CHAPTER 5: KINEMATICS II

Test Your Concepts-I (Based on Curvilinear Motion)

1. $\vec{v} = v_x\hat{i} + v_y\hat{j}$ and $\vec{a} = a_x\hat{i} + a_y\hat{j}$

$$\vec{v}\cdot\vec{a} = v_x a_x + v_y a_y$$

Further $v = \sqrt{v_x^2 + v_y^2}$

$$\Rightarrow \quad v^2 = v_x^2 + v_y^2$$

$$\Rightarrow \quad 2v\frac{dv}{dt} = 2v_x\frac{dv_x}{dt} + 2v_y\frac{dv_y}{dt}$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{v_x a_x + v_y a_x}{v} \qquad \ldots(1)$$

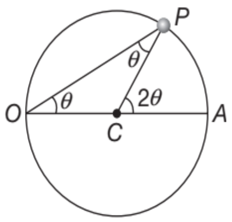
$$\Rightarrow \quad \frac{dv}{dt} = \frac{\vec{a}\cdot\vec{v}}{v} = \frac{v_x a_x + v_y a_y}{v} = a_T$$

Because from our knowledge of vectors we know that component of $\vec{A}$ along $\vec{B}$ is $\frac{\vec{A}\cdot\vec{B}}{B}$. So, $\frac{\vec{a}\cdot\vec{v}}{v}$ is the component of $\vec{a}$ along $\vec{v}$ and this happens to be the tangential acceleration, a_T.

2. Let, O be a point on a circle and P be the position of the particle at any time t, such that

$$\angle POA = \theta$$

Then $\angle PCA = 2\theta$



Here, C is the centre of the circle

Angular velocity of P about O is

$$\omega_0 = \frac{d\theta}{dt}$$

Angular velocity of P about C is

$$\omega_c = \frac{d}{dt}(2\theta) = 2\frac{d\theta}{dt}$$

$$\Rightarrow \quad \omega_c = 2\omega_0$$

3. The angular speed is

$$\omega = \frac{v}{r} = \frac{2}{0.25} = 8 \text{ rads}^{-1}$$

4. The tangential acceleration of the particle is

$$a_T = \frac{dv}{dt} = \frac{4-2}{4} = 0.5 \text{ ms}^{-2}$$

The angular acceleration is

$$\alpha = \frac{a_T}{r} = \frac{0.5}{0.25} = 2 \text{ rads}^{-2}$$

5. $a = \sqrt{a_T^2 + a_N^2} = \sqrt{\left(\frac{dv}{dt}\right)^2 + \left(\frac{v^2}{R}\right)^2}$

$$\Rightarrow \quad a = \sqrt{(8)^2 + \left(\frac{256}{50}\right)^2} = 9.5 \text{ ms}^{-2}$$

6. Since $y = u_y t + \frac{1}{2}a_y t^2$, where $u_y = 0$ and $a_y = g$, $y = h$.

So $h = \frac{1}{2}gt^2$

$$\Rightarrow \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(20)}{10}} = 2 \text{ s}$$

Further since $x = u_x t + \frac{1}{2}a_x t^2$, where $u_x = v$ (say), $a_x = 0$, $x = 10$ m, so

$$10 = vt = v(2)$$

$$\Rightarrow \quad v = 5 \text{ ms}^{-1}$$

Since $a = \frac{v^2}{R}$

$$\Rightarrow \quad a = \frac{(5)^2}{0.5} = \frac{25}{0.5} = 50 \text{ ms}^{-2}$$

$$\Rightarrow \quad a = 50 \text{ ms}^{-2}$$

7. $1.5t + 0.7t = 2\pi R = 14\pi$

$$\Rightarrow \quad t = \frac{14\pi}{2\cdot 2} = \frac{14\left(\frac{22}{7}\right)}{2\cdot 2} = 20 \text{ s}$$

$$a = \frac{v_B^2}{R} = \frac{(1\cdot 5)^2}{7} = \frac{2\cdot 25}{7} = 0\cdot 32 \text{ ms}^{-2}$$

8. When $t = 3$ s, the motorcycle travels at a speed of

$$v = \frac{1}{5}(3^2) = 1.8 \text{ ms}^{-1}$$

The tangential acceleration is

$$a_T = \dot{v} = \frac{2t}{5} = (0.4t) \text{ ms}^{-2}. \text{ When } t = 3 \text{ s},$$

$$a_T = 0.4(3) = 1.2 \text{ ms}^{-2}$$

Since, $a_N = \frac{v^2}{r} = \frac{(1.8)^2}{50} = 0.0648\ \text{ms}^{-2}$

Thus, the magnitude of acceleration is

$$a = \sqrt{a_T^2 + a_N^2} = \sqrt{(1.2)^2 + (0.0648)^2} \cong 1.2\ \text{ms}^{-2}$$

9. Since the speed of the race car is constant, its tangential component of acceleration is zero, i.e., $a_T = 0$. Thus,

$$a = a_N = \frac{v^2}{r}$$

$$\Rightarrow\quad 7.5 = \frac{v^2}{200}$$

$$\Rightarrow\quad v = 38.7\ \text{ms}^{-1}$$

10. Since the automobile is travelling at a constant speed,

$$a_T = 0$$

Thus, $a_N = a = 5\ \text{ms}^{-2}$

Since $a_N = \frac{v^2}{r}$, so we get

$$v = \sqrt{a_N r} = \sqrt{800(5)} \cong 63\ \text{ms}^{-1}$$

11. $a_T = \left(\frac{2000\ \text{km}}{h^2}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)^2 = 0.1543\ \text{ms}^{-2}$

$$v = \left(\frac{60\ \text{km}}{h}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 16.67\ \text{ms}^{-1}$$

$$a_N = \frac{v^2}{r} = \frac{16.67^2}{600} = 0.463\ \text{ms}^{-2}$$

$$\Rightarrow\quad a = \sqrt{a_T^2 + a_N^2} = \sqrt{(0.1543)^2 + (0.463)^2} = 0.488\ \text{ms}^{-2}$$

12. Since the car is travelling with a constant speed, its tangential component of acceleration is zero, i.e., $a_T = 0$. Thus,

$$a = a_N = \frac{v^2}{r}$$

$$\Rightarrow\quad 3 = \frac{25^2}{r}$$

$$\Rightarrow\quad r = 208\ \text{m}$$

13. Here, the car's tangential component of acceleration of $a_T = -3\ \text{ms}^{-2}$. Thus,

$$a = \sqrt{a_T^2 + a_N^2}$$

$$\Rightarrow\quad 5 = \sqrt{(-3)^2 + a_N^2}$$

$$\Rightarrow\quad a_N = 4\ \text{ms}^{-2}$$

Since $a_N = \frac{v^2}{r}$

$$4 = \frac{20^2}{r}$$

$$\Rightarrow\quad r = 100\ \text{m}$$

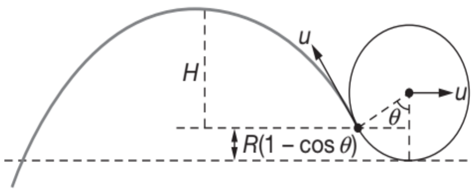
14. Velocity of the particle at an angle θ as it leaves the rim is

$$\vec{v} = (u - u\cos\theta)\hat{i} + (u\sin\theta)\hat{j}$$

$$\Rightarrow\quad \vec{v} = u(1-\cos\theta)\hat{i} + (u\sin\theta)\hat{j}$$

Maximum height from ground attained by a particle

$$h = R(1-\cos\theta) + H = R(1-\cos\theta) + \frac{u^2\sin^2\theta}{2g} \quad \ldots(1)$$



For h to be maximum, we have

$$\frac{dh}{d\theta} = 0$$

$$\Rightarrow\quad \sin\theta\left(\frac{u^2}{g}\cos\theta + R\right) = 0$$

$$\Rightarrow\quad \text{Either } \sin\theta = 0 \text{ or } \cos\theta = -\frac{Rg}{u^2} = -\frac{3}{5}$$

$$\Rightarrow\quad \theta = 0^\circ \text{ or } \theta = \cos^{-1}\left(-\frac{3}{5}\right)$$

Also we observe that $\left[\frac{d^2h}{d\theta^2}\right] < 0$; for $\theta = \cos^{-1}\left(-\frac{3}{5}\right)$

and $\frac{d^2h}{d\theta^2} > 0$; for $\theta = 0^\circ$

So h is maximum, when

$$\cos\theta = -\frac{3}{5}$$

$$\Rightarrow\quad h_{max} = \frac{3}{2}\left(1 + \frac{3}{5}\right) + \frac{25}{20}\frac{16}{25} = 3.2\ \text{m}$$

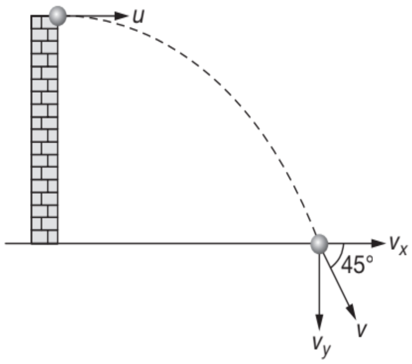
Test Your Concepts-II (Based on Horizontal Projectile)

1. $u_y = 0$ and $a_y = g = 9.8\ \text{ms}^{-2}$

Since $y = u_y t + \frac{1}{2}a_y t^2$

$$\Rightarrow\quad y = (0)(3) + \frac{1}{2}(9.8)(3)^2$$

$$\Rightarrow\quad y = 44.1\ \text{m}$$



Further, $v_y = u_y + a_y t = 0 + (9.8)(3)$

$\Rightarrow \quad v_y = 29.4 \text{ ms}^{-1}$

As the resultant velocity v makes an angle of $45°$ with the horizontal so

$$\tan 45° = \frac{v_y}{v_x}$$

$\Rightarrow \quad 1 = \frac{29.4}{v_x}$

$\Rightarrow \quad v_x = 29.4 \text{ ms}^{-1}$

Therefore, the speed with which the body was projected (horizontally) is 29.4 ms^{-1}.

2. $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times (36 - 16.4)}{9.8}} = 2 \text{ s}$

Since $x = ut$

$\Rightarrow \quad u = \frac{x}{t} = \frac{40}{2} = 20 \text{ ms}^{-1}$

3. $u = \sqrt{2gh}$

$\Rightarrow \quad u = \sqrt{(2)(10)(5)} = 10 \text{ ms}^{-1}$

Let the water strike the ground in time t. Then

$x = ut = 10t \qquad \ldots(1)$

Further since $y = u_y t + \frac{1}{2} a_y t^2$

$\Rightarrow \quad 2.5 = \frac{1}{2}(10)t^2 \qquad \{\because u_y = 0\}$

$\Rightarrow \quad t = 0.7 \text{ s}$

$\Rightarrow \quad x = (10)(0.7) \text{ m}$

$\Rightarrow \quad x = 7 \text{ m}$

4. Let the ball clear the net at time t_1. Then

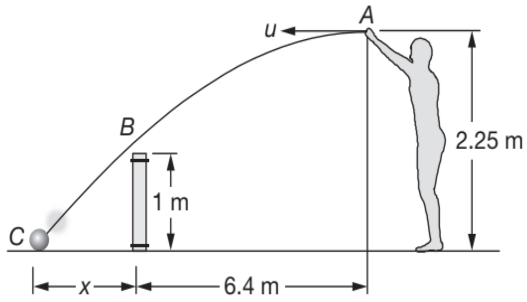
$\Delta y = 2.25 - 1 = 1.25 \text{ m}$

Since $\Delta y = u_y t + \frac{1}{2} a_y t^2$

$\Rightarrow \quad 1.25 = 0 + \frac{1}{2}(10)t_1^2$

$\Rightarrow \quad t_1^2 = 0.25$

$\Rightarrow \quad t_1 = 0.5 \text{ s}$



Let the ball clear the net and then strike the ground in total time t_2 (say). Then

$2.25 = 0 + \frac{1}{2}(10)t_2^2$

$\Rightarrow \quad t_2 = 0.67 \text{ s}$

Further the net is at a horizontal distance of

$7 = u(0.5)$

$\Rightarrow \quad u = 14 \text{ ms}^{-1}$

Now for the ball to fall after the net, let the horizontal distance be x. Then total horizontal distance covered by the ball from the beginning is $(7 + x)$, in a time $t_2 = 0.67 \text{ s}$. So,

$(7 + x) = (14)(0.67)$

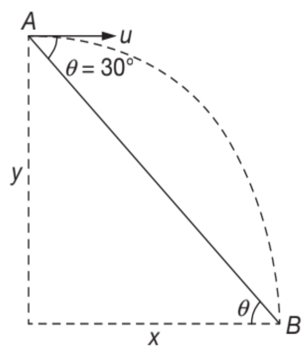
$\Rightarrow \quad 7 + x = 9.4$

$\Rightarrow \quad x = 2.4 \text{ m}$

5. $y = \frac{1}{2} g t^2,$

$x = ut$

$\tan\theta = \frac{y}{x} = \frac{gt}{2u}$



$\Rightarrow \quad u = \dfrac{gt}{2\tan\theta} = \dfrac{(10)(4)}{(2)\left(\dfrac{1}{\sqrt{3}}\right)} = 20\sqrt{3}\ \text{ms}^{-1}$

$\Rightarrow \quad u \cong 34\ \text{ms}^{-1}$

6. Let the ball clear the point C at time t_1. Then

$$y = u_y t + \frac{1}{2}a_y t^2$$

$\Rightarrow \quad (1+3.9) = 0 + \dfrac{1}{2}(9.8)t_1^2 \qquad \{\because u_y = 0\}$

$\Rightarrow \quad 4.9 = \dfrac{1}{2}(9.8)t_1^2$

$\Rightarrow \quad t_1 = 1\ \text{s}$

Since $BC = u_x t$

$\Rightarrow \quad 6 = u(1)$

$\Rightarrow \quad u = 6\ \text{ms}^{-1}$

Let the ball hit the ground at D in time t.
Then

$6 + x = 6t \qquad \ldots(1)$

Further $(1+3.9+14.7) = \dfrac{1}{2}(9.8)t^2$

$\Rightarrow \quad t = 2\ \text{s}$

$\Rightarrow \quad 6 + x = 12$

$\Rightarrow \quad x = 6\ \text{m}$

7. $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2\times 0.8}{10}} = 0.4\ \text{s}$

$u_{\text{MIN}} = \dfrac{(120-30+10)\times 10^{-3}}{0.4} = 0.25\ \text{ms}^{-1}$

and $u_{\text{MAX}} = \dfrac{(120+30-10)\times 10^{-3}}{0.4} = 0.35\ \text{ms}^{-1}$

$\Rightarrow \quad u_{MIN} = 25\ \text{cms}^{-1}$ and $u_{MAX} = 35\ \text{cms}^{-1}$

Test Your Concepts-III (Based on Oblique Projectile)

1. Since, $R = H$

$\Rightarrow \quad \dfrac{u^2\sin(2\theta)}{g} = \dfrac{u^2\sin^2\theta}{2g}$

$\Rightarrow \quad 2\sin\theta\cos\theta = \dfrac{\sin^2\theta}{2}$

$\Rightarrow \quad \dfrac{\sin\theta}{\cos\theta} = 4$

$\Rightarrow \quad \tan\theta = 4$

$\Rightarrow \quad \theta = \tan^{-1}(4)$

$\Rightarrow \quad \theta = 76^\circ$

2. Maximum horizontal range is obtained when $\theta = 45^\circ$, so

$$R_{\text{max}} = \frac{u^2}{g}$$

Maximum height attained i.e., at $\theta = 45^\circ$ is

$$H_{\text{max}} = \frac{u^2\sin^2(45^\circ)}{2g} = \frac{u^2}{4g} = \frac{R_{\text{max}}}{4}$$

$\Rightarrow \quad R_{\text{max}} = 4H_{\text{max}}$

3. There are two launch angles θ and $90^\circ - \theta$ for which the horizontal range R is same. So,

$H_1 = \dfrac{u^2\sin^2\theta}{2g}$ and

$H_2 = \dfrac{u^2\sin^2(90^\circ - \theta)}{2g} = \dfrac{u^2\cos^2\theta}{2g}$

$\Rightarrow \quad H_1 + H_2 = \dfrac{u^2}{2g}(\sin^2\alpha + \cos^2\alpha) = \dfrac{u^2}{2g}$

Clearly the sum of the maximum heights for these two launch angles is independent of them

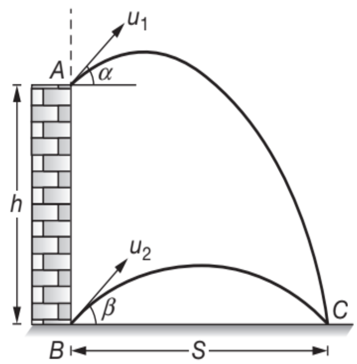
4. Done in theory.

5. Let, height of tower be h.

$-h = (u_1\sin\alpha)t - \dfrac{1}{2}gt^2$ and

$0 = (u_2\sin\beta)t - \dfrac{1}{2}gt^2$

$\Rightarrow \quad (u_1\sin\alpha)t + h = (u_2\sin\beta)t$



Also, $S = (u_1\cos\alpha)t = (u_2\cos\beta)t$

$\Rightarrow \quad t = \dfrac{S}{u_1\cos\alpha} = \dfrac{S}{u_2\cos\beta}$

$$\Rightarrow \quad u_1 \sin\alpha\left(\frac{S}{u_1\cos\alpha}\right)+h=\left(\frac{u_2\sin\beta}{u_2\cos\beta}\right)S$$

$$\Rightarrow \quad h+S\tan\alpha=S\tan\beta$$

$$\Rightarrow \quad h=S(\tan\beta-\tan\alpha)$$

6. (a) Since $v_y^2-u_y^2=2a_y y$

$$\Rightarrow \quad u_y^2=v_y^2-2a_y y$$

where

$$v_y=6.1\text{ ms}^{-1}$$

$$a_y=-9.8\text{ ms}^{-1}$$

$$y=9.1\text{ m}$$

$$\Rightarrow \quad u_y=14.7\text{ ms}^{-1}$$

Since, at maximum height $v_y=0$

$$\Rightarrow \quad 0^2-u_y^2=2(-g)H$$

$$\Rightarrow \quad H=\frac{u_y^2}{2g}=\frac{(14.7)^2}{2(9.8)}\cong 11\text{ m}$$

(b) Range $R=\dfrac{2u_x u_y}{g}=\dfrac{2\times7.6\times14.7}{9.8}\approx 23\text{ m}$

(c) At the instant when the ball hits the ground, we have

$$\text{Speed}=\sqrt{u_y^2+u_x^2}=\sqrt{(14.7)^2+(7.6)^2}\approx 17\text{ ms}^{-1}$$

$$\theta=\tan^{-1}\left(\frac{u_y}{u_x}\right) \qquad \text{\{with horizontal\}}$$

$$\Rightarrow \quad \theta=\tan^{-1}\left(\frac{14.7}{7.6}\right)\approx 63^\circ$$

7. Range $=\dfrac{u^2\sin2\theta}{g}=\dfrac{(20)^2\sin90^\circ}{9.8}=40.82\text{ m}$

Since $T=\dfrac{2u\sin\theta}{g}=\dfrac{(2)(20)\sin(45^\circ)}{9.8}=2.9\text{ s}$

So, speed of player is

$$v=\frac{\Delta x}{T}=\frac{50-40.82}{2.9}=3.16\text{ ms}^{-1}$$

{towards the coach}

8. $t=\dfrac{2v_r}{g}=\dfrac{2\times20}{9.8}=4.1\text{ s}$

If s be the distance travelled by the car in this time then

$$s=ut$$

$$\Rightarrow \quad s=\left(30\times\frac{5}{18}\right)(4.1)=34\text{ m}$$

If R be the range of the projectile, then

$$R=\frac{2}{g}(u_x)(u_y)$$

$$\Rightarrow \quad R=\frac{2}{g}(u)(v_r)=\frac{2}{9.8}\left(30\times\frac{5}{18}\right)(20)=34\text{ m}$$

Now since we observe both R and s to be the same i.e., 34 m, so the projectile will land on the vehicle at the tube location.

9. $u_x=30\cos30^\circ=15\sqrt{3}\text{ ms}^{-1}$

$$u_y=30\sin30^\circ=15\text{ ms}^{-1}$$

(a) Since $y=u_y t+\dfrac{1}{2}a_y t^2$

$$\Rightarrow \quad -50=15t-\frac{1}{2}(10)t^2$$

$$\Rightarrow \quad t=5\text{ s}$$

(b) $x=u_x t=75\sqrt{3}\text{ m}\cong 130\text{ m}$

(c) Since $v^2=u^2+2gh$

$$\Rightarrow \quad v^2=(30)^2+2\times10\times50$$

$$\Rightarrow \quad v=43.6\text{ ms}^{-1}$$

10. **METHOD I**

Let the balls collide at time t (say), then

For ball A, $x_A=(v_0\cos\theta)t$ ...(1)

For ball B, $x_B=(kv_0\cos\phi)t$ ...(2)

Further at the point of collision, we have

$$x_A=x_B$$

$$\Rightarrow \quad \cos\theta=k\cos\phi$$

$$\Rightarrow \quad k=\frac{\cos\theta}{\cos\phi}$$

METHOD II

The balls will collide if component of relative velocity in horizontal direction is zero, or

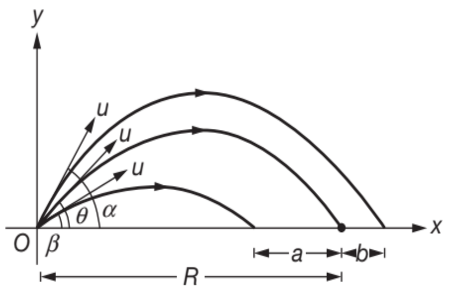
$$kv_0\cos\phi=v_0\cos\theta$$

$$\Rightarrow \quad k=\frac{\cos\theta}{\cos\phi}$$

11. Let θ be the proper angle of projection. Then the actual range is

$$R=\frac{u^2\sin(2\theta)}{g}$$

CHAPTER 5



For the first projectile,

$$\frac{u^2\sin(2\theta)}{g}-a=\frac{u^2\sin(2\alpha)}{g} \quad \ldots(1)$$

For the second projectile

$$\frac{u^2\sin(2\theta)}{g}+b=\frac{u^2\sin(2\beta)}{g} \quad \ldots(2)$$

Using $(1)b+(2)a$, we get

$$(a+b)\left(\frac{u^2\sin(2\theta)}{g}\right)=\frac{bu^2\sin(2\alpha)}{g}+\frac{au^2\sin(2\beta)}{g}$$

$$\Rightarrow \quad \sin(2\theta)=\frac{b\sin(2\alpha)+a\sin(2\beta)}{a+b}$$

$$\Rightarrow \quad 2\theta=\sin^{-1}\left(\frac{b\sin(2\alpha)+a\sin(2\beta)}{a+b}\right)$$

$$\Rightarrow \quad \theta=\frac{1}{2}\sin^{-1}\left(\frac{b\sin(2\alpha)+a\sin(2\beta)}{a+b}\right)$$

12. Let the velocities of the two particles be parallel at time t. If the angles made by the respective velocities be θ and ϕ respectively, then for them being parallel,

$$\theta=\phi$$

$$\Rightarrow \quad \tan\theta=\tan\phi$$

$$\Rightarrow \quad \frac{u\sin\alpha-gt}{u\cos\alpha}=\frac{v\sin\beta-gt}{v\cos\beta}$$

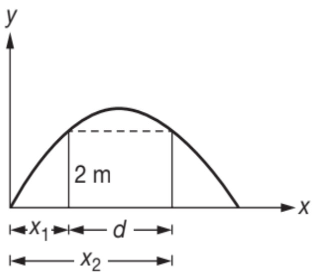
$$\Rightarrow \quad uv\sin\alpha\cos\beta-gt(v\cos\beta)=uv\cos\alpha\sin\beta-gt(u\cos\alpha)$$

$$\Rightarrow \quad uv\sin(\alpha-\beta)=gt(v\cos\beta-u\cos\alpha)$$

$$\Rightarrow \quad t=\frac{uv\sin(\alpha-\beta)}{(v\cos\beta-u\cos\alpha)}$$

13. Since $y=x\tan\theta-\dfrac{gx^2}{2u^2\cos^2\theta}$

Now $y=2$ m, $x=?$, $u=10\text{ ms}^{-1}$, $\theta=45^\circ$



$$\Rightarrow \quad 2=x-\frac{10\times x^2}{(2)(100)\left(\frac{1}{2}\right)}$$

$$\Rightarrow \quad 2=x-\frac{x^2}{10}$$

$$\Rightarrow \quad x^2-10x+20=0$$

$$\Rightarrow \quad x=\frac{10+\sqrt{100-80}}{2} \text{ OR } x=\frac{10-\sqrt{100-80}}{2}$$

$$\Rightarrow \quad d=\frac{10+\sqrt{20}}{2}-\frac{10-\sqrt{20}}{2}=\sqrt{20}=4.5\text{ m}$$

So, $x_1=\dfrac{10-\sqrt{20}}{2}=2.75\text{ m}$

14. $t_A=\dfrac{2u_{Ay}}{g}$, $t_B=\dfrac{2u_{By}}{g}$

$$\Rightarrow \quad t_A^2-t_B^2=\frac{4}{g^2}\left(u_{Ay}^2-u_{By}^2\right)=\frac{4}{g^2}(2gh)$$

$$\Rightarrow \quad g=\frac{8h}{t_A^2-t_B^2}$$

15. Range $=800\text{ mm}=0.8\text{ m}$

$$\Rightarrow \quad \frac{u^2\sin(2\theta)}{g}=0.8\text{ m} \quad \ldots(1)$$

Maximum height $H=500\text{ mm}=0.5\text{ m}$

$$\Rightarrow \quad \frac{u^2\sin^2\theta}{2g}=0.5\text{ m} \quad \ldots(2)$$

Solving equations (1) and (2), we get

$$\theta=68.2^\circ \text{ and } u=3.374$$

$$\Rightarrow \quad v=u\cos\theta=1.253\text{ ms}^{-1}$$

16. Let us take the x-y coordinate system so that its origin coincides with point A.

***x*-Motion:** Here, $x=20-0=20$ m

Since $x=u_xt+\dfrac{1}{2}a_xt^2$

$$\Rightarrow \quad 20=(v_A\cos30^\circ)t \quad \ldots(1)$$

y-Motion: Here, $y = 10 - 1.8 = 8.2$ m

$$(v_A)_y = v_A \sin 30^\circ \text{ and } a_y = -g = -9.8 \text{ ms}^{-2}$$

Since

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow \quad 8.2 = (v_A \sin 30^\circ) t + \frac{1}{2}(-9.8)(t)^2$$

