

**Test Your Concepts-I (Based on Principle of Homogeneity: Verification)**

$$1. [Q] = \frac{(ML^2)(MLT^{-2})(L^2T^{-2})}{(ML^2T^{-2})(L^3)}$$

$$\Rightarrow [Q] = MT^{-2}$$

So, dimensions of Q are 1 in mass, zero in length and -2 in time.

2. Yes, like work and torque, pressure and stress, surface tension and spring constant, angular momentum and Planck's constant etc.

$$3. ML^2T^{-2}K^{-1}, M^{-1}L^3T^{-2}$$

$$4. [V] = L^3T^{-1}$$

$$\left[ \frac{\pi r^4 P}{8 \eta \ell} \right] = \left( \frac{L^4}{ML^X T^{-1}} \right) \left( \frac{ML^X T^{-2}}{L} \right) = L^3 T^{-1}$$

$$6. \left[ \frac{EJ^2}{M^5 G^2} \right] = \frac{(ML^2 T^{-2})(ML^2 T^{-1})^2}{(M^5)(M^{-1}L^3 T^{-2})^2} = M^0 L^0 T^0$$

$$7. [x] = [\text{Capacitance}] = \left[ \frac{Q}{V} \right] = \left[ \frac{Q}{\frac{W}{Q}} \right]$$

$$\Rightarrow [x] = M^{-1}L^{-2}T^2Q^2$$

$$[z] = [B] = \left[ \frac{F}{qv} \right] = MT^{-1}Q^{-1}$$

$$\text{Since } x = 3yz^2$$

$$\Rightarrow [x] = [y][z^2]$$

$$\Rightarrow M^{-1}L^{-2}T^2Q^2 = [y](M^2T^{-2}Q^{-2})$$

$$\Rightarrow [y] = M^{-3}L^{-2}T^4Q^4$$

8. Yes,  $\frac{e^2}{2h\epsilon_0 c}$  is actually the Fine Structure Constant, which happens to be a pure number having a value  $\frac{1}{137}$ .

$$9. \text{ Here } [\mu] = \frac{\text{Velocity of light in air}}{\text{Velocity of light in glass}}$$

$$\text{Since } [\mu] = \text{a dimensionless number} = M^0L^0T^0$$

$$\Rightarrow [A] = [\mu] = \text{a dimensionless number} = M^0L^0T^0$$

$$\text{Also } \left[ \frac{B}{\lambda^2} \right] = [\mu]$$

$$\Rightarrow [B] = [\mu][\lambda^2] = (M^0L^0T^0)L^2 = L^2$$

Hence A, being dimensionless, has no units and SI unit of B is  $m^2$ .

10. Since trigonometric functions are dimensionless, so

$$[\tan \theta] = M^0L^0T^0$$

$$\text{But } [v] = LT^{-1}$$

$\therefore$  Dimensions of LHS  $\neq$  Dimensions of RHS

Hence the given relation is dimensionally wrong.

This relation can be corrected by dividing RHS by the speed  $u$  of the rainfall. So the corrected relation is

$$\tan \theta = \frac{v}{u}$$

**Test Your Concepts-II (Based on Principle of Homogeneity: Conversion)**

$$1. n_2 = n_1 \left( \frac{M_1}{M_2} \right)^1 \left( \frac{L_1}{L_2} \right)^2 \left( \frac{T_1}{T_2} \right)^{-2}$$

$$\Rightarrow n_2 = 4.18 \left( \frac{1 \text{ kg}}{\alpha \text{ kg}} \right) \left( \frac{1 \text{ m}}{\beta \text{ m}} \right)^2 \left( \frac{1 \text{ s}}{\gamma \text{ s}} \right)^{-2}$$

$$\Rightarrow n_2 = 4.18 \alpha^{-1} \beta^{-1} \gamma^2$$

So, 4.18 J = 4.18  $\alpha^{-1} \beta^{-1} \gamma^2$  new units

$$2. 100 \text{ W} = 100 \text{ Js} = 10^9 \text{ erg} \quad \{ \because 1 \text{ J} = 10^7 \text{ erg} \}$$

3. 10 poise.

$$4. F = 20 \text{ N}, E = 200 \text{ J}, v = 5 \text{ ms}^{-1}$$

$$\text{Since } E = FL$$

$$\Rightarrow L = \frac{E}{F} = \frac{200}{20} = 10 \text{ m}$$

$$\text{Since } v = LT^{-1}$$

$$\Rightarrow 5 = 10 T^{-1}$$

$$\Rightarrow T = 2 \text{ s}$$

$$\text{Further } F = MLT^{-2}$$

$$\Rightarrow 20 = M(10)(2)^{-2}$$

$$\Rightarrow M = \frac{80}{10} = 8 \text{ kg}$$

## H.6 JEE Advanced Physics: Mechanics - I

$$5. \quad n_2 = n_1 \left( \frac{M_1}{M_2} \right) \left( \frac{L_1}{L_2} \right)^{-3}$$

