CHAPTER 1: WORK, ENERGY, POWER AND CONSERVATION OF ENERGY

Test Your Concepts-I (Based on Work Done by a Constant and Variable Force)

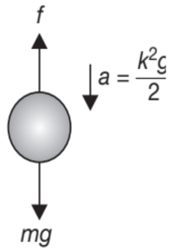
1. **Method I:**

Let a be the net downward acceleration of the ball. Then

$$v^2 - u^2 = 2ah$$

$$\Rightarrow \quad k^2(gh) - 0^2 = 2ah$$

$$\Rightarrow \quad a = \frac{k^2 g}{2}$$



If f be the air drag, then using Newton's Second Law, we get

$$mg - f = ma$$

$$\Rightarrow \quad f = mg - ma$$

$$\Rightarrow \quad f = mg\left(1 - \frac{k^2}{2}\right)$$

Now $W_f = fh\cos(180°)$

$$\Rightarrow \quad W_f = -mgh\left(1 - \frac{k^2}{2}\right)$$

Method II:

Since we know that

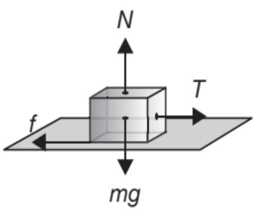
$$W_{nc} = \Delta U + \Delta k$$

$$\Rightarrow \quad W_{\text{air drag}} = (0 - mgh) + \left(\frac{1}{2}mv^2 - 0\right)$$

$$\Rightarrow \quad W_{\text{air drag}} = -mgh + \frac{1}{2}mk^2(gh)$$

$$\Rightarrow \quad W_{\text{air drag}} = -mgh\left(1 - \frac{k^2}{2}\right)$$

2. $W_T = T\ell_0$



$W_N = 0 \qquad \{\because \theta = 90°\}$

$W_{mg} = 0 \qquad \{\because \theta = 90°\}$

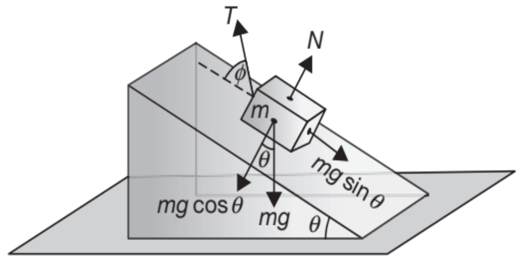
$W_f = f\ell_0\cos(180°) = -f\ell_0$

3. $W_T = Tl\cos\phi$

$W_N = 0 \qquad \left\{\because \theta = \frac{\pi}{2}\right\}$

$F_f = -fl \qquad \{\because \theta = \pi\}$

$W_{mg} = -mgl\sin\theta$



4. (a) $W_1 = \int_{x_1}^{x_2}(-kx)dx = \frac{k}{2}(x_1^2 - x_2^2)$

$$W_1 = \frac{500}{2}\left[(0.15)^2 - (0.08)^2\right] = 4.03 \text{ J}$$

(b) $W_2 = -(mg\sin\theta)(x_2 - x_1)$

$$\Rightarrow \quad W_2 = -(6)(9.8)(\sin 15°)(0.23) = -3.5 \text{ J}$$

5. (a) $W_1 = \int_{(0.1,\,0)}^{(0.1,\,0.4)} \vec{F}\cdot d\vec{r}$

where $\vec{F} = -\alpha x^2\hat{i}$ and $d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$\Rightarrow \quad W_1 = \int_{(0.1,\,0)}^{(0.1,\,0.4)} -\alpha x^2 dx = 0$$

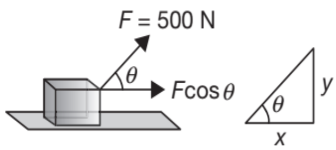
(b) $W_2 = \int_{(0.1,\,0)}^{(0.3,\,0)} -\alpha x^2 dx = -\frac{\alpha}{3}(x^3)\Big|_{(0.1,\,0)}^{(0.3,\,0)} = -\frac{\alpha}{3}(0.026)$

(c) $W_3 = \int_{(0.3,\,0)}^{(0.1,\,0)} -\alpha x^2 dx = -\frac{\alpha}{3}(x^3)\Big|_{(0.3,\,0)}^{(0.1,\,0)} = \frac{\alpha}{3}(0.026)$

6. Using our knowledge of constraints we have

$$x = 8\cot\theta$$

$$\Rightarrow \quad dx = -8\operatorname{cosec}^2\theta\, d\theta$$



Since, $W = \int_{\theta=30^\circ}^{60^\circ} (F\cos\theta)dx$

$\Rightarrow \quad W = \int_{30^\circ}^{60^\circ} (500\cos\theta)(-8\text{cosec}^2\theta)d\theta$

$\Rightarrow \quad W = -4000\int_{30^\circ}^{60^\circ} \cos\theta\,\text{cosec}^2\theta d\theta$

But $\int \cos\theta\,\text{cosec}^2\theta d\theta = \int \text{cosec}\theta\cot\theta d\theta$

$\Rightarrow \quad \int \text{cosec}\theta\cot\theta d\theta = -\text{cosec}\theta$

$\Rightarrow \quad W = -4000(-\text{cosec}\theta)\Big|_{30^\circ}^{60^\circ}$

$\Rightarrow \quad W = 4000[\text{cosec}(60^\circ) - \text{cosec}(30^\circ)]$

$\Rightarrow \quad W = 4000[1.155 - 2]$

$\Rightarrow \quad W = 4000(-0.845)$

$\Rightarrow \quad W = -3380 \text{ J}$

7. $W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$

where $d\vec{r} = \hat{i}\,dx + \hat{j}dy + \hat{k}dx$

$\Rightarrow \quad W = \int_{\vec{r}_1}^{\vec{r}_2} (3x^2\hat{i} + 2y\hat{j})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$

$\Rightarrow \quad W = \int_{\vec{r}_1}^{\vec{r}_2} (3x^2dx + 2ydy) = (x^3 + y^2)\Big|_{(2,3)}^{(4,6)}$

$\Rightarrow \quad W = 83 \text{ J}$

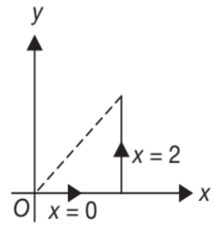
8. (a) $W_1 = \int_{(0,0)}^{(2,2)} -\alpha y^2 dy = \int_{(0,0)}^{(2,2)} -\alpha y^3 dy$

(Along the path $x = y$)

$\Rightarrow \quad W_1 = -\frac{\alpha}{4}(y^4)\Big|_{(0,0)}^{(2,2)} = -4\alpha \; \{\because \alpha = 1.5 \text{ Nm}^{-3}\}$

$\Rightarrow \quad W_1 = -6 \text{ J}$

(b) Initially when we move along x-axis, the work done is zero because the force is acting along y-axis. Then along the path parallel to y-axis, we can put $x = 2$ to get



$W_2 = \int_{(2,0)}^{(2,2)} -2\alpha y^2 dy$

$\Rightarrow \quad W_2 = -\frac{2\alpha}{3}(y^3)\Big|_{(2,0)}^{(2,2)}$

$\Rightarrow \quad W_2 = -8 \text{ J} \qquad \{\because \alpha = 1.5 \text{ Nm}^{-3}\}$

(c) As $W_1 \neq W_2$, force is non-conservative in nature.

9. Since work done is equal to the area under a curve in F-x graph. So

$W = \frac{1}{2}(-2)(-10) + \frac{1}{2}(-2)(10) + (10)(2) + \frac{1}{2}(2)(10)$

$\Rightarrow \quad W = 10 - 10 + 20 + 10$

$\Rightarrow \quad W = 30 \text{ J}$

10. The work from $x = 0$ to $x = 8$ m is the area under the curve.

$W = (10\times 2) + \frac{1}{2}(10)(4-2) + 0 + \frac{1}{2}(-5)(8-6)$

$\Rightarrow \quad W = 25 \text{ J}$

11. (a) W = Area under F-x graph

$\Rightarrow \quad W_{0\to 3} = \frac{1}{2}\times 2\times 2 + 1\times 2 = 4 \text{ J}$

(b) $W_{3\to 4} = 0$

(c) $W_{4\to 7} = -\frac{1}{2}\times 2\times 1 = -1 \text{ J}$

$W_{4\to 7}$ is negative, because force is in negative x-direction while displacement is towards positive x-axis.

(d) $W_{0\to 7} = 4 + 0 - 1 = 3 \text{ J}$

12. Work done = Area under F-s graph

$\Rightarrow \quad W = \frac{1}{2}\times 4\times 10 + (8-4)\times 10$

$\Rightarrow \quad W = 20 + 40$

$\Rightarrow \quad W = 60 \text{ J}$

13. Since, $dW = \vec{F}\cdot d\vec{r}$, where

$$d\vec{r} = (dx)\hat{i} + (dy)\hat{j}$$

$$\Rightarrow \quad dW = -k(ydx + xdy)$$

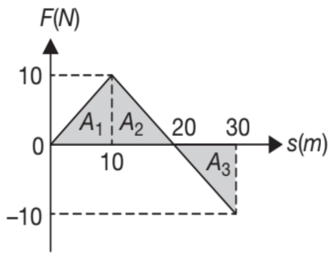
Since, $d(xy) = x\,dy + y\,dx$

$$\Rightarrow \quad dW = -kd(xy)$$

$$\Rightarrow \quad W = \int dW = -k\int_{(0,0)}^{(a,a)} d(xy)$$

$$\Rightarrow \quad W = -k(xy)\Big|_{(0,0)}^{(a,a)} = -ka^2 - 0 = -ka^2$$

14. Since we know that work done is equal to the area under F-s graph.



(a) Work done at the end of 10 m is the area under the curve from 0 to 10 m.

$$\Rightarrow \quad W = A_1 = \frac{1}{2}(10)(10)\ \text{J} = 50\ \text{J}$$

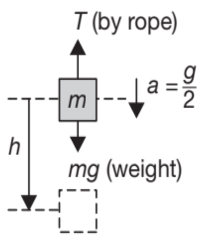
(b) Work done at the end of 20 m is the area under the curve from 0 to 20 m.

$$\Rightarrow \quad W = A_1 + A_2 = 50 + 50 = 100\ \text{J}$$

(c) Work done at the end of 30 m is the area under the curve from 0 to 30 m.

$$\Rightarrow \quad W = A_1 + A_2 + A_3 = 50 + 50 - 50 = 50\ \text{J}$$

15. The FBD of the block placed in the lift is shown in Figure.



According to Newton's Second Law, we have

$$mg - T = ma = \frac{mg}{2}$$

$$\Rightarrow \quad T = \frac{mg}{2}$$

Work done by the gravitational force is

$$W_{mg} = (mg)(h)\cos(0°) = mgh$$

Work done by the tension force is

$$W_T = Th\cos(180°) = -\frac{mgh}{2}$$

Test Your Concepts-II (Based on Kinetic Energy, Potential Energy and Power)

1. $P = Fv$

$$\Rightarrow \quad F = \frac{P}{v} \qquad \ldots(1)$$

Now, we have

$$F - f = ma = m\left(\frac{dv}{dt}\right)$$

At maximum speed, $\frac{dv}{dt} = 0$

$$\Rightarrow \quad F = f$$

So, from (1), we get $\frac{P}{v_{\max}} = f$

$$\Rightarrow \quad v_{\max} = \frac{P}{f}$$

Further, since $F = \frac{P}{v}$

$$\Rightarrow \quad m\left(\frac{dv}{dt}\right) = \frac{P}{v}$$

$$\Rightarrow \quad \int_0^{P/2r} vdv = \left(\frac{P}{m}\right)\int_0^t dt$$

$$\Rightarrow \quad \left(\frac{P}{2r}\right)^2 = \left(\frac{2P}{m}\right)t$$

$$\Rightarrow \quad t = \frac{Pm}{8r^2}$$

2. (a) $P = \text{constant}$

$$\Rightarrow \quad Fv = P$$

$$\Rightarrow \quad mv\frac{dv}{dt} = P \qquad \left\{\because F = \frac{dv}{dt}\right\}$$

$$\Rightarrow \quad mvdv = Pdt$$

$$\Rightarrow \quad m\int_0^v vdv = P\int_0^t dt$$

$$\Rightarrow \quad \frac{mv^2}{2} = Pt$$

$$\Rightarrow \quad v = \sqrt{\frac{2Pt}{m}}$$

(b) Since, $v=\sqrt{\dfrac{2Pt}{m}}$

$$\Rightarrow \frac{dx}{dt}=\sqrt{\frac{2P}{m}}t^{1/2} \qquad \left\{\because v=\frac{dx}{dt}\right\}$$

$$\Rightarrow dx=\sqrt{\frac{2P}{m}}t^{1/2}dt$$

$$\Rightarrow \int_0^x dx=\sqrt{\frac{2P}{m}}\int_0^t t^{1/2}dt$$

$$\Rightarrow x=\left(\frac{2}{3}\sqrt{\frac{2P}{m}}\right)t^{3/2} \qquad \left\{\because \int t^{1/2}dt=\frac{2}{3}t^{3/2}\right\}$$

$$\frac{ds}{dt}=\left(\frac{2Pt}{m}\right)^{1/2} \text{ i.e., } \int_0^s ds=\int_0^t\left(\frac{2Pt}{m}\right)^{1/2}dt$$

which on integration gives

$$s=\left(\frac{2P}{m}\right)^{1/2}\frac{2}{3}t^{3/2} \text{ or } s=\left(\frac{8P}{9m}\right)^{1/2}t^{3/2}$$

3. From Work-Energy Theorem, we have

$$W=\Delta KE$$

$$\Rightarrow Pt=\frac{1}{2}m\left(v^2-u^2\right) \qquad \{\because W=Pt\}$$

$$\Rightarrow t=\frac{m}{2P}\left(v^2-u^2\right) \qquad \ldots(1)$$

Further, $Fv=P$

$$\Rightarrow m\left(\frac{dv}{ds}\right)v=P$$

$$\Rightarrow kv^2\frac{dx}{dt}=P \qquad \left\{\because \frac{dv}{dt}=v\frac{dv}{dx}\right\}$$

$$\Rightarrow \int_u^v v^2dv=\frac{P}{m}\int_0^x dx$$

$$\Rightarrow \left(v^3-u^3\right)=\frac{3P}{m}x$$

$$\Rightarrow \frac{m}{P}=\frac{3x}{v^3-u^3}$$

Substituting in Equation (1), we get

$$t=\frac{3x(u+v)}{2\left(u^2+v^2+uv\right)}$$

4. **For** $t\le 0.2$ s

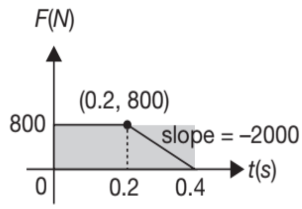
$$F=800\text{ N and } v=\left(\frac{20}{0.4}\right)t=50\text{ t}$$

$$\Rightarrow P=Fv=(40000\text{ t})\text{watt}=40\text{ t kW}$$

For $t>0.2$ s

A straight line with slope $m=-\dfrac{800}{0.4}=-2000$ passing through the point $(x_1y_1)=(0.2,\ 800)$ is

$$y-y_1=m(x-x_1)$$



Here x-axis is time axis and y-axis is force axis

So, we get

$$F-800=-2000(t-0.2)$$

$$\Rightarrow F=800-2000t+400$$

$$\Rightarrow F=-2000t+1200$$

Further $v=\left(\dfrac{20}{0.4}\right)t=50$ t

$$\Rightarrow P=Fv=(-2000t+1200)(50t)\text{ watt}$$

$$\Rightarrow P=Fv=(-20t+12)5t\text{ kW}$$

$$\text{So, } P=\begin{cases}40t & \text{for } t\le 0.2\text{ s}\\ -100t^2+60t & \text{for } t>0.2\text{ s}\end{cases}$$

Now, $P=\dfrac{dW}{dt}$

$$\Rightarrow W=\int Pdt$$

$$\Rightarrow W=\int_0^{0.2}40tdt+\int_{0.2}^{0.4}\left(-100t^2+60t\right)dt$$

$$\Rightarrow W=\left[40\left(\frac{t^2}{2}\bigg|_0^{0.2}\right)-100\left(\frac{t^3}{3}\bigg|_{0.2}^{0.4}\right)+60\left(\frac{t^2}{2}\bigg|_{0.2}^{0.4}\right)\right]\text{kJ}$$

$$\Rightarrow W=\left[20(0.04)-\frac{100}{3}(0.056)+30(0.12)\right]\times 1000\text{ J}$$

$$\Rightarrow W=(800-1867+3600)\text{ J}$$

$$\Rightarrow W=2533\text{ J}$$

5. Since the collar is lifted with a constant speed of 0.6 ms^{-1}. So

$$F\cos\theta-mg=0$$

$$\Rightarrow F\cos\theta=5(10)=50$$

$$\Rightarrow F=\frac{50}{\cos\theta} \qquad \ldots(1)$$

Now $P = \vec{F}\cdot\vec{v}$

$\Rightarrow \quad P = \left(\dfrac{50}{\cos\theta}\right)(v\cos\theta)$

$\Rightarrow \quad P = 50v$

$\Rightarrow \quad P = 30\text{ W} \qquad \{\because v = 0.6\text{ ms}^{-1}\}$

6. At constant speed, there is no acceleration, so the forces acting on the train are in equilibrium.

$\Rightarrow \quad F = R = 3\times10^4\text{ N}$

Since we know that $P = Fv$

$\Rightarrow \quad P = 3\times10^4\times40 = 1.2\times10^6$ watt

7. Since, $F \propto x^{-1/2}$

$\Rightarrow \quad a = v\dfrac{dv}{dx} = kx^{-1/2}$

$\Rightarrow \quad vdv = kx^{-1/2}dx$

Integrating both sides, we get

$$\int_0^v vdv = \int_0^x x^{-1/2}dx$$

$\Rightarrow \quad \dfrac{v^2}{2} = 2x^{\frac{1}{2}}$

$\Rightarrow \quad v \propto x^{\frac{1}{4}}$

Since power is defined as

$P = Fv$

$\Rightarrow \quad P \propto \dfrac{x^{\frac{1}{4}}}{x^{\frac{1}{2}}} = x^{\frac{1}{4}-\frac{1}{2}} = x^{-\frac{1}{4}}$ i.e., $P \propto x^{-\frac{1}{4}}$

$\Rightarrow \quad n = -\dfrac{1}{4}$

8. Here the only force acting is $\vec{F} = (2+3x)\hat{i}$. So, body starts moving along x-axis. According to Work-Energy Theorem,

$$W = \int \vec{F}\cdot d\vec{x} = \Delta K$$

$\Rightarrow \quad \displaystyle\int_0^5 (2+3x)dx = \frac{1}{2}(5)v^2 - 0$

$\Rightarrow \quad \left(2x + \dfrac{3x^2}{2}\right)\Bigg|_0^5 = \dfrac{5}{2}v^2$

$\Rightarrow \quad 10 + \dfrac{75}{2} - 0 = \dfrac{5}{2}v^2$

$\Rightarrow \quad v = \sqrt{19}\text{ ms}^{-1}$

9. According to Work-Energy Theorem, we have

$$W = \Delta K = \frac{1}{2}m(v^2 - u^2)$$

Given that $m = 0.5$ kg and $v = ax\sqrt{x} = 5x^{3/2}$

Initial velocity u i.e. velocity at $x = 0$ is

$u = a(0) = 0$

Final velocity v i.e. velocity at $x = 2$ is

$v = 5\left(2^{\frac{3}{2}}\right) = 10\sqrt{2}\text{ ms}^{-1}$

$\Rightarrow \quad W = \Delta K = \dfrac{1}{2}m(v^2 - u^2) = \dfrac{1}{2}(0.5)(200)$

$\Rightarrow \quad W = 50$ J

10. Since $P = Fv$, where $F = v\dfrac{dm}{dt}$

If ρ be the density of wind, then $\dfrac{dm}{dt} = \rho\left(\dfrac{dV}{dt}\right)$

where, $\dfrac{dV}{dt}$ is the volume of wind flowing per second through the generator.

Since $\dfrac{dV}{dt} = A\left(\dfrac{dx}{dt}\right) = Av$

where A is the area of the generator through which wind flows

$\Rightarrow \quad \dfrac{dm}{dt} = \rho\left(\dfrac{dV}{dt}\right) = Av\rho$

$\Rightarrow \quad F = v\left(\dfrac{dm}{dt}\right) = v(Av\rho)$

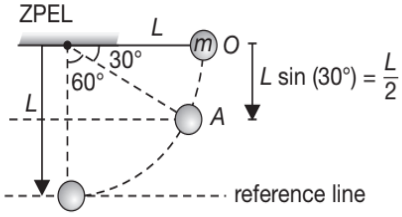
$\Rightarrow \quad F = Av^2\rho$

$\Rightarrow \quad P = Fv = Av^3\rho$

$\Rightarrow \quad P \propto v^3$

So, $n = 3$

11.



Let the reference line pass through point of support as shown.

$\Rightarrow \quad U_O = 0$

$\Rightarrow \quad U_A = -mg\left(\dfrac{L}{2}\right)$ and $U_B = -mgL$

$\Rightarrow \quad U_O - U_A = \dfrac{mgL}{2} = U_A - U_B$

$\Rightarrow \quad \dfrac{U_O - U_A}{U_A - U_B} = 1$

12. Potential energy (PE) for a stretch of 20 cm is

$U_1 = \dfrac{1}{2}k(0.2)^2 = 30 \text{ J}$

$\Rightarrow \quad k = \dfrac{30 \times 2}{0.04} = 1500 \text{ Nm}^{-1}$

PE acquired for a stretch of $20 + 40 = 60 \text{ cm}$ is

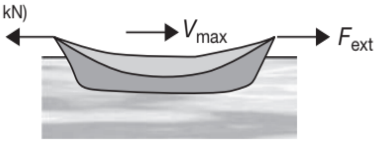
$U_2 = \dfrac{1}{2} \times 1500 \times (0.6)^2 = 1500 \times 0.18$

$\Rightarrow \quad U_2 = 270 \text{ J}$

Additional work required is

$W = U_2 - U_1 = 270 - 30 = 240 \text{ J}$

13. $F_{\text{resistive}} = 20 \, v_{\text{max}}$
(in kN)



Power of engine is

$P = 50 \text{ HP} = 50 \times 746 = 37300 \text{ W}$

Let v_{max} be the maximum speed attained by the boat.

At $v = v_{\text{max}}$, acceleration of boat is zero, so net force acting on boat is also zero.

$\Rightarrow \quad F_{\text{ext}} - F_{\text{resistive}} = 0$

$\Rightarrow \quad F_{\text{ext}} = (20 v_{\text{max}}) 1000 \text{ N}$

Since, $P = Fv$

$\Rightarrow \quad 37300 = (20000 v_{\text{max}}) v_{\text{max}}$

$\Rightarrow \quad v_{\text{max}}^2 = \dfrac{37300}{20000}$

$\Rightarrow \quad v_{\text{max}} = \sqrt{\dfrac{373}{200}} = 1.36 \text{ ms}^{-1} = 4.9 \text{ kmh}^{-1}$

14. When mass is suspended from the spring, then in equilibrium position, let extension in spring be x_1.

$\Rightarrow \quad mg = kx_1$

$\Rightarrow \quad k = \dfrac{mg}{x_1} = \dfrac{4 \times 10}{(0.5 - 0.3)} = 200 \text{ Nm}^{-1}$

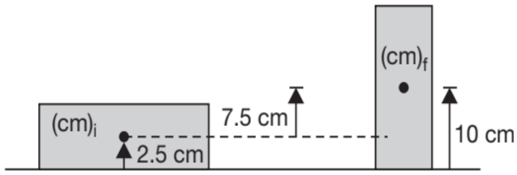
Total stretch in the spring in the final state is

$x_2 = 0.6 - 0.3 = 0.3 \text{ m}$

So, elastic energy stored in the final position is

$U = \dfrac{1}{2}kx_2^2 = \dfrac{1}{2} \times 200 \times (0.3)^2 = 9 \text{ J}$

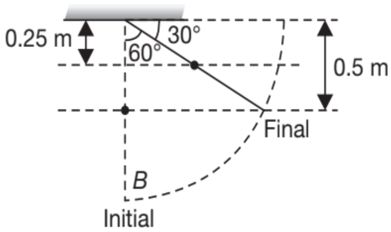
15.



Since centre of mass of brick rises by 7.5 cm, so

$\Delta U = mgh = (5)(10)\left(\dfrac{7.5}{100}\right) = 3.75 \text{ J}$

16.



Initially, the centre of mass of the metre scale lies 0.5 m below the hinge and finally 0.25 m below the hinge when the scale is turned to a position making $30°$ with horizontal. Therefore, centre of mass of the scale rises by

$h = 0.5 - 0.25 = 0.25 \text{ m}$

Hence work done by gravity is $W = mgh\cos 180°$

$\Rightarrow \quad W = 0.1 \times 10 \times 0.25 \times (-1) = -0.25 \text{ J}$

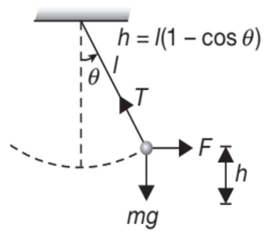
Test Your Concepts-III (Based on Conservation of Energy and Work Energy Theorem)

1. In this case three forces are acting on the object

(i) tension (T)

(ii) weight (mg) and

(iii) applied force (F)



Using Work-Energy Theorem

$W_{\text{net}} = \Delta K.E.$

$\Rightarrow \quad W_T + W_{mg} + W_F = 0 \qquad \ldots(1)$

as $\Delta K.E. = 0$ because $K_i = K_f = 0$

Further, $W_T = 0$, as tension is always perpendicular to displacement.

$$W_{mg} = -mgh$$

$$\Rightarrow \quad W_{mg} = -mg\ell(1-\cos\theta)$$

Substituting these values in Equation (1), we get

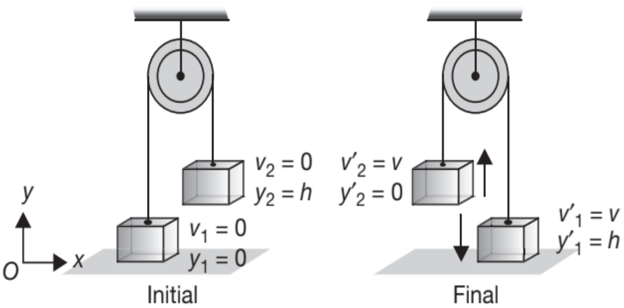
$$W_F = mg\ell(1-\cos\theta)$$

Conceptual Note(s)

Here the applied force F is variable. So, if we do not apply the work energy theorem, we will first find the magnitude of F at different locations and then integrate $dW\left(=\vec{F}\cdot d\vec{r}\right)$ with proper limits.

2. The initial and final configurations are shown in the figure.

It is convenient to set $U_g = 0$ at the floor. Initially, only m_2 has potential energy. As it falls, it loses potential energy and gains kinetic energy. At the same time, m_1 gains potential energy and kinetic energy. Just before m_2 lands, it has only kinetic energy. Let v the final speed of each mass. Then, using the Law of Conservation of Mechanical Energy.



$$K_f + U_f = K_i + U_i$$

$$\Rightarrow \quad \frac{1}{2}(m_1+m_2)v^2 + m_1gh = 0 + m_2gh$$

$$\Rightarrow \quad v^2 = \frac{2(m_2-m_1)gh}{m_1+m_2}$$

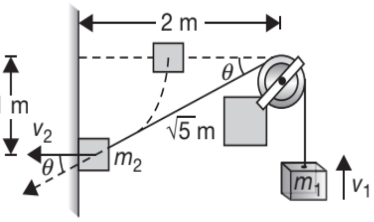
Substitute $m_1 = 3$ kg, $m_2 = 5$ kg, $h = 5$ m and $g = 10\ \text{ms}^{-2}$.

$$\Rightarrow \quad v^2 = \frac{2(5-3)(10)(5)}{5+3}$$

$$\Rightarrow \quad v = 5\ \text{ms}^{-1}$$

3. From conservation of energy,

$$m_2gh_2 = m_1gh_1 + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$



Using our knowledge of constraint relations, we get

$$v_1 = v_2\cos\theta$$

$$\Rightarrow \quad 2\times10\times1 = (0.5)(10)(\sqrt{5}-2) + \frac{1}{2}\times0.5\times v_2^2\times\left(\frac{2}{\sqrt{5}}\right)^2 + \frac{1}{2}\times2\times v_2^2$$

$$\Rightarrow \quad 20 = 1.18 + 0.2v_2^2 + v_2^2$$

$$\Rightarrow \quad v_2 = 3.96\ \text{ms}^{-1} \text{ and } v_1 = v_2\cos\theta = \frac{2}{\sqrt{5}}\times3.96$$

$$\Rightarrow \quad v_1 = 3.54\ \text{ms}^{-1}$$

4. According to Work Energy Theorem, applied on A to B, we have

$$\begin{pmatrix}\text{Work done by force}\\ \text{from A to B}\end{pmatrix} = \begin{pmatrix}\text{Change in KE}\\ \text{from A to B}\end{pmatrix}$$

$$\Rightarrow \quad W_{AB} = \frac{1}{2}m\left(v_B^2 - v_A^2\right)$$

$$\Rightarrow \quad (2T - mg\sin\theta)(AB) = \frac{1}{2}\,\text{m}\left(v_B^2 - v_A^2\right)$$

$$\Rightarrow \quad \left(1150 - 130\times10\times\frac{5}{13}\right)(3) = \frac{1}{2}\times130\left(v_B^2 - 9\right)$$

$$\Rightarrow \quad (650)(3) = 65\left(v_B^2 - 9\right)$$

$$\Rightarrow \quad v_B^2 - 9 = 30$$

$$\Rightarrow \quad v_B = \sqrt{39} = 6.25\ \text{ms}^{-1}$$

While using work-energy theorem we are concerned with speed not the velocity.

5. (a) For spring 1, $X_{10} = \dfrac{(m_1+m_2)g}{k_1} = \dfrac{30}{1500} = 0.02$ m

For spring 2, $X_{20} = \dfrac{m_2g}{k_2} = \dfrac{10}{500} = 0.02$ m

$$\Rightarrow \quad U = \frac{1}{2}k_1x_{01}^2 + \frac{1}{2}k_2x_{02}^2 = (750+250)(0.02)^2$$

$$\Rightarrow \quad U = 0.4\ \text{J}$$

CHAPTER 1

(b) Let Δx_1 and Δx_2 be the additional elongation due to pulling m_2 downwards by $\ell = 0.08$ m. Additional forces on m_2 are equal in magnitude and opposite in direction.

$\Rightarrow \quad k_1\Delta x_1 = k_2\Delta x_2$

$$\Rightarrow \quad \frac{\Delta x_1}{\Delta x_2} = \frac{1}{3} \qquad \ldots(1)$$

Also, $\Delta x_1 + \Delta x_2 = 0.08$ ...(2)

Solving Equation (1) and (2), we get

$\Delta x_1 = 0.02$ m and $\Delta x_2 = 0.06$ m

According to Work Energy Theorem

$W_{\text{total}} = \Delta K$

Since, m_2 is pulled down slowly, so

$\Delta K = 0$

$\Rightarrow \quad W_{mg} + W_{\text{spring}} + W_F = \Delta KE = 0$

$$\Rightarrow \quad W_F = -\left(W_{mg} + W_{\text{spring}}\right) \qquad \ldots(3)$$

Now, $W_{mg} = m_1 g\Delta x_1 + m_2 g\left(\Delta x_2 + \Delta x_1\right) = 1.2$ J

Also, $W_{\text{spring}} = -\left(U_f - U_i\right) = U_i - U_f$

where, $U_i = 0.4$ J and

$$U_f = \frac{1}{2}k_1\left(x_{01} + \Delta x_1^2\right) + \frac{1}{2}k_2\left(x_{02} + \Delta x_2\right)^2 = 2.8 \text{ J}$$

$\Rightarrow \quad W_{\text{spring}} = -2.4$ J

Since, from (3), we have

$$W_F = -\left(W_{mg} + W_{\text{spring}}\right) = -(1.2 - 2.4) \text{ J} = 1.2 \text{ J}$$

6. The lower block will bounce when extension in spring,

$kx = Mg$

$$\Rightarrow \quad x = \frac{Mg}{k} \qquad \ldots(1)$$

By Law of Conservation of Energy, loss in gravitational potential energy (GPE) equals gain in elastic potential energy (EPE) of spring, so

$$mgh - mgx = \frac{1}{2}kx^2$$

$$\Rightarrow \quad mg\left(h - \frac{Mg}{k}\right) = \frac{1}{2}k\left(\frac{Mg}{k}\right)^2$$

$$\Rightarrow \quad h = \frac{\dfrac{M^2g^2}{2k} + \dfrac{Mmg^2}{k}}{mg} = \frac{M^2 g}{2km} + \frac{Mg}{k}$$

7. Let downward velocity of sphere be v_s and horizontal velocity of wedge be v_w. From our knowledge of constraint relations, the constraint equation for velocity can be written as,

$$\frac{v_s}{v_w} = \tan\alpha$$

$$\Rightarrow \quad v_s = v_w \tan\alpha \qquad \ldots(1)$$

When the sphere hits the ground, its centre will still be at a height R above the ground. Using Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in PE}\\ \text{of sphere}\end{pmatrix} = \begin{pmatrix}\text{Gain in KE}\\ \text{of sphere}\end{pmatrix} + \begin{pmatrix}\text{Gain in KE}\\ \text{of wedge}\end{pmatrix}$$

$$\Rightarrow \quad mg(2R) - mgR = \frac{1}{2}mv_s^2 + \frac{1}{2}mv_w^2$$

$$\Rightarrow \quad 2mgR = m\left(1 + \tan^2\alpha\right)v_w^2 = mv_w^2 \sec^2\alpha$$

$$\Rightarrow \quad v_w = \sqrt{2gR}\cos\alpha \text{ and } v_s = v_w \tan\alpha = \sqrt{2gR}\sin\alpha$$

"Please note that the centre of mass of wedge stays at same distance from the ground both in the initial and final situations, so we have not accounted for change in potential energy of wedge, because that happens to be zero".

