CHAPTER 3: ROTATIONAL DYNAMICS

Test Your Concepts-I (Based on Moment of Inertia and Applications)

1. Axis A: $I_A = m_1 d_1^2 + m_2 d_2^2 = (3\text{ kg})(1\text{ m})^2 + (5\text{ kg})(2\text{ m})^2 = 23\text{ kgm}^2$

Axis B: $I_B = m_1(0) + m_2(d_1 + d_2)^2 = 45\text{ kgm}^2$

Axis C: $I_C = m_1(d_1 + d_2)^2 + m_2(0) = 27\text{ kgm}^2$

Axis D: $I_D = 0$

I_D is zero because we treated the masses as point particles and perpendicular distances are equal to zero.

2. For each mass we need its perpendicular distance from the axis. For each axis, two masses do not contribute to the moment of inertia. The other two are at the same distance

$$3\sin(53°) = 2.4\text{ m}$$

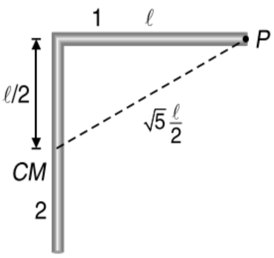
$$I_A = (4\text{ kg})(2.4\text{ m})^2 + (2\text{ kg})(2.4\text{ m})^2$$

$$\Rightarrow \quad I_A = 34.6\text{ kgm}^2$$

$$I_B = (1\text{ kg})(2.4\text{ m})^2 + (3\text{ kg})(2.4\text{ m}^2) = 23\text{ kgm}^2$$

3. Moment of inertia of rod 1 about axis P is

$$I_1 = \frac{ml^2}{3}$$



Moment of inertia of rod 2 about axis P,

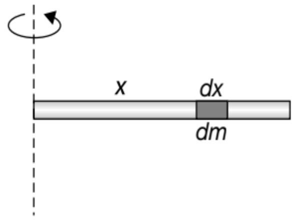
$$I_2 = \frac{ml^2}{12} + m\left(\sqrt{5}\frac{l}{2}\right)^2$$

So, moment of inertia of a system about axis P,

$$I = I_1 + I_2 = \frac{ml^2}{3} + \frac{ml^2}{12} + m\left(\sqrt{5}\frac{l}{2}\right)^2$$

$$\Rightarrow \quad I = \frac{ml^2}{3}$$

4. (a) $dI = x^2(dm) = x^2(\lambda_0 x)dx \qquad \{\because dm = \lambda dx\}$



$$\Rightarrow \quad I = \lambda_0 \int_0^{2L} x^3 dx$$

$$\Rightarrow \quad I = \lambda_0 \left(\frac{x^4}{4} \Big|_0^{2L} \right)$$

$$\Rightarrow \quad I = \frac{\lambda_0}{4}(16L^4)$$

$$\Rightarrow \quad I = 4\lambda_0 L^4 \qquad \ldots(1)$$

Now $dm = \lambda dx = \lambda_0 x dx$

$$\Rightarrow \quad M = \int dm = \lambda_0 \int_0^{2L} x dx$$

$$\Rightarrow \quad M = \lambda_0 \left(\frac{x^2}{2} \Big|_0^{2L} \right)$$

$$\Rightarrow \quad M = 2\lambda_0 L^2$$

So, $I = 2ML^2$

(b) Similarly, in this case

$$I = \frac{1}{3}ML^2$$

5. $I_1 = \frac{1}{12}ML^2 \qquad \ldots(1)$

Now, $L = 2\pi r$

$$\Rightarrow \quad r = \frac{L}{2\pi}$$

$$I_2 = 2Mr^2 = \frac{ML^2}{2\pi^2}$$

$$\Rightarrow \quad \frac{I_1}{I_2} = \left(\frac{\cancel{ML^2}}{12}\right)\left(\frac{2\pi^2}{\cancel{ML^2}}\right) = \frac{\pi^2}{6}$$

6. Using parallel axis theorem

$$I_1 = I_C + ma^2 \quad \ldots(1)$$

$$I_2 = I_C + mb^2 \quad \ldots(2)$$

From (1) and (2)

$$I_1 - I_2 = m(a^2 - b^2)$$

7. (a) $I = 0 + m(2d)^2 + m(\sqrt{2}d)^2 + m(\sqrt{2}d)^2$

$$\Rightarrow \quad I = 4md^2 + 2md^2 + 2md^2$$

$$\Rightarrow \quad I = 8md^2$$

(b) $I = 0 + m\ell^2 + m\ell^2 + m(\sqrt{2}\ell)^2$

$$\Rightarrow \quad I = 4m\ell^2$$

8. $I = I_{\text{total}} = I_{\text{can}} + 2I_{\text{lids}}$

$$\Rightarrow \quad I = MR^2 + 2\left(\frac{1}{2}mR^2\right)$$

$$\Rightarrow \quad I = (2\pi Rh\sigma)R^2 + (\pi R^2\sigma)R^2$$

$$\Rightarrow \quad I = \sigma\pi R^3(2h + R)$$

9. $I = I_{\text{total}} = I_{\text{rod}} + I_{\text{spheres}}$

$$\Rightarrow \quad I = \frac{1}{12}\left[m(3R)^2\right] + 2\left[\frac{2}{5}mR^2 + m\left(\frac{3R}{2} + R\right)^2\right]$$

$$\Rightarrow \quad I = \frac{3}{4}mR^2 + 2\left(\frac{2}{5}mR^2 + \frac{25}{4}mR^2\right)$$

$$\Rightarrow \quad I = mR^2\left(\frac{3}{4} + \frac{4}{5} + \frac{25}{2}\right)$$

$$\Rightarrow \quad I = \frac{mR^2}{20}(15 + 16 + 250)$$

$$\Rightarrow \quad I = \frac{mR^2}{20}(281)$$

$$\Rightarrow \quad I \cong 14mR^2$$

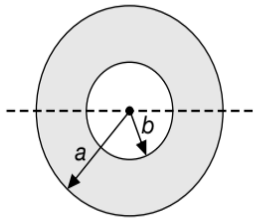
10. Given $M = \rho\left(\frac{4}{3}\pi a^3 - \frac{4}{3}\pi b^3\right)$

$$\Rightarrow \quad M = \frac{4}{3}\pi\rho(a^3 - b^3) \quad \ldots(1)$$

$$I = I_{\substack{\text{sphere of}\\ \text{radius } a}} - I_{\substack{\text{sphere of}\\ \text{radius } b}}$$

$$\Rightarrow \quad I = \frac{2}{5}\left[\left(\frac{4}{3}\pi a^3\right)\rho\right]a^2 - \frac{2}{5}\left[\left(\frac{4}{3}\pi b^3\right)\rho\right]b^2$$

$$\Rightarrow \quad I = \left(\frac{2}{5}\right)\left(\frac{4}{3}\pi\rho\right)(a^5 - b^5)$$



From (1), $\frac{4}{3}\pi\rho = \frac{M}{a^3 - b^3}$

$$\Rightarrow \quad I = \frac{2}{5}M\left(\frac{a^5 - b^5}{a^3 - b^3}\right)$$

11. Let σ be mass per unit area i.e., surface mass density, then

$$\sigma = \frac{M}{\pi(R^2 - a^2)} \quad \ldots(1)$$

Also $I = I_{\text{disc}} - I_{\text{cavity}}$

Now, about the said axis, we have

$$\Rightarrow \quad I_{\text{disc}} = \frac{1}{2}(\pi R^2\sigma)R^2 \text{ and}$$

By Parallel Axis Theorem, we have

$$I_{\text{cavity}} = \frac{1}{2}(\pi a^2\sigma)a^2 + (\pi a^2\sigma)b^2$$

$$\Rightarrow \quad I_{\text{cavity}} = \pi a^2\sigma\left(\frac{a^2}{2} + b^2\right)$$

$$\Rightarrow \quad I = \frac{1}{2}(\pi R^2\sigma)R^2 - \pi a^2\sigma\left(\frac{a^2}{2} + b^2\right)$$

$$\Rightarrow \quad I = \frac{1}{2}\pi\sigma(R^4 - a^4 - 2a^2b^2)$$

Using (1), we get

$$I = \frac{1}{2}M\left(\frac{R^4 - a^4 - 2a^2b^2}{R^2 - a^2}\right)$$

12. Moment of inertia of a semi-circular disc about an axis passing through centre and perpendicular to plane of disc, $I = \frac{MR^2}{2}$

Using parallel axis theorem $I = I_{\text{cm}} + Md^2$, d is the perpendicular distance between two parallel axis passing through centre C and CM.

$$I = \frac{MR^2}{2}, \; d = \frac{4R}{3\pi}$$

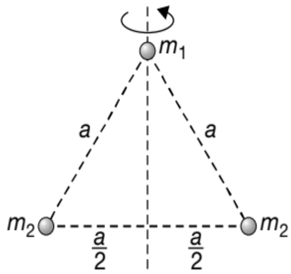
$$\Rightarrow \quad \frac{MR^2}{2} = I_{\text{cm}} + M\left(\frac{4R}{3\pi}\right)^2$$

$$\Rightarrow \quad I_{cm} = \left[\frac{MR^2}{2} - M\left(\frac{4R}{3\pi}\right)^2\right]$$

$$\Rightarrow \quad I_{cm} = MR^2\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)$$

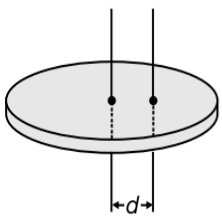
13. $I = m_1(0)^2 + m_2\left(\frac{a}{2}\right)^2 + m_3\left(\frac{a}{2}\right)^2$

$$\Rightarrow \quad I = \frac{a^2}{4}(m_1 + m_2)$$



14. $k = R$

$$\Rightarrow \quad I = Mk^2 = MR^2$$



By Parallel Axis Theorem, we have

$$I = I_G + Md^2$$

$$\Rightarrow \quad MR^2 = \frac{1}{2}MR^2 + Md^2$$

$$\Rightarrow \quad Md^2 = \frac{1}{2}MR^2$$

$$\Rightarrow \quad d = \frac{R}{\sqrt{2}}$$

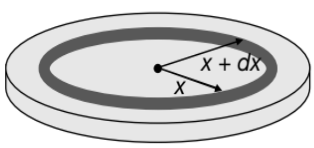
15. $dm = \sigma dA$

$$\Rightarrow \quad dm = (kx^2)(2\pi x dx) \qquad \ldots(1)$$

$$\Rightarrow \quad M = \int dm = 2\pi k\int_0^R x^3 dx$$

$$\Rightarrow \quad M = 2\pi k\left(\frac{R^4}{4}\right)$$

$$\Rightarrow \quad M = \frac{\pi k R^4}{2} \qquad \ldots(2)$$



If dI be the moment of inertia of element, then

$$dI = x^2 dm$$

$$\Rightarrow \quad dI = x^2(2\pi k x^3 dx)$$

$$\Rightarrow \quad I = 2\pi kR\int_0^R x^5 dx$$

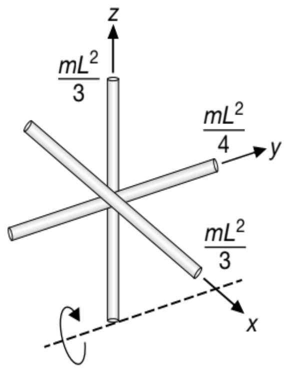
$$\Rightarrow \quad I = 2\pi k\left(\frac{R^6}{6}\right) = \frac{\pi k R^6}{3}$$

But from (2) $\pi k R^4 = 2M$

$$\Rightarrow \quad I = \frac{2MR^2}{3}$$

16. So, $I = \frac{mL^2}{3} + \frac{mL^2}{4} + \frac{mL^2}{3}$

$$\Rightarrow \quad I = \frac{11}{12}mL^2$$



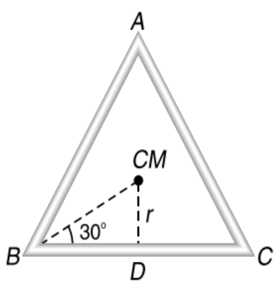
17. Moment of inertia of rod BC about an axis perpendicular to plane of triangle ABC and passing through the mid-point of rod BC (i.e., D) is

$$I_1 = \frac{m\ell^2}{12}$$

According to Parallel Axis Theorem, moment of inertia of this rod about the asked axis is

$$I_2 = I_1 + mr^2$$

$$\Rightarrow \quad I_2 = \frac{m\ell^2}{12} + m\left(\frac{\ell}{2\sqrt{3}}\right)^2 = \frac{m\ell^2}{6}$$



So, moment of inertia of the system is

$$I = 3I_2 = 3\left(\frac{m\ell^2}{6}\right)$$

$$\Rightarrow \quad I = \frac{m\ell^2}{2}$$

Since, $I = (M_{\text{total}})K^2 = \dfrac{m\ell^2}{2}$

$$\Rightarrow \quad (3m)K^2 = \frac{m\ell^2}{2}$$

$$\Rightarrow \quad K = \frac{\ell}{\sqrt{6}}$$

Test Your Concepts-II (Based on Rotational Kinematics, Combined Effect of Rotation and Translation Motion)

1. Angular velocity $\omega = \dfrac{d\theta}{dt} = \dfrac{d}{dt}(4t - 3t^2 + t^3)$

$$\Rightarrow \quad \omega = 4 - 6t + 3t^2$$

(a) At $t = 2$ s

$$\omega = 4 - (6)(2) + 3(2)^2$$

$$\Rightarrow \quad \omega = 4 \text{ rads}^{-1}$$

At $t = 4$ s

$$\omega = 4 - (6)(4) + 3(4)^2$$

$$\Rightarrow \quad \omega = 28 \text{ rads}^{-1}$$

(b) Average angular acceleration

$$\alpha_{av} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{28 - 4}{4 - 2}$$

$$\Rightarrow \quad \alpha_{av} = 12 \text{ rads}^{-2}$$

(c) Instantaneous angular acceleration is,

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(4 - 6t + 3t^2)$$

$$\Rightarrow \quad \alpha = -6 + 6t$$

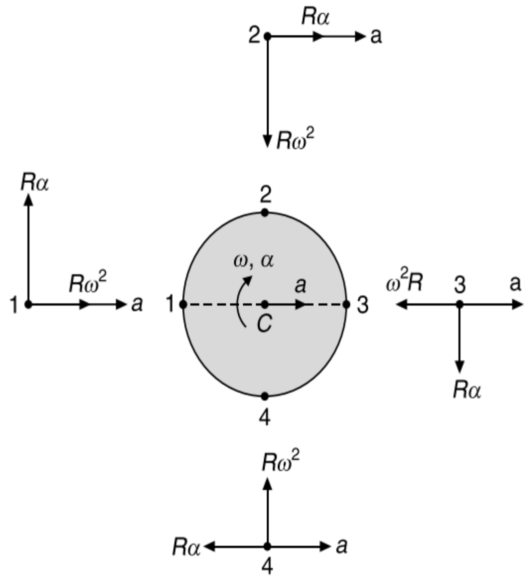
At $t = 2$ s, $\alpha = -6 + (6)(2) = 6 \text{ rads}^{-2}$

At $t = 4$ s, $\alpha = -6 + (6)(4) = 18 \text{ rads}^{-2}$

2. The motion of disc is both translational and rotational, so the acceleration of any point P on the disc can be expressed by

$$\vec{a}_P = \vec{a}_{PC} + \vec{a}_C$$

For all the specified points 1, 2, 3 and 4, the accelerations are shown in Figure.



The acceleration of point 1 is

$$\vec{a}_1 = \vec{a}_{1C} + \vec{a}_C$$

But $\vec{a}_{1C} = (\vec{a}_{1C})_{\text{tangential}} + (\vec{a}_{1C})_{\text{radial}}$

$$\Rightarrow \quad \vec{a}_{1C} = (R\alpha)\hat{j} + (R\omega^2)\hat{i} \text{ and } \vec{a}_C = a\hat{i}$$

$$\Rightarrow \quad \vec{a}_1 = (a + R\omega^2)\hat{i} + (R\alpha)\hat{j}$$

Similarly, the acceleration of point 2 is

$$\vec{a}_2 = \vec{a}_{2C} + \vec{a}_C$$

where, $\vec{a}_{2C} = (R\alpha)\hat{i} - (R\omega^2)\hat{j}$ and $\vec{a}_C = a\hat{i}$

$$\Rightarrow \quad \vec{a}_2 = (a + R\alpha)\hat{i} - (R\omega^2)\hat{j}$$

Acceleration for the point 3 is

$$\vec{a}_3 = \vec{a}_{3C} + \vec{a}_C$$

where, $\vec{a}_{3C} = -(R\omega^2)\hat{i} - (R\alpha)\hat{j}$ and $\vec{a}_C = a\hat{i}$

$$\Rightarrow \quad \vec{a}_3 = (a - R\omega^2)\hat{i} - (R\alpha)\hat{j}$$

Acceleration for the point 4 is

$$\vec{a}_4 = \vec{a}_{4C} + \vec{a}_C$$

where, $\vec{a}_{4C} = -(R\alpha)\hat{i} + (R\omega^2)\hat{j}$ and $\vec{a}_C = a\hat{i}$

$$\Rightarrow \quad \vec{a}_4 = (a - R\alpha)\hat{i} + (R\omega^2)\hat{j}$$

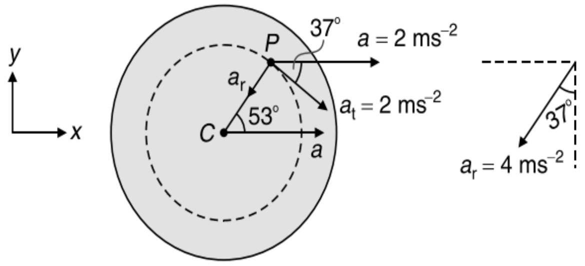
3. For particle at P, $r = CP = 1$ m

$$\Rightarrow \quad \alpha = \frac{d\omega}{dt} = \frac{d}{dt}(2t) = 2 \text{ rads}^{-2}$$

At $t = 1$ s, $\omega = 2 \text{ rads}^{-1}$ and $\alpha = 2 \text{ rads}^{-2}$

So, $a_t = r\alpha = 2 \text{ ms}^{-2}$, $a_r = r\omega^2 = 4 \text{ ms}^{-2}$, $a = 2 \text{ ms}^{-2}$

Net acceleration of P is the vector sum of three terms a, a_r and a_t as shown in Figure below.



$$\Rightarrow \quad \vec{a}_P = 2\hat{i} + \left(2\cos 37°\hat{i} - 2\sin 37°\hat{j}\right) + \left(-4\sin 37°\hat{i} - 4\cos 37°\hat{j}\right)$$

$$\Rightarrow \quad \vec{a}_P = 2\hat{i} + 1.6\hat{i} - 1.2\hat{j} - 2.4\hat{i} - 3.2\hat{j}$$

$$\Rightarrow \quad \vec{a}_P = \left(1.2\hat{i} - 4.4\hat{j}\right)\ ms^{-2}$$

4. Here $\alpha = \pi\ rads^{-2}$

$\omega_0 = 0$

$t = 4\ s$

(a) $\omega = 0 + \left(\pi\ rads^{-2}\right) \times 4\ s = 4\pi\ rads^{-1}$

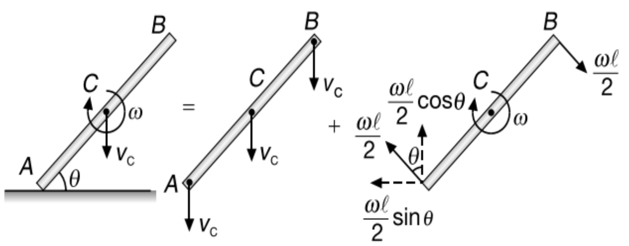
(b) $\theta = \omega_0 t + \frac{1}{2}\left(\pi\ rads^{-2}\right) \times \left(16\ s^2\right) = 8\pi\ rad$

(c) Let the number of turns be n

$\Rightarrow \quad n(2\pi)\text{rad} = 8\pi\ \text{rad}$

$\Rightarrow \quad n = 4$

5. The motion of the rod can be considered as super position of pure translation and pure rotation.



As end A, the rod slide along horizontal surface, the component of its velocity normal to ground should be zero. So, we get

$$-v_C + \frac{\omega l}{2}\cos\theta = 0$$

$$\Rightarrow \quad v_C = \frac{\omega l}{2}\cos\theta$$

6. (a) The tangential acceleration is constant and given by

$$a_T = \alpha r = \left(60\ rads^{-2}\right)(0.2\ m) = 12\ ms^{-2}$$

In order to calculate the radial acceleration, we first need to find the angular velocity at the given time. So,

$$\omega = \omega_0 + \alpha t = 0 + \left(60\ rads^{-2}\right)(0.15\ s)$$

$$\Rightarrow \quad \omega = 9\ rads^{-1}$$

Since $a_C = r\omega^2$, so

$$a_C = \omega^2 r = \left(81\ rad^2 s^{-2}\right)(0.2\ m) = 16.2\ ms^{-2}$$

The magnitude of the net linear acceleration is

$$a = \sqrt{a_C^2 + a_T^2} = 20.2\ ms^{-2}$$

(b) Since $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(60\ rads^{-2}\right)(0.25\ s)^2$

$$\Rightarrow \quad \theta = 1.88\ rad$$

$$\Rightarrow \quad n = \frac{\theta}{2\pi} = \frac{1.88}{2\pi} = 0.3\ rev$$

7. Given that $\omega_0 = 0.2\ rads^{-1}$, $R = 10\ cm = 0.1\ m$. Since $\vec{r}$ rotates relative to O with constant angular velocity ω_0. So angular velocity relative to C is

$$\omega_c = 2\omega_0 = 0.4\ rads^{-1}$$

Modulus of velocity $|\vec{v}| = R\omega_c = (0.1)(0.4)$

$$\Rightarrow \quad |\vec{v}| = 0.04\ ms^{-1} = 4\ cms^{-1}$$

Modulus of total acceleration is

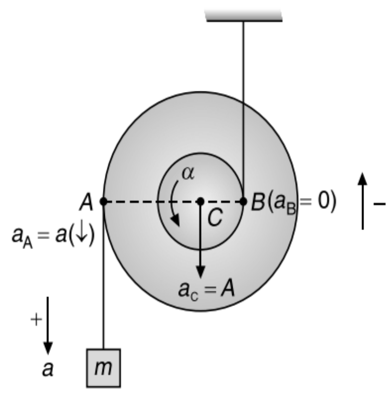
$$|\vec{a}| = R\omega_c^2$$

$$\Rightarrow \quad |\vec{a}| = (0.1)(0.4)^2 = 0.016\ ms^{-2}$$

$$\Rightarrow \quad |\vec{a}| = 1.6\ cms^{-2}$$

The direction of its total acceleration (centripetal acceleration) will be towards centre C.

8. Let the angular acceleration of the bobbin is α as shown in Figure.



Point B is directly connected with roof through string. It means net acceleration of point B should be zero.

$$\vec{a}_B = \vec{a}_{BC} + \vec{a}_C$$

$$\Rightarrow \quad 0 = -\alpha r + A$$

$$\Rightarrow \quad \alpha = \frac{A}{r} \qquad \ldots(1)$$

Similarly, point A on the bobbin is directly connected with block. This means that the acceleration of point A should be same as the acceleration of the block. So, we have

$$\vec{a}_A = \vec{a}_{AC} + \vec{a}_C$$

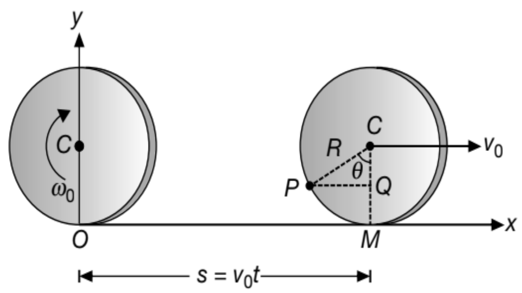
$$\Rightarrow \quad a = \alpha r + A \qquad \ldots(2)$$

From equations (1) and (2), we get

$$a = \left(\frac{A}{r}\right)R + A$$

$$\Rightarrow \quad \frac{A}{a} = \frac{r}{R+r}$$

9. At time t the bottommost point will rotate an angle $\theta = \omega_0 t$ with respect to the centre of the disc C. The centre C will travel a distance $s = v_0 t$.



From the figure, $PQ = R\sin\theta = R\sin(\omega_0 t)$

and $CQ = R\cos\theta = R\cos(\omega_0 t)$

Coordinates of point P at time t are

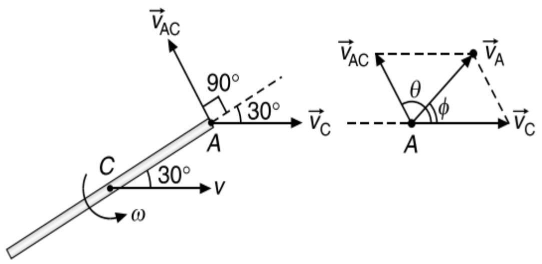
$$x = OM - PQ = v_0 t - R\sin\omega_0 t$$

and $y = CM - CQ = R - R\cos\omega_0 t$

So, $P(x, y) \equiv \left(v_0 t - R\sin(\omega_0 t),\, R - R\cos(\omega_0 t)\right)$

10. The point A is translating as well as rotating, so the velocity of the end A is the combined effect of rotation and translation. The combined velocity of end A is given by

$$\vec{v}_A = \vec{v}_{AC} + \vec{v}_C$$



where, $\omega = \dfrac{2v}{l}$, $v_C = v_{cm} = v$ and

$$v_{AC} = \frac{l\omega}{2} = \left(\frac{l}{2}\right)\left(\frac{2v}{l}\right) = v$$

So, the resultant velocity of A is given by

$$v_A = \sqrt{v_{AC}^2 + v_C^2 + 2v_{AC}v_C\cos\theta}$$

where, $\theta = 90° + 30° = 120°$

$$\Rightarrow \quad v_A = \sqrt{v^2 + v^2 + 2v^2\cos 120°} = v$$

If $\vec{v}_A$ makes angle ϕ with the direction of $\vec{v}_C$, then

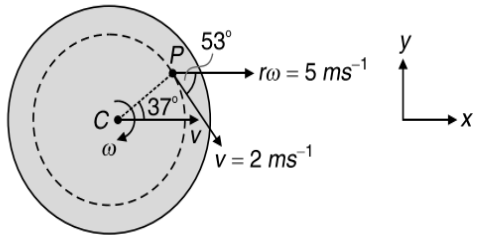
$$\phi = \tan^{-1}\left(\frac{v_{AC}\sin\theta}{v_{AC}\cos\theta + v_C}\right)$$

$$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{v\sin 120°}{v\cos 120° + v}\right) = \frac{\pi}{3}$$

11. For particle P, $r = CP = 1$ m

$$\Rightarrow \quad r\omega = (1)(5) = 5 \text{ ms}^{-1}$$

Net velocity of P is the vector sum of v and $r\omega$ as shown in Figure.



$$\vec{v}_P = 2\hat{i} + \left(5\cos 53°\hat{i} - 5\sin 53°\hat{j}\right)$$

$$\Rightarrow \quad \vec{v}_P = 2\hat{i} + 3\hat{i} - 4\hat{j} = \left(5\hat{i} - 4\hat{j}\right) \text{ ms}^{-1}$$

Test Your Concepts-III (Based on Instantaneous Axis of Rotation, Pure Rolling and Conservation of Energy)

1. For pure rolling, we have

$$\frac{K_R}{K_T} = \frac{k^2}{r^2} = \frac{2}{5}$$

So, translational kinetic energy at topmost point is

$$K_T = \frac{5}{7}mg(h - 2R)$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{5}{7}mg(h - 2R)$$

$$\Rightarrow \quad v^2 = \frac{10}{7}g(h - 2R) \qquad \ldots(1)$$

For the ball not to leave the track, we have

$$mg = \frac{mv^2}{R} \qquad \ldots(2)$$

Solving, equation (1) and (2), we have

$$h = 2.7R$$

In case of frictionless track, there will be no rotational kinetic energy, so

$$\frac{1}{2}mv^2 = mg(h - 2R)$$

$$\Rightarrow \quad v^2 = 2g(h - 2R) \quad \ldots(3)$$

Solving equation (2) and (3), we get

$$h = 2.5R$$

2. If m be the mass of the ball, then total kinetic energy at B is $E_B = mgh$

The ratio of rotational to translational kinetic energy would be,

$$\frac{K_R}{K_T} = \frac{r^2}{k^2} = \frac{2}{5}$$

$$\Rightarrow \quad K_R = \frac{2}{7}mgh \text{ and } K_T = \frac{5}{7}mgh$$

In portion BC, friction is absent, so rotational kinetic energy will remain constant and translational kinetic energy will get converted to potential energy. So, if H be the height to which ball climbs in BC, then

$$mgH = K_T$$

$$\Rightarrow \quad mgH = \frac{5}{7}mgh$$

$$\Rightarrow \quad H = \frac{5}{7}h$$

3. Since speed of rod is v, so speed of centre of mass of the cylinder and the sphere is $v/2$.
For a body rolling without slipping, the kinetic energy is

$$K = K_T + K_R = \frac{1}{2}mv_{cm}^2\left(1 + \frac{R^2}{K^2}\right)$$

So, for the rolling sphere and the cylinder, we have

$$K_{cylinder} = \frac{1}{2}m_2\left(\frac{v}{2}\right)^2\left(1 + \frac{1}{2}\right) = \frac{3}{16}m_2v^2$$

and $K_{sphere} = \frac{1}{2}m_1\left(\frac{v}{2}\right)^2\left(1 + \frac{2}{5}\right) = \frac{7}{40}m_1v^2$

The motion of the rod is purely translational, so kinetic energy of rod is

$$K_{rod} = \frac{1}{2}mv^2$$

The total kinetic energy of the system is

$$K_{system} = K_{rod} + K_{cylinder} + K_{sphere}$$

$$\Rightarrow \quad K_{system} = \frac{1}{2}mv^2 + \frac{3}{16}m_2v^2 + \frac{7}{40}m_1v^2$$

$$\Rightarrow \quad K_{system} = \frac{1}{2}mv^2 + \frac{3}{16}(4m)v^2 + \frac{7}{40}(5m)v^2$$

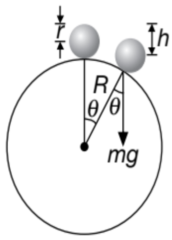
$$\Rightarrow \quad K_{system} = \frac{1}{2}mv^2 + \frac{3}{4}mv^2 + \frac{7}{8}mv^2$$

$$\Rightarrow \quad K_{system} = \frac{1}{2}mv^2\left(1 + \frac{3}{2} + \frac{7}{4}\right) = \frac{17}{8}mv^2$$

4. The equation of motion for the centre of the sphere at the moment of breaking off, $N = 0$ is

$$\frac{mv^2}{R + r} = mg\cos\theta \quad \ldots(1)$$

where v is the speed of the centre of the sphere at that moment and θ is the corresponding angle. The speed v can be found by using the Law of Conservation of Energy, according to which



$$mgh = \frac{mv^2}{2} + \frac{I\omega^2}{2}$$

where $I = \frac{2}{5}mr^2$, $v = r\omega$

and $h = (R + r)(1 - \cos\theta)$

From these equations we get

$$\omega = \sqrt{\frac{10g(R + r)}{17r^2}}$$

5. When the bead comes to its lowest position, then change in gravitational potential energy of the bead is

$$\Delta U = -mgh = -mg(2R) = -2mgR \quad \ldots(1)$$

Similarly, change in kinetic energy will take place both for the bead and the disc, so we have

$$\Delta K = \Delta K_{bead} + \Delta K_{disc}$$

$$\Rightarrow \quad \Delta K = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\omega^2$$

Since $\omega = \frac{v}{R}$

$$\Rightarrow \quad \Delta K = \frac{mv^2}{2} + \frac{mv^2}{4}$$

$$\Rightarrow \quad \Delta K = \frac{3}{4}mv^2 \quad \ldots(2)$$

By Law of Conservation of Energy, we have

$$\Delta U + \Delta K = 0$$

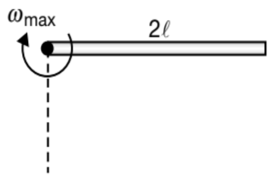
CHAPTER 3

$$\Rightarrow \quad -2mgR + \frac{3}{4}mv^2 = 0$$

$$\Rightarrow \quad v = \sqrt{\frac{8}{3}gR}$$

6. Angular velocity will be maximum, when the rod is in the vertical position. Then by Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in Gravitational}\\ \text{Potential Energy of}\\ \text{Centre of Mass of}\\ \text{the Rod}\end{pmatrix} = \begin{pmatrix}\text{Gain in Rotational}\\ \text{Kinetic Energy}\\ \text{of the}\\ \text{Rod}\end{pmatrix}$$



$$mgh = \frac{1}{2}I\omega_{max}^2$$

where, $h = \frac{2\ell}{2} = \ell$

$$\Rightarrow \quad mg(\ell) = \frac{1}{2}\frac{(m)(2\ell)^2}{3}\omega_{max}^2$$

$$\Rightarrow \quad \omega_{max} = \sqrt{\frac{3g}{2\ell}}$$

7. By Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in Gravitational}\\ \text{Potential Energy}\\ \text{of the point mass}\end{pmatrix} = \begin{pmatrix}\text{Gain in Rotational}\\ \text{Kinetic Energy of}\\ point\,mass + Disc\end{pmatrix}$$

$$\Rightarrow \quad mg(R) = \frac{1}{2}\left(\frac{1}{2}MR^2 + mR^2\right)\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{4mg}{(2m+M)R}}$$

8. If v_{cm} be the velocity of the centre of mass of the rod, then for end A we have

$$v_A = v = \left(\frac{l}{2}\right)\omega - v_{cm} \quad \ldots(1)$$

Similarly, for end B, we have

$$v_B = 2v = v_{cm} + \left(\frac{l}{2}\right)\omega \quad \ldots(2)$$

Solving Equations (1) and (2), we have

$$\omega = \frac{3v}{l} \text{ and } v_C = \frac{v}{2}$$

Angular velocity of the rod can also be directly calculated as

$$\omega = \frac{v_{rel}}{r_\perp} = \frac{3v}{l}$$

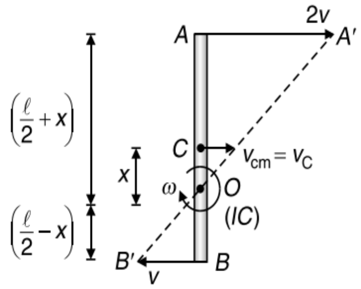
Since, we know that the kinetic energy

$$K = K_T + K_R = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

$$\Rightarrow \quad K = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}\left(\frac{ml^2}{12}\right)\left(\frac{3v}{l}\right)^2$$

$$\Rightarrow \quad K = \frac{1}{2}mv^2$$

Alternatively, we can calculate angular velocity of the rod and velocity of centre of mass by IC method. Let the IC of the rod be at a distance x from the centre of mass of the rod as shown in Figure.



Since the rod will be in pure rotation about the IC so for end A, we have

$$v = \left(\frac{l}{2} - x\right)\omega \quad \ldots(3)$$

and for end B, we have

$$2v = \left(\frac{l}{2} + x\right)\omega \quad \ldots(4)$$

From equations (3) and (4), we get

$$x = \frac{l}{6} \text{ and } \omega = \frac{3v}{l}$$

Also, the velocity of centre of mass is given by

$$v_C = x\omega = \left(\frac{l}{6}\right)\left(\frac{3v}{l}\right) = \frac{v}{2}$$

Rotational kinetic energy of the rod is

$$K_R = \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}\left(\frac{ml^2}{12}\right)\left(\frac{3v}{l}\right)^2 = \frac{3}{8}mv^2$$

Translational kinetic energy of the rod is

$$K_T = \frac{1}{2}mv_{cm}^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{1}{8}mv^2$$

Total kinetic energy of the rod is

$$K = K_R + K_T$$

$$\Rightarrow \quad K = \frac{3}{8}mv^2 + \frac{1}{8}mv^2 = \frac{1}{2}mv^2$$

Total kinetic energy of the rod also equals the rotational kinetic energy of the rod about the instantaneous centre of zero velocity (IC) and is given by

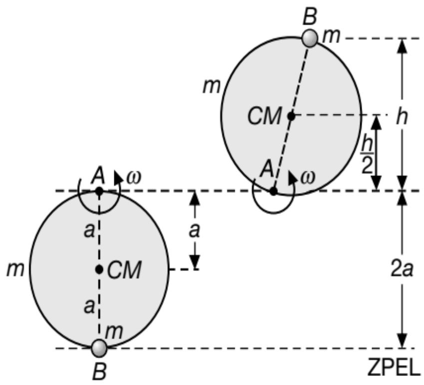
$$K = \frac{1}{2} I_{IC}\omega^2 = \frac{1}{2}\left(I_{cm} + mx^2\right)\omega^2$$

$$\Rightarrow \quad K = \frac{1}{2}\left[\frac{ml^2}{12} + m\left(\frac{l}{6}\right)^2\right]\left(\frac{3v}{l}\right)^2$$

$$\Rightarrow \quad K = \frac{1}{2}mv^2$$

9. The initial angular velocity imparted to the system is

$$\omega = \frac{v}{2a} = \frac{3\sqrt{ga}}{2a} = \frac{3}{2}\sqrt{\frac{g}{a}}$$



By Law of Conservation of Mechanical Energy, we have

$$\begin{pmatrix}\text{Loss in}\\ \text{RKE of}\\ \text{System}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{GPE of}\\ \text{Particle}\end{pmatrix} + \begin{pmatrix}\text{Gain in}\\ \text{GPE of}\\ \text{CM of Ring}\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}\left(m(2a)^2 + 2ma^2\right)\left(\frac{3}{2}\sqrt{\frac{g}{a}}\right)^2 = mg(2a+h) + mg\left(a + \frac{h}{2}\right)$$

$$\Rightarrow \quad \frac{27}{4}mga = 3mga + \frac{3mgh}{2}$$

$$\Rightarrow \quad h = 2.5a$$

10. $mgh = K_R + K_T = \frac{3}{4}mv^2$

where $h = s\sin\theta$

$$\Rightarrow \quad gs\sin\theta = \frac{3}{4}v^2$$

$$\Rightarrow \quad s = \frac{3v^2}{4g\sin\theta} = \frac{(3)(4)}{(4)(10)\left(\frac{1}{2}\right)} = 0.6 \text{ m}$$

11. By Law of Conservation of Energy, we get

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega_0^2$$

Since $v = R\omega_0$ and $I = \frac{1}{2}mR^2$, so we get

$$kx^2 = m\left(R^2\omega_0^2\right) + \left(\frac{1}{2}mR^2\right)\omega_0^2$$

$$\Rightarrow \quad kx^2 = \frac{3}{2}mR^2\omega_0^2$$

$$\Rightarrow \quad x = R\omega_0\sqrt{\frac{3m}{2k}}$$

12. In rolling without sliding on a stationary ground, work done by friction is zero. Hence

$$\begin{pmatrix}\text{Work done by}\\ \text{Applied Force}\end{pmatrix} = \begin{pmatrix}\text{Change in KE of}\\ \text{Cradle and Disks}\end{pmatrix}$$

$$\Rightarrow \quad (30)(0.25) = \frac{1}{2}(9)(v^2) + 2\left(\frac{1}{2}(6)(v^2) + \frac{1}{2}\left(\frac{1}{2}\times 6\times r^2\right)\frac{v^2}{r^2}\right)$$

$$\Rightarrow \quad 7.5 = 13.5v^2$$

$$\Rightarrow \quad v = 0.745 \text{ ms}^{-1}$$

Test Your Concepts-IV (Based on Torque and Applications)

1. Decrease in the height of centre of mass of the rod

$$h = \frac{\ell}{2}(\cos 37^\circ - \cos 60^\circ)$$

$$\Rightarrow \quad h = \frac{\ell}{2}(0.8 - 0.5) = 0.15\ell$$

By Law of Conservation of Mechanical Energy, we get

$$mgh = \frac{1}{2}I_O\omega^2 = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$$

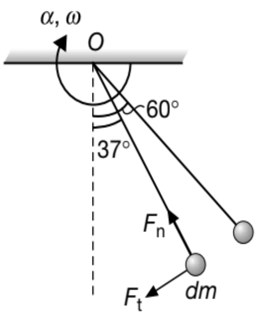
$$\Rightarrow \quad 0.15\ell mg = \left(\frac{m\ell^2}{6}\right)\omega^2$$

$$\Rightarrow \quad a_n = \ell\omega^2 = 0.9g \qquad \ldots(1)$$

Angular acceleration,

$$\alpha = \frac{\tau_O}{I_O} = \frac{\left(mg\frac{\ell}{2}\right)\sin(37^\circ)}{\frac{1}{3}m\ell^2}$$

$$\Rightarrow \quad a_t = \ell\alpha = 0.9g \qquad \ldots(2)$$



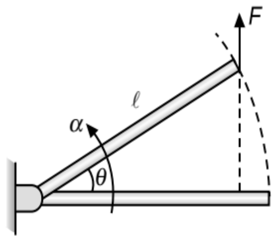
Normal force, $F_n = (dm)\ell\omega^2 = 0.9g dm = 9dm$

Tangential force, $F_t = (dm)\ell\alpha = 0.9g dm = 9dm$

So, net force, $F = \sqrt{F_n^2 + F_t^2} = 9\sqrt{2}dm$

2. $\tau = F\ell\cos\theta$

Since, $\alpha = \frac{\tau}{I} = \frac{F\ell\cos\theta}{m\ell^2/3} = \frac{3F\cos\theta}{m\ell}$



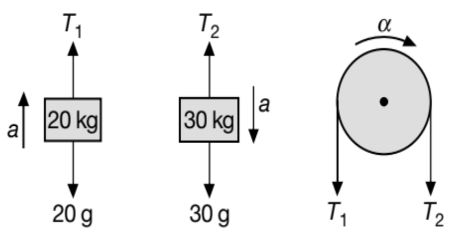
$$\Rightarrow \quad \omega\frac{d\omega}{d\theta} = \frac{3F\cos\theta}{m\ell}$$

$$\Rightarrow \quad \omega d\omega = \frac{3F\cos\theta}{m\ell}d\theta$$

$$\Rightarrow \quad \int_0^{\omega}\omega d\omega = \frac{3F}{m\ell}\int_0^{\phi}\cos\theta d\theta$$

$$\Rightarrow \quad \omega = \sqrt{\frac{6F\sin\phi}{m\ell}}$$

3. The free body diagrams of the bodies and the pulley are shown in Figure, then



for 20 kg mass, we get

$$T_1 - 20g = 20a \quad \ldots(1)$$

for 30 kg mass, we get

$$30g - T_2 = 30a \quad \ldots(2)$$

for pulley, we get

$$\alpha = \frac{(T_2 - T_1)R}{\frac{1}{2}mR^2} = \frac{2(T_2 - T_1)}{mR}$$

$$\Rightarrow \quad \alpha = \frac{2(T_2 - T_1)}{5R} = \frac{0.4(T_2 - T_1)}{R} \quad \ldots(3)$$

For no slipping, $a = R\alpha$ ...(4)

Solving the above equations, we have

$a = 1.87 \text{ ms}^{-2}$, $T_1 = 233.5$ N and

$T_2 = 238.2$ N

$$\Rightarrow \quad v = \sqrt{2as} = \sqrt{2\times1.87\times2} = 2.73 \text{ ms}^{-1}$$

$$\Rightarrow \quad \omega = \frac{v}{R} = \frac{2.73}{0.1} = 27.3 \text{ rads}^{-1}$$

$$\Rightarrow \quad t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times2}{1.87}} = 1.47 \text{ s}$$

4. At angle θ, we have by Law of Conservation of Energy

$$\frac{1}{2}I\omega^2 = mg\frac{\ell}{2}(1-\cos\theta)$$

$$\Rightarrow \quad \frac{1}{2}\frac{m\ell^2}{3}\omega^2 = \frac{mg\ell}{2}(1-\cos\theta)$$

$$\Rightarrow \quad \omega^2 = \frac{3g}{\ell}(1-\cos\theta) \quad \ldots(1)$$

Since $\alpha = \frac{\tau}{I}$

$$\Rightarrow \quad \alpha = \frac{mg\left(\frac{\ell}{2}\sin\theta\right)}{\frac{m\ell^2}{3}} = \frac{3g\sin\theta}{2\ell} \quad \ldots(2)$$

Since, $a_n = r\omega^2$

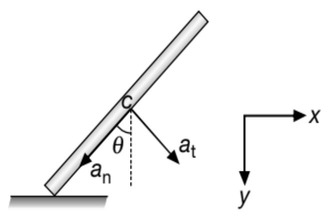
$$\Rightarrow \quad a_n = \frac{\ell}{2}\omega^2 = \frac{3g}{2}(1-\cos\theta)$$

and $a_t = r\alpha = \frac{\ell}{2}\alpha = \frac{3}{4}g\sin\theta$

$$f = ma_x = m(a_t\cos\theta - a_n\sin\theta)$$

$$\Rightarrow \quad f = m\left(\frac{3}{4}g\sin\theta\cos\theta - \frac{3g\sin\theta}{2}(1-\cos\theta)\right)$$

$$\Rightarrow \quad f = \frac{3}{2}mg\sin\theta\left(\frac{3}{2}\cos\theta - 1\right)$$



Further, $mg - N = ma_y$

$$\Rightarrow \quad N = m(g - a_y)$$

$$\Rightarrow \quad N = m[g - (a_t \sin\theta + a_n \cos\theta)]$$

$$\Rightarrow \quad N = m\left[g - \frac{3}{4}g\sin^2\theta - \frac{3g\cos\theta}{2}(1-\cos\theta)\right]$$

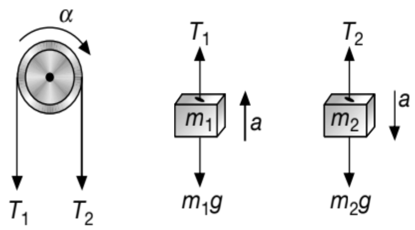
$$\Rightarrow \quad N = \frac{mg}{4}(4 - 3\sin^2\theta - 6\cos\theta + 6\cos^2\theta)$$

$$\Rightarrow \quad N = \frac{mg}{4}(1 - 3\cos\theta)^2$$

So, $N = 0$ at $\theta = \cos^{-1}\left(\frac{1}{3}\right)$

Hence, it will certainly slip beyond that.

5. Let α be the angular acceleration of the cylinder and a be the linear acceleration of two bodies



For mass m_1

$$T_1 - m_1 g = m_1 a \qquad \ldots(1)$$

For mass m_2

$$m_2 g - T_2 = m_2 a \qquad \ldots(2)$$

For cylinder

$$\alpha = \frac{(T_2 - T_1)R}{\frac{1}{2}mR^2} \qquad \ldots(3)$$

For no slipping condition, we have

$$a = R\alpha \qquad \ldots(4)$$

Solving these equations, we get

$$\alpha = \frac{2(m_2 - m_1)g}{(2m_1 + 2m_2 + m)R}$$

and $\dfrac{T_1}{T_2} = \dfrac{m_1(m + 4m_2)}{m_2(m + 4m_1)}$

6. (a) $F\cos\theta = Mg\sin\theta$

$$\Rightarrow \quad F = Mg\tan\theta$$

(b) $f = 0$

7. $\alpha = \dfrac{2mga}{\frac{1}{2}ma^2} = \dfrac{4g}{a}$

Since $\ell = r\theta$

$$\Rightarrow \quad \theta = \frac{4a}{a} = \frac{1}{2}\alpha t^2 = \frac{2g}{a}t^2 \qquad \{\because \ell = 4a\}$$

$$\Rightarrow \quad t = \sqrt{\frac{2a}{g}}$$

Since, $\omega = \alpha t = \dfrac{4g}{a}\sqrt{\dfrac{2a}{g}}$

$$\Rightarrow \quad \omega = 4\sqrt{\frac{2g}{a}}$$

8. Equations of motion are,

$$m_2 g\sin\theta - T = m_2 a \qquad \ldots(1)$$

$$\alpha = \frac{TR}{\frac{1}{2}m_1R^2} = \frac{2T}{m_1R} \qquad \ldots(2)$$

and $a = R\alpha \qquad \ldots(3)$

Solving these equations, we get

$$a = \left(\frac{2m_2 g\sin\theta}{2m_2 + m_1}\right) \text{ and } T = \left(\frac{m_1 m_2 g\sin\theta}{2m_2 + m_1}\right)$$

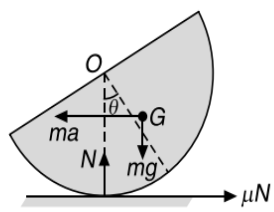
Total energy of system when m_2 is at a height h is mgh. When m_2 reaches the bottom total energy, due to energy conservation will still be mgh.

However, this mgh gets distributed to pulley as rotational kinetic energy $\left(= \frac{1}{2}I\omega^2\right)$ and to mass m_2 as translation kinetic energy $\left(= \frac{1}{2}m_2 v^2\right)$, so speed at bottom is

$$v = \sqrt{2as} = \sqrt{\left(\frac{4m_2 g\sin\theta}{2m_2 + m_1}\right)\left(\frac{h}{\sin\theta}\right)}$$

$$\Rightarrow \quad v = \sqrt{\frac{2gh}{1 + (m_1/2m_2)}}$$

9. Let a be the acceleration of truck. The free body diagram of hemisphere in the frame of reference of the truck is shown in Figure.



The centre of gravity is at G such that

$$OG = \frac{3r}{8}$$

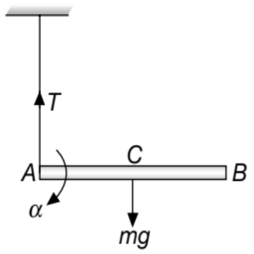
CHAPTER 3

The hemisphere is in equilibrium in this frame of reference. Hence, net moment of all the forces about bottommost point must be zero.

$$\Rightarrow \quad mg\left(\frac{3r}{8}\sin\theta\right) = ma\left(r - \frac{3r}{8}\cos\theta\right)$$

$$\Rightarrow \quad a = \frac{3g\sin\theta}{8 - 3\cos\theta}$$

10. $\alpha = \frac{\tau}{I} = \frac{mg\left(\frac{\ell}{2}\right)}{\frac{m\ell^2}{3}} = \frac{3}{2}\frac{g}{\ell}$



(a) $a_B = \ell\alpha = \frac{3g}{2}$

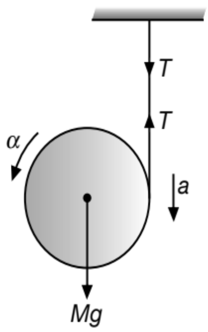
(b) $a_C = \frac{\ell}{2}\alpha = \frac{3g}{4}$

(c) Since $mg - T = ma_C$

$$\Rightarrow \quad mg - T = m\left(\frac{3g}{4}\right)$$

$$\Rightarrow \quad T = \frac{mg}{4}$$

11. From the free body diagram, we get



$$a = \frac{Mg - T}{M} \quad \ldots(1)$$

$$\alpha = \frac{TR}{\frac{1}{2}MR^2} = \frac{2T}{MR} \quad \ldots(2)$$

$$a = R\alpha \quad \ldots(3)$$

Solving these three equations, we get

$$a = \frac{2g}{3} \text{ and } T = \frac{Mg}{3}$$

12. $\alpha = \frac{\tau}{I} = \frac{4mga}{\frac{1}{2}ma^2} = \frac{8g}{a}$

Since $\omega = \omega_0 + \alpha t$

$$\Rightarrow \quad \omega = \alpha t = \frac{32g}{a} \qquad \{\because \omega_0 = 0\}$$

13. (a) $W = \Delta K$

$$\Rightarrow \quad \tau\theta = \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad (40 \times 0.25)\left(\frac{5}{0.25}\right) = \frac{1}{2} \times 4 \times \omega^2$$

$$\Rightarrow \quad 3\omega^2 = 200$$

$$\Rightarrow \quad \omega = 8.16 \text{ rads}^{-1}$$

(b) Mass of the block is $m = \frac{40}{10} = 4$ kg

By Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in}\\ \text{GPE of}\\ \text{4 kg mass}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{KE of}\\ \text{4 kg mass}\end{pmatrix} + \begin{pmatrix}\text{Gain in}\\ \text{RKE}\\ \text{of pulley}\end{pmatrix}$$

$$\Rightarrow \quad mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad mgh = \frac{1}{2}m(R\omega)^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad (4)(10)(5) = \frac{1}{2}(4)\left(\frac{1}{16}\right)\omega^2 + \frac{1}{2}(6)\omega^2$$

$$\Rightarrow \quad 200 = \frac{\omega^2}{8} + 3\omega^2$$

$$\Rightarrow \quad \omega^2 = 64$$

$$\Rightarrow \quad \omega = 8 \text{ rads}^{-1}$$

The difference in answers is due to the fact that in the first case $T = 40$ N and in the second case $T < 40$ N.

14. Equations of motion for the discs are,

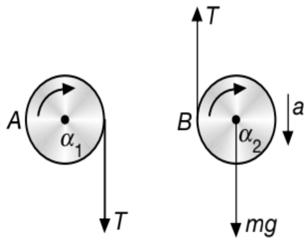
$$a = \frac{mg - T}{m} \quad \ldots(1)$$

$$\alpha_2 = \frac{TR}{mR^2/2} = \frac{2T}{mR} \quad \ldots(2)$$

Similarly, we get

$$\alpha_1 = \frac{TR}{mR^2/2} = \frac{2T}{mR} \quad \ldots(3)$$

$$\Rightarrow \quad \alpha_1 = \alpha_2$$



For no slipping, $a = R\alpha_2$ ...(4)

Further, $a - R\alpha_2 = R\alpha_1$ ...(5)

Solving these equations, we get

$$T = \frac{mg}{5},\ a = \frac{4g}{5} \text{ and } \alpha_2 = \frac{2g}{5R}$$

15. By Law of Conservation of Mechanical Energy,

$$\begin{pmatrix}\text{Loss in}\\ \text{GPE of}\\ \text{CM of}\\ \text{Ring}\end{pmatrix} + \begin{pmatrix}\text{Loss in}\\ \text{GPE of}\\ \text{Particle}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{RKE of}\\ \text{Ring and}\\ \text{Particle}\end{pmatrix}$$

$$\Rightarrow \quad mgr\sin 60° + mg(2r\sin 60°) = \frac{1}{2}\left(\frac{3mr^2}{2} + m(2r)^2\right)\omega^2$$

$$\Rightarrow \quad \frac{3\sqrt{3}mgr}{2} = \frac{11}{4}mr^2\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{6g\sqrt{3}}{11r}}$$

Since, $\tau = mgr\cos(60°) + mg2r\cos(60°)$

$$\Rightarrow \quad \tau = \frac{3}{2}mgr$$

$$\Rightarrow \quad \alpha = \frac{\tau}{I} = \frac{\frac{3}{2}mgr}{\left(\frac{3mr^2}{2} + m(2r)^2\right)}$$

$$\Rightarrow \quad \alpha = \frac{3mgr/2}{11mr^2/2} = \frac{3g}{11r}$$

16. Let us calculated the torque about O, so

$$\tau_0 = I_0\alpha$$

$$\Rightarrow \quad Mgx = \left(\frac{M\ell^2}{12} + Mx^2\right)\alpha$$

$$\Rightarrow \quad \alpha = \frac{gx}{k^2 + x^2}, \text{ where } k^2 = \frac{\ell^2}{12}$$

For α to be maximum, we have

$$\frac{d\alpha}{dx} = 0$$

$$\Rightarrow \quad x(-1)(k^2 + x^2)^{-2}(2x) + (k^2 + x^2)^{-1}(1) = 0$$

$$\Rightarrow \quad \frac{2x^2}{k^2 + x^2} = 1$$

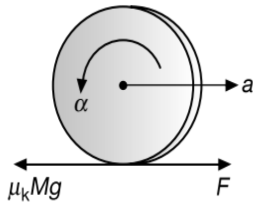
$$\Rightarrow \quad 2x^2 = \frac{\ell^2}{12} + x^2 \qquad \left\{\because k^2 = \frac{\ell^2}{12}\right\}$$

$$\Rightarrow \quad x = \frac{\ell}{2\sqrt{3}}$$

$$\Rightarrow \quad \alpha_{max} = \alpha\Big|_{x=\frac{\ell}{2\sqrt{3}}} = \frac{g\sqrt{3}}{\ell}$$

Test Your Concepts-V (Based on Uniform and Accelerated Pure Rolling)

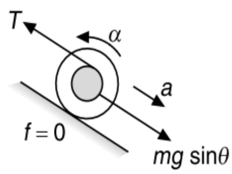
1.



$$a = \frac{F - \mu_k Mg}{M} = \frac{F}{M} - \mu_k g$$

$$\text{and } \alpha = \frac{\tau}{I} = \frac{(F - \mu_k Mg)R}{\frac{1}{2}MR^2} = \frac{2}{R}\left(\frac{F}{M} - \mu_k g\right)$$

2.



$$a = \frac{mg\sin\theta - T}{m} \quad ...(1)$$

$$\alpha = \frac{Tr}{I} \quad ...(2)$$

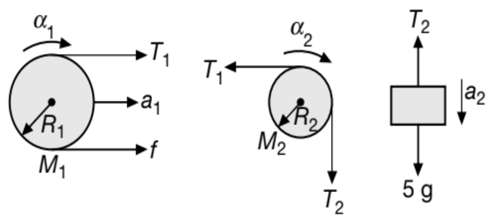
$$a = r\alpha \quad ...(3)$$

Solving these three equations, we get

$$a = \frac{g\sin\theta}{1 + \frac{I}{mr^2}}$$

NOTE: Compare the above result with rolling on a rough ground and think what are the similarities between the two.

3. Let us first draw the free body diagrams for cylinder, pulley and mass. Then



for 5 kg mass, $5g - T_2 = 5a_2$ ...(1)

for pulley, $\dfrac{(T_2 - T_1)R_2}{\frac{1}{2}M_2R_2^2} = \alpha_2$ ...(2)

for cylinder, $T_1 + f = M_1a_1$ ...(3)

$\dfrac{(T_1 - f)R_1}{\frac{1}{2}M_1R_1^2} = \alpha_1$ ...(4)

Also, $a_1 = R_1\alpha_1$ ...(5)

$a_2 = R_2\alpha_2 = a_1 + R_1\alpha_1$ ...(6)

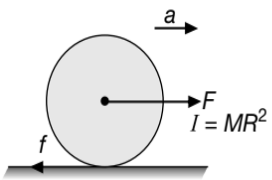
Solving above equations, we get

$$a_2 = \frac{4}{11}g$$

$$\Rightarrow \quad v = a_2t = \frac{4gt}{11}$$

4. For thin walled lawn roller, we have

$$I = MR^2$$



For pure rolling to take place, we have

$$a = R\alpha$$

Also $F - f = Ma$

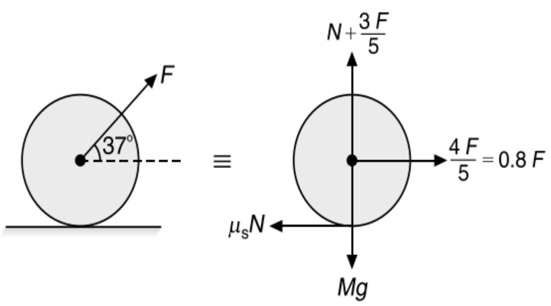
where $a = R\alpha = R\left(\dfrac{\tau}{I}\right)$

$$\Rightarrow \quad a = R\left(\frac{fR}{MR^2}\right) = \frac{f}{M}$$

$$\Rightarrow \quad f = \frac{F}{2}$$

and $a = \dfrac{F - f}{M} = \dfrac{F}{2M}$

5. The free body diagram for cylindrical wheel is shown in Figure (b).



(a) (b)

$$N = Mg - \frac{3F}{5} = Mg - 0.6F$$

For pure rolling $a = R\alpha$

$$\Rightarrow \quad \frac{0.8F - \mu_sN}{M} = \frac{(R)(\mu_sN)(R)}{\frac{1}{2}MR^2} = \frac{2\mu_sN}{M}$$

$$\Rightarrow \quad 0.8F = 3(0.4)(Mg - 0.6F)$$

$$\Rightarrow \quad F = 0.79Mg$$

Therefore, maximum value of F is $0.79Mg$

6. $\dfrac{I}{mR^2} = \dfrac{2}{5} = 0.4$ for sphere

$\dfrac{I}{mR^2} = \dfrac{1}{2} = 0.5$ for disc and $= 1$ for hoop

$$s = \frac{2}{\sin(30°)} = 4 \text{ m}$$

For sphere, we have

$$a = \frac{g\sin\theta}{1 + \dfrac{I}{mR^2}} = \frac{(9.8)\left(\frac{1}{2}\right)}{1 + 0.4} = 3.5 \text{ ms}^{-1}$$

$$\Rightarrow \quad v = \sqrt{2as} = \sqrt{2 \times 3.5 \times 4} = 5.29 \text{ ms}^{-1}$$

$$f = \frac{mg\sin\theta}{1 + \dfrac{mR^2}{I}} = \frac{3 \times 9.8 \times \frac{1}{2}}{1 + \left(\dfrac{1}{0.4}\right)} = 4.2 \text{ N}$$

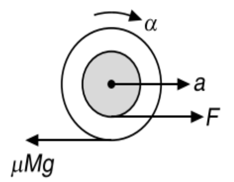
$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 4}{3.5}} = 1.51 \text{ s}$$

Similarly, the values for disc and hoop can be obtained.

7. Force of friction in this case will be backwards. For pure rolling,

$$a = R\alpha$$

$$\Rightarrow \quad \frac{1}{M}(F - \mu Mg) = R\left(\frac{\mu MgR - Fb}{\frac{1}{2}MR^2}\right)$$



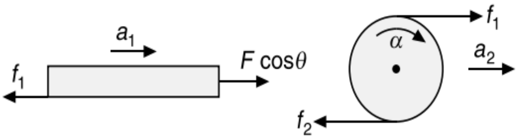
Solving this equation, we get

$$F = \frac{3\mu MgR}{R + 2b}$$

Thus, maximum value of F can be

$$F_{\max} = \frac{3\mu MgR}{R + 2b}$$

8. The free body diagrams for plank and cylinder are shown in Figure. Also



for plank, we have

$$F\cos\theta - f_1 = ma_1 \quad \ldots(1)$$

for cylinder, we have

$$f_1 - f_2 = Ma_2 \quad \ldots(2)$$

$$\alpha = \frac{(f_1 + f_2)R}{\frac{1}{2}MR^2} \quad \ldots(3)$$

$$a_2 = R\alpha \quad \ldots(4)$$

and $a_1 = a_2 + R\alpha$ ...(5)

Solving these equations, we get

$$a_2 = \frac{4F\cos\theta}{3M + 8m},\ f_1 = \frac{3MF\cos\theta}{3M + 8m}$$

and $f_2 = \dfrac{MF\cos\theta}{3M + 8m}$

9. Equation of motion for the block is

$$mg\sin\theta - T = ma$$

$\Rightarrow$ $(1)(9.8)\sin 30° - T = (1)a$

$\Rightarrow$ $T + a = 4.9$ ...(1)

Equation of motion for disk is

$$\frac{TR}{\frac{1}{2}MR^2} = \alpha$$

$\Rightarrow$ $\alpha = \dfrac{2T}{MR}$

For no slipping, we have

$$R\alpha = a = \frac{2T}{M}$$

$\Rightarrow$ $a = \dfrac{2T}{5}$

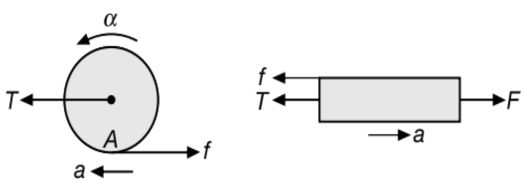
$\Rightarrow$ $T = 2.5a$

Substituting in equation (1), we get

$$3.5a = 4.9$$

$\Rightarrow$ $a = 1.4\ \text{ms}^{-2}$

10. Free body diagram for cylinder and plank are shown below, then



for plank, we have

$$F - T - f = Ma \quad \ldots(1)$$

for cylinder, we have

$$T - f = ma \quad \ldots(2)$$

$$\alpha = \frac{fR}{\frac{1}{2}mR^2} = \frac{2f}{mR} \quad \ldots(3)$$

for no slipping at A, we have

$$R\alpha - a = a$$

$\Rightarrow$ $R\alpha = 2a$ ...(4)

Solving these equations, we get

$$a = \frac{F}{3m + M}$$

11. At P, we have

$$h = (R - r)(1 - \cos\theta) \quad \ldots(1)$$

By Law of Conservation of Energy, we have

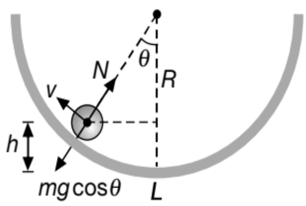
$$\begin{pmatrix}\text{Loss in KE of}\\ \text{Rolling}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{GPE}\end{pmatrix}$$

$\Rightarrow$ $\left(\frac{1}{2}mv_0^2 - \frac{1}{2}mv^2\right)\left(\frac{k^2}{R^2}\right) = mgh$

$\Rightarrow$ $\left(\frac{1}{2}mv_0^2 - \frac{1}{2}mv^2\right)\left(1 + \frac{2}{5}\right) = mgh$

$\Rightarrow$ $\frac{1}{2}mv_0^2 - \frac{1}{2}mv^2 = \frac{5}{7}mgh$

$\Rightarrow$ $v^2 = v_0^2 - \frac{10}{7}mgh$



Equation of motion at angle θ is,

$$N - mg\cos\theta = \frac{mv^2}{(R - r)}$$

$\Rightarrow$ $N = \dfrac{m}{(R - r)}\left(v_0^2 - \dfrac{10}{7}gh\right) + mg\cos\theta$

Substitute value of h from (1), we get

$$N = \left(\frac{m}{R - r}\right)\left(v_0^2 - \frac{10g}{7}(R - r)(1 - \cos\theta)\right) + mg\cos\theta$$

$\Rightarrow$ $F_n = N = \dfrac{mg}{7}(17\cos\theta - 10) + \dfrac{mv_0^2}{(R - r)}$

Tangential force is the force of friction given by

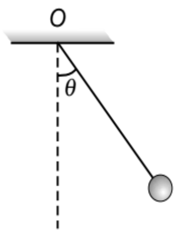
$$f = \frac{mg\sin\theta}{1+(k^2/r^2)} = \frac{mg\sin\theta}{1+(5/2)}$$

$$\Rightarrow \quad F_t = f = \frac{2}{7}mg\sin\theta$$

Test Your Concepts-VI (Based on Angular Momentum and its Conservation)

1. Let v_0 be the initial velocity and v its velocity in the position when the thread forms an angle $\frac{\pi}{2}$ with the vertical. The ball experiences two forces, the gravitational force and the tension of the thread. Moment of these forces is zero relative to the vertical axis passing through the point O. Consequently, the angular momentum of the ball relative to the given axis is constant. So, we have

$$mv_0\ell\sin\theta = mv\ell \quad \ldots(1)$$



The ball moves in the Earth's gravitational field under the influence of an external force, the tension of the thread. That force is always perpendicular to the velocity vector of the sphere and, therefore, does not perform any work. It follows that mechanical energy of the ball remains constant

$$\frac{mv_0^2}{2} = \frac{mv^2}{2} + mg\ell\cos\theta \quad \ldots(2)$$

Solving these two equations, we get

$$v_0 = \sqrt{2g\ell\sec\theta}$$

2. (a) Applying the Law of Conservation of Angular Momentum about the pivot, we get

$$\sum mvr_\perp = (I_{\text{system}})\omega$$

$$\Rightarrow \quad m_{\text{bullet}}v\left(\frac{L}{2}\right) = \left[\frac{m_{\text{rod}}L^2}{3} + m_{\text{bullet}}\left(\frac{L}{2}\right)^2\right]\omega$$

If m be the mass of the rod, then

$$m_{\text{bullet}} = \frac{m_{\text{rod}}}{6} = \frac{m}{6}$$

$$\Rightarrow \quad \left(\frac{m}{6}\right)v\left(\frac{L}{2}\right) = \left(\frac{mL^2}{3} + \frac{mL^2}{24}\right)\omega$$

$$\Rightarrow \quad \frac{mvL}{12} = \frac{9mL^2}{24}\omega$$

$$\Rightarrow \quad \omega = \frac{2v}{9L}$$

(b) The desired ratio is,

$$\frac{K_{\text{system}}}{K_{\text{bullet}}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}\left(\frac{m}{6}\right)v^2} = \frac{\left(\frac{9mL^2}{24}\right)\left(\frac{2v}{9L}\right)^2}{\left(\frac{mv^2}{6}\right)}$$

$$\Rightarrow \quad \frac{K_{\text{system}}}{K_{\text{bullet}}} = \frac{1}{9}$$

3. Since, net external torque on the system is zero. Therefore, angular momentum will remain conserved. So,

$$I_1\omega_1 = I_2\omega_2$$

$$\Rightarrow \quad \omega_2 = \frac{I_1\omega_1}{I_2}$$

where, $I_1 = I$, $\omega_1 = \omega_0$, $I_2 = I + mR^2$

$$\Rightarrow \quad \omega_2 = \frac{I\omega_0}{I+mR^2}$$

4. By Law of Conservation of Angular Momentum about centre of disc, we have

$$I_1\omega_1 = I_2\omega_2$$

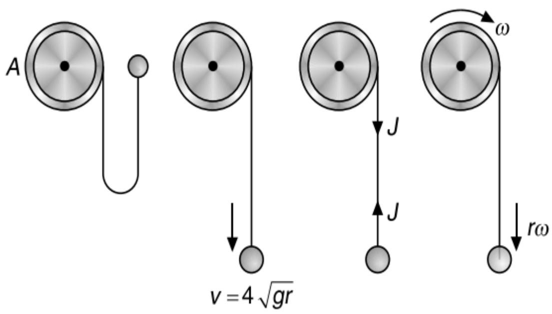
$$\Rightarrow \quad \omega_2 = \left(\frac{I_1}{I_2}\right)\omega_1 = \left[\frac{\frac{1}{2}ma^2}{\frac{1}{2}ma^2 + 2m\left(\frac{a}{2}\right)^2}\right]\omega$$

$$\Rightarrow \quad \omega_2 = \frac{\omega}{2}$$

5. Applying Law of Conservation of Angular Momentum about axis of rotation, we get

$$mvr = (mr\omega)r + I\omega$$

where, $I = \frac{1}{2}(2m)r^2$



$$\Rightarrow \quad \left(4m\sqrt{gr}\right)r = (mr\omega)r + \frac{1}{2}(2m)r^2\omega = 2mr^2\omega$$

$\Rightarrow \quad \omega = 2\sqrt{\frac{g}{r}}$

Considering the sudden change in the linear momentum of the particle caused by the impulsive tension J, we have, by

Impulse – Momentum Theorem,

Impulse = Change in Momentum

$\Rightarrow \quad J = -mr\omega - \left(-4m\sqrt{gr}\right)$

$\Rightarrow \quad J = 2m\sqrt{gr}$

6. (a) By Law of Conservation of Angular Momentum, we get

$$I_1\omega_1 = I_2\omega_2$$

$$\Rightarrow \quad \omega_2 = \left(\frac{I_1}{I_2}\right)\omega_0 = \frac{\left(\frac{1}{2}MR^2 + mR^2\right)}{\left(\frac{1}{2}MR^2\right)}\omega_0$$

$$\Rightarrow \quad \omega_2 = \left(1 + \frac{2m}{M}\right)\omega_0$$

(b) $W = E_f - E_i = \frac{1}{2}I_2\omega_2^2 - \frac{1}{2}I_1\omega_1^2$

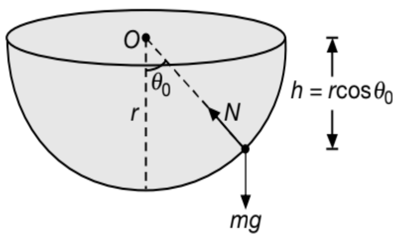
$$\Rightarrow \quad W = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(1 + \frac{2m}{M}\right)^2\omega_0^2 - \frac{1}{2}\left(\frac{1}{2}MR^2 + mR^2\right)\omega_0^2$$

$$\Rightarrow \quad W = \frac{1}{2}m\omega_0^2R^2\left(1 + \frac{2m}{M}\right)$$

7. The following two forces are acting on the particle.

(i) Normal Reaction (N)

(ii) Weight (mg)



The torque due to the normal reaction N about centre O is zero. The torque due to the weight is not zero, but its component along a vertical axis passing through O is zero. Hence, angular momentum of the particle about the same axis will remain conserved. So, we have

$$mv_0 r\sin\theta_0 = mvr \quad \ldots(1)$$

where, v is the speed of the particle at the topmost position.

Since friction is absent, so, mechanical energy will remain conserved

$$\Rightarrow \quad v^2 = v_0^2 - 2gh \quad \ldots(2)$$

Solving equation (1) and (2), we get

$$v_0 = \sqrt{2gr\sec\theta_0}$$

$$\Rightarrow \quad (v_0)_{min} = \sqrt{2gr\sec\theta_0}$$

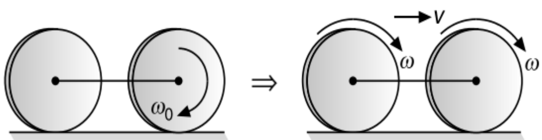
8. According to Angular Impulse – Angular Momentum Theorem, we have

$$J\ell = \left(I_{system}\right)\omega = \frac{m(2\ell)^2}{3}\omega$$

$$\Rightarrow \quad J = \frac{4m\ell\omega}{3}$$

9. By Law of Conservation of Angular Momentum, we have

$L_i = L_f$ {about bottom most point}



$$\Rightarrow \quad I\omega_0 = 2(I\omega + mRv)$$

$$\Rightarrow \quad \left(\frac{1}{2}mR^2\right)\omega_0 = 2\left(\frac{1}{2}mR^2\omega + mR(\omega R)\right)$$

$$\Rightarrow \quad \omega = \frac{\omega_0}{6}$$

$$\Rightarrow \quad v = \omega R = \frac{\omega_0 R}{6}$$

10. Applying Law of Conservation of Angular Momentum, Angular Velocity of disc is

$$\sum mvr_\perp = \left(I_{system}\right)\omega$$

$$\Rightarrow \quad m_1vR = \left(\frac{1}{2}m_2R^2 + m_1R^2\right)\omega$$

$$\Rightarrow \quad \omega = \frac{m_1R}{\frac{1}{2}m_2R^2 + m_1R^2}v = \left(\frac{2m_1}{m_2 + 2m_1}\right)\left(\frac{v}{R}\right)$$

where v is the relative speed of man with respect to disc.

So, $\frac{v}{R} = \omega_r$ (the relative angular speed of man)

$$\Rightarrow \quad \omega = \frac{2m_1}{(m_2 + 2m_1)}\cdot\omega_r$$

$$\Rightarrow \quad \frac{d\phi}{dt} = \left(\frac{2m_1}{m_2 + 2m_1}\right)\frac{d\theta}{dt}$$

Integrating we get,

$$\phi = \left(\frac{2m_1}{m_2 + 2m_1}\right)\theta$$

11. Applying conservation of angular momentum, we get

$$Mv_1R = I\left(\frac{v_2}{R}\right) + Mv_2R$$

$$\Rightarrow \quad v_2 = \frac{v_1}{1 + \frac{I}{MR^2}}$$

$$\Rightarrow \quad \omega = \frac{v_2}{R} = \frac{v_1}{R\left(1 + \frac{I}{MR^2}\right)}$$

12. Let J be the linear impulse applied at the end B and ω the angular speed of rod, then by Impulse – Momentum Theorem, we have

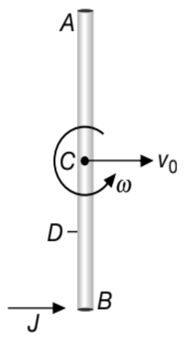
$$J = mv_0 \qquad \ldots(1)$$

Also, on applying Angular Impulse – Angular Momentum theorem, we get

$$J\left(\frac{\ell}{2}\right) = I\omega = \frac{m\ell^2}{12}\omega \qquad \ldots(2)$$

Solving these two equations, we get

$$\omega = \frac{6v_0}{\ell}$$



Linear speed of D (mid-point of CB i.e., centre of mass of lower half) relative to C is

$$v = \omega\left(\frac{\ell}{4}\right) = \frac{3}{2}v_0$$

Force exerted by upper half on the lower half,

$$F = \frac{mv^2/2}{l/4}$$

Substituting $v = \frac{3}{2}v_0$, we get

$$F = \frac{9mv_0^2}{2\ell}$$

13. Linear momentum, angular momentum and kinetic energy are conserved in the process.
Applying Law of Conservation of Linear Momentum, we get

$$MV = mv$$

$$\Rightarrow \quad V = \frac{m}{M}v \qquad \ldots(1)$$

Applying Law of Conservation of Angular Momentum about the centre of the rod, we get

$$\sum mvr = \left(I_{\text{system}}\right)\omega$$

$$\Rightarrow \quad mvd = \left(\frac{M\ell^2}{12}\right)\omega$$

$$\Rightarrow \quad \omega = \left(\frac{12mvd}{M\ell^2}\right) \qquad \ldots(2)$$

Since the collision is elastic, so $e = 1$

$$\Rightarrow \quad \begin{pmatrix}\text{Relative Speed}\\ \text{of Approach}\end{pmatrix} = \begin{pmatrix}\text{Relative Speed}\\ \text{of Separation}\end{pmatrix}$$

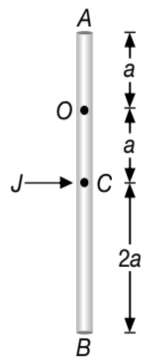
$$\Rightarrow \quad v = V + \omega d$$

Substituting the values, we get

$$v = \frac{m}{M}v + \frac{12mvd^2}{M\ell^2}$$

$$\Rightarrow \quad m = \frac{M\ell^2}{12d^2 + \ell^2}$$

14. Let ω be the angular velocity of rod just after the impulse is applied. Applying Angular Impulse Angular Momentum Theorem, we get



$$Ja = I\omega = \left(\frac{m(4a)^2}{12} + m\left(a^2\right)\right)\omega$$

$$\Rightarrow \quad \omega = \frac{3J}{7ma}$$

For describing the complete revolution

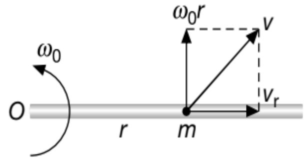
$$\frac{1}{2}I\omega^2 > mgh, \text{ where } h = 2a$$

⇒ $\frac{1}{2}\left(\frac{7}{3}ma^2\right)\left(\frac{3J}{7ma}\right)^2 > mg(2a)$

⇒ $J > 2m\sqrt{\frac{7ga}{3}}$

15. In the process of motion of the given system the kinetic energy and the angular momentum relative to the axis of rotation do not vary. So, by Law of Conservation of Energy, we have

$$\frac{1}{2}I\omega_0^2 = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$



⇒ $I\omega_0^2 = I\omega^2 + mv^2$ ...(1)

By Law of Conservation of Angular Momentum, we have

$I\omega_0 = \left(I + mr^2\right)\omega$ ...(2)

where $v^2 = v_r^2 + \omega_0^2 r^2$ ...(3)

Solving these equations, we get

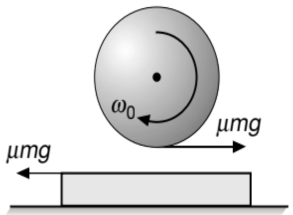
$$v_r = \frac{\omega_0 r}{\sqrt{1 + \frac{mr^2}{I}}}$$

Test Your Concepts-VII (Based on Rolling with Slipping)

1. Let, a_1 be the linear acceleration of sphere (towards right), a_2 be the linear acceleration of plank (towards left) and α be the angular retardation of sphere, then

$$a_1 = a_2 = \frac{\mu mg}{m} = \mu g$$

$$\alpha = \frac{\mu mg}{2mr^2/5} = \frac{5\mu g}{2r}$$



Let pure rolling start after time t, then for pure rolling to occur, we have

$\omega r - v = v$

⇒ $\omega r = 2v$

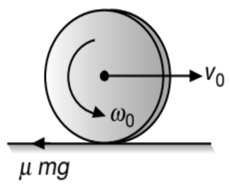
⇒ $(\omega_0 - \alpha t)r = 2(a_1 t)$

Substituting the values, we get

$$t = \frac{2r\omega_0}{9\mu g}$$

⇒ $s = \frac{1}{2}a_2 t^2 = \frac{2\omega_0^2 r^2}{81\mu g}$

2. Initially there is forward slipping so, the friction is backwards and maximum.



Let velocity becomes zero in time t_1 and angular velocity becomes zero in time t_2. Then, $0 = v_0 - at_1$

⇒ $t_1 = \frac{v_0}{a} = \frac{v_0}{\mu g}$ ...(1)

and $0 = \omega_0 - \alpha t_2$

⇒ $t_2 = \frac{\omega_0}{\alpha}$

where, $\alpha = \frac{\mu mgR}{mR^2/2} = \frac{2\mu g}{R}$

⇒ $t_2 = \frac{\omega_0 R}{2\mu g}$ ...(2)

Disk will return when

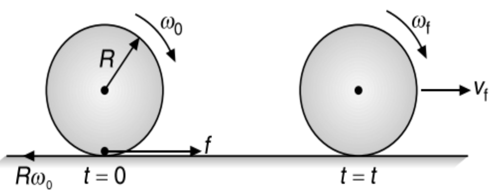
$t_2 > t_1$

⇒ $\frac{\omega_0 R}{2\mu g} > \frac{v_0}{\mu g}$

⇒ $\omega_0 > \frac{2v_0}{R}$

⇒ $(\omega_0)_{min} = \frac{2v_0}{R}$

3. The situation given in the problem is shown in Figure.



In this case sliding friction acts on cylinder in forward direction which increases its linear speed and decreases its angular speed. Let pure rolling start after time t, when final velocity be v_f and final angular velocity be ω_f, then we must have

$v_f = R\omega_f$ (as pure rolling starts)

For translational motion, applying the impulse momentum equation, we get

$$ft = mv_f - 0$$

Initial momentum of the cylinder is taken as zero as it does not have any translational speed.

$$\Rightarrow \quad \mu mgt = mv_f \qquad \ldots(1)$$

For rotational motion, applying the angular impulse angular momentum equation, we get

$$I\omega_0 - fRt = I\omega_f$$

$$\Rightarrow \quad \left(\frac{1}{2}mR^2\right)\omega_0 - \mu mgRt = \left(\frac{1}{2}mR^2\right)\frac{v_f}{R} \qquad \ldots(2)$$

Dividing equations (1) and (2), we get

$$\frac{\mu gt}{R\omega_0 - 2\mu gt} = 1$$

$$\Rightarrow \quad \mu gt = R\omega_0 - 2\mu gt$$

$$\Rightarrow \quad t = \frac{\omega_0 R}{3\mu g}$$

During sliding, friction is the only force acting on the cylinder and we know that the work done by the friction is always negative or loss in kinetic energy of system.