Thus, $8.2 = \left(\frac{20 \sin 30^\circ}{\cos 30^\circ}\right) - 4.9t^2 \qquad \left\{\because t = \frac{20}{V_A \cos(30^\circ)}\right\}$

$$\Rightarrow \quad t = 0.83 \text{ s}$$

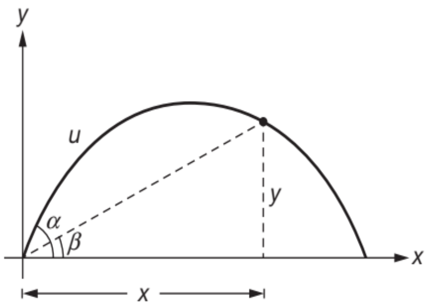
So that

$$v_A = \frac{20}{\cos 30^\circ (0.83)} \cong 28 \text{ ms}^{-1}$$

17. $\tan\beta = \frac{y}{x} = \frac{(u\sin\alpha)t - \frac{1}{2}gt^2}{(u\cos\alpha)t}$

$$\Rightarrow \quad u(\sin\alpha - \cos\alpha\tan\beta) = \frac{gt}{2}$$

$$\Rightarrow \quad u = \frac{gt}{2(\sin\alpha - \cos\alpha\tan\beta)} = \frac{gt\cos\beta}{2\sin(\alpha-\beta)}$$



18. Since $\tan\beta = \frac{u\sin\alpha - gt}{u\cos\alpha}$

$$\Rightarrow \quad \frac{\sin\beta}{\cos\beta} = \frac{u\sin\alpha - gt}{u\cos\alpha}$$

$$\Rightarrow \quad u\sin\beta\cos\alpha = u\sin\alpha\cos\beta - gt\cos\beta$$

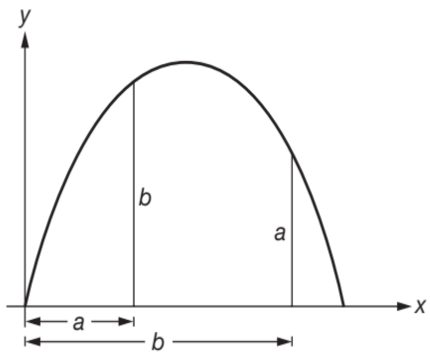
$$\Rightarrow \quad u(\sin\alpha\cos\beta - \sin\beta\cos\alpha) = gt\cos\beta$$

$$\Rightarrow \quad u\sin(\alpha-\beta) = gt\cos\beta$$

19. Since $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} = x\tan\theta\left(1 - \frac{x}{R}\right)$

For the particle clearing the wall of height b at distance a, we have $x = a$, $y = b$. So

$$b = a\tan\theta\left(1 - \frac{a}{R}\right) \qquad \ldots(1)$$



For the particle clearing the wall of height a at a distance b, we have $x = b$, $y = a$. So

$$a = b\tan\theta\left(1 - \frac{b}{R}\right) \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{b}{a} = \frac{a}{b}\left(\frac{R-a}{R-b}\right) \qquad \{\text{Divide (1) by (2)}\}$$

$$\Rightarrow \quad \frac{b^2}{a^2} = \frac{R-a}{R-b}$$

$$\Rightarrow \quad Rb^2 - b^3 = Ra^2 - a^3$$

$$\Rightarrow \quad R(b^2 - a^2) = b^3 - a^3$$

$$\Rightarrow \quad R = \frac{b^3 - a^3}{b^2 - a^2} = \frac{(a^2 + b^2 + ab)(b-a)}{(a+b)(b-a)}$$

$$\Rightarrow \quad R = \frac{a^2 + b^2 + ab}{a+b} \qquad \ldots(3)$$

Substituting (3) in (1) or (2), we get

$$b = a\tan\theta\left(1 - \frac{a(a+b)}{a^2 + ab + b^2}\right)$$

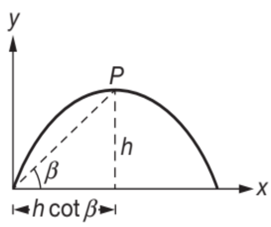
$$\Rightarrow \quad b = a\tan\theta\left(\frac{b^2}{a^2 + ab + b^2}\right)$$

$$\Rightarrow \quad \tan\theta = \frac{a^2 + ab + b^2}{ab}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{a^2 + ab + b^2}{ab}\right)$$

20. Coordinates of the point P are $(h\cot\beta, h)$.

Substituting in $y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$



CHAPTER 5

$$\Rightarrow \quad h = (h\cot\beta)\tan\theta - \frac{gh^2\cot^2\beta}{2u^2}(1+\tan^2\theta)$$

This is quadratic in $\tan\theta$. For θ to be real discriminant of this equation ≥ 0, which gives

$$u \geq \sqrt{gh(1+\operatorname{cosec}\beta)}$$

$$\Rightarrow \quad u_{\text{MIN}} = \sqrt{gh(1+\operatorname{cosec}\beta)}$$

21. Let u be the velocity of projection in both the cases. In the first case, we have

$$T_1 = \frac{2u\sin\alpha}{g} \text{ and} \qquad \ldots(1)$$

$$R_1 = (u\cos\alpha + v)T_1$$

$$\Rightarrow \quad R_1 = \frac{2u^2\sin\alpha\cos\alpha}{g} + \frac{2uv\sin\alpha}{g} \qquad \ldots(2)$$

In the second case, we have

$$T_2 = \frac{2u\sin\alpha}{g} = T_1 \qquad \ldots(3)$$

$$R_2 = (u\cos\alpha - v)T_2$$

$$\Rightarrow \quad R_2 = \frac{2u^2\sin\alpha\cos\alpha}{g} - \frac{2uv\sin\alpha}{g} \qquad \ldots(4)$$

From (2) and (4), we get

$$R_1 - R_2 = \frac{4uv\sin\alpha}{g} \qquad \ldots(5)$$

and $\dfrac{R_1}{R_2} = \dfrac{u\cos\alpha + v}{u\cos\alpha - v}$

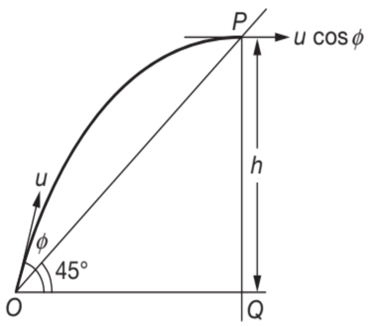
$$\Rightarrow \quad u = \frac{v(R_1+R_2)}{\cos\alpha(R_1-R_2)} \qquad \ldots(6)$$

From equations (5) and (6), we have

$$\tan\alpha = \frac{g(R_1-R_2)^2}{4v^2(R_1+R_2)}$$

Test Your Concepts-IV (Based on Projectile on an Inclined Plane)

1. (a) Let the particle be projected from O with velocity u and strike the plane at a point P horizontally. Then



$$PQ = OQ$$

$$\Rightarrow \quad \text{Maximum height} = \frac{\text{Horizontal range}}{2}$$

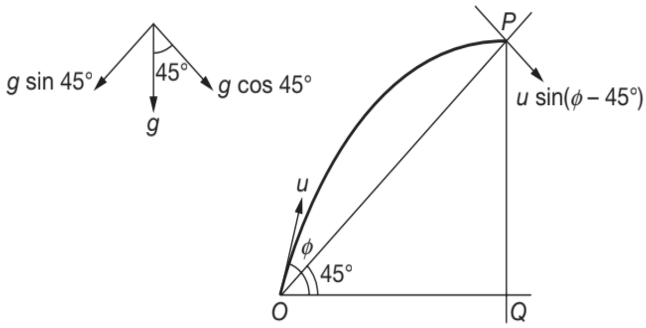
$$\Rightarrow \quad \frac{u^2\sin^2\phi}{2g} = \frac{u^2\sin 2\phi}{2g} = \frac{u^2\sin\phi\cos\phi}{g}$$

$$\Rightarrow \quad \tan\phi = 2$$

(b) At time $t = T = \dfrac{2u\sin(\phi-45^\circ)}{g\cos 45^\circ}$

component of velocity along the plane is zero.

$$\Rightarrow \quad 0 = u\cos(\phi-45^\circ) - (g\sin 45^\circ)t$$



$$\Rightarrow \quad u\cos(\phi-45^\circ) = (g\sin 45^\circ)\left(\frac{2u\sin(\phi-45^\circ)}{g\cos 45^\circ}\right)$$

$$\Rightarrow \quad 2\tan(\phi-45^\circ) = \cot 45^\circ = 1$$

$$\Rightarrow \quad 2\left(\frac{\tan\phi - \tan 45^\circ}{1+\tan\phi\tan 45^\circ}\right) = 1$$

$$\Rightarrow \quad \tan\phi = 3$$

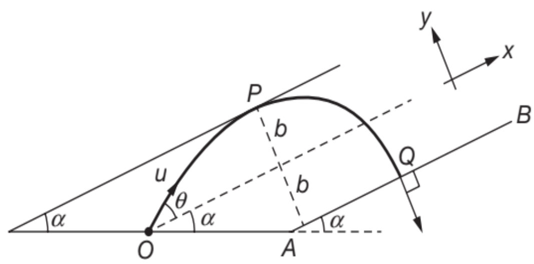
2. Consider the motion of the particle from O to P. The velocity v_y at P is zero.

$$v_y^2 = u_y^2 + 2a_y y$$

$$\Rightarrow \quad 0 = (u\sin\theta)^2 - 2(g\cos\alpha)b$$

$$\Rightarrow \quad b = \frac{u^2\sin^2\theta}{2g\cos\alpha} \qquad \ldots(1)$$

Now, consider the motion of the particle from O to Q. The particle strikes the point Q at 90° to AB, i.e., its velocity along x-direction is zero



Using $v_x = u_x + a_x t$, we get

$$0 = u\cos\theta - (g\sin\alpha)t$$

$$\Rightarrow \quad t = \frac{u\cos\theta}{g\sin\alpha} \qquad \ldots(2)$$

For motion along y-direction, we have

$$y = u_y t + \frac{1}{2}a_y t^2$$

$$\Rightarrow \quad -b = u\sin\theta\left(\frac{u\cos\theta}{g\sin\alpha}\right) + \frac{1}{2}(-g\cos\alpha)\left(\frac{u\cos\theta}{g\sin\alpha}\right)^2 \qquad \ldots(3)$$

From equations (1) and (3), we get

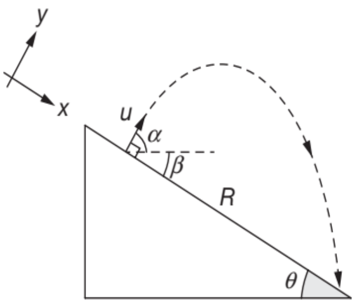
$$-\frac{u^2\sin^2\theta}{2g\cos\alpha} = \frac{u^2\sin\theta\cos\theta}{g\sin\alpha} - \frac{gu^2\cos\alpha\cos^2\theta}{2g^2\sin^2\alpha}$$

$$\Rightarrow \quad -\frac{\sin^2\theta}{2\cos\alpha} = \frac{\sin\theta\cos\theta}{\sin\alpha} - \frac{\cos\alpha\cos^2\theta}{2\sin^2\alpha}$$

Solving, we get

$$\tan\theta = (\sqrt{2} - 1)\cot\alpha$$

3. From the theory for motion of projectile down an inclined plane, we observe that here $\alpha = 90 - \theta$ and $\beta = \theta$.

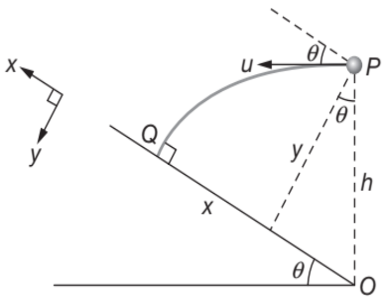


So, we get from, $R = \dfrac{2u^2\sin(\alpha+\beta)\cos\alpha}{g\cos^2\beta}$

$$R = \frac{2u^2\sin(90)\cos(90-\theta)}{g\cos^2\theta}$$

$$\Rightarrow \quad R = \frac{2u^2\tan\theta\sec\theta}{g}$$

4. $u_x = u\cos\theta$, $u_y = u\sin\theta$, $a_x = -g\sin\theta$, $a_y = g\cos\theta$



At Q, we have $v_x = 0$

$$\Rightarrow \quad u_x + a_x t = 0$$

$$\Rightarrow \quad t = \frac{u\cos\theta}{g\sin\theta} \qquad \ldots(1)$$

$$y = h\cos\theta$$

$$\Rightarrow \quad u_y t + \frac{1}{2}a_y t^2 = h\cos\theta$$

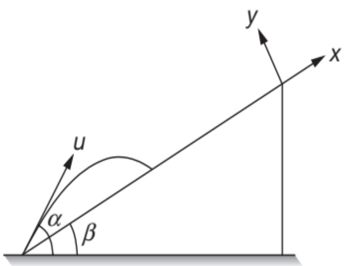
$$\Rightarrow \quad (u\sin\theta)\left(\frac{u\cos\theta}{g\sin\theta}\right) + \frac{1}{2}(g\cos\theta)\left(\frac{u\cos\theta}{g\sin\theta}\right)^2 = h\cos\theta$$

Solving this equation we get,

$$u = \sqrt{\frac{2gh}{2+\cot^2\theta}}$$

5. Particle will retrace its path if it strikes at right angles to the plane i.e.,

$$v_x = 0 \text{ at } T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$



Further, $R = \dfrac{u^2}{g\cos^2\beta}\left[\sin(2\alpha-\beta) - \sin\beta\right] \qquad \ldots(1)$

Substituting in $v_x = u_x + a_x t$, we get

$$0 = u\cos(\alpha-\beta) - g\sin\beta\left\{\frac{2u\sin(\alpha-\beta)}{g\cos\beta}\right\} \qquad \ldots(2)$$

Eliminating α from equations (1) and (2) we get,

$$u = \sqrt{\frac{gR(1+3\sin^2\beta)}{2\sin\beta}}$$

Alternatively, same kind of problem has already been done in theory portion.

6. Consider motion of B along the plane
initial velocity along the incline is $u\cos(\alpha+\beta)$
acceleration is $g\sin\alpha$

$$\Rightarrow \quad OP = u\cos(\alpha+\beta)t + \frac{1}{2}g\sin(\alpha)t^2 \qquad \ldots(1)$$

For motion of particle A along the plane, initial velocity is zero and acceleration is $g\sin\alpha$

$$\Rightarrow \quad OP = \frac{1}{2}(g\sin\alpha)t^2 \qquad \ldots(2)$$

From equations (1) and (2), we get

$$u\cos(\alpha+\beta)t = 0$$

So, either $t = 0$ or $\alpha+\beta = \frac{\pi}{2}$

Thus, the condition for the particles to collide again is

$$\alpha+\beta = \frac{\pi}{2}$$

Single Correct Choice Type Questions

1. $\omega = \frac{(v_\perp)_{rel}}{r_\perp} = \left[\frac{\sqrt{3}\sin(60^\circ) - 8\sin(30^\circ)}{2.5}\right]$

$$\Rightarrow \quad \omega = 1\text{ rads}^{-1}$$

Hence, the correct answer is (D).

2. Since $0 = (v\sin\theta)t + \frac{1}{2}(-a)t^2$

$$\Rightarrow \quad t = \frac{2v\sin\theta}{a}$$

Also, $h = (v\cos\theta)t + \frac{1}{2}gt^2$

$$\Rightarrow \quad h = \frac{2v^2}{a}\sin\theta\left(\cos\theta + \frac{g}{a}\sin\theta\right)$$

Hence, the correct answer is (D).

3. $v_H = \sqrt{\frac{2}{5}}\left(v_{\frac{H}{2}}\right) \qquad \ldots(1)$

Since speed at a point at height h is

$$v = \sqrt{u^2 - 2gh}$$

Since $H = \frac{u^2\sin^2\theta}{2g}$

$$\Rightarrow \quad v_H = \sqrt{u^2 - 2gh} = u\cos\theta \text{ and}$$

$$v_{\frac{H}{2}} = \sqrt{u^2 - 2g\left(\frac{H}{2}\right)} = \sqrt{u^2 - gH}$$

$$\Rightarrow \quad v_{\frac{H}{2}} = \sqrt{u^2 - \frac{u^2\sin^2\theta}{2}}$$

From (1)

$$v_H^2 = \frac{2}{5}(v)_{\frac{H}{2}}^2$$

$$\Rightarrow \quad u^2\cos^2\theta = \frac{2}{5}\left(u^2 - \frac{u^2\sin^2\theta}{2}\right)$$

$$\Rightarrow \quad 5\cos^2\theta = 2 - \sin^2\theta$$

$$\Rightarrow \quad 5 - 5\sin^2\theta = 2 - \sin^2\theta$$

$$\Rightarrow \quad 4\sin^2\theta = 3$$

$$\Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \theta = 60^\circ$$

Hence, the correct answer is (D).

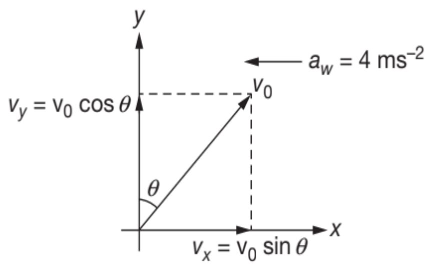
4. If T be the time of flight of the ball, then

$$T = \frac{2v_y}{g} = \frac{2(v_0\cos\theta)}{g}$$

$$\Rightarrow \quad T = \frac{2(20\cos\theta)}{10} = 4\cos\theta \qquad \ldots(1)$$

For the ball to return to the boy's hand, we have $x = 0$.

$$\Rightarrow \quad 0 = v_xT + \frac{1}{2}a_xT^2$$



$$\Rightarrow \quad 0 = (v_0\sin\theta)T + \frac{1}{2}(-4)T^2$$

$$\Rightarrow \quad 0 = 20\sin\theta - 2T$$

$$\Rightarrow \quad T = 10\sin\theta \qquad \ldots(2)$$

From (1) and (2), we get

$$4\cos\theta = 10\sin\theta$$

$$\Rightarrow \quad \cot\theta = 2.5$$

$$\Rightarrow \quad \theta = \cot^{-1}(2.5)$$

Hence, the correct answer is (D).

5. Given $a_N = a_T$

$$\Rightarrow \quad \frac{v^2}{R} = \frac{dv}{dt}$$

$$\Rightarrow \quad v^{-2}dv = \frac{dt}{R}$$

$$\Rightarrow \quad \int_{v_0}^{v} v^{-2}dv = \frac{1}{R}\int_0^t dt$$

$$\Rightarrow \left.\frac{v^{-2+1}}{-2+1}\right|_{v_0}^{v} = \left.\frac{1}{R}t\right|_0^t$$

$$\Rightarrow \frac{1}{v_0} - \frac{1}{v} = \frac{t}{R}$$

$$\Rightarrow v = \frac{v_0 R}{R - v_0 t}$$

$$\Rightarrow \frac{dr}{dt} = \frac{v_0 R}{R - v_0 t}$$

$$\Rightarrow \int_0^{2\pi R} dr = v_0 R \int_0^t \frac{dt}{R - v_0 t}$$

$$\Rightarrow 2\pi R = v_0 R\left[\left. -\frac{1}{v_0}\log_e(R - v_0 t)\right|_0^t\right]$$

$$\Rightarrow 2\pi R = -R\log_e\left(\frac{R - v_0 t}{R}\right)$$

$$\Rightarrow -2\pi = \log_e\left(\frac{R - v_0 t}{R}\right)$$

$$\Rightarrow \frac{R - v_0 t}{R} = e^{-2\pi}$$

$$\Rightarrow t = \frac{R}{v_0}\left(1 - e^{-2\pi}\right)$$

Hence, the correct answer is (D).

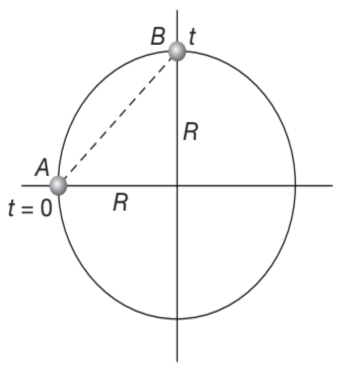
6. $R_{max} = \frac{v^2}{g}$ at $\theta = 45°$

Maximum Area $= \pi(R_{max})^2 = \frac{\pi v^4}{g^2}$

Hence, the correct answer is (B).

7. $\vec{v}_{av} = \frac{\Delta\vec{x}}{\Delta t} = \frac{\text{Displacement}}{\text{Time lapsed}}$

$$\Rightarrow |\vec{v}_{av}| = \frac{|\text{Displacement}|}{\text{Time lapsed}} = \frac{\sqrt{2}R}{t}$$



where $t = \sqrt{\frac{2\theta}{\alpha}}$ $\left\{\because \theta = \frac{1}{2}\alpha t^2\right\}$

Since $\theta = \frac{\pi}{2}$ radian and $\alpha = \frac{\pi}{4}$ rads^{-2}

$$\Rightarrow t = \sqrt{\frac{2\left(\frac{\pi}{2}\right)}{\left(\frac{\pi}{4}\right)}} = 2\text{ s}$$

$$\Rightarrow |\vec{v}_{av}| = \frac{\sqrt{2}R}{2} = \frac{\sqrt{2}\sqrt{2}}{2} = 1\text{ ms}^{-1}$$

Hence, the correct answer is (A).

8. $R_\phi = R_{90-\phi} = 4\sqrt{h_\phi h_{90-\phi}}$

Hence, the correct answer is (A).

9. Considering the vertical motion, if t be the time taken by the particle to go from A to B, then

$$h = \frac{1}{2}gt^2$$

$$\Rightarrow t = \sqrt{\frac{2h}{g}}$$

During this time the particle must make an integral number of rotations (say n).
Now, considering the horizontal motion, then

$$ut = (2\pi r)n$$

$$\Rightarrow n = \frac{u}{2\pi r}\sqrt{\frac{2h}{g}}$$

Hence, the correct answer is (A).

10. $R = (a+b) = \frac{u^2\sin(2\alpha)}{g} = \frac{u^2}{g}$ $\{\because \alpha = 45^o\}$

$$\Rightarrow u^2 = g(a+b) \quad \ldots(1)$$

Since, $y = a\tan\alpha - \frac{ga^2}{2u^2\cos^2\alpha}$

$$\Rightarrow h = a\tan\alpha - \frac{ga^2}{2g(a+b)\cos^2\alpha}$$

$$\Rightarrow h = a\tan 45 - \frac{ga^2}{2g(a+b)\cos^2 45}$$

$$\Rightarrow h = \frac{ab}{a+b}$$

Hence, the correct answer is (D).

11. $y = \dfrac{gx^2}{2u^2}$

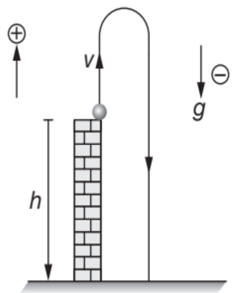
$\Rightarrow \quad 20 - 12 = \dfrac{(9.8)(30)^2}{2u^2}$

$\Rightarrow \quad u^2 = \dfrac{(9.8)900}{8}$

$\Rightarrow \quad u = 23.5 \text{ ms}^{-1}$

Hence, the correct answer is (A).

12. Taking upward direction as ⊕ , we get



$-h = vt + \dfrac{1}{2}(-g)t^2$

$\Rightarrow \quad t^2 - \dfrac{2v}{g}t - \dfrac{2h}{g} = 0$

$\Rightarrow \quad t = \dfrac{1}{2}\left(\dfrac{2v}{g} + \sqrt{\dfrac{4v^2}{g^2} + 4\left(\dfrac{2h}{g}\right)}\right)$

$\Rightarrow \quad t = \dfrac{1}{2}\left(\dfrac{2v}{g} + \dfrac{2v}{g}\sqrt{1 + \dfrac{2hg}{v^2}}\right)$

$\Rightarrow \quad t = \dfrac{v}{g}\left(1 + \sqrt{1 + \dfrac{2hg}{v^2}}\right)$

Hence, the correct answer is (A).

13. At maximum height, radius of curvature is

$R = \dfrac{v_T^2}{a_C} = \dfrac{(u\cos\theta)^2}{g} = \dfrac{u^2\cos^2\theta}{g} = \dfrac{u_x^2}{g}$

$\Rightarrow \quad R = \dfrac{\left(72 \times \dfrac{5}{18}\right)^2}{10}$

$\Rightarrow \quad R = \dfrac{400}{10} = 40 \text{ m}$

Hence, the correct answer is (C).

14. Given $a_y = \dfrac{d^2y}{dt^2} = \alpha$ and $\dfrac{d^2x}{dt^2} = 0$

Since $y = \beta x^2$

$\Rightarrow \quad \dfrac{dy}{dt} = 2\beta x\left(\dfrac{dx}{dt}\right)$

$\Rightarrow \quad \dfrac{d^2y}{dt^2} = 2\beta\left[x\left(\dfrac{d^2x}{dt^2}\right) + \left(\dfrac{dx}{dt}\right)^2\right]$

$\Rightarrow \quad \alpha = 2\beta v_x^2$

$\Rightarrow \quad v_x = \sqrt{\dfrac{\alpha}{2\beta}}$

Hence, the correct answer is (B).