8. (a) According to Work Energy Theorem, we have

$W_{\text{total}} = \Delta K$

$$\Rightarrow \quad \frac{1}{2}mv^2 = W_F + W_{\text{spring}} = Fx - \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{1}{2} \times 0.5 \times v^2 = 20 \times 0.25 - \frac{1}{2} \times 40 \times (0.25)^2$$

$\Rightarrow \quad v = 3.87 \text{ ms}^{-1}$

(b) Total mechanical energy at point B is also equal to work done by the external force F. So, we have

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = Fx = 20 \times 0.25 = 5 \text{ J}$$

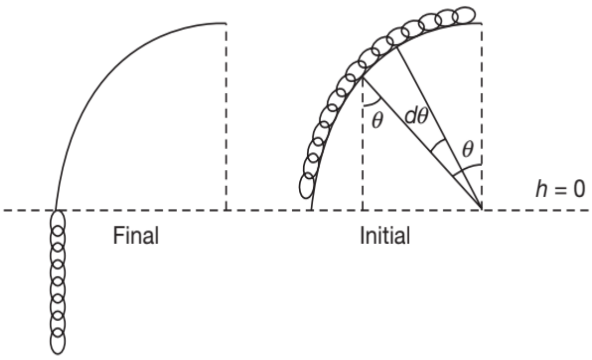
If x_f be the final maximum extension in the spring, then

$$\frac{1}{2}kx_f^2 = 5 \text{ J}$$

$$\Rightarrow \quad x_f = \sqrt{\frac{10}{k}} = \sqrt{\frac{10}{40}} = 0.5 \text{ m}$$

Hence, closest distance from wall is $d = 0.6 - x_f = 0.1$ m

9.



Initial potential energy is

$$U_i = \int_{\theta=0^\circ}^{\theta=\pi/2} (rd\theta)(\lambda)(g)(r\cos\theta)$$

$$U_i = (\lambda g r^2)\sin\theta\Big|_0^{\pi/2} = \lambda g r^2$$

Finally, the centre of mass of chain is $\frac{\pi(r/2)}{2}$ below the last link or the slot. So, final potential energy is given by

$$U_f = \left(\frac{\pi r}{2}\times\lambda\right)(g)\left(-\frac{\pi r/2}{2}\right) = -\frac{\pi^2 r^2 \lambda g}{8}$$

$$\Rightarrow \quad \Delta U = U_f - U_i = -r^2\lambda g\left(1+\frac{\pi^2}{8}\right)$$

By Law of Conservation of Energy

Loss in PE = Gain in KE

$$\Rightarrow \quad -\Delta U = \Delta K$$

$$\Rightarrow \quad r^2\lambda g\left(1+\frac{\pi^2}{8}\right) = \frac{1}{2}\left(\frac{\pi r}{2}\right)(\lambda)v^2$$

$$\Rightarrow \quad v = \sqrt{4rg\left(\frac{1}{\pi}+\frac{\pi}{8}\right)} = \sqrt{rg\left(\frac{4}{\pi}+\frac{\pi}{2}\right)}$$

10. Let T be the tension in the cable, then

$$T - mg = ma$$

$$T - 80g = 80a$$

$$\Rightarrow \quad T = 800 + 80 = 880 \text{ N}$$

(a) $W_T = Tx\cos(0^\circ) = (880)(15) = 13.2$ kJ

(b) $W_{mg} = mgx\cos(180^\circ) = -(880)(15) = -12$ kJ

(c) $KE = W_{\text{total}} = 13.2 - 12 = 1.2$ kJ

(d) $\frac{1}{2}mv^2 = 1200$

$$\Rightarrow \quad 40v^2 = 1200$$

$$\Rightarrow \quad v = 5.47 \text{ ms}^{-1}$$

11. When the block descends 12 mm, spring will further stretch 24 m. So total extension in the spring is $(76+24)$ mm i.e., 100 mm

$$\begin{pmatrix}\text{Decrease}\\ \text{in PE}\\ \text{of Block}\end{pmatrix} = \begin{pmatrix}\text{Increase}\\ \text{in KE}\\ \text{of Block}\end{pmatrix} + \begin{pmatrix}\text{Increase in}\\ \text{Elastic PE}\\ \text{of spring}\end{pmatrix}$$

$$\Rightarrow \quad (50)(10)(0.012) = \frac{1}{2}(1000)\left[(0.1)^2 - (0.076)^2\right] + \frac{1}{2}(50)v^2$$

$$\Rightarrow \quad 6 = 25v^2 + 2.5$$

$$\Rightarrow \quad v = 0.374 \text{ ms}^{-1}$$

12. (a) From Work Energy Theorem, we have

$$\Delta K = W = \int_0^2 \vec{F}\cdot d\vec{r}$$

where $d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$\Rightarrow \quad W = \int_0^2 (2.5 - x^2)dx = 2.33 \text{ J}$$

$$\Rightarrow \quad W = \Delta K = 2.33 \text{ J}$$

(b) Speed of block at x is given by

$$\frac{1}{2}\times 1.5\times v^2 = \int_0^x (2.5 - x^2)dx$$

$$\Rightarrow \quad v^2 = 1.33\left(2.5x - \frac{x^3}{3}\right)$$

For maximum K.E., $\frac{dv^2}{dx} = 0$

$$\Rightarrow \quad \frac{d}{dx}\left(2.5x - \frac{x^3}{3}\right) = 0$$

$$\Rightarrow \quad 2.5 - x^2 = 0$$

$$\Rightarrow \quad x^2 = 2.5$$

$$\Rightarrow \quad x = 1.58 \text{ m}$$

$$\Rightarrow \quad K_{\max} = \frac{1}{2}\times 1.5\times 1.33\left(2.5\times 1.58 - \frac{(1.58)^3}{3}\right)$$

$$\Rightarrow \quad K_{\max} = 2.635 \text{ J}$$

13. (a) $U(x) = -W_{\text{spring}} = \int_0^x (\alpha x + \beta x^2)dx = \frac{\alpha x^2}{2} + \frac{\beta x^3}{3}$

(b) By Law of Conservation of Energy, we get

$$\begin{pmatrix}\text{Gain in KE}\\ \text{of Mass}\end{pmatrix}=\begin{pmatrix}\text{Loss in PE}\\ \text{of Spring}\end{pmatrix}$$

$$\Rightarrow \frac{1}{2}mv^2 = U_i - U_f = U(1\text{ m}) - U(0.5\text{ m})$$

Since $U(x)=\dfrac{\alpha x^2}{2}+\dfrac{\beta x^3}{3}$

$$\Rightarrow U(1\text{ m})=\frac{80(1)}{2}+\frac{24(1)}{3}=48\text{ J}$$

$$\Rightarrow U(0.5\text{ m})=\frac{80(0.5)^2}{2}+\frac{24(0.5)^3}{3}=11\text{ J}$$

$$\Rightarrow \frac{1}{2}(2)v^2=48-11$$

$$\Rightarrow v^2=48-11$$

$$\Rightarrow v=\sqrt{37}\text{ ms}^{-1}\approx 6\text{ ms}^{-1}$$

14. Block will begin to move when,

$$kx = mg\sin\theta+\mu_s gm\cos\theta$$

$$\Rightarrow x=\frac{mg(\sin\theta+\mu_s\cos\theta)}{k}$$

$$\Rightarrow U=\frac{1}{2}kx^2=\frac{[mg(\sin\theta+\mu_s\cos\theta)]^2}{2k}$$

15. Using our knowledge of constrained motion of connected bodies, we observe that if $x_A = h$ (say), then $x_B = 2h$. Also, $v_A=\dot{x}_A = 2\text{ ms}^{-1}$

$$\Rightarrow v_B=\dot{x}_B=2v_A=4\text{ ms}^{-1}$$

Using Law of Conservation of Energy, we get

$$\begin{pmatrix}\text{Decrease}\\ \text{in PE of A}\end{pmatrix}=\begin{pmatrix}\text{Increase}\\ \text{in PE of B}\end{pmatrix}+\begin{pmatrix}\text{Increase in KE}\\ \text{of both A and B}\end{pmatrix}$$

$$\Rightarrow (30)(10)h=(5)(10)(2h)+\frac{1}{2}(30)(2)^2+\frac{1}{2}(5)(4)^2$$

$$\Rightarrow 300h=100h+60+40$$

$$\Rightarrow 200h=100$$

$$\Rightarrow h=0.5\text{ m}$$

16. To use $E_f=E_i$ we would need to assign the initial heights of the blocks arbitrary values h_1 and h_2. The corresponding potential energies, m_1gh_1 and m_2gh_2 would appear in both E_i and E_f and hence would cancel.
We avoid this process by using the form $\Delta K+\Delta U = 0$ instead, since it does not require $U=0$ reference level.

(a) At the maximum extension x_{max}, the blocks come to rest, and thus $\Delta K = 0$. Next, we must find the changes in U_g and U_s. When m_2 falls by x_{max}, the spring extends by x_{max} and m_1 rises by $x_{max}\sin\theta$

$$\Rightarrow \Delta K+\Delta U_g+\Delta U_s=0$$

$$\Rightarrow 0+(-m_2gx_{max}+m_1gx_{max}\sin\theta)+\frac{1}{2}kx_{max}^2=0$$

Thus, $x_{max}=\dfrac{2g}{k}(m_2-m_1\sin\theta)=0.98\text{ m}$

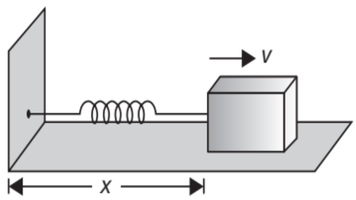
(b) In this case the change in kinetic energy is $\Delta K=\dfrac{1}{2}(m_1+m_2)v^2$.

The change in potential energy has the same form as in part (a), but with x_{max} replaced by $x=0.5\text{ m}$, so

$$\frac{1}{2}(m_1+m_2)v^2+(-m_2gx+m_1gx\sin\theta)+\frac{1}{2}kx^2=0$$

Substituting values we get $v=1.39\text{ ms}^{-1}$.

17. When the block is released, the spring pushes it towards right. The velocity of the block increases till the spring acquires its natural length. Thereafter, the block loses contact with the spring and moves with constant velocity.



Initially, the compression in the spring is $\dfrac{L_0}{2}$. When the distance of the block from the wall becomes x, where $x<L_0$, then compression is (L_0-x). Using the Law of Conservation of Energy,

$$\frac{1}{2}k\left(\frac{L_0}{2}\right)^2=\frac{1}{2}k(L_0-x)^2+\frac{1}{2}mv^2$$

$$\Rightarrow v=\sqrt{\frac{k}{m}\left[\frac{L_0^2}{4}-(L_0-x)^2\right]}$$

When the spring acquires its natural length i.e., $x=L_0$, then velocity becomes $v=\left(\sqrt{\dfrac{k}{m}}\right)\dfrac{L_0}{2}$. Thereafter, the block continues to move with this velocity.

18. Since all surfaces are smooth. According to Law of Conservation of Mechanical Energy, we have

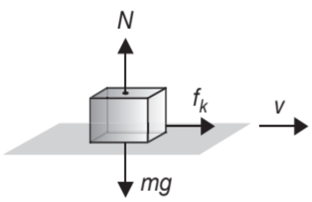
$$\begin{pmatrix}\text{Decrease in P.E. of}\\ \text{both the blocks}\end{pmatrix}=\begin{pmatrix}\text{Increase in K.E. of}\\ \text{both the blocks}\end{pmatrix}$$

$$\Rightarrow\quad (mgr)+(2mg)\left(\frac{\pi r}{2}\right)=\frac{1}{2}(m+2m)v^2$$

$$\Rightarrow\quad v=\sqrt{\frac{2}{3}(1+\pi)gr}$$

Test Your Concepts-IV (Based on Work Energy Theorem for Non-conservative Systems)

1. (a) When the box is first placed on the belt there will be slipping between the two. But the force of friction on the box and its displacement are in the same direction. Consequently, the work done by kinetic friction is positive. Since the final speed of the box is v,



$$W_f=\Delta K=+\frac{1}{2}mv^2 \qquad \ldots(1)$$

(b) The force of friction is $f=\mu_k N=\mu_k mg$ and $W_f=+fd$, so from equation (1), we get

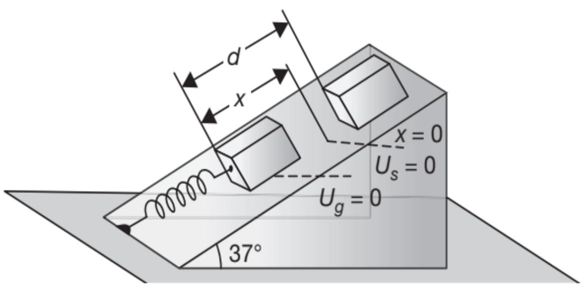
$$+\mu_k mgd=+\frac{1}{2}mv^2 \qquad \ldots(2)$$

$$\Rightarrow\quad d=\frac{v^2}{2\mu_k g}$$

(c) If the box takes a time t to reach speed v, then $v=at$ where a is the acceleration of box. In this time it will move $d=\frac{1}{2}at^2=\frac{1}{2}vt$. Since the belt's speed is fixed, in time t it moves a distance $vt=2d$.

The belt moves twice as far as the box while the box is accelerating.

2.



(a) K_i and K_f both are zero, so $E_i=\frac{1}{2}kx_o^2$ and $E_f=mgd\sin\theta$. From Work Energy Theorem,

$$W_{nc}=E_f-E_i=\Delta U+\Delta K$$

$$\Rightarrow\quad mgd\sin\theta-\frac{1}{2}kx_0^2=-fd$$

$$\Rightarrow\quad f=0.82\text{ N}$$

(b) Initial energy mechanical E_i is still the same as above, but final mechanical energy at $x=0$ is

$$E_f=\frac{1}{2}mv^2+mgx_o\sin\theta$$

From Work Energy Theorem, we have

$$W_{nc}=E_f-E_i=\Delta U+\Delta K$$

$$\Rightarrow\quad \frac{1}{2}mv^2+mgx_0\sin\theta-\frac{1}{2}kx_0^2=-fx_0$$

$$\Rightarrow\quad v=2.45\text{ ms}^{-1}$$

3. (a) Work done by friction $=-\mu Mg\ell$

(b) Work done by the spring force $=-\frac{1}{2}k\ell^2$

(c) Gravitational force and normal reaction of the table do not work as they act in a direction perpendicular to displacement.

(d) Total work done on the block $=-\left(\mu Mg\ell+\frac{1}{2}k\ell^2\right)$

(e) According to Work Energy Theorem

$$W_{\text{total}}=\Delta K$$

$$\Rightarrow\quad W_C+W_{nC}=\Delta K$$

$$\Rightarrow\quad -\Delta U+W_{nC}=\Delta K$$

$$\Rightarrow\quad 0-\frac{1}{2}Mv_0^2=-\left(\mu Mg\ell+\frac{1}{2}k\ell^2\right)$$

$$\Rightarrow\quad \ell^2+\frac{2\mu Mg\ell}{k}-\frac{Mv_0^2}{k}=0$$

$$\Rightarrow\quad \ell=\frac{1}{k}\left[\sqrt{\mu^2M^2g^2+Mkv_0^2}-\mu Mg\right]$$

4. Net retarding force $F=kx+\mu N$

$$\Rightarrow\quad F=kx+Mgbx$$

If a be the net retardation, then

$$a=-\frac{vdv}{dx}=\frac{F}{m}=\left(\frac{k+Mgb}{M}\right)x$$

$$\Rightarrow\quad \int_{v_0}^{0}vdv=-\left(\frac{k+Mgb}{M}\right)\int_0^x xdx$$

CHAPTER 1

$$\Rightarrow \quad -\frac{v_0^2}{2} = -\left(\frac{k + Mgb}{M}\right)\frac{x^2}{2}$$

$$\Rightarrow \quad x = v_0\sqrt{\frac{M}{k + Mgb}}$$

Loss in Mechanical Energy is $-\Delta E = E_i - E_f$

$$\Rightarrow \quad -\Delta E = (U + K)_{\text{initial}} - (U + K)_{\text{final}}$$

$$\Rightarrow \quad \text{Loss} = \left(0 + \frac{1}{2}Mv_0^2\right) - \left(\frac{1}{2}kx^2 + 0\right)$$

$$\Rightarrow \quad \text{Loss} = \frac{1}{2}Mv_0^2 - \frac{1}{2}kx^2 \text{, where } x = v_0\sqrt{\frac{M}{k + Mgb}}$$

$$\Rightarrow \quad \text{Loss} = \left(\frac{M^2v_0^2}{2}\right)\left(\frac{gb}{k + Mgb}\right)$$

5. From B to C

$$0 = (4)^2 - 2(\mu_K g)(4)$$

$$\Rightarrow \quad \mu_K = \frac{16}{(2)(10)(4)} = 0.2$$

From A to B

Since we know that

$$W_{nc} = \Delta U + \Delta K$$

$$\Rightarrow \quad W_{\text{friction}} = -mgR + \frac{1}{2}mv_B^2$$

$$\Rightarrow \quad W_{\text{friction}} = -(0.2)(10)(2) + \frac{1}{2}(0.2)(16)$$

$$\Rightarrow \quad W_{\text{friction}} = -4 + 1.6 = -2.4 \text{ J}$$

6. (a) $100 = kx = 2000x$

$$\Rightarrow \quad x = 0.05 \text{ m}$$

Since, $\frac{1}{2}mv^2 + \mu_k mgx = \frac{1}{2}kx^2$

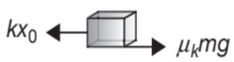
$$\Rightarrow \quad v = \sqrt{\frac{k}{m}x^2 - 2\mu_K gx}$$

$$\Rightarrow \quad v = \sqrt{\left(\frac{2000}{5}\right)(0.05)^2 - 2(0.36)(10)(0.05)}$$

$$\Rightarrow \quad v = 0.8 \text{ ms}^{-1}$$

(b) Velocity is maximum where

$$kx_0 = \mu_K mg$$



$$\Rightarrow \quad x_0 = \frac{\mu_K mg}{k} = \frac{0.36 \times 5 \times 10}{2000} = 0.009 \text{ m}$$

Since $\frac{1}{2}mv_{\max}^2 + \mu_K mg(x - x_0) = \frac{1}{2}k(x^2 - x_0^2)$

$$\Rightarrow \quad v_{\max} = \sqrt{\frac{k}{m}(x^2 - x_0^2) - 2\mu_K g(x - x_0)}$$

Substituting, $k = 2000 \text{ Nm}^{-1}$, $m = 5$ kg, $x = 0.05$ m, $x_0 = 0.009$ m, $\mu_k = 0.36$

$$\Rightarrow \quad v_{\max} = 0.82 \text{ ms}^{-1}$$

7. From Work Energy Theorem, we have

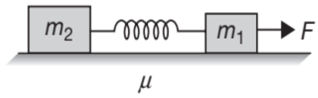
$$W_{mg} + W_{\text{air}} = \Delta KE$$

$$\Rightarrow \quad W_{\text{air}} = \Delta KE - W_{mg}$$

$$\Rightarrow \quad W_{\text{air}} = \frac{1}{2} \times 5 \times (10)^2 - (5)(10)(20) = -750 \text{ J}$$

Negative sign indicates that work is done against the push of the air.

8. Block m_2 will shift when $kx = \mu m_2 g$ ...(1)



For m_1, using Work Energy Theorem, we get

$$Fx = \frac{1}{2}kx^2 + \mu m_1 xg$$

$$\Rightarrow \quad kx = 2F - 2\mu m_1 g \qquad \text{...(2)}$$

Equating Equation (1) and (2), we get

$$F = \left(m_1 + \frac{m_2}{2}\right)\mu g$$

9. For horizontal portion, retardation $a_1 = \mu g = 1.5 \text{ ms}^{-2}$

$$\Rightarrow \quad v^2 = 2a_1 s_1 = 2 \times 1.5 \times 0.5 = 1.5 \text{ m}^2\text{s}^{-2}$$

$$\Rightarrow \quad v = 1.22 \text{ ms}^{-1}$$

For inclined portion, acceleration is given by

$$a_2 = g\sin\theta - \mu g\cos\theta = \left(\frac{1}{2} - 0.15 \times \frac{\sqrt{3}}{2}\right) \times 10$$

$$\Rightarrow \quad a_2 = 3.7 \text{ ms}^{-2}$$

$$\Rightarrow \quad s_2 = \frac{v^2}{2a_2} = \frac{1.5}{2 \times 3.7} = 0.2 \text{ m}$$

Work performed by friction forces is

$$W_{\text{friction}} = W_{\text{case I}} + W_{\text{case II}}$$

$$\Rightarrow \quad W_f = -\mu mg s_1 - (\mu mg\cos\theta)s_2$$

$$\Rightarrow \quad W_f = -0.15 \times 0.05 \times 10\left(0.5 + 0.2 \times \frac{\sqrt{3}}{2}\right) = -0.044 \text{ J}$$

10. From constraint relations, we can see that

$$v_A = 2v_B$$

Therefore, $v_A = 2(0.3) = 0.6\ \text{ms}^{-1}$

as $v_B = 0.3\ \text{ms}^{-1}$ {given}

Since $W_{nc} = \Delta U + \Delta K$

$$\Rightarrow \quad -\mu m_A g x_A = -m_B g x_B + \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$\Rightarrow \quad x_A = 2x_B = 2\ \text{m}$, since $x_B = 1\ \text{m}$, $m_A = 4\ \text{kg}$, $m_B = 1\ \text{kg}$, so we get

$$-80\mu = -10 + 0.72 + 0.045$$

$$\Rightarrow \quad 80\mu = 9.235$$

$$\Rightarrow \quad \mu = 0.115$$

11. Since the applied force is always parallel to the track, normal reaction will always be $mg\cos\theta$. In this case, work done against friction is

$$W = \mu mg \times \text{horizontal distance covered}$$

$$\Rightarrow \quad W = \mu mgL$$

$$\Rightarrow \quad W = 0.5 \times 1 \times 10 \times 1 = 5\ \text{J}$$

Test Your Concepts-V (Based on Relation Between Conservative Force and Potential Energy and Types of Equilibrium)

1. We use $\vec{F} = -\nabla U = -\left(2y\hat{i} + (2x+z)\hat{j} + y\hat{k}\right)$

2. Since, $\vec{F} = -\vec{\nabla}U = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right)$

$$F_x = -\frac{\partial U}{\partial x} = -U_0 \exp(-az)\frac{\partial}{\partial x}[\cos(ax)]$$

$$\Rightarrow \quad F_x = \frac{\partial U}{\partial x} = +aU_0 \sin(ax)\exp(-az)$$

Since, U has no dependence on y, so

$$F_y = \frac{\partial U}{\partial y} = 0$$

$$F_z = -\frac{\partial U}{\partial z} = -U_0\cos(ax)\frac{\partial}{\partial z}[\exp(-az)]$$

$$\Rightarrow \quad F_z = -\frac{\partial U}{\partial z} = aU_0\cos(ax)\exp(-az)$$

$$\Rightarrow \quad F = F_x\hat{i} + F_z\hat{k} \qquad \{\because F_y = 0\}$$

$$\Rightarrow \quad \vec{F} = aU_0\exp(-az)\left[\sin(ax)\hat{i} + \cos(ax)\hat{k}\right]$$

3. Since we know that

$$dU = -\vec{F}\cdot d\vec{l}$$

$$\Rightarrow \quad dU = -\left[2axy\hat{i} + a\left(x^2 - y^2\right)\hat{j}\right]\cdot\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$\Rightarrow \quad dU = -\left[2axydx + a\left(x^2 - y^2\right)dy\right]$$

$$\Rightarrow \quad \int_{(0,0)}^{(x,y)} dU = -\int_{(0,0)}^{(x,y)} 2axydx + a\left(x^2 - y^2\right)dy$$

$$\Rightarrow \quad U_{(x,y)} - U_{(0,0)} = -\int_{(0,0)}^{(x,y)} \left(2axy\,dx + ax^2\,dy\right) - ay^2dy$$

Let $U_{(x,y)} = U$ and $U_{(0,0)} = U_0$, then we have

$$U - U_0 = -\int_{(0,0)}^{(x,y)} \left\{d\left(ax^2y\right) - d\left(\frac{y^3}{3}\right)\right\}$$

$$\Rightarrow \quad U - U_0 = -\int_{(0,0)}^{(x,y)} d\left(ax^2y - \frac{y^3}{3}\right)$$

$$\Rightarrow \quad U - U_0 = \left[\left(-ax^2y + \frac{ay^3}{3}\right)\Bigg|_{(0,0)}^{(x,y)}\right]$$

$$\Rightarrow \quad U = U_0 - ax^2y + \frac{ay^3}{3}$$

4. Since $\Delta U = -\int \vec{F}\cdot d\vec{l}$

where, $dl = \hat{i}dx + \hat{j}dy + \hat{k}dz$

$$\Rightarrow \quad \Delta U = \int_{(0,0)}^{(2,3)} 4\left(y\hat{i} + x\hat{j}\right)\cdot\left(\hat{i}dx + \hat{j}dy + \hat{k}dz\right)$$

$$\Rightarrow \quad U - (-4) = 4\int_{(0,0)}^{(2,3)} ydx + xdy$$

Since, we know that

$$d(xy) = xdy + ydx$$

$$\Rightarrow \quad U - (-4) = 4\int_{(0,0)}^{(2,3)} d(xy)$$

$$\Rightarrow \quad U - (-4) = 4(xy)\Big|_{(0,0)}^{(2,3)}$$

$$\Rightarrow \quad U - (-4) = 4[(2)(3) - (0)(0)] = 24$$

$$\Rightarrow \quad U = 24 - 4 = 20\ \text{J}$$

5. Potential energy of the particle is given as

$$U = 2 - 20x + 5x^2 \quad \ldots(1)$$

Therefore, its potential energy at $x = -3$ is

$$U_1 = 2 - 20\times(-3) + 5(-3)^2 = 2 + 60 + 45 = 107 \text{ J}$$

Since the particle is released from $x = -3$ m, its kinetic energy at this position is zero and therefore its total energy is $107 + 0 = 107$ J.

Particle will again come to rest at a position where its potential energy once again becomes 107 J.

Substituting $U = 107$ J in equation (1), we get

$$2 - 20x + 5x^2 = 107$$

$$\Rightarrow 5x^2 - 20x - 105 = 0$$

$$\Rightarrow x^2 - 4x - 21 = 0$$

$$\Rightarrow x = -3 \text{ m and } x = 7 \text{ m}$$

Hence the particle released from $x = -3$ m will come to rest again at $x = 7$ m

6. Since $F = -\dfrac{dU}{dx}$

$$\Rightarrow dU = -Fdx$$

$$\Rightarrow U(x) = -\int_0^x (-kx + ax^3)dx = \frac{kx^2}{2} - \frac{ax^4}{4}$$

$$\Rightarrow U(x) = \frac{kx^2}{2}\left(1 + \sqrt{\frac{a}{2k}}x\right)\left(1 - \sqrt{\frac{a}{2k}}x\right)$$

Now, $U(x) = 0$ at $x = 0$ and $x = \sqrt{\dfrac{2k}{a}}$

Also, $U(x)$ is negative for $x > \sqrt{\dfrac{2k}{a}}$

For $0 < x < \sqrt{\dfrac{2k}{a}}$, $U(x)$ first increases and then decreases.

From the given function we can see that $F = 0$ at $x = 0$ i.e., slope of U-x graph is zero at $x = 0$. Therefore, the most appropriate OPTION is (D).

7. Since, $F = -\dfrac{dU}{dx}$

$$\Rightarrow \int_0^{U(x)} dU = -\int_0^x Fdx = -\int_0^x (kx)dx$$

$$\Rightarrow U(x) = -\frac{kx^2}{2} \qquad \{\because U(0) = 0\}$$

Test Your Concepts-VI (Based on Vertical Circle)

1. For the body to be able to successfully complete the loop, its velocity at the bottom of the loop must be greater than or equal to $\sqrt{5gr}$.

Now from Law of Conservation of Energy, we get

$$mgh = \frac{1}{2}mv^2$$

$$\Rightarrow v^2 = 2gh \geq 5gR$$

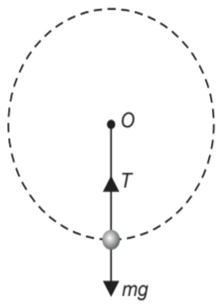
$$\Rightarrow h \geq \frac{5}{2}R$$

So, the minimum height required is $\dfrac{5}{2}R$

2. Let T be the tension in the string at the lowest point of circle. Then the net force on the stone is $(T - mg)$, which is directed towards the centre of the circle and provides necessary centripetal force to the particle to revolve in a circle of radius r. So

$$T - mg = \frac{mv^2}{r}$$

where v is the speed of the stone at the lowest point of the circle of radius r.



$$\Rightarrow T = \frac{mv^2}{r} + mg = m\left(\frac{v^2}{r} + g\right)$$

$$\Rightarrow T = 0.5\left(\frac{3^2}{0.40} + 9.8\right) = 16.15 \text{ N}$$

3. (a) Speed of the particle at the lowest point is first calculated by the Law of Conservation of Energy, so

$$(U + K)_A = (U + K)_B$$

$$\Rightarrow mgL(1 - \cos\theta) + \frac{1}{2}mu^2 = 0 + \frac{1}{2}mv^2$$

$$\Rightarrow v^2 = u^2 + 2gL(1 - \cos\theta) \quad \ldots(1)$$

Now, tension at the lowest point is

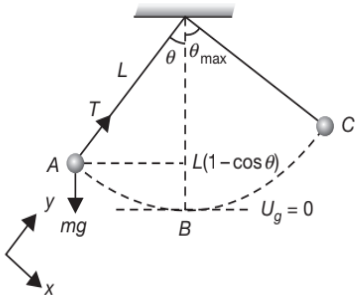
$$T = \frac{m}{L}(v_B^2 + gL), \text{ where } v_B = v$$

$$\Rightarrow \quad T = \frac{m}{L}\left(u^2 + 3gL - 2gL\cos\theta\right)$$

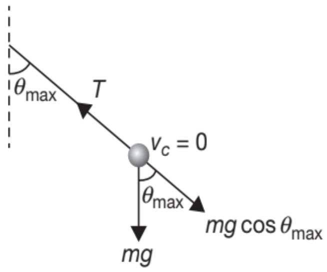
Now, according to question, we have

$$u = 1\ \text{ms}^{-1},\ g = 10\ \text{ms}^{-2},\ L = 2\ \text{m},\ m = 2\ \text{kg}$$

$$\Rightarrow \quad T = \left(\frac{2}{2}\right)(1 + 60 - 32) = 29\ \text{N}$$



(b) At the highest point, we have $T_C = mg\cos\theta_{max}$



Now, please do not confuse θ_{max} with θ as both may be different. So, let us first calculate θ_{max}. For this purpose, we shall try to calculate the total mechanical energy of the particle at A, which is

$$E_A = U_A + K_A$$

$$\Rightarrow \quad E_A = mgL(1 - \cos\theta) + \frac{1}{2}mu^2$$

$$\Rightarrow \quad E_A = (2)(10)(2)\left(1 - \frac{4}{5}\right) + \frac{1}{2}(2)(1)^2$$

$$\Rightarrow \quad E_A = 8 + 1 = 9\ \text{J}$$

So, now at maximum height, we have $v_C = 0$ and by Law of Conservation of Energy, total energy at C should also be 9 J

$$\Rightarrow \quad mgL(1 - \cos\theta_{max}) = 9$$

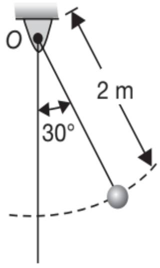
$$\Rightarrow \quad (2)(10)(2)(1 - \cos\theta_{max}) = 9$$

$$\Rightarrow \quad 1 - \cos\theta_{max} = \frac{9}{40}$$

$$\Rightarrow \quad \cos\theta_{max} = \frac{31}{40}$$

$$\Rightarrow \quad T_C = (2)(10)\left(\frac{31}{40}\right) = 15.5\ \text{N}$$

4. Since, $T - mg\cos\theta = \dfrac{mv^2}{\ell}$



$$\Rightarrow \quad 2.5mg - mg\cos 30° = \frac{mv^2}{2}$$

$$\Rightarrow \quad v = 5.66\ \text{ms}^{-1}$$

$$\Rightarrow \quad a = \sqrt{\left(\frac{v^2}{\ell}\right)^2 + (g\sin\theta)^2}$$

$$\Rightarrow \quad a = \sqrt{\left(\frac{5.66 \times 5.66}{2}\right)^2 + (9.8 \times \sin 30°)^2}$$

$$\Rightarrow \quad a = 16.75\ \text{ms}^{-2}$$

5. (a) $T_1 = \dfrac{mv_1^2}{R_1} = \dfrac{(0.8)(1)^2}{0.4} = 2\ \text{N}$

(b) $T_2 = \dfrac{mv_2^2}{R} = \dfrac{(0.8)(4)^2}{0.2} = 64\ \text{N}$

(c) $W = \dfrac{1}{2}m\left(v_2^2 - v_1^2\right) = \dfrac{1}{2} \times 0.8(4 - 1) = 1.2\ \text{J}$

6. Using Work Energy Theorem, for the rough track, we get

$$W_{nc} = \Delta U + \Delta K$$

$$\Rightarrow \quad -\mu mg\ell = \left(0 - \frac{1}{2}k\delta^2\right) + \left(\frac{1}{2}mv^2 - 0\right)$$

For looping the loop, we have $v^2 = 5gR$

$$\Rightarrow \quad \frac{1}{2}k\delta^2 = \frac{1}{2}m(5gR) + \mu_k mgR$$

$$\Rightarrow \quad \delta = \sqrt{\frac{mgR}{k}}\sqrt{5 + 2\mu_k}$$

7. Since, $h = r(1 + \sin\theta)$ ...(1)

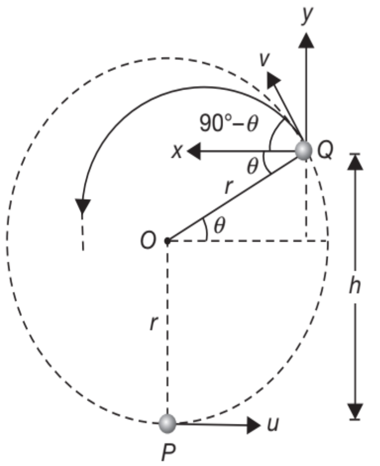
Also, by Law of Conservation of Energy, we get

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mgh$$

$$\Rightarrow \quad v^2 = u^2 - 2gh$$

$$\Rightarrow \quad v^2 = u^2 - 2gr(1 + \sin\theta) \quad \text{...(2)}$$

Coordinates of point P with Q as origin (x and y axes as shown), are $P(r\cos\theta, -r-r\sin\theta)$



At Q, we have, $T=0$ and $v \neq 0$, so motion of particle ceases to be circular and the particle becomes a projectile, launched with initial velocity v and launch angle α such that its x and y coordinates are related to each other as

$$y = x\tan\alpha - \frac{gx^2}{2v^2\cos^2\alpha}, \text{ where } \alpha = 90-\theta$$

$$\Rightarrow \quad -r(1+\sin\theta) = (r\cos\theta)\tan(90^\circ-\theta) - \frac{gr^2\cos^2\theta}{2v^2\cos^2(90-\theta)}$$

$$\Rightarrow \quad (1+\sin\theta) = \frac{gr\cot^2\theta}{2v^2} - \frac{\cos^2\theta}{\sin\theta} \qquad \ldots(3)$$

Substituting value of v from equation (2) in (3), we get

$$\theta = 30^\circ \text{ and } u = \sqrt{\frac{7rg}{2}}$$

8. (a) Applying conservation of energy

$$mgh = \frac{1}{2}m\left(\sqrt{3gL}\right)^2$$

$$\Rightarrow \quad h = \frac{3L}{2}$$

(b) Since $\sqrt{3gL}$ lies between $\sqrt{2gL}$ and $\sqrt{5gL}$, the string will slack in upper half of the circle. Assuming that string slacks when it makes an angle θ with horizontal, then we get

$$mg\sin\theta = \frac{mv^2}{L} \qquad \ldots(1)$$

$$\Rightarrow \quad v^2 = \left(\sqrt{3gL}\right)^2 - 2gL(1+\sin\theta) \qquad \ldots(2)$$

Solving Equation (1) and (2), we get

$$\sin\theta = \frac{1}{3} \text{ and } v^2 = \frac{gL}{3}$$

Maximum height of the bob from starting point, is

$$H = L(1+\sin\theta) + \frac{v^2\sin^2(90^\circ-\theta)}{2g}$$

$$\Rightarrow \quad H = L\left(1+\frac{1}{3}\right) + \left(\frac{gL}{6g}\right)\left(\frac{8}{9}\right) = \frac{4L}{3} + \frac{4L}{27}$$

$$\Rightarrow \quad H = \frac{40}{27}L$$

9. Here tension in the rod at the top most point of circle can be zero or negative for completing the loop. So, velocity at the top most point is zero.
From energy conservation

$$\frac{1}{2}mv^2 + \frac{1}{2}m\frac{v^2}{4} = mg(2\ell) + mg(4\ell) + 0$$

$$\Rightarrow \quad v = \sqrt{\frac{48g\ell}{5}}$$

10. When the motorcyclist is at the highest point of the death-well, the normal reaction N on the motorcyclist by the ceiling of the chamber acts downwards. His weight mg also act downwards.

$$F_{\text{net}} = ma_C$$

$$\Rightarrow \quad N + mg = \frac{mv^2}{r} \qquad \ldots(1)$$

Here v is the speed of the motorcyclist and m is the mass of the motorcyclist (including the mass of the motor cycle). Because of the balancing of the forces, the motorcyclist does not fall down.

