CHAPTER 3: SIMPLE HARMONIC MOTION

Test Your Concepts-I (Based on SHM Properties)

1. (a) From graph, we observe that

$A = 0.08 \text{ m} = 8 \text{ cm}$

(b) Since, $T = 4$ s

$\Rightarrow \quad \omega = \frac{2\pi}{T} = \frac{\pi}{2} = 1.57 \text{ rads}^{-1}$

(c) $\omega = \sqrt{\frac{k}{m}}$

$\Rightarrow \quad k = m\omega^2 = (0.8)\left(\frac{\pi}{2}\right)^2 = 1.97 \text{ Nm}^{-1}$

(d) Since, $x = 0.08\sin(\omega t) = 0.08\sin\left(\frac{\pi t}{2}\right) \qquad \left\{\because \omega = \frac{\pi}{2}\right\}$

$\Rightarrow \quad v = \frac{dx}{dt} = (0.08)\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi t}{2}\right)$

At $t = 1$ s, $v = 0 \qquad \left\{\because \cos\left(\frac{\pi}{2}\right) = 0\right\}$

(e) At $t = 1.0$ s, $x = 0.08$ m

$\Rightarrow \quad |a| = \omega^2 x = \left(\frac{\pi}{2}\right)^2 \times (0.08) = 0.197 \text{ ms}^{-2}$

2. Differentiating with respect to time, we get

$$\frac{dx}{dt} = iA\omega e^{i\omega t}$$

Differentiating again with respect to time, we get

$$\frac{d^2x}{dt^2} = i^2\omega^2\left(Ae^{i\omega t}\right) = -\omega^2 x \qquad \left\{\because x = Ae^{i\omega t}\right\}$$

Since x satisfies, $\frac{d^2x}{dt^2} + \omega^2 x = 0$, so $x = Ae^{i\omega t}$ represents equation of an SHM.

3. (a) $F = -2(x-2)^3$

$F = 0$ at $x = 2$

Force is along negative x-direction for $x > 2$ and it is along positive x-direction for $x < 2$. Thus, the motion of the particle is oscillatory (but not simple harmonic) about $x = 2$.

(b) $F = 0$ for $x = 2$, but force is always along negative x-direction for any value of x except at $x = 2$. Thus, the motion of the particle is rectilinear along negative x-direction.

(c) Let, us take $x - 2 = X$, then the given force can be written as,

$F = -2X$

This is the equation of SHM. Hence, the particle oscillates simple harmonically about $X = 0$ or $x = 2$.

4. Amplitude, $A = 6$ cm

$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \text{ rads}^{-1}$

$\Rightarrow \quad x = A\cos(\omega t) = 6\cos(\pi t)$

Differentiating with respect to time, we get

$v = -6\pi\sin(\pi t)$

Again, differentiating with respect to time,

$a = -6\pi^2\cos(\pi t)$

5. Time taken in SHM from one end to other is

$$\Delta t = \frac{T}{2} = \frac{\pi}{\omega}$$

Mean velocity is $v_{mean} = \frac{\Delta s}{\Delta t} = \frac{2A}{\pi/\omega} = \frac{2A\omega}{\pi}$

Maximum velocity in SHM is

$v_{max} = A\omega$

$\Rightarrow \quad \frac{v_{mean}}{v_{max}} = \frac{2}{\pi}$

Mean acceleration from one end to centre is

$$a_{mean} = \frac{\Delta v}{\Delta t} = \frac{A\omega}{T/4} = \frac{A\omega}{\pi/2\omega} = \frac{2A\omega^2}{\pi}$$

and maximum acceleration is

$a_{max} = A\omega^2$

$\Rightarrow \quad \frac{a_{mean}}{a_{max}} = \frac{2}{\pi}$

6. (a) $\frac{1}{2}kA^2 = (9-5) = 4$ J

$\Rightarrow \quad k = \frac{8}{A^2} = \frac{8}{(10^{-2})^2} = 8 \times 10^4 \text{ Nm}^{-1}$

Since, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2.0}{8\times 10^4}}$

$\Rightarrow \quad T = 0.03145 = 3.14 \times 10^{-2}$ s

(b) $E = -\frac{dV}{dx} = -16x$

$\Rightarrow \quad F = mE = -16mx$

$\Rightarrow \quad a = -16x$

$\Rightarrow \quad T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{1}{16}} = 1.571$ s

7. According to the statement, we have

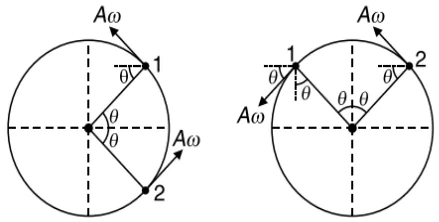
$A\omega\sin\theta = 1.6 \qquad$ ...(1)

$A\omega\cos\theta = 1.2 \qquad$ ...(2)

Squaring and adding equations (1) and (2), we get

$A\omega = v_{max} = 2 \text{ ms}^{-1}$

Further, $\sin\theta = \frac{1.6}{A\omega} = \frac{1.6}{2} = \frac{4}{5}$



So, phase difference between the particles is

$$\Delta\phi = 2\theta = 2\sin^{-1}\left(\frac{4}{5}\right)$$

8. We know in one complete oscillation i.e., in period T, a particle covers a distance $4A$ and in first one quarter of its period it goes from its mean position to its extreme position. Since it starts from mean position, so the distance travelled by the particle in time $\frac{5T}{4}$ is $5A$.

9. (a) Since, $v = \omega\sqrt{A^2 - x^2}$

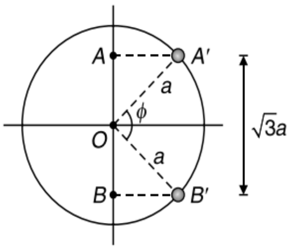
Given $A = 0.100$ m, $x = 0.060$ m, $v = 0.360$ m, we get ω and hence T, because $T = \frac{2\pi}{\omega}$

(b) $v = \omega\sqrt{A^2 - x^2}$

(c) $\mu g = a_{max} = \omega^2 A$

10. The phase difference between SHMs of A and B is given as

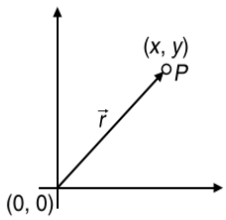
$$\phi = 2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 2\times\frac{\pi}{3} = \frac{2\pi}{3}$$



11. Since $\vec{F} = -kx\hat{i} - ky\hat{j}$, so $\vec{F} = 0$ at $(0,\ 0)$

When it is displaced to a point P whose position vector is given by

$$\vec{r} = x\hat{i} + y\hat{j}$$



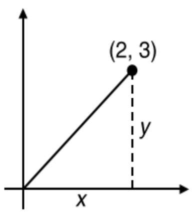
Then, force on it is given by

$$\vec{F} = -k\left(x\hat{i} + y\hat{j}\right) = -k\vec{r}$$

Since, $\vec{F} \propto -\vec{r}$, motion is simple harmonic. At $t = 0$ particle is at $(2,\ 3)$

So, $\frac{y}{x} = \frac{3}{2}$

$\Rightarrow\ 2y - 3x = 0$



i.e., the particle will oscillate simple harmonically along this line.

12. (a) Since $x = 4\sin(2t)$, so $x_{max} = 4$ m

When, $\sin(2t) = 1$

$\Rightarrow\ 2t = \frac{\pi}{2}$

$\Rightarrow\ t = \frac{\pi}{4}$ second

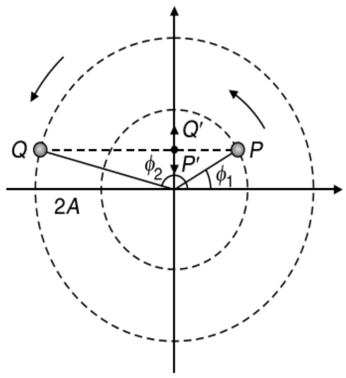
(b) $v = \frac{dx}{dt} = 8\cos(2t)$

$\Rightarrow\ v(0) = 8\ ms^{-1}$

(c) $a = \frac{dv}{dt} = -16\sin(2t)$

$\Rightarrow\ |a_{max}| = 16\ ms^{-2} a(0) = 0$

13. The two corresponding particles of circular motion for the two mentioned particles in SHM are shown in Figure.



Let particle P be going up and particle Q be going down. The respective phase differences of particles P' and Q' are

$$\phi_1 = \sin^{-1}\left(\frac{A/3}{A}\right)$$

and $\phi_2 = \pi - \sin^{-1}\left(\frac{A/3}{2A}\right)$

Hence phase difference between two SHMs is

$$\Delta\phi = \phi_2 - \phi_1 = \pi - \sin^{-1}\left(\frac{1}{6}\right) - \sin^{-1}\left(\frac{1}{3}\right)$$

14. (a) $v = \omega A = \left(\frac{2\pi}{T}\right)A = \left(\frac{2\pi}{1.8}\right)(0.1)$

$\Rightarrow\ v = 0.35\ ms^{-1}$

(b) $a = \omega^2 x = \left(\frac{2\pi}{T}\right)^2 (0.05)$

$\Rightarrow\ a = \left(\frac{2\pi}{1.8}\right)^2 (0.05) = 0.61\ ms^{-2}$

(c) $0.05 = 0.1\sin(\omega t)$

where, t is the time taken by the particle to move from equilibrium position to 0.05 m

$\Rightarrow\ \omega t = \frac{\pi}{6}$

$\Rightarrow\ \left(\frac{2\pi}{1.8}\right)t = \frac{\pi}{6}$

$\Rightarrow\ t = 0.15$ s

So, total time $= 2\times 0.15 = 0.3$ s

(d) $kA = mg$

$$\Rightarrow \quad A = \frac{g}{\omega^2} = \frac{gT^2}{4\pi^2}$$

Substituting the values,

$$A = 0.80 \text{ m}$$

15. At mean position, potential energy is minimum and kinetic energy is maximum. Hence,

$$U_{min} = 50 \text{ J} \qquad \text{\{at mean position\}}$$

and $K_{max} = E - U_{min} = 200 - 50$

$$K_{max} = 150 \text{ J} \qquad \text{\{at mean position\}}$$

At extreme positions, kinetic energy is zero and potential energy is maximum, so

$$U_{max} = E$$

$$\Rightarrow \quad U_{max} = 200 \text{ J} \qquad \text{\{at extreme position\}}$$

16. At mean position, kinetic energy is

$$K = \frac{1}{2}m\omega^2 A^2 = 8 \times 10^{-3}$$

$$\Rightarrow \quad \frac{1}{2}(0.1)\omega^2(0.1)^2 = 8 \times 10^{-3}$$

$$\Rightarrow \quad \omega^2 = 16$$

$$\Rightarrow \quad \omega = 4 \text{ rads}^{-1}$$

So, general equation of SHM is

$$y = A\sin(\omega t + \phi) = 0.1\sin\left(4t + \frac{\pi}{4}\right) \text{ metre}$$

17. $F = -\dfrac{dU}{dx} = -U_0 b \sin(bx)$

For small oscillations, $\sin(bx) \approx bx$

$$\Rightarrow \quad F = -U_0 b^2 x$$

$$\Rightarrow \quad ma = m\ddot{x} = -U_0 b^2 x$$

$$\Rightarrow \quad T = 2\pi\sqrt{\left|\frac{x}{\ddot{x}}\right|} = 2\pi\sqrt{\frac{m}{b^2 U_0}}$$

18. $\mu mg = m|a_{max}| = m\omega^2 A$

$$\Rightarrow \quad \mu g = (2\pi f)^2 A$$

$$\Rightarrow \quad \mu = \frac{4\pi^2 f^2 A}{g} = \frac{4\pi^2(2)^2(5.0 \times 10^{-2})}{9.8} = 0.806$$

19. $\dfrac{1}{2}mv^2 = \dfrac{1}{2}kA^2$

$$\Rightarrow \quad A = \left(\sqrt{\frac{m}{k}}\right)v = \left(\sqrt{\frac{10^{-2}}{124}}\right)(8.00) = 0.072 \text{ m} = 7.2 \text{ cm}$$

20. $T = 8$ s

$$\Rightarrow \quad \omega = \frac{2\pi}{T} = \frac{\pi}{4} \text{ rads}^{-1}$$

$$x = A\sin\left(\frac{\pi}{4}\right)t$$

(a) In first 4 second it travels a distance $2A$. In next 4 second it again travels a distance $2A$.

(b) In first 2 second it travels a distance A and in next 2 second it again travels a distance A.

(c) Distance travelled in first 1 second is

$$x_1 = A\sin\left(\frac{\pi}{4}\right)(1) = \frac{A}{\sqrt{2}}$$

Hence distance travelled in next 1 second is

$$x_2 = A - x_1 = A - \frac{A}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{x_1}{x_2} = \frac{\frac{A}{\sqrt{2}}}{A - \frac{A}{\sqrt{2}}} = \frac{1}{\sqrt{2}-1} \approx 2.4$$

$$\Rightarrow \quad x_1 = 2.4x_2$$

21. $E = \dfrac{1}{2}kA^2$ and $K = \dfrac{1}{2}k(A^2 - x^2)$

$$\Rightarrow \quad \frac{K}{E} = \frac{A^2 - x^2}{A^2}$$

At $x = \dfrac{A}{2}$, we get $\dfrac{K}{E} = \dfrac{A^2 - \frac{A^2}{4}}{A^2} = \dfrac{3}{4}$

Since, $U = \dfrac{1}{2}kx^2$

Equating $U = K$, we get $x^2 = A^2 - x^2$

$$\Rightarrow \quad x = \frac{A}{\sqrt{2}}$$

22. (a) $y = a(1 - \cos\omega t)$

$$\Rightarrow \quad \frac{d^2y}{dt^2} = a\omega^2\cos(\omega t)$$

Since, $N - mg = m\dfrac{d^2y}{dt^2}$

$$\Rightarrow \quad N = mg + ma\omega^2\cos(\omega t)$$

$$\Rightarrow \quad N = m(g + a\omega^2\cos\omega t)$$

(b) $\left(\dfrac{d^2y}{dt^2}\right) = a\omega^2$

$$\Rightarrow \quad a\omega^2 = g$$

$$\Rightarrow \quad a = \frac{g}{\omega^2} = \frac{980}{(11)^2} = 8.01 \text{ cm}$$

23. (a) $f = \dfrac{\omega}{2\pi} = \dfrac{5\pi}{2\pi} = 2.5$ Hz

$$T = \frac{1}{f} = 0.4 \text{ s}$$

(b) Amplitude $= 3.0$ m

(c) At $t = 0$, $x = 3\cos 3\pi = -3$ m

and at $t = \dfrac{1}{2}$ s, $x = 3\cos\left(\dfrac{5\pi}{2} + \pi\right) = 0$

Test Your Concepts-II
(Based on Spring Mass Systems)

1. (a) The total mechanical energy of the block and spring before the lump of putty is dropped is

$$E_1 = \frac{1}{2}kA_1^2$$

Since, the block is at the equilibrium position, $U = 0$ and the energy is purely kinetic. Let v_1 be the speed of the block at the equilibrium position, we get

$$E_1 = \frac{1}{2}Mv_1^2 = \frac{1}{2}kA_1^2$$

$$\Rightarrow \quad v_1 = \sqrt{\frac{k}{M}}A_1$$

During the process, the momentum of the system in horizontal direction is conserved. Let v_2 be the speed of the combined mass, then

$$(M+m)v_2 = Mv_1$$

$$\Rightarrow \quad v_2 = \frac{M}{M+m}v_1$$

Now, let A_2 be the amplitude afterwards, then

$$E_2 = \frac{1}{2}kA_2^2 = \frac{1}{2}(M+m)v_2^2$$

Substituting the values, we get

$$A_2 = A_1\sqrt{\frac{M}{M+m}}$$

We observe that $E_2 < E_1$ because some energy is lost into heating up the block and putty.

Further, $T_2 = 2\pi\sqrt{\dfrac{M+m}{k}}$

(b) When the putty drops on the block, the block is instantaneously at rest. All the mechanical energy is stored in the spring as potential energy. Again, the momentum in horizontal direction is conserved during the process. But now it is zero just before and after putty is dropped. So, in this case, adding the extra mass of the putty has no effect on the mechanical energy, i.e.,

$$E_2 = E_1 = \frac{1}{2}kA_1^2$$

and the amplitude is still A_1. So, $A_2 = A_1$

and $T_2 = 2\pi\sqrt{\dfrac{M+m}{k}}$

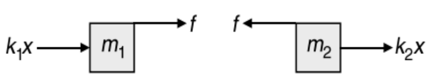
2. (a) $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k_{\text{eff}}}{\text{total mass}}} = \dfrac{1}{2\pi}\sqrt{\dfrac{k_1+k_2}{m_1+m_2}}$

(b) Suppose the system is displaced towards left by a distance x.

Restoring force on m_1 is

$$F = m_1\omega^2 x \qquad \text{(towards right)}$$

$$F = m_1\left(\frac{k_1+k_2}{m_1+m_2}\right)x$$



Friction f on it will be towards right if,

$$k_1x < F$$

$$\Rightarrow \quad k_1x < m_1\left(\frac{k_1+k_2}{m_1+m_2}\right)x$$

$$\Rightarrow \quad \frac{k_1}{k_2} < \frac{m_1}{m_2}$$

(c) $k_1A_m + \mu m_2 g = m_1\left(\dfrac{k_1+k_2}{m_1+m_2}\right)A_m$

$$\Rightarrow \quad A_m\left(\frac{m_1k_1+m_1k_2}{m_1+m_2} - k_1\right) = \mu m_2 g$$

$$\Rightarrow \quad A_m = \frac{\mu(m_1+m_2)m_2 g}{m_1k_2 - m_2k_1}$$

3. For a spring block system, we have

$$T = 2\pi\sqrt{\frac{M}{k}}$$

where, k is the spring constant of the spring

In the first case, $2 = 2\pi\sqrt{\dfrac{M}{k}}$ ...(1)

In the second case, $3 = 2\pi\sqrt{\dfrac{M+2}{k}}$ ...(2)

Dividing (2) by (1), we get

$$\frac{9}{4} = \frac{M+2}{M} = 1 + \frac{2}{M}$$

$$\Rightarrow \quad M = 1.6 \text{ kg}$$

4. $T = 2\pi\sqrt{\dfrac{\mu}{k}} = 2\pi\sqrt{\dfrac{mM}{k(M+m)}}$

5. (a) $v = \omega A = \left(\dfrac{2\pi}{T}\right)A = \left(\dfrac{2\pi}{1.8}\right)(0.1)$

$$\Rightarrow \quad v = 0.35 \text{ ms}^{-1}$$

(b) $a = \omega^2 x = \left(\dfrac{2\pi}{T}\right)^2(0.05)$

$$\Rightarrow \quad a = \left(\frac{2\pi}{1.8}\right)^2(0.05) = 0.61 \text{ ms}^{-2}$$

(c) $0.05 = 0.1\sin(\omega t)$

where, t is the time taken by the particle to move from equilibrium position to 0.05 m

$$\Rightarrow \quad \omega t = \frac{\pi}{6}$$

$$\Rightarrow \quad \left(\frac{2\pi}{1.8}\right)t = \frac{\pi}{6}$$

$$\Rightarrow \quad t = 0.15 \text{ s}$$

So, total time $= 2 \times 0.15 = 0.3$ s

(d) $kA = mg$

$$\Rightarrow \quad A = \frac{g}{\omega^2} = \frac{gT^2}{4\pi^2}$$

Substituting the values,

$$A = 0.80 \text{ m}$$

6. $\frac{1}{2}mv^2 = \frac{1}{2}kA^2$

$$\Rightarrow \quad A = \left(\sqrt{\frac{m}{k}}\right)v = \left(\sqrt{\frac{10^{-2}}{124}}\right)(8.00)$$

$$\Rightarrow \quad A = 0.072 \text{ m} = 7.2 \text{ cm}$$

7. (a) $\mu_s m_2 g \ge m_2 |a_{\max}|$

$$\Rightarrow \quad \mu_s \ge \frac{\omega^2 A}{g} \ge \frac{kA}{(m_1+m_2)g}$$

(b) A and $E\left(=\frac{1}{2}kA^2\right)$ will remain unchanged ω will decrease as

$$\Rightarrow \quad \omega = \sqrt{\frac{k}{m}}$$

$$\Rightarrow \quad \omega_f = \left(\sqrt{\frac{m_1}{m_1+m_2}}\right)\omega_i$$

T will increase, because $T \propto \frac{1}{\omega}$

$$\Rightarrow \quad T_f = \left(\sqrt{\frac{m_1+m_2}{m_1}}\right)T_i$$

8. Let F be the restoring force (extra tension) on block m when displaced by x from its equilibrium position.

$$x = 2x_1 + 2x_2 = 2\left[\frac{2F}{k_1} + \frac{2F}{k_2}\right] = 4F\left(\frac{k_1+k_2}{k_1k_2}\right)$$

$$\Rightarrow \quad F = -\frac{k_1k_2}{4(k_1+k_2)}x$$

$$\Rightarrow \quad a = -\frac{k_1k_2}{4m(k_1+k_2)}x$$

$$\Rightarrow \quad \omega = \sqrt{\frac{k_1k_2}{4m(k_1+k_2)}}$$

9. (a) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$$\Rightarrow \quad m = \frac{k}{(2\pi f)^2} = \frac{1800}{(2\pi\times 5.5)^2} = 1.5 \text{ kg}$$

(b) $kx = mg$

$$\Rightarrow \quad x = \frac{mg}{k} = \frac{(1.5)(9.8)}{1800} = 0.0082 \text{ m} = 0.82 \text{ cm}$$

(c) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{1.5}} = 34.6 \text{ rads}^{-1}$

$$\Rightarrow \quad x = A\cos(\omega t) = 2.5\cos(34.6t) \text{ cm}$$

$$\Rightarrow \quad v = \frac{dx}{dt} = -86.5\sin(34.6t) \text{ cms}^{-1}$$

$$\Rightarrow \quad a = \frac{dv}{dt} = -29.9\cos(34.6t) \text{ ms}^{-2}$$

10. In the present case,

$$x = 2\left(\frac{2F}{k_1}\right) + \left(\frac{F}{k_2}\right)$$

$$\Rightarrow \quad F = -\frac{x}{\left(\frac{4}{k_1}+\frac{1}{k_2}\right)}$$

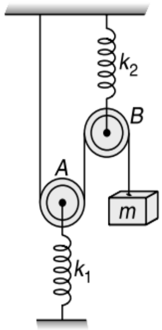
$$\Rightarrow \quad a = \frac{-x}{m\left(\frac{4}{k_1}+\frac{1}{k_2}\right)}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{m(k_1+4k_2)}{k_1k_2}}$$

11. Frequency will remain unchanged, whereas the mean position will change.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

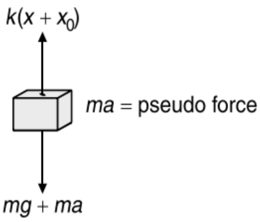
Free body diagram of block w.r.t. ground is shown in figure.



For equilibrium of block, we have

$$kx = mg \qquad \ldots(1)$$

Free body diagram of block w.r.t. elevator is shown in figure.



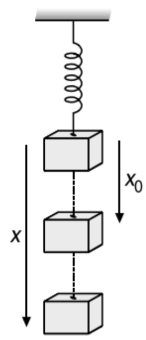
For equilibrium of block, we have

$$k(x+x_0) = mg + ma \qquad \ldots(2)$$

From equations (1) and (2), we get

$$x_0 = \text{amplitude} = \frac{ma}{k}$$

12. (a) $mgx = \frac{1}{2}kx^2$



$$\Rightarrow \quad x = \frac{2mg}{k} = \frac{2\times 1.0\times 9.8}{150}$$

$$\Rightarrow \quad x = 0.13 \text{ m} = 13 \text{ cm}$$

(b) $mg = kx_0$

$$\Rightarrow \quad x_0 = \frac{mg}{k} = \frac{x}{2} = 6.5 \text{ cm}$$

(c) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.0}{150}} = 0.51 \text{ s}$

(d) $U = \frac{1}{2}k(x-x_0)^2 = \frac{1}{2}\times 150\times(0.065)^2$

$$\Rightarrow \quad U = 0.32 \text{ J}$$

(e) By Law of Conservation of Energy, we have

$$mgx_0 = \frac{1}{2}kx_0^2 + \frac{1}{2}mv^2$$

Substituting the values, we get

$$\frac{1}{2}mv^2 = 0.32 \text{ J}$$

$$\Rightarrow \quad v = \sqrt{\frac{0.32 \times 2}{m}} = \sqrt{\frac{0.64}{1.0}} = 0.8 \text{ ms}^{-1}$$

$$t = \frac{T}{4} = \frac{0.51}{4} = 0.13 \text{ s}$$

(f) $$t = \sqrt{\frac{2x_0}{g}} = \sqrt{\frac{2 \times 0.065}{9.8}} = 0.12 \text{ s} \qquad \left\{x_0 = \frac{1}{2}gt^2\right\}$$

$$v = \sqrt{2gx_0} = \sqrt{2 \times 9.8 \times 0.065} = 1.13 \text{ ms}^{-1}$$

13. $kx = mg$

$$\Rightarrow \quad k = \frac{mg}{x} = \frac{2.8 \times 9.8}{0.018} = 1524.4 \text{ Nm}^{-1}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\Rightarrow \quad m = \frac{k}{4\pi^2 f^2} = \frac{1524.4}{4\pi^2(3)^2} = 4.3 \text{ kg}$$

14. When both the blocks stick to each other, then by law of conservation of linear momentum, we get

$$mu + 0 = (m+m)v$$

$$\Rightarrow \quad v = \frac{u}{2}$$

This velocity is the velocity of combined blocks at the mean position, so

$$v = \frac{u}{2} = A\omega$$

where, A is the amplitude of oscillation having

$$\omega = \sqrt{\frac{k}{2m}}$$

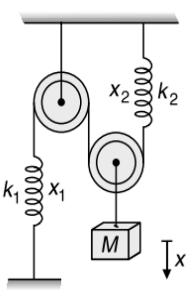
$$\Rightarrow \quad \frac{u}{2} = A\sqrt{\frac{k}{2m}}$$

$$\Rightarrow \quad A = \frac{u}{2}\sqrt{\frac{2m}{k}} = u\sqrt{\frac{m}{2k}}$$

The time taken by combined system to reach extreme position is

$$t = \frac{T}{4} = \frac{2\pi}{4\omega} = \frac{\pi}{2\omega} = \frac{\pi}{2}\sqrt{\frac{k}{2m}}$$

15. Let x be the displacement of the mass from the equilibrium value and let x_1 and x_2 be the deformation of spring k_1 and k_2, respectively from the equilibrium. Then, using constraint relation,



$$\frac{x_1 + x_2}{2} = x$$

$$\Rightarrow \quad x_1 + x_2 = 2x \qquad \ldots(1)$$

Also, tension in string should be same

$$\Rightarrow \quad k_1x_1 = k_2x_2 \qquad \ldots(2)$$

Solving (1) and (2), we get

$$x_1 = \frac{2k_2}{k_1 + k_2}x, \; x_2 = \frac{2k_1}{k_1 + k_2}x \qquad \ldots(3)$$

The energy of the system is $E = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2 + \frac{1}{2}mv^2$

$$\Rightarrow \quad E = \frac{2k_1k_2}{k_1 + k_2}x^2 + \frac{1}{2}mv^2 \qquad \{\text{from (3)}\}$$

Differentiating w.r.t. time, we get

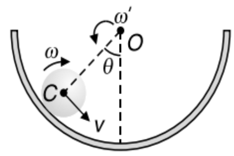
$$\frac{dE}{dt} = \frac{4k_1k_2}{k_1 + k_2}x\frac{dx}{dt} + mv\frac{dv}{dt} = 0$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \left[\frac{4k_1k_2}{(k_1 + k_2)m}\right]x = 0$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{(k_1 + k_2)m}{4k_1k_2}}$$

Test Your Concepts-III (Based on Rotational SHM)

1. For pure rolling, $v = R\omega$ and $a = R\alpha$. Let, ω' be the angular velocity of CM of sphere C about O, then



$$\omega' = \frac{v}{4R} = \frac{R\omega}{4R} = \frac{\omega}{4}$$

$$\Rightarrow \quad \frac{d\omega'}{dt} = \frac{1}{4}\frac{d\omega}{dt}$$

$$\Rightarrow \quad \alpha' = \frac{\alpha}{4}$$

Since, for pure rolling $\alpha = \frac{a}{R}$, where

$$a = \frac{g\sin\theta}{1 + (I/mR^2)} = \frac{5g\sin\theta}{7} \qquad \left\{\because I = \frac{2}{5}mR^2\right\}$$

$$\Rightarrow \quad \alpha' = \frac{\alpha}{4} = \frac{5g\sin\theta}{28R}$$

For small θ, $\sin\theta \approx \theta$

$$\Rightarrow \quad \alpha' = -\frac{5g}{28R}\theta$$

Negative sign shows restoring nature of α'

$$\Rightarrow \quad \ddot{\theta} + \left(\frac{5g}{28R}\right)\theta = 0$$

$$\Rightarrow \quad T = 2\pi\sqrt{\left|\frac{\theta}{\alpha'}\right|} = 2\pi\sqrt{\frac{28R}{5g}}$$

2. In the displaced position, total energy is

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}k(2x)^2$$

where, $I = \frac{1}{2}mR^2$ and $\omega = \frac{v}{R}$

$$\Rightarrow \quad E = \frac{3}{4}mv^2 + 2kx^2$$

Since E = constant, so $\frac{dE}{dt} = 0$

$$\Rightarrow \quad \frac{3}{2}mv\frac{dv}{dt}+4kx\frac{dx}{dt}=0$$

Substituting, $\frac{dx}{dt}=v$ and $\frac{dv}{dt}=a$

$$\Rightarrow \quad a=-\left(\frac{8k}{3m}\right)x$$

Comparing with, $a=-\omega^2 x$, we get

$$\omega=\sqrt{\frac{8k}{3m}}=\sqrt{\frac{8(1000)}{3(100/9.8)}}=16.16\text{ rads}^{-1}$$

$$\Rightarrow \quad \theta=\theta_0\cos(\omega t)=0.4\cos(16.16t)$$

3. Let x_0 be the initial extension in spring at equilibrium, then

$$\Sigma\tau_0=0$$

$$\Rightarrow \quad (kx_0)b=(mg)a \qquad \ldots(1)$$

When mass m is displaced downwards, then

$$m\ddot{x}=-(T-mg) \qquad \ldots(2)$$

For the light (i.e., massless) rod, we have

$$k\left(x_0+\frac{bx}{a}\right)b=Ta \qquad \ldots(3)$$

Substituting value of T from equation (3) in (1), we get

$$m\ddot{x}=-\left[\frac{k}{a}\left(x_0+\frac{bx}{a}\right)b-mg\right]$$

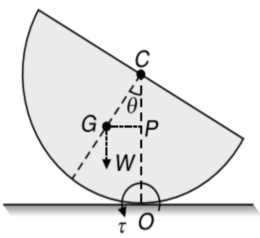
$$\Rightarrow \quad m\ddot{x}=-\left(\frac{kb^2}{a^2}\right)x$$

$$\Rightarrow \quad \ddot{x}+\left(\frac{kb^2}{ma^2}\right)x=0$$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{x}{|\ddot{x}|}}=2\pi\sqrt{\frac{ma^2}{kb^2}}=\frac{2\pi a}{b}\sqrt{\frac{m}{k}}$$

4. Restoring torque is given by

$$\tau=-W(GP)=-W(GC\sin\theta)$$



For small oscillations $\sin\theta\approx\theta$

$$\Rightarrow \quad \tau=I\alpha=-W\left(\frac{4r}{3\pi}\right)\theta \qquad \ldots(1)$$

where, I is the moment of inertia about the point O. So, we have

$$I=\frac{1}{2}mr^2+m\left(r-\frac{4r}{3\pi}\right)^2-m\left(\frac{4r}{3\pi}\right)^2$$

$$\Rightarrow \quad I\alpha=-mg\left(\frac{4r}{3\pi}\right)^2-m\left(\frac{4r}{3\pi}\right)^2$$

Further, $I\alpha=-mg\left(\frac{4r}{3\pi}\right)\theta \qquad \{\because \text{ of } (1)\}$

Since, α is proportional to $-\theta$, motion is simple harmonic in nature.

$$T=2\pi\sqrt{\left|\frac{\theta}{\alpha}\right|}$$

Substituting the proper values, we get

$$T=0.963\text{ second}$$

5. Let the rod be given a small angular displacement from the mean position, so restoring torque is

$$\tau=-mg(l\theta)+2k(b\theta)b$$

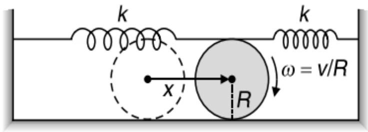
$$\Rightarrow \quad \tau=-(mgl+2kb^2)\theta$$

$$\Rightarrow \quad \ddot{\theta}+\left(\frac{mgl+2kb^2}{I}\right)\theta=0 \qquad \{\because \tau=I\ddot{\theta}\}$$

where, $I=ml^2$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{\theta}{|\ddot{\theta}|}}=2\pi\sqrt{\frac{ml^2}{mgl+2kb^2}}$$

6. Let cylinder be displaced to right, then after being released, at some instant let it have a speed v, angular speed ω and is at a distance x from the mean position. In this situations, one springs will have extension $2x$ and other will have a compression $2x$. Also, for no slipping between cylinder and ground, we have $v=R\omega$.



