CHAPTER 4: MECHANICAL WAVES

Test Your Concepts-I (Based On Wave Equation & Properties)

1. The amplitude of wave is, $A = 0.06$ m

   Also, from diagram, $\frac{5}{2}\lambda = 0.2$ m

   $\Rightarrow \lambda = 0.08$ m

   $\Rightarrow f = \frac{v}{\lambda} = \frac{300}{0.08} = 3750$ Hz

   $\Rightarrow k = \frac{2\pi}{\lambda} = 78.5\ \text{m}^{-1}$

   $\Rightarrow \omega = 2\pi f = 23562\ \text{rads}^{-1}$

   At $t = 0, x = 0, \frac{dy}{dx} =$ positive

   and the given curve is a sine curve.
   Hence, equation of wave travelling along positive x-direction has the form,

   $y(x, t) = A\sin(\omega t - kx)$

   Substituting the values, we get

   $y(x, t) = (0.06\ \text{m})\sin(23562t - 78.5x)$

2. Since, $y = A\sin(\omega t - kx + \phi)$

   At some given instant $y = 0$ at $x = 0$

   $\Rightarrow \omega t + \phi = 0$

   Now, $-0.01\sqrt{3} = 0.02\sin(-0.1k)$

   and $0.01\sqrt{3} = 0.02\sin(-0.8k)$

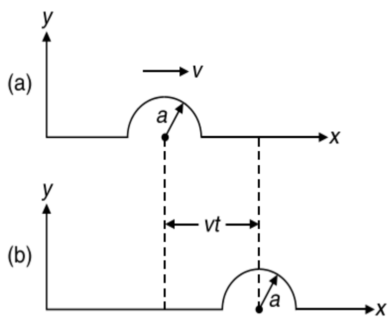
   Satisfying these two conditions, we get

   $k = \frac{20\pi}{3}\ \text{m}^{-1}, \omega = 2\pi f = 1000\pi\ \text{rads}^{-1}$

   $v = \frac{\omega}{k} = 150\ \text{ms}^{-1}$

3. Since, $y^2 = a^2 - x^2$ i.e., $x^2 + y^2 = a^2$

   So, the pulse is semicircular in shape as shown in figure (a). For the wave travelling in positive x-direction $y(x,t) = f(x - vt)$, so



$$y(x,t) = \begin{cases} \sqrt{a^2 - (x - vt)^2}, & \text{when } |x - vt| \le a \\ 0, & \text{when } |x - vt| \ge 0 \end{cases}$$

   Above equation reduces to equation given in the problem for $t = 0$. The shape of pulse at a later time t is shown in Figure (b).

4. $\frac{\partial^2 y}{\partial t^2} = -\omega^2 a\sin(\omega t - kx)$ ...(1)

   and $\frac{\partial^2 y}{\partial x^2} = -k^2 a\sin(\omega t - kx)$ ...(2)

   From (1) and (2), we get

   $\frac{\partial^2 y}{\partial t^2} = \left(\frac{\omega^2}{k^2}\right)\frac{\partial^2 y}{\partial x^2}$

   $\Rightarrow \frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$ $\left\{\because v = \frac{\omega}{k}\right\}$

   The negative sign between ωt and kx implies that wave is travelling along positive x-direction.

5. We know that for a wave $v = f\lambda$

   $\Rightarrow \lambda = \frac{v}{f} = \frac{360}{500} = 0.72$ m

   Given $\Delta\phi = 60° = \frac{\pi}{180°} \times 60° = \frac{\pi}{3}$ rad

   Since phase difference, $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

   $\Rightarrow \Delta x = \frac{\lambda}{2\pi}\Delta\phi = \frac{0.72}{2\pi} \times \frac{\pi}{3} = 0.12$ m

6. Although all the four functions are written in the form $f(ax \pm bt)$, only (d) among the four functions is finite everywhere at all times. Hence $e^{-(x-vt)^2}$ represents a wave.

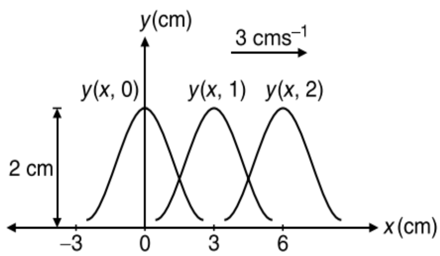
7. Since $y(x, t) = \frac{2}{(x - 3t)^2 + 1}$, so wave speed is $v = 3\ \text{cms}^{-1}$ and wave amplitude (the maximum value of y) is $A = 2$ cm. Since,

   $y(x, 0) = \frac{2}{x^2 + 1}$

   $y(x, 1) = \frac{2}{(x - 3)^2 + 1}$

   and $y(x, 2) = \frac{2}{(x - 6)^2 + 1}$

   The corresponding graph are shown in figure.



8. Since, $k_{eff} = 2k = 1\ \text{Nm}^{-1}$

   $\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m}} = \frac{10}{2\pi}\ \text{s}^{-1}$

   $v = 0.1\ \text{ms}^{-1}, A = 0.02$ m

$$\Rightarrow \quad \lambda = \frac{v}{f} = \frac{0.1}{\left(\frac{10}{2\pi}\right)} = \frac{2\pi}{100} \text{ m}$$

Since, $y = A\cos\left(\frac{2\pi}{\lambda}\right)(vt - x)$

$$\Rightarrow \quad y = 0.02\cos 100(0.1t - x)$$

$$\Rightarrow \quad y = 0.02\cos(10t - 100x) \text{ metre}$$

Distance between two successive maxima is

$$\lambda = \frac{2\pi}{100} = 0.0628 \text{ m}$$

9. Comparing given equation with standard equation $y = A\sin(kx - \omega t) = A\sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$, we get, $A = 2$ cm, $f = \frac{1}{T} = \frac{1}{0.01} = 100$ Hz, $\lambda = 30$ cm and $v = f\lambda = 3000 \text{ cms}^{-1} = 30 \text{ ms}^{-1}$

10. Comparing this with $y = \frac{a}{b + (x + vt)^2}$, we get $a = 10$, $b = 5$, $v = 2 \text{ ms}^{-1}$ along $-x$ direction. Also, the amplitude (at $x = 0$ and $t = 0$) is $y = \frac{a}{b} = \frac{10}{5} = 2$ units.

11. On comparing the given expression with

$$y = f(x - vt)$$

We get the velocity of the wave as

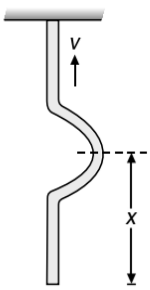
$$v = 4 \text{ ms}^{-1}$$

Since these occurs negative sign between x and t in the given expression, the wave propagates **along the +ve x-axis**.

12. The negative sign between $3x$ and $4t$ implies that y_1 is travelling along positive x-direction and positive sign between them means y_2 is travelling along negative x-direction. They will cancel each other when $y_1 + y_2 = 0$. Substituting the values of y_1 and y_2 in above equation, we get $t = 0.75$ s and $x = 1$ m.

Test Your Concepts-II (Based On Transverse Wave in a String & Properties)

1. Since, $v = \frac{dx}{dt} = \sqrt{\frac{T}{\mu}}$



$$\Rightarrow \quad T = g\int_0^x \mu dx = g\int_0^x \mu_0 x dx = \frac{g\mu_0 x^2}{2}$$

$$\Rightarrow \quad \frac{T}{\mu} = \frac{g\mu_0 x^2}{2\mu_0 x} = \frac{gx}{2}$$

$$\Rightarrow \quad \frac{dx}{dt} = \sqrt{\frac{gx}{2}}$$

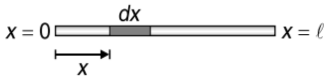
$$\Rightarrow \quad \sqrt{\frac{g}{2}}\int_0^t dt = \int_0^{\ell_0} \frac{dx}{\sqrt{x}}$$

$$\Rightarrow \quad t = \sqrt{\frac{8\ell_0}{g}}$$

2. Speed of transverse wave is given by

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{100}{\mu_0 + \alpha x}} = \frac{10}{\sqrt{\mu_0 + \alpha x}} = \frac{dx}{dt}$$

$$\Rightarrow \quad \int_0^l (\mu_0 + \alpha x)^{\frac{1}{2}} dx = \int_0^t 10 dt$$



$$\Rightarrow \quad \left.\frac{(\mu_0 + \alpha x)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)\alpha}\right|_0^l = 10(t - 0)$$

$$\Rightarrow \quad \frac{2}{3\alpha}\left[(\mu_0 + \alpha l)^{\frac{3}{2}} - \mu_0^{\frac{3}{2}}\right] = 10t$$

$$\Rightarrow \quad t = \frac{1}{15\alpha}\left[(\mu_0 + \alpha l)^{\frac{3}{2}} - \mu_0^{\frac{3}{2}}\right]$$

3. Tension is string AB is $T_{AB} = 6.4 \text{ g} = 64$ N

So, speed of transverse waves in string AB is

$$v_{AB} = \sqrt{\frac{T_{AB}}{\mu_{AB}}} = \sqrt{\frac{64}{10 \times 10^{-3}}}$$

$$\Rightarrow \quad v_{AB} = \sqrt{6400} = 80 \text{ ms}^{-1}$$

Tension in strong CD is $T_{CD} = 3.2 \text{ g} = 32$ N

So, speed of transverse waves in string CD is

$$v_{CD} = \sqrt{\frac{T_{CD}}{\mu_{CD}}} = \sqrt{\frac{32}{8 \times 10^{-3}}}$$

$$\Rightarrow \quad v_{CD} = \sqrt{4000} = 63.24 \text{ ms}^{-1}$$

4. Already Done in Theory.

5. Let the tension at point 1 and 2 be T_1 and T_2 respectively, then, $T_1 = 2g$, $T_2 = 6g$. If v_1 and v_2 are the speeds of wave at ends 1 and 2 respectively, then

$$\frac{v_1}{v_2} = \frac{f\lambda_1}{f\lambda_2} = \sqrt{\frac{T_1/\mu}{T_2/\mu}} = \sqrt{\frac{T_1}{T_2}}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{2g}{6g}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \lambda_2 = \sqrt{3}\lambda_1 = 0.06\sqrt{3} \text{ m}$$

6. The linear mass density is

$$\mu = \frac{5 \times 10^{-3}}{50 \times 10^{-2}} = 1 \times 10^{-2} \text{ kgm}^{-1}$$

Since, $v = \sqrt{\frac{T}{\mu}}$ i.e., $T = \mu v^2$

$$\Rightarrow \quad T = (1 \times 10^{-2}) \times 6400 = 64 \text{ N}$$

Also, young's modulus is given by

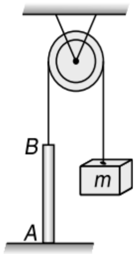
$$Y = \frac{T/A}{\Delta L/L}$$

$$\Rightarrow \quad \Delta L = \frac{TL}{AY} = \frac{64\times 0.50}{1\times 10^{-6}\times 16\times 10^{11}} = 0.02 \text{ mm}$$

7. Tension in the rubber tube AB is

$$T = mg$$

$$\Rightarrow \quad T = (5)(9.8) = 49 \text{ N}$$



Mass per unit length of rubber tube is

$$\mu = \frac{0.9}{12} = 0.075 \text{ kgm}^{-1}$$

Speed of wave on the tube is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{49}{0.075}} = 25.56 \text{ ms}^{-1}$$

The required time is

$$t = \frac{AB}{v} = \frac{12}{25.56} = 0.47 \text{ s}$$

8. The velocity of the wave is $v_1 = \sqrt{\frac{T_1}{\mu}}$

On increasing the tension to $4T_1$, velocity v_2 is given by

$$v_2 = \sqrt{\frac{4\pi}{\mu}} = 2v_1$$

Also $\omega_1 = 2\pi v_1$ and $\omega_2 = 2\pi\frac{v_1}{4} = \frac{\omega_1}{4}$

Since, $P_1 = \frac{1}{2}\mu\omega_1^2 v_1 A^2$

$$\Rightarrow \quad P_2 = \frac{1}{2}\mu\omega_1^2 v_2 A^2 = \frac{1}{2}\mu\left(\frac{\omega_1}{4}\right)^2(2v_1)A^2$$

$$\Rightarrow \quad P_2 = \frac{1}{8}\left(\frac{1}{2}\mu\omega_1^2 v_1 A^2\right) = \frac{P_1}{8}$$

9. Wave speed on a wire is $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T\ell}{m}}$

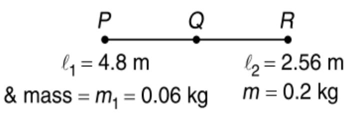
$$\Rightarrow \quad v = \sqrt{\frac{8\times 0.64}{5\times 10^{-3}}} = 32 \text{ ms}^{-1}$$

10. Given that, $v_1 = 2v_2$

$$\Rightarrow \quad \sqrt{\frac{T}{A\rho_1}} = 2\sqrt{\frac{T}{A\rho_2}} \qquad \left\{\because v = \sqrt{\frac{T}{A\rho}}\right\}$$

$$\Rightarrow \quad \frac{\rho_1}{\rho_2} = \frac{1}{4} = 0.25$$

11. The tension is maintained at 80 N and amplitude of incident wave is given to be 3.5 cm.



The mass per unit length of PQ and QR is

$$\mu_1 = \frac{0.06}{4.8} = \frac{1}{80} \text{ kgm}^{-1} \text{ and}$$

$$\mu_2 = \frac{0.2}{2.56} = \frac{1}{12.8} \text{ kgm}^{-1}$$

Speed of wave in PQ is

$$v_1 = \sqrt{\frac{T}{\mu_1}} = \sqrt{\frac{80}{1/80}} = 80 \text{ ms}^{-1}$$

Speed of wave in QR is

$$v_2 = \sqrt{\frac{T}{\mu_2}} = \sqrt{\frac{80}{1/12.8}} = 32 \text{ ms}^{-1}$$

(a) Time taken by the wave pulse to move from P to R is given by

$$t = \frac{l_1}{v_1} + \frac{l_2}{v_2} = \frac{4.8}{80} + \frac{2.56}{32} = 0.14 \text{ s}$$

(b) Since, $A_r = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)A_i = \left(\frac{32-80}{32+80}\right)3.5$

$$\Rightarrow \quad A_r = -1.5 \text{ cm}$$

Also, $A_t = \left(\frac{2v_2}{v_2 + v_1}\right)A_i = \left(\frac{2\times 32}{32+80}\right)3.5$

$$\Rightarrow \quad A_t = 2 \text{ cm}$$

12. Speed of a transverse wave on a string is $v = \sqrt{\frac{T}{\mu}}$ i.e., $v \propto \frac{1}{\sqrt{\mu}}$

Given that $\mu_2 > \mu_1$, $v_2 < v_1$

i.e., medium 2 is denser and medium 1 is rarer.

13. Tension in string is

$$T = kx = 160\times 0.01 = 1.6 \text{ N}$$

So, wave speed is $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T\ell}{m}}$

$$\Rightarrow \quad v = \sqrt{\frac{1.6\times 0.4}{0.01}} = 8 \text{ ms}^{-1}$$

Time taken by pulse is $t = \frac{\ell}{v} = \frac{0.4}{8} = 0.05$ s

Test Your Concepts-III (Based On Sound Waves & Properties)

1. (a) At a distance r from a point source of power P, the intensity of the sound is

$$I = \frac{P}{4\pi r^2} = \frac{0.8}{(4\pi)(1.5)^2}$$

$$\Rightarrow \quad I = 2.83\times 10^{-2} \text{ Wm}^{-2} \qquad \ldots(1)$$

Further, the intensity of sound in terms of $(\Delta P)_m$, ρ and v is given by

$$I = \frac{(\Delta P)_m^2}{2\rho v} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$(\Delta P)_m = \sqrt{2\times 2.83\times 10^{-2}\times 1.29\times 340}$$

$$\Rightarrow \quad (\Delta P)_m = 4.98 \text{ Nm}^{-2}$$

(b) Pressure oscillation amplitude $(\Delta P)_m$ and displacement oscillation amplitude A are related by the equation

$$(\Delta P)_m = BAk$$

Since, $v = \sqrt{\dfrac{B}{\rho}}$

Substituting $B = \rho v^2$, $k = \dfrac{\omega}{v}$ and $\omega = 2\pi f$, we get

$$(\Delta P)_m = 2\pi A \rho v f$$

$$\Rightarrow \quad A = \frac{(\Delta P)_m}{2\pi \rho v f} = \frac{4.98}{(2\pi)(1.29)(340)(600)}$$

$$\Rightarrow \quad A = 3 \times 10^{-6} \text{ m}$$

2. Since hydrogen is a diatomic gas, so $\gamma_{H_2} = \dfrac{7}{5}$

Speed of sound in hydrogen is $v_{H_2} = \sqrt{\dfrac{\gamma p}{\rho}}$

RMS speed of hydrogen molecules is

$$(v_{\text{rms}})_{H_2} = \sqrt{\frac{3P}{\rho}}$$

$$\Rightarrow \quad \frac{v_{H_2}}{(v_{\text{rms}})_{H_2}} = \sqrt{\frac{\gamma}{3}} = \sqrt{\frac{7/5}{3}} = \sqrt{\frac{7}{15}}$$

3. Since power is distributed uniformly in a hemisphere, intensity at a distance of 5 m from the source will be

$$I = \frac{P}{A} = \frac{P}{4\pi r^2/2} = \frac{10^{-3}}{2\pi(5)^2} = 6.37\ \mu\text{Wm}^{-2}$$

$$\Rightarrow \quad L = 10\log_{10}\left(\frac{I}{I_0}\right) = 10\log_{10}\left(\frac{6.37 \times 10^{-6}}{10^{-12}}\right)$$

$$\Rightarrow \quad L = 10(\log_{10} 6.37 + 6\log_{10} 10)$$

$$\Rightarrow \quad L = 10(0.80 + 6) = 68 \text{ dB}$$

If there are five dogs barking at the same time and at the same level, then $I_2 = 5I_1$. So

$$L_2 - L_1 = 10\log_{10}\left(\frac{I_2}{I_1}\right) = 10\log_{10}\left(\frac{5I_1}{I_1}\right)$$

$$\Rightarrow \quad L_2 = L_1 + 10\log_{10} 5$$

$$\Rightarrow \quad L_2 = 68 + 10(0.7) = 75 \text{ dB}$$

4. Speed of sound wave

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{(2\times 10^9)}{10^3}} = 1414 \text{ ms}^{-1}$$

Wavelength, $\lambda = \dfrac{v}{f} = 5.84$ m

5. Since, $\dfrac{v_2}{v_1} = \sqrt{\dfrac{T_2}{T_1}} = \sqrt{\dfrac{273+10}{273+27}} = \sqrt{\dfrac{283}{300}} = 0.97$

As frequency remains unchanged, so

$$\frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1} = 0.97 \qquad \{\because v = f\lambda\}$$

So, percentage change in wavelength is

$$\frac{\lambda_1 - \lambda_2}{\lambda_1} \times 100 = \left(1 - \frac{\lambda_2}{\lambda_1}\right) \times 100$$

So, percentage change is $(1 - 0.97) \times 100 = 3\%$

6. Given that, $L = 10\log_{10}(I/I_0) = 60$

$$\Rightarrow \quad I/I_0 = 10^6$$

$$\Rightarrow \quad I = 10^{-12} \times 10^6 = 1\ \mu\text{Wm}^{-2}$$

Power entering the room is $P = IA$

$$\Rightarrow \quad P = 1 \times 10^{-6} \times 2 = 2\ \mu\text{W}$$

Energy collected in a day is $E = Pt$

$$\Rightarrow \quad E_{1\text{ day}} = 2 \times 10^{-6} \times 86400 = 0.173 \text{ J}$$

7. $v = \sqrt{\dfrac{\gamma RT}{M}}$

Since, $v_{H_2} = v_{O_2}$

$$\Rightarrow \quad \sqrt{\frac{\gamma_{H_2} R T_{H_2}}{M_{H_2}}} = \sqrt{\frac{\gamma_{O_2} R T_{O_2}}{M_{O_2}}}$$

Further $\gamma_{H_2} = \gamma_{O_2}$ $\{\because$ both are diatomic$\}$

$$\Rightarrow \quad T_{H_2} = \left(\frac{M_{H_2}}{M_{O_2}}\right)(T_{O_2}) = \left(\frac{1}{16}\right)(100 + 273)$$

$$\Rightarrow \quad T_{H_2} = 23.31 \text{ K}$$

$$\Rightarrow \quad T_{H_2} \approx -249.7\ ^\circ\text{C}$$

8. Since, $v = \sqrt{\dfrac{B}{\rho}} = \sqrt{\dfrac{75 \times 10^9}{2.70 \times 10^3}} = 10^3\sqrt{\dfrac{75}{2.7}}$

$$\Rightarrow \quad v = 10^3\sqrt{\frac{75}{2.7}} = 5.27 \times 10^3 \text{ ms}^{-1}$$

9. Since, $I = \dfrac{(\Delta P)^2_{\max}}{2\rho v}$

$$\Rightarrow \quad (\Delta P)_{\max} = \sqrt{2\rho v \text{I}}$$

$$\Rightarrow \quad (\Delta P)_{\max} = \sqrt{2 \times 1.293 \times 340 \times 10^{-3}}$$

$$\Rightarrow \quad (\Delta P)_{\max} = 0.94 \text{ Nm}^{-2}$$

10. (a) Since $L = 10\log_{10}(I/I_0)$

$$\Rightarrow \quad 20 = 10\log_{10}\left(\frac{I}{10^{-12}}\right)$$

$$\Rightarrow \quad I = 10^{-10} \text{ Wm}^{-2}$$

Further, $v = \sqrt{\dfrac{B}{\rho}} = 200 \text{ ms}^{-1}$

Intensity of a sound wave is given by

$$I = \frac{(\Delta P_{\max})^2}{2\rho v}$$

$$\Rightarrow \quad (\Delta P)_{\max} = \sqrt{2I\rho v}$$

$$\Rightarrow \quad (\Delta P)_{\max} = \sqrt{2 \times 10^{-10} \times 1 \times 200}$$

$$\Rightarrow \quad (\Delta P)_{max} = 2\times 10^{-4}\ \text{Nm}^{-2}$$

(b) Since, $I = 2\pi^2 f^2 A^2 \rho v = \frac{1}{2}\omega^2 A^2 \rho v$

$$\Rightarrow \quad A = \sqrt{\frac{2I}{\omega^2 \rho v}} = \sqrt{\frac{2\times 10^{-10}}{(500)^2(1)(200)}}$$

$$\Rightarrow \quad A = 2\times 10^{-9}\ \text{m}$$

Further, $k = \frac{\omega}{v} = \frac{500}{200} = \frac{5}{2}\ \text{m}^{-1}$ $\left\{\because v = \frac{\omega}{k}\right\}$

So, equation of the wave is

$$y = 2\times 10^{-9}\sin\left(500t - \frac{5}{2}x\right)$$

11. Since, $v_{H_2} = \sqrt{\frac{\gamma P}{\rho_{H_2}}}$ at 0 °C and

$$\rho_{mix} = \frac{(2V)\rho_{H_2} + V\rho_{N_2}}{3V}$$

$$\Rightarrow \quad \rho_{mix} = \frac{16\rho_{H_2}}{3} \qquad \{\because \rho_{N_2} = 14\rho_{H_2}\}$$

$$\Rightarrow \quad v_{mix} = \sqrt{\frac{\gamma P}{\rho_{mix}}} \text{ at } 0\ °\text{C}$$

$$\Rightarrow \quad v_{mix} = \frac{1300\sqrt{3}}{4} = 325\sqrt{3}\ \text{ms}^{-1}$$

So, velocity of sound in the mixture at 27 °C is given by

$$v = \left(\sqrt{\frac{300}{273}}\right)(325\sqrt{3}) \qquad \{\because v \propto \sqrt{T}\}$$

$$\Rightarrow \quad v = 591\ \text{ms}^{-1}$$

12. Intensity, $I = \frac{(\Delta P)_m^2}{2\rho v}$

where ΔP_m is the pressure amplitude

$$\Rightarrow \quad I = \frac{(6\times 10^{-5})^2}{2\times 1.2\times 344} = 4.4\times 10^{-12}\ \text{Wm}^{-2}$$

13. Since, $I = \frac{(\Delta P)_m^2}{2\rho v}$

$$\Rightarrow \quad (\Delta P)_m = \sqrt{2I\rho v} = \sqrt{2(10^{-12})(1.3)(332)}$$

$$\Rightarrow \quad (\Delta P)_m = 2.94\times 10^{-5}\ \text{Nm}^{-2}$$

Also, $(\Delta P)_m = BAK = \rho v^2 A\left(\frac{\omega}{v}\right) = \rho v A\omega$

$$\Rightarrow \quad A = \frac{(\Delta P)_m}{\rho v\omega} = \frac{2.94\times 10^{-5}}{1.3\times 332\times 2p\times 10^3}$$

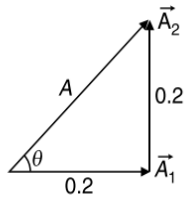
$$\Rightarrow \quad A = 1.1\times 10^{-11}\ \text{m}$$

Test Your Concepts-IV (Based On Interference)

1. $y = y_1 + y_2$

$$\Rightarrow \quad y = 0.2\sin(x-3t) + 0.2\sin\left(x - 3t + \frac{\pi}{2}\right)$$

$$\Rightarrow \quad y = A\sin(x - 3t + \theta)$$



where, $A = \sqrt{(0.2)^2 + (0.2)^2} = 0.28$ m

and $\theta = \frac{\pi}{4}$

$$\Rightarrow \quad y = 0.28\sin\left(x - 3t + \frac{\pi}{4}\right)$$

Since the amplitude of the resulting wave is 0.32 m and $A = 0.2$ m, so we get

$$0.32 = 0.2\sqrt{1 + 1 + 2\cos\phi}$$

$$\Rightarrow \quad \phi = \pm 1.29\ \text{rad}$$

2. Given, $\frac{A_1}{A_2} = \frac{3}{5}$

Since $I \propto A^2$, so $\sqrt{\frac{I_1}{I_2}} = \frac{3}{5}$

Intensity is maximum when

$\cos\phi = 1$ and $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

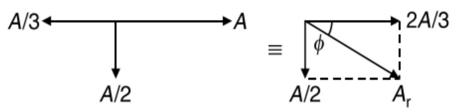
Intensity is minimum when

$\cos\phi = -1$ and $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$$\Rightarrow \quad \frac{I_{min}}{I_{max}} = \left(\frac{\sqrt{I_1} - \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{I_1/I_2} - 1}{\sqrt{I_1/I_2} + 1}\right)^2$$

$$\Rightarrow \quad \frac{I_{min}}{I_{max}} = \left(\frac{\frac{3}{5} - 1}{\frac{3}{5} + 1}\right)^2 = \frac{4}{64} = \frac{1}{16}$$

3. The phasor diagram for the situation discussed is drawn below



Resultant amplitude $A_r = \sqrt{\left(\frac{2A}{3}\right)^2 + \left(\frac{A}{2}\right)^2}$

$$\Rightarrow \quad A_r = \frac{5}{6}A$$

Also, $\tan\phi = -\frac{A/2}{2A/3} = -\frac{3}{4}$

$$\Rightarrow \quad \phi = -\tan^{-1}\left(\frac{3}{4}\right)$$

4. Path Difference $= ACP - ABP = \Delta x = 11.5$ cm

Since there is complete silence at P, so

$$\Delta x = (2n-1)\frac{\lambda}{2}, \text{ where } n = 1, 2, 3, ...$$

$$\Rightarrow \quad \frac{\lambda}{2} = 11.5\ \text{cm}$$

$$\Rightarrow \quad \lambda = 23\ \text{cm} = 0.23\ \text{m}$$

$$\Rightarrow \quad f_{min} = \frac{v}{\lambda} = \frac{331.2}{0.23} = 1440\ \text{Hz}$$

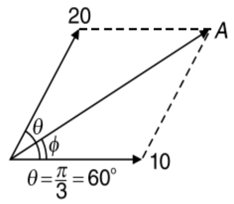
5. $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$

$$\Rightarrow \quad A = \sqrt{(10)^2 + (20)^2 + (2)(10)(20)\cos(60°)}$$

$\Rightarrow \quad A = 26.46 \text{ m}$

$$\tan\phi = \frac{20\sin(60°)}{10+20\cos(60°)} = 0.866$$

$\Rightarrow \quad \phi = 41° = 0.714 \text{ rad}$



Therefore, phase of the resultant wave is

$$(5x + 25t + 0.714) \text{ rad}$$

6. Since $v = f\lambda$

$$\Rightarrow \quad \lambda = \frac{v}{f} = \frac{330}{220} = 1.5 \text{ m}$$

Further $\phi = \frac{2\pi}{\lambda}\Delta x$

$$\Rightarrow \quad \phi_1 = \left(\frac{2\pi}{\lambda}\right)(\Delta x_1) = \left(\frac{2\pi}{1.5}\right)(0.75) = \pi$$

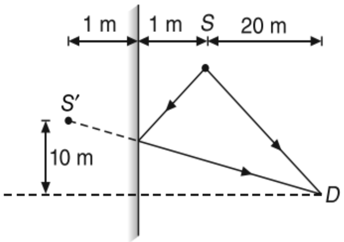
$$\Rightarrow \quad \phi_2 = \left(\frac{2\pi}{\lambda}\right)(\Delta x_2) = \left(\frac{2\pi}{1.5}\right)(3) = 4\pi$$

Since, they are out of phase at P, therefore destructive interference must be taking place. So, the resultant power at P is

$$P_1 - P_2 = (1.8\times10^{-3} - 1.2\times10^{-3}) \text{ watt}$$

$\Rightarrow \quad P_1 - P_2 = 0.6\times10^{-3} \text{ watt}$

7. Two sound waves reach point D. One reaches D directly from S and the other after reflection from the wall as shown in Figure.



The reflected wave appears to come from the virtual image S' of the source (remember that sound waves follows the ordinary laws of reflection and refraction). This image S' is distant 1 m from wall just like the source. The two path lengths, now are

$$SD = \sqrt{20^2 + 10^2} = 22.36 \text{ m}$$

and $S'D = \sqrt{22^2 + 10^2} = 24.17 \text{ m}$

Path difference between two waves is

$$\Delta x = 24.17 - 22.36 = 1.81 \text{ m}$$

When path difference of 1.81 m equals $n\lambda$, a maxima is formed at D.

$\Rightarrow \quad 1.81 = n\lambda$ where $n = 0, 1, 2, \ldots\ldots$

$$\Rightarrow \quad 1.81 = \frac{n(340)}{f} \qquad \{\because v = f\lambda\}$$

$$\Rightarrow \quad f = n\left(\frac{340}{1.81}\right) = n(188) \text{ Hz}$$

$\Rightarrow \quad f = 188 \text{ Hz}, 376 \text{ Hz}, 564 \text{ Hz}, \ldots\ldots,$

8. Path difference $\Delta x = S_1P - S_2P$

$$\Rightarrow \quad \Delta x = \sqrt{(2)^2 + (4)^2} - \sqrt{(1)^2 + (4)^2}$$

$\Rightarrow \quad \Delta x = 4.47 - 4.12$

$\Rightarrow \quad \Delta x = 0.35 \text{ m}$

(a) Constructive interference occurs when

$$\Delta x = (2n)\frac{\lambda}{2} = \frac{nv}{f}$$

$$\Rightarrow \quad f = \frac{n(v)}{\Delta x}, \text{ where } n = 1, 2, 3, \ldots.$$

$$\Rightarrow \quad f = \frac{350}{0.35}, \frac{2\times350}{0.35}, \frac{3\times350}{0.35}, \ldots$$

$\Rightarrow \quad f = 1000 \text{ Hz}, 2000 \text{ Hz}, 3000 \text{ Hz}, \ldots.$

(b) Destructive interference occurs when

$$\Rightarrow \quad \Delta x = (2n-1)\frac{\lambda}{2}, \text{ where } n = 1, 2, 3, \ldots.$$

$$\Rightarrow \quad \Delta x = (2n-1)\frac{v}{2f} \qquad \{\because v = f\lambda\}$$

$$\Rightarrow \quad f = \frac{(2n-1)v}{2\Delta x}$$

$$\Rightarrow \quad f = \frac{350}{2\times0.35}, \frac{3\times350}{2\times0.35}, \frac{5\times350}{2\times0.35}, \ldots.$$

$\Rightarrow \quad f = 500 \text{ Hz}, 1500 \text{ Hz}, 2500 \text{ Hz}, \ldots.$

9. The path difference between the waves moving along the straight path and the semicircular path is

$$\Delta x = \pi r - 2r = (\pi - 2)r$$

For minima this path difference should be at least $\lambda/2$ i.e., 20 cm. Hence, we have, for the minimum value of r

$$(\pi - 2)r = 20 \text{ cm}$$

$$\Rightarrow \quad r = \frac{20}{\pi - 2} = 17.54 \text{ cm}$$

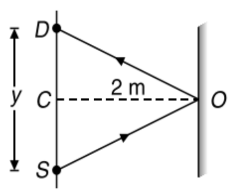
10. (a) $\lambda = \frac{v}{f} = \frac{360}{180} = 2 \text{ m}$

A maximum will be observed at D, if

$$\Delta x = \sqrt{y^2 + 16} - y = \lambda = 2 \text{ m}$$

Solving this equation, we get

$$y = 3 \text{ m}$$



(b) Similarly, for minima

$$\Delta x = \frac{3\lambda}{2} = 3 \text{ m}$$

Let the obstacle be shifted to the right by a distance z. Then

$$2\sqrt{\left(\frac{y}{2}\right)^2+(2+z)^2}-y=3$$

$$\Rightarrow \quad 2\sqrt{\left(\frac{3}{2}\right)^2+(2+z)^2}-3=3$$

$$\Rightarrow \quad z=0.6 \text{ m}$$

11. Let x be the distance travelled by the wave when it reaches the respective point.
For waves arriving at A from P and Q, phase difference is

$$\Delta\phi_A=(\phi_P-\phi_Q)+\frac{2\pi}{\lambda}(x_P-x_Q)$$

When the waves reach A, then Q is ahead of P in terms of path because wave emitted by Q reaches A before the wave emitted by P and also phase of P is ahead that of Q by 90°, so we have

$$\phi_P-\phi_Q=+\frac{\pi}{2} \text{ and } x_P-x_Q=-5 \text{ m}$$

$$\Rightarrow \quad \Delta\phi_A=+\frac{\pi}{2}+\frac{2\pi}{20}(-5)=0$$

Since, $I_R=I_1+I_2+2\sqrt{I_1I_2}\cos\Delta\phi$

$$\Rightarrow \quad I_A=I+I+2I\cos(0°)=4I$$

For waves arriving at C from P and Q, phase difference is

$$\Delta\phi_C=(\phi_P-\phi_Q)+\frac{2\pi}{\lambda}(x_P-x_Q)$$

When the waves reach C, then P is ahead of Q in terms of path, because wave emitted by P reaches C before the wave emitted by Q and also phase of P is ahead that of Q by 90°, so we have

$$\phi_P-\phi_Q=+\frac{\pi}{2} \text{ and } x_P-x_Q=+5 \text{ m}$$

$$\Rightarrow \quad \Delta\phi_C=+\frac{\pi}{2}+\frac{2\pi}{20}(+5)=\pi$$

$$\Rightarrow \quad I_C=I+I+2I\cos(\pi)=0$$

For waves arriving at B from P and Q, phase difference is

$$\Delta\phi_B=(\phi_P-\phi_Q)+\frac{2\pi}{\lambda}(x_P-x_Q)$$

When the waves reach B, then P and Q have same path and phase of P is ahead that of Q by 90°, so we have

$$\phi_P-\phi_Q=+\frac{\pi}{2} \text{ and } x_P-x_Q=0$$

$$\Rightarrow \quad \Delta\phi_B=+\frac{\pi}{2}+0=\frac{\pi}{2}$$

$$\Rightarrow \quad I_B=I+I+2I\cos(\pi/2)=2I$$

Hence, $I_A:I_B:I_C=4I:2I:0=2:1:0$

12. $\lambda=\frac{v}{f}=\frac{343}{21.5}\approx 16 \text{ m}$

Path difference between two sound waves reaching at A is given by

$$\Delta x=S_1A-S_2A=(9-1)=8 \text{ m}$$

Since, we observe that

$$\Delta x=\frac{\lambda}{2}=(\text{Odd Multiple})\frac{\lambda}{2}$$

So, the minimum intensity is observed at A.

Since, $S_1=(-5,\ 0)$ and $S_2=(5,\ 0)$

For intensity to be minimum

$$S_1P-S_2P=(2n-1)\frac{\lambda}{2} \qquad n=1,2,3,\ldots$$

$$\Rightarrow \quad \sqrt{(x+5)^2+y^2}-\sqrt{(x-5)^2+y^2}=(16n-8) \quad n=1,2,3,\ldots$$

This is the desired relation between x and y when intensity is minimum.

