iCON

CHAPTER 2

Measurements and General Physics

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Physical Quantity and its Measurement
(b) Fundamental and Derived Units
(c) Dimensional Analysis
(d) Principle of Homogeneity and its uses
(e) Limitations of Dimensional Analysis
(f) Least Count, Significant Figures and Rounding off
(g) Errors and their Propagation
(h) Measurements done using Vernier Calliper (VC)
(i) Measurements done using Screw Gauge (SG).

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

SCIENTIFIC PROCESS

The history of science reveals that it has evolved through a series of steps. Let us have a small discussion of the steps involved.

STEP-1: Observation
STEP-2: Proposing/propounding a theory based on those observations.
STEP-3: Verification of the theory as applied to those observations.
STEP-4: Modification in the theory, if at all necessary.

OBSERVATION

Observations are basically of two types

Subjective Observation

An observation that varies from person to person is called **Subjective Observation**. Physics never deals with subjective observations like beauty, emotion, personality etc.

Objective Observation

An observation that remains identical for all the observers (persons) is called an **Objective Observation**.

EXAMPLE:
Four observers viewing the same painting may feel differently about the "**Beauty**" of the painting, where as they shall report identical length, breadth or area of the painting, when asked. So, Beauty is not an **Objective Observation** as it cannot be assigned a numerical value along with some appropriate unit (that could have measured it).
Physics always deals with **Objective Observation**.

PHYSICAL QUANTITY

The objective quantities to which a numerical value can be attached along with some unit (specified to measure it) are called Physical Quantities. Else, they may also be defined as the quantities through which (through the help of which) we can describe the Laws of Physics appropriately. A physical quantity can be completely specified if it has

(a) only magnitude (some constants or some ratios) e.g., specific gravity, dielectric constant, strain, refractive index etc.
(b) magnitude along with a unit (scalars) e.g., mass, length, time, energy, current etc.
(c) magnitude along with a unit in a specified direction (vectors) such that laws of vector algebra are obeyed by this quantity e.g., displacement, force, momentum, torque, angular momentum, electric field, magnetic field etc.
(d) magnitude and phase (phasors) e.g., superposition of mechanical wave, AC voltages and currents, SHM etc.
(e) magnitude of varying values in different directions (having no specified directions) (are called Tensors) e.g., Moment of Inertia (cannot be defined without specifying the axis of rotation).

In few anisotropic media the physical quantity(ies) like density, refractive index, dielectric constant, stress, strain, electric conductivity all become Tensors.

MEASUREMENT OF A PHYSICAL QUANTITY

The process of measurement is basically a comparison process. To measure a physical quantity we need to have the following two things.

(a) Firstly, a unit u (of same nature) to measure that physical quantity.
(b) Secondly, the number of times (n) this selected unit is contained in the required physical quantity.

e.g., if we are asked to measure the length and breadth of a room with the help of a scale that is half a metre in length (half metre scale) and we observe that this unit (half metre scale) is contained 10 times in the length of the room and 8 times in its width, then

$$\text{Length of the room} = 10\left(\frac{1}{2}\text{ m}\right) = 5\text{ m}$$

$$\text{Breadth of the room} = 8\left(\frac{1}{2}\text{ m}\right) = 4\text{ m}$$

So, measurement of a Physical Quantity $= nu$
where, u is the unit selected (of same nature) to measure the physical quantity.
n is the number of times this selected unit is contained in it.
For a particular measurement, irrespective of the system in which the unit is selected, measurement of a physical quantity is a constant.

$$\Rightarrow \quad nu = \text{constant}$$

$$\Rightarrow \quad n_1u_1 = n_2u_2 = n_3u_3 = \ldots\ldots\ldots\ldots$$

where,
$u_1, u_2, u_3, \ldots\ldots$ are the units selected to measure a physical quantity in system 1, 2, 3, …….. respectively.
$n_1, n_2, n_3, \ldots\ldots$ are the numerical values that the measured physical quantity contains corresponding to the respective systems.
Also, we observe that

$$n \propto \frac{1}{u}$$

i.e., the bigger the unit selected to measure the physical quantity, the smaller the numerical values vice-versa.

FUNDAMENTAL AND DERIVED UNITS

The exact specification of the measurement of a physical quantity requires

(a) the standard or unit in which the quantity is measured and
(b) the numerical value representing the number of times the quantity contains that unit.

The physical quantities which do not depend upon other quantities are called **fundamental quantities.** In M.K.S. system the fundamental quantities are mass, length and time, while in more general Standard International (S.I.) system the fundamental quantities are **mass, length, time, temperature, illuminating power (or luminous intensity), current and amount of substance.** The units of fundamental quantities are called **fundamental units**. The units of physical quantities which may be derived from fundamental units are called **derived units**.

Conceptual Note(s)

Measurement of a Physical Quantity means a comparison of it with some reference standard, also called as unit, which does not change (under any circumstances). So, these standards must be

(a) invariable
(b) easily accessible
(c) precise and
(d) universally agreed.

SYSTEM OF UNITS

Following principal systems of units are used commonly.

C.G.S. System

In this system the unit of length is centimetre (cm), that of mass is gram (g) and that of time is second (s).

F.P.S. System

In this system the unit of length is foot, weight is pound (lb) and time is second.

Conceptual Note(s)

(a) In fps system the unit pound is the unit of weight and not of mass.

(b) 1 pound = 0.4536 kgwt

i.e. 1 pound is equivalent to the weight of a body having a mass of 0.4536 kg.

(c) The unit of mass in fps system is slug.

M.K.S. System

In this system the units of length is metre (m), mass is kilogram (kg) and time is second(s).

S.I. System

In this system there are seven fundamental quantities whose units and symbols are as follows:

Fundamental Quantity	Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Temperature	kelvin	K
Luminous Intensity	candela	cd
Electric Current	ampere	A
Amount of Substance	mole	mol

(*Continued*)

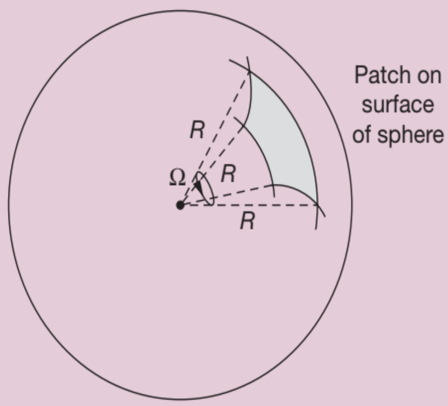
In S.I. system there are two supplementary units.

(a) Radian (rad): Unit of plane angle.
(b) Steradian (sr): Unit of solid angle.

Conceptual Note(s)

To understand the concept of solid angle, let us consider a football having black and white patches on it. Now consider any one patch, say black. Then on the boundary of this patch lie infinite number of points. If you join all these points with the centre of the football, then you observe all these points to be at equal distance from it and this distance happens to be the **radius of the football**. Now, if you join all these points to the centre of the football then the angle enclosed by this patch at the centre of the football is called the Solid Angle, just like the angle at the end of an empty ice-cream cone. Now this solid angle, denoted by Ω, has a value given by

$$\Omega = \frac{\text{Area of patch on the surface of sphere}}{(\text{Radius of sphere})^2}$$



Prefixes for Power of Ten

Prefix	Abbreviation	Power of Ten
atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}
peta	P	10^{15}
exa	E	10^{18}

(Continued)

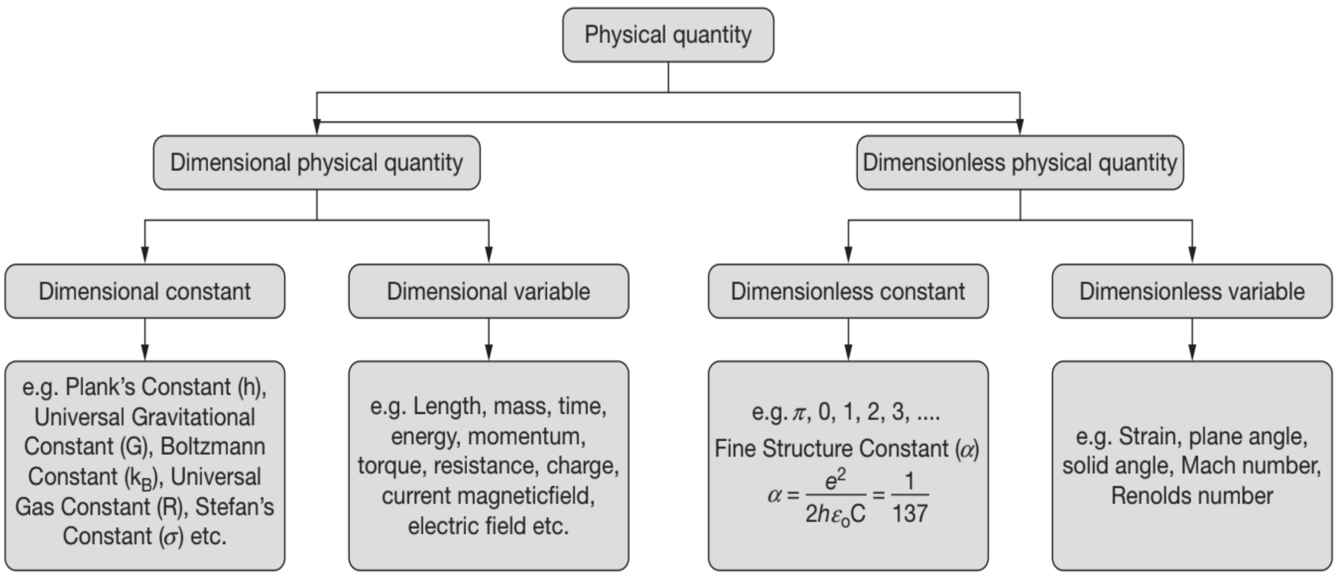
EXAMPLES:

Derived units

1 micro second $= 1\,\mu s = 10^{-6}$ s

1 nano second $= 1$ ns $= 10^{-9}$ s

1 kilo-metre $= 1$ km $= 10^{-3}$ m



Some Important Commonly Used Units

(a) micrometer $(\mu m) = 1\mu m = 10^{-6}$ m

(b) Angstrom $1\text{Å} = 10^{-10}$ m

(c) Astronomical unit

This is the mean distance of earth from sun

$$1\,\text{AU} = 1.496 \times 10^{11}\,\text{m} \approx 1.5 \times 10^{11}\,\text{m}$$

(d) Light year

It is the distance traversed by light in vacuum in 1 year,

$$1\text{light year} = 365 \times 24 \times 60 \times 60 \times 3 \times 10^{8} = 9.45 \times 10^{15}\text{m}$$

(e) parsec

It is the distance at which an arc of length one Astronomical unit subtends an angle of 1 second i.e. 1″.

Since $l = r\theta$

$$\Rightarrow \quad r = \frac{l}{\theta}$$

where $l = 1\,\text{AU}$ and $\theta = 1'' = \left(\frac{1}{3600}\right)\left(\frac{\pi}{180}\right)$rad

$$\Rightarrow \quad r = 1 \text{ parsec} = \frac{1.496 \times 10^{11}\text{m}}{\frac{1}{3600} \times \frac{\pi}{180}(\text{rad})}$$

$\Rightarrow$ $r = 1$ parsec $= 3.07 \times 10^{16}$ m

$\Rightarrow$ 1 parsec $= 3.26$ light years

(f) X-ray Unit (XU) $= 1$ XU $= 10^{-13}$ m

(g) 1 Bar $= 10^5$ Nm^{-2} $= 10^5$ pascal

(h) 1 atmosphere (atm) $= 1.013 \times 10^5$ Pa

(i) 1 torr $= 1$ mm of Hg $= 133.3$ Pa

(j) 1 barn $= 10^{-28}$ m^2

(k) 1 horse power $= 746$ watt

(l) 1 ft $= 0.3048$ m

(m) 1 pound $= 453.6$ g $= 0.4536$ kg

(n) 1 slug $= 14.57$ kg

(o) 1 poiseuille (PI) $= 10$ poise

(p) 1 metricton $= 10$ quintal $= 1000$ kg

(q) 1 Chandra Shekhar Limit (CSL) $= 1.4\, M_s$

where M_s = mass of sun

(r) 1 Shake $= 10^{-8}$ s

DIMENSIONS

The powers to which the fundamental units of mass, length and time are raised so as to get the required physical quantity.

EXAMPLE:

To get the physical quantity "force", mass has to be raised to the power 1, length to the power 1 and time to the power –2. So, dimensions of force are 1 in mass, 1 in length and –2 in time.

DIMENSIONAL FORMULA

$M^a L^b T^c$ is the dimensional formula of a physical quantity which has dimensions a, b and c in mass, length and time respectively. So, dimensional formula of force is MLT^{-2}. Whenever a physical quantity is written in square brackets, it just means that dimensional formula of the physical quantity has to be taken.

DIMENSIONAL EQUATION

Whenever a physical quantity is equated to its dimensional formula, we get a dimensional equation. So, dimensional equation for force is

$$[F] = MLT^{-2}$$

In general, any physical quantity X, having dimensional formula $M^a L^b T^c$, has a dimensional equation

$$[X] = M^a L^b T^c$$

DIMENSIONS OF SOME PHYSICAL QUANTITIES

Sl.	Physical Quantity and Symbol	Relationship with Other Physical Quantities	SI Unit	Dimensional Formula
1.	Area (A)	Length × Breadth	m^2	$M^0L^2T^0$
2.	Volume (V)	Length × Breadth × Height	m^3	$M^0L^3T^0$

(Continued)

Sl.	Physical Quantity and Symbol	Relationship with Other Physical Quantities	SI Unit	Dimensional Formula
3.	Mass density (ρ)	$\frac{\text{Mass}}{\text{Volume}}$	kgm^{-3}	$ML^{-3}T^{0}$
4.	Surface Mass Density (σ)	$\frac{\text{Mass}}{\text{Area}}$	kgm^{-2}	$ML^{-2}T^{0}$
5.	Linear Mass Density (λ)	$\frac{\text{Mass}}{\text{Length}}$	kgm^{-1}	$ML^{-1}T^{0}$
6.	Frequency (v)	$\frac{1}{\text{Time period}}$	Hz	$M^{0}L^{0}T^{-1}$
7.	Velocity, Speed (v)	$\frac{\text{Displacement}}{\text{Time}}, \frac{\text{Distance}}{\text{Time}}$	ms^{-1}	$M^{0}LT^{-1}$
8.	Acceleration (a)	$\frac{\text{Velocity}}{\text{Time}}$	ms^{-2}	$M^{0}LT^{-2}$
9.	Force (F)	$\text{Mass} \times \text{Acceleration}$	newton (N)	MLT^{-2}
10.	Impulse (I)	$\text{Force} \times \text{Time}$	Ns	MLT^{-1}
11.	Work, Energy (W, E)	$\text{Force} \times \text{Distance}$	joule (J)	$ML^{2}T^{-2}$
12.	Power (P)	$\frac{\text{Work}}{\text{Time}}$	watt (W)	$ML^{2}T^{-3}$
13.	Momentum (p)	$\text{Mass} \times \text{Velocity}$	kgms^{-1}	MLT^{-1}
14.	Kinetic energy $(K \text{ or K.E.})$	$\left(\frac{1}{2}\right) \times \text{Mass} \times (\text{Velocity})^{2}$	J	$ML^{2}T^{-2}$
15.	Potential energy (U)	$\text{Mass} \times \text{Acceleration due to gravity} \times \text{Height}$	J	$ML^{2}T^{-2}$
16.	Spring Constant (k)	$\frac{\text{Force}}{\text{Extension}}$	Nm^{-1}	$ML^{0}T^{-2}$
17.	Elastic Potential Energy (U)	$\frac{1}{2}(\text{Spring constant})(\text{Extension})^{2}$	J	$ML^{2}T^{-2}$
18.	Angle, Angular displacement (θ)	$\frac{\text{Arc}}{\text{Radius}}$	radian	$M^{0}L^{0}T^{0}$

(Continued)