The total energy of system at this instant is

$$E=\frac{1}{2}Mv^2\left(1+\frac{1}{2}\right)+2\left[\frac{1}{2}k(2x)^2\right]$$

Since $E=$ constant, So $\frac{dE}{dt}=0$

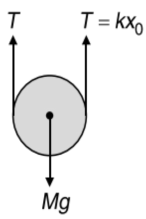
$$\Rightarrow \quad \frac{3M}{4}\left(2v\frac{dv}{dt}\right)+2k(2x)(2v)=0$$

$$\Rightarrow \quad \frac{3M}{4}\ddot{x}+4kx=0$$

$$\Rightarrow \quad \ddot{x}+\left(\frac{16k}{3M}\right)x=0$$

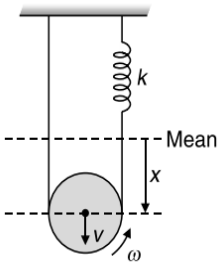
$$\Rightarrow \quad T=2\pi\sqrt{\frac{x}{|\ddot{x}|}}=2\pi\sqrt{\frac{3M}{16k}}$$

7. Initially in equilibrium, let extension in the spring be x_0, then $T=kx_0$. Also, free body diagram of pulley at equilibrium is shown in Figure.



So, $2T=Mg$

$$\Rightarrow \quad 2kx_0=Mg \qquad \ldots(1)$$



Let the pulley be now displaced slightly through x from its mean position as shown in Figure.
Due to a further pull by x, the spring extends by $x_0 + 2x$. If at this instant it is going down with a speed v and rotating with angular speed ω, then due to non-slipping of string on pulley surface, we have $\omega = \frac{v}{R}$.

Total mechanical energy of system is

$$E = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}k(x_0 + 2x)^2 - Mgx$$

$$\Rightarrow \quad E = \frac{v^2}{2}\left(M + \frac{I}{R^2}\right) + \frac{1}{2}k(x_0 + 2x)^2 - Mgx$$

Since E is constant, so $\frac{dE}{dt} = 0$

$$\Rightarrow \quad \left(M + \frac{I}{R^2}\right)\frac{vdv}{dt} + k(x_0 + 2x)2v - Mgv = 0$$

$$\Rightarrow \quad \left(M + \frac{I}{R^2}\right)\ddot{x} + 2k(x_0 + 2x) - Mg = 0$$

$$\Rightarrow \quad \left(M + \frac{I}{R^2}\right)\ddot{x} + 2kx_0 + 4kx - Mg = 0$$

Since $Mg = 2kx_0$

$$\Rightarrow \quad \ddot{x} + \left(\frac{4k}{M + \frac{I}{R^2}}\right)x = 0$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{M + \frac{I}{R^2}}{4k}}$$

8. For equilibrium of T shaped rod, if h is the initial elongation in spring, then taking torque about O to be zero, we get

$$\left(\frac{2Mg}{3}\right)(a) + \left(\frac{Mg}{3}\right)(2a) = (kh)a$$

$$\Rightarrow \quad 4Mg = 3kh$$

On slightly displacing the rod by θ, the restoring τ is given by

$$\tau = -[k(h + a\theta)a - kh] = -ka^2\theta$$

Since $\tau = I\alpha$, where, I is the moment of inertia about the point O.

$$I = \left[\frac{1}{3}\left(\frac{2M}{3}\right)(2a)^2 + \frac{1}{12}\left(\frac{M}{3}\right)a^2 + \frac{M}{3}(2a)^2\right]$$

$$\Rightarrow \quad I = \left(\frac{8}{9} + \frac{1}{36} + \frac{4}{3}\right)Ma^2 = \frac{81}{36}Ma^2 = \frac{9}{4}Ma^2$$

$$\Rightarrow \quad -ka^2\theta = \left(\frac{9}{4}Ma^2\right)\alpha$$

$$\Rightarrow \quad \alpha = -\left(\frac{4k}{9M}\right)\theta$$

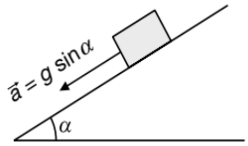
$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\theta}{|\ddot{\theta}|}} = 2\pi\sqrt{\frac{9M}{4k}}$$

Test Your Concepts-IV (Based on Pendulum Systems)

1. $T_0 = 2\pi\sqrt{\frac{\ell}{g}}$ ...(1)

When the cart rolls down the slope (without friction), acceleration of the cart.

$$a = g\sin\alpha$$



Now $\vec{g}_{\text{eff}} = \vec{g} - \vec{a}$

We can show that $|\vec{g}_{\text{eff}}| = g\cos\alpha$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\ell}{g\cos\alpha}} \quad \text{...(2)}$$

From equations (1) and (2), we get

$$T = \frac{T_0}{\sqrt{\cos\alpha}}$$

2. $$\frac{T_A}{T_B} = \frac{2\pi\sqrt{\frac{(md^2/3)}{(mg)(d/2)}}}{2\pi\sqrt{\frac{d}{g}}} = \sqrt{\frac{2}{3}} = 0.816$$

3. Since, $g_{\text{eff}} = \frac{\text{Weight} - \text{Upthrust}}{\text{Mass}}$

$$\Rightarrow \quad g_{\text{eff}} = \frac{V\rho g - V(\rho/3)g}{V_\rho} = \frac{2g}{3}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\ell}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{0.2 \times 3}{2 \times 9.8}} = 1.1 \text{ s}$$

4. $g_{\text{eff}} = \sqrt{a^2 + g^2} = \sqrt{(7)^2 + (9.8)^2}$

$$\Rightarrow \quad g_{\text{eff}} = 12.0 \text{ ms}^{-2}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\ell}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{0.5}{12}} = 12.8 \text{ s}$$

5. Since $\theta = \theta_0\sin(\omega t)$, so time t_0 required to move from equilibrium position to wall for bob is given by

$$\alpha = \beta\sin\left(\sqrt{\frac{g}{l}}t_0\right)$$

$$\Rightarrow \quad t_0 = \sqrt{\frac{l}{g}}\left[\sin^{-1}\left(\frac{\alpha}{\beta}\right)\right]$$

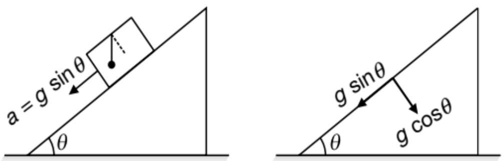
Since collision is elastic, so ball will return at same speed and hence total oscillation time period is

$$T = \pi\sqrt{\frac{l}{g}} + 2t_0$$

$$\Rightarrow \quad T = \pi\sqrt{\frac{l}{g}} + 2\sqrt{\frac{l}{g}}\sin^{-1}\left(\frac{\alpha}{\beta}\right)$$

$$\Rightarrow \quad T = \sqrt{\frac{l}{g}}\left[\pi + 2\sin^{-1}\left(\frac{\alpha}{\beta}\right)\right]$$

6. Here, point of suspension has an acceleration. $\vec{a} = g\sin\theta$ (down the plane). Further, $\vec{g}$ can be resolved into two components $g\sin\theta$ (along the plane) and $g\cos\theta$ (perpendicular to plane).



Since, $\vec{g}_{\text{eff}} = \vec{g} - \vec{a}$

and $\vec{g}_{\text{eff}} = g\cos\theta$ {perpendicular to plane}

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\ell}{|\vec{g}_{\text{eff}}|}} = 2\pi\sqrt{\frac{\ell}{g\cos\theta}}$$

Observation: If $\theta = 0°$, $T = 2\pi\sqrt{\frac{\ell}{g}}$

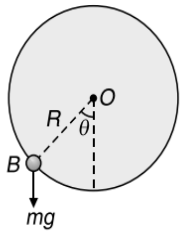
7. $\frac{f_1}{f_2} = \frac{15}{10} = \sqrt{\frac{\ell_2}{\ell_1}}$

$$\Rightarrow \quad \frac{\ell_1}{\ell_2} = \frac{4}{9}$$

8. $\tau = -mgR\sin\theta$

$$\Rightarrow \quad I\alpha \approx -mgR\theta \qquad \{\because \sin\theta \approx \theta\}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\left|\frac{\theta}{\alpha}\right|} = 2\pi\sqrt{\frac{I}{mgR}}$$



where, $I = mR^2 + \frac{1}{2}MR^2$

Substituting the values, we get

$$T = 2\pi\sqrt{\frac{\left(m + \frac{M}{2}\right)R}{mg}}$$

Further, $2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{\left(m + \frac{M}{2}\right)R}{mg}}$

$$\Rightarrow \quad \ell = \frac{\left(m + \frac{M}{2}\right)R}{m}$$

9. (a) We are given $\theta_0 = 0.1\pi$ rad, $\phi = \frac{\pi}{6}$ rad and $\omega = 2\pi$ rads^{-1}. Since $\omega^2 = \left(\frac{g}{L}\right)$, we have

$$L = \frac{g}{\omega^2} = \frac{9.8 \text{ ms}^{-2}}{\left(2 \times 3.14 \text{ rads}^{-2}\right)^2} = 0.25 \text{ m}$$

(b) Since $s = L\theta$, the velocity of the bob, $v = \frac{ds}{dt}$, is

$$v = L\frac{d\theta}{dt} = (0.25 \text{ m})(0.1\pi)(2\pi)\cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$$

$$\Rightarrow \quad v = -0.123 \text{ ms}^{-1}$$

10. This is the case of a physical pendulum, the time period of which is,

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

where, I = moment of inertia of the ring about point of suspension

$$\Rightarrow \quad I = mr^2 + mr^2 = 2mr^2$$

and l = distance of point of suspension from centre of gravity

So, $l = r$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{2mr^2}{mgr}} = 2\pi\sqrt{\frac{2r}{g}}$$

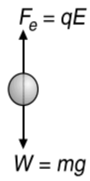
Since, angular frequency $\omega = \frac{2\pi}{T}$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{2r}}$$

Test Your Concepts-V (Based on SHM in Other Physical Systems, Composition of SHM, Damped Oscillations, Forced Oscillations & Resonance)

1. The two forces acting on the bob are shown in figure. So, effective value of gravity in this case is

$$g_{\text{eff}} = \frac{W - F_e}{m}$$



$$\Rightarrow \quad g_{\text{eff}} = \frac{mg - qE}{m} = g - \frac{qE}{m}$$

Since, $T = 2\pi\sqrt{\frac{\ell}{g_{\text{eff}}}}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\ell}{g - \frac{qE}{m}}}$$

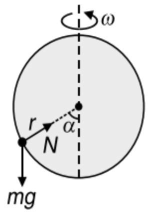
2. Let, ω =angular speed of wire frame, then

In equilibrium, $N\cos\alpha = mg$ ...(1)

and $N\sin\alpha = m(r\sin\alpha)\omega^2$ ...(2)

From these two equations, we get,

$$\omega^2 = \frac{g}{r\cos\alpha}$$



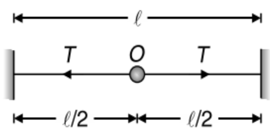
Since, $T = 2\pi\sqrt{\frac{r}{g_{\text{eff}}}}$

where, $g_{\text{eff}} = \sqrt{g^2 + (r\omega^2)^2} = \sqrt{g^2 + \left(\frac{g}{\cos\alpha}\right)^2}$

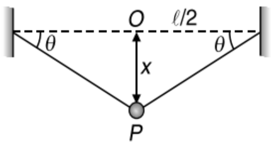
$$\Rightarrow \quad g_{\text{eff}} = \frac{g}{\cos\alpha}\sqrt{1 + \cos^2\alpha}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{r\cos\alpha}{g\left(1 + \cos^2\alpha\right)^{1/2}}}$$

3. The situation is shown in Figure. The ball is in equilibrium at point O as shown in Figure.



Now it is displaced from O by a distance x in horizontal plane to a position P as shown in Figure.



The components of tensions on ball towards the mean position O are $T\sin\theta$ and $T\sin\theta$, hence the restoring force on ball toward mean position is

$$F = -2T\sin\theta$$

Negative sign tells restoring nature of force.

$$\Rightarrow \quad F = -\frac{2Tx}{\sqrt{x^2+\frac{l^2}{4}}}$$

Since, x is very small compared to ℓ, so we can neglect its square compared to $\frac{l^2}{4}$.

$$\Rightarrow \quad F = -\left(\frac{4T}{l}\right)x$$

If a is the acceleration of ball toward mean position, then

$$a = \frac{F}{m} = -\left(\frac{4T}{ml}\right)x$$

We observe that acceleration is directly proportional to x and is directed towards the mean position, so motion of ball is simple harmonic with angular frequency ω, given by

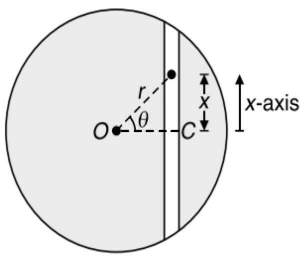
$$\omega = \sqrt{\frac{4T}{ml}}$$

$$\Rightarrow \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{ml}{4T}} = 3.14\sqrt{\frac{(0.04)(1)}{10}} = 0.2 \text{ s}$$

4. For the elastic wire of area A, length l and young's modulus Y, the equivalent spring constant is $k = \frac{YA}{l}$. Since $T = 2\pi\sqrt{\frac{m}{k}}$

$$\Rightarrow \quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{YA}{ml}}$$

5. Let at some instant the particle be at radial distance r from centre of earth O. Since the particle is constrained to move along the tunnel, we define its position as distance x from C.



Hence, equation of motion of the particle is,

$$ma_x = F_x$$

The gravitational force on mass m at distance r is,

$$F = \frac{GMmr}{R^3} \qquad \{\text{towards } O\}$$

Therefore, $F_x = -F\sin\theta$

$$\Rightarrow \quad F_x = -\frac{GMmr}{R^3}\left(\frac{x}{r}\right)$$

$$\Rightarrow \quad F_x = -\left(\frac{GMm}{R^3}\right)x$$

Since, $F_x \propto -x$, motion is simple harmonic in nature. Further,

$$ma_x = -\left(\frac{GMm}{R^3}\right)x$$

$$\Rightarrow \quad a_x = -\left(\frac{GM}{R^3}\right)x$$

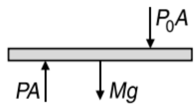
So, the period of oscillation is,

$$T = 2\pi\sqrt{\left|\frac{x}{a_x}\right|} = 2\pi\sqrt{\frac{R^3}{GM}}$$

The time taken by particle to go from one end to the other is $\frac{T}{2}$

$$\Rightarrow \quad t = \frac{T}{2} = \pi\sqrt{\frac{R^3}{GM}}$$

6. At equilibrium if gas pressure is P, then we have



$$PA = Mg + P_0A$$

$$\Rightarrow \quad P = \frac{Mg}{A} + P_0$$

Let the piston be displaced down by a distance x and since the system is isolated, so we have

$$PV^\gamma = \text{constant}$$

$$\Rightarrow \quad d\left(PV^\gamma\right) = 0$$

$$\Rightarrow \quad P\left(\gamma V^{\gamma-1}\right)dV + V^\gamma dP = 0$$

$$\Rightarrow \quad dP = -\gamma\left(\frac{dV}{V}\right)P$$

Now the restoring force developed due to this excess pressure is

$$F = AdP = -\gamma\left(\frac{dV}{V}\right)PA$$

where, $dV = Ax$ and $V = V_0$

$$\Rightarrow \quad M\ddot{x} = -\left(\frac{\gamma A^2 P}{V_0}\right)x$$

$$\Rightarrow \quad \ddot{x} + \left(\frac{\gamma PA^2}{MV_0}\right)x = 0$$

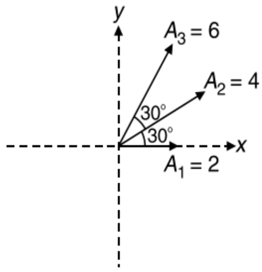
$$\Rightarrow \quad T = 2\pi\sqrt{\frac{x}{|\ddot{x}|}} = 2\pi\sqrt{\frac{MV_0}{\gamma PA^2}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{MV_0}{\gamma A\left(P_0A + Mg\right)}}$$

7. The resultant equation is,

$$x = A\sin\left(\omega t + \phi\right)$$

$$\Sigma A_x = 2 + 4\cos\left(30^\circ\right) + 6\cos\left(60^\circ\right) = 8.46$$



and $\Sigma A_y = 4\sin(30°) + 6\cos(30°) = 7.2$

$$\Rightarrow \quad A = \sqrt{(\Sigma A_x)^2 + (\Sigma A_y)^2} = \sqrt{(8.46)^2 + (7.2)^2}$$

$$\Rightarrow \quad A = 11.25 \text{ and } \tan\phi = \frac{\Sigma A_y}{\Sigma A_x} = \frac{7.2}{8.46} = 0.85$$

$$\Rightarrow \quad \phi = \tan^{-1}(0.85) = 40.4°$$

Thus, the displacement equation of the combined motion is,

$$x = 11.25\sin(\omega t + \phi), \text{ where } \phi = 40.4°$$

8. In 25 oscillations amplitude is reduced to half the initial value of 5 cm. So, in 50 oscillations the amplitude becomes $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ times the initial value, i.e.,1.25 cm.

9. Since $b \ll \sqrt{km}$, so we have $T = 2\pi\sqrt{\frac{m}{k}}$

$$\Rightarrow \quad m = \frac{T^2 k}{4\pi^2} = \frac{(0.5)^2(200)}{4(3.14)^2} \approx 1.25 \text{ kg}$$

10. In one minute, the amplitude becomes half the original value. Since the amplitude decays exponentially, so after three minutes, it should become $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ times the original value. Hence,

$$x = 8$$

Single Correct Choice Type Questions

1. Since, $T = 2\pi\sqrt{\frac{M}{K}}$ and $T' = 2\pi\sqrt{\frac{M+m}{K}}$

By Conservation of Linear Momentum, we have

$$Mv = (M+m)v'$$

$$\Rightarrow \quad M(A\omega) = (M+m)(A'\omega')$$

$$\Rightarrow \quad A' = A\sqrt{\frac{M}{M+m}}$$

Hence, the correct answer is (B).

2. $v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{1 \times 12}{3} = 4 \text{ ms}^{-1}$

Energy of oscillation $E = \frac{1}{2}m_1 v_1^2 - \frac{1}{2}(m_1 + m_2)v_{cm}^2$

Since $E = \frac{1}{2}kA^2$

$$\Rightarrow \quad A = \sqrt{\frac{m_1 v_1^2 - (m_1 + m_2)v_{cm}^2}{k}}$$

$$\Rightarrow \quad A = \sqrt{\frac{(1)(0.12)^2 - (3)(0.04)^2}{24}} = \frac{2}{100} \text{ m} = 2 \text{ cm}$$

Hence, the correct answer is (B).

3. Since, $y = x_0 \sin^2 \omega t = x_0\left(\frac{1 - \cos 2\omega t}{2}\right)$

$$\Rightarrow \quad y = \frac{x_0}{2} - \frac{x_0}{2}\cos 2\omega t$$

$$\Rightarrow \quad y - \frac{x_0}{2} = -\frac{x_0}{2}\cos 2\omega t = \frac{x_0}{2}\sin\left(2\omega t - \frac{\pi}{2}\right)$$

Comparing with $y - y_0 = A\sin(\omega' t + \phi_0)$, we get

$$\omega' = 2\omega$$

$$\Rightarrow \quad \frac{2\pi}{T'} = 2\omega$$

$$\Rightarrow \quad T' = \frac{\pi}{\omega}$$

So, the function $x = x_0 \sin^2(\omega t)$ represents SHM of amplitude $\frac{x_0}{2}$ about the mean position at $\frac{x_0}{2}$ and having a period of $\frac{\pi}{\omega}$

Hence, the correct answer is (B).

4. $A_R = \sqrt{(\sqrt{3})^2 + (1)^2} = 2$

$$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$$

$$\Rightarrow \quad y = 2\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$\Rightarrow \quad \frac{d^2 y}{dt^2} = a = -2\omega^2 \sin\left(\omega t + \frac{\pi}{3}\right)$$

$$\Rightarrow \quad |a_{max}| = 2\omega^2 = g$$

For this maximum acceleration, the mass just breaks off the plank, so $\omega = \sqrt{\frac{g}{2}}$. This will happen for the first time when

$$\omega t + \frac{\pi}{3} = \frac{\pi}{2}$$

$$\Rightarrow \quad \omega t = \frac{\pi}{6}$$

$$\Rightarrow \quad t = \frac{\pi}{6\omega} = \frac{\pi}{6}\sqrt{\frac{2}{g}} = \frac{\sqrt{2}}{6}\left(\frac{\pi}{g}\right)$$

Hence, the correct answer is (A).

5. Since, $\frac{x}{a} = \cos(\omega t)$ and $\frac{y}{a} = \cos\left(\omega t + \frac{\pi}{6}\right)$

$$\Rightarrow \quad \frac{y}{a} = \cos(\omega t)\cos\left(\frac{\pi}{6}\right) - \sin(\omega t)\sin\left(\frac{\pi}{6}\right)$$

$$\Rightarrow \quad \frac{y}{a} = \cos(\omega t)\left(\frac{\sqrt{3}}{2}\right) - \sin(\omega t)\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad \frac{y}{a} = \frac{\sqrt{3}}{2}\frac{x}{a} - \frac{1}{2}\sqrt{1 - \frac{x^2}{a^2}}$$

$$\Rightarrow \quad \frac{1}{2}\sqrt{1 - \frac{x^2}{a^2}} = \frac{\sqrt{3}}{2}\frac{x}{a} - \frac{y}{a}$$

$$\Rightarrow \frac{1}{4}\left(1-\frac{x^2}{a^2}\right)=\left(\frac{\sqrt{3}}{2}\frac{x}{a}-\frac{y}{a}\right)^2$$

$$\Rightarrow \frac{1}{4}\left(1-\frac{x^2}{a^2}\right)=\frac{3}{4}\frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{\sqrt{3}xy}{a^2}$$

$$\Rightarrow a^2-x^2=3x^2+4y^2-4\sqrt{3}xy$$

$$\Rightarrow a^2=4x^2+4y^2-4\sqrt{3}xy$$

$$\Rightarrow x^2+y^2-\sqrt{3}xy=\frac{a^2}{4}$$

Hence, the correct answer is (D).

6. Potential energy of the particle is

$$U=x^2-4x+4$$

$$\Rightarrow F=-\frac{dU}{dx}=-(2x-4)=-2x+4$$

So F varies linearly with x and is directed towards the mean position, hence the particle performs SHM. At equilibrium i.e. at the mean position

$$F=0$$

$$\Rightarrow -2x+4=0$$

$$\Rightarrow x=2$$

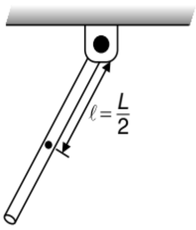
So, particle execute SHM with its equilibrium position at $x=2$

Hence, the correct answer is (C).

7. $T_1=2\pi\sqrt{\frac{L}{g}}$ and $T_2=2\pi\sqrt{\frac{I}{Mgl}}$ (physical pendulum)

$$\Rightarrow T_2=2\pi\sqrt{\frac{ML^2/3}{Mg(L/2)}}=2\pi\sqrt{\frac{2L}{3g}}$$

$$\Rightarrow \frac{T_1}{T_2}=\sqrt{\frac{3}{2}}$$



Hence, the correct answer is (C).

8. Since, $x=A\sin\omega t$

Over the interval $0\le t\le\frac{\pi}{6\omega}$, the particle gets displaced from $x=0$ to $x=A\sin\left(\omega\times\frac{\pi}{6\omega}\right)=\frac{A}{2}$

So, average speed of particle in the given interval is

$$v_{av}=\frac{\text{distance}}{\text{time}}=\frac{A/2}{\pi/6\omega}=\frac{3A\omega}{\pi}$$

Hence, the correct answer is (C).

9. $U(6\text{ m})=10+(6-2)^2=26\text{ J}$

$U(-2\text{ m})=10+(-2-2)^2=26\text{ J}=U(6\text{ m})$

On negative x-axis particle travels upto $x=-2$ m

Mean position of the particle is $x=2$ m

$$U(2\text{ m})=10\text{ J}$$

$$\Rightarrow K(2\text{ m})=(26-10)\text{ J}=16\text{ J}=K_{max}$$

Substitute $x-2=X$

$$\Rightarrow U=10+X^2$$

$$\Rightarrow F=-\frac{dU}{dX}=-2X$$

$$\Rightarrow a=\frac{F}{m}=-2X \qquad \{\because m=1\text{ kg}\}$$

$$\Rightarrow \omega=\sqrt{2}=\frac{2\pi}{T}$$

$$\Rightarrow T=\sqrt{2}\pi$$

Hence, the correct answer is (D).

10. The amplitude of SHM represented by the equation $y=a\sin(\omega t)+y_0$ is a and mean position is at y_0. So, amplitude of SHM is 2 and mean position is at 1.

Hence, the correct answer is (C).

11. Since, $\omega=\sqrt{\frac{k}{\Sigma m}}=\sqrt{\frac{54}{6}}=3\text{ rads}^{-1}$

Maximum friction between 1 kg and 2 kg blocks can be $0.6\times1\times10=6$ N. Therefore, maximum acceleration of 1 kg block can be $\frac{6}{1}=6\text{ ms}^{-2}$. Maximum force of friction between 2 kg and 3 kg blocks can be $0.4\times3\times10=12$ N. Therefore, maximum acceleration of 1 kg and 2 kg blocks jointly can be $\frac{12}{3}=4\text{ ms}^{-2}$.

So, maximum acceleration of the whole system, so that there is no slipping between any of blocks is 4 ms^{-2}.

Now, $\omega^2A_{max}=4$

$$\Rightarrow A_{max}=\frac{4}{\omega^2}=\frac{4}{9}\text{ m}$$

Hence, the correct answer is (C).

12. With increase in k, the angular frequency ω will increase. Hence, A_{max} will decrease.

Hence, the correct answer is (C).

13. Let $BC=CD=x$, then according to the problem, we have

$$\frac{1}{2}m\omega^2(R^2-x^2)=\frac{1}{4}\left(\frac{1}{2}m\omega^2R^2\right)$$

$$\Rightarrow x=\pm\frac{\sqrt{3}R}{2}$$

$$\Rightarrow BD=2x=\sqrt{3}R$$

Hence, the correct answer is (C).

14. Effective force constant in Case (3) and (4) is

$$k_{eff}=2k+2k=4k$$

Therefore, $T_1=T_2=2\pi\sqrt{\frac{m}{k}}$

and $T_3=T_4=2\pi\sqrt{\frac{m}{4k}}=\pi\sqrt{\frac{m}{k}}$

Hence, the correct answer is (B).

15. Since, $\frac{1}{k_s}=\frac{1}{k}+\frac{1}{2k}+\frac{1}{4k}+\frac{1}{8k}+\ldots$

$$\Rightarrow \frac{1}{k_s}=\frac{1}{k}\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\right)$$

CHAPTER 3

$\Rightarrow \quad \dfrac{1}{k_s}=\dfrac{1}{k}\left(\dfrac{1}{1-1/2}\right)=\dfrac{2}{k}$

$\Rightarrow \quad k_s=\dfrac{k}{2}$

$\Rightarrow \quad T=2\pi\sqrt{\dfrac{m}{k_s}}=2\pi\sqrt{\dfrac{2m}{k}}$

Hence, the correct answer is (B).

16. When simple pendulum executes SHM with angular amplitude α and its amplitude is A. Its time period is T. In second case, the angular amplitude of oscillation is $\dfrac{\alpha}{2}$ and hence its amplitude is $\dfrac{A}{2}$.

Since time taken by a particle (executing SHM) to reach the position $\dfrac{A}{2}$ from mean position is $\dfrac{T}{12}$, so its time period is

$$T'=4\left(\frac{T}{12}\right)=\frac{T}{3}$$

$\Rightarrow \quad \dfrac{T}{T'}=\dfrac{3}{1}$

Hence, the correct answer is (B).

17. Since, $T=2\pi\sqrt{\dfrac{m}{k}}T=2\pi\sqrt{\dfrac{m}{F/x}}$ $\quad\left\{\because k=\dfrac{F}{x}\right\}$

$\Rightarrow \quad T=2\pi\sqrt{\dfrac{mx}{mg}}=2\pi\sqrt{\dfrac{x}{g}}$ $\quad\{\because F=mg\}$

$\Rightarrow \quad \dfrac{T_1}{T_2}=\sqrt{\dfrac{x_1}{x_2}}=\sqrt{\dfrac{5}{10}}=\dfrac{1}{\sqrt{2}}$

Hence, the correct answer is (A).

18. Equations can be written as $x_1=A\cos(\omega t)$ and $x_2=A\sin\left(\omega t-\dfrac{\pi}{6}\right)$, where $\omega=\dfrac{2\pi}{T}$.

Equating $x_1=x_2$, we get $t=\dfrac{T}{6}$

Hence, the correct answer is (D).

19. $K_{eff}=\dfrac{(2K)(2K)}{2K+2K}+K+K$

$\Rightarrow \quad K_{eff}=3K$

$\Rightarrow \quad f=\dfrac{1}{2\pi}\sqrt{\dfrac{3K}{M}}$

Hence, the correct answer is (A).

20. Time taken by particle to move from extreme $x=a$ to $x=\dfrac{a}{2}$ is $\dfrac{T}{6}$. So, we have average velocity given by

$$v_{av}=\frac{\text{displacement}}{\text{time interval}}=\frac{a/2}{T/6}=\frac{3a}{T}$$

Hence, the correct answer is (C).

21. At equilibrium position, $\rho=\dfrac{\rho_0}{2}(\alpha+\beta h_0)$

$\Rightarrow \quad h_0=\dfrac{2-\alpha}{\beta}=A$, amplitude of oscillation

Net force at (h_0+x) is $F_{net}=F_b-W$ $\quad$ {upwards}

$\Rightarrow \quad F_{net}=Vg\left[\dfrac{\rho_0}{2}\{\alpha+\beta(h_0+x)\}\right]-Vg\rho_0$

Since, $\rho=\dfrac{\rho_0}{2}(\alpha+\beta h_0)$, so $F=-\left(\dfrac{Vg\rho_0\beta}{2}\right)x$

$F\propto -x$

So, motion is simple harmonic with acceleration

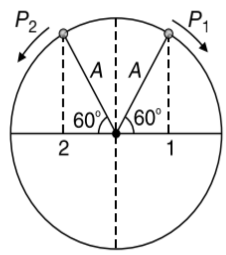
$$a=\frac{F}{m}=\frac{F}{\rho_0 V}=-\left(\frac{\beta g}{2}\right)x$$

$\Rightarrow \quad \omega=\sqrt{\dfrac{\beta g}{2}}$

$\Rightarrow \quad v_{max}=A\omega=\left(\dfrac{2-\alpha}{\sqrt{2\beta}}\right)\sqrt{g}$

Hence, the correct answer is (C).

22. Points on the circle corresponding to particles 1 and 2 are P_1 and P_2 as shown in Figure.



Particles 1 and 2 will collide when P_1 and P_2 will collide i.e., $\theta_1+\theta_2=60°+180°+60°$

$\Rightarrow \quad \omega t+\omega t=\dfrac{\pi}{3}+\pi+\dfrac{\pi}{3}$

$\Rightarrow \quad 2\omega t=2\left(\dfrac{2\pi}{T}\right)t=\dfrac{5\pi}{3}$

$\Rightarrow \quad t=5T/12$

Hence, the correct answer is (B).

23. Let potential energy at mean position be U_0. At the extreme position, the potential energy equals the total energy (because kinetic energy is zero at extreme). So,

$$U_0+\frac{1}{2}kA^2=200$$

$\Rightarrow \quad U_0=200-\dfrac{1}{2}\times 3\times 10^4\times 0.1\times 0.1=50\text{ J}$

Hence, potential energy varies between 50 J and 200 J whereas kinetic energy varies between 0 and 150 J.

Hence, the correct answer is (D).

24. For the ball after collision $h=\dfrac{1}{2}gt^2$, $d=ut$

$\Rightarrow \quad u=d\sqrt{\dfrac{g}{2h}}$

The velocity of recoil of M, $v=\dfrac{md}{M}\sqrt{\dfrac{g}{2h}}$

Since $v=A\omega$, so $A=\dfrac{v}{\omega}=\dfrac{md}{M}\sqrt{\dfrac{Mg}{k(2h)}}$

Hence, the correct answer is (A).

25. Since acceleration is $a=-\omega^2x$

$\Rightarrow \quad \Delta a=-\omega^2\Delta x$

$\Rightarrow \quad \omega^2=-\frac{\Delta a}{\Delta x}=-\frac{27-9}{(-3)-(-1)}=9$

$\Rightarrow \quad \omega=3=\frac{2\pi}{T}$

$\Rightarrow \quad T=\frac{2\pi}{3}\text{ s}$

Hence, the correct answer is (C).

26. Since, $a=0$, at $x=2$, so, $x=2$ is the mean position.

Let $x-2=X$

$\Rightarrow \quad a=-\beta x$

$\Rightarrow \quad a\propto -X$

Since, the oscillations are simple harmonic, so time period of the oscillations is given by

$$T=2\pi\sqrt{\left|\frac{x}{a}\right|}=2\pi\sqrt{\frac{1}{\beta}}$$

Hence, the correct answer is (B).