Test Your Concepts-V (Based On Stationary Waves & Beats)

1. The given wave can be written as the sum of two component waves given by

$$y(x,t)=0.2\sin(0.5x-30t)+0.2\sin(0.5x+30t)$$

So, wave is made of two component waves

(i) $y_1(x,t)=0.2\sin(0.5x-30t)$, travelling along positive x-direction and

(ii) $y_2(x,t)=0.2\sin(0.5x+30t)$, travelling along negative x-direction.

Now, $\omega=30 \text{ rads}^{-1}$ and $k=0.5 \text{ cm}^{-1}$

So, frequency, $f=\frac{\omega}{2\pi}=\frac{15}{\pi}$ Hz

amplitude $A=0.2$ cm

and wave speed, $v=\frac{\omega}{k}=\frac{30}{0.5}=60 \text{ cms}^{-1}$

Particle velocity

$$v_P(x,\ t)=\frac{\partial y}{\partial t}=-12\sin(0.5x)\sin(30t)$$

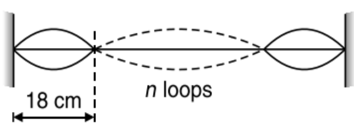
$$\Rightarrow \quad v_P\Big|_{x=2.4\text{ cm},\, t=0.8\text{ s}}=-12\sin(1.2)\sin(24)$$

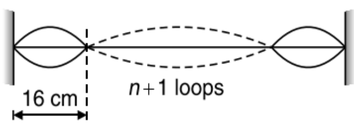
$$\Rightarrow \quad v_P=10.12 \text{ cms}^{-1}$$

2. (a) Let ℓ be the length of the string. Then

$$18n=\ell \qquad \ldots(1)$$

$$16(n+1)=\ell \qquad \ldots(2)$$





From Equations (1) and (2), we get

$$n=8 \text{ and } \ell=144 \text{ cm}$$

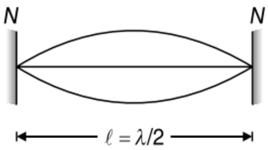
Therefore, the minimum possible length of the string can be 144 cm.

(b) For fundamental frequency, $\ell = \dfrac{\lambda}{2}$

$\Rightarrow \quad \lambda = 2\ell = 288 \text{ cm} = 2.88 \text{ m}$

Speed of wave on the string

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{10}{4 \times 10^{-3}}} = 50 \text{ ms}^{-1}$$



So, fundamental frequency is given by

$$f = \frac{v}{\lambda} = \frac{50}{2.88} = 17.36 \text{ Hz} \approx 17 \text{ Hz}$$

3. (a) $A(x) = (0.5 \text{ cm})\sin\left[(1.57 \text{ cm}^{-1})x\right]$

At $x = 5.66$ cm, $A(x) = 2.56$ cm

This is also the maximum displacement at $x = 5.66$ cm.

So, $A_{max} = 2.56$ mm

(b) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{1.57} = 4 \text{ cm}$

$$v = \frac{\omega}{k} = \frac{314}{1.57} = 200 \text{ cms}^{-1} = 2 \text{ ms}^{-1}$$

(c) $v_p = \dfrac{\partial y}{\partial t} = 157\sin(1.57x)\cos(314t)$

At, $x = 5.66$ cm and $t = 2$ sec, we get

$$v_P = 78.5 \text{ cms}^{-1}$$

(d) Nodes are the points where $A(x) = 0$

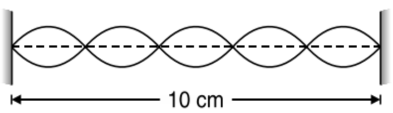
$\Rightarrow \quad x = 0, \dfrac{\lambda}{2}, \lambda, \dfrac{3\lambda}{2}, \ldots$

i.e., at $x = 0$, 2 cm, 4 cm,…, etc., we get NODES

At antinodes, $A(x)$ is a maximum

i.e., at $x = \dfrac{\lambda}{4}, \dfrac{3\lambda}{4}, \dfrac{5\lambda}{4}, \ldots$, etc.

i.e., at $x = 1$ cm, 3 cm, 5 cm,…, etc., we get ANTINODES



So, number of loops is $p = \dfrac{\ell}{\lambda/2} = \dfrac{10}{2} = 5$

4. (a) $f = \dfrac{v}{2\ell} = \dfrac{1}{2\ell}\sqrt{\dfrac{Y}{\rho}}$

If S be the cross sectional area of the wire, then, $\rho = \dfrac{\mu}{S}$

$\{\because \mu = \rho S\}$

$\Rightarrow \quad f = \dfrac{1}{2\ell}\sqrt{\dfrac{YS}{\mu}} = 500\sqrt{2}$ Hz

(b) $T = YA\alpha\Delta\theta = (2 \times 10^{11})(10^{-6})(1.21 \times 10^{-5})(20)$

$\Rightarrow \quad T = 48.4$ N

$\Rightarrow \quad f = \dfrac{v}{2\ell} = \dfrac{1}{2\ell}\sqrt{\dfrac{T}{\mu}} = 11$ Hz

5. For a specific tension, the fundamental frequency of vibration $f \propto \dfrac{1}{L}$. So, for having fundamental frequencies in the ratio of $1:3:4$, the vibrating lengths should be in the ratio

$L_1 : L_2 : L_3 = 1 : \dfrac{1}{3} : \dfrac{1}{4}$

$\Rightarrow \quad L_1 : L_2 : L_3 = 12 : 4 : 3$

Since $L_1 + L_2 + L_3 = 114$ cm, so

$114 = 12x + 4x + 3x = 19x$

$\Rightarrow \quad x = 6$

So, $L_1 = 12x = 72$ cm, $L_2 = 4x = 24$ cm, $L_3 = 3x = 18$ cm

6. Given that, $\mu = 0.05 \text{ gcm}^{-1} = 0.005 \text{ kgm}^{-1}$

So, $420 = n\left(\dfrac{v}{2\ell}\right)$ …(1)

and $490 = (n+1)\left(\dfrac{v}{2\ell}\right)$ …(2)

From Equations (1) and (2), we get $n = 6$

$\Rightarrow \quad 420 = 6\left(\dfrac{1}{2\ell}\sqrt{\dfrac{T}{\mu}}\right)$

$\Rightarrow \quad \ell = 6\left(\dfrac{1}{840}\sqrt{\dfrac{450}{0.005}}\right) = 2.142$ m

7. Given that, $\dfrac{A_{max}}{A_{min}} = \dfrac{A_i + A_r}{A_i - A_r} = \dfrac{3}{2}$

$\Rightarrow \quad 2A_i + 2A_r = 3A_i - 3A_r$

$\Rightarrow \quad 5A_r = A_i$

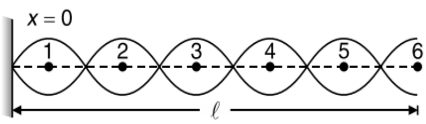
$\Rightarrow \quad \dfrac{A_r}{A_i} = \dfrac{1}{5}$

$\Rightarrow \quad \dfrac{I_r}{I_i} = \left(\dfrac{A_r}{A_i}\right)^2 = \left(\dfrac{1}{5}\right)^2 = \dfrac{1}{25}$

$\Rightarrow \quad I_r = 0.04 I_i$

So, 4% of the incident energy is reflected or 96% energy passes across the obstacle.

8. (a)



From diagram, $\ell = 5\left(\dfrac{\lambda}{2}\right) + \left(\dfrac{\lambda}{4}\right) = \dfrac{11}{4}\lambda$

$\Rightarrow \quad \lambda = \dfrac{4\ell}{11} = \dfrac{4 \times 1}{11} = 0.36$ m

(b) $v = \sqrt{\dfrac{Y}{\rho}} = \sqrt{\dfrac{6.4 \times 10^{10}}{5 \times 10^3}} = 3578 \text{ ms}^{-1}$

$\Rightarrow \quad f = \dfrac{v}{\lambda} = \dfrac{3578}{0.36} = 9939$ Hz

(c) Since, $A(x) = A\sin(kx)$

$$\Rightarrow \quad A(x=0.5) = (4\times10^{-6})\sin\left(\frac{2\pi}{0.36}\right)\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad A(x=0.5) = 2.57\times10^{-6}\ \text{m}$$

$$\Rightarrow \quad y|_{\text{at } x=0.5\text{ m}} = 2.57\times10^{-6}\sin(\omega t+\phi)$$

where, $\omega = 2\pi f = 6.24\times10^4\ \text{rads}^{-1}$

$$\Rightarrow \quad y = 2.57\times10^{-6}\sin(62400t+\phi)$$

9. (a) The equation of standing wave is

$$y = A\sin(kx-\omega t) + A\sin(kx+\omega t)$$

$$\Rightarrow \quad y = 2A\sin(kx)\cos(\omega t)$$

Particle velocity at location x is given by

$$v_P = \frac{\partial y}{\partial t} = -2A\omega\sin(kx)\cos(\omega t) \qquad \ldots(1)$$

The kinetic energy of an element dx is

$$dK = \frac{1}{2}(\mu dx)v_P^2 \qquad \ldots(2) \qquad \{\because dm = \mu dx\}$$

So, the kinetic energy contained between two nodes at any instant t is given by

$$K = \int_0^{\frac{\lambda}{2}} dK$$

Put $k = \frac{2\pi}{\lambda}$, $v = \frac{\omega}{k} = \sqrt{\frac{T}{\mu}}$ in (1) and then substitute (1) in (2), we get

$$K = \pi k A^2 T\sin^2(\omega t)$$

(b) $K_{max} = \pi k A^2 T$ when $\sin^2(\omega t) = 1$

(c) Since, $\langle\sin^2\omega t\rangle_{\text{one cycle}} = \frac{1}{2}$

$$\Rightarrow \quad \langle K\rangle_{\text{one cycle}} = \frac{\pi k A^2 T}{2}$$

10. For first and second wave respectively $\lambda_1 = 204$ cm and $\lambda_2 = 208$ cm. Let velocity of sound in gas be v cms^{-1} then frequencies of first and second wave are

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{204} \text{ and } f_2 = \frac{v}{\lambda_2} = \frac{v}{208}$$

Beat frequency is $f_b = \frac{20}{6}$ Hz

$$\Rightarrow \quad f_1 - f_2 = \frac{v}{204} - \frac{v}{208} = \frac{20}{6}$$

$$\Rightarrow \quad v = \frac{20}{6}\left(\frac{204\times208}{4}\right)\text{cms}^{-1} = 35360\ \text{cms}^{-1}$$

$$\Rightarrow \quad v = 353.6\ \text{ms}^{-1}$$

11. Fundamental frequency of an open pipe is

$$f_1 = \frac{v}{2\ell_0}$$

$$\Rightarrow \quad \ell_0 = \frac{v}{2f_1} = \frac{330}{2\times300} = 0.55\text{ m} = 55\text{ cm}$$

Since, $(f_{\text{closed}})_{\text{1st overtone}} = (f_{\text{open}})_{\text{1st overtone}}$

$$\Rightarrow \quad 3\left(\frac{v}{4\ell_c}\right) = 2\left(\frac{v}{2\ell_0}\right)$$

$$\Rightarrow \quad \ell_c = \frac{3\ell_0}{4} = \frac{3}{4}(0.55) = 0.4125\text{ m}$$

$$\Rightarrow \quad \ell_c = 41.25\text{ cm}$$

12. Fundamental frequency of closed pipe is

$$f_0 = \frac{v}{4\ell} = \frac{330}{4(1)} = 82.5\text{ Hz}$$

At resonance, fundamental frequency of stretched wire equals the frequency of the air column, so we have

$$\Rightarrow \quad \frac{1}{2l}\sqrt{\frac{T}{\mu}} = 82.5$$

$$\Rightarrow \quad T = \left(\frac{0.01}{0.3}\right)(2\times0.3\times82.5)^2$$

$$\Rightarrow \quad T = 81.675\text{ N}$$

13. Let ℓ_1 and ℓ_2 be the lengths of closed and open pipes respectively. Fundamental frequency of closed organ pipe $f_1 = \frac{v}{4\ell}$, where $v = 330$ ms^{-1}. Given that $f_1 = 110$ Hz

$$\Rightarrow \quad \frac{v}{4\ell_1} = 110\text{ Hz}$$

$$\Rightarrow \quad \ell_1 = \frac{v}{4\times110} = \frac{330}{4\times110}\text{ m} = 0.75\text{ m}$$

So, the first overtone of a closed organ pipe will be given by

$$f_3 = 3f_1 = 3(110)\text{ Hz} = 330\text{ Hz}$$

This produces a beat frequency of 2.2 Hz with first overtone of open organ pipe.
Therefore, first overtone frequency of open organ pipe is either

$(330+2.2)$ Hz $= 332.2$ Hz OR

$(330-2.2)$ Hz $= 327.8$ Hz

If it is 332.2 Hz, then $2\left(\frac{v}{2\ell_2}\right) = 332.2$ Hz

$$\Rightarrow \quad \ell_2 = \frac{v}{332.2} = \frac{330}{332.2}\text{ m} = 0.99\text{ m}$$

If it is 327.8 Hz, then $2\left(\frac{v}{2\ell_2}\right) = 327.8$ Hz

$$\Rightarrow \quad \ell_2 = \frac{v}{327.8}\text{ m} = \frac{330}{327.8}\text{ m} = 1.0067\text{ m}$$

So, the length of the closed organ pipe is $\ell_1 = 0.75$ m while length of open pipe is either $\ell_2 = 0.99$ m or 1.0067 m.

14. (a) Fundamental frequency when the pipe is open at both ends is

$$f_1 = \frac{v}{2\ell} = \frac{340}{2\times0.6} = 283.33\text{ Hz}$$

(b) Let the hole be uncovered at a length ℓ from the mouth-piece, then the fundamental frequency will be $f_1 = \frac{v}{2\ell}$

$\Rightarrow \quad \ell = \dfrac{v}{2f_1} = \dfrac{340}{2 \times 330}$

$\Rightarrow \quad \ell = 0.515 \text{ m}$

$\Rightarrow \quad \ell = 51.5 \text{ cm}$

Conceptual Note(s)

Opening holes at the sides effectively shortens the length of the resonance column, thus increasing the frequency.

15. Number of beats $f_b = |f_1 - f_2|$

$\Rightarrow \quad f_1 = f_2 + f_b$ or $f_1 = f_2 - f_b$

$\Rightarrow \quad f_1 = 364 + 3$ or $f_1 = 364 - 3$

$\Rightarrow \quad f_1 = 367 \text{ Hz}$ or $f_1 = 361 \text{ Hz}$

Loading a fork with wax decreases its frequency. On loading the first fork, number of beats produced per second decreases, so

$f_1 = 367 \text{ Hz}$

16. Frequency of fundamental mode of closed pipe is $f_1 = \dfrac{v}{4\ell} = 200 \text{ Hz}$

Since, decreasing the tension in string will decrease the beat frequency, so the first overtone frequency of the string should be 208 Hz (not 192 Hz).

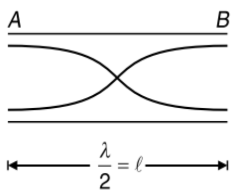
$\Rightarrow \quad 208 = \dfrac{1}{\ell}\sqrt{\dfrac{T}{\mu}}$

$\Rightarrow \quad T = (208\ell)^2 \mu = (208 \times 0.25)^2 \left(\dfrac{2.5 \times 10^{-3}}{0.25}\right)$

$\Rightarrow \quad T = 27.04 \text{ N}$

17. Since $\dfrac{\lambda}{2} = \ell = 0.8 \text{ m}$, so $\lambda = 1.6 \text{ m}$

$\Rightarrow \quad k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{1.6} = 3.93 \text{ m}^{-1}$



$\Rightarrow \quad \omega = vk = (330)(3.93) = 1297 \text{ s}^{-1} \qquad \left\{\because v = \dfrac{\omega}{k}\right\}$

$\Rightarrow \quad y = A\cos(kx)\sin(\omega t)$

$\Rightarrow \quad y = A\cos(3.93x)\sin(1297t)$

Test Your Concepts-VI (Based On Doppler's Effect)

1. Given $f_1 - f_2 = 3$

$\Rightarrow \quad \left(\dfrac{v}{v - v_s}\right)f - \left(\dfrac{v}{v + v_s}\right)f = 3$

$\Rightarrow \quad \left[\dfrac{1}{\left(1 - \dfrac{v_s}{v}\right)} - \dfrac{1}{\left(1 + \dfrac{v_s}{v}\right)}\right]f = 3$

$\Rightarrow \quad \left[\left(1 - \dfrac{v_s}{v}\right)^{-1} - \left(1 + \dfrac{v_s}{v}\right)^{-1}\right]f = 3$

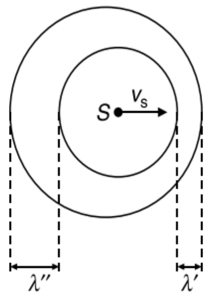
Since, difference in apparent frequencies is very small (3 Hz). So, we may conclude that speed of source $(v_s) \ll$ speed of sound (v). So, we can neglect higher terms of $\dfrac{v_s}{v}$.

$\Rightarrow \quad \left[\left(1 + \dfrac{v_s}{v}\right) - \left(1 - \dfrac{v_s}{v}\right)\right]f = 3$

$\Rightarrow \quad \dfrac{2v_s f}{v} = 3$

$\Rightarrow \quad v_s = \dfrac{3v}{2f} = \dfrac{(3)(340)}{(2)(340)} = 1.5 \text{ ms}^{-1}$

2. (a) Due to motion of the source, the wavelength is changed from λ to λ' (in the direction of v_s) and λ to λ'' (in the direction opposite to v_s), where



$\lambda' = \dfrac{v - v_s}{f} = \dfrac{332 - 32}{1000} = 0.3 \text{ m}$

and $\lambda'' = \dfrac{v + v_s}{f} = \dfrac{332 + 32}{1000} = 0.364 \text{ m}$

(b) The number of waves arriving at the reflecting surface is the same as the number of waves received by an observer moving towards the source, i.e.,

$f' = \left(\dfrac{v + v_0}{v - v_s}\right)f$

$\Rightarrow \quad f' = \left(\dfrac{332 + 64}{332 - 32}\right) \times 1000 = 1320 \text{ Hz}$

OR $f' = \dfrac{v + v_0}{\lambda'} = \dfrac{332 + 64}{0.3} = 1320 \text{ Hz}$

(c) Speed of a wave depends only on the characteristics of the medium. So, the speed of reflected wave is 332 ms^{-1}

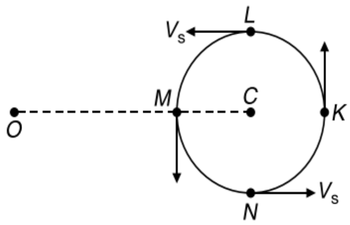
(d) Wavelength of the reflected wave is calculated in the similar manner as was calculated in part (a), i.e.,

$\lambda_r = \dfrac{v - v_0}{f'} = \dfrac{332 - 64}{1320} \approx 0.2 \text{ m}$

3. Apparent frequency will be minimum when the source is at N and moving away from the observer

$f_{\min} = \left(\dfrac{v}{v + v_s}\right)f = \left(\dfrac{330}{330 + 30}\right)(540)$

$\Rightarrow \quad f_{\min} = 495 \text{ Hz}$



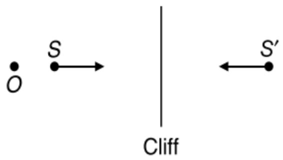
Frequency will be maximum when source is at L and approaching the observer

$$f_{\max} = \left(\frac{v}{v - v_s}\right) f = \left(\frac{330}{330 - 30}\right)(540)$$

$\Rightarrow \quad f_{\max} = 594$ Hz

Further when source is at M and K, angle between velocity of source and line joining source and observer is 90°, so $v_s \cos\theta = v_s \cos(90°) = 0$. Hence, there will be no change in the apparent frequency.

4. The situation is as shown in figure.



(a) Frequency of sound reaching directly to us (by S)

$$f_1 = \left(\frac{v}{v + v_s}\right) f = \left(\frac{330}{330 + 10}\right)(1000)$$

$\Rightarrow \quad f_1 = 970.6$ Hz

(b) Frequency of sound which is reflected off from the cliff (from S')

$$f_2 = \left(\frac{v}{v - v_s}\right) f = \left(\frac{330}{330 - 10}\right)(1000)$$

$\Rightarrow \quad f_2 = 1031.3$ Hz

(c) Beat frequency, $f_b = f_2 - f_1$

$\Rightarrow \quad f_b = 60.7$ Hz

5. (a) Initially the velocity of source is zero, so

$$\lambda_i = \frac{v}{f} = \frac{350}{1000} = 0.35 \text{ m}$$

(b) Instantaneous velocity of the source

$v_s = at = 10t$

$$\Rightarrow \quad f' = \left(\frac{v}{v - v_s}\right) f = \left(\frac{350}{350 - 10t}\right)(1000)$$

The average frequency over a time $t_0 = 1$ sec is given by

$$\langle f' \rangle = \frac{1}{t_0}\int_0^{t_0} f' dt = 1014.5 \text{ Hz}$$

6. Assuming that the airplane is approaching the observer with velocity v_a, then velocity of image of source from which the reflected wave is being produced is

$$v_s = 2v_a \qquad \ldots(1)$$

Just similar to the case of plane mirror, where if the mirror moves with a velocity v, then the image moves with a velocity $2v$. In this case,

$$f' = f\left(\frac{c}{c - v_s}\right) = f\left(1 - \frac{v_s}{c}\right)^{-1} \approx f\left(1 + \frac{v_s}{c}\right)$$

$$\Rightarrow \quad f' = f + f\left(\frac{v_s}{c}\right)$$

Since, $f' - f = 2.6 \times 10^3$ Hz, $n = 7.8 \times 10^9$ Hz and $c = 3 \times 10^8$ ms^{-1}, so we get

$$2.6 \times 10^3 = 7.8 \times 10^9 \left(\frac{v_s}{3 \times 10^8}\right) = 26 v_s$$

$\Rightarrow \quad v_s = 100$ ms^{-1}

Using equation (1), we get

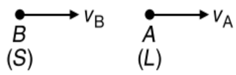
$$v_a = \frac{v_s}{2} = 25 \text{ ms}^{-1} = 180 \text{ kmh}^{-1}$$

7. At $t = 10$ sec, velocity of A is given by

$v_A = u_A + a_A t$

$\Rightarrow \quad v_A = (2) + (2)(10)$

$\Rightarrow \quad v_A = 22$ ms^{-1}



Here B is the source and A is the listener. Hence

$$f' = f\left(\frac{v - v_A}{v - v_B}\right)$$

$$\Rightarrow \quad f = \left(\frac{v - v_B}{v - v_A}\right) f'$$

$$\Rightarrow \quad f = \left(\frac{330 - 2}{330 - 22}\right) \times 352$$

$\Rightarrow \quad f = 374.8$ Hz

8. (a) $$\lambda' = \left(\frac{v + w}{f}\right) = \left(\frac{v + w}{v}\right)\lambda$$

$$\Rightarrow \quad \frac{\lambda' - \lambda}{\lambda} = \frac{w}{v}$$

So, percentage increase in wavelength is

$$\frac{w}{v} \times 100 = 10\%$$

(b) There will be no change in the frequency.

(c) $$f' = \left(\frac{v + v_0}{v}\right) f$$

$\Rightarrow \quad v_0 = 0.1v$

$\Rightarrow \quad f' = 1.1f$

So, the percentage increase in frequency is 10%

9. Since, $f = \dfrac{1}{2\ell}\sqrt{\dfrac{T}{\mu}} = \dfrac{1}{2 \times 0.1}\sqrt{\dfrac{64}{\left(\dfrac{10^{-3}}{0.1}\right)}} = 400$ Hz

CHAPTER 4

Also, $f' = \left(\frac{v}{v+v_s}\right)f$ where $v = 300\ \text{ms}^{-1}$

$\Rightarrow\ f' < f$

Since, $f - f' = 1$ Hz

$\Rightarrow\ 400 - \left(\frac{300}{300+v_s}\right)400 = 1$

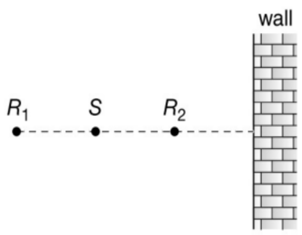
Solving this we get

$v_s = 0.752\ \text{ms}^{-1}$

10. $f_b = \left(\frac{v}{v-v_s}\right)f - \left(\frac{v}{v+v_s}\right)f$

$\Rightarrow\ f_b = \left(\frac{340}{340-0.17} - \frac{340}{340+0.17}\right)1000$ Hz

$\Rightarrow\ f_b = 1$ Hz



There are two cases possible.

Case 1: When S moves towards wall.

Case 2: When S moves away from the wall.

But in both the cases only R_1 will register the beats.

11. Since $v = \left(\frac{\Delta\lambda}{\lambda}\right)c$

$\Rightarrow\ v = \left(\frac{0.5}{100}\right)(3\times10^8) = 1.5\times10^6\ \text{ms}^{-1}$

12. Maximum value of the listener (v_L) or the source (v_s) is given by

$v_L = v_s = A\omega = 4\pi\ \text{ms}^{-1}$

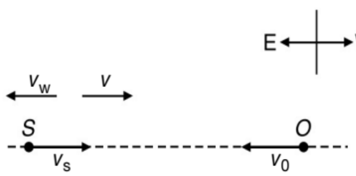
So, $f_{min} = f_{recede} = \left(\frac{340-4\pi}{340+4\pi}\right)(300)$

$f_{min} = 278.62$ Hz

and $f_{max} = f_{approach} = \left(\frac{340+4\pi}{340-4\pi}\right)(300)$

$f_{max} = 323$ Hz

13. Normally, we take east direction pointing to the right and west to the left. However, to be able to determine the sign of different velocities in the most general formula for Doppler's effect, we shall take east on the left, so that the source (i.e., the engine) is on the left of the observer. The directions of all the velocities are marked in the Figure.



The most general formulae to determine apparent frequency is

$$f' = f\left(\frac{v+v_w-v_0}{v+v_w-v_s}\right)$$

Here, $f = 1200$ Hz, $v = 330\ \text{ms}^{-1}$,

$v_0 = 72\ \text{kmh}^{-1} = 20\ \text{ms}^{-1}$

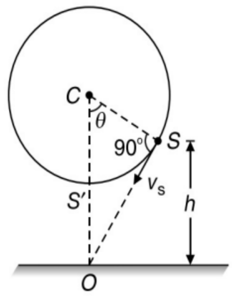
$\Rightarrow\ f = 1200\left(\frac{330+(-10)-(-15)}{330+(-10)-20}\right)$

$\Rightarrow\ f = 1200\left(\frac{330-10+15}{330-10-20}\right)$

$\Rightarrow\ f = \frac{1200\times335}{300} = 1340$ Hz

14. Let the speed of the plane (source) be v_s. The maximum frequency is observed by the observer when v_s is along SO. The observer receives maximum frequency when the plane is nearest to him. That is as soon as the wave pulse reaches from S to O with speed v the plane reaches from S to S' with speed v_s.

Hence, $t = \frac{SO}{v} = \frac{SS'}{v_s}$



$\Rightarrow\ v_s = \left(\frac{SS'}{SO}\right)v$

$\Rightarrow\ v_s = \frac{R\theta}{\sqrt{h^2-R^2}}v$

where $\cos\theta = \frac{R}{h}$ i.e., $\theta = \cos^{-1}\left(\frac{R}{h}\right)$

$\Rightarrow\ v_s = \frac{Rv\cos^{-1}(R/h)}{\sqrt{h^2-R^2}}$

Single Correct Choice Type Questions

1. At $t = 0$, $x = 5$ cm, initial phase of insect

$\phi_1 = (20\pi)(5) - (50\pi)(0) = 100\pi$

After 5 s insect will most a distance $a = vt = 25$ cm

So, it will be at $x = 30$ cm

Phase at this instant is

$\phi_i = (20\pi)(30) - (50\pi)(5) = 350\pi$

$\Rightarrow\ \Delta\phi = \phi_f - \phi_i = 250\pi$

Hence, the correct answer is (B).

2. $f - 5 \propto \sqrt{100}$ $\qquad\{\because f \propto \sqrt{T}\}$

$f + 5 \propto \sqrt{121}$

$\Rightarrow\ \frac{f+5}{f-5} = \frac{11}{10}$

$\Rightarrow\ 10f + 50 = 11f - 55$

$\Rightarrow\ f = 105$ Hz

Hence, the correct answer is (A).

3. $\nu_A = \frac{v}{4\ell} = \frac{v}{0\cdot 60}$

$\nu_B = \frac{v}{2\ell} = \frac{v}{0\cdot 61}$

Since, $\nu_A - \nu_B = 5$

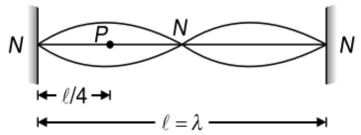
$\Rightarrow \frac{v}{0\cdot 60} - \frac{v}{0\cdot 61} = 5$

$\Rightarrow v = 183 \text{ ms}^{-1}$

$\Rightarrow \nu_A = \frac{183}{0\cdot 6} = 305 \text{ Hz and } \nu_B = 300 \text{ Hz}$

Hence, the correct answer is (C).

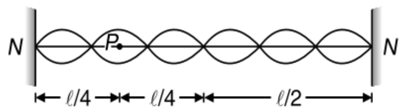
4. In the first case



Point P is an antinode i.e., the string is vibrating in its second harmonic. Let f_0 be the fundamental frequency. Then $2f_0 = 100$ Hz

$\Rightarrow f_0 = 50$ Hz

Now P is an antinode (at length $\ell/4$ from one end) so centre should be a node. So, next higher frequency will be sixth harmonic or $6f_0$ which is equal to 300 Hz as shown below:



Hence, the correct answer is (D).

5. Frequency $f \propto \sqrt{mg}$, so $f \propto \sqrt{g}$

In water $f_w = 0.8 f_{\text{air}}$

$\Rightarrow \frac{g'}{g} = (0.8)^2 = 0.64$

$\Rightarrow 1 - \frac{\rho_w}{\rho_m} = 0.64$

$\Rightarrow \frac{\rho_w}{\rho_m} = 0.36$ ...(1)

where, ρ_w is relative density of water i.e. 1 and ρ_m is relative density of mass

In liquid, $\frac{g'}{g} = (0.6)^2 = 0.36$

$\Rightarrow 1 - \frac{\rho_L}{\rho_m} = 0.36$

$\Rightarrow \frac{\rho_L}{\rho_m} = 0.64$ ...(2)

where, ρ_L is relative density of liquid.

From equations (1) and (2), we get

$\frac{\rho_L}{\rho_w} = \frac{0.64}{0.36}$

$\Rightarrow \rho_L = 1.77$ $\{\because \rho_w = 1\}$

Hence, the correct answer is (B).

6. For pipe P_1, $3f_c = 3\left(\frac{v}{4\ell_c}\right)$

For pipe P_2, $4f_0 = 4\left(\frac{v}{2\ell_0}\right)$

where, ℓ_c and ℓ_0 are lengths of closed and open organ pipe respectively. Since both are in resonance, so

$3\left(\frac{v}{4\ell_c}\right) = 4\left(\frac{v}{2\ell_0}\right)$

$\Rightarrow \frac{\ell_c}{\ell_0} = \frac{3}{8}$

Hence, the correct answer is (B).

7. Since frequency remains constant, so $v \propto \lambda$

$\frac{\lambda_t}{\lambda_0} = \frac{v_t}{v_0}$

$\Rightarrow \lambda_t = \sqrt{\frac{t+273}{273}}\lambda_0 = \left(1 + \frac{t}{273}\right)^{\frac{1}{2}} \lambda_0$

For $-1 < x < 1$, we have $(1+x)^n \approx 1 + nx$

$\Rightarrow \lambda_t \approx \left(1 + \frac{t}{546}\right)\lambda_0 = \left(1 + \frac{25}{546}\right)110 = 115 \text{ cm}$

Hence, the correct answer is (B).

8. Since, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{35}{3.5 \times 10^3}} = 100 \text{ ms}^{-1}$, so $\lambda = \frac{v}{f} = 1$ m

$\Rightarrow k = \frac{2\pi}{\lambda} = 2\pi \text{ radm}^{-1}$

$\Rightarrow \omega = vk = 200\pi \text{ rads}^{-1}$

At $x = 0$, $v_p = \text{maximum} = A\omega$

Since, $A\omega = (\text{Wave Velocity})(\text{Slope})$

$\Rightarrow A = \frac{\text{slope} \times \text{wave velocity}}{\omega} = \frac{(\pi/20) \times (100)}{200\pi}$

$\Rightarrow A = 0.025$ m

Hence, the correct answer is (D).

9. Since, $f = f_0\left(\frac{v + v_0}{v}\right)$

$\Rightarrow f = f_0\left(\frac{v + at}{v}\right) = f_0 + \left(\frac{f_0 a}{v}\right)t$

This is an equation of a straight line with positive intercept f_0 and positive slope $\frac{f_0 a}{v}$, where v is the speed of sound in air.

Hence, the correct answer is (A).

10. $y(x,t) = \frac{a}{(x \pm vt)^2 + b}$ is another form of progressive wave equation propagating with a speed v. Negative sign to be taken for propagation along $+x$-axis and positive sign to be taken for propagation along $-x$-axis.

Hence, the correct answer is (D).

CHAPTER 4

11. Let, Δx be the path difference, then

$$\Delta x = L_2P - L_1P$$

$$\Rightarrow \quad \Delta x = \sqrt{40^2 + 9^2} - 40 = 41 - 40 = 1 \text{ m}$$

For first maximum $\Delta x = (2n)\dfrac{\lambda}{2}$, where $n = 1$

$$\Rightarrow \quad 1 = 2(1)\left(\frac{\lambda}{2}\right)$$

$$\Rightarrow \quad \lambda = 1 \text{ m}$$

$$\Rightarrow \quad f = \frac{v}{\lambda} = 330 \text{ Hz}$$

Hence, the correct answer is (B).

12. Since, $\lambda = \dfrac{330}{500}$ m

So, first resonance length is

$$\ell_1 = \frac{\lambda}{4} = \frac{33}{200} \text{ m}$$

Second resonance length is

$$\ell_2 = \frac{3\lambda}{4} = \frac{66}{200} \text{ m}$$

Third resonance length is

$$\ell_3 = \frac{5\lambda}{4} = \frac{33}{40} \text{ m}$$

Fourth resonance length is

$$\ell_4 = \frac{7\lambda}{4} = \frac{231}{200} \text{ m}$$

which is greater than 1 m. Hence only three resonances are obtained.

Hence, the correct answer is (D).

13. Since $v = \sqrt{\dfrac{T}{\mu}}$, where T is tension in string, μ is mass per unit length i.e., $\mu = A\rho$, where A is the area of cross section of the wire.

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{Al\rho/l}} = \sqrt{\frac{T}{A\rho}}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{T_1A_2}{T_2A_1}} = \sqrt{4} = 2$$

Hence, the correct answer is (B).

14. Since, $f = \left(\dfrac{v + v_0}{v}\right)f_0 = f_0 + \dfrac{f_0v_0}{v}$, where $v_0 = gt$

$$\Rightarrow \quad f = f_0 + \left(\frac{f_0g}{v}\right)t$$

i.e., f-t graph is a straight line of slope $\dfrac{f_0g}{v}$

$$\Rightarrow \quad \frac{f_0g}{v} = \text{slope}$$

$$\Rightarrow \quad v = \frac{f_0g}{\text{slope}} = \frac{(10^3)(10)}{10^3/30} = 300 \text{ ms}^{-1}$$

Hence, the correct answer is (C).

15. According to Snell's Law of Refraction

$$\frac{v_1}{v_2} = \frac{\sin\phi}{\sin\phi}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{BD/AD}{AC/AD} = \frac{BD}{AC}$$

Hence, the correct answer is (C).

16. Since, $f_0 - f_c = 2$

$$\Rightarrow \quad \frac{v}{2\ell} - \frac{v}{4\ell} = \frac{v}{4l} = 2$$

$$\Rightarrow \quad \frac{v}{\ell} = 8 \qquad \ldots(1)$$

When length of open organ pipe is halved and that of closed organ pipe is doubled, beat frequency will be

$$f_0' - f_c' = \frac{v}{\ell} - \frac{v}{8\ell} = \frac{7}{8}\frac{v}{\ell}$$

Substituting $\dfrac{v}{\ell} = 8$ from equation (1), we get

$$f_b = 7$$

Hence, the correct answer is (B).

17. For Listener in the front

$$f' = f\left(\frac{v}{v - v_s}\right)$$

$$\Rightarrow \quad \lambda' = \frac{v}{f'} = \left(\frac{v - v_s}{f}\right) = \frac{345 - 30}{500} = 0.63 \text{ m}$$

For Listener at the back

$$f' = f\left(\frac{v}{v + v_s}\right)$$

$$\Rightarrow \quad \lambda' = \frac{v}{f'} = \frac{v + v_s}{f} = \frac{345 + 30}{500} = 0.75 \text{ m}$$

Hence, the correct answer is (B).