The minimum speed required to perform a vertical loop is given by equation (1), when $N=0$

$$\Rightarrow \quad mg = \frac{mv_{\min}^2}{r}$$

$$\Rightarrow \quad v_{\min}^2 = gr$$

$$\Rightarrow \quad v_{\min} = \sqrt{gr} = \sqrt{9.8\times 25} \text{ ms}^{-1} = 15.65 \text{ ms}^{-1}$$

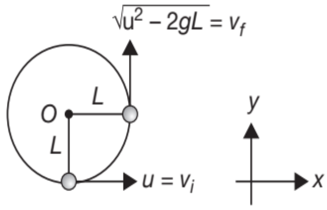
11. Let v_i be the initial velocity i.e., velocity at lowest point

$$\Rightarrow \quad \vec{v}_i = u\hat{i}.$$

Let v_f be the final velocity i.e., velocity when the string becomes vertical

$$v_f^2 - v_i^2 = 2(-g)L$$

$$\Rightarrow \quad v_f = \sqrt{u^2 - 2gL} \text{ (in magnitude)}$$



$\Rightarrow \quad \vec{v}_f=\left(\sqrt{u^2-2gL}\right)\hat{j}$

So, $\Delta\vec{v}=\vec{v}_f-\vec{v}_i=\left(\sqrt{u^2-2gL}\right)\hat{j}-u\hat{i}$

$\Rightarrow \quad |\Delta\vec{v}|=\sqrt{u^2-2gL+u^2}=\sqrt{2\left(u^2-gL\right)}$

$\Rightarrow \quad |\Delta\vec{v}|=\sqrt{2\left[(10)^2-(10)(5)\right]}$

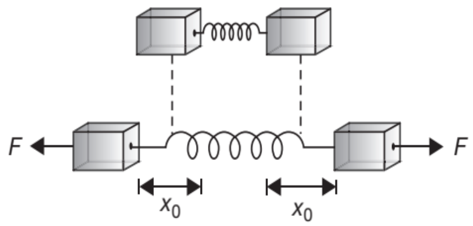
$\Rightarrow \quad |\Delta\vec{v}|=\sqrt{2(100-50)}=\sqrt{100}=10\text{ ms}^{-1}$

Single Correct Choice Type Questions

1. $U_i=0$ and $U_f=\frac{1}{2}k(2x_0)^2$

$\Rightarrow \quad U_f=2kx_0^2$

Since we know that, $W_C=-\Delta U$



If W be the work done by the spring on earth mass, then

$W_C=2W=0-2kx_0^2$

$\Rightarrow \quad W=-kx_0^2$

Hence, the correct answer is (D).

2. Force needed to pull the block down by a distance y from equilibrium position is

$F=k(x+y)-mg=ky$

Therefore, work done to extend the spring is

$W=\frac{1}{2}ky^2$

At equilibrium, $mg=kx$

$\Rightarrow \quad k=\frac{mg}{x}=\frac{2\times 10}{0.06}=\frac{1000}{3}\text{ Nm}^{-1}$

When pulled down by additional distance 10 cm, external work done $W=\frac{1}{2}\times k\times(0.1)^2$

$\Rightarrow \quad W=\frac{1}{2}\times\frac{1000}{3}\times 0.01=1.67\text{ J}$

Hence, the correct answer is (D).

3. By Law of Conservation of Energy

$mg(h+x)=\frac{1}{2}kx^2$

$\Rightarrow \quad x=\frac{mg+\sqrt{m^2g^2+2mghk}}{k}$

Hence, the correct answer is (A).

4. From Work Energy Theorem $W=\Delta\text{KE}$

$\Rightarrow \quad W=K_f-K_i=\frac{1}{2}m\left(v_f^2-v_i^2\right)$

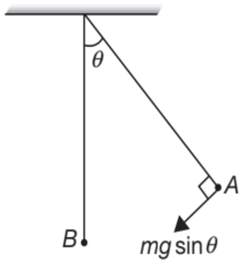
$\Rightarrow \quad W=\frac{1}{2}(2)(400-100)=300\text{ J}$

Hence, the correct answer is (A).

5. $P_{\text{inst}}=\vec{F}\cdot\vec{v}=50-30+120=140\text{ W}$

Hence, the correct answer is (C).

6.



At extreme position A, since the ball is at rest so, net acceleration is equal to the tangential acceleration.

$\Rightarrow \quad a_A=g\sin\theta$

At lowermost position B, net acceleration is centripetal acceleration, i.e.,

$a_B=\frac{v^2}{L}$ where $v=\sqrt{2gL(1-\cos\theta)}$

$\Rightarrow \quad a_B=2g(1-\cos\theta)$

Since, $a_A=a_B$

$\Rightarrow \quad g\sin\theta=2g(1-\cos\theta)$

$\Rightarrow \quad 2g\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)=2g\left(1-\left(1-2\sin^2\frac{\theta}{2}\right)\right)$

Since $\sin\theta=2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$ and

$1-\cos\theta=2\sin^2\left(\frac{\theta}{2}\right)$

$$\Rightarrow \quad 2g\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)=2g\times 2\sin^2\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \quad \tan\frac{\theta}{2}=\frac{1}{2}$$

$$\Rightarrow \quad \frac{\theta}{2}=26.5^\circ$$

$$\Rightarrow \quad \theta=53^\circ$$

Hence, the correct answer is (C).

7. $v=\dfrac{dx}{dt}=3-8t+3t^2$

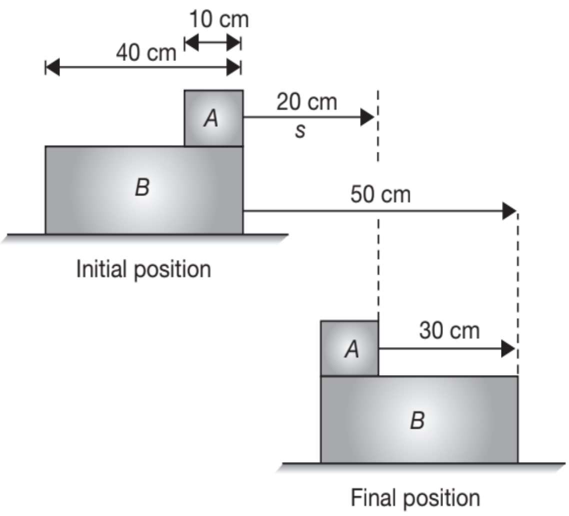
Initial velocity (velocity at $t=0$) is $u=3\text{ ms}^{-1}$. Final velocity (velocity at $t=4s$) is $v=19\text{ ms}^{-1}$

Since, $W=\dfrac{1}{2}m(v^2-u^2)=\dfrac{1}{2}\left(\dfrac{3}{1000}\right)(361-9)$

$$\Rightarrow \quad W=\frac{528}{1000}\text{ J}=528\text{ mJ}$$

Hence, the correct answer is (B).

8.



The displacement of block A in ground frame towards right is s (as shown in figure).

$$\Rightarrow \quad s=0.5-0.3=0.2\text{ m}$$

Kinetic friction acting on block A acting towards right is

$$f_k=\mu mg=0.2\times 45\times 10=90\text{ N}$$

So, work done by friction force in the ground frame is

$$W=fs\cos(0^\circ)=90\times 0.2\times 1=18\text{ J}$$

Hence, the correct answer is (B).

9. $a=\left(\dfrac{m_2-\mu m_1}{m_1+m_2}\right)g$

$$\Rightarrow \quad a=\left[\frac{4-(0.2)(10)}{4+10}\right]9.8$$

$$\Rightarrow \quad a=1.4\text{ ms}^{-2}$$

Since, $v^2-0^2=2as$

$$\Rightarrow \quad v^2=2(1.4)(4)$$

$$\Rightarrow \quad v^2=11.2$$

$$\Rightarrow \quad v\cong 3.5\text{ ms}^{-1}$$

Hence, the correct answer is (D).

10. Since, $\vec{F}$ is a constant force, so work done will not depend on path. So,

$$W=\int_{(0,0,0)}^{(2,2,2)}\vec{F}\cdot d\vec{\ell}$$

$$\Rightarrow \quad W=\int_{(0,0,0)}^{(2,2,2)}\left(2\hat{i}+5\hat{j}+\hat{k}\right)\cdot\left(dx\hat{i}+dy\hat{j}+dz\hat{k}\right)$$

$$\Rightarrow \quad W=\int_{(0,0,0)}^{(2,2,2)}\left(2dx+5dy+dz\right)$$

$$\Rightarrow \quad W=\left(2x+5y+z\right)\Big|_{(0,0,0)}^{(2,2,2)}=16\text{ J}$$

Hence, the correct answer is (D).

11. Given force is a constant force and work done is path independent under a constant force. Hence, $W_1=W_2$.

Hence, the correct answer is (A).

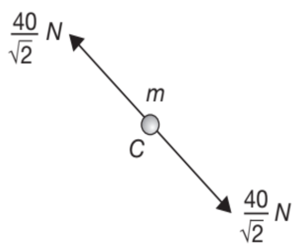
12. Since $\Delta U=-\int\vec{F}\cdot d\vec{l}$

$$\Rightarrow \quad U-U_0=\int 40(ydx+xdy)=40\int d(xy)$$

$$\Rightarrow \quad U-U_0=40xy$$

$$\Rightarrow \quad U=U_0+40xy$$

At the mid point (C) of the rod, the particle is in equilibrium.



Also, we must note that from A till mid point C, the force is towards A, so the particle faces opposition from force to reach the mid point C. However after C i.e., between C and B, the force is directed towards B. This makes us conclude that if we give the particle a sufficient speed just to reach the mid point C, then after crossing C it will automatically reach B. Hence by Law of Conservation of Energy applied between A and C we have

$$\begin{pmatrix}\text{Loss in KE}\\ \text{of particle}\end{pmatrix} = \begin{pmatrix}\text{Gain in PE}\\ \text{of particle}\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}mv_0^2 = 40\left(\frac{2}{2}\right)\left(\frac{2}{2}\right) - 40(0)(2)$$

$$\Rightarrow \quad \frac{1}{2}(5)v_0^2 = 40$$

$$\Rightarrow \quad v_0 = \sqrt{\frac{80}{5}} = \sqrt{16} = 4\text{ ms}^{-1}$$

Hence, the correct answer is (D).

13. Method-I:

$$dW = \vec{F}\cdot d\vec{\ell} = 2dx + 3dy$$

Along the path $3y + kx = 5$, we have

$$3dy + kdx = 0 \qquad \ldots(1)$$

Since $W = 0$ i.e., $\int dW = 0$

$$\Rightarrow \quad \int 2dx - kdx = 0 \qquad \{\because \text{ from (1), } 3dy = -kdx\}$$

$$\Rightarrow \quad k = 2$$

Method-II:

Force is parallel to a line $y = \frac{3}{2}x + c$

The equation of given line can be written as

$$y = -\frac{k}{3}x + \frac{5}{3}$$

Work done will be zero, when force is perpendicular to the displacement i.e., the above two lines are perpendicular or

$$m_1 m_2 = -1$$

$$\Rightarrow \quad \left(\frac{3}{2}\right)\left(-\frac{k}{3}\right) = -1$$

$$\Rightarrow \quad k = 2$$

Hence, the correct answer is (A).

14. In the first case, speed imparted to the marble by the spring is

$$\frac{1}{2}kd^2 = \frac{1}{2}mu^2$$

$$\Rightarrow \quad u = \sqrt{\frac{k}{m}}d$$

Since horizontal range of marble is uT, where T is the time of flight. So, we get

$$R - r = uT = \left(\sqrt{\frac{k}{m}}d\right)T \qquad \ldots(1)$$

Let the required compression in the spring for landing the ball in the box be x', then

$$R = \left(\sqrt{\frac{k}{m}}x'\right)T \qquad \ldots(2)$$

Please note that time of flight is independent of the velocity of the ball and therefore same in both the cases. From (1) and (2), we get

$$\frac{R}{R-r} = \frac{x'}{d}$$

$$\Rightarrow \quad x' = \frac{Rd}{R-r}$$

Hence, the correct answer is (D).

15. Since $W_{nc} = \Delta U + \Delta K = mgh - (-mgh) + 0 = 2mgh$

Hence, the correct answer is (B).

16. According to Work Energy Theorem, $W = \Delta K$, so kinetic energy of block at $x = x$ is

$$K = W = \int_0^x F dx = \int_0^x (4 - x^2)dx$$

$$\Rightarrow \quad K = 4x - \frac{x^3}{3}$$

For K to be maximum $\frac{dK}{dx} = 0$

$$\Rightarrow \quad 4 - x^2 = 0$$

$$\Rightarrow \quad x = \pm 2\text{ m}$$

At $x = +2$ m, we have $\frac{d^2K}{dx^2}$ to be negative

So, kinetic energy (K) is maximum at $x = +2$ m

$$\Rightarrow \quad K_{max} = (4)(2) - \frac{(2)^3}{3} = \frac{16}{3}\text{ J} \approx 5.33\text{ J}$$

Hence, the correct answer is (C).

17. Kinetic energy of particle at $x = x$ is

$$K = 4x - \frac{x^3}{3}$$

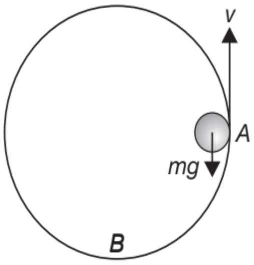
At maximum x-displacement i.e., at point of reversal of motion, we have

$$v = 0,\text{ so } K = 0$$

$$\Rightarrow \quad x = 2\sqrt{3}\text{ m}$$

Hence, the correct answer is (A).

18.



Velocity will be vertical at position A as shown in Figure. At this position, tangential acceleration is due to weight. Therefore,

$$ma_t = mg$$

$$\Rightarrow \quad a_t = g$$

As the particle is just able to complete the circle, speed at position A is

$$v = \sqrt{3gr}$$

Therefore, centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{3gr}{r} = 3g$$

Hence, total acceleration of the particle is

$$a_{\text{total}} = \sqrt{a_c^2 + a_t^2} = \sqrt{(3g)^2 + g^2}$$

$$\Rightarrow \quad a_{\text{total}} = \sqrt{10}\text{g}$$

Hence, the correct answer is (C).

19. Since,

$$\vec{F} = -\vec{\nabla}U$$

$$\Rightarrow \quad \vec{F} = -\lambda(\hat{i} + \hat{j})$$

Further, $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

$$\Rightarrow \quad \Delta\vec{r} = (2\hat{i} + 3\hat{j}) - (\hat{i} + \hat{j}) = \hat{i} + 2\hat{j}$$

So, $W = \vec{F}\cdot\Delta\vec{r} = -\lambda - 2\lambda$

$$\Rightarrow \quad W = -3\lambda$$

Hence, the correct answer is (B).

20. Work done = Area under F-s graph

Area $= 36 + (-6) = 30$ J

Hence, the correct answer is (B).

21. Let the maximum extension in the spring be $x_{\max} = x_m$, then by Applying Work-Energy Theorem from the frame attached to point A, we get

$$(ma)x_m - \frac{1}{2}kx_m^2 = 0$$

$$\Rightarrow \quad x_m = \frac{2ma}{k}$$

Hence, the correct answer is (C).

22. $\vec{v} = \alpha\hat{i} + \beta t\hat{j}$

$$\Rightarrow \quad \vec{a} = \frac{d\vec{v}}{dt} = \beta\hat{j}$$

So, total acceleration $|\vec{a}| = \beta$

The speed, $v = \sqrt{\alpha^2 + \beta^2 t^2}$

So, tangential acceleration, a_T is

$$a_T = \frac{dv}{dt} = \frac{1}{2}(\alpha^2 + \beta^2 t^2)^{1/2}(2\beta^2 t)$$

At $t = \dfrac{\sqrt{3}\alpha}{\beta}$

Tangential acceleration, $a_T = \dfrac{\sqrt{3}\beta}{2}$

and normal acceleration, $a_N = \sqrt{\beta^2 - \dfrac{3\beta^2}{4}} = \dfrac{\beta}{2}$

Hence, the correct answer is (A).

23. $\sin\theta = \dfrac{5}{10} = \dfrac{1}{2}$

$$\Rightarrow \quad \theta = 30°$$

$$W = fs\cos 180 = -fs$$

$$\Rightarrow \quad W = -(\mu mg\cos 30)s$$

$$\Rightarrow \quad W = -(0.1)(0.5)(10)\left(\frac{\sqrt{3}}{2}\right)(10+10)$$

$$\Rightarrow \quad W = -5\sqrt{3}\text{ J}$$

Hence, the correct answer is (D).

24. Let x be the extension in the spring when 2 kg block leaves the contact with ground. Then,

$$kx = 2\text{g}$$

$$\Rightarrow \quad x = \frac{2g}{k} = \frac{2\times 10}{40} = \frac{1}{2}\text{ m}$$

Now by Law of Conservation of Energy, we have

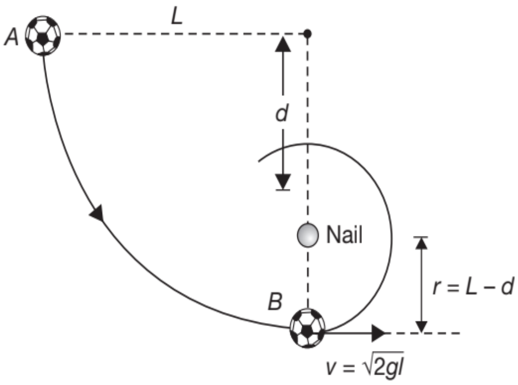
$$mgx = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 \qquad \{\text{where } m = 5\text{ kg}\}$$

$$\Rightarrow \quad v = \sqrt{2gx - \frac{kx^2}{m}}$$

$$\Rightarrow \quad v = \sqrt{2\times 10\times\frac{1}{2} - \frac{40}{4\times 5}} = 2\sqrt{2}\text{ ms}^{-1}$$

Hence, the correct answer is (B).

25.



The ball rotates in a circle around the nail after it goes past the point B.

At point B, velocity of the ball is $v = \sqrt{2gL}$ ...(1)

Since the ball just completes the vertical circle about the nail, its speed at the lowest point should be

$$v = \sqrt{5gr} = \sqrt{5g(L-d)} \quad ...(2)$$

Equating (1) and (2), we get

$$\sqrt{2gL} = \sqrt{5g(L-d)}$$

$\Rightarrow \quad 2L = 5L - 5d$

$\Rightarrow \quad d = 0.6L$

Hence, the correct answer is (A).

26. $-fs = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

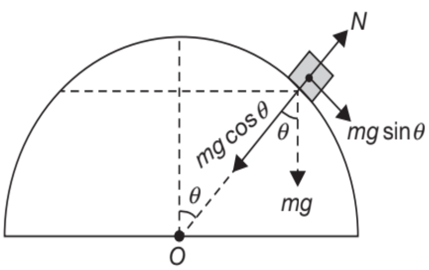
$\Rightarrow \quad -f\left(\frac{20}{100}\right) = \frac{1}{2}\left(\frac{60}{1000}\right)\left[(300)^2 - (600)^2\right]$

$\Rightarrow \quad f\left(\frac{20}{100}\right) = \frac{1}{2}\left(\frac{60}{1000}\right)(10^4)(36-9)$

$\Rightarrow \quad f = 1.5\times10^3 \times 27 = 40.5\times10^3 \text{ N} = 40.5 \text{ kN}$

Hence, the correct answer is (A).

27.



When the particle looses its contact with the sphere, then we know that

$$\cos\theta = \frac{2}{3}$$

$\Rightarrow \quad \sin\theta = \frac{\sqrt{5}}{3}$

So, tangential acceleration at this position is

$$a_T = g\sin\theta = \frac{\sqrt{5}g}{3}$$

Hence, the correct answer is (B).

28. The cable does not work on the bob.
So, the bob's energy is conserved

$\Rightarrow \quad \frac{1}{2}mv_0^2 = mgh$ ...(1)

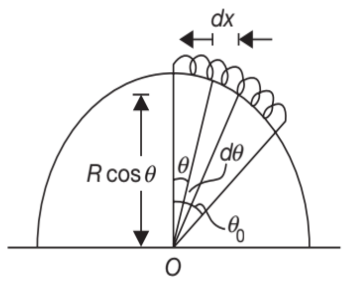
Where $h = \ell(1-\cos\theta)$ ...(2)

From equations (1) and (2), we get

$$v_0 = 7 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

29. Reference level at the centre of the sphere means gravitational potential energy is zero at centre.
Consider an element of length dx subtending an angle $d\theta$ at an angle θ with the vertical. The vertical height of this element is $R\cos\theta$.



If dU be the potential energy of the element, then

$dU = (dm)gh$, where dm is mass of element.

$\Rightarrow \quad dm = \left(\frac{m}{\ell}\right)dx = \frac{m}{\ell}(Rd\theta) \quad \{\because dx = Rd\theta\}$

So, $\quad dU = \frac{m}{\ell}(Rd\theta)g(R\cos\theta) = \frac{m}{\ell}R^2 g\cos\theta\, d\theta$

$\Rightarrow \quad U = \int dU = \frac{mgR^2}{\ell}\int_0^{\theta_0}\cos\theta\, d\theta = \frac{mgR^2}{\ell}\left[\sin\theta|_0^{\theta_0}\right]$

$\Rightarrow \quad U = \frac{mgR^2}{\ell}[\sin\theta_0] = \frac{mgR^2}{\ell}\sin\left(\frac{\ell}{R}\right) \quad \left\{\because \theta_0 = \frac{\ell}{R}\right\}$

Hence, the correct answer is (A).

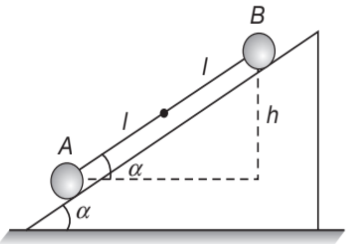
30. $h = 2\ell\sin\alpha$

A is the lowest point and B the highest point. At B, in critical case tension is zero. Let velocity of particle at B at this instant be v_B. Then

$$mg\sin\alpha = \frac{mv_B^2}{\ell}$$

CHAPTER 1

$\Rightarrow \quad v_B^2 = g\ell \sin\alpha$



Now by Law of Conservation of Energy, we have

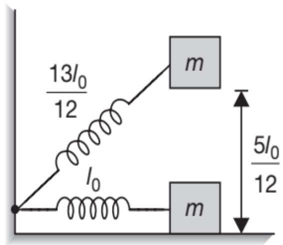
$$v_A^2 = v_B^2 + 2gh$$

$$\Rightarrow \quad v_A^2 = (g\ell\sin\alpha) + 2g(2\ell\sin\alpha)$$

$$\Rightarrow \quad v_A = \sqrt{5g\ell\sin\alpha}$$

Hence, the correct answer is (D).

31.



Elongation in spring is

$$x = \frac{13l_0}{12} - l_0 = \frac{l_0}{12}$$

Work done by lifting force is

$$W_{\text{ext}} = \Delta U + \Delta K = \Delta U_g + \Delta U_e + 0$$

$$\Rightarrow \quad W_{\text{ext}} = \Delta U_g + \Delta U_e$$

$$\Rightarrow \quad W_{\text{ext}} = mg\left(\frac{5l_0}{12}\right) + \frac{1}{2}k\left(\frac{l_0}{12}\right)^2$$

$$\Rightarrow \quad W = \frac{5mgl_0}{12} + \frac{kl_0^2}{288}$$

Hence, the correct answer is (A).

32. If R be the radius of the circumference, then

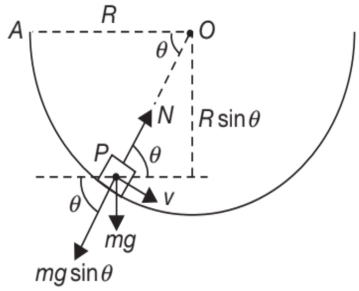
$$OP = OQ\cos 60^\circ = (2R)\left(\frac{1}{2}\right) = R$$

$$\Rightarrow \quad h_1 = OP\cos 60^\circ = \frac{R}{2} \text{ and } h_2 = 2R$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{\sqrt{2gh_1}}{\sqrt{2gh_2}} = \sqrt{\frac{h_1}{h_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

Hence, the correct answer is (C).

33.



By Law of Conservation of Energy applied between A and P, we get

$$\frac{1}{2}mv^2 = mgR\sin\theta$$

$$\Rightarrow \quad v = \sqrt{2gR\sin\theta}$$

At point P, $N = mg\sin\theta + \dfrac{mv^2}{R}$

$$\Rightarrow \quad N = mg\sin\theta + 2mg\sin\theta = 3mg\sin\theta$$

Centripetal force is given by

$$F_C = \frac{mv^2}{R} = 2mg\sin\theta$$

So, the required ratio is

$$\frac{F_C}{N} = \frac{2}{3}$$

Hence, the correct answer is (C).

34. $a = \left(\dfrac{m_2 - m_1\sin\theta - \mu m_1\cos\theta}{m_1 + m_2}\right)g$

$$\Rightarrow \quad a = \left\{\frac{10 - 4\sin 30 - (0.2)4\cos 30}{14}\right\}10$$

$$\Rightarrow \quad a = 5.2 \text{ ms}^{-2}$$

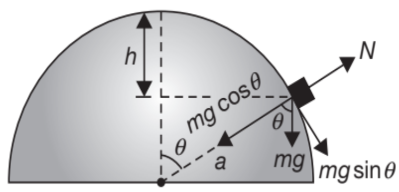
Since, $v^2 - 0^2 = 2as$

$$\Rightarrow \quad v^2 = 2(5.2)(4)$$

$$\Rightarrow \quad v \approx 6.5 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

35.



$$mg\cos\theta - N = \frac{mv^2}{a}$$

The body breaks off the sphere when $N = 0$

$$\Rightarrow \quad mg\cos\theta = \frac{mv^2}{a}$$

$$\Rightarrow \quad v^2 = ga\cos\theta \qquad \ldots(1)$$

Applying Law of Conservation of Mechanical Energy, we get

$$\frac{1}{2}mv^2 = mgh$$

$$\Rightarrow \quad v^2 = 2gh \qquad \ldots(2)$$

From (1) and (2), we get

$$ga\cos\theta = 2gh$$

$$\Rightarrow \quad \cos\theta = 2h/a$$

Hence, the correct answer is (B).

36. $W = \int_{(0,\,0)}^{(1,\,1)} \vec{F}\cdot d\vec{\ell}$

where $d\vec{\ell} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$\Rightarrow \quad W = \int_{(0,\,0)}^{(1,\,1)} \left(x^2dy + ydx\right)$$

$$\Rightarrow \quad W = \int_{(0,\,0)}^{(1,\,1)} \left(y^2dy + xdx\right) \qquad \{\because\ x = y\}$$

$$\Rightarrow \quad W = \left(\frac{y^3}{3} + \frac{x^2}{2}\right)\Bigg|_{(0,\,0)}^{(1,\,1)} = \frac{5}{6}\text{ J}$$

Hence, the correct answer is (B).

37. $P_{\text{input}} = \dfrac{mg(h_2 - h_1)}{t} = \dfrac{(2000)(10)(500)}{1}$

$$\Rightarrow \quad P_{\text{input}} = 10\text{ MW}$$

Since $\eta = \dfrac{P_{\text{output}}}{P_{\text{input}}}$

$$\Rightarrow \quad 0.8 = \frac{x}{10}$$

$$\Rightarrow \quad x = 8\text{ MW}$$

Hence, the correct answer is (A).

38. For the particle just to complete the vertical circle, speed at topmost point is

$$v_0 = \sqrt{gr}$$

Speed when the string becomes horizontal is

$$v = \sqrt{u^2 + 2gr} = \sqrt{gr + 2gr} = \sqrt{3gr}$$

$$\Rightarrow \quad v = \sqrt{3}v_0$$

Tension, when string becomes horizontal, is

$$T = \frac{mv^2}{r} = \frac{m\left(\sqrt{3}v_0\right)^2}{r}$$

$$\Rightarrow \quad T = \frac{3mv_0^2}{r} = \frac{3m(gr)}{r} = 3mg$$

Hence, the correct answer is (D).

39. From Work Energy Theorem, $W = \Delta K$

$$\Rightarrow \quad Pt = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2Pt}{m}} \qquad \ldots(1)$$

$$\Rightarrow \quad v = \frac{ds}{dt} = \sqrt{\frac{2P}{m}}t^{1/2}$$

$$\Rightarrow \quad \int_0^s ds = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}dt$$

$$\Rightarrow \quad s = \frac{2}{3}\sqrt{\frac{2P}{m}}t^{3/2} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{s}{v} = \frac{2}{3}t$$

$$\Rightarrow \quad \frac{s}{v} \propto t$$

i.e., graph between $\dfrac{s}{v}$ and t is a straight line passing through origin.

Hence, the correct answer is (B).

40. $u^2 = gL(2 + 3\sin\phi) = \dfrac{7}{2}gL$

$$\Rightarrow \quad \sin\phi = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 90° + \phi = 120°$$

Hence, the correct answer is (C).

41. The centre of mass of the rope is at a height $\dfrac{h}{2}$. So, total work done is the sum of work done to pull the bucket of mass M to a height h and the work done to pull the CM of the rope of mass m to a height $\dfrac{h}{2}$. So,

$$W = Mgh + mg\frac{h}{2}$$

$$\Rightarrow \quad W = \left(M + \frac{m}{2}\right)gh$$

Hence, the correct answer is (B).

42. Initial velocity $=$ final velocity $= 0$

But displacement $\Delta x \neq 0$

$\{\because \Delta x = \text{Area under } v\text{-}t \text{ graph}\}$

From Work Energy Theorem, $W = \Delta KE = 0$

Hence, the correct answer is (B).

43. $a = \dfrac{|\vec{F}|}{m} = \dfrac{5}{2} = 2.5 \text{ ms}^{-2}$

Velocity at $t = 20$ s is $v = at$

$$\Rightarrow \quad v = (2.5)(20) = 50 \text{ ms}^{-1}$$

$$\Rightarrow \quad \text{K.E.} = \frac{1}{2}(2)(2500)$$

$$\Rightarrow \quad \text{K.E.} = 2500 \text{ J}$$

Hence, the correct answer is (C).