27. Since, $k_s=\frac{k_1k_2}{k_1+k_2}=\frac{(k/5)(k/4)}{(k/5)+(k/4)}=\frac{k}{9}$

$\Rightarrow \quad T=2\pi\sqrt{\frac{m}{k_s}}=2\pi\sqrt{\frac{9m}{k}}=6\pi\sqrt{\frac{m}{k}}$

$\Rightarrow \quad f=\frac{1}{T}=\frac{1}{6\pi}\sqrt{\frac{k}{m}}$

Hence, the correct answer is (A).

28. Since, $k_P=\frac{k}{5}+\frac{k}{4}=\frac{9k}{20}$

$\Rightarrow \quad T=2\pi\sqrt{\frac{m}{9k/20}}=\frac{2\pi}{3}\sqrt{20}\sqrt{\frac{m}{k}}$

$\Rightarrow \quad f=\frac{1}{T}=\frac{3}{2\pi\sqrt{20}}\sqrt{\frac{k}{m}}$

Hence, the correct answer is (B).

29. Let the piston be displaced slightly through x. Considering this process to take place gradually, we apply the equation obeying isothermal process.

$\Rightarrow \quad PV=\text{constant}$

$\Rightarrow \quad P\Delta V+V\Delta P=0$

$\Rightarrow \quad \Delta P=-\frac{P\Delta V}{V}=-\frac{P(Ax)}{Ah}=-\frac{Px}{h} \qquad \ldots(1)$

This excess pressure is responsible for providing the restoring force to the piston of mass M.

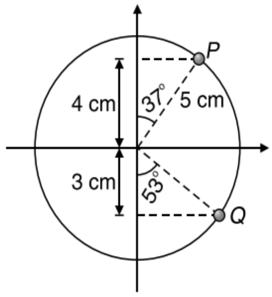
$\Rightarrow \quad F=M\ddot{x}=A\Delta P=-\left(\frac{PA}{h}\right)x \qquad \{\because \text{ of } (1)\}$

$\Rightarrow \quad \ddot{x}+\frac{PA}{Mh}x=0$

So, $\omega=\sqrt{\frac{PA}{Mh}}$ and hence $T=2\pi\sqrt{\frac{Mh}{PA}}$

Hence, the correct answer is (A).

30. The mapping of SHM on the circle is shown in Figure.



From figure, we see that the angle between the radii vectors when the particle goes from P to Q is 90°. Therefore, time taken is

$$t=\left(\frac{90}{360}\right)T=\frac{20}{4}=5\text{ s}$$

Hence, the correct answer is (C).

31. Since, $\phi=\omega_s t-\omega_\ell t$

$\Rightarrow \quad \phi=\frac{2\pi}{T}\frac{5T}{4}-\frac{2\pi}{(5T/4)}\frac{5T}{4}$

$\Rightarrow \quad \phi=2\pi\left(\frac{5}{4}-1\right)=2\pi\left(\frac{1}{4}\right)=\frac{\pi}{2}$

Hence, the correct answer is (D).

32. Since, $T=2\pi\sqrt{\frac{m}{k_P}}=2\pi\sqrt{\frac{m}{2k}}$ and $T'=2\pi\sqrt{\frac{m}{k}}=\frac{T}{\sqrt{2}}$

Hence, the correct answer is (A).

33. Since, $F=-kx$, so the displacement and force are out of phase $(\Delta\phi=\pi)$ in SHM. Therefore, the correct graph will be (D).

Hence, the correct answer is (D).

34. Phase difference between the two SHMs is 90°. So, resultant amplitude is

$$A_R=\sqrt{2}A$$

$\Rightarrow \quad E=\frac{1}{2}m\omega^2A_R^2=\frac{1}{2}m\omega^2\left(\sqrt{2}A\right)^2=m\omega^2A^2$

Hence, the correct answer is (B).

35. Potential energy of the particle is

$$U=mV=8\times10^5x^2\text{ erg}$$

If A is the amplitude, then at the extreme position, total energy equals the potential energy and hence we get

$$8\times10^7=8\times10^5A^2$$

$\Rightarrow \quad A=10\text{ cm}$

Also, $F=-\frac{dU}{dx}=-16\times10^5x$

$\Rightarrow \quad k=\frac{F}{x}=16\times10^5\text{ dynecm}^{-1}$

So, angular frequency is

$$\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{16\times10^5}{10}}=400\text{ rads}^{-1}$$

Position of the particle is given by

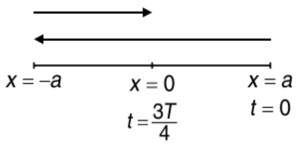
$$x=A\sin(\omega t+\phi)=10\sin(400t+\phi)\text{ cm}$$

Hence, the correct answer is (A).

36. Comparing the given equation with $x = a\cos(\omega t)$, we get $\omega = \frac{\pi}{2}$

$\Rightarrow \quad \frac{2\pi}{T} = \frac{\pi}{2}$

$\Rightarrow \quad T = 4\text{ s}$



The given time is $t = 3$ s

$\Rightarrow \quad t = \frac{3T}{4}$

At time $t = 0$ particle is at $x = a$ (at extreme position) and at $t = 3$, i.e., $t = \frac{3T}{4}$ it will be at mean position $x = 0$. So, distance covered will be $3a$.

Hence, the correct answer is (B).

37. At A, $v = 0$ i.e., particle is at extreme position. At B, v is maximum i.e., particle is at mean position, so acceleration of particle is zero. At C, $v = 0$ and after some time its velocity is negative. Hence, particle is at $x = +A$ i.e., acceleration of particle is maximum in negative directions.

Hence, the correct answer is (C).

38. In case (a), MI of system about point of suspension is

$$I_1 = mL^2 + mL^2 = 2mL^2$$

and centre of mass of system from point of suspension is $d_1 = L$. Therefore, frequency of the system is

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{mgd_1}{I_1}} = \frac{1}{2\pi}\sqrt{\frac{mgL}{2mL^2}} = \frac{1}{2\pi}\sqrt{\frac{g}{2L}}$$

In case (b), MI of system about point of suspension is

$$I_2 = m\left(\frac{L}{2}\right)^2 + mL^2 = \frac{5}{4}mL^2$$

and centre of mass of system from point of suspension is

$$d_2 = \frac{m(L/2) + mL}{m + m} = \frac{3L}{4}$$

Frequency of the system is

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{mgd_2}{I_2}} = \frac{1}{2\pi}\sqrt{\frac{mg\left(\frac{3}{4}L\right)}{\frac{5}{4}mL^2}} = \frac{1}{2\pi}\sqrt{\frac{3g}{5L}}$$

Ratio of frequencies in configuration (b) to (a) is

$$\frac{f_b}{f_a} = \sqrt{\frac{3g}{5L}} \Big/ \sqrt{\frac{g}{2L}} = \sqrt{\frac{6}{5}}$$

Hence, the correct answer is (A).

39. Since, $\omega = \sqrt{\frac{k}{m}}$ and general equation of motion is

$$x = 2\ell\cos(\omega t)$$

$\Rightarrow \quad \ell = 2\ell\cos(\omega t)$

$\Rightarrow \quad \omega t = \frac{\pi}{3}$

$\Rightarrow \quad \sqrt{\frac{k}{m}}t = \frac{\pi}{3}$

$\Rightarrow \quad t = \frac{\pi}{3}\sqrt{\frac{m}{k}}$

$\Rightarrow \quad T = 2t = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$

Hence, the correct answer is (B).

40. Starting from rest means starting from extreme, so we have $y = A\cos(\omega t)$

At $t = 1$ s, $y = A\cos\omega$

At $t = 2$ s, $y = A\cos(2\omega)$

and at $t = 0$ s, $y = A$

$\Rightarrow \quad a = A - A\cos\omega$ ...(1)

and $\quad a + b = A - A\cos 2\omega$ ...(2)

$\cos\omega = 1 - \frac{a}{A}$ $\quad$ {from equation (1)}

$\Rightarrow \quad a + b = A - A(2\cos^2\omega - 1) = A - A\left[2\left(1 - \frac{a}{A}\right)^2 - 1\right]$

Solving this equation, we get

$$A = \frac{2a^2}{3a - b}$$

Hence, the correct answer is (C).

41. If the two limbs of the U-tube make angles θ_1 and θ_2 respectively with the vertical, then the time period of oscillation of the liquid is given by

$$T = 2\pi\sqrt{\frac{m}{S\rho(\cos\theta_1 + \cos\theta_2)g}}$$

In the given case, we have $\theta_1 = 0°$ and $\theta_2 = 60°$, so we get

$$T = 2\pi\sqrt{\frac{m}{S\rho(\cos 0° + \cos 60°)g}} = 2\pi\sqrt{\frac{2m}{3\rho g S}}$$

Hence, the correct answer is (B).

42. $F = Kx + K(x\cos 45)\cos 45 + K(x\cos 45)\cos 45 = 2Kx$

$\Rightarrow \quad K_{eff} = 2K$

$\Rightarrow \quad T = 2\pi\sqrt{m/2K}$

Hence, the correct answer is (B).

43. Since, $\omega_s t - \omega_\ell t = 2\pi$

$\Rightarrow \quad \frac{2\pi}{T}t - \frac{2\pi}{4T}t = 2\pi$ $\quad$ $\{\because T \propto \sqrt{\ell}\}$

$\Rightarrow \quad t\left(1 - \frac{1}{4}\right) = T$

$\Rightarrow \quad t = 4T/3$

Hence, the correct answer is (C).

44. Given that, $E_1 = \frac{1}{2}kx^2$ and $E_2 = \frac{1}{2}ky^2$

$\Rightarrow \quad E = \frac{1}{2}k(x + y)^2 = \frac{1}{2}k\left(\sqrt{\frac{2E_1}{k}} + \sqrt{\frac{2E_2}{k}}\right)^2$

$\Rightarrow \quad E = \left(\sqrt{E_1} + \sqrt{E_2}\right)^2$

Hence, the correct answer is (D).

45. If ℓ_0 is the extension of spring in equilibrium, then $K\ell_0 = mg$. If y is downward displacement of mass from equilibrium, then spring is stretched upward by y. So, restoring force is

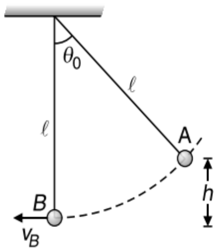
$F = ma = -Ky$ i.e., $a = -\frac{K}{m}y$.

Hence $\omega^2 = \frac{K}{m}$ and time period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$

Hence, the correct answer is (B).

46. Maximum tension is in position B and minimum at A. Let ℓ be the length of string and m the mass of bob. Then

$h = \ell(1-\cos\theta_0)$



Applying Law of Conservation of Energy, we get

$$\frac{1}{2}mv^2 = mg\ell(1-\cos\theta_0)$$

$\Rightarrow$ $v_B^2 = 2gh = 2g\ell(1-\cos\theta_0)$

Also, $T_A = mg\cos\theta_0$ ...(1)

and $T_B - mg = \frac{mv_B^2}{\ell} = 2mg(1-\cos\theta_0)$

$\Rightarrow$ $T_B = mg(3-2\cos\theta_0)$ ...(2)

Given that $T_B = 2T_A$ ...(3)

From equations (1), (2) and (3), we get

$$\cos\theta_0 = \frac{3}{4}$$

Hence, the correct answer is (B).

47. Given that, $v^2 = \frac{1}{4}(5^2 - x^2)$

Comparing with $v^2 = \omega^2(A^2 - x^2)$, we get

$$\omega = \frac{1}{2}\text{ rads}^{-1}$$

$\Rightarrow$ $T = 4\pi$

Hence, the correct answer is (C).

48. Since, $T = 2\pi\sqrt{\frac{\ell}{g}}$

$\Rightarrow$ $\frac{T'}{T} = \sqrt{\frac{g}{g'}}$ ...(1)

Since, $g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

$\Rightarrow$ $\frac{g'}{g} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$

$\Rightarrow$ $\frac{T'}{T} = \left(1+\frac{h}{R}\right)$ {From (1)}

$\Rightarrow$ $T' = T\left(1+\frac{h}{R}\right)$

Since, $T' > T$, the clock will lose the time, so

$$\Delta T = T' - T = T\left(\frac{h}{R}\right)$$

So, time lost in $t = 1$ day is

$$\Delta t = \left(\frac{\Delta T}{T'}\right)t = \frac{t\left(\frac{h}{R}\right)}{\left(1+\frac{h}{R}\right)} \approx t\left(\frac{h}{R}\right)$$

$\Rightarrow$ $\Delta t = \frac{(24\times3600)(200)}{6.4\times10^6}\text{ s} = 2.7\text{ s}$

Hence, the correct answer is (B).

49. Two springs in parallel give a force constant of $2k$ which is in series with k.

$$k_{\text{net}} = \frac{(2k)(k)}{2k+k} = \frac{2k}{3}$$

$\Rightarrow$ $f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{net}}}{m}} = \frac{1}{2\pi}\sqrt{\frac{2k}{3m}}$

Hence, the correct answer is (B).

50. Given that, $y = a\sin(\omega t + \theta)$

At $t = 0$, $y = a\sin\theta = 3$ ...(1)

Also, $v = a\omega\cos(\omega t + \theta)$

At $t = 0$, $v = a\omega\cos\theta = 1.5\pi$

$\Rightarrow$ $a(0.5\pi)\cos\theta = 1.5\pi$

$\Rightarrow$ $a\cos\theta = 3$ ...(2)

From equation (1) and (2), we get

$$\theta = 45° \text{ and } a = 3\sqrt{2}\text{ cm}$$

Hence, the correct answer is (C).

51. Since, $F = ma = -\frac{dU}{dx} = -8\sin(2x)$

$\Rightarrow$ $a = \frac{F}{m} = -8\sin(2x)$ $\{\because m = 1\text{ kg}\}$

For small oscillations $\sin(2x) \approx 2x$, so $a = -16x$

Hence the oscillations are simple harmonic in nature

$\Rightarrow$ $T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{1}{16}} = \frac{\pi}{2}\text{ s}$

Hence, the correct answer is (C).

52. $v^2 = \omega^2(a^2 - x^2)$

Since $v_{\max} = a\omega$

$\Rightarrow$ $v^2 = v_{\max}^2\left(1 - \frac{x^2}{a^2}\right)$

$\Rightarrow$ $2500 = 10000\left(1 - \frac{x^2}{a^2}\right)$

CHAPTER 3

$\Rightarrow \quad x = \frac{\sqrt{3}}{2}a = 5\sqrt{3}$ cm

Hence, the correct answer is (B).

53. At $x = \frac{\sqrt{3}}{2}A$, kinetic energy increases by $\frac{1}{2}m\omega^2A^2$, so the new total energy is

$$E_{\text{new}} = E + \frac{1}{2}m\omega^2A^2$$

$$\Rightarrow \quad E_{\text{new}} = \frac{1}{2}m\omega^2A^2 + \frac{1}{2}m\omega^2A^2 = m\omega^2A^2$$

If A_{new} is the new amplitude, then

$$\frac{1}{2}m\omega^2A_{\text{new}}^2 = m\omega^2A^2$$

$$\Rightarrow \quad A_{\text{new}} = \sqrt{2}A$$

Hence, the correct answer is (C).

54. Block of mass m_2 shorts off carrying some kinetic energy away from the system.

Applying Law of Conservation of Mechanical Energy

$$\begin{pmatrix}\text{Potential Energy} \\ \text{of Spring}\end{pmatrix} = \begin{pmatrix}\text{Maximum Kinetic} \\ \text{Energy of Blocks}\end{pmatrix}$$

$$\Rightarrow \quad \frac{kd^2}{2} = (m_1 + m_2)\frac{v^2}{2} \qquad \{k = \text{force constant of spring}\}$$

$$\Rightarrow \quad v^2 = \frac{kd^2}{m_1 + m_2}$$

With m_1 alone on the spring, we have

$$\begin{pmatrix}\text{Maximum Potential} \\ \text{Energy}\end{pmatrix} = \begin{pmatrix}\text{Maximum Kinetic} \\ \text{Energy of } m_1\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}kA^2 = \frac{1}{2}m_1v^2$$

$$\Rightarrow \quad kA^2 = \frac{km_1d^2}{m_1 + m_2}$$

$$\Rightarrow \quad A = d\sqrt{\frac{m_1}{m_1 + m_2}}$$

Hence, the correct answer is (A).

55. From Trigonometry

$$\cos^2\theta - \sin^2\theta = \cos(2\theta)$$

$$\Rightarrow \quad \cos^2\left(\frac{\pi t}{2}\right) - \sin^2\left(\frac{\pi t}{2}\right) = \cos(\pi t)$$

$$\Rightarrow \quad y = 0.4\cos(\pi t)$$

which indicates that motion is harmonic with amplitude 0.4 m.

Hence, the correct answer is (B).

56. Equivalent spring constant of the combination is

$$\frac{1}{k_{\text{eq}}} = \frac{1}{k} + \frac{1}{2k} = \frac{3}{2k}$$

$$\Rightarrow \quad k_{\text{eq}} = \frac{2k}{3}$$

The reduced mass is $\mu = \frac{m_1m_2}{m_1 + m_2} = \frac{m}{2}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\mu}{k_{\text{eq}}}} = 2\pi\sqrt{\frac{m/2}{2k/3}} = 2\pi\sqrt{\frac{3m}{4k}}$$

Hence, the correct answer is (B).

57. Amplitude of motion $A = 2x_0$. Time to cover from extreme position to mean position (i.e., from compressed position to normal position) is $\frac{T}{4}$. Time taken to cover distance x_0 from mean position is calculated using $y = A\sin(\omega t)$

$$\Rightarrow \quad x_0 = 2x_0\sin\left(\frac{2\pi t}{T}\right)$$

$$\Rightarrow \quad \frac{2\pi t}{T} = \frac{\pi}{6}$$

$$\Rightarrow \quad t = \frac{T}{12}$$

So, total time taken to hit the wall

$$t = \frac{T}{4} + \frac{T}{12} = \left(\frac{3+1}{12}\right)T = \frac{T}{3} = \frac{1}{3}\left(2\pi\sqrt{\frac{m}{K}}\right)$$

Hence, the correct answer is (D).

58. Since, $k_{\text{eff}} = 2k$, so $T = 2\pi\sqrt{\frac{M}{2k}}$

Period (or k_{eff}) is independent of θ.

Hence, the correct answer is (A).

59. Time period of pendulum is given by

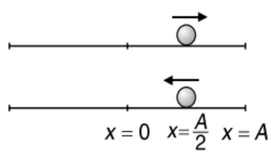
$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1}{\pi^2}} = 2 \text{ s}$$

On the right side, it completes half an oscillation, whereas on the left side, it only goes from mean to half the amplitude and comes back. Therefore, time of oscillation is

$$T' = \frac{T}{2} + 2\left(\frac{T}{12}\right) = \frac{2}{3}T = \frac{4}{3} \text{ s}$$

Hence, the correct answer is (B).

60. The displacement time and velocity-time equations in this situation can be written as



$$x = A\sin(\omega t + \phi)$$

$$\Rightarrow \quad v = A\omega\cos(\omega t + \phi)$$

at $\quad t = 0, x = \frac{A}{2}$

$$\Rightarrow \quad \phi = \frac{\pi}{6} \text{ and } \frac{5\pi}{6}$$

If $\phi = \frac{\pi}{6}$, displacement and velocity both are positive at $t = 0$.

When, $\phi = \frac{5\pi}{6}$, displacement is positive but velocity is negative.

Displacement-time equations of the two particles can be written as

$$x_1 = A\sin\left(\omega t + \frac{\pi}{6}\right) \text{ and } x_2 = A\sin\left(\omega t + \frac{5\pi}{6}\right)$$

$$\Rightarrow \quad \Delta\phi = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$$

Hence, the correct answer is (D).

61. In SHM $v_{\max} = a\omega$

$$\text{Since, } \langle v \rangle = \frac{\text{distance travelled in one oscillation}}{\text{time period}}$$

$$\Rightarrow \quad \langle v \rangle = \frac{4a}{T} = \frac{4a}{2\pi} = \frac{2a\omega}{\pi} = \frac{2v_{\max}}{\pi}$$

Hence, the correct answer is (D).

62. Since, $T = 2\pi\sqrt{\frac{I}{mgd}}$

where $I = I_{cm} + md^2 = mR^2 + mR^2 = 2mR^2$ is the moment of inertial if the ring about the point of suspension and $d = R$ is the distance of separation between the point of suspension and the centre of mass of the disc.

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{2mR^2}{mgR}} = 2\pi\sqrt{\frac{2R}{g}}$$

Hence, the correct answer is (B).

63. Force constant of a spring is inversely proportional to its natural length i.e., $k \propto \frac{1}{l}$ i.e., force constant of two halves will become $2k$ each, where k is force constant of complete spring, hence $k_{\text{eff}} = 2k + 2k = 4k$.

Since $T \propto \frac{1}{\sqrt{k}}$, so $T' = \frac{T}{2}$

Hence, the correct answer is (C).

64. $a_{\max} = \omega^2 A$ and $v_{\max} = A\omega$

If ω is doubled and amplitude is halved, $v_{\max}$ remains constant while $a_{\max}$ becomes two times.

Hence, the correct answer is (C).

65. Since, $\omega = \frac{v}{R} = 4 \text{ rads}^{-1}$

$$\Rightarrow \quad \frac{\text{displacement}}{\text{acceleration}} = \frac{1}{\omega^2} = \frac{1}{16}(\text{second})^2$$

Hence, the correct answer is (A).

66. When the bead is attached at the middle of the spring of length L, effectively the spring behaves as a parallel combination of two springs each of length $\frac{L}{2}$ and spring constant $2K$. Therefore, effective spring constant is

$$K_{\text{eff}} = 200 + 200 = 400 \text{ Nm}^{-1}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{40/1000}{400}} = \frac{\pi}{50}$$

Hence, the correct answer is (A).

67. Since $T = 2\pi\sqrt{\frac{M}{k}}$ and $\frac{5T}{4} = 2\pi\sqrt{\frac{M+m}{k}}$

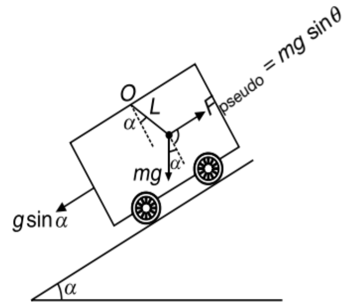
$$\Rightarrow \quad \frac{5}{4} = \sqrt{\frac{M+m}{M}}$$

$$\Rightarrow \quad \frac{25}{16} = 1 + \frac{m}{M}$$

$$\Rightarrow \quad \frac{m}{M} = \frac{9}{16}$$

Hence, the correct answer is (C).

68. Since the vehicle is moving down the frictionless incline, so the acceleration of the vehicle is $g\sin\alpha$ along the incline. So, the bob will experience a pseudo force $mg\sin\theta$ in the backward direction, as shown. The weight of the bob is mg acting vertically downwards. If a_{net} is the net acceleration of the bob then,



$$a_{net} = \sqrt{g^2 + g^2\sin^2\alpha + 2(g)(g\sin\alpha)\cos(90+\alpha)}$$

$$\Rightarrow \quad a_{net} = \sqrt{g^2 + g^2\sin^2\alpha - 2g^2\sin^2\alpha}$$

$$\Rightarrow \quad a_{net} = \sqrt{g^2 - g^2\sin^2\alpha} = g\sqrt{1-\sin^2\alpha}$$

$$\Rightarrow \quad a_{net} = g\cos\alpha$$

Further $T = 2\pi\sqrt{\frac{L}{a_{net}}}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$

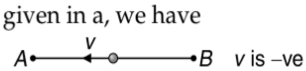
As a short cut if we think $\alpha \to 0$, then

$$T = 2\pi\sqrt{\frac{L}{g}}$$

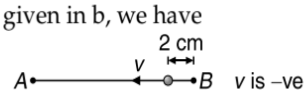
i.e., the time period of a simple pendulum

Hence, the correct answer is (B).

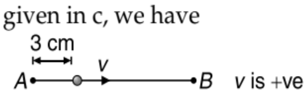
69. Since direction from A to B is positive, therefore for position given in a, we have



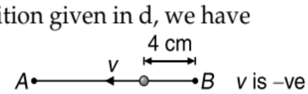
for position given in b, we have



for position given in c, we have



and for position given in d, we have



Hence, the correct answer is (B).

70. If y is downward displacement of mass, then stretching of spring $= 2y$

Tension $T = F = K(2y)$

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Also, $mg = 2T$

Restoring force $F' = 2T = 4Ky$

$\Rightarrow \quad ma = -4Ky$

$\Rightarrow \quad m\ddot{y} = -4Ky$

$\Rightarrow \quad \ddot{y} + \left(\frac{4K}{m}\right)y = 0$

$\Rightarrow \quad t = 2\pi\sqrt{\frac{y}{|\ddot{y}|}} = 2\pi\sqrt{\frac{m}{4K}} = \pi\sqrt{\frac{m}{K}}$

Hence, the correct answer is (C).

71. Since, we know that time period of a simple pendulum is proportional to the square root of the length.

$\Rightarrow \quad T \propto \sqrt{\ell}$

$\Rightarrow \quad \frac{T_s}{T_\ell} = \frac{1}{4}$

$\Rightarrow \quad 4T_s = 1\,T_\ell$

So, four oscillations of shorter pendulum equals one oscillation of longer pendulum.

Hence, the correct answer is (B).

72. Since, the sphere floats half immersed in the liquid of density ρ, so we have

$$\left(\begin{array}{c}\text{Mass of}\\ \text{the sphere}\end{array}\right) = \left(\begin{array}{c}\text{Mass of liquid}\\ \text{displaced}\end{array}\right) = \frac{2}{3}\pi R^3 \rho$$

When pushed down by a small distance y, excess upthrust (restoring force) generated is

$$F = (\pi R^2 y)\rho g$$

Therefore, force constant of SHM is

$$k = \frac{F}{y} = (\pi R^2)\rho g$$

Hence, frequency of oscillations is

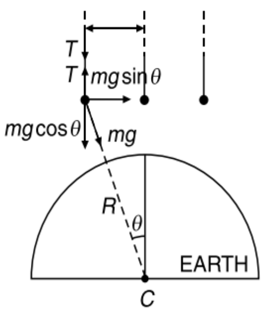
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{\pi R^2 \rho g}{\frac{2}{3}\pi R^3 \rho}} = \frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$$

Hence, the correct answer is (B).

73. $T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{5}{10}} = \sqrt{2}\pi$

Hence, the correct answer is (D).

74. If the length of the pendulum is infinite, the bob would move along the arc of a circle of infinite radius, that is, along a straight line. If the amplitude of oscillation is small compared to the radius of the earth, the bob will always be at a distance R from the centre of the earth.



Restoring force F is given by

$$F = -mg\sin\theta$$

$\Rightarrow \quad m\ddot{x} = -mg\sin\theta$

$\Rightarrow \quad \ddot{x} + \left(\frac{g}{R}\right)x = 0 \qquad \left\{\because \sin\theta \simeq \theta = \frac{x}{R}\right\}$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{x}{|\ddot{x}|}} = 2\pi\sqrt{\frac{R}{g}}$

Hence, the correct answer is (B).

75. At $t = 0$, the particle is at mean position, so displacement of the particle at time t is

$$x = a\sin(\omega t) = a\sin\left(\frac{2\pi}{8}t\right) = a\sin\left(\frac{\pi}{4}t\right)$$

Let x_1 be the displacement of particle from $0 \rightarrow 1$ s

and x_2 be the displacement of particle from $0 \rightarrow 2$ s

Then, $x_1 = a\sin\frac{\pi}{4} = \frac{a}{\sqrt{2}}$ and $x_2 = a$

Displacement of the particle from 1 s to 2 s is

$$x_2 - x_1 = a\left(1 - \frac{1}{\sqrt{2}}\right)$$

$\Rightarrow \quad \frac{x_1}{x_2 - x_1} = \frac{a/\sqrt{2}}{a\left(1 - \frac{1}{\sqrt{2}}\right)} = \frac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$

Hence, the correct answer is (D).

76. Let speed of each block just before colliding be v. Then on colliding, if v' be the velocity of the combined mass, then by Law of Conservation of Linear Momentum, we have

$$2mv - mv = (2m + m)v'$$

$\Rightarrow \quad v' = \frac{v}{3}$

Since, this maximum velocity of the combined mass is one-third of that when amplitude was R, so the new amplitude will be $\frac{R}{3}$ because the angular frequency remains unchanged.

Hence, the correct answer is (C).

77. At the mean position a particle possesses minimum potential energy and maximum kinetic energy.

Hence, the correct answer is (C).

78. $(\text{K.E.})_{max} = \frac{1}{2}(2)(100)^2\left(\frac{6}{100}\right)^2$

$\Rightarrow \quad (\text{K.E.})_{max} = 36$ J

Hence, the correct answer is (B).

79. $v^2 = \omega^2(a^2 - x^2)$

$\Rightarrow \quad v^2 + \omega^2 x^2 = \omega^2 a^2$

$\Rightarrow \quad \frac{v^2}{a^2\omega^2} + \frac{x^2}{a^2} = 1$

which is the equation of an ellipse

Hence, the correct answer is (C).

80. Since $T \propto \ell^{\frac{1}{2}}$, so we have $\frac{\Delta T}{T} = \frac{1}{2}\left(\frac{\Delta \ell}{\ell}\right) = 1\%$

$$\Rightarrow \quad \frac{T'-T}{T} = \frac{1}{100}$$

$$\Rightarrow \quad T' = T + 0.01T = 1.01T$$

Hence, the correct answer is (D).

81. Since, $g' = \dfrac{GM'}{R'^2} = \dfrac{G(2M_e)}{(2R_e)^2} = \dfrac{g}{2}$ $\qquad \left\{\because g = \dfrac{GM_e}{R_e^2}\right\}$

$$\Rightarrow \quad T' = 2\pi\sqrt{\frac{\ell}{g'}} = \sqrt{2}\left(2\pi\sqrt{\frac{\ell}{g}}\right) = \sqrt{2}T$$

For a second's pendulum, we have $T = 2$ s, so

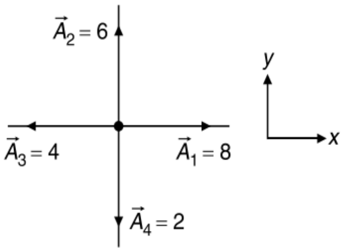
$$\Rightarrow \quad T' = 2\sqrt{2}\text{ s}$$

Hence, the correct answer is (D).

82. Initial phase of the block is zero.
Hence, the correct answer is (B).

83. For a spring $T = 2\pi\sqrt{\dfrac{m}{k}}$. Since liquid is non-viscous, hence the time period remains unaltered.
Hence, the correct answer is (C).

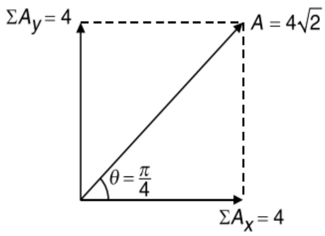
84. The resulting amplitude and corresponding phase difference can be calculated by vector method as follows:



$$\Sigma A_x = 8 - 4 = 4$$

and $\Sigma A_y = 6 - 2 = 4$

Therefore, resulting amplitude is $4\sqrt{2}$ and phase difference with x_1 is $\phi = \dfrac{\pi}{4}$.



Hence, the correct answer is (D).

85. Let two SHMs be represented as

$$x_1 = a\sin(2\pi ft) \text{ and } x_2 = a\sin(2\pi ft + \phi)$$

Then, separation between the particles is

$$x = x_2 - x_1 = a\sin(2\pi ft + \phi) - a\sin(2\pi ft)$$

Since, $\sin C - \sin D = 2\cos\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)$

$$\Rightarrow \quad x = 2a\cos\left(2\pi ft + \frac{\phi}{2}\right)\sin\left(\frac{\phi}{2}\right)$$

Given that the maximum separation between the particles is $a\sqrt{2}$, so we have

$$2a\sin\left(\frac{\phi}{2}\right) = a\sqrt{2}$$

$$\Rightarrow \quad \sin\left(\frac{\phi}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \phi = 90°$$

Hence, the correct answer is (B).

86. For equilibrium of $(M+m)$, we have

$$x_1 = \frac{(M+m)g}{k}$$

and for equilibrium of m, we have

$$x_2 = \frac{mg}{k}$$

So, amplitude of oscillation is given by

$$A = x_1 - x_2 = \frac{Mg}{k}$$

Hence, the correct answer is (A).

87. Since, $F = \left(\dfrac{YA}{\ell}\right)x$, so effective force constant of wire is $k = \dfrac{YA}{\ell}$ and hence $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{m\ell}{YA}}$.

Hence, the correct answer is (C).

88. Since, $0.5 = 2\pi\sqrt{\dfrac{m}{k}}$ and $0.6 = 2\pi\sqrt{\dfrac{m+m'}{k}}$

$$\Rightarrow \quad \frac{0.36}{4\pi^2} = \frac{m}{k} + \frac{m'}{k} = \frac{0.25}{4\pi^2} + \frac{m'}{k}$$

$$\Rightarrow \quad \frac{m'}{k} = \frac{0.11}{4\pi^2}$$

Let x be the additional extension, then

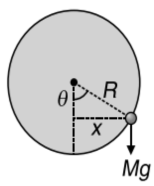
$$m'g = kx$$

$$\Rightarrow \quad x = \frac{m'g}{k} = \frac{(0.11)(9.8)}{4\pi^2} = 2.69 \text{ cm}$$

Hence, the correct answer is (B).

