ICON

CHAPTER 2: CENTRE OF MASS, CONSERVATION OF LINEAR MOMENTUM AND COLLISIONS

CHAPTER 2

Test Your Concepts-I (Based on Centre of Mass)

1. Let the new centre of mass be at C', then its distance from C equals the displacement of the centre of mass, i.e. CC' given by

$$\Delta x = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2} = \frac{m_1 \ell_1 + m_2 \ell_2}{m_1 + m_2}$$

$$\Rightarrow \quad CC' = \Delta x = \frac{m_1 \ell_1 + m_2 \ell_2}{m_1 + m_2}$$

2. For a two particle system, the centre of mass is given by

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Let the masses be placed on x-axis, then according to the problem, we have

$$\Delta x_{cm} = 0$$

$$\Rightarrow \quad \Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2} = 0$$

$$\Rightarrow \quad 0 = \frac{m_1 \Delta x_1 + m_2 x}{m_1 + m_2}$$

$$\Rightarrow \quad \Delta x_1 = -\left(\frac{m_2 x}{m_1}\right)$$

Thus m_1 should be displaced by $\frac{m_2 x}{m_1}$, opposite to the displacement of m_2.

3. The position vector of cm of the three particles will be given by

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3}{m_1 + m_2 + m_3}$$

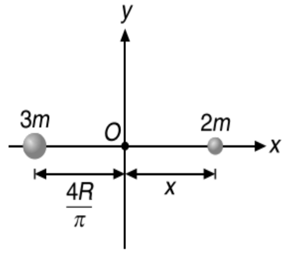
Substituting the values, we get

$$\vec{r}_{cm} = \frac{(1)(\hat{i} + 4\hat{j} + \hat{k}) + (2)(\hat{i} + \hat{j} + \hat{k}) + 3(2\hat{i} - \hat{j} - 2\hat{k})}{1+2+3}$$

$$\Rightarrow \quad \vec{r}_{cm} = \frac{9\hat{i} + 3\hat{j} - 3\hat{k}}{6}$$

$$\Rightarrow \quad \vec{r}_{cm} = \frac{1}{2}(3\hat{i} + \hat{j} - \hat{k}) \text{ m}$$

4. The ring can be substituted by a particle of mass $3m$ at a distance $\frac{2(2R)}{\pi} = \frac{4R}{\pi}$ from the origin.

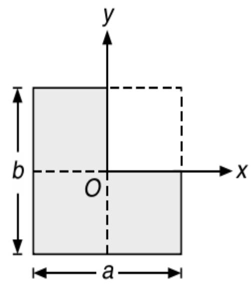


Now, the centre of mass of the two particle system is at origin, so we have

$$3m\left(-\frac{4R}{\pi}\right) + 2mx = 0$$

$$\Rightarrow \quad x = \frac{6R}{\pi}$$

5. The centre of mass of the remaining system is



$$x_{cm} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} = \frac{(ab)(0) - (ab/4)(a/4)}{ab - (ab/4)}$$

$$\Rightarrow \quad x_{cm} = -\frac{a}{12}$$

Similarly, we have

$$y_{cm} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{(ab)(0) - (ab/4)(b/4)}{ab - (ab/4)}$$

$$\Rightarrow \quad y_{cm} = -\frac{b}{12}$$

$$\Rightarrow \quad (x_{cm}, y_{cm}) = \left(-\frac{a}{12}, -\frac{b}{12}\right)$$

6. The mass distribution is symmetrical about x-axis, hence y-coordinate of centre of mass will be zero. So, we have

$$x_{cm} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2}$$

$$\Rightarrow \quad x_{cm} = \frac{(2r^2)\frac{r}{2} - \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right)}{(2r^2) - \left(\frac{\pi r^2}{2}\right)} = \frac{2r}{3(4-\pi)}$$

Hence, coordinates of centre of mass of system are

$$\left(\frac{2r}{3(4-\pi)}, 0\right)$$

7. Since we know that

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{3}{5}$$

$$\Rightarrow \quad \frac{(1)(1) + (2)(4) + m(-3)}{1 + 2 + m} = \frac{3}{5}$$

$$\Rightarrow \quad \frac{9 - 3m}{3 + m} = \frac{3}{5}$$

$$\Rightarrow \quad 45 - 15m = 9 + 3m$$

$$\Rightarrow \quad 18m = 36$$

$$\Rightarrow \quad m = 2 \text{ kg}$$

Also, we have

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = 2$$

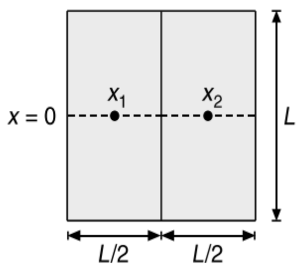
$$\Rightarrow \quad \frac{(1)(2) + (2)(6) + m(-2)}{1 + 2 + m} = 2$$

$$\Rightarrow \quad 14 - 2m = 6 + 2m$$

$$\Rightarrow \quad 4m = 8$$

$$\Rightarrow \quad m = 2 \text{ kg}$$

8. The centre of mass of each half is located at the geometrical centre of that half.



$$\Rightarrow \quad x_{cm} = \frac{\rho_1A_1x_1 + \rho_2A_2x_2}{\rho_1A_1 + \rho_2A_2}$$

$$\Rightarrow \quad x_{cm} = \frac{\rho_1\left(\frac{L^2}{2}\right)\left(\frac{L}{4}\right) + \rho_2\left(\frac{L^2}{2}\right)\left(\frac{3L}{4}\right)}{\rho_1\left(\frac{L^2}{2}\right) + \rho_2\left(\frac{L^2}{2}\right)}$$

$$\Rightarrow \quad x_{cm} = \frac{\left(\frac{\rho_1}{8} + \frac{3\rho_2}{8}\right)L}{\left(\frac{\rho_1}{2} + \frac{\rho_2}{2}\right)} = \frac{(\rho_1 + 3\rho_2)L}{4(\rho_1 + \rho_2)}$$

9. The masses and coordinates of centre of mass of the three sheets are

Square, σl^2 at $\left(\frac{l}{2}, \frac{l}{2}\right)$

Triangle, $\sigma\left(\frac{\sqrt{3}}{4}l^2\right)$ at $\left[\frac{l}{2}, l\left(1 + \frac{1}{2\sqrt{3}}\right)\right]$

Disc, $\sigma\left(\frac{\pi l^2}{4}\right)$ at $\left(\frac{3l}{2}, \frac{l}{2}\right)$

So, coordinates of centre of mass of system are

$$x_{cm} = \frac{\sigma\left[l^2\left(\frac{l}{2}\right) + \left(\frac{\sqrt{3}l^2}{4}\right)\left(\frac{l}{2}\right) + \left(\frac{\pi l^2}{4}\right)\left(\frac{3l}{2}\right)\right]}{\sigma\left(l^2 + \frac{\sqrt{3}l^2}{4} + \frac{\pi l^2}{4}\right)}$$

$$\Rightarrow \quad x_{cm} = \frac{l}{2}\left(\frac{4 + \sqrt{3} + 3\pi}{4 + \sqrt{3} + \pi}\right)$$

Similarly, we have

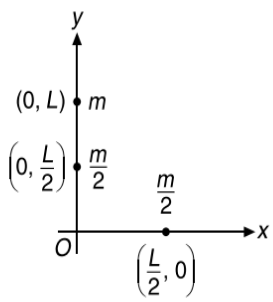
$$y_{cm} = \frac{l^2\left(\frac{l}{2}\right) + \left(\frac{\sqrt{3}l^2}{4}\right)\left(l + \frac{l}{2\sqrt{3}}\right) + \left(\frac{\pi l^2}{4}\right)\left(\frac{l}{2}\right)}{l^2 + \frac{\sqrt{3}l^2}{4} + \frac{\pi l^2}{4}}$$

$$\Rightarrow \quad y_{cm} = \frac{l}{2}\left(\frac{5 + 2\sqrt{3} + \pi}{4 + \sqrt{3} + \pi}\right)$$

10. $$x_{cm} = \frac{\int_0^\ell x\,dm}{\int_0^\ell dm} = \frac{\int_0^\ell (\lambda dx)x}{\int_0^\ell \lambda dx} = \frac{\int_0^\ell \left(\lambda_0\frac{x^2}{\ell^2}\right)x\,dx}{\int_0^\ell \left(\lambda_0\frac{x^2}{\ell^2}\right)dx}$$

$$\Rightarrow \quad x_{cm} = \frac{3\ell}{4}$$

11. Consider the rod to be a combination of two point masses $\frac{m}{2}, \frac{m}{2}$ placed at $x = \frac{L}{2}$ and $y = \frac{L}{2}$. Then this system consists of three particles having masses and positions given by $\frac{m}{2}$ at $\left(\frac{L}{2}, 0\right)$, $\frac{m}{2}$ at $\left(0, \frac{L}{2}\right)$ and m at $(0, L)$ as shown in Figure.

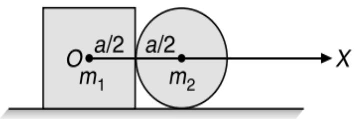


$$\Rightarrow \quad x_{cm} = \frac{\left(\frac{m}{2}\right)\left(\frac{L}{2}\right)+\left(\frac{m}{2}\right)(0)+m(0)}{\frac{m}{2}+\frac{m}{2}+m} = \frac{L}{8}$$

$$\Rightarrow \quad y_{cm} = \frac{\left(\frac{m}{2}\right)(0)+\left(\frac{m}{2}\right)\left(\frac{L}{2}\right)+mL}{\frac{m}{2}+\frac{m}{2}+m} = \frac{5L}{8}$$

So, the position of centre of mass is $\left(\frac{L}{8}, \frac{5L}{8}\right)$

12. Let σ be the surface mass density, i.e. the mass per unit area. The square can be treated as a point mass $m_1 = a^2\sigma$ placed at its geometric centre $(0,0)$ and the disc is treated as another point mass $m_2 = \frac{\pi a^2 \sigma}{4}$ placed at its centre, i.e. at $(a,0)$ as shown in Figure.



The position of centre of mass is given by

$$x_{cm} = \frac{m_1x_1+m_2x_2}{m_1+m_2} = \frac{(a^2\sigma)(0)+(\pi a^2\sigma/4)a}{a^2\sigma+(\pi a^2\sigma/4)}$$

$$\Rightarrow \quad x_{cm} = \frac{\pi a/4}{1+(\pi/4)} = \frac{\pi a}{\pi+4}$$

Hence, the centre of mass is at a distance $\left(\frac{\pi a}{\pi+4}\right)$ from the centre of square on the line joining the centres of the two objects.

13. For a uniform disc, its mass is proportional to area, so if σ is the surface mass density, then we have

$$m_1 = \sigma\left(\pi R^2 - \frac{R^2}{2}\right) = \left(\frac{2\pi-1}{2}\right)\sigma R^2$$

If l be the length of the square plate of diagonal R removed from the disc, then we have

$$l^2 + l^2 = R^2$$

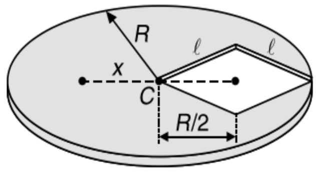
$$\Rightarrow \quad l = \frac{R}{\sqrt{2}}$$

The mass of square plate cut from the disc is

$$m_2 = \sigma l^2 = \sigma\left(\frac{R}{\sqrt{2}}\right)^2 = \frac{\sigma R^2}{2}$$

Also, we must note that the common centre of mass of m_1 and m_2 must be at the centre of the full disc. So, if centre of mass of given object is at a distance x from C, then for a two particle system, we have

$$m_1x_1 = m_2x_2$$

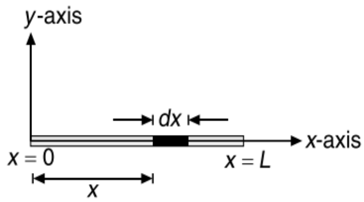


$$\Rightarrow \quad m_1 x = m_2 \frac{R}{2}$$

$$\Rightarrow \quad \left[\left(\frac{2\pi-1}{2}\right)(\sigma R^2)\right]x = \left(\frac{\sigma R^2}{2}\right)\left(\frac{R}{2}\right)$$

$$\Rightarrow \quad x = \frac{R}{2(2\pi-1)}$$

14. Let the rod be placed along the x axis as shown in Figure.



Consider an infinitesimal element of mass dm, length dx at a distance x from the lighter end of the rod (assuming lighter end of the rod to be placed at the origin of the axis). If linear mass density of the rod is λ, then we have

$$\lambda = \frac{dm}{dx}$$

According to the problem, the linear mass density of the rod increases linearly from λ_1 to λ_2, so we can write the mathematical expression for λ as

$$\frac{\lambda-\lambda_1}{x} = \frac{\lambda_2-\lambda_1}{L}$$

$$\Rightarrow \quad \lambda = \lambda_1 + \left(\frac{\lambda_2-\lambda_1}{L}\right)x$$

$$\Rightarrow \quad \lambda = a + bx \qquad \ldots(1)$$

where, $a = \lambda_1$ and $b = \frac{\lambda_2-\lambda_1}{L}$

The position of centre of mass is given by

$$x_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^L x(\lambda dx)}{\int_0^L \lambda dx} = \frac{\int_0^L (ax+bx^2)dx}{\int_0^L (a+bx)dx}$$

$$\Rightarrow \quad x_{cm} = \frac{\frac{aL^2}{2}+\frac{bL^3}{3}}{aL+\frac{bL^2}{2}} = \frac{(3a+2bL)L}{(2a+bL)3}$$

$$\Rightarrow \quad x_{cm} = \left(\frac{\lambda_1+2\lambda_2}{\lambda_1+\lambda_2}\right)\frac{L}{3}$$

Test Your Concepts-II (Based on Motion of Centre of Mass)

1. Suppose the centre of mass of the ice is a distance x_1 above the bottom and that of the tray is a distance x_2 above the bottom. The height of the centre of mass of the ice-tray system is

$$x = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

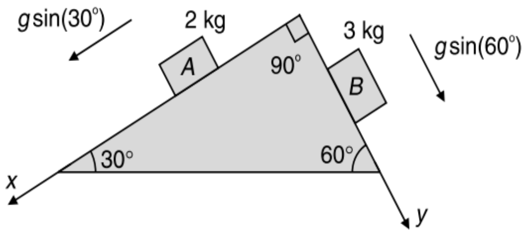
When the ice melts, the water of mass m_1 spreads on the surface of the tray. As the tray is large, the height of water is negligible. The centre of mass of the water is then on the surface of the tray and is at a distance $\left(x_1 - \frac{L}{2}\right)$ above the bottom. The new centre of mass of the ice-tray system will be at the height

$$x' = \frac{m_1\left(x_1 - \frac{L}{2}\right) + m_2x_2}{m_1 + m_2}$$

The shift in the centre of mass is

$$x - x' = \frac{m_1L}{2(m_1 + m_2)}$$

2. As shown in figure, blocks A slides with acceleration $\frac{g}{2}$ and block B slides with acceleration $\frac{\sqrt{3}g}{2}$.



Now the acceleration of centre of mass of the system of blocks A and B is

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2} \quad \ldots(1)$$

where, $\vec{a}_1 = (g\sin 30^\circ)\hat{i} = 5\hat{i}$ and

$$\vec{a}_2 = (g\sin 60^\circ)\hat{j} = (5\sqrt{3})\hat{j}$$

Substituting these values in equation (1), we get

$$\vec{a}_{cm} = \frac{(2)(5\hat{i}) + (3)(5\sqrt{3}\hat{j})}{2+3} = 2\hat{i} + 3\sqrt{3}\hat{j}$$

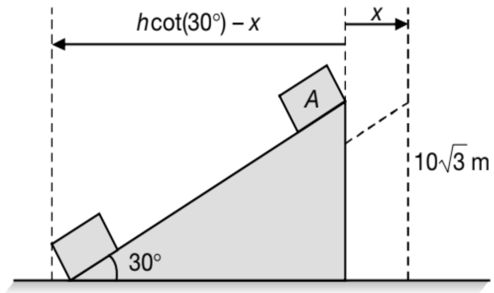
$$\Rightarrow \quad a_{cm} = \sqrt{(2)^2 + (3\sqrt{3})^2} = \sqrt{31}\ \text{ms}^{-2}$$

3. Applying $m_Rx_R = m_Lx_L$

$$\Rightarrow \quad 30x = 5(0.5 - x)$$

$$\Rightarrow \quad x = 0.0714\ \text{m} = 71.4\ \text{mm}$$

When block reaches the bottom i.e., moves to the left, then wedge moves to the right say by x. So, effective displacement of block is $\Delta x_L = h\cot\theta - x$



where $h = 10\sqrt{3}$ m and $\theta = 30^\circ$

$$\Rightarrow \quad m_R\Delta x_R = m_L\Delta x_L$$

$$\Rightarrow \quad 25x = 5[10\sqrt{3}\cot(30^\circ) - x]$$

$$\Rightarrow \quad 25x = 5[10\sqrt{3}(\sqrt{3}) - x]$$

$$\Rightarrow \quad 25x = 5(30 - x)$$

$$\Rightarrow \quad 30x = 150$$

$$\Rightarrow \quad x = 5\ \text{m}$$

4. Since, we know that the acceleration of the centre of mass of the system is

$$a_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$$

along the direction of motion of the heavier mass

$$\Rightarrow \quad a_{cm} = \left(\frac{4-2}{4+2}\right)^2(10) = \frac{10}{9}\ \text{ms}^{-2}$$

$$\Rightarrow \quad s_{cm} = \frac{1}{2}a_{cm}t^2 = \frac{1}{2}\left(\frac{10}{9}\right)(9)^2 = 45\ \text{m}$$

Similarly, we have

$$v_{cm} = u_{cm} + a_{cm}t$$

$$\Rightarrow \quad v_{cm} = 0 + \left(\frac{10}{9}\right)(9) = 10\ \text{ms}^{-1}$$

5. In the horizontal direction, there is no force on the system (dog + boat). Therefore, the centre of mass of the system does not move in the horizontal direction. Since the dog moves towards the shore, the boat moves away from the shore to keep centre of mass stationary. Let d be the distance by which the boat moves backwards and let x be the initial distance of the boat from the shore. The initial x-coordinate of the centre of mass

$$(x_{cm})_{\text{initial}} = \frac{(10)(20) + 40x}{10 + 40}$$

The final x-coordinate of the centre of mass

$$(x_{cm})_{\text{final}} = \frac{10(20 - 8 + d) + 40(x + d)}{10 + 40}$$

Equating the two, we get

$$d = \frac{8}{5}\text{ m}$$

So, from the shore, the dog is at a distance of

$$s = 20 - 8 + \frac{8}{5} = 13.6\text{ m}$$

6. Since no external force acts on the system in the horizontal direction, i.e. along the x direction, so we have

$$\Delta x_{cm} = 0$$

Let wedge displace to the right with respect to ground through x. Taking rightwards direction as positive, we get

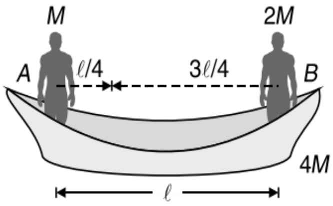
$$\Delta x_{cm} = \frac{m(l\cos 60^\circ + x) + (4m)x - m(l - x)}{m + 4m + m}$$

$$\Rightarrow \quad 0 = \frac{ml}{2} - ml + 6mx$$

$$\Rightarrow \quad x = \frac{l}{12}$$

7. Let the boat be displaced through a distance x towards right, then the horizontal displacement of centre of mass of system must be zero. Taking rightward direction as positive, we get

$$\Delta x_{boat} = +x,\ \Delta x_A = \frac{l}{4} + x,\ \Delta x_B = -\left(\frac{3l}{4} - x\right)$$



$$0 = \frac{M(l/4 + x) - 2M(3l/4 - x) + 4Mx}{7M}$$

$$\Rightarrow \quad \frac{Ml}{4} + 5Mx = \frac{3Ml}{2} - 2Mx$$

$$\Rightarrow \quad 7x = \frac{3l}{2} - \frac{l}{4} = \frac{5l}{4}$$

$$\Rightarrow \quad x = \frac{5l}{28}$$

8. (a) The given time is of no consequence since v_{cm} is constant for all times.

From equation, in component form

$$(v_{cm})_x = \frac{m_1 v_{1x} + m_2 v_{2x}}{M}$$

$$\Rightarrow \quad (v_{cm})_x = \frac{(3)(-5\cos 37^\circ) + (5)(0)}{8}$$

$$\Rightarrow \quad (v_{cm})_x = -1.5\text{ ms}^{-1}$$

Also, $(v_{cm})_y = \dfrac{m_1 v_{1y} + m_2 v_{2y}}{M}$

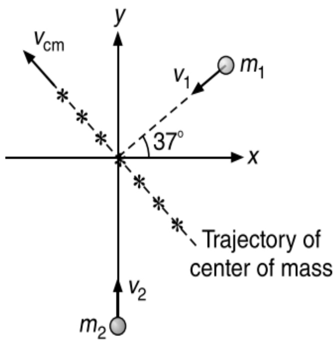
$$\Rightarrow \quad (v_{cm})_y = \frac{(3)(-5\sin 37^\circ) + (5)(5)}{8}$$

$$\Rightarrow \quad (v_{cm})_y = 2\text{ ms}^{-1}$$

So, $\vec{v}_{cm} = \left(1.5\hat{i} + 2\hat{j}\right)\text{ ms}^{-1}$

(b) Since the collision occurs at the origin, the position of the centre of mass 2 s later is

$$\vec{r}_{cm} = (\vec{v}_{cm})t = \left(-3\hat{i} + 4\hat{j}\right)\text{ m}$$



9. Since no external force acts on the system in the horizontal direction, i.e. along the x direction, so we have

$$\Delta x_{cm} = 0$$

Let wedge displace to the left with respect to ground through x, then the block displaces to the right through $R\sin\theta - x$, such that

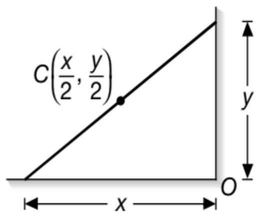
$$m(R\sin\theta - x) = Mx$$

$$\Rightarrow \quad x = \frac{mR\sin\theta}{m + M}$$

10. Since, $y = \sqrt{L^2 - x^2}$

$$\Rightarrow \quad \frac{dy}{dt} = -\frac{x}{\sqrt{L^2 - x^2}}\frac{dx}{dt}$$

$$\Rightarrow \quad v_y = \frac{dy}{dt} = -\frac{3\times 2}{4} = -\frac{3}{2}\text{ ms}^{-1}$$



At any time, the coordinates of centre of mass of rod are $\left(\frac{x}{2}, \frac{y}{2}\right)$

$$\Rightarrow \quad (v_{cm})_x = \frac{d}{dt}\left(\frac{x}{2}\right) = \frac{1}{2}\left(\frac{dx}{dt}\right) = \frac{1}{2}v_x$$

and $\left(v_{\text{cm}}\right)_y = \dfrac{d}{dt}\left(\dfrac{y}{2}\right) = \dfrac{1}{2}\left(\dfrac{dy}{dt}\right) = \dfrac{1}{2}v_y$

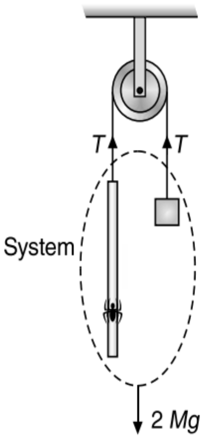
$\Rightarrow \quad \vec{v}_{\text{cm}} = \left(v_{\text{cm}}\right)_x \hat{i} + \left(v_{\text{cm}}\right)_y \hat{j}$

$\Rightarrow \quad \vec{v}_{\text{cm}} = \dfrac{1}{2}\left(v_x\hat{i} + v_y\hat{j}\right)$

$\Rightarrow \quad \left|\vec{v}_{\text{cm}}\right| = \dfrac{1}{2}\sqrt{v_x^2 + v_y^2} = \dfrac{1}{2}\sqrt{4 + \dfrac{9}{4}}$

$\Rightarrow \quad \left|\vec{v}_{\text{cm}}\right| = 1.25 \text{ ms}^{-1}$

11. (a) Considering insect, rod and counter weight as a system, the external forces acting on the system are tension in the string and gravitational force as shown in Figure.



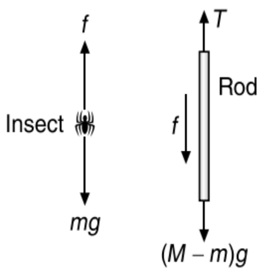
When insect is at rest, we have

$$\Sigma F_{\text{system}} = 2T - 2Mg = 0$$

$$\Rightarrow \quad T = Mg$$

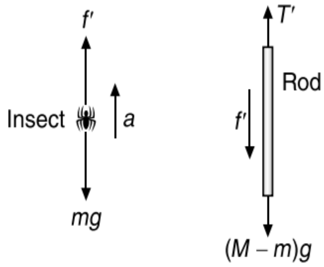
Hence, no net external force is acting on the system.

(b) When we consider free body diagram of the insect, then we observe that the friction force is balancing the weight of insect when it is at rest.



However, when the insect moves with constant velocity, then too the net force on insect is zero and hence the magnitude of friction force will remain unchanged. This simply means that the tension in the string will not change. So, no external force acts on the system. Therefore, the acceleration of the centre of mass of the system will be zero.

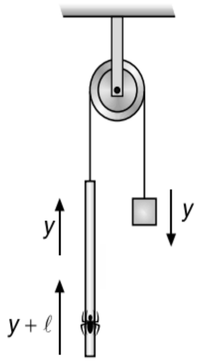
(c) When the insect moves up with acceleration, the friction force between the insect and rod increases. This increased friction increases the tension in the string $(T > Mg)$.



Now in this case, there will be net force on the system in upward direction. Hence the centre of mass of the system will accelerate in upward direction.

(d) Let displacement of the rod when the insect reaches to the other end of the rod be y upward. Hence the displacement of the counter weight M will be y downward as the rod and counter weight both are connected together by a single string. During this time the displacement of the insect will be $(y + l)$.

We know



$$\Delta y_{\text{cm}} = \frac{m_1\Delta y_1 + m_2\Delta y_2 + m_3\Delta y_3}{(m_1 + m_2 + m_3)}$$

$$\Rightarrow \quad \Delta y_{\text{cm}} = \frac{m(y + l) + (M - m)y + M(-y)}{m + (M - m) + M}$$

$$\Rightarrow \quad \Delta y_{\text{cm}} = \frac{ml}{2M}, \text{ upwards}$$

12. (a) $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$

$$\Rightarrow \quad 3 = \frac{m_1(0) + (0.1)(12)}{0.1 + m_1} = \frac{1.2}{0.1 + m_1}$$

$$\Rightarrow \quad m_1 = 0.3 \text{ kg}$$

(b) $\vec{p} = (m_1 + m_2)\vec{v}_{\text{cm}} = (0.3 + 0.1)(6\hat{j}) \text{ kgms}^{-1}$

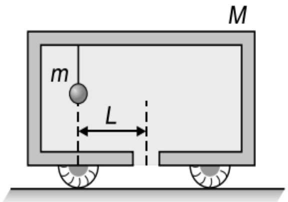
$$\Rightarrow \quad \vec{p} = 2.4\hat{j} \text{ kgms}^{-1}$$

(c) $\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$

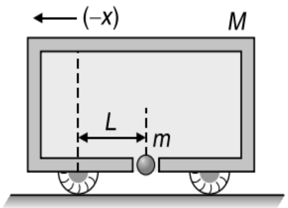
$\Rightarrow \quad 6\hat{j} = \dfrac{(0.3)\vec{v}_1 + (0.1)(0)}{0.4}$

$\Rightarrow \quad \vec{v}_1 = 8\hat{j}\ \text{ms}^{-1}$

13. Let displacement of the cart be x towards left



Then displacement of the bob will be $(L - x)$ such that



$$m_1\Delta x_1 = m_2\Delta x_2$$

$$\Rightarrow \quad Mx = m(L - x)$$

$$x = \frac{ML}{m + M}$$

14. (a) $h_1 = \dfrac{1}{2}gt_1^2 = \dfrac{1}{2} \times 10 \times (0.4)^2 = 0.8\ \text{m}$

$h_2 = \dfrac{1}{2}gt_2^2 = \dfrac{1}{2} \times 10 \times (0.2)^2 = 0.2\ \text{m}$

Since, $h_{cm} = \dfrac{m_1h_1 + m_2h_2}{m_1 + m_2}$

$\Rightarrow \quad h_{cm} = \dfrac{m(0.8) + 2m(0.2)}{3m} = 0.4\ \text{m}$

$\Rightarrow \quad h_{cm} = 40\ \text{cm}$

(b) $v_1 = gt_1 = (10)(0.4) = 4\ \text{ms}^{-1}$

$v_2 = gt_2 = (10)(0.2) = 2\ \text{ms}^{-1}$

Since, $v_{cm} = \dfrac{mv_1 + 2mv_2}{3m}$

Substituting the values, we get

$$v_{cm} = \frac{8}{3}\ \text{ms}^{-1} = 2.67\ \text{ms}^{-1}$$

15. When ball is to enter the hole, then the ball travels a horizontal distance l with respect to the car. During this motion, to maintain the position of centre of mass the car moves towards left, say by a distance x w.r.t. ground. So, distance travelled by the ball towards right w.r.t. ground is $(l - x)$ such that

$$m(l - x) = Mx$$

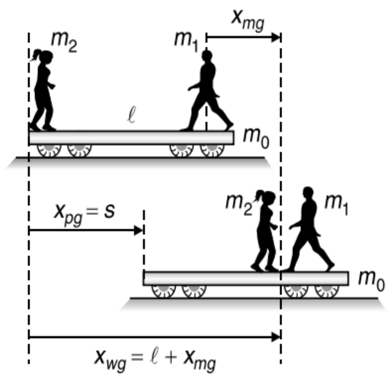
$$\Rightarrow \quad x = \frac{ml}{M + m}$$

16. Since it is given that x_1 is the displacement of the man with respect to the platform, so we have

$$x_{mp} = x_{mg} - x_{pg} = -x_1$$

$$\Rightarrow \quad x_{mg} = x_{pg} - x_1 = s - x_1$$

Also, we know that no external force is acting on the system, so the displacement of the centre of mass is zero. The displacements of the man (m), woman (w) and the platform (p) all w.r.t. ground (g) are shown in Figure.



Since, $\Delta x_{cm} = 0$

$$\Rightarrow \quad m_1x_{mg} + m_2x_{wg} + m_0x_{pg} = 0$$

where x_{mg}, x_{wg} and x_{pg} are the respective displacements of the man w.r.t. ground, woman w.r.t. ground and platform w.r.t. ground.

$$\Rightarrow \quad m_0s + m_1(s - x_1) + m_2(l + s - x_1) = 0$$

$$\Rightarrow \quad s = \frac{(m_1 + m_2)x_1 - m_2l}{m_1 + m_2 + m_0}$$

17. If we take ball 1 and ball 2 as system, there is no external force acting on the system. Hence the velocity of the centre of mass of the system will be constant. From given figure, we can calculate the velocity of the centre of mass, which will also be the velocity of centre of mass 3 s before the collision. x component of velocity of centre of mass

$$(v_{cm})_x = \frac{m_1v_{1x} + m_2v_{2x}}{m_1 + m_2}$$

$$\Rightarrow \quad (v_{cm})_x = \frac{(3)(-5\cos 37°) + (5)(0)}{8\ \text{kg}} = -1.5\ \text{ms}^{-1}$$

y component of velocity of centre of mass

$$(v_{cm})_y = \frac{m_1v_{1y} + m_2v_{2y}}{m_1 + m_2}$$

$$\Rightarrow \quad (v_{cm})_y = \frac{(3)(-5\sin 37°)+(5\times 5)}{8\text{ kg}} = +2\text{ ms}^{-1}$$

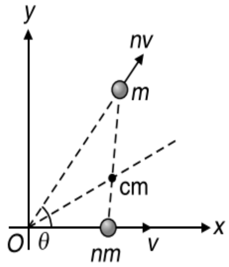
$$\Rightarrow \quad \vec{v}_{cm} = -1.5\hat{i} + 2\hat{j}\text{ ms}^{-1}$$

As collision occurs at the origin $(\vec{r}_i = 0)$, so the position of centre of mass at the time of collision will be at origin. The position of the centre of mass 2 s after the collision is

$$\vec{r}_{cm} = \vec{r}_i + \vec{v}_{cm}t = \vec{v}_{cm}t \qquad \{\because \vec{r}_i = \vec{0}\}$$

$$\Rightarrow \quad \vec{r}_{cm} = (-1.5\hat{i} + 2\hat{j})\times 2 = (-3\hat{i} + 4\hat{j})\text{ m}$$

18. Let the mass $m_1 = m$ and $m_2 = nm$ be moving as shown in Figure.



Since, $\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_1\vec{v}_2}{m_1+m_2}$

$$\Rightarrow \quad \vec{v}_{cm} = \frac{(nm)v\hat{i} + m(nv)(\cos\theta\hat{i} + \sin\theta\hat{j})}{m+nm}$$

$$\Rightarrow \quad \vec{v}_{cm} = \frac{nmv(1+\cos\theta)\hat{i} + (nmv\sin\theta)\hat{j}}{m+nm}$$

$$\Rightarrow \quad |\vec{v}_{cm}| = \frac{nmv\sqrt{(1+\cos\theta)^2 + (\sin\theta)^2}}{m(1+n)}$$

$$\Rightarrow \quad |\vec{v}_{cm}| = \frac{nv\sqrt{1+\cos^2\theta + 2\cos\theta + \sin^2\theta}}{1+n}$$

$$\Rightarrow \quad |\vec{v}_{cm}| = \frac{nv\sqrt{2+2\cos\theta}}{1+n} = \frac{\sqrt{2}nv\sqrt{1+\cos\theta}}{1+n}$$

Since $1+\cos\theta = 2\cos^2\left(\dfrac{\theta}{2}\right)$

$$\Rightarrow \quad |\vec{v}_{cm}| = \frac{2nv\cos\left(\dfrac{\theta}{2}\right)}{1+n}$$

Test Your Concepts-III (Based on Work Energy Theorem for System of Particles and its Applications from Centre of Mass Reference Frame)

1. The kinetic energy of a two particle system with respect to the centroidal frame is

$$K_{cm} = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)|\vec{u}_{rel}|^2$$

$$\Rightarrow \quad K_{cm} = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)|\vec{u}_1 - \vec{u}_2|^2$$

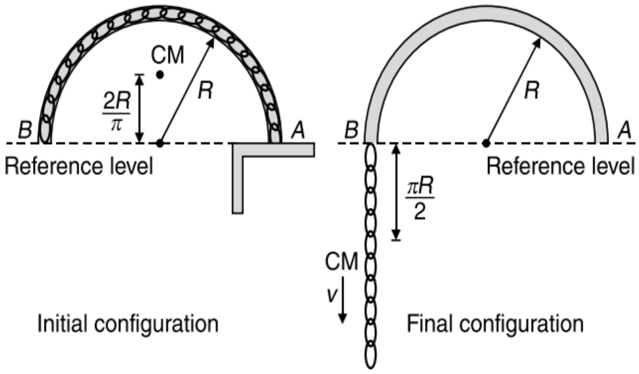
Since $\vec{u}_1$ and $\vec{u}_2$ are perpendicular to each other, so

$$|\vec{u}_1 - \vec{u}_2| = \sqrt{u_1^2 + u_2^2} = \sqrt{(5\sqrt{3})^2 + 5^2} = 10\text{ ms}^{-1}$$

$$\Rightarrow \quad K_{cm} = \frac{1}{2}\left(\frac{0.6\times 0.3}{0.6+0.3}\right)(10)^2 = 10\text{ J}$$

2. The centre of mass of the semi-circular chain will be at a distance $\left(\dfrac{2R}{\pi}\right)$ from the centre and the mass of the chain will be $\lambda\pi R$, where λ is the linear mass density of the chain.

Let us consider the horizontal line passing through points A and B to be the Zero Potential Energy Level (ZPEL) to calculate the gravitational potential energy in the initial and final configurations of the chain as shown in Figure.



Initial potential energy of chain is

$$U_{\text{initial}} = mgh_i$$

$$\Rightarrow \quad U_{\text{initial}} = (\lambda\pi R)g\left(\frac{2R}{\pi}\right) = 2\lambda gR^2$$

Final potential energy of chain is

$$U_{\text{final}} = mg(-h_f)$$

$$\Rightarrow \quad U_{\text{final}} = mg(-h_f) = (\lambda\pi R)g\left(\frac{-\pi R}{2}\right)$$

$$\Rightarrow \quad U_{\text{final}} = -\frac{\lambda\pi^2 gR^2}{2}$$

When the chain starts slipping from the tube, then its initial kinetic energy $K_{\text{initial}} = 0$ and when it slips off completely from the tube, then all the links of the chain move with same common velocity v due to which its kinetic energy in the final configuration is

$$K_{\text{final}} = \frac{1}{2}mv^2 = \frac{1}{2}(\lambda\pi R)v^2$$

Since all surfaces are frictionless, so there will not be any loss of mechanical energy and hence by Law of Conservation of Mechanical Energy, we get

$$U_{\text{initial}} + K_{\text{initial}} = U_{\text{final}} + K_{\text{final}}$$

$$\Rightarrow \quad 2\lambda g R^2 + 0 = -\frac{\lambda \pi^2 g R^2}{2} + \frac{1}{2}(\lambda \pi R)v^2$$

$$\Rightarrow \quad \frac{1}{2}(\lambda \pi R)v^2 = 2\lambda g R^2 + \frac{\lambda \pi^2 g R^2}{2}$$

$$\Rightarrow \quad v^2 = \frac{2\lambda g R^2}{\lambda \pi R}\left(2 + \frac{\pi^2}{2}\right)$$

$$\Rightarrow \quad v = \sqrt{2gR\left(\frac{2}{\pi} + \frac{\pi}{2}\right)}$$

3. Applying Work Energy Theorem from the centre of mass reference frame, we get

$$W_{\text{total}} = \Delta K_{\text{rel}} + \Delta K_{\text{cm}}$$

As no external forces acting, the velocity of centre of mass will be constant. Hence $\Delta K_{\text{cm}} = 0$

$$\Rightarrow \quad W_{\text{total}} = W_{\text{int}} + W_{\text{ext}} = \Delta K_{\text{rel}}$$

where, $W_{\text{int}} = 0$ and

$$W_{\text{ext}} = W_{\text{gravity}} = -\Delta U = -(3m)gh$$

Also, $(\Delta K)_{\text{rel}} = \frac{1}{2}\mu\left(v_{\text{rel}}^2 - u_{\text{rel}}^2\right)$

where, μ is reduced mass of system given by

$$\mu = \frac{(6m)(3m)}{6m + 3m} = 2m$$

So, we have

$$-3mgh = \frac{1}{2}(2m)\left[(0)^2 - (3v_0)^2\right]$$

$$\Rightarrow \quad h = \frac{3v_0^2}{g}$$

4. When the extension in the spring is maximum, both the blocks move with a common velocity. Hence the final relative velocity of the blocks is zero. Applying Work Energy Theorem from the centre of mass reference frame, we get

$$W = W_{\text{int}} + W_{\text{ext}} = (\Delta K)_{\text{cm}} = \frac{1}{2}\mu\left(v_{\text{rel}}^2 - u_{\text{rel}}^2\right)$$

$$W_{\text{int}} = (\Delta K)_{\text{cm}} = \frac{1}{2}\mu v_{\text{rel}}^2 - \frac{1}{2}\mu u_{\text{rel}}^2$$

where, μ is reduced mass of system given by

$$\mu = \frac{(m)(3m)}{m + 3m} = \frac{3m}{4}$$

Since, $W_{\text{int}} = W_{\text{spring}} = -\Delta U = -\frac{1}{2}kx_{\max}^2$

$$\Rightarrow \quad -\frac{1}{2}kx_{\max}^2 = 0 - \frac{1}{2}\left(\frac{3m}{4}\right)(v_0 + 3v_0)^2$$

$$\Rightarrow \quad kx_{\max}^2 = \left(\frac{3m}{4}\right)(16v_0^2)$$

$$\Rightarrow \quad x_{\max} = 2v_0\sqrt{\frac{3m}{k}}$$

5. In the centre of mass reference frame, i.e. a reference frame travelling with the velocity of the centre of mass of the system, the initial mechanical energy of the system is equal to the elastic potential energy of the spring (because initial velocity of both blocks is zero). When the spring regains its natural length, then this energy gets converted into kinetic energy, so we have

$$\frac{1}{2}kx^2 = \frac{1}{2}\mu\left(v_r^2 - 0^2\right) \qquad \ldots(1)$$

where, μ is reduced mass of system given by

$$\mu = \frac{(6m)(3m)}{6m + 3m} = 2m$$

and v_r is the final relative velocity of the blocks when the spring comes to its natural length.

So, from equation (1), we get

$$kx^2 = \mu v_r^2$$

$$\Rightarrow \quad kx^2 = (2m)v_r^2$$

$$\Rightarrow \quad v_r = x\sqrt{\frac{k}{2m}}$$

6. At maximum extension in the spring, let the displacements of the blocks m_1 and m_2 with respect to CM be x_1 (towards left) and x_2 (towards right) respectively. So, the maximum extension in the spring is

$$x = x_1 + x_2 \qquad \ldots(1)$$

From centre of mass reference frame, we can write,

$$-m_1x_1 + m_2x_2 = 0$$

$$\Rightarrow \quad m_1x_1 = m_2x_2 \qquad \ldots(2)$$

From equation (1) and (2), we get

$$x_1 = \frac{m_2 x}{m_1 + m_2} \quad \text{and} \quad x_2 = \frac{m_1 x}{m_1 + m_2}$$

Applying Work Energy Theorem from the centre of mass reference frame, we get

$$W = W_{\text{int}} + W_{\text{ext}} = \Delta K_{\text{cm}}$$

$$\Rightarrow \quad W = \frac{1}{2}\mu\left(v_{\text{rel}}^2 - u_{\text{rel}}^2\right) \qquad \ldots(3)$$

Since both the particles m_1 and m_2 start from rest, so $u_{\text{rel}} = 0$

At maximum extension in the spring both the masses move with a common velocity, so $v_{\text{rel}} = 0$

Hence, from equation (3), we get

$$W_{\text{int}} + W_{\text{ext}} = 0 \qquad \ldots(4)$$

In this case, we must understand that since the centre of mass frame is accelerating, so we do have pseudo force acting on the centre of mass. The work done due to pseudo force in the centre of mass frame is $W_{\text{pseudo}} = 0$. So, we have

$$W_{\text{int}} = W_{\text{spring}} + W_{\text{pseudo}} = W_{\text{spring}}$$

Since, $W_{spring} = \Delta U = -\frac{1}{2}kx^2$, so we have

$$W_{int} = W_{spring} = -\frac{kx^2}{2} \quad \ldots(5)$$

Since F_1 and F_2 are constant external forces, so the work done by these external forces is

$$W_{ext} = F_1x_1 + F_2x_2 \quad \ldots(6)$$

So, from equation (4), we get

$$F_1x_1 + F_2x_2 - \frac{1}{2}kx^2 = 0 \quad \ldots(7)$$

Substituting the values of x_1 and x_2 in equation (7), we get

$$F_1\left(\frac{m_2x}{m_1+m_2}\right) + F_2\left(\frac{m_1x}{m_1+m_2}\right) = \frac{kx^2}{2}$$

$$\Rightarrow \quad x = \frac{2(m_1F_2 + m_2F_1)}{k(m_1+m_2)}$$

7. When the slipping stops, then both the block and plank move with same common velocity. Applying Work Energy Theorem from the centre of mass reference frame, we get

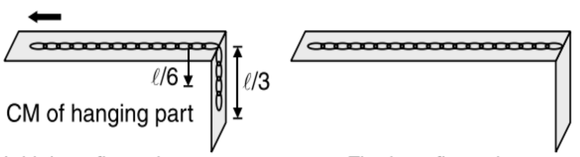
$$W = W_{int} + W_{ext} = (\Delta K)_{cm} = \frac{1}{2}\mu\left(v_{rel}^2 - u_{rel}^2\right)$$

where, μ is reduced mass of system given by

$$\mu = \frac{(m)(3m)}{m+3m} = \frac{3m}{4}$$

$$\Rightarrow \quad W_{friction} = -\frac{1}{2}\left(\frac{3m}{4}\right)v_0^2 = -\frac{3}{8}mv_0^2$$

8. The initial and final situation is shown in Figure.



Initial configuration Final configuration

The work done in slowly pulling the complete chain up the table is

$$W_{ext} = \Delta U_{cm} \quad \ldots(1)$$

Assuming ZPEL to be at the table, then the initial and final potential energy of the part of the chain hanging from the table are given by

$$U_{initial} = mg(-h_{cm}) = -\left(\frac{m}{3}\right)g\left(\frac{l}{6}\right) = -\frac{mgl}{18}$$

and $U_{final} = 0$

So, from equation (1), we get

$$W_{ext} = 0 - \left(-\frac{mgl}{18}\right) = \frac{mgl}{18}$$

9. At maximum extension in the spring, both the rings move with same common velocity. Applying Work Energy Theorem from the centre of mass reference frame, we get

$$W = W_{int} + W_{ext} = (\Delta K)_{cm} = \frac{1}{2}\mu\left(v_{rel}^2 - u_{rel}^2\right)$$

where, μ is reduced mass of system given by

$$\mu = \frac{(6m)(12m)}{6m+12m} = 4m$$

Since no external forces are there, so $W_{ext} = 0$ and due to extension of the spring, we have

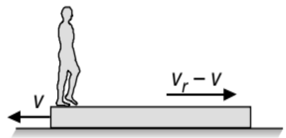
Since, $W_{int} = W_{spring} = -\Delta U = -\frac{1}{2}kx_{max}^2$

$$\Rightarrow \quad -\frac{1}{2}kx_{max}^2 = -\frac{1}{2}(4m)v_0^2 - 0$$

$$\Rightarrow \quad x_{max} = 2v_0\sqrt{\frac{m}{k}}$$

Test Your Concepts-IV (Based on Conservation of Linear Momentum)

1. Absolute velocity of man i.e., velocity of man w.r.t. ground is $v_r - v$ where v is the recoil velocity of platform. Taking the platform and the man as a system, net external force on the system in horizontal direction is zero. The linear momentum of the system remains constant. Initially both the man and the platform were at rest.