15. At maximum height, we have

$u\cos\theta = \dfrac{u}{2}$

$\Rightarrow \quad \cos\theta = \dfrac{1}{2}$

$\Rightarrow \quad \theta = 60°$

So, $R = \dfrac{\sqrt{3}u^2}{2g}$

Hence, the correct answer is (C).

16. Since the projectile is at same height at times t_1 and t_2. As studied, the sum of these times equals the time of flight of the projectile. So,

$T = t_1 + t_2 = 3 + 5 = 8 \text{ s}$

Hence, the correct answer is (D).

17. $h^2 = r^2 - x^2$

$\tan\theta = \dfrac{h}{x}$

$h = r\sin\theta$

$\Rightarrow \quad x = h\cot\theta$

$\Rightarrow \quad \dfrac{dx}{dt} = -h\,\text{cosec}^2\,\theta\left(\dfrac{d\theta}{dt}\right)$

$\Rightarrow \quad \dfrac{dx}{dt} = -(8000)\text{cosec}^2(60)(-0.0.25)$

$\Rightarrow \quad \dfrac{dx}{dt} = (8000)\left(\dfrac{4}{3}\right)(0.025)$

$\Rightarrow \quad \dfrac{dx}{dt} = \dfrac{800}{3} \text{ ms}^{-1}$

$\Rightarrow \quad \dfrac{dx}{dt} = \dfrac{800}{3} \times \dfrac{18}{5}$

$\Rightarrow \quad v = 960 \text{ kmh}^{-1}$

Hence, the correct answer is (B).

18. Taking upward direction as positive, we get

$-80 = 12t + \dfrac{1}{2}(-10)t^2$

$\Rightarrow \quad -80 = 12t - 5t^2$

$\Rightarrow \quad 5t^2 - 12t - 80 = 0$

$$\Rightarrow \quad t = \frac{12 \pm \sqrt{144+1600}}{2(5)}$$

$$\Rightarrow \quad t \cong 5.5 \text{ s}$$

Hence, the correct answer is (B).

19. $4 = (u\cos\alpha)t$ ...(1)

$$2 = \frac{u^2 \sin^2\alpha}{2g} \quad ...(2)$$

$$-3 = (u\sin\alpha)t - \frac{1}{2}gt^2 \quad ...(3)$$

From (2)

$$u\sin\alpha = 2\sqrt{g} \quad ...(4)$$

Substituting in (3), we get

$$-3 = 6.3t - 4.9t^2$$

$$\Rightarrow \quad 4.9t^2 - 6.3t - 3 = 0$$

$$\Rightarrow \quad t = \frac{6.3 \pm \sqrt{39.7 - 4(4.9)(-3)}}{2(4.9)}$$

$$\Rightarrow \quad t = \frac{6.3 \pm \sqrt{98.5}}{9.8}$$

$$\Rightarrow \quad t = \frac{6.3 + 9.9}{9.8} = 8.1 \text{ s}$$

So, from (1), $4 = (u\cos\alpha)(8.1)$

$$\Rightarrow \quad u\cos\alpha = \frac{4}{8} \cong \frac{1}{2} \quad ...(5)$$

From (4) and (5), we get

$$u^2\sin^2\alpha + u^2\cos^2\alpha = 40 + \frac{1}{4}$$

$$\Rightarrow \quad u = \sqrt{40.25} = 6.3 \text{ ms}^{-1}$$

Hence, the correct answer is (A).

20. $\tan\theta = \tan\alpha + \tan\beta$

$$\Rightarrow \quad \tan\alpha + \tan\beta = \tan 60$$

$$\Rightarrow \quad \tan\alpha + \tan\beta = \sqrt{3}$$

Hence, the correct answer is (D).

21. The radius of curvature at the instant the projectile has a velocity v making an angle β with the horizontal is

$$R = \frac{v^2}{g\cos\beta}$$

where $\beta = 30°$ and v has to be found. Since horizontal motion is non-accelerated motion, so

$$v_x = u_x$$

$$\Rightarrow \quad v\cos\beta = u\cos\theta$$

$$\Rightarrow \quad v\cos(30°) = 20\cos(60°)$$

$$\Rightarrow \quad v\left(\frac{\sqrt{3}}{2}\right) = 20\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad v = \frac{20}{\sqrt{3}} \text{ ms}^{-1}$$

$$\Rightarrow \quad R = \frac{\left(\frac{20}{\sqrt{3}}\right)^2}{g\cos(30°)} = \frac{\frac{400}{3}}{10\left(\frac{\sqrt{3}}{2}\right)} = \frac{80}{3\sqrt{3}} = 15.4 \text{ m}$$

Hence, the correct answer is (C).

22. If T be the time of flight, then

$$T = \frac{2u\sin(\alpha - 30°)}{g\cos(30°)}$$

For the projectile to hit the plane normally, $v_x = 0$

$$\Rightarrow \quad u_x + a_x T = 0$$

$$\Rightarrow \quad u\cos(\alpha - 30°) - g\sin(30°)T = 0$$

$$\Rightarrow \quad u\cos(\alpha - 30°) = g\sin(30°)\left(\frac{2u\sin(\alpha - 30°)}{g\cos(30°)}\right)$$

$$\Rightarrow \quad \tan(\alpha - 30°) = \frac{1}{2}\cot(30°)$$

$$\Rightarrow \quad \tan(\alpha - 30°) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad (\alpha - 30°) = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad \alpha = 30° + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

In general, for the particle to hit the incline normally, we have

$$2\tan(\alpha - \beta) = \cot\beta$$

OR

$$\cot(\alpha - \beta) = 2\tan\beta$$

Hence, the correct answer is (D).

23. $y = 9x^2$

$$\Rightarrow \quad \frac{dy}{dt} = v_y = 18x\frac{dx}{dt} = 18x\left(\frac{1}{3}\right)$$

$$\Rightarrow \quad \frac{dy}{dt} = v_y = 6x$$

$\Rightarrow \quad a_y = \frac{d^2y}{dt^2} = \frac{dv_y}{dt} = 6\left(\frac{dx}{dt}\right) = 6\left(\frac{1}{3}\right)$

$\Rightarrow \quad a_y = 2\ ms^{-2}$ along y-axis

Hence $\vec{a}_y = (2\ ms^{-2})\hat{j}$

Hence, the correct answer is (D).

24. $\omega_{\text{centre}} = \frac{d\theta}{dt}$

$\omega_{\text{circumference}} = \frac{d}{dt}\left(\frac{\theta}{2}\right) = \frac{1}{2}\omega_{\text{centre}}$

$\Rightarrow \quad \omega_{\text{centre}} = 2\omega_{\text{circumference}}$

$\Rightarrow \quad \omega_{\text{centre}} = 4\ \text{rads}^{-1}$

Now $v = R\omega_{\text{centre}} = (2)(4) = 8\ \text{ms}^{-1}$

Hence, the correct answer is (D).

25. Since we know that ranges are equal for complimentary angles i.e., ϕ and $90-\phi$. Also, we know that

$R_\phi = R_{90-\phi} = 4\sqrt{H_\phi H_{90-\phi}}$

$\Rightarrow \quad R_\phi = R_{90-\phi} = 4\sqrt{(4)(16)}$

$\Rightarrow \quad R_\phi = R_{90-\phi} = 32$ m

Hence, the correct answer is (D).

26. When the particle is moving in a circular path with uniform speed, then the tangential acceleration is zero. However, radial acceleration is non-zero.
Hence, the correct answer is (C).

27. $y = \alpha x - \beta x^2$

Comparing with standard equation of projectile

$y = x\tan\theta - \frac{ax^2}{2u^2\cos^2\theta}$ we get

$\alpha = \tan\theta$ and $\beta = \frac{a}{2u^2}\sec^2\theta$

Since $\sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \quad \beta = \frac{a}{2u^2}(1+\alpha^2)$

$\Rightarrow \quad u = \sqrt{\frac{a}{2\beta}(1+\alpha^2)}$

Hence, the correct answer is (A).

28. For a horizontal projectile, we have

$y = \frac{gx^2}{2u^2}$. So,

$$h = \frac{g(250)^2}{2v^2} \quad \ldots(1)$$

Similarly,

$$4h = \frac{gx^2}{2(v/2)^2} \quad \ldots(2)$$

Dividing (1) & (2), we get

$x = 250\ m$

Hence, the correct answer is (A).

29. $x^2 + y^2 = l^2$

$\Rightarrow \quad 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\Rightarrow \quad v_b = \frac{x}{y}\frac{dx}{dt} = 10\ \text{ms}^{-1} \qquad (x = y \text{ at } 45^\circ)$

Hence, the correct answer is (D).

30. $b = a\tan\alpha - \frac{ga^2}{2u^2\cos^2\alpha}$

and $a = b\tan\alpha - \frac{gb^2}{2u^2\cos^2\alpha}$

$\Rightarrow \quad \frac{a\tan\alpha - b}{b\tan\alpha - a} = \frac{a^2}{b^2}$

$\Rightarrow \quad ab^2\tan\alpha - b^3 = ba^2\tan\alpha - a^3$

$\Rightarrow \quad ab^2\tan\alpha - ba^2\tan\alpha = b^3 - a^3$

$\Rightarrow \quad ab(b-a)\tan\alpha = (b-a)(b^2+a^2+ab)$

$\Rightarrow \quad \tan\alpha = \frac{b^2+a^2+ab}{ab}$

$\Rightarrow \quad \tan\alpha = \frac{b^2+a^2+2ab+ab-2ab}{ab}$

$\Rightarrow \quad \tan\alpha = \frac{(b-a)^2+3ab}{ab}$

$\Rightarrow \quad \tan\alpha = 3 + \frac{(b-a)^2}{ab}$

Since, $(b-a)^2 > 0$

$\Rightarrow \quad \tan\alpha > 3$

$\Rightarrow \quad (\tan\alpha)_{\min} = 3$

Hence, the correct answer is (B).

31. $H_1 = \frac{u^2\sin^2\alpha}{2g}$

$H_2 = \frac{u^2\sin^2(90-\alpha)}{2g} = \frac{u^2\cos^2\alpha}{2g}$

{Because range is same for complimentary angles}

Further $H_1 - H_2 = \dfrac{H_1 + H_2}{2}$

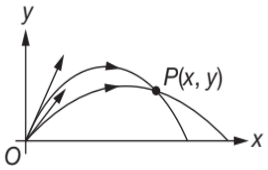
$\Rightarrow \quad H_1 = 3H_2$

$\Rightarrow \quad \tan\alpha = \sqrt{3}$

$\Rightarrow \quad \alpha = 60^o$

Hence, the correct answer is (C).

32.



Let the particles pass through the common point $P(x, y)$ at times t_1 and t_2.

$$u_1 t_1 = u_2 t_2 \qquad \ldots(1)$$

$$v_1 t_1 - \frac{1}{2} g t_1^2 = v_2 t_2 - \frac{1}{2} g t_2^2 \qquad \ldots(2)$$

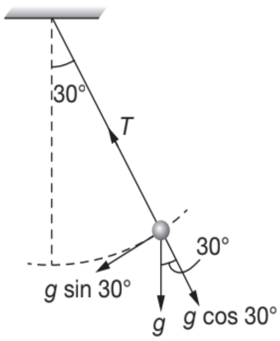
Rearranging and solving equations (1) and (2) to get value of $(t_1 - t_2)$.

$$\Rightarrow \quad (t_1 - t_2) = \frac{2}{g}\left(\frac{v_1 u_2 - v_2 u_1}{u_1 + u_2}\right)$$

Hence, the correct answer is (D).

33. Rate of change of speed is

$$\frac{dv}{dt} = \text{tangential acceleration}$$



$$\Rightarrow \quad \frac{dv}{dt} = g\sin(30°)$$

$$\Rightarrow \quad \frac{dv}{dt} = 10\left(\frac{1}{2}\right) \text{ ms}^{-2} = 5 \text{ ms}^{-2}$$

Hence, the correct answer is (B).

34. $T_0 = \dfrac{2u\sin\theta}{g}$ (is time of flight when no drag exists)

$$T = \frac{u\sin\theta}{g + \frac{g}{10}} + \frac{u\sin\theta}{g - \frac{g}{10}}$$

$$T = \frac{u\sin\theta}{g}\left(\frac{10}{11} + \frac{10}{9}\right)$$

$$T = \frac{2u\sin\theta}{g}\left[5\left(\frac{20}{99}\right)\right]$$

$$T = T_0 \frac{100}{99}$$

$\Rightarrow$ Percentage increase $= \left(\dfrac{100}{99} - 1\right) \times 100 = \dfrac{100}{99}\%$

$\Rightarrow$ Percentage increase $\simeq 1\%$

Hence, the correct answer is (A).

35. Coordinates of the point P are $(R, -h)$. Since the point P lies on the trajectory, so $(R, -h)$ must satisfy

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$$

$$\Rightarrow \quad -h = R\tan\theta - \frac{gR^2}{2(2ag)}(1 + \tan^2\theta)$$

$$\Rightarrow \quad R^2\tan^2\theta - 4aR\tan\theta + (R^2 - 4ah) = 0$$

For θ to be real, we must have

DISCRIMINANT ≥ 0

$$(4aR)^2 - 4R^2(R^2 - 4ah) \geq 0$$

$\Rightarrow \quad 4a^2 \geq R^2 - 4ah$

$\Rightarrow \quad R^2 \leq 4a(a + h)$

$\Rightarrow \quad R \leq 2\sqrt{a(a + h)}$

$\Rightarrow \quad R_{MAX} = 2\sqrt{a(a + h)}$

Hence, the correct answer is (D).

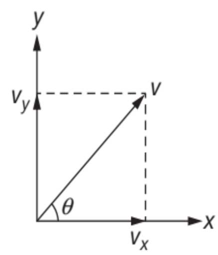
36. $\vec{r} = (2t)\hat{i} + (2t^2)\hat{j}$

$\Rightarrow \quad x = 2t$ and $y = 2t^2$

$\Rightarrow \quad \dfrac{dx}{dt} = v_x = 2$ and $\dfrac{dy}{dt} = v_y = 4t$

Since θ is the angle which $\vec{v} = v_x\hat{i} + v_y\hat{j}$ makes with the positive x-axis, so

$$\tan\theta = \frac{v_y}{v_x} = \frac{4t}{2} = 2t \qquad \ldots(1)$$



Differentiating both sides w.r.t. t we get

$$\sec^2\theta\left(\frac{d\theta}{dt}\right)=2$$

$$\Rightarrow \quad \frac{d\theta}{dt}=\frac{2}{\sec^2\theta}=\frac{2}{1+\tan^2\theta}$$

From (1),

$$\tan\theta=2t$$

$$\Rightarrow \quad \frac{d\theta}{dt}=\frac{2}{1+4t^2}$$

So, $$\left.\frac{d\theta}{dt}\right|_{t=0.5\text{ s}}=\frac{2}{1+4(0.25)}=1\text{ rads}^{-1}$$

Hence, the correct answer is (D).

37. Done in theory.
Hence, the correct answer is (B).

38. For Q

Since acceleration due to gravity has no component along AB. So motion of the particle along AB is non-accelerated motion with uniform velocity v. If t_Q is the time taken by particle Q to go from A to B, then

$$t_Q=\frac{AB}{v} \qquad \ldots(1)$$

For P

Since motion of particle P from A to C is accelerated and that from C to B is retarded. So from A to C the horizontal component of velocity gradually increases to attain a maximum value at the lowest point which further decreases gradually to attain the same value v at point B.

That is the motion from A to B to C experiences a horizontal velocity which has a value more than v. So $t_Q>t_P$.

Hence, the correct answer is (A).

39. Tangential acceleration, for a simple pendulum is

$$a_T=g\sin\theta$$

So, at $\theta=30°$, we have

$$a_T=g\sin(30°)=\frac{g}{2}=5\text{ ms}^{-2}$$

Further we know that $a_C=\ell\omega^2$

$$\Rightarrow \quad a_C=\left(\frac{200}{1000}\right)(9)^2$$

$$\Rightarrow \quad a_C=\left(\frac{2}{10}\right)81=16.2\text{ ms}^{-2}$$

Since the total acceleration is

$$a=\sqrt{a_C^2+a_T^2}$$

$$\Rightarrow \quad a=\sqrt{(16.2)^2+(5)^2}$$

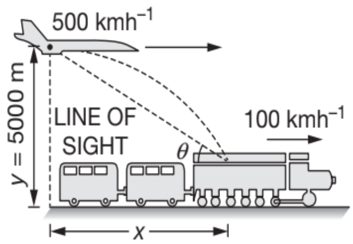
$$\Rightarrow \quad a=17\text{ ms}^{-2}$$

Hence, the correct answer is (D).

40. Velocity of train relative to bomber is u_r, then

$$u_r=500-100=400\text{ kmh}^{-1}$$

$$\Rightarrow \quad u_r=400\times\frac{5}{18}\text{ ms}^{-1}=\frac{1000}{9}\text{ ms}^{-1}$$



Since, $y=\dfrac{gx^2}{2u_r^2}$

$$\Rightarrow \quad 5000=\frac{gx^2}{2\left(\frac{1000}{9}\right)^2}$$

$$\Rightarrow \quad x=3513.65\text{ m}$$

Further $\tan\theta=\dfrac{y}{x}$

$$\Rightarrow \quad \tan\theta=\frac{5000}{3513.65}$$

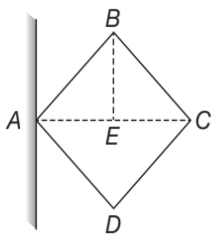
$$\Rightarrow \quad \tan\theta=1.423$$

$$\Rightarrow \quad \theta\simeq 55^o$$

Hence, the correct answer is (A).

41. Let $AC=x$ and $BE=y$. Then

$$BE^2+AE^2=\ell^2$$



$$\Rightarrow \quad y^2+\left(\frac{x}{2}\right)^2=\ell^2$$

$$\Rightarrow \quad 2y\left(\frac{dy}{dt}\right)+\frac{x}{2}\left(\frac{dx}{dt}\right)=0$$

$$\Rightarrow \quad -\frac{dy}{dt}=\frac{1}{2}\left(\frac{x}{2y}\right)\left(\frac{dx}{dt}\right)$$

When the rhombus is a square, then $x = 2y$

$$\Rightarrow \quad v_B=\frac{1}{2}v_c=\frac{v}{2}$$

Hence, the correct answer is (B).

42. $v=\sqrt{v_r^2+v_\theta^2}$

Here $v_r=\frac{dr}{dt}=2\text{ ms}^{-1}$

Since $v_\theta = r\cdot\frac{d\theta}{dt}=(2\times 1)(4)=8\text{ ms}^{-1}$

$$\Rightarrow \quad v=\sqrt{64+4}=\sqrt{68}=8.25\text{ ms}^{-1}$$

Hence, the correct answer is (C).

43. $a_T=\frac{v^2-u^2}{2\ell}=\frac{(15)^2-(5)^2}{2(200)}=0.5\text{ ms}^{-2}$

$a_C=\frac{v_{\text{final}}}{r}=\frac{(15)^2}{\left(\frac{400}{\pi}\right)}$ $\qquad \left\{\because\ l=\frac{2\pi r}{4}=200\right\}$

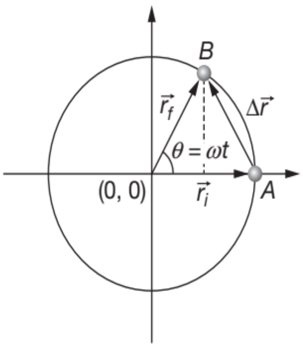
$$\Rightarrow \quad a_C=1.8\text{ ms}^{-2}$$

$$\Rightarrow \quad a=\sqrt{(1.8)^2+(0.5)^2}$$

$$\Rightarrow \quad a=1.9\text{ ms}^{-2}$$

Hence, the correct answer is (C).

44. For a particle moving in a circle with constant angular velocity ω, the figure shows the discussion at time t.



Initial position vector of the particle is

$$\vec r_i = a\hat i$$

Final position vector of the particle at time t is

$$\vec r_f=(a\cos\theta)\hat i+(a\sin\theta)\hat j$$

So, displacement of the particle is

$$\overrightarrow{AB}=\Delta\vec r=\vec r_f-\vec r_i$$

$$\Rightarrow \quad \Delta\vec r=(a\cos\theta-a)\hat i+(a\sin\theta)\hat j$$

$$\Rightarrow \quad |\Delta\vec r|=a\sqrt{(\cos\theta-1)^2+\sin^2\theta}$$

$$\Rightarrow \quad |\Delta\vec r|=a\sqrt{\cos^2\theta+1-2\cos\theta+\sin^2\theta}$$

Since $\cos^2\theta+\sin^2\theta=1$

$$\Rightarrow \quad |\Delta\vec r|=a\sqrt{1+1-2\cos\theta}=\sqrt{2}a\sqrt{1-\cos\theta}$$

Further $1-\cos\theta=2\sin^2\left(\frac{\theta}{2}\right)$

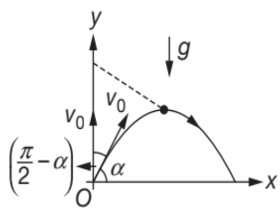
$$\Rightarrow \quad |\Delta\vec r|=\sqrt{2}a\sqrt{2\sin^2\left(\frac{\theta}{2}\right)}=2\sin\left(\frac{\theta}{2}\right)$$

Since $\theta=\omega t$

$$\Rightarrow \quad |\Delta\vec r|=2a\sin\left(\frac{\omega t}{2}\right)$$

Hence, the correct answer is (D).

45. Since both bodies are moving under influence of gravity hence relative acceleration is zero i.e. both move with constant velocity relative to each other



$\vec v_{12}$ = Relative velocity of 1 with respect to 2

$$\vec v_{12}=\vec v_1-\vec v_2$$

$$|\vec v_{12}|=\sqrt{v_1^2+v_2^2-2v_1v_2\cos\left(\frac{\pi}{2}-\alpha\right)}$$

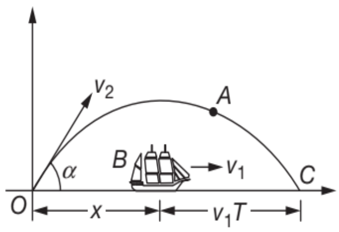
$$|\vec v_{12}|=v_0\sqrt{2(1-\sin\alpha)}=2v_0\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$$

$$|\vec r_{12}|=|\vec v_{12}|t$$

$$|\vec r_{12}|=2v_0t\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$$

Hence, the correct answer is (C).

46.



For shell to hit the boat, time taken by the boat to go from B to C with uniform velocity v_1 equals the time taken by shell to go from O to A to C i.e. $\frac{2v_2\sin\alpha}{g}=T$. Further, we observe

$$x+v_1T=R=\frac{2}{g}(v_2\sin\alpha)(v_2\cos\alpha)$$

$$\Rightarrow\quad x=\frac{2}{g}(v_2\sin\alpha)(v_2\cos\alpha-v_1)$$

Hence, the correct answer is (C).

47. When range is maximum, the $h=\frac{R}{4}$. So $h=100\ m$.
Velocity of the projectile is minimum at the maximum height. So this point has coordinates (200, 100)
Hence, the correct answer is (B).

48. $y=x\tan\theta-\frac{gx^2}{2u^2\cos^2\theta}$

$$y=mx-\frac{gx^2}{2u^2}(1+m^2),\text{ where } m=\tan\theta$$

$$\Rightarrow\quad\left(\frac{gx^2}{2u^2}\right)m^2-xm+\left(\frac{gx^2}{2u^2}+y\right)=0\qquad\ldots(1)$$

Equation (1) is a quadratic in m with two roots m_1 and m_2.

These two roots m_1 and m_2 of equation (1) give us the two firing angles for the two trajectories shown (in the statement) such that the projectiles pass through a common point A. This point A will approach the envelope E as the two roots approach equality i.e. the DISCRIMINANT for quadratic in m must equal zero.

$$x^2-4\left(\frac{gx^2}{2u^2}\right)\left(\frac{gx^2}{2u^2}+y\right)=0$$

$$\Rightarrow\quad 1-\left(\frac{2g}{u^2}\right)\left(\frac{gx^2}{2u^2}+y\right)=0$$

$$\Rightarrow\quad\frac{u^2}{2g}-\frac{gx^2}{2u^2}-y=0$$

$$\Rightarrow\quad y=\frac{u^2}{2g}-\frac{gx^2}{2u^2}$$

is the equation of the envelope E.
Hence, the correct answer is (A).

49. For both the particles launched simultaneously to hit the point C, we have

$$t_A=t_B$$

$$\frac{2u\sin\theta}{g}=\sqrt{\frac{2h}{g}}$$

$$\Rightarrow\quad\frac{2(10)(\sin(60^\circ))}{10}=\sqrt{\frac{2h}{10}}$$

$$\Rightarrow\quad 2\frac{\sqrt{3}}{2}=\sqrt{\frac{2h}{10}}$$

$$\Rightarrow\quad\sqrt{3}=\sqrt{\frac{2h}{10}}$$

$$\Rightarrow\quad h=15\text{ m}$$

Hence, the correct answer is (C).

50. $y=x\tan\alpha-\frac{gx^2}{2u^2\cos^2\alpha}$

For y to be maximum

$$\Rightarrow\quad\frac{dy}{d\alpha}=0$$

$$\Rightarrow\quad\tan\alpha=\frac{u^2}{gx}$$

Hence, the correct answer is (C).

51. $y=x\tan\alpha-\frac{gx^2}{2u^2\cos^2\alpha}$

$$\Rightarrow\quad y=x\tan\alpha-\frac{gx^2}{2u^2}(1+\tan^2\alpha)$$

$$\Rightarrow\quad y_{\max}=\frac{u^2}{2g}-\frac{gx^2}{2u^2}=h\qquad\left\{\because\tan\alpha=\frac{u^2}{gx}\right\}$$

Hence, the correct answer is (D).