89. **Method-1 (Using restoring torque method)**

Let the arrangement be given a small angular displacement θ from the mean position as shown in Figure.



The restoring torque acting on the system is

$$\tau = -mgR\sin\theta = -mgR\theta$$

(because for small θ, we have $\sin\theta \approx \theta$)

$$\Rightarrow \quad I\left(\frac{d^2\theta}{dt^2}\right) = -mgR\theta \qquad \ldots(1)$$

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where I is the moment of inertia of the system about the axis of rotation, given by

$$I=\frac{1}{2}MR^2+mR^2=\frac{1}{2}(2)(5)^2+1(5)^2=50\text{ kgm}^2$$

From equation (1), we get

$$\omega=\frac{2\pi}{T}=\sqrt{\frac{mgR}{I}}=\sqrt{\frac{(1)(10)(5)}{50}}=1\text{ rads}^{-1}$$

$\Rightarrow\quad T=(2\pi)$ second

Method-2 (Using the concept of Physical Pendulum)

Distance of centre of mass of the system from the axis of oscillation is

$$d=\frac{(2)(0)+(1)(5)}{2+1}=\frac{5}{3}\text{ m}$$

Also, MI of the system about axis of rotation is

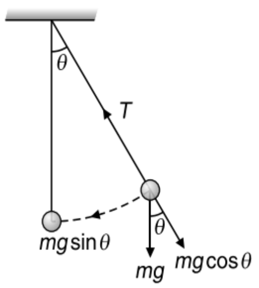
$$I=\frac{1}{2}MR^2+mR^2=\frac{1}{2}(2)(5)^2+1(5)^2=50\text{ kgm}^2$$

$$\Rightarrow\quad T=2\pi\sqrt{\frac{I}{Mgd}}=2\pi\sqrt{\frac{50}{(3)(10)(5/3)}}=2\pi\text{ s}$$

Hence, the correct answer is (A).

90. Initially when water starts draining out of the sphere its cg falls down i.e., the effective length of the pendulum increases so that the time period is increased. As the level has considerably fallen the effective cg starts moving towards the centre thus indicating a decrease in the effective length and hence the time period. Finally, when the sphere becomes empty the CG moves to the centre and the new time period will be same as that of the completely filled sphere.
Hence, the correct answer is (D).

91. Free body diagram of pendulum is shown in Figure.



Since $T=mg\cos\theta$, net force is $F_{\text{net}}=mg\sin\theta$

For small θ, $\sin\theta\approx\tan\theta\approx\theta$, so acceleration is

$$\Rightarrow\quad a=g\sin\theta\approx g\theta=(10)\left(\frac{0.2}{4}\right)=0.5\text{ ms}^{-2}$$

Hence, the correct answer is (C).

92. $T=2\pi\sqrt{\frac{I}{mgl}}$, where, $I=\frac{3}{2}mR^2$ and $l=R$

$$\Rightarrow\quad T=2\pi\sqrt{\frac{3R}{2g}}$$

Hence, the correct answer is (D).

93. Time period of linear oscillations of a spring mass system is independent of any constant force acting on the block.
Hence, the correct answer is (D).

94. On the planet $s=\frac{1}{2}at^2$ i.e., $8=\frac{1}{2}a(4)$

$\Rightarrow\quad a=4\text{ ms}^{-2}$

So, $T=2\pi\sqrt{\frac{\ell}{a}}=2\pi\sqrt{\frac{1}{4}}=\pi$ s

Hence, the correct answer is (B).

95. Energy of oscillation is $E=\alpha A^4$

So, kinetic energy of mass at $x=x$ is

$$K=E-U=\alpha\left(A^4-X^4\right)$$

Given, $K=3U$

$$\Rightarrow\quad \alpha\left(A^4-x^4\right)=3\alpha x^4$$

$$\Rightarrow\quad x=\pm\frac{A}{\sqrt{2}}$$

Hence, the correct answer is (B).

96. Beyond point P, length of pendulum becomes $\frac{\ell}{4}$.

Since $T\propto\sqrt{\ell}$, so beyond P time period will become $T=\frac{T}{2}$.
Hence desired time is

$$t=\frac{T}{2}+\frac{T'}{2}=\frac{T}{2}+\frac{T}{4}=\frac{3T}{4}$$

Hence, the correct answer is (B).

97. Mean position is $\frac{6+(-2)}{2}=2$ cm

Amplitude is $a=\frac{6-(-2)}{2}=4$ cm

Therefore, equation of SHM is

$$x=2+4\sin(\omega t+\phi)$$

where $\omega=\frac{2\pi}{T}=\frac{2\pi}{0.5}=4\pi\text{ rads}^{-1}$

Now, at $t=0$, $x=2+4\sin\phi=4$ {given}

$$\Rightarrow\quad \sin\phi=\frac{1}{2}$$

$$\Rightarrow\quad \phi=30°, 150°, \ldots\ldots$$

Also, velocity is $v=4\omega\cos(\omega t+\phi)$

At $t=0$, $v=4\omega\cos\phi=$ positive {given}

So, ϕ must lie in first or fourth quadrant, i.e. $\phi=30°$

$$\Rightarrow\quad x=2+4\sin(4\pi t+30°)$$

Also, we see that at $x=4$ cm, acceleration of particle will be towards mean position, i.e. towards negative x-direction.
Hence, the correct answer is (A).

98. Force constant is $k=\frac{F}{x}=\frac{\pi^2}{16}$

So, the angular frequency of SHM is

$$\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{\pi^2/16}{4}}=\frac{\pi}{8}\text{ rads}^{-1}$$

Since, particle passes through mean position at $t = 2$ s, hence velocity of particle is given by

$$v = A\omega\cos[\omega(t-2)]$$

According to the problem, we have

$$4\sqrt{2} = A\left(\frac{\pi}{8}\right)\left|\cos\left[\frac{\pi}{8}(10-2)\right]\right| = \frac{\pi A}{8}|\cos\pi|$$

$$\Rightarrow \quad A = \frac{32\sqrt{2}}{\pi} \text{ m}$$

Hence, the correct answer is (A).

99. Since, the particle starts from its equilibrium position i.e., mean position, so $x = A\sin(\omega t)$, where A is the amplitude.

At time $t = \frac{T}{12}$, its displacement from the mean position is

$$x = A\sin\left(\frac{2\pi}{T}\frac{T}{12}\right) = \frac{A}{2}$$

Now, $KE = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{3}{4}\left(\frac{1}{2}m\omega^2A^2\right)$

and $PE = \frac{1}{2}m\omega^2x^2 = \frac{1}{4}\left(\frac{1}{2}m\omega^2A^2\right)$

$$\Rightarrow \quad \frac{KE}{PE} = \frac{3}{1}$$

Hence, the correct answer is (B).

100. Velocity of particle performing SHM is

$$v = \omega\sqrt{A^2 - x^2}$$

$$v^2 = \omega^2A^2 - \omega^2x^2 \qquad \ldots(1)$$

Acceleration of particle performing SHM is

$$a = -\omega^2 x \text{ i.e. } x = -\frac{a}{\omega^2}$$

Substituting the value x in equation (1), we get

$$v^2 = \omega^2A^2 - \omega^2\left(\frac{a^2}{\omega^4}\right) = -\frac{a^2}{\omega^2} + \omega^2A^2$$

Therefore, graph between v^2 and a^2 is a straight line with negative slope and positive intercept.

Hence, the correct answer is (D).

101. When the plank is displaced downwards by x upthrust due to lower liquid will increase while due to upper liquid will decrease. The difference in these two upthrust will become net restoring force. Thus, net restoring force

$$F = -(\text{Extra Upthrust})$$

$$\Rightarrow \quad F = -(Ax)(2\rho - \rho)g = -(\rho Ag)x$$

$$\Rightarrow \quad \text{Acceleration } a = \frac{F}{m} = -\left(\frac{\rho Ag}{m}\right)x$$

$$\Rightarrow \quad T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{m}{\rho Ag}}$$

Hence, the correct answer is (A).

102. The energy is given by $E = \frac{1}{2}mv^2 + mgy$

$$\Rightarrow \quad E = \frac{1}{2}mv^2 + mg\left(\frac{x^2}{40}\right) \qquad \left\{\because\ y = \frac{x^2}{40}\right\}$$

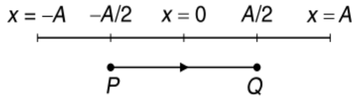
For SHM E = constant, so $\frac{dE}{dt} = 0$

$$\Rightarrow \quad a = -\left(\frac{g}{20}\right)x$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{20}}$$

Hence, the correct answer is (C).

103. For minimum time, the particle must move along the path PQ shown in Figure.



Time taken to go from P to Q is

$$t = \frac{T}{12} + \frac{T}{12} = \frac{T}{6} = \frac{2\pi/\omega}{6} = \frac{\pi}{3\omega}$$

Hence, the correct answer is (A).

104. Since $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the liquid in one of the limbs. However, if L is taken to be the length of the liquid column then length of liquid in each limb is $\frac{L}{2}$ and in that case, $T = 2\pi\sqrt{\frac{L}{2g}}$.

Also $M = (AL)d$, so $L = \frac{M}{Ad}$.

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{M}{2Agd}}$$

Hence, the correct answer is (D).

105. Since $\omega = 2\pi f = \sqrt{\frac{k}{m}}$, so $k = (2\pi f)^2 m$

Total energy of oscillation is $E = (0.5 + 0.4) = 0.9$ J

$$\Rightarrow \quad 0.9 = \frac{1}{2}kA^2$$

$$\Rightarrow \quad A = \sqrt{\frac{1.8}{k}} = \sqrt{\frac{1.8}{(2\pi f)^2 m}} = \frac{1}{2\pi f}\sqrt{\frac{1.8}{0.2}}$$

$$\Rightarrow \quad A = \frac{1}{2\pi(25/\pi)}\sqrt{\frac{1.8}{0.2}} = \frac{3}{50} \text{ m} = 6 \text{ cm}$$

Hence, the correct answer is (A).

106. Mean position will be $x = \frac{mg}{k}$

$$\Rightarrow \quad x = \frac{mg}{k} = \frac{8 \times 10}{200} = \frac{2}{5} \text{ m} = 0.4 \text{ m}$$

This is also the amplitude of oscillation, so

$$A = 0.4 \text{ m}$$

Now, $v_{max} = A\omega = A\sqrt{\frac{k}{m}}$

$$\Rightarrow \quad v_{max} = (0.4)\sqrt{\frac{200}{8}} = 2 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

CHAPTER 3

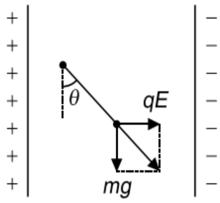
107. The block remains in contact with the spring as long as spring is compressed, i.e. for half the oscillation of system.

Also, $\mu = \dfrac{m_1 m_2}{m_1 + m_2} = \dfrac{2 \times 2}{2+2} = 1 \text{ kg}$

$\Rightarrow \quad t = \dfrac{T}{2} = \pi\sqrt{\dfrac{\mu}{k}} = \pi\sqrt{\dfrac{1}{\pi^2}} = 1 \text{ s}$

Hence, the correct answer is (C).

108.



Since, $a_{\text{net}} = \dfrac{1}{m}\sqrt{(qE)^2 + (mg)^2} = \sqrt{g^2 + \left(\dfrac{qE}{m}\right)^2}$

$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{L}{a_{\text{net}}}}$

Hence, the correct answer is (D).

109. Since, $\dfrac{1}{2}kA^2 = (9-5) = 4 \text{ J}$

$\Rightarrow \quad k = \dfrac{8}{A^2} = \dfrac{8}{(0.01)^2} = 8 \times 10^4 \text{ Nm}^{-1}$

$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{2}{8 \times 10^4}} = \dfrac{\pi}{100} \text{ s}$

Hence, the correct answer is (D).

110. Since, $T' = 2\pi\sqrt{\dfrac{l}{g_{\text{eff}}}} = 2\pi\sqrt{\dfrac{l}{g\left(1 - \dfrac{\rho_{\text{liquid}}}{\rho_{\text{bob}}}\right)}}$

$\Rightarrow \quad T' = T\sqrt{\dfrac{\rho_{\text{bob}}}{\rho_{\text{bob}} - \rho_{\text{liquid}}}}$

Given that $T' = 2T$, we get

$\rho_{\text{body}} = \dfrac{4}{3}\rho_{\text{water}} = 1.33 \text{ gcc}^{-1}$

Hence, the correct answer is (B).

111. Since $E = \dfrac{1}{2}mA^2\omega^2$, so $E \propto (A\omega)^2$

$\Rightarrow \quad (A_1\omega_1)^2 = (A_2\omega_2)^2$

$\Rightarrow \quad A_1\omega_1 = A_2\omega_2$

$\Rightarrow \quad 4 \times 10 = 5 \times \omega$

$\Rightarrow \quad \omega = 8 \text{ unit}$

Hence, the correct answer is (D).

112. At $t = 0$, $x = \dfrac{A}{2}$

$\Rightarrow \quad \dfrac{A}{2} = A\sin\phi$

$\Rightarrow \quad \sin\phi = \dfrac{1}{2}$

$\Rightarrow \quad \phi = \dfrac{\pi}{6} \text{ OR } \phi = \dfrac{5\pi}{6}$

Since, $v = \dfrac{dx}{dt} = A\omega\cos(\omega t + \phi)$

At $t = 0$, $v = A\omega\cos(\omega t + \phi)$

At $t = 0$, $v = A\omega\cos\phi$

Now, $\cos\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

and $\cos\left(\dfrac{5\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$

Since, v is negative at $t = 0$, so ϕ must be $\dfrac{5\pi}{6}$.

Hence, the correct answer is (D).

113. $T = 2\pi\sqrt{\dfrac{\ell}{g_{\text{eff}}}}$

where, $g_{\text{eff}} = \left(1 - \dfrac{1}{2}\right)g = \dfrac{g}{2}$

$\Rightarrow \quad T' = \sqrt{2}T = 2\sqrt{2} \text{ s}$

Hence, the correct answer is (C).

114. If L be the length of the pendulum, then

$T_1 = 2\pi\sqrt{\dfrac{L}{g}}, T_2 = 2\pi\sqrt{\dfrac{L}{g+a}}, T_3 = 2\pi\sqrt{\dfrac{L}{g-a}}$

$\Rightarrow \quad \dfrac{1}{T_2^2} + \dfrac{1}{T_3^2} = \dfrac{1}{4\pi^2}\left(\dfrac{g+a}{L}\right) + \dfrac{1}{4\pi^2}\left(\dfrac{g-a}{L}\right)$

$\Rightarrow \quad \dfrac{1}{T_2^2} + \dfrac{1}{T_3^2} = 2\dfrac{1}{4\pi^2}\dfrac{g}{L} = \dfrac{2}{T_1^2}$

$\Rightarrow \quad T_1 = \sqrt{2}\dfrac{T_2 T_3}{\sqrt{T_2^2 + T_3^2}}$

Hence, the correct answer is (C).

115. Net weight W' of pendulum is $W' = W - U$

$W' = V\rho g - \dfrac{V\rho g}{10} = \dfrac{9}{10}W$

So, $g_{\text{eff}} = \dfrac{9}{10}g$

$\Rightarrow \quad T' = 2\pi\sqrt{\dfrac{l}{g'}} = \sqrt{\dfrac{10}{9}}T$

Hence, the correct answer is (C).

116. Displacement-time equation of the particle will be

$x = A\cos(\omega t)$

So, $x_1 = A\cos\omega$, $x_2 = A\cos(2\omega)$ and $x_3 = A\cos(3\omega)$

Now, $\dfrac{x_1 + x_3}{2x_2} = \dfrac{A(\cos\omega + \cos(3\omega))}{2A\cos(2\omega)}$

$\Rightarrow \quad \dfrac{x_1 + x_3}{2x_2} = \dfrac{2A\cos(2\omega)\cos\omega}{2A\cos(2\omega)} = \cos\omega$

$\Rightarrow \quad \omega = \cos^{-1}\left(\frac{x_1 + x_3}{2x_2}\right) = \frac{2\pi}{T}$

$\Rightarrow \quad T = \frac{2\pi}{\theta}$ where, $\theta = \cos^{-1}\left(\frac{x_1 + x_3}{2x_2}\right)$

Hence, the correct answer is (A).

117. At $t = 0$, $x = A\sin\left(\frac{\pi}{6}\right) - A\cos\left(\frac{\pi}{6}\right) = -\text{ve}$

So, the acceleration is positive. Similarly, we can find sign of v. Now $\frac{dx}{dt}$ at $t = 0$

$\Rightarrow \quad v\Big|_{t=0} = A\cos\left(\frac{\pi}{6}\right) + A\sin\left(\frac{\pi}{6}\right) > 0$

Hence, the correct answer is (D).

118. Using, $s = \frac{1}{2}at^2 = \frac{1}{2}(g\sin\theta)t^2$

For left wedge, $s = \frac{0.2}{\sin 30°} = 0.4$ m and time of ascent or descent is $0.4 = \frac{1}{2}(10\sin 30°)t_1^2$

$\Rightarrow \quad t_1 = 0.4$ s

Since the energy is conserved, so the block will also rise to a height of 20 cm on the right wedge. Hence for the right wedge, we have time of ascent or time of descent given by

$\frac{0.2}{\sin 60°} = \frac{1}{2}(10\sin 60°)t_2^2$

$\Rightarrow \quad t_2 = \frac{0.4}{\sqrt{3}}$ s

So, total time of oscillation is

$$T = 2(t_1 + t_2) = 0.8\left(1 + \frac{1}{\sqrt{3}}\right) \text{ s}$$

Hence, the correct answer is (B).

119. Since, $(v_M)_{max} = (v_N)_{max}$

$\Rightarrow \quad \omega_M A_M = \omega_N A_N$

$\Rightarrow \quad \frac{A_M}{A_N} = \frac{\omega_N}{\omega_M} = \sqrt{\frac{k_2}{k_1}} \qquad \left\{\because \omega = \sqrt{\frac{k}{m}}\right\}$

Hence, the correct answer is (B).

120. Maximum Acceleration = $A\omega^2$

$\Rightarrow \quad \left(\frac{\pi}{2}\right)^2 = 1\left(\frac{2\pi}{T}\right)^2$

$\Rightarrow \quad T^2 = \frac{4\pi^2}{(\pi^2/4)}$

$\Rightarrow \quad T = 4$ s

Hence, the correct answer is (B).

121. $\frac{T_\ell}{T_s} = \frac{11}{10} \qquad \{\because T \propto \sqrt{\ell}\}$

$\Rightarrow \quad 10T_\ell = 11T_s$

$\Rightarrow \quad \begin{pmatrix}\text{10 oscillations} \\ \text{of LONGER} \\ \text{pendulum}\end{pmatrix} \equiv \begin{pmatrix}\text{11 oscillations} \\ \text{of SHORTER} \\ \text{pendulum}\end{pmatrix}$

Hence, the correct answer is (B).

122. Maximum velocity is $v_{max} = A\omega$

$\Rightarrow \quad v = A\sqrt{\frac{g}{L}}$

$\Rightarrow \quad A = v\sqrt{\frac{L}{g}}$

Hence, the correct answer is (A).

123. Since, $T = 2\pi\sqrt{\frac{\ell}{g}}$

$\Rightarrow \quad \frac{\Delta T}{T} = -\frac{1}{2}\left(\frac{\Delta g}{g}\right) = -\frac{1}{2}(0.02) = -0.01$

$\Rightarrow \quad T' - T = -0.01T = 0.99T$

Hence, the correct answer is (C).

124. On reaching the steady state the frequency of the driven damped oscillator equals the frequency of the driver.
Hence, the correct answer is (B).

125. If a pendulum is oscillating inside a container, filled with liquid, placed in a lift accelerating down with retardation a_0, then effective gravity for the pendulum bob is

$$g_{eff} = (g + a_0)\left(1 - \frac{\rho_{liquid}}{\rho_{bob}}\right)$$

$\Rightarrow \quad T' = 2\pi\sqrt{\frac{L}{(g + a_0)\left(1 - \frac{\rho_{liquid}}{\rho_{bob}}\right)}}$

$\Rightarrow \quad T' = 2\pi\sqrt{\frac{L}{(g + a_0)\left(1 - \frac{\sigma}{\rho}\right)}}$

Hence, the correct answer is (C).

126. The given equation is a combination of two equations

$x = x_1 + x_2$

where $x_1' = 4\sin(\omega t)$

and $x_2 = 3\sin\left(\omega t + \frac{\pi}{3}\right)$

where, $A_1 = 4$ m, $A_2 = 3$ m and $\phi = \frac{\pi}{3}$

$\Rightarrow \quad A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$\Rightarrow \quad A = \sqrt{(4)^2 + (3)^2 + 2(4)(3)\frac{\cos\pi}{3}}$

$\Rightarrow \quad A = \sqrt{37} \approx 6$ cm

Hence, the correct answer is (C).

127. Maximum acceleration in SHM is

$a_{max} = \omega^2 A$

this will be provided to the block by friction.

Hence, $a_{max} = \mu g$

$$\Rightarrow \quad A = \frac{\mu g}{\omega^2} = \frac{\left(\frac{1}{2}\right)(10)}{(10)^2} = 0.05 \text{ m} = 5 \text{ cm}$$

Hence, the correct answer is (B).

128. Since, $y = a\sin\omega t + a\cos\omega t$

The amplitude of oscillation is $A = \sqrt{2}a$

Therefore, mechanical energy is

$$E = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}m\omega^2\left(\sqrt{2}a\right)^2 = m\omega^2 a^2$$

Hence, the correct answer is (B).

129. Since $T = 2\pi\sqrt{\frac{m}{k}}$

$$\Rightarrow \quad 2 = 2\pi\sqrt{\frac{0.1+0.3}{k}} = 2\pi\sqrt{\frac{0.4}{k}} \qquad \ldots(1)$$

When 0.3 kg is also removed, then

$$T' = 2\pi\sqrt{\frac{0.1}{k}} \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{T'}{2} = \sqrt{\frac{0.1}{0.4}} \qquad \text{\{Dividing (2) by (1)\}}$$

$$\Rightarrow \quad T' = 1 \text{ s}$$

Hence, the correct answer is (A).

130. $T = 2\pi\sqrt{\frac{\ell_{immersed}}{g}} = 2\pi\sqrt{\frac{0.80}{9.8}} = \frac{4\pi}{7}$

Hence, the correct answer is (A).

131. Time period of a simple pendulum of length L comparable to radius of earth R is

$$T = 2\pi\sqrt{\frac{LR}{(L+R)g}}$$

Since, $L = R$ (given)

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{R\times R}{(R+R)g}} = 2\pi\sqrt{\frac{R}{2g}}$$

Hence, the correct answer is (D).

132. Since, $a = -\omega^2 x$

$$\Rightarrow \quad |a| = \omega^2 x$$

$$\Rightarrow \quad 64 = \omega^2(4)$$

$$\Rightarrow \quad \omega = 4$$

$$\Rightarrow \quad T = \frac{\pi}{2} \text{ s}$$

Hence, the correct answer is (A).

133. Let $x = a\cos(\omega t)$

$$\Rightarrow \quad y = a\cos\left(2\omega t + \frac{\pi}{2}\right)$$

$$\Rightarrow \quad y = -a\sin(2\omega t)$$

$$\Rightarrow \quad y = -2a\sin(\omega t)\cos(\omega t)$$

$$\Rightarrow \quad y = -2a\sqrt{1-\frac{x^2}{a^2}}\,\frac{x}{a}$$

$$\Rightarrow \quad a^2y^2 = 4x^2\left(a^2 - x^2\right)$$

Hence, the correct answer is (C).

134. Maximum acceleration of the platform is

$$a_{max} = \omega^2 A = \left(2\pi\times\frac{2}{\pi}\right)^2 \times 0.1 = 1.6 \text{ ms}^{-2}$$

Hence, reading of the balance can be

$$N = 60(9.8 \pm 1.6) = (588 \pm 96) \text{ newton}$$

i.e., the reading fluctuates between 492 N and 684 N or between 50 kg and 70 kg approximately.

Hence, the correct answer is (D).

135. Since, $U = \frac{1}{2}m\omega^2 A^2$

$$\Rightarrow \quad 1 = \frac{1}{2}(2)\left(\frac{2\pi}{T}\right)^2(0.4)^2$$

$$\Rightarrow \quad T = \frac{4\pi}{5} \text{ s}$$

Hence, the correct answer is (D).

136. Since, $y_1 = \frac{1}{2}\sin(\omega t) + \frac{\sqrt{3}}{2}\cos(\omega t) = \sin\left(\omega t + \frac{\pi}{3}\right)$

and $y_2 = \sin(\omega t) + \cos(\omega t) = \sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right)$

$$\Rightarrow \quad \Delta\phi = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

Hence, the correct answer is (C).

137. $3 = 2\pi\sqrt{\frac{m}{k}}$ and $3 + 1 = 2\pi\sqrt{\frac{m+1}{k}}$

$$\Rightarrow \quad m = \frac{9}{7} \text{ kg}$$

Hence, the correct answer is (B).

138. Since the potential energy U is given by

$$U = \frac{1}{2}m\omega^2 x^2$$

$$\Rightarrow \quad \frac{dU}{dx} = \frac{1}{2}m\omega^2 2x = m\omega^2 x$$

Hence, slope of the graph is proportional to $m\omega^2$. Since slope for A is more than that for B and both masses are the same, so $\omega_A > \omega_B$.

Hence, the correct answer is (A).

139. For object not to break contact with the board, maximum acceleration equals g. So, we have

$$A\omega^2 = g$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{A}{g}}$$

Hence, the correct answer is (D).

140. Acceleration of a particle executing SHM is given by

$$a = -\omega^2 x$$

So, from the graph, we get $\omega^2 = \tan 45° = 1$

$\Rightarrow \quad T = \dfrac{2\pi}{\omega} = 2\pi$ s

Hence, the correct answer is (B).

141. Since. $\frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$

$\Rightarrow \quad x^2 = A^2 - x^2$

$\Rightarrow \quad x = \pm\dfrac{A}{\sqrt{2}}$

Hence, the correct answer is (C).

142. Since, $U = a + bx^2$

So, restoring force is $F = m\left(\dfrac{d^2x}{dt^2}\right) = -\dfrac{dU}{dx} = -2bx$

$\Rightarrow \quad \dfrac{d^2x}{dt^2} + \left(\dfrac{2b}{m}\right)x = 0$

$\Rightarrow \quad f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{2b}{m}}$

Hence, the correct answer is (C).

143. Mean position of the particle is $\dfrac{mg}{k}$ distance below the unstretched position of spring. Therefore, amplitude of oscillation is $A = \dfrac{mg}{k}$

$\Rightarrow \quad \omega = \sqrt{\dfrac{k}{m}} = 2\pi f = 20\pi \qquad \{\because f = 10 \text{ Hz}\}$

$\Rightarrow \quad \dfrac{m}{k} = \dfrac{1}{400\pi^2}$

Therefore, the maximum speed of particle is

$$v_{max} = A\omega = \left(\dfrac{g}{400\pi^2}\right)(20\pi) = \dfrac{1}{2\pi} \text{ ms}^{-1}$$

Hence, the correct answer is (D).

144. Since, $v^2 = \omega^2(a^2 - x^2)$

$\Rightarrow \quad v_1^2 = \omega^2(a^2 - y_1^2) \qquad \ldots(1)$

$\Rightarrow \quad v_2^2 = \omega^2(a^2 - y_2^2) \qquad \ldots(2)$

From (1) and (2), we get

$$T = 2\pi\sqrt{\dfrac{y_1^2 - y_2^2}{v_2^2 - v_1^2}}$$

Hence, the correct answer is (D).

145. Since, $T = 2\pi\sqrt{\dfrac{m}{k}}$

$\Rightarrow \quad k = \dfrac{4\pi^2 m}{T^2} = \dfrac{(4\pi^2)(2)}{(2\pi)^2} = 2 \text{ Nm}^{-1}$

Now $mg = kx_0$

$\Rightarrow \quad x_0 = \dfrac{mg}{k} = \dfrac{2\times g}{2} = g$ metre $= 10$ m

Hence, the correct answer is (A).

146. Total energy, $E = U_0 + \frac{1}{2}kA^2$

$\Rightarrow \quad 9 = 5 + \frac{1}{2}k(0.01)^2$

$\Rightarrow \quad k = 8\times10^4 \text{ Nm}^{-1}$

$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{2}{8\times10^4}} = \dfrac{\pi}{100}$ s

Hence, the correct answer is (D).

147. $U_{max} = E_{max} = E_0$

Hence, the correct answer is (C).

148. By Law of Conservation of Mechanical Energy, we have

$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2$$

$\Rightarrow \quad A = v\sqrt{\dfrac{m}{k}} = 6\sqrt{\dfrac{0.1}{10}}$

$\Rightarrow \quad A = \dfrac{6}{10} = 0.6$ m

Hence, the correct answer is (A).

149. Velocity of the body executing SHM is

$$v^2 = \omega^2(A^2 - y^2)$$

Acceleration of the body is

$$|a| = \omega^2 y$$

$\Rightarrow \quad a^2 = \omega^4 y^2$

Given that, $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{\pi} = 2 \text{ rads}^{-1}$ and $v = a$ at 10 cm from mean position, so we have

$$v^2 = a^2$$

$\Rightarrow \quad \omega^2(A^2 - y^2) = \omega^4 y^2$

$\Rightarrow \quad A^2 - y^2 = \omega^2 y^2$

$\Rightarrow \quad A^2 = (\omega^2 + 1)y^2 = (2^2 + 1)\times10^2 = 500$

$\Rightarrow \quad A = 10\sqrt{5}$ cm

Hence, the correct answer is (D).

150. $T = 2\pi\sqrt{\dfrac{\ell}{g}}$

$T' = 2\pi\sqrt{\dfrac{\ell}{g+g}} = \dfrac{T}{\sqrt{2}}$

Hence, the correct answer is (D).

151. Given, $y = 5\cos\left(2\pi t + \dfrac{\pi}{3}\right)$

Speed is maximum at mean position, i.e., when

$$y = 0$$

$\Rightarrow \quad 2\pi t + \dfrac{\pi}{3} = \dfrac{\pi}{2}$

$\Rightarrow \quad t = \dfrac{1}{12}$ s

Hence, the correct answer is (C).

152. $T = 2\pi\sqrt{\frac{x}{g}}$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{9.8/100}{9.8}}$

$\Rightarrow \quad T = \frac{2\pi}{10}$

Hence, the correct answer is (A).

153. On standing the effective length of the pendulum measured from the point of suspension decreases.
Hence, the correct answer is (A).

Multiple Correct Choice Type Questions

1. For equilibrium, we have
Weight of block = Upthrust on the immersed part of the block

$\Rightarrow \quad mg = (Ad)\rho g$

$\Rightarrow \quad m = A\rho d$

$\Rightarrow \quad d = \frac{m}{A\rho} = \frac{m}{\pi r^2 \rho}$

$$\begin{pmatrix}\text{Restoring} \\ \text{force}\end{pmatrix} = \begin{pmatrix}\text{Upthrust experienced by the} \\ \text{additional part immersed}\end{pmatrix}$$

$\Rightarrow \quad F = -Ax\rho g$

$\Rightarrow \quad m\ddot{x} = -Ax\rho g$

$\Rightarrow \quad (A\sigma\ell)\ddot{x} = -Ax\rho g \qquad \{\because m = (A\ell)\sigma\}$

$\Rightarrow \quad \ddot{x} + \frac{\rho g}{\sigma\ell}x = 0$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{\sigma\ell}{\rho g}}$

Since $m = (\pi r^2)\ell\sigma$

$\Rightarrow \quad \ell\sigma = \frac{m}{\pi r^2}$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{\pi r^2 \rho g}} = \frac{2\pi}{r}\sqrt{\frac{m}{\pi\rho g}}$

Hence, (B) and (D) are correct.

2. $x_{max} = 3A$, when $\cos(\omega t) = -1$
and $x_{min} = A$, when $\cos(\omega t) = 1$
Therefore, the particle oscillates between $x = 3A$ to $x = A$
Since, $x = 2A - A\cos(\omega t)$

$\Rightarrow \quad v = \frac{dx}{dt} = A\omega\sin(\omega t)$

Now $v = v_{max} = A\omega$ at $\omega t = \frac{\pi}{2}$

$\Rightarrow \quad x = 2A$ at $\omega t = \frac{\pi}{2}$

Further,

$T = \frac{2\pi}{\omega}$

$t_{3A\to A} = \frac{T}{2} = \frac{\pi}{\omega}$ and $t_{A\to 2A} = \frac{T}{4} = \frac{\pi}{2\omega}$

Hence, (B), (C) and (D) are correct.

3. Given that $y = 0.05\sin 4\pi(5t + 0.4)$

$\Rightarrow \quad y = 0.05\sin(20\pi t + 1.6\pi)$

$\Rightarrow \quad \omega = 20\pi$

$\Rightarrow \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{20\pi} = 0.1 \text{ s}$

Total energy of SHM is

$$E = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}(0.1)(20\pi)^2(0.05)^2 = 0.05\pi^2 \text{ joule}$$

Maximum acceleration of particle is

$$a_{max} = \omega^2 A = (20\pi)^2 \times (0.05) = 20\pi^2 \text{ ms}^{-2}$$

Force acting on the particle is zero at the mean position and maximum at the extreme position.
Hence, (A) and (C) are correct.