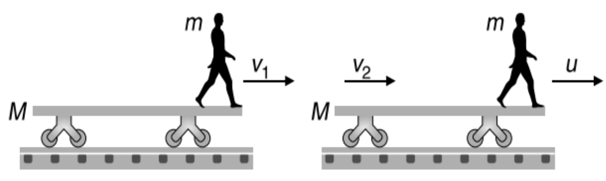


Hence, $0 = m_1(v_r - v) - m_2v$

$$\Rightarrow \quad v = \frac{m_1v_r}{m_1+m_2}$$

2. Let the velocity of the car just after jumping of man be v. The net velocity of the man with respect to the ground will be $u - v$, as u is its velocity with respect to the car.

If we take man and car as system. There is no external force act on it in horizontal direction. It means the linear momentum of the system should be conserved in horizontal direction.



Just before jumping Just after jumping

Initially, the system was at rest, thus according to momentum conservation, momentum after jump must be zero. Hence,

$$m(u-v)=Mv$$

$$\Rightarrow \quad v=\frac{mu}{m+M}$$

3. At the lowest point, when m leaves the surface of M, it has a horizontal velocity v_1 (say). If v_2 be velocity of M, then by Law of Conservation of Linear Momentum, we have

$$mv_1 = Mv_2 \qquad \ldots(1)$$

By Law of Conservation of Mechanical Energy, we have

$$mgR=\frac{1}{2}mv_1^2+\frac{1}{2}Mv_2^2 \qquad \ldots(2)$$

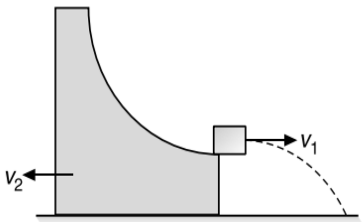
Also, using concepts of Projectile Motion, we have

$$\frac{R}{2}=\frac{1}{2}gt^2$$

$$\Rightarrow \quad t=\sqrt{\frac{R}{g}} \qquad \ldots(3)$$

The desired distance is

$$s=(v_1+v_2)t \qquad \ldots(4)$$

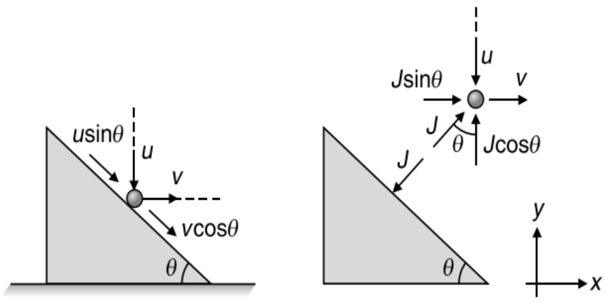


Solving equation (1) and (2) for v_1 and v_2 and substituting in equation (4), we get

$$s=R\sqrt{\frac{2(M+m)}{M}}$$

4. Velocity of the particle just before impact with the plane

$$u=\sqrt{2gh}$$



In the direction normal to impulse, the linear momentum of the particle just after impact will not change. Hence velocity of the particle parallel to inclined plane just before collision and just after collision is the same.

$$\Rightarrow \quad v\cos\theta = u\sin\theta$$

$$\Rightarrow \quad v = u\tan\theta$$

$$\Rightarrow \quad v=\sqrt{2gh}\tan\theta$$

The particle just after collision started moving in horizontal direction. Hence the component of the velocity of the ball in vertical direction after impact should be zero. Applying impulse momentum equation to the particle in vertical direction, we get

$$\vec{J}=(\vec{p}_{\text{final}})_y-(\vec{p}_{\text{initial}})_y$$

$$\Rightarrow \quad J\cos\theta = 0-(-mu)=mu$$

$$\Rightarrow \quad J=\frac{mu}{\cos\theta}=mu\sec\theta$$

$$\Rightarrow \quad J=m\sqrt{2gh}\sec\theta$$

Loss in kinetic energy due to impact is

$$-\Delta K = K_{\text{initial}}-K_{\text{final}}=\frac{1}{2}mu^2-\frac{1}{2}mv^2$$

$$\Rightarrow \quad -\Delta K=\frac{1}{2}m\left[\left(\sqrt{2gh}\right)^2-\left(\sqrt{2gh}\right)^2\tan^2\theta\right]$$

$$\Rightarrow \quad -\Delta K=\frac{1}{2}m\left[2gh(1-\tan^2\theta)\right]$$

$$\Rightarrow \quad -\Delta K = mgh(1-\tan^2\theta)$$

5. Velocity of projectile at highest point is $24\cos 60^\circ$ or 12 ms^{-1} (horizontally). Applying Law of Conservation of Linear Momentum, we get

$$mv_B\hat{j}+m\vec{v}_A=2m(12\hat{i})$$

$$\Rightarrow \quad \vec{v}_A=24\hat{i}-v_B\hat{j}$$

Since they fall 45 m apart, so

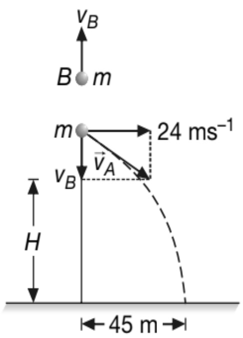
$$x=(v_A)_x t$$

$$\Rightarrow \quad 45=24t$$

$$\Rightarrow \quad t=1.875 \text{ s}$$

Since, $H=vt+\frac{1}{2}gt^2$

$$\Rightarrow \quad \frac{(24)^2\sin^2 60^\circ}{2(10)}=v_B(1.875)+\frac{1}{2}g(1.875)^2$$



Solving this equation, we get

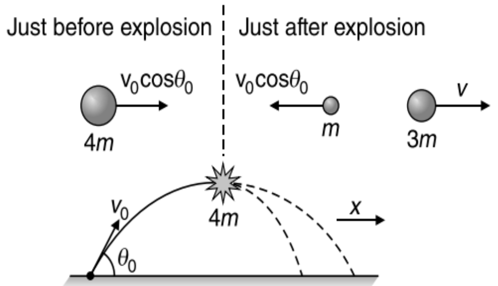
$$\Rightarrow \quad v_B = 2.55 \text{ ms}^{-1}$$

$\Rightarrow \quad \vec{v}_A = 24\hat{i} - 2.55\hat{j}$

$\Rightarrow \quad |\vec{v}_A| = \sqrt{(24)^2 + (2.55)^2}$

$\Rightarrow \quad |\vec{v}_A| = 24.1 \text{ ms}^{-1}$

6. Let the projectile be of mass $4m$. The projectile just before explosion is at the highest point of its path and moving with horizontal velocity of $v_0 \cos\theta_0$. The lighter mass m formed after explosion retraces its path and hence its velocity at the top is just reversed due to explosion. So, m moves leftwards with a velocity $v_0 \cos\theta_0$. At this moment, the heavier part of mass $3m$ is moving rightwards with velocity v.



The linear momentum just before explosion

$$\vec{p}_{\text{initial}} = (4mv_0\cos\theta_0)\hat{i}$$

The linear momentum just after explosion

$$\vec{p}_{\text{final}} = -(mv_0\cos\theta_0)\hat{i} + (3m)\vec{v}$$

Applying Law of Conservation of Linear Momentum along horizontal x-axis, we get

$$(4mv_0\cos\theta_0)\hat{i} = -(mv_0\cos\theta_0)\hat{i} + (3m)\vec{v}$$

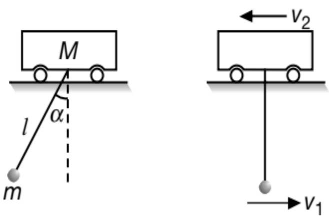
$$\Rightarrow \quad (3m)\vec{v} = (5mv_0\cos\theta_0)\hat{i}$$

$$\Rightarrow \quad \vec{v} = \left(\frac{5v_0\cos\theta_0}{3}\right)\hat{i}$$

Hence the velocity of the heavier part just after the explosion is $v = \frac{5}{3}v_0\cos\theta_0$, along x direction.

7. Applying Law of Conservation of Linear Momentum, we get

$$mv_1 = Mv_2 \quad \ldots(1)$$



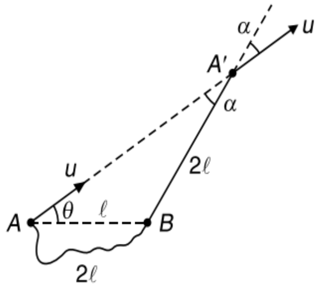
By Law of Conservation of Energy, we get

$$mg\ell(1-\cos\alpha) = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2 \quad \ldots(2)$$

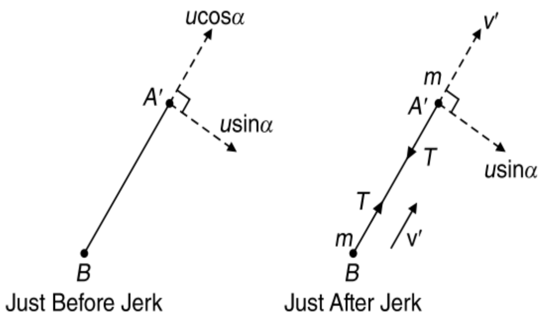
Solving these two equations, we get velocity of wagon

$$v_2 = 2m\sin\left(\frac{\alpha}{2}\right)\sqrt{\frac{g\ell}{(M+m)M}}$$

8. Let us solve the problem for a general case when direction of u makes an angle θ to the line AB as shown in Figure.



At position A', the string gets taut and the components of velocities along and perpendicular to the string just before and after the jerk are shown in Figure.



Applying sine rule to $\Delta AA'B$, we get

$$\frac{\sin\theta}{2l} = \frac{\sin\alpha}{l}$$

$$\Rightarrow \quad \sin\alpha = \frac{1}{2}\sin\theta = \frac{1}{2}\sqrt{1-\cos^2\theta} \quad \ldots(1)$$

Since, velocity of A perpendicular to the string remains constant during jerk, so applying the Impulse Momentum theorem, we get

$$\text{For } A, \int -T dt = mv' - mu\cos\alpha \quad \ldots(2)$$

$$\text{For } B, \int T dt = mv' - 0 \quad \ldots(3)$$

Adding equations (2) and (3), we get

$$0 = mv' + mv' - mu\cos\alpha$$

$$\Rightarrow \quad v' = \frac{u\cos\alpha}{2} \quad \ldots(4)$$

$$\text{So, Impulse} = \int T dt = mv' = \frac{mu\cos\alpha}{2} \quad \ldots(5)$$

For (a), u is along BA, i.e. $\theta = \pi$ and $\alpha = 0°$

$$\Rightarrow \quad v' = \frac{u}{2} \text{ and } \int T dt = \frac{mu}{2}$$

For (b), u makes an angle of $120°$ with AB, i.e. $\theta = \frac{2\pi}{3}$ and $\alpha = \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

$$\Rightarrow \quad v' = \frac{u\sqrt{13}}{8} \text{ and } \int T dt = \frac{mu\sqrt{13}}{8}$$

For (c), u makes an angle of $90°$ with AB, i.e. $\theta = \frac{\pi}{2}$ and $\alpha = \frac{\pi}{6}$

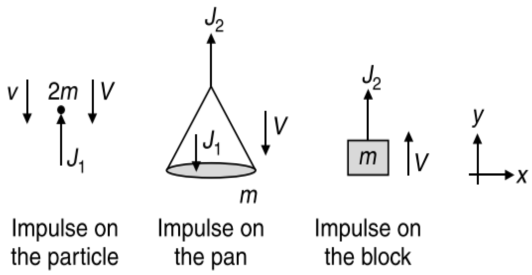
$$\Rightarrow \quad v' = \frac{u\sqrt{3}}{4} \text{ and } \int T dt = \frac{mu\sqrt{3}}{4}$$

9. When the smaller block reaches A, then both the blocks will have same horizontal velocity. Applying Law of Conservation of Linear Momentum (along axis perpendicular to gravity), we get

$$mv = (m + M)v_x$$

$$\Rightarrow \quad v_x = \frac{mv}{m + M}$$

10. When the particle strikes the pan, the normal reaction between the pan and the particle provides the impulse. Hence tension in the string is impulsive in nature. Let the speed of the system just after collision be V. The impulse diagram is shown in Figure.



From impulse momentum equation, we have

$$\vec{J} = \vec{p}_{\text{final}} - \vec{p}_{\text{initial}}$$

Taking downward direction as positive, then for the particle, we have

$$-J_1 = 2m(V - v) \quad \ldots(1)$$

for the pan, we have

$$J_1 - J_2 = mV \quad \ldots(2)$$

and for the block, we have

$$-J_2 = -mV$$

$$\Rightarrow \quad J_2 = mV \quad \ldots(3)$$

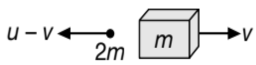
From equations (1), (2) and (3), we get

$$V = \frac{v}{2}$$

11. (a) By Law of Conservation of Linear Momentum, we get

$$Mv = 2m(u - v)$$

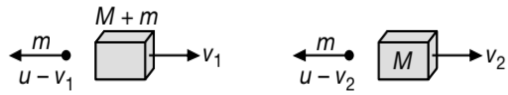
$$\Rightarrow \quad v = \frac{2mu}{M + 2m}$$



(b) Again, applying Law of Conservation of Linear Momentum, but one by one, we get

$$(M + m)v_1 = m(u - v_1)$$

$$\Rightarrow \quad v_1 = \frac{mu}{M + 2m}$$



Further, $Mv_2 = m(u - v_1) + m(u - v_2)$

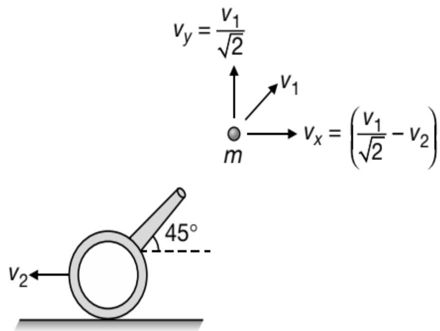
$$\Rightarrow \quad (M + m)v_2 = (M + m)\left(\frac{mu}{M + 2m}\right) + mu$$

$$\Rightarrow \quad v_2 = \frac{mu}{M + 2m} + \frac{mu}{M + m}$$

12. If shell is fired at muzzle at speed v_1, then by conservation of linear momentum, we have

$$m\left(\frac{v_1}{\sqrt{2}} - v_2\right) = kmv_2$$

$$\Rightarrow \quad v_2 = \frac{v_1}{\sqrt{2}(k + 1)}$$



The ratio of kinetic energies is given by

$$\frac{K_{\text{shell}}}{K_{\text{gun}}} = \frac{\frac{1}{2}m\left[\frac{v_1^2}{2} + \left(\frac{v_1}{\sqrt{2}} - v_2\right)^2\right]}{\frac{1}{2}kmv_2^2}$$

$$\Rightarrow \quad \frac{K_{\text{shell}}}{K_{\text{gun}}} == \frac{\frac{v_1^2}{2} + \frac{k^2 v_1^2}{2(k + 1)^2}}{\frac{kv_1^2}{2(k + 1)^2}}$$

$$\Rightarrow \quad \frac{K_{\text{shell}}}{K_{\text{gun}}} = \frac{2k^2 + 2k + 1}{k}$$

13. Applying Law of Conservation of Linear Momentum and Mechanical Energy we get,

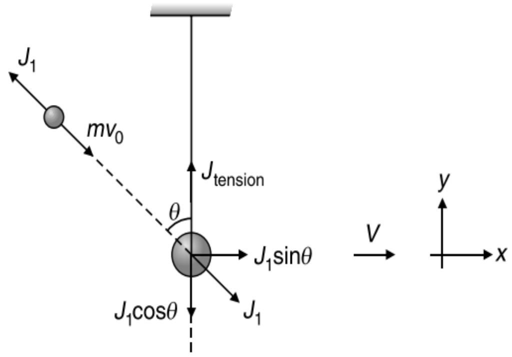
$$mv = MV \quad \ldots(1)$$

and $$mgR = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 \quad \ldots(2)$$

Solving (1) and (2), we get velocity of cube to be

$$v = \sqrt{\frac{2MgR}{M+m}}$$

14. Since tension in the string will be impulsive in nature. The impulse diagram of the situation is shown in Figure.



Applying impulse momentum equation to the particle, we get

$$-J_1 = 0 - mv_0$$

$$\Rightarrow \quad J_1 = mv_0$$

As the string is inextensible, so the bob cannot move in vertical direction. Hence,

$$J_{\text{tension}} = J_1 \cos\theta$$

$$\Rightarrow \quad J_{\text{tension}} = mv_0 \cos\theta$$

The horizontal component of impulse $(J_1 \sin\theta)$ will be responsible for motion of the bob in horizontal direction, hence

$$J_1 \sin\theta = MV$$

$$\Rightarrow \quad V = \frac{J_1 \sin\theta}{M}$$

$$\Rightarrow \quad V = \frac{mv_0 \sin\theta}{M}$$

So, we observe that after collision, the bob starts moving in the circular path.

15. Just after the string becomes taut, velocity of M (upwards) is given by

$$(M+m)v = m\sqrt{2gh}$$

$$\Rightarrow \quad v = \frac{m\sqrt{2gh}}{(M+m)}$$

Retardation,

$$a = \left(\frac{M-m}{M+m}\right)g$$

$$\Rightarrow \quad t = \frac{2v}{a} = \frac{2m\sqrt{2gh}}{(M+m)} \times \frac{M+m}{(M-m)g}$$

$$\Rightarrow \quad t = \frac{2m}{M-m}\sqrt{\frac{2h}{g}}$$

Loss in kinetic energy is given by

$$-\Delta K = K_i - K_f$$

$$\Rightarrow \quad -\Delta K = mgh - \frac{1}{2}(M+m)v^2$$

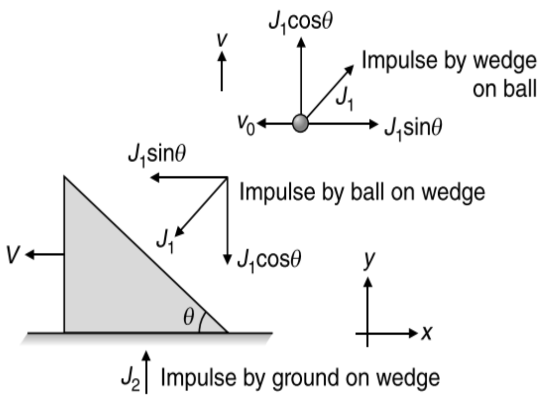
$$\Rightarrow \quad -\Delta K = mgh - \frac{1}{2}(M+m)\frac{m^2(2gh)}{(M+m)^2}$$

$$\Rightarrow \quad -\Delta K = \frac{Mmgh}{M+m}$$

16. In this case both the normal reactions, i.e. between the ball and wedge and between wedge and ground will be impulsive in nature and since impulse is

$$\vec{J} = \int_0^{\Delta t} \vec{F} dt$$

Hence the direction of impulse will be same as the direction of impulsive force which in this case is the normal reaction. Making impulsive diagram of the situation (impulsive diagram will be same as FBD in laws of motion) is shown in Figure.



Applying impulse momentum equation for ball along the x-direction, we get

$$(J_1)_x = (\vec{p}_{\text{final}})_x - (\vec{p}_{\text{initial}})_x$$

Since the final linear momentum of the ball in x-direction will be zero because the ball starts moving in upward direction, so we get

$$J_1 \sin\theta = 0 - (-mv_0) = mv_0 \quad \ldots(1)$$

From equation (1), we get

$$J_1 = \frac{mv_0}{\sin\theta} = mv_0 \operatorname{cosec}\theta \quad \ldots(2)$$

Since the wedge is not moving in the vertical direction, so from impulse diagram for the wedge, we see that

$$J_2 = J_1 \cos\theta = mv_0 \cot\theta$$

Since the ball just after collision starts moving up, so we can apply impulse momentum equation along the y-direction and get

$$\left(\vec{J}_1\right)_y = \left(\vec{p}_{\text{final}}\right)_y - \left(\vec{p}_{\text{initial}}\right)_y$$

$$\Rightarrow \quad J_1 \cos\theta = 0 - (-mv) = mv$$

$$\Rightarrow \quad v = \frac{J_1 \cos\theta}{m} = \frac{mv_0 \cot\theta}{m} \qquad \{\because \text{ of } (2)\}$$

$$\Rightarrow \quad v = v_0 \cot\theta$$

Just after collision the wedge starts moving towards left. Applying impulse-momentum equation to the wedge along the horizontal direction, we get

$$\left(\vec{J}_1\right)_x = \left(\vec{p}_{\text{final}}\right)_x - \left(\vec{p}_{\text{initial}}\right)_x$$

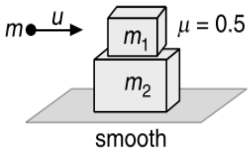
$$-J_1 \sin\theta = -MV - 0$$

$$\Rightarrow \quad V = \frac{J_1 \sin\theta}{M} = \frac{mv_0}{M}$$

Since the normal reaction between ground and wedge is impulsive, hence this means that the friction force between the wedge and ground will also be impulsive. So, impulse due to frictional force will be

$$J_{\text{friction}} = \mu J_2 = \mu\left(mv_0 \cot\theta\right)$$

17. Given $m = 0.25$ kg, $m_1 = 37.5$ kg, $m_2 = 12.5$ kg and $u = 302$ ms^{-1}



Let v be the common velocity then applying Conservation of Linear Momentum, we get

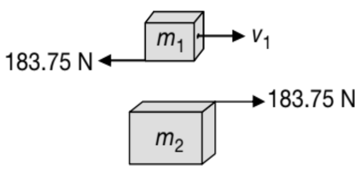
$$mu = \left(m + m_1 + m_2\right)v$$

$$\Rightarrow \quad v = \frac{mu}{m + m_1 + m_2} = \frac{0.25 \times 302}{0.25 + 37.5 + 12.5}$$

$$\Rightarrow \quad v = 1.5 \text{ ms}^{-1}$$

Let v_1 be the velocity of $(m + m_1)$ just after collision. Applying Conservation of Linear Momentum, we get

$$v_1 = \frac{mu}{m + m_1} = \frac{0.25 \times 302}{0.25 \times 37.5} = 2 \text{ ms}^{-1}$$



Force of friction between m_1 and m_2 till it stops is

$$f_1 = \mu m_1 g = 183.75 \text{ N}$$

$$\Rightarrow \quad a_r = \frac{183.75}{m_1} + \frac{183.75}{m_2} = 19.6 \text{ ms}^{-2}$$

Since, $v_r = v_1 = 2$ ms^{-1}

So, displacement of m_1 or m_2 is

$$s_r = \frac{v_r^2}{2a_r} = \frac{(2)^2}{2 \times 19.6} = 0.1 \text{ m}$$

18. When m leaves M, both would be moving at same speed horizontally, so we have

$$mu = (m + M)v$$

$$\Rightarrow \quad v = \frac{mu}{m + M}$$

The mass m will also have a vertical speed v_y due to which it rises to a maximum height H. Applying the Work Energy Theorem, we get

$$\frac{1}{2}mu^2 - mgH = \frac{1}{2}(m + M)\left(\frac{mu}{m + M}\right)^2$$

$$\Rightarrow \quad u^2 - 2gH = \frac{mu^2}{m + M}$$

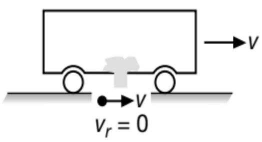
$$\Rightarrow \quad 2gH = u^2 - \frac{mu^2}{m + M} = \frac{Mu^2}{m + M}$$

$$\Rightarrow \quad H = \frac{Mu^2}{2g(M + m)}$$

Test Your Concepts-V (Based on Variable Mass Systems)

1. Since the sand spills through a hole in the bottom of the cart, so the relative velocity of the sand v_r will be zero because it will acquire the same velocity as that of the cart at all the instants. So thrust force is

$$F_t = 0 \qquad \left\{\text{as } F_t = v_r \frac{dm}{dt}\right\}$$



Hence the net force will only be F

$$\Rightarrow \quad F_{\text{net}} = F$$

$$\Rightarrow \quad m\left(\frac{dv}{dt}\right) = F \qquad \ldots(1)$$

But $m = m_0 - \mu t$

$$\Rightarrow \quad \left(m_0 - \mu t\right)\frac{dv}{dt} = F$$

$$\Rightarrow \quad \int_0^v dv = \int_0^t \frac{F dt}{m_0 - \mu t}$$

CHAPTER 2

$$\Rightarrow \quad v=\frac{F}{-\mu}\left[\log_e\left(m_0-\mu t\right)\right]\Big|_0^t$$

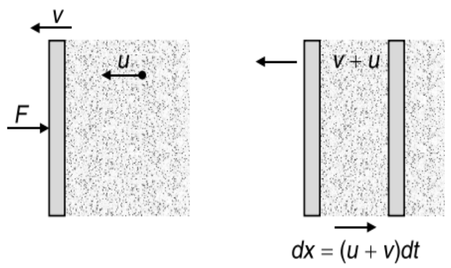
$$\Rightarrow \quad v=\frac{F}{\mu}\log_e\left(\frac{m_0}{m_0-\mu t}\right)$$

From equation (1), we get acceleration of the cart to be

$$a=\frac{dv}{dt}=\frac{F}{m}$$

$$\Rightarrow \quad a=\frac{F}{m_0-\mu t}$$

2. Velocity of plate relative to the dust particles is $(u+v)$, rightwards as shown in Figure.



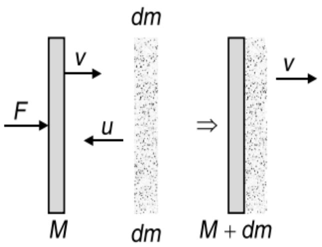
In time dt, the plate comes in contact with dust particles stored in volume given by

$$dV=Adx=A(u+v)dt$$

So, mass of dust striking the plate in time dt is

$$dm=\rho dV=\rho A(v+u)dt \qquad \ldots(1)$$

It we consider a plate of dust of mass dm as a system, and apply Impulse Momentum Theorem over time dt, then we get



$$Fdt=(M+dm)v-(Mv-udm)$$

$$\Rightarrow \quad Fdt=Mv+vdm-Mv+udm$$

$$\Rightarrow \quad F=(u+v)\left(\frac{dm}{dt}\right) \qquad \ldots(2)$$

Substituting equation (1) in (2), we get

$$F=\rho A(v+u)^2$$

3. (a) (i) To just lift the rocket off the launching pad, we have

Weight = Thrust Force

$$\Rightarrow \quad mg=v_r\left(\frac{-dm}{dt}\right)$$

$$\Rightarrow \quad \left(\frac{-dm}{dt}\right)=\frac{mg}{v_r}$$

Substituting the values, we get

$$\left(\frac{-dm}{dt}\right)=\frac{(450+50)(10)}{2\times10^3}=2.5\ \text{kgs}^{-1}$$

(ii) Net acceleration $a=20\ \text{ms}^{-2}$

Since we have $ma=F_t-mg$

$$\Rightarrow \quad a=\frac{F_t}{m}-g$$

$$\Rightarrow \quad a=\frac{v_r}{m}\left(\frac{-dm}{dt}\right)-g$$

$$\Rightarrow \quad \left(-\frac{dm}{dt}\right)=\frac{m(g+a)}{v_r}$$

$$\Rightarrow \quad -\frac{dm}{dt}=\frac{(450+50)(10+20)}{2\times10^3}$$

$$\Rightarrow \quad -\frac{dm}{dt}=7.5\ \text{kgs}^{-1}$$

(b) The rate of fuel consumption is $10\ \text{kgs}^{-1}$. So, the time for the consumption of entire fuel is

$$t=\frac{450}{10}=45\ \text{second}$$

Since, we know that

$$v=u-gt+v_r\log_e\left(\frac{m_0}{m}\right)$$

In this problem, we have

$u=0, v_r=2\times10^3\ \text{ms}^{-1}, m_0=500\ \text{kg}$ and $m=50\ \text{kg}$

Substituting these values, we get,

$$v=-(10)(45)+\left(2\times10^3\right)\log_e\left(\frac{500}{50}\right)$$

$$\Rightarrow \quad v=-450+4606=4156\ \text{ms}^{-1}$$

$$\Rightarrow \quad v=4.156\ \text{kms}^{-1}$$

4. The thrust acting on the container due to ejection of water is bv_0 in the upward direction. Let us first write the equations of motion for mass and container by using the Newton's Second Law.

For mass, we have

$$m_1g-T=m_1a \qquad \ldots(1)$$

For container, we have

$$T+bv_0-(m_0-bt)g=(m_0-bt)a \qquad \ldots(2)$$

Adding equations (1) and (2), we get

$$m_1g+bv_0-m_0g+btg=(m_0+m_1-bt)a$$

$$\Rightarrow \quad a=\frac{(m_1-m_0+bt)g+bv_0}{m_0+m_1-bt}$$

5. For v to remain constant, net pulling force on the chain should be zero. This is possible only when

$$\begin{pmatrix}\text{Thrust}\\ \text{Force}\end{pmatrix}+\begin{pmatrix}\text{Weight}\\ \text{of Left}\\ \text{Side of}\\ \text{Chain}\end{pmatrix}=\begin{pmatrix}\text{Pull}\\ P\end{pmatrix}+\begin{pmatrix}\text{Weight}\\ \text{of Right}\\ \text{Side of}\\ \text{Chain}\end{pmatrix}$$

$$\Rightarrow \quad (\lambda v^2) + (\lambda h)g = P + (\lambda y)g$$

$$\Rightarrow \quad P = \lambda v^2 + \lambda g(h - y)$$

6. Rocket will start to lift when upthrust on rocket will balance its weight. So, we have

$$kv_0 = (M - kt)g$$

$$\Rightarrow \quad t = \frac{Mg - kv_0}{kg} = \frac{M}{k} - \frac{v_0}{g}$$

7. Since, $a = \dfrac{\text{Upward Force}}{\text{Mass}}$

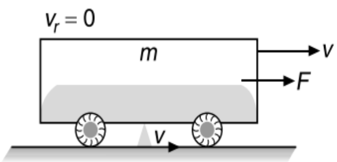
$$\Rightarrow \quad a = \frac{-u\left(\dfrac{dm}{dt}\right)}{m}$$

$$\Rightarrow \quad \frac{dm}{m} = -\left(\frac{a}{u}\right)dt$$

$$\Rightarrow \quad \int_{m_0}^{m} \frac{dm}{m} = -\left(\frac{a}{u}\right)\int_0^t dt$$

$$\Rightarrow \quad m = m_0 e^{-\left(\frac{a}{u}\right)t}$$

8. In this problem the sand spills through a hole at the bottom of the cart. Hence, the relative velocity of the sand v_r will be zero because it will acquire the same velocity as that of the cart at the moment.



Since the thrust force is given by

$$F_{\text{thrust}} = v_r \frac{dm}{dt}$$

$$\Rightarrow \quad F_{\text{thrust}} = 0$$

So, net force equals the external force F, hence

$$F_{\text{net}} = F$$

$$\Rightarrow \quad m\left(\frac{dv}{dt}\right) = F$$

Since, $m = m_0 - \mu t$

$$\Rightarrow \quad (m_0 - \mu t)\frac{dv}{dt} = F$$

$$\Rightarrow \quad \int_0^v dv = \int_0^t \frac{F dt}{m_0 - \mu t}$$

$$\Rightarrow \quad v = -\frac{F}{\mu}\ln(m_0 - \mu t)\Big|_0^t$$

$$\Rightarrow \quad v = \frac{F}{\mu}\ln\left(\frac{m_0}{m_0 - \mu t}\right)$$

Since the acceleration is given by

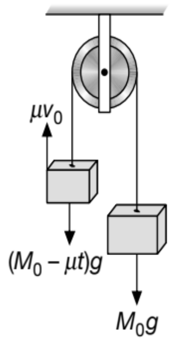
$$a = \frac{dv}{dt} = \frac{F}{m}$$

$$\Rightarrow \quad a = \frac{F}{m_0 - \mu t}$$

9. Net pulley force at time t is,

$$F = \left[M_0 g + v_0\left(\frac{dm}{dt}\right)\right] - \left[M_0 - t\frac{dm}{dt}\right]g$$

$$\Rightarrow \quad F = (M_0 g + \mu v_0) - (M_0 - \mu t)g = \mu(v_0 + tg)$$



Total mass of system at time t is $M = 2M_0 - \mu t$

Since $F = M\dfrac{dv}{dt}$

$$\Rightarrow \quad (2M_0 - \mu t)\frac{dv}{dt} = \mu(v_0 + tg)$$

$$\Rightarrow \quad \int_0^v dv = \mu\int_0^t \left(\frac{v_0 + gt}{2M_0 - \mu t}\right)dt$$

Integrating, we get

$$v = \left(\frac{2M_0 g + \mu v_0}{\mu}\right)\log_e\left(\frac{2M_0}{2M_0 - \mu t}\right) + gt$$

Test Your Concepts-VI (Based on Head-On Collisions)

1. Since $v_1 = \left(\dfrac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\dfrac{(1+e)m_2}{m_1 + m_2}\right)u_2$

and $v_2 = \left(\dfrac{(1+e)m_1}{m_1 + m_2}\right)u_1 + \left(\dfrac{m_2 - em_1}{m_1 + m_2}\right)u_2$

So, for $m_1 = m_2 = m$ (say) and $u_2 = 0$, we have

$$v_1 = \left(\frac{1+e}{2}\right)u \text{ and } v_2 = \left(\frac{1-e}{2}\right)u$$

Given that $K_f = \dfrac{3}{4}K_i$

$\Rightarrow \quad \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{3}{4}\left(\frac{1}{2}mu^2\right)$

Substituting the value, we get

$$\left(\frac{1+e}{2}\right)^2 + \left(\frac{1-e}{2}\right)^2 = \frac{3}{4}$$

$\Rightarrow \quad (1+e)^2 + (1-e)^2 = 3$

$\Rightarrow \quad 2 + 2e^2 = 3$

$\Rightarrow \quad e^2 = \frac{1}{2}$

$\Rightarrow \quad e = \frac{1}{\sqrt{2}}$

2. $u_1 = 12 \text{ ms}^{-1}$

$\Rightarrow \quad m_1 = 4 \text{ kg}$

$\Rightarrow \quad u_2 = 4 \text{ ms}^{-1}$

$\Rightarrow \quad m_2 = 8 \text{ kg}$

Let v_1 and v_2 be the velocities after impact
Conservation of momentum

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$

$\Rightarrow \quad 4v_1 + 8v_2 = 80 \qquad \ldots(1)$

Newton's experimental law

$$v_1 - v_2 = -e(u_1 - u_2)$$

$$v_1 - v_2 = -0.5(12-4) = -4 \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$v_1 = 4 \text{ ms}^{-1},\ v_2 = 8 \text{ ms}^{-1}$$

Loss in kinetic energy is

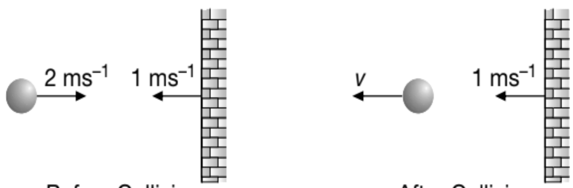
$$-\Delta K = K_i - K_f$$

$\Rightarrow \quad -\Delta K = \frac{1}{2}\left(m_1u_1^2 + m_2u_2^2 - m_1v_1^2 - m_2v_2^2\right)$

$\Rightarrow \quad -\Delta K = \frac{1}{2}[4(144) + 8(16) - 4(16) - 8(64)]$

$\Rightarrow \quad -\Delta K = 64 \text{ J}$

3. The speed of wall will not change after the collision. So, let v be the velocity of the ball after collision in the direction shown in figure. Since collision is elastic $(e=1)$,



Before Collision　　After Collision

$$\left(\begin{array}{c}\text{Velocity of Approach of}\\ \text{Bodies just Before Impact}\end{array}\right) = \left(\begin{array}{c}\text{Velocity of Separation of}\\ \text{Bodies just After Impact}\end{array}\right)$$

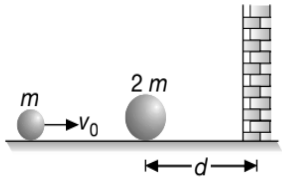
$\Rightarrow \quad 2 + 1 = v - 1$

$\Rightarrow \quad v = 4 \text{ ms}^{-1}$

4. $v_{2m} = \left(\dfrac{m + \dfrac{m}{4}}{m + 2m}\right)v_0 = \dfrac{5v_0}{12}$

$$v_m = \left(\dfrac{m - \dfrac{2m}{4}}{m + 2m}\right)v_0 = \dfrac{v_0}{6}$$

So, $t_1 = \dfrac{d}{\dfrac{5v_0}{12}} = \dfrac{2.4d}{v_0}$



In this time, the ball m will move a distance.

$$s_1 = \left(\frac{v_0}{6}\right)\left(\frac{2.4d}{v_0}\right) = 0.4d$$

Next collision will take place after a time

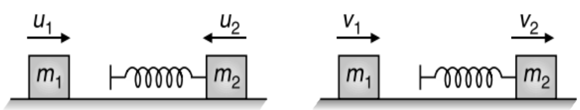
$$t_2 = \frac{0.6d}{\left(\dfrac{v_0}{6}\right) + \left(\dfrac{5v_0}{48}\right)} = \frac{2.22d}{v_0}$$

In this time, the ball $2m$ will move a distance

$$s_2 = \left(\frac{5v_0}{48}\right)\left(\frac{2.22d}{v_0}\right) \text{ from the wall}$$

$\Rightarrow \quad s_2 = 0.23d$

5. If we take the 'block + spring' as a system, no external forces acting on the system. Hence linear momentum of the system will remain conserved. Let the velocities of the blocks when the spring is again relaxed be v_1 and v_2 respectively. Now using conservation of linear momentum



Just before collision　　Just after collision

$$m_1u_1 - m_2u_2 = m_1v_1 + m_2v_2$$

$\Rightarrow \quad 2 \times 4 - 1 \times 2 = 2 \times v_1 + 1 \times v_2$

$\Rightarrow \quad 2v_1 + v_2 = 6 \qquad \ldots(1)$

The collision of the spring block system can be treated as perfectly elastic collision. Hence, we can use the coefficient of restitution equation

$$e = \frac{v_2 - v_1}{u_1 - u_2} = 1$$

where, $e = 1$, $u_1 = 4 \text{ ms}^{-1}$, $u_2 = -2 \text{ ms}^{-1}$

$$\Rightarrow \quad 1 = \frac{v_2 - v_1}{4-(-2)}$$

$$\Rightarrow \quad v_2 - v_1 = 6 \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$v_1 = 0 \text{ and } v_2 = 6 \text{ ms}^{-1}$$

6. Since the collision is elastic between the blocks, the two identical blocks will exchange their velocities. First block will come to rest while the second will start moving with velocity v_0. The retardation in this block is μg. Hence, the velocity of this block before colliding with the wall will be given by

$$v^2 = v_0^2 - 2\mu g d$$

$$\Rightarrow \quad v = \sqrt{v_0^2 - 2\mu g d} \qquad \ldots(1)$$

Now, let v' be the velocity of this block in opposite direction after colliding with the wall, then as the collision is elastic $(e=1)$, hence, relative speed of separation is equal to the relative speed of approach.

$$v' - u = v + u$$

$$\Rightarrow \quad v' = (2u + v) \qquad \ldots(2)$$

Now, to avoid the second collision between the blocks

$$0 = (v')^2 - 2\mu g d$$

$$\Rightarrow \quad (2u + v) = \sqrt{2\mu g d}$$

$$\Rightarrow \quad u = \frac{\sqrt{2\mu g d} - v}{2} = \frac{\sqrt{2\mu g d} - \sqrt{v_0^2 - 2\mu g d}}{2}$$

Therefore, the maximum values of u is

$$\frac{\sqrt{2\mu g d} - \sqrt{v_0^2 - 2\mu g d}}{2}$$

7. Let velocities of sphere 1 and 2 after collision be v_1 and v_2 respectively, then by conservation of linear momentum, we have

$$mv_2 + mv_1 = mu \qquad \ldots(1)$$

Also, $e = \frac{v_2 - v_1}{u}$ ...(2)

From equations (1) and (2), we get

$$v_2 = \left(\frac{1+e}{2}\right)u$$

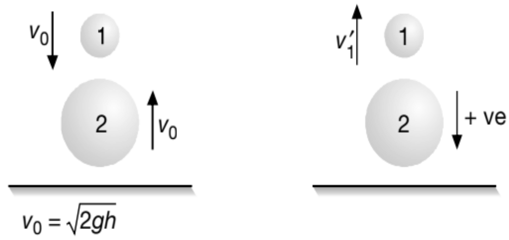
Now, when sphere 2 collides with sphere 3, then after collision velocity of 3 will be

$$v_3 = \left(\frac{1+e}{2}\right)^2 u \text{ and so on.}$$

Similarly, velocity of nth sphere will be

$$v_n = \left(\frac{1+e}{2}\right)^{n-1} u$$

8. Let us assume that the balls are slightly separated when the superball hits the floor.
Let v_1 be the velocity of small ball (1) just after collision.
Let v_1' be the velocity of small ball 1 just after collision with big ball 2. Then,



$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2$$

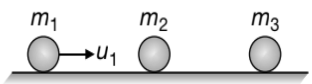
where $v_1 = v_0$, $v_2 = -v_0$ and $\frac{m_1}{m_2} = \frac{m}{M} \approx 0$, we get

$$v_1' = -3v_0 = -3\sqrt{2gh}$$

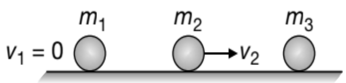
So, height gained by small ball after collision, is

$$h_1 = \frac{v_1'^2}{2g} = \frac{9(2gh)}{2g} = 9h$$

9. After first impact $v_1 = 0$



Before first impact

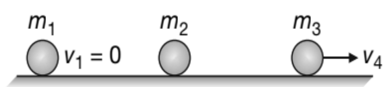


After first impact

$$\Rightarrow \quad v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 = 0$$

$$\Rightarrow \quad m_1 = em_2 \qquad \ldots(1)$$

After second impact, $v_3 = 0$



After second impact

$$\Rightarrow \quad \left(\frac{m_2 - em_3}{m_2 + m_3}\right)v_2 = 0$$

$$\Rightarrow \quad m_2 = em_3 \qquad \ldots(2)$$

From equations (1) and (2), we get

$$m_2 = \sqrt{m_1 m_3}$$

10. By Law of Conservation of Mechanical Energy, we get

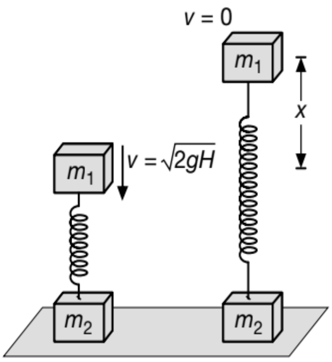
$$\frac{1}{2}m_1 v^2 = \frac{1}{2}kx^2 + m_1 g x$$

Since, $v = \sqrt{2gH}$

$$\Rightarrow \quad 2m_1gH = kx^2 + 2m_1gx \qquad \ldots(1)$$

The lower block will rebounce only when

$$x > \frac{m_2g}{k} \qquad \{\because kx = m_2g\}$$



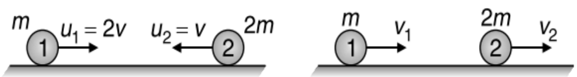
Substituting, $x = \dfrac{m_2g}{k}$ in Equation (1), we get

$$2m_1gH = k\left(\frac{m_2g}{k}\right)^2 + 2m_1g\left(\frac{m_2g}{k}\right)$$

$$\Rightarrow \quad H = \frac{m_2g}{k}\left(\frac{m_2 + 2m_1}{2m_1}\right)$$

$$\Rightarrow \quad H_{\text{min}} = \frac{m_2g}{k}\left(\frac{m_2 + 2m_1}{2m_1}\right)$$

11. Let us assume the final velocities of the particles m and $2m$ be v_1 and v_2, respectively, as shown in Figure.



Just before collision Just after collision

Applying conservation of linear momentum, we get

$$m(2v) + 2m(-v) = m(v_1) + 2m(v_2)$$

$$\Rightarrow \quad 0 = mv_1 + 2mv_2$$

$$\Rightarrow \quad v_1 + 2v_2 = 0 \qquad \ldots(1)$$

Since collision is elastic, so relative velocity of approach of bodies equals the relative velocity of separation of bodies. Hence, we get

$$v_2 - v_1 = 2v - (-v)$$

$$\Rightarrow \quad v_2 - v_1 = 3v \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$v_1 = -2v \text{ and } v_2 = v$$

This simply means that both masses reverse their direction of motion after the collision as shown in Figure.

