CHAPTER 6: NEWTON'S LAWS OF MOTION

Test Your Concepts-I (Based on Impulse Momentum)

1. Since $\Delta\vec{p} = \vec{F}\Delta t$

Now, $\Delta p_y = m(v_y - u_y) = mv[\cos(30°) - \cos(30°)]$

$\Rightarrow \quad \Delta p_y = 0$

Similarly, $\Delta p_x = m(v_x - u_x)$

$\Rightarrow \quad \Delta p_x = mv[-\sin(30°) - \sin(30°)]$

$\Rightarrow \quad \Delta p_x = -2mv\sin(30°) = -30 \text{ kgms}^{-1}$

So, $F_{av} = \dfrac{30}{0.2} = 150 \text{ N}$

2. Velocity of the ball just before hitting the floor is

$v_1 = \sqrt{2gh_1}$, downwards $\quad \{\because v_1^2 - 0^2 = 2gh_1\}$

Velocity of the ball just after impact with the floor is

$v_2 = \sqrt{2gh_2}$, upwards $\quad \{\because 0^2 - v_2^2 = 2(-g)h_2\}$

From Impulse Momentum theorem,

Impulse = Change in Momentum

$\Rightarrow \quad I = m(v_2 + v_1)$

$\Rightarrow \quad I = \dfrac{150}{1000}\left(\sqrt{2(10)(20)} + \sqrt{2(10)(5)}\right)$

$\Rightarrow \quad I = \dfrac{150}{1000}(20+10) = 4.5 \text{ kgms}^{-1}$

So, impulse is 4.5 kgms^{-1}, upwards

3. (a) Since impulse is equal to the area under the F-t graph

$I = \int F dt$ = Area under F-t graph

$\Rightarrow \quad I = \dfrac{1}{2}(3.5-1)(10^{-3})(16000)$

$\Rightarrow \quad I = 20 \text{ Ns}$

(b) Since we know that $F_{av}\Delta t = \Delta I$

$\Rightarrow \quad F_{av}(2.5\times10^{-3}) = 20$

$\Rightarrow \quad F_{av} = 8000 \text{ N} = 8 \text{ kN}$

(c) From the graph we observe that the peak force exerted on the ball is $16000 \text{ N} = 16 \text{ kN}$.

4. $F = \dfrac{\Delta p_{\text{water}}}{\Delta t} = \dfrac{m(v-u)}{\Delta t}$

$\Rightarrow \quad F = (0.6)(25-0) \quad \left\{\because \dfrac{m}{\Delta t} = 0.6\right\}$

$\Rightarrow \quad F = 15 \text{ N}$

Now due to Newton's Third Law, the water exerts a force of equal magnitude back on the hose, hence the gardener must apply a 15 N force in the direction of the velocity of water stream to hold the hose in its position (stationary).

5. When the diver falls freely, then the velocity of the diver just before he hits the surface of water is

$u = \sqrt{2gh} = \sqrt{2(9.8)(10)} = 14 \text{ ms}^{-1}$

Now $|F| = \left|\dfrac{\Delta p}{\Delta t}\right| = \left|\dfrac{m(v-u)}{\Delta t}\right|$

$\Rightarrow \quad |F| = \left|\dfrac{(60)(4-14)}{1}\right|$

$\Rightarrow \quad |F| = 600 \text{ N}$

Test Your Concepts-II (Based on Constraints)

1. As already done in an Illustration, we obtained

$a_B + a_C + 2a_A = 0$

Similarly, we can find

$v_B + v_C + 2v_A = 0$

Taking, upward direction as positive we are given:

$v_A = v_B = 1 \text{ ms}^{-1}$

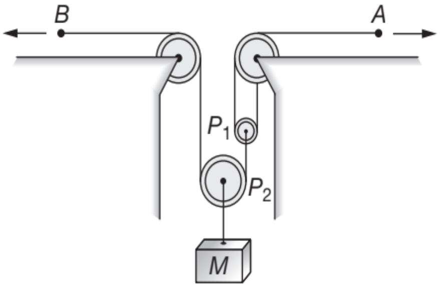
$\Rightarrow \quad v_C = -3 \text{ ms}^{-1}$

i.e., velocity of block C is 3 ms^{-1} (downwards).

2. $v_A = 2 \text{ ms}^{-1}$ $\quad$ {towards right}

$\Rightarrow \quad v_{P_1} = \dfrac{v_A}{2} = 1 \text{ ms}^{-1}$ $\quad$ {upwards}

$v_B = 2 \text{ ms}^{-1}$ $\quad$ {towards left}



Now $2v_{P_2} = v_B + v_{P_1}$

$$\Rightarrow \quad v_{P_2} = \frac{v_B + v_{P_1}}{2} = \frac{2+1}{2} = 1.5 \text{ ms}^{-1}$$

3. Let B and C both move upwards (alongwith their pulleys) with speeds v_B and v_C, then we observe that, A will move downward with speed, $2v_B + 2v_C$. So, with sign we write velocity of A as

$$v_A = 2v_B - 2v_C$$

$$\Rightarrow \quad v_B = -\frac{v_A}{2} - v_C$$

Substituting $v_A = -2 \text{ ms}^{-1}$ and $v_C = 1 \text{ ms}^{-1}$, we get

$$v_B = 0$$

4. $OB = x = \left(L^2 - y^2\right)^{1/2}$

Since, $v_x = \dfrac{dx}{dt} = \dfrac{(-2y)\left(\dfrac{dy}{dt}\right)}{2\left(L^2-y^2\right)^{1/2}} = -\dfrac{yv_A}{\left(L^2-y^2\right)^{1/2}}$

$$\Rightarrow \quad a_x = \frac{dv_x}{dt}$$

$$\Rightarrow \quad a_x = -\left[\frac{\sqrt{L^2-y^2}\left(y\dfrac{dv_A}{dt} + v_A\dfrac{dy}{dt}\right) - \dfrac{yv_A(-2y)\dfrac{dy}{dt}}{2\left(L^2-y^2\right)^{1/2}}}{\left(L^2-y^2\right)}\right]$$

Substituting

$$\frac{dv_A}{dt} = 0 \text{ and } \frac{dy}{dt} = v_A \qquad \{\because v_A = \text{constant}\}$$

$$\Rightarrow \quad a_x = -\frac{L^2 v_A^2}{\left(L^2-y^2\right)^{3/2}}$$

5. $a_A = \dfrac{d^2y}{dt^2} = \dfrac{1}{2} \text{ ms}^{-2}$

From constraint equations, we get $a_B = 8a_A$ (in opposite direction). Hence, if a_A is $\dfrac{1}{2}$ ms^{-2} upwards, a_B will be 4 ms^{-2} downwards.

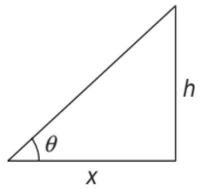
6. Since $h = x\tan\theta$

$$\Rightarrow \quad x = h\cot\theta$$

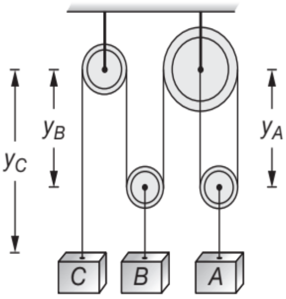
$$\Rightarrow \quad \frac{dx}{dt} = -h\,\text{cosec}^2\theta\left(\frac{d\theta}{dt}\right)$$

$$\Rightarrow \quad \left|\frac{d\theta}{dt}\right| = \omega = \frac{1}{h\,\text{cosec}^2\theta}\frac{dx}{dt}$$

$$\Rightarrow \quad \omega = \frac{v}{h}\sin^2\theta \qquad \left\{\because v = \frac{dx}{dt}\right\}$$



7. Length of cable $L = 2y_A + 2y_B + y_C + \text{constant}$



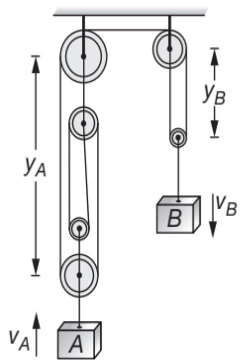
$$0 = 2\dot{y}_A + 2\dot{y}_B + \dot{y}_C$$

$$0 = 2\ddot{y}_A + 2\ddot{y}_B + \ddot{y}_C$$

So, $2a_A + 2a_B + a_C = 0$

2 degrees of freedom

8. Total length of cable to within constants is



$$L = 4y_A + 2y_B$$

$$0 = 4\dot{y}_A + 2\dot{y}_B$$

$$0 = 4\ddot{y}_A + 2\ddot{y}_B$$

Upward acceleration of A is

$$a_A = -\ddot{y}_A = \frac{1}{2}\ddot{y}_B = \frac{1}{2}a_B$$

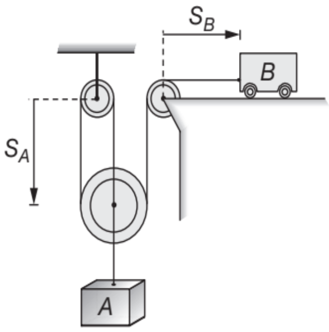
$$v_B = \frac{t^2}{2} + \frac{t^3}{6},\ a_B = \dot{v}_B = t + \frac{t^2}{2}$$

For $t = 2$ s, $a_B = 2 + \frac{4}{2} = 4$ ms^{-2}

So, $a_A = \frac{1}{2} a_B = 2$ ms^{-2}

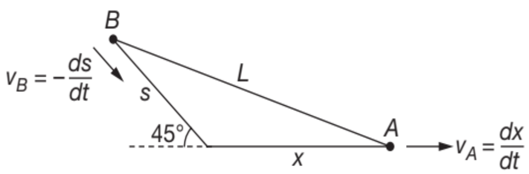
9. Length of cable $L = S_B + 3S_A + \text{constant}$

$$0 = v_B + 3v_A,\ v_A = -\frac{v_B}{3} = -\left(\frac{-1.2}{3}\right) = 0.4\ \text{ms}^{-1}\ \text{(down)}$$



10. $L^2 = \left(x + \frac{s}{\sqrt{2}}\right)^2 + \left(\frac{s}{\sqrt{2}}\right)^2 = x^2 + \frac{2xs}{\sqrt{2}} + s^2$

$$\Rightarrow\ 2x\frac{dx}{dt} + \frac{2\frac{dx}{dt}s}{\sqrt{2}} + \frac{2x\frac{ds}{dt}}{\sqrt{2}} + 2s\frac{ds}{dt} = 0$$



$$\Rightarrow\ xv_A + \frac{s}{\sqrt{2}}v_A - \frac{x}{\sqrt{2}}v_B - sv_B = 0$$

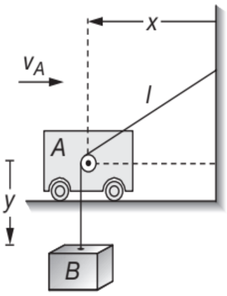
$$\Rightarrow\ v_B\left(s + \frac{x}{\sqrt{2}}\right) = v_A\left(x + \frac{s}{\sqrt{2}}\right)$$

$$\Rightarrow\ v_B = \frac{x + \frac{s}{\sqrt{2}}}{s + \frac{x}{\sqrt{2}}}v_A$$

$$\Rightarrow\ v_B = \frac{s + \sqrt{2}x}{x + \sqrt{2}s}v_A$$

11. $l^2 = x^2 + h^2$

$$\Rightarrow\ l\frac{dl}{dt} = x\frac{dx}{dt} \qquad \{\because h = \text{Constant}\}$$



and $\dot{y} = -\dot{l} = -\frac{x}{l}\dot{x}$

But $v_A = -\frac{dx}{dt}$

So, $(v_B)_y = \frac{dy}{dt} = \frac{x}{l}v_A$

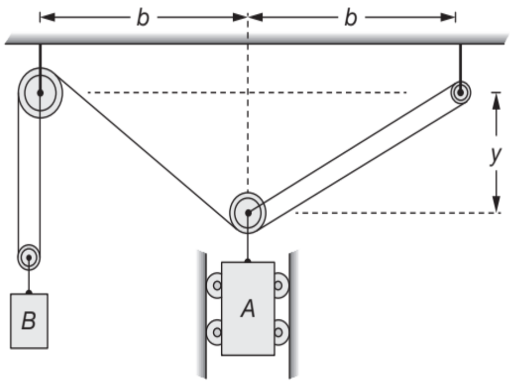
$$(v_B)_x = \frac{dx}{dt} = -v_A$$

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = v_A\sqrt{1 + \frac{x^2}{l^2}}$$

$$\Rightarrow\ v_B = v_A\sqrt{\frac{2x^2 + h^2}{x^2 + h^2}}$$

12. The total length of the cable is

$$L = 2x + 3\sqrt{y^2 + b^2} + \text{constant}$$



Differentiate to obtain

$$\frac{dL}{dt} = 0 = 2\frac{dx}{dt} + 3\frac{y}{\sqrt{y^2 + b^2}}\left(\frac{dy}{dt}\right)$$

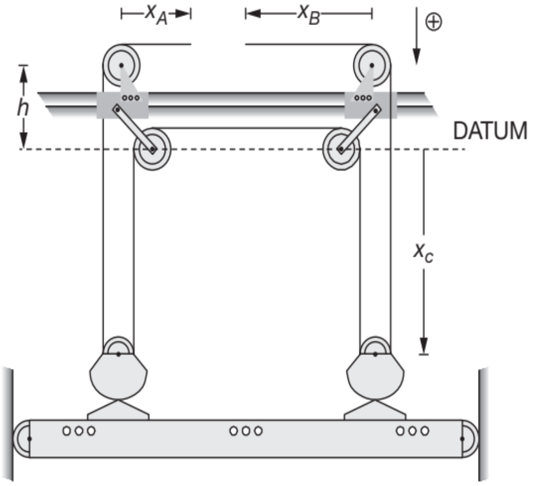
where $\frac{dx}{dt} = v_B$ and $\frac{dy}{dt} = v_A$

So, we get

$$v_B = -\frac{3y}{2\sqrt{y^2 + b^2}}v_A$$

13. Make the constraint equation. 1.8 ms^{-1}, down

14. Datum is established as shown.



The position of point A and B and load C with respect to datum are x_A, x_B and x_C, respectively.

$$4x_C + x_A + x_B + 2h = l \quad \ldots(1)$$

Since h is a constant, taking the time derivative equation (1), we get

$$4v_C + v_A + v_B = 0 \quad \ldots(2)$$

Since $v_A = 60\text{ cms}^{-1}$ and $v_B = 120\text{ cms}^{-1}$ from equation (2), we get

$$4v_C + 60 + 120 = 0$$

$$v_C = -45\text{ cms}^{-1} = 45\text{ cms}^{-1}\text{, upwards}$$

15. $2s_A + (h - s_C) = l$

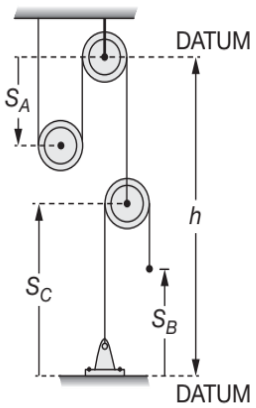
$2v_A = v_C$

$s_C + (s_C - s_B) = l$

$2v_C = v_B$

$v_B = 4v_A$

$a_B = 4a_A$



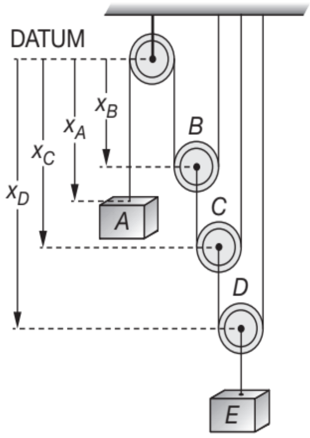
Thus, $-4 = 4v_A$

$$v_A = -1\text{ ms}^{-1} = 1\text{ ms}^{-1}\text{, upwards}$$

$$2 = 4a_A$$

$$a_A = 0.5\text{ ms}^{-2} = 0.5\text{ ms}^{-2}\text{, downwards}$$

16. Datum is located at fixed pulley. The position of point A, pulley B, C and D (or block E) with respect to datum are x_A, x_B, x_C and x_D respectively. Since the system consists of three cords, three position-coordinate equations can be developed.



$$2x_B + x_A = l_1 \quad \ldots(1)$$

$$x_C + (x_c - x_B) = l_2 \quad \ldots(2)$$

$$x_D + (x_D - x_C) = l_3 \quad \ldots(3)$$

Eliminating x_C and x_B from equations (1), (2) and (3), we have

$$x_A + 8x_D = l_1 + 2l_2 + 4l_3$$

Taking the time derivative of the above equation yields

$$v_A + 8v_D = 0 \quad \ldots(4)$$

Since $v_A = 2\text{ ms}^{-1}$, from equation (3) and taking downward direction as positive, we get

$$2 + 8v_D = 0$$

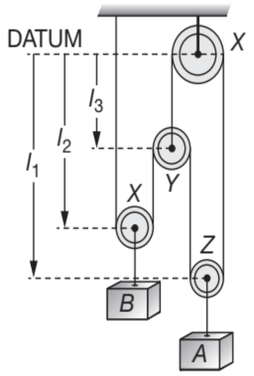
$$\Rightarrow \quad v_D = -0.25\text{ ms}^{-1} = 0.25\text{ ms}^{-1}\text{, upwards}$$

Since the block E is connected directly to the pulley D, so

$$v_E = v_D = 0.25\text{ ms}^{-1}\text{, upwards}$$

17. (a) $l_2 + (l_2 - l_3) + (l_1 - l_3) + l_1 + l_3 = \text{constant}$

$$\Rightarrow \quad 2l_2 + 2l_1 - l_3 = \text{constant}$$



$$\Rightarrow \quad 2\left(\frac{dl_2}{dt}\right)+2\left(\frac{dl_1}{dt}\right)-\frac{dl_3}{dt}=0$$

$$\Rightarrow \quad 2v_B+2v_A-v_y=0 \qquad \ldots(1)$$

For $v_B = 1\ \text{ms}^{-1}$ and $v_y = 2\ \text{ms}^{-1}$, we get

$$v_A = 0$$

(b) From (1), we get

$$2a_B+2a_A-a_y=0$$

Taking downward direction as positive, we get

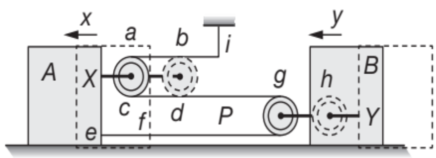
$$a_B=-3\ \text{ms}^{-2} \text{ and } a_y = 4\ \text{ms}^{-2}$$

$$\Rightarrow \quad 2(-3)+2a_A-4=0$$

$$\Rightarrow \quad 2a_A=10$$

$$\Rightarrow \quad a_A = 5\ \text{ms}^{-2}$$

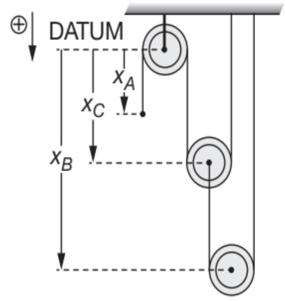
18. Analysis of the problem is shown in figure. If the block A moves towards left by a distance x, the string lengths $ab+cd+ef=3x$ will be pulled towards right and to provide this length block B has to move toward left by a distance y such that the string lengths on the two sides of the pulley Y (twice of ab) i.e., $2y$ will slack to tight the string $3x$. Thus we have $3x=2y$.



As here we are required to find the displacement of the point P, we can start from either end of string e or i. If we start from i, we can see that from i to P there is only pulley X which pulls the thread by a distance $2x$, thus, P will move toward left by $2x$.

Alternatively we can start from e. As we can see that from e to P there is only pulley Y, which slacks the string by $2y$ and point e is pulled toward left by a distance x, thus point P will move toward left by a distance $2y-x$, which is again $2x$. Thus the displacement of point P is twice of x which is 6 m.

19. Let the datum be passing through the fixed pulley D. The position of point A, block B and pulley C with respect to datum are x_A, x_B and x_C, respectively. Since the system consists of two cords, two position coordinate equations can be developed.



$$2x_C+x_A=l_1 \qquad \ldots(1)$$

$$x_B+(x_B-x_C)=l_2 \qquad \ldots(2)$$

Eliminating s_C from equations (1) and (2) yields

$$x_A+4x_B=l_1+2l_2 \qquad \ldots(3)$$

Taking the time derivative of equation (3), we get

$$v_A+4v_B=0 \qquad \ldots(4)$$

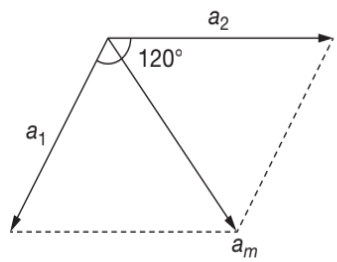
Since $v_A = 2\ \text{ms}^{-1}$, from equation (3)

$$\Rightarrow \quad 2+4v_B=0$$

$$v_B=-0.5\ \text{ms}^{-1}=0.5\ \text{ms}^{-1}\text{, upwards}$$

20. $v_A = 2v_0$, downwards

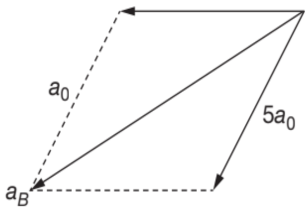
21. As m is also moving down along the incline with M, we can find the net acceleration of m using vector addition of the two acceleration in m, shown in figure.



$$a_m=\sqrt{a_1^2+a_2^2+2a_1a_2\cos(120°)}$$

$$\Rightarrow \quad a_m=\sqrt{25+9-15}=\sqrt{19}=4.36\ \text{ms}^{-2}$$

22. When A moves to the left through x, then a portion $5x$ of the string is loosened which makes the block B to go down by $5x$. So, acceleration of B w.r.t A is $5a_0$. Now, acceleration of B w.r.t. ground is calculated from the diagram



$$\Rightarrow \quad a_B = a_0\sqrt{1+25+10\left(\frac{1}{2}\right)}$$

$$\Rightarrow \quad a_B = \sqrt{31}a_0$$

23. If the distance of mass M from the ceiling is y and the distance of M from each pulley is x and the distance between the two pulleys is l. Then u will be the rate at which x is decreasing. If v is the velocity of M upward, it is the rate at which y is decreasing. Thus we have

$$u = -\frac{dx}{dt}$$

and $v = -\dfrac{dy}{dt}$

Now we find the relation in x and y as

$$x^2 + \frac{l^2}{4} = y^2$$

On differentiating with respect to t

$$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

$$xu = yv$$

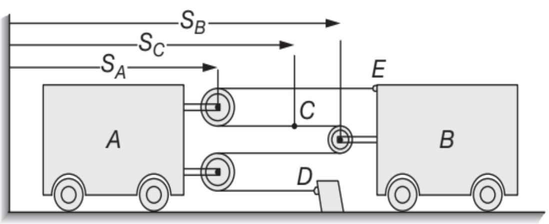
$$v = u\frac{dx}{dt} = u\sec\theta$$

25. Cable length $L = 3(S_B - S_A) + (S_D - S_A)$

$$\Rightarrow \quad 0 = 3v_B - 4v_A \text{ and } 0 = 3a_B - 4a_A$$

$$\Rightarrow \quad v_A = \frac{3}{4}v_B = \frac{3}{4}(2) = 1.5 \text{ ms}^{-1}$$

$$\Rightarrow \quad a_A = \frac{3}{4}a_B = \frac{3}{4}(3) = 2.25 \text{ ms}^{-2}$$



Since $v_{B/A} = v_B - v_A = 2 - (1.5) = 0.5 \text{ ms}^{-1}$

$$a_{B/A} = a_B - a_A = 3 - (2.25) = 0.75 \text{ ms}^{-2}$$

Length of cable between points E and C is

$$L' = (S_B - S_A) + (S_C - S_A) + \text{constants}$$

$$\Rightarrow \quad 0 = v_B - 2v_A + v_C$$

$$\Rightarrow \quad v_C = 2v_A - v_B$$

$$\Rightarrow \quad v_C = 2(1.5) - 2 = 1 \text{ ms}^{-1}$$

(All answers are quantities directed to the right)

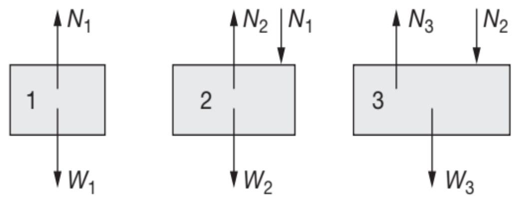
Test Your Concepts-III (Based on F.B.D.)

1. Since, N_1 = normal reaction between 1 and 2

N_2 = normal reaction between 2 and 3

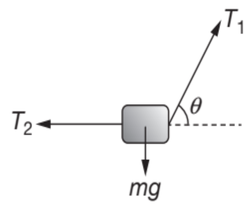
N_3 = normal reaction between 3 and ground

Free body diagrams of 1, 2 and 3 are shown below:

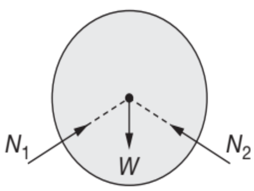


FBD of 1 FBD of 2 FBD of 3

2. The free body diagram of the block is as shown in figure.

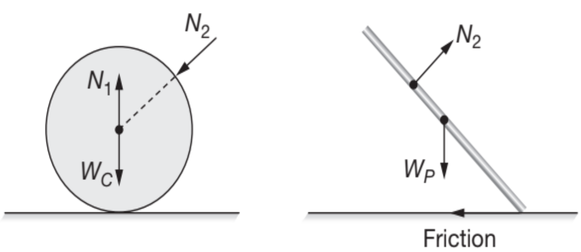


3. The free body diagram of the ball is shown in figure.

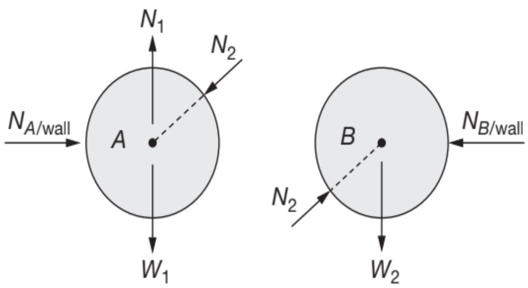


Here, W is weight of ball acting downwards and N_1 and N_2 are the normal reactions between the ball and the two inclined walls.

4.



5.



Test Your Concepts-IV (Based on Newton's Laws of Motion: Accelerated Systems)

1. Since the pull diminishes uniformly at a rate of 1 kgwt per metre for the rope wound up. So, if the length x of the rope is pulled up, then the pull P is given by

$$P = 250\text{ g} - (1)(x)g$$

{$\because$ pull decreases by 1 kgwt per metre pulled up}

So, if F is the net force required to pull the weight of 200 kg, then

$$F = P - 200\text{ g}$$

$$\Rightarrow \quad F = 250\text{ g} - xg - 200\text{ g}$$

$$\Rightarrow \quad F = (50 - x)g$$

$$\Rightarrow \quad 200a = (50 - x)(10)$$

$$\Rightarrow \quad a = \frac{dv}{dt} = \frac{50 - x}{20}$$

$$\Rightarrow \quad v\frac{dv}{dx} = \frac{5}{2} - \frac{x}{20}$$

$$\Rightarrow \quad vdv = \frac{5}{2}dx - \frac{1}{20}xdx$$

$$\Rightarrow \quad \int_0^v vdv = \frac{5}{2}\int_0^{20} dx - \frac{1}{20}\int_0^{20} x\,dx$$

$$\Rightarrow \quad \frac{v^2}{2} = \left(\frac{5}{2}x - \frac{x^2}{40}\right)\Bigg|_0^{20}$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{5}{2}(20) - \frac{400}{40}$$

$$\Rightarrow \quad \frac{v^2}{2} = 50 - 10$$

$$\Rightarrow \quad v^2 = 80$$

$$\Rightarrow \quad v = 4\sqrt{5}\text{ ms}^{-1}$$

2. Normal reaction between A and B is $N = mg\cos\theta$. Its horizontal component is $N\sin\theta$. Therefore, tension in cord CD is equal to this horizontal component.

Hence, $T = N\sin\theta = (mg\cos\theta)(\sin\theta)$

$$\Rightarrow \quad T = \frac{mg}{2}\sin(2\theta)$$

3. Suppose acceleration of m_1 down the plane is a, then acceleration of m_2, vertically up will be $2a$. Since the pulleys are massless, so, if T is the tension in the string connected to m_2, then the tension in the string connected to mass m_1, is 2T. The equations of motion of the two masses m_1 and m_2 are written for calculating value of a.

For mass m_1,

$$m_1 g\sin\alpha - 2T = m_1 a \qquad \ldots(1)$$

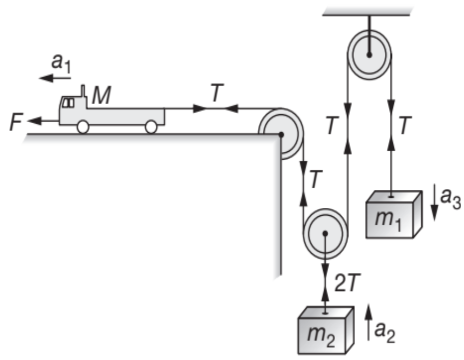
For mass m_2,

$$T - m_2 g = m_2(2a) \qquad \ldots(2)$$

Solving these two equations, we get

$$a = \left(\frac{m_1\sin\alpha - 2m_2}{m_1 + 4m_2}\right)g \text{ and } 2a = 2\left(\frac{m_1\sin\alpha - 2m_2}{m_1 + 4m_2}\right)g$$

4. Tensions in different branches of the string are shown in figure. The direction and magnitude of acceleration, initially assumed by us are also shown.



Let the mass m_2 moves up by a distance x_2, pulley attached to it will also move up by x_2, which will result a slackness of $2x_2$ in the string attached to truck and m_1. If in the same duration, truck moves by x_1 and mass m_1 moves down by x_3, we have $x_1 + x_3 = 2x_2$. Same relation we have among the acceleration of the respective bodies as

$$a_1 + a_3 = 2a_2 \qquad \ldots(1)$$

Writing the equations of the three bodies, we get for truck, we have

$$F - T = Ma_1 \qquad \ldots(2)$$

for mass m_1, we have

$$m_1 g - T = m_1 a_3 \qquad \ldots(3)$$

for mass m_2

$$2T - m_2 g = m_2 a_2 \quad \ldots(4)$$

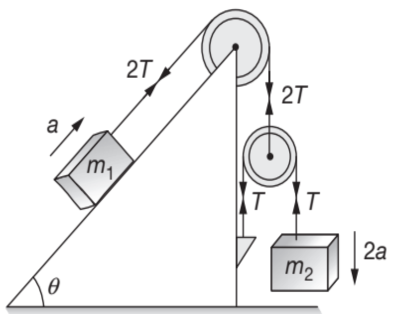
Solving these equations for the given data, we get

$$T = (\text{force acting on truck towards right}) = 4.21\text{ N}$$

$$a_3\,(= \text{acceleration of } m_1) = 5.79\text{ ms}^{-2}\text{, downwards}$$

and $a_2\,(= \text{acceleration of } m_2) = 6.84\text{ ms}^{-2}$, upwards

5. As discussed earlier that for a movable pulley fixed at one end, the acceleration of the free end is twice the acceleration of the pulley. So, acceleration of m_2 is two times that of m_1 (attached directly to the pulley through thread). So, we assume if m_1 is moving up the inclined plane with an acceleration a, the acceleration of mass m_2 going down is $2a$. The tensions in different strings are shown in figure.



The dynamic equations can be written as

For mass m_1

$$2T - m_1 g \sin\theta = m_1 a \quad \ldots(1)$$

For mass m_2

$$m_2 g - T = m_2 (2a) \quad \ldots(2)$$

Substituting $m_2 = \eta m_1$ and solving equations (1) and (2), we get acceleration of m_2 as

$$2a = \frac{2g(2\eta - \sin\theta)}{4\eta + 1}$$

6. From our knowledge of constraint relations we get that the acceleration of the rod is double than that of the acceleration of the ball. If ball is going up with an acceleration a, rod will be coming down with the acceleration $2a$, thus, the relative acceleration of the ball with respect to rod is $3a$ in upward direction. If it takes time t second to reach the upper end of the rod, we have

$$2l = \frac{1}{2}(3a)t^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2l}{3a}} \quad \ldots(1)$$

Let mass of ball be m and that of rod is M, the dynamic equations of these are

For rod $Mg - T = M(2a)$ $\quad \ldots(2)$

For ball $2T - mg = ma$ $\quad \ldots(3)$

Substituting $m = \eta M$ and solving equations (2) and (3), we get

$$a = \left(\frac{2-\eta}{\eta+4}\right)g$$

Substituting the value of a in equation (1), we get

$$t = \sqrt{\frac{2l(\eta+4)}{3g(2-\eta)}}$$

7. (a) In this case net pulling force is

$$F = m_A g\sin(60°) + m_B g\sin(60°) - m_C g\sin(30°)$$

$$\Rightarrow \quad F = (1)(10)\frac{\sqrt{3}}{2} + (3)(10)\left(\frac{\sqrt{3}}{2}\right) - (2)(10)\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad F = 24.64\text{ N}$$

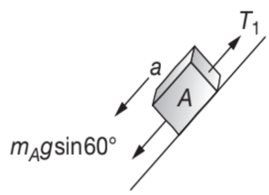
Total mass being pulled $= 1 + 3 + 2 = 6$ kg

So, acceleration of the system is

$$a = \frac{24.64}{6} = 4.1\text{ ms}^{-2}$$

(b) For the tension in the string between A and B

F.B.D. of A

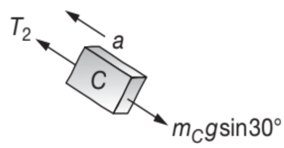


$$m_A g\sin 60° - T_1 = (m_A)(a)$$

$$\Rightarrow \quad T_1 = m_A g\sin(60°) - m_A a = m_A\left(g\sin(60°) - a\right)$$

$$\Rightarrow \quad T_1 = (1)\left(10\times\frac{\sqrt{3}}{2} - 4.1\right) = 4.56\text{ N}$$

For the tension in the string between B and C



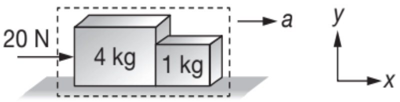
F.B.D. of C

$$T_2 - m_C g \sin 30° = m_C a$$

$$\Rightarrow \quad T_2 = m_C\left(a + g\sin(30°)\right)$$

$$\Rightarrow \quad T_2 = 2\left[4.1 + 10\left(\frac{1}{2}\right)\right] = 18.2 \text{ N}$$

8. (a) Since, both the blocks will move with same acceleration (say a) in horizontal direction.



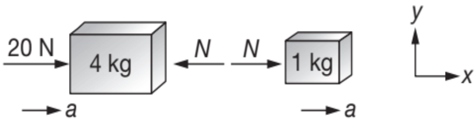
Let us take both the blocks as a system. Net external force on the system is 20 N in horizontal direction.

Using $\Sigma F_x = ma_x$

$$20 = (4+1)a = 5a$$

$$\Rightarrow \quad a = 4 \text{ ms}^{-2}$$

(b) The free body diagram of both the blocks are as shown in figure



Using $\Sigma F_x = ma_x$, we get

For 4 kg block

$$20 - N = 4a = 4 \times 4$$

$$N = 20 - 16 = 4 \text{ N}$$

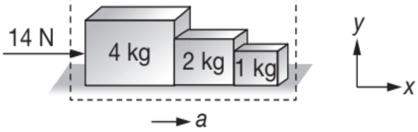
For 1 kg block

$$N = 1a = 1 \times 4 = 4 \text{ N}$$

Here, N is the normal reaction between the two blocks.

Please note that in free body diagram of the blocks we have not shown the forces acting on the blocks in vertical direction, because normal reaction between the blocks and acceleration of the system can be obtained without using $\Sigma F_y = 0$.

9. Since, all the blocks will move with same acceleration (say a) in horizontal direction. Let us take all the blocks as a system.



Net external force on the system is 12 N in horizontal direction.

Using $\Sigma F_x = ma_x$, we get

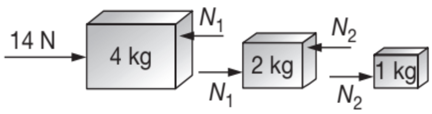
$$14 = (4+2+1)a = 7a$$

$$\Rightarrow \quad a = \frac{14}{7} = 2 \text{ ms}^{-2}$$

Now, let F be the net force on 2 kg block in x-direction, then using $\Sigma F_x = ma_x$ for 2 kg block, we get

$$F = (2)(2) = 4 \text{ N}$$

Also drawing the free body diagram for the arrangement will given same result.



Since, $14 - N_1 = 4(2)$

$$\Rightarrow \quad N_1 = 6 \text{ N}$$

Also, $N_1 - N_2 = 2(2)$

$$\Rightarrow \quad 6 - N_2 = 4$$

$$\Rightarrow \quad N_2 = 2 \text{ N}$$

Please note that here net force F on 2 kg block is the resultant of N_1 and $N_2 (N_1 > N_2)$

where N_1 = normal reaction between 4 kg and 2 kg block.

and N_2 = normal reaction between 2 kg and 1 kg block.

Thus, $F = N_1 - N_2 = 4$ N.

10. Block B will fall vertically downwards and A along the plane.
Writing the equations of motion, we get
for block B, we have

$$m_B g - N = m_B a_B$$

$$\Rightarrow \quad 60 - N = 6a_B \qquad \text{...(1)}$$

for block A, we have

$$(N + m_A g)\sin(30°) = m_A a_A$$

$$\Rightarrow \quad (N + 150) = 30a_A \qquad \text{...(2)}$$

Further $a_B = a_A \sin(30°)$

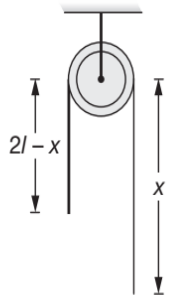
$$\Rightarrow \quad a_A = 2a_B \qquad \text{...(3)}$$

Solving equations, (1), (2) and (3), we get

$$a_A = 6.36 \text{ ms}^{-2}$$

$$a_{BA} = a_A \cos(30°) = 5.5 \text{ ms}^{-2}$$

11. Net pulling force is actually due to the difference of the weights of the rope on both sides of the pulley. So,



$$F_{\text{net}} = \left[\left(\frac{m}{2l}\right)(x) - \left(\frac{m}{2l}\right)(2l - x)\right]g = \frac{mg}{l}(x - l)$$

$$\Rightarrow \quad ma = \frac{g}{l}(x - l)$$

$$\Rightarrow \quad v\frac{dv}{dx} = \frac{g}{l}(x - l)$$

$$\Rightarrow \quad \int_0^v v\,dv = \frac{g}{l}\int_{(l+c)}^{x}(x - l)\,dx$$

Since $\int (x - a)\,dx = \frac{(x - a)^2}{2}$, so

$$\frac{v^2}{2} = \frac{g}{2l}\left[(x - l)^2\right]_{l+c}^{x}$$

$$\Rightarrow \quad v^2 = \frac{g}{l}\left[(x - l)^2 - c^2\right]$$

$$\Rightarrow \quad v = \frac{dx}{dt} = \sqrt{\frac{g}{l}\left[(x - l)^2 - c^2\right]}$$

$$\Rightarrow \quad \int_0^t dt = \int_{(l+c)}^{2l}\frac{dx}{\sqrt{\frac{g}{l}\left[(x - l)^2 - c^2\right]}}$$

$$\Rightarrow \quad t = \sqrt{\frac{l}{g}}\ln\left[\frac{l + \sqrt{l^2 - c^2}}{c}\right]$$

12. Let absolute acceleration of m and M be a_1 and a_2 respectively.

Writing equations of motion

for m, $mg\cos\alpha - N = ma_1$ ...(1)

and for M, $N\sin\alpha = Ma_2$ ...(2)

Constraint equation can be written as,

$a_1 = a_2\sin\alpha$ ...(3)

Solving above three equations, we get

Acceleration of rod is

$$a_1 = \frac{mg\cos\alpha\sin\alpha}{\left(m\sin\alpha + \frac{M}{\sin\alpha}\right)}$$

and acceleration of wedge is

$$a_2 = \frac{mg\cos\alpha}{m\sin\alpha + \frac{M}{\sin\alpha}}$$

Please note that here $a_2 \neq a_1\sin\alpha$.

13. $a = \frac{3T - mg\sin\theta}{m} = \frac{(3)(250) - (100)(10)\sin(30°)}{100}$

$$\Rightarrow \quad a = \frac{750 - 500}{100}$$

$$\Rightarrow \quad a = \frac{250}{100} = 2.5\text{ ms}^{-2}$$

14. As done already, from constraint relations we can see that $2a_A = 3a_B$

$$\Rightarrow \quad a_A = \frac{3}{2}a_B$$

So let $a_B = a$ then $a_A = 1.5a$

for block A, we have

$2T = 70a_A = 105a$ ...(1)

for block B, we have

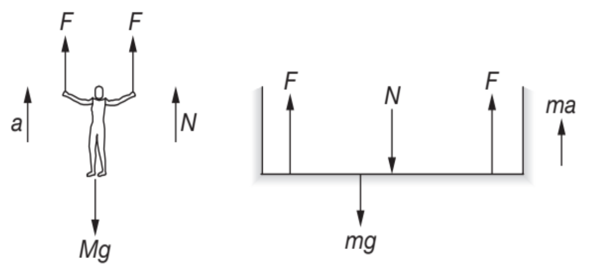
$300 - 3T = 35a_B = 35a$ ...(2)

Solving equations (1) and (2), we get

$T = 81.8$ N and $a = 1.558\text{ ms}^{-2}$

$\Rightarrow \quad T = 81.8$ N, $a_A = 2.34\text{ ms}^{-2}$ and $a_B = 1.558\text{ ms}^{-2}$

15. Let N be the force exerted by the painter on the crate (downwards). Corresponding the same force will be exerted by the crate on the painter (upwards). The free body diagram for both is shown here.



For painter, $2F - Mg + N = Ma$ ...(1)

For crate, $2F - mg - N = ma$ ...(2)

Adding (1) and (2), we get

$$4F - (M + m)g = (M + m)a$$

$$\Rightarrow \quad a = \frac{4F - (M + m)g}{M + m}$$

16. Time taken by lighter man to reach the pulley is given by

$$h = \frac{1}{2}a_1t^2 \qquad \ldots(1)$$

For lighter man,

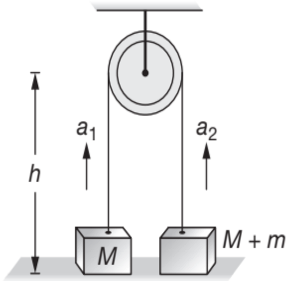
$$T - Mg = Ma_1 \qquad \ldots(2)$$

For heavier man,

$$T - (M+m)g = (M+m)a_2 \qquad \ldots(3)$$

From equations (2) and (3), we get

$$a_2 = \left(\frac{M}{M+m}\right)a_1 - \left(\frac{m}{M+m}\right)g \qquad \ldots(4)$$



In time t, the heavier mass will move a distance, given by

$$s = \frac{1}{2}a_2t^2$$

$$\Rightarrow \quad s = \frac{1}{2}t^2\left[\left(\frac{M}{M+m}\right)a_1 - \left(\frac{m}{M+m}\right)g\right] \qquad \{\because \text{ of } (4)\}$$

Since, $h = \frac{1}{2}a_1t^2$ so, we have

$$s = \frac{Mh}{M+m} - \left(\frac{m}{M+m}\right)\frac{gt^2}{2}$$

$$\Rightarrow \quad h - s = \frac{m}{M+m}\left(\frac{gt^2}{2} + h\right)$$

17. Since $a_0(=12 \text{ ms}^{-2}) > g(=10 \text{ ms}^{-2})$, so the block will leave contact will the floor of the lift and starts falling with an acceleration $a = g = 10 \text{ ms}^{-2}$, so we have

$$s = \frac{1}{2}at^2 = \frac{1}{2}\times 10 \times (0.2)^2 = 0.2 \text{ m}$$

$$\Rightarrow \quad s = 20 \text{ cm}$$

18. This motion is just similar to the motion of a projectile moving under the influence of an acceleration $g\sin(45°)$, instead of g.

So, after 2 seconds, we have

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\Rightarrow \quad v = \sqrt{\left(10\sin(45°) - \frac{g}{\sqrt{2}}\times 2\right) + (10\cos 45°)^2}$$

$$\Rightarrow \quad v = 10 \text{ ms}^{-1}$$

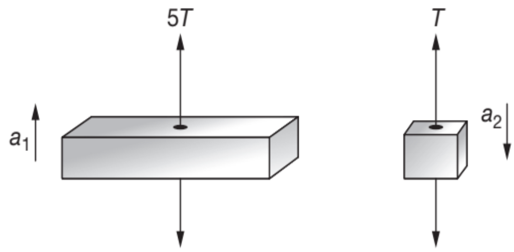
19. Writing equations of motion

for M, $5T - Mg = Ma_1 \qquad \ldots(1)$

for m, $mg - T = ma_2 \qquad \ldots(2)$

Also, from constraint equation, we get

$$a_2 = 5a_1 \qquad \ldots(3)$$



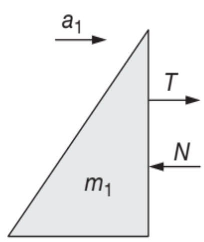
FBD of M FBD of m

Solving these equations, the acceleration of M and m are given by

$$a_1 = \left(\frac{5m - M}{25m - M}\right)g \text{ and } a_2 = 5\left(\frac{5m - M}{25m - M}\right)g$$

20. FBD of m_1 (showing only the horizontal forces)

Equation of motion for m_1 is, $T - N = m_1a_1 \qquad \ldots(1)$

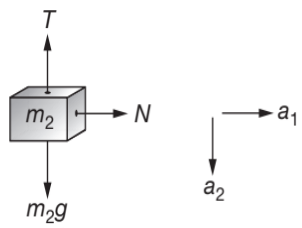


FBD of m_1

Equations of motion for m_2 are given by

$$N = m_2a_1 \text{ and} \qquad \ldots(2)$$

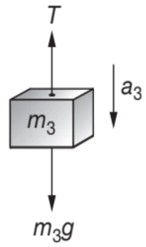
$$m_2g - T = m_2a_2 \qquad \ldots(3)$$



FBD of m_2

Equation of motion for m_3 is

$$m_3g - T = m_3a_3 \quad \ldots(4)$$



FBD of m_3

Further from constraint equation we get

$$a_1 = a_2 + a_3 \quad \ldots(5)$$

Solving the five unknowns a_1, a_2, a_3, T and N, we get

$$a_1 = \frac{2m_1m_3g}{(m_2+m_3)(m_1+m_2)+m_2m_3}$$

21. In mode (a), the man applies a force equal to 25 kg wt in upward direction. According to Newton's Third Law of motion, there will be a downward force of reaction on the floor.

So, total action on the floor by the man is

$$N_1 = 50 \text{ kg wt} + 25 \text{ kg wt} = 75 \text{ kg wt}$$

$$N_1 = 75 \times 9.8 \text{ N} = 735 \text{ N}$$

In mode (b), the man applies a downward force equal to 25 kg wt. According to Newton's Third Law, the reaction will be in the upward direction.

So, total action on the floor by the man is

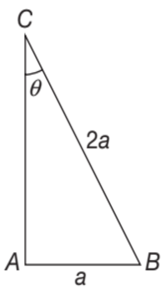
$$N_2 = 50 \text{ kg wt} - 25 \text{ kg wt} = 25 \text{ kg wt}$$

$$N_2 = 25 \times 9.8 \text{ N} = 245 \text{ N}$$

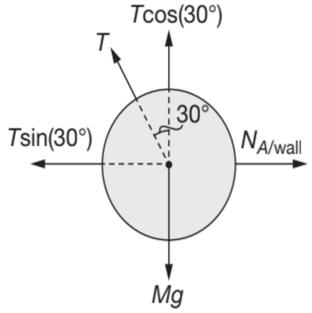
As the floor yields to a downward force of 700 N, so the man should adopt mode (b).

Test Your Concepts-V (Based on Equilibrium)

1. Since $\sin\theta = \frac{a}{2a} = \frac{1}{2}$



$$\Rightarrow \quad \theta = 30°$$



For equilibrium,

$$T\cos(30°) = Mg$$

$$\Rightarrow \quad T = \frac{2Mg}{\sqrt{3}} \quad \ldots(1)$$

Since $N_{A/\text{wall}} = T\sin 30° = \frac{T}{2}$

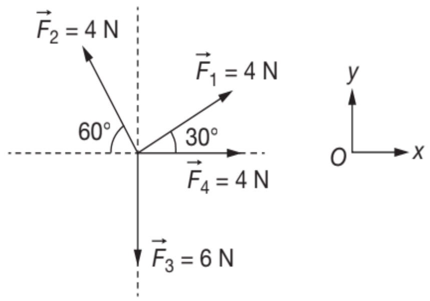
$$\Rightarrow \quad N_{A/\text{wall}} = \frac{Mg}{\sqrt{3}} \quad \ldots(2)$$

2.

	F_1	F_2	F_3	F_4
x	4 cos (30°)	4 cos (120°)	6 cos (90°)	4 cos (0°)
y	4 sin (30°)	4 sin (120°)	6 sin (90°)	4 sin (0°)

$(F_1)_x = 2\sqrt{3}$ N, $(F_2)_x = -2$ N and

$(F_3)_x = 0$ N, $(F_4)_x = 4$ N



Similarly,

$$(F_1)_y = 2 \text{ N}, \; (F_2)_y = 2\sqrt{3} \text{ N}$$

$$(F_3)_y = 6 \text{ N}, \; (F_4)_y = 0 \text{ N}$$

So, $(F_{\text{net}})_x = (F_1)_x + (F_2)_x + (F_3)_x + (F_4)_x$

$$\Rightarrow \quad (F_{\text{net}})_x = 2\sqrt{3} - 2 + 0 + 4 = 2(\sqrt{3}+1) \text{ N}$$

Similarly, $(F_{\text{net}})_y = 8 + 2\sqrt{3} \text{ N} = 2(4+\sqrt{3}) \text{ N}$

$$\Rightarrow \quad \vec{F}_{\text{net}} = 2\left[(\sqrt{3}+1)\hat{i} + (4+\sqrt{3})\hat{j}\right] \text{N}$$

3. **METHOD I**

Since, the block is permanently at rest, it is in equilibrium. Net force on it should be zero. In this case only two forces are acting on the block.

(a) Weight $= mg$ (downwards)

(b) Contact force (resultant of normal reaction and friction force) applied by the wedge on the block. For the block to be in equilibrium these two forces should be equal and opposite.

Therefore, force exerted by the wedge on the block is mg (upwards).