52. Motion of one projectile as seen from other is a straight line inclined at acute angle with the horizontal.
Hence, the correct answer is (D).

53. $(u_y)_1=10\sin(30^\circ)=5\text{ ms}^{-1}$

$$(u_y)_2=\frac{10}{\sqrt{3}}\sin(60^\circ)=5\text{ ms}^{-1}$$

Since $(u_y)_1=(u_y)_2$ and time of flight is $T=\frac{2u_y}{g}$

$$\Rightarrow\quad T_1=T_2$$

i.e., both the particles strike the ground at the same instant.
So, the maximum separation between them is the difference of their horizontal ranges.

$$\Rightarrow\quad\Delta x=R_1-R_2$$

$$\Rightarrow \quad \Delta x = \frac{(20)^2 \sin(60^\circ)}{g} - \frac{\left(\frac{20}{\sqrt{3}}\right)^2 \sin(120^\circ)}{g}$$

$$\Rightarrow \quad \Delta x = 20\sqrt{3} - \frac{20\sqrt{3}}{3}$$

$$\Rightarrow \quad \Delta x = 20\sqrt{3}\left(1 - \frac{1}{3}\right)$$

$$\Rightarrow \quad \Delta x = \frac{40}{\sqrt{3}} = 23 \text{ m}$$

Hence, the correct answer is (C).

54. Since horizontal components of both are the same, so relative horizontal velocity $(v_{12})_x$ is zero and hence $\vec{v}_{12}$ (velocity of 1 w.r.t. 2) has only a vertical component please note that the angles θ_1 and θ_2 are with the vertical and hence the horizontal component of velocities are $u_1 \sin\theta_1$ and $u_2 \sin\theta_2$ which are actually equal.
Hence, the correct answer is (D).

55. Since $y = \frac{gx^2}{2u^2}$ ...(1)

When $x = nb$ and $y = nh$, then

$n = \frac{2hu^2}{gb^2}$ {Put values in (1)}

Hence, the correct answer is (B).

56. METHOD I

$$y = ax - bx^2$$

Compare with

$$y = x\tan\theta - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2$$

we get

$$\tan\theta = a \text{ and } \frac{g}{2u^2\cos^2\theta} = b$$

$$H_{MAX} = \frac{u^2\sin^2\theta}{2g} = \left(\frac{g}{2b\cos^2\theta}\right)\frac{\sin^2\theta}{2g}$$

$$\Rightarrow \quad H_{MAX} = \frac{\tan^2\theta}{4b} = \frac{a^2}{4b}$$

and $\theta = \tan^{-1}(a)$

METHOD II

At maximum height

$$\frac{dy}{dx} = 0$$

$$\Rightarrow \quad a - 2bx = 0$$

$$\Rightarrow \quad x = \frac{a}{2b}$$

$$\Rightarrow \quad y_{MAX} = y\Big|_{x=\frac{a}{2b}} = a\left(\frac{a}{2b}\right) - b\left(\frac{a^2}{4b^2}\right)$$

$$\Rightarrow \quad y_{MAX} = \frac{a^2}{4b}$$

Now $\frac{dy}{dx} = a - 2bx$

So, the slope at the point of launch $(x = 0)$ is $\tan\theta$. Hence

$$\left.\frac{dy}{dx}\right|_{x=0} = a = \tan\theta$$

$$\Rightarrow \quad \theta = \tan^{-1}(a)$$

Hence, the correct answer is (C).

57. $\frac{dx}{dt} = 8\pi\sin(2\pi t)$

$$\Rightarrow \quad dx = 8\pi\sin(2\pi t)dt$$

$$\Rightarrow \quad \int_8^x dx = 8\pi\int_0^t \sin(2\pi t)dt$$

$$\Rightarrow \quad x - 8 = -8\pi\left.\frac{\cos(2\pi t)}{2\pi}\right|_0^t$$

$$\Rightarrow \quad x - 8 = 4\cos(2\pi t - 1)$$

$$\Rightarrow \quad x - 8 = 4\cos(2\pi t) - 4$$

$$\Rightarrow \quad x - 4 = 4\cos(2\pi t) \quad \text{...(1)}$$

Further since, $\frac{dy}{dt} = 8\pi\cos(2\pi t)$

$$\Rightarrow \quad dy = 8\pi\cos(2\pi t)dt$$

$$\Rightarrow \quad \int_0^y dy = 8\pi\int_0^t \cos(2\pi t)dt$$

$$\Rightarrow \quad y = \left.\frac{8\pi\sin(2\pi t)}{2\pi}\right|_0^t$$

$$\Rightarrow \quad y = 4\sin(2\pi t) \quad \text{...(2)}$$

From (1) and (2), we have

$$(x - 4)^2 + y^2 = 4$$

This is the equation of a circle with centre at $(4, 0)$ and radius 2.

Hence, the correct answer is (B).

58. Done Earlier.
Hence, the correct answer is (D).

59. $H = \dfrac{u^2 \sin^2\theta}{2g} = \dfrac{u_y^2}{2g}$

Since u_y and a_y remain the same as previous, so H remains same.

Also, $T = \dfrac{2u\sin\theta}{g} = \dfrac{2u_y}{g}$

So, T also remains the same

Now, $R_{\text{new}} = u_x T + \dfrac{1}{2}a_x T^2$

$$\Rightarrow \quad R_{\text{new}} = R + \frac{1}{2}\left(\frac{g}{2}\right)\left(\frac{4u_y^2}{g^2}\right) = R + \frac{H}{2}$$

Hence, the correct answer is (B).

60. Since $\dfrac{dx}{dt} = \dfrac{dy}{dt} = c$

$$\Rightarrow \quad \frac{d^2x}{dt^2} = \frac{d^2y}{dt^2} = 0$$

Further $z = ax^3 + by^2$

$$\Rightarrow \quad \frac{dz}{dt} = 3ax^2\frac{dx}{dt} + 2by\frac{dy}{dt}$$

$$\Rightarrow \quad \frac{dz}{dt} = 3acx^2 + 2bcy \qquad \left\{\because \frac{dx}{dt} = c = \frac{dy}{dt}\right\}$$

$$\Rightarrow \quad \frac{d^2z}{dt^2} = 6acx\left(\frac{dx}{dt}\right) + 2bc\left(\frac{dy}{dt}\right)$$

$$\Rightarrow \quad \frac{d^2z}{dt^2} = 6ac^2x + 2bc^2$$

Now acceleration of particle is

$$\vec{a} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}$$

$$\Rightarrow \quad \vec{a} = (6ac^2x + 2bc^2)\hat{k}$$

Hence, the correct answer is (B).

61. Since, $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

$$\Rightarrow \quad b = a\tan\alpha - \frac{ga^2}{2u^2\cos^2\alpha}$$

$$\Rightarrow \quad a\tan\alpha - b = \frac{ga^2}{2u^2\cos^2\alpha}$$

$$\Rightarrow \quad \frac{u^2}{g} = \frac{a^2\sec^2\alpha}{2(a\tan\alpha - b)} \qquad \ldots(1)$$

Further, maximum height h is

$$h = \frac{u^2\sin^2\alpha}{2g}$$

$$\Rightarrow \quad h = \frac{a^2\sec^2\alpha\sin^2\alpha}{4(a\tan\alpha - b)}$$

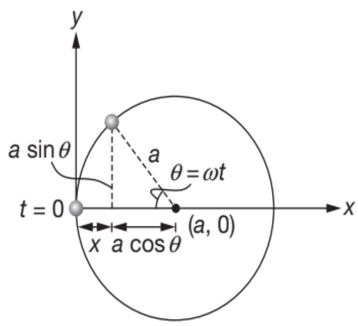
$$\Rightarrow \quad h = \frac{a^2\tan^2\alpha}{4(a\tan\alpha - b)}$$

Hence, the correct answer is (A).

62. In all the cases, we observe that the horizontal component of the velocity of the food packet is same as the horizontal component of the velocity of the aeroplane and due to this, at all the instants, both have the same horizontal displacements.
Hence, the correct answer is (D).

63. $x + a\cos\theta = a$

$$\Rightarrow \quad x = a(1 - \cos\theta) = a[1 - \cos(\omega t)]$$



and $y = a\sin\theta = a\sin(\omega t)$

Also we observe that at $t = 0$, $|\vec{r}| = a$

Hence, the correct answer is (C).

64. Kinetic energy is minimum at the maximum height. So,

$$K_{\text{MIN}} = \frac{1}{2}mu_x^2$$

$$\frac{(K_{MIN})_1}{(K_{MIN})_2} = \frac{(u_x)_1^2}{(u_x)_2^2} = \frac{4}{1}$$

$$\Rightarrow \quad \frac{(u_x)_1}{(u_x)_2} = 2 \qquad \ldots(1)$$

Further, $H = \dfrac{u_y^2}{2g}$

Since, $\dfrac{H_1}{H_2} = \dfrac{9}{1}$

$$\Rightarrow \quad \frac{H_1}{H_2} = \frac{(u_y)_1^2}{(u_y)_2^2} = \frac{9}{1}$$

$$\Rightarrow \quad \frac{(u_y)_1}{(u_y)_2} = \frac{3}{1}$$

Now $R = \frac{2}{g}(u_x)(u_y)$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{(u_x)_1}{(u_x)_2}\frac{(u_y)_1}{(u_y)_2} = (2)(3)$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{6}{1}$$

Hence, the correct answer is (C).

65. $\delta = \frac{1}{2}gt^2 = \frac{gx^2}{2u^2\cos^2\alpha}$ $\left\{\because t = \frac{x}{u\cos\alpha}\right\}$

Hence, the correct answer is (D).

66. $y = \frac{x^2}{2} = \frac{\left(\frac{t^2}{2}\right)^2}{2} = \frac{t^4}{8}$ $\left\{\because x = \frac{t^2}{2}\right\}$

$$\Rightarrow \quad \frac{dy}{dt} = \frac{t^3}{2}$$

$$\Rightarrow \quad \left.\frac{dy}{dt}\right|_{t=2s} = 4 \text{ ms}^{-1}$$

Similarly, for $x = \frac{t^2}{2}$

$$\frac{dx}{dt} = t$$

$$\Rightarrow \quad \left.\frac{dx}{dt}\right|_{t=2s} = 2 \text{ ms}^{-1}$$

$$\Rightarrow \quad \left.\vec{v}\right|_{t=2s} = (4\hat{i} + 2\hat{j}) \text{ ms}^{-1}$$

Hence, the correct answer is (C).

67. $v_x = \frac{dx}{dt} = 8t + 4$

$$v_y = \frac{dy}{dt} = -3t^2 + 12$$

$$a_x = \frac{dv_x}{dt} = 8$$

$$a_y = \frac{dv_y}{dt} = -6t$$

$$\Rightarrow \quad \vec{a} = 8\hat{i} - (6t)\hat{j}$$

$$\Rightarrow \quad \left.\vec{a}\right|_{t=1\text{ s}} = 8\hat{i} - 6\hat{j}$$

$$\Rightarrow \quad |\vec{a}| = 10 \text{ ms}^{-2}$$

Hence, the correct answer is (B).

68. Since $v^2 = v_x^2 + v_y^2$

$$\Rightarrow \quad 2v\frac{dv}{dt} = 2v_x\frac{dv_x}{dt} + 2v_y\frac{dv_y}{dt}$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{v_x a_x + v_y a_y}{v} = \frac{v_x a_x + v_y a_y}{\sqrt{v_x^2 + v_y^2}}$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{(3)(2) + (4)(1)}{5} = 2 \text{ ms}^{-2}$$

Hence, the correct answer is (C).

69. Since horizontal motion is non accelerated, so $u_x = v_x$

$$\Rightarrow \quad u\cos 60 = v\cos 30$$

$$\Rightarrow \quad 10\left(\frac{1}{2}\right) = v\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad v = \frac{10}{\sqrt{3}} \text{ ms}^{-1}$$

Hence, the correct answer is (D).

70. $u_x = v_x$

$u\sin\theta = v\sin\phi$ $\{\theta$ and ϕ are with the vertical$\}$

$$\Rightarrow \quad v = u\sin\theta\,\text{cosec}\,\phi = u\left(\frac{\text{cosec}\,\phi}{\text{cosec}\,\theta}\right)$$

Hence, the correct answer is (D).

71. At $h = 0.4$ m, $\vec{v} = 6\hat{i} + 2\hat{j}$

$\Rightarrow \quad v_x = 6 \text{ ms}^{-1}$ and $v_y = 2 \text{ ms}^{-1}$

Since $v_x = u_x$

$$\Rightarrow \quad u_x = 6 \text{ ms}^{-1}$$

Also, $v_y^2 - u_y^2 = 2a_y y$

$$\Rightarrow \quad 2^2 - u_y^2 = 2(-g)(0.4)$$

$$\Rightarrow \quad u_y^2 = 4 + 8$$

$$\Rightarrow \quad u_y = 2\sqrt{3} \text{ ms}^{-1}$$

If θ is the angle made by the initial velocity with the vertical, then

$u_x = u\sin\theta = 6$ and $u_y = u\cos\theta = 2\sqrt{3}$

$\Rightarrow \quad \dfrac{u\sin\theta}{u\cos\theta} = \dfrac{6}{2\sqrt{3}} = \sqrt{3}$

$\Rightarrow \quad \tan\theta = \sqrt{3}$

$\Rightarrow \quad \theta = 60°$

Hence, the correct answer is (B).

72. Since we know that $\ell = r\theta$

$\Rightarrow \quad 100 = r\left(\dfrac{\pi}{4}\right)$

$\Rightarrow \quad r = \dfrac{400}{\pi}$ m

Since $a = \dfrac{v^2}{r}$

$\Rightarrow \quad a = \dfrac{(20)^2}{\left(\dfrac{400}{\pi}\right)} = \pi \text{ ms}^{-2} \left\{\because v = 72 \text{ kmh}^{-1} = 20 \text{ ms}^{-1}\right\}$

$\Rightarrow \quad a = 3.14 \text{ ms}^{-2}$

Hence, the correct answer is (B).

73. $\tan\beta = \dfrac{u\sin\theta - gt}{u\cos\theta}$

$\Rightarrow \quad \tan(45°) = \dfrac{u\sin\theta - gt}{u\cos\theta}$

$\Rightarrow \quad u\sin\theta - 10 = u\cos\theta \qquad \ldots(1)$

After three more second it is travelling horizontally i.e., at $t = 4$ s $\beta = 0$

$\Rightarrow \quad u\sin\theta - g(4) = 0$

$\Rightarrow \quad u\sin\theta = 40 \qquad \ldots(2)$

Substituting in (1), we get

$u\cos\theta = 30 \qquad \ldots(3)$

Squaring and adding, we get

$u = 50 \text{ ms}^{-1}$

$\tan\theta = \dfrac{4}{3}$

$\Rightarrow \quad \theta = \tan^{-1}\left(\dfrac{4}{3}\right)$

Hence, the correct answer is (C).

74. Since we know that

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$\Rightarrow \quad y = x\tan\theta - \dfrac{gx^2}{2u^2}(1 + \tan^2\theta)$

For the particle to pass through the point (30 m, 40 m), we have

$$40 = 30\tan\theta - \frac{g(30)^2}{2u^2}(1 + \tan^2\theta)$$

$\Rightarrow \quad 900\tan^2\theta - 6u^2\tan\theta + (900 + 8u^2) = 0$

For real values of θ, we must have

$(6u^2)^2 - 4(900)(900 + 8u^2) \geq 0$

$36u^4 - 3600(900 + 8u^2) \geq 0$

$\Rightarrow \quad u^4 - 100(900 + 8u^2) \geq 0$

$\Rightarrow \quad u^4 - 800u^2 - 90000 \geq 0$

$\Rightarrow \quad u^4 - 800u^2 + (400)^2 - (400)^2 - 90000 \geq 0$

$\Rightarrow \quad (u^2 - 400)^2 - 250000 \geq 0$

$\Rightarrow \quad (u^2 - 400)^2 \geq 250000$

$\Rightarrow \quad (u^2 - 400) \geq 500$

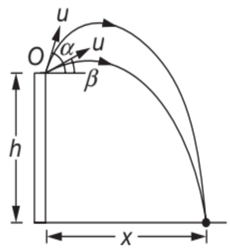
$\Rightarrow \quad u^2 \geq 900$

$\Rightarrow \quad u \geq 30 \text{ ms}^{-1}$

$\Rightarrow \quad u_{MIN} = 30 \text{ ms}^{-1}$

Hence, the correct answer is (D).

75.



$-h = x\tan\alpha - \dfrac{gx^2}{2u^2\cos^2\alpha} \qquad \ldots(1)$

and $-h = x\tan\beta - \dfrac{gx^2}{2u^2\cos^2\beta} \qquad \ldots(2)$

Equating (1) and (2), we get

$x = \dfrac{2u^2}{g}\left(\dfrac{1}{\tan\alpha + \tan\beta}\right) \qquad \ldots(3)$

Substituting (3) in (1), we get

$h = \dfrac{2u^2}{g}\left(\dfrac{1 - \tan\alpha\tan\beta}{\tan\alpha + \tan\beta}\right)$

$\Rightarrow \quad h = \dfrac{2u^2}{g}\cot(\alpha + \beta)$

Hence, the correct answer is (D).

76. $y=\frac{1}{2}gt^2$ and $x=v_0t$

$\Rightarrow\ y=\frac{1}{2}(10)(3)^2$

$\Rightarrow\ y=45$ m

Since $\tan(30°)=\frac{y}{x}$

$\Rightarrow\ x=\frac{y}{\tan(30°)}=45\sqrt{3}$ m

Since $x=v_0t$

$\Rightarrow\ 45\sqrt{3}=v_0(3)$

$\Rightarrow\ v_0=\frac{45\sqrt{3}}{3}$

$\Rightarrow\ v_0=15\sqrt{3}\ \text{ms}^{-1}$

$\Rightarrow\ v_0=25.98\ \text{ms}^{-1}$

$\Rightarrow\ v_0\cong 26\ \text{ms}^{-1}$

Hence, the correct answer is (C).

77. Given $v_T=10\ \text{ms}^{-1}$ and $a=100\ \text{ms}^{-2}$

Also, $a_T=60\ \text{ms}^{-2}$

Since $a^2=a_C^2+a_T^2$

$\Rightarrow\ (100)^2=a_C^2+(60)^2$

$\Rightarrow\ a_C=80\ \text{ms}^{-2}$

$\Rightarrow\ \frac{v_T^2}{r}=80$

$\Rightarrow\ r=\frac{(10)^2}{80}=\frac{100}{80}$

$\Rightarrow\ r=1.25$ m

Hence, the correct answer is (D).

78. $y=2$ m, $x=3$ m

Since $y=\frac{gx^2}{2u^2}$

$\Rightarrow\ 2=\frac{(10)9}{2u^2}$

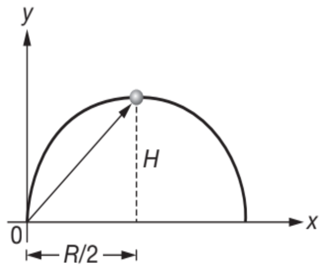
$\Rightarrow\ u^2=\frac{90}{4}$

$\Rightarrow\ u\cong 4.7\ \text{ms}^{-1}$

Hence, the correct answer is (A).

79. Average velocity $=\frac{\text{Displacement}}{\text{Time}}$

$\Rightarrow\ v_{av}=\frac{\sqrt{H^2+\frac{R^2}{4}}}{\left(\frac{T}{2}\right)}$



where $H=\frac{u^2\sin^2\theta}{2g}$

$T=\frac{2u\sin\theta}{g}$

$R=\frac{2u^2\sin\theta\cos\theta}{g}$

$\Rightarrow\ v_{av}=\frac{\sqrt{\frac{u^4\sin^4\theta}{4g^2}+\frac{u^4\sin^2\theta\cos^2\theta}{g^2}}}{\frac{u\sin\theta}{g}}$

$\Rightarrow\ v_{av}=\frac{u}{2}\sqrt{\frac{\sin^4\theta}{\sin^2\theta}+\frac{4\sin^2\theta\cos^2\theta}{\sin^2\theta}}$

$\Rightarrow\ v_{av}=\frac{u}{2}\sqrt{\sin^2\theta+4\cos^2\theta}$

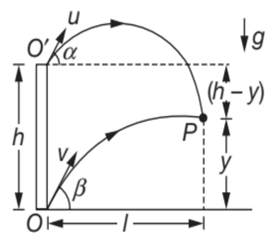
$\Rightarrow\ v_{av}=\frac{u}{2}\sqrt{\sin^2\theta+\cos^2\theta+3\cos^2\theta}$

$\Rightarrow\ v_{av}=\frac{u}{2}\sqrt{1+3\cos^2\theta}$

Hence, the correct answer is (C).

80. Let the origin be located at the point O and O'.
For the particles to strike the object placed at P simultaneously

$u\cos\alpha=v\cos\beta$ ...(1)



Also,

$$y = l\tan\beta - \frac{gl^2}{2v^2\cos^2\beta} \quad \ldots(2)$$

and $$-(h-y) = l\tan\alpha - \frac{gl^2}{2u^2\cos^2\alpha} \quad \ldots(3)$$

Subtract (3) from (2) and use (1) we get

$$h = l(\tan\beta - \tan\alpha)$$

Hence, the correct answer is (D).

81. $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

Here $y = 5$ m, $x = 20$ m, $u = ?$, $\theta = 30°$

$\Rightarrow \quad 5 = 20\tan(30°) - \dfrac{(10)(20)^2}{2u^2\left(\frac{3}{4}\right)}$

$\Rightarrow \quad 5 = \dfrac{20}{\sqrt{3}} - \dfrac{5\times 400\times 4}{3u^2}$

$\Rightarrow \quad \dfrac{5\times 400\times 4}{3u^2} = \dfrac{20}{\sqrt{3}} - 5$

$\Rightarrow \quad u^2 = \dfrac{5\times 1600}{3\left(\frac{20}{\sqrt{3}} - 5\right)}$

$\Rightarrow \quad u^2 = \dfrac{5\times 1600}{20\sqrt{3} - 15}$

$\Rightarrow \quad u^2 = \dfrac{8000}{19.64}$

$\Rightarrow \quad u^2 \cong 400$

$\Rightarrow \quad u = 20\ \text{ms}^{-1}$

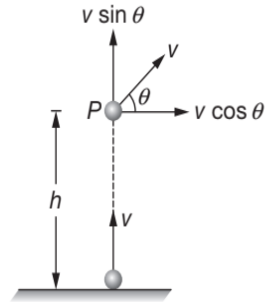
Hence, the correct answer is (A).

82. Relative acceleration between the particles is zero, because both are under influence of gravity.
If s be the relative separation between them at time t, then

$$s^2 = [h - (v - v\sin\theta)t]^2 + [(v\cos\theta)t]^2$$

For s to be MINIMUM or s^2 to be MINIMUM, we have

$$\frac{d}{dt}(s^2) = 0$$



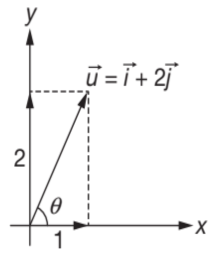
$\Rightarrow \quad -2[h-(v-v\sin\theta)t][v-v\sin\theta]+2(v^2\cos^2\theta)t=0$

$\Rightarrow \quad t = \dfrac{h}{2v}$

Hence, the correct answer is (D).

83. $\vec{u} = \hat{i} + 2\hat{j}$

$|\vec{u}| = \sqrt{5}$ at an angle $\theta = \tan^{-1}(2)$ with the horizontal



Since $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

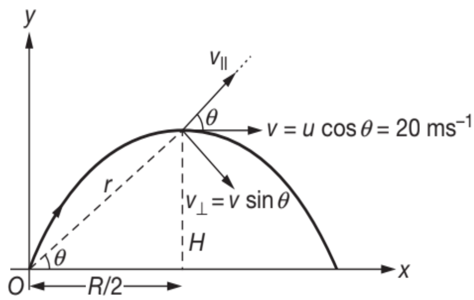
$\Rightarrow \quad y = x\tan\theta - \dfrac{gx}{2u^2}(1+\tan^2\theta)$

$\Rightarrow \quad y = 2x - \dfrac{(10)x^2}{2(5)}(1+4)$

$\Rightarrow \quad y = 2x - 5x^2$

Hence, the correct answer is (D).

84. $H = \dfrac{u^2\sin^2\theta}{2g} = \dfrac{(20\sqrt{2})^2\sin^2(45°)}{2(10)} = 20$ m



Since at maximum height $x = \frac{R}{2}$, so we get

$$r^2 = \frac{R^2}{4} + H^2$$

where $R = \frac{u^2 \sin(2\theta)}{g} = 80$ m

$$\Rightarrow \quad r = 20\sqrt{5} \text{ m}$$

Now $\omega = \frac{v_\perp}{r} = \frac{v \sin\theta}{r}$

where $\sin\theta = \frac{H}{r}$

$$\Rightarrow \quad \omega = \frac{vH}{r^2} = \frac{(20)(20)}{400(5)} = \frac{1}{5} = 0.2 \text{ rads}^{-1}$$

Hence, the correct answer is (B).

85. $x = \frac{2u^2 \sin(\alpha - \beta)\cos\alpha}{g\cos^2\beta}$ where $\alpha = 60°$ and $\beta = 30°$

$$\Rightarrow \quad x = \frac{2(30)^2 \sin(30°)\cos(60°)}{g\cos^2(30°)}$$

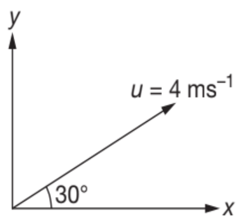
$$\Rightarrow \quad x = \frac{(2)(900)(0.5)(0.5)}{(10)\left(\frac{3}{4}\right)} = 60 \text{ m}$$

Hence, the correct answer is (C).