12. Consider the two blocks plus the spring to be the system. No external force acts on this system in horizontal direction. Hence, the linear momentum will remain constant. Suppose, the block of mass M moves with a speed V and the other block with a speed v after losing contact with the spring. From Law of Conservation of Linear Momentum in horizontal direction, we get

$$MV - mv = 0$$

$$\Rightarrow \quad V = \frac{m}{M}v \qquad \ldots(1)$$

The initial energy of the system is $E_i = \dfrac{1}{2}kx^2$

The final energy of the system is

$$E_f = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

Since there is no friction so, mechanical energy will remain conserved.

$$\Rightarrow \quad \frac{1}{2}mv^2 + \frac{1}{2}MV^2 = \frac{1}{2}kx^2 \qquad \ldots(2)$$

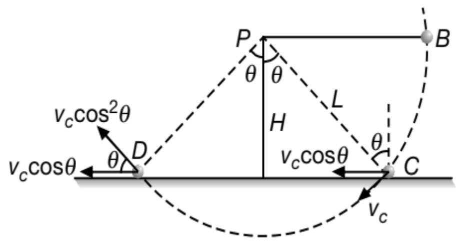
Solving equations (1) and (2), we get

$$v = x\sqrt{\frac{kM}{m(M+m)}} \quad \text{and}$$

$$V = x\sqrt{\frac{km}{M(M+m)}}$$

Test Your Concepts-VII (Based on Oblique Collisions)

1. In perfectly inelastic collision with the horizontal surface, the component parallel to the surface will remain unchanged.



$$\cos\theta = \frac{H}{L}$$

By Law of Conservation of Energy, we have

$$v_c = \sqrt{2gH}$$

Similarly, when the string becomes taut again, the component perpendicular to its length will remain unchanged. So,

$$v_c\cos^2\theta = \left(\sqrt{2gH}\right)\frac{H^2}{L^2} = v \qquad \{\text{say}\}$$

Again, using Law of Conservation of Energy, we get

$$h = \frac{v^2}{2g} = \frac{(2gH)\dfrac{H^4}{L^4}}{2g} = \frac{H^5}{L^4}$$

2. Let $\vec{v}$ be the velocity of the sphere after impact. To find $\vec{v}$ we must separate the velocity components parallel and perpendicular to the wall.
Since, after impact the component of velocity parallel to the wall remains unchanged while component perpendicular to the wall becomes e times in opposite direction. So, we have

$$\vec{v} = -\frac{3}{2}\hat{i} + \hat{j}$$

Therefore, the velocity of the sphere after impact is $-\frac{3}{2}\hat{i} + \hat{j}$

The loss in kinetic energy is

$$-\Delta K = K_i - K_f$$

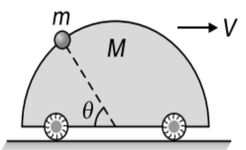
$$\Rightarrow \quad -\Delta K = \frac{1}{2}m\left(3^2 + 1^2 - \left(\frac{3}{2}\right)^2 - 1^2\right) = \frac{27m}{8}\text{ J}$$

Since impulse $\vec{J} = \Delta\vec{p}$, so we have

$$\vec{J} = m\left(-\frac{3}{2}\hat{i} + \hat{j}\right) - m\left(3\hat{i} + \hat{j}\right) = -\left(\frac{9m}{2}\right)\hat{i}$$

3. Since the collision is perfectly inelastic, so the putty wedge system of mass $(M+m)$ moves as a single body with a velocity V. Applying the conservation of linear momentum, we get

$$mv_0\cos\theta = (M+m)V$$



$$\Rightarrow \quad V = \frac{mv_0\cos\theta}{M+m}$$

The loss in kinetic energy of the system is

$$\text{Loss} = -\Delta K = K_i - K_f$$

$$\Rightarrow \quad -\Delta K = \frac{1}{2}mv_0^2 - \frac{1}{2}(M+m)V^2$$

where, $V = \dfrac{mv_0\cos\theta}{M+m}$

$$\Rightarrow \quad -\Delta K = \frac{1}{2}mv_0^2 - \frac{1}{2}(M+m)\left(\frac{mv_0\cos\theta}{M+m}\right)^2$$

$$\Rightarrow \quad -\Delta K = \frac{mv_0^2}{2}\left(1 - \frac{m\cos^2\theta}{M+m}\right)$$

$$\Rightarrow \quad -\Delta K = \frac{mv_0^2}{2}\left(\frac{M+m\sin^2\theta}{M+m}\right)$$

4. Since impulse equals change in momentum, so we have

$$\vec{J} = m\left(\hat{i} + 3\hat{j}\right) - m\left(4\hat{i} - \hat{j}\right)$$

$$\Rightarrow \quad \vec{J} = m\left(-3\hat{i} + 4\hat{j}\right)$$

To find the coefficient of restitution we require the velocity components, before and after impact, in the direction of impulse i.e., along the line of impact, i.e., in the direction $-3\hat{i} + 4\hat{j}$. The unit vector in the direction of $\vec{J}$ is $\frac{1}{5}\left(-3\hat{i} + 4\hat{j}\right)$. So, the magnitudes of the velocity components in this direction just before impact is

$$\left(4\hat{i} - \hat{j}\right)\cdot\frac{1}{5}\left(-3\hat{i} + 4\hat{j}\right) = -\frac{16}{5}$$

and just after impact is

$$\left(\hat{i} + 3\hat{j}\right)\cdot\frac{1}{5}\left(-3\hat{i} + 4\hat{j}\right) = \frac{9}{5}$$

The significance of the negative sign in $-\frac{16}{5}$ is simply an indication that this component is in a direction opposite to that of $\vec{J}$.
The speed of approach to the wall is therefore, $\frac{16}{5}$ and the speed of separation is $\frac{9}{5}$. Since

$$e = \frac{\text{Speed of Separation}}{\text{Speed of Approach}}$$

$$\Rightarrow \quad e = \frac{9/5}{16/5} = \frac{9}{16}$$

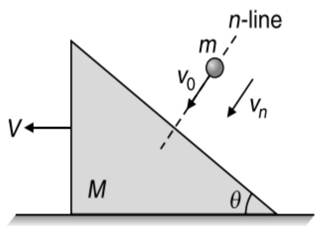
5. Applying Law of Conservation of Linear Momentum in vector form, we get

$$mu\hat{i} = 0 + mv\hat{j} + m\vec{v}'$$

$$\Rightarrow \quad \vec{v}' = u\hat{i} - v\hat{j}$$

$$\Rightarrow \quad |\vec{v}'| = \sqrt{u^2 + v^2}$$

6. **Method I: By using conservation of linear momentum**
If we take wedge and ball as a system, then no external force is acting along the horizontal direction, so linear momentum is conserved along the horizontal direction. Since the ball is moving along the normal direction, so it has no component of velocity along the inclined surface, i.e. the common tangent direction or along the t-line.



Let velocity of the ball and wedge after collision be v_n and V respectively, then by conservation of linear momentum applied along the horizontal direction, we have

$$mv_0\sin\theta = MV + mv_n\sin\theta \qquad \ldots(1)$$

Also, $v_2 - v_1 = e(u_1 - u_2)$

$$\Rightarrow \quad V\sin\theta - v_n = e(v_0 - 0) \qquad \ldots(2)$$

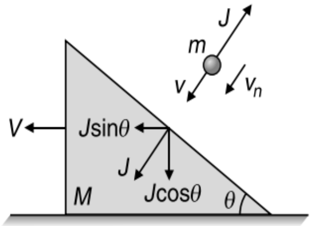
Solving equations (1) and (2), we get

$$V = \left[\frac{(1+e)m\sin\theta}{M + m\sin^2\theta}\right]v_0$$

and $v_n = V\sin\theta - ev_0 = \left(\dfrac{m\sin^2\theta - Me}{M + m\sin^2\theta}\right)v_0$

Method 2: By using the impulse method

The impulse acting on the ball and wedge is shown in Figure.



Since impulse equals the change in momentum of the body, so

for the wedge, we have

$$J\sin\theta = MV \qquad \ldots(1)$$

for the ball, we have

$$-J = mv_n - mv_0 \qquad \ldots(2)$$

Also, $v_2 - v_1 = e(u_1 - u_2)$

$$\Rightarrow \quad V\sin\theta - v_n = e(v_0 - 0) \qquad \ldots(3)$$

Solving equations (1), (2) and (3), we get

$$V = \left[\frac{(1+e)m\sin\theta}{M + m\sin^2\theta}\right]v_0 \text{ and}$$

$$v_n = \left(\frac{m\sin^2\theta - Me}{M + m\sin^2\theta}\right)v_0$$

7. Vertical component of velocity and hence the time of flight becomes e times. Since,

$$\text{Range} = u_H T$$

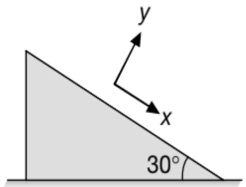
where, u_H is the horizontal component of velocity which remains unchanged due to collision. Hence the new range will become e times i.e.,

$$L_2 = eL_1$$

8. $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{4 + \left(\dfrac{R}{\sqrt{3}}\right)}{g}}$

Since, $R = v_A t$

$$\Rightarrow \quad R = 2\sqrt{\frac{4 + \left(\dfrac{R}{\sqrt{3}}\right)}{g}}$$



Squaring and solving, we get

$$R \approx 2 \text{ m} \text{ and } t \approx 1 \text{ s}$$

Now, $u_x = 2\cos(30°) = \sqrt{3}\ \text{ms}^{-1}$ and

$$u_y = 2\sin(30°) = 1\ \text{ms}^{-2}$$

$$a_x = g\sin(30°) = 4.9\ \text{ms}^{-2} \text{ and}$$

$$a_y = -g\cos 30° = -4.9\sqrt{3}\ \text{ms}^{-2}$$

Speed after bouncing is

$$v = \sqrt{v_x^2 + (ev_y)^2}$$

$$\Rightarrow \quad v = \sqrt{(\sqrt{3} + 4.9\times 1)^2 + (0.6(1 - 4.9\sqrt{3}\times 1))^2}$$

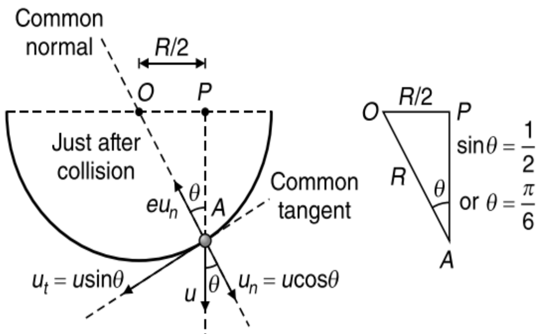
$$\Rightarrow \quad v \cong 8\ \text{ms}^{-1}$$

9. If after collision, the particle moves along the track, then this means that there is no normal component of velocity and hence $e = 0$.

Just before collision, the components of velocity along the normal and along the tangent are

$$u_n = u\cos\theta \text{ and } u_t = u\sin\theta$$

When the velocity of particle becomes horizontal just after collision with the track, then we have



The component of velocity along the common tangent direction will remain unchanged because in this direction there is no impulse. Hence

$$v_t = u_t = u\sin\theta \qquad \ldots(1)$$

Also, speed of the particle just after impact along the n-line is e times the speed of particle before collision along the n-line.

$$v_n = eu_n = eu\cos\theta \qquad \ldots(2)$$

Since after collision, the particle moves in horizontal direction, so vertical component of particle velocity should be zero. For this condition to be obeyed, we have

$$eu_n \cos\theta = v_t \sin\theta \quad \ldots(3)$$

Solving equations (1), (2) and (3), we get

$$(eu\cos\theta)\cos\theta = (u\sin\theta)\sin\theta$$

$$\Rightarrow \quad e = \tan^2\theta = \tan^2\left(\frac{\pi}{6}\right) = \frac{1}{3}$$

10. (a) $u_r = 0$, $a_r = g$

$$\Rightarrow \quad v_r = \sqrt{2gh_1}$$

After collision relative velocity $v_r' = e\sqrt{2gh_1}$ and relative retardation is still g (downwards). So,

$$h_2 = \frac{(v_r')^2}{2g} = e^2 h_1$$

(b) $u_r = 0$, $a_r = g + \frac{g}{4} = \frac{5g}{4}$

So, just before collision, we have

$$v_r = \sqrt{2\left(\frac{5g}{4}\right)h_1}$$

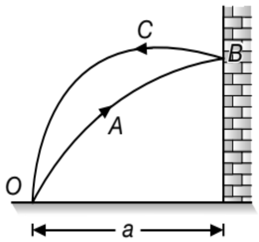
Just after collision, $v_r' = ev_r$

Relative retardation is still $\frac{5g}{4}$

Hence, $h_2 = \dfrac{(v_r')^2}{2\left(\frac{5g}{4}\right)} = e^2 h_1$

11. The horizontal component of the velocity of ball during the path OAB is $u\cos\alpha$ while in its return journey BCO it is $eu\cos\alpha$. The time of flight T also remains unchanged. Hence,

$$T = t_{OAB} + t_{BCO}$$



$$\Rightarrow \quad \frac{2u\sin\alpha}{g} = \frac{a}{u\cos\alpha} + \frac{a}{eu\cos\alpha}$$

$$\Rightarrow \quad \frac{a}{eu\cos\alpha} = \frac{2u\sin\alpha}{g} - \frac{a}{u\cos\alpha}$$

$$\Rightarrow \quad \frac{a}{eu\cos\alpha} = \frac{2u^2\sin\alpha\cos\alpha - ag}{gu\cos\alpha}$$

$$\Rightarrow \quad e = \frac{ag}{2u^2\sin\alpha\cos\alpha - ag}$$

$$\Rightarrow \quad e = \frac{1}{\left(\frac{u^2\sin(2\alpha)}{ag} - 1\right)}$$

Single Correct Choice Type Questions

1. Horizontal and vertical components of initial velocity are

$$u_x = 20\sqrt{2}\cos 45° = 20 \text{ ms}^{-1}$$

and $u_y = 20\sqrt{2}\sin 45° = 20 \text{ ms}^{-1}$

After 1 s, horizontal component remains unchanged while the vertical component becomes

$$v_y = u_y - gt$$

$$\Rightarrow \quad v_y = 20 - (10)(1) = 10 \text{ ms}^{-1}$$

Due to explosion one part comes to rest. Hence, from Conservation of Linear Momentum, vertical component of second part will become $v_y' = 20 \text{ ms}^{-1}$. Therefore, maximum height attained by the second part will be

$$H = h_1 + h_2$$

Where, h_1 be the height attained in 1 s.

$$\Rightarrow \quad h_1 = (20)(1) - \frac{1}{2}(10)(1)^2 = 15 \text{ m}$$

and h_2 be the height attained after 1 s.

$$\Rightarrow \quad h_2 = \frac{v_y'^2}{2g} = \frac{(20)^2}{2\times 10} = 20 \text{ m}$$

$$\Rightarrow \quad H = 20 + 15 = 35 \text{ m}$$

Hence, the correct answer is (B).

2. Let speed of block be V. Then by Law of Conservation of Linear Momentum (applied along the horizontal direction), velocity of cylinder will be v in opposite direction, then

$$mv - MV = 0$$

$$\Rightarrow \quad \frac{Mv}{2} = MV$$

$$\Rightarrow \quad v = 2V$$

By Law of Conservation of Mechanical Energy, we have

$$mgh = \frac{1}{2}MV^2 + \frac{1}{2}m(2V)^2$$

where, $h = R - r = 1$ m

Substituting the values, we get

$$(1)(10)(1) = \frac{1}{2}(2)(V^2) + \frac{1}{2}(1)(4V^2)$$

$$\Rightarrow \quad 3v^2 = 10$$

$$\Rightarrow \quad v = \sqrt{\frac{10}{3}} \text{ ms}^{-1}$$

Hence, the correct answer is (A).

3. Since centre of mass of 1 g, 2 g and 3 g is at $(2, 2, 2)$

$\Rightarrow$ a mass of $(1+2+3)$ g $= 6$ g is placed at $(2, 2, 2)$.

CHAPTER 2

Let the new mass of 4 g be placed at $\vec{r}$, such that the new centre of mass is new at origin $(0, 0, 0)$. Hence

$$(0,0,0)=\frac{6(2,\ 2,\ 2)+4\vec{r}}{10}$$

$\Rightarrow\quad \vec{r}=(-3,\ -3,\ -3)$

Hence, the correct answer is (C).

4. Let the displacement of wedge be x (leftwards). Horizontal displacement of A and B with respect to wedge is $10\cos 45°$ or $5\sqrt{2}$ cm (rightwards) or the horizontal displacement of A and B with respect to ground is $(5\sqrt{2}-x)$ cm rightwards. The centre of mass of the whole system will not move in horizontal direction. So,

$$(2m)x=(5\sqrt{2}-x)(m+2m)$$

$\Rightarrow\quad 5mx=15\sqrt{2}$ m

$\Rightarrow\quad x=3\sqrt{2}$ cm

Hence, the correct answer is (B).

5. $m=m_0e^{-\lambda t}$

$\Rightarrow\quad \left(-\frac{dm}{dt}\right)=m_0\lambda e^{-\lambda t}$

Now thrust force $F=u\left(-\frac{dm}{dt}\right)=um_0\lambda e^{-\lambda t}$

$\Rightarrow\quad m\left(\frac{dv}{dt}\right)=um_0\lambda e^{-\lambda t}$

$\Rightarrow\quad (m_0e^{-\lambda t})\frac{dv}{dt}=um_0\lambda e^{-\lambda t}$

$\Rightarrow\quad dv=u\lambda dt$

$\Rightarrow\quad \int_0^v dv=u\lambda\int_0^t dt$

$\Rightarrow\quad v=u\lambda t$

Hence, the correct answer is (B).

6. By Work-Energy Theorem

$$\frac{1}{2}m(v^2-u^2)=Fs \qquad \ldots(1)$$

For the second case

$$\frac{1}{2}m(v'^2-u'^2)=Fs \qquad \ldots(2)$$

where F is the retarding force offered by the plank to the bullet.

Equating (1) and (2), we get

$$v^2-u^2=v'^2-u'^2$$

$\Rightarrow\quad (150)^2-(125)^2=v'^2-(90)^2$

$\Rightarrow\quad v'^2=(35)^2$

$\Rightarrow\quad v'=35\ \text{ms}^{-1}$

Hence, the correct answer is (B).

7. Let J be the impulse and v the common speed. Since, Impulse = Change in Linear Momentum

So, for mass m, we have

$$J=mv_0-mv \qquad \ldots(1)$$

where, $v_0=\sqrt{2gh}$

and for mass 2m, we have

$$J=2mv \qquad \ldots(2)$$

From equations (1) and (2), we get

$$v=\frac{\sqrt{2gh}}{3}$$

Hence, the correct answer is (D).

8. Before explosion, particle was moving along x-axis, i.e., it has no y-component of velocity. Therefore, the centre of mass will not move in y-direction

$\Rightarrow\quad y_{cm}=0$

$\Rightarrow\quad y_{cm}=\frac{m_1y_1+m_2y_2}{m_1+m_2}$

$\Rightarrow\quad 0=\frac{\left(\frac{m}{4}\right)(+15)+\left(\frac{3m}{4}\right)(y)}{\left(\frac{m}{4}+\frac{3m}{4}\right)}$

$\Rightarrow\quad y=-5$ cm

Hence, the correct answer is (A).

9. Since $\vec{F}_{ext}\neq 0$

$\vec{a}_{cm}=a_o,\ \vec{v}_{cm}=v_o$ at any time t.

Since $\vec{a}_{cm}=\frac{\vec{F}_{ext}}{M}$ so $a_0\neq 0$

$v_0=0$ and $a_0\neq 0$

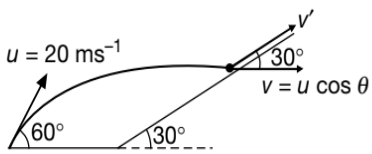
Hence, the correct answer is (B).

10. Speed of ball before collision is

$$v=u\cos 60°=(20)\left(\frac{1}{2}\right)=10\ \text{ms}^{-1}$$

Since, collision is perfectly inelastic $(e=0)$, the ball will not bounce. It will move along the plane with velocity

$$v'=v\cos 30°=\frac{10\sqrt{3}}{2}=5\sqrt{3}\ \text{ms}^{-1}$$



Maximum height attained by the ball is

$$H=\frac{u^2\sin^2 60°}{2g}+\frac{v'^2}{2g}$$

$$\Rightarrow \quad H = \frac{(20)^2\left(\frac{\sqrt{3}}{2}\right)^2}{20} + \frac{(5\sqrt{3})^2}{20}$$

$$\Rightarrow \quad H = \frac{300}{20} + \frac{75}{20} = \frac{375}{20} = 18.75 \text{ m}$$

Hence, the correct answer is (B).

11. Retardation due to friction

$$a = \mu g = (0.25)(10) = 2.5 \text{ ms}^{-2}$$

Since collision is elastic, i.e., after collision first block comes to rest and the second block acquires the velocity of first block. Distance travelled by it will be

$$s = \frac{v^2}{2a} = \frac{(5)^2}{(2)(2.5)} = 5 \text{ m}$$

So, final separation x is

$$x = 5 - 2 = 3 \text{ m}$$

Hence, the correct answer is (C).

12. In perfectly inelastic collision between two particles, linear momentum is conserved. If θ be the angle between the velocities of the two particles before collision, then

$$p^2 = p_1^2 + p_2^2 + 2p_1p_2\cos\theta$$

$$\Rightarrow \quad \left(2m\frac{v}{2}\right)^2 = (mv)^2 + (mv)^2 + 2(mv)(mv)\cos\theta$$

$$\Rightarrow \quad 1 = 1 + 1 + 2\cos\theta$$

$$\Rightarrow \quad \cos\theta = -\frac{1}{2}$$

$$\Rightarrow \quad \theta = 120°$$

Hence, the correct answer is (D).

13. In one dimensional collision of two particles, the velocities are interchanged when collision is elastic and masses are equal.

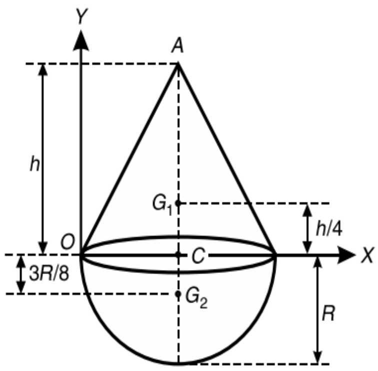
Hence, the correct answer is (A).

14. $M_1 = \text{Mass of cone} = \frac{1}{3}\pi R^2 h\rho$

$M_2 = \text{Mass of hemisphere} = \frac{2}{3}\pi R^3 \rho$

Since $CG_1 = \frac{h}{4}$ therefore $AG_1 = \frac{3h}{4}$

Also $CG_2 = \frac{3R}{8}$



The position vectors of points G_1 and G_2 are

$$\vec{r}_1 = R\hat{i} + \frac{h}{4}\hat{j} \text{ and}$$

$$\vec{r}_2 = R\hat{i} + \frac{3R}{8}(-\hat{j}) = R\hat{i} - \frac{3R}{8}\hat{j}$$

According to the problem the centre of mass lies at C and hence $\vec{r}_{cm} = R\hat{i}$

Since $\vec{r}_{cm} = \dfrac{M_1\vec{r}_1 + M_2\vec{r}_2}{M_1 + M_2}$

Substituting values to get $\frac{h}{R} = \sqrt{3}$

Hence, the correct answer is (A).

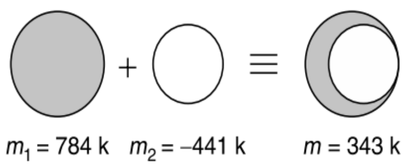
15. $\begin{pmatrix}\text{Kinetic energy}\\ \text{of a system}\\ \text{of particles}\end{pmatrix} = \begin{pmatrix}\text{KE of}\\ \text{centre of}\\ \text{mass}\end{pmatrix} + \begin{pmatrix}\text{KE of different}\\ \text{particles in the}\\ \text{frame of reference}\\ \text{of centre of mass}\end{pmatrix}$

Here, KE of centre of mass is $\frac{1}{2}mv^2$

So, KE of the system of particles $\geq \frac{1}{2}mv^2$

Hence, the correct answer is (C).

16. This problem can be done by using the concept of "Negative Mass being added to a system".



$$r_{cm} = \frac{(784k)(0) + (-441k)7}{784k + (-441k)}$$

$$\Rightarrow \quad r_{cm} = 9 \text{ cm}$$

Hence, the correct answer is (B).

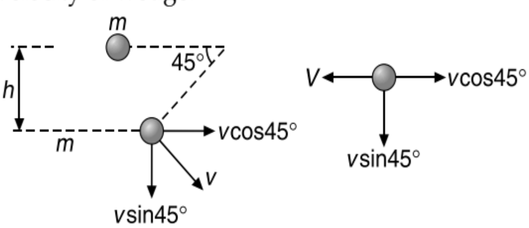
17. Fraction of energy retained $= \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)^2$

$$\Rightarrow \quad \text{Fraction} = \left(\frac{m_{\text{nucleus}} - m_n}{m_{\text{nucleus}} + m_n}\right)^2$$

$$\Rightarrow \quad \text{Fraction} = \left(\frac{Am_n - m_n}{Am_n + m_n}\right)^2 = \left(\frac{A-1}{A+1}\right)^2$$

Hence, the correct answer is (B).

18. Let v be the velocity of ball w.r.t. wedge and V be the velocity of wedge.



Applying conservation of linear momentum, we get

$$mV = m(v\cos 45^\circ - V)$$

$$\Rightarrow \quad V = \frac{v}{2\sqrt{2}}$$

$$\Rightarrow \quad v = 2\sqrt{2}V$$

Applying conservation of energy, we get

$$\frac{1}{2}m\left[(v\cos 45^\circ - V)^2 + (v\sin 45^\circ)^2 + V^2\right] = mgh$$

$$\Rightarrow \quad \left(\frac{v}{\sqrt{2}} - V\right)^2 + \left(\frac{v}{\sqrt{2}}\right)^2 + V^2 = 2gh$$

$$\Rightarrow \quad \left(\frac{2\sqrt{2}V}{\sqrt{2}} - V\right)^2 + \left(\frac{2\sqrt{2}V}{\sqrt{2}}\right)^2 + V^2 = 2g\left(\frac{g}{\sqrt{2}}\right)$$

$$\Rightarrow \quad V^2 + 4V^2 + V^2 = gR\sqrt{2}$$

$$\Rightarrow \quad 6V^2 = gR\sqrt{2}$$

$$\Rightarrow \quad V = \sqrt{\frac{gR}{6}\sqrt{2}}$$

$$\Rightarrow \quad V = \sqrt{\frac{gR}{3\sqrt{2}}}$$

Hence, the correct answer is (A).

19. By Law of Conservation of Momentum

along *x*-axis

$$mv = 2mV\cos\theta$$

$$\Rightarrow \quad V\cos\theta = \frac{v}{2} \qquad \ldots(1)$$

along *y*-axis

$$mv = 2mV\sin\theta$$

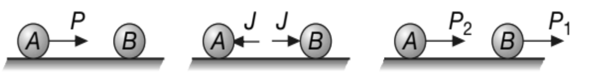
$$\Rightarrow \quad V\sin\theta = \frac{v}{2} \qquad \ldots(2)$$

Squaring (1) and (2) and adding, we get

$$V = \frac{v}{\sqrt{2}}$$

Hence, the correct answer is (C).

20. Let P_1 and P_2 be the momenta of A and B after collision.



Before collision After collision

Since, Impulse = Change in Linear Momentum, so we get

For B: $J = P_1 \qquad \ldots(1)$

For A: $J = P - P_2$

$$\Rightarrow \quad P_2 = P - J \qquad \ldots(2)$$

Coefficient of restitution $e = \dfrac{P_1 - P_2}{P}$

$$\Rightarrow \quad e = \frac{P_1 - P + J}{P}$$

$$\Rightarrow \quad e = \frac{J - P + J}{P} = \frac{2J}{P} - 1$$

Hence, the correct answer is (B).

21. $e = \dfrac{v_1' - v_2'}{u_2 - u_1} = \dfrac{v_1 - v_2}{u - 3u}$

$$\Rightarrow \quad 2eu = v_2 - v_1$$

Hence, the correct answer is (C).

22. $\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$

Since $m_1 = m_2 = m$ {say}

$$\Rightarrow \quad \vec{v}_{cm} = \frac{\vec{v}_1 + \vec{v}_2}{2} \qquad \{m_1 = m_2\}$$

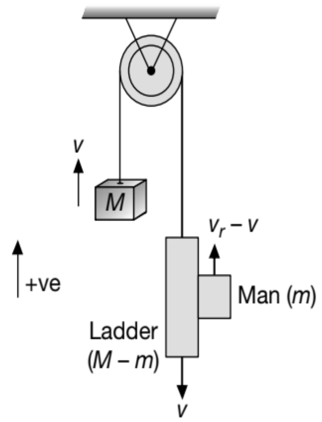
$$\Rightarrow \quad \vec{v}_{cm} = (\hat{i} + \hat{j})\ \text{ms}^{-1}$$

Similarly, $\vec{a}_{cm} = \dfrac{\vec{a}_1 + \vec{a}_2}{2} = \dfrac{3}{2}(\hat{i} + \hat{j})\ \text{ms}^{-2}$

Since, $\vec{v}_{cm}$ is parallel to $\vec{a}_{cm}$, so the path will be a straight line.

Hence, the correct answer is (C).

23. The rope tension is the same both on the left and right hand side at every instant, and consequently momentum of both sides are equal



$$\Rightarrow \quad Mv = (M - m)(-v) + m(v_r - v)$$

$$\Rightarrow \quad v = \frac{m}{2M}v_r$$

Momentum of the centre of mass is

$$P = P_1 + P_2$$

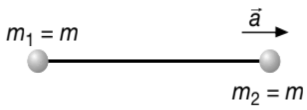
$$\Rightarrow \quad 2Mv_{com} = Mv + Mv$$

$$\Rightarrow \quad v_{com} = v = \frac{m}{2M}v_r$$

Hence, the correct answer is (B).

24. $\vec{a}_{cm} = \dfrac{m(0) + m\vec{a}}{m + m}$

$\vec{a}_{cm} = \dfrac{\vec{a}}{2}$



Hence, the correct answer is (B).

25. At maximum extension, velocity of both the blocks will be same. Let v be the common velocity of the blocks (towards right). By Law of Conservation of Linear Momentum, we get

$6(2) + 3(-1) = (3 + 6)\, v$

$\Rightarrow \quad v = 1 \text{ ms}^{-1}$

If x be the maximum extension in the spring, then by Law of Conservation of Mechanical Energy, we get

$\frac{1}{2}(3)(1)^2 + \frac{1}{2}(6)(2)^2 = \frac{1}{2}(200)x^2 + \frac{1}{2}(9)(1)^2$

$\Rightarrow \quad 3 + 24 = 200x^2 + 9$

$\Rightarrow \quad x = \sqrt{\dfrac{18}{200}}$

$\Rightarrow \quad x = 0.3 \text{ m}$

$\Rightarrow \quad x = 30 \text{ cm}$

Hence, the correct answer is (D).

26. Since, both have equal mass and collision is elastic, so pendulum will have a velocity v after collision

By Law of Conservation of Energy

$\frac{1}{2}mv^2 = mgh$

$\Rightarrow \quad h = \dfrac{v^2}{2g}$

Hence, the correct answer is (B).

27. For completely inelastic collision, the ball will stick to the pendulum

$\Rightarrow \quad mv = (m + m)V$

$\Rightarrow \quad V = \dfrac{v}{2}$

$\Rightarrow \quad h = \dfrac{V^2}{2g} = \dfrac{v^2}{8g}$

Hence, the correct answer is (D).

28. In an elastic oblique collision if two particles are of equal masses and the second particle is at rest, then after collisions the particles scatter at right angles.

Hence, the correct answer is (A).

29. On breaking off from the wall the speed of m_1 is zero and the spring is unstretched. If m_2 has a speed v_2 at that instant, then by Law of Conservation of Energy.

$\begin{pmatrix}\text{Loss in Elastic} \\ \text{Potential Energy}\end{pmatrix} = \begin{pmatrix}\text{Gain in Kinetic} \\ \text{Energy of } m_2\end{pmatrix}$

$\Rightarrow \quad \frac{1}{2}kx^2 = \frac{1}{2}m_2v_2^2$

$\Rightarrow \quad \frac{1}{2}(5)\left(\frac{4}{100}\right)^2 = \frac{1}{2}(5)v_2^2$

$\Rightarrow \quad v_2 = 0.04 \text{ ms}^{-1}$

So, $v_{CM} = \dfrac{m_1v_1 + m_2v_2}{m_1 + m_2} = \dfrac{0 + 5(0.04)}{10 + 5} = \dfrac{0.04}{3} \text{ ms}^{-1}$

$\Rightarrow \quad v_{CM} = \dfrac{4}{3} \text{ cms}^{-1}$

Hence, the correct answer is (B).

30. $v_1 = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)u_1$

Since, $m_1 = \frac{4}{3}\pi r_1^3$ and $m_2 = \frac{4}{3}\pi r_2^3$

$\Rightarrow \quad v_1 = \left(\dfrac{r_1^3 - r_2^3}{r_1^3 + r_2^3}\right)u_1 \quad \ldots(1)$

Similarly, $v_2 = \left(\dfrac{2m_1}{m_1 + m_2}\right)u_1$

$\Rightarrow \quad v_2 = \left(\dfrac{2r_1^3}{r_1^3 + r_2^3}\right)u_1 \quad \ldots(2)$

Put $r_1 = 4$ cm and $r_2 = 2$ cm in (1) and (2), we get

$v_1 = \left(\dfrac{64 - 8}{64 + 8}\right)81 = 63 \text{ cms}^{-1}$

$v_2 = \dfrac{2(64)}{64 + 8} \times 81 = 144 \text{ cms}^{-1}$

Hence, the correct answer is (A).

31. $(1)(12) + (2)(-24) = 1v_1 + 2v_2$

$\Rightarrow \quad v_1 + 2v_2 = -36 \quad \ldots(1)$

Further $e = -\left(\dfrac{v_2 - v_1}{u_2 - u_1}\right)$

$\Rightarrow \quad \dfrac{2}{3} = -\left(\dfrac{v_2 - v_1}{-24 - 12}\right)$

$\Rightarrow \quad \dfrac{2}{3} = \dfrac{v_2 - v_1}{36}$

$\Rightarrow \quad v_2 - v_1 = 24 \quad \ldots(2)$

Add (1) and (2)

$3v_2 = -12$

$\Rightarrow \quad v_2 = -4 \text{ ms}^{-1}$

$\Rightarrow \quad v_1 = -28 \text{ ms}^{-1}$

$\Rightarrow$ Loss = Total Initial – Total Final

Since

Total Initial $= \frac{1}{2}(1)(144) + \frac{1}{2}(2)(576)$

$\Rightarrow \quad E_i = 72 + 576 = 648$ J

Similarly, $E_f = \frac{1}{2}(1)(784) + \frac{1}{2}(2)(16)$

$\Rightarrow \quad E_f = 392 + 16$

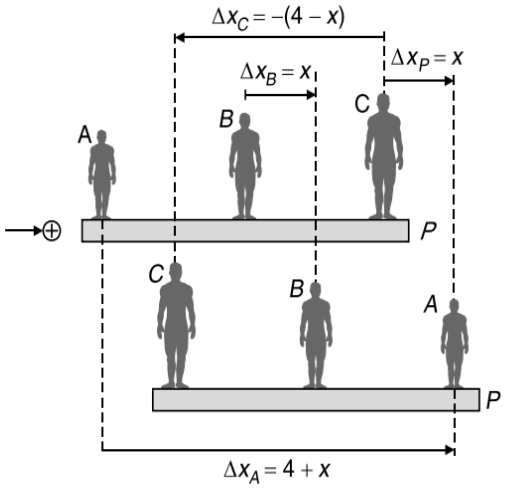
$\Rightarrow \quad E_f = 408$ J

$\Rightarrow \quad \text{Loss} = 648 - 408$

$\Rightarrow \quad \text{Loss} = 240$ J

Hence, the correct answer is (C).

32. When A and C exchange their position, then displacements of A, B, C and plank (P) are shown in Figure.



Since x_{cm} system remains fixed, so

$$\Delta x_{cm} = 0$$

$\Rightarrow \quad m_A \Delta x_A + m_B \Delta x_B + m_C \Delta x_C + m_P \Delta x_P = 0$

$\Rightarrow \quad 40(4+x) + 50x - 60(4-x) + 90x = 0$

$\Rightarrow \quad 240x = 80$

$\Rightarrow \quad x = +\frac{1}{3}$ m

So, B shifts $\frac{1}{3}$ m, rightwards

Hence, the correct answer is (B).

33. $4Mx = M(10R - x)$

$\Rightarrow \quad 4Mx = 10MR - Mx$

$\Rightarrow \quad x = 2R$

Hence, the correct answer is (B).

34. In an inelastic collision, only the momentum of system (ball and earth) may remain conserved. Some energy can be lost in the form of heat, sound etc.

Hence, the correct answer is (C).

35. Let x be the displacement of bead. Displacement of particle with respect to bead is $L(1-\cos\theta)$, i.e., displacement of particle with respect to ground will be $L(1-\cos\theta) - x$. Since net force in horizontal direction on the system is zero. Therefore, the centre of mass will not move in horizontal direction.

$\Rightarrow \quad 2mx = m[L(1-\cos\theta) - x]$

$\Rightarrow \quad 3mx = mL(1-\cos\theta)$

$\Rightarrow \quad x = \frac{L}{3}(1-\cos\theta)$

Hence, the correct answer is (D).

36. $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$

$\Rightarrow \quad v_1 = 0$

Hence, the correct answer is (D).

37. $\begin{pmatrix}\text{Relative} \\ \text{velocity of} \\ \text{separation}\end{pmatrix} = \begin{pmatrix}\text{Relative} \\ \text{velocity of} \\ \text{approach}\end{pmatrix} = v \qquad \{\text{as } e = 1\}$

$\Rightarrow \quad \text{Time of next collision} = \frac{2\pi r}{v}$

Hence, the correct answer is (B).

38. By Law of Conservation of Momentum

$$2.9(0) + 0.1(150) = (2.9 + 0.1)v$$

$\Rightarrow \quad v = \frac{(0.1)(150)}{3}$

$\Rightarrow \quad v = 5 \text{ ms}^{-1}$

By Law of Conservation of Energy

$$\begin{pmatrix}\text{Loss in K.E. of} \\ \text{combined system}\end{pmatrix} = \begin{pmatrix}\text{Gain in P.E. of} \\ \text{combined system}\end{pmatrix}$$

$\Rightarrow \quad \frac{1}{2}(M+m)v^2 = (M+m)gh$

$\Rightarrow \quad v^2 = 2gh$

But $h = \ell(1-\cos\theta)$

$\Rightarrow \quad v^2 = 2g\ell(1-\cos\theta)$

$\Rightarrow \quad 25 = 2(10)(2\cdot5)(1-\cos\theta)$

$\Rightarrow \quad \cos\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = 60^\circ$

Hence, the correct answer is (C).

39. Fraction Transferred

Fraction Lost $= \frac{4m_1m_2}{(m_1+m_2)^2}$

$\Rightarrow \quad \text{Fraction} = \frac{4n}{(n+1)^2}$

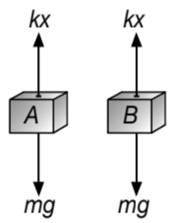
Hence, the correct answer is (D).

40. At maximum extension

$$\frac{1}{2}kx^2 = mgx$$

$\Rightarrow \quad x = \frac{2mg}{k} \qquad \ldots(1)$

Free body diagram of both the blocks at this instant is shown in Figure.



So, $a_{cm} = \dfrac{2kx - 2mg}{2m} = \dfrac{kx - mg}{m}$ ...(2)

$\Rightarrow \quad a_{cm} = g$

Hence, the correct answer is (A).