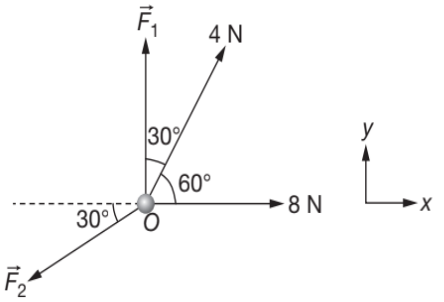
METHOD II

The normal force on the block is $N = mg\cos\theta$ and the friction force on the block is $f = mg\sin\theta = \mu mg\cos\theta$. These two forces are mutually perpendicular. So, net contact force would be

$$\sqrt{N^2 + f^2} = \sqrt{(mg\cos\theta)^2 + (mg\sin\theta)^2} = mg$$

4. The object is in equilibrium. Hence,

$$\sum F_x = 0 \text{ and } \sum F_y = 0$$



$$\Rightarrow \quad 8 + 4\cos(60°) - F_2\cos(30°) = 0 \quad \left\{\because \sum F_x = 0\right\}$$

$$\Rightarrow \quad 8 + 2 - F_2\frac{\sqrt{3}}{2} = 0$$

$$\Rightarrow \quad F_2 = \frac{20}{\sqrt{3}} \text{ N}$$

$$\Rightarrow \quad F_1 + 4\sin(60°) - F_2\sin(30°) = 0 \quad \left\{\because \sum F_y = 0\right\}$$

$$\Rightarrow \quad F_1 + \frac{4\sqrt{3}}{2} - \frac{F_2}{2} = 0$$

$$\Rightarrow \quad F_1 = \frac{F_2}{2} - 2\sqrt{3} = \frac{10}{\sqrt{3}} - 2\sqrt{3}$$

$$\Rightarrow \quad F_1 = \frac{4}{\sqrt{3}} \text{ N}$$

5. $\vec{F}_{\text{net}} = 0$, i.e., $\vec{F}$ and $m\vec{g}$ should be equal and opposite i.e., balance each other, so we have

$$|\vec{F}| = |m\vec{g}|$$

$$\Rightarrow \quad vA\sin\theta = mg$$

$$\Rightarrow \quad v = \frac{mg}{A\sin\theta}$$

v will be MINIMUM, when $\sin\theta$ is maximum. So $\sin\theta = \text{MAXIMUM} = 1$

$$\Rightarrow \quad \theta = 90° \text{ and } V_{\min} = \frac{mg}{A}$$

6. Various forces acting on the ball are as shown in figure. The three concurrent forces are in equilibrium. Using Lami's theorem

$$\frac{T_1}{\sin(150°)} = \frac{T_2}{\sin(120°)} = \frac{10}{\sin(90°)}$$

$$\Rightarrow \quad \frac{T_1}{\sin(30°)} = \frac{T_2}{\sin(60°)} = \frac{10}{1}$$

So, $T_1 = 10\sin(30°) = 10 \times 0.5 = 5$ N and

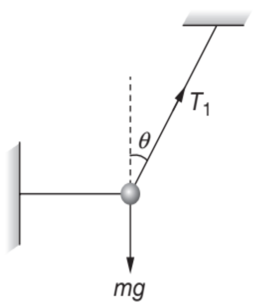
$$T_2 = 10\sin(60°) = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3} \text{ N}$$

7. In the first case, ball is in equilibrium. Therefore, the net force on ball in any direction should be zero. So,

$$(\Sigma\vec{F}) \text{ in vertical direction} = 0$$

$$\Rightarrow \quad T_1\cos\theta = mg$$

$$\Rightarrow \quad T_1 = \frac{mg}{\cos\theta}$$



Substituting $m_1 = 1$ kg, $g = 10$ ms^{-2} and $\theta = 45°$

$$\Rightarrow \quad T_1 = 10\sqrt{2} \text{ N}$$

In the second case ball is not in equilibrium (temporary rest). After few seconds it will move in a direction perpendicular to OQ. Therefore, net force on the ball at Q is perpendicular to OQ, or net force along $OQ = 0$. So, we get

$$T_2 = mg\cos\theta$$

Substituting the values, we get

$$T_2 = 5\sqrt{2}\ \text{N}$$

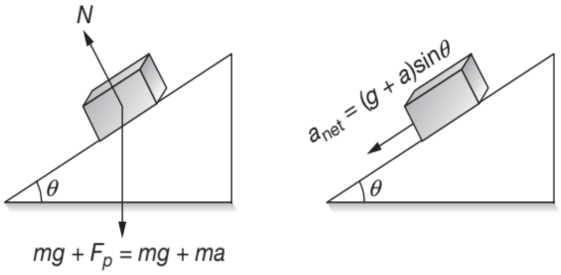
Also, we observe that

$$T_1 \neq T_2$$

Here we deliberately resolved all the forces in vertical direction because component of the tension along the vertical, in the branch RP is zero. Although, since, the ball is in equilibrium, net force on it in any direction is zero. But in a direction other than vertical we will have to consider component of tension in RP also, which will unnecessarily increase the calculation.

Test Your Concepts-VI (Based on Non-inertial Frames: Pseudo Force)

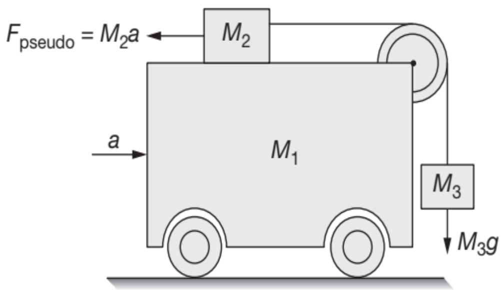
1. Since, the acceleration of block w.r.t. wedge, which happens to be an accelerating or non-inertial frame of reference, is to be find out. So, let us draw the F.B.D. of block w.r.t. wedge is shown in figure.



The acceleration would had been $g\sin\theta$ (down the plane) if the lift were stationary or when only weight (i.e., mg) acts downwards. In this case, however, the downward force is $m(g+a)$.

So, the acceleration of the block w.r.t. wedge will be $(g+a)\sin\theta$, down the plane.

2. The F.B.D. of M_2 and M_3 in accelerated frame of reference is shown in figure



NOTE: Only the necessary forces have been shown.

Mass M_3 will neither rise nor fall if net pulling force is zero.

i.e., $M_2a = M_3g$

$$\Rightarrow \quad a = \frac{M_3}{M_2}g$$

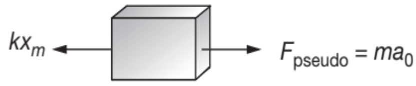
$$\Rightarrow \quad F = (M_1+M_2+M_3)a = (M_1+M_2+M_3)\frac{M_3}{M_2}g$$

3. In equilibrium, we get

$$kx_m = ma_0$$

$$\Rightarrow \quad x_m = \frac{ma_0}{k}$$

Here, x_m is maximum compression in spring, which is also equal to its amplitudes of oscillation.



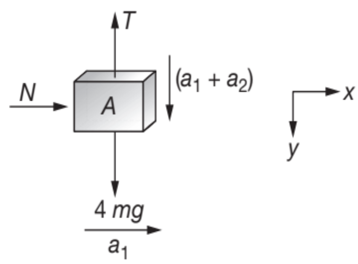
Since $(v_{rel})_{max} = (x_m)\omega$

$$\Rightarrow \quad (v_{rel})_{max} = \frac{ma_0}{k}\sqrt{\frac{k}{m}}$$

$$\Rightarrow \quad (v_{rel})_{max} = a_0\sqrt{\frac{m}{k}}$$

4. Let, acceleration of block C be a_1 (rightwards) and acceleration of block B be a_2 (leftwards)

Then, acceleration of A will be (a_1+a_2) downwards and a_1 rightwards.

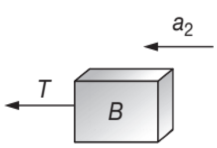


Drawing the free body diagram of A and using $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$, we get

$$N = 4m(a_1) \quad \ldots(1)$$

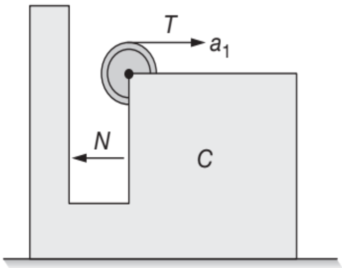
and $4mg - T = 4m(a_1+a_2)$ ...(2)

Drawing the free body diagram of B (showing horizontal forces only) and using $\Sigma F_x = ma_x$, we get



$$T = 3ma_2 \qquad \ldots(3)$$

Again, drawing the free body diagram of C (showing horizontal forces only) and using $\Sigma F_x = ma_x$, we get



$$T - N = 8ma_1 \qquad \ldots(4)$$

Now, we have four unknowns a_1, a_2, T and N. Solving these four equations, we get

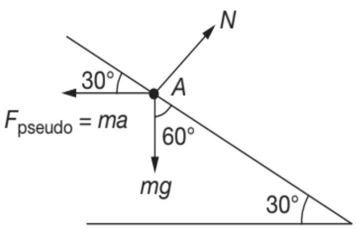
$$a_1 = \frac{g}{8} \text{ and } a_2 = \frac{g}{2}$$

$$\Rightarrow \quad a_1 + a_2 = \frac{5}{8}g$$

So, acceleration of A is $\frac{g}{8}$ in horizontal direction and $\frac{5g}{8}$ in vertical direction.

Acceleration of B is $\frac{g}{2}$ in horizontal direction (leftwards) and acceleration of C is $\frac{g}{8}$ in horizontal direction (rightwards).

5. A is in equilibrium under three concurrent forces shown in figure, so applying Lami's theorem, we get



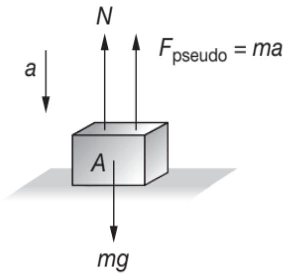
$$\frac{ma}{\sin(90° + 60°)} = \frac{mg}{\sin(90° + 30°)}$$

$$\Rightarrow \quad a = \frac{g\cos(60°)}{\cos(30°)} = 5.66 \text{ ms}^{-2}$$

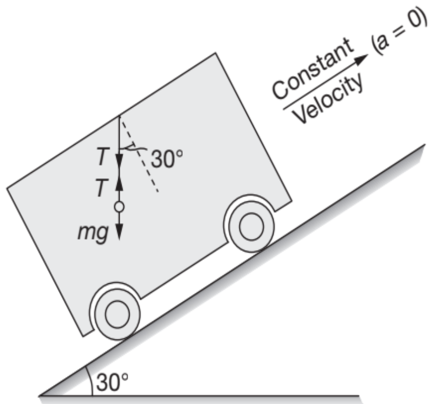
6. Since $N + ma = mg$

$$\frac{mg}{4} + ma = mg$$

$$\Rightarrow \quad a = \frac{3g}{4}$$



7. Since the car is moving up with constant velocity. So it has zero acceleration.



Hence,

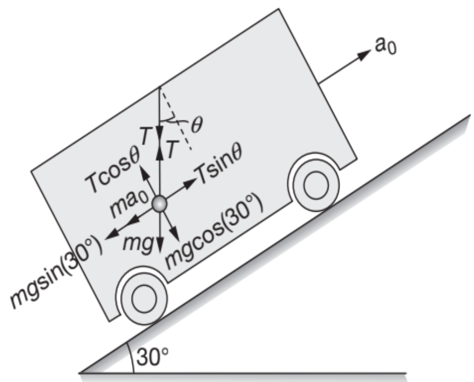
$$\theta = 30°$$

$$T = mg = (2)(10) = 20 \text{ N}$$

8. Let the bob make an angle θ with the normal to the ceiling. Then in the equilibrium state of the bob,

$$T\sin\theta = ma_0 + mg\sin(30°) \text{ and}$$

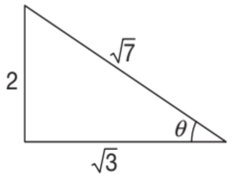
$$T\cos\theta = mg\cos(30°)$$



$$\Rightarrow \quad \tan\theta = \frac{a_0 + g\sin(30°)}{g\cos(30°)} = \frac{\frac{g}{2} + \frac{g}{2}}{\sqrt{3}\frac{g}{2}} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

Since $T\cos\theta = mg\cos(30°)$



$$\Rightarrow \quad T\cos\theta = mg\left(\frac{\sqrt{3}}{2}\right)$$

Also, $\tan\theta = \dfrac{2}{\sqrt{3}}$

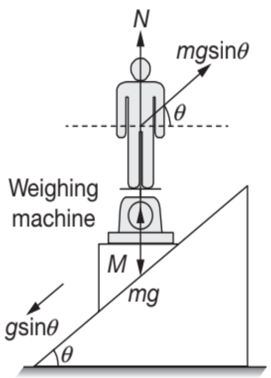
So, we get

$$\cos\theta = \frac{\sqrt{3}}{\sqrt{7}}$$

$$\Rightarrow \quad T\left(\frac{\sqrt{3}}{\sqrt{7}}\right) = (2)(10)\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad T = 10\sqrt{7}\ \text{N}$$

9. If the wedge is in free fall, its acceleration must be $g\sin\theta$. Here we are required to find the weight shown by the weighing machine i.e., the normal force acting between man and the weighing machine. Let us solve the problem in the reference frame of the wedge. The forces acting on man in the reference of wedge block are shown in figure.

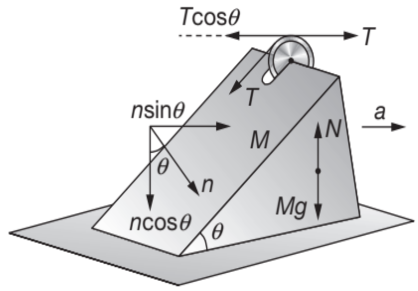


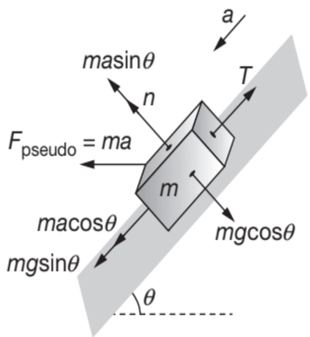
With respect to wedge frame man will experience a pseudo force $mg\sin\theta$ as shown opposite to the acceleration of the wedge. Now for equilibrium of man relative to wedge we have

$$N + mg\sin^2\theta = mg$$

$$\Rightarrow \quad N = mg - mg\sin^2\theta = mg\cos^2\theta$$

10. Using our knowledge of constrained analysis we observe that if the wedge moves toward right by a distance x, then the small bar will travel equal distance x down the inclined plane of wedge. Thus both the bodies will move with same acceleration, wedge towards right on earth and bar downward along incline on wedge. As bar is taken on wedge (a non inertial frame) a pseudo force is applied on bar towards left as shown in its free body diagram. Now we write down the motion equations of m and M according to forces shown in their F.B.D.s in figure.





For M

$$T + n\sin\theta - T\cos\theta = Ma \qquad \ldots(1)$$

$$N = Mg + n\cos\theta + T\sin\theta \qquad \ldots(2)$$

For m

$$mg\sin\theta + ma\cos\theta - T = ma \qquad \ldots(3)$$

$$ma\sin\theta + n = mg\cos\theta \qquad \ldots(4)$$

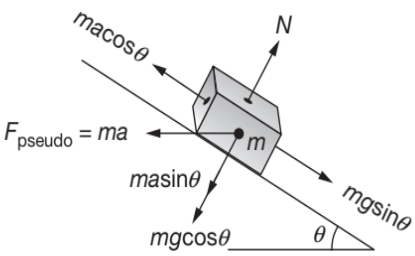
From (3), $T = mg\sin\theta + ma(\cos\theta - 1)$

From (4), $n = mg\cos\theta - ma\sin\theta$

Substitute values of T and n in (1), we get

$$a = \frac{mg\sin\theta}{M + 2m(1-\cos\theta)}$$

11. For the smaller block m, not to slip onsurface of m_0, we have



FBD m

$$mg\sin\theta = ma\cos\theta$$

$$\Rightarrow \quad a = g\tan\theta \qquad \ldots(1)$$

$$N = ma\sin\theta + mg\cos\theta$$

Substituting $a = g\tan\theta$, we get

$$N = mg\sec\theta \qquad \ldots(2)$$

For M

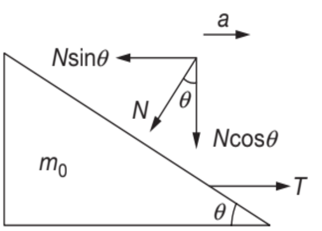
$$Mg - T = ma \qquad \ldots(3)$$

For m

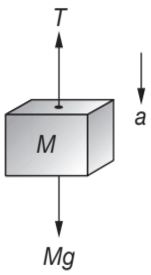
$$T - N\sin\theta = m_0 a \qquad \ldots(4)$$

Adding (3) and (4), we get

$$Mg - N\sin\theta = (M + m_0)a$$



FBD m_0



FBD M

$$\Rightarrow \quad Mg - mg\frac{\sin\theta}{\cos\theta} = (M + m_0)g\tan\theta$$

$$\Rightarrow \quad M - m\tan\theta = (M + m_0)\tan\theta$$

$$\Rightarrow \quad M - m\tan\theta = M\tan\theta + m_0\tan\theta$$

$$\Rightarrow \quad M(1 - \tan\theta) = (m_0 + m)\tan\theta$$

$$\Rightarrow \quad M = \frac{(m_0 + m)\tan\theta}{1 - \tan\theta}$$

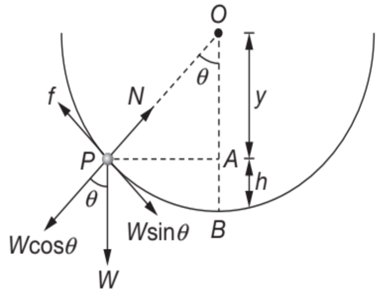
$$\Rightarrow \quad M = \frac{m_0 + m}{\cot\theta - 1}$$

Test Your Concepts-VII (Based on Friction)

1. Let the insect crawl up the bowl upto point P as shown. In doing so, the insect rises through a height $BA = h$, above the bottom B of the bowl of radius r. If W is the weight of insect, then for equilibrium at P,

$$N = W\cos\theta \text{ and}$$

$$f = W\sin\theta$$



From these two equations, we get

$$\tan\theta = \frac{f}{N}$$

In limiting case, we have

$$f = \mu N$$

$$\Rightarrow \quad \frac{f}{N} = \mu$$

Therefore, $\tan\theta = \mu = \dfrac{1}{\sqrt{3}}$

$$\Rightarrow \quad \theta = 30^\circ$$

Since, $y = OP\cos(30^\circ) = \dfrac{\sqrt{3}r}{2}$

and $h = BA = OB - OA = r - y$

$$\Rightarrow \quad h = r - \frac{\sqrt{3}}{2}r = 0.134r$$

2. The given angle 30° is actually the angle of repose α. So, we get

$$\mu = \tan(30^\circ) = \frac{1}{\sqrt{3}}$$

3. Since, $\mu_1 < \mu_2$, acceleration of 2 kg block down the plane will be more than the acceleration of 4 kg block if allowed to move separately. But as the 2 kg block is behind the 4 kg block both of them will move with same acceleration say a. Considering both the blocks as a system, the force down the plane on the system is

$$F_{down} = (4 + 2)g\sin(30^\circ)$$

$$\Rightarrow \quad F_{down} = (6)(10)\left(\frac{1}{2}\right) = 30\text{ N}$$

Similarly, force up the plane on the system is

$$F_{up} = \mu_1(2)(g)\cos(30^\circ) + \mu_2(4)(g)\cos(30^\circ)$$

$$\Rightarrow \quad F_{up} = (2\mu_1 + 4\mu_2)g\cos 30^\circ$$

$\Rightarrow \quad F_{up} = (2\times0.2+4\times0.3)(10)(0.86)$

$\Rightarrow \quad F_{up} = 13.76 \text{ N}$

So, net force down the plane is

$$(F_{net})_{down} = 30-13.76 = 16.24 \text{ N}.$$

Acceleration of both the blocks down the plane will be given by

$$a = \frac{(F_{net})_{down}}{4+2} = \frac{16.24}{6} = 2.7 \text{ ms}^{-2}$$

4. As the box is lying an accelerated frame, so, it experiences a backward force given by

$$F = ma$$

Motion of the box is opposed by the frictional force given by

$$f = \mu mg$$

So, net force on the box in the backward direction is

$$F_{net} = F - f = ma - \mu mg$$

$\Rightarrow \quad F_{net} = 40(2-(0.1)(10)) = 40 \text{ N}$

Acceleration produced in the box relative to truck in the backward direction is

$$a_r = \frac{F_{net}}{m} = \frac{40}{40} = 1 \text{ ms}^{-2}$$

Since $s = \frac{1}{2}a_r t^2$

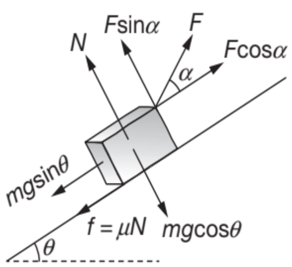
$\Rightarrow \quad t = \sqrt{\frac{2s}{a_r}}$

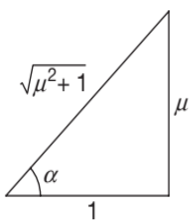
$\Rightarrow \quad t = \sqrt{\frac{2\times5}{1}} = 4.34 \text{ s}$

$\Rightarrow \quad t = 3.16 \text{ s}$

5. In the diagram drawn, we have

$$F\cos\alpha = mg\sin\theta + \mu N \qquad \ldots(1)$$





and $N + F\sin\alpha = mg\cos\theta \qquad \ldots(2)$

$\Rightarrow \quad F\cos\alpha = mg\sin\theta + \mu N$

$\Rightarrow \quad F\cos\alpha = mg\sin\theta + \mu(mg\cos\theta - F\sin\alpha)$

$\Rightarrow \quad F = \frac{mg\sin\theta + \mu mg\cos\theta}{\cos\alpha + \mu\sin\alpha}$

For F to be minimum,

$\cos\alpha + \mu\sin\alpha = z$ (say) should be maximum

$\Rightarrow \quad \frac{dz}{d\alpha} = 0$

$\Rightarrow \quad \alpha = \tan^{-1}(\mu)$

and $F_{min} = \frac{mg(\sin\theta+\mu\cos\theta)}{\left(\frac{1}{\sqrt{\mu^2+1}} + \frac{\mu^2}{\sqrt{\mu^2+1}}\right)}$

$\Rightarrow \quad F_{min} = \frac{mg(\sin\theta+\mu\cos\theta)}{\sqrt{\mu^2+1}}$

6. Retardation offered by the belt to the package is given by

$$a = \mu_k g = 0.15\times9.8 = 1.47 \text{ ms}^{-2}$$

Let s be the distance travelled before the sliding stops then

$$s = \frac{v^2}{2a} = \frac{(5)^2}{2\times1.47} \approx 8.5 \text{ m}$$

7. Limiting friction between A and wedge

$$f_L = \mu_s m_A g\cos\theta = (0.25)(60)(10)\cos(30°)$$

$\Rightarrow \quad f_L = 130 \text{ N}$

and kinetic friction is

$$f_K = \mu_K m_A g\cos\theta = 104 \text{ N}$$

The system will remain stationary when

$$\frac{W_B}{2} + f_L > W_A\sin30°$$

$\Rightarrow \quad 100+130 > 300$

Since this is not happening, so the system will move. Also, we observe that the acceleration a_A down the plane is two times a_B upwards.

Equations of motion are,

$$W_A\sin(30°) - T - f_K = m_A a_A \qquad \ldots(1)$$

$\Rightarrow \quad 300 - T - 104 = 60a_A$

$$2T - W_B = m_B a_B$$

$\Rightarrow \quad 2T - 200 = 20a_B \qquad \ldots(2)$

and $a_A = 2a_B \qquad \ldots(3)$

Solving these three equations, we get

$$T = 107.74 \text{ N}$$

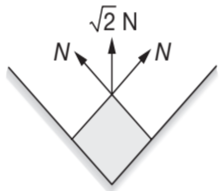
$$a_A = 1.489 \text{ ms}^{-2}$$

and $a_B = 0.744 \text{ ms}^{-2}$

8. Since the block has no motion perpendicular to both the inclines, so we have

$$\sqrt{2}N = mg\cos\theta$$

$$\Rightarrow \quad N = \frac{mg\cos\theta}{\sqrt{2}}$$



Cross sectional view

Down the incline, the acceleration of the block is

$$a = \frac{mg\sin\theta - 2\mu_K N}{m} = g\sin\theta - \sqrt{2}\mu_K g\cos\theta$$

$$\Rightarrow \quad a = g\left(\sin\theta - \sqrt{2}\mu_K\cos\theta\right)$$

9. $a = v\dfrac{dv}{dx} = \dfrac{\text{Net force}}{\text{Mass}} = \dfrac{F - \mu_K\rho(L-x)g}{\rho L}$

$$\Rightarrow \quad \int_0^v v\,dv = \int_0^L \frac{F - \mu_K\rho(L-x)g}{\rho L}dx$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{F}{\rho} - \mu_K gL + \frac{\mu_K gL}{2}$$

$$\Rightarrow \quad v = \sqrt{\frac{2F}{\rho} - \mu_K gL}$$

10. (a) Since $u = 0$, so $v = a_1 t_1 = 2.6 \text{ ms}^{-1}$ $\quad \{\because v = u + at\}$

$$s_1 = \frac{1}{2}a_1 t_1^2 = \frac{1}{2}\times 2\times(1.3)^2 = 1.69 \text{ m} \quad \left\{\because s = ut + \frac{1}{2}at^2\right\}$$

So, $s_2 = (2.2 - 1.69) = 0.51$ m

Now, $s_2 = \dfrac{v^2}{2a_2}$

$$\Rightarrow \quad a_2 = \frac{v^2}{2s_1} = \frac{(2.6)^2}{2\times 0.51} = 6.63 \text{ ms}^{-2}$$

and $t_2 = \dfrac{v}{a_2} = 0.4$ s

(b) Acceleration of package will be 2 ms^{-2} while retardation will be $\mu_K g = 2.5 \text{ ms}^{-2}$.

For the package, we have, as already calculated,

$$v = a_1 t_1 = 2.6 \text{ ms}^{-1}$$

$$s_1 = \frac{1}{2}a_1 t_1^2 = 1.69 \text{ m}$$

So, $s_2' = vt_2 - \dfrac{1}{2}a_2^1 t_2^2 = 2.6\times 0.4 - \dfrac{1}{2}\times 2.5\times(0.4)^2$

$$s_2' = 0.84 \text{ m}$$

So, total displacement of package $= 2.53$ m

Hence, displacement of package w.r.t. belt is

$$s_r = (2.53 - 2.21) \text{ m} = 0.33 \text{ m}$$

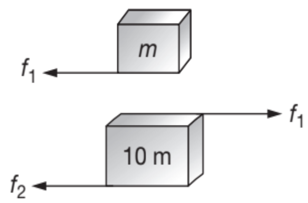
11. (a) Force of friction at different contacts are shown in figure.

Here, $f_1 = \mu_2 mg$ and $f_2 = \mu_1(11mg)$.

Given that $\mu_2 > 11\mu_1$, so

$$f_1 > f_2$$

Retardation of upper block is $a_1 = \dfrac{f_1}{m} = \mu_2 g$



Acceleration of lower block is

$$a_2 = \frac{f_1 - f_2}{m} = \frac{(\mu_2 - 11\mu_1)g}{10}$$

Relative retardation of upper block w.r.t. lower block is

$$a_r = a_1 + a_2$$

$$\Rightarrow \quad a_r = \frac{11}{10}(\mu_2 - \mu_1)g$$

Now, $0 - v_{\text{min}}^2 = 2(-a_r)l \quad \{\because v_r^2 - u_r^2 = 2a_r l_r\}$

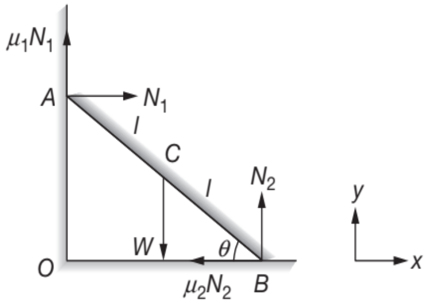
$$\Rightarrow \quad v_{\text{min}} = \sqrt{2a_r l} = \sqrt{\frac{22(\mu_2 - \mu_1)gl}{10}}$$

(b) $0 = v_m - a_r t \quad \{\because v_r = u_r + at\}$

$$\Rightarrow \quad t = \frac{v_m}{a_r} = \sqrt{\frac{20l}{11(\mu_2 - \mu_1)g}}$$

12. For equilibrium of rod, we have

$$\Sigma F_x = 0$$



$$\Rightarrow \quad N_1 = \mu_2 N_2 \qquad \ldots(1)$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad W = N_2 + \mu_1 N_1 \qquad \ldots(2)$$

Also $\Sigma \tau_B = 0$

$$\Rightarrow \quad Wl\cos\theta = N_1(2l\sin\theta) + \mu_1 N_1(2l\cos\theta)$$

$$\Rightarrow \quad \tan\theta = \frac{W - 2\mu_1 N_1}{2N_1} \qquad \ldots(3)$$

From equations (1) and (2), we get

$$N_1 = \frac{\mu_2 W}{1 + \mu_1 \mu_2}$$

Substituting in equation (3), we get

$$\tan\theta = \frac{1 + \mu_1\mu_2}{2\mu_2} - \mu_1$$

Substituting the values of $\mu_1 = \frac{1}{2}$ and $\mu_2 = \frac{1}{4}$, we get

$$\theta = \tan^{-1}\left(\frac{7}{4}\right)$$

13. Assuming that mass of truck $\gg$ mass of crate.
Retardation of truck due to friction is

$$a_1 = (0.9)g = 9 \text{ ms}^{-2},$$

Retardation of crate due to friction is

$$a_2 = (0.7)g = 7 \text{ ms}^{-2}$$

So, relative acceleration of crate is $a_r = 2 \text{ ms}^{-2}$.

Time taken by the truck to stop is t_1 (say), so

$$0 = u + (-a_1)t_1$$

$$\Rightarrow \quad t_1 = \frac{u}{a_1} = \frac{15}{9} \text{ s}$$

Truck will stop after time $t_1 = \frac{15}{9} = 1.67$ s and crate will strike the wall at

$$t_2 = \sqrt{\frac{2s}{a_r}} = \sqrt{\frac{2 \times 3.2}{2}} = 1.78 \text{ s}$$

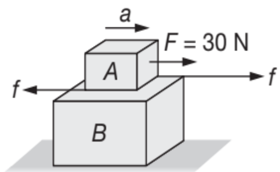
As $t_2 > t_1$, crate will come to rest after travelling a distance

$$s = \frac{1}{2}a_r t_1^2 = \frac{1}{2} \times 2 \times \left(\frac{15}{9}\right)^2 = 2.77 \text{ m}$$

14. Limiting friction between A and B is

$$f_L = \mu N = 0.4 \times 100 = 40 \text{ N}$$

(a) Both the blocks will have a tendency to move together with same acceleration (say a). So, the force diagram is as shown.



Equations of motion are,

$$30 - f = 10 \times a \qquad \ldots(1)$$

$$f = 25 \times a \qquad \ldots(2)$$

Solving these two equations, we get

$$a = 0.857 \text{ ms}^{-2} \text{ and } f = 21.42 \text{ N}$$

Since this force is less than f_L, both the blocks will move together with same acceleration,

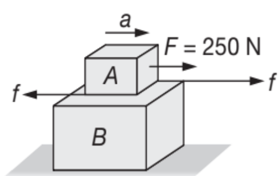
$$a_A = a_B = 0.857 \text{ ms}^{-2}$$

(b) $250 - f = 10a$

$$f = 25a$$

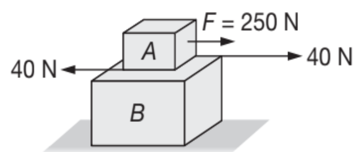
Solving equations (3) and (4), we get

$$f = 178.6 \text{ N}$$



Since, $f > f_L$ so, slipping will take place between the two blocks and hence we have

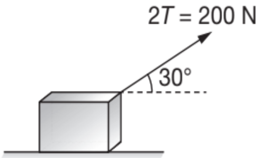
$$f = f_L = 40 \text{ N}$$



$\Rightarrow \quad a_A = \dfrac{250-40}{10} = 21 \text{ ms}^{-2}$

$\Rightarrow \quad a_B = \dfrac{40}{25} = 1.6 \text{ ms}^{-2}$

15. $a = \dfrac{2T\cos(30^\circ) - \mu_K N}{m}$



$$\Rightarrow \quad a = \frac{200\cos(30^\circ) - 0.4(400 - 200 \times \sin(30^\circ))}{40}$$

$\Rightarrow \quad a = 1.33 \text{ ms}^{-2}$

16. (a) Common acceleration,

$$a = \frac{F\cos\alpha}{m_A + m_B} = \frac{F\cos(30^\circ)}{1+2}$$

$$\Rightarrow \quad a = \frac{F}{2\sqrt{3}}$$

(b) Normal reaction between A and B,

$$N = m_A g + F\sin\alpha = \left(10 + \frac{F}{2}\right)$$

Therefore, maximum friction between A and B is,

$$f_{\max} = \mu N = \frac{1}{2}\left(10 + \frac{F}{2}\right) = \left(5 + \frac{F}{4}\right)$$

The friction helps the block B to move. Hence, maximum acceleration of B can be

$$a_{\max} = \frac{f_{\max}}{m_B} = \frac{5 + \left(\frac{F}{4}\right)}{2} = 2.5 + 0.125F$$

This is also the maximum common acceleration.

Hence, $F\cos(30^\circ) = (m_A + m_B)a_{\max}$

$$\Rightarrow \quad \frac{\sqrt{3}F}{2} = 3(2.5 + 0.125F)$$

$\Rightarrow \quad F = 15.27N$

17. $a = \dfrac{m_A g}{m_A + M + m}$

For the equilibrium of B,

$$mg = \mu N = \mu(ma) = \frac{\mu m m_A g}{m_A + M + m}$$

$$\Rightarrow \quad m_A = \frac{(M+m)m}{(\mu - 1)m}$$

$$\Rightarrow \quad m_A = \frac{(M+m)}{\mu - 1}$$

Please note that since $m_A > 0$

So, $\mu > 1$

18. Equations of motion are,

$$T - \mu M_1 g = M_1 a_1 \quad \ldots(1)$$

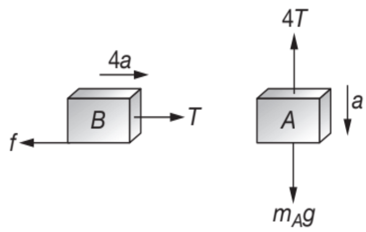
$$T - \mu M_2 g = M_2 a_2 \quad \ldots(2)$$

$$M_3 g - 2T = M_3\left(\frac{a_1 + a_2}{2}\right) \quad \ldots(3)$$

We have three unknowns T, a_1 and a_2. Solving these three equations, we get

$$T = \frac{(\mu + 1)g}{\dfrac{2}{M_3} + \dfrac{1}{2M_1} + \dfrac{1}{2M_2}}$$

19. Let T be the tension in the string attached to block B. Then tension in the string connected to block A would be $4T$.



Similarly, if a be the acceleration of block A (downwards), then acceleration of block B towards right will be $4a$.

Writing equations of motion

for block A, $m_A g - 4T = m_A a$

$$\Rightarrow \quad 50 - 4T = 5a \quad \ldots(1)$$

for block B, $T - f = 10(4a)$

$$\Rightarrow \quad T - (0.1)(10)(10) = 40a$$

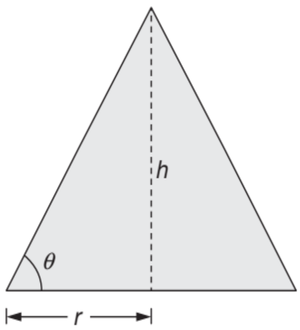
$$\Rightarrow \quad T - 10 = 40a \quad \ldots(2)$$

Solving equations (1) and (2), we get

$$a = 0.06 \text{ ms}^{-2}$$

20. $\tan\theta = \dfrac{h}{r}$ ...(1)

Here θ cannot be greater than the angle of repose $\alpha = \tan^{-1}(\mu)$.



Thus, maximum value of θ can be $\tan^{-1}(\mu)$. In critical case, we have

$$\theta = \tan^{-1}(\mu)$$

$\Rightarrow$ $\tan\theta = \mu$ ...(2)

From equations (1) and (2), we get

$$h = \mu r$$

So, maximum volume is

$$V_{max} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2(\mu r)$$

$$\Rightarrow \quad V_{max} = \frac{1}{3}\pi\mu r^3$$

21. Let, f_1 be the limiting friction between A and ground so, $f_1 = \mu(m_1 + m_2)g$

Let f_2 be the limiting friction between A and B so, $f_2 = \mu(m_2)g$

Net pulling force

$$F = m_3 g - f_1 - f_2$$

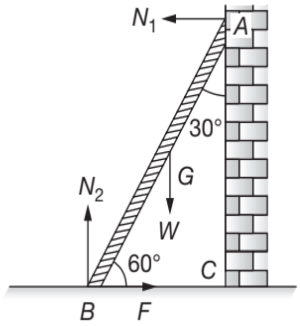
$$\Rightarrow \quad F = m_3 g - \mu(m_1 + 2m_2)g$$

$$\Rightarrow \quad a_A = a_C = \frac{F}{m_1 + m_3} = \frac{g(m_3 - \mu(2m_2 + m_1))}{m_1 + m_3}$$

22. In figure AB is a ladder of weight W which acts at its centre of gravity G. Let N_1 be the reaction of the wall and N_2 the reaction of the ground.

$$\angle ABC = 60°$$

$$\Rightarrow \quad \angle BAC = 30°$$



Force of friction f between the ladder and the ground acts along BC.

For horizontal equilibrium $f = N_1$ ...(1)

For vertical equilibrium $N_2 = W$ ...(2)

Taking moments of the forces about B, we get, for equilibrium,

$$N_1(4\cos 30°) - W(2\cos 60°) = 0 \quad ...(3)$$

where, $W = 250$ N

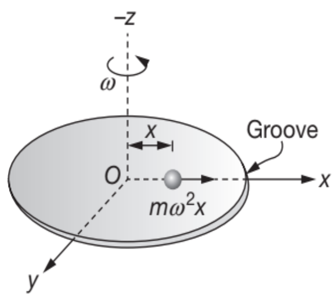
Solving these three equations, we get

$$f = 72.17 \text{ N and } N_2 = 250 \text{ N}$$

$$\Rightarrow \quad \mu = \frac{f}{N_2} = \frac{72.17}{250} = 0.288$$

Test Your Concepts-VIII (Based on Circular Motion)

1. The situation is shown in figure. Let us work from the frame of reference of the table. Now take the origin at the centre of rotation O and the x-axis along the groove. The y-axis is along the line perpendicular to OX, coplanar with the surface of the table and the z-axis is along the vertical. Let at time t the particle in the groove be at a distance x from the origin and is moving along the x-axis with a speed v. The forces acting on the particle (including the pseudo force) are:



(a) weight mg vertically downward,

(b) normal contact force N_1 vertically upward by the bottom surface of the groove,

(c) normal contact force N_2 parallel to the y-axis by the side walls of the groove,

(d) centrifugal force $m\omega^2 x$ along the x-axis.

As the particle can only move in the groove, its acceleration is along the x-axis. The only force along the x-axis is the centrifugal force $m\omega^2 x$. All the other forces are perpendicular to the x-axis and have no components along the x-axis.
Thus, the acceleration along x-axis is

$$a = \frac{F}{m} = \omega^2 x$$

$$\Rightarrow \quad v\frac{dv}{dx} = \omega^2 x$$

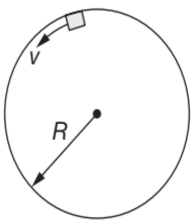
$$\Rightarrow \quad vdv = \omega^2 xdx$$

$$\Rightarrow \quad \int_0^v v\,dv = \int_a^L \omega^2 xdx$$

$$\Rightarrow \quad \left(\frac{1}{2}v^2\right)\Big|_0^v = \left(\frac{1}{2}\omega^2 x^2\right)\Big|_a^L$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{1}{2}\omega^2(L^2 - a^2)$$

$$\Rightarrow \quad v = \omega\sqrt{L^2 - a^2}$$



2. (a) $N = \dfrac{mv^2}{R}$

(b) $f = \mu N = \dfrac{\mu m v^2}{R}$

(c) $a_t = \dfrac{f}{m} = \dfrac{\mu v^2}{R}$

(d) $v\dfrac{dv}{ds} = -\dfrac{\mu v^2}{R}$

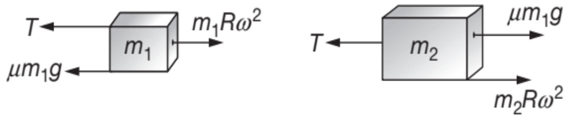
Here negative sign indicates the retarding nature due to friction.

$$\Rightarrow \quad \int_{v_0}^{v} \frac{dv}{v} = -\frac{\mu}{R}\int_0^{2\pi R} ds$$

$$\Rightarrow \quad \log_e\left(\frac{v}{v_0}\right) = -2\pi\mu$$

$$\Rightarrow \quad v = v_0 e^{-2\pi\mu}$$

3. Free body diagrams of m_1 and m_2 from rotating frames are as shown for convenience



Please note that only horizontal forces have been shown.
For equilibrium, we have

$$m_1R\omega^2 = T + \mu m_1 g \qquad \ldots(1)$$

$$T = m_2R\omega^2 + \mu m_1 g \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$\omega = \sqrt{\frac{2m_1\mu g}{(m_1 - m_2)R}}$$

Substituting the values, we get

$$\omega_{\text{min}} = 6.32 \text{ rads}^{-1}$$

Since from (2), we have

$$T = m_2R\omega^2 + \mu m_1 g$$

$$\Rightarrow \quad T = (1)(0.5)(6.32)^2 + (0.5)(2)(10) \cong 30 \text{ N}$$

4. (a) $mL\omega^2 \le \mu mg$

$$\Rightarrow \quad \omega \le \sqrt{\frac{\mu g}{L}}$$

$$\Rightarrow \quad \omega_{MAX} = \sqrt{\frac{\mu g}{L}}$$

(b) $\mu g = \sqrt{a_T^2 + a_C^2} = \sqrt{(L\alpha)^2 + (L\omega^2)^2}$

$$\Rightarrow \quad L^2\omega^4 = \mu^2 g^2 - L^2\alpha^2$$

$$\Rightarrow \quad \omega = \left(\left(\frac{\mu g}{L}\right)^2 - \alpha^2\right)^{\frac{1}{4}}$$

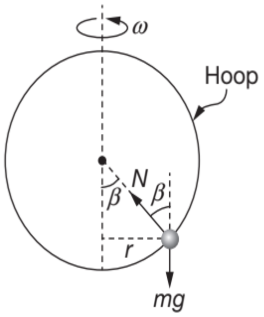
5. (a) $f = \dfrac{mv^2}{R} = \dfrac{85 \times (9)^2}{25} = 275.4 \text{ N}$

(b) $N = mg = 833 \text{ N}$

So, net force on bicycle from the road is

$$F_{\text{net}} = \sqrt{N^2 + f^2} = 877.3 \text{ N}$$

6. (a) From the diagram, we get



$$N\sin\beta = mr\omega^2 = m(R\sin\beta)\omega^2 \quad \ldots(1)$$

$$N\cos\beta = mg \quad \ldots(2)$$

From equations (1) and (2), we get

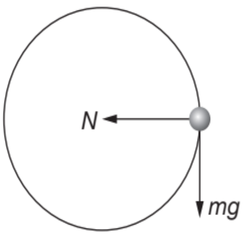
$$\beta = \cos^{-1}\left(\frac{g}{R\omega^2}\right)$$

Since, $\omega = 2\pi f = 6\pi \text{ rads}^{-1}$

$$\Rightarrow \quad \beta = \cos^{-1}\left(\frac{10}{0.1\times 36\times \pi^2}\right)$$

$$\beta \cong 74^\circ$$

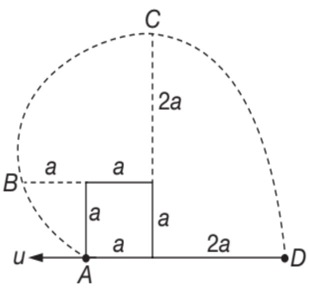
(b) At the same level as the centre of the hoop, component of N in vertical direction is zero, so the bead will fall down due to weight mg and hence it is not possible for the bead to stay at this level.



7. Time $t = \dfrac{AB+BC+CD}{u}$

$$\Rightarrow \quad t = \frac{\left\{\left(\frac{2\pi a}{4}\right)+\frac{2\pi(2a)}{4}+\frac{2\pi(3a)}{4}\right\}}{u}$$

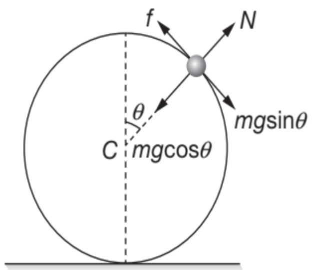
$$\Rightarrow \quad t = \frac{3\pi a}{u}$$



8. (a) $mg = \dfrac{mv_{\min}^2}{R}$

$$\Rightarrow \quad v_{\min} = \sqrt{gR}$$

(b) Velocity at angle θ with respect to point C is $v = R\omega$



Now, $mg\cos\theta - N = \dfrac{mv^2}{R}$

$$\Rightarrow \quad N = mg\cos\theta - \frac{mv^2}{R} \quad \ldots(1)$$

The pebble slides down, when

$$f \le mg\sin\theta$$

$$\Rightarrow \quad \mu N = mg\sin\theta \quad \ldots(2)$$

From equations (1) and (2), we get for $\mu = 1$,

$$\theta = \cos^{-1}\left(\frac{v^2}{\sqrt{2}Rg}\right) - \frac{\pi}{4}$$

9. Since $\tan\theta = \dfrac{v^2}{Rg}$

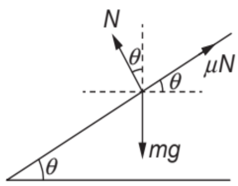
$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{v^2}{Rg}\right) = \tan^{-1}\left(\frac{100}{200}\right) = \tan^{-1}\left(\frac{1}{2}\right)$$

For $v_{\min}$:

Friction is outwards. Hence the equations of motion are,

$$N\sin\theta - \mu N\cos\theta = \frac{mv^2}{R} \quad \ldots(1)$$

$$N\cos\theta + \mu N\sin\theta = mg \quad \ldots(2)$$



From equations (1) and (2), we get

$$v_{\min} = \sqrt{gR\left(\frac{\tan\theta - \mu}{1+\mu\tan\theta}\right)}$$

Substituting the values, we get

$$v_{min} = \sqrt{10 \times 20 \left(\frac{\frac{1}{2} - 0.4}{1 + 0.4 \times \frac{1}{4}} \right)} = 4.08 \text{ ms}^{-1}$$

For v_{min}:

In this case friction is inwards. Hence the equations of motion are,

$$N \sin\theta + \mu N \cos\theta = \frac{mv_{max}^2}{R} \quad \ldots(3)$$

$$N \cos\theta - \mu N \sin\theta = mg \quad \ldots(4)$$

From equations (3) and (4), we get

$$v_{max} = \sqrt{gR\left(\frac{\tan\theta + \mu}{1 - \mu \tan\theta}\right)}$$

$$\Rightarrow \quad v_{max} = \sqrt{10 \times 20 \left(\frac{\frac{1}{2} + 0.4}{1 - 0.4 \times \frac{1}{2}} \right)} = 15 \text{ ms}^{-1}$$

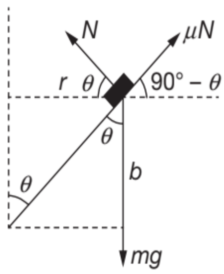
10. $\frac{b}{r} = \cot\theta$

$$\Rightarrow \quad r = b \tan\theta \quad \ldots(1)$$

For ω_{min}, we have

$$N \cos\theta - \mu N \sin\theta = mr\omega^2 \quad \ldots(2)$$

$$N \sin\theta + \mu N \cos\theta = mg \quad \ldots(3)$$



From the above equations, we get

$$\omega_{min} = \sqrt{\frac{g(1 - \mu \tan\theta)}{b \tan\theta(\mu + \tan\theta)}}$$

For ω_{max}, friction μN will act inwards. Equations of motion are,

$$N \cos\theta + \mu N \sin\theta = mr\omega^2 \quad \ldots(4)$$

$$\text{and} \quad N \sin\theta - \mu N \cos\theta = mg \quad \ldots(5)$$

Solving equations (1), (4) and (5), we get

$$\omega_{max} = \sqrt{\frac{g(1 + \mu \tan\theta)}{b \tan\theta(\tan\theta - \mu)}}$$

11. The maximum speed without slipping is given by

$$v_{MAX} = \sqrt{\mu r g} = \sqrt{(0.9) \times (40) \times (9.8)} = 18.8 \text{ ms}^{-1}$$

The maximum speed without overturning

$$v_1 = \sqrt{\frac{gra}{h}}$$

When the car turns right, then

$$a = \frac{1.15}{2} + 0.075 = 0.65 \text{ m}$$

So, for the first bend

$$v_1 = \sqrt{\frac{(9.8)(40)(0.65)}{0.6}} = 20.6 \text{ ms}^{-1}$$

When the car turns left, then

$$a = \frac{1.15}{2} - 0.075 = 0.5 \text{ m}$$

For second bend the maximum speed without overturning

$$v_2 = \sqrt{\frac{9.8 \times 0.5 \times 40}{0.6}} = 18.1 \text{ ms}^{-1}$$

As for the first bend, $v_{MAX} < v_1$, so the car skids.

For second bend, $v_2 < v_{MAX}$, so the car overturns.

12. $N = \frac{mv^2}{R}$

$$f_{max} = \mu N = \frac{\mu m v^2}{R}$$

So, retardation $a = \frac{f_{max}}{m} = \frac{\mu v^2}{R}$

$$\Rightarrow \quad \left(-\frac{dv}{dt}\right) = \frac{\mu v^2}{R}$$

$$\Rightarrow \quad \int_{v_0}^{v} \frac{dv}{v^2} = -\frac{\mu}{R}\int_0^t dt$$

$$\Rightarrow \quad v = \frac{v_0}{\left(1 + \frac{\mu v_0 t}{R}\right)}$$

13. Figure shows the funnel with mass m.

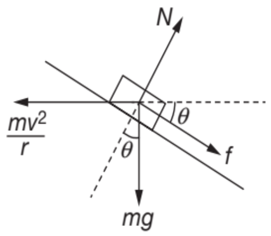
The different forces on mass m (w.r.t. funnel) are:

(i) weight mg acting vertically downwards,

(ii) centrifugal reaction force $\left(\frac{mv^2}{r}\right)$ directed horizontally outwards,

(iii) normal reaction N offered by the wall and

(iv) frictional force f directed along the incline (when the revolution is fast enough, the mass may slide upwards).



Since, the mass m does not move, so we get

$$N - \frac{mv^2}{r}\sin\theta - mg\cos\theta = 0 \qquad \ldots(1)$$

$$f + mg\sin\theta - \frac{mv^2}{r}\cos\theta = 0 \qquad \ldots(2)$$

From equation (1),

$$N = mg\cos\theta + \left(\frac{mv^2}{r}\right)\sin\theta$$

Now, $f = \mu N = \mu\left\{mg\cos\theta + \left(\frac{mv^2}{r}\right)\sin\theta\right\}$ ...(3)

Substituting the value of f from equation (3) in equation (2), we get

$$\mu\left[mg\cos\theta + \frac{mv^2}{r}\sin\theta\right] - \frac{mv^2}{r}\cos\theta = -mg\sin\theta$$

$$\Rightarrow \frac{mv^2}{r}(\mu\sin\theta - \cos\theta) = -mg(\sin\theta + \mu\cos\theta)$$

$$\Rightarrow \frac{v^2}{r}(\cos\theta - \mu\sin\theta) = g(\sin\theta + \mu\cos\theta)$$

$$\Rightarrow \frac{v^2}{r} = \frac{g(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)} \qquad \ldots(4)$$

Since

$$v = r\omega = 2\pi nr \qquad \{\because \omega = 2\pi n\}$$

$$\Rightarrow \frac{4\pi^2 n^2 r^2}{r} = \frac{g(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}$$

$$\Rightarrow n^2 = \frac{1}{4\pi^2 r}\frac{g(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}$$

$$\Rightarrow n = \frac{1}{2\pi}\sqrt{\frac{g(\sin\theta + \mu\cos\theta)}{r(\cos\theta - \mu\sin\theta)}} \qquad \ldots(5)$$

This must be the maximum frequency allowed.