86. Let us first calculate the components of the velocity of ball relative to the lift.

$u_x = 4\cos(30°) = 2\sqrt{3}$ ms^{-1} and

$u_y = 4\sin(30°) = 2$ ms^{-1}



The acceleration of the ball with respect to the lift is $(10 + 2) = 12$ ms^{-2} in the negative y-direction (i.e., vertically downwards)

So, the time of flight T is

$$T = \frac{2u_y}{a_y} = \frac{2(2)}{12} = \frac{1}{3} \text{ s}$$

Hence, the correct answer is (C).

87. Since $R = u_x T$

$$\Rightarrow \quad R = (2\sqrt{3})\left(\frac{1}{3}\right) = \frac{2}{\sqrt{3}} \text{ m}$$

Hence, the correct answer is (B).

88. $x_{\text{AIRCRAFT}} = x_{\text{BULLET}}$

$$\Rightarrow \quad (277)t = (540\cos\theta)t$$

$$\{\because 1000 \text{ kmh}^{-1} = 277 \text{ ms}^{-1}\}$$

$$\Rightarrow \quad \cos\theta = \frac{277}{540}$$

$$\Rightarrow \quad \theta \cong 59°$$

Just think that $\frac{277}{540} \cong \frac{270}{540} = \frac{1}{2}$, so θ must be an angle close to $60°$.

Hence, the correct answer is (C).

89. $v_x = 8t - 2$

$$\Rightarrow \quad \frac{dx}{dt} = 8t - 2$$

$$\Rightarrow \quad \int_{14}^{x} dx = 8\int_{2}^{t} t\,dt - 2\int_{2}^{t} dt$$

$$\Rightarrow \quad x - 14 = 4(t^2 - 4) - 2(t - 2)$$

$$\Rightarrow \quad x - 14 = 4t^2 - 16 - 2t + 4$$

$$\Rightarrow \quad x = 4t^2 - 2t + 2 \qquad \ldots(1)$$

Since $v_y = 2$

$$\Rightarrow \quad \frac{dy}{dt} = 2$$

$$\Rightarrow \quad \int_{4}^{y} dy = 2\int_{2}^{t} dt$$

$$\Rightarrow \quad y - 4 = 2(t - 2)$$

$$\Rightarrow \quad y = 2t \qquad \ldots(2)$$

$$\Rightarrow \quad t = \frac{y}{2}$$

Substituting in (1), we get

$$x = 4\left(\frac{y}{2}\right)^2 - 2\left(\frac{y}{2}\right) + 2$$

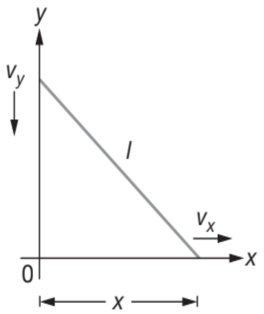
$$\Rightarrow \quad x = y^2 - y + 2$$

Hence, the correct answer is (A).

CHAPTER 5

90. The situation discussed is shown at time t. Then we observe that at any instant

$$\ell^2 = x^2 + y^2 \quad \ldots(1)$$



$$\Rightarrow \quad 0 = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\Rightarrow \quad \frac{dx}{dt} = -\frac{y}{x}\left(\frac{dy}{dt}\right)$$

From (1), $x = \sqrt{\ell^2 - y^2}$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{yv_0}{\sqrt{\ell^2 - y^2}} \qquad \left\{\because \frac{dy}{dt} = -v_0\right\}$$

So, the speed of the lower end decreases and vanishes when $y \to 0$.

Hence, the correct answer is (B).

91. Initial velocity, $\vec{u} = (u\cos\theta)\hat{i} + (u\sin\theta)\hat{j}$

Final velocity, $\vec{v} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$

So, $\Delta\vec{v} = \vec{v} - \vec{u} = -(gt)\hat{j}$

$$\Rightarrow \quad \Delta\vec{v} = -(20)\hat{j}$$

$$\Rightarrow \quad |\Delta\vec{v}| = 20 \text{ ms}^{-1} \text{ along } -y \text{ direction}$$

Hence, the correct answer is (C).

92. To catch the ball, the horizontal relative velocity of the boy relative to the ball must be zero.

$$\Rightarrow \quad v - u\cos(90 - \alpha) = 0$$

$$\Rightarrow \quad v = u\sin\alpha$$

Hence, the correct answer is (B).

93. $\omega = 2000 \text{ rpm} = 2000 \times \dfrac{2\pi}{60} \text{ rads}^{-1}$

$$r = \frac{5}{2} \text{ m} = 2.5 \text{ m}$$

$$a = r\omega^2 = (2.5)\left(2000 \times \frac{2\pi}{60}\right)^2$$

$$\Rightarrow \quad a = (2.5)\frac{16\pi^2}{36}(10000)$$

$$\Rightarrow \quad a = 1.1 \times 10^5 \text{ ms}^{-2}$$

Hence, the correct answer is (B).

94. $\vec{u} = a\hat{i} + b\hat{j}$ and

$$R = 2H \quad \ldots(1)$$

Since $\vec{u} = (u\cos\theta)\hat{i} + (u\sin\theta)\hat{j}$

$$\Rightarrow \quad u\cos\theta = u_x = a \text{ and}$$

$$u\sin\theta = u_y = b$$

Also from (1), we have

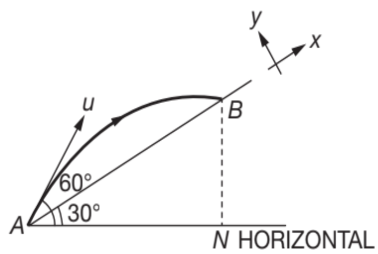
$$\frac{2}{g}(u_x)(u_y) = 2\left(\frac{u_y^2}{2g}\right)$$

$$\Rightarrow \quad 2ab = b^2$$

$$\Rightarrow \quad b = 2a$$

Hence, the correct answer is (C).

95. Time taken by the particle to go from A to N (along horizontal) is also t.



$$AN = (u\cos(60°))t = ut\cos(60°)$$

In ΔABN

$$\cos 30 = \frac{AN}{AB}$$

$$\Rightarrow \quad AB = \frac{AN}{\cos 30} = \frac{ut\cos(60°)}{\cos(30°)}$$

$$\Rightarrow \quad AB = \frac{ut}{\sqrt{3}}$$

Hence, the correct answer is (A).

96. $H = \dfrac{1}{2}g(2t)^2 = 2gt^2 \quad \ldots(1)$

Let stone hit the wall at height h, then

$$h = H - \frac{1}{2}gt^2 \quad \ldots(2)$$

From (1) and (2), we get

$$h = H - \frac{H}{4} = \frac{3H}{4}$$

Hence, the correct answer is (C).

97. $R = u\sqrt{\frac{2h}{g}} = 12\sqrt{\frac{10}{10}} = 12 \text{ m}$

$\Rightarrow \quad S = \sqrt{R^2 + r^2} = 13 \text{ m}$

Hence, the correct answer is (B).

98. For two angles of projection α and β ranges are same, so the angles must be complimentary i.e. $\alpha + \beta = \frac{\pi}{2}$.
Hence, the correct answer is (D).

99. As $\theta < 45°$, body never moves perpendicular to its initial direction of motion. Since $T = \frac{2u\sin\theta}{g}$

$\Rightarrow \quad T\left(=\frac{u}{g}\right) < t\left(=\frac{2u}{g}\right)$, which is impossible.

Hence, the correct answer is (D).

100. Since $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$\Rightarrow \quad y = 80 \times \frac{3}{4} - \frac{10 \times 80 \times 80 \times 25}{2 \times 50 \times 50 \times 16} = 40 \text{ m}$

So, distance from point of projection is

$$r = \sqrt{(80)^2 + (40)^2} \text{ m} = 40\sqrt{5} \text{ m}$$

Hence, the correct answer is (C).

103. $\tan(45°) = \frac{20\sin(60°) - 10t}{20\cos(60°)}$

$\Rightarrow \quad 10t = 20\sin(60°) - 20\cos(60°)$

$\Rightarrow \quad t = (\sqrt{3} - 1) \text{ s}$

Hence, the correct answer is (D).

104. Since $y = (u\sin\theta)t - \frac{1}{2}gt^2$

$\Rightarrow \quad 15 = 52\left(\frac{5}{13}\right)t - 5t^2$

$\Rightarrow \quad 15 = 20t - 5t^2$

$\Rightarrow \quad t^2 - 4t + 3 = 0$

$\Rightarrow \quad (t-1)(t-3) = 0$

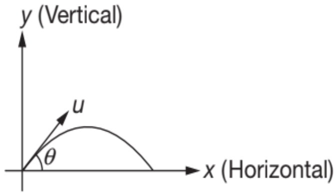
$\Rightarrow \quad t = 1 \text{ s and } t = 3 \text{ s}$

So, time for which the ball is at least 15 m above the ground is

$$\Delta t = 3 - 1 = 2 \text{ s}$$

Hence, the correct answer is (B).

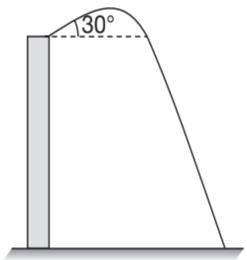
105. According to law of independence of directions motion of a body along three mutually perpendicular directions is independent of each other.



Hence, the correct answer is (D).

106. Since, $y = u_y t + \frac{1}{2}a_y t^2$

$\Rightarrow \quad 70 = -(50\sin 30°)t + \frac{1}{2} \times 10 \times t^2$



$$y = v_y t + \frac{1}{2}a_y t^2$$

$\Rightarrow \quad 70 = -25t + 5t^2$

$\Rightarrow \quad t^2 - 5t - 14 = 0$

$\Rightarrow \quad t^2 - 7t + 2t - 14 = 0$

$\Rightarrow \quad (t-7)(t+2) = 0$

$\Rightarrow \quad t = 7 \text{ s}$ (as negative time is not possible)

Hence, the correct answer is (C).

107. Standard equation of projectile is

$$y = (\tan\theta)x - \frac{g}{2u^2\cos^2\theta}x^2 \quad \ldots(1)$$

and the given equation is

$$y = \sqrt{3}x - \frac{g}{2}x^2 \quad \ldots(2)$$

Comparing (1) and (2), we get

$$\tan\theta = \sqrt{3}$$

$\Rightarrow \quad \theta = 60°$

and $2u^2\cos^2\theta = 2$

$$\Rightarrow \quad u = \frac{1}{\cos\theta} = \frac{1}{\cos 60^\circ} = 2\ \text{ms}^{-1}$$

Hence, the correct answer is (B).

108. $\dfrac{u_x}{u_y} = \cot(30^\circ) = \sqrt{3}$

$$\Rightarrow \quad u_x = 80\sqrt{3}\ \text{ms}^{-1}$$

Since, $T = \dfrac{2u_y}{g}$

$$\Rightarrow \quad T = 16\ \text{s}$$

At $t = \dfrac{T}{4} = 4\ \text{s}$, $v_x = 80\sqrt{3}\ \text{ms}^{-1}$

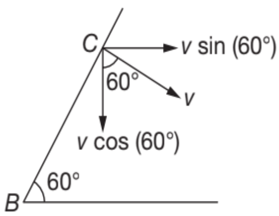
Since, $v_y = 80 - 10 \times 4 = 40\ \text{ms}^{-1}$

$$\Rightarrow \quad v = \sqrt{(80\sqrt{3})^2 + (40)^2} = 140\ \text{ms}^{-1}$$

Hence, the correct answer is (C).

109. Let v be the velocity at the time of collision.

Then, $u\sqrt{2}\cos(45^\circ) = v\sin(60^\circ)$



$$\Rightarrow \quad (u\sqrt{2})\left(\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{3}v}{2}$$

$$\Rightarrow \quad v = \frac{2}{\sqrt{3}}u$$

Hence, the correct answer is (C).

110. $T = \dfrac{2 \times 20\sin(37^\circ)}{10} = \dfrac{12}{5}\ \text{s}$

Range is given by $R = (20\cos 37^\circ + 10)T$

$$\Rightarrow \quad R = \frac{12}{5} \times \left(20 \times \frac{4}{5} + 10\right) = 26 \times \frac{12}{5} = 62.4\ \text{m}$$

Hence, the correct answer is (D).

111. $H = \dfrac{u^2\cos^2\beta}{2g}$

$$\Rightarrow \quad u\cos\beta = \sqrt{2gH}$$

So, $t = \dfrac{T}{2} = \dfrac{u\cos\beta}{g} = \sqrt{\dfrac{2H}{g}}$

Hence, the correct answer is (B).

112. Since, time of flight does not depend on horizontal acceleration, so

$$T = \frac{2u\sin\theta}{g} = \frac{2(20)\left(\frac{1}{\sqrt{2}}\right)}{10} = 2\sqrt{2}\ \text{s}$$

Since $x = u_x t + \dfrac{1}{2}a_x t^2$

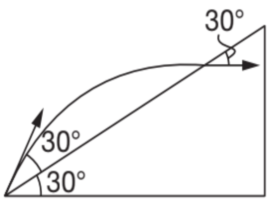
$$\Rightarrow \quad R = u_x T + \frac{1}{2}a_x T^2$$

$$\Rightarrow \quad R = (20\cos 45^\circ)(2\sqrt{2}) + \frac{1}{2}(2)(2\sqrt{2})^2$$

$$\Rightarrow \quad R = 40 + 8 = 48\ \text{m}$$

Hence, the correct answer is (B).

113.



Since, particle hits the inclined plane at an angle of 30°, which means that the particle hits the plane horizontally. So, this is the highest point of trajectory.

$$\Rightarrow \quad t = \frac{T}{2} = \frac{u\sin\theta}{g} = \frac{20\sin(60^\circ)}{g}$$

Hence, the correct answer is (A).

114. Since, $v_x = \dfrac{dx}{dt} = 2\ \text{ms}^{-1}$

$$\Rightarrow \quad x = 2t \qquad \ldots(1)$$

Also, $v_y = \dfrac{dy}{dt} = 5x$

$$\Rightarrow \quad dy = xdt = 5(2t)dt \qquad \{\text{from (1)}\}$$

$$\Rightarrow \quad dy = 10t\,dt$$

$$\Rightarrow \quad y = \frac{10t^2}{2} = 5t^2 \qquad \ldots(2)$$

From (1) and (2), we get

$$y = 5\left(\frac{x}{2}\right)^2$$

$$\Rightarrow \quad y = \frac{5}{4}x^2$$

Hence, the correct answer is (A).

115. Since, $\frac{u^2\sin^2\theta}{2g} = 5$ m

$\Rightarrow \frac{(20)^2\sin^2\theta}{20} = 5$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 30°$

So, $R = \frac{u^2\sin(2\theta)}{g} = \frac{(20)^2\sin(60°)}{10} = 20\sqrt{3}$

Hence, the correct answer is (D).

117. Since, $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$\Rightarrow h = d\tan\theta - \frac{gd^2}{2u^2\cos^2\theta}$

$\Rightarrow \frac{gd^2}{2u^2\cos^2\theta} = (d\tan\theta - h)$

$\Rightarrow u^2 = \frac{gd^2}{2\cos^2\theta(d\tan\theta - h)}$

$\Rightarrow u = \frac{d}{\cos\theta}\sqrt{\frac{g}{2(d\tan\theta - h)}}$

Hence, the correct answer is (B).

118. Apply formula of time of flight on inclined plane.

Hence, the correct answer is (B).

119. $T = \frac{2u\sin\theta}{g}$

$\Rightarrow R = \frac{u^2\sin(2\theta)}{g} - \frac{2u^2\sin^2\theta}{g}$

R will be maximum when, $\frac{dR}{d\theta} = 0$

$\Rightarrow 2\cos(2\theta) - 2\sin(2\theta) = 0$

$\Rightarrow \tan(2\theta) = 1$

$\Rightarrow 2\theta = \frac{\pi}{4}$

$\Rightarrow \theta = \frac{\pi}{8}$ radian

Hence, the correct answer is (B).

120. $u = 20$ ms^{-1}, $\theta = 45°$

Given $h = Ax - Bx^2$

Comparing with, $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

We get

$A = \tan\theta = \tan 45° = 1$

and $B = \frac{b}{2u^2\cos^2\theta} = \frac{10}{2(20)^2\left(\frac{1}{2}\right)} = \frac{1}{40}$

$\Rightarrow \frac{A}{B} = \frac{40}{1}$

Hence, the correct answer is (D).

121. $v_x = \frac{dx}{dt} = 2 + 8t$

$\Rightarrow a_x = \frac{dv_x}{dt} = 8$

$v_y = \frac{dy}{dt} = 5$

$\Rightarrow a_y = \frac{dv_y}{dt} = 0$

Hence, the correct answer is (C).

122. $v_x = \frac{dx}{dt} = v_0$

$\Rightarrow x = v_0 t$

Since, $v_y = \frac{dy}{dt} = a\omega\cos(\omega t)$

$\Rightarrow y = \int_0^t a\omega\cos(\omega t)dt = \frac{a\omega\sin(\omega t)}{\omega}$

$\Rightarrow y = a\sin(\omega t) = a\sin\left(\frac{\omega x}{v_0}\right)$

Hence, the correct answer is (C).

123. $\frac{\frac{1}{2}mu_1^2\cos^2\theta_1}{\frac{1}{2}mu_2^2\cos^2\theta_2} = \frac{4}{1}$

$\Rightarrow \frac{u_1\cos\theta_1}{u_2\cos\theta_2} = 2$ ...(1)

and $\frac{u_1^2\sin^2\theta_1}{u_2^2\sin^2\theta_2} = \frac{4}{1}$

$\Rightarrow \frac{u_1\sin\theta_1}{u_2\sin\theta_2} = \frac{2}{1}$ ...(2)

From equations (1) and (2), we get

$\frac{(u_1\sin\theta_1)(u_1\cos\theta_1)}{(u_2\sin\theta_2)(u_2\cos\theta_2)} = \frac{4}{1}$

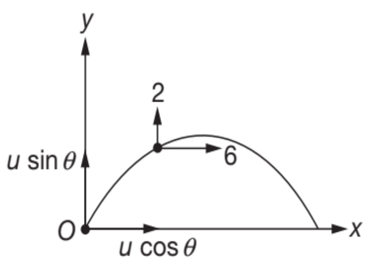
$$\Rightarrow \frac{\frac{gR_1}{2}}{\frac{gR_2}{2}} = \frac{4}{1}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{4}{1}$$

Hence, the correct answer is (B).

124. $2 = \sqrt{u^2\sin^2\theta - 2hg}$

$$\Rightarrow 4 = u^2\sin^2\theta - 2(0.4)(10)$$



$$\Rightarrow u^2\sin^2\theta = 12$$

$$\Rightarrow u\sin\theta = 2\sqrt{3} \text{ and } u\cos\theta = 6$$

$$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = 30°$$

Hence, the correct answer is (C).

125. $u_x = 14.7 \text{ ms}^{-1}$,

$u_y = 9.8 \times 2 = 19.6 \text{ ms}^{-1}$

$$\Rightarrow u = \sqrt{(14.7)^2 + (19.6)^2} = 24.5 \text{ ms}^{-1}$$

$$\Rightarrow \tan\theta = \frac{19.6}{14.7} = \frac{4}{3}$$

$$\Rightarrow \theta = 53°$$

Hence, the correct answer is (D).

126. Since, both projectiles have equal height, so

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{u_y^2}{2g}$$

So, time of flight of both will be same.

$$\Rightarrow T_1 = T_2 = \frac{2u\sin\theta}{g} = \frac{2u_y}{g}$$

Since Range is greater for second, i.e.

$$R_2 > R_1$$

Thus $(u_x)_2 > (u_x)_1$

$$\Rightarrow u_2 > u_1$$

Hence, the correct answer is (D).

Multiple Correct Choice Type Questions

1. Acceleration of the bead down the wire is $g\cos\theta$.

Hence $v^2 = 0^2 + 2(g\cos\theta)(2R\cos\theta)$

Where $P_1P_2 = 2R\cos\theta$

$$\Rightarrow v = 2\left(\sqrt{gR}\right)\cos\theta$$

Since, $t = \frac{v}{a} = \frac{2\sqrt{gR}\cos\theta}{g\cos\theta} = 2\sqrt{\frac{R}{g}}$

Hence, (B), (C) and (D) are correct.

2. $y = x\tan\theta\left(1 - \frac{x}{R}\right)$ ...(1)

where $x = (u\cos\theta)t$ and $R = (u\cos\theta)T$ ...(2)

Also, we observe that

$$R = \frac{2u^2\sin\theta\cos\theta}{g} \text{ and } H = \frac{u^2\sin^2\theta}{2g}$$

$$\Rightarrow \frac{H}{R} = \frac{\sin^2\theta}{4\sin\theta\cos\theta} = \frac{1}{4}\tan\theta$$

$$\Rightarrow \tan\theta = \frac{4H}{R} \quad ...(3)$$

So, we get

$$y = \frac{4Hx}{R}\left(1 - \frac{x}{R}\right)$$

Substitute x and R from (2), we get

$$y = 4H\left(\frac{t}{T}\right)\left(1 - \frac{t}{T}\right)$$

Hence, (A) and (C) are correct.

3. $\vec{u} = (u\cos\theta)\hat{i} + (u\sin\theta)\hat{j}$ ...(1)

Let the velocity at point Q, at time t be $\vec{v}$, then

$$\vec{v} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$$

Since $\vec{v} \perp \vec{u}$, so

$$\vec{v}\cdot\vec{u} = 0$$

$$\Rightarrow u^2 - ugt\sin\theta = 0$$

$$\Rightarrow \quad t = \frac{u}{g\sin\theta} = \left(\frac{u}{g}\right)\text{cosec}\,\theta$$

Speed of the particle at Q is $|\vec{v}|$, where

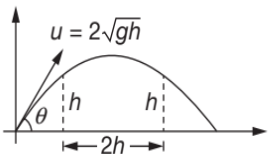
$$|\vec{v}| = \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$$

Substituting the value of t, we get

$$|\vec{v}| = u\cot\theta$$

Hence, (C) and (D) are correct.

4.



$$h = 2\sqrt{hg}\,(\sin\theta)t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad t^2 - 4\sqrt{\frac{h}{g}}(\sin\theta)t + \frac{2h}{g} = 0$$

$$\Rightarrow \quad t = \frac{4\sqrt{\frac{h}{g}} \pm \sqrt{16\left(\frac{h}{g}\right)\sin^2\theta - \frac{8h}{g}}}{2}$$

$$\Rightarrow \quad t = \frac{4\sqrt{\frac{h}{g}} \pm \sqrt{\frac{8h}{g}}\sqrt{2\sin^2\theta - 1}}{2}$$

$$\Rightarrow \quad t_1 = \frac{4\sqrt{\frac{h}{g}} - \sqrt{\frac{8h}{g}}\sqrt{2\sin^2\theta - 1}}{2}$$

$$\Rightarrow \quad t_2 = \frac{4\sqrt{\frac{h}{g}} + \sqrt{\frac{8h}{g}}\sqrt{2\sin^2\theta - 1}}{2}$$

$$\Rightarrow \quad (t_2 - t_1) = \sqrt{\frac{8h}{g}}\sqrt{2\sin^2\theta - 1}$$

Further horizontal distance travelled during this time is $2h$

$$\Rightarrow \quad 2h = 2\sqrt{hg}\cos\theta(t_2 - t_1)$$

$$\Rightarrow \quad 2h = 2\sqrt{hg}\cos\theta\sqrt{\frac{8h}{g}}\sqrt{2\sin^2\theta - 1}$$

Solving we get $\cos\theta = \frac{1}{2}$

$\theta = 60^\circ$ with horizontal or 30° with vertical

$$\Rightarrow \quad (t_2 - t_1) = 2\sqrt{\frac{h}{g}}$$

Hence, (A) and (D) are correct.

5. $\vec{u} = 3\hat{i}$ and $\vec{a} = -\hat{i} - 0.5\hat{j}$

$$\Rightarrow \quad u_x = 3\text{ ms}^{-1},\ a_x = -1\text{ ms}^{-2} \text{ and } a_y = -0.5\text{ ms}^{-2}$$

Since u_x and a_x have opposite sign, so firstly the particle must reverse its direction of motion. Let this be done when the coordinate of the particle is x. Then

$$0^2 - u_x^2 = 2a_x x$$

$$\Rightarrow \quad -(3)^2 = 2(-1)x$$

$$\Rightarrow \quad x = 4.5\text{ m}$$

Also, this will happen at say time t, then

$$v_x = u_x + a_x t \text{ gives, } 0 = 3 + (-1)t$$

$$\Rightarrow \quad t = 3\text{ s}$$

At this time t, the y coordinate of the particle is given by

$$y = u_y t + \frac{1}{2}a_y t^2$$

$$\Rightarrow \quad y = 0 + \frac{1}{2}(-0.5)(3)^2$$

$$\Rightarrow \quad y = -\frac{9}{4}m = -2.25\text{ m}$$

Also, $v_y = 0 + a_y t = -(0.5)(3) = -1.5\text{ ms}^{-1}$

$$\Rightarrow \quad \vec{v} = v_x\hat{i} + v_y\hat{j} = 0\hat{i} + (-1.5)\hat{j} = -1.5\hat{j}\text{ ms}^{-1}$$

and $\vec{r} = x\hat{i} + y\hat{j} = \left(4.5\hat{i} - 2.25\hat{j}\right)$ m

Hence, (A) and (D) are correct.

7. Since the particles have same ranges.

Hence $\alpha + \beta = \frac{\pi}{2}$

$$R = \frac{2u^2\sin\alpha\cos\alpha}{g} = \frac{2u^2\sin\beta\cos\beta}{g}$$

$$h_1 = \frac{u^2\sin^2\alpha}{2g}$$

$$h_2 = \frac{u^2\sin^2\beta}{2g} = \frac{u^2\cos^2\alpha}{2g}$$

$$\Rightarrow \quad R = 4\sqrt{h_1h_2}$$

$$t_1 = \frac{2u\sin\alpha}{g}$$

$$t_2 = \frac{2u\sin\beta}{g} = \frac{2u\cos\alpha}{g}$$

$$\Rightarrow \quad \frac{t_1}{t_2} = \tan\alpha \text{ and } \frac{h_1}{h_2} = \tan^2\alpha$$

$\Rightarrow \quad \tan\alpha = \sqrt{\dfrac{h_1}{h_2}}$

Hence, (A), (B), (C) and (D) are correct.