At $t = 0$, the kinetic energy of the cylinder is

$$E_i = \frac{1}{2}I\omega_0^2$$

$$\Rightarrow \quad E_i = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega_0^2 = \frac{1}{4}mR^2\omega_0^2 \qquad \ldots(3)$$

Finally, when pure rolling starts, kinetic energy of cylinder is

$$E_f = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 \qquad \ldots(4)$$

During sliding the acceleration of cylinder due to friction $(f = \mu mg)$ is $a = \frac{f}{m} = \mu g$, thus after time t it gains a velocity

$$v_f = \mu gt = \frac{1}{3}\omega_0 R$$

At this moment pure rolling starts, thus its angular velocity at this instant is

$$\omega_f = \frac{v_f}{R} = \frac{\mu gt}{R} = \frac{1}{3}\omega_0$$

From equation (4), final kinetic energy is

$$E_f = \frac{1}{2}m\left(\frac{R\omega_0}{3}\right)^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{\omega_0}{3}\right)^2$$

$$\Rightarrow \quad E_f = \frac{1}{12}m\omega_0^2R^2 \qquad \ldots(5)$$

According to Work Energy theorem, work done by friction is

$$W_{\text{friction}} = E_f - E_i$$

Substituting the values of E_i and E_f from equations (3) and (5), we get

$$W_{\text{friction}} = \frac{1}{12}m\omega_0^2R^2 - \frac{1}{4}m\omega_0^2R^2$$

$$\Rightarrow \quad W_{\text{friction}} = -\frac{1}{6}m\omega_0^2R^2$$

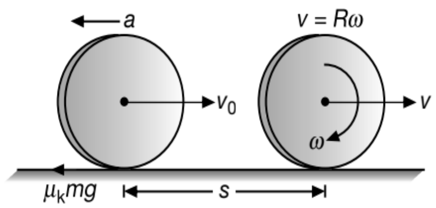
4. Applying Law of Conservation of Angular Momentum about bottommost point, we get

$$L_i = L_f$$

$$\Rightarrow \quad mv_0R = mvR + \left(\frac{2}{5}mR^2\right)\omega$$

$$\Rightarrow \quad mv_0R = mvR + \frac{2}{5}mvR \qquad \left\{\because \ \omega = \frac{v}{R}\right\}$$

$$\Rightarrow \quad v = \frac{5}{7}v_0$$



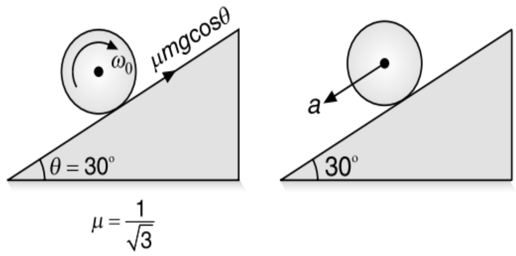
Since, $a = \frac{\mu_k mg}{m} = \mu_k g$ and $v^2 = v_0^2 - 2as$

$$\Rightarrow \quad s = \frac{v_0^2 - v^2}{2a} = \frac{v_0^2 - \frac{25}{49}v_0^2}{2\mu_k g}$$

$$\Rightarrow \quad s = \frac{12v_0^2}{49\mu_k g}$$

5. Till the pure rolling starts, friction (f_1) is upwards and it is maximum. Linear acceleration for $\mu = 1/\sqrt{3}$ is

$$a = \frac{\mu mg\cos 30^\circ - mg\sin 30^\circ}{m} = 0$$



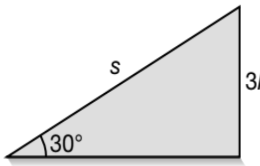
i.e., the cylinder will keep on rotating at the same place till its angular velocity becomes zero. Angular retardation, for $\mu = \frac{1}{\sqrt{3}}$ and $\theta = 30^\circ$ is

$$\alpha = \frac{(\mu mg\cos\theta)R}{mR^2/2} = \frac{g}{R}$$

If t_1 be the time when the angular velocity of cylinder becomes zero, then

$$0 = \omega_0 - \alpha t_1$$

$$\Rightarrow \quad t_1 = \frac{\omega_0}{\alpha} = \frac{\omega_0 R}{g} \qquad \ldots(1)$$



Once, the angular velocity becomes zero, cylinder starts rolling downwards with an acceleration

$$a = \frac{g\sin\theta}{1+(I/mR^2)} = \frac{g\sin 30^\circ}{1+(1/2)} = \frac{g}{3}$$

Also, $s = 3R\,\text{cosec}(30^\circ) = 6R$

Let t_2 be the time taken by the cylinder to reach the bottom, then

$$t_2 = \sqrt{\frac{2s}{a}} = \sqrt{\frac{12R}{g/3}} = 6\sqrt{\frac{R}{g}}$$

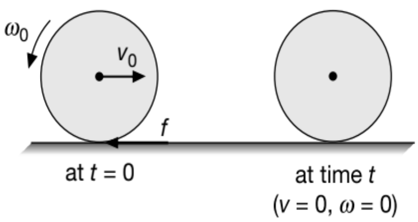
So, total time $t = t_1 + t_2$

$$\Rightarrow \quad t = \frac{\omega_0 R}{g} + 6\sqrt{\frac{R}{g}}$$

6. $a = \frac{f}{m} = \mu g$

$$\alpha = \frac{\tau}{I} = \frac{fR}{\frac{2}{5}MR^2} = \frac{\mu(Mg)R}{\frac{2}{5}MR^2}$$

$$\Rightarrow \quad \alpha = \frac{5\mu g}{2R}$$



Since $v = v_0 + (-\mu g)t$

$$\Rightarrow \quad 0 = v_0 - (\mu g)t$$

$$\Rightarrow \quad t = \frac{v_0}{\mu g} \qquad \ldots(1)$$

Also, $\omega = \omega_0 + \alpha t$

$$\Rightarrow \quad 0 = \omega_0 + \left(-\frac{5\mu g}{2R}\right)t$$

$$\Rightarrow \quad t = \frac{2R\omega_0}{5\mu g} \qquad \ldots(2)$$

Distance travelled is

$$s = v_0 t + \frac{1}{2}(-\mu g)t^2$$

$$\Rightarrow \quad s = v_0\left(\frac{v_0}{\mu g}\right) - \frac{1}{2}(\mu g)\frac{v_0^2}{\mu^2 g^2}$$

$$\Rightarrow \quad s = \frac{v_0^2}{2\mu g}$$

Equating (1) and (2), we get

$$\frac{v_0}{\mu g} = \frac{2R\omega_0}{5\mu g}$$

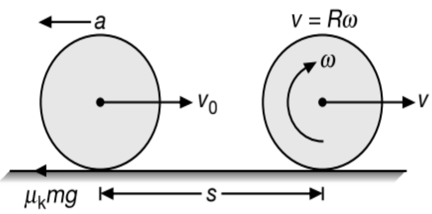
$$\Rightarrow \quad \omega_0 = \frac{5v_0}{2R}$$

7. From conservation of angular momentum about bottommost point.

$$L_i = L_f$$

$$\Rightarrow \quad mv_0R = mvR + \frac{2}{5}mR^2\cdot\omega$$

$$\Rightarrow \quad mv_0R = mvR + \frac{2}{5}mvR \qquad \left\{\because\ \omega = \frac{v}{R}\right\}$$



$$\Rightarrow \quad v = \frac{5}{7}v_0$$

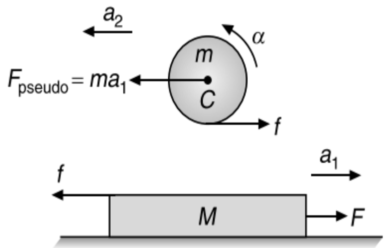
Since, $a = \frac{\mu_k mg}{m} = \mu_k g$

So, from $v^2 = v_0^2 - 2as$

We get $s = \frac{v_0^2 - v^2}{2a} = \frac{v_0^2 - \frac{25}{49}v_0^2}{2\mu_k g}$

$$\Rightarrow \quad s = \frac{12v_0^2}{49\mu_k g}$$

8. Since the cylinder does not slip over the plank, so it is the case of pure rolling and hence we can take friction on cylinder in any direction, say towards right as shown in Figure.



Since friction on cylinder is acting towards right, so it must act on the plank towards left. Let plank moves toward right with an acceleration a_1, due to which the cylinder will experience a pseudo force ma_1 towards left, and hence it will roll towards left with respect to plank with an acceleration a_2.

Since we have used the concept of pseudo force, so we must say that the acceleration a_2 must be with respect to the plank. Let the angular acceleration of the cylinder during rolling be α, so we have

$$a_2 = R\alpha$$

For translational motion of plank, we have

$$F - f = Ma_1 \quad \ldots(1)$$

For translational motion of cylinder with respect to plank we have

$$ma_1 - f = ma_2 \quad \ldots(2)$$

For rotational motion of cylinder with respect to plank, we have

$$fR = I\alpha$$

$$\Rightarrow \quad fR = \left(\frac{1}{2}mR^2\right)\left(\frac{a_2}{R}\right)$$

$$\Rightarrow \quad f = \frac{1}{2}ma_2 \quad \ldots(3)$$

From equation (2) and (3), we get

$$ma_1 - \frac{1}{2}ma_2 = ma_2$$

$$\Rightarrow \quad a_1 = \frac{3}{2}a_2 \quad \ldots(4)$$

Using equations (1), (2) and (3), we get

$$F - \frac{1}{2}ma_2 = \frac{3}{2}Ma_2$$

$$\Rightarrow \quad a_2 = \frac{2F}{3M + m}$$

From equation (4), we get

$$a_1 = \frac{3F}{3M + m}$$

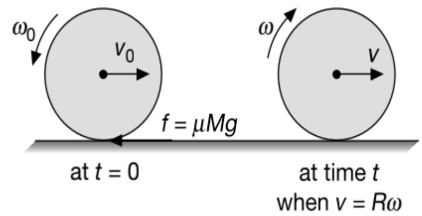
Since, a_2 is acceleration of cylinder relative to the plank, so net acceleration of the cylinder is

$$a_{net} = a_1 - a_2 = \frac{F}{3M + m}$$

Please note that in the equation $a_2 = R\alpha$, a_2 is the acceleration of cylinder with respect to plank. So,

$$\alpha = \frac{a_1 - a_2}{R} = \frac{F}{R(3M + m)}$$

9. Let us draw the situations at $t = 0$ and at time t.



Taking forward direction as positive and CW sense as positive, we get

$$a = \mu g$$

$$\alpha = \frac{\tau}{I} = \frac{\mu MgR}{\frac{2}{5}MR^2} = \frac{5\mu g}{2R}$$

Now, at time t, we have

$$v = v_0 - (\mu g)t \quad \ldots(1)$$

and $$\omega = -\omega_0 + \left(\frac{5\mu g}{2R}\right)t \quad \ldots(2)$$

Since at time t, we have

$$v = R\omega$$

$$\Rightarrow \quad v_0 - (\mu g)t = R\left[-\omega_0 + \left(\frac{5\mu g}{2R}\right)t\right]$$

$$\Rightarrow \quad t = \frac{2}{7}\left(\frac{v_0 + R\omega_0}{\mu g}\right)$$

So, speed of ball at time t is

$$v = v_0 - (\mu g)t$$

$$\Rightarrow \quad v = v_0 - \frac{2}{7}(v_0 + R\omega_0)$$

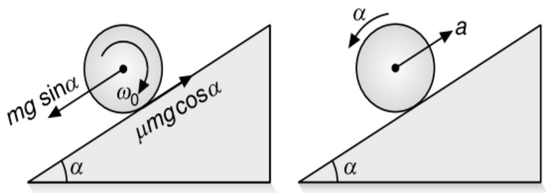
$$\Rightarrow \quad v = \frac{1}{7}(5v_0 + 2R\omega_0)$$

10. Given $\mu > \tan\alpha$

$$\Rightarrow \quad \mu mg\cos\alpha > mg\sin\alpha$$

Since, $a=(\mu g\cos\alpha - g\sin\alpha)$

Also, $\alpha=\dfrac{\tau}{I}=\dfrac{(\mu mg\cos\alpha)r}{\frac{1}{2}mr^2}=\dfrac{2\mu g\cos\alpha}{r}$



Slipping will stop say at time t, then

$$v=r\omega$$

$$\Rightarrow\quad at=r(\omega_0-\alpha t)$$

$$\Rightarrow\quad t=\frac{r\omega_0}{a+r\alpha}=\left(\frac{r\omega_0}{3\mu g\cos\alpha-g\sin\alpha}\right)$$

If d_1 is the distance travelled during this time t, then

$$d_1=\frac{1}{2}at^2=\frac{r^2\omega_0^2(\mu\cos\alpha-\sin\alpha)}{2g(3\mu\cos\alpha-\sin\alpha)^2}$$

Also, $v=at=\dfrac{r\omega_0(\mu\cos\alpha-\sin\alpha)}{(3\mu\cos\alpha-\sin\alpha)}$

Once slipping has stopped, pure rolling continues if

$$\mu>\frac{\tan\alpha}{1+(mR^2/I)}$$

$$\Rightarrow\quad \mu>\frac{\tan\alpha}{1+2}$$

$$\Rightarrow\quad \mu>\frac{\tan\alpha}{3}$$

Since, in the question it is already given that $\mu>\tan\alpha$, so the retardation of cylinder is given by

$$a'=\frac{g\sin\alpha}{1+(I/mr^2)}=\frac{g\sin\alpha}{1+1/2}=\frac{2}{3}g\sin\alpha$$

If it travels a further distance d_2 under this new retardation a', then

$$d_2=\frac{v^2}{2a'}=\frac{3r^2\omega_0^2(\mu\cos\alpha-\sin\alpha)^2}{(3\mu\cos\alpha-\sin\alpha)^2(4g\sin\alpha)}$$

So, $d_{max}=d_1+d_2$

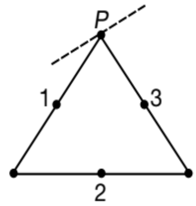
$$\Rightarrow\quad d_{max}=\left(\frac{r^2\omega_0^2(\mu\cos\alpha-\sin\alpha)}{2g(3\mu\cos\alpha-\sin\alpha)^2}\right)\times\left(1+\frac{3(\mu\cos\alpha-\sin\alpha)}{2\sin\alpha}\right)$$

$$\Rightarrow\quad d_{max}=\frac{r^2\omega_0^2(\mu\cos\alpha-\sin\alpha)}{4g\sin\alpha(3\mu\cos\alpha-\sin\alpha)}$$

Single Correct Choice Type Questions

1. Since axis passes through ends of two rods and is perpendicular to their length, so

$$I_{1P}=I_{3P}=\frac{1}{3}m\ell^2$$



Moment of inertia of rod 2 about it CG is $\frac{1}{12}m\ell^2$ and point P is at a distance $\frac{\sqrt{3}}{2}\ell$, so by Parallel Axis Theorem

$$I_{2P}=\frac{1}{12}m\ell^2+m\left(\frac{\sqrt{3}}{2}\ell\right)^2$$

$$\Rightarrow\quad I_{2P}=\frac{1}{12}m\ell^2+\frac{3}{4}m\ell^2$$

$$\Rightarrow\quad I_{2P}=\frac{10}{12}m\ell^2$$

$$\Rightarrow\quad I_{total}=\left(\frac{10}{12}+\frac{1}{3}+\frac{1}{3}\right)m\ell^2$$

$$\Rightarrow\quad I_{total}=\left(\frac{10+4+4}{12}\right)m\ell^2$$

$$\Rightarrow\quad I_{total}=\frac{3}{2}m\ell^2$$

Further $I_{total}=(3m)k^2$

$$\Rightarrow\quad (3m)k^2=\frac{3}{2}m\ell^2$$

$$\Rightarrow\quad k=\frac{\ell}{\sqrt{2}}$$

Hence, the correct answer is (C).

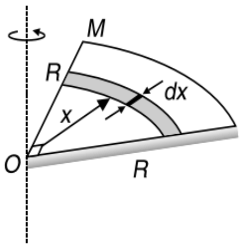
2. If we assume complete disc to be present, then it would have a mass 4 times the mass of the sector. Then, moment of inertia of the complete disc is

$$I_{disc}=\frac{1}{2}M_{disc}R^2=\frac{1}{2}(4M_{sector})R^2$$

Hence, $I_{sector}=\dfrac{I_{disc}}{4}$

$$\Rightarrow\quad I_{sector}=\frac{1}{2}MR^2$$

Calculus Method:



Consider an element of mass dm, thickness dx and radius x of the sector.

$$dm = \frac{M}{\left(\frac{\pi R^2}{4}\right)}\frac{2\pi x dx}{4} = \frac{2M}{R^2}xdx$$

If dI be the moment of inertia of the element, then $dI = x^2 dm$

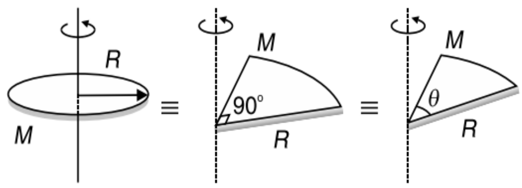
$$\Rightarrow \quad dI = x^2\left(\frac{2M}{R^2}x\,dx\right)$$

$$\Rightarrow \quad dI = \frac{2M}{R^2}\left(x^3\,dx\right)$$

$$\Rightarrow \quad I = \frac{2M}{R^2}\int_0^R x^3\,dx$$

$$\Rightarrow \quad I = \frac{1}{2}MR^2$$

General Funda:



$I_1 = \frac{1}{2}MR^2$ $\quad I_2 = \frac{1}{2}MR^2$ $\quad I_3 = \frac{1}{2}MR^2$

(Moment of Inertia of a DISC of mass M with radius R) ≡ (Moment of Inertia of a SECTOR of mass M with radius R)

Hence, the correct answer is (A).

3. Since, Impulse = Change in Momentum, so

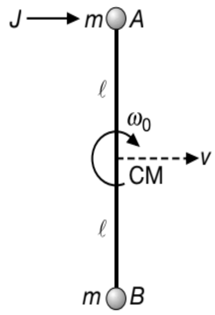
$$J = (2m)v$$

$$\Rightarrow \quad v = \frac{J}{2m}$$

Also, (Angular Impulse) = (Change in Angular Momentum)

$$\Rightarrow \quad J\ell = \left(I_{\text{system}}\right)\omega$$

$$\Rightarrow \quad J\ell = \left(2m\ell^2\right)\omega$$



$$\Rightarrow \quad \omega = \frac{J\ell}{2m\ell^2} = \frac{J}{2m\ell}$$

So, $v_A = v + \ell\omega$

$$\Rightarrow \quad v_A = \frac{J}{2m} + \frac{J}{2m} = \frac{J}{m}$$

Hence, the correct answer is (C).

4. Work done $W = \frac{1}{2}I\omega^2$

If x is the distance of mass 0.3 kg from the centre of mass, we will have,

$$I = (0.3)x^2 + (0.7)(1.4 - x)^2$$

For work to be minimum, the moment of inertia (I) should be minimum

$$\Rightarrow \quad \frac{dI}{dx} = 0$$

$$\Rightarrow \quad 2(0.3x) - 2(0.7)(1.4 - x) = 0$$

$$\Rightarrow \quad (0.3)x = (0.7)(1.4 - x)$$

$$\Rightarrow \quad x = \frac{(0.7)(1.4)}{0.3 + 0.7} = 0.98 \text{ m}$$

Hence, the correct answer is (C).

5. Assuming mass of body to be m, the mechanical energy is conserved in both the cases, so

Hence, $\frac{1}{2}m\left(\frac{5}{4}v_0\right)^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega^2$

$$\Rightarrow \quad \frac{25}{16}v_0^2 = v_0^2 + \frac{I}{m}\left(\frac{v_0}{R}\right)^2$$

If K be the radius of gyration, then

$$I = mK^2, \text{ so}$$

$$\Rightarrow \quad \frac{25}{16} = 1 + \frac{mK^2}{mR^2}$$

$$\Rightarrow \quad K^2 = \frac{9}{16}R^2$$

$$\Rightarrow \quad K = \frac{3}{4}R$$

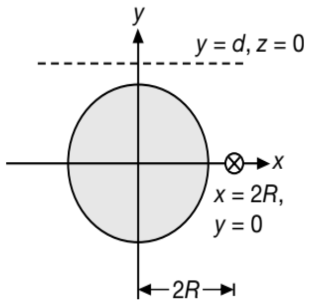
Hence, the correct answer is (B).

6. Moment of inertia about the axis passing through $x = 2R$, $y = 0$ shown by ⊗ direction in figure is

$$I_1 = \frac{1}{2}mR^2 + m(2R)^2$$

$$\Rightarrow \quad I_1 = \frac{9}{2}mR^2 \qquad \ldots(1)$$

Moment of inertia about the axis passing through $y = d$, $z = 0$, shown as dotted line in figure is



$$I_2 = \frac{1}{4}mR^2 + md^2 \qquad \ldots(2)$$

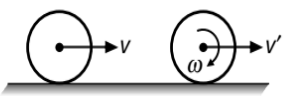
Equating (1) and (2), we get

$$\frac{1}{4}mR^2 + md^2 = \frac{9}{2}mR^2$$

$$\Rightarrow \quad d = \frac{\sqrt{17}}{2}R$$

Hence, the correct answer is (B).

7. Let v be the velocity of centre of mass of ring just after the impulse is applied and v' be its velocity when pure rolling starts. Angular velocity ω of the ring at this instant will be $\omega = \frac{v'}{r}$



Since, Impulse = Change in Linear Momentum, so we have

$$J = mv$$

$$\Rightarrow \quad v = \frac{J}{m} \qquad \ldots(1)$$

Between the two positions shown in figure, force of friction on the ring acts backwards. Angular momentum of the ring about bottommost point will be conserved, so

$$L_i = L_f$$

$$\Rightarrow \quad mvr = mv'r + I\omega$$

$$\Rightarrow \quad mvr = mv'r + (mr^2)\frac{v'}{r} = 2mv'r$$

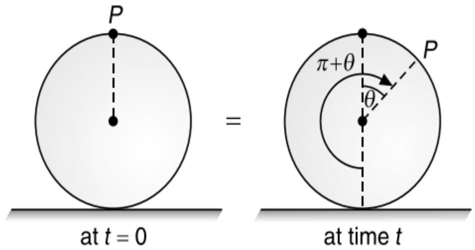
So, from (1), we get

$$\Rightarrow \quad v' = \frac{v}{2} = \frac{J}{2m}$$

Hence, the correct answer is (B).

8. Since at $t = 0$, P is at the top of rim, so

$$v_P = 2v_0 \sin\left(\frac{\pi + \theta}{2}\right)$$



$$\Rightarrow \quad v_p = 2v_0 \cos\left(\frac{\theta}{2}\right) = 2v_0 \cos\left(\frac{v_0 t}{2R}\right)$$

$$\Rightarrow \quad v_P^2 = 4v_0^2 \cos^2\left(\frac{v_0 t}{2R}\right)$$

At $t = 0$, $v_P^2 = 4v_0^2$

Also, $v_P = 0$ at point of contact i.e., when P reaches the bottom i.e., in a time $t = \frac{T}{2} = \frac{\pi R}{v_0}$

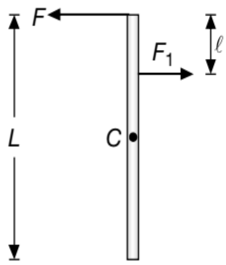
Hence, the correct answer is (B).

9. Let L be the length of the rod and F_1 the magnitude of other force. If C is the centre of the rod, then

$$F_1 - F = ma$$

$$\Rightarrow \quad F_1 = F + ma \qquad \ldots(1)$$

Since the rod moves translationally, so the net torque about C is zero.



$$\Rightarrow \quad F\left(\frac{L}{2}\right) = F_1\left(\frac{L}{2} - \ell\right)$$

$$\Rightarrow \quad F\left(\frac{L}{2}\right) = (F + ma)\left(\frac{L}{2} - \ell\right)$$

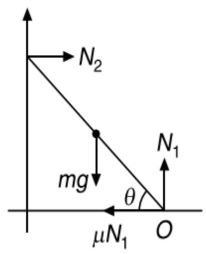
$$\Rightarrow \quad F\left(\frac{L}{2}\right) = (F + ma)\ell$$

$\Rightarrow \quad L = \dfrac{2(F+ma)\ell}{ma}$

$\Rightarrow \quad L = 2\ell\left(1+\dfrac{F}{ma}\right)$

Hence, the correct answer is (B).

10. The free body diagram of arrangement is shown in Figure.



$mg = N_1$

Also, $\mu N_1 = N_2$

$\Rightarrow \quad N_2 = \mu mg$

Taking moment of forces about O, we get

$\mu mgl\sin\theta = mg\dfrac{l}{2}\cos\theta$

$\Rightarrow \quad \tan\theta = \dfrac{1}{2\mu}$

$\Rightarrow \quad \theta = \tan^{-1}\left(\dfrac{1}{2\mu}\right)$

Hence, the correct answer is (D).

11. Given, that I_0 is the moment of inertia of table and gun and m the mass of bullet.

Initial angular momentum of system about centre

$L_i = \left(I_0 + mr^2\right)\omega_0$ ...(1)

Let ω be the angular velocity of table after the bullet is fired. Final angular momentum

$L_f = I_0\omega - m(v - r\omega)r$ ...(2)

where $(v - r\omega)$ is absolute velocity of bullet to the right, then

From (1) and (2), we get

$(\omega - \omega_0) = \dfrac{mvr}{I_0 + mr^2}$

This is also the increase in angular velocity

Hence, the correct answer is (B).

12. The rod will rotate about point A with angular acceleration given by

$\alpha = \dfrac{\tau}{I} = \dfrac{Fx}{\left(\dfrac{m\ell^2}{3}\right)} = \dfrac{3Fx}{m\ell^2}$

$\Rightarrow \quad a = \dfrac{\ell}{2}\alpha = \dfrac{3}{2}\dfrac{Fx}{m\ell}$

$\Rightarrow \quad a \propto x$

So, a-x graph is a straight line passing through origin with slope $\dfrac{3F}{2m\ell}$.

Hence, the correct answer is (B).

13. The angular momentum of a body $\vec{L}$ may be expressed as the sum of two parts,

(a) one arising from the motion of the centre of mass of the body and

(b) the other from the motion of the body with respect to its centre of mass.

i.e. $\vec{L}_{\text{total}} = \vec{L}_{\text{C.M.}} + \vec{r}_{\text{C.M.}} \times \vec{p}$

$\Rightarrow \quad \vec{L}_{\text{total}} = \vec{L}_{\text{C.M.}} + M\left(\vec{r}_{\text{C.M.}} \times \vec{v}_{\text{C.M.}}\right)$

For this Problem

$L_{\text{C.M.}} = I\omega = \dfrac{1}{2}MR^2\omega$ and

$M\left(\vec{r}_{\text{C.M.}} \times \vec{v}_{\text{C.M.}}\right) = MRv_{CM} = MR(R\omega)$

$\Rightarrow \quad M\left(\vec{r}_{\text{C.M.}} \times \vec{v}_{\text{C.M.}}\right) = MR^2\omega$

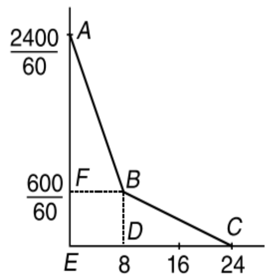
$\Rightarrow \quad L_{\text{total}} = \dfrac{1}{2}MR^2\omega + MR^2\omega = \dfrac{3}{2}MR^2\omega$

Hence, the correct answer is (B).

14. Total Revolutions is equal to area under the curve.

Area= $Ar\Delta ABF + Ar\Delta BCD + Ar$

Rectangle $BDEF$



$\Rightarrow \quad \text{Area} = \dfrac{1}{2}\dfrac{(1800)}{60}(8) + \dfrac{1}{2}(16)\dfrac{(600)}{60} + 8 \times \dfrac{600}{60}$

$\Rightarrow \quad \text{Area} = \dfrac{7200 + 4800 + 4800}{60} = 280$

Hence, the correct answer is (B).

15. Applying conservation of energy, we get

$\dfrac{1}{2}mv^2 = \dfrac{1}{2}M\left(\dfrac{mv}{M}\right)^2 + \dfrac{1}{2}\left(\dfrac{ML^2}{12}\right)(\omega)^2$ ...(1)

Applying conservation of angular momentum, we get

$$mv\left(\frac{L}{2}\right)=\frac{ML^2}{12}\omega$$

$$\Rightarrow \quad \omega=\frac{6mv}{ML} \qquad \ldots(2)$$

From (1) and (2), we get

$$\frac{1}{2}mv^2=\frac{1}{2}\frac{m^2v^2}{M}+\frac{1}{24}ML^2\left(\frac{36m^2v^2}{M^2L^2}\right)$$

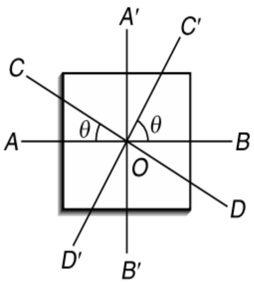
$$\Rightarrow \quad 1=\frac{m}{M}+\frac{3m}{M}$$

$$\Rightarrow \quad 1=\frac{4m}{M}$$

$$\Rightarrow \quad m=\frac{M}{4}$$

Hence, the correct answer is (B).

16.



$$I_{AB}=I_{A'B'}=I \text{ and } I_{CD}=I_{C'D'}$$

If I_0 be the moment of inertia of the square plate about an axis passing through O and perpendicular to the plate, then by perpendicular axis theorem

$$I_0=I_{AB}+I_{A'B'}=2I_{AB} \qquad \ldots(1)$$

OR

$$I_0=I_{CD}+I_{C'D'}=2I_{CD} \qquad \ldots(2)$$

From (1) and (2)

$$I_{CD}=I_{AB}=I \qquad \textbf{\{OPTION (A)\}}$$

Hence, the correct answer is (A).

17. Net torque about O due to weights acting at midpoints of the respective rods should be zero. Hence,

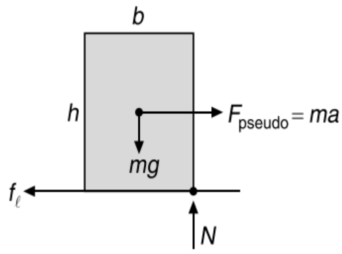
$$mg\left(\frac{\ell}{2}\sin 60°\right)=Mg\left(\frac{\ell}{2}\sin 30°\right)$$

$$\Rightarrow \quad \frac{M}{m}=\frac{\sin 60°}{\sin 30°}=\sqrt{3}$$

Hence, the correct answer is (C).

18. Let a be the acceleration of the truck, then from FBD of the block, taking torque about right edge from where normal reaction is passing, we get

$$ma\left(\frac{h}{2}\right)\le mg\left(\frac{b}{2}\right)$$



$$\Rightarrow \quad a\le\frac{b}{h}g$$

Final velocity of truck is zero, so we have

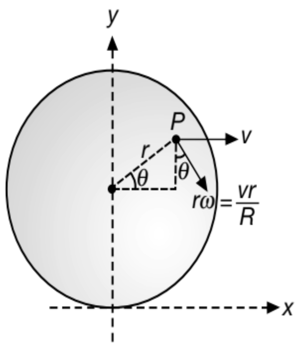
$$0=v^2-2\left(\frac{b}{h}g\right)l$$

$$\Rightarrow \quad l=\frac{hv^2}{2bg}$$

Hence, the correct answer is (B).

19. Since, for pure rolling, we have

$$\omega=\frac{v}{R}$$



Velocity of point P is resultant of two velocity vectors as shown in figure. So,

$$\vec{v}_P=v_x\hat{i}+v_y\hat{j}$$

$$\Rightarrow \quad \vec{v}_P=\left(v+\frac{vr\sin\theta}{R}\right)\hat{i}-\left(\frac{vr\cos\theta}{R}\right)\hat{j}$$

$$\Rightarrow \quad \vec{v}_P=\omega\left[(R+r\sin\theta)\hat{i}-(r\cos\theta)\hat{j}\right]$$

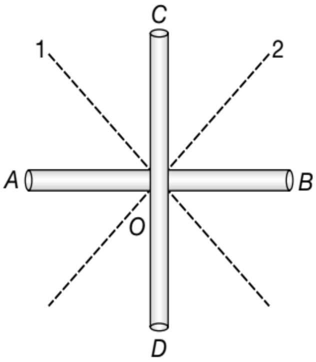
Hence, the correct answer is (C).

20. Moment of inertia of both the rods about an axis passing through their point of intersection O and perpendicular to the plane of the rods is

$$I_0=\frac{m\ell^2}{12}+\frac{m\ell^2}{12}=\frac{m\ell^2}{6}$$

Now axis 1 and axis 2 are two mutually perpendicular axes passing through O lying in the plane of two rods. Then by symmetry, we have

$$I_1=I_2=I \qquad \text{\{say\}}$$



Now, according to Perpendicular Axis Theorem, we have

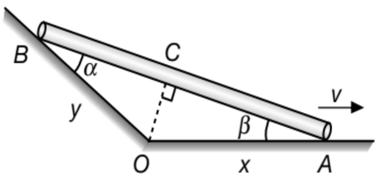
$$I_1 + I_2 = I_0$$

$$\Rightarrow \quad 2I_1 = I_0 = \frac{m\ell^2}{6}$$

$$\Rightarrow \quad I_1 = \frac{m\ell^2}{12}$$

Hence, the correct answer is (C).

21. Let $OA = x$ and $OB = y$ as shown in figure



Then $BC + CA = \ell$

$$\Rightarrow \quad y\cos\alpha + x\cos\beta = \ell \qquad \ldots(1)$$

Differentiating both sides of (1) w.r.t. time t, we get

$$\left(\frac{dy}{dt}\right)\cos\alpha + \left(\frac{dx}{dt}\right)\cos\beta = 0$$

Since, $\dfrac{dx}{dt} = v_A = v$

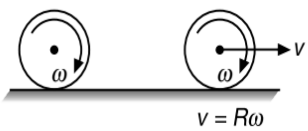
So, $\left(\dfrac{dy}{dt}\right) = v_B = -\dfrac{v\cos\beta}{\cos\alpha}$

Here, negative sign implies that y decreases as t increases.

Hence, the correct answer is (D).

22. Applying Conservation of Angular Momentum about point of contact, we get

$$I\omega_0 = I\omega + mvR$$



$$\Rightarrow \quad I\omega_0 = I\left(\frac{v}{R}\right) + mvR$$

$$\Rightarrow \quad v = \frac{I\omega_0}{\frac{I}{R} + mR}$$

$$\Rightarrow \quad v = \frac{\omega_0}{\frac{1}{R} + \frac{mR}{I}}$$

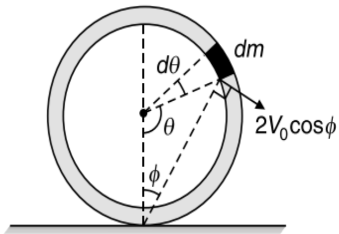
Now $I_{\text{solid sphere}} < I_{\text{hollow}}$

$$\Rightarrow \quad v_{\text{solid}} < v_{\text{hollow}}$$

$$\Rightarrow \quad v_1 < v_2$$

Hence, the correct answer is (C).

23. Considering an elemental mass dm on ring as shown in Figure. Then



$$dm = \left(\frac{M}{2\pi R}\right)(Rd\theta) = \frac{Md\theta}{2\pi}$$

Velocity of this elemental mass is

$$v = 2v_0\cos\phi$$

$$\Rightarrow \quad v = 2v_0\cos\phi = 2v_0\sin\left(\frac{\theta}{2}\right)$$

Kinetic energy of element is

$$dK = \frac{1}{2}(dm)v^2 = \frac{1}{2}(dm)\left(4v_0^2\sin^2\frac{\theta}{2}\right)$$

$$\Rightarrow \quad dK = \frac{Mv_0^2}{2\pi}(1-\cos\theta)d\theta$$

Total kinetic energy of segment ACB is

$$K = K_{ACB} = \frac{Mv_0^2}{2\pi}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}(1-\cos\theta)d\theta$$

$$\Rightarrow \quad K = \frac{Mv_0^2}{2} - \frac{Mv_0^2}{\pi}$$

Hence, the correct answer is (A).

24. $\tau = \dfrac{dL}{dt} = \dfrac{d}{dt}(I\omega)$

$$\Rightarrow \quad \tau = \frac{d}{dt}\left(\frac{ML^2}{3} + mx^2\right)\omega$$

$\Rightarrow \quad \tau = 2mx\dfrac{dx}{dt}\omega$

Now, $x = vt$

$\Rightarrow \quad \tau \propto t$

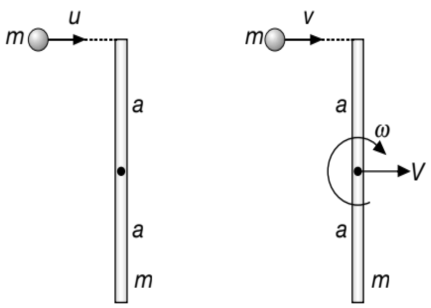
Finally, torque becomes zero.
Hence, the correct answer is (B).

25. No external torque is acting on the system, so angular momentum is conserved. Further there exists no non-conservative forces in the system, so total energy is also conserved.
Hence, the correct answer is (B).

26. By Law of Conservation of Linear Momentum, we have

$$mu = mv + mV$$

$$\Rightarrow \quad u = v + V \qquad \ldots(1)$$



Just Before Impact — Just After Impact

By Law of Conservation of Angular Momentum about centre of rod, we have

$$mua = mva + \left(\frac{ma^2}{3}\right)\omega$$

$$\Rightarrow \quad u = v + \frac{a\omega}{3} \qquad \ldots(2)$$

Further applying the definition of coefficient of restitution $(e = 1)$ at the point of impact, we get

Relative speed of approach = Relative speed of separation

$$\Rightarrow \quad u = (V + a\omega) - v \qquad \ldots(3)$$

Solving these three equations (1), (2) and (3) we get,

$$V = \frac{2}{5}u \text{ and } \omega = \frac{6u}{5a}$$

So, kinetic energy of rod is

$$K = \frac{1}{2}\times m \times \left(\frac{2}{5}u\right)^2 + \frac{1}{2}\times\frac{ma^2}{3}\times\left(\frac{6u}{5a}\right)^2$$

$$\Rightarrow \quad K = \frac{8}{25}mu^2$$

Hence, the correct answer is (D).

27. $\left(\dfrac{1}{2}MR^2 + M_bR^2\right)\omega = \dfrac{1}{2}MR^2\omega'$

(Since the boy reaches the centre, so final angular momentum of boy is zero).

$$\Rightarrow \quad \left(\frac{1}{2}M + M_b\right)\omega = \frac{1}{2}M\omega'$$

$$\Rightarrow \quad (100 + 50)\omega = 100\omega'$$

$$\Rightarrow \quad \omega' = \frac{3\omega}{2} = 15 \text{ rpm}$$

Hence, the correct answer is (C).

28. For the plank P

$$(\text{KE})_{\text{Plank}} = \frac{1}{2}m(2v)^2$$

$$(\text{KE})_{\text{Plank}} = 2mv^2$$

For the cylinder

$$(\text{KE})_{\text{Cylinder}} = \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$$

$$\Rightarrow \quad (\text{KE})_{\text{Cylinder}} = \frac{1}{2}mv^2\left(1 + \frac{1}{1}\right) = mv^2$$

$$\Rightarrow \quad \frac{(\text{KE})_{\text{Plank}}}{(\text{KE})_{\text{Cylinder}}} = \frac{2}{1}$$

Hence, the correct answer is (B).

29. Let M be the mass of the square plate before the holes are cut. If m be the mass of one hole, then

$$m = \left(\frac{M}{16R^2}\right)\pi R^2 = \frac{\pi M}{16}$$

Moment of inertia of remaining portion is

$$I = I_{\text{square}} - 4I_{hole}$$

$$\Rightarrow \quad I = \frac{M}{12}(16R^2 + 16R^2) - 4\left[\frac{mR^2}{2} + m(2R^2)\right]$$

$$\Rightarrow \quad I = \frac{8}{3}MR^2 - 10mR^2$$

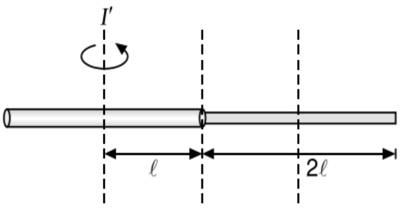
$$\Rightarrow \quad I = \left(\frac{8}{3} - \frac{10\pi}{16}\right)MR^2$$

Hence, the correct answer is (C).

30. **Conceptual the correct answer is (B).**

31. Applying angular momentum conservation, we get

$$2\left[\frac{m(2l)^2}{12}\right]\omega_0 = I'\omega' \qquad \ldots(1)$$



where, $I' = \frac{m(2l)^2}{12} + \left[\frac{m(2l)^2}{12} + m(2l)^2\right]$

$\Rightarrow \quad I' = \frac{ml^2}{3} + \frac{13ml^2}{3} = \frac{14ml^2}{3}$

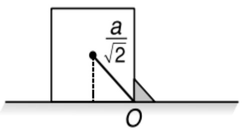
So, equation (1) becomes

$$\frac{2}{3}ml^2\omega_0 = \frac{14}{3}ml^2\omega'$$

$\Rightarrow \quad \omega' = \frac{\omega_0}{7}$

Hence, the correct answer is (B).

32. By Law of Conservation of Angular Momentum



$$mv\left(\frac{a}{2}\right) = \left(I_{\substack{\text{system}\\ \text{about O}}}\right)\omega$$

$$mv\left(\frac{a}{2}\right) = \left[\frac{1}{6}Ma^2 + M\left(\frac{a}{\sqrt{2}}\right)^2\right]\omega$$

$\Rightarrow \quad \omega = \frac{3v}{4a}$

Hence, the correct answer is (A).

33. By Law of Conservation of Energy

Loss in Potential energy = Gain in Kinetic energy

$$mg\ell(1-\cos\theta) = \frac{1}{2}mv^2$$

$\Rightarrow \quad \frac{mv^2}{\ell} = 2mg(1-\cos\theta)$

At point of breaking

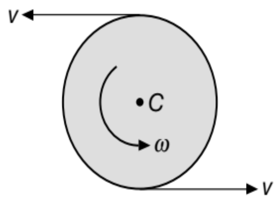
$$mg\cos\theta = \frac{mv^2}{\ell}$$

$\Rightarrow \quad mg\cos\theta = 2mg(1-\cos\theta)$

$\Rightarrow \quad \cos\theta = \frac{2}{3}$

Hence, the correct answer is (B).

34.



$$2(mR^2)(2\pi-\theta) = \frac{1}{2}MR^2\theta$$

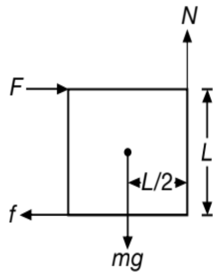
$\Rightarrow \quad 4\pi m - 2m\theta = \frac{M}{2}\theta$

$\Rightarrow \quad \theta = \frac{8\pi m}{4m+M}$

Hence, the correct answer is (D).

35. At the critical condition, normal reaction N will pass through point P. In this condition

$\tau_N = 0 = \tau_f$ (about P)



the block will topple when

$\tau_F > \tau_{mg}$

$\Rightarrow \quad FL > (mg)\frac{L}{2}$

$\Rightarrow \quad F > \frac{mg}{2}$

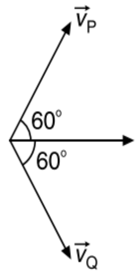
Therefore, the minimum force required to topple the bock is

$$F_{\min} = \frac{mg}{2}$$

Hence, the correct answer is (B).

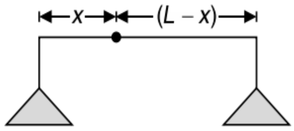
36. Relative velocity is

$$|\vec{v}_P - \vec{v}_Q| = 2R\omega\cos 30^\circ = \sqrt{3}R\omega$$



Hence, the correct answer is (D).

37. When m is placed in left pan, m_1 is placed in right pan. So,



$$mgx = m_1 g(L-x) \qquad ...(1)$$

When m is placed in right pan, m_2 is placed in left pan. So, $m_2 gx = mg(L-x)$ ...(2)

Divide (1) by (2), we get

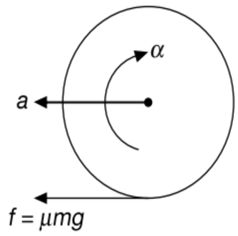
$$m^2 = m_1 m_2$$

$$\Rightarrow \quad m = \sqrt{m_1 m_2}$$

Hence, the correct answer is (B).

38. $f = \mu mg,\ a = \mu g$

$$\alpha = \frac{\tau}{I} = \frac{\mu mgR}{\frac{2}{5}mR^2} = \frac{5}{2}\frac{g\mu}{R}$$



Now, $t = \dfrac{v_0}{a} = \dfrac{\omega_0}{\alpha}$

$$\Rightarrow \quad \frac{a}{\alpha} = \frac{v_0}{\omega_0}$$

$$\Rightarrow \quad \frac{2R}{5} = \frac{v_0}{\omega_0}$$

$$\Rightarrow \quad 5v_0 = 2R\omega_0$$

Hence, the correct answer is (B).

39. $T = \dfrac{mv^2}{r}$

$$\Rightarrow \quad 16 = \frac{(16)v^2}{144}$$

$$\Rightarrow \quad v = 12\ \text{ms}^{-1}$$

Hence, the correct answer is (D).

40. Axis of pure rotation will be the point of intersection of the lines perpendicular to both the straight lines.
A line perpendicular to $2x-3y=2$ and passing through $(4,\ 2)$ is $2y+3x=16$ and a line perpendicular to $3x+4y=7$ and passing through $(1,\ 1)$ is $3y-4x=-1$

The intersection of $2y+3x=16$ and $3y-4x=-1$ is

$$\left(\frac{50}{17}, \frac{61}{17}\right)$$

Hence, the correct answer is (D).

41. Since, retarding torque is constant, so, angular retardation say α will also be constant. Applying $\omega^2 = \omega_0^2 - 2\alpha\theta$ we get,

$$\left(\frac{\omega_0}{2}\right)^2 = \omega_0^2 - 2\alpha\theta_1 \qquad ...(1)$$

$$\text{and } 0 = \left(\frac{\omega_0}{2}\right)^2 - 2\alpha\theta_2 \qquad ...(2)$$

Solving equations (1) and (2), we get

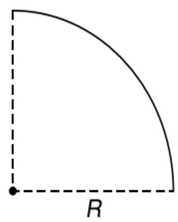
$$\theta_2 = \frac{\theta_1}{3}$$

So, the disc will make $\dfrac{n}{3}$ more rotations before it comes to rest.

Hence, the correct answer is (B).

42. $\ell = \dfrac{2\pi R}{4}$

$$\Rightarrow \quad R = \frac{2\ell}{\pi}$$



Since, $I = mR^2 = m\left(\dfrac{2\ell}{\pi}\right)^2$

$$\Rightarrow \quad I = \frac{2m\ell^2}{5} \qquad \{\because \pi^2 \approx 10\}$$

Hence, the correct answer is (D).

43. $\begin{pmatrix}\text{MI of}\\ \text{Given}\\ \text{Shape}\end{pmatrix} = \begin{pmatrix}\text{MI of Complete}\\ \text{Disc of}\\ \text{Radius } R\end{pmatrix} - \begin{pmatrix}\text{MI of Removed}\\ \text{Portion of}\\ \text{Radius } R/3\end{pmatrix}$

$$\Rightarrow \quad I_{\text{remaining}} = I_{\text{whole}} - I_{\text{removed}}$$

$$\Rightarrow \quad I = \frac{1}{2}(9M)(R)^2 - \left[\frac{1}{2}m\left(\frac{R}{3}\right)^2 + \frac{1}{2}m\left(\frac{2R}{3}\right)^2\right] \qquad ...(1)$$

where, $m = \sigma a = \dfrac{9M}{\pi R^2} \times \pi\left(\dfrac{R}{3}\right)^2 = M$

Substituting in equation (1), we get

$$I = 4MR^2$$

Hence, the correct answer is (A).

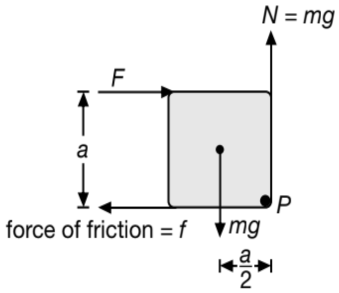
44. If m be the mass of cube and a be the side of cube, then the cube will slide when

$$F > \mu mg \quad \ldots(1)$$

and it will topple if torque due to F about P, is greater than torque due to mg about P i.e.,

$$Fa > \left(\frac{a}{2}\right)mg$$

$$\Rightarrow \quad F > \frac{1}{2}mg \quad \ldots(2)$$



From equations (1) and (2), we observe that cube will topple before sliding if

$$\mu > \frac{1}{2}$$

Hence, the correct answer is (D).

45. Mass of cotton pad after time t is

$$m = \mu t$$

Applying conservation of angular momentum, we get

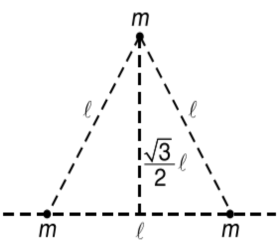
$$\left(\frac{m_0 r^2}{2}\right)\omega = \left(\frac{m_0 r^2}{2} + \mu t r^2\right)\frac{\omega}{2}$$

$$\Rightarrow \quad m_0 r^2 = \frac{m_0 r^2}{2} + \mu t r^2$$

$$\Rightarrow \quad t = \frac{m_0}{2\mu}$$

Hence, the correct answer is (B).

46.



$$I = m(0)^2 + m(0)^2 + m\left(\frac{\sqrt{3}}{2}\ell\right)^2$$

$$\Rightarrow \quad I = \frac{3}{4}m\ell^2$$

Hence, the correct answer is (B).

47. When the string is cut the weight Mg of rod acting at centre produces the torque about P.

$$\Rightarrow \quad \tau = Mg\left(\frac{L}{2}\right)$$

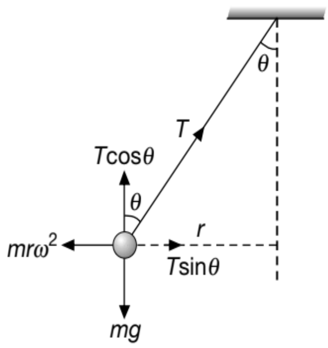
Also, $\tau = I\alpha$

$$\Rightarrow \quad Mg\left(\frac{L}{2}\right) = \left(\frac{ML^2}{3}\right)\alpha$$

$$\Rightarrow \quad \alpha = \frac{3g}{2L}$$

Hence, the correct answer is (D).

48. $T\sin\theta = m(L\sin\theta)\omega^2$



$$\Rightarrow \quad 324 = (0.5)(0.5)\omega^2$$

$$\Rightarrow \quad \omega^2 = \frac{324}{0.5 \times 0.5}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{324}{0.5 \times 0.5}}$$

$$\Rightarrow \quad \omega = \frac{18}{0.5} = 36 \text{ rads}^{-1}$$

Hence, the correct answer is (B).

49. For equilibrium of rod (considering limiting friction), we have

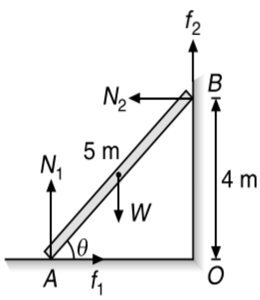
$$f_1 = N_2$$

$$\Rightarrow \quad \mu N_1 = N_2 \quad \ldots(1)$$

Also, $-W + N_1 + f_2 = 0$

$$\Rightarrow \quad N_1 + \mu N_2 = W$$

$$\Rightarrow \quad N_1 + \mu^2 N_1 = W \quad \ldots(2)$$



Balancing all torques about point A, we get

$\Rightarrow \quad -W\left(\frac{3}{2}\right)+f_2(3)+N_2(4)=0$

$\Rightarrow \quad 3f_2+4N_2=\frac{3W}{2}$

$\Rightarrow \quad 3\mu N_2+4N_2=\frac{3W}{2}$

$\Rightarrow \quad \mu N_1(3\mu+4)=\frac{3}{2}N_1(1+\mu^2)$

$\Rightarrow \quad 2(3\mu^2+4\mu)=3+3\mu^2$

$\Rightarrow \quad 6\mu^2+8\mu=3+3\mu^2$

$\Rightarrow \quad 3\mu^2+8\mu-3=0$

$\Rightarrow \quad 3\mu^2+9\mu-\mu-3=0$

$\Rightarrow \quad 3\mu(\mu+3)-1(\mu+3)=0$

$\Rightarrow \quad (3\mu-1)(\mu+3)=0$

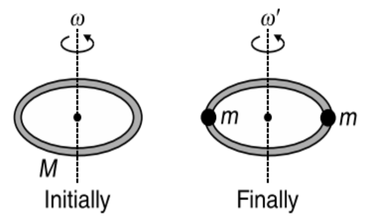
Since $\mu \neq -3$

$\Rightarrow \quad \mu=\frac{1}{3}$

Hence, the correct answer is (B).

50. By Law of Conservation of Angular Momentum

$$(MR^2)\omega = MR^2\omega' + 2mR^2\omega'$$



$\Rightarrow \quad \omega'=\left(\frac{M}{M+2m}\right)\omega$

Hence, the correct answer is (B).

51. By Law of Conservation of Angular Momentum

$$\sum mvr = (I_{\text{system}})\omega$$

$\Rightarrow \quad mv\frac{\ell}{2}=\frac{(2m)(2\ell)^2}{12}\omega=\frac{2m(4\ell^2)}{12}\omega$

$\Rightarrow \quad \omega=\frac{3v}{4\ell}(\text{anticlockwise})$

Hence, the correct answer is (A).

52. $t=\frac{v_0}{a}=\frac{\omega_0}{\alpha}$

$\Rightarrow \quad v_0=\left(\frac{a}{\alpha}\right)\omega_0$

where $\alpha=\frac{\tau}{I}=\frac{fR}{I}=\frac{(\mu mg)R}{\frac{1}{2}mR^2}=\frac{2\mu g}{R}$

$\Rightarrow \quad v_0=\frac{(\mu g)}{\frac{2\mu g}{R}}\omega_0=\frac{r\omega_0}{2}$

Since $v_0=\frac{r\omega_0}{2}$ i.e., $v_0<r\omega_0$, so f acts such that it decreases v_0 and hence acts opposite to v_0 i.e., backwards. Also, the point of contact of disc with the ground has non-zero velocity as $v_0<r\omega_0$.

Hence, the correct answer is (B).

53. At the topmost point of the loop minimum value of linear speed of centre of sphere is

$$v=\sqrt{gR}$$

So, translational kinetic energy at topmost point is

$$K_T=\frac{1}{2}mv^2=\frac{1}{2}mgR$$

In case of pure rolling of a solid sphere the ratio of rotational to translational kinetic energy is

$$\frac{K_R}{K_T}=\frac{k^2}{r^2}=\frac{2}{5}$$

So, total kinetic energy at topmost point is

$$K=\left(\frac{5+2}{5}\right)K_T=\frac{7}{5}\left(\frac{1}{2}mgR\right)=\frac{7}{10}mgR$$

Now by Law of Conservation of Mechanical Energy, we have

$$\frac{7}{10}mgR=mg(h-2R)$$

$\Rightarrow \quad h=2.7\text{R}$

Hence, the correct answer is (D).

54. For pure rolling, $a_{\text{cm}}=R\alpha$, where $\alpha=\frac{\tau}{I}=\frac{F(h-R)}{\frac{2}{5}mR^2}$

$\Rightarrow \quad \frac{F}{m}=R\left[\frac{F(h-R)}{\frac{2}{5}mR^2}\right]$

$\Rightarrow \quad h-R=0.4R$

$\Rightarrow \quad h=1.4R$

Hence, the correct answer is (C).

55. For a given perimeter area of a circle is maximum, i.e., distribution of mass is at a maximum distance from the axis in case of a circle.

Hence, the correct answer is (A).

56. Mass of the loop = $M = L\rho$

Further if r is the radius of the loop, then

$$2\pi r = L$$

$$\Rightarrow \quad r = \frac{L}{2\pi}$$

Moment of Inertia about XX' is $I = \frac{3}{2}Mr^2$

$$\Rightarrow \quad I = \frac{3}{2}(L\rho)\frac{L^2}{(2\pi)^2} = \frac{3\rho L^3}{8\pi^2}$$

Hence, the correct answer is (D).

57. By Law of Conservation of Angular Momentum

$$mvr = \left(I_{\text{system}}\right)\omega$$

$$\Rightarrow \quad mvr = \left(I + mR^2\right)\omega$$

$$\Rightarrow \quad \omega = \frac{mvr}{I + mR^2}$$

Hence, the correct answer is (A).

58. Mass of the element dx is $m = \frac{M}{L}dx$

This element needs centripetal force for rotation, so

$$dF = mx\omega^2 = \left(\frac{M}{L}x\omega^2 dx\right)$$

$$\Rightarrow \quad F = \int_0^L dF = \frac{m}{L}\omega^2\int_0^L x\,dx = \frac{M\omega^2 L}{2}$$

This is the force exerted by the liquid at the other end.

Hence, the correct answer is (A).

59. Friction is not sufficient for pure rolling. Therefore, maximum friction will act upwards in all the three bodies. So, linear acceleration a of all the three bodies will be same and equal to $(g\sin\theta - \mu g\cos\theta)$. Therefore, time taken by all the three will be same.

Hence, the correct answer is (D).

60. Linear velocity of all three will be same. However, ring having maximum moment of inertia will have least angular acceleration and hence least rotational kinetic energy.

Hence, the correct answer is (A).

61. At any angle θ,

$$T = \frac{mv^2}{r} + mg\cos\theta$$

At mean position $\theta = 0°$

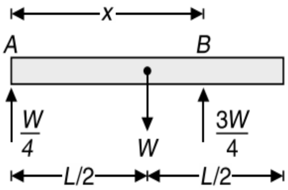
And $v = \sqrt{2gr}$

$$\Rightarrow \quad T = \frac{m(2gr)}{r} + mg$$

$$\Rightarrow \quad T = 3mg$$

Hence, the correct answer is (B).

62.



By Law of Conservation of Moments,

$$\begin{pmatrix}\text{Total Clockwise}\\\text{Moments}\end{pmatrix} = \begin{pmatrix}\text{Total Anticlockwise}\\\text{Moments}\end{pmatrix}$$

$$\Rightarrow \quad W\frac{L}{2} = \frac{W}{4}(0) + \left(\frac{3W}{4}\right)x$$

$$\Rightarrow \quad x = \frac{2L}{3}$$

Hence, the correct answer is (C).

63. $\frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2$

$$\Rightarrow \quad \frac{1}{2}(3)(9) = \frac{1}{2}(27)v^2$$

$$\Rightarrow \quad v = 1 \text{ ms}^{-1}$$

Hence, the correct answer is (A).

64. Since $L = I\omega$

$$\Rightarrow \quad L = \left(mr^2\right)\omega$$

when r becomes $\frac{r}{2}$, L becomes $\frac{L}{4}$.

Hence, the correct answer is (A).

65. If v be the linear velocity of centre of mass of the spherical body and ω be its angular velocity about centre of mass, then

$$\omega = \frac{v}{2R}$$

KE of spherical rolling body is

$$K_1 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad K_1 = \frac{1}{2}mv^2 + \frac{1}{2}\left(2mR^2\right)\left(\frac{v^2}{4R^2}\right)$$

$$\Rightarrow \quad K_1 = \frac{3}{4}mv^2 \quad \ldots(1)$$

Speed of the block will be

$$v' = v_{cm} + R\omega = 2R\omega + R\omega = 3R\omega$$

$$\Rightarrow \quad v' = 3R\omega = (3R)\left(\frac{v}{2R}\right) = \frac{3}{2}v$$

So, KE of block is $K_2 = \frac{1}{2}mv'^2$

$$\Rightarrow \quad K_2 = \frac{1}{2}m\left(\frac{3}{2}v\right)^2 = \frac{9}{8}mv^2 \quad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{K_1}{K_2} = \frac{2}{3}$$

Hence, the correct answer is (B).

66. $T = Ar^n$

$\Rightarrow \quad Ar^n = \dfrac{mv^2}{r}$

Further, by Law of Conservation of Angular Momentum

$L = mvr = \text{constant}$

$\Rightarrow \quad v = \dfrac{L}{mr}$

$\Rightarrow \quad Ar^n = \dfrac{m}{r}\left(\dfrac{L}{mr}\right)^2$

$\Rightarrow \quad Ar^n = \dfrac{L^2}{m} r^{-3}$

$\Rightarrow \quad n = -3 \qquad \left\{\because \dfrac{L^2}{m} = \text{constant}\right\}$

Hence, the correct answer is (D).

67. According to Angular Impulse – Angular Momentum Theorem, we have

$\vec{\tau}_{av}\Delta t = \Delta\vec{L} \qquad \ldots(1)$

where, Δt = time of flight = $\dfrac{2u\sin\theta}{g}$

$|\Delta\vec{L}| = |\vec{L}_f - \vec{L}_i|$ about point of projection

$\Rightarrow \quad \Delta\vec{L} = (mu\sin\theta)(\text{range}) - 0$

$\Rightarrow \quad \Delta\vec{L} = \dfrac{(mu\sin\theta)(u^2\sin 2\theta)}{g}$

$\Rightarrow \quad \Delta\vec{L} = \dfrac{mu^3\sin\theta\sin 2\theta}{g}$

$\Rightarrow \quad |\vec{\tau}_{av}| = \dfrac{\Delta\vec{L}}{\Delta t} = \left(\dfrac{mu^3\sin\theta\sin 2\theta}{g}\right)\left(\dfrac{g}{2u\sin\theta}\right)$

$\Rightarrow \quad |\vec{\tau}_{av}| = \dfrac{mu^2\sin(2\theta)}{2}$

Hence, the correct answer is (C).

68. Equilibrium of m gives

$T = mg \qquad \{T = \text{Tension in string}\}$

Net torque about point of contact of spool should be zero. Hence,

$(2R)(Mg\sin\alpha) = TR$

$\Rightarrow \quad 2Mg\sin\alpha = mg$

$\Rightarrow \quad m = 2M\sin\alpha$

Hence, the correct answer is (A).

69. $M_1 = M_2$

$\Rightarrow \quad \left[(\pi R_1^2)t\right]d_1 = \left[(\pi R_2^2)t\right]d_2$

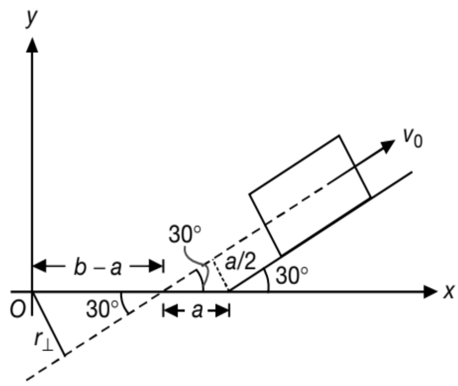
where t is thickness of both discs.

$\Rightarrow \quad \dfrac{R_1^2}{R_2^2} = \dfrac{d_2}{d_1}$

$\Rightarrow \quad \dfrac{\frac{1}{2}MR_1^2}{\frac{1}{2}MR_2^2} = \dfrac{d_2}{d_1}$

Hence, the correct answer is (B).

70. $r_\perp = (b-a)\sin 30° = \dfrac{b-a}{2}$



$\Rightarrow \quad L = mv_0 r_\perp = \dfrac{mv_0(b-a)}{2}$

Hence, the correct answer is (D).

71. $Mg = m\ell\omega^2$

$\Rightarrow \quad f = \dfrac{1}{2\pi}\sqrt{\dfrac{Mg}{m\ell}}$

Hence, the correct answer is (A).

72. $a = \dfrac{g}{\left(1 + \dfrac{I}{mr^2}\right)}$

Since $v^2 - u^2 = 2as$

$\Rightarrow \quad v^2 = 2ah$

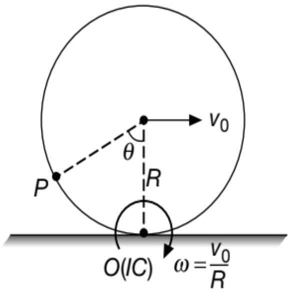
$\Rightarrow \quad v^2 = \dfrac{2(mgr^2)h}{I + mr^2}$

$\Rightarrow \quad v = r\sqrt{\dfrac{2\,mgh}{I + mr^2}}$

$\Rightarrow \quad \omega = \dfrac{v}{r} = \sqrt{\dfrac{2\,mgh}{I + mr^2}}$

Hence, the correct answer is (B).

73. The combined effect of translation and rotation is equivalent to a single effect of rotation about the point having zero velocity i.e., IC just like the hoop being pinned at IC and rotating about IC.



So, this can be assumed as a pure rotation about point of contact say O (i.e., IC) with angular velocity $\omega = \frac{v}{R}$, where R is the radius of hoop. Speed of P will be

$$v_P = (OP)\omega = \left(2R\sin\frac{\theta}{2}\right)\omega$$

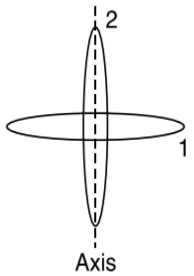
$$\Rightarrow \quad v_P = (2R\omega)\sin\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \quad v_P = 2v_0\sin\left(\frac{\theta}{2}\right)$$

Hence, the correct answer is (B).

74. $I_1 = mr^2$

I_2 =moment of inertia about any one of the diameters



$$= \frac{1}{2}mr^2$$

$$\Rightarrow \quad I = I_1 + I_2 = \frac{3}{2}mr^2$$

Hence, the correct answer is (D).

75. $\vec{L} = m(\vec{r}\times\vec{v})$

$$\Rightarrow \quad \vec{L} = 2\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 0\\ 2 & -2 & 2\end{vmatrix}$$

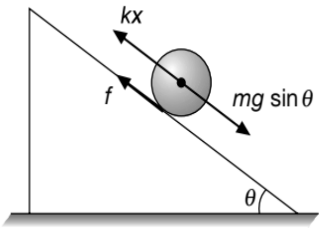
$$\vec{L} = 2\left(2\hat{i} - 2\hat{j} - 4\hat{k}\right)$$

$$\Rightarrow \quad \vec{L} = 4\hat{i} - 4\hat{j} - 8\hat{k}$$

$$\Rightarrow \quad L_z = -8 \text{ kgm}^2\text{s}^{-1}$$

Hence, the correct answer is (D).

76. Initially the spring force kx is less than $mg\sin\theta$, i.e., the cylinder is accelerated downward or force of friction f is upwards. It will reverse its direction when $kx > mg\sin\theta$.

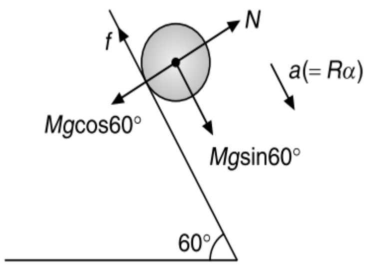


Hence, the correct answer is (D).

77. $\omega = \frac{v}{R}$ and $v_c = 0$

Hence, the correct answer is (C).

78. Let friction act on cylinder in backward direction



Since, $a = R\alpha$ ...(1)

$$\Rightarrow \quad mg\sin 60^\circ - f = Ma \qquad \text{...(2)}$$

Since $fR = I\alpha$

$$\Rightarrow \quad fR = \frac{MR^2}{2}\left(\frac{a}{R}\right)$$

$$\Rightarrow \quad f = \frac{Ma}{2} \qquad \text{...(3)}$$

$$\Rightarrow \quad Mg\sin 60^\circ = \frac{3}{2}Ma$$

$$\Rightarrow \quad a = \frac{2}{3}g\sin 60^\circ = \frac{g}{\sqrt{3}}$$

$$\Rightarrow \quad f = \frac{Mg}{2\sqrt{3}}$$

For sliding, we have

$$\Rightarrow \quad f = \mu Mg\cos 60^\circ$$

$$\Rightarrow \quad \frac{Mg}{2\sqrt{3}} = \left(\frac{2-3x}{\sqrt{3}}\right)\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad 2 - 3x = 1$$

$$\Rightarrow \quad 3x = 1$$

$$\Rightarrow \quad x = \frac{1}{3}m$$

Hence, the correct answer is (A).

79. Since spheres are smooth, so no transfer of angular momentum takes place from A to B. However, sphere A only transfers its linear velocity v to sphere B and stops. Hence, we conclude that A stops but continues to rotate with same angular speed ω and B moves with speed of A but with zero angular speed.
Hence, the correct answer is (B).

80. Consider the origin at $x=0$. Let the equilibrium be established at a distance r from origin.
Then by Law of Conservation of Moments

$$\left[1^2+2^2+3^2+.......+(100)^2\right]=(1+2+3+.......+100)r$$

$$\Rightarrow \frac{(100)(100+1)(200+1)}{6}=\frac{(100)(100+1)}{2}r$$

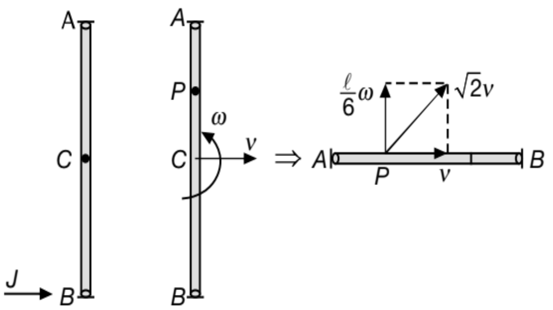
$$\Rightarrow r=\frac{201}{3}$$

$$\Rightarrow r=67 \text{ cm}$$

Hence, the correct answer is (B).

81. Angular momentum of system cannot remain conserved as some external unbalanced torque is present due to forces at axles. Kinetic energy is not conserved, because slipping is there and work is done against friction.
Hence, the correct answer is (C).

82. Let v and ω be the linear and angular speeds of the rod after applying an impulse J at B. Then
Impulse = Change in Momentum



$$\Rightarrow mv=J$$

$$\Rightarrow v=\frac{J}{m} \quad ...(1)$$

Also, $\begin{pmatrix}\text{Angular}\\ \text{Impulse}\end{pmatrix}=\begin{pmatrix}\text{Change in Angular}\\ \text{Momentum}\end{pmatrix}$

$$\Rightarrow I\omega=J\left(\frac{\ell}{2}\right)$$

$$\Rightarrow \frac{m\ell^2}{12}\omega=J\left(\frac{\ell}{2}\right)$$

$$\Rightarrow \omega=\frac{6J}{m\ell} \quad ...(2)$$

After the given time $t=\frac{\pi m\ell}{12J}$, the rod will rotate an angle

$$\theta=\omega t=\left(\frac{6J}{m\ell}\right)\left(\frac{\pi m\ell}{12J}\right)=\frac{\pi}{2}$$

$$\Rightarrow v=\left(\frac{\ell}{6}\right)\omega=\left(\frac{\ell}{6}\right)\left(\frac{6J}{m\ell}\right)=\frac{J}{m}$$

$$\Rightarrow |\vec{v}_P|=\sqrt{2}v=\sqrt{2}\frac{J}{m}$$

Hence, the correct answer is (C).

83. Net external torque is zero. Therefore, angular momentum of system will remain conserved, i.e.,

$$L_i=L_f$$

Initial angular momentum $L_i=0$, so final angular momentum should also be zero

$$\begin{pmatrix}\text{Angular}\\ \text{Momentum of}\\ \text{Man in CW sense}\end{pmatrix}+\begin{pmatrix}\text{Angular Momentum}\\ \text{of Platform in}\\ \text{CCW sense}\end{pmatrix}=0$$

$$\Rightarrow mvr-(I_{\text{platform}})\omega=0$$

$$\Rightarrow \omega=\frac{mv_0r}{I}=\frac{(70)(1)(2)}{200}$$

$$\Rightarrow \omega=0.7 \text{ rads}^{-1}$$

Hence, the correct answer is (B).

84. Angular velocity of man relative to platform is

$$\omega_r=\omega+\frac{v_0}{r}=0.7+\frac{1}{2}=1.2 \text{ rads}^{-1}$$

If t be the time taken by man to complete one round around the platform, then

$$t=\frac{2\pi}{\omega_r}$$

$$\Rightarrow t=\frac{2\pi}{1.2} \text{ s}$$

Angle rotated with respect ground in this time is

$$\theta=\left(\frac{v_0}{r}\right)t=\left(\frac{1}{2}\right)\left(\frac{2\pi}{1.2}\right)=\frac{5\pi}{6}$$

Hence, the correct answer is (B).

85. Angular retardation $\alpha=\frac{\tau}{I}=\frac{\mu mgR}{mR^2}=\frac{\mu g}{R}$
Since, $\omega=\omega_0-\alpha t$

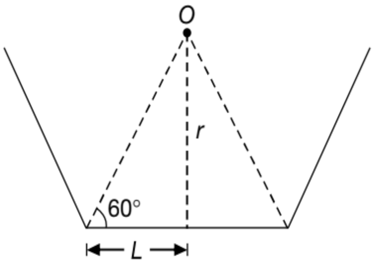
$$\Rightarrow t=\frac{\omega_0-\omega}{\alpha}$$

$$\Rightarrow t=\frac{\omega_0-\frac{\omega_0}{2}}{\frac{\mu g}{R}}=\frac{\omega_0 R}{2\mu g}$$

Hence, the correct answer is (D).

86. Length of each side is $2L$, so if r is the perpendicular distance of wire from the centre of mass (in this case, the geometrical centre), then

$$r = \sqrt{3}L$$



The moment of inertia about O is

$$I = 6(I_{\text{one side}})$$

$$\Rightarrow \quad I = 6\left(\frac{m(2L)^2}{12} + mr^2\right)$$

$$\Rightarrow \quad I = 6\left(\frac{mL^2}{3} + 3mL^2\right) = 20\ \text{mL}^2$$

Hence, the correct answer is (B).

87. Moment of inertia about z-axis is $\frac{1}{2}$ MR^2

Perpendicular distance on $y = x + c$ from origin is $r_\perp = d = \left|\frac{0-0+c}{\sqrt{2}}\right| = \frac{c}{\sqrt{2}}$

According to Parallel Axis Theorem, we have

$$I = I_G + Md^2$$

$$\Rightarrow \quad \frac{1}{2}MR^2 = \frac{1}{4}MR^2 + M\left(\frac{c}{\sqrt{2}}\right)^2$$

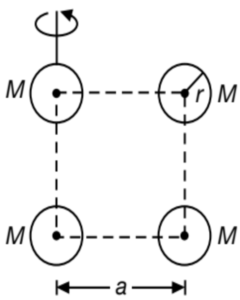
$$\Rightarrow \quad c = \frac{R}{\sqrt{2}}$$

Hence, the correct answer is (A).

88. $I_{\text{system}} = 4I_{\text{sphere}} + 2Ma^2$

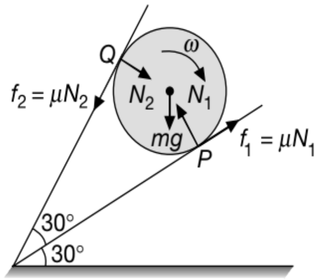
$$\Rightarrow \quad I_{\text{system}} = 4\left(\frac{2}{5}Mr^2\right) + 2Ma^2$$

$$\Rightarrow \quad I_{\text{system}} = \frac{2}{5}M\left(4r^2 + 5a^2\right)$$



Hence, the correct answer is (D).

89. Let μ be the friction coefficient between sphere and each wall. Free body diagram of sphere is



Net force on the sphere in horizontal direction is zero, so

$$N_1 \cos 60° + \mu N_2 \cos 60° = N_2 \cos 30° + \mu N_1 \cos 30°$$

$$\Rightarrow \quad N_1 + \mu N_2 = \sqrt{3}(N_2 + \mu N_1)$$

$$\Rightarrow \quad N_1(1 - \sqrt{3}\mu) = N_2(\sqrt{3} - \mu)$$

$$\Rightarrow \quad \frac{N_1}{N_2} = \frac{\sqrt{3} - \mu}{1 - \sqrt{3}\mu}$$

Substituting $\mu = \frac{1}{3}$ we get,

$$\frac{N_1}{N_2} = \frac{\sqrt{3} - \frac{1}{3}}{1 - \frac{\sqrt{3}}{3}} = \frac{3\sqrt{3} - 1}{3 - \sqrt{3}} = 1 + \frac{4}{\sqrt{3}}$$

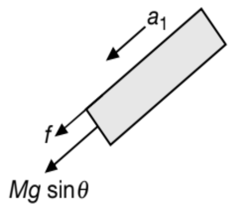
Since, $\frac{f_1}{f_2} = \frac{\mu N_1}{\mu N_2} = 1 + \frac{4}{\sqrt{3}}$

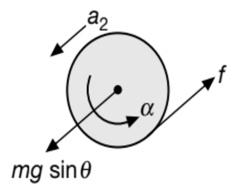
Hence, the correct answer is (B).

90. For Plank, $Mg\sin\theta + f = Ma_1$ ...(1)

For Sphere, $mg\sin\theta - f = ma_2$ ...(2)

where $a_2 - R\alpha = a_1$ ...(3)





Since, $fR = \left(\frac{2}{5}mR^2\right)\alpha$ ...(4)

Solving equations (1), (2), (3) and (4), we get

$$f = 0$$

Hence, the correct answer is (A).

91. Since $\tau = I\alpha$

$$\Rightarrow \quad \tau = \left[\frac{m(2\ell)^2}{12}\right]\left(\frac{\omega}{t}\right)$$

$$\Rightarrow \quad \tau = \frac{m\ell^2\omega}{3t}$$

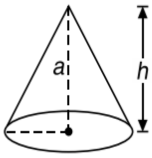
Hence, the correct answer is (B).

92. $KE=\frac{1}{2}m\left(v_0^2\right)+\frac{1}{2}\left(mR^2\right)\left(\frac{v_0}{R}\right)^2$

$+\frac{1}{2}(3m)\left(\sqrt{2}v_0\right)^2+\frac{1}{2}(m)\left(2v_0\right)^2=6mv_0^2$

Hence, the correct answer is (B).

93. $I=\frac{3}{10}Ma^2$



Hence, the correct answer is (D).

94. From conservation of angular momentum $(I\omega=\text{constant})$, angular velocity will remain half. As,

$$K=\frac{1}{2}I\omega^2$$

The rotational kinetic energy will become half.
Hence, the correct answer is (B).

95. Moment of inertia of semi-circular portions about x and y axes are same. But moment of inertia of straight portions about x-axis is zero. So, we have

$$I_x<I_y$$

$$\Rightarrow \frac{I_x}{I_y}<1$$

Hence, the correct answer is (B).

96. By Law of Conservation of Angular Momentum

$$\left(\sum mvr\right)_{\text{about }0}=\left(\mathrm{I}_{\substack{\text{system}\\ \text{about }0}}\right)\omega$$

$$\Rightarrow MV\frac{L}{4}=\left(\mathrm{I}_{\text{rod about }0}+\mathrm{I}_{\text{insect about }0}\right)\omega$$

$$\Rightarrow MV\frac{L}{4}=\left[\frac{ML^2}{12}+M\left(\frac{L}{4}\right)^2\right]\omega$$

$$\Rightarrow MV\frac{L}{4}=ML^2\left(\frac{1}{12}+\frac{1}{16}\right)\omega$$

$$\Rightarrow MV\frac{L}{4}=ML^2\left(\frac{4+3}{48}\right)\omega$$

$$\Rightarrow \omega=\frac{12}{7}\frac{V}{L}$$

Hence, the correct answer is (B).

97. $\text{R.K.E.}=\frac{1}{2}\mathrm{I}\omega^2$

$$\Rightarrow \text{R.K.E.}=\frac{1}{2}\left(2mr^2\right)(2\pi n)^2$$

$$\Rightarrow \text{R.K.E.}=4\pi^2n^2\ \mathrm{mr}^2$$

Hence, the correct answer is (B).

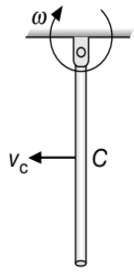
98. The rod will rotate about A. Therefore, by Law of Conservation of Mechanical Energy, we get

$$\begin{pmatrix}\text{Decrease in}\\ \text{Gravitational}\\ \text{Potential Energy}\end{pmatrix}=\begin{pmatrix}\text{Increase in Rotational}\\ \text{Kinetic Energy}\\ \text{about } A\end{pmatrix}$$

$$\Rightarrow mg\left(\frac{\ell}{2}\right)=\frac{1}{2}I_A\omega^2$$

$$\Rightarrow mg\left(\frac{\ell}{2}\right)=\frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$$

$$\Rightarrow \omega^2=\frac{3g}{\ell} \qquad \ldots(1)$$



Centripetal force on CM of rod in this position is

$$F_C=m\left(\frac{\ell}{2}\right)\omega^2=\frac{3mg}{2} \qquad \{\text{towards } A\}$$

Let F be the force exerted by the hinge on the rod upwards, then

$$F-mg=ma_C=F_C$$

$$\Rightarrow F-mg=\frac{3mg}{2}$$

$$\Rightarrow F=\frac{5}{2}mg$$

So, the force exerted by the rod on the hinge is $\frac{5}{2}mg$, downwards
Hence, the correct answer is (B).

99. Assume mass of cylinder to be m, radius R, then linear acceleration of cylinder is zero when

$mg\sin\theta=$ frictional force (f) upwards

Now, angular acceleration about C is $\alpha=\frac{\tau}{I}$

$$\Rightarrow \alpha=\frac{fR}{\frac{1}{2}mR^2}=\frac{2f}{mR}$$

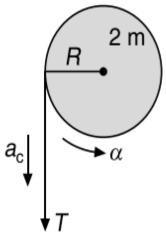
$$\Rightarrow \alpha=\frac{2mg\sin\theta}{mR}=\frac{2g\sin\theta}{R}$$

For no slipping between cylinder and plank, we have

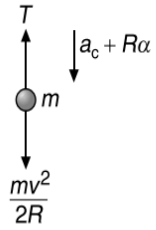
$$a=R\alpha=2g\sin\theta$$

Hence, the correct answer is (C).

100. If a_C is initial acceleration of centre of mass of disc and α is its initial angular acceleration, then we have



FBD of disc



FBD of particle

$$T = (2m)a_C$$

Also, $TR = \frac{1}{2}(2m)R^2\alpha$

$$T = mR\alpha$$

$$\Rightarrow \quad \frac{m\left(\frac{J}{m}\right)^2}{2R} - T = m(a_c + R\alpha)$$

$$\Rightarrow \quad T + \frac{T}{2} + T = \frac{J^2}{2mR}$$

$$\Rightarrow \quad \frac{5T}{2} = \frac{J^2}{2mR}$$

$$\Rightarrow \quad T = \frac{J^2}{5mR}$$

$$\Rightarrow \quad a_C = \frac{T}{2m} = \frac{J^2}{5mR(2m)} = \frac{J^2}{10m^2R}$$

Substituting all the values, we get

$$a_c = 4 \text{ ms}^{-2}$$

Hence, the correct answer is (B).

101. $mgh = \frac{1}{2}mv_{CM}^2\left(1 + \frac{K^2}{R^2}\right)$

$$\Rightarrow \quad \frac{7}{10}mv_{CM}^2 = mgh$$

$$\Rightarrow \quad h = \frac{7}{10}\left(\frac{v_{CM}^2}{g}\right)$$

Hence, the correct answer is (B).

102. KE of ball in position $B = mg(R - r)$

Since, it rolls without slipping the ratio of rotational to translational kinetic energy will be $\frac{k^2}{r^2} = \frac{2}{5}$

$$\Rightarrow \quad \frac{K_R}{K_T} = \frac{k^2}{r^2} = \frac{2}{5}$$

$$\Rightarrow \quad K_T = \frac{5}{7}mg(R - r)$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{5}{7}mg(R - r)$$

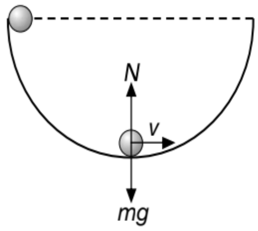
$$\Rightarrow \quad v = \sqrt{\frac{10g(R - r)}{7}}$$

$$\Rightarrow \quad \omega = \frac{v}{R - r} = \sqrt{\frac{10g}{7(R - r)}}$$

Hence, the correct answer is (B).

103. By Law of Conservation of Mechanical Energy, we get

$$mg(R - r) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{7}{10}mv^2 \qquad \text{...(1)}$$



Also, $N - mg = \frac{mv^2}{R - r}$ ...(2)

Solving equations (1) and (2), we get

$$N = \frac{17}{7}\text{ mg}$$

Hence, the correct answer is (D).

104. Since, $a = \dfrac{g\sin\theta}{1 + \frac{k^2}{R^2}}$

$$\Rightarrow \quad a = \frac{g\sin(60^\circ)}{1 + \frac{1}{2}}$$

$$\Rightarrow \quad a = \frac{g}{\sqrt{3}}$$

Hence, the correct answer is (C).

105. According to Parallel Axis Theorem, we have

$$I = I_G + md^2$$

In case of a circle, the distance d is equal from centre of mass of the rigid body for all points lying on it.

Hence, the correct answer is (B).

106. Since, $\tau_{ext} = 0$

$$\Rightarrow \quad I\omega = \text{constant}$$

$$\Rightarrow \quad \Delta(I\omega) = 0$$

$$\Rightarrow \quad \omega\Delta I + I\Delta\omega = 0$$

$$\Rightarrow \quad \frac{\Delta\omega}{\omega} = -\frac{\Delta I}{I}$$

Now, volume $V = \frac{4}{3}\pi R^3$

$\Rightarrow \quad \frac{\Delta V}{V} \times 100 = 3\frac{\Delta R}{R} \times 100$

Percentage increase in volume is 3%, so percentage increase in radius will be 1%

Further, $I = \frac{2}{5}mR^2$

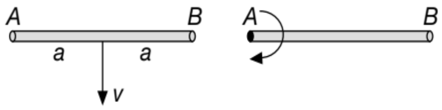
$\Rightarrow \quad \frac{\Delta I}{I} = \frac{2\Delta R}{R}$

$\Rightarrow \quad \frac{\Delta \omega}{\omega} = -\frac{\Delta I}{I} = -\frac{2\Delta R}{R} = -2\%$

Hence, the correct answer is (A).