18. In front of the locomotive, effective value of velocity of sound is $v' = v + v_w$

So, $f' = f\left[\dfrac{(v + v_w) - 0}{(v + v_w) - v_s}\right]$

$$\Rightarrow \quad \lambda' = \frac{(345 + 10) - 30}{500} \qquad \left\{\because \lambda' = \frac{v'}{f'} = \frac{v + v_w}{f'}\right\}$$

$$\Rightarrow \quad \lambda' = \frac{325}{500} = 0.65 \text{ m}$$

At the back of the train the effective value of velocity of sound is $v' = v - v_w$

$$\Rightarrow \quad f' = f\left[\frac{v - v_w}{(v - v_w) + v_s}\right]$$

$$\Rightarrow \quad \lambda' = \left[\frac{(345 - 10) + 30}{500}\right] \qquad \left\{\because \lambda' = \frac{v'}{f'} = \frac{v - v_w}{f'}\right\}$$

$$\Rightarrow \quad \lambda' = \frac{365}{500} = 0.73 \text{ m}$$

Hence, the correct answer is (A).

19. The given pulse is of the form

$$y=\frac{a}{b+(x-vt)^2} \qquad \ldots(1)$$

where v is the wave velocity
Given equation is

$$y=\frac{1}{1+(x-1)^2} \text{ at } t=2\text{ s} \qquad \ldots(2)$$

Comparing equations (1) and (2), we get

$$vt=1$$

$$\Rightarrow \quad v=\frac{1}{t}=\frac{1}{2}=0.5\text{ ms}^{-1}$$

Hence, the correct answer is (B).

20. Standard form of a sine wave is

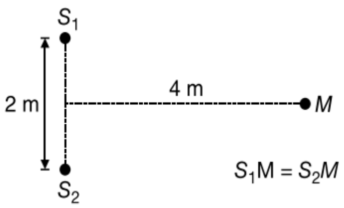
$$y=A\sin(\omega t-kx) \qquad \ldots(1)$$

Here, $\omega=\dfrac{2\pi}{T}$, $k=\dfrac{2\pi}{\lambda}$

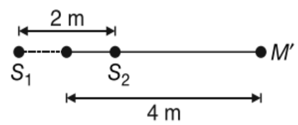
and $v=\dfrac{\omega}{k}=\dfrac{\lambda}{T}$

With these substitutions we can see that equation (1) can be re-written as OPTIONS (A), (C) and (D).
Hence, the correct answer is (B).

21. Path Difference $=S_1M'-S_2M'$



When the box is rotated



$$\Rightarrow \quad \Delta x=5-3=2\,m$$

For MAXIMA

Path Difference $=(\text{Even multiple})\dfrac{\lambda}{2}$

$$\Delta x=(2n)\frac{\lambda}{2}$$

For 5 maximum responses

$$\Rightarrow \quad 2=2(5)\frac{\lambda}{2} \qquad \left\{\because \Delta x=(2n)\frac{\lambda}{2}\right\}$$

$$\Rightarrow \quad \lambda=\frac{2}{5}=0.4\text{ m}$$

Hence, the correct answer is (B).

22. Since, $\omega_1=1000\pi$ and $\omega_2=1008\pi$

$$\Rightarrow \quad f_1=500\text{ Hz and } f_2=504\text{ Hz}$$

$$\Rightarrow \quad f_b=f_2-f_1=4\text{ Hz}$$

Hence, the correct answer is (A).

23. Since, $P=\dfrac{1}{2}\rho\omega^2A^2sv$ and $\dfrac{\lambda_1}{\lambda_2}=\dfrac{1}{2}$

$$\Rightarrow \quad \frac{f_1}{f_2}=\frac{\omega_1}{\omega_2}=\frac{\lambda_2}{\lambda_1}=\frac{2}{1} \qquad \left\{\because f\propto\frac{1}{\lambda}\right\}$$

Given that $P_1=P_2$, so $\omega_1A_1=\omega_2A_2$

$$\Rightarrow \quad \frac{A_1}{A_2}=\frac{\omega_2}{\omega_1}=\frac{1}{2}$$

Now pressure amplitude

$$P_0=B_0Ak$$

$$\Rightarrow \quad \frac{(P_0)_1}{(P_0)_2}=\left(\frac{A_1}{A_2}\right)\left(\frac{k_1}{k_2}\right)=\left(\frac{A_1}{A_2}\right)\left(\frac{\lambda_2}{\lambda_1}\right)=\left(\frac{1}{2}\right)\left(\frac{2}{1}\right)=1$$

Hence, the correct answer is (A).

24. Since $f=\dfrac{100}{60}$ Hz, so $T=\dfrac{60}{100}$ s

$$\Rightarrow \quad d=\frac{vT}{2}=\frac{330(60/100)}{2}=99\text{ m}$$

Hence, the correct answer is (B).

25. At $t=0$, $x=0$ displacement $y=0$. Therefore, OPTIONS (A) or (B) may be correct. Secondly slope at $x=0$ at $t=0$ is positive i.e., particle velocity is in negative y-direction because particle velocity is $v_p=-v(\partial y/\partial x)$. So, particle at $x=0$ is travelling in negative y-direction.
Hence, the correct answer is (B).

26. $f'=f\left[\dfrac{v-(-v_s)}{v-v_s}\right]=f\left(\dfrac{v+v_s}{v-v_s}\right)$

$$\Rightarrow \quad f'=1000\left(\frac{340+15}{340-15}\right)=1092\text{ Hz}$$

Hence, the correct answer is (C).

27. Initially frequency of vibrations of closed organ pipe is 10 kHz.

$$f=\frac{v}{4\ell}=\frac{\sqrt{\gamma RT/M}}{4\ell}$$

$$\Rightarrow \quad f\propto\sqrt{T}$$

$$\frac{f'}{f}=\sqrt{\frac{T'}{T}}=\left(1+\frac{1}{300}\right)^{1/2}\approx 1+\frac{1}{600}$$

$$\Rightarrow \quad f'=f\left(1+\frac{1}{600}\right)$$

$$\Rightarrow \quad \Delta f=f'-f=\frac{f}{600}$$

$$\Rightarrow \quad \Delta f=\frac{10\times10^3}{600}=16.67\text{ Hz}$$

So, number of beats produced $=16.67$ Hz.
Hence, the correct answer is (C).

28. According to Hook's Law, $F=T=k\Delta x$

$$\Rightarrow \quad v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{k(2l-l)}{M/3l}}=\sqrt{\frac{kl}{M/3l}}$$

Also, $v' = \sqrt{\dfrac{k(4l)}{M/6l}} = 2\sqrt{2}v$

Hence, the correct answer is (D).

29. Power (which is directly proportional to intensity) is given by

$$P = \frac{1}{2}\mu\omega^2 A^2 v$$

where, μ is mass per unit length, A is amplitude, v is wave speed.

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{1}{36} = \frac{A_1^2\omega_1^2}{A_2^2\omega_2^2}$$

$$\Rightarrow \quad \frac{1}{36} = \frac{A_1^2}{A_2^2}\frac{1}{4}$$

$$\Rightarrow \quad \frac{A_1}{A_2} = \frac{1}{3}$$

Hence, the correct answer is (A).

30. For not hearing the echo the time interval between the beats of drum must be equal to time of echo.

$$\Rightarrow \quad t_1 = \frac{2d}{v} = \frac{60}{40} \qquad \ldots(1)$$

and $t_2 = \dfrac{2(d-90)}{v} = 1$

$$\Rightarrow \quad 2d - 180 = v \qquad \ldots(2)$$

From (1), we get

$$2d = \frac{3}{2}v.$$

Substituting in (2), we get

$$\Rightarrow \quad \frac{3}{2}v - 180 = v$$

$$\Rightarrow \quad 180 = \frac{v}{2}$$

$$\Rightarrow \quad v = 360 \text{ ms}^{-1}$$

$$\Rightarrow \quad \frac{2(d)}{360} = \frac{3}{2}$$

$$\Rightarrow \quad d = 270 \text{ m}$$

Hence, the correct answer is (D).

31. $y_1 = 4\sin[2\pi(200)t]$ and $y_2 = 3\sin[2\pi(202)t]$

Number of beats per second = 2 bps

$$\Rightarrow \quad \frac{I_{max}}{I_{min}} = \left(\frac{4+3}{4-3}\right)^2 = 49$$

Hence, the correct answer is (B).

32. $y_1 = a_1\sin\left(\omega t - \dfrac{2\pi x}{\lambda}\right)$

$y_2 = a_2\sin\left(\omega t - \dfrac{2\pi x}{\lambda} + \phi + \dfrac{\pi}{2}\right) \qquad \{\because \sin(90+\theta) = \cos\theta\}$

$$\Rightarrow \quad \Delta\phi = \phi + \frac{\pi}{2}$$

Since, $\Delta\phi = \dfrac{2\pi}{\lambda}\Delta x$

$$\Rightarrow \quad \Delta x = \frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$$

Hence, the correct answer is (B).

33. Since, $f' = f\left(\dfrac{c-v}{c+v}\right)$

$$\Rightarrow \quad f' - f = \left(\frac{2v}{c-v}\right)f$$

where, c is velocity of waves emitted by radar i.e. c is velocity of light.

$$\Rightarrow \quad f' - f \cong \frac{2v}{c}f \qquad \{\because c - v \simeq c\}$$

$$\Rightarrow \quad 2\cdot 6\times 10^3 = \frac{2v}{3\times 10^8}\times 780\times 10^6$$

$$\Rightarrow \quad v = \frac{(3\times 10^8)(2.6\times 10^3)}{2\times 780\times 10^6} = 0.5\times 10^3 \text{ ms}^{-1} = 0.5 \text{ kms}^{-1}$$

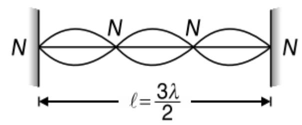
Hence, the correct answer is (B).

34. Since, $\ell = \dfrac{3\lambda}{2}$

$$\Rightarrow \quad \lambda = \frac{2\ell}{3} = \frac{(2)(0.6)}{3} \text{ m} = 0.4 \text{ m}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{80}{0.2}} = 20 \text{ ms}^{-1}$$

$$\Rightarrow \quad f = \frac{v}{\lambda} = \frac{20}{0.4} \text{ Hz} = 50 \text{ Hz}$$



Since, $v_{max} = (a_{max})\omega$

$$\Rightarrow \quad v_{max} = (0.5\times 10^{-2} \text{ m})(2\pi)(50) \text{ ms}^{-1} = \frac{\pi}{2} \text{ ms}^{-1}$$

$$\Rightarrow \quad v_{max} = 1.57 \text{ ms}^{-1}$$

Hence, the correct answer is (A).

35. Since, $f = \dfrac{c}{4\ell}$

$$\Rightarrow \quad \left|\frac{df}{dt}\right| = \frac{c}{4\ell^2}\left|\frac{d\ell}{dt}\right| = \frac{cv}{4\ell^2}$$

Hence, the correct answer is (B).

36. Since, $f = n\left(\dfrac{1}{2l}\sqrt{\dfrac{T}{\mu}}\right) = 4\left[\dfrac{1}{2(0.8)}\sqrt{\dfrac{0.4\times 9\cdot 8}{10^{-3}/0.8}}\right]$

$$\Rightarrow \quad f = \frac{40}{16}\sqrt{4\times 98\times 8} = \frac{40}{16}\times 4\times 14 = 160 \text{ Hz}$$

Hence, the correct answer is (C).

37. Since, $f = n\left(\dfrac{1}{2\ell}\sqrt{\dfrac{T}{\mu}}\right)$

$$\Rightarrow \quad Tn^2 = \text{constant}$$

where n is the number of loops

$$\Rightarrow \quad (0.4)(16) = (25)T$$

$\Rightarrow \quad T=(0\cdot4)\left(\frac{4}{5}\right)^2$

Hence, the correct answer is (B).

38. Velocity of longitudinal wave $v_1=\sqrt{\frac{Y}{\rho}}$ and velocity of transverse wave is $v_2=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{T}{\rho S}}$

$$\frac{v_1}{v_2}=\sqrt{\frac{YS}{T}}=\sqrt{\frac{Y}{T/S}}=\sqrt{\frac{Y}{Y(\Delta l/l)}}=\sqrt{\frac{1}{1/n}}=\sqrt{n}$$

Since, $f\propto v$, so $\frac{f_1}{f_2}=\frac{v_1}{v_2}=\sqrt{n}$

In the above equations ρ is density of string, S is area of cross-section of string, Y is Young's modulus of elasticity, so $\frac{T}{S}=\text{Stress}=Y(\text{Strain})=Y\left(\frac{\Delta l}{l}\right)$

Hence, the correct answer is (C).

39. At $x_1=\frac{\pi}{3k}$ and $x_2=\frac{3\pi}{2k}$

$\Rightarrow \quad \sin kx_1$ is not zero and $\sin kx_2$ is not zero.

Therefore, neither of x_1 or x_2 is a node

$\Rightarrow \quad \Delta x=x_2-x_1=\left(\frac{3}{2}-\frac{1}{3}\right)\frac{\pi}{k}=\frac{7\pi}{6k}$

Since $\frac{\pi}{k}<\Delta x<\frac{2\pi}{k}$, so $\frac{\lambda}{2}<\Delta x<\lambda$ $\quad \left\{\because k=\frac{2\pi}{\lambda}\right\}$

Therefore $\phi_1=\pi$ and $\phi_2=k\Delta x=\frac{7\pi}{6}$

$\Rightarrow \quad \frac{\phi_1}{\phi_2}=\frac{6}{7}$

Hence, the correct answer is (D).

40. Since, $f\propto\sqrt{T}$

$\Rightarrow \quad \frac{f_{\text{air}}}{f_{\text{water}}}=\sqrt{\frac{w_{\text{air}}}{w_{\text{water}}}}=\sqrt{\frac{V\rho g}{V\rho g-V\rho_w g}}$

$\Rightarrow \quad \frac{f}{f/2}=2=\sqrt{\frac{\rho}{\rho-\rho_w}}$

$\Rightarrow \quad 4\rho-4\rho_w=\rho$

$\Rightarrow \quad \rho=\frac{4}{3}\rho_w \quad \ldots(1)$

Similarly, in second case

$\frac{f}{f/3}=\sqrt{\frac{\rho}{\rho-\rho_L}}$

$\Rightarrow \quad 3=\sqrt{\frac{\frac{4}{3}\rho_w}{\frac{4}{3}\rho_w-\rho_L}}=\sqrt{\frac{4}{4-3\frac{\rho_L}{\rho_w}}}$

where, $\frac{\rho_L}{\rho_w}=$ specific gravity (say s)

$\Rightarrow \quad 9=\frac{4}{4-3s}$

$\Rightarrow \quad 36-27s=4$

$\Rightarrow \quad s=\frac{32}{27}$

Hence, the correct answer is (D).

41. From the given equation wave number

$k=\frac{2\pi}{\lambda}=(10\pi)(0.01)$

$\Rightarrow \quad k=0.1\pi\, m^{-1}$

Since, phase difference $\Delta\phi=\left(\frac{2\pi}{\lambda}\right)\Delta x$

$\Rightarrow \quad \Delta\phi=(0.1\pi)(10)=\pi$

Hence, the correct answer is (B).

42. $\frac{v_{H_2}}{v_{CO_2}}=\sqrt{\frac{\gamma_{H_2}M_{CO_2}}{\gamma_{CO_2}M_{H_2}}}=\sqrt{\frac{(7/5)(44)}{(9/7)(2)}}=\sqrt{\frac{49}{45}\times22}$

Hence, the correct answer is (A).

43. Since, 45 cm $=5(9\text{ cm})$ and 99 cm $=11(9\text{ cm})$

So, two other length between these two values are $7(9\text{ cm})$ and $9(9\text{ cm})$ i.e. 63 cm and 81 cm respectively. So, the fundamental length is 9 cm

$\Rightarrow \quad 9=\frac{\lambda}{4}$ $\quad$ {for a closed organ pipe}

$\Rightarrow \quad \lambda=36$ cm

Hence, the correct answer is (D).

44. Since, $f_{\text{approach}}=\left(\frac{v}{v-v_s}\right)f_0=f_1$ and

$f_{\text{recede}}=\left(\frac{v}{v+v_s}\right)f_0=f_2(<f_1)$

Hence, the correct answer is (B).

45. On getting reflected from a rigid boundary the wave suffers an additional phase change of π (According to Stokes Law)

$\Rightarrow \quad y=0.8A\sin(\omega t+kx+\pi)$

$\Rightarrow \quad y=-0.8A\sin(\omega t+kx)$

Hence, the correct answer is (B).

46. $y_r=-0.8A\sin(\omega t+kx)$

Hence, the correct answer is (C).

47. For change of pressure, velocity of sound remains the same

$\Rightarrow \quad \frac{v_{30}}{v_{15}}=\frac{v_{30}}{340}=\sqrt{\frac{303}{288}}$

$\Rightarrow \quad v_{30}=340\sqrt{\frac{303}{288}}$

Hence, the correct answer is (A).

48. Since $\frac{I_2}{I_1}=2$ and $L_2-L_1=10\log_{10}\left(\frac{I_2}{I_1}\right)$

$\Rightarrow \quad L_2-L_1=10\log_{10}2=3$ dB $\quad \{\because \log_e 2\cong 0\cdot3\}$

Hence, the correct answer is (C).

CHAPTER 4

49. Let equation of a plane progressive harmonic wave be

$$y = A\sin(\omega t - kx)$$

Wave speed is $v = \frac{\omega}{k}$, maximum particle speed is $v_p = A\omega$.

Given that $v_p < v$

$\Rightarrow \quad A\omega < \frac{\omega}{k}$

$\Rightarrow \quad A < \frac{1}{k}$

$\Rightarrow \quad A < \frac{\lambda}{2\pi}$

Hence, the correct answer is (A).

50. Since, $\frac{f_{\text{approach}} - f_{\text{recede}}}{f} = 2\%$

$\Rightarrow \quad \frac{2vv_s}{v^2 - v_s^2} = 0.02$

$v^2 >> v_s^2$

$\Rightarrow \quad \frac{2vv_s}{v^2} = 0.02$

$\Rightarrow \quad 2\frac{v_s}{v} = 0.02$

$\Rightarrow \quad 2v_s = (0.02)(300) = 3 \text{ ms}^{-1}$

Hence, the correct answer is (D).

51. This problem is a Doppler effect analogy where $f = 20 \text{ min}^{-1}$, $v = 300 \text{ mmin}^{-1}$ and $v_s = 30 \text{ mmin}^{-1}$

Since, $f' = f\left(\frac{v}{v - v_s}\right)$

$\Rightarrow \quad f' = (20)\left(\frac{300}{300 - 30}\right) = 22.22 \text{ min}^{-1}$

Hence, the correct answer is (C).

52. Since, $f_0 = \frac{v}{2\ell}$ ...(1)

Now beat frequency $f_b = f_1 - f_2$

$\Rightarrow \quad f_b = \frac{v}{2\left(\frac{\ell}{2} - \Delta\ell\right)} - \frac{v}{2\left(\frac{\ell}{2} + \Delta\ell\right)}$

$\Rightarrow \quad f_b = \frac{v}{2}\left[\frac{1}{\frac{\ell}{2} - \Delta\ell} - \frac{1}{\frac{\ell}{2} + \Delta\ell}\right]$

$\Rightarrow \quad f_b = (f_0\ell)\left[\frac{2}{\ell - 2\Delta\ell} - \frac{2}{\ell + 2\Delta\ell}\right]$

$\Rightarrow \quad f_b = 2f_0\ell\left[\frac{\ell + 2\Delta\ell - \ell + 2\Delta\ell}{\ell^2 - 4(\Delta\ell)^2}\right]$

$\Rightarrow \quad f_b \approx 2f_0\ell\left(\frac{4\Delta\ell}{\ell^2}\right) \approx \frac{8f_0\Delta\ell}{\ell}$

Hence, the correct answer is (A).

53. Let the third note of fixed frequency be f. Let f_1 and f_2 be the frequency of notes. Then

$$f_1 = \frac{v}{\lambda_1} = \frac{195v}{80} \text{ and } f_2 = \frac{v}{\lambda_2} = \frac{193v}{80}$$

Also, $f_1 - f = 5$ and $f - f_2 = 5$

$\Rightarrow \quad f_1 - f_2 = 10$

$\Rightarrow \quad \frac{2v}{80} = 10$

$\Rightarrow \quad v = 400 \text{ ms}^{-1}$

Hence, the correct answer is (D).

54. Fundamental frequency of open organ pipe is

$$f_0 = \frac{v}{2\ell} = \frac{330}{2 \times 0.3} = 550 \text{ Hz}$$

The given frequency

$$f = 1.1 \text{ kHz} = 1100 \text{ Hz} = 2f_0$$

Therefore, the given frequency corresponds to 2nd harmonic.

Hence, the correct answer is (A).

55. Since $f \propto \sqrt{T}$, so $\frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta T}{T} = \frac{1}{2}\left(\frac{1}{100}\right)$

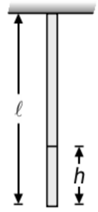
Given that $\Delta f = \frac{3}{2} = 1\cdot5$

$\Rightarrow \quad \frac{1\cdot5}{f} = \frac{1}{200}$

$\Rightarrow \quad f = 300 \text{ Hz}$

Hence, the correct answer is (C).

56. Let ℓ be the length of rope. Then tension in the string at height h will be



$$T = \frac{m}{\ell}hg$$

Since, $v = \sqrt{\frac{T}{\mu}}$

where, μ = mass per unit length $= \frac{m}{\ell}$

$\Rightarrow \quad v = \sqrt{gh}$

$\Rightarrow \quad v^2 = gh$

i.e., v versus h graph is a parabola.

Hence, the correct answer is (C).

57. Since, $\frac{f'}{f} = \sqrt{\frac{T + \Delta T}{T}} \approx 1 + \frac{\Delta T}{2T}$

$\Rightarrow \quad \frac{\Delta T}{T} = 2\frac{\Delta f}{f} = 2\left(\frac{5}{500}\right) = \frac{2}{100}$

So, %age change is 2%

Hence, the correct answer is (C).

58. Since $f = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$ and $2f = \frac{1}{2\ell}\sqrt{\frac{T + W}{m}}$

$\Rightarrow \quad 4 = \frac{T + W}{T} = \frac{m_0 + m}{m}$

$\Rightarrow \quad m = 3m_0 = 12 \text{ kg}$

Hence, the correct answer is (C).

59. A monosyllabic sound reaches listener after reflection in $\frac{1}{5}$ s, so distance between source and listener is

$$d = v\left(\frac{t}{2}\right) = 330\left(\frac{1}{10}\right) = 33 \text{ m}$$

Hence, the correct answer is (B).

60. Let n_1 and n_2 be the number of loops in the wires. Also, they vibrate with same frequency, so

$$\frac{n_1}{2\ell}\sqrt{\frac{T}{\pi r^2 \rho}} = \frac{n_2}{2\ell}\sqrt{\frac{T}{\pi(2r)^2 \rho}}$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{1}{2}$$

Hence, the correct answer is (C).

61. Since, $\frac{\Delta\lambda}{\lambda} = +\frac{v_s}{c}$

$$\Rightarrow \quad \Delta\lambda = \frac{\lambda v_s}{c}$$

When a star recedes the λ increases (as f decreases)

Hence, the correct answer is (D).

62. $f' = f\left(\frac{v-0}{v-v_s}\right) = 600\left(\frac{330}{300}\right) = 660$ Hz

Hence, the correct answer is (B).

63. $R^2 = a^2 + a^2 + 2a^2\cos\phi$

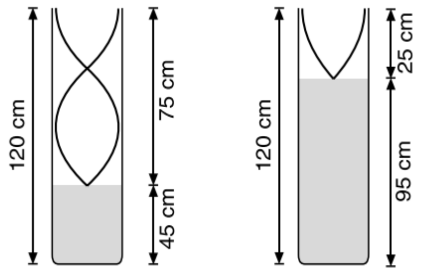
$$\Rightarrow \quad a^2 = a^2 + a^2 + 2a^2\cos\phi$$

$$\Rightarrow \quad \cos\phi = -\frac{1}{2}$$

$$\Rightarrow \quad \phi = \frac{2\pi}{3}$$

Hence, the correct answer is (D).

64. $\ell_0 = \frac{v}{4f_0} = \frac{340}{4(340)} = 25$ cm



$f_3 = 1020$ Hz $\quad f_1 = 340$ Hz

Fundamental length = 25 cm

$$\left(\begin{array}{l}\text{Length of organ pipe/air}\\ \text{column for third harmonic}\end{array}\right) = \ell_3 = 75 \text{ cm}$$

(As organ pipe is closed, so even harmonics are absent)

$$\left(\begin{array}{l}\text{Length of organ pipe/air}\\ \text{column for fifth harmonic}\end{array}\right) = \ell_5 = 125 \text{ cm}$$

(as $\ell_5 > 120$ cm so this value is not permissible)

As water is being poured in the organ pipe it starts rising and gives the resonance firstly for 75 cm. Hence length of water column $= 120 - 75 = 45$ cm.

This is due to the fact that while pouring water f_3 comes first and f_1 comes later.

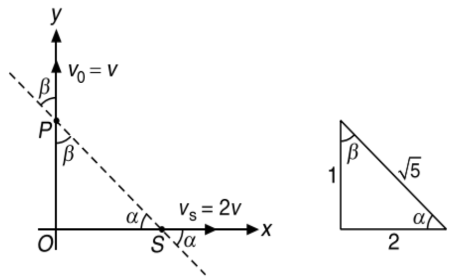
Hence, the correct answer is (B).

65. Let speed of observer be v along y-axis and speed of source is $2v$ along the x-axis.

$OS = 2(OP) \qquad \{\because v_s = 2v_0\}$

$$\Rightarrow \quad \cos\alpha = \frac{2}{\sqrt{5}} \text{ and } \cos\beta = \frac{1}{\sqrt{5}}$$

Now, $f = f_0\left(\frac{V - v_0\cos\beta}{V + v_s\cos\alpha}\right)$



$$\Rightarrow \quad f = f_0\left(\frac{V - \frac{v}{\sqrt{5}}}{V + \frac{4v}{\sqrt{5}}}\right) \quad \ldots(1)$$

where, V is the speed of sound

From equation (1) we can see that f is constant but less than f_0.

Hence, the correct answer is (B).

66. $\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$ and $\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$

$$\Rightarrow \quad \frac{V_1}{V_2} = \frac{v_1}{v_2}$$

Hence, the correct answer is (B).

67. Since, $(\Delta P)_m = 2\pi f \rho v A$

$$\Rightarrow \quad A = \frac{(\Delta P)_m}{2\pi f \rho v}$$

$$\Rightarrow \quad A = \frac{30}{2\pi \times 10^3 \times 300 \times 1.5} = \frac{10^{-4}}{3\pi} \text{ m}$$

Hence, the correct answer is (D).

68. Since $y = 4\sin\left(\pi t + \frac{\pi x}{16}\right)$, where $\omega = \pi$, $k = \frac{2\pi}{\lambda} = \frac{\pi}{16}$

$$\Rightarrow \quad v = \frac{\omega}{k} = 16 \text{ cms}^{-1}$$

Also $\omega = \pi$, so $2\pi f = \pi$

$$\Rightarrow \quad f = 0.5 \text{ Hz and } \lambda = 32 \text{ cm}$$

Hence, the correct answer is (A).

69. $\Delta\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x = \frac{2\pi}{32}(v\Delta t) \qquad \{\because \Delta x = v\Delta t\}$

$$\Rightarrow \quad \Delta\phi = \frac{2\pi}{32}(16 \times 0.4) = 0.4\pi$$

Hence, the correct answer is (B).

70. $\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{32}(12) = \frac{3\pi}{4}$

Hence, the correct answer is (C).

71. Since $v_A - v_B = 2 \quad \ldots(1)$

and $v_B - v_A = 2 \quad \ldots(2)$

On putting wax on prongs of B, frequency of B decreases and number of bps decreases to (1) bps. This condition is satisfied by equation (2). Hence

$$v_B - 100 = 2$$

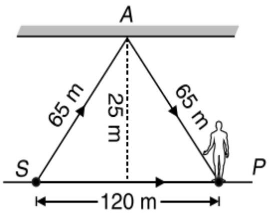
$$\Rightarrow \quad v_B = 102 \text{ Hz}$$

Hence, the correct answer is (D).

72. Path Difference $\Delta x = (SA + AP) - SP$

$$\Rightarrow \quad \Delta x = (65 + 65) - 120$$

$$\Rightarrow \quad \Delta x = 10 \text{ m}$$



So, net path difference between waves arriving at the person is $\Delta x = 10$ m

For maxima, i.e. constructive interference, we have

$$\Delta x = (2n)\frac{\lambda}{2}; \; n = 0, 1, 2, \ldots\ldots$$

$$\Rightarrow \quad 10 = (2n)\frac{\lambda}{2}; \; n = 0, 1, 2, \ldots\ldots$$

$$\Rightarrow \quad 10 = n\lambda; \; n = 0, 1, 2, \ldots\ldots$$

$$\Rightarrow \quad \lambda = \frac{10}{n}; \; n = 0, 1, 2, \ldots\ldots$$

$$\Rightarrow \quad \lambda = 10, 5, \frac{10}{3}, \ldots\ldots\ldots\ldots\ldots$$

Hence, the correct answer is (B).

73. When the source approaches the observer the apparent frequency f' is increased and since, $I \propto f^2$ intensity of sound wave get increased.

Hence, the correct answer is (A).

74. Since $f' = f\left(\dfrac{v - v_\ell}{v - v_s}\right)$, so $\dfrac{f}{f'} = \left(\dfrac{v - v_s}{v - v_\ell}\right)$

Hence, the correct answer is (B).

75. Let ℓ be the length of the pipes and v the speed of sound. Then frequency of open organ pipe of nth overtone is

$$f_1 = (n+1)\frac{v}{2\ell}$$

and frequency of closed organ pipe of nth overtone is

$$f_2 = (2n+1)\frac{v}{4\ell}$$

The desired ratio is $\dfrac{f_1}{f_2} = \dfrac{2(n+1)}{2n+1}$

Hence, the correct answer is (B).

76. Since, $f \propto \dfrac{\sqrt{T/\mu}}{\ell} = \dfrac{\sqrt{T/\rho S}}{\ell}$

where $\mu = \rho S$ and S is cross-sectional area of wire.

$$\Rightarrow \quad f \propto \frac{\sqrt{T/\rho r^2}}{\ell}$$

$$\Rightarrow \quad f \propto \frac{\sqrt{T/\rho}}{r\ell}$$

$$\Rightarrow \quad \frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}}\sqrt{\frac{\rho_2}{\rho_1}}\frac{r_2\ell_2}{r_1\ell_1} = (\sqrt{2})\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{4}\right)$$

$$\Rightarrow \quad f_2 = 4f_1$$

Hence, the correct answer is (C).

77. Velocity of wave pulse at distance x from the bottom

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{(\rho x s)g}{(\rho s)}} = \sqrt{xg}$$

As velocity is independent of ρ, time taken by both are same.

Hence, the correct answer is (C).

78. Beat frequency $f_b = f_1 - f_2$

$$\Rightarrow \quad f_b = \frac{v}{2\ell} - \frac{v}{2(\ell + x)}$$

$$\Rightarrow \quad f_b = \frac{v}{2\ell}\left[1 - \left(1 + \frac{x}{\ell}\right)^{-1}\right]$$

$$\Rightarrow \quad f_b = \frac{v}{2\ell}\left[1 - 1 + \frac{x}{\ell}\right] = \frac{vx}{2\ell^2}$$

Hence, the correct answer is (C).

79. Let density of Hydrogen be ρ, then that of Oxygen is 16ρ.

$$\rho_{\text{mixture}} = \frac{(4V)\rho + V(16\rho)}{4V + V}$$

$$\Rightarrow \quad \rho_{\text{mixture}} = 4\rho$$

Given that, $v_{H_2} = 1270 = \sqrt{\dfrac{\gamma p}{\rho}}$

So, $v_{\text{mixure}} = \sqrt{\dfrac{\gamma p}{4\rho}} = \dfrac{1}{2}(1270) = 635 \text{ ms}^{-1}$

Hence, the correct answer is (B).

80. Since $(v_p)_{\max} = A\omega$ and wave velocity, $v = \dfrac{\omega}{k}$

For both to be equal, $\lambda = 2\pi A$

Hence, the correct answer is (B).

81. Since, $f = f_0\left(\dfrac{v}{v - u}\right)\left(\dfrac{v + u}{v}\right) = 8\left(\dfrac{320 + 10}{320 - 10}\right) = 8\left(\dfrac{33}{21}\right)$

$$\Rightarrow \quad f = 8.5 \text{ kHz}$$

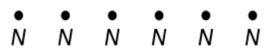
Hence, the correct answer is (A).

82. Since $n = 2\left(\dfrac{v}{2\ell}\right)$, so $n' = 3\left(\dfrac{v}{4\ell}\right) = \dfrac{3n}{4}$

Hence, the correct answer is (C).

83. $f = 1000$ Hz

Distance between two consecutive nodes is $\dfrac{\lambda}{2}$ so, distance between six nodes is $(6-1)\dfrac{\lambda}{2} = \dfrac{5\lambda}{2}$.



$$\Rightarrow \quad \frac{5\lambda}{2} = 82 \cdot 5$$

$\Rightarrow \quad \lambda = 33$ cm

Since, $v = f\lambda$

$$\Rightarrow \quad v = \left(\frac{33}{100}\right)(1000) = 330 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

84. $I \propto A^2 \propto \frac{1}{r^2}$ (Inverse Square Law)

$$\Rightarrow \quad A \propto \frac{1}{r}$$

$$\Rightarrow \quad A' = \frac{A}{2}$$

Hence, the correct answer is (C).

85. $n = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

$$n = \frac{1}{2\ell}\sqrt{\frac{T}{\pi D^2 \rho/4}}$$

$$\Rightarrow \quad n = \frac{1}{\ell D}\sqrt{\frac{T}{\pi\rho}} \qquad \left\{\because \mu = \frac{\pi(D/2)^2 \ell\rho}{\ell}\right\}$$

$$\Rightarrow \quad n' = \frac{\sqrt{2}}{4}n = \frac{n}{2\sqrt{2}}$$

Hence, the correct answer is (D).

86. $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{YA\Delta\ell}{A\rho}} = 35$ Hz $\qquad \left\{\because Y = \frac{T/A}{\Delta\ell/\ell}\right\}$

Hence, the correct answer is (A).

87. Since, $f = \frac{1}{2\ell D}\sqrt{\frac{T}{\pi\rho}}$

So fD = constant and hence f becomes $\frac{f}{3}$

Hence, the correct answer is (B).

88. Since, $f_b = \frac{v}{2\ell} - \frac{v}{2(\ell + \Delta\ell)}$

$$\Rightarrow \quad f_b = \frac{v}{2\ell}\left[1 - \left(1 + \frac{\Delta\ell}{\ell}\right)^{-1}\right]$$

Since $\frac{\Delta\ell}{\ell} << 1$, so from Binomial Theorem, we get

$$f_b \approx \frac{v}{2\ell}\left[1 - \left(1 - \frac{\Delta\ell}{\ell}\right)\right] \approx \frac{v}{2\ell}\left(\frac{\Delta\ell}{\ell}\right) = \frac{v\Delta\ell}{2\ell^2}$$

Hence, the correct answer is (D).

89. Let v be the speed of sound and u the speed of train.

Then $v_s = v_0 = u$

and $f' = f\left(\frac{v + v_0\cos\theta}{v + v_s\cos\theta}\right)$

$$\Rightarrow \quad f' = f\left(\frac{v + u\cos\theta}{v + u\cos\theta}\right) = f$$

Hence, the correct answer is (C).

90. All of the options except (D) form equation of a progressive wave.

Hence, the correct answer is (D).

91. Since $v = \sqrt{\frac{T}{\mu}}$, where $\mu = \rho S$

$$\Rightarrow \quad v = \sqrt{\frac{T}{\rho S}} = \sqrt{\frac{1000}{100 \times 10 \times 10^{-6}}} = 10^3 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

92. Since $L_2 - L_1 = 10\log_{10}\left(\frac{I_2}{I_1}\right)$

$$\Rightarrow \quad 20 - 40 = 10\log_{10}\left(\frac{r_1^2}{r_2^2}\right) = 10\log_{10}\left(\frac{1}{r_2^2}\right)$$

$$\Rightarrow \quad -2 = \log_{10}\left(\frac{1}{r_2^2}\right) = -2\log_{10}(r_2)$$

$$\Rightarrow \quad r_2 = 10 \text{ m}$$

Hence, the correct answer is (C).

93. Since two coherent waves have different amplitude, so $I_{min} = (a_1 - a_2)^2 \neq 0$

Hence, the correct answer is (C).