4. Let the displacement equation of particle is

$$x = a\sin(\omega t)$$

Time period of particle is given by

$$T = (t_{PA} + t_{AP}) + (t_{PB} + t_{BP})$$

$\Rightarrow \quad T = (0.5 \text{ s}) + (1.5 \text{ s}) = 2 \text{ s} = \frac{2\pi}{\omega}$

$\Rightarrow \quad \omega = \pi \text{s}^{-1}$

$\Rightarrow \quad x = a\sin(\pi t) \qquad \ldots(1)$

and $v = (a\pi)\cos\pi t \qquad \ldots(2)$

Let $t_{OP} = t$ then $t_{OAP} = t + \frac{1}{2}$

Since, it is given that

$$3 = a\pi\cos(\pi t) = \left|a\pi\cos\pi\left(t + \frac{1}{2}\right)\right|$$

$\Rightarrow \quad 3 = \left|a\pi\cos\left(\frac{\pi}{2} + \pi t\right)\right| = a\pi\sin(\pi t)$

$\Rightarrow \quad \pi t = \frac{\pi}{4}$

$\Rightarrow \quad a\pi = \frac{3}{\cos\left(\frac{\pi}{4}\right)} = 3\sqrt{2} \text{ ms}^{-1} = v_{max}$

Also, $x = a\sin(\pi t) = a\sin\left(\frac{\pi}{4}\right) = \frac{a}{\sqrt{2}}$

$\Rightarrow \quad \frac{AP}{BP} = \frac{a - x}{a + x} = \frac{a - \frac{a}{\sqrt{2}}}{a + \frac{a}{\sqrt{2}}} = \frac{\sqrt{2} - 1}{\sqrt{2} + 1}$

Hence, (A) and (C) are correct.

5. Time period of spring block system does not depend on the effective value of g. But in case of simple pendulum

$$T \propto \frac{1}{\sqrt{g}}$$

Hence, (B) and (C) are correct.

6. Relative error in measurement of time $= \frac{1\text{ s}}{40\text{ s}} = \frac{1}{40}$

Time period = 2 s

So, error in measurement of time period is

$$\Delta T = 2 \times \frac{1}{40} = \frac{1}{20}\text{ s} = 0.05\text{ s}$$

$$g \propto \frac{1}{T^2}$$

$$\Rightarrow \quad \frac{\Delta g}{g} = \frac{2\Delta T}{T} = \frac{2 \times 1}{40} = \frac{1}{20}$$

$$\Rightarrow \quad \frac{\Delta g}{g} \times 100 = 5\%$$

Hence, (A) and (C) are correct.

7. $\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{16}{1}} = 4\text{ rads}^{-1}$

$$\frac{1}{2}mv_{\text{max}}^2 = 8$$

$$\Rightarrow \quad \frac{1}{2} \times 1 \times (\omega A)^2 = 8$$

$$\Rightarrow \quad \frac{1}{2} \times 1 \times 16 \times A^2 = 8$$

$$\Rightarrow \quad A = 1\text{ m}$$

At half the amplitude $x = \frac{A}{2} = 0.5\text{ m}$, potential energy stored in the spring will be

$$U = \frac{1}{2}Kx^2 = \frac{1}{2} \times 16 \times \left(\frac{1}{2}\right)^2 = 2\text{ J}$$

Only when spring block system is horizontal. At half the amplitude

$$v = \omega\sqrt{A^2 - x^2}$$

$$\Rightarrow \quad v = 4\sqrt{(1)^2 - (0.5)^2} = 2\sqrt{3}\text{ ms}^{-1}$$

$$\Rightarrow \quad KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 1 \times (2\sqrt{3})^2 = 6\text{ J}$$

Hence, (A), (B) and (C) are correct.

8. From the graph, we see that the mean position is at $x = 20$ cm i.e., where the potential energy is minimum.

Also, we know that $U = \frac{1}{2}k(\Delta x)^2$, where Δx is the displacement of the particle form the mean position. Also, from the graph, we see that at $x = 14$ cm, the potential energy is 8 J, so we have

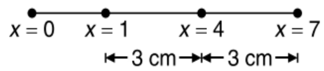
$$8 = \frac{1}{2}k(0.2 - 0.14)^2$$

$$\Rightarrow \quad k = 4444.4\text{ Nm}^{-1}$$

Also, a. is correct, because at the extreme positions, i.e. at $x = 20\text{ cm} \pm 6\text{ cm}$, potential energy is equal to total energy (because kinetic energy at the extreme position is zero).

Hence, (A), (B) and (D) are correct.

9. The motion of the particle is somewhat like the minimum value of x can be

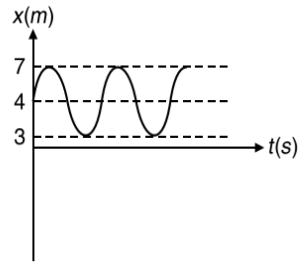


$$4 - 3 = 1\text{ cm}$$

and maximum value of x can be

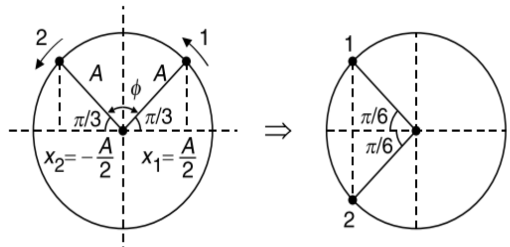
$$4 + 3 = 7\text{ cm}$$

i.e., the particle oscillates simple harmonically about point $x = 4$ cm with amplitude 3 cm. The x-t graph will be as shown.



Hence, (A) and (C) are correct.

10. From the figure we can see that phase difference between them is $\phi = \frac{\pi}{3}$



They will collide when the perpendiculars drawn on the diameter coincide. This will happen when any of the particles rotates an angle $\theta = \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$.

$$\Rightarrow \quad \omega t = \frac{\pi}{2}$$

$$\Rightarrow \quad t = \frac{\pi}{2\omega}$$

Hence, (A) and (C) are correct.

11. The situation is similar as if a block of mass m is suspended from a vertical spring and a constant force mg acts downwards. Therefore, in this case also block will execute SHM with time period

$$T = 2\pi\sqrt{\frac{m}{k}}$$

At compression x, we have

$$F = kx$$

$$\Rightarrow \quad x = \frac{F}{k}$$

This is also the amplitude of oscillations. Hence,

$$A = \frac{F}{k}$$

At mean position speed of the block will be maximum. Applying Work Energy Theorem

$$Fx = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2Fx - kx^2}{m}}$$

Hence, (A), (C) and (D) are correct.

12. Since we know that $\vec{v}$ is either parallel or antiparallel to $\vec{r}$, so $\vec{v}\cdot\vec{r}$ can be positive for $\theta = 0°$ (parallel) or negative for $\theta = 180°$ (anti-parallel). Hence (a) is false.

Also, $\vec{v}\times\vec{r} = \vec{0}$, for both $\theta = 0°$ and $180°$. Hence (c) is correct.

Since, $\vec{F} = -k\vec{r}$, so $\vec{F}$ is anti-parallel to $\vec{r}$. Hence $\vec{F}\cdot\vec{r}$ is always negative and $\vec{F}\times\vec{r}$ will always be zero, hence (b) and (d) are correct.

Hence, (B), (C) and (D) are correct.

13. At $t = 0$, kinetic energy is $\frac{1}{4}$th the maximum kinetic energy i.e., speed of the particle is half the maximum speed, so $v = \frac{\pm A\omega}{2}$ at $t = 0$

$v = \frac{-A\omega}{2}$ in OPTION (A)

$v = \frac{A\omega}{2}$ in OPTION (B)

$v = \frac{-A\omega}{2}$ in OPTION (C)

and $v = \frac{A\omega}{2}$ in OPTION (D)

Therefore, all the OPTIONS are correct.

Hence, (A), (B), (C) and (D) are correct.

14. Restoring force $F = -(\rho A g)x$

$\Rightarrow \quad F \propto -x$

This is just like a spring-block system of force constant

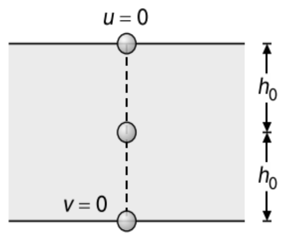
$K = \rho A g$

Hence, (A), (C) and (D) are correct.

15. Net force on the ball will be zero at $\rho = \rho_0$

$\Rightarrow \quad \alpha h_0 = \rho_0$

$\Rightarrow \quad h_0 = \frac{\rho_0}{\alpha}$



i.e., the mean position is at a depth $h_0 = \frac{\rho_0}{\alpha}$

Net force at a depth $(h_0 + x)$ is given by

$F = (\rho - \rho_0)Vg$ {upwards}

$\Rightarrow \quad F = \alpha x V g$ {upwards}

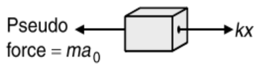
F is proportional to $-x$

Thus, motion of the ball is simple harmonic.

$$h_{max} = 2h_0 = \frac{2\rho_0}{\alpha}$$

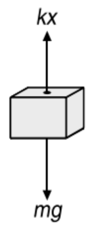
Hence, (A) and (C) are correct.

16. Free body diagram of the truck from non-inertial frame of reference is shown in figure.



This is similar to a situation when a block is suspended from a vertical spring.

Therefore, the block will execute simple harmonically with time period $T = 2\pi\sqrt{\frac{m}{k}}$



Amplitude will be given by

$A = x = \frac{ma_0}{k}$ $\{\because\ ma_0 = kx\}$

Energy of oscillation will be

$$E = \frac{1}{2}kA^2 = \frac{1}{2}k\left(\frac{ma_0}{k}\right)^2 = \frac{m^2a_0^2}{2k}$$

Hence, (A), (B) and (C) are correct.

17. Restoring Force = Extra Upthrust

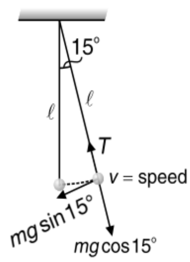
$\Rightarrow \quad F = -Ax\rho g$

$$\Rightarrow \quad a = \frac{F}{m} = -\left(\frac{A\rho g}{m}\right)x$$

$$\Rightarrow \quad T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{m}{A\rho g}}$$

Hence, (A), (C) and (D) are correct.

18. Simple pendulum of length 1 m is called second's pendulum whose time period is $T = 2$ s. But this time period is for small oscillations. In this case angular amplitude is $30°$. Therefore, time period will not be 2 s.



At angular displacement $15°$

$$T - mg\cos(15°) = \frac{mv^2}{\ell}$$

$$\Rightarrow \quad T > mg\cos(15°)$$

and tangential acceleration $a_t = g\sin(15°) =$ rate of change of speed.

Hence, (B) and (C) are correct.

19. Let $x = a\sin\omega t$

Then, $v = \frac{dx}{dt} = a\omega\cos\omega t$

So, potential energy is

$$PE = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2a^2\sin^2\omega t \qquad \{\because k = m\omega^2\}$$

$$\Rightarrow \quad U = U_{avg} = \frac{1}{2}m\omega^2a^2(\sin^2\omega t)_{avg}$$

Since, $(\sin^2\omega t)_{avg} = (\cos^2\omega t)_{avg} = \frac{1}{2}$

$$\Rightarrow \quad U = \frac{1}{4}m\omega^2a^2 \qquad \ldots(1)$$

Similarly, $KE = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2a^2\cos^2\omega t$

$$\Rightarrow \quad K = (KE)_{avg} = \frac{1}{2}m\omega^2a^2(\cos^2\omega t)_{avg}$$

$$\Rightarrow \quad K = \frac{1}{4}m\omega^2a^2 \qquad \ldots(2)$$

From (1) and (2), we get

$$K = U$$

Since total energy (TE) is $TE = PE + KE$

$$\Rightarrow \quad (TE)_{avg} = (PE)_{avg} + (KE)_{avg}$$

$$\Rightarrow \quad E = \frac{1}{2}m\omega^2a^2 = 2U = 2K$$

Hence, (B) and (C) are correct.

20. Given $x = x_0 + a\sin(\omega t)$

$$\Rightarrow \quad v = \frac{dx}{dt} = a\omega\cos(\omega t) \qquad \{\text{for block } A\}$$

at $t = 0$, $x = x_0$ and $v = a\omega$

i.e., block A is at $x = x_0$ (main position) and its velocity is $a\omega$ is positive x-direction. It collides elastically with an identical block B moving towards negative x-direction with velocity v_0. So, the blocks will interchange their velocities.

$$\Rightarrow \quad v_A = v_0 \qquad \{\text{in negative } x\text{-direction}\}$$

and $v_B = a\omega$ {in positive x-direction}

Let A be the new amplitude of block A, then by Law of Conservation of Mechanical Energy, we get

$$\frac{1}{2}mv_A^2 = \frac{1}{2}kA^2$$

$$\Rightarrow \quad A = v_A\sqrt{\frac{m}{k}} = v_0\sqrt{\frac{m}{k}}$$

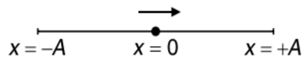
New displacement time equation is

$$x = x_0 - v_0\sqrt{\frac{m}{k}}\sin(\omega t)$$

Hence, (A) and (C) are correct.

21. At 1, acceleration is positive therefore, displacement is negative $(a \propto -x)$.

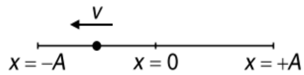
At 2, acceleration of particle is zero but after some time acceleration is negative. Therefore, velocity of particle is positive. This is shown in figure below



At 3, acceleration of particle is maximum. Therefore, potential energy of maximum.

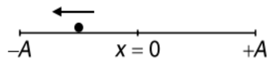
At 4, acceleration of particle is positive and its is increasing in magnitude.

Therefore, speed of particle is decreasing as shown below:



Hence, (A), (B) (C) and (D) are correct.

22. At position 1, velocity of particle is negative but afterwards speed of particle is decreasing. It implies that displacement of particle is negative as shown below



At position 2, velocity of particle is zero, but after some time its velocity is positive i.e., in this position displacement is negative.

At position 3, velocity of particle is positive and still increasing, so acceleration is positive.

At position 4, velocity of particle is maximum i.e., acceleration of particle is zero.

Hence, (B) and (C) are correct.

23. OPTIONS (B) and (C) can be true if we choose zero potential energy at extreme position.

Hence, (A), (B) and (C) are correct.

24. Reading will be maximum when platform accelerates up and minimum when it accelerates down. So,

$$R_{max} = m(g + a\omega^2) \qquad \{\because \text{ maximum acceleration} = a\omega^2\}$$

$$\Rightarrow \quad R_{max} = 60[10 + 8]$$

$$\Rightarrow \quad R_{max} = 1080 \text{ N}$$

$$\Rightarrow \quad \text{Reading} = 108 \text{ kg}$$

Similarly, $R_{min} = m(g = a\omega^2)$

$$\Rightarrow \quad R_{min} = 60(10 - 8)$$

$$\Rightarrow \quad R_{min} = 120 \text{ N}$$

$$\Rightarrow \quad \text{Reading} = 12 \text{ kg}$$

Hence, (A) and (C) are correct.

25. Since $U = 5x(x - 4) = 5x^2 - 20x$

Force acting is

$$F = -\frac{dU}{dx} = -10x + 20$$

Since the force depends on position, so it is not constant. Also, F is linearly related with x and is restoring. So, motion is SHM with mean position i.e., equilibrium position i.e., when $F = 0$ at

CHAPTER 3

$-10x + 20 = 0$

$\Rightarrow \quad x = 2 \text{ m}$

Hence, (B), (C) and (D) are correct.

26. Comparing the given equation with standard velocity-displacement equation of SHM i.e.,

$$v = \omega\sqrt{A^2 - x^2}$$

$$\Rightarrow \quad v^2 = A^2\omega^2 - \omega^2 x^2$$

We observe that $\omega = 3$

$$\Rightarrow \quad \frac{2\pi}{T} = 3$$

$$\Rightarrow \quad T = \frac{2\pi}{3} \text{ units}$$

Similarly, amplitude of oscillations is

$$A = \frac{v_{max}}{\omega} = \sqrt{\frac{A^2\omega^2}{\omega^2}}$$

$$\Rightarrow \quad A = \sqrt{\frac{A^2\omega^2}{\omega^2}} = \sqrt{\frac{144}{9}} = \frac{12}{3} = 4 \text{ unit}$$

Acceleration of the particle at a distance x from the mean position is given by

$$a = \omega^2 x = (9)(3) \qquad \{\because x = 3 \text{ unit}\}$$

$$\Rightarrow \quad a = 27 \text{ unit}$$

Hence, (A), (B) and (C) are correct.

27. Let, $x_1 = -A\cos(\omega t)$ and $x_2 = A\sin(\omega t)$

Equating $x_1 = x_2$ at the point of crossing, we get

$$-A\cos(\omega t) = A\sin(\omega t)$$

$$\Rightarrow \quad \tan(\omega t) = -1$$

$$\Rightarrow \quad \omega t = \frac{3\pi}{4}$$

$$\Rightarrow \quad \left(\frac{2\pi}{T}\right)t = \frac{3\pi}{4}$$

$$\Rightarrow \quad t = \frac{3T}{8}$$

$$\Rightarrow \quad x_2 = A\sin\left(\frac{3\pi}{4}\right) = \frac{A}{\sqrt{2}}$$

Hence, (B) and (D) are correct.

28. Only along y-axis force is restoring in nature. Further oscillations are simple harmonic in nature only for small oscillations. **Hence, (C) and (D) are correct.**

29. $T_1 = 2\pi\sqrt{\frac{m}{k_1}}$ and $T_2 = 2\pi\sqrt{\frac{m}{k_2}}$

$$\Rightarrow \quad T_s = 2\pi\sqrt{\frac{m}{k_s}},\ k_s = \frac{k_1 k_2}{k_1 + k_2}$$

$$\Rightarrow \quad T_p = 2\pi\sqrt{\frac{m}{k_p}},\ k_p = k_1 + k_2$$

Hence, (A), (B) and (C) are correct.

30. Since, $F = -\frac{dU}{dx}$

$$\Rightarrow \quad F = -U_0 a\sin(ax) \qquad \ldots(1)$$

For small oscillations,

$$\sin(ax) \approx ax$$

$$\Rightarrow \quad m\ddot{x} = -U_0 a(ax)$$

$$\Rightarrow \quad \ddot{x} + \left(\frac{U_0 a^2}{m}\right)x = 0$$

Compare with $\ddot{x} + \omega^2 x = 0$, we get

$$\omega = \sqrt{\frac{U_0 a^2}{m}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{U_0 a^2}}$$

The speed of the particle is maximum i.e., kinetic energy is maximum and hence potential energy is minimum i.e., $\frac{dU}{dx} = 0$

From (1), we get $\frac{dU}{dx} = 0$ at $x = 0$

Hence, (B), (C) and (D) are correct.

31. $U_{min} = U_{max} - K_{max} = 20 \text{ J}$

Since, $K_{max} = 40 \text{ J}$

$$\Rightarrow \quad \frac{1}{2}mv_{max}^2 = \frac{1}{2}m(A^2\omega^2) = 40 \text{ J}$$

Since, $m\omega^2 = k$

$$\Rightarrow \quad K_{max} = \frac{1}{2}(m\omega^2)A^2 = \frac{1}{2}kA^2 = 40 \text{ J}$$

$$\Rightarrow \quad \frac{1}{2}k\left(\frac{A}{2}\right)^2 = 10 \text{ J}$$

$$\Rightarrow \quad U_{A/2} = U_{min} + \frac{1}{2}k\left(\frac{A}{2}\right)^2 = 30 \text{ J}$$

$$K_{A/2} = E - U_{A/2} = 60 - 30 = 30 \text{ J}$$

Hence, (A) and (B) are correct.

Reasoning Based Questions

1. From the definition of SHM a particle executes SHM when it goes to and fro about a mean position under the restoring force. Hence, Statement-1 is true.
The earth completes one revolution around the sun after a fixed interval of 1 year. Therefore, it is a periodic motion but not harmonic Statement-2 is also true.
Hence, the correct answer is (B).

2. In initial position, the ball is completely filled with water, CG of the ball is at the centre. When water flows out of the ball the centre of gravity goes below the centre of the ball. So, the effective length of the ball increases, hence the time period of pendulum also increases. After draining out the water continuously and when ball is emptied more than half then centre of gravity rises so that effective length decreases and so the time period of pendulum also decreases. Hence, the time period does not remain constant. We also know that time period of a simple pendulum does not depend upon the mass of bob as is obvious from relation $T = 2\pi\sqrt{\frac{\ell}{g}}$.

Hence, the correct answer is (B).

3. From the relation

$$T = 2\pi\sqrt{\frac{m}{k}}, \text{ where } k \text{ is the spring constant.}$$

$$\Rightarrow \quad T \propto \frac{1}{\sqrt{k}} \qquad \ldots(1)$$

As we know that the spring constant of hard spring is large in comparison to that of soft spring, therefore from equation (1) the time period will be less for hard spring in comparison to that of soft spring.

Hence, the correct answer is (B).

4. From the relation, potential energy of SHM is given as

$$\text{PE} = \frac{1}{2}m\omega^2 y^2$$

and $\text{KE} = \frac{1}{2}m\omega^2\left(A^2 - y^2\right)$

where y is the displacement and A is the amplitude. By the condition KE = PE,

$$\frac{1}{2}m\omega^2\left(A^2 - y^2\right) = \frac{1}{2}m\omega^2 y^2$$

$$\Rightarrow \quad m\omega^2 y^2 = \frac{1}{2}m\omega^2 A^2$$

$$\Rightarrow \quad y = \frac{A}{\sqrt{2}} \qquad \ldots(1)$$

Also, we know that total energy is

$$E = \text{KE} + \text{PE}$$

Hence, if PE is maximum then KE will be zero.

Hence, the correct answer is (B).

5. As velocity, $\frac{dy}{dt} = \omega\sqrt{A^2 - y^2}$

and acceleration, $\frac{d^2y}{dt^2} = -\omega^2 y$

When $y = 0$ i.e., at mean position

Velocity, $\frac{dy}{dt} = \omega A$ (maximum)

and acceleration, $\frac{d^2y}{dt^2} = 0$ (minimum)

When $y = \pm A$, i.e., at extreme position

$$v = \frac{dy}{dt} = 0 \text{ (minimum)}$$

and acceleration, $\frac{d^2y}{dt^2} = \mp\omega^2 A$ (maximum)

Hence, acceleration of particle executing SHM is zero (where velocity is maximum) at mean position and maximum at extreme position where velocity is minimum.

Now, let $y = a\sin(\omega t + \phi)$

Then velocity

$$\frac{dy}{dt} = a\omega\cos(\omega t + \phi) = a\omega\sin\left\{\left(\omega t + \frac{\phi}{2}\right) + \frac{\pi}{2}\right\}$$

Thus, displacement and velocity of SHM differ by a phase of $\frac{\pi}{2}$.

Hence, the correct answer is (D).

6. From the relation of time period

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\Rightarrow \quad T \propto \frac{1}{\sqrt{g}}$$

When the satellite is orbiting the earth, the value of g inside it is zero, the time period of pendulum in a satellite will be infinity and it is also clear that time period of pendulum is inversely proportional to square root of acceleration due to gravity g.

Hence, the correct answer is (A).

7. In SHM, the body does a to and fro motion about a fixed point called the mean position. At the extreme position, velocity of body is zero but acceleration is maximum which tends it to move back to mean position. Thus, acceleration is always directed towards mean position.

Also, in SHM, $a \propto -x$ which states that acceleration is in opposite direction of displacement i.e., towards mean position.

Hence, the correct answer is (A).

8. As $T \propto \sqrt{\ell}$

$$\frac{\Delta T}{T} = \frac{1}{2}\left(\frac{\Delta\ell}{\ell}\right) = \frac{1}{2} \times 4\% = 2\%$$

As length increases, the time period will increase by 2% it will not decrease.

Hence, the correct answer is (D).

9. If length of the pendulum is large, g no longer remains vertical but will be directed towards the centre of the earth and expression of the time period is given by

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l} + \frac{1}{R}\right)}}$$

Here, R is the radius of earth.

When $\ell \to \infty$, $\frac{1}{\ell} \to 0$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{R}{g}} = 84.6 \text{ min}$$

In general, time period of a simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \quad T \propto \sqrt{l}$$

Hence, the correct answer is (D).

10. In simple harmonic motion,

$$v = \omega\sqrt{(a^2 - y^2)}$$

As y changes, velocity v will also change. So, simple harmonic motion is not a uniform motion.
Simple harmonic motion may be defined as the projection of uniform circular motion along any one or two mutually perpendicular diameters of the circle.
Hence, the correct answer is (B).

11. From the relations,

$$PE = \frac{1}{2}m\omega^2y^2 \quad ...(1)$$

and $$KE = \frac{1}{2}m\omega^2(A^2 - y^2) \quad ...(2)$$

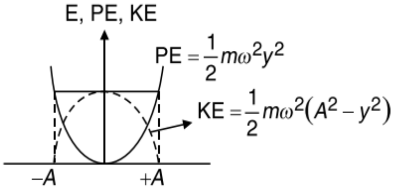


Figure shows the variation of total energy (E), potential energy (PE) and kinetic energy (KE) with displacement y.
Thus, the graph is a parabola.
Statement-2 says "PE and KE do not vary linearly" i.e., this portion is correct in the manner that KE and PE vary non-linearly but then how do they vary is missing. So, Statement-2 is not the correct explanation to Statement-1.
Hence, the correct answer is (B).

12. The period of the liquid executing SHM in a U-tube does not depend upon the density of the liquid. Therefore, time period will be the same, when mercury is filled up to the same height as the water in the U-tube. In fact, the period of the liquid is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where, L = Length to which the liquid is filled in one limb of the U-tube.
Now, as the pendulum oscillates, it drags air along with it. Therefore, its kinetic energy is dissipated in overcoming viscous drag due to air and hence, its amplitude goes on decreasing.
Hence, the correct answer is (D).

13. In nature, many types of dissipating forces such as friction, air resistance etc., are operating. Therefore, as the oscillator oscillates, a part of its energy is used up in overcoming these dissipating forces. Such oscillations are damped oscillations.
Hence, the correct answer is (A).

14. When the earth contracts, its moment of inertia decreases. Therefore, from the relation

$$I\omega = L$$

$$\Rightarrow \quad I\omega = \text{constant}$$

with decrease of moment of inertia, angular velocity of earth increases. Thus, time period $T = \frac{2\pi}{\omega}$ will decrease.

Hence, duration of the day will decrease.
Hence, the correct answer is (C).

15. The ball will not go out of the other end of the hole, because it will execute SHM. On reaching the other end of the hole, its velocity become zero and acceleration of ball will be maximum and will be directed towards the centre of earth. The ball will have a time period of 84.6 minute and hence will be seen at the other end at half the time period i.e., 42.3 minute.
Hence, the correct answer is (A).

16. While executing SHM, from relation,

$$v = \omega\sqrt{A^2 - y^2}$$

or $$v^2 = \omega^2(A^2 - y^2)$$

$$v^2 = \omega^2A^2 - \omega^2y^2$$

Dividing both side by ω^2A^2

$$\frac{v^2}{\omega^2A^2} + \frac{y^2}{A^2} = 1 \quad ...(1)$$

It is understood that Equation (1) represents the equation of an ellipse so the graph between v (velocity) and y (displacement) is not a parabola.
Again, from Equation (1), as displacement (y) increases, the velocity decreases, becomes zero at maximum displacement and again increases with decrease in displacement. Thus, in SHM, the velocity does not change uniformly.
Hence, the correct answer is (D).

17. The total energy of the harmonic oscillator is given by

$$E = \frac{1}{2}m\omega^2A^2$$

$$\Rightarrow \quad E \propto A^2$$

Hence, $$\frac{E_2}{E_1} = \left(\frac{2A}{A}\right)^2$$

$$\Rightarrow \quad E_2 = 4E_1 = 4E$$

Also, total energy must become 4 times when amplitude is doubled.
Hence, the correct answer is (D).

18. Time period of a liquid column

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\times\frac{22}{7}\times\sqrt{\frac{0.3}{9.8}} = 1.1\text{ s}$$

Hence, the correct answer is (A).

19. From the relation,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \quad T^2 = \frac{4\pi^2 l}{g}$$

$\Rightarrow \quad T^2 \propto \dfrac{1}{g}$

Therefore, the graph between T^2 and g is hyperbola.

Hence, the correct answer is (C).

20. When the man falls from the top of a tower, he will be in the state of weightlessness. A spring wound watch runs on the basis of spring action i.e., on the basis of the potential energy stored in the wound spring. Since, acceleration due to gravity does not play any role, the watch will give correct time, when the man falls from the top of a tower.
Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. Given: Mass of each block A and B, $m = 0.1$ kg

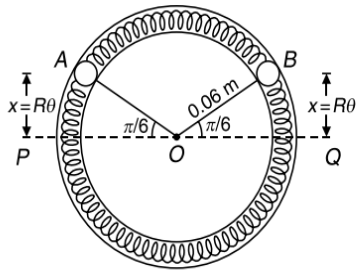
Radius of circle, $R = 0.06\,m$

Natural length of spring $\ell_o = 0.06\pi = \pi R$ (half circle) and spring constant, $k = 0.1\ \text{Nm}^{-1}$.

In the stretched position elongation in each spring

$x = R\theta$

Let draw FBD of A:
Spring in lower side is stretched by $2x$ and on upper side compressed by $2x$. Therefore, each spring will exert a force $2kx$ on each block.
Hence, a restoring force, $F = -4kx$ will act on A in the direction shown in Figure.



Restoring force of this force about origin

$\tau = -F.R = -(4kx)R = -(4kR\theta)R$

$\Rightarrow \quad \tau = -4kR^2\theta \qquad \ldots(1)$

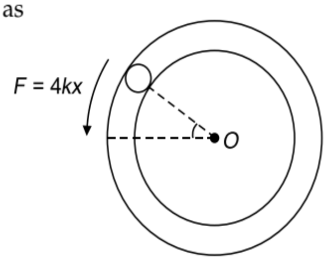
Since $\tau \propto -\theta$, each ball executes angular SHM about origin O.

Equation (1), can be written as

$I\alpha = -4kR^2\theta$

$\Rightarrow \quad (mR^2)\alpha = -4kR^2\theta$

$\Rightarrow \quad \alpha = -\left(\dfrac{4k}{m}\right)\theta$



So, frequency of oscillation

$$f = \frac{1}{2\pi}\sqrt{\left|\frac{\text{Acceleration}}{\text{Displacement}}\right|} = \frac{1}{2\pi}\sqrt{\left|\frac{\alpha}{\theta}\right|}$$

$\Rightarrow \quad f = \dfrac{1}{2\pi}\sqrt{\dfrac{4k}{m}}$

Substituting the values, we have

$$f = \frac{1}{2\pi}\sqrt{\frac{4\times 0.1}{0.1}} = \frac{1}{\pi}\ \text{Hz}$$

Hence, the correct answer is (C).

2. In stretched position, potential energy of the system is

$$\text{PE} = 2\left\{\frac{1}{2}k\right\}(2x)^2 = 4kx^2$$

And in mean position, both the blocks have kinetic energy only. Hence,

$$\text{KE} = 2\left\{\frac{1}{2}mv^2\right\} = mv^2$$

From energy conservation, PE = KE

$\therefore \quad 4kx^2 = mv^2$

$\therefore \quad v = 2x\sqrt{\dfrac{k}{m}} = 2R\theta\sqrt{\dfrac{k}{m}}$

Substituting the values

$v = 2(0.06)(\pi/6)\sqrt{\dfrac{0.1}{0.1}}$

or $\quad v = 0.0628\ \text{ms}^{-1}$

Hence, the correct answer is (B).

3. Total energy of the system is E equal to potential energy in stretched position.
or $\quad$ KE in mean position

$\Rightarrow \quad E = mv^2 = (0.1)(0.0628)^2\ \text{J}$

$\Rightarrow \quad E = 39\times 10^{-4}\ \text{J}$

Hence, the correct answer is (C).

4. $F = -4x + 8$

Let us write $x = (X + 2)$

Then, $F = -4X$

This is the equation of SHM.

Further $F = 0$ as $X = 0$ or $x = 2$ m

Hence, mean position is $x = 2$ m

Hence, the correct answer is (D).

5. $F = -4X$

$\Rightarrow \quad k = 4\ \text{Nm}^{-1}$

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2\times 18}{4}} = 3\ \text{m}$$

Mean position of the particle is $x = 2$ m

Hence, the extreme points are 5 m and –1 m

Hence, the correct answer is (B).

6. $T_0 = 2\pi\sqrt{\dfrac{m}{2k}}$

$T = 2\pi\sqrt{\dfrac{m}{k}} = \sqrt{2}T_0$

Hence, the correct answer is (B).

CHAPTER 3

7. Since, the velocity of the block remains unchanged, so

$$\omega_0\sqrt{A_0^2-\frac{A_0^2}{4}}=\omega\sqrt{A^2-\frac{A_0^2}{4}}$$

$$\Rightarrow \sqrt{\frac{2k}{m}}\sqrt{\frac{3A_0^2}{4}}=\sqrt{\frac{k}{m}}\sqrt{A^2-\frac{A_0^2}{4}}$$

$$\Rightarrow A=\frac{\sqrt{7}A_0}{2}$$

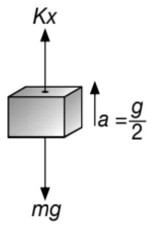
Hence, the correct answer is (C).

