CHAPTER 4: KINEMATICS I

Test Your Concepts-I (Based on Displacement, Velocity, Acceleration, Average Speed and Velocity)

1. $a = \frac{dv}{dt} = 2t - 1$

$\Rightarrow \int_2^v dv = \int_0^t (2t-1)dt$

$\Rightarrow v = t^2 - t + 2$

Since $v = \frac{dx}{dt}$

$\Rightarrow \int_1^x dx = \int_0^t (t^2 - t + 2)dt$

$\Rightarrow x = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2t + 1$

When $t = 6$ s,

$v = 32 \text{ ms}^{-1}$

$s = 67$ m

Since $v \neq 0$ at all the times, so

$d = 67 - 1 = 66$ m

2. Since $s = \frac{1}{2}at^2$

$\Rightarrow t = \sqrt{\frac{2s}{a}}$

So, average velocity $= \frac{s}{t} = \frac{s}{\sqrt{\frac{2s}{a}}} = \sqrt{\frac{as}{2}}$

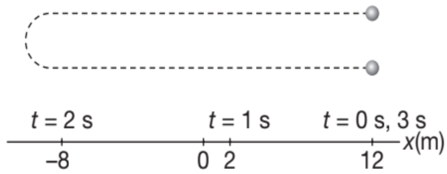
3. $|\vec{v}_{av}| = \frac{AB}{\Delta t} = \frac{2}{1} \text{ ms}^{-1} = 2 \text{ ms}^{-1}$

4. **Velocity:** $v = \frac{dx}{dt} = \frac{d}{dt}(12 - 15t^2 + 5t^3)$

$\Rightarrow v = -30t + 15t^2 \text{ ms}^{-1}$

The velocity of the particle changes direction at the instant when it is momentarily brought to rest. So,

$v = -30t + 15t^2 = 0$



$\Rightarrow t(-30 + 15t) = 0$

$\Rightarrow t = 0$ and 2 s

Position: The positions of the particle at $t = 0$ s, 1 s, 2 s and 3 s are

$x\big|_{t=0\text{ s}} = 12 - 15(0^2) + 5(0^3) = 12$ m

$x\big|_{t=1\text{ s}} = 12 - 15(1^2) + 5(1^3) = 2$ m

$x\big|_{t=2\text{ s}} = 12 - 15(2^2) + 5(2^3) = -8$ m

$x\big|_{t=3\text{ s}} = 12 - 15(3^2) + 5(3^3) = 12$ m

Using the above results, the path of the particle is shown. From this figure, the distance travelled by the particle during the time interval $t = 1$ s to $t = 3$ s is

$x_{\text{Tot}} = (2 + 8) + (8 + 12) = 30$ m

The average speed of the particle during the same time interval is

$v_{\text{avg}} = \frac{x_{\text{Tot}}}{\Delta t} = \frac{30}{3-1} = 15 \text{ ms}^{-1}$

5. (a) $c \equiv \frac{x}{t^2}$

Unit of c is ms^{-2}

Since, $b \equiv \frac{x}{t^3}$

Unit of b is ms^{-3}

(b) $x = 3t^2 - 2t^3$ ...(1)

For x to be maximum, $\frac{dx}{dt} = 0$

$\Rightarrow 6t - 6t^2 = 0$

$\Rightarrow t = 0, 1$ s

Further $\frac{d^2x}{dt^2} = 6 - 12t$

At $t = 1\text{ s}$, $\dfrac{d^2x}{dt^2} = -6$

So, at $t = 1\text{ s}$, x is maximum.

(c) $x|_{t=0} = 0$

$x|_{t=1\text{ s}} = 1\text{ m}$

$x|_{t=4\text{ s}} = -80\text{ m}$

$\Rightarrow \quad x_{\text{Total}} = 1 + 1 + 80 = 82\text{ m}$

At $t = 4\text{ s}$, displacement $x = -80\text{ m}$

(d) From equation (1), we get

$v = 6t - 6t^2$ and $a = 6 - 12t$. So,

at $t = 0$, $v = 0$ and $a = 6\text{ ms}^{-2}$

at $t = 1\text{ s}$, $v = 0$ and $a = -6\text{ ms}^{-2}$

at $t = 2\text{ s}$, $v = -12\text{ ms}^{-1}$ and $a = -18\text{ ms}^{-2}$

at $t = 3\text{ s}$, $v = -36\text{ ms}^{-1}$ and $a = -30\text{ ms}^{-2}$

at $t = 4\text{ s}$, $v = -72\text{ ms}^{-1}$ and $a = -42\text{ ms}^{-2}$

6. $v = \dfrac{dx}{dt} - 3t^2 - 18t + 15$

$a = \dfrac{dv}{dt} = 6t - 18$

Acceleration at $t = 0$ is -18 ms^{-2} at $t = 3\text{ s}$ it is zero. Then it continuously goes on increasing. Hence acceleration and velocity both are maximum at $t = 10\text{ s}$. So,

$v_{\max} = 3 \times (10)^2 - 18(10) + 15 = 135\text{ ms}^{-1}$ and

$a_{\max} = 6 \times 10 - 18 = 42\text{ ms}^{-2}$

7. $\vec{v}_{av} = \dfrac{\vec{r}_f - \vec{r}_i}{\Delta t} = \dfrac{(5.1\hat{i} + 0.4\hat{j}) - (5\hat{i})}{0.02} = 5\hat{i} + 20\hat{j}$

$|\vec{v}_{av}| = \sqrt{(20)^2 + (5)^2} = 20.6\text{ ms}^{-1}$

$\tan\theta = \dfrac{v_y}{v_x} = \dfrac{20}{5}$

$\Rightarrow \quad \theta = \tan^{-1}\left(\dfrac{20}{5}\right) = 76°$

8. The displacement from A to C is

$\Delta x = x_C - x_A = -6 - (-8) = 2\text{ m}$

So, average velocity is

$v_{\text{avg}} = \dfrac{\Delta_x}{\Delta t} = \dfrac{2}{4+5} = 0.222\text{ ms}^{-1}$

The distances travelled from A to B and B to C are $x_{A\to B} = 8 + 3 = 11\text{ m}$ and $x_{B\to C} = 3 + 6 = 9\text{ m}$, respectively. Then, the total distance travelled is

$x_{\text{Total}} = x_{A\to B} + x_{B\to C} = 11 + 9 = 20\text{ m}$

$(v_{sp})_{avg} = \dfrac{x_{\text{Total}}}{\Delta t} = \dfrac{20}{4+5} = 2.22\text{ ms}^{-1}$

9. (a) $a = \dfrac{dv}{dt} = 6t - 6$

At $t = 2\text{ s}$, $v = 0$

From $t = 0$ to $t = 2\text{ s}$, particle travels along negative x-direction. Then it moves towards positive x-axis. Since

$v = \dfrac{dx}{dt} = 3t^2 - 6t$

$\Rightarrow \quad \int_0^x dx = \int_0^t (3t^2 - 6t)\,dt$

$\Rightarrow \quad x = t^3 - 3t^2$

At $t = 2\text{ s}$, $x = -4\text{ m}$

At $t = 3.5\text{ s}$, $x = 6.125\text{ m}$

Distance travelled, $x_{\text{Total}} = 4 + 4 + 6.125 = 14.125\text{ m}$

(b) Average velocity $= \dfrac{6.125}{3.5} = 1.75\text{ ms}^{-1}$ and

Average speed $= \dfrac{14.125}{3.5} = 4.03\text{ ms}^{-1}$

10. **Position:** The position of the particle when $t = 6\text{ s}$ is

$x\Big|_{t=6\text{ s}} = 1.5(6^3) - 13.5(6^2) + 22.5(6) = -27\text{ m}$

Total Distance Travelled: The velocity of the particle can be determined by applying.

$v = \dfrac{dx}{dt} = 4.5t^2 - 27t + 22.5$

The times when the particle stops are obtained by substituting $v = 0$. So,

$4.5t^2 - 27t + 22.5 = 0$

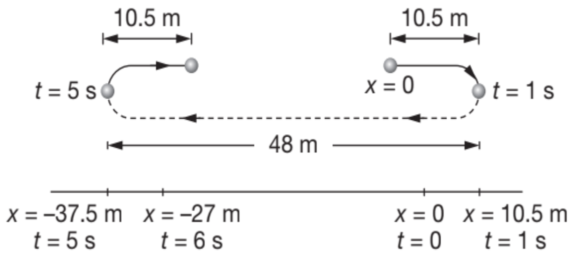
$\Rightarrow \quad t = 1\text{ s}$ and $t = 5\text{ s}$

The position of the particle at $t = 0\text{ s}$, 1 s and 5 s are

$x\Big|_{t=0\text{ s}} = 1.5(0^3) - 13.5(0^2) + 22.5(0) = 0$

$$x\Big|_{t=1\text{ s}} = 1.5(1^3) - 13.5(1^2) + 22.5(1) = 10.5\text{ m}$$

$$x\Big|_{t=5\text{ s}} = 1.5(5^3) - 13.5(5^2) + 22.5(5) = -37.5\text{ m}$$



From the particle's path, the total distance travelled is

$$x_{\text{total}} = (10.5) + [10.5 - (-37.5)] + (10.5)$$

$$x_{\text{total}} = 10.5 + 48 + 10.5 = 69\text{ m}$$

You can also try obtaining the same result by using

$$x = \frac{\left|\int_0^1 v\,dt\right| + \left|\int_1^5 v\,dt\right|}{6} = 69\text{ m}$$

11. According to the problem, we are given that

$$\frac{dv}{dx} = -3\text{ ms}^{-1}\text{ per metre}$$

Since $a = \frac{dv}{dt} = v\frac{dv}{dx}$

$$\Rightarrow \quad a = \frac{dv}{dt} = v\frac{dx}{dt} = (10)(-3) = -30\text{ ms}^{-2}$$

Test Your Concepts-II (Based on Constant Acceleration)

1. Since, $s_{n^{\text{th}}} = u + \frac{a}{2}(2n-1)$

$$\Rightarrow \quad x = u + \frac{a}{2}(2p-1) \qquad \ldots(1)$$

$$\Rightarrow \quad y = u + \frac{a}{2}(2q-1) \qquad \ldots(2)$$

$$\Rightarrow \quad z = u + \frac{a}{2}(2r-1) \qquad \ldots(3)$$

Subtracting equation (3) from equation (2), we get

$$y - z = \frac{a}{2}(2q-2r)$$

$$\Rightarrow \quad q - r = \frac{y-z}{a}$$

$$\Rightarrow \quad (q-r)x = \frac{1}{a}(yx - zx) \qquad \ldots(4)$$

Similarly, we can show that

$$(r-p)y = \frac{1}{a}(zy - xy) \text{ and} \qquad \ldots(5)$$

$$(p-q)z = \frac{1}{a}(xz - yz) \qquad \ldots(6)$$

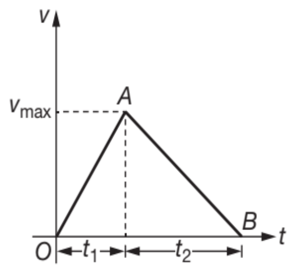
Adding equation (4), (5) and (6), we get

$$(q-r)x + (r-p)y + (p-q)z = 0$$

2. (a) Let the car accelerates for time t_1 and decelerates for time t_2. Then

$$t = t_1 + t_2 \qquad \ldots(1)$$

and corresponding velocity-time graph will be as shown in figure.



Since

$$v_{\max} = \alpha t_1$$

$$\Rightarrow \quad t_1 = \frac{v_{\max}}{\alpha} \qquad \ldots(2)$$

and $0 = v_{\max} + (-\beta)t_2$

$$\Rightarrow \quad t_2 = \frac{v_{\max}}{\beta} \qquad \ldots(3)$$

From equations (1), (2) and (3), we get

$$\frac{v_{\max}}{\alpha} + \frac{v_{\max}}{\beta} = t$$

$$\Rightarrow \quad v_{\max}\left(\frac{\alpha+\beta}{\alpha\beta}\right) = t$$

$$\Rightarrow \quad v_{\max} = \left(\frac{\alpha\beta}{\alpha+\beta}\right)t$$

(b) Total distance or displacement is the area under v-t graph

$$\Rightarrow \quad s = \frac{1}{2}v_{\max}t$$

$$\Rightarrow \quad s = \frac{1}{2}\left(\frac{\alpha\beta t}{\alpha+\beta}\right)t$$

$$\Rightarrow \quad \text{Distance} = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2$$

CHAPTER 4

3. As acceleration is constant, from $s = ut + \frac{1}{2}at^2$, we have

$$x = \frac{1}{2}at^2 \qquad (\text{as } u = 0) \qquad \ldots(1)$$

Now, if it travels a distance y in next t s then the total distance travelled in $(2t)$ sec will be $x + y$, so

$$x + y = \frac{1}{2}a(2t)^2 \qquad \ldots(2)$$

Dividing equation (2) by (1),

$$\frac{x+y}{x} = 4$$

$$\Rightarrow \quad y = 3x$$

4. Let s be the distance covered by each car. Let the times taken by the two cars to complete the journey be t_A and t_B and their velocities at the finishing point be v_A and v_B respectively. According to the problem,

$$v_A - v_B = v \text{ and } t_B - t_A = t$$

Now, $\frac{v}{t} = \frac{v_A - v_B}{t_B - t_A}$

Where $v_A = \sqrt{2a_1 s}$ and $v_B = \sqrt{2a_2 s}$

$$t_A = \sqrt{\frac{2s}{a_1}} \text{ and } t_B = \sqrt{\frac{2s}{a_2}}$$

$$\Rightarrow \quad \frac{v}{t} = \frac{\sqrt{2a_1 s} - \sqrt{2a_2 s}}{\sqrt{\frac{2s}{a_2}} - \sqrt{\frac{2s}{a_1}}} = \frac{\sqrt{a_1} - \sqrt{a_2}}{\sqrt{\frac{1}{a_2}} - \sqrt{\frac{1}{a_1}}} = \sqrt{a_1 a_2}$$

$$\Rightarrow \quad v = t\sqrt{a_1 a_2}$$

5. Since the race ends in a dead heat i.e., the two cars reach their destination in the same time say t. If the length of the course be s. Then

$$s = v_1 t + \frac{1}{2}a_1 t^2 = v_2 t + \frac{1}{2}a_2 t^2$$

$$\Rightarrow \quad t\left[\frac{1}{2}(a_1 - a_2)t + (v_1 - v_2)\right] = 0$$

$$\Rightarrow \quad t = 0 \text{ or } t = -\frac{2(v_1 - v_2)}{(a_1 - a_2)}$$

But $t \neq 0$, as it corresponds to initial position. So,

$$t = -\frac{2(v_1 - v_2)}{(a_1 - a_2)}$$

Substituting t in the equation, $s = v_1 t + \frac{1}{2}a_1 t^2$, we get

$$s = -\frac{2v_1(v_1 - v_2)}{(a_1 - a_2)} + \frac{1}{2}a_1 \frac{4(v_1 - v_2)^2}{(a_1 - a_2)^2}$$

$$\Rightarrow \quad s = \frac{2(v_1 - v_2)}{(a_1 - a_2)^2}\left[-v_1(a_1 - a_2) + a_1(v_1 - v_2)\right]$$

$$\Rightarrow \quad s = \frac{2(v_1 - v_2)(v_1 a_2 - v_2 a_1)}{(a_1 - a_2)^2}$$

6. (a) $(15)^2 - v_1^2 = 2\left(\frac{5}{3}\right)(60)$ $\qquad \{\because v^2 - u^2 = 2as\}$

$$\Rightarrow \quad v_1 = 5 \text{ ms}^{-1}$$

(b) Since $v = u + at$

$$\Rightarrow \quad 15 = v_1 + 6a \qquad \ldots(1)$$

Since $s = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad 60 = 6v_1 + \frac{1}{2} \times a \times (6)^2$$

$$\Rightarrow \quad 10 = v_1 + 3a \qquad \ldots(2)$$

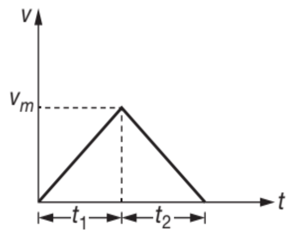
Solving equation (1) and (2), we get

$$a = \frac{5}{3} = 1.67 \text{ ms}^{-2}$$

(c) $v_1^2 = 2as$

$$\Rightarrow \quad s = \frac{v_1^2}{2a} = \frac{25}{\frac{10}{3}} = 7.5 \text{ m}$$

7. The corresponding v-t graph is shown in figure



Given $t_1 + t_2 = 4$ min $\qquad \ldots(1)$

Since, $\frac{v_m}{t_1} = x$ and $\frac{v_m}{t_2} = y$

$$\Rightarrow \quad t_1 = \frac{v_m}{x} \text{ and } t_2 = \frac{v_m}{y}$$

Since $s_1 = \left(\frac{0 + v_m}{2}\right)t_1$

and $s_2 = \left(\frac{v_m + 0}{2}\right)t_2$

$$\Rightarrow \quad \frac{1}{2}(t_1+t_2)(v_m)=4 \text{ km}$$

$$\Rightarrow \quad \frac{1}{2}\times 4\times v_m=4$$

$$\Rightarrow \quad v_m=2 \text{ kmmin}^{-1}$$

Substituting these values in equation (1), we get

$$\frac{2}{x}+\frac{2}{y}=4$$

$$\Rightarrow \quad \frac{1}{x}+\frac{1}{y}=2$$

Please see that this problem can also be done by using the concept of graphs.

8. Let t_0 be the reaction time and a the magnitude of deceleration. Then in the first case,

$$0=\left(80.5\times\frac{5}{18}\right)^2-2a\left(56.7-80.5\times\frac{5}{18}\times t_0\right) \quad \ldots(1)$$

In the second case, we have

$$0=\left(48.3\times\frac{5}{18}\right)^2-2a\left(24.4-48.3\times\frac{5}{18}\times t_0\right) \quad \ldots(2)$$

Solving these two equations, we get

$$t_0=0.74 \text{ s and } a=6.2 \text{ ms}^{-2}$$

9. $s=\frac{1}{2}at_0^2$ and $v=at_0$

Since t is measured from the beginning of motion, so we have for the particle to return to the initial position,

$$-s=v(t-t_0)-\frac{1}{2}a(t-t_0)^2$$

$$\Rightarrow \quad -\frac{1}{2}at_0^2=at_0(t-t_0)-\frac{1}{2}a(t-t_0)^2$$

$$\left\{\because s=\frac{1}{2}at_0^2 \text{ and } v=at_0\right\}$$

$$\Rightarrow \quad -\frac{1}{2}at_0^2=at_0t-at_0^2-\frac{1}{2}at^2-\frac{1}{2}at_0^2+at_0t$$

$$\Rightarrow \quad -\frac{1}{2}at^2+2at_0t-at_0^2=0$$

$$\Rightarrow \quad t^2-4t_0t-2t_0^2=0$$

$$\Rightarrow \quad t=\frac{4t_0\pm\sqrt{16t_0^2+8t_0^2}}{2}$$

$$\Rightarrow \quad t=\frac{4t_0+\sqrt{24}t_0}{2}$$

$$\Rightarrow \quad t=\frac{4t_0+2\sqrt{6}t_0}{2}$$

$$\Rightarrow \quad t=t_0+\sqrt{6}t_0$$

$$\Rightarrow \quad t=3.45t_0$$

10. Squaring the given equation, we get

$$v^2=4+4x$$

Now, comparing it with

$$v^2=u^2+2as$$

we get, $u=2 \text{ ms}^{-1}$ and $a=2 \text{ ms}^{-2}$

So, displacement at $t=2\,s$ is given by

$$s=ut+\frac{1}{2}at^2$$

$$\Rightarrow \quad s=(2)(2)+\frac{1}{2}(2)(2)^2$$

$$\Rightarrow \quad s=8 \text{ m}$$

11. Average Speed $=v_{av}=\frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$

$$\Rightarrow \quad v_{av}=\frac{l+2l+3l}{t_1+t_2+t_3} \quad \ldots(1)$$

For *OA*

$l=$ Area $\Delta(OAM)$

$$\Rightarrow \quad l=\frac{1}{2}v_{\max}t_1$$

$$\Rightarrow \quad t_1=\frac{2l}{v_{\max}} \quad \ldots(2)$$

For *AB*

$2l=$ Area(ABNM)

$$\Rightarrow \quad 2l=v_{\max}t_2$$

$$\Rightarrow \quad t_2=\frac{2l}{v_{\max}} \quad \ldots(3)$$

For *BC*

$3l=$ Area $\Delta(BCN)$

$$\Rightarrow \quad 3l=\frac{1}{2}v_{\max}t_3$$

$$\Rightarrow \quad t_3=\frac{6l}{v_{\max}} \quad \ldots(4)$$

Substituting (2), (3) and (4) in (1), we get

$$\frac{v_{av}}{v_{max}} = \frac{3}{5}$$

12. Let a be the uniform acceleration of α-particle. According to the problem $s = 2$ m, $v = 9000 \text{ ms}^{-1}$ and $u = 1000 \text{ ms}^{-1}$

Since $v^2 - u^2 = 2as$, so we get

$$a = \frac{v^2 - u^2}{2s} = \frac{(9000)^2 - (1000)^2}{2 \times (2)}$$

$$\Rightarrow \quad a = \frac{8 \times 10^7}{4} = 2 \times 10^7 \text{ ms}^{-2}$$

Let the particle remains in the tube for time t. Then

$$v = u + at$$

$$\Rightarrow \quad t = \frac{v - u}{a} = \frac{9000 - 1000}{2 \times 10^7} = 4 \times 10^{-4} \text{ s}$$

13. (a) Suppose the time of retardation of sports car be t hours. Thus the time during which the sports car moves with constant speed before being overtaken is also t. Since

$$s_{total} = 1 \text{ km}$$

$$\Rightarrow \quad \left(\frac{60 + 40}{2}\right)t + 40t = 1 \text{ km}$$

$$\Rightarrow \quad t = \frac{1}{90} \text{ h}$$

Time taken by police car to reach sports car is

$$2t = \frac{1}{45} \text{ h} = 80 \text{ s}$$

(b) Let the speed of the police car be v, when it overtakes the sports car. Then

$$\frac{v}{2} \times 2t = 1 \text{ km}$$

$$\Rightarrow \quad vt = 1 \text{ km}$$

$$\Rightarrow \quad \frac{v}{90} = 1 \text{ km}$$

$$\Rightarrow \quad v = 90 \text{ kmh}^{-1}$$

(c) Let a be the acceleration of police car, then

$$90 = \frac{a}{45}$$

$$\Rightarrow \quad a = 90 \times 45 \text{ kmh}^{-2}$$

If the retardation of the sports car be a', then

$$40 = 60 - \frac{a'}{90}$$

$$\Rightarrow \quad a' = 1800 \text{ kmh}^{-2}$$

Let t_1 be the required time, then

$$90 \times 45 \times t_1 = 60 - 1800t_1$$

$$\Rightarrow \quad t_1 = \frac{60}{5850} \text{ h} = 37 \text{ s}$$

14. (a) Time of acceleration, say t_1 is

$$t_1 = \frac{v_{max}}{a_1} = \frac{90 \text{ kmh}^{-1}}{3 \text{ kmh}^{-1} \text{ per second}} = 30 \text{ s} = 0.5 \text{ min}$$

Time for deceleration, say t_2 is

$$t_2 = \frac{v_{max}}{a_2} = \frac{90 \text{ kmh}^{-1}}{3 \text{ kmh}^{-1} \text{ per second}}$$

$$t_2 = 18 \text{ s} = 0.3 \text{ min}$$

Now we shall calculate the distance covered during acceleration and deceleration

Distance covered during acceleration is

$$s_1 = \left(\frac{0 + v_{max}}{2}\right)t_1$$

$$\Rightarrow \quad s_1 = \frac{1}{2}\left(90 \times \frac{5}{18}\right) = 375 \text{ m}$$

Distance covered during deceleration is

$$s_3 = \left(\frac{v_{max} + 0}{2}\right)t_2$$

$$\Rightarrow \quad s_3 = \frac{1}{2}\left(90 \times \frac{5}{18}\right)(18) = 225 \text{ m}$$

Distance covered with maximum speed is

$$s_2 = 50000 - 375 - 225 = 49400 \text{ m}$$

$$\Rightarrow \quad s_2 = 49.4 \text{ km}$$

So, total time for a non-stop train is

$$T = 0.5 \text{ min} + \frac{49.4 \times 60}{90} \text{ min} + 0.3 \text{ min}$$

$$\Rightarrow \quad T = 33.7 \text{ min}$$

(b) 12 intermediate stations mean that the train has to accelerate and decelerate at 13 stations (including the start). So, combined time of deceleration and acceleration is $t_1 + t_3 = 0.8 \times 13 \text{ min} = 10.4 \text{ min}$. Distance covered in acceleration and deceleration.

$$s = (375 + 225) \times 13 = 7800 \text{ m} = 7.8 \text{ km}$$

Time for uniform speed is $t_2 = (50 - 7.8)\left(\frac{60}{90}\right)$

$$\Rightarrow \quad t_2 = 28.13 \text{ min}$$

Time in motion $T = 10.4 + 28.13 = 38.53$ min

Halting time at 12 stations is $t_{halt} = 12 \times \frac{1}{2} = 6$ min

Toal time $T = t_1 + t_2 + t_3 + t_{halt}$

$\Rightarrow \quad T = (38.53 + 6)$ min $= 44.53$ min

15. Let x be the distance travelled by the truck when truck and car are side by side. The distance travelled by the car will be $(x + 150)$ as the car is 150 metre behind the truck.

Let the car overtake the truck at time t, then

For Truck

$$x = \frac{1}{2} \times (1.5) t^2 \qquad \ldots(1)$$

For Car

$$(x + 150) = \frac{1}{2} \times (2) t^2 \qquad \ldots(2)$$

From equations (1) and (2), we have

$$\frac{x + 150}{x} = \frac{2}{1.5}$$

$\Rightarrow \quad x = 450$ metre (truck)

and $x + 150 = 600$ metre (car)

Substituting the value of x in equation (1), we get

$$450 = \frac{1}{2}(1.5) t^2$$

$$\Rightarrow \quad t = \sqrt{\frac{450 \times 2}{1.5}}$$

$$\Rightarrow \quad t = \sqrt{600} = 24.5 \text{ s}$$

16. Let x_1 and x_2 be the distances travelled by the car before they stop under deceleration.

Since $v^2 - u^2 = 2as$, so we get

$$(0)^2 - (10)^2 = 2(-2) x_1$$

and $(0)^2 - (12)^2 = 2(-2) x_2$

Solving we get $x_1 = 25$ m and $x_2 = 36$ m

Total distance covered by the two cars

$$x_1 + x_2 = 25 + 36 = 61 \text{ metre}$$

Distance between the two cars when they stop

$$x = 150 - 61 = 89 \text{ m}$$

17. Let car A take t_1 hour for first half and t_2 hour for second half.

Since first half distance = second half distance

$\Rightarrow \quad 30 t_1 = 60 t_2$

$\Rightarrow \quad t_1 = 2 t_2$

further $t_1 + t_2 = 2$

$\Rightarrow \quad t_1 = \frac{4}{3}$ h and $t_2 = \frac{2}{3}$ h

Total distance $x = 30\left(\frac{4}{3}\right) + 60\left(\frac{2}{3}\right) = 80$ km

Let a be the acceleration of car B. Since $s = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad 80 = 0 \times 2 + \frac{1}{2} \times a \times (2)^2$$

$\Rightarrow \quad a = 40$ kmh^{-2}

To have the **same velocity**, we have

In interval 1, $at_1' = v_1$

$\Rightarrow \quad 40 t_1' = 30$

$\Rightarrow \quad t_1' = \frac{3}{4}$ h $\qquad \left(t_1 < \frac{4}{3}\right)$

and in interval 2, $at_2' = v_2$

$40 t_2' = 60$

$\Rightarrow \quad t_2' = \frac{3}{2}$ h $\qquad \left(\frac{4}{3} < t_2 < 2\right)$

To overtake, the distance travelled by both the cars in the first half is the same

$$\Rightarrow \quad \frac{1}{2}(40) t^2 = 30t$$

$$t = \frac{3}{2}h \left(> \frac{4}{3}h \right)$$

So no over taking in first half.

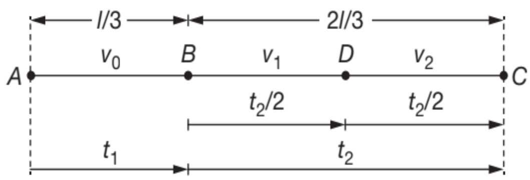
For second half, again we have $s_A = s_B$

$$\Rightarrow \quad 30\left(\frac{4}{3}\right) + 60\left(t - \frac{4}{3}\right) = \frac{1}{2}(40) t^2$$

$\Rightarrow \quad t = 1h \left(< \frac{4}{3}h \right)$ not possible and $t = 2$, i.e., at the end of the journey

Hence there is no overtaking during the entire journey.

18. The situation is shown in figure



$$\text{Since } v_{av} = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$$

$$\Rightarrow \quad v_{av} = \frac{\frac{l}{3} + \frac{2l}{3}}{t_1 + t_2} \qquad \ldots(1)$$

For AB

$$\frac{l}{3} = v_0 t_1$$

$$\Rightarrow \quad t_1 = \frac{l}{3v_0} \qquad \ldots(2)$$

For BC

$$\frac{2l}{3} = v_1\left(\frac{t_2}{2}\right) + v_2\left(\frac{t_2}{2}\right)$$

$$\Rightarrow \quad t_2 = \frac{4l}{3(v_1+v_2)} \qquad \ldots(3)$$

Substituting (2) and (3) in (1), we get

$$v_{av} = \frac{3v_0(v_1+v_2)}{4v_0+v_1+v_2}$$

19. Let the man take t s to get the door. The distance moved by the man in t s should therefore be 6 m more than the distance moved by the bus in the same time.

Distance moved by the man in t s is $x_{man} = 4t$

Distance moved by the bus in t s is

$$x_{bus} = \frac{1}{2} \times 1.2 \times t^2 = 0.6t^2$$

Since $x_{bus} = x_{man}$

$$\Rightarrow \quad 4t = 6 + 0.6t^2$$

Solving, we get

$$t = 4.387 \text{ s}$$

$$\Rightarrow \quad 2.27 \text{ s}$$

If the man is 10 m behind the door, then $4t = 10 + 0.6t^2$. This equation has imaginary roots.

20. For car A, $u_1 = 8 \text{ ms}^{-1}$, $a_1 = 1 \text{ ms}^{-2}$

For car B, $u_2 = 5 \text{ ms}^{-1}$, $a_2 = 1.1 \text{ ms}^{-2}$

Let x be the length of the track, then

$$x = 8t + \left(\frac{1}{2}\right)(1)t^2 \text{ and } x = 5t + \frac{1}{2}(1.1)t^2$$

$$\Rightarrow \quad t = 60 \text{ s}$$

Now, $x = 8 \times 60 + \left(\frac{1}{2}\right)(1)(60)^2$

$$\Rightarrow \quad x = 2280 \text{ m}$$

Further, before 10 s, i.e., in $60 - 10 = 50$ s

Car A has travelled a distance $s_A = 1650$ m and car B has travelled a distance $s_B = 1625$ m

21. $25 = 0 + (1.5)t_1$

$$\Rightarrow \quad t_1 = \frac{50}{3} \text{ s}$$

$$\Rightarrow \quad s_1 = 0 + \frac{1}{2}(1.5)\left(\frac{2500}{9}\right) = \frac{1875}{9} \text{ m}$$

$$\Rightarrow \quad s_1 = \frac{1875}{9} \text{ m}$$

Distance covered in the next 60 seconds is

$$s_2 = (25)(60) = 1500 \text{ m}$$

$$\text{Average Speed} = \frac{s_1+s_2}{t_1+t_2} = \frac{1500 + \frac{1875}{9}}{\frac{50}{3} + 60}$$

$$\Rightarrow \quad v_{av} = \frac{1708.33}{76.67}$$

$$\Rightarrow \quad v_{av} = 22.3 \text{ ms}^{-1}$$

22. The corresponding v-t graph is shown in figure.

For accelerated interval,

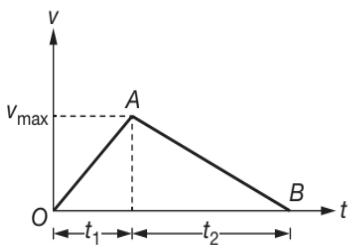
$$v_{max} = 0 + (0.2)t_1$$

$$\Rightarrow \quad t_1 = 5 \text{ v}_{max} \qquad \ldots(1)$$

For decelerated interval,

$$0 = v_{max} + (-0.1)t_2$$

$$\Rightarrow \quad t_2 = 10 \text{ v}_{max} \qquad \ldots(2)$$



Further,

$$s_1 + s_2 = 14$$

$$\Rightarrow \quad \left(\frac{0+v_{max}}{2}\right)t_1 + \left(\frac{v_{max}+0}{2}\right)t_2 = 14$$

$$\Rightarrow \quad \frac{1}{2}(t_1+t_2)v_{max} = 14$$

$$\Rightarrow \quad \frac{1}{2}(15 \text{ v}_{max}^2) = 14$$

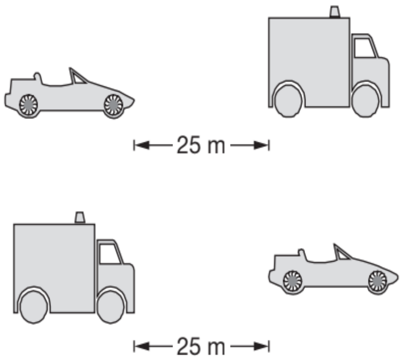
$$\Rightarrow \quad v_{max}^2 = \frac{28}{15}$$

$\Rightarrow \quad v_{max} = 1.37 \text{ ms}^{-1}$

This v_{max} happens to be less than the maximum attainable speed of 2.5 ms^{-1}. So,

$$t_{MIN} = t_1 + t_2 = 15\, v_{max} = 20.6 \text{ ms}^{-1}$$

23.



(a) $s_{Car} = s_{Truck} + 25 + 25 + \begin{pmatrix}\text{length}\\ \text{of car}\end{pmatrix} + \begin{pmatrix}\text{length}\\ \text{of truck}\end{pmatrix}$

$$\Rightarrow \quad 20t + \frac{1}{2}(0.6)t^2 = (20t) + 50 + 5 + 20$$

$$\Rightarrow \quad t = 15.8 \text{ s}$$

(b) $s_{Car} = (20)(15.8) + \frac{1}{2}(0.6)(15.8)^2 = 391 \text{ m}$

(c) $v_{Car} = 20 + (0.6)(15.8) = 29.5 \text{ ms}^{-1}$

24. Velocity of train after 30 second,

$$v_T = at = 30 \times 3 \times 10^{-2} = 0.9 \text{ ms}^{-1}$$

Let v_C be the velocity of the car in opposite direction. Then, in the next 60 s,

$$348 = \begin{vmatrix}\text{displacement}\\ \text{of train}\end{vmatrix} + \begin{vmatrix}\text{Displacement of the car}\\ \text{in opposite direction}\end{vmatrix}$$

$$\Rightarrow \quad 348 = (0.9)(60) + \frac{1}{2}(3 \times 10^{-2})(3600) + 60v_c$$

$$\Rightarrow \quad v_C = 4 \text{ ms}^{-1}$$

Thus velocity of the car should be 4 ms^{-1}, opposite to the direction of motion of the train.

25. Let the initial velocity of the particle (i.e., at $t = 0$) be u and acceleration be a. Since at $t = 0$, displacement measured from a convenient fixed position is 2 m and at $t = 10$ s it is zero. So displacement from $t = 0$ to $t = 10$ s is -2 m. Since

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad -2 = 10u + \frac{1}{2}a(10)^2$$

$$\Rightarrow \quad 5u + 25a = -1 \qquad \ldots(1)$$

Further the particle reverses its direction of motion at $t = 6$ s, so

$$0 = u + 6a$$

$$5u + 30a = 0 \qquad \ldots(2)$$

Subtracting (1) from (2), we get

$$5a = 1$$

$$\Rightarrow \quad a = \frac{1}{5}\text{ms}^{-2} = 0.2\text{ms}^{-2} \text{ and } u = -6a = -1.2 \text{ ms}^{-1}$$

So, if v be the velocity at $t = 10$ s, then

$$v = u + at$$

$$\Rightarrow \quad v = -1.2 + (0.2)(10) = 0.8 \text{ ms}^{-1}$$

Test Your Concepts-III (Based on Variable Acceleration)

1. Since $a = \frac{dv}{dt}$

$$\Rightarrow \quad 30 - \frac{v}{5} = \frac{dv}{dt}$$

$$\Rightarrow \quad \frac{dv}{\left(30 - \frac{v}{5}\right)} = dt$$

$$\Rightarrow \quad \int_0^{30} \frac{dv}{30 - \frac{v}{5}} = \int_0^t dt$$

$$\Rightarrow \quad -5\log_e\left(30 - \frac{v}{5}\right)\Bigg|_0^{30} = t\Big|_0^t$$

$$\Rightarrow \quad -5\log_e\left(\frac{24}{30}\right) = t$$

$$\Rightarrow \quad t = 5\log_e\left(\frac{5}{4}\right) \text{ s}$$

2. $a = 2s$

$$\Rightarrow \quad v\frac{dv}{ds} = 2s$$

$$\Rightarrow \quad vdv = 2s\,ds$$

$$\Rightarrow \quad \int_0^v vdv = \int_0^s 2s\,ds$$

$$\Rightarrow \quad \left(\frac{v^2}{2}\right)\Bigg|_0^v = \left(s^2\right)\Big|_0^s$$

$$\Rightarrow \quad \frac{v^2}{2} = s^2$$

$\Rightarrow \quad v = \pm\sqrt{2}s$

This is the desired velocity-displacement equation.

3. Since $v = v_0 - kx$

$$\Rightarrow \quad \frac{dx}{dt} = v_0 - kx$$

$$\Rightarrow \quad \int_0^x \frac{dx}{v_0 - kx} = \int_0^t dt$$

$$\Rightarrow \quad -\frac{1}{k}\log_e(v_0 - kx)\Big|_0^x = t\Big|_0^t$$

$$\Rightarrow \quad -\frac{1}{k}\log_e\left(\frac{v_0 - kx}{v_0}\right) = t$$

$$\Rightarrow \quad \frac{v_0 - kx}{v_0} = e^{-kt}$$

$$\Rightarrow \quad x = \frac{v_0}{k}\left(1 - e^{-kt}\right) \qquad \ldots(1)$$

Since $v = \dfrac{dx}{dt} = \dfrac{d}{dt}\left[\dfrac{v_0}{k}\left(1 - e^{-kt}\right)\right]$

$$\Rightarrow \quad v = v_0 e^{-kt}$$

Also, $a = \dfrac{dv}{dt} = \dfrac{d}{dt}\left(v_0 e^{-kt}\right)$

$$\Rightarrow \quad a = -kv_0 e^{-kt}$$

4. Given $a = -4s$

$$\Rightarrow \quad v\frac{dv}{ds} = -4s$$

$$\Rightarrow \quad \int_{v_0}^0 v\,dv = -4\int_0^s s\,ds$$

$$\Rightarrow \quad -\frac{v_0^2}{2} = -4\frac{s^2}{2}$$

$$\Rightarrow \quad s = 2v_0$$

Students remain often confused that the equation $s = 2v_0$ is not dimensionally correct. Please do not miss to write dimensions of proportionality constant.