Sl.	Physical Quantity and Symbol	Relationship with Other Physical Quantities	SI Unit	Dimensional Formula
19.	Trigonometric ratio ($\sin\theta$, $\cos\theta$, $\tan\theta$, etc.)	$\frac{\text{Length}}{\text{Length}}$	No Units	$M^0L^0T^0$
20.	Angular velocity (ω)	$\frac{\text{Angle}}{\text{Time}}$	rads^{-1}	$M^0L^0T^{-1}$
21.	Angular acceleration (α)	$\frac{\text{Angular velocity}}{\text{Time}}$	rads^{-2}	$M^0L^0T^{-2}$
22.	Radius of gyration (k)	Distance	m	M^0LT^0
23.	Moment of inertia (I)	$\text{Mass}\times(\text{Radius of gyration})^2$	kgm^2	ML^2T^0
24.	Angular momentum (L)	$\text{Moment of inertia}\times\text{Angular velocity}$	$\text{kgm}^2\text{s}^{-1}$	ML^2T^{-1}
25.	Moment of force, moment of couple (τ)	$\text{Force}\times\text{Distance}$	Nm	ML^2T^{-2}
26.	Torque (τ)	$\frac{\text{Angular momentum}}{\text{Time}}$ or $\text{Force}\times\text{Distance}$	Nm	ML^2T^{-2}
27.	Angular frequency (ω)	$2\pi\times\text{Frequency}$	rads^{-1}	$M^0L^0T^{-1}$
28.	Rotational kinetic energy (RKE)	$\frac{1}{2}\times\text{Moment of inertia}\times(\text{Angular velocity})^2$	J	ML^2T^{-2}
29.	Angular impulse (J)	$\text{Torque}\times\text{Time}$	Js	ML^2T^{-2}
30.	Centripetal acceleration (a_c)	$\frac{(\text{Velocity})^2}{\text{Radius}}$	ms^{-2}	M^0LT^{-2}
31.	Pressure (P)	$\frac{\text{Force}}{\text{Area}}$	Nm^{-2}	$ML^{-1}T^{-2}$
32.	Stress	$\frac{\text{Restoring force}}{\text{Area}}$	Nm^{-2}	$ML^{-1}T^{-2}$
33.	Strain	$\frac{\text{Change in dimension}}{\text{Original dimension}}$	No units	$M^0L^0T^0$
34.	Modulus of elasticity (E)	$\frac{\text{Stress}}{\text{Strain}}$	Nm^{-2}	$ML^{-1}T^{-2}$

(Continued)

Sl.	Physical Quantity and Symbol	Relationship with Other Physical Quantities	SI Unit	Dimensional Formula
35.	Surface tension $(T \text{ or } \sigma)$	$\dfrac{\text{Force}}{\text{Length}}$	Nm^{-1}	ML^0T^{-2}
36.	Surface energy	$\dfrac{\text{Energy}}{\text{Area}}$	Jm^{-2} $(=\text{Nm}^{-1})$	ML^0T^{-2}
37.	Speed gradient	$\dfrac{\text{Speed}}{\text{Distance}}$	s^{-1}	$M^0L^0T^{-1}$
38.	Pressure gradient	$\dfrac{\text{Pressure}}{\text{Distance}}$	Nm^{-3}	$ML^{-2}T^{-2}$
39.	Pressure energy	$\text{Pressure} \times \text{Volume}$	$\text{Nm}(=\text{J})$	ML^2T^{-2}
40.	Fluid flow rate (V)	$\left(\dfrac{\pi}{8}\right)\dfrac{(\text{Pressure})\times(\text{Radius})^4}{(\text{Viscosity coefficient})\times(\text{Length})}$	m^3s^{-1}	$M^0L^3T^{-1}$
41.	Bulk modulus (B) or $(\text{Compressibility})^{-1}$	$\dfrac{\text{Volume}\times(\text{Change in pressure})}{(\text{Change in volume})}$	Nm^{-2}	$ML^{-1}T^{-2}$
42.	Coefficient of viscosity (η)	$\dfrac{\text{Force}}{\text{Area}\times\text{Speed gradient}}$	$\text{kgm}^{-1}\text{s}^{-1}$	$ML^{-1}T^{-1}$
43.	Wavelength (λ)	Distance	m	M^0LT^0
44.	Hubble constant (H)	$\dfrac{\text{Recession speed}}{\text{Distance}}$	s^{-1}	$M^0L^0T^{-1}$
45.	Solid Angle (Ω)	$\dfrac{\text{Area of patch on surface of sphere}}{(\text{Radius of sphere})^2}$	steradian	$M^0L^0T^0$
46.	Intensity of wave (I)	$\dfrac{\left(\dfrac{\text{Energy}}{\text{Time}}\right)}{\text{Area}}$	Wm^{-2}	ML^0T^{-3}
47.	Radiation pressure (P)	$\dfrac{\text{Intensity of wave}}{\text{Speed of light}}$	Wm^{-3}s	$ML^{-1}T^{-2}$
48.	Energy density (u)	$\dfrac{\text{Energy}}{\text{Volume}}$	Jm^{-3}	$ML^{-1}T^{-2}$

(Continued)

Sl.	Physical Quantity and Symbol	Relationship with Other Physical Quantities	SI Unit	Dimensional Formula
49.	Critical velocity (v_c)	$\frac{\text{Renold's number} \times \text{Coefficient of viscocity}}{\text{Mass density} \times \text{Diameter}}$	ms^{-1}	M^0LT^{-1}
50.	Escape velocity (v_e)	$\sqrt{2 \times \text{Acceleration due to gravity} \times \text{Earth's radius}}$	ms^{-1}	M^0LT^{-1}
51.	Heat energy, internal energy (Q, U)	$\text{Work}(= \text{Force} \times \text{Distance})$	J	ML^2T^{-2}
52.	Efficiency (η)	$\frac{\text{Output work or energy}}{\text{Input work or energy}}$		$M^0L^0T^0$
53.	Gravitational constant (G)	$\frac{\text{Force} \times (\text{Distance})^2}{\text{Mass} \times \text{Mass}}$	Nm^2kg^{-2}	$M^{-1}L^3T^{-2}$
54.	Planck constant (h)	$\frac{\text{Energy}}{\text{Frequency}}$	Js	ML^2T^{-1}
55.	Heat capacity (C), entropy (S)	$\frac{\text{Heat energy}}{\text{Temperature}}$	JK^{-1}	$ML^2T^{-2}K^{-1}$
56.	Specific heat capacity (c)	$\frac{\text{Heat Energy}}{\text{Mass} \times \text{Temperature}}$	$Jkg^{-1}K^{-1}$	$M^0L^2T^{-2}K^{-1}$
57.	Latent heat (L)	$\frac{\text{Heat energy}}{\text{Mass}}$	Jkg^{-1}	$M^0L^2T^{-2}$
58.	Thermal expansion coefficient or Thermal expansivity $(\alpha, \beta$ or $\gamma)$	$\frac{\text{Change in dimension}}{\text{Original dimension} \times \text{temperature}}$	K^{-1}	$M^0L^0K^{-1}$
59.	Thermal conductivity (κ)	$\frac{\text{Heat energy} \times \text{Thickness}}{\text{Area} \times \text{Temperature} \times \text{Time}}$	$Js^{-1}m^{-1}K^{-1}$ $(= Wm^{-1}K^{-1})$	$MLT^{-3}K^{-1}$
60.	Stefan's constant (σ)	$\frac{\left(\frac{\text{Energy}}{\text{Area} \times \text{Time}}\right)}{(\text{Temperature})^4}$	$Wm^{-2}K^{-4}$	$ML^0T^{-3}K^{-4}$
61.	Wien constant (b)	$\text{Wavelength} \times \text{Temperature}$	Km	M^0LT^0K
62.	Boltzmann constant (k_B)	$\frac{\text{Energy}}{\text{Temperature}}$	JK^{-1}	$ML^2T^{-2}K^{-1}$

(Continued)

Sl.	Physical Quantity and Symbol	Relationship with Other Physical Quantities	SI Unit	Dimensional Formula
63.	Universal gas constant (R)	$\frac{\text{Pressure} \times \text{Volume}}{\text{Mole} \times \text{Temperature}}$	$JK^{-1}mol^{-1}$	$ML^2T^{-2}K^{-1}mol^{-1}$
64.	Charge (Q)	Current × Time	C (coulomb)	M^0L^0TA
65.	Current density (J)	$\frac{\text{Current}}{\text{Area}}$	Am^{-2}	$M^0L^{-2}T^0A$
66.	Voltage, electric potential, electromotive force $(V \text{ or } E)$	$\frac{\text{Work}}{\text{Charge}}$	V (volt)	$ML^2T^{-3}A^{-1}$
67.	Resistance (R)	$\frac{\text{Potential difference}}{\text{Current}}$	ohm (Ω)	$ML^2T^{-3}A^{-2}$
68.	Capacitance (C)	$\frac{\text{Charge}}{\text{Potential difference}}$	farad (F)	$M^{-1}L^{-2}T^4A^2$
69.	Electrical resistivity (ρ) or $\left(\text{Electrical conductivity}\right)^{-1}$	$\frac{\text{Resistance} \times \text{Area}}{\text{Length}}$	Ωm	$ML^3T^{-3}A^{-2}$
70.	Electric field $(\vec{E})$	$\frac{\text{Electrical force}}{\text{Charge}}$	NC^{-1} (or Vm^{-1})	$MLT^{-3}A^{-1}$
71.	Electric flux (ϕ_E)	Electric field × Area	$NC^{-1}m^2$ (or Vm)	$ML^3T^{-3}A^{-1}$
72.	Electric dipole moment $(\vec{p})$	$\frac{\text{Torque}}{\text{Electric field}}$	Cm (Coulomb metre)	M^0LTA
73.	Electric field strength or electric intensity $(\vec{E})$	$\frac{\text{Potential difference}}{\text{Distance}}$	Vm^{-1}	$MLT^{-3}A^{-1}$
74.	Magnetic field, magnetic flux density, magnetic induction $(\vec{B})$	$\frac{\text{Force}}{\text{Current} \times \text{Length}}$	tesla (T)	$ML^0T^{-2}A^{-1}$
75.	Magnetic flux (ϕ_B)	Magnetic field × Area	weber $(= Tm^2)$	$ML^2T^{-2}A^{-1}$

(Continued)

Sl.	Physical Quantity and Symbol	Relationship with Other Physical Quantities	SI Unit	Dimensional Formula
76.	Inductance (L)	$\frac{\text{Magnetic flux}}{\text{Current}}$	henry (H)	$ML^2T^{-2}A^{-2}$
77.	Magnetic dipole moment $(\vec{m})$	$\frac{\text{Torque}}{\text{Magnetic field}}$ or $\text{Current} \times \text{Area}$	Am^2	$M^0L^2T^0A$
78.	Magnetic field strength, magnetic intensity or magnetic moment density $(\vec{B} \text{ or } \vec{H})$	$\frac{\text{Magnetic moment}}{\text{Volume}}$	Am^{-1}	$M^0L^{-1}T^0A$
79.	Permittivity constant (of free space) (ε_0)	$\frac{\text{Charge} \times \text{Charge}}{4\pi \times \text{Electric force} \times (\text{Distance})^2}$	$C^2N^{-1}m^{-2}$	$M^{-1}L^{-3}T^4A^2$
80.	Permeability constant (of free space) (μ_0)	$\frac{2\pi \times \text{Force} \times \text{Distance}}{\text{Current} \times \text{Current length}}$	NA^{-2}	$MLT^{-2}A^{-2}$
81.	Refractive index (μ)	$\frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$		$M^0L^0T^0$
82.	Faraday constant (F)	$\text{Avogadro constant} \times \text{Elementary charge}$	$Cmol^{-1}$	$M^0L^0T\,A\,mol^{-1}$
83.	Wave number $(\bar{\lambda})$	$\frac{2\pi}{\text{Wavelength}}$	m^{-1}	$M^0L^{-1}T^0$
84.	Radiant flux, Radiant power	$\frac{\text{Energy emitted}}{\text{Time}}$	W	ML^2T^{-3}
85.	Luminosity of radiant flux or radiant intensity	$\frac{\text{Radiant power of radiant flux of source}}{\text{Solid angle}}$	Wsr^{-1}	ML^2T^{-3}
86.	Luminous power or luminous flux of source	$\frac{\text{Luminous energy emitted}}{\text{Time}}$	W	ML^2T^{-3}
87.	Luminous intensity or illuminating power of source	$\frac{\text{Luminous flux}}{\text{Solid angle}}$	Wsr^{-1}	ML^2T^{-3}
88.	Intensity of illumination or luminance	$\frac{\text{Luminous intensity}}{(\text{Distance})^2}$	$Wm^{-2}sr^{-1}$	ML^0T^{-3}

(Continued)

Sl.	Physical Quantity and Symbol	Relationship with Other Physical Quantities	SI Unit	Dimensional Formula
89.	Relative luminosity	$\dfrac{\left(\text{Luminous flux of a source of given wavelength}\right)}{\left(\text{Luminous flux of peak sensitivity wavelength (555 nm) source of same power}\right)}$		$M^0L^0T^0$
90.	Luminous efficiency	$\dfrac{\text{Total luminous flux}}{\text{Total radiant flux}}$		$M^0L^0T^0$
91.	Illuminance or illumination	$\dfrac{\text{Luminous flux incident}}{\text{Area}}$	Wm^{-2}	ML^0T^{-3}
92.	Mass defect (Δm)	(Sum of masses of nucleons) – (Mass of the nucleus)	kg	ML^0T^0
93.	Binding energy of nucleus (BE)	Mass defect × (Speed of light in vacuum)^2	J	ML^2T^{-2}
94.	Decay constant (λ)	$\dfrac{0.693}{\text{Half life}}$	s^{-1}	$M^0L^0T^{-1}$
95.	Resonant frequency	$(\text{Inductance} \times \text{Capacitance})^{-\frac{1}{2}}$	Hz	$M^0L^0A^0T^{-1}$
96.	Quality factor of Q-factor of coil (Q)	$\dfrac{\text{Resonant frequency} \times \text{Inductance}}{\text{Resistance}}$		$M^0L^0T^0$
97.	Power of lens (P)	$(\text{Focal length})^{-1}$	dioptre (D)	$M^0L^{-1}T^0$
98.	Magnification (m)	$\dfrac{\text{Image distance}}{\text{Object distance}}$		$M^0L^0T^0$
99.	Capacitive reactance (X_C)	$(\text{Angular frequency} \times \text{Capacitance})^{-1}$	ohm(Ω)	$ML^2T^{-3}A^{-2}$
100.	Inductive reactance (X_L)	(Angular frequency × Inductance)	ohm(Ω)	$ML^2T^{-3}A^{-2}$

QUANTITIES HAVING SAME DIMENSIONS

Physical Quantities	Dimensional Formula
Frequency, angular frequency, angular velocity, velocity gradient and decay constant	$[M^0L^0T^{-1}]$
Work, internal energy, potential energy, kinetic energy, torque, moment of force.	$[M^1L^2T^{-2}]$

(Continued)

Physical Quantities	Dimensional Formula
Pressure, stress, Young's modulus, bulk modulus, modulus of rigidity, energy density.	$[M^1L^{-1}T^{-2}]$
Momentum, impulse.	$[M^1L^1T^{-1}]$
Acceleration due to gravity, gravitational field intensity.	$[M^0L^1T^{-2}]$
Thrust, force, weight, energy gradient.	$[M^1L^1T^{-2}]$
Angular momentum and Planck's constant	$[M^1L^2T^{-1}]$
Surface tension, Surface energy (energy per unit area).	$[M^1L^0T^{-2}]$
Strain, refractive index, relative density, angle, solid angle, distance gradient, relative permittivity (dielectric constant), relative permeability etc.	$[M^0L^0T^0]$
Latent heat and gravitational potential.	$[M^0L^2T^{-2}]$
Thermal capacity, gas constant, Boltzmann constant and entropy.	$[ML^2T^{-2}Q^{-1}]$
$\sqrt{\frac{l}{g}}$, $\sqrt{\frac{m}{k}}$, $\sqrt{\frac{R}{g}}$, where l = length, g = acceleration due to gravity, m = mass, k = spring constant, R = Radius of earth.	$[M^0L^0T^1]$
$\frac{L}{R}$, $\sqrt{LC}$, RC where L = inductance, R = resistance, C = capacitance.	$[M^0L^0T^1]$
I^2Rt, $\frac{V^2}{R}t$, VIt, qV, LI^2, $\frac{q^2}{C}$, CV^2 where I = current, t = time, q = charge, L = inductance, C = capacitance, R = resistance	$[ML^2T^{-2}]$

SYMBOLS

Following table gives the international symbols for SI units. In addition to the symbols of fundamental units, the symbols of derived units have also been included in this table.

Unit	Symbol	Unit	Symbol
metre	m	joule	J
kilogram	kg	watt	W
second	s	coulomb	C

(Continued)

Unit	Symbol	Unit	Symbol
ampere	A	weber	Wb
kelvin	K	ohm	ohm Ω
candela	cd	volt	V
radian	rad	farad	F
steradian	sr	henry	H
newton	N	siemen	S
hertz	Hz	tesla	T

Following points must be noted regarding the symbolic representation of various units.