94. $\frac{a_1}{a_2} = \frac{1}{3}$

$$I_{max} = (a_1 + a_2)^2$$

$$\Rightarrow \quad I_{max} = (a_1 + 3a_1)^2 = 16a_1^2 = 16I$$

Hence, the correct answer is (A).

95. $I_{min} = (a_1 - a_2)^2$

$$\Rightarrow \quad I_{min} = (a_1 - 3a_1)^2 = 4a_1^2 = 4I$$

Hence, the correct answer is (C).

96. $f_{approach} = 1000 = \left(\frac{v - 0}{v - v_s}\right)f = \frac{350}{300}f$

$$\Rightarrow \quad f_{recede} = \left(\frac{v - 0}{v + v_s}\right)f = \frac{350}{400} \times \frac{300}{350} \times 1000$$

$$\Rightarrow \quad f_{recede} = 750 \text{ Hz}$$

Hence, the correct answer is (A).

97. Intensity $I = \frac{\text{Power}}{4\pi r^2} = 2\pi^2 f^2 A^2 \rho v$

$$\Rightarrow \quad A = \frac{1}{2\pi f r}\sqrt{\frac{\text{Power}}{2\pi\rho v}}$$

$$\Rightarrow \quad A = \frac{1}{2\pi(1000)(10)}\sqrt{\frac{10}{2\pi(1.29)(340)}} \text{ m} \approx 0.96\ \mu\text{m}$$

Hence, the correct answer is (D).

98. Since, $\nu_A - \nu_B = 5$ ...(1)

or $\nu_B - \nu_A = 5$ ...(2)

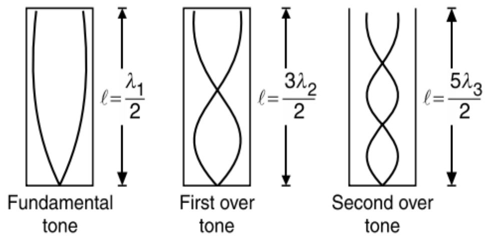
On filing A, the number of bps increases which is satisfied by (1). So

$$\nu_A - 512 = 5$$

$$\Rightarrow \quad \nu_A = 517 \text{ Hz}$$

Hence, the correct answer is (C).

99.



Since, $\ell = \dfrac{\lambda_1}{2}$

$\Rightarrow \quad \lambda_1 = 2\ell$

Similarly, $\ell = \dfrac{3\lambda_2}{2}$

$\Rightarrow \quad \lambda_2 = \dfrac{2\ell}{3}$

Also, $\ell = \dfrac{5\lambda_2}{2}$

$\Rightarrow \quad \lambda_3 = \dfrac{2\ell}{5}$

$\Rightarrow \quad \lambda_1 : \lambda_2 : \lambda_3 = 2\ell : \dfrac{2\ell}{3} : \dfrac{2\ell}{5} = 1 : \dfrac{1}{3} : \dfrac{1}{5}$

Hence, the correct answer is (B).

100. $f \propto \sqrt{T}$

$$f + \frac{f}{2} \propto \sqrt{T'}$$

$\Rightarrow \quad \dfrac{T'}{T} = \dfrac{9}{4}$

$\Rightarrow \quad \dfrac{T' - T}{T} = \left(\dfrac{9}{4} - 1\right) \times 100 = 125\%$

Hence, the correct answer is (C).

101. Speed of sound in a gas $v_1 = \sqrt{\dfrac{\gamma RT}{M}}$

while r.m.s. speed of gas molecules is

$$v_2 = \sqrt{\frac{3RT}{M}}$$

Since, γ is always less than 3

$$v_1 < v_2$$

Hence, the correct answer is (B).

102. $f\ell = \text{constant}$

$(256)\ell = (512)(0 \cdot 5)$

$\Rightarrow \quad \ell = 1\text{ m}$

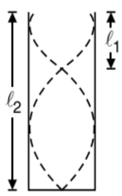
Hence, the correct answer is (C).

103. An open pipe and a closed pipe of equal length cannot have the same frequency at any harmonic.

Hence, the correct answer is (D).

104. $\ell_1 + x = \dfrac{\lambda}{4}$

$$\ell_2 + x = \frac{3\lambda}{4f}$$



$\Rightarrow \quad x = \dfrac{\ell_2 - 3\ell_1}{2}$

Hence, the correct answer is (C).

105. Since, $f = \dfrac{v}{\lambda}$

Given, $f_1 - f_2 = 6$

$\Rightarrow \quad \dfrac{v}{1} - \dfrac{v}{1.02} = 6$

Solving this we get, $v = 306\text{ ms}^{-1}$

Hence, the correct answer is (B).

106. $\dfrac{I_1}{I_2} = \dfrac{16}{1} = \dfrac{a_1^2}{a_2^2}$

$\Rightarrow \quad \dfrac{a_1}{a_2} = 4$

$$\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{4+1}{4-1}\right)^2 = \frac{25}{9}$$

Hence, the correct answer is (D).

107. $v \propto \sqrt{T}$ and $T \propto \Delta\ell$

$\Rightarrow \quad \dfrac{v_2}{v_1} = \sqrt{\dfrac{(\Delta\ell)_2}{(\Delta\ell)_1}}$

$\Rightarrow \quad \dfrac{v_2}{v_1} = \sqrt{\dfrac{(L/10)}{(L/20)}} = \sqrt{2}$

$\Rightarrow \quad v_2 = \sqrt{2}v$

Hence, the correct answer is (C).

108. Let $a =$ amplitude due to s_1 and s_2 individually

Loudness due to $s_1 = I_1 = ka^2$ $\quad\{k = \text{constant}\}$

Loudness due to $s_1 + s_2 = I = k(2a)^2 = 4I_1$

$\Rightarrow \quad n = 10\log_{10}\left(\dfrac{4I_1}{I_1}\right)$

$\Rightarrow \quad n = 10\log_{10}(4) = 10 \times 0.6 = 6$

Hence, the correct answer is (D).

109. Since, $\dfrac{\Delta f}{f} = 2.5\%$

$\Rightarrow \quad \dfrac{f' - f}{f} = \dfrac{2.5}{100} = \dfrac{1}{40}$

Since $f' = f\left(\dfrac{v}{v - v_s}\right)$

$\Rightarrow \quad \dfrac{f\left(\dfrac{v}{v - v_s}\right) - f}{f} = \dfrac{1}{40}$

$\Rightarrow \quad \dfrac{v - v + v_s}{v - v_s} = \dfrac{1}{40}$

$\Rightarrow \quad \dfrac{v_s}{320 - v_s} = \dfrac{1}{40}$

$\Rightarrow \quad 40v_s = 320 - v_s$

$\Rightarrow \quad 41v_s = 320$

$\Rightarrow \quad v_s = \dfrac{320}{41} = 7.8 \text{ ms}^{-1}$

Hence, the correct answer is (C).

110. $f = \dfrac{1}{2\ell}\sqrt{\dfrac{T}{\mu}}$

T becomes $4T$, so f becomes $2f$ i.e. all harmonics and overtones are multiplied by a factor of 2.
Hence, the correct answer is (A).

111. Let $y_1 = A\sin(\omega t)$, then

$\Rightarrow \quad y_2 = A\sin\left(\omega t + \dfrac{\pi}{2}\right)$

$\Rightarrow \quad R^2 = A^2 + A^2 + 2A^2\cos\left(\dfrac{\pi}{2}\right)$

$\Rightarrow \quad R = \sqrt{2}A$

However, both will have the same frequency on superimposing.
Hence, the correct answer is (D).

112. Path difference

$\Delta x = s_2D - s_1D = 5 - 4 = 1\text{ m}$

The corresponding phase difference will be

$\phi = \left(\dfrac{2\pi}{\lambda}\right)\Delta x = \left(\dfrac{2\pi}{4}\right)(1) = \dfrac{\pi}{2}$

Using $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$, we get

$I = I_0 + I_0 + 2\sqrt{I_0 I_0}\cos\left(\dfrac{\pi}{2}\right) = 2I_0$

Hence, the correct answer is (C).

113. Since it is being plucked at a distance of $\dfrac{\ell}{4}$. So, we must get two 100 ps. Hence

$f = 2f_1 = 2\left(\dfrac{1}{2\ell}\sqrt{\dfrac{T}{m}}\right)$

$\Rightarrow \quad f = \sqrt{\dfrac{\dfrac{20}{0\cdot 5}}{1000}}$

$\Rightarrow \quad f = 200\text{ Hz}$

Hence, the correct answer is (B).

114. Since $v_0 = \sqrt{\dfrac{\gamma RT}{M}}$, $v = \sqrt{\dfrac{3RT}{M}}$

$\Rightarrow \quad v_0 = v\sqrt{\dfrac{\gamma}{3}}$

Hence, the correct answer is (C).

115. Since, $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{200} = 0.01\ \pi\text{cm}^{-1}$ and $v = \dfrac{\omega}{k}$

$\Rightarrow \quad \omega = vk = (100\text{ cms}^{-1})(0.01\ \pi\text{ cm}^{-1}) = \pi\text{s}^{-1}$

$\Rightarrow \quad y = 10\sin(0.01\ \pi x - \pi t)$ is the desired equation

because the particle at $x = 0$ is moving downwards at $t = 0$.
If we substitute $x = 0$ in this equation we get $y = -10\sin(\pi t)$, which is an equation of a SHM moving along negative of direction at $t = 0$.
Hence, the correct answer is (A).

116. Pressure variation has no effect on the speed of sound
Hence, the correct answer is (D).

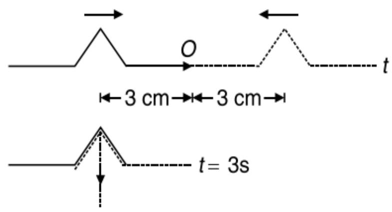
117. The maximum particle velocity is $a\omega$, a is the amplitude

$\Rightarrow \quad v_{\max} = (0.1)2\pi(300)$

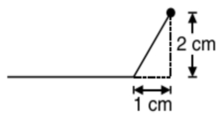
$\Rightarrow \quad v_{\max} = 60\pi\text{ cms}^{-1}$

Hence, the correct answer is (A).

118. The formation of the reflected pulse from a free support is similar to the overlap of two pulses of same nature travelling in opposite directions as shown below.

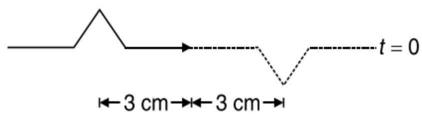


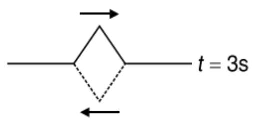
Therefore, net displacement of all points will become twice as shown below:



Hence, the correct answer is (D).

119. The formation of the reflected pulse from a fixed support is similar to the overlap of two inverted pulses travelling in opposite directions as shown below:





Hence, at $t = 3$ s, net displacement of all particles of the string will be zero i.e., string will be straight as shown below:

Hence, the correct answer is (A).

120. For 1st reading of oscillator

$f_A = (514 \pm 2)$ Hz

$\Rightarrow \quad f_A = 516$ Hz or 512 Hz

For 2nd reading of oscillator

$f_A = (510 \pm 6)$ Hz

$\Rightarrow \quad f_A = 516$ Hz or 504 Hz

A has a frequency of 516 Hz
Hence, the correct answer is (C).

121. $\frac{I_1}{I_2}=\left(\frac{a_1}{a_2}\right)^2$ only when other properties like density of medium, speed of wave etc. remains the same $\left(I=\frac{1}{2}\rho v\omega^2 a^2\right)$.

In incident and reflected waves medium is same. Therefore, other properties are same. Hence,

$$\frac{I_i}{I_r}=\left(\frac{a_i}{a_r}\right)^2$$

Hence, the correct answer is (A).

122. $v=v_0=\sqrt{\frac{T}{\mu}}$

Hence, the correct answer is (D).

123. $L_1=10\log\left(\frac{10}{I_0}\right)$ and $L_2=10\log\left(\frac{500}{I_0}\right)$

$\Rightarrow \quad L_2-L_1=10\log\left(\frac{500}{10}\right)=1.7$ dB

Hence, the correct answer is (B).

124. Next Resonance length after the fundamental is $3\ell_1=48$ cm

Hence, the correct answer is (C).

125. The maximum difference between the frequencies is

$$f_b=(n+1)-(n-1)=2\text{ Hz}$$

Hence, the correct answer is (C).

126. Since, $\frac{\Delta\lambda}{\lambda}=0.01\%$

$\Rightarrow \quad \frac{v}{c}=\frac{0.01}{100}$

$\Rightarrow \quad v=\frac{0.01}{100}\times 3\times 10^8$

$\Rightarrow \quad v=3\times 10^4\ \text{ms}^{-1}$

$\Rightarrow \quad v=30\ \text{kms}^{-1}$

Hence, the correct answer is (C).

127. Since, $v\propto\sqrt{T}$

$\Rightarrow \quad \frac{2v}{v}=2=\sqrt{\frac{T_2}{T_1}}$

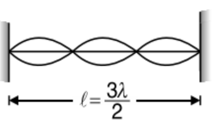
$\Rightarrow \quad T_2=4\,T_1=4(273)$

$\Rightarrow \quad T_2=1092\text{ K}=819\text{ °C}$

Hence, the correct answer is (C).

128. Wave number $k=\frac{2\pi}{\lambda}=0.6\text{ cm}^{-1}$

$\Rightarrow \quad \frac{\lambda}{2}=\frac{\pi}{0.6}$ cm



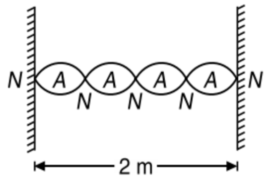
$\Rightarrow \quad \ell=\frac{3\lambda}{2}=3\left(\frac{\pi}{0.6}\right)$ cm $=15.7$ cm

Hence, the correct answer is (D).

129. Since it vibrates in 4 segments, so we must get 5 Nodes and 4 Antinodes.

$\Rightarrow \quad 2=2\lambda$

$\Rightarrow \quad \lambda=1$ m



Distance between two consecutive nodes is $\frac{\lambda}{2}$

Hence, the correct answer is (A).

130. Two syllables in an echo means that time is $2\left(\frac{1}{5}\right)$ s

$\Rightarrow \quad d=v\left(\frac{t}{2}\right)=\left(\frac{330}{2}\right)\left(\frac{2}{5}\right)=66$ m

Hence, the correct answer is (C).

131. $y=ae^{x\pm vt}$ is one more form of progressive wave equation propagating with speed v.

Hence, the correct answer is (B).

132. $\Delta\phi=\left(\frac{2\pi}{\lambda}\right)\Delta x$

$\Rightarrow \quad \frac{\pi}{3}=\frac{2\pi}{(360/500)}\Delta x \qquad \left\{\because \lambda=\frac{v}{f}=\frac{360}{500}\right\}$

$\Rightarrow \quad \Delta x=\frac{120}{1000}$ m $=12$ cm

Hence, the correct answer is (B).

133. Third overtone is the fourth harmonic i.e.

$$f_4=4f_1=400\text{ Hz}$$

Hence, the correct answer is (C).

134. $f_1=\frac{v}{4\ell}=\frac{340}{0\cdot 80}$

$\Rightarrow \quad f_1=425$ Hz

Since in a closed organ pipe only odd harmonics are present. So, possible vibrations are $f_1,\ 3f_1,\ 5f_1,$......

$\Rightarrow \quad 425,\ 1275,\ 2125,$ .....

Hence, the correct answer is (B).

135. If the last fork gives octave of the first, then frequency of last must be 2 times frequency of first. So, the series becomes

$$\nu,\ \nu+4,\ \nu+8,\text{........}2\nu$$

This is an Arithmetic Progression with common difference 4.

$\Rightarrow \quad 2\nu=\nu+(65-1)(4)$

$\Rightarrow \quad \nu=256$ Hz

Hence, the correct answer is (B).

136. Since $n=\frac{v}{2\ell}$, so $n'=\frac{v}{4\ell}=\frac{n}{2}$

Hence, the correct answer is (A).

137. Since $v=\sqrt{\frac{\gamma p}{\rho}}$, so $\frac{v_{\text{monatomic}}}{v_{\text{triatomc}}}=\sqrt{\frac{\gamma_{\text{monatomic}}}{\gamma_{\text{triatomic}}}}=\sqrt{\frac{5/3}{4/3}}$

$$\Rightarrow \frac{v_{\text{monatomic}}}{v_{\text{triatomc}}} = \sqrt{\frac{5}{4}} = 1.12$$

Hence, the correct answer is (A).

138. $\omega = 50\pi$ and $k = 10\pi$

$$\text{Speed} = \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5 \text{ ms}^{-1}$$

Hence, the correct answer is (C).

139. $350 = \frac{v}{2\ell}$

$$\Rightarrow 350 = \frac{350}{2\ell}$$

$$\Rightarrow \ell = 0\cdot5 \text{ m}$$

Hence, the correct answer is (B).

140. Since $f \propto \frac{1}{\ell}$, so $f+5 \propto \frac{1}{1}$ and $f-5 \propto \frac{1}{1\cdot05}$

$$\Rightarrow \frac{f+5}{f-5} = \frac{105}{100}$$

$$\Rightarrow 100f + 500 = 105f - 525$$

$$\Rightarrow 5f = 1025$$

$$\Rightarrow f = 205 \text{ Hz}$$

Hence, the correct answer is (D).

141. $$f' = \left[\frac{v - \left(-\frac{v}{10}\right)}{v - \frac{v}{10}}\right] f = \frac{11}{9} f = 1.22 f$$

Hence, the correct answer is (B).

142. Second overtone is the third harmonic i.e., 3 loops and hence 4 nodes and 3 antinodes.

Hence, the correct answer is (C).

143. $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$$\Rightarrow I_R = I + 4I + 0$$

$$\Rightarrow I_R = 5I$$

Hence, the correct answer is (B).

144. Fundamental frequency of a closed pipe is given by

$$f_0 = \frac{v}{4\ell}$$

Here, v = speed of sound in air

Length ℓ of air column will first decrease and then becomes constant (when rate of inflow = rate of outflow). Therefore, f_0 will first increase and then become constant.

Hence, the correct answer is (B).

145. Since, $L = 10\log_{10}\left(\frac{I}{I_0}\right)$

$$\Rightarrow L = 10\log_{10}\left(\frac{2\times10^{-8}}{10^{-12}}\right) = 10\log_{10}(2\times10^4)$$

$$\Rightarrow L = 10\left[\log_{10}2 + \log_{10}(10^4)\right]$$

$$\Rightarrow L = 10(4.3) = 43 \text{ dB}$$

Hence, the correct answer is (C).

146. Since $f \propto \sqrt{T}$, so $2f \propto \sqrt{T'}$

$$\Rightarrow \frac{T'}{T} = 4$$

$$\Rightarrow \frac{T'-T}{T} = \frac{3}{4}$$

Hence, the correct answer is (D).

147. Apparent frequency for reflector (will act as an observer) would be

$$f_1 = \left(\frac{v+u}{v}\right) f$$

where f = actual frequency of source.
After being reflected the apparent frequency will further change and the reflector will now behave as a source. The apparent frequency will not become

$$f_2 = \left(\frac{v}{v-u}\right) f_1$$

Substituting value of f_1, we get

$$f_2 = \left(\frac{v+u}{v-u}\right) f$$

Hence, the correct answer is (C).

148. $$\left(f_{\text{approach}}\right)_A = 5.5 \text{ kHz} = \left(\frac{v+v_A}{v}\right)5 \quad \ldots(1)$$

$$\left(f_{\text{approach}}\right)_B = 6 \text{ kHz} = \left(\frac{v+v_B}{v}\right)5 \quad \ldots(2)$$

where v is the velocity of sound

$$\Rightarrow 5.5 = \left(1 + \frac{v_A}{v}\right)5$$

$$\Rightarrow \frac{v_A}{v} = 0.1 \quad \ldots(3)$$

Similarly, $6 = \left(1 + \frac{v_B}{v}\right)5$

$$\Rightarrow \frac{v_B}{v} = 0.2 \quad \ldots(4)$$

$$\Rightarrow \frac{v_B}{v_A} = 2$$

Hence, the correct answer is (B).

149. Time interval between compressional maximum and rarefactional maximum is $\frac{T}{2}$

$$\Rightarrow t = \frac{T}{2} = \frac{1}{2f} = \frac{1}{1000} \text{ s}$$

Hence, the correct answer is (C).

150. Separation $= \frac{\lambda}{2} = \frac{v}{2f} = \frac{350}{1000} = 0.35$ m

Hence, the correct answer is (C).

151. Since, $I \propto A^2\omega^2$

$$\Rightarrow \frac{I_1}{I_2} = \left(\frac{1}{4}\right)(16)$$

$\Rightarrow \quad \dfrac{I_1}{I_2} = 4$

$\Rightarrow \quad \dfrac{I_2}{I_1} = \dfrac{1}{4}$

Hence, the correct answer is (C).

152. Distance between two adjacent antinodes is $\dfrac{\lambda}{2}$ or $\dfrac{\pi}{k}$, volume of string between two adjacent antinodes $V = \dfrac{\pi}{k}a$

$$E = (V)(u_1 + u_2)$$

Here, u is the energy density (energy per unit volume) which is equal to $\dfrac{1}{2}\rho A^2\omega^2$

$$\Rightarrow \quad E = \left(\frac{\pi}{k}\right)(s)\left[\frac{1}{2}\rho a^2\omega^2 + \frac{1}{2}\rho(2a)^2\omega^2\right] = \frac{5\pi s\rho a^2\omega^2}{2k}$$

Hence, the correct answer is (C).

153. $L_2 - L_1 = 10\log_{10}\left(\dfrac{I_2}{I_1}\right) = 10\log_{10}(10^5) = 50 \text{ dB}$

Hence, the correct answer is (D).

154. Since $\dfrac{I_1}{I_2} = \dfrac{4}{1}$, so $a_1 = 2a_2$

$$\Rightarrow \quad \frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{2+1}{2-1}\right)^2 = 9$$

So, $L_1 - L_2 = 10\log_{10}\left(\dfrac{I_{max}}{I_{min}}\right) = 10\log_{10}(9)$

$\Rightarrow \quad L_1 - L_2 = 20\log_{10}(3)$

Hence, the correct answer is (B).

155. Since, $L = 10\log_{10}\left(\dfrac{I}{I_0}\right)$, where $I_0 = 10^{-12} \text{ Wm}^{-2}$

$\Rightarrow \quad 70 = 10\log_{10}\left(\dfrac{I}{I_0}\right)$

$\Rightarrow \quad \dfrac{I}{I_0} = 10^7$

$\Rightarrow \quad I = 10^{-5} \text{ Wm}^{-2}$

Hence, the correct answer is (A).

156. $f_1 = 900\left(\dfrac{300}{300 + v_1}\right) \approx 900\left(1 + \dfrac{v_1}{300}\right)^{-1}$

$\Rightarrow \quad f_1 = 900 - 3v_1$

Similarly, $f_2 = 900\left(\dfrac{300}{300 + v_2}\right) = 900 - 3v_2$

Given, $f_2 - f_1 = 6$

$\Rightarrow \quad 3(v_1 - v_2) = 6$

$\Rightarrow \quad v_1 - v_2 = 2 \text{ ms}^{-1}$

Hence, the correct answer is (B).

157. $f_{approach} = \left(\dfrac{v - 0}{v - v_s}\right)f = \dfrac{340}{320}f$

Also, $f_{recede} = \left(\dfrac{v - 0}{v + v_s}\right)f = \left(\dfrac{340}{360}\right)f$

$\Rightarrow \quad \dfrac{f_{approach}}{f_{recede}} = \dfrac{360}{320} = \dfrac{9}{8}$

Hence, the correct answer is (A).

158. $P = \dfrac{1}{2}\mu\omega^2A^2v$ where $v = \sqrt{\dfrac{T}{\mu}}$

Hence, the correct answer is (C).

159. When the source approaches the observer

$$f_1 = f\left(\frac{v}{v - v_s}\right) = f\left(1 - \frac{v_s}{v}\right)^{-1} \approx f\left(1 + \frac{v_s}{v}\right)$$

$$\Rightarrow \quad \left(\frac{f_1 - f}{f}\right) \times 100 = \frac{v_s}{v} \times 100 = 10 \qquad \ldots(1)$$

In the second case when the source recedes the observer.

$$f_2 = f\left(\frac{v}{v + v_s}\right) = f\left(1 + \frac{v_s}{v}\right)^{-1} \approx f\left(1 - \frac{v_s}{v}\right)$$

$$\Rightarrow \quad \left(\frac{f_2 - f}{f}\right) \times 100 = -\frac{v_s}{v} \times 100 = -10 \qquad \{\text{from (1)}\}$$

In the first case observed frequency increase by 10% while in the second case observed frequency decrease by 10%.

Hence, the correct answer is (D).

160. Since on decreasing the temperature, velocity of sound also decreases $(v \propto \sqrt{T})$ and hence frequency of air column also decreases. Since beat frequency also decreases hence it is obvious that the frequency of air column must be higher than that of fork. Let f be the frequency of fork, then

$$\frac{f+4}{f+1} = \frac{v_{51}}{v_{16}} = \sqrt{\frac{273+51}{273+16}} = \frac{18}{17}$$

$\Rightarrow \quad f = 50 \text{ Hz}$

Hence, the correct answer is (A).

161. $\Rightarrow f_1 = \left(\dfrac{340}{340 - 34}\right)f = \dfrac{10}{9}f$

and $f_2 = \left(\dfrac{340}{340 - 17}\right)f = \dfrac{20}{19}f$

$\Rightarrow \quad \dfrac{f_1}{f_2} = \dfrac{10/9}{20/19} = \dfrac{19}{18}$

Hence, the correct answer is (D).

162. Since there is no relative motion between the source and listener, so apparent frequency equals original frequency.

Hence, the correct answer is (A).

163. Since $f_b = \dfrac{1}{t_b} = 2 \text{ Hz}$, so $t_b = 0.5 \text{ s}$

Hence, the correct answer is (A).

164. Since, $90 - 40 = 10\log_{10}\left(\dfrac{I_{90}}{I_{40}}\right)$

$\Rightarrow \quad \dfrac{I_{90}}{I_{40}} = 10^5$

Hence, the correct answer is (D).

165. Since $fD = \text{constant}$

$\Rightarrow f_1D_1 = f_2D_2$

$\Rightarrow fD = (4f)D'$

$\Rightarrow D' = \frac{1}{4}D$

Hence, the correct answer is (B).

166. Given, $3\left(\frac{v}{4\ell_c}\right) = 2\left(\frac{v}{2\ell_0}\right)$

$\Rightarrow \frac{\ell_c}{\ell_0} = \frac{3}{4}$

Now, $n\left(\frac{v}{4\ell_c}\right) = m\left(\frac{v}{2\ell_0}\right)$

$\Rightarrow \frac{n}{m} = 2\left(\frac{\ell_v}{\ell_0}\right) = \frac{6}{4} = \frac{3}{2}$

$\Rightarrow \frac{n}{m} = \frac{3}{2} = \frac{9}{6}$

So, $n = 9$ and $m = 6$.

Hence, the correct answer is (C).

167. Given that $\frac{\Delta f}{f} = 25\% = \frac{1}{4}$

$\Rightarrow \frac{f' - f}{f} = \frac{1}{4}$

$\Rightarrow 4f' - 4f = f$

$\Rightarrow 4f' = 5f$

$\Rightarrow \frac{f'}{f} = \frac{5}{4}$

$\Rightarrow \frac{v}{v - v_s} = \frac{5}{4}$

$\Rightarrow 4v = 5v - 5v_s$

$\Rightarrow v_s = \frac{v}{5}$

Hence, the correct answer is (A).

168. For open pipe, the fundamental frequency is $f = \frac{v}{2l}$.

When dipped half in water, it becomes a closed organ pipe of length $l/2$, so fundamental frequency is

$$f' = \frac{v}{4(l/2)} = f$$

Hence, the correct answer is (B).

169. Fundamental tone of a closed organ pipe is less than the fundamental tone of an open organ pipe of same length. Given that $f_o - f_c = 4$

When the length of open organ pipe is increased f_0 will decrease, because $f \propto \frac{1}{l}$, so $|f_o \sim f_c|$ or beat frequency may increase or decrease.

Hence, the correct answer is (D).

170. Length of string $\ell = (\text{number of loops})\left(\frac{\lambda}{2}\right)$

Since, length is constant, wavelength λ will become half when the number of loops become two times.

Further frequency $f = \frac{v}{\lambda} = \text{consant}$

So, v should also become $\frac{1}{2}$ times $\{\because v \propto \sqrt{T}\}$

Therefore, speed v will become $\frac{1}{2}$ times when mass on the pan is reduced to $\frac{M}{4}$

Hence, the correct answer is (B).

171. Since, $425 : 595 : 765 \equiv 5 : 7 : 9$

So, pipe must be a closed organ pipe

Hence, $\frac{5v}{4\ell} = 425$

$\Rightarrow \frac{5(340)}{4\ell} = 425$

$\Rightarrow \ell = 1 \text{ m}$

Hence, the correct answer is (A).

172. $f_0 = \frac{v}{4\ell} = \frac{340}{4} = 85 \text{ Hz}$

Hence, the correct answer is (D).

173. $v = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma\left(\frac{RT}{M}\right)}$

$\Rightarrow \frac{v_{H_2}}{v_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}}$

$\Rightarrow \frac{v_{H_2}}{v_{O_2}} = 4$

Hence, the correct answer is (C).

174. Due to weight of the rope, the tension will increase along the rope from the lower end to the upper end. Hence, the pulse will travel with increasing speed as

$$v = \sqrt{\frac{T}{\mu}}$$

Hence, the correct answer is (B).

175. $v = f\lambda$

$\Rightarrow 25 = f(100)$

$\Rightarrow f = 0.25 \text{ Hz}$

$\Rightarrow T = \frac{1}{f} = 4 \text{ s}$

Hence, the correct answer is (A).

176. $y = 2a\sin(\omega t)\cos(kx)$

$\Rightarrow k = \frac{\pi}{15}$

$\Rightarrow \frac{2\pi}{\lambda} = \frac{\pi}{15}$

$\Rightarrow \lambda = 30 \text{ cm}$

Distance between a node and antinode is $\frac{\lambda}{4}$.

So, $d = 7\cdot5$ cm

Hence, the correct answer is (A).

177. Since, the frequencies are in the ratio of $5:7:9$, so, it is a closed pipe

$$425 \text{ Hz} = 5\left(\frac{v}{4\ell}\right)$$

$$\Rightarrow \quad l = \frac{5v}{4\times425} = \frac{5\times340}{4\times425} \text{ m} = 1 \text{ m}$$

Hence, the correct answer is (C).

178. Let a_i and a_r be the amplitudes of incident and reflected waves. Then

$$\frac{a_i + a_r}{a_i - a_r} = n \qquad \{\text{given}\}$$

$$\Rightarrow \quad \frac{a_r}{a_i} = \left(\frac{n-1}{n+1}\right)$$

So, fraction of energy reflected is

$$\frac{E_r}{E_i} = \left(\frac{a_r}{a_i}\right)^2 = \left(\frac{n-1}{n+1}\right)^2$$

Hence, the correct answer is (B).

179. $I_{\text{total}} = n^2 a^2 = n^2 I_0$

$$\Rightarrow \quad I_{\text{total}} = (10)^2 I_0 = 100 I_0$$

$$\Rightarrow \quad I_{av} = \langle I \rangle = \frac{100 I_0}{10} = 10 I_0$$

Hence, the correct answer is (C).

180. $f_1 = 50$ Hz

Second overtone in a closed organ pipe is the fifth harmonic (as even harmonics are absent), So

$$f_5 = 5f_1 = 250 \text{ Hz}$$

Hence, the correct answer is (D).

181. $f' = f\left(\frac{v-0}{v-v_s}\right) = 90\frac{v}{\left(v-\frac{v}{10}\right)} = 90\times\frac{10}{9} = 100 \text{ Hz}$

Hence, the correct answer is (A).

182. Since, $f_b = 6$ bps $= 6$ Hz

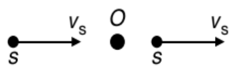
$$\Rightarrow \quad 6 = \frac{v}{50} - \frac{v}{50\cdot4}$$

$$\Rightarrow \quad v = 378 \text{ ms}^{-1}$$

Hence, the correct answer is (C).

183. Since, $\Delta f = f_1 - f_2$

$$\Rightarrow \quad \Delta f = f\left(\frac{v}{v-v_s}\right) - f\left(\frac{v}{v+v_s}\right)$$



$$\Rightarrow \quad \Delta f = f\left[\left(1-\frac{v_s}{v}\right)^{-1} - \left(1+\frac{v_s}{v}\right)^{-1}\right]$$

$$\Rightarrow \quad \Delta f \approx f\left[\left(1+\frac{v_s}{v}\right) - \left(1-\frac{v_s}{v}\right)\right] = \frac{2fv_s}{v}$$

Hence, the correct answer is (A).

Multiple Correct Choice Type Questions

1. $v = \sqrt{\frac{T}{\mu}}$

For equilibrium $Mg = mg\sin30 = T$

$$\Rightarrow \quad M = \frac{m}{2}$$

$$\Rightarrow \quad 100 = \sqrt{\frac{Mg}{9.8\times10^{-3}}} = \sqrt{\frac{M(9.8)}{9.8\times10^{-3}}}$$

$$\Rightarrow \quad 100 = \sqrt{M(1000)}$$

$$\Rightarrow \quad M = 10 \text{ kg and } m = 20 \text{ kg}$$

Hence, (A) and (D) are correct.

2. $y = 10^{-4}\sin(60t + 2x)$

$A = 10^{-4}$ m, $\omega = 60$ rads^{-1}, $k = 2$ m^{-1}

Speed of wave $v = \frac{\omega}{k} = 30$ ms^{-1}

Frequency, $f = \frac{\omega}{2\pi} = \frac{30}{\pi}$ Hz. Wavelength, $\lambda = \frac{2\pi}{k} = \pi m$

Further, $60t$ and $2x$ are of same sign. Therefore, the wave should travel in negative x-direction.

Hence, (A), (B), (C) and (D) are correct.

3. $y = a\sin(\alpha t - \beta x)$

Compare with the standard equation of progressive wave i.e.

$$y = a\sin(\omega t - kx)$$

$$\Rightarrow \quad \omega = \alpha \text{ and } k = \beta$$

$$\Rightarrow \quad v = \frac{\omega}{k} = \frac{\alpha}{\beta} \qquad \{\text{OPTION (D)}\}$$

$$\Rightarrow \quad 2\pi f = \alpha$$

$$\Rightarrow \quad f = \frac{\alpha}{2\pi}$$

$$\Rightarrow \quad T = \frac{2\pi}{\alpha} \qquad \{\text{OPTION (B)}\}$$

Since, $k = \frac{2\pi}{\lambda} = \beta$

$$\Rightarrow \quad \lambda = \frac{2\pi}{\beta} \qquad \{\text{OPTION (C)}\}$$

Hence, (B), (C) and (D) are correct.

4. Let the progressive wave equation be

$$y = a\sin(\omega t - kx)$$

$$\Rightarrow \quad \frac{\partial y}{\partial t} = a\omega\cos(\omega t - kx) \qquad \ldots(1)$$

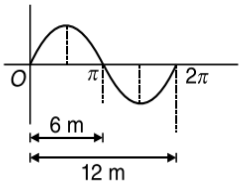
$$\Rightarrow \quad \frac{\partial^2 y}{\partial t^2} = -a\omega^2\sin(\omega t - kx) \qquad \ldots(2)$$

Also $\frac{\partial y}{\partial x} = -ak\cos(\omega t - kx) \qquad \ldots(3)$

$$\Rightarrow \frac{\partial^2 y}{\partial x^2} = ak^2 \sin(\omega t - kx) \quad \ldots(4)$$

Hence, (A) and (B) are correct.

5. $\lambda = \frac{v}{f} = \frac{300}{25} = 12 \text{ m}$



Amplitude difference = zero = A

Phase Difference = $\pi = \phi$

Hence, (A) and (D) are correct.

6. Maximum speed of any point on the string is $v_{max} = a\omega$

$$\Rightarrow v_{max} = a(2\pi f)$$

$$\Rightarrow v_{max} = \frac{v}{10} = \frac{10}{10} = 1 \quad \{\text{Given: } v = 10 \text{ ms}^{-1}\}$$

$$\Rightarrow 2\pi af = 1$$

$$\Rightarrow f = \frac{1}{2\pi a}$$

Since, $a = 10^{-3}$ m

$$\Rightarrow f = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi} \text{ Hz}$$

Speed of wave, $v = f\lambda$

$$\Rightarrow (10 \text{ ms}^{-1}) = \left(\frac{10^3}{2\pi} \text{ s}^{-1}\right)\lambda$$

$$\Rightarrow \lambda = 2\pi \times 10^{-2} \text{ m}$$

Hence, (A) and (C) are correct.