5. Since $a = \dfrac{dv}{dt}$

$$\Rightarrow \quad v\,dv = a\,dx \qquad \left\{\because\ a = v\frac{dv}{dx}\right\}$$

$$\Rightarrow \quad \int_0^v v\,dv = \int_0^x (8 - 2x)\,dx$$

$$\Rightarrow \quad \frac{v^2}{2}\Big|_0^v = \left(8x - x^2\right)\Big|_0^x$$

$$\Rightarrow \quad v = \sqrt{16x - 2x^2}\ \text{ms}^{-1} \qquad \ldots(1)$$

At $x = 2$ m,

$$v\Big|_{x=2\text{ m}} = \sqrt{16(2) - 2(2^2)} = \pm 2\sqrt{6}\ \text{ms}^{-1}$$

When the velocity is maximum then

$$\frac{dv}{dx} = 0$$

$$\Rightarrow \quad \frac{dv}{dx} = \frac{16 - 4x}{2\sqrt{16x - 2x^2}} = 0$$

$$\Rightarrow \quad 16 - 4x = 0$$

$$\Rightarrow \quad x = 4\text{ m}$$

Substituting $x = 4$ m in (1), we get

$$|v_{MAX}| = \sqrt{16(4) - 2(16)} = \sqrt{64 - 32}$$

$$\Rightarrow \quad |v_{MAX}| = \sqrt{32} = \pm 4\sqrt{2}\text{ m}$$

6. $a = \dfrac{dv}{dt}$

$$\Rightarrow \quad \frac{k}{v} = \frac{dv}{dt}$$

$$\Rightarrow \quad v\,dv = k\,dt$$

$$\Rightarrow \quad \int_{v_0}^v v\,dv = k\int_0^t dt$$

$$\Rightarrow \quad \frac{v^2}{2}\Big|_{v_0}^v = kt\Big|_0^t$$

$$\Rightarrow \quad v^2 - v_0^2 = 2kt$$

$$\Rightarrow \quad v = \sqrt{v_0^2 + 2kt}$$

7. Initial Acceleration $= \dfrac{dv}{dt}\Big|_{t=0} = A - B(0) = A$

$$a = 0$$

$$\Rightarrow \quad A - Bv = 0$$

$$\Rightarrow \quad v = \frac{A}{B}$$

Since $\dfrac{dv}{dt} = A - Bv$

$$\Rightarrow \quad \int_0^v \frac{dv}{A - Bv} = \int_0^t dt$$

$$\Rightarrow \quad -\frac{1}{B}\log_e(A-Bv)\Big|_0^v = t$$

$$\Rightarrow \quad \log_e\left(\frac{A-Bv}{A}\right) = -Bt$$

$$\Rightarrow \quad A - Bv = Ae^{-Bt}$$

$$\Rightarrow \quad v = \frac{A}{B}\left(1-e^{-Bt}\right)$$

Check-Point

At $t=0$, $v=0$

At $t\to\infty$, $v\to\frac{A}{B}$

8. $v = -4x^2$

$$\Rightarrow \quad \frac{dx}{dt} = -4x^2$$

$$\Rightarrow \quad \int_2^x x^{-2}dx = \int_0^t -4dt$$

$$\Rightarrow \quad -x^{-1}\Big|_2^x = -4t\Big|_0^t$$

$$\Rightarrow \quad t = \frac{1}{4}\left(x^{-1}-0.5\right)$$

$$\Rightarrow \quad x = \frac{2}{8t+1}$$

$$v = \frac{dx}{dt} = \frac{(8t+1)(0)-(2)\frac{d}{dt}(8t+1)}{(8t+1)^2} = -\frac{16}{(8t+1)^2}$$

$$a = \frac{dv}{dt} = -\left(\frac{(8t+1)^2(0)-16\frac{d}{dt}(8t+1)^2}{(8t+1)^4}\right)$$

$$\Rightarrow \quad a = \frac{16(2)(8)(8t+1)}{(8t+1)^4}$$

$$\Rightarrow \quad a = \frac{dv}{dt} = \frac{256}{(8t+1)^3}\ \text{ms}^{-2}$$

9. (a) $\frac{dv}{dt} = 3-2t$

$$\Rightarrow \quad \int_{v_0}^v dv = \int_0^t (3-2t)dt$$

$$\Rightarrow \quad v = v_0 + 3t - t^2 \quad \ldots(1)$$

Since $v = \frac{dx}{dt}$

$$\Rightarrow \quad \int_{x_0}^x dx = \int_0^t \left(v_0+3t-t^2\right)dt$$

$$\Rightarrow \quad x = x_0 + v_0t + \frac{3}{2}t^2 - \frac{t^3}{3} \quad \ldots(2)$$

Since $x\Big|_{t=0} = x\Big|_{t=5}$

$$\Rightarrow \quad x_0 = x_0 + 5v_0 + \frac{75}{2} - \frac{125}{3}$$

$$\Rightarrow \quad 5v_0 = \frac{125}{3} - \frac{75}{2}$$

$$\Rightarrow \quad v_0 = \frac{25}{3} - \frac{15}{2}$$

$$\Rightarrow \quad v_0 = \frac{50-45}{6} = \frac{5}{6}\ \text{ms}^{-1}$$

(b) From (1), we get velocity at $t = 5$ s

$$v = \frac{5}{6} + (3)(5) - (5)^2$$

$$\Rightarrow \quad v = \frac{5}{6} + 15 - 25$$

$$\Rightarrow \quad v = \frac{5+90-150}{6}$$

$$\Rightarrow \quad v = -\frac{55}{6}\ \text{ms}^{-1}$$

10. Since $dv = adt$ $\quad\left\{\because a = \frac{dv}{dt}\right\}$

$$\Rightarrow \quad \int_0^v dv = 5\int_0^t e^t dt$$

$$\Rightarrow \quad v = 5\left(e^t-1\right)\ \text{ms}^{-1}$$

Further $v = \frac{dx}{dt}$

$$\Rightarrow \quad dx = vdt$$

$$\Rightarrow \quad \int_0^x dx = 5\int_0^t \left(e^t-1\right)dt$$

$$\Rightarrow \quad x = 5\left(e^t - t - 1\right)\ \text{m}$$

11. $v = 200x$

$$\Rightarrow \quad \frac{dv}{dx} = 200\frac{d}{dx}(x)$$

$$\Rightarrow \frac{dv}{dx} = 200$$

Since $a = \frac{dv}{dt} = v\left(\frac{dv}{dx}\right)$

$$\Rightarrow a = (200x)(200)$$

$$\Rightarrow a = 4 \times 10^4 x \text{ mms}^{-2}$$

When $x = 2000$ mm, then

$$a = (4 \times 10^4)(2000) \text{ mms}^{-2}$$

$$\Rightarrow a = 80 \text{ kms}^{-2}$$

Further $v = 200x$

$$\Rightarrow \frac{dx}{dt} = 200x$$

$$\Rightarrow \int_{500}^{x} \frac{dx}{x} = 200\int_0^t t$$

$$\Rightarrow \log_e\left(\frac{x}{500}\right) = 200t$$

$$\Rightarrow t = \frac{1}{200}\log_e\left(\frac{x}{500}\right)$$

At $x = 2000$ mm,

$$t = \frac{1}{200}\log_e(4)$$

$$\Rightarrow t = 6.93 \times 10^{-3} \text{ s}$$

$$\Rightarrow t = 6.93 \text{ ms}$$

12. Please note that here acceleration is a function of time, i.e., acceleration is not constant. So, we cannot apply

$$\vec{v} = \vec{u} + \vec{a}t$$

Since $\vec{a} = \frac{d\vec{v}}{dt}$

$$\Rightarrow d\vec{v} = \vec{a}dt$$

$$\Rightarrow \int_0^{\vec{v}} d\vec{v} = \int_0^2 \vec{a}dt$$

$$\Rightarrow \vec{v} = \int_0^2 (2t\hat{i} + 3t^2\hat{j})dt = (t^2\hat{i} + t^3\hat{j})\Big|_0^2$$

$$\Rightarrow \vec{v} = (4\hat{i} + 8\hat{j}) \text{ ms}^{-1}$$

Hence, velocity of particle at time $t = 2$ s is $(4\hat{i} + 8\hat{j})$ ms^{-1}

13. Since $a = \frac{dv}{dt} = v\frac{dv}{dx}$

$$\Rightarrow -(g + kv^2) = v\frac{dv}{dx}$$

$$\Rightarrow -\int_{v_0}^{v} \frac{v\,dv}{g + kv^2} = \int_0^x dx \qquad \ldots(1)$$

Let us calculate $\int \frac{v\,dv}{g + kv^2} = I$ (say)

Substitute $g + kv^2 = z$

$$\Rightarrow 2kv\,dv = dz$$

$$\Rightarrow v\,dv = \frac{dz}{2k}$$

So, $I = \frac{1}{2k}\int \frac{dz}{z}$

$$\Rightarrow I = \frac{1}{2k}\log_e(z)$$

Substituting value of z, we get

$$I = \frac{1}{2k}\log_e(g + kv^2)$$

So, from (1), we get

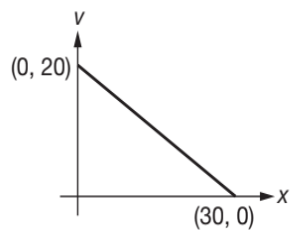
$$-\frac{1}{2k}\log_e(g + kv^2)\Big|_{v_0}^{v} = x\Big|_0^x$$

$$\Rightarrow x = -\frac{1}{2k}\log_e\left(\frac{g + kv^2}{g + kv_0^2}\right) = \frac{1}{2k}\log_e\left(\frac{g + kv_0^2}{g + kv^2}\right)$$

Now, the particle achieves its maximum height, which is also the point of reversal of motion when $v = 0$. Thus,

$$h_{max} = \frac{1}{2k}\log_e\left(\frac{g + kv_0^2}{g}\right)$$

14. The equation or the relation that predicts the motion specified in the problem is



$$v - 20 = \left(\frac{20 - 0}{0 - 30}\right)(x - 0)$$

$$\Rightarrow \quad v = 20 - \frac{2}{3}x$$

$$\frac{dv}{dt} = -\frac{2}{4}\frac{dx}{dt} = -\frac{2}{3}v \quad \ldots(1)$$

At $x = 15$ m, $v = 20 - \frac{2}{3} \times 15 = 10$ ms^{-1}

So, acceleration $\frac{dv}{dt} = -\frac{2}{3} \times 10 = -\frac{20}{3}$ ms^{-2}

From equation (1), we get

$$\int_{20}^{V} \frac{dv}{v} = -\frac{2}{3}\int_{0}^{t} dt$$

$$\Rightarrow \quad v = 20e^{-2/3t}$$

$$\Rightarrow \quad dx = 20e^{-2/3t}dt \quad \left\{\because v = \frac{dx}{dt}\right\}$$

$$\Rightarrow \quad \int_0^x dx = 20\int_0^t e^{-2/3t}dt$$

$$\Rightarrow \quad x = 30\left(1 - e^{-2/3t}\right) \text{ m}$$

Note that $x \to 30$ m when $t \to \infty$.

15. $dv = a\,dt \quad \left\{\because a = \frac{dv}{dt}\right\}$

$$\Rightarrow \quad \int_0^v dv = \int_0^t \left(12t - 3t^{1/2}\right)dt$$

$$\Rightarrow \quad v\Big|_0^v = \left(6t^2 - 2t^{3/2}\right)\Big|_0^t$$

$$\Rightarrow \quad v = \left(6t^2 - 2t^{3/2}\right) \text{ ms}^{-1}$$

Using this result and the initial condition $x_0 = 15\ m$ at $t = 0$ s,

$$dx = v\,dt \quad \left\{\because v = \frac{dx}{dt}\right\}$$

$$\Rightarrow \quad \int_{x_0}^x dx = \int_0^t \left(6t^2 - 2t^{3/2}\right)dt$$

$$\Rightarrow \quad x\Big|_{15\text{ m}}^x = \left(2t^3 - \frac{4}{5}t^{5/2}\right)\Bigg|_0^t$$

$$\{\because \text{ at } t = 0,\ x_0 = 15 \text{ m}\}$$

$$\Rightarrow \quad x = \left(2t^3 - \frac{4}{5}t^{5/2} + 15\right) \text{ m}$$

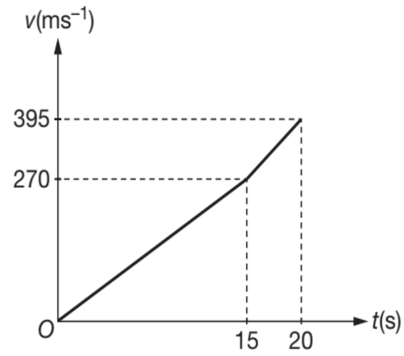
Test Your Concepts-IV (Based on Graph)

1. At $t = 15$ s, $v = 0 + (18)(15) = 270$ ms^{-1}

At $t = 20$ s, $v = 270 + (25)(20 - 15) = 395$ ms^{-1}

For stage 1, $v = 18t \quad 0 \le t \le 15$ s

For stage 2, $v = 270 + 25(t - 15) \quad 15 < t \le 20$ s



From v-t graph, we have

at $t = 15$ s, $s = \frac{1}{2}(15)(270) = 2025$ m

at $t = 20$ s,

$$s = 2025 + 270(20 - 15) + \frac{1}{2}(395 - 270)(20 - 15)$$

$$\Rightarrow \quad s = 3687.5 \text{ m}$$

For stage 1, i.e.,

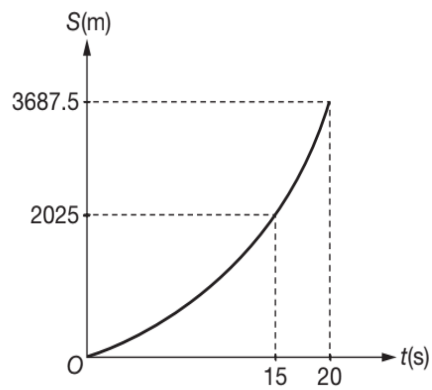
$0 \le t \le 15$ s, we have

$a_1 = 18$ and $v = 18t$

$$\Rightarrow \quad s = s_0 + v_0 t + \frac{1}{2}a_1 t^2 = 0 + 0 + 9t^2 = 9t^2$$

So, at $t = 15$ s, we have

$$s = 9(15)^2 = 2025 \text{ m}$$



For stage 2, i.e.,

$15 \le t \le 20$ s, we have

$a_2 = 25$ and $v = 270 + 25(t - 15)$

CHAPTER 4

$$\Rightarrow \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2 =$$

$$\Rightarrow \quad s = 2025 + 270(t-15) + \frac{1}{2}(25)(t-15)^2$$

So, at $t = 20$ s, we have

$$v = 395 \text{ ms}^{-1} \text{ and } s = 3687.5 \text{ m}$$

2. For $0 \le x < 3$ m,

$$a = \frac{dv}{dt} = v\frac{dv}{dx} = (2x+4)(2) = (4x+8) \text{ ms}^{-2}$$

At $x = 0$ m and 3 m,

$$a\Big|_{x=0 \text{ m}} = 4(0) + 8 = 8 \text{ ms}^{-2}$$

$$a\Big|_{x=3 \text{ m}} = 4(3) + 8 = 20 \text{ ms}^{-2}$$

For $3m < x \le 6$ m,

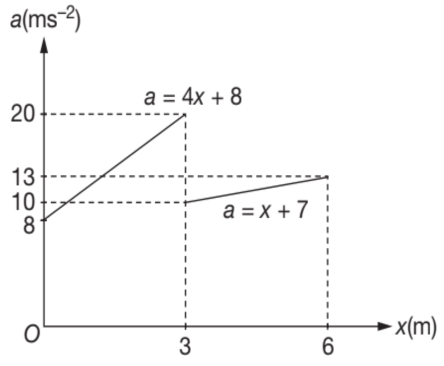
$$a = \frac{dv}{dt} = v\frac{dv}{dx} = (x+7)(1) = (x+7) \text{ ms}^{-2}$$

At $x = 3$ m and 6 m,

$$a\Big|_{x=3 \text{ m}} = 3 + 7 = 10 \text{ ms}^{-2}$$

$$a\Big|_{x=6 \text{ m}} = 6 + 7 = 13 \text{ ms}^{-2}$$

So, we observe that at $x = 3$ m, the acceleration suddenly switches from 20 ms^{-2} to 10 ms^{-2}. The a-x graph is shown as per the data calculated.



3. For $0 \le x < 225$ m,

$$a = \frac{dv}{dt} = v\frac{dv}{dx}$$

$$\Rightarrow \quad a = \left(5\sqrt{x}\right)\left(\frac{5}{2\sqrt{x}}\right) = 12.5 \text{ ms}^{-2}$$

For $225 \text{ m} < x \le 525$ m,

$$a = \frac{dv}{dt} = v\frac{dv}{dx}$$

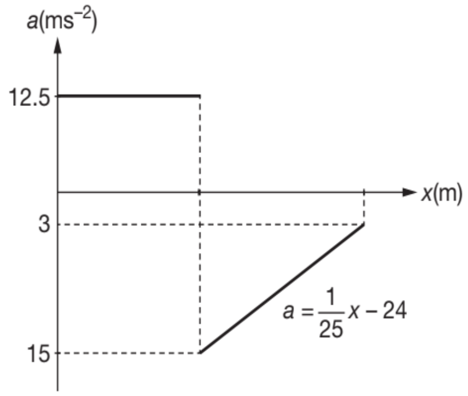
$$\Rightarrow \quad a = (-0.2x + 120)(-0.2) = (0.04x - 24) \text{ ms}^{-2}$$

At $x = 225$ m and 525 m

$$a\Big|_{x=225 \text{ m}} = 0.04(225) - 24 = -15 \text{ ms}^{-2}$$

$$a\Big|_{x=525 \text{ m}} = 0.04(525) - 24 = -3 \text{ ms}^{-2}$$

The a-x graph is shown in figure.



4. For $0 \le t < 6$, $dv = a\,dt$

$$\Rightarrow \quad \int_0^v dv = \int_0^t \frac{1}{6}t^2 dt$$

$$\Rightarrow \quad v = \frac{1}{18}t^3 \qquad \ldots(1)$$

Since $dx = v\,dt$ $\qquad \left\{\because v = \frac{dx}{dt}\right\}$

$$\Rightarrow \quad \int_0^x dx = \int_0^t \frac{1}{18}t^3 dt$$

$$\Rightarrow \quad x = \frac{1}{72}t^4$$

So, when $t = 6$ s, $v = 12 \text{ ms}^{-1}$, $x = 18$ m

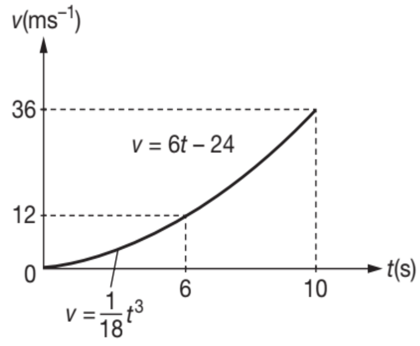
For $6 < t \le 10$, $dv = a\,dt$

$$\Rightarrow \quad \int_{12}^v dv = \int_6^t 6\,dt$$

$$\Rightarrow \quad v = 6t - 24$$

Since $dx = v\,dt$

$$\Rightarrow \quad \int_{18}^x dx = \int_6^t (6t - 24)\,dt$$



$$x = 3t^2 - 24t + 54$$

When $t = 10$ s, $v = 36\ \text{ms}^{-1}$ and $x = 114$ m

5. For $0 \le x < 200$ m, the initial condition is $v = 0$ at $x = 0$

$$vdv = adx$$

Since $a = v\dfrac{dv}{dx}$

$$\Rightarrow \int_0^v vdv = \int_0^x (0.1x + 5)dx$$

$$\left.\frac{v^2}{2}\right|_0^v = \left.(0.05x^2 + 5x)\right|_0^x$$

$$v = \left(\sqrt{0.1x^2 + 10x}\right)\ \text{ms}^{-1}$$

At $x = 200$ m,

$$\left. v\right|_{x=200\text{ m}} = \sqrt{0.1(200^2) + 10(200)}$$

$$= 77.46\ \text{ms}^{-1} = 77.5\ \text{ms}^{-1}$$

For $200\text{ m} < x \le x_0$, the initial condition is

$$v = 77.46\ \text{ms}^{-1} \text{ at } x = 200\text{ m}$$

Again, since $vdv = adx$ $\left\{\because a = \dfrac{vdv}{dx}\right\}$

$$\Rightarrow \int_{77.46\text{ ms}^{-1}}^{v} v\,dv = \int_{200\text{ m}}^{x} -15dx$$

$$\Rightarrow \left.\frac{v^2}{2}\right|_{77.46\text{ ms}^{-1}}^{v} = \left.-15x\right|_{200\text{ m}}^{x}$$

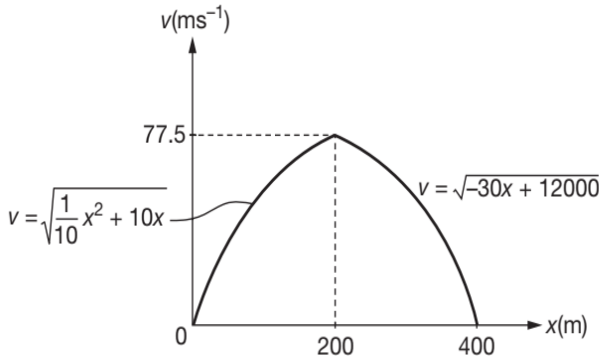
$$\Rightarrow v = \sqrt{-30x + 12000}\ \text{ms}^{-1}$$

When $v = 0$ i.e., at $x = x_0$, we have

$$0 = \sqrt{-30x_0 + 12000}$$

$$\Rightarrow x_0 = 400\text{ m}$$

The v-x graph is shown in figure.



6. $x = t^3 - 3t^2 + 2t$

$$\Rightarrow v = \frac{dx}{dt} = 3t^2 - 6t + 2$$

$$\Rightarrow a = \frac{dv}{dt} = 6t - 6$$

To draw the x-t graph, let us first calculate the points of reversal of motion i.e., the points or the times when $v = 0$. So, $v = 0$ gives

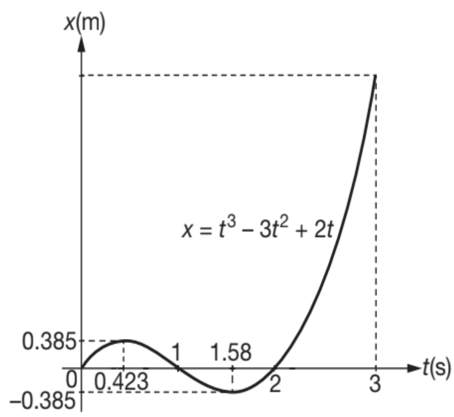
$$3t^2 - 6t + 2 = 0$$

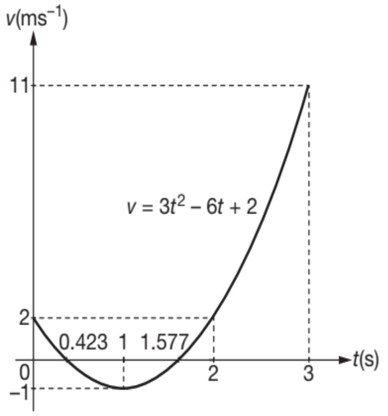
$$\Rightarrow t = \left(\frac{6 - \sqrt{12}}{6}\right)\text{ s and } t = \left(\frac{6 + \sqrt{12}}{6}\right)\text{ s}$$

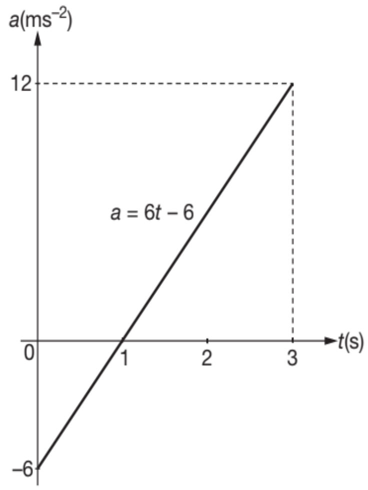
$$\Rightarrow t = 1.577\text{ s and } t = 0.423\text{ s}$$

$$\left. x\right|_{t=1.577} = -0.386\text{ m}$$

$$\left. x\right|_{t=0.4226} = 0.385\text{ m}$$







7. For $0 \le x < 625$ m, we observe

$$a = \frac{dv}{dt} = v\frac{dv}{dx}$$

$$\Rightarrow \quad a = \left(0.6x^{3/4}\right)\left[\frac{3}{4}(0.6)x^{-1/4}\right] = \left(0.27x^{1/2}\right) \text{ ms}^{-2}$$

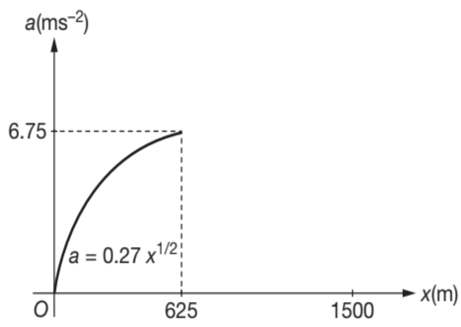
At $x = 625$ m,

$$a\Big|_{x=625\text{ m}} = 0.27\left(625^{1/2}\right) = 6.75 \text{ ms}^{-2}$$

For $625 \text{ m} < x < 1500$ m

$$a = \frac{dv}{dt} = v\frac{dv}{dx} = 75(0) = 0 \qquad \left\{\because \frac{dv}{dx} = 0\right\}$$

The a-x graph is shown in figure



8. From the graph, $a = 22.5 - \frac{22.5}{150}x$

$$\Rightarrow \quad \int_0^v v\,dv = \int_0^{60}\left(22.5 - \frac{22.5}{150}x\right)dx$$

$$\Rightarrow \quad \frac{v^2}{2} = (22.5)(60) - \left(\frac{22.5}{150}\right)\frac{(60)^2}{2}$$

$$\Rightarrow \quad v = 46.47 \text{ ms}^{-1}$$

9. **At** $x = 50$ m

Since $a = \frac{dv}{dt} = v\frac{dv}{dx}$

$$\Rightarrow \quad a = (20)\left(\frac{40}{100}\right) = 8 \text{ ms}^{-2}$$

At $x = 150$ m

$$a = \frac{dv}{dt} = v\frac{dv}{dx}$$

$$\Rightarrow \quad a = (45)\left(\frac{10}{100}\right) = 4.5 \text{ ms}^{-2}$$

10. Since, $a = \frac{dv}{dt} = v\frac{dv}{dx}$

$$\Rightarrow \quad v\,dv = a\,dx$$

$$\Rightarrow \quad \frac{v^2}{2} = \text{area under } a\text{-}x \text{ graph} \qquad \{\text{since } v_{\text{initial}} = 0\}$$

$$\Rightarrow \quad v = \sqrt{2(\text{area of } a\text{-}x \text{ graph})}$$

At $x = 40$ m, we have

$$v = \sqrt{2 \times 40 \times 2} = 12.7 \text{ ms}^{-1}$$

At $x = 90$ m, we have

$$v = \sqrt{2 \times (100 + 40 \times 4)} = 22.8 \text{ ms}^{-1}$$

At $x = 200$ m, we have

$$v = \sqrt{2 \times (100 + 400 + 150)} = 36.1 \text{ ms}^{-1}$$

11. (a) From the graph $a = 2t - 2$

$$\Rightarrow \quad \int_0^V dv = \int_0^t (2t - 2)\,dt$$

$$\Rightarrow \quad v = t^2 - 2t$$

(b) $$\int_0^x dx = \int_2^4 \left(t^2 - 2t\right)dt \qquad \left\{\because v = \frac{dx}{dt}\right\}$$

$\Rightarrow \quad x = \left(\frac{t^3}{3} - t^2\right)\Bigg|_2^4 = \left(\frac{64}{3} - 16 - \frac{8}{3} + 4\right)$ m

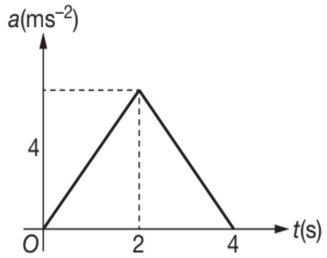
$\Rightarrow \quad x = 6.67$ m

12. $dv = adt$

$$\Rightarrow \int_{v_i}^{v_f} dv = \int_0^t a\,dt$$

$$\Rightarrow \Delta v = \int_0^t adt$$

$\Rightarrow$ Change in velocity = Area under a-t graph



$$\Rightarrow v_f - v_i = \frac{1}{2}(4)(4) = 8 \text{ ms}^{-1}$$

$$\Rightarrow v_f = v_i + 8 = (2+8) \text{ ms}^{-1}$$

$$\Rightarrow v_f = 10 \text{ ms}^{-1}$$

13. Displacement = Area under velocity-time graph

Hence, $x_{OA} = \frac{1}{2} \times 2 \times 10 = 10$ m

$x_{AB} = 2 \times 10 = 20$ m

$\Rightarrow x_{OAB} = 10 + 20 = 30$ m

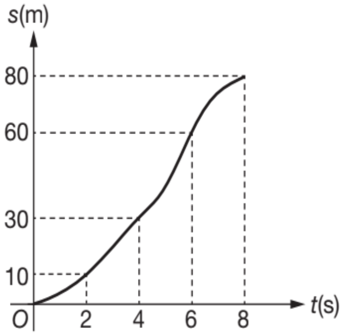
$x_{BC} = \frac{1}{2} \times 2(10+20) = 30$ m

$\Rightarrow x_{OABC} = 30 + 30 = 60$ m

and $x_{CD} = \frac{1}{2} \times 2 \times 20 = 20$ m

$\Rightarrow x_{OABCD} = 60 + 20 = 80$ m

Between 0 to 2 s and 4 s to 6 s motion is accelerated, hence displacement-time graph is a parabola. Between 2 s to 4 s motion is uniform, so displacement-time graph will be a straight line. Between 6 s to 8 s motion is decelerated hence displacement-time graph is again a parabola but inverted in shape. At the end of 8 s velocity is zero, therefore, slope of displacement-time graph should be zero. The corresponding graph is shown in figure.



Test Your Concepts-V (Based on Motion Under Gravity)

1. (a) A particle thrown upwards has its velocity in opposite direction to its acceleration (g, downwards).

(b) When the particle is released from rest from a certain height, its velocity is zero, while acceleration is g downwards. Similarly, at the extreme position of a pendulum velocity is zero, while acceleration is not zero.

(c) In uniform circular motion velocity is perpendicular to its radial or centripetal acceleration.

2. Since $a = \frac{dv}{dt} = v\frac{dv}{dy}$

$$\Rightarrow v\frac{dv}{dy} = -\frac{g_0 R^2}{(R+y)^2}$$

$$\Rightarrow \int_u^0 vdv = -g_0 R^2 \int_0^\infty \frac{dy}{(R+y)^2}$$

$$\Rightarrow u^2 = 2g_0 R^2 \left(\frac{1}{R+y}\right)\Bigg|_\infty^0 = 2g_0 R$$

$$\Rightarrow u = \sqrt{2g_0 R}$$

This velocity is also called as the escape velocity, v_e

Substituting the values, we get $v_e = 11.2 \text{ kms}^{-1}$

3. Let the particles meet t_0 second after the projection of the first particle. Further suppose that they meet at a height h from the ground

For first particle,

$$h = ut_0 - \frac{1}{2}gt_0^2 \quad \ldots(1)$$

where u ms^{-1} is the velocity of the particle

For second particle,

$$h = u(t_0 - t) - \frac{1}{2}g(t_0 - t)^2 \quad \ldots(2)$$

From equations (1) and (2), we get

$$ut_0 - \frac{1}{2}gt_0^2 = u(t_0 - t) - \frac{1}{2}g(t_0 - t)^2$$

$$\Rightarrow \quad ut_0 - \frac{1}{2}gt_0^2 = ut_0 - ut - \frac{1}{2}gt_0^2 + gt_0t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad ut - gt_0t + \frac{1}{2}gt^2 = 0$$

$$\Rightarrow \quad u - gt_0 + \frac{1}{2}gt = 0$$

$$\Rightarrow \quad u - gt_0 + \frac{1}{2}gt = 0$$

$$\Rightarrow \quad t_0 = \left(\frac{t}{2} + \frac{u}{g}\right) \text{ second}$$

Velocity of first particle is v_0. So

$$\Rightarrow \quad v_1 = u - gt_0 = u - g\left(\frac{t}{2} + \frac{u}{g}\right)$$

$$\Rightarrow \quad v_1 = -\frac{1}{2}gt \text{ (downwards)}$$

Velocity of second particle is v_2. So

$$v_2 = u - g(t_0 - t) = u - g\left(\frac{t}{2} + \frac{u}{g} - t\right)$$

$$\Rightarrow \quad v_2 = -\frac{1}{2}gt \text{ (upwards)}$$

4. Let h be the maximum height. Then applying

$$v^2 - u^2 = 2as$$

Taking upward direction as positive, we get

$$v = 0,\ u = 100 \text{ ms}^{-1},\ a = -g = -10 \text{ ms}^{-2}$$

$$\Rightarrow \quad 0^2 - (100)^2 = 2(-10)h$$

$$\Rightarrow \quad h = 500 \text{ m}$$

Hence, velocity at height $h = 250$ m will be

$$v^2 - (100)^2 = 2(-10)(250)$$

$$\Rightarrow \quad v^2 = 10000 - 5000 = 5000$$

$$\Rightarrow \quad v = 10\sqrt{50} \text{ ms}^{-1}$$

5. Let u be the velocity of projection of the particle. Then

$$h = uT - \frac{1}{2}gT^2$$

$$\Rightarrow \quad uT = h + \frac{1}{2}gT^2$$

$$\Rightarrow \quad u = \frac{h}{T} + \frac{1}{2}gT = \frac{2h + gT^2}{2T}$$

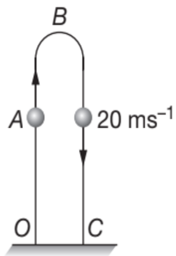
If H be the greatest height reached, then

$$0 = u^2 - 2gH$$

$$\Rightarrow \quad 0 = \left(\frac{2h + gT^2}{2T}\right)^2 - 2gH$$

$$\Rightarrow \quad H = \frac{(2h + gT^2)^2}{8gT^2}$$

6. In the graphs, $v_A = at_{O\to A} = (4)(5) = 20 \text{ ms}^{-1}$

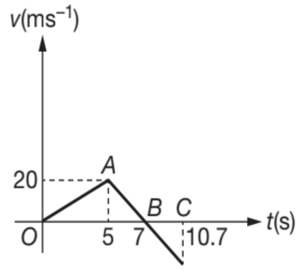


$$v_B = 0 = v_A - gt_{A\to B}$$

$$\Rightarrow \quad t_{A\to B} = \frac{v_A}{g} = \frac{20}{10} = 2 \text{ s}$$

$$\Rightarrow \quad t_{O\to A\to B} = (5 + 2)\text{s} = 7 \text{ s}$$

Now, $s_{O\to A\to B}$ = area under v-t graph between 0 to 7 s



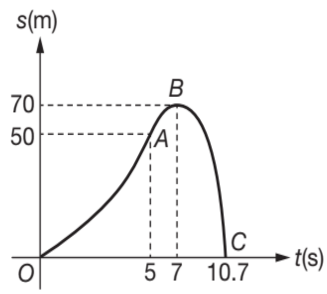
$$\Rightarrow \quad s_{O\to A\to B} = \frac{1}{2}(7)(20) = 70 \text{ m}$$

Further $|s_{OAB}| = |s_{BC}| = \frac{1}{2}g(t_{B\to C})^2$

$$\Rightarrow \quad 70 = \frac{1}{2}(10)(t_{B\to C})^2$$

$$\Rightarrow \quad t_{B\to C} = \sqrt{14} = 3.7 \text{ s}$$

$$\Rightarrow \quad t_{O\to A\to B\to C} = 7 + 3.7 = 10.7 \text{ s}$$



Also $s_{O\to A}$ = area under v-t graph between OA

$$s_{O\to A} = \frac{1}{2}(5)(20) = 50 \text{ m}$$

7. The second stone will catch up the first stone when the distance covered by it in $(t-n)$ second will be equal to the distance covered by the first stone in t second. Distance covered by first stone in t second

$$s_1 = \frac{1}{2}gt^2 \quad \ldots(1)$$

Distance covered by second stone in $(t-n)$ second

$$s_2 = u(t-n) + \frac{1}{2}g(t-n)^2 \quad \ldots(2)$$

From equations (1) and (2), we have

$$\frac{1}{2}gt^2 = u(t-n) + \frac{1}{2}g(t-n)^2$$

$$\Rightarrow \frac{1}{2}g\left[t^2 - (t-n)^2\right] = u(t-n)$$

$$\Rightarrow \frac{1}{2}g\left[2tn - n^2\right] = u(t-n)$$

$$\Rightarrow gtn - \frac{1}{2}gn^2 = ut - un$$

$$\Rightarrow t(gn-u) = n\left(\frac{1}{2}gn - u\right)$$

$$\Rightarrow t = \frac{n\left(\frac{1}{2}gn-u\right)}{(gn-u)}$$

The distance covered by the first stone in this time

$$s_1 = \frac{1}{2}g \times \left[\frac{n\left(\frac{1}{2}gn-u\right)}{(gn-u)}\right]^2 = \frac{gn^2}{8}\left(\frac{gn-2u}{gn-u}\right)^2$$

Thus the second stone will overtake the first stone at a distance

$$\frac{gn^2}{8}\left(\frac{gn-2u}{gn-u}\right)^2, \text{ below the top of the cliff.}$$

8. Here the speed of the body is attenuated due to air drag. The speed becomes zero under the effect of gravity. Due to air drag, let effective value of g be g'. Then

$$0 = u_0^2 - 2g'h$$

$$\Rightarrow u_0^2 = 2g'h \quad \ldots(1)$$

$$\text{Here } g' = g + \left(\frac{cu_0^2}{m}\right) \quad \ldots(2)$$

because cu_0^2 is the force with which air drags and produces a retardation of $\left(\frac{cu_0^2}{m}\right)$

From equation (1)

$$h = \frac{u_0^2}{2g'} = \frac{u_0^2}{2\left[g + \left(\frac{cu_0^2}{m}\right)\right]}$$

$$\Rightarrow h = \frac{u_0^2}{2g\left(1+\frac{cu_0^2}{mg}\right)} \quad \ldots(3)$$

As the body comes down, it acquires the velocity u'

Now $u'^2 = 2gh$

$$\Rightarrow h = \frac{u'^2}{2g} \quad \ldots(4)$$

From equation (3) and (4), we get

$$\frac{u_0^2}{2g\left(1+\frac{cu_0^2}{mg}\right)} = \frac{u'^2}{2g}$$

$$\Rightarrow u'^2 = \frac{u_0^2}{\left(1+\frac{cu_0^2}{mg}\right)}$$

$$\Rightarrow u' = \frac{u_0}{\sqrt{1+\frac{cu_0^2}{mg}}}$$

9. $a = \dfrac{\Delta v}{\Delta t} = \dfrac{v_2 - v_1}{t_2 - t_1}$

Take downward direction as positive, we get

$$v_1 = \sqrt{2gh_1} \text{ and } v_2 = -\sqrt{2gh_2}$$

$$\Rightarrow a = -\frac{\sqrt{2gh_2} - \sqrt{2gh_1}}{\Delta t}$$

$$\Rightarrow a = -\left(\frac{\sqrt{2gh_2} + \sqrt{2gh_1}}{\Delta t}\right)$$

Negative sign with a indicates that it is in the upward direction. So,

$$|a| = \frac{\sqrt{2(9.8)(4)} + \sqrt{2(9.8)(2)}}{12\times 10^{-3}}$$

$$\Rightarrow |a| = \frac{8.85 + 6.26}{12\times 10^{-3}} = 1.26\times 10^3 \text{ ms}^{-2}$$

10. For the motion of the rocket for first minute

$u_1 = 0$, $a_1 = 30 \text{ msec}^{-2}$

$\Rightarrow$ $t_1 = 60$ sec.