8. $$v=A\omega=\sqrt{\frac{k}{m}}\frac{\sqrt{7}A_0}{2}=\frac{\sqrt{7}A_0}{2}\times\frac{\sqrt{2}\pi}{T_0}$$

$$\Rightarrow v=A\omega=\frac{\sqrt{14}\pi A_0}{2T_0}$$

Hence, the correct answer is (D).

9. In mean position of oscillations, spring will be elongated. Let x be the extension, then

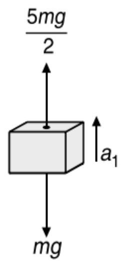


$$kx-mg=ma=\frac{mg}{2}$$

$$\Rightarrow x=\frac{3mg}{2k}$$

Hence, the correct answer is (D).

10. **For a_1:**

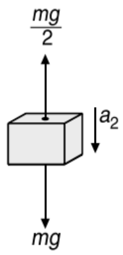


$$a_1=\frac{\frac{5mg}{2}-mg}{m}=\frac{3}{2}g$$

Amplitude $A=\frac{5mg}{2k}-\frac{3mg}{2k}=\frac{mg}{k}$

So, minimum extension $=\frac{3mg}{2k}-\frac{mg}{k}=\frac{mg}{2k}$

For a_2:



$$a_2=\frac{mg-\frac{mg}{2}}{m}=\frac{g}{2}$$

Hence, the correct answer is (B).

11. $p^2\propto x$

Hence, the correct answer is (D).

12. $p_1=2p_2$

$$\Rightarrow \frac{E_2}{E_1}=\frac{p_2^2}{p_1^2}=\frac{1}{4}$$

$$\Rightarrow E_1=4E_2$$

Hence, the correct answer is (C).

13. Due to the viscous drag offered by the water, the momentum will not be same but less when it returns.

Hence, the correct answer is (B).

14. Since the particle starts from the extreme position, so

$$x=a\cos\omega t$$

$$\Rightarrow \frac{a}{2}=a\cos\omega t$$

$$\Rightarrow \cos\omega t=\frac{1}{2}=\cos\frac{\pi}{3}$$

$$\Rightarrow t=\frac{T}{6}=\frac{0.60}{6}=0.10\text{ s}$$

Hence, the correct answer is (C).

15. $$\langle v\rangle=\frac{a/2}{T/6}=\frac{3a}{T}$$

$$\Rightarrow \langle v\rangle=\frac{3\times10\times10^{-2}}{0.60}\text{ ms}^{-1}=0.50\text{ ms}^{-1}$$

Hence, the correct answer is (A).

16. Since the particle starts from the mean position, so

$$x=a\sin\omega t$$

$$\Rightarrow \frac{a}{2}=a\sin\omega t$$

$$\Rightarrow \frac{1}{2}=\sin\omega t\sin\frac{\pi}{6}=\sin\omega t$$

$$\Rightarrow \frac{\pi}{6}=\frac{2\pi}{T}t$$

$$\Rightarrow t=\frac{T}{12}=\frac{0.60}{12}\text{ s}=0.05\text{ s}$$

Hence, the correct answer is (D).

17. $$\langle v\rangle=\frac{a/2}{T/12}=\frac{6a}{T}$$

$$\Rightarrow \langle v\rangle=\frac{6\times10\times10^{-2}}{0.60}\text{ ms}^{-1}=1\text{ ms}^{-1}$$

Hence, the correct answer is (B).

18. Since $F=-4x$, so $k=4\text{ Nm}^{-1}$

$$\Rightarrow a=\frac{F}{m}=-8x$$

$$\Rightarrow \omega=\sqrt{8}=2\sqrt{2}\text{ rads}^{-1}$$

Since the total energy of the particle is 10 J and the amplitude of the oscillations is 2 m, so the potential energy of the particle at the specified instance is

$$U=\frac{1}{2}kA^2=\frac{1}{2}\times4\times4=8\text{ J}$$

So, potential energy at mean position is

$$U=E-\frac{1}{2}kA^2=2\text{ J}$$

Hence, the correct answer is (D).

19. At time $t=0$, $a=16\text{ ms}^{-2}$, so $a=-\omega^2x=-8x$

$$\Rightarrow x=A=2\text{ m}$$

So, the equation of motion is $x = 2\cos\left(2\sqrt{2}t\right)$

Hence, the correct answer is (D).

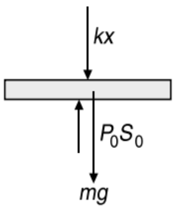
20. $\frac{1}{2}kx^2 = \frac{1}{2}\times 4\times 1 = 2\text{ J}$

Potential energy at mean position is 2 J

Therefore, $U = (2+2) = 4\text{ J}$ and $K = E - U = 6\text{ J}$

Hence, the correct answer is (D).

21.



$kx = P_0S_0 - mg$

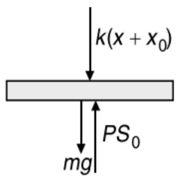
$\Rightarrow \quad x = \frac{P_0S_0 - mg}{k}$

Hence, the correct answer is (C).

22. From the diagram, we observe that

$$k(x + x_0) + mg - PS_0 = ma$$

For an adiabatic process, we have



$$PV^\gamma = \text{constant}$$

$$\Rightarrow \quad P_0(\ell S_0)^\gamma = P((\ell + x)S_0)^\gamma$$

$$\Rightarrow \quad P = P_0\left(1 - \frac{\gamma x}{\ell}\right)$$

$$\Rightarrow \quad kx + kx_0 + mg - P_0S_0 + \frac{P_0\gamma x S_0}{\ell} = ma$$

$$\Rightarrow \quad \left(k + \frac{\gamma P_0 S_0}{\ell}\right)x = ma$$

$$\Rightarrow \quad a = \underbrace{\left(\frac{k\ell + \gamma P_0 S_0}{m\ell}\right)}_{\omega^2} x$$

$$\Rightarrow \quad \omega = \sqrt{\frac{k\ell + \gamma P_0 S_0}{m\ell}}$$

Hence, the correct answer is (D).

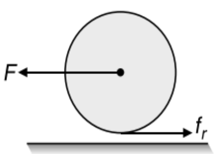
23. $\omega = \sqrt{\frac{\gamma P_1 S_0}{m\ell_1}}$

So only restoring force is due to pressure.

Hence, the correct answer is (A).

24. $F - f_r = Ma$

Since, $\tau = f_r R = I\alpha$



$\Rightarrow \quad f_r = \frac{Ia}{R^2}$

Solving these two, we get

$$a = \frac{F}{M + \frac{I}{r^2}}$$

Also, $F = -2kx$

$$\Rightarrow \quad a = \frac{-2kx}{M + \frac{I}{r^2}}$$

$$\Rightarrow \quad f_r = \frac{I}{R^2} \times \left(\frac{-2kx}{M + \frac{I}{r^2}}\right)$$

For disc $I = \frac{1}{2}MR^2$

$$\Rightarrow \quad f_r = -\frac{4kx}{3}$$

Hence, the correct answer is (D).

25. $a = -\frac{2kx}{M + \frac{I}{r^2}}$

Since, $a = -\omega^2 x$, which is comparable to the characteristic equation of SHM, so

$$\omega = \sqrt{\frac{|a|}{|x|}}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2k}{M + \frac{I}{r^2}}} = \sqrt{\frac{4k}{3M}}$$

Hence, the correct answer is (B).

26. $f_r = \frac{Ia}{R^2} \le \mu Mg$

and $\frac{Iv_0\omega}{R^2} \le \mu mg$ $\qquad \{\because a_{max} = v_0\omega\}$

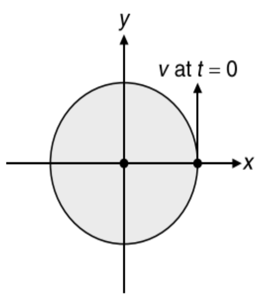
$$\Rightarrow \quad \frac{MR^2}{2}\frac{v_0\omega}{R^2} \le \mu mg$$

$$\Rightarrow \quad v_0 \le 2\mu g\sqrt{\frac{3M}{4k}}$$

$$\Rightarrow \quad v_0 \le \mu g\sqrt{\frac{3M}{k}}$$

Hence, the correct answer is (C).

27.



At any time t, particle is at $\frac{T_0}{\sqrt{2}}$ and $\omega r = v$

$$v_x = -v\sin\omega t$$

$$v_y = v\cos\omega t$$

Hence, the correct answer is (D).

CHAPTER 3

28. $F_y = -F_C \sin(\omega t)$

$\Rightarrow \quad F_y = -\dfrac{mv^2}{r}\sin(\omega t)$

Hence, the correct answer is (B).

29. $\theta = \omega t$

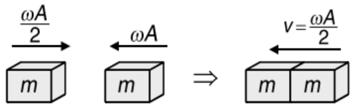
$\theta = \dfrac{v}{r}t$

Hence, the correct answer is (A).

30. They will collide at their mean positions because time period of both are same and that is $2\pi\sqrt{\dfrac{m}{K}}$. After collision combined mass is 2 m and $K_{eff} = 2K$. Hence, time period remains unchanged.

Hence, the correct answer is (C).

31. Applying Law of Conservation of Linear Momentum, the velocity of combined mass just after collision is $v = \dfrac{\omega A}{4}$.



Before collision Just after collision

Since, this is the velocity at mean position, so

$v = \omega' A'$

$\Rightarrow \quad \dfrac{\omega A}{4} = \omega' A'$

$\Rightarrow \quad A' = \dfrac{A}{4}$ because $\omega' = \omega = \sqrt{\dfrac{K}{m}} = \sqrt{\dfrac{2K}{2m}}$

Hence, the correct answer is (A).

32. $E = \dfrac{1}{2}(2m)v^2 = \dfrac{1}{2}(2m)\left(\dfrac{\omega A}{4}\right)^2$

$\Rightarrow \quad E = \dfrac{m\omega^2 A^2}{16} = \dfrac{kA^2}{16} \qquad \left\{\because\ \omega^2 = \dfrac{k}{m} \text{ or } \dfrac{2k}{2m}\right\}$

$\Rightarrow \quad E = \dfrac{1}{2}(2k)\left(\dfrac{A}{4}\right)^2 = \dfrac{kA^2}{16}$

Hence, the correct answer is (D).

33. $x = a\cos(\omega t)$

$\Rightarrow \quad \dfrac{a}{2} = a\cos(\omega t)$

$\Rightarrow \quad \cos^{-1}\left(\dfrac{1}{2}\right) = \dfrac{2\pi}{T}t$

$\Rightarrow \quad t = \dfrac{T}{6} = \dfrac{0.6}{6} = 0.1 \text{ s}$

Hence, the correct answer is (C).

34. $v_{average} = \dfrac{\text{total displacement}}{\text{total time}} = \dfrac{\frac{a}{2}}{t} = \dfrac{5 \text{ cm}}{0.1} = 0.5 \text{ ms}^{-1}$

Hence, the correct answer is (A).

35. $\omega = \sqrt{\dfrac{K}{m}} = \sqrt{\dfrac{200}{2}}$

$\omega = 10 \text{ sec}^{-1}$

Hence, the correct answer is (B).

36. $x = A\sin(\omega t + \delta)$ ...(1)

$v = A\omega\cos(\omega t + \delta)$ ...(2)

From (1), $0.05 = A\sin\delta$

From (2), $1 = A\omega\cos\delta$

$0.1 = A\cos\delta \qquad \{\because\ \omega = 10\}$

$\Rightarrow \quad 0.1^2 + 0.05^2 = A^2$

$\Rightarrow \quad A = \sqrt{\dfrac{1}{10^2} + \dfrac{25}{100^2}}$

$\Rightarrow \quad A = \dfrac{1}{100}\sqrt{125} = 0.112 \text{ m}$

$\Rightarrow \quad \sin\delta = \dfrac{0.05}{0.112}$

$\Rightarrow \quad \delta = \sin^{-1}(0.446)$

Hence, the correct answer is (C).

37. $(10t + \delta) = \dfrac{\pi}{2}$

$\Rightarrow \quad 10t = \dfrac{\pi}{2} - 8$

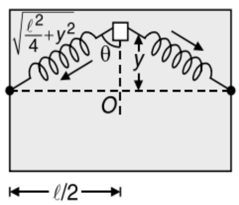
$\Rightarrow \quad t = \dfrac{\pi}{20} - \dfrac{\sin^{-1}(0.446)}{10}$

$\Rightarrow \quad t = 0.157 - \dfrac{2\pi}{360} \times 26.5 \times \dfrac{1}{10}$

$\Rightarrow \quad t = 0.157 - 0.0462 \approx 0.11 \text{ s}$

Hence, the correct answer is (A).

38. Extension in spring, $\sqrt{y^2 + \dfrac{\ell^2}{4}} - \dfrac{\ell}{2}$



So, spring force due to one spring is

$F = k\left(\sqrt{\dfrac{\ell^2}{4} + y^2} - \dfrac{\ell}{2}\right)$

Net spring force due to both the springs is

$F_{net} = 2F\cos\theta$ along negative Y-axis

$F_{net} = -2k\left(\sqrt{\dfrac{\ell^2}{4} + y^2} - \dfrac{\ell}{2}\right)\left(\dfrac{y}{\sqrt{\dfrac{\ell^2}{4} + y^2}}\right)$

$\Rightarrow \quad F_{net} = 2ky\left(\dfrac{\frac{\ell}{2}}{\sqrt{\dfrac{\ell^2}{4} + y^2}} - 1\right)\hat{j}$

Hence, the correct answer is (C).

39. Since, $U = \frac{1}{2}k_{\text{net}}(\Delta y)^2$

where $k_{\text{net}} = 2k$ and $\Delta y = \sqrt{y^2 + \frac{\ell^2}{4}} - \frac{\ell}{2}$

Hence, the correct answer is (B).

40. Extension in the spring is

$$x = \sqrt{5^2 + (\sqrt{11})^2} - 5 = 1 \text{ m}$$

Then by law of conservation of energy, we have

$$2\left[\frac{1}{2}(100)(1)^2\right] = \frac{1}{2}(2)v^2$$

$\Rightarrow \quad v = 10 \text{ ms}^{-1}$

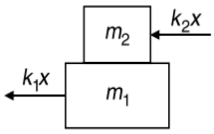
Hence, the correct answer is (C).

41. Since both the blocks moves together and springs are in parallel, so we have

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m_1 + m_2}}$$

Hence, the correct answer is (B).

42. As springs are in parallel, so we have



$$a = \frac{F_{\text{net}}}{\text{Total mass}} = \frac{(k_1 + k_2)x}{m_1 + m_2}$$

We can also answer the same question by taking help from the free body diagram as shown in figure. So, we have

$$a = \frac{F_{\text{net}}}{\text{Total mass}} = \left(\frac{k_1 + k_2}{m_1 + m_2}\right)x$$

Hence, the correct answer is (C).

43. Frictional force on m_2 will act in direction of displacement if

$$k_2 x > m_2 a$$

$\Rightarrow \quad k_2 x > m_2\left(\frac{k_1 + k_2}{m_1 + m_2}\right)x$

$\Rightarrow \quad \frac{m_1}{m_2} > \frac{k_1}{k_2}$

Hence, the correct answer is (A).

44. Again, if amplitude of oscillation is A and $a_{\max}$ is the maximum acceleration, then we have

$$a_{\max} = A\omega^2$$

Now, for the said condition to be met, we have

$$k_2 A - f = m_2(\omega^2 A)$$

$\Rightarrow \quad f = A(k_2 - m_2\omega^2)$

$\Rightarrow \quad A_{\max} = \left(\frac{\mu m_2 g}{k_2 - m_2\omega^2}\right)$

where $\omega^2 = \frac{k_1 + k_2}{m_1 + m_2}$

Hence, the correct answer is (A).

45. Particle 1 is oscillating between $(8+3)$ m and $(8-3)$ m i.e., between 11 m and 5 m. While the particle 2 is oscillating between 4 m and –4 m so, they will not collide.

Hence, the correct answer is (D).

46. At time $t = 0$, x_1 is at +8 m and moving towards positive x-axis and x_2 is at +4 m and is moving towards negative x-axis. At this time they are at shortest distance. Next time they will be at minimum distance after time, $t = T = \frac{2\pi}{\omega} = 2$ s. Time period of both is same.

Hence, the correct answer is (B).

47. Maximum KE = Total Energy

Total energy $= 20 + 20 + (5-2)^2$

$\Rightarrow \quad E = 40 + 9 = 49$ J

Hence, the correct answer is (A).

48. $F = -\frac{dU}{dx} = -2(x-2) = 0$

$\Rightarrow \quad x = 2$ m

Hence, the correct answer is (A).

49. $F = 2x - 4 = 2x$

$\Rightarrow \quad F = \omega^2 x$

So, $\omega = \sqrt{2}$

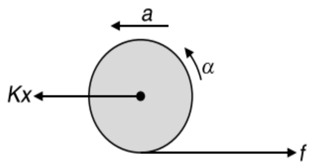
Hence, the correct answer is (A).

50. The forces on the disc at displacement x of the spring are shown in Figure.

For pure rolling to take place, we have

$$a = R\alpha$$

$\Rightarrow \quad \frac{kx - f}{m} = \frac{R(fR)}{\frac{1}{2}mR^2} = \frac{2f}{m}$



$\Rightarrow \quad f = \left(\frac{k}{3}\right)x$

The cylinder is starting from $x = A$, so the x-t equation would be

$$x = A\cos(\omega t)$$

$\Rightarrow \quad f = \frac{kA}{3}\cos(\omega t)$

$\Rightarrow \quad f$-t curve will be a cosine curve.

Hence, the correct answer is (C).

51. Energy of oscillation, $E = \frac{1}{2}kA^2 = \frac{1}{2} \times 10 \times 4 = 20$ J

At mean position this is totally kinetic energy (translational + rotational). In case of pure rolling ratio of rotational and translational kinetic energy is $\frac{1}{2}$ in case of disc.

Hence, $K_T = \frac{1}{2}mv^2 = \frac{2}{3} \times 20 = \frac{40}{3}$

$\Rightarrow \quad v^2 = \frac{80}{3 \times 2} = \frac{40}{3} \text{ m}^2\text{s}^{-2}$

$\Rightarrow \quad v = \sqrt{\frac{40}{3}} \text{ ms}^{-1}$

Hence, the correct answer is (A).

52. $x = a\sin^2\left(\omega t - \frac{\pi}{4}\right)$

$$x = \frac{a}{2}\left[1 - \cos\left\{2\left(\omega t - \frac{\pi}{4}\right)\right\}\right]$$

CHAPTER 3

$\Rightarrow$ Amplitude is $\frac{a}{2}$

Hence, the correct answer is (B).

53. $x=\frac{a}{2}\left[1-\cos\left(2\omega t-\frac{\pi}{2}\right)\right]$

$\Rightarrow$ $2\omega=\frac{2\pi}{T}$

$\Rightarrow$ $T=\frac{\pi}{\omega}$

Hence, the correct answer is (A).

54. Conceptual

Hence, the correct answer is (A).

55. $x=a\sin^2\left(\omega t-\frac{\pi}{4}\right)$ ...(1)

The plot of x versus ωt follows from equation (1)

Differentiating equation (1)

$$\frac{dx}{dt}=2a\omega\sin\left(\omega t-\frac{\pi}{4}\right)\cos\left(\omega t-\frac{\pi}{4}\right) \quad ...(2)$$

Again, $x=a\sin^2\left(\omega t-\frac{\pi}{4}\right)$

$$ax=a^2\sin^2\left(\omega t-\frac{\pi}{4}\right)$$

$\Rightarrow$ $\sqrt{ax}=a\sin\left(\omega t-\frac{\pi}{4}\right)$

$\Rightarrow$ $\sin\left(\omega t-\frac{\pi}{4}\right)=\sqrt{\frac{x}{a}}$

Also, $\cos\left(\omega t-\frac{\pi}{4}\right)=\sqrt{1-\frac{x}{a}}$

From equation (2),

$$v_x=2a\omega\sqrt{\frac{x}{a}}\sqrt{1-\frac{x}{a}}$$

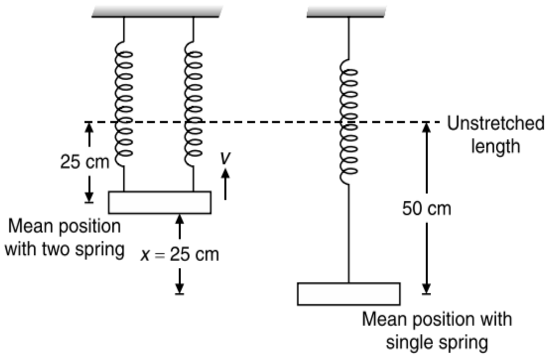
$\Rightarrow$ $v_x^2=4a^2\omega^2\frac{x}{a}\left(1-\frac{x}{a}\right)$

$\Rightarrow$ $v_x^2=4\omega^2x(a-x)$

The plot of v_x versus x follows from this equation.

Hence, the correct answer is (B).

56. Let x_1 be the elongation in equilibrium with two springs and x_2 with single spring.



Then, $2kx_1=mg$

$\Rightarrow$ $(2)(10)(x_1)=\frac{1}{2}(10)$

$\Rightarrow$ $x_1=25$ cm

Further, $kx_2=mg$

$\Rightarrow$ $(10)(x_2)=\frac{1}{2}(10)$

$\Rightarrow$ $x_2=50$ cm

The spring breaks when stretched by 25 cm, so the speed of the block at this instant is

$$v=\omega A=\left(\sqrt{\frac{2k}{m}}\right)(0.25)$$

$\Rightarrow$ $v=\left(\sqrt{\frac{20}{1/2}}\right)(0.25)=\frac{\sqrt{10}}{2}$ ms^{-1}

For single mass, we have

$$\omega=\sqrt{\frac{k}{m}}=\sqrt{20}\text{ rads}^{-1}$$

Also, the block has the above velocity when it is at a displacement $x=25$ cm $=0.25$ m from its mean position. Hence, from the equation

$$v=\omega\sqrt{A^2-x^2}$$

$\Rightarrow$ $v^2=\omega^2(A^2-x^2)$, we have

$\Rightarrow$ $\left(\frac{\sqrt{10}}{2}\right)^2=20\left(A_1^2-(0.25)^2\right)$

$\Rightarrow$ $A_1=0.433$ m

$\Rightarrow$ $A_1=43.3$ cm

Hence, the correct answer is (C).

57. Let x_3 be the extension with two springs and $\frac{m}{2}$ be the mass.

Then, $2kx_3=\frac{m}{2}g$

$\Rightarrow$ $2\times10\times x_3=\frac{1}{4}\times10$

$\Rightarrow$ $x_3=\frac{1}{8}$ m $=12.5$ cm

$\Rightarrow$ $\omega=\sqrt{\frac{2k}{m/2}}=\sqrt{\frac{20}{1/4}}=\sqrt{80}\text{ rads}^{-1}$

Since, $v^2=\omega^2(A^2-x^2)$

$\Rightarrow$ $\left(\frac{\sqrt{10}}{2}\right)^2=80\left(A_2^2-(0.125)^2\right)$

$\Rightarrow$ $A_2=21.6$ cm

Hence, the correct answer is (C).

Matrix Match/Column Match Type Questions

1. A $\rightarrow$ (q); B $\rightarrow$ (r); C $\rightarrow$ (s); D $\rightarrow$ (t)

The given equation, $y=A\sin(\omega t)+A\sin\left(\omega t+\frac{2\pi}{3}\right)$ can also be written as.

$$y=2A\sin\left(\omega t+\frac{\pi}{3}\right)\cdot\cos\left(\frac{\pi}{3}\right)$$

$$y = A\sin\left(\omega t + \frac{\pi}{3}\right)$$

Now, we can see that this is SHM with amplitude A and initial phase $\frac{\pi}{3}$.

$$v = A\omega\cos\left(\omega t + \frac{\pi}{3}\right)$$

$\Rightarrow \quad v_{max} = A\omega$

2. A → (r); B → (p); C → (q); D → (s)

Frequency of oscillation of kinetic energy is $2f$.

Since, $\omega = 12\pi$

$\Rightarrow \quad 2\pi f = 12\pi$

$\Rightarrow \quad 2f = 12$

$$v = \frac{dx}{dt} = 12\pi\cos(12\pi t)$$

So, v is MAXIMUM, when $12\pi t = 0, \pi, 2\pi, 3\pi, \ldots$

$\Rightarrow \quad 12\pi t = \pi$

$\Rightarrow \quad t = \frac{1}{12}$ s

$$U_{max} = \frac{1}{2}kA^2 = \frac{1}{2}(m\omega^2)A^2$$

$\Rightarrow \quad U_{max} = \frac{1}{2}\left(\frac{1}{4}\right)(144\pi^2)(1)^2 = 18\pi^2$

Also, $k = m\omega^2$

$\Rightarrow \quad k = \left(\frac{1}{4}\right)(144\pi^2) = 36\pi^2$

3. A → (r); B → (q); C → (p); D → (q)

Time period is given by

$$T = 2\pi\sqrt{\frac{m}{k}} = \frac{\pi}{20}\text{ s}$$

Displacement of particle is given by (taking mean position as origin) is

$$x = -A\cos(\omega t) = -10\cos\left(\frac{\pi t}{20}\right)$$

Kinetic Energy is

$$K = \frac{1}{2}m\omega^2A^2\sin^2(\omega t) = K_{max}\sin^2\left(\frac{\pi t}{20}\right)$$

Potential Energy is

$$U = \frac{1}{2}m\omega^2A^2\cos^2(\omega t) = U_{max}\cos^2(\omega t)$$

$\Rightarrow \quad U = U_{max}\cos^2\left(\frac{\pi t}{20}\right)$

Now, $x = -5$ cm

$\Rightarrow \quad t = \frac{T}{6}$

$x = 0$

$\Rightarrow \quad t = \frac{T}{4}$

Time taken to travel the first 5 cm distance is

$$t_1 = \frac{T}{6} = \frac{\pi}{120}\text{ sec}$$

Time taken to travel the next 5 cm distance is

$$t_2 = \frac{T}{4} - \frac{T}{6} = \frac{\pi}{240}\text{ sec}$$

$U = K$

$\Rightarrow \quad \omega t = \frac{\pi}{4}$

$\Rightarrow \quad t = \frac{\pi}{160}$ sec

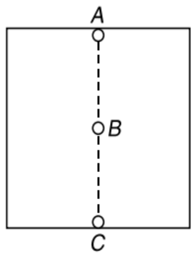
$$K = \frac{K_{max}}{4}$$

$\Rightarrow \quad \omega t = \frac{\pi}{12}$

$\Rightarrow \quad t = \frac{\pi}{240}$ sec

4. A → (r); B → (r); C → (s); D → (s)

Ball will oscillate simple harmonically. The mean position in at depth



$\rho = \alpha h$

$\Rightarrow \quad h = \frac{\rho}{\alpha} = AB$

$$h_{max} = \frac{2\rho}{\alpha} = AC$$

Amplitude $= \frac{\rho}{\alpha} = AB$ or BC

From A to B : $\rho > \rho_l$, weight > upthrust

At B, $\rho = \rho_l$, weight = upthrust

From B to C, $\rho_e > \rho$, upthrust > weight.

From A to C → upthrust will increase and gravitational potential energy will decrease.

From C to A, upthrust will decrease and gravitational potential energy will increase.

From A to B, speed will increase.

From B to C, speed will decrease.

5. A → (q); B → (p); C → (s); D → (q)

Since $F = 8 - 2x$, so we have $m\omega^2 = 2$

$\Rightarrow \quad \omega = 1\text{ rads}^{-1}$

At mean position $F = 0$, so $x = 4$ m

Since particle is released from $x = 7$ m, so the amplitude of motion is

$A = 7 - 4 = 3$ m

Total mechanical energy is

$$E = \frac{1}{2}(m\omega^2)A^2 = 9\text{ J}$$

Velocity of particle at mean position i.e., $x = 4$ m is

$$v_{max} = A\omega = 3\text{ ms}^{-1}$$

The particle will oscillate between $x = 1$ m to $x = 7$ m with mean position at $x = 4$ m, so when the particle is at $x = 2.5$ m, then its distance from the mean position is

$$x = 4 - 2.5 = 1.5 = A/2$$

So, time taken by particle to go from 2.5 m to 4 m is

$$t = \frac{T}{12} = \frac{2\pi}{12} \approx 0.5\text{ s}$$

6. A → (q); B → (s); C → (p); D → (r)

From x-t graph, $T = 8$ s

$$\Rightarrow \quad \omega = \frac{2\pi}{T} = \frac{\pi}{4} \text{ rads}^{-1}$$

Since, $F = -m\omega^2 x$

So, from F-x graph

$$10 = -m\left(\frac{\pi^2}{16}\right)(-1)$$

$$\Rightarrow \quad m = \frac{160}{\pi^2}$$

Also, from x-t graph, we observe that

$$x = 4\sin(\omega t) = 4\sin\left(\frac{\pi t}{4}\right)$$

$$\Rightarrow \quad v = \frac{dx}{dt} = \pi\cos\left(\frac{\pi t}{4}\right)$$

$$\Rightarrow \quad v_{\max} = \pi$$

$$\Rightarrow \quad K_{\max} = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}\left(\frac{160}{\pi^2}\right)(\pi^2) = 80$$

Further, spring constant, $k = m\omega^2$

$$\Rightarrow \quad k = \left(\frac{160}{\pi^2}\right)\left(\frac{\pi^2}{16}\right) = 10$$

7. A → (q); B → (s); C → (p); D → (r)

$\omega = \sqrt{\dfrac{k}{\mu}}$. Here, μ = reduced mass $= \dfrac{m_1 m_2}{m_1 + m_2} = \dfrac{2}{3}$ kg

$$\Rightarrow \quad \omega = \sqrt{\frac{6}{2/3}} = 3 \text{ rads}^{-1}$$

Amplitude 12 cm distributes in the inverse ratio of mass, so we get

$$A_1 = 8 \text{ cm and } A_2 = 4 \text{ cm}$$

Now, maximum kinetic energy for 1 kg block is

$$K_1 = \frac{1}{2}m_1(v_1)_{\max}^2 = \frac{1}{2}m_1\omega^2 A_1^2$$

$$\Rightarrow \quad K_1 = \frac{1}{2}\times 1\times 9\times(8\times 10^{-2})^2 = 28.8 \text{ mJ}$$

Maximum kinetic energy for 2 kg block is

$$K_2 = \frac{1}{2}\times 2\times 9\times(4\times 10^{-2})^2 = 14.4 \text{ mJ}$$

8. A → (q); B → (r); C → (q); D → (q)

Suppose $x = A\sin(\omega t)$

then $v = \dfrac{dx}{dt} = \omega A\cos(\omega t)$ and $a = \dfrac{dv}{dt} = -\omega^2 A\sin(\omega t)$

9. Conceptual

A → (r); B → (p); C → (s); D → (p), (q)

10. A → (q); B → (s); C → (s); D → (s)

Since, $a = -\omega^2 x$, so a-x graph is straight line passing through origin.

If $v = v_0\sin(\omega t)$

then $a = \dfrac{dv}{dt} = v_0\omega\cos(\omega t) = v_0\omega\sqrt{1-\sin^2\omega t}$

$$\Rightarrow \quad a = \frac{dv}{dt} = v_0\omega\sqrt{1-\frac{v^2}{v_0^2}} = \omega\sqrt{v_0^2 - v^2}$$

So, a-v graph is neither a straight line nor a parabola. Further, acceleration and velocity time graphs are sine or cosine functions.