7. Wavelength depends on length which is fixed. Therefore, wavelength does not change.

Further $v = \sqrt{\frac{T}{m}}$

$$\Rightarrow v \propto T^{1/2}$$

$$\Rightarrow (\% \text{ change in } v) = \frac{1}{2}(\% \text{ change in } T)$$

$$\Rightarrow (\% \text{ change in } v) = \frac{1}{2}(2) = 1\%$$

i.e., speed and hence frequency will change by 1%

Change in frequency is 15 Hz which is 1% of 1500 Hz

Therefore, original frequency should be 1500 Hz.

Hence, (B), (C) and (D) are correct.

8. $T_1 > T_2$

$$\Rightarrow v_1 > v_2$$

$$\Rightarrow f_1 > f_2 \text{ and } f_1 - f_2 = 6 \text{ Hz}$$

Now, if T_1 is increased, f_1 will increase or $f_1 - f_2$ will increase. Therefore (D) OPTION is wrong.

If T_1 is decreased f_1 will decrease and it may be possible that now $f_2 - f_1$ become 6 Hz. Therefore (C) OPTION is correct.

Similarly, when T_2 is increased, f_2 will increase and again $f_2 - f_1$ may become equal to 6 Hz. Therefore (C) OPTION is correct.

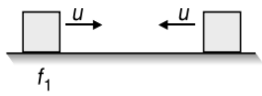
Similarly, when T_2 is increased, f_2 will increase and again $f_2 - f_1$ may become equal to 6 Hz. So, (B) is also correct but (A) is wrong.

Hence, (B) and (C) are correct.

9. Conceptual

Hence, (B) and (C) are correct.

10. $f_2 = \left(\frac{V - w + u}{V - w - u}\right) f_1$



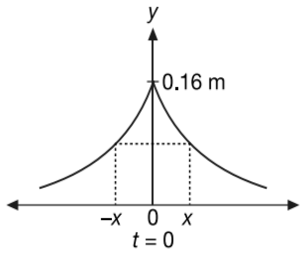
So, $f_2 > f_1$

$$f_2 = \left(\frac{V + w + u}{V + w - u}\right) f_1$$

So, $f_2 > f_1$

Hence, (A) and (B) are correct.

11. The shape of pulse at $x = 0$ and $t = 0$ would be as shown, in Figure.



$$y(0, 0) = \frac{0.8}{5} = 0.16 \text{ m}$$

From the figure it is clear that $y_{max} = 0.16$ m

Pulse will be symmetric (Symmetry is checked about y_{max}) if at $t = 0$

$$y(x) = y(-x)$$

From the given equation

$$y(x) = \frac{0.8}{16x^2 + 5}$$

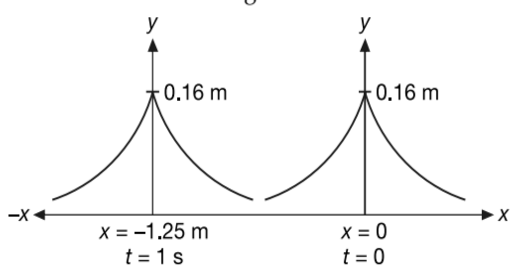
$$\text{and } y(-x) = \frac{0.8}{16x^2 + 5} \quad \{\text{at } t = 0\}$$

$$\Rightarrow y(x) = y(-x)$$

Therefore, pulse is symmetric.

Speed of pulse

At $t = 1$ s and $x = -1.25$ m value of y is again 0.16 m, i.e., pulse has travelled a distance of 1.25 m in 1 s in negative x-direction or we can say that the speed of pulse is 1.25 ms^{-1} and it is travelling in negative x-direction. Therefore, it will travel a distance of 2.5 m in 2 s. The above statement can be better understood from Figure.



Alternate Method:

If equation of a wave pulse is

$$y = f(ax \pm bt)$$

The speed of wave is $\frac{b}{a}$ in negative x-direction for $y = f(ax+bt)$ and positive x-direction for $y = f(ax-bt)$. Comparing this from given equation we can find that speed of wave is $\frac{5}{4} = 1.25\ \text{ms}^{-1}$ and it is travelling in negative x-direction.

Hence, (B), (C) and (D) are correct.

12. For closed organ pipe,

$$f = n\left(\frac{v}{4\ell}\right) \text{ where, } n = 1, 3, 5, \ldots..$$

$$\Rightarrow \quad \ell = \frac{nv}{4f}$$

For $n = 1$, $\ell_1 = \frac{(1)(330)}{4 \times 264} \times 100\ \text{cm} = 31.25\ \text{cm}$

For $n = 3$, $\ell_3 = 3\ell_1 = 93.75\ \text{cm}$

For $n = 5$, $\ell_5 = 5\ell_1 = 156.25\ \text{cm}$

Hence, (A) and (C) are correct.

13. The fundamental frequency of oscillation is

$$f = \frac{1}{2\ell}\sqrt{\frac{T}{m}} = \frac{1}{2\ell}\sqrt{\frac{T}{\rho S}}$$

Here, m = mass per unit length of wire $= \rho S$

where S = area of cross-section of wire

$$\frac{T}{S} = \text{stress} = Y(\text{strain}) = Y\left(\frac{\Delta\ell}{\ell}\right) = Y \propto t$$

$$\Rightarrow \quad f = \frac{1}{2\ell}\sqrt{\frac{Y\alpha t}{\rho}}$$

$$\Rightarrow \quad f \propto \frac{1}{\ell} \propto \sqrt{Y} \propto \sqrt{t} \text{ and } f \propto \sqrt{\frac{\alpha}{\rho}}$$

Hence, (A), (B), (C) and (D) are correct.

14. $\omega = 15\pi$, $k = 10\pi$

Speed of wave, $v = \frac{\omega}{k} = 1.5\ \text{ms}^{-1}$

Wavelength of wave $\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = 0.2\ \text{m}$

$10\pi x$ and $15\pi t$ have the same sign. Therefore, wave is travelling in negative x-direction.
Hence, (B) and (C) are correct.

15. In plane progressive harmonic wave particles execute SHM and in SHM phase difference is

$\Delta\phi = \pi$ between displacement and acceleration

$\Delta\phi = \frac{\pi}{2}$ between displacement and velocity

and $\Delta\phi = \frac{\pi}{2}$ between velocity and acceleration

Hence, (B), (C) and (D) are correct.

16. For a plane wave intensity (energy crossing per unit area per unit time) is constant at all points.

But for a spherical wave, intensity at a distance r from a point source of power P (energy transmitted per unit time) is given by

$$I = \frac{P}{4\pi r^2}$$

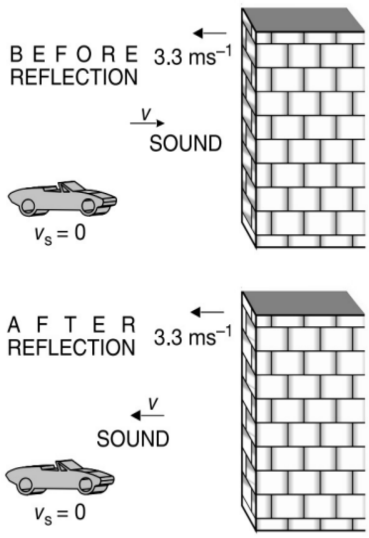
$$\Rightarrow \quad I \propto \frac{1}{r^2}$$

For a line source $I \propto \frac{1}{r}$ because $I = \frac{P}{\pi r \ell}$

Hence, (A), (C) and (D) are correct.

17. Conceptual
Hence, (B), (C) and (D) are correct.

18. $f' = f\left(\frac{v + v_{\text{wall}}}{v - v_{\text{wall}}}\right)$



$$\Rightarrow \quad f' = 1000\left(\frac{340 + 3.3}{340 - 3.3}\right) \approx 1020\ \text{Hz}$$

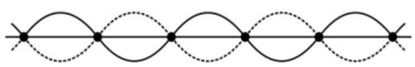
$$\Rightarrow \quad \frac{f' - f}{f} \times 100 = \frac{\Delta f}{f} \times 100 = 2\%$$

Hence, (A) and (C) are correct.

19. For fifth harmonic, number of loops formed is 5, so

No. of nodes = 6

$$\lambda = \frac{2\pi}{k} = \frac{2 \times 3.14}{62.8} = 0.1\ \text{m}$$



Length $= \frac{5\lambda}{2} = 0.25\ \text{m}$

The mid-point is antinode. Its maximum displacement $= 0.01\ \text{m}$

$$\Rightarrow \quad f = \frac{v}{2\ell} = \frac{\omega}{k \times 2\ell} = 20\ \text{Hz}$$

Hence, (B) and (C) are correct.

20. Velocity of particle is given by $v_p = -v\left(\frac{dy}{dx}\right)$

Here, v is wave speed and $\frac{dy}{dx}$ the slope.

At point E slope is positive, therefore, v_p will be along negative x-direction. Similarly, slope at D is zero. Therefore, v_p at D will be zero.

Excess pressure $dP = -B \cdot \frac{dy}{dx}$

At C slope is negative. Therefore, dP is positive i.e., particles located near C are under compression.

At point D, slope is zero i.e., $dP = 0$.

Hence, (A), (B), (C) and (D) are correct.

21. Standing waves can be produced only when two similar type of waves (same frequency and speed, but amplitude may be different) travel in opposite directions.
Hence, (A), (B) and (C) are correct.

22. Any function $y = f(at \pm bx)$ represents a wave if it is finite everywhere and at all times. The function

$$y = \frac{1}{4t + 3x}$$

is not defined at $x = 0$ and $t = 0$

Hence, (A) and (B) are correct.

23. The wave travels from left to right. Therefore, points lying leftwards are always ahead in phase. Further

Particle velocity $= -(\text{wave speed})(\text{slope})$

Slope at A is positive, while at B is negative i.e., particle velocity at A is negative and at B is positive. Therefore, A is moving downwards while B is moving upwards.
Hence, (A) and (D) are correct.

24. Velocity of wave is $v = \frac{\omega}{k} = \frac{4}{5} = 0.8 \text{ ms}^{-1}$

The displacement of a particle of the string at $t = 0$ and $x = \frac{\pi}{30}$ m from the mean position is

$$y = 8\sin\left(\frac{5\pi}{30} - 0\right) = 8\sin\left(\frac{\pi}{6}\right) = 8\left(\frac{1}{2}\right) = 4 \text{ m}$$

Hence, (A) and (B) are correct.

25. From the given expression for y:

amplitude $A = 0.02$ m

angular frequency $\omega = 50\pi$ rads^{-1}

and wave number $k = 10\pi$ m^{-1}

Now wave speed $v = \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5 \text{ ms}^{-1}$

Therefore, OPTION (D) is wrong

Displacement node occurs at $10\pi x = \frac{\pi}{2}, \frac{3\pi}{2}$ etc.

$$\Rightarrow \quad x = \frac{1}{20}, \frac{3}{20}$$

$\Rightarrow \quad x = 0.05$ m and 0.15 m

Displacement antinode occurs at

$10\pi x = 0, \pi, 2\pi, 3\pi$ etc.

$\Rightarrow \quad x = 0, 0.1$ m, 0.2 m and 0.3 m

Wavelength $\lambda = 2$ (distance between two consecutive nodes or antinodes)

$\Rightarrow \quad \lambda = 2(0.1) = 0.2$ m

Hence, (A), (B) and (C) are correct.

26. ABC has negative slope hence, represents compression, while CDE has positive slope hence, represents rarefaction.

Hence, (A) and (D) are correct.

27. The number of waves encountered by the moving plane per unit time is given by

$$n = \frac{\text{distance travelled}}{\text{wavelength}}$$

$$\Rightarrow \quad n = \frac{c+v}{\lambda} = \frac{c}{\lambda}\left(1 + \frac{v}{c}\right) = f\left(1 + \frac{v}{c}\right) \qquad \text{\{OPTION (A)\}}$$

The stationary observer meets the frequency f' of the incident wave and receives the reflected wave of frequency f'' emitted by the moving platform as

$$f'' = \frac{f'}{1 - \frac{v}{c}} = \frac{f\left(1 + \frac{v}{c}\right)}{1 - \frac{v}{c}} = \frac{f(c+v)}{(c-v)} \qquad \text{\{OPTION (C)\}}$$

$$\text{Wavelength, } \lambda'' = \frac{c}{f''} = \frac{c}{f}\left(\frac{c-v}{c+v}\right) \qquad \text{\{OPTION (B)\}}$$

Beat frequency, $f_b = f'' - f$

$$\Rightarrow \quad f_b = \frac{f\left(1 + \frac{v}{c}\right)}{\left(1 - \frac{v}{c}\right)} - f = f\left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} - 1\right)$$

$$\Rightarrow \quad f_b = \frac{\left(1 + \frac{v}{c}\right)f}{\left(1 - \frac{v}{c}\right)} = \frac{2vf}{c - v}$$

Hence, (A), (B) and (C) are correct.

28. Conceptual
Hence, (A), (B) and (D) are correct.

29. As $f_1 : f_2 : f_3 = 3:5:7$, string is fixed at one end. Its fundamental frequency is $f_0 = \frac{f_1}{3} = \frac{105}{3} = 35$ Hz.

Hence, (B) and (C) are correct.

30. Comparing the given equation with standard equation of stationary wave

$$y = 2a\sin(kx)\cos(\omega t)$$

we have $a = 2$ mm

$$k = \frac{2\pi}{\lambda} = 3.14$$

$$\Rightarrow \quad \frac{\lambda}{2} = 1 \text{ m}$$

So, the smallest possible length is $\frac{\lambda}{2}$ or 1 m.

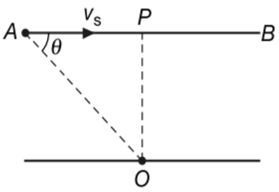
Hence, (A) and (D) are correct.

31. $T=\left(\frac{M}{L}\right)xg=\left(\begin{array}{c}\text{weight of part of rope hanging}\\ \text{below the point under}\\ \text{consideration.}\end{array}\right)$

Since $v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{\left(\frac{M}{L}\right)xg}{\left(\frac{M}{L}\right)}}=\sqrt{xg}$

Hence, (B) and (D) are correct.

32. The graph shows the situation shown in figure. The observed frequency will initially be more than the natural frequency. When the source is at P, natural frequency i.e., 2000 Hz.



For region $AP: f=f_0\left(\frac{v}{v-v_s\cos\theta}\right)$

For $PB: f=f_0\left(\frac{v}{v+v_s\cos\theta}\right)$

Minimum value of f will be

$$f_{\min}=f_0\left(\frac{v}{v+v_s}\right) \quad \text{when } \cos\theta=1$$

$$\Rightarrow \quad 1800=2000\left(\frac{300}{300+v_s}\right)$$

Solving this we get,

$v_s=33.33\ \text{ms}^{-1}$ and maximum value of f can be

$$f_{\max}=f_0\left(\frac{v}{v-v_0}\right) \quad \text{when } \cos\theta=1$$

$$\Rightarrow \quad f_{\max}=2000\left(\frac{300}{300-33.33}\right)=2250\ \text{Hz}$$

Hence, (C) and (D) are correct.

33. For closed pipe, $f=n\left(\frac{v}{4\ell}\right)n=1,3,5.....$

For $n=1$, $f_1=\frac{v}{4\ell}=\frac{320}{4\times1}=80\ \text{Hz}$

For $n=3$, $f_3=3f_1=240\ \text{Hz}$

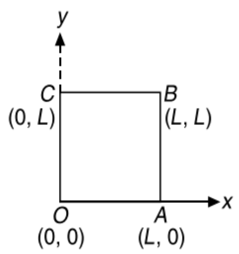
For $n=5$, $f_5=5f_1=400\ \text{Hz}$

Hence, (A), (B) and (D) are correct.

34. Conceptual

Hence, (C) and (D) are correct.

35. Since, the edges are clamped, displacement of the edges $u(x,\ y)=0$ for



Line OA i.e., $y=0,\ 0\le x\le L$

AB i.e., $x=L,\ 0\le y\le L$

BC i.e., $y=L\ \ 0\le x\le L$

OC i.e., $x=0,\ 0\le y\le L$

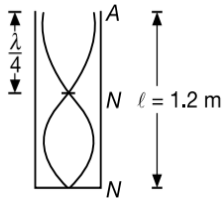
The above conditions are satisfied only in alternatives (B) and (C).

Note that $u(x,\ y)=0$, for all four values e.g., in alternative (D), $u(x,\ y)=0$ for $y=0$, $y=L$ but it is not zero for $x=0$ or $x=L$. Similarly, in OPTION (A) $u(x,\ y)=0$ at $x=L$, $y=L$ but it is not zero for $x=0$ or $y=0$ while in OPTIONS (B) and (C), $u(x,\ y)=0$ for $x=0$, $y=0$, $x=L$ and $y=L$.

Hence, (B) and (C) are correct.

36. $\ell=\frac{3\lambda}{4}$

$\Rightarrow \quad \frac{\lambda}{4}=\frac{\ell}{3}=0.4\ \text{m}$



Pressure variation will be maximum at displacement nodes i.e., at 0.4 m from the open end and at closed end.

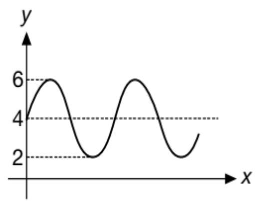
Hence, (B) and (C) are correct.

37. In case of sound wave, y can represent pressure and displacement, while in case of an electromagnetic wave it represents electric and magnetic fields.

NOTE: In general, y is any general physical quantity which is a made to oscillate at one place and these oscillations are propagated to other places also.

Hence, (A), (B), (C) and (D) are correct.

38. The shape of the equation is as follows:



From this we can see that amplitude of wave is 2 units.

Wave speed $v=\frac{\text{coefficient of } t}{\text{coefficient of } x}$

$\Rightarrow \quad v=\frac{6}{3}=2$ units

Hence, (B) and (C) are correct.

Reasoning Based Questions

1. In resonance tube, the vibrations are set-up in the air column which only depends on the length of the resonance column and velocity of sound. The liquid in the tube only reflects the waves which upon superposition produce stationary waves. Therefore, if oil of density higher than that of water is used, there will be no effect on the frequency of waves set-up.

Hence, the correct answer is (D).

2. It is known that all the stars are moving away from each other. Therefore, apparent frequency of light emitted from a star as received by an observer on the earth is less than its actual frequency. Since, wavelength is inversely proportional to the frequency, the apparent wavelength of the light from the star is more than the actual wavelength. In other

words, wavelength of light shifts towards longer end i.e., towards red end of the visible spectrum, this is red shift.
Hence, the correct answer is (A).

3. The gases possess only volume elasticity (gases do not possess shear elasticity). As such, only longitudinal waves can propagate in gases. On the other hand, the solids, possess both volume and shear elasticity and likewise both the longitudinal and transverse waves can propagate through the solids.
Hence, the correct answer is (B).

4. If a closed pipe of length L is in resonance with a tuning fork of frequency ν, then

$$\nu = \frac{v}{4L}$$

An open pipe of same length L produces vibrations of frequency $\frac{v}{2L}$. Obviously, it cannot be in resonance with the given tuning fork of frequency $\nu\left(=\frac{v}{4L}\right)$.

Hence, the correct answer is (C).

5. For an isothermal process, $PV = \text{constant}$.
Differentiating both sides, we get

$$PdV + VdP = 0$$

$$\Rightarrow \quad PdV = -VdP$$

$$\Rightarrow \quad B_{\text{isot}} = -\frac{dP}{\left(\frac{dV}{V}\right)} = P \qquad \ldots(1)$$

For an adiabatic process, $PV^{\gamma} = \text{constant}$
Differentiating both sides,

$$P\left(\gamma V^{\gamma-1}\right)dV + V^{\gamma}dP = 0$$

$$\Rightarrow \quad \gamma PdV + VdP = 0$$

$$\Rightarrow \quad B_{\text{ad}} = -\frac{dP}{\left(\frac{dV}{V}\right)} = \gamma P \qquad \ldots(2)$$

From equation (1) and (2)

$$B_{\text{ad}} = \gamma B_{\text{isot}}$$

As $\gamma > 1$, therefore $B_{\text{ad}} > B_{\text{isot}}$

Hence, the correct answer is (B).

6. Only transverse waves can be polarised. Sound waves (mechanical wave) cannot be polarised as they are longitudinal in nature whereas light waves can be polarised as they are transverse in nature.
Hence, the correct answer is (A).

7. According to Newton, speed of sound in gases,

$$v = \sqrt{\frac{K_{\text{iso}}}{\rho}} = \sqrt{\frac{P}{\rho}}$$

Laplace pointed out that since the wave propagates quickly in the medium and the changes taking place in the gases due to the propagation of sound cannot be isothermal but are adiabatic in nature, he corrected the Newton's formula accordingly, i.e.,

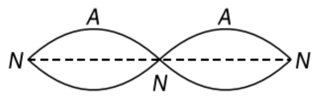
$$v = \sqrt{\frac{K_{\text{adia}}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

Hence, the correct answer is (A).

8. For open organ pipe fundamental frequency is expressed as $n = \frac{v}{2\ell}$ here v is the velocity and ℓ is the length of organ pipe. Since, the velocity increases rapidly with the increase of temperature in comparison with the increase in its length. Therefore, it is clear that the frequency increases with the increase of temperature.
Hence, the correct answer is (A).

9. Intensity of sound at any point is energy flowing per second per unit area around that point. So, it will not change due to motion of listener.
The apparent wavelength changes with the motion of listener.
Hence, the correct answer is (B).

10. Stationary wave is represented as shown in figure.



From the figure we observe that at nodes the amplitude is zero and velocity of particle is also zero and at antinodes the amplitude is maximum. Hence the velocity of particle is also maximum and all particles cross mean position between two successive nodes.
Hence, the correct answer is (A).

11. The wood offers high damping to the sound waves. For making the bells the low damping is required and for low damping the metals are most suitable. Hence, the bells are made of metals.
Hence, the correct answer is (A).

12. Sound has greater speed in solid than in air. Hence, when ear is placed on the rails the sound of train coming from some distance is heard. Hence, Statement-1 is true and Statement-2 is false.
Hence, the correct answer is (C).

13. Sound coming from the different sources can be recognised by virtue of their quality which is characteristics of sound. That is why we recognise the voices of our friends.
Hence, the correct answer is (A).

14. The speed of sound in gaseous medium is given by

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma\left(\frac{RT}{M}\right)} \qquad \ldots(1)$$

At constant temperature

$$PV = \text{constant} \qquad \ldots(2)$$

If V is the volume of one mole of gas, then density of gas $\rho = \frac{M}{V}$ or $V = \frac{M}{\rho}$

where M is the molecular weight of the gas.

CHAPTER 4

Therefore, Equation (2) becomes, $\frac{PM}{\rho}=\text{constant}$

$\Rightarrow \quad \frac{P}{\rho}=\text{constant as } M \text{ is a constant}$

$\Rightarrow \quad v=\text{constant}$

Thus, change in air pressure does not affect the speed of sound.
Statement-2 is clear from Equation (1).
Hence, the correct answer is (D).

15. In stationary wave, total energy associated with it is twice the energy of each of incidence and reflected wave.
Large amount of energy are stored equally in standing waves and is trapped within the waves. Hence, there is no transmission of energy through the waves.
Hence, the correct answer is (B).

16. When the source of sound (engines) moves towards the listener, the apparent frequency,

$$\nu_1=\frac{v}{v-v_s}\nu$$

Or the other hand, if the source of sound moves away from the listener, the apparent frequency

$$\nu_2=\frac{v}{v+v_s}\nu$$

If follow that $v_1>v_2$. Since pitch of a sound depends on frequency, the whistle of the approaching engine is shriller that the receding engine.
Hence, the correct answer is (C).

17. Doppler's effect is observed readily in sound waves due to the larger wavelength. This effect is not followed with light due to shorter wavelength. The velocity of light is $3\times10^8\ \text{ms}^{-1}$ and $332\ \text{ms}^{-1}$ velocity of sound.
Hence, the correct answer is (B).

18. At the point where a compression and a rarefaction meet, the displacement is minimum and it is called displacement node. At this point, the pressure difference is maximum i.e., at the same time, it is a pressure antinode. On the other hand, at the mid-point of a compression or a rarefaction, the displacement variation is maximum i.e., such a point is displacement antinode. However, such a point is pressure node, as pressure variation is minimum at such a point.
Hence, the correct answer is (B).

19. The resultant amplitude of two waves is given by

$$A=\sqrt{a_1^2+a_2^2+2a_1a_2\cos\theta}$$

Here, $a_1=a_2=A=a$

$$\frac{1}{2}=1+\cos\theta$$

$$\Rightarrow \quad \cos\theta=-\frac{1}{2}$$

$$\Rightarrow \quad \theta=120^o$$

Statement-2 is incorrect as it is just opposite to the Principle of Superposition of waves.
Hence, the correct answer is (C).

20. Experimental value of velocity of sound in air is $332\ \text{ms}^{-1}$
Theoretical value of velocity of sound from Newton's formula is $280\ \text{ms}^{-1}$
Difference between the two values is $332-280=52\ \text{ms}^{-1}$

$$\text{Percentage error}=\frac{332-280}{332}\times100=16\%$$

Hence, the correct answer is (C).

Linked Comprehension Type Questions

1. $y=4\sin\frac{\pi x}{15}\cos(96\pi t)=A_x\cos(96\pi t)$

Here, $A_x=4\sin\frac{\pi x}{15}$

at $x=5\ \text{cm}$, $A_x=4\sin\frac{5\pi}{15}=4\sin\frac{\pi}{3}=2\sqrt{3}\ \text{cm}$

This is the amplitude or maximum displacement at $x=5$ cm.
Hence, the correct answer is (C).

2. Nodes are located where $A_x=0$

or $\frac{\pi x}{15}=0,\ \pi,\ 2\pi,\ .....$

or $x=0,\ 15\ \text{cm},\ 30\ \text{cm}$ etc.

Hence, the correct answer is (B).

3. Velocity of particle,

$$v_p=\left.\frac{\partial y}{\partial t}\right|_{x=\text{constant}}=-384\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$$

At $x=7.5$ cm and $t=0.25$ s

$$v_p=-384\pi\sin\left(\frac{\pi}{2}\right)\sin(24\pi)=0$$

Hence, the correct answer is (A).

4. Amplitude of components waves is $A=\frac{4}{2}=2\ \text{cm}$

$$\omega=96\pi \text{ and } k=\frac{\pi}{15}$$

Component waves are, $y_1=2\sin\left(\frac{\pi}{15}x-96\pi t\right)$ and $y_2=2\sin\left(\frac{\pi}{15}x+96\pi t\right)$

Hence, the correct answer is (D).

5. Since $f_{\text{app}}=\left(\frac{v}{v-v_s}\right)f_0$

$$\Rightarrow \quad 2.2\times10^3=\frac{300}{300-30}f_0$$

$$\Rightarrow \quad f_0=\frac{2.2\times10^3\times270}{300}\ \text{Hz}$$

$$\Rightarrow \quad f_0=1980\ \text{Hz}=1.98\ \text{kHz}$$

Hence, the correct answer is (B).

6. $f_{\text{app}}=2.2\times10^3=\frac{300}{300-v_s}f_0$ ...(1)

$f_{\text{recede}}=1.8\times10^3=\frac{300}{300+v_s}f_0$ ...(2)

Dividing equation (1) by (2), we get

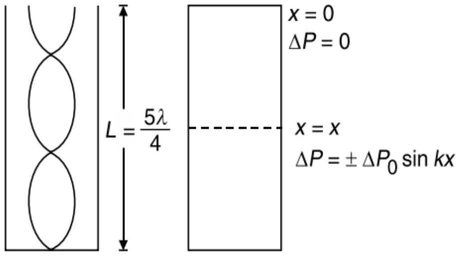
$$\frac{22}{18}=\frac{300-v_s}{300+v_s}$$

$$\Rightarrow \quad 40v_s=(22-18)300$$

$$\Rightarrow \quad v_s=\frac{1200}{40}\text{ ms}^{-1}=30\text{ ms}^{-1}$$

Hence, the correct answer is (D).

7. Frequency of second overtone of the closed pipe



$$\lambda_5=5\left(\frac{v}{4L}\right)=440$$

$$\Rightarrow \quad L=\frac{5v}{4\times 440}\text{ m}$$

Substituting v = speed of sound in air = 330 ms^{-1}

$$L=\frac{5\times 330}{4\times 440}=\frac{15}{16}\text{ m}$$

Hence, the correct answer is (B).

8. $\lambda=\frac{4L}{5}=\frac{4\left(\frac{15}{16}\right)}{5}=\frac{3}{4}$ m

Hence, the correct answer is (D).

9. Open end is displacement antinode. Therefore, it would be a pressure node.

or at $x=0$, $\Delta P=0$

Pressure amplitude at $x=x$, can be written as

$$\Delta P=\pm\Delta P_o\sin kx$$

Where $k=\frac{2\pi}{\lambda}=\frac{2\pi}{3/4}=\frac{8\pi}{3}\text{ m}^{-1}$

Therefore, pressure amplitude at $x=\frac{L}{2}=\frac{15/16}{2}$ m or $\left(\frac{15}{32}\right)$ m will be

$$\Delta P=\pm\Delta P_o\sin\left(\frac{8\pi}{3}\right)\left(\frac{15}{32}\right)=\pm\Delta P_o\sin\left(\frac{5\pi}{4}\right)$$

$$\Delta P=\pm\frac{\Delta P_o}{\sqrt{2}}$$

Hence, the correct answer is (C).

10. Open end is a pressure node i.e., $\Delta P=0$.

Hence, $P_{\max}=P_{\min}$ = Mean pressure (P_o)

Hence, the correct answer is (C).

11. Closed end is a displacement node or pressure antinode.

Therefore, $P'_{\max}=P_o+\Delta P_o$

and $P'_{\min}=P_o-\Delta P_o$

Hence, the correct answer is (B).

12. E is the mean position of oscillation of the particle

So, the kinetic energy is maximum at E

Hence, the correct answer is (B).

13. The stretching of string is maximum at E

So, the elastic potential energy is maximum at E.

Hence, the correct answer is (B).

14. B is moving up and D is moving down.

Hence, the correct answer is (C).

15. The wave reflected from denser medium will suffer an additional phase change of π. So

$$y_r=A\cos(\omega t-kx+\phi+\pi)$$

$$\Rightarrow \quad y_r=-A\cos(\omega t-kx+\phi)$$

Hence, the correct answer is (D).

16. Conceptual

Hence, the correct answer is (C).

17. Conceptual

Hence, the correct answer is (D).

18. **The correct answer is (B).**

19. **The correct answer is (A).**

Combined solution to 18 and 19

$$y_i+y_r=y_t\text{ (at point }P)$$

$$\Rightarrow \quad A_i+A_r=A_t \qquad \ldots(1)$$

Also, $\frac{\partial y_i}{\partial x}+\frac{\partial y_r}{\partial x}=\frac{\partial y_t}{\partial x}$ (at point P)

$$\Rightarrow \quad A_i+A_r=A_t\frac{k_t}{k_i} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$A_r=\frac{(k_i-k_t)}{(k_i+k_t)}A_i\text{ and }A_r=\frac{2k_i}{(k_i+k_t)}A_i$$

Since, tension T is same in both the strings, so we have

$$v=\frac{\omega}{k}$$

$$\Rightarrow \quad \sqrt{\frac{T}{\mu}}=\frac{\omega}{k}$$

As T and ω are same in both the strings, so

$$k\propto\sqrt{\mu}$$

$$\Rightarrow \quad A_r=\frac{(\sqrt{\mu_1}-\sqrt{\mu_2})}{(\sqrt{\mu_1}+\sqrt{\mu_2})}A_i\text{ and }A_t=\frac{2(\sqrt{\mu_1})}{(\sqrt{\mu_1}+\sqrt{\mu_2})}A_i$$

20. Path difference between the waves reaching D is

$$\Delta x=\frac{3\pi R}{2}-\frac{\pi R}{2}=\frac{2\pi R}{2}=\pi R$$

Since, $I_R=I_1+I_2+2\sqrt{I_1I_2}\cos\phi$

$\Rightarrow \quad I_R = \left(\sqrt{I_1}+\sqrt{I_2}\right)^2$ when $\phi = 2n\pi$, $n = 0, 1, 2, 3, \ldots$

Since the wave generated is divided equally in two parts, so we have

$$I_1 = I_2 = \frac{I_0}{2}$$

$$\Rightarrow \quad I_R = \left(\sqrt{\frac{I_0}{2}}+\sqrt{\frac{I_0}{2}}\right)^2$$

$$\Rightarrow \quad I_R = 2I_0$$

Hence, the correct answer is (C).

21. Since $\Delta x = \pi R$

For maxima, we have

$$\Delta\phi = (2n)\pi, \text{ where } n = 0, 1, 2, 3, \ldots$$

$$\Rightarrow \quad \left(\frac{2\pi}{\lambda}\right)(\pi R) = (2n)\pi$$

$$\Rightarrow \quad \lambda_{max} = \frac{\pi R}{n_{min}} = \frac{\pi R}{1}$$

$$\Rightarrow \quad \lambda_{max} = \pi R$$

Hence, the correct answer is (A).

22. Similarly, for minima, we have

$$\Delta\phi = (2n+1)\pi, \text{ where } n = 0, 1, 2, 3, \ldots$$

$$\Rightarrow \quad \left(\frac{2\pi}{\lambda}\right)(\pi R) = (2n+1)\pi$$

$$\Rightarrow \quad \lambda_{max} = \frac{2\pi R}{2n+1}$$

$$\Rightarrow \quad \lambda_{max} = 2\pi R$$

Hence, the correct answer is (B).

23. Since, $y = a\sin\left[\frac{2\pi}{\lambda}(vt - x)\right]$

From the graph, $a = 10$ mm $= 1$ cm

$$\lambda = 8 \text{ m}, v = 1 \text{ ms}^{-1}$$

$$\Rightarrow \quad y = 1\sin\left[\frac{2\pi}{8}(t-x)\right] \text{ cm}$$

Now $v_P = \frac{\partial y}{\partial t} = \frac{\pi}{4}\cos\left[\frac{\pi}{4}(t-x)\right]$

$$\Rightarrow \quad v_P(x = 4 \text{ m}, t = 0 \text{ s}) = \frac{\pi}{4} \text{ cms}^{-1}$$

Hence, the correct answer is (B).

24. At the instant and location asked, the two waves will superimpose, so the net displacement is sum of displacement due to individual pulses.

$$y_1(x = 9 \text{ m}, t = 8 \text{ s}) = \frac{1}{\sqrt{2}} \text{ cm and } y_2 = 1 \text{ cm}$$

$$\Rightarrow \quad y_{net} = y_1 + y_2 = \left(\frac{1}{\sqrt{2}}+1\right) \text{ cm}$$

Hence, the correct answer is (A).

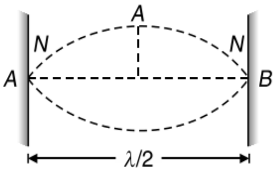
25. Net velocity is sum of velocity due to individual pulses.

Since, $v_1 = \frac{\pi}{4}\cos\left[\frac{\pi}{4}(t-x)\right]$

$$\Rightarrow \quad v_1(x = 9 \text{ m}, t = 8 \text{ s}) = -\frac{\pi}{4\sqrt{2}}$$

Hence, the correct answer is (C).

26.



Hence, the correct answer is (A).

27. $\frac{\omega}{v} = k$

$$v = \sqrt{\frac{40\times 2}{0.1}} = 20\sqrt{2}$$

$$\omega = \frac{\pi}{2}\left(20\sqrt{2}\right)$$

$$\Rightarrow \quad \omega \approx 44 \text{ rads}^{-1}$$

Hence, the correct answer is (A).

28. Conceptual

Hence, the correct answer is (A).

29. Velocity of particle undergoing wave motion is given by

$$\frac{\partial y}{\partial t} = -u\frac{\partial y}{\partial x}$$

At $t = 0$ and $x = 2$, we have

$$\frac{\partial y}{\partial t} = \frac{-1\times 10\times 10^{-3}}{4\times 10^{-2}} = -\frac{1}{4} = -0.25 \text{ ms}^{-1}$$

Hence, the correct answer is (D).

30. Displacement and velocity of particle is vector sum of displacements and velocity due to individual pulses.

So, displacement at $x = 8$ cm and $t = 6$ sec is

$$y = 5 \text{ mm} - 5 \text{ mm} = 0$$

Hence, the correct answer is (D).

31. Velocity at $x = 8$ cm and $t = 6$ sec is

$$v = -0.25 + 0.125$$

$$v = -0.125 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

32. Wavelength of incident wave $= \frac{2\pi}{a}$

Hence, the correct answer is (C).

33. Frequency of incident wave $= \frac{b}{2\pi}$

Hence, the correct answer is (B).