$$\Rightarrow n_2 = 8 \left( \frac{1 \text{ g}}{20 \text{ g}} \right) \left( \frac{1 \text{ cm}}{5 \text{ cm}} \right)^{-3}$$

$$\Rightarrow n_2 = 8 \times \frac{1}{20} \times 5 \times 5 \times 5$$

$$\Rightarrow n_2 = 50$$

So,  $8 \text{ gcc}^{-1} = 50 \text{ new units}$

$$6. \quad 1 \text{ hp} = 746 \text{ W}$$

$$7. \quad [p] = MLT^{-1} \text{ and } [\sigma] = MT^{-2}$$

Since  $[v] = LT^{-1}$ ,  $[\rho] = ML^{-3}$ ,  $[v] = T^{-1}$

So, let

$$\rho v^4 = ML^{-3} L^4 T^{-4}$$

$$\Rightarrow \rho v^4 = MLT^{-4}$$

$$\Rightarrow \frac{\rho v^4}{v^3} = \frac{MLT^{-4}}{T^{-3}} = MLT^{-1}$$

$$\Rightarrow p = \rho v^4 v^3$$

Similarly,

$$\rho v^3 = MT^{-3}$$

$$\Rightarrow \frac{\rho v^3}{v} = \frac{MT^{-3}}{T^{-1}} = MT^{-2}$$

So,  $p \propto \rho v^4 v^3$  and  $\sigma \propto \frac{\rho v^3}{v}$

$$8. \quad \text{Speed of light} = 1 \text{ new unit of length/s}$$

Time = 8 min 20 s =  $8 \times 60 + 20 = 500 \text{ s}$

Distance between the earth and the sun is

$$d = \text{Speed of light} \times \text{time} = 1 \times 500$$

$$\Rightarrow d = 500 \text{ New units of length.}$$

$$9. \quad F = MLT^{-2}$$

$$\Rightarrow M = FT^2 L^{-1} = \frac{FT^2}{L}$$

$$\Rightarrow M = \frac{(1000 \text{ N})(100 \text{ s})^2}{(1000 \text{ m})}$$

$$\Rightarrow M = 10^4 \text{ kg}$$

$$10. \quad \text{Given that,}$$

$$S = 2 \text{ cal cm}^{-2} \text{ min}$$

But as  $1 \text{ cal} = 4.18 \text{ J}$ ,  $1 \text{ cm} = 10^{-2} \text{ m}$  and  $1 \text{ min} = 60 \text{ s}$

$$S = \frac{2 \times 4.18 \text{ J}}{(10^{-2} \text{ m})^2 (60 \text{ s})} \approx 1.4 \times 10^3 \frac{\text{J}}{\text{m}^2 \text{s}}$$

$$\Rightarrow S = 1.4 \text{ kWm}^{-2}$$

## Test Your Concepts-III (Based on Principle of Homogeneity: Dependence)

$$1. \quad T \propto d^a r^b S^c$$

$$\Rightarrow T = kd^a r^b S^c \quad \dots(1)$$

where  $k$  is a dimensionless constant.

Taking dimensions on both sides of (1) and applying the Principle of Homogeneity we get

$$T = (ML^{-3})^a (L)^b (MT^{-2})^c$$

$$\Rightarrow T = M^{a+c} L^{-3a+b} T^{-2c}$$

So, we get  $-2c = 1$

$$\Rightarrow c = -\frac{1}{2}$$

$$a + c = 0$$

$$\Rightarrow a = \frac{1}{2}$$

$$-3a + b = 0$$

$$\Rightarrow b = \frac{3}{2}$$

$$\Rightarrow T = kd^{\frac{1}{2}} r^{\frac{3}{2}} S^{-\frac{1}{2}}$$

$$\Rightarrow T = k \sqrt{\frac{r^3 d}{S}}$$

$$2. \quad t \propto p^a d^b E^c$$

$$\Rightarrow t = kp^a d^b E^c$$

where  $k$  is a dimensionless constant

$$\Rightarrow T = (ML^{-1} T^{-2})^a (ML^{-3})^b (ML^2 T^{-2})^c$$

$$\Rightarrow T = M^{a+b+c} L^{-a-3b+2c} T^{-2a-2c}$$

Using Principle of Homogeneity we get

$$a + b + c = 0 \quad \dots(1)$$

$$-a - 3b + 2c = 0 \quad \dots(2)$$

$$-2a - 2c = 1 \quad \dots(3)$$

From (3), we get

$$a + c = -\frac{1}{2}$$

Substituting in (1)