11. A → (q); B → (t); C → (p); D → (t)

On a satellite and at centre of earth $g' = 0$, so

$$T \to \infty$$

At pole, value of g is more than the normal value. Hence,

$$T > 2 \text{ s}$$

Integer/Numerical Answer Type Questions

1. m = mass of block = 2 kg

In equilibrium, $kx_0 = mg$ ...(1)

When displaced further by x,

$$E = \frac{1}{2}kv^2 + \frac{1}{2}I\omega^2 - mgx + \frac{1}{2}k(x+x_0)^2$$

$$\Rightarrow \quad E = \frac{1}{2}mv^2 + \frac{1}{2}(0.6MR^2)\left(\frac{v^3}{R^2}\right) - mgx + \frac{1}{2}k(x+x_0)^2$$

$$\Rightarrow \quad E = \frac{1}{2}(m+0.6M)v^2 - mgx + \frac{1}{2}k(x+x_0)^2$$

Since, E = constant

$$\Rightarrow \quad \frac{dE}{dt} = 0$$

$$\Rightarrow \quad v(m+0.6M)\frac{dv}{dt} - mg\left(\frac{dx}{dt}\right) + k(x_0+x)\frac{dx}{dt} = 0$$

Substituting $\dfrac{dx}{dt} = v$, $\dfrac{dv}{dt} = a$ and $kx_0 = mg$

We get $(m+0.6M)a = -kx$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\left|\frac{a}{x}\right|} = \frac{1}{2\pi}\sqrt{\frac{k}{m+0.6M}}$$

$$\Rightarrow \quad \omega = 2\pi f = \sqrt{\left|\frac{a}{x}\right|} = \sqrt{\frac{k}{m+0.6M}}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{20}{2+(0.6)(5)}} = \sqrt{\frac{20}{5}} = 4 \text{ rads}^{-1}$$

2. (a) $mg = kx$

$$\Rightarrow \quad x = \frac{mg}{k} = \frac{1.1\times 9.8}{120} \approx 9.0\times 10^{-2} \text{ m} = 9 \text{ cm}$$

(b) $\dfrac{1}{2}mv^2 + \dfrac{1}{2}kx_1^2 = \dfrac{1}{2}kx_2^2$

where, $x_1 = 9.0\times 10^{-2}$ m $= 0.09$ m

and $x_2 = 0.2 + 0.09 = 0.29$ m

$$\Rightarrow \quad v = \sqrt{\frac{k}{m}(x_2^2 - x_1^2)}$$

$$\Rightarrow \quad v = \sqrt{\frac{120}{1.1}(0.29\times 0.29 - 0.09\times 0.09)}$$

$$\Rightarrow \quad v = 2.88 \text{ ms}^{-1} = 3 \text{ ms}^{-1}$$

3. In equilibrium, let the spring is stretched by x_0, then

$$T = mg \quad \ldots(1)$$

Also, $T + mg\sin\theta = kx_0 \quad \ldots(2)$

$\Rightarrow \quad mg + mg\sin\theta = kx_0 \quad \ldots(3)$

Let the spring is further stretched by x then total mechanical energy of the system in this position is,

$$E = \frac{1}{2}(m+m)v^2 + \frac{1}{2}k(x+x_0)^2 - mg(x + x\sin\theta)$$

Since, $E =$constant

$\Rightarrow \quad \frac{dE}{dt} = 0$

$\Rightarrow \quad 0 = 2mv\frac{dv}{dt} + k(x+x_0)\frac{dx}{dt} - mg(1+\sin\theta)\frac{dx}{dt}$

Substituting, $\frac{dx}{dt} = v$, $\frac{dv}{dt} = a$ and $kx_0 = mg(1+\sin\theta)$, we get

$$a = -\frac{k}{2m}x$$

Comparing with $a = -\omega^2 x$, we get

$$\omega = \sqrt{\frac{k}{2m}}$$

$\Rightarrow \quad \omega = \sqrt{\frac{20}{2(2.5)}} = \sqrt{\frac{20}{5}} = 2 \text{ rads}^{-1}$

4. Since, $\vec{g}_{\text{eff}} = \vec{g} - \vec{a}_0$

$\Rightarrow \quad g_{\text{eff}} = \sqrt{a_0^2 + g^2 - 2a_0 g\cos\beta}$

$\Rightarrow \quad \vec{g}_{\text{eff}}\sqrt{\left(\frac{g}{2}\right)^2 + g^2 - 2\times\frac{g}{2}\times g\times\cos 120°}$

$\Rightarrow \quad \vec{g}_{\text{eff}} = \frac{\sqrt{7}}{2}g = 12.96 \text{ ms}^{-2}$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{\ell}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{0.21}{12.96}} = 0.8 \text{ s}$

$\Rightarrow \quad 10T = 8 \text{ s}$

5. Applying Law of Conservation of Momentum, velocity of $(\text{ball} + \text{shell})$ just after collision is

$$v_1 = \frac{v_0}{2} = 3 \text{ ms}^{-1}$$

At highest point the whole system has same horizontal velocity (say v_2), where

$$v_2 = \frac{(m+m)v_1}{(m+m+M)} = \frac{2\times 3}{6} = 1 \text{ ms}^{-1}$$

Applying Law of Conservation of Energy, we get

$$\frac{1}{2}(2m)v_1^2 = \frac{1}{2}(2m+M)v_2^2 + 2mgh$$

$\Rightarrow \quad \frac{1}{2}\times 2\times 9 = \frac{1}{2}\times 6\times 1 + 2\times 10\times h$

$\Rightarrow \quad h = 0.3 \text{ m}$

Further $h = \ell(\ell - \cos\theta)$

$\Rightarrow \quad 0.3 = 1.5(1-\cos\theta)$

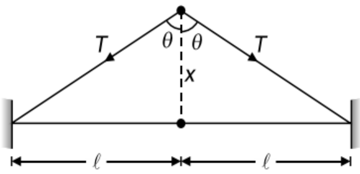
$\Rightarrow \quad \cos\theta = \frac{3}{5}$

$\Rightarrow \quad x = 3$ and $y = 5$

6. Net restoring force $F = -2T\cos\theta$

$\Rightarrow \quad F = -\frac{2Tx}{\sqrt{x^2+\ell^2}}$

If $x \ll \ell$, then $x^2 + \ell^2 \approx \ell^2$



$\Rightarrow \quad F = -\frac{2Tx}{\ell}$

$\Rightarrow \quad a = -\frac{2Tx}{m\ell}$

as $a \propto -x$, motion is simple harmonic.

$\Rightarrow \quad T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2x\sqrt{\frac{m\ell}{2T}}$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{40\times 10^{-3}\times 0.5}{2\times 10}} = 0.2 \text{ second}$

$\Rightarrow \quad 5T = 1 \text{ s}$

7. In the displaced position.

$$E = \frac{1}{2}kx^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

where $I = \frac{1}{2}mR^2$ and $\omega = \frac{v}{R}$

$\Rightarrow \quad E = \frac{1}{2}kx^2 + \frac{3}{4}mv^2$

Since $E =$constant

$\Rightarrow \quad \frac{dE}{dt} = 0$

$\Rightarrow \quad 0 = kx\frac{dx}{dt} + \frac{3}{2}kv\frac{dv}{dt}$

Substituting $\frac{dx}{dt} = v$ and $\frac{dv}{dt} = a$

$$\frac{3}{2}ma = -kx$$

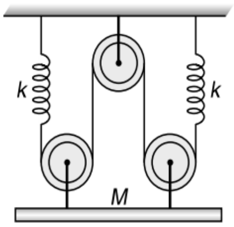
Since, $a \propto -x$, motion is simple harmonic with period given by

$$T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{3m}{2k}}$$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{3(4)}{2(3)}} = 2\pi\sqrt{2} \text{ s}$

So, $* = 2$

8. If the mass M is displaced by x from its mean position each spring further stretches by $2x$.



Net restoring force is

$$F = -8kx$$

$$\Rightarrow \quad Ma = -8kx$$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\left|\frac{a}{x}\right|}$$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{8k}{M}} = \frac{1}{\pi}\sqrt{\frac{2k}{M}}$$

$$\Rightarrow \quad x = 2$$

9. Since, $kx - mg = ma$

$$\Rightarrow \quad x = \frac{m(g+a)}{k} = \frac{(5)(9.8+2.2)}{1200}$$

$$\Rightarrow \quad x = 5\times10^{-2}\text{ m} = 5\text{ cm}$$

10. $mgx = \frac{1}{2}kx^2$

$$\Rightarrow \quad \frac{m}{k} = \frac{x}{2g}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{x}{2g}} = 2\pi\sqrt{\frac{4.9\times10^{-2}}{2\times9.8}}$$

$$T = 0.26\text{ s}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{1}{400}}$$

$$\Rightarrow \quad T = \frac{2\pi}{20}$$

$$\Rightarrow \quad T = \frac{\pi}{10}\text{ s}$$

$$\Rightarrow \quad * = 10$$

11. Since, $2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{I}{mgR}} = 2\pi\sqrt{\frac{7mR^2}{5mgR}}$

$$\Rightarrow \quad \ell = \frac{7}{5}R$$

$$\Rightarrow \quad \ell = \frac{7}{5}(5\text{ cm})$$

$$\Rightarrow \quad \ell = 7\text{ cm}$$

12. (a) In equilibrium, $k\ell = mg$

$$\Rightarrow \quad k = \frac{mg}{\ell}$$

Substituting the proper values, we have

$$k = \frac{(3)(9.8)}{0.2} = 147\text{ Nm}^{-1}$$

(b) $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$\Rightarrow \quad \omega = \frac{2\pi}{T} = \sqrt{\frac{g}{\ell}} = \sqrt{\frac{9.8}{0.2}} = 7\text{ rads}^{-1}$$

13. Restoring torque, $\tau = -k\ell\theta - k\frac{\ell}{2}\theta = -\frac{3}{2}k\ell\theta$

$$\Rightarrow \quad \left(\frac{m\ell^2}{3}\right)\alpha = -\frac{3}{2}k\ell\theta$$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\left|\frac{\alpha}{\theta}\right|} = \frac{1}{2\pi}\sqrt{\frac{9k}{2m\ell}}$$

$$\Rightarrow \quad * = 9$$

14. (a) $T = 2\pi\sqrt{\frac{m}{k}}$

$$\Rightarrow \quad k = \frac{4\pi^2 m}{T^2} = 4\pi^2 mf^2 = 4\pi^2(0.1)(20)^2$$

$$\Rightarrow \quad k = 1600\text{ Nm}^{-1}$$

(b) $a_{max} = \omega^2 A = \left(\frac{k}{m}\right)(A) = \left(\frac{1600}{0.1}\right)(0.5\times10^{-2})$

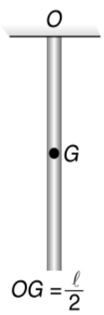
$$\Rightarrow \quad a_{max} = 80\text{ ms}^{-2}$$

(c) $E = \frac{1}{2}m\omega^2A^2 = \frac{1}{2}kA^2$

$$\Rightarrow \quad E = \frac{1}{2}\times1600\times(0.5\times10^{-2})^2 = 0.02\text{ J}$$

$$\Rightarrow \quad E = 20\times10^{-3}\text{ J} = 20\text{ mJ}$$

15. Since, $T = 2\pi\sqrt{\frac{I_0}{mg(OG)}}$



where, $I_0 = \frac{1}{3}m\ell^2$ and $OG = \frac{\ell}{2}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\left(\frac{1}{3}m\ell^2\right)}{(m)(g)\left(\frac{\ell}{2}\right)}} = 2\pi\sqrt{\frac{2\ell}{3g}}$$

$$\Rightarrow \quad \omega = \frac{2\pi}{T} = \sqrt{\frac{3g}{2\ell}} = \sqrt{\frac{3(9.8)}{2(0.147)}} = 10\text{ rads}^{-1}$$

16. Since, $kA = ma$

$$\Rightarrow \quad A = \frac{ma}{k} = \frac{(1)(2)}{100} = 0.02\text{ m} = 2\text{ cm}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{1}} = 10\text{ rads}^{-1}$$

17. In equilibrium, $mg\sin\theta = kx_0$

When further displaced by x, then by Law of Conservation of Mechanical Energy, we get

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}k(x+x_0)^2 - mgx\sin\theta$$

Since, $E =$ constant

$$\Rightarrow \quad \frac{dE}{dt} = 0$$

$$\Rightarrow \quad 0 = mv\left(\frac{dv}{dt}\right) + I\omega\left(\frac{d\omega}{dt}\right) + k(x + x_0)\frac{dx}{dt} - mg\sin\theta\frac{dx}{dt}$$

Substituting, $\frac{dv}{dt} = a$, $\omega = \frac{v}{R}$, $I = \frac{1}{2}mR^2$

$\frac{d\omega}{dt} = \alpha = \frac{a}{R}$, $\frac{dx}{dt} = v$ and $kx_0 = mg\sin\theta$, we get

$$3ma = -2kx$$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\left|\frac{a}{x}\right|} = \frac{1}{2\pi}\sqrt{\frac{2k}{3m}}$$

$$\Rightarrow \quad \omega = 2\pi f = \sqrt{\frac{2k}{3m}}$$

Substituting the values, we get

$$\omega = \sqrt{\frac{2(300)}{3(0.5)}} = \sqrt{400} = 20 \text{ rads}^{-1}$$

18. By Law of Conservation of Mechanical Energy, we have

$$\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx_f^2 - mg(x_0 - x_f)\sin(30^\circ)$$

$$\Rightarrow \quad \frac{1}{2}\times 310\times(0.31)^2 = \frac{1}{2}\times(1.7)v^2 + \frac{1}{2}\times 310\times(0.14)^2 - (1.7)(9.8)(0.31-0.14)\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad 14.9 = 0.85v^2 + 3.0 - 1.4$$

$$\Rightarrow \quad v = 3.95 \text{ ms}^{-1}$$

$$\Rightarrow \quad v = 4 \text{ ms}^{-1}$$

19. Since, $T = 2\pi\sqrt{\frac{m/\rho g S}{\sin\theta_1 + \sin\theta_2}}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{0.2/(13.6\times 10^3\times 9.8\times 0.5\times 10^{-4})}{(\sin 90^\circ + \sin 60^\circ)}}$$

$$\Rightarrow \quad T = 0.8 \text{ s}$$

$$\Rightarrow \quad 10T = 8 \text{ s}$$

20. (a) $g' = g\left(1 - \frac{h}{R}\right)$

$$\Rightarrow \quad \frac{T'}{T} = \sqrt{\frac{g}{g'}} = \left(1 - \frac{h}{R}\right)^{-\frac{1}{2}} \approx 1 + \frac{h}{2R}$$

$$\Rightarrow \quad T' - T = \frac{Th}{2R} = \Delta T$$

Time lost per day $= \frac{\Delta T}{T'}\times t$

$$\Rightarrow \quad \frac{\Delta T}{T'}\times t = \left(\frac{\frac{h}{2R}}{1 + \frac{h}{2R}}\right)(24\times 3600) \text{ second}$$

$$\Rightarrow \quad (\text{Time Lost Per Day}) = 54 \text{ second}$$

(b) $g' \approx g\left(1 - \frac{2h}{R}\right)$

Proceeding in the similar manner we can show that time lost per day to be

$$\frac{\Delta T}{T'}\times t = \left(\frac{\frac{h}{R}}{1 + \frac{h}{R}}\right)(24\times 3600) \text{ sec}$$

$$\Rightarrow \quad (\text{Time Lost Per Day}) = 107.9 \text{ sec} = 108 \text{ s}$$

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1. (A) $F = ma$ and $a = -\omega^2 x$

At $\frac{3T}{4}$, is $x = 0$, so $a = 0$ i.e., $F = 0$

(B) at $t = T$, $x = A$ i.e., x is maximum, so acceleration is maximum

(C) Since $v = \omega\sqrt{A^2 - x^2}$

$\Rightarrow$ $v_{max} = A\omega$ at $x = 0$

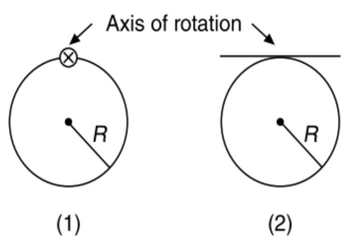
At $t = \frac{T}{4}$, $x = 0$, So $v = v_{max}$

(D) For KE = PE, we have $x = \frac{A}{\sqrt{2}}$

At $t = \frac{T}{2}$, $x = -A$ (So not possible)

Hence, the correct answer is (D).

2. Moment of inertia in case (1) is $I_1 = 2MR^2$



Moment of inertia in case (2) is $I_2 = \frac{3}{2}MR^2$

$$\Rightarrow \quad T_1 = 2\pi\sqrt{\frac{I_1}{Mgd}} \text{ and } T_2 = 2\pi\sqrt{\frac{I_2}{Mgd}}$$

$$\Rightarrow \quad \frac{T_1}{T_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{2MR^2}{\frac{3}{2}MR^2}} = \frac{2}{\sqrt{3}}$$

Hence, the correct answer is (A).

3. An elastic wire can be treated as a spring with spring constant

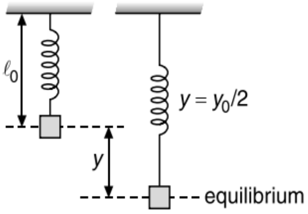
$$k = \frac{YA}{l}$$

Since, $T = 2\pi\sqrt{\frac{m}{k}}$

$\Rightarrow \quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$

Hence, the correct answer is (A).

4. Since, $y = y_0 \sin^2 \omega t$



$\Rightarrow \quad y = \frac{y_0}{2}(1 - \cos 2\omega t)$

$\Rightarrow \quad y - \frac{y_0}{2} = -\frac{y_0}{2}\cos(2\omega t)$

So, amplitude is $\frac{y_0}{2}$ and time period is $\frac{2\pi}{2\omega} = \frac{\pi}{\omega} = \frac{T}{2}$ such that

$mg\left(\frac{y_0}{2}\right) = \frac{1}{2}k\left(\frac{y_0}{2}\right)^2$

$\Rightarrow \quad y_0 = \frac{2mg}{k}$ ...(1)

$\Rightarrow \quad \frac{y_0}{2} = \frac{mg}{k}$

$\Rightarrow \quad 2\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2g}{y_0}}$

From (1), we get

$$\omega = \sqrt{\frac{g}{2y_0}}$$

Hence, the correct answer is (B).

5. Time for 10 oscillations is $\frac{10}{5} = 2$ s

Since, $A = A_0 e^{-kt}$

$\Rightarrow \quad \frac{1}{2} = e^{-2k}$

$\Rightarrow \quad \ell n 2 = 2k$

Also, $10^{-3} = e^{-kt}$

$\Rightarrow \quad 3\ell n 10 = kt$

$\Rightarrow \quad t = \frac{3\ell n 10}{k} = \frac{3\ell n 10}{\ell n 2} \times 2$

$\Rightarrow \quad t = 6 \times \frac{2.3}{0.69} \approx 20$ s

Hence, the correct answer is (D).

6. Since, $T = 2\pi\sqrt{\frac{I}{g_{\text{eff}}}}$

$\Rightarrow \quad \frac{T'}{T} = \sqrt{\frac{g_{\text{eff}}}{g'_{\text{eff}}}}$, where $g'_{\text{eff}} = g - \frac{g}{16} = \frac{15}{16}$

$\Rightarrow \quad \frac{T'}{T} = \sqrt{\frac{16}{15}}$

Hence, the correct answer is (B).

7. Amplitude at $(t = 0)$ is $A_0 = e^{-0.1 \times 0} = 1$

$\Rightarrow \quad$ at $t = t$ if $A = \frac{A_0}{2}$

$\Rightarrow \quad \frac{1}{2} = e^{-0.1t}$

$\Rightarrow \quad t = 10 \ln 2 \simeq 7$ s

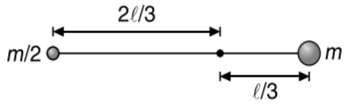
Hence, the correct answer is (A).

8. Since, $W_{\text{man}} = Mg_{\text{eff}}\ell$, where $g_{\text{eff}} = g\left(1 + \theta_0^2\right)$

$\Rightarrow \quad W_{\text{man}} = Mg\ell\left(1 + \theta_0^2\right)$

Hence, the correct answer is (B).

9. Since, $I = \frac{m\ell^2}{9} + \frac{m}{2}\left(\frac{4\ell^2}{9}\right) = \frac{m\ell^2}{3}$



PE at (θ_0) is $U = \frac{1}{2}k\theta_0^2$

$\Rightarrow \quad \frac{1}{2}k\theta_0^2 = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$

$\Rightarrow \quad \frac{3k\theta_0^2}{m\ell^2} = \omega^2$

$\Rightarrow \quad T = m\omega^2\frac{\ell}{3} = \frac{m\left(3k\theta_0^2\right)}{m\ell^2}\left(\frac{\ell}{3}\right) = \frac{k\theta_0^2}{\ell}$

Hence, the correct answer is (B).

10. Since, $I_1 = \frac{M(2L)^2}{12} = \frac{ML^2}{3}$

Now, $I_2 = I_1 + 2\left(\frac{mL^2}{4}\right) = \frac{ML^2}{3} + \frac{mL^2}{2}$

Since $T = 2\pi\sqrt{\frac{I}{C}}$

$\Rightarrow \quad \omega \propto \frac{1}{\sqrt{I}}$

$\Rightarrow \quad \frac{\omega_1}{\omega_2} = \frac{1}{0.8} = \sqrt{\frac{\frac{M}{3} + \frac{m}{2}}{M/3}}$

$\Rightarrow \quad \frac{m}{M} = 0.375$

Hence, the correct answer is (C).

11. Since KE = PE

$\Rightarrow \quad \frac{1}{2}m\omega^2\left(A^2 - x^2\right) = \frac{1}{2}m\omega^2 x^2$

$\Rightarrow \quad A^2 - x^2 = x^2$

$\Rightarrow \quad x = \pm\frac{A}{\sqrt{2}}$

Hence, the correct answer is (A).

12. Since, $a = -\omega^2 x = -\omega^2 \times 4$

Also, $v = \omega\sqrt{A^2 - x^2} = \omega \times \sqrt{5^2 - 4^2} = 3\omega$

Given that $|v| = |a|$

$\Rightarrow \quad 3\omega = 4\omega^2$

$\Rightarrow \quad \omega = \frac{3}{4}\ \text{rads}^{-1}$

$\Rightarrow \quad T = \frac{2\pi}{3/4} = \frac{8\pi}{3}$

Hence, the correct answer is (C).

13. $$\frac{KE}{PE} = \frac{\frac{1}{2}kA^2 - \frac{1}{2}kA^2\sin^2\frac{\pi t}{90}}{\frac{1}{2}kA^2\sin^2\frac{\pi t}{90}} = \frac{1}{3} \qquad \left\{\because \frac{\pi t}{90} = \frac{7\pi}{3}\right\}$$

*No given option is correct.

14. Since, $T = 2\pi\sqrt{\frac{I}{g_{\text{eff}}}}$

$\Rightarrow \quad \omega = \sqrt{\frac{g_{\text{eff}}}{I}}$

$\Rightarrow \quad \frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta g_{\text{eff}}}{g_{\text{eff}}} = \frac{1}{2}\frac{(2\omega^2 A)}{g}$

$\Rightarrow \quad \frac{\Delta\omega}{\omega} = 10^{-3}\ \text{rads}^{-1}$

Hence, the correct answer is (A).

15. Since, $T = 2\pi\sqrt{\frac{\ell}{g}}$

$\Rightarrow \quad \frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$

$\Rightarrow \quad T_2 = 2\sqrt{\frac{g_1}{g_2}}$

Since, $g_2 = \frac{3GM}{(3R)^2} = \frac{g}{3}$ and $g_1 = g$

$\Rightarrow \quad T_2 = 2\sqrt{3}$ s

Hence, the correct answer is (D).

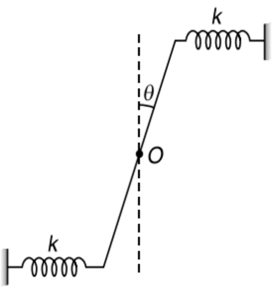
16. Since, $\tau = I\alpha$

$\Rightarrow \quad \frac{M\ell^2}{12}\alpha = 2k\left(\frac{\ell}{2}\right)\left(\frac{\ell}{2}\right)\theta$

$\Rightarrow \quad \frac{M\ell^2}{12}\alpha = \frac{-k\ell^2}{2}\theta$

$\Rightarrow \quad \omega = \sqrt{\frac{6k}{m}}$

$\Rightarrow \quad v = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}$



Hence, the correct answer is (D).

17. Since, $y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$

$\Rightarrow \quad y = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

So, $A = 10$ cm, $\frac{2\pi}{T} = 3\pi$

$\Rightarrow \quad T = \frac{2}{3}$ s

Hence, the correct answer is (A).

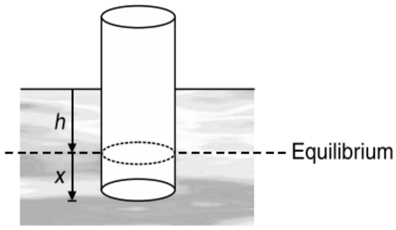
18. In equilibrium, $mg = U = (\rho Ah)g$

When displaced x from equilibrium position, then

$F_{\text{restoring}} = U - mg$

$\Rightarrow \quad F_{\text{restoring}} = \rho A(h + x)g - \rho Ahg$

$\Rightarrow \quad ma = -\rho Agx$



$\Rightarrow \quad a = \ddot{x} = -\left(\frac{\rho Ag}{m}\right)x$

$\Rightarrow \quad \ddot{x} + \omega^2 x = 0$

$\Rightarrow \quad \omega = \sqrt{\frac{10^3 \times A \times g}{310 \times 10^{-3}}} = \sqrt{\frac{10^3 \times \pi \times 6.25 \times 10^{-4} \times 10}{310 \times 10^{-3} \times 100}}$

$\Rightarrow \quad \omega = \sqrt{62.5} \approx 8\ \text{rads}^{-1}$

*No given option is correct.

19. Frequency of a particle executing SHM is

$v = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$\Rightarrow \quad k = 4\pi^2 \times v^2 \times m$

where, $v = 10^{12}\ \text{s}^{-1}$, $m = \frac{108}{6.02 \times 10^{23}} \times 10^{-3}$ kg, $k = ?$

$\Rightarrow \quad k = 4(3.14)^2(10^{12})^2\left(\frac{108 \times 10^{-3}}{6.02 \times 10^{23}}\right) = 7.1\ \text{Nm}^{-1}$

Hence, the correct answer is (B).

20. For $A = B$, $a = b$ and $\delta = \frac{\pi}{2}$, we get

$x^2 + y^2 = A^2$ i.e., a circle

For $A \neq B$, $a = b$ and $\delta = 0$, we get

$y = \left(\frac{B}{A}\right)x$

i.e., a straight line

For $A = B$, $a = 2b$ and $\delta = \frac{\pi}{2}$, we get

$x^2 + y^2 = A^2[\cos^2(2bt) + \sin^2 bt]$

For $A \neq B$, $a = b$ and $\delta = \frac{\pi}{2}$, we get

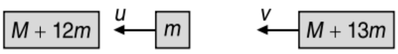
$x = A\sin\left(at + \frac{\pi}{2}\right)$ and $y = B\sin(at)$

$\Rightarrow \quad \dfrac{x^2}{A^2}+\dfrac{y^2}{B^2}=1$ i.e., an ellipse

Hence, the correct answer is (D).

21. In first collision, momentum mu will be imparted to system. In second collision, when momentum of $(M+m)$ is in opposite direction with momentum of particle mu will make its momentum zero.

On 13th collision,



Applying momentum conservation, we get

$$mu=(M+13m)v$$

$$\Rightarrow \quad v=\frac{mu}{M+13m}=\frac{u}{15}$$

Also, $v=A\omega$

$$\Rightarrow \quad \frac{u}{15}=A\sqrt{\frac{k}{M+13m}}$$

$$\Rightarrow \quad A=\frac{1}{15}\sqrt{\frac{75}{1}}=\frac{1}{\sqrt{3}}$$

Hence, the correct answer is (A).

22. Different positions of a particle executing simple harmonic motion is given by

$$a=A\sin\omega t_0,\ b=A\sin 2\omega t_0,\ c=A\sin 3\omega t_0$$

Now, $a+c=A(\sin\omega t_0+\sin 3\omega t_0)=2A\sin 2\omega t_0\cos\omega t_0$

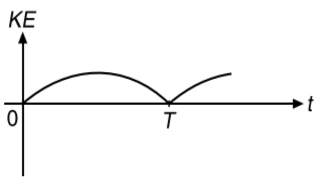
$$\Rightarrow \quad \frac{a+c}{b}=2\cos\omega t_0$$

$$\Rightarrow \quad \omega=\frac{1}{t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$$

$$\Rightarrow \quad \nu=\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$$

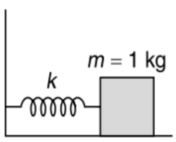
Hence, the correct answer is (D).

23. $\text{KE}=\dfrac{1}{2}m\omega^2A^2\cos^2(\omega t)$



Hence, the correct answer is (D).

24. The 1 kg block attached to a spring vibrates with a frequency of 1 Hz, so

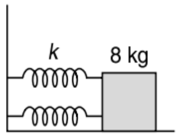


$$\nu=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=1\text{ Hz}$$

$$\Rightarrow \quad k=4\pi^2\text{ Nm}^{-1}$$

When two springs are attached in parallel to an 8 kg block, then

$$k_{eq}=k+k=2k$$



Frequency, $\nu'=\dfrac{1}{2\pi}\sqrt{\dfrac{k_{eq}}{m'}}$

$$\Rightarrow \quad \nu'=\frac{1}{2\pi}\sqrt{\frac{2k}{8}}=\frac{1}{2\pi}\sqrt{\frac{2\times 4\pi^2}{8}}=\frac{1}{2}\text{ Hz}$$

Hence, the correct answer is (B).

25. For simple harmonic motion,

$$\frac{\text{Maximum acceleration}}{\text{Maximum velocity}}=10$$

$$\Rightarrow \quad \frac{A\omega^2}{A\omega}=10$$

$$\Rightarrow \quad \omega=10$$

At $t=0$, displacement is $x=5$ cm

Since, $x=A\sin(\omega t+\phi)$

$$\Rightarrow \quad 5=A\sin\left(0+\frac{\pi}{4}\right)$$

$$\Rightarrow \quad 5=A\sin\frac{\pi}{4}$$

$$\Rightarrow \quad A=5\sqrt{2}\text{ m}$$

Maximum acceleration is

$$a_{max}=A\omega^2=10^2\times 5\sqrt{2}=500\sqrt{2}\text{ ms}^{-2}$$

Hence, the correct answer is (A).

26. Given that, $m=0.1$ kg, $k=640\text{ Nm}^{-1}$

$$b=10^{-2}\text{ kgs}^{-1},\ E=\frac{E_0}{2},\ t=?$$

Amplitude of damped oscillation is $A=A_0e^{\frac{-bt}{2m}}$

Total energy of the system is $E=E_0e^{\frac{-bt}{m}}$

$$\Rightarrow \quad \frac{bt}{m}=\ln\left(\frac{E_0}{E}\right)$$

$$\Rightarrow \quad t=\frac{m}{b}\ln\left(\frac{E_0}{E}\right)=\frac{0.1}{10^{-2}}\ln(2)$$

$$\Rightarrow \quad t=10\times 0.693=6.93\text{ s}\approx 7\text{ s}$$

Hence, the correct answer is (C).

27. Since, $v^2=\omega^2\left(A^2-\left(\dfrac{2A}{3}\right)^2\right)$ ...(1)

where A is initial amplitude and ω is angular frequency. Let new amplitude be A', then

$$(3v)^2=\omega^2\left(A'^2-\left(\frac{2A}{3}\right)^2\right) \quad ...(2)$$

From equation (1) and equation (2), we get

$$\frac{1}{9}=\frac{A^2-\dfrac{4A^2}{9}}{A'^2-\dfrac{4A^2}{9}}$$

$$\Rightarrow \quad A'=\frac{7A}{3}$$

Hence, the correct answer is (A).

28. Since, $T = 2\pi\sqrt{\frac{l}{g}}$ and $T_M = 2\pi\sqrt{\frac{l+\Delta l}{g}}$

$$\Rightarrow \frac{T_M}{T} = \sqrt{\frac{l+\Delta l}{l}}$$

$$\Rightarrow \left(\frac{T_M}{T}\right)^2 = 1 + \frac{\Delta l}{l}$$

Also, $Y = \frac{F/A}{\Delta\ell/\ell} = \frac{Mg/A}{\Delta\ell/\ell}$

$$\Rightarrow \frac{\Delta l}{l} = \frac{Mg}{AY}$$

$$\Rightarrow \left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{AY}$$

$$\Rightarrow \frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$$

Hence, the correct answer is (A).

29. For a simple pendulum in harmonic motion,
(i) at the mean position, KE is maximum and PE is minimum.
(ii) at the extreme position, PE is maximum and KE is minimum.

Hence, the correct answer is (B).

30. Given that, $x = a\sin\omega t$ and $y = a\sin(2\omega t)$

$$\Rightarrow y = 2a\sin\omega t\cos\omega t$$

$$\Rightarrow y = 2x\sqrt{1-\frac{x^2}{a^2}} \qquad \left\{\because \sin(\omega t) = \frac{x}{a}\right\}$$

$$\Rightarrow y = \frac{2}{a}x\sqrt{(a-x)(a+x)}$$

So, $y = 0$ at $x = 0$ and at $x = \pm a$

Hence, the correct answer is (C).

31. It is given that oscillator is at rest at $t = 0$ i.e., at $t = 0$, $v = 0$.

So, we can check options for $v = \frac{dx}{dt} = 0$ by substituting $t = 0$ in value of v.

$$\Rightarrow v = \int a\,dt = \int \frac{F}{m}dt$$

$$\Rightarrow v = \frac{1}{m}\int \sin t\,dt$$

$$\Rightarrow v = \frac{1}{m}(-\cos t)\Big|_0^t = \frac{1}{m}(1-\cos t)$$

(A) If $x \propto \sin t + \frac{1}{2}\sin 2t$

$v \propto \cos t + \frac{1}{2}\times 2\cos 2t$ at $t = 0$, $v \propto 1 + 1 = 2 \neq 0$

(B) If $x \propto \sin t + \frac{1}{2}\cos 2t$

$v \propto \cos t + \frac{1}{2}\times 2(-\sin 2t)$ at $t = 0$, $v \propto 1 - 0 \neq 0$

(C) If $x \propto \cos t - \frac{1}{2}\sin 2t$

$v \propto -\sin t - \frac{1}{2}\times 2\cos 2t$ at $t = 0$, $v \propto -1 \neq 0$

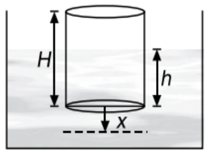
(D) If $x \propto \sin t - \frac{1}{2}\sin 2t$

$v \propto \cos t - \frac{1}{2}\times 2\cos 2t$ at $t = 0$, $v \propto 1 - 1 = 0$

So, in OPTION (D) $v = 0$, at $t = 0$

Hence, the correct answer is (B).

