Superposition, we have

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

where,

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{\sqrt{10} \times 10^{-6}}{10}\left(-\frac{\hat{i} + 3\hat{j}}{\sqrt{10}}\right)$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{(-25\times10^{-6})}{25}\left(-\frac{4\hat{i}+3\hat{j}}{5}\right)$$

$$\Rightarrow \quad \vec{E} = 9\times10^9\times10^{-6}\left[-\frac{\hat{i}+3\hat{j}}{10} - \left(-\frac{4\hat{i}+3\hat{j}}{5}\right)\right]$$

$$\Rightarrow \quad \vec{E} = \frac{9\times10^3}{10}\left(-\hat{i}+3\hat{j}+8\hat{i}-6\hat{j}\right)$$

$$\Rightarrow \quad \vec{E} = 9\times10^2\left(+7\hat{i}-3\hat{j}\right)$$

$$\Rightarrow \quad \vec{E} = \left(63\hat{i}-27\hat{j}\right)\times10^2\ \text{Vm}^{-1}$$

Hence, the correct answer is (A).

17. $Q = \int_0^R 4\pi r^2\left(\frac{A}{r^2}e^{-\frac{2r}{a}}\right)dr$

$$\Rightarrow \quad Q = \frac{4\pi Aa}{-2}e^{\frac{-2r}{a}}\Bigg|_0^R = 2\pi aA\left(1-e^{-\frac{2R}{a}}\right)$$

$$\Rightarrow \quad Q = e^{-\frac{2R}{a}} = 1 - \frac{Q}{2\pi aA}$$

$$\Rightarrow \quad e^{\frac{2R}{a}} = \frac{1}{\left(1-\frac{Q}{2\pi aA}\right)}$$

$$\Rightarrow \quad R = \frac{a}{2}\ln\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$$

Hence, the correct answer is (A).

18. $V \propto \frac{1}{d^2}$

$$\Rightarrow \quad \frac{4qa}{x^2} = \frac{2qa}{(R-x)^2}$$

$$\Rightarrow \quad (R-x) = \frac{x}{\sqrt{2}}$$

$$\Rightarrow \quad x = \frac{\sqrt{2}R}{(\sqrt{2}+1)}$$

Hence, the correct answer is (B).

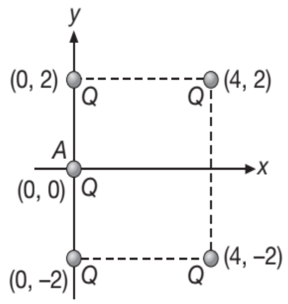
19. $Q = 4\pi\sigma\left(a^2+b^2+c^2\right)$

$$\Rightarrow \quad V = \frac{\sigma}{4\pi\varepsilon_0}\left(\frac{4\pi a^2}{a}+\frac{4\pi b^2}{b}+\frac{4\pi c^2}{c}\right)$$

$$\Rightarrow \quad V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{a+b+c}{a^2+b^2+c^2}\right)$$

Hence, the correct answer is (A).

20.



$$V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{2}+\frac{Q}{2}+\frac{q}{\sqrt{2^2+4^2}}+\frac{Q}{\sqrt{2^2+4^2}}\right)$$

$$\Rightarrow \quad V_A = \frac{1}{4\pi\varepsilon_0}\left(Q+\frac{2Q}{2\sqrt{5}}\right)$$

So, potential energy is given by

$$U = \frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$$

Hence, the correct answer is (D).

21. $F = \frac{p}{4\pi\varepsilon_0 y^3}Q$, for electric dipole

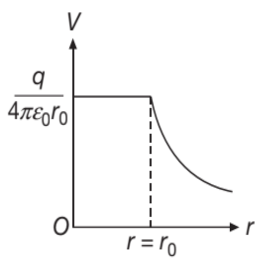
and $F' = \frac{p}{4\pi\varepsilon_0\left(\frac{y}{3}\right)^3}Q = \frac{27pQ}{4\pi\varepsilon_0 y^3}$

$$\Rightarrow \quad F' = 27F$$

Hence, the correct answer is (C).

22. For spherical shell $V = \frac{q}{4\pi\varepsilon_0 r_0}$ for $r \le r_0$

and $V = \frac{q}{4\pi\varepsilon_0 r}$ for $r > r_0$



Hence, the correct answer is (A).

23. $0 = U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{a}+\frac{Qq}{a}+\frac{Qq}{\sqrt{2}a}\right)$

$$\Rightarrow \quad -q = Q\left(1+\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad Q = \frac{-q\sqrt{2}}{\sqrt{2}+1}$$

Hence, the correct answer is (A).

24. $U = -\vec{p}\cdot\vec{E}$

$\Rightarrow \quad U = -pE\cos(45°)$

$\Rightarrow \quad U = -10^{-29}\times 10^{3}\times\dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad U = -7\times 10^{-27}$ J

Hence, the correct answer is (C).

25. By Law of Conservation of Energy, we get

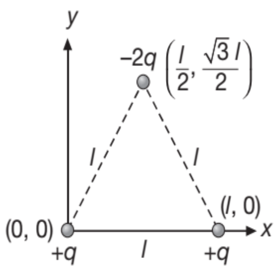
$$\frac{Q^2}{4\pi\varepsilon_0 R_0} = \frac{Q^2}{4\pi\varepsilon_0 R} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2Q^2}{4\pi\varepsilon_0 m}\left(\frac{1}{R_0} - \frac{1}{R}\right)}$$

Hence v increases and attains a finite value after a large time.

Hence, the correct answer is (D).

26.



Since $p_x = q_1x_1 + q_2x_2 + q_3x_3$

$\Rightarrow \quad p_x = (q)(0) + (q)(\ell) + (-2q)\left(\dfrac{\ell}{2}\right)$

$\Rightarrow \quad p_x = 0$

Similarly, $p_y = q_1y_1 + q_2y_2 + q_3y_3$

$\Rightarrow \quad p_y = (q)(0) + (q)(0) + (-2q)\left(\dfrac{\sqrt{3}\ell}{2}\right)$

$\Rightarrow \quad p_y = -\sqrt{3}q\ell$

$\Rightarrow \quad \vec{p} = p_x\hat{i} + p_y\hat{j}$

$\Rightarrow \quad \vec{p} = -\sqrt{3}q\ell\hat{j}$

Hence, the correct answer is (C).

27. Since $E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{A\varepsilon_0}$

$\Rightarrow \quad Q = \varepsilon_0 AE = (8.85\times 10^{-12})(1)(100)$

$\Rightarrow \quad Q = 8.85\times 10^{-10}$ C

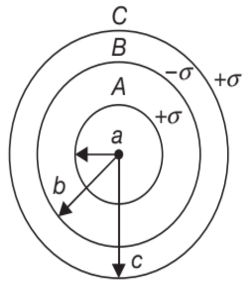
Hence, the correct answer is (A).

28. The potential of the shell B,

$$V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_A}{r_b} + \frac{q_B}{r_b} + \frac{q_C}{r_c}\right)$$

$$\Rightarrow \quad V_B = \frac{4\pi}{4\pi\varepsilon_0}\left(\frac{\sigma\times a^2}{b} - \frac{\sigma\times b^2}{b} + \frac{\sigma\times c^2}{c}\right)$$

$$\Rightarrow \quad v_B = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2 - b^2}{b} + c\right)$$



Hence, the correct answer is (B).

29. Charged particle can be considered at the centre of a cube of side a, and the given surface represents its one side.

So, flux through each face $\phi = \dfrac{Q}{6\varepsilon_0}$

Hence, the correct answer is (B).

30. Charge density on given solid ball varies as

$$\rho = \rho_0\left(1 - \frac{r}{R}\right), \text{ for } 0 \le r \le R$$

Electric field outside the ball is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad \ldots(1)$$

Now, $dq = \rho dV = \rho(4\pi r^2)dr$

$$\Rightarrow \quad q = \int dq = \int_0^R \rho_0\left(1 - \frac{r}{R}\right)(4\pi r^2)dr$$

$$\Rightarrow \quad q = (4\pi\rho_0)\left[\frac{r^3}{3} - \frac{1}{R}\times\frac{r^4}{4}\right]_0^R = 4\pi\rho_0\left(\frac{R^3}{3} - \frac{R^3}{4}\right)$$

$$\Rightarrow \quad q = 4\pi\rho_0\left(\frac{R^3}{12}\right) \qquad \ldots(2)$$

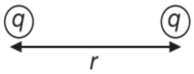
From (1) and (2), $E = \dfrac{\rho_0 R^3}{12\varepsilon_0 r^2}$

Hence, the correct answer is (A).

31. Initially force between spheres A and B, $F=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$

When A and C are touched, charge on both will be $\frac{q}{2}$. Again C is touched with B the charge on B is given by

$$q_B=\frac{\frac{q}{2}+q}{2}=\frac{3q}{4}$$



Required force between spheres A and B is given by

$$F'=\frac{1}{4\pi\varepsilon_0}\frac{q_Aq_B}{r^2}=\frac{1}{4\pi\varepsilon_0}\frac{\frac{q}{2}\times\frac{3q}{4}}{r^2}=\frac{3}{8}\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}\right)=\frac{3}{8}F$$

Hence, the correct answer is (A).

32. Equilibrium position will shift to a point where resultant force is zero. So,

$$kx_{eq}=qE$$

$$\Rightarrow\quad x_{eq}=\frac{qE}{k}$$

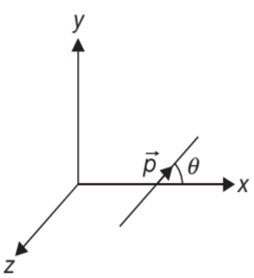
Total energy of the system is

$$E=\frac{1}{2}m\omega^2A^2+\frac{1}{2}\frac{q^2E^2}{k}$$

Hence, the correct answer is (D).