When the revolution is slow enough, the block may slide down. In this case the frictional force f will be directed along the incline upwards.

So, replacing μ by $-\mu$ in equation (5) the minimum frequency allowed will be

$$n = \frac{1}{2\pi}\sqrt{\frac{g(\sin\theta - \mu\cos\theta)}{r(\cos\theta + \mu\sin\theta)}} \qquad \ldots(6)$$

Hence, $n_{max} = \frac{1}{2\pi}\sqrt{\frac{g(\sin\theta + \mu\cos\theta)}{r(\cos\theta - \mu\sin\theta)}}$

$$n_{min} = \frac{1}{2\pi}\sqrt{\frac{g(\sin\theta - \mu\cos\theta)}{r(\cos\theta + \mu\sin\theta)}}$$

Single Correct Choice Type Questions

1. $m = 2\ kg$

$\vec{v} = 2\hat{i}$

$\vec{F} = 3\hat{j}$

$$\Rightarrow \vec{a} = \frac{\vec{F}}{m} = \frac{3}{2}\hat{j}$$

$$t = 2\ s$$

Since $\vec{s} = \vec{v}t + \frac{1}{2}\vec{a}t^2$

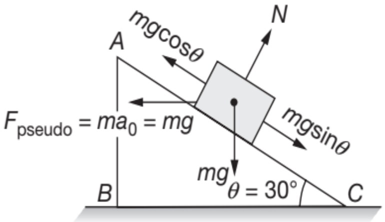
$$\Rightarrow \vec{s} = (2\hat{i})2 + \frac{1}{2}\left(\frac{3}{2}\right)(4)\hat{j}$$

$$\Rightarrow \vec{s} = 4\hat{i} + 3\hat{j}$$

$$\Rightarrow |\vec{s}| = \sqrt{4^2 + 3^2} = 5\ \text{m}$$

Hence, the correct answer is (B).

2. Drawing free body diagram of block with respect to plane.



Acceleration of the block up the plane is

$$a = \frac{mg\cos(30°) - mg\sin(30°)}{m}$$

$$\Rightarrow \quad a = \left(\frac{\sqrt{3}-1}{2}\right)g = 3.66 \text{ ms}^{-2}$$

Since, $s = \frac{1}{2}at^2$

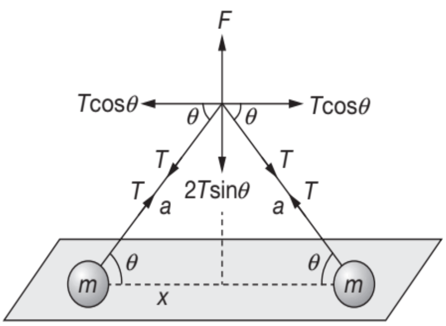
$$\Rightarrow \quad t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times 1}{3.66}} = 0.74 \text{ s}$$

Hence, the correct answer is (B).

3. Since the string is light, so at the point of application of force F, and when the threads make an angle θ with the horizontal, we have

$$F - 2T\sin\theta = 0$$

$$\Rightarrow \quad F = 2T\sin\theta$$



For mass m to move, say with acceleration A, we have

$$T\cos\theta = mA$$

$$\Rightarrow \quad \left(\frac{F}{2\sin\theta}\right)\cos\theta = mA$$

$$\Rightarrow \quad A = \left(\frac{F}{2m}\right)\cot\theta$$

where $\tan\theta = \frac{\sqrt{a^2 - x^2}}{x}$

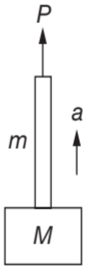
$$\Rightarrow \quad A = \frac{F}{2m}\left(\frac{x}{\sqrt{a^2 - x^2}}\right)$$

Hence, the correct answer is (C).

4. Let f be the force exerted by the rope on the block M. Then

$$f - Mg = Ma \qquad \ldots(1)$$

We must note that once P acts on free end of the rope, then tension due to pull P at the end of rope attached to M is zero. So, M will move up only due to the interaction force f.



Now, for mass m, we have

$$P - f - mg = ma \qquad \ldots(2)$$

Adding (1) and (2), we get

$$P - (M+m)g = (M+m)a$$

$$\Rightarrow \quad P = (M+m)(g+a)$$

$$\Rightarrow \quad a = \frac{P}{M+m} - g$$

$$f = M(g+a)$$

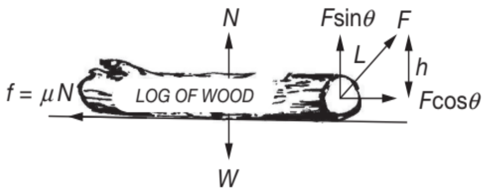
$$\Rightarrow \quad f = \frac{MP}{M+m}$$

Hence, the correct answer is (D).

5. $N + F\sin\theta = W$ and

$$F\cos\theta = \mu N$$

$$\Rightarrow \quad \mu = \frac{F\cos\theta}{N}$$



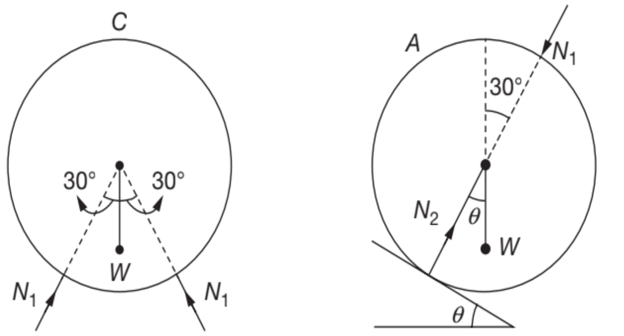
$$\Rightarrow \quad \mu = \frac{F\cos\theta}{W - F\sin\theta}$$

Since $\sin\theta = \frac{h}{L}$ and $\cos\theta = \frac{\sqrt{L^2 - h^2}}{L}$

$$\Rightarrow \quad \mu = \frac{F\sqrt{L^2 - h^2}}{WL - Fh}$$

Hence, the correct answer is (A).

6. Let us draw the free body diagrams of C and A.



W = Weight of each sphere

N_2 = Normal reaction between A and inclined plane

N_1 = Normal reaction between A and C

N_1 = Normal reaction between B and C

Free body diagram of *C*

Resolving vertically $2N_1 \cos 30° = W$

$$\Rightarrow \quad N_1 = \frac{W}{\sqrt{3}} \qquad \ldots(1)$$

When the arrangement is on the point of collapsing, the reaction between A and B is zero.

Free body diagram of *A*

Resolving horizontally and vertically

$$N_2 \sin\theta = N_1 \sin 30°$$

$$\Rightarrow \quad N_2 \sin\theta = \frac{W}{2\sqrt{3}} \qquad \ldots(2)$$

$$N_2 \cos\theta = W + N_1 \cos 30° = \frac{3W}{2} \qquad \ldots(3)$$

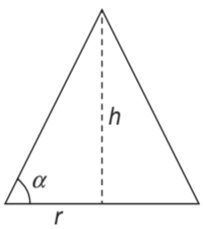
Dividing (2) by (3), we get

$$\tan\theta = \frac{1}{3\sqrt{3}}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)$$

Hence, the correct answer is (C).

7. As soon as the inclination of slanted side reaches the angle of friction, sand begins to slide down.



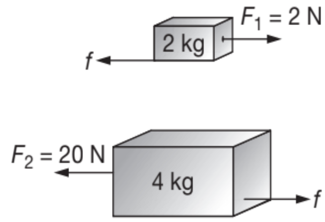
For maximum height, we have

$$\mu = \tan\alpha = \frac{h}{R}$$

$$\Rightarrow \quad h = \mu R$$

Hence, the correct answer is (C).

8. Free body diagram of the two bodies are as follows



Maximum friction between the two blocks is

$$f_{max} = \mu mg \text{ where } m = 2 \text{ kg}$$

$$\Rightarrow \quad f_{max} = (0.5)(2)(10) = 10 \text{ N}$$

Let acceleration of both the blocks be a towards left. Then

$$a = \frac{f-2}{2} = \frac{20-f}{4}$$

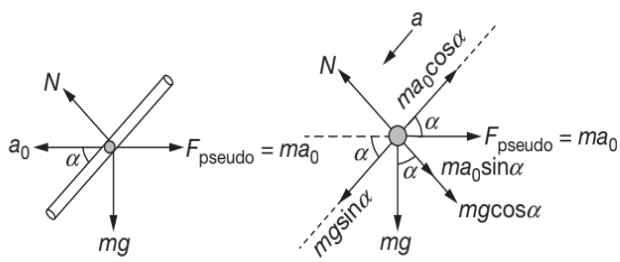
$$\Rightarrow \quad 2f - 4 = 20 - f$$

$$\Rightarrow \quad f = 8 \text{ N}$$

Now since $f < f_{max}$, i.e., static region of 2 kg on 4 kg, hence friction force between the two blocks is 8 N.

Hence, the correct answer is (C).

9.



$$mg \sin\alpha - ma_0 \cos\alpha = ma$$

$$\Rightarrow \quad a = g\sin\alpha - a_0 \cos\alpha$$

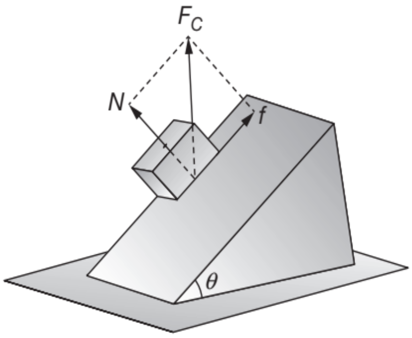
Hence, the correct answer is (D).

10. $\sin\theta = \frac{3}{5}$

$$\Rightarrow \quad \cos\theta = \frac{4}{5}$$

$$mg\sin\theta = (10)(10)\left(\frac{3}{5}\right) = 60 \text{ N}$$

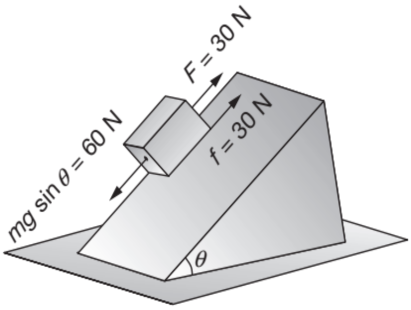
$$\mu mg\cos\theta = \left(\frac{3}{4}\right)(10)(10)\left(\frac{4}{5}\right) = 60 \text{ N}$$



i.e., when a force of 30 N is applied in upward direction friction force of 30 N acts upwards. Normal reaction is perpendicular to plane. Therefore, resultant force will be along OB.

Hence, the correct answer is (B).

11. $mg\sin\theta = 60$ N, F = 30 N and f = 30 N. So, from the free body diagram, we observe that the net force on the block is zero and hence the acceleration is zero.



Hence, the correct answer is (D).

12. For upward motion

$$-Fh = -\frac{1}{2}mv^2 + mgh$$

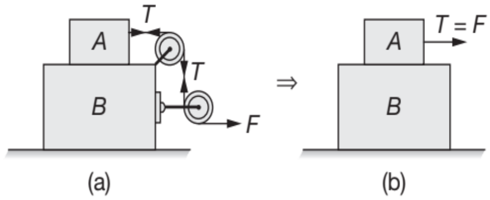
For downward motion

$$-Fh = \frac{1}{2}mv'^2 - mgh$$

$$\Rightarrow \quad v' = v\sqrt{\frac{mg - F}{mg + F}}$$

Hence, the correct answer is (C).

13. Net horizontal force on block B is zero.
Hence, the given figure (a) can be replaced by figure (b)



Maximum value of friction

$$f_{max} = \mu m_A g = (0.5)(2)(10) = 10 \text{ N}$$

Block B moves due to friction
Therefore, maximum common acceleration of the two blocks can be

$$a_{max} = \frac{f_{max}}{m_B} = \frac{10}{2} = 5 \text{ ms}^{-2}$$

$$\Rightarrow \quad F_{max} = (m_A + m_B)a_{max}$$

$$\Rightarrow \quad F_{max} = (2+2)(5) = 20 \text{ N}$$

Hence, the correct answer is (D).

14. $2T\cos\theta = \sqrt{2}\,Mg$ ...(1)

$T\sin\theta$ and $T\sin\theta$ cancel out as both are equal in magnitude and are directed opposite to each other. Further,

$$T = Mg \qquad \text{...(2)}$$

$$\Rightarrow \quad 2\,Mg\cos\theta = \sqrt{2}\,Mg$$

$$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \theta = 45°$$

Hence, the correct answer is (C).

15. Mass m_1 moves with constant velocity if tension in the lower string

$$T_1 = m_1 g = (1)(10) = 10 \text{ N} \qquad \text{...(1)}$$

So, tension in the upper string is

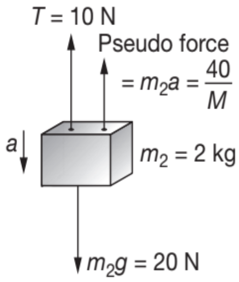
$$T_2 = 2T_1 = 20 \text{ N} \qquad \text{...(2)}$$

Acceleration of block M is therefore,

$$a = \frac{T_2}{M} = \frac{20}{M} \qquad \text{...(3)}$$

This is also the acceleration of pulley 2

Absolute acceleration of mass m_1 is zero. Thus, acceleration of m_1 relative to pulley 2 is a upwards or acceleration of m_2 with respect to pulley 2 is a downwards. Drawing free body diagram of m_2 with respect to pulley 2.



Equation of motion gives

$$20-\frac{40}{M}-10=2a=\frac{40}{M}$$

Solving this we get $M=8$ kg

Hence, the correct answer is (C).

17. Taking upward direction as positive

Given $a_A=1\text{ ms}^{-2}$, $a_B=7\text{ ms}^{-2}$ and $a_C=2\text{ ms}^{-2}$

Let acceleration of pulley P is a upwards, then acceleration of pulley Q will be a downwards

$\Rightarrow\ a_P=a$

and $a_Q=-a$

Now $a_{AP}=-a_{BP}$

$\Rightarrow\ a_A-a_P=a_P-a_B$

$\Rightarrow\ 1-a=a-7$

$\Rightarrow\ 2a=8$

$\Rightarrow\ a=4\text{ ms}^{-2}$

Further $a_{CQ}=-a_{DQ}$

$\Rightarrow\ a_C-a_Q=a_Q-a_D$

$\Rightarrow\ 2-(-a)=-a-a_D$

$\Rightarrow\ a_D=-2a-2=-10\text{ ms}^{-2}$

So, acceleration of D is 10 ms^{-2} downwards.

Hence, the correct answer is (C).

18. $\text{Thrust}=2T=\left(\frac{4mM}{M+m}\right)g$

$$\Rightarrow\ \text{Thrust}=\frac{4mg}{1+\frac{m}{M}}$$

Since, $\frac{m}{M}\to 0$

$\Rightarrow\ \text{Thrust}\approx 4mg$

Hence, the correct answer is (C).

19. From symmetrical behaviour of the arrangement, we observe that tension in both the branches BC and CD to be equal.

$\Rightarrow\ 2T\cos\beta=mg$

$$\Rightarrow\ T=\frac{mg}{2\cos\beta}=\frac{mg}{2}\sec\beta$$

$$\Rightarrow\ T=\frac{mg}{2}\sqrt{1+\tan^2\beta}$$

$$\Rightarrow\ T=\frac{mg}{2}\sqrt{1+\left(\frac{12}{5}\right)^2}$$

$$\Rightarrow\ T=\frac{13}{10}mg$$

Hence, the correct answer is (B).

20. Block starts sliding over the surface when F is the maximum static force of friction

$\Rightarrow\ Kt=\mu_s mg$

$$\Rightarrow\ t=\frac{\mu_s mg}{K}$$

for $t>\frac{\mu_s mg}{K}$, net acceleration of block is

$$a=\frac{F-\mu_k mg}{m}=\frac{Kt}{m}-\mu_k g$$

which is a straight line with positive slope and negative intercept.

So, for $t\le\frac{\mu_s mg}{K}$, acceleration of particle is zero

and for $t>\frac{\mu_s mg}{K}$, acceleration is $a=\frac{Kt}{m}-\mu_k g$.

Hence, the correct answer is (D).

21. $a=\dfrac{F}{m_1+m_2+m_3}$

For equilibrium $m_1a=m_2g$

$$\Rightarrow\ m_1\left(\frac{F}{m_1+m_2+m_3}\right)=m_2g$$

$$\Rightarrow\ F=\frac{m_2g}{m_1}(m_1+m_2+m_3)$$

Hence, the correct answer is (C).

22. $kx_1=2g$

$$\Rightarrow\ x_1=\frac{2g}{k} \qquad \ldots(1)$$

{k = force constant of spring}

$$\frac{3g-kx_2}{3}=\frac{3g-2g}{5}$$

$$\Rightarrow \quad x_2=\frac{12}{5}\frac{g}{k} \qquad \ldots(2)$$

$$\frac{kx_3-g}{1}=\frac{2g-1g}{3}$$

$$\Rightarrow \quad x_3=\frac{4g}{3k} \qquad \ldots(3)$$

From (1), (2) and (3) we see that

$$x_2 > x_1 > x_3$$

Hence, the correct answer is (B).

23. Impulse = $\left(\begin{array}{c}\text{Area under force time graph}\\ \text{under the specified interval}\end{array}\right)$

Hence, the correct answer is (A).

24. From constraint relations we can see that acceleration of block B is

$$a_B=\left(\frac{a_A+a_C}{2}\right) \text{ with proper signs}$$

Hence $a_B=\left(\frac{3-12t}{2}\right)=1.5-6t$

$$\Rightarrow \quad \frac{dv_B}{dt}=1.5-6t$$

$$\Rightarrow \quad \int_0^{v_B} dv_B=\int_0^t (1.5-6t)\,dt$$

$$\Rightarrow \quad v_B=1.5t-3t^2$$

$$\Rightarrow \quad v_B=0 \text{ at } t=\frac{1}{2}\text{ s}$$

Hence, the correct answer is (D).

25. Let T be the tension in the string then

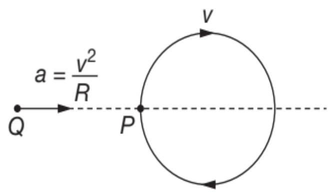
For body $A: F=T+\mu mg$

For body $B: T=\mu mg+\mu(m+M)g$

$$\Rightarrow \quad F=\mu(3m+M)g$$

Hence, the correct answer is (D).

26. At the moment shown in figure net force on P observed by Q is zero.



Hence, the correct answer is (C).

27. $$\frac{g}{2}=\frac{n-(16-n)\left(\frac{1}{3}\right)}{16}g$$

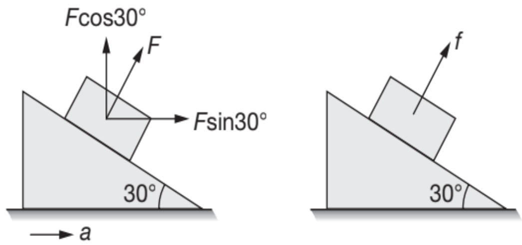
$$\Rightarrow \quad 8=\frac{3n-(16-n)}{3}$$

$$\Rightarrow \quad 24=3n-16+n$$

$$\Rightarrow \quad n=10$$

Hence, the correct answer is (C).

28. Since the surfaces in contact are smooth, so force of friction between the block and the plane is zero. Hence, contact force is really the normal reaction between the two.



In the first case $F\sin 30° = ma$

and $F\cos 30° = mg \Rightarrow F=\dfrac{mg}{\cos 30°}$

and in the second case

$$f=mg\cos 30°$$

$$\Rightarrow \quad \frac{F}{f}=\frac{1}{\cos^2 30°}=\frac{4}{3}$$

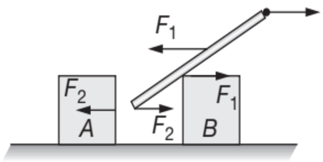
Hence, the correct answer is (B).

29. $$\left(\begin{array}{c}\text{Scale}\\ \text{reading}\\ \text{at time } t\end{array}\right)=\left(\begin{array}{c}\text{Average}\\ \text{force}\\ \text{exerted}\\ \text{by pebbles}\end{array}\right)+\left(\begin{array}{c}\text{Weight}\\ \text{of pebbles}\\ \text{collected}\\ \text{in box at time } t\end{array}\right)$$

$$\left(\begin{array}{c}\text{Scale}\\ \text{reading}\\ \text{at time } t\end{array}\right)=F_{av}+(mg)(nt)=mn\left(\sqrt{2gh}+gt\right)$$

Hence, the correct answer is (D).

30. Considering translational equilibrium of rod in horizontal direction.



$F_1 - F_2 = F \Rightarrow F_1 > F_2$

Therefore, block B will tend to move first.

Hence, the correct answer is (B).

31. Acceleration of body down the plane is

$a = (g \sin 30°)(\cos 60°)$

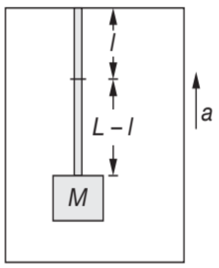
$\Rightarrow \quad a = 10\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 2.5 \text{ ms}^{-2}$

$\Rightarrow \quad t = \sqrt{\frac{2l}{a}} = \sqrt{\frac{2 \times 5}{2.5}} = 2 \text{ s} \qquad \left\{\because l = \frac{1}{2}at^2\right\}$

Hence, the correct answer is (B).

32. Let us consider A and B as a system. Since, there is no vertical force in the upward direction to support their weight, hence the system cannot stay in equilibrium.
Hence, the correct answer is (D).

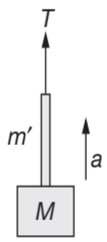
33. Let a be the acceleration of the elevator. Drawing free body diagram of the lower portion.



Let the mass of lower portion of string be $m' = \frac{m}{L}(L - l)$

Equation of motion gives

$T - Mg - m'g = (M + m')a$



$\Rightarrow \quad T - Mg - \frac{mg}{L}(L - l) = \left(M + \frac{m}{L}(L - l)\right)a$

$\Rightarrow \quad a = \frac{T}{\left(M + m - \frac{ml}{L}\right)} - g$

Hence, the correct answer is (C).

34. $T\cos\theta = mg\cos\alpha$ and ...(1)

$T\sin\theta = ma_0 + mg\sin\alpha$ ...(2)

$\Rightarrow \quad \tan\theta = \frac{a_0 + g\sin\alpha}{g\cos\alpha}$

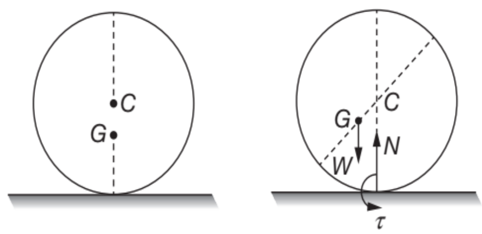
$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{a_0 + g\sin\alpha}{g\cos\alpha}\right)$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{\frac{g}{2} + g\sin(30°)}{g\cos(30°)}\right)$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Hence, the correct answer is (B).

35. When centre of gravity lies vertically below its centre, the restoring torque while displaced from its equilibrium position, brings it back to its equilibrium position as shown below.



C = centre of sphere

G = centre of gravity

and τ = restoring torque

Hence, the correct answer is (B).

36. $mg = 2T\sin 45$

$\Rightarrow \quad mg = \sqrt{2}T$ ...(1)

$T_1\cos\theta = T\cos 45$

$\Rightarrow \quad T_1\cos\theta = \frac{T}{\sqrt{2}} = \frac{mg}{2} \qquad \left\{\because T = \frac{mg}{\sqrt{2}}\right\}$

Further, $Mg + T\cos 45 = T_1\sin\theta$

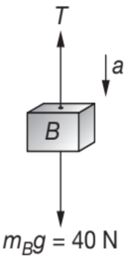
$\Rightarrow \quad T_1\sin\theta = Mg + \frac{mg}{\sqrt{2}}\frac{1}{\sqrt{2}}$

$\Rightarrow \quad T_1\sin\theta = Mg + \frac{mg}{2}$ ...(2)

$$\Rightarrow \quad \tan\theta = \frac{Mg + \frac{mg}{2}}{\frac{mg}{2}} = 1 + \frac{2M}{m} \qquad \{\text{Divide (2) by (1)}\}$$

Hence, the correct answer is (A).

37.



From constraint relations we can see that

$$a_A = 2a_B$$

so, let $a_B = a$ then $a_A = 2a$

Free body diagram of B gives

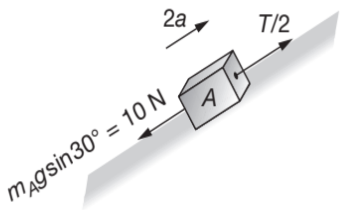
$$40 - T = 4a \qquad \ldots(1)$$

Free body diagram of A is

$$\frac{T}{2} - 10 = 4a$$

$$\Rightarrow \quad T - 20 = 8a \qquad \ldots(2)$$

Solving equations (1) and (2) we get



$$2a = \frac{10}{3}\ \text{ms}^{-2}$$

$$\Rightarrow \quad a_A = \frac{10}{3}\ \text{ms}^{-2}$$

Hence, the correct answer is (A).

38. Let acceleration of A be a_1 downwards and that of B is a_2 upwards and let T be the tension in the string. Then equations of motion of A and B gives

$$m_1 g - T = m_1 a_1 \qquad \ldots(1)$$

$$2T - m_2 g = m_2 a_2 \qquad \ldots(2)$$

Equilibrium of pulley C gives

$$2T - T = 0$$

$$\Rightarrow \quad T = 0$$

Substituting in (1) and (2)

$$a_1 = g \text{ and } a_2 = -g$$

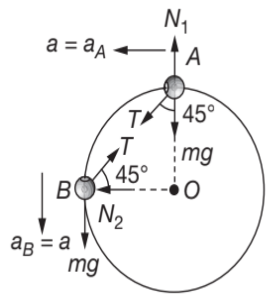
So, both the blocks are falling freely.

Hence, the correct answer is (D).

39. $T\sin 45 = ma$ (For A) ...(1)

$mg - T\sin 45 = ma$ (For B) ...(2)

$$\Rightarrow \quad T = \frac{mg}{2\sin 45} = \frac{mg}{\sqrt{2}}$$



Hence, the correct answer is (C).

40. For the first 3 second, the acceleration of the combined system is

$$a = \left(\frac{5-m}{5+m}\right)g \qquad \ldots(1)$$

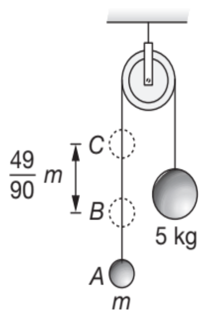
Velocity gained by m (starting from rest) at the end of 3 second is

$$v = 0 + at$$

$$\Rightarrow \quad v = 3\left(\frac{5-m}{5+m}\right)g \qquad \ldots(2)$$

At $t = 3s$, the string is cut (i.e., at position B).

As soon as the string is cut the mass m starts moving under the retardation g and covers a further distance of $\frac{49}{90}$m before its velocity becomes zero $(\text{at } c)$ and then it begins to descend.



$$\Rightarrow \quad 0 - v^2 = 2(-g)\frac{49}{90}$$

$\Rightarrow \quad 9\left(\frac{5-m}{5+m}\right)^2 g^2 = -2g\left(\frac{49}{90}\right)$

$\Rightarrow \quad \left(\frac{5-m}{5+m}\right)^2 = \frac{1}{81}$

$\Rightarrow \quad \frac{5-m}{5+m} = \pm\frac{1}{9}$

$\Rightarrow \quad m = 4 \text{ kg}$ or $m = 6.25 \text{ kg}$

But $m = 6.25 \text{ kg}$ is not acceptable, as the value of lighter mass cannot exceed 5 kg.

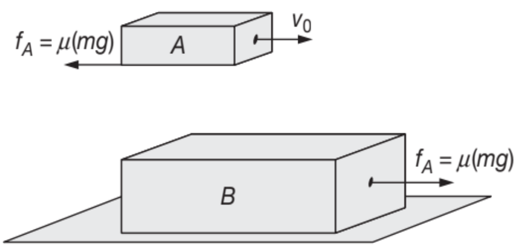
Hence, the correct answer is (A).

41. Force of friction between the two is

$f_A = \mu mg$

Retardation of A is $a_A = \frac{\mu mg}{m} = \mu g$

and acceleration of B is $a_B = \frac{\mu mg}{2m} = \frac{\mu g}{2}$



Hence, acceleration of B relative to A is

$$a_{BA} = a_A + a_B = \frac{3\mu g}{2}$$

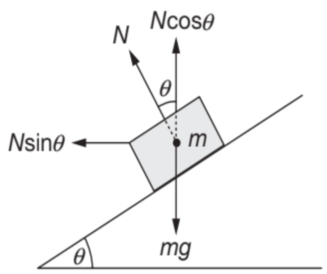
{∵ both are in opposite direction}

Since $\mu = \frac{1}{2}$

$\Rightarrow \quad a_{BA} = \frac{3g}{4}$

Hence, the correct answer is (C).

42. From the free body diagram, we observe that the component $N\sin\theta$ provides the necessary centripetal force, where as the component $N\cos\theta$ equals mg.



So, $N\cos\theta = mg$ ...(1)

$N\sin\theta = \frac{mv^2}{R}$ ...(2)

So, $N = mg\sec\theta$ $\{\because$ of $(1)\}$

Hence, the correct answer is (C).

43. Blocks A and C both move due to friction and less friction is available to A as compared to C because normal reaction between A and B is less. Maximum friction between A and B can be

$$f_{max} = \mu m_A g = \left(\frac{1}{2}\right)mg$$

Maximum acceleration of A can be

$$a_{max} = \frac{f_{max}}{m} = \frac{g}{2}$$

Further $a_{max} = a_D$, because no slipping of A and C has to be on B (which moves forward with an acceleration a_D).

$\Rightarrow \quad a_{max} = \frac{m_D g}{3m + m_D}$

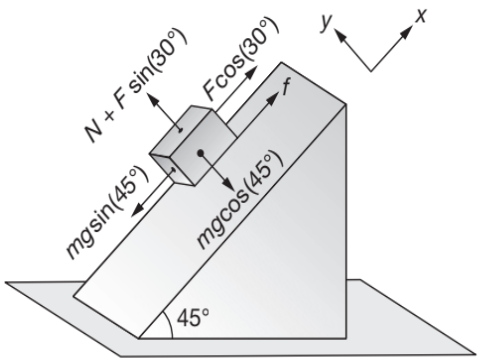
$\Rightarrow \quad \frac{g}{2} = \frac{m_D g}{3m + m_D}$

$\Rightarrow \quad m_D = 3m$

Hence, the correct answer is (A).

44. Drawing free body diagram of block, we have

$\Sigma F_y = 0$



$\Rightarrow \quad N + F\sin 30° = mg\cos 45°$

$\Rightarrow \quad N = mg\cos 45° - F\sin 30°$

$\Rightarrow \quad N = (4)(10)\left(\frac{1}{\sqrt{2}}\right) - (10)\left(\frac{1}{2}\right)$

$\Rightarrow \quad N = 23.28 \text{ N}$ ...(1)

$$f_{max} = \mu N = (0.6)(23.28) = 13.97 \text{ N}$$

$$mg\sin(45°) = (4)(10)\left(\frac{1}{\sqrt{2}}\right) = 28.28 \text{ N}$$

and $F\cos(30°) = (10)\left(\frac{\sqrt{3}}{2}\right) = 8.66 \text{ N}$

Since, $mg\sin 45° > f_{max} + F\cos 30°$, so the block will slide down and friction will be maximum. Therefore, net contact force is

$$F_C = \sqrt{N^2 + (f_{max})^2}$$

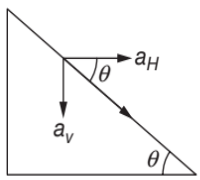
$$\Rightarrow \quad F_C = \sqrt{(23.28)^2 + (13.97)^2}$$

$$\Rightarrow \quad F_C = 27.15 \text{ N}$$

Hence, the correct answer is (C).

45. For no slipping, both A and B move together down the incline with an acceleration $a = g\sin\theta$, down the plane as no slipping exists between them. Horizontal component of this acceleration is $a_H = a\cos\theta$ and vertical component is $a_V = a\sin\theta$

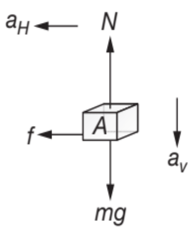
where $a = g\sin\theta$



$$\Rightarrow \quad a_H = a\cos\theta = g\sin\theta\cos\theta \text{ and}$$

$$a_V = a\sin\theta = g\sin^2\theta$$

FBD for A



Let N be the normal reaction between A and B. Equations of motion in horizontal and vertical directions give

$$mg - N = ma_V$$

$$\Rightarrow \quad N = mg - ma_V = mg - mg\sin^2\theta$$

$$\Rightarrow \quad N = mg\cos^2\theta$$

If f be the frictional force between A and B, then

$$ma_H \le \mu N$$

$$\Rightarrow \quad mg\sin\theta\cos\theta \le \mu mg\cos^2\theta$$

$$\Rightarrow \quad \theta \le \tan^{-1}(\mu)$$

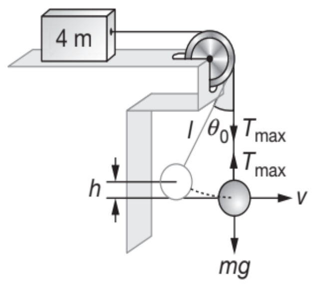
Hence, the correct answer is (D).

46. The tension is maximum in the string when it is in the lowest position. So,

$$h = l(1 - \cos\theta_0)$$

Speed of mass m in its lowest position is

$$v^2 = 2gh = 2gl(1 - \cos\theta_0)$$



Further since, $T_{max} - mg = \dfrac{mv^2}{l}$

$$\Rightarrow \quad T_{max} = mg + \frac{mv^2}{l}$$

$$\Rightarrow \quad T_{max} = mg + 2mg(1 - \cos\theta_0)$$

$$\Rightarrow \quad T_{max} = mg(3 - 2\cos\theta_0)$$

The block of mass $4m$ will not move, when

$$T_{max} \le \mu(4m)g$$

$$\Rightarrow \quad mg(3 - 2\cos\theta_0) \le 4\,\mu mg$$

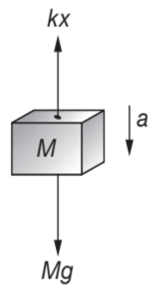
$$\Rightarrow \quad \mu \ge \frac{3 - 2\cos\theta_0}{4}$$

$$\Rightarrow \quad \mu_{MIN} = \frac{3 - 2\cos\theta_0}{4}$$

Hence, the correct answer is (D).

47. Net pulling force on the system is $Mg + mg - mg$ or simply Mg. Total mass being pulled is $M + 2m$. Hence acceleration of system is

$$a = \frac{Mg}{M + 2m}$$



Now since $a < g$, there should be an upward force on M so that its acceleration becomes less than g. Hence for any value of M spring will be elongated.
Hence, the correct answer is (D).

48. Net pulling force $= 2g - 1g = 10$ N

Mass being pulled $= 2 + 1 = 3$ kg

So, acceleration of the system is $a = \frac{10}{3}$ ms^{-2}

Velocity of both the blocks at $t = 1$ s will be

$$v_0 = at = \left(\frac{10}{3}\right)(1) = \frac{10}{3} \text{ ms}^{-1}$$

Now, at this moment velocity of 2 kg block becomes zero, while that of 1 kg block is $\frac{10}{3}$ ms^{-1} upwards.
Hence, string becomes tight again when, displacement of 1 kg block = displacement of 2 kg block

$$\Rightarrow \quad v_0 t - \frac{1}{2}gt^2 = \frac{1}{2}gt^2$$

$$\Rightarrow \quad t = \frac{v_0}{g} = \frac{\frac{10}{3}}{10} = \frac{1}{3} \text{ s}$$

Hence, the correct answer is (D).

49. Limiting force of friction between A and B is

$$f_1 = \mu_1 m_A g = (0.3)(30)(10) = 90 \text{ N}$$

Limiting force of friction between B and C is

$$f_2 = \mu_2 (m_A + m_B) g$$

$$f_2 = (0.2)(30 + 10)(10) = 80 \text{ N}$$

and limiting force of friction between C and ground is

$$f_3 = \mu_3 (m_A + m_B + m_C) g$$

$$f_3 = (0.1)(30 + 10 + 20)(10) = 60 \text{ N}$$

As F is gradually increased the force of friction between A and B will increase.

When $F = 60$ N block A will exert a horizontal force of 60 N on B. There will be no relative motion between A and B since this force is less than 90 N. B will also exert a horizontal force of 60 N on C. Hence C will be on the point of motion.

Hence, the least value of F is 60 N. All the three blocks will be on the point of motion as a single body.
Hence, the correct answer is (A).

50. Since $a_{\text{down}} = g(\sin\theta - \mu\cos\theta)$

$$\Rightarrow \quad a_{\text{down}} = a = 5\left(1 - 0.5\sqrt{3}\right) = 0.67 \text{ ms}^{-2}$$

So, minimum speed while reaching the bottom can be

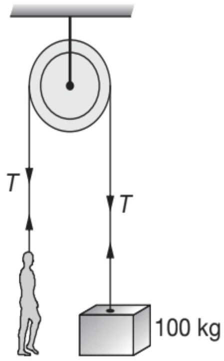
$$v^2 = u^2 + 2as$$

$$\Rightarrow \quad v = (6)^2 + 2(0.67)(15) = 56.1$$

$$\Rightarrow \quad v = 7.49 \text{ ms}^{-1} \cong 7.5 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

51. Let T be the tension in the rope and a the acceleration of rope. The absolute acceleration of man is therefore $\left(\frac{5g}{4} - a\right)$. Equations of motion for mass and man gives



$$T - 100g = 100a \quad \ldots(1)$$

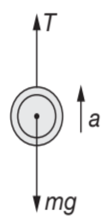
$$T - 60g = 60\left(\frac{5g}{4} - a\right) \quad \ldots(2)$$

Solving (1) and (2) we get

$$T = 1218 \text{ N}$$

Hence, the correct answer is (C).

52. Let T be the tension in the string. The upward force exerted on the clamp is $T\sin 30° = \frac{T}{2}$.



Given $\frac{T}{2} = 40$ N

$$\Rightarrow \quad T = 80 \text{ N} \quad \ldots(1)$$

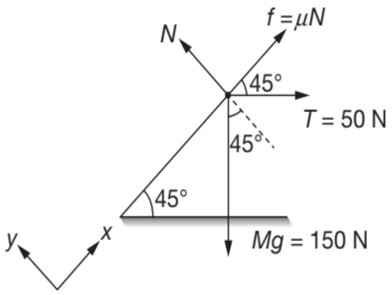
If a is the acceleration of monkey in upward direction. Equation of monkey in upward direction is

$$a = \frac{T - mg}{m}$$

$$a = \frac{80 - (5)(10)}{5} = 6 \text{ ms}^{-2}$$

Hence, the correct answer is (C).

53. The string is under tension, hence there is limiting friction between the block and the plane. Drawing free body diagram of the block.



$$\Sigma F_x = 0$$

$$\Rightarrow \quad \mu N + 50\cos 45° = 150 \sin 45° \qquad \ldots(1)$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad N = 50 \sin 45° + 150 \cos 45° \qquad \ldots(2)$$

Solving (1) and (2) we get

$$\mu = \frac{1}{2}$$

Hence, the correct answer is (A).

54. Acceleration of the system and hence tension (T) on the string attached with the pulley is same in both the cases. Hence

$$F_1 = F_2 = 2T$$

Hence, the correct answer is (C).

55. As long as $mg\sin\theta < \mu mg\cos\theta$, the block will remain stationary.

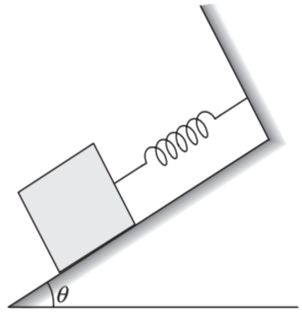
So, $x = 0$, till $\tan\theta < \mu$.

As soon as $\tan\theta > \mu$, then the spring of force constant k gets extended by say x.

So, we have, for equilibrium

$$kx + \mu mg\cos\theta = mg\sin\theta$$

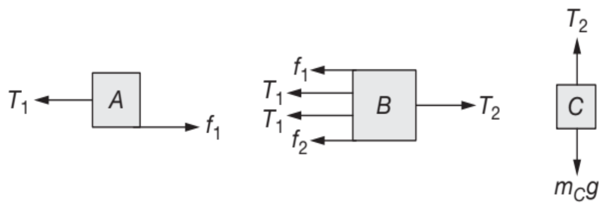
$$\Rightarrow \quad x = \frac{mg\sin\theta - \mu mg\cos\theta}{k}$$



As $\theta \to \frac{\pi}{2}$, $x = \frac{mg}{k}$ $\qquad \{\because N = 0\}$

Hence, the correct answer is (A).

56. Maximum friction that can be obtained between A and B is



$$f_1 = \mu m_A g = (0.3)(100)(10) = 300 \text{ N}$$

and maximum friction between B and ground is

$$f_2 = \mu(m_A + m_B)g$$

$$f_2 = (0.3)(100 + 140)(10) = 720 \text{ N}$$

Drawing free body diagrams of A, B and C in limiting case

Equilibrium of A gives

$$T_1 = f_1 = 300 \text{ N} \qquad \ldots(1)$$

Equilibrium of B gives

$$2T_1 + f_1 + f_2 = T_2$$

$$\Rightarrow \quad T_2 = 2(300) + 300 + 720 = 1620 \text{ N} \qquad \ldots(2)$$

and equilibrium of C gives

$$m_C g = T_2$$

$$\Rightarrow \quad 10 m_C = 1620$$

$$\Rightarrow \quad m_C = 162 \text{ kg}$$

Hence, the correct answer is (D).

57. For equilibrium

$$\begin{pmatrix}\text{Weight of} \\ \text{hanging part} \\ \text{of chain}\end{pmatrix} = \begin{pmatrix}\text{Force of friction} \\ \text{on the part resting} \\ \text{on table}\end{pmatrix}$$

Let l be the length of the chain hanging

$$\Rightarrow \left(\frac{M}{L}l\right)g = \mu\left[\frac{M}{L}(L-l)g\right]$$

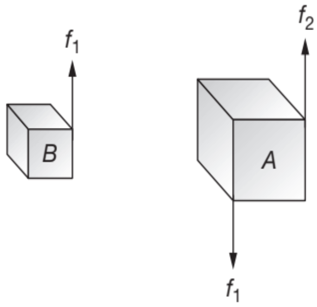
$$\Rightarrow l = \mu(L-l)$$

$$\Rightarrow (\mu+1)l = \mu L$$

$$\Rightarrow l = \frac{\mu L}{\mu+1}$$

Hence, the correct answer is (A).

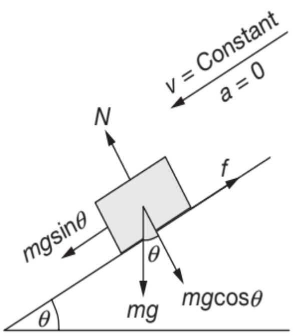
58. The directions of friction forces at different surfaces are shown in figure.



Here, f_1 is the force of friction between A and B and f_2 is the force of friction between A and vertical wall.

Hence, the correct answer is (B).

59. Block slides down with constant velocity. Hence, net force on the block is zero.



So, net contact force F_C on the body is

$$F_C = \sqrt{f^2 + N^2}$$

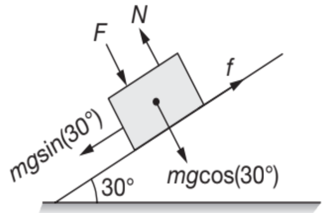
where $f = mg\sin\theta$ $\qquad \{\because a = 0 \Rightarrow f = mg\sin\theta\}$

and $N = mg\cos\theta$

$$\Rightarrow F_C = \sqrt{m^2g^2(\cos^2\theta + \sin^2\theta)} = mg$$

Hence, the correct answer is (D).

60. Since, we observe that $mg\sin(30°) > \mu mg\cos(30°)$



So, the block has a tendency to slip downwards. Let F be the minimum force applied on it, such that it does not slip. Then

$$N = F + mg\cos(30°)$$

Also, $mg\sin(30°) = \mu N = \mu(F + mg\cos 30°)$

$$\Rightarrow F = \frac{mg\sin(30°)}{\mu} - mg\cos(30°)$$

$$\Rightarrow F = \frac{(2)(10)\left(\frac{1}{2}\right)}{0.5} - (2)(10)\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow F = 20 - 17.32$$

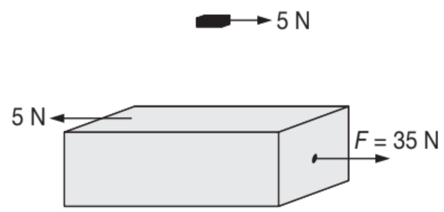
$$\Rightarrow F = 2.68 \text{ N}$$

Hence, the correct answer is (C).

61. Maximum frictional force between the block and the plank is

$$f_{\max} = \mu mg = (0.5)(1)(10) = 5 \text{ N}$$

The free body diagrams of the block and the plank are shown for convenience to solve the problem



Acceleration of block is

$$a_B = \frac{5}{1} = 5 \text{ ms}^{-2}$$

Acceleration of plank is

$$a_P = \frac{35-5}{2} = 15 \text{ ms}^{-2}$$

So, relative acceleration of block with respect to the plank is

$$a_{BP} = a_B - a_P = -10 \text{ ms}^{-2}$$

$$\Rightarrow a_{BP} = 10 \text{ ms}^{-2}\text{, backwards}$$

Since, $s = \frac{1}{2}at^2$, where $s = 1.25$ m, backwards

$$\Rightarrow \quad t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{(2)(-1.25)}{(-10)}}$$

$$\Rightarrow \quad t = \sqrt{\frac{2.5}{10}} = 0.5 \text{ s}$$

Hence, the correct answer is (B).

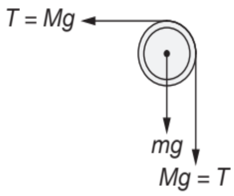
62. $a = g(\sin\theta - \mu\cos\theta)$

$$\Rightarrow \quad a = g(\sin\theta - kx\cos\theta)$$

At $x = 0$, $a = g\sin\theta$. As x increases a decreases and becomes zero at $x_0 = \frac{1}{k}\tan\theta$. For $x > x_0$, the acceleration becomes negative.

Hence, the correct answer is (D).

63.



Three forces are acting on the pulley T (horizontally), T (vertically downward) and the weight of the pulley (also acting vertically downwards) i.e. Force Mg acts horizontally and $(M+m)g$ acts vertically downwards. So, total force acting on the pulley is

$$F_{net} = \sqrt{M^2g^2 + (M+m)^2 g^2}$$

$$\Rightarrow \quad F_{net} = \left[\sqrt{M^2 + (M+m)^2}\right]g$$

Hence, the correct answer is (D).

64. $h = R\sin\theta$

Speed of particle at point B is

$$v^2 = 2gh$$

$$\Rightarrow \quad v^2 = 2gR\sin\theta$$

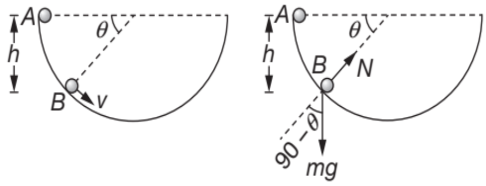
The centripetal force is given by

$$F = \frac{mv^2}{R} = 2mg\sin\theta \quad \ldots(1)$$

Further since,

$$\Rightarrow \quad N - mg\sin\theta = \frac{mv^2}{R} = 2mg\sin\theta$$

$$\Rightarrow \quad N = 3mg\sin\theta \quad \ldots(2)$$

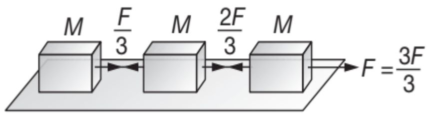


The desired ratio is

$$x = \frac{F}{N} = \frac{2}{3} = \text{constant}$$

Hence, the correct answer is (D).

65.



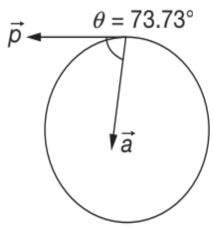
Hence, the correct answer is (A).

66. Angle θ between $\vec{a}$ and $\vec{p}$ (or $\vec{v}$) is given by

$$\theta = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{p}}{|\vec{a}||\vec{p}|}\right)$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{32-18}{\sqrt{(16+9)}\,\sqrt{64+36}}\right)$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{14}{50}\right)$$



$$\Rightarrow \quad \theta = 73.73°$$

Since $0° < \theta < 90°$, the motion is an accelerated motion.

Hence, the correct answer is (A).

67. $|\vec{F}| = vA\sin\theta$

where θ is angle between $\vec{v}$ and $\vec{A}$.

Particle moves undeflected if $\vec{F}$ is equal and opposite to the weight of the particle. i.e.,

$$|\vec{F}| = mg$$

$$\Rightarrow \quad vA\sin\theta = mg$$

$$\Rightarrow \quad v = \frac{mg}{A\sin\theta} \quad \ldots(1)$$

At $\theta = 90°$, we have $\sin\theta = 1 = \text{MAXIMUM}$, so

$$v_{min} = \frac{mg}{A}$$

Now weight is in negative z-direction i.e., $\vec{F}$ should be in positive z-direction. Hence velocity should be in negative y-direction

$$\Rightarrow \quad \vec{v}_{\min} = -\frac{mg}{A}\hat{j}$$

Hence, the correct answer is (D).

68. $a = \left(\frac{M-m}{M+m}\right)g$

$$\Rightarrow \quad 1.4 = \frac{1}{2}\left(\frac{M-m}{M+m}\right)g(2)^2$$

$$\Rightarrow \quad \frac{m}{M} = \frac{13}{15}$$

Hence, the correct answer is (A).

69. $F_{\text{applied}} - f = ma$

$$\Rightarrow \quad F_{\text{applied}} = \mu mg + ma$$

$$\Rightarrow \quad F_{\text{applied}} = m(\mu g + a) \quad \ldots(1)$$

Since $v^2 - u^2 = 2as$ ...(2)

$$a = \frac{v^2 - u^2}{2s}$$

Put value of a as calculated from (2) in equation (1)
Hence, the correct answer is (C).

70. On displacing m through x towards right, M gets displaced downward by $2x$.
For equilibrium

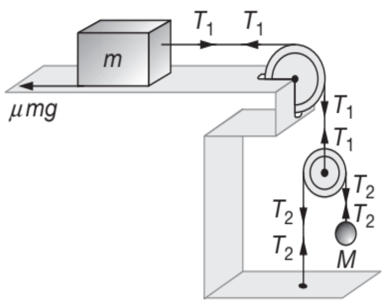
$$T_1 = \mu mg \quad \ldots(1)$$

$$T_1 = 2T_2 \quad \ldots(2)$$

$$T_2 = Mg \quad \ldots(3)$$

$$\Rightarrow \quad 2(Mg) = \mu mg$$

$$\Rightarrow \quad M = \frac{1}{2}\mu m$$



Hence, the correct answer is (A).