32. Let the block of height H be floating with depth h inside the liquid.
Then at equilibrium



$$M_{Block}g = F_{up}$$

$$(AH\rho_B)g = (Ah)\rho_L g \qquad \ldots(1)$$

When block depressed slightly by distance x then

$$F_{Net} = Mg - F'_{up} = AH\rho_B g - A(h+x)\rho_L g$$

$$\Rightarrow F_{net} = -Ax\rho_L g \qquad \{\text{Using equation (1)}\}$$

$$H\rho_B\frac{d^2x}{dt^2} = -x\rho_L g$$

$$\frac{d^2x}{dt^2} = -\frac{\rho_L g}{H\rho_B}x = -\omega^2 x$$

$$\Rightarrow \omega^2 = \frac{\rho_L g}{H\rho_B}$$

For simple pendulum $\omega^2 = \frac{g}{l}$

$$\Rightarrow l = \frac{H\rho_B}{\rho_L} = \frac{650\times 54}{900} = 39\text{ cm}$$

Hence, the correct answer is (C).

33. Amplitude in damped oscillation is given by $A = A_0e^{-\beta t}$

Energy, $E \propto A^2$

$\Rightarrow \sqrt{E} = \sqrt{E_0}e^{-\beta t}$ where E_0 is initial energy

Here, $E_0 = 45$ J, $T = 1$ s, $E = 15$ J

$$t = nT = 15\times 1 = 15\text{ s}$$

Then, $\sqrt{15} = \sqrt{45}e^{-\beta\times 15}$

$$3^{-\frac{1}{2}} = e^{-15\beta}$$

Taking log on both sides $-\frac{1}{2}\ln(3) = -15\beta$

$$\beta = \frac{\ln 3}{30}$$

Hence, the correct answer is (A).

34. Since the particle starts from rest i.e., at $t = 0$, $v = 0$, so it must be starting from the extreme position and hence we have

$$x = A\cos(\omega t)$$

where x is displacement of particle from the mean position and A is the amplitude. Now at $t = \tau$, the particle travels a distance a, so

CHAPTER 3

$$x = A - a = A\cos(\omega\tau) \quad \ldots(1)$$

and similarly

$$x = A - 3a = A\cos(2\omega\tau) \quad \ldots(2)$$

$\Rightarrow \quad a = A[1-\cos(\omega\tau)]$ {from (1)}

$\Rightarrow \quad 3a = A[1-\cos(2\omega\tau)]$ {from (2)}

$$\Rightarrow \quad \frac{1}{3} = \frac{1-\cos(\omega\tau)}{1-\cos(2\omega\tau)}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{1-\cos(\omega\tau)}{2\sin^2(\omega\tau)}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{1-\cos(\omega\tau)}{2[1-\cos^2(\omega\tau)]}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{1-\cos(\omega\tau)}{2[1+\cos(\omega\tau)][1-\cos(\omega\tau)]}$$

$$\Rightarrow \quad 1+\cos(\omega\tau) = \frac{3}{2}$$

$$\Rightarrow \quad \cos(\omega\tau) = \frac{1}{2} \text{ i.e., } \omega\tau = \frac{\pi}{3}$$

$$\Rightarrow \quad \left(\frac{2\pi}{T}\right)\tau = \frac{\pi}{3} \text{ i.e., } T = 6\tau$$

Hence, the correct answer is (D).

35. The amplitude of a damped oscillator at a given instant of time t is given by $A = A_0 e^{-\frac{bt}{2m}}$

where A_0 is its amplitude in the absence of damping, b is the damping constant?

As per question

After 5 s (i.e., $t = 5$ s), its amplitude becomes

$$0.9A_0 = A_0 e^{-\frac{b(5)}{2m}} = A_0 e^{-\frac{5b}{2m}}$$

$$\Rightarrow \quad 0.9 = e^{-\frac{5b}{2m}} \quad \ldots(1)$$

After 10 more second (i.e., $t = 15$ s), its amplitude becomes

$$\alpha A_0 = A_0 e^{\frac{-b(15)}{2m}} = A_0 e^{-\frac{15b}{2m}}$$

$$\alpha = \left(e^{-\frac{5b}{2m}}\right)^3 = (0.9)^3 \quad \text{\{using (1)\}}$$

$$\Rightarrow \quad \alpha = 0.729$$

Hence, the correct answer is (D).

36. $PV^\gamma =$ constant

Differentiating, we get

$$\frac{dP}{P} + \gamma\frac{dV}{V} = 0$$

$$\Rightarrow \quad dP = -\frac{\gamma P\, dV}{V} = -\frac{\gamma PAx}{V}$$

Taking $P = P_0$, $V = V_0$, we get $F_{\text{restoring}} = -\dfrac{\gamma PA^2 x}{V}$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0 A^2}{V_0 M}}$$

Hence, the correct answer is (C).

37. According to the problem, we are given that

$$a = -bv$$

The equation of motion of a damped harmonic oscillator is written as

$$F = -ky - b'v$$

The damping force is $F = -b'v$, where b' is the damping constant

$$\Rightarrow \quad \frac{d^2y}{dt} + \left(\frac{b'}{m}\right)\frac{dy}{dt} + \frac{k}{m}y = 0$$

$$\Rightarrow \quad \frac{d^2y}{dt} + b\frac{dy}{dt} + \frac{k}{m}y = 0 \quad \left\{\therefore\ b = \frac{b'}{m}\right\}$$

Actually, we take damping constant as b with force, but here in the problem it is given to be b with acceleration, so, we have written the actual damping constant as b'.

For this case of a damped oscillator, the amplitude is not constant and varies with time as

$$A' = Ae^{-\left(\frac{b'}{2m}\right)t} = Ae^{-\left(\frac{b}{2}\right)t} \quad \left\{\because\ b = \frac{b'}{m}\right\}$$

According to the problem, we have

$$A' = \frac{A}{e}$$

$$\Rightarrow \quad \frac{A}{e} = Ae^{-bt/2}$$

$$\Rightarrow \quad e^{bt/2} = e$$

$$\Rightarrow \quad \frac{bt}{2} = 1$$

$$\Rightarrow \quad t = \frac{2}{b}$$

Hence, the correct answer is (C).

38. Let $x_1 = A\sin(\omega t)$ and $x_2 = A\sin(\omega t + \phi)$

$$\Rightarrow \quad x_1 - x_2 = 2A\sin\left(\frac{\phi}{2}\right)\cos\left(\omega t + \frac{\phi}{2}\right)$$

Given that, $2A\sin\left(\dfrac{\phi}{2}\right) = A$

$$\Rightarrow \quad \frac{\phi}{2} = \frac{\pi}{6}$$

$$\Rightarrow \quad \phi = \frac{\pi}{3}$$

Hence, the correct answer is (B).

39. Since, $T_1 = 2\pi\sqrt{\dfrac{M}{k}}$ ...(1)

When a mass m is placed on mass M, the new system is of mass $(M+m)$ attached to the spring. So, new time period of oscillation is

$$T_2 = 2\pi\sqrt{\frac{(m+M)}{k}} \quad \ldots(2)$$

If v_1 is the velocity of mass M passing through mean position and v_2 is the velocity of mass $(m+M)$ passing through mean position, then using law of conservation of linear momentum, we get

$$Mv_1 = (m+M)v_2$$

where $v_1 = A_1\omega_1$ and $v_2 = A_2\omega_2$

$$\Rightarrow \quad M(A_1\omega_1) = (m+M)(A_2\omega_2)$$

$$\Rightarrow \quad \frac{A_1}{A_2} = \frac{(m+M)}{M}\frac{\omega_2}{\omega_1} = \left(\frac{m+M}{M}\right)\times\frac{T_1}{T_2}$$

$$\Rightarrow \quad \frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}}$$

Hence, the correct answer is (D).

40. For a particle executing simple harmonic motion, we have

Acceleration, $a = -\omega^2 x$ where ω is a constant $= \frac{2\pi}{T}$

$$\Rightarrow \quad a = -\frac{4\pi^2}{T^2}x$$

$$\Rightarrow \quad \frac{aT}{x} = -\frac{4\pi^2}{T}$$

Since, the period of oscillation T is a constant, so

$$\frac{aT}{x} = \text{constant}$$

Hence, the correct answer is (B).

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Single Correct Choice Type Problems

1. For the block, $T = \frac{2\pi}{\omega} = 6$ s

For block, the position in 1 s will be

$$x = 0.2\cos\omega t$$

At $t = 1$ s, $x = 0.1$ m i.e., at $t = 1$ s, block will be at a distance $4.9\text{ m} + 0.1\text{ m} = 5\text{ m}$ from wall

So, range of pebble is also $R = 5$ m

$$\Rightarrow \quad R = (v\cos 45°)(1\text{ s}) = 5\text{ m}$$

$$\Rightarrow \quad v = 5\sqrt{2} = \sqrt{50}\text{ ms}^{-1}$$

Hence, the correct answer is (A).

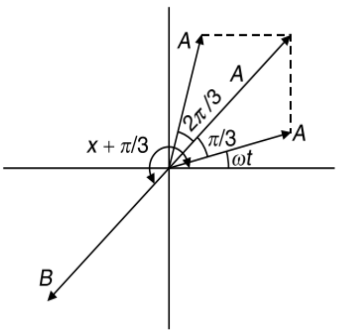
2. The frequency or time period of SHM depends on variable force. It does not depend on constant external force. Constant external force can only change the mean position. In the given problem, the mean position is at natural length of spring in the absence of electric field, whereas in the presence of electric field mean position will be obtained after a compression of x_0, where x_0 is given by

$$Kx_0 = QE$$

$$\Rightarrow \quad x_0 = \frac{QE}{K}$$

Hence, the correct answer is (A).

3. The situation is shown in the phasor diagram drawn.



For the mass to be at complete rest, we have

$$B = A \text{ and } \phi = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

Hence, the correct answer is (B).

4. From graph $x = 1\sin\left(\frac{2\pi}{8}t\right)$

Since $a = -\omega^2 A\sin(\omega t)$

At $t = \frac{4}{3}$ s, $a = -\left(\frac{\pi^2}{16}\right)(1)\left(\sin\frac{2\pi}{8}\right)\left(\frac{4}{3}\right)$

$$\Rightarrow \quad a = -\frac{\sqrt{3}}{32}\pi^2\text{ cms}^{-2}$$

Hence, the correct answer is (B).

5. Internal forces in the springs are same, so we have

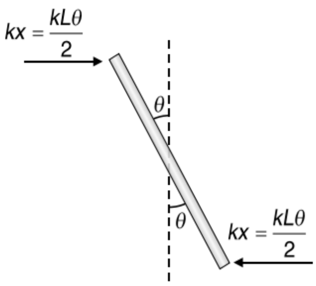
$$k_1x_1 = k_2x_2$$

Also, $x_1 + x_2 = A$

$$\Rightarrow \quad x_1 = \frac{k_2A}{k_1+k_2}$$

Hence, the correct answer is (B).

6.



Restoring torque is

$$\tau = -\left(k\frac{L}{2}\theta\right)\frac{L}{2}\times 2$$

$$\Rightarrow \quad \tau = -\frac{kL^2\theta}{2}$$

$$\Rightarrow \quad \alpha = \frac{\tau}{I} = \frac{-kL^2\theta/2}{ML^2/12}$$

$$\Rightarrow \quad \alpha = -\frac{6k\theta}{M} = -\omega^2\theta$$

$$\Rightarrow \quad \omega = \sqrt{\frac{6k}{M}} = 2\pi f$$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}$$

Hence, the correct answer is (C).

7. Angular frequency of the system is

$$\omega = \sqrt{\frac{k}{m+m}} = \sqrt{\frac{k}{2m}}$$

Maximum acceleration of the system will be

$$\omega^2 A = \frac{kA}{2m}$$

This maximum acceleration to the lower block is provided by friction, so we have

$$f_{\max} = ma_{\max} = m\omega^2 A = m\left(\frac{kA}{2m}\right) = \frac{kA}{2}$$

Hence, the correct answer is (A).

8. Since, $y = Kt^2$

$$\Rightarrow \quad \frac{d^2y}{dt^2} = 2K$$

$$\Rightarrow \quad a_y = 2\ \text{ms}^{-2} \qquad \{\because K = 1\ \text{ms}^{-2}\}$$

Since, $T_1 = 2\pi\sqrt{\frac{\ell}{g}}$ and $T_2 = 2\pi\sqrt{\frac{\ell}{g+a_y}}$

$$\Rightarrow \quad \frac{T_1^2}{T_2^2} = \frac{g+a_y}{g} = \frac{10+2}{10} = \frac{6}{5}$$

Hence, the correct answer is (A).

9. Potential energy is minimum (in this case zero) at mean position $(x=0)$ and maximum at extreme positions $(x=\pm A)$.

At time $t=0$, $x=A$. Hence, PE should be maximum. Therefore, graph I is correct. Further in graph III, PE is minimum at $x=0$. Hence, this is also correct.

Hence, the correct answer is (A).

10. In SHM, velocity of particle also oscillates simple harmonically. Speed is more near the mean position and less near the extreme positions. Therefore, the time taken for the particle to go from 0 to $\frac{A}{2}$ will be less than the time taken to go it from $\frac{A}{2}$ to A, or $T_1 < T_2$.

Conceptual Note(s)

From the equations of SHM we can show that

$$T_1 = T_{0\to A/2} = \frac{T}{12}$$

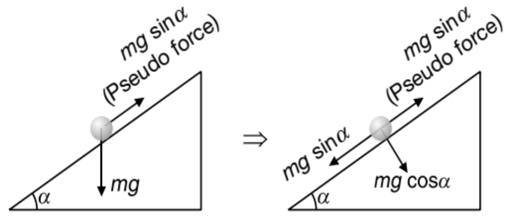
and $t_2 = T_{A/2\to A} = \frac{T}{6}$

So, $T_1 + T_2 = T_{0\to A} = \frac{T}{4}$

Hence, the correct answer is (B).

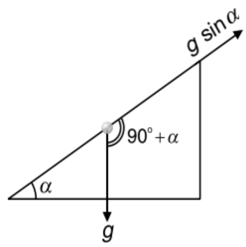
11. Free body diagram of bob of the pendulum with respect to the accelerating frame of reference is as shown in figure.

Net force on the bob is $F_{\text{net}} = mg\cos\alpha$



So, net acceleration of the bob is $g_{\text{eff}} = g\cos\alpha$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$



Conceptual Note(s)

Whenever point of suspension is accelerating take

$$T = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}}$$

where, $\vec{g}_{\text{eff}} = \vec{g} - \vec{a}$

$\vec{a}$ = Acceleration of point of suspension

In this question $\vec{a} = g\sin\alpha$ (down the plane)

$$\Rightarrow \quad |\vec{g} - \vec{a}| = g_m$$

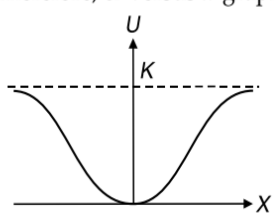
$$\Rightarrow \quad g_{\text{eff}} = g\sqrt{1+\sin^2\alpha + 2\sin\alpha\cos(90°+\alpha)}$$

$$\Rightarrow \quad g_{\text{eff}} = g\cos\alpha$$

Hence, the correct answer is (B).

12. Since, $U(x) = k\left(1 - e^{-x^2}\right)$

It is an exponentially increasing graph of potential energy (U) with x^2. Therefore, U versus x graph will be as shown.



From the graph it is clear that at origin.

Potential energy U is minimum (therefore, kinetic energy will be maximum) and force acting on the particle is also zero because

$$F = \frac{-dU}{dx} = -(\text{slope of } U\text{-}x \text{ graph}) = 0$$

Therefore, origin is the stable equilibrium position. Hence, particle will oscillate simple harmonically about $x=0$ for small displacements. Therefore, correct OPTION is (D).

(A), (B) and (C) OPTIONS are wrong due to following reasons:

(a) At equilibrium position $F = \frac{-dU}{dx} = 0$ i.e., slope U-x graph should be zero and from the graph we can see that slope is zero at $x = 0$ and $x = \pm\infty$.
Now among these equilibriums stable equilibrium position is that where U is minimum (Here $x = 0$).
Unstable equilibrium position is that where U is maximum (Here none).
Neutral equilibrium position is that where U is constant (Here $x = \pm\infty$)
Therefore, OPTION (A) is wrong.

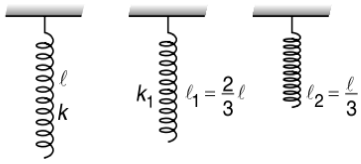
(b) For any finite non-zero value of x, force is directed towards the origin because origin is in stable equilibrium position. Therefore, OPTION (B) is incorrect.

(c) At origin, potential energy is minimum, hence kinetic energy will be maximum. Therefore, OPTION (C) is also wrong.

Hence, the correct answer is (D).

13. Since for a spring of force constant k and natural length ℓ, we have

$k\ell = \text{constant}$



Now, since $\ell_1 = 2\ell_2$

$\Rightarrow \quad \ell_1 = \frac{2}{3}\ell$

$\Rightarrow \quad k_1 = \frac{3}{2}k$

Hence, the correct answer is (B).

14. Given that, $U(x) = k|x|^3$

$$\Rightarrow \quad [k] = \frac{[U]}{[x^3]} = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}]$$

Now, time period may depend on mass, amplitude and k, so

$T \propto (\text{mass})^x (\text{amplitude})^y (k)^z$

$\Rightarrow \quad [M^0 L^0 T] = [M]^x [L]^y [ML^{-1}T^{-2}]^z$

$\Rightarrow \quad [M^0 L^0 T] = [M^{x+z} L^{y-z} T^{-2z}]$

Applying the Principle of Homogeneity and equating the powers, we get

$-2z = 1$ and $y - z = 0$ or $z = -\frac{1}{2}$

$\Rightarrow \quad z = -\frac{1}{2}$ and $y = z = -\frac{1}{2}$

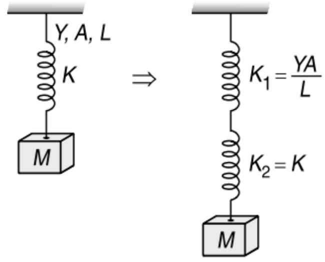
$\Rightarrow \quad T \propto (\text{amplitude})^{-1/2}$

$\Rightarrow \quad T \propto (a)^{-1/2}$

$\Rightarrow \quad T \propto \frac{1}{\sqrt{a}}$

Hence, the correct answer is (A).

15. Since, $K_{eq} = \frac{K_1K_2}{K_1 + K_2} = \frac{L}{\frac{YA}{L} + K} = \frac{YAK}{YA + LK}$



$$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{m(YA + LK)}{YAK}}$$

Conceptual Note(s)

Equivalent force constant for a wire is given by $k = \frac{YA}{L}$. Since, in case of a wire, $F = \left(\frac{YA}{L}\right)x$ and in case of spring, $F = kx$. Comparing these two, we find the equivalent spring constant of the wire is $k = \frac{YA}{L}$.

Hence, the correct answer is (B).

16. Modulus of Rigidity, $\eta = \frac{F}{A\theta}$

where, $A = L^2$ and $\theta = \frac{x}{L}$

Therefore, restoring force, $F = -\eta A\theta = -\eta Lx$

$\Rightarrow \quad$ Acceleration, $a = \frac{F}{M} = -\frac{\eta L}{M}x$

Since, $a \propto -x$, oscillations are simple harmonic in nature, time period of which is given by

$$T = 2\pi\sqrt{\left|\frac{\text{displacement}}{\text{acceleration}}\right|} = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{M}{\eta L}}$$

Hence, the correct answer is (D).

17. When cylinder is displaced by an amount x from its mean position, spring force and upthrust both will increase. Hence,

$$\begin{pmatrix}\text{Net Restoring} \\ \text{Force}\end{pmatrix} = \begin{pmatrix}\text{Extra Spring} \\ \text{Force}\end{pmatrix} + \begin{pmatrix}\text{Extra} \\ \text{Upthrust}\end{pmatrix}$$

$\Rightarrow \quad F = -(kx + Ax\rho g)$

$\Rightarrow \quad a = -\left(\frac{k + \rho A g}{M}\right)x$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\left|\frac{a}{x}\right|} = \frac{1}{2\pi}\sqrt{\frac{k + \rho A g}{M}}$$

Hence, the correct answer is (B).

18. Since, $(v_{\max})_A = (v_{\max})_B$

$\Rightarrow \quad A_1\omega_1 = A_2\omega_2$

$\Rightarrow \quad A_1\left(\frac{2\pi}{T_1}\right) = A_2\left(\frac{2\pi}{T_2}\right)$

$\Rightarrow \quad \frac{A_1}{A_2} = \frac{T_1}{T_2} = \sqrt{\frac{K_2}{K_1}} \qquad \left\{\because T = 2\pi\sqrt{\frac{m}{k}}\right\}$

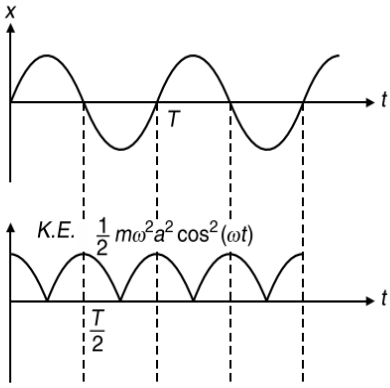
Hence, the correct answer is (D).

19. Let $x = a\sin(\omega t)$

$\Rightarrow \quad \dot{x} = v = a\omega\cos(\omega t)$

$\Rightarrow \quad \text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2a^2\cos^2(\omega t)$

$\Rightarrow \quad \text{PE} = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2a^2\sin^2(\omega t)$



So, x has a period T i.e., frequency f and kinetic energy has a period $\frac{T}{2}$ i.e., frequency $2f$

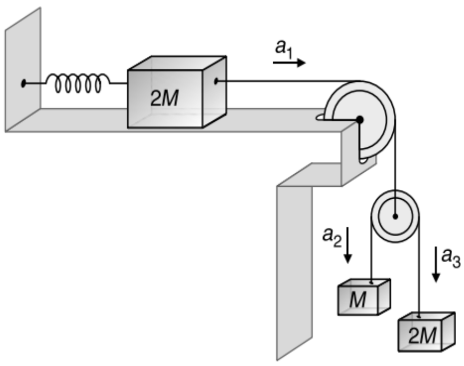
Hence, the correct answer is (C).

Multiple Correct Choice Type Problems

1. Using constraint relations, we get

$2a_1 = a_2 + a_3$

$\Rightarrow \quad a_1 - a_3 = a_2 - a_1$



Also $2mg - T = 2ma_3$ ...(1)

$mg - T = ma_2$ ...(2)

and $2T - kx = 2ma_1$ ...(3)

Solving the above equations, we get

$T = \frac{4mg}{7} + \frac{2kx}{7}$ and $a_1 = \frac{4g}{7} - \frac{3kx}{14m}$

For $x = \frac{x_0}{4} = \frac{4mg}{3k}$, we get

$a_1 = \frac{4g}{7} - \frac{2g}{7} = \frac{2g}{7}$

For the oscillation of mass $2m$ about the mean position, we have

$a_1 = 0$

$\Rightarrow \quad x = \frac{8mg}{3k} = \text{Amplitude}$

$\Rightarrow \quad x_0 = 2A = \frac{16mg}{3k}$

Since $\omega = \sqrt{\frac{3k}{14m}}$

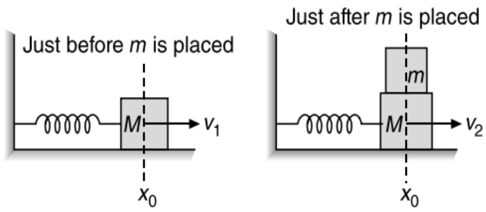
Also, at mean i.e., at $x = \frac{x_0}{2}$, we have

$v = v_{max} = A\omega$

$\Rightarrow \quad v = \frac{x_0}{2}\sqrt{\frac{3k}{14m}} = \frac{8mg}{3k}\sqrt{\frac{3k}{14m}}$

Hence, the correct answer is (C).

2. **IN CASE-I**



$Mv_1 = (M+m)v_2$

$\Rightarrow \quad v_2 = \left(\frac{M}{M+m}\right)v_1$, where $v_2 = A_2\omega_2$ and $v_1 = A_1\omega_1$

i.e., velocity decreases at equilibrium position

$\Rightarrow \quad \sqrt{\frac{k}{M+m}}A_2 = \left(\frac{M}{M+m}\right)\sqrt{\frac{k}{M}}A_1$

$\Rightarrow \quad A_2 = \sqrt{\frac{k}{M+m}}A_1$

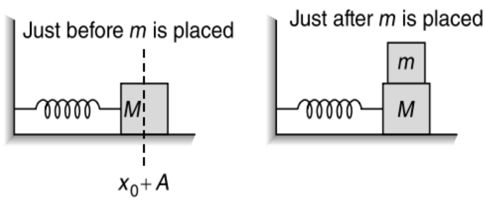
$\Rightarrow \quad E_2 = \frac{1}{2}(M+m)v_2^2$

$\Rightarrow \quad E_2 = \frac{1}{2}kA^2\left(\frac{M}{M+m}\right)$

IN CASE-II

$\omega_2 = \sqrt{\frac{k}{M+m}}$

No energy loss, so $A_1 = A_2$



$T' = 2\pi\sqrt{\frac{M+m}{k}}$, in both cases.

$v_2 = A\omega_2 = A\sqrt{\frac{k}{M+m}}$

Total energy decreases in first case whereas remain same in second case. Instantaneous speed at x_0 decreases in both cases (because ω decreases in both).

Hence, (A), (B) and (D) are correct.

3. **For First Oscillator**

$p = 0$ at $x = a$

So, a is the amplitude of oscillation A_1.

At $x = 0$ i.e., mean position, $p = b$

$$\Rightarrow \quad mv_{max} = b$$

$$\Rightarrow \quad v_{max} = \frac{b}{m}$$

$$\Rightarrow \quad E_1 = \frac{1}{2}mv_{max}^2 = \frac{m}{2}\left(\frac{b}{m}\right)^2 = \frac{b^2}{2m}$$

Also, $A_1\omega_1 = v_{max} = \frac{b}{m}$

$$\Rightarrow \quad \omega_1 = \frac{b}{ma} = \frac{1}{mn^2} \qquad \left\{\because A_1 = a,\ \frac{a}{b} = n^2\right\}$$

For Second Oscillator

$p = 0$ at $x = R$

$$\Rightarrow \quad A_2 = R$$

At $x = 0$ i.e., mean position, $p = R$

$$\Rightarrow \quad v_{max} = \frac{R}{m}$$

$$\Rightarrow \quad E_2 = \frac{1}{2}mv_{max}^2 = \frac{m}{2}\left(\frac{R}{m}\right)^2 = \frac{R^2}{2m}$$

Also, $A_2\omega_2 = \frac{R}{m}$

$$\Rightarrow \quad \omega_2 = \frac{R}{mR} = \frac{1}{m}$$

From above calculations, we see that

$$\frac{\omega_2}{\omega_1} = \frac{1/m}{1/mn^2} = n^2$$

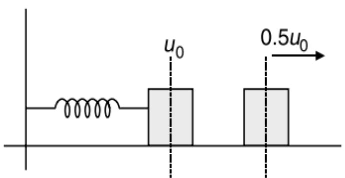
$$\omega_1\omega_2 = \frac{1}{mn^2} \times \frac{1}{m} = \frac{1}{m^2n^2}$$

$$\frac{E_1}{\omega_1} = \frac{b^2/2m}{1/mn^2} = \frac{b^2n^2}{2} = \frac{a^2}{2n^2} = \frac{R^2}{2}$$

$$\frac{E_2}{\omega_2} = \frac{R^2/2m}{1/m} = \frac{R^2}{2}$$

$$\Rightarrow \quad \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$$

Hence, (B) and (D) are correct.

4. 

OPTION (A) is correct due to Law of Conservation of Energy

OPTION (B)

$$x = A\sin(\omega t)$$

$$\Rightarrow \quad \frac{dx}{dt} = A\omega\cos\omega t = \frac{u_0}{2} = u_0\cos\omega t$$

$$\Rightarrow \quad \cos(\omega t) = \frac{1}{2}$$

$$\Rightarrow \quad \omega t = \frac{\pi}{3}$$

$$\Rightarrow \quad t = \frac{\pi}{3\omega}$$

$$\Rightarrow \quad \Delta t = \frac{2\pi}{3\omega} = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$$

So, this is incorrect

OPTION (C)

Maximum compression will occur at time

$$t = \frac{2\pi}{3}\sqrt{\frac{m}{k}} + \frac{\pi}{2}\sqrt{\frac{m}{k}}$$

$$\Rightarrow \quad t_1 = \frac{7\pi}{6}\sqrt{\frac{m}{k}}$$

So, this is incorrect

OPTION (D)

Second time at equilibrium $t' = \frac{2\pi}{3}\sqrt{\frac{m}{k}} + \pi\sqrt{\frac{m}{k}}$

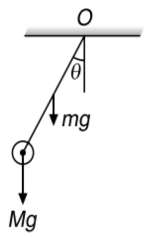
$$\Rightarrow \quad t' = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$$

So, this is correct

Hence, (A) and (D) are correct.

5. Restoring torque about O is given by

$$\tau_A = \tau_B = \left(mg\frac{L}{2}\sin\theta + MgL\sin\theta\right)$$



In case A, moment of inertia will be more. Hence, angular acceleration $\left(\alpha = \frac{\tau}{I}\right)$ will be less. Therefore, angular frequency will be less.

Hence, (A) and (D) are correct.

6. For $A = -B$ and $C = 2B$, we get

$$x = B\cos 2\omega t + B\sin 2\omega t$$

$$\Rightarrow \quad x = \sqrt{2}B\sin\left(2\omega t + \frac{\pi}{4}\right)$$

This is equation of SHM of amplitude $\sqrt{2}B$.

If $A = B$ and $C = 2B$, then $x = B + B\sin 2\omega t$

This is also equation of SHM about the point $x = B$. Function oscillates between $x = 0$ and $x = 2B$ with amplitude B.

Hence, (B) and (D) are correct.

7. Applying the Principle of Superposition, we get

$$y = y_1 + y_2 + y_3$$

$\Rightarrow \quad y = a\sin(\omega t) + a\sin(\omega t + 45^\circ) + a\sin(\omega t + 90^\circ)$

$\Rightarrow \quad y = a[\sin(\omega t) + \sin(\omega t + 90^\circ)] + a\sin(\omega t + 45^\circ)$

$\Rightarrow \quad y = 2a\sin(\omega t + 45^\circ)\cos(45^\circ) + a\sin(\omega t + 45^\circ)$

Hence, (A) and (C) are correct.

8. $K_{max} = \frac{1}{2}(m\omega^2)A^2 = \frac{1}{2}kA^2 = \frac{1}{2}\times 2\times 10^6 \times (10^{-2}) = 100 \text{ J}$

This is basically the energy of oscillation of the particle. K, U and E at mean position $(x = 0)$ and extreme position $(x = \pm A)$ are shown in Figure.

Hence, (B) and (C) are correct.

Linked Comprehension Type Questions

1. For motion to be periodic, it must reverse its path i.e., KE should become zero for a finite value of x.

Now $K + U = E$

$\Rightarrow \quad K = E - U$

$U_{max} = V_0$

$\Rightarrow \quad K_{min} = E - V_0$

If $K_{min} > 0$, particle will escape

So, $E - V_0 < 0$

$\Rightarrow \quad E < V_0$

Also $E = K + U > 0$

Hence, the correct answer is (C).

2. Since, $V = \alpha x^4$

$\Rightarrow \quad [\alpha] = ML^{-2}T^{-2}$

By methods of dimensional analysis, we have

$$\left[\frac{1}{A}\sqrt{\frac{m}{\alpha}}\right] = M^0L^0T$$

Hence, the correct answer is (B).

3. For $X > X_0$, potential energy is constant.

Since, $F = -\frac{dU}{dx}$, so $F = 0$

Hence, the correct answer is (D).

Matrix Match Type Questions

1. A → (p, s); B → (q, r, s); C → (s); D → (q)

For A

Potential energy of a simple pendulum varies curvilinearly with displacement and is minimum at a certain value.

For B

q and r correspond to zero acceleration, while s corresponds to accelerated motion p is not possible as velocity is negative corresponding to initial portion of the graph.

For C

$$R = \frac{u^2\sin(2\theta)}{g}$$

$\Rightarrow \quad R \propto u^2$

For D

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$\Rightarrow \quad T^2 \propto \ell$

2. A → (p); B → (q, r); C → (p); D → (q, r)

(A) Compare with the standard equation of SHM

$$v = \omega\sqrt{A^2 - x^2}$$

We see that the given motion is SHM with,

$\omega = C_1$ and $A^2 = C_2$

(B) The equation shows that the object does not change its direction and kinetic energy of the object keeps on decreasing.

(C) A pseudo force (with respect to elevator) will start acting on the object. Its means position is now changed and it starts SHM.

(D) The given velocity is greater than the escape velocity $v_e = \sqrt{\frac{2GM_e}{R_e}}$. Therefore, it keeps on moving towards infinity with decreasing speed.

Integer/Numerical Answer Type Questions

1. Since $k = \frac{YA}{L}$ and so,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{YA}{lm}} = \sqrt{\frac{(n\times 10^9)(4.9\times 10^{-7})}{1\times 0.1}}$$

Substituting, $\omega = 140 \text{ rads}^{-1}$, we get

$n = 4$