33. $\vec{p}=p\cos\theta\hat{i}+p\sin\theta\hat{j}$

$$\vec{E}_1=E\hat{i}$$



$$\Rightarrow\quad \vec{T}_1=\vec{p}\times\vec{E}_1$$

$$\Rightarrow\quad \vec{T}_1=\left(p\cos\theta\hat{i}+p\sin\theta\hat{j}\right)\times E\left(\hat{i}\right)$$

$$\Rightarrow\quad \tau\hat{k}=pE\sin\theta\left(-\hat{k}\right)\qquad\ldots(1)$$

$$\vec{E}_2=\sqrt{3}E_1\hat{j}$$

$$\vec{T}_2=\left(p\cos\theta\hat{i}+p\sin\theta\hat{j}\right)\times\sqrt{3}E_1\hat{j}$$

$$\Rightarrow\quad -\tau\hat{k}=\sqrt{3}pE_1\cos\theta\hat{k}\qquad\ldots(2)$$

From (1) and (2), we get

$$pE\sin\theta=\sqrt{3}pE\cos\theta$$

$$\Rightarrow\quad \tan\theta=\sqrt{3}$$

$$\Rightarrow\quad \theta=60^\circ$$

Hence, the correct answer is (C).

35. $\phi=\frac{q_{\text{enclosed}}}{\varepsilon_0}$

For S_1, $\phi_1=\frac{2q}{\varepsilon_0}$

For S_2, $\phi_2=\frac{3q-q}{\varepsilon_0}=\frac{2q}{\varepsilon_0}$

For S_3, $\phi_3=\frac{q+q}{\varepsilon_0}=\frac{2q}{\varepsilon_0}$

For S_4, $\phi_4=\frac{8q-6q}{\varepsilon_0}=\frac{2q}{\varepsilon_0}$

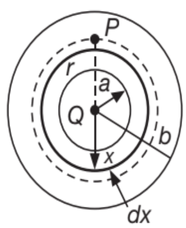
So, $\phi_1=\phi_2=\phi_3=\phi_4=\frac{2q}{\varepsilon_0}$

Hence, the correct answer is (A).

36. Applying Gauss's theorem for radius r, we get

$$\int\vec{E}\cdot d\vec{A}=\frac{1}{\varepsilon_0}(Q+q)$$

$$\Rightarrow\quad E\left(4\pi r^2\right)=\frac{1}{\varepsilon_0}(Q+q)\qquad\ldots(1)$$



where q is charge enclosed between $x=a$ and $x=r$.

$$q=\int_a^r\frac{A}{x}4\pi x^2dx=4\pi A\int_a^r xdx=4\pi A\left(\frac{x^2}{2}\right)_a^r$$

$$\Rightarrow\quad q=2\pi A\left(r^2-a^2\right)$$

Substituting the value of q in equation (1), we get

$$E\left(4\pi r^2\right)=\frac{1}{\varepsilon_0}\left[Q+2\pi A\left(r^2-a^2\right)\right]$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r^2} + 2\pi A - \frac{2\pi A a^2}{r^2}\right)$$

E will be constant if it is independent of r

$$\Rightarrow \quad \frac{Q}{r^2} = \frac{2\pi A a^2}{r^2}$$

$$\Rightarrow \quad A = \frac{Q}{2\pi a^2}$$

Hence, the correct answer is (A).

37. Since, $V(z) = \begin{cases} 30 - 5z^2 \text{ for } |z| \le 1\text{ m} \\ 35 - 10|z| \text{ for } |z| \ge 1\text{ m} \end{cases}$

and $E(z) = -\frac{dV}{dz} = 10z$ for $|z| \le 1$ m and

$E(z) = 10$ for $|z| \ge 1$ m

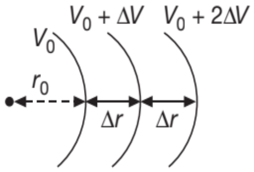
So, the source is an infinite non-conducting thick plate of thickness 2 m.

$$\Rightarrow \quad E = \frac{q}{2A\varepsilon_0} \text{ since } \rho_0 = \frac{q}{At} = \frac{2E}{t}\varepsilon_0$$

$$\Rightarrow \quad \rho_0 = \frac{2\times 10}{2}\varepsilon_0 = 10\varepsilon_0$$

Hence, the correct answer is (B).

38. Since, $E = -\left(\frac{\Delta V}{\Delta r}\right)$



where, ΔV and Δr are same for any pair of surfaces and hence, $E =$ constant

Since, electric field inside the spherical charge distribution is given by

$$E = \frac{\rho}{3\varepsilon_0} r = \text{constant}$$

$$\Rightarrow \quad \rho r = \text{constant}$$

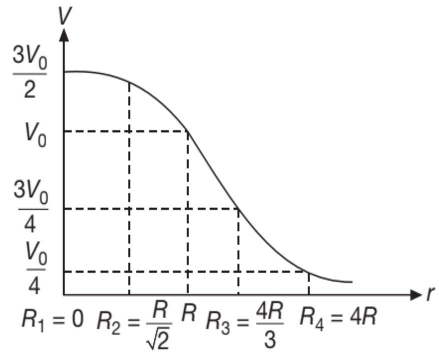
$$\Rightarrow \quad \rho(r) \propto \frac{1}{r}$$

Hence, the correct answer is (C).

39. $V = \begin{cases} \dfrac{Q}{8\pi\varepsilon_0 R^3}(3R^2 - r^2), & \text{for } r < R \\ \dfrac{Q}{4\pi\varepsilon_0 r}, & \text{for } r \ge R \end{cases}$

So, $V_{\text{surface}} = V_0 = \frac{Q}{4\pi\varepsilon_0 R}$,

$V_{\text{centre}} = \frac{3V_0}{2}$, so $R_1 = 0$ ...(1)



Now, $V = \frac{5V_0}{4} = \frac{Q}{8\pi\varepsilon_0 R^3}(3R^2 - R_2^2)$

$$\Rightarrow \quad \frac{5}{2} = 3 - \frac{R_2^2}{R^2}$$

$$\Rightarrow \quad R_2 = \frac{R}{\sqrt{2}} \qquad \text{...(2)}$$

Similarly, at R_3, $V = \frac{3V_0}{4} = \frac{Q}{4\pi\varepsilon_0 R_3}$

$$\Rightarrow \quad R_3 = \frac{4R}{3} \qquad \text{...(3)}$$

Again, at R_4, $V = \frac{V_0}{4} = \frac{Q}{4\pi\varepsilon_0 R_4}$

$$\Rightarrow \quad R_4 = 4R \qquad \text{...(4)}$$

So, we observe from (1), (2), (3) and (4), that

$$R_1 = 0,\ R_4 > 2R \text{ and } R_4 - R_3 = 4R - \frac{4R}{3} = \frac{8R}{3} > R_2$$

Hence, the correct answer is (C) and (D).

40. Electric field lines must originate from the positive charge and end at the negative charge. Also they must exit and enter a surface normally, must be smooth and continuous.

41. Electric field due to complete disc $(R = 2a)$,

$$E_1 = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right) = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{h}{\sqrt{4a^2 + h^2}}\right)$$

$$\Rightarrow \quad E_1 = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{h}{2a}\right) \qquad \{\because h \ll a\}$$

Electric field due to disc of radius $R = a$ is

$$E_2 = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{h}{a}\right)$$

CHAPTER 1

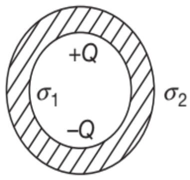
Hence, electric field due to given annular disc is

$$\Rightarrow \quad E = E_1 - E_2 = \frac{\sigma h}{4\varepsilon_0 a}$$

$$\Rightarrow \quad C = \frac{\sigma}{4a\varepsilon_0}$$

Hence, the correct answer is (C).

42.



On outer surface there will be no charge

So $Q_2 = \sigma_2 = 0$ on inner surface total charge will be zero, however, charge distribution will be there, so

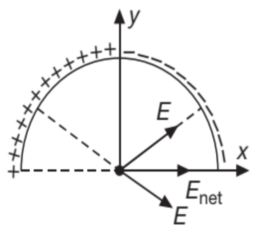
$Q_1 = 0$ and $\sigma_1 \neq 0$

Hence, the correct answer is (C).

43. Due to quarter ring electric field intensity is

$$E = \frac{\lambda}{2\pi\varepsilon_0 R}\sin\left(\frac{\theta}{2}\right)$$

So, due to each quarter section, $\theta = \frac{\pi}{2}$, so field intensity is



$$E = \frac{\lambda}{2\pi\varepsilon_0 R}\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 R}$$

$$\Rightarrow \quad \vec{E}_{net} = \sqrt{2}E\ \hat{i} = \frac{\sqrt{2}\sqrt{2}\lambda}{4\pi\varepsilon_0 R}\hat{i} = \frac{2\lambda}{4\pi\varepsilon_0 R}\hat{i}$$

Since $\lambda = \frac{Q}{\left(\frac{\pi R}{2}\right)} = \frac{2Q}{\pi R}$

$$\Rightarrow \quad \vec{E}_{net} = \frac{2(2Q)}{4\pi\varepsilon_0 \pi R^2}\hat{i} = \left(\frac{4Q}{4\pi^2\varepsilon_0 R^2}\right)\hat{i}$$

where $Q = 10^3\varepsilon_0 C$

and $\pi R = L = 20$ cm

$$\Rightarrow \quad \vec{E}_{net} = \left(\frac{4\times10^3\varepsilon_0}{4\pi^2\varepsilon_0 R^2}\right)\hat{i} = \left(\frac{4\times10^3}{4L^2}\right)\hat{i}$$

$$\Rightarrow \quad \vec{E}_{net} = \left(\frac{4\times10^3}{4\times(0.2)^2}\right)\hat{i} = \left(\frac{4\times10^3}{4\times0.04}\right)\hat{i}$$

$$\Rightarrow \quad \vec{E}_{net} = \left(25\times10^3\ \text{NC}^{-1}\right)\hat{i}$$

Hence, the correct answer is (B).