44. From Work Energy Theorem

$$\Delta KE = W_{\text{net}}$$

$$\Rightarrow \quad K_f - K_i = \int P dt$$

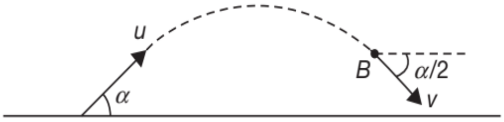
$$\Rightarrow \quad \frac{1}{2}mv^2 = \int_0^2 \left(\frac{3}{2}t^2\right)dt \qquad \{\because m = 2 \text{ kg}\}$$

$$\Rightarrow \quad v^2 = \left(\frac{t^3}{2}\right)\Bigg|_0^2$$

$$\Rightarrow \quad v = 2 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

45. Work done by gravitational force is equal to the change in kinetic energy.
Let v be the velocity of particle at point B is v



The horizontal component of velocity remains constant, so

$$u\cos\alpha = v\cos\left(\frac{\alpha}{2}\right)$$

$$\Rightarrow \quad v = \frac{u\cos\alpha}{\cos(\alpha/2)}$$

$$\Rightarrow \quad W = \frac{1}{2}mv^2 - \frac{1}{2}m(u\cos\alpha)^2$$

$$\Rightarrow \quad W = \frac{1}{2}m\left[\frac{u^2\cos^2\alpha}{\cos^2(\alpha/2)} - u^2\cos^2\alpha\right]$$

$$\Rightarrow \quad W = \frac{1}{2}mu^2\cos^2\alpha\left[\sec^2\left(\frac{\alpha}{2}\right) - 1\right]$$

Since $1 + \tan^2\theta = \sec^2\theta$

$$\Rightarrow \quad W = \frac{1}{2}mu^2\cos^2\alpha\tan^2\left(\frac{\alpha}{2}\right)$$

Hence, the correct answer is (C).

46. Increase in P.E. is the increase in P.E. of C.M.

$$\Delta U = mg\ell_{CM}(1 - \cos\theta)$$

$$\Rightarrow \quad \Delta U = \left(\frac{100}{1000}\right)(10)\left(\frac{1}{2}\right)\left(1 - \frac{1}{2}\right)$$

$$\Rightarrow \quad \Delta U = 0.25 \text{ J}$$

Hence, the correct answer is (A).

47. $(\text{K.E.})_{\text{man}} = \dfrac{1}{2}(\text{K.E.})_{\text{boy}}$

$$\Rightarrow \quad \frac{1}{2}Mv_m^2 = \frac{1}{2}\left[\frac{1}{2}\left(\frac{M}{2}\right)v_b^2\right]$$

$$\Rightarrow \quad v_m = \frac{v_b}{2} \qquad \ldots(1)$$

Further,

$$\frac{1}{2}M(v_m + 1)^2 = \frac{1}{2}\left(\frac{M}{2}\right)v_b^2$$

$$\Rightarrow \quad v_m + 1 = \frac{v_b}{\sqrt{2}} \qquad \ldots(2)$$

Solving (1) & (2), we get

$$v_b = 2\left(\sqrt{2} + 1\right) = 4.82 \text{ ms}^{-1}$$

$$v_m = \sqrt{2} + 1 = 2.41 \text{ ms}^{-1}$$

Hence, the correct answer is (C).

48. Acceleration $= a = \dfrac{v_0}{t_0}$

$$P_{\text{inst}} = Fv_t = (ma)v_t = m\left(\frac{v_0}{t_0}\right)(at)$$

$$\Rightarrow \quad P_{\text{inst}} = m\left(\frac{v_0}{t_0}\right)^2 t$$

Hence, the correct answer is (D).

49. In the initial position, the length of the stretched spring is

$$L = \frac{h}{\cos 37^\circ} = \frac{5h}{4} = 1.25h$$

So, extension in the spring is

$$x = L - h = 0.25h$$

PE stored in the spring is

$$U = \frac{1}{2}kx^2 = \frac{1}{2}k(0.25h)^2 = \frac{kh^2}{32}$$

When the spring becomes vertical, it acquires natural length and hence the PE gets converted into kinetic energy of the ring

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{kh^2}{32}$$

$$\Rightarrow \quad v = \frac{h}{4}\sqrt{\frac{k}{m}}$$

Hence, the correct answer is (D).

50. When the particle at x_1 is displaced slightly towards left or right, the force is in the same direction. Hence, the equilibrium is unstable, whereas at x_2 the force is directed opposite to displacement.

Hence, the correct answer is (B).

51. Since, $a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)g$ and $v^2 - u^2 = 2as$

$$\Rightarrow \quad v^2 - 0^2 = 2\left(\frac{m_2 - m_1}{m_2 + m_1}\right)gh$$

$$\Rightarrow \quad v = \sqrt{2\left(\frac{m_2 - m_1}{m_2 + m_1}\right)gh}$$

Hence, the correct answer is (D).

52. Acceleration down the plane is $a = g(\sin\theta - \mu\cos\theta)$

velocity of block at time t is $v = at = gt(\sin\theta - \mu\cos\theta)$

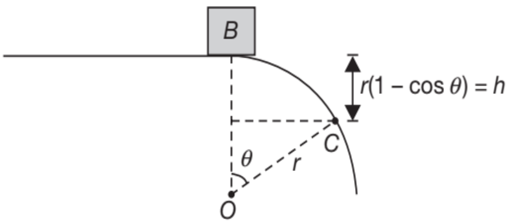
Force of friction is $f = \mu mg\cos\theta$

So, rate of work done by force of friction will be

$$P = fv = \mu mg^2 t\cos\theta(\sin\theta - \mu\cos\theta)$$

Hence, the correct answer is (C).

53.



Let velocity of block at C be v

By Law of Conservation of Energy, we get

$$\frac{1}{2}mv_0^2 + mgr(1 - \cos\theta) = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{v_0^2 + 2gr(1 - \cos\theta)}$$

When block leaves the surface, we have

$$N = 0$$

$$\Rightarrow \quad mg\cos\theta = \frac{mv^2}{r}$$

$$\Rightarrow \quad rg\cos\theta = v_0^2 + 2gr(1 - \cos\theta)$$

$$\Rightarrow \quad 3rg\cos\theta = \frac{rg}{4} + 2rg$$

$$\Rightarrow \quad 3\cos\theta = \frac{9}{4}$$

$$\Rightarrow \quad \cos\theta = \frac{3}{4}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{3}{4}\right)$$

Hence, the correct answer is (B).

54. x = Elongation in spring due to mass 10 kg

$$x = \frac{mg}{k} = \frac{10 \times 10}{100} = 1 \text{ m}$$

$$W_F = \frac{1}{2} \times 100 \times [(3)^2 - (1)^2] - 10 \times 10 \times 2 = 200 \text{ J}$$

Hence, the correct answer is (A).

55. By Law of Conservation of Energy

$$mgh = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v^2 = 2gh = 2(10)(5)$$

$$\Rightarrow \quad v = 10 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

56. $F \propto x^{-1/3}$

i.e., acceleration $a \propto x^{-1/3}$

$$\Rightarrow \quad v\frac{dv}{dx} = Kx^{-1/3}$$

$$\Rightarrow \quad v^2 \propto x^{2/3}$$

$$\Rightarrow \quad v \propto x^{1/3}$$

Now $P = Fv$

$$\Rightarrow \quad P \propto x^{-1/3} \times x^{1/3}$$

$$\Rightarrow \quad P \propto x^0$$

So, power is independent of x

Hence, the correct answer is (D).

57. Since forces are conservative in nature, so the line integral of their sum or their difference in a closed path should also be zero. Hence $W_1 = W_2 = 0$

Hence, the correct answer is (C).

58. Three forces are acting on the particle with respect to the sphere

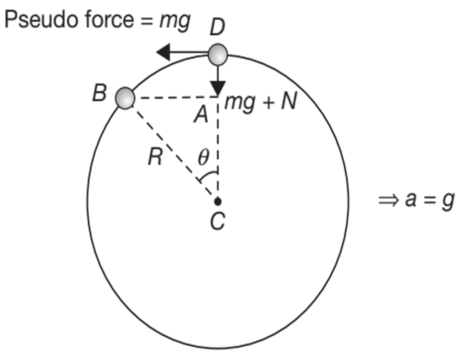
(i) pseudo force $= ma = mg$ $\{as\ a = g\}$

(ii) weight $= mg$

(iii) normal reaction $= N$

Of these first two forces are constant and do work. The third force is not constant but it does not perform any work. Applying Work Energy Theorem, in non-inertial frame, we get

$$\frac{1}{2}mv_r^2 = \begin{pmatrix}\text{work done by}\\ \text{pseudo force}\end{pmatrix} + \begin{pmatrix}\text{work done by}\\ \text{force of gravity}\end{pmatrix}$$



Where $v_r =$ speed of particle relative to sphere

$$\Rightarrow \quad \frac{1}{2}mv_r^2 = mg(AB) + mg(AD)$$

$$\Rightarrow \quad \frac{1}{2}v_r^2 = g[R\sin\theta + R(1-\cos\theta)]$$

$$\Rightarrow \quad \frac{1}{2}v_r^2 = gR(1+\sin\theta-\cos\theta)$$

$$\Rightarrow \quad v_r = \sqrt{2gR(1+\sin\theta-\cos\theta)}$$

Hence, the correct answer is (D).

59. $P_{output} = \dfrac{(100)(10)(9)}{12}$

$$\Rightarrow \quad P_{output} = 750 \text{ W}$$

Since, $\eta = 0.8$

$$\Rightarrow \quad 0.8 = \frac{750}{P_{input}}$$

$$\Rightarrow \quad P_{input} = \frac{750}{0.8} = 937.5 \text{ W}$$

$$\Rightarrow \quad P_{input} \simeq 0.94 \text{ kW}$$

Hence, the correct answer is (C).

60. Let the initial amplitude decreases to a_1 to the other side i.e., after the first sweep;

$$\begin{pmatrix}\text{Decrease in Elastic}\\ \text{Potential Energy}\end{pmatrix} = \begin{pmatrix}\text{Work Done Against}\\ \text{Friction}\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}ka^2 - \frac{1}{2}ka_1^2 = \mu mg(a+a_1)$$

$$\Rightarrow \quad \frac{1}{2}k(a+a_1)(a-a_1) = \mu mg(a+a_1)$$

$$\Rightarrow \quad a - a_1 = \frac{2\mu mg}{k} \qquad \ldots(1)$$

Similarly $a_1 - a_2 = \dfrac{2\mu mg}{k}$ $\ldots(2)$

$$a_{n-1} - a_n = \frac{2\mu mg}{k} \qquad \ldots(3)$$

Adding all the above equations, we get

$$a - a_n = \frac{2n\mu mg}{k} \qquad \ldots(4)$$

Now, the block stops when

$$\mu mg = ka_n$$

$$\Rightarrow \quad a_n = \frac{\mu mg}{k}$$

Substituting in the above equation (4), we get

$$(2n+1)\left(\frac{\mu mg}{k}\right) = a$$

$$\Rightarrow \quad (2n+1) = \frac{ka}{\mu mg} = \frac{(20)(0.3)}{(0.04)(1)(10)} = 15$$

$$\Rightarrow \quad n = 7$$

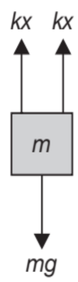
Hence, the correct answer is (C).

61. $\eta = \dfrac{P_{output}}{P_{input}} \times 100 = \dfrac{100 \times 10 \times 25}{30,000} \times 100$

$$\Rightarrow \quad \eta = 83.3\%$$

Hence, the correct answer is (C).

62. Speed is maximum at equilibrium, so when the block has descended by x, then both springs apply a force $2kx$ (upwards) on the block as shown in figure.



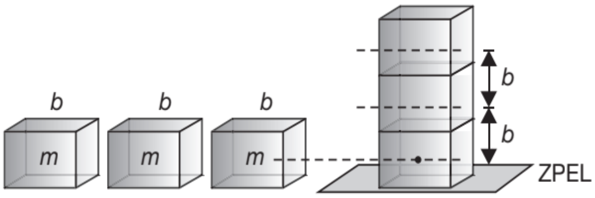
For equilibrium, we have

$$2kx = mg$$

$$x = \frac{mg}{2k}$$

Hence, the correct answer is (A).

63.



Work done = change in potential energy

$$W = mgb + mg(2b) = 3mgb$$

Hence, the correct answer is (C).

64. At break up, $F = 0$

$$\Rightarrow \quad -\frac{dU}{dx} = 0 \text{ at } x = x_0 \quad \text{(say)}$$

$$\Rightarrow \quad \frac{d}{dx}(ax^{-12} - bx^{-6}) = 0$$

$$\Rightarrow \quad a(-12)x_0^{-13} - (b)(-6)x_0^{-7} = 0$$

$$\Rightarrow \quad x_0^6 = \frac{2a}{b}$$

$$\Rightarrow \quad U(x_0) = \frac{a}{x_0^{12}} - \frac{b}{x_0^6}$$

$$\Rightarrow \quad U(x_0) = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)}$$

$$\Rightarrow \quad U(x_0) = \frac{b^2}{4a} - \frac{b^2}{2a}$$

$$\Rightarrow \quad U(x_0) = -\frac{b^2}{4a}$$

This is the energy with which two atoms are associated. So, dissociation energy equals $\frac{b^2}{4a}$.

Hence, the correct answer is (C).

65. $a = \frac{10g - 5g - 25}{15} = \frac{100 - 50 - 25}{15} = \frac{5}{3} \text{ ms}^{-2}$

Since, $v^2 - 0^2 = 2as$

$$\Rightarrow \quad v^2 = 2\left(\frac{5}{3}\right)(2) = \frac{20}{3}$$

$$\Rightarrow \quad v = 2.6 \text{ ms}^{-1}$$

Hence, the correct answer is (A).

66. Power $P = Fv = \frac{K}{v}v = K = \text{constant}$

$$\Rightarrow \quad W = Pt = Kt$$

Hence, the correct answer is (C).

67. For month of June (having 30 days), each day motor is operated for 8 hrs. So total time (t) for which motor is operational in June is

$$t = (8)(30) = 240 \text{ hrs}$$

So, total energy consumed in the month of June is equal to work done $(1 \text{ HP} = 746 \text{ W})$

$$W = Pt = (2 \times 746)(240) \text{ watt-hour}$$

Since 1 unit of electricity $(= 1 \text{ kw-hr})$ costs ₹6,

So, bill for the month of June is

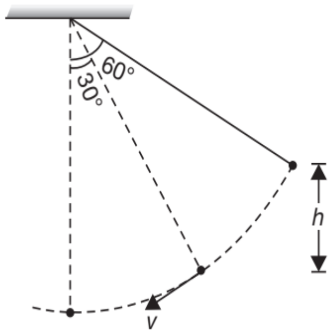
$$\frac{2 \times 746 \times 240}{1000} \times 6 = ₹2148$$

Hence, the correct answer is (A).

68. Given $\ell = 1$ m

$$\Rightarrow \quad h = \ell(\cos 30° - \cos 60°)$$

$$\Rightarrow \quad h = 1(0.86 - 0.5) = 0.36 \text{ m}$$



Since, $v = \sqrt{2gh} = \sqrt{2(10)(0.36)}$

$$\Rightarrow \quad v = 2.68 \text{ ms}^{-1}$$

Also, $P = Fv = (mg\sin\theta)v$

$$\Rightarrow \quad P = (mg\sin 30°)v = (1)(10)\left(\frac{1}{2}\right)(2.68)$$

$$\Rightarrow \quad P = 13.4 \text{ W}$$

Hence, the correct answer is (B).

69. Velocity of ball to just reach the top of the tube should be given by Law of Conservation of Energy, according to which

$$(U + K)_i = (U + K)_f$$

$$\Rightarrow \quad mgh + \frac{1}{2}mv^2 = mg(2R) + \frac{1}{2}m(0)^2$$

$$\Rightarrow \quad v = \sqrt{2g(2R - h)}$$

Hence, the correct answer is (D).

70. Since velocity is constant so the kinetic energy does not change. If W_F be the work done by external force and W_f be the work done by friction, then

$$W_F + W_f = \Delta K = 0$$

$$\Rightarrow \quad W_F = -W_f$$

$$\Rightarrow \quad W_F = 0.5\mu_k Mgx + \mu_k Mgx + 0.5\mu_k Mgx$$

$$\Rightarrow \quad W_F = 2\mu_k Mgx$$

Hence, the correct answer is (B).

71. According to Work Energy Theorem, we have

$$W_{\text{total}} = \Delta K$$

$$\Rightarrow \quad W_{\text{ext}} + W_{nc} = \Delta U + \Delta K$$

$$\Rightarrow \quad Fx - \mu m_1 gx = \frac{1}{2}kx^2 + 0$$

$$\Rightarrow \quad Fx - \mu m_1 gx - \frac{1}{2}kx^2 = 0$$

But $kx = \mu m_2 g$, for just shifting m_2

$$\Rightarrow \quad Fx - \mu m_1 gx = \frac{1}{2}(\mu m_2 g)x$$

$$\Rightarrow \quad F = \mu\left(m_1 + \frac{m_2}{2}\right)g$$

$$\Rightarrow \quad F = 0.4\left(1 + \frac{2}{2}\right)(10) = 8 \text{ N}$$

Hence, minimum constant force is 8 N.

Hence, the correct answer is (A).

72. When the brick just reaches at the required height, the man saves the excess kinetic energy possessed by the brick.

So, percentage energy saved is

$$\frac{KE}{KE+PE} \times 100 = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv^2 + mgh} \times 100\%$$

$$\% \text{ saved} = \frac{v^2}{v^2 + 2gh} \times 100\%$$

$$\% \text{ saved} = \frac{(10)^2}{(10)^2 + (2)(10)(5)} \times 100 = 50\%$$

Hence, the correct answer is (D).

73. Work done = Change in potential energy $= \Delta U$

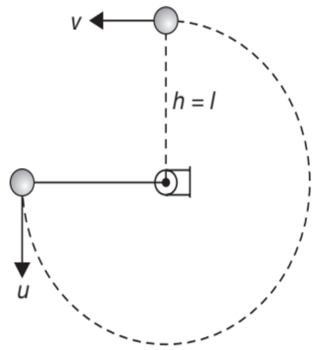
$$\Rightarrow \quad \Delta U = mgL - mgL\cos\theta$$

$$\Rightarrow \quad \Delta U = mgL(1 - \cos\theta)$$

Hence, the correct answer is (A).

74. In the critical case, velocity at topmost point should be zero i.e.,

$$v = 0$$



Using Law of Conservation of Energy, we get

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mgh$$

$$\Rightarrow \quad v^2 = u^2 - 2gh$$

$$\Rightarrow \quad 0 = u^2 - 2g\ell \qquad \{\because h = \ell\}$$

$$\Rightarrow \quad u = \sqrt{2g\ell}$$

Hence, the correct answer is (B).

75. For a simple pendulum, when string is vertical, tension in the string is

$$T_1 = mg + \frac{mv^2}{L}$$

In extreme position $(v = 0)$, so tension is

$$T_2 = mg\cos\theta$$

Given that, $T_1 = 2T_2$

$$\Rightarrow \quad mg + \frac{mv^2}{L} = 2mg\cos\theta$$

The speed (v) at the lowermost point is given by

$$v = \sqrt{2gL(1-\cos\theta)}$$

$$\Rightarrow \quad mg + \frac{m}{L}\left[2gL(1-\cos\theta)\right] = 2mg\cos\theta$$

$$\Rightarrow \quad 1 + 2 - 2\cos\theta = 2\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{3}{4}$$

Hence, the correct answer is (D).

76. Total mechanical energy $= mgH$

Given that $\frac{PE}{KE} = \frac{1}{2}$

$$\Rightarrow \quad PE = \frac{1}{3}mgH \text{ and } KE = \frac{2}{3}mgH$$

So, height from the ground at this instant is $h = \frac{H}{3}$ and speed of particle is

$$v=\sqrt{2g\left(\frac{2H}{3}\right)}=2\sqrt{\frac{gH}{3}}$$

Hence, the correct answer is (C).

77. Decrease in potential energy = Increase in kinetic energy

$$\Rightarrow \quad \frac{1}{2}mv^2=mg\left(\ell-\frac{\ell}{2}\right)$$

$$\Rightarrow \quad v=\sqrt{g\ell}$$

Hence, the correct answer is (B).

78. If x_i be the compression in the spring initially, then, in equilibrium,

$$F+m_1g=kx_i$$

$$\Rightarrow \quad x_i=\frac{F+m_1g}{k} \quad \ldots(1)$$

Let x_f be the final elongation in spring when lower block just lifts. For this to happen, we have

$$kx_f=m_2g$$

$$\Rightarrow \quad x_f=\frac{m_2g}{k} \quad \ldots(2)$$

Now by Law of Conservation of Energy

$$\left(\begin{array}{c}\text{Increase in}\\ \text{Gravitational}\\ \text{Potential}\\ \text{Energy of } m_1\end{array}\right)=\left(\begin{array}{c}\text{Decrease in}\\ \text{Elastic Potential}\\ \text{Energy of Spring}\end{array}\right)$$

$$\Rightarrow \quad m_1g\left(x_i+x_f\right)=\frac{1}{2}k\left(x_i^2-x_f^2\right)$$

Substituting the values of x_i and x_f from equations (1) and (2), we get

$$F=\left(m_1+m_2\right)g$$

Hence, the correct answer is (C).

79. $W_{nc}=W_{\text{friction}}=\Delta U+\Delta K$

$$\Delta U=(\Delta U)_{A\to B}+(\Delta U)_{B\to C}$$

$$\Rightarrow \quad \Delta U=U_B-U_A+U_C-U_B$$

$$\Rightarrow \quad \Delta U=U_C-U_A=0-(2)(10)(1)$$

$$\Rightarrow \quad \Delta U=-20\text{ J}$$

$$\Rightarrow \quad \Delta K=(\Delta K)_{A\to B}+(\Delta K)_{B\to C}$$

$$\Rightarrow \quad \Delta K=K_B-K_A+K_C-K_B$$

$$\Rightarrow \quad \Delta K=K_C-K_A=0-0$$

$$\Rightarrow \quad \Delta K=0$$

So, $W_{nc}=-20$ J

Hence, the correct answer is (B).

80. Since, $\frac{3}{4}$th energy is lost i.e., $\frac{1}{4}$th kinetic energy is left with the particle. Hence, its velocity becomes $\frac{v_0}{2}$ under a retardation of μg in time t_0.

$$\Rightarrow \quad \frac{v_0}{2}=v_0-\mu g t_0$$

$$\Rightarrow \quad \mu g t_0=\frac{v_0}{2}$$

$$\Rightarrow \quad \mu=\frac{v_0}{2gt_0}$$

Hence, the correct answer is (B).

81. $F_{down}=mg\sin\theta=\frac{3}{5}mg$

$$f_{\max}=\mu mg\cos\theta=\left(\frac{3}{4}\right)\left(\frac{4}{5}\right)mg=\frac{3}{5}mg$$

$$\Rightarrow \quad f_{\max}=mg\sin\theta=\frac{3}{5}mg$$

Block will move upwards when

$$kx_0=f_{\max}+mg\sin\theta=\frac{6}{5}mg \quad \ldots(1)$$

By Law of Conservation of Energy, we have

$$Mgx_0=\frac{1}{2}kx_0^2$$

$$\Rightarrow \quad M=\frac{kx_0}{2g}=\frac{\frac{6}{5}mg}{2g}=\frac{3\text{ m}}{5}$$

Hence, the correct answer is (A).

82. Retardation $\propto x$

$$\Rightarrow \quad a=-kx$$

$$\Rightarrow \quad v\frac{dv}{dx}=-kx$$

$$\Rightarrow \quad v\,dv=-kx\,dx$$

$$\Rightarrow \quad \int v\,dv=-k\int x\,dx$$

$$\Rightarrow \quad \frac{v^2}{2}=-\frac{kx^2}{2}$$

$$\Rightarrow \quad \frac{mv^2}{2}=\text{K.E.}=-km\frac{x^2}{2}$$

$$\Rightarrow \quad \text{K.E.}\propto x^2$$

Hence, the correct answer is (B).

83. Potential energy of particle at $x=\sqrt{\frac{2E}{k}}$ i.e., for $x>0$ is zero

So, Total Energy = Kinetic Energy $=E$

$$\Rightarrow \frac{1}{2}mv^2=E$$

$$\Rightarrow v=\sqrt{\frac{2E}{m}}$$

Hence, the correct answer is (B).

84. $P=(F_{net})v$

where

$$F_{net}=mg+0.1mg+0.02mg$$

$$\Rightarrow F_{net}=1.12\text{mg}$$

$$\Rightarrow P=(1.12)(1000)(10)(10)$$

$$\Rightarrow P=112\text{ kW}$$

Hence, the correct answer is (A).

85. Unit vector corresponding to $\hat{i}+\hat{j}+\hat{k}$ is $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

So, $\vec{F}=\frac{30}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})=10\sqrt{3}(\hat{i}+\hat{j}+\hat{k})$

Since, $W=\vec{F}\cdot\Delta\vec{r}=\vec{F}\cdot(\vec{r}_2-\vec{r}_1)$

$$\Rightarrow W=10\sqrt{3}(\hat{i}+\hat{j}+\hat{k})\cdot(2\hat{i}+\hat{k})$$

$$\Rightarrow W=10\sqrt{3}(2+1)=30\sqrt{3}\ J$$

Hence, the correct answer is (B).

86. By Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in Gravitational}\\ \text{Potential Energy}\end{pmatrix}=\begin{pmatrix}\text{Gain in Elastic}\\ \text{Potential Energy}\end{pmatrix}$$

$$\Rightarrow mg(h+x)=\frac{1}{2}kx^2$$

Putting values to solve the Quadratic Equation, we get

$$x=0.1\text{ m}$$

Hence, the correct answer is (A).

87. Tension is maximum at the lowest point, so

$$T_L=\frac{m}{r}(u^2+gr)$$

Tension is minimum at the highest point, so

$$T_H=\frac{m}{r}(u^2-5gr)$$

Given that $\frac{T_L}{T_H}=4$

$$\Rightarrow \frac{T_L}{T_H}=\frac{\frac{m}{r}(u^2+gr)}{\frac{m}{r}(u^2-5gr)}=4$$

$$\Rightarrow 4u^2-20gr=u^2+gr$$

$$\Rightarrow 3u^2=21gr$$

$$\Rightarrow u=\sqrt{7gr}$$

Since $v^2=u^2-2gh$

$$\Rightarrow v=\sqrt{7gr-2g(2r)}=\sqrt{3gr}$$

$$\Rightarrow v=\sqrt{3(10)\left(\frac{10}{3}\right)}=10\text{ ms}^{-1}$$

Hence, the correct answer is (D).

88. Velocity of bob at the lowest point just before hitting the pin is

$$v^2=2gL(1-\cos\alpha)$$

It will rise further to a height h (say), then by Law of Conservation of Energy, we get

$$mgh=\frac{1}{2}mv^2$$

$$\Rightarrow h=\frac{v^2}{2g}=L(1-\cos\alpha)$$

Now $h=(L-\ell)(1-\cos\theta)$

$$\Rightarrow (L-\ell)(1-\cos\theta)=L(1-\cos\alpha)$$

$$\Rightarrow \theta=\cos^{-1}\left(\frac{L\cos\alpha-\ell}{L-\ell}\right)$$

Hence, the correct answer is (C).

89. $W_{ext}=W_{F_0}=\int_0^{\frac{\pi}{2}}\vec{F}_0\cdot d\vec{r}$

$$\Rightarrow W_{F_0}=\frac{\pi F_0 l}{2}$$

Applying Law of Conservation of Mechanical Energy, we get

$$\frac{\pi F_0 l}{2}=\frac{1}{2}mv_1^2+mgl$$

$$\Rightarrow v_1=\sqrt{\frac{l}{m}(\pi F_0-2mg)}$$

Hence, the correct answer is (A).

90. In this case, the particle has both tangential and centripetal accelerations. Given that

$$a_T=\alpha t$$

$\Rightarrow \dfrac{dv}{dt} = \alpha t$

$\Rightarrow \int_0^v dv = \alpha \int_0^t t dt$

$\Rightarrow v = \dfrac{\alpha t^2}{2}$

Since, $a_{\text{total}} = \sqrt{a_C^2 + a_T^2}$

Since a_{total} is equally inclined to a_c and a_T, so $\beta = 45°$

$\Rightarrow \tan\beta = \tan 45° = \dfrac{a_T}{a_C} = 1$

$\Rightarrow \dfrac{\alpha t}{\left(\dfrac{v^2}{r}\right)} = \dfrac{\alpha t \times 4 \times 2}{\alpha^2 t^4} = 1$

At $t = 2$ s, we have

$\dfrac{\alpha(2) \times 4 \times 2}{\alpha^2 (2)^4} = 1$

$\Rightarrow \alpha = 1 \text{ ms}^{-3}$

Hence, the correct answer is (B).

91. Since the car is moving with a constant velocity, so
Applied Force = Force of Friction

$\Rightarrow F = 50$ N

$\Rightarrow W = Fs$

$\Rightarrow W = (50)(1000)$

$\Rightarrow W = 5 \times 10^4$ J

Hence, the correct answer is (A).

92. When it is lowered gradually then to attain the equilibrium, we have

$mg = kd$ ...(1)

Further when it is allowed to fall suddenly, and if x is the maximum stretching in the spring, then by Law of Conservation of Energy

$\begin{pmatrix}\text{Loss in Gravitational} \\ \text{Potential Energy} \\ \text{of Body}\end{pmatrix} = \begin{pmatrix}\text{Gain in Elastic} \\ \text{Potential Energy} \\ \text{of Spring}\end{pmatrix}$

$\Rightarrow mgx = \dfrac{1}{2}kx^2$

$\Rightarrow mg = \dfrac{1}{2}kx$ ...(2)

Equating (1) and (2), we get

$x = 2d$

Hence, the correct answer is (B).

93. By Law of Conservation of Energy

$\dfrac{1}{2}mv^2 = \dfrac{1}{2}kx^2$

$\Rightarrow \dfrac{1}{2}(4)(1.6)^2 = \dfrac{1}{2}(2)x^2$

$\Rightarrow x = 2.26$ m

Hence, the correct answer is (C).

94. $a = g(\sin 30 - \mu \cos\theta)$

$\Rightarrow a = 9.8\left(\dfrac{1}{2} - 0.2\dfrac{\sqrt{3}}{2}\right)$

$\Rightarrow a = 9.8(0.3268)$

$\Rightarrow a = 3.2 \text{ ms}^{-2}$

Since $v^2 - (10)^2 = 2(3.2)(10)$

$\Rightarrow v^2 = 164$

$\Rightarrow v \cong 13 \text{ ms}^{-1}$

Hence, the correct answer is (B).

95. Speed will be maximum at equilibrium, i.e., when

$F = kx$

$\Rightarrow x = \dfrac{F}{k}$

Applying $MWET$, we get

$W_{ext} = \Delta U + \Delta K$

$\Rightarrow Fx = \dfrac{1}{2}kx^2 + \dfrac{1}{2}mv^2$

$\Rightarrow F\left(\dfrac{F}{k}\right) = \dfrac{1}{2}k\left(\dfrac{F}{k}\right)^2 + \dfrac{1}{2}mv^2$

$\Rightarrow \dfrac{1}{2}mv^2 = \dfrac{F^2}{2k}$

$\Rightarrow v = \dfrac{F}{\sqrt{mk}}$

Hence, the correct answer is (A).

96. $\begin{pmatrix}\text{Loss in Kinetic} \\ \text{Energy}\end{pmatrix} = \begin{pmatrix}\text{Gain in Gravitational} \\ \text{Potential Energy}\end{pmatrix}$

$\Rightarrow \dfrac{1}{2}mV_0^2 - \dfrac{1}{2}mV^2 = mgh$

$\Rightarrow V^2 = V_0^2 - 2gh$

$\Rightarrow V = \sqrt{V_0^2 - 2gh}$

Hence, the correct answer is (C).

97. Potential energy of cube at position 1,

$$U_1 = Mg(4R) = 4MgR$$

Applying Law of Conservation of Energy between 1 and 2, we get

$$K_1 + U_1 = K_2 + U_2$$

$$\Rightarrow \quad 4MgR = 2MgR + \frac{1}{2}Mv^2$$

$$\Rightarrow \quad 4MgR = Mv^2$$

$$\Rightarrow \quad v^2 = 4gR$$

Since, $N = \frac{Mv^2}{R} - Mg$

$$\Rightarrow \quad N = \frac{M}{R}(4Rg) - Mg = 3Mg$$

Hence, the correct answer is (A).

98. Let l be the length of spring at the situation shown in Figure.

$$\Rightarrow \quad \cos(37°) = \frac{h}{l}$$

$$\Rightarrow \quad l = \frac{h}{\cos(37°)} = \frac{5h}{4}$$

Extension x in spring is

$$x = l - h = \frac{5h}{4} - h = \frac{h}{4}$$

By Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in EPE}\\ \text{of springs}\end{pmatrix} = \begin{pmatrix}\text{Gain in KE}\\ \text{of Bead}\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}kx^2 + \frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \quad 2\left(\frac{1}{2}(1000)\frac{h^2}{16}\right) = \frac{1}{2}(5)v^2$$

$$\Rightarrow \quad v^2 = 25h^2$$

$$\Rightarrow \quad v = 5h$$

Hence, the correct answer is (A).

99. Speed of ball is maximum at equilibrium, so

$$x_{eq} = \frac{mg}{k}$$

Hence, the correct answer is (A).

Multiple Correct Choice Type Questions

1. According to Work-Energy Theorem for a non-conservative system, we have

$$W_c + W_{nc} = \Delta k \qquad \text{\{OPTION (C)\}}$$

Also, we know that the work done by a conservative force equals the decrease in potential energy, so

$$W_c = -\Delta U \qquad \text{\{OPTION (B)\}}$$

$$\Rightarrow \quad W_{nc} - \Delta U = \Delta k \qquad \text{\{OPTION (A)\}}$$

Hence, (A), (B) and (C) are correct.