71. Acceleration of block till the slipping continues is $a = \frac{f_{\max}}{m}$

$$\Rightarrow \quad a = \frac{\mu mg}{m} = \mu g = (0.5)(10) = 5 \text{ ms}^{-2}$$

Slipping will continue till the velocity of the block also becomes 3 ms^{-1}. So, using $v = u + at$, we get

$$3 = 0 + 5t \quad \{\because u = 0\}$$

$$\Rightarrow \quad t = 0.6 \text{ s}$$

During this time, the displacement of the block is

$$s_1 = \frac{1}{2}at^2 = \frac{1}{2}(5)(0.6)^2 = 0.9 \text{ m}$$

and the displacement of the belt is

$$s_2 = vt = (3)(0.6) = 1.8 \text{ m}$$

Hence displacement of the block w.r.t., the belt is

$$|s_1 - s_2| = 0.9 \text{ m}$$

Hence, the correct answer is (A).

72. $mg \sin\alpha - T - f = m\ddot{x}$

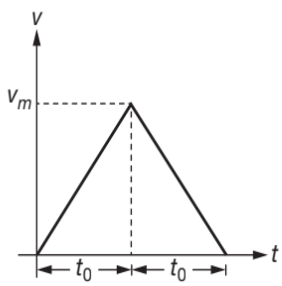
$$\Rightarrow \quad mg \sin\alpha - \frac{\lambda(l - l_0)}{l_0} - k\dot{x} = m\ddot{x}$$

where, k is any known constant.
At Equilibrium Position, $\dot{x} = \ddot{x} = 0$

$$\Rightarrow \quad l = l_0\left(1 + \frac{mg}{\lambda}\sin\alpha\right)$$

Hence, the correct answer is (B).

73. The maximum acceleration and maximum retardation for the car can be $\mu g = \left(\frac{1}{2}\right)(10) = 5 \text{ ms}^{-2}$. The corresponding velocity-time graph is as follows



Let t_0 be the time taken by the car to accelerate. The time taken by it to decelerate is also t_0. So,

$$v_m = (\mu g)t_0 = 5t_0$$

Since, displacement = area under v-t graph. So, we get

$$500 = \frac{1}{2}(2t_0)(5t_0)$$

$\Rightarrow$ $t_0 = 10$ s

Hence total time of journey is $t = 2t_0 = 20$ s.

Hence, the correct answer is (C).

74. Impulse is the area under F-t graph, as well as the change in momentum. So,

$$mu = \text{Area} = \frac{1}{2}F_0T$$

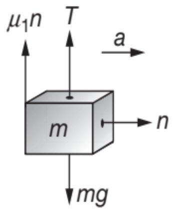
$$\Rightarrow F_0 = \frac{2mu}{T}$$

Hence, the correct answer is (C).

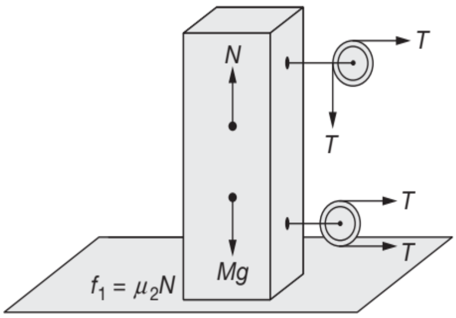
75. For equilibrium of the system i.e., M and m, we have

$$T + \mu_1 n = mg \quad ...(1)$$

$$n = ma \quad ...(2)$$



Since the mass m is at rest, so $n = 0$. This is a justified and an expected answer $(f_2 = 0)$, because when M does not move, then no force is exerted by M on m and hence $n = 0$. Let us now draw free body diagram for M, without taking into account the frictional force on M due to m (as it is zero). So, for M to be at rest we have



$$N = Mg + T \quad ...(3)$$

$$3T = \mu_2 N \quad ...(4)$$

Also, from (1), we get

$$T = mg \quad \{\because n = 0\}$$

From (3), we get

$$N = (M + m)g$$

From (4), we get

$$3mg = \mu_2(M + m)g$$

$$\Rightarrow 3m = \mu_2 M + \mu_2 m$$

$$\Rightarrow m = \frac{\mu_2 M}{3 - \mu_2}$$

$$\Rightarrow m = \frac{\left(\frac{1}{3}\right)(8)}{3 - \left(\frac{1}{3}\right)} = 1 \text{ kg}$$

Hence, the correct answer is (C).

76. For $a \le \mu g$, the blocks will remain stationary with respect to the surface of the plank, so $x = 0$.

For $a \ge \mu g$, both the blocks will move with respect to the surface of the plank with an acceleration $(a - \mu g)$. However at any instant no relative motion between the blocks exists, so still they don't stretch the spring.

Hence, the correct answer is (D).

77. Since the block is about to slide, so we have

$$mg\sin\theta = \mu N$$

$$\Rightarrow \cancel{m}g\sin\theta = \mu(\cancel{m}g\cos\theta)$$

$$\Rightarrow \mu = \tan\theta$$

This condition is independent of the mass, so it will still be on the verge of motion down the incline.

Hence, the correct answer is (D).

78. In CASE-1, sliding starts when

$$mg\sin\theta \ge \mu mg\cos\theta$$

$$\Rightarrow \tan\theta \ge \mu \quad ...(1)$$

While in CASE-2, sliding starts when

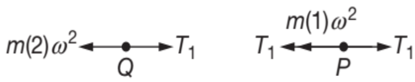
$$m(g + a_0)\sin\alpha \ge \mu m(g + a_0)\cos\alpha$$

$$\Rightarrow \tan\alpha \ge \mu \quad ...(2)$$

From (1) and (2), we get

$$\theta = \alpha$$

Hence, the correct answer is (D).

79. 

$$T_1 = m(2)\omega^2 \quad ...(1)$$

($\because$ centrifugal force acting on Q is $2m\omega^2$ radially outwards)

$$T_1 + m(1)\omega^2 = T_2 \quad \ldots(2)$$

$$\Rightarrow \quad T_2 = 3m\omega^2 \quad \ldots(3)$$

$$\Rightarrow \quad \frac{T_1}{T_2} = \frac{2}{3}$$

Hence, the correct answer is (C).

80. $T\cos\theta = mg\cos\alpha$ and $\quad \ldots(1)$

$$T\sin\theta = ma_0 + mg\sin\alpha \quad \ldots(2)$$

$$\Rightarrow \quad \tan\theta = \frac{a_0 + g\sin\alpha}{g\cos\alpha}$$

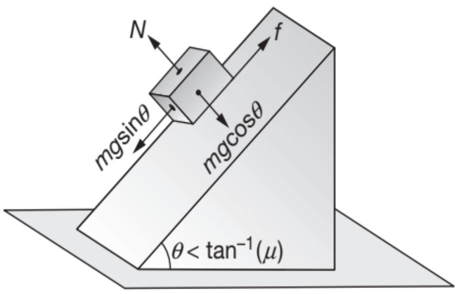
Hence, the correct answer is (D).

81. $\tan\theta = \dfrac{v^2}{rg} = 1$

$$\Rightarrow \quad \theta = 45°$$

Hence, the correct answer is (C).

82. As long as $\tan\theta \le \mu$, the block will not slide down, so we have the contact force $F_C = \sqrt{N^2 + f^2}$



where $N = mg\cos\theta$ and $f = mg\sin\theta$

So, for $\tan\theta \le \mu$, we have

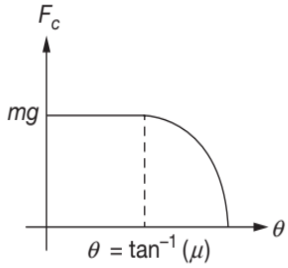
$$F_C = \sqrt{(mg\cos\theta)^2 + (mg\sin\theta)^2} = mg$$

However, when $\tan\theta > \mu$, then the block starts sliding down and now $mg\sin\theta > f$. So,

$$F_C = \sqrt{N^2 + f^2} = \sqrt{N^2 + (\mu N)^2}$$

$$\Rightarrow \quad F_C = N\sqrt{1+\mu^2} = mg\cos\theta\sqrt{1+\mu^2}$$

Since, $\cos\theta$ decreases as θ increases hence, the contact force starts decreasing and finally it becomes zero at $\theta = 90°$. Therefore, graph between F_C and θ will be as follows



Hence, the correct answer is (D).

83. Impulse is the area under F-t graph, as well as the change in momentum. So,

$$mu = \pi\left(\frac{F_0}{2}\right)\left(\frac{T}{2}\right)$$

$$\Rightarrow \quad u = \frac{\pi F_0 T}{4m}$$

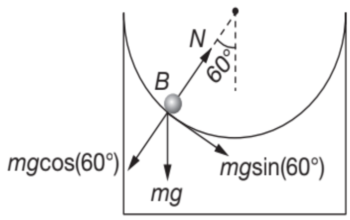
Hence, the correct answer is (C).

84. At position B, speed of ball is $v^2 = 2gh$

where $h = R\cos(60°) = (0.5)\left(\dfrac{1}{2}\right) = 0.25$ m

$$\Rightarrow \quad v^2 = (2)(10)(0.25) = 5 \text{ m}^2\text{s}^{-2} \quad \ldots(1)$$

Let N be the normal reaction between the ball and the wedge in this position. Then



Since, we have

$$N - mg\cos(60°) = \frac{mv^2}{R}$$

$$\Rightarrow \quad N = \frac{mv^2}{R} + mg\cos(60°)$$

Since $v^2 = 5 \quad \{\because \text{ of } (1)\}$

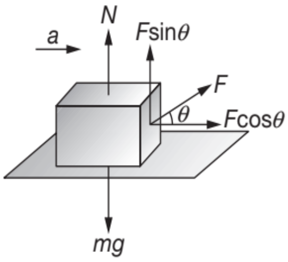
$$\Rightarrow \quad N = \frac{(1)(5)}{(0.5)} + (1)(10)\left(\frac{1}{2}\right) = 15 \text{ N}$$

Now for horizontal equilibrium of wedge
Force exerted by vertical wall on wedge is

$$F = N\sin(60°) = \frac{15\sqrt{3}}{2} \text{ N}$$

Hence, the correct answer is (C).

85.



$$\Rightarrow \quad F\cos\theta = ma$$

$$\Rightarrow \quad \frac{mg}{3}\cos\theta = m\frac{dv}{dt}$$

$$\Rightarrow \quad \frac{mg}{3}\cos(ks) = m\frac{dv}{ds}\frac{ds}{dt}$$

$$\Rightarrow \quad \frac{mg}{3}\cos(ks) = mv\frac{dv}{ds}$$

$$\Rightarrow \quad vdv = \frac{g}{3}\cos(ks)ds$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{g}{3k}\sin(ks)$$

$$\Rightarrow \quad v = \sqrt{\frac{2g}{3k}\sin\theta}$$

Hence, the correct answer is (C).

86. Since, the radii of both the balls is same, so the force of buoyancy is also the same.
When the balls fall with a uniform velocity, then condition of equilibrium is attained, so we get

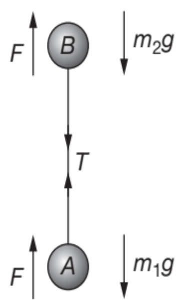
For A, $F + T = m_1 g$ ...(1)

For B, $m_2 g + T = F$ ...(2)

From (1), equating the value of T, we get

$$2F = m_1 g + m_2 g$$

$$\Rightarrow \quad F = \left(\frac{m_1 + m_2}{2}\right)g \qquad \text{...(3)}$$



Substitute (3) either in (1) or in (2), we get

$$T = m_1 g - \left(\frac{m_1 + m_2}{2}\right)g$$

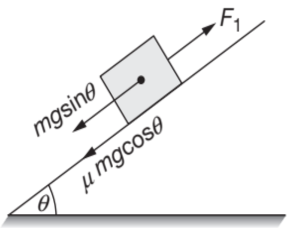
$$\Rightarrow \quad T = \left(\frac{m_1 - m_2}{2}\right)g$$

Hence, the correct answer is (B).

87. Let the mass of the block be m. Force required to drag it up along the plane is

$$F_1 = (mg\sin\theta + \mu mg\cos\theta)$$

and force required to lift it is



$$F_2 = mg$$

Since it is given that $F_1 < F_2$

$$\Rightarrow \quad mg\sin\theta + \mu mg\cos\theta < mg$$

$$\Rightarrow \quad \mu < \frac{1 - \sin\theta}{\cos\theta}$$

$$\Rightarrow \quad \mu < \frac{1 - \sin 30^\circ}{\cos 30^\circ}$$

$$\Rightarrow \quad \mu < \frac{1}{\sqrt{3}}$$

So, $\mu_{MAX} = \dfrac{1}{\sqrt{3}}$

Hence, the correct answer is (C).

88. If $m_1 = m_2$, block will slide down when

$$\mu g\cos\theta < g\sin\theta$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right) < \frac{1}{2} \qquad \text{\{which is true\}}$$

If $m_1 < m_2$, block will slide down when

$$\mu(g - a)\cos\theta < (g - a)\sin\theta$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) < \frac{1}{2}$$

Here $a = \left(\dfrac{m_2 - m_1}{m_1 + m_2}\right)g$

If $m_1 > m_2$, block will slide down when

$$\mu(g + a)\cos\theta < (g + a)\sin\theta$$

$\Rightarrow \quad \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right) < \frac{1}{2}$

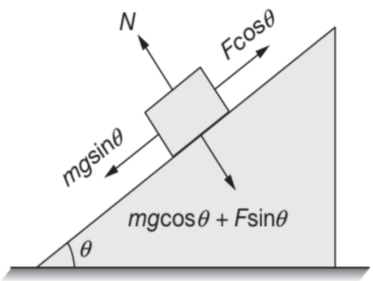
Here $a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$

So, block can't remain stationary when $m_1 > m_2$.

Therefore, block will slide down under all conditions.

Hence, the correct answer is (D).

89. The free body diagram of the block is shown here.



So, we get

$$N = mg\cos\theta + F\sin\theta$$

and $F_{net} = F\cos\theta - mg\sin\theta$ {up the plane}

Hence, the correct answer is (D).

90. Let x be the maximum displacement of block downwards. Then from conservation of mechanical energy, we have the decrease in potential energy of 2 kg block must be equal to the increase in elastic potential energy of both the springs. So, we have

$$mgx = \frac{1}{2}(k_1 + k_2)x^2$$

$$\Rightarrow \quad x = \frac{2mg}{k_1 + k_2} = \frac{(2)(4)(10)}{100 + 300} = 0.2 \text{ m}$$

Acceleration of block in this position is

$$a = \frac{(k_1 + k_2)x - mg}{m} \quad \text{\{upwards\}}$$

$$\Rightarrow \quad a = \frac{(400)(0.2) - (4)(10)}{4} = \frac{80 - 40}{4}$$

$$\Rightarrow \quad a = 10 \text{ ms}^{-2} \quad \text{\{upwards\}}$$

Hence, the correct answer is (D).

91. Let T be the tension in the rope, then for the block, we have

$$T = Mg \quad \ldots(1)$$

If N is the normal reaction between the man and the ground, then for the man, we have

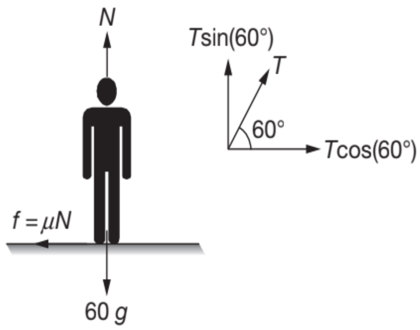
$$N + T\sin(60°) = 60g$$

$$\Rightarrow \quad N = 60g - T\sin(60°) \quad \ldots(2)$$

Also $T\cos(60°) = \mu N$ ...(3)

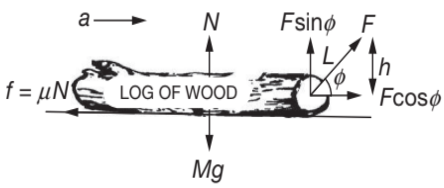
Solving these three equations we get

$$M = 32.15 \text{ kg} \approx 32 \text{ kg}$$



Hence, the correct answer is (D).

92.



$$F\cos\phi - \mu N = Ma \quad \ldots(1)$$

$$F\sin\phi + N = Mg$$

$$\Rightarrow \quad N = Mg - F\sin\phi \quad \ldots(2)$$

Substituting (2) in (1), we get

$$F\cos\phi - \mu Mg + \mu F\sin\phi = Ma$$

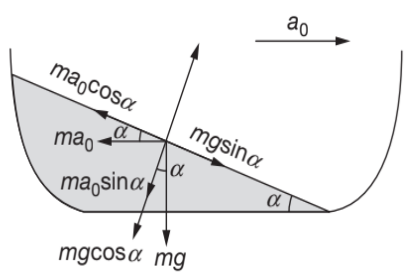
$$\Rightarrow \quad a = \frac{F}{M}(\cos\phi + \mu\sin\phi) - \mu g$$

Hence, the correct answer is (D).

93. According to Newton's Third Law to every action there is equal and opposite reaction and both act on two different bodies. But in this case both are acting on the same body and hence they cancel so that the boat remains stationary.

Hence, the correct answer is (D).

95.



$$ma_0 \cos\alpha = mg \sin\alpha$$

$$\Rightarrow \quad a_0 = g\tan\alpha$$

$$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{a_0}{g}\right)$$

Hence, the correct answer is (A).

96. Thrust on the block $= F = v\dfrac{dm}{dt}$

$$\Rightarrow \quad F = (5)(1) = 5N$$

Acceleration of the block $= \dfrac{F}{M} = \dfrac{5}{2}\ \text{ms}^{-2}$

Hence, the correct answer is (B).

97. $F = \dfrac{mv_x^2}{R} = \dfrac{mv_0^2\cos^2\alpha}{R}$

Hence, the correct answer is (B).

98. $kx = 2mg$

By Law of Conservation of Energy

$$Mgx = \frac{1}{2}kx^2$$

$$\Rightarrow \quad kx = 2Mg$$

$$\Rightarrow \quad M = m$$

Hence, the correct answer is (B).

99. $mg - U = ma_0$ ...(1)

and $U - (m - \Delta m)g = (m - \Delta m)a_0$ ...(2)

$\Rightarrow \quad \Delta m = \left(\dfrac{2a_0}{g + a_0}\right)m$ {Adding (1) and (2)}

Hence, the correct answer is (C).

100. Since windows are closed and the balloon is filled with helium which is lighter than air.
The concept of pseudo force cannot be applied here as in this case the density of body is much less than the density of surrounding air. So when car takes a left turn the expected answer is that Helium balloon will be pushed to the right whereas actually it will be pushed to the left.

Hence, the correct answer is (B).

101. The laws of Physics are the same in all inertial frames or the frames moving with constant velocity with respect to an inertial frame.

Hence, the correct answer is (B).

102. $F = \sqrt{T^2 + T^2 - 2T^2\cos 2\theta_0}$

$$\Rightarrow \quad F = \sqrt{2}T\sqrt{1 - \cos 2\theta_0}$$

$$\Rightarrow \quad F = \sqrt{2}T\sqrt{2}\sin\theta_0$$

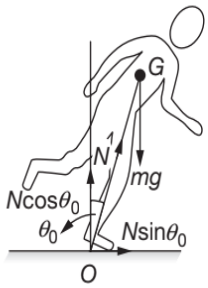
$$\Rightarrow \quad F = 2T\sin\theta_0$$

Hence, the correct answer is (C).

103. For the man not to slip i.e. for rotational equilibrium to exist, the algebraic sum of the moments of the forces acting on the man must be zero. Let N be the normal reaction exerted by the earth on the man. The following forces are acting on the man.

(a) Weight mg acting vertically downwards from the centre of gravity G of the man.

(b) Normal reaction N exerted by earth on the man along OG for rotational equilibrium to exist.



For the man not to slip.

$N\sin\theta_0 \le f_{\text{limiting}}$ ...(1)

But $f_{\text{limiting}} = \mu(N\cos\theta_0)$

$$\Rightarrow \quad N\sin\theta_0 \le \mu N\cos\theta_0$$

$$\tan\theta_0 \le \mu$$

Hence, the correct answer is (B).

104. The vertically hanging curtains bend forward thus indicating that the pseudo force acting on the curtain points in the forward direction. For this to happen the plane must be slowing down for landing.

Hence, the correct answer is (C).

105. $F\cos\theta = \mu N$ and ...(1)

$N + F\sin\theta = mg$

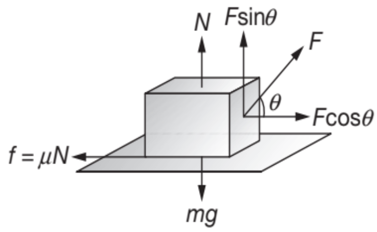
$\Rightarrow \quad N = mg - F\sin\theta$ ...(2)

Substituting (2) in (1), we get

$$F\cos\theta = \mu mg - \mu F\sin\theta$$

$\Rightarrow \quad F = \dfrac{\mu mg}{\cos\theta + \mu\sin\theta}$ ...(3)

Let Z = Denominator = $\cos\theta + \mu\sin\theta$



For F to be MINIMUM, Z must be MAXIMUM

$$\Rightarrow \quad \frac{dZ}{d\theta} = 0$$

$$\Rightarrow \quad -\sin\theta + \mu\cos\theta = 0$$

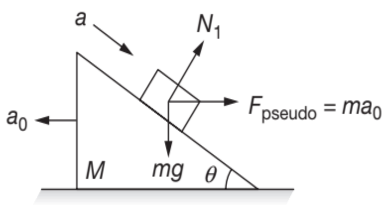
$$\Rightarrow \quad \tan\theta = \mu$$

$$\Rightarrow \quad \theta = \tan^{-1}\mu$$

and $F_{\min} = \dfrac{\mu mg}{\sqrt{1+\mu^2}}$

Hence, the correct answer is (C).

106. Let a be the acceleration of the block with respect to prism. Let the prism move backward with acceleration a_0, then a fictitious (pseudo) force acts on the mass (see figure).



For the mass to move down with acceleration a, we have

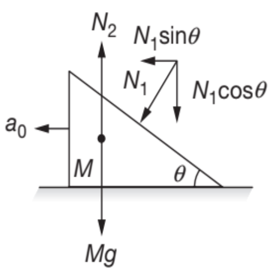
$$ma_0 \cos\theta + mg\sin\theta = ma$$

$$\Rightarrow \quad a = a_0\cos\theta + g\sin\theta \qquad \ldots(1)$$

Also the block is in equilibrium perpendicular to the incline i.e.

$$N_1 + ma_0\sin\theta = mg\cos\theta \qquad \ldots(2)$$

For the prism, we have that it is placed on the ground (an inertial frame) with no fictitious force acting on the prism. Following forces are acting on the prism.



(a) Weight of the prism Mg.

(b) Normal reaction N_2 exerted by the surface on the prism.

(c) Normal reaction N_1 exerted by the block placed on the prism.

$N_1\sin\theta$ is the component responsible for horizontal motion of the prism. So,

$$N_1\sin\theta = Ma_0$$

$$\Rightarrow \quad N_1 = \frac{Ma_0}{\sin\theta} \qquad \ldots(3)$$

Substituting (3) in (2), we get

$$\frac{Ma_0}{\sin\theta} + ma_0\sin\theta = mg\cos\theta$$

$$\Rightarrow \quad a_0 = \left(\frac{m\cos\theta}{M + m\sin^2\theta}\right)g\sin\theta \qquad \ldots(4)$$

Substituting a_0 in (1) we get

$$a = \left(\frac{M+m}{M+m\sin^2\theta}\right)g\sin\theta$$

Hence, the correct answer is (C).

OBJECTIVE TRICK

If $\theta \to \frac{\pi}{2}$, then m (i.e. the block) will fall freely with acceleration g. This is satisfied by both (A) and (C).
To check which one of the two we can further think that if M is very heavy as compared to m, then M must not move and acceleration of m w.r.t. M is $g\sin\theta$. This is satisfied by (C) only. If we can directly think the second way first, then we can even solve the problem without any cumbersome steps.

107. Since $x = kt^2 = 1.732t^2$

$$\Rightarrow \quad x = \sqrt{3}t^2$$

$$\Rightarrow \quad \dot{x} = 2\sqrt{3}t$$

$$\Rightarrow \quad \ddot{x} = a = 2\sqrt{3}\ \text{ms}^{-2}$$

$$\Rightarrow \quad a = g(\sin 45 - \mu\cos 45) = 2\sqrt{3}$$

$$\Rightarrow \quad g\frac{1}{\sqrt{2}}(1-\mu) = 2\sqrt{3}$$

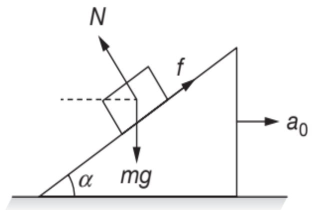
$$\Rightarrow \quad 1-\mu = \frac{2\sqrt{6}}{g}$$

$$\Rightarrow \quad \mu = 1 - \frac{2\sqrt{6}}{g} = 1 - \frac{2(2.45)}{9.8}$$

$$\Rightarrow \quad \mu = 0.5$$

Hence, the correct answer is (A).

108. Consider the block not to slide, then it must have an acceleration same as that of the plane. Then



$$f\cos\alpha - N\sin\alpha = ma_0 \quad \ldots(1)$$

and $f\sin\alpha + N\cos\alpha = mg \quad \ldots(2)$

From (1) and (2), we get

$$f = m(a_0\cos\alpha + g\sin\alpha) \quad \ldots(3)$$

and $N = m(g\cos\alpha - a_0\sin\alpha) \quad \ldots(4)$

$$\Rightarrow \quad \frac{f}{N} = \frac{a_0\cos\alpha + g\sin\alpha}{g\cos\alpha - a_0\sin\alpha}$$

$$\Rightarrow \quad \frac{f}{N} = \frac{a_0 + g\tan\alpha}{a_0 - g\tan\alpha} \quad \ldots(5)$$

Further $\frac{f}{N} = \mu_s = \tan\theta$ in the absence of slipping

$$\Rightarrow \quad \frac{a_0 + g\tan\alpha}{a_0 - g\tan\alpha} \le \tan\theta$$

$$\Rightarrow \quad a_0 \le g\left[\frac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}\right]$$

$$\Rightarrow \quad a_0 \le g\tan(\theta - \alpha) \quad \{\text{for no slipping}\}$$

So for the block to slide $a_0 > g\tan(\theta - \alpha)$.

Hence, the correct answer is (C).

109. Let N_1 be normal reaction between the man and the board

Let N_2 be normal reaction between the board and the floor.

For man : $F + N_1 = Mg \quad \ldots(1)$

For board : $mg + N_1 = N_2 \quad \ldots(2)$

$$F = \mu N_2 \quad \ldots(3)$$

Hence, the correct answer is (D).

111. $mg(\sin\theta + \mu\cos\theta) = 2mg(\sin\theta - \mu\cos\theta)$

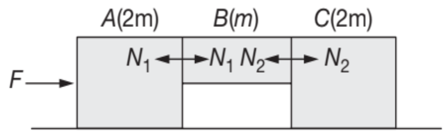
$$\Rightarrow \quad \tan\theta = 3\mu$$

$$\Rightarrow \quad \tan\theta = 3\tan\phi \qquad \{\because \mu = \tan\phi\}$$

Hence, the correct answer is (C).

112. Horizontal acceleration of the system is

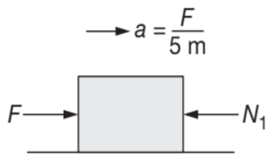
$$a = \frac{F}{2m + m + 2m} = \frac{F}{5m}$$



FBD A

$$F - N_1 = (2m)a$$

$$\Rightarrow \quad N_1 = F - \frac{2F}{5} = \frac{3F}{5}$$

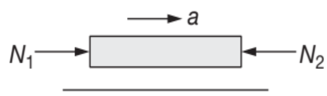


FBD B

$$N_1 - N_2 = ma$$

$$\Rightarrow \quad N_2 = \frac{3F}{5} - \frac{F}{5} = \frac{2F}{5}$$

Now B will not slide downwards if



$$\mu N_2 \ge m_B g$$

$$\Rightarrow \quad (\mu)\left(\frac{2}{5}F\right) \ge mg$$

$$\Rightarrow \quad F \ge \frac{5}{2\mu}mg$$

So, $F_{\min} = \frac{5mg}{2\mu}$

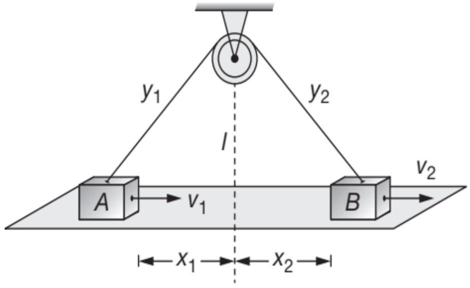
Hence, the correct answer is (B).

113. For A, we have

$$y_1^2 = l^2 + x_1^2$$

$$\Rightarrow \quad 2y_1\frac{dy_1}{dt} = 2x_1\frac{dx_1}{dt}$$

$$\Rightarrow \frac{dy_1}{dt} = \frac{x_1}{y_1}\left(\frac{dx_1}{dt}\right)$$



Since $\frac{x_1}{y_1} = \cos\alpha$

$$\Rightarrow \frac{dy_1}{dt} = v_1 \cos\alpha \quad \ldots(1)$$

Similarly for B, we have

$$y_2^2 = l^2 + x_2^2$$

$$\Rightarrow 2y_2 \frac{dy_2}{dt} = 2x_2 \frac{dx_2}{dt}$$

$$\Rightarrow \frac{dy_2}{dt} = \frac{x_2}{y_2}\left(\frac{dx_2}{dt}\right)$$

Since $\frac{x_2}{y_2} = \cos\beta$

$$\Rightarrow \frac{dy_2}{dt} = v_2 \cos\beta \quad \ldots(2)$$

Also from our knowledge of constrained motion, we observe that

$$y_1 + y_2 = \text{constant}$$

$$\Rightarrow \frac{dy_1}{dt} + \frac{dy_2}{dt} = 0$$

$$\Rightarrow \left|\frac{dy_1}{dt}\right| = \left|\frac{dy_2}{dt}\right|$$

$$\Rightarrow v_1 \cos\alpha = v_2 \cos\beta$$

$$\Rightarrow \frac{v_2}{v_1} = \frac{\cos\alpha}{\cos\beta}$$

Hence, the correct answer is (D).

SHORTCUT

We could have directly equated the components of the velocities of blocks A and B along the thread to get $v_1 \cos\alpha = v_2 \cos\beta$

114. Since one end of pulley P_1 is fixed and the other end moves with an acceleration of 6 ms^{-2}. So, the acceleration of pulley is

$$a_{P_1} = \frac{6+0}{2} = 3\text{ ms}^{-2}\text{, towards left}$$

Also, if P_1 moves, say left by x, then B moves up by $\frac{x}{3}$. Hence, acceleration of B must be one third acceleration of P_1. Hence

$$a_B = \frac{(a)_{P_1}}{3} = 1\text{ ms}^{-2}$$

Hence, the correct answer is (D).

115. $\frac{1}{K_s} = \frac{1}{K} + \frac{1}{3K} + \frac{1}{9K} + \frac{1}{27K} + \ldots$

$$\Rightarrow \frac{1}{K_s} = \frac{1}{K}\left(1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots\right)$$

$$\Rightarrow \frac{1}{K_s} = \frac{1}{K}\left(\frac{1}{1-\frac{1}{3}}\right)$$

$$\Rightarrow K_s = \frac{2K}{3}$$

Hence, the correct answer is (C).

116. $a = \left(\frac{10 - 4\sin 30 - (0.2)(4)\cos 30}{10+4}\right)g$

$$\Rightarrow a = \frac{(10-2-0.7)}{14}g$$

$$\Rightarrow a = \left(\frac{7.3}{14}\right)9.8$$

$$\Rightarrow a = 5.11\text{ ms}^{-2}$$

Since $v^2 - u^2 = 2as$

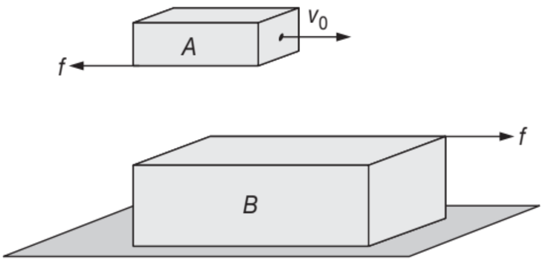
$$\Rightarrow v^2 - 0^2 = 2(5.11)(4)$$

$$\Rightarrow v \simeq 6.5\text{ ms}^{-1}$$

Hence, the correct answer is (A).

117. Friction force between A and $B(=\mu mg)$ will accelerate B and retard A till slipping stops between A and B. Since mass of both are equal, So we have acceleration of $B =$ retardation of $A = \mu g$.

$$\Rightarrow v_A = v_0 - (\mu g)t \text{ and } v_B = (\mu g)t$$



After slipping ceases, the common velocity of both becomes $\frac{v_0}{2}$.

This can also be obtained from conservation of linear momentum also (to be studied later).

Hence, the correct answer is (B).

119. $\sin\theta = \frac{h}{l}$

$$Mg - T = Ma \text{ and}$$

$$\Rightarrow \quad T - mg\sin\theta = ma$$

$$\Rightarrow \quad a = \left(\frac{M - m\sin\theta}{M+m}\right)g$$

Since $v^2 - 0^2 = 2as$, where s is the distance travelled by m before M is detached. After detaching M, m will move the remaining distance of $(l-s)$ up the incline, under a retardation $g\sin\theta$.

$$\Rightarrow \quad 0^2 = v^2 - 2(g\sin\theta)(l-s)$$

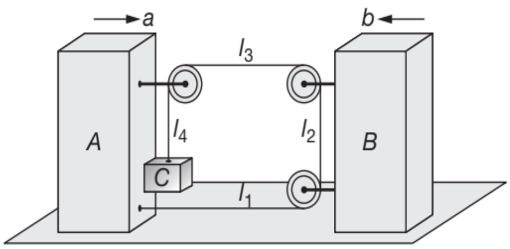
$$\Rightarrow \quad 0^2 = 2as - 2g\sin\theta(l-s)$$

$$\Rightarrow \quad s = \frac{M+m}{M}\left(\frac{hl}{h+l}\right)$$

Hence, the correct answer is (D).

120. Let the initial lengths of the respective segments of strings be l_1, l_2, l_3 and l_4. Then

$$\left(\begin{array}{c}\text{Total length of the}\\ \text{string at } t=0\end{array}\right) = \left(\begin{array}{c}\text{Total length of the}\\ \text{string at time } t\end{array}\right)$$



Let A move forward by x, B move backwards (towards left) by y and correspondingly C moves down, say by z. Then

$$\cancel{l_1} + \cancel{l_2} + \cancel{l_3} + \cancel{l_4} = \left[\cancel{l_1} - (x+y)\right] + \cancel{l_2} + \left[\cancel{l_3} - (x+y)\right] + \left(\cancel{l_4} + z\right)$$

$$z = 2(x+y)$$

$$\Rightarrow \quad \ddot{z} = 2(\ddot{x} + \ddot{y})$$

$$\Rightarrow \quad a_C = 2(a+b), \text{ downwards}$$

Also, as A moves forward, C also moves forward, hence

$$\vec{a}_C = a\hat{i} - 2(a+b)\hat{j}$$

$$\Rightarrow \quad |\vec{a}_C| = \sqrt{a^2 + 4(a+b)^2}$$

$$\Rightarrow \quad |\vec{a}_C| = \sqrt{5a^2 + 4b^2 + 8ab}$$

Hence, the correct answer is (D).

121. $a = \mu g$

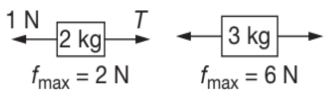
$$s = 0 + \frac{1}{2}(\mu g)t_{MIN}^2$$

$$\Rightarrow \quad t_{MIN} = \sqrt{\frac{2s}{\mu g}}$$

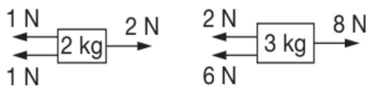
$$\Rightarrow \quad t_{MIN} \propto \frac{1}{\sqrt{\mu}}$$

Hence, the correct answer is (B).

122. FBD of blocks is shown



Net force without friction on system is 7 N towards right side, so maximum friction will first come on 3 kg block



$$\Rightarrow \quad f_1 = 1\text{ N},\ f_2 = 6\text{ N},\ T = 2\text{ N}$$

Hence, the correct answer is (C).

123. As no force is acting on A,

$$T = 0 \text{ and } a_B = \frac{F}{m_B}$$

Hence, the correct answer is (A).

124. As block moves to the right, N decreases.

Hence, the correct answer is (C).

126. Only magnitude remains constant and direction changes

Hence, the correct answer is (A).

127. $x = 2t$

$$\Rightarrow \quad v_x = \frac{dx}{dt} = 2$$

$y = 2t^2$

$$\Rightarrow \quad v_y = \frac{dy}{dx} = 4t$$

$$\Rightarrow \quad \tan\theta = \frac{v_y}{v_x} = \frac{4t}{2} = 2t$$

Differentiating with respect to time we get,

$$(\sec^2\theta)\frac{d\theta}{dt} = 2$$

$$\Rightarrow \quad (1+\tan^2\theta)\frac{d\theta}{dt} = 2$$

$$\Rightarrow \quad (1+4t^2)\frac{d\theta}{dt} = 2$$

$$\Rightarrow \quad \frac{d\theta}{dt} = \frac{2}{1+4t^2}$$

So, $\frac{d\theta}{dt}$ at $t = 2$ s is

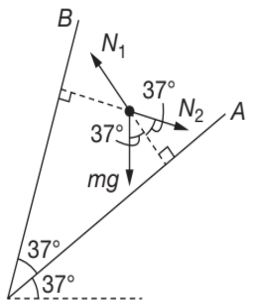
$$\frac{d\theta}{dt} = \frac{2}{1+4(2)^2} = \frac{2}{17} \text{ rads}^{-1}$$

Hence, the correct answer is (A).

128. $2kx\cos(30°) = \left(\frac{4m_1m_2}{m_1+m_2}\right)g$

Hence, the correct answer is (A).

129. Using Lami's Theorem, we get



$$\frac{mg}{\sin(180° - 37°)} = \frac{N_2}{\sin(180° - 37°)}$$

$$\Rightarrow \quad N_2 = mg$$

Hence, the correct answer is (A).

130. Limiting friction on block B is

$$f_l = (0.5)(2+8)(10) = 50 \text{ N}$$

So, block B will not move.

Hence, the correct answer is (A).

131. Retardation $a = g(\sin\theta + \mu\cos\theta) = 5(\sqrt{3}+\mu)$

Since $v = u - at$

$$\Rightarrow \quad 0 = u - at$$

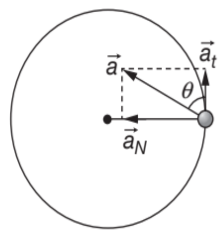
$$\Rightarrow \quad a = \frac{u}{t}$$

$$\Rightarrow \quad 5(\sqrt{3}+\mu) = 10$$

$$\Rightarrow \quad \mu = 0.27$$

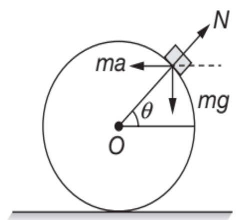
Hence, the correct answer is (C).

132. $\tan\theta = \frac{a_N}{a_t}$



Hence, the correct answer is (C).

134.



Drawing F.B.D of m, we get

$$N\sin\theta = mg \text{ and } N\cos\theta = ma$$

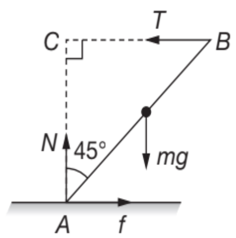
$$\Rightarrow \quad \tan\theta = \frac{g}{a}$$

$$\Rightarrow \quad a = g\cot\theta$$

So, $F = (m+M)g\cot\theta$

Hence, the correct answer is (C).

136.



$$T = f \quad \ldots(1)$$

$$N = mg \quad \ldots(2)$$

Taking torque about A, we get

$$T \times \frac{L}{\sqrt{2}} = mg \times \frac{L/2}{\sqrt{2}}$$

$$\Rightarrow \quad T = \frac{mg}{2} = 50 \text{ N}$$

Hence, the correct answer is (A).

137. $F_{\text{net}} = F - f_k = 2t - \mu mg$

Hence, the correct answer is (C).

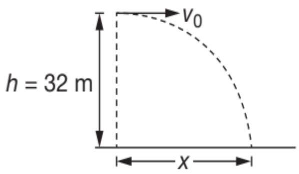
138. $R = u\sqrt{\frac{2h}{g}} = 12\sqrt{\frac{2 \times 5}{10}} = 12 \text{ m}$

So, distance from origin is

$$r = \sqrt{5^2 + 12^2} = 13 \text{ m}$$

Hence, the correct answer is (C).

139.



$$x = v_0 t \text{ and } h = \frac{1}{2}gt^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 32}{10}} = \frac{8}{\sqrt{10}}$$

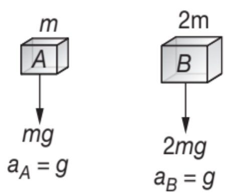
Since, $v_0 = \omega r = \frac{2\pi}{T} \times 12 = \frac{2\pi}{1.2} \times 12 = 20\pi$

$$\Rightarrow \quad x = 20\pi\left(\frac{8}{\sqrt{10}}\right) = \frac{160 \times 3.14}{3.162}$$

$$\Rightarrow \quad x = 160 \text{ m}$$

Hence, the correct answer is (D).

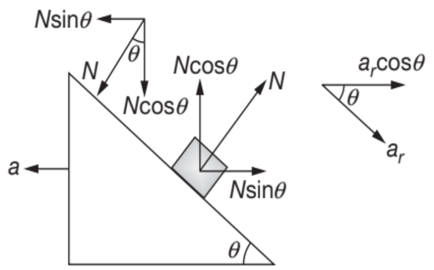
140. In this case spring force is zero initially. F.B.D of A and B are shown below.



Hence, the correct answer is (A).

141. Let acceleration of mass m relative to wedge down the plane is a_r. Its absolute acceleration in horizontal direction is

$$(a_r \cos 60° - a) \quad \text{(towards right)}$$



Let N be the normal reaction between the mass and the wedge. Then, we have

for the block, $N\sin\theta = m(a_r \cos\theta - a)$ ...(1)

for the wedge, $N\sin\theta = Ma$ ...(2)

So, from equations (1) and (2), we get

$$N\sin\theta = Ma = m(a_r\cos\theta - a)$$

$$\Rightarrow \quad a_r = \frac{(M+m)a}{m\cos\theta} = \frac{(M+m)a}{m\cos(60°)} = \frac{2(M+m)a}{m}$$

Hence, the correct answer is (C).

142. For both Case (i) and Case (ii), the blocks move together if friction between the blocks is static or limiting. Since we need to calculate the maximum force for which the blocks move together, so friction between the blocks will be limiting in nature.
So, for Case (i), we have

$$F - f_l = ma \quad \ldots(1)$$

$$f_l = Ma \quad \ldots(2)$$

Also, $f_l = \mu mg$ ...(3)

From (1), (2) and (3), we get

$$F_{\max} = \mu mg\left(1 + \frac{m}{M}\right)$$

Similarly for Case (ii), we have

$$(F_{\text{pseudo}}) \le f_l$$

$$\Rightarrow \quad ma \le \mu mg$$

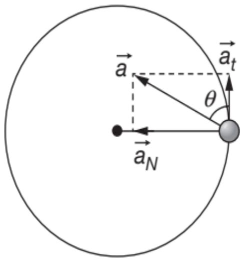
$$\Rightarrow \quad m\frac{F}{M+m} \le \mu mg$$

$$\Rightarrow \quad F \le \mu(M+m)g$$

$$\Rightarrow \quad F_{\max} = \mu(M+m)g$$

Hence, the correct answer is (C).

144.



$$\tan\theta = \frac{a_c}{a_t}$$

$$\Rightarrow \quad \frac{a_c}{a_t} = \tan(30^\circ) = \frac{1}{\sqrt{3}}$$

Hence, the correct answer is (C).

145. Since, $\vec{a} = 6\hat{i} - 8\hat{j}$

$$\Rightarrow \quad a_r = 8 \text{ and } a_t = 6$$

$$\Rightarrow \quad r\omega^2 = 8 \text{ and } r\alpha = 6$$

$$\Rightarrow \quad \omega = 2 \text{ rads}^{-1} \text{ and } \alpha = 3 \text{ rads}^{-2}$$

Since the body is rotating in clockwise sense, so by using Right Hand Thumb Rule, we get

$$\vec{\omega} = -2\hat{k} \text{ rads}^{-1} \text{ and}$$

$$\vec{\alpha} = -3\hat{k} \text{ rads}^{-2}$$

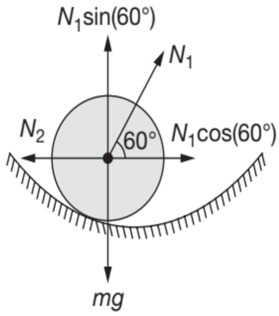
Hence, the correct answer is (A).

146. Since supporting plane is lowered slowly, so

$$N = mg - kx$$

Hence, the correct answer is (C).

147.



$$N_1\cos 60^\circ = N_2$$

$$\Rightarrow \quad \frac{N_1}{N_2} = 2$$

Hence, the correct answer is (B).

149. Force required to stop the body from just sliding down the plane is

$$F_2 = (mg\sin\theta - \mu mg\cos\theta)$$

Force required to just pull the body up the plane is

$$F_1 = (mg\sin\theta + \mu mg\cos\theta)$$

Since $F_1 = 2F_2$

$$\Rightarrow \quad mg(\sin\theta + \mu\cos\theta) = 2mg(\sin\theta - \mu\cos\theta)$$

$$\Rightarrow \quad \sin\theta + \mu\cos\theta = 2\sin\theta - 2\mu\cos\theta$$

$$\Rightarrow \quad \sin\theta = 3\mu\cos\theta$$

$$\Rightarrow \quad \tan\theta = 3\mu$$

Hence, the correct answer is (D).

150. Since $r \ll R$, so mass/length can be given by

$$\lambda = \frac{m}{2r} = \left(\frac{\frac{4}{3}\pi r^3\rho}{2r}\right)\left(\frac{2}{3}\pi r^2\rho\right)$$

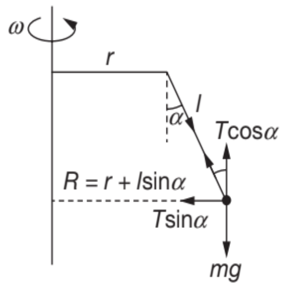
Now, tension in tube is given by

$$T = (\lambda R)(R\omega^2) = \left(\frac{2}{3}\pi r^2\rho\right)(\omega^2)(R^2)$$

$$\Rightarrow \quad T = \frac{2}{3}\pi\rho\omega^2 r^2 R^2$$

Hence, the correct answer is (D).

151.



$$T\cos\alpha = mg$$

$$T\sin\alpha = m(r + l\sin\alpha)\omega^2$$

$$\Rightarrow \quad \omega^2 = \frac{g\tan\alpha}{r + l\sin\alpha}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g\tan\alpha}{r + l\sin\alpha}}$$

Hence, the correct answer is (A).

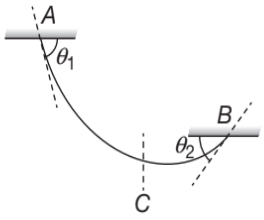
152. Since $T = mg$, so for pulley P, we have

$$T_{\text{net}} = \sqrt{T^2 + T^2 + 2(T)(T)\cos(120^\circ)}$$

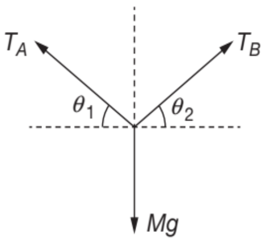
$$\Rightarrow \quad T = mg$$

Hence, the correct answer is (A).

153.



Free body diagram of rope AB is

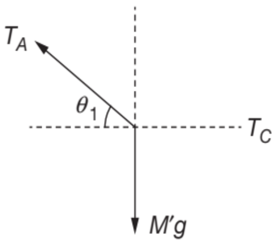


$$T_A \cos\theta_1 = T_B \cos\theta_2$$

$$T_A \sin\theta_1 + T_B \sin\theta_2 = Mg$$

$$\Rightarrow \quad T_A = \frac{Mg\cos\theta_2}{\sin(\theta_1+\theta_2)} \text{ and } T_B = \frac{Mg\cos\theta_1}{\sin(\theta_1+\theta_2)}$$

Free body diagram of portion AC of the rope is



For horizontal equilibrium, we have

$$T_A \cos\theta_1 = T_C$$

$$\Rightarrow \quad T_C = \frac{Mg\cos\theta_1\cos\theta_2}{\sin(\theta_1+\theta_2)}$$

So, tension will be maximum at A and minimum at C.

Hence, the correct answer is (C).

154. Net force on m_3 is $F_{net} = \sqrt{(30)^2 + (40)^2} = 50 \text{ N}$

and limiting friction on m_3 is $f_\ell = \mu m_3 g = 60 \text{ N}$

Since the applied force is less than the limiting force of friction, so the system remains in equilibrium and friction between m_3 and the surface is equal to the net applied force i.e. 50 N.

Hence, the correct answer is (C).

155. Friction force between A and $B(= \mu mg)$ will accelerate B and retard A till slipping is stopped between the two and since mass of both are equal, so

Acceleration of B is equal to Retardation of A. i.e.

$$|a_B| = |a_A| = \mu g$$

$$\Rightarrow \quad v_1 = v_0 - \mu gt \text{ and } v_2 = \mu gt$$

Hence, the correct graph is (B).

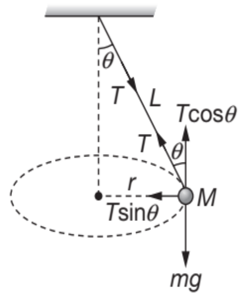
After slipping has ceased, the common velocity of both will become $\frac{v_0}{2}$, which can be also be obtained from Conservation of Linear Momentum also.

Hence, the correct answer is (B).

156. We cannot choose (A) or (B) because the centripetal acceleration vector is not constant it continuously changes in direction. Of the remaining choices, only (C) gives the correct perpendicular relationship between a_c and v.

Hence, the correct answer is (C).

157.



$$T\cos\theta = Mg \qquad \ldots(1)$$

$$T\sin\theta = Mr\omega^2 = M(L\sin\theta)\omega^2$$

$$\Rightarrow \quad T = ML\omega^2 \qquad \ldots(2)$$

Since, $\omega = \frac{2}{\pi}$ rev per sec

$$\Rightarrow \quad \omega = 4 \text{ rads}^{-1}$$

$$\Rightarrow \quad T = M\omega^2 L = M(4)^2 L = 16ML$$

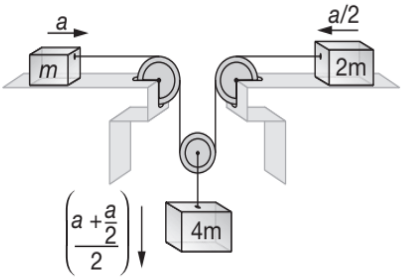
Hence, the correct answer is (D).

158. Free body diagram of "wedge + mass" system is

$$\Rightarrow \quad N = (m+M)g$$

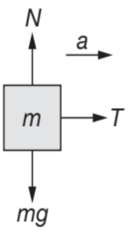
Hence, the correct answer is (B).

159.



Let a be the acceleration of block of mass m, then acceleration of $2m$ will be $\frac{a}{2}$ and that of $4m$ will be $\left(\frac{a+\frac{a}{2}}{2}\right)=\frac{3a}{4}$.

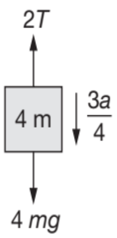
For m, we have



$N = mg$ and

$T = ma$ ...(1)

For $4m$, we have



$4mg - 2T = 4m\left(\frac{3a}{4}\right)$ ...(2)

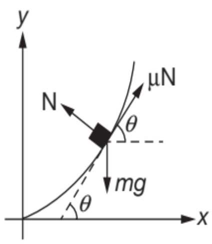
$\Rightarrow \quad a = \frac{4g}{5}$

Hence, the correct answer is (B).