44. Since $\Delta V = -\int \vec{E}\cdot d\vec{r}$

$$\Rightarrow \quad V_2 - V_1 = -\int_1^2 \vec{E}\cdot d\vec{r}$$

where, $d\vec{r} = dx\hat{i} + dy\hat{j}$ and

$$\vec{E} = \left(25\hat{i} + 30\hat{j}\right)\text{NC}^{-1}$$

$$\Rightarrow \quad \Delta V = V - 0 = -\int_{(0,0)}^{(2,2)}\left(25\hat{i}+30\hat{j}\right)\cdot\left(dx\hat{i}+dy\hat{j}\right)$$

$$\Rightarrow \quad V = -\left(\int_0^2 25dx + \int_0^2 30dy\right)$$

$$\Rightarrow \quad V = -\left(25(x)_0^2 + 30(y)_0^2\right)$$

$$\Rightarrow \quad V = -(25\times2 + 30\times2)\ \ V = -110\text{ V} = -110\text{ J/C}$$

Hence, the correct answer is (D).

45. $V_A - V_O = -\int_O^A E_x dx$

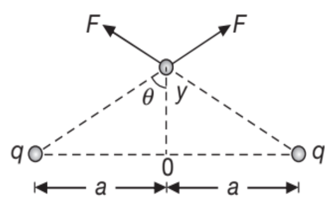
$$\Rightarrow \quad V_A - V_O = -30\int_0^2 x^2 dx$$

$$\Rightarrow \quad V_A - V_O = -30\left(\left.\frac{x^3}{3}\right|_0^2\right) = -10(8)$$

$$\Rightarrow \quad V_A - V_O = -80\text{ V}$$

Hence, the correct answer is (C).

46.



$$F_{net} = 2F\cos\theta$$

$$\Rightarrow \quad F_{net} = \frac{2\times kq^2}{2\left(a^2+y^2\right)}\times\frac{y}{\left(a^2+y^2\right)^{1/2}}$$

So, $F \approx \frac{kq^2y}{a^3}$ (for $y \ll a$)

$\Rightarrow \quad F \propto y$

Hence, the correct answer is (A).

47. $V = \int \frac{kdq}{L+x}$ where $k = \frac{1}{4\pi\varepsilon_0}$

$\Rightarrow \quad V = \frac{kQ}{L}\int_O^L \frac{dL}{L+x} = \frac{kQ}{L}\ln 2$

$\Rightarrow \quad V = \frac{Q}{4\pi\varepsilon_0 L}\ln 2$

Hence, the correct answer is (D).

48. Potential at the centre of the sphere, $V_C = \frac{R^2\rho}{2\varepsilon_0}$

Potential at the surface of the sphere, $V_S = \frac{1}{3}\frac{R^2\rho}{\varepsilon_0}$

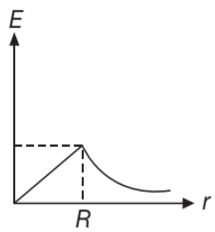
When a charge q is taken from the centre to the surface, the change in potential energy is.

$$\Delta U = (V_C - V_S)q = \left(\frac{R^2\rho}{2\varepsilon_0} - \frac{1}{3}\frac{R^2\rho}{\varepsilon_0}\right)q = \frac{1}{6}\frac{R^2\rho q}{\varepsilon_0}$$

Statement 1 is false. Statement 2 is true.

Hence, the correct answer is (B).

49. For uniformly charged sphere



$E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}$ (For $r < R$)

$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$ (For $r = R$)

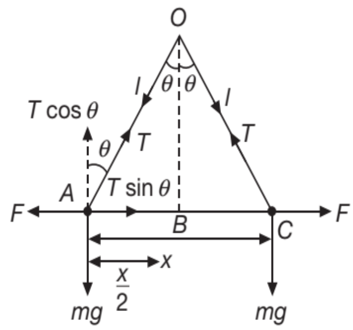
$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ (For $r > R$)

The variation of E with distance r from the centre is as shown in figure.

Hence, the correct answer is (B).

50. Method-I

Figure shows equilibrium positions of the two spheres, so $T\cos\theta = mg$ and



$$T\sin\theta = F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2}$$

$\Rightarrow \quad \tan\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2 mg}$

$\Rightarrow \quad \frac{x}{2\ell} = \frac{q^2}{4\pi\varepsilon_0 x^2 mg}$ $\quad \left\{\because \tan\theta = \frac{x}{2\ell}\right\}$

$\Rightarrow \quad \frac{x}{2\ell} \propto \frac{q^2}{x^2}$

$\Rightarrow \quad q^2 \propto x^3$

$\Rightarrow \quad q \propto x^{\frac{3}{2}}$

when charge begins to leak from both the spheres at a constant rate, then

$$\frac{dq}{dt} \propto \frac{3}{2}x^{\frac{1}{2}}\frac{dx}{dt}$$

$\Rightarrow \quad v \propto x^{-\frac{1}{2}}$ $\quad \left\{\because \frac{dq}{dt} = \text{constant}\right\}$

Method-II

At any instant, loss in electrostatic potential energy is equal to gain in kinetic energy.

So, $\frac{q^2}{4\pi\varepsilon_0 x} = 2\left(\frac{1}{2}mv^2\right)$

$\Rightarrow \quad v \propto x^{-\frac{1}{2}}$

Hence, the correct answer is (A).

51. $\phi = ar^2 + b$

Electric field, $E = -\frac{d\phi}{dr} = -2ar$ $\quad$ ...(1)

According to Gauss's theorem, $\oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{inside}}}{\varepsilon_0}$

$\Rightarrow \quad (-2ar)4\pi r^2 = \frac{q_{\text{inside}}}{\varepsilon_0}$ $\quad$ {using (1)}

$\Rightarrow \quad q_{\text{inside}} = -8\varepsilon_0 a\pi r^3$

Charge density inside the ball is

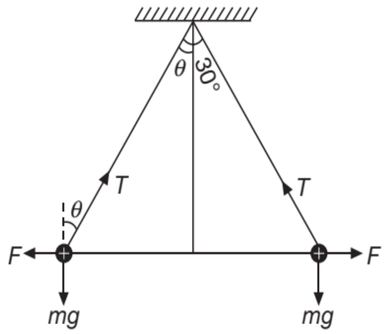
$$\rho_{\text{inside}} = \frac{q_{\text{inside}}}{\frac{4}{3}\pi r^3}$$

$$\Rightarrow \quad \rho_{\text{inside}} = \frac{-8\varepsilon_0 a\pi r^3}{\frac{4}{3}\pi r^3}$$

$$\Rightarrow \quad \rho_{\text{inside}} = -6a\varepsilon_0$$

Hence, the correct answer is (D).

52.



Initially, the forces acting on each ball are

(i) Tension T

(ii) Weight mg

(iii) Electrostatic force of repulsion F

For its equilibrium along vertical

$$T\cos\theta = mg \qquad \ldots(1)$$

and along horizontal

$$T\sin\theta = F \qquad \ldots(2)$$

Dividing equation (2) by (1), we get

$$\tan\theta = \frac{F}{mg} \qquad \ldots(3)$$

When the balls are suspended in a liquid of density σ and dielectric constant K, the electrostatic force will become $(1/K)$ times, i.e. $F' = (F/K)$, whereas weight

$$mg' = mg - \text{upthrust}$$

$$\Rightarrow \quad mg' = mg - V\sigma g \qquad \{\because \text{Upthrust} = V\sigma g\}$$

$$\Rightarrow \quad mg' = mg\left(1 - \frac{\sigma}{\rho}\right) \qquad \left\{\because V = \frac{m}{\rho}\right\}$$

For equilibrium of balls, we have

$$\tan\theta' = \frac{F'}{mg'} = \frac{F}{Kmg\left(1-(\sigma/\rho)\right)} \qquad \ldots(4)$$

According to given problem, we have $\theta' = \theta$

From equations (3) and (4), we get $K = \dfrac{1}{\left(1-\dfrac{\sigma}{\rho}\right)}$

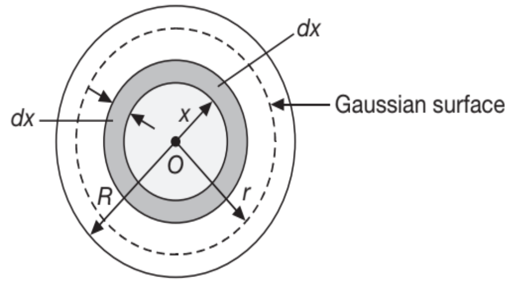
$$\Rightarrow \quad K = \frac{\rho}{(\rho-\sigma)} = \frac{1.6}{(1.6-0.8)} = 2$$

Hence, the correct answer is (D).

53. Consider a thin spherical shell of radius x and thickness dx as shown in the figure.

Volume of the shell,

$$dV = 4\pi x^2 dx$$



Let us draw a Gaussian surface of radius $r\,(r < R)$ as shown in the figure above.

Total charge enclosed (Q_{enc}) inside the Gaussian surface is

$$Q_{\text{enc}} = \int_0^r \rho\, dV = \int_0^r \rho_0\left(\frac{5}{4} - \frac{x}{R}\right)4\pi x^2 dx$$

$$\Rightarrow \quad Q_{\text{enc}} = 4\pi\rho_0 \int_0^r \left(\frac{5}{4}x^2 - \frac{x^3}{R}\right)dx$$

$$\Rightarrow \quad Q_{\text{enc}} = 4\pi\rho_0\left(\frac{5}{12}x^3 - \frac{x^4}{4R}\right)_0^r = 4\pi\rho_0\left(\frac{5}{12}r^3 - \frac{r^4}{4R}\right)$$

$$\Rightarrow \quad Q_{\text{enc}} = \frac{4\pi\rho_0}{4}\left(\frac{5}{3}r^3 - \frac{r^4}{R}\right) = \pi\rho_0\left(\frac{5}{3}r^3 - \frac{r^4}{R}\right)$$

According to Gauss's Law, $E(4\pi r^2) = \dfrac{Q_{enc}}{\varepsilon_0}$

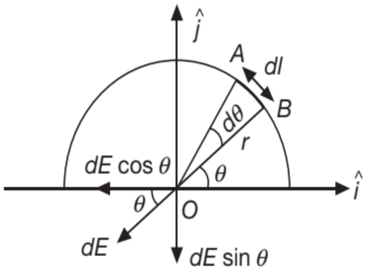
$$\Rightarrow \quad E(4\pi r^2) = \frac{\pi\rho_0}{\varepsilon_0}\left(\frac{5}{3}r^3 - \frac{r^4}{R}\right)$$

$$\Rightarrow \quad E = \frac{\pi\rho_0 r^3}{4\pi r^2\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right) = \frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$$

Hence, the correct answer is (C).