2. Tension in the string will be maximum when mass m is at point B, so

$$T = mg + \frac{mv^2}{\ell}$$

By Law of Conservation of Energy,

$$\frac{1}{2}mv^2 = mgh = mg\ell(1 - \cos\theta)$$

$$\Rightarrow \quad v^2 = 2g\ell(1 - \cos(60°))$$

$$\Rightarrow \quad T = mg + \frac{2mg\ell}{\ell}\left(1 - \frac{1}{2}\right)$$

$$\Rightarrow \quad T = 2mg$$

For mass $4m$ to be stationary, $f \geq T$

$$\Rightarrow \quad \mu(4mg) \geq 2mg$$

$$\Rightarrow \quad \mu \geq 0.5$$

Hence, **(A)** is correct

When m moves from A to B it has vertical displacement downwards, i.e., along the direction of gravitational force.

So, **(B)** is correct.

Tension is always perpendicular to the velocity, so power delivered by the tension is zero.

So, **(C)** is correct.

According to Work Energy Theorem,

$$W_{\text{total}} = \Delta K$$

Since work done by tension is zero therefore work is done only by the gravitational force. So, **(D)** is also correct.

Hence, (A), (B), (C) and (D) are correct.

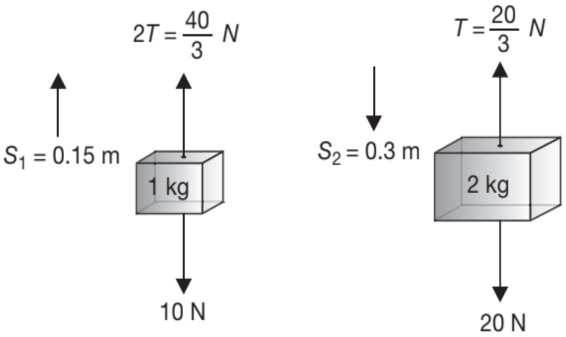
3. $P =$ (Total Frictional Force) (velocity)

$$\Rightarrow \quad P = (mf)v$$

$$\Rightarrow \quad P_{extra} = mg(\sin\theta)v = \frac{mghv}{s}$$

Hence, (A), (B) and (C) are correct.

4. Using the concepts of Laws of Motion and Kinematics we can draw following diagrams and then using the definition of work, we can find corresponding work done.



Hence, (A), (B), (C) and (D) are correct.

5. For a light string pulled by a constant Force $F = 40$ N, tension (T) in the string is also 40 N. So, $T = 40$ N

Since $W = 40 \text{ J} = Fs$ where $F = 40 - 20 \text{ N} = 20 \text{ N}$

$\Rightarrow \quad 40 = 20 \text{ s}$

$\Rightarrow \quad s = 2 \text{ m}$

Since acceleration is constant, so $v \propto t$ and hence $P = Fv$

$\Rightarrow \quad P \propto t$

Also, $v = \sqrt{2ax}$

$\Rightarrow \quad P \propto \sqrt{x}$.

So, all options (A), (B), (C) and (D) are correct.

Hence, (A), (B), (C) and (D) are correct.

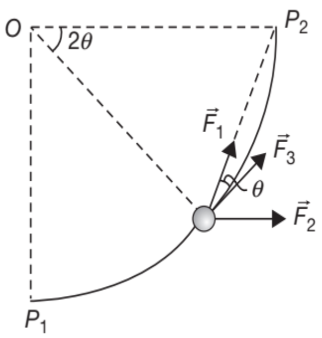
6. Work done by $\vec{F}_1$ is

$$W_1 = \int_{P_1}^{P_2} F_1 \cos\theta ds$$

Where, $ds = (6)d(-2\theta) = -12d\theta$ and $F_1 = 20$ N

$$\Rightarrow \quad W_1 = -240\int_{\pi/4}^{0} \cos\theta d\theta$$

$$\Rightarrow \quad W_1 = 240 \sin\left(\frac{\pi}{4}\right) = 120\sqrt{2} \text{ J}$$



$\vec{F}_1$ is conservative because it is always directed towards a fixed point P_2. Therefore, W_1 can be directly calculated as

$$W_1 = F_1(P_1P_2) = (20)(6\sqrt{2}) = 120\sqrt{2} \text{ J}$$

Similarly, $W_2 = F_2(OP_2) = (30)(6) = 180$ J

and $W_3 = \int_0^{6(\pi/2)} F_3 ds = \int_0^{3\pi} 15 ds$

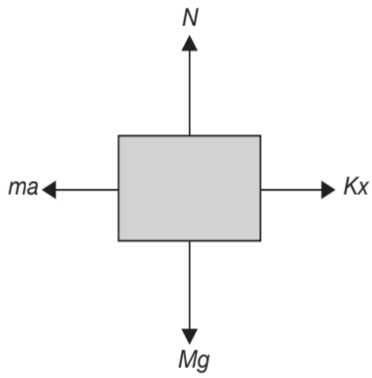
$$W_3 = 15s\Big|_0^{3\pi} = 45\pi \text{ J}$$

Hence, (A), (C) and (D) are correct.

7. According to Work Energy Theorem, work done by all the forces is equal to change in kinetic energy, so

$$W_{total} = \Delta K$$

$$\Rightarrow \quad W_{ext} + W_{nc} = \Delta U + \Delta K$$



Since, $W_{nc} = 0$, because friction is absent

So, $W_{ext} = \Delta U + \Delta K$

$$\Rightarrow \quad (Ma)x_{max} = \frac{1}{2}kx_{max}^2$$

$$\Rightarrow \quad x_{max} = \frac{2Ma}{k}$$

The block will execute SHM and its initial position will be the one of the extreme positions i.e., block returns to its initial position after maximum elongation in the spring and compression in the spring is zero.
So, **(D)** is also correct.

Hence, (B) and (D) are correct.

8. $W = \int \vec{F} \cdot d\vec{r} = \int (\hat{i}F_x + \hat{j}F_y) \cdot (\hat{i}dx + \hat{j}dy)$

$$W = \int (7\hat{i} - 6\hat{j}) \cdot (\hat{i}dx + \hat{j}dy)$$

$$\Rightarrow \quad W = 7\int_0^{-3} dx - 6\int_0^{4} dy = -21 - 24 = -45 \text{ J}$$

Displacements along the direction of X and Y axes are opposite to the direction of force acting along the respective directions, hence the particle must have some velocity at $(0, 0)$.

Hence, (B) and (D) are correct.

9. In region AB, $s \propto t$

$\Rightarrow \quad v = \text{constant}$

$\Rightarrow \quad \Delta K = 0$

$\Rightarrow \quad W_{A\to B} = \Delta K = 0$

Similarly, from B to C, slope of s-t graph is decreasing, so v is also decreasing and hence

$W = \Delta K = \text{NEGATIVE}$

Hence, (B) and (C) are correct.

10. Let m be the mass of the block
Initial elongation of the spring will be

$$x_i = \frac{mg}{k} \quad \ldots(1)$$

On application of F work done by F and gravity is used to increase the elastic potential energy of spring. So,

$$(F + mg)x_0 = \frac{1}{2}k(x_i + x_0)^2 - \frac{1}{2}kx_i^2 \quad \ldots(2)$$

From (1) and (2) we get,

$$x_0 = \frac{2F}{k}$$

Work done by applied force F is $W = Fx_0$

Hence, (C) and (D) are correct.

11. Since $P = \vec{F}\cdot\vec{v}$. At 1 s and 3 s, P is positive. Hence, angle is acute. At 7 s power is negative. Hence, angle is obtuse.
Area under P-t graph is the work done which equals the change in kinetic energy.
Hence, (A), (C) and (D) are correct.

12. In a conservative field, the work done W_C by the conservative force is always equal to the decrease in potential energy.

$\Rightarrow \quad W_c = U_{\text{initial}} - U_{\text{final}} = -\Delta U$

$\Rightarrow \quad W_c = U_A - U_B$

Decrease in potential energy implies $U_A > U_B$

Hence, (B) and (C) are correct.

13. Conceptual (B) and (D) are correct.

14. By Work-Energy Theorem

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \vec{F}\cdot\vec{s}$$

K.E. increasing with time means $v > u$

$\Rightarrow \quad$ Either $\theta = 0°$

OR $\quad 0 < \theta \le 90°$ i.e. θ is acute

OR $\quad P = \frac{W}{t} \ne 0$, because

$W \ne 0$

Since, K.E. is increasing, so P.E. must be decreasing and hence height above the ground must be decreasing.
Hence, (A), (C), (D) and (E) are correct.

15. Force on the particle is given by

$$\vec{F} = -\nabla U$$

$$\Rightarrow \quad \vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} = -(6xy^2 + 6)\hat{i}$$

Now, for initial acceleration i.e. $t = 0$ and $x = 1$, $y = 1$, we have

$$\vec{a} = \frac{\vec{F}}{M} = 12\hat{i} - 6\hat{j}$$

$$\Rightarrow \quad |\vec{a}| = 6\sqrt{5}\ \text{ms}^{-2}$$

Since particle is at rest at $x = 1$, $y = 1$ so, the total mechanical energy (E) is

$E = U + K$

$\Rightarrow \quad U(1, 1) + K(1, 1) = E$

$\Rightarrow \quad E = 9\ \text{J}$

According to modified work-Energy Theorem, we have

$W_{\text{ext}} = \Delta U + \Delta K$

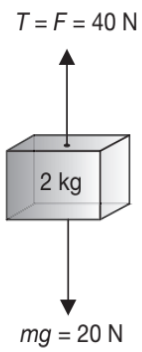
Since the particle is moving slowly, so $\Delta K = 0$

$\Rightarrow \quad W_{\text{ext}} = \Delta U = U(0, 0) - U(1, 1)$

$\Rightarrow \quad W_{\text{ext}} = -9\ \text{J}$

Hence, (A) and (C) are correct.

16. Free body diagram of block is as shown in figure.



From Work Energy Theorem

$W_{\text{net}} = \Delta\text{KE}$

$\Rightarrow \quad (40-20)s = 40$

$\Rightarrow \quad s = 2\text{ m}$

Work done by gravity is

$$W_g = -20\times 2 = -40\text{ J}$$

and work done by tension is

$$W_T = 40\times 2 = 80\text{ J}$$

Hence, (A), (B) and (D) are correct.

17. From 0 to t_1, slope is increasing, so speed is increasing. Hence, work done is positive.

From t_1 to t_2, slope is decreasing, so speed is decreasing. Hence, work done is negative.

From t_2 to t_3, slope of s-t graph is zero, so speed is zero. Hence, work done is zero.

From t_3 to t_4, slope is constant, so speed is constant. Hence, work done is zero.

Hence, (A), (B) and (C) are correct.

18. As soon as the block hits the wall, the suspension point B comes to a stop, whereas the particle C keeps moving with a velocity v_0 towards left. In order that it complete a full circle, it must have kinetic energy enough to take it to the top of the circle.

$\Rightarrow \quad \frac{1}{2}mv_0^2 = mg(2l)$

$\Rightarrow \quad v_0 = \sqrt{4gl}$

Hence, (A) and (C) are correct.

19. **Conceptual (B), (C) and (D) are correct.**

20. $K.E. \propto t$

$\Rightarrow \quad \frac{1}{2}mv^2 = kt$

$\Rightarrow \quad v = \sqrt{\frac{2k}{m}}\,t^{1/2} \qquad \ldots(1)$

$\Rightarrow \quad a = \frac{dv}{dt} = \sqrt{\frac{2k}{m}}\left(\frac{1}{2\sqrt{t}}\right)$

Since $F = ma$

$\Rightarrow \quad F \propto \frac{1}{\sqrt{t}}$

From (1), we get

$$\frac{1}{\sqrt{t}} = \sqrt{\frac{2k}{m}}\frac{1}{v}$$

$\Rightarrow \quad a = \sqrt{\frac{2k}{m}}\left(\frac{1}{2}\sqrt{\frac{2k}{m}}\frac{1}{v}\right) = \frac{2k}{m}\left(\frac{1}{2}\right)\left(\frac{1}{v}\right)$

$\Rightarrow \quad a = \frac{k}{mv}$

$\Rightarrow \quad F \propto \frac{1}{v}$

Hence, (B) and (D) are correct.

21. Motion of block is non-uniform circular motion, so, acceleration is not constant throughout.

At B:

$$v^2 = 2gR$$

$\Rightarrow \quad a = \frac{v^2}{R} = 2g$

Hence, (A) and (C) are correct.

22. $\vec{F} = -\vec{\nabla}U$

$\Rightarrow \quad \vec{F} = 7\hat{i} - 24\hat{j}$

$\Rightarrow \quad m\vec{a} = 7\hat{i} - 24\hat{j}$

$\Rightarrow \quad \vec{a} = \frac{7}{5}\hat{i} - \frac{24}{5}\hat{j}$

$\Rightarrow \quad |\vec{a}| = 5\text{ ms}^{-2}$

Also, $\vec{a}\cdot\vec{u} = 0$

$\Rightarrow \quad \vec{a} \perp \vec{u}$

Further we have

$$\vec{v} = \vec{u} + \vec{a}t$$

$\Rightarrow \quad \vec{v} = (14.4\hat{i} + 4.2\hat{j}) + \left(\frac{7}{5}\hat{i} - \frac{24}{5}\hat{j}\right)4$

$\Rightarrow \quad \vec{v} = (14.4\hat{i} + 4.2\hat{j}) + \left(\frac{28}{5}\hat{i} - \frac{96}{5}\hat{j}\right)$

$\Rightarrow \quad \vec{v} = (14.4\hat{i} + 4.2\hat{j}) + (5.6\hat{i} - 19.2\hat{j})$

$\Rightarrow \quad \vec{v} = 20\hat{i} - 15\hat{j}$

$\Rightarrow \quad |\vec{v}| = \sqrt{20^2 + 15^2} = 25\text{ ms}^{-1}$

Hence, (A), (B) and (C) are correct.

23. By Law of Conservation of Energy, we have

$$\frac{1}{2}mu^2 + 0 = 0 + mg\ell(1-\cos\theta)$$

Since $1-\cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$

$\Rightarrow \quad u^2 = 2g\ell\left[2\sin^2\left(\frac{\theta}{2}\right)\right]$

$\Rightarrow \quad \frac{\theta}{2} = \sin^{-1}\left(\frac{u}{2\sqrt{g\ell}}\right)$

$$\Rightarrow \quad \theta = 2\sin^{-1}\left(\frac{u}{2\sqrt{g\ell}}\right)$$

Hence, (B) and (C) are correct.

24. At the highest point, if T' and v' be the respective tension and velocity, then

$$mg + T' = \frac{mv'^2}{\ell}$$

Since $T' = 2mg$

$$\Rightarrow \quad \frac{mv'^2}{\ell} = mg + 2mg$$

$$v' = \sqrt{3g\ell}$$

By Law of Conservation of Energy

$$\frac{1}{2}mv'^2 + mg(2\ell) = \frac{1}{2}mv^2$$

where v is the velocity at the lowest point

$$\Rightarrow \quad v = \sqrt{7g\ell}$$

Hence, (B) and (D) are correct.

25. $W_{Sp} = U_i - U_f = -\frac{1}{2}k\left(x_2^2 - x_1^2\right)$

Since $W_{Sp} = \frac{1}{2}kx^2$

So, $x_2 = 0$ and $x_1 = x$ i.e., either the spring was initially stretched by a distance x and finally was in its natural length or the spring was initially compressed by a distance x and finally was in its natural length.

Hence, (A) and (C) are correct.

26. In equilibrium $F = 0$

Since, $F = -\frac{dU}{dr}$

$$\Rightarrow \quad \frac{dU}{dr} = 0$$

Either U is zero or it is a non-zero constant.
So, for equilibrium, we have

Either $F = 0$, $U = 0$

or $F = 0$, U is a constant (but not zero).

Hence, (A) and (C) are correct.

27. In simple harmonic motion, the body is accelerated at all points except at mean position.
At the starting point, spring force is zero. The only force is mg (downwards). Hence, acceleration is maximum, i.e., g (downwards). From mean position to initial position, retardation is due to forces, spring force and force of gravity. When spring is detached only gravity is left i.e., retardation has reduced and so the block rises to a height greater than from where it was released more height.

Hence, (A), (C) and (D) are correct.

28. $\vec{a} = \frac{\vec{F}}{m} = -\frac{1}{m}\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j}\right)$ $\quad \{m = 1 \text{ kg}\}$

$$\vec{a} = \frac{\vec{F}}{m} = -\left(3\hat{i} + 4\hat{j}\right) = \text{constant}$$

$$\Rightarrow \quad a_x = -3 \text{ ms}^{-2} \text{ and } a_y = -4 \text{ ms}^{-2}$$

$$\Rightarrow \quad a = \sqrt{a_x^2 + a_y^2} = 5 \text{ ms}^{-2} = \text{constant}$$

Hence, **(A)** is correct

$$v_x = -3t \text{ and } v_y = -4t$$

$$x = 6 + \frac{1}{2}a_x t^2 = 6 - \frac{3}{2}t^2$$

and $y = 4 + \frac{1}{2}a_y t^2 = 4 - 2t^2$

Particle crosses the y-axis when

$$x = 0 \text{ or } t = 2 \text{ s}$$

speed of particle at this moment

$$v = at = 5 \times 2 = 10 \text{ ms}^{-1}$$

co-ordinates of particle at $t = 1$ s are

$$(x, y) = (4.5, 2)$$

Hence **(C)** and **(D)** are also correct.

Hence, (A), (C) and (D) are correct.

29. Work done against friction on ice is zero and work done against friction on the road is $(\mu mg)\ell$. So, average work done is

$$\frac{0 + (\mu mg)\ell}{2} = (\mu mg)\frac{\ell}{2}$$

Thus, indicating that the effective length of the sledge that has to be dragged so that it just gets completely on the road is $\frac{\ell}{2}$.

Distance covered by the sledge on the road before coming to rest is $\frac{v_0^2}{2\mu g}$. $\quad \left\{\because 0^2 - v_0^2 = 2(-\mu g)s\right\}$

So total distance moved by the sledge is

$$\left(\frac{v_0^2}{2\mu g} + \frac{\ell}{2}\right)$$

Distance covered by the sledge on the road is

$$\ell-\left(\frac{v_0^2}{2\mu g}+\frac{\ell}{2}\right)=\left(\frac{v_0^2}{2\mu g}-\frac{\ell}{2}\right)$$

Hence, (B), (C) and (D) are correct.

30. In uniformly accelerated motion

$$v=u+at$$

and $v^2=u^2+2as$

$\Rightarrow \quad v=\sqrt{u^2+2as}$

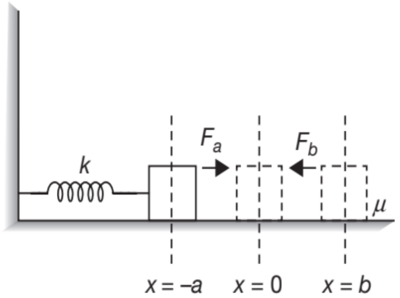
So, power, $P=Fv=F(u+at)$

also $P=F\sqrt{u^2+2as}$

i.e., power varies linearly with time and parabolically with displacement.
So, **(B)** and **(C)** are incorrect

Hence, (B) and (C) are correct.

31.



Work done by spring on block is

$$W=\frac{1}{2}ka^2-\frac{1}{2}kb^2=\frac{1}{2}k\left(a^2-b^2\right)$$

According to *MWET*, we have $W_{nc}=\Delta U+\Delta K$

Since $\Delta K=0$

$\Rightarrow \quad -\mu mg(a+b)=\frac{1}{2}k\left(b^2-a^2\right)$

$\Rightarrow \quad \mu mg(a+b)=\frac{1}{2}k\left(a^2-b^2\right)$

$\Rightarrow \quad \mu=\frac{k(a-b)}{2mg}$

Hence, (B) and (C) are correct.

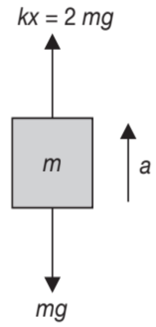
32. Work done by spring in parts (A) and (C) is $\frac{1}{2}kx^2$ and work done by spring in parts (B) and (D) is $-\frac{1}{2}kx^2$.

Hence, (A) and (C) are correct.

33. At maximum extension, x, we have

$$mgx=\frac{1}{2}kx^2$$

$\Rightarrow \quad x=\frac{2mg}{k}$



When $x'=\frac{x}{2}$, $F_{\text{net}}=0$

When $x'=x$, then

$$kx-mg=ma$$

$\Rightarrow \quad a=g$ (upwards)

Hence, (B), (C) and (D) are correct.

34. $\vec{F}=-\vec{\nabla}U=-\left(\hat{i}\frac{\partial U}{\partial x}+\hat{j}\frac{\partial U}{\partial y}+\hat{k}\frac{\partial U}{\partial z}\right)$

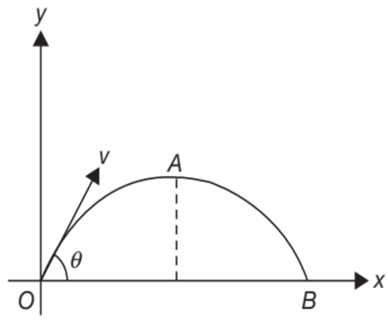
$\Rightarrow \quad \vec{F}=k\left(y\hat{i}+x\hat{j}\right)$

Work done by this force field is independent of the path followed between any two points and hence this force field is conservative in nature.

Hence, (A) and (C) are correct.

35. Power, $P=\vec{F}\cdot\vec{v}$

Between O and A angle between $\vec{F}(=m\vec{g})$ and $\vec{v}$ is greater than $90°$. Hence, power is negative. At A angle is $90°$. Hence, power is zero. From A to B angle is less than $90°$ therefore, power is positive.



At any instant, t

$$\vec{F}=-mg\,\hat{j}$$

$$\vec{v}=(u\cos\theta)\hat{i}+(u\sin\theta-gt)\hat{j}$$

$\Rightarrow \quad P=\vec{F}\cdot\vec{v}=mg(gt-u\cos\theta)$

$\Rightarrow \quad P=mg(gt-u\cos\theta)$

i.e., P varies linearly with t

Therefore, OPTIONS **(B)** and **(D)** are correct

Hence, (B) and (D) are correct.

36. Momentum, $p=\sqrt{2mK}$

Therefore, without knowing the mass we cannot compare the momentum.

From Work Energy Theorem, more work will be done to stop B.

Distance travelled before stopping is $s=\frac{v^2}{2a}$, where $a=\mu g$.

Again, distance cannot be compared unless v is known and for v mass should be known to us.

Hence, (B) and (D) are correct.

37. In region OA particle is accelerated, in region AB particle has uniform velocity while in region BC particle is decelerating. Therefore, work done is positive in region OA, zero in region AB and negative in region BC.

Hence, (B) and (C) are correct.

38. For $x>0$, $F=-\text{ve}$

For $x<0$, $F=+\text{ve}$

From $x=3$ to $x=1$, displacement is in negative direction.

From $x=-1$ to $x=-3$ also displacement is in negative direction.

Now work done is positive if force and displacement are parallel to each other and work done is negative if they are antiparallel.

Hence, (A) and (D) are correct.

39. $$\text{Power Loss}=\frac{\frac{1}{2}(dm)v^2}{dt}=\frac{1}{2}\frac{(A\,dx\,\rho)v^2}{dt}=\frac{1}{2}\rho A v^3$$

Further, Power $=Fv=\frac{1}{2}\rho A v^3$

$$\Rightarrow \quad F=\frac{1}{2}\rho A v^2$$

Hence, (A) and (C) are correct.

40. $Fx_0=\frac{1}{2}kx_0^2$

$$\Rightarrow \quad x_0=\frac{2F}{k}$$

Work done by applied force is

$$W=Fx_0$$

Hence, (C) and (D) are correct.

Reasoning Based Questions

1. Work done by force of gravity is path independent near the surface of Earth, for one reason that the force of gravity is constant near the surface of Earth. Also work done by force of gravity is path independent at greater height as the force of gravity is a conservative force.

Hence, the correct answer is (D).

2. According to Work-Energy Theorem, work done equals change in kinetic energy, so

$$W=Fx=\frac{1}{2}mv^2=\Delta K$$

Since for both the cars of different mass, ΔK will be different, so W is also different.

Hence, the correct answer is (D).

3. Since $m_{car}<m_{truck}$, so retardation on the car is more than the truck and hence the car will stop earlier in lesser distance.

Hence, the correct answer is (D).

4. $F=-\frac{dU}{dx}$

$F=0$ at points B and C.

Hence, the correct answer is (D).

5. At the topmost point, the velocity is $\sqrt{gR}\,(\neq 0)$, because to just complete the circle, velocity at lowest point is $\sqrt{5gR}$. At the highest point velocity is zero, however acceleration is non-zero equal to g.

Hence, the correct answer is (D).

6. $\frac{1}{2}mv^2=mgh_1$

$$\Rightarrow \quad h_1=\frac{v^2}{2g}$$

Also, $\frac{1}{2}(2m)v^2=(2m)gh_2$

$$\Rightarrow \quad h_2=\frac{v^2}{2g}$$

Hence, the correct answer is (A).

7. Work done and power developed is zero in uniform circular motion, because $\vec{F}\perp\vec{r}$. Also $P=\frac{W}{t}$, so Statement-2 is the correct explanation to Statement-1.

Hence, the correct answer is (A).

8. $W(mg)+W(sp)+W(Man)=0$

$$\Rightarrow \quad -\Delta U+E+W=0$$

$$\Rightarrow \quad \Delta U=E+W$$

Work done by spring force is positive when a compressed spring is released or stretched spring released.

Hence, the correct answer is (C).

9. By Law of Conservation of Momentum, we have

$$O = m_1v_1 - m_2v_2$$

and $K_1 + K_2 = E$ the energy released in explosion

$$\Rightarrow \frac{K_1}{K_2} = \frac{m_2}{m_1}$$

$$\Rightarrow P = \sqrt{2mE}$$

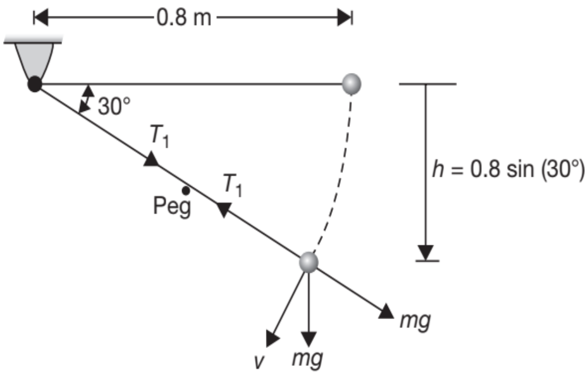
Hence, the correct answer is (C).

10. Statement-1 is false because normal force cannot do any work in the absence of any displacement. The Statement-2 is correct because an external force alone can shift the position of centre of mass.

Hence, the correct answer is (D).

Linked Comprehension Type Questions

1. Just before the cord comes in contact with the peg, we have



$$h = 0.8 \sin 30° = 0.4 \text{ m}$$

$$\Rightarrow v^2 = 2gh$$

So, $T_1 - mg\sin(30°) = \dfrac{mv^2}{R_1}$ $\{R_1 = 0.8 \text{ m}\}$

$$\Rightarrow T_1 = \frac{mg}{2} + \frac{m(2g)(0.4)}{0.8} = \frac{3mg}{2}$$

Hence, the correct answer is (C).

2. Just after the cord comes in contact with the peg, we have

$$h = 0.8\sin(30°) = 0.4$$

$$\Rightarrow v^2 = 2gh$$

So, $T_2 - mg\sin(30°) = \dfrac{mv^2}{R_2}$

$$\Rightarrow T_2 = \frac{mg}{2} + \frac{m(2g)(0.4)}{0.4}$$

$$\Rightarrow T_2 = \frac{5mg}{2}$$

Hence, the correct answer is (A).

3. At maximum extension (situation of momentary rest),

$$v_A = v_B = zero$$

By Law of Conservation of Energy

$$\begin{pmatrix}\text{Loss of Gravitational}\\ \text{Potential Energy}\\ \text{of block B}\end{pmatrix} = \begin{pmatrix}\text{Gain in Elastic}\\ \text{Potential Energy}\\ \text{of the spring}\end{pmatrix}$$

$$\Rightarrow (2m)gx_{max} = \frac{1}{2}kx_{max}^2$$

$$\Rightarrow x_{max} = \frac{4mg}{k}$$

Hence, the correct answer is (D).

4. At $x = \dfrac{x_{max}}{2} = \dfrac{2\text{ mg}}{\text{k}}$, we have

$v_A = v_B = v$ (say)

{Because A and B are connected by a rigid string}

Then by Law of Conservation of Energy

$$\begin{pmatrix}\text{Decrease}\\ \text{in GPE}\\ \text{of B}\end{pmatrix} = \begin{pmatrix}\text{Increase in}\\ \text{EPE of}\\ \text{the spring}\end{pmatrix} + \begin{pmatrix}\text{Increase}\\ \text{in KE of}\\ \text{both blocks}\end{pmatrix}$$

$$\Rightarrow (2m)gx = \frac{1}{2}kx^2 + \frac{1}{2}(m+2m)v^2$$

$$\Rightarrow (2mg)\left(\frac{2mg}{k}\right) = \frac{1}{2}k\left(\frac{2mg}{k}\right)^2 + \frac{1}{2}(3m)v^2$$

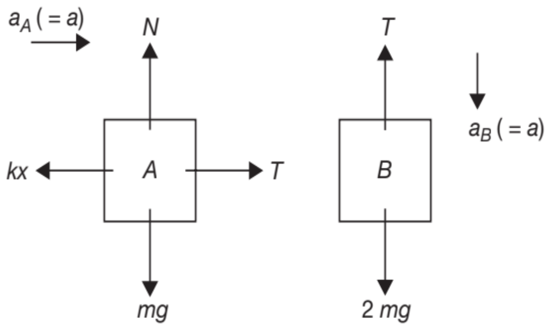
$$\Rightarrow \frac{8m^2g^2}{k} = \frac{4m^2g^2}{k} + 3mv^2$$

$$\Rightarrow 3mv^2 = \frac{4m^2g^2}{k}$$

$$\Rightarrow v = 2g\sqrt{\frac{m}{3k}}$$

Hence, the correct answer is (D).

5. At $x = \dfrac{x_{max}}{4} = \dfrac{mg}{k}$, we draw the free body diagrams for the blocks A and B, below



Since A and B are connected with a rigid thread, so at any instant $a_A = a_B = a$ (say)

$\Rightarrow \quad T - kx = ma$

$\Rightarrow \quad T - mg = ma \quad \ldots(1) \quad \left\{\because x = \frac{mg}{k}\right\}$

Also, $2mg - T = 2ma \quad \ldots(2)$

Solving these two equations, we get

$$mg = 3ma$$

$$\Rightarrow \quad a = \frac{g}{3}$$

Hence, the correct answer is (C).

6. By Law of Conservation of Energy

$$\frac{1}{2}mv^2 = mgH - mgh$$

$$\Rightarrow \quad v = \sqrt{2g(H-h)}$$

Hence, the correct answer is (D).

7. From concepts of projectile motion, we have

$$h = \frac{1}{2}gt^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2h}{g}}$$

Hence, the correct answer is (B).

8. $s = vt$

$$\Rightarrow \quad s = \sqrt{2g(H-h)}\sqrt{\frac{2h}{g}}$$

$$\Rightarrow \quad s = 2\sqrt{h(H-h)} \quad \ldots(1)$$

For s to be MAXIMUM, s^2 has to be MAXIMUM.

So, $\frac{d(s^2)}{dh} = 0$

$$\Rightarrow \quad \frac{d}{dh}[4h(H-h)] = 0$$

$$\Rightarrow \quad H - 2h = 0$$

$$\Rightarrow \quad h = \frac{H}{2}$$

Hence, the correct answer is (B).

9. $s_{max} = 2\sqrt{\left(\frac{H}{2}\right)\left(\frac{H}{2}\right)}$

$$\Rightarrow \quad s_{max} = H$$

Hence, the correct answer is (D).

10. Total height $h = 2 + \frac{u_B^2 \sin^2\theta}{2g}$

$$\Rightarrow \quad h = 2 + \frac{2g(h_A - h_B)\sin^2\theta}{2g}$$

$$\Rightarrow \quad h = 2 + 4\sin^2\theta$$

We can see that $h = 6$ m only when $\theta = 90°$

Hence, the correct answer is (B).

11. Since, $h = 2 + 4\sin^2\theta$, so at $\theta = 30°$, $h = 3$ m

Hence, the correct answer is (A).

12. $dW = \vec{F}\cdot d\vec{\ell}$

$$\Rightarrow \quad dW = k\left(x^2y^3dx + x^3y^2dy\right)$$

$$\Rightarrow \quad dW = k\left[y^3 d\left(\frac{x^3}{3}\right) + x^3 d\left(\frac{y^3}{3}\right)\right]$$

$$\Rightarrow \quad dW = k\left[d\left(\frac{x^3y^3}{3}\right)\right]$$

$$\Rightarrow \quad W = \int dW = \frac{k}{3}\left(x^2y^3\right)$$

$$W_{(0,\,0)\to(a,\,0)} = \frac{k}{3}\left(x^3y^3\right)\Big|_{(0,\,0)}^{(a,\,0)} = \frac{24}{3}(0-0) = \text{zero}$$

Hence, the correct answer is (A).

13. $W_{(a,0)\to(a,\,a)} = \frac{k}{3}\left(x^3y^3\right)\Big|_{(a,0)}^{(a,\,a)} = \frac{9}{3}\left(a^6 - 0\right) = 3a^6$

Hence, the correct answer is (C).

14. $W_{(0,0)\to(a,\,a)} = \frac{k}{3}\left(x^3y^3\right)\Big|_{(0,\,0)}^{(a,\,a)} = \frac{k}{3}\left(a^6 - 0\right) = \frac{ka^6}{3}$

Hence, the correct answer is (C).

15. Since we observe that the work done in all the three paths is same i.e., $\frac{ka^6}{3}$. So, work done due to this force is independent of the path followed between $(0,\ 0)$ to $(a,\ a)$. Hence $\vec{F}$ is a conservative force.