34. Intensity of reflected wave has become 0.64 times. But since $I \propto A^2$ amplitude of reflected wave will become 0.8 times. a and b will remain as it is. But direction of velocity of wave will become opposite. Further there will be a phase change of π, as it is reflected by an obstacle (denser medium). Therefore, equation of reflected wave would be

$$y_r = 0.8A\cos[ax - bt + \pi] = -0.8A\cos(ax - bt)$$

Hence, the correct answer is (C).

35. The equation of resultant wave will be,

$$y = y_i + y_r = A\cos(ax + bt) - 0.8A\cos(ax - bt)$$

Particle velocity

$$v_p = \frac{\partial y}{\partial t} = -Ab\sin(ax + bt) - 0.8Ab\sin(ax - bt)$$

Maximum particle speed can be $1.8Ab$, where,

$$\sin(ax + bt) = \pm 1 \text{ and } \sin(ax - bt) = \pm 1$$

and minimum particle speed can be zero, where

$\sin(ax + bt)$ *and* $\sin(ax - bt)$ both and zero

Hence, the correct answer is (D).

36. Since, there is no relative motion along y-axis, so we have

$$f_{\text{app}} = f_0\left(\frac{v}{v + v_s}\right)$$

$$\Rightarrow \quad f_{\text{app}} = 960\left(\frac{310}{310 + 10}\right) = 930 \text{ Hz}$$

Hence, the correct answer is (B).

37. Throughout the apparent frequency will remain same.
Hence, the correct answer is (D).

38. Both the source and detector fall simultaneously on ground and relative velocity exists only along the horizontal.
Hence, the correct answer is (A).

39. Since, $(x - vt)$ is a pulse travelling in $+ve$ x-direction, so y_1 is along $+x$-axis.

Similarly, $(x + vt)$ is a pulse travelling in $-$ve x direction, so y_2 is along $-x$ axis.

Hence, the correct answer is (C).

40. Two waves cancel when

$$|y_1| = |y_2|$$

$$\Rightarrow \quad \frac{5}{(3x - 4t)^2 + 2} = \frac{+5}{(3x + 4t - 6)^2 + 2}$$

$$\Rightarrow \quad (3x - 4t)^2 + 2 = (3x + 4t - 6)^2 + 2$$

$$\Rightarrow \quad 3x - 4t = 3x + 4t - 6$$

$$\Rightarrow \quad 6 = 8t$$

$$\Rightarrow \quad t = \frac{6}{8} = \frac{3}{4} = 0.75 \text{ s}$$

Hence, the correct answer is (D).

41. At $t = 0$

$$y_1 = \frac{5}{(3x)^2 + 2} = \frac{5}{9x^2 + 2}$$

$$y_2 = \frac{-5}{(3x - 6)^2 + 2}$$

For, $|y_1| = |y_2|$, we have

$$\frac{1}{9x^2 + 2} = \frac{1}{9x^2 + 36 - 36x + 2}$$

$$\Rightarrow \quad 9x^2 + 2 = 9x^2 + 36 - 36x + 2$$

$$\Rightarrow \quad 36(x - 1) = 0$$

$$\Rightarrow \quad x = 1 \text{ m}$$

Hence, the correct answer is (D).

42. $v = \sqrt{\dfrac{B}{\rho}}$

$$\Rightarrow \quad \rho v^2 = B$$

$$\Rightarrow \quad \rho = \frac{B}{v^2}$$

But from the given equation, we have

$$\omega = 6000\pi \text{ and } k = 15\pi$$

$$\Rightarrow \quad v = \frac{\omega}{k} = 400 \text{ ms}^{-1}$$

$$\Rightarrow \quad \rho = \frac{1.6 \times 10^5}{(400)^2}$$

$$\Rightarrow \quad \rho = 1 \text{ kgm}^{-3}$$

Hence, the correct answer is (A).

43. $I_R = 2I = \dfrac{2P_0^2}{2\rho v} = \dfrac{2 \times (24\pi)^2}{2 \times 1 \times 400} = 1.44\pi^2$

Hence, the correct answer is (B).

44. For wave in string

$$I \propto A^2 v^2$$

$$\frac{I_1}{I_2} = \frac{A_1^2 v_1^2}{A_2^2 v_2^2} = (2)^2 (2)^2 = 16$$

Hence, the correct answer is (C).

45. Assuming phase at $x = 0$ to be zero

Phase of point P on string 1 is $\dfrac{\pi}{2}$

Phase of point P' on string 2 is π

So, phase difference $= \pi - \dfrac{\pi}{2} = \dfrac{\pi}{2}$

Hence, the correct answer is (A).

46. Phase of particle at $x = 20$ cm on string $1 = 2\pi$

Phase of particle at $x = 20$ cm on string $2 = \pi$

So, phase difference $= 2\pi - \pi = \pi$

Hence, the correct answer is (D).

47. **The correct answer is (A).**

48. **The correct answer is (B).**

49. **The correct answer is (C).**

Combined solution to 47, 48 and 49

Since, $\mu_2 = 4\mu_1$ and $v = \sqrt{\dfrac{T}{\mu}}$

$$\Rightarrow \quad v_2 = \frac{v_1}{2}$$

Also, $k = \dfrac{\omega}{v}$ $\left\{\because v = \dfrac{\omega}{k}\right\}$

$\Rightarrow k_2 = 2k_1$

Also, $A_r = \left(\dfrac{v_2 - v_1}{v_2 + v_1}\right)A_i$

$A_t = \left(\dfrac{2v_2}{v_2 + v_1}\right)A_i$

Hence, the correct answer is (B).

50. $y(x=0,\ t) = 8\sin(4t) = A\sin(\omega t)$

$y(x,t) = A\sin(\omega t - kx)$

$\omega = kv = \left(\dfrac{2\pi}{\lambda}\right)v$

Since, $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{1}{0.2}} = \sqrt{5}\ \text{ms}^{-1}$

$\Rightarrow k = \dfrac{\omega}{v} = \dfrac{4}{\sqrt{5}}\ \text{m}^{-1}$

$\Rightarrow y(x,t) = 8\sin\left(4t - \dfrac{4}{\sqrt{5}}x\right)$

Hence, the correct answer is (B).

51. Average power is given by

$P_{av} = \dfrac{1}{2}\mu v\omega^2 A^2$

$\Rightarrow P_{av} = \dfrac{1}{2}(0.2)\sqrt{5}(4)^2(0.08)^2$

$\Rightarrow P_{av} = 0.1 \times \sqrt{5} \times 16 \times 64 \times 10^{-2}$

$\Rightarrow P_{av} = 0.02$ watt

Hence, the correct answer is (A).

52. Power transferred to bath is

$P' = 50\%$ of $P_{av} = \dfrac{0.02}{2} = 0.01$ W

Since, $dQ = mcdT$

$\Rightarrow \dfrac{dQ}{dt} = mc\left(\dfrac{dT}{dt}\right) = mc\left(\dfrac{\Delta T}{\Delta t}\right) = 0.01$

$\Rightarrow \Delta t = \dfrac{mc\Delta T}{0.01}$

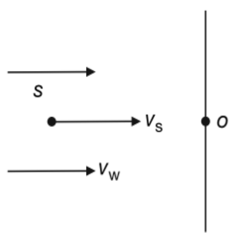
$\Rightarrow \Delta t = \dfrac{(1)(1)(4200)(1)}{0.01}$

$\Rightarrow \Delta t = 4.2 \times 10^5$ s

Hence, the correct answer is (A).

53. Given: $v_s = v_w = 40\ \text{kmh}^{-1}$ and

$v = 1200\ \text{kmh}^{-1}$ = Speed of sound



Frequency observed by observer

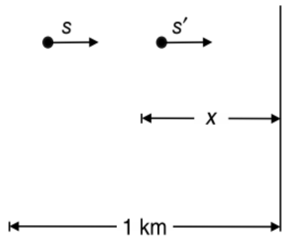
$f' = f\left(\dfrac{v + v_w}{v + v_w - v_s}\right)$

Substituting the values, we have

$f' = 580\left(\dfrac{1200 + 40}{1200 + 40 - 40}\right) = 599.33$ Hz

Hence, the correct answer is (B).

54. Let x be the distance of the source from the hill at which echo is heard of the sound which was produced when source was at a distance 1 km from the hill. Then, time taken by the source to reach from s to s' = time taken by the sound to reach from s to hill and then from hill to s'. Thus,



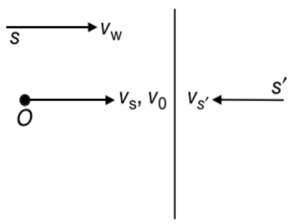
$$\frac{1-x}{40} = \frac{1}{1200+40} + \frac{x}{1200-40}$$

Solving this equation, we get

$x = 0.935$ km

Hence, the correct answer is (A).

55.



Frequency heard by the driver of the reflected wave

$f'' = f\left[\dfrac{v - v_w + v_o}{v - v_w - v_{s'}}\right] = 580\left[\dfrac{1200 - 40 + 40}{1200 - 40 - 40}\right]$

$f'' = 621.43$ Hz

Hence, the correct answer is (D).

Matrix Match/Column Match Type Questions

1. A → (p, r); B → (q, s); C → (q, r); D → (p, s)

For the incident wave in (A), two cases arise

$y_i = A\sin(kx - \omega t)$

For reflection at flexible support,

$y_r = A\sin(kx + \omega t)$

$\Rightarrow y = y_i + y_r = 2A\sin(kx)\cos(\omega t)$

$y_i = A\sin(kx - \omega t)$

For reflection at rigid support,

$y_r = -A\sin(kx + \omega t)$

$\Rightarrow y = y_i + y_r = 2A\cos(kx)\sin(\omega t)$

For the incident wave in (B), two cases arise

$y_i = A\cos(kx - \omega t)$

For reflection at flexible support,

$y_r = A\cos(kx + \omega t)$

$\Rightarrow \quad y = y_i + y_r = 2A\cos(kx)\cos(\omega t)$

$y_i = A\cos(kx - \omega t)$

For reflection at rigid support,

$y_r = -A\cos(kx + \omega t)$

$\Rightarrow \quad y = y_i + y_r = 2A\sin(kx)\sin(\omega t)$

When, $x = 0$ is the rigid support then nodes are formed at $x = 0$ and at the nodes, $y = 0$.
This is satisfied by the equations

$y = 2A\sin(kx)\cos(\omega t)$ and $y = 2A\sin(kx)\sin(\omega t)$

When, $x = 0$ is the flexible support then antinodes are formed at $x = 0$ and at the antinodes, $y = \pm A$
This is satisfied by the equations

$y = 2A\cos(kx)\sin(\omega t)$ and $y = 2A\cos(kx)\cos(\omega t)$

2. A → (r); B → (p); C → (q); D → (s)

$y = A\sin(\omega t - kx)$

$\Rightarrow \quad \omega = 10 \text{ rads}^{-1}, k = 5 \text{ m}^{-1}$

Since, $v = \dfrac{\omega}{k}$

$\Rightarrow \quad v = 2 \text{ ms}^{-1}$

Since, $k = 5$

$\Rightarrow \quad \dfrac{2\pi}{\lambda} = 5$

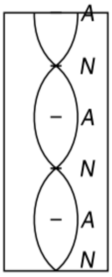
$\Rightarrow \quad \lambda = 0.4\pi$

$v_P = \dfrac{\partial y}{\partial t} = A\omega\cos(\omega t - kx)$

$\Rightarrow \quad (v_P)_{\max} = A\omega = Avk$

$\Rightarrow \quad (v_P)_{\max} = (0.02)(5)(2) = 0.2 \text{ ms}^{-1}$

3. A → (s); B → (p); C → (q); D → (r)
As discussed, third overtone frequency of closed pipe means seventh harmonic, which is 7 times the fundamental frequency. So, $x = 7$.
The second overtone, as shown, corresponds to fifth harmonic and has 3 Nodes and 3 Antinodes



Similarly, the third overtone has 4 Nodes and 4 Antinodes.

H	O
1	x
3	1
5	2
7	3
9	4
⋮	⋮
n	$\dfrac{n-1}{2}$
$(2n+1)$	n

4. A → (r); B → (p); C → (p); D → (p)
Frequency is the property of source. It remains unchanged.
In rarer medium, speed of wave is more.

$\lambda = \dfrac{v}{f}$

$\Rightarrow \quad \lambda \propto v$

$A_t = \left(\dfrac{2v_2}{v_1 + v_2}\right)A_i$

Since, $v_2 > v_1$

$\Rightarrow \quad A_t > A_i$

5. A → (r); B → (r); C → (s); D → (q)

OPTION (A)

At $t = \dfrac{T}{2}$, we have

$y = A\cos kx \cos\left(\dfrac{2\pi}{T} \times \dfrac{T}{2}\right) = -A\cos kx$

Hence plot is (r)

OPTION (B)

$v_p = \dfrac{\partial y}{\partial t} = -A\omega\cos(kx)\sin(\omega t)$

At $t = \dfrac{T}{4}$, $v_P = -A\omega\cos(kx)\sin\left(\dfrac{\pi}{2}\right) = -A\omega\cos(kx)$

Hence plot is (r)

OPTION (C)

$\Delta P = -B\dfrac{\partial y}{\partial x}$, where B is the Bulks Modulus. So,

$\Delta P = +BkA\sin(kx)\cos(\omega t)$

At $t = 0$, $\Delta P = +BkA\sin(kx)$
Hence plot is (s)

OPTION (D)

Pressure at any point is $P = P_0 \pm \Delta P$

Since, $\Delta P = (BAk)\sin(kx)$

Also, density $\rho = \dfrac{PM}{RT}$

$\Rightarrow \quad \rho = \dfrac{M}{RT}(P_0 \pm \Delta P)$

$\Rightarrow \quad \rho = \rho_0 \pm \Delta\rho$, where $\Delta\rho = \dfrac{M\Delta P}{RT}$

$\Rightarrow \quad \rho = \rho_0 \pm \dfrac{M}{RT}\Delta P$

Now $\Delta P = BkA\sin(kx)\cos(\omega t)$

At $t = \dfrac{T}{2}$, we have

$\Delta P = -BAk\sin(kx)$

$\Rightarrow \quad \rho = \rho_0 \pm \dfrac{M}{RT}[-(BAk)\sin(kx)]$

$\Rightarrow \quad \rho = \rho_0 \pm \rho_0'[-\sin(kx)]$ where $\rho_0' = BAk$

Hence plot is (q)

6. A → (r); B → (p); C → (q); D → (r)

(A) $f' = f\left(\dfrac{v+v_0'}{v+v_s}\right) = f$

(B) $f' = f\left(\dfrac{v+v_0}{v-v_s}\right) > f$

(C) $f' = f\left(\dfrac{v-v_0}{v+v_f}\right) < f$

(D) $f' = f\left(\dfrac{v-v_0}{v-v_s}\right) = f$

7. A → (q); B → (p); C → (p); D → (r)

$$u = \frac{1}{2}\rho\omega^2A^2$$

ω depends on source, which is same for both, so

$$\frac{u_1}{u_2} = \frac{\rho_1}{\rho_2} \times \left(\frac{A_1}{A_2}\right)^2 = \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)^2 = \frac{1}{16}$$

$$P = \frac{1}{2}\rho\omega^2A^2sv = usv$$

$\Rightarrow \quad P \propto uv$

Since, $v \propto \dfrac{1}{\sqrt{\mu}}$

$\mu_2 = 4\mu_1$

$\Rightarrow \quad v_2 = \dfrac{v_1}{2}$

$\Rightarrow \quad \dfrac{v_1}{v_2} = 2$

Now, $\dfrac{P_1}{P_2} = \dfrac{u_1}{u_2} \times \dfrac{v_1}{v_2} = \dfrac{1}{16} \times 2 = \dfrac{1}{8}$

$I = \dfrac{P}{S}$

$\Rightarrow \quad I \propto P$

$\Rightarrow \quad \dfrac{I_1}{I_2} = \dfrac{P_1}{P_2} = \dfrac{1}{8}$

8. A → (q); B → (t); C → (s); D → (t)

$k = \dfrac{2\pi}{\lambda} = 2\pi a$

$\Rightarrow \quad \lambda = \dfrac{1}{a}$

$\omega = \dfrac{2\pi}{\lambda} = 2\pi b$

$\Rightarrow \quad T = \dfrac{1}{b}$ or $f = b$

$\Delta\phi = \dfrac{2\pi}{\lambda}\Delta x = (2\pi a)\left(\dfrac{1}{4a}\right) = \dfrac{\pi}{2}$

$\Delta\phi = \dfrac{2\pi}{T}\Delta t = (2\pi b)\left(\dfrac{1}{8b}\right) = \dfrac{\pi}{4}$

9. A → (r); B → (p, r); C → (q, r); D → (p, r, s)

Conceptual

10. A → (q); B → (r); C → (q); D → (s)

Since, $v = \sqrt{\dfrac{3RT}{M}}$

Change in pressure has no effect on speed of sound, where as

$v \propto \sqrt{T}$ $\quad \left\{\because v = \sqrt{\dfrac{3RT}{M}}\right\}$

11. A → (s); B → (q); C → (r); D → (p)

$y = 2A\sin(kx)\cos(\omega t)$

$\Rightarrow \quad 2A = 0.06$

$\Rightarrow \quad A = 0.03$ m

At nodes, $y = 0$

$\Rightarrow \quad \sin(2\pi x) = 0$

$\Rightarrow \quad x = 0.5$ m

At antinodes, $y =$ maximum

$\Rightarrow \quad \sin(2\pi x) = 1$

$\Rightarrow \quad x = 0.25$ m

$|y| = \left|2A\sin\left(2\pi \times \dfrac{3}{4}\right)\right|$

$\Rightarrow \quad |y| = |y_{max}| = 0.06$ m

12. A → (p); B → (p); C → (t); D → (r)

For closed pipe, $100 = \dfrac{v}{4\ell_c}$

$\Rightarrow \quad \ell_c = \dfrac{v}{400} = \dfrac{330}{400} = 0.825$ m

For open pipe, $200 = \dfrac{v}{2\ell_0}$

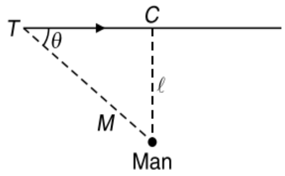
$\Rightarrow \quad \ell_0 = \dfrac{v}{400} = 0.825$ m

Different frequencies of closed pipes will be 100 Hz, 300 Hz, 500 Hz, etc., and different frequencies of open pipe are 200 Hz, 400 Hz, 600 Hz etc. i.e., none of them matches with each other.

So, the asked ratio is $\dfrac{600}{300} = 2$

Integer/Numerical Answer Type Questions

1. Let the velocity of truck at T when it blows the whistle be v_s. Then



$$600 = \left(\frac{v}{v - v_s\cos\theta}\right)500 \qquad \ldots(1)$$

During this time, speed of truck gets doubled, so

$$2v_s = v_s + at$$

$$\Rightarrow \quad v_s = at \qquad \ldots(2)$$

Now, $vt = AM = \dfrac{\ell}{\sin\theta}$ $\qquad \ldots(3)$

Also, $AB = \ell\cot\theta = v_s t + \dfrac{1}{2}at^2$ $\qquad \ldots(4)$

$$\Rightarrow \quad \frac{\ell\cos\theta}{\sin\theta} = at^2 + \frac{1}{2}at^2 = \frac{3}{2}at^2$$

$$\Rightarrow \quad \frac{\ell\cos\theta}{\sin\theta} = \frac{3}{2}a\left(\frac{\ell}{v\sin\theta}\right)^2$$

$$\Rightarrow \quad \frac{a\ell}{v} = \left(\frac{2}{3}\sin\theta\cos\theta\right)v$$

From Equation (1), we get

$$\frac{6}{5} = \frac{u}{v - at\cos\theta} = \frac{v}{v - \dfrac{a\ell}{v}\left(\dfrac{\cos\theta}{\sin\theta}\right)} \qquad \left\{\because t = \frac{\ell}{v\sin\theta}\right\}$$

$$\Rightarrow \quad \frac{6}{5} = \frac{v}{v - \left(\dfrac{2}{3}\sin\theta\cos\theta\right)\left(\dfrac{\cos\theta}{\sin\theta}\right)v}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 60^\circ$$

2. Let the frequency of the tuning fork be f Hz. Let the frequency of the note from the open organ pipe at 20 °C and 30 °C be f_1 and f_2 respectively.

At 20 °C unison takes place. So,

$$f = f_1 \qquad \ldots(1)$$

At 30 °C, 5 beats per second are produced. So,

$$f_2 - f = 5 \qquad \ldots(2)$$

Since $f \propto \sqrt{T}$

$$\Rightarrow \quad \frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{293}{303}} = 0.983 \qquad \ldots(3)$$

Solving above three equations (1), (2) and (3), we get

$$f = 296.2 \text{ Hz} \approx 296 \text{ Hz}$$

3. Frequency of sound reflected by the car is

$$f = \left(\frac{v + v_c}{v - v_c}\right)f_0$$

Since $v_c \ll v$

$$\Rightarrow \quad f = f_0\left(1 + \frac{v_c}{v}\right)\left(1 - \frac{v_c}{v}\right)^{-1} \approx \left(1 - \frac{2v_c}{v}\right)f_0$$

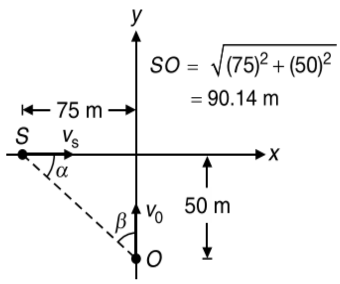
$$\Rightarrow \quad \Delta f = \frac{-2f_0}{v}\Delta v_c$$

$$\Rightarrow \quad \frac{1.2}{100}f_0 = \frac{-2f_0}{v}\Delta v_c$$

Difference $|\Delta v_c| = \dfrac{1.2}{100} \times \dfrac{300}{2}$ ms^{-1}

$$\Rightarrow \quad |\Delta v_c| = \frac{1.2}{100} \times \frac{330}{2} \times \frac{18}{5} \approx 7 \text{ kmh}^{-1}$$

4. After 5 seconds the observer will move a distance of 50 m while the source moves a distance of 25 m.



The apparent frequency is given by

$$f' = f\left(\frac{v + v_0\cos\beta}{v - v_s\cos\alpha}\right)$$

$$\Rightarrow \quad f' = (1000)\left(\frac{330 + 10 \times \dfrac{50}{90.14}}{330 - 5 \times \dfrac{75}{90.14}}\right)$$

$$\Rightarrow \quad f' = 1030 \text{ Hz}$$

5. Since, $y = A\sin\dfrac{2\pi}{\lambda}(vt - x)$

$$\Rightarrow \quad \frac{y}{A} = \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

In the first case, $\dfrac{y_1}{A} = \sin 2\pi\left(\dfrac{t}{T} - \dfrac{x_1}{\lambda}\right)$

where, $y_1 = +6$, $A = 8$, $x_1 = 10$ cm

$$\Rightarrow \quad \frac{6}{8} = \sin 2\pi\left(\frac{t}{T} - \frac{10}{\lambda}\right) \qquad \ldots(1)$$

Similarly, in the second case, we get

$$\frac{4}{8} = \sin 2\pi\left(\frac{t}{T} - \frac{25}{\lambda}\right) \qquad \ldots(2)$$

From Equation (1), we get

$$2\pi\left(\frac{t}{T} - \frac{10}{\lambda}\right) = \sin^{-1}\left(\frac{6}{8}\right) = 0.85 \text{ rad}$$

$$\Rightarrow \quad \frac{t}{T} - \frac{10}{\lambda} = 0.14 \qquad \ldots(3)$$

Similarly, from Equation (2), we get

$$2\pi\left(\frac{t}{T} - \frac{25}{\lambda}\right) = \sin^{-1}\left(\frac{4}{8}\right) = \frac{\pi}{6} \text{ rad}$$

$$\Rightarrow \quad \frac{t}{T} - \frac{25}{\lambda} = 0.08 \qquad \ldots(4)$$

Subtracting Equation (4) from Equation (3), we get

$$\frac{15}{\lambda} = 0.06$$

$$\Rightarrow \quad \lambda = 250 \text{ cm}$$

6. When incident wave and reflected wave superimpose to produce stationary wave, the ratio of amplitudes at anti-node and at node is given by,

$$\frac{A_{max}}{A_{min}} = \frac{A_i + A_r}{A_i - A_r} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{A_r}{A_i} = \frac{1}{5}$$

Since, $I \propto A^2$

$$\Rightarrow \quad \frac{I_r}{I_i} = \left(\frac{A_r}{A_i}\right)^2 = \frac{1}{25}$$

$$\Rightarrow \quad I_r = 0.04 I_i$$

Hence, 4% of the incident energy is reflected i.e., 96% energy passes across the obstacle.

7. If speed of astronaut is v, then apparent frequency of the echo received is

$$f' = f\left(\frac{c+v}{c-v}\right)$$

$$\Rightarrow \quad f' = f\left(1 + \frac{v}{c}\right)\left(1 - \frac{v}{c}\right)^{-1}$$

Since $v \ll c$, so we get

$$f' = f\left(1 + \frac{2v}{c}\right)$$

$$\Rightarrow \quad f' - f = \frac{2fv}{c}$$

$$\Rightarrow \quad v = \frac{c|\Delta f|}{2f} = \frac{(3\times10^8)(10^3)}{2(5\times10^9)} = 30 \text{ ms}^{-1}$$

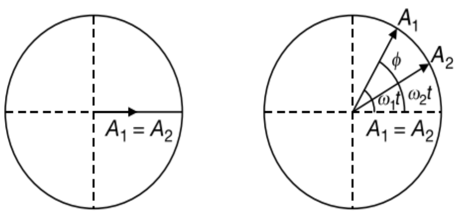
8. Since, $f \propto \frac{1}{\ell}$

$$\Rightarrow \quad \frac{\ell_2}{\ell_1} = \frac{f_1}{f_2}$$

$$\Rightarrow \quad \ell_2 = \left(\frac{f_1}{f_2}\right)\ell_1 = \left(\frac{124}{186}\right)(90) = 60 \text{ cm}$$

Thus, the string should be pressed at 60 cm from one end.

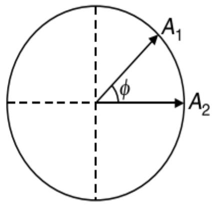
9. The problem can be represented in pictorial form as shown in figure.



This can also be seen as if A_2 is stationary and A_1 rotating with angular speed $(\omega_1 - \omega_2)$, so that phase difference between them at time t is

$$\phi = (\omega_1 - \omega_2)t$$

So, $T = \dfrac{2\pi}{(\omega_1 - \omega_2)} = \dfrac{1}{f_1 - f_2} = \dfrac{1}{10^3}$ sec



Now, it is given that the detector can detect signals only when resultant intensity $\geq 2I_0$ or resultant amplitude $\geq \sqrt{2}A_0$, because $I \propto A^2$. We can see that resultant amplitude is less than $\sqrt{2}A_0$ from $\phi = \frac{\pi}{2}$ to $\frac{3\pi}{2}$ i.e., for each half cycle.

Hence, required time for which the detector remains idle is

$$t = \frac{T}{2} = \frac{(1/10^3)}{2} = 5\times10^{-4} \text{ s} = 500\ \mu\text{s}$$

10. Difference in sound level is $L_2 - L_1$ given by

$$\Rightarrow \quad |\Delta L| = 10\log_{10}\left(\frac{I_2}{I_1}\right) = 10\log_{10}\left(\frac{r_1^2}{r_2^2}\right) \qquad \left\{\because I \propto \frac{1}{r^2}\right\}$$

Since $\dfrac{I_2}{I_1} = \left(\dfrac{r_1}{r_2}\right)^2 = \dfrac{1}{4}$ $\qquad \{\because r_2 = 2r_1\}$

$$\Rightarrow \quad |\Delta L| = \left|10\log_{10}\left(\frac{1}{4}\right)\right| = 6 \text{ dB}$$

11. (a) The given condition is possible only when

$$\sin(kx) = \cos(kx)$$

$$\Rightarrow \quad kx = \frac{\pi}{4}$$

Since, $A(x) = A\sin kx$

$$\Rightarrow \quad A = \frac{A(x)}{\sin(kx)} = \frac{3\sqrt{2}}{1/\sqrt{2}} = 6 \text{ mm}$$

(b) Since, $kx = \frac{\pi}{4}$

$$\Rightarrow \quad \left(\frac{2\pi}{\lambda}\right)\left(\frac{20}{2}\right) = \frac{\pi}{4}$$

$$\Rightarrow \quad \lambda = 80 \text{ cm}$$

$$\Rightarrow \quad \frac{\lambda}{2} = 40 \text{ cm}$$

So, number of loops is $p = \dfrac{\ell}{\lambda/2} = 6$

i.e., the string is vibrating in its fifth overtone mode.

12. Speed of a transverse wave on a wire is given by

$$v = \sqrt{\frac{T}{\mu}} \qquad \text{...(1)}$$

Differentiating with respect to tension, we have

$$\frac{dv}{dT} = \frac{1}{2\sqrt{\mu T}} \qquad \text{...(2)}$$

Dividing Equation (2) by Equation (1), we get

$$\frac{dv}{v}=\frac{1}{2}\frac{dT}{T}$$

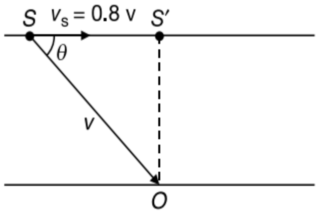
$$\Rightarrow \quad dT=(2T)\frac{dv}{v}$$

Substituting the values, we get

$$dT=\frac{(2)(50)(312-300)}{300}=4\text{ N}$$

So, tension must be increased by 4 N

13. (a) Assume that the pulse which is emitted when the source is at S, reaches the observer O in the same time in which the source reaches from S to S', then



$$\cos\theta=\frac{SS'}{SO}=\frac{v_s t}{vt}=\frac{v_s}{v}=0.8$$

Since, $f'=\left(\frac{v}{v-v_s\cos\theta}\right)f$

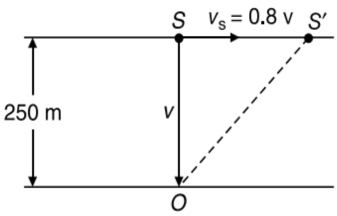
$$\Rightarrow \quad f'=\left(\frac{v}{v-(0.8v)(0.8)}\right)(900)$$

$$\Rightarrow \quad f'=\left(\frac{1}{1-0.64}\right)(900)=2500\text{ Hz}$$

(b) The observer will observe no change in the frequency when the source is at S as shown in figure. In the time when the wave pulse reaches from S to O, the source will reach from S to S'. Hence

$$t=\frac{SO}{v}=\frac{SS'}{v_s}$$

$$\Rightarrow \quad SS'=\left(\frac{v_s}{v}\right)SO=(0.8)(250)=200\text{ m}$$



Therefore, distance of observer from source at this instant is

$$S'O=\sqrt{(SO)^2+(SS')^2}$$

$$\Rightarrow \quad S'O=\sqrt{(250)^2+(200)^2}=320\text{ m}$$

14. (a) Intensity due to a point source varies with distance r from it as $I\propto\frac{1}{r^2}$. So, $\frac{I_2}{I_1}=\frac{r_1^2}{r_2^2}$

$$\Rightarrow \quad L_2-L_1=10\log_{10}\left(\frac{I_2}{I_1}\right)=10\log\left(\frac{r_1^2}{r_2^2}\right)$$

Since $L_1=30$ dB, $r_1=20$ m and $r_2=10$ m, so

$$L_2-30=10\log\left(\frac{20}{10}\right)^2=10\log_{10}(4)\approx 6$$

$$\Rightarrow \quad L_2=36\text{ dB}$$

(b) Since, $L_2-L_1=10\log_{10}\left(\frac{I_2}{I_1}\right)=10\log_{10}\left(\frac{r_1^2}{r_2^2}\right)$

Sound is not heard at a point where $L_2=0$

$$\Rightarrow \quad 30-0=10\log_{10}\left(\frac{r_1^2}{r_2^2}\right)$$

$$\Rightarrow \quad \left(\frac{r_2}{r_1}\right)^2=1000$$

$$\Rightarrow \quad \frac{r_2}{r_1}=31.62$$

$$\Rightarrow \quad r_2=(31.62)(20)\approx 632\text{ m}$$

15. $\omega=2\pi f=2\pi(125)=785\text{ rads}^{-1}$

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{49\text{ N}}{0.0625\text{ kgm}^{-1}}}=28\text{ ms}^{-1}$$

$$\Rightarrow \quad P=\frac{1}{2}\mu\omega^2vA^2$$

$$\Rightarrow \quad P=\frac{1}{2}\times 0.0625\times 785^2\times 28\times(9\times10^{-3})^2$$

$$\Rightarrow \quad P=43.7\text{ W}$$

16. Since, $\lambda=\frac{v}{f}=\frac{\sqrt{T/\mu}}{f}$ $\left\{\because v=\sqrt{\frac{T}{\mu}}\right\}$

$$\Rightarrow \quad \lambda\propto\sqrt{T}$$

$$\Rightarrow \quad \frac{\lambda_{\text{Top}}}{\lambda_{\text{Bottom}}}=\sqrt{\frac{T_{\text{Top}}}{T_{\text{Bottom}}}}$$

$$\Rightarrow \quad \lambda_{\text{Top}}=(0.06)\sqrt{\frac{6+2}{2}}=0.12\text{ m}=12\text{ cm}$$

17. Since, $\frac{\lambda}{2}=(85-60)=25$ cm

$$\Rightarrow \quad \lambda=50\text{ cm}=0.5\text{ cm}$$

$$\Rightarrow \quad v=f\lambda=(500)(0.5)=350\text{ ms}^{-1}$$

18. Velocity of transverse wave in a string is $v=\sqrt{\frac{T}{\mu}}$

where, $T=mg=150$ N and

$$\mu=\frac{M}{L}=\frac{3\times10^{-2}}{2}=1.5\times10^{-2}\text{ kgm}^{-1}$$

$$\Rightarrow \quad v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{150}{1.5\times10^{-2}}}=100\text{ ms}^{-1}$$

19. Since, $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

$\Rightarrow f \propto \frac{1}{\ell}$

$\Rightarrow f_1 : f_2 : f_3 = 1:2:3$

$\Rightarrow \ell_1 : \ell_2 : \ell_3 = \frac{1}{1}:\frac{1}{2}:\frac{1}{3} = 6:3:2$

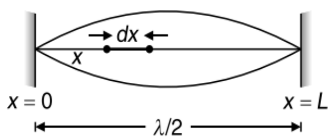
$\ell_1 = (1.1)\left(\frac{6}{6+3+2}\right) = 0.6 \text{ m} = 60 \text{ cm}$

$\ell_2 = (1.1)\left(\frac{3}{6+3+2}\right) = 0.3 \text{ m} = 30 \text{ cm}$

and $\ell_3 = (1.1)\left(\frac{2}{6+3+2}\right) = 0.2 \text{ m} = 20 \text{ cm}$

Therefore, one bridge should be placed at 60 cm from one end and the other should be placed at 20 cm from the other end.

20. Mass of string is $m = \mu\ell$



Since, $\frac{\lambda}{2} = \ell$

$\Rightarrow \lambda = 2\ell$

$\Rightarrow k = \frac{2\pi}{\lambda} = \frac{\pi}{\ell}$

Now, $A(x) = A\sin(kx)$

$\Rightarrow dE = \frac{1}{2}(\mu dx)\omega^2 A^2 \sin^2 kx$

$\Rightarrow dE = \frac{1}{4}(\mu\omega^2 A^2)(1-\cos(2kx))dx$

$\Rightarrow E = \int_{x=0}^{x=\ell} dE = \frac{\mu\omega^2 A^2}{4}\left(x - \frac{\sin 2kx}{2}\right)\Bigg|_0^{\ell}$

$\Rightarrow E = \frac{\mu\ell\omega^2 A^2}{4} = \frac{m\omega^2 A^2}{4}$

21. Since, $v = \sqrt{\frac{T}{\mu}} v = 40 \text{ ms}^{-1}$

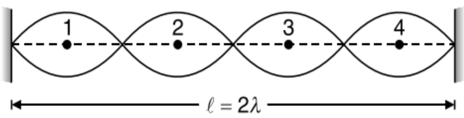
$\omega = 2\pi f = 377 \text{ rads}^{-1}$

and $P = \frac{1}{2}\mu\omega^2 A^2 v = 512$ watt

22. Four antinodes means four loops, so $\ell = 2\lambda$

$\Rightarrow \frac{\ell}{2} = \lambda$

$\Rightarrow k = \frac{2\pi}{\lambda} = \frac{4\pi}{\ell}$



Since, $v = \frac{\omega}{k}$

$\Rightarrow 400 = \frac{(2\pi)(800)(\ell)}{(4\pi)}$

$\Rightarrow \ell = 1 \text{ m}$

23. Since, strain $= \frac{\Delta\ell}{\ell} = \frac{0.1}{100} = 10^{-3}$

So, tension in the wire is

$T = Y(\text{strain})(S) = 10^{-3}YS$

Since, $f = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{T}{\rho S}} = \frac{1}{2\ell}\sqrt{\frac{10^{-3}Y}{\rho}}$

$\Rightarrow f = \frac{1}{2\times 1}\sqrt{\frac{(10^{-3})(20\times 10^{10})}{8000}} = 79 \text{ Hz}$

24. Third overtone of closed organ pipe means seventh harmonic. Given that

$(f_7)_{\text{clsoed}} = (f_4)_{\text{open}}$

$\Rightarrow 7\left(\frac{v}{4\ell_c}\right) = 4\left(\frac{v}{2\ell_0}\right)$

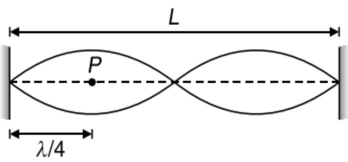
$\Rightarrow \frac{\ell_c}{\ell_0} = \frac{7}{8}$

$\Rightarrow \ell_0 = \frac{8}{7}\ell_c = \frac{8}{7}(7 \text{ cm}) = 8 \text{ cm}$

25. (a) $v_{O_2} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{1.41\times 10^5}{1.43}} = 314 \text{ ms}^{-1}$

(b) $v_{He} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{1.7\times 10^5}{0.18}} = 972 \text{ ms}^{-1}$

26. Given that $\frac{L}{4} = \frac{\lambda}{4}$, so we have $L = \lambda$



In the next higher mode there will be total 6 loops and the desired frequency is $\left(\frac{6}{2}\right)(100) = 300 \text{ Hz}$

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1. Since, $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

For identical strings, l and μ will be same

$$f \propto \sqrt{T}$$

$$\Rightarrow \quad \frac{450}{300} = \sqrt{\frac{T_X}{T_Z}}$$

$$\Rightarrow \quad \frac{T_X}{T_Z} = \frac{9}{4} = 2.25$$

Hence, the correct answer is (C).