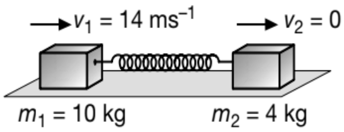
41. $x_{cm} = \dfrac{m\left(\dfrac{R}{2}\right) + 2m(x)}{m + 2m}$

$\Rightarrow \quad R = \dfrac{\dfrac{R}{2} + 2x}{3}$

$\Rightarrow \quad x = \dfrac{5}{4}R$

Hence, the correct answer is (C).

42.



$v_{cm} = \dfrac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

$\Rightarrow \quad v_{cm} = \dfrac{10 \times 14 + 4 \times 0}{10 + 4} = \dfrac{10 \times 14}{14} = 10 \text{ ms}^{-1}$

Hence, the correct answer is (C).

43. $v_{cm} = \dfrac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \dfrac{10 \times 14 + 4 \times 0}{10 + 4} = 10 \text{ ms}^{-1}$

Hence, the correct answer is (C).

44. The system is stationary in air implies equilibrium, i.e. net force is zero. If the man now climbs up to the balloon using the rope, the centre of mass of the 'man plus balloon' system will remain stationary

Hence, the correct answer is (A).

45. Let $m_A = m$, then $m_B = 2m$. If f be the friction between the two blocks, then net force on block A is

$F_1 = mg\sin\theta - f$ {down the plane}

Similarly, net force on block B is

$F_2 = 2mg\sin\theta + f$ {down the plane}

Where, $f = \mu mg\cos\theta$

$\Rightarrow \quad a_{cm} = \dfrac{F_1 + F_2}{m + 2m} = \dfrac{3mg\sin\theta}{3m}$

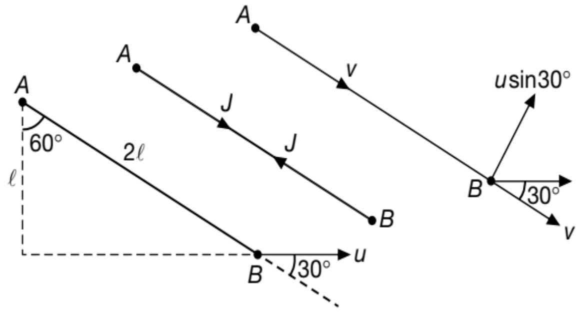
$\Rightarrow \quad a_{cm} = g\sin\theta$

Hence, the correct answer is (D).

46. When the string becomes taut both particles begin to move with velocity components v in the direction AB. Applying Law of Conservation of Linear Momentum in the direction AB, we get

$mu\cos 30° = mv + mv$

$\Rightarrow \quad v = \dfrac{u\sqrt{3}}{4}$



Hence, the velocity of ball A just after the string becomes taut is $\dfrac{u\sqrt{3}}{4}$

Hence, the correct answer is (A).

47. Since this situation is satisfied only for an elastic collision, so both momentum and energy are conserved.

Hence, the correct answer is (D).

48. By Law of Conservation of Momentum

$0 = mu + M(-v)$

$\Rightarrow \quad v = \dfrac{mu}{M}$

is the velocity of the man in the upward direction.

Since, no gravity is existing hence ball will reach the floor in a time $t = \dfrac{h}{u}$

During this time the man will move up by a distance $d = vt$

$\Rightarrow \quad d = \left(\dfrac{mu}{M}\right)\dfrac{h}{u} = \dfrac{mh}{M}$

So, total distance of man from floor when ball reaches the floor is

$s = d + h = h\left(1 + \dfrac{m}{M}\right)$

Hence, the correct answer is (A).

49. $v_1 = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)u_1$

$\Rightarrow \quad \dfrac{2}{3} = \dfrac{m_1 - m_2}{m_1 + m_2}$

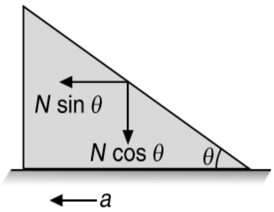
$\Rightarrow \quad 2m_1 + 2m_2 = 3m_1 - 3m_2$

$\Rightarrow \quad 5m_2 = m_1$

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{5}{1}$$

Hence, the correct answer is (C).

50. Let a be the acceleration of wedge leftwards and a_r the relative acceleration of block down the plane. Then absolute acceleration of block in horizontal direction will be $(a_r \cos\theta - a)$ towards right. Net force on the system in horizontal direction is zero. Therefore, acceleration of COM in horizontal direction will be zero or acceleration of wedge towards left is equal to the acceleration of block towards right.



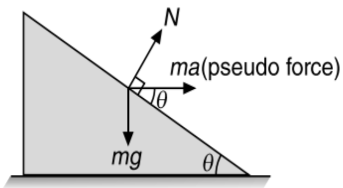
So, $a_r \cos\theta - a = a$

$$\Rightarrow \quad 2a = a_r \cos\theta \qquad \ldots(1)$$

Now let N be the normal reaction between the block and the wedge. Then free body diagram of wedge gives

$$N\sin\theta = ma \qquad \ldots(2)$$

Free body diagram of block with respect to wedge is

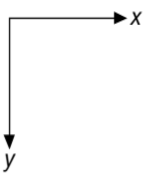


Net force on block perpendicular to plane is zero.

Hence, $N + ma\sin\theta = mg\cos\theta \qquad \ldots(3)$

Solving equations (1), (2) and (3), we get

$$a_r = \frac{2g\sin\theta}{1+\sin^2\theta}$$



Acceleration of block vertically downwards is

$$a_y = a_r \sin\theta$$

$$\Rightarrow \quad a_y = \frac{2g\sin^2\theta}{1+\sin^2\theta}$$

So, acceleration of COM is

$$a_{cm} = \frac{a_y}{2} = \frac{g\sin^2\theta}{(1+\sin^2\theta)}$$

Hence, the correct answer is (C).

51. If the shape of the body is spherical with its centre at origin then

$$R_{cm} = (0,0)$$

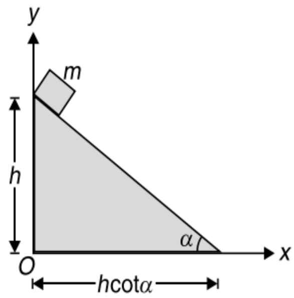
and $R_{cm} \frac{1}{M}\int r_i dm_i$

$$R_{cm} = R$$

$$\Rightarrow \quad R_{cm} \le R$$

Hence, the correct answer is (B).

52. Let the origin O be located just below the top position of m. Let the centre of mass of M be located at point (x, y) before m is released.
When m comes down to the floor, let the centre of mass of M move to the left by ℓ. The centre of mass of m has travelled through $(h\cot\alpha - \ell)$ to the right.



Initially, $(x_{cm})_{\text{initial}} = \dfrac{Mx+0}{M+m} = \dfrac{Mx}{M+m} \qquad \ldots(1)$

Finally, $(x_{cm})_{\text{final}} = \dfrac{M(x-\ell)+m(h\cot\alpha-\ell)}{M+m} \qquad \ldots(2)$

Further no external force is acting on the system and the system $(M \text{ plus } m)$ is initially at rest, so we must conclude that the position of centre of mass remains fixed.

$$\Rightarrow \quad \frac{Mx}{M+m} = \frac{M(x-\ell)+m(h\cot\alpha-\ell)}{M+m}$$

$$\Rightarrow \quad 0 = -\frac{M\ell}{M+m} + \frac{m}{M+m}(h\cot\alpha - \ell)$$

$$\Rightarrow \quad \ell = \frac{mh\cot\alpha}{M+m}$$

Hence, the correct answer is (C).

53. When the car C accelerates from rest to a velocity v_0 relative to the double-boat system, the two boats accelerate to the left, so we have

$$v_C(\text{to right}) + v_A(\text{to left}) = v_0$$

By conservation of momentum, we get

$$mv_C = 2Mv_A$$

Solving, we get

$$v_A = \frac{mv_0}{m+2M}, \quad v_C = \frac{2Mv_0}{m+2M}$$

After the car applies brakes to come to rest, then the tension in the string connecting A, B also becomes zero. Applying conservation of momentum to A and C, we get

$$mv_C - Mv_A = (m+M)v'_A$$

So, velocity of A (to right) is

$$v'_A = \frac{mMv_0}{(m+M)(m+2M)}$$

Hence, the correct answer is (A).

54. $F = \rho a v^2 = \dfrac{\rho (av)^2}{a}$

where, $\rho = 10^3\ \text{kgm}^{-3}$, $av = 20\times 10^{-6}\ \text{m}^3\text{s}^{-1}$

and $a = \dfrac{\pi d^2}{4} = \dfrac{\pi}{4}(10^{-2})^2$

Substituting the values, we get

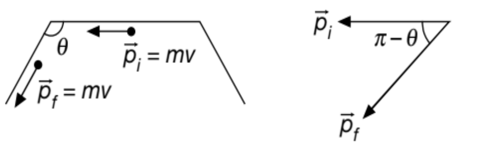
$$F = \frac{(10^3)(20\times 10^{-6})^2}{\frac{\pi}{4}\times 10^{-4}} = 5.1\times 10^{-3}\ \text{N}$$

Hence, the correct answer is (D).

55. Angle $\theta = \left(\pi - \dfrac{\pi}{n}\right)$

$$\Rightarrow \quad \pi - \theta = \frac{\pi}{n}$$

Impulse = change in linear momentum $= \vec{p}_f - \vec{p}_i$



$$\Rightarrow \quad \Delta p = \sqrt{(mv)^2 + (mv)^2 - 2(mv)(mv)\cos(\pi - \theta)}$$

$$\Rightarrow \quad \Delta p = mv\sqrt{2(1+\cos\theta)} = 2mv\cos\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \quad \Delta p = 2mv\cos\left(\frac{\pi}{2} - \frac{\pi}{2n}\right) = 2mv\sin\left(\frac{\pi}{2n}\right)$$

Hence, the correct answer is (D).

56. When two bodies of equal mass collide elastically head-on, then they exchange their velocity.
Hence, the correct answer is (A).

57. Let v' be the velocity of block. Then by Law of Conservation of Linear Momentum, we have

$$mu = mv + mnv'$$

$$\Rightarrow \quad v' = \left(\frac{u-v}{n}\right)$$

Velocity of bullet relative to block is

$$v_r = v - v' = v - \left(\frac{u-v}{n}\right)$$

$$\Rightarrow \quad v_r = \frac{(1+n)v - u}{n}$$

Hence, the correct answer is (C).

58. $T = \dfrac{d}{v/\sqrt{2}} + \dfrac{d}{ev/\sqrt{2}} = \left(1 + \dfrac{1}{e}\right)\dfrac{\sqrt{2}d}{v}$

where $T = \dfrac{2v\sin(45^\circ)}{g}$

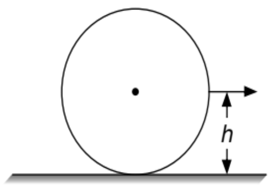
$$\Rightarrow \quad \frac{2v/\sqrt{2}}{g} = \left(1+\frac{1}{e}\right)\frac{\sqrt{2}d}{v}$$

$$\Rightarrow \quad e = \frac{gd}{v^2 - gd}$$

Hence, the correct answer is (C).

59. $\vec{F} = ma_{cm}$

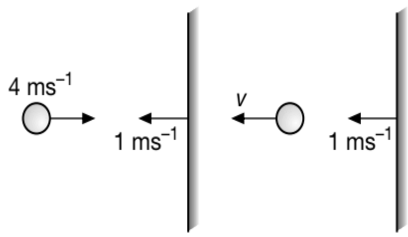
$$a_{cm} = \frac{\vec{F}}{m}$$



which is independent of h.
Hence, the correct answer is (D).

60. Let v be the velocity of ball after an elastic collision with the wall, so

$$e = 1$$



Before Collision   After Collision

$$\Rightarrow \quad \begin{pmatrix}\text{Relative velocity}\\ \text{of separation}\end{pmatrix} = \begin{pmatrix}\text{Relative velocity}\\ \text{of approach}\end{pmatrix}$$

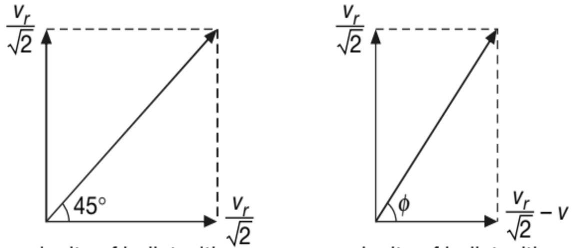
$$\Rightarrow \quad v - 1 = 4 + 1$$

$$\Rightarrow \quad v = 6\ \text{ms}^{-1} \qquad \text{\{away from the wall\}}$$

Hence, the correct answer is (D).

61. Let v_r be the speed of bullet with respect to gun and v the velocity of gun. Then from the two figures it is clear that $\phi > 45^\circ$.

CHAPTER 2



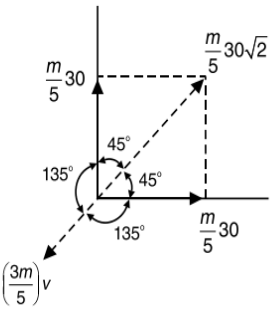
velocity of bullet with respect to gun — velocity of bullet with respect to ground

Hence, the correct answer is (C).

62. If mass is non-uniformly distributed, then centre of mass of ring may lie from origin to circumference. Hence, $0 \le b \le a$.
Hence, the correct answer is (D).

63. $0 = \frac{m}{5}(30\sqrt{2}) + \frac{3m}{5}v$

$\Rightarrow \quad v = -10\sqrt{2}\ ms^{-1}$

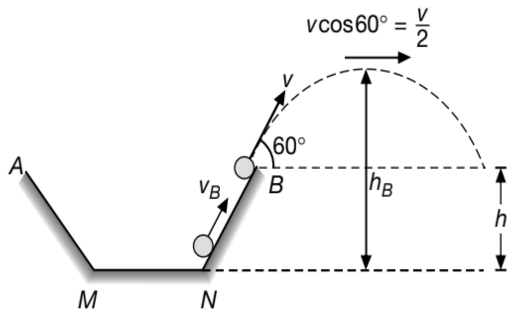


Hence, the correct answer is (B).

64. After collision balls exchange their velocities, so

$$v_A = \sqrt{2gh} \text{ and } v_B = \sqrt{2g(4h)} = 2\sqrt{2gh}$$

Height attained by A will be $h_A = \frac{v_A^2}{2g} = h$



But path of B will be first straight line and then parabolic as shown in figure. For N to B, Loss in KE = Gain in PE

$\Rightarrow \quad \frac{1}{2}mv_B^2 + \frac{1}{2}mv^2 = mgh$

$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{2}m(8gh) - mgh$

$\Rightarrow \quad \frac{1}{2}mv^2 = 3mgh$

$\Rightarrow \quad v = \sqrt{6gh}$

So, $h_B = h + \frac{v^2\sin^2(60)}{2g}$

$\Rightarrow \quad h_B = h + \frac{9h}{4} = \frac{13h}{4}$

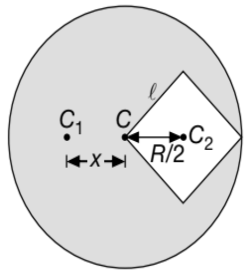
$\Rightarrow \quad \frac{h_A}{h_B} = \frac{4}{13}$

Hence, the correct answer is (C).

65. Let σ be surface mass density. Then

$$m_{\text{disc}} = (\pi R^2)\sigma$$

$$m_{\text{cavity}} = -\ell^2\sigma$$



where ℓ is length of square. Now,

$$\ell\cos(45°) = \frac{R}{2}$$

$\Rightarrow \quad \ell = \frac{R}{\sqrt{2}}$

$\Rightarrow \quad m_{\text{cavity}} = -\frac{R^2\sigma}{2}$

Now assume origin to be at centre of disc, then

$$r_{\text{CM remainder}} = \frac{(\pi R^2\sigma)0 - \left(\frac{R^2\sigma}{2}\right)\left(\frac{R}{2}\right)}{\pi R^2\sigma - \frac{R^2\sigma}{2}}$$

$\Rightarrow \quad r_{\text{CM remainder}} = -\frac{R}{2(2\pi - 1)}$

Negative sign indicates that CM of remained is to the left of centre of disc i.e., origin.
Hence, the correct answer is (C).

66. Thrust $= v\frac{dm}{dt}$

$\Rightarrow \quad \text{Thrust} = (2)\frac{\left(\frac{2}{1000}\right)}{5}$

$\Rightarrow \quad \text{Thrust} = 8 \times 10^{-4}$ N

Hence, the correct answer is (D).

67. By Law of Conservation of Momentum

$$mu - mv = m(0) + mv'$$

$\Rightarrow \quad v' = u - v \qquad \ldots(1)$

Further

$$e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right)$$

$\Rightarrow \quad e = -\left(\frac{v' - 0}{-v - u}\right)$

$\Rightarrow \quad v' = e(v + u) \qquad \ldots(2)$

Equate (1) and (2), we get

$$u - v = ev + eu$$

$\Rightarrow \quad (1-e)u = (1+e)v$

$\Rightarrow \quad \frac{u}{v} = \frac{1+e}{1-e}$

Hence, the correct answer is (A).

68. At the highest point velocity before explosion is $v\cos 60$ along $+x$-axis. By Law of Conservation of Momentum

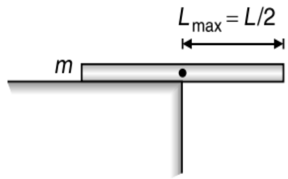
$$(mv\cos 60)\hat{i} = \frac{m}{3}(100\hat{j}) + \frac{m}{3}(-100\hat{j}) + \frac{mv'}{3}$$

$\Rightarrow \quad v' = \frac{3v}{2}\hat{i} = \frac{3(200)}{2}\hat{i}$

$\Rightarrow \quad v' = 300\hat{i}\ \text{ms}^{-1}$

Hence, the correct answer is (B).

69. Let us first consider maximum offset/overhang of one brick as shown in Figure.

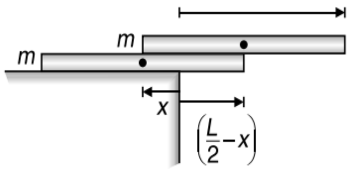


For One Brick

So, for one brick, we have

$$L_{max} = \frac{L}{2}(1) = \frac{L}{2}$$

Now let us consider two bricks and calculate maximum offset (or overhang) for combination.



For Two Bricks

By Law of Conservation of Moments, we have

$$mgx = mg\left(\frac{L}{2} - x\right)$$

$\Rightarrow \quad 2x = \frac{L}{2}$

$\Rightarrow \quad x = \frac{L}{4}$

So, maximum offset is

$$L_{max} = \left(\frac{L}{2} - x\right) + \frac{L}{2} = \frac{3L}{4}$$

For two bricks this can be generalised as

$$L_{max} = \frac{L}{2}\left(1 + \frac{1}{2}\right)$$

For three bricks, we have

$$L_{max} = \frac{L}{2}\left(1 + \frac{1}{2} + \frac{1}{3}\right) = \frac{11L}{12}$$

For four bricks, we have

$$L_{max} = \frac{L}{2}\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) \text{ and so on.}$$

Hence, the correct answer is (D).

70. $p^2 = p_x^2 + p_y^2$

$\Rightarrow \quad p^2 = 12^2 + 5^2$

$\Rightarrow \quad p = 13\ \text{kgms}^{-1}$

Hence, the correct answer is (C).

71. $v = \frac{dx}{dt} = 6t^2$

Impulse $(J) = \Delta p = m(v_f - v_i)$

$\Rightarrow \quad J = 2(6 \times 1 - 0) = 12\ \text{Ns}$

Hence, the correct answer is (B).

72. When the man moves forward with a velocity v with respect to the plank, then the plank moves backwards with a velocity V with respect to ground. So, velocity of the man with respect to ground in the forward direction is $(v - V)$. Applying conservation of linear momentum, we get

$$MV = m(v - V)$$

$\Rightarrow \quad V = \frac{mv}{m+M} = \left(\frac{m}{m+M}\right)\left(\frac{L}{t}\right)$

Hence, the correct answer is (B).

73. By Law of Conservation of Mechanical Energy

$$\frac{1}{2}kx^2 = \frac{1}{2}\mu v_r^2 \qquad \ldots(1)$$

Here, μ = reduced mass of the blocks, so

$$\mu = \frac{(m)(2m)}{m+2m} = \frac{2}{3}\ \text{m}$$

and v_r = relative velocity of the two blocks

Substituting in equation (1), we get

$$kx^2 = \frac{2}{3}mv_r^2$$

CHAPTER 2

$\Rightarrow \quad v_r = \left(\sqrt{\frac{3k}{2m}}\right)x$

Hence, the correct answer is (A).

74. $\left|\vec{v}_{b,w}\right| = \left|\vec{v}'_{b,w}\right| = 2\sqrt{2}\ \text{ms}^{-1}$

So, $\left|\vec{v}'_b\right| = \left|\vec{v}'_{b,w} + \vec{v}_w\right|$

$\Rightarrow \quad \left|\vec{v}'_b\right| = \sqrt{(2)^2 + (2\sqrt{2})^2 + 2(2)(2\sqrt{2})\cos 45°}$

$\Rightarrow \quad \left|\vec{v}'_b\right| = \sqrt{4+8+8} = \sqrt{20}$

$\Rightarrow \quad \left|\vec{v}'_b\right| = 2\sqrt{5}\ \text{ms}^{-1}$

Hence, the correct answer is (D).

75. $M_0 g = \text{Thrust} = v\dfrac{dM}{dt}$

$\Rightarrow \quad \dfrac{dM}{dt} = \dfrac{M_0 g}{v}$

$\Rightarrow \quad \dfrac{dM}{dt} = \dfrac{60000}{1000} = 60\ \text{kgs}^{-1}$

Hence, the correct answer is (B).

76. $F - Mg = Ma$

$\Rightarrow \quad F = M(g+a)$

$\Rightarrow \quad F = (6000)(10+20)$

$\Rightarrow \quad F = 18 \times 10^4\ N$

Further

$F = v\dfrac{dM}{dt}$

$\Rightarrow \quad 18 \times 10^4 = (1000)\dfrac{dM}{dt}$

$\Rightarrow \quad \dfrac{dM}{dt} = 180\ \text{kgs}^{-1}$

Hence, the correct answer is (D).

77. Since the system is free from external force, hence $a_{cm} = 0$ and since initially they are at rest, so

$V_{cm} = 0$

Hence, the correct answer is (A).

78. After collision $v_2 = \left(\dfrac{1+e}{2}\right)u$

and $v_1 = \left(\dfrac{1-e}{2}\right)u$

Here, u = initial speed of ball A

Now, $v_2 = 2v_1$

$\Rightarrow \quad e = \dfrac{1}{3}$

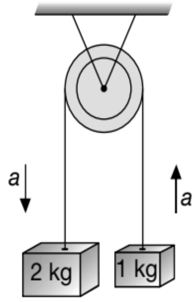
Hence, the correct answer is (B).

79. By Law of Conservation of Linear Momentum

Hence, the correct answer is (C).

80. $a = \dfrac{2g - 1g}{2+1} = \dfrac{10}{3}\ \text{ms}^{-2}$

$\Rightarrow \quad a_{cm} = \dfrac{(2)(a) - (1)(a)}{2+1}$



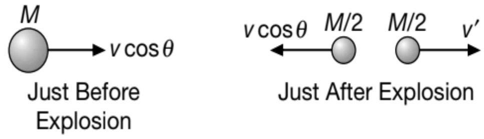
$\Rightarrow \quad a_{cm} = \dfrac{a}{3} = \dfrac{10}{9}\ \text{ms}^{-2}$ {downwards}

Since, $s_{cm} = \dfrac{1}{2}a_{cm}t^2$

$\Rightarrow \quad s_{cm} = \dfrac{1}{2}\left(\dfrac{10}{9}\right)(2)^2 = \dfrac{20}{9}\ \text{m} = 2.22\ \text{m}$

Hence, the correct answer is (B).

81. At the highest point velocity of the shell is $v\cos\theta$. At the highest point, it explodes into two pieces of equal masses out of which one piece retraces the path i.e. has a velocity $-v\cos\theta$.



So, by Law of Conservation of Momentum

$M(v\cos\theta) = \dfrac{M}{2}(-v\cos\theta) + \dfrac{Mv'}{2}$

$\Rightarrow \quad v' = 3v\cos\theta$

Hence, the correct answer is (A).

82. Since Impulse = Change in linear momentum

$\Rightarrow \quad \vec{F}\Delta t = m(\vec{v}_f - \vec{v}_i)$

$\Rightarrow \quad (2\hat{i} + \hat{j} + 3\hat{k})(2) = (1)(\vec{v}_f - (2\hat{i} + \hat{j}))$

$\Rightarrow \quad \vec{v}_f = 6\hat{i} + 3\hat{j} + 6\hat{k}$

$\Rightarrow \quad \left|\vec{v}_f\right| = \sqrt{(6)^2 + (3)^2 + (6)^2} = 9\ \text{ms}^{-1}$

Hence, the correct answer is (C).

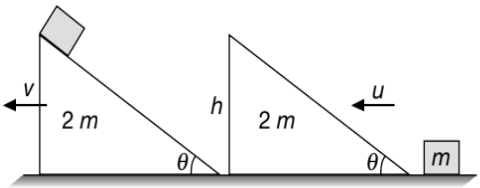
83. When the block just reaches the top of the wedge then the velocity of block with respect to wedge at the top of the wedge is zero. Let v be the horizontal velocity of both at this instant. By Law of Conservation of Linear Momentum, we have

$$(2m+m)v = mu$$

$$\Rightarrow \quad v = \frac{u}{3}$$

By Law of Conservation of Mechanical Energy, we get

$$\frac{1}{2}mu^2 = \frac{1}{2}(3m)v^2 + mgh$$



$$\Rightarrow \quad u^2 = 3\left(\frac{u^2}{9}\right) + 2gh$$

$$\Rightarrow \quad \frac{2}{3}u^2 = 2gh$$

$$\Rightarrow \quad u = \sqrt{3gh}$$

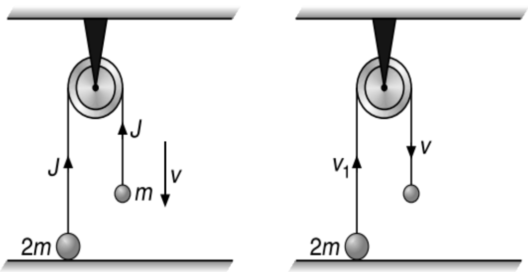
Hence, the correct answer is (B).

84. Fraction Lost $= \dfrac{4m_1m_2}{(m_1+m_2)^2} = \dfrac{8}{9}$

Hence, the correct answer is (D).

85. The centre of mass cannot move towards left. It will always move towards right. Since wedge has a tendency to move towards left so the only external force on the system in horizontal direction is friction which acts towards right.
Hence, the correct answer is (D).

86. Velocity of mass m just before string becomes taut is $v = \sqrt{2gh} = \sqrt{4gl} = 2\sqrt{gl}$



Since, impulse e quals the change in momentum, so we have

for mass $2m$, $J = 2mv_1$

and for mass m, $J = mv - mv_1$

$$\Rightarrow \quad mv - J = mv_1$$

$$\Rightarrow \quad mv - 2mv_1 = mv_1$$

$$\Rightarrow \quad 3mv_1 = mv$$

$$\Rightarrow \quad v_1 = \frac{v}{3} = \frac{2\sqrt{gl}}{3}$$

Hence, the correct answer is (B).

87. $y_{cm} = \dfrac{m\left(-\right) \quad M(\ )}{m+M} = \dfrac{mL}{(m+M)} = y_{initial}$

Finally, when ice melts $y_{final} = \text{zero}$

Shift $= \Delta y = -\dfrac{mL}{2(m+M)}$

Hence, the correct answer is (C).

88. If v_{cm} be the velocity of the centre of mass, then

$$v_{cm} = \frac{mu+0}{m+\eta m} = \frac{u}{1+\eta}$$

Further by Law of Conservation of Energy

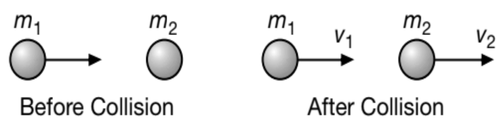
$$\begin{pmatrix}\text{Loss in} \\ \text{K.E. of m}\end{pmatrix} = \begin{pmatrix}\text{Gain in K.E. of} \\ \text{centre of mass} \\ \text{of system}\end{pmatrix} + \begin{pmatrix}\text{Gain in} \\ \text{P.E. of m}\end{pmatrix}$$

$$\frac{1}{2}mu^2 = \frac{1}{2}(m+\eta m)v_{cm}^2 + mgh$$

$$\Rightarrow \quad u = \sqrt{2gh\left(1+\frac{1}{\eta}\right)}$$

Hence, the correct answer is (C).

89. Let v_1 and v_2 be the velocities of the two masses after collision in the same direction. Then



$$m_1v = m_1v_1 + m_2v_2 \quad \text{...(1)}$$

and $v_2 - v_1 = ev$ ...(2)

Solving equations (1) and (2) we get,

$$v_1 = \frac{v(m_1 - em_2)}{m_1+m_2}$$

For v_1 to be positive $m_1 > em_2$

$$\Rightarrow \quad \frac{m_1}{m_2} > e$$

Hence, the correct answer is (B).

90. Since, the lower end moves towards positive x-axis the force of friction will be along negative x-direction. Therefore, centre of mass of the rod will finally fall at $x < 0$. Hence, the lower end will be at $x < \dfrac{\ell}{2}$.

Hence, the correct answer is (C).

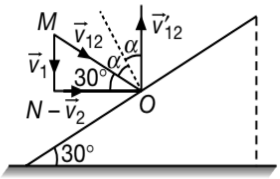
91. $\vec{a}_{CM} = \dfrac{M\vec{a}_1 + (2M)\vec{a}_2}{3M} = \dfrac{M\left(\dfrac{\vec{F}_{12}}{M}\right) + 2M\left(\dfrac{\vec{F}_{21}}{2M}\right)}{3M}$

$\Rightarrow \quad \vec{a}_{CM} = \dfrac{\vec{F}_{12} + \vec{F}_{21}}{3M} = 0$

Hence, the correct answer is (A).

92. In the figure let, $\vec{v}_{12}$ = velocity of ball w.r.t. wedge before collision and

$\vec{v}_{12}$ = velocity of ball w.r.t. wedge after collision, which must be in vertically upward direction as shown.



In elastic collision $\vec{v}_{12}$ and $\vec{v}'_{12}$ will make equal angle (say α) with the normal to the plane.

We can show that $\alpha = 30°$

$\Rightarrow \quad \angle MON = 30°$

Now, $\dfrac{v_2}{v_1} = \cot 30° = \sqrt{3}$

Hence, the correct answer is (A).

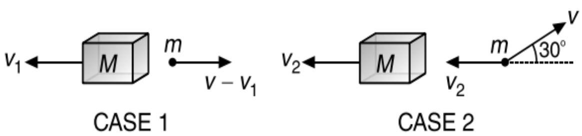
93. $r_{cm} = \dfrac{\int x\,dm}{\int dm}$

$\Rightarrow \quad r_{cm} = \dfrac{\int_0^L x\left(m_0 \dfrac{x^2}{L}\right)dx}{\int_0^L \dfrac{m_0 x^2}{L}dx} = \dfrac{\dfrac{L^4}{4}}{\dfrac{L^3}{3}}$

$\Rightarrow \quad r_{cm} = \dfrac{3L}{4}$

Hence, the correct answer is (D).

94. Let mass of gun to be M and that of shell to be m. The two cases are shown in figure as below



Here, v_1 and v_2 are the recoil speeds of the gun in two cases. Applying Law of Conservation of Linear Momentum in horizontal direction in two cases, we get

CASE-1: $m(v - v_1) = Mv_1$

$\Rightarrow \quad v_1 = \dfrac{mv}{M+m}$ ...(1)

CASE-2: $m(v\cos 30° - v_2) = Mv_2$

$\Rightarrow \quad v_2 = \dfrac{\sqrt{3}mv}{2(M+m)}$ ...(2)

From equations (1) and (2), we get

$\dfrac{v_1}{v_2} = \dfrac{2}{\sqrt{3}}$

Hence, the correct answer is (B).

95. $(0)A = 4v + (A-4)V$

$\Rightarrow \quad V = -\dfrac{4v}{A-4}$

Negative sign indicates the recoil speed.

Hence, the correct answer is (A).

96. If m be mass of each block, then acceleration of system is

$a = \dfrac{mg\sin(60°) - mg\sin(30°)}{2m}$

$\Rightarrow \quad a = \left(\dfrac{\sqrt{3}-1}{4}\right)g$

Now $\vec{a}_{cm} = \dfrac{m\vec{a}_1 + m\vec{a}_2}{2m} = \dfrac{\vec{a}_1 + \vec{a}_2}{2}$

Here, $\vec{a}_1$ and $\vec{a}_2$ are $\left(\dfrac{\sqrt{3}-1}{4}\right)g$ directed at right angles to each other.

$\Rightarrow \quad |\vec{a}_{cm}| = \dfrac{\sqrt{a^2 + a^2}}{2} = \left(\dfrac{\sqrt{3}-1}{4\sqrt{2}}\right)g$

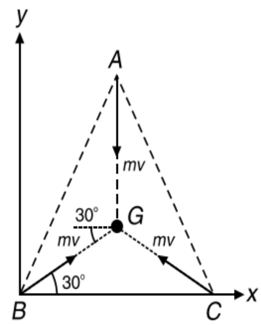
Hence, the correct answer is (A).

97. Let h be the height of each step. Then

$e = \dfrac{v}{u} = \dfrac{\sqrt{2gh}}{\sqrt{2g(2h)}} = \dfrac{1}{\sqrt{2}}$

Hence, the correct answer is (B).

98. Let Origin be placed at B and x-axis along BC. By Law of Conservation of Momentum



along x-axis

$0 + mv\cos 30 - mv\cos 30 = 0 - mv\cos 30 + mv_x$

$\Rightarrow \quad v_x = v\cos 30$ ...(1)

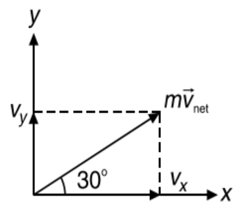
along y-axis

$-mv + mv\sin 30 + mv\sin 30 = 0 - mv\sin 30 + mv_y$

$\Rightarrow \quad v_y = v\sin 30$ ...(2)

Squaring (1) and (2) and adding, we get

$$v_{net} = \sqrt{v_x^2 + v_y^2} = v \text{ at}$$



Hence, the particle C will have the same magnitude of velocity but directed along the original direction (direction before collision) of particle B.

Hence, the correct answer is (C).

99. By Law of Conservation of Momentum

$$0 = 8(6) + 4v$$

$\Rightarrow \quad v = -12 \text{ ms}^{-1}$

Negative sign indicates the recoil of second piece

$$\text{KE} = \frac{1}{2}(4)(12)^2 = 288 \text{ J}$$

Hence, the correct answer is (D).

100. $\vec{F} = \frac{d\vec{p}}{dt} = 2\vec{B}t$

When $\vec{a}$ and $\vec{u}$ are at 45°, $\vec{F}$ and $\vec{p}$ will also be inclined at 45°. This will happen at $t = \sqrt{\frac{A}{B}}$

$\Rightarrow \quad \vec{F} = 2\vec{B}\sqrt{\frac{A}{B}}$

Hence, the correct answer is (C).

101. Net external force on the two blocks (whether they move with same retardation or not) is

$$F_{ext} = (0.2)(2+1)(10) = 6 \text{ N}$$

$\Rightarrow \quad a_{cm} = \frac{F_{ext}}{2+1} = 2 \text{ ms}^{-2}$

Hence, the correct answer is (B).

102. K.E. Retained $= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 E$

$\Rightarrow \quad$ K.E. Retained $= \left(\frac{8-2}{8+2}\right)^2 E$

$\Rightarrow \quad$ K.E. Retained $= 0.36E$

Hence, the correct answer is (C).

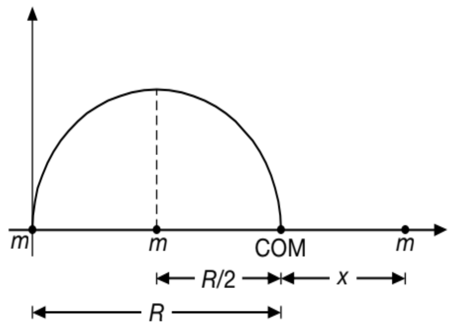
103. Change in linear momentum $\Delta\vec{p} = \vec{F}\Delta t$

Here, $\vec{F}$ is weight, i.e., mg

$\Rightarrow \quad |\Delta\vec{p}| = (mg)(\Delta t) = (1)(10)(1) = 10 \text{ kgms}^{-1}$

Hence, the correct answer is (C).

104.



The centre of mass will follow the same path, so

$$mx = m\frac{R}{2} + mR$$

$\Rightarrow \quad x = \frac{3R}{2} = \frac{3 \times 100}{2} = 150 \text{ m}$

Hence, the desired distance is $(100 + 150)$ m = 250 m

Hence, the correct answer is (C).

105. $6(9) = (12 + 6)v$

$\Rightarrow \quad v = 3 \text{ ms}^{-1}$

Hence, the correct answer is (A).

106. $\hat{j}$ component, i.e., component of velocity parallel to wall remains unchanged while $\hat{i}$ component will become $\left(-\frac{1}{2}\right)(2\hat{i}) = -\hat{i}$. Therefore, velocity vector of the sphere after it hits the wall is $-\hat{i} + 2\hat{j}$.

Hence, the correct answer is (B).

107. Since $\Delta x_{cm} = \frac{m_1\Delta x_1 + m_2\Delta x_2}{m_1 + m_2} = 0$

$\Rightarrow \quad M(4L - x) = 5Mx$

$\Rightarrow \quad x = \frac{2}{3}L$ to left

Hence, the correct answer is (D).

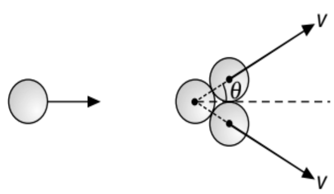
108. $M\frac{d^2\vec{R}}{dt^2} = M\vec{g}$

$\Rightarrow \quad \frac{d^2\vec{R}}{dt^2} = \vec{g}$

Hence, the correct answer is (C).

109. $\sin\theta = \frac{r}{2r} = \frac{1}{2}$

$\Rightarrow \quad \theta = 30°$



By Law of Conservation of Linear Momentum

$$mu = 2mv\cos(30°)$$

$$\Rightarrow \quad v = \frac{u}{\sqrt{3}} \qquad \ldots(1)$$

Now $e = -\left[\dfrac{(v_2)_n - (v_1)_n}{(u_2)_n - (u_1)_n}\right]$

$$\Rightarrow \quad e = \frac{v}{u\cos(30°)} = \frac{\frac{u}{\sqrt{3}}}{\frac{u\sqrt{3}}{2}} = \frac{2}{3}$$

Hence, the correct answer is (C).

110. Unless $m_1 = m_3$ the COM of all the four particles can never be at the centre of the square.
Hence, the correct answer is (D).

111. Let u be the velocity of ball before collision. Speed of ball after collision will become.

$$v = \sqrt{\left(\frac{u}{\sqrt{2}}\right)^2 + \left(\frac{u}{2\sqrt{2}}\right)^2} = \sqrt{\frac{5}{8}}u$$

Fraction of KE lost in collision $= \dfrac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2}$

$$\Rightarrow \quad \text{Fraction Lost } = 1 - \left(\frac{v}{u}\right)^2 = 1 - \frac{5}{8} = \frac{3}{8}$$

Hence, the correct answer is (C).

112. $\left|(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2)\right| = \Delta\vec{p}$

$$\Rightarrow \quad \Delta\vec{p} = |\text{change in momentum of the two particles}|$$

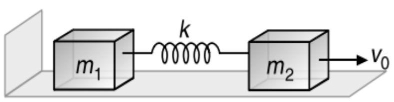
$$\Rightarrow \quad \Delta\vec{p} = \left|\begin{matrix}\text{External force on}\\ \text{the system}\end{matrix}\right| \times \text{time interval}$$

$$\Rightarrow \quad \Delta\vec{p} = (m_1 + m_2)g(2t_0)$$

$$\Rightarrow \quad \Delta\vec{p} = 2(m_1 + m_2)gt_0$$

Hence, the correct answer is (C).

113. Let common velocity of both blocks after spring is completely extended be v, then applying conservation of linear momentum, we get



$$m_2v_0 = (m_1 + m_2)v$$

$$\Rightarrow \quad v = \frac{m_2v_0}{m_1 + m_2}$$

Applying conservation of energy, we get

$$\frac{1}{2}m_2v_0^2 - \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{1}{2}kx^2 = \frac{1}{2}m_2v_0^2 - \frac{1}{2}(m_1 + m_2)\frac{m_2^2v_0^2}{(m_1 + m_2)^2}$$

$$\Rightarrow \quad kx^2 = m_2v_0^2 - \frac{m_2^2v_0^2}{m_1 + m_2}$$

$$kx^2 = \frac{m_1m_2v_0^2 + m_2^2v_0^2 - m_2^2v_0^2}{m_1 + m_2} = \left(\frac{m_1m_2}{m_1 + m_2}\right)v_0^2$$

$$\Rightarrow \quad x = v_0\sqrt{\frac{m_1m_2}{(m_1 + m_2)k}}$$

Hence, the correct answer is (D).

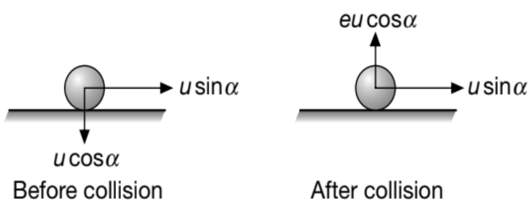
114. Velocity of man with respect to ground is 1 ms^{-1} in opposite direction.

$$\Rightarrow \quad v_{cm} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$$\Rightarrow \quad v_{cm} = \frac{40\times2 - 80\times1}{40 + 80} = 0$$

Hence, the correct answer is (A).

115. Impulse = Change in Linear Momentum



$$\Rightarrow \quad Ft = m(eu\cos\alpha + u\cos\alpha)$$

$$\Rightarrow \quad F = \frac{mu\cos\alpha(1 + e)}{t}$$

Hence, the correct answer is (D).

116. $v_1 = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\dfrac{2m_2}{m_1 + m_2}\right)u_2$

For $m_1 << m_2$

$$m_1 - m_2 \quad -m_2$$

$$m_1 + m_2 \simeq m_2$$

$$\Rightarrow \quad v_1 = -u_1 + 2u_2$$

$$\Rightarrow \quad v_1 = -12 + 2(10)$$

$$\Rightarrow \quad v_1 = 8 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

117. Since, $h \ll$ radius of earth, acceleration due to gravity g can be assumed to be constant. Let v be the velocity of block at height $\frac{h}{2}$. Then by Law of Conservation of Linear Momentum, velocity of earth will be

$$MV - \frac{Mv}{3} = 0$$

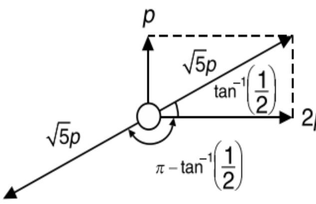
By Law of Conservation of Mechanical Energy, we get

$$\left(\frac{M}{3}\right)g\left(\frac{h}{2}\right)=\frac{1}{2}\left(\frac{M}{3}\right)v^2+\frac{1}{2}(M)\left(\frac{v}{3}\right)^2$$

$$\Rightarrow \quad v=\frac{\sqrt{3gh}}{2}$$

Hence, the correct answer is (C).