Hence, the correct answer is (A).

16. $W_{mg} = mgh\cos 180°$

$$\Rightarrow \quad W_{mg} = (2)(10)(2)(-1) = -40 \text{ J}$$

From Work Energy Theorem, we get

$$W_{mg} + W_{upthrust} = \Delta KE$$

$$\Rightarrow \quad -40 \text{ J} + W_{upthrust} = 16$$

$$\Rightarrow \quad W_{upthrust} = 56 \text{ J}$$

Hence, the correct answer is (D).

17. Let ρ be the density of ball, then

$$56 = (\text{upthrust force}) \times (\text{displacement}) \times \cos 0^\circ$$

$$\Rightarrow \quad 56 = \left(\frac{m}{\rho}\right)\rho_w g \times 2 = \frac{2}{\rho} \times 1000 \times 10 \times 2$$

$$\Rightarrow \quad \rho = \left(\frac{40}{56}\right) \times 10^3 \text{ kgm}^{-3}$$

$$\Rightarrow \quad \rho = \frac{5}{7} \times 10^3 \text{ kgm}^{-3}$$

Hence, the correct answer is (A).

18. At equilibrium r_0

$$F = \frac{dU}{dr} = 0$$

$$\Rightarrow \quad F|_{r=r_0} = \left.\frac{dU}{dr}\right|_{r=r_0} = -\frac{2A}{r_0^3} + \frac{B}{r_0^2} = 0$$

$$\Rightarrow \quad r_0 = \frac{2A}{B}$$

Hence, the correct answer is (B).

19. Since, $\frac{dU}{dr} = -\frac{2A}{r^3} + \frac{B}{r^2}$

$$\Rightarrow \quad \frac{d^2U}{dr^2} = \frac{6A}{r^4} - \frac{2B}{r^3}$$

$$\Rightarrow \quad \left.\frac{d^2U}{dr^2}\right|_{r_0 = \frac{2A}{B}} = \frac{6A}{\left(\frac{2A}{B}\right)^4} - \frac{2B}{\left(\frac{2A}{B}\right)^3}$$

$$\Rightarrow \quad \left.\frac{d^2U}{dr^2}\right|_{r_0 = \frac{2A}{B}} = \text{A POSITIVE VALUE}$$

So, U is minimum at equilibrium position r_0. So, the equilibrium is stable.

Hence, the correct answer is (A).

20. Work done by a conservative force equals the decrease in potential energy of the particle. So,

$$W = U_i - U_f = U_{\text{at equilibrium}} - U_{\text{at } \infty}$$

Now $U_i = U_{\text{at } r_0} = \frac{A}{r_0^2} - \frac{B}{r_0} = -\frac{B^2}{4A}$

and $U_\infty = 0$

So, work done by external agent will be $\frac{B^2}{4A}$, so that the particle is taken from equilibrium position to infinity.

Hence, the correct answer is (B).

21. For velocity to be zero, i.e., kinetic energy has to be zero, so the total energy must be purely potential.

$$\Rightarrow \quad E = -\frac{3B^2}{16A} = \frac{A}{r^2} - \frac{B}{r}$$

Also, we observe that at $r = \frac{2r_0}{3} = \frac{4A}{B}$, we have

$$E = \frac{A}{\left(\frac{4A}{B}\right)^2} - \frac{B}{\left(\frac{2A}{B}\right)}$$

$$\Rightarrow \quad E = \frac{B^2}{16A} - \frac{B^2}{2A}$$

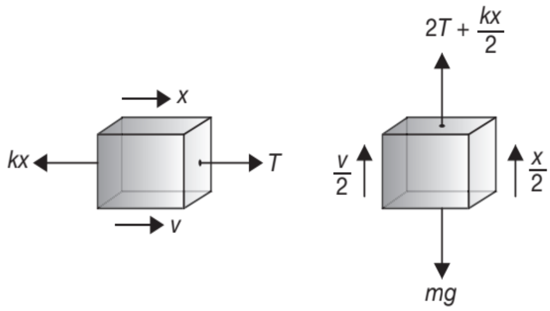
$$\Rightarrow \quad E = -\frac{3B^2}{16A}$$

Hence, the correct answer is (C).

22. In equilibrium $(F_{\text{net}} = 0)$

$$T = kx \qquad \ldots(1)$$

$$2T + \frac{kx}{2} = mg \qquad \ldots(2)$$



Solving these equations, we have

$$x = \frac{2mg}{5k} = \frac{2 \times 1 \times 10}{5 \times 10} = 0.4 \text{ m}$$

Hence, the correct answer is (B).

23. Let v be the speed of block placed horizontally, then by Law of Conservation of Energy, we have

$$\frac{1}{2}mv^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}kx^2 + \frac{1}{2}k\left(\frac{x}{2}\right)^2 = mg\left(\frac{x}{2}\right)$$

$$\Rightarrow \quad \frac{3}{8}mv^2 = \frac{mgx}{2} - \frac{3}{8}kx^2$$

$$\Rightarrow \quad v = \sqrt{\frac{8}{3}\left(\frac{gx}{2} - \frac{3kx^2}{8m}\right)}$$

Substituting the values, we have

$$v = \sqrt{\frac{8}{3}\left(\frac{10 \times 0.4}{2} - \frac{3 \times 10 \times 0.16}{8 \times 1}\right)}$$

$$\Rightarrow \quad v = 1.93 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

24. $KE = \frac{1}{2}(4)(20)^2$

$\Rightarrow \quad KE = 800\text{ J}$

Hence, the correct answer is (C).

25. Work done from OAB

Area $= \frac{1}{2}(4)(12) = 24\text{ J}$

Since the force is non-conservative from 0 to A, so KE must decrease and hence KE at $x = 4\text{ m}$ is $(800-24)\text{ J}$

$\Rightarrow \quad KE|_{x=4\text{ m}} = 776\text{ J}$

Hence, the correct answer is (C).

26. Gain in KE from BCDE is 60 J

So, new KE is $(776+60)\text{ J} = 836\text{ J}$

Hence, the correct answer is (B).

27. $A \to O$ is a non conservative force, so again negative work will be done while going back.

Hence, the correct answer is (C).

28. $a_t = g\sin 60° = 5\sqrt{3}\text{ ms}^{-2}$

$$a_n = \frac{v^2}{R} = 16\text{ ms}^{-2}$$

$\Rightarrow \quad a = \sqrt{a_t^2 + a_n^2} = 18.2\text{ ms}^{-2}$

Hence, the correct answer is (C).

29. Height from bottom at when $\theta = 60°$ is

$$h_1 = R(1-\cos 60°) = \frac{R}{2} = 0.5\text{ m}$$

Further height raised,

$$h_2 = \frac{v^2}{2g} = \frac{16}{20} = 0.8\text{ m}$$

Now since $h_1 + h_2 = 1.3\text{ m} > R$

The bob will rise to $\theta = 90°$, but there string will slack and total height will be less than 1.3 m.

Hence, the correct answer is (C).

30. $a = g\sin\theta - \mu g\cos\theta$

$$a = 10\times\frac{3}{5} - 0.5\times 10\times\frac{4}{5} = 2\text{ ms}^{-2}$$

Velocity of block at B or at C is

$$v = \sqrt{2as} = \sqrt{2\times 2\times\frac{h}{\sin\theta}} = \sqrt{\frac{20h}{3}}$$

For the block, not to leave contact anywhere

$$v \le \sqrt{2gR}$$

$\Rightarrow \quad \sqrt{\frac{20h}{3}} \le \sqrt{2gR}$

$\Rightarrow \quad h \le 3R$

$\Rightarrow \quad h \le 1.5\text{ m}$

Hence, the correct answer is (D).

31. Since, $g\sin\theta - \mu g\cos\theta = 2\text{ ms}^{-2}$

$\Rightarrow \quad g\sin\theta > \mu g\cos\theta$

Block will never stop on rough inclined surface. So, it will cross point C infinite number of times.

Hence, the correct answer is (A).

32. Potential energy of the projectile at any instant will be

$$U = \frac{1}{2}mv_0^2 - \frac{1}{2}mv^2$$

$\Rightarrow \quad \frac{dU}{dt} = 0 - mv\frac{dv}{dt}$

Since, $m = 1\text{ kg}$, $\frac{dv}{dt} = a_t = \frac{\vec{a}\cdot\vec{v}}{v}$ = tangential acceleration

$\Rightarrow \quad \left|\frac{dU}{dt}\right| = \vec{a}\cdot\vec{v}$

Hence, the correct answer is (B).

33. At height h, we have

$$K = \frac{1}{2}mv_0^2 - mgh$$

and $U = mgh$

$\Rightarrow \quad \frac{K}{U} = \frac{v_0^2}{2gh} - 1 = \text{constant}$

Hence, $\frac{K}{U}$ versus $\frac{1}{h}$ graph is a straight line not passing through the origin.

Hence, the correct answer is (B).

34. $\text{KE}|_{x=5} = \frac{1}{2}(2)(4)^2 = 16\text{ J}$

$\text{PE}|_{x=5} = 20 + (5-2)^2 = 29\text{ J}$

$\Rightarrow$ Total Energy = Mechanical Energy

So, $E = U + K = 16 + 29 = 45\text{ J}$

$\Rightarrow \quad E = 45\text{ J}$

Hence, the correct answer is (C).

35. KE possessed by the particle will be MAXIMUM, when PE is MINIMUM i.e. at $x = 2\text{ m}$

So, $\text{PE}|_{\text{MIN}} = 20\text{ J}$

$\Rightarrow$ $KE|_{MAX} = 45 - 20 = 25$ J

Hence, the correct answer is (B).

36. The particle will be in equilibrium i.e. $F = 0$

$$\Rightarrow \frac{dU}{dx} = 0$$

$$\Rightarrow x_0 - 2 = 0$$

$$\Rightarrow x_0 = 2 \text{ m}$$

Hence, the correct answer is (B).

37. Let lower spring compresses maximum by x metre, then by Law of Conservation of Mechanical Energy, we have

$$\begin{pmatrix}\text{Decreases in} \\ \text{potential energy} \\ \text{of block}\end{pmatrix} = \begin{pmatrix}\text{Increases in elastic} \\ \text{potential energy of} \\ \text{both the springs}\end{pmatrix}$$

$$\Rightarrow (2)(10)(x+1) = \frac{1}{2}\times 10 \times x^2 + \frac{1}{2}\times 10 \times (x+1)^2$$

$$\Rightarrow 20x + 20 = 5x^2 + 5x^2 + 5 + 10x$$

$$\Rightarrow 10x^2 - 10x - 15 = 0$$

$$\Rightarrow 2x^2 - 2x - 3 = 0$$

$$\Rightarrow x = \frac{2 \pm \sqrt{4+24}}{4} = 1.82 \text{ m}$$

So, maximum extension in the upper spring is

$$x_{max} = 1 + x = 2.82 \text{ m}$$

Hence, the correct answer is (C).

38. At equilibrium position net force on the block should be zero. So, let it is at distance y from where it was released.
Then,

$$mg = Ky + K(y-1)$$

$$\Rightarrow 20 = 10y + 10(y-1)$$

$$\Rightarrow 20y = 30$$

$$\Rightarrow y = 1.5 \text{ m}$$

Hence, the correct answer is (D).

39. Equation can be written as,

$$v^2 = (9 + 4s)$$

Comparing this with $v^2 = u^2 + 2as$, we get

$$u = 3 \text{ ms}^{-1} \text{ and } a = 2 \text{ ms}^{-1}$$

At $t = 0$, velocity is 3 ms^{-1} and at $t = 2$ s, velocity is 7 ms^{-1}. From Work Energy Theorem, we have

$$W = K_f - K_i = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{1}{2}(2)(49-9)$$

$$\Rightarrow W = 40 \text{ J}$$

Hence, the correct answer is (D).

40. $P_{av} = P_{inst}$

$$\Rightarrow \frac{W}{t} = Fv_{inst}$$

$$\Rightarrow \frac{1}{2}m\frac{\left(v_f^2 - v_i^2\right)}{t} = \frac{3}{4}(mav)$$

$$\Rightarrow (3+2t)^2 - (3)^2 = \frac{3}{2}atv$$

$$\Rightarrow 9 + 4t^2 + 12t - 9 = \frac{3}{2}(2)(t)(3+2t)$$

$$\Rightarrow 4t^2 + 12t = 9t + 6t^2$$

$$\Rightarrow 2t^2 = 3t$$

$$\Rightarrow t = 1.5 \text{ s}$$

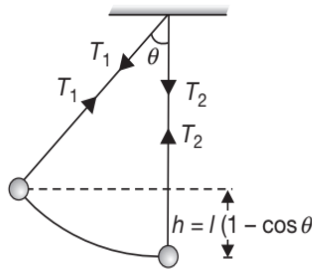
Hence, the correct answer is (A).

41. $T_1 = mg\cos\theta$ ...(1)

$$T_2 - mg = \frac{mv^2}{\ell} = \frac{m}{\ell}(2gh)$$

$$\Rightarrow T_2 - mg = \frac{2mg}{\ell}(1-\cos\theta)\ell$$

$$\Rightarrow T_2 - mg = 2mg(1-\cos\theta)$$



$\Rightarrow T_2 = mg + 2mg(1-\cos\theta)$ ...(2)

Given $T_2 = 2T_1$

$$\Rightarrow mg + 2mg(1-\cos\theta) = 2mg\cos\theta$$

$$\Rightarrow \cos\theta = \frac{3}{4}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{3}{4}\right)$$

Hence, the correct answer is (D).

42. Tension is maximum at bottommost point, so

$$a_{max} = \frac{2mg - mg}{m} = g$$

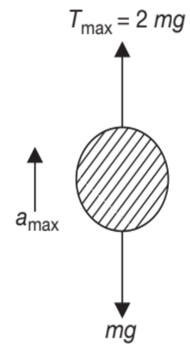
$$\Rightarrow \quad \frac{v^2}{\ell} = g$$

$$\Rightarrow \quad \frac{2\ell(1-\cos\theta)}{\ell} = 1$$

$$\Rightarrow \quad 2 - 2\cos\theta = 1$$

$$\Rightarrow \quad 2\cos\theta = 1$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$



Hence, the correct answer is (D).

43. Let t be the time taken by the ball from B to C.

$$R = \frac{1}{2}gt^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2R}{g}}$$

Since, $x = vt$

$$\Rightarrow \quad 2R = v\sqrt{\frac{2R}{g}}$$

$$\Rightarrow \quad v = \sqrt{2gR}$$

Applying Energy Conservation at points A and B, we get

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mg(2R)$$

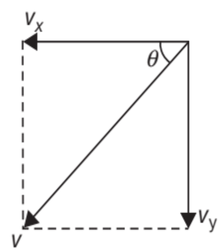
$$\Rightarrow \quad v_0 = \sqrt{6gR}$$

Hence, the correct answer is (D).

44. $v_y = gt$

$$\Rightarrow \quad v_y = g\sqrt{\frac{2R}{g}} = \sqrt{2gR}$$

and $v_x = \sqrt{2gR}$



$$\tan\theta = \frac{v_y}{v_x} = 1$$

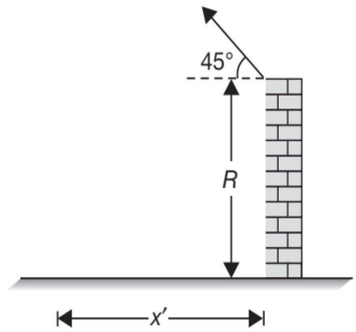
$$\Rightarrow \quad \theta = 45°$$

Hence, the correct answer is (B).

45. Since all surfaces are frictionless, so collision is elastic, hence

$$v' = \sqrt{v_x^2 + v_y^2} = \sqrt{2gR + 2gR}$$

$$\Rightarrow \quad v' = 2\sqrt{gR}$$



Since, $-R = v'\sin(45°)t - \frac{1}{2}gt^2$

$$\Rightarrow \quad t = \sqrt{\frac{2R}{g}}\left(1+\sqrt{2}\right)$$

$$x' = \left(v'\cos(45°)\right)t$$

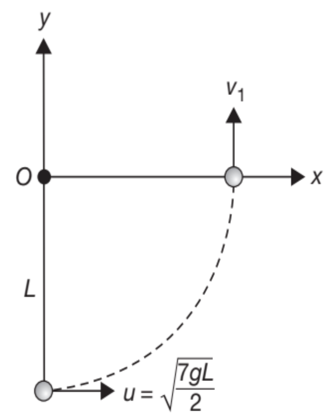
$$\Rightarrow \quad x' = 2\left(1+\sqrt{2}\right)R$$

Hence, the correct answer is (C).

46. By Law of Conservation of Energy, we get

$$\frac{1}{2}m\frac{7gL}{2} = mgL + \frac{1}{2}mv_1^2$$

$$\Rightarrow \quad 7gL = 4gL + 2v_1^2$$



$$\Rightarrow \quad v_1^2 = \frac{3}{2}gL$$

$$\Delta\vec{v} = \vec{v}_1 - \vec{u}$$

$$\Rightarrow \quad \Delta\vec{v} = \left(\sqrt{\frac{3}{2}gL}\right)\hat{j} - \left(\sqrt{\frac{7}{2}gL}\right)\hat{i}$$

$$\Rightarrow \quad |\Delta\vec{v}| = \sqrt{\frac{3}{2}gL + \frac{2gL}{2}}$$

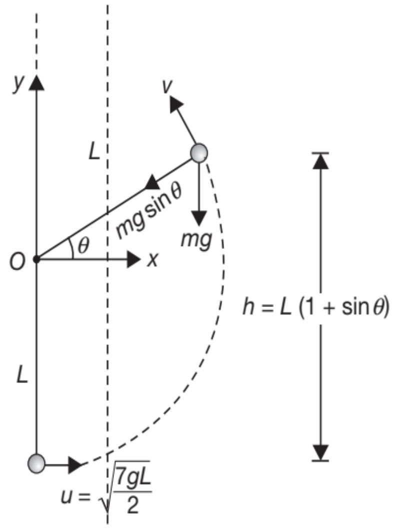
$$\Rightarrow \quad |\Delta\vec{v}| = \sqrt{5gL}$$

Hence, the correct answer is (D).

47. Since $\frac{1}{2}m\left(\frac{7gL}{2}\right) = mgL(1+\sin\theta) + \frac{1}{2}mv^2$

$\Rightarrow \quad 7gL = 4gL(1+\sin\theta) + 2v^2$

$\Rightarrow \quad 2v^2 = gL(3-4\sin\theta) \qquad \ldots(1)$



Also, when the motion ceases to be circular, then $T = 0$

$\Rightarrow \quad \frac{mv^2}{L} = mg\sin\theta$

$\Rightarrow \quad v^2 = gL\sin\theta$

$\Rightarrow \quad 2gL\sin\theta = gL(3-4\sin\theta)$

$\Rightarrow \quad 6\sin\theta = 3$

$\Rightarrow \quad \theta = 30^\circ$

Hence, the correct answer is (A).

48. $v^2 = gL\sin(30^\circ) = \frac{gL}{2}$

$\Rightarrow \quad \vec{v} = -v\cos\theta\hat{i} + v\sin\theta\hat{j}$

$\Rightarrow \quad \vec{v} = \sqrt{\frac{gL}{2}}\left(-\sqrt{3}\hat{i} + \hat{j}\right)$

Hence, the correct answer is (C).

49. Work done by all the force = change in KE

Hence, the correct answer is (D).

50. Gravitational force is constant.

Hence, the correct answer is (C).

51. Work done by kinetic friction in this case is negative because force of friction is acting in the opposite direction of motion.

Hence, the correct answer is (C).

52. Velocity at B is

$v_B = \sqrt{2gh} = \sqrt{2g(2R)}$

$\Rightarrow \quad v_B = 2\sqrt{gR}$

The normal force N_B at B is

$N_B = \frac{mv_B^2}{R} = 4mg$

Hence, the correct answer is (C).

53. $v_C = \sqrt{2gh'} = \sqrt{2g(3R)}$

$\Rightarrow \quad v_C = \sqrt{6gR}$

The normal force N_C at C is $N_C = mg + \frac{mv_C^2}{R}$

$\Rightarrow \quad N_C = mg + \frac{m(6gR)}{R} = 7mg$

Hence, the correct answer is (D).

54. $v_D = \sqrt{2g(2R)} = 2\sqrt{gR}$

Let the final velocity when its stops be $v = 0$.

Retardation offered beyond D is

$a = -\left[g\sin(30^\circ) + \mu_k\cos(30^\circ)\right]$

$\Rightarrow \quad a = -\frac{g}{2}\left(1+\sqrt{3}\mu_k\right)$

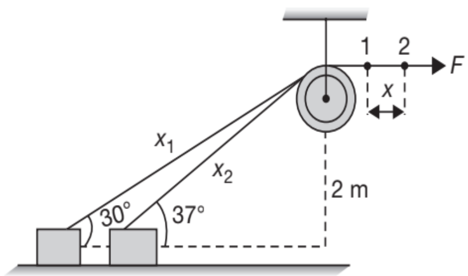
Since $v^2 - v_D^2 = 2ax$

$\Rightarrow \quad 0^2 - (4gR) = -\frac{2g}{2}\left(1+\sqrt{3}\mu_k\right)x$

$\Rightarrow \quad x = \frac{4R}{1+\sqrt{3}\mu_k}$

Hence, the correct answer is (C).

55.



Let us consider the motion of a point on the free portion of string on which F is being applied. Then

$x = x_1 - x_2$

$\Rightarrow \quad x = \frac{2}{\sin(30^\circ)} - \frac{2}{\sin(37^\circ)}$

$\Rightarrow \quad x = \frac{2}{\left(\frac{1}{2}\right)} - \frac{2}{\left(\frac{3}{5}\right)} = 4 - \frac{10}{3}$

$\Rightarrow \quad x = \frac{2}{3}$ m

So, work done is

$$W = Fx = 50\left(\frac{2}{3}\right) = \frac{100}{3} \text{ J}$$

Hence, the correct answer is (C).

56. From Work-Energy Theorem, we have

$$W = \Delta K$$

$$\Rightarrow \quad \frac{100}{3} = \frac{1}{2}mv^2 = 5v^2$$

$$\Rightarrow \quad v = \sqrt{\frac{20}{3}} \text{ ms}^{-1}$$

Hence, the correct answer is (C).

57. Initial Acceleration is $a_1 = \frac{F\cos(30°)}{m}$

Final Acceleration is $a_2 = \frac{F\cos(37°)}{m}$

So, ratio $\frac{a_1}{a_2} = \frac{\cos(30°)}{\cos(37°)}$

$$\Rightarrow \quad \frac{a_1}{a_2} = \frac{5\sqrt{3}}{8}$$

Hence, the correct answer is (B).

58. As the particle moves straight from $(0, 0)$ to (a, a), then it must move along the line

$$y = x$$

$$\Rightarrow \quad \frac{dy}{dx} = 1$$

$$\Rightarrow \quad dy = dx$$

Since, work done is

$$W = \int \vec{F} \cdot d\vec{\ell} = \int \left(y\hat{i} + x\hat{j}\right)\left(dx\hat{i} + dy\hat{j}\right)$$

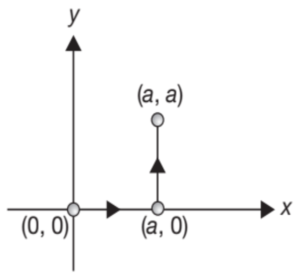
$$\Rightarrow \quad W = \int \left(ydx + xdy\right)$$

Taking $y = x$, $dy = dx$ and the limits from $(0, 0)$ to (a, a), we get

$$W = \int_0^a 2xdx = \left(x^2\right)\Big|_0^a = a^2$$

Hence, the correct answer is (A).

59. The path followed is shown in the Figure.



For the path between $(0, 0)$ to $(a, 0)$, $y = 0$, so $dy = 0$.

$$\Rightarrow \quad W_{(0,\,0)\to(a,\,0)} = W_1 = \int ydx + xdy = 0$$

For the path between $(a, 0)$ to (a, a), $x = a$, so $dx = 0$.

$$\Rightarrow \quad W_{(a,\,0)\to(a,\,a)} = W_2 = \int ydx + xdy$$

$$\Rightarrow \quad W_{(a,\,0)\to(a,\,a)} = W_2 = \int_0^a ady = a^2$$

So, the total work done is

$$W = W_1 + W_2 = a^2$$

Hence, the correct answer is (A).

60. Since, $dW = \vec{F} \cdot d\vec{\ell} = ydx + xdy$

Also, we know that $d(xy) = ydx + xdy$

$$\Rightarrow \quad dW = d(xy)$$

Integrating, we get

$$W = \int d(xy)$$

$$\Rightarrow \quad W = (xy)\Big|_{(x_1,\, y_1)}^{(x_2,\, y_2)}$$

$$\Rightarrow \quad W = x_2y_2 - x_1y_1$$

In this case, we observe that the work done by the force depends only on (x_1, y_1), the initial position and (x_2, y_2), the final position. So, the work done by the force depends only on initial and final values of x and y.

Also, when the object returns to the original position, then we have $(x_2, y_2) = (x_1, y_1)$, and hence

$$W = 0.$$

Hence, (A) and (D) are correct.

Matrix Match/Column Match Type Questions

1. A → (q)
B → (s)
C → (r)
(D) → (r)

Since $W_{\text{total}} = \Delta K$ and $W_C = -\Delta U$

and $W_{\text{ext}} = \Delta U + \Delta K = (U_f + K_f) - (U_i + K_i)$

2. A → (s)
B → (r)
C → (p)
D → (q)

$v_B^2 = u_A^2 - 2gh_{AB} = (9g\ell) - (2g\ell) = 7g\ell$

$\Rightarrow \quad v_B = \sqrt{7g\ell}$

$\Rightarrow \quad \dfrac{v_B^2}{g\ell} = 7$

Further, $T_B = \dfrac{mv_B^2}{\ell} = 7mg$

$\Rightarrow \quad \dfrac{T_B}{2mg} = \dfrac{7}{2}$

Again, $v_C^2 = v_A^2 - 2gh_{AC} = (9g\ell) - 2g(2\ell) = 5g\ell$

$\Rightarrow \quad v_C = \sqrt{5g\ell}$

$\Rightarrow \quad \dfrac{v_C^2}{g\ell} = 5$

Further, $T_C + mg = \dfrac{mv_C^2}{\ell}$

$\Rightarrow \quad T_C = 4mg$

$\Rightarrow \quad \dfrac{T_C}{mg} = 4$

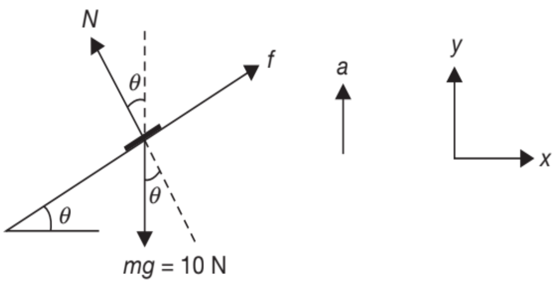
3. A → (t)
B → (p)
C → (s)
D → (q)

At $t = 4$ s,

$v = at = 8 \text{ ms}^{-1}$

and $s = \dfrac{1}{2}at^2 = 16m$

$KE = \dfrac{1}{2}mv^2 = 32$ J



From Work-Energy Theorem, we have

Work done by all the forces $= \Delta KE = 32$ J

Work done by gravity

$= -mgh = -(1)(10)(16) = -160$ J

Writing equation of motion, we have, $\sum F_y = ma$

$N\cos 30° + f\sin 30° - 10 = ma = 2$

$\Rightarrow \quad \sqrt{3}\text{ N} + f = 24 \qquad \ldots(1)$

$\sum F_x = 0$

$\Rightarrow \quad N\sin 30° = f\cos 30°$

$\Rightarrow \quad N = \sqrt{3}f \qquad \ldots(2)$

Solving equations (1) and (2), we have

$f = 6$ N

and $N = \dfrac{18}{\sqrt{3}} = 6\sqrt{3}$ N

So, $W_N = (N\cos\theta)(s) = (6\sqrt{3})\left(\dfrac{\sqrt{3}}{2}\right)(16) = 144$ J

and $W_f = (f\sin\theta)(s) = (6)\left(\dfrac{1}{2}\right)(16) = 48$ J

4. A → (r)
B → (q)
C → (p)
D → (t)

$v_f - v_i =$ area under a-x graph $= 12 \text{ ms}^{-1}$

$\Rightarrow \quad v_f = 12 + 4 = 16 \text{ ms}^{-1}$

$\Rightarrow \quad \Delta KE = \dfrac{1}{2}m\left(v_f^2 - v_i^2\right) = 120$ J

Work done by all the forces $= \Delta KE = 120$ J

$K_f = \dfrac{1}{2}mv_f^2 = 128$ J

Work done by conservative forces $= U_i - U_f = 240$ J

Work done by external force i.e. W_{ext} is given by

$$W_{ext} = \begin{pmatrix}\text{Total}\\ \text{work}\\ \text{done}\end{pmatrix} - \begin{pmatrix}\text{Work done by}\\ \text{conservative}\\ \text{forces}\end{pmatrix}$$

$$\Rightarrow \quad W_{ext} = -112 \text{ J}$$

5. $A \to (p, q)$
$B \to (t)$
$C \to (r)$
$D \to (s)$

Angle between net force and the string can never be obtuse. It is $90°$ at A, $0°$ at B and acute in between.

6. $A \to (p, q, r)$
$B \to (p, q, r)$
$C \to (q, r)$
$D \to (p, r)$

Conceptual

7. $A \to (r)$
$B \to (p)$
$C \to (q, s)$
$D \to (q, s)$

Conceptual

8. $A \to (p, s)$
$B \to (q, r)$
$C \to (p, s)$
$D \to (q, r)$

Conceptual

9. $A \to (s)$
$B \to (r)$
$C \to (p)$
$D \to (q)$

$$\frac{v^2}{R} = 9t^2$$

$$\Rightarrow \quad v^2 = (16)(9t^2)$$

$$\Rightarrow \quad v = 12t$$

$$\Rightarrow \quad a_t = \frac{dv}{dt} = 12$$

$$\Rightarrow \quad F_t = ma_t = \frac{1}{2}(12) = 6 \text{ N}$$

At $t = 1$ s, $a_n = \frac{v^2}{R} = 9$

$$\Rightarrow \quad F_n = ma_n = \frac{9}{2} \text{ N}$$

$$\Rightarrow \quad F = \sqrt{F_t^2 + F_n^2} = \sqrt{36 + \frac{81}{4}}$$

$$\Rightarrow \quad F = \sqrt{\frac{225}{4}} = \frac{15}{2} = 7.5 \text{ N}$$

$$P = \vec{F} \cdot \vec{v}$$

$$\Rightarrow \quad P = (F_t)(v)$$

$$\Rightarrow \quad P = (6)(12) = 72 \text{ watt}$$

$$P_{av} = \frac{W}{t}$$

$$\Rightarrow \quad P_{av} = F_t(v_{av})$$

But $v_{av} = \dfrac{\int_0^1 v\,dt}{\int_0^1 dt} = \dfrac{12\int_0^1 t\,dt}{1}$

$$\Rightarrow \quad v_{av} = \left.\frac{12t^2}{2}\right|_0^1 = 6 \text{ ms}^{-1}$$

$$\Rightarrow \quad P_{av} = (6)(6) = 36 \text{ watt}$$

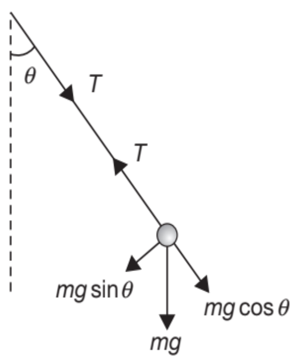
10. $A \to (s)$
$B \to (r)$
$C \to (q)$
$D \to (p)$

Just after the vehicle is stopped, the bob swings out to an angle of $60°$. So, by Law of Conservation of Mechanical Energy, we have

$$mgh = \frac{1}{2}mv_0^2$$

$$\Rightarrow \quad \frac{1}{2}mv_0^2 = mg\ell(1 - \cos\theta)$$

$$\Rightarrow \quad v_0 = \sqrt{2(10)(5)\left(1 - \frac{1}{2}\right)} = 5\sqrt{2} \text{ ms}^{-1}$$



So, net force on the bob at the lowest point is

$$F = \frac{mv_0^2}{\ell} = \frac{(2)(50)}{5} = 20 \text{ N}$$

Acceleration of the bob at lowest point is

$$a = \frac{F}{m} = \frac{v_0^2}{\ell} = 10\ \text{ms}^{-2}$$

At the highest point, net force is

$$F = mg\sin(60^\circ) = 10\sqrt{3}\ \text{N}$$

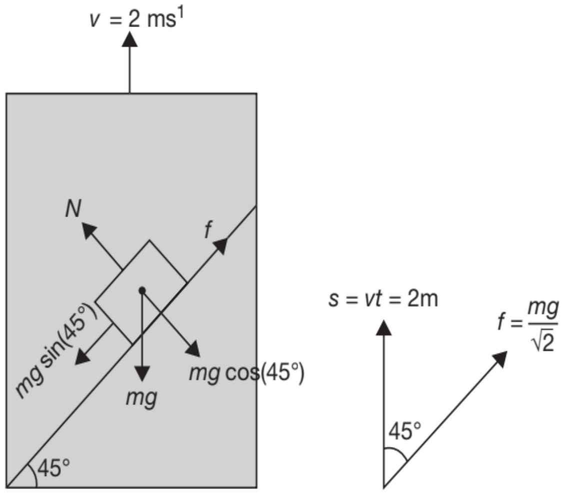
So, acceleration at highest point is

$$a = \frac{F}{m} = 5\sqrt{3}\ \text{ms}^{-2}$$

Integer/Numerical Answer Type Questions

1. Since the block does not slide on the wedge. So

$$f = mg\sin(45^\circ) = \frac{(1)(10)}{\sqrt{2}} = \frac{10}{\sqrt{2}}\ \text{N}$$



Now $W_f = \vec{f}\cdot\vec{s}$

$$\Rightarrow\ W_f = (f\cos 45^\circ)(2)$$

$$\Rightarrow\ W_f = \left(\frac{10}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)(2)$$

$$\Rightarrow\ W_f = 10\ \text{J}$$

2. Since, $P = Fv$

$$\Rightarrow\ F = \frac{P}{v} = \frac{8\times10^5}{20} = 4\times10^4\ \text{N}$$

At constant speed, the forces acting on the train are in equilibrium. Resolving the forces parallel to the hill

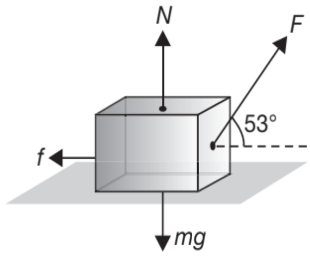
$$F = R + (2\times10^5)\times g\times\left(\frac{1}{50}\right)$$

$$\Rightarrow\ 4\times10^4 = R + 39200$$

$$\Rightarrow\ R = 800\ \text{N}$$

Therefore, the resistance is 800 N.