Let v_1 be the velocity of the rocket after one minute (60 sec.), then

$v_1 = u_1 + a_1 t_1 = 0 + 30 \times 60 = 1800 \text{ msec}^{-1}$

Height h_1 attained is

$h_1 = \frac{1}{2} a_1 t_1^2 = \frac{1}{2} \times 30 \times (60)^2$

$\Rightarrow$ $h_1 = 54000$ metre

When the fuel is used up,

$u_2 = 1800 \text{ msec}^{-1}$, $v_2 = 0$ and $a_2 = -10 \text{ msec}^{-2}$

$\Rightarrow$ $h_2 = \frac{(1800)^2}{2+10} = 162000$ metre

(a) Maximum height reached $= h_1 + h_2$

$\Rightarrow$ $h_{max} = 54000 + 162000$

$\Rightarrow$ $h_{max} = 216{,}000$ metre

(b) Let t_2 be the time for the journey when fuel is used up, then

$t_2 = \frac{v_2 - u_2}{a_2} = \frac{1800}{10} = 180$ sec.

Again let t_3 be the time for descent, then using the formula $h = \frac{1}{2} g t_3^2$, we have

$216000 = \frac{1}{2} \times 10 \times t_3^2$

$\Rightarrow$ $t_3^2 = 43200$

$\Rightarrow$ $t_3 = (43200)^{1/2} = 207.8$ sec.

Hence total time elapsed is

$T = t_1 + t_2 + t_3 = 60 + 180 + 207.8$

$\Rightarrow$ $T = 447.8$ sec.

11. Take upward direction as positive, then

$v^2 - u^2 = 2as$

$\Rightarrow$ $(5^2) - u^2 = 2(-10)(15)$

$\Rightarrow$ $u^2 = 325 \text{ m}^2\text{s}^{-2}$

$\Rightarrow$ $u = 5\sqrt{13} \text{ ms}^{-1}$

(a) $h = \frac{u^2}{2g} = \frac{325}{20} = 16.25$ m

(b) $t = \frac{u}{g} = \frac{5\sqrt{13}}{10} = 1.8$ s

12. Let h be the height from the ground at which both balls collide. Suppose the two balls collide after t second. Here the distance travelled by the first ball in t second must be the same as the distance travelled by the second ball in $(t-2)$ second, because second ball is thrown after 2 second of the first.

For the first,

$u = 39.2 \text{ msec}^{-1}$, $g = 9.8 \text{ msec}^{-2}$ and $s = h$

Using, $h = ut + \frac{1}{2} g t^2$, we get

$$h = 39.2t - \frac{1}{2} \times 9.8 \times t^2 \quad \ldots(1)$$

For the second ball, we have

$$h = 39.2(t-2) - \frac{1}{2} \times 9.8 (t-2)^2 \quad \ldots(2)$$

From equations (1) and (2), we get

$39.2t - \frac{1}{2} \times 9.8 \times t^2 = 39.2(t-2) - \frac{1}{2} \times 9.8 \times (t-2)^2$

$\Rightarrow$ $39.2t - 4.9t^2 = 39.2t - 78.4 - 4.9t^2 + 19.6t - 19.6$

Solving, we get $t = 5.02$ second

Putting this value of t in equation (1), we have

$h = 73.302$ metre

13. $s_1(t) = s_2(t - t_0)$

Taking upward direction as positive, we get

$v_0 t - \frac{1}{2} g t^2 = v_0 (t - t_0) - \frac{1}{2} g (t - t_0)^2$

On evaluation, we get

$t = \frac{v_0}{g} + \frac{t_0}{2}$

14. $\sqrt{\frac{2h}{g}} - \sqrt{\frac{2 \times 5}{g}} = \sqrt{\frac{2 \times (h - 25)}{g}}$

$\Rightarrow$ $h = 45$ m

15. (a) $h_{max} = \frac{v_{net}^2}{2g} + 28 + 2 = \frac{(20+10)^2}{2(10)} + 30 = 75$ m

(b) Using $s = ut + \frac{1}{2} a t^2$ {relative to lift}

$\Rightarrow$ $-2 = 20t - \frac{1}{2}(10)t^2$

$-4 = 40t - 10t^2$

$\Rightarrow$ $10t^2 - 40t - 4 = 0$

$\Rightarrow$ $t = \frac{40 \pm \sqrt{1600 + 160}}{20}$

$$\Rightarrow \quad t = \frac{40 + 41.95}{20}$$

$$\Rightarrow \quad t \cong 4.1 \text{ s}$$

16. Let h be the height of the tower and v be the velocity of the ball at the top of tower. Since the boy catches the ball 3 second after the ball first passes him, the displacement of the ball during the time is zero.

Using the formula, $h = ut + \frac{1}{2}gt^2$, we have

$$0 = u(3) - \frac{1}{2}(9.8)(3)^2$$

Solving, we get $v = 14.7 \text{ ms}^{-1}$

So when the boy catches the ball, its velocity will be 14.7 ms^{-1} in the downward direction.

Now, using the formula,

$v^2 = u^2 + 2gh$, we have

$$(14.7)^2 = (24.5)^2 - 2 \times 9.8 \times h$$

Solving, we get $h = 19.6$ metre

17. $s_1(t) - s_2(t-1) = 10$

$$\Rightarrow \quad \frac{1}{2} \times 10 \times t^2 - \frac{1}{2} \times 10(t-1)^2 = 10$$

$$\Rightarrow \quad t = 1.5 \text{ s}$$

Test Your Concepts-VI (Based on Planar Motion)

1. $u_x = 0$, $a_x = 4 \text{ ms}^{-2}$ and $a_y = 2 \text{ ms}^{-2}$

$$x = u_x t + \frac{1}{2}a_x t^2 = \frac{1}{2}a_x t^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2x}{a_x}} = \sqrt{\frac{2 \times 32}{4}} = 4 \text{ s}$$

(a) $y = u_y t + \frac{1}{2}a_y t^2 = (8)(4) + \frac{1}{2}(2)(4)^2 = 48 \text{ m}$

(b) $v_x = a_x t = (4)(4) = 16 \text{ ms}^{-1}$

$$v_y = u_y + a_y t = 8 + (2)(4) = 16 \text{ ms}^{-1}$$

$$\Rightarrow \quad v = \sqrt{v_x^2 + v_y^2} \approx 22 \text{ ms}^{-1}$$

$$\Rightarrow \quad v = \sqrt{16^2 + 16^2} = 16\sqrt{2} \text{ ms}^{-1}$$

2. $v_x = \frac{dx}{dt} = 5t$

$$\Rightarrow \quad \int_0^x dx = \int_0^t 5t\,dt \qquad \{\because \text{ at } t = 0,\ x = 0 \text{ and } y = 0\}$$

$$\Rightarrow \quad x = \frac{5t^2}{2}$$

$$\Rightarrow \quad y = 0.5x^2 = \frac{25t^4}{8}$$

$$\Rightarrow \quad v_y = \frac{dy}{dt} = \frac{25}{2}t^3$$

At $t = 1$ s, $x = \frac{5}{2}$ m and $y = \frac{25}{8}$ m

So, distance from the origin is

$$s = \sqrt{x^2 + y^2} \cong 4 \text{ m}$$

Now, $a_x = \frac{dv_x}{dt} = 5 \text{ ms}^{-2}$ and $a_y = \frac{dv_y}{dt} = \frac{75}{2}t^2$

At $t = 1$ s we have, $a_y = \frac{75}{2} \text{ ms}^{-2}$

$$\Rightarrow \quad a = \sqrt{a_x^2 + a_y^2} = 37.8 \text{ ms}^{-2}$$

3. $\frac{dx}{dt} = v_x = k\omega\cos(\omega t)$ and

$$\frac{dy}{dt} = v_y = k\omega\sin(\omega t)$$

Now, speed $v = \sqrt{(v_x^2 + v_y^2)} = k\omega$

$$\Rightarrow \quad s = vt = k\omega t$$

4. (a) $x = kt$ ...(1)

$y = kt(1 - \alpha t)$ ...(2)

Equation of trajectory is given by

$$y = kt - k\alpha t^2 = x - k\alpha\frac{x^2}{k^2} \quad \left\{\because \text{ from } (1),\ t = \frac{x}{k}\right\}$$

$$\Rightarrow \quad y = x - \alpha\frac{x^2}{k}$$

This equation happens to be a parabola.

(b) $v_x = \frac{dx}{dt} = k$ and

$$v_y = \frac{dy}{dt} = k - 2k\alpha t = k(1 - 2\alpha t)$$

$$\Rightarrow \quad \vec{v} = v_x\hat{i} + v_y\hat{j} = k\hat{i} + k(1 - 2\alpha t)\hat{j}$$

$$\Rightarrow \quad v = \sqrt{k^2 + k^2(1 - 2\alpha t)^2} = k\sqrt{1 + (1 - 2\alpha t)^2}$$

Acceleration $\vec{a}$ is given by

$$\vec{a} = \frac{d\vec{v}}{dt} = -2\alpha k\hat{j} = \text{constant}$$

5. Since $v^2 = v_x^2 + v_y^2$

$$\Rightarrow \frac{d}{dt}(v^2) = \frac{d}{dt}(v_x^2) + \frac{d}{dt}(v_y^2)$$

$$\Rightarrow 2v\frac{dv}{dt} = 2v_x\frac{dv_x}{dt} + 2v_y\frac{dv_y}{dt}$$

$$\Rightarrow \frac{dv}{dt} = \frac{v_x a_x + v_y a_y}{v} \quad \ldots(1)$$

Now, if $\vec{a} = a_x\hat{i} + a_y\hat{j}$ and $\vec{v} = v_x\hat{i} + v_y\hat{j}$, then

$$\vec{a}\cdot\vec{v} = a_x v_x + a_y v_y \quad \ldots(2)$$

From (1) and (2), we get

$$\frac{dv}{dt} = \frac{\vec{a}\cdot\vec{v}}{v} = \frac{\vec{a}\cdot\vec{v}}{|\vec{v}|}$$

6. $(y-40)^2 = 160x$

Differentiating w.r.t. time, we get

$$2(y-40)\frac{dy}{dt} = 160\frac{dx}{dt}$$

$$\Rightarrow v_x = \frac{dx}{dt} = \frac{1}{80}(y-40)v_y \quad \ldots(1)$$

Since $y = 80$ m

$$\Rightarrow v_x = \frac{1}{80}(80-40)(180) = 90 \text{ ms}^{-1}$$

$$\Rightarrow v = \sqrt{v_x^2 + v_y^2} = \sqrt{(90)^2 + (180)^2} = 201.25 \text{ ms}^{-1}$$

Again differentiating (1) w.r.t. time, we get

$$a_x = \frac{1}{80}\left[(y-40)a_y + v_y^2\right]$$

At $a_y = 0$ and $v_y = 180 \text{ ms}^{-1}$, we get

$$a_x = \frac{1}{80}(180)^2 = 405 \text{ ms}^{-2}$$

7. Here, acceleration $\vec{a} = (4\hat{i} + 2\hat{j}) \text{ ms}^{-2}$ is constant. So,

$$\vec{v} = \vec{u} + \vec{a}t \text{ and } \vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

Substituting the proper values, we get

$$\vec{v} = (2\hat{i} + 3\hat{j}) + (2)(4\hat{i} + 2\hat{j}) = (10\hat{i} + 7\hat{j}) \text{ ms}^{-1} \text{ and}$$

$$\vec{s} = (2)(2\hat{i} + 3\hat{j}) + \frac{1}{2}(2)^2(4\hat{i} + 2\hat{j}) = (12\hat{i} + 10\hat{j}) \text{ m}$$

Therefore, velocity and displacement of particle at $t = 2$ s are $(10\hat{i} + 7\hat{j}) \text{ ms}^{-1}$ and $(12\hat{i} + 10\hat{j})$ m respectively.

8. $u_x = 20 \text{ kmh}^{-1} = \frac{50}{9} \text{ ms}^{-1}$

$$u_y = 12 \text{ kmh}^{-1} = \frac{10}{3} \text{ ms}^{-1}$$

Using, $s_y = u_y t + \frac{1}{2}a_y t^2$

Taking downward direction as positive, we get

$$s_y = 50 \text{ m}, \; u_y = -\frac{10}{3} \text{ ms}^{-1}, \; a_y = 10 \text{ ms}^{-2}$$

$$\Rightarrow 50 = -\frac{10}{3}t + \frac{1}{2}(10)t^2$$

$$\Rightarrow 10 = -\frac{2}{3}t + t^2$$

$$\Rightarrow 3t^2 - 2t - 30 = 0$$

$$\Rightarrow t = \frac{2 + \sqrt{4 + 360}}{2(3)} = \frac{2 + 19}{6}$$

$$\Rightarrow t = \frac{21}{6} = \frac{7}{2} = 3.5 \text{ s}$$

Speed with which the bag strikes the ground is v given by

$$v^2 = u^2 + 2gh \text{ where } u^2 = u_x^2 + u_y^2$$

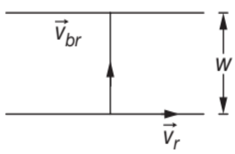
$$\Rightarrow v = \sqrt{u_x^2 + u_y^2 + 2gh}$$

$$\Rightarrow v = \sqrt{\frac{2500}{81} + \frac{100}{9} + (2)(10)(50)}$$

$$\Rightarrow v \cong 32 \text{ ms}^{-1}$$

Test Your Concepts-VII (Based on Relative Velocity)

1. Done already.

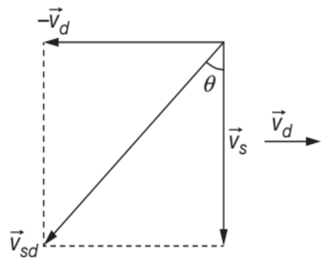


(a) $t = \frac{w}{v_{br}} = \frac{400}{10} = 40$ s

(b) $x = v_r t = 80$ m

2. v_s = Velocity of snow flakes $= 8 \text{ ms}^{-1}$,

v_d = Velocity of driver $= 50 \text{ kmh}^{-1} = 13.9 \text{ ms}^{-1}$



Since we know,

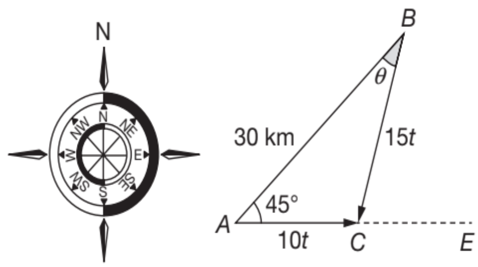
$\vec{v}_{sd} = \vec{v}_s - \vec{v}_d$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{13.9}{8}\right) = 60°$

3. Done already, see theory.

4. The situation is shown in figure

Let A and B be the positions of two ships respectively. The ship A at 9 am is 30 km south-west of B, i.e., $AB = 30$ km. AB makes an angle $45°$ with the east. A travels in the direction AC with velocity 10 kmhr^{-1}. Let B travels in the direction BC and interception takes place at point C after a time t. Then $AC = 10t$ and $BC = 15t$.



(a) Let BC makes an angle θ with BA

In triangle ABC, we have from Lami's Theorem

$\Rightarrow \quad \frac{10t}{\sin\theta} = \frac{15t}{\sin 45°}$

$\Rightarrow \quad \sin\theta = \frac{10t \sin 45°}{15t}$

$\Rightarrow \quad \sin\theta = \frac{10}{15} \times \frac{1}{\sqrt{2}} = 0.4714$

$\Rightarrow \quad \theta = 28°$

Now, $\angle BCA = 180° - (45° + 28°) = 107°$

(b) From triangle ABC, again using Lami's Theorem, we get

$\frac{15t}{\sin 45°} = \frac{30}{\sin 107°}$

$\Rightarrow \quad 15t = \frac{30 \sin 45°}{\sin 107°}$

$\Rightarrow \quad t = \frac{2\sin 45°}{\sin 107°} = 2\left(\frac{0.7070}{0.9563}\right) = 1.48$ hour

5. Let t be the time after turning back of the motorboat when it again meets with the raft.

Throughout the journey raft moves with v_r (absolute velocity of river) while the boatman travels with $(v_{br} + v_r)$ for 1 hour (downstream) and with $(v_{br} - v_r)$ for t hours (upstream). Since,

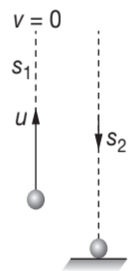
Displacement of raft = displacement of motorboat = 6 km

$\Rightarrow \quad v_r(1+t) = (v_{br}+v_r)(1) - (v_{br} - v_r)(t) = 6$ km

Solving, we get

$t = 1$ hr and $v_r = 3$ kmhr^{-1}

6. (a) If we consider elevator at rest, then relative acceleration of the bolt is $a_r = 9.8 + 1.2 = 11$ ms^{-1} (downwards).



After 2 second velocity of the elevator is $v = at = (1.2)(2) = 2.4$ ms^{-1}. Therefore initial velocity of the bolt is also 2.4 ms^{-1} and it gets accelerated with relative acceleration 11 ms^{-2}. With respect to elevator, initial velocity of bolt is zero and it has to travel 2.7 m with 11 ms^{-2}. Thus, time taken can be directly given as

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 2.7}{11}} = 0.7 \text{ s}$$

(b) Displacement of bolt relative to ground in $t = 0.7$ s is given by

$s = ut + \frac{1}{2}at^2$

$\Rightarrow \quad s = (2.4)(0.7) + \frac{1}{2}(-9.8)(0.7)^2$

$\Rightarrow \quad s = -0.72$ m

Velocity of bolt will become zero after a time, say t_0. Then

$t_0 = \frac{u}{g} \qquad \{v = u - gt\}$

$\Rightarrow \quad t_0 = \frac{2.4}{9.8} = 0.245$ s

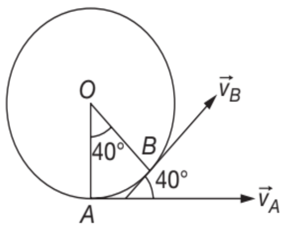
CHAPTER 4

Therefore, distance travelled by the bolt is $s_{\text{bolt}} = s_1 + s_2$

$$\Rightarrow \quad s_{\text{bolt}} = \frac{u^2}{2g} + \frac{1}{2}g(t - t_0)^2$$

$$\Rightarrow \quad s_{\text{bolt}} = \frac{(2.4)^2}{2(9.8)} + \frac{1}{2}(9.8)(0.7 - 0.245)^2 = 1.3 \text{ m}$$

7. Here, $|\vec{v}_A| = |\vec{v}_B| = v$



Change in velocity, $\Delta\vec{v} = \vec{v}_B - \vec{v}_A = \vec{v}_B + (-\vec{v}_A)$

Angle between $\vec{v}_A$ and $\vec{v}_B$ is $\theta = 40°$

Since

$$|\Delta\vec{v}| = |\vec{v}_B - \vec{v}_A| = \sqrt{v_B^2 + v_A^2 + 2v_A v_B \cos(180 - \theta)}$$

$$\Rightarrow \quad |\Delta\vec{v}| = |\vec{v}_B - \vec{v}_A| = \sqrt{v^2 + v^2 - 2v_2 \cos(40°)}$$

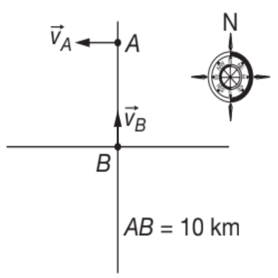
$$\Rightarrow \quad |\Delta\vec{v}| = |\vec{v}_B - \vec{v}_A| = \sqrt{2}v\sqrt{1 - \cos(40°)}$$

$$\Rightarrow \quad |\Delta\vec{v}| = |\vec{v}_B - \vec{v}_A| = \sqrt{2}v\sqrt{2}\sqrt{\sin^2\left(\frac{40°}{2}\right)}$$

$$\left\{\because\ 1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)\right\}$$

$$\Rightarrow \quad |\Delta\vec{v}| = 2v\sin(20°)$$

8. Ships A and B are moving with same speed 20 kmhr^{-1} in the directions shown in figure. It is a two dimensional, two body problem with zero acceleration. Let us find $\vec{v}_{BA}$

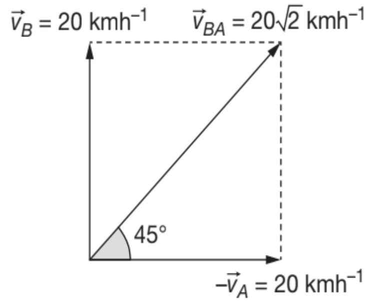


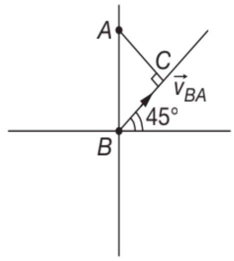
$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

Here, $|\vec{v}_{BA}| = \sqrt{(20)^2 + (20)^2} = 20\sqrt{2} \text{ kmhr}^{-1}$

i.e., $\vec{v}_{BA}$ is $20\sqrt{2} \text{ kmhr}^{-1}$ at an angle of $45°$ from east towards north. Thus, the given problem can be simplified as:

A is at rest and B is moving with $\vec{v}_{BA}$ in the direction shown in figure.





Therefore, the minimum distance between the two is

$$s_{\min} = AC = AB\sin 45° = 10\left(\frac{1}{\sqrt{2}}\right) \text{ km} = 5\sqrt{2} \text{ km}$$

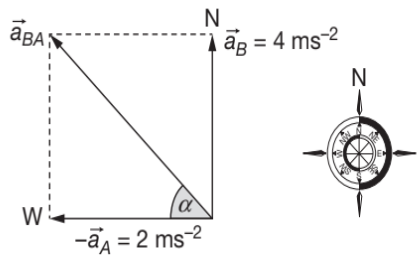
and the desired time is

$$t = \frac{BC}{|\vec{v}_{BA}|} = \frac{5\sqrt{2}}{20\sqrt{2}} \qquad \{BC = AC = 5\sqrt{2} \text{ km}\}$$

$$\Rightarrow \quad t = \frac{1}{4} \text{ hr} = 15 \text{ minutes}$$

9. This is a case of two dimensional motion. Therefore, acceleration of car B with respect to car A is

$$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A$$



Here, $\vec{a}_B$ = acceleration of car $B = 4 \text{ ms}^{-2}$ (due north)

and $\vec{a}_A$ = acceleration of car $A = 2 \text{ ms}^{-2}$ (due east)

$$|\vec{a}_{BA}| = \sqrt{(4)^2 + (2)^2} = 2\sqrt{5} \text{ ms}^{-2}$$

and $\alpha = \tan^{-1}\left(\frac{4}{2}\right) = \tan^{-1}(2)$

Thus, $\vec{a}_{BA}$ is $2\sqrt{5} \text{ ms}^{-2}$ at an angle of $\alpha = \tan^{-1}(2)$ NW

i.e., at an angle $\alpha = \tan^{-1}(2)$ towards the north of west.

10. Time for which train A accelerates is t_1 (say). Then

$$80 = 0 + 6t_1$$

$$\Rightarrow \quad t_1 = \frac{40}{3} \text{ s}$$

Distance travelled is say x_1, given by

$$(80)^2 - 0^2 = 2(6)x_1$$

$$\Rightarrow \quad x_1 = \frac{6400}{12} = \frac{1600}{3} \text{ m}$$

In time t_1, train B must have moved forward by

$$x_2 = 60\left(\frac{40}{3}\right) = 800 \text{ m}$$

So, total remaining distance is

$$x = 6000 - (x_1 + x_2)$$

$$\Rightarrow \quad x = 6000 - \left(\frac{1600}{3} + 800\right)$$

$$\Rightarrow \quad x = \frac{14000}{3} \text{ m}$$

If the trains now meet in time t_2, then

$$t_2 = \frac{x}{v_{\text{rel}}} = \frac{\frac{14000}{3}}{140}$$

$$\Rightarrow \quad t_2 = \frac{100}{3} \text{ s}$$

$$\Rightarrow \quad t = t_1 + t_2 = \frac{140}{3} = 46.67 \text{ s}$$

So, the distance travelled by A, when both meet is

$$x_A = x_1 + v_A t_2 = \frac{1600}{3} + (80)\left(\frac{100}{3}\right)$$

$$\Rightarrow \quad x_A = \frac{9600}{3} = 3200 \text{ m}$$

11. Let a_r be the relative acceleration of lift upwards. Then

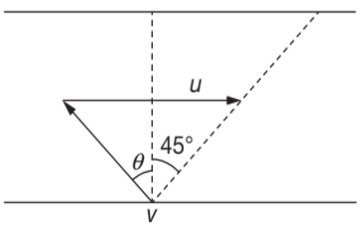
$$t = \frac{2u}{a_r} = \frac{2u}{a+g}$$

$$\Rightarrow \quad a = \frac{2u - gt}{t}$$

12. $v_x = u - v\sin\theta$ and $v_y = v\cos\theta$

Absolute velocity of boatman is along AB i.e., it makes an angle 45 degrees with x-axis. Hence

$$\tan 45° = \frac{v_y}{v_x} = 1$$



$$\Rightarrow \quad v_y = v_x$$

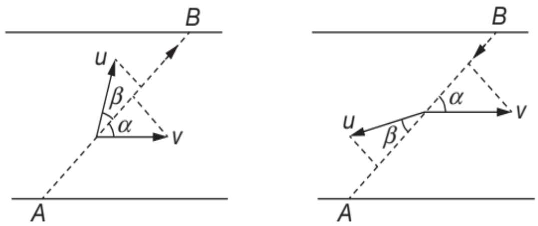
$$\Rightarrow \quad u - v\sin\theta = v\cos\theta$$

$$\Rightarrow \quad v = \frac{u}{\sin\theta + \cos\theta} = \frac{u}{\sqrt{2}\sin(\theta + 45°)}$$

The minimum value of v is when $\theta = 45°$. So

$$v_{\text{MIN}} = \frac{u}{\sqrt{2}}, \text{ for } \theta = 45°$$

13. In order that the moving launch is always on the straight line AB, the components of velocity of the current and of the launch in the direction perpendicular to AB should be equal, so,



$$u\sin\beta = v\sin\alpha \qquad \ldots(1)$$

Also, $S = AB = (u\cos\beta + v\cos\alpha)t_1$ ...(2)

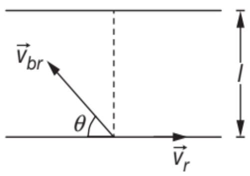
Further $BA = (u\cos\beta - v\cos\alpha)t_2$ ...(3)

$$t_1 + t_2 = t \qquad \ldots(4)$$

Solving these equations, we get

$$u = 8 \text{ ms}^{-1} \text{ and } \beta = 12°$$

14. (a) $v_r > v_{br}$, the man cannot reach the point directly opposite to his starting point just by rowing. He will experience a non-zero drift and hence has to walk to reach the destination.



In the situation shown in figure, if t_1 is the time taken to cross the river, then

$$t_1 = \frac{l}{v_{br}\cos\theta} = \frac{120}{3\cos\theta} = \frac{40}{\cos\theta}$$

Drift, $x = (v_r - v_{br}\sin\theta)t = (4 - 3\sin\theta)\left(\frac{40}{\cos\theta}\right)$

If, v_w is the walking speed of the man $(= 1 \text{ ms}^{-1})$,

then $t_2 = \dfrac{x}{v_w}$ is the time taken by him to walk and reach the point opposite to the point of start.

$\Rightarrow \quad t_2 = \dfrac{40}{\cos\theta} = (4 - 3\sin\theta)$

Total time $t = t_1 + t_2$

$\Rightarrow \quad t = \dfrac{40}{\cos\theta}(5 - 3\sin\theta)$

For t to be minimum, $\dfrac{dt}{d\theta} = 0$

$\Rightarrow \quad -3\cos^2\theta + (5 - 3\sin\theta)\sin\theta = 0$

$\Rightarrow \quad 5\sin\theta = 3$

$\Rightarrow \quad \theta = \sin^{-1}\left(\dfrac{3}{5}\right)$

So, he should row at an angle $90° + \sin^{-1}\left(\dfrac{3}{5}\right)$ upstream.

(b) $t_{\min} = \left(\dfrac{40}{4/5}\right)\left(5 - 3 \times \dfrac{3}{5}\right) = 50\left(\dfrac{16}{5}\right) = 160 \text{ s}$

15. (a) Let the two meet at a distance s from ground. Then

$$s - 12 = 18t - \frac{1}{2}(9.8)t^2 \qquad \ldots(1)$$

and $s - 5 = 2t \qquad \ldots(2)$

Solving these two equations, we get

$t = 3.65 \text{ s}$ and $s = 12.3 \text{ m}$

(b) $v_b = 18 - (9.8)(3.65) = -17.8 \text{ ms}^{-1}$

i.e., velocity of ball is 17.8 ms^{-1} (downwards) at the time of impact.

So, the relative velocity is 19.8 ms^{-1} (downwards)

16. $t = \dfrac{80}{1.6} = 50 \text{ s}$

(a) $v_r = \dfrac{40}{50} = 0.8 \text{ ms}^{-1}$

(b) $v = \sqrt{v_r^2 + v_{sr}^2} = \sqrt{(0.8)^2 + (1.6)^2} = 1.79 \text{ ms}^{-1}$

(c) $\theta = \sin^{-1}\left(\dfrac{v_r}{v_{sr}}\right) = \sin^{-1}\left(\dfrac{0.8}{1.6}\right) = \sin^{-1}\left(\dfrac{1}{2}\right) = 30°$

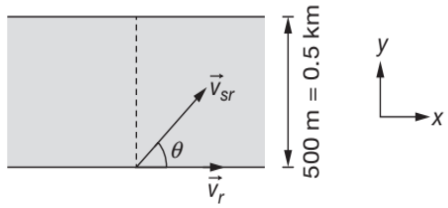
Hence the swimmer should head at $(90° + 30°) = 120°$ upstream.

17. Time taken by the pedestrian to cross the track, $t = \dfrac{d}{v}$

So, minimum distance required for cyclist is $s_{\min} = Vt = \dfrac{Vd}{v}$

18. (a) $(v_s)_y = (v_{sr})_y = 3\sin\theta \text{ kmh}^{-1}$

$\Rightarrow \quad t = \dfrac{w}{(v_s)_y} = \dfrac{0.5}{3\sin\theta} \text{ hour} = \dfrac{10}{\sin\theta} \text{ minute}$



(b) Time is shortest at $\sin\theta = 1$ or $\theta = 90°$

$t_{\min} = 10$ minute

(c) Since, $v_r > v_{sr}$, so he can not reach the other shore at the point directly opposite to his starting point. If he starts at angle θ as shown in figure he will reach a distance.

$$x = \left(\frac{w}{v_{sr}\sin\theta}\right)(v_r + v_{sr}\cos\theta)$$

For x to be minimum $\dfrac{v_r + v_{sr}\cos\theta}{\sin\theta}$ should be minimum

$\Rightarrow \quad \dfrac{d}{d\theta}\left\{\dfrac{v_r + v_{sr}\cos\theta}{\sin\theta}\right\} = 0$

$\Rightarrow \quad -v_{sr}\sin^2\theta - (v_r + v_{sr}\cos\theta)(\cos\theta) = 0$

$\Rightarrow \quad v_r\cos\theta = -v_{sr}$

$\Rightarrow \quad \cos\theta = -\dfrac{v_{sr}}{v_r} = -\dfrac{3}{5}$

$\Rightarrow \quad \sin\theta = \dfrac{4}{5}$

$\Rightarrow \quad x_{\min} = \dfrac{\left(\frac{1}{2}\right)}{3\sin(127°)}(5 + 3\cos(127°))$

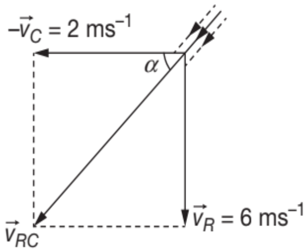
$\Rightarrow \quad x_{\min} = \dfrac{\left(\frac{1}{2}\right)}{\left(3 \times \frac{4}{5}\right)}\left(5 - \dfrac{9}{5}\right) = \dfrac{2}{3} \text{ km}$

19. $\vec{v}_{RC}$ = velocity of rain w.r.t. cart $= \vec{v}_R - \vec{v}_C$

From figure, we observe that

$$\tan\alpha = \frac{6}{2}$$

$$\Rightarrow \quad \alpha = \tan^{-1}(3)$$



20. Time taken by the mirrors to collide is $T = \frac{d}{v+v} = \frac{d}{2v}$

Speed of the particle with respect to the approaching mirror is $(v+3v) = 4v$.

Time taken for the first trip is $t_1 = \frac{d}{4v}$

New separation between the mirrors just after first trip i.e., at $t_1 = \frac{d}{4v}$ is

$$x_1 = d - (v+v)t_1$$

$$\Rightarrow \quad x_1 = d - \frac{d}{2} = \frac{d}{2}$$

So, if t_2 is the time taken by the particle for the second trip, then

$$t_2 = \frac{\frac{d}{2}}{v+3v} = \frac{d}{8v}$$

New separation between the mirrors just after second trip i.e., at $t_2 = \frac{d}{8v}$ is

$$x_2 = \frac{d}{2} - (v+v)t_2$$

$$\Rightarrow \quad x_2 = \frac{d}{2} - (2v)\left(\frac{d}{8v}\right)$$

$$\Rightarrow \quad x_2 = \frac{d}{2} - \frac{d}{4} = \frac{d}{4}$$

So, if t_3 is the time taken by the particle for the third trip, then

$$t_3 = \frac{\frac{d}{4}}{v+3v} = \frac{d}{16v}$$

and so time taken for nth trip is $t_n = \frac{d}{2^{n+1}v}$

However, we observe that

$$t_1 + t_2 + t_3 + ... + t_n = T$$

$$\Rightarrow \quad \frac{d}{4v} + \frac{d}{8v} + \frac{d}{16v} + ... + \frac{d}{2^{n+1}v} = \frac{d}{2v}$$

$$\Rightarrow \quad \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ... + \frac{1}{2^n} = 1$$

$$\Rightarrow \quad \frac{\frac{1}{2}\left[1 - \left(\frac{1}{2}\right)^n\right]}{1 - \frac{1}{2}} = 1 \qquad \left\{\because S_n = \frac{a(1-r^n)}{1-r}\right\}$$

$$\Rightarrow \quad 1 - \left(\frac{1}{2}\right)^n = 1 \Rightarrow \left(\frac{1}{2}\right)^n = 0$$

$$\Rightarrow \quad n \to \infty$$

Total distance travelled by the particle is

$$s_{\text{total}} = vT = (3v)\left(\frac{d}{2v}\right) = \frac{3d}{2}$$

Single Correct Choice Type Questions

1.

> **MISCONCEPTION**
>
> Taking east direction as positive, we have
>
> $u = +9\ \text{ms}^{-1}$ and $a = -3\ \text{ms}^{-2}$
>
> $$s_{5^{th}} = u + \frac{a}{2}(2(5) - 1)$$
>
> $$\Rightarrow \quad s_{5^{th}} = 9 - \frac{2}{2}(9) = 0$$
>
> Actually this is not the correct answer, because we must observe that the particle reverses its direction of motion $(v = 0)$ at $t = 4.5$ s which lies in the specified interval. So, we will proceed other way round.

Since the total distance is

$$ds = vdt$$

$$\Rightarrow \quad s_{\text{total}} = \int v\,dt$$

$$\Rightarrow \quad s_{\text{total}} = \left|\int_4^{4.5} v\,dt\right| + \left|\int_{4.5}^{5} v\,dt\right|$$

Now $\int v\,dt = \int (u + at)dt = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad s_{\text{total}} = \left|9(4.5-4)+\frac{1}{2}(-2)\left[\left(\frac{9}{2}\right)^2-(4)^2\right]\right|+\left|9(5-4.5)+\frac{1}{2}(-2)\left[(5)^2-\left(\frac{9}{2}\right)^2\right]\right|$$

$$\Rightarrow \quad s_{\text{total}} = |4.5-4.25|+|4.5-4.25| = 0.5\text{ m}$$

Hence, the correct answer is (B).

2. $a=\dfrac{dv}{dt}=8-2t$

$$\Rightarrow \quad a=8-2(5)$$

$$\Rightarrow \quad a=-2\text{ ms}^{-2}$$

Hence, the correct answer is (D).

3. For constant speed, $a_T=0$

$$\Rightarrow \quad a=a_C=a_N=\frac{V_T^2}{R}$$

$$\Rightarrow \quad 0.8g=\frac{v^2}{(200)}$$

$$\Rightarrow \quad v^2=(0.8)(10)(200)$$

$$\Rightarrow \quad v=40\text{ ms}^{-1}$$

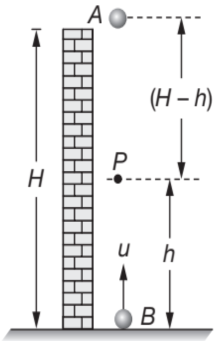
$$\Rightarrow \quad v=40\times\frac{18}{5}\text{ kmh}^{-1}$$

$$\Rightarrow \quad v=144\text{ kmh}^{-1}$$

Hence, the correct answer is (C).

4. Let the balls A and B collide at the point P a distance h from the ground. Let u be the velocity of launch of B and the launch velocity of A is zero. At the point P, final velocity of A is

$$v_A=\sqrt{2g(H-h)} \qquad \ldots(1)$$



and that of B is

$$v_B=\sqrt{u^2-2gh} \qquad \ldots(2)$$

Also, for the balls to collide at P, time taken by A to reach P equals time taken by B to reach P. So we have

$$(H-h)=\frac{1}{2}gt^2 \qquad \ldots(3)$$

$$h=ut-\frac{1}{2}gt^2 \qquad \ldots(4)$$

From (3), $t=\sqrt{\dfrac{2(H-h)}{g}}$

Substitute in (4), we get

$$h=u\sqrt{\frac{2(H-h)}{g}}-\frac{1}{2}g\frac{2(H-h)}{g}$$

$$\Rightarrow \quad h=u\sqrt{\frac{2(H-h)}{g}}-(H-h)$$

$$\Rightarrow \quad h=u\sqrt{\frac{2(H-h)}{g}}-H+h$$

$$\Rightarrow \quad u=H\sqrt{\frac{g}{2(H-h)}} \qquad \ldots(5)$$

Substitute (5) in (2), we get

$$v_B=\sqrt{\frac{gH^2}{2(H-h)}-2gh} \qquad \ldots(6)$$

Since $v_A=2v_B$

$$\Rightarrow \quad v_A^2=4v_B^2$$

$$\Rightarrow \quad 2g(H-h)=\frac{4H^2g}{2(H-h)}-8gh$$

$$\Rightarrow \quad 2gH-2gh=\frac{2H^2g}{H-h}-8gh$$

$$\Rightarrow \quad H-h=\frac{H^2}{H-h}-4h$$

$$\Rightarrow \quad \frac{H^2}{H-h}-H=3h$$

$$\Rightarrow \quad \frac{H^2-H^2+Hh}{H-h}=3h$$

$$\Rightarrow \quad \frac{H}{H-h}=3$$

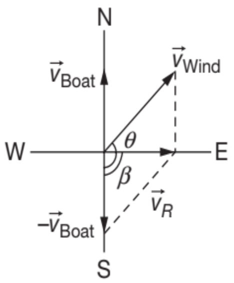
$$\Rightarrow \quad H=3H-3h$$

$$\Rightarrow \quad 2H=3h$$

$$\Rightarrow \quad h=\frac{2H}{3}$$

Hence, the correct answer is (D).