(a) Small letters are used as symbols of units. However, if the symbol is derived from a proper name, then capital letter is used. As an example, the symbol of newton is "N" and **not** "n". It may be pointed out here only that if a unit is derived from the name of a person, only the symbol will be represented by capital letter. The unit itself will start with small letter. Thus, the unit of force will be written as "newton" and **not** as "Newton".

(b) The symbols of units are regarded as algebraic symbols. They are not followed by full stop, dots, dashes etc. Thus, SI unit of impulse will be represented by Ns and **not** N-s or N s.

(c) Some space is always left between the number and the symbol of the unit. Thus, it will be **incorrect** to write 2.4 kg. The **correct** representation is 2.4 kg.

(d) Even if the unit is in the plural form, "s" is not mentioned at the end of the unit. The same is true for its symbolic representation. Thus, it will be **incorrect** to write "metres". This will be written as "metre". In the same way, the symbol will be m and **not** ms.

PRINCIPLE OF HOMOGENEITY AND USES OF DIMENSIONAL ANALYSIS

According to this principle, the dimensions of all the terms of the two sides of an equation must be the same. If

$$X = A \pm (BC)^2 \pm \sqrt{\frac{DE}{F}}$$

then according to Principle of Homogeneity

$$[X] = [A] = \left[(BC)^2\right] = \left[\sqrt{\frac{DE}{F}}\right] = M^a L^b T^c$$

Taking help from the Principle of Homogeneity and knowing the dimensional formulae of various physical quantities, dimensional analysis can be employed to

(a) check the dimensional correctness of a physical relation.

(b) convert units from one system to another.

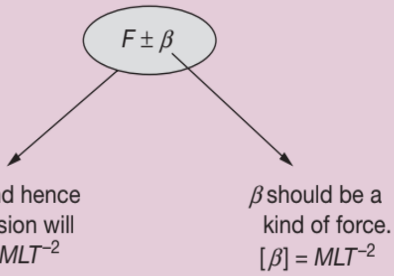
(c) find dependency of a physical quantity on other physical quantities.

Conceptual Note(s)

(a) Suppose in any formula, $(L \pm \alpha)$ term is coming (where L is length). As length can be added only with a length, so α should also be a kind of length.

$$\Rightarrow \quad [\alpha] = M^0 L^1 T^0 = L$$

(b) Similarly consider a term $(F \pm \beta)$ where F is force. A force can be added/subtracted with a force only and give rise to force. So β should be a kind of force and its result $(F \pm \beta)$ should also be a kind of force.



So, only like Physical Quantities (scalars with scalars and vectors with vectors) can be added or subtracted. However there is no restriction on multiplication and division of Physical Quantities.

ILLUSTRATION 1

Check the dimensional correctness of $Fs = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$.

SOLUTION

Given $\quad Fs = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \qquad \ldots(1)$

According to Principle of Homogeneity

$$[Fs] = \left[\frac{1}{2}mv^2\right] = \left[\frac{1}{2}mv_0^2\right]$$

$$\Rightarrow \quad (MLT^{-2})L = M(L^2T^{-2}) = M(L^2T^{-2})$$

$$\Rightarrow \quad ML^2T^{-2} = ML^2T^2 = ML^2T^{-2}$$

ILLUSTRATION 2

A student finds that pressure can be expressed as $P = \frac{4FV^2}{\pi t^2 x}$, where F is force, V is velocity, t is time and x is distance. However his teacher is doubtful about the expression. Express the way his teacher confirms the correctness of expression.

SOLUTION

Dimension of LHS $=[P]=M^1L^{-1}T^{-2}$

Dimension of RHS is $\left[\dfrac{4FV^2}{\pi t^2 x}\right]=\dfrac{[4][F][v^2]}{[\pi][t^2][x]}$

$$\Rightarrow \left[\frac{4FV^2}{\pi t^2 x}\right]=\frac{(M^1L^1T^{-2})(L^2T^{-2})}{(T^2)(L)}=M^1L^2T^{-6}$$

Dimension of LHS and RHS are not same. So the relation cannot be correct.

ILLUSTRATION 3

For n moles of gas, Vander Waal's equation is $\left(P+\dfrac{a}{V^2}\right)(V-b)=nRT$. Find the dimensions of a and b, where P is gas pressure. V is volume of gas and T is temperature of gas.

SOLUTION

$$\underbrace{\left(P+\frac{a}{V^2}\right)}_{\text{pressure}}\times\underbrace{(V-b)}_{\text{volume}}=nRT$$

$$\Rightarrow [P]=\left[\frac{a}{V^2}\right] \text{and } [b]=[V]=L^3$$

$$\Rightarrow \frac{[a]}{[V^2]}=M^1L^{-1}T^{-2}$$

$$\Rightarrow \frac{[a]}{(L^3)^2}=M^{-1}L^{-1}T^{-2}$$

$$\Rightarrow [a]=M^1L^5T^{-2} \text{ and } [b]=L^3$$

ILLUSTRATION 4

If, $\dfrac{\alpha}{t^2}=Fv+\dfrac{\beta}{x^2}$, then find dimension formula for α and β, ω here t is time, F is force, V is velocity, x is distance.

SOLUTION

Since $[Fv]=M^1L^2T^{-3}$

So, $\left[\dfrac{\beta}{x^2}\right]$ should also be $M^1L^2T^{-3}$

$$\Rightarrow \frac{[\beta]}{[x^2]}=M^1L^2T^{-3}$$

$$\Rightarrow [\beta]=M^1L^4T^{-3}$$

and $\left[Fv+\dfrac{\beta}{x^2}\right]$ will also have dimension $M^1L^2T^{-3}$

$$\Rightarrow \frac{[\alpha]}{[t^2]}=M^1L^2T^{-3}$$

$$\Rightarrow [\alpha]=M^1L^2T^{-1}$$

Test Your Concepts-I

Based on Principle of Homogeneity and Verification

(Solutions on page H.5)

1. If a composite physical quantity in terms of moment of inertia I, force F, velocity v, work W and length L is defined as, $Q=\left(\dfrac{IFv^2}{WL^3}\right)$. Find the dimensions of Q.
2. Can two physical quantities have same dimensions? Explain with example.
3. Find dimensions of universal gas constant R, universal gravitational constant G.
4. The rate of flow (V) of a liquid flowing through a pipe of radius r and a pressure gradient $\left(\dfrac{P}{\ell}\right)$ is given by Poiseuille's equation:
$$V=\frac{\pi}{8}\frac{\mathrm{Pr}^4}{\eta l}$$
Check the dimensional consistency of this equation.
5. Check the correctness of the equation: $y=a\sin(\omega t+\phi)$, where y = displacement, a = amplitude, ω = angular frequency and ϕ is an angle.
6. If E, M, J and G respectively denote energy, mass, angular momentum and gravitational constant, calculate the dimensions of $\dfrac{EJ^2}{M^5G^2}$.

7. In the formula $x = 3yz^2$, x and z are the dimensions of capacitance and magnetic induction respectively. Find the dimensions of y in MKSQ system.
8. State whether the following statement is true or false. Give very brief reason in support of your answer.

   The quantity $\frac{e^2}{2\varepsilon_0 hc}$ is dimensionless. Here e, h and c are electronic charge, Planck's constant and velocity of light respectively and ε_0 is the permittivity constant of free space.
9. When white light travels through glass, the refractive index of glass (μ = velocity of light in air/velocity of light in glass) is found to vary with wavelength as $\mu = A + \frac{B}{\lambda^2}$. Using the particle of homogeneity of dimensions, find the SI unit in which the constants A and B must be expressed.
10. A man walking briskly in rain with speed v must slant his umbrella forward making an angle θ with the vertical. A student derives the following relation between θ and v as $\tan\theta = v$ and checks that the relation has a correct limit: as $v \to 0$, $\theta \to 0$, as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct? If not, guess at the correct relation.

CONVERSION OF UNITS FROM ONE SYSTEM TO ANOTHER

The measure of a physical quantity is given by

$$nu = \text{constant}$$

If a physical quantity X has dimensional formula $M^aL^bT^c$ and if (derived) units of that physical quantity in two systems are $M_1^aL_1^bT_1^c$ and $M_2^aL_2^bT_2^c$, respectively and n_1 and n_2 be the numerical values in the two systems respectively, then

$$n_1[u_1] = n_2[u_2]$$

$$\Rightarrow \quad n_1\left(M_1^aL_1^bT_1^c\right) = n_2\left(M_2^aL_2^bT_2^c\right)$$

$$\Rightarrow \quad n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$$

a, b and c are the respective dimensions in mass, length and time of the physical quantity to be converted.

M_1, L_1 and T_1 are fundamental units of mass, length and time in the **first (known)** system and M_2, L_2 and T_2 are fundamental units of mass, length and time in the **second (unknown)** system.

Thus knowing the values of fundamental units in two systems and numerical value in one system, the numerical value in other system may be evaluated.

ILLUSTRATION 5

If 1 hp is 746 watt, comment on the statement "1 hp is 550 ft lb/s".

SOLUTION

hp is unit of power, so dimensional formula will be ML^2T^{-3}. Hence $a = 1$, $b = 2$ and $c = -3$.

So 1 hp = 746 watt = 746 kg m^{2}s^{-3}

Now $M_1 = 1$ kg $\quad M_2 = 1$ lb

$L_1 = 1$ m $\quad L_2 = 1$ ft

$T_1 = 1$ sec $\quad T_2 = 1$ sec

$n_1 = 746 \quad n_2 = ?$

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$$

where $a = 1$, $b = 2$, $c = -3$

$$n_2 = 746\left(\frac{1\text{ kg}}{0.4536\text{ kg}}\right)^1\left(\frac{1\text{ m}}{0.3048\text{ m}}\right)^2\left(\frac{1\text{ sec}}{1\text{ sec}}\right)^{-3}$$

$$\Rightarrow \quad n_2 = 746 \times 23.73$$

{as 1 lb = 0.4536 kg, 1 ft = 0.3048 m}

$$\Rightarrow \quad n_2 = 17702.55$$

As $n_1u_1 = n_2u_2$

$$1\text{ hp} = 746\text{ watt} = 17702.55\text{ lbft}^2\text{s}^{-3}$$

Since $(1\ \text{lb})\times\left(1\ \frac{\text{ft}}{\text{s}^2}\right)=1$ poundal

$$17702.55\ \text{lbft}^2\text{s}^{-3}=17702.55\ \frac{\text{foot-poundal}}{\text{sec}}$$

Since 1 lb = 32.2 poundal

$$\Rightarrow\ 1\ \text{hp}=\frac{17702.55}{32.2}\ \frac{\text{ftlb}}{\text{s}}$$

$$\Rightarrow\ 1\ \text{hp}\approx 550\ \text{ftlbs}^{-1}$$

Conceptual Note(s)

The poundal is a non-SI unit of force and is a part of FPS system of units. It is equal to $1\ \frac{(\text{lb})(\text{ft})}{\text{s}^2}$.

ILLUSTRATION 6

If velocity of light in air $(=3\times10^8\ \text{ms}^{-1})$, acceleration due to gravity $(=9.8\ \text{ms}^{-2})$ and density of mercury at $0°\text{C}$ $(13600\ \text{kgm}^{-3})$ be chosen as fundamental units, find the unit of mass, length and time.

SOLUTION

Given

$$LT^{-1}=3\times10^8\ \text{ms}^{-1} \quad \ldots(1)$$

$$LT^{-2}=9.8\ \text{ms}^{-2} \quad \ldots(2)$$

$$ML^{-3}=13600\ \text{kgm}^{-3} \quad \ldots(3)$$

Dividing (1) by (2)

$$\frac{LT^{-1}}{LT^{-2}}=\frac{3\times10^8}{9.8}$$

$$\Rightarrow\ T=3.061\times10^7$$

Substituting for T in equation (1)

$$L\left(3.061\times10^7\right)^{-1}=3\times10^8$$

$$\Rightarrow\ L=3\times3.061\times10^{15}$$

$$\Rightarrow\ L=9.918\times10^{15}\,\text{m}$$

Substituting for L in equation (3)

$$M\left(9.183\times10^{15}\right)^{-3}=13600$$

$$\Rightarrow\ M=\frac{13600}{\left(9.183\times10^{15}\right)^{-3}}$$

$$\Rightarrow\ M=13600\times(9.183)^3\times10^{45}$$

$$\Rightarrow\ M=1.29\times10^{51}\ \text{kg}$$

So, $M=1.29\times10^{51}\,\text{kg}$,

$L=9.183\times10^{15}\,\text{m}$,

$T=3.061\times10^7\,\text{s}$

ILLUSTRATION 7

In two systems of relations between velocity, acceleration and force are respectively given by

$$v_2=\frac{\alpha^2}{\beta}v_1,\ a_2=\alpha\beta a_1 \text{ and } F_2=\frac{F_1}{\alpha\beta}.$$

If α and β are constants then find the relations between mass, length and time in two systems.

SOLUTION

$$v_2=v_1\frac{\alpha^2}{\beta}$$

$$\Rightarrow\ \left(L_2T_2^{-1}\right)=\left(L_1T_1^{-1}\right)\frac{\alpha^2}{\beta} \quad \ldots(1)$$

$$a_2=a_1\alpha\beta$$

$$\Rightarrow\ \left(L_2T_2^{-2}\right)=\left(L_1T_1^{-2}\right)\alpha\beta \quad \ldots(2)$$

Since $F_2=\frac{F_1}{\alpha\beta}$

$$\Rightarrow\ \left(M_2L_2T_2^{-2}\right)=\left(M_1L_1T_1^{-2}\right)\frac{1}{\alpha\beta} \quad \ldots(3)$$

Dividing equation (3) by equation (2) we get

$$M_2=\frac{M_1}{(\alpha\beta)\alpha\beta}=\frac{M_1}{\alpha^2\beta^2}$$

Squaring equation (1) and dividing by equation (2) we get

$$L_2=L_1\frac{\alpha^3}{\beta^3}$$

Dividing equation (1) by equation (2) we get

$$T_2=T_1\frac{\alpha}{\beta^2}$$

Test Your Concepts-II

Based on Principle of Homogeneity: Conversion

(Solutions on page H.5)

1. Calorie is the unit of heat energy and its value is 4.18 J. Suppose we use a new system of units in which the unit of mass is α kg, unit of length is β m and that of time is γ s. Find the value of calorie in terms of the new system of units.
2. Express the power of 100 W bulb in CGS unit with proper prefix.
3. The *CGS* unit of viscosity is poise (*P*). Find how many poise are there in 1 MKS unit of viscosity called poiseuille (*Pl*)?
4. If the units of force, energy and velocity are 20 N, 200 J and 5 ms^{-1}, find the units of length, mass and time.
5. Density of a material in the cgs system is 8 gcc^{-1}. In a system of units in which the unit of mass is 20 g and that of length is 5 cm, what is the density of the material in this new system of units.
6. Given that 1 pound = 1 lb = 0.4536 kgwt, 1 foot = 0.3048 m, then by dimensional analysis find the value of 1 horse power. Given that 1 hp = 550 foot lbs^{-1}.
7. Calculate the dimensions of linear momentum and surface tension in terms of velocity *v*, density ρ and frequency ν as fundamental quantities.
8. A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the sun and earth in terms of the new unit if light take 8 min and 20 s to cover this distance.
9. If the unit of force is 1 kN, unit of length 1 km and the unit of time is 100 s, what will be the unit of mass.
10. It is estimated that per minute each cm^2 of earth receives about 2 calorie of heat energy from the sun. This constant is called solar constant *S*. Express solar constant in SI units.

TO DERIVE THE NEW RELATIONS

If a physical quantity X depends on other physical quantities P, Q and R (say), then we may write

$$X \propto P^a Q^b R^c \qquad \ldots(1)$$

respectively.
Then writing dimensional formula for X, P, Q and R and equating the dimensions on either sides give the values of a, b and c. The substitution of these values in (1) gives the new dimensional relation.

ILLUSTRATION 8

The time period (T) of a simple pendulum depends upon the length of the thread (l), mass of bob (m), acceleration due to gravity (g) and the angle of swing (θ). Find the relation of t with other physical quantities.