160. Block A moves in horizontal direction only due to friction.

Hence, the correct answer is (B).

161. Since $\frac{dy}{dx} = \frac{x}{10}$



$\Rightarrow \quad \tan\theta = \frac{x}{10}$ ...(1)

Equilibrium of mass in horizontal direction gives the equation

$\mu N\cos\theta = N\sin\theta$

$\Rightarrow \quad \tan\theta = \mu = \frac{1}{2}$ ...(2)

From (1) and (2), we get

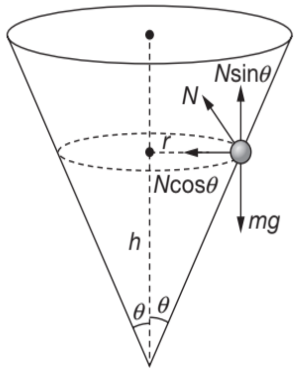
$\frac{x}{10} = \frac{1}{2}$

$\Rightarrow \quad x = 5\text{ m}$

$\Rightarrow \quad y = \frac{x^2}{20} = \frac{25}{20} = 1.25\text{ m}$

Hence, the correct answer is (B).

162.



$N\sin\theta = mg$

$N\cos\theta = \frac{mv^2}{r}$

$\Rightarrow \quad \tan\theta = \frac{gr}{v^2}$

Since $\tan\theta = \frac{r}{h}$

$\Rightarrow \quad \frac{r}{h} = \frac{rg}{v^2}$

$\Rightarrow \quad h = \frac{v^2}{g} = 2.5\text{ cm}$

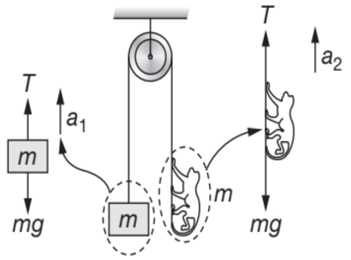
Hence, the correct answer is (D).

163. $\frac{mv^2}{r} = \mu mg$

$\Rightarrow \quad v = \sqrt{\mu rg}$

Hence, the correct answer is (D).

164.



For Bananas, we have

$$T - mg = ma_1$$

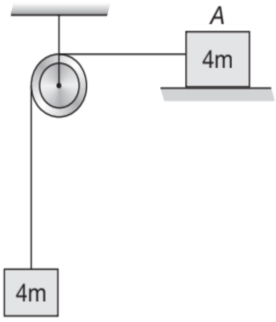
For Monkey, we have

$$T - mg = ma_2$$

Since $a_1 = a_2$ so both bananas and monkey have same acceleration and hence they move up at speed v.

Hence, the correct answer is (D).

165. The system can be redrawn as

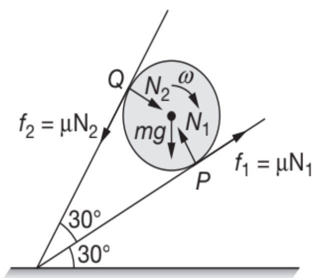


$$\Rightarrow \quad a = \frac{4mg}{8m} = \frac{g}{2} = 5 \text{ ms}^{-2}$$

$$\Rightarrow \quad v = u + at = 2 \times 5 = 10 \text{ ms}^{-1}$$

Hence, the correct answer is (C).

166. Let μ be the friction coefficient between sphere and each wall. Free body diagram of sphere is



Net force on the sphere in horizontal direction is zero. So,

$$N_1 \cos 60° + \mu N_2 \cos 60° = N_2 \cos 30° + \mu N_1 \cos 30°$$

$$\Rightarrow \quad N_1 + \mu N_2 = \sqrt{3}(N_2 + \mu N_1)$$

$$\Rightarrow \quad N_1(1 - \sqrt{3}\mu) = N_2(\sqrt{3} - \mu)$$

$$\Rightarrow \quad \frac{N_1}{N_2} = \frac{\sqrt{3} - \mu}{1 - \sqrt{3}\mu}$$

Substituting $\mu = \frac{1}{3}$ we get,

$$\frac{N_1}{N_2} = \frac{\sqrt{3} - \frac{1}{3}}{1 - \frac{\sqrt{3}}{3}} = \frac{3\sqrt{3} - 1}{3 - \sqrt{3}} = 1 + \frac{4}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{f_1}{f_2} = \frac{\mu N_1}{\mu N_2} = 1 + \frac{4}{\sqrt{3}}$$

Hence, the correct answer is (A).

167. Blocks A and C both move due to friction. But less friction is available to A as compared to C because normal reaction between A and B is less. Maximum friction between A and B can be

$$f_{\max} = \mu m_A g = \left(\frac{1}{2}\right) mg$$

So, maximum acceleration of A can be

$$a_{\max} = \frac{f_{\max}}{m} = \frac{g}{2}$$

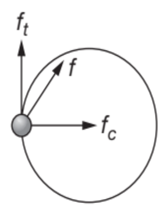
Further $a_{\max} = \dfrac{m_D g}{3m + m_D}$

$$\Rightarrow \quad \frac{g}{2} = \frac{m_D g}{3m + m_D}$$

$$\Rightarrow \quad m_D = 3m$$

Hence, the correct answer is (C).

168.



Let f be the friction force, then

$$f = \sqrt{F_C^2 + F_T^2}$$

After 2 seconds, $\omega = \alpha t = \frac{2}{3}$ rads^{-1}

$$\Rightarrow \quad F_T = ma_T = m(\alpha R) = (36 \times 10^{-5})\left(\frac{1}{3}\right)\left(\frac{1}{4}\right) \text{N}$$

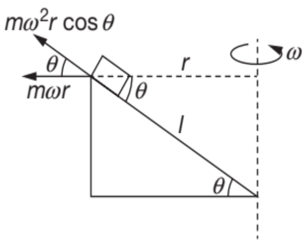
$\Rightarrow \quad F_T = 3\times10^{-5}$ N

$$F_C = m\omega^2 R = (36\times10^{-5})\left(\frac{4}{9}\right)\left(\frac{1}{4}\right) = 4\times10^{-5}\text{ N}$$

$\Rightarrow \quad f = \sqrt{F_C^2 + F_T^2} = 5\times10^{-5}\text{ N} = 50\ \mu\text{N}$

Hence, the correct answer is (C).

169.



$$m\omega^2 r\cos\theta = mg\sin\theta$$

$\Rightarrow \quad m\omega^2 l\cos^2\theta = mg\sin\theta \qquad \{\because\ r = \ell\cos\theta\}$

$$\Rightarrow \quad \omega = \sqrt{\frac{g\sin\theta}{l\cos^2\theta}} = \sqrt{\frac{g\sin\theta}{l}}\sec\theta$$

Hence, the correct answer is (A).

170. $a = \dfrac{a_A + a_B}{2} = \dfrac{3+4}{2} = \dfrac{7}{2} = 3.5\text{ ms}^{-2}$

Hence, the correct answer is (C).

171. Reading of spring balance is

$$\text{Reading } = \frac{\text{Thrust}}{g} = \frac{4m_1m_2}{m_1+m_2} = \frac{10}{3}\text{ kg}$$

$\Rightarrow \quad$ Reading $= 3.33$ kg $\ (< 6\text{ kg})$

Hence, the correct answer is (B).

172. Net horizontal force on wedge

$$F_H = mg\cos\theta\sin\theta$$

Net normal reaction from the ground is

$$N = 2mg + mg\cos^2\theta$$

Since, $\mu N = F_H$

$\Rightarrow \quad \mu = 0.20$

Hence, the correct answer is (B).

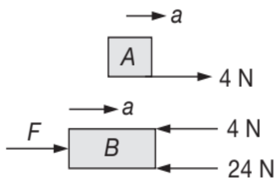
173. Maximum frictional force between A and B is

$$f_1 = \mu_1 m_A g = (0.2)(2)(10)\text{ N}$$

$\Rightarrow \quad f_1 = 4$ N

Limiting friction between B and ground is

$$f_2 = (0.4)(6)(10) = 24\text{ N}$$



For A, $4 = 2a_A = 2a$

$\Rightarrow \quad a = 2\text{ ms}^{-2}$

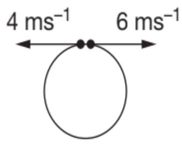
For B, $F - 28 = 4a$

$\Rightarrow \quad F - 28 = 8$

$\Rightarrow \quad F = 36$ N

Hence, the correct answer is (B).

174.



Since $S_1 + S_2 = 2\pi r$

$\Rightarrow \quad 4t + 6t = 2\pi r$

$$\Rightarrow \quad t = \frac{2\pi r}{10} = \frac{2\times3.14\times4}{10} = 2.5\text{ s}$$

Hence, the correct answer is (B).

175. Since $a = g\tan\theta$

$\Rightarrow \quad Mg - T = Ma \qquad \ldots(1)$

$T = (m + M')a \qquad \ldots(2)$

Put (2) in (1), we get

$$Mg - (m+M')a = Ma$$

$\Rightarrow \quad Mg = (m + M + M')g\tan\theta$

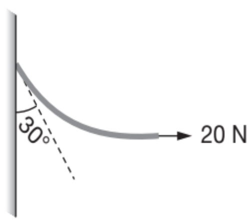
$\Rightarrow \quad Mg(1-\tan\theta) = (m+M')(g\tan\theta)$

$$\Rightarrow \quad M = \frac{(m+M')\tan\theta}{1-\tan\theta}$$

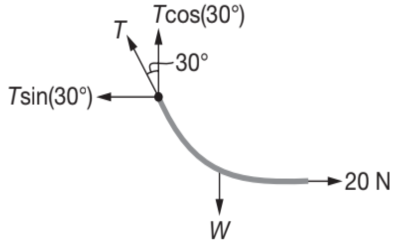
$$\Rightarrow \quad M = \frac{m+M'}{\cot\theta - 1}$$

Hence, the correct answer is (D).

177.



Free body diagram



For equilibrium, we have

$$T\sin(30°) = 20$$

$$T\cos(30°) = W$$

$$\Rightarrow \quad \tan(30°) = \frac{20}{W}$$

$$\Rightarrow \quad W = 20\cot(30°)$$

$$\Rightarrow \quad W = 20\sqrt{3} = 20 \times 1.732$$

So, weight, $W = 34.64$ N

$\Rightarrow$ Mass, $m = \dfrac{W}{g} = 3.5$ kg

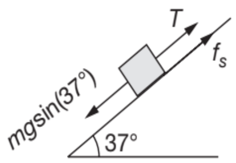
Hence, the correct answer is (C).

179. Since $\mu = 0.8 = \dfrac{4}{5}$

$\Rightarrow$ Angle of repose $\alpha = 39°$

Since angle which incline makes with the horizontal is $37°(<39°)$, so the block will not slide down the incline and friction between the block and the incline will be static in nature. So,

$$f_s = F_{\text{applied}} = mg\sin(37°)$$



Now, if T be the tension in the string, then

$$T + f_s = mg\sin(37°)$$

$$\Rightarrow \quad T = 0 \qquad \{\because f_s = mg\sin(37°)\}$$

Hence, the correct answer is (D).

180. Mass of hanging part of chain is

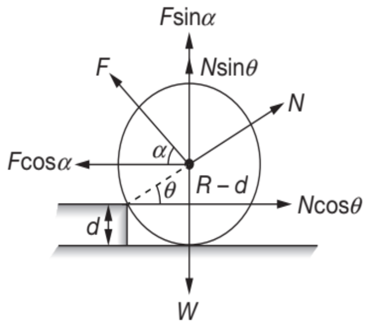
$$m' = \frac{m}{\left(\frac{3\pi R}{2}\right)}\left(\frac{3\pi R}{2} - \pi R\right) = \frac{m}{3}$$

So, acceleration of chain is given by

$$a = \frac{2mg - \frac{mg}{3}}{m} = \frac{5g}{3}$$

Hence, the correct answer is (D).

182.



Since, $F\cos\alpha - N\cos\theta = 0$

$$\Rightarrow \quad F\cos\alpha = N\cos\theta \qquad \ldots(1)$$

Also, $N\sin\theta + F\sin\alpha - W = 0$

$$\Rightarrow \quad N\sin\theta = W - F\sin\alpha \qquad \ldots(2)$$

From (1), $N = \dfrac{F\cos\alpha}{\cos\theta}$, put in (2), we get

$$F\cos\alpha\tan\theta = W - F\sin\alpha$$

$$\Rightarrow \quad F\left(\cos\alpha\frac{\sin\theta}{\cos\theta} + \sin\alpha\right) = W$$

$$\Rightarrow \quad F\sin(\alpha + \theta) = W\cos\theta$$

$$\Rightarrow \quad F = \frac{W\cos\theta}{\sin(\theta + \alpha)}$$

Now $F = F_{\min}$ when $\theta + \alpha = 90°$

$$\Rightarrow \quad F_{\min} = W\cos\theta$$

Hence, the correct answer is (A).

183. METHOD I

For A, $f = ma$

$$\Rightarrow \quad \mu mg = ma$$

$$\Rightarrow \quad a = \mu g \qquad \ldots(1)$$

For B, $T - \mu mg = 2ma \qquad \ldots(2)$

For C, $mg - T = ma \qquad \ldots(3)$

From (1), (2) and (3), we get

$$mg - \mu mg = 3m(\mu mg)$$

$$\Rightarrow \quad 4\mu mg = mg$$

$$\Rightarrow \quad \mu = \frac{1}{4}$$

METHOD II

Acceleration of the system is

$$a_{system} = \frac{mg}{4m} = \frac{g}{4}$$

For Block A not to slip on B, we have

$$F_{pseudo} = f$$

$$\Rightarrow \quad m\left(\frac{g}{4}\right) = \mu mg$$

$$\Rightarrow \quad \mu = \frac{1}{4}$$

Hence, the correct answer is (C).

184. If common maximum acceleration is 'a' then,

$$(m_A + m_B)a = f$$

$$\Rightarrow \quad (2+4)a = 2t$$

$$\Rightarrow \quad 3a = t \qquad \ldots(1)$$

For block B, we have

$$f_L = m_B \times a$$

$$\Rightarrow \quad 0.5 \times 2 \times 10 = 4a$$

$$\Rightarrow \quad a = 2.5 \text{ ms}^{-2}$$

From equation (1), we get

$$t = 7.5 \text{ sec}$$

Hence, the correct answer is (B).

185. Since M is at rest, so the tension in the string is

$$T = \frac{Mg}{2}$$

Let acceleration of m and m' be a, so one will move downward and other will move upward.
For m, we have

$$mg - \frac{Mg}{2} = ma \qquad \ldots(1)$$

For m', we have

$$\frac{Mg}{2} - m'g = m'a \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$\frac{4}{M} = \frac{1}{m} + \frac{1}{m'}$$

Hence, the correct answer is (A).

186. $T = 4g + 5(10+2)$

$$\Rightarrow \quad T = 40 + 60 = 100 \text{ N}$$

Hence, the correct answer is (B).

188. For the sliding not to occur, we have $\tan\theta \le \mu$.

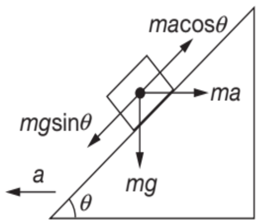
Since $\tan\theta = \frac{dy}{dx} = \frac{2x}{a} = \frac{2\sqrt{ya}}{a} = 2\sqrt{\frac{y}{a}}$

$$\Rightarrow \quad 2\sqrt{\frac{y}{a}} \le \mu$$

$$\Rightarrow \quad y \le \frac{a\mu^2}{4}$$

Hence, the correct answer is (C).

189.



Acceleration of block relative to wedge, is

$$a' = a\cos\theta - g\sin\theta \text{ (up the plane)}$$

$$\Rightarrow \quad a' = 10(\sqrt{3})\left(\frac{\sqrt{3}}{2}\right) - 10\left(\frac{1}{2}\right) = 10 \text{ ms}^{-2}$$

Since $l = \frac{1}{2}a't^2$

$$\Rightarrow \quad t = \sqrt{\frac{2l}{a'}} = \sqrt{\frac{2.5}{10}} = \frac{1}{2} \text{ sec}$$

Hence, the correct answer is (D).

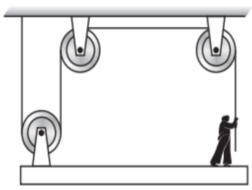
190. Since, acceleration of body w.r.t. lift is $a_{net} = g + a$.

So, time of flight of body, $t = \frac{2u}{g+a}$

$$\Rightarrow \quad a = \frac{2u - gt}{t} = \frac{(2\times5)-(10\times0.8)}{0.8} = 2.5 \text{ ms}^{-2}$$

Hence, the correct answer is (C).

191. For the system to move at constant velocity, force should be zero. So, we have

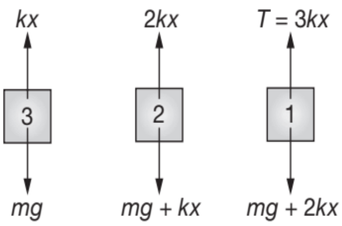


$$3T = 750$$

$$\Rightarrow \quad T = 250 \text{ N}$$

Hence, the correct answer is (B).

192. In equilibrium,



When thread is cut, $T = 0$ so, a_1 is maximum but $a_2 = a_3 = 0$.

Hence, the correct answer is (D).

193. $N = mg\sin\theta + \dfrac{mv^2}{r}$ and

$$v^2 = 2gr\sin\theta$$

Hence, the correct answer is (A).

194. $N = m\ell\omega^2$

Since $f = \mu N$

$$\Rightarrow \quad \mu ml\omega^2 = mg$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{\mu l}}$$

Hence, the correct answer is (A).

195. Let the thickness of each plank be x.
Then, we have

$$0 = u^2 - 2a(2x)$$

$$\Rightarrow \quad u^2 = 4ax \qquad \ldots(1)$$

When velocity is doubled, let n planks be required to stop the bullet, then

$$0 = (2u)^2 - 2a(nx)$$

$$\Rightarrow \quad n = \frac{4u^2}{2ax} = \frac{4(4ax)}{2ax} = 8$$

Hence, the correct answer is (C).

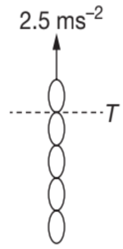
196. Acceleration of system $a = \dfrac{Mg}{M + nm}$

Tension in the last string is

$$T = ma = \frac{mMg}{M + nm}$$

Hence, the correct answer is (A).

197.



$$T = mg + ma$$

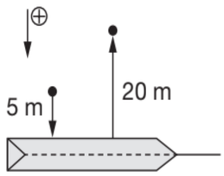
$$\Rightarrow \quad T = 0.4g + 0.4a$$

$$\Rightarrow \quad T = 0.4(g + a)$$

$$\Rightarrow \quad T = 0.4(9.8 + 2.5) = 4.92 \text{ N}$$

Hence, the correct answer is (B).

198.



Velocity of ball just before collision is

$$v_1 = \sqrt{2gh_1} = \sqrt{2 \times 10 \times 5} = 10 \text{ ms}^{-1}$$

Velocity of ball just after collision is

$$v_2 = \sqrt{2gh_2} = \sqrt{2 \times 10 \times 20} = 20 \text{ ms}^{-1}$$

$$\Rightarrow \quad \Delta p = p_2 - p_1 = -m(v_1 + v_2)$$

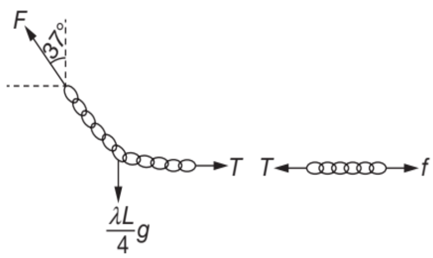
$$\Rightarrow \quad F_{av} = \frac{\Delta p}{\Delta t} = \frac{m(v_1 + v_2)}{\Delta t}$$

$$\Rightarrow \quad 100 = \frac{(0.4)(10 + 20)}{\Delta t}$$

$$\Rightarrow \quad \Delta t = 0.12 \text{ sec}$$

Hence, the correct answer is (A).

199. Let λ be the mass per unit length of the chain, then



$$F\cos(37^\circ) = \frac{\lambda L}{4}g$$

Also, $F\sin(37^\circ) = T = f$

$\Rightarrow \quad f = \dfrac{3\lambda L g}{16} \le \mu N$

$\Rightarrow \quad \mu \ge \dfrac{1}{4}$

Hence, the correct answer is (B).

200. Acceleration $a = \dfrac{dv}{dt}$

$\Rightarrow \quad a = 2bt$

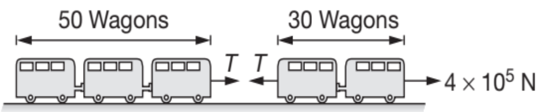
For block to just slide on the plate, we have

$\mu mg = m(2bt)$

$\Rightarrow \quad t = \dfrac{\mu g}{2b}$

Hence, the correct answer is (D).

201.



Acceleration of train is

$$a = \frac{4\times 10^5}{80\times 5\times 10^3} = 1\ \text{ms}^{-2}$$

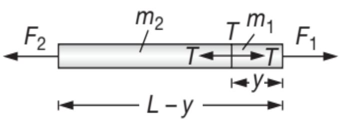
Thus, tension between 30th and 31st wagon is

$T = ma = 50\times 5\times 10^3 \times 1$

$\Rightarrow \quad T = 25\times 10^4\ \text{N}$

Hence, the correct answer is (A).

202.



$m_1 = \dfrac{M}{L}y$ and $m_2 = \dfrac{M}{L}(L-y)$

Since, $a = \dfrac{F_1 - F_2}{M}$

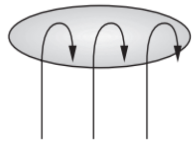
$F_1 - T = m_1 a$

$\Rightarrow \quad T = F_1 - m_1 a = F_1 - \left(\dfrac{M}{L}y\right)\left(\dfrac{F_1 - F_2}{M}\right)$

$\Rightarrow \quad T = F_1\left(1 - \dfrac{y}{L}\right) + F_2\left(\dfrac{y}{L}\right)$

Hence, the correct answer is (A).

203. $F_{av} = \dfrac{\Delta p}{\Delta t} = 2mnv$



$\Rightarrow \quad 2mnv = Mg$

$\Rightarrow \quad v = \dfrac{Mg}{2mn} = \dfrac{(10\times 10^{-3})\times 9.8}{2(5\times 10^{-3})\times 10}$

$\Rightarrow \quad v = 0.98\ \text{ms}^{-1} = 98\ \text{cms}^{-1}$

Hence, the correct answer is (B).

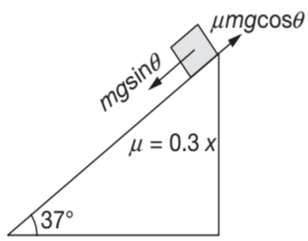
204. Since area under the curve $= \Delta p$

$$\Rightarrow \quad \frac{1}{2}(2)(10) + (2)(10) + \frac{1}{2}(30)(2) + \frac{1}{2}(4)(20) = 2(v_2 - 0)$$

$\Rightarrow \quad v_2 = 50\ \text{ms}^{-1}$

Hence, the correct answer is (D).

205. After some time, friction becomes more than $mg\sin\theta$, then body will retard. Thus speed is maximum when, total force or acceleration is zero.



$\Rightarrow \quad mg\sin\theta - \mu mg\cos\theta = 0$

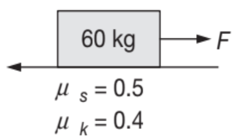
$\Rightarrow \quad \mu = \tan\theta$

$\Rightarrow \quad 0.3x = \dfrac{3}{4}$

$\Rightarrow \quad x = 2.5\ \text{m}$

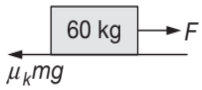
Hence, the correct answer is (D).

206. Force required to start the motion is



$F = \mu_s mg$

Once the body starts sliding, friction becomes kinetic, so



$$a = \frac{F - \mu_k mg}{m} = \frac{\mu_s mg - \mu_k mg}{m}$$

$$\Rightarrow \quad a = (\mu_s - \mu_k)g = 1\ \text{ms}^{-2}$$

Hence, the correct answer is (D).

207. Since external force on each block $= mg\sin\theta$ and acceleration of each block $= g\sin\theta$, so force on each block due to another block is zero.

Hence, the correct answer is (B).

208. Thrust force on the rocket,

$$F = v\frac{dm}{dt} = 600 \times 1 = 600\ \text{N}$$

So, acceleration of rocket

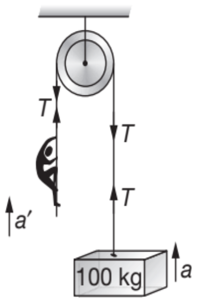
$$a = \frac{F}{M}$$

$$\Rightarrow \quad a = \frac{600}{120} = 5\ \text{ms}^{-2}$$

Hence, the correct answer is (B).

209. If a be the acceleration of body w.r.t. ground, then acceleration of man w.r.t. ground is

$$a' = \frac{5}{4}g - a$$



For body,

$$T - 100g = 100a \qquad \ldots(1)$$

For man,

$$T - 60g = 60\left(\frac{5}{4}g - a\right)$$

$$\Rightarrow \quad T = 135g - 60a \qquad \ldots(2)$$

By solving equations (1) and (2), we get

$$T = 1218\ \text{N}$$

Hence, the correct answer is (C).

210. Change in momentum of bead is

$$\Delta p = 2mv$$

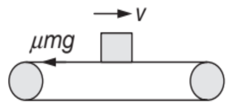
and time to cover the half cycle is

$$\Delta t = \frac{\text{Distance}}{\text{Speed}} = \frac{\pi r}{v} = \frac{\pi d}{2v}$$

$$\Rightarrow \quad F = \frac{\Delta p}{\Delta t} = \frac{2mv}{\pi d/2v} = \frac{4mv^2}{\pi d}$$

Hence, the correct answer is (B).

211. Relative to belt, $a = \frac{\mu mg}{m} = \mu g = 5\ \text{ms}^{-2}$

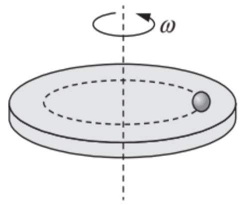


$$\Rightarrow \quad 0 = u^2 - 2as$$

$$\Rightarrow \quad s = \frac{u^2}{2a} = \frac{(3)^2}{2 \times 5} = 0.9\ \text{m}$$

Hence, the correct answer is (C).

212.

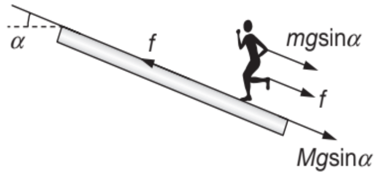


$$mR\omega^2 \le \mu mg$$

$$\Rightarrow \quad R \le \frac{\mu g}{\omega^2}$$

Hence, the correct answer is (D).

213. F.B.D. of man and plank are



For plank to be at rest, applying Newton's Second Law on the plank along the incline, we get

$$Mg\sin\alpha = f \qquad \ldots(1)$$

and applying Newton's Second Law on man along the incline, we get

$$Mg\sin\alpha + f = ma \qquad \ldots(2)$$

$$\Rightarrow \quad a = g\sin\alpha\left(1 + \frac{M}{m}\right), \text{ down the incline}$$

Hence, the correct answer is (B).

214. Limiting friction between A and $B = 60$ N

Limiting friction between B and $C = 90$ N

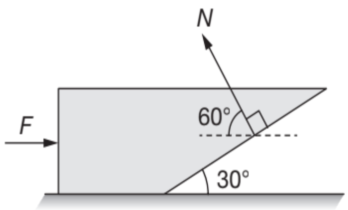
Limiting friction between C and ground $= 50$ N

Since limiting friction is least between C and ground, slipping will occur at first between C and ground. This will occur when $F = 50$ N.

Hence, the correct answer is (D).

215. Acceleration of two mass system is $a = \frac{F}{2m}$ leftwards

FBD of block A is drawn, so



$$N\cos(60°) - F = ma = \frac{mF}{2m}$$

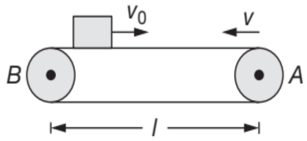
$\Rightarrow \quad N = 3F$

Hence, the correct answer is (D).

216. $f = ma_2$

Hence, the correct answer is (B).

217.



Maximum amount of heat is liberated when the friction between the block and belt is kinetic, i.e., when the block moves with a velocity relative to the belt.

Since, $a = \mu g$

Also, $v^2 = u^2 + 2as$

$\Rightarrow \quad 0 = v_0^2 + 2(-\mu g)\ell$

$\Rightarrow \quad v_0 = \sqrt{2\mu g \ell}$

Hence, the correct answer is (B).

218. Since, $a_A = 6a_B$

$$\Rightarrow \quad a_B = \frac{a_A}{6} = \frac{12}{6} = 2 \text{ ms}^{-2}$$

Hence, the correct answer is (B).

219. Let acceleration of block A and B be a_1 and a_2 respectively and a_p be acceleration of pulley P and let tension in string be T. Then, according to principle of virtual work, we have

$$(3T)a_p = Ta_1 + (8T)a_2$$

$$\Rightarrow \quad a_p = \frac{9g}{3} = 3g \text{ (upwards)}$$

Hence, the correct answer is (D).

220. For first $\frac{H}{3}$, we have

$$a_1 = \frac{2mg}{3m} = \frac{2g}{3}$$

$$\Rightarrow \quad v_1^2 = 2\left(\frac{2g}{3}\right)\left(\frac{H}{3}\right)$$

$$\Rightarrow \quad v_1^2 = \frac{4gH}{9}$$

For next H, we have

$$a_2 = \frac{mg}{2m} = \frac{g}{2}$$

$$\Rightarrow \quad v^2 = v_1^2 + 2a_2H$$

$$\Rightarrow \quad v^2 = \frac{4gH}{9} + 2\left(\frac{g}{2}\right)H = \frac{13}{9}gH$$

$$\Rightarrow \quad v = \frac{\sqrt{13gH}}{3}$$

Hence, the correct answer is (C).

221. Acceleration of A w.r.t. ground (G) is

$$\vec{a}_{AG} = a_0\cos\theta\hat{i} + a_0\sin\theta\hat{j}$$

Acceleration of B w.r.t. ground (G) is

$$\vec{a}_{BG} = a_0\sin\theta\hat{j}$$

So, acceleration of A w.r.t. B is

$$\vec{a}_{AB} = \vec{a}_{AG} - \vec{a}_{BG} = a_0\cos\theta\hat{i}$$

Hence, the correct answer is (C).

222. $2V_A\cos\theta = V_0$

Differentiating, we get

$$2a_A\cos\theta - 2V_A\sin\theta\frac{d\theta}{dt} = 0 \quad \ldots(1)$$

Since $Y_A = b\cot\theta$

$$\Rightarrow \quad V_A = \frac{dY_A}{dt} = (b\,\text{cosec}^2\theta)\frac{d\theta}{dt}$$

Put V_A in (1), we get

$$a_A = \frac{V_0^2}{4b}$$

Hence, the correct answer is (C).

Multiple Correct Choice Type Questions

1. (A) is correct when the bicycle is being pedalled because during pedalling, the force is acting in the backward direction on the rear wheel and thus frictional force acts in the forward direction.

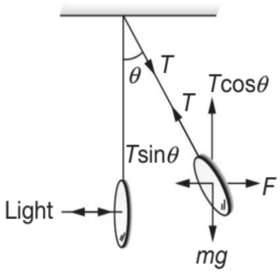
   (D) is correct when bicycle is not pedalled.

   Hence, (A) and (D) are correct.

2. All accelerated frames are Non-inertial frames. Since earth rotates about its own axis and revolves around the sun, so it is a Non-Inertial frame (STRICTLY SPEAKING), whereas for a good number of cases we assume the earth to be an Inertial frame of reference as the value of acceleration is extremely small.
   Hence, (B) and (D) are correct.

3. Let F be the force exerted on the mirror by the photon. Since a photon will be reflected from the mirror with the same value of momentum so, change in momentum $\Delta p = p-(-p) = 2p = \frac{2E}{c}$.

   (Because for a photon $E = pc$)



$$\Rightarrow \quad F = \frac{\Delta p}{\Delta t}$$

$$\Rightarrow \quad F = \frac{2E}{c\Delta T} \qquad \ldots(1)$$

Also

$$T\cos\theta = mg \text{ and} \qquad \ldots(2)$$

$$T\sin\theta = F = \frac{2E}{c\Delta T} \qquad \ldots(3)$$

$$\Rightarrow \quad \tan\theta = \frac{2E}{mgc\Delta T} \qquad \ldots(4)$$

Further by definition intensity of a photon beam is defined as the energy incident per unit area of a surface per unit time.

$$\Rightarrow \quad I = \frac{E}{A\Delta t}$$

$$\Rightarrow \quad \frac{E}{\Delta t} = IA \qquad \ldots(5)$$

Substituting (5) in (4), we get

$$\tan\theta = \frac{2IA}{mgc}$$

Also from (1), we get

$$F = \frac{2}{c}(IA)$$

$$\Rightarrow \quad \text{Radiation Pressure} = \frac{F}{A} = \frac{2I}{c}$$

Hence, (A) and (C) are correct.

4. When P is gradually increased, so till the moment P becomes equal to f_0, mass A will not move and hence the string will not develop any tension due to pull P. So, $T = 0$ for $P < f_0$. Further for A and B to move collectively with acceleration a (say), we must have

$$a = \frac{P-(f_1)_{\lim}-(f_2)_{\lim}}{2m}$$

$$\Rightarrow \quad a = \frac{P-2f_0}{2m} \qquad \ldots(1)$$

So, the system will not move till $P > 2f_0$, but T will be developed as soon as P becomes greater than f_0. Hence

$$P = T + f_0$$

$$\Rightarrow \quad T = P - f_0 \text{ for } f_0 < P < 2f_0$$

So, **(B)** is also correct.

Once $P > 2f_0$, then for

BLOCK *A*

$$P - T - f_0 = ma \qquad \ldots(2)$$

$$\Rightarrow \quad P - T - f_0 = m\left(\frac{P-2f_0}{2m}\right) \qquad \{\because \text{of}(1)\}$$

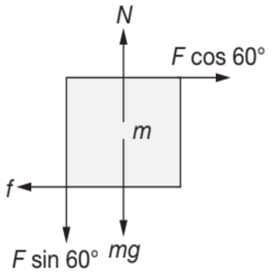
$$\Rightarrow \quad P - T - f_0 = \frac{P}{2} - f_0$$

$$\Rightarrow \quad P - T = \frac{P}{2}$$

$$\Rightarrow \quad T = \frac{P}{2}$$

Hence, (A), (B) and (C) are correct.

5.



For no motion $F_{\text{applied}} \le f\,(=\mu N)$

$$\Rightarrow \quad F\cos 60^\circ \le \mu\left(\text{mg} + F\sin 60^\circ\right)$$

$$\Rightarrow \quad \frac{F}{2} \le \frac{1}{2\sqrt{3}}\left(\sqrt{3}g + \frac{F\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad \frac{F}{2} \le g$$

$$\Rightarrow \quad F_{\max} = 20\text{ N}$$

Hence, (A) is correct.

6. If T is the tension in the string and N is the normal reaction exerted by B on A, then from FBD of A, we get

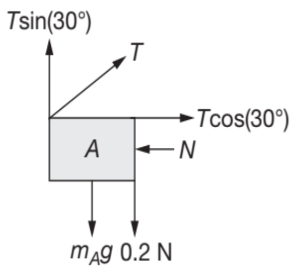
$$T\sin(30^\circ) = m_A g + 0.2\text{ N} \qquad \ldots(1)$$

$$T\cos(30^\circ) = N \qquad \ldots(2)$$

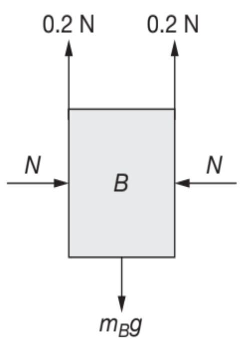
$$\Rightarrow \quad T\sin(30^\circ) = m_A g + 0.2T\cos(30^\circ)$$

$$\Rightarrow \quad T\left[\sin(30^\circ) - 0.2\cos(30^\circ)\right] = 100$$

$$\Rightarrow \quad T \cong 306\ N \text{ and } N \cong 265\ N$$



Free body diagram for B is shown here. If a be the acceleration of B, then



$$m_B g - 2(0.2\text{ N}) = m_B a$$

$$\Rightarrow \quad a = \frac{200 - (0.4)(265)}{20}$$

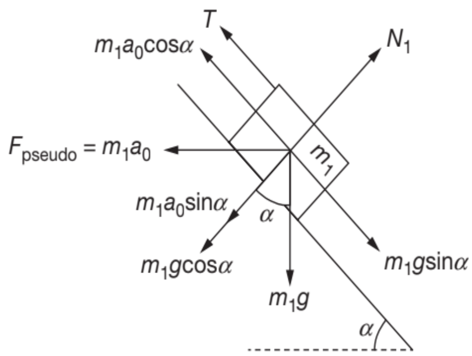
$$\Rightarrow \quad a = 4.7\text{ ms}^{-2}$$

Hence, (A), (C) and (D) are correct.

7. **For m_1**

$$T + m_1 a_0\cos\alpha = m_1 g\sin\alpha$$

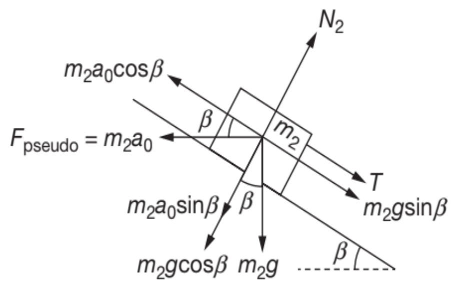
$$\Rightarrow \quad T = m_1 g\sin\alpha - m_1 a_0\cos\alpha \qquad \ldots(1)$$



For m_2

$$T + m_2 g\sin\beta = m_2 a_0\cos\beta$$

$$\Rightarrow \quad T = m_2 a_0\cos\beta - m_2 g\sin\beta \qquad \ldots(2)$$



Equating (1) and (2), we get

$$m_1 g\sin\alpha - m_1 a_0\cos\alpha = m_2 a_0\cos\beta - m_2 g\sin\beta$$

$$\Rightarrow \quad \left(m_1\sin\alpha + m_2\sin\beta\right)g = \left(m_1\cos\alpha + m_2\cos\beta\right)a_0$$

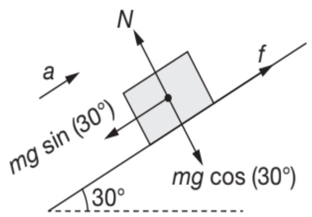
$$\Rightarrow \quad a_0 = \left(\frac{m_1\sin\alpha + m_2\sin\beta}{m_1\cos\alpha + m_2\cos\beta}\right)g \qquad \ldots(3)$$

Substituting (3) either in (2) or in (1), we get

$$T = \left[\frac{m_1 m_2}{m_1\cos\alpha + m_2\cos\beta}\right]g\sin(\alpha - \beta) \qquad \ldots(4)$$

Hence, (B) and (D) are correct.

8. Free body diagram for the block is shown for convenience



According to Newton's Second Law, we have

$$f - mg\sin(30°) = ma$$

$$\Rightarrow \quad f = (1)(10)\left(\frac{1}{2}\right) + (1)(1) = 6\text{ N}$$

Also, we have $N = mg\cos(30°) = 5\sqrt{3}$ N.

So, net contact force between the block and the belt is

$$F_c = \sqrt{N^2 + f^2} = \sqrt{75 + 36} = \sqrt{111}\text{ N}$$

$$\Rightarrow \quad F_c \cong 10.5\text{ N}$$

Hence, (A), (B) and (C) are correct.

9. $a = g\tan\alpha$ ...(1)

Since, $\sin\alpha = \dfrac{1}{x}$

$$\Rightarrow \quad \tan\alpha = \frac{1}{\sqrt{x^2 - 1}}$$

$$\Rightarrow \quad a = \frac{g}{\sqrt{x^2 - 1}} \qquad \{\text{Using (1)}\}$$

and $N = \dfrac{mg}{\cos\alpha} = \dfrac{mgx}{\sqrt{x^2 - 1}}$

Hence, (B), (C) and (D) are correct.

10. From our knowledge of constraint equations, we observe that $a_A = a_B = a$ (say). Also, since the string is light and inextensible, so tension throughout the string is also the same, say T.

For A,

$$T + mg\sin(30°) = ma$$

$$\Rightarrow \quad T + \frac{mg}{2} = ma \qquad \text{...(1)}$$

For B,

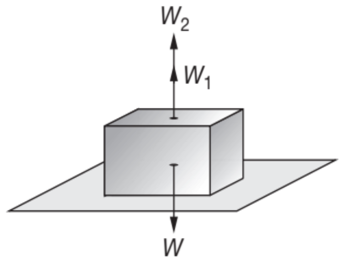
$$mg - T = ma \qquad \text{...(2)}$$

Solving (1) and (2), we get

$$a = \frac{3g}{4} \text{ and } T = \frac{mg}{4}$$

Hence, (B) and (D) are correct.

11. The readings of the spring balance (under various situations discussed) and the reading of the weighing machine are equal to the forces exerted by them on the block.



Hence, (A), (B) and (D) are correct.

12. Let m be the mass per unit length of the chain. Then

$$(mx)g - T = (mx)a \qquad \text{...(1)}$$

$$T = m(L - x)a \qquad \text{...(2)}$$

$$\Rightarrow \quad (mx)g - m(L - x)a = (mx)a$$

$$\Rightarrow \quad xg = La$$

$$\Rightarrow \quad a = g\frac{x}{L}$$

$$\Rightarrow \quad v\frac{dv}{dx} = g\frac{x}{L}$$

$$\Rightarrow \quad vdv = \frac{g}{L}xdx$$

$$\Rightarrow \quad \int_0^v vdv = \frac{g}{L}\int_0^x xdx$$

$$\Rightarrow \quad \frac{v^2}{\not{2}} = \frac{g}{L}\frac{x^2}{\not{2}}$$

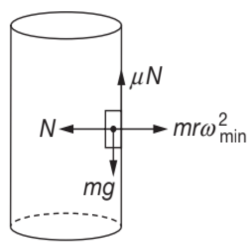
$$\Rightarrow \quad v = x\sqrt{\frac{g}{L}}$$

Hence, (A) and (C) are correct.

13. $mg = \mu N$ and

$$N = mr\omega_{\min}^2$$

$$\Rightarrow \quad mg = \mu mr\omega_{\min}^2$$



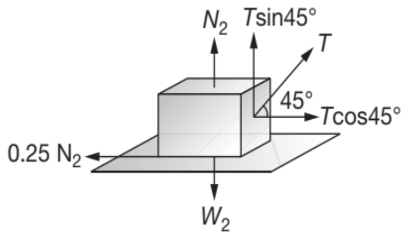
$$\Rightarrow \quad \omega_{min} = \sqrt{\frac{g}{\mu R}} \text{ and } v_{min} = R\omega_{min} = \sqrt{\frac{gR}{\mu}}$$

Hence, (A) and (D) are correct.

14. For W_2:

$$N_2 + T\sin 45 = W_2 = 100 \quad \ldots(1)$$

$$T\cos 45 = 0.25N_2 \quad \ldots(2)$$

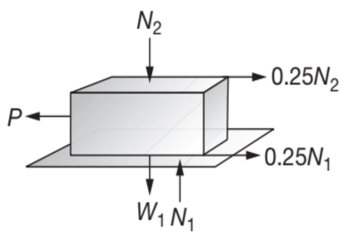


FBD for W_2

For W_1:

$$P = 0.25(N_1 + N_2) \quad \ldots(3)$$

$$N_2 + W_1 = N_1 \quad \ldots(4)$$

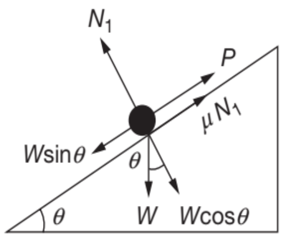


FBD for W_1

Solving, we get $P = 90$ N, $T = 20\sqrt{2}$ N

Hence, (B) and (D) are correct.

15.



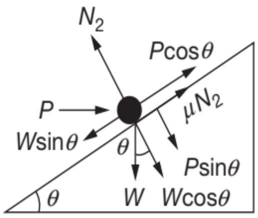
$$W\sin\theta = P + \mu N_1 \text{ and } N_1 = W\cos\theta$$

$$\Rightarrow \quad W\sin\theta = P + \mu W\cos\theta$$

$$\Rightarrow \quad P = W(\sin\theta - \mu\cos\theta)$$

$$\Rightarrow \quad P = W(\sin\theta - \tan\phi\cos\theta) \quad \{\because \mu = \tan\phi\}$$

$$\Rightarrow \quad \frac{P}{W} = \frac{\sin(\theta-\phi)}{\cos\phi} \quad \ldots(1)$$



$$P\cos\theta + \mu N_2 = W\sin\theta$$

$$W\cos\theta + P\sin\theta = N_2$$

$$\Rightarrow \quad P\cos\theta + \mu(W\cos\theta + P\sin\theta) = W\sin\theta$$

$$\Rightarrow \quad P(\cos\theta + \mu\sin\theta) = W(\sin\theta - \mu\cos\theta)$$

$$\Rightarrow \quad P(\cos\theta\cos\phi + \sin\theta\sin\phi) = W(\sin\theta\cos\phi - \cos\theta\sin\phi)$$

$$\Rightarrow \quad P\cos(\theta-\phi) = W\sin(\theta-\phi)$$

$$\Rightarrow \quad \frac{P}{W} = \tan(\theta-\phi) \quad \ldots(2)$$

Equating (1) and (2), we get

$$\cos\phi = \cos(\theta-\phi)$$

$$\Rightarrow \quad \theta - \phi = \phi$$

$$\Rightarrow \quad \theta = 2\phi$$

$$\Rightarrow \quad \frac{P}{W} = \tan(2\phi - \phi)$$

$$\Rightarrow \quad \frac{P}{W} = \tan\phi$$

Hence, (A) and (D) are correct.

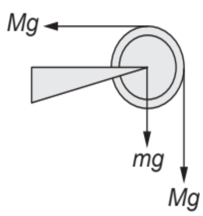
16. $mg\cos\alpha = N \quad \ldots(1)$

$$mg\sin\alpha = \mu N \quad \ldots(2)$$

$$\Rightarrow \quad \cot\alpha = \frac{1}{\mu} = 3$$

Hence, (A) is correct.

17. $T = Mg$



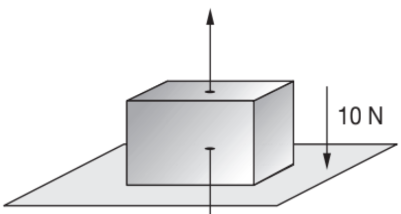
Also the weight of pulley mg is acting vertically downward. So total downward force is $(m+M)g$ and horizontal force is $T = Mg$. The resultant of these two is

$$\sqrt{M^2 + (m+M)^2}\,g$$

Hence, (D) is correct.

18. Reading of the spring balance is equal to the tension in the rope which equals the force of friction between the rope and the boy as well as the force exerted by the rope and the boy on each other.
Hence, (A), (C) and (D) are correct.

19. Due to Newton's Third Law, the block will exert an equal and opposite force on the table. Since $F > W$, so the block will have an upward acceleration.



Hence, (B) and (C) are correct.

20. Net torque must be zero, so

$$\vec{\tau}_N + \vec{\tau}_{fr} = \vec{0}$$

$$\Rightarrow \quad \vec{\tau}_N = -\vec{\tau}_{fr}$$

Hence, the correct answer is (D).

21. When the entire incline is smooth, then both the blocks move down with an acceleration $g\sin(30°) = 5 \text{ ms}^{-2}$. However, if A is smooth and B is rough then initial acceleration of A will be $g\sin\theta$, whereas that of B is less than $g\sin\theta$ and hence there will be elongation in the spring.
Hence, (A) and (D) are correct.

22. Newton's Laws are valid in all inertial frames i.e., frames either at rest or frames moving with constant velocity. Further a frame moving with constant velocity with respect to an inertial frame is also Inertial. Earth is not an inertial frame due to rotation about its axis and revolution round the sun.
Hence, (B) and (C) are correct.

23. $F - fy - Mg = Ma$ {at a distance y}

$$\Rightarrow \quad a = \frac{F - fy - Mg}{M}$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{F - fy - Mg}{M}$$

$$\Rightarrow \quad v\frac{dv}{dy} = \frac{F - fy - Mg}{M}$$

$$\Rightarrow \quad \int_0^v v\,dv = \frac{1}{M}\int_0^s (F - fy - Mg)\,dy$$

$$\Rightarrow \quad v = \sqrt{\frac{2s}{M}\left(F - Mg - \frac{fs}{2}\right)}$$

Hence, (A) and (C) are correct.

24. When lift moves with uniform velocity no pseudo force exists and hence $f = \mu mg\,(A\,\&\,B)$. When lift moves down with acceleration of 4.8 ms^{-2} then $N = m(9.8 - 4.8) = 5$ m.

So, frictional force $= \mu N = 5\,\mu m$

Hence, (A), (B), (C) and (D) are correct.

25. The block with lower value of μ will tend to have greater acceleration down the slope.
Hence, (A) and (B) are correct.

26. $2T\cos\theta = \sqrt{2}\,Mg$ ...(1)

$$\Rightarrow \quad 2Mg\cos\theta = \sqrt{2}\text{ Mg}$$

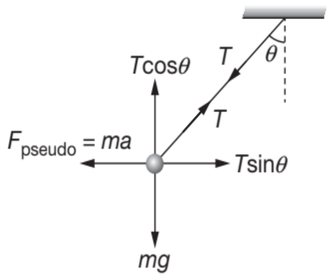
$$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \theta = 45°$$

Hence, (C) is correct.

27. For equilibrium, we get

$$T\cos\theta = mg \text{ and } T\sin\theta = ma$$



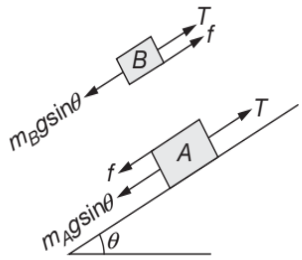
$$\Rightarrow \quad \tan\theta = \frac{a}{g}$$

Also, $T = \dfrac{mg}{\cos\theta} = mg\sec\theta = mg\sqrt{1+\tan^2\theta}$

$$\Rightarrow \quad T = m\sqrt{a^2 + g^2}$$

Hence, (B) and (D) are correct.

28. Since friction always has a tendency to stop the relative motion between the blocks, so when $m_B > m_A$, then for a frictional force f between A and B, we have



$$m_B g\sin\theta = T + f \qquad \ldots(1)$$

$$m_A g\sin\theta = T - f \qquad \ldots(2)$$

Subtracting (2) from (1), we get

$$(m_B - m_A)g\sin\theta = 2f$$

$$\Rightarrow \quad f = \frac{1}{2}(m_B - m_A)g\sin\theta$$

Now, $f = 0$, when $m_A = m_B$

So, (B) is correct.

Also, for $f \le f_{\max}$, the blocks will remain stationary.

$$\Rightarrow \quad \frac{1}{2}(m_B - m_A)g\sin\theta \le \mu(m_B g\cos\theta)$$

$$\Rightarrow \quad \mu \ge \left(\frac{m_B - m_A}{2m_B}\right)\tan\theta$$

Hence the acceleration of the system is zero if $\mu \ge \left(\frac{m_B - m_A}{2m_B}\right)\tan\theta$ and $m_B > m_A$.