107. Applying Law of Conservation of Angular momentum about A, we get

$L_i = L_f$



$\Rightarrow \quad mva = I\omega$

$\Rightarrow \quad mva = \frac{m(2a)^2}{3}\omega$

$\Rightarrow \quad \omega = \frac{3v}{4a}$

Hence, the correct answer is (D).

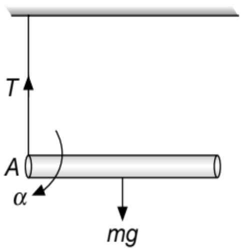
108. $K = K_T + K_R$

$\Rightarrow \quad K = \frac{1}{2}(2m)(v^2) + \frac{1}{2}\left(mR^2 + \frac{(2R)^2}{12}\right)\left(\frac{v}{R}\right)^2$

$\Rightarrow \quad K = \frac{5}{3}mv^2$

Hence, the correct answer is (D).

109. The rod will rotate about point A. Let a be the linear acceleration of centre of mass of the rod and α the angular acceleration of the rod about A. Then



$mg - T = ma \quad \ldots(1)$

Also, $\alpha = \frac{\tau}{I} = \frac{mg\frac{\ell}{2}}{\frac{m\ell^2}{3}} = \frac{3g}{2\ell} \quad \ldots(2)$

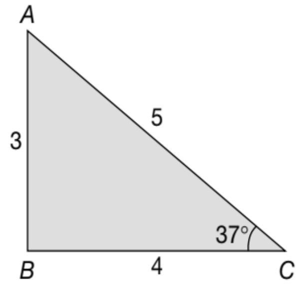
$\Rightarrow \quad a = \frac{\ell}{2}\alpha \quad \ldots(3)$

Solving equations (1), (2) and (3), we get

$T = \frac{mg}{4}$

Hence, the correct answer is (A).

110. $AC > BC > AB$



Moment of inertia of a body depends on distribution of mass from the axis of rotation. The mass is farthest from the axis AB. So, I_1 is maximum and nearest from the axis AC. So, I_3 is minimum or $I_3 < I_2 < I_1$.

Hence, the correct answer is (C).

111. In case of pure rolling, we have

$\frac{K_R}{K_T} = \frac{k^2}{R^2}$

for solid sphere $\frac{K_R}{K_T} = \frac{2}{5}$ and

for solid cylinder $\frac{K_R}{K_T} = \frac{1}{2}$

At the bottom, both have the same linear velocities, i.e., they have the same translational kinetic energies, so

$\frac{5}{7}mgh_1 = \frac{2}{3}mgh_2$

$\Rightarrow \quad \frac{h_1}{h_2} = \frac{14}{15}$

Hence, the correct answer is (C).

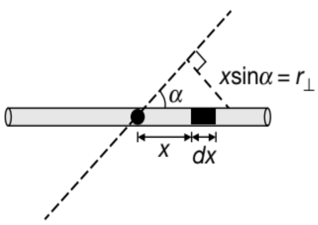
112. $\int \tau dt = \Delta L$

Since, $\tau = Fr_\perp = (2t)(R + r)$

$\Rightarrow \quad L = \int_0^t 2t(R+r)dt = (R+r)t^2$

Hence, the correct answer is (C).

113. The desired moment of inertia is



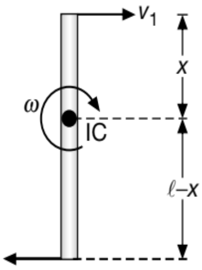
$$I = \int_{x=-\ell}^{x=+\ell} dI = \int_{-\ell}^{\ell} dm(r_\perp)^2$$

$$I = \int_{-\ell}^{\ell}\left(\frac{m}{2\ell}dx\right)(x\sin\alpha)^2$$

$$I = \frac{m\ell^2}{3}\sin^2\alpha$$

Hence, the correct answer is (B).

114.



$$\omega = \frac{v_1}{x} = \frac{v_2}{\ell - x}$$

$$\Rightarrow \quad x = \frac{v_1}{v_1 + v_2}\ell$$

Hence, the correct answer is (C).

115. By Law of Conservation of Angular Momentum

$$I\omega = \text{constant}$$

Since R goes to $\frac{R}{n}$, so I goes to $\frac{I}{n^2}$, hence ω goes to $n^2\omega$

$\{\because\ I\omega = \text{constant}\}$

$$\Rightarrow \quad \omega' = n^2\omega$$

$$\Rightarrow \quad \frac{2\pi}{T'} = n^2\frac{2\pi}{T}$$

$$\Rightarrow \quad T' = \frac{T}{n^2} = \frac{24}{n^2}$$

So, here we formulate a rule to calculate the new time period. If Radius of earth shrinks by $\frac{1}{n}(n>1)$ then new time period is $\frac{T}{n^2}$ i.e. $\frac{24}{n^2}$. So new time period of rotation also decreases.

Hence, the correct answer is (B).

116. Angular retardation $\alpha = \frac{\tau}{I} = \frac{(\mu mgR)}{\frac{2}{5}mR^2}$

$$\Rightarrow \quad \alpha = \frac{5\mu g}{2R}$$

$$\Rightarrow \quad \alpha = \frac{5\times 0.1\times 10}{2\times 1}$$

$$\Rightarrow \quad \alpha = 2.5\ \text{rads}^{-2}$$

Now, $0 = \omega_0 - \alpha t$

$$\Rightarrow \quad t = \frac{\omega_0}{\alpha} = \frac{40}{2.5} = 16\ \text{s}$$

Hence, the correct answer is (C).

117. Since $\theta = \frac{1}{2}\alpha t^2$

$$\Rightarrow \quad \theta = \frac{1}{2}(2)(10)^2$$

$$\Rightarrow \quad \theta = 100\ \text{rads}^{-1}$$

$$\Rightarrow \quad n = \frac{\theta}{2\pi} = \frac{100}{2\pi}$$

$$\Rightarrow \quad n \simeq 16$$

Hence, the correct answer is (B).

118. $\frac{I_2}{I_1} = 4$

$$\Rightarrow \quad \frac{M_2R_2^2}{M_1R_1^2} = 4$$

Let λ be the linear mass density. Then

$$M_1 = (2\pi R_1)\lambda \text{ and}$$

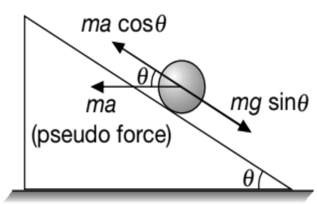
$$M_2 = (2\pi R_2)\lambda$$

$$\Rightarrow \quad \frac{R_2^3}{R_1^3} = 4$$

$$\Rightarrow \quad \frac{R_2}{R_1} = 4^{\frac{1}{3}}$$

Hence, the correct answer is (B).

119.



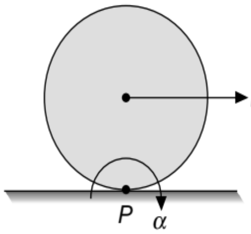
The sphere will continue pure rolling when

$$ma\cos\theta = mg\sin\theta$$

$$\Rightarrow \quad a = g\tan\theta$$

Hence, the correct answer is (D).

120. In case of rolling without slipping, the point of contact P on the ground is at rest. So, the cylinder will rotate about P with angular acceleration



$$\alpha = \frac{\tau_P}{I_P} = \frac{FR}{\frac{3}{2}MR^2}$$

$$\Rightarrow \quad \alpha = \frac{2F}{3MR}$$

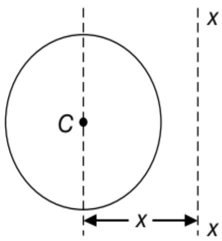
Hence, the correct answer is (B).

121. If m be the mass of sphere, then

$$I(x) = I = I_c + mx^2$$

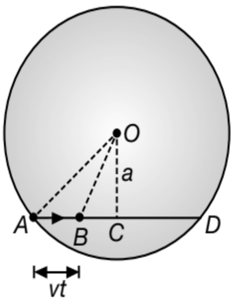
$I = I_c$ at $x = 0$

Therefore, I versus x graph is a parabola with minimum value of $I = I_c$ at $x = 0$. Therefore, the correct graph is (B).



Hence, the correct answer is (B).

122. Since, there is no external torque, angular momentum will remain conserved. The moment of inertia will first decrease till the tortoise moves from A to C and then increase as it moves from C and D. Therefore, ω will initially increase and then decrease.



Let R be the radius of platform, m the mass of disc and M is the mass of platform.

Moment of inertia when the tortoise is at A

$$I_1 = mR^2 + \frac{MR^2}{2}$$

and moment of inertia when the tortoise is at B

$$I_2 = mr^2 + \frac{MR^2}{2}$$

Here, $r^2 = a^2 + \left[\sqrt{R^2 - a^2} - vt\right]^2$

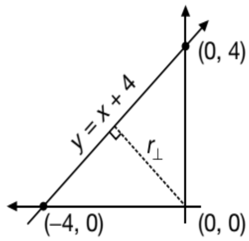
From conservation of angular momentum

$$\omega_0 I_1 = \omega(t) I_2$$

Substituting the values, we can see that variation of $\omega(t)$ is non-linear.

Hence, the correct answer is (B).

123.



$$L = mvr_{\perp}$$

where $r_{\perp} = \frac{|0 - 0 - 4|}{\sqrt{2}} = \frac{4}{\sqrt{2}}$

$$\Rightarrow \quad L = (5)(3\sqrt{2})\left(\frac{4}{\sqrt{2}}\right)$$

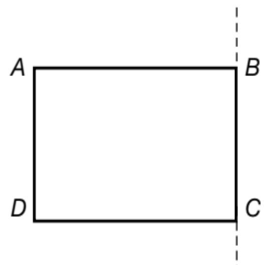
$$\Rightarrow \quad L = 60 \text{ cgs unit}$$

Hence, the correct answer is (D).

124. **Conceptual the correct answer is (C).**

125. $\frac{AB}{BC} = 2$

$$\Rightarrow \quad AB = DC = \frac{\ell}{3} \text{ and } BC = AD = \frac{\ell}{6}$$



Similarly, $m_{AB} = m_{DC} = \frac{m}{3}$

and $m_{BC} = m_{AD} = \frac{m}{6}$

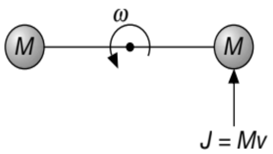
Now $I = 2I_{AB} + I_{AD} + I_{BC}$

$$\Rightarrow \quad I = 2\left\{\left(\frac{1}{3}\right)\left(\frac{m}{3}\right)\left(\frac{\ell}{3}\right)^2\right\} + \left(\frac{m}{6}\right)\left(\frac{\ell}{3}\right)^2 + 0$$

$$\Rightarrow \quad I = \frac{189}{4374}m\ell^2 = \frac{7}{162}m\ell^2$$

Hence, the correct answer is (D).

126. Let ω be the angular velocity of the rod. Applying, angular impulse equals change in angular momentum about centre of mass of the system, we get



$$J\left(\frac{L}{2}\right) = I_c\omega$$

$$\Rightarrow \quad (Mv)\left(\frac{L}{2}\right) = (2)\left(\frac{ML^2}{4}\right)\omega$$

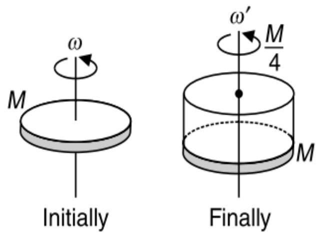
$$\Rightarrow \quad \omega = \frac{v}{L}$$

Hence, the correct answer is (A).

127. $mg\sin\theta$ component is always down the plane whether it is rolling up or rolling down. Therefore, for no slipping, sense of angular acceleration should also be same in both the cases. Therefore, force of friction f always act upwards.

Hence, the correct answer is (B).

128.



By Law of Conservation of Angular Momentum

$$\left(\frac{1}{2}MR^2\right)\omega = \left(\frac{1}{2}MR^2 + \frac{1}{2}\left(\frac{M}{4}\right)R^2\right)\omega'$$

$$\Rightarrow \quad \omega = \frac{5}{4}\omega'$$

$$\Rightarrow \quad \omega' = \frac{4\omega}{5}$$

Hence, the correct answer is (B).

129. Force of friction passes through point P. Hence, its torque about P will be zero. Hence, only $mg\sin\theta$ will provide torque to ring about P.

Since, $\tau t = \Delta L$

$$\Rightarrow \quad (mgR\sin\theta)t = L$$

Hence, the correct answer is (B).

130. Let m be the mass of the disc. Then translational kinetic energy of the disc is

$$K_T = \frac{1}{2}mv^2 \qquad \ldots(1)$$

As it ascends on a smooth track its rotational kinetic energy will remain same while translational kinetic energy will go on decreasing. At highest point.

$$K_T = mgh$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = mgh$$

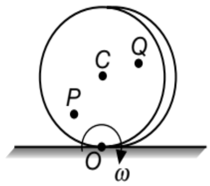
$$\Rightarrow \quad h = \frac{v^2}{2g} = \frac{(6)^2}{2\times 10} = 1.8 \text{ m}$$

Hence, the correct answer is (B).

131. Since, net force on centre of mass is zero so, the centre of mass will not move at all. Hence, the body can only rotate about centre of mass.

Hence, the correct answer is (B).

132. In case of pure rolling bottommost point is the instantaneous centre of zero velocity.



Velocity of any point on the disc, $v = r\omega$, where r is the distance of point from O.

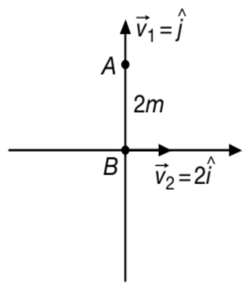
$$r_Q > r_C > r_P$$

$$\Rightarrow \quad v_Q > v_C > v_P$$

Hence, the correct answer is (A).

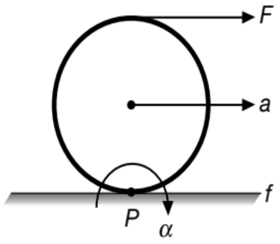
133. At $t = 2$ s, A and B are at $(0,\ 2\text{ m})$ and $(0,\ 0)$ as shown. Component of relative velocity perpendicular to AB is 2 ms^{-1}. So,

$$\omega = \frac{2}{2} = 1 \text{ rads}^{-1}$$



Hence, the correct answer is (B).

134. Let f be the friction on the ring towards right, a be the linear acceleration and α be the angular acceleration of the ring about centre of mass.



Point of contact P is momentarily at rest, about Instantaneous Centre of Zero Velocity IC (located at P), i.e., ring will rotate about P, so

$$\alpha = \frac{\tau_P}{I_P} = \frac{F(2R)}{2MR^2} = \frac{F}{MR}$$

For translational motion of ring, we have

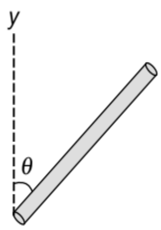
$$F + f = Ma = MR\alpha = F$$

$\Rightarrow \quad f = 0$

Hence, the correct answer is (B).

135. Since moment of inertia of a rod about an axis passing through one end, making angle θ with it is

$$I = \frac{1}{3}m\ell^2 \sin^2\theta$$



So, $I_x = I_y = 2\left\{\frac{m\ell^2}{3}\sin^2 45°\right\} = \frac{m\ell^2}{3}$

$$I_z = 2\left(\frac{m\ell^2}{3}\right) = \frac{2}{3}m\ell^2$$

Hence, $I_x = I_y < I_z$

Hence, the correct answer is (B).

136. $\begin{pmatrix}\text{Loss in} \\ \text{Gravitational} \\ \text{P.E. of} \\ \text{C.M.}\end{pmatrix} = \begin{pmatrix}\text{Gain in} \\ \text{Rotational} \\ \text{K.E. of} \\ \text{C.M.}\end{pmatrix} + \begin{pmatrix}\text{Gain in} \\ \text{Translational} \\ \text{K.E. of} \\ \text{C.M.}\end{pmatrix}$

$$\Rightarrow \quad MgR - M'gR' = \left(\frac{1}{2}I'\omega^2\right) + \frac{1}{2}M'v_{cm}^2$$

$$\left\{\text{where } R' = \frac{R}{2} \ \& \ M' = \pi R'^2 \ell\rho = \frac{\pi R^2 \ell\rho}{4} = \frac{M}{4}\right\}$$

$$\Rightarrow \quad \frac{7}{8}MgR = \frac{1}{2}\left(\frac{1}{2}M'R'^2\right)\left(\frac{v_{cm}}{R'}\right)^2 + \left(\frac{1}{2}\left(\frac{M}{4}\right)v_{cm}^2\right)$$

$$\Rightarrow \quad \frac{7}{8}MgR = \frac{1}{2}\left(\frac{M}{8}\right)v_{cm}^2 + \left(\frac{M}{8}\right)v_{cm}^2$$

$$\Rightarrow \quad \frac{7}{8}MgR = \left(\frac{M}{8}\right)\left(\frac{3}{2}\right)v_{cm}^2$$

$$\Rightarrow \quad v_{cm} = \sqrt{\frac{14}{3}gR}$$

Hence, the correct answer is (B).

137. Tangential Force $= F_T = ma = m(\alpha L) = N$

$\therefore$ Limiting value of friction $(f_s)_{max} = \mu N = \mu F_T$

$\Rightarrow \quad (f_s)_{max} = \mu N = \mu m\alpha L \qquad \ldots(1)$

Further if ω is the angular velocity at time t, then

$\omega = \alpha t \qquad \ldots(2)$

Also, the centripetal force is

$F_C = mL\omega^2 = mL(\alpha^2 t^2) \qquad \ldots(3)$

For bead to slide $F_C > (f_s)_{max}$

$$\Rightarrow \quad mL(\alpha^2 t^2) > \mu m\alpha L$$

$$\Rightarrow \quad t > \sqrt{\frac{\mu}{\alpha}}$$

So, the minimum time after which the bead begins to start is $\sqrt{\frac{\mu}{\alpha}}$

Hence, the correct answer is (A).

138. In pure rolling mechanical energy remains conserved, therefore speed will be same in both the cases. Acceleration of the sphere, a, down the plane is proportional to $\sin\theta$. So,

$$a \propto \sin\theta$$

i.e., acceleration and hence, time of descend will be different.

Hence, the correct answer is (B).

139. $\vec{v} = \vec{\omega} \times \vec{r}$

$$\Rightarrow \quad \vec{v} = (2\hat{k}) \times (2\hat{i} + 2\hat{j}) = 4(\hat{j} - \hat{i})$$

Hence, the correct answer is (B).

140. In uniform circular motion the only force acting on the particle is centripetal (towards centre). Torque of this force about the centre is zero. Hence, angular momentum about centre remain conserved.

Hence, the correct answer is (A).

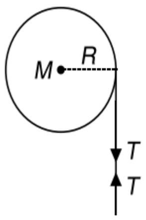
141. For Angular Momentum to be conserved $\vec{\tau}_{ext} = \vec{0}$

$\Rightarrow \quad \frac{a}{2} = \frac{3}{-6} = \frac{6}{-12}$

$\Rightarrow \quad a = -1$

Hence, the correct answer is (B).

142. Torque $\tau = TR$



$\Rightarrow \quad I\alpha = TR$

$\Rightarrow \quad \left(\frac{1}{2}MR^2\right)\alpha = TR$

$\Rightarrow \quad T = \frac{1}{2}MR\alpha$

$\Rightarrow \quad T = \frac{1}{2}(50)(0.5)(4\pi)$

$\Rightarrow \quad T = 157 \text{ N}$

Hence, the correct answer is (B).

143. $\tau = FR = I\alpha$

$\Rightarrow \quad FR = \left(\frac{1}{2}MR^2\right)\alpha$

$\Rightarrow \quad \alpha = \frac{2F}{MR}$

$\Rightarrow \quad a = \frac{2F}{M}$

$s = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{2F}{M}\right)t^2$

$\Rightarrow \quad s = \left(\frac{F}{M}\right)t^2$ and $\theta = \frac{1}{2}\alpha t^2$

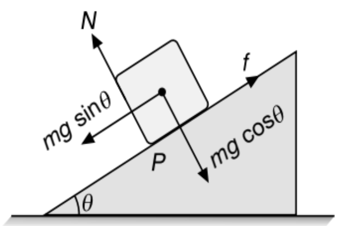
$\Rightarrow \quad \theta = \left(\frac{F}{MR}\right)t^2$

Hence, the correct answer is (C).

144. If m be the mass of the cube and a be the side of the cube, then the cube will slide when

$mg\sin\theta > \mu mg\cos\theta$

$\Rightarrow \quad \tan\theta > \mu \qquad \ldots(1)$



The cube will topple if torque due to $mg\sin\theta$ about P is greater than torque due to $mg\cos\theta$ about P i.e.,

$$(mg\sin\theta)\left(\frac{a}{2}\right) > (mg\cos\theta)\left(\frac{a}{2}\right)$$

Since f, the force of friction passes through P, so torque due to f about P is zero.

$\Rightarrow \quad \tan\theta > 1 \qquad \ldots(2)$

From equations (1) and (2), we observe that cube slides before toppling if

$\mu < 1$

Hence, the correct answer is (B).

145. MI of the sphere about the diameter parallel to PQ is

$$I = \frac{2}{5}MR^2$$

By parallel axis theorem, MI of the sphere about the centre of mass is

$$I = I_{cm} + Md^2$$

$\Rightarrow \quad I_{cm} = \frac{2}{5}MR^2 - M\left(\frac{3R}{8}\right)^2 \qquad \ldots(1)$

The MI about the axis PQ at a distance $d' = \frac{3R}{4} - \frac{3R}{8} = \frac{3R}{8}$ from the CM is

$$I_{PQ} = I_{cm} + M(d')^2$$

$\Rightarrow \quad I_{PQ} = \left[\frac{2}{5}MR^2 - M\left(\frac{3R}{8}\right)^2\right] + M\left(\frac{3R}{8}\right)^2$

$\Rightarrow \quad I_{PQ} = \frac{2}{5}MR^2 = MK^2$

$\Rightarrow \quad K = R\sqrt{\frac{2}{5}}$

Hence, the correct answer is (D).

146. $\vec{L} = m(\vec{r} \times \vec{v})$

Direction of $(\vec{r} \times \vec{v})$, hence the direction of angular momentum remains the same.

Hence, the correct answer is (B).

147. By Law of Conservation of Energy

$$Mgh = \frac{1}{2}Mv_{CM}^2\left(1 + \frac{K^2}{R^2}\right)$$

Hence, the correct answer is (C).

148. $R.K.E. = \frac{1}{2}I\omega^2 = 1500$

$\Rightarrow \quad \frac{1}{2}I(\alpha t)^2 = 1500$

$\Rightarrow \quad (1.2)(25)^2 t^2 = 3000$

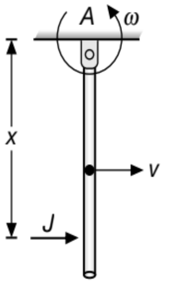
$\Rightarrow \quad t^2 = 4$

$\Rightarrow \quad t = 2$

Hence, the correct answer is (B).

149. Since, Impulse = Change in Momentum, so we get

$$J = mv \qquad \ldots(1)$$



where v is the linear speed of centre of mass of rod. Also, Angular Impulse = Change in Angular Momentum

$$\Rightarrow \quad Jx = I\omega = \left(\frac{m\ell^2}{3}\right)\omega \qquad \ldots(2)$$

$$\Rightarrow \quad v = \frac{\ell}{2}\omega \qquad \ldots(3)$$

where, ω is the angular speed of rod about point A.

Solving these three equations, we get

$$x = \frac{2}{3}\ell$$

Hence, the correct answer is (B).

150. Let the extension produced in the spring due to rotation be x. Then, for the body to be in equilibrium

$$F_{\text{spring}} = F_{\text{centrifugal}}$$

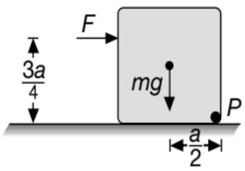
$$\Rightarrow \quad kx = m(\ell + x)\omega^2$$

$$\Rightarrow \quad x = \frac{m\ell\omega^2}{k - m\omega^2}$$

Hence, the correct answer is (B).

151. For the cube to topple about P, we must have

$$\tau_F > \tau_{\text{mg}}$$



$$\Rightarrow \quad F\left(\frac{3a}{4}\right) > (mg)\frac{a}{2}$$

$$\Rightarrow \quad F > \frac{2}{3}mg$$

So, the minimum value of F is $\frac{2}{3}mg$

Hence, the correct answer is (C).

152. $I_{\text{RIM}} = 2MR^2$

$$I_{\text{DIAMETER}} = \frac{1}{2}MR^2$$

$\Rightarrow$ Required Ratio $= 4:1$

Hence, the correct answer is (B).

153. Conceptual the correct answer is (D).

154. $Pd = I_A\omega$ where $I_A = \frac{1}{3}m\ell^2$

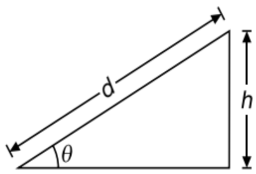
Since the distance from A to C is $\frac{\ell}{2}$, we get $v_C = \frac{\omega\ell}{2}$.

$$\Rightarrow \quad m\left(\frac{\omega\ell}{2}\right)d = \left[\frac{1}{3}m\ell^2\right]\omega$$

$$\Rightarrow \quad d = \frac{2}{3}\ell$$

Hence, the correct answer is (B).

155. $\sin\theta = \frac{h}{d}$



Also, condition for banking is

$$\tan\theta = \frac{v^2}{rg}$$

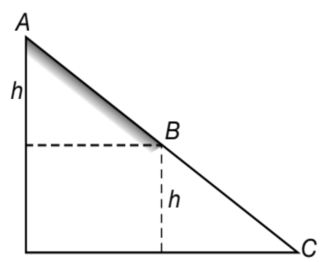
$$\Rightarrow \quad \tan\left[\sin^{-1}\left(\frac{h}{d}\right)\right] = \frac{v^2}{rg}$$

Hence, the correct answer is (B).

156. In case of pure rolling upto B, we have for a cylinder

$$\frac{K_T}{K_R} = 2$$

where $K_T = \frac{2}{3}mgh$ and $K_R = \frac{1}{3}mgh$



After B: Rotational kinetic energy is constant (because no torque is provided due to absence of friction) however translational kinetic energy increases.

At C: $K_T = \frac{2}{3}mgh + mgh = \frac{5}{3}mgh$

while $K_R = \frac{1}{3}mgh$

So, $\frac{K_T}{K_R} = 5$

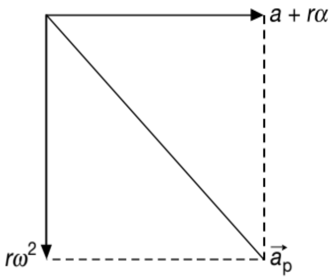
Hence, the correct answer is (B).

157. $\vec{a}_P = \vec{a}_{P_0} + \vec{a}_0$

Here, $\vec{a}_{P_0}$ = acceleration of P with respect to O, so

$$\vec{a}_{P_0} = \vec{a}_{P_0 t} + \vec{a}_{P_0 n}$$

$$\Rightarrow \quad \vec{a}_P = \left(\vec{a}_{P_0 t} + \vec{a}_{P_0 n}\right) + \vec{a}_0$$



where, $\vec{a}_{P_0 t}$ = tangential component of $\vec{a}_{P_0}$

and $\vec{a}_{P_0 n}$ = normal component of $\vec{a}_{P_0}$

So, $\left|\vec{a}_0 + \vec{a}_{P_0 t}\right| = a + r\alpha$ and

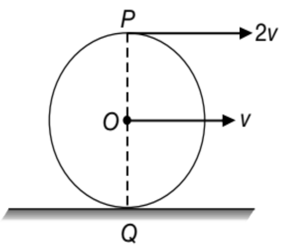
$$\left|\vec{a}_{P_0 n}\right| = r\omega^2$$

$$\Rightarrow \quad \left|\vec{a}_P\right| = \sqrt{(a + r\alpha)^2 + \left(r\omega^2\right)^2}$$

Hence, the correct answer is (A).

158. In case (i) work done by friction is zero, while in case (ii) it is non-zero.
Hence, the correct answer is (B).

159.

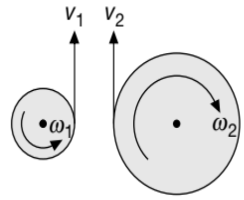


$$v_r = v_{PQ} = 2v$$

$$\Rightarrow \quad a_r = \frac{v_r^2}{2R} = \frac{4v^2}{2R} = \frac{2v^2}{R}$$

Hence, the correct answer is (B).

160. Let ω_1 and ω_2 be the final angular velocities when the slipping has ceased. Then



$$v_1 = v_2$$

$$\Rightarrow \quad \omega_1 R = \omega_2 (2R)$$

$$\Rightarrow \quad \omega_2 = \frac{\omega_1}{2} \qquad \ldots(1)$$

Since, $\begin{pmatrix}\text{Angular}\\ \text{Impulse}\end{pmatrix} = \begin{pmatrix}\text{Change in Angular}\\ \text{Momentum}\end{pmatrix}$, so

$$JR = I\omega_1 \qquad \ldots(2)$$

and $J2R = 4I\left(\omega_0 - \omega_2\right) \qquad \ldots(3)$

where, J is the linear impulse due to friction which acts tangentially and is equal for both the cylinders.

Solving equations (1), (2) and (3), we get

$$\omega_1 = \omega_0 \text{ and } \omega_2 = \frac{\omega_0}{2}$$

Hence, the correct answer is (B).

161. The frictional force μmg is the only horizontal force acting on the two bodies. So, each body has an acceleration $\frac{\mu mg}{m} = \mu g$ in opposite direction. So, relative acceleration is $2\mu g$.

Hence, the correct answer is (B).

162. $a = \dfrac{g}{1 + \dfrac{M}{2m}}$

So, $v^2 = 2ah$

$\Rightarrow$ v is independent of R.

Hence, the correct answer is (D).

163. $I = Mk^2 = (40)(0.5)^2$

$$\Rightarrow \quad I = 40(0.25) = 10 \text{ kgm}^2$$

$$\omega_0 = 1800 \times \frac{2\pi}{60} = 60\pi \text{ rads}^{-1}$$

$$\Rightarrow \quad \alpha = \frac{\omega_0}{t} = \frac{60\pi}{30} = 2\pi \text{ rads}^{-2}$$

Also, $\tau = I\alpha$

$$\Rightarrow \quad \tau = 20\pi$$

Hence, the correct answer is (B).

164. By Law of Conservation of Energy

$$Mgh = \frac{1}{2}Mv_{CM}^2\left(1+\frac{K^2}{R^2}\right)$$

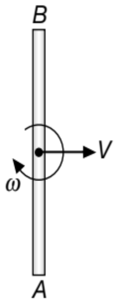
For a cylinder $\frac{K^2}{R^2} = \frac{1}{2}$

$$\Rightarrow \quad v_{CM} = \sqrt{\frac{4}{3}gh}$$

Hence, the correct answer is (B).

165. Let V be the linear velocity of centre of mass of rod just after collision and ω be its angular velocity, then by Law of Conservation of Linear Momentum, we have

$$mv_0 = Mv \qquad \ldots(1)$$



By Law of Conservation of Angular Momentum about centre of rod, we get

$$mv_0x = \left(\frac{ML^2}{12}\right)\omega \qquad \ldots(2)$$

For A to be at rest just after collision, we have

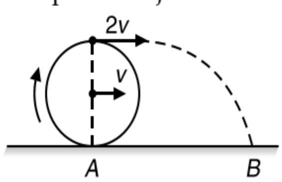
$$\frac{L}{2}\omega = V \qquad \ldots(3)$$

Solving equations (1), (2) and (3), we get

$$x = \frac{L}{6}$$

Hence, the correct answer is (B).

166. From the concepts of Projectile Motion.



$$2r = \frac{1}{2}gt^2$$

$$\Rightarrow \quad t = \sqrt{\frac{4r}{g}}$$

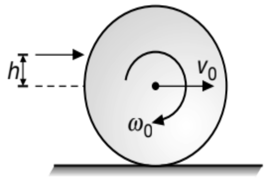
Since $x = AB = (2v)t$

$$\Rightarrow \quad AB = 4v\sqrt{\frac{r}{g}}$$

Hence, the correct answer is (C).

167. Since $L = mvr_\perp$ and as $r_\perp$ is constant, so L is constant.
Hence, the correct answer is (B).

168. Let J be the linear impulse imparted to the ball, then



Impulse =Change in Momentum

$$\Rightarrow \quad J = mv_0 \qquad \ldots(1)$$

Also, $\begin{pmatrix}\text{Angular}\\\text{Impulse}\end{pmatrix} = \begin{pmatrix}\text{Change in Angular}\\\text{Momentum}\end{pmatrix}$

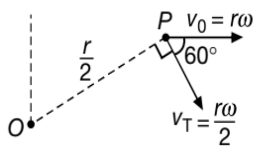
$$\Rightarrow \quad Jh = I\omega_0 = \frac{2}{5}mr^2\omega_0 \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\omega_0 = \frac{5}{2}\frac{v_0h}{r^2}$$

Hence, the correct answer is (B).

169. The point P will have a velocity v_0 and a tangential velocity $v_T = \left(\frac{r}{2}\right)\omega = \frac{v_0}{2}$ inclined to each other at 60°.

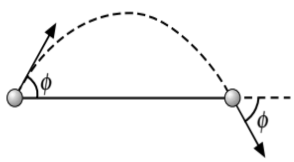


So, $v_P^2 = v_0^2 + \frac{v_0^2}{4} + 2(v_0)\left(\frac{v_0}{2}\right)\cos(60^\circ)$

$$\Rightarrow \quad v_P = \frac{v_0\sqrt{7}}{2}$$

Hence, the correct answer is (C).

170.



$$\langle\omega\rangle = \frac{\Delta\phi}{\Delta t} = \frac{2\phi}{T} = \frac{2\phi}{\left(\frac{2v\sin\phi}{g}\right)} = \frac{g\phi}{v\sin\phi}$$

Hence, the correct answer is (D).

171. Angular momentum, $L = I\omega$

For the said axis, $I = \frac{1}{3}m\ell^2$

$$\Rightarrow \quad L = \frac{m\ell^2}{3}\omega$$

Hence, the correct answer is (B).

Multiple Correct Choice Type Questions

1. $I_1 = \frac{1}{2}MR^2 + \frac{1}{12}M\ell^2$

$\Rightarrow\ I_1 = \frac{1}{2}MR^2 + \frac{1}{12}M(4R^2)$

$\Rightarrow\ I_1 = \frac{1}{2}MR^2 + \frac{1}{3}MR^2 = \frac{5}{6}MR^2$

Also, $I_2 = \frac{1}{2}MR^2 + \frac{1}{3}M(4R^2)$

$\Rightarrow\ I_2 = \frac{1}{2}MR^2 + \frac{4}{3}MR^2 = \frac{11}{6}MR^2$

$\Rightarrow\ I_2 > I_1$ and $I_2 - I_1 = MR^2$

Hence, (A) and (B) are correct.

2. Due to Law of Conservation of Angular Momentum $\vec{L}$ =constant

$\Rightarrow\ \vec{L}\cdot\vec{L} = \text{constant}$

$\Rightarrow\ \frac{d}{dt}(\vec{L}\cdot\vec{L}) = 0$

$\Rightarrow\ 2\vec{L}\cdot\frac{d\vec{L}}{dt} = 0$

$\Rightarrow\ \vec{L} \perp \frac{d\vec{L}}{dt}$

Since $\vec{\tau} = \vec{A}\times\vec{L}$

$\Rightarrow\ \frac{d\vec{L}}{dt} = \vec{A}\times\vec{L}$

$\Rightarrow\ \frac{d\vec{L}}{dt}$ must be perpendicular to $\vec{A}$ as well as $\vec{L}$

Further component of $\vec{L}$ along $\vec{A}$ is $\frac{\vec{A}\cdot\vec{L}}{A} = x$(say). Since $\vec{A} \perp \frac{d\vec{L}}{dt}$ and $\frac{d\vec{A}}{dt} = \vec{0}$

$\Rightarrow\ \frac{d}{dt}(\vec{A}\cdot\vec{L}) = \vec{A}\cdot\frac{d\vec{L}}{dt} + \vec{L}\cdot\frac{d\vec{A}}{dt} = 0$

$\Rightarrow\ \vec{A}\cdot\vec{L} = \text{constant}$

$\Rightarrow\ \frac{\vec{A}\cdot\vec{L}}{A} = x = \text{constant}$

Since $\frac{d\vec{L}}{dt}$ (or $\vec{\tau}$) is perpendicular to $\vec{L}$, hence it cannot change magnitude of $\vec{L}$ but can surely change direction of $\vec{L}$.

Hence, (A), (B) and (C) are correct.

3. By Law of Conservation of Linear Momentum, we have

$$mv\hat{i} + mv\hat{j} + mv(-\hat{j}) + \vec{0} = \vec{p}_{\substack{\text{Triangular}\\ \text{Wedge } ABC}} + \vec{0}$$

$$\Rightarrow\ \vec{p}_{\substack{\text{Triangular}\\ \text{Wedge } ABC}} = (mv)\hat{i}$$

Since the net linear momentum imparted to the triangular wedge is along x-axis and is non-zero, so the centre of mass of wedge ABC will move along x-axis.

Hence, the correct answer is (B).

4. In the frame of rod, the small vertical rods will experience centrifugal forces which forms a couple in clockwise direction in the state given in problem. To balance this couple force by hinge at A on the rod must be downward and the force by hinge at B must be upward.

The angular momenta of the vertical rod particles about point O will be inclined to rod hence option (D) is also correct.

Hence, (A), (B) and (D) are correct.

5. $V_c = \frac{2m(-v) + m(2v) + 8m(0)}{2m + m + 8m} = 0$ {OPTION **(A)**}

Further, by Law of Conservation of Angular Momentum

$$\left(I_{\substack{\text{system}\\ \text{about } O}}\right)\omega = \sum_{\substack{\text{system}\\ \text{about O}}} mvr_{\perp}$$

$$I_{\substack{\text{system}\\ \text{about } O}} = \left[\frac{1}{12}(8m)(6a)^2 + m(2a)^2 + 2ma^2\right] = 30ma^2$$

$$\sum_{\substack{\text{system}\\ \text{about } O}} mvr_{\perp} = (8m)(0)(0) + 2m(-v)(-a) + m(2v)(2a)$$

$$\Rightarrow\ \sum_{\substack{\text{system}\\ \text{about } O}} mvr_{\perp} = 6mva$$

$$\Rightarrow\ (30ma^2)\omega = 6mva$$

$\Rightarrow\ \omega = \frac{v}{5a}$ {OPTION **(C)**}

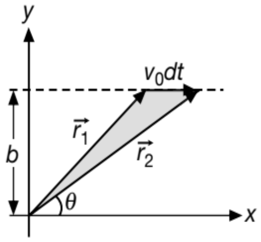
Further total energy of the system E is

$E = \frac{1}{2}I\omega^2 = \frac{3}{5}mv^2$ {OPTION **(D)**}

Hence, (A), (C) and (D) are correct.

6. $L = mv_0 r\sin\theta$

$L = mv_0 b = \text{constant}$



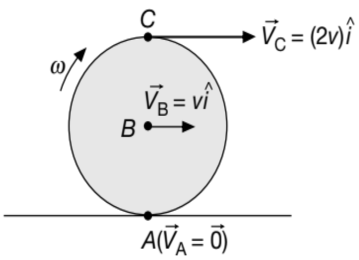
Also, $|\vec{r}_1| \approx |\vec{r}_2| = r$

Since, $dA = \frac{1}{2}(v_0 dt) r\sin\theta$

$$\Rightarrow \quad \frac{dA}{dt} = \frac{mv_0 r\sin\theta}{2m} = \frac{L}{2m}$$

Hence, (B) and (D) are correct.

7.



Hence, (B) and (C) are correct.

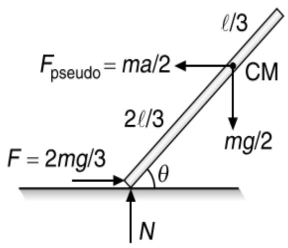
8. The mass M of the rod is

$$M = \int_0^l \lambda dr = \int_0^l \left(\frac{\lambda_0}{l^2} r\right) dr = \frac{\lambda_0}{2}$$

Position of centre of mass of rod from end lying on the surface is

$$r_{cm} = \frac{\int_0^l \left(\frac{\lambda_0}{l^2} rdr\right) r}{\int_0^l \frac{\lambda_0}{l^2} rdr} = \frac{2l}{3}$$

Free body diagram of rod is shown in Figure.



Given that $F = \frac{4W}{3} = \frac{4Mg}{3} = \frac{2\lambda_0 g}{3}$

If $N = \frac{\lambda_0 g}{2}$, then for equilibrium of the rod, we have $\Sigma\tau = 0$ and $\Sigma F = 0$.

$\Sigma F = 0$ gives, $F_{\text{applied}} = F_{\text{pseudo}} = Ma$

$$\Rightarrow \quad \frac{2\lambda_0 g}{3} = \frac{\lambda_0 a}{2}$$

$$\Rightarrow \quad a = \frac{4}{3}g$$

Similarly, $\Sigma\tau = 0$ gives $F\sin\theta = N\cos\theta$

$$\Rightarrow \quad \tan\theta = \frac{g}{a} = \frac{3}{4}$$

$$\Rightarrow \quad \theta = 37°$$

Hence, (A), (B) and (C) are correct.

9. $F\hat{i} = 3m\vec{a} \quad \{\because \text{Net force} = (\text{Total mass})(\text{acceleration})\}$

$$\Rightarrow \quad \vec{a} = \vec{a}_0 = \frac{1}{3}\frac{F}{m}\hat{i}$$

Since $\tau = I_0\alpha$

$$\Rightarrow \quad Fb = (3mr^2)\alpha$$

$$\Rightarrow \quad \alpha = \frac{Fb}{3mr^2}$$

Hence, (A) and (C) are correct.

10. $L_1 = I\omega = MK^2\omega \quad \ldots(1)$

$L_2 = I\omega + MvR$

$\Rightarrow \quad L_2 = MK^2\omega + MR(\omega R) \quad \{\because v = R\omega\}$

$\Rightarrow \quad L_2 = M\omega(K^2 + R^2) \quad \ldots(2)$

From equations (1) and (2), we can see that

$L_2 = 2L_1$, when $K = R$ and

$L_2 > 2L_1$, when $K > R$

Hence, (B) and (D) are correct.

11. **Conceptual (C) and (D) are correct.**

12. $\omega = \omega_0 - a\phi$

$$\Rightarrow \quad \frac{d\phi}{dt} = \omega_0 - a\phi$$

$$\Rightarrow \quad \int_0^\phi \frac{d\phi}{\omega_0 - a\phi} = \int_0^t dt$$

$$\Rightarrow \quad -\frac{1}{a}\log_e(\omega_0 - a\phi)\Big|_0^\phi = t$$

$$\Rightarrow \quad \log_e(\omega_0 - a\phi) - \log_e \omega_0 = -at$$

$$\Rightarrow \quad \frac{\omega_0 - a\phi}{\omega_0} = e^{-at}$$

$$\Rightarrow \quad \phi = \frac{\omega_0}{a}\left(1-e^{-at}\right) \qquad \text{\{OPTION (A)\}}$$

Further, $\omega = \dfrac{d\phi}{dt} = \dfrac{\omega_0}{a}\dfrac{d}{dt}\left[1-e^{-at}\right]$

$$\Rightarrow \quad \omega = \frac{\omega_0}{a}\left[-e^{-at}(-a)\right]$$

$$\Rightarrow \quad \omega = \omega_0 e^{-at} \qquad \text{\{OPTION (C)\}}$$

Hence, (A) and (C) are correct.

13. When cylinder comes down, at the point where string leaves contact with the cylinder is point of instantaneous rest, thus string does zero work.

$$\frac{K_R}{K_T} = \frac{\frac{1}{2}I_C\omega^2}{\frac{1}{2}mv^2} = \frac{\frac{1}{2}\left(\frac{mR^2}{2}\right)\left(\frac{v}{R}\right)^2}{\frac{1}{2}mv^2} = \frac{1}{2}$$

Hence, (A) and (C) are correct.

14. The farther the mass is distributed from the Axis of Rotation (AOR), the more the Moment of Inertia.

Here C must be towards heavier side and hence lies between O and B.

$$\Rightarrow \quad I_A > I_O > I_C > I_B$$

Hence, (A) and (C) are correct.

15. Since $L = J\ell$

Where $J = mv$ is the impulse.

Also, $L = I\omega$

$$\Rightarrow \quad \omega = \frac{L}{I} = \frac{J\ell}{\frac{1}{3}m\ell^2}$$

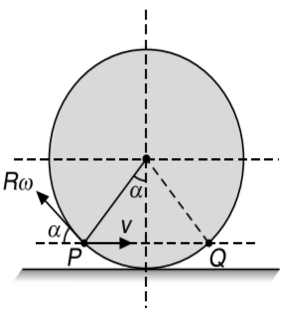
$$\Rightarrow \quad \omega = \frac{3J}{m\ell}$$

Kinetic energy $= \dfrac{L^2}{2I} = \dfrac{3J^2}{2m}$

and $v_C = \omega\dfrac{\ell}{2} = \dfrac{3J}{2m}$

Hence, (A), (B), (C) and (D) are correct.

16. Particle P will have a velocity in vertical direction, when



$$R\omega\cos\alpha = v$$

$$\Rightarrow \quad \alpha = \cos^{-1}\left(\frac{v}{R\omega}\right)$$

So, the required angle θ is

$$\theta = \pi - \cos^{-1}\left(\frac{v}{R\omega}\right) \text{ and } \theta = \pi + \cos^{-1}\left(\frac{v}{R\omega}\right)$$

Point P corresponds to $\theta = \pi - \cos^{-1}\left(\dfrac{v}{R\omega}\right)$ and it has velocity in vertically upward direction, while Q corresponds to $\theta = \pi + \cos^{-1}\left(\dfrac{v}{R\omega}\right)$ and it has velocity in vertically downward direction.

Hence, (C) and (D) are correct.

17. If v_{cm} be the velocity of centre of mass of the cylinder, ω be the angular velocity of the cylinder, then for no sliding between plank A and cylinder C, we have

$$R\omega - v_{cm} = v \qquad \ldots(1)$$

Similarly, for no sliding between plank B and cylinder C, we have

$$v_{cm} + R\omega = 2v \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$v_{cm} = \frac{v}{2} \text{ and } \omega = \frac{3v}{2R}, \text{ clockwise}$$

Below the centre of the cylinder at a distance x (say), we observe a point which happens to be at instantaneous rest and this point is called the instantaneous centre (IC) of zero velocity and the axis passing through it is called the Instantaneous Axis of Rotation. At this point, we have

$$v_{cm} - x\omega = 0$$

$$\Rightarrow \quad x = \frac{v_{cm}}{\omega} = \frac{v/2}{3v/2R} = \frac{R}{3}$$

The translational kinetic energy of the system is

$$K_T = K_A + K_B + (K_C)_T$$

$$\Rightarrow \quad K_T = \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)v^2 + \frac{1}{2}(8m)\left(\frac{v}{2}\right)^2$$

$$\Rightarrow \quad K_T = 4mv^2$$

Since the cylinder is in pure rolling, so the kinetic energy of the system is

$$K_T = K_A + K_B + (K_C)_{\text{rolling}}$$

The kinetic energy in rolling is the sum of translational kinetic energy and rotational kinetic energy and is given by

$$K_{\text{rolling}} = \frac{1}{2}Mv_{cm}^2\left(1+\frac{K^2}{R^2}\right)$$

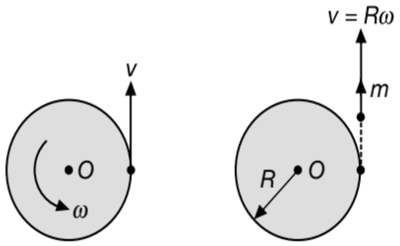
So, $(K_C)_{\text{rolling}} = \frac{1}{2}(8m)\left(\frac{v}{2}\right)^2\left(1+\frac{1}{2}\right) = \frac{3}{2}mv^2$

$$\Rightarrow \quad K = \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)v^2 + \frac{3}{2}mv^2$$

$$\Rightarrow \quad K_T = \frac{9}{2}mv^2$$

Hence, (A), (B), (C) and (D) are correct.

18. Since, the broken away piece of mass m (say) has the same linear speed as it had before detaching from the disc, the angular speed of the remaining disc will remain stay unchanged. Further, the linear speed of the piece will first decrease and then increase due to gravity. So, its angular speed about an axis passing through O will first decrease and then increase.



Hence, (C) and (D) are correct.

19. Since $\begin{pmatrix}\text{Angular}\\\text{Impulse}\end{pmatrix} = \begin{pmatrix}\text{Change in Angular}\\\text{Momentum}\end{pmatrix}$, so

$$L = \tau t$$

$$\Rightarrow \quad L = F(2R)t$$

i.e., L varies linearly with time.

Hence, (B) and (C) are correct.

20. Velocity of particle is $\vec{v} = \vec{\omega}\times\vec{r}$ and centripetal acceleration is

$$\vec{a}_c = \vec{\omega}\times\vec{v} = \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

Hence, (B) and (D) are correct.

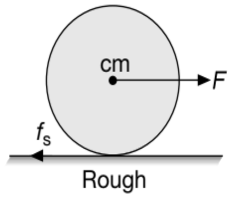
21. On a smooth horizontal surface, it can roll without slipping if $v = R\omega$ and no external force is acting on it.

Hence, (A), (C) and (D) are correct.

22. On melting, moment of inertia increases and hence ω decreases.

Hence, (B) and (D) are correct.

23. Since cylinder will roll towards right and required torque about centre of mass can be provided by friction if friction acts in the backward direction.



Now $F - f_s = ma$ ...(1)

Also, $\tau = f_s R = I_{\text{cm}}\alpha = \left(\frac{1}{2}mR^2\right)\frac{a}{R}$

$$\Rightarrow \quad f_s = \frac{1}{2}ma \qquad \text{...(2)}$$

Solving (1) and (2), we get

$a = \frac{2F}{3M}$ and $f_s = \frac{F}{3}$

Hence, (B), (C) and (D) are correct.

24. Point of contact P is at rest. Taking torque due to all the forces about point P we observe that

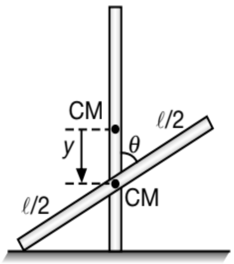
Torque is clockwise if F_1 is applied i.e., spool will rotate clockwise.

Torque due to F_2 is zero i.e., spool will not rotate if F_2 is applied, because F_2 passes through P.

Torque due to F_3 and F_4 is anticlockwise i.e., spool will rotate anticlockwise if only F_3 or F_4 is applied.

Hence, (B) and (D) are correct.

25. Since surface is frictionless, the centre of mass of the rod will fall along the straight line



$$y_{\text{cm}} = \frac{\ell}{2}(1-\cos\theta)$$

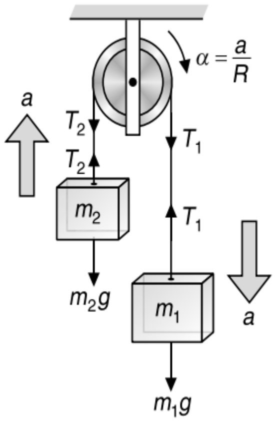
$$\Rightarrow \quad \frac{dy}{dt} = \left(\frac{\ell}{2}\sin\theta\right)\frac{d\theta}{dt} = \frac{\ell\omega}{2}\sin\theta$$

$$\Rightarrow \quad a = \frac{d^2y}{dt^2} = \frac{\ell}{2}\omega^2\cos\theta + \left(\frac{\ell}{2}\sin\theta\right)\alpha$$

$$\Rightarrow \quad a = \frac{\omega^2\ell}{2}\cos\theta + \frac{\ell\alpha}{2}\sin\theta$$

Hence, (A) and (D) are correct.

26. Since the pulley has mass, so tension in both branches of string must be different.



Assume T_1 to be absent then T_2 provides a torque T_2R (in anticlockwise sense).

Now assume T_2 to be absent then T_1 provides a torque T_1R (in clockwise sense). So, Net Torque

$$\tau = T_1R - T_2R$$

$$\Rightarrow \quad I\alpha = (T_1 - T_2)R \qquad \{\because \tau = I\alpha\}$$

$$\Rightarrow \quad I\left(\frac{a}{R}\right) = (T_1 - T_2)R \qquad \left\{\because \alpha = \frac{a}{R}\right\}$$

$$\Rightarrow \quad (T_1 - T_2) = \frac{Ia}{R^2} \qquad \ldots(1)$$

Further for m_1

$$m_1g - T_1 = m_1a \qquad \ldots(2)$$

and for m_2

$$T_2 - m_2g = m_2a \qquad \ldots(3)$$

Adding (2) and (3) and using (1), we get

$$a = \left[\frac{m_1 - m_2}{m_1 + m_2 + (I/R^2)}\right]g$$

Hence, (A), (B), (C) and (D) are correct.

27. When the centre of mass moves a distance S, then the distance covered by the point of application of force F is $2S$, therefore work done is $W = F(2S)$

From Work Energy Theorem, we have

$$W = 2FS = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad v = \sqrt{\frac{20FS}{7M}}$$

Hence, (A) and (B) are correct.

28. Moment of inertia will decrease but radius of gyration will increase because the distance of remaining mass from the axis has been increased. Radius of gyration depends on the distribution of mass from the axis.

Hence, (A), (B) and (D) are correct.

29. From symmetry we have

$$I_1 = I_2 \text{ and } I_4 = I_3 \qquad \ldots(1)$$

Also, by perpendicular Axis Theorem

$$I_0 = I_1 + I_2 = I_4 + I_3 \qquad \ldots(2)$$

From (2), we get

$$2I_1 = 2I_3 \qquad \{\because \text{ of } (1)\}$$

$$\Rightarrow \quad I_1 = I_3$$

$$\Rightarrow \quad I_0 = I_1 + I_2 = I_4 + I_3 = I_1 + I_3$$

Hence, (A), (B) and (C) are correct.

30. $\vec{L}_{\text{total}} = \vec{L}_{\text{about cm}} + \vec{L}_{\text{cm about } O}$

$$\Rightarrow \quad |\vec{L}_O| = \underbrace{(I_{\text{cm}})\omega}_{CCW} + \underbrace{Mv_{\text{cm}}(r_{\perp \text{ cm from } O})}_{CW}$$

Taking CW as positive, we get

$$|\vec{L}_O| = -\left(\frac{1}{2}MR^2\right)\omega + M(R\omega)(3R)$$

$$\Rightarrow \quad |\vec{L}_O| = \left(\frac{5}{2}MR^2\right)\omega$$

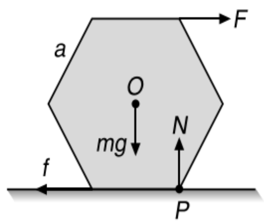
Similarly, angular momentum about A is

$$\vec{L}_A = \vec{L}_{\text{about cm}} + \vec{L}_{\text{cm about } A}$$

$$\Rightarrow \quad |\vec{L}_A| = \left(\frac{1}{2}mR^2\right)\omega + 0 = \left(\frac{1}{2}mR^2\right)\omega$$

Hence, (A) and (C) are correct.

31. When the body is on the verge of toppling, then we can apply the equilibrium condition to get the desired results. Also, if the body topples about the point P, then the normal reaction will shift to this point as shown in Figure.



So, we have $\Sigma F = 0$ and $\Sigma\tau = 0$

For translational equilibrium of the body

$$N = mg \text{ and } F = f_l = \mu N = \mu mg$$

For rotational equilibrium of the body, taking torque about P, we get

$$F\sqrt{3} = mg\left(\frac{a}{2}\right)$$

$$\Rightarrow \quad F = \frac{mg}{2\sqrt{3}}$$

For rotational equilibrium of the body, taking torque about O, we get

$$\frac{mg}{2\sqrt{3}}\left(\frac{\sqrt{3}a}{2}\right)+\mu mg\left(\frac{\sqrt{3}a}{2}\right)=mg\left(\frac{a}{2}\right)$$

$$\Rightarrow \quad \mu=\frac{1}{2\sqrt{3}}$$

When $\mu=2\mu_{\min}=\frac{1}{\sqrt{3}}$ and $F=\frac{mg}{\sqrt{3}}$, taking torque about P, we get

$$\frac{mg}{\sqrt{3}}(\sqrt{3}a)-mg\left(\frac{a}{2}\right)=\left(\frac{17}{12}ma^2\right)\alpha$$

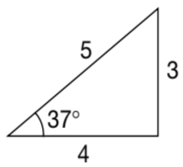
$$\Rightarrow \quad \alpha=\frac{6g}{17a}$$

Hence, (A) and (C) are correct.

32. Relative velocity between A and B along AB should be zero. Hence

$$v_A\sin\theta=v_B\cos\theta$$

$$\Rightarrow \quad v_A=v_B\cot\theta=\frac{4v_0}{3}$$



$$\text{Since, } \omega=\frac{\left(\begin{array}{c}\text{component of relative velocity}\\ \text{perpendicular to } AB\end{array}\right)}{AB}$$

$$\Rightarrow \quad \omega=\frac{v_A\cos\theta+v_B\sin\theta}{\ell}$$

$$\Rightarrow \quad \omega=\frac{\left(\frac{4v_0}{3}\right)\left(\frac{4}{5}\right)+v_0\left(\frac{3}{5}\right)}{\ell}=\frac{5v_0}{3\ell}$$

Hence, (B) and (D) are correct.

33. Conceptual (A), (B), (C) and (D) are correct.

34. In case of pure rolling, $\frac{K_R}{K_T}=\frac{k^2}{k^2+r^2}$, so

$$\frac{K_R}{K_T}=1 \text{ for a ring and } \frac{K_R}{K_T}=\frac{2}{5} \text{ for a solid sphere.}$$

where, K_R = rotational kinetic energy and

K_T = translational kinetic energy

Therefore, fraction of its total energy associated with rotation is

$$\alpha=\frac{1}{1+1}=\frac{1}{2} \text{ for ring and}$$

$$\beta=\frac{2}{2+5}=\frac{2}{7} \text{ for solid sphere}$$

Hence, (A) and (D) are correct.

35. If F_1 or F_2 is applied, the centre of mass of spool will move towards right and it will rotate anticlockwise about centre of mass. Hence, friction will be towards left.

If F_3 is applied, centre of mass of spool will move towards left and it will rotate anticlockwise about centre of mass. Hence, friction will be towards left.

If F_4 is applied, then the centre of mass of spool will move towards left and it will rotate counter clockwise and F_4 in itself has a tendency of both. So, friction may be zero, leftwards or rightwards depending upon the moment of inertia, mass and radius.

Hence, (A), (B) and (D) are correct.

36. Since no external torque is acting on the system hence $\vec{L}$ is conserved.

$$\left(\vec{L}_{\text{student+stool}}\right)_{\text{initial}}+\left(\vec{L}_{\text{wheel}}\right)_{\text{initial}}=\left(\vec{L}_{\text{student+stool}}\right)_{\text{final}}+\left(\vec{L}_{\text{wheel}}\right)_{\text{final}}$$

$$\Rightarrow \quad 0+\vec{L}_0=\left(\vec{L}_{\text{student+stool}}\right)_{\text{final}}+\left(-\vec{L}_0\right)$$

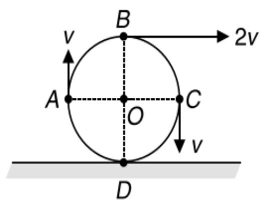
$$\Rightarrow \quad \left(\vec{L}_{\text{student+stool}}\right)_{\text{final}}=2\vec{L}_0$$

Hence, (A) and (C) are correct.

37. Speed of the bottommost point is zero but acceleration is not zero. Friction force may be there if it is an accelerated motion but work done by friction is always zero.

Hence, (A) and (D) are correct.

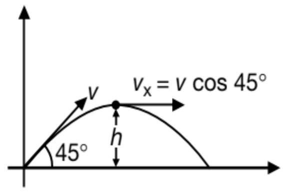
38. This figure makes us conclude that section AB and BC have equal kinetic energy. Also, section ABC has greater kinetic energy than section ADC and BC has greater kinetic energy than CD.



Hence, (A) and (B) are correct.

39. $L = mv_x h$

$$\Rightarrow \quad L = m(v\cos 45)\left(\frac{v^2 \sin^2 45}{2g}\right)$$



$$\Rightarrow \quad L = \frac{mv^3}{4\sqrt{2}\,g} \qquad \left\{\because h\frac{v^2\sin^2 45}{2g}\right\}$$

Also, from $h = \dfrac{v^2}{4g}$

$$\Rightarrow \quad v = 2\sqrt{gh}$$

Also, $L = \dfrac{mv}{\sqrt{2}}h$

$$L = \frac{m\left(2\sqrt{gh}\right)h}{\sqrt{2}}$$

$$\Rightarrow \quad L = m\sqrt{2gh^3}$$

Hence, (B) and (D) are correct.

40. When the net force on a rigid body is zero, then the summation of moments about any point is constant.

Hence, (A) and (C) are correct.

41. By Law of Conservation of Angular Momentum

$$\left(\frac{1}{2}MR^2\right)\omega = \left(\frac{1}{2}MR^2 + MR^2\right)\omega'$$

$$\Rightarrow \quad \omega' = \frac{\omega}{3}$$

Loss in Kinetic energy= $K_i - K_f$

Where $K_i = \dfrac{1}{2}I\omega^2 = \dfrac{1}{4}MR^2\omega^2$

$$K_f = \frac{1}{2}(I_{\text{total}})\omega'^2$$

$$\Rightarrow \quad K_f = \frac{1}{2}\left(\frac{3}{2}MR^2\right)\left(\frac{1}{9}\omega^2\right)$$

$$\Rightarrow \quad K_f = \frac{1}{12}MR^2\omega^2$$

$$\Rightarrow \quad \text{Loss} = \left(\frac{1}{4} - \frac{1}{12}\right)MR^2\omega^2$$

$$\Rightarrow \quad \text{Loss} = \frac{1}{6}MR^2\omega^2$$

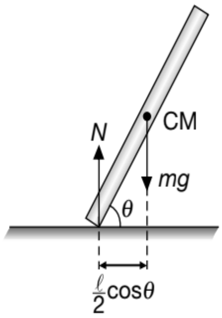
$$\Rightarrow \quad \text{Loss} = \frac{2}{3}\left(\frac{1}{4}MR^2\omega^2\right)$$

$$\Rightarrow \quad \text{Loss} = \frac{2}{3}K_i$$

Obviously, the lost energy must change to heat and actually this energy is lost due to the presence of friction which helps the two bodies attain a common angular velocity.

Hence, (A), (B) and (D) are correct.

42. Initially, when rod is vertical, magnitude of normal reaction is mg and only gravitational force is acting on the centre of mass of the rod in the vertical downward direction.



Now consider the situation in the figure, then

$$Mg - N = Ma \qquad \ldots(1)$$

Also, $N\left(\dfrac{\ell}{2}\cos\theta\right) = I\alpha = \left(\dfrac{M\ell^2}{12}\right)\alpha \qquad \ldots(2)$

$$\Rightarrow \quad \alpha\frac{\ell}{2} = a\cos\theta \qquad \ldots(3)$$

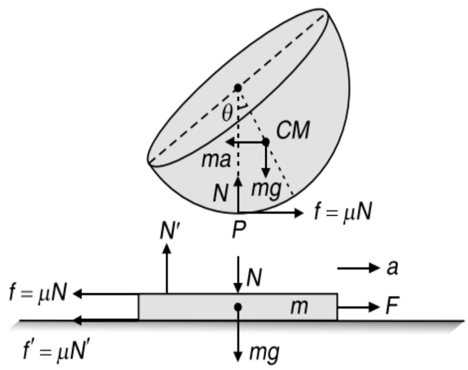
After solving (1), (2) and (3), we get

$$N = \frac{Mg}{4}$$

Hence, (A), (B) and (D) are correct.

43. When seen from the frame attached to the plank, net torque about P is zero, so we have

$$ma\left(R - \frac{R}{2}\cos\theta\right) = mg\frac{R}{2}\sin\theta$$



$$\Rightarrow \quad a = \frac{g\sin\theta}{2 - \cos\theta}$$

When $\theta = 37^\circ$, we get $a = 5\ \text{ms}^{-2}$

Also, $f = \mu N = ma$

$\Rightarrow \quad \mu mg = ma \quad \ldots(1)$

$\Rightarrow \quad \mu_{\min} = \frac{a}{g} = \frac{1}{2}$

For the plank, we have

$F - f - f' = ma$

$\Rightarrow \quad F - \mu mg - 2\mu mg = ma \quad \ldots(2)$

Adding equations (1) and (2), we get

$F - 2\mu mg = 2ma$

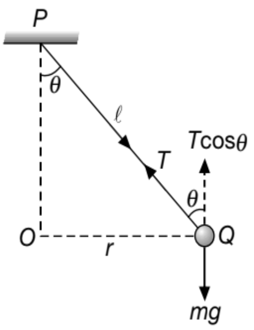
$\Rightarrow \quad F = 100\ \text{N}$

Hence, (B) and (D) are correct.

44. $T\cos\theta = mg$ and $T\sin\theta = \frac{mv^2}{r}$

$\Rightarrow \quad \frac{v^2}{rg} = \tan\theta = \tan(30^\circ) = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad v = \sqrt{\frac{rg}{\sqrt{3}}} = \sqrt{\frac{0.5 \times 10}{1.731}} \approx 1.7\ \text{ms}^{-1}$



Now, $PO = \ell\cos\theta$

$\Rightarrow \quad PO = (1)(0.86) = 0.86\ \text{m}$

Also, $r = \ell\sin\theta = (1)\left(\frac{1}{2}\right) = 0.5\ \text{m}$

Horizontal component of angular momentum is

$L_H = (m)(v)(PO)$

$\Rightarrow \quad L_H = (2)(1.7)(0.86) = 2.9\ \text{kgm}^2\text{s}^{-1}$

Vertical component of angular momentum is

$L_V = (m)(v)(OQ)$

$L_V = (2)(1.7)(0.5) = 1.7\ \text{kgm}^2\text{s}^{-1}$

Also, $\left|\frac{d\vec{L}}{dt}\right| = |\vec{\tau}| = mgr = (2)(10)(0.5)$

$\Rightarrow \quad \frac{d\vec{L}}{dt} = 10\ \text{kgm}^2\text{s}^{-2}$

Hence, (A), (B) and (C) are correct.

Reasoning Based Questions

1. When earth shrinks its angular momentum remains constant i.e.,

$$L = I\omega = \frac{2}{5}mR^2 \times \frac{2\pi}{T} = \text{constant}$$

$T \propto I \propto R^2$ it means if size of earth changes then it's moment of inertia changes. If radius is half so time period will become $\frac{T}{4} = \frac{24}{4} = 6$ hr.

Hence, the correct answer is (A).

2. In the case of smooth surface, $f_r = 0$, whereas in pure rolling motion friction can be present or absent.

Hence, the correct answer is (B).

3. Perpendicular axis theorem is not valid for a sphere.

Hence, the correct answer is (D).

4. If angular velocity is constant then frictional force acting on sphere is zero. In case of pure rolling velocity of contact point is zero.

Hence, the correct answer is (B).

5. If system has only rotational kinetic energy than momentum may be zero.

Hence, the correct answer is (D).

6. As the person climbs up, normal reaction and friction between the ladder and the wall both increases. This decreases normal reaction from the floor, decreasing limiting value of friction there. This increases the possibility of the ladder slipping.

Hence, the correct answer is (C).

7. Since net torque is zero angular velocity remains constant.

Hence, the correct answer is (A).

8. Moment of inertia is then sum of mr^2 terms. We cannot change all the r's, keep m's the same, and expect $\Sigma m_i r_i^2$ to remain unchanged.

Hence, the correct answer is (A).

9. Velocity of point of contact

$v = v_{cm} - R\omega$

When pure rolling occurs $v_{cm} = R\omega$

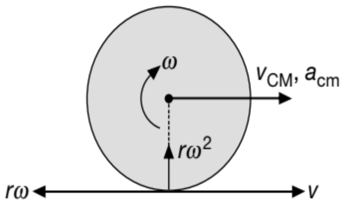
$\Rightarrow \quad V = 0$

Also, frictional force provides torque which further helps in achieving the pure rolling condition.

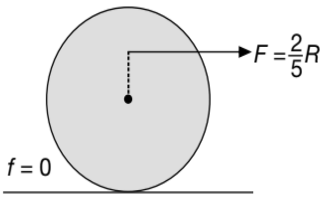
Hence, the correct answer is (B).

CHAPTER 3

10.

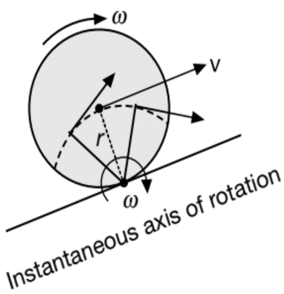


During the motion of rolling there is radial acceleration toward the centre. Hence contact point moves vertically upward.



Hence, the correct answer is (D).

11.



Hence, the correct answer is (A).

12. In sliding down, the entire potential energy is converted into kinetic energy. While in rolling down same part of potential energy is converted into kinetic energy of rotation, therefore linear velocity acquired is less.

Hence, the correct answer is (A).

13. Frictional force on an inclined plane, for a disc is

$$f = \frac{1}{3} g \sin\alpha$$

Hence, the correct answer is (B).

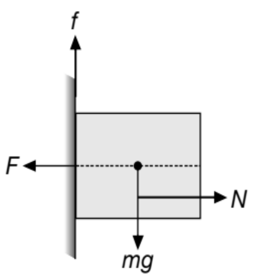
14. As the block remains stationary

$\Sigma fx = 0$ i.e., $F = N$

$\Sigma fy = 0$ i.e., $f = mg$

and $\Sigma\tau = 0$

$\Rightarrow \quad \vec{\tau}_f + \vec{\tau}_N = 0$



Since $\vec{\tau}_f \neq 0$

$\Rightarrow \quad \vec{\tau}_N \neq 0$ and torque by friction and normal reaction will be in opposite.

Hence, the correct answer is (A).

15. $I_1\omega_1 = I_2\omega_2$

If $I_2 < I_1$

$\Rightarrow \quad \omega_2 > \omega_1$

The diver does work by pulling his limps and thus, ω increases or rotational kinetic energy increases.

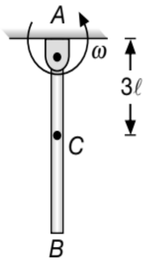
Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. By Law of Conservation of Mechanical Energy, we have

$$(3m)(g)(2\ell) = \frac{1}{2}I\omega^2 = \frac{1}{2}\left[\frac{(3m)(4\ell)^2}{3}\right]\omega^2 = 8m\ell^2\omega^2$$

$$\Rightarrow \quad \omega = \frac{1}{2}\sqrt{\frac{3g}{\ell}}$$



Applying, $\begin{pmatrix}\text{Angular}\\ \text{Impulse}\end{pmatrix} = \begin{pmatrix}\text{Change in}\\ \text{Angular}\\ \text{Momentum}\end{pmatrix}$, we get

$$J_1(3\ell) = I\omega$$

$$\Rightarrow \quad 3J_1\ell = (16m\ell^2)\left(\frac{1}{2}\sqrt{\frac{3g}{\ell}}\right)$$

$$\Rightarrow \quad J_1 = \frac{8}{3}m\ell\sqrt{\frac{3g}{\ell}}$$

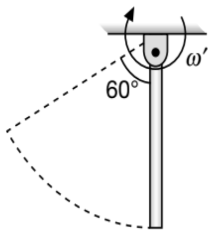
$$\Rightarrow \quad J_1 = \frac{8}{3}m\sqrt{3g\ell} = 8m\sqrt{\frac{g\ell}{3}}$$

Hence, the correct answer is (C).

2. Let ω' be the angular speed in opposite direction. Again, applying Law of Conservation of Mechanical Energy, we get

$$(3m)(g)(\ell) = \frac{1}{2}I(\omega')^2 = 8m\ell^2(\omega')^2$$

$$\Rightarrow \quad \omega' = \frac{1}{2\sqrt{2}}\sqrt{\frac{3g}{\ell}}$$



Since, $\begin{pmatrix}\text{Angular}\\ \text{Impulse}\end{pmatrix} = \begin{pmatrix}\text{Change in Angular}\\ \text{Momentum}\end{pmatrix}$, so

$$J_2(3\ell) = I(\omega+\omega') = (16m\ell^2)\frac{1}{2}\sqrt{\frac{3g}{\ell}}\left(1+\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow\ J_2 = \frac{4}{3}m\sqrt{6g\ell}\left(\sqrt{2}+1\right) = 4m\left(\sqrt{2}+1\right)\sqrt{\frac{2g\ell}{3}}$$

Hence, the correct answer is (B).

3. Torque about point A is

$$\tau = F(2R) = I\alpha$$

$$\Rightarrow\ F(2R) = \left(\frac{3mR^2}{2}\right)\alpha$$

$$\Rightarrow\ \alpha = \frac{4F}{3mR}$$

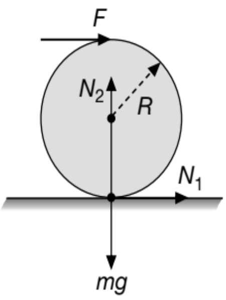
So tangential acceleration of center of mass is

$$a_t = R\alpha$$

$$\Rightarrow\ a_t = \frac{4F}{3m}$$

Hence, the correct answer is (D).

4.



$N_2 = mg$

Hence, the correct answer is (B).

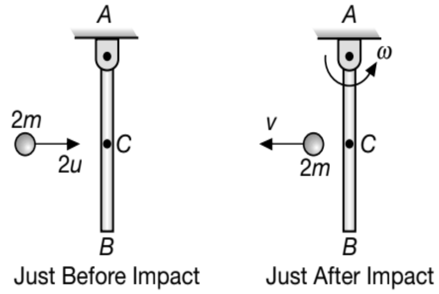
5. $F + N_1 = ma_t$

$$\Rightarrow\ N_1 = \frac{F}{3} \qquad \left\{\because\ a_t = \frac{4F}{3m}\right\}$$

Hence, the correct answer is (C).

6. By Law of Conservation of Angular Momentum, we have

$$(2m)(2u)(\ell) = \frac{(3m)(2\ell)^2}{3}\omega - (2m)(v_1)(\ell)$$



$$\Rightarrow\ 4u = 4\ell\omega - 2v_1$$

$$\Rightarrow\ 2u = 2\ell\omega - v_1 \qquad \ldots(1)$$

Since, for an elastic collision we have, $e = 1$, so

$\begin{pmatrix}\text{Relative}\\ \text{speed of}\\ \text{approach}\end{pmatrix} = \begin{pmatrix}\text{Relative}\\ \text{speed of}\\ \text{separation}\end{pmatrix}$, at point C

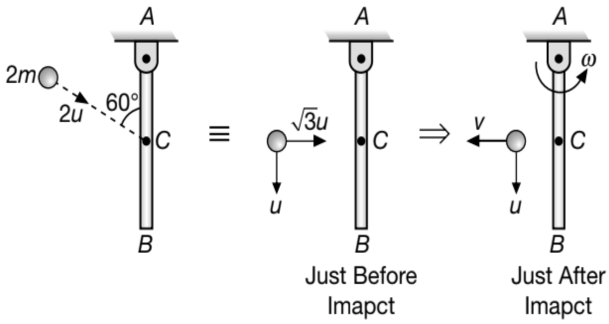
$$\Rightarrow\ 2u = v_1 + \ell\omega \qquad \ldots(2)$$

From equation (1) and (2), we get

$$v_1 = \frac{2u}{3}$$

Hence, the correct answer is (C).

7. In this case, the component of velocity along AC remains unchanged. Proceeding in the same manner, we can just replace $2u$ by $2u\cos(30°) = \sqrt{3}u$ and find v_2.

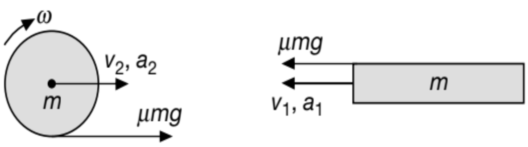


Hence, $v = \left(\frac{2}{3}\right)\frac{\sqrt{3}}{2}u = \frac{u}{\sqrt{3}}$

Speed of particle after impact is

$$v_2 = \sqrt{v^2+u^2} = \sqrt{\frac{u^2}{3}+u^2} = \frac{2}{\sqrt{3}}u$$

Hence, the correct answer is (B).

8. 

$$a_1 = \frac{\mu mg}{m} = \mu g$$

$$a_2 = \frac{\mu mg}{m} = \mu g$$

If v_1 and v_2 be the velocities of plank and cylinder at time t, then relative velocity of centre of mass of sphere w.r.t. plank is $(v_1 + v_2)$. So, when pure rolling begins, then

$$\omega = \frac{v_1 + v_2}{r}$$

where $v_1 = (\mu g)t$ and $v_2 = (\mu g)t$

Now, $\alpha = \frac{\tau}{I} = \frac{(\mu mg)r}{\frac{2}{5}mr^2}$

$\Rightarrow \quad \alpha = \frac{5}{2}\left(\frac{\mu g}{r}\right)$

when pure rolling begins at time t, then

$$v_1 + v_2 = r\omega$$

$\Rightarrow \quad (2\mu g)t = r(\omega_0 - \alpha t)$

$\Rightarrow \quad (2\mu g)t = r\omega_0 - \left(\frac{5}{2}\mu g\right)t$

$\Rightarrow \quad t = \frac{2}{9}\left(\frac{r\omega_0}{\mu g}\right)$

Hence, the correct answer is (B).

9. Since $v_2 = (\mu g)t$

$\Rightarrow \quad v_2 = \frac{2}{9} r\omega_0$

Hence, the correct answer is (A).

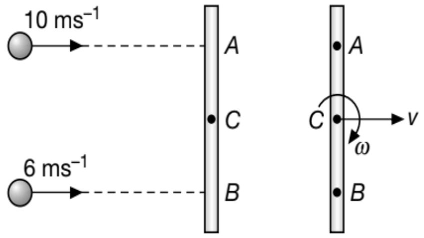
10. $x_P = \frac{1}{2}a_1 t^2$

$\Rightarrow \quad x_P = \frac{1}{2}(\mu g)\left(\frac{2}{9}\frac{r\omega_0}{\mu g}\right)^2$

$\Rightarrow \quad x_P = \frac{2}{81}\left(\frac{r^2\omega_0^2}{\mu g}\right)$

Hence, the correct answer is (D).

11. Let v be the velocity of centre of mass (also at C) of rod and two particles and ω the angular velocity of the system.



By Law of Conservation of Linear Momentum, we have

$$(0.08)(10 + 6) = (0.08 + 0.08 + 0.16)v$$

$\Rightarrow \quad v = 4 \text{ ms}^{-1}$

Hence, the correct answer is (D).

12. Also, $AB = CB = 0.5$ m

Similarly applying Conservation of Angular Momentum about the point C, we get

$$(0.08)(10)(0.5) - (0.08)(6)(0.5) = I_{system}\omega \quad \dots(1)$$

where, $I_{system} = I_{rod} + I_{two\ particles} = \frac{(1.6)(\sqrt{3})^2}{12} + 2(0.08)(0.5)^2$

$\Rightarrow \quad I_{system} = 0.08 \text{ kgm}^2$

Substituting in equation (1), we get

$$\omega = 2 \text{ rads}^{-1}$$

Hence, the correct answer is (C).

13. Loss of kinetic energy $= -\Delta K = K_i - K_f$

$\Rightarrow \quad -\Delta K = \frac{1}{2}(0.08)(10)^2 + \frac{1}{2}(0.08)(6)^2 - \frac{1}{2}(0.08 + 0.08 + 0.16)(4)^2 - \frac{1}{2}(0.08)(2)^2$

$\Rightarrow \quad -\Delta K = 4 + 1.44 - 2.56 - 0.16$

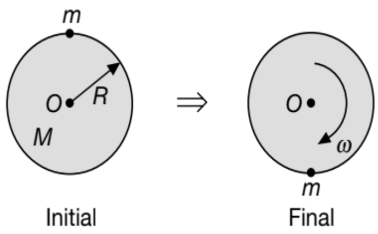
$\Rightarrow \quad -\Delta K = 2.72$ J

Hence, the correct answer is (B).

14. By Law of Conservation of Mechanical Energy, we have

$$mg(2R) = \frac{1}{2}\left(\frac{1}{2}MR^2 + mR^2\right)\omega^2$$

$\Rightarrow \quad \omega = \sqrt{\frac{8mg}{(2m + M)R}}$



Since, $m = \dfrac{M}{2}$, so we have

$$\omega = \sqrt{\frac{2g}{R}}$$

Hence, the correct answer is (C).

15. Let F be the force exerted by disk on the particle (upwards). Then,

$$F - mg = mR\omega^2$$

$$\Rightarrow \quad F = mg + \frac{8m^2g}{(2m+M)}$$

$$\Rightarrow \quad F = \frac{mg(10m+M)}{(2m+M)}$$

Since, $m = \dfrac{M}{2}$, so we have

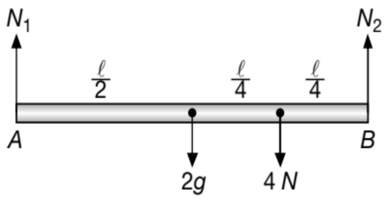
$$F = \frac{3}{2}Mg$$

Hence, the correct answer is (D).

16. Since the rod is in equilibrium, so

$$N_1 + N_2 = (2)(10) + 4$$

$$\Rightarrow \quad N_1 + N_2 = 24\text{ N} \qquad \ldots(1)$$



Taking torque due to all forces about A, we get

$$N_1(0) + \underbrace{(20)\left(\frac{\ell}{2}\right)}_{CW} + \underbrace{4\left(\frac{3\ell}{4}\right)}_{CW} + \underbrace{N_2\ell}_{CCW} = 0$$

$$\Rightarrow \quad 10\ell + 3\ell - N_2\ell = 0$$

$$\Rightarrow \quad N_2 = 13\text{ N} \qquad \ldots(2)$$

From (1), we get

$$N_1 = 11\text{ N}$$

Hence, the correct answer is (B).

17. Just after hinge B breaks (and F is removed too) the rod will not be in equilibrium. Let the new reaction at A be N, then

$$2g - N = 2a \qquad \ldots(1)$$

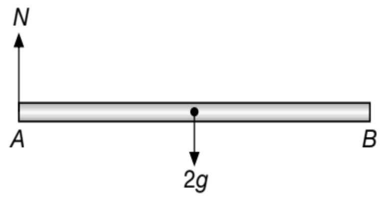
$$\tau = I\alpha$$

$$\Rightarrow \quad (2g)\frac{\ell}{2} = \left(\frac{(2)\ell^2}{3}\right)\alpha$$

$$\Rightarrow \quad g\ell = \left(\frac{2\ell^2}{3}\right)\alpha$$

$$\Rightarrow \quad \alpha = \frac{3g}{2\ell}$$

Since $a = \left(\dfrac{\ell}{2}\right)\alpha$



$$\Rightarrow \quad a = \frac{\ell}{2}\left(\frac{3g}{2\ell}\right)$$

$$\Rightarrow \quad a = \frac{3g}{4}$$

So, from (1), we get

$$20 - N = 2\left(\frac{3g}{4}\right)$$

$$\Rightarrow \quad 20 - N = \frac{60}{4}$$

$$\Rightarrow \quad N = 5\text{ N}$$

Hence, the correct answer is (D).

18. By Law of Conservation of Energy

$$\begin{pmatrix}\text{Loss in GPE} \\ \text{of CM of Rod}\end{pmatrix} = \begin{pmatrix}\text{Gain in RKE} \\ \text{of Rod}\end{pmatrix}$$

$$\Rightarrow \quad (2)g\left(\frac{\ell}{2}\right) = \frac{1}{2}\left(\frac{(2)\ell^2}{3}\right)\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3g}{\ell}}$$

So, acceleration is

$$a_n = \ell\omega^2 = 3g = 30\text{ ms}^{-2}$$

Hence, the correct answer is (C).

19. Applying Conservation of Angular Momentum, we get

$$\sum mvr_\perp = (I_{\text{system}})\omega$$

$$\Rightarrow \quad (mu)L = \left(\frac{1}{3}ML^2 + mL^2\right)\omega$$

$$\Rightarrow \quad \omega = \frac{3mu}{(M+3m)L}$$

Since $m \ll M$, so, we get

$$\omega \cong \frac{3mu}{ML} \qquad \ldots(1)$$

Hence, the correct answer is (C).