54. Linear charge density, $\lambda = \frac{q}{\pi r}$

Consider a small element AB of length $d\ell$ subtending an angle $d\theta$ at the centre O as shown in the figure.



Charge dq on the element is $dq = \lambda d\ell$

$$\Rightarrow \quad dq = \lambda(rd\theta) \qquad \left\{\because d\theta = \frac{d\ell}{r}\right\}$$

the electric field at the centre O due to the charge element is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{\lambda r d\theta}{4\pi\varepsilon_0 r^2}$$

Resolving dE into two rectangular components, we get by symmetry,

$$\int dE\cos\theta = 0$$

So, the net electric field at O is

$$\vec{E} = \int_0^{\pi} dE\sin\theta(-\hat{j}) = \int_0^{\pi}\frac{\lambda r d\theta}{4\pi\varepsilon_0 r^2}\sin\theta(-\hat{j})$$

$$\Rightarrow \quad \vec{E} = -\int_0^{\pi}\frac{qr\sin\theta d\theta}{4\pi^2\varepsilon_0 r^3}\hat{j} \qquad \left\{\because \lambda = \frac{q}{\pi r}\right\}$$

$$\Rightarrow \quad \vec{E} = -\int_0^{\pi}\frac{q\sin\theta d\theta}{4\pi^2\varepsilon_0 r^2}\hat{j} = -\frac{q}{4\pi^2\varepsilon_0 r^2}(-\cos\theta)\Big|_0^{\pi}\hat{j}$$

$$\Rightarrow \quad \vec{E} = -\left(\frac{q}{2\pi^2\varepsilon_0 r^2}\right)\hat{j}$$

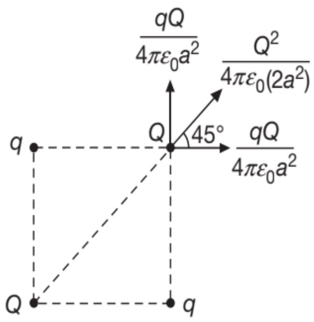
Hence, the correct answer is (D).

55. For net force on Q to be zero

$$\frac{qQ}{4\pi\varepsilon_0 a^2} + \frac{Q^2}{4\pi\varepsilon_0(2a^2)}\cos 45 + \frac{qQ}{4\pi\varepsilon_0 a^2}\cos 90 = 0$$

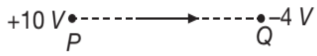
$$\Rightarrow \quad Q = -2\sqrt{2}q$$

$$\Rightarrow \quad \frac{Q}{q} = -2\sqrt{2}$$



Hence, the correct answer is (A).

56.



Work done in moving 100 negative charges i.e. electrons from the positive to the negative potential i.e. from P to Q is

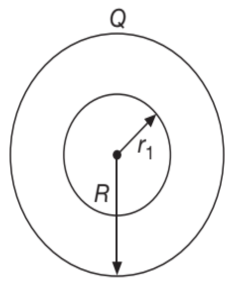
$$W = (100e^-)(V_Q - V_P) = (-100\times1.6\times10^{-19})(-14\text{ V})$$

$$\Rightarrow \quad W = 2.24\times10^{-16}\text{ J}$$

Hence, the correct answer is (D).

57. If the charge density, $\rho = \frac{Q}{\pi R^4}r$,

The electric field at the point P distant r_1 from the centre, according to Gauss's theorem is



$$E(4\pi r_1^2) = \frac{q_{enc}}{\varepsilon_0}$$

$$\Rightarrow \quad E(4\pi r_1^2) = \frac{1}{\varepsilon_0}\int\rho dV \qquad \left\{\because q_{enc} = \int\rho dV\right\}$$

$$\Rightarrow \quad E(4\pi r_1^2) = \frac{1}{\varepsilon_0}\int_0^{r_1}\frac{Qr}{\pi R^4}(4\pi r^2 dr)$$

$$\Rightarrow \quad E = \frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$$

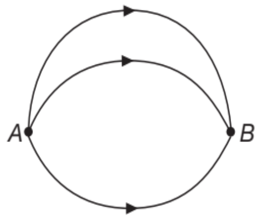
Hence, the correct answer is (C).

58. Work done = Potential difference × charge

$$\Rightarrow \quad W = (V_B - V_A)\times q,$$

V_A and V_B only depend on the initial and final positions and not on the path. Electrostatic force is a conservative force. if the loop is completed, $V_A - V_A = 0$. No network is done as the initial and final potentials are the same.

Both the statements are true but Statement-2 is not the reason for Statement-1.



Hence, the correct answer is (C).

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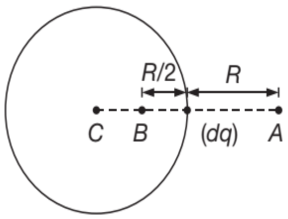
Single Correct Choice Type Problems

1. For uniformly distributed charged shell, the surface charge density (σ) is

$$\sigma = \frac{Q}{4\pi R^2}$$

So, charge of small area $dA = \alpha 4\pi R^2$ is

$$dq = \sigma dA = \alpha Q$$



Since potential at the surface before removing the charge dq is V_0, so

$$V_0 = \frac{Q}{4\pi\varepsilon_0 R}$$

When dq is removed, then potential at the centre is

$$V_{\text{centre}} = V_0 - V_{(dq)}$$

$$\Rightarrow \quad V_{\text{centre}} = \frac{Q}{4\pi\varepsilon_0 R} - \frac{\alpha Q}{4\pi\varepsilon_0 R} = V_0(1-\alpha)$$

Similarly, potential at the point B (after removing dq) is

$$V_B = V_0 - \frac{\alpha Q}{4\pi\varepsilon_0\left(\frac{R}{2}\right)} = V_0(1-2\alpha)$$

$$\Rightarrow \quad \frac{V_{\text{centre}}}{V_B} = \frac{V_C}{V_B} = \frac{1-\alpha}{1-2\alpha}$$

According to principle of superposition, we get the electric field at A as

$$E_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{(2R)^2} - \frac{\alpha Q}{(R)^2}\right) = E_{\text{shell}} - \frac{\alpha V_0}{R}$$

$$\Rightarrow \quad \Delta E_A = -\frac{\alpha V_0}{R}$$

So, at A, magnitude of electric field decreases by $\frac{\alpha V_0}{R}$

Similarly, $(E_C)_{\text{initial}} = 0$ and $(E_C)_{\text{final}} = \frac{\alpha Q}{4\pi\varepsilon_0 R^2}$

$$\Rightarrow \quad \Delta E_C = \frac{\alpha Q}{4\pi\varepsilon_0 R^2} = \frac{\alpha V_0}{R}$$

Hence, field at C increases by $\frac{\alpha V_0}{R}$

Hence, the correct answer is (D).

2. The sphere with cavity can be assumed as a complete sphere with positive charge of radius $R_1 +$ another complete sphere with negative charge and radius R_2.

$E_+ \rightarrow \vec{E}$ due to total positive charge

$E_- \rightarrow \vec{E}$ due to total negative charge.

$E = E_+ + E_-$

If we calculate it at P, then E_- comes out to be zero.

So, $E = E_+$

and $E_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{R_1^3}(OP)$, in the direction of OP.

Here, q is total positive charge on whole sphere.

It is in the direction of OP or a.

Now, inside the cavity electric field comes out to be uniform at any point. This is a standard result.

Hence, the correct answer is (D).

3. At the shown position, net force on both charges is zero. Hence they are in equilibrium. But equilibrium of $+q$ is stable equilibrium. So, it will start oscillations when displaced from this position. These small oscillations are simple harmonic in nature. While equilibrium of $-q$ is unstable. So, it continues to move in the direction of its displacement.

Hence, the correct answer is (C).

4. $E_1 = \frac{kQ}{R^2}$, where $k = \frac{1}{4\pi\varepsilon_0}$

$$E_2 = \frac{k(2Q)}{R^2}$$

$$\Rightarrow E_2 = \frac{2kQ}{R^2}$$

$$E_3 = \frac{k(4Q)R}{(2R)^3}$$

$$\Rightarrow E_3 = \frac{kQ}{2R^2}$$

Hence, the correct answer is (C).

5. For inside points $(r \le R)$

$$E = 0 \Rightarrow V = \text{constant} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

For outside points $(r \ge R)$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \text{ or } E \propto \frac{1}{r^2}$$

and $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ or $V \propto \frac{1}{r}$

On the surface $(r = R)$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

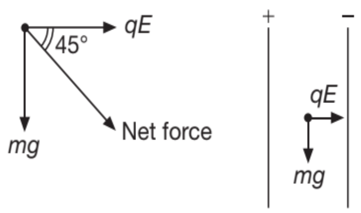
$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} = \frac{\sigma}{\varepsilon_0}$$

where, $\sigma = \frac{q}{4\pi R^2}$ = surface charge density

corresponding to above equations the correct graphs are shown in option (D).

Hence, the correct answer is (D).

6.



Net force is at $45°$ from vertical.

$$\Rightarrow qE = mg$$

$$\Rightarrow \frac{qX}{d} = mg \qquad \left\{\because E = \frac{X}{d}\right\}$$

$$\Rightarrow X = \frac{mgd}{q}$$

$$\Rightarrow X = \frac{(1.67\times10^{-27})(9.8)(10^{-2})}{(1.6\times10^{-19})}$$

$$\Rightarrow X \approx 1\times10^{-9}\text{ V}$$

Hence, the correct answer is (C).