118.



Hence, the correct answer is (D).

119. Since the compartment (including passengers) is stationary, so the combined centre of mass of compartment and passengers is fixed i.e. C_2 is fixed. When the passengers move here and there in the compartment then in an attempt to keep C_2 fixed C_1 has to move.
Hence, the correct answer is (C).

120. By Law of Conservation of Momentum
along x-axis

$$m(9)+m(0)=mv_1\cos 30+mv_2\cos 30$$

$$\Rightarrow \quad v_1+v_2=\frac{18}{\sqrt{3}}=6\sqrt{3}$$

along y-axis

$$0=mv_1\sin 30-mv_2\sin 30$$

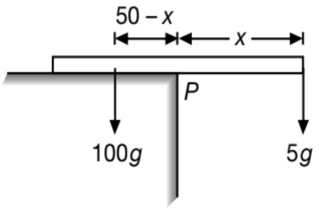
$$\Rightarrow \quad v_1=v_2$$

$$\Rightarrow \quad v_1=v_2=3\sqrt{3}\text{ ms}^{-1}$$

Hence, the correct answer is (D).

121. Transfer of energy is the maximum when both bodies are of equal masses.
Hence, the correct answer is (A).

122. For the rule not to tilt about P, anticlockwise torque about $P>$ clockwise torque about P. In critical case these two torques are equal



$$\Rightarrow \quad 100g(50-x)=5gx$$

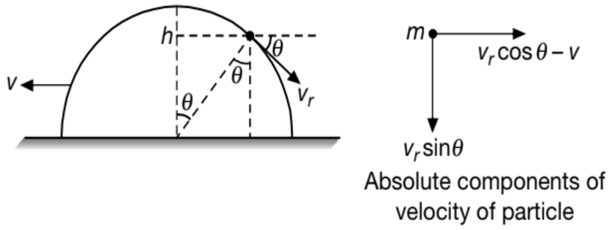
$$\Rightarrow \quad 105x=50\times 100$$

$$\Rightarrow \quad x=47.6\text{ cm}$$

Hence, the correct answer is (D).

123. Centre of mass will move in a vertical line if $v_1\cos\theta_1=v_2\cos\theta_2$. Else, for any other values, it will follow a parabolic path.
Hence, the correct answer is (B).

124. Let v_r be the velocity of particle relative to hemisphere and v the linear velocity of hemisphere at this moment. Then from Law of Conservation of Linear Momentum, we have



$$4mv=m(v_r\cos\theta-v)$$

$$\Rightarrow \quad 5v=v_r\cos\theta$$

$$\Rightarrow \quad v_r=\frac{5v}{\cos\theta}$$

$$\Rightarrow \quad \omega=\frac{v_r}{R}=\frac{5v}{R\cos\theta}$$

Hence, the correct answer is (C).

125. $$a_{cm}=\frac{F}{m+2m}=\frac{F}{3m}$$

$$\Rightarrow \quad x_{cm}=\frac{1}{2}a_{cm}t^2=\frac{Ft^2}{6m}$$

Since, $x_{cm}=\dfrac{m(x)+2m(x')}{m+2m}$

$$\Rightarrow \quad \frac{Ft^2}{6m}=\frac{x+2x'}{3}$$

$$\Rightarrow \quad x'=\frac{Ft^2}{4m}-\frac{x}{2}$$

Hence, the correct answer is (B).

126. $$MV=m(0)+(M-m)v$$

$$\Rightarrow \quad v=\frac{MV}{M-m}$$

Hence, the correct answer is (C).

127. Range of stone $=\dfrac{u^2\sin 2\theta}{g}=\dfrac{(10)^2\sin 90^\circ}{10}=10\text{ m}$

Net force on system in horizontal direction is zero. Therefore, centre of mass will remain stationary in horizontal direction. Hence,

$$(60+40)x=(1)(10)$$

Where, x is the displacement of boy + platform

$$\Rightarrow \quad x=\frac{10}{100}=0.1\text{ m}=10\text{ cm}$$

Hence, the correct answer is (B).

128. Since both particles move under the influence of gravity, so relative acceleration of one as seen by other is zero. Therefore, the relative motion between the two is uniform. Relative velocity of the two is $v_r=(10+5)\text{ ms}^{-1}=15\text{ ms}^{-1}$ in horizontal direction. Therefore, the collision will take place after a time

$$t=\frac{60}{15}=4\text{ s}$$

Net linear momentum in horizontal direction of the two particles is zero. Therefore, after collision the combined mass will fall vertically downwards. The desired distance from A would be

$$x=v_rt=10\times4=40\text{ m}$$

Hence, the correct answer is (A).

Multiple Correct Choice Type Questions

1. Horizontal velocity of the ball B at the time when it strikes the ground is v_B, then $10=v_B\times t$

$$\Rightarrow \quad 10=v_B\times\sqrt{\frac{2\times5}{g}}=v_B\sqrt{\frac{10}{10}}=v_B$$

$$\Rightarrow \quad v_B=10\text{ ms}^{-1} \qquad \ldots(1)$$

Since collision is perfectly elastic and mass of ball A and that of B are same, therefore velocity of ball A, before collision is also 10 ms^{-1}.

So, OPTION **(C)** is correct.

$$\text{KE}=\frac{1}{2}mv^2=\frac{1}{2}m\left[\sqrt{(10)^2+(10)^2}\right]^2$$

$$\Rightarrow \quad \text{KE}=\frac{1}{2}m\times2\times100=(m\times100)\text{ J}$$

So, OPTION **(B)** is also correct.

Hence, (B) and (C) are correct.

2. By Law of Conservation of Linear Momentum,

$$\left(\frac{m}{2}\right)u=\left(m+\frac{m}{2}\right)v$$

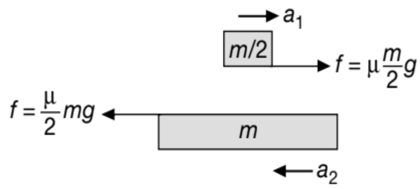
$\Rightarrow$ –

Work done against friction $=E_i-E_f=W_f$

$$\Rightarrow \quad W_f=\frac{1}{2}\left(\frac{m}{2}\right)u^2-\frac{1}{2}\left(\frac{3m}{2}\right)\left(\frac{u}{3}\right)^2$$

$$\Rightarrow \quad W_f=\frac{1}{6}mu^2=\frac{2}{3}\left(\frac{1}{4}mu^2\right)$$

Force of friction on the two blocks before the blocks reach a common velocity is as shown in Figure.



$$a_1=\mu g \text{ and } a_2=\frac{\mu}{2}g$$

$$\Rightarrow \quad a_r=\frac{3}{2}\mu g$$

Hence, (A), (B) and (C) are correct.

3. When the string becomes taut, speed of balls is given by

$$V=\frac{mv_0}{2m}=\frac{v_0}{2}$$

If CM having velocity v_0 is raised up by height h, then we have

$$(2m)gh=\frac{1}{2}(2m)\left(\frac{v_0}{2}\right)^2$$

$$\Rightarrow \quad h=\frac{v_0^2}{8g}$$

When CM is at highest point, then

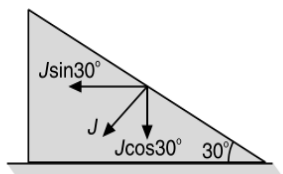
$$K_{\text{system}}=\frac{mv_0^2}{2}-2mgh=\frac{mv_0^2}{2}-\frac{mv_0^2}{4}=\frac{mv_0^2}{4}$$

Hence, (A) and (D) are correct.

4. Since collision is elastic so, the kinetic energy will be conserved. Let v be the speed of ball after collision. Then,

$$\frac{1}{2}(1)(10)^2=\frac{1}{2}(4)(4)^2+\frac{1}{2}(1)v^2$$

$$\Rightarrow \quad v=6\text{ ms}^{-1}$$



Let J be the impulse between the ball and wedge during collision.

Since Impulse = Change in Linear Momentum

$$\Rightarrow \quad J\sin30^\circ=(4)(4)$$

$$\Rightarrow \quad J=32\text{ Ns}$$

Hence, (A) and (D) are correct.

5. Since initially the system is at rest so the centre of mass remains fixed.

$$x_{\text{cm}}=\frac{mx_1+Mx_2}{m+M}$$

$$\Rightarrow \quad \Delta x_{cm} = \frac{m\Delta x_1 + M\Delta x_2}{M+m} = 0$$

$$\Rightarrow \quad m\Delta x_1 + M\Delta x_2 = 0$$

$$\Rightarrow \quad m(L-D) + MD = 0$$

$$\Rightarrow \quad D = \frac{mL}{M+m} < L$$

Hence, (B) and (D) are correct.

6. The impulse imparted to m is equal to change in its momentum, so we have

$$\text{Impulse} = \Delta p = m(v_f - v_i)$$

Impulse received by m is

$$\Delta \vec{p}_1 = m\left[\left(-2\hat{i}+\hat{j}\right) - \left(3\hat{i}+2\hat{j}\right)\right]$$

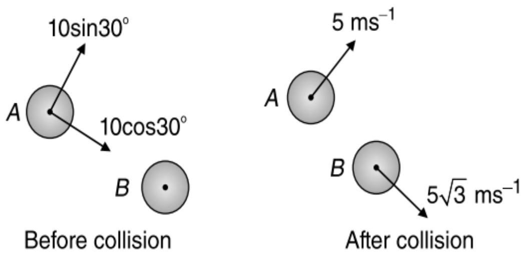
$$\Rightarrow \quad \Delta \vec{p}_1 = m\left(-5\hat{i}-\hat{j}\right)$$

Impulse imparted to M is Δp_2 given by

$$\Delta \vec{p}_2 = -\Delta \vec{p}_1$$

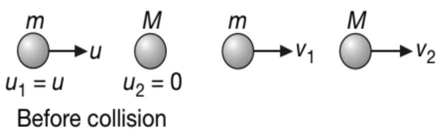
$$\Rightarrow \quad \Delta \vec{p}_2 = m\left(5\hat{i}+\hat{j}\right)$$

Hence, (B) and (D) are correct.

7. 

Velocity components in common tangent direction will remain unchanged while velocity components in common normal direction are interchanged in case of an elastic collision. Hence, both A and B move at right angles after collision with $v_A = 5\text{ ms}^{-1}$ and $v_B = 5\sqrt{3}\text{ ms}^{-1}$. Kinetic energy is conserved in an elastic collision, whether it is a head on or an oblique collision.

Hence, (A), (B) and (D) are correct.

8. 

Since $v_1 = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)u_1$

$$\Rightarrow \quad v_1 = \left(\frac{m-M}{m+M}\right)u$$

$$\Rightarrow \quad v_1 = \left(\frac{1-x}{1+x}\right)u \qquad \ldots(1)$$

Where, $x = \dfrac{M}{m}$

Now speed of ball 1 after impact is one third its initial speed, so

$$v_1 = \pm\frac{u}{3}$$

Substituting this in equation (1), we get

$$x = \frac{M}{m} = \frac{1}{2} \text{ OR } 2$$

Hence, (B) and (C) are correct.

9. Since impulse equals change in momentum, so we have

$$\Delta \vec{p} = 2(\vec{v}_2 - \vec{v}_1)$$

$$\Rightarrow \quad \Delta \vec{p} = 2\left(3\hat{i}-\hat{j}\right)$$

As impulse is in the direction normal to the colliding surface, so we have

$$\tan\theta = \frac{1}{3}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{3}\right)$$

$$\Rightarrow \quad \alpha = 90^\circ + \tan^{-1}\left(\frac{1}{3}\right)$$

Hence, (B) and (D) are correct.

10. Since, $v_{cm} = \dfrac{mv_o + M(0)}{m+M} = \dfrac{mv_o}{m+M}$

So, OPTION **(A)** is correct.

At the time of maximum compression both the blocks will be moving with same common velocity. Applying Law of Conservation of Linear Momentum and Conservation of Mechanical Energy, we get

$$mv_o = (M+m)v \qquad \ldots(1)$$

$$\frac{1}{2}mv_o^2 = \frac{1}{2}(M+m)v^2 + \frac{1}{2}kx_{max}^2 \qquad \ldots(2)$$

Solving (1) and (2), we get

$$x_{max} = v_o\sqrt{\frac{mM}{(m+M)}\frac{1}{k}}$$

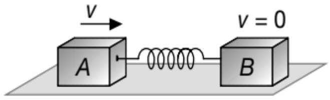
So, OPTION **(C)** is also correct.

In the state of maximum compression, both the blocks are moving with the same velocity. Therefore, velocity of centre of mass in the CM frame is zero.

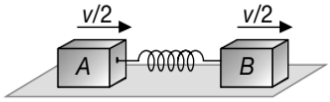
Hence OPTION **(D)** is also correct.

Hence, (A), (C) and (D) are correct.

11. Here, at maximum compression x_{max}, we introduce the concept of reduced mass μ of the system (details discussed in Rotational Dynamics).



Initially



At maximum compression

$$\frac{1}{\mu}=\frac{1}{m}+\frac{1}{m}$$

$$\Rightarrow \quad \mu=\frac{m}{2}$$

Further, by Law of Conservation of Energy

$$\frac{1}{2}\mu v^2=\frac{1}{2}kx_{\max}^2$$

$$\Rightarrow \quad x_{\max}=v\sqrt{\frac{m}{2k}} \text{ and}$$

$$\text{KE}=\frac{1}{2}\mu v^2=\frac{1}{2}\left(\frac{m}{2}\right)v^2$$

$$\Rightarrow \quad \text{KE}=\frac{mv^2}{4}$$

Hence, (B) and (D) are correct.

12. Since external force i.e., gravitational force acts on system, therefore momentum of system is not conserved. With the passage of time, mass keeps on decreasing, so acceleration keeps on increasing.
Hence, (B), (C) and (D) are correct.

13. By Law of Conservation of Momentum

$$mv=(M+m)V$$

Momentum of combined system is $(M+m)V=mv$

$$\text{KE}=\frac{1}{2}(M+m)V^2$$

$$\Rightarrow \quad \text{KE}=\frac{m^2v^2}{2(M+m)}$$

Hence, (B) and (C) are correct.

14. B will give impulse $-J$ to A and

$$e=\frac{J-(P-J)}{P}=\frac{2J}{P}-1$$

Hence, (A), (B) and (D) are correct.

15. At minimum separation, the common velocity attained by the bodies equals the velocity of centre of mass. So,

$$v_{\text{cm}}=v_0=\frac{3\times 2}{1+3}=1.5 \text{ ms}^{-1}$$

At time t, for body A, we have

$$v_A=u_A+a_At=0+\left(\frac{F}{m_A}\right)t$$

$$\Rightarrow \quad v_A=6t \qquad \ldots(1)$$

At time t, for body B, we have

$$v_B=u_B+a_Bt=2-\left(\frac{6}{3}\right)t$$

$$\Rightarrow \quad v_B=2-2t \qquad \ldots(2)$$

Since $v_A=v_B$, so we get

$$6t=2-2t$$

$$\Rightarrow \quad t=0.25 \text{ s}$$

Change in kinetic energy is given by

$$\Delta K=\frac{1}{2}(1+3)(1.5)^2-\frac{1}{2}(3)(2)^2=-1.5 \text{ J}$$

During this time let displacement of A be Δs, then from Work-Energy Theorem, work done equals change in kinetic energy, so

$$W=\Delta K=F\Delta s\cos\theta=F\Delta s\cos(180^\circ)$$

$$\Rightarrow \quad -1.5=6\Delta s(-1)$$

$$\Rightarrow \quad \Delta s=\frac{1.5}{6}=0.25 \text{ m}$$

Minimum separation is $1-0.25=0.75$ m
Hence, (B) and (D) are correct.

16. $$a_{\text{cm}}=\frac{\text{Force on centre of mass}}{\text{Mass}}=\frac{M_{\text{cm}}g}{M_{\text{cm}}}=g$$

Hence, (A) and (C) are correct.

17. Before collision: $v_A=\sqrt{2gH}$ and $v_B=0$

If collision is elastic, then velocities after collision will be

$$v_A'=\left(\frac{m_A-m_B}{m_A+m_B}\right)v_A$$

$$\Rightarrow \quad v_A'=\left(\frac{M-3M}{M+3M}\right)\sqrt{2gH}=-\frac{\sqrt{2gH}}{2}$$

$$\text{and} \quad v_B'=\left(\frac{2m_A}{m_A+m_B}\right)v_A$$

$$\Rightarrow \quad v_B'=\left(\frac{2M}{M+3M}\right)\sqrt{2gH}=\frac{\sqrt{2gH}}{2}$$

So, $h_A=\frac{v_A'^2}{2g}=\frac{H}{4}$ and $h_B=\frac{v_B'^2}{2g}=\frac{H}{4}$

If collision in perfectly inelastic, velocity of combined mass after collision

$$v=\frac{m_Av_A}{m_A+m_B}=\frac{M\sqrt{2gH}}{M+3M}=\frac{\sqrt{2gH}}{4}$$

The combined mass will rise to a height

$$h = \frac{v^2}{2g} = \frac{H}{16}$$

Hence, (A), (B) and (D) are correct.

18. Since velocity remains same along the common tangent line, so we have

$$u\cos\theta = v\cos\phi \quad \ldots(1)$$

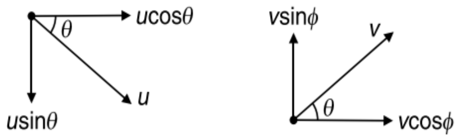
Also, along the line of impact, we have

$$v\sin\phi = eu\sin\theta$$

$$\Rightarrow \quad eu\sin\theta = v\sin\phi \quad \ldots(2)$$

From equations (1) and (2), we get

$$\tan\phi = e\tan\theta$$



Momentum or velocity changes only in vertical direction.

$$\Rightarrow \quad |\text{Impulse}| = |\Delta p|$$

$$\Rightarrow \quad |\text{Impulse}| = m(u\sin\theta + eu\sin\theta)$$

$$\Rightarrow \quad |\text{Impulse}| = m(1+e)u\sin\theta$$

Also, $v = \sqrt{(v\cos\phi)^2 + (v\sin\phi)^2}$

$$\Rightarrow \quad v = \sqrt{(u\cos\theta)^2 + (eu\sin\theta)^2}$$

$$\Rightarrow \quad v = \sqrt{u^2(\cos^2\theta + e^2\sin^2\theta)}$$

$$\Rightarrow \quad v = u\sqrt{1-(1-e^2)\sin^2\theta}$$

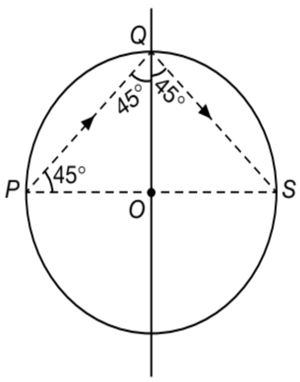
The ratio of final kinetic energy K_f to initial kinetic energy K_i is

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mu^2} = \frac{v^2}{u^2}$$

$$\Rightarrow \quad \frac{K_f}{K_i} = \cos^2\theta + e^2\sin^2\theta$$

Hence, (A), (B), (C) and (D) are correct.

19.



We observe that, $\angle PQS = 90^\circ$

Since, angle made by the body with the normal OQ before collision and after collision are same, therefore, collision is elastic,

Hence, (B) and (C) are correct.

20. In case of perfectly inelastic collision the velocity of combined mass will be $\frac{v}{2}$, so

$$\text{KE} = \frac{1}{2}(2m)\left(\frac{v^2}{4}\right)$$

$$\Rightarrow \quad (0.2) = \frac{1}{2}(0.2)\left(\frac{v^2}{4}\right)$$

$$\Rightarrow \quad v = 2\sqrt{2}\ \text{ms}^{-1}$$

In case of perfectly elastic collision, first particle stops and the second acquires the same velocity v

$$\text{KE} = \frac{1}{2}mv^2$$

$$\Rightarrow \quad 0.2 = \frac{1}{2}(0.1)v^2$$

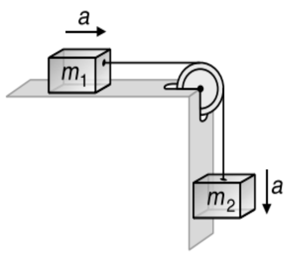
$$\Rightarrow \quad v = 2\ \text{ms}^{-1}$$

Therefore, minimum value of v is $2\ \text{ms}^{-1}$ in case of perfectly elastic collision and maximum value of v is $2\sqrt{2}\ \text{ms}^{-1}$ in case of perfectly inelastic collision.

Hence, (B) and (C) are correct.

21. Acceleration a of the blocks is

$$a = \frac{\text{Net pulling force}}{\text{Total mass}} = \frac{m_2 g}{m_1 + m_2}$$

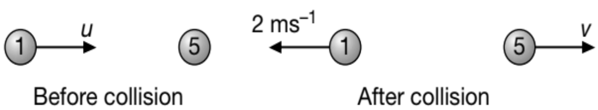


$$\Rightarrow \quad (a_{\text{cm}})_x = \frac{m_1 a}{m_1 + m_2} = \frac{m_1 m_2 g}{(m_1 + m_2)^2}$$

and $(a_{\text{cm}})_y = \frac{m_2 a}{m_1 + m_2} = \left(\frac{m_2}{m_1 + m_2}\right)^2 g$

Hence, (B) and (C) are correct.

22.



Since collision is elastic, so $e = 1$

$\Rightarrow$ Velocity of Approach = Velocity of Separation

$\Rightarrow \quad u = v + 2 \quad \ldots(1)$

By Law of Conservation of Linear Momentum, we have

$$(1)u = (5)v - (1)(2)$$

$\Rightarrow \quad u = 5v - 2$

$\Rightarrow \quad v + 2 = 5v - 2$

$\Rightarrow \quad v = 1 \text{ ms}^{-1}$ and $u = 3 \text{ ms}^{-1}$

Momentum of system $= (1)(3) = 3 \text{ kgms}^{-1}$

Momentum of 5 kg after collision $= (5)(1) = 5 \text{ kgms}^{-1}$

So, kinetic energy of centre of mass is

$$K_{cm} = \frac{1}{2}(m_1 + m_2)\left(\frac{m_1 u}{m_1 + m_2}\right)^2$$

$$\Rightarrow \quad K_{cm} = \frac{1}{2}(1+5)\left(\frac{1 \times 3}{6}\right)^2$$

$\Rightarrow \quad K_{cm} = 0.75 \text{ J}$

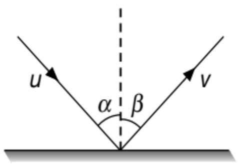
Total kinetic energy $= K = \frac{1}{2} \times 1 \times 3^2$

$\Rightarrow \quad K = 4.5 \text{ J}$

Hence, (A) and (C) are correct.

23. Let velocity of ball before collision and after collision be u and v respectively, then

$$e = \frac{v\cos\beta}{u\cos\alpha} \quad \ldots(1)$$



and by Law of Conservation of Momentum along horizontal direction, we get

$$u\sin\alpha = v\sin\beta \quad \ldots(2)$$

From (1) and (2), we get

$$e = \frac{\tan\alpha}{\tan\beta}$$

So, OPTION **(A)** is correct.

If $\alpha < \beta$, then $e < 1$, implies that collision is inelastic.

So, OPTION **(B)** is correct.

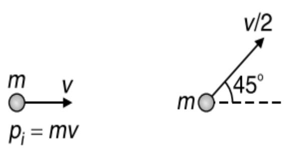
If $\alpha = \beta$, then $e = 1$, collision is elastic.

So, OPTION **(C)** is correct.

Hence, (A), (B) and (C) are correct.

24. By Law of Conservation of Momentum, we have

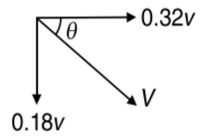
$$\vec{p}_i = \vec{p}_f$$



$$\Rightarrow \quad (mv)\hat{i} = \left(\frac{mv}{2\sqrt{2}}\hat{i} + \frac{mv}{2\sqrt{2}}\hat{j}\right) + (2m)\vec{V}$$

$$2\vec{V} = v\left(1 - \frac{1}{2\sqrt{2}}\right)\hat{i} - \frac{v}{2\sqrt{2}}\hat{j}$$

$$\Rightarrow \quad \vec{V} = (0.32v)\hat{i} - (0.18v)\hat{j}$$



$\Rightarrow \quad V = 0.37v$

$$\Rightarrow \quad \tan\theta = \frac{0.18v}{0.32v} = 0.5625$$

$\Rightarrow \quad \theta = 29.35°$

Since $(\theta + 45°) < 90°$. Therefore, the angle of divergence between particles after collision is less than 90°. Further

$$K_i = \frac{1}{2}mv^2 \text{ and } K_f = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}(2m)(0.37v)^2$$

$\Rightarrow \quad K_f < K_i$

So, collision is inelastic.

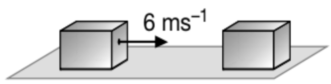
Hence, (B) and (D) are correct.

25. By Law of Conservation of Linear Momentum, we get

$$1(6) + 2(0) = 1\,v + 2(4)$$

$\Rightarrow \quad v = -2 \text{ ms}^{-1}$

Negative sign indicates, that the 1 kg block has velocity leftwards after impact.



Before Collision

After Collision

$$e = \frac{\text{relative velocity of separation}}{\text{relative velocity of approach}} = \frac{v_2 - v_1}{u_1 - u_2}$$

$$\Rightarrow \quad e = \frac{4 - (-2)}{6 - 0} = \frac{6}{6} = 1$$

From $0 < t < 1$ s, we have

$$v_{cm} = \frac{m_1 v_2 + m_2 v_2}{m_1 + m_2} = \frac{1 \times 6 + 2 \times 0}{1 + 2} = 2 \text{ ms}^{-1}$$

For $t > 1$ s, we have

$$v_{cm} = \frac{1(-2) + 2(4)}{1 + 2} = 2 \text{ ms}^{-1}$$

Hence, (B), (C) and (D) are correct.

26. Velocity of ball just before striking the ground is

$$v_0 = \sqrt{2gh} = 10\sqrt{2}\ \text{ms}^{-1}$$

After collision, its speed becomes

$$v_1 = ev_0$$

Since the ball rises to a height of 2.5 m after the first collision, so we have

$$h = \frac{v_1^2}{2g} = \frac{e^2 v_0^2}{2g} = \frac{e^2(200)}{20} = 2.5$$

$$\Rightarrow \quad e^2 = 0.25$$

$$\Rightarrow \quad e = 0.5$$

Impulse equals the change in momentum, so we have

$$|J| = |\Delta p| = |m(v_1 - v_0)| = mv_0(1+e)$$

$$\Rightarrow \quad J = \left(\frac{10}{100}\right)(10\sqrt{2})(1+0.5) \approx 2.1\ \text{kgms}^{-1}$$

Hence, (A), (B) and (C) are correct.

27. Since, $v_1 = \left(\dfrac{m_2 - em_1}{m_1 + m_2}\right)u_1 + \left[\dfrac{(1+e)m_2}{m_1+m_2}\right]u_2$

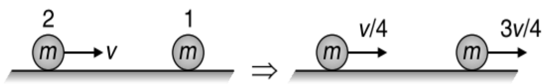
$$\Rightarrow \quad v_1 = \left(\frac{m_1 - m/2}{m+m}\right)v + 0 = \frac{v}{4}$$

Also, $v_2 = \left[\dfrac{(1+e)m_1}{m_1+m_2}\right]u_1 + \left(\dfrac{m_1 - em_2}{m_1+m_2}\right)u_2$

$$\Rightarrow \quad v_2 = \frac{(1+1/2)m}{2m} = \frac{3v}{4}$$

$$|\text{Impulse}| = |\Delta p_1| = |\Delta p_2|$$

$$\Rightarrow \quad |\text{Impulse}| = m\left(\frac{3}{4}v\right) = \frac{3mv}{4}$$



Loss in kinetic energy is $K_i - K_f = -\Delta K$

$$\Rightarrow \quad -\Delta K = \frac{1}{2}mv^2 - \left[\frac{1}{2}m\left(\frac{3v}{4}\right)^2 + \frac{1}{2}m\left(\frac{v}{4}\right)^2\right]$$

$$\Rightarrow \quad -\Delta K = \frac{3}{16}mv^2$$

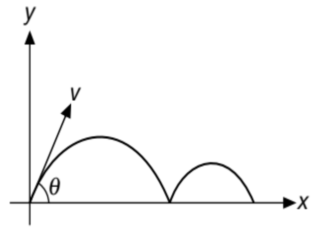
Hence, (B) and (C) are correct.

28. If A and B are free to move, no external forces are acting and hence p and E both are conserved. But when B is fixed, (with the help of an external force) then E is conserved but p is not conserved.

Hence, (A) and (C) are correct.

29. As discussed and studied, the horizontal component u_x remains unchanged while the vertical component u_y becomes eu_y. Since

$$T = \frac{2u\sin\theta}{g} = \frac{2u_y}{g}$$



$$\Rightarrow \quad T \propto u_y$$

$$\Rightarrow \quad a = \frac{T_1}{T_2} = \frac{u_y}{u'_y} = \frac{1}{e}$$

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{u_y^2}{2g}$$

$$\Rightarrow \quad H \propto u_y^2$$

$$\Rightarrow \quad b = \frac{H_1}{H_2} = \frac{u_y^2}{u_y'^2} = \frac{1}{e^2}$$

$$R = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{2u_x u_y}{g}$$

$$\Rightarrow \quad R \propto u_y$$

$$\Rightarrow \quad c = \frac{R_1}{R_2} = \frac{u_y}{u'_y} = \frac{1}{e}$$

Hence, (A), (B) and (D) are correct.

30. Just after collision, we have

$$v_m = \left(\frac{m - 5m}{m + 5m}\right)\sqrt{2gl} = -\frac{2}{3}\sqrt{2gl}$$

and $v_{5m} = \left(\dfrac{2\times m}{m+5m}\right)\sqrt{2gl} = \dfrac{\sqrt{2gl}}{3}$

Also, $T - mg = \dfrac{mv_m^2}{l} = \dfrac{m}{l}\left(\dfrac{8gl}{9}\right)$

$$\Rightarrow \quad T = \frac{17mg}{9}$$

By Law of Conservation of Energy, we get

$$h_m = \frac{v_m^2}{2g} = \frac{4l}{9}$$

Hence, (B), (C) and (D) are correct.

31. Momentum of the system is always conserved. Potential energy of the system is maximum when both the particles move with same velocity i.e., mass becomes double while momentum is constant.

$$K = \frac{P^2}{2m} \quad \text{or} \quad K \propto \frac{1}{m}$$

i.e., at this moment KE becomes half the original i.e., $\dfrac{K}{2}$. The rest is in the form of elastic potential energy.

Therefore, minimum kinetic energy of system is $\frac{K}{2}$ and at the same moment, elastic potential energy of the system is maximum i.e., $\frac{K}{2}$.

Hence, (B) and (D) are correct.

32. $a_{cm} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$

$$a_{cm} = \frac{(m)(g)+(m)(g)}{2m} = g$$

i.e., acceleration of centre of mass of particles is g downwards

Horizontal and vertical components of velocity of centre of mass will be 10 ms^{-1} each. So,

$$H = \frac{v_y^2}{2g}$$

Where, v_y is the vertical component of velocity of centre of mass

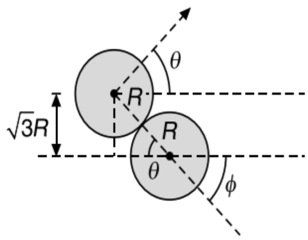
$$\Rightarrow \quad H = \frac{(10)^2}{2\times 10} = 5 \text{ m}$$

Therefore, maximum height of centre of mass from the ground will be $(20+5)$ or 25 m.

Hence, (B) and (C) are correct.

33. For elastic collision oblique collision in which bodies are of equal mass and target body is at rest, then

$$\theta + \phi = 90°$$



From figure, we see that

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \theta = 60°$$

$$\Rightarrow \quad \phi = 30°$$

Hence, (B) and (C) are correct.

34. Let after collision, the velocity of $2m$ and m be and v_1 respectively, then

$$e = \frac{v_2 - v_1}{v}$$

$$\Rightarrow \quad v_2 - v_1 = ev \qquad \ldots(1)$$

Relative velocity $v_r = v_2 - v_1$

Time after which the next collision takes place is

$$t = \frac{2\pi R}{v_r} = \frac{2\pi R}{v_2 - v_1} = \frac{2\pi R}{ev}$$

$$\Rightarrow \quad t = \frac{2\pi R}{v} \qquad \{\because \text{ collision is elastic } e = 1\}$$

So, OPTION **(A)** is correct.

Time after which next collision takes place is $\frac{2\pi R}{v}$, which is independent of mass.

So, OPTION **(D)** is correct.

Hence, (A) and (D) are correct.

35. Out of two blocks, one block of mass m is also moving in vertical direction (downwards). Therefore, CM is moving vertically downwards and momentum of the system is not conserved in vertical direction because of application of force. However, since no force is acting on system in horizontal direction, so momentum of system is conserved along the horizontal direction.

Hence, (C) and (D) are correct.

36. Since, $v_{cm} = \frac{mv + mv}{2m} = v$ and

$$a_{cm} = \frac{mg + mg}{2m} = g$$

Electrostatic forces between the particles are equal and opposite. On the lower mass it will be downwards and on upper mass it will be upwards. Therefore, both the particles will always lie on a vertical line and the horizontal displacement of the lower particle will be less and the upper particle will be more than the value which would had been in the absence of charges on them because time of flight of lower mass will decrease while that of upper mass will increase.

Hence, (A), (B), (C) and (D) are correct.

37. If the man walks along the rails with velocity v relative to car, then some velocity say V is also imparted to car. If M be the mass of car, then by Law of Conservation of Linear Momentum, we get

$$MV = m(v - V)$$

$$\Rightarrow \quad V = \frac{mv}{m+M}$$

So, work done by man is W_{man} given by

$$W_{man} = \frac{1}{2}m(v-V)^2 + \frac{1}{2}mV^2$$

$$W_{man} = \frac{1}{2}\left(\frac{mM}{m+M}\right)v^2 < \frac{1}{2}mv^2$$

Hence, OPTION **(B)** is correct.

If the man moves normal to the rails, then car will not move. Hence, work done by him in this case will just be $\frac{1}{2}mv^2$ and hence OPTION **(C)** is also correct.

Hence, (B) and (C) are correct.

Reasoning Based Questions

1. $E = \frac{p^2}{2m}$

During firing of bullet from gun, momentum is conserved.

So, $p = \text{constant}$

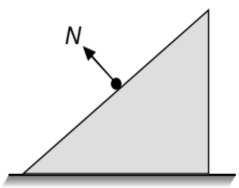
$\Rightarrow \quad E \propto \frac{1}{m}$

$\Rightarrow \quad \frac{E_{gun}}{E_{bullet}} = \frac{m_{bullet}}{m_{gun}}$

Hence, the correct answer is (A).

2. Centre of mass is the property of mass distribution and it is possible that no mass lies at the centre of mass e.g., a ring, a shell.

Hence, the correct answer is (D).

3. 

During collision force exerted by wedge on particle is perpendicular to the inclined face. So linear momentum of wedge is conserved along the face of wedge, because the impact force has got no component along the wedge surface.

Hence, the correct answer is (A).

4. By Law of Conservation of Linear Momentum and Kinetic Energy, we have

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 = \frac{2u_1}{\left(1 + \frac{m_2}{m_1}\right)}$$

Hence, the correct answer is (C).

5. When man moves 4 m w.r.t. boat, let boat move x in opposite direction. So, net displacement of man w.r.t. ground is $(4 - x)$. So,

$60(40 - x) = 60x$

$\Rightarrow \quad 240 - 120 = 0$

$\Rightarrow \quad x = 2\text{ m}$ is distance moved by boat.

Hence, the correct answer is (D).

6. Net external force is zero,

$\Rightarrow \quad \vec{a}_{cm} = \vec{0}$

So, centre of mass may be at rest or may move with a constant velocity.

Hence, the correct answer is (D).

7. Net force acting on the wedge and block system is gravity therefore centre of mass is accelerated in downward direction.

If external force acting on the system is zero, centre of mass may be in rest or moving with constant velocity.

Hence, the correct answer is (C).

8. If m be mass of man, then $M_{\text{ladder}} = M - m$

$$\Delta y_{cm} = \frac{My + (M - m)(-y) + m(h - y)}{2M}$$

$\Rightarrow \quad \Delta y_{cm} = \frac{mh}{2M}$ (upward)

Hence, the correct answer is (D).

9. $(F_{\text{ext}})\Delta t = \text{Impulse} = \Delta p$

Internal forces are always zero, in a system because they always form an action-reaction pair, so no need to say "must be zero".

Hence, the correct answer is (C).

10. The momentum of a system of particles from any frame is given by

$\vec{p} = m\vec{v}_{cm}$

From the centre of mass frame, $\vec{v}_{cm} = \vec{0}$

$\Rightarrow \quad \vec{p} = \vec{0}$

Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. Since the collision is elastic velocity of the centre of mass will remain unchanged.

$\Rightarrow \quad v_{\text{cm}} = \frac{Mv_0}{M + m}$

Hence, the correct answer is (B).

2. Since m remains stationary wrt ground, so

$t_1 = \frac{L}{v_0}$

Hence, the correct answer is (A).

3. Applying conservation of linear momentum, we get

$$Mv_0 = Mv_1 + mv_2 \quad \ldots(1)$$

Since the collision is elastic, so we have

$$\frac{v_2 - v_1}{v_0} = 1 \quad \ldots(2)$$

$\Rightarrow \quad v_2 - v_1 = v_0$

$\Rightarrow \quad v_1 = v_2 - v_0$

CHAPTER 2

$\Rightarrow \quad Mv_0 = M(v_2 - v_0) + mv_2$

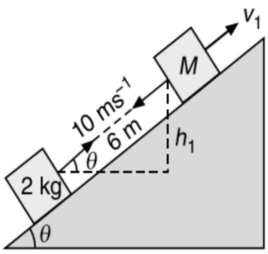
$\Rightarrow \quad v_2 = \dfrac{2Mv_0}{M+m}$

Since $v_{cm} = \dfrac{Mv_0}{m+M}$, so velocity of bead in the centre of mass frame is

$$v = v_2 - v_{cm} = \frac{Mv_0}{m+M}$$

Hence, the correct answer is (B).

4. Let v_1 be the velocity of block 2 kg just before collision, v_2 be the velocity of block 2 kg just after collision and v_3 be the velocity of block M just after collision. Applying work energy theorem, according to which, change in kinetic energy equals the work done by all forces at different stages as shown in Figure.



$$\Delta KE = W_{\text{friction}} + W_{\text{gravity}}$$

$\Rightarrow \quad \left[\dfrac{1}{2}m\left(v_1^2 - (10)^2\right)\right] = -6\mu mg\cos\theta - mgh_1$

Since, $m = 2$ kg, so we get

$$v_1^2 - 100 = -2[6\mu g\cos\theta + gh_1]$$

where, $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-(0.05)^2} \approx 0.99$

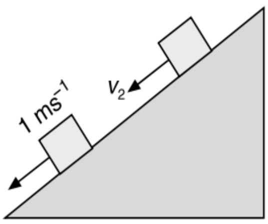
$\Rightarrow \quad v_1^2 = 100 - 2[(6)(0.25)(10)(0.99) + (10)(0.3)]$

$\Rightarrow \quad v_1 \approx 8 \text{ ms}^{-1}$

Hence, the correct answer is (B).

5. Since, $\Delta K = W_{\text{friction}} + W_{\text{gravity}}$

$\Rightarrow \quad \dfrac{1}{2}m\left[(1)^2 - \left(v_2^2\right)\right] = -6\mu mg\cos\theta + mgh_1$



$\Rightarrow \quad 1 - v_2^2 = 2(-6\mu g\cos\theta + gh_1)$

$\Rightarrow \quad 1 - v_2^2 = 2[(-6)(0.25)(10)(0.99) + (10)(0.3)]$

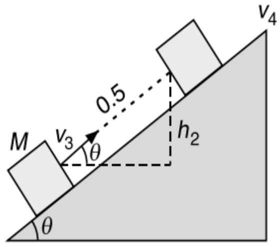
$\Rightarrow \quad 1 - v_2^2 = -23.7$

$\Rightarrow \quad v_2^2 = 24.7$

$\Rightarrow \quad v_2 \approx 5 \text{ ms}^{-1}$

Hence, the correct answer is (C).

6.



Since, $\Delta K = W_{\text{friction}} + W_{\text{gravity}}$

$\Rightarrow \quad \dfrac{1}{2}M\left(0 - v_3^2\right) = -(0.5)(\mu)(M)g\cos\theta - Mgh_2$

$\Rightarrow \quad -v_3^2 = -\mu g\cos\theta - 2gh_2$

$\Rightarrow \quad v_3^2 = (0.25)(10)(0.99) + 2(10)(0.025)$

$\Rightarrow \quad v_3^2 = 2.975$

$\Rightarrow \quad v_3 \approx 1.72 \text{ ms}^{-1}$

Hence, the correct answer is (C).

7. Coefficient of restitution e is given by

$$e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

$\Rightarrow \quad e = \dfrac{v_2 + v_3}{v_1} = \dfrac{5 + 1.72}{8} = \dfrac{6.72}{8}$

$\Rightarrow \quad e \approx 0.84$

Hence, the correct answer is (D).

8. Applying conservation of linear momentum before and after collision, we get

$$2v_1 = Mv_3 - 2v_2$$

$\Rightarrow \quad M = \dfrac{2(v_1 + v_2)}{v_3} = \dfrac{2(8+5)}{1.72} = \dfrac{26}{1.72}$

$\Rightarrow \quad M \approx 15.12$ kg

Hence, the correct answer is (D).

9. Since, $v_1 = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\dfrac{2m_2}{m_1 + m_2}\right)u_2$

and $v_2 = \left(\dfrac{2m_1}{m_1 + m_2}\right)u_1 + \left(\dfrac{m_2 - m_1}{m_1 + m_2}\right)u_2$

where

$$m_1 = m$$

$$m_2 = 2m$$

$$u_1 = -\sqrt{2gH}$$

$$u_2 = \sqrt{2gH}$$

$$\Rightarrow \quad v_1 = \left(\frac{m-2m}{3m}\right)\left(-\sqrt{2gH}\right) + \frac{2(2m)}{3m}\sqrt{2gH}$$

$$\Rightarrow \quad v_1 = \frac{5}{3}\sqrt{2gH}$$

Similarly

$$v_2 = \left(\frac{2m}{3m}\right)\left(-\sqrt{2gH}\right) + \left(\frac{2m-m}{3m}\right)\sqrt{2gH}$$

$$\Rightarrow \quad v_2 = -\frac{2}{3}\sqrt{2gH} + \frac{1}{3}\sqrt{2gH}$$

$$\Rightarrow \quad v_2 = -\frac{1}{3}\sqrt{2gH}$$

Hence, the correct answer is (B).

10. $h_2 = \frac{v_2^2}{2g} = \frac{\frac{1}{9}(2gH)}{2g} = \frac{H}{9}$

Hence, the correct answer is (A).

11. $h_1 = \frac{v_1^2}{2g} = \frac{\frac{25}{9}(2gH)}{2g} = \frac{25H}{9}$

Hence, the correct answer is (C).

12. Since the centre of mass of system remains at rest, so we have

$$m(5l - x) = Mx$$

$$\Rightarrow \quad x = \frac{5ml}{M+m}$$

Hence, the correct answer is (D).

13. By Conservation of Linear Momentum, we get

$$Mv_M - mv_m = 0 \qquad \ldots(1)$$

By Conservation of Energy, we get

$$\frac{1}{2}kl^2 = \frac{1}{2}mv_m^2 + \frac{1}{2}Mv_M^2 \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$v_M = l\sqrt{\frac{km}{M(M+m)}}$$

Hence, the correct answer is (C).