8. $x = a\cos(pt)$

$y = b\sin(pt)$

$\Rightarrow \quad \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$\Rightarrow$ Trajectory is an Ellipse

$\Rightarrow \quad \ddot{x} = -ap^2\cos(\omega t)$

$\Rightarrow \quad \ddot{x} = -p^2 x$

Similarly $\ddot{y} = -p^2 y$

so acceleration is directed towards the focus.

Hence, (A), (B) and (C) are correct.

9. $0 = \sqrt{2gh}(\cos\alpha)t - \dfrac{1}{2}(g\cos\alpha)t^2$

$\Rightarrow \quad t = \sqrt{\dfrac{8h}{g}}$ after it strikes the incline at A

$s = \sqrt{2gh}(\sin\alpha)t + \dfrac{1}{2}(g\sin\alpha)t^2$

Substituting value of t, we get $s = 8h\sin\alpha$

Hence, (A) and (D) are correct.

10. $\vec{u} = 4\hat{i} + 3\hat{j} + 3\sqrt{3}\hat{k}$

$\Rightarrow \quad |\vec{u}| = 2\sqrt{13}\ \text{ms}^{-1}$

Velocity at highest point is $\vec{v} = 4\hat{i} + 3\hat{j}$

{because at the highest point $v_Z = 0$}

$\Rightarrow \quad |\vec{v}| = 5\ \text{ms}^{-1}$

Hence, (B) and (D) are correct.

11. Let shells collide in mid-air at point $P(x, y)$.

For the 1st shell

$x = (u\cos\alpha)T \qquad \ldots(1)$

$y = (u\sin\alpha)T - \dfrac{1}{2}gT^2 \qquad \ldots(2)$

For the 2nd shell

$x = u\cos\beta(T - t) \qquad \ldots(3)$

$y = (u\sin\beta)(T - t) - \dfrac{1}{2}g(T - t)^2 \qquad \ldots(4)$

Equating (1) and (3), we get

$T\cos\alpha = (T - t)\cos\beta$ {OPTION **(A)**}

$\Rightarrow \quad \dfrac{\cos\alpha}{\cos\beta} = 1 - \dfrac{t}{T}$

$\Rightarrow \quad \dfrac{\cos\alpha}{\cos\beta} < 1$

$\Rightarrow \quad \cos\alpha < \cos\beta$

$\Rightarrow \quad \alpha > \beta$ {OPTION **(B)**}

Equating (2) and (4), we get

$(u\sin\alpha)T - \dfrac{1}{2}gT^2 = (u\sin\beta)(T - t) - \dfrac{1}{2}g(T - t)^2$

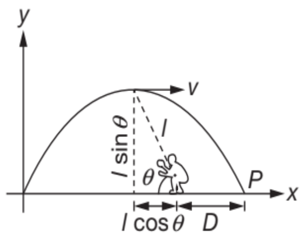
{OPTION **(D)**}

Hence, (A), (B) and (D) are correct.

13. Let the projectile reach the point of impact in time t. Then

$l\sin\theta = \dfrac{1}{2}gt^2$

$\Rightarrow \quad t = \sqrt{\dfrac{2\ell\sin\theta}{g}} \qquad \ldots(1)$



Further

$l\cos\theta + D = vt$

$\Rightarrow \quad D = vt - l\cos\theta$

$\Rightarrow \quad D = v\sqrt{\dfrac{2l\sin\theta}{g}} - l\cos\theta \qquad \ldots(2)$

For projectile to pass over the observer's head

$D > 0$

$\Rightarrow \quad v\sqrt{\dfrac{2l\sin\theta}{g}} > l\cos\theta$

$\Rightarrow \quad v^2\left(\dfrac{2l\sin\theta}{g}\right) > l^2\cos^2\theta$

$\Rightarrow \quad l < \dfrac{2v^2\tan\theta\sec\theta}{g}$

Hence, (B) and (C) are correct.

14. Since, we know that the range is same for complimentary angles i.e., $R_\theta = R_{90-\theta} = \dfrac{2u^2\sin\theta\cos\theta}{g}$

Then, $T_1 = \frac{2u\sin\theta}{g}$ and $T_2 = \frac{2u\cos\theta}{g}$

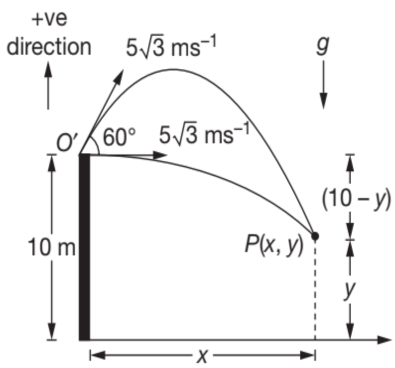
$\Rightarrow \quad T_1T_2 \propto R$ and

$$\frac{T_1}{T_2} = \tan\theta$$

Hence, (C) and (D) are correct.

15. Let the second shell reach the point $P(x, y)$ at time t_2 and the first shell be fired at a time

$$(t_2 - \Delta t) = t_1.$$



For shell 1

$$x = (5\sqrt{3})t_1 \quad \ldots(1)$$

$$-(10 - y) = -\frac{1}{2}gt_1^2 \quad \ldots(2)$$

For shell 2

$$x = (5\sqrt{3}\cos 60)t_2 \quad \ldots(3)$$

$$-(10 - y) = (5\sqrt{3}\sin 60)t_2 - \frac{1}{2}gt_2^2 \quad \ldots(4)$$

Equating (1) and (3), we get

$$t_2 = 2t_1 \quad \ldots(5)$$

Equating (2) and (4), we get

$$-\frac{1}{2}gt_1^2 = (5\sqrt{3}\sin 60)t_2 - \frac{1}{2}gt_2^2$$

$$\Rightarrow \quad -\frac{1}{2}gt_1^2 = \frac{15}{2}(2t_1) - \frac{1}{2}g(2t_1)^2$$

$$\Rightarrow \quad -\frac{1}{2}gt_1^2 = 15t_1 - 2gt_1^2$$

$$\Rightarrow \quad -5t_1^2 = 15t_1 - 20t_1^2$$

$$\Rightarrow \quad 15t_1^2 = 15t_1$$

$$\Rightarrow \quad 15t_1^2 - 15t_1 = 0$$

$$\Rightarrow \quad 15t_1(t_1 - 1) = 0$$

$$\Rightarrow \quad t_1 = 1s$$

$$\Rightarrow \quad t_2 = 2s \text{ i.e. } \Delta t = 1s$$

$$\Rightarrow \quad x = 5\sqrt{3}\ m \text{ and } y = 5\ m$$

Hence, (A), (B), (C) and (D) are correct.

16. Since $R = \frac{u^2\sin(2\theta)}{g}$

$$\Rightarrow \quad R = kg^{-1} \quad \ldots(1)$$

$$\Rightarrow \quad dR = -kg^{-2}dg \quad \ldots(2)$$

$$\Rightarrow \quad \frac{dR}{R} = \frac{-kg^{-2}dg}{kg^{-1}}$$

$$\Rightarrow \quad \frac{dR}{R} = -\frac{dg}{g} = -\left(\frac{g_p - g_e}{g}\right)$$

$$\Rightarrow \quad \text{Fractional Decrement} = \frac{dR}{R} = \frac{1}{291}$$

Range at equator is $R_e = \frac{u^2\sin(2\theta)}{g_e}$

Range at pole is $R_p = \frac{u^2\sin(2\theta)}{g_p}$

$$R = \left(\begin{array}{c}\text{Original range with}\\ \text{no variation of } g\\ \text{taken into account}\end{array}\right) = \frac{u^2\sin(2\theta)}{g}$$

$$\frac{dR}{R} = \left(\begin{array}{c}\text{Fractional}\\ \text{Decrement}\end{array}\right) = \frac{R_e - R_p}{R}$$

$$\Rightarrow \quad \frac{dR}{R} = \frac{u^2\sin(2\theta)\left(\frac{1}{g_e} - \frac{1}{g_p}\right)}{u^2\sin(2\theta)\frac{1}{g}}$$

$$\Rightarrow \quad \frac{dR}{R} = -g\left(\frac{1}{g_p} - \frac{1}{g_e}\right)$$

Hence, (B) and (D) are correct.

17. For shell to hit the boat the time lapsed equals the time of flight of the shell $= \frac{2v_2\sin\alpha}{g}$

$$\Rightarrow \quad \text{Distance travelled by boat} = v_1\left(\frac{2v_2\sin\alpha}{g}\right)$$

Distance of boat from the cannon at the instant the shell is fired is $x = R - \left(\frac{2v_1v_2\sin\alpha}{g}\right)$

$$\Rightarrow \quad x=\frac{2}{g}(v_2\sin\alpha)(v_2\cos\alpha)-\frac{2v_1v_2\sin\alpha}{g}$$

$$\Rightarrow \quad x=\frac{2}{g}(v_2\sin\alpha)(v_2\cos\alpha-v_1)$$

and the distance of boat from the cannon when the shell hits the boat is equal to the range of the shell

$$R=\frac{2}{g}(v_2\sin\alpha)(v_2\cos\alpha)$$

Hence, (A), (B), (C) and (D) are correct.

18. $u_x=u\cos(45^\circ)=40\text{ ms}^{-1}$ and

$u_y=u\sin(45^\circ)=40\text{ ms}^{-1}$

At $t=2$ s,

$$v_x=u_x=40\text{ ms}^{-1} \qquad \ldots(1)$$

$$v_y=u_y-gt=40-(10)(2)=20\text{ ms}^{-1}$$

So, (B) is correct

Further $x=u_xt=80$ m

and $y=u_yt+\frac{1}{2}a_yt^2$

$$\Rightarrow \quad y=(40)(2)+\frac{1}{2}(-10)(4)$$

$$\Rightarrow \quad y=60\text{ m}$$

Hence (D) is correct

So, displacement of the particle is $r=\sqrt{x^2+y^2}$

$$\Rightarrow \quad r=\sqrt{(80)^2+(60)^2}=100\text{ m}$$

Hence (A) is also correct.

Also, we observe that if α is the angle made by v with the horizontal, then

$$\tan\alpha=\frac{v_y}{v_x}=\frac{20}{40}=\frac{1}{2}$$

Hence (C) is not correct.

Hence, (A), (B) and (D) are correct.

19. $x_0+ut=vt$

$$\Rightarrow \quad (6\hat{i}+8\hat{j})+(2t)\hat{i}=(a\hat{i}+b\hat{j})t$$

At $t=2$ s, we get

$$\Rightarrow \quad 10\hat{i}+8\hat{j}=2a\hat{i}+2b\hat{j}$$

$$\Rightarrow \quad 2a=10 \text{ and } 2b=8$$

$$\Rightarrow \quad a=5 \text{ and } b=4$$

Hence, (B) and (C) are correct.

20. $y=h-x\tan\alpha \qquad \ldots(1)$

Further $y=(v_0\sin\theta)t-\frac{1}{2}gt^2 \qquad \ldots(2)$

and $x=(v_0\cos\theta)t \qquad \ldots(3)$

Equating (1) and (2) and put value of x from (3) to get

$$\Rightarrow \quad t^2-\frac{2v_0}{g}(\cos\theta\tan\alpha+\sin\theta)t+\frac{2h}{g}=0$$

$$\Rightarrow \quad t^2-\left[\frac{2v_0\sin(\theta+\alpha)}{g\cos\alpha}\right]t+\frac{2h}{g}=0 \qquad \ldots(4)$$

For t to be MINIMUM

$$\frac{dt}{d\theta}=0$$

$$\Rightarrow \quad 2t\frac{dt}{d\theta}-\frac{2v_0}{g\cos\alpha}\frac{d}{d\theta}[t\sin(\alpha+\theta)]+0=0$$

$$\Rightarrow \quad 0-\frac{2v_0}{g\cos\alpha}\left[\sin(\alpha+\theta)\frac{dt}{d\theta}+t\cos(\alpha+\theta)\right]=0$$

$$\Rightarrow \quad 0-\frac{2v_0t\cos(\alpha+\theta)}{g\cos\alpha}=0 \qquad \left\{\because \frac{dt}{d\theta}=0\right\}$$

$$\Rightarrow \quad \cos(\theta+\alpha)=0 \qquad \{\because t\neq 0\}$$

$$\Rightarrow \quad \theta+\alpha=\frac{\pi}{2} \qquad \text{\{OPTION (A)\}}$$

Substituting $\theta+\alpha=\frac{\pi}{2}$ in (4), we get

$$t_{\text{min}}^2-\frac{2v_0}{g\cos\alpha}t_{\text{min}}+\frac{2h}{g}=0$$

$$\Rightarrow \quad t_{\text{min}}=\frac{v_0-\sqrt{v_0^2-2gh\cos^2\alpha}}{g\cos\alpha} \qquad \text{\{OPTION (C)\}}$$

For t_{min} to be real $v_0^2>2gh\cos^2\alpha$

$$\Rightarrow \quad v_0>\sqrt{2gh}\cos\alpha$$

So if $v_0<\sqrt{2gh}\cos\alpha$, then projectile never reaches the roof. {OPTION **(D)**}

Hence, (A), (C) and (D) are correct.

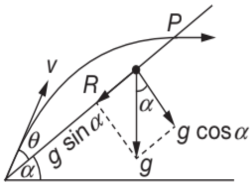
21. As studied and discussed in theory, we observe that all (A), (B), (C) and (D) are correct.

Hence, (A), (B), (C) and (D) are correct.

22. The vertical velocity at P is zero.

Hence $v\sin(\theta+\alpha)-gt=0$

$$\Rightarrow \quad t=\frac{v\sin(\theta+\alpha)}{g} \qquad \ldots(1)$$



Since displacement perpendicular to the plane is zero hence

$$0=(v\sin\theta)t+\frac{1}{2}(-g\cos\alpha)t^2$$

$\Rightarrow \quad t=\dfrac{2v\sin\theta}{g\cos\alpha}$ ...(2)

Equating (1) & (2) and rearranging, we get

$$\frac{v\sin(\theta+\alpha)}{g}=\frac{2v\sin\theta}{g\cos\alpha}$$

$\Rightarrow \quad 2\sin\theta=\cos\alpha(\sin\theta\cos\alpha+\cos\theta\sin\alpha)$

$\Rightarrow \quad 2\sin\theta=\cos^2\alpha\sin\theta+\cos\theta\sin\alpha\cos\alpha$

$\Rightarrow \quad (2-\cos^2\alpha)\sin\theta=\cos\theta\sin\alpha\cos\alpha$

$\Rightarrow \quad \tan\theta=\dfrac{\sin\alpha\cos\alpha}{2-\cos^2\alpha}$

$\Rightarrow \quad \tan\theta=\dfrac{\sin\alpha\cos\alpha}{(1-\cos^2\alpha)+1}=\dfrac{\sin\alpha\cos\alpha}{1+\sin^2\alpha}$

$\Rightarrow \quad \tan\theta=\dfrac{\tan\alpha}{1+2\tan^2\alpha}$

Hence, (A) and (D) are correct.

23. Time of ascent $=2+1=3$ s

If α is the angle of projection then

$$\frac{u\sin\alpha}{g}=3$$

$\Rightarrow \quad u\sin\alpha=30$ ...(1)

If $\beta=30^\circ$ is the angle which projectile makes with horizontal at $t=2$ s and v is the velocity at this instant of time then

$v\cos\beta=u\cos\alpha$ ...(2)

and $v\sin\beta=u\sin\alpha-g(2)$

$\Rightarrow \quad v\sin\beta=30-20=10$

$\Rightarrow \quad v\sin 30=10$

$\Rightarrow \quad v=20\ \text{ms}^{-1}$

Substituting $\beta=30^\circ$ and $v=20\ \text{ms}^{-1}$ in (2), we get

$$v\frac{\sqrt{3}}{2}=u\cos\alpha$$

$u\cos\alpha=10\sqrt{3}$ ...(3)

Squaring (1) & (3) and adding

$$u^2=1200$$

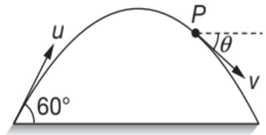
$\Rightarrow \quad u=20\sqrt{3}\ \text{ms}^{-1}$

and $\tan\alpha=\sqrt{3}$

$\Rightarrow \quad \alpha=60^\circ$ with horizontal or 30° with vertical

Hence, (B) and (C) are correct.

24. At the instant shown, we have



$$v\cos\theta=u\cos(60^\circ)=\frac{u}{2}$$

$\Rightarrow \quad v=\dfrac{u}{2\cos\theta}$ ...(1)

The radius of curvature is given by

$$R=\frac{v^2}{g\cos\theta}=\frac{u^2}{4g\cos^3\theta}$$

$\Rightarrow \quad R_{min}=\dfrac{u^2}{4g}$, when $\theta=0^\circ$

Since, $R=\dfrac{8}{3\sqrt{3}}R_{min}$

$\Rightarrow \quad \dfrac{u^2}{4g\cos^3\theta}=\dfrac{8}{3\sqrt{3}}\left(\dfrac{u^2}{4g}\right)$

$\Rightarrow \quad \cos^3\theta=\dfrac{3\sqrt{3}}{8}=\left(\dfrac{\sqrt{3}}{2}\right)^3$

$\Rightarrow \quad \cos\theta=\dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad \theta=30^\circ$

So, from equation (1), we get

$$v=\frac{u}{\sqrt{3}}$$

Hence, (B) and (C) are correct.

25. $y=3x+4$

$\Rightarrow \quad \dfrac{dy}{dt}=\dfrac{3dx}{dt}$

$\Rightarrow \quad v_y=3v_x=3$ $\{\because v_2=1\ \text{ms}^{-1}\}$

$$\Rightarrow \quad \frac{dy_y}{at} = \frac{3dv_x}{dt}$$

$$\Rightarrow \quad a_y = 3a_x = 3 \qquad \{\because a_x = 1\ \text{ms}^{-1}\}$$

nothing can be said about velocity and acceleration as the given values are at particular instant.

Hence, (A) and (B) are correct.

26. $x = 2 + 2t + 4t^2$ and $y = 4t + 8t^2$

$$\Rightarrow \quad y = 2 + 2x$$

So, the particle moves along the straight line.

Since, $\frac{dx}{dt} = 2 + 8t$ and $\frac{dy}{dt} = 4 + 16t$

$$\Rightarrow \quad \frac{d^2x}{dt^2} = 8 \text{ and } \frac{d^2y}{dt^2} = 16$$

$$\Rightarrow \quad \vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = (2+8)\hat{i} + (4+16t)\hat{j}$$

$$\Rightarrow \quad \vec{a} = 8\,\hat{i} + 16\,\hat{j} = \text{(constant)}$$

Hence, (A) and (B) are correct.

27. $\vec{v} = 2t\,\hat{i} + t^2\,\hat{j}$

$$\Rightarrow \quad \frac{d\vec{v}}{dt} = 2\hat{i} + 2t\,\hat{j}$$

$$\Rightarrow \quad |\vec{a}_{net}| = \sqrt{2^2 + 2^2} = 2\sqrt{2}$$

Since, tangential acceleration is defined as the rate of change of speed, which is

$$|\vec{v}| = \sqrt{(2t)^2 + (t^2)^2} = \sqrt{4t^2 + t^4}$$

So, $a_T = \frac{d|\vec{v}|}{dt} = \frac{1}{2\sqrt{4t^2 + t^4}}(8t + 4t^3)$

$$\Rightarrow \quad a_T = \frac{1}{2 \times \sqrt{5}} \times 12 = \frac{6}{\sqrt{5}}\ \text{ms}^{-2}$$

Since, we know that, for a particle moving in a curvilinear path with variable speed, the particle possesses both tangential acceleration (a_T) and centripetal acceleration (a_C), so we have

$$a_{\text{net}}^2 = a_C^2 + a_T^2$$

$$\Rightarrow \quad a_C^2 = a_{\text{net}}^2 - a_T^2$$

$$\Rightarrow \quad a_C^2 = \left(2\sqrt{2}\right)^2 - \left(\frac{6}{\sqrt{5}}\right)^2 = 8 - \frac{36}{5} = \frac{4}{5}$$

$$\Rightarrow \quad a_C = \frac{2}{\sqrt{5}}\ \text{ms}^{-2}$$

Hence, (B) and (C) are correct.

Reasoning Based Questions

1. $R = H$

$$\Rightarrow \quad \frac{2u^2 \sin\theta\cos\theta}{g} = \frac{u^2 \sin^2\theta}{2g}$$

$$\Rightarrow \quad \tan\theta = 4$$

$$R_{\max}\big|_{\theta = 45^\circ} = \frac{u^2}{g}$$

Hence, the correct answer is (B).

2. In case of non-uniform circular motion, net acceleration will not be directed towards centre.

Hence, the correct answer is (D).

3. Direction of acceleration changes.

Hence, the correct answer is (D).

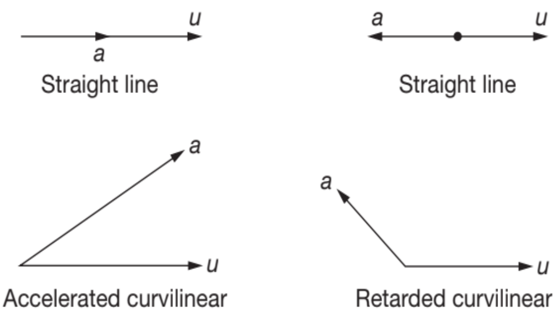
4. The particle will move in a parabolic path till acceleration (due to gravity) is constant. For this the particle should be near the surface of the Earth and air resistance should be negligible.

Hence, the correct answer is (D).

5. In uniform circular motion direction of acceleration is always along the radial direction. As particle is rotating so radial vector keeps on changing.

Hence, the correct answer is (D).

6.



Hence, the correct answer is (A).

7. In uniform circular motion, the magnitude of velocity and acceleration remains same, but due to change in direction of motion, the direction of velocity and acceleration changes. Furthermore, the centripetal acceleration is given by $a = \omega^2 r$.

Hence, the correct answer is (B).

8. For the observer sitting in train, the initial horizontal velocity of coin and train are same, thus observer will find its path as a straight line. Whereas the observer on the ground will see the coin to follow a parabolic path (as of a horizontal projectile).

Hence, the correct answer is (D).

9. $u_x = 0$, $a_x = 0$, $u_y = u\sin\theta$ and $a_y = -g$

$\Rightarrow \quad v_x = u_x + a_x T = u_x \qquad \left\{\text{where } T = \dfrac{2u\sin\theta}{g}\right\}$

Also $v_y = u\sin\theta - gt$

$\Rightarrow \quad v_y = u\sin\theta - g\left(\dfrac{2u\sin\theta}{g}\right)$

$\Rightarrow \quad v_y = -u\sin\theta$

Hence, the correct answer is (A).

10. $H = \dfrac{u^2\sin^2\theta}{2g}$ i.e., it is independent of mass of projectile.

Hence, the correct answer is (A).

11. As $T = \dfrac{2u\sin\theta}{g}$

$\Rightarrow \quad T' = \dfrac{2nu\sin\theta}{g}$

$\Rightarrow \quad T' = nT$

$R = u\cos\theta T$

$\Rightarrow \quad R' = (nu\cos\theta)(nT)$

$\Rightarrow \quad R' = n^2 R$

Hence, the correct answer is (C).

12. Since acceleration due to gravity acts vertically downwards, so it has no component along the horizontal and hence does not affect the horizontal velocity.

Hence, the correct answer is (A).

13. Since, $R_{max} = \dfrac{u^2}{g} \qquad \{\text{For } \theta = 45°\}$

$\Rightarrow \quad H = \dfrac{u^2\sin^2(45°)}{2g} = \dfrac{u^2}{4g}$

So, H is 25% of range.

Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. $x = kt$, $y = kt(1-\alpha t)$

Equation of trajectory,

$$y = kt - k\alpha t^2 = k\left(\frac{x}{k}\right) - k\alpha\left(\frac{x^2}{k^2}\right)$$

$\Rightarrow \quad y = x - \alpha\dfrac{x^2}{k} \rightarrow \text{parabola}$

Hence, the correct answer is (C).

2. $v_x = \dfrac{dx}{dt} = k$

$v_y = \dfrac{dy}{dt} = k - 2k\alpha t$

$v_y = k(1-2\alpha t)$

$\Rightarrow \quad \vec{v} = \hat{i}v_x + \hat{j}v_y = \hat{i}k + \hat{j}k(1-2\alpha t)$

$\Rightarrow \quad |\vec{v}| = v = \sqrt{k^2 + k^2(1-2\alpha t)^2}$

$\Rightarrow \quad v = k\sqrt{1+(1-2\alpha t)^2}$

$\Rightarrow \quad v^2 = k^2\left[1+(1-2\alpha t)^2\right]$

For v to be MINIMUM OR v^2 to be MINIMUM, we have

$$\frac{d}{dt}(v^2) = 0$$

$\Rightarrow \quad t = \dfrac{1}{2\alpha}$

Hence, the correct answer is (B).

3. Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \hat{j}(-2\alpha)k = -(2k\alpha)\hat{j} = \text{constant}$$

Hence, the correct answer is (D).

4. Let t_0 be the instant when $\vec{v}$ and $\vec{a}$ make an angle $\dfrac{\pi}{4}$ with each other; the condition is

$$\vec{v}\cdot\vec{a} = va\cos\left(\frac{\pi}{4}\right)$$

$\Rightarrow \quad 0 + k(1-2\alpha t)(-2\alpha k) = k\sqrt{1-(1-2\alpha t)^2}\,(-2\alpha k)\cdot\dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad \sqrt{2}(1-2\alpha t_0) = \sqrt{1+(1-2\alpha t_0)^2}$

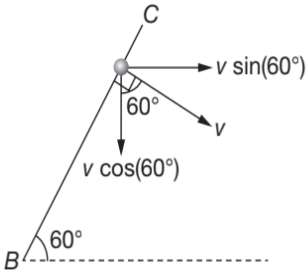
$\Rightarrow \quad t_0 = \dfrac{1}{\alpha}$

Hence, the correct answer is (A).