7. Electric flux, $\phi = \vec{E}\cdot\vec{A}$

$$\Rightarrow \phi = EA\cos\theta$$

Here, θ is the angle between $\vec{E}$ and $\vec{A}$

In this question $\theta = 45°$, because $\vec{A}$ is perpendicular to surface

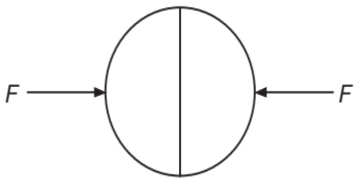
$E = E_0$ and

$$A = (\sqrt{2}a)(a) = \sqrt{2}a^2$$

$$\Rightarrow \phi = (E_0)(\sqrt{2}a^2)\cos(45°) = E_0a^2$$

Hence, the correct answer is (C).

8.



Electrostatics repulsive force :

$$F_{ele} = \left(\frac{\sigma^2}{2\varepsilon_0}\right)\pi R^2$$

$$F = F_{ele} = \frac{\sigma^2\pi R^2}{2\varepsilon_0}$$

Hence, the correct answer is (A).

9. In equilibrium, $mg = qE$

In absence of electric field, $mg = 6\pi\eta r v$

$$\Rightarrow qE = 6\pi\eta rv$$

$$m = \frac{4}{3}\pi R r^3 d = \frac{qE}{g}$$

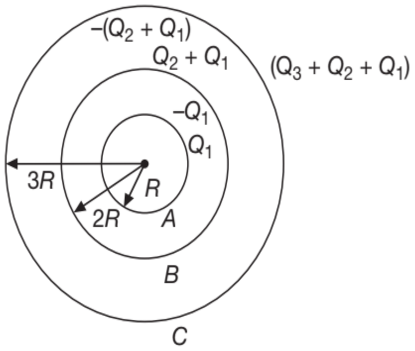
$$\frac{4}{3}\pi\left(\frac{qE}{6\pi\eta v}\right)^3 d = \frac{qE}{g}$$

After substituting value we get,

$$q = 8\times10^{-19}\text{ C}$$

Hence, the correct answer is (D).

10. $Q_1 = \sigma(4\pi R^2) = 4\pi R^2\sigma$



$$Q_2 + Q_1 = 16\pi R^2\sigma$$

$$\Rightarrow \quad Q_2 = 16\pi R^2\sigma - Q_1 = 12\pi R^2\sigma$$

$$Q_3 + Q_2 + Q_1 = 36\pi R^2\sigma$$

$$\Rightarrow \quad Q_3 = 36\pi R^2\sigma - 16\pi R^2\sigma = 20\pi R^2\sigma$$

$$\Rightarrow \quad Q_1 : Q_2 : Q_3 = 1:3:5$$

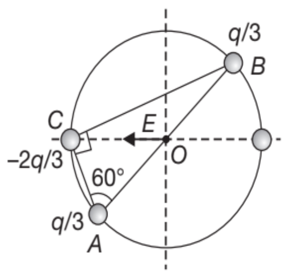
Hence, the correct answer is (B).

11. From Gauss's Law, we have flux equal to

$$\phi = \frac{\Sigma Q_{enc}}{\varepsilon_0} = \frac{\left(\frac{8C}{4}\right) - 7C + \left(\frac{6C}{2}\right)}{\varepsilon_0} = -\frac{2C}{\varepsilon_0}$$

Hence, the correct answer is (A).

12. Net electric field due to both charges $\frac{q}{3}$, will get cancelled. Electric field due to $\left(-\frac{2q}{3}\right)$ will be directed in –ve axis



$$E = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{2q}{3}\right)}{R^2}$$

$$\Rightarrow \quad E = \frac{q}{6\pi\varepsilon_0 R^2}$$

$$\text{P.E. of system} = \frac{1}{4\pi\varepsilon_0}\left(\frac{\left(\frac{q}{3}\right)^2}{2R} + \frac{\frac{q}{3}\left(-\frac{2q}{3}\right)}{2R\sin 60^\circ} + \frac{\frac{q}{3}\left(-\frac{2q}{3}\right)}{2R\cos 60^\circ}\right)$$

P.E. of system $\neq 0$

Force between B and C

$$F = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{2q}{3}\right)\left(\frac{q}{3}\right)}{(2R\sin 60^\circ)^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\times 2q^2}{9\times 4\times 3R^2}$$

$$\Rightarrow \quad F = \frac{2q^2}{9\times 3\times 4\pi\varepsilon_0 R^2} \text{ (attractive)}$$

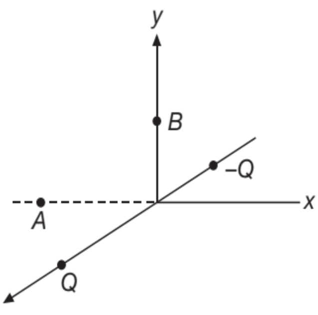
$$\Rightarrow \quad F = \frac{1}{54}\frac{q^2}{\pi\varepsilon_0 R^2}$$

Potential at O is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q}{3} + \frac{q}{3} - \frac{2q}{3}\right)}{R} = 0$$

Hence, the correct answer is (C).

13. $A \equiv (-a, 0, 0)$, $B \equiv (0, a, 0)$



Since point charge is moved from A to B and

$$V_A = V_B = 0$$

$$\Rightarrow \quad W = 0$$

Hence, the correct answer is (C).

14. Charge will be induced in the conducting sphere, but net charge on it will be zero.

Hence, the correct answer is (D).

15. Inside the cavity, field at any point is uniform and non-zero.

Hence, the correct answer is (B).

16. There will be an electric field between two cylinders (using Gauss theorem). This electric field will produce a potential difference.

Hence, the correct answer is (A).

17. All the three plates will produce electric field at P along negative z-axis. Hence ,

$$\vec{E}_P = \left[\frac{\sigma}{2\varepsilon_o} + \frac{2\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o}\right](-\hat{k}) = -\frac{2\sigma}{\varepsilon_o}\hat{k}$$

The correct answer is (B).

Hence, the correct answer is (B).

18. At any point over the spherical Gaussian surface, net electric field is the vector sum of electric fields due to $+q_1, -q_1$ and q_2.

Don't confuse with the electric flux which is zero (net) passing over the Gaussian surface as the net charge enclosing the surface is zero.
The correct answer is (C).

Hence, the correct answer is (C).

19. According to option (D), the electric field due to P and S and due to Q and T add to zero. While due to U and R will be added up. Hence, the correct answer is (D).

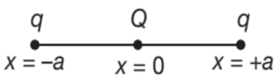
Hence, the correct answer is (D).

20. Electric field is zero everywhere inside a metal or conductor i.e. field lines do not enter a metal. Also, field lines must enter or leave the conductor surface at right angles because it is an equipotential surface.

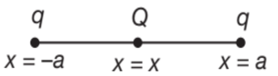
Hence, the correct answer is (C).

21. $U_i = \dfrac{2Qq}{4\pi\varepsilon_0 a}$

and $U_f = \dfrac{Qq}{4\pi\varepsilon_0}\left[\dfrac{1}{a+x} + \dfrac{1}{a-x}\right]$



Initial position



Final position

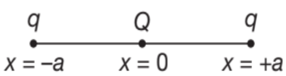
$$\Delta U = U_f - U_i$$

$$\Rightarrow \quad |\Delta U| = \frac{2Qqx^2}{4\pi\varepsilon_0 a^3} \text{ for } x \ll a$$

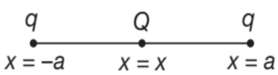
$$\Rightarrow \quad \Delta U \propto x^2$$

Hence, the correct answer is (B).

22.



Initial position



Final position

$$U_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Qq}{a} + \frac{q^2}{2a}\right)$$

and $U_f = \dfrac{Qq}{4\pi\varepsilon_0}\left[\dfrac{1}{a+x} + \dfrac{1}{a-x}\right] + \dfrac{q^2}{4\pi\varepsilon_0(2a)}$

$$\Rightarrow \quad \Delta U = U_f - U_i$$

$$\Rightarrow \quad |\Delta U| = \frac{2KQqx^2}{a^3}$$

For $x \ll a$, we have

$$\Delta U \propto x^2$$

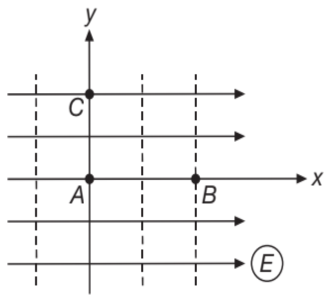
Hence, the correct answer is (B).

23. Electrostatic lines of force never form a closed loop. Therefore, options (B) and (D) are wrong, Electrostatic lines of force emanate from positive charge and terminate on negative charge, therefore, option (A) is also wrong.

Hence, the correct answer is (C).

24. Potential decreases in the direction of electric field. Dotted lines are equipotential lines. So

$$V_A = V_C \text{ and } V_A > V_B$$



Hence, the correct answer is (B).

25. Net electrostatic energy of the configuration will be

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{a} + \frac{Qq}{\sqrt{2}a} + \frac{Qq}{a}\right] = 0$$

$$\Rightarrow \quad Q = \frac{-2q}{2+\sqrt{2}}$$

Hence, the correct answer is (B).

26. $V = \dfrac{q}{4\pi\varepsilon_0 x_0}\left(1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \dfrac{1}{5} - \dfrac{1}{6}.........\right)$

$$\Rightarrow \quad V = \frac{q}{4\pi\varepsilon_0 x_0}\log_e 2$$

Hence, the correct answer is (D).

27. $s = \dfrac{1}{2}at^2$

where $a = \dfrac{qE}{m}$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{qE}{m_e}\right)t_1^2 = \frac{1}{2}\left(\frac{qE}{m_p}\right)t_2^2$$

CHAPTER 1

$$\Rightarrow \quad \frac{t_2}{t_1}=\left(\frac{m_p}{m_e}\right)^{\frac{1}{2}}$$

Hence, the correct answer is (B).

28. $-\int_{\ell=\infty}^{\ell=0}\vec{E}\cdot d\vec{\ell}$ is the line integral of electric field which is also called electrostatic potential. Since, charge is distributed non-uniformly, so $E \neq 0$ at the centre.