5. $\tan\beta = \dfrac{v_{wind}\sin\theta}{v_{boat} + v_{wind}\cos\theta}$



$\Rightarrow \quad \tan\beta = \dfrac{72\sin(135°)}{51 + 72\cos(135°)}$

$\Rightarrow \quad \tan\beta \to \infty$

$\Rightarrow \quad \beta \approx 90°$

i.e., direction of flag is towards east

Hence, the correct answer is (A).

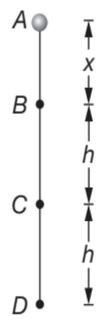
6. $L = \dfrac{1}{2}gt^2 - \dfrac{1}{2}g(t-T)^2$

Solve $t = \dfrac{T}{2} + \dfrac{L}{gT}$

Hence, the correct answer is (B).

7. Let the point B be situated at a distance x below A. If the particle reaches B in time t, then

$$x = \frac{1}{2}gt^2 \quad \ldots(1)$$



For AC, $x + h = \dfrac{1}{2}g(t+2)^2 \quad \ldots(2)$

For AD, $x + 2h = \dfrac{1}{2}g(t+3)^2 \quad \ldots(3)$

Solving (1), (2) and (3), we get

$t = 0.5$ s

Hence, the correct answer is (B).

8. At the maximum height, $v = 0$

$\Rightarrow \quad 0^2 - u^2 = 2(-g)(h)$

$\Rightarrow \quad u = \sqrt{2gh}$

$\Rightarrow \quad u = \sqrt{2(10)(20)} = 20 \text{ ms}^{-1}$

If t be the time taken by the ball to reach the maximum height, then

$0 = u - gt$

$\Rightarrow \quad 0 = 20 - (10)t$

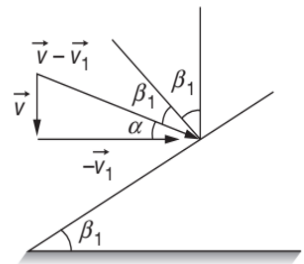
$\Rightarrow \quad t = 2$ s

So, time taken by the ball to return to juggler's hand is 4 s. To maintain a proper distance between them, the balls must be thrown up at an interval of

$$\Delta t = \frac{4}{4}s = 1 \text{ s}$$

Hence, the correct answer is (B).

10. Let hail fall along the vertical with a velocity $\vec{v}$. In the reference frame fixed to the car, the angle of incidence of hailstones on the windscreen is equal to the angle of reflection. The velocity of the hailstone before striking the windscreen is $\vec{v} - \vec{v}_1$ (by triangle law of vectors). Further the hailstones bounce vertically upwards (from the viewpoint of the driver) after reflection from wind screen of the car. If angle of incidence and hence the angle of reflection both equal β_1 then $\alpha + 2\beta_1 = 90°$.



Further $\tan\alpha = \dfrac{v}{v_1}$

$\Rightarrow \quad \tan(90 - 2\beta_1) = \dfrac{v}{v_1}$

$\Rightarrow \quad \dfrac{v}{v_1} = \cot(2\beta_1)$ {for 1st car}

Similarly for second car

$\Rightarrow \quad \dfrac{v}{v_2} = \cot(2\beta_2)$

$$\frac{v_1}{v_2} = \frac{\cot(2\beta_2)}{\cot(2\beta_1)} = \frac{\cot(30)}{\cot(60)} = \frac{\sqrt{3}}{1/\sqrt{3}} = 3$$

Hence, the correct answer is (A).

CHAPTER 4

11. Let v be the velocity of escalator and u be the velocity of the person. Then

$$t_1 = \frac{l}{u} \quad \text{...(1)}$$

Further $t_2 = \frac{l}{v}$ ...(2)

If t be the time taken by him to walk up the escalator, then

$$t = \frac{l}{u+v}$$

From (1) and (2)

$$u = \frac{l}{t_1} \text{ and } v = \frac{l}{t_2}$$

$$\Rightarrow \quad t = \frac{l}{u+v} = \frac{l}{\frac{l}{t_1}+\frac{l}{t_2}}$$

$$\Rightarrow \quad t = \frac{t_1 t_2}{t_1 + t_2}$$

Hence, the correct answer is (D).

12. Since both are falling under gravity, so relative acceleration of one stone w.r.t. other is zero. So,

$$\Delta x = (u_1 + u_2)t = ut \qquad \{\because u_1 = 0\}$$

So, the graph is a straight line passing through the origin.

Hence, the correct answer is (C).

13. Velocity of wind is $10\ ms^{-1}$ from South to North i.e.

$$\vec{v}_{\text{wind}} = 10\,\hat{j}$$

But, to the cyclist it appears to blow from the East at $10\ \text{ms}^{-1}$. So, velocity of wind relative to the cyclist is $10\ \text{ms}^{-1}$ from East to West i.e.

$$\vec{v}_{\text{wind/cyclist}} = -10\,\hat{i}$$

Since $\vec{v}_{\text{wind/cyclist}} = \vec{v}_{\text{wind}} - \vec{v}_{\text{cyclist}}$

$$\Rightarrow \quad -10\,\hat{i} = 10\,\hat{j} - \vec{v}_{\text{cyclist}}$$

$$\Rightarrow \quad \vec{v}_{\text{cyclist}} = 10\,\hat{i} + 10\,\hat{j}$$

Hence, the correct answer is (B).

14. $$\frac{v}{t} = \frac{v_1 - v_2}{t_2 - t_1} = \frac{\sqrt{2a_1 s} - \sqrt{2a_2 s}}{\sqrt{\frac{2s}{a_2}} - \sqrt{\frac{2s}{a_1}}}$$

$$\Rightarrow \quad v = \left(\sqrt{a_1 a_2}\right)t$$

$$\Rightarrow \quad v = \sqrt{(2g)(8g)}\,t$$

$$\Rightarrow \quad v = 4gt$$

Hence, the correct answer is (A).

15. The graph shows the following v-s relation i.e., $v = s$

$$\Rightarrow \quad \frac{dv}{ds} = 1$$

Since $a = \frac{vdv}{ds}$

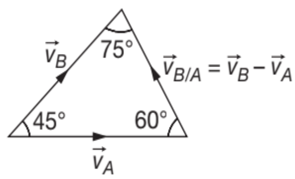
$$\Rightarrow \quad a = v(1)$$

$$\Rightarrow \quad a = v$$

So, a-v graph is again a straight line passing through the origin and inclined to the x-axis at an angle of $45°$.

Hence, the correct answer is (C).

16. According to Law of sines or Lami's Theorem



$$\Rightarrow \quad \frac{v_A}{\sin 75^o} = \frac{v_B}{\sin 60^o} = \frac{v_{B/A}}{\sin 45^o}$$

$$\Rightarrow \quad v_B = 717\ \text{km}\,h^{-1}$$

Hence, the correct answer is (C).

17. Since, $h = \frac{1}{2}gt^2$ ...(1)

and $\frac{9h}{16} = \frac{1}{2}g(t-1)^2$ ...(2)

$$\Rightarrow \quad \frac{3}{4} = \frac{t-1}{t}$$

$$\Rightarrow \quad 3t = 4t - 4$$

$$\Rightarrow \quad t = 4\ \text{s}$$

$$\Rightarrow \quad h = 80\ \text{m}$$

Hence, the correct answer is (C).

18. The graph shown is actually for

$$s = ut - \frac{1}{2}at^2$$

Lets see this, by making the LHS a perfect square.

$$\Rightarrow \quad s = -\frac{a}{2}\left(t^2 - \frac{2ut}{a}\right)$$

$$\Rightarrow \quad s = -\frac{a}{2}\left[t^2 - \left(\frac{2u}{a}\right)t + \frac{u^2}{a^2} - \frac{u^2}{a^2}\right]$$

$$\Rightarrow \quad s = -\frac{a}{2}\left[\left(t - \frac{u}{a}\right)^2 - \frac{u^2}{a^2}\right]$$

$$\Rightarrow \quad s - \frac{u^2}{2a} = -\frac{a}{2}\left(t - \frac{u}{a}\right)^2$$

$$\Rightarrow \quad S = -\frac{a}{2}T^2$$

where $S = \frac{s - u^2}{2a}$ and $T = t - \frac{u}{a}$ and the origin at $(T = 0,\ S = 0) \equiv \left(\frac{u}{a}, \frac{u^2}{2a}\right)$

So, $\frac{u}{a} = 2.5$ and $\frac{u^2}{2a} = 100$

$$\Rightarrow \quad \frac{u^2}{2a} = \frac{u}{2}\left(\frac{u}{a}\right) = 100$$

$$\Rightarrow \quad \frac{u}{2}(2.5) = 100$$

$$\Rightarrow \quad u = \frac{200}{2.5} = 80 \text{ ms}^{-1}$$

and $a = \frac{80}{2.5} = 32 \text{ ms}^{-2}$

Hence, the correct answer is (C).

19. $s_1 = \frac{1}{2}at^2$

$$s_2 = (at)t - \frac{1}{2}a\frac{t^2}{4}$$

$$\Rightarrow \quad s_2 = at^2 - \frac{at^2}{8}$$

$$\Rightarrow \quad s_2 = \frac{7at^2}{8}$$

Hence, the correct answer is (A).

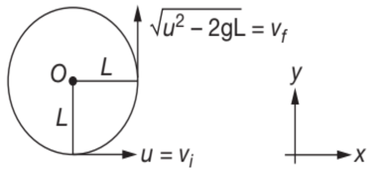
20. v_i be the initial velocity i.e. velocity at lowest point

$$\Rightarrow \quad \vec{v}_i = u\hat{i}$$

v_f be the final velocity i.e. velocity when the string becomes vertical. So, we have

$$v_f^2 - v_i^2 = 2(-g)L$$

$$\Rightarrow \quad v_f = \sqrt{u^2 - 2gL} \qquad \text{(in magnitude)}$$



$$\Rightarrow \quad \vec{v}_f = \left(\sqrt{u^2 - 2gL}\right)\hat{j}$$

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = \left(\sqrt{u^2 - 2gL}\right)\hat{j} - u\hat{i}$$

$$\Rightarrow \quad |\Delta\vec{v}| = \sqrt{u^2 - 2gL + u^2} = \sqrt{2(u^2 - gL)}$$

Hence, the correct answer is (D).

21. $v = \frac{dx}{dt} = 3at^2 + 2bt + c$

$$\Rightarrow \quad u = v\Big|_{t=0} = c \qquad \ldots(1)$$

$$\Rightarrow \quad a = \frac{dv}{dt} = 6at + 2b$$

$$\Rightarrow \quad a\Big|_{t=0} = 2b$$

$$\Rightarrow \quad \frac{a\Big|_{t=0}}{v\Big|_{t=0}} = \frac{2b}{c}$$

Hence, the correct answer is (B).

22. $a = \frac{P}{mv}$

$$\Rightarrow \quad v\frac{dv}{dx} = \frac{P}{mv}$$

$$\Rightarrow \quad v^2 dv = \frac{P}{m}dx$$

$$\Rightarrow \quad \int_{v_1}^{v_2} v^2 dv = \frac{P}{m}\int_0^x dx \qquad \{\because P = \text{constant}\}$$

$$\Rightarrow \left.\frac{v^3}{3}\right|_{v_1}^{v_2} = \frac{P}{m}\left(\left. x\right|_0^x\right)$$

$$\Rightarrow v_2^3 - v_1^3 = \frac{3Px}{m}$$

$$\Rightarrow x = \frac{m}{3P}\left(v_2^3 - v_1^3\right)$$

Hence, the correct answer is (C).

23. $\frac{dv}{dt} = a - bv$

$$\Rightarrow \int_0^v \frac{dv}{a-bv} = \int_0^t dt$$

$$\Rightarrow -\frac{1}{b}\left[\log_e(a-bv) - \log_e(a)\right] = t$$

$$\Rightarrow \frac{a-bv}{a} = e^{-bt}$$

$$\Rightarrow v = \frac{a}{b}\left(1 - e^{-bt}\right)$$

At $t = 0$ i.e. initially, velocity must be zero (as stated in the Problem) and this condition is met only by Option (A).

Hence, the correct answer is (A).

24. Initial velocity of dropping = zero

Let v_1 be velocity at end of 10 s.

$$\Rightarrow v_1 = gt$$

$$\Rightarrow v_1 = 100 \text{ ms}^{-1}$$

Distance travelled during this time is

$$h_1 = \frac{v_1^2}{2g} = \frac{(100)^2}{2(10)}$$

$$\Rightarrow h_1 = 500 \text{ m}$$

So, a remaining distance of $2495 - 500 = 1995\ m$ has to be travelled with a retardation of 2.5 ms^{-2}. Let the parachutist strike the ground with velocity v. Then

$$v^2 - v'^2 = 2a(h - h')$$

$$\Rightarrow v^2 - (100)^2 = 2(-2.5)(1995)$$

$$\Rightarrow v^2 = 10000 - 9975$$

$$\Rightarrow v^2 = 25$$

$$\Rightarrow v = 5 \text{ ms}^{-1}$$

Hence, the correct answer is (A).

25. Since $s = \left(\frac{u+v}{2}\right)t$

$$4 = \left(\frac{1000 + 9000}{2}\right)t$$

$$\Rightarrow t = \frac{8}{10000}$$

$$\Rightarrow t = 8 \times 10^{-4} \text{ s}$$

Hence, the correct answer is (B).

26. $s_A = s_B$

$$8t + \frac{1}{2}(2)t^2 = 4t + \frac{1}{2}(4)t^2$$

$$\Rightarrow 8 + t = 4 + 2t$$

$$\Rightarrow t = 4 \text{ s}$$

Since $s_A = 8t + \frac{1}{2}(2)t^2$

$$\Rightarrow s_A = 8(4) + (4)^2$$

$$\Rightarrow s_A = 48 \text{ m}$$

Hence, the correct answer is (C).

27. $v_y = \frac{dy}{dt} = \frac{1}{2}\frac{dx}{dt} = \frac{1}{2}v_x$

$$\Rightarrow v_y = \frac{dy}{dt} = 2 - t \qquad \{\because v_x = 4 - 2t\}$$

$$a_x = -2 \text{ ms}^{-2} \text{ and } a_y = -1 \text{ ms}^{-2}$$

$$u_x = v_x\big|_{t=0} = 4 \text{ ms}^{-1} \text{ and } u_y = v_y\big|_{t=0} = 2 \text{ ms}^{-1}$$

Since we observe that a_x and a_y both are negative but u_x and u_y are positive, so motion is retarded initially and accelerated once the direction of motion is reversed.

Hence, the correct answer is (D).

28. Let the ball be at height h at time t and time $(t + \Delta t)$. Then

$$h = ut - \frac{1}{2}gt^2 \text{ and} \qquad \ldots(1)$$

$$h = u(t + \Delta t) - \frac{1}{2}g(t + \Delta t)^2 \qquad \ldots(2)$$

Equating (1) and (2), we get

$$t = \frac{2u - g\Delta t}{2g} \qquad \ldots(3)$$

Substituting (3) in (1), we get

$$h = \frac{4u^2 - g^2(\Delta t)^2}{8g}$$

$$\Rightarrow \quad u = \frac{1}{2}\sqrt{8gh + g^2(\Delta t)^2}$$

Hence, the correct answer is (C).

29. $v_x = \frac{dx}{dt} = 30\cos(6t)$ and

$$v_y = \frac{dy}{dt} = 30\sin(6t)$$

$$\Rightarrow \quad v = \sqrt{v_x^2 + v_y^2} = 30 \text{ ms}^{-1}$$

So, we observe that the speed is constant. Hence distance

$$x = vt$$

$$\Rightarrow \quad x = (30)(4) = 120 \text{ m}$$

Hence, the correct answer is (D).

30. Let the acceleration be A and initial velocity be U. Then

$$a = Up + \frac{1}{2}Ap^2 \qquad \ldots(1)$$

$$a + b = U(p+q) + \frac{1}{2}A(p+q)^2$$

$$\Rightarrow \quad b = Uq + \frac{1}{2}Aq^2 + Apq \qquad \ldots(2)$$

$q(1) - p(2)$ gives

$$qa - pb = \frac{1}{2}Ap^2q - \frac{1}{2}Aq^2p - Ap^2q$$

$$\Rightarrow \quad qa - pb = -\frac{1}{2}Aq^2p - \frac{1}{2}Ap^2q$$

$$\Rightarrow \quad pb - qa = \frac{1}{2}Apq(p+q)$$

$$\Rightarrow \quad A = \frac{2(pb - qa)}{pq(p+q)} = \frac{2(bp - aq)}{pq(p+q)}$$

Hence, the correct answer is (D).

31. $v_x = \frac{dx}{dt} = -a\omega\sin(\omega t)$

$$a_x = \frac{dv_x}{dt} = -a\omega^2\cos(\omega t)$$

Similarly

$$v_y = \frac{dy}{dt} = a\omega\cos(\omega t)$$

$$a_y = \frac{dv_y}{dt} = -a\omega^2\sin(\omega t)$$

So, acceleration is $\sqrt{a_x^2 + a_y^2}$

$$\Rightarrow \quad \text{Acceleration} = a\omega^2$$

Hence, the correct answer is (C).

32. **For** $0 \le t \le 2s$

$$a_1 = 2 \text{ ms}^{-2}$$

$$\Rightarrow \quad \frac{dv_1}{dt} = 2$$

$$\Rightarrow \quad \int_0^{v_1} dv_1 = 2\int_0^t dt$$

$$\Rightarrow \quad v_1 = 2t \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{dx_1}{dt} = 2t$$

$$\Rightarrow \quad dx_1 = 2t\,dt$$

$$\Rightarrow \quad \int_0^{x_1} dx_1 = 2\int_0^2 t\,dt$$

$$\Rightarrow \quad x_1 = 2\frac{t^2}{2}\Big|_0^2$$

$$\Rightarrow \quad x_1 = 4 \text{ m}$$

For $2s \le t \le 4s$

$$a_2 = 1 \text{ ms}^{-2}$$

$$\Rightarrow \quad \frac{dv_2}{dt} = 1$$

$$\Rightarrow \quad \int_4^{v_2} dv^2 = \int_2^t dt \qquad \{\because \text{ of } (1)\}$$

$$\Rightarrow \quad v_2 - 4 = t - 2$$

$$\Rightarrow \quad v_2 = 2 + t \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{dx_2}{dt} = 2 + t$$

$$\Rightarrow \quad \int_4^{x_2} dx_2 = \int_2^4 (2+t)dt$$

$$\Rightarrow \quad x_2 - 4 = 2(2) + \frac{1}{2}(4^2 - 2^2)$$

$$\Rightarrow \quad x_2 - 4 = 4 + 6$$

$\Rightarrow \quad x_2 = 14 \text{ m}$

Now $v\Big|_{t=4} = 6 \text{ ms}^{-1}$

With this initial velocity and a retardation of 1.5 ms^{-2}, the particle will come to rest in a time t (say). Then

$$0 = 6 - (1.5)t$$

$\Rightarrow \quad t = 4 \text{ s}$

So, for a further 4 s, $\dfrac{dv_3}{dt} = -1.5$

$$dv_3 = -1.5dt$$

$$\Rightarrow \quad \int_6^{v_3} dv_3 = -1.5\int_4^t dt$$

$$\Rightarrow \quad v_3 - 6 = -1.5(t-4)$$

$$\Rightarrow \quad v_3 - 6 = -1.5t + 6$$

$$\Rightarrow \quad v_3 = -1.5t + 12 \qquad \ldots(3)$$

$$\Rightarrow \quad \frac{dx_3}{dt} = -1.5t + 12$$

$$\Rightarrow \quad \int_{14}^{x} dx_3 = -1.5\int_4^8 tdt + 12\int_4^8 dt$$

$$\Rightarrow \quad x - 14 = -1.5\left(\frac{t^2}{2}\Bigg|_4^8\right) + 12\left(t\Big|_4^8\right)$$

$$\Rightarrow \quad x - 14 = -1.5(32-8) + 12(4)$$

$$\Rightarrow \quad x - 14 = 12$$

$$\Rightarrow \quad x = 26 \text{ m}$$

Please be careful while taking the limits in each interval.

Hence, the correct answer is (B).

33. $80 \text{ kmh}^{-1}\, NE \equiv 80\left(\dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{j}\right) = \vec{v}_A$

$60 \text{ kmh}^{-1}\, SE \equiv 60\left(\dfrac{1}{\sqrt{2}}\hat{i} - \dfrac{1}{\sqrt{2}}\hat{j}\right) = \vec{v}_B$

Since $\vec{v}_{A/B}$ = velocity of A relative to $B = \vec{v}_A - \vec{v}_B$

$$\Rightarrow \quad \vec{v}_{A/B} = \vec{v}_A - \vec{v}_B = 10\sqrt{2}\,\hat{i} + 70\sqrt{2}\,\hat{j}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{7}\right).$$

Hence, the correct answer is (D).

34. $s = \dfrac{t^2}{2} + \dfrac{t^3}{3}$

$$v_T = \frac{ds}{dt} = t + t^2$$

$$\Rightarrow \quad a_T = 1 + 2t$$

$$\Rightarrow \quad a_T\big|_{t=2s} = 5 \text{ ms}^{-2}$$

Now $a_C = \dfrac{v_T^2}{r} = \dfrac{\left(v_T\big|_{t=2}\right)^2}{r}$

$$\Rightarrow \quad a_C = \frac{(6)^2}{3} = 12 \text{ ms}^{-2}$$

Since $a^2 = a_C^2 + a_T^2$

$$\Rightarrow \quad a^2 = 144 + 25$$

$$\Rightarrow \quad a = 13 \text{ ms}^{-2}$$

Hence, the correct answer is (D).

35. $v = \sqrt{2gh}$

$$\Rightarrow \quad v = \sqrt{2(10)5}$$

$$\Rightarrow \quad v = 10 \text{ ms}^{-1}$$

$$t = \sqrt{\frac{2h}{g}}$$

$$\Rightarrow \quad t = \sqrt{\frac{2(5)}{10}} = 1 \text{ s}$$

So, total time to hit the bottom from the surface is $t' = 5 - 1 = 4$ s

$$\Rightarrow \quad d = vt'$$

$$\Rightarrow \quad d = (10)(4)$$

$$\Rightarrow \quad d = 40 \text{ m}$$

Hence, the correct answer is (D).

36. Whenever a particle separated from a body moving with some velocity and having some acceleration, then the particle separated always retains the velocity of the body but does not retains its acceleration. So, in this case we have the velocity of the balloon at $t = 8$ s is

$$v = 0 + (1.25)(8) = 10 \text{ ms}^{-1} \text{ and}$$

$$s_{\text{balloon}} = \frac{1}{2}(1.25)(64) = 40 \text{ m}$$

When released the stone has a velocity of 10 ms^{-1} up. So, for the stone, we have

$$s = ut + \frac{1}{2}at^2$$

$\Rightarrow \quad -40 = (10)t + \frac{1}{2}(-10)t^2$

$\Rightarrow \quad -4 = t - \frac{t^2}{2}$

$\Rightarrow \quad -8 = 2t - t^2$

$\Rightarrow \quad t^2 - 2t - 8 = 0$

$\Rightarrow \quad t^2 - 4t + 2t - 8 = 0$

$\Rightarrow \quad t(t-4) + 2(t-4) = 0$

$\Rightarrow \quad t = -2 \text{ s or } t = 4 \text{ s}$

Hence, the correct answer is (C).

37. $v = \sqrt{2gh}$ and $v' = \sqrt{2g(h+x)}$

$\Rightarrow \quad v' = \sqrt{2gh}\left(1 + \frac{x}{h}\right)^{\frac{1}{2}}$

Since $x \ll h$

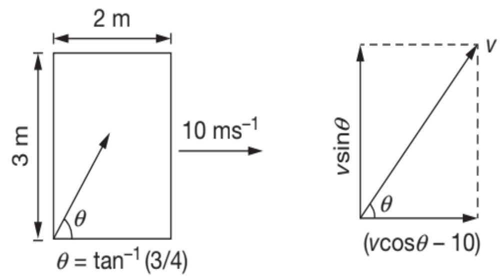
$\therefore \quad v' = v\left(1 + \frac{x}{2h}\right)$

$\therefore$ Increase in velocity $= \frac{vx}{2h}$

Hence, the correct answer is (A).

38. $2 = (v\cos\theta - 10)t$ and

$3 = (v\sin\theta)t$



$\Rightarrow \quad t = \frac{1}{10}\left(\frac{3}{\tan\theta} - 2\right)$

$\Rightarrow \quad t = \frac{1}{10}\left(\frac{3}{\left(\frac{3}{4}\right)} - 2\right)$

$\Rightarrow \quad t = \frac{1}{10}(4-2)$

$\Rightarrow \quad t = 0.2 \text{ s}$

Hence, the correct answer is (A).

39. Since slope of v-t graph gives acceleration, so we observe that at all the instants acceleration of P is greater than Q.

Hence, the correct answer is (D).

40. Since, $t = \sqrt{\frac{2h}{g}}$ and $t' = \sqrt{\frac{2(h+x)}{g}}$

$\Rightarrow \quad t' = \sqrt{\frac{2h}{g}}\left(1 + \frac{x}{h}\right)^{\frac{1}{2}}$

$\Rightarrow \quad t' - t \approx \frac{xt}{2h}$

Hence, the correct answer is (C).

41. $a = -0.6 - 0.001\,v^2$

$v\frac{dv}{dx} = -0.6 - 0.001v^2$

$\Rightarrow \quad \int_{60}^{0} \frac{vdv}{0.6 + 0.001v^2} = -\int_{0}^{s} dx$

$\Rightarrow \quad \frac{1000}{2}\int_{60}^{0} \frac{2vdv}{v^2 + 600} = -s$

$\Rightarrow \quad s = -\frac{1000}{2}\log_e(v^2 + 600)\Big|_{60}^{0}$

$\Rightarrow \quad s = -500\log_e\left[\frac{600}{(60)^2 + 600}\right]$

$\Rightarrow \quad s = -500\log_e\left(\frac{600}{4200}\right)$

$\Rightarrow \quad s = 500\log_e(7)$

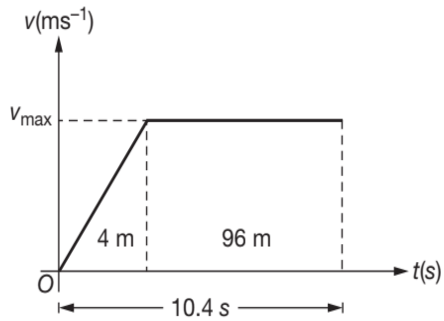
$\Rightarrow \quad s = 500(1.946)$

$\Rightarrow \quad s = 973 \text{ m}$

Hence, the correct answer is (C).

42. $s_1 = 4 = \frac{1}{2}(v_{max})t_1$

$s_2 = 96 = v_{max}t_2$



Since $t_1 + t_2 = 10.4$ s

$\Rightarrow \quad \frac{8}{v_{max}} + \frac{96}{v_{max}} = 10.4$

$\Rightarrow \quad \dfrac{8+96}{v_{max}} = 10.4$

$\Rightarrow \quad v_{max} = \dfrac{104}{10.4} = 10\ ms^{-1}$

Hence, the correct answer is (A).

43. $\vec{v} = \dot{\vec{r}} = \dfrac{d\vec{r}}{dt} = -a\omega\sin(\omega t)\hat{i} + a\omega\cos(\omega t)\hat{j} + b\hat{k}$

$|\vec{v}| = \sqrt{a^2\omega^2 + b^2}$

Distance moved by the particle in one full turn of helix is given by

$s = |\vec{v}|T = \dfrac{2\pi}{\omega}\sqrt{a^2\omega^2 + b^2}$

Hence, the correct answer is (B).

44. Area under a v-t graph gives the distance travelled. So

$s = (20)(6) + \dfrac{1}{2}(20+45)3 + \dfrac{1}{2}(3)(45)$

$\Rightarrow \quad s = 120 + 165$

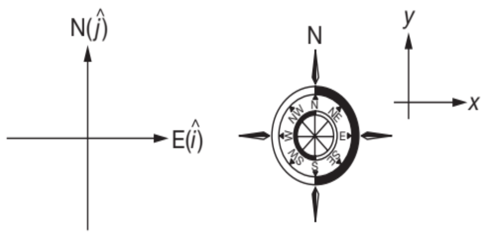
$\Rightarrow \quad s = 285\ m$

Hence, the correct answer is (C).

45. $\vec{v}_w = \dfrac{v}{\sqrt{2}}\hat{i} + \dfrac{v}{\sqrt{2}}\hat{j}$

$\vec{v}_m = (at)\hat{j}$

$\Rightarrow \quad \vec{v}_{wm} = \vec{v}_w - \vec{v}_m = \dfrac{v}{\sqrt{2}}\hat{i} + \left(\dfrac{v}{\sqrt{2}} - at\right)\hat{j}$



It appears due east when, the y component vanishes. So

$\dfrac{v}{\sqrt{2}} - at = 0$

$\Rightarrow \quad t = \dfrac{v}{\sqrt{2}a}$

Hence, the correct answer is (C).

46. From graph

$v_f^2 = (\text{final velocity})^2 = 900\ km^2h^{-2}$

$v_i^2 = (\text{initial velocity})^2 = 3600\ km^2h^{-2}$

s = Distance covered = 0.6 km

Since, $v_f^2 - v_i^2 = 2as$ we get $a = -2250\ kmh^{-2}$

Hence, the correct answer is (C).

47. $a = \dfrac{v-u}{t} = \dfrac{10-25}{3} = -5\ ms^{-2}$ {Retardation}

$s = ut + \dfrac{1}{2}at^2$

$\Rightarrow \quad s = 25(3) + \dfrac{1}{2}(-5)(9)$

$\Rightarrow \quad s = 75 - 22.5$

$\Rightarrow \quad s = 52.5\ m$

Acceleration is constant, so (C) happens to be incorrect.

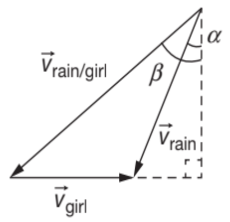
Hence, the correct answer is (A).

48. $v_{av} = \dfrac{l+l}{\dfrac{l}{2v_1} + \dfrac{l}{2v_2}}$

$\Rightarrow \quad v_{av} = \dfrac{2v_1v_2}{v_1+v_2} = 48\ kmh^{-1}$

Hence, the correct answer is (D).

49.



$\tan\beta = \dfrac{v_{girl} + v_{rain}\sin\alpha}{v_{rain}\cos\alpha}$

$\Rightarrow \quad \tan\beta = \dfrac{8 + 10\sin\alpha}{10\cos\alpha}$

Hence, the correct answer is (B).

50. The particle strikes the point B, when the velocity of the particle with respect to the platform is along AB i.e., the component of relative velocity along AD must be zero.

$\Rightarrow \quad v - 2v\cos\theta = 0$

$\Rightarrow \quad \cos\theta = \dfrac{1}{2}$

$\Rightarrow \quad \theta = 60°$

Hence, the correct answer is (C).

51. In this problem, we must check that the distance travelled by the elevator while accelerating plus the distance travelled by the elevator while decelerating must equal 100 m. If this is not satisfied then the remaining distance must have been travelled with a uniform speed equal to the maximum speed attained by the elevator at the end of accelerated motion.

Hence, the correct answer is (B).

52. Let v be the velocity of the river and u be the velocity of swimmer with respect to river. Then

$$t_1 = \frac{2d}{\sqrt{u^2 - v^2}} \qquad \ldots(1)$$

Further

$$t_2 = \frac{d}{u+v} + \frac{d}{u-v} = \frac{2ud}{u^2 - v^2} \qquad \ldots(2)$$

and $t_3 = \dfrac{2d}{u}$ $\qquad \ldots(3)$

From (1), (2) and (3), we get

$$t_1^2 = t_2 t_3$$

Hence, the correct answer is (D).

53. On passing through a plank of thickness x, the velocity of bullet becomes $\frac{19}{20}u$, where u is the initial velocity.

$$\Rightarrow \left(\frac{19u}{20}\right)^2 - u^2 = 2ax \qquad \ldots(1)$$

Let the bullet further pass through n planks. Hence total number of planks $= (n+1)$

$$0^2 - u^2 = 2a(n+1)x \qquad \ldots(2)$$

Solving we get, $(n+1) \approx 11$

Hence, the correct answer is (B).

54. $v = \dfrac{6 \text{ km}}{2(1\, hr)} = 3 \text{ kmh}^{-1}$

Hence, the correct answer is (A).

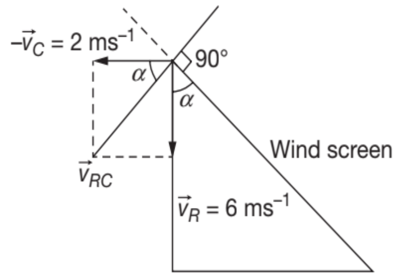
55. For rain drops to strike the wind screen normally, velocity of rain with respect to car $\vec{v}_{RC} = \vec{v}_R - \vec{v}_C$ should be perpendicular to the wind screen.

So, components of $\vec{v}_R$ and $-\vec{v}_C$ parallel to wind screen should cancel each other.

$$\Rightarrow \quad 6\cos\alpha = 2\sin\alpha$$

$$\Rightarrow \quad \tan\alpha = 3$$

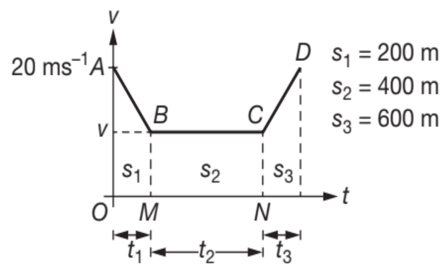
$$\Rightarrow \quad \alpha = \tan^{-1}(3)$$



Hence, the correct answer is (D).

56. For AB

$$200 = \left(\frac{20+v}{2}\right)t_1 \qquad \ldots(1)$$



($\because$ 72 kmh^{-1} = 20 ms^{-1})

For BC

$$400 = vt_2 \qquad \ldots(2)$$

For CD

$$600 = \left(\frac{20+v}{2}\right)t_3 \qquad \ldots(3)$$

$$\Rightarrow \quad t_1 = \frac{1}{3}t_3$$

Since, $t_2 = t_1 + t_3 = 4t_1$

$$\Rightarrow \quad 400 = 4vt_1$$

$$\Rightarrow \quad vt_1 = 100 \qquad \ldots(4)$$

Divide (1) and (4), we have

$$2 = \frac{20+v}{2v}$$

$$\Rightarrow \quad 4v = 20 + v$$

$$\Rightarrow \quad v = \frac{20}{3} \text{ ms}^{-1}$$

$$\Rightarrow \quad t_1 = 15\, s,\ t_2 = 60\, s,\ t_3 = 45\, s$$

Hence, $t = t_1 + t_2 + t_3$

$$\Rightarrow \quad t = 120 \text{ s} = 2 \text{ minute}$$

Hence, the correct answer is (C).

57. From $O \rightarrow A$ we have acceleration varying linearly with time hence

$\Rightarrow \quad a-0=\frac{5}{6}(t-0)$

$\Rightarrow \quad a=\frac{5}{6}t$

$\Rightarrow \quad \frac{dv}{dt}=\frac{5}{6}t$

$\Rightarrow \quad \int_0^{v_1} dv=\frac{5}{6}\int_0^t t\,dt$

$\Rightarrow \quad v_1=\frac{5}{12}t^2$

Further $v_1=\frac{dx_1}{dt}=\frac{5}{12}t^2$

$\Rightarrow \quad \int_0^{x_1} dx_1=\frac{5}{12}\int_0^6 t^2dt \Rightarrow x_1=\frac{5}{36}t^3\Big|_0^6$

$\Rightarrow \quad x_1=\frac{5}{36}(6^3)=30\text{ m}$

For $A \to B$ the acceleration is constant and have a value of 5 ms^{-2}. Now velocity at point A is $v_1=\frac{5}{12}(36)=15\text{ ms}^{-1}$

$\Rightarrow \quad x_2=v_1(6)+\frac{1}{2}(5)(6)^2$

$\Rightarrow \quad x_2=180\text{ m}$

So, total distance travelled is $x=30+180=210\text{ m}$

Hence, the correct answer is (B).

58. Let u be the initial velocity and a be the acceleration. Then

$$2=u(2)+\frac{1}{2}a(2)^2$$

$\Rightarrow \quad u+a=1 \qquad \ldots(1)$

$$4.2=u(2+4)+\frac{1}{2}a(2+4)^2$$

$\Rightarrow \quad 4.2=6u+18a$

$\Rightarrow \quad 0.7=u+3a$

$\Rightarrow \quad u+3a=0.7 \qquad \ldots(2)$

From (1) and (2), we get

$$2a=-0.3$$

$$a=-0.15\text{ ms}^{-2}$$

and $u=1.15\text{ ms}^{-1}$

$$v\big|_{t=7s}=u+7a$$

$\Rightarrow \quad v\big|_{t=7s}=1.15-7(0.15) \Rightarrow v\big|_{t=7s}=1.15-1.05$

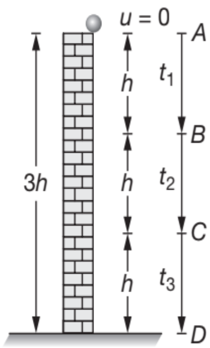
$\Rightarrow \quad v\big|_{t=7s}=0.1\text{ ms}^{-1}$

Hence, the correct answer is (A).

59. For $A \to B$, $h=\frac{1}{2}gt_1^2 \qquad \ldots(1)$

For $A \to C$, $2h=\frac{1}{2}g(t_1+t_2)^2 \qquad \ldots(2)$

For $A \to D$, $3h=\frac{1}{2}g(t_1+t_2+t_3)^2 \qquad \ldots(3)$



Please note that $u=0$ for all the intervals that start from A.

From (1), (2) and (3), we get

$$t_1:t_2:t_3=1:(\sqrt{2}-1):(\sqrt{3}-\sqrt{2})$$

Hence, the correct answer is (C).

60. Initially, when both are thrown up simultaneously, then acceleration of A w.r.t. B is zero (as both fall under the influence of gravity). So, relative position versus time plot is a straight line. Now the particle A reach the ground and stops, so its acceleration becomes zero and hence the relative acceleration of B w.r.t. A is g.

Therefore the relative position vs t graph is now parabolic. So, the variation plot is first linear and then parabolic.

Hence, the correct answer is (D).

61. Initially let the dog and cat be at A and B respectively with their velocities perpendicular to each other as shown

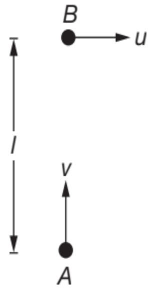


FIGURE 1: Positions of dog and cat at $t = 0$

The cat has a velocity constant in magnitude and direction but dog has a velocity continuously aimed at cat i.e. constant in magnitude but varying in direction.