20. By Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in RKE}\\ \text{of Ball-Rod}\\ \text{System}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{GPE of}\\ \text{CM of Rod}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{GPE of}\\ \text{Ball}\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{1}{3}ML^2 + mL^2\right)\omega^2 = Mg\left(\frac{L}{2}\right)(1-\cos\theta) + mgL(1-\cos\theta)$$

$$(M+3m)L\omega^2 = 3g(1-\cos\theta)(M+2m)$$

Since $M \gg m$, so

$$M + 3m \cong M \text{ and } M + 2m \cong M$$

$$\Rightarrow \quad \omega \cong \sqrt{\frac{3g}{L}(1-\cos\theta)}$$

$$\Rightarrow \quad \frac{3mu}{ML} \cong \sqrt{\frac{3g}{L}\left[2\sin^2\left(\frac{\theta}{2}\right)\right]}$$

$$\Rightarrow \quad u \cong \frac{M}{m}\sqrt{\frac{2}{3}gL}\sin\left(\frac{\theta}{2}\right)$$

Hence, the correct answer is (C).

21. Since angular momentum is conserved about the hinge so there will be no change in angular momentum of system, however the rod is hinged at support and due to this the linear momentum changes. So,

$$\Delta p = p_f - p_i$$

$$\Rightarrow \quad \Delta p = \left[m(L\omega) + M\left(\frac{L\omega}{2}\right)\right] - mu$$

$$\Rightarrow \quad \Delta p = \left(m + \frac{M}{2}\right)L\omega - mu$$

$$\Rightarrow \quad \Delta p = \left(m + \frac{M}{2}\right)\left(\frac{3mu}{M}\right) - mu$$

$$\Rightarrow \quad \Delta p = \left(\frac{m}{M} + \frac{1}{2}\right)3mu - mu$$

$$\Rightarrow \quad \Delta p \simeq \frac{3}{2}mu - mu$$

$$\Rightarrow \quad \Delta p \cong \frac{1}{2}mu$$

But $u = \dfrac{M}{m}\sqrt{\dfrac{2}{3}gL}\sin\left(\dfrac{\theta}{2}\right)$

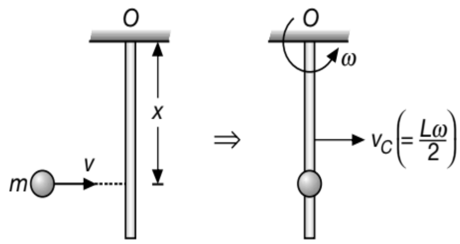
$$\Rightarrow \quad \Delta p \cong \frac{1}{2}(m)\left(\frac{M}{m}\right)\sqrt{\frac{2}{3}gL}\sin\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \quad \Delta p \cong M\sqrt{\frac{gL}{6}}\sin\left(\frac{\theta}{2}\right)$$

Hence, the correct answer is (D).

22. Applying Law of Conservation of Angular Momentum about the hinge we get

$$mvx = I\omega = \left(\frac{ML^2}{3}\right)\omega \qquad \ldots(1)$$



Also, by Law of Conservation of Linear Momentum, we get

$$P_i = P_f$$

$$\Rightarrow \quad mv = Mv_c = M\left(\frac{L}{2}\omega\right) \qquad \ldots(2)$$

From equations (1) and (2), we get

$$x = \frac{2L}{3}$$

Hence, the correct answer is (C).

23. $I = \dfrac{m\ell^2}{3}$

$$\Rightarrow \quad m\ell^2 = 3I \qquad \ldots(1)$$

Further, let R be the radius of ring, then

$$\ell = 2\pi R$$

$$\Rightarrow \quad R = \frac{\ell}{2\pi}$$

$$I_1 = mR^2 = m\left(\frac{\ell^2}{4\pi^2}\right) = \left(\frac{3I}{4\pi^2}\right)$$

Hence, the correct answer is (B).

24. $\ell = 2(2\pi R)$

$$\Rightarrow \quad R' = \frac{\ell}{4\pi}$$

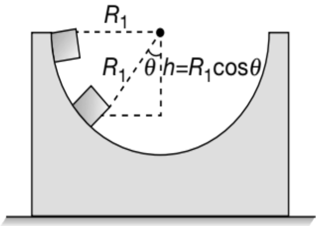
$$I_2 = 2m(R')^2 = \frac{2m\ell^2}{16\pi^2} = \left(\frac{3}{8\pi^2}\right)I$$

Hence, the correct answer is (D).

25. For block

$$v(\theta) = \sqrt{2gR_1\cos\theta}$$

$$\Rightarrow R_1\left(-\frac{d\theta}{dt}\right) = \sqrt{2gR_1\cos\theta}$$



$$\Rightarrow \frac{-d\theta}{\sqrt{\cos\theta}} = \sqrt{\frac{2g}{R_1}}dt$$

$$\Rightarrow \sqrt{\frac{2g}{R_1}}\int_0^{t_1} dt = \int_{\pi/2}^{0^\circ} \frac{-d\theta}{\sqrt{\cos\theta}}$$

$$\Rightarrow t_1 = \sqrt{\frac{R_1}{2g}}\int_{\pi/2}^{0^\circ} \frac{-d\theta}{\sqrt{\cos\theta}} \qquad \ldots(1)$$

For the ball, we have, $\frac{K_R}{K_T} = \frac{2}{5}$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{5}{7}E$$

where E is the total mechanical energy, so

$E = mgh$, where $h = gR_2\cos\theta$

$$\Rightarrow v(\theta) = \sqrt{\frac{10}{7}gR_2\cos\theta}$$

Now again proceeding in the similar manner, we get

$$t_2 = \sqrt{\frac{7R_2}{10g}}\int_{\pi/2}^{0^\circ} \frac{-d\theta}{\sqrt{\cos\theta}} \qquad \ldots(2)$$

From equations (1) and (2), on dividing and using $R_1 = 2R_2$, we get

$$\frac{t_1}{t_2} = \sqrt{\frac{10}{7}}$$

$$\Rightarrow t_1 > t_2$$

So, the ball will reach the bottom of the track first.

Hence, the correct answer is (B).

26. From equations (1) and (2), already calculated, when

$t_1 = t_2$

then $\frac{R_1}{2} = \frac{7R_2}{10}$

$$\Rightarrow \frac{R_1}{R_2} = \frac{7}{5}$$

Hence, the correct answer is (C).

27. $\tau_{av} = \left|\frac{\Delta L}{\Delta t}\right| = \frac{1}{2} = 0.5$ Nm

Hence, the correct answer is (C).

28. $\theta = \left(\frac{\omega + \omega_0}{2}\right)t = \left(\frac{L + L_0}{2I}\right)t$

$$\Rightarrow \theta = \left[\frac{3+2}{2(0.4)}\right](2) = 12.5 \text{ rad}$$

So, number of revolutions is

$$N = \frac{\theta}{2\pi} = \frac{12.5}{2(3.14)} = 1.99 \approx 2$$

Hence, the correct answer is (A).

29. $W = \Delta K = \frac{L^2}{2I} - \frac{L_0^2}{2I}$

$$\Rightarrow W = \frac{1}{2(0.4)}(4-9) = -6.25 \text{ J}$$

$$\Rightarrow |W| = 6.25 \text{ J}$$

Hence, the correct answer is (D).

30. C is the centre of mass of rod and O the combined centre of mass of system (ball + rod) after collision. If we consider the origin at the centre of the rod, C, then the combined centre of mass of ball + rod is at a distance $\frac{x}{2}$ from C, as shown. Conserving angular momentum about point O.

$$mv_0\left(\frac{x}{2}\right) = I_0\omega = \left[m\left(\frac{x}{2}\right)^2 + \frac{m\ell^2}{12} + m\left(\frac{x}{2}\right)^2\right]\omega$$

$$\Rightarrow \omega = \frac{v_0\left(\frac{x}{2}\right)}{\left(\frac{\ell^2}{12}\right) + \left(\frac{x^2}{2}\right)} = \frac{6v_0x}{(\ell^2 + 6x^2)}$$

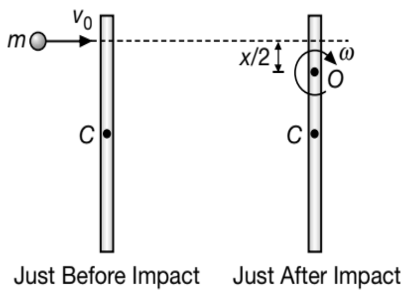
$$\Rightarrow \omega = \frac{6v_0}{\left(6x + \frac{\ell^2}{x}\right)}$$

For ω to be maximum or minimum quantity $\left(6x + \frac{\ell^2}{x}\right)$ should be minimum or maximum

$$\frac{d}{dx}\left(6x + \frac{\ell^2}{x}\right) = 0$$

$$\Rightarrow 6 - \frac{\ell^2}{x^2} = 0$$

CHAPTER 3



$$\Rightarrow \quad x = \frac{\ell}{\sqrt{6}}$$

Further $\dfrac{d^2\left(6x + \dfrac{\ell^2}{x}\right)}{dx^2}$ at $x = \dfrac{\ell}{\sqrt{6}}$ is positive, hence at $x = \dfrac{\ell}{\sqrt{6}}$, the denominator is minimum and hence ω is maximum

Further at $x = 0$, $\omega = 0$, therefore ω will first increase and then it will decrease.

Hence, the correct answer is (B).

31. Impulse $|J| = \Delta p = |p_f - p_i|$

$$\Rightarrow \quad |J| = p_i - p_f \left(\text{as } p_i > p_f\right)$$

Now, $|J|$ will be maximum when p_f or v_f for ball is least. v_f will be least when the ball strikes at centre of rod, because in that case $v_f = \dfrac{v_0}{2}$ (by Law of Conservation of Linear Momentum). In all other cases, one more term $r\omega$, will be added to this.

So, $|J|_{\max} = m\left(v_0 - \dfrac{v_0}{2}\right) = \dfrac{mv_0}{2}$

Hence, the correct answer is (A).

32. Translational kinetic energy in both cases will be equal but rotational kinetic energy in CASE-1 will be less. This is because perpendicular distance of linear impulse from the centre is less. So, angular impulse imparted to the sphere will be less or ω will be less.

Hence, the correct answer is (C).

33. Friction will be in forwards direction in the case of backward slipping just after being hit by the cue, i.e.,

$$R\omega > v$$

By Angular Impulse – Angular Momentum Theorem, we have

$$Jh = I\omega$$

where J is the linear impulse

$$\Rightarrow \quad R\left(\frac{Jh}{I}\right) > \frac{J}{m}$$

$$\Rightarrow \quad h > \frac{I}{mR}$$

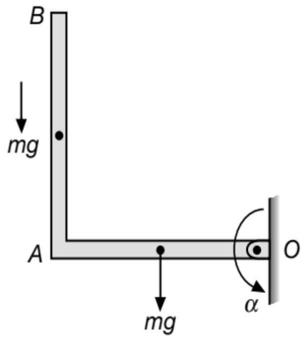
$$\Rightarrow \quad h > \frac{\frac{2}{5}mR^2}{mR}$$

$$\Rightarrow \quad h > 0.4R$$

i.e., in CASE-2

Hence, the correct answer is (D).

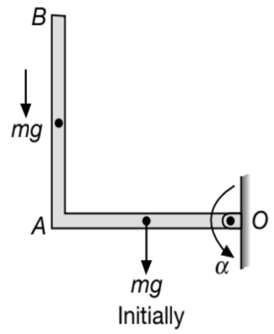
34.

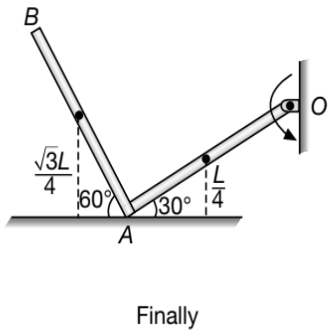


$$\alpha = \frac{\tau_0}{I_0} = \frac{mg\frac{L}{2} + mgL}{\frac{mL^2}{3} + mL^2} = \frac{9g}{8L}$$

Hence, the correct answer is (B).

35. Let ω be the angular speed just before striking with ground.





$$\begin{pmatrix}\text{Decrease in}\\ \text{Gravitational}\\ \text{Potential Energy}\end{pmatrix} = \begin{pmatrix}\text{Increase in}\\ \text{Rotational}\\ \text{Kinetic Energy}\end{pmatrix}$$

$$\Rightarrow \quad mg\left(\frac{L}{2} - \frac{L}{4}\right) + mg\left(L - \frac{\sqrt{3}L}{4}\right) = \frac{1}{2}\left(\frac{mL^2}{3} + mL^2\right)\omega^2$$

Solving this equation, we get,

$$\omega = 1 \cdot 1\sqrt{\frac{g}{L}}$$

$$\Rightarrow \quad v_A = (OA)\omega = L\omega = 1 \cdot 1\sqrt{gL}$$

Hence, the correct answer is (D).

36. The distance of centre of mass of the system from point A is

$$r = \frac{\ell}{\sqrt{3}}$$

Therefore, the magnitude of horizontal force exerted by the hinge on the body is

$$F = \text{centripetal force}$$

$$\Rightarrow \quad F = (3m)r\omega^2$$

$$\Rightarrow \quad F = (3m)\left(\frac{\ell}{\sqrt{3}}\right)\omega^2$$

$$\Rightarrow \quad F = \sqrt{3}\, m\ell\omega^2$$

Hence, the correct answer is (C).

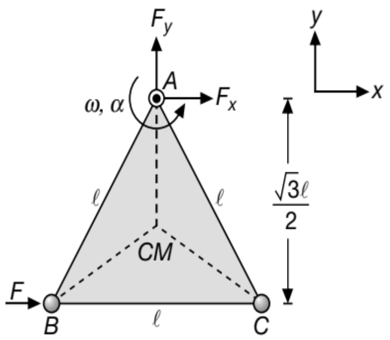
37. Angular acceleration of system about point A is

$$\alpha = \frac{\tau_A}{I_A} = \frac{(F)\left(\frac{\sqrt{3}}{2}\ell\right)}{2m\ell^2} = \frac{\sqrt{3}F}{4m\ell}$$

Now, acceleration of centre of mass along x-axis is

$$a_x = r\alpha = \left(\frac{\ell}{\sqrt{3}}\right)\left(\frac{\sqrt{3}F}{4m\ell}\right)$$

$$\Rightarrow \quad a_x = \frac{F}{4m}$$



Now, let F_x be the force applied by the hinge along x-axis. Then,

$$F_x + F = (3m)a_x$$

$$\Rightarrow \quad F_x + F = (3m)\left(\frac{F}{4m}\right)$$

$$\Rightarrow \quad F_x + F = \frac{3}{4}F$$

$$\Rightarrow \quad F_x = -\frac{F}{4}$$

$$\Rightarrow \quad F_x = \frac{F}{4}, \text{ along negative } x\text{-axis}$$

Hence, the correct answer is (D).

38. If F_y be the force applied by the hinge along y-axis. Then,

$$F_y = \text{Centripetal Force}$$

$$\Rightarrow \quad F_y = \sqrt{3}\, m\ell\omega^2$$

Hence, the correct answer is (C).

39. If ground is smooth the body will rotate about O, so

$$K = \frac{1}{2}I_0\omega^2$$

$$\Rightarrow \quad K = \frac{1}{2}I_0(\alpha t)^2$$

$$\Rightarrow \quad K = \frac{1}{2}I_0\left(\frac{\tau}{I_0}\right)^2 t^2$$

$$\Rightarrow \quad K = \frac{\tau^2 t^2}{2I_0}$$

where $\tau = F(2R) - F(R) = FR$

$$\Rightarrow \quad K = \frac{F^2R^2t^2}{2I_0}$$

Substituting the values, we get

$$K = \frac{(10)^2(1)^2(2)^2}{2\times 4} = 50 \text{ J}$$

Hence, the correct answer is (B).

40. In this case the body will rotate about bottommost point, so

$$K = \frac{\tau^2 t^2}{2I}$$

$$\Rightarrow \quad K = \frac{[F(4R) - F(3R)]^2 t^2}{2\left[I_0 + m(2R)^2\right]}$$

$$\Rightarrow \quad K = \frac{F^2R^2t^2}{2\left[I_0 + 4mR^2\right]}$$

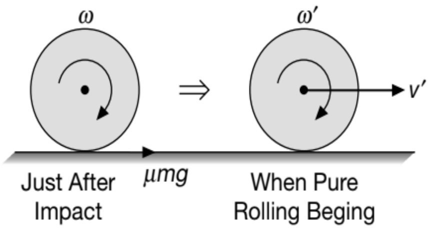
$$\Rightarrow \quad K = \frac{(10)^2(1)^2(6)^2}{2[4+8]}$$

$$\Rightarrow \quad K = 150 \text{ J}$$

Hence, the correct answer is (D).

41. Since both spheres are of equal masses and the second sphere is at rest, so just after collision the first sphere comes to rest and it will have only ω i.e., it will slip backwards. So, friction will be maximum and in forward direction. Let v' be its linear speed and ω' its angular speed when it again starts pure rolling. Friction is passing through its bottommost point, so we can conserve angular momentum about an axis passing through its bottommost point and perpendicular to plane of motion. So,

CHAPTER 3



$$L_i = L_f$$

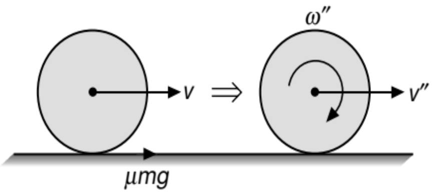
$$\Rightarrow \frac{2}{5}(mR^2)\omega = \frac{2}{5}(mR^2)\omega' + mv'R$$

$$\Rightarrow \frac{2}{5}(mR^2)\omega = \frac{2}{5}\left(mR^2\left(\frac{v'}{R}\right) + mv'R\right)$$

$$\Rightarrow v' = \frac{2}{7}R\omega$$

Hence, the correct answer is (B).

42. Thinking on the same concept as discussed above, we again have



$$L_i = L_f$$

$$\Rightarrow mvR = \frac{2}{5}(mR^2)\omega'' + m(\omega''R)R$$

$$\Rightarrow \omega'' = \frac{5}{7}\frac{v}{R}$$

Now, angular impulse imparted to the sphere is

$$\Delta L = I\omega'' = \left(\frac{2}{5}MR^2\right)\left(\frac{5}{7}\frac{v}{R}\right)$$

$$\Rightarrow \Delta L = \frac{2}{7}mvR$$

Hence, the correct answer is (A).

43. In case of pure rolling

$$a = \frac{g\sin\theta}{1 + \dfrac{I}{mR^2}}$$

a is minimum when I is maximum which is for ring equal to mR^2. So,

$$a_{min} = \frac{g\sin\theta}{2} = \frac{10}{4}\ \text{ms}^{-2}$$

$$\Rightarrow t_{max} = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times 5}{\left(\frac{10}{4}\right)}} = 2\text{ s}$$

Hence, the correct answer is (D).

44. Minimum value of μ required in this case is

$$\mu_{min} = \frac{\tan\theta}{1 + \dfrac{mR^2}{I}}$$

I is maximum for ring, hence minimum value of μ required for pure rolling will have maximum value for ring. So, it will be the first to start slipping.

Hence, the correct answer is (A).

45. $$M = \int_0^a \frac{\rho_0(a+x)}{a}dx = \frac{\rho_0}{a}\left(ax + \frac{x^2}{2}\right)\Bigg|_0^a = \frac{3a\rho_0}{2}$$

Hence, the correct answer is (D).

46. $$x_{cm} = \frac{\int_0^a (dm)x}{M} = \frac{\int_0^a \frac{\rho_0(a+x)}{a}x\,dx}{\frac{3a\rho_0}{2}}$$

$$\Rightarrow x_{cm} = \frac{2}{3a}\left(\frac{ax^2}{2} + \frac{x^3}{2}\right)\Bigg|_0^a = \frac{5a}{9}$$

Hence, the correct answer is (C).

47. $$I_A = \int_0^a (dm)x^2 = \int_0^a \frac{\rho_0(a+x)}{a}x^2dx = \frac{\rho_0}{a}\left(\frac{ax^3}{3} + \frac{x^4}{4}\right)\Bigg|_0^a$$

$$\Rightarrow I_A = \frac{7\rho_0 a^3}{12}$$

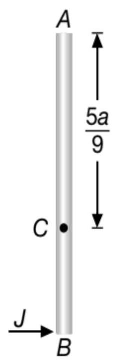
$$\Rightarrow I_A = \frac{7Ma^2}{18} \qquad \left\{\because M = \frac{3a\rho_0}{2}\right\}$$

Hence, the correct answer is (A).

48. Since, $\begin{pmatrix}\text{Angular}\\ \text{Impulse}\end{pmatrix} = \begin{pmatrix}\text{Change in Angular}\\ \text{Momentum}\end{pmatrix}$, so we get

$$Ja = I_A\omega = \frac{7Ma^2}{18}\omega$$

$$\Rightarrow \omega = \frac{18J}{7Ma}$$



B will pass through a point vertically above A when

$$\frac{1}{2}I_A\omega^2 > Mg(2AC)$$

$$\Rightarrow \frac{1}{2}\left(\frac{7Ma^2}{18}\right)\left(\frac{18J}{7Ma}\right)^2 > 2Mg\left(\frac{5a}{9}\right)$$

$$\Rightarrow J > \frac{M}{9}\sqrt{70ag}$$

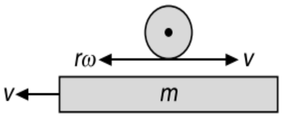
$$\Rightarrow J_{min} = \frac{M}{9}\sqrt{70ag}$$

Hence, the correct answer is (B).

49. At the time of pure rolling, we have

$$-v = v - r\omega$$

$$\Rightarrow v = \frac{r\omega}{2}$$



Also, we have

$$\frac{1}{2}\left(\frac{1}{2}I_{cm}\omega_0^2\right) = \frac{1}{2}mv^2 + \left(\frac{1}{2}mv^2 + \frac{1}{2}I_{cm}\omega^2\right)$$

$$\Rightarrow \frac{r^2\omega_0^2}{8} = 2v^2$$

$$\Rightarrow v = \frac{r\omega_0}{4}$$

Hence, the correct answer is (A).

50. When slipping ceases, we have

$$0 = v - (\mu g)t$$

$$\Rightarrow v = (\mu g)t$$

$$\Rightarrow t = \frac{r\omega_0}{4\mu g}$$

Hence, the correct answer is (D).

51. $L_i = \left(\frac{1}{2}mr^2\right)\omega_0$

$$L_f = \left(\frac{1}{2}mr^2\right)\omega$$

Since $\omega = \frac{\omega_0}{4}$, so

$$|\Delta L| = |L_f - L_i| = \frac{3}{8}mr^2\omega_0$$

Hence, the correct answer is (D).

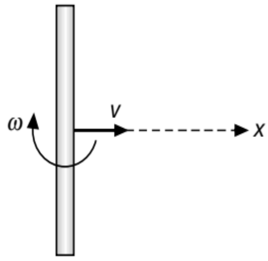
52. Since $0^2 - v^2 = 2(-\mu g)s$

$$\Rightarrow s = \frac{v^2}{2\mu g}$$

$$\Rightarrow s = \frac{r^2\omega_0^2}{32\mu g}$$

Hence, the correct answer is (D).

53. $v = \frac{J}{m} = \frac{10}{2} = 5 \text{ ms}^{-1}$



$$\omega = \frac{\tau}{I} = \frac{J\left(\frac{\ell}{2}\right)}{I} = \frac{J\ell}{2\left(\frac{m\ell^2}{12}\right)} = \frac{6J}{m\ell}$$

$$\Rightarrow \omega = \frac{6\times 10}{2\times 2} = 15 \text{ rads}^{-1}$$

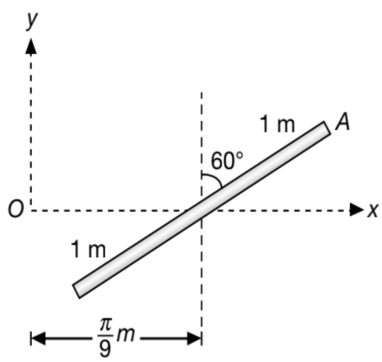
At point P, which is say at distance r from centre where if, v and $r\omega$ cancel each other then that point will be at rest just after impact.

Therefore, $r = \frac{v}{\omega} = \frac{5}{15} = \frac{1}{3}$ m

Hence, the correct answer is (C).

54. In the given time angle through which the rod rotates is

$$\theta = \omega t = 15\left(\frac{\pi}{45}\right) = \frac{\pi}{3} = 60°$$



The displacement of centre of mass from origin O is

$$s = vt = (5)\left(\frac{\pi}{45}\right) = \frac{\pi}{9} \text{ m}$$

$\Rightarrow \quad x_A = \dfrac{\pi}{9} + (1)\sin(60°) = \left(\dfrac{\pi}{9} + \dfrac{\sqrt{3}}{2}\right)$ m

$\Rightarrow \quad y_A = 1\cos(60°) = \dfrac{1}{2}$ m

Hence, the correct answer is (D).

55. Acceleration of block $a_B = g\sin(45°) - \mu g\cos(45°)$

$\Rightarrow \quad a_B = \dfrac{g}{\sqrt{2}} - \dfrac{g}{2\sqrt{2}} = \dfrac{g}{2\sqrt{2}}$

and acceleration of cylinder $a_C = \dfrac{g\sin 45°}{1 + \dfrac{I}{2mR^2}}$ ...(1)

where, $I = \dfrac{1}{2}(2m)R^2$

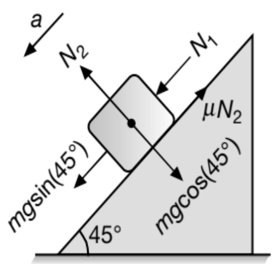
$\Rightarrow \quad \dfrac{I}{2mR^2} = \dfrac{1}{2}$

Substituting in equation (1), we get

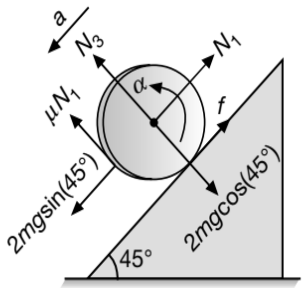
Acceleration of cylinder $a_C = \dfrac{\sqrt{2}g}{3}$

Hence, the correct answer is (D).

56. The free body diagrams of the block and the cylinder are as shown.



FBD of Block



FBD of Cylinder

Equations of motion are,

$N_2 = mg\cos(45°) + \mu N_1$ ...(1)

$mg\sin(45°) + N_1 - \mu N_2 = ma$ ...(2)

$\mu N_1 + N_3 = 2mg\cos(45°)$ ...(3)

$2mg\sin(45°) - N_1 - f = 2ma$ ...(4)

Also, $\alpha = \dfrac{(f - \mu N_1)R}{\dfrac{1}{2}(2m)R^2}$ ...(5)

For no slipping, we have

$a = R\alpha$ ...(6)

Solving these equations, we get

$a = \dfrac{3g}{5\sqrt{2}}$

Hence, the correct answer is (B).

57. In process A, $T = 5$ N

In process B, $mg - T' = ma$

$\Rightarrow \quad T' = (5 - ma) < T$

So, tangential acceleration will be greater in process A than that in B.

Hence, the correct answer is (B).

58. The mechanical energy is conserved in both A and B.

Hence, the correct answer is (A).

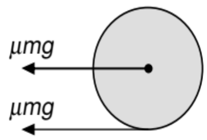
59. Let the a be the acceleration of C.M. w.r.t. plank, then

$2\mu mg = ma$

$\Rightarrow \quad a = 2\mu g$ ...(1)

Further, $\tau = \mu(mg)r = \dfrac{mr^2}{2}\alpha$

$\Rightarrow \quad \alpha = \dfrac{2\mu g}{r}$



Let a_0 be the acceleration of point of contact due to rotation.

$a_0 = r\alpha$

$\Rightarrow \quad a_0 = 2\mu g$ ...(2)

Acceleration a' of point of contact w.r.t. the plank is

$a' = a + a_0$

$\Rightarrow \quad a' = 4\mu g$

$\Rightarrow \quad a' = 4 \text{ ms}^{-2}$

Hence, the correct answer is (C).

60. $t = \dfrac{v}{a'} = \dfrac{v}{4\mu g}$

$\Rightarrow \quad t = 2.5$ s

Hence, the correct answer is (B).

61. Distance travelled by the cylinder w.r.t. plank in 2.5 s is

$s = -vt + \dfrac{1}{2}(2\mu g)t^2$

$\Rightarrow \quad s = -18.75$ m

$\Rightarrow \quad |s| = 18.75$ m

After $t = 2.5$ s the velocity of cylinder is

$v' = -v + (2\mu g)(2.5)$

$\Rightarrow \quad v' = -5 \text{ ms}^{-1}$

Remaining distance = 40 – 18.75 = 21.25 m

Time taken to cover 21.25 m is

$$t' = \frac{21.25}{5} = 4.25 \text{ s}$$

So, total time $T = 2.5 + 4.25 = 6.75$ s

Hence, the correct answer is (D).

62. Let v_{cm} be the velocity of centre of mass, then

$$v_P = v_{cm} - \frac{R\omega}{2}$$

$$\Rightarrow \quad v_P = R\omega - \frac{R\omega}{2} = \frac{R\omega}{2} = \frac{v_{cm}}{2}$$

Since $v_P = 2 \text{ ms}^{-1}$

$$\Rightarrow \quad v_{cm} = R\omega = 4 \text{ ms}^{-1}$$

$$\Rightarrow \quad \omega = 4 \text{ rads}^{-1} \qquad \{\because R = 1 \text{ m}\}$$

Now, acceleration of point P is

$$a_P = 10 = \sqrt{a_T^2 + a_N^2}$$

$$\Rightarrow \quad 10 = \sqrt{a_T^2 + \left(\omega^2 \frac{R}{2}\right)^2}$$

$$\Rightarrow \quad a_T = \sqrt{36} = 6 \text{ ms}^{-2}$$

If a_{cm} be the acceleration of the centre of mass then

$$a_T = a_{cm} - \frac{R\alpha}{2} = 6$$

$$\Rightarrow \quad a_{cm} - \frac{a_{cm}}{2} = 6$$

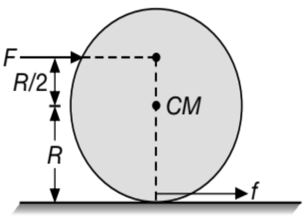
$$\Rightarrow \quad a_{cm} = 12 \text{ ms}^{-2}$$

Hence, the correct answer is (C).

63. If f be the force of friction, then

$$F\left(\frac{R}{2}\right) - fR = I\alpha = \left(\frac{1}{2}mR^2\right)\alpha$$

$$\Rightarrow \quad \frac{F}{2} - f = (5)(12) = 60 \qquad \ldots(1)$$



Also, $F + f = ma_{cm} = 10 \times 12 = 120 \qquad \ldots(2)$

From (1) and (2), we get

$$\frac{3F}{2} = 180$$

$$\Rightarrow \quad F = (180)\left(\frac{2}{3}\right) = 120 \text{ N}$$

Hence, the correct answer is (C).

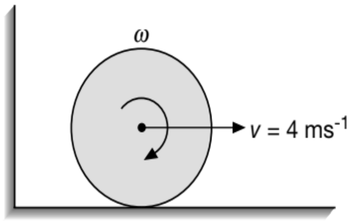
64. Let N be normal force between the wall and sphere at the time of impact and Δt the time of impact, then

$$N\Delta t = \Delta p = 8 \text{ m} \qquad \ldots(1)$$

$$\{\because \Delta v = 8 \text{ ms}^{-1}\}$$

Further $R(\mu N\Delta t) = \Delta(I\omega) = \left(\frac{2}{5}mR^2\right)(5) \qquad \ldots(2)$

$$\{\because \Delta\omega = 5 \text{ rads}^{-1}\}$$



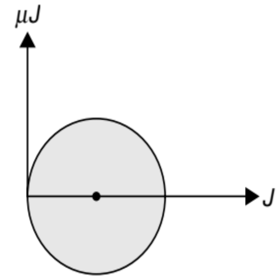
Dividing equation (2) by (1), we get

$$\mu = \frac{1}{4}$$

Hence, the correct answer is (A).

65. Net linear impulse is $J_{net} = \sqrt{J^2 + \mu^2 J^2}$

$$\Rightarrow \quad J_{net} = \left(\sqrt{1+\mu^2}\right) J$$



where, $J = \Delta P = 16$ Ns

So, net impulse is $J_{net} = \left(\sqrt{1 + \frac{1}{16}}\right)(16) = 4\sqrt{17}$ Ns

Hence, the correct answer is (C).

66. $$\frac{\text{Rotational KE}}{\text{Translational KE}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}Mv^2} = \frac{K^2}{R^2} \qquad \{\because I = MK^2, v = R\omega\}$$

Hence, the correct answer is (D).

67. $$\frac{\text{RKE}}{\text{TE}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}I\omega^2\left(1 + \frac{R^2}{K^2}\right)} = \frac{K^2}{R^2 + K^2}$$

Hence, the correct answer is (D).

68. $\dfrac{\text{TKE}}{\text{TE}} = \dfrac{\frac{1}{2}mv^2}{\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)} = \dfrac{R^2}{R^2+K^2}$

Hence, the correct answer is (B).

69. At maximum height, both the sphere and wedge have same horizontal velocity say v, then by Impulse – Momentum Theorem, we have

$$(\text{Im}\,pulse) = \left(\begin{matrix}\text{Total Change in}\\ \text{Momentum of Wedge}\end{matrix}\right) + (\text{sphere})$$

$\Rightarrow \quad P = (4m)v + (m)v$

$\Rightarrow \quad P = 5mv$

Just after sharp impulse is given to the sphere, let ω_0 be the angular velocity and v_0 be the velocity of centre of mass of sphere, then by Law of Conservation of Energy, we have

$$\frac{1}{2}mv_0^2 + \frac{1}{2}I_{\text{cm}}\omega_0^2 = \left(\frac{1}{2}mv^2 + \frac{1}{2}I_{\text{cm}}\omega_0^2\right) + \frac{1}{2}(4m)v^2$$

$$\Rightarrow \quad \frac{1}{2}mv_0^2 = \frac{1}{2}(5m)v^2 + mgh$$

$$\Rightarrow \quad h = \frac{4P^2}{10m^2g} = \frac{2P^2}{5m^2g}$$

Hence, the correct answer is (A).

70. $KE = \left(\frac{1}{2}mv^2 + \frac{1}{2}I_{\text{cm}}\omega_0^2\right) + \frac{1}{2}(4m)v^2$

$$\Rightarrow \quad KE = \frac{1}{2}(5m)v^2 + \frac{1}{2}I_{\text{cm}}\omega_0^2 \quad \ldots(1)$$

Now, by Angular Impulse – Angular Momentum Theorem, we have

$$Ph = I_{\text{cm}}\omega_0, \text{ where } h = 0.4r = \frac{2}{5}r$$

$$\Rightarrow \quad \omega_0 = \frac{Ph}{I_{\text{cm}}} = \frac{P\left(\frac{2}{5}r\right)}{\frac{2}{5}mr^2}$$

$$\Rightarrow \quad \omega_0 = \frac{P}{mr}$$

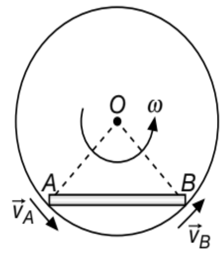
So, from (1), we get

$$KE = \frac{1}{2}(5m)\left(\frac{P}{5m}\right)^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{P}{mr}\right)^2$$

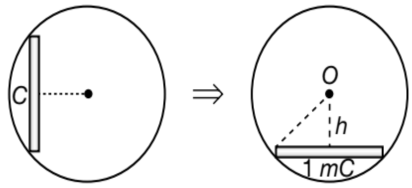
$$\Rightarrow \quad KE = \frac{3P^2}{10m}$$

Hence, the correct answer is (C).

71. Direction of velocities of two points of the rod A and B are tangential as shown.



When we draw perpendicular on them, they meet at O, the centre of sphere. So, the rod can be assumed in pure rotation about an axis passing through O and perpendicular to plane of paper (also called $IAOR$). Since, the surface is smooth mechanical energy will remain conserved. So, decrease in gravitational potential energy of CM of rod = Increase in rotational kinetic energy about $IAOR$



$$\Rightarrow \quad mgh = \frac{1}{2}I_{IAOR}\omega^2 = \frac{1}{2}\left(\frac{m\ell^2}{12} + mh^2\right)\omega^2$$

where, $h = \sqrt{4-1} = \sqrt{3}$ m

$$\Rightarrow \quad 4\times 10\times\sqrt{3} = \left(\frac{1}{2}\frac{(4)(4)}{12} + (4)(3)\right)\omega^2$$

$$\Rightarrow \quad \omega = 3.2 \text{ rads}^{-1}$$

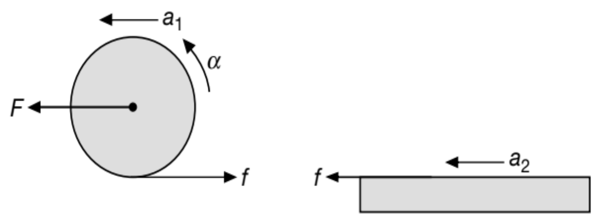
Hence, the correct answer is (A).

72. $v_c = h\omega = (\sqrt{3})(3.2) = 5.5 \text{ ms}^{-1}$

Hence, the correct answer is (B).

73-76. The correct answer is 73(A), 74(C), 75(B) and 76(A).

Combined solution to 73, 74, 75, 76



$$F - f = ma_1 \quad \ldots(1)$$

$$f = ma_2 \quad \ldots(2)$$

Also, $\alpha = \dfrac{\tau}{I} = \dfrac{fR}{\frac{1}{2}mR^2} = \dfrac{2f}{mR}$ $\quad \ldots(3)$

and $a_1 - R\alpha = a_2$ $\quad \ldots(4)$

Solving equations, (1), (2), (3) and (4), we get

$$a_1 = \frac{3F}{4m}, a_2 = \frac{F}{4m}, f = \frac{F}{4}, \alpha = \frac{F}{2mR}$$

77. Moment of inertia of the system about the given axis,

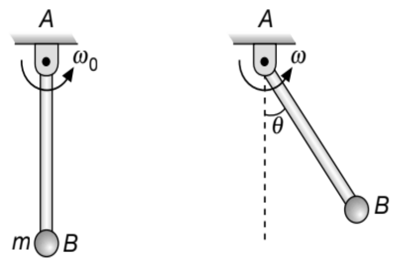
$$I = \frac{(3m)(2\ell)^2}{3} + m(2\ell)^2 = 8m\ell^2 \quad \ldots(1)$$

By Law of Conservation of Mechanical Energy in the two positions shown in figure, we get

$$\frac{1}{2}I\left(\sqrt{\frac{kg}{\ell}}\right)^2 = \frac{1}{2}I\omega^2 + mg2\ell(1-\cos\theta) + 3mg\ell(1-\cos\theta)$$

Substituting the value of I from equation (1), we get

$$4\ell\omega^2 = 4kg - 5g(1-\cos\theta) \quad \ldots(2)$$

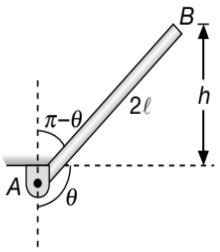


Initial Position General Position

When the rod comes to instantaneous rest, $\omega = 0$, so, we get

$$4kg - 5g(1-\cos\theta) = 0$$

Substituting, $k = 2$, we get $\cos\theta = -\frac{3}{5}$



In this position, θ is an obtuse angle, so the height h of B above A is given by,

$$h = 2\ell\cos(\pi-\theta) = 2\ell\left(\frac{3}{5}\right) = \frac{6\ell}{5}$$

Hence, B rises to $\frac{6\ell}{5}$ above the level of A.

Hence, the correct answer is (C).

78. Since the particle is attached to a rigid rod, its motion is restricted to a circular path. It will therefore, describe complete circles if its speed at the highest point is greater than zero. Since, $4\ell\omega^2 = 4kg - 5g(1-\cos\theta)$ has a real ω at $\theta = \pi$, when

$$4kg - 5g(1-\cos\pi) > 0$$

The particle will therefore, describe complete circle, only when

$$k > \frac{5}{2}$$

$$\Rightarrow \quad k_{min} = \frac{5}{2}$$

Hence, the correct answer is (B).

79. $1g - T = 1a \quad \ldots(1)$

Also, $T(1) = \frac{1}{2}(2)a$

$\Rightarrow \quad T = a \quad \ldots(2)$

$\Rightarrow \quad 1g - 1a = 1a$

$\Rightarrow \quad a = \frac{g}{2} = 5 \text{ ms}^{-2}$

$\Rightarrow \quad T = a = 5\text{ N} \quad \{\because R = 1\text{ m}\}$

Hence, the correct answer is (B).

80. $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(5)(16) = 40 \text{ rad}$

Hence, the correct answer is (A).

81. $W = \tau\theta = (5)(40) = 200\text{ J}$

Hence, the correct answer is (D).

82. $\Delta K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}(2)(1)^2\right)(5\times 4)^2$

$\Rightarrow \quad \Delta K = 200\text{ J}$

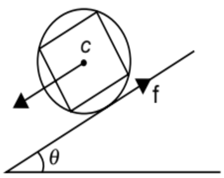
Hence, the correct answer is (C).

83. $I = 4\left\{\frac{M(R\sqrt{2})^2}{12} + M\left(\frac{R}{\sqrt{2}}\right)^2\right\} + mR^2$

$\Rightarrow \quad I = \frac{8}{3}MR^2 + mR^2 = 20\text{ kgm}^2$

Hence, the correct answer is (B).

84.



$$(4M+m)g\sin\theta - f = (4M+m)a \quad \ldots(1)$$

Also, $\tau = fR = I\left(\frac{a}{R}\right)$ ...(2)

Solving (1) and (2)

$$a = \frac{7g}{24}$$

Hence, the correct answer is (D).

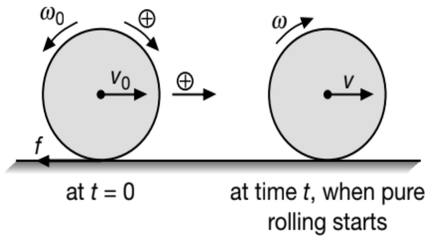
85. From above we get

$$f = 20\,a = \mu \times 28g\cos(30^\circ)$$

$$\Rightarrow \quad \mu = \frac{5}{12\sqrt{3}}$$

Hence, the correct answer is (D).

86.



Let pure rolling start at time t, then

$$I\omega - I(-\omega_0) = (fR)t$$

$$\Rightarrow \quad I\omega + I\omega_0 = (fR)t$$

$$\Rightarrow \quad \frac{1}{2}MR^2\left(\frac{v}{R} + \frac{v_0}{R}\right) = fRt$$

$$\Rightarrow \quad Mv + Mv_0 = 2ft \quad \ldots(1)$$

where $f = \mu Mg$

Also, we have

$$Mv - Mv_0 = -ft \quad \ldots(2)$$

So, from (1) and (2), we get

$$t = \frac{2v_0}{3\mu g}$$

Hence, the correct answer is (C).

87. Applying Law of Conservation of Angular Momentum about the point of contact, because net torque on the disc about the point of contact is zero, so

$$L_{\text{initial}} = L_{\text{final}}$$

Since, for a body in combined effect of rotation and translation, we have

$$\vec{L} = \vec{L}_{\text{cm}} + m(\vec{r} \times \vec{v})$$

So, $L_{\text{initial}} = Mv_0R - I_{\text{cm}}\omega_0$ and

$$L_{\text{final}} = MvR + I_{\text{cm}}(0)$$

$$\Rightarrow \quad Mv_0R - \left(\frac{1}{2}MR^2\right)\frac{v_0}{R} = MvR$$

$$\Rightarrow \quad \frac{Mv_0R}{2} = MvR$$

$$\Rightarrow \quad v = \frac{v_0}{2}$$

Hence, the correct answer is (B).

88. Since $Mv - Mv_0 = -ft$

$$\Rightarrow \quad Mv - Mv_0 = -\mu(Mg)t$$

$$\Rightarrow \quad v - v_0 = -\mu g t \quad \ldots(1)$$

Now, since pure rolling begins at $t = \frac{2v_0}{3\mu g}$

$$\Rightarrow \quad v - v_0 = -\mu g\left(\frac{2v_0}{3\mu g}\right)$$

$$\Rightarrow \quad v = v_0 - \frac{2v_0}{3}$$

$$\Rightarrow \quad v = \frac{v_0}{3}$$

This result can also be obtained by using Law of Conservation of Angular Momentum, according to which

$$Mv_0R - I_{cm}\omega_0 = MvR + I_{cm}\omega$$

$$\Rightarrow \quad Mv_0R - \left(\frac{1}{2}MR^2\right)\frac{v_0}{R} = MvR + \left(\frac{1}{2}MR^2\right)\frac{v}{R}$$

$$\Rightarrow \quad Mv_0R - \frac{Mv_0R}{2} = MvR + \frac{MvR}{2}$$

$$\Rightarrow \quad \frac{Mv_0R}{2} = \frac{3MvR}{2}$$

$$\Rightarrow \quad v = \frac{v_0}{3}$$

Hence, the correct answer is (C).

89. $L = mvr\sin\theta$

$$\Rightarrow \quad L = (2)(4)(3)\sin 30$$

$$\Rightarrow \quad L = 12\ \text{kgm}^2\text{s}^{-1}$$

Direction is found by Right Hand Thumb Rule

Hence, the correct answer is (A).

90. $\tau = Fr\sin\theta$

$$\Rightarrow \quad \tau = (2)(3)\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad \tau = 3\ \text{Nm}$$

Direction is found by Right Hand Thumb Rule.

Hence, the correct answer is (D).

Matrix Match/Column Match Type Questions

1. A → (r)
B → (p)
C → (p)
D → (s)

$$mv - mv_0 = -\int F dt = -J \quad \ldots(1)$$

$$mV = J \quad \ldots(2)$$

$$mv_0 = mv + mV$$

$$\Rightarrow \quad v_0 = v + V \quad \ldots(3)$$

$$e = 1 = -\left[\frac{\left(V + \frac{\ell\omega}{2}\right) - v}{0 - v_0}\right]$$

$$\Rightarrow \quad V + \frac{\ell\omega}{2} - v = v_0$$

$$\Rightarrow \quad v + v_0 = V + \frac{\ell w}{2} \quad \ldots(4)$$

Also, $mv_0\left(\frac{\ell}{2}\right) = mv\left(\frac{\ell}{2}\right) + \left(\frac{m\ell^2}{12}\right)\omega$

$$\Rightarrow \quad v_0 = v + \frac{\ell\omega}{6} \quad \ldots(5)$$

From (3) and (5), we get

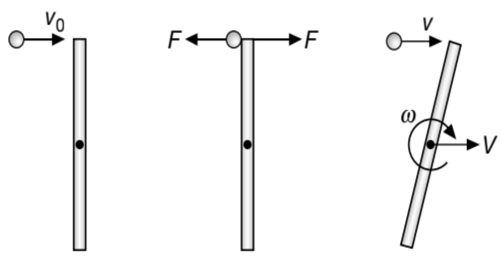
$$V = \frac{\ell\omega}{6}$$

So, from (4), we get

$$v + v_0 = \frac{2}{3}\ell\omega \quad \ldots(6)$$

Solving (5) and (6), we get

$$\omega = \frac{12v_0}{5\ell}$$



Just Before Impact | During Impact | Just After Impact

So, we get

$$v = \frac{3v_0}{5}, V = \frac{2v_0}{5}$$

$$\Rightarrow \quad \frac{\text{Final KE of Ball}}{\text{Initial KE of Ball}} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv_0^2} = \frac{9}{25}$$

$$\Rightarrow \quad \frac{\text{Impulse Delivered to Rod }(J)}{\text{Initial Momentum of Ball }(mv_0)} = \frac{2}{5}$$

$$\frac{(L_f)_{\text{rod}}}{(L_i)_{\text{ball about CM}}} = \frac{I\omega}{mv_0\left(\frac{\ell}{2}\right)}$$

$$\Rightarrow \quad \frac{(L_f)_{\text{rod}}}{(L_i)_{\text{ball}}} = \frac{\left(\frac{m\ell^2}{12}\right)\omega}{mv_0\left(\frac{\ell}{2}\right)} = \frac{2}{5}$$

$$\Rightarrow \quad \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mV^2} = \frac{\left(\frac{m\ell^2}{12}\right)\omega^2}{mV^2}$$

$$\Rightarrow \quad \frac{(K_{\text{rod}})_{\text{rotation}}}{(K_{\text{rod}})_{\text{translation}}} = \frac{\ell^2\omega^2}{12V^2} = 3$$

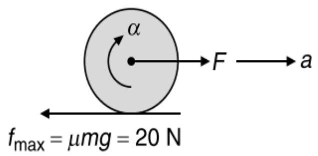
2. A → (q, r)
B → (p)
C → (s)
D → (q)

From graph in question, we see that

$$F = 10t$$

Since $a = R\alpha = R\left(\frac{\tau}{I}\right)$, where $a = \frac{F - f_\ell}{m}$

$$\Rightarrow \quad \frac{F - 20}{10} = (1)\left(\frac{20 \times 1}{\frac{1}{2} \times 10 \times 1^2}\right)$$



$$\Rightarrow \quad F = 60 \text{ N}$$

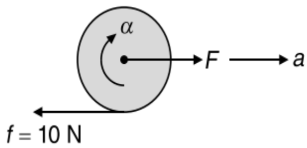
So, when $F = 60$ N, friction reaches its maximum value, i.e. slipping will start and F becomes 60 N at 6 s.

When $f = 10$ N, then

$$\frac{F - 10}{10} = (1)\left(\frac{10 \times 1}{\frac{1}{2} \times 10 \times 1^2}\right)$$

$$\Rightarrow \quad F = 30 \text{ N}$$

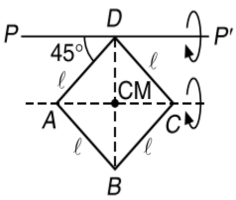
Since $F = 10t$, so F becomes 30 N at 3 s



3. A → (r, s)
B → (p, q)
C → (s)
D → (p, q, r, s)
Conceptual

4. A → (s)
B → (q)
C → (p)
D → (r)

(A) Moment of inertia of the frame about an axis passing through AC



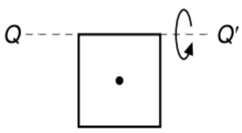
$$I_{AC} = 4\left(\frac{1}{3}ml^2 \sin^2 45°\right) = \frac{2}{3}ml^2$$

Using parallel axis theorem, we get

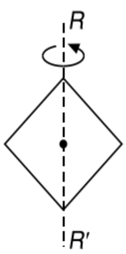
$$I_{PP'} = I_{AC} + 4m\left(\frac{l}{\sqrt{2}}\right)^2 = \frac{2}{3}ml^2 + 2ml^2$$

$$\Rightarrow \quad I_{PP'} = \frac{8}{3}ml^2$$

(B) $I_{QQ'} = \dfrac{2ml^2}{3} + ml^2 = \dfrac{5ml^2}{3}$

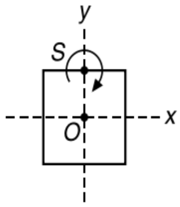


(C) $I_{RR'} = 4 \times \dfrac{1}{3}ml^2 \sin^2 45° = \dfrac{2}{3}ml^2$



(D) $I_x = I_y = 2\left[\dfrac{1}{12}ml^2 + m\left(\dfrac{l}{2}\right)^2\right] = \dfrac{2ml^2}{3}$

Since, $I_z = I_x + I_y$



$$\Rightarrow \quad I_z = \frac{4ml^2}{3}$$

Moment of inertia about an axis passing through D

$$I_D = I_z + 4m\left(\frac{l}{2}\right)^2 = \frac{4}{3}ml^2 + ml^2 = \frac{7ml^2}{3}$$

5. A → (q)
B → (p)
C → (s)
D → (r)

In general, as $v_P = 2v_0 \sin\left(\dfrac{\theta}{2}\right)$

6. A → (r)
B → (q)
C → (t)
D → (p)

For (A), $I = \dfrac{5}{4}MR^2$

For (B), $I = \dfrac{7}{5}MR^2$

For (C), $I = \dfrac{3}{2}MR^2$

For (D), $I = \dfrac{1}{2}MR^2$

7. A → (s)
B → (r)
C → (q)
D → (p)

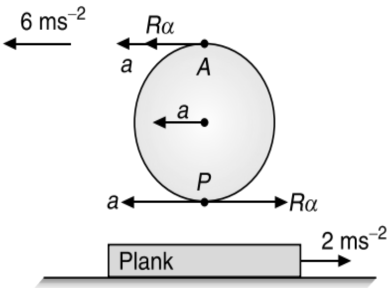
For rolling without slipping, acceleration of point P at bottom of cylinder equals the acceleration of plank, i.e. 2 ms^{-2}. So, we get

$\Rightarrow \quad R\alpha - a = 2$ ...(1)

For rolling without slipping, acceleration of point A at the top of cylinder equals acceleration of the block B, i.e. 6 ms^{-2}. So, we get

$\Rightarrow \quad R\alpha + a = 6$ ...(2)

where a is the acceleration of centre of mass of cylinder towards left.



Solving (1) and (2), we get

$a = 2\ ms^{-2}$ and $\alpha = 1\ rads^{-2}$

$\Rightarrow\ 2a = 4\ ms^{-2}$ and $\alpha = 1\ rads^{-2}$

Further we have, for B,

$m_B g - T = m_B a_B$

$\Rightarrow\ m_B g - T = m_B(6)$

For cylinder, we have

$T = (m_{cyl})a = 2m_{cyl}$

$\Rightarrow\ m_B g - 2m_{cyl} = 6m_B$

$\Rightarrow\ 2m_{cyl} = 4m_B$

$\Rightarrow\ \dfrac{m_{cyl}}{m_B} = 2$

Also, $\ell = \ell_0 + \dfrac{1}{2}a't^2$

where a' is the acceleration at which the thread unwraps from the cylinder, so

$a' = R\alpha$

$\Rightarrow\ \ell = 20 + \dfrac{1}{2}(R\alpha)(2)^2$

$\Rightarrow\ \ell = 20 + 8$

$\Rightarrow\ \ell = 28$ m

$\Rightarrow\ \Delta\ell = \ell - \ell_0 = 28 - 20 = 8$ m

8. A → (p, s)
B → (p, r)
C → (q, r)
D → (p, r)

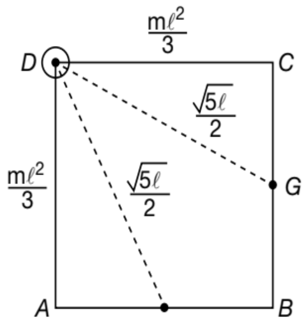
For the case of pure rolling (upwards or downwards), the required value of friction acts in upward direction and in case of slip (forward or backward) maximum friction will act in backward or forward direction.

9. A → (q)
B → (s)
C → (q)
D → (r)

$$I_1 = 2\left(\frac{m\ell^2}{12}\right) + 2(m)\left(\frac{\ell}{2}\right)^2 = \frac{2}{3}m\ell^2$$

$$I_2 = 0 + 2\left(\frac{m\ell^2}{3}\right) + m\ell^2 = \left(\frac{5}{3}\right)m\ell^2$$

$$I_3 = 4\left[\frac{m\ell^2}{3}\sin^2 45°\right] = \frac{2}{3}m\ell^2 = I_1$$



Note that $I_1 = I_3$ {∵ of symmetry}

MI of rods AB and BC about D i.e., 4 is

$$(I_{AB})_4 = (I_{BC})_4 = \frac{m\ell^2}{12} + m\left(\frac{\sqrt{5}\ell}{2}\right)^2 = \frac{4}{3}m\ell^2$$

$$\Rightarrow\ I_4 = 2\left(\frac{m\ell^2}{3}\right) + 2\left(\frac{4}{3}m\ell^2\right) = \frac{10}{3}m\ell^2$$

10. A → (p, q, r)
B → (p, q, r)
C → (p, q, r)
D → (s)
Conceptual

11. A → (r)
B → (p)
C → (p)
D → (q)

If total mass of disc be M, then mass of cut out portion is $m_1 = \dfrac{M}{4}$

So, $I_1 = \dfrac{1}{2}MR^2 - \dfrac{1}{2}m_1\left(\dfrac{R}{2}\right)^2 = \dfrac{15}{32}MR^2$

Also, by parallel axis theorem, we have

$$I_2 = I_1 + \frac{3M}{4}\left(\frac{R}{2}\right)^2 = \frac{21}{32}MR^2$$

By perpendicular axis theorem, we have

$I_3 + I_4 = I_2$

Since, $I_2 = I_1 + \dfrac{3M}{4}\left(\dfrac{R}{2}\right)^2$

and $I_3 = \dfrac{I_1}{2} + \dfrac{3M}{4}\left(\dfrac{R}{2}\right)^2$ $\left\{\because I_{diameter} = \dfrac{I_1}{2}\right\}$

$\Rightarrow\ I_2 - I_3 = \dfrac{I_1}{2}$

12. A → (p)
B → (s)
C → (p)
D → (r)

For all cases if J is the linear impulse, then

$$v = \frac{J}{m}$$

Also, $\omega = \frac{Jr_\perp}{I_E}$

where, $r_\perp$ is the perpendicular distance from centre E and v is rightwards.

For pure rolling ω should be clockwise and hence J should be applied at A.

If it is below A, angular velocity is anticlockwise and it will cause forward slip.

13. A → (q)
B → (r)
C → (p)
D → (r)

Since, $\tau = 0$, so $L = \text{constant}$, $K = \frac{L^2}{2I}$ and $I\omega = \text{constant}$.

Insect first moves away from the axis, then towards it. Hence, I will first increase and then decrease.

14. A → (t)
B → (s)
C → (p)
D → (p)

$a = \dfrac{g\sin\theta}{1+\dfrac{I}{mR^2}}$ but $\dfrac{I}{mR^2} = 1$ for a thin walled cylindrical shell

$$\Rightarrow \quad a = \frac{g\sin\theta}{2} = \frac{g}{4}$$

$$\Rightarrow \quad \alpha = \frac{a}{R} = \frac{g\sin\theta}{2R} = \frac{g}{4R}$$

$$K_T = \frac{1}{2}mv^2 = \frac{1}{2}m(at)^2 = \frac{1}{2}m\left(\frac{g\sin\theta}{2}\right)^2(t^2)$$

$$\Rightarrow \quad K_T = \frac{mg^2t^2\sin^2\theta}{2} = \frac{mg^2t^2}{8}$$

$$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}I(\alpha t)^2 = \frac{1}{2}(mR^2)\left(\frac{g\sin\theta}{2R}\right)^2(t^2)$$

$$\Rightarrow \quad K_R = \frac{mg^2t^2\sin^2\theta}{2} = \frac{mg^2t^2}{8}$$

15. A → (q)
B → (s)
C → (q)
D → (p)

Since $\ell = \frac{1}{2}at^2$

$$\Rightarrow \quad t = \sqrt{\frac{2S}{a}}, \text{ where } a = \frac{g\sin\theta}{1+\dfrac{I}{mR^2}}$$

Moment of inertia I of hollow sphere is maximum, hence a is minimum and t is maximum

Total kinetic energy equals mgh for all

$\frac{K_R}{K_T} = \frac{2}{5}$, for solid sphere

$\frac{K_R}{K_T} = \frac{2}{3}$, for hollow sphere

$\frac{K_R}{K_T} = \frac{1}{2}$, for cylinder

So, rotational kinetic energy is maximum for hollow sphere.

Since $\frac{K_T}{K_R}$ is maximum for solid sphere, hence it has maximum translational kinetic energy.

16. A → (p, r, s)
B → (q, r, s)
C → (q, r, s)
D → (p, r, s)

Conceptual

17. A → (q)
B → (s)
C → (p)
D → (s)

For A: At 2, $K_{\text{Total}} = mgh = E$ (say)

Since, $\frac{K_R}{K_T} = \frac{2}{5}$

$$\Rightarrow \quad K_R = \frac{2}{7}E = \frac{2}{7}mgh$$

For B: At 3, $K_{\text{Total}} = mg\left(\frac{h}{2}\right)$

Since, $\frac{K_R}{K_T} = \frac{2}{5}$

$$\Rightarrow \quad K_T = \frac{5}{7}E = \frac{5}{7}\left(mg\frac{h}{2}\right) = \frac{5}{14}mgh$$

For C: From 3 to 4, K_R will be constant, so

At 4, $K_R = \frac{2}{7}\left(mg\frac{h}{2}\right) = \frac{1}{7}mgh$

For D: At 4, $K_T = K_{\text{Total}} - K_R$

$$\Rightarrow \quad K_T = mgh - \frac{1}{7}mgh = \frac{6}{7}mgh$$

18. A $\rightarrow$ (p, r)
B $\rightarrow$ (p, r)
C $\rightarrow$ (q, s)
D $\rightarrow$ (p, s)
Conceptual

19. A $\rightarrow$ (s)
B $\rightarrow$ (p)
C $\rightarrow$ (p)
D $\rightarrow$ (q, r)
Conceptual

20. A $\rightarrow$ (q)
B $\rightarrow$ (r)
C $\rightarrow$ (p)
D $\rightarrow$ (s)
Acceleration of 1 w.r.t. centre of mass is $\vec{a}_{1/cm} = r\alpha\hat{i} - \omega^2 r\hat{j}$

Since $\vec{a}_{1/cm^-} = \vec{a}_1 - \vec{a}_{cm}$

$\Rightarrow \quad \vec{a}_1 = \vec{a}_{1/cm} + \vec{a}_{cm}$ where $\vec{a}_{cm} = (R\alpha)\hat{i}$

$\Rightarrow \quad \vec{a}_1 = r\alpha\hat{i} - \omega^2 r\hat{j} + R\alpha\hat{i} = (R+r)\alpha\hat{i} - \omega^2 r\hat{j}$

Similarly, we have

$$\vec{a}_2 = -r\alpha\hat{j} - \omega^2 r\hat{i} + R\alpha\hat{i} = (R\alpha - \omega^2 r)\hat{i} - r\alpha\hat{j}$$

$$\vec{a}_3 = -r\alpha\hat{i} + \omega^2 r\hat{j} + R\alpha\hat{i} = (R\alpha - r\alpha)\hat{i} + \omega^2 r\hat{j}$$

$$\vec{a}_4 = r\alpha\hat{j} + \omega^2 r\hat{i} + R\alpha\hat{i} = (R\alpha + \omega^2 r)\hat{i} + r\alpha\hat{j}$$

21. A $\rightarrow$ (q)
B $\rightarrow$ (s)
C $\rightarrow$ (s)
D $\rightarrow$ (p)

Since $K = \sqrt{\dfrac{I}{m}}$

For axis 1, $K = \sqrt{\dfrac{m(2a)^2}{3}} = \dfrac{2}{\sqrt{3}}a$

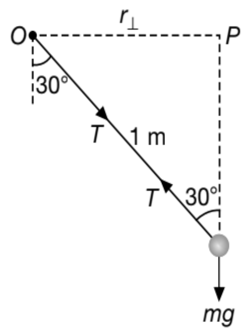
For axis 2, $K = \sqrt{\dfrac{m(2a)^2}{12}} = \dfrac{a}{\sqrt{3}}$

For axis 3, $K = \sqrt{\dfrac{ma^2}{3}} = \dfrac{a}{\sqrt{3}}$

For axis 4, $K = \sqrt{\dfrac{ma^2}{12}} = \dfrac{a}{\sqrt{12}}$

Integer/Numerical Answer Type Questions

1. Two forces are acting on the ball.
(i) tension (T)
(ii) weight (mg)



Since the tension passes through O, so torque due to tension about point O is zero.

So, $\tau_{mg} = F \times r_\perp$

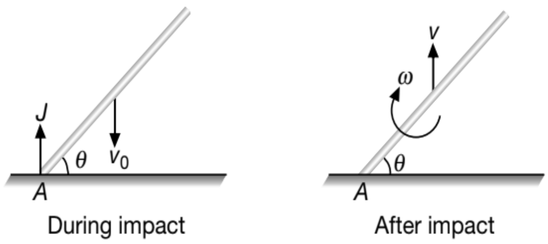
where, $r_\perp = OP = 1\sin(30°) = 0.5$ m

$\Rightarrow \quad \tau_{mg} = (mg)(0.5)$

$\Rightarrow \quad \tau_{mg} = (1)(10)(0.5)$

$\Rightarrow \quad \tau_{mg} = 5$ Nm

2. If v be the linear velocity of rod after impact (upwards), ω be the angular velocity of rod and J be the linear impulse at A during impact, then by



Impulse – Momentum Theorem, we have

$J = \Delta p = p_f - p_i$

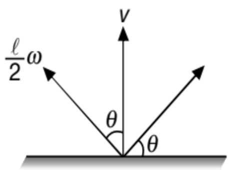
$\Rightarrow \quad J = mv - (-mv_0)$

$\Rightarrow \quad J = m(v + v_0) \qquad \ldots(1)$

Further by (Angular Impulse) – (Angular Momentum Theorem), we have

$J\left(\dfrac{\ell}{2}\cos\theta\right) = I\omega = \dfrac{m\ell^2}{12}\omega \qquad \ldots(2)$

Since the collision is elastic, so at the point of impact, $e = 1$.



$\Rightarrow$ (Relative speed of approach) = (Relative speed of separation)

$$\Rightarrow \quad v_0 = v + \frac{\ell}{2}\omega\cos\theta \qquad \ldots(3)$$

Solving equations (1), (2) and (3), we get

$$\omega = \frac{6v_0\cos\theta}{\ell\left(1+3\cos^2\theta\right)}$$

For ω to be maximum, we have

$$\frac{d\omega}{d\theta} = 0$$

$$\Rightarrow \quad \frac{6v_0}{\ell}\frac{d}{d\theta}\left(\frac{\cos\theta}{1+3\cos^2\theta}\right) = 0$$

$$\Rightarrow \quad \frac{\left(1+3\cos^2\theta\right)(-\sin\theta) - \cos\theta(-6\sin\theta\cos\theta)}{\left(1+3\cos^2\theta\right)^2} = 0$$

$$\Rightarrow \quad 1+3\cos^2\theta = 6\cos^2\theta$$

$$\Rightarrow \quad 3\cos^2\theta = 1$$

$$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{3}}$$

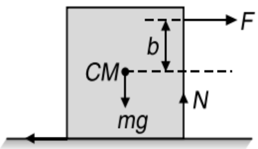
$$\Rightarrow \quad * = 3$$

3. For the block to be in equilibrium

$$F = f = \mu mg \qquad \ldots(1)$$

For block not to topple, we have

$$F\left(b+\frac{a}{2}\right) < mg\frac{a}{2} \qquad \ldots(2)$$



$$\Rightarrow \quad \mu mg\left(b+\frac{a}{2}\right) < mg\left(\frac{a}{2}\right)$$

$$\Rightarrow \quad \left(b+\frac{a}{2}\right)\mu < \frac{a}{2}$$

$$\Rightarrow \quad 0.4 = \mu < \frac{a}{2b+a}$$

$$\Rightarrow \quad 0.8b + 0.4a < a$$

$$\Rightarrow \quad 0.8b < 0.6a$$

$$\Rightarrow \quad \frac{b}{a} < \frac{3}{4}$$

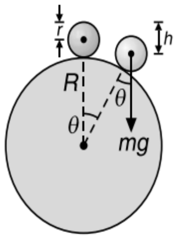
So, $\left(100\frac{b}{a}\right)_{\max} = 75$

4. The equation of motion for the centre of the sphere at the moment of breaking off, $N = 0$ is

$$\frac{mv^2}{R+r} = mg\cos\theta \qquad \ldots(1)$$

where v is the speed of the centre of the sphere at that moment and θ is the corresponding angle. The speed v can be found by using the Law of Conservation of Energy, according to which

$$mgh = \frac{mv^2}{2} + \frac{I\omega^2}{2}$$



where $I = \frac{2}{5}mr^2$, $v = r\omega$ and $h = (R+r)(1-\cos\theta)$

From these equations we get

$$\omega = \sqrt{\frac{10g(R+r)}{17r^2}}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{10g\left(R+\frac{R}{16}\right)}{17\left(\frac{R}{16}\right)^2}}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{10\times10\times17\times16}{17\times1}}$$

$$\Rightarrow \quad \omega = 40 \text{ rads}^{-1}$$

5. $I = 2\left(\frac{2}{5}MR^2\right) + 2\left(\frac{2}{5}MR^2 + Md^2\right)$

$$\Rightarrow \quad I = \frac{8}{5}MR^2 + 2md^2$$

$$\Rightarrow \quad I = \left[\frac{8}{5}(0.5)\left(\frac{\sqrt{5}}{2}\right)^2 + 2(0.5)(4)(2)\right]10^{-4}$$

$$\Rightarrow \quad I = \left(\frac{5}{5}+8\right)\times10^{-4}$$

$$\Rightarrow \quad I = 9\times10^{-4}$$

$$\Rightarrow \quad N = 9$$

6. $I = \frac{1}{2}\times40\times(0.5)^2 = 5 \text{ kgm}^2$

Given, $\omega = \frac{240}{\pi}\times\frac{2\pi}{60} = 8 \text{ rads}^{-1}$

Since, $\omega = \alpha t_0$

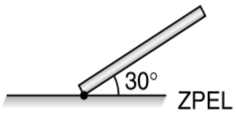
$$\Rightarrow \quad \alpha = \frac{\omega}{t_0} = \frac{8}{2} = 4 \text{ rads}^{-2}$$

$$\Rightarrow \quad \tau = I\alpha = (5)(4) = 20 \text{ Nm}$$

Further, $\alpha' = \frac{FR}{I} = \frac{10 \times 0.5}{5} = 1 \text{ rads}^{-2}$

$$\Rightarrow \quad t = \frac{\omega}{\alpha'} = \frac{8}{1} = 8 \text{ s}$$

7. From mechanical energy conservation, we get



$$U_i + K_i = U_f + K_f$$

$$\Rightarrow \quad mg\left(\frac{l}{2}\sin 30°\right) + 0 = 0 + \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad mg\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + 0 = 0 + \frac{1}{2}\left[\frac{m(1)^2}{3}\right]\omega^2$$

$$\Rightarrow \quad \omega^2 = \frac{3g}{2} = 15$$

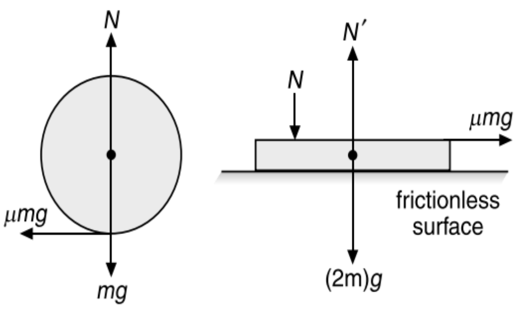
$$\Rightarrow \quad \omega = \sqrt{15}$$

$$\Rightarrow \quad n = 15$$

8. Initially the cylinder will slip on the plank, therefore kinetic friction will act between the cylinder and the plank. If a_p be the acceleration of plank, a_c and α_c be the linear and angular acceleration of the cylinder, then

$$a_c = -\frac{\mu mg}{m} = -\mu g$$

$$a_p = +\frac{\mu g}{2}$$



$$\alpha_c = +\frac{(\mu mg)(R)}{\left(\frac{mR^2}{2}\right)} = +\frac{2\mu g}{R}$$

For pure rolling, we have

$$(v_{\text{net}})_{\text{cylinder}} = v_{\text{plank}}$$

$$\Rightarrow \quad v_c - R\omega_c = v_p$$

$$\Rightarrow \quad \frac{\mu g}{2}t = v_0 - \mu g t - (R)\left(\frac{2\mu g}{R}\right)(t)$$

$$\Rightarrow \quad t = \frac{v_0}{3.5\mu g} = \frac{7}{3.5 \times 0.1 \times 10} = 2 \text{ s}$$

Now, $s_c - s_p = \left(v_0 t - \frac{1}{2}\mu g t^2\right) - \frac{1}{2}\left(\frac{\mu g}{2}\right)t^2$

$$\Rightarrow \quad s_c - s_p = (7 \times 2) - \frac{1}{2}(0.1)(10)(4) - \frac{1}{2}\left(\frac{0.1 \times 10}{2}\right)(4)$$

$$\Rightarrow \quad s_c - s_p = 11 \text{ m}$$

Also, $v_c - v_p = (v_0 - \mu g t) - \left(\frac{\mu g}{2}\right)(t)$

$$\Rightarrow \quad v_c - v_p = 7 - 0.1 \times 10 \times 2 - \frac{0.1 \times 10 \times 2}{2}$$

$$\Rightarrow \quad v_c - v_p = 4 \text{ ms}^{-1}$$

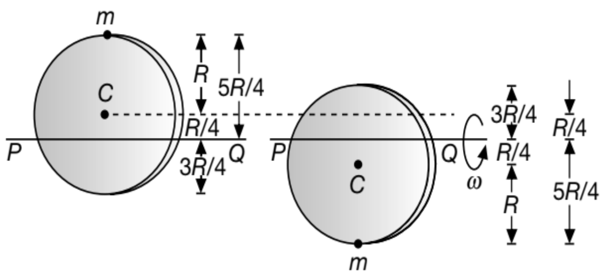
Hence, the remaining distance $(12 - 11 = 1 \text{ m})$ is travelled in a time,

$$t' = \frac{1}{4} = 0.25 \text{ s}$$

So, total time equals $T = 2 + 0.25 = 2.25$ s

9. Initial and final positions are shown in the figure. By Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in}\\ \text{GPE}\\ \text{of m}\end{pmatrix} + \begin{pmatrix}\text{Loss in}\\ \text{GPE of}\\ \text{CM of disc}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{RKE of}\\ \text{mass + disc}\end{pmatrix} \quad \ldots(1)$$



Loss in gravitational potential energy of mass m is

$$mg\left(2 \times \frac{5R}{4}\right) = \frac{5mgR}{2}$$

Loss in gravitational potential energy of CM of disc is

$$mg\left(2 \times \frac{R}{4}\right) = \frac{mgR}{2}$$

Therefore, total decrease in gravitational potential energy of system is

$$-\Delta U = \frac{5mgR}{2} + \frac{mgR}{2} = 3mgR \quad \ldots(2)$$

Gain in rotational kinetic energy of system (mass + disc) is $\Delta K = \frac{1}{2}I\omega^2$

where I is the moment of inertia of system (disc + mass) about axis PQ

$\Rightarrow$ I = moment of inertia of disc + moment of inertia of mass

$$\Rightarrow \quad I = \left(\frac{mR^2}{4} + m\left(\frac{R}{4}\right)^2\right) + m\left(\frac{5R}{4}\right)^2$$

$$\Rightarrow \quad I = \frac{15mR^2}{8}$$

$$\Rightarrow \quad \Delta K = \frac{1}{2}\left(\frac{15mR^2}{8}\right)\omega^2 \qquad \ldots(3)$$

Substituting (2) and (3) in (1), we get

$$3mgR = \frac{1}{2}\left(\frac{15mR^2}{8}\right)\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{16g}{5R}}$$

Therefore, linear speed of particle at its lowest point is

$$v = \left(\frac{5R}{4}\right)\omega = \frac{5R}{4}\sqrt{\frac{16g}{5R}}$$

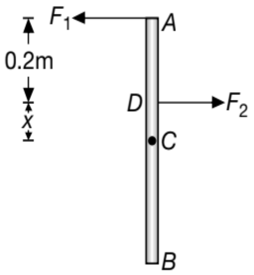
$$\Rightarrow \quad v = \sqrt{5gR}$$

$$\Rightarrow \quad x = 5$$

10. Let x be the distance of centre point C of rod from D. Then,

$$F_2 - F_1 = ma$$

$$\Rightarrow \quad F_1 = 3 \text{ N}$$



Further, $\tau_c = 0$

$$\Rightarrow \quad F_2 x = F_1(0.2 + x)$$

$$\Rightarrow \quad 5x = F_1(0.2 + x)$$

$$\Rightarrow \quad 5x = 3(0.2 + x)$$

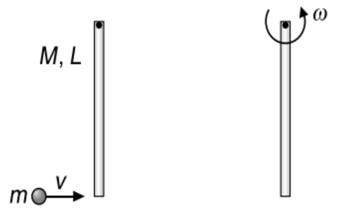
$$\Rightarrow \quad x = 0.3 \text{ m}$$

So, length of rod is $L = 2(x + 0.2) = 1$ m

11. Applying conservation of angular momentum, we get

$$\vec{L}_i = \vec{L}_f$$

$$\Rightarrow \quad mvL = I\omega$$



Before collision After collision

$$\Rightarrow \quad mvL = \left(\frac{mL^2}{3} + mL^3\right)\omega$$

$$\Rightarrow \quad 0.1 \times 80 \times 1 = \left(\frac{0.9 \times 1^2}{3} + 0.1 \times 1^2\right)\omega$$

$$\Rightarrow \quad 8 = \left(\frac{3}{10} + \frac{1}{10}\right)\omega$$

$$\Rightarrow \quad 8 = \frac{4}{10}\omega$$

$$\Rightarrow \quad \omega = 20 \text{ rads}^{-1}$$

12. (a) Angular retardation $\alpha = \frac{\tau}{I} = \frac{4}{16} = 0.25 \text{ rads}^{-2}$

Since α is constant, so

$$0 = \omega_0 - \alpha t_0$$

$$\Rightarrow \quad t_0 = \frac{\omega_0}{\alpha} = \frac{9}{0.25} = 36 \text{ second}$$

(b) $\alpha = -\frac{d\omega}{dt} = \frac{8t}{16}$

$$\Rightarrow \quad \int_0^{t_0} 8t\,dt = -16\int_9^0 d\omega$$

$$\Rightarrow \quad 4t_0^2 = -16(0 - 9)$$

$$\Rightarrow \quad t_0^2 = 4 \times 9$$

$$\Rightarrow \quad t_0 = \sqrt{36}$$

$$\Rightarrow \quad t_0 = 6 \text{ s}$$

13. (a) Maximum frictional force on the block can be

$$f_{max} = \mu mg$$

Therefore, maximum acceleration of block can be

$$a_{max} = \frac{f_{max}}{m} = \frac{\mu mg}{m} = \mu g$$

$$\Rightarrow \quad a_{max} = (0.4)(10) = 4 \text{ ms}^{-2}$$

If acceleration of truck, exceeds this value, then slipping will start between the block and the truck.

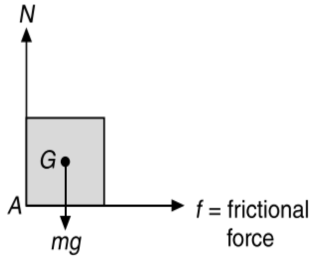
(b) In the critical case when the block is about to topple about A, normal reaction will pass through A. So, the block is on the verge of motion and hence

$$f = ma \qquad \ldots(1)$$

$$N = mg \quad \ldots(2)$$

The block will topple when,

Torque due to f about G > torque due to N about G



Taking the limiting value, we get

$$\tau_f = \tau_N$$

$$\Rightarrow \quad (f)(h) = (N)(b)$$

$$\Rightarrow \quad (ma)(h) = (mg)(b)$$

$$\Rightarrow \quad a = \left(\frac{b}{h}\right)g = \left(\frac{0.6}{1.2}\right)(10) = 5 \text{ ms}^{-2}$$

So, if the acceleration of truck exceeds this value, the block will topple about A.

(c) The maximum acceleration/retardation at which the block is not disturbed, is the smaller of the two values, obtained above, i.e., 4 ms^{-2}

Hence, the maximum retardation can be 4 ms^{-2}

Since, $u = 72 \text{ kmhr}^{-1} = 20 \text{ ms}^{-1}$

So, applying $v^2 = u^2 - 2as$ $\quad \{\because v = 0\}$

$$\Rightarrow \quad s = \frac{u^2}{2a} \quad \{\because a = 4 \text{ ms}^{-2}\}$$

$$\Rightarrow \quad s = \frac{(20)^2}{(2)(4)} = 50 \text{ m}$$

14. $L_i = L_f$

$$\Rightarrow \quad \left(80R^2 + \frac{200R^2}{2}\right)\omega = \left(0 + \frac{200R^2}{2}\right)\omega_1$$

$$\Rightarrow \quad \omega_1 = 1.8\omega_0 = (1.8)(5)$$

$$\Rightarrow \quad \omega_1 = 9 \text{ rpm}$$

15. If α be the angular acceleration of the hoop and a be the acceleration of its centre, acceleration of m would be $a + r\alpha$.

If I be the moment of inertia of the hoop, then $I = Mr^2$

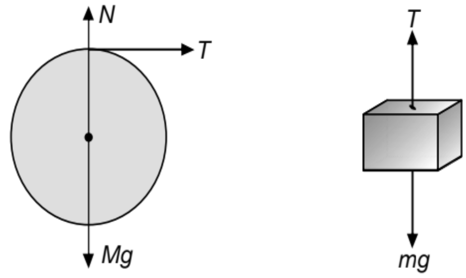
Since, $\tau = Tr = I\alpha$

$$\Rightarrow \quad \alpha = \frac{Tr}{I} = \frac{T}{Mr}$$

and $T = Ma$

For m, we have

$$mg - T = m(a + r\alpha)$$



F.B.D. of hoop F.B.D. of block of mass m

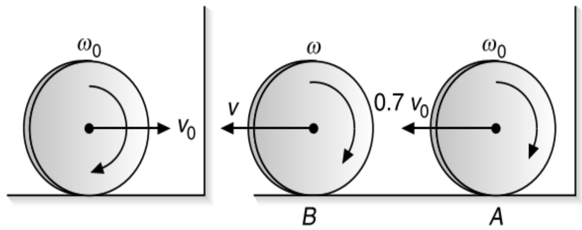
Solving, we get

$$a = \frac{mg}{(M+2m)} = \frac{2}{2} = 1 \text{ ms}^{-2}$$

$$\Rightarrow \quad T = 1 \text{ N}$$

$$\text{and } \alpha = \frac{Tr}{I} = \frac{T}{Mr} = 5 \text{ rads}^{-2}$$

16. Between A and B, there is forward slipping. Therefore, friction will be maximum and backwards (rightwards). At point B where $v = R\omega$, ball starts rolling without slipping and force of friction becomes zero.