SOLUTION

It is found experimentally that T depends upon length of thread (l), mass of bob (m), acceleration due to gravity g, and angle of swing (θ).
So $T = f(l, m, g, \theta)$

If the function is product of power functions of l, m, g, θ

$$T = kl^a m^b g^c \theta^d$$

where k is dimensionless constant

$$T = (L)^a (M)^b (LT^{-2})^c \qquad \{\theta \text{ is dimensionless}\}$$

$$[T] = [M^b L^{a+c} T^{-2c}]$$

$$\Rightarrow \quad T = M^b L^{a+c} T^{-2c}$$

Equating the exponents of similar quantities, we get

$$b = 0;\ a + c = 0;\ -2c = 1$$

Solving for a, b, c we get

$$a = \frac{1}{2},\ b = 0,\ c = -\frac{1}{2}$$

$$\Rightarrow \quad T = Kl^{1/2} g^{-1/2}$$

on experimental grounds

$$K = 2\pi$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{l}{g}}$$

From, this illustration, it is clearly that dimensional analysis does not provide any information about dependence of a physical quantity on dimensionless physical quantities (like θ angle of swing).

ILLUSTRATION 9

Given that the time period t of oscillation of a gas bubble from an explosion under water depends upon the static pressure p, the density of water d and the total energy of explosion E. Find a dimensional relation for t.

SOLUTION

$$t \propto p^a d^b E^c$$

$$\Rightarrow \quad t = kp^a d^b E^C$$

where k is a dimensionless constant

$$\Rightarrow \quad T = \left(ML^{-1}T^{-2}\right)^a \left(ML^{-3}\right)^b \left(ML^2T^{-2}\right)^c$$

$$\Rightarrow \quad T = M^{a+b+c}L^{-a-3b+2c}T^{-2a-2c}$$

Using Principle of Homogeneity we get

$$a + b + c = 0 \quad \ldots(1)$$

$$-a - 3b + 2c = 0 \quad \ldots(2)$$

$$-2a - 2c = 1 \quad \ldots(3)$$

From (3), we get

$$a + c = -\frac{1}{2}$$

Substituting in (1),

$$-\frac{1}{2} + b = 0$$

$$\Rightarrow \quad b = \frac{1}{2}$$

Substituting in (2), we get

$$-a - \frac{3}{2} + 2c = 0$$

$$\Rightarrow \quad -a + 2c = \frac{3}{2} \quad \ldots(4)$$

Adding (3) and (4), we get

$$-3a = \frac{5}{2}$$

$$\Rightarrow \quad a = -\frac{5}{6}$$

Substituting in (3), to get

$$-2\left(-\frac{5}{6}\right) - 2c = 1$$

$$\Rightarrow \quad \frac{5}{3} - 2c = 1$$

$$\Rightarrow \quad 2c = \frac{5}{3} - 1$$

$$\Rightarrow \quad 2c = \frac{2}{3}$$

$$\Rightarrow \quad c = \frac{1}{3}$$

Hence, we get

$$a = -\frac{5}{6},\ b = \frac{1}{2},\ c = \frac{1}{3}$$

$$\Rightarrow \quad T = kp^{-\frac{5}{6}} d^{\frac{1}{2}} E^{\frac{1}{3}}$$

ILLUSTRATION 10

The planets move round the sun in nearly circular orbits. Assuming that the period of rotation depends upon the radius of the orbit, the mass of the Sun and the Universal Gravitational constant. Show that the planet obeys the Kepler's Third Law of Planetary Motion.

SOLUTION

To show that the planet obeys Kepler's Third Law of planetary motion, we have to prove that

$$T^2 \propto r^3$$

Now, according to the problem, we have

$$T \propto r^a M_S^b G^c$$

$$\Rightarrow \quad T = kr^a M_S^b G^c$$

where k is a dimensionless constant.

$$\Rightarrow \quad T = L^a M^b \left(M^{-1}L^3T^{-2}\right)^c$$

$$\Rightarrow \quad T = M^{b-c}L^{a+3c}T^{-2c}$$

Using the Principle of Homogeneity, we get

$$b - c = 0 \quad \ldots(1)$$

$$a + 3c = 0 \quad \ldots(2)$$

$$-2c = 1 \quad \ldots(3)$$

$$\Rightarrow \quad c=-\frac{1}{2},\ b=-\frac{1}{2},\ a=\frac{3}{2}$$

So, we get

$$T=kr^{\frac{3}{2}}M_S^{-\frac{1}{2}}G^{-\frac{1}{2}}$$

$$\Rightarrow \quad T^2=\left(\frac{k^2}{M_S G}\right)r^3$$

Since the product $M_S G$ is constant, so

$$T^2 \propto r^3$$

ILLUSTRATION 11

The height to which the liquid rises in the capillary tube of radius r depends upon in addition to r on the surface tension S of the liquid, the density d of the liquid and the acceleration due to gravity g. Is it possible to obtain dimensionally a relation for h without the experimental information that h is inversely proportional to r? What is the relation?

SOLUTION

No, it would not be possible to obtain dimensionally a relation for h without the additional experimental information that $h \propto \frac{1}{r}$. This is because without the information $h \propto \frac{1}{r}$, we will get four variables to be calculated from three equations which is just impossible. So, we shall be writing

$$h \propto r^{-1}S^a d^b g^c$$

$$\Rightarrow \quad h=kr^{-1}S^a d^b g^c$$

where k is a dimensionless constant

$$\Rightarrow \quad L=L^{-1}\left(MT^{-2}\right)^a\left(ML^{-3}\right)^b\left(LT^{-2}\right)^c$$

$$\Rightarrow \quad L=M^{a+b}L^{-1-3b+c}T^{-2a-2c}$$

Using Principle of Homogeneity, we get

$$a+b=0 \qquad \ldots(1)$$

$$-1-3b+c=1 \qquad \ldots(2)$$

$$-2a-2c=0 \qquad \ldots(3)$$

So, $2\times(1)+(3)$ gives

$$2b-2c=0 \qquad \ldots(4)$$

Again $2\times(2)+(4)$ gives

$$b=-1$$

$$\Rightarrow \quad a=1 \qquad \{\because \text{ of } (1)\}$$

$$\Rightarrow \quad c=-1$$

So, $a=1$, $b=-1$ and $c=-1$

$$\Rightarrow \quad h=kr^{-1}Sd^{-1}g^{-1}$$

$$\Rightarrow \quad h=k\left(\frac{S}{rgd}\right)$$

ILLUSTRATION 12

If the velocity of light (c), gravitational constant (G) and Planck's constant (h) are chosen as fundamental units, then find the dimensional formula of mass in this new system.

SOLUTION

Let $m \propto c^x G^y h^z$ or $m=Kc^x G^y h^z$

By substituting the dimension of each quantity in both sides

$$[M^1L^0T^0]=K[LT^{-1}]^x[M^{-1}L^3T^{-2}]^y[ML^2T^{-1}]^z$$

$$[M^1L^0T^0]=[M^{-y+z}L^{x+3y+2z}T^{-x-2y-z}]$$

By equating the power of M, L and T in both sides:

$$-y+z=1,\ x+3y+2z=0,\ -x-2y-z=0$$

By solving above three equations, we get

$$x=\frac{1}{2},\ y=-\frac{1}{2} \text{ and } z=\frac{1}{2}$$

So, $m \propto c^{1/2}G^{-1/2}h^{1/2}$

Test Your Concepts-III

Based on Principle of Homogeneity: Dependence

(Solutions on page H.6)

1. Given that the time period of oscillation of a small drop of liquid under the influence of surface tension depends upon the density *d*, radius *r* and the surface tension *S*. Find the expression for the time period.
2. Given that the time period *t* of oscillation of a gas bubble from an explosion under water depends upon the static pressure *p*, the density of water *d* and the total energy of explosion *E*. Find a dimensional relation for *t*.
3. The time period of vibration of a stretched string depends upon the mass *m* of the string, tension *f* in the string and the length of the string. Find an expression for the time period of oscillation of the string.
4. A small steel ball of radius *r* is allowed to fall under the gravity through a column of liquid of coefficient of viscosity η. After some time the ball attains a constant velocity called the terminal velocity v_T. This terminal velocity depends upon the weight *W*, the coefficient of viscosity η and the radius of the ball *r*. Find an expression for the terminal velocity.
5. A liquid is flowing steadily through a capillary tube. The rate of flow of the volume of the liquid depends upon the coefficient of viscosity η, radius of the tube *r* and the pressure gradient along the tube *p*. Find an expression for the rate of flow of the volume of the liquid through the tube.
6. The critical angular velocity ω_c of a cylinder inside another cylinder containing a liquid at which its turbulence occurs depends on viscosity η, density ρ and the distance *d* between the walls of the cylinder. Find the expression for ω_c.
7. The height to which the liquid rises in the capillary tube of radius *r* depends upon in addition to *r* on the surface tension *S* of the liquid, the density *d* of the liquid and the acceleration due to gravity *g*. Is it possible to obtain dimensionally a relation for *h* without the experimental information that *h* is inversely proportional to *r*? What is the relation?
8. The planets move round the sun in nearly circular orbits. Assuming that the period of rotation depends upon the radius of the orbit, the mass of the Sun and the Universal Gravitational constant. Show that the planet obeys the Kepler's Third Law of Planetary Motion.
9. If density ρ, acceleration due to gravity *g* and frequency v are the basic quantities, find the dimensions of force.
10. If velocity, force and time are taken to be fundamental quantities find dimensional formula for
    (a) Mass, and
    (b) Energy.

LIMITATIONS OF DIMENSIONAL ANALYSIS

The following are the limitations of dimensional analysis

(a) One has to make a guess about the dependence of the Physical Quantity on other Physical Quantities, which may or may not work.

(b) This method fails to find the dependency on functions other than power functions. As an example we cannot find the relation $s = ut + \frac{1}{2}at^2$, using the method of dimensional analysis. However dimensional analysis does help us to check the physical correctness of this relation.

(c) This method would also fail to find the dependency of a Physical Quantity on the two Physical Quantities with identical dimensional formulae.

(d) It gives no information about dimensionless constants which are to be calculated either by experiments or by actual derivation.

(e) It gives no information about the dependence of a physical quantity on trigonometrical, exponential and logarithmic functions as all are dimensionless i.e. it cannot find dependence on dimensionless physical quantities.

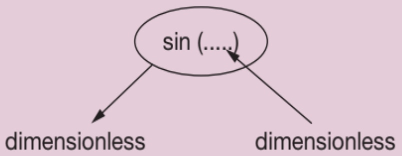
(f) If a physical quantity of mechanics depends on more than three physical quantities all having dimensional formulae, then dimensional analysis cannot be used to derive their relationship.

(g) Dimensional correctness does not establish numerical correctness but reverse is true.

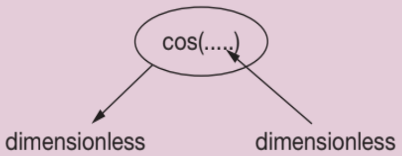
Conceptual Note(s)

Consider the terms, $\sin\theta$, $\cos\theta$, $\tan\theta$ (and their reciprocals) where θ is dimensionless and $\sin\theta = \left(\dfrac{\text{Perpendicular}}{\text{hypotenuse}}\right)$ is also dimensionless. Similarly $\cos\theta$ and $\tan\theta$ are also dimensionless. Whatever comes in sin(.....) is dimensionless and entire [sin(.....)] is also dimensionless. So,

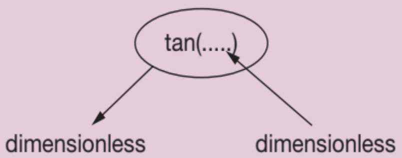
(a)



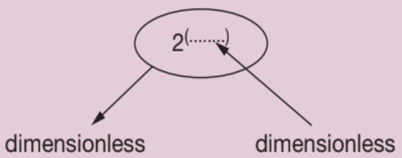
(b)



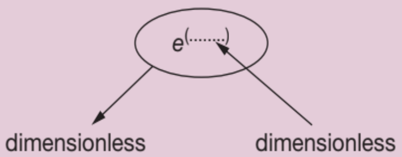
(c)



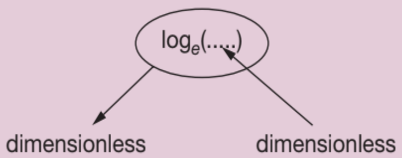
(d)



(e)



(f)



So, to conclude we have

1. $\sin x$, $\cos x$, $\tan x$, $\operatorname{cosec} x$, $\sec x$, $\cot x$, $\log x$, e^x, a^x all are dimensionless.

   i.e., $[\sin x] = [\cos x] = [\tan x] = [\operatorname{cosec} x] =$

   $$[\sec x] = [\log x] = [e^x] = [a^x] = M^0L^0T^0$$

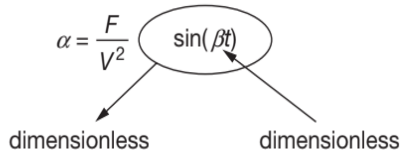
2. The argument of all the functions i.e. x is also dimensionless. Hence

   $$[x] = M^0L^0T^0$$

ILLUSTRATION 13

If V is velocity, F is force, t is time and $\alpha = \dfrac{F}{V^2}\sin(\beta t)$, then find the dimensional formula of α and β.

SOLUTION



$$\Rightarrow \quad [\beta t] = M^0L^0T^0$$

$$\Rightarrow \quad [\beta] = T^{-1}$$

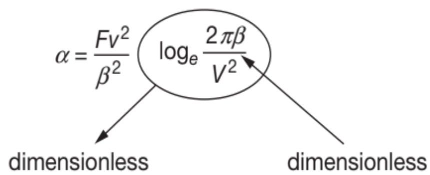
Similarly $[\alpha] = \dfrac{[F]}{[V^2]}$

$$\Rightarrow \quad \alpha = \frac{[M^1L^1T^{-2}]}{[L^1T^{-1}]^2} = M^1L^{-1}T^0$$

ILLUSTRATION 14

If, $\alpha = \dfrac{FV^2}{\beta^2}\log_e\left(\dfrac{2\pi\beta}{V^2}\right)$ where F is force, V is velocity, then find the dimensional formula of α and β.

SOLUTION



$$\Rightarrow \quad [\alpha]=\frac{[F][V^2]}{[\beta^2]} \qquad \ldots(1)$$

Also, $\frac{[2\pi][\beta]}{[V^2]}=1$

$$\Rightarrow \quad \frac{[1][\beta]}{L^2T^{-2}}=1$$

$$\Rightarrow \quad [\beta]=L^2T^{-2}$$

Substituting β in (1), we get

$$[\alpha]=\frac{(M^1L^1T^{-2})(L^2T^{-2})}{(L^2T^{-2})^2}$$

$$\Rightarrow \quad [\alpha]=M^1L^{-1}T^0$$

LEAST COUNT

The minimum measurement that can be actually taken by an instrument is called the **least count**.

The least count of a metre scale graduated in millimetre mark is 1 mm.

The least count of a watch having second's hand is 1 second.

The least count of a stop watch is $\frac{1}{100}$th of a second i.e., $\frac{1}{100}$ s.

Conceptual Note(s)

(a) Least count (LC) of vernier calliper (VC) is given by

$$LC=\begin{bmatrix}\text{Value of one}\\ \text{part on main}\\ \text{scale}\end{bmatrix}-\begin{bmatrix}\text{Value of one}\\ \text{part on vernier}\\ \text{scale}\end{bmatrix}$$

$$\Rightarrow \quad LC = 1\,MSD - 1\,VSD$$

where

MSD = Main Scale Division

VSD = Vernier Scale Division

(b) Least count (LC) of vernier calliper (VC) is also given by

$$LC=\frac{\text{Value of 1 part on main scale }(s)}{\text{Number of parts on vernier scale }(n)}$$

(c) Least count (LC) of screw gauge (SG) is given by

$$LC=\frac{\text{Pitch}(p)}{\text{Number of parts on circular scale}(n)}$$

These have been discussed in detail at the end of the chapter.

(d)

Measurement from Device	Least Count
2.890 m	0.001 m
0.005 kg	0.001 kg
15.01 cm	0.01 cm
327.92 mm	0.01 mm
1111.111 m	0.001 m

SIGNIFICANT FIGURES

Significant figures in the measured value of a physical quantity are the number of digits which are known reliably plus the one additional digit which is uncertain.

Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement. The reverse is also true.

The following rules are observed in counting the number of significant figures in a given measured quantity.

RULE-1

All non-zero digits are significant.

EXAMPLE:

42.3 has three significant figures.
243.4 has four significant figures.
24.123 has five significant figures.

RULE-2

All zeros between two non-zero digits are significant.