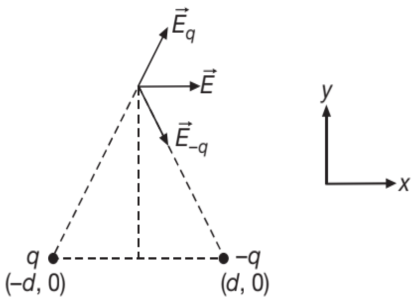
$$\Rightarrow \quad -\int_{\ell=\infty}^{\ell=0}\vec{E}\cdot d\vec{\ell}=\frac{q}{4\pi\varepsilon_0 R}=2$$

Hence, the correct answer is (A).

29. Field lines always enter or leave a surface normal to it.

Hence, the correct answer is (D).

30. The electrical field $\vec{E}$ at all points on the x-axis will not have the same direction.



For $-d \le x \le d$, electric field is along positive x-axis, while for all other points it is along negative x-axis. The electric field $\vec{E}$ at all points on the y-axis will be parallel to the x-axis (i.e. along $\hat{i}$) (OPTION (C)).

The electrical potential at the origin due to both the charges is zero, hence, no work is done in bringing a test charge from infinity to the origin.

Dipole moment is directed from the $-q$ charge to the $+q$ charge (i.e. along $-\hat{i}$ direction).

Hence, the correct answer is (C).

31. $V_1=\dfrac{Q_1}{4\pi\varepsilon_0 a}+\dfrac{Q_2}{4\pi\varepsilon_0\sqrt{2}a}$

$$V_2=\frac{Q_1}{4\pi\varepsilon_0\sqrt{2}a}+\frac{Q_2}{4\pi\varepsilon_0 a}$$

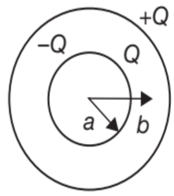
$$\Rightarrow \quad W_{1\to 2}=q(V_2-V_1)$$

$$\Rightarrow \quad W_{1\to 2}=\frac{q(Q_1-Q_2)}{4\pi\varepsilon_0 a}\left(1-\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad W=\frac{q(Q_1-Q_2)(\sqrt{2}-1)}{4\sqrt{2}\pi\varepsilon_0 a}$$

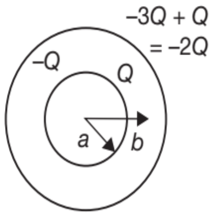
Hence, the correct answer is (B).

32.



$$V=V_{\substack{\text{surface}\\\text{sphere}}}-V_{\substack{\text{outer}\\\text{shell}}}$$

$$\Rightarrow \quad V=\frac{Q}{4\pi\varepsilon_0 a}-\frac{Q}{4\pi\varepsilon_0 b}$$



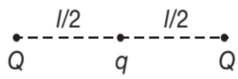
Potential difference $=V'=V_{\substack{\text{surface}\\\text{sphere}}}-V_{\substack{\text{outer}\\\text{shell}}}$

$$\Rightarrow \quad V'=\left(\frac{Q}{4\pi\varepsilon_0 a}+\frac{-2Q}{4\pi\varepsilon_0 b}\right)-\left(\frac{Q}{4\pi\varepsilon_0 b}+\frac{-2Q}{4\pi\varepsilon_0 b}\right)$$

$$\Rightarrow \quad V'=\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)=V$$

Hence, the correct answer is (A).

33. For equilibrium of Q



$$\frac{Qq}{4\pi\varepsilon_0\left(\frac{\ell}{2}\right)^2}+\frac{Q^2}{4\pi\varepsilon_0\ell^2}=0$$

$$\Rightarrow \quad 4q+Q=0$$

$$\Rightarrow \quad q=-\frac{Q}{4}$$

Here, we are asked to calculate equilibrium for system. In this system q is already in equilibrium. So, we are calculating equilibrium condition for Q.

Hence, the correct answer is (B).

34. By symmetry of problem the components of force on Q due to charges at A and B along y-axis will cancel each other while along x-axis will add up and will

be along CO. Under the action of this force charge Q will move towards O. If at any time charge Q is at a distance x from O.

$$F = 2\frac{1}{4\pi\varepsilon_0}\frac{-qQ}{(a^2+x^2)} \times \frac{x}{(a^2+x^2)^{\frac{1}{2}}}$$

i.e., $$F = \frac{1}{4\pi\varepsilon_0}\frac{2qQx}{(a^2+x^2)^{\frac{3}{2}}}$$

As the restoring force F is not linear, motion will be oscillatory (with amplitude $2a$) but not simple harmonic.

Hence, the correct answer is (D).

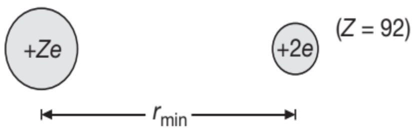
35. The potential at the centre of a hollow sphere is equal to the potential at its surface.

Hence, the correct answer is (B).

36. From conservation of mechanical energy

Decrease in kinetic energy = increase in potential energy

or $$\frac{1}{4\pi\varepsilon_o}\frac{(Ze)(2e)}{r_{\min}} = 5 \text{ MeV} = 5\times1.6\times10^{-13}\text{ J}$$



$$\Rightarrow \quad r_{\min} = \frac{1}{4\pi\varepsilon_o}\frac{2Ze^2}{5\times1.6\times10^{-13}} = \frac{(9\times10^9)(2)(92)(1.6\times10^{-19})^2}{5\times1.6\times10^{-13}}$$

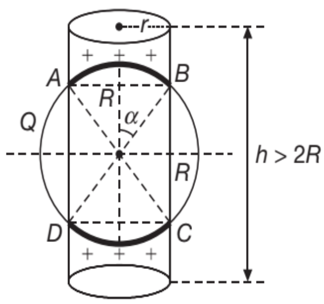
$$\Rightarrow \quad r_{\min} = 5.3\times10^{-14}\text{ m} = 5.3\times10^{-12}\text{ cm}$$

So, $r_{\min}$ is of the order of 10^{-12} cm.

Hence, the correct answer is (C).

Multiple Correct Choice Type Problems

1. **For OPTION (A)**



Since $h > 2R$ and $r = \frac{3R}{5}$, so we get

$$\sin\alpha = \frac{r}{R} = \frac{3}{5}$$

$$\Rightarrow \quad \cos\alpha = \frac{4}{5}$$

Since $$\phi = 2\left[\frac{Q}{2\varepsilon_0}(1-\cos\alpha)\right] \quad \ldots(1)$$

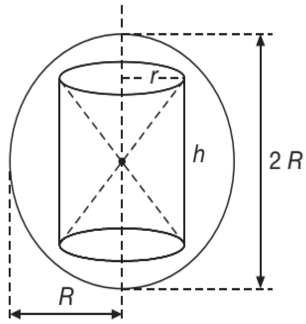
$$\Rightarrow \quad \phi = \frac{Q}{5\varepsilon_0}$$

Please note that a factor of 2 is used in equation (1), because we have two surfaces i.e., AB and CD of the shell that are enclosed by the cylinder.

For OPTION (B)

$$h < \frac{8R}{5} \text{ and } r = \frac{3R}{5}$$

$$h < 2R \text{ and } r < R$$

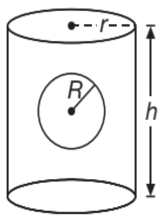


$$\Rightarrow \quad Q_{\text{enclosed}} = 0$$

$$\Rightarrow \quad \phi = 0$$

For OPTION (C)

$$h > 2R \text{ and } r > R$$



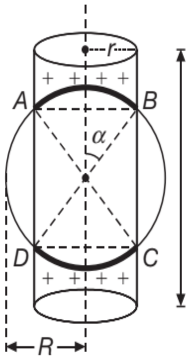
$$\Rightarrow \quad Q_{\text{enclosed}} = Q$$

$$\Rightarrow \quad \phi = \frac{Q}{\varepsilon_0}$$

For OPTION (D)

$$h > 2R \text{ and } r = \frac{4R}{5} \text{ i.e., } r < R$$

CHAPTER 1



$Q_{\text{enclosed}} = Q[1-\cos\alpha]$

Since, $\sin\alpha = \frac{r}{R} = \frac{3}{5}$

$\Rightarrow \quad \cos\alpha = \frac{3}{5}$

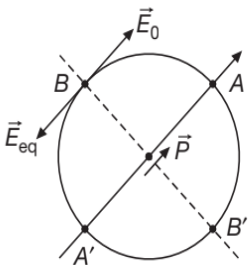
$\Rightarrow \quad \phi = \frac{2Q}{2\varepsilon_0}(1-\cos\alpha)$

$\Rightarrow \quad \phi = \frac{Q}{\varepsilon_0}\left(1-\frac{3}{5}\right)$

$\Rightarrow \quad \phi = \frac{2Q}{5\varepsilon_0}$

Hence, (A), (B) and (C) are correct.

2. E_0 is the external field in the direction of $\vec{p} = \frac{p_0}{\sqrt{2}}(\hat{i}+\hat{j})$



For an equipotential surface of radius r, the point B is the point at the equitorial line of the dipole. Since, on an equipotential surface, no tangential field exists, so we have

$E_0 = E_{eq}$ (in magnitude)

but both in opposite direction (as shown)

$\Rightarrow \quad \frac{p_0}{4\pi\varepsilon_0 R^3} = E_0$

$\Rightarrow \quad R = \left(\frac{p_0}{4\pi\varepsilon_0 E_0}\right)^{\frac{1}{3}}$

Also, A lies at axial line of dipole, so

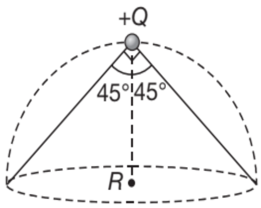
$\vec{E}_A = \frac{2\vec{p}}{4\pi\varepsilon_0 R^3} + \vec{E}_0 = 2\vec{E}_0 + \vec{E}_0 = 3\vec{E}_0$

Since $E_{\text{eq}} = E_0$

$\Rightarrow \quad |\vec{E}_B| = E_{\text{eq}} - E_0 = 0$

Hence, (A) and (C) are correct.