3. (a) The forces acting on the block are shown in the figure.



Clearly, $W_N = 0$ and $W_g = 0$, whereas $W_F = Fs\cos\theta$

$$W_f = -fs = -\mu_k Ns \text{ where } N = mg - F\sin\theta$$

According to Work Energy Theorem,

$$\Delta K = W_{net} = W_F + W_f$$

$$\Rightarrow\ \Delta K = Fs\cos\theta - \mu_k(mg - F\sin\theta)s$$

$$\Rightarrow\ \Delta K = (30)(2)(0.6) - \frac{1}{8}(40-24)(2) = 32\ \text{J}$$

(b) Now $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 32$ J

Since $v_i = 3\ \text{ms}^{-1}$

therefore, $v_f = 5\ \text{ms}^{-1} = 500\ \text{cms}^{-1}$

4. (a) By Law of Conservation of Energy, we get

$$0 + mg(4R) = \frac{1}{2}mv^2$$

$$\Rightarrow\ 4gR = \frac{v^2}{2}$$

At Q, we have

$$N = \frac{mv^2}{R} = \frac{m(8gR)}{R} = 8mg$$

So, net force at Q is

$$F_Q = \sqrt{N^2 + (mg)^2}$$

$$\Rightarrow\ F_Q = \sqrt{64m^2g^2 + m^2g^2} = \sqrt{65}mg$$

$$\Rightarrow\ x = 65$$

(b) For block to exert force (on the top of track) equal to weight, we have

$$N = mg$$

Since $mg + N = \frac{mv^2}{R}$

$$\Rightarrow\ \frac{mv^2}{R} = 2mg$$

$\Rightarrow \quad v^2 = 2Rg$

Applying Law of Conservation of Energy, we get

$$mg(h) = \frac{1}{2}m(2Rg) + mg(2R)$$

$\Rightarrow \quad h = 3R$

$\Rightarrow \quad * = 3$

5. Let m be the mass of B. From its free-body diagram

$$T - \mu N = m(0) = 0$$

where T is the tension in the string and $N = mg$

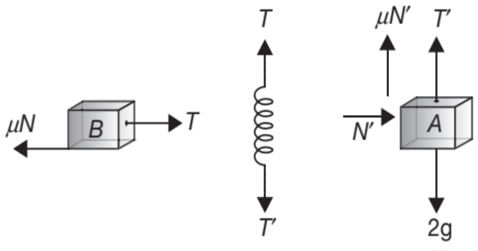
$\Rightarrow \quad T = \mu(mg)$

From the free-body diagram of the spring

$$T - T' = 0$$

where T' is the force exerted by A on the spring

$\Rightarrow \quad T = T' = \mu mg$



(a) (b) (c)

From the free-body diagram of A

$$2g - (T' + \mu N') = 2(0) = 0$$

where N' is the normal reaction of the vertical wall of C on A and $N' = 2(0) = 0$ (because there is no horizontal acceleration of A)

$\Rightarrow \quad 2g = T' = \mu mg$

$\Rightarrow \quad m = \frac{2g}{\mu g} = \frac{2}{0.2} = 10 \text{ kg}$

Tensile force on the spring is given by

$$T = T' = \mu mg = 0.2 \times 10 \times 9.8 = 19.6 \text{ N}$$

Now, for a spring, we have $F = kx$

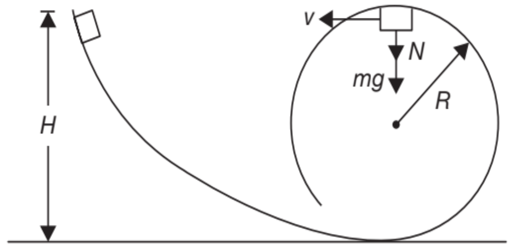
$\Rightarrow \quad 19.6 = 1960x$

$\Rightarrow \quad x = \frac{1}{100} \text{ m}$

$\Rightarrow$ Energy stored in the spring is given by

$$U = \frac{1}{2}kx^2 = \frac{1}{2} \times 1960 \times \left(\frac{1}{100}\right)^2 = 0.098 \text{ J}$$

6. Let v be the velocity of the particle at the highest point.



According to Newton's Second Law, the net force $F_{net} = N + mg$ provides the centripetal force to the body to revolve in a circle of radius R. So

$$N + mg = \frac{mv^2}{R}$$

The body does not fall at the uppermost point of the loop if $N \geq 0$. In the limiting case, $N = 0$. So,

$$mg = \frac{mv^2}{R}$$

$\Rightarrow \quad v^2 = gR \qquad \ldots(1)$

Applying Law of Conservation of Energy at the initial and highest point of the loop, we get

$$mgH = mg(2R) + \frac{1}{2}mv^2$$

From (1), $v^2 = gR$, so, we get

$$mgH = mg(2R) + \frac{1}{2}m(gR)$$

$\Rightarrow \quad H = \frac{5}{2}R = 2.5R$

Since $R = 40$ cm, we get

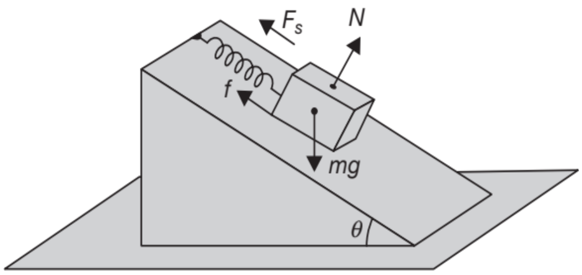
$\Rightarrow \quad H = (2.5)(40) = 100 \text{ cm}$

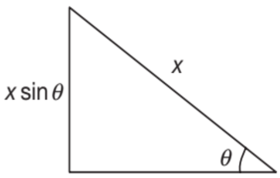
7. Since $W_{nc} + W_{ext} = \Delta U + \Delta K$

Let the block move down the incline through a distance x, then

$$fx\cos(180°) + 0 = \frac{1}{2}kx^2 + (-mgx\sin\theta) + \frac{1}{2}mv^2$$

$\Rightarrow \quad -(\mu_k mg\cos\theta)x = \frac{1}{2}kx^2 - mgx\sin\theta + \frac{1}{2}mv^2$





$\Rightarrow \quad -1 = 1 - 6 + v^2$

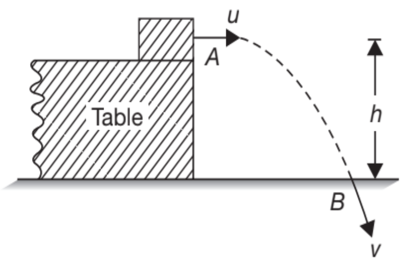
$\Rightarrow \quad v^2 = 4$

$\Rightarrow \quad v = 2\ \text{ms}^{-1}$

8. Let u be the speed of ice block when it leaves the table top. Then

$$\frac{1}{2}mu^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad u^2 = \left(\frac{k}{m}\right)x^2$$



If v be its speed when it reaches the floor, then By Law of Conservation of Energy, we have

$$(U+K)_{\text{at A}} = (U+K)_{\text{at B}}$$

$$\Rightarrow \quad \frac{1}{2}mu^2 + mgh = \frac{1}{2}mv^2 + 0$$

$$\Rightarrow \quad v = \sqrt{u^2 + 2gh} = \sqrt{\frac{kx^2}{m} + 2gh}$$

Now, $x = 5\ \text{cm} = 0.05\ \text{m}$

$k = 2400\ \text{Nm}^{-1}$

$m = 120\ \text{g} = 0.12\ \text{kg}$

$h = 2.5\ \text{m}$

$$\Rightarrow \quad v = \sqrt{\frac{(2400)(0.05)^2}{0.12} + 2(10)(2.5)}$$

$$\Rightarrow \quad v = \sqrt{50+50} = 10\ \text{ms}^{-1}$$

9. (a) Increase in thermal energy of block-floor system is the work done against friction.

$$\Rightarrow \quad W_{\text{friction}} = \mu_K mgs = (0.25)(4)(10)(8)$$

$$\Rightarrow \quad W_{\text{friction}} \approx 80\ \text{J}$$

(b) Maximum kinetic energy of the block is the mechanical energy of spring block system which actually is the energy dissipated as heat $= 80$ J

(c) $80 = \frac{1}{2}kx^2 = \frac{1}{2}(640)x^2$

$\Rightarrow \quad x = 0.5\ \text{m} = 50\ \text{cm}$

10. Let ℓ_0 be the unstretched length of the spring, then by Law of Conservation of Energy, we have

$$(U+K)_{\text{at A}} = (U+K)_{\text{at } B}$$

$$\frac{1}{2}k(3-\ell_0)^2 + \frac{1}{2}mv_A^2 = \frac{1}{2}k(5-\ell_0)^2 + \frac{1}{2}mv_B^2$$

But $v_B = 0$

$$\Rightarrow \quad (3-\ell_0)^2 + 4 = (5-\ell_0)^2$$

$$\Rightarrow \quad (9 + \cancel{\ell_0^2} - 6\ell_0) + 4 = (25 + \cancel{\ell_0^2} - 10\ell_0)$$

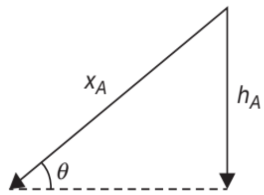
$$\Rightarrow \quad 16 - 4\ell_0 = 4$$

$$\Rightarrow \quad \ell_0 = 3\ \text{m}$$

11. Using our knowledge of constraint relations, we observe that at any instant $x_B = 2x_A$ and $v_B = 2v_A$. So, let $v_A = v$ after $x_A = 1\ \text{m}$. Using Work Energy Theorem for a non-conservative system, we get $W_{nc} = \Delta U + \Delta K$.

Before we make use of the above relation, we must observe that

(i) A move down by x_A, so its potential energy decreases by $m_A g h_A$ where $h_A = x_A \sin\theta = (1)\left(\frac{3}{5}\right) = \frac{3}{5}$ m.



(ii) B moves up by $x_B = h_B$, so its potential energy as well as kinetic energy increases.

$$\Rightarrow \quad f_K x_A \cos(180°) = \Delta U_A + \Delta K_A + \Delta U_B + \Delta K_B$$

where $f_K = \mu_K m_A g\cos\theta$, $\cos\theta = \frac{4}{5}$, $x_A = 1$ m, $x_B = 2$ m, $h_A = 0.6$ m, $m_A = 30$ kg, $m_B = 5$ kg, $g = 10\ \text{ms}^{-2}$.

$$\Rightarrow \quad -(\mu_K m_A g\cos\theta)x_A = -m_A g h_A + m_B g h_B + \frac{1}{2}m_A v^2 + \frac{1}{2}m_B(2v)^2$$

Substituting the values, we get

$$-\left(\frac{1}{5}\right)(300)\left(\frac{4}{5}\right)=-(300)\left(\frac{3}{5}\right)+(50)(2)+\frac{1}{2}(30)v^2+\frac{1}{2}(5)(2v)^2$$

$\Rightarrow \quad -48=-180+100+25\,v^2$

$\Rightarrow \quad v=\sqrt{\frac{32}{25}}=\frac{4\sqrt{2}}{5}$

$\Rightarrow \quad * = 2$

12. $h=\ell\sin\theta=2\sin\theta$

$v^2=v_0^2+2gh=25+2\times10\times2\sin\theta$

$\Rightarrow \quad v^2=25+40\sin\theta$

Further, $T-mg\sin\theta=\frac{mv^2}{\ell}$

Given, $T=2mg$

$\Rightarrow \quad 2mg-mg\sin\theta=\frac{mv^2}{\ell}$

$\Rightarrow \quad 2(10)-10\sin\theta=\frac{25+40\sin\theta}{2}$

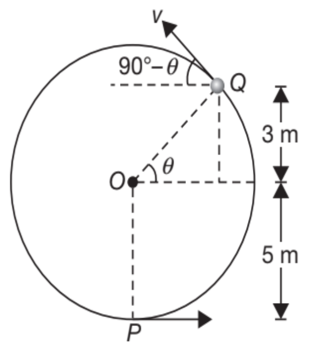
$\Rightarrow \quad \sin\theta=\frac{1}{4}$

$\Rightarrow \quad \theta=\sin^{-1}\left(\frac{1}{4}\right)$

So, $*=4$

13. $\sin\theta=\frac{3}{5}$

At Q, $T=0$ and hence the particle will become a projectile following a parabolic path.



$\Rightarrow \quad \frac{mv^2}{r}=mg\sin\theta$

$\Rightarrow \quad v=\sqrt{rg\sin\theta}=\sqrt{(5)(10)\left(\frac{3}{5}\right)}=\sqrt{30}\text{ ms}^{-1}$

$\Rightarrow \quad *=30$

The maximum height of the projectile is

$$H=\frac{v^2\sin^2(90^\circ-\theta)}{2g}=\frac{v^2\cos^2\theta}{2g}$$

$\Rightarrow \quad H=\frac{(30)\left(\frac{16}{25}\right)}{2(10)}=\frac{24}{25}$

$\Rightarrow \quad H=0.96\text{ m}=96\text{ cm}$

$\Rightarrow \quad H=24x=96$

$\Rightarrow \quad x=4$

14. Speed of each particle at angle θ using Law of Conservation of Energy is

$v=\sqrt{2gh}$, where $h=R(1-\cos\theta)$

$\Rightarrow \quad v=\sqrt{2gR(1-\cos\theta)}$

Also, at angle θ, we have

$N+mg\cos\theta=\frac{mv^2}{R}$

$\Rightarrow \quad N+mg\cos\theta=2mg(1-\cos\theta)$

$\Rightarrow \quad N=2mg-3mg\cos\theta \quad \ldots(1)$

When the tube just breaks its contact with the ground then

$2N\cos\theta=Mg$

Substituting the value of N from (1), we get

$4mg\cos\theta-6mg\cos^2\theta=Mg$

Substituting, $\theta=60^\circ$, we get

$2mg-\frac{3mg}{2}=Mg$

$\Rightarrow \quad \frac{m}{M}=2$

Conceptual Note(s)

Initially, the normal reaction on each ball will be radially outwards and later it will be radially inwards, so that the net normal reaction on the tube is radially outwards to break it off from the ground.

15. Let v be the speed of ball at point B. Then by Law of Conservation of Energy applied between A and B, we get

$U_A+K_A=U_B+K_B$

$\Rightarrow \quad 0+\frac{1}{2}(1)(5)^2=(1)(10)(1)+\frac{1}{2}(1)v^2$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{25}{2} - 10$$

$$\Rightarrow \quad v^2 = 5$$

Force acting on particle at B is

$$F = \sqrt{(mg)^2 + \left(\frac{mv^2}{r}\right)^2}$$

$$\Rightarrow \quad F = \sqrt{(10)^2 + \left(\frac{1\times 5}{1}\right)^2}$$

$$\Rightarrow \quad F = \sqrt{125}$$

$$\Rightarrow \quad F = 5\sqrt{5}\text{ N}$$

$$\Rightarrow \quad x = 5$$

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1. After reading the problem, we conclude that

for $0 \le x \le 15$, $F = 200$ N and

for $15 \le x < 30$, $F = 200 - \frac{100}{15}(x-15)$

$$\Rightarrow \quad F\ (\text{in N}) = \begin{cases} 200 & , \text{for } 0 \le x \le 15 \\ 300 - \frac{100}{15}x & , \text{for } 15 \le x < 30 \end{cases}$$

Since, $W = \int F dx$

$$\Rightarrow \quad W = \int_0^{15} 200 dx + \int_{15}^{30}\left(300 - \frac{100}{15}x\right)dx$$

$$\Rightarrow \quad W = (200)(15) + (300)(15) - \left(\frac{100}{15}\right)\left(\frac{(30^2 - 15^2)}{2}\right)$$

$$\Rightarrow \quad W = 3000 + 4500 - 2250 = 5250\text{ J}$$

Hence, the correct answer is (B).

2. Given that, $U = -\frac{A}{r^6} + \frac{B}{r^{12}}$

$$\Rightarrow \quad F = -\frac{dU}{dr} = -A(-6r^{-7}) + B(-12r^{-13})$$

At equilibrium, $F = 0$

$$\Rightarrow \quad 0 = \frac{6A}{r^7} - \frac{12B}{r^{13}}$$

$$\Rightarrow \quad \frac{6A}{12B} = \frac{1}{r^6}$$

$$\Rightarrow \quad r = \left(\frac{2B}{A}\right)^{1/6}$$

So, $U_{eq} = U|_{r=\left(\frac{2B}{A}\right)^{1/6}} = -\frac{A}{(2B/A)} + \frac{B}{(4B^2/A^2)}$

$$\Rightarrow \quad U_{eq} = -\frac{A^2}{2B} + \frac{A^2}{4B} = -\frac{A^2}{4B}$$

Hence, the correct answer is (C).

3. Since, $P = \text{constant}$. From Work-Energy Theorem

$$W = \frac{1}{2}mv^2$$

Also, we know that $W = Pt$

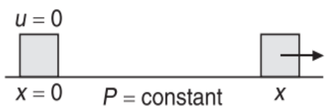
$$\Rightarrow \quad K = Pt = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \frac{ds}{dt} = \sqrt{\frac{2Pt}{m}} = \sqrt{\frac{2P}{m}}t^{1/2}$$

$$\Rightarrow \quad s = \int ds = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}dt = \frac{2}{3}\sqrt{\frac{2P}{m}}t^{3/2}$$

Hence, the correct answer is (D).

4.



Since, $P = \text{constant}$, so from Work-Energy Theorem, we get $W = \frac{1}{2}mv^2$. Also, $W = Pt$

$$\Rightarrow \quad K = Pt = \frac{1}{2}mv^2 \text{ i.e., } v = \sqrt{\frac{2Pt}{m}}$$

$$\Rightarrow \quad v = \frac{dx}{dt} = \sqrt{\frac{2Pt}{m}} = \sqrt{\frac{2P}{m}}t^{1/2}$$

$$\Rightarrow \quad x = \int dx = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}dt = \frac{2}{3}\sqrt{\frac{2P}{m}}t^{3/2}$$

$$\Rightarrow \quad x = \frac{2}{3}\sqrt{\frac{2(1)}{2}}(9)^{3/2} = \frac{2}{3}(1)(27) = 18\text{ m}$$

Hence, the correct answer is (18).

5. Since, $W_F = \frac{1}{2}mv^2 = mgh$

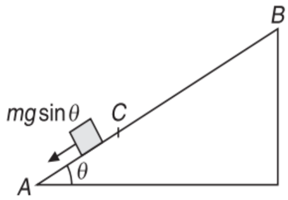
$$\Rightarrow \quad Fs = mgh$$

$$\Rightarrow \quad F(0.2) = (0.15)(10)(20)$$

$$\Rightarrow \quad F = 150\text{ N}$$

Hence, the correct answer is (150).

6. Applying Work-Energy Theorem, we get



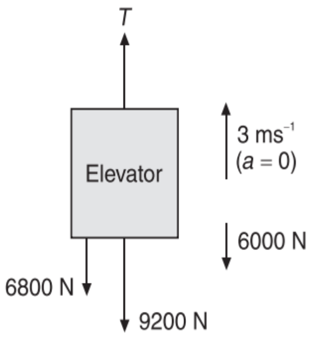
$$mg\sin\theta(AC+2AC)-(\mu mg\cos\theta)AC=0$$

$\Rightarrow$ $\mu=3\tan\theta$

$\Rightarrow$ $k=3$

Hence, the correct answer is (3).

7. Since the elevator is moving with a constant speed, so we have



$$T=6800+9200+6000=22000\text{ N}$$

So, power delivered by the motor is

$$P=Tv=22000\times3=66000\text{ W}$$

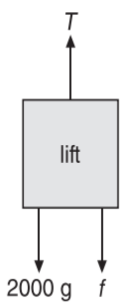
Hence, the correct answer is (C).

8. $W=\int_{\vec r_i}^{\vec r_g}\vec F\cdot d\vec r=\int_1^0-xdx+\int_0^1 ydy=\left(-\frac{x^2}{2}\right)\Big|_1^0+\left(\frac{y^2}{2}\right)\Big|_0^1$

$\Rightarrow$ $W=-\left(\frac{0^2}{2}-\frac{1^2}{2}\right)+\left(\frac{1^2}{2}-\frac{0^2}{2}\right)=1\text{ J}$

Hence, the correct answer is (B).

9. Let the elevator be moving up with a constant speed v. Then frictional force f will be downwards. So, tension in the cable is



$$T=2000g+f=2000+4000$$

$\Rightarrow$ $T=24000\text{ N}$

Power of the electric motor of the elevator is

$$P=Fv=Tv$$

$\Rightarrow$ $60\times746=(24000)v$

$\Rightarrow$ $v=\frac{60\times746}{24000}=1.865\approx1.9\text{ ms}^{-1}$

Hence, the correct answer is (1.9).

10. Applying Law of Conservation of Mechanical Energy between A and B, we get

$$U_A+K_A=U_B+K_B$$

$\Rightarrow$ $mg(2)=mg(1)+K_2$

$\Rightarrow$ $K_2=mg=10\text{ J}$

Hence, the correct answer is (10).

11. $W=\Delta K=$ Area under F-x graph

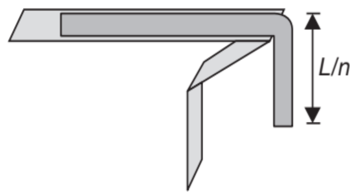
$\Rightarrow$ $\Delta K=W=\frac{1}{2}\times(3+2)\times(3-2)+2\times2=2.5+4$

$\Rightarrow$ $\Delta K=6.5\text{ J}$

Hence, the correct answer is (D).

12. Mass of hanging portion is

$$m_1=\frac{M}{n}$$



So, to pull the complete hanging portion up, the centre of mass of hanging portion has to be pulled through $\frac{L}{2n}$.

$\Rightarrow$ $W=\left(\frac{Mg}{n}\right)\left(\frac{L}{2n}\right)$

$\Rightarrow$ $W=\frac{MgL}{2n^2}$

Hence, the correct answer is (C).

13. Initial equilibrium position of block is at $x=0$

Speed of the block will be maximum at equilibrium position i.e., when $F=kx_{eq}$

$\Rightarrow$ $x_{eq}=\frac{F}{k}$

Applying Law of Conservation of Energy, we get

$$(U+K)_i=(U+K)_f$$

$$\Rightarrow \quad Fx_{eq}=\frac{1}{2}kx_{eq}^2+\frac{1}{2}mv^2$$

$$\Rightarrow \quad F\left(\frac{F}{k}\right)=\frac{1}{2}k\left(\frac{F}{k}\right)^2+\frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}mv^2=\frac{F^2}{2k}$$

$$\Rightarrow \quad v=\frac{F}{\sqrt{mk}}$$

Hence, the correct answer is (D).

14. $v=\frac{dx}{dt}=6t$

$u=v(t=0)=0$

$v=v(t=5\text{ s})=30\text{ ms}^{-1}$

$$\Rightarrow \quad W=\Delta KE=\frac{1}{2}(2)(30)^2=900\text{ J}$$

Hence, the correct answer is (B).

15. $N-mg=\frac{mg}{2}$

$$\Rightarrow \quad N=\frac{3mg}{2}$$

Since, $s=\frac{1}{2}\left(\frac{g}{2}\right)t^2$

So, work done by normal reaction is

$$W_N=Ns\cos(0°)=Ns$$

$$\Rightarrow \quad W=\left(\frac{3mg}{2}\right)\left(\frac{g}{4}t^2\right)$$

$$\Rightarrow \quad W=\frac{3mg^2t^2}{8}$$

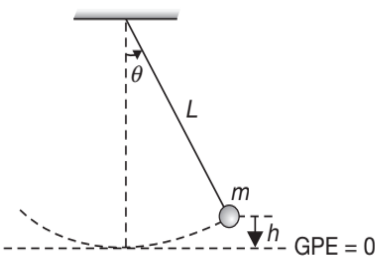
Hence, the correct answer is (A).

16. $W=K_f-K_i=(3\hat{i}-12\hat{j})\cdot 4\hat{i}=12\text{ J}$

$$\Rightarrow \quad K_f=12+K_i=12+3=15\text{ J}$$

Hence, the correct answer is (A).

17.



By Law of Conservation of Energy,

$$\begin{pmatrix}\text{Loss in GPE}\\ \text{of pendulum}\end{pmatrix}=\begin{pmatrix}\text{Gain in KE}\\ \text{of pendulum}\end{pmatrix}$$

$$\Rightarrow \quad mgh=\frac{1}{2}mv^2$$

$$\Rightarrow \quad K_1=mgh=mgL(1-\cos\theta)$$

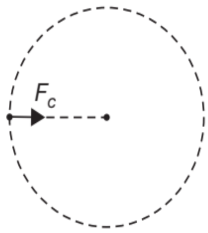
Now, when L is doubled, keeping θ same, then Kinetic energy is also doubled. So

$$K_2=2K_1$$

Hence, the correct answer is (A).

18. Force on particle in the given potential field is given as

$$\vec{F}=-\left(\frac{dU}{dr}\right)\hat{r}=\frac{k}{r^3}\hat{r}$$



For body to move in a circle, we have

$$F_C=\frac{k}{r^3}=\frac{mv^2}{r}$$

$$\Rightarrow \quad mv^2=\frac{k}{r^2}$$

$\Rightarrow$ The kinetic energy of particle is

$$KE=\frac{1}{2}mv^2=\frac{k}{2r^2}$$

Total energy of particle $TE=KE+PE$

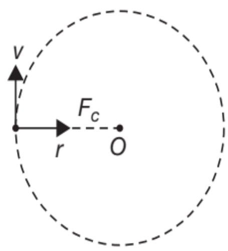
$$\Rightarrow \quad E=\frac{k}{2r^2}-\frac{k}{2r^2}=0$$

Hence, the correct answer is (C).

19. For particle to move at speed v in a circular orbit, we have

$$F_C=\left|\frac{16}{r}+r^3\right|=\frac{mv^2}{r}$$

$$\Rightarrow \quad mv^2=\left(\frac{16}{r}+r^3\right)(r)=16+r^4$$



So, kinetic energy of particle is

$$K=\frac{1}{2}mv^2=\frac{1}{2}(16+r^4)$$

Ratio of kinetic energy of the two particles is given by

$$\frac{K_1}{K_2}=\frac{\frac{1}{2}(16+(1)^4)}{\frac{1}{2}(16+(4)^4)}=\frac{16+1}{16+256}$$

$$\Rightarrow \frac{K_1}{K_2}=\frac{17}{272}=0.06$$

Hence, the correct answer is (B).

20. $v=a\sqrt{s}$

$$\Rightarrow \frac{ds}{dt}=a\sqrt{s}$$

$$\Rightarrow \int_0^s \frac{ds}{\sqrt{s}}=\int_0^t a\,dt$$

$$\Rightarrow 2\sqrt{s}=at$$

$$\Rightarrow s=\frac{a^2t^2}{4}$$

Velocity at any time t is given by

$$v=\frac{ds}{dt}=\frac{a^2t}{2}$$

Applying Work Energy theorem, we get

$$W=\Delta K$$

$$\Rightarrow W=\frac{1}{2}mv^2-0=\frac{1}{2}m\left(\frac{a^4t^2}{4}\right)=\frac{1}{8}ma^4t^2$$

Hence, the correct answer is (C).

21. Since, $\frac{K_f}{K_i}=\frac{\frac{1}{8}mv_0^2}{\frac{1}{2}mv_0^2}=\frac{1}{4}$

$$\Rightarrow \frac{v_f}{v_i}=\frac{1}{2}$$

$$\Rightarrow v_f=\frac{v_i}{2}=\frac{v_0}{2}$$

Since, $F=-kv^2=\frac{mdv}{dt}$

$$\Rightarrow \int_{v_0}^{\frac{v_0}{2}} \frac{dv}{v^2}=\int_0^{t_0}\frac{-kdt}{m}$$

$$\Rightarrow \left(-\frac{1}{v}\right)\Bigg|_{v_0}^{\frac{v_0}{2}}=-\frac{k}{m}t_0$$

$$\Rightarrow \frac{1}{v_0}-\frac{2}{v_0}=-\frac{k}{m}t_0$$

$$\Rightarrow -\frac{1}{v_0}=-\frac{k}{m}t_0$$

$$\Rightarrow k=\frac{m}{v_0t_0}$$

$$\Rightarrow k=\frac{10^{-2}}{10\times 10}$$

$$\Rightarrow k=10^{-4}\ \text{kgm}^{-1}$$

Hence, the correct answer is (C).

22. $F=6t=m\frac{dv}{dt}$ where $m=1$ kg

$$\Rightarrow \int_0^v dv=\int_0^1 6t\,dt$$

$$\Rightarrow v=6\left(\frac{t^2}{2}\right)\Bigg|_0^1$$

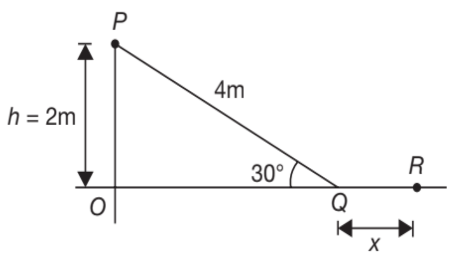
$$\Rightarrow v=3\ \text{ms}^{-1}$$

According to Work Energy Theorem, we have

$$W=\Delta KE=\frac{1}{2}(1)(9)=4.5\ \text{J}$$

Hence, the correct answer is (A).

23.



Energy lost over path PQ is

$$E_1=\mu mg\cos\theta\times 4$$

Energy lost over path QR is

$$E_2=\mu mgx$$

Since $E_1=E_2$

$$\Rightarrow \mu mgx=\mu mg\cos\theta\times 4$$

$$\Rightarrow x=\cos\theta\times 4=4\cos(30^\circ)$$

$$\Rightarrow x=2\sqrt{3}=3.45\ \text{m}$$

From Q to R energy loss is half of the total energy loss, so

$$\mu mgx = \frac{1}{2}(mgh)$$

$\Rightarrow \quad \mu = 0.29$

Hence, the correct answer is (D).

24. Work done against gravity in lifting the mass 1000 times is

$$W = (1000)(mgh)$$

$\Rightarrow \quad W = 10 \times 9.8 \times 10^3$

$\Rightarrow \quad W = 9.8 \times 10^4$ Joule

Let M kg of fat be used up in the process. Then energy supplied by fat is

$$E = (3.8 \times 10^7 \text{ M})\frac{20}{100} \text{ J}$$

This energy supplied by the fat must equal to the work done by him against gravity. So,

$$\frac{20}{100}(3.8 \times 10^7 \text{ M}) = 9.8 \times 10^4$$

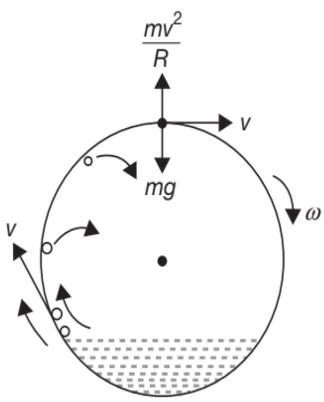
$\Rightarrow \quad M = 12.89 \times 10^{-3}$ kg

Hence, the correct answer is (A).

25. In the frame of rotating drum, at the topmost point the mixture will not loose contact with the drum wall if

$$\frac{mv^2}{r} \geq mg$$

$\Rightarrow \quad v \geq \sqrt{gR}$



If $v \geq \sqrt{gR}$, then proper mixing will not take place

Thus, for proper mixing $v < \sqrt{gR}$

$\Rightarrow \quad \omega = \frac{v}{R} = \sqrt{\frac{g}{R}} = \sqrt{\frac{10}{1.25}}$ rads^{-1}

ω in rpm is $\omega_{RPM} = \frac{60}{2\pi}\sqrt{\frac{10}{1.25}} \approx 27$ rpm

So, if then before reaching top, ingredients fall and mix proper.

Hence, the correct answer is (A).