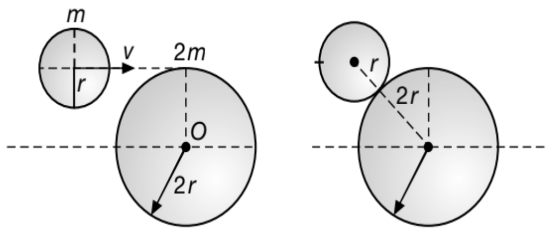
14. Since $v_m = \left(\frac{M}{m}\right)v_M$

$$\Rightarrow \quad v_m = \frac{M}{m}\sqrt{\frac{km}{M(M+m)}}\,l = \sqrt{\frac{kM}{m(M+m)}}\,l$$

$$\Rightarrow \quad KE = \frac{1}{2}mv_m^2 = \frac{kMl^2}{2(M+m)}$$

Hence, the correct answer is (B).

15.



$$r_{cm} = \frac{m(3r)}{3m} = r$$

Hence, the correct answer is (A).

16. $v_{cm} = \frac{mv}{3m} = \frac{v}{3}$

Hence, the correct answer is (D).

17. $I_{cm} = \frac{2}{5}mr^2 + m(2r)^2 + \frac{2}{5}(2m)(2r)^2 + 2mr^2$

$$\Rightarrow \quad I_{cm} = \frac{18}{5}mr^2 + 6mr^2 = \frac{48}{5}mr^2$$

Hence, the correct answer is (C).

18-20. The correct answer is 18(D), 19(C) and 20(C).

Combined solution to 18, 19, 20

Since $\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$

So, velocity of 1 and 2 w.r.t. centre of mass frame are given by

$$\vec{v}_{1/cm} = \vec{v}_1 - \vec{v}_{cm} = \frac{m_2(\vec{v}_1 - \vec{v}_2)}{m_1 + m_2}$$

$$\vec{v}_{2/cm} = \vec{v}_2 - \vec{v}_{cm} = \frac{m_1(\vec{v}_2 - \vec{v}_1)}{m_1 + m_2}$$

Total momentum of system $(\vec{p}_{system})$ equals

$$\vec{p}_{system} = m_1\vec{v}_{1/cm} + m_2\vec{v}_{2/cm}$$

$$\Rightarrow \quad \vec{p}_{system} = \frac{m_1 m_2(\vec{v}_1 - \vec{v}_2)}{m_1 + m_2} + \frac{m_2 m_1(\vec{v}_2 - \vec{v}_1)}{m_1 + m_2} = 0$$

Please note that the total momentum of a system in centre of mass reference frame, i.e. C-frame is always zero

$$K_{1/cm} = \frac{1}{2}m_1 v_{1/cm}^2 \text{ and } K_{2/cm} = \frac{1}{2}m_2 v_{2/cm}^2$$

In C-frame, we have

$$K = \frac{1}{2}\frac{m_1 m_2^2}{(m_1+m_2)^2}(\vec{v}_1 - \vec{v}_2)^2 + \frac{1}{2}\frac{m_2 m_1^2}{(m_1+m_2)^2}(\vec{v}_2 - \vec{v}_1)^2$$

$$\Rightarrow \quad K = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(\vec{v}_1 - \vec{v}_2)^2$$

CHAPTER 2

Consider two particles of mass m_1 and m_2 having velocity $\vec{v}_1$ and $\vec{v}_2$ respectively in ground frame. Let velocity of centre of mass equals $\vec{v}_{cm}$ in ground frame, then

$$\vec{v}_{1/cm} = \vec{v}_1 - \vec{v}_{cm}$$

$$\vec{v}_{2/cm} = \vec{v}_2 - \vec{v}_{cm}$$

Kinetic energy in ground frame

$$K = \frac{1}{2}m_1\left(v_{1/cm}\right)^2 + \frac{1}{2}m_2\left(v_{2/cm}\right)^2$$

$$\Rightarrow \quad K = \frac{1}{2}m_1\left(v_1^2 + v_{cm}^2 - 2\vec{v}_1\cdot\vec{v}_{cm}\right) + \frac{1}{2}m_2\left(v_2^2 + v_{cm}^2 - 2\vec{v}_2\cdot\vec{v}_{cm}\right)$$

$$\Rightarrow \quad K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}(m_1+m_2)v_{cm}^2 - (m_1\vec{v}_1 + m_2\vec{v}_2)\cdot\vec{v}_{cm}$$

$$\Rightarrow \quad K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{1}{2}(m_1+m_2)v_{cm}^2$$

$$\Rightarrow \quad K_{\text{in C-frame}} = K_{\text{in ground frame}} - \frac{1}{2}(m_1+m_2)v_{cm}^2$$

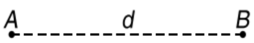
Since $m_1 + m_2 = m$ and $v_{cm} = v$

$$\Rightarrow \quad K_{\text{in ground frame}} = K_{\text{in cm frame}} + \frac{1}{2}mv^2$$

21-23. The correct answer is 21(B), 22(C) and 23(D).

Combined solution to 21, 22, 23

Applying Law of Conservation of Linear Momentum, we get



$$40v + 4\times 5 = 0$$

$$\Rightarrow \quad v = -\frac{1}{2}\ \text{ms}^{-1}$$

$$\Rightarrow \quad |\vec{v}| = 0.5\ \text{ms}^{-1}$$

Applying Law of Conservation of Linear Momentum, we get

$$20 = 44v$$

$$\Rightarrow \quad v = \frac{10}{22} = \frac{5}{11}\ \text{ms}^{-1}$$

24. If v_1 and v_2 are the velocities of object of mass m and block of mass $4m$, just after collision then by conservation of momentum,

$$mv = mv_1 + 4mv_2, \text{ i.e., } v = v_1 + 4v_2 \quad \ldots(1)$$

Further, as collision is elastic

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}4mv_2^2, \text{ i.e., } v^2 = v_1^2 + 4v_2^2 \quad \ldots(2)$$

Solving, these two equations we get either

$$v_2 = 0 \text{ or } v_2 = \frac{2}{5}v$$

Therefore, $v_2 = \frac{2}{5}v$

Substituting in equation (1), $v_1 = \frac{3}{5}v$

When $v_2 = 0$, $v_1 = v_2$, but it is physically unacceptable. Now, after collision the block B will start moving with velocity v_2 to the right. Since, there is no friction between blocks A and B the upper block A will stay at its position and will topple if B moves a distance s such that

$$s > 2d \quad \ldots(3)$$

(Toppling will take place if line of action of weight does not pass through the base area in contact)

However, the motion of B is retarded by frictional force $f = \mu(4m + 2m)g$ between table and its lower surface. So, the distance moved by B till it stops

$$0 = v_2^2 - 2\left(\frac{6\mu mg}{4m}\right)s \text{ , i.e., } s = \frac{v_2^2}{3\mu g}$$

Substituting this value of s in equation (3), we find that for toppling of A

$$v_2^2 > 6\mu gd \text{ or } \frac{2}{5}v > \sqrt{6\mu gd} \qquad \left\{\text{as } v_2 = \frac{2v}{5}\right\}$$

i.e., $v > \frac{5}{2}\sqrt{6\mu gd}$ or $v_{min} = v_o = \frac{5}{2}\sqrt{6\mu gd}$

Hence, the correct answer is (C).

25. If $v = 2v_o = 5\sqrt{6\mu gd}$, the object will rebound with speed

$$v_1 = \frac{3}{5}v = 3\sqrt{6\mu gd}$$

and as time taken by it to fall down

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2d}{g}} \qquad \{\because \text{ as } h = d\}$$

The horizontal distance moved by it to the left of P in this time

$$x = v_1t = 6d\sqrt{3\mu}$$

Hence, the correct answer is (B).

26. $mu_x = \frac{mu_x}{2} + Mv_x$

$$\Rightarrow \quad \frac{mu\cos\theta}{2} = Mv_x$$

$$\Rightarrow \quad v_x = \frac{mu\cos\theta}{2M}$$

Hence, the correct answer is (C).

27. Loss in kinetic energy of bullet block system is

$$-\Delta K = \frac{1}{2}mu_x^2 - \left[\frac{1}{2}m\left(\frac{ux}{2}\right)^2 + \frac{1}{2}Mv_x^2\right]$$

$\Rightarrow \quad -\Delta K = \frac{1}{2}mu_x^2 - \frac{1}{8}mu_x^2 - \frac{1}{2}Mv_x^2$

$\Rightarrow \quad -\Delta K = \frac{3}{8}mu_x^2 - \frac{1}{2}Mv_x^2$

$\Rightarrow \quad -\Delta K = \frac{3mu^2\cos^2\theta}{8} - \frac{1}{2}M\left(\frac{m^2u^2\cos^2\theta}{4M^2}\right)$

$\Rightarrow \quad -\Delta K = \frac{3mu^2\cos^2\theta}{8} - \frac{m^2u^2\cos^2\theta}{8M}$

$\Rightarrow \quad -\Delta K = \frac{3mMu^2\cos^2\theta}{8M} - \frac{mu^2\cos^2\theta}{8M}$

$\Rightarrow \quad -\Delta K = \frac{m(3M-m)u^2\cos^2\theta}{8M}$

Hence, the correct answer is (C).

28. $|\Delta v_x| = v_0\cos\theta$

Hence, the correct answer is (C).

29. Since collision is elastic, so for the particle

$|\Delta v_x| = v_0\cos\theta$

Hence, the correct answer is (C).

30. During collision,

$r_\perp = 0$, so $L = mvr_\perp = 0$

Hence, the correct answer is (B).

31. By Newton's Law

$$F - \mu v = (M+\mu t)\frac{dv}{dt} \quad \ldots(1)$$

$\Rightarrow \quad \int_0^v \frac{dv}{F-\mu v} = \int_0^t \frac{dt}{M+\mu t}$

$\Rightarrow \quad v = \frac{Ft}{M_0+\mu t}$

Hence, the correct answer is (D).

32. For constant velocity, $\frac{dv}{dt} = 0$

So, from (1), we get $F = \mu v$

$\Rightarrow \quad P = Fv = \mu v^2$

Hence, the correct answer is (C).

33. $K = \frac{1}{2}Mv^2$

For constant velocity, v = constant

$\Rightarrow \quad \frac{dK}{dt} = \frac{v^2}{2}\frac{dM}{dt}$

Since $M = M_0 + \mu t$

$\Rightarrow \quad \frac{dM}{dt} = \mu$

$\Rightarrow \quad \frac{dK}{dt} = \frac{1}{2}\mu v^2$

Hence, the correct answer is (D).

34. No additional force is required.

Hence, the correct answer is (C).

35. $v_1 = \left(\frac{m_1-m_2}{m_1+m_2}\right)u_1$

$v_2 = \left(\frac{2m_1}{m_1+m_2}\right)u_1$

For $m_1 \gg m_2$

$v_1 \simeq u_1$

$v_2 \cong 2u_1$

Hence, the correct answer is (C).

36. If the two bodies are of equal mass and second body is at rest, then after impact they exchange their velocities i.e. body one comes to rest and body two moves with velocity of body 1 and hence the maximum transfer of momentum takes place when $m_1 = m_2$ and m_2 is at rest

$\Rightarrow \quad 3 - x^2 + x = 1$

$\Rightarrow \quad x^2 - x - 2 = 0$

$\Rightarrow \quad x^2 - 2x + x - 2 = 0$

$\Rightarrow \quad x(x-2) + 1(x-2) = 0$

$\Rightarrow \quad (x+1)(x-2) = 0$

$\Rightarrow \quad x = -1$ or $x = 2$

Hence, the correct answer is (B, D).

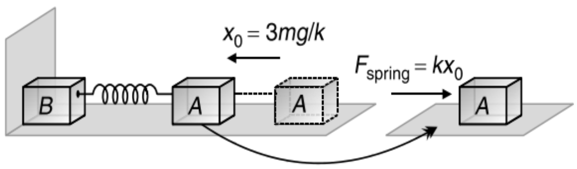
37. Fraction of energy lost by m_1 is $f = \frac{\frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2}{\frac{1}{2}m_1u_1^2}$

Put $v_1 = \left(\frac{m_1-m_2}{m_1+m_2}\right)u_1$, we get

$f = \frac{4m_1m_2}{(m_1+m_2)^2}$

Hence, the correct answer is (A).

38. The normal reaction by the wall provides external force to the system of blocks A and B. The block B will not move but the block A will move due to initial compression of spring.



Acceleration of block A just after release is

$$a_A = \frac{F_{sp}}{m} = \frac{k(3mg/k)}{m}$$

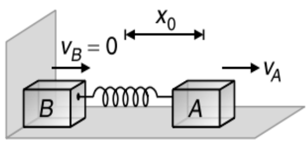
$\Rightarrow$ $a_A = 3g$, towards right

and acceleration of block B is zero. Hence acceleration of the system is

$$a_{cm} = \frac{m_A a_A + m_B a_B}{m_A + m_B} = \frac{m(3g) + m(0)}{(m+m)} = \frac{3}{2}g$$

Hence, the correct answer is (C).

39. The block B will lose contact with wall when block is moving towards right and spring attains its natural length.



For velocity of block A, using $\Delta K + \Delta U = 0$

$$\left(\frac{1}{2}mv_A^2 - 0\right) + \left(0 - \frac{1}{2}kx_0^2\right) = 0$$

$$\Rightarrow \quad v_A^2 = \frac{k}{m}x_0^2$$

$$\Rightarrow \quad v_A = \sqrt{\frac{k}{m}}x_0 = \left(\sqrt{\frac{k}{m}}\right)\left(\frac{3mg}{k}\right)$$

$$\Rightarrow \quad v_A = 3g\sqrt{\frac{m}{k}}$$

The velocity of block B just at the time of losing contact with wall will be zero. So, we have

$$v_{cm} = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{m\left(3g\sqrt{\frac{m}{k}}\right)}{m+m} = \frac{3g}{2}\sqrt{\frac{m}{k}}$$

Hence, the correct answer is (C).

40. When both the blocks move with same velocity, the extension spring will be maximum. The common velocity of the blocks is equal to the velocity of CM. Applying Conservation of Mechanical Energy to the initial and the final state when extension is maximum, we get

$$\Delta K + \Delta U = 0$$

$$\Rightarrow \quad \left(\frac{1}{2}(m+m)v_{cm}^2 - 0\right) + \left(\frac{1}{2}kx^2 - \frac{1}{2}kx_0^2\right) = 0$$

$$\Rightarrow \quad \frac{1}{2}(2m)\left(\frac{9g^2m}{4k}\right) + \frac{1}{2}k\left(x^2 - \frac{9m^2g^2}{k^2}\right) = 0$$

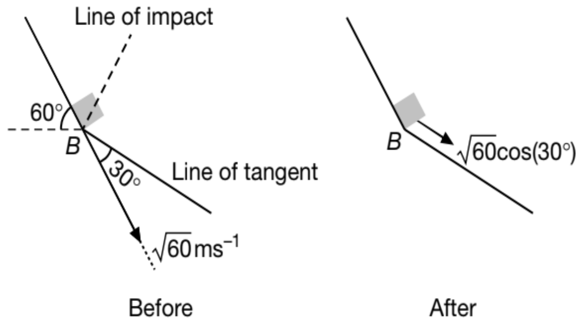
$$\Rightarrow \quad \frac{9m^2g^2}{4k} + \left(\frac{1}{2}kx^2 - \frac{9m^2g^2}{2k}\right) = 0$$

$$\Rightarrow \quad kx^2 = \frac{9m^2g^2}{2k}$$

$$\Rightarrow \quad x = \frac{3mg}{k\sqrt{2}}$$

Hence, the correct answer is (C).

41.



Let speed of block just before it strikes the second inclined plane be v then,

$$\frac{1}{2}mv^2 = mg\left(\sqrt{3}\tan 60°\right)$$

$$\Rightarrow \quad v = \sqrt{60}\ \text{ms}^{-1}$$

Speed of block immediately after it strikes the second incline is $\sqrt{45}\ \text{ms}^{-1}$. because in perfectly inelastic collision the component of velocity along line of impact becomes zero.

Hence, the correct answer is (B).

42. By Conservation of Mechanical Energy

$$\frac{1}{2}mv_C^2 = \frac{1}{2}m\left(\sqrt{45}\right)^2 + mg\left(3\sqrt{3}\tan 30°\right)$$

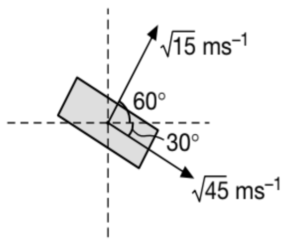
$$\Rightarrow \quad v_C^2 = 45 + 60 = 105$$

$$\Rightarrow \quad v_C = \sqrt{105}\ \text{ms}^{-1}$$

Hence, the correct answer is (B).

43. If collision is completely elastic, then vertical component of velocity becomes,

$$\sqrt{45}\sin 30° - \sqrt{15}\sin 60° = \frac{\sqrt{45}}{2} - \frac{\sqrt{15}\times\sqrt{3}}{2} = 0$$



Hence, the correct answer is (C).

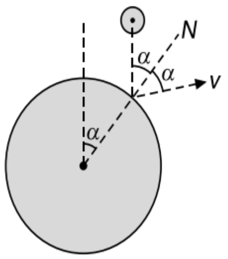
44. Maximum potential energy is stored when the relative velocities of the balls become zero (i.e., both have the same velocities) and the potential energy is stored at the cost of kinetic energy.

Since, in both the cases, when the relative velocity becomes zero, the balls have same velocities.
Hence, the correct answer is (C).

45. Since, there is no external force, center of mass will move with uniform speed all the time.
Hence, the correct answer is (B).

46. Since, the collision is inelastic kinetic energy will not be conserved but the velocity of center of mass will not change because there is no external force.
Hence, the correct answer is (B).

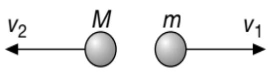
47. Since collision is elastic, so angle of reflection is same as that of incidence angle. In frame of big sphere, small sphere will rebound at an angle 2α with vertical as shown in Figure.



Hence, the correct answer is (B).

48. In frame of larger sphere, the smaller sphere is travelling with $2v_0$ before second collision. Since larger sphere is massive in comparison to smaller sphere, so the smaller sphere will rebound with same velocity $2v_0$ as collision is elastic and surface is frictionless. The small sphere will reflect at same angle.
Hence, the correct answer is (C).

49. At an instant when m is just leaving M, let v_1 and v_2 be the velocities of m and M (both horizontal).



By Law of Conservation of Linear Momentum, we have

$$1 \times v_1 = 4v_2 \qquad \ldots(1)$$

By Law of Conservation of Mechanical Energy, we have

$$(1)(10)(4-2) = \frac{1}{2}(1)v_1^2 + \frac{1}{2}(4)v_2^2$$

$$\Rightarrow \quad v_1^2 + 4v_2^2 = 40 \qquad \ldots(2)$$

Solving these two equations, we get

$$v_2 = \sqrt{2} \text{ ms}^{-1} \text{ and } v_1 = 4\sqrt{2} \text{ ms}^{-1}$$

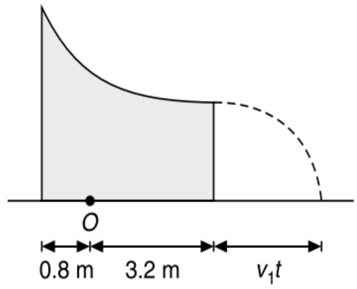
Hence, the correct answer is (D).

50. Let d be the displacement of M, when m just leaves M, then position of centre of mass remains fixed, so

$$\sum m_R x_L = \sum m_L x_L$$

$$\Rightarrow \quad 4d = 1(4-d)$$

$$\Rightarrow \quad d = 0.8 \text{ m}$$



i.e., m is already at $x = 3.2$ m at this instant. Time taken by m to fall to ground from this instant will be

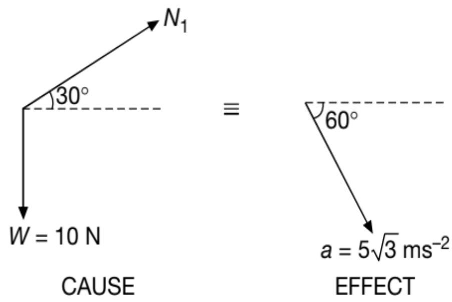
$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 2}{10}} = \sqrt{\frac{2}{5}} \text{ s}$$

The x-coordinate of m where it strikes the ground will be

$$x = 3.2 + v_1 t = 3.2 + 4\sqrt{2} \times \sqrt{\frac{2}{5}} \approx 6.8 \text{ m}$$

Hence, the correct answer is (C).

51. Let N be normal reaction between M and m. Then free body diagram of m will be as shown below.



$$\sum F_H = ma_H$$

$$\Rightarrow \quad N\cos 30^\circ = (1)5\sqrt{3}\cos(60^\circ)$$

$$\Rightarrow \quad N = 5 \text{ Newton}$$

Hence, the correct answer is (B).

52. Since, $N' = Mg + N\sin(30^\circ)$

$$\Rightarrow N' = 40 + 2.5$$

$$\Rightarrow N' = 42.5 \text{ N}$$

Hence, the correct answer is (C).

53. Relative acceleration of the bullet is zero

Relative velocity of the bullet is 100 ms^{-1}

$$\Rightarrow \quad t = \frac{\text{Initial Separation}}{\text{Relative Velocity of Approach}} = \frac{100}{100} = 1 \text{ s}$$

Hence, the correct answer is (A).

54. $|\Delta\vec{p}| = F\Delta t = (m_1 + m_2)g\Delta t$

$\Rightarrow \quad |\Delta\vec{p}| = \left(\dfrac{30+20}{1000}\right)(10)(10^{-4})$

$\Rightarrow \quad |\Delta\vec{p}| = 5\times10^{-5}$ Ns

Hence, the correct answer is (B).

55. Since collision is perfectly inelastic, so

$(20)(90)-(30)(10)=(20+30)v$

$\Rightarrow \quad v = 30\ \text{ms}^{-1}$

Hence, the correct answer is (D).

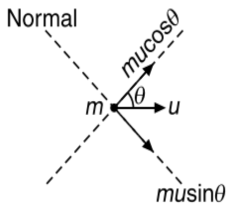
56. The block is not free to move along horizontal or vertical. It can move along the inclined surface when the bullet collides, jerk or impulse will be generated by the inclined surface. As the surface is smooth, only normal impulse will be generated. So, the forces along the inclined will not be impulsive and conservation of linear momentum is applicable along the inclined.

Hence, the correct answer is (D).

57. As discussed above, the impulse by inclined plane will be along normal to surface. The impulsive force between bullet and block will be internal. So net impulse will be along normal to the incline.

Hence, the correct answer is (C).

58. By impulse momentum theorem, $I = \Delta p$



Initial momentum along the normal is

$p_i = -mu\sin\theta$

Also, $p_f = 0$

$\Rightarrow \quad I = \Delta p = mu\sin\theta$

Hence, the correct answer is (A).

59. Applying conservation of linear momentum along the incline, we get

$mu\cos\theta = (m+M)u'$

$\Rightarrow \quad u' = \dfrac{mu\cos\theta}{m+M}$

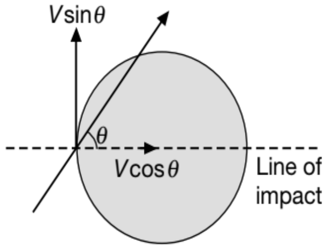
Stopping distance $s = \dfrac{u^2}{2a}$

$\Rightarrow \quad s = \left(\dfrac{mu\cos\theta}{m+M}\right)^2 \times \dfrac{1}{2g\sin\theta}$

$\Rightarrow \quad s = \dfrac{m^2u^2\cos^2\theta}{2g\sin\theta(m+M)^2}$

Hence, the correct answer is (C).

60. For the following collision line of impact is as shown in the Figure.



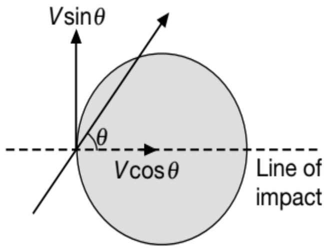
So, velocity of approach along line of impact is

$v_n = V\cos\theta \qquad \ldots(1)$

Hence, the correct answer is (C).

61. Let V_{cm} be the velocity of cm of the disc after collision and V' be velocity of ball after collision, then

$e = \dfrac{V_{cm} - V}{V\cos\theta} \qquad \ldots(2)$



Conserving Linear Momentum along the line of impact, we get

$MV\cos\theta = MV_{cm} + MV'$

From equation (2), we get

$MV\cos\theta = MV_{cm} + M(-eV\cos\theta + V_{cm})$

$\Rightarrow \quad V_{cm} = \dfrac{V\cos\theta(1+e)}{2}$

Hence, the correct answer is (D).

62. Speed of ball after collision is

$V' = V\sqrt{\dfrac{\cos^2\theta(1-e)^2}{4} + \sin^2\theta}$

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. A → (p); B → (r); C → (s); D → (r)

(A) $K = \dfrac{p^2}{2m}$

$K' = \dfrac{(3p)^2}{2m} = 9\left(\dfrac{p^2}{2m}\right) = 9K$

% increase in $K = 800\% = 8(100\%)$

(B) $p=\sqrt{2\text{ mK}}$

$p'=\sqrt{2(4K)m}=2\sqrt{2Km}=2p$

% increase in $p=100\%=1(100\%)$

(C) $K=\dfrac{p^2}{2m}$

For small percentage changes,
$\%\,\text{increase in}\,K=2\,(\%\,\text{increase in}\,p)=2\%=4(0.5\%)$

(D) $p=\sqrt{2mK}$

% increase in $p=\dfrac{1}{2}$ (% increase in K) $=0.5\%=1(0.5\%)$

2. A → (p); B → (p); C → (q); D → (r)

$$\frac{1}{2}mv^2=\mu mgx$$

$$\Rightarrow\quad v=\sqrt{2Mgx}=\sqrt{2\times0.5\times10\times2.5}=5\text{ ms}^{-1}$$

Apply Law of Conservation of Energy, we get

$$\frac{1}{2}mv^2=mgR(1-\cos\theta)$$

$$\Rightarrow\quad v=5\text{ ms}^{-1}$$

Since, $v^2=u^2+2as$

$$\Rightarrow\quad v^2=25-2\times\mu g\times1.6=3\text{ ms}^{-1}$$

Since, $e=\dfrac{v_2-v_1}{u_1-u_2}$

$$\Rightarrow\quad e=1$$

3. A → (q); B → (q); C → (p); D → (s)

The time of flight of the particle launched from the ground is

$$t_1=\frac{2u}{g}=\frac{2\times20}{10}=4\text{ s}$$

The time taken by the second particle dropped from a height of 180 m is

$$t_2=\sqrt{\frac{2h}{g}}=\sqrt{\frac{2(180)}{10}}=6\text{ s}$$

Taking the downward direction as positive, we have at $t=0$

$$a_{\text{CM}}=\frac{m_1a_1+m_2a_2}{m_1+m_2}=\frac{(m)(10)+(m)(10)}{2m}=10\text{ ms}^{-2}$$

$$v_{\text{CM}}=\frac{m_1v_1+m_2v_2}{m_1+m_2}=\frac{(m)(-20)+(m)(0)}{2m}=-10\text{ ms}^{-1}$$

At $t=5$ s, the first particle will come to rest whereas the second particle will still be moving with a downward velocity

$$v_2=gt=50\text{ ms}^{-1}$$

So, at $t=5$ s, we have

$$a_{\text{CM}}=\frac{(m)(0)+(m)(10)}{2m}=5\text{ ms}^{-2}$$

$$v_{\text{CM}}=\frac{(m)(0)+(m)(50)}{2m}=25\text{ ms}^{-1}$$

4. A → (q); B → (s); C → (r); D → (p)

$$\vec{F}_{\text{cm}}=\vec{F}_1+\vec{F}_2$$

$$\vec{F}_{\text{cm}}=m_1\vec{a}_1+m_2\vec{a}_2=\left(2\hat{i}+8\hat{j}\right)$$

$$\Rightarrow\quad\left|\vec{F}_{\text{cm}}\right|=\sqrt{4+64}=\sqrt{68}\text{ unit}$$

$$\vec{V}_{\text{cm}}=\frac{m_1\vec{v}_1+m_2\vec{v}_2}{m_1+m_2}$$

$$\Rightarrow\quad\vec{V}_{\text{cm}}=\frac{(1)\left(4\hat{i}\right)+(2)\left(4\hat{j}\right)}{3}=\frac{4\hat{i}+8\hat{j}}{3}$$

$$\Rightarrow\quad\left|\vec{v}_{\text{cm}}\right|=\frac{1}{3}\sqrt{16+64}=\frac{\sqrt{80}}{3}\text{ unit}$$

$$\Rightarrow\quad m\left|\vec{v}_{\text{cm}}\right|=\sqrt{80}\text{ unit}$$

Now, $\vec{r}_1=\displaystyle\int_0^2\vec{v}_1dt=\left(4\hat{i}\right)$

and $\vec{r}_2=\displaystyle\int_0^2\vec{v}_2dt=\left(\frac{8}{3}\hat{j}\right)$

$$\Rightarrow\quad\vec{r}_{\text{cm}}=\frac{m_1\vec{r}_1+m_2\vec{r}_2}{m_1+m_2}=\frac{(1)\left(4\hat{i}\right)+(2)\left(\frac{8}{3}\hat{j}\right)}{3}$$

$$\Rightarrow\quad\vec{r}_{cm}=\left(\frac{4}{3}\hat{i}+\frac{16}{9}\hat{j}\right)$$

$$\Rightarrow\quad\left|\vec{r}_{\text{cm}}\right|=\sqrt{\frac{16}{9}+\frac{256}{81}}=\frac{20}{9}$$

5. A → (p, r, s); B → (p, r, s); C → (p); D → (p, q)

Since, $v_{\text{cm}}=\dfrac{\text{Total momentum}}{\text{Total mass}}=\dfrac{\Sigma p}{\Sigma m}$

$$\Rightarrow\quad v_{\text{cm}}=\frac{2\times3}{3+6}=\frac{2}{3}\text{ ms}^{-1}=\text{constant}$$

At maximum deformation in the spring, both the blocks move together with the velocity that equals the velocity of centre of mass, so we have

$$v_{3\text{ kg}}=v_{6\text{ kg}}=v_{\text{cm}}=\frac{2}{3}\text{ ms}^{-1}$$

Up to the maximum extension in spring, the spring force on 6 kg is towards left. So 6 kg block will accelerate and its velocity will be maximum.

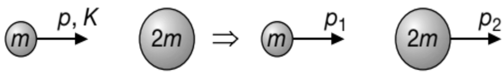
6. A → (r); B → (p); C → (q); D → (s)

$$p_1 + p_2 = p \quad \dots(1)$$

Further, $K_1 + K_2 = K$

$$\Rightarrow \frac{p_1^2}{2m} + \frac{p_2^2}{4m} = \frac{p^2}{2m}$$

$$\Rightarrow 2p_1^2 + p_2^2 = 2p^2 \quad \dots(2)$$



Before Collision After Collision

Solving these two equations we get,

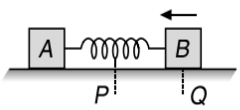
$$p_2 = \frac{4}{3}p$$

and $p_1 = -\frac{p}{3}, K_1 = \frac{K}{9}$

and $K_2 = \frac{8K}{9}$

7. A → (r); B → (q); C → (s); D → (p)

Let the spring be compressed to state P from its natural state at Q as shown in Figure.



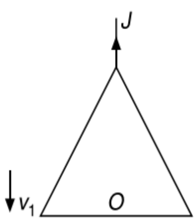
When released, the block B goes from P to Q, so we have

$$a_B = \frac{kx}{m_B}$$

and $a_{CM} = \frac{m_B a_B}{m_A + m_B} = \frac{(m_B)(kx)}{m_A + m_B}$

When the block goes from P to Q, we see that the compression x decreases, so a_{CM} decreases. After Q, we observe that A leaves contact with the wall and spring comes to its natural length, hence net force on system is zero. Therefore, a_{CM} is zero. Also, we see that from P to Q, the velocity of B increases and hence velocity of centre of mass also increases. After this the acceleration of centre of mass, i.e. a_{CM} becomes zero and hence v_{CM} becomes constant.

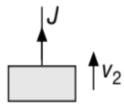
8. A → (q); B → (r); C → (p); D → (p)



According to Impulse - Momentum Theorem, we have

$$J = \vec{P}_2 - \vec{P}_1$$

$$\Rightarrow -J = (0.5 + 1.5)v_2 - 0.5 \times 16 \quad \dots(1)$$



and $J = 3 \times v_2 \quad \dots(2)$

Solving (1) and (2), we get

$$v_2 = 1.6 \text{ ms}^{-1}$$

Since, $a = \frac{(m_2 - m_1)g}{(m_1 + m_2)} = \left(\frac{3-2}{3+2}\right)g = \frac{g}{5}$

$$\Rightarrow a = 2 \text{ ms}^{-2}$$

Also, $h_{max} = \frac{v^2}{2a} = \frac{1.6^2}{2 \times 2} = 0.64 \text{ m}$

and $t = \frac{24}{a} = 0.64 \text{ s}$

9. A → (p, r); B → (p, r); C → (q); D → (q)

The velocity of the centre of mass is

$$v_{cm} = \frac{\text{Total momentum}}{\text{Total mass}} = \frac{\Sigma p}{\Sigma m}$$

$$\Rightarrow v_{cm} = \frac{(1)(10) + (2)(-5)}{3} = 0 = \text{constant}$$

$$\Rightarrow p_{cm} = Mv_{cm} = 0 = \text{constant}$$

Finally, both blocks the will stop.

10. A → (q); B → (p, r); C → (p, r); D → (r, s)

Since, $\vec{F} = \frac{Md\vec{v}_{cm}}{dt} = M\vec{a}_{cm}$

$$\Rightarrow M\frac{d\vec{v}_{cm}}{dt} = M\vec{a}_{cm} = \vec{0}$$

$$\Rightarrow \vec{a}_{cm} = \vec{0}$$

So, (A) → (q)

But $\vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt} = \vec{0}$

$$\Rightarrow \vec{v}_{cm} = \text{constant}$$

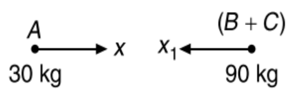
and this constant value may be zero also.

So, (B) → (p, r) and similarly (C) → (p, r)

No comments can be given about velocities of individual particles. So, (D) → (r, s)

11. A → (s); B → (p); C → (p); D → (r)

(a) When A moves x towards right, then B and C will move towards left as shown in Figure.

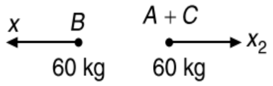


Under these conditions the centre of mass of the system will remain fixed, so we have

$$30x = 90x_1$$

$$\Rightarrow \quad x_1 = \frac{x}{3}$$

(b) When B moves x towards left, then A and C will move towards right as shown in Figure

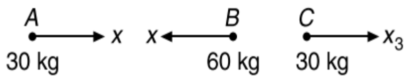


Under these conditions the centre of mass of the system will remain fixed, so we have

$$60x = 60x_2$$

$$\Rightarrow \quad x_2 = x$$

(c) When A moves x towards right and B moves x towards left, then C will move towards right as shown in Figure.

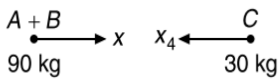


Under these conditions the centre of mass of the system will remain fixed, so we have

$$30x + 30x_3 = 60x$$

$$\Rightarrow \quad x_3 = x$$

(d) When A and B both move x towards right, then C will move towards left as shown in Figure.



Under these conditions the centre of mass of the system will remain fixed, so we have

$$90x = 30x_4$$

$$\Rightarrow \quad x_4 = 3x$$

12. A → (s); B → (s); C → (p, q, r); D → (p, q, r)

Conceptual

13. A → (r); B → (s); C → (p); D → (q)

By conservation of linear momentum, we have

$$mv = mv_A + mv_B$$

$$\Rightarrow \quad v_A + v_B = v \qquad \ldots(1)$$

Also, applying the definition of e, we get

$$e = -\left(\frac{v_A - v_B}{0 - v}\right)$$

$$\Rightarrow \quad v_A - v_B = ev \qquad \ldots(2)$$

From equations (1) and (2), we get

$$v_B = \left(\frac{1-e}{2}\right)v \text{ and } v_A = \left(\frac{1+e}{2}\right)v$$

For elastic collision, $e = 1$, so $v_A = v$

For perfectly inelastic collision, $e = 0$, so $v_A = \frac{v}{2}$

For inelastic collision having $e = \frac{1}{2}$, $v_A = \frac{3v}{4}$

For inelastic collision having $e = \frac{1}{4}$, $v_A = \frac{5v}{8}$

14. A → (q); B → (q); C → (q); D → (p)

$$v_{cm} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = 1 \text{ ms}^{-1}$$

So, (A) → (q)

At maximum compression by Law of Conservation of Linear Momentum, we get

$$(2)(2) + (2)(0) = (2+2)\, v_{combined}$$

$$\Rightarrow \quad v_{combined} = 1 \text{ ms}^{-1}$$

So, (B) → (q)

Now, $U_{max} = K_i - K_f$

$$\Rightarrow \quad U_{max} = \frac{1}{2} \times 2 \times (2)^2 - \frac{1}{2} \times 4 \times (1)^2 = 2 \text{ J}$$

So, (D) → (p)

Since, $\frac{1}{2}Kx_{max}^2 = 2$ J

$$\Rightarrow \quad x_{max} = 1 \text{ m}$$

So, (C) → (q)

15. A → (r); B → (r, s); C → (p, q); D → (p)

(a) Let T be the tension in string connecting m_1 to m_2, then tension in the string held by the man is $2T$. For equilibrium of m_2, we have

$$T = m_2g = 200 \text{ N}$$

For equilibrium of $(M + m_1)$, we have

$$3T = W_{man} + m_1g$$

$$\Rightarrow \quad 600 = W_{man} + 100$$

$$\Rightarrow \quad W_{man} = 500 \text{ N}$$

(b) The net upward force acting on the system is $4T$ and the net downward force acting on the system is the weight of all. For the centre of mass to accelerate upwards, we must have

$$4T > 200 + 100 + 500$$

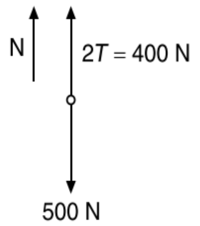
$$\Rightarrow \quad T > 200 \text{ N}$$

(c) For the centre of mass to accelerate downwards, we must have

$$4T < 200 + 100 + 500$$

$$\Rightarrow \quad T < 200 \text{ N}$$

(d) When in equilibrium, the forces acting on the man are shown in Figure.



Since, $\Sigma F = 0$

$\Rightarrow \quad N + 400 = 500$

$\Rightarrow \quad N = 100 \text{ N}$

16. A → (q); B → (p); C → (r); D → (r)
Apply conservation of linear momentum in horizontal direction and conservation of mechanical energy.

17. A → (q, r); B → (q, r); C → (p, s); D → (p)
In the absence of external force, the state of centre of mass will remain unchanged and the linear momentum of system remains conserved e.g. a man walking on a platform which lies on a frictionless horizontal surface.
Centre of mass of a body can lie inside or outside it. Centre of mass of a solid cube will lie inside the cube.

18. A → (r); B → (r); C → (q); D → (p)

For (A), $p = \sqrt{2mK} = \sqrt{2 \times 0.5 \times 4} = 2 \text{ kgms}^{-1}$

For (B), $p_{CM} = p_A + p_B = 0 + 2 = 2 \text{ kgms}^{-1}$

For (C), i.e. at maximum compression the blocks move with a common velocity and will have momentum equal to half the momentum of centre of mass of the particles.

$$\Rightarrow \quad p_A = p_B = \frac{p_{CM}}{2} = 1 \text{ kgms}^{-1}$$

For (D), since we know that for an elastic collision between two equal masses, the velocities are interchanged. Hence, B comes to rest and A starts moving.

19. A → (q); B → (s); C → (p); D → (r)

Using $x_{cm} = \dfrac{m_1x_1 + \ldots + m_nx_n}{m_1 + \ldots + m_n}$

and $y_{cm} = \dfrac{m_1y_1 + \ldots + m_ny_n}{m_1 + \ldots + m_n}$

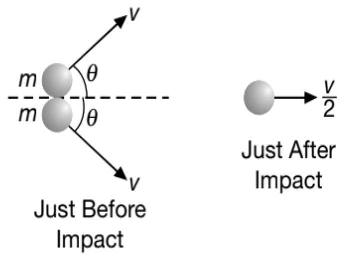
Integer/Numerical Answer Type Questions

1. $mv\cos\theta + mv\cos\theta = (m+m)\dfrac{v}{2}$

$2mv\cos\theta = mv$

$\Rightarrow \quad \cos\theta = \dfrac{1}{2}$

$\Rightarrow \quad \theta = 60^\circ$



So, angle between the two before collision is

$\Rightarrow \quad \phi = 2\theta = 120^\circ$

$\Rightarrow \quad \dfrac{\phi}{20} = 6^\circ$

2. $v'_A = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_A = \left(\dfrac{3-2}{3+2}\right)(2) = 0.4 \text{ ms}^{-1}$

$v'_B = \left(\dfrac{2m_A}{m_A + m_B}\right)v_B = \left(\dfrac{6}{3+2}\right)(2) = 2.4 \text{ ms}^{-1}$

Retardation of both the blocks will be,

$$a = \mu_K g = 0.4 \times 10 = 4 \text{ ms}^{-2}$$

$$s_A = \frac{v_A'^2}{2a} \text{ and } s_B = \frac{v_B'^2}{2a}$$

Therefore, the desired distance is,

$$d = s_B - s_A = \frac{v_B'^2 - v_A'^2}{2a} = \frac{(2.4)^2 - (0.4)^2}{8}$$

$$d = \frac{5.6}{8} = 0.7 \text{ m} = 70 \text{ cm}$$

3. For stable circular motion of centre of mass of rope, we use

$$T = m\left(\frac{l}{2}\right)\omega^2$$

For the rope not to break, we must have $T \le 40$ N

$\Rightarrow \quad m\left(\dfrac{l}{2}\right)\omega^2 \le 40$

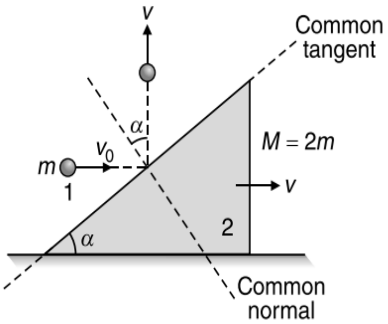
$\Rightarrow \quad (0.4)\left(\dfrac{2}{2}\right)\omega^2 \le 40$

$\Rightarrow \quad \omega^2 \le 100$

$\Rightarrow \quad \omega \le 10 \text{ rads}^{-1}$

$\Rightarrow \quad \omega_{max} = 10 \text{ rads}^{-1}$

4. Along common tangent direction, the speed of the particle remains unchanged. So, we have



$$v_0 \cos\alpha = v \sin\alpha$$

$$\Rightarrow \quad v = v_0 \cot\alpha \qquad \ldots(1)$$

Also, we can conserve linear momentum of the 'particle + wedge' system along horizontal direction. So,

$$mv_0 + 0 = 2mV$$

$$\Rightarrow \quad V = \frac{v_0}{2} \qquad \ldots(2)$$

Coefficient of restitution

$$e = \frac{(\text{Relative velocity of separation})_{n\text{-line}}}{(\text{Relative velocity of approach})_{n\text{-line}}}$$

$$\Rightarrow \quad e = \frac{V\sin\alpha + v\cos\alpha}{v_0 \sin\alpha} = \frac{V}{v_0} + \frac{v}{v_0}\cot\alpha$$

Using (1) and (2), we get

$$e = \frac{1}{2} + \cot^2\alpha$$

Given that $\tan\alpha = 2$

$$\Rightarrow \quad \cot\alpha = \frac{1}{2}$$

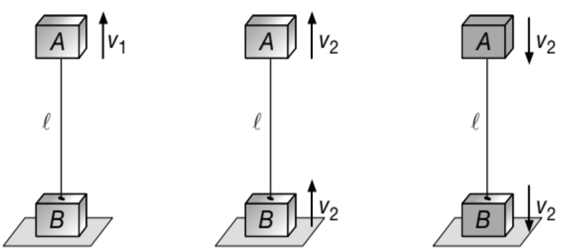
$$\Rightarrow \quad e = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} = 0.75$$

5. String becomes taut when A moves upwards by a distance ℓ under influence of gravity. Let v_1 be the velocity of A at this moment, then

$$v_1^2 = \left(\sqrt{10gl}\right)^2 - 2gl = 8gl$$

$$\Rightarrow \quad v_1 = \sqrt{8gl}$$

Let v_2 be the common velocities of both A and B, just after string becomes taut. Then from Law of Conservation of Linear Momentum, we have



$$m\sqrt{8g\ell} + m(0) = (m+m)v_2$$

$$\Rightarrow \quad v_2 = \frac{v_1}{2} = \frac{\sqrt{8gl}}{2}$$

Both particles return to their original height with the same speed v_2 and the string becomes loose as soon as B strikes the ground and the speed v with which A strikes the ground is,

$$v^2 = v_2^2 + 2g\ell = \frac{8g\ell}{4} + 2g\ell$$

$$\Rightarrow \quad v^2 = 4g\ell$$

$$\Rightarrow \quad v = 2\sqrt{g\ell} = \sqrt{4g\ell}$$

$$\Rightarrow \quad x = 4$$

6. Since no external force acts in z direction, hence z coordinate of the centre of mass of the ball should be zero. To make z coordinate zero other ball should fall symmetrically with respect to z axis.
Hence, z coordinate of other ball is $z_2 = -5$ m.
The balls do not have any external force in x direction. Hence in x direction the centre of mass should move with constant velocity. The x coordinate of centre of mass at $t = 2.0$ s is

$$x_{cm} = 200 \times 2 = 400 \text{ m}$$

Since, $x_{cm} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$

$$\Rightarrow \quad 400 = \frac{20 \times 250 + 20x_2}{20 + 20}$$

$$\Rightarrow \quad x_2 = 800 - 250 = 550 \text{ m}$$

Height fallen by centre of mass at $t = 2$ s is

$$h = \frac{1}{2} \times 10 \times (2)^2 = 20 \text{ m}$$

Hence, y-coordinate of centre of mass is

$$y_{cm} = 30 - 20 = 10 \text{ m}$$

Since, $y_{cm} = \dfrac{m_1y_1 + m_2y_2}{m_1 + m_2}$

$$\Rightarrow \quad 10 = \frac{20 \times 0 + 20 \times y_2}{20 + 20}$$

$$\Rightarrow \quad y_2 = 20 \text{ m}$$

$$\Rightarrow \quad x_2 + y_2 + z_2 = -5 + 20 + 20 = 35$$

7. Assuming wedge, block 1 and block 2 as a system.
Since no external force acts on the system in horizontal direction, so both the linear momentum and mechanical energy will be conserved for the system.
Final step: Let velocity of block 1 in vertical direction be v. Hence velocity of block 2 with respect to wedge will be v towards left. Let velocity of wedge towards right be V.