3. (a) $\Omega = 2\pi(1-\cos\theta)$, where $\theta = 45°$



$\Rightarrow \quad \phi = -\frac{\Omega}{4\pi} \times \frac{Q}{\varepsilon_0}$

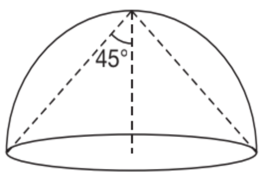
$\Rightarrow \quad \phi = -\frac{2\pi(1-\cos\theta)}{4\pi}\frac{Q}{\varepsilon_0}$

$\Rightarrow \quad \phi = -\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$

The negative sign signifies that the flux is due to field lines entering a surface.

(b) The component of the electric field perpendicular to the flat surface will decrease so we move away from the centre as the distance increases (magnitude of electric field decreases) as well as the angle between the normal and electric field will increase. Hence, the component of the electric field normal to the flat surface is not constant.

(c) Total flux ϕ due to charge Q is $\frac{Q}{\varepsilon_0}$.



So, ϕ through the curved and flat surface will be less than $\frac{Q}{\varepsilon_0}$.

(d) Since, the circumference is equidistant from Q it will be equipotential $V = \frac{Q}{4\pi\varepsilon_0\sqrt{2}R}$.

Hence, (A) and (D) are correct.

4. $\frac{Q}{4\pi\varepsilon_0 r_0^2} = \frac{\lambda}{2\pi\varepsilon_0 r_0} = \frac{\sigma}{2\varepsilon_0}$

$\Rightarrow \quad Q = 2\pi\sigma r_0^2$

(A) is incorrect, $r_0 = \dfrac{\lambda}{\pi\sigma}$

(B) is incorrect, $E_1\left(\dfrac{r_0}{2}\right) = 4E_1(r_0)$

As $E_1 \propto \dfrac{1}{r^2}$

$\Rightarrow \quad E_2\left(\dfrac{r_0}{2}\right) = 2E_2(r_0)$ as $E_2 \propto \dfrac{1}{r}$

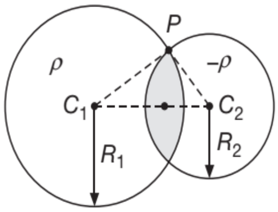
(C) is correct

$$E_3\left(\frac{r_0}{2}\right) = E_3(r_0) = E_2(r_0)$$

as $E_3 \propto r^0$

Hence, option (C) is correct.

5.



For electrostatic field,

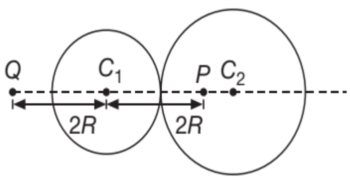
$$E_P = E_1 + E_2 = \frac{\rho}{3\varepsilon_0}C_1P + \frac{(-\rho)}{3\varepsilon_0}C_1P$$

$$\Rightarrow \quad E_P = \frac{\rho}{3\varepsilon_0}(C_1P + PC_2)$$

$$\Rightarrow \quad E_P = \frac{\rho}{3\varepsilon_0}C_1C_2$$

Hence, (C) and (D) are correct.

6. **At point P**



If resultant electric field is zero, then

$$\frac{KQ_1}{4R^2} = \frac{KQ_2}{8R^3}R$$

$$\Rightarrow \quad \frac{\rho_1}{\rho_2} = 4$$

At point Q

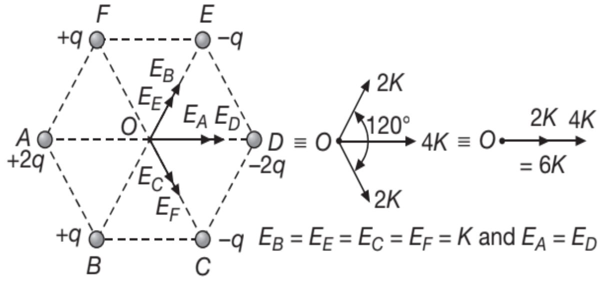
If resultant electric field is zero then

$$\frac{KQ_1}{4R^2} + \frac{KQ_2}{25R^2} = 0$$

$$\Rightarrow \quad \frac{\rho_1}{\rho_2} = -\frac{32}{25} \qquad (\rho_1 \text{ must be negative})$$

Hence, (B) and (D) are correct.

7. (A)



$E_B = E_E = E_C = E_F = K$ and $E_A = E_D = 2K$

Resultant of $2K$ and $2K$ (at $120°$) is also $2K$ towards $4K$. Therefore, net electric field is $6K$.

(B) $V_0 = \dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{q_A}{L} + \dfrac{q_B}{L} + \dfrac{q_C}{L} + \dfrac{q_D}{L} + \dfrac{q_E}{L} + \dfrac{q_F}{L}\right]$

$$V_0 = \frac{1}{4\pi\varepsilon_0 L}(q_A + \ldots + q_F) = 0$$

Because $q_A + q_B + q_C + q_D + q_E + q_F = 0$

(C) Only line PR, potential is same $(=0)$.

Hence, (A), (B) and (C) are correct.

8. OPTION (A) is correct due to symmetry.

OPTION (B) is wrong again due to symmetry.

OPTION (C) is correct because as per Gauss's theorem, net electric flux passing through any closed surface $= \dfrac{q_{in}}{\varepsilon_0}$

Here, $q_{in} = 3q - q - q = q$

So, net electric flux $= \dfrac{q}{\varepsilon_0}$

OPTION (D) is incorrect because there is no symmetry in two given planes.

Hence, (A) and (C) are correct.

9. If charges are of opposite signs, then the two fields are along the same direction. So, they cannot be zero. Hence, the charges should be of same sign.

Therefore, OPTION (C) is correct.

Further, $\begin{pmatrix}\text{Work done by} \\ \text{external force}\end{pmatrix} = \begin{pmatrix}\text{Change in electrostatic} \\ \text{potential energy}\end{pmatrix}$

$$\Rightarrow \quad W_{A\to B} = q(\Delta V) = (+1)(V_B - V_A)$$

$$\Rightarrow \quad W_{A\to B} = V_B - V_A$$

So, OPTION (D) is also correct.
Hence the correct OPTIONS are (C) and (D).
Hence, (C) and (D) are correct.

10. Inside a conducting shell electric field is always zero. Therefore, OPTION (A) is correct. When the two are connected, their potentials become the same.

$\Rightarrow \quad V_A = V_B$

$\Rightarrow \quad \dfrac{Q_A}{R_A} = \dfrac{Q_B}{R_B} \qquad \left\{\because V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R}\right\}$

Since, $R_A > R_B$

$\Rightarrow \quad Q_A > Q_B$

So, OPTION (B) is correct.

Potential is also equal to, $V = \dfrac{\sigma R}{\varepsilon_0}$

Since $V_A = V_B$

$\Rightarrow \quad \sigma_A R_A = \sigma_B R_B$

$\Rightarrow \quad \dfrac{\sigma_A}{\alpha_B} = \dfrac{R_B}{R_A}$

$\Rightarrow \quad \sigma_A < \sigma_B$

So, OPTION (C) is correct.

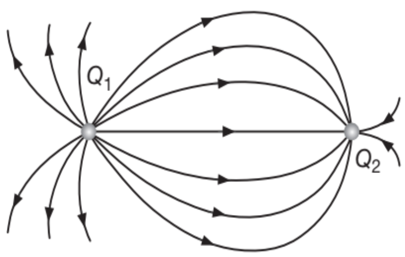
Electric field on surface, $E = \dfrac{\sigma}{\varepsilon_0}$

$\Rightarrow \quad E \propto \sigma$

Since, $\sigma_A < \sigma_B \Rightarrow E_A < E_B$

So, OPTION (D) is also correct.
Hence the correct OPTIONS are (A), (B), (C) and (D).
Hence, (A), (B), (C) and (D) are correct.

11.



From the diagram, it can be observed that Q_1 is positive, Q_2 is negative.

Number of lines on Q_1 is greater and number of lines is directly proportional to magnitude of charge.

So, $|Q_1| > |Q_2|$

Electric field will be zero to the right of Q_2 as it has small magnitude and opposite sign to that of Q_1.

Hence, (A) and (D) are correct.

12. Net torque on $(-q)$ about a point (say P) lying over $+Q$ is zero. Therefore, angular momentum of $(-q)$ about point P should remain constant.

Hence, option (A) is correct.

13. The given graph is of charged conducting sphere of radius. The entire charge distributes on the surface of the sphere.

Hence, (A), (B), (C) and (D) are correct.

14. Charge density at A < charge density at B. Since field inside cavity is zero, hence
Potential at A = potential at B = a constant value.
By Gauss Theorem,

$$\begin{pmatrix}\text{Total electric}\\ \text{flux}\end{pmatrix} = \frac{1}{\varepsilon_0}\begin{pmatrix}\text{charge}\\ \text{enclosed}\end{pmatrix} = \left(\frac{q}{\varepsilon_0}\right)$$

Hence, (C) and (D) are correct.

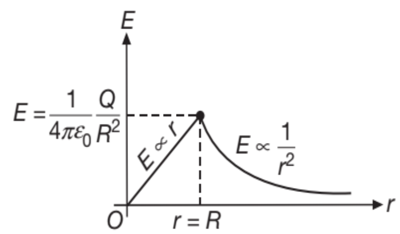
15. Inside the sphere i.e. for $r < R$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R^3}r$$

$\Rightarrow \quad E \propto r$

i.e., E at centre is $E_{\text{centre}} = E|_{r=0} = 0\,(r=0)$

and E at surface is $E_{\text{surface}} = E|_{r=R} = \dfrac{1}{4\pi\varepsilon_o}\dfrac{Q}{R^2}$



Outside the sphere, i.e. for $r > R$

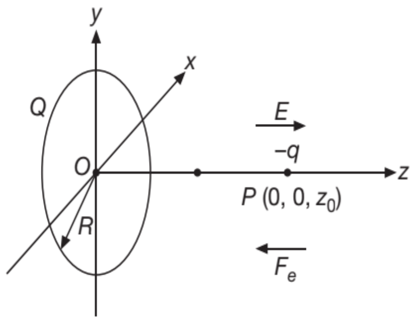
$$E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$$

$\Rightarrow \quad E \propto \dfrac{1}{r^2}$

Thus, variation of electric field (E) with distance (r) from the centre will be as follows.