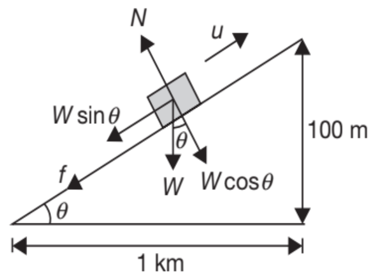
26. Inclination of road, $\theta = \tan^{-1}\left(\frac{100}{1000}\right) = \tan^{-1}\left(\frac{1}{10}\right)$

$\Rightarrow \quad \tan\theta = \frac{1}{10}$

$\Rightarrow \quad \tan\theta \approx \sin\theta = \frac{1}{10}$ (for very small value of θ)

When car is moving uphill with a speed of u=10 ms^{-1}, then

$$F = W\sin\theta + f$$



$\Rightarrow \quad P = Fu = (W\sin\theta + f)u$

$\Rightarrow \quad P = \left(\frac{W}{10} + \frac{W}{20}\right) \times 10$

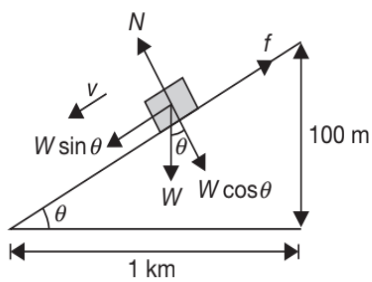
$\Rightarrow \quad P = \frac{3W}{20} \times 10 = \frac{3W}{2}$

When car is moving downhill with a constant speed, say v, then

$$F' = W\sin\theta - f$$

$\Rightarrow \quad \frac{P}{2} = F'v = (W\sin\theta - f)v$

$\Rightarrow \quad \frac{3W}{4} = \left(\frac{W}{10} - \frac{W}{20}\right)v$

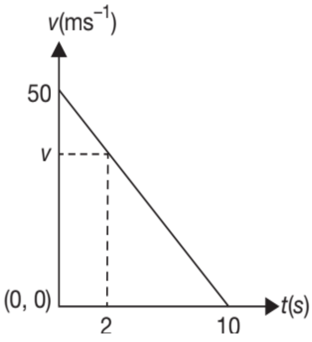


$\Rightarrow \quad \frac{3W}{4} = \frac{W}{20}v$

$\Rightarrow \quad v = 15$ ms^{-1}

Hence, the correct answer is (C).

27.



Given that, $m = 10$ kg, $t = 2$ s

$$u = 50 \text{ ms}^{-1} \text{ at } t = 0 \text{ s}$$

Since, $a = \dfrac{\Delta v}{\Delta t} = \dfrac{50-0}{0-10} = -5 \text{ ms}^{-2}$

So, speed of the body at $t = 2$ s is given by

$$v = u + at = 50 + (-5)\times 2 = 40 \text{ ms}^{-1}$$

Using Work-Energy theorem i.e. $W = \Delta K$, we get

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}(10)(40)^2 - \frac{1}{2}(10)(50)^2$$

$$\Rightarrow \quad W = 5(40^2 - 50^2) = 5\times(40-50)(40+50)$$

$$\Rightarrow \quad W = -4500 \text{ J}$$

Hence, the correct answer is (C).

28. Centripetal acceleration is $a_c = n^2Rt^2 = \dfrac{v_T^2}{R}$

$$\Rightarrow \quad v_T^2 = n^2R^2t^2$$

$$\Rightarrow \quad v_T = nRt$$

Tangential force on the particle is

$$F_T = M\frac{dv_T}{dt} = MnR$$

Power delivered to the particle is

$$P = F_Tv_T = (MnR)(nRt)$$

$$\Rightarrow \quad P = Mn^2R^2t$$

Hence, the correct answer is (B).

29. The block comes momentarily to rest means that its velocity at that point was also 3 ms^{-1}.

$$\begin{pmatrix}\text{Loss in EPE}\\ \text{of Spring}\end{pmatrix} = \begin{pmatrix}\text{Gain in KE}\\ \text{of Block}\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}k\left(x^2 - \frac{x^2}{4}\right) = \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}kx^2\left(1 - \frac{1}{4}\right) = \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}kx^2 = U_i = \frac{4}{6}mv^2 = \frac{2}{3}mv^2$$

$$\Rightarrow \quad U_i = \frac{2}{3}(0.1)(9) = 0.6 \text{ J}$$

Hence, the correct answer is (B).

30. Centripetal force $F = \alpha r^2$

$$\Rightarrow \quad \frac{mv^2}{r} = \alpha r^2$$

So, kinetic energy of the particle is

$$K = \frac{1}{2}mv^2 = \frac{\alpha r^3}{2}$$

Potential energy of the particle is

$$U = \int_0^r F dr = \int_0^r \alpha r^2 dr = \frac{\alpha r^3}{3}$$

Total energy of the particle is

$$E = K + U = \frac{\alpha r^3}{2} + \frac{\alpha r^3}{3}$$

$$\Rightarrow \quad E = \frac{5}{6}\alpha r^3$$

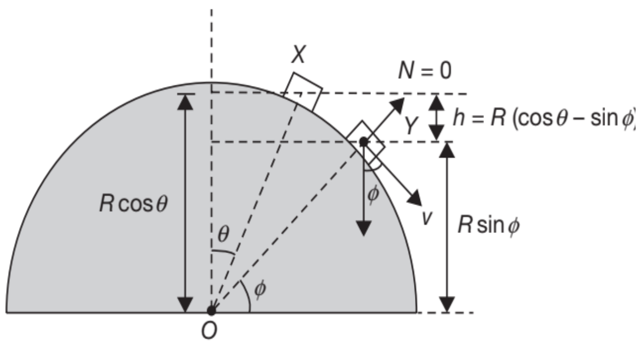
Hence, the correct answer is (D).

31. $W = \displaystyle\int_0^L F dx = \int_0^L (ax + bx^2)dx$

$$\Rightarrow \quad W = \frac{aL^2}{2} + \frac{bL^3}{3}$$

Hence, the correct answer is (C).

32.



Applying Energy Theorem, we get

$$mgh = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{2gR(\cos\theta - \sin\phi)}$$

At point Y, we have along radial direction in circular motion

$$\frac{mv^2}{R} = mg\sin\phi$$

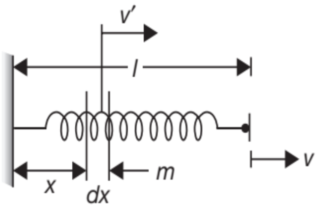
$$\Rightarrow \quad 2mg(\cos\theta - \sin\phi) = mg\sin\phi$$

$$\Rightarrow \quad 2\cos\theta = 3\sin\phi$$

Hence, the correct answer is (C).

33. To find KE of spring, we consider an elemental part of spring as shown.

Mass of element is $dm = \frac{m}{\ell}dx$



As spring is uniformly stretched, the speed v' of element is

$$v' = \left(\frac{v}{\ell}\right)x$$

Kinetic energy of element is $dK = \frac{1}{2}dmv'^2$

$$\Rightarrow \quad dK = \frac{1}{2}\left(\frac{m}{\ell}dx\right)\left(\frac{v}{\ell}x\right)^2$$

$$\Rightarrow \quad \int dK = \frac{1}{2}\frac{mv^2}{\ell^3}\int_0^\ell x^2dx$$

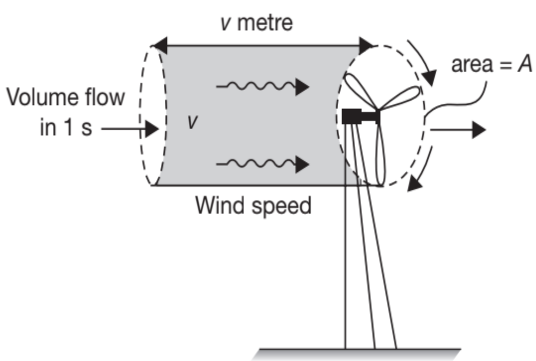
Total kinetic energy of spring is given as

$$K = \frac{1}{2}\frac{mv^2}{\ell^3}\left(\frac{\ell^3}{3}\right) = \frac{1}{6}mv^2$$

Hence, the correct answer is (D).

34. Wind volume flowing through the cross-sectional area of blades (rotating) of wind mill per unit time is given by

$$V = Av$$



If ρ be the density of air, then mass of wind intercepting the blades per second is

$$m = \rho Av$$

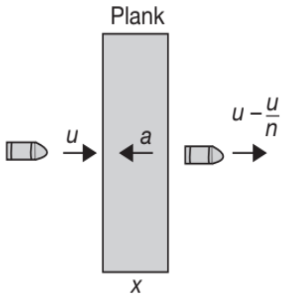
Kinetic energy of wind mass intercepting the section of blades per second is

$$\frac{dK}{dt} = \frac{1}{2}mv^2 = \frac{1}{2}\rho Av^3$$

$\Rightarrow$ Electrical power $\propto v^3$

Hence, the correct answer is (D).

35. If plank thickness is x, then, we have

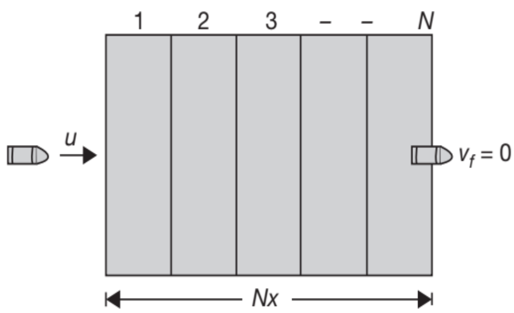


$$u^2\left(1 - \frac{1}{n}\right)^2 = u^2 - 2ax$$

$$\Rightarrow \quad 2ax = u^2\left[1 - \left(\frac{n-1}{n}\right)^2\right]$$

$$\Rightarrow \quad 2ax = u^2\left[\frac{2n-1}{n^2}\right]$$

Using constant retardation, we have



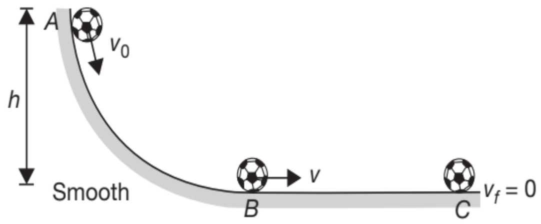
$$0 = u^2 - 2a(Nx)$$

$$\Rightarrow \quad u^2 = Nu^2\left[\frac{2n-1}{n^2}\right]$$

$$\Rightarrow \quad N = \frac{n^2}{2n-1}$$

Hence, the correct answer is (A).

36. Applying Work-Energy Theorem from point A to C, we get



$$\frac{1}{2}mv_0^2 + mgh - \mu mgL = 0$$

$$\Rightarrow \quad L = \frac{v_0^2}{2\mu g} + \frac{h}{\mu}$$

Hence, the correct answer is (B).

37. Maximum energy loss is

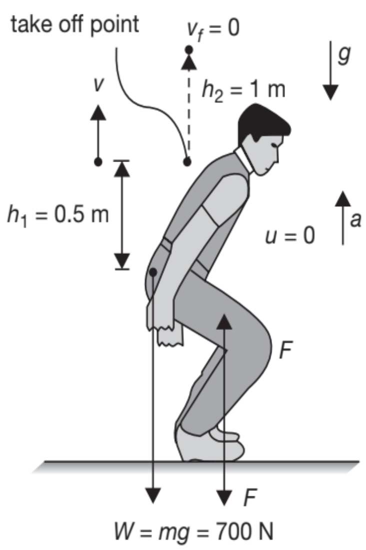
$$\frac{1}{2}\frac{Mm}{M+m}(v-0)^2 = \frac{M}{M+m}\left(\frac{1}{2}mv^2\right)$$

So, Statement-I is wrong.

Hence, the correct answer is (D).

38. If a is upward acceleration of man before leaping, then

$$v = \sqrt{2ah_1} = \sqrt{2gh_2}$$



$$\Rightarrow \quad a = \frac{gh_2}{h_1} = \frac{10\times 1}{0.5} = 20 \text{ ms}^{-2}$$

Velocity at take-off is

$$v = \sqrt{2\times 10\times 1} = \sqrt{20} \text{ ms}^{-1}$$

Power delivered by muscles is

$$P = Fv = mav$$

$$\Rightarrow \quad P = 70\times 20\times\sqrt{20}$$

$$\Rightarrow \quad P = 6.26\times 10^3 \text{ W}$$

Hence, the correct answer is (B).

39. For the same force, $F = k_1x_1 = k_2x_2$ ...(1)

Work done on spring S_1 is

$$W_1 = \frac{1}{2}k_1x_1^2 = \frac{(k_1x_1)^2}{2k_1} = \frac{F^2}{2k_1} \qquad \text{\{using (1)\}}$$

Work done on spring S_2 is

$$W_2 = \frac{1}{2}k_2x_2^2 = \frac{(k_2x_2)^2}{2k_2} = \frac{F^2}{2k_2} \qquad \text{\{using (1)\}}$$

$$\Rightarrow \quad \frac{W_1}{W_2} = \frac{k_2}{k_1}$$

Since, $W_1 > W_2$

$$\Rightarrow \quad k_2 > k_1$$

$$\Rightarrow \quad k_1 < k_2$$

Statement 2 is true.

For the same extension, i.e. for $x_1 = x_2 = x$ ...(2)

Work done on spring S_1 is $W_1 = \frac{1}{2}k_1x_1^2 = \frac{1}{2}k_1x^2$

Work done on spring S_2 is $W_2 = \frac{1}{2}k_2x_2^2 = \frac{1}{2}k_2x^2$

$$\Rightarrow \quad \frac{W_1}{W_2} = \frac{k_1}{k_2}$$

Since, $k_1 < k_2$

$$\Rightarrow \quad W_1 < W_2$$

Statement 1 is false.

Hence, the correct answer is (D).

40. By Work Energy Theorem

$$W = \vec{F}\cdot\Delta\vec{r} = \Delta K$$

$$\Rightarrow \quad \Delta K = (7\hat{i} + 4\hat{j} + 3\hat{k})\cdot(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\Rightarrow \quad \Delta K = 14 + 12 + 12 = 38 \text{ J}$$

Hence, the correct answer is (A).

41. As $x \to \infty$, $U(x) = 0$

At equilibrium, force on atoms is $F = 0$

$$\Rightarrow \quad F = -\frac{dU}{dx} = \frac{12a}{x^{13}} - \frac{6b}{x^7} = 0$$

$$\Rightarrow \quad x^6 = \frac{2a}{b}$$

$$\Rightarrow \quad x = \left(\frac{2a}{b}\right)^{\frac{1}{6}}$$

Potential energy at equilibrium is given by

$$U_{eqbm} = \frac{a}{\left(\frac{2a}{b}\right)^{\frac{12}{6}}} - \frac{b}{\left(\frac{2a}{b}\right)^{\frac{6}{6}}}$$

$$\Rightarrow \quad U_{eqbm} = \frac{a}{\left(\frac{4a^2}{b^2}\right)} - \frac{b}{\frac{2a}{b}}$$

$$\Rightarrow \quad U_{eqbm} = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$$

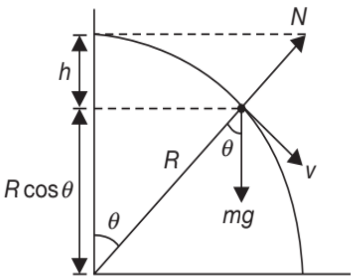
$$\Rightarrow \quad D = U_{\infty} - U_{eqbm} = 0 - \left(-\frac{b^2}{4a}\right) = \frac{b^2}{4a}$$

Hence, the correct answer is (D).

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

1. Let at any instant, the bead make an angle θ with the vertical, as shown in Figure.



Since $h = R - R\cos\theta$

Applying Law of Conservation of energy, we get

$$mgR(1-\cos\theta) = \frac{1}{2}mv^2$$

According to Newton's Second Law, we have

$$\sum F_{\text{towards the centre}} = ma_c = \frac{mv^2}{R}$$

$$\Rightarrow \quad mg\cos\theta - N = \frac{mv^2}{R}$$

where, N is the normal force on bead by the wire

$$\Rightarrow \quad N = mg\cos\theta - \frac{mv^2}{R}$$

Since $v^2 = 2gR(1-\cos\theta)$

$$\Rightarrow \quad N = mg(3\cos\theta - 2)$$

Now, $N = 0$ at $\cos\theta = \frac{2}{3}$

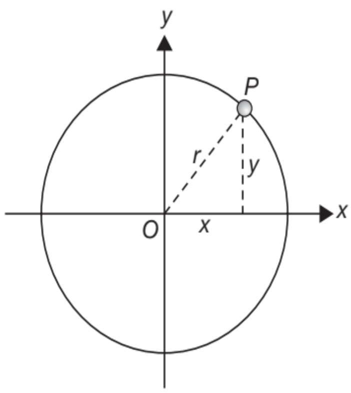
The normal force N will act radially outwards on bead, when

$\cos\theta > \frac{2}{3}$ and normal force N will act radially inwards on bead, when

$$\cos\theta < \frac{2}{3}$$

So, force acting on ring is opposite to normal force acting on bead.

Hence, the correct answer is (C).

2. 

Since, $\vec{r} = \overrightarrow{OP} = x\hat{i} + y\hat{j}$

$$\Rightarrow \quad \vec{F} = \frac{k}{(x^2+y^2)^{\frac{3}{2}}}(x\hat{i} + y\hat{j}) = \frac{k}{r^3}(\vec{r})$$

$$\Rightarrow \quad \vec{F} \parallel \vec{r}$$

Since, $\vec{F}$ is along $\vec{r}$ or in radial direction, so work done is zero.

Hence, the correct answer is (D).

3. Area under F-t graph (A) is equal to change in momentum. So,

$$p = \sqrt{2mK}$$

$\Rightarrow \quad K=\frac{p^2}{2m}=\frac{A^2}{2m}$

$\Rightarrow \quad K=\frac{A^2}{2m}$

Area $(A)=\frac{1}{2}(3)(4)-\frac{1}{2}(1.5)(2)$

$\Rightarrow \quad A=6-1.5=4.5\ \text{kgms}^{-1}$

$\Rightarrow \quad K=\frac{A^2}{2m}=\frac{(4.5)^2}{2(2)}$

$\Rightarrow \quad K=5.0625\ \text{J}$

Hence, the correct answer is (C).

4. Applying Law of Conservation of Energy, we get

$\frac{1}{2}kx^2=\frac{1}{2}(4k)y^2$

$\Rightarrow \quad \frac{y}{x}=\frac{1}{2}$

Hence, the correct answer is (C).

5. Since horizontal velocity at A is sufficient enough to make the bob reach to B, so

$v=\sqrt{5gL}$ ...(1)

When speed of bob is half this speed, then

$\left(\frac{v}{2}\right)^2=v^2-2gh$ ...(2)

Since, $h=L(1-\cos\theta)$ ...(3)

Solving equations (1), (2) and (3), we get

$\cos\theta=-\frac{7}{8}$

$\Rightarrow \quad \theta=\cos^{-1}\left(-\frac{7}{8}\right)=151^\circ$

i.e., $\frac{3\pi}{4}<\theta<\pi$

Hence, the correct answer is (D).

6. Since, $F=-\frac{dU}{dx}$

$\Rightarrow \quad \int_0^{U(x)} dU=-\int_0^x F dx=-\int_0^x (kx)dx$

$\Rightarrow \quad U(x)=-\frac{kx^2}{2} \qquad \{\because U(0)=0\}$

Hence, the correct answer is (A).

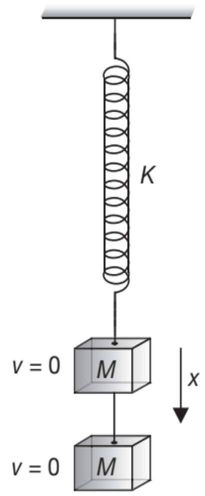
7. Since, gravitational field is a conservative force field in which the work done is independent of path followed between two points, so

$W_1=W_2=W_3$

Hence, the correct answer is (B).

8. Let x be the maximum extension of the spring. Then, by Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Decrease in}\\ \text{Gravitational}\\ \text{Potential Energy}\end{pmatrix}=\begin{pmatrix}\text{Increase in}\\ \text{Elastic}\\ \text{Potential Energy}\end{pmatrix}$$



$\Rightarrow \quad Mgx=\frac{1}{2}kx^2$

$\Rightarrow \quad x=\frac{2Mg}{k}$

Hence, the correct answer is (B).

9. $F=-\frac{dU}{dx}$

$\Rightarrow \quad dU=-Fdx$

$\Rightarrow \quad U(x)=-\int_0^x(-kx+ax^3)dx$

$\Rightarrow \quad U(x)=\frac{kx^2}{2}-\frac{ax^4}{4}$

Now, $U(x)=0$ at $x=0$ and $x=\sqrt{\frac{2k}{a}}$

$\Rightarrow \quad U(x)$ is negative for $x>\sqrt{\frac{2k}{a}}$

From the given function we can see that $F=0$ at $x=0$ i.e., slope of U-x graph is zero at $x=0$. Therefore, the most appropriate OPTION is (D).

Hence, the correct answer is (D).

10. Since $P = Fv$

In this case, $F = v\dfrac{dm}{dt}$

If ρ be the density of wind, then

$$\frac{dm}{dt} = \rho\left(\frac{dV}{dt}\right)$$

where $\dfrac{dV}{dt}$ is the volume of wind flowing per second through the generator.

Since $\dfrac{dV}{dt} = A\left(\dfrac{dx}{dt}\right) = Av$

where A is the area of the generator through which wind flows

$$\frac{dm}{dt} = \rho\left(\frac{dV}{dt}\right) = Av\rho$$

$$\Rightarrow \quad F = v\left(\frac{dm}{dt}\right) = v(Av\rho)$$

$$\Rightarrow \quad F = Av^2\rho$$

$$\Rightarrow \quad P = Fv = Av^3\rho$$

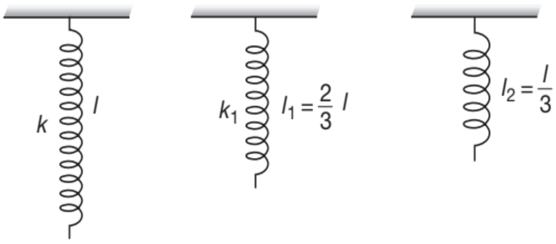
$$\Rightarrow \quad P \propto v^3$$

Hence, the correct answer is (C).

11. $\ell_1 = 2\ell_2$

If ℓ is the total natural length of the spring, then

$$\therefore \quad \ell_1 = \frac{2}{3}\ell \text{ and } \ell_2 = \frac{\ell}{3}$$



Since for a spring $k\ell$ = constant

$$\Rightarrow \quad k_1\ell_1 = k_2\ell_2 = k\ell$$

$$\Rightarrow \quad k_1 = \frac{3}{2}k \text{ and } k_2 = 3k$$

Hence, the correct answer is (B).

13. Since

$d W = \vec{F}\cdot d\vec{r}$, where

$d\vec{r} = (dx)\hat{i} + (dy)\hat{j}$

$$\Rightarrow \quad dW = -k(ydx + xdy)$$

$$\Rightarrow \quad dW = -kd(xy) \qquad \{\because d(xy) = xdy + ydx\}$$

$$\Rightarrow \quad W = \int dW = -k\int_{(0,0)}^{(a,a)} d(xy)$$

$$\Rightarrow \quad W = -k(xy)\Big|_{(0,0)}^{(a,a)}$$

$$\Rightarrow \quad W = -ka^2 - 0$$

$$\Rightarrow \quad W = -ka^2$$

Hence, the correct answer is (C).

14. $a_c = k^2rt^2$

$$\Rightarrow \quad \frac{v^2}{r} = k^2rt^2$$

$$\Rightarrow \quad v = krt$$

So, tangential acceleration, $a_T = \dfrac{dv}{dt} = kr$

$\Rightarrow$ Tangential force, $F_T = ma_T = mkr$

Since, only the tangential force does work, so

Power $= F_Tv = (mkr)(krt)$

$\Rightarrow$ Power $= mk^2r^2t$

Hence, the correct answer is (B).

15. The centre of mass of part hanging is at $\dfrac{L}{6}$. At this point a mass of $\dfrac{M}{3}$ exists.

So, work done to drag a weight $\dfrac{Mg}{3}$ up is

$$W = \left(\frac{Mg}{3}\right)\left(\frac{L}{6}\right)$$

$$\Rightarrow \quad W = \frac{MgL}{18}$$

Hence, the correct answer is (D).

17. $p = \sqrt{2Km}$

$$\Rightarrow \quad p \propto \sqrt{m}$$

Since, $\dfrac{m_1}{m_2} = \dfrac{1}{4}$

$$\Rightarrow \quad \frac{p_1}{p_2} = \frac{1}{2}$$

Hence, the correct answer is (C).

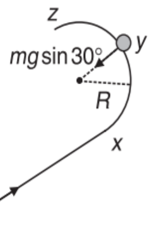
18. **The correct answer is (B).**

Multiple Correct Choice Type Problems

1. At point y, we have

$$mg\sin 30° = \frac{mv^2}{R}$$

$$\Rightarrow \quad v^2 = \frac{gR}{2}$$



Applying conservation of mechanical Energy, we get

$$\frac{1}{2}mv_0^2 = mgh + \frac{1}{2}mv^2$$

$$\Rightarrow \quad v_0^2 - 2gh = \frac{gR}{2}$$

Along the path xyz, speed is maximum at points x and z due to which maximum centripetal force will be required.

Hence, (A) and (D) are correct.

2. The given case is of uniform circular motion. In which speed or kinetic energy remain constant. However, the direction of velocity and acceleration keep on changing although its magnitude remains constant.

Hence, (C) and (D) are correct.

Reasoning Based Questions

1. **For Statement 1**, decrease in mechanical energy in CASE-1 will be

$$\Delta U_1 = \frac{1}{2}mv^2$$

However, decrease in mechanical energy in CASE-2 will be

$$\Delta U_2 = \frac{1}{2}mv^2 - mgh$$

$$\Rightarrow \quad \Delta U_2 < \Delta U_1$$

So, Statement 1 is True.

For Statement 2, coefficient of friction will not change and hence this statement is False.

Hence, the correct answer is (C).

Matrix Match/Column Match Type Questions

1. (A) → (p, q, r, t); (B) → (q, s); (C) → (p, q, r, s); (D) → (p, r, t)

METHOD-1: ANALYTICAL METHOD

(A) $\vec{F} = -\left(\frac{dU}{dx}\right)\hat{i} = -\frac{U_0}{2}2\left(1-\left(\frac{x}{a}\right)^2\right)\times\left[-2\left(\frac{x}{a}\right)\times\frac{1}{a}\right]\hat{i}$

$$\Rightarrow \quad F = 2U_0\left[1-\left(\frac{x}{a}\right)^2\right]\left(\frac{x}{a^2}\right)$$

When $x = 0$, $\vec{F} = \frac{U_0}{2}[2(1)\times 0] = \vec{0}$ and $U = \frac{U_0}{2}$

When $x = a$, $\vec{F} = \vec{0}$ and $U = 0$

When $x = -a$, $\vec{F} = \vec{0}$ and $U = 0$

(B) $\vec{F} = -\left(\frac{dU}{dx}\right)\hat{i} - \frac{U_0}{2}\times 2\left(\frac{x}{a}\right)\times\frac{1}{a}\hat{i} = -\frac{U_0 x}{a^2}\hat{i}$

When $x = 0$, $\vec{F} = 0$ and $U = 0$

When $x = a$, $\vec{F} = -\frac{U_0}{a}\hat{i}$ and $U = \frac{U_0}{2}$

When $x = -a$, $\vec{F} = +\frac{U_0}{a}\hat{i}$ and $U = \frac{U_0}{2}$

(C) $\vec{F} = -\left(\frac{dU}{dx}\right)\hat{i}$

$$\Rightarrow \quad F = \frac{U_0 e^{-\frac{x^2}{a^2}}}{a^4}x\left(x^2 - a^2\right)$$

When $x = 0$, $F = 0$ and $U = 0$

When $x = \pm a$, $F = 0$ and $U = \frac{U_0}{2e}$

For $|x| < a$ i.e., $-a < x < a$, the force is attractive in nature.

(D) $\vec{F} = -\left(\frac{dU}{dx}\right)\hat{i}$

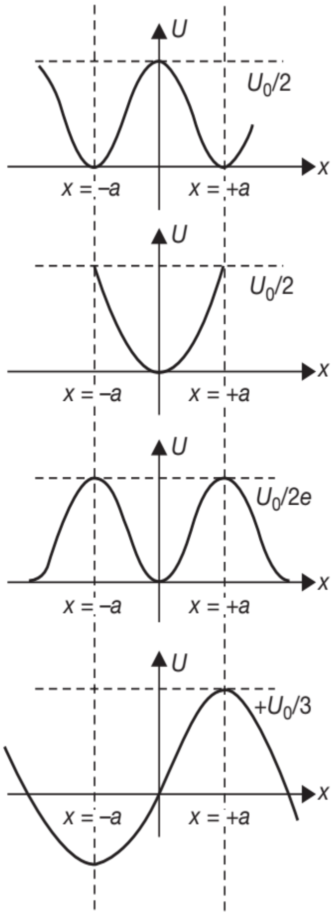
$$\Rightarrow \quad F = -\frac{U_0}{2}\left(\frac{1}{a} - \frac{x^2}{a^3}\right)$$

$$\Rightarrow \quad F = -\frac{U_0}{2a}\left(1 - \frac{x^2}{a^2}\right) = \frac{U_0}{2a^3}\left(x^2 - a^2\right)$$

At $x = 0$, $F = -\frac{U_0}{2a}$ and $U = 0$

At $x = \pm a$, $F = 0$ and $U = \frac{U_0}{3}$

METHOD-2: GRAPHICAL METHOD



Linked Comprehension Type Questions

1. Height fallen up to point Q

$$h = R\sin 30° = 40 \times \frac{1}{2} = 20 \text{ m}$$

Since, $W_{nc} = \Delta U + \Delta K$, so we get

$$W_{nc} = -mgh + \frac{1}{2}mv^2$$

$$\Rightarrow \quad W_{nc} = -(1)(10)(20) + \frac{1}{2}(1)v^2$$

Given that $W_{nc} = -150$ J

$$\Rightarrow \quad -150 = -200 + \frac{v^2}{2}$$

$$\Rightarrow \quad v^2 = 100$$

$$\Rightarrow \quad v = 10 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

2. At point Q, component of weight along PQ (radially outwards) is $mg\cos 60°$ i.e., $\frac{mg}{2}$.

Normal reaction is radially inwards, so we have

$$N - \frac{mg}{2} = \frac{mv^2}{R}$$

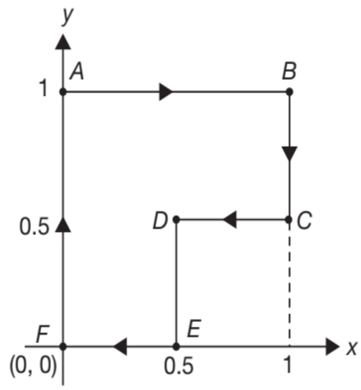
$$\Rightarrow \quad N = \frac{mg}{2} + \frac{mv^2}{R}$$

$$\Rightarrow \quad N = \frac{1\times 10}{2} + \frac{1\times(10)^2}{40} = 7.5 \text{ N}$$

Hence, the correct answer is (A).

Integer/Numerical Answer Type Questions

1. Given that, $\vec{F} = \alpha y\hat{i} + 2\alpha x\hat{j}$ where $\alpha = -1$



Since Work done is given by

$$W = \int \vec{F}\cdot d\vec{\ell}$$

$$\Rightarrow \quad dW = \alpha(ydx + 2xdy)$$

For $A \to B$, $y = 1$, so $dy = 0$

$$\Rightarrow \quad W_{A\to B} = \alpha\int ydx = \alpha\int_0^1 dx = \alpha$$

For $B \to C$, $x = 0$, so $dx = 0$

$$\Rightarrow \quad W_{B\to C} = 2\alpha\int_1^{0.5} dy = 2\alpha(-0.5) = -\alpha$$

For $C \to D$, $y = 0.5$, so $dy = 0$

$$\Rightarrow \quad W_{C\to D} = 0.5\alpha\int_1^{0.5} dx = -\frac{\alpha}{4}$$

For $D \to E$, $x = 0.5$, so $dx = 0$

$$\Rightarrow \quad W_{D\to E} = 2\alpha(0.5)\int_{0.5}^{0} dy = -\frac{\alpha}{2}$$

For $E \to F$, $x = 0$, $y = 0$, so $dx = 0$ and $dy = 0$

$\Rightarrow \quad W_{E\to F} = 0$

For $F \to A$, $x = 0$, so $dx = 0$

$\Rightarrow \quad W_{F\to A} = 0$

Hence total work done is

$$W_{total} = \alpha - \alpha - \frac{\alpha}{4} - \frac{\alpha}{2}$$

$$\Rightarrow \quad W_{total} = -\frac{3\alpha}{4} = \frac{-3(-1)}{4} = \frac{3}{4}\ \text{J} = 0.75\ \text{J}$$

2. From Modified Work-Energy Theorem, we have

$$W_{ext} = \Delta U + \Delta K$$

$$\Rightarrow \quad W_F = \Delta U + \Delta K$$

$$\Rightarrow \quad (18)(5) = (1)(10)(4) + (K_f - 0)$$

$$\Rightarrow \quad 90 = 40 + K_f$$

$$\Rightarrow \quad K_f = 50\ \text{J}$$

$$\Rightarrow \quad K_f = 5(10\ \text{J})$$

$$\Rightarrow \quad n = 5$$

$$\Rightarrow \quad W_F + W_{mg} = K_f - K_i$$

$$18 \times 5 + (1 \times 10)(-4) = K_f$$

$$90 - 40 = K_f$$

$$\Rightarrow \quad K_f = 50\ \text{J} = 5 \times 10\ \text{J}$$

$$\Rightarrow \quad n = 5$$

3. Velocity of first bob at highest point (when it just completes the circle) is

$$v_1 = \sqrt{g\ell_1}$$

Since collision of first bob with second bob is elastic and both have equal mass, so first bob will stop and second will move with velocity of first such that this velocity is sufficient enough for second bob to just complete the circle of radius ℓ_2. So

$$\sqrt{g\ell_1} = \sqrt{5g\ell_2}$$

$$\Rightarrow \quad \frac{\ell_1}{\ell_2} = 5$$

4. $W = \frac{1}{2}mv^2$

$$\Rightarrow \quad Pt = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2Pt}{m}} = \sqrt{\frac{2 \times 0.5 \times 5}{0.2}} = 5\ \text{ms}^{-1}$$

5. According to Work-Energy Theorem, we have

$$\frac{1}{2}mV^2 = (\mu mg)(0.06) + \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{1}{2}(0.18)V^2 = (0.1)(0.18)(10)(0.06) + \frac{1}{2}(2)(0.06)^2$$

$$\Rightarrow \quad V = 0.4\ \text{ms}^{-1}$$

$$\Rightarrow \quad 0.4 = \frac{N}{10}$$

$$\Rightarrow \quad N = 4$$