Also, from (1) and (2), we get

$$T = \left(\frac{m_A + m_B}{2}\right)g\sin\theta$$

$$\Rightarrow \quad T = m_A g\sin\theta = mg\sin\theta$$

Hence, (A), (B) and (D) are correct.

29. $a = \dfrac{m_B g}{m_A + m_B} = \dfrac{2}{7}(9.8)\text{ ms}^{-2}$

$$\Rightarrow \quad a = 2.8\text{ ms}^{-2} = 280\text{ cms}^{-2}$$

Since the initial velocity of A is towards left whereas acceleration is towards the right. So this acceleration acts as retardation for motion of A towards left. (Take left to right as positive).

$$v|_{t=1s} = -50 + (280)(1)$$

$$\Rightarrow \quad v|_{t=1s} = +230\text{ cms}^{-1} \text{ (towards right) and}$$

$$s|_{t=1s} = -(50)(1) + \frac{1}{2}(280)(1)$$

$$\Rightarrow \quad s|_{t=1s} = +90\text{ cm} \qquad \text{\{displacement towards right\}}$$

So distance from pulley = 200 – 90 = 110 cm

Hence, (B) and (D) are correct.

30. $a = \left(\dfrac{M - m}{M + m}\right)g$,

$$T = \left(\frac{2Mm}{M + m}\right)g$$

Thrust $= 2T = \left(\dfrac{4Mm}{M + m}\right)g$ and

$$(M + m)g - 2T = (M + m)a_{cm}$$

$$\Rightarrow \quad a_{cm} = \left(\frac{M - m}{M + m}\right)^2 g$$

or $\quad a_{cm} = \dfrac{Ma + m(-a)}{M + m} \qquad \left\{\because a_{cm} = \dfrac{m_1 a_1 + m_2 a_2}{m_1 + m_2}\right\}$

$$\Rightarrow \quad a_{cm} = \left(\frac{M - m}{M + m}\right)a = \left(\frac{M - m}{M + m}\right)^2 g$$

Hence, (A), (B), (C) and (D) are correct.

31. Since the surfaces are smooth, so for any value of θ both A and B will have the same acceleration downwards, as a result of which neither of them exerts a force on each other.

Hence, (B) and (D) are correct.

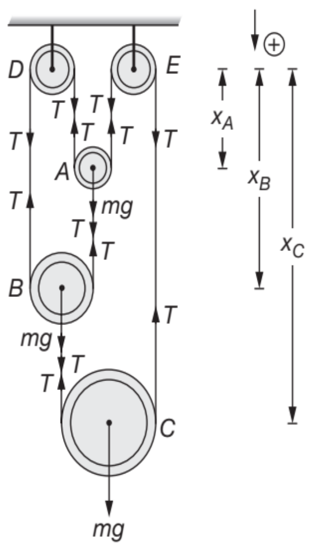
32. According to Impulse-Momentum Theorem

Impulse = Change in momentum

$$\Rightarrow \quad F(\Delta t) = m[v - (-u)]$$

Hence, (A) and (B) are correct.

33.
- Take downward as positive direction.
- The total length of the thread is a constant.
- Starting from centre of A and going to centre of B we have



$$(x_B - x_A) + x_B + 2x_A + x_C + (x_C - x_B) = \text{constant}$$

$\Rightarrow \quad x_B + x_A + 2x_C = \text{constant}$

$\Rightarrow \quad \ddot{x}_B + \ddot{x}_A + 2\ddot{x}_C = 0$

$\Rightarrow \quad a_B + a_A + 2a_C = 0 \quad \ldots(1)$

Applying cause-effect equations we get

$mg - 2T = ma_C$ (For C) $\quad \ldots(2)$

$mg + T - 2T = ma_B$ (For B)

$\Rightarrow \quad mg - T = ma_B \quad \ldots(3)$

$mg + T - 2T = ma_A$ (For A)

$\Rightarrow \quad mg - T = ma_A \quad \ldots(4)$

From (3) and (4), $a_A = a_B$ so (1) gives

$\Rightarrow \quad a_A = a_B = -a_C \quad \ldots(5)$

Adding (3) and (4), we get

$2mg - 2T = 2ma_A \quad \ldots(6)$

Subtract (2) from (6), we get

$mg = 2ma_A - ma_C$

$\Rightarrow \quad mg = 2ma_A + ma_A \quad \{\because a_A = -a_C\}$

$\Rightarrow \quad mg = 3ma_A$

$\Rightarrow \quad a_A = \dfrac{g}{3}$

So, $a_A = a_B = \dfrac{g}{3}$; Downwards

$a_C = -\dfrac{g}{3}$ Upwards

Also we get T by putting value of a_A in (4) i.e. $T = \dfrac{2}{3}$mg.

Hence, (A) and (B) are correct.

34. $200 - kx = 20a_1$

and $kx = 10(12) = 120$

$\Rightarrow \quad a_1 = 4 \text{ ms}^{-2}$

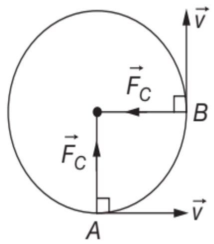
$$a = \frac{F}{m_1} + \frac{F}{m_2} = \frac{F}{\left(\dfrac{m_1 m_2}{m_1 + m_2}\right)} = \frac{F}{\mu}$$

$\Rightarrow \quad a = \dfrac{200}{(20/3)}$

$\Rightarrow \quad a = 30 \text{ ms}^{-2}$

Hence, (B), (C) and (D) are correct.

35. Centripetal force is a force which is directed radially inwards, but this does not imply that is a force constant in magnitude and direction.



So $\quad |\vec{F}_C| = \dfrac{mv^2}{r}$

Also in uniform circular motion magnitude of velocity is constant but velocity is not constant as it changes its direction continuously. Further, $\vec{F} \perp \vec{v}$ implies work done is zero, so according to **Work Energy Theorem**

Work done = Change in K.E.

$\Rightarrow \quad 0 = \Delta(K.E.)$

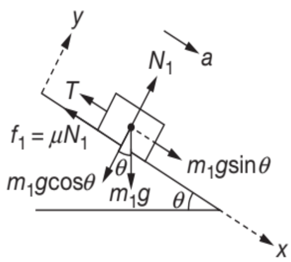
$\Rightarrow \quad$ K.E. = constant

Hence, (C) and (D) are correct.

36. Concept of Pseudo-Force.

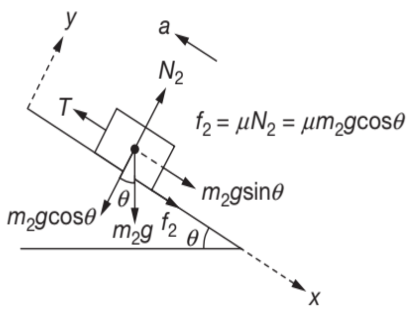
Hence, (A), (C) and (D) are correct.

37.



$m_1 g \sin\theta - T - \mu N_1 = m_1 a$

$\Rightarrow \quad m_1 g \sin\theta - T - \mu m_1 g \cos\theta = m_1 a \quad \ldots(1)$



$T - m_2 g \sin\theta - \mu N_2 = m_2 a$

$\Rightarrow \quad T - m_2 g \sin\theta - \mu m_2 g \cos\theta = m_2 a \quad \ldots(2)$

Adding (1) and (2), we get

$$a=\left[\frac{(m_1-m_2)\sin\theta-\mu(m_1+m_2)\cos\theta}{m_1+m_2}\right]g$$

for m_1 downwards along the incline.

So for m_2 acceleration is directed up the incline i.e.

$$a_2=-a$$

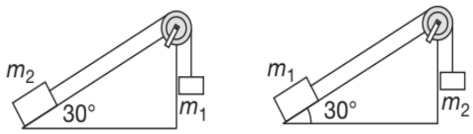
Hence, (A) and (B) are correct.

OBJECTIVE TRICK

If $\theta=\frac{\pi}{2}$ and no friction exists i.e. $\mu=0$, then 'a' must be that for an ideal Atwood's machine i.e. $\left(\frac{m_1-m_2}{m_1+m_2}\right)g$ for m_1 downwards.

This condition is met by option **(A)** for m_1 and option **(B)** for m_2.

38.



$$\left(\begin{matrix}\text{Time taken by}\\ m_2 \text{ to reach}\\ \text{maximum height}\end{matrix}\right)=\frac{1}{2}\left(\begin{matrix}\text{Time taken by}\\ m_1 \text{ to reach}\\ \text{maximum height}\end{matrix}\right)$$

$$a_1=\left(\frac{m_1-m_2\sin 30}{m_1+m_2}\right)g \text{ and}$$

$$a_2=\left(\frac{m_2-m_1\sin 30}{m_1+m_2}\right)g$$

Since $l=\frac{1}{2}a_1t_1^2=\frac{1}{2}a_2t_2^2$

$$\Rightarrow \frac{1}{2}\left(\frac{2m_1-m_2}{m_1+m_2}\right)\frac{t_2^2}{4}=\frac{1}{2}\left(\frac{2m_2-m_1}{m_1+m_2}\right)t_2^2$$

$$\Rightarrow \left(\frac{2m_1-m_2}{m_1+m_2}\right)\frac{1}{4}=\left(\frac{2m_2-m_1}{m_1+m_2}\right)$$

$$\Rightarrow 8m_2-4m_1=2m_1-m_2$$

$$\Rightarrow 9m_2=6m_1$$

$$\Rightarrow \frac{m_1}{m_2}=\frac{3}{2}$$

Hence, (A) and (C) are correct.

40. $T_{upper}=(2.9+0.2+1.9)(9.8+0.2)$

$\Rightarrow$ $T_{upper}=50\text{ N}$ and

$T_{lower}=(1.9+0.1)(9.8+0.2)$

$\Rightarrow$ $T_{lower}=20\text{ N}$

Hence, (A) and (D) are correct.

41. $4500-500-T=5\times10^4a$ ...(1)

$T-400=4\times10^4a$ ...(2)

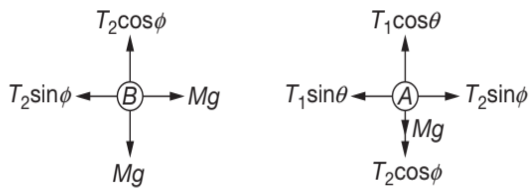
Adding (1) and (2), we get

$$a=\frac{3600}{9\times10^4}$$

$\Rightarrow$ $a=0.04\text{ ms}^{-2}$ and $T=2000\text{ N}$

Hence, (A) and (C) are correct.

42.



$T_2\cos\phi=Mg$ ...(1)

$T_2\sin\phi=Mg$ ...(2)

$T_1\sin\theta=T_2\sin\phi$ ...(3)

$T_1\cos\theta=T_2\cos\phi+Mg$ ...(4)

From (1) and (2), we get

$\tan\phi=1$ {OPTION **(A)**}

$T_2=\sqrt{2}Mg$ {OPTION **(C)**}

From (3), we get

$$T_1\sin\theta=\sqrt{2}Mg\frac{1}{\sqrt{2}}$$

$T_1\sin\theta=Mg$ ...(5)

From (4), we get

$$T_1\cos\theta=\sqrt{2}Mg\left(\frac{1}{\sqrt{2}}\right)+Mg$$

$\Rightarrow$ $T_1\cos\theta=2Mg$ ...(6)

From (5) and (6), we get

$\tan\theta=\frac{1}{2}=0.5$ {OPTION **(B)**}

and $T_1=\sqrt{5}Mg$ {OPTION **(D)**}

Hence, (A), (B), (C) and (D) are correct.

43. Concept of Pseudo Force.

Hence, (A) and (C) are correct.

44. $a_{BA} = 3a_0$ {using constraint relations}

$$a_{BG} = \sqrt{a_0^2 + 9a_0^2 + 6a_0^2\cos\theta}$$

$$a_0\sqrt{10 + 6\cos\theta}$$

Hence, (A) and (D) are correct.

45. With respect to wedge, the acceleration of block is same as that of wedge with respect to ground.
Hence, (A) and (B) are correct.

46. $F = kt$

$$\Rightarrow \quad m\frac{dv}{dt} = kt$$

$$\Rightarrow \quad mv = \frac{kt^2}{2} \qquad \ldots(1)$$

Since, $a = \dfrac{F}{m}$

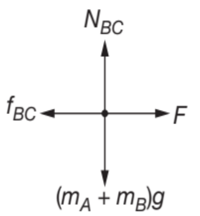
$$\Rightarrow \quad a = \frac{k}{m}t \qquad \ldots(2)$$

From (1) and (2), we get

$$mv = \frac{k}{2}\times\left(\frac{ma}{k}\right)^2$$

Hence, (A), (B) and (C) are correct.

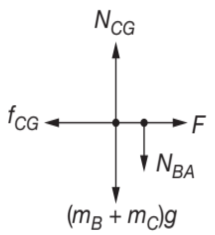
48. Free body diagram of $(A+B)$ system



$$f_{BC} = F \neq 0$$

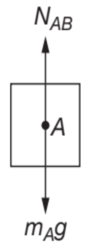
$$\Rightarrow \quad f_{CB} \neq 0$$

Free body diagram of $(B+C)$ system



$$f_{CG} = F = f_{BC}$$

Free body diagram of A is



No force acts on A parallel to the surface in contact, so $f_{AB} = 0$

Hence, (B) and (C) are correct.

49. (A) Magnitude of velocity is changing hence acceleration is present

(C) Velocity is changing hence it can happen by change in direction also as in a uniform circular motion. Hence acceleration is present.

Hence, (A) and (C) are correct.

50. Since, $v = 2t$ and if the radius of circular path is r, then we have

$$a_T = \frac{dv}{dt} = 2$$

$$a_r = \frac{v^2}{r} = \frac{4t^2}{r}$$

$$\Rightarrow \quad a = \sqrt{a_T^2 + a_r^2}$$

$$\Rightarrow \quad a = \sqrt{4 + \frac{16t^4}{r^2}}$$

Hence, (B) and (D) are correct.

51. $N_A = mg\cos\theta$ and $N_B = mg\cos(53^\circ)$

Since, $\dfrac{N_A}{N_B} = \dfrac{4}{3}$

$$\Rightarrow \quad \frac{\cos\theta}{\cos 53^\circ} = \frac{4}{3}$$

$$\Rightarrow \quad \cos\theta = \frac{4}{3}\times\frac{3}{5}$$

$$\Rightarrow \quad \cos\theta = \frac{4}{5}$$

$$\Rightarrow \quad a = \frac{mg[\sin(53^\circ) - \sin\theta]}{2m} = \frac{g}{10}$$

Hence, (A) and (C) are correct.

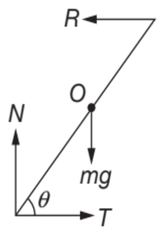
52. $R = T$ ...(1)

$N = mg$ ...(2)

For equilibrium, we have

$$\Sigma\vec{\tau} = \vec{0}$$

So, lets calculate torque about centre of mass of ladder i.e., O.



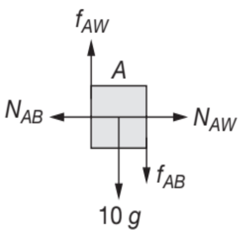
$$N \times \frac{L}{2}\cos\theta = R \times \frac{L}{2}\sin\theta + T \times \frac{L}{2}\sin\theta$$

$$\Rightarrow \quad \frac{mg}{2}\cos\theta = T\sin\theta$$

$$\Rightarrow \quad T = \frac{mg}{2}\cot\theta$$

Hence, (A) and (B) are correct.

53. Free body diagram of block A:

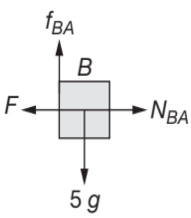


Friction on A due to B, f_{AB} acts downwards and friction on A due to Wall (W), f_{AW} acts upwards. So, we have

$f_{AW} = f_{AB} + 10\text{ g}$ ...(1)

$N_{AW} = N_{AB}$ ...(2)

Free body diagram of block B:



$f_{BA} = 5\text{ g}$ ...(3)

$N_{BA} = F$ ...(4)

From (1) and (3), we get

$$f_{AW} = 150\text{ N}$$

From (2) and (4), we get

$$N_{AW} = F$$

$$\Rightarrow \quad \mu N_{AW} \ge 150\text{ N}$$

$$\Rightarrow \quad 0.4 \times F \ge 150\text{ N}$$

$$\Rightarrow \quad F \ge 375\text{ N}$$

Hence, (A), (B) and (C) are correct.

54. Acceleration of block after application of force $a = \frac{2t}{m} - \mu g$

Hence, (B), (C) and (D) are correct.

55. Given that, $a_T = a_N$

$$\Rightarrow \quad a_T = \frac{v^2}{R}$$

$$\Rightarrow \quad -\frac{dv}{dt} = \frac{v^2}{R}$$

$$\Rightarrow \quad dv = -\frac{v^2}{R}dt$$

$$\Rightarrow \quad \int_{v_0}^{v}\frac{dv}{v^2} = -\frac{1}{R}\int_0^t dt$$

$$\Rightarrow \quad -\frac{1}{v}\Big|_{v_0}^{v} = -\frac{1}{R}t$$

$$\Rightarrow \quad -\frac{1}{v}\Big|_{v_0}^{v} = -\frac{1}{R}t$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{v_0} + \frac{t}{R}$$

$$\Rightarrow \quad v = \frac{v_0}{1 + \frac{v_0 t}{R}}$$

Since, $a_T = a_N = \frac{v^2}{R}$ and $a_T = v\frac{dv}{dS}$

$$\Rightarrow \quad v\frac{dv}{dS} = -\frac{v^2}{R}$$

$$\Rightarrow \quad \int_{v_0}^{v}\frac{dv}{v} = -\frac{1}{R}\int_0^S dS$$

$$\Rightarrow \quad \log_e v\Big|_{v_0}^{v} = -\frac{S}{R}$$

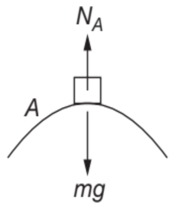
$$\Rightarrow \quad \log_e\left(\frac{v}{v_0}\right) = -\frac{S}{R}$$

$$\Rightarrow \quad \frac{v}{v_0} = e^{-\frac{S}{R}}$$

$$\Rightarrow \quad v = v_0 e^{-\frac{S}{R}}$$

Hence, (A) and (B) are correct.

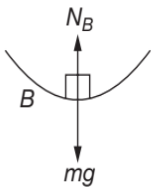
56. At *A*:



$$mg - N_A = \frac{mv^2}{r_A}$$

$$N_A = mg - \frac{mv^2}{r_A}$$

At *B*:



$$N_B - mg = \frac{mv^2}{r_B}$$

$$N_B = mg + \frac{mv^2}{r_B}$$

At *C*:

$$N_C = mg - \frac{mv^2}{r_C}$$

At *D*:

$$N_D = mg + \frac{mv^2}{r_D}$$

From figure $r_B < r_D$, so $N_B > N_D$

Hence N_B is greatest

Since, $r_C < r_A$, so $N_C < N_A$

Hence N_C is least

At A and C; $N_A < mg$

$N_C < mg$

At B and D; $N_B > mg$

$N_D > mg$

Hence, (A), (B) and (C) are correct.

57. For 20 kg block, we have

$$20g\sin(37°) = \mu[20g\cos(37°)] + T$$

$$\Rightarrow \quad 20 \times 10 \times \frac{3}{5} = 0.5 \times 20 \times 10 \times \frac{4}{5} + T$$

$$\Rightarrow \quad 120 = 80 + T$$

$$\Rightarrow \quad T = 40 \text{ N}$$

Also, $2mg\cos(37°) = T$

$$\Rightarrow \quad 2m \times 10 \times \frac{4}{5} = 40$$

$$\Rightarrow \quad m = 2.5 \text{ kg}$$

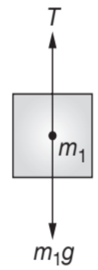
Also, force applied by 20 kg block on inclined is

$$R = \sqrt{N^2 + f^2} = 20g\cos(37°)\sqrt{1+\mu^2} = 178.8 \text{ N}$$

Hence, (A) and (C) are correct.

58. Free body diagram of blocks are drawn.

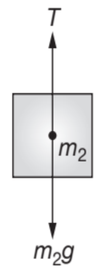
For m_1:



$$a_1 = \frac{m_1 g - T}{m_1}$$

$$\Rightarrow \quad a_1 = g - \frac{T}{m_1}$$

For m_2:



$$a_2 = \frac{m_2 g - T}{m_2}$$

$\Rightarrow \quad a_2 = g - \dfrac{T}{m_2}$

For $t = 0, T = 0$ $\quad \{\because x = 0\}$

$\Rightarrow \quad a_1 = a_2$

For $t > 0, T > 0$

$\Rightarrow \quad a_1 < a_2$

Hence, (A) and (D) are correct.

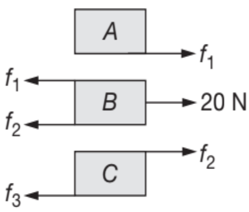
59. $\mu m_3 g = \sqrt{(m_1 g)^2 + (m_2 g)^2}$

Hence, (A), (C) and (D) are correct.

60. (f_1) maximum $= 0.2 \times 10 = 2$ N

(f_2) maximum $= 0.8 \times 20 = 16$ N

(f_3) maximum $= 0.3 \times 30 = 9$ N



So, acceleration of the lowest block is

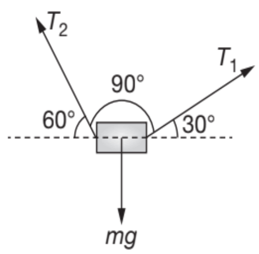
$$a_3 = \frac{f_2 - f_3}{1} = 4.5 \text{ ms}^{-2}$$

Acceleration of upper most block is

$$a_1 = \frac{f_1}{1} = 2 \text{ ms}^{-2}$$

Hence, (A), (B) and (D) are correct.

63.



Applying Lami's Theorem, we get

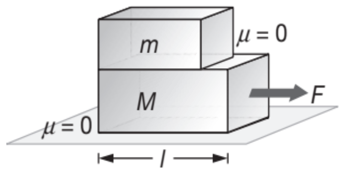
$$\frac{mg}{\sin 90^\circ} = \frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 120^\circ}$$

$\Rightarrow \quad T_1 = mg \sin(150^\circ) = 50 \times \dfrac{1}{2} = 25$ N

$\Rightarrow \quad T_2 = mg \sin 120^\circ = 50 \times \dfrac{\sqrt{3}}{2} = 25\sqrt{3}$ N

Hence, (A) and (D) are correct.

64.



Since friction is absent everywhere, so

$$a = \frac{F}{M}$$

Since $s = \dfrac{1}{2} a t^2$

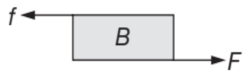
$\Rightarrow \quad l = \dfrac{1}{2}\left(\dfrac{F}{M}\right) t^2$

$\Rightarrow \quad t = \sqrt{\dfrac{2lM}{F}}$

Hence, (B) and (D) are correct.

65.

$$f = m_A a = 40 \text{ N}$$

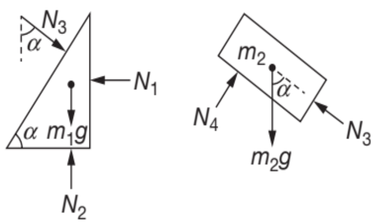


$$F - f = m_B a = 50 \times 3 \text{ N} = 150 \text{ N}$$

$\Rightarrow \quad F = 150 \text{ N} + 40 \text{ N} = 190$ N

Hence, (A), (B) and (C) are correct.

66. The normal reaction on the wedge can be due to the wall, ground and the block



From free body diagrams of m_1 and m_2, we get

$$N_3 = m_2 g \cos\alpha$$

$$N_3 \sin\alpha = N_1$$

$$N_2 = m_1 g + N_3 \cos\alpha$$

$\Rightarrow \quad N_1 = m_2 g \sin\alpha \cos\alpha$

$\Rightarrow \quad N_2 = m_1 g + m_2 g \cos^2 \alpha$

Hence, (A), (B) and (C) are correct.

Reasoning Based Questions

1. (i) $F = \dfrac{dp}{dt}$ (Newton's Second Law)

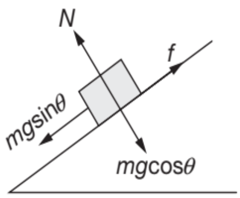
(ii) Newton's Third Law

Hence, the correct answer is (B).

2. $f = mg\sin\theta$ ...(1)

$M = mg\cos\theta$

$R = \sqrt{N^2 + f^2} = mg$ ...(2)



Hence, the correct answer is (A).

3. Conceptual

Hence, the correct answer is (D).

4. Law of inertia.

Hence, the correct answer is (A).

5. Force needed when breaks are applied

$$f_1 = ma = \frac{mv^2}{d}$$

$\{v : \text{initial speed},\ d : \text{distance from wall}\}$

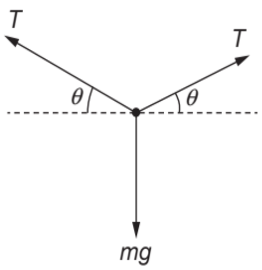
when turn is taken

$$f_2 = ma = \frac{mv^2}{d}$$

Hence brakes must be applied.

Hence, the correct answer is (B).

7. $2T\sin\theta = mg$



$T = \dfrac{mg}{2\sin\theta} > mg$ for $\sin\theta < \dfrac{1}{2}$ i.e., $\theta < 30°$

Hence, the correct answer is (A).

8. Here the Statement-1 is based on the idea that if you substitute $\vec{F} = \vec{0}$ in $\vec{F} = m\vec{a}$ (because there happens to be a particle on which net force $\vec{F} = \vec{0}$)

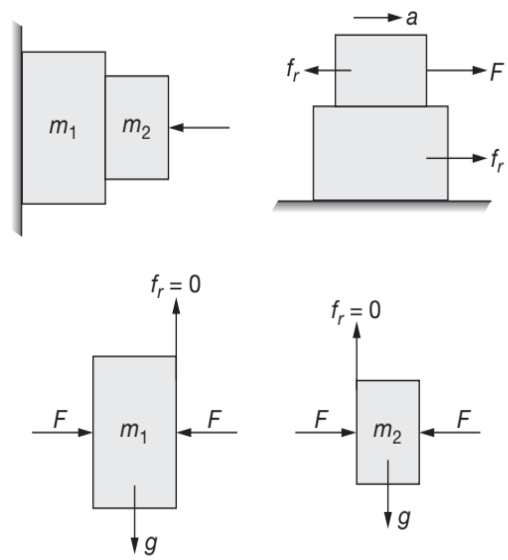
You get $\vec{a} = \vec{0}$

$\Rightarrow \quad \vec{v} = \text{constant}$

$\Rightarrow$ such a particle will move with constant speed along a fixed direction which is Newton's First Law. But the point is, you cannot employ $\vec{F} = m\vec{a}$, without first ascertaining that it is valid in this form (i.e., without pseudo forces). And that can be done only by applying Newton's First Law and checking the behaviour of your frame against the description laid down in the First Law.

Hence, the correct answer is (D).

10.



Hence, the correct answer is (B).

11. According to Newton's Third Law of motion action and reactions are equal and opposite.

Hence, the correct answer is (D).

12. Horizontal range also depends upon the velocity of projection.

Hence, the correct answer is (D).

13. In the direction of normal reaction net acceleration is zero. Hence forces in this direction will be balanced. Hence $N = mg\cos\theta$.

Hence, the correct answer is (A).

14. By the definition of inertial and non-inertial frame.

Hence, the correct answer is (B).

15. Coefficient of friction $\mu = \tan(\theta)$. The value of $\tan\theta$ way exceed unity.

Hence, the correct answer is (C).

16. $F = \dfrac{\Delta P}{\Delta t}$

If Δt is more, then F will be less.

Hence, the correct answer is (A).

17. $a_1 = g - \dfrac{F}{M}$

$a_2 = g - \dfrac{F}{m}$

$\Rightarrow \quad a_1 > a_2$

Hence, the correct answer is (A).

18. While running boy pushes the ground in backward direction and available friction pushed him in forward direction.

Hence, the correct answer is (D).

19. In equilibrium, net force on the body = 0, therefore, it acceleration $a = 0$. If the body is at rest it will remain at rest. If the body is moving with constant speed along a straight line, it will continue to do so.

Hence, the correct answer is (D).

20. Inertia is the property by virtue of which the body is unable to change by itself not only the state of rest but also the state of motion.

Hence, the correct answer is (A).

21. Due to change in normal reaction pulling is easier.

Hence, the correct answer is (D).

22. Contact force is sum of friction and normal reaction.

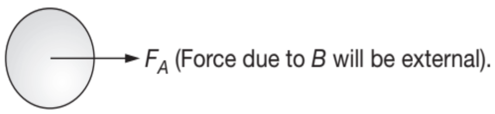
Hence, the correct answer is (D).

23. Static frictional force is self adjusting.

Hence, the correct answer is (A).

24.

F.B.D. of A

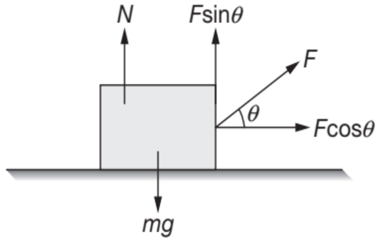


Hence, the correct answer is (B).

25. For pulling condition:

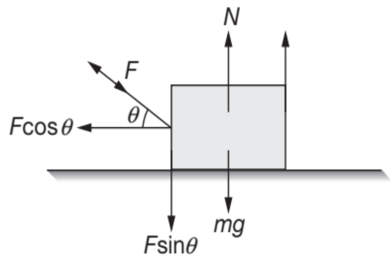
$$N + F\sin\theta = mg$$

$$N = Mg - F\sin\theta \qquad \ldots(1)$$



For pushing condition:

$$N = F\sin\theta + Mg \qquad \ldots(2)$$



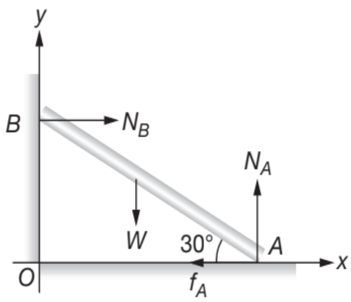
Hence, the correct answer is (D).

Linked Comprehension Type Questions

1. **The correct answer is (B).**
2. **The correct answer is (A).**
3. **The correct answer is (B).**

Combined solution to 1, 2, 3

Let length of the rod be $2l$. Using the three conditions of equilibrium. Anticlockwise moment is taken as positive.



For translational equilibrium, we have

$\Sigma F_x = 0$	$\Sigma F_y = 0$
$\Rightarrow \quad N_B - f_A = 0$	$\Rightarrow \quad N_A - W = 0$
$\Rightarrow \quad N_B = f_A \qquad \ldots(1)$	$\Rightarrow \quad N_A = W \qquad \ldots(2)$

For rotational equilibrium, we have

$$\Sigma\tau_0 = 0$$

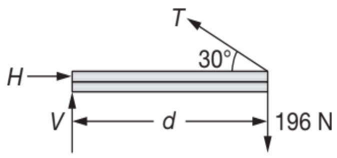
$$\Rightarrow \quad N_A(2l\cos 30°) - N_B(2l\sin 30°) - W(l\cos 30°) = 0$$

$$\Rightarrow \quad \sqrt{3}N_A - N_B - \frac{\sqrt{3}}{2}W = 0 \qquad \ldots(3)$$

Solving these three equations, we get

$$f_A = \frac{\sqrt{3}}{2}W,\ N_A = W,\ N_B = \frac{\sqrt{3}}{2}W$$

4. Consider the torques about an axis perpendicular to the page and through the left end of the horizontal beam.



$$\sum \tau = +(T\sin(30°))d - (196\text{ N})d = 0$$

$$\Rightarrow \quad T = 392\text{ N}$$

Hence, the correct answer is (D).

5. Since $\sum F_x = 0$

$$H - T\cos(30°) = 0$$

$$\Rightarrow \quad H = (392)\cos(30°) = 339\text{ N} \text{ to the right}$$

Hence, the correct answer is (C).

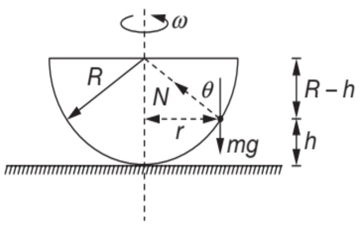
6. Since $\sum F_y = 0$

$$V + T\sin(30°) - 200 = 0$$

$$\Rightarrow \quad V = 196 - (392)\sin(30°) = 0$$

Hence, the correct answer is (A).

7. FBD of particle in ground frame of reference is shown in figure. We observe that



$$\tan\theta = \frac{r}{R-h}$$

Also, $N\cos\theta = mg$ and $\qquad \ldots(1)$

$$N\sin\theta = mr\omega^2 \qquad \ldots(2)$$

Dividing equation (2) by (1), we get

$$\tan\theta = \frac{r\omega^2}{g}$$

$$\Rightarrow \quad \frac{r}{R-h} = \frac{r\omega^2}{g}$$

$$\Rightarrow \quad \omega^2 = \frac{g}{R-h} \qquad \ldots(3)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{R-h}}$$

Hence, the correct answer is (D).

8. From equation (3),

$$h = R - \frac{g}{\omega^2}$$

For non-zero value of h, we must have

$$R > \frac{g}{\omega^2}$$

$$\Rightarrow \quad \omega > \sqrt{g/R}$$

So, minimum value of ω should be

$$\omega_{\min} = \sqrt{\frac{g}{R}} = \sqrt{\frac{9.8}{0.1}}\text{ rads}^{-1}$$

$$\Rightarrow \quad \omega_{\min} = 9.9\text{ rads}^{-1}$$

Hence, the correct answer is (C).

9. Since, $h = R - \frac{g}{\omega^2}$

If R and ω are known precisely, then

$$\Delta h = -\frac{\Delta g}{\omega^2}$$

$$\Rightarrow \quad \Delta g = \omega^2 \Delta h \qquad \text{(neglecting the negative sign)}$$

$$\Rightarrow \quad (\Delta g)_{\min} = (\omega_{\min})^2 \Delta h = (9.89)^2 \times 10^{-4}\text{ ms}^{-2}$$

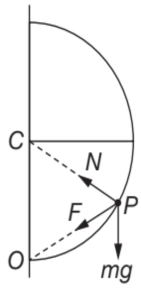
$$\Rightarrow \quad (\Delta g)_{\min} = 9.78 \times 10^{-3}\text{ ms}^{-2}$$

Hence, the correct answer is (B).

10. $CP = CO =$ Radius of circle (R)

$\therefore \quad \angle CPO = \angle POC = 60°$

$\therefore \quad \angle OCP$ is also $60°$.



Therefore, ΔOCP is an equilateral triangle.

Hence, $OP = R$

Natural length of spring is $3R/4$.

So, extension in the spring is

$$x = R - \frac{3R}{4} = \frac{R}{4}$$

$\Rightarrow$ Spring force, $F = kx = \left(\frac{mg}{R}\right)\left(\frac{R}{4}\right) = \frac{mg}{4}$

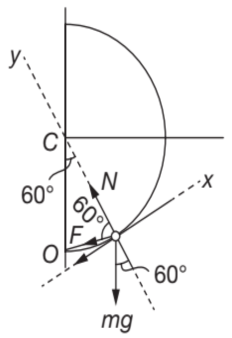
The free body diagram of the ring will be as shown.

Here, $F = k\Delta x = \frac{mg}{4}$ $\left\{\because \Delta x = \frac{R}{4}\right\}$

And $N =$ normal reaction.

Hence, the correct answer is (C).

11. Tangential acceleration a_T :



The ring will move towards the x-axis just after the release. So, net force along x-axis i.e. the tangential force is

$$F_T = F_x = F\sin 60° + mg\sin 60°$$

$$\Rightarrow \quad F_T = \left(\frac{mg}{4}\right)\frac{\sqrt{3}}{2} + mg\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad F_T = \frac{5\sqrt{3}}{8}mg$$

$$\Rightarrow \quad a_T = a_x = \frac{F_x}{m} = \frac{5\sqrt{3}}{8}g$$

Hence, the correct answer is (D).

12. Since, $N + F\cos 60° = mg\cos 60°$

$$\Rightarrow \quad N = mg\cos 60° - F\cos 60°$$

$$\Rightarrow \quad N = \frac{mg}{2} - \frac{mg}{4}\left(\frac{1}{2}\right) = \frac{mg}{2} - \frac{mg}{8}$$

$$\Rightarrow \quad N = \frac{3}{8}mg$$

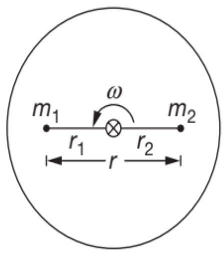
Hence, the correct answer is (C).

13. Given $m_1 = 10$ kg, $m_2 = 5$ kg, $\omega = 10$ rads^{-1}

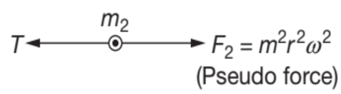
$r = 0.3$ m, $r_1 = 0.124$ m

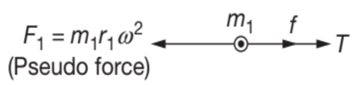
$$\Rightarrow \quad r_2 = r - r_1 = 0.176 \text{ m}$$

Masses m_1 and m_2 are at rest with respect to rotating table. Let f be the friction between mass m_1 and table.



Free body diagram of m_1 and m_2 with respect to table (non-inertial frame of reference are shown in figures).





Equilibrium of m_2 gives

$$T = m_2 r_2 \omega^2 \qquad \ldots(1)$$

Since, $m_2 r_2 \omega^2 < m_1 r_1 \omega^2$

Therefore, $m_1 r_1 \omega^2 > T$

and friction of M_1 will be inward (toward centre)
Equilibrium of m_1 gives

$$f + T = m_1 r_1 \omega^2 \qquad \ldots(2)$$

From equations (1) and (2), we get

$$f = m_1 r_1 \omega^2 - m_2 r_2 \omega^2 \qquad \ldots(3)$$

$$f = (m_1 r_1 - m_2 r_2)\omega^2$$

$$\Rightarrow \quad f = (10 \times 0.124 - 5 \times 0.176)(10)^2 \text{ N} = 36 \text{ N}$$

Therefore, frictional force on m_1 is 36 N (inwards)

Hence, the correct answer is (D).

14. From equation (3),

$$f = (m_1r_1 - m_2r_2)\omega^2$$

Masses will start slipping when this force is greater than f_{max} or $(m_1r_1 - m_2r_2)\omega^2 > f_{max} > \mu m_1 g$.

Minimum values of ω is

$$\omega_{min} = \sqrt{\frac{\mu m_1 g}{m_1r_1 - m_2r_2}} = \sqrt{\frac{0.5\times 10\times 9.8}{10\times 0.124 - 5\times 0.176}}$$

$$\Rightarrow \quad \omega_{min} = 11.67 \text{ rads}^{-1}$$

Hence, the correct answer is (B).

15. From equation (3), frictional force $f = 0$ where $m_1r_1 = m_2r_2$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{m_2}{m_1} = \frac{5}{10} = \frac{1}{2}$$

and $r = r_1 + r_2 = 0.3$ m

$$\Rightarrow \quad r_1 = 0.1 \text{ m and } r_2 = 0.2 \text{ m}$$

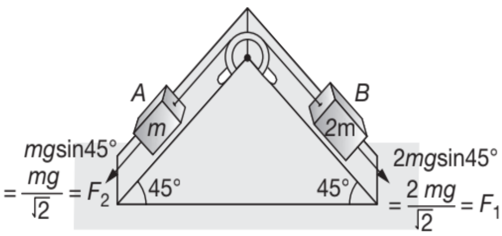
i.e. mass m_2 should placed at 0.2 m and m_1 at 0.1 m from the centre O.

Hence, the correct answer is (C).

16. Acceleration of block A

Maximum friction force that can be obtained at A is

$$(f_{max})_A = \mu_A(mg\cos 45°) = \frac{2}{3}\left(\frac{mg}{\sqrt{2}}\right) = \frac{\sqrt{2}\,mg}{3}$$



Similarly,

$$(f_{max})_B = \mu_B(2mg\cos 45°) = \frac{1}{3}\left(\frac{2mg}{\sqrt{2}}\right)$$

$$\Rightarrow \quad (f_{max})_B = \frac{\sqrt{2}\,mg}{3}$$

Therefore, maximum value of friction that can be obtained on the system is

$$(f_{max}) = (f_{max})_A + (f_{max})_B = \frac{2\sqrt{2}\,mg}{3} \qquad \ldots(1)$$

Net pulling force on the system is

$$F = F_1 - F_2 = \frac{2mg}{\sqrt{2}} - \frac{mg}{\sqrt{2}} = \frac{mg}{\sqrt{2}} \qquad \ldots(2)$$

From (1) and (2), we can see that

Net pulling force $< f_{max}$

Therefore, the system will not move or the acceleration of block A will be zero.

Hence, the correct answer is (D).

17. The correct answer is (B).

18. The correct answer is (C).

Combined solution to 17, 18

Tension in the string and friction at A

Net external force on the system (block A and B)

$$F = F_1 - F_2 = \frac{mg}{\sqrt{2}}$$

Therefore, total friction force on the blocks should also be equal to $\frac{mg}{\sqrt{2}}$

$$\Rightarrow \quad f_A + f_B = F = \frac{mg}{\sqrt{2}}$$

Now since the blocks will start moving from block B first (if they move), therefore, f_B will reach its limiting value first and if still some force is needed, it will be provided by f_A.

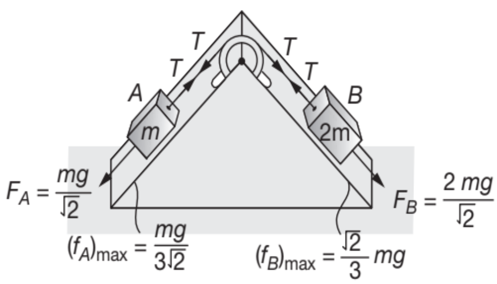
Here $(f_{max})_B < F$.

Therefore, f_B will be in its limiting value and rest will be provided by f_A.

$$\text{Hence } f_B = (f_{max})_B = \frac{\sqrt{2}\,mg}{3}$$

$$\text{and } f_A = F - f_B = \frac{mg}{\sqrt{2}} - \frac{\sqrt{2}\,mg}{3} = \frac{mg}{3\sqrt{2}}$$

The FBD of the whole system will be as shown in the figure.



Therefore, friction on A is

$$f_A = \frac{mg}{3\sqrt{2}} \qquad \text{(down the plane)}$$

Now for tension T in the string, we may consider either equilibrium of A or B.
Equilibrium of A gives

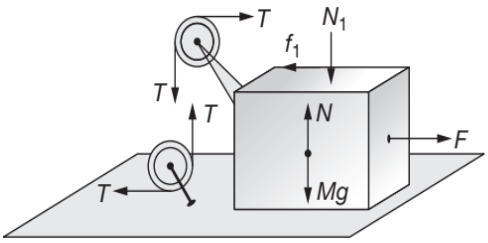
$$T = F_2 + f_A = \frac{mg}{\sqrt{2}} + \frac{mg}{3\sqrt{2}} = \frac{4mg}{3\sqrt{2}} \text{ or } \frac{2\sqrt{2}mg}{3}$$

Similarly, equilibrium of B gives $T + f_B = F_1$.

$$T = F_1 - f_B = \frac{2\ mg}{\sqrt{2}} - \frac{\sqrt{2}\,mg}{3} = \frac{4\ mg}{3\sqrt{2}} = \frac{2\sqrt{2}}{3}\ \text{mg}$$

Therefore, tension in the string is $\frac{2\sqrt{2}}{3}$ mg.

19. Free body diagram of mass M is



Hence, the correct answer is (B).

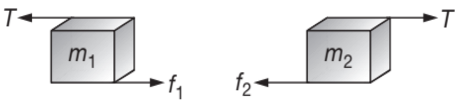
20. The maximum value of f_1 is

$$(f_1)_{max} = (0.3)(20)(10) = 60\ \text{N}$$

The maximum value of f_2 is

$$(f_2)_{max} = (0.3)(5)(10) = 15\ \text{N}$$

Forces on m_1 and m_2 in horizontal direction are as follows



Now there are only two possibilities

(1) Either both m_1 and m_2 will remain stationary (with respect to ground) or

(2) Both m_1 and m_2 will move (with respect to ground)

First case is possible when

$$T \le (f_1)_{max} \text{ i.e. } T \le 60\ \text{N}$$

and $T \le (f_2)_{max}$ i.e. $T \le 15$ N

These conditions will be satisfied when $T \le 15$ N

say $T = 14$ N then $f_1 = f_2 = 14$ N

Therefore, the condition $f_1 = 2f_2$ will not be satisfied.
So, m_1 and m_2 both can't remain stationary.
In the second case, when m_1 and m_2 both move

$$f_2 = (f_2)_{max} = 15\ \text{N}$$

$$\Rightarrow \quad f_1 = 2f_2 = 30\ \text{N}$$

Now since $f_1 < (f_2)_{max}$, there is no relative motion between m_1 and M. i.e., all the masses move with same acceleration, say a.

$$\Rightarrow \quad f_2 = 15\ \text{N}$$

and $f_1 = 30$ N

Hence, the correct answer is (D).

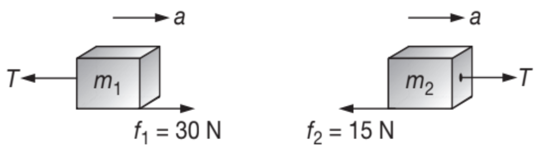
21. The correct answer is (A).

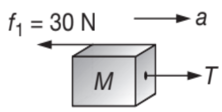
22. The correct answer is (D).

23. The correct answer is (C).

Combined solution to 21, 22, 23

Free body diagrams and equations of motion are as shown





For m_1, $30 - T = 20a$ ...(1)

For m_2, $T - 15 = 5a$ ...(2)

For M, $F - 30 = 50a$ ...(3)

Solving these three equations, we get

$$F = 60\ \text{N}$$

$$T = 18\ \text{N} \text{ and } a = \frac{3}{5}\ \text{ms}^{-2}$$

24. $F_x = \frac{d}{dt}(p_x) = -2\sin t$

$$F_y = \frac{d}{dt}(p_y) = 2\cos t$$

$$\Rightarrow \quad F = \sqrt{F_x^2 + F_y^2}$$

$\Rightarrow \quad F = \sqrt{4\sin^2 t + 4\cos^2 t}$

$\Rightarrow \quad F = \sqrt{4(\sin^2 t + \cos^2 t)} = 2$ unit

Hence, the correct answer is (B).

25. $\vec{F} = -2\sin t\hat{i} + 2\cos t\hat{j}$

$\vec{p} = 2\cos t\hat{i} + 2\sin t\hat{j}$

$\vec{F}\cdot\vec{p} = (-2\sin t\hat{i} + 2\cos t\hat{j})\cdot(2\cos t\hat{i} + 2\sin t\hat{j})$

$\Rightarrow \quad \vec{F}\cdot\vec{p} = -4\sin t\cos t + 4\sin t\cos t = 0$

So, the angle between $\vec{F}$ and $\vec{p}$ is $90°$.

Hence, the correct answer is (D).

26. $p = \sqrt{p_x^2 + p_y^2}$

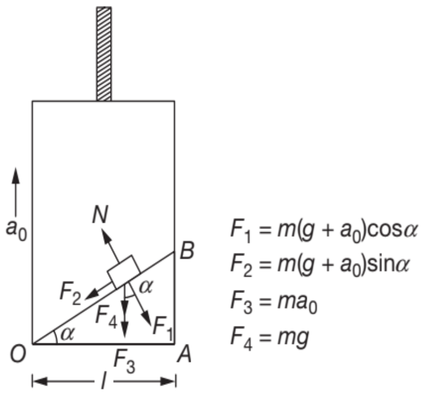
$p = \sqrt{4\cos^2 t + 4\sin^2 t} = 2$ units

Hence, the correct answer is (B).

27. $N = m(g + a_0)\cos\alpha$ and

$ma = m(g + a_0)\sin\alpha$

$\Rightarrow \quad a = (g + a_0)\sin\alpha$



Hence, the correct answer is (B).

29. Since $OB = \frac{l}{\cos\alpha} = \frac{1}{2}at^2$

$\Rightarrow \quad \frac{l}{\cos\alpha} = \frac{1}{2}(g + a_0)\sin\alpha t^2$

$\Rightarrow \quad T = \sqrt{\frac{2l}{(g + a_0)\sin\alpha\cos\alpha}}$

$\Rightarrow \quad T = \sqrt{\frac{4l}{(g + a_0)\sin 2\alpha}}$

Hence, the correct answer is (B).

31. **The correct answer is (A).**

32. **The correct answer is (B).**

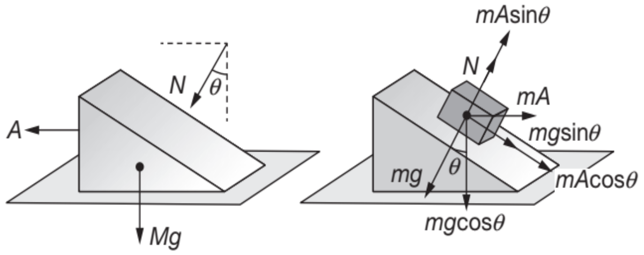
33. **The correct answer is (B).**

Combined solution to 31, 32, 33

$N\sin\theta = MA$ ...(1)

$N + mA\sin\theta = mg\cos\theta$ ...(2)

$mg\sin\theta + mA\cos\theta = ma$ ...(3)



From (1) and (2)

$\frac{MA}{\sin\theta} + mA\sin\theta = mg\cos\theta$

$\Rightarrow \quad A\left(\frac{M}{\sin\theta} + m\sin\theta\right) = mg\cos\theta$

$\Rightarrow \quad A = \frac{mg\sin\theta\cos\theta}{M + m\sin^2\theta}$

From (3)

$a = g\sin\theta + A\cos\theta$

$\Rightarrow \quad a = g\sin\theta + \frac{mg\sin\theta\cos^2\theta}{M + m\sin^2\theta}$

$\Rightarrow \quad a = \frac{g\sin\theta(M + m\sin^2\theta + m\cos^2\theta)}{M + m\sin^2\theta}$

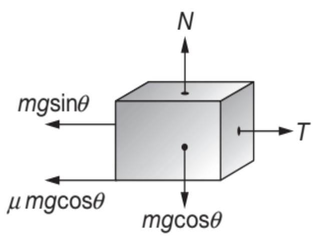
$\Rightarrow \quad a = \frac{(M + m)g\sin\theta}{M + m\sin^2\theta}$

$\Rightarrow \quad N = \frac{MA}{\sin\theta} = \frac{Mmg\cos\theta}{M + m\sin^2\theta}$

34. **The correct answer is (B).**

35. **The correct answer is (D).**

Combined solution to 34, 35



The block is moving with constant velocity, so

$T = mg(\sin\theta + \mu\cos\theta)$

Since, $P = Tv$

$\Rightarrow \quad 5\times10^3 = 250\times10\left(\frac{1}{2}+\mu\frac{\sqrt{3}}{2}\right)\times2$

$\Rightarrow \quad 5\times10^3 = 1250\left(1+\sqrt{3}\mu\right)\times2$

$\Rightarrow \quad 1+\sqrt{3}\,\mu = \frac{5000}{1250} = 4$

$\Rightarrow \quad \mu = \frac{1}{\sqrt{3}}$

$F_C = \sqrt{N^2+(\mu N)^2}$

$\Rightarrow \quad F_C = N\sqrt{1+\mu^2} = mg\cos\theta\sqrt{1+\mu^2}$

$\Rightarrow \quad F_C = 250\times10\times\frac{\sqrt{3}}{2}\sqrt{1+\frac{1}{3}}$

$\Rightarrow \quad F_C = 2500\text{ N}$

36. $T = mg(\sin\theta+\mu\cos\theta)$

$\Rightarrow \quad T = 2500\left(\frac{1}{2}+\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{2}\right)$

$\Rightarrow \quad T = 2500\text{ N}$

Hence, the correct answer is (C).