Hence, (A) and (C) are correct.

16. Let Q be the charge on the ring, the negative charge $-q$ is released from point $P(0,\ 0,\ z_0)$. The electric field at P due to the charged ring will be along positive z-axis and its magnitude will be



$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qz_0}{(R^2+z_0^2)^{\frac{3}{2}}}$$

$E = 0$ at centre of the ring because $z_0 = 0$

Force on charge at P will be towards centre as shown, and its magnitude is

$$F_e = qE = \frac{1}{4\pi\varepsilon_0}\frac{Qqz_0}{(R^2+z_0^2)^{\frac{3}{2}}} \quad \ldots(1)$$

Similarly, when it crosses the origin, the force is again towards centre O.

Thus, the motion of the particle is periodic for all values of z_0 lying between 0 and ∞.

Secondly, if $z_0 \ll R$, then $(R^2+z_0^2)^{\frac{3}{2}} \approx R^3$

$$\Rightarrow \quad F_e \approx \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^3}z_0 \quad \text{\{From equation (1)\}}$$

i.e. the restoring force $F_e \propto -z_0$. So, the motion of the particle will be simple harmonic. (The negative sign implies that the force is towards the mean position).

Reasoning Based Questions

1. Statement-1 is also practical experience based; so it is true.
   Statement-2 is also true but is not the correct explanation of Statement-1. Correct explanation is "there is increase in normal reaction when the object is pushed and there is decrease in normal reaction when object is pulled".
   Hence, the correct answer is (B).

Linked Comprehension Type Questions

1. After hitting the top plate, the balls will get negatively charged and will now get attracted to the bottom plate which is positively charged. The motion of the balls will be periodic but not SHM.
   Hence, the correct answer is (A).

2. As the balls keep on carrying charge from one plate to another, current will keep on flowing even in steady state. When at bottom plate, if all balls attain charge q, then

$$\frac{kq}{r} = V_0$$

$$\Rightarrow \quad q = \frac{V_0 r}{k}$$

Inside the cylinder, electric field is

$$E = [V_0 - (-V_0)]h$$

$$\Rightarrow \quad E = 2V_0 h$$

$\Rightarrow$ Acceleration of each ball, $a = \frac{qE}{m} = \left(\frac{2hr}{km}\right)V_0^2$

$\Rightarrow$ Time taken by balls to reach other plate is

$$t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2hmk}{2hrV_0^2}} = \frac{1}{V_0}\sqrt{\frac{mk}{r}}$$

If there are n balls, then

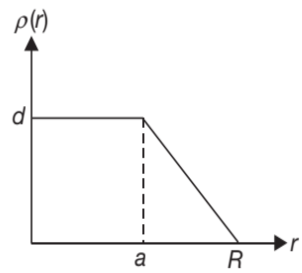
Average current, $i_{av} = \frac{nq}{t} = n \times \frac{V_0 r}{k} \times V_0\sqrt{\frac{r}{mk}}$

$$\Rightarrow \quad i_{av} \propto V_0^2$$

Hence, the correct answer is (B).

3. Electric field at $r = R$ is

$$E = \frac{Q}{4\pi\varepsilon_0 R^2}$$



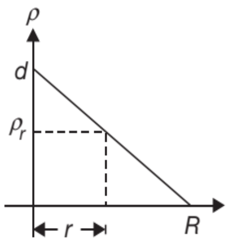
Q = Total charge within the nucleus = Ze

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Ze}{R^2}$$

Hence the electric field is independent of a.

Hence, the correct answer is (A).

4. $Q = \int \rho_r 4\pi r^2 dr$ for $a = 0$, and the figure for the distribution looks like as shown



Slope $= \frac{d}{R} = \frac{\rho_r}{R-r}$

$$\Rightarrow \quad \rho_r = \frac{d}{R}(R-r)$$

$$\Rightarrow \quad Q = \int_0^R \frac{d}{R}(R-r)4\pi r^2 dr$$

$$\Rightarrow \quad Q = \frac{4\pi d}{R}\left(R\int_0^R r^2 dr - \int_0^R r^3 dr\right) = \frac{4\pi d}{R}\left(\frac{R^4}{3} - \frac{R^4}{4}\right)$$

$$Q = \frac{\pi d R^3}{3}$$

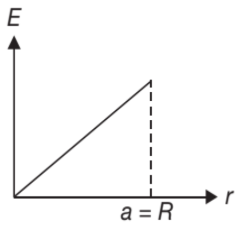
$$\Rightarrow \quad Q = Ze = \frac{\pi d R^3}{3} \qquad \{\because Q = Ze\}$$

$$\Rightarrow \quad d = \frac{3Ze}{\pi R^3}$$

Hence, the correct answer is (B).

5. Let us review the formula of uniformly (volume) charged solid sphere according to which we have

$$E = \frac{\rho r}{3\varepsilon_0}$$



$\Rightarrow \quad E \propto r$, ρ should be constant throughout the volume of nucleus.

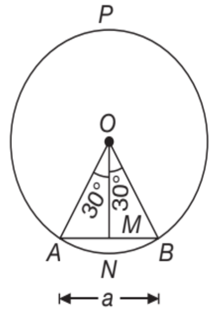
This will be possible only when $a = R$.

Hence, the correct answer is (C).

Integer/Numerical Answer Type Questions

1. ANBP is cross-section of a cylinder of length L. The line charge passes through the centre O and perpendicular to paper.

$$AM = \frac{a}{2}, \quad MO = \frac{\sqrt{3}a}{2}$$



$$\Rightarrow \quad \angle AOM = \tan^{-1}\left(\frac{AM}{OM}\right)$$

$$\Rightarrow \quad \angle AOM = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$

Electric flux passing from the whole cylinder

$$\phi_1 = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

Electric flux passing through $ABCD$ plane surface (shown only AB) = Electric flux passing through cylindrical surface ANB. So

$$\phi = \left(\frac{60^\circ}{360^\circ}\right)(\phi_1)$$

$$\Rightarrow \quad \phi = \frac{\lambda L}{6\varepsilon_0}$$

$$\Rightarrow \quad n = 6$$

2. Volume of cylinder per unit length $(\ell = 1)$ is

$$V = \pi R^2 \ell = (\pi R^2)$$

So, charge per unit length is

$$\lambda = \begin{pmatrix}\text{Volume per}\\ \text{unit length}\end{pmatrix} \times \begin{pmatrix}\text{Volume charge}\\ \text{density}\end{pmatrix} = (\pi R^2 \rho)$$

Now at P, $E_{total} = E_{remainder} + E_{cavity}$

$$\Rightarrow \quad E_R = E_T - E_C$$

Where R = Remainder portion

T = Total portion and

C = cavity portion

$$\Rightarrow \quad E_R = \frac{\lambda}{2\pi\varepsilon_0(2R)} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{(2R)^2}$$

where Q is charge on sphere given by

$$Q = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho = \frac{\pi R^3 \rho}{6}$$

Substituting the values, we get

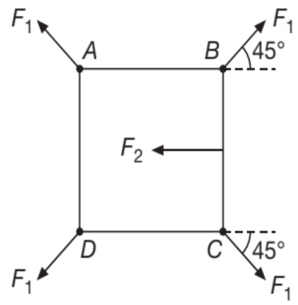
$$E_R = \frac{(\pi R^2 \rho)}{4\pi\varepsilon_0 R} - \frac{1}{4\pi\varepsilon_0}\frac{\left(\pi R^3 \frac{\rho}{6}\right)}{4R^2}$$

$$\Rightarrow \quad E_R = \frac{23\rho R}{96\varepsilon_0} = \frac{23\rho R}{(16)(6)\varepsilon_0}$$

$$\Rightarrow \quad k = 6$$

3. $F_1 =$ Net electrostatic force on any one charge due to rest of three charges. So

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right)$$



$F_2 =$ Force due to surface tension $= \gamma a$

If we see the equilibrium of line BC, then

$$2F_1 \cos(45°) = F_2$$

$$\Rightarrow \quad \sqrt{2}F_1 = F_2$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\left(2 + \frac{1}{\sqrt{2}}\right) = \gamma a$$

$$\Rightarrow \quad a^3 = \frac{1}{4\pi\varepsilon_0}\left(2 + \frac{1}{\sqrt{2}}\right)\frac{q^2}{\gamma}$$

$$\Rightarrow \quad a = \left(\frac{1}{4\pi\varepsilon_0}\left(2 + \frac{1}{\sqrt{2}}\right)\right)^{\frac{1}{3}}\left(\frac{q^2}{\gamma}\right)^{\frac{1}{3}} = k\left(\frac{q^2}{\gamma}\right)^{\frac{1}{3}}$$

where, $k = \left(\frac{1}{4\pi\varepsilon_0}\left(2 + \frac{1}{\sqrt{2}}\right)\right)^{\frac{1}{3}}$

$$\Rightarrow \quad N = 3$$

4. Total charge

$$Q = \int \rho dV = \int_{r=0}^{r=R} (Kr^a)(4\pi r^2 dr) = \frac{4\pi k}{a+3}(R^{a+3})$$

$$Q' = \int \rho dV = \int_{r=0}^{r=\frac{R}{2}} (Kr^a)(4\pi r^2 dr) = \frac{4\pi k}{a+3}\left(\frac{R}{2}\right)^{a+3}$$

According to question

$$\frac{1}{4\pi\varepsilon_0}\frac{Q'}{\left(\frac{R}{2}\right)^2} = \frac{1}{8}\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\right)$$

Putting the value of Q and Q' get

$$a = 2$$