2. Given that, $\rho_{wire} = 9 \times 10^{-3}$ kgcm^{-3} = 9000 kgm^{-3}

$$\Delta l/l = 4.9 \times 10^{-4},\ Y = 9 \times 10^{10}\ \text{Nm}^{-2}$$

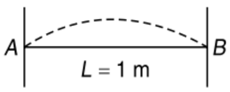
For lowest frequency, $L = 1\ \text{m} = \frac{\lambda}{2}$

$$\Rightarrow \quad \lambda = 2\ \text{m}$$

$$\Rightarrow \quad v = \sqrt{\frac{T}{\mu}} = f\lambda \qquad \text{...(1)}$$

If A be the cross-sectional area of the wire, then

$$\mu = \frac{Al\rho}{l} = A\rho \text{ and } Y = \frac{T/A}{\Delta l/l}$$



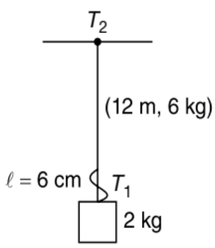
$$\Rightarrow \quad T = YA(\Delta l/l)$$

$$\Rightarrow \quad \sqrt{\frac{YA(\Delta l/l)}{A\rho}} = (2)f \qquad \text{\{from equation (1)\}}$$

$$\Rightarrow \quad f = \frac{1}{2}\sqrt{\frac{Y(\Delta l/l)}{\rho}} = \frac{1}{2}\sqrt{\frac{9\times10^{10}\times4.9\times10^{-4}}{9000}} = 35\ \text{Hz}$$

Hence, the correct answer is 35.

3. Since $v \propto \lambda$ i.e., $\sqrt{T/\mu} \propto \lambda$



Now, $T_2 = 8g$ and $T_1 = 2g$

$$\Rightarrow \quad \lambda_2 = \lambda_1\sqrt{\frac{T_2}{T_1}} = \lambda_1\sqrt{\frac{8g}{2g}} = (6)(2) = 12\ \text{cm}$$

Hence, the correct answer is (B).

4. The separation between two crests is

$$n_1\lambda = 5$$

and separation between crest and a trough is

$$(2n_2+1)\frac{\lambda}{2} = 1.5$$

where, n_1 and n_2 are positive integers.

$$\Rightarrow \quad \frac{1.5}{5} = \frac{(2n_2+1)}{2n_1}$$

$$\Rightarrow \quad n_1 = \frac{5}{3}(2n_2+1)$$

For n_1 and n_2 both to be integers, we must have

$$2n_2 + 1 = 3,\ 9,\ 15,\ 21,\ ...$$

because then only we get, $n_1 = 5,\ 15,\ 25,\ 35,\ ...$

So, $\lambda = \frac{5}{n_1} = \frac{3}{2n_2+1} = 1,\ \frac{1}{3},\ \frac{1}{5},\ ...$

Hence, the correct answer is (D).

5. $\lambda = 2(l_2 - l_1) = 2(24.5 - 17) = 2(7.5) = 15$ cm

Also, $v = f\lambda$

$$\Rightarrow \quad 330 = \lambda \times 15 \times 10^{-2}$$

$$\Rightarrow \quad \lambda = \frac{330}{15}\times 100 = \frac{1100\times100}{5} = 2200\ \text{Hz}$$

Hence, the correct answer is (C).

6. Since, $\Delta p = BAk = Bsk$ $\quad \{\because A = s\}$

$$\Rightarrow \quad \Delta p = (\rho v^2)s\left(\frac{\omega}{v}\right) \qquad \left\{\because v = \sqrt{\frac{B}{\rho}}\right\}$$

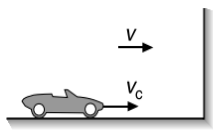
$$\Rightarrow \quad s = \frac{\Delta p}{\rho v\omega} \approx \frac{10}{1\times300\times1000}\ \text{m} = \frac{1}{30}\ \text{mm} = \frac{3}{100}\ \text{mm}$$

Hence, the correct answer is (B).

7. Frequency heard by wall is

$$f_1 = f\left(\frac{v}{v - v_c}\right) \qquad \text{...(1)}$$

If f_2 be the frequency heard by driver after sound is reflected from the wall, then



$$f_2 = f_1\left(\frac{v+v_c}{v}\right) = f\left(\frac{v+v_c}{v-v_c}\right) \qquad \text{...(2)}$$

From equations (1) and (2), we get

$$\frac{f_2}{f} = \frac{48}{44} = \frac{v+v_c}{v-v_c}$$

$$\Rightarrow \quad 12(v+v_c) = 11(v-v_c)$$

$$\Rightarrow \quad 23v_c = v$$

$$\Rightarrow \quad v_c = \frac{v}{23} = \frac{345}{23} = 15\ \text{ms}^{-1} = 54\ \text{kmh}^{-1}$$

Hence, the correct answer is (D).

8. Let x be the distance travelled by the wave when it reaches the respective point.

For waves arriving at A from P and Q, phase difference is

$$\Delta\phi_A = (\phi_P - \phi_Q) + \frac{2\pi}{\lambda}(x_P - x_Q)$$

When the waves reach A, then Q is ahead of P in terms of path because wave emitted by Q reaches A before the wave emitted by P and also phase of P is ahead that of Q by 90°, so we have

$$\phi_P - \phi_Q = +\frac{\pi}{2} \text{ and } x_P - x_Q = -5 \text{ m}$$

$$\Rightarrow \quad \Delta\phi_A = +\frac{\pi}{2} + \frac{2\pi}{20}(-5) = 0$$

Since, $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$

$$\Rightarrow \quad I_A = I + I + 2I\cos(0°) = 4I$$

For waves arriving at C from P and Q, phase difference is

$$\Delta\phi_C = (\phi_P - \phi_Q) + \frac{2\pi}{\lambda}(x_P - x_Q)$$

When the waves reach C, then P is ahead of Q in terms of path, because wave emitted by P reaches C before the wave emitted by Q and also phase of P is ahead that of Q by 90°, so we have

$$\phi_P - \phi_Q = +\frac{\pi}{2} \text{ and } x_P - x_Q = +5 \text{ m}$$

$$\Rightarrow \quad \Delta\phi_C = +\frac{\pi}{2} + \frac{2\pi}{20}(+5) = \pi$$

$$\Rightarrow \quad I_C = I + I + 2I\cos(\pi) = 0$$

For waves arriving at B from P and Q, phase difference is

$$\Delta\phi_B = (\phi_P - \phi_Q) + \frac{2\pi}{\lambda}(x_P - x_Q)$$

When the waves reach B, then P and Q have same path and phase of P is ahead that of Q by 90°, so we have

$$\phi_P - \phi_Q = +\frac{\pi}{2} \text{ and } x_P - x_Q = 0$$

$$\Rightarrow \quad \Delta\phi_B = +\frac{\pi}{2} + 0 = \frac{\pi}{2}$$

$$\Rightarrow \quad I_B = I + I + 2I\cos(\pi/2) = 2I$$

Hence, $I_A : I_B : I_C = 4I : 2I : 0 = 2 : 1 : 0$

Hence, the correct answer is (D).

9. Since, $\nu' = \nu\left(\frac{v_{\text{sound}}}{v_{\text{sound}} - v\cos\theta}\right)$

Initially θ will be less, so $\cos\theta$ is more. Hence ν' is more and then decreases.

Hence, the correct answer is (D).

10. Since, $v = \sqrt{\frac{T}{\mu}}$

$$\Rightarrow \quad 90 = \sqrt{\frac{YA(\Delta\ell/\ell)}{m/\ell}} = \sqrt{\frac{16\times10^{11}\times10^{-6}\times\Delta\ell}{6\times10^{-3}}}$$

$$\Rightarrow \quad \Delta\ell = \frac{8100\times3}{8}\times10^{-8} = 0.03 \text{ mm}$$

Hence, the correct answer is (C).

11. Since, $\nu_{\text{approach}} = \left(\frac{v}{v - v_s}\right)\nu_0$ and $\nu_{\text{recede}} = \left(\frac{v}{v + v_s}\right)\nu_0$

Given that beat frequency is $\Delta\nu = \nu_1 - \nu_2 = 2$ bps

$$\Rightarrow \quad \Delta\nu = \nu_{\text{app}} - \nu_{\text{rec}} = \nu_0\left(\frac{1}{c - v} - \frac{1}{c + v}\right)v$$

$$\Rightarrow \quad \Delta\nu = \left(\frac{2vv_s}{v^2 - v_s^2}\right)\nu_0$$

Since, $v_s \ll v$, so $\Delta\nu \approx 2\left(\frac{v_s}{v}\right)\nu_0 = 2$

$$\Rightarrow \quad 2\left(\frac{v_s}{350}\right)1400 = 2$$

$$\Rightarrow \quad v_s = \frac{1}{4} \text{ ms}^{-1}$$

Hence, the correct answer is (D).

12. Since, $v_s = \sqrt{\frac{\gamma P}{\rho}}$

$$\Rightarrow \quad \frac{v_{\text{gas}}}{v_{\text{air}}} = \sqrt{\frac{\rho_{\text{air}}}{\rho_{\text{gas}}}}$$

$$\Rightarrow \quad \frac{v_{\text{gas}}}{300} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad v_{\text{gas}} = \frac{300}{\sqrt{2}} = 150\sqrt{2} \text{ ms}^{-1}$$

$$\Rightarrow \quad f_2 - f_1 = \frac{v_{\text{gas}}}{2\ell} = \frac{150\sqrt{2}}{2(1)} = 75\sqrt{2} \text{ Hz} \approx 106 \text{ Hz}$$

Hence, the correct answer is 106.

13. Velocity of transverse wave $v \propto \sqrt{T}$

$$\Rightarrow \quad \frac{v'}{v} = \frac{v/2}{v} = \sqrt{\frac{T'}{T}}$$

$$\Rightarrow \quad T' = \frac{T}{4} = \frac{2.06\times10^4}{4} = 5.15\times10^3 \text{ N}$$

Hence, the correct answer is (B).

14. Let amplitude of each wave be A, then resultant wave equation is given by

$$x = A\sin\omega t + A\sin\left(\omega t - \frac{\pi}{4}\right) + A\sin\left(\omega t + \frac{\pi}{4}\right)$$

$$\Rightarrow \quad x = A\sin\omega t + \sqrt{2}A\sin\omega t = (\sqrt{2}+1)A\sin\omega t$$

Resultant wave amplitude is $R = (\sqrt{2}+1)A$

Since $I \propto A^2$, so $\frac{I}{I_0} = (\sqrt{2}+1)^2$

$$\Rightarrow \quad I = 5.8I_0$$

Hence, the correct answer is (A).

15. Given that $\frac{nv}{2l} = 420$ ...(1)

and $\frac{(n+1)v}{2l} = 490$ ...(2)

From equation (1) and (2), we get

$$\frac{n}{n+1} = \frac{6}{7}$$

$$\Rightarrow \quad 7n = 6n + 6$$

$\Rightarrow \quad n = 6$

Substituting in (1), we get

$$\frac{v}{2l} = 70, \text{ where } v = \sqrt{\frac{T}{\mu}}$$

$$\Rightarrow \quad \ell = \frac{v}{140} = \frac{1}{140}\sqrt{\frac{540}{6\times10^{-3}}} = \frac{1}{140}\sqrt{90\times10^{3}}$$

$$\Rightarrow \quad \ell = \frac{300}{140} = 2.142 \text{ m}$$

Hence, the correct answer is (D).

16. Since $f = n\left(\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}\right)$, where $\mu = \rho A = \pi r^2 \rho$

$$\Rightarrow \quad f \propto \frac{n}{r}$$

Since frequency is same so, we have

$$\frac{p}{r_A} = \frac{q}{r_B}$$

$$\Rightarrow \quad \frac{p}{q} = \frac{r_A}{r_B} = \frac{1}{2}$$

Hence, the correct answer is (B).

17. $v = \frac{\omega}{k} = \frac{1000}{3} \text{ ms}^{-1}$

Since $v \propto \sqrt{T}$

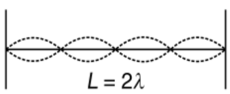
$$\Rightarrow \quad \frac{dv}{v} = \frac{1}{2}\frac{dT}{T}$$

$$\Rightarrow \quad \frac{8\times3}{3\times1000} = \frac{1}{2}\times\frac{dT}{273}$$

$$\Rightarrow \quad dT = \frac{273\times2\times8}{1000} = 4.36\ ^{\circ}\text{C}$$

Hence, the correct answer is (A).

18. Since $k = 0.157 = \frac{2\pi}{\lambda}$



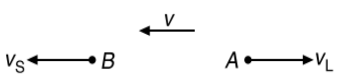
$$\Rightarrow \quad \lambda = \frac{2\pi}{0.157} = 40 \text{ m}$$

In fourth harmonic show in Figure, we have

$$L = \frac{4\lambda}{2} = 2\lambda = 80 \text{ m}$$

Hence, the correct answer is (D).

19.



Since, $f' = f\left(\frac{v - v_L}{v - v_S}\right) = f\left(\frac{v - 20}{v - (-20)}\right)$

$$\Rightarrow \quad 2000 = \frac{320}{360} f$$

$$\Rightarrow \quad f = \frac{2000\times9}{8} = 2250 \text{ Hz}$$

Hence, the correct answer is (D).

20. For third harmonic, we have

$$\frac{3\lambda}{2} = L = 2$$

$$\Rightarrow \quad \lambda = \frac{4}{3} \text{ m}$$

$$\Rightarrow \quad v = f\lambda = \frac{4}{3}\times240 = 320 \text{ ms}^{-1}$$

Also, for third harmonic $f_3 = nf_1$

$$\Rightarrow \quad f_1 = \frac{f_3}{3} = \frac{240}{3} = 80 \text{ Hz}$$

Hence, the correct answer is (B).

21. Frequency of source is $f = 500$ Hz

When observer is moving away from the source with speed v_1, then apparent frequency is

$$f_{\text{recede}} = f_1 = 480 = f\left(\frac{v - v_1}{v}\right) \quad \ldots(1)$$

$$\Rightarrow \quad 480 = 500\left(\frac{300 - v_1}{300}\right)$$

$$\Rightarrow \quad v_1 = 12 \text{ ms}^{-1}$$

When observer is moving towards the source with speed v_2, then apparent frequency is

$$f_{\text{approach}} = f_2 = 530 = f\left(\frac{v + v_2}{v}\right) \quad \ldots(2)$$

$$\Rightarrow \quad 530 = 500\left(\frac{300 + v_2}{300}\right)$$

$$\Rightarrow \quad v_2 = 18 \text{ ms}^{-1}$$

Hence, the correct answer is (D).

22. Beat frequency $f_b = |f_1 - f_2| = 11 - 9 = 2$ Hz

Hence, beat time i.e., time interval between two consecutive maxima or minima is

$$t_b = \frac{1}{f_b} = \frac{1}{2} \text{ s}$$

Hence, the correct answer is (A).

23. Since, $f_{\text{approach}} = 1000 = f\left(\frac{v - 0}{v - v_S}\right) = f\left(\frac{350}{350 - 50}\right)$

$$\Rightarrow \quad 1000 = f\left(\frac{350 - 0}{350 - 50}\right)$$

$$\Rightarrow \quad f = 1000\left(\frac{300}{350}\right) \text{ Hz}$$

On receding from the listener, we have

$$f_{\text{recede}} = f\left(\frac{v - 0}{v + v_S}\right) = f\left(\frac{350}{350 + 50}\right) = f\left(\frac{350}{400}\right)$$

$$\Rightarrow \quad f_{\text{recede}} = 1000\left(\frac{300}{350}\right)\left(\frac{350}{400}\right) = 750 \text{ Hz}$$

Hence, the correct answer is (D).

24. Let v_1 be the frequency received by A, then

$$v_1 = v_0\left(\frac{1500 - 5}{1500 - 7.5}\right) = v_0\left(\frac{1495}{1492.5}\right)$$

Frequency received by B is

$$v_2 = v_1\left(\frac{v + v_L}{v + v_S}\right) = v_0\left(\frac{1495}{1492.5}\right)\left(\frac{1507.5}{1505}\right)$$

CHAPTER 4

$\Rightarrow \quad \nu_2 = \nu = 502$ Hz

Hence, the correct answer is (D).

25. Since, $y = A\sin(kx - \omega t + \phi)$

At $t = 0$ and $x = 0$, we observe that $y = 0$. So, particle is at mean position and will proceed in positive y direction.
Hence, the correct answer is (A).

26. Since, $120 = 10\log_{10}\left(\dfrac{I}{10^{-12}}\right)$

$\Rightarrow \quad \dfrac{I}{10^{-12}} = 10^{12}$

$\Rightarrow \quad I = 1\ \text{Wm}^{-2}$

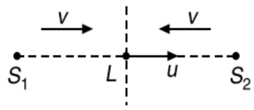
$\Rightarrow \quad \dfrac{2}{4\pi r^2} = 1$

$\Rightarrow \quad r = \sqrt{\dfrac{2}{4\pi}}\ \text{m} = 0.399\ \text{m} = 40\ \text{cm}$

Hence, the correct answer is (B).

27. From Figure, we get

$f_1 = \left(\dfrac{v-u}{v}\right)f$ and $f_2 = \left(\dfrac{v+u}{v}\right)f$



$\Rightarrow \quad f_2 - f_1 = \left(\dfrac{2u}{v}\right)f$

$\Rightarrow \quad 10 = \left(\dfrac{2u}{330}\right)660$

$\Rightarrow \quad u = 2.5\ \text{ms}^{-1}$

Hence, the correct answer is (C).

28. Since, $\ell_1 = 30$ cm, $\ell_2 = 70$ cm

$\Rightarrow \quad \dfrac{\lambda}{2} = (\ell_2 - \ell_1) = 40$ cm

$\Rightarrow \quad \lambda = 80$ cm

$\Rightarrow \quad v = f\lambda = (480)(0.8) = 384\ \text{ms}^{-1}$

Hence, the correct answer is (B).

29. Since, $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{mg}{\mu}}$

Also, $v' = \sqrt{\dfrac{m\sqrt{g^2 + a^2}}{\mu}}$

$\Rightarrow \quad \dfrac{v'}{v} = \sqrt{\dfrac{\sqrt{g^2+a^2}}{g}}$

$\Rightarrow \quad a \approx 1.83\ \text{ms}^{-2} \approx \dfrac{g}{5}$

Hence, the correct answer is (B).

30. For second harmonic, we have

$\lambda = L = 0.50$ m

$\Rightarrow \quad f = \dfrac{v}{\lambda} = 330 \times 2 = 660$ Hz

From Doppler's effect of sound, we get

$$f' = f\left(\dfrac{330 + \dfrac{50}{18}}{330}\right) \approx 666\ \text{Hz}$$

Hence, the correct answer is (D).

31. Since, $f_1 = f\left(\dfrac{340}{340-34}\right)$ and $f_2 = f\left(\dfrac{340}{340-17}\right)$

$\Rightarrow \quad \dfrac{f_1}{f_2} = \dfrac{340-17}{340-34} = \dfrac{19}{18}$

Hence, the correct answer is (D).

32. Since, $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{8\times 1}{5\times 10^{-3}}} = 40\ \text{ms}^{-1}$

$\Rightarrow \quad \lambda = \dfrac{v}{f} = \dfrac{40}{100} = 0.4$ m

Separation between successive nodes is $\dfrac{\lambda}{2}$

$\Rightarrow \quad \Delta x = 0.2\ \text{m} \approx 20$ cm

Hence, the correct answer is (D).

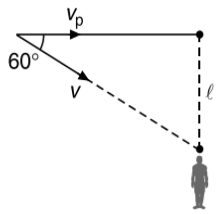
33. Since, $v = \dfrac{\omega}{k} = \dfrac{450}{9} = 50\ \text{ms}^{-1}$

$\Rightarrow \quad v = 50 = \sqrt{\dfrac{T}{\mu}}$

$\Rightarrow \quad T = 2500 \times 5 \times 10^{-3} = 12.5$ N

Hence, the correct answer is (D).

34. Since, $\dfrac{l\,\text{cosec}(60°)}{v} = \dfrac{l\cot(60°)}{v_p}$



$\Rightarrow \quad v_p = \dfrac{v}{2}$

Hence, the correct answer is (C).

35. Since, $y(x,\ t) = 10^{-3}\sin(50t + 2x)$

$\Rightarrow \quad v = \dfrac{\omega}{k} = \dfrac{50}{2} = 25\ \text{ms}^{-1}$

Also, wave is travelling along negative x direction.
Hence, the correct answer is (A).

36. Since, $\dfrac{\lambda_1}{4} = l_0 + 11$ and $\dfrac{\lambda_2}{4} = l_0 + 27$

$\Rightarrow \quad \dfrac{\lambda_2 - \lambda_1}{4} = 16$ cm

$\Rightarrow \quad v\left(\dfrac{1}{256} - \dfrac{1}{512}\right) = 0.64$ m

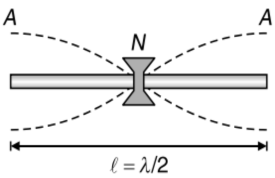
$\Rightarrow \quad v = (512)(0.64) = 328\ \text{ms}^{-1}$

Hence, the correct answer is (C).

37. Velocity of wave, $v=\sqrt{\frac{Y}{\rho}}=\sqrt{\frac{9.27\times10^{10}}{2.7\times10^{3}}}$

$\Rightarrow\quad v=\sqrt{3.433\times10^{7}}=10^{3}\sqrt{34.33}=5.86\times10^{3}\ \text{ms}^{-1}$

Since rod is clamped at the middle, shape of fundamental wave is shown in Figure.



$\Rightarrow\quad \frac{\lambda}{2}=L=\frac{60}{100}\ \text{m}$

$\Rightarrow\quad \lambda=1.2\ \text{m}$

So fundamental frequency is given by

$$f=\frac{v}{\lambda}=\frac{5.86\times10^{3}}{1.2}=4.88\times10^{3}\ \text{Hz}=5\ \text{kHz}$$

Hence, the correct answer is (A).

38. Frequency of organ pipe will either be 257 Hz or 255 Hz. If frequency of tuning fork is 257 Hz, then

$$257=\frac{3v}{2\ell}$$

$\Rightarrow\quad \ell=\frac{3\times340}{2\times257}\ \text{m}=1.98\ \text{m}=198\ \text{cm}$

When frequency of tuning fork is 255 Hz, then

$$255=\frac{3v}{2\ell}$$

$\Rightarrow\quad \ell=\frac{3\times340}{2\times255}\ \text{m}=2\ \text{m}=200\ \text{cm}$

So, length of pipe should be 200 cm

Hence, the correct answer is (C).

39. Frequency of a sonometer wire is $f=\frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$

$\Rightarrow\quad f\propto\frac{1}{L}$

Given that, $\frac{f_1}{f_2}=\frac{1}{0.95}=\frac{100}{95}$ ...(1)

and $f_1-f_2=10$ Hz ...(2)

From equations (1) and (2), we get

$$f_1=200\ \text{Hz},\ f_2=190\ \text{Hz}$$

So, frequency of tuning fork is

$$f=f_1-5=f_2+5=195\ \text{Hz}$$

Hence, the correct answer is (C).

40. Frequency of sitar string B is either 420 Hz or 430 Hz. As tension in string B is increased, its frequency will increase. If the frequency is 430 Hz, then beat frequency will increase and if the frequency is 420 Hz, then beat frequency will decrease. Hence correct frequency is 420 Hz.

Hence, the correct answer is (C).

41. Given that $e=1$ cm

For first resonance, $\frac{\lambda}{4}=\ell_1+e=11$ cm

For second resonance, $\frac{3\lambda}{4}=\ell_2+e$

$\Rightarrow\quad \ell_2=(3)(11)-1=32$ cm

Hence, the correct answer is (D).

42. The fundamental frequency in a stretched string is

$$f=\frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

where, n is number of antinodes (or number of loops), μ is the mass per unit length.
At the midpoint O of the two bridges, the node of stationary wave lies, hence length of two wires are equal, i.e., $L_1=L_2=L$.
Also, frequency remains same for both wires, i.e., $f_1=f_2$

$\Rightarrow\quad \frac{n_1}{2L}\sqrt{\frac{T}{\pi r^2\rho_1}}=\frac{n_2}{2L}\sqrt{\frac{T}{\pi r^2\rho_2}}$

$\Rightarrow\quad \frac{n_1}{\sqrt{\rho_1}}=\frac{n_2}{\sqrt{\rho_2}}$

$\Rightarrow\quad \frac{n_1}{n_2}=\sqrt{\frac{\rho_1}{\rho_2}}=\sqrt{\frac{\rho_1}{4\rho_1}}=\frac{1}{2}\qquad\{\because\ \rho_2=4\rho_1\}$

Hence, the correct answer is (B).

43. Given, $y(x,t)=0.5\left(\frac{5\pi}{4}x\right)\sin\cos(200\pi t)$

Comparing, this equation with standard equation of standing wave i.e., $y(x,t)=2a\sin kx\cos\omega t$, we get

$$k=\frac{5\pi}{4}\ \text{radm}^{-1}\ \text{and}\ \omega=200\pi\ \text{rads}^{-1}$$

So, speed of the travelling wave is

$$v=\frac{\omega}{k}=\frac{200\pi}{5\pi/4}=160\ \text{ms}^{-1}$$

Hence, the correct answer is (B).

44. Velocity at point P is

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{(\mu x)g}{\mu}}=\sqrt{gx}$$

$\Rightarrow\quad \frac{dx}{dt}=\sqrt{gx}$

$\Rightarrow\quad \int_0^{20}\frac{dx}{\sqrt{x}}=\int_0^{t}\sqrt{g}\,dt$

$\Rightarrow\quad t=2\sqrt{2}$ sec

Hence, the correct answer is (D).

45. Given that, $f_{\text{open}}=f=\frac{v}{2L}$

When pipe is half dipped in water, then

$$f'=f_{\text{closed}}=\frac{v}{4L'}=\frac{v}{4(L/2)}=f$$

Hence, the correct answer is (A).

CHAPTER 4

46. Frequency heard by the driver of second engine

$$f' = f\left(\frac{v+v_L}{v-v_S}\right)$$

$$\Rightarrow \quad f' = 540\left(\frac{330+30}{330-30}\right) = 540\left(\frac{360}{300}\right) = 648 \text{ Hz}$$

Hence, the correct answer is (D).

47. Given that $f_1 - f_2 = 5$ bps ...(1)

Apparent frequency heard by the observer directly is

$$f_1 = f\left(\frac{v}{v-v_S}\right) = f\left(\frac{340}{340-5}\right) = f\left(\frac{340}{335}\right)$$

Apparent frequency heard by the observer after sound gets reflected from the wall is

$$f_2 = f\left(\frac{v}{v+v_S}\right) = f\left(\frac{340}{340+5}\right) = f\left(\frac{340}{345}\right)$$

Substituting in equation (1), we get

$$f\left(\frac{340}{335} - \frac{340}{345}\right) = 5$$

$$\Rightarrow \quad f = \frac{5\times 335\times 345}{340\times 10} = 169.96 \text{ Hz} \approx 170 \text{ Hz}$$

Hence, the correct answer is (D).

48. Since, $f_{\text{approach}} = f\left(\frac{v}{v-v_S}\right)$ and $f_{\text{recede}} = f\left(\frac{v}{v+v_s}\right)$

$$\Rightarrow \quad \Delta f = f_{\text{approach}} - f_{\text{recede}} = \left(\frac{2vv_S}{v^2-v_S^2}\right)f \approx \left(\frac{2v_S}{v}\right)f$$

$$\Rightarrow \quad \frac{\Delta f}{f}\times 100\% = \left(\frac{2\times 20}{320}\right)\times 100\% = 12.5\%$$

Hence, the correct answer is (B).

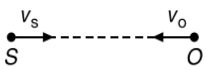
49. Apparent frequency heard by bat is

$$f' = f\left(\frac{v+v_{\text{bat}}}{v-v_{\text{bat}}}\right) = 8000\left(\frac{320+10}{320-10}\right)$$

$$\Rightarrow \quad f' = \frac{330}{310}\times 8000 = 8516 \text{ Hz}$$

Hence, the correct answer is (B).

50. For the given situation, frequency heard by observer is given by



$$f = f_0\left(\frac{v+v_0}{v-v_s}\right) = \frac{f_0 v}{v-v_s} + \frac{f_0 v_0}{v-v_s}$$

Comparing this equation with the equation of a straight line i.e., $y = mx + c$, we observe that slope of graph is $m = \frac{f_0}{v-v_s}$.

Hence, the correct answer is (A).

51. Since pipe is closed from one end, so it behaves like a closed organ pipe in which frequencies are given by

$$f = (2n-1)\left(\frac{v}{4L}\right), \text{ where } n = 1, 2, 3, 4, \ldots$$

According to question, $f < 1250$ Hz

$$\Rightarrow \quad (2n-1)\left(\frac{v}{4L}\right) < 1250$$

$$\Rightarrow \quad (2n-1)\left(\frac{340}{4\times 0.85}\right) < 1250$$

$$\Rightarrow \quad (2n-1) < 12.5$$

Hence possible value of n are

$n = 1, 2, 3, 4, 5, 6$

So, number of possible natural frequencies that lie below 1250 Hz is 6.

Hence, the correct answer is (D).

52. Since, $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2L}\sqrt{\frac{\text{Stress}}{\text{Density}}}$ ...(1)

Also, $Y = \frac{\text{Stress}}{\text{Strain}}$ i.e., Stress $= Y(\text{Strain})$

Therefore, equation (1) becomes

$$f = \frac{1}{2L}\sqrt{\frac{Y(\text{Strain})}{\text{Density}}}$$

$$\Rightarrow \quad f = \frac{1}{2(1.5)}\sqrt{\frac{(2.2\times 10^{11})(1/100)}{7.7\times 10^3}}$$

$$\Rightarrow \quad f = \frac{1}{3}\sqrt{\frac{2}{7}\times 10^6} = \frac{1000}{3}\times\frac{\sqrt{2}}{\sqrt{7}} \approx 178.2 \text{ Hz}$$

Hence, the correct answer is (B).

53. Given, $y(x,t) = e^{-(ax^2+bt^2+2\sqrt{ab}xt)}$

$$\Rightarrow \quad y(x,t) = e^{-(\sqrt{a}x+\sqrt{b}t)^2} \qquad ...(1)$$

Comparing equation (1) with standard equation i.e.

$$y(x,t) = f(ax+bt)$$

Since there is positive sign between x and t terms, so wave travels along $-x$ direction such that wave speed is

$$v = \left|\frac{\text{Coefficient of } t}{\text{Coefficient of } x}\right| = \sqrt{\frac{b}{a}}$$

Hence, the correct answer is (B).

54. Wave velocity, $v = \frac{\omega}{k} = \frac{2\pi/0.04}{2\pi/0.5}$ ms^{-1}

Also $v = \sqrt{\frac{T}{\mu}}$, where T is the tension in the string and μ is the linear mass density given by $\mu = 0.04$ kgm^{-1}

$$\Rightarrow \quad v = \frac{\omega}{k} = \sqrt{\frac{T}{\mu}}$$

$$\Rightarrow \quad T = \frac{\mu\omega^2}{k^2} = \mu v^2$$

$$T = 0.04\left(\frac{2\pi/0.04}{2\pi/0.5}\right)^2 = 6.25 \text{ N}$$

Hence, the correct answer is (A).

55. The number of beats produced per second is

$$\Delta\nu = |(\nu+1)-(\nu-1)| = 2$$

Hence, the correct answer is (C).

56. Since source is at rest and observer is moving away from source, so

$$f' = f\left(\frac{v-v_L}{v-0}\right) = f\left(\frac{v-v_L}{v}\right)$$

$$\Rightarrow \left(\frac{f'}{f}\right)v = v - v_L$$

$$\Rightarrow v\left(1-\frac{f'}{f}\right) = v_L$$

$$\Rightarrow 330(1-0.94) = v_L$$

$$\Rightarrow v_L = 330\times 0.06 = 19.80 \text{ ms}^{-1}$$

$$\Rightarrow s = \frac{v_L^2 - 0^2}{2a} = \frac{(19.80)^2}{2\times 2} = 98 \text{ m}$$

Hence, the correct answer is (B).

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

1. Since, $\lambda = 4(L+e)$

$$\Rightarrow L = \frac{\lambda}{4} - e$$

$$\Rightarrow L = \frac{\lambda}{4} - 0.3d = 15.2 \text{ cm}$$

Hence, the correct answer is (B).

2. For hollow pipe, fundamental frequency is

$$f = \frac{v}{4\ell} = \frac{320}{4\times 0.8}$$

For string in 2^{nd} harmonic

$$f = \frac{1}{\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{\ell}\sqrt{\frac{T\ell}{m}} = \frac{1}{0.5}\sqrt{\frac{50\times 0.5}{m}}$$

Equating, we get

$$m = 0.01 \text{ kg} = 10 \text{ g}$$

Hence, the correct answer is (B).

3. $v = 10 \text{ cms}^{-1}$

$\lambda = 0.5$ m

$A = 10 \text{ cm} = 0.1$ m

From the figure, we observe that the equation of the wave must be

$$y = A\sin(kx - \omega t) \quad \ldots(1)$$

where $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.5} = 4\pi \text{ m}^{-1}$

Since, $v = \frac{\omega}{k}$

$$\Rightarrow \omega = vk$$

$$\Rightarrow \omega = \left(\frac{10}{100}\right)(4\pi) = 0.4\pi \text{ rads}^{-1}$$

So, equation (1) becomes

$$y = 0.1\sin(4\pi x - 0.4\pi t)\hat{j} \quad \ldots(2)$$

Now $v_P = \frac{\partial y}{\partial t} = (0.1)(0.4\pi)\cos(4\pi x - 0.4\pi t) \quad \ldots(3)$

Further, velocity at point P at displacement $y = 5$ cm is calculated first by substituting $y = \frac{5}{100}$ m in equation (2). So

$$0.05 = 0.1\sin(4\pi x - 0.4\pi t)$$

$$\Rightarrow 4\pi x - 0.4\pi t = \frac{\pi}{6} \quad \ldots(4)$$

$$\Rightarrow v_P = (0.1)(0.4\pi)\cos\left(\frac{\pi}{6}\right)$$

$$\Rightarrow v_P = (0.1)(0.4\pi)\frac{\sqrt{3}}{2} = \frac{4\pi\sqrt{3}}{200} = \frac{\pi\sqrt{3}}{50} \text{ ms}^{-1}$$

Hence, the correct answer is (C).

4. With increase in tension, frequency of vibrating string will increase. Since number of beats are decreasing. Therefore, frequency of vibrating string or third harmonic frequency of closed pipe should be less than the frequency of tuning fork by 4, so we have frequency of tuning fork (n) given by

$$n = f_3 + 4$$

$$\Rightarrow n = 3\left(\frac{v}{4l}\right) + 4 = 3\left(\frac{340}{4\times 0.75}\right) + 4 = 344 \text{ Hz}$$

Hence, the correct answer is (A).