By Law of Conservation of Angular Momentum between points A and B about bottommost point (because torque of friction about this point is zero), we get

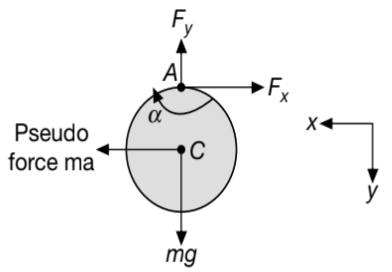
$$L_A = L_B$$

$$\Rightarrow \quad m(0.7v_0)R - I\omega_0 = mvR + I\omega$$

Substituting $\omega_0 = \dfrac{v_0}{R}$, $\omega = \dfrac{v}{R}$ and $I = \dfrac{2}{5}mR^2$, we get

$$v = \frac{3}{14}v_0 = \left(\frac{3}{14}\right)(7) \text{ ms}^{-1} = 1.5 \text{ ms}^{-1}$$

17. FBD of plate w.r.t. truck is shown in figure.



Equations of motion for centre of mass of plate is

$$a_x = \frac{ma - F_x}{m}$$

$$a_y = 0$$

$\Rightarrow \quad F_y = mg = 100\text{ N}$

Angular acceleration about A is

$$\alpha = \frac{maR}{\frac{3}{2}mR^2} = \frac{2a}{3R}$$

Since $a_x = R\alpha$

$$\Rightarrow \quad \frac{ma - F_x}{m} = \frac{2}{3}a$$

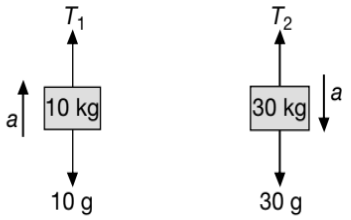
$$\Rightarrow \quad F_x = \frac{1}{3}ma = \frac{1}{3}\left(\frac{100}{10}\right)(0.9) = 3\text{ N}$$

Absolute acceleration of centre of mass is

$$a_C = \frac{F_x}{m} = \frac{3}{10} = 0.3\text{ ms}^{-2} = 30\text{ cms}^{-2}$$

Since, $\alpha = \frac{2}{3}\frac{a}{R} = \frac{2}{3}\left(\frac{0.9}{0.6}\right) = 1\text{ rads}^{-2}$

18. The free body diagrams of the bodies and the pulley are shown in figure, then

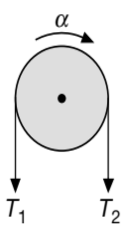


for 10 kg mass, we have

$$T_1 - 10g = 10a \quad \ldots(1)$$

for 30 kg mass, we have

$$30g - T_2 = 30a \quad \ldots(2)$$



for the pulley, we have

$$\alpha = \frac{(T_2 - T_1)R}{\frac{1}{2}mR^2} = \frac{2(T_2 - T_1)}{mR}$$

$$\Rightarrow \quad \alpha = \frac{2(T_2 - T_1)}{20R} = \frac{0.1(T_2 - T_1)}{R} \quad \ldots(3)$$

For no slipping, $a = R\alpha$ ...(4)

Solving the above equations, we get

$$a = 4\text{ ms}^{-2},\ T_1 = 500\text{ N and } T_2 = 180\text{ N}$$

$$v = \sqrt{2as} = \sqrt{(2)(4)(2)} = 4\text{ ms}^{-1}$$

$$\omega = \frac{v}{R} = \frac{4}{0.1} = 40\text{ rads}^{-1}$$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times 2}{4}} = 1\text{ second}$$

19. If moment of inertia of bigger disc is $I = \frac{MR^2}{2}$

So, MI of small disc is $I_2 = \frac{M(R/2)^2}{2} = \frac{I}{4}$

By conservation of angular momentum, we have

$$I\omega_1 + \frac{I}{4}(0) = I\omega_2 + \frac{I}{4}\omega_2$$

$$\Rightarrow \quad \omega_2 = \frac{4\omega_1}{5}$$

Initial kinetic energy is $K_i = \frac{1}{2}I\omega_1^2$

Final kinetic energy K_f is

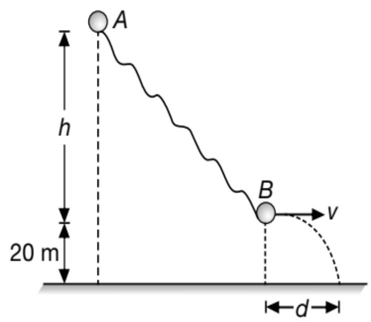
$$K_f = \frac{1}{2}\left(I + \frac{I}{4}\right)\left(\frac{4\omega_1}{5}\right)^2 = \frac{1}{2}I\omega_1^2\left(\frac{4}{5}\right)$$

$$\Rightarrow \quad p\% = \left(\frac{K_i - K_f}{K_i}\right)100\% = \frac{1 - \frac{4}{5}}{1}\times 100 = 20\%$$

20. $h = (48 - 20)\text{ m} = 28\text{ m}$

Total kinetic energy of the sphere at B is

$$K = mgh$$



In case of pure rolling, we have $\frac{K_T}{K_R} = \frac{r^2}{k^2} = \frac{5}{2}$, for a sphere

$$\Rightarrow \quad K_T = \frac{1}{2}mv^2 = \frac{5}{7}mgh$$

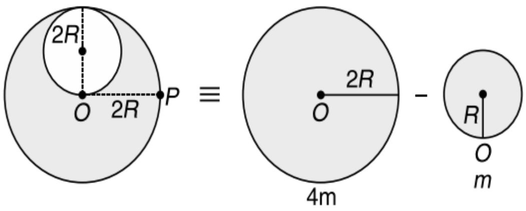
$$\Rightarrow \quad v = \sqrt{\frac{10}{7}gh} = \sqrt{\left(\frac{10}{7}\right)(10)(28)} = 20\text{ ms}^{-1}$$

Now, $t = \sqrt{\frac{2h}{g}}$

$$\Rightarrow \quad t = \sqrt{\frac{2(20)}{10}} = 2 \text{ s}$$

$$\Rightarrow \quad d = vt = (20)(2) = 40 \text{ m}$$

21.



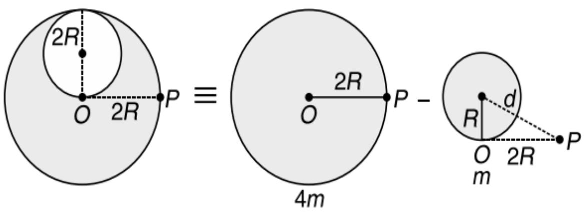
$$I_O = \frac{(4m)(2R)^2}{2} - \frac{3}{2}mR^2$$

$$\Rightarrow \quad I_O = mR^2\left(8 - \frac{3}{2}\right)$$

$$\Rightarrow \quad I_O = \frac{13}{2}mR^2$$

$$I_P = \frac{3}{2}(4m)(2R)^2 - \left(\frac{mR^2}{2} + md^2\right)$$

where $d^2 = R^2 + (2R)^2$

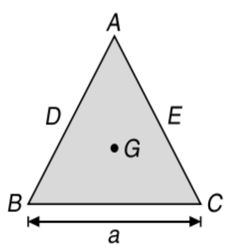


$$\Rightarrow \quad I_P = 24mR^2 - \frac{11}{2}mR^2$$

$$\Rightarrow \quad I_P = \frac{37}{2}mR^2$$

$$\Rightarrow \quad \frac{I_P}{I_O} = \frac{\frac{37}{2}}{\frac{13}{2}} = \frac{37}{13} \approx 3$$

22. Let the side of triangle be a and its mass be m



MI of plate ABC about centroid G is

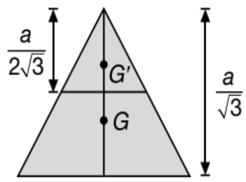
$$I = \frac{ma^2}{12}$$

Triangle ADE is also an equilateral triangular of side $\frac{a}{2}$.

Let moment of inertia of triangular plate ADE about its centroid (G') be I_1 and its mass be m_1

$$m_1 = \frac{m}{(\sqrt{3}a^2/4)} \times \frac{\sqrt{3}}{4}\left(\frac{a}{2}\right)^2 = \frac{m}{4}$$

$$\Rightarrow \quad I_1 = \frac{m_1}{12}\left(\frac{a}{2}\right)^2 = \frac{m}{4 \times 12}\frac{a^2}{4} = \frac{ma^2}{192}$$



Distance $GG' = \frac{a}{\sqrt{3}} - \frac{2}{2\sqrt{3}} = \frac{a}{2\sqrt{3}}$

So, MI of part ADE about centroid G is

$$I_2 = I_1 + m_1\left(\frac{a}{2\sqrt{3}}\right)^2 = \frac{ma^2}{192} + \frac{m}{4}\left(\frac{a^2}{12}\right)$$

$$\Rightarrow \quad I_2 = \frac{5ma^2}{192}$$

So, MI of remaining part is

$$= \frac{ma^2}{12} - \frac{5ma^2}{192} = \frac{11ma^2}{12 \times 16} = \frac{11I_0}{16}$$

$$\Rightarrow \quad N = 11$$

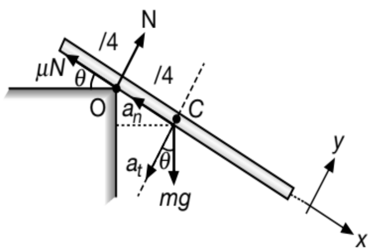
23. $\frac{1}{2}I_0\omega^2 = mg\left(\frac{\ell}{4}\sin\theta\right)$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{m(4a)^2}{12} + m\left(\frac{\ell}{4}\right)^2\right)\omega^2 = mg\left(\frac{\ell}{4}\sin\theta\right)$$

$$\Rightarrow \quad \frac{7}{6}\left(\frac{\ell}{4}\right)\omega^2 = g\sin\theta$$

$$\Rightarrow \quad \omega = \sqrt{\frac{24g\sin\theta}{7\ell}}$$

Since, $\alpha = \frac{\tau}{I_0} = \frac{mg\left(\frac{\ell}{4}\cos\theta\right)}{\frac{7}{3}m\left(\frac{\ell}{4}\right)^2} = \frac{12g\cos\theta}{7\ell}$



Now $\sum F_y = ma_y$

$\Rightarrow \quad mg\cos\theta - N = ma_t$

$\Rightarrow \quad N = mg\cos\theta - ma_t$

$\Rightarrow \quad N = mg\cos\theta - m\left(\frac{\ell}{4}\alpha\right) \qquad \left\{\because a_t = \frac{\ell}{4}\alpha\right\}$

$\Rightarrow \quad N = mg\cos\theta - m\left(\frac{3g\cos\theta}{7}\right) = \frac{4}{7}mg\cos\theta$

Rod begins to slip, when

$\mu N - mg\sin\theta = ma_n$

$\Rightarrow \quad \frac{4}{7}\mu mg\cos\theta - mg\sin\theta = m\left(\frac{\ell}{4}\omega^2\right)$

$\Rightarrow \quad \frac{4\mu mg\cos\theta}{7} - mg\sin\theta = \frac{6mg\sin\theta}{7}$

$\Rightarrow \quad \frac{4}{7}\mu mg\cos\theta = \frac{13}{7}mg\sin\theta$

$\Rightarrow \quad \tan\theta = \frac{4\mu}{13}$

So, $* = 4$

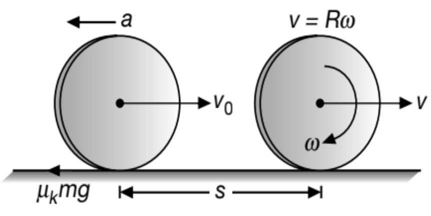
24. Applying Law of Conservation of Angular Momentum about bottommost point, we get

$L_i = L_f$

$\Rightarrow \quad mv_0R = mvR + \left(\frac{2}{5}mR^2\right)\omega$

$\Rightarrow \quad mv_0R = mvR + \frac{2}{5}mvR \qquad \left\{\because \omega = \frac{v}{R}\right\}$

$\Rightarrow \quad v = \frac{5}{7}v_0$



Since, $a = \frac{\mu_k mg}{m} = \mu_k g$ and $v^2 = v_0^2 - 2as$

$\Rightarrow \quad s = \frac{v_0^2 - v^2}{2a} = \frac{v_0^2 - \frac{25}{49}v_0^2}{2\mu_k g}$

$\Rightarrow \quad s = \frac{12v_0^2}{49\mu_k g}$ Substituting the values, we get

$s = \frac{(12)(12)^2}{(49)(0.75)(9.8)} = 4.8 \text{ m}$

25. Equation of motion for the block is

$mg\sin\theta - T = ma$

$\Rightarrow \quad (1)(10)\sin 30^\circ - T = (1)a$

$\Rightarrow \quad T + a = 5 \qquad \ldots(1)$

Equation of motion for disk is

$\frac{TR}{\frac{1}{2}MR^2} = \alpha$

$\Rightarrow \quad \alpha = \frac{2T}{MR}$

For no slipping, we get

$R\alpha = a = \frac{2T}{M}$

$\Rightarrow \quad a = \frac{2T}{3}$

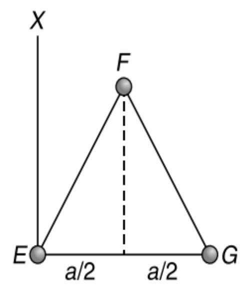
$\Rightarrow \quad T = 1.5a$

Substituting in equation (1), we have

$2.5a = 5$

$\Rightarrow \quad a = 2 \text{ ms}^{-2}$

26. Since, $I = 0 + m\left(\frac{a}{2}\right)^2 + ma^2$



$\Rightarrow \quad I = \frac{5}{4}ma^2 = \frac{25}{20}ma^2$

$\Rightarrow \quad N = 25$

27. (a) By Law of Conservation of Mechanical Energy, we have

$\begin{pmatrix}\text{Loss in} \\ \text{Gravitational} \\ \text{Potential Energy} \\ \text{of Particle}\end{pmatrix} = \begin{pmatrix}\text{Gain in} \\ \text{Rotational} \\ \text{Kinetic Energy of} \\ \text{disc + Particle}\end{pmatrix}$

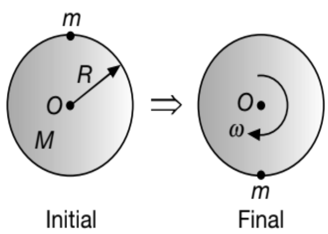
$\Rightarrow \quad mg(2R) = \frac{1}{2}\left(\frac{1}{2}MR^2 + mR^2\right)\omega^2$

$\Rightarrow \quad \omega = \sqrt{\frac{8mg}{(2m+M)R}}$

$\Rightarrow \quad \omega = \sqrt{\frac{(8)(5)(10)}{(10+40)0.5}} = 4 \text{ rads}^{-1}$

(b) If F be the force exerted by disk on the particle (upwards), then

$$F - mg = mR\omega^2$$



$$\Rightarrow \quad F = mg + \frac{8m^2g}{(2m+M)}$$

$$\Rightarrow \quad F = \frac{mg(10m+M)}{(2m+M)}$$

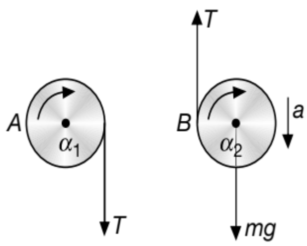
$$\Rightarrow \quad F = \frac{(5)(10)(50+40)}{(10+40)}$$

$$\Rightarrow \quad F = 90 \text{ N}$$

28. Equations of motion for the discs are,

$$a = \frac{mg - T}{m} \quad \ldots(1)$$

$$\alpha_2 = \frac{TR}{\frac{1}{2}mR^2} = \frac{2T}{mR} \quad \ldots(2)$$



Similarly, $\alpha_1 = \dfrac{TR}{\frac{1}{2}mR^2} = \dfrac{2T}{mR}$ …(3)

$\Rightarrow \quad \alpha_1 = \alpha_2$

For no slipping $a = R\alpha_2$ …(4)

Further, $a - R\alpha_2 = R\alpha_1$ …(5)

Solving these equations, we get

$$T = \frac{mg}{5},\ a = \frac{4}{5}g \text{ and } \alpha_2 = \frac{2g}{5R}$$

$$\Rightarrow \quad T = 4 \text{ N},\ a = 8 \text{ ms}^{-2},\ \alpha_2 = 8 \text{ rads}^{-2}$$

29. (a) $\omega = \dfrac{250}{60} \times 2\pi = 26.18 \text{ rads}^{-1}$

$$\text{I} = \frac{1}{2} \times 30 \times (0.2)^2 = 0.6 \text{ kgm}^2$$

Now, $\omega = \alpha t$

$$\Rightarrow \quad \alpha = \frac{\omega}{t} = 2.618 \text{ rads}^{-2}$$

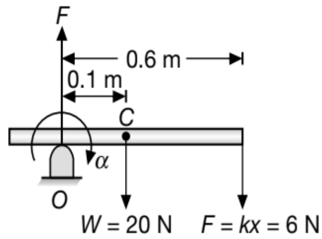
and $\tau = \text{I}\alpha = 1.57$ Nm

(b) $\theta = \dfrac{1}{2}\alpha t^2 = \dfrac{1}{2} \times (2.618)(100) = 131$ rad

(c) $W = \dfrac{1}{2} \times \text{I} \times \omega^2 = \dfrac{1}{2} \times (0.6) \times (26.18)^2$

$\Rightarrow \quad W \approx 206$ J

30. Just after the thread is burnt, the forces acting on the rod are as shown in Figure.



Torque due to forces about O is

$$\tau = (20)(0.1) + (6)(0.6) = 5.6 \text{ N}$$

Angular acceleration about O is given by

$$\alpha = \frac{\tau}{\text{I}} = \frac{5.6}{\left(\frac{(2)(1)^2}{12} + (2)(0.1)^2\right)}$$

$\Rightarrow \quad \alpha \cong 30 \text{ rads}^{-2}$

Now, $a_c = r\alpha = (0.1)\alpha$

$\Rightarrow \quad a_c = 3 \text{ ms}^{-2}$ {downwards}

Since $W + kx - F = ma_C$

$\Rightarrow \quad F = W + kx - ma_C$

$\Rightarrow \quad F = 20 + 6 - (2)(3)$

$\Rightarrow \quad F = 20$ N

31. $\omega_0 = 600 \text{ rpm} = 600 \times \dfrac{2\pi}{60} \text{ rads}^{-1}$

{revolutions per minute}

$\Rightarrow \quad \omega_0 = 20\pi \text{ rads}^{-1}$

Let α be the constant angular retardation, then

$$\omega = \omega_0 - \alpha t$$

$\Rightarrow \quad 0 = (20\pi) - 3(\alpha)$

$\Rightarrow \quad \alpha = \dfrac{20}{3}\pi \text{ rads}^{-2}$

Further, $\alpha = \dfrac{\tau}{I}$

If R be the radius of fly wheel and F be the tangential force exerted by the brake lining on fly wheel, then

$$\tau = \mu FR$$

where $I = \frac{1}{2}mR^2$

From the above equations, we get

$$\alpha = \frac{20}{3}\pi = \frac{\mu FR}{\frac{1}{2}mR^2} = \frac{2\mu F}{mR}$$

$$\Rightarrow \quad F = \frac{10\pi mR}{3\mu}$$

Substituting the values, we get

$$F = \frac{10 \times 22 \times 20 \times 0.12}{3 \times 7 \times 0.1}$$

$$\Rightarrow \quad F = 251.43 \text{ N}$$

32. By Law of Conservation of Angular Momentum, we get

$$I_1\omega_1 = I_2\omega_2$$

$$\Rightarrow \quad I_1\left(\frac{2\pi}{T_1}\right) = (I_1 + \Delta I)\left(\frac{2\pi}{T_2}\right)$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{I_1 + \Delta I}{I_1} = 1 + \frac{\Delta I}{I_1}$$

$$\Rightarrow \quad \Delta T = (T_2 - T_1) = T_1\left(\frac{\Delta I}{I_1}\right) \quad \ldots(1)$$

where, $I_1 = \frac{2}{5}MR^2$ and $\Delta I = \frac{2}{3}mR^2$

where, M is the mass of earth, m is the mass of ice. Substituting the values in equation (1), we get

$$\Delta T = \frac{(24 \times 3600)\left(\frac{2}{3} \times 3 \times 10^{19}\right)}{\left(\frac{2}{5} \times 6 \times 10^{24}\right)} = 0.7 \text{ s}$$

33. Since, $\vec{\tau} = (\vec{r}_2 - \vec{r}_1) \times \vec{F}$

$$\Rightarrow \quad \vec{\tau} = \left[(4\hat{i} + 3\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + \hat{k})\right] \times \vec{F}$$

$$\Rightarrow \quad \vec{\tau} = (3\hat{i} + \hat{j} - 2\hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\Rightarrow \quad \vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix} = 7\hat{i} - 11\hat{j} + 5\hat{k}$$

$$\Rightarrow \quad |\vec{\tau}| = \sqrt{195} \text{ Nm}$$

$$\Rightarrow \quad x = 195$$

34. (a) By Law of Conservation of Angular Momentum, we get

$$I_1\omega_2 = I_2\omega_2$$

where $\omega_1 = (0.5)(2\pi)$ rads^{-1}

$$I_1 = 1.6 + 2 \times 4 \times (0.9)^2 = 8.08 \text{ kgm}^2$$

$$I_2 = 1.6 + 2 \times 4 \times (0.15)^2 = 1.78 \text{ kgm}^2$$

and $\omega_1 = 0.5$ rads^{-1}

$$\Rightarrow \quad \omega_2 = \left(\frac{8.08}{1.78}\right)(0.5)(2\pi) = 14.3 \text{ rads}^{-1}$$

(b) $E_i = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}(8.08)(2\pi \times 0.5)^2 \approx 40$ J

$$E_f = \frac{1}{2}I_2\omega_2^2 = \frac{1}{2}(1.78)(2\pi \times 2.27)^2 = 181 \text{ J}$$

(c) $W = E_f - E_i \approx 141$ J

35. For equilibrium of cylinder in horizontal direction, we have

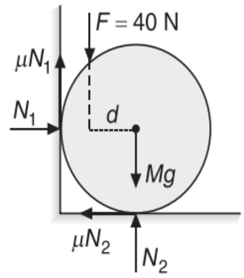
$$N_1 = \mu N_2 \quad \ldots(1)$$

For equilibrium of cylinder in vertical direction, we have

$$N_2 + \mu N_1 = F + Mg \quad \ldots(2)$$

Solving these two equations with $F = 40$ N, $M = 2$ kg and $\mu = \frac{1}{3}$, we get

$$N_1 = 18 \text{ N and } N_2 = 54 \text{ N}$$



Since, $Fd = \mu(N_1 + N_2)r$

$$\Rightarrow \quad d = \frac{\mu(N_1 + N_2)r}{F} = \frac{\left(\frac{1}{3}\right)(18 + 54)(0.1)}{40}$$

$$\Rightarrow \quad d = 0.06 \text{ m} = 6 \text{ cm}$$

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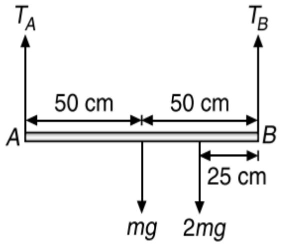
1. For equilibrium, $\tau_B = 0$ (torque about point B is zero)



$$T_A(100) - (mg)(50) - (2mg)(25) = 0$$

$\Rightarrow$ $100T_A = 100mg$

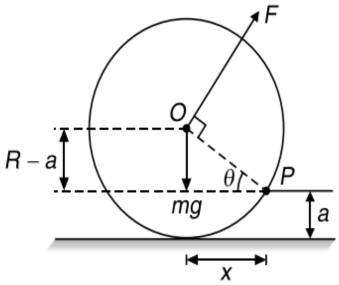
$\Rightarrow$ $T_A = 1mg$

Hence, the correct answer is (D).

2. For equilibrium, $(\tau)_P = 0$

$\Rightarrow$ $FR - mgx = 0$

$\Rightarrow$ $F = mg\left(\frac{x}{R}\right)$



Since, $x = \sqrt{R^2 - (R-a)^2}$

$\Rightarrow$ $F = mg\sqrt{1 - \left(\frac{R-a}{R}\right)^2}$

Hence, the correct answer is (D).

3. Since, both discs are rotating in same sense, so applying conservation of an angular momentum to the system, we get

$$L_{\text{initial}} = L_{\text{final}}$$

$\Rightarrow$ $I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f$

$\Rightarrow$ $(0.1)(10) + (0.2)(5) = (0.1 + 0.2)(\omega_f)$

$\Rightarrow$ $\omega_f = \frac{20}{3}$

Kinetic energy of combined disc system is

$$\text{RKE} = \frac{1}{2}(I_1 + I_2)\omega_f^2 = \frac{1}{2}(0.1 + 0.2)\left(\frac{20}{3}\right)^2$$

$\Rightarrow$ $\text{RKE} = \left(\frac{0.3}{2}\right)\left(\frac{400}{9}\right) = \frac{120}{18} = \frac{20}{3}\ \text{J}$

Hence, the correct answer is (D).

4. Angular momentum conservation gives

$$mv\ell = \left(\frac{M\ell^2}{3}\right)\omega + m\ell^2\omega$$

$\Rightarrow$ $\omega = \frac{(1)(6)(1)}{\frac{2}{3} + 1} = \frac{18}{5}\ \text{rads}^{-1}$

Energy conservation gives

$$\frac{1}{2}\left(\frac{M\ell^2}{3}\right)\omega^2 + \frac{1}{2}(m\ell^2)\omega^2 = (m+M)r_{\text{cm}}(1 - \cos\theta)$$

where, $r_{\text{cm}} = \frac{m\ell + (M\ell/2)}{m+M}$

$\Rightarrow$ $\frac{\ell^2\omega^2}{2}\left(\frac{M}{3} + m\right) = (m+M)\left(\frac{m\ell + \frac{M\ell}{2}}{m+M}\right)g(1 - \cos\theta)$

$\Rightarrow$ $\left(\frac{5}{6}\right)\left(\frac{18}{5}\right)^2 = 20(1 - \cos\theta)$

$\Rightarrow$ $1 - \cos\theta = \left(\frac{18}{5}\right)\left(\frac{3}{20}\right)$

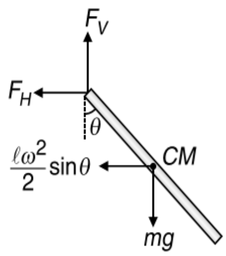
$\Rightarrow$ $\cos\theta = 1 - \frac{27}{50} = \frac{23}{50}$

$\Rightarrow$ $\theta \approx 63°$

Hence, the correct answer is (B).

5. From free body diagram, we see that $F_V = mg$ and

$$F_H = \frac{m\ell}{2}\omega^2\sin\theta$$



Net torque about CM due to F_V and F_H is

$$\tau_{\text{net}} = F_V\left(\frac{\ell}{2}\sin\theta\right) - F_H\left(\frac{\ell}{2}\cos\theta\right)$$

According to problem, $\tau_{\text{net}} = \frac{m\ell^2\omega^2}{12}\sin\theta\cos\theta$

$\Rightarrow$ $mg\left(\frac{\ell}{2}\sin\theta\right) - \left(m\omega^2\frac{\ell}{2}\sin\theta\right)\left(\frac{\ell}{2}\cos\theta\right) = \frac{m\ell^2\omega^2}{12}\sin\theta\cos\theta$

$\Rightarrow$ $\cos\theta = \frac{3g}{2\omega^2\ell}$ ...(2)

Hence, the correct answer is (B).

6. $I_1 = \dfrac{MR^2}{2} = \dfrac{\rho(\pi R^2)tR^2}{2}$

$\Rightarrow \quad I \propto R^4$

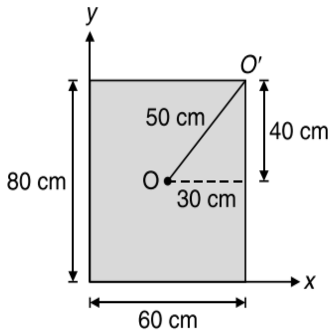
$\Rightarrow \quad \dfrac{I_1}{I_2} = \dfrac{R_1^4}{R_2^4} = \dfrac{1}{16}$

$\Rightarrow \quad \dfrac{R_1}{R_2} = \dfrac{1}{2}$

Hence, the correct answer is (B).

7. Moment of inertia about O is

$$I_O = \frac{M}{12}(L^2 + B^2) = \frac{M}{12}(80^2 + 60^2)$$



Moment of inertia about O' is obtained by applying parallel axis theorem, so we have

$$I_{O'} = I_0 + Md^2$$

$$\Rightarrow \quad I_{O'} = \frac{M}{12}(80^2 + 60^2) + M(50)^2$$

$$\Rightarrow \quad \frac{I_O}{I_{O'}} = \frac{\frac{M}{12}(80^2 + 60^2)}{\frac{M}{12}(80^2 + 60^2) + M(50)^2} = \frac{1}{4}$$

Hence, the correct answer is (B).

8. Loss in KE is given by

$$\text{Loss} = -\Delta K = \frac{1}{2}\left(\frac{I_1 I_2}{I_1 + I_2}\right)(\omega_1 - \omega_2)^2$$

$$\Rightarrow \quad \text{Loss} = \frac{1}{2}\left[\frac{(I)(3I)}{4I}\right]\omega^2$$

$$\Rightarrow \quad \text{Loss} = \left(\frac{1}{2}I\omega^2\right)\frac{3}{4}$$

$$\Rightarrow \quad \frac{\text{Loss}}{K_i} = \frac{3}{4}$$

Hence, the correct answer is (C).

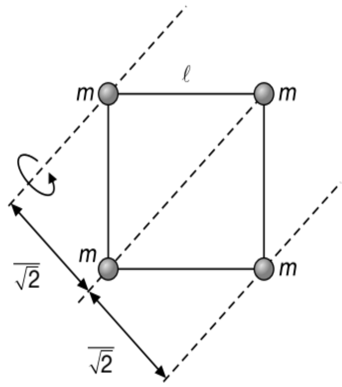
9. Moment of inertia of the cone about the specified axis is

$$I = \frac{MR^2}{2}$$

Hence, the correct answer is (A).

10. The moment of inertia of system about the specified axis is

$$I = m(0)^2 + (2)m\left(\frac{\ell}{\sqrt{2}}\right)^2 + m\left(\sqrt{2}\ell\right)^2$$



$$\Rightarrow \quad I = \frac{2m\ell^2}{2} + 2m\ell^2 = 3m\ell^2$$

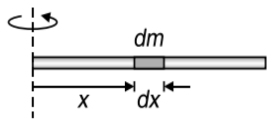
Angular momentum of system about the specified axis is

$$L = I\omega = (3m\ell^2)\omega$$

Hence, the correct answer is (B).

11. Consider an infinitesimal element of mass dm, length dx at a distance x from the specified axis of rotation, then

$$I = \int r^2 dm = \int x^2 \lambda dx$$



$$\Rightarrow \quad I = \int_0^L x^2 \lambda_0 \left(1 + \frac{x}{L}\right) dx$$

$$\Rightarrow \quad I = \lambda_0 \int_0^L \left(x^2 + \frac{x^3}{L}\right) dx = \lambda\left(\frac{L^3}{3} + \frac{L^3}{4}\right)$$

$$\Rightarrow \quad I = \frac{7L^3\lambda_0}{12} \qquad \text{...(1)}$$

Since $M = \int_0^L \lambda dx = \int_0^L \lambda_0 \left(1 + \frac{x}{L}\right) dx$

$$\Rightarrow \quad M = \lambda_0 \left(L + \frac{L}{2}\right) = \frac{3\lambda_0 L}{2}$$

$$\Rightarrow \quad \lambda_0 L = \frac{2}{3}M \qquad \text{...(2)}$$

From (1) and (2), we get

$$I = \frac{7}{12}\left(\frac{2}{3}M\right)L^2 = \frac{7ML^2}{18}$$

Hence, the correct answer is (D).

12. $mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v^2}{r^2}\right) = \frac{3}{4}mv^2$

$\Rightarrow \quad u = \sqrt{\frac{4}{3}gh}$

$\Rightarrow \quad \omega = \frac{u}{r} = \frac{1}{r}\sqrt{\frac{4gh}{3}}$

Hence, the correct answer is (C).

13. $I = I_{cm} + md^2$

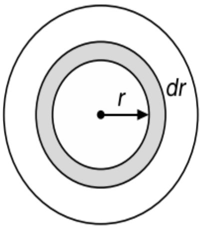
$\Rightarrow \quad m\frac{l^2}{12} + m\frac{l^2}{16} = mk^2$

$\Rightarrow \quad \frac{7l^2}{48} = k^2$

$\Rightarrow \quad k = l\sqrt{\frac{7}{48}}$

Hence, the correct answer is (B).

14. Since $dI = r^2 dm$



$\Rightarrow \quad dI = \sigma(2\pi r dr)r^2$

$\Rightarrow \quad dI = 2\pi(A + Br)r^3 dr$

$\Rightarrow \quad \int dI = 2\pi\int_0^a (Ar^3 + Br^4)dr$

$\Rightarrow \quad I = 2\pi a^4\left(\frac{A}{4} + \frac{aB}{5}\right)$

Hence, the correct answer is (A).

15. Rotational KE is $\text{RKE} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$\Rightarrow \quad \text{RKE} = \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right) = 8.75 \times 10^{-4}\ \text{J}$

Hence, the correct answer is (A).

16. From parallel axis theorem, we get

$$I_0 = 3 \times \left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + M\left(\frac{d}{\sqrt{3}}\right)^2\right] = \frac{13}{10}Md^2$$

Now, $I_A = I_0 + 3M\left(\frac{d}{\sqrt{3}}\right)^2 = \frac{13}{10}Md^2 + Md^2$

$\Rightarrow \quad I_A = \frac{23}{10}Md^2$

$\Rightarrow \quad \frac{I_0}{I_A} = \frac{13}{23}$

Hence, the correct answer is (A).

17. Since $a = \frac{(m_1 - m_2)g}{m_1 + m_2 + \frac{I}{R^2}} = \frac{(m_1 - m_2)gR^2}{(m_1 + m_2)R^2 + I}$

$\Rightarrow \quad v = \sqrt{2ah}\sqrt{2\left[\frac{(m_1 - m_2)gR^2}{(m_1 + m_2)R^2 + J}\right]h}$

$\Rightarrow \quad \omega = \frac{v}{R} = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}}$

Hence, the correct answer is (B).

18. Mass of plate is

$$M = \int_0^R \rho_0 r(2\pi r dr) = \frac{2\pi\rho_0 R^3}{3}$$

$\Rightarrow \quad I_C = \int (dm)r^2 = \int_0^R \rho_0 r(2\pi r dr)r^2 = \frac{2\pi\rho_0 R^5}{5}$

$\Rightarrow \quad I = I_C + MR^2 = 2\pi\rho_0 R^5\left(\frac{1}{3} + \frac{1}{5}\right) = \frac{16\pi\rho_0 R^5}{15}$

$\Rightarrow \quad I = \frac{8}{5}\left(\frac{2}{3}\pi\rho_0 R^3\right)R^2 = \frac{8}{5}MR^2$

Hence, the correct answer is (D).

19. If α be initial angular acceleration (before it slips off), then

$$mg\left(\frac{\ell}{2}\right) = \text{I}\alpha = \left(\frac{m\ell^2}{3}\right)\alpha$$

$\Rightarrow \quad \alpha = \frac{3g}{2\ell}$

Angular speed acquire by the box in time $\tau = 0.01$ s is

$$\omega = \alpha t = \left(\frac{3g}{2\ell}\right)(0.01) = \frac{3 \times 10 \times 0.01}{2 \times 0.3} = \frac{1}{2}\ \text{rads}^{-1}$$

The angle by which it would rotate when it hits the ground is $\theta = \omega t'$, where, $\omega = \text{constant}$ and $t' = \text{time of fall} = \sqrt{\frac{2H}{g}} = 1$ sec

$\Rightarrow \quad \theta = \frac{1}{2}$ radian

Hence, the correct answer is (D).

20. For sphere, we have $mgh_{sph} = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$

$\Rightarrow \quad mgh_{sph} = \frac{7}{10}mv^2$ ...(1)

For cylinder, we have

$$mgh_{\text{cylinder}}=\frac{1}{2}mv^2+\frac{1}{2}\left(\frac{mR^2}{2}\right)\left(\frac{v}{R}\right)^2=\frac{3}{4}mv^2$$

$$\Rightarrow \quad \frac{h_{\text{sph}}}{h_{\text{cylinder}}}=\frac{7\times4}{10\times3}=\frac{14}{15}$$

Hence, the correct answer is (D).

21. Since, $\tau=\dfrac{dE}{d\theta}$

$$\Rightarrow \quad 2k\theta=I\alpha$$

$$\Rightarrow \quad \alpha=\frac{2k\theta}{I}$$

Hence, the correct answer is (A).

22. Since, $mgh=\dfrac{1}{2}I_p\omega^2$

For ring, $h_1=\dfrac{1}{2}\dfrac{(2mr'^2)}{mg}\dfrac{v^2}{(r')^2}=\dfrac{v^2}{g}$

For cylinder, $h_2=\dfrac{1}{2}\left(\dfrac{3}{2}mr'^2\right)\dfrac{v^2}{(r')^2}=\dfrac{3}{4}\left(\dfrac{v^2}{g}\right)$

For sphere, $h_3=\dfrac{1}{2}\times\dfrac{\frac{7}{5}m(r')^2}{g}\dfrac{v^2}{(r')^2}=\dfrac{7}{10}\left(\dfrac{v^2}{g}\right)$

$$\Rightarrow \quad h_1:h_2:h_3=2:\frac{3}{2}:\frac{14}{10}=20:15:14$$

No given option is correct.

23. Applying conservation of angular momentum, we get

$$\frac{ML^2}{12}\omega_0=\left(\frac{ML^2}{12}+2\frac{mL^2}{4}\right)\omega$$

$$\Rightarrow \quad \omega=\frac{M\omega_0}{M+6m}$$

Hence, the correct answer is (C).

24. Since, $\omega=\alpha t$

Also, $E=\dfrac{1}{2}I\omega^2=1200$

$$\Rightarrow \quad \frac{1}{2}\times1.5(20t)^2=1200\text{ J}$$

$$\Rightarrow \quad t=2\text{ s}$$

Hence, the correct answer is (B).

25. Applying conservation of angular momentum, we get

$$(I_1+I_2)\omega_{\text{common}}=I_1\omega_1+I_2\omega_2$$

$$\Rightarrow \quad \omega_{\text{common}}=\omega_c=\frac{I_1\omega_1+\frac{I_1\omega_1}{4}}{I_1+\frac{I_1}{2}}=\left(\frac{5}{4}\times\frac{2}{3}\right)\omega_1$$

$$\Rightarrow \quad \omega_c=\frac{5\omega_1}{6}$$

Loss in $KE=\left(\dfrac{1}{2}I_1\omega_1^2+\dfrac{1}{2}I_2\omega_2^2\right)-\dfrac{1}{2}(I_1+I_2)\omega_c^2$

$$\Rightarrow \quad \Delta KE=-\frac{I_1\omega_1^2}{24}$$

Also, loss in kinetic energy is

$$\text{Loss}=-\Delta K=\frac{1}{2}\left(\frac{I_1I_2}{I_1+I_2}\right)(\omega_1-\omega_2)^2$$

$$\Rightarrow \quad -\Delta K=\frac{1}{2}\left(\frac{I_1}{3}\right)\left(\frac{\omega_1^2}{4}\right)=\frac{I_1\omega_1^2}{24}$$

$$\Rightarrow \quad \Delta E=\Delta K=-\frac{I_1\omega_1^2}{24}$$

Hence, the correct answer is (D).

26. Since, $x=x_0+a\cos\omega_1t$ and $y=y_0+b\sin\omega_2t$

$$\Rightarrow \quad v_x=-a\omega_1\sin(\omega_1t),\ v_y=b\omega_2\cos(\omega_2t)$$

$$\Rightarrow \quad a_x=-a\omega_1^2\cos(\omega_1t),\ a_y=-b\omega_2^2\sin(\omega_2t)$$

At $t=0$, $x=x_0+a$, $y=y_0$

$$a_x=-a\omega_1^2\text{ and }a_y=0$$

$$\Rightarrow \quad \vec{\tau}=m(-a\omega_1^2)\times y_0(-\hat{k})$$

$$\Rightarrow \quad \vec{\tau}=(my_0a\omega_1^2)\hat{k}$$

Hence, the correct answer is (C).

27. Surface mass density is $\sigma=kr^2$

Mass of disc, $M=\displaystyle\int_0^R(kr^2)2\pi r dr$

$$\Rightarrow \quad M=2\pi k\frac{R^4}{4}=\frac{\pi kR^4}{2}$$

Moment of inertia of the disc about the axis is

$$I=\int dI=\int(dm)r^2=\int\sigma dAr^2$$

$$\Rightarrow \quad I=\int(Kr^2)(2\pi rdr)r^2$$

$$\Rightarrow \quad I=\int_0^R 2\pi kr^5dr=\frac{\pi kR^6}{3}=\frac{2}{3}MR^2$$

Hence, the correct answer is (D).

28. Since, $\vec{r}=2t\hat{i}-3t^2\hat{j}$

$$\Rightarrow \quad \vec{v}=\frac{d\vec{r}}{dt}=2\hat{i}-6t\hat{j}$$

At $t=2$ s, $\vec{r}=4\hat{i}-12\hat{j}$ and $\vec{v}=2\hat{i}-12\hat{j}$

$\Rightarrow \quad \vec{L} = m(\vec{r} \times \vec{v}) = 2(4\hat{i} - 12\hat{j}) \times (2\hat{i} - 12\hat{j}) = -48\hat{k}$

Hence, the correct answer is (D).

29. Since $\tau = I\alpha$ and $\omega = \omega_0 + \alpha t$

$\Rightarrow \quad 25 \times 2\pi = (\alpha)5$

$\Rightarrow \quad \alpha = 10\pi$

$\Rightarrow \quad \tau = \left(\frac{5}{4}mR^2\right)\alpha \approx \left(\frac{5}{4}\right)(5 \times 10^{-3})(10^{-4})10\pi$

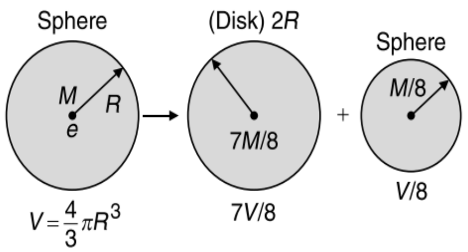
$\Rightarrow \quad \tau = 2.0 \times 10^{-5}$ Nm

Hence, the correct answer is (C).

30. For small sphere, $\frac{4}{3}\pi r^3 \rho = \frac{M}{8}$ $\quad \left\{\because r = \left(\frac{R}{2}\right)\right\}$

For sphere, $I_1 = \left(\frac{7M}{8}\right)(2R)^2 \frac{1}{2} = \frac{14}{8}MR^2$

For disc, $I_2 = \frac{2}{5}\left(\frac{M}{8}\right)r^2$



$\Rightarrow \quad I_2 = \frac{2}{5}\left(\frac{M}{8}\right)\left(\frac{R^2}{4}\right) = \frac{MR^2}{80}$

$\Rightarrow \quad \frac{I_1}{I_2} = \frac{14 \times 80}{(8)} = 140$

Hence, the correct answer is (A).

31. Since, $\sigma = \frac{\sigma_0}{r}$

$\Rightarrow \quad dm = \frac{\sigma_0}{r} 2\pi r dr$

$\Rightarrow \quad m = \int dm = \sigma_0 2\pi (b - a)$

$\Rightarrow \quad I = 2\pi\sigma_0 \int_a^b r^2 dr = \frac{2\pi\sigma_0}{3}(b^3 - a^3)$

Since, $I = mk^2$

$\Rightarrow \quad 2\pi\sigma_0 (b - a)k^2 = \frac{2\pi\sigma_0}{3}(b^3 - a^3)$

$\Rightarrow \quad k^2 = \frac{1}{3}(b^2 + ab + a^2)$

$\Rightarrow \quad k = \sqrt{\frac{1}{3}\frac{(b^3 - a^3)}{b - a}}$

Hence, the correct answer is (D).

32. Applying conservation of mechanical energy, we get

$$mg\left(\frac{\ell}{2}\sin 30^\circ\right) = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$$

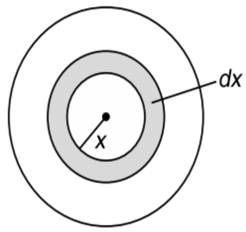
$\Rightarrow \quad \omega^2 = \frac{3g}{2\ell} = \frac{30}{1}$

$\Rightarrow \quad \omega = \sqrt{30}$ rads^{-1}

Hence, the correct answer is (B).

33. The force per unit area P is

$$P = \frac{F}{\pi R^2}$$



Torque due to frictional force is

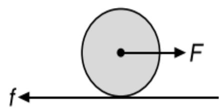
$$\tau = \mu \int (dN)x = \frac{\mu F}{\pi R^2} \int_0^R 2\pi x^2 dx$$

$\Rightarrow \quad \tau = \frac{\mu F \times 2\pi R^3}{3\pi R^2} = \frac{2}{3}\mu FR$

Hence, the correct answer is (D).

34. Since, $F - f = Ma$

Also, $fR = \frac{MR^2}{2}\frac{a}{R}$



$\Rightarrow \quad f = \frac{Ma}{2}$

$\Rightarrow \quad F = \frac{3Ma}{2}$

$\Rightarrow \quad a = \frac{2F}{3M}$

$\Rightarrow \quad \alpha = \frac{a}{R} = \frac{2F}{3MR}$

Hence, the correct answer is (B).

35. Since, $\tau = I_p \alpha$

$\Rightarrow \quad 5M_0 g\ell - 4M_0 g\ell = \left[2M_0(2\ell)^2 + 5M_0\ell^2\right]\alpha$

$\Rightarrow \quad M_0 g\ell = 13M_0\ell^2\alpha$

$$\Rightarrow \quad \alpha = \frac{g}{13\ell}$$

Hence, the correct answer is (C).

36. Since, $I = I_{\text{spheres}} + I_{\text{rod}}$

where, $I_{\text{rod}} = \frac{M \times 4R^2}{12} = \frac{MR^2}{3}$

$$I_{\text{spheres}} = 2\left(\frac{2}{5}MR^2 + 4MR^2\right) = \frac{44}{5}MR^2$$

$$\Rightarrow \quad I = \left(\frac{44}{5} + \frac{1}{3}\right)MR^2$$

$$\Rightarrow \quad I = \frac{137}{15}MR^2$$

Hence, the correct answer is (D).

37. Let $I_0 = KML^2$

$$\Rightarrow \quad I_1 = K\frac{M}{4}\left(\frac{L}{2}\right)^2 = K\frac{ML^2}{16}$$

So, $I = I_0 - I_1$

$$\Rightarrow \quad I = \frac{15}{16}KML^2 = \frac{15}{16}I_0$$

Hence, the correct answer is (B).

38. Since, $\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2$

where, $\vec{\tau}_1 = (2\hat{i} + 3\hat{j}) \times F\hat{k} = F(3\hat{i} - 2\hat{j})$

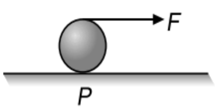
$$\vec{\tau}_2 = 6\hat{j} \times F(-\sin 30°\hat{i} - \cos 30°\hat{j}) = 3F\hat{k}$$

$$\Rightarrow \quad \vec{\tau} = F(3\hat{i} - 2\hat{j} + 3\hat{k})$$

Hence, the correct answer is (A).

39. Since $\tau_p = I_p\alpha$

$$\Rightarrow \quad F(2R) = 2MR^2\alpha$$



$$\Rightarrow \quad \alpha = \frac{F}{MR} = \frac{40}{0.5 \times 5}$$

$$\Rightarrow \quad \alpha = 16 \text{ rads}^{-2}$$

Hence, the correct answer is (A).

40. Since $\tau = Fr\sin\theta$

$\Rightarrow \quad 2.5 = 1 \times 5\sin\theta$

$\Rightarrow \quad \sin\theta = 0.5$

$$\Rightarrow \quad \theta = \frac{\pi}{6}$$

Hence, the correct answer is (B).

41. Since $I = I_1 + I_2 + I_3$

$$\Rightarrow \quad I = \frac{MR^2}{2} + 2\left(\frac{MR^2}{4} + MR^2\right) = 3MR^2$$

Hence, the correct answer is (A).

42. Since, $\frac{M}{2}(R_1^2 + R_2^2) = MR^2$

$$\Rightarrow \quad R = \sqrt{\frac{R_1^2 + R_2^2}{2}} = \sqrt{\frac{100 + 400}{2}} = \sqrt{250}$$

$\Rightarrow \quad R \approx 16$ cm

Hence, the correct answer is (C).

43. According to parallel axis theorem, we have

$$I(x) = I_0 + mx^2$$

Hence, the correct answer is (C).

44. Since, $L = mv_0r$

Also, $\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + mgh$

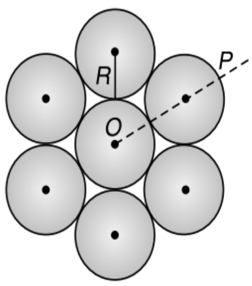
$\Rightarrow \quad v_0^2 = 25 + 2 \times 10 \times 10 = 225$

$\Rightarrow \quad v_0 = 15 \text{ ms}^{-1}$

$\Rightarrow \quad L = 20 \times 10^{-3} \times 15 \times 20 = 6 \text{ kgm}^2\text{s}^{-1}$

Hence, the correct answer is (D).

45. Moment of inertia of one of the outer disc about an axis passing through point O and perpendicular to the plane



$$I_1 = \frac{1}{2}MR^2 + M(2R)^2 = \frac{9}{2}MR^2$$

Moment of inertia of the system about point O,

$$I_O = \frac{1}{2}MR^2 + 6I_1 = \frac{1}{2}MR^2 + 6\left(\frac{9}{2}MR^2\right) = \frac{55}{2}MR^2$$

Required moment of inertia of the system about point P is

$$I_P = I_O + 7M(3R)^2 = \frac{55}{2}MR^2 + 63MR^2 = \frac{181}{2}MR^2$$

Hence, the correct answer is (D).

46. Mass per unit area of disc is $\sigma = \dfrac{9M}{\pi R^2}$

Mass of removed portion is $M' = \dfrac{9M}{\pi R^2} \times \pi \left(\dfrac{R}{3}\right)^2 = M$

Let moment of inertia of removed portion be I_1

$$\Rightarrow \quad I_1 = \frac{M}{2}\left(\frac{R}{3}\right)^2 + M\left(\frac{2R}{3}\right)^3 = \frac{MR^2}{2}$$

Let I_2 be the moment of inertia of the whole disc, so

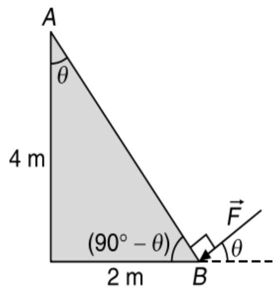
$$I_2 = \frac{9MR^2}{2}$$

Moment of inertia of remaining disc is $I = I_2 - I_1$

$$\Rightarrow \quad I = \frac{9MR^2}{2} - \frac{MR^2}{2} = \frac{8MR^2}{2} = 4MR^2$$

Hence, the correct answer is (D).

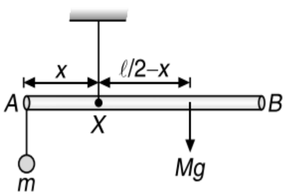
47. Moment of force will be maximum when line of action of force is perpendicular to line AB.



$$\tan\theta = \frac{2}{4} = \frac{1}{2}$$

Hence, the correct answer is (A).

48.



Balancing torque about point of suspension X, we get

$$mgx = Mg\left(\frac{\ell}{2} - x\right)$$

$$\Rightarrow \quad mx = M\frac{\ell}{2} - Mx$$

$$\Rightarrow \quad m = \left(M\frac{\ell}{2}\right)\frac{1}{x} - M$$

This is equation of straight line with variables m and $\dfrac{1}{x}$.

Hence, the correct answer is (D).

49. As coin is at rest on rotating disc, centripetal force is provided by the friction force between the coin and disc.

$$f = m\omega^2 R$$

$$\Rightarrow \quad \mu mg = m\omega^2 r$$

$$\Rightarrow \quad \mu = \frac{\omega^2 r}{g} = \frac{(2\pi v)^2 r}{g}$$

$$\Rightarrow \quad \mu = \frac{4\pi^2 (3.5)^2 \times 1.25 \times 10^{-2}}{10} = 604 \times 10^{-3} \approx 0.6$$

Hence, the correct answer is (D).

50. Applying conservation of energy, we get

$$mgh = mg\ell\sin\alpha = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$$

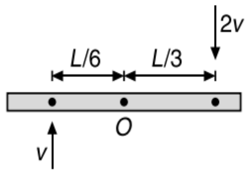
$$\Rightarrow \quad 6g\ell\sin\alpha = v^2$$

$$\Rightarrow \quad v = \sqrt{6g\ell\sin\alpha}$$

$$\Rightarrow \quad v \propto \sqrt{\sin\alpha}$$

Hence, the correct answer is (D).

51. Applying law of conservation of angular momentum, i.e. $L_i = L_f$, we get



$$m(2v)\left(\frac{L}{3}\right) + 2m(v)\left(\frac{L}{6}\right) = I\omega$$

$$\Rightarrow \quad mvL = \left[\frac{1}{12}(8m)L^2 + m\left(\frac{L}{3}\right)^2 + 2m\left(\frac{L}{6}\right)^2\right]\omega$$

$$\Rightarrow \quad v = L\left(\frac{2}{3} + \frac{1}{9} + \frac{1}{18}\right)\omega = \frac{5}{6}\omega L$$

$$\Rightarrow \quad \omega = \frac{6v}{5L}$$

Hence, the correct answer is (A).

52. Moment of inertia about z-axis is $I_z = \dfrac{mR^2}{2}$

Moment of inertia about z' axis is $I_{z'} = I_z + mR^2 = \dfrac{3}{2}mR^2$

$$\Rightarrow \quad I_z : I_{z'} = 1:3$$

Hence, the correct answer is (B).

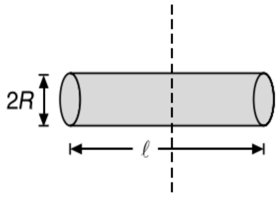
53. Since, $I = \dfrac{mR^2}{4} + \dfrac{m\ell^2}{12}$

$$\Rightarrow \quad I = \frac{m}{4}\left(R^2 + \frac{\ell^2}{3}\right)$$

If V be volume of cylinder, then $V = (\pi R^2)\ell$

$$\Rightarrow \quad I = \frac{m}{4}\left(\frac{V}{\pi \ell} + \frac{\ell^2}{3}\right)$$

For I to be minimum, we have



$$\frac{dI}{d\ell} = \frac{m}{4}\left(\frac{-V}{\pi \ell^2} + \frac{2\ell}{3}\right) = 0$$

$$\Rightarrow \quad \frac{V}{\pi \ell^2} = \frac{2\ell}{3}$$

$$\Rightarrow \quad V = \frac{2\pi \ell^3}{3}$$

$$\Rightarrow \quad \pi R^2 \ell = \frac{2\pi \ell^3}{3}$$

$$\Rightarrow \quad \frac{\ell^2}{R^2} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{\ell}{R} = \sqrt{\frac{3}{2}}$$

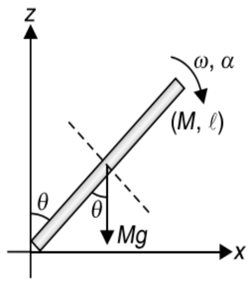
Hence, the correct answer is (D).

54. Torque at angle θ is

$$\tau = F r_{\perp} = Mg\left(\frac{\ell}{2}\sin\theta\right)$$

Since, $\tau = I\alpha$

$$\Rightarrow \quad I\alpha = \frac{Mg\ell}{2}\sin\theta, \text{ where } I = \frac{M\ell^2}{3}$$



$$\Rightarrow \quad \left(\frac{M\ell^2}{3}\right)\alpha = \frac{Mg\ell}{2}\sin\theta$$

$$\Rightarrow \quad \frac{\ell\alpha}{3} = g\left(\frac{\sin\theta}{2}\right)$$

$$\Rightarrow \quad \alpha = \frac{3g\sin\theta}{2\ell}$$

Hence, the correct answer is (A).

55. From figure, we conclude

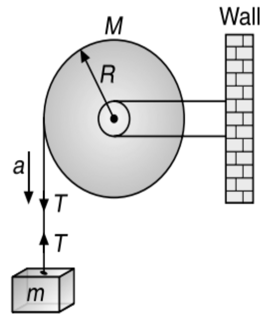
$$mg - T = ma \qquad \ldots(1)$$

Moment of inertia of a uniform disc is $I = \frac{MR^2}{2}$ and an acceleration is $a = \alpha R$

Since, $\tau = TR = I\alpha$

$$\Rightarrow \quad TR = \left(\frac{MR^2}{2}\right)\left(\frac{a}{R}\right)$$

$$\Rightarrow \quad T = \frac{Ma}{2}$$



Substituting this value in equation (1)

$$mg - \frac{Ma}{2} = ma$$

$$\Rightarrow \quad mg = a\left(m + \frac{M}{2}\right)$$

$$\Rightarrow \quad a = \frac{2mg}{M + 2m}$$

Hence, the correct answer is (D).

56. $W_1 \ell_1 = W_2 \ell_2$, i.e. $W \propto \frac{1}{\ell}$

Hence, the correct answer is (A).

57. Moment of inertia of the disc about the given axis

$$I_D = \frac{MR^2}{2}$$

Mass of removed portion is $M' = \frac{M}{\pi R^2} \times \pi\left(\frac{R}{4}\right)^2 = \frac{M}{16}$

Moment of inertia of removed portion about the given axis, using parallel axes theorem is

$$I_R = \frac{1}{2}\left(\frac{M}{16}\right)\left(\frac{R^2}{16}\right) + \left(\frac{M}{16}\right)\left(\frac{9R^2}{16}\right) = \frac{19MR^2}{512}$$

Required moment of inertia

$$I = I_D - I_R = \frac{1}{2}MR^2 - \frac{19MR^2}{512} = \frac{237}{512}MR^2$$

Hence, the correct answer is (D).

58. Since, $\vec{L} = \vec{r} \times \vec{p} = rp\sin\theta\hat{n}$

$\Rightarrow \quad \vec{L} = pr_{\perp}\hat{n}$

For D to A

$$\vec{L} = \frac{R}{\sqrt{2}}mv(-\hat{k})$$

For A to B

$$\vec{L} = \frac{R}{\sqrt{2}}mv(-\hat{k})$$

For C to D

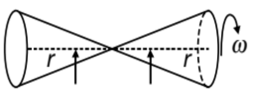
$$\vec{L} = \left(\frac{R}{\sqrt{2}} + a\right)mv(\hat{k})$$

For B to C

$$\vec{L} = \left(\frac{R}{\sqrt{2}} + a\right)mv(\hat{k})$$

Hence, (A) and (C) are correct.

59. Let the distance of central line from instantaneous axis of rotation be r, then r from the point on left becomes lesser than that for right.



So, v_{left} point $= \omega r' < \omega r = v_{\text{right}}$ point

So, the roller will turn to left.

Hence, the correct answer is (D).

60. Since no external torque acts on the system, therefore total angular momentum of the system about point O remains constant. So, $L_i = L_f$

$$\Rightarrow \quad mv\left(\frac{a}{2}\right) = I\omega$$

$$\Rightarrow \quad \omega = \frac{mva}{2I}$$

where, I is the moment of inertia of cube about its edge

$$\Rightarrow \quad I = m\frac{a^2}{6} + m\left(\frac{\sqrt{2}a}{2}\right)^2 = \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{2ma^2}{3}$$

$$\Rightarrow \quad \omega = \frac{mva \times 3}{2 \times 2ma^2} = \frac{3v}{4a} = \frac{3 \times 2}{4 \times 0.3} = 5 \text{ rads}^{-1}$$

Hence, the correct answer is (D).

61. For just one complete rotation, speed of the drum at top position,

$v = \sqrt{Rg}$, where $R = 1.25$ m

Angular velocity of the drum is

$$\omega = \frac{v}{R} = \sqrt{\frac{g}{R}}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{10}{1.25}} \text{ rads}^{-1} = \frac{60}{2\pi}\sqrt{\frac{10}{1.25}} \text{ rpm} = 27 \text{ rpm}$$

Hence, the correct answer is (A).

62. (Diameter of Sphere) = (Diagonal of the Cube)

$$\Rightarrow \quad 2R = \sqrt{3}a$$

$$\Rightarrow \quad a = \frac{2R}{\sqrt{3}}$$

Mass of Cube $= m = \rho a^3 = \left(\dfrac{M}{\frac{4}{3}\pi R^3}\right)a^3$

$$\Rightarrow \quad m = \frac{3M}{4\pi R^3}\left(\frac{2R}{\sqrt{3}}\right)^3$$

$$\Rightarrow \quad m = \frac{2M}{\sqrt{3}\pi}$$

Since MI of cube is $I = \frac{1}{6}ma^2$

$$\Rightarrow \quad I = \frac{4MR^2}{9\sqrt{3}\pi}$$

Hence, the correct answer is (C).

63. Equation of motion for solid cylinder is

$$F - f = ma \quad \ldots(1)$$

Also, $\tau = fR = I\alpha$

For pure rolling, we have $a = \alpha R$

$$\Rightarrow \quad fR = \frac{mR^2}{2} \cdot \frac{a}{R}$$

$$\Rightarrow \quad f = \frac{ma}{2} \quad \ldots(2)$$

From equations (1) and (2), we get

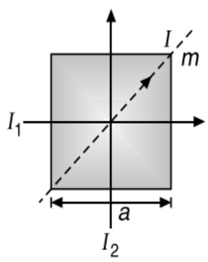
$$F - \frac{ma}{2} = ma$$

$$\Rightarrow \quad F = \frac{3}{2}ma$$

Hence, the correct answer is (C).

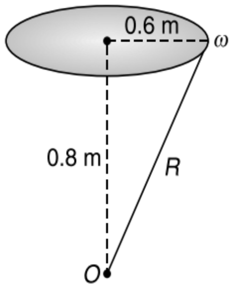
64. For a thin uniform square sheet

$$I_1 = I_2 = I = \frac{ma^2}{12}$$



Hence, the correct answer is (C).

65. $R=\sqrt{(0.8)^2+(0.6)^2}=1\text{ m},\ v=r\omega=(0.6)\omega$



Angular momentum of the particle about point O is

$$L=mvR\sin 90^\circ=m(r\omega)R$$

$$\Rightarrow\ L=(2)(0.6)(12)(1)=14.4\text{ kgm}^2\text{s}^{-1}$$

Hence, the correct answer is (D).

66. In this case L changes in direction but not in magnitude.

Hence, the correct answer is (C).

67. Since, $mg-T=ma$ ...(1)

Also, $\tau=TR=I\alpha$

$$\Rightarrow\ TR=(mR^2)\left(\frac{a}{R}\right)$$

$$\Rightarrow\ T=ma \quad \text{...(2)}$$

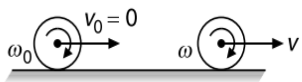
Substituting (2) in (1), we get

$$mg-ma=ma$$

$$\Rightarrow\ a=\frac{g}{2}$$

Hence, the correct answer is (B).

68. Applying conservation of angular momentum about a point on ground



$$mr^2\omega_r=mr^2\omega+mvr$$

$$\Rightarrow\ mr^2\omega_0=2mvr$$

$$\Rightarrow\ v=\frac{r\omega_0}{2}$$

Hence, the correct answer is (C).

69. Torque exerted on pulley is $\tau=FR$

$$\Rightarrow\ \alpha=\frac{FR}{I} \qquad \left\{\because\ \alpha=\frac{\tau}{I}\right\}$$

Given that, $F=(20t-5t^2)$, $R=2$ m, $I=10$ kgm^{-2}

$$\Rightarrow\ \alpha=\frac{(20t-5t^2)\times 2}{10}$$

$$\Rightarrow\ \alpha=(4t-t^2)$$

$$\Rightarrow\ \frac{d\omega}{dt}=(4t-t^2)dt$$

$$\Rightarrow\ d\omega=(4t-t^2)dt$$

On integrating, we get

$$\omega=2t^2-\frac{t^3}{3}$$

At direction of reversal of motion, $\omega=0$

$$\Rightarrow\ t=6\text{ s}$$

Since, $\omega=\dfrac{d\theta}{dt}=2t^2-\dfrac{t^3}{3}$

$$\Rightarrow\ d\theta=\left(2t^2-\frac{t^3}{3}\right)dt$$

On integrating, we get

$$\theta=\frac{2t^3}{3}-\frac{t^4}{12}$$

At, $t=6$ s, $\theta=36$ rad

$$\Rightarrow\ 2\pi n=36$$

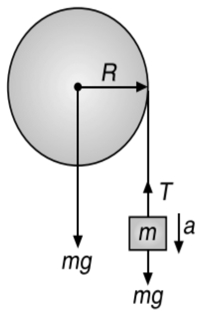
$$\Rightarrow\ n=\frac{36}{2\pi}<6$$

Hence, the correct answer is (B).

70. The free body diagram of pulley and mass

$$mg-T=ma$$

$$\Rightarrow\ a=\frac{mg-T}{m} \quad \text{...(1)}$$



As per question, pulley to be consider as a circular disc. So, angular acceleration of disc is

$$\alpha=\frac{\tau}{I}=\frac{TR}{I} \quad \text{...(2)}$$

where, $I=\dfrac{1}{2}mR^2$ {for circular disc}

$$\Rightarrow\ T=\frac{mR\alpha}{2} \quad \text{\{using (2)\}}$$

$$\Rightarrow\ a=\frac{mg-\dfrac{mR\alpha}{2}}{m} \quad \text{\{using (1)\}}$$

$$\Rightarrow \quad ma = mg - \frac{ma}{2} \qquad \left\{ \because \ \alpha = \frac{a}{R} \right\}$$

$$\Rightarrow \quad a = \frac{2g}{3}$$

Hence, the correct answer is (C).

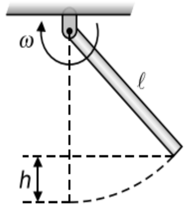
71. By conservation of angular momentum, $I\omega = \text{constant}$

Since I firstly decreases and then increases

So, L first increases and then decreases

Hence, the correct answer is (D).

72. The uniform rod of length ℓ and mass m is swinging about an axis passing through the end. When the centre of mass is raised through h, the increase in potential energy is mgh. This is equal to the kinetic energy $\frac{1}{2}I\omega^2$.



$$\Rightarrow \quad mgh = \frac{1}{2}\left(m\frac{\ell^2}{3}\right)\omega^2$$

$$\Rightarrow \quad h = \frac{\ell^2\omega^2}{6g}$$

Hence, the correct answer is (D).

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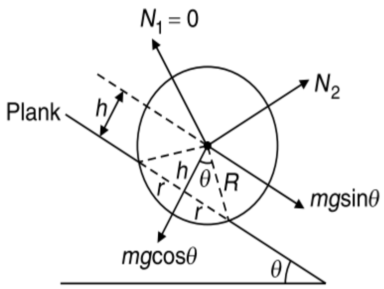
Single Correct Choice Type Problems

1. At the verge of toppling, $N_1 = 0$

$$(mg\sin\theta)h = (mg\cos\theta)r$$



Also, $\cos\theta = \frac{h}{R}$

$$\Rightarrow \quad (mg\sin\theta)h = mg\left(\frac{h}{R}\right)r$$

$$\Rightarrow \quad \sin\theta = \frac{r}{R}$$

Hence, the correct answer is (A).

2. Let the other mass at this instant is at a distance of x from the centre O. Applying Law of Conservation of Angular Momentum, we get

$$I_1\omega_1 = I_2\omega_2$$

$$\Rightarrow \quad (MR^2)(\omega) = \left[MR^2 + \frac{M}{8}\left(\frac{3}{5}R\right)^2 + \frac{M}{8}x^2\right]\left(\frac{8}{9}\omega\right)$$

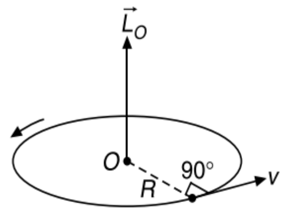
$$x = \frac{4}{5}R$$

Hence, the correct answer is (D).

3. Angular momentum of a particle about a point is

$$\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$

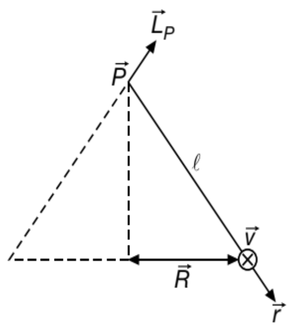
For L_O



$$|\vec{L}| = (mvr\sin\theta) = m(R\omega)(R)\sin 90° = \text{constant}$$

Direction of $\vec{L}_O$ is always upwards. Therefore, complete $\vec{L}_O$ is constant, both in magnitude as well as direction.

For L_P



$$|\vec{L}_P| = (mvr\sin\theta) = (m)(R\omega)(\ell)\sin 90° = (mR\ell\omega)$$

Magnitude of $\vec{L}_P$ will remain constant but direction of $\vec{L}_P$ keeps on changing.

Hence, the correct answer is (C).

4. Since P has most of its mass concentrated at surface, so

$$I_P > I_Q$$

In case of pure rolling, $a = \dfrac{g\sin\theta}{1+\dfrac{I}{mR^2}}$

$\Rightarrow$ $a_Q > a_P$ (as its moment of inertia is less)

Therefore, Q reaches first with more linear speed and more translational kinetic energy.

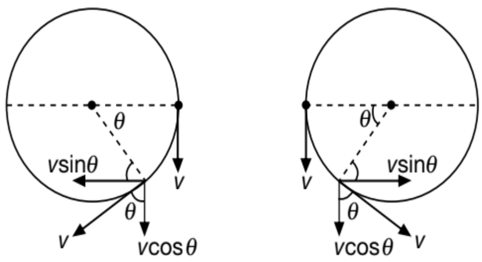
Further, $\omega = \dfrac{v}{R}$

$\Rightarrow$ $\omega \propto v$

$\Rightarrow$ $\omega_Q > \omega_P$ as $v_P > v_Q$

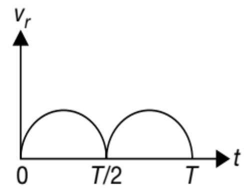
Hence, the correct answer is (D).

5.



$$v_r = |2v\sin\theta|$$

$\Rightarrow$ $v_r = |2v\sin(\omega t)|$



Hence, the correct answer is (D).

6. $\tau = \dfrac{dL}{dt} = \dfrac{d}{dt}(I\omega)$

$\Rightarrow$ $\tau = \dfrac{d}{dt}\left(\dfrac{ML^2}{3} + mx^2\right)\omega$

$\Rightarrow$ $\tau = 2mx\dfrac{dx}{dt}\omega$

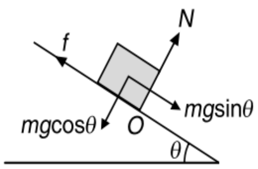
Now, $x = vt$

$\Rightarrow$ $\tau \propto t$

Finally, torque becomes zero.

Hence, the correct answer is (B).

7. Condition of sliding is



$$mg\sin\theta > \mu mg\cos\theta$$

$\Rightarrow$ $\tan\theta > \mu$

$\Rightarrow$ $\tan\theta > \sqrt{3}$ ...(1)

Condition of toppling is

$$\left(\tau_{mg\sin\theta}\right)_{\text{about } O} > \left(\tau_{mg\cos\theta}\right)_{\text{about } O}$$

$\Rightarrow$ $(mg\sin\theta)\left(\dfrac{15}{2}\right) > (mg\cos\theta)\left(\dfrac{10}{2}\right)$

$\Rightarrow$ $\tan\theta > \dfrac{2}{3}$ ...(2)

With increase in value of θ, condition of sliding is satisfied first.

Hence, the correct answer is (D).

8. $\dfrac{1}{2}mv^2 + \dfrac{1}{2}I\left(\dfrac{v}{R}\right)^2 = mg\left(\dfrac{3v^2}{4g}\right)$

$\Rightarrow$ $I = \dfrac{1}{2}mR^2$

$\Rightarrow$ Body is disc.

Hence, the correct answer is (D).

9. $\dfrac{2}{5}MR^2 = \dfrac{1}{2}Mr^2 + Mr^2$

$\Rightarrow$ $\dfrac{2}{5}MR^2 = \dfrac{3}{2}Mr^2$

$\Rightarrow$ $r = \dfrac{2}{\sqrt{15}}R$

Hence, the correct answer is (A).

10. On smooth part BC, due to zero torque, angular velocity and hence the rotational kinetic energy remains constant. While moving from B to C translational kinetic energy converts into gravitational potential energy.

Hence, the correct answer is (D).

11. $\begin{pmatrix}\text{MI of}\\ \text{Given}\\ \text{Shape}\end{pmatrix} = \begin{pmatrix}\text{MI of}\\ \text{Complete Disc}\\ \text{of Radius R}\end{pmatrix} - \begin{pmatrix}\text{MI of}\\ \text{Removed Portion}\\ \text{of Radius } R/3\end{pmatrix}$

$\Rightarrow$ $I_{\text{remaining}} = I_{\text{whole}} - I_{\text{removed}}$

$\Rightarrow$ $I = \dfrac{1}{2}(9M)(R)^2 - \left[\dfrac{1}{2}m\left(\dfrac{R}{3}\right)^2 + \dfrac{1}{2}m\left(\dfrac{2R}{3}\right)^2\right]$ ...(1)

where, $m = \sigma a = \dfrac{9M}{\pi R^2} \times \pi\left(\dfrac{R}{3}\right)^2 = M$

Substituting in equation (1), we get

$$I = 4MR^2$$

Hence, the correct answer is (D).

12. $\vec{L} = m(\vec{r} \times \vec{v})$

Direction of $(\vec{r} \times \vec{v})$, hence the direction of angular momentum remains the same.

Hence, the correct answer is (D).

13. From conservation of angular momentum $(I\omega = \text{constant})$, angular velocity will remain half. As,

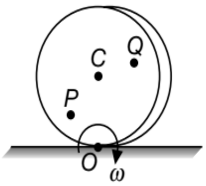
$$K = \frac{1}{2}I\omega^2$$

The rotational kinetic energy will become half.

Hence, the correct answer is (B).

14. In case of pure rolling bottommost point is the instantaneous centre of zero velocity.

Velocity of any point on the disc, $v = r\omega$, where r is the distance of point from O.



$$r_Q > r_C > r_P$$

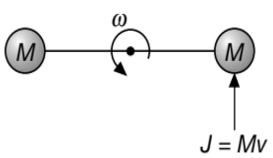
$$\Rightarrow \quad v_Q > v_C > v_P$$

Hence, the correct answer is (A).

15. In uniform circular motion the only force acting on the particle is centripetal (towards centre). Torque of this force about the centre is zero. Hence, angular momentum about centre remain conserved.

Hence, the correct answer is (A).

16. Let ω be the angular velocity of the rod. Applying, angular impulse equals change in angular momentum about centre of mass of the system, we get



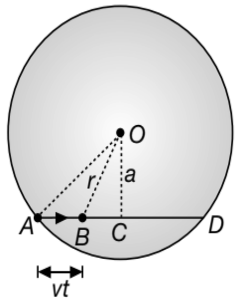
$$J\left(\frac{L}{2}\right) = I_c\omega$$

$$\Rightarrow \quad (Mv)\left(\frac{L}{2}\right) = (2)\left(\frac{ML^2}{4}\right)\omega$$

$$\Rightarrow \quad \omega = \frac{v}{L}$$

Hence, the correct answer is (A).

17. Since, there is no external torque, angular momentum will remain conserved. The moment of inertia will first decrease till the tortoise moves from A to C and then increase as it moves from C and D. Therefore, ω will initially increase and then decrease.



Let R be the radius of platform, m the mass of disc and M is the mass of platform.

Moment of inertia when the tortoise is at A

$$I_1 = mR^2 + \frac{MR^2}{2}$$

and moment of inertia when the tortoise is at B

$$I_2 = mr^2 + \frac{MR^2}{2}$$

Here, $r^2 = a^2 + \left[\sqrt{R^2 - a^2} - vt\right]^2$

From conservation of angular momentum

$$\omega_0 I_1 = \omega(t) I_2$$

Substituting the values, we can see that variation of $\omega(t)$ is non-linear.

Hence, the correct answer is (D).

18. $mg\sin\theta$ component is always down the plane whether it is rolling up or rolling down. Therefore, for no slipping, sense of angular acceleration should also be same in both the cases. Therefore, force of friction f always act upwards.

Hence, the correct answer is (B).

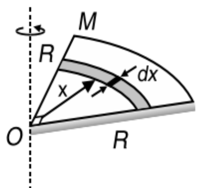
19. If we assume complete disc to be present, then it would have a mass 4 times the mass of the sector. Then, moment of inertia of the complete disc is

$$I_{\text{disc}} = \frac{1}{2}M_{\text{disc}}R^2 = \frac{1}{2}(4M_{\text{sector}})R^2$$

Hence, $I_{\text{sector}} = \frac{I_{\text{disc}}}{4}$

$$\Rightarrow \quad I_{\text{sector}} = \frac{1}{2}MR^2$$

Calculus Method:



CHAPTER 3

Consider an element of mass dm, thickness dx and radius x of the sector.

$$dm = \frac{M}{\left(\frac{\pi R^2}{4}\right)}\frac{2\pi x dx}{4} = \frac{2M}{R^2}x dx$$

If dI be the moment of inertia of the element, then $dI = x^2 dm$

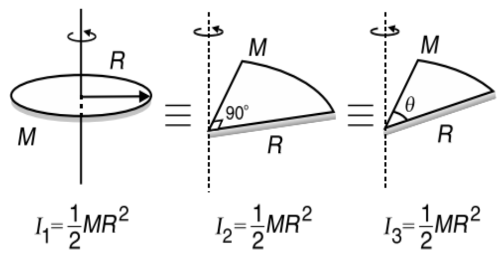
$$\Rightarrow \quad dI = x^2\left(\frac{2M}{R^2}x\,dx\right)$$

$$\Rightarrow \quad dI = \frac{2M}{R^2}\left(x^3\,dx\right)$$

$$\Rightarrow \quad I = \frac{2M}{R^2}\int_0^R x^3\,dx$$

$$\Rightarrow \quad I = \frac{1}{2}MR^2$$

General Funda:



$$\begin{pmatrix}\text{Moment of Inertia}\\ \text{of a DISC of mass}\\ M \text{ with radius } R\end{pmatrix} \equiv \begin{pmatrix}\text{Moment of Inertia}\\ \text{of a SECTOR of}\\ \text{mass } M \text{ with radius } R\end{pmatrix}$$

Hence, the correct answer is (A).

20. Mass of the loop $= M = L\rho$

Further if r is the radius of the loop, then

$$2\pi r = L$$

$$\Rightarrow \quad r = \frac{L}{2\pi}$$

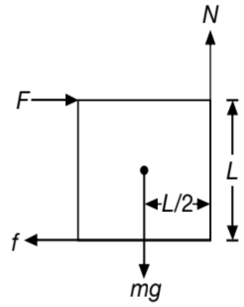
Moment of Inertia about XX' is $I = \frac{3}{2}Mr^2$

$$\Rightarrow \quad I = \frac{3}{2}(L\rho)\frac{L^2}{(2\pi)^2} = \frac{3\rho L^3}{8\pi^2}$$

Hence, the correct answer is (D).

21. At the critical condition, normal reaction N will pass through point P. In this condition

$$\tau_N = 0 = \tau_f \qquad \text{(about } P\text{)}$$



the block will topple when

$$\tau_F > \tau_{mg}$$

$$\Rightarrow \quad FL > (mg)\frac{L}{2}$$

$$\Rightarrow \quad F > \frac{mg}{2}$$

Therefore, the minimum force required to topple the bock is

$$F_{\min} = \frac{mg}{2}$$

Hence, the correct answer is (D).

22. No external torque is acting on the system, so angular momentum is conserved. Further there exists no non-conservative forces in the system, so total energy is also conserved.
Hence, the correct answer is (B).

23. The angular momentum of a body $\vec{L}$ may be expressed as the sum of two parts,
(a) one arising from the motion of the centre of mass of the body and
(b) the other from the motion of the body with respect to its centre of mass.

i.e. $\vec{L}_{\text{total}} = \vec{L}_{C.M.} + \vec{r}_{C.M.} \times \vec{p}$

$$\Rightarrow \quad \vec{L}_{\text{total}} = \vec{L}_{C.M.} + M\left(\vec{r}_{C.M.} \times \vec{v}_{C.M.}\right)$$

For this Problem

$$L_{C.M.} = I\omega = \frac{1}{2}MR^2\omega \text{ and}$$

$$M\left(\vec{r}_{C.M.} \times \vec{v}_{C.M.}\right) = MRv_{CM} = MR(R\omega)$$

$$\Rightarrow \quad M\left(\vec{r}_{C.M.} \times \vec{v}_{C.M.}\right) = MR^2\omega$$

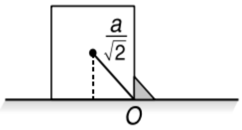
$$\Rightarrow \quad L_{\text{total}} = \frac{1}{2}MR^2\omega + MR^2\omega = \frac{3}{2}MR^2\omega.$$

Hence, the correct answer is (C).

24. Since spheres are smooth, so no transfer of angular momentum takes place from A to B. However, sphere A only transfers its linear velocity v to sphere B and stops. Hence, we conclude that A stops but continues to rotate with same angular speed ω and B moves with speed of A but with zero angular speed.
Hence, the correct answer is (C).

25. By Law of Conservation of Angular Momentum

$$mv\left(\frac{a}{2}\right)=\left(I_{\substack{\text{system}\\ \text{about O}}}\right)\omega$$

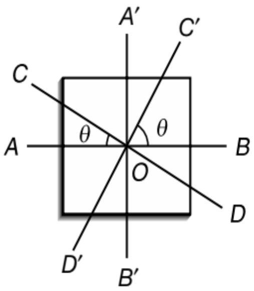


$$\Rightarrow \quad mv\left(\frac{a}{2}\right)=\left[\frac{1}{6}Ma^2+M\left(\frac{a}{\sqrt{2}}\right)^2\right]\omega$$

$$\Rightarrow \quad \omega=\frac{3v}{4a}$$

Hence, the correct answer is (A).

26.



$I_{AB}=I_{A'B'}=I$ and $I_{CD}=I_{C'D'}$

If I_0 be the moment of inertia of the square plate about an axis passing through O and perpendicular to the plate, then by perpendicular axis theorem

$$I_0=I_{AB}+I_{A'B'}=2I_{AB} \quad \ldots(1)$$

OR

$$I_0=I_{CD}+I_{C'D'}=2I_{CD} \quad \ldots(2)$$

From (1) and (2)

$$I_{CD}=I_{AB}=I \quad \text{\{OPTION (A)\}}$$

Hence, the correct answer is (A).

27. Since $L=mvr_{\perp}$ and as $r_{\perp}$ is constant, so L is constant.

Hence, the correct answer is (B).

28. Work done $W=\frac{1}{2}I\omega^2$

If x is the distance of mass 0.3 kg from the centre of mass, we will have,

$$I=(0.3)x^2+(0.7)(1.4-x)^2$$

For work to be minimum, the moment of inertia (I) should be minimum

$$\Rightarrow \quad \frac{dI}{dx}=0$$

$$\Rightarrow \quad 2(0.3x)-2(0.7)(1.4-x)=0$$

$$\Rightarrow \quad (0.3)x=(0.7)(1.4-x)$$

$$\Rightarrow \quad x=\frac{(0.7)(1.4)}{0.3+0.7}=0.98 \text{ m}$$

Hence, the correct answer is (D).

29. Mass of the element dx is $m=\frac{M}{L}dx$

This element needs centripetal force for rotation, so

$$dF=mx\omega^2=\left(\frac{M}{L}x\omega^2dx\right)$$

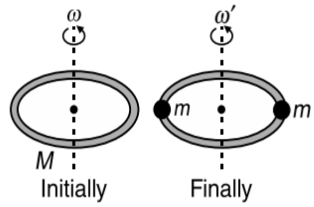
$$\Rightarrow \quad F=\int_0^L dF=\frac{m}{L}\omega^2\int_0^L xdx=\frac{M\omega^2L}{2}$$

This is the force exerted by the liquid at the other end.

Hence, the correct answer is (D).

30. By Law of Conservation of Angular Momentum

$$(MR^2)\omega=MR^2\omega'+2mR^2\omega'$$



$$\Rightarrow \quad \omega'=\left(\frac{M}{M+2m}\right)\omega$$

Hence, the correct answer is (C).

Multiple Correct Choice Type Problems

1. Applying conservation of angular momentum about hinge, we get

$$mvx=\left(mx^2+\frac{mL^2}{3}\right)\omega$$

$$\Rightarrow \quad \omega=\frac{3vx}{L^2+3x^2}$$

For ω to be maximum, we have

$$\frac{d\omega}{dx}=0$$

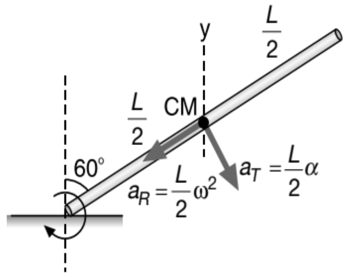
$$\Rightarrow \quad x_M=\frac{L}{\sqrt{3}}$$

$$\Rightarrow \quad \omega_M=\frac{\sqrt{3}v}{2L}$$

Hence, (A), (C) and (D) are correct.

2. Applying conservation of energy (as the friction is acting at the point of no slipping), we get

$$\frac{MgL}{2}(1-\cos 60°)=\frac{1}{2}I_0\omega^2$$



Since $I_0 = \dfrac{ML^2}{3}$

$$\Rightarrow \quad \omega = \sqrt{\frac{3g}{2L}}$$

$$\Rightarrow \quad a_R = \frac{L}{2}\omega^2 = \frac{3g}{4}$$

Also $\tau_0 = Mg\dfrac{L}{2}\sin(60^\circ) = I_0\alpha$

$$\Rightarrow \quad \alpha = \frac{3\sqrt{3}g}{4L}$$

For CM of the rod, we have

$$Mg - N = Ma_y$$

where, $a_y = a_T\sin(60^\circ) + a_R\cos(60^\circ)$

$$\Rightarrow \quad a_y = \frac{L}{2}\alpha\sin(60^\circ) + \frac{\omega^2 L}{2}\cos(60^\circ)$$

$$\Rightarrow \quad a_y = \frac{\sqrt{3}}{4}\left(\frac{3\sqrt{3}g}{4}\right) + \left(\frac{3g}{2L}\right)\left(\frac{L}{2}\right)\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad a_y = \left(\frac{9}{16} + \frac{3}{8}\right)g = \frac{15g}{16}$$

$$\Rightarrow \quad Mg - N = M\left(\frac{15g}{16}\right)$$

$$\Rightarrow \quad N = \frac{Mg}{16}$$

Hence, (B), (C) and (D) are correct.