37. $f_k = \mu mg\cos\theta$

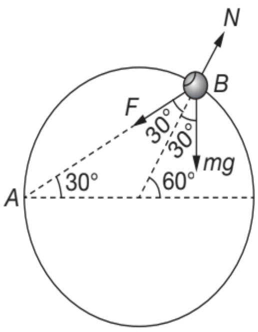
$\Rightarrow \quad f_k = \frac{1}{\sqrt{3}}\times2500\frac{\sqrt{3}}{2}$

$\Rightarrow \quad f_k = 1250\text{ N}$

Hence, the correct answer is (C).

38. Extension in the spring is

$x = AB - R = 2R\cos(30^\circ) - R = (\sqrt{3}-1)R$



The spring force is given by

$F = kx = \frac{(\sqrt{3}+1)\text{ mg}}{R}(\sqrt{3}-1)R = 2\text{mg}$

Free body diagram of bead is shown here.

$N = (F+mg)\cos(30^\circ)$

$\Rightarrow \quad N = (2mg+mg)\frac{\sqrt{3}}{2}$

$\Rightarrow \quad N = \frac{3\sqrt{3}mg}{2}$

Hence, the correct answer is (D).

39. Tangential force

$F_t = F\sin(30^\circ) - mg\sin(30^\circ)$

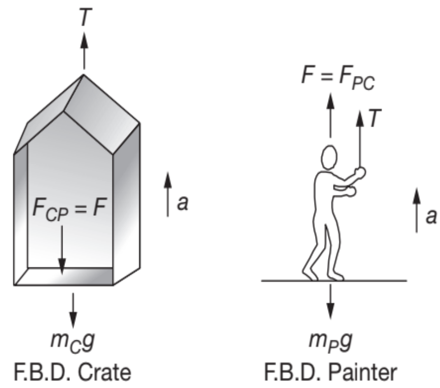
$\Rightarrow \quad F_t = (2mg-mg)\sin(30^\circ) = \frac{mg}{2}$

So, tangential acceleration is given by

$a_t = \frac{F_t}{m} = \frac{g}{2}$

Hence, the correct answer is (A).

40.



$T + F - m_P g = m_P a$ (For Painter) ...(1)

$T - F - m_C g = m_C a$ (For crate) ...(2)

Subtract (2) from (1), we get

$2F + (m_C - m_P)g = (m_P - m_C)a$

$\Rightarrow \quad 900 + (25-100)(10) = 75a$

$\Rightarrow \quad 900 - 750 = 75a$

$\Rightarrow \quad a = 2\text{ ms}^{-2}$

Hence, the correct answer is (C).

41. Substituting in (2), we get

$T = 750\text{ N}$

Hence, the correct answer is (D).

42. $a = 0$

$\Rightarrow \quad T + F - Mg = 0$ {from (1)}

and $T - F - mg = 0$ [from (2)}

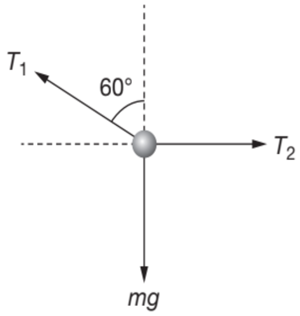
Subtracting, we get

$$2F = (M - m)g$$

$$\Rightarrow \quad F = \frac{1}{2}(M - m)g$$

Hence, the correct answer is (D).

43.



$$T_1 \cos(60°) = mg$$

$$\Rightarrow \quad T_1 = 2\text{mg}$$

Hence, the correct answer is (A).

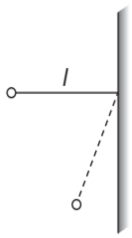
44. $T_1 \sin(60°) = T_2$

$$\Rightarrow \quad 2mg\frac{\sqrt{3}}{2} = T_2$$

$$\Rightarrow \quad T_2 = \sqrt{3}\,mg$$

Hence, the correct answer is (B).

45. Loss in PE = Gain in KE



$$\Rightarrow \quad mg\ell = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{2g\ell}$$

Hence, the correct answer is (B).

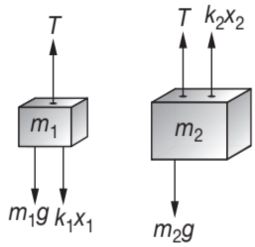
46. For equilibrium of m_1 and m_2, we have

$$T = m_1 g + k_1 x_1 \quad \ldots(1)$$

$$T + k_2 x_2 = m_2 g \quad \ldots(2)$$

From (1) and (2), we get

$$m_1 g + k_1 x_1 = m_2 g - k_2 x_2$$



$$\Rightarrow \quad k_1 x_1 = (m_2 - m_1)g - k_2 x_2$$

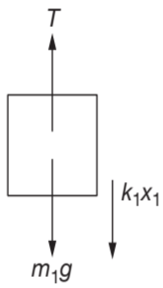
$$\Rightarrow \quad k_1 x_1 = (8 - 2)(10) - (70)(0.5)$$

$$\Rightarrow \quad k_1 x_1 = 25$$

$$\Rightarrow \quad x_1 = \frac{25}{50} = 0.5 \text{ m}$$

Hence, the correct answer is (C).

47. Since, $T = m_1 g + k_1 x_1$ $\{\because$ of $(1)\}$



$$\Rightarrow \quad T = 50 \times 0.5 + 20$$

$$\Rightarrow \quad T = 45 \text{ N}$$

Hence, the correct answer is (A).

49. No friction between M and m_0, so acceleration of m_0 is zero

Hence, the correct answer is (B).

50. $\mu(M + m_0)g = mg$

$$\Rightarrow \quad \mu = \frac{m}{M + m_0}$$

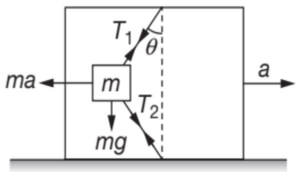
Hence, the correct answer is (B).

51. $a = \dfrac{mg - \mu(M + m_0)g}{M + m}$

$$a = \left[\frac{m - \mu(M + m_0)}{M + m}\right]g$$

Hence, the correct answer is (C).

52. For minimum force F, the lower string just remains tight i.e. $T_2 \to 0$.



$\Rightarrow$ $T_1\cos\theta = mg$ and $T_1\sin\theta = ma$

$\Rightarrow$ $a = g\tan\theta$

$\Rightarrow$ $F = (m+M)g\tan\theta$

Hence, the correct answer is (C).

53. When the lower string is just tight, then

$T_2 \to 0$

$\Rightarrow$ $T_1\cos\theta = mg$

$\Rightarrow$ $T_1 = \dfrac{mg}{\cos\theta}$

Hence, the correct answer is (B).

54. $T_1\cos\theta = mg + T_2\cos\theta$

$\Rightarrow$ $(T_1 - T_2)\cos\theta = mg$ ...(1)

Also $(T_1 + T_2)\sin\theta = ma$ ...(2)

From (1) and (2), we get

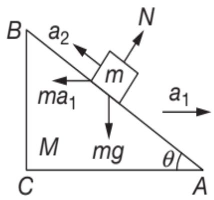
$$\left(\frac{T_1+T_2}{T_1-T_2}\right)\tan\theta = \frac{a}{g}$$

Since $T_1 = 2T_2$

$\Rightarrow$ $a = 3g\tan\theta$

Hence, the correct answer is (A).

55.



Let a_2 be acceleration of m w.r.t. M

Then, $ma_1\cos\theta - mg\sin\theta = ma_2$

$$\frac{a_1}{\sqrt{2}} - \frac{g}{\sqrt{2}} = a_2$$

$\Rightarrow$ $a_1 - g = a_2\sqrt{2}$ ...(1)

Also, $N = ma_1\sin\theta + mg\cos\theta$ ...(2)

and $F - N\sin\theta = Ma_1$ ...(3)

Put (2) in (3), we get

$$F - ma_1\sin^2\theta - mg\sin\theta\cos\theta = Ma_1$$

$\Rightarrow$ $F - mg\sin\theta\cos\theta = (M + m\sin^2\theta)a_1$

$\Rightarrow$ $210 - 20\left(\dfrac{1}{2}\right) = \left[9 + 2\left(\dfrac{1}{2}\right)\right]a_1$

$\Rightarrow$ $a_1 = 20\ \text{ms}^{-2}$

Hence, the correct answer is (C).

56. Since, $AB = l = \dfrac{1}{2}a_2t^2$, where

$$a_2 = \frac{a_1 - g}{\sqrt{2}} = \frac{20-10}{\sqrt{2}} = \frac{10}{\sqrt{2}}\ \text{ms}^{-2}$$

$\Rightarrow$ $\dfrac{10}{\cos(45^\circ)} = \dfrac{1}{2}\left(\dfrac{10}{\sqrt{2}}\right)t^2$

$\Rightarrow$ $10\sqrt{2} = \dfrac{10}{2\sqrt{2}}t^2$

$\Rightarrow$ $t = 2\ \text{s}$

Since $v = u + at$

$\Rightarrow$ $v = a_2t = \left(\dfrac{10}{\sqrt{2}}\right)^2 = \dfrac{20}{\sqrt{2}} = 10\sqrt{2}\ \text{ms}^{-1}$

Hence, the correct answer is (C).

57. $l' = \dfrac{1}{2}a_1t^2$

$\Rightarrow$ $l' = \dfrac{1}{2}(20)(4) = 40\ \text{m}$

Hence, the correct answer is (D).

58. $F\sin\theta + f_2 = mg$

$\Rightarrow$ $F\sin\theta + \mu F\cos\theta = mg$

$\Rightarrow$ $F = \dfrac{mg}{\sin\theta + \mu\cos\theta}$

Hence, the correct answer is (B).

59. $F\sin\theta = mg + f_2$

$\Rightarrow$ $F\sin\theta - \mu F\cos\theta = mg$

$\Rightarrow$ $F = \dfrac{mg}{\sin\theta - \mu\sin\theta}$

Hence, the correct answer is (C).

60. $F\sin\theta = mg$

Hence, the correct answer is (B).

61. Since there is no force on upper block of mass 10 kg and no tendency of slipping over 20 kg block therefore friction on it will be zero.

Hence, the correct answer is (A).

62. Maximum acceleration or retardation upto which both blocks will move together is

$$a_{max} = \mu_1 g = 5\ ms^{-2}$$

$\Rightarrow \quad \mu_2(m_1 + m_2)g = (m_1 + m_2)a$

$\Rightarrow \quad \mu_2 = 0.5$

Also kinetic friction on $20\ kg$ is

$$f_k = \mu_2(m_1 + m_2)g = 150\ N$$

Hence, the correct answer is (A).

63. Since, retardation is $5\ ms^{-2}$, so applying

$v^2 = u^2 - 2as$, we get

$0^2 = 400 - 2 \times 5 \times s$

$\Rightarrow \quad s = 100\ m$

Hence, the correct answer is (B).

64. $\omega = at^2$

Linear speed $v = \omega r = 1 \times 2 \times t^2 = (2\ ms^{-3})(t^2)$

Since, $a_T = \dfrac{dv}{dt} = \dfrac{d(2t^2)}{dt} = 4t$ and

$$a_C = \frac{v^2}{r} = \omega^2 r$$

Net acceleration is

$$a = a_{net} = \sqrt{a_T^2 + a_C^2}$$

Force $= ma = m\sqrt{a_T^2 + a_C^2}$

Hence, the correct answer is (B).

66. Since rate of change of speed is tangential acceleration, so we have

$$a_T = \frac{a}{\sqrt{2}}$$

$\Rightarrow \quad a_T^2 + a_C^2 = a^2$

$\Rightarrow \quad \dfrac{a^2}{2} + a_C^2 = a^2$

$\Rightarrow \quad a_C = \dfrac{a}{\sqrt{2}}$

$\Rightarrow \quad a_C = a_T = \dfrac{a}{\sqrt{2}}$

$\Rightarrow \quad 4t = 4t^3$

$\Rightarrow \quad t = 1\ s$

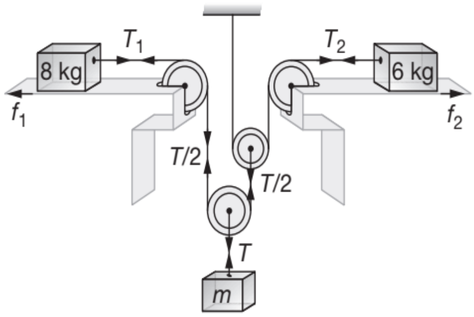
Hence, the correct answer is (B).

67. The correct answer is (B).

68. The correct answer is (B).

69. The correct answer is (C).

Combined solution to 67, 68, 69



For equilibrium of

8 kg block, $T_1 \le 40\ N$

6 kg block, $T_2 \le 24\ N$

Also, $T_1 = \dfrac{T}{2}$ and $T_2 = \dfrac{T}{4}$

So, for equilibrium of

8 kg block, we have $T \le 80\ N$ and

6 kg block, we have $T \le 96\ N$

However, system will be equilibrium if all the three blocks are in equilibrium. Hence

$T \le 80\ N$

$\Rightarrow \quad m \le 8\ kg$

The $6\ kg$ block will start moving if

$T_2 > 96\ N$

$\Rightarrow \quad m > 9.6\ kg$

When $9\ kg$ block is hanged, $6\ kg$ block remains at rest, but $8\ kg$ block moves. Let acceleration of $9\ kg$ block be a, then acceleration of $8\ kg$ block will be $2a$.

For 9 kg block, we have

$$90 - T = 9a \qquad \ldots(1)$$

For 8 kg block, we have

$$\frac{T}{2} - f = 8(2a)$$

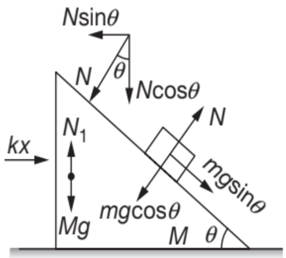
$$\Rightarrow \quad \frac{T}{2} - 40 = 8(2a) \qquad \ldots(2)$$

From (1) and (2), we get

$$\frac{90 - 9a}{2} = 16a + 40$$

$\Rightarrow \quad a = \dfrac{10}{41}\ ms^{-2}$

70.



$N\sin\theta = kx$

$N = mg\cos\theta$

$\Rightarrow \quad x = \dfrac{N\sin\theta}{k} = \dfrac{mg\sin\theta\cos\theta}{k} = \dfrac{mg\sin(2\theta)}{2k}$

Hence, the correct answer is (C).

71. $N\cos\theta + Mg = N_1$

$\Rightarrow \quad N_1 = (mg\cos\theta)\cos\theta + Mg$

$\Rightarrow \quad N_1 = mg\cos^2\theta + Mg$

$\Rightarrow \quad N_1 = (m\cos^2\theta + M)g$

Hence, the correct answer is (D).

72. Net unbalanced force on the block is $mg\sin\theta$.

Hence, the correct answer is (A).

73. Friction force will be static in nature. So

$f = 30 \text{ N}$

Hence, the correct answer is (C).

74. Slipping will start if acceleration a of system is

$a > \dfrac{0.3\times 25\times 10}{5}$

$\Rightarrow \quad \left(\dfrac{M}{M+30}\right)g > \dfrac{0.3\times 25\times 10}{5}$

Hence, the correct answer is (D).

75. $a = \left(\dfrac{M}{M+30}\right)g = 6 \text{ ms}^{-2}$

Hence, the correct answer is (A).

76. $v = kt_1$

Tangential acceleration is

$a_T = \dfrac{dv}{dt} = k$

Centripetal acceleration is

$a_C = \dfrac{v^2}{r}$

$\Rightarrow \quad \dfrac{v^2}{r} = k$

$\Rightarrow \quad \dfrac{k^2t_1^2}{r} = k$

$\Rightarrow \quad t_1 = \sqrt{\dfrac{r}{k}}$

Hence, the correct answer is (C).

77. Net force $F = \sqrt{mk^2 + mk^2} = \sqrt{2}mk$

Hence, the correct answer is (B).

78. $\tan\theta = \dfrac{a_T}{a_C} = \dfrac{K}{\left(\dfrac{v^2}{r}\right)} = \dfrac{K}{\left(\dfrac{a_T^2t^2}{r}\right)}$

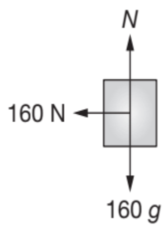
$\Rightarrow \quad \tan\theta = \dfrac{K}{\left(\dfrac{4}{3}\right)\dfrac{K^2t_1^2}{r}} = \dfrac{3}{4}$

$\Rightarrow \quad \tan\theta = \dfrac{3}{4}$

$\Rightarrow \quad \theta = 37^\circ$

Hence, the correct answer is (B).

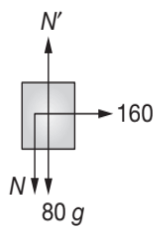
79. Free body diagram of block



$\Rightarrow \quad a_{\text{block}} = \dfrac{160}{160} = 1 \text{ ms}^{-2}$ {leftwards}

Free body diagram (platform + mass) system

Assuming no friction between platform and surface, we get acceleration of pulley to be equal to acceleration of platform. So



$a_{\text{pulley}} = \dfrac{160}{80} = 2 \text{ ms}^{-2}$ {rightwards}

Relative acceleration of pulley and block is

$(2+1) = 3 \text{ ms}^{-2}$

So, time taken by block to reach pulley is

$$l = \frac{1}{2} a_{\text{rel}} t^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2 \times 6}{3}} = 2 \text{ s}$$

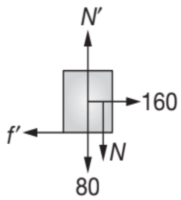
Hence, the correct answer is (B).

80. Length of string pulled out in 1 s is four times the distance moved by block relative to pulley. So

$$l = 4 \times 1.5 = 6 \text{ m}$$

Hence, the correct answer is (B).

81. Assuming friction between platform and surface,



$$f' = 160$$

$$\Rightarrow \quad \mu N' \geq 160$$

$$\Rightarrow \quad \mu \geq \frac{160}{2400} = \frac{2}{30} = 0.066$$

Hence, the correct answer is (D).

83. $mg = 5v$

$$\Rightarrow \quad 50 = 5v$$

$$\Rightarrow \quad v = 10 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

84. At any, instant, $a = g - kv$, where k is constant

$$\Rightarrow \quad \frac{dv}{dt} = g - kv$$

$$\Rightarrow \quad \int_0^v \frac{dv}{g - kv} = \int_0^v dt$$

$$\Rightarrow \quad \frac{\left[ln(g - kv) \right]_0^v}{-k} = t$$

$$\Rightarrow \quad \ln(g - kv) - \ln(g) = -kt$$

$$\Rightarrow \quad \ln\left[1 - \frac{kv}{g}\right] = -kt$$

$$\Rightarrow \quad 1 - \frac{kv}{g} = e^{-kt}$$

$$\Rightarrow \quad v = \frac{g}{k}\left(1 - e^{-kt}\right)$$

Hence, the correct answer is (B).

85. $a = \dfrac{m_3 g + (m_1 + m_2) g \sin\theta - F}{m_1 + m_2 + m_3}$

Hence, the correct answer is (A).

86. Elongation in string is maximum when relative velocity of 1 w.r.t. 2 is zero.

Hence, the correct answer is (B).

87. $a_3 = \dfrac{m_3 g + m_2 g \sin\theta - kx_{\max}}{m_2 + m_3}$

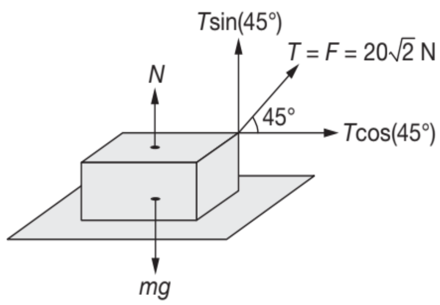
Hence, the correct answer is (A).

Matrix Match/Column Match Type Questions

1. A → (q)
B → (r)
C → (s)
D → (p)

$N + T\sin(45°) = mg$, where $T = 20\sqrt{2}$ N

$$\Rightarrow \quad N = 40 - 20 = 20 \text{ N}$$



So, $f_s = \mu_s N = 16$ N and $f_k = \mu_k N = 12$ N

Since $T\cos(45°) = 20$ N which happens to be greater than f_s. So, the block will move and hence we have

$$T\cos(45°) - f_k = ma$$

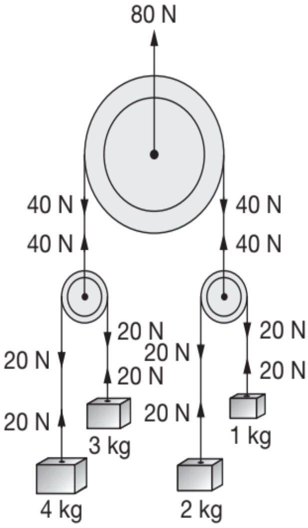
$$\Rightarrow \quad 20 - 12 = 4a$$

$$\Rightarrow \quad a = 2 \text{ ms}^{-2} = 200 \text{ cms}^{-2}$$

2. A → (p, u)
B → (t)
C → (r, s)
D → (q, r)

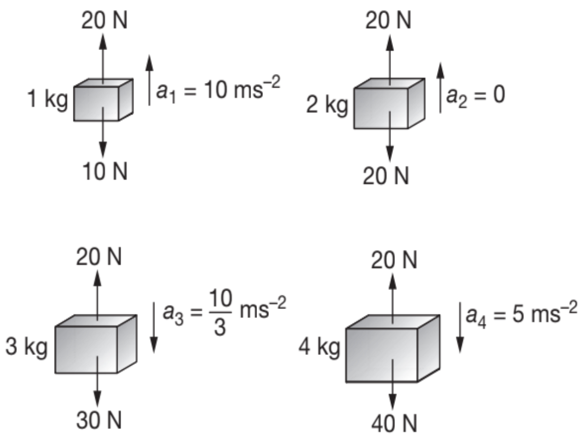
Since the pulleys are smooth and massless, so net force on each pulley (even when accelerating) is zero (because mass of each pulley tends to be zero).

Let acceleration of 1 kg, 2 kg, 3 kg and 4 kg blocks be a_1, a_2, a_3 and a_4 respectively. Then drawing the FBD's of all we get the values of a_1, a_2, a_3 and a_4.



So, $a_1 = 10\text{ ms}^{-2}$ (up), $a_2 = 0$,

$a_3 = \dfrac{10}{3}\text{ ms}^{-2}$ (down), $a_4 = 5\text{ ms}^{-2}$ (down)



3. $A \rightarrow (r)$
$B \rightarrow (s)$
$C \rightarrow (t)$
$D \rightarrow (q)$

If a be the acceleration of the system, then

$$a = \frac{60 - 18 - (1+2+3)g\sin(30^\circ)}{1+2+3}$$

$\Rightarrow \quad a = 2\text{ ms}^{-2}$

Net force on the 3 kg block is $F = m_3 a = 6\text{ N}$

So, (A) $\rightarrow$ (r) and (B) $\rightarrow$ (s)

Let N_{21} be the force on 2 due to 1 and N_{32} be the force on 3 due to 2.

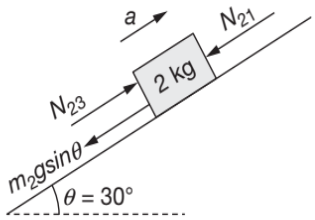
$$N_{23} - N_{21} - m_2 g\sin\theta = m_2 a \quad \text{...(1)}$$

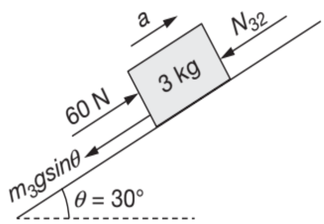
$$60 - N_{32} - m_3 g\sin\theta = m_3 a \quad \text{...(2)}$$

Since $a = 2\text{ ms}^{-2}$, so from (2), we get

$$60 - N_{32} - 30\left(\frac{1}{2}\right) = 3(2)$$

$\Rightarrow \quad N_{32} = 60 - 15 - 6 = 39\text{ N} = N_{23}$





Substituting the value of N_{32} in equation (1), we get

$$39 - N_{21} - (2)(10)\left(\frac{1}{2}\right) = (2)(2)$$

$\Rightarrow \quad N_{21} = 25\text{ N}$

4. $A \rightarrow (t)$
$B \rightarrow (r)$
$C \rightarrow (p)$
$D \rightarrow (s)$
$E \rightarrow (q)$

Pseudo force comes into play, when the frame of reference (from where the phenomenon is being observed) is accelerating. This pseudo force is then directed opposite to the acceleration of the observation frame. So for the observation frames A, B, C, D and E, respective values of accelerations are

$\vec{a}_A = 8\hat{i}$, $\vec{a}_B = \vec{0}$

$\vec{a}_C = -4\hat{j}$, $\vec{a}_D = -3\hat{i}$ and

$\vec{a}_E = 5\hat{j}$

Hence, the pseudo force on A as observed by B is zero.

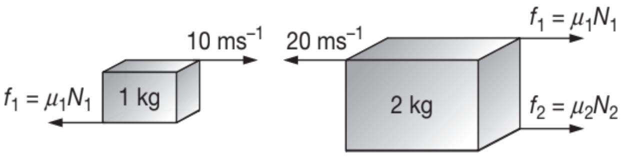
The pseudo force on B as observed by C is along $+y$ direction.

The pseudo force on C as observed by D is along $+x$ direction.

The pseudo force on D as observed by E is along $-y$ direction.

The pseudo force on E as observed by A is along $-x$ direction.

5. A $\rightarrow$ (q)
B $\rightarrow$ (s)
C $\rightarrow$ (p, t)
D $\rightarrow$ (r)



$$f_1 = \mu_1 N_1 = (0.4)(1)(10) = 4 \text{ N}$$

$$f_2 = \mu_2 N_2 = (0.6)(1+2)(10) = 18 \text{ N}$$

So, net force acting on 1 is 4 N, hence acceleration of 1 is $a_1 = \frac{f_1}{1} = 4 \text{ ms}^{-2}$, towards left.

Similarly, net force acting on 2 is $f_1 + f_2 = 22$ N.

So, acceleration of 2 is

$$a_2 = \frac{f_1 + f_2}{2} = 11 \text{ ms}^{-2}, \text{ towards right}$$

Finally, relative acceleration of one w.r.t. other is 15 ms^{-2}.

6. A $\rightarrow$ (r, s)
B $\rightarrow$ (p, s)
C $\rightarrow$ (p, s)
D $\rightarrow$ (p, r, s)

For an object moving along $+x$ axis with constant velocity, there may be no net force acting on the object or there may not be any force acting along $+x$ axis. So, (A) $\rightarrow$ (r, s).

For both the accelerated motions expressed in (B) and (C), we have either the net force on the object to act along $+x$ axis or also no net force along $+x$ axis such that components of force along $+x$ axis happen to be zero. Hence (B) $\rightarrow$ (p, s) and (C) $\rightarrow$ (p, s).

Finally, for the motion of object along $+x$ axis (accelerated or non-uniform) all (p, r, s) satisfy the condition. So (D) $\rightarrow$ (p, r, s).

7. A $\rightarrow$ (q, r)
B $\rightarrow$ (p, s)
C $\rightarrow$ (p, s)
D $\rightarrow$ (q, r)

As per the question, we are given that

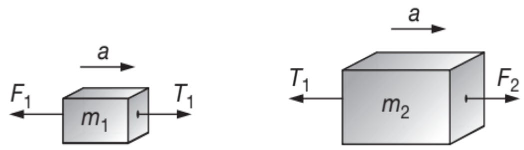
$$a_1 = \frac{F_1}{m_1} \text{ and } a_2 = \frac{F_2}{m_2}$$

For CASE-1 and 4:

If a be the combined acceleration of the system, then

$$T_1 - F_1 = m_1 a \text{ and } F_2 - T_1 = m_2 a$$

$$\Rightarrow \quad a = \frac{F_2 - F_1}{m_1 + m_2}$$



Substituting in either of the equations, we get

$$T_1 = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_1}{m_1} + \frac{F_2}{m_2}\right) = \frac{m_1 m_2}{m_1 + m_2}(a_1 + a_2)$$

So, (A) $\rightarrow$ (q, r)

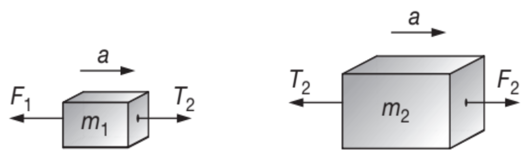
Also,

$$N_2 = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_1}{m_1} + \frac{F_2}{m_2}\right) = \frac{m_1 m_2}{m_1 + m_2}(a_1 + a_2)$$

So, (D) $\rightarrow$ (q, r)

For CASE-2 and 3:

If a be the combined acceleration of the system, then



$$F_1 + T_2 = m_1 a \text{ and } F_2 - T_2 = m_2 a$$

$$\Rightarrow \quad a = \frac{F_1 + F_2}{m_1 + m_2}$$

Substituting in either of the equations, we get

$$T_2 = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_2}{m_2} - \frac{F_1}{m_1}\right) = \frac{m_1 m_2}{m_1 + m_2}(a_2 - a_1)$$

So, (C) $\rightarrow$ (p, s)

Also,

$$N_1 = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_2}{m_2} - \frac{F_1}{m_1}\right) = \frac{m_1 m_2}{m_1 + m_2}(a_2 - a_1)$$

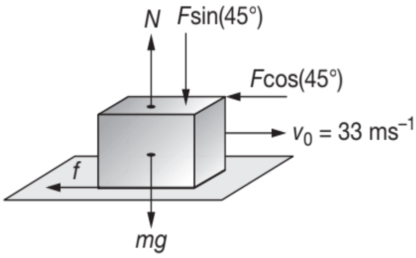
So, (B) $\rightarrow$ (p, s)

8. A $\rightarrow$ (r)
B $\rightarrow$ (t)
C $\rightarrow$ (q)
D $\rightarrow$ (p)

$$N = mg + F\sin(45^\circ)$$

$$\Rightarrow \quad N = 50 + (20\sqrt{2})\frac{1}{\sqrt{2}} = 70 \text{ N}$$

$$\Rightarrow \quad f_{max} = \mu N = (0.5)(70) = 35 \text{ N}$$



So, retardation offered to the block is given by

$$a = \frac{F\cos(45^\circ) + f}{m} = \frac{35 + 20}{5} = 11 \text{ ms}^{-2}$$

Since $v = u + at$, so the block attains zero velocity, say at time t_0, we have

$$0 = 33 - 11t_0$$

$$\Rightarrow \quad t_0 = 3 \text{ s}$$

Till $t < t_0 (= 3 \text{ s})$, we have the following two facts

(a) the force of friction acting on the block is $f_{max} = \mu N_1 = 35$ N

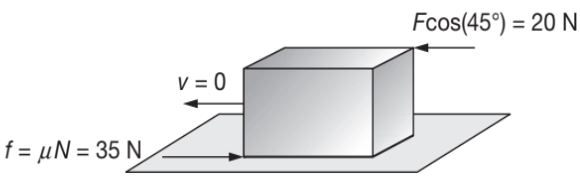
(b) and the net retarding force acting on the block is

$$(F_{net})_{retarding} = F\cos(45^\circ) + f_{max} = 55 \text{ N}$$

So, we have (A) → (r) and (C) → (q)

Now, for $t = 4.5$ s, i.e., $t > t_0$, since the block has zero velocity at $t_0 = 3$ s. At this very instant the friction will be opposite to the applied force and happens to be more than the applied force, so the block will remain at rest after $t_0 = 3$ s and in the **Static Region**, we have the force of static friction to self adjust to a value equal to the applied force, so for $t > t_0$, i.e., $t = 4.5$ s, we have

$$f_{static} = F_{applied} = F\cos(45^\circ) = 20 \text{ N}$$



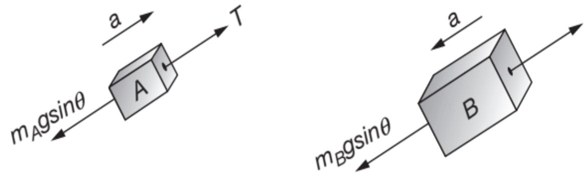
Hence (B) → (t)

Also, at $t = 4.5$ s, the net force on the block is zero as it happens to be in static mode. Hence (D) → (p)

9. A → (r)
B → (t)
C → (p)
D → (q)

For A, $T - m_A g\sin\theta = m_A a$ ...(1)

For B, $m_B g\sin\theta - T = m_B a$ ...(2)



Add (1) and (2), we get

$$(m_B - m_A)g\sin\theta = (m_A + m_B)a$$

$$\Rightarrow \quad a = \left(\frac{m_B - m_A}{m_A + m_B}\right)g\sin\theta$$

$$\Rightarrow \quad a = \left(\frac{4-1}{4+1}\right)(10)\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad a = 3 \text{ ms}^{-2}$$

So, (A) → (r)

Substitute the value of a in (1), we get

$$T - (10)\left(\frac{1}{2}\right) = (1)(3)$$

$$\Rightarrow \quad T = 8 \text{ N}$$

So, Thrust $= 2T = 16$ N, i.e., (B) → (t)

Force exerted by A on B is $N_A = m_A g\cos\theta = 5\sqrt{3}$ N and force exerted by B on incline is $N_B = (m_A + m_B)g\cos\theta = 25\sqrt{3}$ N

Hence (C) → (p) and (D) → (q)

10. A → (s)
B → (p)
C → (t)
D → (q)

For 10 kg block, $10g - T = 10a$ ...(1)

For A, $f = \mu N$ and $N = 4a$ ...(2)

$$\Rightarrow \quad f = (0.5)(4)a$$

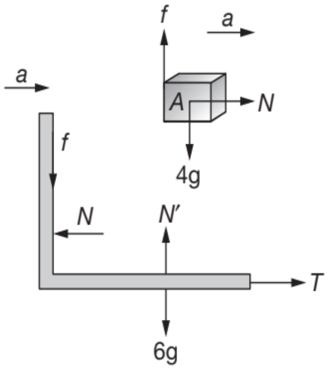
$$\Rightarrow \quad f = 2a \quad \text{...(3)}$$

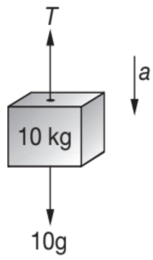
For L shaped block B,

$$T - N = 6a \qquad \ldots(4)$$

From (2) and (4), we get

$$T = 10a \qquad \ldots(5)$$





Substituting in (1), we get

$$100 - 10a = 10a$$

$\Rightarrow \quad a = 5 \text{ ms}^{-2}$

$\Rightarrow \quad f = 10 \text{ N}$

$\Rightarrow \quad N = 20 \text{ N}$, i.e., normal reaction between A and B.
Also, from diagram, we see the normal reaction on the block B by the table is N' given by

$$N' = 6g + f$$

$\Rightarrow \quad N' = 60 + 10 = 70 \text{ N}$

and finally from (5), we get

$$T = 50 \text{ N}$$

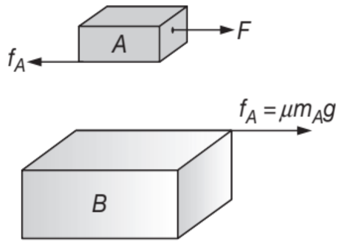
Please note that, the block A will have both the horizontal and vertical accelerations however the asked questions have nothing to do with the vertical acceleration of A, which happens to be

$$a_y = \frac{m_A g - f}{m_A} = \frac{40 - 10}{4} = 7.5 \text{ ms}^{-2}$$

11. A → (t, u)
B → (p, u)
C → (q, u)
D → (s, u)

Since $f_A = \mu m_A g = (0.5)(10)(10) = 50 \text{ N}$

So, friction on B due to A is 50 N, towards right
Hence (A) → (t, u)



Now, when $F = 35 \text{ N} < f_A$, so no slipping of A on B and since B lies on a frictionless surface, hence both A and B move together with a common acceleration a (say). Then

$$a = \frac{F}{m_A + m_B} = \frac{35}{35} = 1 \text{ ms}^{-2}, \text{ towards right}$$

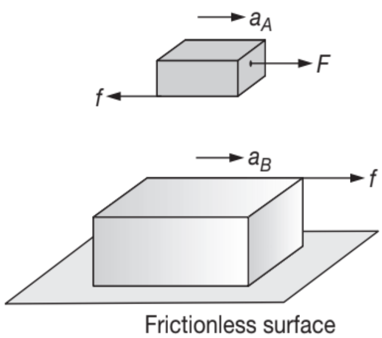
So, we get (B) → (p, u)
Now, when $F = 250 \text{ N} > 50 \text{ N}$, then for block A

$$F - \mu m_A g = m_A a_A$$

$\Rightarrow \quad 250 - 50 = (10) a_A$

$\Rightarrow \quad a_A = 20 \text{ ms}^{-2}$, towards right

So, (C) → (q, u)



Also, for B, we get

$$f = m_B a_B$$

$\Rightarrow \quad 50 = 25 a_B$

$\Rightarrow \quad a_B = 2 \text{ ms}^{-2}$, towards right

Hence, finally (D) → (s, u)

12. A → (p, q, r)
B → (p, q, r)
C → (p, q)
D → (s)
The directions of $\vec{F}$ and $\vec{p}$ may be different, so $\vec{p}$ may change its direction.
If direction of $\vec{F}$ and $\vec{p}$ are the same, then direction of $\vec{p}$ would not change.

If $\vec{F}$ is non-zero in magnitude then magnitude of $\vec{p}$ will change.

13. A → (s)
B → (r)
C → (p)
D → (q)

Since $N = m(g + a_y) = 20(10+10) = 400\text{ N}$

and $F_{\text{pseudo}} = m\sqrt{(3t)^2 + (4t)^2} = 100t$

$\Rightarrow \quad f_\ell = \mu N = 0.5 \times 400 = 200\text{ N}$

Slipping starts when, $100t = 200$

$\Rightarrow \quad t = 2\text{ s}$

So, $\vec{a}$ at $t = 2$ s is $\vec{a} = (6\hat{i} + 10\hat{j} + 8\hat{k})\text{ ms}^{-2}$

$\Rightarrow \quad |\vec{a}| = 10\sqrt{2}\text{ ms}^{-2}$

Total force F applied by the surface on the block at $t = 1$ s is $\sqrt{N^2 + f_l^2}$ (also called as Contact Force (C.F.)

$\Rightarrow$ Contact Force $= \sqrt{(400)^2 + (100)^2}$

$\Rightarrow$ C.F. $= 100\sqrt{17}$ N

15. A → (r)
B → (p)
C → (q)
D → (s)

$a_1 \cos(37°) = a_2$

$\Rightarrow \quad 4a_1 = 5a_2 \quad \ldots(1)$

Since, $100 - T = 10a_2 \quad \ldots(2)$

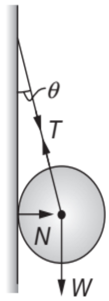
and $T\cos(37°) = 5a_1$

$\Rightarrow \quad T = \frac{25}{4}a_1 \quad \ldots(3)$

Force on pulley by the string is

$$F = \sqrt{T^2 + T^2 + 2T^2\cos(53°)}$$

16. A → (p)
B → (r)
C → (q)
D → (s)



Since the system is in equilibrium, so

$$\vec{T} + \vec{W} + \vec{N} = 0$$

$\Rightarrow \quad \vec{T} = -\vec{W} - \vec{N}$

Also, $T\cos\theta = W$ and $T\sin\theta = N$

$\Rightarrow \quad T^2 = N^2 + W^2$ and $N = W\tan\theta$

17. A → (q, s)
B → (p, s)
C → (r)
D → (r)

$(f_1)_{\max} = (f_1)_{\lim} = (0.8)(50) = 40\text{ N}$

$(f_2)_{\max} = (f_2)_{\lim} = (0.2)(150) = 30\text{ N}$

$F_1 = 35\text{ N}$, $F_2 = 0$: Block A will not slip over block B, because $F_{\text{app}} < (f_1)_{\lim}$

$F_1 = 50\text{ N}$, $F_2 = 0$: Block B will slip over horizontal surface, because $F_{\text{app}} > (f_1)_{\lim}$

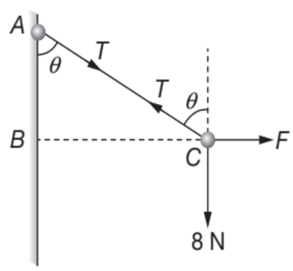
$F_1 = 0$, $F_2 = 30\text{ N}$: Block B will be at rest, because $F_{\text{app}} = (f_2)_{\lim}$

$F_1 = 10\text{ N}$, $F_2 = 50\text{ N}$: Block A have no tendency of slipping over block B.

Integer/Numerical Answer Type Questions

1. $AC = 0.5$ m, $BC = 0.3$ m

$\Rightarrow \quad AB = 0.4$ m



Let $\angle BAC = \theta$. Then

$$\cos\theta = \frac{AB}{AC} = \frac{0.4}{0.5} = \frac{4}{5} \text{ and } \sin\theta = \frac{BC}{AC} = \frac{0.3}{0.5} = \frac{3}{5}$$

We observe that the object is in equilibrium under three concurrent forces. So, applying Lami's theorem, we get

$$\frac{F}{\sin(180^\circ - \theta)} = \frac{8}{\sin(90^\circ + \theta)} = \frac{T}{\sin 90^\circ}$$

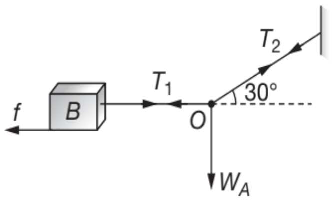
$$\Rightarrow \frac{F}{\sin\theta} = \frac{8}{\cos\theta} = T$$

$$\Rightarrow T = \frac{8}{\cos\theta} = \frac{8}{4/5} = 10 \text{ N and}$$

$$F = \frac{8\sin\theta}{\cos\theta} = \frac{(8)\left(\frac{3}{5}\right)}{\left(\frac{4}{5}\right)} = 6 \text{ N}$$

2. Equilibrium of B gives,

$$T_1 = f_1 = \mu W_B = 0.25 \times 692 = 173 \text{ N} \qquad \ldots(1)$$



Equation of knot (Point O) gives,

$$T_1 \cos(30^\circ) = T_1 \text{ and} \qquad \ldots(2)$$

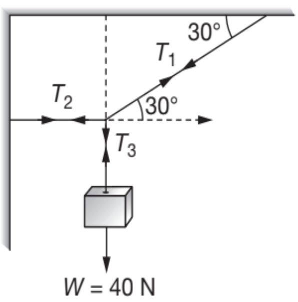
$$T_2 \sin(30^\circ) = W_A \qquad \ldots(3)$$

From equations (2) and (3), we get

$$W_A = T_1 \tan(30^\circ) = \frac{173}{\sqrt{3}} = \frac{173}{1.73} = 100 \text{ N}$$

$$\Rightarrow W_A = 100 \text{ N} = 10 \text{ kgwt}$$

3. Resolving the tension T_1 along horizontal and vertical directions. As the body is in equilibrium,



$$T_1 \sin(30^\circ) = T_3 = W = 40 \text{ N} \qquad \ldots(1)$$

$$\Rightarrow T_3 = 40 \text{ N and } T_1 = 80 \text{ N}$$

Also, $T_1 \cos(30^\circ) = T_2$ ...(2)

$$\Rightarrow T_2 = 80\cos(30^\circ)$$

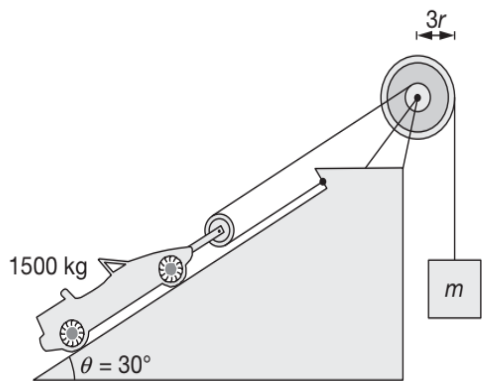
$$\Rightarrow T_2 = 80\frac{\sqrt{3}}{2} = 40\sqrt{3} \text{ N}$$

$$\Rightarrow T_2 = 40(1.7) = 68 \text{ N}$$

So, $T_1 = 80$ N, $T_2 = 68$ N and $T_3 = 40$ N

4. $\sum \tau = 0 = mg(3r) - Tr$

$$2T - Mg\sin(30^\circ) = 0$$



$$\Rightarrow T = \frac{Mg\sin(30^\circ)}{2}$$

$$\Rightarrow T = \frac{(1500)(10)}{2}\left(\frac{1}{2}\right) = 3750 \text{ N}$$

$$m = \frac{T}{3g} = \frac{3750}{30} = 125 \text{ kg}$$

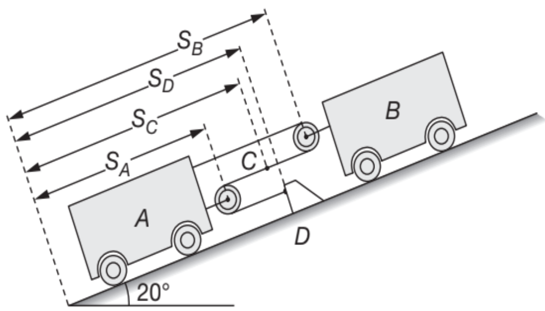
5. The cable length is

$$L = 2(S_B - S_A) + S_D - S_A + \text{constants}$$

Differentiating,

$$0 = 2v_B - 3v_A$$

$$0 = 2a_B - 3a_A$$



So, $v_A = \frac{2}{3}v_B = \frac{2}{3}(3) = 2 \text{ ftsec}^{-1}$

$$a_A = \frac{2}{3}a_B = \frac{2}{3}(6) = 4 \text{ ftsec}^{-2}$$

$$v_{B/A} = v_B - v_A = 3 - 2 = 1 \text{ ftsec}^{-1}$$

$$a_{B/A} = a_B - a_A = 6 - 4 = 2 \text{ ftsec}^{-2}$$

The length of cable between A and C is

$$L' = (S_B - S_A) + (S_B - S_C) = 2S_B - S_A - S_C + \text{constants}$$

$\Rightarrow \quad 0 = 2v_B - v_A - v_C$

$$v_C = 2v_B - v_A = 2(3) - 2 = 4 \text{ ftsec}^{-1}$$

(All answers are quantities directed up incline).

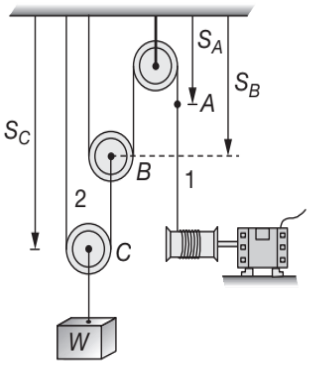
6. Let A be a point on cable 1

$$L_1 = S_A + 2S_B$$

$$0 = v_A + 2v_B \quad \text{...(1)}$$

$$L_2 = S_C + (S_C - S_B)$$

$$0 = 2v_C - v_B \quad \text{...(2)}$$

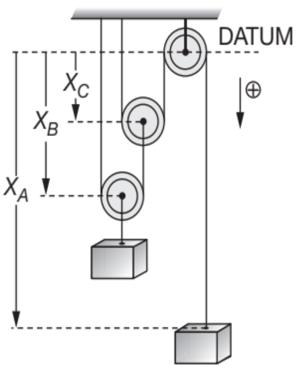


Combine (1) and (2) to obtain

$$v_C = -\frac{1}{4}v_A = -\frac{1}{4}(320) = -80 \text{ mms}^{-1}$$

So, W rises by $h = 80(5) = 400$ mm

7. Let the datum be passing through the fixed pulley as shown



The length of the two ropes in terms of the position coordinates x_A, x_B and x_C are

$$x_A + 2x_C = l_1 \quad \text{...(1)}$$

and $x_B + (x_B - x_C) = l_2 \quad \text{...(2)}$

Eliminating s_C from equations (1) and (2),

$$4x_B + x_A = 2l_2 + l_1 \quad \text{...(3)}$$

Taking the time derivative of equation (3), we get

$$4v_B + v_A = 0$$

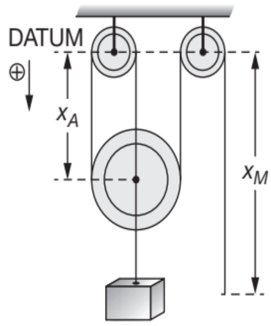
Since $v_A = 4 \text{ ms}^{-1}$, so we get

$$4v_B + 4 = 0$$

$\Rightarrow \quad v_B = -1 \text{ ms}^{-1} = 1 \text{ ms}^{-1}$, upwards

$\Rightarrow \quad v_B = 100 \text{ cms}^{-1}$, upwards

8. Let the datum be passing through both the fixed pulleys as shown.



The length of the rope in terms of the position coordinates x_A and x_M is

$$3x_A + x_M = l \quad \text{...(1)}$$

Taking the time derivative of equation (1), we get

$$3v_A + v_M = 0$$

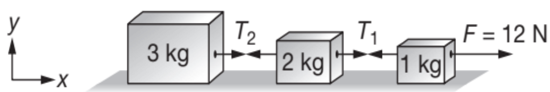
Taking downward direction as positive, so we have

$$v_M = 10 \text{ ms}^{-1}$$

$\Rightarrow \quad 3v_A + 9 = 0$

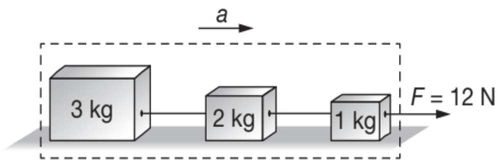
$\Rightarrow \quad v_A = -3 \text{ ms}^{-1} = 3 \text{ ms}^{-1}$, upwards.

9. (a) Let a be the acceleration of each block and T_1 and T_2 be the tensions, in the two strings as shown in figure.



Taking the three blocks and the two strings as the system.

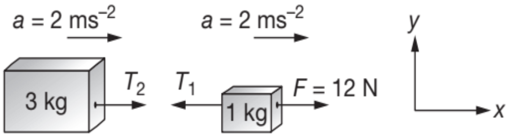
Using $\Sigma F_x = ma_x$



$\Rightarrow \quad 12 = (3+2+1)a$

$\Rightarrow \quad a = \dfrac{12}{6} = 2\text{ ms}^{-2}$

(b) Free body diagram (showing the forces in x-direction only) of 3 kg block and 1 kg block are shown in figure.