EXAMPLE:

5.03 has three significant figures.
5.004 has four significant figures.
140.004 has six significant figures.

RULE-3

Leading zeros or the zeros placed to the left of the number are never significant

EXAMPLE:

0.543 has three significant figures.
0.045 has two significant figures.
0.006 has one significant figures.

RULE-4

Zeros that occur at the end of a number (i.e., on the right-hand side) without an expressed decimal point are ambiguous (i.e., we have no information on whether they are significant or not) and hence they are not considered to be significant.

EXAMPLE:

575000 has three significant figures.
3000 has one significant figure.

Conceptual Note(s)

(a) The measurements like 190 km may have 2 or 3 significant figures and 50800 calorie may have 3, 4 or 5 significant figures.

(b) The potential ambiguity here can be avoided by making use of Scientific Notation. Depending on whether 3, 4 or 5 significant figures are correct, we can write 50800 calorie as:

5.08×10^4 calorie has 3 significant figures.
5.080×10^4 calorie has 4 significant figures.
5.0800×10^4 calorie has 5 significant figures.

RULE-5

Trailing zeros or the zeros placed to the right of the number are significant.

EXAMPLE:

4.330 has four significant figures.
433.00 has five significant figures.
343.000 has six significant figures.

RULE-6

In exponential notation (or the results expressed in powers of 10), the numerical portion gives the number of significant figures i.e., the powers of 10 are not to be counted as significant figures.

EXAMPLE:

1.32×10^{-2} has three significant figures.
1.32×10^{4} has three significant figures.

RULE-7

The number of significant figures does not depend upon the system of units used to measure the quantity.

EXAMPLE:

164 mm, 16.4 cm, 0.164 m, 0.000164 km, 164×10^{-6} km all have three significant figures.

RULE-8

There are certain measurements that are exact, like "If the number of students sitting in a class is say 125". Then the number of students sitting in the class is 125.00000000000000000 … and so this type of measurement is infinitely accurate and possesses infinite significant figures.

EXAMPLE:

Number of apples in a pack is 12,
Number of spheres in a box is 25
Both have infinite significant figures.

ROUNDING OFF

While rounding off measurements, we use the following rules by convention:

(a) *If the digit to be dropped is less than 5, then the preceding digit is left unchanged.*

EXAMPLE:

7.82 rounded off to one decimal place (RO1DP) is 7.8
3.94 rounded off to one decimal place (RO1DP) is 3.9
47.833 rounded off to one decimal place (RO1DP) is 47.8
47.862 rounded off to two decimal place (RO2DP) is 47.86

(b) *If the digit to be dropped is more than 5, then the preceding digit is raised by one.*

EXAMPLE:

6.87 rounded off to one decimal place (RO1DP) is 6.9
12.78 rounded off to one decimal place (RO1DP) is 12.8

47.862 rounded off to one decimal place (RO1DP) is 47.9

6.758 rounded off to two decimal place (RO2DP) is 6.76

(c) *If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one.*

EXAMPLE:

16.351 rounded off to one decimal place (RO1DP) is 16.4

6.758 rounded off to one decimal place (RO1DP) is 6.8

(d) *If digit to be dropped is 5 or 5 followed by zeros, then preceding digit is left unchanged, if it is even.*

EXAMPLE:

3.250 rounded off to one decimal place (RO1DP) is 3.2

12.6850 rounded off to two decimal place (RO2DP) is 12.68

12.6850 rounded off to one decimal place (RO1DP) is 12.7

(e) *If digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one, if it is odd.*

EXAMPLE:

3.750 rounded off to one decimal place (RO1DP) is 3.8

16.150 rounded off to one decimal place (RO1DP) is 16.2

PRECISION AND ACCURACY OF A MEASUREMENT

Precision

Precision of measurement is the uncertainity in number. The precision of measurement is determined by the least count of measuring instrument. The smaller is the least count, larger is the precision of measurement.

Accuracy

The accuracy of a measurement is also determined by the number of significant figures. Larger the number of significant figures, more accurate is the measurement.

EXAMPLE:

If you find a length to be between values 3.375×10^7 m and 3.3765×10^7 m. The standard statement of your result is:

$$\text{Length} = (3.3760 \pm 0.0005)\times10^7 \text{ m}$$

The length 3.376×10^7 has an accuracy of four significant figures and precision of 0.0005×10^7 m.

SIGNIFICANT FIGURES IN CALCULATIONS: FEW EXAMPLES

In most of the experiments, the observations of various measurements are to be combined mathematically, i.e., added, subtracted, multiplied or divided as to achieve the final result. Since, all the observations in measurements do not have the same precision, it is natural that the final result cannot be more precise than the least precise measurement. The following two rules should be followed to obtain the proper number of significant figures in any calculation.

1. In addition or subtraction of the numbers having different precisions, the result should be reported to the same number of decimal places as **are present in the number having the least number of decimal places**. In other words, the final result cannot be more precise than the least precise measurement. The rule is illustrated by the following examples:

(i)
$$\begin{array}{l} 33.3 \leftarrow (\text{has only one decimal place}) \\ +3.11 \\ +0.313 \\ \hline 36.723 \leftarrow \begin{pmatrix}\text{answer should be reported} \\ \text{to one decimal place}\end{pmatrix} \\ \hline \end{array}$$

Answer = 36.7

(ii)
$$\begin{array}{l} 3.1421 \\ +0.241 \\ +0.09 \leftarrow (\text{has 2 decimal places}) \\ \hline 3.4731 \leftarrow \begin{pmatrix}\text{answer should be reported} \\ \text{to 2 decimal places}\end{pmatrix} \\ \hline \end{array}$$

Answer = 3.47

(iii)
$$\begin{array}{l} 62.831 \leftarrow (\text{has 3 decimal places}) \\ -24.5492 \\ \hline 38.2818 \leftarrow \begin{pmatrix}\text{answer should be reported} \\ \text{to 3 decimal places after} \\ \text{rounding off}\end{pmatrix} \\ \hline \end{array}$$

Answer = 38.2818

2. In multiplication and division of numbers having different precisions, the final result should be reported to the same number of significant figures as that of the original number with **minimum number of significant figures**. In other words the result in multiplication or division **cannot be more accurate** than the **least accurate** measurement.

(i) 142.06

$\times 0.23 \quad \leftarrow (\text{two significant figures})$

32.6738

Answer $= 33$

(ii) 51.028

$\times 1.31 \quad \leftarrow (\text{three significant figures})$

66.84668

Answer $= 66.8$

(iii) $\dfrac{0.90}{4.26} = 0.2112676$

Answer $= 0.21$

ORDER OF MAGNITUDE: REVISITED

For determining this power, the value of the quantity has to be rounded off. While rounding off, we ignore the last digit which is less than 5. If the last digit is 5 or more than five, the preceding digit is increased by one. For example,

(a) Speed of light in vacuum $= 3 \times 10^8 \text{ ms}^{-1} \approx 10^8 \text{ ms}^{-1}$ (ignoring $3 < 5$)

(b) Mass of electron $= 9.1 \times 10^{-31}$ kg $\approx 10^{-30}$ kg (as $9.1 > 5$)

Conceptual Note(s)

(a) Change in units of measurement of a quantity does not affect the number of significant figures.

(b) Significant figures are quoted for a measurement not for a pure number.

(c) Greater is number of significant figures in a measurement, smaller is the percentage error.

ORDER OF MAGNITUDE

In Physics, measurement of all things from atom to universe is done, so we have to deal with very small and very large magnitudes. Hence, we often talk about order of magnitude. In scientific notation the numbers are expressed as, Number $= M \times 10^x$. Where M is a number lies between 1 and 10 and x is integer. Order of magnitude of quantity is the power of 10 required to represent the quantity. Order of magnitude is expressed in terms of powers of 10 and is taken to be 10° if $M \le \sqrt{10}$ and 10^1 if $M > \sqrt{10}$ (where $\sqrt{10} = 3.16$).

EXAMPLE:

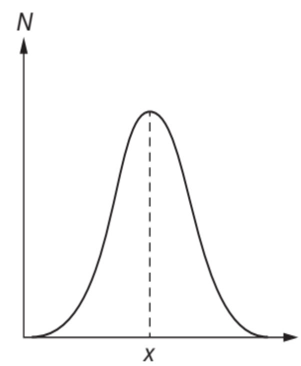
Speed of light is $3 \times 10^8 \text{ ms}^{-1}$, its order of magnitude is and mass of electron is 9.1×10^{-31} kg, its order of magnitude is $10 \times 10^{-31} \approx 10^{-30}$ kg.

ERRORS IN A REPEATED MEASUREMENT

If we take a measurement experimentally, it necessarily involves errors, due to two factors.

(a) Human errors, which may be due to reaction time or carelessness.

(b) Experimental errors, which are due to least count of measuring instruments. For given measuring instruments, the human errors may be reduced by repeating experiment for a large number of times. If a graph is plotted between number of observations and the observed quantity x, the graph is shown in figure. Such a curve is called **Gaussian distribution** or **normal distribution curve**.



MEAN VALUE

If $x_1, x_2, x_3 ... x_n$ are n measured values of a physical quantity, then the mean value is given by

$$x_{av} = \langle x \rangle = \bar{x} = \frac{x_1 + x_2 + x_3 + ... x_n}{N} = \frac{\sum_{i=1}^{N} x_i}{N}$$

STANDARD DEVIATION (σ)

The spread of the experimental data is measured by the quantity called **Standard Deviation, defined as**

$$\sigma = \sqrt{\frac{1}{N-1} \sum_i (x_i - x_{av})^2}$$

STANDARD ERROR IN THE MEAN

The standard error (also called probable error of the mean) α, for a given set of readings (data) is

$$\alpha = \frac{\sigma}{\sqrt{N}}$$

The larger is the number of readings, the smaller is the error.

EXAMPLE:

$10^{\circ} \times 10^{8} = 10^{8}\ \text{ms}^{-1}$

A student takes 100 readings and standard error is *e*. If he takes 400 readings, then the error will be $\frac{e}{\sqrt{4}} = \frac{e}{2}$. So, on taking 400 readings the standard error will be halved.

ABSOLUTE ERRORS

The positive difference between arithmetic mean value and the measured value in the ith observation is called as the absolute error of that observation. The arithmetic mean value is also called as true value.

Absolute error is

$$|(\text{mean value or true value}) - (i\text{th measured value})|$$

$$\Rightarrow \quad \Delta a_i = |a_{av} - a_i|$$

where Δa_i is absolute error in the ith observation. Then clearly

$$\Delta a_1 = |a_{av} - a_1|$$

$$\Delta a_2 = |a_{av} - a_2|$$

....

....

$$\Delta a_n = |a_{av} - a_n|$$

The arithmetic mean of all the absolute errors is called as mean absolute error and is given by

$$(\Delta a)_{av} = \langle \Delta a \rangle = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + ... + |\Delta a_n|}{n}$$

$$(\Delta a)_{av} = \langle \Delta a \rangle = \frac{1}{n} \sum_{i=1}^{n} |\Delta a_i|$$

And if we take the single measurement then the result of measurement will be $\langle a \rangle \pm \langle \Delta a \rangle = a_{av} + (\Delta a)_{av}$

RELATIVE AND PERCENTAGE ERROR

The relative error is defined as the ratio of the mean absolute error to the mean value or the true value. Mathematically,

$$\text{Relative error} = \frac{(\Delta a)_{av}}{a} = \frac{\langle \Delta a \rangle}{a}$$

$$\Rightarrow \quad \text{Percentage relative error} = \frac{\langle \Delta a \rangle}{a} \times 100\%$$

ILLUSTRATION 15

The length of a rod as measured in an experiment is recorded as 2.50 m, 2.54 m, 2.49 m, 2.58 m, 2.49 m, 2.57 m respectively. Find the mean/true length, absolute error in each case, mean absolute error and the percentage error.

SOLUTION

Mean length or true length

$$a_{av} = \langle a \rangle = \frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6}{6}$$

$$\langle a \rangle = \frac{2\cdot 50 + 2.54 + 2.49 + 2.58 + 2.49 + 2.57}{6}$$

$$\langle a \rangle = \frac{15.17}{6} = 2.528$$

$$\Rightarrow \quad \langle a \rangle \cong 2.53\ \text{m}$$

$$\Delta a_1 = |a_{av} - a_1| = |2.53 - 2.50| = 0.03$$

$$\Delta a_2 = |a_{av} - a_2| = |2.53 - 2.54| = 0.01$$

$$\Delta a_3 = |a_{av} - a_3| = |2.53 - 2.49| = 0.04$$

$$\Delta a_4 = |a_{av} - a_4| = |2.53 - 2.58| = 0.05$$

$$\Delta a_5 = |a_{av} - a_5| = |2.53 - 2.49| = 0.04$$

$$\Delta a_6 = |a_{av} - a_6| = |2.53 - 2.57| = 0.04$$

Δa_1, Δa_2, Δa_3, Δa_4, Δa_5, Δa_6 are the absolute errors in each case.

Mean absolute error i.e., $(\Delta a)_{av} = \langle \Delta a \rangle$ is

$$(\Delta a)_{av} = \langle \Delta a \rangle = \frac{\Delta a_1 + \Delta a_2 + \Delta a_3 + \Delta a_4 + \Delta a_5 + \Delta a_6}{6}$$

$$\Rightarrow \quad (\Delta a)_{av} = \langle \Delta a \rangle = \frac{0.03+0.01+0.04+0.05+0.04+0.04}{6}$$

$$\Rightarrow \quad (\Delta a)_{av} = \langle \Delta a \rangle = \frac{0.21}{6} = 0.035$$

So, mean length $= (2.53 \pm 0.035)$ m

$$\Rightarrow \quad \text{Percentage error} = \frac{\langle \Delta a \rangle}{a} \times 100\%$$

$$\Rightarrow \quad \%\text{age error } = \frac{0.035}{2.53} \times 100\%$$

$$\Rightarrow \quad \%\text{age error} = 1.38\%$$

COMBINATION OR PROPAGATION OF ERRORS

The formula used in an experiment may involve addition, subtraction, multiplication or division etc. of different quantities measured in the experiment. There may be some error in each of the reading. But all the errors will not affect the final result to the same extent. Different errors will affect the final result differently. So the final result will depend upon the way these errors are combined through mathematical operations. We shall calculate the maximum possible error in all the cases.

When the Result Involves the Sum of Two Observed Quantities

We suppose that the result X is given as the sum of two observed quantities A and B, i.e.

$$X = A + B$$

Let ΔA and ΔB the absolute errors in A and B. Then the values of A and B should be recorded as $A \pm \Delta A$ and $B \pm \Delta B$. If ΔX be the absolute error in the final result X, then

$$X \pm \Delta X = (A \pm \Delta A) + (B \pm \Delta B)$$

$$\Rightarrow \quad X \pm \Delta X = (A + B) \pm (\Delta A + \Delta B)$$

$$\Rightarrow \quad \pm \Delta X = \pm(\Delta A + \Delta B)$$

So, maximum possible error in X is

$$\Delta X = \Delta A + \Delta B$$

Thus when two quantities are added, the absolute error in the final result is the sum of the absolute errors of the quantities.

When the Result Involves the Difference of Two Observed Values

Let the result X be given as the different of two observed quantities A and B i.e.

$$X = A - B$$

Let ΔA and ΔB be absolute errors in A and B.
If ΔX is the absolute error in the final result, then

$$X \pm \Delta X = (A \pm \Delta A) - (B \pm \Delta B)$$

$$\Rightarrow \quad X \pm \Delta X = (A + B) \pm (\Delta A \pm \Delta B)$$

$$\Rightarrow \quad \pm \Delta X = \pm(\Delta A \pm \Delta B)$$

But the error in X will be maximum if

$$\Delta X = \Delta A + \Delta B$$

Thus when the two quantities are subtracted, the absolute error (simply we may call it as error) in the final result is again the sum of absolute errors of the two quantities.