5. Since, $f \propto v \propto \sqrt{T}$

Given that, $f_{AB} = 2f_{CD}$

$$\Rightarrow T_{AB} = 4T_{CD} \quad \ldots(1)$$

Further $\sum \tau_p = 0$

$$\Rightarrow T_{AB}(x) = T_{CD}(l-x)$$

$$\Rightarrow 4x = l - x \quad \{\because T_{AB} = 4T_{CD}\}$$

$$\Rightarrow x = \frac{l}{5}$$

Hence, the correct answer is (A).

6. Given, $\frac{\lambda}{2} = (63.2 - 30.7)$ cm

$$\Rightarrow \lambda = 0.65 \text{ m}$$

So, the speed of sound observed is

$$v_0 = f\lambda = 512\times 0.65 = 332.8 \text{ ms}^{-1}$$

Error in calculating velocity of sound is $\Delta v = 2.8 \text{ ms}^{-1}$

$$\Rightarrow \Delta v = 280 \text{ cms}^{-1}$$

Hence, the correct answer is (D).

7. $f_1 = \frac{v}{\ell}$ $\{2^{nd}$ harmonic of open pipe$\}$

$f_2 = n\left(\frac{v}{4\ell}\right)$ $\{n^{th}$ harmonic of closed pipe$\}$

Here, n is odd and $f_2 > f_1$

It is possible when $n = 5$ because with $n = 5$

CHAPTER 4

$$f_2 = \frac{5}{4}\left(\frac{v}{\ell}\right) = \frac{5}{4}f_1$$

Hence, the correct answer is (C).

8. The frequency is a characteristic of source. It is independent of the medium.

Hence, the correct answer is (D).

9. $f_c = f_0$ {both first overtone}

$$\Rightarrow \quad 3\left(\frac{v_c}{4L}\right) = 2\left(\frac{v_0}{2L_0}\right)$$

$$\Rightarrow \quad \ell_0 = \frac{4}{3}\left(\frac{v_0}{v_c}\right)L = \frac{4}{3}\sqrt{\frac{\rho_1}{\rho_2}}L \qquad \left\{\because v \propto \frac{1}{\sqrt{p}}\right\}$$

Hence, the correct answer is (C).

10. Let $\Delta\ell$ be the end correction.

Given that, fundamental tone for a length 0.1 m = first overtone for the length 0.35 m.

$$\frac{v}{4(0.1+\Delta\ell)} = \frac{3v}{4(0.35+\Delta\ell)}$$

Solving this equation, we get $\Delta\ell = 0.025$ m = 2.5 m

Hence, the correct answer is (B).

11. The motorcyclist observes no beats. So, the apparent frequency observed by him from the two source must be equal.

$$f_1 = f_2$$

$$\Rightarrow \quad 176\left(\frac{330-v}{330-22}\right) = 165\left(\frac{330+v}{330}\right)$$

Solving this equation, we get

$$v = 22 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

12. $f = \dfrac{p}{2\ell}\sqrt{\dfrac{T}{\mu}}$

$$\Rightarrow \quad \frac{5}{2\ell}\sqrt{\frac{9g}{\mu}} = \frac{3}{2\ell}\sqrt{\frac{Mg}{\mu}}$$

$$\Rightarrow \quad 5(3) = 3\sqrt{M}$$

$$\Rightarrow \quad M = 25 \text{ kg}$$

Hence, the correct answer is (A).

13. Using the formula $f' = f\left(\dfrac{v+v_0}{v}\right)$

we get, $5.5 = 5\left(\dfrac{v+v_A}{v}\right)$ ...(1)

and $6 = 5\left(\dfrac{v+v_B}{v}\right)$ ...(2)

Here, v = speed of sound

v_A = speed of train A

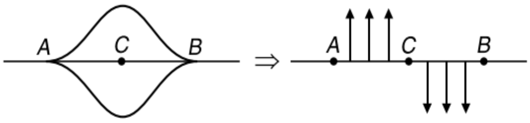
v_B = speed of train B

Solving equations (1) and (2), we get

$$\frac{v_B}{v_A} = 2$$

Hence, the correct answer is (B).

14. After two seconds both the pulse will move 4 cm towards each other. So, by their superposition, the resultant displacement at every point will be zero. Therefore, total energy will be purely in the form of kinetic. Half of the particles will be moving upwards and half downwards.



Hence, the correct answer is (B).

15. Energy $E \propto (\text{amplitude})^2(\text{frequency})^2$

Amplitude (A) is same in both the cases, but frequency 2ω in the second case is two times the frequency (ω) in the first case.

$\Rightarrow \quad E_2 = 4E_1$

Hence, the correct answer is (C).

16. Fundamental frequency v is given by

$$v = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} \qquad \text{\{with both the ends fixed\}}$$

So, fundamental frequency is

$$v \propto \frac{1}{\ell\sqrt{\mu}} \qquad \text{\{for same tension in both strings\}}$$

where μ is mass per unit length of wire

$\Rightarrow \quad \mu = \rho A = \rho(\pi r^2)$, where ρ is density of wire

$$\Rightarrow \quad \sqrt{\mu} \propto r$$

$$\Rightarrow \quad v \propto \frac{1}{r\ell}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \left(\frac{r_2}{r_1}\right)\left(\frac{\ell_2}{\ell_1}\right) = \left(\frac{r}{2r}\right)\left(\frac{2L}{L}\right) = 1$$

Hence, the correct answer is (D).

17. Since, $f_1 = f\left(\dfrac{v}{v-v_s}\right) = f\left(\dfrac{340}{340-34}\right) = f\left(\dfrac{340}{306}\right)$

and $f_2 = f\left(\dfrac{340}{340-17}\right) = f\left(\dfrac{340}{323}\right)$

$$\Rightarrow \quad \frac{f_1}{f_2} = \frac{323}{306} = \frac{19}{18}$$

Hence, the correct answer is (D).

18. Speed of sound in gases is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$\Rightarrow \quad v \propto \frac{1}{\sqrt{M}}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}$$

Hence, the correct answer is (B).

19. Speed of sound in an ideal gas is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$\Rightarrow \quad v \propto \sqrt{\frac{\gamma}{M}} \qquad \{T \text{ is mass for both the gases}\}$$

$$\Rightarrow \quad \frac{v_{N_2}}{v_{He}} = \sqrt{\frac{\gamma_{N_2}}{\gamma_{He}} \frac{M_{He}}{M_{N_2}}} = \sqrt{\frac{7/5}{5/3}\left(\frac{4}{28}\right)} = \frac{\sqrt{3}}{5}$$

Because, $\gamma_{N_2} = r_{\text{diatomic}} = \frac{7}{5}$ and $\gamma_{He} = r_{\text{monatomic}} = \frac{5}{3}$

Hence, the correct answer is (C).

20. Mass per unit length of the string

$$m = \frac{10^{-2}}{0.4} = 2.5 \times 10^{-2} \text{ kgm}^{-1}$$

So, velocity of wave in the string $v = \sqrt{\frac{T}{m}}$

$$\Rightarrow \quad v = \sqrt{\frac{1.6}{2.5 \times 10^{-2}}} = 8 \text{ ms}^{-1}$$

For constructive interference between successive pulses

$$\Delta t_{\text{min}} = \frac{2\ell}{v} = \frac{(2)(0.4)}{8} = 0.10 \text{ s}$$

Hence, the correct answer is (B).

21. This is an equation of a travelling wave in which particles of the medium are in SHM and maximum particle velocity in SHM is $A\omega$, where A is the amplitude and ω the angular velocity.

Hence, the correct answer is (A).

22. Source is moving towards the observer

$$f' = f\left(\frac{v}{v - v_s}\right)$$

$$\Rightarrow \quad f' = 450\left(\frac{330}{330 - 33}\right)$$

$$\Rightarrow \quad f' = 500 \text{ Hz}$$

Hence, the correct answer is (D).

23. From Hooke's Law

Tension in a string $(T) \propto$ extension (x)

and speed of sound in string $v = \sqrt{Tm}$

$$\Rightarrow \quad v \propto \sqrt{T}$$

Therefore, $v \propto \sqrt{x}$

x is increased to 1.5 times i.e., speed will increase by $\sqrt{1.5}$ times or 1.22 times. Therefore, speed of sound in new position will be 1.22 v.

Hence, the correct answer is (A).

24. Length of the organ pipe is same in both the cases. Fundamental frequency of open pipe is $f_1 = \frac{v}{2\ell}$ and frequency of third harmonic of closed pipe will be

$$f_2 = 3\left(\frac{v}{4\ell}\right)$$

Given that $f_2 = f_1 + 100$

$$\Rightarrow \quad f_2 - f_1 = 100$$

$$\Rightarrow \quad \frac{3}{4}\left(\frac{v}{\ell}\right) - \left(\frac{1}{2}\right)\left(\frac{v}{\ell}\right) = 100$$

$$\Rightarrow \quad \frac{v}{4\ell} = 100 \text{ Hz}$$

$$\Rightarrow \quad \frac{v}{2\ell} = 200 \text{ Hz}$$

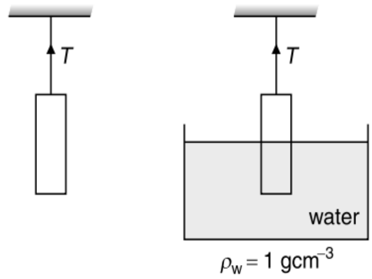
$$\Rightarrow \quad f_1 = 200 \text{ Hz}$$

Therefore, fundamental frequency of the open pipe is 200 Hz.

Hence, the correct answer is (A).

25. The diagrammatic representation of the given problem is shown in figure. The expression of fundamental frequency is

$$v = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$



In air $T = mg = (V\rho)g$

$$\Rightarrow \quad v = \frac{1}{2\ell}\sqrt{\frac{A\rho g}{\mu}} \qquad \ldots(1)$$

When the object is half immersed in water

$$T' = mg - \text{upthrust} = V_\rho g - \left(\frac{V}{2}\right)\rho_w g$$

$$\Rightarrow \quad T' = \left(\frac{V}{2}\right)g(2\rho - \rho_w)$$

The new fundamental frequency

$$v' = \frac{1}{2\ell} \times \sqrt{\frac{T}{m}} = \frac{1}{2\ell}\sqrt{\frac{(V_g/2)(2\rho - \rho_w)}{\mu}} \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{v'}{v} = \left(\sqrt{\frac{2\rho - \rho_w}{2\rho}}\right)$$

$$\Rightarrow \quad v' = v\left(\frac{2\rho - \rho_w}{2\rho}\right)^{1/2} = 300\left(\frac{2\rho - 1}{2\rho}\right)^{1/2} \text{ Hz}$$

Hence, the correct answer is (A).

26. The given equation can be written as

$$y = 2\left(2\cos^2\frac{t}{2}\right)\sin(1000t)$$

$$\Rightarrow \quad y = 2(\cos t + 1)\sin(1000t)$$

$$\Rightarrow \quad y = 2\cos t \sin 1000t + 2\sin(1000t)$$

$$\Rightarrow \quad y = \sin(1001t) + \sin(999t) + 2\sin(1000t)$$

i.e., the given expression is a result of superposition of three independent harmonic motions of angular frequencies 999, 1000 and 1001 rads^{-1}.

Hence, the correct answer is (B).

27. Since $(f_1)_{closed} = (f_3)_{open}$

$$\Rightarrow \quad \frac{v}{4\ell_1} = 3\left(\frac{v}{2\ell_2}\right)$$

$$\Rightarrow \quad \frac{\ell_1}{\ell_2} = \frac{1}{6}$$

Hence, the correct answer is (C).

28. Since the point $x = 0$ is a node and reflection is taking place from point $x = 0$. This means that reflection must be taking place from the fixed end and hence the reflected ray must suffer an additional phase change of π or a path change of $\frac{\lambda}{2}$.

So, if $y_{incident} = a\cos(kx - \omega t)$

$$\Rightarrow \quad y_{reflected} = a\cos(-kx - \omega t + \pi)$$

$$\Rightarrow \quad y_{reflected} = -a\cos(\omega t + kx)$$

Hence, the correct answer is (B).

29. Only first equation is the equation of standing wave. The condition for a function of x and t to represent a wave is

$$\frac{\partial^2 y}{\partial x^2} = (\text{constant})\frac{\partial^2 y}{\partial t^2}$$

Only first equation satisfies the condition.

Hence, the correct answer is (A).

30. $f_c = \frac{v}{4\ell} = 512 \text{ Hz}$

$$f_0 = \frac{v}{2\ell} = 2f_c = 1024 \text{ Hz}$$

Hence, the correct answer is (A).

31. $(v_P)_{max} = a\omega = y_0\omega$

Since, $(v_P)_{max} = 4v$

$$\Rightarrow \quad y_0\omega = 4\left(\frac{\omega}{k}\right)$$

$$\Rightarrow \quad y_0 = \frac{4}{k} = \frac{4}{(2\pi/\lambda)}$$

$$\Rightarrow \quad \lambda = \frac{\pi y_0}{2}$$

Hence, the correct answer is (B).

32. Initially the tube was open at both ends and then it is closed. Further

$$f_0 = \frac{v}{2\ell_0} \text{ and } f_c = \frac{v}{4\ell_c}$$

Since, tube is half dipped in water, $\ell_c = \frac{\ell_0}{2}$

$$\Rightarrow \quad f_c = \frac{v}{4\left(\frac{\ell_0}{2}\right)} = \frac{v}{2\ell_0} = f_0 = f$$

Hence, the correct answer is (C).

Multiple Correct Choice Type Questions

1. Let F be tension in the rope of linear mass density (ρ), then $v = \sqrt{\frac{F}{\mu}}$. Hence, speed at any position will be same for both pulses. Therefore, time taken by both pulses will be same.

Since, $\lambda = \frac{v}{f}$ and $v \propto \sqrt{F}$

$$\Rightarrow \quad \lambda \propto v \propto \sqrt{F}$$

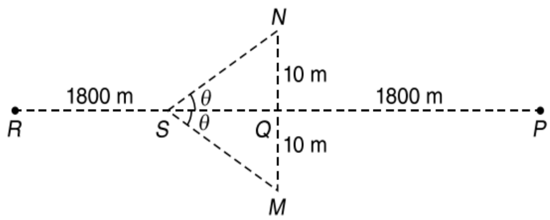
When pulse 1 reaches at A tension F decreases, so speed decreases so, λ decreases.

Conceptual Note(s)

If we refer velocity by magnitude only, then option (A, C, D) will be correct, else only (A, C) will be correct.

Hence, (A), (C) and (D) are correct.

2. Speed of car, $v_C = 60 \text{ kmh}^{-1} = \frac{500}{3} \text{ ms}^{-1}$



At a point S, between P and Q

$$v'_M = v_M\left(\frac{v + v_C\cos\theta}{v}\right) \text{ and } v'_N = v_N\left(\frac{v + v_C\cos\theta}{v}\right)$$

$$\Rightarrow \quad \Delta v = (v_N - v_M)\left(1 + \frac{v_C\cos\theta}{v}\right)$$

Similarly, between Q and R, we have

$$\Delta v = (v_N - v_M)\left(1 - \frac{v_C\cos\theta}{v}\right)$$

$$\Rightarrow \quad \frac{d(\Delta v)}{dt} = \pm(v_N - v_M)\frac{v_C}{v}\sin\theta\left(\frac{d\theta}{dt}\right)$$

At P and R, the car is very far from M and N, so $\theta \approx 0°$. Hence, slope of graph is zero.

At Q, we have $\theta = 90°$, so $\sin\theta$ is maximum. Also value of $\frac{d\theta}{dt}$ is maximum as $\left(\frac{d\theta}{dt}\right)_Q = \frac{v}{r_{min}} = \frac{v}{10}$ constant

Hence, slope is maximum at Q.

At P, $v_P = \Delta v = (v_N - v_M)\left(1 + \frac{v_C}{v}\right)$ $\{\because \theta = 0°\}$

At R, $v_R = \Delta v = (v_N - v_M)\left(1 - \frac{v_C}{v}\right)$ $\{\because \theta = 0°\}$

At Q, $v_Q = \Delta v = v_N - v_M$ $\{\because \theta = 90°\}$

From these equations, we get

$$v_P + v_R = 2v_Q$$

Hence, (A), (B) and (C) are correct.

3. Minimum resonance length is $\ell_1 = \frac{\lambda}{4}$

$$\Rightarrow \quad \lambda = 4\ell_1$$

$$\Rightarrow \quad v = f\lambda = (244)(4\ell_1)$$

where $\ell_1 = (0.350 \pm 0.005)$ m

So, v lies between 336.7 ms^{-1} to 346.5 ms^{-1}

Now, $v = \sqrt{\dfrac{\gamma RT}{M \times 10^{-3}}}$, here M is molar mass of gas in gram

$\Rightarrow \quad v = \sqrt{100\gamma RT} \times \sqrt{\dfrac{10}{M}}$

For monoatomic gas i.e., for neon and argon, $\gamma = 1.67$

$\Rightarrow \quad v = 640 \times \sqrt{\dfrac{10}{M}}$

For diatomic gas i.e., for oxygen and nitrogen, $\gamma = 1.4$

$\Rightarrow \quad v = 590 \times \sqrt{\dfrac{10}{M}}$

$v_{Ne} = 640 \times \dfrac{7}{10} = 448\ ms^{-1}$, $v_{Ar} = 640 \times \dfrac{17}{32} = 340\ ms^{-1}$

$v_{O_2} = 590 \times \dfrac{9}{16} = 331.8\ ms^{-1}$, $v_{N_2} = 590 \times \dfrac{3}{5} = 354\ ms^{-1}$

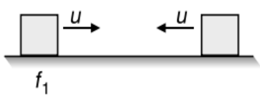
Only possible answer is Argon.
Hence, the correct answer is (D).

4. There should be a node at $x = 0$ and antinode at $x = 3$ m.
Also, $v = \dfrac{\omega}{k} = 100\ ms^{-1}$

Hence $y = 0$ at $x = 0$ and $y = \pm A$ at $x = 3$ m

Only (A), (C) and (D) satisfy the condition.
Hence, (A), (C) and (D) are correct.

5. $f_2 = \left(\dfrac{V - w + u}{V - w - u}\right) f_1$



So, $f_2 > f_1$

$f_2 = \left(\dfrac{V + w + u}{V + w - u}\right) f_1$

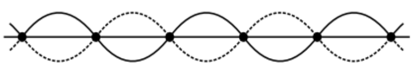
So, $f_2 > f_1$

Hence, (A) and (B) are correct.

6. At open end phase of pressure wave changes by 180°. So, compression returns as rarefaction. At closed end there is no phase change. So, compression returns as compression returns as compression and rarefaction as rarefaction.
Hence, (A), (B) and (D) are correct.

7. For fifth harmonic, number of loops formed is 5, so
No. of nodes = 6

$$\lambda = \frac{2\pi}{k} = \frac{2 \times 3.14}{62.8} = 0.1 \text{ m}$$



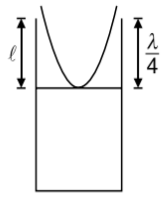
Length $\ell = \dfrac{5\lambda}{2} = 0.25$ m

The mid-point is antinode, so its maximum displacement is 0.01 m

$\Rightarrow \quad f = \dfrac{v}{2\ell} = \dfrac{\omega}{k(2\ell)} = 20$ Hz

Hence, (B) and (C) are correct.

8. Since $\ell < \dfrac{\lambda}{4}$

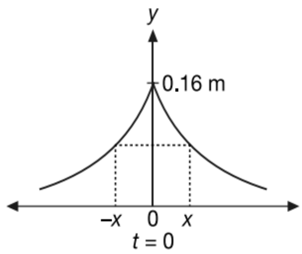


Further, larger the length of air column, weaker is the intensity.
Hence, (A) and (D) are correct.

9. Standing waves can be produced only when two similar type of waves (same frequency and speed, but amplitude may be different) travel in opposite directions.
Hence, (A), (B) and (C) are correct.

10. In case of sound wave, y can represent pressure and displacement, while in case of an electromagnetic wave it represents electric and magnetic fields.
NOTE: In general, y is any general physical quantity which is a made to oscillate at one place and these oscillations are propagated to other places also.
Hence, (A), (B), (C) and (D) are correct.

11. The shape of pulse at $x = 0$ and $t = 0$ would be as shown, in Figure.



$$y(0,\ 0) = \frac{0.8}{5} = 0.16 \text{ m}$$

From the figure it is clear that $y_{max} = 0.16$ m
Pulse will be symmetric (Symmetry is checked about y_{max}) if at $t = 0$

$$y(x) = y(-x)$$

From the given equation

$$y(x) = \frac{0.8}{16x^2 + 5}$$

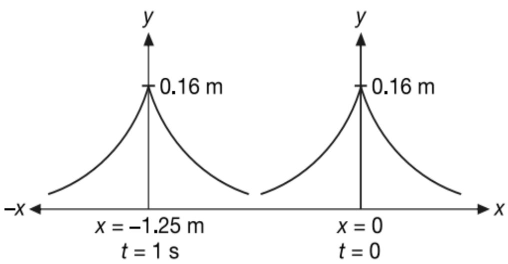
and $y(-x) = \dfrac{0.8}{16x^2 + 5}$ {at $t = 0$}

$\Rightarrow \quad y(x) = y(-x)$

Therefore, pulse is symmetric.

Speed of pulse

At $t = 1$ s and $x = -1.25$ m value of y is again 0.16 m, i.e., pulse has travelled a distance of 1.25 m in 1 s in negative x-direction or we can say that the speed of pulse is 1.25 ms^{-1} and it is travelling in negative x-direction. Therefore, it will travel a distance of 2.5 m in 2 s. The above statement can be better understood from Figure.



Alternate Method:

If equation of a wave pulse is

$$y = f(ax \pm bt)$$

The speed of wave is $\frac{b}{a}$ in negative x-direction for $y = f(ax+bt)$ and positive x-direction for $y = f(ax-bt)$. Comparing this from given equation we can find that speed of wave is $\frac{5}{4} = 1.25 \text{ ms}^{-1}$ and it is travelling in negative x-direction.

Hence, (B), (C) and (D) are correct.

12. For a plane wave intensity (energy crossing per unit area per unit time) is constant at all points.
But for a spherical wave, intensity at a distance r from a point source of power P (energy transmitted per unit time) is given by

$$I = \frac{P}{4\pi r^2} \text{ i.e., } I \propto \frac{1}{r^2}$$

For a line source $I \propto \frac{1}{r}$, because $I = \frac{P}{\pi r \ell}$

Hence, (A), (C) and (D) are correct.

13. Maximum speed of any point on the string is $v_{max} = a\omega$

$\Rightarrow \quad v_{max} = a(2\pi f)$

$\Rightarrow \quad v_{max} = \frac{v}{10} = \frac{10}{10} = 1 \qquad$ {Given: $v = 10 \text{ ms}^{-1}$}

$\Rightarrow \quad 2\pi a f = 1$

$\Rightarrow \quad f = \frac{1}{2\pi a}$

Since, $a = 10^{-3}$ m

$\Rightarrow \quad f = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi}$ Hz

Speed of wave, $v = f\lambda$

$\Rightarrow \quad (10 \text{ ms}^{-1}) = \left(\frac{10^3}{2\pi} \text{ s}^{-1}\right)\lambda$

$\Rightarrow \quad \lambda = 2\pi \times 10^{-2}$ m

Hence, (A) and (C) are correct.

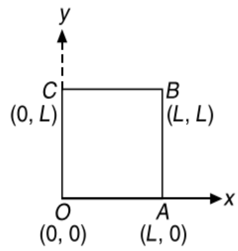
14. Since, the edges are clamped, displacement of the edges $u(x, y) = 0$ for

Line OA i.e., $y = 0, 0 \le x \le L$

AB i.e., $x = L, 0 \le y \le L$

BC i.e., $y = L\ 0 \le x \le L$

OC i.e., $x = 0, 0 \le y \le L$



The above conditions are satisfied only in alternatives (B) and (C).

Note that $u(x, y) = 0$, for all four values e.g., in alternative (D), $u(x, y) = 0$ for $y = 0$, $y = L$ but it is not zero for $x = 0$ or $x = L$.
Similarly, in OPTION (A) $u(x, y) = 0$ at $x = L$, $y = L$ but it is not zero for $x = 0$ or $y = 0$ while in OPTIONS (B) and (C), $u(x, y) = 0$ for $x = 0$, $y = 0$, $x = L$ and $y = L$.

Hence, (B) and (C) are correct.

15. $y = 0.02\cos(10\pi x)\cos\left(50\pi t + \frac{\pi}{2}\right)$

At node, amplitude = 0

$\Rightarrow \quad \cos(10\pi x) = 0$

$\Rightarrow \quad 10\pi x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\Rightarrow \quad x = \frac{1}{20} = 0.05 \text{ m}, 0.15 \text{ m}.......$

At antinode, amplitude is maximum

$\Rightarrow \quad \cos(10\pi x) = \pm 1$

$\Rightarrow \quad x = 0, \pi, 2\pi...$

$\Rightarrow \quad x = 0, 0.1 \text{ m}, 0.2 \text{ m}......$

Now λ is twice the distance between two nodes or antinodes

$\Rightarrow \quad \lambda = 2(0.1) = 0.2$ m and $\frac{2\pi v t}{\lambda} = 50\pi t$

$\Rightarrow \quad v = 25\lambda = 25 \times 0.2 = 5 \text{ ms}^{-1}$

Hence, (A), (B), (C) and (D) are correct.

16. The number of waves encountered by the moving plane per unit time is given by

$$n = \frac{\text{distance travelled}}{\text{wavelength}}$$

$\Rightarrow \quad n = \frac{c+v}{\lambda} = \frac{c}{\lambda}\left(1+\frac{v}{c}\right) = f\left(1+\frac{v}{c}\right) \qquad$ {OPTION (A)}

The stationary observer meets the frequency f' of the incident wave and receives the reflected wave of frequency f'' emitted by the moving platform as

$$f'' = \frac{f'}{1-\frac{v}{c}} = \frac{f\left(1+\frac{v}{c}\right)}{1-\frac{v}{c}} = \frac{f(c+v)}{(c-v)} \qquad \text{{OPTION (C)}}$$

Wavelength, $\lambda'' = \frac{c}{f''} = \frac{c}{f}\left(\frac{c-v}{c+v}\right) \qquad$ {OPTION (B)}

Beat frequency, $f_b = f'' - f$

$\Rightarrow \quad f_b = \frac{f\left(1+\frac{v}{c}\right)}{\left(1-\frac{v}{c}\right)} - f = f\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}} - 1\right)$

$$\Rightarrow \quad f_b = \frac{\left(1+\frac{v}{c}\right)f}{\left(1-\frac{v}{c}\right)} = \frac{2vf}{c-v}$$

Hence, (A), (B) and (C) are correct.

17. $T_1 > T_2$

$\Rightarrow \quad v_1 > v_2$

$\Rightarrow \quad f_1 > f_2$ and $f_1 - f_2 = 6$ Hz

Now, if T_1 is increased, f_1 will increase or $f_1 - f_2$ will increase. Therefore (D) OPTION is wrong.

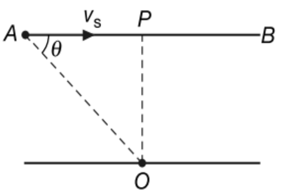
If T_1 is decreased f_1 will decrease and it may be possible that now $f_2 - f_1$ become 6 Hz. Therefore (C) OPTION is correct.

Similarly, when T_2 is increased, f_2 will increase and again $f_2 - f_1$ may become equal to 6 Hz. Therefore (C) OPTION is correct.

Similarly, when T_2 is increased, f_2 will increase and again $f_2 - f_1$ may become equal to 6 Hz. So, (B) is also correct but (A) is wrong.

Hence, (B) and (C) are correct.

18. The graph shows the situation shown in figure. The observed frequency will initially be more than the natural frequency. When the source is at P, natural frequency i.e., 2000 Hz.



For region $AP: f = f_0\left(\frac{v}{v - v_s\cos\theta}\right)$

For $PB: f = f_0\left(\frac{v}{v + v_s\cos\theta}\right)$

Minimum value of f will be

$$f_{min} = f_0\left(\frac{v}{v+v_s}\right) \qquad \text{when } \cos\theta = 1$$

$$\Rightarrow \quad 1800 = 2000\left(\frac{300}{300+v_s}\right)$$

Solving this we get,

$v_s = 33.33$ ms^{-1} and maximum value of f can be

$$f_{max} = f_0\left(\frac{v}{v-v_0}\right) \qquad \text{when } \cos\theta = 1$$

$$\Rightarrow \quad f_{max} = 2000\left(\frac{300}{300-33.33}\right) = 2250 \text{ Hz}$$

Hence, (C) and (D) are correct.

19. For closed pipe, $f = n\left(\frac{v}{4\ell}\right) n = 1, 3, 5.....$

For $n = 1$, $f_1 = \frac{v}{4\ell} = \frac{320}{4\times 1} = 80$ Hz

For $n = 3$, $f_3 = 3f_1 = 240$ Hz

For $n = 5$, $f_5 = 5f_1 = 400$ Hz

Hence, (A), (B) and (D) are correct.

20. For closed organ pipe,

$$f = n\left(\frac{v}{4\ell}\right) \text{ where, } n = 1, 3, 5, ...$$

$$\Rightarrow \quad \ell = \frac{nv}{4f}$$

For $n = 1$, $\ell_1 = \frac{(1)(330)}{4\times 264} \times 100$ cm $= 31.25$ cm

For $n = 3$, $\ell_3 = 3\ell_1 = 93.75$ cm

For $n = 5$, $\ell_5 = 5\ell_1 = 156.25$ cm

Hence, (A) and (C) are correct.

21. $y = 10^{-4}\sin(60t + 2x)$

$A = 10^{-4}$ m, $\omega = 60$ rads^{-1}, $k = 2$ m^{-1}

Speed of wave $v = \frac{\omega}{k} = 30$ ms^{-1}

Frequency, $f = \frac{\omega}{2\pi} = \frac{30}{\pi}$ Hz

Wavelength, $\lambda = \frac{2\pi}{k} = \pi m$

Further, $60t$ and $2x$ are of same sign. Therefore, the wave should travel in negative x-direction.

Hence, (A), (B), (C) and (D) are correct.

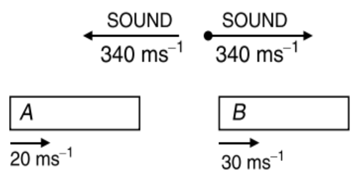
Linked Comprehension Type Questions

1. Velocity of sound w.r.t. passengers in train A is

$V_{SA} = 340 + 20 = 360$ ms^{-1}

Velocity of sound w.r.t. passengers in train B is

$V_{SB} = 340 - 30 = 310$ ms^{-1}



Hence, the correct answer is (B).

2. For the passengers in train A, there is no relative motion between source and observer, as both are moving with velocity 20 ms^{-1}. Therefore, there is no change in observed frequencies and correspondingly there is no change in their intensities.

Hence, the correct answer is (A).

3. For the passengers in train B, observer is receding with velocity 30 ms^{-1} and source is approaching with velocity 20 ms^{-1}.

$$\Rightarrow \quad f_1' = 800\left(\frac{340-30}{340-20}\right) = 775 \text{ Hz}$$

and $f_2' = 1120\left(\frac{340-30}{340-20}\right) = 1085$ Hz

Spread of frequency $\Delta f' = f_2' - f_1' = 310$ Hz

Hence, the correct answer is (A).

4. In one second number of maximas is called the beat frequency. Hence,

$$f_b = f_1 - f_2$$

$$\Rightarrow \quad f_b = \frac{100\pi}{2\pi} - \frac{92\pi}{2\pi}$$

$$\Rightarrow \quad f_b = 4 \text{ Hz}$$

Hence, the correct answer is (A).

5. Speed of wave $v = \frac{\omega}{k}$

$$\Rightarrow \quad v = \frac{100\pi}{0.5\pi} = \frac{92\pi}{0.46\pi}$$

$$\Rightarrow \quad v = 200 \text{ ms}^{-1}$$

Hence, the correct answer is (A).

6. At $x = 0$, $y = y_1 + y_2$

$$x = 2A\cos(96\pi t)\cos(4\pi t)$$

Frequency of $\cos(96\pi t)$ function is 48 Hz and that of $\cos(4\pi t)$ function is 2 Hz.

In one second cos function becomes zero at $2f$ times, where f is the frequency. Therefore, first function will become zero at 96 times and the second at 4 times. But second will not overlap with first. Hence, net y will become zero 100 times in 1 s.

Hence, the correct answer is (C).

Matrix Match Type Questions

1. A → (p, r, s, q); B → (q, p, r, t); C → (q, s, r, p); D → (p, q, t, s)

(1) $f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

(2) $f_1 = \frac{1}{2L_0}\sqrt{\frac{T_0}{2\mu}}$

(3) $f_2 = \frac{1}{2L_0}\sqrt{\frac{T_0}{3\mu}}$

(4) $f_3 = \frac{1}{2L_0}\sqrt{\frac{T_0}{4\mu}}$

For all highest fundamental is when length is L_0

2. A → (p, q, r, t); B → (p, q, t, u); C → (p, r, t, u); D → (t, q, r, u)

(1) $f_0 = \frac{1}{2L_0} \times \sqrt{\frac{T_1}{\mu}} = \frac{1}{2L}\sqrt{\frac{T_0}{\mu}}$

(2) $f_0 = \frac{3 \times 2}{2 \times 3L_0}\sqrt{\frac{T_2}{2\mu}}$

$$\Rightarrow \quad T_2 = \frac{T_0}{2}$$

(3) $f_0 = \frac{5 \times 2}{5L_0}\sqrt{\frac{T_3}{3\mu}}$

$$\Rightarrow \quad T_3 = \frac{3T_0}{16}$$

(4) $f_0 = \frac{14 \times 4}{2 \times 7L_0}\sqrt{\frac{T_4}{4\mu}}$

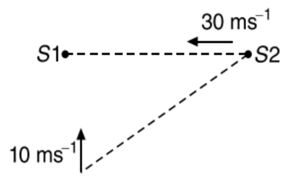
$$\Rightarrow \quad T_4 = \frac{T_0}{16}$$

3. A → (p, t); B → (p, s); C → (q, s); D → (q, r)

In organ pipes, longitudinal waves exist. In strings, transverse waves exist. Open end is an antinode, fixed end is a node. The least distance between node and antinode is $\frac{\lambda}{4}$ and between two nodes is $\frac{\lambda}{2}$.

Integer/Numerical Answer Type Questions

1. $v_{\text{sound}} = 330 \text{ ms}^{-1}$



$$f_1 = 120\left[\frac{330 + 10\sin 53°}{330 - 30\cos 37°}\right] \text{Hz}$$

$$f_2 = 120\left[\frac{330 + 10}{330}\right] \text{Hz}$$

$$\Delta f = 120 \times \left[\frac{336}{306} - \frac{34}{33}\right] = 8.13 \text{ Hz}$$

2. Frequency observed at car

If v is speed of sound and v_C is speed of car, then

$$f_1 = f_0\left(\frac{v + v_C}{v}\right)$$

Frequency of reflected sound as observed at the source

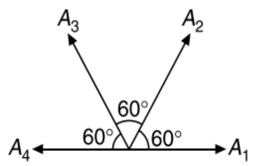
$$f_2 = f_1\left(\frac{v}{v - v_C}\right) = f_0\left(\frac{v + v_C}{v - v_C}\right)$$

Beat frequency $\Delta f = f_2 - f_0$

$$\Rightarrow \quad \Delta f = f_0\left(\frac{v + v_C}{v - v_C} - 1\right) = f_0\left(\frac{2v_C}{v - v_C}\right)$$

$$\Rightarrow \quad \Delta f = 492 \times \frac{2 \times 2}{328} = 6 \text{ Hz}$$

3. Let individual amplitudes are A_0 each. Amplitudes can be added by vector method.



$$A_1 = A_2 = A_3 = A_4 = A_0$$

Resultant of A_1 and A_4 is zero. Resultant of A_2 and A_3 is

$$A = \sqrt{A_0^2 + A_0^2 + 2A_0A_0\cos 60°} = \sqrt{3}A_0$$

This is also the net resultant.

Now, $I \propto A^2$

$\Rightarrow$ Net intensity will become $3I_0$

4. $A = \sqrt{4^2 + 3^2 + 2 \times 4 \times 3 \times \cos\left(\frac{\pi}{2}\right)} = 5$

5. $\ell = 20$ cm, $m = 1$ g, $T = 0.5$ N, $f = 100$ Hz

According to Melde's Formula, we have

$$f = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$$

$$\Rightarrow \quad 100 = \frac{n}{2 \times 20 \times 10^{-2}}\sqrt{\frac{0.5}{10^{-3}/(20 \times 10^{-2})}}$$

$$\Rightarrow \quad 1 = \frac{n}{40}\sqrt{0.5 \times 20 \times 10} = \frac{n}{40} \times 10$$

$$\Rightarrow \quad n = 4$$

So, $l = \frac{n\lambda}{2}$

Separation between successive nodes on a string is

$$\frac{\lambda}{2} = \frac{20}{4} = 5 \text{ cm}$$