5. Let v be the velocity at the time of collision.

Then, $u\sqrt{2}\cos 45° = v\sin 60°$

$\Rightarrow \quad (u\sqrt{2})\left(\dfrac{1}{\sqrt{2}}\right) = \dfrac{\sqrt{3}v}{2}$



$$\Rightarrow \quad v = \frac{2}{\sqrt{3}} u$$

Hence, the correct answer is (D).

6. Substituting the proper values in $v_y = u_y + a_x t$, we get,

$$-v\cos 60° = \left(u\sqrt{2}\sin 45°\right) - gt$$

$$\Rightarrow \quad t = \frac{u}{g}\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$$

Hence, the correct answer is (C).

7. Since $\vec{v} = 20\hat{i} + 10\hat{j}$

$$\Rightarrow \quad v_x = u_x = 20 \text{ ms}^{-1}$$

Since, $v_y = 10 \text{ ms}^{-1}$ and $v_y^2 - u_y^2 = 2a_y y$

$$\Rightarrow \quad (10)^2 - u_y^2 = 2(-10)(15)$$

$$\Rightarrow \quad u_y^2 = 400$$

$$\Rightarrow \quad u_y = 20 \text{ ms}^{-1}$$

So, $u = \sqrt{u_x^2 + u_y^2} = 20\sqrt{2} \text{ ms}^{-1}$

Hence, the correct answer is (B).

8. Since $\tan\theta = \dfrac{u_y}{u_x} = \dfrac{20}{20} = 1$

$$\Rightarrow \quad \theta = 45° = \frac{\pi}{4} \text{ radian}$$

Hence, the correct answer is (B).

9. $H = \dfrac{u_y^2}{2g} = \dfrac{400}{20} = 20 \text{ m}$

$$R = \frac{u^2 \sin(2\theta)}{g} = \frac{\left(20\sqrt{2}\right)^2 \sin(90)}{10}$$

$$\Rightarrow \quad R = 80 \text{ m}$$

Now, the coordinate where the maximum height is attained is $\left(\dfrac{R}{2}, H\right)$. So, we get $(40, 20)$ m as the answer.

Hence, the correct answer is (D).

10. For minimum horizontal velocity, i.e., for the particle just to land at A, we have

$$x = 3a \text{ and } y = 4a$$

$$\Rightarrow \quad 4a = g\frac{(3a)^2}{2u_{MIN}^2} \qquad \left\{\because y = \frac{gx^2}{2u^2}\right\}$$

$$\Rightarrow \quad u_{MIN}^2 = \frac{9}{8}ag$$

$$\Rightarrow \quad u_{MIN} = \sqrt{\frac{9}{8}ag}$$

Hence, the correct answer is (A).

11. Since $y = \dfrac{1}{2}a_y t^2$

$$\Rightarrow \quad 4a = \frac{1}{2}gt^2$$

$$\Rightarrow \quad t = \sqrt{\frac{8a}{g}}$$

Hence, the correct answer is (D).

12. For maximum horizontal, velocity i.e., when the particle lands at B, we have

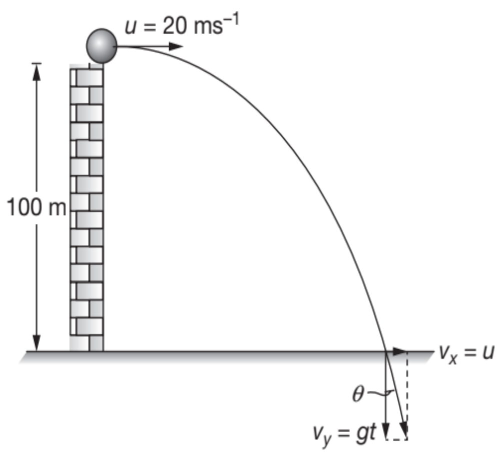
$$x = 3a + 2a = 5a \text{ and } y = 4a$$

So, $4a = \dfrac{g(5a)^2}{2u_{MAX}^2}$

$$\Rightarrow \quad u_{MAX} = \sqrt{\frac{25}{8}ag}$$

Hence, the correct answer is (B).

13. $100 = \dfrac{1}{2}gt^2 \qquad \left\{\because y = \dfrac{1}{2}gt^2\right\}$



$\Rightarrow \quad t = \sqrt{\frac{200}{10}} = \sqrt{20} = 2\sqrt{5}$ s

Hence, the correct answer is (C).

14. $x = ut = 20(2\sqrt{5})$

$\Rightarrow \quad x = 40\sqrt{5}$ m

Hence, the correct answer is (B).

15. $\tan\theta = \frac{v_x}{v_y}$

Since $v_x = u$ and $v_y = gt$

$\Rightarrow \quad \tan\theta = \frac{u}{gt} = \frac{20}{(10)(2\sqrt{5})}$

$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{5}}$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$

Hence, the correct answer is (C).

16. $x = 6t \quad y = 8t - 5t^2$

Compare with equations of motion of the projectile i.e.

$x = (u\cos\theta)t$ and $y = (u\sin\theta)t - \frac{1}{2}gt^2$, we get

$u_x = u\cos\theta = 6$ and $u_y = u\sin\theta = 8$

and $g = 10\text{ ms}^{-2}$ (acting along $-y$ axis)

Hence, the correct answer is (C).

17. Since acceleration has no component along x-axis, so $u_x = \text{constant} = 6\text{ ms}^{-1}$.

Hence, the correct answer is (C).

18. $u_y = u\sin\theta = 8\text{ ms}^{-1}$

Hence, the correct answer is (C).

19. The velocity of projection is

$u = \sqrt{u_x^2 + u_y^2}$

$\Rightarrow \quad u = \sqrt{6^2 + 8^2}$

$\Rightarrow \quad u = 10\text{ ms}^{-1}$

Hence, the correct answer is (D).

20. $t_{\text{ascent}} = \frac{u_y}{g}$

$\Rightarrow \quad t_{\text{ascent}} = \frac{8}{10} = 0.8$ s

Hence, the correct answer is (D).

21. $H = \frac{u_y^2}{2g}$

$\Rightarrow \quad H = \frac{(8)^2}{2(10)}$

$\Rightarrow \quad H = 3.2$ m

Hence, the correct answer is (D).

22. $R = \frac{2}{g}(u_x)(u_y)$

$\Rightarrow \quad R = \frac{2}{10}(6)(8)$

$\Rightarrow \quad R = 9.6$ m

Hence, the correct answer is (D).

23. $H = \frac{u^2\sin^2\theta}{2g} = \frac{u_y^2}{2g}$

$\Rightarrow \quad 8 = \frac{u_y^2}{2g}$

$\Rightarrow \quad u_y = 4\sqrt{g}$

Hence, the correct answer is (D).

24. Since, $R = \frac{2}{g}(u_x)(u_y)$

$\Rightarrow \quad 24 = \frac{2}{g}(u_x)(4\sqrt{g})$

$\Rightarrow \quad u_x = 3\sqrt{g}$

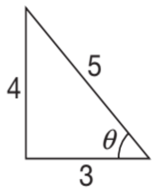
Hence, the correct answer is (B).

25. $u = \sqrt{u_x^2 + u_y^2}$

$\Rightarrow \quad u = 5\sqrt{g}$

Hence, the correct answer is (D).

26.



$\tan\theta = \frac{u_y}{u_x}$

$\Rightarrow \quad \tan\theta = \frac{4\sqrt{g}}{3\sqrt{g}}$

$\Rightarrow \quad \tan\theta = \frac{4}{3}$

$\Rightarrow \quad \sin\theta = \frac{4}{5} = 0.8$

Hence, the correct answer is (B).

27.

Along x	Along y
$a_x = 6t$	$a_y = 8t$
$\frac{dv_x}{dt} = 6t$	$\frac{dv_y}{dt} = 8t$
$\int_0^{v_x} dv_x = \int_0^3 6t\,dt$	$\int_0^{v_y} dv_y = \int_0^3 8t\,dt$
$v_x = \frac{6t^2}{2}\Big\vert_0^3 = 27\text{m}$	$v_y = \frac{8t^2}{2}\Big\vert_0^3 = 36\text{ m}$

$\Rightarrow \quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{27^2 + 36^2}$

$\Rightarrow \quad v = \sqrt{729 + 1296} = 45 \text{ ms}^{-1}$

Hence, the correct answer is (A).

28. $|\vec{a}| = 10 \text{ ms}^{-2}$

Since, we know that,

$$s = \frac{1}{2}at^2$$

$\Rightarrow \quad s = \frac{1}{2} \times 10 \times 9$

$\Rightarrow \quad s = 45 \text{ m}$

Hence, the correct answer is (D).

29. Path is a straight line.
Hence, the correct answer is (A).

Matrix Match/Column Match Type Questions

1. A → (r)
B → (t)
C → (q)
D → (p)
E → (s)

$$\vec{u} = u\left(\cos(45^\circ)\hat{i} + \sin(45^\circ)\hat{j}\right)$$

$\Rightarrow \quad \vec{u} = 20\hat{i} + 20\hat{j} \text{ ms}^{-1}$ and $\vec{a} = -10\hat{j}$

Since

$$\vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$\Rightarrow \quad \vec{r} = \left(20\hat{i} + 20\hat{j}\right)t + \frac{1}{2}\left(-10\hat{j}\right)t^2$

$\Rightarrow \quad \vec{r} = (20t)\hat{i} + (20 - 5t)\hat{j}$

So, at $t = 1$ s, we get

$$\vec{r} = 20\hat{i} + 15\hat{j}$$

$\Rightarrow \quad$ Average Velocity at $t = 1$ s is

$$\vec{v}_{av} = \frac{\vec{r}}{t} = 20\hat{i} + 15\hat{j}$$

$\Rightarrow \quad |\vec{v}_{av}| = \sqrt{400 + 225} = 25 \text{ ms}^{-1}$

Further $\vec{v} = \vec{u} + \vec{a}t$

$\Rightarrow \quad \vec{v} = 20\hat{i} + 20\hat{j} - \left(10\hat{j}\right)t$

$\Rightarrow \quad \vec{v} = 20\hat{i} + (20 - 10t)\hat{j}$

$\Rightarrow \quad \Delta\vec{v} = \vec{v} - \vec{u} = -(10t)\hat{j}$

$\Rightarrow \quad \Delta\vec{v} = -20\hat{j}$ {at $t = 2$ s}

So, average acceleration at $t = 1$ s, 2 s, 3 s and 4 s

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = -10\hat{j} \qquad \text{\{as expected\}}$$

Radius of curvature at $t = 0$ is

$$r = \frac{u^2}{(g\cos\theta)}$$

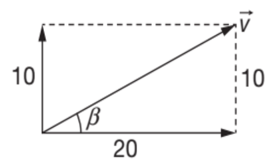
$\Rightarrow \quad r = \frac{\left(20\sqrt{2}\right)^2}{\left(\frac{10}{\sqrt{2}}\right)} = 80\sqrt{2} \text{ m}$

Radius of curvature at $t = 1$ s is

$$r = \frac{v^2}{g\cos\beta}$$

where v is the velocity at $t = 1$ s and β is the angle which $v(\text{at } t = 1\text{ s})$ makes with the x-axis

Since $v = 20\hat{i} + 10\hat{j}$, so $\tan\beta = \frac{10}{20}$



$\Rightarrow \quad \cos\beta = \frac{20}{\sqrt{20^2 + 10^2}} = \frac{20}{10\sqrt{5}} = \frac{2}{\sqrt{5}}$

So, $r = \dfrac{\left(\sqrt{20^2+10^2}\right)^2}{(10)\left(\dfrac{2}{\sqrt{5}}\right)}$

$\Rightarrow \quad r = \dfrac{500\sqrt{5}}{20}$

$\Rightarrow \quad r = 25\sqrt{5}$ m

Radius of curvature at $t = 2$ s is

$$r = \frac{(u\cos\theta)^2}{g}$$

$$\Rightarrow \quad r = \frac{\left[(20\sqrt{2})\left(\frac{1}{\sqrt{2}}\right)\right]^2}{10} = \frac{(20)^2}{10} = 40 \text{ m}$$

2. A → (r)
B → (s)
C → (p)
D → (q)

Since $v = 2t$

So, distance travelled is

$$s = \int_0^2 v dt = \int_0^2 2t dt = 2\left(\frac{t^2}{2}\Big|_0^2\right)$$

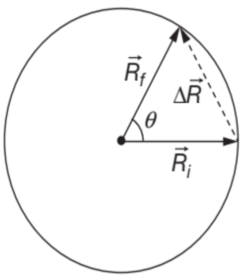
$\Rightarrow \quad s = 4$ m

Hence average speed is

$$v_{av} = \frac{s}{t} = \frac{4}{2} = 2 \text{ ms}^{-1}$$

So, (A) → (r), (C) → (p)
Further since the particle is moving in a circular path, so we have

$$\omega = \frac{v}{r} = 2t \text{ rads}^{-1}$$



$R = 1$ m

Since

$$\theta = \int_0^2 \omega dt$$

$$\Rightarrow \quad \theta = 2\int_0^2 t\, dt$$

$\Rightarrow \quad \theta = 4$ rad

$\Delta\vec{R} = \text{Displacement} = \vec{R}_f - \vec{R}_i$

$\Rightarrow \quad |\text{Displacement}| = \sqrt{R_f^2 + R_i^2 + 2R_fR_i\cos(180-\theta)}$

Since $|\vec{R}_f| = |\vec{R}_i| = R$

$\Rightarrow \quad |\text{Displacement}| = \sqrt{R^2 + R^2 - 2R^2\cos\theta}$

$\Rightarrow \quad |\text{Displacement}| = 2R\sin\left(\dfrac{\theta}{2}\right) = 2\sin(2)$

So, average velocity

$$|\vec{v}_{av}| = \frac{|\text{Displacement}|}{\text{Time}} = \frac{2\sin(2)}{2} = \sin(2)$$

Hence (B) → (s) and (D) → (q)

3. A → (r)
B → (s)
C → (p)
D → (q)

Since $\vec{a} = \dfrac{dv}{dt}$

$\Rightarrow \quad \vec{a} = 2\hat{i} + (2t)\hat{j}$

So, at $t = 1$ s, we get

$\vec{v} = 2\hat{i} + \hat{j}$ and $\vec{a} = 2\hat{i} + 2\hat{j}$

$\Rightarrow \quad |\vec{a}| = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ ms^{-2}

Hence (C) → (p)
Now, since the tangential acceleration is

$$a_T = \frac{\vec{a}\cdot\vec{v}}{|\vec{v}|}$$

$$\Rightarrow \quad a_T = \frac{(2\hat{i}+2\hat{j})\cdot(2\hat{i}+\hat{j})}{\sqrt{2^2+1^2}}$$

$$\Rightarrow \quad a_T = \frac{4+2}{\sqrt{5}} = \frac{6}{\sqrt{5}} \text{ ms}^{-2}$$

So, (A) → (r)
Now since we know that

$a^2 = a_C^2 + a_T^2$, where a_C is the radial acceleration

$\Rightarrow \quad a_C = \sqrt{a^2 - a_T^2}$

$\Rightarrow \quad a_C = \sqrt{8 - \dfrac{36}{5}}$

$$\Rightarrow \quad a_C = \frac{2}{\sqrt{5}} \text{ ms}^{-2}$$

So, (B) → (s)

Finally, the radius of curvature is given by

$$r = \frac{v^2}{a_C} = \frac{5}{\frac{2}{\sqrt{5}}} = \frac{5\sqrt{5}}{2} \text{ m}$$

So, (D) → (q)

4. A → (q)
B → (s)
C → (p)
D → (r)

Let us choose the x and y directions along OB and OA respectively. Then

$$u_x = u = 10\sqrt{3} \text{ ms}^{-1}, \; u_y = 0$$

$$a_x = -g\sin(60°) = -5\sqrt{3} \text{ ms}^{-2}$$

and $a_y = -g\cos(60°) = -5 \text{ ms}^{-2}$

At point Q, x-component of velocity is zero. Hence, substituting in

$$v_x = u_x + a_x t$$

$$\Rightarrow \quad 0 = 10\sqrt{3} - 5\sqrt{3}t$$

$$\Rightarrow \quad t = \frac{10\sqrt{3}}{5\sqrt{3}} = 2s$$

So, (A) → (q)

At point Q, $v = v_y = u_y + a_y t$

$$\Rightarrow \quad v = 0 - (5)(2) = -10 \text{ ms}^{-1}$$

Here, negative sign implies that velocity of particle at Q is along negative y direction. So, $v = 10 \text{ ms}^{-1}$, along $-y$ direction.

Hence (B) → (s)

$$\text{Distance } PO = \left| \begin{matrix}\text{Displacement of particle} \\ \text{along } y\text{-direction}\end{matrix} \right| = |s_y|$$

Since, $s_y = u_y t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}(5)(2)^2 = -10 \text{ m}$

$$\Rightarrow \quad PO = 10 \text{ m}$$

Since, $h = PO\sin(30°) = (10)\left(\frac{1}{2}\right)$

$$\Rightarrow \quad h = 5 \text{ m}$$

So, (C) → (p)

$$\text{Distance } OQ = \left| \begin{matrix}\text{Displacement of particle} \\ \text{along } x\text{-direction}\end{matrix} \right| = |s_x|$$

Since, $s_x = u_x t + \frac{1}{2}a_x t^2$

$$\Rightarrow \quad s_x = (10\sqrt{3})(2) - \frac{1}{2}(5\sqrt{3})(2)^2 = 10\sqrt{3} \text{ m}$$

$$\Rightarrow \quad OQ = 10\sqrt{3} \text{ m}$$

Since, $PQ = \sqrt{(PO)^2 + (OQ)^2}$

$$\Rightarrow \quad PQ = \sqrt{(10)^2 + (10\sqrt{3})^2} = \sqrt{400}$$

$$\Rightarrow \quad PQ = 20 \text{ m}$$

5. A → (q)
B → (r)
C → (s)
D → (t)
E → (p)

$$t = \frac{x}{\alpha} \qquad \ldots(1)$$

$$y = \alpha\left(\frac{x}{\alpha}\right)\left[1 - \beta\left(\frac{x}{\alpha}\right)\right]$$

$$\Rightarrow \quad y = x - \left(\frac{\beta}{\alpha}\right)x^2 \qquad \ldots(2)$$

$y = 0$ at $x = R$

$$\Rightarrow \quad 0 = R - \frac{\beta}{\alpha}R^2$$

$$\Rightarrow \quad R\left(1 - \frac{\beta}{\alpha}R\right) = 0$$

$$\Rightarrow \quad R = \frac{\alpha}{\beta} = 2\left(\frac{\alpha}{2\beta}\right)$$

So, (A) → (q)

At maximum height, $\frac{dy}{dx} = 0$

$$\Rightarrow \quad 1 - \frac{\beta}{\alpha}(2x) = 0$$

$$\Rightarrow \quad x = \frac{\alpha}{2\beta}$$

$$H_{max} = \frac{\alpha}{2\beta} - \frac{\beta}{\alpha}\left(\frac{\alpha^2}{4\beta^2}\right)$$

$$\Rightarrow \quad H_{max} = \frac{\alpha}{2\beta} - \frac{\alpha}{4\beta} = \frac{\alpha}{4\beta} = 4\left(\frac{\alpha}{16\beta}\right)$$

So, (B) → (r)

$$T = \frac{R}{u_x} \qquad \left\{\text{or from (1), we have } t = \frac{x}{\alpha}\right\}$$

Since $x = \alpha t$, so $v_x = \frac{dx}{dt} = \alpha = \text{constant}$

$\Rightarrow \quad v_x = u_x = \alpha$

$$\Rightarrow \quad T = \frac{\left(\frac{\alpha}{\beta}\right)}{\alpha} = \frac{1}{\beta} = 8\left(\frac{1}{8\beta}\right)$$

So, (C) → (s)

Also, we observe that $y = \alpha t - (\alpha\beta)t^2$

$$\Rightarrow \quad \frac{dy}{dt} = \alpha - 2\alpha\beta t = \alpha(1 - 2\beta t)$$

$\Rightarrow \quad v_y = \alpha(1 - 2\beta t)$

$\Rightarrow \quad \vec{v} = v_x\hat{i} + v_y\hat{j}$

$\Rightarrow \quad \vec{v} = \alpha\hat{i} + \alpha(1 - 2\beta t)\hat{j}$

Now, $a_x = \frac{dv_x}{dt} = 0$ and $a_y = -2(\alpha\beta)\hat{j} = \text{constant}$

$\Rightarrow \quad \vec{a} = a_x\hat{i} + a_y\hat{j} = -(2\alpha\beta)\hat{j}$

$$\Rightarrow \quad |\vec{a}| = 2\alpha\beta = 16\left(\frac{\alpha\beta}{8}\right)$$

So, (D) → (t)

Now at $t = \frac{1}{2\beta}$, we have

$$\vec{v} = \alpha\hat{i} + \alpha\left[1 - (2\beta)\left(\frac{1}{2\beta}\right)\right]\hat{j} = \alpha\hat{i}$$

$\Rightarrow \quad |\vec{v}| = \alpha = 1(\alpha)$

So, (E) → (p)

7. A → (q)
B → (p)
C → (r)
D → (s)

Since range is same for complimentary angles, so

$R_1 = R_2$

$$\Rightarrow \quad \frac{R_1}{R_2} = 1$$

$$\frac{H_1}{H_2} = \frac{u^2\sin^2(30^\circ)}{u^2\sin^2(60^\circ)} = \frac{1}{3}$$

$$\frac{T_2}{T_1} = \frac{u\sin(60^\circ)}{u\sin(30^\circ)} = \sqrt{3}$$

$$\frac{T_1H_1R_1}{T_2H_2R_2} = \frac{T_1H_1}{T_2H_2} = \frac{1}{3}\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{3\sqrt{3}}$$

8. A → (q)
B → (r)
C → (p)
D → (s)

$\vec{u} = 20\hat{i} + 20\hat{j}$, $\vec{a} = -10\hat{j}$ and $t = 1$ s

Since, $\Delta\vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2 = 20\hat{i} + 15\hat{j}$ ...(1)

and $\vec{v} = \vec{u} + \vec{a}t = 20\hat{i} + 10\hat{j}$ ...(2)

$$\Rightarrow \quad |\vec{v}_{av}| = \left|\frac{\Delta\vec{r}}{t}\right| = \sqrt{(20)^2 + (15)^2} = 25 \text{ ms}^{-1} \quad \{\text{from (1)}\}$$

$\Rightarrow \quad |\Delta\vec{v}| = |\vec{v} - \vec{u}| = 10 \text{ ms}^{-1}$ {from (2)}

$\Rightarrow \quad |\vec{v}_{\text{inst}}| = |\vec{v}| = \sqrt{(20)^2 + (10)^2} = 10\sqrt{5} \text{ ms}^{-1}$

$\Rightarrow \quad \Delta|\vec{v}| = |\vec{v}| - |\vec{u}| = 20\sqrt{2} - 10\sqrt{5} = 6 \text{ ms}^{-1}$

9. A → (r)
B → (r)
C → (p)
D → (q)

Since, $y = x - \frac{x^2}{80}$

Comparing with the standard equation of projectile, i.e.

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}, \text{ we get}$$

$\tan\theta = 1$

$\Rightarrow \quad \theta = 45^\circ$

and $\frac{1}{80} = \frac{g}{2u^2\cos^2\theta}$

$\Rightarrow \quad u = 20\sqrt{2} \text{ ms}^{-1}$

Since, $\vec{v} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j} = 20\hat{i} - 20\hat{j}$

When $x = R$, $y = 0$

$$\Rightarrow \quad 0 = R - \frac{R^2}{80}$$

$\Rightarrow \quad R = 80$ m

10. A $\rightarrow$ (p, r)
B $\rightarrow$ (q, s)
C $\rightarrow$ (p, q, r, s)
D $\rightarrow$ (p, q, r, s)

For a particle to move in circular path, $\vec{a} \perp \vec{v}$
For a particle to undergo projectile motion, angle between $\vec{a}$ and $\vec{v}$ must be acute.

11. A $\rightarrow$ (q)
B $\rightarrow$ (r)
C $\rightarrow$ (s)
D $\rightarrow$ (p)

(A) Change in momentum is $\Delta p = (mg)t$ in time t.
(B) Angle at highest point is $0°$.
(C) Kinetic energy of body is minimum at highest point.
(D) Horizontal component remains unchanged.