Let at any instant t, v makes an angle θ with u. Then let us resolve u parallel to v and perpendicular to v as shown in Figure (2). Component parallel to v is $u\cos\theta$ and perpendicular to v is $u\sin\theta$

Relative velocity of dog towards cat is

$$(v - u\cos\theta) = -\frac{dx}{dt}$$

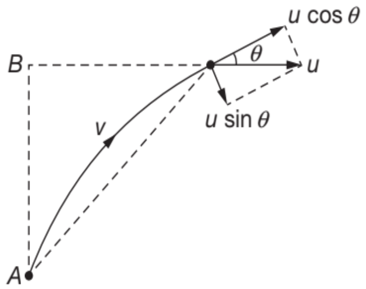


FIGURE 2: Positions of dog and cat at time t

The negative sign on RHS indicates that the distance between cat and dog decreases if the dog has to catch the cat.

$$\Rightarrow \quad -dx = (v - u\cos\theta)dt$$

$$\Rightarrow \quad -\int_{\ell}^{0} dx = \int_0^t v\,dt - \int_0^t (u\cos\theta)dt$$

$$\Rightarrow \quad l = \int_0^t v\,dt - \int_0^t (u\cos\theta)dt \qquad \ldots(1)$$

For dog to catch the cat we must have

$$\begin{pmatrix}\text{Horizontal}\\ \text{displacement}\\ \text{of CAT}\end{pmatrix} = \begin{pmatrix}\text{Horizontal}\\ \text{displacement}\\ \text{of DOG}\end{pmatrix}$$

i.e., $$ut = \int_0^t (v\cos\theta)dt \qquad \ldots(2)$$

From (2) $\int_0^t (\cos\theta)dt = \frac{ut}{v}$

Substituting in (1), we get

$$l = vt - u\left(\frac{ut}{v}\right)$$

$$\Rightarrow \quad l = \frac{(v^2 - u^2)t}{v}$$

$$\Rightarrow \quad t = \frac{vl}{v^2 - u^2}$$

Also for dog to catch the cat $v > u$.

Hence, the correct answer is (A).

62. $v^2 \propto s$

$\Rightarrow$ acceleration is constant.

Hence, the correct answer is (A).

63. $AB = \left(100 \times \frac{5}{18}\right)t$ metre

$CD = \left(60 \times \frac{5}{18}\right)t$ metre

For BC, $v_C = v_B + at$

$$\Rightarrow \quad 60 \times \frac{5}{18} = 100 \times \frac{5}{18} + a(4)$$

$$\Rightarrow \quad 4a = -40 \times \frac{5}{18}$$

$$\Rightarrow \quad a = -\frac{50}{18} = -\frac{25}{9}\ \text{ms}^{-2}$$

So the distance BC is

$$BC = 100 \times \frac{5}{18} \times 4 + \frac{1}{2}\left(-\frac{25}{9}\right)(4)^2$$

$$BC = \frac{1000}{9} - \frac{200}{9} = \frac{800}{9}\ \text{m}$$

However $AB + BC + CD = 3000$ m

$$\Rightarrow \quad (160)\left(\frac{5}{18}\right)t + \frac{800}{9} = 3000$$

$$\Rightarrow \quad \frac{400}{9}t + \frac{800}{9} = 3000$$

$$\Rightarrow \quad 400t + 800 = 27000$$

$$\Rightarrow \quad 400t = 26200$$

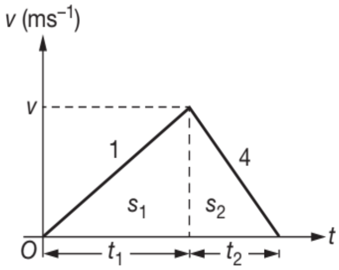
$$\Rightarrow \quad t = 65.5\ \text{s}$$

Hence, the correct answer is (C).

CHAPTER 4

64. If we take the maximum attainable velocity as $24\ \text{ms}^{-1}$, then to accelerate with $1\ \text{ms}^{-2}$ (from rest), the time is

$t_1 = 24\ \text{s}$ $\quad\{\because 24 = 0 + (1)t_1\}$



During this time the distance covered is observed to be

$$s_1 = \frac{1}{2}a_1 t_1^2$$

$$\Rightarrow \quad s_1 = \frac{1}{2}(1)(24)^2 = 288\ \text{m} > 200\ \text{m}$$

which is not possible.

So, the car must attain a $v_{\max} < 24\ \text{ms}^{-1}$

Let that maximum velocity be v. Then

$t_1 + t_2 = t$ (say) and $s_1 + s_2 = 200$ m

$$\Rightarrow \quad \frac{v}{1} + \frac{v}{4} = t$$

$$\Rightarrow \quad v = \frac{4t}{5} \qquad \ldots(1)$$

Now $s_1 + s_2 = 200$

$$\Rightarrow \quad \frac{v^2}{2(1)} + \frac{v^2}{2(4)} = 200$$

$$\Rightarrow \quad \frac{16t^2}{50} + \frac{16t^2}{200} = 200$$

$$\Rightarrow \quad 16t^2\left(\frac{5}{200}\right) = 200$$

$$\Rightarrow \quad t^2 = 500$$

$$\Rightarrow \quad t \cong 22.4\ \text{s}$$

You could have directly applied the formula

$$s = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2$$

where $s = 200$ m, $t = ?$

$\alpha = 1\ \text{ms}^{-2}$ and $\beta = 4\ \text{ms}^{-2}$

$\Rightarrow \quad t = 22.4$ s

Hence, the correct answer is (A).

65. Let the length of the escalator be ℓ. If u be the velocity of person, then

$$90 = \frac{l}{u} \qquad \ldots(1)$$

If v be the velocity of the escalator, then

$$60 = \frac{l}{v} \qquad \ldots(2)$$

If t be the time taken by him to walk up the moving escalator, then

$$t = \frac{l}{u+v} \qquad \ldots(3)$$

From (1), $u = \dfrac{l}{90}$

From (2), $v = \dfrac{l}{60}$

Substituting in (3), we get

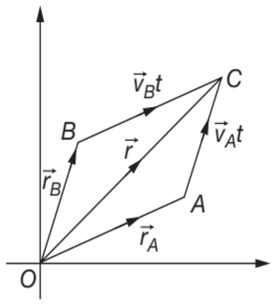
$$t = \frac{l}{\dfrac{l}{90} + \dfrac{l}{60}}$$

$$\Rightarrow \quad t = \frac{(60)(90)}{60+90}$$

$$\Rightarrow \quad t = 36\ \text{s}$$

Hence, the correct answer is (A).

66. Let the particles collide at time t.



For particles to collide at C

$$\vec{r}_B + \vec{v}_B t = \vec{r}_A + \vec{v}_A t$$

$$\Rightarrow \quad \vec{r}_B - \vec{r}_A = (\vec{v}_A - \vec{v}_B)t \qquad \ldots(1)$$

$$\Rightarrow \quad |\vec{r}_B - \vec{r}_A| = |\vec{v}_A - \vec{v}_B|t$$

$$\Rightarrow \quad t = \frac{|\vec{r}_B - \vec{r}_A|}{|\vec{v}_B - \vec{v}_A|} \qquad \ldots(2)$$

Substituting (2) in (1) and rearrange to get (D)

Hence, the correct answer is (D).

67. Since, $\dfrac{dv}{dt} = (g - cx^2)$

$$\Rightarrow \quad v\frac{dv}{dx} = g - cx^2$$

$$\Rightarrow \int_{v_0}^{0} v\,dv = \int_{0}^{x_m} (g - cx^2)\,dx$$

$$\Rightarrow -\frac{v_0^2}{2} = gx_m - \frac{cx_m^3}{3}$$

$$\Rightarrow \frac{cx_m^3}{3} = gx_m + \frac{v_0^2}{2}$$

$$\Rightarrow c = \frac{6gx_m + 3v_0^2}{2x_m^3}$$

Hence, the correct answer is (D).

68. $s_{n^{th}} = u + \frac{a}{2}(2n-1)$

$$\Rightarrow s_{n^{th}} = 0 + \frac{a}{2}(2n-1)$$

$$\Rightarrow s_{n^{th}} = \frac{a}{2}(2n-1)$$

$$\Rightarrow s_1 : s_2 : s_3 \equiv 1:3:5$$

Hence, the correct answer is (C).

69. $\vec{v} = \vec{u} + \vec{a}t$

$$\Rightarrow \vec{v} = (6\hat{i} + 8\hat{j}) + (0.8\hat{i} + 0.6\hat{j})10$$

$$\Rightarrow \vec{v} = (6+8)\hat{i} + (8+6)\hat{j}$$

$$\Rightarrow \vec{v} = 14\hat{i} + 14\hat{j}$$

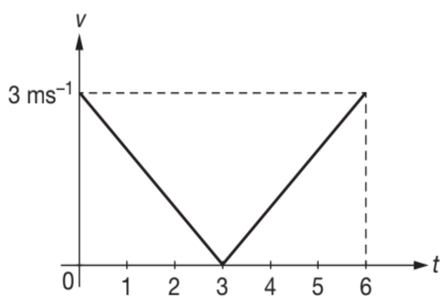
$$\Rightarrow |\vec{v}| = 14\sqrt{2}\ \text{ms}^{-2}$$

Hence, the correct answer is (C).

70. Distance moved by the car is the area under v-t graph. So,

$$s_{\text{total}} = \frac{1}{2}(3)(3) + \frac{1}{2}(3)(3)$$

$$\Rightarrow s_{\text{total}} = 9\text{ m}$$



Hence, the correct answer is (C).

72. For OA, we have $a = \frac{5}{4}t$

$$\Rightarrow dv = \frac{5}{4}t\,dt$$

$$\Rightarrow v_A = \int dv = \frac{5}{4}\int_0^4 t\,dt = 10\text{ ms}^{-1}$$

Otherwise, we can also observe that Δv = Area under a-t graph. So,

$$\Delta v = v = 10\text{ ms}^{-1}$$

For AB, we have

$$v_B = v_A + at$$

$$\Rightarrow v_B = 10 + (5)(4) = 30\text{ ms}^{-1}$$

Hence, the correct answer is (A).

73. For constant acceleration, assuming $u = 0$, then

$v^2 \propto x$ or $s \propto t^2$

Hence, the correct answer is (D).

74. s = Area under the graph

$$\Rightarrow s = \frac{1}{2}(8)(2) + \frac{1}{2}(12)(3)$$

$$\Rightarrow s = 8 + 18$$

$$\Rightarrow s = 26\text{ m}$$

Hence, the correct answer is (C).

75. Maximum acceleration is from C to D. So

$$a = \frac{60-20}{1-0.75}$$

$$\Rightarrow a = \frac{40}{0.25}$$

$$\Rightarrow a = 160\text{ kmh}^{-2}$$

Hence, the correct answer is (B).

76. $\vec{v}_{\text{man}} = \frac{v}{\sqrt{2}}\hat{i} + \frac{v}{\sqrt{2}}\hat{j}$

Let $\vec{v}_{\text{wind}} = a\hat{i} + b\hat{j}$

$$\Rightarrow \vec{v}_{\text{wind/man}} = \left(a - \frac{v}{\sqrt{2}}\right)\hat{i} + \left(b - \frac{v}{\sqrt{2}}\right)\hat{j}$$

$$\Rightarrow \tan\theta = \frac{b - \frac{v}{\sqrt{2}}}{a - \frac{v}{\sqrt{2}}} = \tan(270)$$

$$\Rightarrow a - \frac{v}{\sqrt{2}} = 0$$

CHAPTER 4

$\Rightarrow \quad a = \frac{v}{\sqrt{2}}$

$\Rightarrow \quad \vec{v}_{\text{wind}} = \frac{v}{\sqrt{2}}\hat{i} + b\hat{j}$

When the man doubles his speed

$$\vec{v}'_{\text{man}} = 2\left(\frac{v}{\sqrt{2}}\hat{i} + \frac{v}{\sqrt{2}}\hat{j}\right) = \sqrt{2}\left(v\hat{i} + v\hat{j}\right)$$

$\Rightarrow \quad \vec{v}'_{\text{wind/man}} = \left(\frac{v}{\sqrt{2}} - \sqrt{2}v\right)\hat{i} + \left(b - \sqrt{2}v\right)\hat{j}$

$\Rightarrow \quad \tan\theta' = \frac{b - \sqrt{2}v}{\frac{v}{\sqrt{2}} - \sqrt{2}v} = \frac{2v - \sqrt{2}b}{v}$

But $\theta' = 270 - \cot^{-1}(2)$

$\Rightarrow \quad \tan\left[270 - \cot^{-1}(2)\right] = \frac{2v - \sqrt{2}b}{v}$

$\Rightarrow \quad \cot\left[\cot^{-1}(2)\right] = \frac{2v - \sqrt{2}b}{v}$

$\Rightarrow \quad 2v = 2v - \sqrt{2}b$

$\Rightarrow \quad b = 0$

$\therefore \quad \vec{v}_{\text{wind}} = \frac{v}{\sqrt{2}}\hat{i}$

Hence, the correct answer is (B).

77. Distance between two cars leaving from station x is,

$$d = \left(\frac{1}{6}\right) \times 60 = 10 \text{ km}$$

Man meets the first car after time,

$$t_1 = \frac{60}{60+60} = \frac{1}{2} \text{ h}$$

He will meet the next car after time,

$$t_2 = \frac{10}{60+60} = \frac{1}{12} \text{ h}$$

In the remaining half an hour, number of cars he will meet again is, $n = \frac{1/2}{1/12} = 6$

So, total number of cars would be meet on route will be 7.

Hence, the correct answer is (B).

78. $h = \frac{1}{2}gt^2 \qquad \ldots(1)$

$\frac{h}{2} = \frac{1}{2}g(t-1)^2 \qquad \ldots(2)$

Solving (1) and (2), we get

$t = 2 \pm \sqrt{2}$ s

But $2 - \sqrt{2} < 1$ s

$\Rightarrow \quad t = 2 + \sqrt{2}$ s

$\Rightarrow \quad t = 3.41$ s

Hence, the correct answer is (B).

79. $\frac{dv}{A - Bv} = dt$

$\Rightarrow \quad \int_0^v \frac{dv}{A - Bv} = \int_0^t dt$

$\Rightarrow \quad -\frac{1}{B}\log_e(A - Bv)\Big|_0^v = t$

$\Rightarrow \quad \log_e\left(\frac{A - Bv}{A}\right) = -Bt$

$\Rightarrow \quad \frac{A - Bv}{A} = e^{-Bt}$

$\Rightarrow \quad v = \frac{A}{B}\left(1 - e^{-Bt}\right)$

Hence, the correct answer is (C).

80. Let the distance travelled by each car be s, then

$v_1 - v_2 = v_0$

$\Rightarrow \quad \sqrt{2a_1 s} - \sqrt{2a_2 s} = v_0$ and

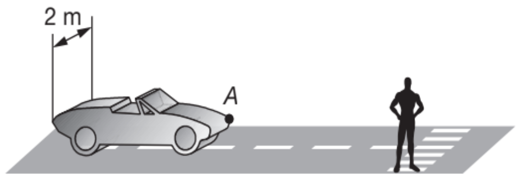
$t_2 - t_1 = t_0$

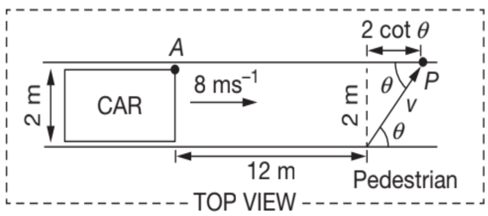
$\Rightarrow \quad \sqrt{\frac{2s}{a_2}} - \sqrt{\frac{2s}{a_1}} = t_0$

$\Rightarrow \quad \frac{v_0}{t_0} = \frac{\sqrt{a_1} - \sqrt{a_2}}{\frac{1}{\sqrt{a_2}} - \frac{1}{\sqrt{a_1}}} = \sqrt{a_1 a_2}$

Hence, the correct answer is (D).

81. For safely crossing the road the pedestrian must cross the road by the time the car travels a distance $(12 + 2\cot\theta)$ (see figure).





So $\dfrac{12+2\cot\theta}{8}=\dfrac{2}{v\sin\theta}$

$\Rightarrow \quad v=\dfrac{8}{6\sin\theta+\cos\theta}$

For v to be MINIMUM

$\Rightarrow \quad \dfrac{dv}{d\theta}=0$

$\Rightarrow \quad \tan\theta=6$ {Answer to PROBLEM 82}

$\Rightarrow \quad v=\dfrac{8}{\cos\theta(6\tan\theta+1)}=\dfrac{8}{\cos\theta(36+1)}$

$\Rightarrow \quad v=\dfrac{8}{37\left(\dfrac{1}{\sqrt{37}}\right)}$ $\{\because \tan\theta=6\}$

$\Rightarrow \quad v=\dfrac{8}{\sqrt{37}}\simeq\dfrac{8}{6}$

$\Rightarrow \quad v\simeq\dfrac{4}{3}\,\text{ms}^{-1}$ {Answer to PROBLEM 81}

Hence, the correct answer is (A).

82. See PROBLEM 81.
Hence, the correct answer is (C).

83. For pedestrian to cross the road safely, the time taken by the pedestrian to reach P equals the time taken by the edge A of the car to reach P.
To reach P total distance $(12+2\cot\theta)$ is travelled by the car with a velocity of 8 ms^{-1}. Hence

$$t=\frac{12+2\cot\theta}{8}$$

$\Rightarrow \quad t=\dfrac{12+2\left(\dfrac{1}{6}\right)}{8}=\dfrac{\left(12+\dfrac{1}{3}\right)}{8}$

$\Rightarrow \quad t=\dfrac{37}{24}\text{ s}$

Hence, the correct answer is (B).

84. $a=\dfrac{v^2}{r}$

$\Rightarrow \quad \dfrac{a}{g}=\dfrac{v^2}{rg}$

$\Rightarrow \quad \dfrac{a}{g}=\dfrac{10^{14}}{(0.8)(10)}$

$\Rightarrow \quad \dfrac{a}{g}=1.25\times10^{13}$

Hence, the correct answer is (C).

85. Let the separation between the particles be 10 m at time t. Then $10=\dfrac{1}{2}gt^2-\dfrac{1}{2}g(t-1)^2$

$\Rightarrow \quad 2=t^2-(t-1)^2$

$\Rightarrow \quad 2=t^2-t^2-1+2t$

$\Rightarrow \quad 3=2t$

$\Rightarrow \quad t=1.5\text{ s}$

Hence, the correct answer is (B).

86. Taking downward direction as positive
CASE-1

$$h=ut_1+\frac{1}{2}gt_1^2 \quad \ldots(1)$$

CASE-2

$$h=-ut_2+\frac{1}{2}gt_2^2 \quad \ldots(2)$$

$(1)t_2+(2)t_1$ gives

$$h(t_1+t_2)=\frac{1}{2}gt_1^2t_2+\frac{1}{2}gt_2^2t_1$$

$\Rightarrow \quad h=\dfrac{1}{2}gt_1t_2 \quad \ldots(3)$

If t is the time taken by it to fall freely to the ground from the same height, then

$$h=\frac{1}{2}gt^2 \quad \ldots(4)$$

Equating (3) and (4), we get

$$t=\sqrt{t_1t_2}$$

Hence, the correct answer is (D).

87. $a=kv$

$v\dfrac{dv}{dx}=kv$ $\left\{\because a=\dfrac{dv}{dt}=v\dfrac{dv}{dx}\right\}$

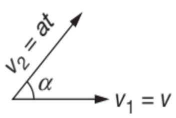
$\Rightarrow \quad \dfrac{dv}{dx}=k$

$\Rightarrow$ Slope of velocity-displacement graph is a constant
Hence, the correct answer is (C).

88. Velocity of 1st particle at time t is $v_1=v$
Velocity of 2nd particle at time t is $v_2=at$

Relative velocity $v_r=\sqrt{v_1^2+v_2^2-2v_1v_2\cos\alpha}$

$\Rightarrow \quad v_r^2=v^2+a^2t^2-2v(at)\cos\alpha$



For v_r to be least or v_r^2 to be least we must have

$$\frac{d}{dt}\left(v_r^2\right)=0$$

$$\Rightarrow \quad 0+2a^2t-2av\cos\alpha=0$$

$$\Rightarrow \quad t=\frac{v\cos\alpha}{a}$$

Hence, the correct answer is (B).

89. $$v_r=\sqrt{v^2+a^2\left(\frac{v^2\cos^2\alpha}{a^2}\right)-2v\left(\frac{v\cos\alpha}{a}\right)a\cos\alpha}$$

$$\Rightarrow \quad v_r=\sqrt{v^2-v^2\cos^2\alpha}$$

$$\Rightarrow \quad v_r=v\sin\alpha$$

Hence, the correct answer is (A).

90. $v_A^2=2g(h+x)$ and

$$v_B^2=u^2+2gx$$

Since $v_A=v_B$

$$\Rightarrow \quad u^2+2gx=2gh+2gx$$

$$\Rightarrow \quad u^2=2gh$$

$$\Rightarrow \quad u=\sqrt{2gh}$$

Hence, the correct answer is (C).

91. $v_1+v_2=4$

$$v_1-v_2=\frac{4}{10}=0.4$$

$$\Rightarrow \quad 2v_1=4.4$$

$$\Rightarrow \quad v_1=2.2\text{ ms}^{-1} \text{ and } v_2=1.8\text{ ms}^{-1}$$

Hence, the correct answer is (A).

92. $$\frac{g}{2}(2n-1)=\frac{1}{2}g(3)^2$$

$$\Rightarrow \quad 2n=10$$

$$\Rightarrow \quad n=5\text{ s}$$

Hence, the correct answer is (A).

93. $$t=\frac{a}{v-v\cos\left(\frac{2\pi}{n}\right)}=\frac{a}{2v\sin^2\left(\frac{\pi}{n}\right)}$$

where n is the number of sides in a symmetrical polygon. For this problem $n=6$

$$\Rightarrow \quad t=\frac{a}{2v\sin^2\left(\frac{\pi}{6}\right)}=\frac{2a}{v}$$

Hence, the correct answer is (A).

94. METHOD I

$$v=kx+v_0$$

Since $a=\dfrac{dv}{dt}$

$$\Rightarrow \quad a=k\left(\frac{dx}{dt}\right)$$

$$\Rightarrow \quad a=kv=k^2x+kv_0$$

$$\Rightarrow \quad a=\alpha x+\text{constant}$$

METHOD II

Since $v=kx+v_0$

$$\Rightarrow \quad \frac{dv}{dx}=k$$

Now $a=\dfrac{dv}{dt}=v\left(\dfrac{dv}{dx}\right)$

$$\Rightarrow \quad a=(kx+v_0)k$$

$$\Rightarrow \quad a=k^2x+kv_0$$

$$\Rightarrow \quad a=\alpha x+\text{constant}$$

So, we observe that acceleration increases linearly with x.

Hence, the correct answer is (D).

95. Area under a-t graph equals the change in velocity (Δv)

$$\Rightarrow \quad \Delta v=\frac{1}{2}(3)(4)=6\text{ ms}^{-1}$$

$$\Rightarrow \quad v_f-v_i=6$$

$$\Rightarrow \quad v_f-2=6$$

$$\Rightarrow \quad v_f=8\text{ ms}^{-1}$$

Hence, the correct answer is (D).

96. From ground to maximum height, we have

$$h=\frac{u^2}{2g}=\frac{(20)^2}{2(9.8)}$$

$$\Rightarrow \quad h=\frac{400}{19.6}=20.4\text{ m}$$

$$\Rightarrow \quad t_1=\frac{v}{g}=\frac{20.4}{9.8}=2.08\text{ s}$$

Now the ball is caught 5 m above the point of launch in its downward motion. So, it is travelled 15 m down in time t_2 (say).

So, $15 = 0 + \frac{1}{2}gt_2^2$

$\Rightarrow \quad 15 = 4.9t_2^2$

$\Rightarrow \quad t_2 = \sqrt{3.06}$

$\Rightarrow \quad t_2 = 1.75 \text{ s}$

So, total time $T = t_1 + t_2$

$\Rightarrow \quad T = 2.08 + 1.75$

$\Rightarrow \quad T = 3.83 \text{ s}$

Please observe that if the options given are not close to each other, then you can also take $g = 10 \text{ ms}^{-2}$ and the answer will be 3.73 s, which is very close to 3.8 s.
DO NOT APPLY THIS WHEN ANSWERS ARE VERY CLOSE IN VALUES.

Hence, the correct answer is (B).

97. At time t, the separation between the particles is

$$x = x_1 - x_2 = 2vt - \frac{1}{2}at^2$$

For x to be MAXIMUM,

$$\frac{dx}{dt} = 0$$

$$2v - at = 0$$

$\Rightarrow \quad t = \frac{2v}{a}$

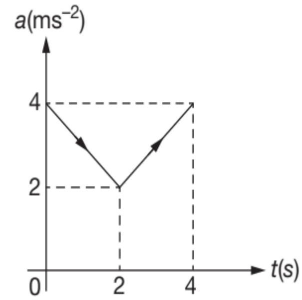
$$x_{MAX} = (2v)\left(\frac{2v}{a}\right) - \frac{1}{2}a\left(\frac{4v^2}{a^2}\right)$$

$\Rightarrow \quad x_{MAX} = \frac{4v^2}{a} - \frac{2v^2}{a} = \frac{2v^2}{a}$

Hence, the correct answer is (C).

98. Since, area under a-t graph is equal to the change in velocity, so

$$\Delta v = \frac{1}{2}(2+4)(2) + \frac{1}{2}(2+4)(2) = 12 \text{ ms}^{-1}$$



$\Rightarrow \quad \Delta v = v - u = 12 \text{ ms}^{-1}$

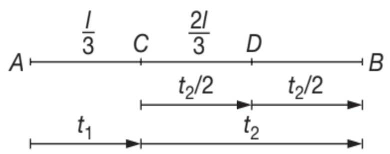
Since $u = 0$

$\Rightarrow \quad v = 12 \text{ ms}^{-1}$

Hence, the correct answer is (B).

99. $v_{av} = \dfrac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$

$\Rightarrow \quad v_{av} = \dfrac{\frac{l}{3} + \frac{2l}{3}}{t_1 + t_2}$ ...(1)



For AC, $\frac{l}{3} = 4t_1$

$\Rightarrow \quad t_1 = \frac{l}{12}$ ...(2)

For CB, $\frac{2l}{3} = 2\left(\frac{t_2}{2}\right) + 6\left(\frac{t_2}{2}\right)$

$\Rightarrow \quad \frac{2l}{3} = t_2 + 3t_2$

$\Rightarrow \quad t_2 = \frac{l}{6}$ ...(3)

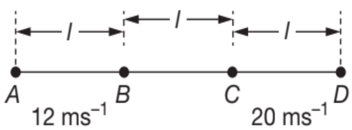
Substituting (2) and (3) in (1), we get

$$v_{av} = \frac{l}{\frac{l}{12} + \frac{2l}{12}} = \frac{l}{\left(\frac{3l}{12}\right)}$$

$\Rightarrow \quad v_{av} = 4 \text{ ms}^{-1}$

Hence, the correct answer is (B).

100. Let velocities at A, B, C and D be v_A, v_B, v_C and v_D.



Since, $\ell = (v_{av})t$

For AB, $12 = \dfrac{v_A + v_B}{2} \Rightarrow v_A + v_B = 24$ ...(1)

For CD, $20 = \dfrac{v_C + v_D}{2} \Rightarrow v_C + v_D = 40$ ...(2)

For AB, $v_B^2 - v_A^2 = 2al$ ...(3)

For CD, $v_D^2 - v_C^2 = 2al$ ...(4)

$\Rightarrow \quad (v_B - v_A)24 = (v_D - v_C)40$

(Equating (3) & (4) and using (1) & (2))

$\Rightarrow \quad \dfrac{v_B - v_A}{v_D - v_C} = \dfrac{5}{3}$

Lets assume

$v_B - v_A = 5k$ ...(5)

$v_D - v_C = 3k$ ...(6)

Solving (1) and (5), we get

$v_B = \dfrac{24 + 5k}{2}$; $v_A = \dfrac{24 - 5k}{2}$ ...(7)

Solving (2) and (6), we get

$v_D = \dfrac{40 + 3k}{2}$; $v_C = \dfrac{40 - 3k}{2}$ ...(8)

Further for interval *BC*

$v_C^2 - v_B^2 = 2al$ ...(9)

Equating (9) and (3) and using (7) and (8) and solving we get, a quadratic in *k* i.e.

$k^2 + 60k - 64 = 0$

We get $k = -30 + 2\sqrt{241}$

(Rejecting negative sign)

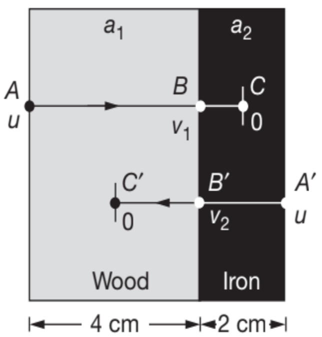
$\therefore$ Average velocity in interval BC is

$v_{av} = \dfrac{v_B + v_C}{2} = \dfrac{24 + 40 + 2k}{4}$

$\Rightarrow \quad v_{av} = (1 + \sqrt{241})\ \text{ms}^{-1}$

Hence, the correct answer is (C).

101. Let a_1 and a_2 be the retardations offered to the bullet by wood and iron respectively.



For A → B → C

$v_1^2 - u^2 = 2a_1(4)$, and

$0^2 - v_1^2 = 2a_2(1)$

Adding, we get

$-u^2 = 2(4a_1 + a_2)$ ...(1)

For $A' \to B' \to C'$

$v_2^2 - u^2 = 2a_2(2)$, and

$0^2 - v_2^2 = 2a_1(2)$

Adding, we get

$-u^2 = 2(2a_1 + 2a_2)$ ...(2)

Equating (1) and (2) and solving, we get

$4a_1 + a_2 = 2a_1 + 2a_2$

$\Rightarrow \quad a_2 = 2a_1$

Hence, the correct answer is (B).

102. $v_x = 30\ \text{ms}^{-1}$

Since, $h = \dfrac{1}{2}gt^2$

$\Rightarrow \quad 80 = \dfrac{1}{2}(10)t^2$

$\Rightarrow \quad t = 4\ \text{s}$

i.e., the ball hits the ground 4 s after the launch.

Now, since $v_y = gt = (10)(4) = 40\ \text{ms}^{-1}$

$\Rightarrow \quad v = \sqrt{v_x^2 + v_y^2}$

$\Rightarrow \quad v = \sqrt{(30)^2 + (40)^2}$

$\Rightarrow \quad v = 50\ \text{ms}^{-1}$

Hence, the correct answer is (B).

103. $v_1 = -\sqrt{2gd}$ and $v_2 = +\sqrt{2g\left(\dfrac{d}{2}\right)}$

Hence, the correct answer is (A).

104. Distance covered in the last one second i.e., distance covered in the nth second if the ball takes n seconds to hit the ground is

$x_1 = s_{n^{th}} = 0 + \dfrac{g}{2}(2n - 1)$ ...(1)

Distance covered in the first three seconds is

$x_2 = \dfrac{1}{2}g(3)^2 = \dfrac{9g}{2}$ ...(2)

Since $x_1 = x_2$

$\Rightarrow \quad \dfrac{g}{2}(2n - 1) = \dfrac{9g}{2}$

$\Rightarrow \quad 2n - 1 = 9$

$\Rightarrow \quad n = 5\text{ s}$

So, total time taken by the ball to hit the ground is 5 s. So,

$$h = \frac{1}{2}gt^2$$

$$\Rightarrow \quad h = \frac{1}{2}(10)(5)^2 = 5(25) = 125\text{ m}$$

$$\Rightarrow \quad h = 125\text{ m}$$

Hence, the correct answer is (C).

105. Let $OP = k$

$OQ = kr$

$OR = kr^2$ and so on

then $v_P^2 = 2a(OP)$ ...(1)

$v_Q^2 = 2a(OQ)$ ...(2)

$v_R^2 = 2a(OR)$ ...(3)

and so on.

$$\Rightarrow \quad v_P^2 : v_Q^2 : v_R^2 : \ldots\ldots \equiv 1 : r : r^2 : \ldots\ldots$$

$$\Rightarrow \quad v_P : v_Q : v_R : \ldots\ldots \equiv 1 : r^{\frac{1}{2}} : r : \ldots\ldots$$

Hence, the correct answer is (B).

106. $0^2 - u^2 = 2(-g)H$

$\Rightarrow \quad u = \sqrt{2gH}$ ...(1)

$$\Rightarrow \quad h = ut - \frac{1}{2}gt^2$$

$$\Rightarrow \quad h = \left(\sqrt{2gH}\right)t - \frac{1}{2}gt^2$$

Quadratic in t solve t getting two values for time whose ratio is $\frac{t_1}{t_2} = \frac{1}{3}$ (given). Substituting values we get

$4h = 3H$

Hence, the correct answer is (A).

107. $v_b =$ velocity of boat $= v$ (say)

$v_r =$ velocity of river $= \eta v$

To start with, we must observe that velocity of river flow is η times greater than the velocity of boat. So, the boat has to drift and in this problem we are to minimise the drift. Let, the velocity of the boat make an angle θ with the river velocity. Then $v_x = v\cos\theta$ is the component of the velocity of boat along the river flow, whereas $v_y = v\sin\theta$ is the component of the velocity of boat perpendicular to river flow. Due to this the net velocity of boat downstream will become $v_r + v_x = \eta v + v\cos\theta = V$ (say)

If t is the time taken by the boat to cross the river of width L, then

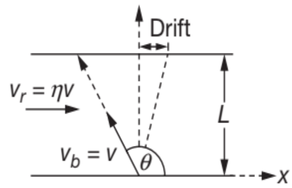
$$t = \frac{L}{v_y} = \frac{L}{v\sin\theta}$$

During this time the drift equals (say x)

$$x = Vt$$

$$x = (\eta v + v\cos\theta)\frac{L}{v\sin\theta}$$

$$x = (\eta\,\mathrm{cosec}\,\theta + \cot\theta)L$$



To MINIMISE x, we must have

$$\frac{dx}{d\theta} = 0$$

$$\eta(-\mathrm{cosec}\,\theta\cot\theta) - \mathrm{cosec}^2\theta = 0$$

$$\Rightarrow \quad \eta\,\mathrm{cosec}\,\theta\cot\theta = -\mathrm{cosec}^2\theta$$

$$\Rightarrow \quad \eta\cot\theta = -\mathrm{cosec}\,\theta$$

$$\Rightarrow \quad \eta\frac{\cos\theta}{\sin\theta} = -\frac{1}{\sin\theta}$$

$$\Rightarrow \quad \cos\theta = -\frac{1}{\eta}$$

$$\Rightarrow \quad -\cos\theta = \frac{1}{\eta}$$

$$\Rightarrow \quad \sin\left(\theta - \frac{\pi}{2}\right) = \frac{1}{\eta}$$

$$\Rightarrow \quad \theta - \frac{\pi}{2} = \sin^{-1}\left(\frac{1}{\eta}\right)$$

$$\Rightarrow \quad \theta = \frac{\pi}{2} + \sin^{-1}\left(\frac{1}{\eta}\right)$$

Hence, the correct answer is (C).

108. Relative to P, the path of S should be along the line SP. So perpendicular to this path, relative velocity should be zero.

Hence, the correct answer is (C).

Multiple Correct Choice Type Questions

1. $$\frac{dv}{dt} = -\alpha v \quad \ldots(1)$$

$$\Rightarrow \quad v\frac{dv}{dx} = -\alpha v$$

$$\Rightarrow \quad \frac{dv}{dx} = -\alpha$$

$$\Rightarrow \quad \int_{v_0}^{0} dv = -\alpha \int_0^s dx$$

$$\Rightarrow \quad 0 - v_0 = -\alpha s$$

$$\Rightarrow \quad s = \frac{v_0}{\alpha}$$

Also from (1),

$$\frac{dv}{v} = -\alpha dt$$

$$\Rightarrow \quad \int_{v_0}^{v} \frac{dv}{v} = -\alpha \int_0^t dt$$

$$\Rightarrow \quad v = v_0 e^{-\alpha t} \quad \ldots(2)$$

$v = 0$, when $t \to \infty$

Also, $v = \frac{v_0}{2}$

$$\Rightarrow \quad \frac{v_0}{2} = v_0 e^{-\alpha t}$$

$$\Rightarrow \quad e^{\alpha t} = 2$$

$$\Rightarrow \quad t = \frac{\log_e(2)}{\alpha} = \frac{7}{10\alpha} \quad \{\because \log_e(2) = 0.7\}$$

Hence, (A), (C) and (D) are correct.

2. $$s = ut + \frac{1}{2}at^2$$

Average velocity $= v_{av} = \frac{s}{t} = u + \frac{1}{2}at$

$$\Rightarrow \quad v_{av} = \frac{2u + at}{2} = \frac{u + (u + at)}{2}$$

$$\Rightarrow \quad v_{av} = \frac{u + v}{2}$$

Hence, (A) and (D) are correct.

3. Acceleration $= \frac{dv}{dt} = \dot{v} = 0 + k\dot{x} \quad \left\{\because \dot{x} = \frac{dx}{dt} = v\right\}$

$$\Rightarrow \quad \dot{v} = a = kv = k(v_0 + kx)$$

Further,

$$a = \frac{dv}{dt} = kv$$

$$\Rightarrow \quad \frac{dv}{dt} = kv$$

$$\Rightarrow \quad \frac{dv}{v} = kdt$$

$$\Rightarrow \quad \int_{v_0}^{v_1} \frac{dv}{v} = k\int_0^t dt$$

$$\Rightarrow \quad t = \frac{1}{k}\log_e\left(\frac{v_1}{v_0}\right)$$

Since, $v = v_0 + kx$. Hence slope of velocity displacement curve is $\frac{dv}{dx} = k$

Hence, (A), (B) and (C) are correct.

4. The speed with which the lower end of the rod moves is

$$v_x = \frac{dx}{dt}$$

$$\Rightarrow \quad v_x = \frac{dy}{dt}\left(\frac{dx}{dy}\right)$$

Since, $x = \sqrt{l^2 - y^2}$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{-y}{\sqrt{l^2 - y^2}}$$

$$\Rightarrow \quad v_x = \frac{-y}{\sqrt{l^2 - y^2}}\frac{dy}{dt}$$

$$\Rightarrow \quad v_x = \frac{y}{\sqrt{l^2 - y^2}}\left(-\frac{dy}{dt}\right)$$

$$\Rightarrow \quad v_x = \frac{yv}{\sqrt{l^2 - y^2}}$$

At $y = 0$, $v_x = 0$

Hence, (C) and (D) are correct.

5. Since $v^2 = v_x^2 + v_y^2$

$$\Rightarrow \quad 2v\frac{dv}{dt} = 2v_x\left(\frac{2v_x}{dt}\right) + 2v_y\left(\frac{dv_y}{dt}\right)$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{v_x a_x + v_y a_y}{v} = \frac{\vec{a} \cdot \vec{v}}{v}$$

This is also equal to the projection of $\vec{a}$ along $\vec{v}$.

Hence, (C) and (D) are correct.