Using $\Sigma F_x = ma_x$

For 1 kg block, $F - T_1 = (1)(a)$

$\Rightarrow \quad 12 - T_1 = (1)(2) = 2$

$\Rightarrow \quad T_1 = 12 - 2 = 10\text{ N}$

For 3 kg block,

$T_2 = (3)(a)$

$\Rightarrow \quad T_2 = (3)(2) = 6\text{ N}$

10. Let a be the acceleration with which the masses move and T_1 and T_2 be the tensions in left and right strings. Friction on mass A is $\mu mg = 8\text{ N}$. Then equations of motion of masses A, B and C are

For mass *A*

$T_1 - 8 = 4a \qquad \ldots(1)$

For mass *B*

$T_2 = 8a \qquad \ldots(2)$

For mass *C*

$200 - T_1 - T_2 = 20a \qquad \ldots(3)$

Adding the above three equations, we get

$32a = 192$

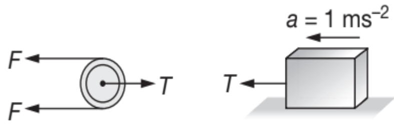
$\Rightarrow \quad a = 6\text{ ms}^{-2}$

From equations (1) and (2), we have

$T_2 = 48\text{ N}$ and $T_1 = 32\text{ N}$

11. When force F is applied at the end of the string, the tension in the lower part of the string is also F as shown. If T is the tension in string connecting the pulley and the block, then,

$T = 2F$



Also, $T = ma = (200)(1) = 200\text{ N}$

$\Rightarrow \quad 2F = 200\text{ N}$

$\Rightarrow \quad F = 100\text{ N}$

12. Free body diagram of crate A w.r.t. ground is shown in figure.

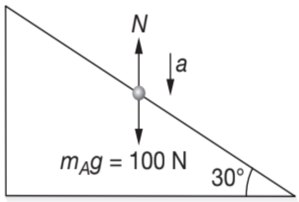
Equation of motion is given by

$100 - N = 10a_A \qquad \ldots(1)$

Further, we have $a_A \operatorname{cosec} 30° =$ acceleration of crate

$\Rightarrow \quad 2a_A = 2$

$\Rightarrow \quad a_A = 1\text{ ms}^{-2}$



Substituting in equation (1) we get

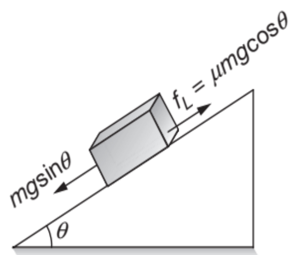
$N = 90\text{ N}$

13. Let θ be the required angle, then maximum retardation is

$$a = \frac{f_L - mg\sin\theta}{m}$$

$$\Rightarrow \quad a = \frac{\mu mg\cos\theta - mg\sin\theta}{m}$$

$$\Rightarrow \quad a = \mu g\cos\theta - g\sin\theta = g(\mu\cos\theta - \sin\theta)$$



Now, using $v^2 = u^2 - 2as$, we get

$$0 = (10)^2 - 2g(\mu\cos\theta - \sin\theta)(5)$$

$$\Rightarrow \quad \mu g\cos\theta - g\sin\theta = 10$$

$$\Rightarrow \left(\frac{4}{3}\right)(10)\cos\theta-(10)\sin\theta=10$$

$$\Rightarrow 4\cos\theta-3\sin\theta=3$$

$$\Rightarrow 4\cos\theta=3(1+\sin\theta)$$

$$\Rightarrow 16\cos^2\theta=9\left(1+\sin^2\theta+2\sin\theta\right)$$

$$\Rightarrow 16\left(1-\sin^2\theta\right)=9\left(1+\sin^2\theta+2\sin\theta\right)$$

$$\Rightarrow 25\sin^2\theta+18\sin\theta-7=0$$

$$\Rightarrow \sin\theta=\frac{-18\pm\sqrt{324+700}}{50}$$

$$\Rightarrow \sin\theta=\frac{\sqrt{1024}-18}{50}=\frac{32-18}{50}=\frac{14}{50}$$

{∵ Rejecting negative sign}

$$\Rightarrow \theta\approx 16^\circ$$

14. Since the man is standing stationary w.r.t. the belt, so acceleration of the man equals the acceleration of the belt i.e. $a=1\text{ ms}^{-2}$

Net force on the man is $F_{net}=Ma=65\times1=65\text{ N}$

Limiting friction is

$$f_L=\mu mg$$

Let the man remains stationary with respect to the belt for maximum acceleration a_0, then

$$ma_0=f_L=\mu mg$$

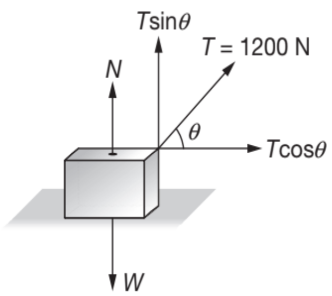
$$\Rightarrow a_0=\mu g=(0.2)(10)=2\text{ ms}^{-2}$$

$$\Rightarrow a_0=200\text{ cms}^{-2}$$

15. Equations of motion in this case are,

$$T\cos\theta=\mu N \quad \ldots(1)$$

and $N=W-T\sin\theta$ …(2)



(a) From equations (1) and (2), we get

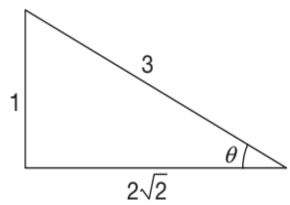
$$W=\frac{T(\cos\theta+\mu\sin\theta)}{\mu} \quad \ldots(3)$$

(b) For W to be maximum

$$\frac{d}{d\theta}(\cos\theta+\mu\sin\theta)=0$$

$$\Rightarrow \theta=\tan^{-1}(\mu)=\tan^{-1}\left(\frac{1}{2\sqrt{2}}\right)$$

Since $\sin(10^\circ)\cos(10^\circ)=\frac{1}{6}$



$$\Rightarrow 2\sin(10^\circ)\cos(10^\circ)=\frac{1}{3}$$

$$\Rightarrow \sin(20^\circ)=\frac{1}{3}$$

$$\Rightarrow \tan(20^\circ)=\frac{1}{2\sqrt{2}}$$

So, $\theta=20^\circ$

From equation (3), we get

$$W=\frac{1200\left(\cos(20^\circ)+\frac{1}{2\sqrt{2}}\sin(20^\circ)\right)}{\frac{1}{2\sqrt{2}}}$$

$$\Rightarrow W=\frac{1200\left(\frac{2\sqrt{2}}{3}+\frac{1}{2\sqrt{2}}\frac{1}{3}\right)}{\frac{1}{2\sqrt{2}}}$$

$$\Rightarrow W=400\left(2\sqrt{2}\right)\left(2\sqrt{2}+\frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow W=400(8+1)$$

$$\Rightarrow W=3600\text{ N}$$

16. (a) When the truck accelerates eastwards, force of friction on mass is eastwards, because the block has a tendency to move westwards w.r.t. the truck. So,

$$f_{required}=\text{mass}\times\text{acceleration}=30\times1.8=54\text{ N}$$

Since it is less than $\mu_s mg$, hence

$$f=54\text{ N (eastwards)}$$

(b) When the truck accelerates westwards then similar to the argument given earlier, force of friction is westwards. So,

$$f_{\text{required}} = \text{Mass} \times \text{Acceleration} = 30 \times 3.8 = 114 \text{ N}$$

Since it is greater than $\mu_s mg$, hence

$$f = f_k = \mu_k mg \approx 59 \text{ N (westwards)}$$

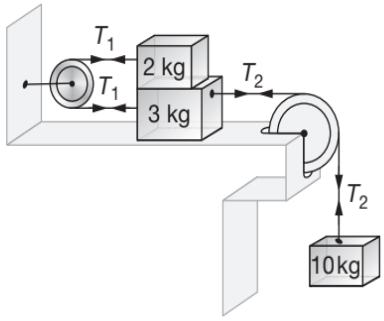
17. $100 = \dfrac{mv^2}{r} = \dfrac{(4)(v^2)}{(1)}$

$\Rightarrow \quad v = 5 \text{ ms}^{-1}$

Now, $5 = (\mu_k g)t \qquad \{\because v = at\}$

$\Rightarrow \quad t = \dfrac{5}{(0.1)(10)} = 5 \text{ s}$

18.



For 10 kg, $10g - T_2 = 10a \qquad \ldots(1)$

For 3 kg, $T_2 - (T_1 + 0.6g) = 3a \qquad \ldots(2)$

For 2 kg, $T_1 - 0.6g = 2a \qquad \ldots(3)$

Solving these equations, we get

$$a = 6 \text{ ms}^{-2}$$

19. Since, $v_C^2 = v_A^2 + 2a_T s$

Tangential acceleration in interval AC is

$$a_T = \frac{v_C^2 - v_A^2}{2s} = 1.447 \text{ ms}^{-2}$$

Centripetal i.e. Normal acceleration a_N at C is

$$a_N = \frac{v^2}{r_C} = 2.41 \text{ ms}^{-2}$$

Tangential force F_T at C is

$$F_T = ma_T = 1500 \times 1.447 = 2170 \text{ N}$$

Normal force F_N at C is

$$F_N = ma_N = 1500 \times 2.41 = 3620 \text{ N}$$

Total force at C is

$$F = \sqrt{F_N^2 + F_T^2}$$

$\Rightarrow \quad F = \sqrt{(2170)^2 + (3620)^2}$

$\Rightarrow \quad F = 4220 \text{ N}$

20. $R = \dfrac{v^2}{\mu g} = \dfrac{5 \times 5}{0.5 \times 10} = 5 \text{ m}$

21. $m_A = m_B = m_C = m$

Since, $F = (m_A + m_B + m_C) g \sin\theta$ and $N = m_C g \sin\theta$

$\Rightarrow \quad N = \dfrac{m_C F}{m_A + m_B + m_C} = \left(\dfrac{m}{3m}\right) 15 = 5 \text{ N}$

22. There is no tendency of relative slipping between blocks.

23. For equilibrium,

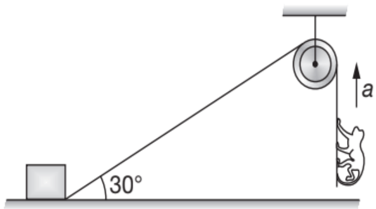
$10 = 8 + T \qquad \ldots(1)$

$T + f_2 = 20 \qquad \ldots(2)$

$\Rightarrow \quad f_2 = 18 \text{ N}$

24. Let T be the tension in the string. The upward force exerted on the clamp is

$$T \sin(30°) = \frac{T}{2}$$



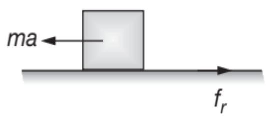
Now $\dfrac{T}{2} = 40 \text{ N}$

$\Rightarrow \quad T = 80 \text{ N}$

$\Rightarrow \quad a = \dfrac{T - mg}{m} = \dfrac{80 - 50}{5} = 6 \text{ ms}^{-2}$

25. $v = 2t^2$

$\Rightarrow \quad a = 4t$



At $t = 1$ s, slipping occurs, so we have

$\Rightarrow \quad ma = \mu_s mg$

$\Rightarrow \quad 4t = \mu_s g$

$\Rightarrow \quad \mu_s = 0.4$

So in the interval from $t = 1$ to $t = 3$ s, we observe that the slipping occurs between $t = 1$ and $t = 2$ s.

$$\Rightarrow \quad 4t - \mu_k g = \frac{dv}{dt}$$

Integrating, we get

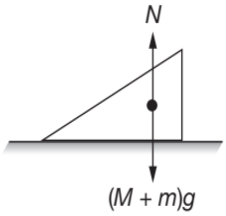
$$v = \left(2t^2 - \mu_k g t\right)\Big|_1^2$$

$$\Rightarrow \quad \mu_k = 0.3$$

$$\Rightarrow \quad \frac{3\mu_s}{\mu_k} = 4$$

26. $a = \dfrac{160-120}{40} = 1\ \text{ms}^{-2}$

27. Since wedge is stationary and block is slipping down with a constant velocity, so normal force on the wedge due to the table is



$$N = (m+M)g = 8\ \text{N}$$

28. Let tension the thread connecting m_2 to the pulley P_1 be T and acceleration of m_2 be a. Then tension in the thread connecting lowest pulley P_3 to m_1 is $4T$ and acceleration of m_1 is $\dfrac{a}{4}$.

For m_1: $40 - 4T = 4\left(\dfrac{a}{4}\right)$

For m_2: $T = 1a$

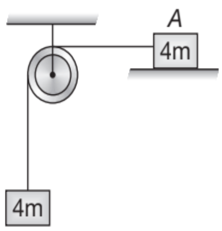
$$\Rightarrow \quad 40 - 4a = a$$

$$\Rightarrow \quad 5a = 40$$

$$\Rightarrow \quad a = 8\ \text{ms}^{-2}$$

29. Pseudo force is zero w.r.t. an observer in an Inertial or ground frame.

30. The system can be redrawn as



$$\Rightarrow \quad a = \frac{4mg}{8m} = \frac{g}{2} = 5\ \text{ms}^{-2}$$

Since, $v = u + at$

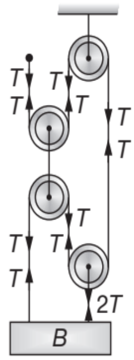
$$\Rightarrow \quad v = 0 + (2)(5) = 10\ \text{ms}^{-1}$$

$\Rightarrow \quad v = 1$ decametre per second

31. According to Principle of Virtual Work, we have

$$Tx = 3Tx_B$$

$$\Rightarrow \quad x = 3x_B$$



$$\Rightarrow \quad \frac{dx}{dt} = 3\frac{dx_B}{dt}$$

$$\Rightarrow \quad v = 3v_B$$

$$\Rightarrow \quad v_B = \frac{v}{3}$$

32. By constraint relation knowledge, we have

$$V_A \sin(60^\circ) = V_{P_2}\left[1 + \cos(60^\circ)\right] + V_B$$

$$\Rightarrow \quad V_{P_2} = 4\ \text{ms}^{-1}$$

33. $a = \dfrac{2\times 10}{2 + \frac{1}{3} + 1} = 6\ \text{ms}^{-2}$

Also, $2\times g - T = 2\times 6$

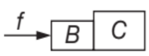
$$\Rightarrow \quad T = 8\ \text{N}$$

34. Since the block is not moving, so frictional force acting on the block is opposite to $\vec{F}_{\text{pseudo}} = -(ma)\hat{i}$

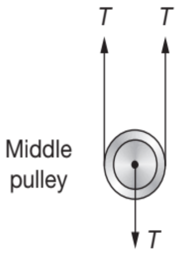
$$\Rightarrow \quad \vec{f} = -\vec{F}_{\text{pseudo}} = (2\hat{i})\ \text{newton}$$

35. $a = \dfrac{10}{5} = 2\ \text{ms}^{-2}$

$$f = (M_B + M_C)\times a = 8\ \text{N}$$



36. Free body diagram for the middle pulley is shown.



So, we observe that

$$2T - T = 0 \qquad \{\because \text{ pulley is light, so } m_{pulley} \cong 0\}$$

$$\Rightarrow \quad T = 0$$

37. For B, $mg - T = ma$ ...(1)

For A, $T + mg\sin\theta = ma$ ...(2)

Adding, (1) and (2), we get

$$a = \frac{g}{2}(1 + \sin\theta)$$

$$\Rightarrow \quad a = \frac{3g}{4}$$

Put $a = \frac{3g}{4}$ in (1), we get

$$mg - T = \frac{3mg}{4}$$

$$\Rightarrow \quad T = \frac{mg}{4} = 5 \text{ N}$$

38. $a_C = \dfrac{a_A + a_B}{2}$

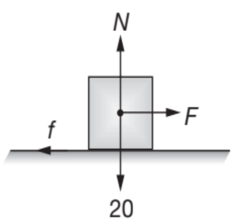
$$\Rightarrow \quad a_C = \frac{7-3}{2} = 2 \text{ ms}^{-2}$$

39. $f_l = (0.2)(20) = 4$ N

So, till $2t = 4 \Rightarrow t = 2$ s, block will be at rest.

For $2 \le t \le 4$ s, the block will move with an acceleration

$$a = \frac{F - f_l}{m}$$



$$\Rightarrow \quad \frac{dv}{dt} = (2t - 4)$$

$$\Rightarrow \quad \int_0^v dv = 2\int_2^4 t\,dt - 4\int_2^4 dt$$

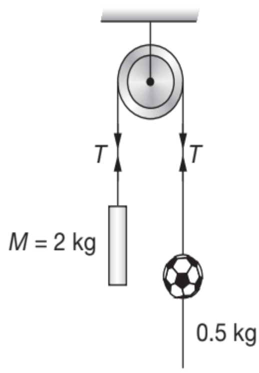
$$\Rightarrow \quad v = 2\left(\frac{t^2}{2}\Bigg|_2^4\right) - 4\left(t\Big|_2^4\right)$$

$$\Rightarrow \quad v = (16-4) - 4(4-2)$$

$$\Rightarrow \quad v = 12 - 8$$

$$\Rightarrow \quad v = 4 \text{ ms}^{-1}$$

40.



For rod of mass 2 kg

$$20 - T = 2a_1 \qquad ...(1)$$

For ball

$$5 - f = 5a_2 \qquad ...(2)$$

Since thread is mass less

$$T = f \qquad ...(3)$$

$$l = 0.3 = \frac{1}{2}a_{rel}t^2$$

41. If C doesn't move, then

$$a_A = 4a_B \qquad ...(1)$$

For *A*

$$P - T - \mu mg = m(4a_B) \qquad ...(2)$$

For *B*

$$4T - mg = ma_B \qquad ...(3)$$

$$\Rightarrow \quad \frac{P}{mg} = 5$$

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1. Since $P = \frac{F}{A}$

$$\Rightarrow \quad P = \frac{N \times (2mv)}{A\Delta t}$$

$$\Rightarrow \quad P = \frac{10^{22} \times 2 \times 10^{-26} \times 10^{4}}{1 \times 1} = 2\ \text{Nm}^{-2}$$

Hence, the correct answer is (B).

2. $p = mV = m\sqrt{V_0^2 - 2gh}$

Direction of momentum changes at top most point

Hence, the correct answer is (D).

3. Retardation of the particle

$$a = -\left(g + \gamma v^2\right)$$

For $H_{\max}$, $v = 0$

$$\Rightarrow \quad \int_{V_0}^{0} \frac{-dv}{g + \gamma v^2} = \int_0^t dt$$

$$\Rightarrow \quad \frac{1}{\sqrt{\gamma g}} \tan^{-1}\left(\frac{\sqrt{\gamma} V_0}{\sqrt{g}}\right) = t$$

Hence, the correct answer is (C).

4. $a = \left(\frac{F - f}{M + m}\right)$

$$\Rightarrow \quad a = \frac{F - (0.2)4 \times 10}{4} = \left(\frac{F - 8}{4}\right)$$

Since $a \le \mu g$

$$\Rightarrow \quad \frac{F - 8}{4} \le (0.2)10$$

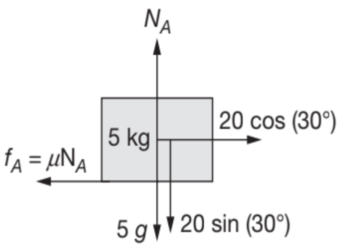
$$\Rightarrow \quad F - 8 \le 8$$

$$\Rightarrow \quad F \le 16$$

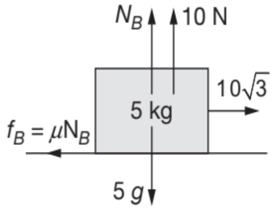
Hence, the correct answer is (C).

5. $N_A = 5g + 20\sin(30^\circ) = 60\ \text{N}$

Since, $f_A = 0.2 \times 60 = 12\ \text{N}$



$$\Rightarrow \quad a_A = \frac{\left(\frac{20\sqrt{3}}{2} - 12\right)}{5} = \frac{5.3}{5} = 1.06\ \text{ms}^{-2}$$



For B,

$$N_B + 10 = 50$$

$$\Rightarrow \quad N_B = 40\ \text{N}$$

Since $f_B = \mu N_B$

$$\Rightarrow \quad f_B = 8\ \text{N}$$

$$\Rightarrow \quad \frac{20\sqrt{3}}{2} - 8 = 5a_B$$

$$\Rightarrow \quad a_B = \frac{17.3 - 8}{5} = \frac{9.3}{5} = 1.86\ \text{ms}^{-2}$$

$$\Rightarrow \quad a_B - a_A = 0.8\ \text{ms}^{-2}$$

Hence, the correct answer is (D).

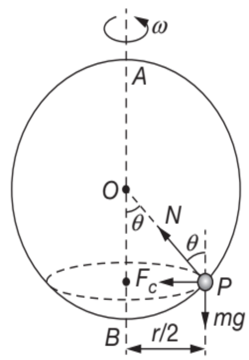
6. $l_1 = nl_2$

Since $k \propto \frac{1}{l}$

$$\Rightarrow \quad \frac{k_1}{k_2} = \frac{l_2}{l_1} = \frac{1}{n}$$

Hence, the correct answer is (D).

7.



Let θ be the angle with the vertical, then

$$\sin\theta = \frac{r/2}{r} = \frac{1}{2}$$

$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{1}{2}\right) = 30°$

$N\cos\theta = mg \quad \ldots(1)$

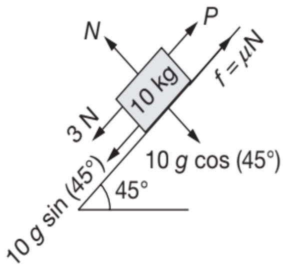
$N\sin\theta = F_c = \frac{m\omega^2 r}{2} \quad \ldots(2)$

$\Rightarrow \quad \tan\theta = \frac{\omega^2 r}{2g}$

$\Rightarrow \quad \omega^2 = \frac{2g\tan\theta}{r} = \frac{2g}{r\sqrt{3}}$

Hence, the correct answer is (C).

8.



Friction force should be acting upward along the plane. For the state of impending motion, we have

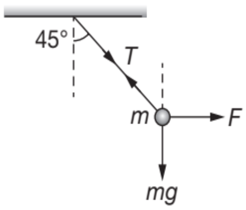
$$3 + (10)(10)\frac{1}{\sqrt{2}} = P + \left(\frac{6}{10}\right)(10)(10)\frac{1}{\sqrt{2}}$$

$\Rightarrow \quad P = 73.71 - 42.42$

$\Rightarrow \quad P = 31.28 \approx 32 \text{ N}$

Hence, the correct answer is (B).

9.



$T\cos(45°) = mg$

$T\sin(45°) = F$

$\Rightarrow \quad F = mg$

$\Rightarrow \quad F = 10 \times 10 = 100 \text{ N}$

Hence, the correct answer is (A).

10. $F = kt$

Since $F = \frac{dp}{dt}$

$\Rightarrow \quad \frac{dp}{dt} = kt$

$\Rightarrow \quad \int_p^{3p} dp = k\int_0^T t\,dt$

$\Rightarrow \quad 2p = \frac{kT^2}{2}$

$\Rightarrow \quad T = 2\sqrt{\frac{p}{k}}$

Hence, the correct answer is (B).

11. Since, $\vec{r}_{\text{final}} = \vec{r}_{\text{initial}} + \vec{u}t + \frac{1}{2}\vec{a}t^2$

$\Rightarrow \quad \vec{r}_{\text{final}} = (2\hat{i} + 4\hat{j}) + (5\hat{i} + 4\hat{j}) \times 2 + \frac{1}{2}[4\hat{i} + 4\hat{j}] \times 2^2$

$\Rightarrow \quad \vec{r}_{\text{final}} = (20\hat{i} + 20\hat{j}) \text{ m}$

$\Rightarrow \quad |\vec{r}_{\text{final}}| = 20\sqrt{2} \text{ m}$

Hence, the correct answer is (A).

12. Since the block remains at rest on the rough incline upto a maximum force of 2 N down the incline, so we get

$2 + mg\sin(30°) = \mu N$

$\Rightarrow \quad 2 + \frac{mg}{2} = \mu mg\frac{\sqrt{3}}{2} \quad \ldots(1)$

When only, 10 N is applied up the incline, then we have

$\frac{mg}{2} + \mu mg\frac{\sqrt{3}}{2} = 10 \quad \ldots(2)$

From (1) and (2), we get

$2 + mg = 10$

$\Rightarrow \quad mg = 8$

From (1), we get

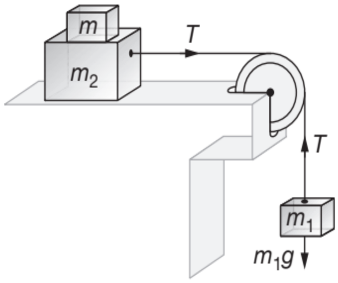
$6 = \mu \times 8 \times \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \mu = \frac{2 \times 6}{8\sqrt{3}}$

$\Rightarrow \quad \mu = \frac{\sqrt{3}}{2}$

Hence, the correct answer is (B).

13. To stop the moving blocks, the frictional force between m_2 and surface is increased by placing some extra mass m on top of mass m_2.



Condition for stopping moving blocks is $f \geq T$

$\Rightarrow \quad \mu N \geq T$

$\Rightarrow \quad \mu(m+m_2)g \geq m_1 g$

For minimum value of m, we have

$$\mu(m+m_2)g = m_1 g$$

$$\Rightarrow \quad m = \frac{m_1}{\mu} - m_2 = \frac{5}{0.15} - 10$$

$$\Rightarrow \quad m = 33.33 - 10 = 23.33 \text{ kg}$$

From given options the best suitable answer will be 27.3 kg.

Hence, the correct answer is (B).

14. Time taken to slide along smooth surface is given by

$$l = \frac{1}{2}[g\sin(45°)]t_1^2$$

$$\Rightarrow \quad t_1 = \sqrt{\frac{2\sqrt{2}l}{g}}$$

Time taken to slide along rough surface is given by

$$l = \frac{1}{2}g[\sin(45°) - \mu\cos(45°)]t_2^2$$

$$\Rightarrow \quad t_2 = \sqrt{\frac{2\sqrt{2}l}{g(1-\mu)}}$$

As per question, $t_2 = nt_1$

$$\Rightarrow \quad t_2^2 = n^2 t_1^2$$

$$\Rightarrow \quad \frac{2\sqrt{2}l}{g(1-\mu)} = n^2 \times \frac{2\sqrt{2}l}{g}$$

$$\Rightarrow \quad 1-\mu = \frac{1}{n^2}$$

$$\Rightarrow \quad \mu = 1 - \frac{1}{n^2}$$

Hence, the correct answer is (B).

15. Downward acceleration on the incline is

$$a = g(\sin 30° - \mu\cos 30°)$$

Since body moves up with an acceleration a upwards due to external force F, so we have

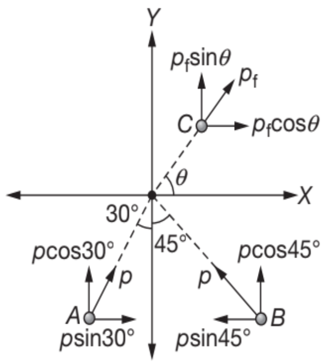
$$F - mg(\sin 30° + \mu\cos 30°) = ma$$

$$\Rightarrow \quad F = mg(\sin 30° + \mu\cos 30°) + mg(\sin 30° - \mu\cos 30°)$$

$$\Rightarrow \quad F = 2mg\sin 30° = 2\times 2\times 10\times\frac{1}{2} = 20 \text{ N}$$

Hence, the correct answer is (D).

16. During completely inelastic collision both particles A and B stick together.



Here, p_i = initial momentum of each particle

p_f = final momentum of the system

Using conservation of linear momentum,

Along X-axis, $p_f\cos\theta = p\sin 30° - p\sin 45°$ …(1)

Along Y-axis, $p_f\sin\theta = p\cos 30° + p\cos 45°$ …(2)

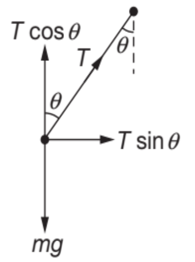
Dividing equation (2) by equation (1), we get

$$\frac{\sin\theta}{\cos\theta} = \frac{\cos 30° + \cos 45°}{\sin 30° - \sin 45°} = \frac{\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}}{\frac{1}{2} - \frac{1}{\sqrt{2}}}$$

$$\Rightarrow \quad \tan\theta = \frac{\sqrt{3}+\sqrt{2}}{1-\sqrt{2}}$$

Hence, the correct answer is (A).

17. FBD of the pendulum is shown in the figure. So, we have



$$T\sin\theta = \frac{mv^2}{r} \text{ and}$$

$$T\cos\theta = mg$$

$$\Rightarrow \quad \tan\theta = \frac{v^2}{rg}$$

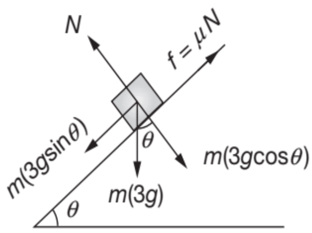
Since, $\theta = 45°$, $r = 0.4$ m

$$\Rightarrow \quad v^2 = rg$$

$$\Rightarrow \quad v = \sqrt{rg} = \sqrt{0.4\times 10} = 2 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

18. Since the rocket is moving vertically upwards with acceleration $2g$, therefore the apparent acceleration experienced by the point object is $g + 2g = 3g$ vertically downwards.



From figure, we have $N = 3mg\cos\theta$

Point object does not move on inclined surface, so

$$\mu N = 3mg\sin\theta$$

$$\Rightarrow \quad \mu(3mg\cos\theta) = 3mg\sin\theta$$

$$\Rightarrow \quad \mu = \tan\theta$$

Hence, the correct answer is (B).

19. Here, $F = \frac{R}{t^2}v(t)$

$$\Rightarrow \quad m\frac{dv}{dt} = \frac{R}{t^2}v(t)$$

$$\Rightarrow \quad \frac{dv}{v(t)} = \frac{R\,dt}{m\,t^2}$$

Integrating both sides,

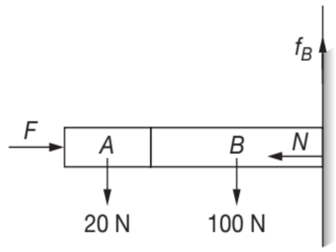
$$\int \frac{dv}{v(t)} = \frac{R}{m}\int \frac{dt}{t^2}$$

$$\Rightarrow \quad \ln v = -\left(\frac{R}{m}\right)\left(\frac{1}{t}\right) + C$$

Graph between $\ln v$ and $\left(\frac{1}{t}\right)$ is a straight line.

Hence, the correct answer is (A).

20. Various forces acting on the system are shown in the figure.



For vertical equilibrium of the system,

$$f_B = 100 \text{ N} + 20 \text{ N} = 120 \text{ N}$$

Hence, the correct answer is (A).

21. System (block + bullet) comes to rest after moving

2 m, $s = 2$ m, $v_2 = 0$, $v_1 = ?$

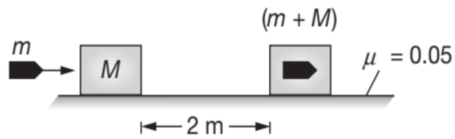
$$a = -\mu g = -0.05\times 10 = -0.5 \text{ ms}^{-2}$$

Using $v^2 = u^2 + 2as$, we get

$$0^2 = v_1^2 - 2(0.5)\times 2$$

$$\Rightarrow \quad v_1 = \sqrt{2} \text{ ms}^{-1}$$

By Conservation of Linear Momentum, we have



$$\begin{pmatrix}\text{Momentum of} \\ \text{the system} \\ \text{after collision}\end{pmatrix} = \begin{pmatrix}\text{Momentum of} \\ \text{the system} \\ \text{before collision}\end{pmatrix}$$

$$mv + 0 = (M + m)V$$

$$\Rightarrow \quad \sqrt{2}(10 + 50\times 10^{-3}) = (50\times 10^{-3})\times v + 0$$

$$\Rightarrow \quad v = \frac{\sqrt{2}\times 10}{50\times 10^{-3}} = 200\sqrt{2} \text{ ms}^{-1}$$

$$\Rightarrow \quad \frac{v}{10} = 20\sqrt{2} \text{ ms}^{-1}$$

For a freely falling body, to acquire

$v' = \frac{v}{10} = 20\sqrt{2} \text{ ms}^{-1}$, we have

$$v'^2 = 2gH$$

$$\Rightarrow \quad H = \frac{v'^2}{2g}$$

$$\Rightarrow \quad H = \frac{800}{20} = 40 \text{ m} = 0.04 \text{ km}$$

Hence, the correct answer is (C).

22. Since collisions are elastic and masses are equal, velocities of colliding particles get exchanged.
Change in momentum Δp in each collision with the support is $\Delta p = 2mv$.
Time interval between consecutive collisions with one support

$$\Delta t = \frac{(L - 2nr) \times 2}{v}$$

So, average force experienced by each support is

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{2mv}{\frac{(L-2nr)2}{v}} = \frac{mv^2}{L - 2nr}$$

Hence, the correct answer is (B).

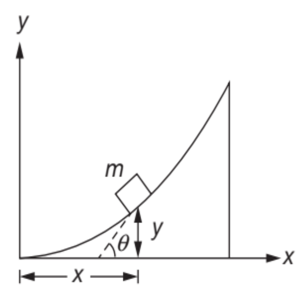
23. Block is under limiting friction, so

$$\mu = \tan\theta \quad \ldots(1)$$

Equation of the surface is

$$y = \frac{x^3}{6}$$

So, slope is $\frac{dy}{dx} = \frac{x^2}{2}$. ...(2)



From equations (1) and (2), we get

$$\mu = \frac{x^2}{2}$$

$$\Rightarrow \quad 0.5 = \frac{x^2}{2}$$

$$\Rightarrow \quad x^2 = 1$$

$$\Rightarrow \quad x = 1$$

$$\Rightarrow \quad y = \frac{1}{6} \text{ m}$$

Hence, the correct answer is (B).

24. Centripetal acceleration, $a_c = \omega^2 r$, where $\omega = \frac{2\pi}{T}$

Since, $T_1 = T_2$

$$\Rightarrow \quad \omega_1 = \omega_2$$

$$\Rightarrow \quad \frac{a_{c_1}}{a_{c_2}} = \frac{r_1}{r_2}$$

Hence, the correct answer is (B).

25. $F(t) = F_0 e^{-bt}$ {Given}

$$\Rightarrow \quad ma = F_0 e^{-bt}$$

$$\Rightarrow \quad a = \frac{F_0}{m} e^{-bt}$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{F_0}{m} e^{-bt}$$

$$\Rightarrow \quad dv = \frac{F_0}{m} e^{-bt} dt$$

Integrating both sides, we get

$$\int_0^v dv = \int_0^t \frac{F_0}{m} e^{-bt} dt$$

$$\Rightarrow \quad v = \frac{F_0}{m}\left(\frac{e^{-bt}}{-b}\right)\Bigg|_0^t = \frac{F_0}{mb}\left(1 - e^{-bt}\right)$$

Hence, the correct answer is (B).

26. The acceleration of the body down the smooth inclined plane is $a = g\sin\theta$ along the inclined plane, where θ is the angle of inclination.
The vertical component of acceleration a is

$$a_{(\text{along vertical})} = (g \sin\theta)\sin\theta = g\sin^2\theta$$

For block A

$$a_{A(\text{along vertical})} = g\sin^2(60^\circ)$$

For block B

$$a_{B(\text{along vertical})} = g\sin^2(30^\circ)$$

The relative vertical acceleration of A with respect to B is

$$a_{AB(\text{along vertical})} = a_{A(\text{along vertical})} - a_{B(\text{along vertical})}$$

$\Rightarrow \quad a_{AB(\text{along vertical})} = g\sin^2(60°) - g\sin^2(30°)$

$\Rightarrow \quad a_{AB(\text{along vertical})} = g\left(\left(\dfrac{\sqrt{3}}{2}\right)^2 - \left(\dfrac{1}{2}\right)^2\right)$

$\Rightarrow \quad a_{AB(\text{along vertical})} = \dfrac{g}{2} = 4.9 \text{ ms}^{-2}$

Hence, the correct answer is (A).

27. Since position-time $(x\text{-}t)$ graph is a straight line, so motion is uniform. Because of impulse, direction of velocity changes as can be seen from slopes of the graph.
From graph,

Initial velocity, $u = \dfrac{(2-0)}{(2-0)} = 1 \text{ ms}^{-1}$

Final velocity, $v = \dfrac{(0-2)}{(4-2)} = -1 \text{ ms}^{-1}$

So, initial momentum, $p_i = mu = 0.4 \times 1 = 0.4$ Ns and

Final momentum, $p_f = mv = 0.4 \times (-1) = -0.4$ Ns

$\Rightarrow$ Impulse = Change in momentum $= p_f - p_i$

$\Rightarrow$ Impulse $= -0.4 - (0.4)$ Ns $= -0.8$ Ns

$\Rightarrow$ $|\text{Impulse}| = 0.8$ Ns

Hence, the correct answer is (C).

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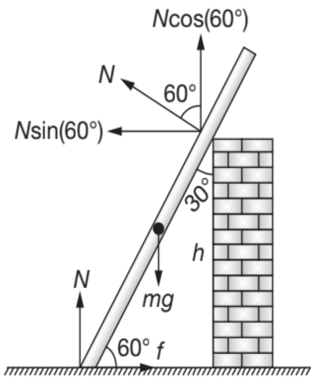
Single Correct Choice Type Problems

1.



For Translational Equilibrium of the stick, we have

$$N + N\cos(60°) = mg \qquad \ldots(1)$$

$$f = N\sin(60°) \qquad \ldots(2)$$

$\Rightarrow \quad N = \dfrac{2mg}{3}$

For Rotational Equilibrium of stick about the lowest point, we have

$$mg\left[\frac{l}{2}\cos(60°)\right] = N\left(\frac{h}{\sin 60°}\right)$$

$\Rightarrow \quad \dfrac{mgl}{4} = \dfrac{2mg}{3}\left(\dfrac{2h}{\sqrt{3}}\right)$

$\Rightarrow \quad \dfrac{h}{l} = \dfrac{3\sqrt{3}}{16}$

Also, $f = N\sin(60°)$

$\Rightarrow \quad f = \dfrac{2mg}{3}\dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad f = \dfrac{mg}{\sqrt{3}} = \dfrac{(1.6)(10)}{\sqrt{3}} = \dfrac{16}{\sqrt{3}} \text{ N} = \dfrac{16\sqrt{3}}{3} \text{ N}$

Hence, the correct answer is (D).

2. Block will not slip if

$(m_1 + m_2)g\sin\theta \le \mu m_2 g\cos\theta$

$\Rightarrow \quad 3\sin\theta \le \left(\dfrac{3}{10}\right)(2)\cos\theta$

$\quad \tan\theta \le \dfrac{1}{5}$

$\Rightarrow \quad \theta \le 11.5°$

For (P), $\theta = 5°$ friction is static

$\quad f = (m_1 + m_2)g\sin\theta$

For (Q), $\theta = 10°$ friction is static

$\quad f = (m_1 + m_2)g\sin\theta$

For (R), $\theta = 15°$ friction is kinetic

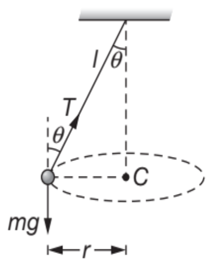
$\quad f = \mu m_2 g\cos\theta$

For (S), $\theta = 20°$ friction is kinetic

$\Rightarrow \quad f = \mu m_2 g\cos\theta$

Hence, the correct answer is (D).

3. $T\cos\theta$ component will cancel mg.



$T\sin\theta$ component will provide necessary centripetal force to the ball towards centre C.

$\Rightarrow \quad T\sin\theta = mr\omega^2 = m(l\sin\theta)\omega^2$

$\Rightarrow \quad T = ml\omega^2$

$\Rightarrow \quad \omega = \sqrt{\dfrac{T}{ml}}$

$\Rightarrow \quad \omega_{max} = \sqrt{\dfrac{T_{max}}{ml}}$

$\Rightarrow \quad \omega_{max} = \sqrt{\dfrac{324}{0.5\times 0.5}}$

$\Rightarrow \quad \omega_{max} = 36 \text{ rads}^{-1}$

Hence, the correct answer is (D).

4. When $P = mg(\sin\theta - \mu\cos\theta)$

$f = \mu mg\cos\theta$ {upwards}

when $P = mg\sin\theta$; $f = 0$

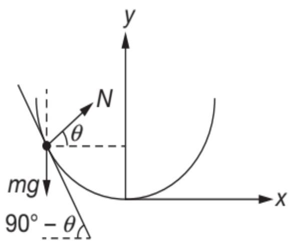
and when $P = mg(\sin\theta + \mu\cos\theta)$

$f = \mu mg\cos\theta$ {downwards}

Hence, friction is first positive, then zero and then negative.

Hence, the correct answer is (A).

5.



$N\sin\theta = mg$

$N\cos\theta = ma$

$\Rightarrow \quad \tan\theta = \dfrac{g}{a}$

$\Rightarrow \quad \cot\theta = \dfrac{a}{g} = \tan(90° - \theta) = \dfrac{dy}{dx}$

$\Rightarrow \quad \cot\theta = \dfrac{dy}{dx}$ ...(1)

Since $y = kx^2$

$\Rightarrow \quad \dfrac{dy}{dx} = 2kx$

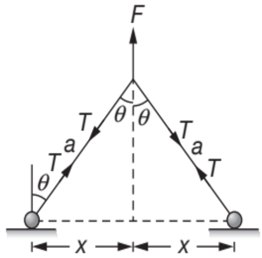
$\Rightarrow \quad \dfrac{a}{g} = 2kx$

$\Rightarrow \quad x = \dfrac{a}{2\text{ kg}}$

Hence, the correct answer is (B).

6. $2T\cos\theta = F$

$\Rightarrow \quad T = \dfrac{F}{2}\sec\theta$



Acceleration of particle a_0 is given by

$a_0 = \dfrac{T\sin\theta}{m}$

$\Rightarrow \quad a_0 = \dfrac{F\tan\theta}{2m}$

$\Rightarrow \quad a_0 = \dfrac{F}{2m}\dfrac{x}{\sqrt{a^2 - x^2}}$

Hence, the correct answer is (B).

7. Initially under equilibrium of mass m

$T = mg$

Now, the string is cut. Therefore, $T = mg$ force is decreased on mass m upwards and downwards on mass $2m$.

$\Rightarrow \quad a_m = \dfrac{mg}{m} = g$ (downwards) and

$a_{2m} = \dfrac{mg}{2m} = \dfrac{g}{2}$ (upwards)

Hence, the correct answer is (A).

8. This is the equilibrium of coplanar forces. Hence,

$\Sigma F_x = 0$

$\Rightarrow \quad F = N$

$\Rightarrow \quad \Sigma F_y = 0$,

$f = mg$

$\Sigma\tau_c = 0$

$\Rightarrow \quad \vec{\tau}_N + \vec{\tau}_f = 0$

Since, $\vec{\tau}_f \neq 0$

$\Rightarrow \quad \vec{\tau}_N \neq 0$

Hence, the correct answer is (D).

9. Angular frequency of the system,

$$\omega = \sqrt{\frac{k}{m+m}} = \sqrt{\frac{k}{2m}}$$

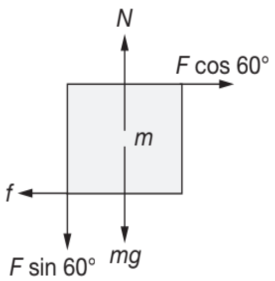
Maximum acceleration of the system will be, $\omega^2 A$ or $\frac{kA}{2m}$.

This acceleration to the lower block is provided by friction.

Hence, $f_{\max} = ma_{\max} = m\omega^2 A = m\left(\frac{kA}{2m}\right) = \frac{kA}{2}$

Hence, the correct answer is (A).

10.



For no motion $F_{\text{applied}} \le f\,(=\mu N)$

$$\Rightarrow \quad F\cos 60° \le \mu\left(\text{mg} + F\sin 60°\right)$$

$$\Rightarrow \quad \frac{F}{2} \le \frac{1}{2\sqrt{3}}\left(\sqrt{3}g + \frac{F\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad \frac{F}{2} \le g$$

$$\Rightarrow \quad F_{\max} = 20\text{ N}$$

Hence, the correct answer is (A).

11. $2T\cos\theta = \sqrt{2}\,Mg$ ...(1)

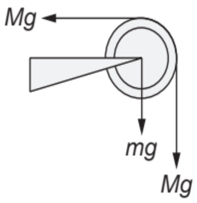
$$\Rightarrow \quad 2Mg\cos\theta = \sqrt{2}\text{ Mg}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \theta = 45°$$

Hence, the correct answer is (C).

12. $T = Mg$



Also the weight of pulley mg is acting vertically downward. So total downward force is $(m+M)g$ and horizontal force is $T = Mg$. The resultant of these two is

$$\sqrt{M^2 + (m+M)^2}\,g$$

Hence, the correct answer is (D).

13. $mg\cos\alpha = N$ ...(1)

$mg\sin\alpha = \mu N$ ...(2)

$$\Rightarrow \quad \cot\alpha = \frac{1}{\mu} = 3$$

Hence, the correct answer is (A).

14. At the highest point all blocks will possess equal speeds (as we assume friction to be absent) so, at the highest point

$$mg + N = \frac{mv'^2}{R}$$

$$\Rightarrow \quad N = \frac{mv'^2}{R} - mg$$

R is MINIMUM in the first case, so

N is MAXIMUM.

Hence, the correct answer is (A).

15. Tangential Force = $F_T = ma = m(\alpha L) = N$

Limiting value of friction $(f_s)_{\max} = \mu N = \mu F_T$

$\Rightarrow \quad (f_s)_{\max} = \mu N = \mu m\alpha L$ ...(1)

Further if ω is the angular velocity at time t, then

$\omega = \alpha t$ ...(2)

Also, the centripetal force is

$F_C = mL\omega^2 = mL\left(\alpha^2 t^2\right)$ ...(3)

For bead to slide $F_C > (f_s)_{\max}$

$$\Rightarrow \quad mL\left(\alpha^2 t^2\right) > \mu m\alpha L$$

$$\Rightarrow \quad t > \sqrt{\frac{\mu}{\alpha}}$$

So, the minimum time after which the bead begins to start is $\sqrt{\frac{\mu}{\alpha}}$.

Hence, the correct answer is (A).

16. Limiting value of static friction is given by,

$$f_s = \mu_s N = (0.5)(5) = 2.5 \text{ N}$$

Weight of the block = 0.98 N

Since the block remains stationary, we have

Force of friction = Weight of block

$\Rightarrow \quad f = 0.98 \text{ N}$

Hence, the correct answer is (B).

17. $\tan\theta = \dfrac{v^2}{rg} = 1$

$\Rightarrow \quad \theta = 45°$

Hence, the correct answer is (C).

19. For equilibrium $f = mg\sin\theta = (2)(9.8)\sin 30$

$\Rightarrow \quad f = 9.8 \text{ N}$

Here, we have not used the formula $f = \mu mg\cos\theta$ as we are provided with coefficient of static friction which may or may not be the limiting value of static friction.

Hence, the correct answer is (A).

20. $a = \dfrac{5\times 10^4}{3\times 10^7} \text{ ms}^{-2}$

$\Rightarrow \quad a = \dfrac{5}{3}\times 10^{-3} \text{ ms}^{-2}$

Since $v^2 - u^2 = 2as$

$$v^2 - 0^2 = 2\left(\frac{5}{3}\times 10^{-3}\right)3$$

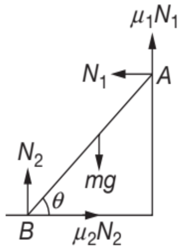
$\Rightarrow \quad v^2 = 10^{-2}$

$\Rightarrow \quad v = 0.1 \text{ m}$

Hence, the correct answer is (C).

Multiple Correct Choice Type Problems

1. μ_2 can never be zero for equilibrium.



When $\mu_1 = 0$, we have

$$N_1 = \mu_2 N_2 \quad \ldots(1)$$

$$N_2 = mg \quad \ldots(2)$$

$$\tau_B = 0$$

$\Rightarrow \quad mg\dfrac{L}{2}\cos\theta = N_1 L\sin\theta$

$\Rightarrow \quad N_1 = \dfrac{mg\cot\theta}{2}$

$\Rightarrow \quad N_1\tan\theta = \dfrac{mg}{2}$

When $\mu_1 \neq 0$ we have

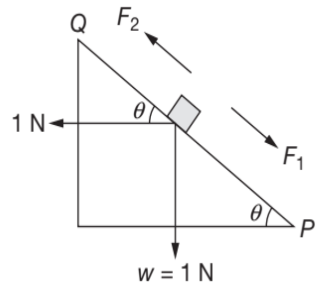
$$\mu_1 N_1 + N_2 = mg \quad \ldots(3)$$

$$\mu_2 N_2 = N_1 \quad \ldots(4)$$

$\Rightarrow \quad N_2 = \dfrac{mg}{1+\mu_1\mu_2}$

Hence, (C) and (D) are correct.

2. $w = mg = 0.1 \times 10 = 1 \text{ N}$



$F_1 =$ component of weight $= 1\sin\theta = \sin\theta$

$F_2 =$ component of applied force $= 1\cos\theta = \cos\theta$

Now, at $\theta = 45° : F_1 = F_2$ and block remains stationary without the help of friction.

For $\theta > 45°$, $F_1 > F_2$, so friction will act towards Q.

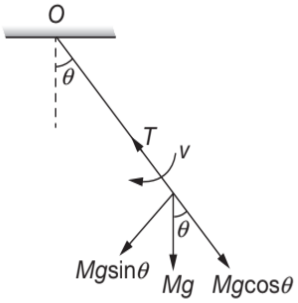
For $\theta < 45°$, $F_2 > F_1$ and friction will act towards P.

Hence, (A) and (C) are correct.

3. Motion of pendulum is the part of a circular motion. In circular motion it is better to resolve the forces in two perpendicular directions. First along radius (towards centre) and second along tangential. Along radius, net force should be equal to mv^2/R and along tangent it should be equal to ma_T, where a_T is the tangential acceleration in the figure.

$$T - Mg\cos\theta = \frac{Mv^2}{L} \text{ and } Mg\sin\theta = Ma_T$$

$\Rightarrow \quad a_T = g\sin\theta$



Hence, (B) and (C) are correct.

4. All accelerated frames are Non-inertial frames. Since earth rotates about its own axis and revolves around the sun, so it is a Non-Inertial frame (STRICTLY SPEAKING), whereas for a good number of cases we assume the earth to be an Inertial frame of reference as the value of acceleration is extremely small.
Hence, (B) and (D) are correct.

Integer/Numerical Answer Type Problems

1. Linear impulse, $J = mv_0$

$$\Rightarrow \quad v_0 = \frac{J}{m} = 2.5 \text{ ms}^{-1}$$

$$\Rightarrow \quad v = v_0 e^{-t/\tau}$$

$$\Rightarrow \quad \frac{dx}{dt} = v_0 e^{-t/\tau}$$

$$\Rightarrow \quad \int_0^x dx = v_0 \int_0^\tau e^{-t/\tau} dt$$

$$\Rightarrow \quad x = v_0 \left[\frac{e^{-t/\tau}}{-1/\tau} \right]_0^\tau$$

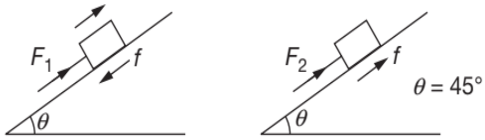
$$\Rightarrow \quad x = 2.5(-4)\left(e^{-1} - e^0\right)$$

$$\Rightarrow \quad x = 2.5(-4)(0.37 - 1)$$

$$\Rightarrow \quad x = 6.30 \text{ m}$$

$$\Rightarrow \quad x = 6 \text{ m}$$

2.



Moving upwards Just remains stationary

$$F_1 = mg\sin\theta + \mu mg\cos\theta$$

$$F_2 = mg\sin\theta - \mu mg\cos\theta$$

Given that $F_1 = 3F_2$

$$\Rightarrow \quad \left(\sin 45° + \mu\cos 45°\right)$$

$$\Rightarrow \quad F_1 = 3\left(\sin 45° - \mu\cos 45°\right)$$

On solving, we get

$$\mu = 0.5$$

$$\Rightarrow \quad N = 10\mu = 5$$

Assertion and Reasoning Type Problems

1. Both Statements are correct. But Statement-II, does not explain correctly, Statement-I.
Correct explanation: There is an increase in normal reaction when the object is pushed and there is a decrease in normal reaction when the object is pulled (but strictly not horizontally).

2. The cloth can be pulled out without dislodging the dishes from the table due to law of inertia, which is Newton's First Law. While, the Statement-II is true, but it is Newton's Third Law.