When the Result Involves the Product of Two Observed Quantities

We suppose that $X = AB$, then

$$X \pm \Delta X = (A \pm \Delta A)(B \pm \Delta B)$$

$$\Rightarrow \quad X \pm \Delta X = AB \pm A\Delta B \pm \Delta AB \pm \Delta A\,\Delta B$$

Dividing both sides of the above equation with X, we get

$$\frac{X \pm \Delta X}{X} = \frac{AB \pm A\Delta B \pm \Delta AB \pm \Delta A\Delta B}{AB}$$

$$\Rightarrow \quad 1 \pm \frac{\Delta X}{X} = 1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \frac{\Delta A \Delta B}{AB}$$

Since ΔA and ΔB are small, so the product $\Delta A \Delta B$ can be neglected. So we get

$$\Rightarrow \quad \pm \frac{\Delta X}{X} = \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B}$$

To have the maximum relative error, we get

$$\frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Alternative Method: The above result can be derived as follows:

$$X = AB$$

$$\Rightarrow \quad \log X = \log A + \log B$$

Differentiating on both the sides

$$\frac{dX}{X} = \frac{dA}{A} + \frac{dB}{B}$$

$$\Rightarrow \quad \frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B} \text{ and}$$

$$\Rightarrow \quad \frac{\Delta X}{X}\% = \frac{\Delta A}{A} \times 100 + \frac{\Delta B}{B} \times 100$$

Thus, the final relative error, when the quantity is the product of two observed quantities is the sum of the relative errors of the two quantities.

When the Result Involves the Quotient of Two Observed Quantities

Suppose that $X = \dfrac{A}{B}$

Then $X \pm \Delta X = \dfrac{A \pm \Delta A}{B \pm \Delta B}$

$$\Rightarrow \quad X \pm \Delta X = (A \pm \Delta A)(B \pm \Delta B)^{-1}$$

$$\Rightarrow \quad X\left(1 \pm \frac{\Delta X}{X}\right) = A\left(1 \pm \frac{\Delta A}{A}\right)B^{-1}\left(1 \pm \frac{\Delta B}{B}\right)^{-1}$$

$$\Rightarrow \quad X\left(1 \pm \frac{\Delta X}{X}\right) = \frac{A}{B}\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \pm \frac{\Delta B}{B}\right)^{-1}$$

$$\Rightarrow \quad \left(1 \pm \frac{\Delta X}{X}\right) = \left(1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B} \pm \frac{\Delta A \Delta B}{AB}\right)$$

Neglecting $\dfrac{\Delta A \Delta B}{AB}$ $\{\because \Delta A$ and ΔB are very small$\}$

$$\Rightarrow \quad \left(1 \pm \frac{\Delta X}{X}\right) = \left(1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B}\right)$$

$$\Rightarrow \quad \pm \frac{\Delta X}{X} = \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B}$$

$$\Rightarrow \quad \frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

So, whether the quantities are being multiplied or divided, then

$$\Rightarrow \quad \begin{pmatrix}\text{Percentage} \\ \text{Error in} \\ \text{value of X}\end{pmatrix} = \begin{pmatrix}\text{Percentage} \\ \text{Error in} \\ \text{value of } A\end{pmatrix} + \begin{pmatrix}\text{Percentage} \\ \text{Error in} \\ \text{value of } B\end{pmatrix}$$

If we are to find the absolute error ΔX, then we have

$$\Delta X = X\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \quad X = AB$$

Now, if, then $\Delta X = B\Delta A + A\Delta B$

and if $X = \dfrac{A}{B}$, then $\Delta X = \dfrac{A}{B}\left(\dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}\right)$

In Case of Power Functions

CASE-1

Suppose $X = kA^n$, where k is a constant

Then $X \pm \Delta X = k(A \pm \Delta A)^n$

$$\Rightarrow \quad X\left(1 \pm \frac{\Delta X}{X}\right) = kA^n\left(1 \pm \frac{\Delta A}{A}\right)^n$$

$$\Rightarrow \quad 1 \pm \frac{\Delta X}{X} = \left(1 \pm \frac{\Delta A}{A}\right)^n$$

Since $\dfrac{\Delta A}{A} \ll 1$, so from Binomial Theorem, we get

$$1 \pm \frac{\Delta X}{X} = 1 \pm n\left(\frac{\Delta A}{A}\right)$$

$$\Rightarrow \quad \pm \frac{\Delta X}{X} = \pm n\left(\frac{\Delta A}{A}\right)$$

$$\Rightarrow \quad \frac{\Delta X}{X} = n\left(\frac{\Delta A}{A}\right)$$

$\{\because$ Error in a measurement is always extremum during measurement$\}$

CASE-2

If $X = k\left(\dfrac{A^{\ell} B^m}{C^n}\right)$

$\{l > 0,\ m > 0,\ n > 0$ and k is a constant$\}$

If ΔA, ΔB and ΔC be the respective absolute errors in calculating the values of A, B and C, then let the corresponding error in calculating X (depending on A, B and C) be ΔX. Then

$$X \pm \Delta X = \frac{(A \pm \Delta A)^l (B \pm \Delta B)^m}{(C \pm \Delta C)^n}$$

$$\Rightarrow \quad X\left(1 \pm \frac{\Delta X}{X}\right) = \frac{A^l\left(1 \pm \frac{\Delta A}{A}\right)^l B^m \left(1 \pm \frac{\Delta B}{B}\right)^m}{C^n\left(1 \pm \frac{\Delta C}{C}\right)^n}$$

$$\Rightarrow \quad \cancel{X}\left(1 \pm \frac{\Delta X}{X}\right) = \frac{\cancel{A^l B^m}}{\cancel{C^n}} \frac{\left(1 \pm \frac{\Delta A}{A}\right)^l \left(1 \pm \frac{\Delta B}{B}\right)^m}{\left(1 \pm \frac{\Delta C}{C}\right)^n}$$

$$\Rightarrow \quad \left(1 \pm \frac{\Delta X}{X}\right) = \left(1 \pm \frac{\Delta A}{A}\right)^l \left(1 \pm \frac{\Delta B}{B}\right)^m \left(1 \pm \frac{\Delta C}{C}\right)^{-n} \quad \ldots(1)$$

Since $\dfrac{\Delta A}{A} \ll 1$, $\dfrac{\Delta B}{B} \ll 1$ and $\dfrac{\Delta C}{C} \ll 1$, so from Binomial Theorem, we have

$$\left(1 \pm \frac{\Delta A}{A}\right)^l \cong 1 \pm l\frac{\Delta A}{A}$$

$$\left(1 \pm \frac{\Delta B}{B}\right)^m \cong 1 \pm m\frac{\Delta B}{B} \text{ and } \left(1 \pm \frac{\Delta C}{C}\right)^{-n} \cong 1 \mp n\frac{\Delta C}{C}$$

So, (1) becomes

$$1 \pm \frac{\Delta X}{X} = \left(1 \pm l\frac{\Delta A}{A}\right)\left(1 \pm m\frac{\Delta B}{B}\right)\left(1 \mp n\frac{\Delta C}{C}\right)$$

$$\Rightarrow \quad \pm\frac{\Delta X}{X} = \pm l\frac{\Delta A}{A} \pm m\frac{\Delta B}{B} \mp n\frac{\Delta C}{C} + \begin{pmatrix}\text{Neglected}\\ \text{Terms}\end{pmatrix}$$

$$\Rightarrow \quad \pm\frac{\Delta X}{X} = \pm l\frac{\Delta A}{A} \pm m\frac{\Delta B}{B} \mp n\frac{\Delta C}{C}$$

Since, we know that during propagation errors are always taken to be the extremum, so we have

$$+\frac{\Delta X}{X} = +l\frac{\Delta A}{A} + m\frac{\Delta B}{B} + n\frac{\Delta C}{C}$$

OR

$$-\frac{\Delta X}{X} = -l\frac{\Delta A}{A} - m\frac{\Delta B}{B} - n\frac{\Delta C}{C}$$

From both these relations, we get

$$\frac{\Delta X}{X} = l\frac{\Delta A}{A} + m\frac{\Delta B}{B} + n\frac{\Delta C}{C}$$

$$\Rightarrow \quad \begin{pmatrix}\%\text{age}\\ \text{Error}\\ \text{in } y\end{pmatrix} = l\begin{pmatrix}\%\text{age}\\ \text{Error}\\ \text{in } A\end{pmatrix} + m\begin{pmatrix}\%\text{age}\\ \text{Error}\\ \text{in } B\end{pmatrix} + n\begin{pmatrix}\%\text{age}\\ \text{Error}\\ \text{in } C\end{pmatrix}$$

Conceptual Note(s)

And if $X = kA^{\pm N}$, where k is a constant and N is any real number.

Then $\dfrac{\Delta X}{X} = N\dfrac{\Delta A}{A}$

$$\Rightarrow \quad \frac{\Delta X}{X}\% = N\left(\frac{\Delta A}{A}\right) \times 100\%$$

ILLUSTRATION 16

If $V = (50 \pm 2)V$ and $I = (5 \pm 0.1)A$, then find the percentage error in measuring the resistance. Also find the resistance with limits of error.

SOLUTION

(a) Given $V = (50 \pm 2)V$ and $I = (5 \pm 0.1)V$

We know that $R = \dfrac{V}{I}$

$$\Rightarrow \quad \frac{\Delta R}{R} = \frac{\Delta V}{V} \pm \frac{\Delta I}{I} = \frac{2}{50} + \frac{0.1}{5}$$

$$\Rightarrow \quad \frac{\Delta R}{R}(\text{in }\%) = \frac{2}{50} \times 100 + \frac{0.1}{5} \times 100 = 6\ \%$$

(b) $R = \dfrac{V}{I} = \dfrac{50}{5} = 10$ ohm

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{2}{50} + \frac{0.1}{5}$$

$$\Rightarrow \quad \Delta R = \frac{2}{50} \times 10 + \frac{0.1}{5} \times 10 = 0.6$$

$$\Rightarrow \quad R \pm \Delta R = (10 \pm 0.8)\text{ ohm}$$

Problem Solving Technique(s)

(a) Δy is always positive i.e. $\Delta y > 0$.

(b) Δy has units same as that of y.

(c) A quantity in terms of absolute error is expressed as $y = (y_t \pm \Delta y)$ units

(d) If least count is not given and a measurement is given, then error in the measurement will be ± 1 in last digit.

EXAMPLE

If $L = 5.216$ metre,

then $\Delta L = \pm\, 0.001$ metre

Also, if $M = 2.50$ Kg,

then $\Delta M = \pm\, 0.01$ kg

(e) The error in a measurement is equal to the least count of the instrument.

(f) For evaluating error in a formula
(i) the powers are changed into multiplication.
(ii) the multiplication and divisions are changed into addition.

(g) In case of addition and subtraction, it is advisable to calculate absolute error first and then relative error.

(h) In case of multiplication, division and power functions, it is advisable to calculate relative error first and then absolute error.

(i) Relative Error or Percentage relative error is dimensionless, hence a measurement can be expressed in terms of percentage relative error as

$$y = y_t \text{ units} \pm \left(\frac{\Delta y}{y_t} \times 100\right)\%$$

(j) Errors never propagate in case of constants.

EXAMPLE

If $V = \frac{4}{3}\pi r^3$, then

$$\frac{\Delta V}{V} = \frac{3\Delta r}{r} \qquad \left\{\text{Only when } \frac{\Delta r}{r} << 1\right\}$$

(k) In general if $y = kx^n$, then

$$\frac{\Delta y}{y} = n\left(\frac{\Delta x}{x}\right) \qquad \left\{\text{Only when } \frac{\Delta x}{x} << 1\right\}$$

irrespective of value of k.

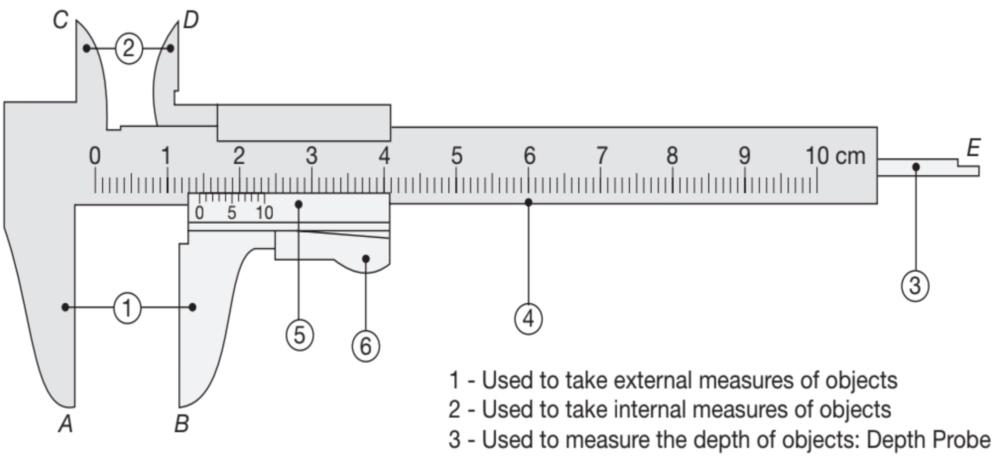
VERNIER CALLIPER

Vernier Calliper is a device used to measure the internal or external diameter as well as the depth of a vessel. The maximum precision up to which a scale can measure is 1 mm. For measuring lengths that are less than 1 mm, vernier calliper is used. Vernier calliper is also called as Slide Calliper.

A Vernier calliper consists of two scales (See Figure)

(a) **A Main Scale:** It is calibrated in cm on one side and in inches on the other side. The value of one main scale division (MSD) is 1 mm or 0.1 cm.

(b) **A Vernier Scale:** It is the movable scale that can slide on the main scale. Generally 10 vernier scale divisions (VSD) is equal to 9 main scale divisions (MSD). Therefore the value of one vernier scale division (VSD) is equal to $\frac{9}{10}$ mm.



1 - Used to take external measures of objects
2 - Used to take internal measures of objects
3 - Used to measure the depth of objects: Depth Probe
4 - Main scale (cm)
5 - Vernier (cm)
6 - Used to block movable part: Retainer

The upper two jaws C and D are used to find the internal diameter of a pipe, test tube and that of hollow cylinder. The lower two jaws A and B are used to measure the length and diameter of a cylinder or the diameter of a sphere by placing the object between the lower jaws. A thin metallic strip E projects at one end of the calliper which is used to measure the depth of the vessel. The smallest length up to which the vernier calliper can measure is called **Least Count (LC)** or **Vernier Constant**.

Least count of vernier calliper is defined as the ratio of the value of one MSD to the number of divisions on the vernier scale called Vernier Scale Division (VSD)

$$\text{Least count} = \frac{S}{N} = \frac{\text{Value of 1 MSD}}{\left(\begin{array}{c}\text{Number of divisions on}\\ \text{the vernier scale}\end{array}\right)}$$

Least count of vernier callipers is also defined as the difference between the value of one MSD and the value of one VSD.

$\Rightarrow$ Least count = 1 MSD – 1 VSD

It is observed that N VSD (divisions of vernier scale) coincide with $(N-1)$ MSD (divisions of the main scale)

$$N \text{ VSD} = (N-1) \text{ MSD}$$

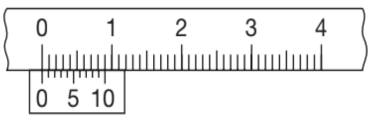
$$\Rightarrow 1 \text{ VSD} = \frac{(N-1)}{N} \text{MSD}$$

Least count = 1 MSD – 1 VSD

$$\text{Least Count} = 1 \text{ MSD} - \frac{(N-1)}{N} \text{ MSD} = \frac{1}{N} \text{ MSD}$$

CONCEPT OF ZERO ERROR

When the lower two jaws touch each other, the zero of the vernier scale coincides with the zero of the main scale. When this is the case then there is no zero error. However, if the zero of the vernier scale does not coincide with zero of the main scale (when the jaws are in contact), then error exists and necessary correction has to be applied.



No zero error

Positive Zero Error

The zero of the vernier scale lies to the right of zero of the main scale. When the two jaws are in contact, the distance between them is zero but the reading is positive. In other words, the instrument measures more than the actual distance and therefore zero error is positive. Figure A represents the case of positive zero error.

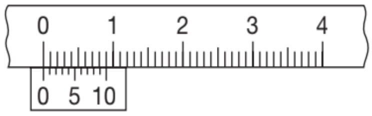


Figure A Positive zero error

Negative Zero Error

The zero of vernier scale lies to the left of zero of the main scale. When the two jaws are in contact, the distance between them is zero but now the reading is negative. In other words, the instrument measures less than the actual distance and therefore zero error is negative. Figure B represents the case of negative zero error.

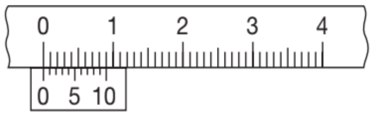


Figure B Negative zero error

CALCULATING ZERO ERROR

To calculate the zero error the two jaws are kept in contact and the number of vernier scale divisions coinciding with some main scale division is noted. For the arrangement shown in Figure A, the fourth vernier division coincides with some division of main scale.