6. $x_t = t^3 - 9t^2 + 6t$

$$v = \frac{dx_t}{dt} = \left(3t^2 - 18t + 6\right) \text{cms}^{-1}$$

For body to be at rest $v = 0$

$\Rightarrow \quad t = \left(3 \pm \sqrt{7}\right)s$

$\Rightarrow \quad t_1 = \left(3 - \sqrt{7}\right)s$ and $t_2 = \left(3 + \sqrt{7}\right)s$

Displacement of particle in travelling from 1st zero of velocity to second zero is –74 cm and $v < 0$ for

$$\left(3 - \sqrt{7}\right) < t < \left(3 + \sqrt{7}\right)$$

Hence, (A), (C) and (D) are correct.

7. The v-t graph for the two cases are shown here

For Case A, we have

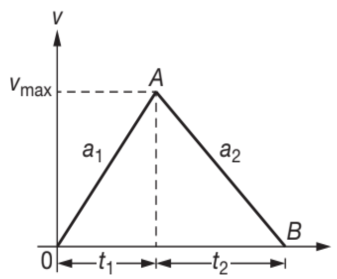
$$v_{max} = a_1 t_1 = a_2 t_2 \qquad \ldots(1)$$

For Case B, we have

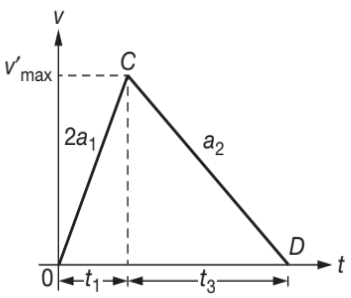
$$v'_{max} = (2a_1)t_1 = a_2 t_3 \qquad \ldots(2)$$

From these equations, we get

$$v'_{max} = 2v_{max} \text{ and } t_3 = 2t_2$$



Case A



Case B

Also, $s_1 = $ (Area under the curve OAB)

$\Rightarrow \quad s_1 = \frac{1}{2} v_{max}\left(t_1 + t_2\right)$

and $s_2 = $ (Area under the curve OCD)

$\Rightarrow \quad s_2 = \frac{1}{2} v'_{max}\left(t_1 + t_3\right)$

$\Rightarrow \quad s_2 = \frac{1}{2}\left(2v_{max}\right)\left(t_1 + 2t_2\right)$

So, the average velocity in Case A, is

$$v_1 = \frac{s_1}{t_1 + t_2} = \frac{v_{max}}{2}$$

The average velocity in Case B, is

$$v_2 = \frac{s_2}{t_1 + t_3} = \frac{1}{2} v'_{max} = v_{max} \qquad \left\{\because v'_{max} = 2v_{max}\right\}$$

So, we observe that $v_2 = 2v_1$ and $2s_1 < s_2 < 4s_1$

Hence, (B) and (C) are correct.

8. $\Rightarrow \quad \frac{dv}{dt} = -\alpha v$

$\Rightarrow \quad v\frac{dv}{dx} = -\alpha v$

$\Rightarrow \quad \int_{v_0}^{0} dv = -\alpha \int_{0}^{x_0} dx$

$\Rightarrow \quad -v_0 = -\alpha x_0$

$\Rightarrow \quad x_0 = \frac{v_0}{\alpha}$

Further, $\frac{dv}{dt} = -\alpha v$

$\Rightarrow \quad \int_{v_0}^{v} \frac{dv}{v} = -\alpha \int_{0}^{t} dt$

$\Rightarrow \quad \log_e v\Big|_{v_0}^{v} = -\alpha t\Big|_{0}^{t}$

$\Rightarrow \quad v = v_0 e^{-\alpha t}$

For $t \to \infty$, $v \to 0$

Hence particle continues to move for a long time span.

Hence, (A) and (B) are correct.

9. For AC, $v^2 - (7)^2 = 2al \qquad \ldots(1)$

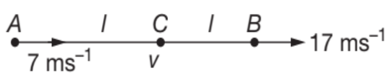
For CB, $(17)^2 - v^2 = 2al \qquad \ldots(2)$

From (1) and (2), we get

$$v^2 = \frac{(7)^2 + (17)^2}{2}$$

$\Rightarrow \quad v^2 = 169$

$\Rightarrow \quad v = 13 \text{ ms}^{-1}$



Let t_1 be the time taken to go from A to C and t_2 be the time taken to go from C to B. Then

$$l=\left(\frac{13+7}{2}\right)t_1 \qquad \text{(for } AC\text{)} \qquad \ldots(3)$$

and $$l=\left(\frac{13+17}{2}\right)t_2 \qquad \text{(for } CB\text{)} \qquad \ldots(4)$$

So, $10t_1=15t_2$

$$\Rightarrow \quad \frac{t_1}{t_2}=\frac{3}{2}$$

Also, we know that, for accelerated motion between two points at separation s, we have

$$s=\left(\frac{u+v}{2}\right)t=v_{av}t$$

So, from (1), we get

$$(v_{av})_{AC}=10\text{ ms}^{-1} \text{ and } (v_{av})_{CB}=15\text{ ms}^{-1}.$$

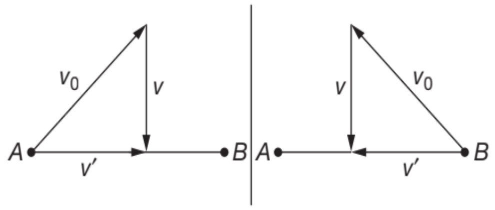
Hence, (A), (B) and (D) are correct.

10. When wind blows along the line AB,

$$t=t_{A\to B}+t_{B\to A}$$

$$\Rightarrow \quad t=\frac{l}{v+v_0}+\frac{l}{v_0-v}$$

$$\Rightarrow \quad t=\frac{2lv_0}{v_0^2-v^2}$$



If wind blows perpendicular to AB

$$t=t_{A\to B}+t_{B\to A}$$

In both the cases, we have

$$v'=\sqrt{v_0^2-v^2}$$

$$t_{A\to B}=\frac{l}{\sqrt{v_0^2-v^2}} \text{ and}$$

$$t_{B\to A}=\frac{l}{\sqrt{v_0^2-v^2}}$$

$$\Rightarrow \quad t=\frac{2l}{\sqrt{v_0^2-v^2}}$$

If the wind were not present then total time taken for the trip would have been $t=\dfrac{2l}{v_0}$

i.e. the total time for the trip increases because of the presence of wind.

Hence, (A), (B) and (D) are correct.

11. $a=4-2x$

$$\Rightarrow \quad v\frac{dv}{dx}=4-2x$$

$$\Rightarrow \quad vdv=(4-2x)dx$$

$$\Rightarrow \quad \int_0^v vdv=\int_0^x(4-2x)dx$$

$$\Rightarrow \quad \frac{v^2}{2}=4x-x^2$$

$$\Rightarrow \quad v^2=8x-2x^2 \qquad \ldots(1)$$

From (1), we observe that $v=0$, when $x=0$ and $x=4$ m.

Now, $a=0$

$$\Rightarrow \quad 4-2x=0$$

$$\Rightarrow \quad x=2\text{ m}$$

So, this happens to be the mean position for the particle to oscillate.

Hence, (B) and (C) are correct.

12. Distance travelled by motor bike at $t=18\ s$

$$s_{\text{bike}}=s_1=\frac{1}{2}(18)(60)=540\text{ m}$$

Distance travelled by car at $t=18\ s$

$$s_{\text{car}}=s_2=(18)(40)=720\text{ m}$$

Therefore, separation between them at $t=18\ s$ is 180 m.

Let separation between them decreases to zero at time t beyond 18 s.

Hence, $s_{\text{bike}}=540+60t$ and $s_{\text{car}}=720+40t$

$$s_{\text{car}}-s_{\text{bike}}=0$$

$$\Rightarrow \quad 720+40t=540+60t$$

$$\Rightarrow \quad t=9\ s \text{ beyond 18 s}$$

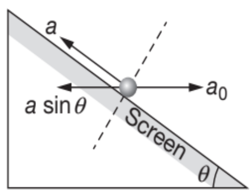
$$\Rightarrow \quad t=(18+9)\ s=27\ s \text{ from start}$$

and distance travelled by both is $s_{\text{bike}}=s_{\text{car}}=1080$ m

Hence, (A) and (C) are correct.

13. $(a_S)_{HOR}=a_0-a\cos\theta$

$(a_S)_{VER}=a_0\sin\theta$



Hence, (B) and (C) are correct.

14. Since particle is decelerated, Hence

$$a = -\alpha\sqrt{v}$$

$$\Rightarrow \quad \frac{dv}{dt} = -\alpha\sqrt{v} \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{dv}{dx}\times\frac{dx}{dt} = -\alpha\sqrt{v}$$

$$\Rightarrow \quad \frac{v}{\sqrt{v}}dv = -\alpha dx$$

$$\Rightarrow \quad \sqrt{v}\,dv = -\alpha\,dx$$

Integrating both sides, we have

$$\int_{v_0}^{0} v^{1/2}\,dv = -\alpha\int_{0}^{x_0} dx$$

$$x_0 = \frac{2}{3}\frac{v_0^{3/2}}{\alpha}$$

Further, from (1), we have

$$\Rightarrow \quad v^{-1/2}dv = -\alpha dt$$

Integrating,

$$\int_{v_0}^{0} v^{-1/2}dv = -\alpha\int_{0}^{t_0} dt$$

$$\Rightarrow \quad t_0 = 2\frac{\sqrt{v_0}}{\alpha}$$

Hence, (A) and (D) are correct.

15. $v_{av} = \dfrac{s_1+s_2+s_3}{t_1+t_2+t_3}$

$$\Rightarrow \quad 20 = \frac{s_1+s_2+s_3}{25}$$

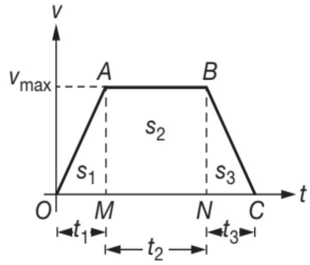
$$\Rightarrow \quad s_1+s_2+s_3 = 500 \qquad \ldots(1)$$

Since car accelerated from $0 \to v_{max}$ in time t_1 with acceleration of $5\ ms^{-2}$ and it decelerated from $v_{max} \to 0$ in time t_3 with a retardation of $5\ ms^{-2}$ hence $t_1 = t_3$ and $s_1 = s_3$. Hence,

$$t_2 = 25 - 2t_1 \qquad \ldots(2)$$

$$\Rightarrow \quad s_1 = s_3 = \frac{1}{2}(5)t_1^2 \text{ and } v_{max} = 5t_1$$

$$\Rightarrow \quad s_2 = v_{max}t_2 = 5t_1t_2$$



Substituting values in (1) and using (2) we get, a quadratic in t_1 which on solving gives $t_1 = 5$ s.

Hence, $t_2 = 15$ s and $v_{max} = 25\ ms^{-1}$

Hence, (A), (B), (C) and (D) are correct.

17. $x_t = 8t^2 - 3t^3$

$$\Rightarrow \quad v_t = 16t - 9t^2$$

For particle to be at rest $v_t = 0$

$$\Rightarrow \quad 16t - 9t^2 = 0$$

$$\Rightarrow \quad t = 0 \text{ or } t = \frac{16}{9}s$$

Substituting $t = \frac{16}{9}s$ in $x_t = 8t^2 - 3t^3$, we get

$$x_t = 8.43 \text{ m}$$

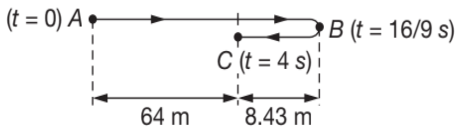
Now $v_{av} = \dfrac{x_{t=4} - x_{t=0}}{4-0}$

$$\Rightarrow \quad v_{av} = \frac{\left[8(4)^2 - 3(4)^3\right] - \left[8(0)^2 - 3(0)^3\right]}{4}$$

$$\Rightarrow \quad v_{av} = \frac{8(16) - 3(64)}{4} = -16\text{ ms}^{-1}$$

Acceleration $= a = \dfrac{dv_t}{dt} = 16 - 18t$

$$\Rightarrow \quad a|_{t=4s} = 16 - 18(4) = -56\text{ ms}^{-2}$$



$$\text{Average speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

Total Distance $= (64 + 8.43 + 8.43)\text{m} = 80.86$ m

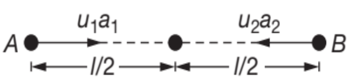
$$\Rightarrow \quad \text{Average speed} = \frac{80.86}{4} = 20.21\text{ ms}^{-1}$$

Hence, (A), (B), (C) and (D) are correct.

18. In general, we have $|d\vec{r}| = dr$ and $v \neq \dfrac{dr}{dt}$. You can think about these cases by taking an example of circular motion.

Hence, (A), (B) and (D) are correct.

19.



$$\frac{l}{2} = u_1t + \frac{1}{2}a_1t^2 \qquad \ldots(1)$$

and $-\dfrac{l}{2} = -u_2t + \dfrac{1}{2}(-a_2)t^2$

$$\Rightarrow \quad \frac{l}{2} = u_2t + \frac{1}{2}a_2t^2 \qquad \ldots(2)$$

Subtracting (1) & (2), we get

$$t = 2\left(\frac{u_2 - u_1}{a_1 - a_2}\right) \qquad \ldots(3)$$

Substituting (3) in (1) or (2) and rearranging, we get

$$l = \frac{4(u_2 - u_1)}{(a_1 - a_2)^2}(a_1u_2 - a_2u_1) \qquad \ldots(4)$$

Since the particles P & Q reach the other ends of A and B with equal velocities say v

For particle P

$$v^2 - u_1^2 = 2a_1l \qquad \ldots(5)$$

For particle Q

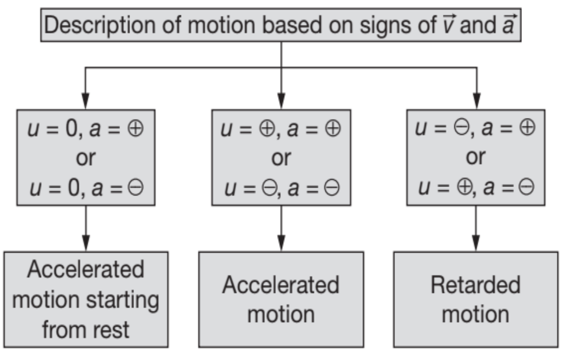
$$v^2 - u_2^2 = 2a_2l \qquad \ldots(6)$$

Subtracting and then substituting value of ℓ and rearranging, we get

$$(u_2 + u_1)(a_1 - a_2) = 8(a_1u_2 - a_2u_1)$$

Hence, (A), (B) and (C) are correct.

20. My dear students, most of you forget to keep in mind the nature of motion, whether accelerated or decelerated, is just not predicted by checking the sign of $\vec{a}$. It also requires the nature of $\vec{v}$ to be defined or mentioned along with. So keep in mind the following table for describing an accelerated or a retarded motion.



Hence, (B), (C) and (D) are correct.

21. Since, $x = 2 + 2t + 4t^2$ and $y = 4t + 8t^2$

$$\Rightarrow \quad x = 2 + \frac{y}{2}$$

$$\Rightarrow \quad y = 2x - 4 \qquad \ldots(1)$$

This is the equation of a straight line

Now, $\dfrac{dx}{dt} = 2 + 8t$ and $\dfrac{dy}{dt} = 4 + 16t$

$$\Rightarrow \quad a_x = \frac{dv_x}{dt} = 8 \text{ and } a_y = \frac{dv_y}{dt} = 16$$

$$\Rightarrow \quad \vec{a} = a_x\hat{i} + a_y\hat{j}$$

$$\Rightarrow \quad \vec{a} = 8\hat{i} + 16\hat{j}$$

$$\Rightarrow \quad \vec{a} = \text{constant}$$

So, the motion is uniformly accelerated along a straight line.

Hence, (C) and (D) are correct.

22. The separation r, between the particle at time t is

$$r^2 = (4 - 2t)^2 + (2t)^2 \qquad \ldots(1)$$

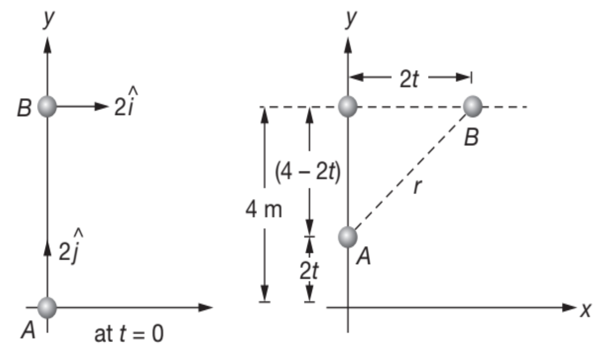
This separation will first decrease and then increase. This r is minimum, when

$$\frac{d}{dt}(r^2) = 0$$

$$\Rightarrow \quad 2(4 - 2t)(-2) + 2(2t)(2) = 0$$

$$\Rightarrow \quad -8 + 4t + 4t = 0$$

$$\Rightarrow \quad t = 1 \text{ s}$$



At this time, we have

$$r_{\min}^2 = (4 - 2)^2 + (2)^2$$

$$\Rightarrow \quad r_{\min}^2 = 8$$

$$\Rightarrow \quad r_{\min} = 2\sqrt{2} \text{ m}$$

Hence, (B), (C) and (D) are correct.

23. This problem can be answered if you think about the situation when the particle reverses its direction of motion, because in that case $a \neq 0$ but $v = 0$, but this is not an always condition.

So, a can be non-zero when $v = 0$. Another answer is (D), because this situation is just like when the v is constant, then $a = 0$.

Hence, (B) and (D) are correct.

24. At time T, the particle reverses its direction of motion, because $v = 0$ and also $a =$ constant. So, (A) and (B) are correct. Also from the graph we observe that the initial and the final speeds of the particle have same value. So, (C) happens to be correct too. Finally, displacement of the particle is actually the area under the v-t graph which is zero. So, (D) is also correct. You can also think about this graph for a body through up with an initial velocity under the influence of gravity. So it will return to the point of start, hence giving zero displacement.

Hence, (A), (B), (C) and (D) are correct.

26. METHOD I

Since $v = \sqrt{x}$

$\Rightarrow \quad v^2 = x$

Comparing with

$v^2 - 0^2 = 2ax$

we observe that $u = 0$ and $2a = 1$

$$\Rightarrow \quad u = 0 \text{ and } a = \frac{1}{2}\text{ ms}^{-2}$$

Please keep this thing in mind that whenever you observe $v^2 \propto x$ OR $v \propto \sqrt{x}$ OR $x \propto t^2$ OR $t \propto \sqrt{x}$ OR $v \propto t$, then **acceleration is constant** or the motion is **uniformly accelerated**.

METHOD II

$v = \sqrt{x}$

$$\Rightarrow \quad \frac{dv}{dt} = \left(\frac{1}{2}x^{\frac{1}{2}-1}\right)\frac{dx}{dt}$$

$$\Rightarrow \quad \frac{dv}{dt} = \left(\frac{1}{2\sqrt{x}}\right)v$$

But $v = \sqrt{x}$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{1}{2}$$

$$\Rightarrow \quad a = \frac{1}{2}\text{ ms}^{-2}$$

Hence, (A), (B) and (C) are correct.

28. Distance covered is $x|_{t\to\infty} - x|_{t=0} = x_0$

However, since the time of motion is infinite, so average speed is zero, because

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$$

$$\Rightarrow \quad \text{Average Speed} = \frac{x_0}{\infty} = 0$$

Hence, (B) and (C) are correct.

29. $\dfrac{dv}{dt} = -v^2 + 2v - 1$

Terminal velocity is a constant velocity and hence when terminal velocity is attained, then acceleration is zero.

$$\Rightarrow \quad \frac{dv}{dt} = 0$$

$$\Rightarrow \quad -v^2 + 2v - 1 = 0$$

$$\Rightarrow \quad (v-1)^2 = 0$$

$$\Rightarrow \quad v = 1\text{ ms}^{-1}$$

Also, $\dfrac{dv}{dt} = -v^2 + 2v - 1 = -(v-1)^2$

$$\Rightarrow \quad \int_0^v \frac{dv}{(v-1)^2} = -\int_0^t dt$$

$$\Rightarrow \quad -\left(\frac{1}{v-1}\right)\Bigg|_0^v = -t$$

$$\Rightarrow \quad \frac{1}{v-1} + 1 = t$$

$$\Rightarrow \quad \frac{1}{v-1} = t - 1$$

$$\Rightarrow \quad v = \frac{t}{t-1}$$

When acceleration is one fourth of its initial value, i.e. $a = -\dfrac{1}{4}$, then we have

$$-v^2 + 2v - 1 = -\frac{1}{4}$$

$$\Rightarrow \quad -4v^2 + 8v - 3 = 0$$

$$\Rightarrow \quad v = \frac{-(8) \pm \sqrt{64 - 4(-4)(-3)}}{2(-4)}$$

$$\Rightarrow \quad v = \frac{-8 \pm 4}{-8} = \frac{1}{2}\text{ ms}^{-1},\ \frac{3}{2}\text{ ms}^{-1}$$

Hence, (A), (B) and (D) are correct.

30. In x-t graph at maxima and minima. $\dfrac{dx}{dt} = 0$. Hence particle comes to rest for 6 times and average velocity for total period is negative.

Hence, (A) and (D) are correct.

CHAPTER 4

31. Since, $v\frac{dv}{dx} = -\alpha v$

$$\Rightarrow \int_{v_0}^{0} dv = -\alpha \int_{0}^{x_0} dx$$

$$\Rightarrow x_0 = \frac{v_0}{\alpha}$$

So, option (A) is correct.

Also, $\frac{dv}{dt} = -\alpha v$

$$\Rightarrow \int_{v_0}^{v} \frac{dv}{v} = -\alpha \int_{0}^{t} dt$$

$$\Rightarrow v = v_0 e^{-\alpha t}$$

$$\Rightarrow v = 0 \text{ for } t \to \infty$$

So, option (C) is also correct.
Hence, (A) and (C) are correct.

32. Equation of straight line is

$$v^2 = 15x + 25$$

Differentiating with respect to x, we get

$$2v\frac{dv}{dx} = 15$$

$$\Rightarrow v\frac{dv}{dx} = a = \frac{15}{2}\text{ ms}^{-2}$$

$$\Rightarrow a = 7.5\text{ ms}^{-2}$$

Since, $v = u + at = 5 + (7.5)(1) = 12.5\text{ ms}^{-1}$

$$\Rightarrow v = 12.5\text{ ms}^{-1}$$

Hence, (B), (C) and (D) are correct.

35. During accelerated motion

$$v = \alpha t_1 \text{ and } v^2 = 2\alpha x$$

During decelerated motion

$$0 = v - \beta t_2$$

$$\Rightarrow \alpha t_1 = \beta t_2$$

$$\Rightarrow \frac{t_1}{t_2} = \frac{\beta}{\alpha}$$

Also, $0^2 = v^2 - 2\beta y$

$$\Rightarrow v^2 = 2\beta y$$

$$\Rightarrow 2\alpha x = 2\beta y$$

$$\Rightarrow \frac{x}{y} = \frac{\beta}{\alpha}$$

Hence, (A), (B) and (C) are correct.

36. Acceleration is the slope of v-t graph, which is negative between 5 s and 6 s.
Between $t = 0$ and $t = 4$ s the particle changes its direction of motion at 1 s and 3 s.
Hence, (C) and (D) are correct.

38. $v_{max} = \left(\frac{a_1 a_2}{a_1 + a_2}\right)t = \left(\frac{2\times 4}{2+4}\right)(6) = 8\text{ ms}^{-1}$

and $s = \frac{1}{2}\left(\frac{a_1 a_2}{a_1 + a_2}\right)t^2 = \frac{1}{2}\left(\frac{2\times 4}{2+4}\right)(6)^2 = 24\text{ m}$

Hence, (A) and (C) are correct.

39. $v = 4t - t^2$

$$\Rightarrow \frac{ds}{dt} = 4t - t^2$$

$$\Rightarrow \int ds = \int_0^5 (4t - t^2)dt$$

$$\Rightarrow s = 2\times 5^2 - \frac{5^3}{3}$$

$$\Rightarrow s = \frac{150 - 125}{3} = \frac{25}{3}\text{ m}$$

Average velocity $= \frac{25}{3\times 5}\text{ ms}^{-1} = \frac{5}{3}\text{ ms}^{-1}$

$$a = \frac{dv}{dt} = (4 - 2t)\text{ ms}^{-2}$$

$$\Rightarrow a = 4\text{ ms}^{-2} \text{ at } t = 0$$

Hence, (C) and (D) are correct.

Reasoning Based Questions

1. Positive slope indicates that acceleration is increasing uniformly with time and is not uniform.
Hence, the correct answer is (D).

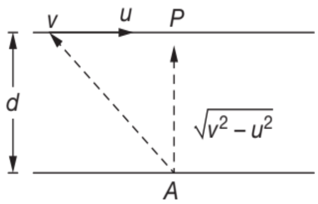
2. Acceleration is the rate of change of velocity i.e., $a = \frac{dv}{dt}$.
Hence, the correct answer is (D).

3. A body has no relative motion with respect to itself. Hence if a frame of reference of the body is fixed, then the bodies will be always at relatively rest in this frame of reference. So, (A) is correct
Hence, the correct answer is (A).

4. As $\vec{v} = \frac{d\vec{r}}{dt}$, hence statement-I is correct.
In most of the cases known instantaneous velocity and average velocity are not equal. They can be equal only when the particle moves with a constant uniform velocity.
Hence, the correct answer is (C).

5. To an observer on the balloon, the ground is a moving frame.
Hence, the correct answer is (D).

6. Time to cross the river $t = \dfrac{d}{\sqrt{v^2 - u^2}}$



Hence, the correct answer is (A).

8. Suppose motion of 1 is viewed from 2 then as 2 is stationary. If 1 is to meet it, then 1 must come straight towards 2 i.e., the relative velocity must be along the line joining 1 and 2.
Hence, the correct answer is (A).

9. $a_r = g - g = 0$

$$\Rightarrow \frac{dv_r}{dt} = 0$$

$$\Rightarrow v_r = \text{constant (say } k_1\text{)}$$

$$\Rightarrow \frac{dx_r}{dt} = k_1$$

$$\Rightarrow x_r = k_1 t$$

Hence, the correct answer is (A).

11. At the highest point of the graph velocity (slope of x-t curve) changes suddenly from a positive to a negative value. This is physically impossible.
Hence, the correct answer is (D).

12. Area under acceleration - time graph gives change in velocity and not average velocity.
Hence, the correct answer is (C).

Linked Comprehension Type Questions

1. **The correct answer is (B).**
2. **The correct answer is (D).**
3. **The correct answer is (C).**

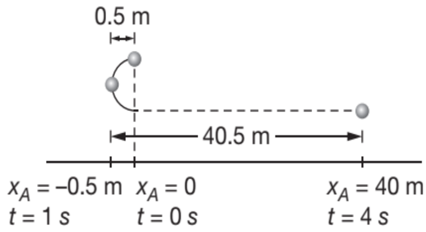
Combined solution to 1, 2, 3

For A

$$dv_A = a_A dt$$

$$\Rightarrow \int_0^{v_A} dv_A = \int_0^t (6t - 3)dt$$

$$\Rightarrow v_A = 3t^2 - 3t$$



For B

$$dv_B = a_B dt$$

$$\Rightarrow \int_0^{v_B} dv_B = \int_0^t (12t^2 - 8)dt$$

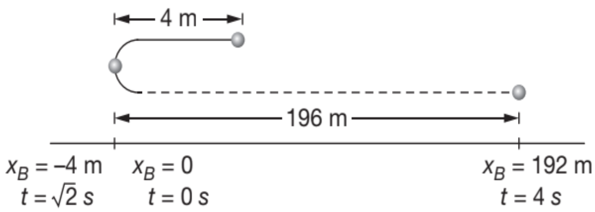
$$\Rightarrow v_B = 4t^3 - 8t$$

Let us now calculate the times when A and B are at rest.

The particle A is at rest $(v_A = 0)$, when

$$3t^2 - 3t = 0$$

$$\Rightarrow t = 0 \text{ s and } t = 1 \text{ s}$$



The particle B is at rest $(v_B = 0)$, when

$$4t^3 - 8t = 0$$

$$\Rightarrow t = 0 \text{ s and } t = \sqrt{2} \text{ s}$$

The position of particles A and B can be determined using

$$v = \frac{dx}{dt}, \text{ so}$$

$$dx_A = v_A dt$$

$$\Rightarrow \int_0^{x_A} dx_A = \int_0^t (3t^2 - 3t)dt$$

$$\Rightarrow x_A = t^3 - \frac{3}{2}t^2$$

Similarly

$$dx_B = v_B dt$$

$$\Rightarrow \int_0^{x_B} dx_B = \int_0^t (4t^3 - 8t)dt$$

$\Rightarrow \quad x_B = t^4 - 4t^2$

The positions of particle A at $t = 1$ s and 4 s are

$$x_A\big|_{t=1\text{ s}} = 1^3 - \frac{3}{2}(1^2) = -0.5 \text{ m}$$

$$x_A\big|_{t=4\text{ s}} = 4^3 - \frac{3}{2}(4^2) = 40 \text{ m}$$

Particle A has travelled a total distance given by

$$d_A = 2(0.5) + 40 = 41 \text{ m}$$

The positions of particle B at $t = \sqrt{2}$ s and 4 s are

$$x_B\big|_{t=\sqrt{2}} = (\sqrt{2})^4 - 4(\sqrt{2})^2 = -4 \text{ m}$$

$$x_B\big|_{t=4} = (4)^4 - 4(4)^2 = 192 \text{ m}$$

Particle B has travelled a total distance given by

$$d_B = 2(4) + 192 = 200 \text{ m}$$

At $t = 4$ s the distance between A and B is

$$\Delta x_{AB} = 192 - 40 = 152 \text{ m}$$

4. Taking, $+x$ direction as positive, we get

$u = 40 \text{ ms}^{-1}$ and $a = -10 \text{ ms}^{-2}$

So, this a acts as retardation and the velocity of the body is first reduced to zero, say at time t_0.
Since

$$v = u + at$$

$\Rightarrow \quad 0 = 40 + (-10)t_0$

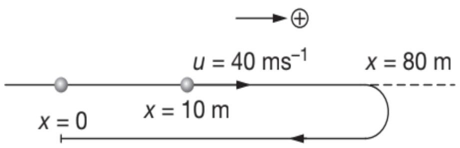
$\Rightarrow \quad t_0 = 4 \text{ s}$

Hence, the correct answer is (B).

5. $0^2 - u^2 = 2ax_0$

$\Rightarrow \quad -(40)^2 = 2(-10)x_0$

$\Rightarrow \quad x_0 = \dfrac{1600}{20} = 80 \text{ m}$



So, the maximum x-coordinate of the particle, along the $+x$ direction is

$$x_{MAX} = x_{Initial} + x_0$$

$\Rightarrow \quad x_{MAX} = 10 + 80 = 90 \text{ m}$

Hence, the correct answer is (C).

6. Further, at the origin, $s = -10$ m. So, using

$v^2 - u^2 = 2as$, we get

$$v^2 - (40)^2 = 2(-10)(-10)$$

$\Rightarrow \quad v^2 = 1800$

$\Rightarrow \quad v = 30\sqrt{2} \text{ ms}^{-1}$, along negative x direction

Hence, the correct answer is (B).

7. Also, using $v = u + at$, we get

$$t = \frac{v-u}{a} = \frac{-30\sqrt{2} - 40}{-10}$$

$\Rightarrow \quad t = (4 + 3\sqrt{2})$ s

Hence, the correct answer is (B).

8. $a = 2t - 4$

$\Rightarrow \quad \dfrac{dv}{dt} = 2t - 4$

$\Rightarrow \quad \displaystyle\int_0^v dv = 2\int_0^t t\,dt - 4\int_0^t dt$

$\Rightarrow \quad v = \left(2\dfrac{t^2}{2} - 4t\right)\Big|_0^t$

$\Rightarrow \quad v = t^2 - 4t \qquad \ldots(1)$

Now, $v = 0$

$\Rightarrow \quad t^2 - 4t = 0$

$\Rightarrow \quad t = 0$ (initially) OR $t = 4$ s

Hence, the correct answer is (D).

9. The velocity of the particle is maximum, when

$$\frac{dv}{dt} = 0$$

$\Rightarrow \quad a = 0$

$\Rightarrow \quad 2t_0 - 4 = 0$

$\Rightarrow \quad t_0 = 2$ s

and $v_{max} = v\big|_{t_0=2\text{ s}} = 2^2 - 4(2)$

$\Rightarrow \quad v_{max} = -4 \text{ ms}^{-1}$

Hence, the correct answer is (B).

10. Since we have calculated

$$v = t^2 - 4t$$

$\Rightarrow \quad v = t^2 - 4t + 4 + 4$

$\Rightarrow \quad v = (t-2)^2 + 4$

$\Rightarrow \quad v-4=(t-2)^2$

This happens to be the equation of the parabola with origin at the point (2, 4)

Hence, the correct answer is (D).

11. Initially, at $t=0$, the body falls from rest, so $v=0$ at $t=0$ and hence $\left.\frac{dv}{dt}\right|_{t=0}=6\ \text{ms}^{-2}$

Hence, the correct answer is (A).

12. Acceleration is zero (when terminal velocity is attained), so

$$0=6-3v_T$$

$$\Rightarrow \quad v_T=2\ \text{ms}^{-1}$$

Hence, the correct answer is (C).

13. $\frac{dv}{dt}=6-3\text{ v}$

$$\Rightarrow \quad \int_0^v \frac{dv}{6-3v}=\int_0^t dt$$

$$\Rightarrow \quad -\frac{1}{3}\log_e(6-3v)\Big|_0^v=t$$

$$\Rightarrow \quad \log_e\left(\frac{6-3v}{6}\right)=-3t$$

$$\Rightarrow \quad v=2\left(1-e^{-3t}\right)$$

Hence, the correct answer is (B).

14. $a=\frac{dv}{dt}=6e^{-3t}$

Hence, the correct answer is (C).

15. $v_x=a\omega\cos(\omega t)$

$$v_y=a\omega\sin(\omega t)$$

Since, $v^2=v_x^2+v_y^2$

$$\Rightarrow \quad v=a\omega$$

Hence, the correct answer is (A).

16. $x=a\sin(\omega t)=2a\sin\left(\frac{\omega t}{2}\right)\cos\left(\frac{\omega t}{2}\right)$

$$y=2a\sin^2\left(\frac{\omega t}{2}\right)$$

$$\Rightarrow \quad \sin\left(\frac{\omega t}{2}\right)=\sqrt{\frac{y}{2a}}$$

$$\Rightarrow \quad \cos\left(\frac{\omega t}{2}\right)=\sqrt{1-\frac{y}{2a}}$$

$$\Rightarrow \quad x=2a\sqrt{\frac{y}{2a}}\sqrt{1-\frac{y}{2a}}$$

$$\Rightarrow \quad x=\sqrt{2ay}\sqrt{1-\frac{y}{2a}}$$

Hence, the correct answer is (B).

17. $a_x=\frac{dv_x}{dt}=-a\omega^2\sin(\omega t)$

$$a_y=\frac{dv_y}{dt}=a\omega^2\cos(\omega t)$$

Since acceleration $=\sqrt{a_x{}^2+a_y{}^2}$

$$\Rightarrow \quad \text{Acceleration}=a\omega^2$$

Hence, the correct answer is (B).

18. Since displacement is zero, so average velocity is also zero.

Hence, the correct answer is (A).

19. From A to B, let time taken be t_1, then

$$t_1=\frac{d}{v}$$

From B to A, let time taken be t_2, then

$$t_2=\frac{d}{\left(v-\frac{v}{3}\right)}=\frac{3d}{2v}$$

So, total time taken is

$$T=t_1+t_2$$

$$\Rightarrow \quad T=\frac{d}{v}+\frac{3d}{2v}=\frac{5d}{2v}$$

Hence, the correct answer is (D).

20. Average speed $(v_{av})=\frac{\text{Total Distance}}{\text{Total Time}}$

$$\Rightarrow \quad v_{av}=\frac{2d}{T}$$

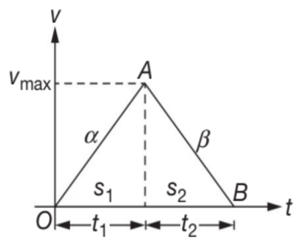
$$\Rightarrow \quad v_{av}=\frac{2d}{\left(\frac{5d}{2v}\right)}$$

$$\Rightarrow \quad v_{av}=\frac{4v}{5}$$

Hence, the correct answer is (D).

21. According to the problem, we have

$$s_1+s_2=\frac{1}{2}v_{\max}t$$



$\Rightarrow \quad 100 = \frac{1}{2}v_{max}(20)$

$\Rightarrow \quad v_{max} = 10 \text{ ms}^{-1}$

Hence, the correct answer is (B).

22. Now $v_{max} = \alpha t_1 = \beta t_2$

$\Rightarrow \quad 10 = 2t_1$

$\Rightarrow \quad t_1 = 5 \text{ ms}^{-1}$

Now $t_1 + t_2 = 20$ s

$\Rightarrow \quad t_2 = 15$ s

Hence, the correct answer is (C).

23. Also $v_{max} = \beta t_2$

$\Rightarrow \quad 10 = \beta(15)$

$\Rightarrow \quad \beta = \frac{2}{3} \text{ ms}^{-2}$

Hence, the correct answer is (B).

24. Average speed $= \frac{\text{Total Distance}}{\text{Total Time}}$

$\Rightarrow \quad v_{av} = \frac{100}{20} = 5 \text{ ms}^{-1}$

Hence, the correct answer is (A).

25. $s_1 = \frac{1}{2}v_{max}t_1$

$\Rightarrow \quad s_1 = \frac{1}{2}(10)(5) = 25$ m

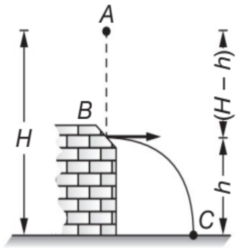
Hence, the correct answer is (A).

26. $s_2 = 100 - 25 = 75$ m

Hence, the correct answer is (B).

27. Let time taken by body to go from A to B be t_1 and that to go from B to C be t_2. Then

$$t_1 = \sqrt{\frac{2(H-h)}{g}} \text{ and } t_2 = \sqrt{\frac{2h}{g}}$$



So, total time taken is

$$t = t_1 + t_2 = \sqrt{\frac{2}{g}}\left[\sqrt{H-h} + \sqrt{h}\right]$$

For t to be maximum $\frac{dt}{dh} = 0$, we have

$$\frac{h}{H} = \frac{1}{2}$$

Hence, the correct answer is (C).

28. Substituting $h = \frac{H}{2}$ from PROBLEM 27, in expression for t we get value of t_{max}

Hence, the correct answer is (A).

29. The correct answer is (C).

30. The correct answer is (A).

Combined solution to 29, 30

Take downward direction as positive, we get

$$-h = -ut_1 + \frac{1}{2}gt_1^2 \quad \text{...(1)}$$

$$-h = -ut_2 + \frac{1}{2}gt_2^2 \quad \text{...(2)}$$

Subtracting (2) from (1), we get

$$0 = u(t_2 - t_1) + \frac{1}{2}g(t_1^2 - t_2^2)$$

$$\Rightarrow \quad u = \frac{1}{2}g(t_1 + t_2) \quad \text{...(3)}$$

{ANSWER TO PROBLEM 30}

Substituting value of u from (3) in (1), we get

$$h = \frac{gt_1t_2}{2}$$

31. $v^2 - u^2 = 2(g)(-h)$

At $h = h_{max}$

$v = 0$

$\Rightarrow \quad h_{max} = \frac{u^2}{2g}$

$\Rightarrow \quad h_{max} = \frac{1}{8}g(t_1 + t_2)^2$

Hence, the correct answer is (C).