Hence velocity of block 2 towards left (with respect to ground) will be $(v-V)$.

Applying conservation of linear momentum for the system in horizontal direction, we get

$$(M+m)V - m(v-V) = 0$$

Since $M = 3m$

$$\Rightarrow \quad v = 5V \qquad \ldots(1)$$

Applying conservation of mechanical energy for the system, we get

$$\Delta K + \Delta U = 0$$

$$\Rightarrow \quad \frac{(M+m)}{2}V^2 + \frac{m}{2}(v-V)^2 + \frac{m}{2}v^2 - 0 + (-mgh) = 0$$

$$\Rightarrow \quad \frac{1}{2}(3m+m)V^2 + \frac{1}{2}m(5V-V)^2 + \frac{1}{2}m(5V)^2 = mgh$$

$$\Rightarrow \quad V = \sqrt{\frac{2gh}{45}}$$

$$\Rightarrow \quad \alpha = 45$$

8. (a) $y_{\text{cm}} = \dfrac{m_1y_1 + m_2y_2}{m_1+m_2}$

$$24 = \frac{(m_1)(0) + \left(\frac{600}{1000}\right)(80)}{m_1 + 0.6} = \frac{48}{m_1+0.6}$$

$$\Rightarrow \quad m_1 = 1.4 \text{ kg}$$

So, total mass $= (1.4+0.6)$ kg $= 2$ kg

(b) $\vec{v}_{\text{cm}} = (6 \text{ ms}^{-3})t^2\hat{j}$

$$\Rightarrow \quad \vec{a}_{\text{cm}} = \frac{d\vec{v}_{\text{cm}}}{dt} = (12 \text{ ms}^{-3})t\hat{j}$$

$$\vec{a}_{\text{cm}}\big|_{t=2\text{ s}} = (12)(2) = 24 \text{ ms}^{-2}$$

(c) $\vec{a}_{\text{cm}}$ at $t = 3$ s is $(36 \text{ ms}^{-2})\hat{j}$

$$\Rightarrow \quad \vec{F}_{\text{cm}} = (m_1+m_2)\vec{a}_{\text{cm}} = (2)(36)\hat{j} = (72 \text{ N})\hat{j}$$

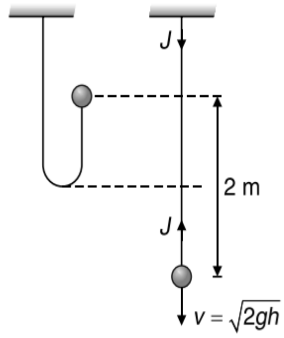
9. The system will be in equilibrium about the centre of mass. Considering origin at the left end of the rod we get

$$x_c = \frac{25\times0 + 8.5\times30 + 10\times50 + 5\times80 + 5\times90}{53.5}$$

$$\Rightarrow \quad x_c = \frac{1605}{53.5} = 30 \text{ cm}$$

10. The string will become taut when the particle will fall through a distance 2 m in downward direction. Applying impulse momentum equation, we get

$$mv - J = 0$$



$$\Rightarrow \quad J = mv = m\sqrt{2gh}$$

$$\Rightarrow \quad J = 1\sqrt{2\times10\times2} = 2\sqrt{10} \text{ kgms}^{-1} = \sqrt{40} \text{ kgms}^{-1}$$

$$\Rightarrow \quad \alpha = 40$$

11. At the bottommost point, just before collision,

$$v_m = \sqrt{4ga} \text{ and } v_{2m} = 0$$

Also, we know that

$$v_1 = \left(\frac{m_1 - em_2}{m_1+m_2}\right)u_1$$

and $v_2 = \left[\dfrac{(1+e)m_1}{m_1+m_2}\right]u_1$

$$\Rightarrow \quad v'_m = \left(\frac{m - \frac{1}{2}\times 2m}{m+2m}\right)\sqrt{4ga} = 0$$

$$\Rightarrow \quad v'_{2m} = \left(\frac{\left(1+\frac{1}{2}\right)\text{m}}{m+2m}\right)\sqrt{4ga} = \frac{1}{2}\sqrt{4ga}$$

After second impact,

$$v''_m = \left(\frac{\left(1+\frac{1}{2}\right)2m}{m+2m}\right)\left(\frac{-\sqrt{4ga}}{2}\right) = -\frac{\sqrt{4ga}}{2} = -\frac{\sqrt{4ga}}{2}$$

$$v''_{2m} = \left(\frac{2m - \frac{1}{2}\times m}{2m+m}\right)\left(-\frac{\sqrt{4ga}}{2}\right) = -\frac{\sqrt{-4ga}}{4}$$

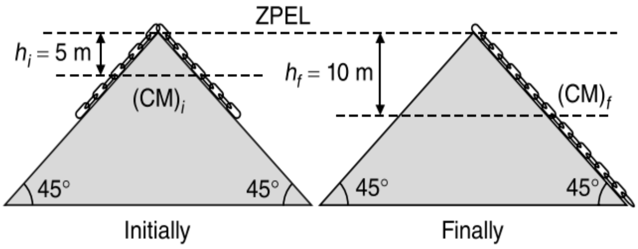
So, $h_m = \dfrac{(v''_m)^2}{2g} = \dfrac{a}{2}$ and $h_{2m} = \dfrac{(v''_{2m})^2}{2g} = \dfrac{a}{8}$

$$\Rightarrow \quad x = 2 \text{ and } y = 8$$

12. When chain leaves the vertex, then P.E. is decreased by the same factor by which K.E. is increased

$$\sin(45°) = \frac{h_f}{10\sqrt{2}}$$

$$\Rightarrow \quad h_f = 10 \text{ m}$$



Since, $U_f = -mgh_f = -10 \times 10 \times 10$

$\Rightarrow \quad U_f = -1000$ J

Also, $U_i = -mgh_i = -10 \times 10 \times 5$

$\Rightarrow \quad U_i = -500$ J

Since $\Delta U + \Delta K = 0$

$$\Rightarrow \quad -500 + \frac{1}{2} \times 10 \times v^2 = 0$$

$\Rightarrow \quad v^2 = 100$

$\Rightarrow \quad v = 10 \text{ ms}^{-1}$

13. Let us consider ball A and B as a system, then the initial height of centre of mass from ground is

$$(y_{\text{cm}})_i = \frac{m(0) + m(40)}{2m} = 20 \text{ m}$$

Initial velocity of centre of mass is

$$u_{\text{cm}} = \frac{m(50) + m(30)}{2m} = 40 \text{ ms}^{-1}$$

Acceleration of centre of mass is

$$a_{\text{cm}} = -g = -10 \text{ ms}^{-2}$$

Since $v_{\text{cm}}^2 - u_{\text{cm}}^2 = 2a_{\text{cm}} \Delta y_{\text{cm}}$

where Δy_{cm} is displacement of centre of mass along y direction (taken vertically upwards).

$\Rightarrow \quad 0^2 - 40^2 = -2 \times 10 \times \Delta y_{\text{cm}}$

$$\Rightarrow \quad \Delta y_{\text{cm}} = \frac{1600}{20} = 80 \text{ m}$$

$\Rightarrow \quad (y_{\text{cm}})_f - (y_{\text{cm}})_i = 80$

$\Rightarrow \quad (y_{\text{cm}})_f = 20 + 80 = 100$ m

Hence, maximum height reach by centre of mass from ground level will be

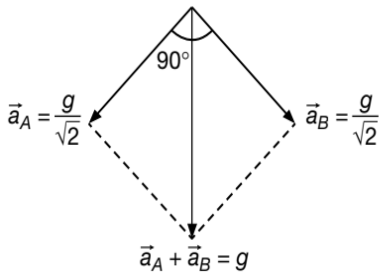
$$h_{\text{max}} = 100 \text{ m}$$

14. Acceleration of both the blocks will be

$$a_1 = a_2 = g \sin 45° = \frac{g}{\sqrt{2}}$$

at right angles to each other. Since,

$$\vec{a}_{cm} = \frac{m_A \vec{a}_A + m_B \vec{a}_B}{m_A + m_B}$$



Also, $m_A = m_B$

$$\Rightarrow \quad \vec{a}_{\text{cm}} = \frac{1}{2}(\vec{a}_A + \vec{a}_B) = \frac{g}{2\sqrt{2}}(\hat{i} + \hat{j}), \text{ downwards}$$

$$\Rightarrow \quad |\vec{a}_{\text{cm}}| = \left(\frac{g}{2\sqrt{2}}\right)(\sqrt{2}) = \frac{g}{2} = 5 \text{ ms}^{-2}$$

15. When bock reaches ground and if B is displaced toward right by x, the displacement of centre of mass of system in horizontal direction must be zero hence we use

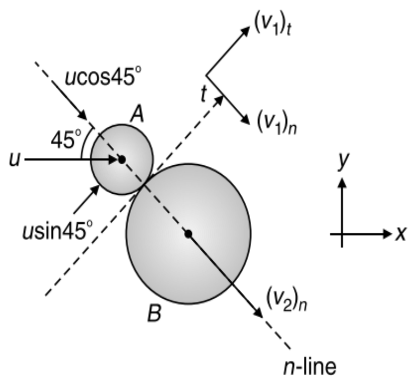
$$4M(l - x) = 20Mx$$

$$x = \frac{l}{6} = \frac{12}{6} = 2 \text{ m}$$

16. Velocity of disc A in tangential direction is

$$(v_1)_t = u \sin 45° = \frac{u}{\sqrt{2}} \qquad \ldots(1)$$

The disc B will move along common normal direction only.



Since both the discs move perpendicular to each other, so disc A should move along common tangent direction.

So, $v_A = \frac{u}{\sqrt{2}}$, along common tangent direction.

Applying conservation of linear momentum along common normal direction, we get

$$m\left(\frac{u}{\sqrt{2}}\right) + 0 = m(0) + (2m)(v_2)_n$$

$$\Rightarrow \quad (v_2)_n = \frac{u}{2\sqrt{2}}$$

Hence, velocity of disc B along common normal is $\dfrac{u}{2\sqrt{2}}$.

Since, $e = \dfrac{(\text{Relative velocity of separation})_{n\text{-line}}}{(\text{Relative velocity of approach})_{n\text{-line}}}$

$\Rightarrow \quad e = \dfrac{(v_2)_n - 0}{u\cos 45^\circ} = \dfrac{u/2\sqrt{2}}{u/\sqrt{2}} = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{1}{e} = 2$

17. Maximum transfer of momentum takes place when both the masses are equal i.e.,

$x^2 - 9x + 21 = 1$

$\Rightarrow \quad x^2 - 9x + 20 = 0$

$\Rightarrow \quad (x-4)(x-5) = 09$

$\Rightarrow \quad x = 4$ and $x = 5$

18. Let ρ be the mass density, then mass of the removed cone is

$$m_1 = \rho\left(\frac{1}{3}\pi\left(\frac{R}{2}\right)^2 R\right) = \frac{\pi R^3 \rho}{12}$$

Also, centre of mass of cone is at height $\dfrac{R}{4}$

Mass of remaining hemisphere is

$$m_2 = \left(\frac{2}{3}\pi R^3 - \frac{\pi R^3}{12}\right)\rho = \frac{7\pi R^3 \rho}{12}$$

Let its centre of mass be at a height y

Since common centre of mass of $m_1 + m_2$ (i.e. a hemisphere) is at a height $\dfrac{3R}{8}$ so we get

$$\frac{3R}{8} = \frac{\rho\left(\frac{\pi R^3}{12}\right)\frac{R}{4} + \rho\left(\frac{7}{12}\pi R^3\right)y}{\rho\left(\frac{2}{3}\pi R^3\right)}$$

$\Rightarrow \quad \dfrac{2}{3}\left(\dfrac{3R}{8}\right) = \dfrac{R}{48} + \dfrac{7y}{12}$

$\Rightarrow \quad \dfrac{7y}{12} = \dfrac{R}{4} - \dfrac{R}{48} = \dfrac{11R}{48}$

$\Rightarrow \quad y = \dfrac{11R}{28} = \dfrac{11}{28}(28) = 11 \text{ cm}$

19. Velocity of B after collision is

$$v_2 = \frac{(1+e)m_1 u_1}{m_1 + m_2} = \left[\frac{(1+e)m}{m+m}\right](10)$$

$\Rightarrow \quad v_2 = 5(1+e)$

Height to which B rises is $h = \dfrac{v_2^2}{2g}$

Now, $\dfrac{v_2^2}{2g} = 3.2$

$\Rightarrow \quad \dfrac{[5(1+e)]^2}{2 \times 10} = 3.2$

$\Rightarrow \quad e = \dfrac{3}{5} = 0.6$

20. Since gases are ejected out during motion we use

$$\int_0^v m\,dv = \int_{m_0}^{m_0/2} u\,dm$$

$\Rightarrow \quad v = u\ln 2 = 2\ln(2)\text{ ms}^{-1} \approx 1.4\text{ ms}^{-1}$

21. According to Work-Energy Theorem, we have

$$\frac{1}{2}mV^2 = (\mu mg)(0.06) + \frac{1}{2}kx^2$$

$\Rightarrow \quad \dfrac{1}{2}(0.18)V^2 = (0.1)(0.18)(10)(0.06) + \dfrac{1}{2}(2)(0.06)^2$

$\Rightarrow \quad V = 0.4\text{ ms}^{-1}$

$\Rightarrow \quad 0.4 = \dfrac{N}{10}$

$\Rightarrow \quad N = 4$

22. Since no force acts along the horizontal direction and initial velocity of centre of mass along a horizontal direction is also zero, so x coordinate of centre of mass remains zero. However, $(a_{cm})_y = -g$

Initial velocity of centre of mass along y-direction is

$$(u_{cm})_y = \frac{m[10\sin(30^\circ)] + m[10\sin(30^\circ)]}{2m}$$

$\Rightarrow \quad (u_{cm})_y = 5\text{ ms}^{-1}$

At $t = 10$ s

$$y_{cm} = (u_{cm})_y t + \frac{1}{2}(a_{cm})_y t^2$$

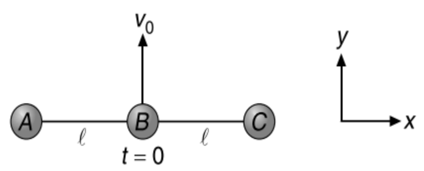
$\Rightarrow \quad y_{cm} = (5)(10) + \dfrac{1}{2}(-10)(10)^2$

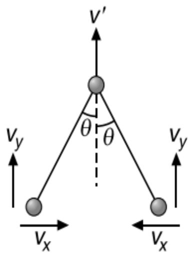
$\Rightarrow \quad y_{cm} = 50 - 500 = -450\text{ m}$

$\Rightarrow \quad x = 0$ and $y = -450$

$\Rightarrow \quad x - y = 450$

23. Initial state at $t = 0$ and final state at time t are shown in Figure.





Applying conservation of momentum at time t, we get

$$mv_0 = mv' + 2mv_y$$

$$\Rightarrow \quad v_0 = v' + 2v_y$$

At the time of collision $\theta = 0$ and $v' = v_y$

$$\Rightarrow \quad v' = v_y = \frac{v_0}{3}$$

Applying conservation of energy, we get

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv'^2 + 2\left[\frac{1}{2}m\left(v_x^2 + v_y^2\right)\right]$$

$$\Rightarrow \quad v_0^2 = \frac{v_0^2}{9} + 2\left(v_x^2 + \frac{v_0^2}{9}\right)$$

$$\Rightarrow \quad v_0^2\left(1 - \frac{1}{9} - \frac{2}{9}\right) = 2v_x^2$$

$$\Rightarrow \quad v_0^2\left(\frac{9-3}{9}\right) = 2v_x^2$$

$$\Rightarrow \quad v_x = \frac{v_0}{\sqrt{3}}$$

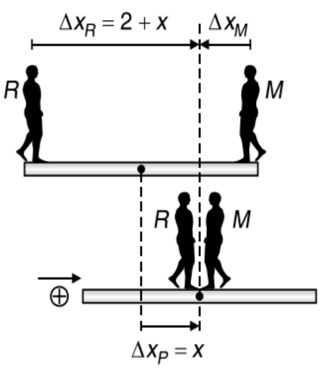
So, net velocity of A at the time of collision is

$$v_A = \sqrt{v_x^2 + v_y^2}$$

$$\Rightarrow \quad v_A = \sqrt{\frac{v_0^2}{3} + \frac{v_0^2}{9}} = v_0\sqrt{\frac{4}{9}} = \frac{2v_0}{3}$$

$$\Rightarrow \quad v_A = \frac{2v_0}{3} = \left(\frac{2}{3}\right)(9) = 6 \text{ ms}^{-1}$$

24. Let displacement of the platform be $\Delta x_p = x$, towards right as shown in Figure.



The displacement of Rahul is $\Delta x_R = (2 + x)$, rightwards

The displacement of Mohit is $\Delta x_M = (2 - x)$, leftwards

Since no external forces are acting on the system, the displacement of the centre of mass of the system should be zero. Hence

$$\Delta x_{cm} = \frac{m_P \Delta x_P + m_R \Delta x_R + m_M \Delta x_M}{m_P + m_R + m_M} = 0$$

$$\Rightarrow \quad \frac{40x + 50(2 + x) - 60(2 - x)}{150} = 0$$

$$\Rightarrow \quad x = \frac{2}{15} \text{ m} \approx 13 \text{ cm}$$

25. $H = \dfrac{u^2 \sin^2\theta}{2g} = \dfrac{(20)^2 \times \frac{3}{4}}{2 \times 10} = 15$ m

i.e., the shell strikes the ball at highest point of its trajectory. Velocity of (ball + shell) just after collision, using Law of Conservation of Linear Momentum is

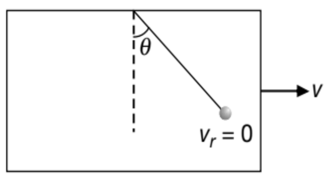
$$v = \frac{u\cos 60°}{2}$$

$$\Rightarrow \quad v = \frac{20}{2 \times 2} = 5 \text{ ms}^{-1}$$

At the highest point, combined mass is at rest relative to the trolley. Let v be the velocity of trolley at this instant. Then by Law of Conservation of Momentum, we get

$$2 \times 5 = (2 + 18)v$$

$$\Rightarrow \quad v = \frac{1}{2} \text{ ms}^{-1}$$



Using Law of Conservation of Mechanical Energy, we get

$$\left(\frac{1}{2}\right)(2)(5)^2 - \left(\frac{1}{2}\right) \times (2 + 18)\left(\frac{1}{2}\right)^2 = (2)(10)(1)(1 - \cos\theta)$$

$$\Rightarrow \quad 25 - 2.5 = 20(1 - \cos\theta)$$

$$\Rightarrow \quad 1 - \cos\theta = 1.125$$

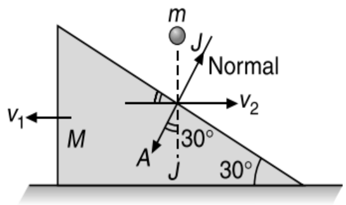
$$\Rightarrow \quad \cos\theta = 0.125$$

$$\Rightarrow \quad \cos\theta = \frac{1}{8}$$

$$\Rightarrow \quad * = 8$$

26. Let v be velocity of the ball after collision along the normal, v' be velocity of the ball after collision along incline and J be impulse on ball, then

$$J = v - (-2\cos 30°) = v + \sqrt{3}$$



Impulse on wedge is

$$J\sin 30° = Mv_1 = 2v_1$$

$$\Rightarrow \frac{v+\sqrt{3}}{2} = 2v_1$$

$$\Rightarrow v = 4v_1 - \sqrt{3} \qquad \ldots(1)$$

Coefficient of restitution is defined as

$$e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right)$$

$$\Rightarrow \frac{1}{2} = \frac{\left(v + \frac{v_1}{2}\right)}{2\cos 30°}$$

$$\Rightarrow v = \frac{\sqrt{3}}{2} - \frac{v_1}{2} \qquad \ldots(2)$$

Solving (1) and (2), we get

$$v_1 = \frac{1}{\sqrt{3}} \text{ ms}^{-1} \text{ and } v = \frac{1}{\sqrt{3}} \text{ ms}^{-1}$$

For the ball, velocity along incline remains constant, so

$$v' = 2\sin(30°) = 1 \text{ ms}^{-1}$$

Hence final velocity of ball is

$$v_2 = \sqrt{1^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2}{\sqrt{3}} \text{ ms}^{-1}$$

$$\Rightarrow \alpha = 2$$

27. If all the particles of the system have same acceleration, $\vec{a}$ say, then acceleration of centre of mass will also be equal to $\vec{a}$. Now, in this problem since both particles are projected in the air, so both particles have acceleration g downwards.

$$\Rightarrow \vec{a}_{CM} = -g\hat{j}$$

Now, if $\vec{a}_{CM}$ remains constant, then equation of motion can be applied to centre of mass. In this question, maximum height attained by the centre of mass is asked, thus we need not to think about the vertical motion of centre of mass.

$$(v_{CM})_y = \frac{5(10\sin 30°) + 2(17\sin 30°)}{5+2}$$

$$\Rightarrow (v_{CM})_y = \frac{25+17}{7} = \frac{42}{7} = 6 \text{ ms}^{-1}$$

Also, $(a_{CM})_y = -10 \text{ ms}^{-2}$

$$\Rightarrow (h_{CM})_{max} = \frac{\left[(v_{CM})_y\right]^2}{2(a_{CM})_y} = \frac{36}{2\times 10} = 1.8 \text{ m}$$

$$\Rightarrow (h_{CM})_y = 180 \text{ cm}$$

28. Before we can draw working diagrams, the velocity components, before and after impact, parallel and perpendicular to the line of centres must be found. To do this we need unit vectors in these two directions. Let the unit vector along the line of centres i.e., along n-line be $\hat{n}$, where

$$\hat{n} = \frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$$

and a unit vector perpendicular to the line of centres i.e., along t-line be $\hat{t}$ where

$$\hat{t} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$$

The velocity components along these directions are calculated in table.

Sphere	Magnitude of component	
	Parallel to $\hat{n}$	Parallel to $\hat{t}$
A	$(\hat{i}+2\hat{j})\cdot\frac{1}{\sqrt{2}}(\hat{i}-\hat{j}) = -\frac{1}{\sqrt{2}}$	$(\hat{i}+2\hat{j})\cdot\frac{1}{\sqrt{2}}(\hat{i}+\hat{j}) = \frac{3}{\sqrt{2}}$
B	$(-\hat{i}+3\hat{j})\cdot\frac{1}{\sqrt{2}}(\hat{i}-\hat{j}) = -2\sqrt{2}$	$(-i+3\hat{j})\cdot\frac{1}{\sqrt{2}}(\hat{i}+\hat{j}) = \sqrt{2}$

Using Law of Conservation of Linear Momentum and the Law of Restitution along the line of impact i.e., n-line, we get

$$m\left(\frac{1}{\sqrt{2}}\right) + 2m(2\sqrt{2}) = mu + 2mv$$

and $\frac{1}{2}\left(2\sqrt{2} - \frac{1}{\sqrt{2}}\right) = u - v$

Solving, we get

$$u = 2\sqrt{2} \text{ and } v = \frac{5}{4}\sqrt{2}$$

The velocity of A after impact is

$$\vec{v}_A = -u\hat{n} + \frac{3}{\sqrt{2}}\hat{t} = (-2\sqrt{2})\frac{1}{\sqrt{2}}(\hat{i}-\hat{j}) + \left(\frac{3}{\sqrt{2}}\right)\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$$

$$\vec{v}_A = \frac{1}{2}(-\hat{i} + 7\hat{j})$$

$\Rightarrow \quad \alpha = 2,\ x_1 = 1 \text{ and } y_1 = 7$

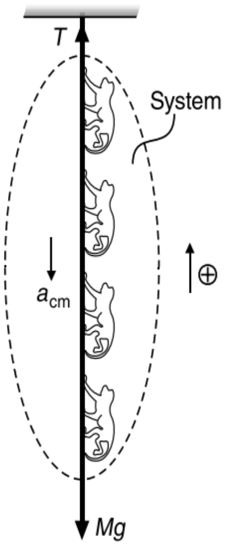
The velocity of B after impact is

$$\vec{v}_B = -v\hat{n} + \sqrt{2}\hat{t} = \left(-\frac{5}{4}\sqrt{2}\right)\frac{1}{\sqrt{2}}\left(\hat{i} - \hat{j}\right) + \left(\sqrt{2}\right)\frac{1}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$$

$$\Rightarrow \quad \vec{v}_B = \frac{1}{4}\left(-\hat{i} + 9\hat{j}\right)$$

$\Rightarrow \quad \beta = 4,\ x_2 = 1 \text{ and } y_2 = 9$

29. Taking four monkeys together as system, the external forces acting on the system is tension and gravitational force.



Acceleration of centre of mass of system is

$$a_{cm} = \frac{m_1a_1 + m_2a_2 + m_3a_3 + m_4a_4}{m_1 + m_2 + m_3 + m_4}$$

Taking upward direction is positive, we get

$$a_{cm} = \frac{15(1) + 25(-3) + 20(0) + 40(0)}{15 + 25 + 20 + 40}$$

$$\Rightarrow \quad a_{cm} = -\frac{60}{100} = \frac{6}{10} \text{ ms}^{-2}\text{, downwards}$$

Since, $\sum F_{ext} = Ma_{cm}$

$\Rightarrow \quad Mg - T = Ma$

$\Rightarrow \quad (100)(9.8) - T = 100(0.6)$

$\Rightarrow \quad T = 980 - 60 = 920 \text{ N}$

30. Mass of gun is $80m - 5m = 75m$

After first shot, if speed of car is v_1, then we have

$$75mv = 5m(100 - v)$$

$$\Rightarrow \quad v_1 = \frac{500}{80} = \frac{100}{6} \text{ ms}^{-1}$$

After second shot, if speed of car is v_2, then we have

$$75m\left(\frac{100}{16}\right) = 70mv_2 - 5m(100 - v_2)$$

$$\Rightarrow \quad 75v_2 = \frac{75 \times 100}{16} + 500$$

$$\Rightarrow \quad v_2 = \frac{100}{16} + \frac{100}{15} \text{ ms}^{-1}$$

$$\Rightarrow \quad v_2 = 100\left(\frac{1}{16} + \frac{1}{15}\right) \text{ ms}^{-1} \approx 13 \text{ ms}^{-1}$$

31. (a) Let v be velocity of block when the bullet emerges from it. Applying Law of Conservation of Linear Momentum, we get

$$(1)v + \left(4 \times 10^{-3}\right)(100) = \left(4 \times 10^{-3}\right)(600)$$

$$\Rightarrow \quad v = 2 \text{ ms}^{-1}$$

Using $v^2 = u^2 - 2as$, we get

$$0 = (2)^2 - 2\left(\mu_k g\right) s$$

$$0 = 4 - 2\mu_k (10)(0.4)$$

$$\Rightarrow \quad \mu_k = 0.5 = \frac{1}{2}$$

$$\Rightarrow \quad * = 2$$

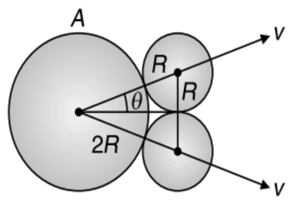
(b) Decrease in kinetic energy of bullet is

$$-\Delta K = \frac{1}{2}\left(4 \times 10^{-3}\right)\left[(600)^2 - (100)^2\right] = 700 \text{ J}$$

(c) Kinetic energy of block

$$K = \frac{1}{2} \times 1 \times (2)^2 = 2 \text{ J}$$

32. Just before collision, the arrangement is shown in Figure.



So, $\cos\theta = \dfrac{2\sqrt{2}R}{2R + R} = \dfrac{2\sqrt{2}}{3}$

Let initial velocity of A be u and final velocity of each small balls be v

Applying conservation of linear momentum, we get

$$mu = 2mv\cos\theta$$

Since, $e = \dfrac{(\text{Relative velocity of separation})_{n\text{-line}}}{(\text{Relative velocity of approach})_{n\text{-line}}}$

$$\Rightarrow \quad e = \frac{v}{u\cos\theta}$$

$$\Rightarrow \quad e = \frac{1}{2\cos^2\theta} = \frac{1}{2\left(\frac{8}{9}\right)} = \frac{9}{16} \approx 0.56$$

33. If final velocity of shuttle is v, then by conservation of momentum, we get

$$M(4000) = \left(\frac{4M}{5}\right)v + \frac{M}{5}(3880)$$

$\Rightarrow \quad \frac{4v}{5} = 4000 - \frac{3880}{5} = 3224$

$\Rightarrow \quad v = 4030$ kph

34. For elastic collision between A and B, we have

$$v_B = \left(\frac{2m}{2m+m}\right)9$$

$\Rightarrow \quad v_B = 6\text{ ms}^{-1}$

Now, for completely inelastic collision between B and C, we have

$$v_C = \frac{2m(v_B)+m(0)}{2m+m}$$

$\Rightarrow \quad v_C = \frac{(2m)(6)}{3m}$

$\Rightarrow \quad v_C = 4\text{ ms}^{-1}$

35. Applying conservation of momentum along x and y direction. Along x direction,

$$m_B v = m_A v_A \cos\theta \qquad \ldots(1)$$

If A moves at an angle θ to initial direction of B which is taken as x axis, then along y direction

$$m_B \frac{v}{2} = m_A v_A \sin\theta \qquad \ldots(2)$$

Dividing (2) by (1), we get

$$\tan\theta = \frac{1}{2}$$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \quad \alpha = 2$

Since collision is elastic, so we have

$$\frac{1}{2}m_B v^2 = \frac{1}{2}m_B\left(\frac{v^2}{4}\right) + \frac{1}{2}m_A v_A^2$$

Using equation (1), we get

$$(m_A v_A \cos\theta)v = (m_A v_A \cos\theta)\frac{v}{4} + m_A v_A^2$$

$\Rightarrow \quad v\cos\theta = \frac{1}{4}v\cos\theta + V_A$

$\Rightarrow \quad V_A = \frac{3}{4}v\cos\theta$

Since, $\tan\theta = \frac{1}{2}$, so $\cos\theta = \frac{2}{\sqrt{5}}$

$\Rightarrow \quad V_A = \left(\frac{3}{4}\right)\left(\frac{2}{\sqrt{5}}\right)v = \frac{3v}{2\sqrt{5}} = \left(\sqrt{\frac{9}{4\times 5}}\right)v = \sqrt{0.45}v$

$\Rightarrow \quad \beta = 0.45$

36. At maximum compression (x_m), velocity of both the blocks is same, say it is v. Applying Law of Conservation of Linear Momentum, we get

$$(m_A + m_B)v = m_B v_0$$

$\Rightarrow \quad (1+1)v = (1)v_0$

$\Rightarrow \quad v = \frac{v_0}{2} = \frac{2}{2} = 1\text{ ms}^{-1}$

By Law of Conservation of Mechanical Energy, we get

$$\frac{1}{2}m_B v_0^2 = \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{2}kx_m^2$$

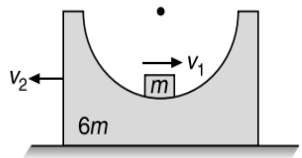
Substituting the values, we get

$$\frac{1}{2}\times(1)\times(2)^2 = \frac{1}{2}\times(1+1)\times(1)^2 + \frac{1}{2}\times(200)\times x_m^2$$

$\Rightarrow \quad 2 = 1 + 100x_m^2$

$\Rightarrow \quad x_m = 0.1\text{ m} = 10\text{ cm}$

37. Let smaller block be moving with speed v_1 relative to bigger block when it reaches the bottommost position and at this instant, bigger block is moving at v_2 (say). Applying conservation of momentum in horizontal direction we get



$$6mv_2 = m(v_1 - v_2) \qquad \ldots(1)$$

Applying conservation of energy, we get

$$mga = \frac{1}{2}m(v_1 - v_2)^2 + \frac{1}{2}(6m)v_2^2 \qquad \ldots(2)$$

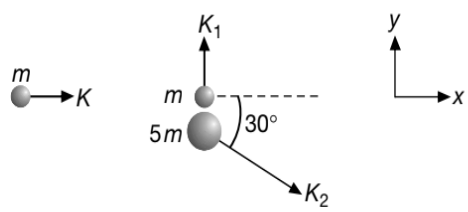
Solving equations (1) and (2), we get

$$2ga = 36v_2^2 + 6v_2^2$$

$\Rightarrow \quad v_2 = \sqrt{\frac{ga}{21}}$

$\Rightarrow \quad * = 21$

38. Let K be the initial kinetic energy of mass m and K_1 and K_2 the kinetic energies after collision in the direction shown in figure. Initial Momentum of the particle is $\sqrt{2mK}$. So,



Applying conservation of linear momentum in vector form, we get,

$$\sqrt{2mK}\,\hat{i} = \sqrt{2mK_1}\,\hat{j} + \sqrt{2MK_2}\left(\cos 30\hat{i} - \sin 30\hat{j}\right)$$

$$\Rightarrow \quad \sqrt{2mK} = \sqrt{10mK_2}\,\frac{\sqrt{3}}{2} \text{ and}$$

$$\sqrt{2mK_1} = \sqrt{10mK_2}\,\frac{1}{2}$$

Solving, we get

$$K_2 = \frac{4}{15}K \text{ and } K_1 = \frac{K}{3}$$

$$\Rightarrow \quad K_1 + K_2 = \frac{9}{15}K = 0.6K$$

So, loss in $\text{KE} = K - 0.6K = 0.4K$

Hence, KE will be decreased by 40%.

39. After time t, let us calculate velocity of centre of mass of system by using impulse momentum equation. As spring force is an internal force of system, so we have

$$p_{\text{system}} = (2+3)v_{\text{cm}} = \int_0^t 5t\,dt = \frac{5t^2}{2} \quad \ldots(1)$$

At $t = 10$ s, from equation (1) we get

$$v_{\text{cm}} = \frac{5(10)^2/2}{5} = 50 \text{ ms}^{-1}$$

If at this instant, let velocity of 2 kg mass be v_2, then we have

$$v_{\text{cm}} = 50 = \frac{(3)(30) + 2v_2}{5}$$

$$\Rightarrow \quad v_2 = \frac{(5)(50) - (3)(30)}{2} = \frac{160}{2} = 80 \text{ ms}^{-1}$$

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CHAPTER 2

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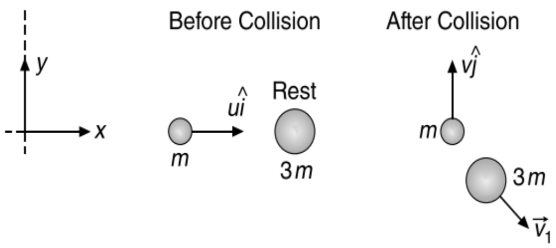
1. Applying Law of Conservation of Momentum, we get

$$\vec{p}_i = \vec{p}_f$$



$$\Rightarrow \quad (m)(u\hat{i}) + (3m)(\vec{0}) = (m)(v\hat{j}) + (3m)\vec{v}_1$$

$$\Rightarrow \quad mui - mvj = 3m\vec{v}_1$$

$$\Rightarrow \quad \vec{v}_1 = \frac{u\hat{i} - v\hat{j}}{3}$$

$$\Rightarrow \quad |\vec{v}_1| = \frac{\sqrt{u^2+v^2}}{3} \quad \ldots(1)$$

$$\Rightarrow \quad v_1^2 = \frac{u^2+v^2}{9}$$

Since collision is perfectly elastic, so we have

$$\Sigma K_{\text{initial}} = \Sigma K_{\text{final}}$$

$$\Rightarrow \quad \frac{1}{2}mu^2 + \frac{1}{2}(3m)(0)^2 = \frac{1}{2}mv^2 + \frac{1}{2}(3m)v_1^2$$

$$\Rightarrow \quad u^2 = v^2 + 3v_1^2$$

Substituting value of v_1 from equation (1), we get

$$u^2 = v^2 + 3\left(\frac{u^2+v^2}{9}\right)$$

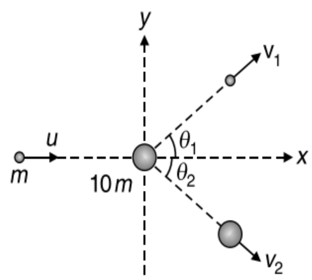
$$\Rightarrow \quad 3u^2 = 3v^2 + u^2 + v^2$$

$$\Rightarrow \quad 2u^2 = 4v^2$$

$$\Rightarrow \quad v = \frac{u}{\sqrt{2}}$$

Hence, the correct answer is (D).

2. Applying momentum conservation along y-axis, we get



$$m_1 v_1 \sin\theta_1 = m_2 v_2 \sin\theta_2$$

$$\Rightarrow \quad m v_1 \sin\theta_1 = 10 m v_2 \sin\theta_2$$

$$\Rightarrow \quad v_1 \sin\theta_1 = 10 v_2 \sin\theta_2 \qquad \ldots(1)$$

Since it is given that

$$\left(K_f\right)_m = \frac{1}{2}\left(K_i\right)_m$$

$$\Rightarrow \quad \frac{1}{2} m v_1^2 = \frac{1}{2}\left(\frac{1}{2} m u^2\right)$$

$$\Rightarrow \quad v_1 = \frac{u}{\sqrt{2}} \qquad \ldots(2)$$

Also, collision is elastic, so we have $K_f = K_i$

$$\Rightarrow \quad \frac{1}{2} m u^2 = \frac{1}{2} m v_1^2 + \frac{1}{2}(10m) v_2^2$$

$$\Rightarrow \quad \frac{1}{2} m u^2 = \frac{1}{2}\left(\frac{1}{2} m u^2\right) + \frac{1}{2}(10m) v_2^2$$

$$\Rightarrow \quad \frac{1}{4} m u^2 = \frac{1}{2}(10m) v_2^2$$

$$\Rightarrow \quad v_2 = \frac{u}{\sqrt{20}} \qquad \ldots(3)$$

Substituting equations (2) and (3) in equation (1), we get

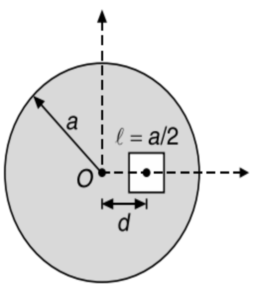
$$\frac{u}{\sqrt{2}} \sin\theta_1 = 10 \frac{u}{\sqrt{20}} \sin\theta_2$$

$$\Rightarrow \quad \sin\theta_1 = \sqrt{10} \sin\theta_2$$

$$\Rightarrow \quad n = 10$$

Hence, the correct answer is (10.00).

3. Since $x_{cm} = \dfrac{m_1 x_1 - m_2 x_2}{m_1 - m_2}$



where, m_1 is mass of complete disc and m_2 is removed mass.

Let σ be the surface mass density of disc material, then

$$x_{cm} = \frac{\sigma\pi a^2 (0) - \sigma\left(a^2/4\right) d}{\sigma\pi a^2 - \sigma\left(a^2/4\right)} = \frac{-a^2 d/4}{\pi a^2 - \left(a^2/4\right)}$$

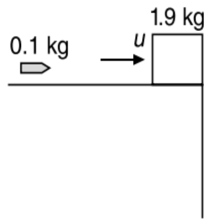
$$\Rightarrow \quad x_{cm} = \frac{-d}{4\pi - 1} = -\frac{a}{2(4\pi - 1)}$$

$$\Rightarrow \quad X = 2(4\pi - 1) = (8\pi - 2) = 23.12$$

So, nearest integer value of $X = 23$

Hence, the correct answer is (23.00).

4. Since, $p_i = p_f$



$$\Rightarrow \quad 0.1 \times 20 = 2v$$

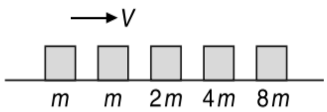
$$\Rightarrow \quad v = 1 \text{ ms}^{-1}$$

So, final kinetic energy is

$$K_f = mgh + \frac{1}{2} m v^2 = 21 \text{ J}$$

Hence, the correct answer is (A).