$\Rightarrow$ Zero error $= +4 \times (\text{LC}) = +4 \times 0.01 = +0.04$ cm

$\Rightarrow$ Zero correction $= -0.04$ cm

For the arrangement shown in Figure B, fifth vernier division coincides with some main scale division. To calculate zero error, the number of division to be considered will be 5.

$\Rightarrow$ Zero error $= -5 \times (\text{LC}) = -5 \times 0.01 = -0.05$ cm

$\Rightarrow$ Zero correction $= +0.05$ cm

So, the correct length is

$$\text{Correct Length} = (\text{Observed Length}) \pm (\text{Zero Error})$$

STEPS TO BE FOLLOWED WHILE TAKING READINGS WITH VERNIER CALLIPERS

(a) Find the Least Count (LC).

(b) For taking the Main Scale Reading (MSR), the division next before the zero of the vernier should be considered.

(c) The division of the vernier scale which coincides with any main scale division should be noted. This is called Vernier Coincidence (VC).

(d) The Correct Reading (CR) is given by

$$CR = MSR + (LC) \times (VC)$$

(e) Prefer using a lens to note Vernier Coincidence (VC).

(f) Vernier principle is also used for measurement of fraction of angles as in sextant.

Precautionary Measures

(a) As a precaution, the jaws (lower) of the callipers should not be pressed too hard on the object placed between them so as to get the actual reading.

(b) Also at any position, the diameter should be measured in two directions at right angles to each other.

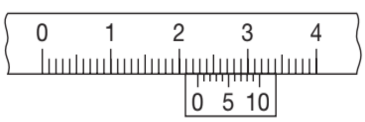
(c) Same units should be used while calculating the result.

(d) Vernier coincidence must be noted without any parallax error by repeating the observations five times at different positions of the object. Slide calliper is called vernier callipers since it was first designed by French mathematician named Vernier.

HOW TO MEASURE

STEP-1: Measure the least count/vernier constant.

STEP-2: Place the object between the jaws and tighten them



STEP-3: Suppose that during this observation, zero of the vernier scale (having vernier constant 0.01 cm) lies between 2.2 cm and 2.3 cm mark. Therefore, in this observation, the length of the object under measurement is 2.2 cm plus some fraction (say y).

STEP-4: The value of the fraction is found by locating the vernier division which coincides with a main scale division. In present case it is 7. Therefore, fraction to be added is given by

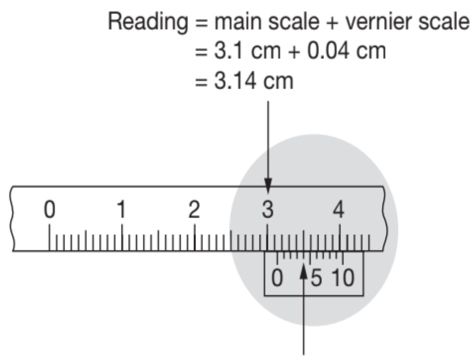
$$\text{Fraction } = 7 \times \text{V.C.} = 7 \times 0.01 \text{ cm} = 0.07 \text{ cm}$$

Hence, the reading of the vernier callipers is

$$2.2 + 0.07 \text{ i.e., } 2.27 \text{ cm.}$$

STEP-5: So, the correct reading is given by

$$CR = MSR + (LC) \times (VC)$$



The fourth vernier mark coincides with a marking on the main scale. This gives a reading of 0.4 mm or 0.04 cm to be added to main scale reading.

ILLUSTRATION 17

Least count of a vernier calliper is 0.01 cm. When the two jaws of the instrument touch each other the 5th division of the vernier scale coincide with a main scale division and the zero of the vernier scale lies to the left of the zero of the main scale. Further more while measuring the diameter of a sphere, the zero mark of the vernier scale lies between 2.4 cm and 2.5 cm and the 6th vernier division coincides with a main scale division. Calculate the diameter of the sphere.

SOLUTION

The instrument has a negative error,

$$e = (-5 \times 0.01) \text{ cm} = -0.05 \text{ cm}$$

Measured reading $= (2.4 + 6 \times 0.01) = 2.46$ cm

True reading = Measured reading $-e$

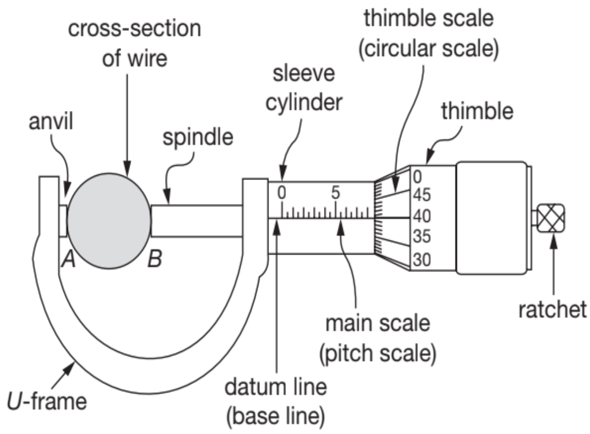
$\Rightarrow$ True Reading $= 2.46 - (-0.05)$

$\Rightarrow$ True Reading $= 2.51$ cm

Therefore, diameter of the sphere is 2.51 cm

SCREW GAUGE

A Screw Gauge is a device used to determine thickness (or diameter) of thin sheet or a wire. With its help one can measure the diameter of very thin wires or similar objects. Its measures accurately up to 0.001 cm and hence is also commonly called **micrometer screw gauge**.



CONSTRUCTION

The screw gauge consists of the following parts

U-Frame

It is a steel frame having a U shape. On one end of U shaped frame a screw is fixed permanently. This fixed screw is commonly called **Stud** or **Anvil** and forms the fixed jaw of the screw gauge. On the other end of U shaped frame is fixed a **Nut** through which slides a screw. The end A of the screw forms the movable jaw of screw gauge.

Nut and Screw

The nut (at B) is threaded from inside and the screw is threaded from outside. The screw can move in and out of nut by circular motion.

Thimble or Circular Cylinder

The screw is connected to a hollow circular cylinder(s) also called **Thimble**, which rotates along with the nut on turning.

Sleeve Cylinder

To the nut is attached a hollow cylinder, commonly called **Sleeve Cylinder**. The spindle of the screw passes through this sleeve cylinder.

Base Line

A reference line or Base Line or Datum Line is graduated in mm and is drawn on the sleeve cylinder parallel to the axis of nut. It is commonly called **Main Scale** or **Sleeve Scale** or **Pitch Scale.**

Circular Scale or Thimble Scale or Head Scale

The hollow cylinder that moves over the sleeve cylinder is tapered from one end. On the tapered end graduations are made, which are either 50 or 100 in number. The scale marked on sleeve is called **circular scale** or **thimble scale** or **head scale**.

Ratchet

The ratchet is attached to screw with the help of a spring. When the flattened end B of the screw comes in contact with the stud A, the ratchet becomes free and makes a rattling noise. Thus, end B of the screw will not be further pushed towards the stud A.

PITCH OF SCREW

The pitch of screw is defined as, the distance between two consecutive threads of screw when measured along the axis of screw

OR

Pitch of screw can also be defined as the forward distance travelled by the tip of screw (end B) when head of screw completes one rotation.

PRINCIPLE OF SCREW GAUGE

It works on fundamental screw principle that rotational motion can be converted into translational motion.

DETERMINATION OF PITCH OF SCREW

To determine the pitch of the screw gauge, the screw is given five complete rotations. The distance moved by the thimble on the main scale is then recorded. The pitch is calculated by the formula

$$\text{Pitch} = \left(\frac{\begin{matrix}\text{Distance moved by thimble} \\ \text{on Main Scale (MS)}\end{matrix}}{\text{Number of rotations of thimble}} \right)$$

ILLUSTRATION 18

If 5 mm is the distance moved by the thimble on the main scale for 5 rotations then calculate the pitch.

SOLUTION

$$\text{Pitch} = \frac{5\text{ mm}}{5} = 1\text{ mm}$$

LEAST COUNT OF SCREW

For a screw gauge, the least count is the smallest distance moved by its tip when the screw turns through division marked on it.

DETERMINATION OF LEAST COUNT

First of all we shall determine the pitch and also count the number of divisions on circular scale. Then least count is determined by the formula

$$\text{Least Count} = \frac{\text{Pitch}}{\begin{pmatrix}\text{Number of divisions on} \\ \text{the Circular Scale (CS)}\end{pmatrix}}$$

For a screw gauge having a pitch of 1 mm and having 100 divisions marked on its thimble, the least count is

$$\text{Least Count} = \frac{1\text{ mm}}{100} = 0.01\text{ mm} = 0.001\text{ cm}$$

BACKLASH ERROR

This type of error is associated with all instruments based on the Screw Principle. In the screw gauge, the screw moves in a nut. If the screw is not fitting properly in the grooves (threads), then there is irregularity in the movement of the screw, when the head is rotated. Error due to this is called **backlash error**. In order to avoid this error, during an observation the screw is always moved in one direction.

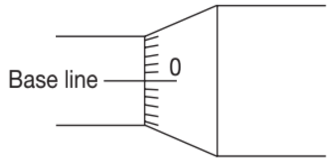
DETERMINATION OF ZERO ERROR

On bringing the screw end B in contact with the stud A, if the zero of the main scale (MS) does not coincide with zero of circular scale (CS), the micrometer or the screw gauge is said to have zero error.

No Zero Error

Before using the screw gauge, we must check for a zero error. Close the screw gauge so that the spindle touches the anvil.

If there is no zero error then the reading will be 0.00 mm, as shown. In this case, the zero of the circular scale coincides with Base Line (or Reference Line).

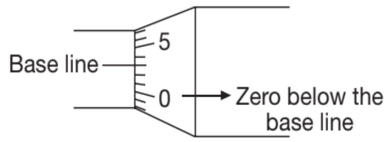


Positive Zero Error with Negative Correction

The zero of circular scale may be left behind the Base Line or below the Base Line (or Reference Line) as shown in figure.

In such a case when the studs are in contact (zero distance), the screw gauge is giving a positive reading. Such a screw gauge measures more and therefore it has got Positive Zero Error or Negative Zero Correction.

From figure we note that the zero of circular scale has been left behind by 3 circular divisions.



So, zero error is

$$\begin{pmatrix}\text{Zero} \\ \text{Error}\end{pmatrix} = +\begin{pmatrix}\text{Number of divisions} \\ \text{left behind}\end{pmatrix} \times \begin{pmatrix}\text{Least} \\ \text{Count}\end{pmatrix}$$

$\Rightarrow$ Zero Error $= +3 \times 0.001$ cm $= +0.003$ cm

$\Rightarrow$ Zero Correction $= -0.003$ cm

So, to conclude we have

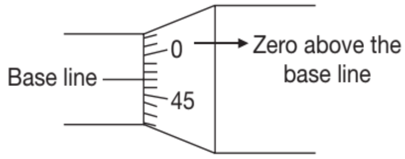
$$\begin{pmatrix}\text{Correct} \\ \text{Length}\end{pmatrix} = \begin{pmatrix}\text{Observed} \\ \text{Length}\end{pmatrix} + \begin{pmatrix}\text{Zero} \\ \text{Correction}\end{pmatrix}$$

OR

$$\text{Correct Length} = \text{Observed Length} - \text{Zero Error}$$

Negative Zero Error with Positive Correction

When the zero line, marked on circular scale lies above the Base Line of the main scale, then there is a negative error and the correction is positive. The zero of circular scale may cross the reference line as shown in figure. In such a case, if we move the screw back so as to make the zero of circular scale to coincide with reference line, a gap will be left between the studs. Thus such a screw gauge gives the zero reading even when there is gap between studs i.e., it measures less. Therefore, the screw gauge possesses negative zero error or positive zero correction.



To determine the zero correction, note the number of divisions through which the zero mark of circular scale has crossed the reference line. From figure, we note that the number of such divisions that cross the zero mark is 3.
So, Zero error is

$$\begin{pmatrix}\text{Zero}\\ \text{Error}\end{pmatrix} = -\begin{pmatrix}\text{Number of}\\ \text{divisions crossed}\end{pmatrix}\times\begin{pmatrix}\text{Least}\\ \text{Count}\end{pmatrix}$$

$\Rightarrow$ Zero Error $= -3 \times 0.001\text{ cm} = -0.003\text{ cm}$

$\Rightarrow$ Zero Correction $= +0.003\text{ cm}$

Here also, we have

$$\begin{pmatrix}\text{Correct}\\ \text{Length}\end{pmatrix} = \begin{pmatrix}\text{Observed}\\ \text{Length}\end{pmatrix} + \begin{pmatrix}\text{Zero}\\ \text{Correction}\end{pmatrix}$$

OR

$$\text{Correct Length} = \text{Observed Length} - \text{Zero Error}$$

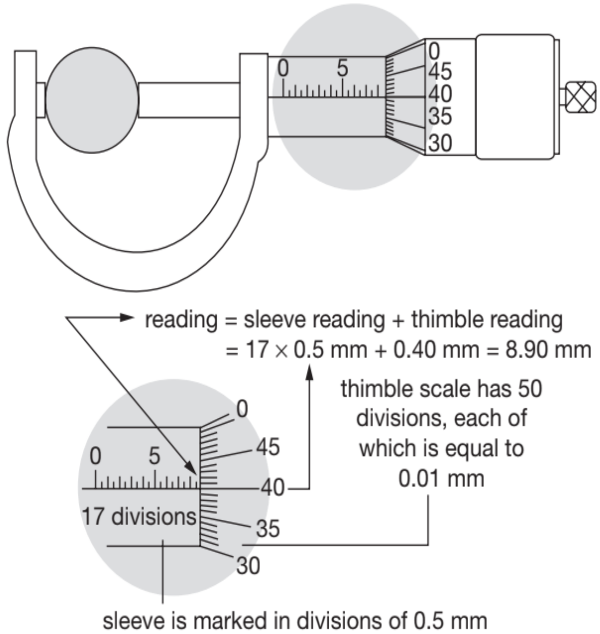
READING A SCREW GAUGE

Place the object say, a piece of wire (whose diameter is to be found) between the two studs of the screw gauge. Turn the screw with the help of ratchet arrangement, till the ratchet becomes free with the sound of a click. It ensures the gentle placing of the wire between the studs.
Let the complete main scale reading (MSR) be read as a and the nth circular scale division (CSD) coinciding with the reference line.
Then, diameter of the wire,

$$d = a \times \text{pitch} + n \times \text{least count}$$

Here $a = 17$, pitch $= 0.5$ mm, $n = 40$,
least count $=$ 0.01 mm
Also, we can say that,

$$\text{Reading} = \text{Sleeve Reading} + \text{Thimble Reading}$$



Conceptual Note(s)

So, in general the correct measurement taken by Screw Gauge is

$$\text{Correct Reading} = \text{MSR} + (\text{CSR} \times \text{LC}) + \text{ZC}$$

OR

$$\text{Correct Reading} = \text{MSR} + (\text{CSR} \times \text{LC}) - \text{ZE}$$

where, **MSR** is Main Scale Reading

CSR is Circular Scale Reading, **LC** is Least Count

ZC is Zero Correction and **ZE** is Zero Error

ILLUSTRATION 19

The pitch of a screw gauge is 1 mm and there are 100 divisions on circular scale. When faces A and B are just touching each without putting anything between the studs 32nd division of the circular scale coincides with the reference line. When a glass plate is placed between the studs, the linear scale reads 4 divisions and the circular scale reads 16 divisions. Find the thickness of the glass plate. Zero of linear scale is not hidden from circular scale when A and B touches each other.

SOLUTION

Least count

$$\text{L.C.} = \frac{\text{pitch}}{\text{number of divisions on circular scale}}$$

$$LC = \frac{1}{100}\text{ mm} = 0.01\text{ mm}$$

As zero is not hidden from circular scale when A and B touches each other. Hence, the screw gauge has positive error.

$$e = +n(\text{L.C.}) = 32 \times 0.01 = 0.32\text{ mm}$$

Linear scale reading $= 4 \times (1\text{ mm}) = 4\text{ mm}$

Circular scale reading $= 16 \times (0.01\text{ mm}) = 0.16\text{ mm}$

$\Rightarrow$ Measured reading $= (4 + 0.16)\text{ mm} = 4.16\text{ mm}$

$\Rightarrow$ Absolute reading $=$ Measured reading $-e$

$\Rightarrow$ Absolute reading $= (4.16 - 0.32)\text{ mm} = 3.84\text{ mm}$

Therefore, thickness of the glass plate is 3.84 mm

ILLUSTRATION 20

The pitch of a screw gauge is 1 mm and there are 100 divisions on its circular scale. When nothing is put in between its jaws, the zero of the circular scale lies 6 divisions below the reference line. When a wire is placed between the jaws, 2 linear scale divisions are clearly visible while 62 divisions on circular scale coincide with the reference line. Determine the diameter of the wire.