3. When the bar makes an angle θ, the height of its CM (mid-point) is $\dfrac{L}{2}\cos\theta$

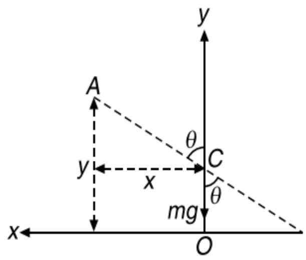
Displacement of CM is

$$y = \frac{L}{2}(1 - \cos\theta)$$

Since, force on CM is only along the vertical direction, hence CM is falling vertically downward. Instantaneous torque about point of contact is

$$\tau = mg\left(\frac{L}{2}\sin\theta\right)$$

$$\Rightarrow \quad \tau \propto \sin\theta$$



Now, $x = \dfrac{L}{2}\sin\theta$ and $y = L\cos\theta$

$$\Rightarrow \quad \frac{x^2}{\left(\frac{L}{2}\right)^2} + \frac{y^2}{L^2} = 1$$

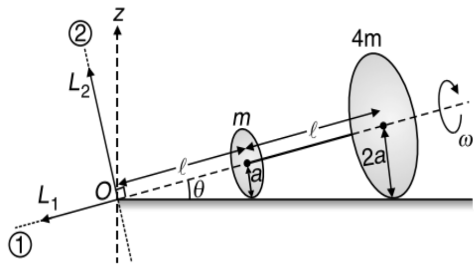
So, path of A is an ellipse.

Hence, (A), (C) and (D) are correct.

4. Let Ω be the angular velocity of assembly perpendicular to axis of the rod, then for pure rolling, we have

$$a\omega = \ell\Omega$$

$$\Rightarrow \quad \Omega = \frac{a\omega}{\ell}$$



So, component of Ω along z-axis is $\Omega\cos\theta$

$$\Rightarrow \quad \Omega\cos\theta = \frac{a\omega}{\ell}\frac{\sqrt{24}}{5} = \frac{\ell\omega}{5\ell} = \frac{\omega}{5} \qquad \left\{\because \ell = \sqrt{24}\,a\right\}$$

So, (A) is correct.

Let L_1 be the angular momentum of assembly about its centre of mass, then

$$L_1 = \left[\frac{ma^2}{2} + \frac{(4m)(2a)^2}{2}\right]\omega = \left(\frac{17}{2}ma^2\right)\omega$$

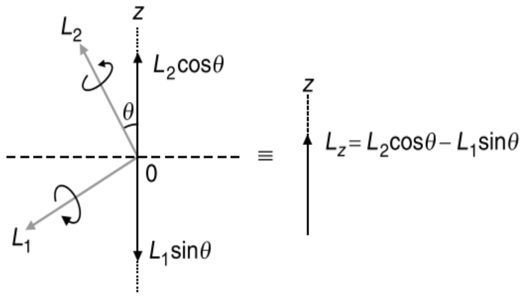
So, (C) is correct

If I_2 be the moment of inertia of assembly about and axis perpendicular to axis of rod (marked as 2), then

$$I_2 = \left(\frac{ma^2}{4} + m\ell^2\right) + \left[\frac{(4m)(2a)^2}{4} + (4m)(2\ell)^2\right]$$

$$\Rightarrow \quad I_2 = \frac{17}{4}ma^2 + 17m\ell^2 = \frac{17}{4}ma^2 + 17m(24a^2)$$

So, angular momentum of assembly about axis 2 is $L_2 = I_2\Omega$



$$L_2 = (17ma^2)\left(\frac{1}{4}+24\right)\frac{a\omega}{\sqrt{24a}}$$

$$\Rightarrow \quad L_2 = (17ma^2)\left(\frac{97}{4}\right)\frac{\omega}{\sqrt{24}} \cong 17\sqrt{24}\,\text{ma}^2\omega$$

$$\Rightarrow \quad L_2\cos\theta = (17\sqrt{24}ma^2\omega)\left(\frac{\sqrt{24}}{5}\right) = \frac{(17)(24)}{5}ma^2\omega$$

Also, $L_1\sin\theta = \left(\frac{17}{2}ma^2\omega\right)\left(\frac{1}{5}\right)$

$$\Rightarrow \quad L_Z = L_2\cos\theta - L_1\sin\theta = \frac{17}{5}ma^2\omega\left(24-\frac{1}{2}\right)$$

$$\Rightarrow \quad L_Z \cong 80ma^2\omega$$

So, (D) is not correct

Angular momentum of CM of assembly about O is

$$L_{CM\text{ about }O} = (5m)(v_{cm})\left(\frac{9\ell}{5}\right)$$

where $v_{cm} = \dfrac{m(a\omega)+4m((2a)\omega)}{5m} = \dfrac{9a\omega}{5}$

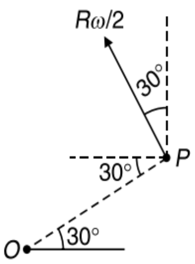
$$L_{CM\text{ about }O} = (5m)\left(\frac{9a\omega}{5}\right)\left(\frac{9\ell}{5}\right) = 81ma^2\omega\left(\frac{\sqrt{24}}{5}\right)$$

So, (B) is not correct

Hence, (A) and (C) are correct.

5. Velocity of point O is $\vec{v}_O = (3R\omega)\hat{i}$

Also, $|\vec{v}_{PO}| = \dfrac{R\omega}{2}$ in the direction shown in Figure.



In vector form,

$$\vec{v}_{PO} = -\frac{R\omega}{2}\sin 30°\hat{i} + \frac{R\omega}{2}\cos 3($$

$$\Rightarrow \quad \vec{v}_{PO} = -\frac{R\omega}{4}\hat{i} + \frac{\sqrt{3}R\omega}{4}\hat{k}$$

Since, $\vec{v}_{PO} = \vec{v}_P - \vec{v}_O$

$$\Rightarrow \quad \vec{v}_P = \vec{v}_{PO} + \vec{v}_O$$

$$\Rightarrow \quad \vec{v}_P = \left(-\frac{R\omega}{4}\hat{i} + \frac{\sqrt{3}R\omega}{4}\hat{k}\right) + 3R\omega\hat{i}$$

$$\Rightarrow \quad \vec{v}_P = \frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$$

Hence, (A) and (B) are correct.

6. The data seems to be incomplete.

Let us assume that friction from ground on ring is not impulsive during impact. From linear momentum conservation along horizontal direction, we get

$$(-2\times 1)+(0.1\times 20) = (0.1\times 0)+(2\times v)$$

Here, v is the velocity of CM of ring after impact.

Solving the above equation, we get $v = 0$

So, CM becomes stationary.

Correct OPTION is (A)

Linear impulse during impact

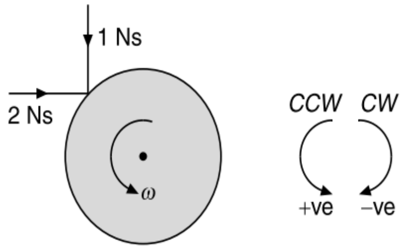
1. Along horizontal direction is

   $J_1 = \Delta p = 0.1\times 20 = 2$ Ns

2. Along vertical direction is

   $J_2 = \Delta p = 0.1\times 10 = 1$ Ns

   Writing the equation (about CM)



Since, angular impulse equals change in angular momentum, so we get

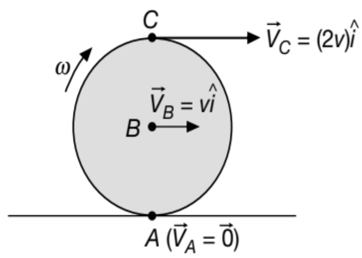
$$1\times\left(\frac{\sqrt{3}}{2}\times\frac{1}{2}\right) - 2\times 0.5\times\frac{1}{2} = 2\times(0.5)^2\left(\omega - \frac{1}{0.5}\right)$$

Solving this equation, ω comes out to be positive i.e. ω is counter clockwise. So just after collision, rightwards slipping is taking place.

Hence, friction is leftwards.

Hence, (A) and (C) are correct.

7.



Hence, (B) and (C) are correct.

8. In case of pure rolling,

$$f = \frac{mg\sin\theta}{1+\frac{mR^2}{I}} \quad \text{(upwards)}$$

$\Rightarrow \quad f \propto \sin\theta$

Therefore, as θ decreases force of friction will also decrease.

Hence, (C) and (D) are correct.

9. Due to Law of Conservation of Angular Momentum $\vec{L}$ =constant

$\Rightarrow \quad \vec{L}\cdot\vec{L} = \text{constant}$

$\Rightarrow \quad \frac{d}{dt}(\vec{L}\cdot\vec{L}) = 0$

$\Rightarrow \quad 2\vec{L}\cdot\frac{d\vec{L}}{dt} = 0$

$\Rightarrow \quad \vec{L} \perp \frac{d\vec{L}}{dt}$

Since $\vec{\tau} = \vec{A}\times\vec{L}$

$\Rightarrow \quad \frac{d\vec{L}}{dt} = \vec{A}\times\vec{L}$

$\Rightarrow \quad \frac{d\vec{L}}{dt}$ must be perpendicular to $\vec{A}$ as well as $\vec{L}$

Further component of $\vec{L}$ along $\vec{A}$ is $\frac{\vec{A}\cdot\vec{L}}{A} = x$(say). Also

$$\frac{d}{dt}(\vec{A}\cdot\vec{L}) = \vec{A}\cdot\frac{d\vec{L}}{dt} + \vec{L}\cdot\frac{d\vec{A}}{dt} = 0$$

$$\left\{\because \vec{A}\perp\frac{d\vec{L}}{dt} \text{ and } \frac{d\vec{A}}{dt} = \vec{0}\right\}$$

$\Rightarrow \quad \vec{A}\cdot\vec{L} = \text{constant}$

$\Rightarrow \quad \frac{\vec{A}\cdot\vec{L}}{A} = x = \text{constant}$

Since $\frac{d\vec{L}}{dt}$ (or $\vec{\tau}$) is perpendicular to $\vec{L}$, hence it cannot change magnitude of $\vec{L}$ but can surely change direction of $\vec{L}$.

Hence, (A), (B) and (C) are correct.

10. From symmetry we have

$$I_1 = I_2 \text{ and } I_4 = I_3 \quad \ldots(1)$$

Also, by perpendicular Axis Theorem

$$I_0 = I_1 + I_2 = I_4 + I_3 \quad \ldots(2)$$

From (2), we get

$$2I_1 = 2I_3 \quad \{\because \text{of}(1)\}$$

$\Rightarrow \quad I_1 = I_3$

$\Rightarrow \quad I_0 = I_1 + I_2 = I_4 + I_3 = I_1 + I_3$

Hence, (A), (B) and (C) are correct.

11. $V_c = \frac{2m(-v)+m(2v)+8m(0)}{2m+m+8m} = 0$ {OPTION **(A)**}

Further, by Law of Conservation of Angular Momentum

$$\left(I_{\substack{\text{system}\\ \text{about O}}}\right)\omega = \sum_{\substack{\text{system}\\ \text{about O}}} mvr_\perp$$

$$I_{\substack{\text{system}\\ \text{about O}}} = \left[\frac{1}{12}(8m)(6a)^2 + m(2a)^2 + 2ma^2\right] = 30ma^2$$

$$\sum_{\substack{\text{system}\\ \text{about O}}} mvr_\perp = (8m)(0)(0) + 2m(-v)(-a) + m(2v)(2a)$$

$\Rightarrow \quad \sum_{\substack{\text{system}\\ \text{about O}}} mvr_\perp = 6mva$

$\Rightarrow \quad (30ma^2)\omega = 6mva$

$\Rightarrow \quad \omega = \frac{v}{5a}$ {OPTION **(C)**}

Further total energy of the system E is

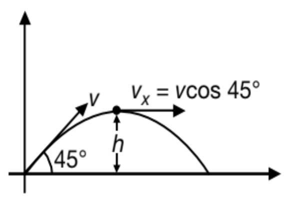
$$E = \frac{1}{2}I\omega^2 = \frac{3}{5}mv^2 \quad \text{\{OPTION (D)\}}$$

Hence, (A), (C) and (D) are correct.

12. $L = mv_x h$

$\Rightarrow \quad L = m(v\cos 45)\left(\frac{v^2\sin^2 45}{2g}\right)$

$\Rightarrow \quad L = \frac{mv^3}{4\sqrt{2}\,g} \quad \left\{\because h\frac{v^2\sin^2 45}{2g}\right\}$



Also, from $h = \frac{v^2}{4g}$

$\Rightarrow \quad v = 2\sqrt{gh}$

$\Rightarrow \quad L = \frac{mv}{\sqrt{2}}h$ gives

$$L = \frac{m(2\sqrt{gh})h}{\sqrt{2}}$$

$\Rightarrow \quad L = m\sqrt{2gh^3}$

Hence, (B) and (D) are correct.

Reasoning Based Questions

1. In case of pure rolling on inclined plane,

$$a = \frac{g\sin\theta}{1+\frac{I}{mR^2}}$$

$$I_{solid} < I_{hollow}$$

$$\Rightarrow \quad a_{solid} > a_{hollow}$$

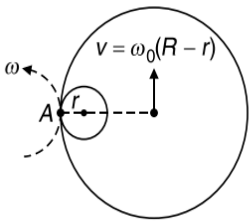
Solid cylinder will reach the bottom first. Further, in case of pure rolling on stationary ground, work done by friction is zero. Therefore, mechanical energy of both the cylinders will remain constant.

$$\Rightarrow \quad (\text{KE})_{\text{Hollow}} = (\text{KE})_{\text{Solid}} = \text{decrease in PE} = mgh$$

Hence, the correct answer is (D).

Linked Comprehension Type Questions

1. The point A will be $IAOR$ (i.e. instantaneous axis of rotation). For no slipping, we have



$$R\omega = (R-r)\omega_0$$

$$\Rightarrow \quad \omega = \omega_0\left(\frac{R-r}{R}\right)$$

$$\Rightarrow \quad \text{KE} = \frac{1}{2}(2mR^2)\omega^2 = m\omega_0^2(R-r)^2$$

Hence, the correct answer is (A).

2. If height of the cone is $h \quad r$

Then, $\mu N = mg$

$$\Rightarrow \quad \mu m(R-r)\omega_0^2 = mg$$

$$\Rightarrow \quad \omega_0 = \sqrt{\frac{g}{\mu(R-r)}}$$

Hence, the correct answer is (C).

3. Force on block along slot is the centripetal force, so

$$m\omega^2 r = ma = m\left(\frac{vdv}{dr}\right)$$

$$\Rightarrow \quad \int_0^v vdv = \int_{\frac{R}{2}}^r \omega^2 rdr$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{\omega^2}{2}\left(r^2 - \frac{R^2}{4}\right)$$

$$\Rightarrow \quad v = \omega\sqrt{r^2 - \frac{R^2}{4}} = \frac{dr}{dt}$$

$$\Rightarrow \quad \int_{\frac{R}{4}}^r \frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \int_0^t \omega dt$$

$$\Rightarrow \quad \ln\left(\frac{r+\sqrt{r^2-\frac{R^2}{4}}}{\frac{R}{2}}\right) - \ln\left(\frac{\frac{R}{2}+\sqrt{\frac{R^2}{4}-\frac{R^2}{4}}}{\frac{R}{4}}\right) = \omega t$$

$$\Rightarrow \quad r + \sqrt{r^2 - \frac{R^2}{4}} = \frac{R}{2}e^{\omega t}$$

$$\Rightarrow \quad r^2 - \frac{R^2}{4} = \frac{R^2}{4}e^{2\omega t} + r^2 - 2r\frac{R}{2}e^{\omega t}$$

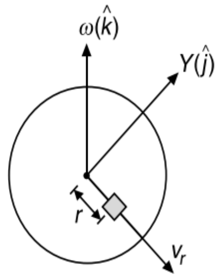
$$\Rightarrow \quad r = \frac{\frac{R^2}{4}e^{2\omega t} + \frac{R^2}{4}}{Re^{\omega t}} = \frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$$

Hence, the correct answer is (C).

4. Since, $\vec{F}_{rot} = \vec{F}_{in} + 2m\left(v_{rot}\hat{i}\right)\times\omega\hat{k} + m\left(\omega\hat{k}\times r\hat{i}\right)\times\omega\hat{k}$

$$\Rightarrow \quad mr\omega^2\hat{i} = \vec{F}_{in} + 2mv_{rot}\omega\left(-\hat{j}\right) + m\omega^2 r\hat{i}$$

$$\Rightarrow \quad F_{in} = 2mv_r\omega\hat{j}$$



Since, $r = \frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$

$$\Rightarrow \quad \frac{dr}{dt} = v_r = \frac{R}{4}\left(\omega e^{\omega t} - \omega e^{-\omega t}\right)$$

$$\Rightarrow \quad \vec{F}_{in} = 2m\frac{R\omega}{4}\left(e^{\omega t} - e^{-\omega t}\right)\omega\hat{j}$$

$$\Rightarrow \quad \vec{F}_{in} = \frac{mR\omega^2}{2}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j}$$

Also, reaction is due to the disc surface, so

$$\vec{F}_{\text{reaction}} = \frac{mR\omega^2}{2}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\hat{k}$$

Hence, the correct answer is (B).

5-6. The correct answer is 5(D), 6(A).

Combined solution to 5, 6

Following points must be kept in mind.

(i) Every particle of the disc is rotating in a horizontal circle.

(ii) Actual velocity of any particle is horizontal.

(iii) Magnitude of velocity of any particle is

$$v = r\omega$$

where, r is the perpendicular distance of that particle from actual axis of rotation (z-axis)

(iv) When it is broken into two parts then actual velocity of any particle is resultant of two velocities

$$v_1 = r_1\omega_1 \text{ and } v_2 = r_2\omega_2$$

Here, r_1 is the perpendicular distance of centre of mass from z-axis, ω_1 is the angular speed of rotation of centre of mass from z-axis, r_2 is the distance of particle from centre of mass and ω_2 is the angular speed of rotation of the disc about the axis passing through centre of mass.

(v) Net v will be horizontal, if v_1 and v_2 both are horizontal. Since, v_1 is already horizontal, because centre of mass is rotating about a vertical z-axis. To make v_2 also horizontal, the second axis should also be vertical.

Hence, the correct answer is (A).

7. $\frac{1}{2}I(2\omega)^2 = \frac{1}{2}kx_1^2$ ...(1)

$$\frac{1}{2}(2I)(\omega)^2 = \frac{1}{2}kx_2^2 \quad \text{...(2)}$$

From equations (1) and (2), we have

$$\frac{x_1}{x_2} = \sqrt{2}$$

Hence, the correct answer is (C).

8. Let ω' be the common velocity. Then applying conservation of angular momentum, we get

$$(I + 2I)\omega' = I(2\omega) + 2I(\omega)$$

$$\Rightarrow \quad \omega' = \frac{4}{3}\omega$$

Applying the concept that angular impulse equals change in angular momentum. For any of the disc, we have

$$\tau t = I(2\omega) - I\left(\frac{4}{3}\omega\right)$$

$$\Rightarrow \quad \tau t = \frac{2I\omega}{3}$$

$$\Rightarrow \quad \tau = \frac{2I\omega}{3t}$$

9. Loss of kinetic energy is $-\Delta K = K_i - K_f$

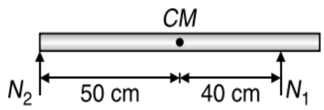
$$\Rightarrow \quad -\Delta K = \left\{\frac{1}{2}I(2\omega)^2 + \frac{1}{2}(2I)(\omega)^2\right\} - \frac{1}{2}(3I)\left(\frac{4}{3}\omega\right)^2$$

$$\Rightarrow \quad -\Delta K = \frac{1}{3}I\omega^2$$

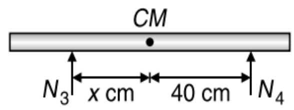
Hence, the correct answer is (B).

Integer/Numerical Answer Type Questions

1. Initially, $40N_1 = 50N_2$



For First Move:



For translational equilibrium, we have

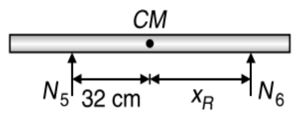
$$\mu_k N_3 = \mu_s N_4 \quad \text{...(1)}$$

For rotational equilibrium, we have

$$xN_3 = 40N_4 \quad \text{...(2)}$$

$$\Rightarrow \quad x = 32 \text{ cm}$$

For Second Move:



For translational equilibrium, we have

$$\mu_s N_5 = \mu_k N_6 \quad \text{...(1)}$$

For rotational equilibrium, we have

$$32(N_5) = X_R N_6 \quad \text{...(2)}$$

$$\Rightarrow \quad X_R = 25.6 \text{ cm}$$

2. In case of pure rolling, mechanical energy remains constant since work done by friction is zero. Further in case of a disc,

$$\frac{\text{Translational Kinetic Energy}}{\text{Rotational Kinetic Energy}} = \frac{K_T}{K_R} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}I\omega^2}$$

$$\Rightarrow \frac{K_T}{K_R} = \frac{mv^2}{\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2} = \frac{2}{1}$$

$$\Rightarrow K_T = \frac{2}{3}(\text{Total KE}) = \frac{2}{3}K$$

So, total kinetic energy is

$$K = \frac{3}{2}K_T = \frac{3}{2}\left(\frac{1}{2}mv^2\right) = \frac{3}{4}mv^2$$

Since, decrease in potential energy equals increase in kinetic energy, so

$$mgh = \frac{3}{4}m\left(v_f^2 - v_i^2\right)$$

$$\Rightarrow v_f = \sqrt{\frac{4}{3}gh + v_i^2}$$

Given that the final velocity in both cases is same, so, value of $v_f = \sqrt{\frac{4}{3}gh + v_i^2}$ should be same in both cases.

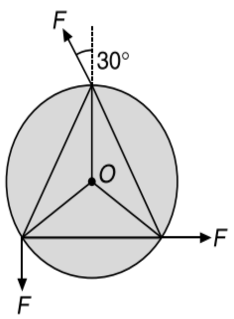
$$\Rightarrow \sqrt{\frac{4}{3}\times 10\times 30 + (3)^2} = \sqrt{\frac{4}{3}\times 10\times 27 + (v_2)^2}$$

$$\Rightarrow v_2 = 7 \text{ ms}^{-1}$$

3. Since angular impulse equals change in angular momentum, so

$$\int \tau dt = I\omega$$

$$\Rightarrow \omega = \frac{\int \tau dt}{I} = \frac{\int_0^t 3F\sin 30° R dt}{I}$$

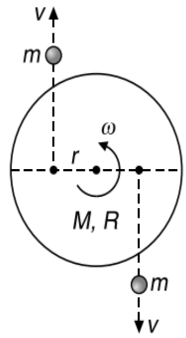


$$\Rightarrow \omega = \frac{3(0.5)(0.5)(0.5)(1)}{1.5(0.5)^2/2} = 2 \text{ rads}^{-1}$$

4. Applying conservation of angular momentum, we get

$$2mvr - \frac{MR^2}{2}\omega = 0$$

$$\Rightarrow \omega = \frac{4mvr}{MR^2}$$



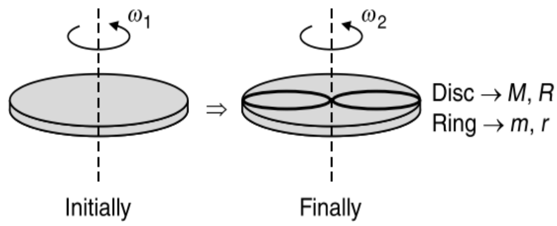
Substituting the values, we get

$$\omega = \frac{(4)(5\times 10^{-2})(9)(0.25)}{45\times 10^{-2}(0.5)^2}$$

$$\Rightarrow \omega = 4 \text{ rads}^{-1}$$

5. By conservation of angular momentum, we have

$$I_1\omega_1 = I_2\omega_2$$

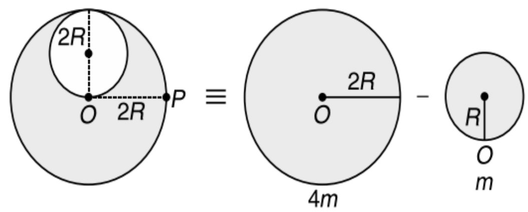


$$\Rightarrow \omega_2 = \left(\frac{I_1}{I_2}\right)\omega_1 = \left(\frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + 2(mr^2)}\right)\omega_1$$

$$\Rightarrow \omega_2 = \left(\frac{50(0.4)^2}{50(0.4)^2 + 8\times(6.25)\times(0.2)^2}\right)(10)$$

$$\Rightarrow \omega_2 = 8 \text{ rads}^{-1}$$

6. Moment of inertia of this lamina about the point O is



$$I_O = \frac{(4m)(2R)^2}{2} - \frac{3}{2}mR^2$$

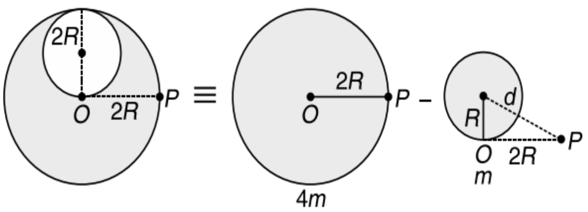
$$\Rightarrow I_O = mR^2\left(8 - \frac{3}{2}\right)$$

$$\Rightarrow I_O = \frac{13}{2}mR^2$$

Similarly, MI about point P is

$$I_P = \frac{3}{2}(4m)(2R)^2 - \left(\frac{mR^2}{2} + md^2\right)$$

where $d^2 = R^2 + (2R)^2$



$$\Rightarrow \quad I_P = 24mR^2 - \frac{11}{2}mR^2$$

$$\Rightarrow \quad I_P = \frac{37}{2}mR^2$$

$$\Rightarrow \quad \frac{I_P}{I_O} = \frac{\frac{37}{2}}{\frac{13}{2}} = \frac{37}{13} \approx 3$$

7. Moment of inertia of system about diagonal of square is

$$I = 2\left(\frac{2}{5}MR^2\right) + 2\left(\frac{2}{5}MR^2 + Md^2\right)$$

$$\Rightarrow \quad I = \frac{8}{5}MR^2 + 2md^2$$

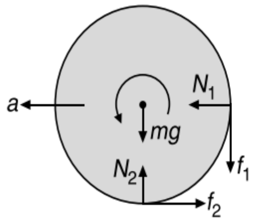
$$\Rightarrow \quad I = \left[\frac{8}{5}(0.5)\left(\frac{\sqrt{5}}{2}\right)^2 + 2(0.5)(4)(2)\right]10^{-4}$$

$$\Rightarrow \quad I = \left(\frac{5}{5} + 8\right) \times 10^{-4}$$

$$\Rightarrow \quad I = 9 \times 10^{-4}$$

$$\Rightarrow \quad N = 9$$

8. There is no slipping between ring and ground, so f_2 is not maximum. However, there is slipping between ring and stick, so, f_1 is maximum.



Moment of inertia of the ring about its centre of mass is

$$I = mR^2 = (2)(0.5)^2 = 0.5 \text{ kgm}^2$$

Since, $N_1 - f_2 = ma$

$$\Rightarrow \quad N_1 - f_2 = (2)(0.3) = 0.6 \text{ N} \qquad \ldots(1)$$

Also, $a = R\alpha = \dfrac{R\tau}{I} = \dfrac{R(f_2 - f_1)R}{I} = \dfrac{R^2(f_2 - f_1)}{I}$

$$\Rightarrow \quad 0.3 = \frac{(0.5)^2(f_2 - f_1)}{(0.5)}$$

$$\Rightarrow \quad f_2 - f_1 = 0.6 \text{ N} \qquad \ldots(2)$$

Since stick applies a force of 2 N on the ring, so we have

$$N_1^2 + f_1^2 = (2)^2 = 4 \qquad \ldots(3)$$

Further, $f_1 = \mu N_1 = \left(\dfrac{P}{10}\right)N_1 \qquad \ldots(4)$

From equations (1), (2), (3) and (4), we get

$$P \approx 3.6$$

CHAPTER 4: GRAVITATION AND SATELLITES

Test Your Concepts-I (Based on Acceleration due to Gravity, Gravitational Field and Applications)

1. Escape velocity from the surface of moon is

$$v_e = \sqrt{\frac{2GM_m}{R_m}}$$

Substituting the values, we have

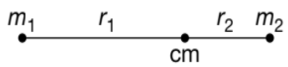
$$v_e = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.4 \times 10^{22}}{1.74 \times 10^6}}$$

$$\Rightarrow \quad v_e = 2.4 \times 10^3 \text{ ms}^{-1} \text{ or } 2.4 \text{ kms}^{-1}$$

2. Both the planet and the sun revolve around their centre of mass with same angular velocity (say ω)

$$r = r_1 + r_2 \qquad \ldots(1)$$

$$m_1 r_1 \omega^2 = m_2 r_2 \omega^2 = \frac{Gm_1 m_2}{r^2} \qquad \ldots(2)$$



Solving these two equations, we get

$$r_1 = r\left(\frac{m_2}{m_1 + m_2}\right)$$

$$r_2 = r\left(\frac{m_1}{m_1 + m_2}\right)$$

and $\omega^2 = \dfrac{G(m_1 + m_2)}{r^3}$

Now, total energy of the system is

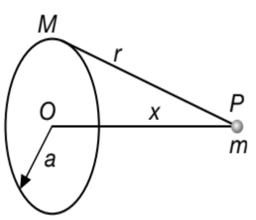
$$E = \text{P.E.} + \text{K.E.}$$

$$\Rightarrow \quad E = -\frac{Gm_1 m_2}{r} + \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2$$

Substituting the values of r_1, r_2 and ω^2, we get

$$E = -\frac{Gm_1 m_2}{2r}$$

3. (a) Gravitational potential at point P due to the ring.



$$V = -\frac{GM}{r} = -\frac{GM}{\sqrt{a^2 + x^2}}$$

$$\Rightarrow \quad U = mV = -\frac{GMm}{\sqrt{a^2 + x^2}}$$

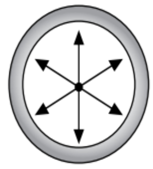
(b) When $x \quad a$, $a^2 + x^2 \approx x^2$

$$\Rightarrow \quad U = -\frac{GMm}{x}$$

{the potential energy of two point masses}

(c) $F_x = -\dfrac{dU}{dx} = -GMm \cdot \dfrac{d}{dx}(a^2 + x^2)^{-1/2}$

$$\Rightarrow \quad F_x = -\frac{GMm \cdot x}{(x^2 + a^2)^{3/2}}$$



(d) When $x \quad a$, $a^2 + x^2 \approx x^2$

and $F_x = -\dfrac{GMm}{x^2}$

{force between two point masses}

(e) At $x = 0$ {centre of the ring}

$F_x = 0$

As the particle is attracted equally from all the four sides.

4. Net force on M due to the pairs $2M$ and $2M$, $4M$ and $4M$, $5M$ and $5M$, $7M$ and $7M$, M and M is zero. So, the only left out mass is $3M$ and net force on M is due to $3M$, given by

$$F = \frac{G(3M)M}{d^2}$$

$$\Rightarrow \quad F = \frac{3GM^2}{d^2}, \text{ along } +x \text{ axis}$$

5. Given that $F = \dfrac{Gm_A m_B}{r^2}$

Since acceleration of A is $a_A = \dfrac{F}{m_A}$

$$\Rightarrow \quad a = a_A = \frac{F}{m_A} = \frac{Gm_B}{r^2} \qquad \ldots(1)$$

Now if, $F = \dfrac{Gm_A m_B}{r^4}$, then

$$a'_A = \frac{F}{m_A} = \frac{Gm_B}{r^4}$$

$$\Rightarrow \quad a'_A = \frac{a}{r^2} \qquad \{\because \text{ of } (1)\}$$

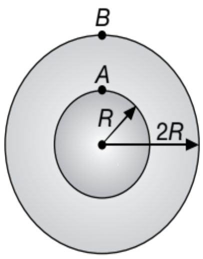
6. Since, $E=\dfrac{F}{m_0}$

$\Rightarrow \quad E=\dfrac{4N}{20\times10^{-3}\text{ kg}}$

$\Rightarrow \quad E=200\text{ Nkg}^{-1}$, along $+x$ direction.

7. Let m_1 be the mass of the core and m_2 the mass of outer shell, then

$g_A=g_B$



$\Rightarrow \quad \dfrac{Gm_1}{R^2}=\dfrac{G(m_1+m_2)}{(2R)^2}$

$\Rightarrow \quad 4m_1=(m_1+m_2)$

$\Rightarrow \quad 4\left(\dfrac{4}{3}\pi R^3\rho_1\right)=\left(\dfrac{4}{3}\pi R^3\right)\rho_1+\left(\dfrac{4}{3}\pi(2R)^3-\dfrac{4}{3}\pi R^3\right)\rho_2$

$\Rightarrow \quad 4\rho_1=\rho_1+7\rho_2$

$\Rightarrow \quad \dfrac{\rho_1}{\rho_2}=\dfrac{7}{3}$

8. $L=\pi R$

$\Rightarrow \quad R=\dfrac{L}{\pi}$

Gravitational field at the centre of a semicircular wire of radius R is

$E=\dfrac{2G\lambda}{R}$

where $\lambda=\dfrac{M}{L}$

$\Rightarrow \quad E=\dfrac{2G\left(\dfrac{M}{L}\right)}{\left(\dfrac{L}{\pi}\right)}$

$\Rightarrow \quad E=\dfrac{2\pi GM}{L^2}$

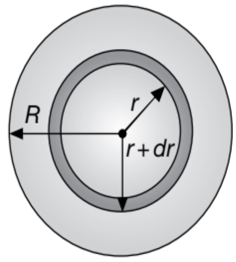
Since $F=mE$ $\qquad \left\{\because E=\dfrac{F}{m}\right\}$

$\Rightarrow \quad F=\dfrac{2\pi GMm}{L^2}$

9. Since $r(=2R)$ lies outside the sphere, so

$E=\dfrac{GM_{\text{total}}}{r^2}$ for $r\ge R$

where M_{total} is the total mass of the sphere.



To calculate M_{total}, we consider an infinitesimal shell element of inner radius r and outer radius $r+dr$. If dm be the mass of this infinitesimal element, then

$dm=(4\pi r^2dr)\rho$

But, $\rho=\dfrac{\rho_0R}{r}$

$\Rightarrow \quad dm=\left(\dfrac{\rho_0R}{r}\right)(4\pi r^2dr)$

$\Rightarrow \quad dm=(4\pi\rho_0R)(rdr)$

$\Rightarrow \quad M_{\text{total}}=\int dm=4\pi\rho_0R\int_0^R rdr$

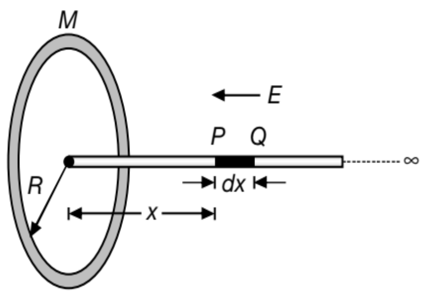
$\Rightarrow \quad M_{\text{total}}=\dfrac{4\pi\rho_0R^3}{2}=2\pi\rho_0R^3$

$\Rightarrow \quad E(r)=\dfrac{GM}{r^2}$

$\Rightarrow \quad E=\dfrac{G(2\pi\rho_0R^3)}{4R^2}=\dfrac{\pi RG\rho_0}{2}$

10. Field strength at the axis at distance x from the centre of the ring is,

$E=\dfrac{GMx}{(R^2+x^2)^{3/2}}$



Consider an infinitesimal mass element PQ of length dx, mass dm at a distance x from centre of ring. Then

$dm=\lambda dx$

Force on this element will be,

$dF=Edm=\dfrac{GM\lambda x}{(R^2+x^2)^{3/2}}dx$

$\Rightarrow \quad F=\int_0^\infty dF=\int_0^\infty\dfrac{GM\lambda xdx}{(R^2+x^2)^{3/2}}$

Since $\int \frac{dx}{\sqrt{R^2+x^2}} = -\frac{1}{\sqrt{R^2+x^2}}$

$$\Rightarrow \quad F = -GM\lambda\left(\frac{1}{\sqrt{R^2+x^2}}\Bigg|_0^\infty\right)$$

$$\Rightarrow \quad F = -GM\lambda\left(\frac{1}{\infty}-\frac{1}{R}\right)$$

$$\Rightarrow \quad F = \frac{GM\lambda}{R}$$

11. At the equator, we have

$$g_e = g - R\omega^2$$

60% of his weight at the pole means

$$g_e = \frac{60}{100}g = \frac{3}{5}g$$

$$\Rightarrow \quad \frac{3}{5}g = g - R\omega^2$$

$$\Rightarrow \quad \frac{2}{5}g = R\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2}{5}\frac{g}{R}}$$

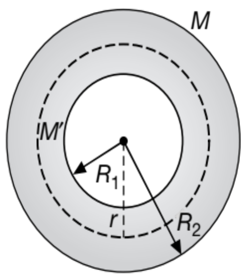
$$\Rightarrow \quad \omega = \sqrt{\frac{2\times 9.8}{5\times 6.4\times 10^6}}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{19.6}{32\times 10^6}}$$

$$\Rightarrow \quad \omega = 7.8\times 10^{-4}\ \text{rads}^{-1}$$

12. For $r < R_1$, we have $E_r = 0$

For $r > R_2$, we have, $E_r = \frac{GM}{r^2}$



Inside i.e., for $R_1 < r < R_2$, we have

$$E_r = \frac{GM'}{r^2}$$

where M' is mass of shell from R_1 to r. So,

$$M' = \frac{4}{3}\pi\left(r^3 - R_1^3\right)\rho$$

where $\rho = \frac{M}{\frac{4}{3}\pi\left(R_2^3 - R_1^3\right)}$ ...(1)

$$\Rightarrow \quad E_r = \frac{G\left[\frac{4}{3}\pi\left(r^3 - R_1^3\right)\rho\right]}{r^2}$$

From (1), we get

$$E_r = \frac{GM}{r^2}\left(\frac{r^3 - R_1^3}{R_2^3 - R_1^3}\right)$$

$$\text{So, } E_r = \begin{cases} 0 & \text{for} \quad r < R_1 \\ \frac{GM}{r^2}\left(\frac{r^3 - R_1^3}{R_2^3 - R_1^3}\right) & \text{for} \quad R_1 < r < R_2 \\ \frac{GM}{r^2} & \text{for} \quad r > R_2 \end{cases}$$

13. Reduced by 36% means the value is 64% the original

$$\Rightarrow \quad g_h = \frac{64}{100}g$$

Since $g_h = \frac{gR^2}{(R+h)^2}$

$$\Rightarrow \quad \frac{64}{100} = \left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \quad \frac{R}{R+h} = \frac{8}{10}$$

$$\Rightarrow \quad 10R = 8R + 8h$$

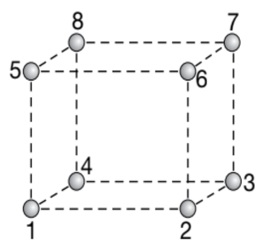
$$\Rightarrow \quad h = \frac{R}{4} = 1600\ \text{km}$$

Test Your Concepts-II (Based on Gravitational Potential, Potential Energy and Applications)

1. First of all let us calculate the total number of interactions between these eight particles. Since, total number of interactions (N) between n particles is

$$N = {}^nC_2 = \frac{n(n-1)}{2}$$

$$\Rightarrow \quad N = \frac{8(8-1)}{2} = 28$$



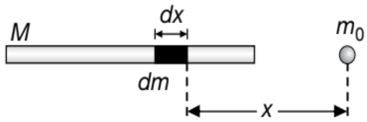
Out of these 28 interactions, 12 interactions are due to masses at separation a, 12 interactions are due to masses at separation $\sqrt{2}a$ (the face diagonal) and 4 interactions are due to masses at separation $\sqrt{3}a$ (the body diagonal). So,

$$U=-12\left(\frac{Gm^2}{a}\right)-12\left(\frac{Gm^2}{\sqrt{2}a}\right)-4\left(\frac{Gm^2}{\sqrt{3}a}\right)$$

$$\Rightarrow\quad U=-\frac{Gm^2}{a}\left(12+\frac{12}{\sqrt{2}}+\frac{4}{\sqrt{3}}\right)$$

2. Consider an infinitesimal element of the rod of length dx at a distance x from m_0. If dm be the mass of the infinitesimal element, then

$$dm=\frac{M}{L}dx$$



The gravitational potential energy of the infinitesimal mass dm and the point mass m_0 is

$$dU=-\frac{Gm_0dm}{x}=-\frac{Gm_0\left(\frac{M}{L}dx\right)}{x}$$

$$\Rightarrow\quad U=\int dU=-\frac{GMm_0}{L}\int_a^{a+L}\frac{dx}{x}$$

$$\Rightarrow\quad U=-\frac{GMm_0}{L}\log_e\left(\frac{a+L}{a}\right)$$

3. $U=-\dfrac{GMm}{R}$

So, the binding energy is $|U|=\dfrac{GMm}{R}$

i.e., this much energy is required to displace the particle from the centre of the ring to infinity.

4. Outside the sphere, we have

$$U=-\frac{GMm}{r}$$

as if the sphere were a point mass concentrated at its centre.

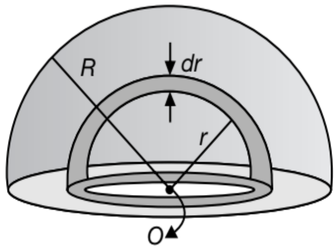
Even if the sphere is replaced by a thin shell, the gravitational potential energy of the particle-shell system will remain the same, because for the shell to the entire mass is concentrated at the centre, so

$$U=-\frac{GMm}{r}$$

5. Consider an elementary hemispherical shell of radius r and thickness dr. If dm is mass of this shell, then

$$dm=\frac{M}{\frac{1}{2}\left(\frac{4}{3}\pi R^3\right)}\frac{1}{2}4\pi r^2dr$$

$$\Rightarrow\quad dm=\frac{3M}{R^3}r^2dr$$



Initial potential energy at O due to this element and m is

$$U_i=\int dU_i=-\int\frac{Gm\,dm}{r}=-\frac{3GMm}{R^3}\int_0^R r\,dr$$

$$\Rightarrow\quad U_i=-\frac{3}{2}\frac{GMm}{R}$$

So, $W_{\text{external}}=U_f-U_i=0-\left(-\frac{3}{2}\frac{GMm}{R}\right)=\frac{3}{2}\frac{GMm}{R}$

So, the work performed in the process by gravitational force is $\left(-\dfrac{3}{2}\dfrac{GMm}{R}\right)$

6. Let gravitational field be zero at a point lying at distance x from M. Then,

$$\frac{GM}{x^2}=\frac{Gm}{(d-x)^2}$$

$$\Rightarrow\quad \frac{d-x}{x}=\sqrt{\frac{m}{M}}$$

$$\Rightarrow\quad \frac{d}{x}-1=\sqrt{\frac{m}{M}}$$

$$\Rightarrow\quad x=\left(\frac{\sqrt{M}}{\sqrt{M}+\sqrt{m}}\right)d \qquad \ldots(1)$$

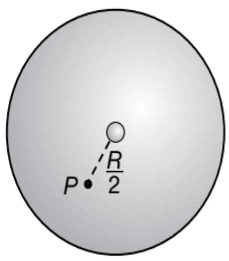
$$\Rightarrow\quad (d-x)=\left(\frac{\sqrt{m}}{\sqrt{M}+\sqrt{m}}\right)d \qquad \ldots(2)$$

Since, $V_P=-\dfrac{Gm}{d-x}-\dfrac{GM}{x}$ ...(3)

Substituting (1) and (2) in (3), we get

$$V_P=-\frac{G}{d}\left(\sqrt{m}+\sqrt{M}\right)^2$$

7. $V_P=\left(\begin{array}{c}\text{Potential due to}\\ \text{shell at P}\end{array}\right)+\left(\begin{array}{c}\text{Potential due to}\\ \text{particle at P}\end{array}\right)$



Since point P lies inside the shell, so the potential inside the shell is constant and equals the value at the surface i.e., $-\frac{GM}{R}$.

$$\Rightarrow \quad V_P = -\frac{GM}{R} - \frac{GM}{(R/2)}$$

$$\Rightarrow \quad V_P = -\frac{3M}{R}$$

8. (a) Since, $V(m) = -\frac{Gm}{R}$

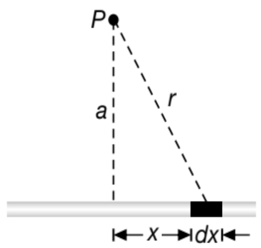
$$\Rightarrow \quad dW = V(dm) = -\left(\frac{Gm}{R}\right)dm$$

$$\Rightarrow \quad W = \int_0^M dW = -\frac{GM^2}{2R} = \text{self energy}$$

(b) See theory.

9. (a) Consider a mass element of length dx, mass dm at a distance x from the centre of rod. Then

$dm = \lambda dx$



The potential at the point P due to this infinitesimal element is

$$dV = -\frac{Gdm}{r} = -\frac{Gdm}{\sqrt{a^2+x^2}}$$

$$\Rightarrow \quad dV = -\frac{G\lambda dx}{\sqrt{a^2+x^2}}$$

$$\Rightarrow \quad V = \int dV = -G\lambda \int_{-\ell}^{\ell} \frac{dx}{\sqrt{a^2+x^2}}$$

Since $\int_{-\ell}^{\ell} f(x)dx = 2\int_0^{\ell} f(x)dx$, iff $f(x)$ is even

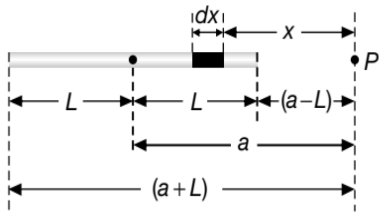
$$\Rightarrow \quad V = -2G\lambda \int_0^{\ell} \frac{dx}{\sqrt{a^2+x^2}}$$

Also, $\int \frac{dx}{\sqrt{a^2+x^2}} = \log_e\left(x+\sqrt{a^2+x^2}\right)$

$$V = -2G\lambda\left[\log_e\left(x+\sqrt{x^2+a^2}\right)\Big|_0^{\ell}\right]$$

$$\Rightarrow \quad V = -2G\lambda \log_e\left(\frac{\ell+\sqrt{\ell^2+a^2}}{a}\right)$$

(b) Again consider an infinitesimal element of length dx, mass dm at a distance x from P. Then $dm = \lambda dx$



If dV be the gravitational potential due to this element at the point P, then

$$dV = -\frac{Gdm}{x}$$

$$\Rightarrow \quad dV = -\frac{G(\lambda dx)}{x}$$

$$\Rightarrow \quad V = -G\lambda \int_{a-L}^{a+L} \frac{dx}{x}$$

$$\Rightarrow \quad V = -G\lambda\left(\log_e x\Big|_{a-L}^{a+L}\right)$$

$$\Rightarrow \quad V = -G\lambda \log_e\left(\frac{a+L}{a-L}\right)$$

Test Your Concepts-III (Based on Relation between Gravitational Field and Potential)

1. Since, $dV = -\vec{E}\cdot d\vec{\ell}$

$$\Rightarrow \quad \int_B^A dV = -\int_{(0,2,4)}^{(2,1,0)} \left(x\hat{i} - 2y\hat{j} + z\hat{k}\right)\cdot\left(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}\right)$$

$$\Rightarrow \quad V_A - V_B = -\int_{(0,2,4)}^{(2,1,0)} (xdx - 2ydy + zdz)$$

$$\Rightarrow \quad V_{AB} = -\left[\left(\frac{x^2}{2} - y^2 + \frac{z^2}{2}\right)\Bigg|_{(0,2,4)}^{(2,1,0)}\right]$$

$$\Rightarrow \quad V_{AB} = 3\ \text{Jkg}^{-1}$$

2. Since, $dV = -\vec{E}\cdot d\vec{\ell}$

$$\Rightarrow \quad dV = -a(ydx + axdy) + b(zdy + ydz)$$

$$\Rightarrow \quad dV = -[ad(xy) + bd(yz)]$$

Integrating we get,

$$V = -(axy + byz) + C$$

CHAPTER 4

3. (a) $dV = -\vec{E}\cdot d\vec{\ell}$

$$\int_B^A dV = -\int_{(1,1,1)}^{(0,0,0)} \left(y\hat{i} + x\hat{j}\right)\cdot\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$\Rightarrow \quad V_A - V_B = -\int_{(1,1,1)}^{(0,0,0)} \left(ydx + xdy\right)$$

$$\Rightarrow \quad V_{AB} = -\int_{(1,1,1)}^{(0,0,0)} d\left(xy\right) \quad \left\{\text{as } y\,dx + xdy = d\left(xy\right)\right\}$$

$$\Rightarrow \quad V_{AB} = -\left[\left(xy\right)\Big|_{(1,1,1)}^{(0,0,0)}\right] = 1\text{ Jkg}^{-1}$$

(b) $dV = -\vec{E}\cdot d\vec{\ell}$

$$\Rightarrow \quad \int_B^A dV = -\int_{(1,1,1)}^{(0,0,0)} \vec{E}\cdot d\vec{\ell}$$

$$\Rightarrow \quad V_A - V_B = -\int_{(1,1,1)}^{(0,0,0)} \left(3x^2 ydx + x^3 dy\right)$$

$$\Rightarrow \quad V_A - V_B = -\int_{(1,1,1)}^{(0,0,0)} d\left(x^3 y\right)$$

$$\Rightarrow \quad V_{AB} = -\left[\left(x^3 y\right)\Big|_{(1,1,1)}^{(0,0,0)}\right] = 1\text{ Jkg}^{-1}$$

Since in both the cases, the line integral of the field is an exact differential and hence both the fields are conservative in nature.

4. Since $\vec{E} = -\left(\hat{i}\dfrac{\partial V}{\partial x} + \hat{j}\dfrac{\partial V}{\partial y} + \hat{k}\dfrac{\partial V}{\partial z}\right)$

where $\dfrac{\partial V}{\partial x} = 6xy$

$\dfrac{\partial V}{\partial y} = 3x^2 + 3y^2 z$

$\dfrac{\partial V}{\partial z} = y^3$

$$\Rightarrow \quad \vec{E} = -\left(6xy\right)\hat{i} - 3\left(x^2 + y^2 z\right)\hat{j} - y^3\hat{k}$$

5. (a) Given, $V = a\left(x^2 - y^2\right)$

So, $\vec{E} = -\vec{\nabla}V = -2a\left(x\hat{i} - y\hat{j}\right)$

(b) Since $V = axy$

So, $\vec{E} = -\vec{\nabla}V = -ay\hat{i} - ax\hat{j}$

6. $V = 5x - 3x^2 y + 2yz^2$

Since $\vec{E} = -\vec{\nabla}V = -\left(\dfrac{\partial V}{\partial x}\hat{i} + \dfrac{\partial V}{\partial y}\hat{j} + \dfrac{\partial V}{\partial z}\hat{k}\right)$

$$\Rightarrow \quad \vec{E} = \left(-5 + 6xy\right)\hat{i} + \left(3x^2 - 2z^2\right)\hat{j} - 4yz\hat{k}$$

$$\Rightarrow \quad \vec{E}\Big|_{(1,\,0,\,-2)} = -5\hat{i} - 5\hat{j}$$

$$\Rightarrow \quad E = \sqrt{E_x^2 + E_y^2 + E_z^2}$$

$$E = \sqrt{(-5)^2 + (-5)^2 + 0^2} = 5\sqrt{2}\text{ Nkg}^{-1}$$

7. Since, $\vec{E}_g = -\left(\hat{i}\dfrac{\partial V}{\partial x} + \hat{j}\dfrac{\partial V}{\partial y}\right)$

where $\dfrac{\partial V}{\partial x} = 20$

and $\dfrac{\partial V}{\partial y} = 20$

$$\Rightarrow \quad \vec{E}_g = -20\left(\hat{i} + \hat{j}\right)$$

Since, $\vec{F} = m\vec{E}_g = -(0.5)(20)\left(\hat{i} + \hat{j}\right)$

$$\Rightarrow \quad \vec{F} = -10\left(\hat{i} + \hat{j}\right)$$

$$\Rightarrow \quad \left|\vec{F}\right| = 10\sqrt{2}\text{ N}$$

8. Given, $V = a\left(x^2 + y^2\right) + bz^2$

Since, $\vec{E} = -\vec{\nabla}V$

$$\Rightarrow \quad \vec{E} = -\left(2ax\hat{i} + 2ay\hat{j} + 2bz\hat{k}\right)$$

Hence $\left|\vec{E}\right| = 2\sqrt{a^2\left(x^2 + y^2\right) + b^2 z^2}$

9. $\vec{E} = -\left(\dfrac{\partial V}{\partial x}\hat{i} + \dfrac{\partial V}{\partial y}\hat{j} + \dfrac{\partial V}{\partial z}\hat{k}\right)$

where, $\dfrac{\partial V}{\partial x} = \dfrac{\partial}{\partial x}\left(2x + 3y - z\right) = 2$

$\dfrac{\partial V}{\partial y} = \dfrac{\partial}{\partial y}\left(2x + 3y - z\right) = 3$

$\dfrac{\partial V}{\partial z} = \dfrac{\partial}{\partial z}\left(2x + 3y - z\right) = -1$

$$\Rightarrow \quad \vec{E} = -2\hat{i} - 3\hat{j} + \hat{k}$$

Test Your Concepts-IV (Based on Conservation Laws, Escape Velocity and Applications)

1. (a) Increase in potential energy is $\Delta U = U_f - U_i$

$$\Rightarrow \Delta U = -\frac{GmM}{R+nR} - \left(-\frac{GmM}{R}\right)$$

$$\Rightarrow \Delta U = \frac{GmM}{R}\left(1 - \frac{1}{1+n}\right)$$

$$\Rightarrow \Delta U = \left(\frac{n}{n+1}\right)\left(\frac{GM}{R^2}\right)mR$$

Since $g = \frac{GM}{R^2}$

$$\Rightarrow \Delta U = \left(\frac{n}{n+1}\right)mgR$$

(b) By Law of Conservation of Energy,

$$\begin{pmatrix}\text{Gain in GPE}\\ \text{of } m\end{pmatrix} = \begin{pmatrix}\text{Loss in KE}\\ \text{of } m\end{pmatrix}$$

$$\Rightarrow \left(\frac{n}{n+1}\right)mgR = \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{\frac{2ngR}{n+1}}$$

2. $(U+K)_{\text{at }\infty} = (U+K)_{\text{surface}}$

$$0 + 0 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{\frac{2GM}{R}}$$

$$\Rightarrow v = \sqrt{2\left(\frac{GM}{R^2}\right)R}$$

$$\Rightarrow v = \sqrt{2gR} = 11.2 \text{ kms}^{-1}$$

3. By Law of Conservation of Energy, we have

$$(U+K)_{\text{at 1 m}} = (U+K)_{\text{at 0.5 m}}$$

$$-\frac{Gm_1m_2}{r_1} = -\frac{Gm_1m_2}{r_2} + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \ldots(1)$$

Further by Law of Conservation of Linear Momentum, we have

$$m_1(0) + m_2(0) = m_1v_1 + m_2(-v_2)$$

$$\Rightarrow m_1v_1 = m_2v_2 \quad \ldots(2)$$

$$\Rightarrow Gm_1m_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2\left(\frac{m_1v_1}{m_2}\right)^2$$

$$\Rightarrow Gm_1m_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{1}{2}m_1v_1^2\left(1 + \frac{m_1}{m_2}\right)$$

$$\Rightarrow v_1 = \sqrt{\frac{2Gm_2^2}{m_1+m_2}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}$$

$$\Rightarrow v_1 = \sqrt{\frac{2G(100)}{30}\left(\frac{1}{0.5} - \frac{1}{1}\right)}$$

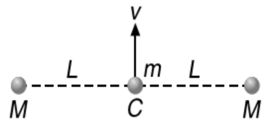
$$\Rightarrow v_1 = \sqrt{\frac{20G}{3}}$$

$$\Rightarrow v_1 = 2.1 \times 10^{-5} \text{ ms}^{-1}$$

From (2), we get

$$v_2 = 4.2 \times 10^{-5} \text{ ms}^{-1}$$

4. Let v is the minimum velocity, then by Law of Conservation of Energy, we have



$$(U+K)_C = (U+K)_\infty$$

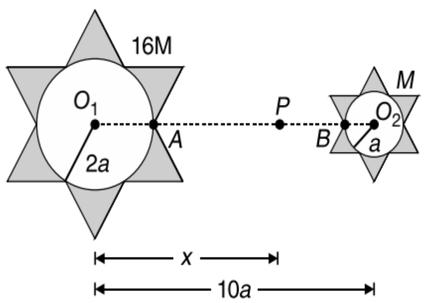
$$\Rightarrow \left(-\frac{GMm}{L}\right)2 + \frac{1}{2}mv^2 = 0 + 0$$

$$\Rightarrow v = 2\sqrt{\frac{GM}{L}}$$

5. Our first job is to find a point where the resultant field due to both is zero. Let the point P be at a distance x from centre of bigger star.

$$\Rightarrow \frac{G(16M)}{x^2} = \frac{GM}{(10a-x)^2}$$

$$\Rightarrow x = 8a \qquad \{\text{from } O_1\}$$



i.e. once the body reaches P the gravitational pull of attraction due to $16M$ vanishes and the gravitation pull due to M takes the lead to make m move towards it automatically.

i.e. a minimum K.E. or velocity has to be imparted to m from surface of $16M$ such that it is just able to overcome the gravitational pull of $16M$.

By Law of Conservation of Energy.

$$\begin{pmatrix}\text{Total Mechanical}\\ \text{Energy at } A\end{pmatrix} = \begin{pmatrix}\text{Total Mechanical}\\ \text{Energy at } P\end{pmatrix}$$

Total mechanical energy at A is

$$E_A = \frac{1}{2}mv_{\text{min}}^2 + \left[-\frac{G(16M)m}{2a} - \frac{GMm}{8a}\right]$$

$$\Rightarrow \quad E_A = \frac{1}{2}mv_{\text{min}}^2 - \frac{GMm}{2a}\left(16 + \frac{1}{4}\right)$$

$$\Rightarrow \quad E_A = \frac{1}{2}mv_{\text{min}}^2 - \frac{65GMm}{8a}$$

Total mechanical energy at B is

$$E_B = \left(-\frac{GMm}{2a} - \frac{G(16M)m}{8a}\right)$$

$$\Rightarrow \quad E_B = -\frac{GMm}{2a}(1+4) = -\frac{5GMm}{2a}$$

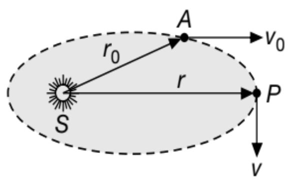
$$\Rightarrow \quad \frac{1}{2}mv_{\text{min}}^2 = \frac{GMm}{8a}(45)$$

$$\Rightarrow \quad v_{\text{min}} = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

6. At minimum and maximum distances, the velocity vector $(\vec{v})$ makes an angle of $90°$ with radius vector. Then, by Law of Conservation of Angular Momentum, we have

$$mv_0 r_0 \sin\phi = mvr \qquad \ldots(1)$$

where, m is the mass of the planet.



Further by Law of Conservation of Energy, we have

$$(U+K)_{\text{at } A} = (U+K)_P$$

$$\Rightarrow \quad \frac{mv_0^2}{2} - \frac{GMm}{r_0} = \frac{mv^2}{2} - \frac{GMm}{r} \qquad \ldots(2)$$

where, M is the mass of the sun.
Solving equations (1) and (2) for r using the concept of quadratic equations we get two values of r, one is r_{max} and another is r_{min}. So,

$$r_{\text{max}} = \frac{r_0}{2-K}\left(1 + \sqrt{1 - K(2-K)\sin^2\phi}\right)$$

and $r_{\text{min}} = \frac{r_0}{2-K}\left(1 - \sqrt{1 - K(2-K)\sin^2\phi}\right)$

where, $K = \frac{v_0^2 r_0^2}{GM}$

7. $E = U + K$

$$\Rightarrow \quad E = -\frac{GMm}{R} + \frac{1}{2}mv_e^2$$

But $v_e^2 = \frac{2GM}{R}$

$$\Rightarrow \quad E = -\frac{GMm}{R} + \frac{1}{2}m\left(\frac{2GM}{R}\right)$$

$$\Rightarrow \quad E = 0$$

8. Given that $v_e = 11.2 \text{ kms}^{-1} = \sqrt{\frac{2GM_e}{R_e}}$

By Law of Conservation of Energy, we have

$$K_i + U_i = K_f + U_f$$

$$\Rightarrow \quad \frac{1}{2}mv_s^2 - \frac{GM_s m}{r} - \frac{GM_e m}{R_e} = 0 + 0$$

where, r is the distance of rocket from sun

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2GM_s}{r}}$$

Since, $M_s = 3\times10^5 M_e$ and $r = 2.5\times10^4 R_e$

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2G(3\times10^5 M_e)}{2.5\times10^4 R_e}}$$

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e}\left(1 + \frac{3\times10^5}{2.5\times10^4}\right)}$$

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e}\times13}$$

$$\Rightarrow \quad v_s \approx 42 \text{ kms}^{-1}$$

9. Since the gravitational potential energy between the disc of radius a, mass m_1 and a particle of mass m_2 placed at axis of disc at a distance ℓ from centre is

$$U_{\text{axis}} = -\frac{2Gm_1m_2}{a^2}\left(\sqrt{a^2+\ell^2} - \ell\right) \qquad \ldots(1)$$

$$\Rightarrow \quad U_{\text{axis}} = -\frac{2Gm_1m_2\ell}{a^2}\left[\left(1 + \frac{a^2}{\ell^2}\right)^{\frac{1}{2}} - 1\right]$$

Since $\frac{a}{\ell}$ 1, $\left(1 + \frac{a^2}{\ell^2}\right)^{1/2}$ $1 + \frac{a^2}{2\ell^2}$

$$\Rightarrow \quad U_{\text{axis}} = -\left(\frac{2Gm_1m_2\ell}{a^2}\right)\left(\frac{a^2}{2\ell^2}\right)$$

$$\Rightarrow \quad U_{\text{axis}} = -\frac{Gm_1m_2}{\ell}$$

Further, when the particle collides with the disc (at its centre i.e., $\ell = 0$), then

$$U_{\text{centre}} = -\frac{2Gm_1m_2}{a} \qquad \{\because \text{ of } (1)\}$$

Applying Law of Conservation of Energy and using the concept of reduced mass we get

$$(U+K)_{\text{axis}} = (U+K)_{\text{centre}}$$

$$\Rightarrow \quad -\frac{Gm_1m_2}{\ell} + 0 = -2\left(\frac{Gm_1m_2}{a}\right) + \frac{1}{2}\mu v_r^2$$

where $\mu = \dfrac{m_1m_2}{m_1+m_2}$

$$\Rightarrow \quad Gm_1m_2\left(\frac{2}{a} - \frac{1}{\ell}\right) = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)v_r^2$$

$$\Rightarrow \quad v_r = \sqrt{2G(m_1+m_2)\left(\frac{2}{a} - \frac{1}{\ell}\right)}$$

10. $v_{es} = \sqrt{\dfrac{2GM}{R}} = \sqrt{\dfrac{2G\left(\frac{4}{3}\pi R^3\right)\rho}{R}} = \sqrt{\dfrac{4G\rho}{3}}R$

$v_{es} \propto R$

Surface area of 1 is $A = 4\pi R_1^2$

Surface area of 2 is $4A = 4\pi R_2^2$

$\Rightarrow \quad R_2 = 2R_1$

Given that mass of planet 3 is $M_3 = M_1 + M_2$

$$\Rightarrow \quad \left(\frac{4}{3}\pi R_3^3\right)\rho = \left(\frac{4}{3}\pi R_1^3\right)\rho + \left(\frac{4}{3}\pi R_2^3\right)\rho$$

$\Rightarrow \quad R_3^3 = R_1^3 + R_2^3$

$\Rightarrow \quad R_3^3 = 9R_1^3$

$\Rightarrow \quad R_3 = 9^{1/3}R_1$

$\Rightarrow \quad R_3 > R_2 > R_1$

$\Rightarrow \quad v_3 > v_2 > v_1$

$\Rightarrow \quad \dfrac{v_3}{v_1} = 9^{1/3}$ and $\dfrac{v_2}{v_1} = 2$

$\Rightarrow \quad \dfrac{v_3^3}{v_1^3} = 9$ and $\dfrac{v_2}{v_1} = 2$

Test Your Concepts-V (Based on Satellites, Kepler's Laws and Applications)

1. $r_A = R + R = 2R$

and $r_B = R + 3R = 4R$

Since, kinetic energy $K = \dfrac{GMm}{2r}$

$\Rightarrow \quad K \propto \dfrac{1}{r}$

$\Rightarrow \quad \dfrac{K_A}{K_B} = \dfrac{r_B}{r_A} = \dfrac{4R}{2R} = 2$

Also, $U = -\dfrac{GMm}{r}$

$\Rightarrow \quad |U| \propto \dfrac{1}{r}$

$\Rightarrow \quad \dfrac{U_A}{U_B} = \dfrac{r_B}{r_A} = 2$

2. (a) Just before the explosion, the orbital velocity is $v_0 = \sqrt{\dfrac{GM}{R}}$ corresponding to which the total energy associated with the satellite in the orbit is NEGATIVE.

When a mass Δm is expelled very rapidly with a speed v, then for the satellite to still remain within the gravitational pull of the planet, the new total energy must still be negative. So

$$\frac{1}{2}(m-\Delta m)(v_0+v_r)^2 - \frac{GM(m-\Delta m)}{R} \le 0 \qquad \ldots(1)$$

$\Rightarrow \quad (v_0+v_r) \le \dfrac{2GM}{R}$

$\Rightarrow \quad v_0 + v_r \le \sqrt{\dfrac{2GM}{R}}$

$\Rightarrow \quad \sqrt{\dfrac{GM}{R}} + v_r \le \sqrt{\dfrac{2GM}{R}} \qquad \left\{\because v_0 = \sqrt{\dfrac{GM}{R}}\right\}$

$\Rightarrow \quad v_r \le (\sqrt{2}-1)\sqrt{\dfrac{GM}{R}}$

(b) Since the term $(m-\Delta m)$ cancels (see equation (1)), so, no modification is needed.

3. Since $T^2 = 4\pi^2\left(\dfrac{r^3}{GM}\right)$

When the satellite is revolving close to the planet, then $r \cong R$

$\Rightarrow \quad T^2 = 4\pi^2\left(\dfrac{R^3}{GM}\right)$

Since $M = \left(\dfrac{4}{3}\pi R^3\right)\rho$

$\Rightarrow \quad T^2 = 4\pi^2 \dfrac{R^3}{G\left(\frac{4}{3}\pi R^3\right)\rho}$

$\Rightarrow \quad \rho T^2 = \text{constant}$

4. For the satellite to revolve in a circular orbit of radius r_0 the orbital velocity is given by

$$v_0 = \sqrt{\frac{GM}{r_0}} \qquad \ldots(1)$$

At maximum or minimum distances, velocity is perpendicular to the radius vector. So, on applying Conservation of Angular Momentum and Mechanical Energy, we get

CHAPTER 4

$$mv_0r_0\cos\alpha = mvr \qquad \ldots(2)$$

and $$-\frac{GMm}{2r_0} = -\frac{GMm}{r} + \frac{1}{2}mv^2 \qquad \ldots(3)$$

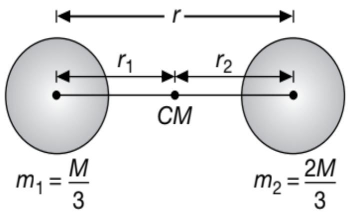
Solving these three equations, we get two values of r i.e., $(1+\sin\alpha)r_0$ and $(1-\sin\alpha)r_0$. Therefore,

$$r_{max} = (1+\sin\alpha)r_0 \text{ and } r_{min} = (1-\sin\alpha)r_0$$

5. Let M be the mass of the sun and r be the distance between the two stars, then

$$r_1 = \frac{m_2}{m_1+m_2}r = \frac{2}{3}r$$

and $$r_2 = \frac{m_1}{m_1+m_2}r = \frac{r}{3}$$



Centripetal force on m_2 is $\dfrac{Gm_1m_2}{r^2} = \dfrac{G\left(\frac{2}{9}\right)M^2}{r^2}$

$$\Rightarrow \frac{2}{9}\frac{GM^2}{r^2} = m_2r_2\omega^2 = \left(\frac{2}{3}M\right)\left(\frac{r}{3}\right)\omega^2$$

$$\Rightarrow \omega^2 = \frac{GM}{r^3}$$

$$\Rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{GM}{r^3}$$

$$\Rightarrow T^2 = \frac{4\pi^2r^3}{GM}$$

Since time period of earth around sun is

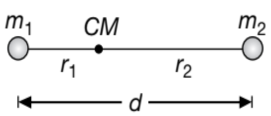
$$T^2 = \frac{4\pi^2R^3}{GM}$$

$$\Rightarrow r = R$$

6. (a) Let the origin be at m_1 and the centre of mass be at a distance r_1 from it. Then

$$r_1 = \frac{m_1(0)+m_2r}{m_1+m_2}$$

$$\Rightarrow r_1 = \left(\frac{m_2}{m_1+m_2}\right)r \text{ and}$$



The centripetal force is provided by gravitational force, so

$$m_1r_1\omega^2 = m_2r_2\omega^2 = \frac{Gm_1m_2}{d^2} \qquad \ldots(1)$$

Solving these equations, we get

$$\omega = \sqrt{\frac{G(m_1+m_2)}{d^3}}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{d^3}{G(m_1+m_2)}}$$

(b) $$\frac{K_1}{K_2} = \frac{\frac{1}{2}I_1\omega^2}{\frac{1}{2}I_2\omega^2} = \frac{I_1}{I_2} = \frac{m_1r_1^2}{m_2r_2^2}$$

$$\Rightarrow \frac{K_1}{K_2} = \left(\frac{m_1}{m_2}\right)\left(\frac{r_1}{r_2}\right)^2$$

Since, $m_1r_1\omega^2 = m_2r_2\omega^2$

$$\Rightarrow m_1r_1 = m_2r_2$$

$$\Rightarrow \frac{K_1}{K_2} = \left(\frac{m_1}{m_2}\right)\left(\frac{m_2}{m_1}\right)^2 = \frac{m_2}{m_1}$$

(c) $$\frac{L_1}{L_2} = \frac{I_1\omega}{I_2\omega} = \frac{I_1}{I_2} = \frac{m_2}{m_1}$$

(d) $$L = L_1 + L_2 = (I_1+I_2)\omega$$

$$\Rightarrow L = (m_1r_1^2 + m_2r_2^2)\omega$$

$$\Rightarrow L = \left[\frac{m_1m_2^2d^2}{(m_1+m_2)^2} + \frac{m_2m_1^2d^2}{(m_1+m_2)^2}\right]\omega$$

$$\Rightarrow L = \mu\omega d^2$$

where $\mu = \dfrac{m_1m_2}{m_1+m_2}$ is the reduced mass

(e) $$K = \frac{1}{2}(I_1+I_2)\omega^2 = \frac{1}{2}\mu\omega^2d^2$$

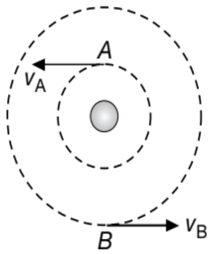
7. For two satellites revolving around same planet, we have

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

$$\Rightarrow T_2 = T_1\left(\frac{r_2}{r_1}\right)^{3/2} = 28\left(\frac{2\times10^4}{10^4}\right)^{3/2} = 56\sqrt{2} \text{ hour}$$

Since, $\omega = \dfrac{2\pi}{T}$

$$\Rightarrow \omega_1 = \frac{2\pi}{28} \text{ radhr}^{-1} \text{ and } \omega_2 = \frac{2\pi}{56\sqrt{2}} \text{ radhr}^{-1}$$



Positions corresponding to maximum separation are shown in figure, so we have

$$v_{BA} = \vec{v}_B - \vec{v}_A = |v_B| + |v_A| \quad \text{\{for shown position\}}$$

$$\Rightarrow \quad v_{BA} = \omega_2 r_2 + \omega_1 r_1$$

$$\Rightarrow \quad v_{BA} = \left[\frac{2\pi}{56\sqrt{2}} \times 2 \times 10^4 + \frac{2\pi}{28} \times 10^4\right] \text{kmhr}^{-1}$$

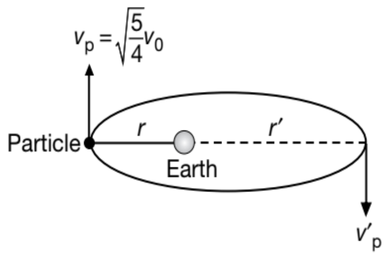
$$\Rightarrow \quad v_{BA} = (0.1587 + 0.2242) \times 10^4 \text{ kmhr}^{-1}$$

$$\Rightarrow \quad v_{BA} = 3829 \text{ kmhr}^{-1}$$

8. Orbital speed of the satellite is $v_0 = \sqrt{\frac{GM}{r}}$, where M is the mass of earth ...(1)

Absolute velocity of particle would be:

$$v_p = v + v_0 = \sqrt{\frac{5}{4}} v_0 = \sqrt{1.25} v_0 \quad \ldots(2)$$



Since, v_p lies between orbital velocity and escape velocity, path of the particle would be an ellipse with r being the minimum distance.

Let r' be the maximum distance and v_p' be the velocity at that moment.

Then applying the Law of Conservation of Angular Momentum and Conservation of Mechanical Energy, we get

$$mv_p r = mv_p' r' \quad \ldots(3)$$

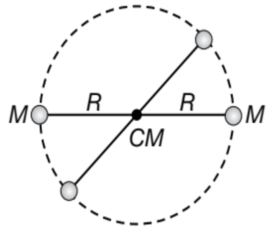
$$\text{and} \quad \frac{1}{2} mv_p^2 - \frac{GMm}{r} = \frac{1}{2} mv_p'^2 - \frac{GMm}{r'} \quad \ldots(4)$$

Solving the above equations (1), (2), (3) and (4), we get

$$r' = \frac{5r}{3} \text{ and } r' = r$$

Hence, the maximum and minimum distances are $\frac{5r}{3}$ and r respectively.

9. (a) $F = \frac{G(M)(M)}{(2R)^2} = \frac{GM^2}{4R^2}$



(b) $F = MR\omega^2$

$$\Rightarrow \quad \frac{GM^2}{4R^2} = MR\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{F}{MR}} = \sqrt{\frac{GM}{4R^3}}$$

$$\Rightarrow \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{4R^3}{GM}} = 4\pi\sqrt{\frac{R^3}{GM}}$$

(c) $E = -\frac{G(M)(M)}{2R} + 2\left(\frac{1}{2}I\omega^2\right) = -\frac{GM^2}{2R} + I\omega^2$

$$\Rightarrow \quad E = -\frac{GM^2}{2R} + (MR^2)\left(\frac{GM}{4R^3}\right) = -\frac{GM^2}{4R}$$

So, binding energy is $|E| = \frac{GM^2}{4R}$

Hence, this much amount of energy will be required to separate the two stars to infinity.

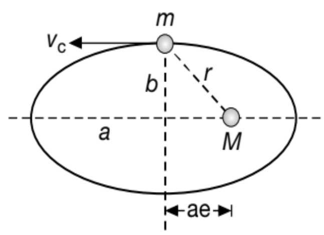
10. Since, we observe that

$$r^2 = a^2e^2 + b^2$$

But $b^2 = a^2(1 - e^2)$

$$\Rightarrow \quad r^2 = a^2e^2 + a^2 - a^2e^2$$

$$\Rightarrow \quad r = a$$



Total energy of a satellite in an elliptical orbit is,

$$E = -\frac{GMm}{2a}$$

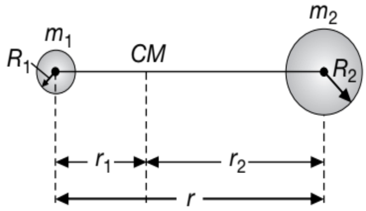
$$\Rightarrow \quad E_C = \frac{1}{2}mv_c^2 - \frac{GMm}{r} = -\frac{GMm}{2a}$$

$$\Rightarrow \quad \frac{1}{2}mv_c^2 - \frac{GMm}{a} = -\frac{GMm}{2a} \quad \{r = a\}$$

$$\Rightarrow \quad v_c = \sqrt{\frac{GM}{a}}$$

$$\Rightarrow \quad v_c = R\sqrt{\frac{g}{a}} \qquad \{\text{as } GM = gR^2\}$$

11. Both the stars rotate about their centre of mass.



For the position of CM let Origin be at m_1 and CM be at distance r_1 from origin. Then

$$r_1 = \frac{m_2 r}{m_1 + m_2} \text{ and } r_2 = \frac{m_1 r}{m_1 + m_2}$$

Also, $m_1 r_1 \omega^2 = \dfrac{G m_1 m_2}{r^2}$ $\qquad \left\{\because \omega = \dfrac{2\pi}{T}\right\}$

Since, $r_1 = \dfrac{m_2 r}{m_1 + m_2}$

$$\Rightarrow \quad \omega^2 = \frac{G(m_1 + m_2)}{r^3}$$

$$\Rightarrow \quad r = \left[\frac{G(m_1+m_2)}{\omega^2}\right]^{1/3} \qquad \ldots(1)$$

Applying Law of Conservation of Mechanical Energy, we get

$$-\frac{G m_1 m_2}{r} = -\frac{G m_1 m_2}{(R_1 + R_2)} + \frac{1}{2}\mu v_r^2 \qquad \ldots(2)$$

where, μ is the reduced mass given by $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$

and v_r is the relative velocity between the two stars

From equation (2), we get

$$v_r^2 = \frac{2 G m_1 m_2}{\mu}\left(\frac{1}{R_1 + R_2} - \frac{1}{r}\right)$$

$$\Rightarrow \quad v_r^2 = \frac{2 G m_1 m_2}{\left(\dfrac{m_1 m_2}{m_1 + m_2}\right)}\left(\frac{1}{R_1 + R_2} - \frac{1}{r}\right)$$

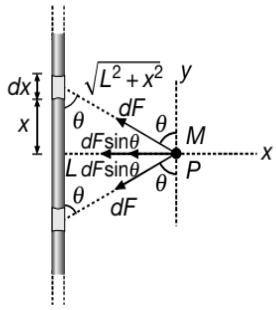
$$\Rightarrow \quad v_r^2 = 2G(m_1 + m_2)\left(\frac{1}{R_1 + R_2} - \frac{1}{r}\right)$$

Substituting the value of r from equation (1), we get

$$v_r = \sqrt{2G(m_1 + m_2)\left[\frac{1}{R_1 + R_2} - \left\{\frac{4\pi^2}{G(m_1 + m_2)T^2}\right\}^{1/3}\right]}$$

Single Correct Choice Type Questions

1.



Let the mass M be placed symmetrically.

$$\Rightarrow \quad F_{\text{net}} = \int_{-\infty}^{\infty} dF \sin\theta = \int_{-\infty}^{\infty} \frac{GM(\lambda\, dx)}{x^2 + L^2}\frac{L}{\sqrt{x^2 + L^2}}$$

$$\Rightarrow \quad F_{\text{net}} = GM\lambda L \int_{-\infty}^{\infty} \frac{dx}{(x^2 + L^2)^{3/2}}$$

$$\Rightarrow \quad F_{\text{net}} = \frac{GM\lambda L}{L^2}(2)$$

$$\Rightarrow \quad F_{\text{net}} = \frac{2GM\lambda}{L}$$

Hence, the correct answer is (C).

2. Let M be the mass of the planet and m the mass of satellite. Then

$$m r \omega^2 = \frac{GMm}{r^2}$$

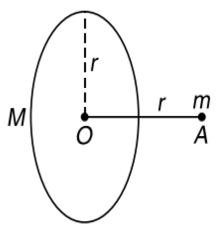
$$\Rightarrow \quad GM = r^2 \omega^2$$

Now, $g = \dfrac{GM}{R^2}$

$$\Rightarrow \quad g = \frac{r^3 \omega^2}{R^2}$$

Hence, the correct answer is (C).

3.



$$(U + K)_A = (U + K)_O$$

$$\Rightarrow \quad -\frac{GMm}{\sqrt{r^2 + r^2}} + 0 = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{GM}{r}\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{2GM}{r}\left(1 - \frac{1}{\sqrt{2}}\right)}$$

Hence, the correct answer is (D).

4. $$F = \int_h^{h+L} \frac{G\left(\frac{M}{L}dx\right)m}{x^2} = \frac{GMm}{L}\int_h^{h+L} x^{-2}dx$$

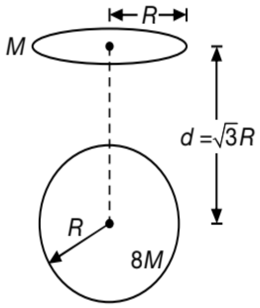
$$\Rightarrow \quad F = \frac{GMm}{L}\left(\frac{x^{-2+1}}{-2+1}\Bigg|_h^{h+L}\right)$$

$$\Rightarrow \quad F = -\frac{GMm}{L}\left(\frac{1}{h+L} - \frac{1}{h}\right)$$

$$\Rightarrow \quad F = \frac{GMm}{h(h+L)}$$

Hence, the correct answer is (C).

5. Gravitational field due to the ring at a distance $d = \sqrt{3}R$ on its axis is



$$E = \frac{GMd}{(R^2 + d^2)^{3/2}} = \frac{\sqrt{3}GM}{8R^2}$$

Force on sphere is $F = (8M)E = \frac{\sqrt{3}GM^2}{R^2}$

Hence, the correct answer is (D).

6. By Law of Conservation of Energy, we get

$$(U + K)_{\text{surface}} = (U + K)_{\text{centre}}$$

Now, for a solid sphere, we have

$$U_{surface} = -\frac{GMm}{R} \text{ and } U_{\text{centre}} = -\frac{3}{2}\frac{GMm}{R}$$

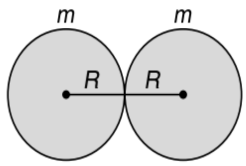
$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}m(0)^2 = -\frac{3}{2}\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = -\frac{GMm}{R} - \left(-\frac{3}{2}\frac{GMm}{R}\right)$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{2}\frac{GMm}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}} = \frac{v_e}{\sqrt{2}}$$

Hence, the correct answer is (D).

7. 

Since $F = \frac{Gm^2}{(2R)^2}$

If ρ be the density of each sphere, then

$$m = \left(\frac{4}{3}\pi R^3\right)\rho$$

$$\Rightarrow \quad F = \frac{G\left(\frac{4}{3}\pi R^3 \rho\right)^2}{4R^2}$$

$$\Rightarrow \quad F = \frac{4}{9}G\pi^2\rho^2R^4$$

$$\Rightarrow \quad F \propto R^4$$

Hence, the correct answer is (C).

8. Since, $g = \frac{GM}{R^2}$ ...(1)

Also, $M = \left(\frac{4}{3}\pi R^3\right)\rho$

$$\Rightarrow \quad R = \left(\frac{3M}{4\pi\rho}\right)^{1/3}$$

Substituting in (1) we get,

$$g = GM\left(\frac{4\pi\rho}{3M}\right)^{2/3}$$

$$\Rightarrow \quad g \propto M^{1/3}\rho^{2/3}$$

$$\Rightarrow \quad g_{\text{planet}} = g(8)^{1/3}(8)^{2/3} = 8g$$

Hence, the correct answer is (C).

9. $F_g \propto \frac{1}{r^2}$ and does not depend on the medium. Hence,

$$F_1' = \frac{F_1}{4}$$

$$F_c \propto \frac{1}{Kr^2}$$

and when air is sucked, force will slightly increase in vacuum, so

$$F_2' > \frac{F_2}{4}$$

Hence, the correct answer is (C).

10. $g\left(1 - \frac{2h}{R}\right) \quad g\left(1 - \frac{d}{R}\right)$

$$\Rightarrow \quad d = 2h$$

Hence, the correct answer is (B).

11. Imagine an inverted hemispherical shell to be placed on this shell so as to complete the spherical shell inside which net gravitational field is zero. Net field can be zero only when field at P is directed along c.

Hence, the correct answer is (C).

12. $(U+K)_{\text{surface}} = (U+K)_{R+7R}$

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{1}{2}mv^2 = -\frac{GMm}{R+7R}+0$$

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{1}{2}mv^2 = -\frac{GMm}{8R}$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{GM}{R}-\frac{GM}{8R} \quad \Rightarrow \quad \frac{v^2}{2} = \frac{7}{8}\left(\frac{GM}{R}\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{7GM}{4R}}$$

Hence, the correct answer is (D).

13. Since, $g_h = g\left(1-\frac{2h}{R}\right)$ $\qquad \{\because h \quad R\}$

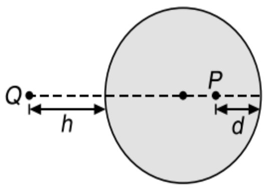
$$\Rightarrow \quad W_h = W\left(1-\frac{2h}{R}\right)$$

Rate of change of weight with height

$$\left|\frac{dW_h}{dh}\right| = \left|0-\frac{2W}{R}\right| = \frac{2mg}{R} \qquad \{\because W = mg\}$$

Hence, the correct answer is (B).