5. Since all collisions are perfectly inelastic, so after the final collision, all blocks are moving together. So, if the final velocity be V, then on applying momentum conservation, we get



$$mv = 16mV$$

$$\Rightarrow \quad V = \frac{v}{16}$$

Now initial energy is $E_i = \dfrac{1}{2} m v^2$

Final energy is $E_f = \dfrac{1}{2}(16m)\left(\dfrac{v}{16}\right)^2$

$$\Rightarrow \quad E_f = \frac{mv^2}{32}$$

Energy loss is $E_i - E_f$ given by

$$E_i - E_f = \frac{1}{2} m v^2 - \frac{1}{2} m \frac{v^2}{16}$$

$$\Rightarrow \quad E_i - E_f = \frac{1}{2} m v^2 \left(1 - \frac{1}{16}\right)$$

$$\Rightarrow \quad E_i - E_f = \frac{1}{2} m v^2 \left(\frac{15}{16}\right)$$

So, percentage loss in KE is

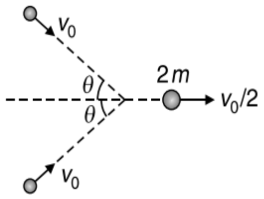
$$\left|\frac{\Delta K}{K}\right| \times 100\% \frac{\text{Energy loss}}{\text{Original energy}} \times 100\%$$

$$\Rightarrow \quad \left|\frac{\Delta K}{K}\right| \times 100\% = \frac{\frac{1}{2}mv^2\left(\frac{15}{16}\right)}{\frac{1}{2}mv^2} \times 100 = 93.75\%$$

$$\Rightarrow \quad p \approx 94$$

Hence, the correct answer is (D).

6. Momentum conservation along x direction, we get



$$2mv_0\cos\theta = (2m)\frac{v_0}{2}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 60^\circ$$

Angle is $2\theta = 120^\circ$

Hence, the correct answer is (120.00).

7. Given that the respective initial velocity of A and B be

$$\vec{u}_1 = \left(\sqrt{3}\hat{i} + \hat{j}\right)\text{ ms}^{-1}\text{-and } \vec{u}_2 = \vec{0}$$

Also, $m_1 = 2m_2$ and after collision, we have

$$\vec{v}_1 = \left(\hat{i} + \sqrt{3}\hat{j}\right)\text{ ms}^{-1} \text{ and } \vec{v}_2 = ?$$

Applying Law of Conservation of Linear Momentum, we get

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\Rightarrow \quad 2m_2\left(\sqrt{3}\hat{i} + \hat{j}\right) + 0 = 2m_2\left(\hat{i} + \sqrt{3}\hat{j}\right) + m_2\vec{v}_2$$

$$\Rightarrow \quad \vec{v}_2 = 2\left(\sqrt{3}\hat{i} + \hat{j}\right) - 2\left(\hat{i} + \sqrt{3}\hat{j}\right)$$

$$\Rightarrow \quad \vec{v}_2 = 2\left(\sqrt{3}\hat{i} - \hat{j}\right) + 2\left(i - \sqrt{3}\hat{j}\right)$$

$$\Rightarrow \quad \vec{v}_2 = 2\left(\sqrt{3} - 1\right)\left(\hat{i} - \hat{j}\right)$$

If θ be the angle between $\vec{v}_1$ and $\vec{v}_2$, then we have

$$\cos\theta = \frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1||\vec{v}_2|} = \frac{2\left(\sqrt{3}-1\right)\left(1-\sqrt{3}\right)}{(2)\left[2\sqrt{2}\left(\sqrt{3}-1\right)\right]}$$

$$\Rightarrow \quad \cos\theta = \frac{1-\sqrt{3}}{2\sqrt{2}} \quad \ldots(1)$$

Since we can write, $\cos(105^\circ) = \cos(60^\circ + 45^\circ)$

$$\Rightarrow \quad \cos(105^\circ) = \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$$

$$\Rightarrow \quad \cos(105^\circ) = \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$

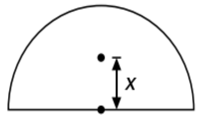
$$\Rightarrow \quad \cos(105^\circ) = \frac{1-\sqrt{3}}{2\sqrt{2}} \quad \ldots(2)$$

So, from equations (1) and (2), we get

$$\theta = 105^\circ$$

Hence, the correct answer is (D).

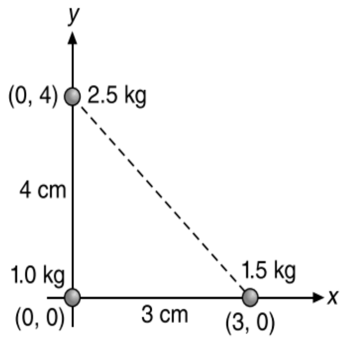
8. Since, $x = \frac{3R}{8} = 3$ cm



$$\Rightarrow \quad x = 3$$

Hence, the correct answer is (3).

9. Let 1 kg be taken as origin and xy axis is shown in Figure.

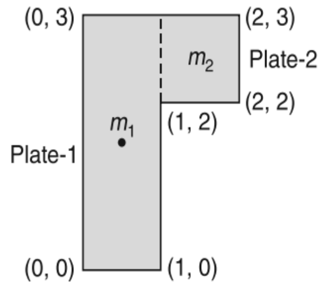


So, $x_{cm} = \frac{1(0) + 1.5(3) + 2.5(0)}{5} = 0.9$ cm and

$$y_{cm} = \frac{1(0) + 1.5(0) + 2.5(4)}{5} = 2 \text{ cm}$$

Hence, the correct answer is (B).

10. Let lamina be divided in two parts such that $m_1 = 3$ kg and $m_2 = 1$ kg



Mass of plate-1 is assumed to be concentrated at $(0.5, 1,.5)$ and mass of plate-2 is assumed to be concentrated at $(1.5, 2.5)$, so we have

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{3 \times 0.5 + 1 \times 1.5}{4} = 0.75 \text{ and}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{3 \times 1.5 + 1 \times 2.5}{4} = 1.75$$

Hence, the correct answer is (D).

11. By conservation of linear momentum, we have

$$(0.1)(3\hat{i})+(0.1)(5\hat{j})=(0.1)(4)(\hat{i}+\hat{j})+(0.1)\vec{v}$$

$\Rightarrow \quad \vec{v}=-\hat{i}+\hat{j}$

Speed of B after collision is $|\vec{v}|=\sqrt{2}$

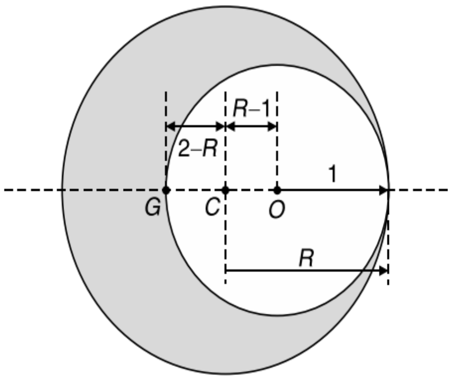
Now, kinetic energy of B is

$$K_B=\frac{1}{2}mv^2=\frac{1}{2}(0.1)(2)=\frac{1}{10}$$

$\Rightarrow \quad x=1$

Hence, the correct answer is (1.00).

12. Applying Law of Conservation of Moments about the centre C of the full disc, we have



$m_1r_1=m_2r_2$, where

$m_1=m_{\text{remainder}}=\frac{4}{3}\pi\left(R^3-(1^3)\right)\rho$, $r_1=2-R$ and

$m_2=m_{\text{cavity}}=\frac{4}{3}\pi(1)^3\rho$, $r_1=R-1$

$\Rightarrow \quad m_{\text{remainder}}(2-R)=m_{\text{cavity}}(R-1)$

$\Rightarrow \quad \left[\frac{4}{3}\pi R^3\rho-\frac{4}{3}\pi(1)^3\rho\right](2-R)=\left[\frac{4}{3}\pi(1)^3\rho\right](R-1)$

$\Rightarrow \quad (R^3-1)(2-R)=R-1$

$\Rightarrow \quad (R^2+R+1)(2-R)=1$

Alternatively, we can also get the desired result by applying the formula

$$x_{\text{cm}}=\frac{m_{\text{full disc}}x_1-m_{\text{removed disc}}x_2}{m_{\text{full disc}}-m_{\text{removed disc}}}$$

Considering origin to be at the centre of the full disc, we have

$x_{\text{cm}}=-(2-R)$, $x_1=0$ and $x_2=R-1$

$\Rightarrow \quad -(2-R)=\dfrac{0-\frac{4}{3}\pi(1)^3\rho(R-1)}{\frac{4}{3}\pi R^3\rho-\frac{4}{3}\pi(1)^3\rho}=\dfrac{-(R-1)}{R^3-(1)^3}$

$\Rightarrow \quad -(2-R)(R^3-1)=-(R-1)$

$\Rightarrow \quad (2-R)(R-1)(R^2+R+1)=(R-1)$

$\Rightarrow \quad (R^2+R+1)(2-R)=1$

Hence, the correct answer is (C).

13. Applying Law of Conservation of Momentum, we get

$$(mu)\hat{i}+mu\left(\frac{\hat{i}+\hat{j}}{2}\right)=(m+m)\vec{v}$$

$\Rightarrow \quad \vec{v}=\frac{3}{4}u\hat{i}+\frac{u}{4}\hat{j}$

$\Rightarrow \quad |\vec{v}|=\frac{u}{4}\sqrt{10}$

So, final kinetic energy is

$$K_{\text{final}}=\frac{1}{2}2m\left(\frac{u}{4}\sqrt{10}\right)^2=\frac{5}{8}mu^2$$

and initial kinetic energy is

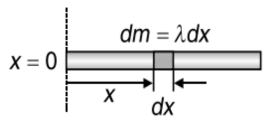
$$K_i=\frac{1}{2}mu^2+\frac{1}{2}m\left(\frac{u}{\sqrt{2}}^2\right)=\frac{6}{8}mu^2$$

So, loss in kinetic energy is

$$-\Delta K=K_i-K_f=\frac{1}{8}mu^2$$

Hence, the correct answer is (B).

14. Since, $x_{\text{cm}}=\dfrac{\int x\,dm}{\int dm}=\dfrac{\int(\lambda dx)x}{\int dm}$

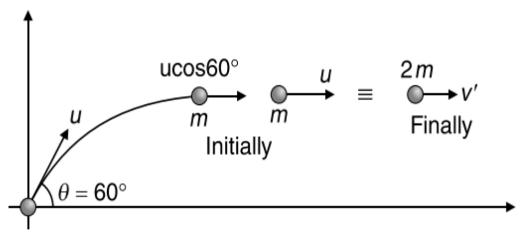


$$\Rightarrow \quad x_{\text{cm}}=\frac{\int_0^L\left(a+\frac{bx^2}{L^2}\right)x\,dx}{\int_0^L\left(a+\frac{bx^2}{L^2}\right)dx}=\frac{\frac{aL^2}{2}+\frac{b}{L^2}\frac{L^4}{4}}{aL+\frac{b}{L^2}\frac{L^3}{3}}$$

$$\Rightarrow \quad x_{\text{cm}}=\frac{\frac{(4a+2b)}{8}}{\frac{(3a+b)}{3}}=\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$$

Hence, the correct answer is (D).

15. Applying Law of Conservation of Momentum along x direction, we get



$$\frac{mu}{2} + mu = 2mv'$$

$$\Rightarrow \quad v' = \frac{3u}{4}$$

Range after collision is

$$R = v't = \left(\frac{3u}{4}\right)t = \frac{3u}{4}\sqrt{\frac{2H}{g}} \qquad \left\{\because H = \frac{1}{2}gt^2\right\}$$

where, $H = \frac{u^2 \sin^2(60°)}{2g}$

$$\Rightarrow \quad R = \frac{3u}{4}\sqrt{\frac{2u^2\sin^2(60°)}{g(2g)}} = \left(\frac{3u}{4}\right)\left(\frac{u\sqrt{3}}{2g}\right)$$

$$\Rightarrow \quad R = \frac{3\sqrt{3}u^2}{8g}$$

Hence, the correct answer is (C).

16. $\vec{a}_{CM} = \frac{(2m)a\hat{j} + 3m \times a\hat{i} + ma(-\hat{i}) + 4m \times a(-\hat{j})}{2m + 3m + 4m + m}$

$$\Rightarrow \quad \vec{a}_{CM} = \frac{2a\hat{i} - 2a\hat{j}}{10} = \frac{a}{5}(\hat{i} - \hat{j})$$

Hence, the correct answer is (D).

17. Applying Conservation of Linear Momentum, we get

$$m_1v_1 + m_2v_2 = m_1v_3 + m_2v_4$$

$$\Rightarrow \quad m_1v_1 + 0.5m_1v_2 = 0.5m_1v_1 + 0.5m_1v_4$$

$$\Rightarrow \quad v_1 = v_4 - v_2$$

Hence, the correct answer is (B).

18. X-coordinate of CM of remaining sheet is

$$x_{cm} = \frac{MX - mx}{M - m}$$

$$\Rightarrow \quad x_{cm} = \frac{(4m)\times\left(\frac{a}{2}\right) - M\left(\frac{3a}{4}\right)}{4m - m} = \frac{5a}{12}$$

Similarly, $y_{cm} = \frac{5b}{12}$

So, centre of mass is located at $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

Hence, the correct answer is (B).

19. Applying Conservation of Linear Momentum, we get

$$2V = \frac{2V}{4} + m_2V_2 \qquad \ldots(1)$$

Since $e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) = 1$

$$\Rightarrow \quad V = V_2 - \frac{V}{4} \qquad \ldots(2)$$

Substituting (2) in (1), we get

$$\frac{3V}{2} = m_2\left(V + \frac{V}{4}\right)$$

$$\Rightarrow \quad m_2 = \frac{6}{5} \text{ kg}$$

Hence, the correct answer is (D).

20. Applying momentum conservation, we get

$$mv = (4m + m)v'$$

So, common speed is $v' = \frac{v}{5}$

Applying energy conservation, we get

$$mgh + \frac{1}{2}5m\left(\frac{v^2}{25}\right) = \frac{1}{2}mv^2$$

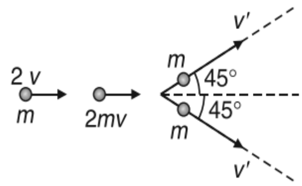
$$\Rightarrow \quad mgh = \frac{1}{2}mv^2\left(1 - \frac{1}{5}\right) = \frac{1}{2}mv^2\frac{4}{5}$$

$$\Rightarrow \quad h = \frac{2mv^2}{5 \times mg} = \frac{2v^2}{5g}$$

Hence, the correct answer is (B).

21. Initial momentum is $p_i = 2mv + 2mv = 4mv$

Let v' be the speed of each splitted particle, then by momentum conservation, we get



$$\Rightarrow \quad 2\frac{mv'}{\sqrt{2}} = 4mv$$

$$\Rightarrow \quad v' = 2\sqrt{2}v$$

Hence, the correct answer is (D).

22 Applying conservation of linear momentum along x and y directions.

Along x direction, initial and final momentum are

$$(\Sigma p_x)_{\text{initial}} = M(10\cos 30°) + 2M(5\cos 45°)$$

$$(\Sigma p_x)_{\text{final}} = 2M(v_1\cos 30°) + M(v_2\cos 45°)$$

Since, $(\Sigma p_x)_{\text{initial}} = (\Sigma p_x)_{\text{final}}$

$$\Rightarrow \quad 5\sqrt{3} + \frac{10}{\sqrt{2}} = \sqrt{3}v_1 + \frac{v_2}{\sqrt{2}}$$

$$\Rightarrow \quad 5\sqrt{3} + 5\sqrt{2} = \sqrt{3}v_1 + \frac{v_2}{\sqrt{2}} \qquad \ldots(1)$$

Along y direction, initial and final momentum are

$$(\Sigma p_y)_{\text{initial}} = 2M(5\sin 45°) - M(10\sin 30°)$$

$$\left(\Sigma p_y\right)_{\text{final}} = 2M\left(v_1 \sin 30°\right) - M\left(v_2 \sin 45°\right)$$

Since, $\left(\Sigma p_y\right)_{\text{initial}} = \left(\Sigma p_y\right)_{\text{final}}$

$$\Rightarrow \quad 5\sqrt{2} - 5 = v_1 - \frac{v_2}{\sqrt{2}} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$\left(\sqrt{3}+1\right)v_1 = 5\sqrt{3} + 10\sqrt{2} - 5$$

$$\Rightarrow \quad v_1 = 6.5 \text{ ms}^{-1} \text{ and } v_2 = 6.3 \text{ ms}^{-1}$$

Hence, the correct answer is (D).

23. Applying momentum conservation, we get

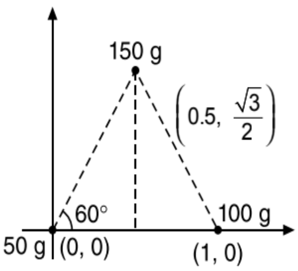
$$50V_1 = 20V_2$$

Also, $V_1 + V_2 = 0.70$

$$\Rightarrow \quad V_1 = 0.20$$

Hence, the correct answer is (A).

24.

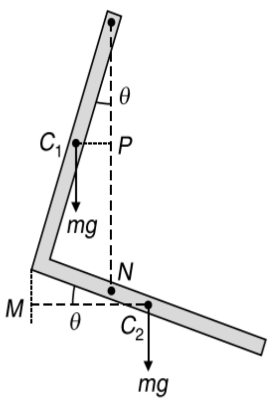


$$x_{CM} = \frac{(m)(0) + (2m)(1) + (3m)\left(\frac{1}{2}\right)}{6m} = \frac{7}{12} \text{ m}$$

$$y_{CM} = \frac{(m)(0) + (2m)(0) + (3m)\left(\frac{\sqrt{3}}{2}\right)}{6m} = \frac{\sqrt{3}}{4} \text{ m}$$

Hence, the correct answer is (D).

25. Let mass of one rod be m, then balancing the torque about one end of the rod, we get



$$mg\left(C_1P\right) = mg\left(C_2N\right)$$

$$\Rightarrow \quad mg\left(\frac{L}{2}\sin\theta\right) = mg\left(\frac{L}{2}\cos\theta = L\sin\theta\right)$$

$$\Rightarrow \quad \frac{3}{2}mgL\sin\theta = \frac{1}{2}mgL\cos\theta$$

$$\Rightarrow \quad \tan\theta = \frac{1}{3}$$

Hence, the correct answer is (C).

26. Since, $\Delta E_1 = \frac{1}{2}\mu v_{\text{rel}}^2$

$$\Rightarrow \quad \Delta E_1 = \frac{1}{2}\left(\frac{m}{2}\right)v^2 = \frac{1}{4}mv^2$$

Velocity after collision is $V = \frac{v}{2}$

$$\Rightarrow \quad \Delta E_2 = \frac{1}{2}\left(\frac{2mM}{2m+M}\right)\left(\frac{v}{2}\right)^2$$

Since, $\Delta E_1 + \Delta E_2 = \frac{5}{6}\left(\frac{1}{2}mv^2\right)$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{m}{2}\right)v^2 + \left(\frac{1}{2}\right)\left(\frac{m}{2}\right)v^2\left(\frac{M}{2m+M}\right) = \frac{5}{6}\left(\frac{1}{2}mv^2\right)$$

$$\Rightarrow \quad 1 + \frac{M}{2m+M} = \frac{10}{6} = \frac{5}{3}$$

$$\Rightarrow \quad \frac{M}{m} = 4$$

Hence, the correct answer is (B).

27. Let bullet meet the piece of wood in time t, then we have

$$100 - \frac{1}{2}10t^2 = 100t - 5t^2$$

$$\Rightarrow \quad t = 1 \text{ s}$$

Applying Law of Conservation of Momentum, we get

$$90(0.02) - 10(0.03) = v(0.05)$$

$$\Rightarrow \quad v = \frac{1.8 - 0.3}{0.05} = \frac{1.5}{0.05} = \frac{150}{5} = 30 \text{ ms}^{-1}$$

$$\Rightarrow \quad s_2 = \frac{30^2}{2 \times 10} = \frac{30 \times 30}{2 \times 10} = 45 \text{ m}$$

So, maximum height above the building is

$$h_{\max} = 45 - 5 = 40 \text{ m}$$

Hence, the correct answer is (B).

28. Initial compression is $x_i = \frac{3 \times 10}{k}$

Since spring constant is high, so initial compression is low. Let v_1 be velocity after collision, then by Conservation of Momentum, we have

$$4v_1 = v_0 \qquad \ldots(1)$$

where, $v_0 = \sqrt{2gh} = \sqrt{2g \times 100}$

$$\Rightarrow \quad \frac{1}{2}(4)v_1^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad x = 2 \text{ cm}$$

*No given option is correct.

29. Since, $u = \omega\theta_0$ and $v = \omega\theta_1$

$$\Rightarrow \quad \frac{u}{v} = \frac{\theta_0}{\theta_1}$$

Also, $v = \left(\frac{M-m}{M+m}\right)u$

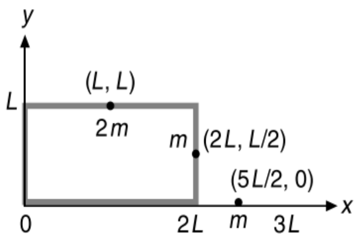
$$\Rightarrow \quad \frac{M+m}{M-m} = \frac{u}{v} = \frac{\theta_0}{\theta_1}$$

$$\Rightarrow \quad \frac{M}{m} = \frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}$$

$$\Rightarrow \quad M = m\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$$

Hence, the correct answer is (A).

30. Let assume linear mass density is λ. The corresponding coordinates of the rod are shown in Figure.



then, $m_1 = 2L\lambda$ and $r_{1\text{ cm}} \equiv (L, L)$

$$m_2 = L\lambda \text{ and } r_{2\text{ cm}} \equiv \left(2L, \frac{L}{2}\right) \text{ and}$$

$$m_3 = L\lambda \text{ and } r_{3\text{ cm}} \equiv \left(\frac{5L}{2}, 0\right)$$

Since, $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$

$$\Rightarrow \quad x_{cm} = \frac{(2m)L + m(2L) + m(5L/2)}{2m + m + m} = \frac{13}{8}L$$

and $y_{cm} = \dfrac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$

$$\Rightarrow \quad x_{cm} = \frac{(2m)L + m(L/2) + m(0)}{2m + m + m} = \frac{5}{8}L$$

Hence, the correct answer is (C).

31.

Applying Conservation of Linear Momentum, we get

$$mv_0 = mv_1 + mv_2$$

$$\Rightarrow \quad v_1 + v_2 = v_0 \quad \text{...(1)}$$

According to the question, we have

$$K_f = \frac{3}{2}K_i \quad \text{...(2)}$$

$$\Rightarrow \quad \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{3}{2}\left(\frac{1}{2}mv_0^2\right)$$

$$\Rightarrow \quad v_1^2 + v_2^2 = \frac{3}{2}v_0^2 \quad \text{...(3)}$$

From equation (1), we have

$$(v_1 + v_2)^2 = v_0^2$$

$$\Rightarrow \quad v_1^2 + v_2^2 + 2v_1v_2 = v_0^2$$

Using equation (3), we get

$$2v_1v_2 = v_0^2 - \frac{3}{2}v_0^2$$

$$\Rightarrow \quad 2v_1v_2 = -\frac{v_0^2}{2}$$

$$\Rightarrow \quad 4v_1v_2 = -v_0^2$$

Since, $(v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2$

$$\Rightarrow \quad (v_1 - v_2)^2 = v_0^2 - (-v_0^2) = 2v_0^2$$

$$\Rightarrow \quad v_1 - v_2 = \sqrt{2}v_0$$

Hence, the correct answer is (B).

32. In an elastic oblique 2-D collision, if two bodies are of equal masses and second body is at rest, then after collision, the bodies scatter at right angles. So, the bodies have equal mass i.e.

$$m_{\text{unknown}} = m$$

Hence, the correct answer is (A).

33. The kinetic energy of an object just after it hits the ground is 50% of K.E. of the object. So, we have

$$\frac{1}{2}mv'^2 = \frac{1}{2}\left(\frac{1}{2}mv^2\right)$$

$$\Rightarrow \quad v' = \frac{v}{\sqrt{2}}$$

By definition, we have

$$e = \frac{\text{Relative velocity after collision}}{\text{Relative velocity before collision}}$$

$$\Rightarrow \quad e = \frac{v'}{v} = \frac{1}{\sqrt{2}}$$

When a ball dropped from a height h, total distance covered when

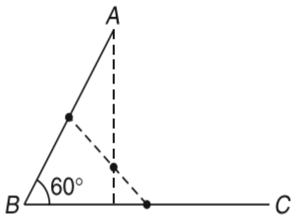
$$t \to \infty \text{ is } \ell = h\left(\frac{1+e^2}{1-e^2}\right)$$

$$\Rightarrow \quad \ell = h\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right) = 3h$$

None of the given OPTIONS is correct.

34. Let the length $AB = p$ and $BC = q$

If λ be the linear mass density of the rod, then according to question, centre of mass of the rod lies vertically below point A, so $x_{cm} = p\cos 60°$



$AB = p$ and $BC = q$

$$\Rightarrow \quad x_{CM} = p\cos 60° = \frac{(\lambda q)\left(\frac{q}{2}\right) + (\lambda p)\left(\frac{p}{2}\right)\cos 60°}{\lambda(p+q)}$$

$$\Rightarrow \quad \frac{p}{2} = \frac{\frac{q^2}{2} + \frac{p^2}{4}}{(p+q)}$$

$$\Rightarrow \quad p^2 + pq = q^2 + \frac{p^2}{2}$$

$$\Rightarrow \quad 1 + \frac{q}{p} = \frac{q^2}{p^2} + \frac{1}{2}$$

$$\Rightarrow \quad \left(\frac{q}{p}\right)^2 - \frac{q}{p} - \frac{1}{2} = 0$$

$$\Rightarrow \quad \frac{q}{p} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)\left(-\frac{1}{2}\right)}}{2\times 1} = \frac{1\pm\sqrt{3}}{2}$$

$$\Rightarrow \quad \frac{q}{p} = \frac{1+\sqrt{3}}{2} = 1.366 \approx 1.37$$

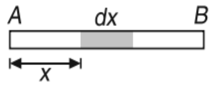
Hence, the correct answer is (C).

35. The centre of mass of a uniform solid cone of height h lies at a distance of $\frac{h}{4}$ from base or $\frac{3h}{4}$ from vertex.

Hence, the correct answer is (B).

36. Consider a small segment dx of the rod at a distance x from A. Mass of this small segment

$$dm = \mu dx = \left(a + \frac{bx}{L}\right)dx$$



Then CM of the rod AB (from A) is given by

$$x_{CM} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^L (\mu dx)x}{\int_0^L \mu dx}$$

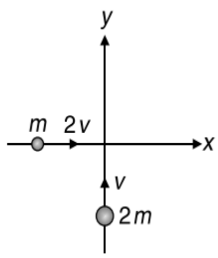
$$\Rightarrow \quad \frac{7}{12}L = \frac{\int_0^L \left(ax + \frac{bx^2}{L}\right)dx}{\int_0^L \left(a + \frac{bx}{L}\right)dx} = \frac{\left(\frac{aL^2}{2} + \frac{bL^2}{3}\right)}{\left(aL + \frac{bL}{2}\right)}$$

$$\Rightarrow \quad \frac{7}{12} = \frac{\frac{a}{2} + \frac{b}{3}}{a + \frac{b}{2}}$$

$$\Rightarrow \quad b = 2a$$

Hence, the correct answer is (C).

37. $$\vec{v}_{combined} = \frac{m(2v)\hat{i} + (2m)\hat{j}}{m + 2m}$$



$$\Rightarrow \quad \vec{v}_{combined} = \frac{2v}{3}(\hat{i} + \hat{j})$$

$$\Rightarrow \quad |\vec{v}_{combined}| = \frac{2\sqrt{2}v}{3}$$

Now loss in KE is

$$-\Delta E = E_i - E_f$$

$$\Rightarrow \quad \text{Loss} = \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)v^2 - \frac{1}{2}(3m)\left(\frac{2\sqrt{2}v}{3}\right)^2$$

$$\Rightarrow \quad \text{Loss} = 3mv^2 - \frac{4}{3}mv^2 = \frac{5}{3}mv^2$$

$$\Rightarrow \quad \%\text{ Loss} = \frac{5mv^2/3}{2mv^2 + mv^2} \times 100\% = 56\%$$

Hence, the correct answer is (C).

38. **Hence, the correct answer is (B).**

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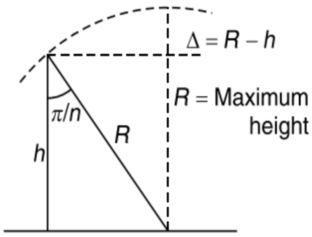
Single Correct Choice Type Problems

1.

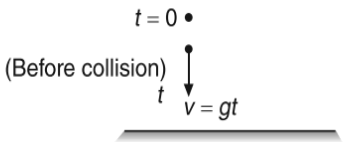


Since, $\cos\left(\frac{\pi}{n}\right)=\frac{h}{R}$

$\Rightarrow \quad \Delta = R-h=\frac{h}{\cos(\pi/n)}-h=h\left[\frac{1}{\cos(\pi/n)-1}\right]$

Hence, the correct answer is (D).

2.



Since, $K=\frac{1}{2}mv^2=\frac{1}{2}mg^2t^2 \qquad \{\because v=gt\}$

$\Rightarrow \quad K \propto t^2$

Therefore, K-t graph is parabola.
However, during collision, retarding force is just like the spring force $(F\propto x)$, therefore kinetic energy first decreases to elastic potential energy and then increases.
Hence, the correct answer is (B).

3. Final momentum of object

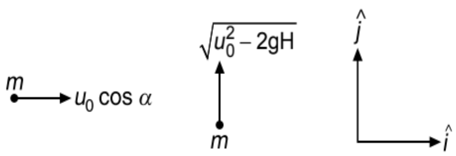
$$p=\frac{E}{c}=\frac{\text{Power}\times\text{Time}}{\text{Speed of light}}=\frac{Pt}{c}$$

$$\Rightarrow \quad p=\frac{30\times10^{-3}\times100\times10^{-9}}{3\times10^{8}}$$

$$\Rightarrow \quad p=1.0\times10^{-17}\ \text{kgms}^{-1}$$

Hence, the correct answer is (B).

4. From momentum conservation, we have



$\Sigma\vec{p}_i=\Sigma\vec{p}_f$

$\Rightarrow \quad m(u_0\cos\alpha)\hat{i}+m\left(\sqrt{u_0^2-2gH}\right)\hat{j}=(2m)\vec{v} \qquad \ldots(1)$

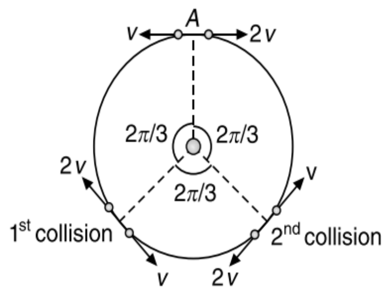
Since, $H=\frac{u_0^2\sin^2\alpha}{2g} \qquad \ldots(2)$

From equations (1) and (2), we get

$$\vec{v}=\left(\frac{u_0\cos\alpha}{2}\right)\hat{i}+\left(\frac{u_0\cos\alpha}{2}\right)\hat{j}$$

Since both x and y components of $\vec{v}$ are equal, so $\vec{v}$ makes an angle of $45°$ with horizontal.
Hence, the correct answer is (A).

5. Before the first collision, the particle having speed $2v$ will rotate $240°$ $\left(\text{or } \frac{4\pi}{3}\right)$ while other particle having speed v will rotate $120°$ $\left(\text{or } \frac{2\pi}{3}\right)$. After first collision, they will exchange their velocities. So, after two collisions they will again reach at point A as shown in Figure.



Hence, the correct answer is (C).

6. Since, $R=u\sqrt{\frac{2h}{g}}$

$\Rightarrow \quad 20=v_1\sqrt{\frac{2\times5}{10}}$ and $100=v_2\sqrt{\frac{2\times5}{10}}$

$\Rightarrow \quad v_1=20\ \text{ms}^{-1},\ v_2=100\ \text{ms}^{-1}$

Applying momentum conservation just before and just after the collision, we get

$$(0.01)(v)=(0.2)(20)+(0.01)(100)$$

$\Rightarrow \quad v=500\ \text{ms}^{-1}$

Hence, the correct answer is (D).

7. Since, $y_{CM}=\frac{m_1y_1+m_2y_2+m_3y_3+m_4y_4+m_5y_5}{m_1+m_2+m_3+m_4+m_5}$

$$\Rightarrow \quad y_{CM}=\frac{(6m)(0)+(m)(a)+m(a)+m(0)+m(-a)}{6m+m+m+m+m}$$

$$\Rightarrow \quad y_{CM}=\frac{a}{10}$$

Hence, the correct answer is (A).

8. On a system of particles if, $\sum F_{ext.} = 0$ then,

$$p_{\text{system}} = \text{constant}$$

Hence, the correct answer is (A).

9. Since, $\vec{p} = A\cos(kt)\hat{i} - A\sin(kt)\hat{j}$

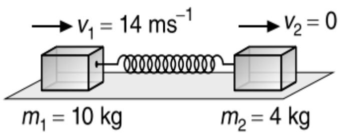
$$\Rightarrow \quad \vec{F} = \frac{d\vec{p}}{dt} = -kA\sin(kt)\hat{i} - kA\cos(kt)\hat{j}$$

Since, $\vec{F}\cdot\vec{p} = 0$

So, angle between $\vec{F}$ and $\vec{p}$ is $90°$

Hence, the correct answer is (D).

10.



Since, $v_{\text{cm}} = \dfrac{m_1v_1 + m_2v_2}{m_1 + m_2}$

$$\Rightarrow \quad v_{\text{cm}} = \frac{10\times 14 + 4\times 0}{10+4} = \frac{10\times 14}{14} = 10 \text{ ms}^{-1}$$

Hence, the correct answer is (C).

11. Change in momentum of two particles is

$$\Delta\vec{p} = \left|(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2)\right|$$

Since, $\Delta\vec{p} = \left|\begin{array}{c}\text{External force on}\\ \text{the system}\end{array}\right| \times \text{time interval}$

$$\Rightarrow \quad \Delta\vec{p} = F_{\text{ext}}\Delta t = (m_1+m_2)g(2t_0)$$

$$\Rightarrow \quad \Delta\vec{p} = 2(m_1+m_2)gt_0$$

Hence, the correct answer is (C).

12. Before explosion, particle was moving along x-axis, i.e., it has no y-component of velocity. Therefore, the centre of mass will not move along the y-direction

$$\Rightarrow \quad y_{\text{cm}} = 0$$

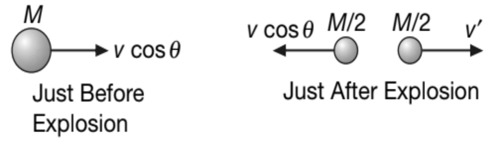
$$\Rightarrow \quad y_{\text{cm}} = \frac{m_1y_1 + m_2y_2}{m_1+m_2}$$

$$\Rightarrow \quad 0 = \frac{\left(\frac{m}{4}\right)(+15) + \left(\frac{3m}{4}\right)(y)}{\left(\frac{m}{4} + \frac{3m}{4}\right)}$$

$$\Rightarrow \quad y = -5 \text{ cm}$$

Hence, the correct answer is (A).

13. At the highest point velocity of the shell is $v\cos\theta$. At the highest point, it explodes into two pieces of equal masses out of which one piece retraces the path i.e., has a velocity $-v\cos\theta$.



So, by Law of Conservation of Momentum

$$M(v\cos\theta) = \frac{M}{2}(-v\cos\theta) + \frac{Mv'}{2}$$

$$\Rightarrow \quad v' = 3v\cos\theta$$

Hence, the correct answer is (A).

14. In an inelastic collision, only the momentum of system (ball and earth) may remain conserved. Some energy can be lost in the form of heat, sound etc.
Hence, the correct answer is (C).

15. Since the system is free from external force, hence $a_{cm} = 0$ and since initially they are at rest, so

$$V_{cm} = 0$$

Hence, the correct answer is (A).

Multiple Correct Choice Type Problems

1. The rate of collision of particle with the piston is

$$f = \frac{1}{2L/v} = \frac{v}{2L}$$

The speed of the particle after collision (when it collides with a speed v) with the piston is $v + 2V$. Assuming that the piston moves inwards by dL, then speed of the particle increases by

$$dv = 2V\left(\frac{dL}{V}\right)\left(\frac{v}{2L}\right)$$

$$\Rightarrow \quad \frac{dv}{v} = \frac{dL}{L}$$

Since L is decreasing, so we have

$$\frac{dv}{v} = -\frac{dL}{L}$$

$$\Rightarrow \quad \int_{v_0}^{v}\frac{dv}{v} = -\int_{L_0}^{L_0/2}\frac{dL}{L}$$

$$\Rightarrow \quad (\log_e v)\Big|_{v_0}^{v} = -(\log_e L)\Big|_{L_0}^{L_0/2}$$

$$\Rightarrow \quad \log_e\left(\frac{v}{v_0}\right) = -\log_e\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad v = 2v_0$$

So, kinetic energy at L_0 is $K_{L_0} = \frac{1}{2}mv_0^2$ and kinetic energy at $L_0/2$ is

$$K_{L_0/2} = \frac{1}{2}m(2v_0)^2 = 4K_{L_0}$$

Hence, (B) and (C) are correct.

2. Since, Δx_{cm} of the block and point mass system is zero i.e., CM coil not move along x-direction .

$\Rightarrow \quad m(x+R)+Mx = 0$

where, x is displacement of the block.

$$\Rightarrow \quad x = -\frac{mR}{M+m}$$

Applying conservation of momentum, we get

$$0 = mv - MV$$

$\Rightarrow \quad mv = MV$

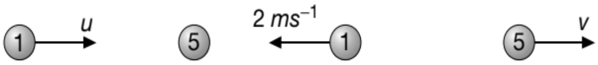
Applying conservation of energy, we get

$$mgR = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

Solving these two equations, we get

$$v = \sqrt{\frac{2gR}{1+\dfrac{m}{M}}}$$

Hence, (A) and (B) are correct.

3. 

Before collision After collision

Since collision is elastic, so $e=1$

$\Rightarrow$ Velocity of Approach = Velocity of Separation

$\Rightarrow \quad u = v+2 \quad \ldots(1)$

By Law of Conservation of Linear Momentum, we get

$$(1)u = (5)v - (1)(2)$$

$\Rightarrow \quad u = 5v - 2 \quad \ldots(2)$

Equating (1) and (2), we get

$$v+2 = 5v-2$$

$\Rightarrow \quad v = 1\ \text{ms}^{-1}$ and $u = 3\ \text{ms}^{-1}$

Momentum of system $p_{system} = (1)(3) = 3\ \text{kgms}^{-1}$

Momentum of 5 kg after collision is

$$p_2 = (5)(1) = 5\ \text{kgms}^{-1}$$

So, kinetic energy of centre of mass is

$$K_{cm} = \frac{1}{2}(m_1+m_2)\left(\frac{m_1u}{m_1+m_2}\right)^2 = \frac{1}{2}(1+5)\left(\frac{1\times 3}{6}\right)^2$$

$\Rightarrow \quad K_{cm} = 0.75\ \text{J}$

Total kinetic energy is $K = \frac{1}{2}\times 1\times 3^2$

$\Rightarrow \quad K = 4.5\ \text{J}$

Hence, (A) and (C) are correct.

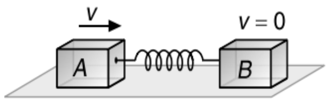
4. Initial momentum of the system is $\Sigma p_{initial} = \vec{p}_1 + \vec{p}_2 = 0$

So, final momentum of system $\Sigma p_{final} = \vec{p}_1' + \vec{p}_2' = 0$

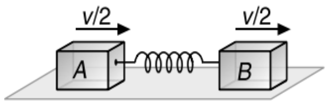
OPTION (B) is allowed because if we put $c_1 = -c_2 \neq 0$, $\vec{p}_1' + \vec{p}_2'$ will be zero. Similarly, we can check other options.

Hence, (A) and (D) are correct.

5. Here, at maximum compression x_{max}, we introduce the concept of reduced mass μ of the system (details discussed in Rotational Dynamics) given by



Initially



At maximum compression

$$\frac{1}{\mu} = \frac{1}{m} + \frac{1}{m}$$

$$\Rightarrow \quad \mu = \frac{m}{2}$$

Further, by Law of Conservation of Energy we get

$$\frac{1}{2}\mu v^2 = \frac{1}{2}kx_{max}^2$$

$$\Rightarrow \quad x_{max} = v\sqrt{\frac{m}{2k}}$$

So, $\quad \text{K.E.} = \frac{1}{2}\mu v^2 = \frac{1}{2}\left(\frac{m}{2}\right)v^2$

$$\Rightarrow \quad \text{K.E.} = \frac{mv^2}{4}$$

Hence, (B) and (D) are correct.

Reasoning Based Questions

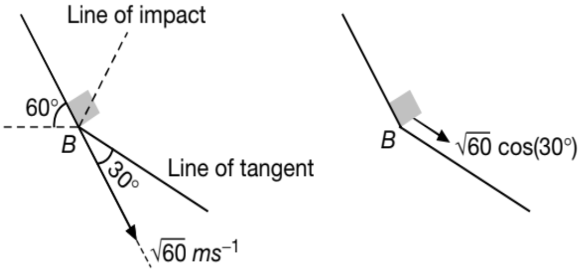
1. If a force is applied at centre of mass of a rigid body, its torque about centre of mass will be zero, but acceleration will be non-zero. Hence, velocity will change.
Hence, the correct answer is (D).

2. In case of elastic collision, coefficient of restitution $e=1$.
Magnitude of relative velocity of approach equals magnitude of relative velocity of separation.

But relative speed of approach is not equal to relative speed of separation.
Hence, the correct answer is (D).

Linked Comprehension Type Questions

1.



Before After

Let speed of block just before it strikes the second inclined plane be v then,

$$\frac{1}{2}mv^2 = mg\left(\sqrt{3}\tan 60^\circ\right)$$

$$\Rightarrow \quad v = \sqrt{60}\ \text{ms}^{-1}$$

Speed of block immediately after it strikes the second incline is $\sqrt{45}\ \text{ms}^{-1}$. Because in perfectly inelastic collision the component of velocity along line of impact becomes zero.
Hence, the correct answer is (B).

2. By Conservation of Mechanical Energy

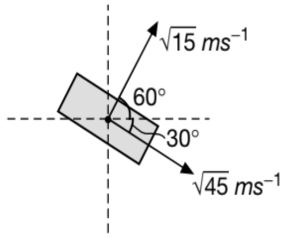
$$\frac{1}{2}mv_C^2 = \frac{1}{2}m\left(\sqrt{45}\right)^2 + mg\left(3\sqrt{3}\tan 30^\circ\right)$$

$$\Rightarrow \quad v_C^2 = 45 + 60 = 105$$

$$\Rightarrow \quad v_C = \sqrt{105}\ \text{ms}^{-1}$$

Hence, the correct answer is (B).

3. If collision is completely elastic, then vertical component of velocity becomes,



$$\sqrt{45}\sin 30^\circ - \sqrt{15}\sin 60^\circ = \frac{\sqrt{45}}{2} - \frac{\sqrt{15}\times\sqrt{3}}{2} = 0$$

Hence, the correct answer is (C).

Integer/Numerical Answer Type Questions

1. For elastic collision between A and B, we have

$$v_B = \left(\frac{2m}{2m+m}\right)9$$

$$\Rightarrow \quad v_B = 6\ \text{ms}^{-1}$$

Now, for completely inelastic collision between B and C, we have

$$v_C = \frac{2m(v_B) + m(0)}{2m+m} = \frac{(2m)(6)}{3m}$$

$$\Rightarrow \quad v_C = 4\ \text{ms}^{-1}$$