32. $v^2 - u^2 = 2g\left(-\frac{h}{2}\right)$

$\Rightarrow \quad v^2 = \frac{1}{4}g^2(t_1+t_2)^2 - gh$

But $h = \frac{1}{2}gt_1t_2$ {From PROBLEM 29}

$\Rightarrow \quad v^2 = \frac{1}{4}g^2(t_1+t_2)^2 - \frac{1}{2}g^2t_1t_2$

$\Rightarrow \quad v^2 = \frac{1}{4}g^2(t_1^2+t_2^2+2t_1t_2-2t_1t_2)$

$\Rightarrow \quad v = \frac{1}{2}g\sqrt{t_1^2+t_2^2}$

Hence, the correct answer is (C).

33. Let v be the velocity of the particle at height $\left(\frac{H}{2}\right)$

$\Rightarrow \quad v^2 = u^2 + 2(-g)\frac{H}{2} = u^2 - gH$

$\Rightarrow \quad v^2 = \frac{g^2}{4}(t_1+t_2)^2 - g\left\{\frac{g}{8}(t_1+t_2)\right\}$

$\Rightarrow \quad v^2 = \frac{g^2}{8}(t_1+t_2)^2$

$\Rightarrow \quad v = \frac{g}{2\sqrt{2}}(t_1+t_2)$

Hence, the correct answer is (C).

34. Slope of line $= -\frac{2}{3}$

Equation of line is $(v-20) = -\frac{2}{3}(s-0)$

$\Rightarrow \quad v = 20 - \frac{2}{3}s$ …(1)

Velocity at s = 15 m i.e.

$$v = \left.\frac{ds}{dt}\right|_{s=15\,m} = 20 - \frac{2}{3}(15) = 10 \text{ ms}^{-1}$$

Differentiate (1) with respect to time

acceleration $= \frac{dv}{dt} = -\frac{2}{3}\frac{ds}{dt}$

$$\therefore \quad \left.\frac{dv}{dt}\right|_{s=15\,m} = -\frac{2}{3}\left.\frac{ds}{dt}\right|_{s=15\,m} = -\frac{20}{3}\text{ ms}^{-2}$$

Hence, the correct answer is (D).

35. From Solution to Problem 60, we have

$v = 20 - \frac{2}{3}s$ {See equation (1)}

$\Rightarrow \quad \frac{ds}{dt} = 20 - \frac{2}{3}s$

$$\frac{ds}{20-\frac{2}{3}s} = dt$$

$$\Rightarrow \quad \int_0^{30}\frac{ds}{\left(20-\frac{2}{3}s\right)} = \int_0^t dt$$

$$\Rightarrow \quad -\frac{1}{\left(\frac{2}{3}\right)}\log_e\left(20-\frac{2}{3}s\right)\Bigg|_0^{30} = t$$

$$\Rightarrow \quad -\frac{3}{2}[\log_e(0) - \log_e(20)] = t$$

$\Rightarrow \quad t \to \infty$ $\{\because \log_e 0 \to \infty\}$

Hence, the correct answer is (D).

36. Since, we know that time interval is

$$t = \frac{\text{Relative Displacement}}{\text{Relative Velocity}} = \frac{\Delta l_r}{v_r}$$

Relative Displacement is the distance travelled by each bus in 5 minutes, so

$$\Delta l_r = \frac{5}{60}\times 60 = 5 \text{ km}$$

Also, relative velocity of the buses is

$$v_r = 60 + 60 = 120 \text{ kmhr}^{-1}$$

$$\Rightarrow \quad t = \frac{5}{120}\text{ hr} = \frac{5}{120}\times 60 \text{ min} = 2.5 \text{ min}$$

Hence, the correct answer is (B).

37. Distance = speed × time $= 60\times\frac{2.5}{60} = 2.5$ km

Hence, the correct answer is (A).

38. Since each bus takes 30 min to reach another city, therefore total number of buses it meets during the journey is

$$N = \frac{30}{2.5} - 1 = 11$$

(Note that the 12th bus just starts the motion)
Hence, the correct answer is (D).

39. Average velocity is defined as

$$v_{av} = \frac{\text{Displacement}}{\text{Time Interval}} = \frac{\Delta x}{\Delta t} = \frac{x_2-x_1}{t_2-t_1}$$

$\Rightarrow \quad v_{av} = \dfrac{\left[12(8)-(8)^2\right]-0}{8-0} = 4\ \text{ms}^{-1}$

Hence, the correct answer is (A).

40. To calculate the average speed, we must first find the instant of reversal of motion, i.e. when $v = 0$. Since

$$v = \frac{dx}{dt} = 12 - 2t$$

$\Rightarrow \quad v = 0$

$\Rightarrow \quad t = 6\ \text{s}$

So distance travelled by the particle from $t = 0$ to $t = 8$ s is

$$l = |36-0| + |32-36| = 40\ \text{m}$$

So average speed is

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$$

$\Rightarrow \quad \text{Average Speed} = \dfrac{40}{8} = 5\ \text{ms}^{-1}$

Hence, the correct answer is (B).

41. Average acceleration is the rate of change of velocity. **Hence, option (C) is correct.**

42. $a = 2t - 4$

$\Rightarrow \quad \dfrac{dv}{dt} = 2t - 4$

$\Rightarrow \quad \displaystyle\int_0^v dv = \int_0^t (2t-4)\,dt$

$\Rightarrow \quad v = t^2 - 4t$

The particle comes to rest when $v = 0$

$\Rightarrow \quad t = 4\ \text{s}$

Hence, the correct answer is (B).

43. The speed of the particle is maximum when

$$\frac{dv}{dt} = 2t - 4 = 0$$

$\Rightarrow \quad t = 2\ \text{s}$

Hence, the correct answer is (C).

44. Since, we have

$\Rightarrow \quad \displaystyle\int_0^x dx = \int_0^4 \left(t^2 - 4t\right)dt$

$\Rightarrow \quad x = \dfrac{4^3}{3} - 2\times 4^2 = \dfrac{64}{3} - 32 = -\dfrac{32}{3}$

$$\text{Distance} = \frac{32}{3}\ \text{m}$$

Hence, the correct answer is (D).

48. $x = 6t^2 - t^3$

$\Rightarrow \quad \dfrac{dx}{dt} = v = 12t - 3t^2$

$\Rightarrow \quad \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2} = a = 12 - 6t$

$\Rightarrow \quad \dfrac{dv}{dt} = 0$ for v to be maximum

$\Rightarrow \quad 12 - 6t$

$\Rightarrow \quad t = 2\ \text{s}$

Hence, the correct answer is (B).

49. The position of the particle at different times are given here.

Position (x)	At Time (t)
$x_3 = 25$ m	$t = 3$ s
$x_4 = 32$ m	$t = 4$ s
$x_5 = 27$ m	$t = 5$ s

So, the distance travelled by the particle in the said interval i.e. from $t = 3$ s to $t = 5$ s is

$$\ell = |32-25| + |27-32| = 12\ \text{m}$$

Hence, the correct answer is (C).

50. The position of the particle at different times are given here

Position (x)	At Time (t)
$x_0 = 0$ m	$t = 0$ s
$x_4 = 32$ m	$t = 4$ s
$x_6 = 0$ m	$t = 6$ s

So, the distance travelled by the particle in the said interval i.e. from $t = 0$ s to $t = 6$ s is

$$l = |32-0| + |0-32| = 64\ \text{m}$$

Since,

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$$

Av. Speed $= \frac{64}{6} = \frac{32}{3}$ ms^{-1}

Hence, the correct answer is (B).

51. Since $a = -6t$

$\Rightarrow \frac{dv}{dt} = -6t$

$\Rightarrow \int_{27}^{v} dv = -\int_{0}^{t} 6t\,dt$

$\Rightarrow v - 27 = -3t^2$

$\Rightarrow v = (27 - 3t^2)$ ms^{-1}

$\Rightarrow \frac{dx}{dt} = 27 - 3t^2$

$\Rightarrow \int_{0}^{x} dx = \int_{0}^{t} (27 - 3t^2)\,dt$

$\Rightarrow x = 27t - t^3$

Now, at $x = 26$ m, let the time be $t = t_0$ (say). So, we have

$26 = 27t_0 - t_0^3$

$\Rightarrow t_0^3 - 27t_0 + 26 = 0$

$\Rightarrow (t_0 - 1)(t_0^2 + t_0 - 26) = 0$

$\Rightarrow t_0 = 1$ s

$\Rightarrow t_0 = \left(\frac{\sqrt{105} - 1}{2}\right)$ s

The earlier time gives the value of time when particle is at $x = 26$ m on forward journey and former time gives the value of time when particle is at $x = 26$ m on return journey.
However, in the second case, the particle would have covered more than 26 m.
So particle covers a distance of 26 m at $t = 1$ s
Hence, velocity of the particle at this time is

$v = [27 - 3(1)^2]$ ms^{-1} $= 24$ ms^{-1}

Hence, the correct answer is (C).

52. $\frac{dv}{dt} = -6t$

For maximum velocity, we have $\frac{dv}{dt} = 0$

$\Rightarrow t = 0$

Since, $\frac{d^2v}{dt^2} = -6 < 0$

So $t = 0$ corresponds to maxima and hence

$v_{max} = 27 - 3(0)^2 = 27$ ms^{-1}

Hence, the correct answer is (C).

53. $\frac{dx}{dt} = 27 - 3t^2$

For maximum displacement, we have

$\frac{dx}{dt} = 0$

$\Rightarrow 27 - 3t^2 = 0$

$\Rightarrow t = 3$ s

Since, $\frac{d^2x}{dt^2} = -6t$ and

$\left.\frac{d^2x}{dt^2}\right|_{t=3} = -18 < 0$

Hence $t = 3$ corresponds to maxima of position. So

$x_{max} = 27(3) - (3)^3 = 81 - 27 = 54$ m

Hence, the correct answer is (A).

Matrix Match/Column Match Type Questions

1. A → (s, u)
B → (s, t)
C → (s, u)
D → (s, t)
E → (q, r)
F → (s, u)
G → (p, s)

Between $t = 0$ and $t = 1$ s, motion is decelerated $(a \neq 0)$.
(A) → (s, u)
Between $t = 1$ s and $t = 2$ s, motion is accelerated $(a \neq 0)$.
So, (B) → (s, t)
Between $t = 2$ s and $t = 3$ s, motion is decelerated $(a \neq 0)$.
So, (C) → (s, u)
Between $t = 3$ s and $t = 4$ s, motion is accelerated $a \neq 0$.
So, (D) → (s, t)
Between $t = 4$ s and $t = 5$ s, motion is uniform i.e., $v \neq 0$ and $a = 0$.
So, (E) → (q, r)
Between $t = 5$ s and $t = 6$ s, motion is decelerated $(a \neq 0)$.
So, (F) → (s, u)
At $t = 1$ s and at $t = 3$ s the motion reverses its direction, so $v = 0$ and simultaneously $a \neq 0$.
So, (G) → (p, s)

2. A → (q)
B → (r)
C → (s)
D → (p)

$$x=-5t^2+20t+10$$

$$\Rightarrow \frac{dx}{dt}=-10t+20$$

$$\Rightarrow v=20-10t \quad \ldots(1)$$

Now $v=0$ at $t=2\text{ s}$, so the distance travelled by the particle from $t=0$ to $t=3\text{ s}$ is

$$x=\left|\int_0^2 vdt\right|+\left|\int_2^3 vdt\right|$$

Now $\int v\,dt=(20-10t)dt$

$$\Rightarrow \int v\,dt=20t-5t^2$$

$$\Rightarrow x=\left|\left(20t-5t^2\right)\Big|_0^2\right|+\left|\left(20t-5t^2\right)\Big|_2^3\right|$$

$$\Rightarrow x=|40-20|+|80-80-40+20|$$

$$\Rightarrow x=|20|+|-20|$$

$$\Rightarrow x=40\text{ m}$$

So, average speed is

$$v_{av}=\frac{x}{t}=\frac{40}{4}=10\text{ ms}^{-1}$$

Displacement of the particle from $t = 0$ to $t = 4$ s is

$$\Delta x=\int_0^4 v\,dt=\int_0^4 (20-10t)dt$$

$$\Rightarrow \Delta x=\left(20t-5t^2\right)\Big|_0^4$$

$$\Rightarrow \Delta x=80-80=0$$

So, average velocity is

$$v_{av}=\frac{\Delta x}{\Delta t}=\frac{0}{4}=0\text{ ms}^{-1}$$

From (1), $a=\dfrac{dv}{dt}=-10\text{ ms}^{-2}$

$$u=v|_{t=0}=20\text{ ms}^{-1}$$

Since, initially $u=\oplus$ and $a=\ominus$, so a happens to be retardation for the motion from $t=0$ to $t=2\text{ s}$ (till its velocity becomes zero momentarily i.e., the particle reverses its direction of motion).

Velocity at $t=4\text{ s}$ is

$$v|_{t=4\text{ s}}=20-10(4)=-20\text{ ms}^{-1}$$

Hence, speed at $t=4\text{ s}$ is 20 ms^{-1}

3. A → (q)
B → (r)
C → (p, s)
D → (t)

$$\frac{d\vec{r}}{dt}=\text{Velocity}$$

$$\frac{d|\vec{v}|}{dt}=\text{Tangential Acceleration}$$

$$\frac{d\vec{v}}{dt}=\frac{d^2\vec{r}}{dt^2}=\text{Acceleration}$$

$$\left|\frac{d\vec{r}}{dt}\right|=\text{Instantaneous speed}$$

4. A → (r, t)
B → (p)
C → (q)
D → (s)

$$x=(t-3)^2$$

$$\Rightarrow x=t^2-6t+9$$

$$\Rightarrow v=\frac{dx}{dt}=2t-6$$

At the point of reversal, $v=0$, so

$$2t-6=0$$

$$\Rightarrow t=3\text{ s}$$

t	0	1	2	3	4	5	6
x	9	4	1	0	1	4	9
v	–6	–4	–2	0	2	4	6
a	2	2	2	2	2	2	2
Nature of Motion	Decele-rating	Decele-rating	Decele-rating	Accele-rating	Accele-rating	Accele-rating	Accele-rating

Displacement of the particle from $t=0$ to $t=6\text{ s}$ is zero.
Distance travelled by the particle from $t=0$ to $t=6$ is $9+9=18\text{ m}$

Average speed of the particle is

$$v_{av} = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$$

$$\Rightarrow \quad v_{av} = \frac{18}{6} = 3 \text{ ms}^{-1}$$

Average Velocity of the particle is

$$|\vec{v}_{av}| = \frac{\text{Displacement}}{\text{Time}} = 0$$

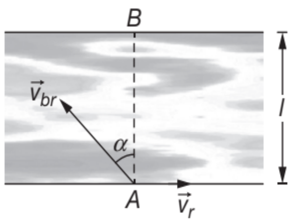
Acceleration of the particle over the entire duration of motion is 2 ms^{-2}.

5. A → (t)
B → (s)
C → (p)
D → (q)
E → (r)

Given, that $v_{br} = 4 \text{ kmhr}^{-1}$ and $v_r = 2 \text{ kmhr}^{-1}$

$$\Rightarrow \quad \alpha = \sin^{-1}\left(\frac{v_r}{v_{br}}\right) = \sin^{-1}\left(\frac{2}{4}\right) = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

Hence, to reach the point directly opposite to starting point he should head the boat at an angle of $30°$ with AB or $90° + 30° = 120°$ with the river flow.



Time taken by the boatman to cross the river for zero drift condition is

$$t_1 = \frac{l}{v_{br} \cos\alpha}$$

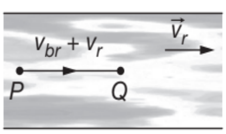
where $l = 4$ km, $v_{br} = 4 \text{ kmh}^{-1}$ and $\alpha = 30°$

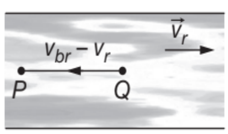
$$\Rightarrow \quad t_1 = \frac{4}{4\cos(30°)} = \frac{2}{\sqrt{3}} \text{ hr}$$

$$\Rightarrow \quad t_1 = \frac{120}{\sqrt{3}} = 40\sqrt{3} \text{ minute}$$

For shortest time $\theta = 0°$ i.e., the man must row the boat perpendicular to the river flow. So $\beta = 90°$

$$\text{and } t_{min} = t_2 = \frac{l}{v_{br}\cos(0°)} = \frac{4}{4} = 1 \text{ hr} = 60 \text{ minute}$$





Hence, he should head his boat perpendicular to the river current for crossing the river in shortest time and this shortest time t_2 of 60 minute.

$$t = t_{P\to Q} + t_{Q\to P}$$

$$\Rightarrow \quad t = \frac{PQ}{v_{br} + v_r} + \frac{QP}{v_{br} + v_r}$$

$$\Rightarrow \quad t = \frac{2}{4+2} + \frac{2}{4-2} = \frac{4}{3} \text{ hr}$$

$$\Rightarrow \quad t = 80 \text{ minute}$$

9. A → (r)
B → (p)
C → (r)
D → (s)

$$v_i = +10 \text{ ms}^{-1} \text{ and } v_f = 0$$

$$\Rightarrow \quad \Delta v = v_f - v_f = -10 \text{ ms}^{-1}$$

$$\Rightarrow \quad a_{av} = \frac{\Delta v}{\Delta t} = \frac{-10}{6} = \frac{-5}{3} \text{ ms}^{-2}$$

Total displacement = area under v-t graph (with sign) and acceleration = slope of v-t graph.

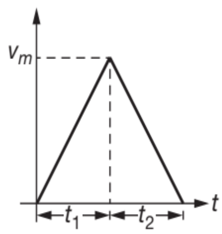
10. A → (r)
B → (p)
C → (s)
D → (q)

After 2 s velocity of balloon and hence the velocity of the particle will be $20 \text{ ms}^{-1} (= \text{at})$ and its height from the ground will be 20 m $\left(= \frac{1}{2}at^2\right)$. Now g, will start acting on the particle.

11. A → (p)
B → (q)
C → (s)
D → (r)

$$\frac{v_m}{t_1} = \alpha \text{ and } \frac{v_m}{t_2} = \beta$$

Also $x + y = \frac{1}{2} \times v_m \times (t_1 + t_2)$



$$\Rightarrow \quad (x+y)=\frac{1}{2}\times v_m\times\left(\frac{v_m}{\alpha}+\frac{v_m}{\beta}\right)$$

$$\Rightarrow \quad v_m=\sqrt{\frac{2(x+y)\alpha\beta}{(\alpha+\beta)}}$$

Now, $v_{av}=\dfrac{x+y}{t_1+t_2}=\dfrac{x+y}{\dfrac{v_m}{\alpha}+\dfrac{v_m}{\beta}}=\dfrac{\alpha\beta}{\alpha+\beta}\cdot\dfrac{x+y}{v_m}$

12. A → (q)
B → (p, q)
C → (p, r)
D → (r, s)

$$v=\frac{dx}{dt}$$

$$\Rightarrow \quad a=\frac{dv}{dt}$$

$$\Rightarrow \quad v=\int_0^t a\,dt$$

$$\Rightarrow \quad x=\int_0^t v\,dt$$

14. A → (r)
B → (q)
C → (s)
D → (p)

$A\to v=Kt$

$$\Rightarrow \quad a=\frac{dv}{dt}=K$$

$B\to v=Kt^2$

$$\Rightarrow \quad a=\frac{dv}{dt}=2Kt$$

$C\to K_1t\to$ First part of graph

$$\Rightarrow \quad a=\frac{dv}{dt}=K_1$$

$\Rightarrow \quad v=K_2\to$ Second part of graph

$$\Rightarrow \quad a=\frac{dv}{dt}=0$$

$D\to v=v_0e^{-Kt}$

$$\Rightarrow \quad a=\frac{dv}{dt}=(-v_0K)e^{-Kt}$$

$$\Rightarrow \quad a=-a_0e^{-Kt}$$

$$\Rightarrow \quad a_0=v_0K$$

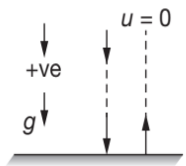
15. A → (r)
B → (s)
C → (p)
D → (q)

In motion M: slope of s-t graph is positive and increasing. Therefore, velocity of the particle is positive and increasing. Hence, it is A type motion. Similarly, N, P and Q can be observed from the slope.

20. A → (s)
B → (r)
C → (q)
D → (p)

Acceleration is constant and is equal to acceleration due to gravity, which is acting vertically downwards i.e. in the positive direction, so (p) represents the a-t graph for the motion.

Since $v=u+at$ and $s=\Delta y=ut+\frac{1}{2}gt^2$



Before collision, $v=gt$ $\{\because\ u=0\}$

So, before collision velocity is increasing linearly with time and is increasing. However, after collision velocity decreases with time linearly and is negative as in (r).

Displacement before collision is $y=\frac{1}{2}gt^2$ and positive, whereas after collision, the displacement is $y=-v_0t+\frac{1}{2}gt^2$ and is positive (a parabola opening upwards). Here is the tricky part, as we have taken the origin at the point from where the ball is dropped and in both the cases of upward and downward motion, the displacement is downwards and hence displacement is positive in both the cases.

The distance-time graph is positive and always increasing.

22. A → (p, q)
B → (p, q)
C → (r)
D → (p, q)

With constant positive acceleration, speed increases, when velocity is positive and speed decreases, when velocity is negative.

Similarly, with constant negative acceleration speed increases, when velocity is negative and speed decreases, when velocity is positive.

23. A → (q)
B → (r)
C → (s)
D → (p)
Solve using relative velocity diagram.

25. A → (q)
B → (r)
C → (r)
D → (p, r)
Particle will change the direction of motion

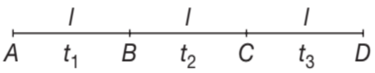
when $\frac{dx}{dt} = 0$

$\Rightarrow \quad -20 + 10t = 0$

$\Rightarrow \quad t = 2\text{ s}$

Integer/Numerical Answer Type Questions

1. Let velocities at A, B, C and D be v_A, v_B, v_C and v_D respectively.



For AB

$$l = \left(\frac{v_A + v_B}{2}\right)t_1 \qquad \ldots(1)$$

For BC

$$l = \left(\frac{v_B + v_C}{2}\right)t_2 \qquad \ldots(2)$$

For CD

$$l = \left(\frac{v_C + v_D}{2}\right)t_3 \qquad \ldots(3)$$

From (1), (2) and (3), we have

$$\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{1}{2l}(v_A + v_B - v_B - v_C + v_C + v_D)$$

$$\Rightarrow \quad \frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{1}{2l}(v_A + v_D) \qquad \ldots(4)$$

For AD

$$3l = \left(\frac{v_A + v_D}{2}\right)(t_1 + t_2 + t_3)$$

$$\Rightarrow \quad \frac{1}{2l}(v_A + v_D) = \frac{3}{t_1 + t_2 + t_3} \qquad \ldots(5)$$

So, from (4) and (5), we get

$$\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{3}{t_1 + t_2 + t_3}$$

$\Rightarrow \quad k = 3$

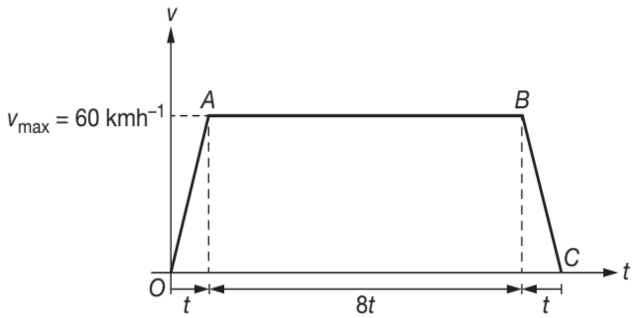
2. Since time to accelerate from zero to 60 kmh^{-1} is equal to the time taken to decelerate from 60 kmh^{-1} to zero. Hence, we can say both OA and BC are identical intervals. So, total distance travelled from O to A to B to C is

$$l = l_1 + l_2 + l_1 = \frac{1}{2}v_{\max}t + (v_{\max})8t + \frac{1}{2}v_{\max}t$$

Since,

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$$

$$\Rightarrow \quad v_{av} = \frac{l_1 + l_2 + l_1}{t + 8t + t}$$



$$\Rightarrow \quad v_{av} = \frac{\frac{1}{2}v_{\max}t + (v_{\max})8t + \frac{1}{2}v_{\max}t}{10t}$$

$$\Rightarrow \quad v_{av} = \frac{9}{10}v_{\max} = \frac{9}{10}(60\text{ kmh}^{-1}) = 54\text{ kmh}^{-1}$$

$$\Rightarrow \quad v_{av} = 54 \times \frac{5}{18}\text{ ms}^{-1} = 15\text{ ms}^{-1}$$

3. $u = 27\text{ ms}^{-1}$ at $t_0 = 0\text{ s}$. Since

$dv = adt$

$$\Rightarrow \quad \int_{27}^{v} dv = \int_{0}^{t} -6t\,dt$$

$$\Rightarrow \quad v = (27 - 3t^2)\text{ ms}^{-1} \qquad \ldots(1)$$

At $v = 0$, from equation …(1)

$$0 = 27 - 3t^2$$

$\Rightarrow \quad t = 3\text{ s}$

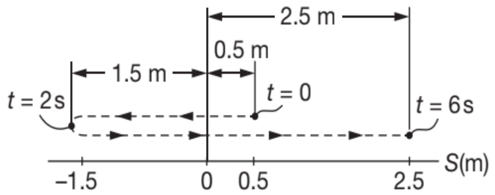
So, it will stop at $t = 3\text{ s}$

Since $dx = vdt$

$$\Rightarrow \quad \int_{0}^{x} dx = \int_{0}^{3} (27 - 3t^2)\,dt$$

$$\Rightarrow \quad x = (27t - t^3)\Big|_0^3 = 27(3) - (3)^3 = 54\text{ m}$$

4. $x_{total} = (0.5 + 1.5 + 1.5 + 2.5) = 6\text{ m}$

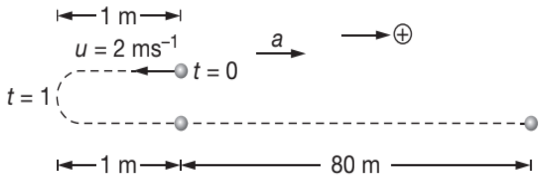


$t = (2 + 4) = 6\text{ s}$

$(v_{sp})_{avg} = \frac{x_{total}}{t} = \frac{6}{6} = 1\text{ ms}^{-1}$

5. (a) $80 = \left(\frac{u+18}{2}\right)10$

$\Rightarrow \quad 16 = u + 18$



$\Rightarrow \quad u = -2\text{ ms}^{-1}$

(b) $18 = -2 + a(10)$

$\Rightarrow \quad a = 2\text{ ms}^{-2}$

(c) $v^2 - u^2 = 2as$

$\Rightarrow \quad 0^2 - (2)^2 = 2(-2)s$

$\Rightarrow \quad s = 1\text{ m}$

$0 = -2 + (2)t$

$\Rightarrow \quad t = 1\text{ s}$

(d) $s_{total} = 80 + |-1| + |1| = 82\text{ m}$

6. For normal driver, the car moves a distance of $x_0 = vt = 44(0.75) = 33\text{ m}$ before he or she reacts and decelerates the car. The stopping distance can be obtained using

$v^2 - u^2 = 2a(x - x_0)$,

where $x = ?$, $x_0 = 33\text{ m}$, $a = -2\text{ ms}^{-2}$, $a = 44\text{ ms}^{-1}$

$\Rightarrow \quad 0^2 - 44^2 = 2(-2)(x - 33)$

$\Rightarrow \quad x = 517\text{ m}$

For a drunk driver, the car moves a distance of $x_0' = vt = 44(3) = 132\text{ m}$ before he or she reacts and decelerates the car. The stopping distance can be obtained again by using

$v^2 - u^2 = 2a(x' - x_0')$

$\Rightarrow \quad 0^2 - 44^2 = 2(-2)(x' - 132)$

$x' = 616\text{ m}$

7. (a) Let the bus overtake the car after a distance x. When the two meet, the time taken (say t) is the same. So, for bus,

$x = 0 + \frac{1}{2}5t^2 \qquad \left\{\because s = ut + \frac{1}{2}at^2\right\}$

For car, $x = 50t$

$\Rightarrow \quad \frac{5}{2}t^2 = 50t$

$\Rightarrow \quad t = 20\text{ s}$

$\Rightarrow \quad x = (50)(20) = 1000\text{ m}$

(b) Let the bus be moving with a velocity v.

So, $v^2 - 0 = 2(5)(1000) \qquad \{\because v^2 - u^2 = 2as\}$

$\Rightarrow \quad v = 100\text{ ms}^{-1}$

8. Velocity of truck is $V = 72\text{ kmh}^{-1} = 20\text{ ms}^{-1}$

Time to overtake the truck, 0.6 km down the road is

$t = \frac{d}{V} = \frac{0.6 \times 10^3}{20} = 30\text{ s}$

If u be the initial velocity of the car, then

$600 = u(30) + \frac{1}{2}(1)(30)^2$

Solving it we have $u = 5\text{ ms}^{-1}$

Velocity of the car at the instant of overtaking is

$v = 5 + (1)(30) = 35\text{ ms}^{-1} \qquad \{\because v = u + at\}$

The relative velocity of car with respect to truck is

$v_r = 35 - 20 = 15\text{ ms}^{-1}$

The distance between them at $t = 50\text{ s}$, i.e., 20 s after overtaking is

$s = (15)(20) = 300\text{ m}$

9. Since $s = \left(\frac{u+v}{2}\right)t$, so for decelerated interval,

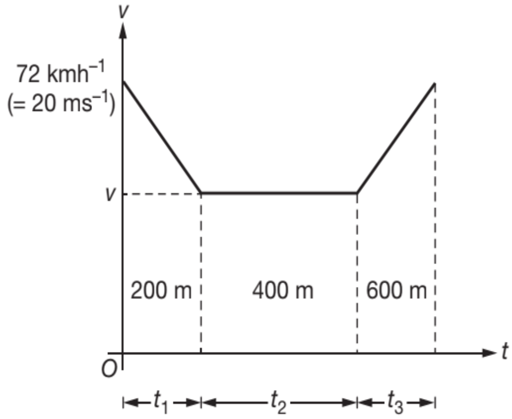
$200 = \left(\frac{20+v}{2}\right)t_1 \qquad \ldots(1)$

For uniform interval,

$400 = vt_2 \qquad \ldots(2)$

and again for accelerated interval, we have

$600 = \left(\frac{20+v}{2}\right)t_3 \qquad \ldots(3)$



Given $t_2 = t_1 + t_3$

$$\Rightarrow \quad \frac{400}{v} = \frac{400}{20+v} + \frac{1200}{20+v}$$

$$\Rightarrow \quad \frac{400}{v} = \frac{1600}{20+v}$$

$$\Rightarrow \quad 20 + v = 4v$$

$$\Rightarrow \quad v = \frac{20}{3}\ \text{ms}^{-1}$$

$$\Rightarrow \quad t_1 = \frac{400}{20+\frac{20}{3}} = \frac{1200}{80}\ \text{s} = 15\ \text{s}$$

$$\Rightarrow \quad t_2 = \frac{400}{v} = \frac{400}{\frac{20}{3}} = \frac{1200}{20} = 60\ \text{s}$$

$$\Rightarrow \quad t_3 = \frac{1200}{20+v} = \frac{1200}{20+\frac{20}{3}} = \frac{3600}{80}\ \text{s} = 45\ \text{s}$$

$$\Rightarrow \quad t = (15+60+45)\ \text{s} = 120\ \text{s}$$

$$\Rightarrow \quad t = 2\ \text{minute}$$

10. Let the truck overtake the car at time $t = 0$ and the car overtake the truck at time t.

For truck, $s = 10t$, and for car, $s = \frac{1}{2}2t^2$

$$10t = \frac{1}{2}(2)t^2$$

$$\Rightarrow \quad t = 10\ \text{s}$$

$$\Rightarrow \quad s = \frac{1}{2}(2)(10)^2 = 100\ \text{m}$$

and $v = (2)(10) = 20\ \text{ms}^{-1}$

11. Let u be the initial velocity of the bullet.

According to the problem $v = \frac{u}{2}$, when $s = 3$ cm

$$\Rightarrow \quad \left(\frac{u}{2}\right)^2 - u^2 = 2a(3)$$

$$\Rightarrow \quad a = \frac{u^2}{8} \qquad \ldots(1)$$

Let the bullet travels a distance x cm before coming to rest so,

$$0 - u^2 = 2ax$$

$$\Rightarrow \quad u^2 = 2\left(\frac{u^2}{8}\right)x$$

$$\Rightarrow \quad x = 4\ \text{cm}$$

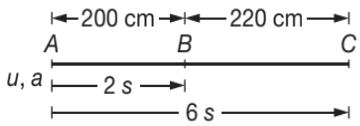
So the bullet will move $4 - 3 = 1$ cm further before coming to rest

12. Let the particle start with initial velocity u and a uniform acceleration a from point A.

For AB

$$200 = u(2) + \frac{1}{2}a(2)^2$$

$$\Rightarrow \quad u + a = 100 \qquad \ldots(1)$$



For AC

$$(200+220) = u(2+4) + \frac{1}{2}a(2+4)^2$$

$$\Rightarrow \quad 420 = 6u + 18a$$

$$\Rightarrow \quad u + 3a = 70 \qquad \ldots(2)$$

Solving (1) and (2), we get

$$a = -15\ \text{cms}^{-2} \text{ and } u = 115\ \text{cms}^{-1}$$

If v be the velocity at the end of seventh second from the start, then using

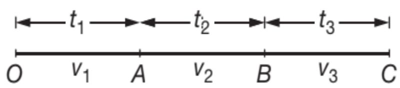
$v = u + at$, we get

$$\Rightarrow \quad v = 115 + (-15)(7)$$

$$\Rightarrow \quad v = 10\ \text{cms}^{-1}$$

13. **For OB**

$$a = \frac{v_2 - v_1}{(t_2+t_1) - 0} = \frac{v_2 - v_1}{t_2 + t_1} \qquad \ldots(1)$$



For AC

$$a = \frac{v_3 - v_2}{(t_3+t_2+t_1) - t_1} = \frac{v_3 - v_2}{t_3 + t_2} \qquad \ldots(2)$$

From (1) and (2), equating the values of a, we get

$$\left(\frac{v_1-v_2}{v_2-v_3}\right)\left(\frac{t_3+t_2}{t_2+t_1}\right)=1$$

14. For $0 \le x < 1000$ m, the initial condition is $v=0$ at $x=0$

Since $vdv = adx$

$$\Rightarrow \int_0^v vdv = \int_0^x 6dx \qquad \{\because a = 6 \text{ ms}^{-2} \text{ from graph}\}$$

$$\Rightarrow \frac{v^2}{2} = 6x$$

$$\Rightarrow v = 2\sqrt{3x} \qquad \ldots(1)$$

When $x = 1000$ m, we get

$$v = 2\sqrt{3000} = 20\sqrt{30} \text{ ms}^{-1}$$

For $1000 \text{ m} < x \le x_0$, the initial condition is

$v = 20\sqrt{30}$ ms^{-1} at $x = 1000$ m

Since $vdv = adx$

$$\Rightarrow \int_{20\sqrt{30}}^{v} vdv = \int_{1000}^{x} -4dx$$

$$\Rightarrow \frac{v^2}{2}\Bigg|_{20\sqrt{30}}^{v} = -4x\Bigg|_{1000}^{x}$$

$$\Rightarrow \frac{v^2}{2} - \frac{12000}{2} = -4(x-1000)$$

$$\Rightarrow v^2 - 12000 = -8x + 8000$$

$$\Rightarrow v^2 = 20000 - 8x$$

$$\Rightarrow v = \left(\sqrt{20000-8x}\right) \text{ ms}^{-1}$$

When $v = 0$, at $x = x_0$, then

$$\Rightarrow 0 = \sqrt{20000 - 8x_0}$$

$$\Rightarrow x_0 = 2500 \text{ m}$$

15. The velocity v of the balloon when it has risen to a height H is given by

$$v^2 = 0 + 2\left(\frac{g}{8}\right)H$$

taking upward direction positive

$$\Rightarrow v = \sqrt{\frac{gH}{4}} = \frac{\sqrt{gH}}{2} \text{ msec}^{-1}$$

This will also be the velocity of the stone in upward direction when it is dropped.

Taking upward direction positive for the stone,

$$-H = \left(\frac{\sqrt{gH}}{2}\right)t - \frac{1}{2}gt^2 \qquad \{\because g = -g \text{ and } h = -H\}$$

$$\Rightarrow gt^2 - \sqrt{gH}\,t - 2H = 0$$

$$\Rightarrow t = \frac{\sqrt{gH} \pm \sqrt{gH + 8gH}}{2g}$$

$$\Rightarrow t = \frac{\sqrt{gH} \pm 3\sqrt{gH}}{2g}$$

$$\Rightarrow t = 2\sqrt{\frac{H}{g}} \qquad \{\text{taking positive value}\}$$

$$\Rightarrow x = 2$$

16. For the first kilometre of the journey,

$$v^2 - u^2 = 2ax$$

where $v = 10$ ms^{-1}, $u = 2$ ms^{-1}, $a+?$, $s = 1000$ m

$$\Rightarrow 100 - 4 = 2a(1000)$$

$$\Rightarrow a = 0.048 \text{ ms}^{-2}$$

For the second kilometre, again applying

$$v^2 - u^2 = 2as$$

where $v = ?$, $u = 10$ ms^{-1}, $a = 0.048$ ms^{-2} and $s = 1000$ m

$$\Rightarrow v = 14 \text{ ms}^{-1}$$

For the whole journey, $v_0 = 2$ ms^{-1}, $v = 14$ ms^{-1} and 0.048 ms^{-2}. So,

$$v = k + at$$

$$\Rightarrow 14 = 2 + 0.048t$$

$$\Rightarrow t = 250 \text{ s}$$

17. $t = \dfrac{AC}{v_1} + \dfrac{CB}{v_2}$

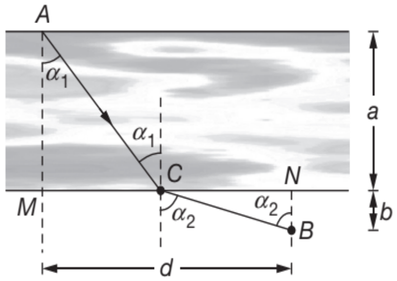
where $AC = a\sec\alpha_1$ and $CB = b\sec\alpha_2$

$$\Rightarrow t = \frac{a\sec\alpha_1}{v_1} + \frac{b\sec\alpha_2}{v_2}$$

For t_1 to be MINIMUM, we have

$$\frac{dt}{d\alpha_1} = 0$$

$$\Rightarrow \frac{a}{v_1}\sec\alpha_1\tan\alpha_1 + \frac{b}{v_2}\sec\alpha_2\tan\alpha_2\left(\frac{d\alpha_2}{d\alpha_1}\right) = 0 \qquad \ldots(1)$$



Please note that the question itself asks us to adjust the values of α_1 and α_2. So both α_1 and α_2 are variables.