Integer/Numerical Answer Type Questions

1. Since $R = \dfrac{2u^2 \sin(\alpha - \beta)\cos\alpha}{g\cos^2\beta}$

$$\Rightarrow \quad R = \frac{2(21)^2 \sin(60° - 30°)\cos(60°)}{9.8\cos^2(30°)}$$

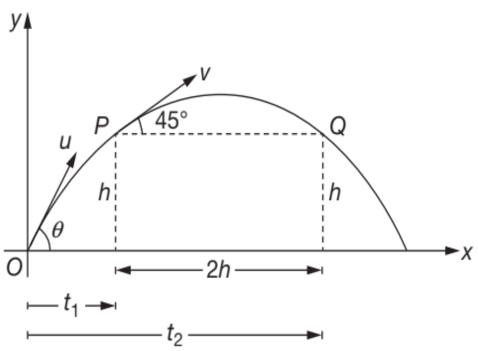
$$\Rightarrow \quad R = \frac{(2)(441)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}{(9.8)\left(\frac{3}{4}\right)} = 30 \text{ m}$$

2. At P, at height h,

$$v^2 = u^2 - 2gh$$

$$\Rightarrow \quad v^2 = 4gh - 2gh \qquad \{\because u = 2\sqrt{gh}\}$$

$$\Rightarrow \quad v = \sqrt{2gh} \qquad \ldots(1)$$



Assuming PQ to be horizontal, then Range $= 2h = PQ$

$$\Rightarrow \quad \frac{v^2\sin(2\theta)}{g} = 2h$$

$$\Rightarrow \quad \frac{2(gh)\sin(2\theta)}{g} = 2h$$

$$\Rightarrow \quad \sin(2\theta) = 1$$

$$\Rightarrow \quad \theta = 45°$$

So, to fly from P to Q, the projectile will take a time t given by

$$t = \frac{2v\sin(45°)}{g} = \left(\frac{2\sqrt{2gh}}{g}\right)\left(\frac{1}{\sqrt{2}}\right) = 2\sqrt{\frac{h}{g}}$$

So, $* = 2$

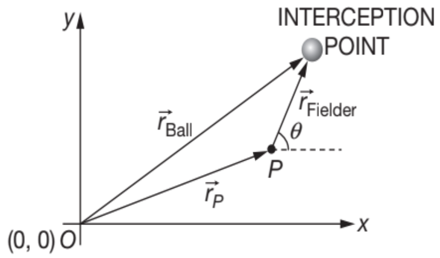
3. From the diagram, we observe that

$$\vec{r}_P + \vec{r}_{\text{Fields}} = \vec{r}_{\text{Ball}} \qquad \ldots(1)$$

where $\vec{r}_P = \left(46\hat{i} + 28\hat{j}\right)$ m $\qquad \ldots(2)$

Let fielder run for the ball making an angle θ with the horizontal.

Then $\vec{r}_{\text{Fielder}} = 5t\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right) \qquad \ldots(3)$



where t is the time taken by the fielder to go from point P to the interception point which equals the time taken by the ball to go from the origin to the interception point with a velocity of $\left(7.5\hat{i} + 10\hat{j}\right)$ ms^{-1}. So

$$\vec{r}_{\text{Ball}} = \left(7.5t\hat{i} + 10t\hat{j}\right) \qquad \ldots(4)$$

Substituting (2), (3) and (4) in (1), we get

$$(46 + 5t\cos\theta)\hat{i} + (28 + 5t\sin\theta)\hat{j} = (7.5t)\hat{i} + (10t)\hat{j}$$

$$\Rightarrow \quad 46 + 5t\cos\theta = 7.5t \qquad \ldots(5)$$

$$28 + 5t\sin\theta = 10t \qquad \ldots(6)$$

From (5) and (6), we get

$$\cos\theta = \frac{7.5t - 46}{5t} \text{ and } \sin\theta = \frac{10t - 28}{5t}$$

Since, $\cos^2\theta + \sin^2\theta = 1$

$$\Rightarrow \quad \left(\frac{7.5t - 46}{5t}\right)^2 + \left(\frac{10t - 28}{5t}\right)^2 = 1$$

Solving to get

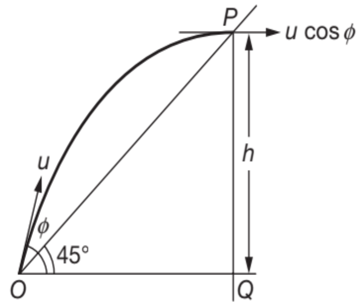
$$t = 4 \text{ s or } t = \frac{116}{21} \text{ s}$$

So, the shortest time for interception is

$$t = 4 \text{ s}$$

4. Let the particle be projected from O with velocity u and strike the plane at a point P horizontally. Then

$PQ = OQ$.



$\Rightarrow$ Maximum height $= \dfrac{\text{Horizontal range}}{2}$

$$\Rightarrow \frac{u^2 \sin^2\phi}{2g} = \frac{u^2 \sin 2\phi}{2g} = \frac{u^2 \sin\phi \cos\phi}{g}$$

$\Rightarrow \tan\phi = 2$

5. The angular speed is

$$\omega = \frac{v}{r} = \frac{2}{0.25} = 8 \text{ rads}^{-1}$$

6. Since $y = u_y t + \frac{1}{2} a_y t^2$, where $u_y = 0$ and $a_y = g$, $y = h$.

So $h = \frac{1}{2} g t^2$

$$\Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(20)}{10}} = 2 \text{ s}$$

Further since $x = u_x t + \frac{1}{2} a_x t^2$ where $u_x = v$ (say), $a_x = 0$, $x = 10$ m, so

$$10 = vt = v(2)$$

$\Rightarrow v = 5 \text{ ms}^{-1}$

Since $a = \dfrac{v^2}{R}$

$$\Rightarrow a = \frac{(5)^2}{0.5} = \frac{25}{0.5} = 50 \text{ ms}^{-2}$$

$\Rightarrow a = 50 \text{ ms}^{-2}$

7. Since the car is travelling with a constant speed, its tangential component of acceleration is zero, i.e., $a_T = 0$. Thus,

$$a = a_N = \frac{v^2}{r}$$

$$\Rightarrow 3 = \frac{25^2}{r}$$

$\Rightarrow r = 208$ m

8. Here, the car's tangential component of acceleration of $a_T = -3 \text{ ms}^{-2}$. Thus,

$$a = \sqrt{a_T^2 + a_N^2}$$

$$\Rightarrow 5 = \sqrt{(-3)^2 + a_N^2}$$

$\Rightarrow a_N = 4 \text{ ms}^{-2}$

Since $a_N = \dfrac{v^2}{r}$

$$\Rightarrow 4 = \frac{20^2}{r}$$

$\Rightarrow r = 100$ m

9. $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2 \times 0.8}{10}} = 0.4$ s

$$u_{\text{MIN}} = \frac{(120 - 30 + 10) \times 10^{-3}}{0.4} = 0.25 \text{ ms}^{-1}$$

and $u_{\text{MAX}} = \dfrac{(120 + 30 - 10) \times 10^{-3}}{0.4} = 0.35 \text{ ms}^{-1}$

$\Rightarrow u_{MIN} = 25 \text{ cms}^{-1}$ and $u_{MAX} = 35 \text{ cms}^{-1}$

10. Let the ball clear the point C at time t_1. Then

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow (1 + 3.9) = 0 + \frac{1}{2}(9.8) t_1^2 \quad \{\because u_y = 0\}$$

$$\Rightarrow 4.9 = \frac{1}{2}(9.8) t_1^2$$

$\Rightarrow t_1 = 1$ s

Since $BC = u_x t$

$\Rightarrow 6 = u(1)$

$\Rightarrow u = 6 \text{ ms}^{-1}$

Let the ball hit the ground at D in time t. Then

$$6 + x = 6t \quad \ldots(1)$$

Further $(1 + 3.9 + 14.7) = \frac{1}{2}(9.8) t^2$

$\Rightarrow t = 2$ s

$\Rightarrow 6 + x = 12$

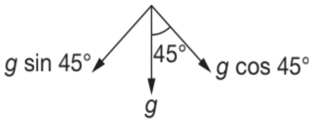
$\Rightarrow x = 6$ m

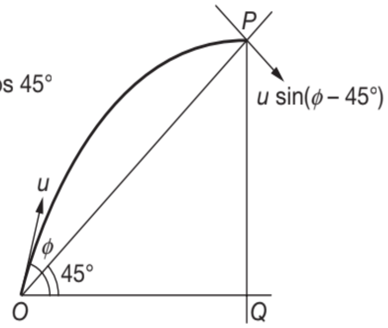
CHAPTER 5

11. At time $t = T = \dfrac{2u\sin(\phi - 45^\circ)}{g\cos 45^\circ}$

component of velocity along the plane is zero.

$\Rightarrow \quad 0 = u\cos(\phi - 45^\circ) - (g\sin 45^\circ)t$





$\Rightarrow \quad u\cos(\phi - 45^\circ) = (g\sin 45^\circ)\left(\dfrac{2u\sin(\phi - 45^\circ)}{g\cos 45^\circ}\right)$

$\Rightarrow \quad 2\tan(\phi - 45^\circ) = \cot 45^\circ = 1$

$\Rightarrow \quad 2\left(\dfrac{\tan\phi - \tan 45^\circ}{1 + \tan\phi \tan 45^\circ}\right) = 1$

$\Rightarrow \quad \tan\phi = 3$

12. As $y = x\tan\theta - \dfrac{gx^2}{2u^2}(1 + \tan^2\theta)$

for (a, b), i.e. when $x = a$ and $y = b$, we have

$$ga^2\tan^2\theta - 2au^2\tan\theta + (ga^2 + 2bu^2) = 0$$

This is a quadratic in $\tan\theta$ and since discriminant of a quadratic must be positive, so we have

$$4a^2u^2 - 4ga^2(ga^2 + 2bu^2) \ge 0$$

Solving, we get

$$u \ge \sqrt{bg + g\sqrt{a^2 + b^2}}$$

On substituting the values, we get

$$u_{min} = 30 \text{ ms}^{-1}$$

13. $v_x = 8\sqrt{2}$ ms^{-1} is the relative velocity along x-axis

$\Rightarrow \quad x = 22 - (8\sqrt{2})t$

$v_y = 6\sqrt{2}$ ms^{-1} is the relative velocity along y-axis

$\Rightarrow \quad y = 9 - (6\sqrt{2})t$

$\Rightarrow \quad r^2 = x^2 + y^2 \qquad \ldots(1)$

For minimum r, we have r^2 to be minimum, so

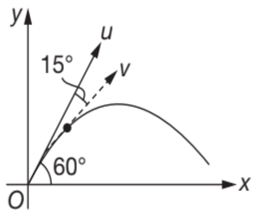
$$\frac{d}{dt}(r^2) = 0$$

$\Rightarrow \quad t = \dfrac{23}{10\sqrt{2}}$ s

Substituting the value of t in equation (1), we get

$$r_{min} = 6 \text{ m}$$

14. $v = \dfrac{u\cos(60^\circ)}{\cos(45^\circ)}$



$\Rightarrow \quad K_f = K_i\left(\dfrac{\cos^2(60^\circ)}{\cos^2(45^\circ)}\right)$

$\Rightarrow \quad \dfrac{K_i}{K_f} = 2$

15. For Range to be maximum, the projectile must be launched at an angle

$$\alpha = \frac{\pi}{4} + \frac{\beta}{2}$$

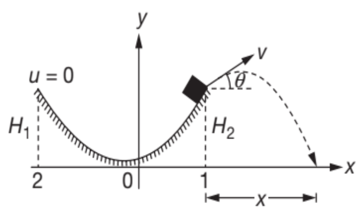
$\Rightarrow \quad \alpha = 45^\circ + 15^\circ = 60^\circ$

16. Since, $H_1 = (2)^2 = 4$ m and

$$H_2 = (1)^2 = 1 \text{ m}$$

By Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in gravitational} \\ \text{potential energy}\end{pmatrix} = \begin{pmatrix}\text{Gain in} \\ \text{kinetic energy}\end{pmatrix}$$



$\Rightarrow \quad mg(H_1 - H_2) = \dfrac{1}{2}mv^2$

$\Rightarrow \quad v = \sqrt{2g(H_1 - H_2)}$

$\Rightarrow \quad v = \sqrt{2 \times g \times (H_2 - H_1)}$

$\Rightarrow \quad v = \sqrt{20 \times 3}$

$\Rightarrow \quad v = \sqrt{60}$

Now $\tan\theta = \left.\frac{dy}{dx}\right|_{x=1} = 2x|_{x=1} = 2$

$\Rightarrow \quad \tan\theta = 2$

Since, $y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$

$\Rightarrow \quad -1 = x(2) - \frac{10x^2}{2v^2}(1+4)$

$\Rightarrow \quad -1 = 2x - \frac{10x^2}{2(60)}(5)$

$\Rightarrow \quad -1 = 2x - \frac{5}{12}x^2$

$\Rightarrow \quad -12 = 24x - 5x^2$

$\Rightarrow \quad 5x^2 - 24x - 12 = 0$

17. Time of flight is

$$T = \frac{2u\sin\theta}{g} = 2\sqrt{2} \text{ s}$$

Range (along north) is

$$R_1 = \frac{u^2\sin 2\theta}{g} = 40 \text{ m}$$

Range (along east) is

$$R_2 = \frac{1}{2}aT^2 = 30 \text{ m}$$

So, range $R = \sqrt{R_1^2 + R_2^2} = \sqrt{30^2 + 40^2} = 50$ m

18. $10 - v\cos(60°) = 0$

$\Rightarrow \quad H = \frac{v^2\sin^2(60°)}{2g} = 15$ m

$\Rightarrow \quad H - 10 = 5$ m

19. Let angle made by $\vec{V}$ initially and after time t be θ and α respectively.

$$\tan(30°) = \frac{u\sin\theta - g \times 2}{u\cos\theta} \quad \ldots(1)$$

$$\Rightarrow \quad \tan(0°) = \frac{u\sin\theta - g \times 3}{u\cos\theta} \quad \ldots(2)$$

$\Rightarrow \quad \theta = 60°$

$\Rightarrow \quad \frac{\theta}{10} = 6$

20. Coordinates of the point C are

$$C\left(\frac{10}{3} + x,\ y\right)$$

Since, $\frac{y}{x} = \tan 37°$

$\Rightarrow \quad y = \frac{3}{4}x$

$\Rightarrow \quad u_y t - \frac{1}{2}gt^2 = \frac{3}{4}\left[u_x t - \frac{10}{3}\right]$

$\Rightarrow \quad t = 1.06$

$\Rightarrow \quad x = \frac{3}{4} \times 10 \times 1.06 - \frac{10}{3} = 4.64$

21. Since, $u_{\min} = \sqrt{g\left(h + \sqrt{d^2 + h^2}\right)} = 10\sqrt{3}$ ms^{-1}

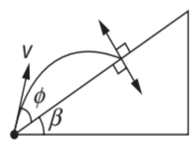
and $\tan\theta = \frac{h + \sqrt{d^2 + h^2}}{d}$

$\Rightarrow \quad \theta = 60°$

$\Rightarrow \quad t = \frac{d}{u\cos\theta} = \frac{10\sqrt{3}}{10\sqrt{3} \times \frac{1}{2}} = 2$ s

22. Since, $\tan\phi = \frac{\cot\beta}{2}$

$\Rightarrow \quad t = \frac{2v}{g\sqrt{1 + 3\sin^2\beta}}$



23. $T = \frac{2u\sin\theta}{g} = \frac{2 \times 10 \times \frac{1}{2}}{10} = 1$ s

CHAPTER 5

ARCHIVE: JEE MAIN

1. Since $R=\dfrac{2u\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$, where $u=2\text{ ms}^{-1}$

$\alpha=30+15=45^\circ$

$\beta=30^\circ$

$\Rightarrow\ R=\dfrac{2(2)^2\sin(15^\circ)\cos(45^\circ)}{g\cos^2(30^\circ)}$

$\Rightarrow\ R=\dfrac{8\sqrt{2}}{15}\sin(15^\circ)$

$\Rightarrow\ R=\dfrac{8\sqrt{2}}{15}\sin(45^\circ-30^\circ)$

$\Rightarrow\ R=\dfrac{8\sqrt{2}}{15}\left(\dfrac{1}{2\sqrt{2}}\right)(\sqrt{3}-1)=\dfrac{4}{15}(\sqrt{3}-1)$

$\Rightarrow\ R\approx 20\text{ cm}$

Hence, the correct answer is (B).

2. For same horizontal range, we must have angles as complimentary. So

For $\theta_1=\theta$ and $\theta_2=(90-\theta)$

$$R=\frac{2u^2\sin\theta\cos\theta}{g}$$

$\Rightarrow\ t_1=\dfrac{2u\sin\theta}{g}$ and $t_2=\dfrac{2u\cos\theta}{g}$

$\Rightarrow\ t_1t_2=\dfrac{u^2 4\sin\theta\cos\theta}{g^2}$

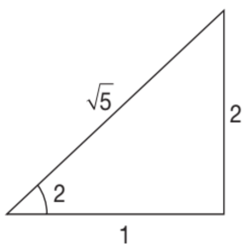
$\Rightarrow\ t_1t_2=\dfrac{2u^2\sin 2\theta}{g^2}=\dfrac{2R}{g}$

Hence, the correct answer is (B).

3. $y=2x-9x^2$

Comparing it with standard equation of trajectory of projectile, i.e.,

$$y=x\tan\theta-\frac{gx^2}{24^2\cos^2\theta}$$



we get, $\tan\theta=2$

and $9=\dfrac{10\times 5}{2v_0^2}$

$\Rightarrow\ v_0=\dfrac{5}{3}\text{ ms}^{-1}$

Hence, the correct answer is (C).

4. For complementary angles, ranges are equal.

So, for $\theta_1=\theta$ and $\theta_2=(90-\theta)$, we have

$$R=\frac{2u^2\sin\theta\cos\theta}{g}$$

Also, $h_1=\dfrac{u^2\sin^2\theta}{2g}$, $h_2=\dfrac{u^2\cos^2\theta}{2g}$

$\Rightarrow\ h_1h_2=\left(\dfrac{2u^2\sin\theta\cos\theta}{g}\right)^2\times\left(\dfrac{1}{16}\right)$

$\Rightarrow\ 16h_1h_2=R^2$

$\Rightarrow\ R^2=16(h_1h_2)$

Hence, the correct answer is (B).

5. Maximum areas will be covered when range is maximum, so

$$R_{max}=\frac{u^2}{g}$$

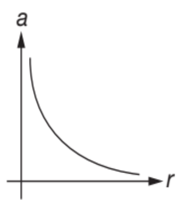
$\Rightarrow\ \dfrac{A_1}{A_2}=\dfrac{\pi(R_{max})_1^2}{\pi(R_{max})_2^2}=\left(\dfrac{u_1^2}{u_2^2}\right)^2=\dfrac{u_1^4}{u_2^4}=\dfrac{1}{16}$

Hence, the correct answer is (D).

6. Speed $v=10\text{ ms}^{-1}$

Since, centripetal acceleration is given by,

$$a=\frac{v^2}{r}$$



and $|\vec{v}|=\text{constant}$

$\Rightarrow\ a\propto\dfrac{1}{r}$

$\Rightarrow \quad ar = \text{constant}$

This represents a rectangular hyperbola.

Hence, the correct answer is (C).

7. Since, $\vec{u} = \hat{i} + 2\hat{j}$

Also, $\vec{u} = u_x\hat{i} + u_y\hat{j}$

$\Rightarrow \quad u_x = 1$ and $u_y = 2$

Now $x = u_x t$ and $y = u_y t - \frac{1}{2}gt^2$

$\Rightarrow \quad x = t$

and $y = 2t - \frac{1}{2} \times 10 \times t^2 = 2t - 5t^2$

$\Rightarrow \quad y = 2x - 5x^2$

So, equation of trajectory is $y = 2x - 5x^2$

Hence, the correct answer is (C).

8. Let u be the velocity of projection of the stone. The maximum height a boy can throw a stone is

$$H_{max} = \frac{u^2}{2g} = 10\text{ m} \qquad \ldots(1)$$

The maximum horizontal distance the boy can throw the same stone is

$$R_{max} = \frac{u^2}{g} = 20\text{ m} \qquad \{\text{Using (1)}\}$$

Hence, the correct answer is (C).

9. $R_{max} = \dfrac{v^2 \sin 90^\circ}{g} = \dfrac{v^2}{g}$

Area $= \pi(R_{max})^2 = \dfrac{\pi v^4}{g^2}$

Hence, the correct answer is (B).

10. The position vector of the particle from the origin at any time t is

$$\vec{r} = v_0 t\cos\theta\hat{i} + \left(v_0 t\sin\theta - \frac{1}{2}gt^2\right)\hat{j}$$

Velocity vector, $\vec{v} = \dfrac{d\vec{r}}{dt}$

$$\Rightarrow \quad \vec{v} = \frac{d}{dt}\left(v_0 t\cos\theta\hat{i} + \left(v_0 t\sin\theta - \frac{1}{2}gt^2\right)\hat{j}\right)$$

$$\Rightarrow \quad \vec{v} = v_0\cos\theta\hat{i} + (v_0\sin\theta - gt)\hat{j}$$

The angular momentum of the particle about the origin is

$$\vec{L} = \vec{r} \times m\vec{v}$$

$$\Rightarrow \quad \vec{L} = m(\vec{r} \times \vec{v})$$

$$\Rightarrow \quad \vec{L} = m\left[\left(v_0 t\cos\theta\hat{i} + \left(v_0 t\sin\theta - \frac{1}{2}gt^2\right)\hat{j}\right) \times \left(v_0\cos\theta\hat{i} + (v_0\sin\theta - gt)\hat{j}\right)\right]$$

$$\Rightarrow \quad \vec{L} = m\left[\left(v_0^2 t\cos\theta\sin\theta - v_0 g t^2\cos\theta\right)\hat{k} + \left(v_0^2 t\sin\theta\cos\theta - \frac{1}{2}gt^2 v_0\cos\theta\right)(-\hat{k})\right]$$

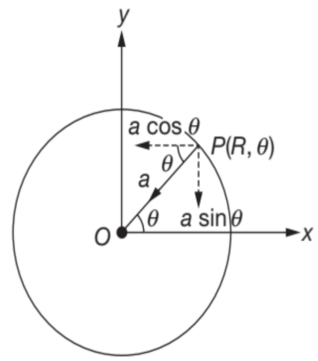
$$\Rightarrow \quad \vec{L} = m\left[v_0^2 t\sin\theta\cos\theta\hat{k} - v_0 g t^2\cos\theta\hat{k} - v_0^2 t\sin\theta\cos\theta\hat{k} + \frac{1}{2}v_0 g t^2\cos\theta\hat{k}\right]$$

$$\Rightarrow \quad \vec{L} = m\left[-\frac{1}{2}v_0 g t^2\cos\theta\hat{k}\right] = -\frac{1}{2}mgv_0 t^2\cos\theta\hat{k}$$

Hence, the correct answer is (D).

11. For a particle in uniform circular motion, acceleration is

$$a = \frac{v^2}{R} \quad \text{(towards the centre)}$$



From figure, we get

$$\vec{a} = -a\cos\theta\hat{i} - a\sin\theta\hat{j} = -\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$$

Hence, the correct answer is (D).

12. $s = t^3 + 3$

$$\Rightarrow \quad v = \frac{ds}{dt} = \frac{d}{dt}(t^3 + 3) = 3t^2$$

Tangential acceleration, $a_t = \dfrac{dv}{dt} = \dfrac{d}{dt}(3t^2) = 6t$

CHAPTER 5

At $t = 2$ s, we have

$$v = 3(2)^2 = 12 \text{ ms}^{-1}$$

and $a_t = 6(2) = 12 \text{ ms}^{-2}$

Also, centripetal acceleration is given by

$$a_c = \frac{v^2}{R} = \frac{(12)^2}{20} = \frac{144}{20} = 7.2 \text{ ms}^{-2}$$

Net acceleration,

$$a = \sqrt{(a_c)^2 + (a_t)^2} = \sqrt{(7.2)^2 + (12)^2} = 14 \text{ ms}^{-2}$$

Hence, the correct answer is (A).

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Integer/Numerical Answer Type Questions

1. Average velocity $\langle v \rangle$ is given by

$$\langle v \rangle = \frac{\Sigma R}{\Sigma T}$$

where $\Sigma R = \left(\frac{2u_0^2 \sin\theta\cos\theta}{g}\right)\left(1 + \frac{1}{\alpha^2} + \frac{1}{\alpha^4} + ...\right)$

$$\Sigma R = \left(\frac{2u_0^2 \sin\theta\cos\theta}{g}\right)\left(\frac{1}{1 - \frac{1}{\alpha^2}}\right)$$

and $\Sigma T = \left(\frac{2u_0 \sin\theta}{g}\right)\left(1 + \frac{1}{\alpha} + \frac{1}{\alpha^2} + ...\right)$

$$\Rightarrow \quad \Sigma T = \left(\frac{2u_0 \sin\theta}{g}\right)\left(\frac{1}{1 - \frac{1}{\alpha}}\right)$$

Since $\langle v \rangle = 0.8V_1$, where $V_1 = u_0 \cos\theta$

$$\Rightarrow \quad 0.8V_1 = \frac{u_0 \cos\theta \left(\frac{1}{1 - \frac{1}{\alpha^2}}\right)}{\frac{1}{1 - \frac{1}{\alpha}}}$$

$$\Rightarrow \quad \left(\frac{\alpha^2}{\alpha^2 - 1}\right)\left(\frac{\alpha - 1}{\alpha}\right) = 0.8$$

$$\Rightarrow \quad \frac{\alpha}{\alpha + 1} = 0.8$$

$$\Rightarrow \quad 5\alpha = 4\alpha + 4$$

$$\Rightarrow \quad \alpha = 4$$

2. $H = \frac{u^2 \sin^2(45^\circ)}{2g} = 120$ m

$$\Rightarrow \quad \frac{u^2}{4g} = 120 \text{ m}$$

If speed is v after the first collision, then speed should remain $\frac{1}{\sqrt{2}}$ times, because kinetic energy has reduced to half.

$$\Rightarrow \quad v = \frac{u}{\sqrt{2}}$$

$$\Rightarrow \quad h_{max} = \frac{v^2 \sin^2(30^\circ)}{2g}$$

$$\Rightarrow \quad h_{max} = \frac{\left(\frac{u}{\sqrt{2}}\right)^2 \sin^2 30^\circ}{2g}$$

$$\Rightarrow \quad h_{max} = \left(\frac{u^2/4g}{4}\right) = \frac{120}{4}$$

$$\Rightarrow \quad h_{max} = 30 \text{ m}$$

3. $T = \frac{2u_y}{g} = \frac{2 \times 10 \sin(60^\circ)}{g} = \sqrt{3}$ s

Since $R = 1.15$ m and

$$R = u_x T - \frac{1}{2} a_x T^2$$

$$\Rightarrow \quad 1.15 = 10\cos(60^\circ)\sqrt{3} - \frac{1}{2} a \left(\sqrt{3}\right)^2$$

$$\Rightarrow \quad a = 5 \text{ ms}^{-2}$$