SOLUTION

$$\text{L.C.} = \frac{p}{N} = \frac{1\text{ mm}}{100} = 0.01\text{ mm}$$

The instrument has a positive zero error.

$$e = +n(\text{L.C.}) = +(6 \times 0.01) = +0.06\text{ mm}$$

Linear scale reading $= 2 \times (1\text{ mm}) = 2\text{ mm}$

Circular scale reading $= 62 \times (0.01\text{ mm}) = 0.62\text{ mm}$

So, measured reading $= 2 + 0.62 = 2.62\text{ mm}$

i.e., True reading $= 2.62 - 0.06 = 2.56\text{ mm}$

Test Your Concepts-IV

Based on Errors, Significant Figures, Vernier Calliper and Screw Gauge

(Solutions on page H.9)

1. In a circuit the potential difference across a resistance is $V = (8 \pm 0.5)$ volt and the current in the circuit is $I = (2 \pm 0.2)$ ampere. Calculate the resistance with absolute and the percentage error.
2. In an experiment for determining the density d of a rectangular block of metal the length, breadth and height are 5.12 cm, 2.56 cm, 0.37 cm and the mass is 39.3 g. Calculate the maximum permissible absolute and relative error in the determination of density.
3. In a vernier callipers N divisions of vernier coincide with $(N-1)$ divisions of main scale in which length of 1 division is 1 mm. What is the least count of the instrument in cm?
4. The length and the radius of a cylinder measured with slide callipers are found to be 4.54 cm and 1.75 cm respectively. Calculate the volume of the cylinder.
5. In the table shown, Column A gives few measured values and Column B has the number of significant figures to be correctly against the measured respective value. Complete the entries of Column B.

Column A	Column B
17236	______
510 m	______
270	______
4.20	______
7042.6	______
0.017	______
6.1×10^{14}	______

6. Use the rules studied for rounding off to give the rounded off values to three significant figures.

Measured Value	Rounded off value to Three Significant Figures
7.364	______
8.3251	______
9.445	______
15.75	______
7.367	______
9.4450	______
15.7500	______

7. Calculate :

(a) $6.789 + 3.45 + 1.2 =$ ________

(b) $12.63 - 10.2 =$ ________

(c) $36.72 \times 1.2 =$ ________

(d) $\dfrac{1100\ \text{ms}^{-1}}{10.2\ \text{ms}^{-1}} =$ ________

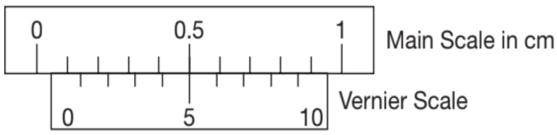
8. The smallest division on main scale of a vernier callipers is 1 mm and 10 vernier divisions coincide with 9 scale divisions. While measuring the length of a line, the zero mark of the vernier scale lies between 10.2 cm and 10.3 cm and the third division of vernier scale coincide with a main scale division.

(a) Determine the least count of the callipers

(b) Find the length of the line.

9. The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. In measuring the diameter of a sphere there are six divisions on the linear scale and forty divisions on circular scale coincides with the reference line. Find the diameter of the sphere.

10. In the diagram shown in figure, 9 MSD coincide with 10 VSD. Find the magnitude and nature of zero error.



SOLVED PROBLEMS

PROBLEM 1

P.A.M. Dirac, a great Physicist of the 20th century found that from the following basic constants a number having dimensional formula same as that of time can be constructed i.e.,

(a) the charge on the electron e
(b) permittivity of free space ε_o
(c) mass of the electron m_e
(d) mass of the proton m_p
(e) speed of light c
(f) Universal Gravitational constant G.

Obtain the relation for this Dirac's number. You are given that the desired number is proportional to m_p^{-1} and m_e^{-2}. What is the significance of this number? Assume constant of proportionality to be 1.

SOLUTION

According to the statement of the problem, we have

$$t \propto e^p \varepsilon_0^q m_e^{-2} m_p^{-1} c^r G^s$$

Taking constant of proportionality as 1 (given in question) and dimensions on both sides, we get

$$T = (AT)^p (M^{-1}L^{-3}A^2T^4)^q (M)^{-2}(M)^{-1} \times (LT^{-1})^r (M^{-1}L^3T^{-2})^s$$

$$\Rightarrow \quad T = M^{-q-3-s} L^{-3q+r+3s} T^{p+4q-r-2s} A^{p+2q}$$

Applying Principle of Homogeneity, we get

$$-q-3-s=0 \qquad \ldots(1)$$
$$-3q+r+3s=0 \qquad \ldots(2)$$
$$p+4q-r-2s=1 \qquad \ldots(3)$$
$$p+2q=0 \qquad \ldots(4)$$

Adding (2) and (3) for eliminating r, we get

$$q+p+s=1 \qquad \ldots(5)$$

From (5) $p=-2q$, put in (5) to get

$$q-2q+s=1$$
$$\Rightarrow \quad -q+s=1 \qquad \ldots(6)$$

Adding (1) and (6), we get

$$-2q=4$$
$$\Rightarrow \quad q=-2$$

So, $p=4$, $q=-2$, $s=-1$, $r=-3$

Hence, we get

$$t = e^4 \varepsilon_0^{-2} m_e^{-2} m_p^{-1} c^{-3} G^{-1}$$

$$\Rightarrow \quad t = \frac{e^4}{\varepsilon_0^2 m_e^2 m_p G c^3} \qquad \ldots(7)$$

Also, you can verify the dimensional correctness of equation (7) by substituting the dimensional formulae of the respective quantities on the right.

Now substituting the values of all constants in equation (7), we get the value of this time t as

$$t = 3.36 \times 10^{18} \text{ s} \cong 10^{11} \text{ years}$$

Dirac estimated this time to be the age of universe.

PROBLEM 2

The mean life of the neutral elementary particle pion is 2×10^{-7} ns. The age of the universe is about 4×10^9 year. Find a time that is midway between these two times on the logarithmic scale.

SOLUTION

Let t be the time half way between these two on the logarithmic scale. Then

$$\log_e t = \frac{\log_e t_1 + \log_e t_2}{2}$$

$$\Rightarrow \quad \log_e t = \frac{1}{2}\log_e(t_1 t_2) = \log_e \sqrt{t_1 t_2}$$

Taking antilog both sides, we get

$$t = \sqrt{t_1 t_2}$$

$$t_1 = 2\times10^{-7} \text{ ns} = 2\times10^{-16} \text{ s}$$

$$t_2 = 4\times10^9 \text{ year} = 4\times10^9 \times 365\times24\times60\times60 \text{ s}$$

$$\Rightarrow \quad t_2 = 1.26\times10^{17} \text{ s}$$

So, $t_1 t_2 \cong 25$

$$\Rightarrow \quad \sqrt{t_1 t_2} = 5 \text{ s}$$

PROBLEM 3

Derive by the method of dimensions, an expression for the volume of a liquid flowing out per second through a narrow pipe. Assume that the rate of flow of liquid depends on

(a) the coefficient of viscosity η of the liquid

(b) the radius r of the pipe and

(c) the pressure gradient $\left(\frac{p}{l}\right)$ along the pipe.

Take $k=\frac{\pi}{8}$

SOLUTION

Let volume flowing out per second through the pipe be given by

$$V=k\eta^a r^b\left(\frac{p}{l}\right)^c \qquad \ldots(1)$$

where $k=a$ dimensionless constant. Dimensions of the various quantities are

$$[V]=\frac{\text{volume}}{\text{time}}=\frac{L^3}{T}=L^3T^{-1}$$

$$[\eta]=ML^{-1}T^{-1},\ [r]=L$$

$$\left[\frac{p}{l}\right]=\frac{\text{pressure}}{\text{length}}=\frac{\text{force}}{\text{area}\times\text{length}}$$

$$\Rightarrow\ \left[\frac{p}{l}\right]=\frac{MLT^{-2}}{L^2.L}=ML^{-2}T^{-2}$$

Substituting these dimensions in equation (1), we get

$$\left[L^3T^{-1}\right]=\left[ML^{-1}T^{-1}\right]^a[L]^b\left[ML^{-2}T^{-2}\right]^c$$

$$\Rightarrow\ M^0L^3T^{-1}=M^{a+c}L^{-a+b-2c}T^{-a-2c}$$

Equating the powers of M, L and T, we get

$$a+c=0,\ -a+b-2c=3,\ -a-2c=-1$$

On solving, $a=-1$, $b=4$, $c=1$.

$$\Rightarrow\ V=k\eta^{-1}r^4\left(\frac{p}{l}\right)^1$$

$$\Rightarrow\ V=\frac{\pi r^4 p}{8\eta l} \qquad \text{[Poiseuille's equation]}$$

PROBLEM 4

Finding dimensions of resistance R and inductance L, speculate what physical quantities $\left(\frac{L}{R}\right)$ and $\left(\frac{1}{2}\right)Li^2$ represents ?

SOLUTION

As $e=L\frac{di}{dt}$ i.e., $L=e\frac{dt}{di}$

So $[L]=\left[\frac{W}{q}\right]\left[\frac{t}{i}\right]=\frac{ML^2T^{-2}}{AT}\times\frac{T}{A}$

i.e., $[L]=ML^2T^{-2}A^{-2}$

and as $V=IR$, i.e., $R=\frac{V}{I}$

So $[R]=\left[\frac{W}{qA}\right]=\frac{ML^2T^{-2}}{ATA}$

$\Rightarrow\ [R]=ML^2T^{-3}A^{-2}$

So $\left[\frac{L}{R}\right]=\frac{ML^2T^{-2}A^{-2}}{ML^2T^{-3}A^{-2}}=\frac{1}{T^{-1}}=T$

and $\left[\frac{1}{2}Li^2\right]=\left(ML^2T^{-2}A^{-2}\right)\left(A^2\right)=ML^2T^{-2}$

Now as $\left(\frac{L}{R}\right)$ has dimensions of time and hence is called time constant of $L-R$ circuit and $\left(\frac{1}{2}\right)Li^2$ has dimensions of work or energy, so it represents magnetic energy stored in a coil.

PROBLEM 5

The Renold's number n_R for a liquid through a pipe depends upon the density of the liquid d, the coefficient of viscosity η, the speed of the liquid v and the radius of the tube r. Obtain by dimensional analysis an expression for n_R. Given $n_R\propto r$.

SOLUTION

As per the statement of the question, we get

$$n_R\propto r \qquad \ldots(1)$$

$$n_R \propto d^a \quad \ldots(2)$$

$$n_R \propto \eta^b \quad \ldots(3)$$

$$n_R \propto v^c \quad \ldots(4)$$

Combining (1), (2), (3) and (4), we get

$$n_R \propto r d^a \eta^b v^c$$

$$\Rightarrow \quad n_R = k r d^a \eta^b v^c \quad \ldots(5)$$

where k is a dimensionless constant

Taking dimensions on both sides of (5), we get

$$M^0L^0T^0 = L\left(ML^{-3}\right)^a\left(ML^{-1}T^{-1}\right)^b\left(LT^{-1}\right)^c$$

$$\Rightarrow \quad M^0L^0T^0 = M^{a+b}L^{-3a-b+c+1}T^{-b-c}$$

So, from Principle of Homogeneity, we get

$$a+b=0,$$

$$-3a-b+c+1=0,$$

$$-b-c=0$$

$$\Rightarrow \quad a=-b \text{ and } c=-b$$

So, we get

$$-3(-b)-b-b+1=0$$

$$\Rightarrow \quad 3b-2b+1=0$$

$$\Rightarrow \quad b=-1$$

Hence $a=1$, $b=-1$ and $c=1$.

So, we get the final relation as

$$n_R = \frac{kvrd}{\eta}$$

PROBLEM 6

Two resistances are expressed as $R_1 = (4 \pm 0.5)\ \Omega$ and $R_2 = (12 \pm 0.5)\ \Omega$. Calculate the net resistance when both are connected in series and in parallel with absolute and %age error.

SOLUTION

$$R_S = R_1 + R_2 = 16\ \Omega$$

$$R_P = \frac{R_1R_2}{R_1+R_2} = \frac{R_1R_2}{R_S} = 3\ \Omega$$

$$\Delta R_S = \Delta R_1 + \Delta R_2 = 1\ \Omega$$

$$\Rightarrow \quad \frac{\Delta R_S}{R_S}\times 100 = \frac{1}{16}\times 100\%$$

$$\Rightarrow \quad \frac{\Delta R_S}{R_S}\times 100 = 6.25\%$$

$$\Rightarrow \quad R_S = 16\ \Omega \pm 6.25\%$$

Similarly

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow \quad R_P^{-1} = R_1^{-1} + R_2^{-1}$$

$$\Rightarrow \quad \Delta\left(R_P^{-1}\right) = \Delta\left(R_1^{-1}\right) + \Delta\left(R_2^{-1}\right)$$

$$\Rightarrow \quad (-1)R_P^{-2}\Delta R_P = (-1)R_1^{-2}\Delta R_1 + (-1)R_2^{-2}\Delta R_2$$

$$\Rightarrow \quad \frac{\Delta R_P}{R_P^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2} \text{ where}$$

$$R_P = \frac{R_1R_2}{R_1+R_2} = 3\ \Omega$$

$$\Rightarrow \quad \frac{\Delta R_P}{R_P} = \left(\frac{0.5}{16} + \frac{0.5}{144}\right)(3)$$

$$\Rightarrow \quad \frac{\Delta R_P}{R_P} = \frac{15}{144}$$

$$\Rightarrow \quad \%RE = 10.42\%$$

PROBLEM 7

In an experiment the following readings were recorded $L = 2.890\text{ m}$, $M = 3.00\text{ kg}$, $l = 0.87\text{ cm}$ and $D = 0.041\text{ cm}$. Taking $g = 9.8\text{ ms}^{-2}$ and using the formula for Young's modulus $Y = \frac{MgL}{\pi r^2 l}$, calculate the maximum permissible error in the value of Y.

SOLUTION

$$Y = \frac{MgL}{\pi r^2 l} = \left(\frac{4g}{\pi}\right)\frac{ML}{D^2 l} \qquad \left\{\because\ r = \frac{D}{2}\right\}$$

$$\Rightarrow \quad \frac{\Delta Y}{Y} = \frac{\Delta M}{M} + \frac{\Delta L}{L} + 2\frac{\Delta D}{D} + \frac{\Delta l}{l}$$

Since maximum permissible error can be equal to the least count of the device taking the measurement, so we have

$$\Delta M = 0.01\text{ kg},\ \Delta L = 0.001\text{ m},$$

$$\Delta D = 0.001\text{ cm and } \Delta l = 0.01\text{ cm}$$

$$\Rightarrow \quad \% \frac{\Delta Y}{Y} = \left[\frac{0.01}{3} + \frac{0.001}{2.89} + 2\left(\frac{0.001}{0.041}\right) + \left(\frac{0.01}{0.87}\right)\right] \times 100$$

$$\Rightarrow \quad \% \frac{\Delta Y}{Y} = \frac{1}{3} + \frac{0.1}{2.890} + \frac{2(0.1)}{0.041} + \frac{1}{0.87}$$

$$\Rightarrow \quad \% \frac{\Delta Y}{Y} = \frac{1}{3} + \frac{10}{289} + \frac{200}{41} + \frac{100}{87}$$

$$\Rightarrow \quad \% \frac{\Delta Y}{Y} = 0.34 + 0.035 + 4.9 + 1.2$$

$$\Rightarrow \quad \% \frac{\Delta Y}{Y} = 6.5\%$$

PROBLEM 8

N divisions on the main scale of a vernier callipers coincide with $N+1$ divisions on the vernier scale. If each division on the main scale is of a units, determine the least count of the instrument.

SOLUTION

$(N+1)$ divisions on the vernier scale $= N$ divisions on main scale

1 division on vernier scale $= \frac{N}{N+1}$ divisions on main scale

Each division on the main scale is of a units.

So, 1 division on vernier scale $= \left(\frac{N}{N+1}\right) a$ unit $= a'$ (say)

Least count $= 1$ MSD $- 1$ VSD

$$\Rightarrow \quad \text{Least count} = a - a' = a - \left(\frac{N}{N+1}\right)a = \frac{a}{N+1}$$