Now, since $MC + CN = d =$ constant

$\Rightarrow \quad a\tan\alpha_1 + b\tan\alpha_2 = \text{constant} \qquad \ldots(2)$

Taking derivative of (2) w.r.t. α_1 on both sides, we get

$$a\sec^2\alpha_1 + b\sec^2\alpha\left(\frac{d\alpha_2}{d\alpha_1}\right) = 0$$

$$\Rightarrow \quad \frac{d\alpha_2}{d\alpha_1} = -\frac{a\sec^2\alpha_1}{b\sec^2\alpha_2} \qquad \ldots(3)$$

Substituting (3) in (1), we get

$$\frac{\cancel{a}}{v_1}\cancel{\sec\alpha_1}\tan\alpha_1 + \frac{\cancel{b}}{v_2}\cancel{\sec\alpha_2}\tan\alpha_2\left(-\frac{\cancel{a}\sec^{\cancel{2}}\alpha_1}{\cancel{b}\sec^{\cancel{2}}\alpha_2}\right) = 0$$

$$\frac{\tan\alpha_1}{v_1} = \frac{\tan\alpha_2\sec\alpha_1}{v_2\sec\alpha_2}$$

$$\frac{1}{v_1} = \frac{\sin\alpha_1}{\cancel{\cos\alpha_1}} = \frac{1}{v_2}\left(\frac{\sin\alpha_2}{\cancel{\cos\alpha_2}}\right)\left(-\frac{\cancel{\cos\alpha_2}}{\cancel{\cos\alpha_1}}\right)$$

$$\Rightarrow \quad \frac{\sin\alpha_1}{\sin\alpha_2} = \frac{v_1}{v_2}$$

Since, $v_1 = 3\sqrt{3}\ \text{ms}^{-1}$, $\alpha_1 = 30°$ and $\alpha_2 = 60°$

$$\Rightarrow \quad \frac{\sin(30°)}{\sin(60°)} = \frac{3\sqrt{3}}{v_2}$$

$$\Rightarrow \quad v_2 = 9\ \text{ms}^{-1}$$

18. v_m = absolute velocity of man,

v_{me} = velocity of man w.r.t. escalator,

v_e = velocity of escalator

If the length of the elevator by ℓ, then from the given condition,

$$v_m = \frac{l}{90}\ \text{ms}^{-1}$$

$$v_e = \frac{l}{60}\ \text{ms}^{-1}$$

Time taken by the person to walk up in the moving escalator is,

$$t = \frac{l}{v_e + v_m} = \frac{l}{\frac{l}{90} + \frac{l}{60}} = \frac{90\times 60}{90 + 60} = 36\ \text{s}$$

Here we observe that the time t does not depend on ℓ the length of escalator.

19. Initial relative velocity of approach of A and B is

$$u_r = (144 - 108)\ \text{kmh}^{-1} = 36\ \text{kmh}^{-1} = 10\ \text{ms}^{-1}$$

The driver of train A applies brakes, so if a is the retardation, then retardation of A w.r.t. B is $a_r = -a$. With this retardation he just manages to avoid the collision, so

$$v_r^2 - u_r^2 = 2a_r l_r$$

$$\Rightarrow \quad O^2 - (10)^2 = 2(-a)(1000)$$

$$\Rightarrow \quad a = \frac{100}{2000} = \frac{1}{20}\ \text{ms}^{-2} = 5\ \text{cms}^{-2}$$

If this retardation is produced for a duration on t (say), then

$$v_r = u_r + a_r t$$

$$\Rightarrow \quad 0 = (10) - \left(\frac{1}{20}\right)t$$

$$\Rightarrow \quad t = 200\ \text{s}$$

20. For convenience, let us take $\hat{i}$ along east, $\hat{j}$ along north and $\hat{k}$ vertically downwards.

Now given, $\vec{v}_b = 8\hat{i}$, $\vec{v}_{sb} = 12\hat{j} + 2\hat{k}$

So, absolute velocity of submarine is

$$\vec{v}_s = \vec{v}_{sb} + \vec{v}_b = 8\hat{i} + 12\hat{j} + 2\hat{k} \qquad \{\because \vec{v}_{sb} = \vec{v}_s - \vec{v}_b\}$$

Further $\vec{v}_{hs} = -4\hat{i} - 5\hat{k}$

$$\Rightarrow \quad \vec{v}_h = \vec{v}_{hs} + \vec{v}_s = 4\hat{i} + 12\hat{j} - 3\hat{k} \qquad \{\because \vec{v}_{hs} = \vec{v}_h - \vec{v}_s\}$$

$$\Rightarrow \quad |\vec{v}_h| = \sqrt{(4)^2 + (12)^2 + (3)^2} = 13\ \text{ms}^{-1}$$

Since, $\vec{v}_{hb} = \vec{v}_h - \vec{v}_b = -4\hat{i} + 12\hat{j} - 3\hat{k}$

$$\Rightarrow \quad |\vec{v}_{hb}| = \sqrt{(-4)^2 + (12)^2 + (-3)^2} = 13\ \text{ms}^{-1}$$

21. (a) Distance travelled by the particle between $t = 0$ to 10 s is given by

$$s = \text{Area of } \Delta OAB = \frac{1}{2}\times 10\times 12 = 60\ \text{m}$$

$$\text{Average speed} = \frac{\text{total distance covered}}{\text{total time taken}}$$

$$\Rightarrow \quad \text{Average speed} = \frac{60}{10} = 6\ \text{ms}^{-1}$$

(b) Acceleration of the particle during journey OA is given by

$v = u + at$ where $v = 12\ ms^{-1}$, $u = 0$ and $t = 5$ s

$\Rightarrow \quad a = +2.4\ ms^{-2}$

Similarly, acceleration of the particle during journey AB is given by

$v = u + at$ where $v = 0$, $u = 12\ ms^{-1}$ and $t = 5$ s

$\Rightarrow \quad a = -2.4\ ms^{-2}$

Velocity of the particle after 2 s from the start is given by

$$v = u + at = 0 + 2.4 \times 2 = 4.8\ ms^{-1}$$

So, distance covered by the particle between $t = 2$ to 5 s (in 3 s) is given by

$$s_1 = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad s_1 = (4.8)(3) + \frac{1}{2}(2.4)(3)^2 = 25.2\text{ m}$$

Distance covered by the particle from $t = 5$ s to 6 s (in 1 s) is given by

$$s_2 = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad s_2 = (12)(1) + \frac{1}{2}(-2.4)(1)^2 = 10.8\text{ m}$$

Total distance travelled from $t = 2$ s to 6 s is

$$s = s_1 + s_2 = 25.2 + 10.8 = 36\text{ m}$$

Average speed in the interval $t = 2$ s to 6 s

$$v_{av} = \frac{\text{total distance covered}}{\text{total time taken}} = \frac{36}{4} = 9\ ms^{-1}$$

22. $\dfrac{dx}{dt} = v_0 - \dfrac{v_0 t}{5}$

$$\Rightarrow \quad x = v_0 t - \frac{v_0 t^2}{10}$$

where x can be either $+10$ cm or -10 cm

For $x = +10$ cm, we have $10t - t^2 = 10$

$$\Rightarrow \quad t = \left(\frac{10 \pm \sqrt{60}}{2}\right)\text{ s}$$

Hence we have

$$t_1 = \left(\frac{10 - \sqrt{60}}{2}\right)\text{ s} \qquad \text{\{First Instant\}}$$

$$\Rightarrow \quad t_2 = \left(\frac{10 + \sqrt{60}}{2}\right)\text{ s} \qquad \text{\{Second Instant\}}$$

For $x = -10$ cm, we have $10t - t^2 = -10$

$$\Rightarrow \quad t = \left(\frac{10 \pm \sqrt{140}}{2}\right)\text{ s}$$

$$\Rightarrow \quad t_3 = \left(\frac{10 + \sqrt{140}}{2}\right)\text{ s} \qquad \text{\{Third Instant\}}$$

However, we will not get fourth instant as time cannot be negative.

Time interval between the second and the third instant is $\Delta t = t_3 - t_2 \approx 2$ s

23. Initial velocity is $v_i = 0$ and final velocity of the athlete is $v_f = 36 \times \dfrac{5}{18} = 10\ ms^{-1}$.

Since the particle is moving with uniform acceleration, so, we have

$$v_{av} = \langle v \rangle = \frac{v_i + v_f}{2} = 5\ ms^{-1}$$

25. $h = ut - \dfrac{1}{2}gt^2$

$$\Rightarrow \quad gt^2 - 2ut + 2h = 0$$

$$\Rightarrow \quad t_1 t_2 = \frac{2h}{g} \text{ and } t_1 + t_2 = \frac{2u}{g} = T$$

Since $(t_2 - t_1)^2 = (t_1 + t_2)^2 - 4t_1 t_2$

$$\Rightarrow \quad 16 = 64 - 4\left(\frac{2h}{g}\right)$$

$$\Rightarrow \quad h = 60\text{ m}$$

26. $(\Delta S)_{\min} = \left(\sqrt{a^2 + \dfrac{a^2}{4}}\right) \times 2 = a\sqrt{5}$

$$\Rightarrow \quad t_{\min} = \frac{(\Delta s)_{\min}}{v} = \frac{a\sqrt{5}}{v}$$

$$\Rightarrow \quad n = 5$$

28. From graph, we have

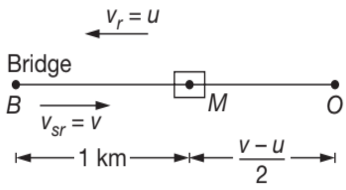
$$v = t + 2$$

$$\Rightarrow \quad x_2 - x_0 = \frac{1}{2} \times (2 + 4) \times 2 = 6\text{ m}$$

$$\Rightarrow \quad x_2 = 9\text{ m}$$

29. Let $v_r = u$ and $v_{sr} = v$, then

Time taken by swimmer to go from M to O and O to B is equal to the time taken by cork to go from M to B.



$$\frac{1}{2}+\frac{1+\frac{v-u}{2}}{v+u}=\frac{1}{u}$$

$$\Rightarrow \frac{1}{2}+\frac{2+v-u}{2(v+u)}=\frac{1}{u}$$

$$\Rightarrow \frac{(v+u+2+v-u)}{2(v+u)}=\frac{1}{u}$$

$$\Rightarrow (2v+2)u=2(v+u)$$

$$\Rightarrow 2vu+2u=2v+2u$$

$$\Rightarrow u=1\text{ kmh}^{-1}$$

30. $s_{A/B}=v_{A/B}t+\frac{1}{2}a_{A/B}t^2$

31. Displacement in last two seconds is

$$s=\Delta x=\frac{1}{2}(1)(2)^2=2\text{ m}$$

$$\Rightarrow v_{av}=\frac{\Delta x}{\Delta t}=1\text{ ms}^{-1}$$

32. $0=16^2-2as$ $\quad\{\because v^2=u^2+2as\}$

$$\Rightarrow a=\frac{16\times 16}{2\times 0.4}=320\text{ ms}^{-2}$$

$$\Rightarrow t=\frac{v}{a}=\frac{16}{320}=5\times 10^{-2}\text{ s}$$

$$\Rightarrow x=5$$

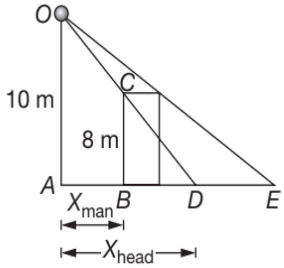
33. $a=\frac{v_f-v_i}{\Delta t}=\frac{35+25}{0.01}=6000\text{ ms}^{-2}$

$$\Rightarrow a=6\text{ kms}^{-2}$$

34. $[v_{av}]_x=\frac{\int_{x_1}^{x_2} v\,dx}{x_2-x_1}=\frac{\int_0^x \sqrt{u^2+2ax}\,dx}{x}=5.6\text{ ms}^{-1}$

35. $x_{man}=vt$

$$\Rightarrow \frac{OA}{AD}=\frac{BC}{BD}$$



$$\Rightarrow \frac{10\text{ m}}{x_{man}}=\frac{2\text{ m}}{x_{head}-x_{man}}$$

$$\Rightarrow x_{head}=\frac{5}{4}x_{man}=\frac{5}{4}vt$$

Velocity of shadow of head $=10\text{ ms}^{-1}$

36. Relative displacement = relative velocity × time

$$\Rightarrow 8\times 3=(10-2)t$$

$$\Rightarrow t=3\text{ s}$$

37. $v=\int_0^4 adt=\int_0^2 adt+\int_2^4 adt$

$$\Rightarrow v=\int_0^2(2-t)dt+\int_2^4(t-2)dt$$

$$\Rightarrow v=4\text{ ms}^{-1}$$

38. $v=u+at$

$$\Rightarrow v=6-\frac{1}{2}\times 10=1\text{ ms}^{-1}$$

39. $a=\frac{30-20}{1.5}=\frac{36.7-30}{\Delta t}$

$$\Rightarrow \Delta t=\frac{6.7\times 1.5}{10}\approx 1\text{ s}$$

40. Area under curve is

$$A=\frac{1}{2}\times 10\times 30=150 \quad \ldots(1)$$

Also, area under a-x curve is equal to

$$A=\frac{v^2-u^2}{2} \quad \ldots(2)$$

From (1) and (2), we get

$$\frac{1}{2}(v^2-u^2)=150$$

$$\Rightarrow v^2=u^2+300$$

$$\Rightarrow v^2=(10)^2+300$$

$$\Rightarrow v=\sqrt{400}=20\text{ ms}^{-1}$$

41. Speed of bike is

$$v_b = 72 \times \frac{5}{18} = 20 \text{ ms}^{-1}$$

Speed of jeep is

$$v_j = 90 \times \frac{5}{18} = 25 \text{ ms}^{-1}$$

Relative velocity of jeep w. r. t. bike is

$$v_{jb} = 25 - 20 = 5 \text{ ms}^{-1}$$

Distance covered by bike in 10 s is

$$s_b = 20 \times 10 = 200 \text{ m}$$

Time taken by Jeep to cover 200 m with velocity 5 ms^{-1} is

$$t = \frac{200}{5} = 40 \text{ s}$$

Therefore distance covered by police jeep in 40 s is

$$s_j = 40 \times 25 = 1000 \text{ m} = 1 \text{ km}$$

42. Time when velocity of ball is zero

$$0 = 9.8 \times gt$$

$$\Rightarrow \quad t = \frac{9.8}{9.8} = 1 \text{ s}$$

So, total time taken by ball to come back is 2 s.
Distance travelled by trolley in 2 s is

$$s = \frac{1}{2}at^2 = \frac{1}{2} \times 1 \times 2^2 = 2 \text{ m}$$

So, ball will fall 2 m behind the boy.

43. $t = \dfrac{2\ell}{v-u} = \dfrac{2 \times 3600}{2}$

$$\Rightarrow \quad t = 3600 \text{ s}$$

$$\Rightarrow \quad t = 1 \text{ h}$$

44. At B, velocity is 2 ms^{-1}

Slope of line AB is $v = -2 \text{ ms}^{-1}$

So, speed of particle at $t = 12.5$ s is 2 ms^{-1}

45. $v_{avg} = \dfrac{2v_0(v_1 + v_2)}{2v_0 + v_1 + v_2}$

$$\Rightarrow \quad v_{avg} = \frac{2 \times 3(4.5 + 7.5)}{6 + 4.5 + 7.5} \text{ ms}^{-1}$$

$$\Rightarrow \quad v_{avg} = \frac{6 \times 12}{18} \text{ ms}^{-1} = 4 \text{ ms}^{-1}$$

46. $vdv = ads$

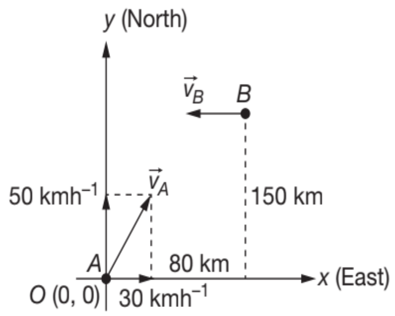
$$\Rightarrow \quad \frac{v^2}{2} = \text{Area of a-s graph}$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{1}{2}$$

$$\Rightarrow \quad v = 1 \text{ ms}^{-1}$$

ARCHIVE: JEE MAIN

1. The arrangement of the ships is shown in the Figure (not drawn to scale). Given that



$$\vec{v}_A = 30\hat{i} + 50\hat{j} \text{ kmhr}^{-1}, \vec{r}_A = 0\hat{i} + 0\hat{j}$$

$$\vec{v}_B = (-10\hat{i})\text{kmhr}^{-1}, \vec{r}_B = (80\hat{i} + 150\hat{j}) \text{ km}$$

$$\Rightarrow \quad \vec{r}_{BA} = (80\hat{i} + 150\hat{j}) \text{ km}$$

$$\Rightarrow \quad \vec{v}_{BA} = \vec{v}_B - \vec{v}_A = -10\hat{i} - 30\hat{i} - 50\hat{i} = -40\hat{i} - 50\hat{j}$$

The time after which distance between the ships is minimum is given by

$$t = \frac{\vec{r}_{BA} \cdot \vec{v}_{BA}}{|\vec{v}_{BA}|^2}$$

$$\Rightarrow \quad t = \frac{(80\hat{i} + 150\hat{j}) \cdot (-40\hat{i} - 50\hat{j})}{|-40\hat{i} - 50\hat{j}|^2}$$

$$\Rightarrow \quad t = \left(\frac{3200 + 7500}{4100}\right) \text{hr}$$

$$\Rightarrow \quad t = \frac{10700}{4100} \text{hr} = 2.6 \text{ hr}$$

Hence, the correct answer is (D).

2. Since $a = \text{constant}$, $u = 0$

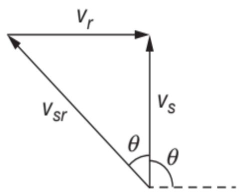
$$\Rightarrow \quad v = at$$

Since particle starts from the origin

$$\Rightarrow \quad x = \frac{1}{2}at^2$$

So, (I), (II) and (IV) are correct graphs.
Hence, the correct answer is (C).

3. Drawing the relative velocity diagram, we get



$$\sin\theta = \frac{v_r}{v_{sr}} = \frac{1}{2}$$

$\Rightarrow \quad \theta = 30°$

$\Rightarrow \quad \phi = 90 + \theta = 120°$

Hence, the correct answer is (D).

4. $\vec{r} = 15t^2\hat{i} + 4\hat{j} - 20t^2\hat{j}$

$$\Rightarrow \quad \frac{d\vec{r}}{dt} = 30t\hat{i} - 40t\hat{j}$$

$$\Rightarrow \quad \frac{d^2\vec{r}}{dt^2} = 30\hat{i} - 40\hat{j}$$

$$\Rightarrow \quad \left|\frac{d^2\vec{r}}{dt^2}\right| = 50 \text{ ms}^{-2}$$

Hence, the correct answer is (A).

5. $x = at + bt^2 - ct^3$

Velocity $\dot{x} = \frac{dx}{dt}$ is given by

$$\dot{x} = a + 2bt - 3ct^2$$

Acceleration $\ddot{x} = \frac{d^2x}{dt^2}$ is given by

$$\ddot{x} = 2b - 6ct$$

For $\ddot{x} = 0$, $t = +\frac{b}{3c}$

$$\Rightarrow \quad v = \dot{x} = a + 2b\left(\frac{+b}{3c}\right) - 3c\left(\frac{b^2}{3c \times 3c}\right)$$

$$\Rightarrow \quad v = -\frac{b^2}{3c} + \frac{2b^2}{3c} + a = a + \frac{b^2}{3c}$$

Hence, the correct answer is (B).

6. $v^2 = u^2 - 2as$

$$\Rightarrow \quad v^2 = (1)^2 - (2)\left(\frac{2.5 \times 10^{-2}}{20 \times 10^{-3}}\right)\left(\frac{20}{100}\right)$$

$$\Rightarrow \quad v^2 = 1 - \frac{1}{2}$$

$$\Rightarrow \quad v = \frac{1}{\sqrt{2}} \text{ ms}^{-1} = 0.7 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

7. $v = \frac{dx}{dt} = b\sqrt{x}$ ...(1)

$$\Rightarrow \quad \int_0^x \frac{dx}{\sqrt{x}} = \int_0^\tau b\,dt$$

$\Rightarrow \quad 2\sqrt{x} = b\tau$ ...(2)

$$\Rightarrow \quad v = b\left(\frac{b\tau}{2}\right) = \frac{b^2\tau}{2}$$

Hence, the correct answer is (C).

8. Since $\frac{dx}{dt} = ky$ and $\frac{dy}{dt} = kx$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x}{y}$$

$$\Rightarrow \quad y\,dy = x\,dx$$

$$\Rightarrow \quad \int y\,dy = \int x\,dx$$

$$\Rightarrow \quad \frac{y^2}{2} = \frac{x^2}{2} + \text{constant}$$

$$\Rightarrow \quad y^2 - x^2 = \text{constant}$$

$$\Rightarrow \quad y^2 = x^2 + \text{constant}$$

Hence, the correct answer is (C).

9. $t_A = t_B - t$

$\Rightarrow \quad v_A = a_1(t_B - t) = a_2 t_B + v$ ...(1)

$$\Rightarrow \quad S = \frac{1}{2}a_1(t_B - t)^2 = \frac{1}{2}a_2 t_B^2$$

$\Rightarrow \quad t_B\left(1 - \sqrt{\frac{a_2}{a_1}}\right) = t$ ...(2)

Solving equations (1) and (2), we get

$$v = t\sqrt{a_1 a_2}$$

Hence, the correct answer is (D).

10. $v_x = \frac{dx}{dt} = -a\omega\sin(\omega t)$

$$v_y = \frac{dy}{dt} = a\omega\cos(\omega t)$$

$$v_z = \frac{dz}{dt} = a\omega$$

$$\Rightarrow \quad v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{2}a\omega$$

Hence, the correct answer is (B).

11. Since displacement is the area under v-t graph. So

$$\text{Area} = \frac{1}{2}\times 2\times 2 + 2\times 2 + 3\times 1$$

$\Rightarrow$ Displacement $= 9$ m

Hence, the correct answer is (A).

12. When moving in same direction

$$t_1 = \frac{\ell_1 + \ell_2}{v_1 - v_2}$$

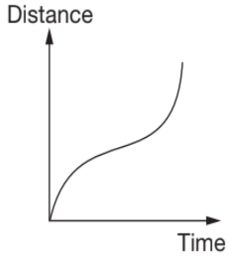
When moving in opposite direction

$$t_2 = \frac{\ell_1 + \ell_2}{v_1 + v_2}$$

$$\Rightarrow \quad \frac{t_1}{t_2} = \frac{v_1 + v_2}{v_1 - v_2} = \frac{80+30}{80-30} = \frac{11}{5}$$

Hence, the correct answer is (B).

13. In OPTIONS (A), (C) and (D), the given graphs represent uniformly decelerated motion of a particle in a straight line with positive initial velocity. Distance-time graph of such a motion is shown here.



Hence, the correct answer is (B).

14. Distance travelled by car in 15 s is

$$s_1 = \frac{1}{2}\times AC \times OC = \frac{1}{2}(45)(15) = \frac{675}{2} \text{ m}$$

Distance travelled by scooter in 15 s is

$$s_2 = v \times t = 30 \times 15 = 450 \text{ m}$$

Required difference in distance is $(s_2 - s_1) = \Delta s$

$$\Rightarrow \quad \Delta s = 450 - \frac{675}{2} = \frac{225}{2} = 112.5 \text{ cm}$$

Let the car catch scooter in time t, then

$$\frac{675}{2} + 45(t-15) = 30t$$

$$\Rightarrow \quad 337.5 + 45t - 675 = 30t$$

$$\Rightarrow \quad 15t = 337.5$$

$$\Rightarrow \quad t = 22.5 \text{ s}$$

Hence, the correct answer is (D).

15. Using, $v^2 = u^2 - 2as$, we get

$$0 = u^2 - 2as$$

$$\Rightarrow \quad s = \frac{u^2}{2a}$$

$$\Rightarrow \quad \frac{s_1}{s_2} = \frac{u_1^2}{u_2^2}$$

$$\Rightarrow \quad s_2 = \left(\frac{u_2}{u_1}\right)^2 s_1$$

$$\Rightarrow \quad s_2 = (2)^2(40) = 160 \text{ m}$$

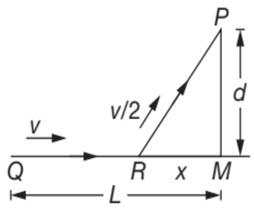
Hence, the correct answer is (C).

16. Time taken by car to reach at location P from location Q is

$$t = \frac{QR}{v} + \frac{RP}{(v/2)}$$

$$\Rightarrow \quad t = \frac{(L-x)}{v} + \frac{2\sqrt{d^2+x^2}}{v}$$

For t to be minimum, we have



$$\frac{dt}{dx} = 0$$

$$\Rightarrow \quad \frac{dt}{dx} = \frac{1}{v}(0-1) + 2\times\left(\frac{1}{2}\right)\frac{1}{v}\times\frac{2x}{\sqrt{d^2+x^2}}$$

$$\Rightarrow \quad \frac{dt}{dx} = \frac{-1}{v} + \frac{2x}{v\sqrt{d^2+x^2}}$$

Since, for minimum value of t

$$\frac{dt}{dx} = 0$$

$$\Rightarrow \quad -\frac{1}{v} + \frac{2x}{v\sqrt{d^2+x^2}} = 0$$

$$\Rightarrow \quad 1 = \frac{2x}{\sqrt{d^2+x^2}}$$

$$\Rightarrow \quad 4x^2 = d^2 + x^2$$

$$\Rightarrow \quad 3x^2 = d^2$$

$$\Rightarrow \quad x = \frac{d}{\sqrt{3}}$$

Hence, the correct answer is (B).

17. Velocity of the body going upwards is given by $v = v_0 - gt$, where v_0 is the initial velocity.

Hence, the graph between velocity and time should be a straight line with negative slope (g) and intercept v_0.

Also, during the whole motion, acceleration of the body is constant i.e., slope should be constant.

Hence, the correct answer is (C).

18. Here, acceleration is given by, $a = -c$

$$\Rightarrow \frac{dv}{dt} = -c$$

$$\Rightarrow \frac{dx}{dt} \cdot \frac{dv}{dx} = -c$$

$$\Rightarrow v\frac{dv}{dx} = -c$$

$$\Rightarrow vdv = -cdx$$

$$\Rightarrow \frac{v^2}{2} = -cx + k$$

$$\Rightarrow x = -\frac{v^2}{2c} + \frac{k}{c}$$

Hence, the correct answer is (A).

20. Acceleration of car, $a_C = 4 \text{ ms}^{-2}$

Acceleration of bus, $a_B = 2 \text{ ms}^{-2}$

Initial separation between the bus and car is

$$s_{CB} = 200 \text{ m}$$

Acceleration of car with respect to bus is

$$a_{CB} = a_C - a_B = 2 \text{ ms}^{-2}$$

Initial velocity of car w.r.t. bus is

$$u_{CB} = 0,\ t = ?$$

Since, $s_{CB} = u_{CB}t + \frac{1}{2}a_{CB}t^2$

$$\Rightarrow 200 = (0)t + \frac{1}{2}(2)t^2$$

$$\Rightarrow t^2 = 200$$

$$\Rightarrow t = 10\sqrt{2} \text{ s}$$

Hence, the correct answer is (C).

21. Using $h = ut + \frac{1}{2}at^2$

For stone 1, we have

$$y_1 = 10t - \frac{1}{2}gt^2$$

and for stone 2, we have

$$y_2 = 40t - \frac{1}{2}gt^2$$

Relative position of the second stone with respect to the first,

$$\Delta y = y_2 - y_1 = 40t - \frac{1}{2}gt^2 - 10t + \frac{1}{2}gt^2$$

$$\Rightarrow \Delta y = 30t$$

After 8 seconds, stone 1 reaches ground, i.e., $y_1 = -240$ m

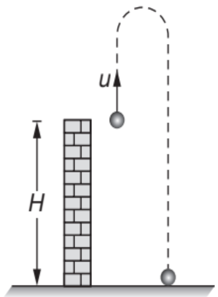
$$\Rightarrow \Delta y = y_2 - y_1 = 40t - \frac{1}{2}gt^2 + 240$$

Therefore, it will be a parabolic curve till other stone reaches ground.

Hence, the correct answer is (A).

22. Time taken by the particle to reach the top most point is,

$$t = \frac{u}{g} \quad \ldots(1)$$



Time taken by the particle to reach the ground is

$$t' = nt$$

Since, $s = ut + \frac{1}{2}at^2$

$$\Rightarrow -H = u(nt) - \frac{1}{2}g(nt)^2$$

Using (1), we get

$$-H = u \times n\left(\frac{u}{g}\right) - \frac{1}{2}gn^2\left(\frac{u}{g}\right)^2$$

$$\Rightarrow -2gH = 2nu^2 - n^2u^2$$

$$\Rightarrow 2gH = nu^2(n-2)$$

Hence, the correct answer is (D).

23. $\frac{dv}{dt} = -2.5\sqrt{v}$

$$\Rightarrow \frac{1}{\sqrt{v}}dv = -2.5dt$$

CHAPTER 4

On integrating, within limits i.e., $v_1 = 6.25\ \text{ms}^{-1}$ to $v_2 = 0$, we get

$$\int_{6.25}^{0} v^{-1/2} dv = -2.5 \int_0^t dt$$

$$\Rightarrow \quad 2\sqrt{v}\Big|_{6.25}^{0} = -(2.5)t$$

$$\Rightarrow \quad t = \frac{-2\sqrt{6.25}}{-2.5} = 2\ \text{s}$$

Hence, the correct answer is (B).

24. Here, $\vec{v} = K(y\hat{i} + x\hat{j})$

$$\Rightarrow \quad \vec{v} = Ky\hat{i} + Kx\hat{j} \qquad \ldots(1)$$

Since, $\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$ ...(2)

Equating equations (1) and (2), we get

$$\frac{dx}{dt} = Ky \text{ and } \frac{dy}{dt} = Kx$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{Kx}{Ky} = \frac{x}{y} \qquad \ldots(3)$$

Integrating both sides of the above equation, we get

$$\int y dy = \int x dx$$

$$\Rightarrow \quad \frac{y^2}{2} = \frac{x^2}{2} + \text{constant}$$

$$\Rightarrow \quad y^2 = x^2 + \text{constant}$$

Hence, the correct answer is (A).

25. $\vec{v} = \vec{u} + \vec{a}t$

$$\Rightarrow \quad \vec{v} = (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j}) \times 10$$

$$\Rightarrow \quad \vec{v} = (3+4)\hat{i} + (4+3)\hat{j}$$

$$\Rightarrow \quad |\vec{v}| = \sqrt{49+49} = \sqrt{98} = 7\sqrt{2} \text{ units}$$

Hence, the correct answer is (B).

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

1. Given line have positive intercept but negative slope. So its equation can be written as

$$v = -mx + v_0 \qquad \ldots(1)$$

where $m = \tan\theta = \frac{v_0}{x_0}$

By differentiating with respect to time we get

$$\frac{dv}{dt} = -m\frac{dx}{dt} = -mv$$

Now substituting the value of v from equation (1) we get

$$\frac{dv}{dt} = -m(-mx + v_0) = m^2x - mv_0$$

$$\Rightarrow \quad a = m^2x - mv_0$$

i.e. the graph between a and x should have positive slope but negative intercept on a-axis. So graph (A) is correct.

Hence, the correct answer is (A).

2. Distance travelled in t^{th} second is,

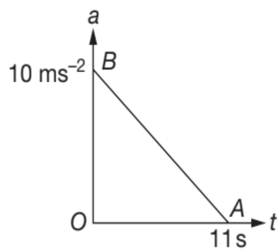
$$s_t = u + \frac{a}{2}(2t-1)$$

Since, $u = 0$, so

$$\Rightarrow \quad \frac{s_n}{s_{n+1}} = \frac{an - \frac{1}{2}a}{a(n+1) - \frac{1}{2}a} = \frac{2n-1}{2n+1}$$

Hence, the correct answer is (C).

3. The area under acceleration time graph gives change in velocity. As acceleration is zero at the end of 11 s



$$\Rightarrow \quad v_{max} = \text{Area of } \Delta OAB$$

$$\Rightarrow \quad v_{max} = \frac{1}{2} \times 11 \times 10 = 55\ ms^{-1}$$

Hence, the correct answer is (B).

4. $v_1 = -\sqrt{2gd}$ and $v_2 = +\sqrt{2g\left(\frac{d}{2}\right)}$

Hence, the correct answer is (A).

5. $|\text{Average Velocity}| = \left|\dfrac{\text{Displacement}}{\text{time}}\right| = \dfrac{AB}{\text{time}} = \dfrac{2}{1} = 2\ \text{ms}^{-1}$

Hence, the correct answer is (B).

6. **For Q**

Since acceleration due to gravity has no component along AB. So motion of the particle along AB is non-accelerated motion with uniform velocity v. If t_Q is the time taken by particle Q to go from A to B, then

$$t_Q = \frac{AB}{v} \quad \ldots(1)$$

For P

Since motion of particle P from A to C is accelerated and that from C to B is retarded. So from A to C the horizontal component of velocity gradually increases to attain a maximum value at the lowest point which further decreases gradually to attain the same value v at point B.

That is the motion from A to B to C experiences a horizontal velocity which has a value more than v. So $t_Q > t_P$

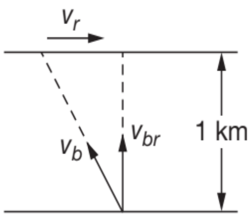
Hence, the correct answer is (A).

7. v_b = velocity of boat in still water $= 5\ \text{kmh}^{-1}$

v_{br} = velocity of boat w.r.t. river

v_r = velocity of river

$$v_{br} = \frac{1\ \text{km}}{\left(\frac{15}{60}\right)h} = 4\ \text{kmh}^{-1}$$



By Pythagoras Theorem

$$v_r^2 + v_{br}^2 = v_b^2$$

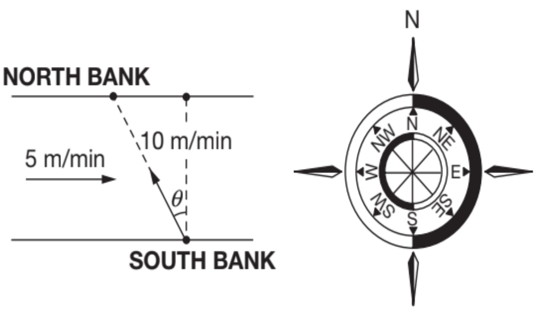
$$\Rightarrow \quad v_r^2 = v_b^2 - v_{br}^2$$

$$\Rightarrow \quad v_r^2 = 5^2 - 4^2$$

$$\Rightarrow \quad v_r = 3\ \text{kmh}^{-1}$$

Hence, the correct answer is (B).

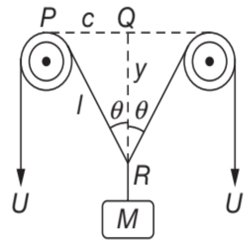
8.



To swim across the river to a point directly opposite in the shortest time, the man should swim at an angle $\theta = \sin^{-1}\left(\frac{5}{10}\right) = 30°$ towards the West of North.

Hence, the correct answer is (C).

9. In the right angle ΔPQR, $l^2 = c^2 + y^2$



c = constant, l and y are variable

Differentiating this equation with respect to time, we get

$$2l\frac{dl}{dt} = 0 + 2y\frac{dy}{dt}$$

or $\quad \left(-\dfrac{dy}{dt}\right) = \dfrac{l}{y}\left(-\dfrac{dl}{dt}\right)$

Here, $-\dfrac{dy}{dt} = v_M$

$\Rightarrow \quad \dfrac{l}{y} = \dfrac{1}{\cos\theta}$

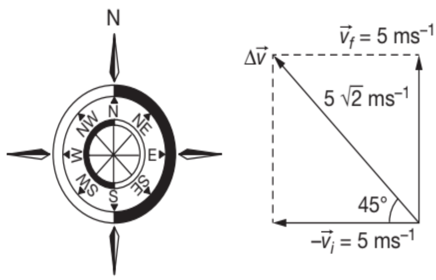
and $\quad \dfrac{-dl}{dt} = U$

Hence, $v_M = \dfrac{U}{\cos\theta}$

Hence, the correct answer is (C).

10. $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$\Delta\vec{v} = 5\sqrt{2}\ \text{ms}^{-1}$ in North-West direction.



$$\vec{a}_{av} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}\ \text{ms}^{-2}\ \text{(in North-West direction)}$$

Hence, the correct answer is (C).

Multiple Correct Choice Type Problems

1. $x = a\cos(pt)$

$y = b\sin(pt)$

$\Rightarrow \quad \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$\Rightarrow$ Trajectory is an Ellipse

$\Rightarrow \quad \ddot{x} = -ap^2\cos(\omega t)$

$\Rightarrow \quad \ddot{x} = -p^2 x$

Similarly $\ddot{y} = -p^2 y$

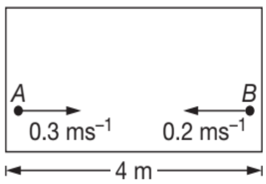
So, acceleration is directed towards the focus.

Hence, (A), (B) and (C) are correct.

2. The body starts from rest at $x = 0$ and then again comes to rest at $x = 1$. It means initially acceleration is positive and then negative.
So, we can conclude that α cannot remains positive for all t in the interval $0 \le t \le 1$ i.e., α must change sign during the motion.
Hence, (A) and (D) are correct.

Integer/Numerical Answer Type Questions

1. Let us consider motion of two balls with respect to rocket.



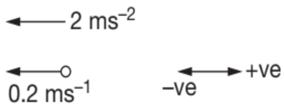
Motion of ball *A* relative to rocket

Maximum distance of ball A from left wall is

$$S_A = \frac{u^2}{2a} = \frac{0.3 \times 0.3}{2 \times 2} = \frac{0.09}{4} \approx 0.02 \text{ m} \left\{\because 0 = u^2 - 2aS\right\}$$

So, collision of two balls will take place very near to left wall.

Motion of ball *B* relative to rocket



$$S = ut + \frac{1}{2}at^2$$

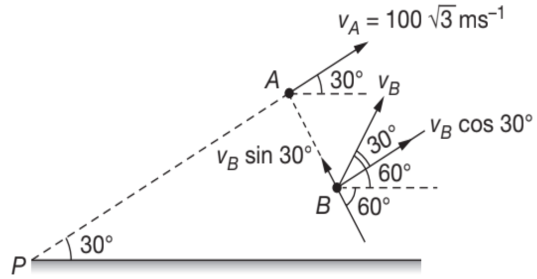
$$\Rightarrow \quad -4 = -0.2t - \left(\frac{1}{2}\right)2t^2$$

Solving this equation, we get

$t = 1.9$ s

Nearest Integer $= 2$ s

2. Relative velocity of B with respect to A is perpendicular to line PA. Therefore, along the line PA, velocity components of A and B should be same.



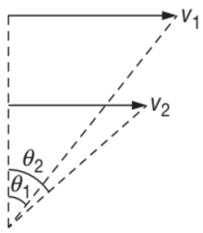
$\Rightarrow \quad v_A = 100\sqrt{3} = v_B \cos 30°$

$\Rightarrow \quad v_B = 200 \text{ ms}^{-1}$

$$\Rightarrow \quad t_0 = \frac{500}{v_B \sin(30)} = \frac{500}{200 \times \frac{1}{2}} = 5 \text{ s}$$

Assertion and Reasoning Type Problems

1.



Statement-II, is the mathematical definition of relative velocity. However, it does not explain Statement-I correctly. The correct explanation of Statement-I is due to visual perception of motion. The object appears to be moving faster, when its angular velocity is greater w.r.t. observer.

Hence, the correct answer is (B).