14.



Given that points P and Q located inside and outside the planet have same acceleration due to gravity equal to $\frac{g}{4}$. Thus

$$g\left(1-\frac{d}{R}\right) = \frac{gR^2}{(R+h)^2} = \frac{g}{4}$$

$$\Rightarrow \quad d = \frac{3R}{4} \text{ and } \frac{R}{R+h} = \frac{1}{2}$$

$$\Rightarrow \quad h = R$$

Maximum separation between P and Q will be when these are diametrically opposite. So, maximum separation is

$$r_{\max} = h+R+(R-d)$$

$$\Rightarrow \quad r_{\max} = R+R+\left(R-\frac{3R}{4}\right) = \frac{9R}{4}$$

Hence, the correct answer is (B).

15. $g' = g - R\omega^2\cos^2\phi$

At equator $\phi = 0°$

$$\Rightarrow \quad g' = g - R\omega^2$$

For g' to be zero

$$\omega = \sqrt{\frac{g}{R}} = \frac{1}{800}\ \text{rads}^{-1}$$

Hence, the correct answer is (C).

16. $g = \frac{GM}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\rho\right)}{R^2} = \frac{4}{3}\pi G\rho R$

$$\Rightarrow \quad g \propto \rho R$$

R is increased by a factor of 2 i.e., to keep the value of g to be the same, the value of ρ has to be changed by a factor of $\frac{1}{2}$.

Hence, the correct answer is (C).

17. $E = \frac{GM}{R^3}r$

i.e., Slope, $\frac{dE}{dr} = \frac{GM}{R^3}$

but $M = \frac{4}{3}\pi R^3\rho$

$$\Rightarrow \quad \frac{M}{R^3} = \frac{4\pi\rho}{3} \quad (\rho = \text{density})$$

$$\Rightarrow \quad \text{Slope } = \frac{4\pi G\rho}{3} = \text{constant}$$

Hence, the correct answer is (A).

18. $\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{i}+\frac{\partial U}{\partial y}\hat{j}\right)$

$$\Rightarrow \quad \vec{F} = -a\hat{i}-b\hat{j}$$

$$\Rightarrow \quad |\vec{F}| = \sqrt{a^2+b^2}$$

$$\Rightarrow \quad |\text{Acc}| = \frac{|\vec{F}|}{m} = \frac{\sqrt{a^2+b^2}}{m}$$

Hence, the correct answer is (C).

19. Let R be the radius of earth and g the acceleration due to gravity on earth's surface. Then the desired ratio (say x) is

$$x = \frac{g\left(1-\frac{h}{R}\right)}{\frac{g}{\left(1+\frac{h}{R}\right)^2}} = \left(1-\frac{h}{R}\right)\left(1+\frac{h}{R}\right)^2$$

Since h R, so $\left(1+\frac{h}{R}\right)^2 \cong 1+\frac{2h}{R}$

$$\Rightarrow \quad x=\left(1-\frac{h}{R}\right)\left(1+\frac{2h}{R}\right)$$

$$\Rightarrow \quad x \approx 1+\frac{h}{R}$$

From this expression we see that x increases linearly with h.

Hence, the correct answer is (C).

20. $E(r)=\left(\frac{GM}{R^3}\right)r$ where $M=\left(\frac{4}{3}\pi R^3\right)\rho$,

$$\Rightarrow \quad E(r)=\frac{Gr}{R^3}\left(\frac{4}{3}\pi R^3\rho\right)=\left(\frac{4}{3}\pi\rho G\right)r$$

Hence, the correct answer is (A).

21. Energy required to raise a satellite to a height h is

$$\Delta E=U_f-U_i=-\frac{GMm}{R+h}-\left(-\frac{GMm}{R}\right)$$

$$\Rightarrow \quad \Delta E=GMm\left(\frac{1}{R}-\frac{1}{R+h}\right)$$

$$\Rightarrow \quad \Delta E=\frac{GMmh}{R(R+h)}$$

Further, if v_0 be the orbital velocity in the orbit, then

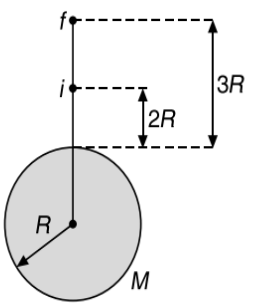
$$v_0=\sqrt{\frac{GM}{R+h}}$$

$$\Rightarrow \quad KE=\frac{1}{2}mv_0^2=\frac{GMm}{2(R+h)}$$

So, $\frac{\Delta E}{KE}=\frac{2h}{R}$

Hence, the correct answer is (C).

22.



Since $U=-\frac{GMm}{R+h}$

So, $U_i=-\frac{GMm}{2R+R}=-\frac{GMm}{3R}$

and $U_f=-\frac{GMm}{R+3R}=-\frac{GMm}{4R}$

Hence, external work done W is given by

$$W=\Delta U=U_f-U_i=\frac{GMm}{12R}=\frac{GM}{R^2}\times\frac{mR}{12}$$

$$\Rightarrow \quad W=\frac{mgR}{12}$$

Hence, the correct answer is (C).

23. Since, $F_1=\frac{GMm}{9R^2}$

Also, $F_2=\frac{GMm}{9R^2}-\frac{G(M/8)m}{(5R/2)^2}$

$$\Rightarrow \quad F_2=\frac{GMm}{9R^2}-\frac{GMm}{50R^2}=\frac{41}{450}\frac{GMm}{R^2}$$

$$\Rightarrow \quad \frac{F_2}{F_1}=\frac{41}{50}$$

Hence, the correct answer is (B).

24. By the principle of superposition of fields

$$\vec{E}=\vec{E}_1+\vec{E}_2$$

where, $\vec{E}$ is net field at the centre of hole due to entire mass, $\vec{E}_1$ is field due to remaining mass and $\vec{E}_2$ is field due to mass in hole $=0$.

Since, $\vec{E}_1=\vec{E}=\left(\frac{GM}{R^3}\right)r$ where $r=\frac{R}{2}$

$$\Rightarrow \quad \vec{E}=\frac{GM}{2R^2}$$

Hence, the correct answer is (C).

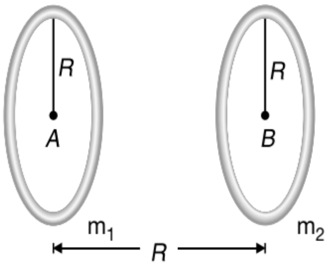
25. Initially field due to both is along positive x-axis. Due to the ring, field will first increase and then decrease to zero at centre. While field due to the solid sphere, will continuously increase in positive x-direction. On the other side of the ring field is now towards negative x-axis.

Hence, the correct answer is (B).

26. Potential due to particle at the surface is $V_1=-\frac{GM}{R}$ and potential due to shell at its own surface is $V_2=-\frac{G(3M)}{R}$

So, total potential is $V=V_1+V_2=-\frac{4GM}{R}$

Hence, the correct answer is (D).

27. 

$$V_A = \begin{pmatrix}\text{Potential at}\\ A \text{ due to } A\end{pmatrix} + \begin{pmatrix}\text{Potential at}\\ A \text{ due to } B\end{pmatrix}$$

$$\Rightarrow \quad V_A = -\frac{Gm_1}{R} - \frac{Gm_2}{\sqrt{2}R} \text{ and}$$

Similarly,

$$V_B = \begin{pmatrix}\text{Potential at}\\ B \text{ due to } A\end{pmatrix} + \begin{pmatrix}\text{Potential at}\\ B \text{ due to } B\end{pmatrix}$$

$$\Rightarrow \quad V_B = -\frac{Gm_2}{R} - \frac{Gm_1}{\sqrt{2}R}$$

Since $W_{A\to B} = m(V_B - V_A)$

$$\Rightarrow \quad W_{A\to B} = \frac{Gm(m_1 - m_2)(\sqrt{2}-1)}{\sqrt{2}R}$$

Hence, the correct answer is (B).

28. $E_g = G\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + \ldots\right]$

$$\Rightarrow \quad E_g = G\left[1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots\right]$$

$$\Rightarrow \quad E_g = G\left(\frac{1}{1-\frac{1}{4}}\right) \qquad \left\{S_\infty = \frac{\text{1st term}}{\text{1-Common Ratio}}\right\}$$

$$\Rightarrow \quad E_g = \frac{4G}{3}$$

Hence, the correct answer is (B).

29. For a satellite revolving near the surface of planet, we have

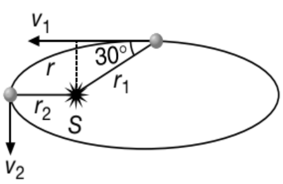
$$T \cong 2\pi\sqrt{\frac{R^3}{GM}} \qquad \{\because r \cong R\}$$

Since, $M = \left(\frac{4}{3}\pi R^3\right)\rho$

$$\Rightarrow \quad T = \sqrt{\frac{3\pi}{\rho G}}$$

i.e., T is independent of R.

Hence, the correct answer is (D).

30. 

By Law of Conservation of Angular Momentum, we get

$$mv_1 r = mv_2 r_2$$

$$\Rightarrow \quad mv_1 r_1 \sin 30° = mv_2 r_2$$

$$\Rightarrow \quad v_1 r_1 = 2v_2 r_2$$

Hence, the correct answer is (C).

31. Kinetic energy of satellite is $K = \frac{GMm}{2r}$ and the magnitude of gravitational potential energy is

$$U = \left|-\frac{GMm}{r}\right| = \frac{GMm}{r}$$

So, $K = \frac{U}{2}$

Hence, the correct answer is (B).

32. Total mechanical energy, at earth's surface is

$$E = U + K = -\frac{GMm}{R} + \frac{1}{2}mv_e^2$$

Since $v_e = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \quad E = -\frac{GMm}{R} + \frac{1}{2}m\left(\frac{2GM}{R}\right) = 0$$

Hence, the correct answer is (D).

33. By Law of Conservation of Mechanical Energy, we get

$$-\frac{GmM}{2R} = -\frac{GmM}{\frac{R}{2}} + \frac{1}{2}k\left(\frac{R}{2}\right)^2$$

$$\Rightarrow \quad \frac{GMm}{R}\left(2 - \frac{1}{2}\right) = \frac{kR^2}{8}$$

$$\Rightarrow \quad k = \frac{12GMm}{R^3}$$

Hence, the correct answer is (D).

34. By Law of Conservation of Energy

$$(U+K)_{\text{surface}} = (U+K)_r$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{r} + 0$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}mk^2v_e^2 = -\frac{GMm}{r}$$

But $v_e = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \quad -\frac{GM}{R} + \frac{1}{2}k^2\left(\frac{2GM}{R}\right) = -\frac{GM}{r}$$

$$\Rightarrow \quad -\frac{1}{R} + \frac{k^2}{R} = -\frac{1}{r}$$

$$\Rightarrow \quad \frac{1}{r} = \frac{1-k^2}{R}$$

$$\Rightarrow \quad r = \frac{R}{1-k^2}$$

Hence, the correct answer is (B).

35. When kinetic energy $< E$, total energy will be negative, when kinetic energy equals E, total energy is zero and when kinetic energy $> E$, total energy is positive.
Hence, the correct answer is (C).

36. For $r \le r_1$

$$V = \frac{Gm_1}{r_1} - \frac{Gm_2}{r_2} = \text{constant}$$

For $r_1 \le r \le r_2$

$$V = -\frac{Gm_2}{r_2} - \frac{Gm_1}{r}$$

Slope of V-r graph $\frac{dV}{dr} = \frac{Gm_1}{r^2}$

For $V = -\frac{G(m_1+m_2)}{r}$, slope of V-r graph is

$$\frac{dV}{dr} = \frac{G(m_1+m_2)}{r^2}$$

At the boundary of outer shell, slope of V-r graph changes from $\frac{Gm_1}{r_2^2}$ to $\frac{G(m_1+m_2)}{r_2^2}$ i.e., slope increases.

Hence, the correct answer is (D).

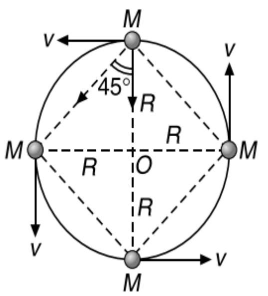
37. $c = \sqrt{\frac{2GM_{star}}{R}}$

$$\Rightarrow \quad 3\times 10^8 = \sqrt{\frac{2(6.67\times 10^{-11})(3\times 2\times 10^{30})}{R}}$$

$$\Rightarrow \quad R \quad 9 \text{ km}$$

Hence, the correct answer is (D).

38. Gravitational force on each particle due to the other three particles will provide the necessary centripetal force.



$$\Rightarrow \quad \sqrt{2}\frac{GM^2}{(\sqrt{2}R)^2}\cos 45° + \frac{GM^2}{(2R)^2} = \frac{Mv^2}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}\left(\frac{2\sqrt{2}+1}{4}\right)}$$

Hence, the correct answer is (D).

39. $dW = \vec{F}\cdot d\vec{\ell} = m_0(\vec{E}\cdot d\vec{\ell})$

$$\Rightarrow \quad dW = m_0(4\hat{i}+\hat{j})\cdot(\hat{i}dx+\hat{j}dy)$$

$$\Rightarrow \quad dW = m_0(4dx+dy)$$

Given that, $W = 0$

$$\Rightarrow \quad \int dW = 0$$

$$\Rightarrow \quad \int (4dx+dy) = 0$$

$$\Rightarrow \quad \int d(4x+y) = 0$$

$$\Rightarrow \quad 4x+y = \text{constant}$$

$\Rightarrow \quad y+4x = 2$, satisfies the condition.

Hence, the correct answer is (A).

40. Total energy of a planet of mass m, in an elliptical orbit is

$$E = -\frac{GMm}{2a} \qquad \{m = \text{mass of planet}\}$$

By Law of Conservation of Energy, we have

$$KE + PE = E$$

$$\Rightarrow \quad \frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a}$$

$$\Rightarrow \quad v = \sqrt{GM\left(\frac{2}{r}-\frac{1}{a}\right)}$$

Hence, the correct answer is (A).

41. Since $u = \frac{3}{4}v_e$

By Law of Conservation of Energy,

$$(U+K)_{\text{surface}} = (U+K)_r$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}mu^2 = \frac{-GMm}{r} + 0$$

$$\Rightarrow \quad \frac{-GMm}{R} + \frac{1}{2}m\left(\frac{9}{16}v_e^2\right) = -\frac{GMm}{r}$$

Since $v_e^2 = \frac{2GM}{R}$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}m\frac{9}{16}\left(\frac{2GM}{R}\right) = -\frac{GMm}{r}$$

$$\Rightarrow \quad -\frac{7}{16}\frac{GMm}{R} = -\frac{GMm}{r}$$

$$\Rightarrow \quad r = \frac{16}{7}R$$

Hence, the correct answer is (B).

42. $\begin{pmatrix}\text{Work}\\ \text{done}\end{pmatrix} = \begin{pmatrix}\text{Increase in Gravitational}\\ \text{Potential Energy}\end{pmatrix}$

Since, $\Delta U = \dfrac{mgh}{1+\dfrac{h}{R}}$

$\Rightarrow \quad W_1 = \dfrac{mgR}{1+\dfrac{R}{R}} = \dfrac{mgR}{2}$

and similarly, $W_2 = \dfrac{mgh}{1+\dfrac{h}{R}}$

Since, $W_1 = 2W_2$

$\Rightarrow \quad \dfrac{mgR}{2} = \dfrac{2mgh}{1+\dfrac{h}{R}}$

$\Rightarrow \quad h = \dfrac{R}{3}$

Hence, the correct answer is (C).

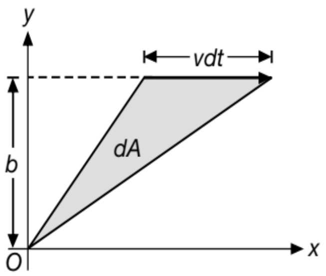
43. $\dfrac{dA}{dt} = \dfrac{1}{2} \times \dfrac{\text{base} \times \text{height}}{dt}$

$\Rightarrow \quad \dfrac{dA}{dt} = \dfrac{1}{2}\dfrac{(vdt)(b)}{dt}$

$\Rightarrow \quad \dfrac{dA}{dt} = \dfrac{bv}{2}$

Since, $v = at$, so

$\Rightarrow \quad \dfrac{dA}{dt} = \dfrac{abt}{2}$



Hence, the correct answer is (B).

44. $\vec{E}_g = -\vec{\nabla} V = -\left(\hat{i}\dfrac{\partial V}{\partial x} + \hat{j}\dfrac{\partial V}{\partial y}\right) = k\left(y\hat{i} + x\hat{j}\right)$

The work done by this field is independent of the path followed between any two states i.e.

$$W_{(0,0)\to(a,a)} = W_{(0,0)\to(0,a)\to(a,a)} = W_{(0,0)\to(a,0)\to(a,a)}$$

Since, $dW = m\vec{E}_g \cdot d\vec{r}$

where $d\vec{r} = \hat{i}dx + \hat{j}dy$

$\Rightarrow \quad dW = mk\left(y\,dx + x\,dy\right)$

$\Rightarrow \quad W = \displaystyle\int dW = mk \int_{(0,0)}^{(a,a)} d(xy) = mka^2$

$\Rightarrow \quad W = mka^2$ {where m is mass of particle}

$$W_{(0,0)\to(a,a)} = W_{(0,0)\to(0,a)\to(a,a)} = W_{(0,0)\to(a,0)\to(a,a)} = mka^2$$

Hence the field is conservative.

Hence, the correct answer is (B).

45. Let σ be the surface density, then

$$M_A = \sigma 4\pi R_A^2,\ m_B = \sigma 4\pi R_B^2$$

Since $V_A = \dfrac{-GM_A}{R_A}$ and $V_B = \dfrac{-GM_B}{R_B}$

$\Rightarrow \quad \dfrac{V_A}{V_B} = \dfrac{M_A}{M_B}\dfrac{R_B}{R_A} = \dfrac{\sigma\left(4\pi R_A^2\right)}{\sigma\left(4\pi R_B^2\right)}\dfrac{R_B}{R_A} = \dfrac{R_A}{R_B}$

Given that, $\dfrac{V_A}{V_B} = \dfrac{R_A}{R_B} = \dfrac{3}{4}$

$\Rightarrow \quad R_B = \dfrac{4}{3}R_A$

Since the new shell formed also has same surface mass density. So, we have

$$\sigma = \frac{M_A}{4\pi R_A^2} = \frac{M_B}{4\pi R_B^2} = \frac{M_A + M_B}{4\pi R^2} \qquad \ldots(1)$$

From equation (1), we get

$$\frac{M_A}{M_B} = \frac{R_A^2}{R_B^2} \qquad \ldots(2)$$

and $\dfrac{M_A + M_B}{M_A} = \dfrac{R^2}{R_A^2}$ …(3)

From (2) and (3), we get

$$R^2 = R_A^2 + R_B^2$$

Now $\dfrac{V}{V_A} = \dfrac{M}{R}\dfrac{R_A}{M_A} = \left(\dfrac{M_A + M_B}{M_A}\right)\dfrac{R_A}{\sqrt{R_A^2 + R_B^2}}$ …(4)

From (3), we have $\dfrac{M_A + M_B}{M_A} = \dfrac{R^2}{R_A^2}$

Substituting in equation (4), we get

$$\frac{V}{V_A} = \frac{\sqrt{R_A^2 + R_B^2}}{R_A} = \frac{5}{3}$$

Hence, the correct answer is (C).

46. Since, $T^2 = kr^3$

$\Rightarrow \quad 2\dfrac{\Delta T}{T} = 3\dfrac{\Delta r}{r}$

$\Rightarrow \quad \dfrac{\Delta T}{T} = \dfrac{3}{2}\dfrac{\Delta r}{r}$

Hence, the correct answer is (A).

47. Net mass, $M = \left[\dfrac{4}{3}\pi(2R)^3 - \dfrac{4}{3}\pi R^3\right]\rho$

where ρ is the density of the material of sphere. So,

$$\rho = \frac{M}{\frac{4}{3}\pi\left[(2R)^3 - (R)^3\right]} = \left(\frac{3M}{28\pi R^3}\right)$$

Now $V = V_{2R \text{ at centre}} - V_{R \text{ at centre}}$ …(1)

$$V_{2R} = -\frac{GM_1}{2R} \text{ and } V_R = -\frac{GM_2}{2R}$$

where, $M_1 = \rho\left(\frac{4}{3}\pi\right)(2R)^3 = \frac{8}{7}M$

and $M_2 = \rho\left(\frac{4}{3}\pi\right)(R)^3 = \frac{M}{7}$

Substituting in equation (1), we get

$$V = -\frac{9GM}{14R}$$

Hence, the correct answer is (B).

48. By Law of Conservation of Energy

$$(U+K)_{\text{surface}} = (U+K)_h$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h} + 0$$

$\{\because$ At maximum height velocity is zero$\}$

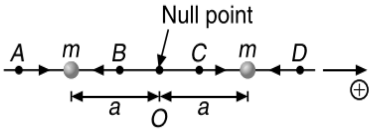
$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}mgR = -\frac{GMm}{R+h}$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}\frac{GMm}{R} = -\frac{GMm}{R+h} \qquad \left\{\because g = \frac{GM}{R^2}\right\}$$

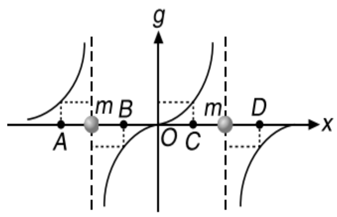
$$\Rightarrow \quad h = R$$

Hence, the correct answer is (B).

49.



The direction of net gravitational intensity at various points is shown above. Taking gravitational intensity towards right as positive, the graph will be



Hence, the correct answer is (A).

50. $\Delta U_1 = -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$

when taken from surface to h, we have

$$\Delta U_1 = -\frac{GMm}{R+h} + \frac{GMm}{R}$$

Now, when taken from h to infinity, we have

$$\Delta U_2 = 0 - \left(-\frac{GMm}{R+h}\right)$$

Since $\Delta U_1 = \Delta U_2$

$$\Rightarrow \quad -\frac{GMm}{R+h} + \frac{GMm}{R} = \frac{GMm}{R+h}$$

$$\Rightarrow \quad 2\left(\frac{GMm}{R+h}\right) = \frac{GMm}{R}$$

$$\Rightarrow \quad 2R = R+h$$

$$\Rightarrow \quad h = R$$

Hence, the correct answer is (A).

51. Since $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$

$$\Rightarrow \quad \frac{T_1^2}{T_2^2} = \left(\frac{R}{4R}\right)^3 = \frac{1}{64}$$

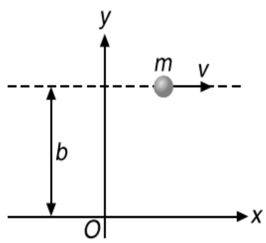
(Three earth radii from the surface means four radii from the centre).

$$\Rightarrow \quad \frac{T_1}{T_2} = \frac{1}{8}$$

$$\Rightarrow \quad T_2 = 8T_1 = 8(83) = 664 \text{ minute}$$

Hence, the correct answer is (C).

52.



Since angular momentum of the particle about origin O, $L = mvb =$ constant, therefore, areal velocity of the particle about origin O is constant. So, it sweeps equal areas in equal intervals of time. Thus, area swept in smaller time interval will also be smaller. Hence

$$A_1 > A_2 \text{ if } (t_4 - t_3) < (t_2 - t_1)$$

Hence, the correct answer is (B).

53. $v_0 = \sqrt{\frac{Gm}{5R}}$

After perfectly inelastic collision, velocity of combined mass will become zero. Therefore, if v be the desired speed, then by Law of Conservation of Mechanical Energy, we have

$$(U+K)_{r=5R} = (U+K)_{\text{surface}}$$

$$-\left(\frac{GmM}{5R}\right) - \left(\frac{GmM}{5R}\right) + 0 = -\frac{GM(2m)}{R} + \frac{1}{2}(2m)v^2$$

$$\Rightarrow \quad \frac{1}{2}(2m)v^2 = -\frac{GM(2m)}{5R} + \frac{GM(2m)}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{8GM}{5R}} = 2\sqrt{2}v_0$$

Hence, the correct answer is (A).

54. Since Areal velocity $= \dfrac{\text{Area Swept}}{\text{Time for one Revolution of Earth about the Sun}}$

Further Areal velocity $= \dfrac{L}{2M}$

$$\Rightarrow \quad \text{Area Swept} = \left(\frac{L}{2M}\right)\left(\begin{array}{c}\text{Time for one}\\ \text{Revolution of Earth}\\ \text{about the Sun}\end{array}\right)$$

$$\text{Area Swept} = \frac{1}{2}(4.4\times10^{15})(365\times24\times60\times60)$$

$$\Rightarrow \quad \text{Area Swept} \quad 7\times10^{22}\ \text{m}^2$$

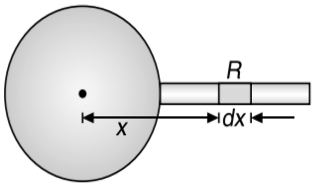
Hence, the correct answer is (D).

55. Force on element dx on rod is

$$dF = \frac{GM(dm)}{x^2}$$

where $dm = \dfrac{M}{2R}dx$

$$\Rightarrow \quad dF = \left(\frac{GM^2}{2R}\right)\frac{dx}{x^2}$$



Total force on the rod is

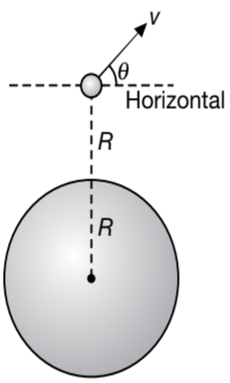
$$F = \int dF = \frac{GM^2}{2R}\int_R^{2R}\frac{1}{x^2}dx$$

$$\Rightarrow \quad F = \frac{GM^2}{2R}\left(-\frac{1}{x}\right)\Bigg|_R^{2R} = -\frac{GM^2}{2R}\left(\frac{1}{2R}-\frac{1}{R}\right)$$

$$\Rightarrow \quad F = \frac{GM^2}{4R^2}$$

Hence, the correct answer is (A).

56. At height $h = R$, the distance of the particle from centre of earth is $r = 2R$.
Let v be the velocity at that point. Then by Law of Conservation of Mechanical Energy, we have



$$(U+K)_{\text{surface}} = (U+K)_{\text{at } h}$$

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{1}{2}mv_e^2 = -\frac{GMm}{R+R}+\frac{1}{2}mv^2$$

Since, $v_e = \sqrt{\dfrac{2GM}{R}}$

$$\Rightarrow \quad \frac{1}{2}mv^2 = -\frac{GMm}{R}+\frac{GMm}{R}+\frac{GMm}{2R}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}}$$

Let θ be the angle of its velocity with horizontal, then by Law of Conservation of Angular Momentum about centre of earth, we have

$$mv_eR\cos(45^\circ) = mvr\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{v_eR\cos(45^\circ)}{vr}$$

$$\Rightarrow \quad \cos\theta = \frac{R\sqrt{\dfrac{2GM}{R}}\dfrac{1}{\sqrt{2}}}{(2R)\sqrt{\dfrac{GM}{R}}} = \frac{1}{2}$$

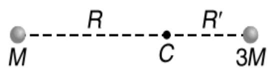
$$\Rightarrow \quad \theta = 60^\circ$$

Hence, the correct answer is (C).

57. $$U(r) = \begin{cases} -\dfrac{GMm}{r}, & r \geq R \\ -\dfrac{GMm}{R}, & r < R \end{cases}$$

Hence, the correct answer is (C).

58. In a double star system, both stars revolve around centre of mass of system (C).



From the definition of centre of mass

$$MR = 3MR'$$

$$\Rightarrow \quad R' = \frac{R}{3}$$

Hence, distance between two stars is

$$r = R+\frac{R}{3} = \frac{4R}{3}$$

The gravitational force between them will provide the centripetal force to revolve in the circle. Therefore, for the smaller star, we have

$$\frac{GM\times3M}{\left(\dfrac{4R}{3}\right)^2} = MR\omega^2$$

$\Rightarrow \quad \omega = \sqrt{\frac{27GM}{16R^3}}$

$\Rightarrow \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{16R^3}{27GM}}$

Hence, the correct answer is (C).

59. By Law of Conservation of Energy, we have

$$(U+K)_{\text{surface}} = (U+K)_h$$

$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}m\left(\frac{v_e}{2}\right)^2 = -\frac{GMm}{R+h} = U_h$

$\Rightarrow \quad U_h = -\frac{GMm}{R} + \frac{1}{8}mv_e^2 = -\frac{GMm}{R} + \frac{1}{8}m\left(\frac{2GM}{R}\right)$

$\Rightarrow \quad U_h = -\frac{3}{4}\frac{GMm}{R}$

Hence, the correct answer is (C).

60. $E_i = -\frac{GMm}{2r}$, $E_f = -\frac{GMm}{2(3r)} = -\frac{GMm}{6r}$

Energy Required is

$$\Delta E = E_f - E_i = \frac{GMm}{3r}$$

Hence, the correct answer is (C).

61. Since, $W = U_f - U_i$

Where, $U_f = -\frac{3Gm^2}{r_f}$ and $U_i = -\frac{3Gm^2}{r_i}$

$\Rightarrow \quad W = U_f - U_i = 3Gm^2\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$

where, $G = 6.67\times10^{-11}\ \text{Nm}^2\text{kg}^{-2}$, $m = 0.1$ kg, $r_f = 0.4$ m and $r_i = 0.2$ m

Substituting the values, we get

$$W = 5\times10^{-12}\ \text{J}$$

Hence, the correct answer is (C).

62. The whole space can be divided into three regions

(i) $0 < r < R_1$, we have $F(r) = 0$

(ii) $R_1 < r < R_2$, we have $F(r) = \frac{4}{3}\pi G\rho m\left(r - \frac{R_1^3}{r^2}\right)$

(iii) $R_2 < r < \infty$, we have $F(r) = \frac{4}{3}\pi G\rho m\left(\frac{R_2^3 - R_1^3}{r^2}\right)$

where, ρ is the density of material of the sphere.
Hence, the correct answer is (C).

63. The weight of the traveler first decreases, then becomes zero and after that again increases.
Hence, the correct answer is (C).

64. Since $E = -\frac{dV}{dx}$

$\Rightarrow \quad -dV = Edx$

$\Rightarrow \quad -dV = Kx^{-3}dx$

$\Rightarrow \quad -\int dV = K\int x^{-3}\,dx$

$\Rightarrow \quad -V = K\left(\frac{x^{-3+1}}{-3+1}\right)$

$\Rightarrow \quad V = \frac{K}{2x^2}$

Hence, the correct answer is (D).

65. Total number of interactions $= {}^nC_2 = \frac{n(n-1)}{2}$

Hence, the correct answer is (D).

66. Gravitational field due to element dx at origin is

$$dE_g = \frac{G(a+bx^2)dx}{x^2}$$

Net field at O is

$$E_g = \int dE_g = G\int_{\alpha}^{l+\alpha}\left(\frac{a}{x^2}+b\right)dx$$

$\Rightarrow \quad E_g = \left(-\frac{Ga}{x}\right)\Big|_{\alpha}^{l+\alpha} + Gbl$

$\Rightarrow \quad E_g = Ga\left(\frac{1}{\alpha} - \frac{1}{l+\alpha}\right) + Gbl$

Hence force on mass m is given by

$$F = mE_g = Gm\left[a\left(\frac{1}{\alpha} - \frac{1}{l+\alpha}\right) + bl\right]$$

Hence, the correct answer is (A).

67. Net gravitational field inside a shell is zero. Hence, net force on the particle will be zero i.e., the particle stays at rest in its original position.
Hence, the correct answer is (D).

68. Angular momentum of earth remains constant even after the hit because the meteor is coming radially inward. However, the MOI increases by mR^2 as meteorite strikes the earth at equator.

$I\omega = \text{constant}$

$\Rightarrow \quad Id\omega + \omega dI = 0$

$\Rightarrow \quad \frac{d\omega}{\omega} = -\frac{dI}{I}$

Since $\omega = \frac{2\pi}{T}$, we get $\frac{dT}{T} = \frac{dI}{I}$

Hence $dT = \frac{dI}{I}T = \frac{mR^2}{\frac{2}{5}MR^2}T = \frac{5mT}{2M}$

Hence, the correct answer is (A).

CHAPTER 4

69. The gravitational potential at the centre of the ring is

$$V=-\frac{GM}{R}$$

irrespective of the distribution of mass

Since, $W_{C\to\infty}=m(V_\infty-V_C)$

$$\Rightarrow\quad W_{C\to\infty}=m\left[0-\left(-\frac{GM}{R}\right)\right]$$

$$\Rightarrow\quad W_{C\to\infty}=\frac{GMm}{R}$$

Hence, the correct answer is (B).

70. When mass of the shell is m, the potential is $V=-\frac{Gm}{R}$

To add an elementary mass dm, the work done is $dW=Vdm=-\frac{Gm}{R}dm$

This, net work done equals the gravitational self energy, so

$$U=-\int_0^M\frac{Gm\,dm}{R}=-\frac{GM^2}{2R}$$

Hence, the correct answer is (D).

71. Given that $v_0=\frac{v_e}{2}$

$$\Rightarrow\quad\sqrt{\frac{gR^2}{R+h}}=\frac{1}{2}\sqrt{2gR}$$

$$\Rightarrow\quad\frac{R}{R+h}=\frac{1}{2}$$

$$\Rightarrow\quad h=R$$

Since the satellite now stops and falls from this height, using conservation of mechanical energy $U_i+K_i=U_f+K_f$

$$\Rightarrow\quad-\frac{GMm}{R+R}+0=-\frac{GMm}{R}+\frac{1}{2}mv^2$$

$$\Rightarrow\quad v=\sqrt{\frac{GM}{R}}=\sqrt{gR}$$

Hence, the correct answer is (A).

72. Potential energy at a height of $2R$ from surface of earth is

$$U=-\frac{GMm}{R+2R}=-\frac{GMm}{3R}$$

and total energy of a satellite at a height h is $E=-\frac{GMm}{2(R+h)}$

Given that $U=E$

$$\Rightarrow\quad-\frac{GMm}{3R}=-\frac{GMm}{2(R+h)}$$

$$\Rightarrow\quad h=\frac{R}{2}$$

Hence, the correct answer is (B).

73. **CONCEPT OF REDUCED MASS** (μ)

Let m_1 be at rest and think that m_2 has been replaced by μ and is moving with velocity v. Then by Law of Conservation of Energy

$$\frac{1}{2}m_1(0)^2+\frac{1}{2}\mu v^2=-\frac{Gm_1m_2}{r}+0$$

where, μ = reduced mass of system = $\frac{m_1m_2}{m_1+m_2}$

$$\Rightarrow\quad v=\sqrt{\frac{2G(m_1+m_2)}{r}}$$

Hence, the correct answer is (B).

74. Since $U=v_O=\sqrt{\frac{GM}{R}}$

By Law of Conservation of Energy, we have

$$(U+K)_{\text{surface}}=(U+K)_{at\ \text{h}}$$

$$\Rightarrow\quad-\frac{GMm}{R}+\frac{1}{2}mu^2=-\frac{GMm}{R+h}+\frac{1}{2}m(0)^2$$

$$\Rightarrow\quad-\frac{GMm}{R}+\frac{GMm}{2R}=-\frac{GMm}{R+h}$$

$$\Rightarrow\quad h=R$$

Hence, the correct answer is (C).

75. Gravitational force between balls is

$$F=\frac{Gm^2}{(a-x)^2}$$

For equilibrium of each ball, we have

$$T\sin\theta=\frac{Gm^2}{(a-x)^2}\qquad\ldots(1)$$

$$T\cos\theta=mg\qquad\ldots(2)$$

$$\Rightarrow\quad\tan\theta=\frac{Gm}{(a-x)^2g}$$

$$\Rightarrow\quad\theta=\tan^{-1}\left[\frac{Gm}{(a-x)^2g}\right]$$

Hence, the correct answer is (B).

76. Since the torque on the planet due to gravitational pull about the sun is zero, so angular momentum of the planet about the sun is constant.

$$\Rightarrow\quad mvr_\perp=\text{constant}$$

Hence, the correct answer is (B).

77. By Law of Conservation of Energy

$$(U+K)_{\text{surface}} = (U+K)_{\infty}$$

$$\Rightarrow \frac{-GMm}{R} + \frac{1}{2}m(3v_e)^2 = 0 + \frac{1}{2}mv^2$$

$$\Rightarrow -\frac{GM}{R} + \frac{9v_e^2}{2} = \frac{1}{2}v^2$$

Since $v_e^2 = \frac{2GM}{R}$

$$\Rightarrow -\frac{v_e^2}{2} + \frac{9v_e^2}{2} = \frac{1}{2}v^2$$

$$\Rightarrow v^2 = 8v_e^2$$

$$\Rightarrow v = 2\sqrt{2}v_e$$

Hence, the correct answer is (D).

78. A lighter body inside a denser medium behaves like a negative mass so far the gravitational force is concerned even the air bubbles with negative masses will attract each other.

Hence, the correct answer is (B).

79. According to Kepler's Laws the planets sweep equal areas in equal intervals of time. So,
for

$$\Delta A_1 = \Delta A_2$$

$$t_1 = t_2$$

Hence, the correct answer is (C).

80. By Kepler's Second Law, we have

$$\frac{\Delta A}{\Delta t} = \frac{L}{2M} = \frac{\pi ab}{T}$$

$$\Rightarrow \frac{L}{2M} = \frac{\pi ab}{T}$$

where a is semi major and b is semi minor axis of elliptical orbit.

$$\Rightarrow L = \frac{m\pi(r_a + r_p)\sqrt{r_a r_p}}{T}$$

Hence, the correct answer is (A).

81. Areal velocity $\frac{dA}{dt} = \frac{L}{2m} = \text{constant}$

Since, $L = mvr\sin\theta$

$$\Rightarrow \frac{dA}{dt} = \frac{vr\sin\theta}{2}$$

$$\Rightarrow \frac{dA}{dt} \propto m^0$$

Hence, the correct answer is (D).

82. Since, $T = \frac{2\pi r}{v}$

$$\Rightarrow \frac{2\pi R}{v_0} = 2 \text{ and } \frac{2\pi R'}{v_0'} = 16$$

This is possible when $R' = 4R$ and $v_0' = \frac{v_0}{2}$

Hence, the correct answer is (D).

83. Distance of second satellite from the centre of earth becomes four times.
Since, according to Kepler's Laws we have

$$T^2 \propto r^3$$

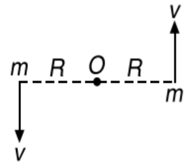
$$\Rightarrow T \propto r^{3/2}$$

So, time period of second satellite must become $(4)^{3/2} = 8$ times

Hence, $T' = 8T = (8)(90) = 720$ min

Hence, the correct answer is (D).

84. The gravitational force provides the necessary centripetal force for circular motion



$$\Rightarrow \frac{Gm^2}{(2R)^2} = \frac{mv^2}{R}$$

$$\Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$$

Hence, the correct answer is (C).

85. Energy of the satellite at the surface of earth is $E_i = -\frac{GMm}{R}$ and energy of satellite at a distance $r = (2R+R) = 3R$ from the centre of earth is

$$E_f = -\frac{GMm}{2(3R)} = -\frac{GMm}{6R}$$

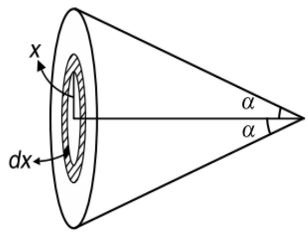
So, the energy required to launch the satellite is

$$E = E_f - E_i = \frac{5GMm}{6R}$$

Hence, the correct answer is (C).

86. $E = -2\pi\sigma G\int_0^{\alpha} \sin\theta\, d\theta$

$$\Rightarrow E = -2\pi\sigma G(1 - \cos\alpha)$$



$$\Rightarrow \quad E = -2\pi\sigma G\left(1 - \frac{r}{\sqrt{r^2 + R^2}}\right)$$

$$V = -2\pi\sigma r G\left(1 - \frac{r}{\sqrt{r^2 + R^2}}\right)$$

Hence, the correct answer is (C).

87. For all points lying in the plane of base of a hemispherical shell, gravitational field is normal to the base and hence this surface is equipotential. So, $V_A = V_B = V_C$
Hence, the correct answer is (D).

88. Since, Total energy $= -$ Kinetic energy $= -E$
So, energy E will have to be supplied to escape the electron to infinity.
Hence, the correct answer is (C).

89. $U_h = -\dfrac{GMm}{R+h}$

$$\Rightarrow \quad U_h = -\frac{GMm}{R}\left(1 + \frac{h}{R}\right)^{-1} = -\frac{GMm}{R}\left(1 - \frac{h}{R}\right) \qquad \{\because h \quad R\}$$

$$\Rightarrow \quad U_h = -\frac{GMm}{R} + mgh \qquad \left\{\because g = \frac{GM}{R^2}\right\}$$

Hence, the correct answer is (B).

90. Since $W = \int \vec{F} \cdot d\vec{l}$

where $\vec{F} = m_0 \vec{I}$ and $d\vec{l} = dx\hat{i} + dy\hat{j}$

$$\Rightarrow \quad W = m_0 \int_{(1,\,1)}^{\left(2,\,\frac{1}{3}\right)} 2dx + 3dy$$

Since $3y + 2x = 5$

$$\Rightarrow \quad 3dy + 2dx = 0$$

$$\Rightarrow \quad W = 0$$

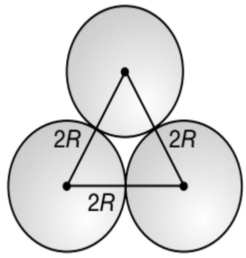
Hence, the correct answer is (A).

91. By Law of Conservation of Energy, we have

$$(U + K)_{\text{initial}} = (U + K)_{\text{final}}$$

$$\Rightarrow \quad -\frac{3Gm^2}{d} + 3\left(\frac{1}{2}m(0)^2\right) = -\frac{3Gm^2}{2R} + 3\left(\frac{1}{2}mv^2\right)$$

$$\Rightarrow \quad 3\left(\frac{1}{2}mv^2\right) = 3\left(\frac{Gm^2}{2R} - \frac{Gm^2}{d}\right)$$



$$\Rightarrow \quad v^2 = Gm\left(\frac{1}{R} - \frac{2}{d}\right)$$

$$\Rightarrow \quad v = \sqrt{Gm\left(\frac{1}{R} - \frac{2}{d}\right)}$$

Hence, the correct answer is (D).

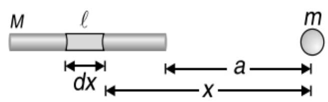
92. $(U + K)_{\text{initial}} = (U + K)_{\infty}$

$$-\frac{GM_e m}{\frac{r}{2}} - \frac{GM_m m}{\frac{r}{2}} + \frac{1}{2}mv^2 = 0$$

$$\Rightarrow \quad v = \sqrt{\frac{4G}{r}(M_e + M_m)}$$

Hence, the correct answer is (A).

93. $dU = -\dfrac{Gm\left(\dfrac{M}{\ell}dx\right)}{x}$



$$\Rightarrow \quad U = \int dU = -\frac{GmM}{\ell}\int_a^{a+\ell}\frac{dx}{x}$$

$$\Rightarrow \quad U = -\frac{GMm}{\ell}\log_e\left(\frac{a+\ell}{a}\right)$$

Hence, the correct answer is (C).

94. By Law of Conservation of Mechanical Energy

$$U_i + K_i = U_f + K_f$$

$$-\frac{GMm}{R + h_i} + 0 = -\frac{GMm}{R + h_f} + \frac{1}{2}mv^2$$

$$-\frac{GM}{R + R} + 0 = -\frac{GM}{R + 0.5R} + \frac{1}{2}v^2$$

$$\Rightarrow \quad -\frac{GM}{2R} + \frac{2GM}{3R} = \frac{1}{2}v^2$$

$\Rightarrow \quad \dfrac{GM}{6R} = \dfrac{1}{2}v^2$

$\Rightarrow \quad v = \sqrt{\dfrac{GM}{3R}}$

Hence, the correct answer is (D).

95. Given that $\dfrac{U}{m} = E$. By Law of Conservation of Energy, we have

$$(U+K)_{\text{surface}} = (U+K)_{\infty}$$

$$-mE + \frac{1}{2}mv_e^2 = 0 \qquad \left\{\because U = -\frac{GmM}{R}\right\}$$

$\Rightarrow \quad v_e = \sqrt{2E}$

Hence, the correct answer is (A).

96. Total initial energy is $E_i = -\dfrac{GMm}{R}$

Total final energy is $E_f = -\dfrac{GMm}{2(2R+R)}$

Energy required $= E_f - E_i$

$\Rightarrow \quad \Delta E = -\dfrac{GMm}{6R} + \dfrac{GMm}{R} = \dfrac{5}{6}\dfrac{GMm}{R}$

$\Rightarrow \quad \Delta E = \dfrac{5}{6}mgR$

Hence, the correct answer is (D).

97. Time period of a satellite above the earth's surface is

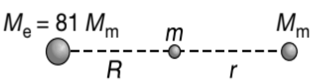
$$T^2 = \frac{4\pi^2 R^3}{GM} = \frac{3\pi}{G\left(\dfrac{M}{\frac{4}{3}\pi R^3}\right)} = \frac{3\pi}{G\rho}$$

$\Rightarrow \quad \rho T^2 = \dfrac{3\pi}{G}$ = a universal constant

Hence, the correct answer is (A).

98. $\displaystyle\int_S \vec{E}_g \cdot d\vec{A} = -4\pi m_0 G$ (Gauss Theorem for Gravitation)

Hence, the correct answer is (B).

99. 

$$U = -Gm\left(\frac{M_e}{R} + \frac{M_m}{r}\right)$$

$\Rightarrow \quad U = -GmM_m\left(\dfrac{81}{R} + \dfrac{1}{r}\right)$

Hence, the correct answer is (C).

100. Since, we know that from Work – Energy Theorem,

$$W_{\text{ext}} = \Delta U + \Delta K$$

$\Rightarrow \quad W_{A\to B} = (U_B - U_A) + (K_B - K_A)$

$\Rightarrow \quad W_{A\to B} = m(V_B - V_A) + (K_B - K_A)$

Substituting the values, we get

$$-10 = 2(V_B - V_A) + 4$$

$\Rightarrow \quad V_B - V_A = -7 \text{ Jkg}^{-1}$

Hence, the correct answer is (D).

101. Gravitational pull on satellite revolving in an orbit of radius $\dfrac{3R}{2}$ is

$$F = \frac{GMm}{\left(\frac{3R}{2}\right)^2} = \frac{4}{9}\frac{GMm}{R^2}$$

As weight of 1 kg body on the surface of earth is

$$\frac{GM \times 1}{R^2} = 10 \text{ N} \qquad \text{(given)}$$

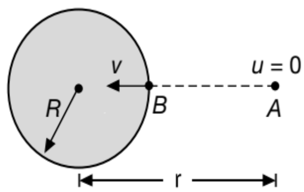
$\Rightarrow \quad F = \dfrac{4}{9}\dfrac{GMm}{R^2} = \dfrac{4}{9} \times 10 \times 200 = \dfrac{8000}{9}$

$\Rightarrow \quad F = 889 \text{ N}$

Hence, the correct answer is (B).

102. Let r be the radius of the satellite. Then

$$v_0^2 = \frac{GM}{r}$$



By Law of Conservation of Mechanical Energy,

$$(U+K)_A = (U+K)_B$$

$\Rightarrow \quad -\dfrac{GMm}{r} + \dfrac{1}{2}m(0)^2 = -\dfrac{GMm}{R} + \dfrac{1}{2}mv^2$

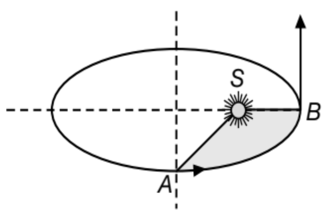
$\Rightarrow \quad v^2 = \dfrac{2GM}{R} - \dfrac{2GM}{r} = v_e^2 - 2v_0^2$

$\Rightarrow \quad v = \sqrt{v_e^2 - 2v_0^2}$

Hence, the correct answer is (D).

103. L is conserved both in magnitude and direction.
Hence, the correct answer is (C).

104. Since, we know that areal velocity of planet is constant. So, we have



$$\frac{\text{Area of Ellipse}}{\text{Period of Revolution}} = \frac{\text{Area } SAB}{t_{AB}}$$

$$\Rightarrow \quad t_{AB} = \left(\frac{\text{Area } SAB}{\text{Area of ellipse}}\right)T$$

$$\Rightarrow \quad t_{AB} = \frac{T\left(\frac{\pi ab}{4} - \frac{1}{2}(b)(ea)\right)}{\pi ab}$$

$$\Rightarrow \quad t_{AB} = T\left(\frac{1}{4} - \frac{e}{2\pi}\right)$$

Hence, the correct answer is (B).

105. Gravitational field inside the cavity is

$$\vec{E} = \frac{4}{3}\pi\rho G\vec{r}$$

where ρ is mass density and $\vec{r}$ is separation between centre of sphere and centre of cavity. Escape velocity from point A is calculated by applying Law of Conservation of Energy according to which

$$(U+K)_A = (U+K)_\infty$$

$$\frac{1}{2}mv_{\text{esc}}^2 - \frac{GMm}{R} + \frac{G\left(\frac{M}{9}\right)m}{\left(\frac{5R}{3}\right)} = 0$$

$$\Rightarrow \quad v_{\text{esc}} = \sqrt{\frac{28GM}{15R}}$$

Hence, the correct answer is (A).

106. $L = mvr$ ...(1)

Also, $\dfrac{mv^2}{r} = \dfrac{GMm}{r^2}$ ...(2)

From equations (1) and (2), we get

$$L = m\sqrt{GMr}$$

$$\Rightarrow \quad L \propto r^{1/2}$$

Hence, the correct answer is (D).

107. Let us consider the shell when a mass m is already piled on it by the agency. If V is the potential on the shell, then

$$V = -\frac{Gm}{R}$$

To add a mass dm further, we have

$$dW = Vdm$$

$$\Rightarrow \quad dW = -\frac{Gm}{R}dm$$

$$\Rightarrow \quad W = -\frac{G}{R}\int_0^M m\,dm$$

$$\Rightarrow \quad W = -\frac{1}{2}\frac{GM^2}{R}$$

$\Rightarrow$ W = Potential Energy of Interaction

Hence, the correct answer is (B).

108. Total energy of satellite in the first case is $E_i = -\dfrac{GMm}{2r}$

and in the second case is $E_f = \dfrac{GMm}{2\left(\frac{3r}{2}\right)} = -\dfrac{GMm}{3r}$

Energy increased $\Delta E = \dfrac{GMm}{r}\left(\dfrac{1}{2} - \dfrac{1}{3}\right) = \dfrac{GMm}{6r}$

So, percentage increase $= \dfrac{\left(\frac{GMm}{6r}\right)}{\left(\frac{GMm}{2r}\right)} \times 100\% = 33.33\%$

Hence, the correct answer is (C).

109. $\Delta E = U_f - U_i = -\dfrac{GM^2}{2(2R)} + \dfrac{GM^2}{2R} = \dfrac{GM^2}{4R}$

Hence, the correct answer is (C).

110. With respect to earth, satellite appears after every 6 hours over same place. Since it is revolving in opposite direction to that of earth, angular speed of satellite relative to earth is

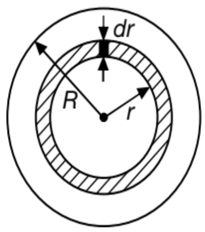
$$\omega_{\text{relative}} = \omega_{\text{satellite}} + \omega_{\text{earth}} = \frac{2\pi}{T} = \frac{2\pi}{6}$$

$$\Rightarrow \quad \frac{2\pi}{24} + \omega_{\text{satellite}} = \frac{2\pi}{6}$$

$$\Rightarrow \quad \omega_{\text{satellite}} = \frac{2\pi}{6} - \frac{2\pi}{24} = \frac{\pi}{4} \text{ radhr}^{-1}$$

Hence, the correct answer is (C).

111. $dU = -\dfrac{Gm\,dm}{r}$



$$\Rightarrow \quad dU = -\frac{G\left(\frac{4}{3}\pi r^3\rho\right)\left(4\pi r^2 dr\rho\right)}{r}$$

$$\Rightarrow \quad dU = -\frac{16\pi^2 G\rho^2}{3}r^4 dr$$

$$\Rightarrow \quad U = -\frac{16}{3}\pi^2 G\left(\frac{M}{\frac{4}{3}\pi R^3}\right)^2 \int_0^R r^4 dr$$

$$\Rightarrow \quad U=\left(-\frac{16}{3}\pi^2 G\right)\left(\frac{M^2}{\frac{16}{9}\pi^2R^6}\right)\left(\frac{R^5}{5}\right)$$

$$\Rightarrow \quad U=-\frac{3}{5}\frac{GM^2}{R}$$

Hence, the correct answer is (C).

112. $\frac{dA}{dt}=\frac{A}{T}=\frac{L}{2m}$

$$\Rightarrow \quad L=\frac{2mA}{T}$$

Hence, the correct answer is (C).

113. $\text{K.E.}=\frac{GMm}{2r}=-E_0$, and $\text{P.E.}=-\frac{GMm}{r}=2E_0$

$$\Rightarrow \quad \text{T.E.}=\text{K.E.}+\text{P.E.}=-\frac{GMm}{2r}=E_0$$

Hence, the correct answer is (C).

114. Since the satellite appears after every 8 hours above the same place on earth, therefore its angular speed relative to earth is $\frac{2\pi}{8}$. Hence

$$\frac{2\pi}{8}=\omega_{\text{satellite}}-\omega_{\text{earth}}$$

$$\Rightarrow \quad \frac{2\pi}{8}=\frac{2\pi}{T_{\text{satellite}}}-\frac{2\pi}{24}$$

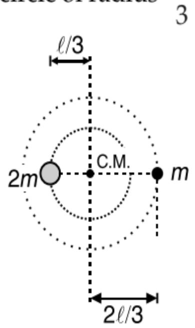
$$\Rightarrow \quad \frac{2\pi}{T_{\text{satellite}}}=\frac{2\pi}{8}+\frac{2\pi}{24}=\frac{2\pi}{6}$$

$$\Rightarrow \quad T_{\text{satellite}}=6\text{ hr}$$

Hence, the correct answer is (C).

115. The system will revolve/rotate about an axis passing through the centre of mass of the combined system. Considering origin at the particle of mass $2m$, we have the centre of mass at a distance $\frac{\ell}{3}$ from $2m$ and $\frac{2\ell}{3}$ from m.

The gravitational force of attraction between $2m$ and m provides the necessary centripetal force to the mass to revolve in a circle of radius $\frac{2\ell}{3}$ for m or $\frac{\ell}{3}$ for $2m$.



$$\Rightarrow \quad m\left(\frac{2\ell}{3}\right)\omega^2=\frac{Gm(2m)}{\ell^2}$$

$$\Rightarrow \quad \omega=\sqrt{\frac{3Gm}{\ell^3}}$$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{\ell^3}{3Gm}}$$

Hence, the correct answer is (C).

116. $W'=\frac{3}{4}$

$$\Rightarrow \quad g_{\text{eff}}=\frac{3}{4}g$$

At equator, we have

$$g_{\text{eff}}=g-R\omega^2=\frac{3}{4}g$$

$$\Rightarrow \quad \omega=\sqrt{\frac{g}{4R}}=\frac{1}{2}\left(\sqrt{\frac{g}{R}}\right)=\frac{1}{2}\times\frac{1}{800}$$

$$\Rightarrow \quad \omega=\frac{1}{1600}\text{ rads}^{-1}$$

Hence, the correct answer is (C).

117. Let gravitational field be zero at a point lying at distance x from M. Then,

$$\frac{GM}{x^2}=\frac{Gm}{(d-x)^2}$$

$$\Rightarrow \quad \frac{d-x}{x}=\sqrt{\frac{m}{M}}$$

$$\Rightarrow \quad \frac{d}{x}-1=\sqrt{\frac{m}{M}}$$

$$\Rightarrow \quad x=\left(\frac{\sqrt{M}}{\sqrt{M}+\sqrt{m}}\right)d \qquad \text{...(1)}$$

$$\Rightarrow \quad (d-x)=\left(\frac{\sqrt{m}}{\sqrt{M}+\sqrt{m}}\right)d \qquad \text{...(2)}$$

Since, $V_P=-\frac{Gm}{d-x}-\frac{GM}{x}$ ...(3)

Substituting (1) and (2) in (3), we get

$$V_P=-\frac{G}{d}\left(\sqrt{m}+\sqrt{M}\right)^2$$

Hence, the correct answer is (D).

118. $\left(\begin{matrix}\text{Total}\\ \text{Mechanical Energy}\end{matrix}\right)_P=\left(\begin{matrix}\text{Total}\\ \text{Mechanical Energy}\end{matrix}\right)_O$

$$\Rightarrow \quad \frac{1}{2}m(0)^2-\frac{GMm}{\sqrt{\left(\sqrt{3}R\right)^2+R^2}}=\frac{1}{2}mv^2-\frac{GMm}{R}$$

$$\Rightarrow \quad -\frac{GMm}{2R}=\frac{1}{2}mv^2-\frac{GMm}{R}$$

CHAPTER 4

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}}$$

Hence, the correct answer is (A).

119. By Law of Conservation of Energy, we have

$$(U+K)_{axis} = (U+K)_C$$

If m_0 be the mass of the particle, then

$$-\frac{Gmm_0}{\sqrt{r^2+r^2}} + \frac{1}{2}m_0(0)^2 = -\frac{Gmm_0}{r} + \frac{1}{2}m_0v^2$$

$$\Rightarrow \quad \frac{1}{2}m_0v^2 = \frac{Gmm_0}{r}\left(1-\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{2Gm}{r}\left(1-\frac{1}{\sqrt{2}}\right)}$$

Hence, the correct answer is (C).

120. $U_i = -\frac{GMm}{R}$ and $U_f = 0$

$$\Rightarrow \quad W = \Delta U = \frac{GMm}{R}$$

Hence, the correct answer is (A).

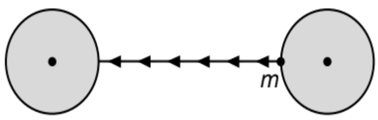
121. Use Gauss Theorem for Gravitation.

Hence, the correct answer is (C).

122. $E = -\frac{dV}{dr}$

Hence, the correct answer is (C).

123. Centre point is the unstable equilibrium position where potential energy is maximum.



Hence, the correct answer is (C).

124. Since, $g = \frac{GM}{R^2} = GMR^{-2}$

Fractional change in g is

$$\frac{\Delta g}{g} = \frac{\Delta M}{M} - \frac{2\Delta R}{R}$$

$$\Rightarrow \quad \frac{\Delta g}{g} = 0.5\% - 2(0.5\%) = -0.5\%$$

So, g will decrease by 0.5%

$$v_e = \sqrt{\frac{2GM}{R}}$$

Fractional change in escape velocity is

$$\frac{\Delta v_e}{v_e} = \frac{1}{2}\frac{\Delta M}{M} - \frac{1}{2}\frac{\Delta R}{R}$$

Percentage change in escape velocity is

$$\frac{\Delta v_e}{v_e} = \frac{1}{2}(0.5-0.5) = 0\%$$

So, escape velocity will remain same.

Potential energy of an object of mass m on the surface of earth is

$$U = -\frac{GMm}{R}$$

Fractional change in potential energy is

$$\frac{\Delta U}{U} = -\frac{\Delta M}{M} + \frac{\Delta R}{R}$$

Percentage change in PE is

$$\frac{\Delta U}{U} - 0.5 + 0.5 = 0\%$$

So, potential energy will remain same.

Hence, the correct answer is (C).

Multiple Correct Choice Type Questions

1. $T^2 = \frac{4\pi^2}{GM}(R_e+h)^3$ and $v_0 = \sqrt{\frac{GM}{R_e+h}}$

If h R_e, then

$$T = 2\pi\sqrt{\frac{R_e}{g}}$$

$$\Rightarrow \quad T = 84.4 \text{ minute}$$

Hence, (B) and (C) are correct.

2. $U_i = \frac{-GMm}{R} = U$ (given)

$$\Delta U = U_f - U_i = \frac{-GMm}{(R+R)} + \frac{GMm}{R}$$

$$\Rightarrow \quad \Delta U = \frac{GMm}{2R} = -\frac{U}{2}$$

Same is the case with potential.

Hence, (A) and (D) are correct.

3. At a point P lying outside both the shells, the field will be due to both the shells as if they were point masses with total mass (m_1+m_2) concentrated at the centre. So, $E = G\left(\frac{m_1+m_2}{r^2}\right)$ for $r > r_1$ and $r > r_2$.

If P lies inside one shell and outside the other. Then field at point P due to the shell enclosing it is zero and is $\frac{Gm_2}{r^2}$ due to the other shell.

If P lies inside both the shells, then $E_p = 0$.

Hence, (A), (B) and (C) are correct.

4. Similarly, as in SOLUTION to PROBLEM 3,

$$V_p = -\frac{G(m_1+m_2)}{r} \text{ for } r > R_1,\ r > R_2$$

Which satisfies OPTION **(A)**

For the point P lying inside shell 1 and outside shell 2, the total potential at P equals the potential at P due to shell 1 plus potential at P due to shell 2.

$$V_p = V_{p1} + V_{p2}$$

$$\Rightarrow \quad V_p = \frac{-Gm_1}{r_1} + \frac{-Gm_2}{r}$$

$$\Rightarrow \quad V_p = -G\left(\frac{m_1}{r_1} + \frac{m_2}{r}\right)$$

Which satisfies OPTION **(C)**

(Since field inside shell 1 is zero, so potential at a point lying inside shell 1 is a constant and that equals the value of the potential at the surface of shell 1 i.e. $-\frac{Gm_1}{r_1}$).

Similarly, for point P lying inside both the shells, we have

$$V = -\frac{Gm_1}{r_1} - \frac{Gm_2}{r_2}$$

$$V = -G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2}\right) \text{ for } r < r_2$$

Which satisfies OPTION **(D)**

Hence, (A), (C) and (D) are correct.

5. **Conceptual (A), (B), (C) and (D) are correct.**

6. Inside the inner sphere field is zero but potential is constant.

Between two, field is due to inner sphere and potential is due to both but it is constant due to outer shell.

Outside the outer shell, field and potential are due to both and it decreases in both the cases.

Hence, (A) and (C) are correct.

7.

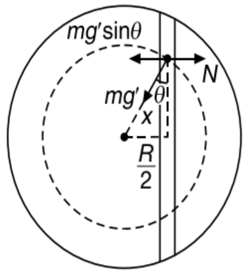
Velocity of Satellite	Nature of Path
$v = v_0$	Circular path around the earth.
$v < v_0$	Elliptical path and body returns to earth.
$v > v_0$ but $< v_e$	Elliptical path around the earth and will not escape.
$v = v_e$	Parabolic path and it escapes from the earth.
$v > v_e$	Hyperbolic path and escapes from earth.

Hence, (A), (B), (C) and (D) are correct.

8. Net force towards centre of earth is mg', where $g' = g\left(\frac{x}{R}\right)$

$$\Rightarrow \quad mg' = \frac{mgx}{R}$$

Normal force $N = mg'\sin\theta$



Since $\sin\theta = \frac{\frac{R}{2}}{x}$

$$\Rightarrow \quad \sin\theta = \frac{R}{2x}$$

$$\Rightarrow \quad N = \left(\frac{mgx}{R}\right)\left(\frac{R}{2x}\right)$$

$$\Rightarrow \quad N = \frac{mg}{2}$$

Constant and independent of x

Tangential force is given by

$$F = ma = mg'\cos\theta$$

$$\Rightarrow \quad a = g'\cos\theta$$

Since $\cos\theta = \frac{\sqrt{\frac{R^2}{4} - x^2}}{x}$

$$\Rightarrow \quad a = \frac{gx}{R}\sqrt{R^2 - 4x^2}$$

So, curve is parabolic and at $x = \frac{R}{2}$ we have $a = 0$

Hence, (B) and (C) are correct.

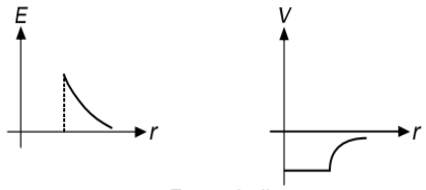
9. $V = -\frac{GM}{R}$

$$\Rightarrow \quad V = -\frac{GM}{R^2}R$$

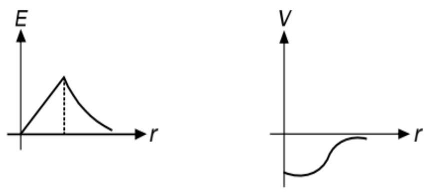
$$\Rightarrow \quad V = -gR$$

Hence, (C) and (D) are correct.

10. E-r and V-r graphs for a spherical shell and a solid sphere are shown here.



For a shell



For a solid sphere

Hence, (A), (B) and (C) are correct.

11. $E = -\dfrac{GMm}{2a}$

So, $E \propto m$

$E \propto m_s$

$E \propto \dfrac{1}{a}$

Hence, (A), (B) and (C) are correct.

12. $T^2 = \dfrac{4\pi^2}{GM}\left(\dfrac{r_A + r_P}{2}\right)^3 \qquad \left\{\because r = \dfrac{r_A + r_P}{2}\right\}$

$\Rightarrow \quad T^2 = \dfrac{\pi^2}{2GM}(r_A + r_P)^3$

By Law of Conservation of Angular Momentum

$mv_A r_A = mv_P r_P$

$\Rightarrow \quad v_A r_A = v_P r_P$

Hence, (B), (C) and (D) are correct.

13. Time period in both the cases is

$T_1 = T_2 = 2\pi\sqrt{\dfrac{R^3}{GM}} \approx 84.6 \text{ min}$

However, $v_1 > v_2$, because the difference in potential energy between the extreme position and mean position will be more in the first case.
Hence, (B) and (D) are correct.

14. $E_g = -\dfrac{dV}{dx}$

If $E_g = 0$, then $V = \text{constant}$ and this constant may also be zero.
Hence, (A) and (C) are correct.

15. At two positions, when the planet is closest to the sun (perigee) and when it is farthest from the sun (apogee), velocity vector is perpendicular to force vector i.e., work done is zero. In one complete revolution work done is zero.
Hence, (A) and (D) are correct.

16. The field inside the shell is zero and so potential inside the shell is constant equal to the value that exists at the surface i.e. $-\dfrac{GM}{a}$.
Hence, (A) and (D) are correct.

17. **Hence, (B) and (D) are correct.**

18. By Law of Conservation of Mechanical Energy, we get

$(U + K)_\infty = (U + K)_r$

$\Rightarrow \quad 0 + 0 = -\dfrac{Gm(4m)}{r} + \dfrac{1}{2}\mu v_r^2$

$\Rightarrow \quad \dfrac{G(m)(4m)}{r} = \dfrac{1}{2}\mu v_r^2 \qquad \ldots(1)$

where, $\mu = \text{reduced mass} = \dfrac{(m)(4m)}{m + 4m} = \dfrac{4m}{5}$ and

$v_r =$ relative velocity of approach

Substituting in equation (1), we get

$\Rightarrow \quad v_r = \sqrt{\dfrac{10Gm}{r}}$

From equation (1), the total kinetic energy is

$K = \dfrac{G(m)(4m)}{r}$

$\Rightarrow \quad K = \dfrac{4Gm^2}{r}$

Net torque of two equal and opposite forces acting on two objects is zero. Therefore, angular momentum will remain conserved. Initially both the objects were stationary i.e., angular momentum about any point was zero. Hence, angular momentum of both the particles about any point will be zero at all instants.
Hence, (A), (B), (C) and (D) are correct.

19. When total force is zero, $g_{eff} = 0$

$V = -\dfrac{Gm}{r}$

For a shell of radius R,

$V_{\underset{\text{shell}}{\text{inside}}} = V_{\text{surface}} = -\dfrac{GM}{R}$

$I_{\text{inside}} = 0$ whereas

$I_{\text{outside}} = \dfrac{GM}{r^2}$

So, plot of V vs r is continuous whereas plot of I vs r is discontinuous
Hence, (A), (B) and (C) are correct.

20. Orbital radius of first satellite is $2R$ and that of second satellite is $8R$.
For a satellite

$K.E. = \dfrac{GMm}{2r}$

$P.E. = -\dfrac{GMm}{r}$

$T.E. = -\dfrac{GMm}{2r}$

Hence, (A), (B) and (C) are correct.

21. **Conceptual (A), (B) and (C) are correct.**

22. According to Kepler's Laws Areal Speed is constant. Also, by Law of Conservation of Angular Momentum L must be constant.
Hence, (B) and (C) are correct.

23. According to Right Hand Thumb Rule curl the fingers of right hand in the direction of rotation then thumb gives the direction of the areal velocity/angular momentum.
Hence, (A) and (C) are correct.

24. At centre, $V = -\frac{3}{2}\frac{GM}{R}$

and $E = 0$

Hence, (A) and (D) are correct.

25. Force on P is $\frac{Gm_1m_3}{r^2}$, attractive i.e. towards O.

Hence, (A), (B) and (D) are correct.

26. $v_0 = \sqrt{\frac{GM}{r}}$

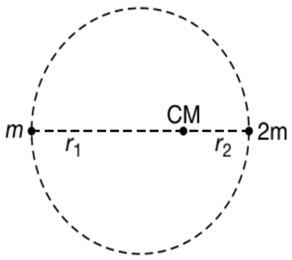
$$T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

$$U = \frac{-GMm}{2r}$$

$$K = \frac{GMm}{2r}$$

Hence, (A) and (D) are correct.

27.



Let CM of system be at a distance r_1 from m and r_2 from $2m$. Then, $mr_1 = 2m(r - r_1)$

$\Rightarrow \quad r_1 = \frac{r}{3}$ and $r_2 = r - r_1 = \frac{2r}{3}$

Since, $T_2^2 = \frac{4\pi^2 r_2^3}{Gm}$

$$\Rightarrow \quad T_2^2 = \frac{32\pi^2 r^3}{27Gm}$$

$\Rightarrow \quad T_2 \propto r^{\frac{3}{2}}$ and $T_2 \propto m^{-\frac{1}{2}}$

Hence, (A) and (D) are correct.

28. Conceptual (A), (B) and (D) are correct.

29. Distance from centre of sun and hence the kinetic energy and potential energy keep changing.
Hence, (C) and (D) are correct.

30. $U_i = -\frac{GMm}{R}$

$$\Rightarrow \quad U_f = -\frac{GMm}{R+h}$$

$$(\text{Work done}) = (\text{Change in G.P.E.})$$

$$\Rightarrow \quad W = U_f - U_i = -GMm\left(\frac{1}{R+h} - \frac{1}{R}\right)$$

$$\Rightarrow \quad W = -\frac{GMm}{R}\left[\left(1 + \frac{h}{R}\right)^{-1} - 1\right]$$

$$\Rightarrow \quad W = -\frac{GMm}{R}\left[1 - \frac{h}{R} - 1\right]$$

$$\Rightarrow \quad W \approx \frac{GMmh}{R^2}$$

$\Rightarrow \quad W \approx mgh$ for $h \quad R$

Also, $U_i = -\frac{GMm}{R}$, and

$$U_f(h = R) = -\frac{GMm}{R+R} = -\frac{GMm}{2R}$$

$$\Rightarrow \quad W = U_f - U_i = \frac{GMm}{2R}$$

$$\Rightarrow \quad W = \frac{1}{2}mgR \qquad \left\{\because g = \frac{GM}{R^2}\right\}$$

Hence, (A) and (D) are correct.

31. $T^2 = \frac{4\pi^2}{GM}(R+h)^3$

Since $h \quad R$

$$\Rightarrow \quad T^2 = \text{MIN} = \frac{4\pi^2 R^3}{GM}$$

Also, $v = \frac{2\pi R}{T}$

$\Rightarrow \quad v$ is MAXIMUM

and total energy is $E = -\frac{GMm}{2R} = \text{MINIMUM}$

Hence, (A), (B) and (C) are correct.

32. Since, $t = nT$

$$\Rightarrow \quad T = \frac{t}{n} = \frac{40}{20}$$

$$\Rightarrow \quad T = 2 \text{ s}$$

Now, $\Delta T = n\Delta T$

$\Rightarrow \quad \dfrac{\Delta t}{t} = \dfrac{\Delta T}{T}$

So, $\dfrac{1}{40} = \dfrac{\Delta T}{2}$

$\Rightarrow \quad \Delta T = 0.05$

Since, time period, $T = 2\pi\sqrt{\dfrac{\ell}{g}}$

$\Rightarrow \quad \dfrac{\Delta T}{T} = -\dfrac{1}{2}\dfrac{\Delta g}{g}$

$\Rightarrow \quad -\dfrac{\Delta g}{g} = 2\dfrac{\Delta T}{T}$

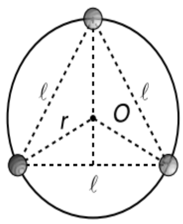
So, percentage error in g is $\dfrac{\Delta g}{g} \times 100\%$

% Error $= -2\dfrac{\Delta T}{T} \times 100\% = -2 \times \dfrac{0.05}{2} \times 100\%$

$\Rightarrow$ % Error $= 5\%$

Hence, (A) and (C) are correct.

33. The force of attraction between any two point masses is responsible for providing the necessary centripetal force to a mass to revolve in a circle of radius r. Using trigonometry, we get



$\cos 30 = \dfrac{\ell/2}{r} = \dfrac{\ell}{2r}$

$\Rightarrow \quad r = \dfrac{\ell}{2\cos 30}$

$\Rightarrow \quad r = \dfrac{\ell}{\sqrt{3}}$

$\Rightarrow \quad mr\omega^2 = \dfrac{Gmm}{\ell^2}$

$\Rightarrow \quad m\dfrac{\ell}{\sqrt{3}}\omega^2 = \dfrac{Gm^2}{\ell^2}$

$\Rightarrow \quad \omega = \sqrt{\dfrac{\sqrt{3}Gm}{\ell^3}}$

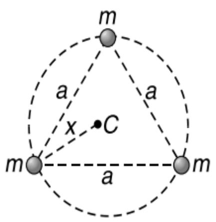
$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{\ell^3}{\sqrt{3}Gm}}$

$\Rightarrow \quad T \propto \ell^{\frac{3}{2}}$ and {OPTION **(A)**}

$T \propto m^{-\frac{1}{2}}$ {OPTION **(D)**}

Hence, (A) and (D) are correct.

34.



Distance of any mass from centre is

$r = \dfrac{a}{\sqrt{3}}$

So, radius of circular path followed is

$r = \dfrac{a}{\sqrt{3}}$

Mass is moving in circular path of radius $r = \dfrac{a}{\sqrt{3}}$

Such that gravitational force on the particle due to other two provides the necessary centripetal force.

$\Rightarrow \quad \dfrac{mv^2}{\left(\dfrac{a}{\sqrt{3}}\right)} = \dfrac{\sqrt{3}Gm^2}{a}$

$\Rightarrow \quad v = \sqrt{\dfrac{Gm}{a}}$

$\Rightarrow \quad T = \dfrac{2\pi\left(\dfrac{a}{\sqrt{3}}\right)}{v} = \sqrt{\dfrac{2\pi a^3}{3Gm}}$

Total kinetic energy of the particles is

$K = 3\left(\dfrac{1}{2}mv^2\right) = \dfrac{3}{2}\dfrac{Gm^2}{a}$

Potential energy U of the system is

$U = -3\left(\dfrac{Gm^2}{a}\right)$

$\Rightarrow$ Total energy E is

$E = U + K = -\dfrac{3}{2}\left(\dfrac{Gm^2}{a}\right)$

$\Rightarrow$ Binding Energy $= \dfrac{3}{2}\left(\dfrac{Gm^2}{a}\right)$

Hence, (B) and (C) are correct.

35. Kinetic energy, $KE = \dfrac{GMm}{2r}$

Potential energy, $PE = -\dfrac{GMm}{r}$

and the total energy, $E = -\dfrac{GMm}{2r}$

Kinetic energy is always positive and $KE \propto \dfrac{1}{r}$

Potential energy is negative and $|PE| \propto \dfrac{1}{r}$

Similarly, total energy is also negative and $|E| \propto \dfrac{1}{r}$

Also $|E| < |PE|$, so from the graph we observe that A is kinetic energy, B is potential energy and C is total energy of the satellite.

Hence, (A), (B) and (D) are correct.

36. At all the places potential will increase by $\frac{GM}{R}$, so

$V_\infty = \frac{GM}{R}$ and $V_{\text{centre}} = -\frac{3GM}{2R} + \frac{GM}{R} = -\frac{GM}{2R}$

Hence, (C) and (D) are correct.

37. **Conceptual (A), (B), (C) and (D) are correct.**

38. Both the stars will revolve about their centre of mass. So, if the centre of mass be at a distance x from $2m$, then

$$x = \frac{2m(0)+mr}{3m} = \frac{r}{3}$$

So, $r_1 = \frac{2r}{3}$ and $r_2 = \frac{r}{3}$

ω and T will be same for both the stars, so

$$K_1 = \frac{1}{2}I_1\omega^2 \text{ and } K_2 = \frac{1}{2}I_2\omega^2$$

$$\Rightarrow \frac{K_1}{K_2} = \frac{I_1}{I_2} = \frac{m\left(\frac{2r}{3}\right)^2}{2m\left(\frac{r}{3}\right)^2} = 2$$

$$L_1 = I_1\omega \text{ and } L_2 = I_2\omega$$

$$\Rightarrow \frac{L_1}{L_2} = \frac{I_1}{I_2} = 2$$

Hence, (A), (B) and (C) are correct.

Reasoning Based Questions

1. $\frac{d\vec{A}}{dt} = \frac{\vec{L}}{2m}$

Since $\vec{L} = \text{constant}$

$$\Rightarrow \frac{d\vec{A}}{dt} = \text{constant}$$

Also, $\frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega$

$$\Rightarrow \frac{dA}{dt} = \frac{mr^2\omega}{2m} = \frac{L}{2m} = \text{constant}$$

Hence, the correct answer is (A).

2. The torque on a body is given by $\vec{\tau} = \frac{d\vec{L}}{dt}$. In case of planet orbiting around Sun no torque is acting on it. So,

$$\frac{d\vec{L}}{dt} = 0$$

$$\Rightarrow \vec{L} = \text{constant}$$

Hence, the correct answer is (A).

3. Using energy conservation $\frac{1}{2}mv_e^2 - \frac{GMm}{R} = 0$

$$\Rightarrow v_e = \sqrt{\frac{2Gm}{R}} = \sqrt{2gR}$$

Hence, the correct answer is (A).

4. Although no gravitational field is produced inside a symmetric shell, it produces a field at points outside of shell.

Hence, the correct answer is (D).

5. Work will be done only in bringing the unit mass from infinity upto the surface of shell because inside shell there is no gravitational field and in moving inside the shell no work will be done.

Hence, the correct answer is (D).

6. $g' = g\left(1 - \frac{2h}{R}\right)$

$$\Rightarrow \Delta g = g - g' = g\left(\frac{2h}{R}\right)$$

$$\Rightarrow \frac{dW}{dh} = \frac{d}{dh}(m\Delta g) = m\frac{d}{dh}(\Delta g) = m\frac{d}{dh}\left(\frac{2gh}{R}\right)$$

$$\Rightarrow \frac{dW}{dh} = \frac{2mg}{R} = \text{constant}$$

Hence, the correct answer is (B).

7. $V_{in} = \frac{GM}{2R^3}(3R^2 - r^2)$

At surface, $r = R$, so $V_s = \frac{GM}{R}$

At centre, $r = 0$, so $V_{\text{centre}} = \frac{3}{2}\frac{GM}{R}$

$$\Rightarrow V_{\text{centre}} = \frac{3}{2}V_S$$

$$\Rightarrow V_{in} > V_S$$

V is not same everywhere as indicated by V_{in}.

Hence, the correct answer is (C).

8. The time period of satellite which is very near to earth is given by $T = 2\pi\sqrt{\frac{R}{g}} = 84 \text{ min} = 1 \text{ hr } 24 \text{ min}$

Hence, the correct answer is (A).

9. Work done in conservative field in cyclic process is zero.

Hence, the correct answer is (A).

10. Acceleration due to gravity is given by $g = \frac{GM}{r^2}$, so it does not depend on mass of body on which it is acting. Also, it is not a constant quantity because it changes with the change in value of both M and r (distance between two bodies).

Hence, the correct answer is (C).

Linked Comprehension Type Questions

1. According to Kepler's Law, we have

$$T^2 \propto r^3$$

$$\Rightarrow \quad \frac{T_m^2}{T_e^2} = \frac{r_m^3}{r_e^3}$$

$$\Rightarrow \quad T_m = \left(\frac{r_m}{r_e}\right)^{3/2} T_e$$

Since $T_e = 1$ yr

$$\Rightarrow \quad T_m = \left(\frac{6 \times 10^{10}}{1.5 \times 10^{11}}\right)^{3/2} (1 \text{ yr})$$

$$\Rightarrow \quad T_m = \left(\frac{6 \times 10^{10}}{15 \times 10^{10}}\right)^{3/2} \text{yr}$$

$$\Rightarrow \quad T_m = \left(\frac{2}{5}\right)^{3/2} \text{yr}$$

Hence, the correct answer is (D).

2. Since orbital velocity v_o is given by

$$v_o = \sqrt{\frac{GM}{r}}$$

$$\Rightarrow \quad (v_o)_m = \sqrt{\frac{GM_{\text{sun}}}{r_m}}$$

$$\Rightarrow \quad (v_o)_e = \sqrt{\frac{GM_{\text{sun}}}{r_e}}$$

$$\Rightarrow \quad \frac{(v_o)_m}{(v_o)_e} = \sqrt{\frac{r_e}{r_m}}$$

$$\Rightarrow \quad \frac{(v_o)_m}{(v_o)_e} = \sqrt{\frac{1.5 \times 10^{11}}{6 \times 10^{10}}} = \sqrt{\frac{15}{6}}$$

$$\Rightarrow \quad \frac{(v_o)_m}{(v_o)_e} = \sqrt{\frac{5}{2}}$$

Hence, the correct answer is (A).

3. $U_i = -\dfrac{GMm}{R} = -\dfrac{GMm}{R + \dfrac{R}{4}}$

$$\Rightarrow \quad \Delta U = U_f - U_i = \frac{1}{5}\frac{GMm}{R}$$

$$\Rightarrow \quad \Delta U = \frac{mgR}{5} \qquad \left\{\because g = \frac{GM}{R^2}\right\}$$

Hence, the correct answer is (C).

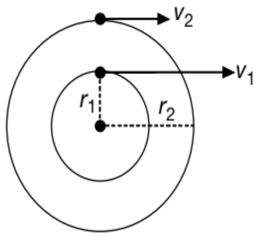
4. By Law of Conservation of Energy

$$\Delta U = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2}{5}gR}$$

Hence, the correct answer is (D).

5. $\dfrac{T_1^2}{T_2^2} = \dfrac{r_1^3}{r_2^3}$



$$\Rightarrow \quad \frac{1}{64} = \frac{r_1^3}{r_2^3}$$

$$\Rightarrow \quad r_2 = 4r_1$$

$$\Rightarrow \quad r_2 = 4 \times 10^4 \text{ km}$$

$$v_1 = \frac{2\pi r_1}{T_1} \text{ and } v_2 = \frac{2\pi r_2}{T_2}$$

$$\Rightarrow \quad v_1 = \frac{2\pi(10^4)}{1} \text{ kmh}^{-1}$$

and $v_2 = \dfrac{2\pi(4 \times 10^4)}{8} = \pi \times 10^4 \text{ kmh}^{-1}$

$\Rightarrow$ Speed of S_2 relative to S_1 is v_{21}

$$v_{21} = -\pi \times 10^4 \text{ kmh}^{-1}$$

Hence, the correct answer is (A).

6. $\omega = \dfrac{v_2 - v_1}{r_2 - r_1}$

Hence, the correct answer is (B).

7. $F = \begin{cases} \dfrac{GMm}{r^2} & , \; r \ge R \\ 0 & , \; r < R \end{cases}$

Hence, the correct answer is (A).

8. $F(r) = \begin{cases} \dfrac{GMm}{r^2} & , \; r \ge R \\ \dfrac{4\pi G\rho r m}{3} & , \; r < R \end{cases}$

where ρ is density of sphere

Hence, the correct answer is (C).

9. Given that $v_o = \dfrac{v_e}{2}$, where $v_e = \sqrt{\dfrac{2GM}{R}}$

Since $v_o = \sqrt{\dfrac{GM}{R+h}} = \dfrac{1}{2}\sqrt{\dfrac{2GM}{R}}$

$$\Rightarrow \quad R + h = 2R$$

$$\Rightarrow \quad h = R$$

Hence, the correct answer is (C).

10. Since the satellite is stopped suddenly, so its kinetic energy in this orbit becomes zero, however its gravitational potential energy in this orbit is

$$U = -\frac{GMm}{R+h} = -\frac{GMm}{2R}$$

Since $(U+K)_{\text{initial}} = (U+K)_{\text{final}}$

$$\Rightarrow \quad -\frac{GMm}{2R} + \frac{1}{2}m(0)^2 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}v^2 = \frac{GM}{2R}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}}$$

Also, we know that $g = \frac{GM}{R^2}$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

Hence, the correct answer is (A).

11. $T = 2\pi\sqrt{\frac{R}{g}}$, substituting $g = \frac{GM}{R^2}$ we get,

$$T = 2\pi\sqrt{\frac{R^3}{GM}}$$

Hence, (A), (B) and (C) are correct.

12. Maximum speed is attained by the particle at centre. Applying Law of Conservation of Mechanical Energy (from surface to centre), we get

$$(U+K)_{\text{surface}} = (U+K)_{\text{centre}}$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}m(0)^2 = -\frac{3}{2}\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$v = \sqrt{2\left(-\frac{GM}{R} + \frac{3GM}{2R}\right)} = \sqrt{\frac{GM}{R}}$$

Hence, the correct answer is (B).

13. $U = -\frac{GmM}{\sqrt{a^2+x^2}}$

Hence, the correct answer is (A).

14. Since $F = \frac{GMmx}{(a^2+x^2)^{3/2}}$

When $x = 0$, $F = 0$

Hence, the correct answer is (D).

15. For small x i.e., $x \quad a$, the force acting on the particle is

$$F \approx -\left(\frac{GMm}{a^3}\right)x$$

The negative sign showing that this force is directed towards the mean position (i.e. position where $F = 0$). Hence the particle will perform oscillations about the mean position i.e. the centre of the ring.

Hence, the correct answer is (B).

16. Given that $r = 3.6 \times 10^7$ m, $m = 5000$ kg,

$v = 4000\ \text{ms}^{-1}$ and $\phi = 30°$

Angular momentum of satellite

$$L = mvr\sin\phi$$

$$\Rightarrow \quad L = 5000 \times 4000 \times 3.6 \times 10^7 \times \sin 30°$$

$$\Rightarrow \quad L = 3.6 \times 10^{14}\ \text{Js}$$

Hence, the correct answer is (D).

17. Energy of the satellite is

$$E = K + U = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(5000)(4000)^2$$

$$\Rightarrow \quad K = 4 \times 10^{10}\ \text{J}$$

$$\Rightarrow \quad U = -\frac{(6.67 \times 10^{-11}) \times (5.97 \times 10^{24}) \times 5000}{3.6 \times 10^7}$$

$$\Rightarrow \quad U = -5.5 \times 10^{10}\ \text{J}$$

$$\Rightarrow \quad E = K + U = -1.5 \times 10^{10}\ \text{J}$$

Hence, the correct answer is (D).

18. Since $E = -\frac{GMm}{2a}$

So, semi-major axis a is given by

$$a = \frac{-GMm}{2E}$$

$$\Rightarrow \quad a = -\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 5000}{2 \times (-1.5 \times 10^{10})}$$

$$\Rightarrow \quad a = 6.6 \times 10^7\ \text{m}$$

Hence, the correct answer is (A).

19. Eccentricity (e) of the orbit is given by

$$e = \left(1 + \frac{2EL^2}{G^2M^2m^3}\right)$$

$$\Rightarrow \quad e = 1 + \frac{2 \times (-1.5 \times 10^{10}) \times (3.6 \times 10^{14})^2}{(6.67 \times 10^{-11})^2 \times (5.97 \times 10^{24})^2 \times (5000)^3}$$

$$\Rightarrow \quad e = 0.804$$

Since, semi-minor axis b is given by

$$b = a\sqrt{1-e^2}$$

$$\Rightarrow \quad b = 6.6 \times 10^7 \sqrt{1-(0.804)^2}$$

$$\Rightarrow \quad b = 3.92 \times 10^7\ \text{m}$$

Hence, the correct answer is (B).

20. Minimum distance r_{min} is

$$r_{\text{min}} = a(1-e)$$

$\Rightarrow \quad r_{min} = 6.6 \times 10^7 \times (1 - 0.804)$

$\Rightarrow \quad r_{min} = 1.29 \times 10^7 \text{ m}$

Hence, the correct answer is (C).

21. Maximum distance r_{max} is

$$r_{max} = a(1+e) = 6.6 \times 10^7 \times (1 + 0.804)$$

$\Rightarrow \quad r_{max} = 11.9 \times 10^7 \text{ m}$

Hence, the correct answer is (C).

22. $F_1 = \dfrac{GMm}{(2R)^2} = \dfrac{GMm}{4R^2}$

Hence, the correct answer is (D).

23. Mass of complete sphere,

$$M = \frac{4}{3}\pi R^3 \rho$$

Mass removed $M' = \dfrac{4}{3}\pi\left[\dfrac{R}{2}\right]^3 \rho = \dfrac{1}{8}M$

Force due to hollow sphere F_2 at P equals force due to solid sphere at P minus force due to removed mass at P.

$\Rightarrow \quad F_2 = \dfrac{GMm}{4R^2} - \dfrac{G(M/8)m}{(3R/2)^2}$

$\Rightarrow \quad F_2 = \dfrac{GMm}{R^2}\left[\dfrac{1}{4} - \dfrac{4}{9 \times 8}\right]$

$\Rightarrow \quad F_2 = \dfrac{GMm}{R^2}\left[\dfrac{1}{4} - \dfrac{1}{18}\right]$

$\Rightarrow \quad F_2 = \dfrac{GMm}{R^2}\left[\dfrac{9-2}{36}\right]$

$\Rightarrow \quad F_2 = \dfrac{7}{36}\dfrac{GMm}{R^2}$

Hence, the correct answer is (D).

24. $\dfrac{F_1}{F_2} = \dfrac{GMm}{4R^2} \times \dfrac{36R^2}{7GMm} = \dfrac{9}{7}$

Hence, the correct answer is (A).

25. The system must revolve about the center of mass.
Hence, the correct answer is (B).

26. If we consider the origin at the heavier mass, then the center of mass is at a distance $\dfrac{\ell}{3}$ from the origin. So, the heavier star revolves in an orbit of radius $\ell/3$.
Hence, the correct answer is (C).

27. For m,

$$\frac{Gm(2m)}{\ell^2} = m\left(\frac{2\ell}{3}\right)\omega^2 \qquad \ldots(1)$$

For $2m$,

$$\frac{Gm(2m)}{\ell^2} = (2m)\left(\frac{\ell}{3}\right)\omega^2 \qquad \ldots(2)$$

Adding (1) and (2), we get

$$\frac{4Gm^2}{\ell^2} = \left(\frac{4m\ell}{3}\right)\omega^2$$

$\Rightarrow \quad \omega = \sqrt{\dfrac{3Gm}{\ell^3}}$

Hence, the correct answer is (C).

28. $(\text{K.E.})_{light} = K_L = \dfrac{1}{2}\text{m}\left(\dfrac{2\ell\omega}{3}\right)^2 = \left(\dfrac{2}{9}m\ell\right)\omega^2$

$(\text{K.E.})_{heavy} = K_H = \dfrac{1}{2}(2m)\left(\dfrac{\ell\omega}{3}\right)^2 = \dfrac{m\ell^2\omega^2}{9}$

$\Rightarrow \quad K_L + K_H = \dfrac{1}{3}m\ell^2\omega^2$

Hence, the correct answer is (A).

29. Acceleration due to gravity near the surface of shell can be assumed to be uniform $(h \quad R)$.

$$g = \frac{G(2M)}{(2R)^2} = \frac{GM}{2R^2}$$

Since $h = \dfrac{1}{2}gt^2$

$\Rightarrow \quad t = \sqrt{\dfrac{2h}{g}} = 2\sqrt{\dfrac{hR^2}{GM}}$

Hence, the correct answer is (A).

30. $U_A = \sqrt{2gh} = \sqrt{2\left(\dfrac{Gm}{2R^2}\right)h} = \sqrt{\dfrac{Gmh}{R^2}}$

From A to B, field due to shell is zero, but field due to sphere is non-zero.

Hence, $t_{AB} < \dfrac{R}{V_A}$

$\Rightarrow \quad t_{AB} < \dfrac{R^2}{\sqrt{GMh}}$

Hence, the correct answer is (C).

31. Since $h \quad R$, so we have $K_A \approx 0$

Potential between A and B due to shell is constant.

Applying the Law of Conservation of Energy, we get

$$K_A + U_A = K_B + U_B$$

$\Rightarrow \quad K_B = U_A - U_B = m(V_A - V_B)$

$\Rightarrow \quad \dfrac{1}{2}mv_B^2 = m(V_A - V_B)$

$\Rightarrow \quad v_B = \sqrt{2(V_A - V_B)}$

$\Rightarrow \quad v_B = \sqrt{2\left[-\frac{GM}{2R} + \frac{GM}{R}\right]}$

$\Rightarrow \quad v_B = \sqrt{\frac{GM}{R}}$

Hence, the correct answer is (D).

32-33. The correct answer is 32(C) and 33(B).

Combined solution to 32 and 33

If m be the mass of satellite, then by Law of Conservation of Angular Momentum, we have

$$mv_1(2R) = mv_2(4R)$$

$\Rightarrow \quad v_1 = 2v_2 \qquad \ldots(1)$

Also, by Law of Conservation of Mechanical Energy, we have

$$(U+K)_{2R} = (U+K)_{4R}$$

$\Rightarrow \quad -\frac{GmM}{2R} + \frac{1}{2}mv_1^2 = -\frac{GmM}{4R} + \frac{1}{2}mv_2^2$

$$\frac{1}{2}(v_1^2 - v_2^2) = \frac{GM}{R}\left(\frac{1}{2} - \frac{1}{4}\right)$$

$\Rightarrow \quad 4v_2^2 - v_2^2 = \frac{GM}{2R}$

$\Rightarrow \quad v_2 = \sqrt{\frac{GM}{6R}}$

Since, $v_1 = 2v_2$

$\Rightarrow \quad v_1 = \sqrt{\frac{2GM}{3R}}$

34. $\frac{mv_1^2}{r_1} = \frac{GmM}{(2R)^2}$

$\Rightarrow \quad r_1 = \frac{4R^2}{GM}v_1^2 = \left(\frac{4R^2}{GM}\right)\left(\frac{2GM}{3R}\right)$

$\Rightarrow \quad r_1 = \frac{8R}{3}$

Hence, the correct answer is (A).

35. Let M be the mass of the planet, m_1 be the mass of the moon 1, m_2 be the mass of moon 2 and m_3 be the mass of moon 3.

$$\frac{GM}{R_2^2} = \omega^2 R^2$$

$\Rightarrow \quad M = \frac{\omega^2 R_2^3}{G}$

$\Rightarrow \quad M = \left(\frac{2\pi}{3\times 10^5}\right)^2 \times (9\times 10^7)^3 \times \frac{1}{6.67\times 10^{-11}}$

$\Rightarrow \quad M = 4\pi^2 \times 81 \times 10^{11} \times \frac{10^{11}}{6.67}$

$\Rightarrow \quad M = 4.8 \times 10^{24}$

Hence, the correct answer is (D).

36. $\frac{Gm_3}{R_3^2} = 0.2$

$\Rightarrow \quad \frac{6.67\times 10^{-11} m_3}{(2\times 10^5)^2} = 0.2$

$\Rightarrow \quad m_3 = \frac{0.2\times 4\times 10^{10}}{6.67\times 10^{-11}}$

$\Rightarrow \quad m_3 \approx 0.12\times 10^{21} = 1.2\times 10^{20}$ kg

Hence, the correct answer is (C).

37. Centripetal Acceleration of Moon II is

$$a_C = \left(\frac{2\pi}{3\times 10^5}\right)^2 \times 9\times 10^7$$

$\Rightarrow \quad a_C = 4\pi^2 \times 0 \times 10^{-3}$

$\Rightarrow \quad a_C \approx 0.04 \text{ ms}^{-2}$

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. A → (p, q, r); B → (r); C → (t); D → (p, q, r, s)

Conceptual

2. A → (q); B → (p); C → (p); D → (q)

$$E = \frac{Gm}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\right)\rho}{R^2}$$

$\Rightarrow \quad E \propto \rho R$

At surface, $V = -\frac{Gm}{R} = -\frac{G\left(\frac{4}{3}\pi R^3 \rho\right)}{R}$

$\Rightarrow \quad V \propto \rho R^2$

$$V_{\text{centre}} = -\frac{3}{2}\frac{GM}{R}$$

At centre of a solid sphere field strength is zero.

3. A → (q); B → (r); C → (q); D → (p); E → (s)

At perihelion position planet is nearest to sun.

4. A → (p, q, r, s); B → (p, q, r, s); C → (q, r, s); D → (p, r)

$T = 2\pi\sqrt{\frac{r^3}{GM}}$, $v_0 = \sqrt{\frac{GM}{r}}$ and $E = -\frac{GMm}{2r}$

5. A → (p); B → (q); C → (q); D → (q)

Due to rotation of earth, variation in value of g is given by

$$g' = g - R\omega^2\cos^2\phi$$

At pole $\phi = 90°$, so there is no effect of rotation of earth.

At equator g' will increase if ω decreases.

Further, T will increase with decrease in ω

From Kepler's Third Law $T^2 \propto r^3$, so r should also increase.

Since, $E = -\dfrac{GMm}{2r}$, so with increase in r, E also increases.

6. A $\to$ (r, t); B $\to$ (s); C $\to$ (p); D $\to$ (q)
Conceptual

7. A $\to$ (p); B $\to$ (q); C $\to$ (s); D $\to$ (t)

$$E = \frac{GM}{R^2},\ V = -\frac{GM}{R}$$

Gravitational field of the earth is actually the measure of acceleration due to gravity.

At height $h = R$:

$$E' = \frac{GM}{(R+h)^2} = \frac{GM}{4R^2} = \frac{E}{4}$$

$$V' = -\frac{GM}{2R}$$

i.e., E' decreases by a factor $\dfrac{1}{4}$ and V' increases by a factor of 2.

At depth $d = \dfrac{R}{2}$:

$$E' = E\left(\phi - \frac{d}{R}\right) = \frac{E}{2}$$

$$V' = -\frac{GM}{2R^3}\left(3R^2 - r^2\right)$$

$$V' = -\frac{GM}{2R^3}\left(3R^2 - \frac{R^2}{4}\right) = -\frac{11}{8}\frac{GM}{R}$$

i.e., E' decreases by a factor $\dfrac{1}{2}$ and V' also decreases by a factor $\dfrac{11}{8}$.

8. A $\to$ (q); B $\to$ (s); C $\to$ (p); D $\to$ (r)
Conceptual

9. A $\to$ (r, q); B $\to$ (s); C $\to$ (p); D $\to$ (r)
Conceptual

10. A $\to$ (s); B $\to$ (r); C $\to$ (p); D $\to$ (p)
Conceptual

11. A $\to$ (r); B $\to$ (q); C $\to$ (p); D $\to$ (r)
For planet revolving around sun in an elliptical orbit, we have from Kepler's Second Law

$$\frac{\Delta A}{\Delta t} = \frac{L}{2m}$$

$$\Rightarrow \frac{\pi ab}{T} = \frac{L}{2m}$$

$$\Rightarrow \frac{2\pi mab}{T} = L = \text{constant}$$

Integer/Numerical Answer Type Questions

1. Area that cannot be covered by the satellite moving in an orbit of radius r as shown in figure is

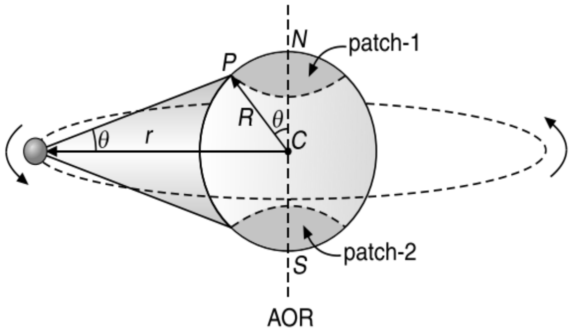
$$A = 4\pi R^2 - \frac{4\pi R^2\sqrt{r^2 - R^2}}{r}$$

$$\Rightarrow A = 2\Omega R^2$$

where, $\Omega = 2\pi(1 - \cos\theta)$

$$\Rightarrow \Omega = 2\pi\left(1 - \frac{\sqrt{r^2 - R^2}}{r}\right)$$

$$\Rightarrow A = 4\pi R^2\left(1 - \frac{\sqrt{r^2 - R^2}}{r}\right)$$



According to the problem, $A = 0.25 \times 4\pi R^2$

$$\Rightarrow 1 - \frac{\sqrt{r^2 - R^2}}{r} = 0.25$$

$$\Rightarrow \frac{\sqrt{r^2 - R^2}}{r} = \frac{3}{4}$$

$$\Rightarrow \frac{r^2 - R^2}{r^2} = \frac{9}{16}$$

$$\Rightarrow 16r^2 - 16R^2 = 9r^2$$

$$\Rightarrow r = \frac{4}{\sqrt{7}}R = 1.515R$$

$$\Rightarrow \frac{r}{R} = \frac{4}{\sqrt{7}} = 1.51$$

2. Since, $dW = \vec{F} \cdot d\vec{\ell}$

$$\Rightarrow dW = m\vec{E}_g \cdot d\vec{\ell}$$

$$\Rightarrow dW = m\left(2\hat{i} + 3\hat{j}\right)\cdot\left(\hat{i}dx + \hat{j}dy\right)$$

$$\Rightarrow dW = m\left(2dx + 3dy\right) \quad \ldots(1)$$

Since the particle is moved along the line

$$3y + 2x = 5$$

$$\Rightarrow d(3y + 2x) = 0$$

$$\Rightarrow 3dy + 2dx = 0$$

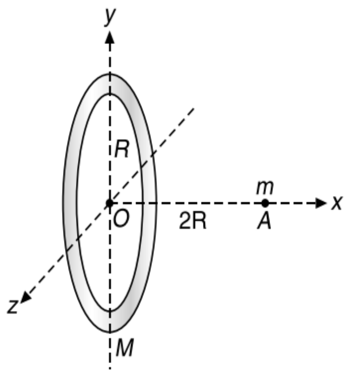
$$\Rightarrow 3dy = -2dx$$

$\Rightarrow \quad dW = 0 \qquad \{\because \text{ of } (1)\}$

$\Rightarrow \quad W = 0$

3. Applying Law of Conservation of Energy, we get

$$(U+K)_A = (U+K)_O$$



$$U_A = -\frac{GMm}{\sqrt{R^2+4R^2}} = -\frac{GMm}{\sqrt{5}R}$$

$$K_A = 0$$

Finally, when m passes through O, we have

$$U_O = -\frac{GMm}{R} \text{ and } K_O = \frac{1}{2}mv^2$$

Since $(U+K)_A = (U+K)_O$

$$\Rightarrow \quad -\frac{GMm}{\sqrt{5}R} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{2(\sqrt{5}-1)GM}{\sqrt{5}R}}$$

$$\Rightarrow \quad v = \left[2\left(1-\frac{1}{\sqrt{5}}\right)\frac{GM}{R}\right]^{\frac{1}{2}}$$

$$\Rightarrow \quad x = 2,\ y = \sqrt{5} \text{ and } z = 2$$

$$\Rightarrow \quad \frac{y^2}{xz} = \frac{(\sqrt{5})^2}{(2)(2)} = \frac{5}{4} = 1.25$$

4. By Law of Conservation of Energy, we have

$$(U+K)_{surface} = (U+K)_{\text{at }\infty}$$

$$\Rightarrow \quad -\frac{GmM}{R} + \frac{1}{2}mu^2 = 0 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad -\frac{2GM}{R} + u^2 = v^2$$

Since $v_e = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \quad -v_e^2 + u^2 = v^2$$

$$\Rightarrow \quad v^2 = -(11.2)^2 + (15)^2 = -125 + 225$$

$$\Rightarrow \quad v = 10 \text{ kms}^{-1}$$

5. By Law of Conservation of Angular Momentum, we have

$$m(v_0 \cos 60^\circ)4R = mvR$$

$$\Rightarrow \quad \frac{v}{v_0} = 2$$

Applying Law of Conservation of Energy, we get

$$-\frac{GMm}{4R} + \frac{1}{2}mv_0^2 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}v^2 - \frac{1}{2}v_0^2 = \frac{3}{4}\frac{GM}{R}$$

$$\Rightarrow \quad v_0 = \frac{1}{\sqrt{2}}\sqrt{64\times10^6} = \frac{8000}{\sqrt{2}} \text{ ms}^{-1}$$

$$\Rightarrow \quad v_0 = 4000\sqrt{2} \text{ ms}^{-1}$$

$$\Rightarrow \quad v_0 = 5656 \text{ ms}^{-1}$$

6. (a) Let x be the displacement of ring, then the displacement of the particle is $(3-x)$ m. Since, no external force is acting on the system and particles are initially at rest, so the centre of mass will not move. Hence,

$$(5.4\times10^9)x = (6\times10^8)(3-x)$$

$$\Rightarrow \quad x = 0.3 \text{ m} = 30 \text{ cm}$$

(b) By Law of Conservation of Linear Momentum, we have

$$0 = 5.4\times10^9 v_1 - 6\times10^8 v_2$$

$$\Rightarrow \quad v_2 = 9v_1 \qquad \ldots(1)$$

By Law of Conservation of Energy, we have

$$(U+K)_{\text{initial}} = (U+K)_{\text{final}}$$

$$\Rightarrow \quad -\frac{Gm_1m_2}{\sqrt{4^2+3^2}} + 0 = -\frac{Gm_1m_2}{4} + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\Rightarrow \quad Gm_1m_2\left(\frac{1}{4}-\frac{1}{5}\right) = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2(81v_1^2)$$

$$\Rightarrow \quad \frac{G(9m_2^2)}{20} = \frac{1}{2}(90m_2)v_1^2$$

$$\Rightarrow \quad v_1 = \sqrt{\frac{Gm_2}{100}} = \sqrt{\frac{6.67\times10^{-11}\times6\times10^8}{100}}$$

$$\Rightarrow \quad v_1 = 0.02 \text{ ms}^{-1} = 2 \text{ cms}^{-1}$$

$$\Rightarrow \quad v_2 = 9v_1 = 18 \text{ cms}^{-1}$$

7. Potential at origin is

$$V_0 = -\frac{2GM}{a}$$

Potential at point $(0,\ 0,\ 2\sqrt{3}a)$ is

$$V_P = -\frac{2GM}{\sqrt{a^2+12a^2}} = -\frac{2}{\sqrt{13}}\frac{GM}{a}$$

By Law of Conservation of Energy, we get

$$(U+K)_O = (U+K)_P$$

$$\Rightarrow \frac{1}{2}\left(\frac{M}{2}\right)v^2 + \left(\frac{M}{2}\right)V_0 = \left(\frac{M}{2}\right)V_P$$

$$\Rightarrow v = \sqrt{2(V_P - V_0)}$$

$$\Rightarrow v = \sqrt{2\left(\frac{2GM}{a} - \frac{2}{\sqrt{13}}\frac{GM}{a}\right)}$$

$$\Rightarrow v = 2\sqrt{\frac{GM}{a}\left(1-\frac{1}{\sqrt{13}}\right)}$$

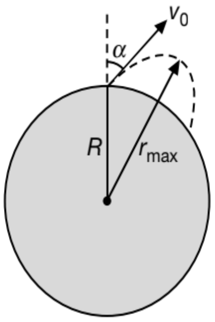
$$\Rightarrow v = \sqrt{\frac{4GM}{a}\left(1-\frac{1}{\sqrt{13}}\right)}$$

$$\Rightarrow \alpha = 4 \text{ and } \beta = 13$$

8. Let v be the speed of the projectile at highest point and r_{max} its distance from the centre of the earth. By Law of Conservation of Angular Momentum and Law of Conservation of Mechanical Energy, we have

$$mv_0 \sin\alpha = mvr_{max} \quad \ldots(1)$$

$$\frac{1}{2}mv_0^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 - \frac{GMm}{r_{max}} \quad \ldots(2)$$



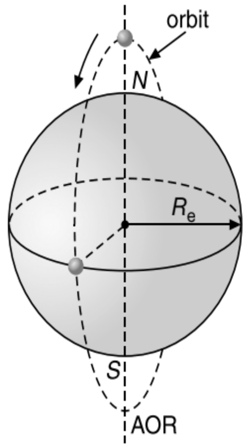
Solving these two equation, we get

$$r_{max} = \frac{3R}{2}$$

So, the maximum height is $h_{max} = r_{max} - R = \frac{R}{2}$

$$\Rightarrow * = 2$$

9. A satellite which rotates with angular speed equal to earth's rotation has angular speed of revolution is given by



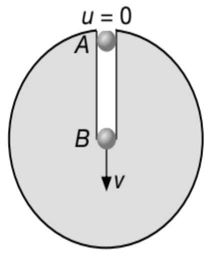
$$\omega = \frac{2\pi}{T}, \text{ where } T = 24 \text{ hr}$$

When satellite moves from a point above north pole to a point above equator, it traverses an angle $\frac{\pi}{2}$. So, this time taken is given by

$$T' = \frac{\pi/2}{(2\pi/T)} = \frac{T}{4} = 6 \text{ hr}$$

10. By Law of Conservation of Energy, we have

$$(U+K)_{surface} = (U+K)_{centre}$$



$$\Rightarrow -\frac{GMm}{R} + \frac{1}{2}m(0)^2 = -\frac{3}{2}\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{2R}$$

where m is the mass of the ball

$$\Rightarrow v = \sqrt{\frac{GM}{R}}$$

Velocity of ball just after collision

$$v' = ev = \frac{1}{2}\sqrt{\frac{GM}{R}}$$

Let r be the distance from the centre upto the point P, where the ball reaches after collision. Then by Law of Conservation of Energy, we have

$$(U+K)_{centre} = (U+K)_P$$

where $U_P = -\frac{GMm}{2R^3}(3R^2 - r^2)$

$\Rightarrow \quad -\frac{3}{2}\frac{GMm}{R}+\frac{1}{2}mv'^2=-\frac{GMm}{2R^3}(3R^2-r^2)+0$

But $v'=\frac{1}{2}\sqrt{\frac{GM}{R}}$

$\Rightarrow \quad -\frac{3}{2}\frac{GMm}{R}+\frac{1}{8}\frac{GMm}{R}=-\frac{GMm}{2R^3}(3R^2-r^2)$

$\Rightarrow \quad -\frac{11}{8}=-\left(\frac{3R^2-r^2}{2R^2}\right)$

$\Rightarrow \quad 22R^2=24R^2-8r^2$

$\Rightarrow \quad 8r^2=2R^2$

$\Rightarrow \quad r^2=\frac{R^2}{4}$

$\Rightarrow \quad r=\frac{R}{2}$

So, the desired distance, is

$$s=R+\frac{R}{2}+\frac{R}{2}=2R$$

$\Rightarrow \quad x=2$

11. At a point P at a distance r from Moon, we have net gravitational field equal to zero

$\Rightarrow \quad g=\frac{GM}{r^2}-\frac{G(81M)}{(D-r)^2}=0$

$\Rightarrow \quad D-r=9r$

$\Rightarrow \quad 10r=D$

So, $r=\frac{D}{10}$ from the Moon and $\frac{9D}{10}$ from the earth.

$\Rightarrow \quad x=9$ and $y=10$

$\Rightarrow \quad y-x=1$

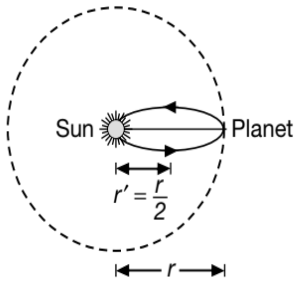
12. Consider an imaginary planet moving along a strongly extended flat ellipse, the extreme points of which are located on the planet's orbit and at the centre of the sun, the semi-major axis of the orbit of such a planet would apparently be half the semi-major axis of the planet's orbit. So, the time period of the imaginary planet T' according to Kepler's Law will be given by:

$$\left(\frac{T'}{T}\right)=\left(\frac{r'}{r}\right)^{3/2}$$

$\Rightarrow \quad T'=T\left(\frac{1}{2}\right)^{3/2} \qquad \left\{\because r'=\frac{r}{2}\right\}$

So, time taken by the planet to fall onto the sun is

$$t=\frac{T'}{2}=\frac{T}{2}\left(\frac{1}{2}\right)^{3/2}$$



$\Rightarrow \quad t=\frac{\sqrt{2}}{8}T=\sqrt{\frac{2}{64}}T=\sqrt{\frac{1}{32}}=\frac{1}{\sqrt{2^5}}$

$\Rightarrow \quad x=5$

13. For point mass at distance $r=3\ell$

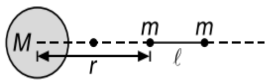
$$\frac{GMm}{(3\ell)^2}-\frac{Gm^2}{\ell^2}=ma \qquad \ldots(1)$$

For point mass at distance $r=4\ell$

$$\frac{GMm}{(4\ell)^2}+\frac{Gm^2}{\ell^2}=ma \qquad \ldots(2)$$

Equating the two equations, we have

$$\frac{GMm}{9\ell^2}-\frac{Gm^2}{\ell^2}=\frac{GMm}{16\ell^2}+\frac{Gm^2}{\ell^2}$$



$$\frac{7GMm}{144}=\frac{2Gm^2}{\ell^2}$$

$$m=\frac{7M}{288}$$

$\Rightarrow \quad k=7$

14. Since, $g_p=\frac{GM_p}{R_p^2}=\frac{4}{3}G\pi R_p\rho_p$

$\Rightarrow \quad \frac{g_p}{g_e}=\frac{R_p\rho_p}{R_e\rho_e}$

Also, $v_e=\sqrt{2gR}$

$\Rightarrow \quad \frac{v_p}{v_e}=\sqrt{\frac{g_pR_p}{g_eR_e}}=\left(\frac{g_p}{g_e}\right)\sqrt{\frac{\rho_e}{\rho_p}}=\frac{\sqrt{12}}{11}\times\sqrt{\frac{3}{4}}$

$\Rightarrow \quad \frac{v_p}{v_e}=\frac{2\sqrt{3}}{11}\times\frac{\sqrt{3}}{2}=\frac{3}{11}$

$\Rightarrow \quad v_p=3\ \text{kms}^{-1}$

15. The gravitational field due to cylinder at a distance x from its axis is $E_g=\frac{2G\lambda}{x}$

$$\Rightarrow \quad m\left(\frac{2G\lambda}{x}\right)=\frac{mv^2}{x}$$

where λ is the linear mass density given by

$$\lambda=\rho\left(\pi R^2\right)$$

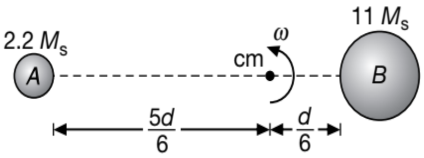
$$\Rightarrow \quad 2G\rho\pi R^2=v^2$$

$$\Rightarrow \quad v=R\sqrt{2G\rho\pi}$$

$$\Rightarrow \quad v=R\left(2G\rho\pi\right)^{\frac{1}{2}}$$

$$\Rightarrow \quad *=2$$

16.



$$\frac{\text{Total angular momentum about cm}}{\text{Angular momentum of B about cm}}=\frac{L}{L_B}$$

$$\Rightarrow \quad \frac{L}{L_B}=\frac{\left(2.2M_S\right)\left(\frac{5\omega d}{6}\right)\left(\frac{5d}{6}\right)+\left(11M_S\right)\left(\frac{\omega d}{6}\right)\left(\frac{d}{6}\right)}{\left(11M_S\right)\left(\frac{\omega d}{6}\right)\left(\frac{d}{6}\right)}$$

$$\Rightarrow \quad \frac{L}{L_B}=6$$

17. The time period T of the satellite is

$$T=2\pi\sqrt{\frac{r^3}{GM}}=2\pi\sqrt{\frac{r^3}{gR^2}}$$

$$\Rightarrow \quad T=2\sqrt{\frac{\pi^2 r^3}{gR^2}}=2\sqrt{\frac{r^3}{R^2}}$$

where, $r=6400+1600=800\text{ km}=8000\times10^3\text{ m}$

and $R=6400\times10^3\text{ m}$

Substituting these values, we get

$$T=2\sqrt{\frac{\left(8000\times10^3\right)^3}{\left(6400\times10^3\right)^2}}=7071\text{ s}$$

Further, orbital speed,

$$v=\sqrt{\frac{GM}{r}}=\sqrt{\frac{gR^2}{r}}$$

$$\Rightarrow \quad v=\sqrt{\left(\frac{9.8}{8000\times10^3}\right)}\times\left(6400\times10^3\right)$$

$$\Rightarrow \quad v=7155\text{ ms}^{-1}$$

Let t be the time interval between two successive moments at which the satellite is overhead to an observer at a fixed position on the equator. Since both satellite and earth are moving in the same sense (i.e. from west to east) with angular speed ω_S and ω_E respectively so, we can write the time of separation (t) as

$$t=\frac{2\pi}{\omega_S-\omega_E}$$

where $\omega_S=\frac{2\pi}{7071}$ and $\omega_E=\frac{2\pi}{86400}$

$$\Rightarrow \quad t=\frac{86400\times7071}{86400-7071}$$

$$\Rightarrow \quad t=\frac{611\times10^6}{79329}$$

$$\Rightarrow \quad t=7702\text{ s}$$

18. Applying conservation of mechanical energy, we have

$$(U+K)_{\text{at }B}=(U+K)_{\text{at }A}$$

$$\Rightarrow \quad U_B+K_B=U_A+K_A$$

$$\Rightarrow \quad U_B+0=U_A+\frac{1}{2}mv_A^2$$

where $U_B=mV_B$ and $U_A=mV_A$, so we have

$$\frac{1}{2}mv_A^2=U_B-U_A=m\left(V_B-V_A\right)$$

$$\Rightarrow \quad v_A=\sqrt{2\left(V_B-V_A\right)} \qquad \ldots(1)$$

Potential at A

$$V_A=\begin{pmatrix}\text{Potential Due}\\\text{to the Complete}\\\text{Sphere at }A\end{pmatrix}-\begin{pmatrix}\text{Potential Due}\\\text{to the Spherical}\\\text{Cavity at }A\end{pmatrix}$$

$$V_A=-\frac{3}{2}\frac{GM}{R}-\left(-\frac{GM'}{r}\right)=\frac{GM'}{r}-\frac{3}{2}\frac{GM}{R}$$

where

$$M=\frac{4}{3}\pi R^3\rho,\ r=\frac{R}{2}\text{ and }M'=\frac{4}{3}\pi r^3\rho=\frac{\pi\rho R^3}{6}$$

Substituting the values, we get

$$V_A=\frac{G}{R}\left(\frac{\pi\rho R^3}{3}-2\pi\rho R^3\right)=-\frac{5}{3}\pi G\rho R^2$$

Potential at B

$$V_B=\begin{pmatrix}\text{Potential Due}\\\text{to the Complete}\\\text{Sphere at }B\end{pmatrix}-\begin{pmatrix}\text{Potential Due}\\\text{to the Spherical}\\\text{Cavity at }B\end{pmatrix}$$

$$\Rightarrow \quad V_B=-\frac{GM}{2R^3}\left(3R^2-r^2\right)-\left(-\frac{3}{2}\frac{GM'}{r}\right)$$

where

$$M=\frac{4}{3}\pi R^3\rho,\ r=\frac{R}{2}\text{ and }M'=\frac{4}{3}\pi r^3\rho=\frac{\pi\rho R^3}{6}$$

$$\Rightarrow \quad V_B = -\frac{11}{8}\frac{GM}{R} + \frac{3GM'}{R}$$

$$\Rightarrow \quad V_B = \frac{G}{R}\left(\frac{\pi\rho R^3}{2} - \frac{11\pi\rho R^3}{6}\right) = -\frac{4}{3}\pi G\rho R^2$$

$$\Rightarrow \quad V_B - V_A = \frac{1}{3}\pi G\rho R^2$$

So, from equation (1)

$$v = \sqrt{\frac{2}{3}\pi G\rho R^2}$$

Also, $v_e = \sqrt{\frac{2GM_{\text{net}}}{R}}$

where $M_{\text{net}} = M - M' = \frac{7}{6}\pi R^3 \rho$

$$\Rightarrow \quad v_e = \sqrt{\frac{2G\left(\frac{7}{6}\pi R^3\right)\rho}{R}}$$

$$\Rightarrow \quad v_e = \sqrt{\frac{7}{3}\pi G\rho R^2}$$

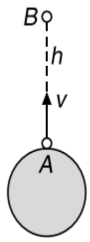
$$\Rightarrow \quad \frac{v_e}{v} = \sqrt{\frac{7}{2}}$$

$$\Rightarrow \quad \frac{v_e^2}{v^2} = \frac{7}{2}$$

$$\Rightarrow \quad \frac{2v_e^2}{v^2} = 7$$

19. At height h, we have

$$g_h = \frac{gR^2}{(R+h)^2} \qquad \ldots(1)$$



Given that $g_h = \frac{g}{9}$

Substituting in equation (1) we get,

$$\frac{1}{9} = \left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \quad h = 2R$$

Applying Law of Conservation of Energy, from A to B, we get

$$(U+K)_A = (U+K)_B$$

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h} + 0$$

Since $h = 2R$, so we have

$$\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{3R}$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{2GM}{3R}$$

$$\Rightarrow \quad v = \sqrt{\frac{4GM}{3R}}$$

Since $v_e = \sqrt{\frac{2GM}{R}}$

Given that $v_e = v\sqrt{N}$

$$\Rightarrow \quad \sqrt{\frac{2GM}{R}} = \sqrt{\frac{4GM}{3R}}\sqrt{N}$$

$$\Rightarrow \quad N = \frac{3}{2} = 1.5$$

20. Let E be the gravitational field at x due to the complete sphere.

If E_1 be the field due to hole and E_2 be the field due to the remaining portion, then we have

$$E = E_1 + E_2$$

$$\Rightarrow \quad E_2 = E - E_1$$

$$\Rightarrow \quad E_2 = \frac{GM}{x^2} - \frac{Gm}{\left(x - \frac{R}{2}\right)^2} \qquad \ldots(1)$$

where, $M = \frac{4}{3}\pi R^3 \rho_0$ and $m = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho_0$

Substituting the values in equation (1), we get

$$E_2 = -\left(\frac{\pi G\rho_0 R^3}{6}\right)\left[\frac{1}{\left(x - \frac{R}{2}\right)^2} - \frac{8}{x^2}\right]$$

$$\Rightarrow \quad E_2 = -\left(\frac{\pi G\rho_0 R^3}{6}\right)\left[\frac{1}{\left(2R - \frac{R}{2}\right)^2} - \frac{8}{(2R)^2}\right]$$

$$\Rightarrow \quad E_2 = -\frac{\pi G\rho_0 R^3}{6}\left(\frac{4}{9R^2} - \frac{2}{R^2}\right)$$

$$\Rightarrow \quad E_2 = -\frac{\pi G\rho_0 R}{6}\left(\frac{4-18}{9}\right)$$

$$\Rightarrow \quad E_2 = \frac{14}{54}\pi G\rho_0 R$$

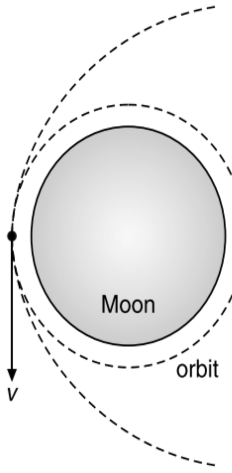
$$\Rightarrow \quad E_2 = \left(\frac{7}{27}\right)\pi G\rho_0 R$$

Since, $E = \left(\frac{a}{a+20}\right)\pi G\rho_0 R$

$$\Rightarrow \quad \frac{a}{a+20} = \frac{7}{27}$$

$$\Rightarrow \quad a = 7$$

21. Figure shows the corresponding situation



Since the spaceship follows a parabolic trajectory tangential to the moon. When at the surface of moon it has speed equal to escape speed

$$v_e = \sqrt{\frac{2GM}{R}}$$

Now to transform it into a circular orbit, its speed should be decreased to orbital speed v_0 given by

$$v_0 = \sqrt{\frac{GM}{R}}$$

So, change in speed is

$$\Delta v = v_e - v_0$$

$$\Rightarrow \quad \Delta v = \left(\sqrt{2}-1\right)\sqrt{\frac{GM}{R}}$$

$$\Rightarrow \quad x = 2$$

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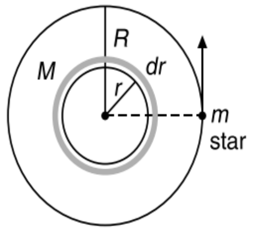
1. Since $dm = \rho dV$, where $dV = 4\pi r^2 dr$



$$\Rightarrow \quad dm = \left(\frac{k}{r}\right)\left(4\pi r^2 dr\right) = 4\pi k r dr$$

$$\Rightarrow \quad M = \int_0^R dm = \int_0^R 4\pi k r dr = 4\pi k \frac{r^2}{2}\Big|_0^R$$

$$\Rightarrow \quad M = 2\pi k\left(R^2 - 0\right) = 2\pi k R^2$$

For circular motion, gravitational force will provide the centripetal force so that

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$\Rightarrow \quad \frac{G\left(2\pi k R^2\right)m}{R^2} = \frac{mv^2}{R}$$

$$\Rightarrow \quad v = \sqrt{2\pi G k R}$$

Time period T is given by

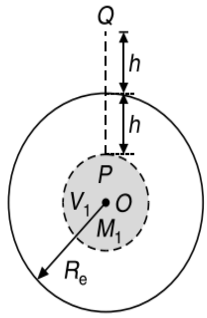
$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{2\pi G k R}} \propto \sqrt{R}$$

$$\Rightarrow \quad T^2 \propto R$$

Hence, the correct answer is (C).

2. Let M be the mass of earth, M_1 be the mass of shaded portion and R be the radius of earth.

At height h, $g_h = \dfrac{GM}{(R+h)^2}$



At depth h, we have

$$M_1 = \rho V_1 = \frac{M}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi(R-h)^3 = \frac{M(R-h)^3}{R^3}$$

Weight of body is same at P and Q, so

$$mg_P = mg_Q$$

$$\Rightarrow \quad g_P = g_Q$$

$$\Rightarrow \quad \frac{GM_1}{(R-h)^2} = \frac{GM}{(R+h)^2}$$

$$\Rightarrow \quad \frac{GM(R-h)^3}{(R-h)^2 R^3} = \frac{GM}{(R+h)^2}$$

$$\Rightarrow \quad (R-h)(R+h)^2 = R^3$$

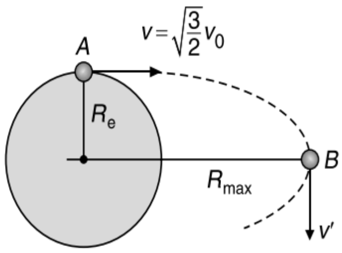
$$\Rightarrow \quad R^3 - hR^2 - h^2R - h^3 + 2R^2h - 2Rh^2 = R^3$$

$$\Rightarrow \quad R^2 - Rh^2 - h^3 = 0$$
$$\Rightarrow \quad R^2 - Rh - h^2 = 0$$
$$\Rightarrow \quad h^2 + Rh - R^2 = 0$$
$$\Rightarrow \quad h = \frac{-R \pm \sqrt{R^2 + 4R^2}}{2}$$
$$\Rightarrow \quad h = \frac{-R + \sqrt{5}R}{2} = \left(\frac{\sqrt{5}-1}{2}\right)R$$

Hence, the correct answer is (A).

3. Since, $v_0 = \sqrt{\frac{GM}{R_e}}$

$$(U+K)_A = (U+K)_B$$



$$\Rightarrow \quad \frac{-GMm}{R_e} + \frac{1}{2}mv^2 = \frac{-GMm}{R_{max}} + \frac{1}{2}mv'^2 \quad \ldots(1)$$

Also, $vR_e = v'R_{max}$ ...(2)

Solving equations (1) and (2), we get

$$R_{max} = 3R_e$$

Hence, the correct answer is (B).

4. According to Gauss Law for gravitation, we have

$$E(4\pi r^2) = \int \rho_0 4\pi r^2 dr$$
$$\Rightarrow \quad Er^2 = 4\pi G \int_0^r \rho_0 \left(1 - \frac{r^2}{R^2}\right) r^2 dr$$
$$\Rightarrow \quad |E| = E = 4\pi G \rho_0 \left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)$$

For E to be maximum, $\frac{dE}{dr} = 0$

$$\Rightarrow \quad r = \sqrt{\frac{5}{9}}R$$

Hence, the correct answer is (B).

5. Given that $E_G = \frac{Ax}{(x^2+a^2)^{3/2}}$ and $V_\infty = 0$

Since, $\int_{V_\infty}^{V_x} dV = -\int_\infty^x \vec{E}_G \cdot d\vec{x}$

$$\Rightarrow \quad V_x - V_\infty = -\int_\infty^x \frac{Ax}{(x^2+a^2)^{3/2}} dx$$

Substitute $x^2 + a^2 = z$ i.e., $2xdx = dz$

$$\Rightarrow \quad V_x - 0 = -\int_\infty^x \frac{Adz}{2(z)^{3/2}} = \left(\frac{A}{z^{1/2}}\right)\Bigg|_\infty^x = \left(\frac{A}{(x^2+a^2)^{1/2}}\right)\Bigg|_\infty^x$$
$$\Rightarrow \quad V_x = \frac{A}{(x^2+a^2)^{1/2}} - 0 = \frac{A}{(x^2+a^2)^{1/2}}$$

Hence, the correct answer is (A).

6. For orbit close to the planet, $R + h \approx R$

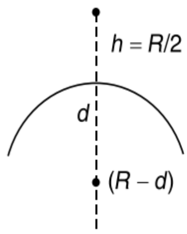
$$v_{orbit} = \sqrt{\frac{GM}{R}}$$

Since, $v_{escape} = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \quad \frac{v_{orbit}}{v_{escape}} = \frac{1}{\sqrt{2}}$$

Hence, the correct answer is (C).

7. Since, $g_1 = \frac{GM}{(R+h)^2} = \frac{GM}{(3R/2)^2}$ ...(1)



Also, $g_2 = \frac{GM(R-d)}{R^3}$ ...(2)

Given that, $g_1 = g_2$

$$\Rightarrow \quad \frac{GM}{(3R/2)^2} = \frac{GM(R-d)}{R^3}$$
$$\Rightarrow \quad \frac{4}{9} = \frac{(R-d)}{R}$$
$$\Rightarrow \quad 4R = 9R - 9d$$
$$\Rightarrow \quad 5R = 9d$$
$$\Rightarrow \quad \frac{d}{R} = \frac{5}{9}$$

Hence, the correct answer is (D).

8. At equator, we have $g_e = g - R\omega^2$

For h R, we have

$$g_2 \approx g\left(1 - \frac{2h}{R}\right) = g - \frac{2gh}{R}$$

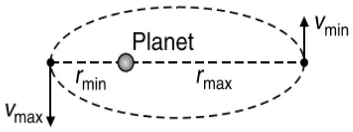
Since, $g_1 = g_2$

$$\Rightarrow \quad R\omega^2 = \frac{2gh}{R}$$

$$\Rightarrow \quad h = \frac{R^2\omega^2}{2g}$$

Hence, the correct answer is (D).

9. Applying conservation of angular momentum, we get



$$r_{min}v_{max} = r_{max}v_{min} \quad \ldots(1)$$

Given that, $v_{min} = \frac{v_{max}}{6}$

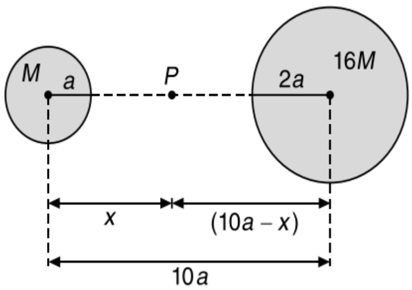
From equation (1), we get

$$\frac{r_{min}}{r_{max}} = \frac{v_{min}}{v_{max}} = \frac{1}{6}$$

Hence, the correct answer is (A).

10. The minimum speed will take the body to the neutral point P after which the bigger planet will automatically attract the body towards itself. So,

$$\frac{GM}{x^2} = \frac{G(16M)}{(10a-x)^2}$$



$$\Rightarrow \quad \frac{1}{x} = \frac{4}{(10a-x)}$$

$$\Rightarrow \quad 4x = 10a - x$$

$$\Rightarrow \quad x = 2a \quad \ldots(1)$$

Applying law of conservation of mechanical energy, we get

$$-\frac{GMm}{8a} - \frac{G(16M)m}{2a} + KE = -\frac{GMm}{2a} - \frac{G(16M)m}{8a}$$

$$\Rightarrow \quad KE = GMm\left(\frac{1}{8a} + \frac{16}{2a} - \frac{1}{2a} - \frac{16}{8a}\right)$$

$$\Rightarrow \quad KE = GMm\left(\frac{1+64-4-16}{8a}\right)$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = GMm\left(\frac{45}{8a}\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{90GM}{8a}} = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

Hence, the correct answer is (B).

11. $W = 196 - mR\omega^2$

Hence, the correct answer is (D).

12. Gravitational field on the surface of a solid sphere

$$I_g = \frac{GM}{R^2}$$

From graph, $\frac{GM_1}{(1)^2} = 2$ and $\frac{GM_2}{(2)^2} = 3$

$$\Rightarrow \quad \frac{M_1}{M_2} = \frac{1}{6}$$

Hence, the correct answer is (D).

13. Initially, the body of mass m is moving in a circular orbit of radius R. So, it must be moving with orbital speed given by

$$v_0 = \sqrt{\frac{GM}{R}}$$

After collision, let the combined mass moves with speed v_1, then by conservation of momentum, we get

$$mv_0 + \left(\frac{m}{2}\right)\left(\frac{v_0}{2}\right) = \left(\frac{3m}{2}\right)v_1$$

$$\Rightarrow \quad v_1 = \frac{5v_0}{6}$$

Since after collision, the speed is not equal to orbitial speed at the point. So, motion cannot be circular. Also, velocity will remain tangential, so it cannot fall vertically towards the planet. Their speed after collision is less than escape speed $\sqrt{2}v_0$, so they cannot escape gravitational field. Hence their motion will be elliptical around the planet.

Hence, the correct answer is (A).

14. The escape velocity is

$$v_e = \sqrt{\frac{2GM}{R}}$$

So, $v_A = \sqrt{\frac{2GM}{R}}$ and $v_B = \sqrt{\frac{2G(M/2)}{R/2}} = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \quad \frac{v_A}{v_B} = 1 = \frac{n}{4}$$

$$\Rightarrow \quad n = 4$$

Hence, the correct answer is (A).

15. Since, $U_1 + K_1 = U_2 + K_2$

$$\Rightarrow \quad -\frac{GM_em}{10R} + \frac{1}{2}mv_0^2 = -\frac{GM_em}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{9}{10}\left(\frac{GM_e m}{R}\right)+\frac{1}{2}mv_0^2=\frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{9}{10}\left(\frac{1}{2}mv_e^2\right)+\frac{1}{2}mv_0^2=\frac{1}{2}mv^2$$

$$\Rightarrow \quad v^2=\frac{9}{10}v_e^2+v_0^2$$

$$\Rightarrow \quad v^2=\frac{9}{10}\times(11.2)^2+(12)^2$$

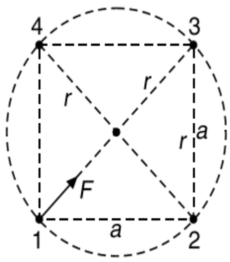
$$\Rightarrow \quad v^2=112.896+144$$

$$\Rightarrow \quad v=16.027\approx 16\text{ kms}^{-1}$$

Hence, the correct answer is 16.

16. Net force on 1 acting towards the centre of circle is

$$F=\frac{GM^2}{a^2}(\sqrt{2})+\frac{GM^2}{2a^2}=\frac{Mv^2}{r}$$



$$\Rightarrow \quad \frac{Mv^2}{\left(\frac{a}{\sqrt{2}}\right)}=\frac{GM^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right) \qquad \left\{\because r=\frac{a}{\sqrt{2}}\right\}$$

$$\Rightarrow \quad v^2=\frac{GM}{a}\left(1+\frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow \quad v=\sqrt{\frac{GM}{a}\left(1+\frac{1}{2\sqrt{2}}\right)}=1.16\sqrt{\frac{GM}{a}}$$

Hence, the correct answer is (B).

17. Let m_0 be the mass of rocket, then

$$E=\frac{GM_e m_0}{R_e} \text{ and } E'=\frac{GM_m m_0}{R_m}$$

Since $V_e=64V_m$

$$\rho R_e^3=64\rho R_m^3$$

$$\Rightarrow \quad R_e=4R_m$$

$$\Rightarrow \quad \frac{E'}{E}=\left(\frac{M_m}{M_e}\right)\left(\frac{R_e}{R_m}\right)=\left(\frac{1}{64}\right)(4)=\frac{1}{16}$$

$$\Rightarrow \quad E'=\frac{E}{16}$$

Hence, the correct answer is (C).

18. $E=\dfrac{GM}{(3a)^2}+\dfrac{2GM}{(3a)^2}$

$$\Rightarrow \quad E=\frac{GM}{3a^2}$$

Hence, the correct answer is (C).

19. $M=\displaystyle\int_0^R (4\pi r^2 dr)\frac{K}{r^2}$

$$\Rightarrow \quad M=4\pi KR$$

$$\Rightarrow \quad G\left(\frac{4\pi KR}{R^2}\right)=\frac{v_0^2}{R}$$

$$\Rightarrow \quad v_0=\sqrt{4\pi GK}$$

Since $T=\dfrac{2\pi R}{v_0}=\dfrac{2\pi R}{\sqrt{4\pi GK}}$

$$\Rightarrow \quad \frac{T}{R}=\text{constant}$$

Hence, the correct answer is (A).

20. Given that $g_h=\dfrac{g}{2}$

Since $g_h=\dfrac{gR^2}{(R+h)^2}$

$$\Rightarrow \quad \frac{g}{2}=g\left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \quad \frac{R}{R+h}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad h=(\sqrt{2}-1)R$$

$$\Rightarrow \quad h=6400\times 0.414$$

$$\Rightarrow \quad h=2649.6\text{ km}$$

$$\Rightarrow \quad h=2.6\times 10^6\text{ m}$$

Hence, the correct answer is (D).

21. Since $T=\dfrac{2\pi r}{v_0}$ and $v_0=\sqrt{\dfrac{GM}{r}}$

$$\Rightarrow \quad T=2\pi r\sqrt{\frac{r}{GM}}=2\pi\sqrt{\frac{r^3}{GM}}$$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{(202)^3\times 10^{12}}{6.67\times 10^{-11}\times 8\times 10^{22}}}\text{ s}$$

$$\Rightarrow \quad T=7812.2\text{ s}$$

$$\Rightarrow \quad T \quad 2.17\text{ hr}$$

So, number of revolutions is

$$N=\frac{24}{T}\approx 11$$

Hence, the correct answer is (C).

22. Given that $\frac{g_e}{g_p}=\frac{9}{4}$

Since $g=\frac{GM}{R^2}$

$\Rightarrow \quad \frac{M_e \times R_p^2}{R_e^2 \times M_p}=\frac{9}{4}$

$\Rightarrow \quad 9\times\left(\frac{R_p}{R_e}\right)^2=\frac{9}{4}$

$\Rightarrow \quad R_p=\frac{R_e}{2}=\frac{R}{2}$

Hence, the correct answer is (D).

23. Areal velocity is given by

$$\frac{\Delta A}{\Delta t}=\frac{L}{2m}$$

Hence, the correct answer is (C).

24. $E_1=U_f-U_i$

$\Rightarrow \quad E_1=\frac{GMm}{R}-\frac{GMm}{(R+h)}$

$\Rightarrow \quad E_1=\frac{GMmh}{R(R+h)}$

KE of satellite in this orbit is

$$KE=E_2=\frac{GMm}{2r}=\frac{GMm}{2(R+h)}$$

$\Rightarrow \quad E_2=\frac{1}{2}\frac{GMm}{(R+h)}$

Given that $E_1=E_2$

$\Rightarrow \quad \frac{h}{R}=\frac{1}{2}$

$\Rightarrow \quad h=\frac{R}{2}$

Hence, the correct answer is (A).

25. Since $U=-2\left(\frac{1}{2}mv^2\right)$

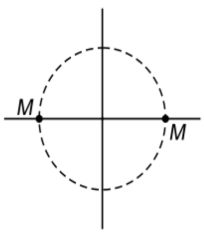
In order to escape, we have $U+K=0$

$\Rightarrow \quad K=mv^2$

Hence, the correct answer is (B).

26. $2R=d=2\times10^{11}$ m

$\Rightarrow \quad R=10^{11}$ m



$\Rightarrow$ Let v_0 be the minimum speed of meteorite at O, then by Law of Conservation of Energy, we have

$$\frac{1}{2}mv_0^2-\frac{2GMm}{R}=0$$

$\Rightarrow \quad v_0=\sqrt{\frac{4GM}{R}}=\sqrt{\frac{4\times6.67\times10^{-11}\times3\times10^{31}}{10^{11}}}$

$\Rightarrow \quad v_0=2.83\times10^5\ \text{ms}^{-1}\approx2.8\times10^5\ \text{ms}^{-1}$

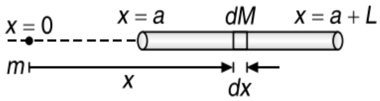
Hence, the correct answer is (A).

27. Since, $v_0=\sqrt{2gR},\ v_e=\sqrt{2gR}$

$\Rightarrow \quad \Delta v=\sqrt{gR}\left(\sqrt{2}-1\right)$

Hence, the correct answer is (B).

28.



$$dF=\frac{GmdM}{x^2}\text{, where } dM=\lambda dx$$

$\Rightarrow \quad dM=\left(A+Bx^2\right)dx$

So, $\quad dF=-Gm\int_a^{L+a}\frac{\left(A+Bx^2\right)dx}{x^2}$

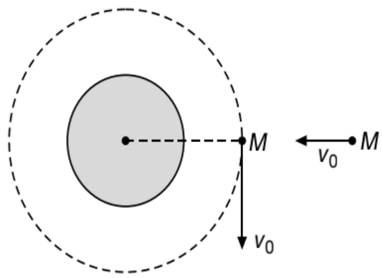
$$F=-Gm\left[-A\left(\frac{1}{L+a}-\frac{1}{a}\right)+BL\right]$$

$\Rightarrow \quad F=-Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)+BL\right]$

Hence, the correct answer is (C).

29. Orbital velocity v_0 is given by

$$v_0=\sqrt{\frac{GM}{R}}$$



After collision, we have

$$mv_0\left(-\hat{j}\right)+mv_0\left(-\hat{i}\right)=2m\vec{v}$$

$\Rightarrow \quad \vec{v}=-\frac{v_0}{2}\hat{i}-\frac{v_0}{2}\hat{j}$

$\Rightarrow \quad |\vec{v}|=\frac{v_0}{\sqrt{2}}=0.7v_0$

$\Rightarrow \quad v<v_0$

So, the path will be elliptical.

Hence, the correct answer is (C).

30. Since $\frac{v^2}{r} = \frac{GM}{r^2}$

$$\Rightarrow \quad v^2 = \frac{GM}{r}$$

$$\Rightarrow \quad \frac{T_A}{T_B} = \frac{\frac{1}{2}mv_A^2}{\frac{1}{2}2mv_B^2} = \frac{1}{2}\left(\frac{v_A}{v_B}\right)^2$$

$$\Rightarrow \quad \frac{T_A}{T_B} = \frac{1}{2}\frac{R_B}{R_A} = 1$$

Hence, the correct answer is (A).

31. Since $U = -\frac{k}{2r^2}$

Force acting on the particle is $F = -\frac{dU}{dr} = \frac{k}{r^3}$

This force provides necessary centripetal force, so

$$\frac{mv^2}{r} = \frac{k}{r^3}$$

$$\Rightarrow \quad mv^2 = \frac{k}{r^2}$$

Kinetic energy of particle, $K = \frac{1}{2}mv^2 = \frac{k}{2r^2}$

Total energy of the particle $= U + K = -\frac{k}{2r^2} + \frac{k}{2r^2} = 0$

Hence, the correct answer is (C).

32. Since, central force is given by

$$F_c \propto \frac{1}{R^n}$$

$$\Rightarrow \quad F_c = k\left(\frac{1}{R^n}\right)$$

$$\Rightarrow \quad m\omega^2 R = k\frac{1}{R^n}$$

$$\Rightarrow \quad m\frac{(2\pi)^2}{T^2} = k\frac{1}{R^{n+1}}$$

$$\Rightarrow \quad T^2 \propto R^{n+1}$$

$$\Rightarrow \quad T \propto R^{\frac{(n+1)}{2}}$$

Hence, the correct answer is (C).

33. Initially, total energy is $E_i = -\frac{GMm}{2R}$

Final total energy is $E_f = -\frac{GM\left(\frac{m}{2}\right)}{2\left(\frac{R}{2}\right)} - \frac{GM\left(\frac{m}{2}\right)}{2\left(\frac{3R}{2}\right)}$

$$\Rightarrow \quad E_f = -\frac{2GMm}{3R}$$

Required difference in energies is $\Delta E = E_f - E_i$

$$\Rightarrow \quad \Delta E = -\frac{GMm}{R}\left(\frac{2}{3} - \frac{1}{2}\right) = -\frac{GMm}{6R}$$

Hence, the correct answer is (D).

34. $F_1 = \frac{GM_eM_m}{r_1^2}$ and $F_2 = \frac{GM_eM_s}{r_2^2}$

$$\Rightarrow \quad \Delta F_1 = -\frac{2GM_eM_m}{r_1^3}\Delta r_1 \text{ and } \Delta F_2 = -\frac{2GM_eM_s}{r_2^3}\Delta r_2$$

$$\Rightarrow \quad \frac{\Delta F_1}{\Delta F_2} = \frac{M_m\Delta r_1}{r_1^3}\frac{r_2^3}{M_s\Delta r_2} = \left(\frac{M_m}{M_s}\right)\left(\frac{r_2^3}{r_1^3}\right)\left(\frac{\Delta r_1}{\Delta r_2}\right)$$

Given that

$\Delta r_1 = \Delta r_2 = 2R_{\text{earth}}$, $M_m = 8\times10^{22}$ kg

$M_s = 2\times10^{30}$ kg, $r_1 = 0.4\times10^6$ km,

$r_2 = 150\times10^6$ km.

$$\Rightarrow \quad \frac{\Delta F_1}{\Delta F_2} = 2$$

Hence, the correct answer is (A).

35. Effect of rotation of earth on acceleration due to gravity is given by $g' = g - R\omega^2\cos^2\phi$

where ϕ is the latitude angle. There will be no change in gravity at poles because $\phi = 90°$ at the poles and at all other points as ω increases, g' will decrease.

Hence, the correct answer is (C).

36. This force provides the centripetal force to the particle to move in a circular orbit.

$$\Rightarrow \quad \frac{mv^2}{r} = \frac{16}{r} + r^3$$

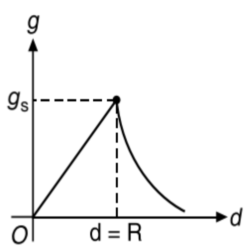
Kinetic energy, $K = \frac{1}{2}mv^2 = \frac{1}{2}(16 + r^4)$

$$\Rightarrow \quad \frac{K_1}{K_2} = \frac{\frac{16+1}{2}}{\frac{16+256}{2}} = \frac{17}{272} = 0.0625$$

$$\Rightarrow \quad \frac{K_1}{K_2} \quad 6\times10^{-2}$$

Hence, the correct answer is (A).

37. Variation of g inside earth surface



CHAPTER 4

For $d < R$, $g = \left(\frac{Gm}{R^2}\right)d$

For $d = R$, $g_s = \frac{Gm}{R^2}$

For $d > R$, $g = \frac{Gm}{d^2}$

Hence, the correct answer is (D).

38. Here, the weight of person on the equator is W. If the earth rotates about its axis, then weight is $\frac{3W}{4}$

Radius of the earth $= 6400$ km

The acceleration due to gravity at the equator is

$$g_e = g - R\omega^2$$

$$\Rightarrow \quad \frac{3}{4}g = g - R\omega^2$$

$$\Rightarrow \quad R\omega^2 = \frac{g}{4}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{4R}} = \sqrt{\frac{10}{4 \times 6400 \times 10^3}} = 0.625 \times 10^{-3} \text{ rads}^{-1}$$

$$\Rightarrow \quad \omega \approx 0.63 \times 10^{-3} \text{ rads}^{-1}$$

Hence, the correct answer is (A).

39. The correct answer is (A).

40. $v_0 = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

So, increase in velocity is

$$\Delta v = \sqrt{gR}\left(\sqrt{2} - 1\right)$$

Hence, the correct answer is (A).

41. Let the area of the ellipse be A. According to Kepler's Second Law, areal velocity of a planet around the sun is constant, i.e., $\frac{dA}{dt} = \text{constant}$, so we have

$$\frac{t_1}{t_2} = \frac{\text{Area of } abcsa}{\text{Area of } adcsa} = \frac{\frac{A}{2} + \frac{A}{4}}{\frac{A}{2} - \frac{A}{4}} = \frac{\frac{3A}{4}}{\frac{A}{4}} = 3$$

$$\Rightarrow \quad t_1 = 3t_2$$

Hence, the correct answer is (C).

42. Gravitational pull on the astronaut $F_G = \frac{GMm}{(R+h)^2}$

Net force on the astronaut is zero.

Hence, the correct answer is (C).

43. $V_P = V_{\text{sphere at } P} - V_{\text{cavity at } P}$

Since, $V = -\frac{GM}{2R^3}\left(3R^2 - r^2\right)$

$$\Rightarrow \quad V_{\text{sphere at P}} = -\frac{GM}{2R^3}\left(3R^2 - \left(\frac{R}{2}\right)^2\right) = -\frac{11GM}{8R}$$

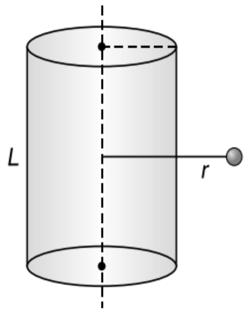
Now, $V_{\text{cavity at P}} = -\frac{3}{2}\frac{G\left(\frac{M}{8}\right)}{\left(\frac{R}{2}\right)} = -\frac{3GM}{8R}$

$$\Rightarrow \quad V_P = -\frac{11GM}{8R} - \left(-\frac{3GM}{8R}\right) = -\frac{GM}{R}$$

Hence, the correct answer is (B).

44. Centripetal force is provided by the gravitational force which is proportional to $\frac{1}{r}$. So, $F \propto \frac{1}{r}$

$$\Rightarrow \quad \frac{mv_0^2}{r} \propto \frac{1}{r}$$



$$\Rightarrow \quad v_0 = \text{constant}$$

Since $T = \frac{2\pi r}{v}$

$$\Rightarrow \quad T \propto r$$

Hence, the correct answer is (C).

45. Potential $V(r)$ due to a large planet of radius R is given by

$$V_0(r) = -\frac{GM}{r} \quad \text{for } r > R$$

$$V(r) = \frac{-GM}{R} \quad \text{for } r = R$$

$$V_{in} = -\frac{3}{2}\frac{GM}{R}\left(1 - \frac{r^2}{3R^2}\right) \quad \text{for } r < R$$

Hence, the correct answer is (B).

46. $F = F_{\text{gravitational force on M}} = \frac{Mv^2}{R}$

$$\Rightarrow \quad F = 2\left(\frac{GM^2}{\left(\sqrt{2}R\right)^2}\frac{1}{\sqrt{2}}\right) + \frac{GM^2}{(2R)^2} = \frac{Mv^2}{R}$$

$$\Rightarrow \quad \frac{GM^2}{\sqrt{2}R^2} + \frac{GM^2}{4R^2} = \frac{Mv^2}{R}$$

$$\Rightarrow \quad v = \frac{1}{2}\sqrt{\frac{GM}{R}\left(1 + 2\sqrt{2}\right)}$$

Hence, the correct answer is (D).

47. At surface, $E_i = -\frac{GMm}{R}$

In orbit, $E_f = -\frac{GMm}{2(3R)} = -\frac{GMm}{6R}$

$\Rightarrow$ Required energy is

$$\Rightarrow \quad \Delta E = E_f - E_i = \frac{5GMm}{6R}$$

Hence, the correct answer is (A).

48. Energy required to make the spaceship reach the free space is

$$\Delta E = \frac{GMm}{R}$$

Since $g = \frac{GM}{R^2}$

$$\Rightarrow \quad \Delta E = gR^2 \times \frac{m}{R}$$

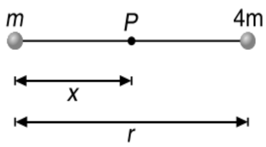
$$\Rightarrow \quad \Delta E = mgR$$

$$\Rightarrow \quad \Delta E = 1000 \times 10 \times 6400 \times 10^3 = 64 \times 10^9 \text{ J}$$

$$\Rightarrow \quad \Delta E = 6.4 \times 10^{10} \text{ J}$$

Hence, the correct answer is (C).

49. Let x be the distance of the point P from the mass m where gravitational field is zero.



$$\Rightarrow \quad \frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2}$$

$$\Rightarrow \quad \left(\frac{x}{(r-x)}\right)^2 = \frac{1}{4}$$

$$\Rightarrow \quad x = \frac{r}{3} \qquad \ldots(1)$$

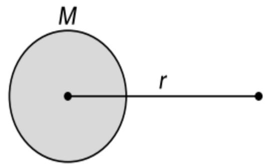
Gravitational potential at a point P i.e. at $x = \frac{r}{3}$ is

$$V = -\frac{Gm}{x} - \frac{G(4m)}{(r-x)} = -\frac{Gm}{\left(\frac{r}{3}\right)} - \frac{G(4m)}{\left(r - \frac{r}{3}\right)}$$

$$\Rightarrow \quad V = -\frac{3Gm}{r} - \frac{3G(4m)}{2r} = -9\frac{Gm}{r}$$

Hence, the correct answer is (D).

50. The acceleration due to gravity at a height h from the ground is given as $\frac{g}{9}$.



$$\frac{GM}{r^2} = \left(\frac{GM}{R^2}\right)\frac{1}{9}$$

$$\Rightarrow \quad r = 3R$$

The height above the ground is $2R$

Hence, the correct answer is (A).

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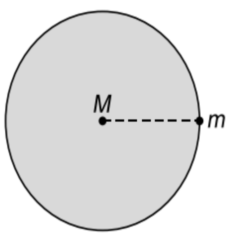
Single Correct Choice Type Problems

1. For a particle revolving in a circular orbit of radius r due to the gravitational attraction of inner cloud of mass M, we have

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\Rightarrow \quad M = \frac{v^2 r}{G} = \frac{2mv^2 r}{2Gm}$$

Since $K = \frac{1}{2}mv^2 = \text{constant}$



$$\Rightarrow \quad mv^2 = 2K$$

$$\Rightarrow \quad M = \frac{2Kr}{Gm}$$

$$\Rightarrow \quad dM = \frac{2Kdr}{Gm} \qquad \ldots(1)$$

Also, we know that

$$dM = \rho(r)dV$$

$$\Rightarrow \quad dM = \rho(r)4\pi r^2 dr$$

So, equation (1) becomes,

$$\rho(r)4\pi r^2 dr = \frac{2Kdr}{Gm}$$

$$\Rightarrow \quad \frac{\rho(r)}{m} = n(r) = \frac{K}{2\pi Gm^2 r^2}$$

Hence, the correct answer is (B).

2. Given that $v_e = 11.2 \text{ kms}^{-1} = \sqrt{\dfrac{2GM_e}{R_e}}$

By Law of Conservation of Energy, we have

$$K_i + U_i = K_f + U_f$$

$$\Rightarrow \frac{1}{2}mv_s^2 - \frac{GM_s m}{r} - \frac{GM_e m}{R_e} = 0 + 0$$

where, r is the distance of rocket from sun

$$\Rightarrow v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2GM_s}{r}}$$

Since, $M_s = 3\times10^5 M_e$ and $r = 2.5\times10^4 R_e$

$$\Rightarrow v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2G3\times10^5 M_e}{2.5\times10^4 R_e}}$$

$$\Rightarrow v_s = \sqrt{\frac{2GM_e}{R_e}\left(1 + \frac{3\times10^5}{2.5\times10^4}\right)}$$

$$\Rightarrow v_s = \sqrt{\frac{2GM_e}{R_e}\times 13}$$

$$\Rightarrow v_s \quad 42 \text{ kms}^{-1}$$

Hence, the correct answer is (C).

3. Given, $R_{\text{planet}} = R = \dfrac{R_{\text{earth}}}{10}$

Since, density $\rho = \dfrac{M_{\text{earth}}}{\frac{4}{3}\pi R_{\text{earth}}^3}$

Also, $\rho = \dfrac{M_{\text{planet}}}{\frac{4}{3}\pi R_{\text{planet}}^3}$

$$\Rightarrow M_{\text{planet}} = \frac{M_{\text{earth}}}{10^3} = \frac{M_e}{1000}$$

Let the acceleration due to gravity at surface of planet and at the surface of earth be g_p and g_e respectively. Then

$$g_p = \frac{GM_{\text{planet}}}{R_{\text{planet}}^2} = \frac{GM_e(10)^2}{(10)^3 R_e^2} = \frac{GM_e}{10R_e^2}$$

$$\Rightarrow g_p = \frac{g_e}{10}$$

The value of g inside the planet at a distance x from centre of the planet is

$$g_{\text{inside}} = g_{\text{surface of planet}}\left(\frac{x}{R}\right) = g_p\left(\frac{x}{R}\right)$$

So, total force acting on wire is

$$F = \int_{\frac{4R}{5}}^{R} (\lambda dx) g_p\left(\frac{x}{R}\right)$$

$$\Rightarrow F = \frac{\lambda g_p}{R}\left(\frac{x^2}{2}\right)\Bigg|_{4R/5}^{R}$$

Substituting the given values, we get

$$F = 108 \text{ N}$$

Hence, the correct answer is (B).

4. In circular orbit of a satellite, potential energy U is

$$U = -2\times(\text{kinetic energy}) = -2\times\frac{1}{2}mv^2 = -mv^2$$

Just to escape from the gravitational pull, its total mechanical energy should be zero. Therefore, its kinetic energy should be $+mv^2$.

Hence, the correct answer is (B).

5. For annular disc, gravitational potential at the point P lying on the axis at a distance x from centre is

$$V_P = -\frac{2GM}{(R_2^2 - R_1^2)}\left(\sqrt{R_2^2 + x^2} - \sqrt{R_1^2 + x^2}\right)$$

where $R_2 = 4R$, $R_1 = 3R$ and $x = 4R$

$$\Rightarrow V_P = -\frac{2GM}{7R^2}\left(4\sqrt{2}R - 5R\right)$$

Also, $V_\infty = 0$

Since $W_{P\to\infty} = m_0(V_\infty - V_P)$, where $m_0 = 1$ unit

$$\Rightarrow W_{P\to\infty} = \frac{2GM}{7R}\left(4\sqrt{2} - 5\right)$$

Hence, the correct answer is (A).

6. For $r \le R$, $\dfrac{mv^2}{r} = \dfrac{GmM}{r^2}$ ...(1)

Here, $M = \left(\dfrac{4}{3}\pi r^3\right)\rho_0$

Substituting in Equation (1), we get $v \propto r$

i.e. v-r graph is a straight line passing through origin.

For $r > R$, we have

$$\frac{mv^2}{r} = \frac{Gm\left(\frac{4}{3}\pi R^3\right)\rho_0}{r^2}$$

$$\Rightarrow v \propto \frac{1}{\sqrt{r}}$$

The corresponding v-r graph will be as shown in option (C).

Hence, the correct answer is (C).

7. In case of binary star system, angular velocity and hence, the time period of both the stars are equal.

Hence, the correct answer is (D).

8. Time period of a satellite very close to earth's surface is 84.6 min. Time period increases as the distance of the satellite from the surface of earth increases. So, time period of spy satellite orbiting a few 100 km above

the earth's surface should be slightly greater than 84.6 min. Therefore, the most appropriate option is (C) or 2 h

Hence, the correct answer is (C).

9. Since $T_1 = 2\pi\sqrt{\dfrac{\ell}{g}}$

$$T_2 = 2\pi\sqrt{\frac{\ell}{g_h}}$$

Now $g_h = \dfrac{gR^2}{(R+h)^2}$

$$\Rightarrow \quad g_h = \frac{g}{4} \qquad \{\because h = R\}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \sqrt{\frac{g}{g/4}} = 2$$

Hence, the correct answer is (D).

10. Force on satellite is always towards earth, therefore, acceleration of satellite S is always directed towards centre of the earth due to which net torque of this gravitational force F about centre of earth is zero. Therefore, angular momentum (both in magnitude and direction) of S about centre of earth is constant throughout. Since, the force F is conservative in nature, therefore mechanical energy of satellite remains constant. Speed of S is maximum when it is nearest to earth and minimum when it is farthest.

Hence, the correct answer is (A).

11. From Kepler's Third Law, we have

$$T^2 \propto r^3$$

$$\Rightarrow \quad T \propto (r)^{3/2}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

$$\Rightarrow \quad T_2 = T_1\left(\frac{r_2}{r_1}\right)^{3/2} = (365)\left(\frac{1}{2}\right)^{3/2}$$

$$\Rightarrow \quad T_2 \approx 129 \text{ days}$$

Hence, the correct answer is (B).

12. $F \propto R^{-5/2}$

This gravitational force of attraction provides necessary centripetal force to planet to revolve about the massive star.

$$\Rightarrow \quad mR\omega^2 \propto R^{-5/2}$$

$$\Rightarrow \quad \omega^2 \propto R^{-7/2}$$

$$\Rightarrow \quad \frac{4\pi^2}{T^2} \propto R^{-7/2}$$

$$\Rightarrow \quad T^2 \propto R^{7/2}$$

Hence, the correct answer is (B).

13. $\Delta U = -\dfrac{GMm}{R+h} - \left(-\dfrac{GMm}{R}\right)$

$$\Rightarrow \quad \Delta U = -\frac{GMm}{2R} + \frac{GMm}{R} \qquad \{\because h = R\}$$

$$\Rightarrow \quad \Delta U = \frac{GMm}{2R} = \frac{1}{2}m\left(\frac{GM}{R^2}\right)R = \frac{mgR}{2}$$

Hence, the correct answer is (A).

14. Since, $g = \dfrac{GM}{R^2}$

$$\Rightarrow \quad g \propto \frac{1}{R^2}$$

$$\Rightarrow \quad \frac{\Delta g}{g} = -2\frac{\Delta R}{R}$$

So, g will increase, if R decreases

$$\Rightarrow \quad \frac{\Delta g}{g} = -2(-1\%) = 2\%$$

Hence, the correct answer is (C).

Multiple Correct Choice Type Problems

1. Gravitational field at a distance r due to mass $m\left(= \dfrac{4}{3}\pi r^3\rho\right)$ is

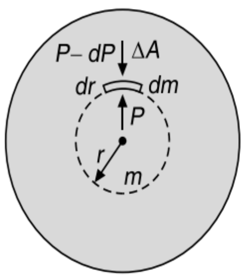
$$E = \frac{G\rho\left(\frac{4}{3}\pi r^3\right)}{r^2} = \frac{4G\rho\pi r}{3}$$

Consider a small element of width dr and area ΔA at a distance r from the centre. Pressure force on this element is due to the gravitational force on dm from m inwards towards the centre.

$$\Rightarrow \quad (dP)\Delta A = E(dm)$$

where $dm = (\Delta A)(dr)\rho$ and $m = \left(\dfrac{4}{3}\pi r^3\right)\rho$

$$\Rightarrow \quad -dP\Delta A = \left(\frac{4}{3}G\pi\rho r\right)(\rho\Delta A dr)$$



$$\Rightarrow \quad -\int_0^P dP = \int_R^r \left(\frac{4G\rho^2\pi}{3}\right) r\,dr$$

$$\Rightarrow \quad -P = \frac{4G\rho^2\pi}{3\times 2}(r^2 - R^2)$$

CHAPTER 4

$\Rightarrow \quad P = \frac{2G\rho^2\pi}{3}(R^2 - r^2)$

$\Rightarrow \quad P = k(R^2 - r^2)$, where $k = \frac{2G\rho^2\pi}{3} = \text{constant}$

For, $r = \frac{3R}{4}$, $P_1 = k\left(R^2 - \frac{9R^2}{16}\right) = k\left(\frac{7R^2}{16}\right)$

For, $r = \frac{2R}{3}$, $P_2 = k\left(R^2 - \frac{4R^2}{9}\right) = k\left(\frac{5R^2}{9}\right)$

$\Rightarrow \quad \frac{P_1}{P_2} = \frac{63}{80}$

For, $r = \frac{3R}{5}$, $P_3 = k\left(R^2 - \frac{9}{25}R^2\right) = k\left(\frac{16R^2}{25}\right)$

For, $r = \frac{2R}{5}$, $P_4 = k\left(R^2 - \frac{4R^2}{25}\right) = k\left(\frac{21R^2}{25}\right)$

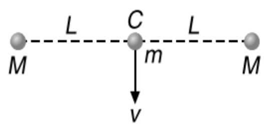
$\Rightarrow \quad \frac{P_3}{P_4} = \frac{16}{21}$

For $r = \frac{R}{2}$, $P_5 = k\left(R^2 - \frac{R^2}{4}\right) = k\left(\frac{3R^2}{4}\right)$

For $r = \frac{R}{3}$, $P_6 = k\left(R^2 - \frac{R^2}{9}\right) = k\left(\frac{8R^2}{9}\right)$

$\Rightarrow \quad \frac{P_5}{P_6} = \frac{27}{32}$

Hence, (B) and (C) are correct.

2. 

Let v is the minimum velocity, then by Law of Conservation of Energy, we have

$(U + K)_C = (U + K)_\infty$

$\Rightarrow \quad \left(-\frac{GMm}{L}\right)2 + \frac{1}{2}mv^2 = 0 + 0$

$\Rightarrow \quad v = 2\sqrt{\frac{GM}{L}}$

Hence, (B) and (D) are correct.

3. $v_{es} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G\left(\frac{4}{3}\pi R^3\right)\rho}{R}} = \sqrt{\frac{4G\rho}{3}}R$

$v_{es} \propto R$

Surface area of P is $A = 4\pi R_P^2$

Surface area of Q is $4A = 4\pi R_Q^2$

$\Rightarrow \quad R_Q = 2R_P$

Mass of R is $M_R = M_P + M_Q$

$\Rightarrow \quad \left(\frac{4}{3}\pi R_R^3\right)\rho = \left(\frac{4}{3}\pi R_P^3\right)\rho + \left(\frac{4}{3}\pi R_Q^3\right)\rho$

$\Rightarrow \quad R_R^3 = R_P^3 + R_Q^3$

$\Rightarrow \quad R_R^3 = 9R_P^3$

$\Rightarrow \quad R_R = 9^{1/3}R_P$

$\Rightarrow \quad R_R > R_Q > R_P$

$\Rightarrow \quad V_R > V_Q > V_P$

Also, $\frac{V_R}{V_P} = 9^{1/3}$ and $\frac{V_P}{V_Q} = \frac{1}{2}$

Hence, (B) and (D) are correct.

4. Gravitational field is the acceleration due to gravity.

So, $g = \begin{cases} \frac{GM}{r^2} & r \geq R \text{ (Outside)} \\ \frac{4}{3}\pi G\rho r & r < R \text{ (Inside)} \end{cases}$

$\Rightarrow \quad \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2} \text{(Outside)}$

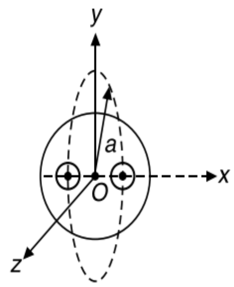
where $r_1 > R$ and $r_2 > R$

and $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ (Inside)

where $r_1 < R$ and $r_2 < R$

Hence, (A) and (B) are correct.

5. The spherical cavities can be assumed to be negative masses placed symmetrically about origin O. So gravitational force due to this object at origin is zero.



The dotted circle is lying in the yz plane and is an equipotential surface (as we can see that two cavities are lying symmetrically on each side of the EPS (Equi-Potential Surface) so gravitational potential is same at all the points lying on the circle $y^2 + z^2 = a^2$ (where $a^2 = \text{constant}$).

Hence, (A), (C) and (D) are correct.