$$-\frac{1}{2} + b = 0$$

$$\Rightarrow b = \frac{1}{2}$$

Substituting in (2), we get

$$-a - \frac{3}{2} + 2c = 0$$

$$\Rightarrow -a + 2c = \frac{3}{2} \quad \dots(4)$$

Adding (3) and (4), we get

$$-3a = \frac{5}{2}$$

$$\Rightarrow a = -\frac{5}{6}$$

Substituting in (3), to get

$$-2\left(-\frac{5}{6}\right) - 2c = 1$$

$$\Rightarrow \frac{5}{3} - 2c = 1$$

$$\Rightarrow 2c = \frac{5}{3} - 1$$

$$\Rightarrow 2c = \frac{2}{3}$$

$$\Rightarrow c = \frac{1}{3}$$

Hence we get

$$a = -\frac{5}{6}, b = \frac{1}{2}, c = \frac{1}{3}$$

$$\Rightarrow T = kp^{-\frac{5}{6}}d^{\frac{1}{2}}E^{\frac{1}{3}}$$

3.  $T \propto m^a f^b \ell^c$

$$T = km^a f^b \ell^c$$

$$T = M^a (MLT^{-2})^b L^c$$

$$\Rightarrow T = M^{a+b} L^{b+c} T^{-2b}$$

Using Principle of Homogeneity, we get

$$a + b = 0 \quad \dots(1)$$

$$b + c = 0 \quad \dots(2)$$

$$-2b = 1 \quad \dots(3)$$

$$\Rightarrow b = -\frac{1}{2}, c = \frac{1}{2}, a = \frac{1}{2}$$

Hence  $T = km^{\frac{1}{2}} f^{-\frac{1}{2}} \ell^{\frac{1}{2}}$

$$\Rightarrow T = k \sqrt{\frac{m\ell}{f}}$$

4.  $v_T \propto W^a \eta^b r^c$

$$\Rightarrow v_T = kW^a \eta^b r^c$$

$$\Rightarrow LT^{-1} = (MLT^{-2})^a (ML^{-1}T^{-1})^b L^c$$

$$\Rightarrow LT^{-1} = M^{a+b} L^{a-b+c} T^{-2a-b}$$

Using Principle of Homogeneity, we get

$$a + b = 0 \quad \dots(1)$$

$$a - b + c = 1 \quad \dots(2)$$

$$-2a - b = -1 \quad \dots(3)$$

Adding (1) and (3), we get

$$a = 1$$

$$\Rightarrow b = -1$$

$$\Rightarrow 1 - (-1) + c = 1 \quad \{\because \text{of (2)}\}$$

$$\Rightarrow c = -1$$

So,  $a = 1, b = -1, c = -1$

$$\Rightarrow v_T = k \left( \frac{W}{\eta r} \right)$$

In Fluid Mechanics, this law is also called the Stokes' Law with the value of  $k = 6\pi$ .

5. Let the rate of flow of volume of the liquid be denoted by  $V$ .

$$\text{Then } [V] = \left[ \frac{\text{Volume}}{\text{Time}} \right] = L^3 T^{-1}$$

Now,  $V \propto \eta^a r^b p^c$

$$\Rightarrow V = k \eta^a r^b p^c$$

where  $k$  is a dimensionless constant and  $p$  is the pressure gradient. So

$$[p] = \left[ \frac{\text{Pressure}}{\text{Length}} \right] = ML^{-2} T^{-2}$$

$$\Rightarrow L^3 T^{-1} = (ML^{-1} T^{-1})^a (L)^b (ML^{-2} T^{-2})^c$$

$$\Rightarrow L^3 T^{-1} = M^{a+c} L^{-a+b-2c} T^{-a-2c}$$

Using Principle of Homogeneity, we get

$$a + c = 0 \quad \dots(1)$$

$$-a + b - 2c = 3 \quad \dots(2)$$

$$-a - 2c = -1 \quad \dots(3)$$

From (1) and (3), we get

$$c = 1$$

$$\Rightarrow a = -1 \quad \{\because \text{of (1)}\}$$

$$\Rightarrow -(-1) + b - 2(1) = 3 \quad \{\because \text{of (2)}\}$$

$$\Rightarrow b = 4$$

So, we get

$$a = -1, b = 4, c = 1$$

$$\Rightarrow V = k \left( \frac{r^4}{\eta} \right) p$$

This expression is also called the Poiseuille's Equation, which governs the flow of a liquid through a horizontal tube across the ends of which a pressure gradient ( $p$ ) is maintained. Also we shall see in hydromechanics that for this case  $k = \frac{\pi}{8}$ . So

$$V = \frac{\pi}{8} \left( \frac{r^4}{\eta} \right) p \quad \{\text{Poiseuille's Equation}\}$$

## H.8 JEE Advanced Physics: Mechanics - I

6.  $\omega_c \propto \eta^a \rho^b d^c$

$$\omega_c = k\eta^a \rho^b d^c$$

where  $k$  is a dimensionless constant

$$\Rightarrow T^{-1} = [ML^{-1}T^{-1}]^a (ML^{-3})^b (L)^c$$

$$\Rightarrow T^{-1} = M^{a+b} L^{-a-3b+c} T^{-a}$$

Using the Principle of Homogeneity, we get

$$a + b = 0 \quad \dots(1)$$

$$-a - 3b + c = 0 \quad \dots(2)$$

$$-a = -1 \quad \dots(3)$$

$$\Rightarrow a = 1, b = -1, c = -2$$

So, we get

$$\omega_c = k \left( \frac{\eta}{\rho d^2} \right)$$

7. No it would not be possible to obtain dimensionally a relation for  $h$  without the additional experimental information that  $h \propto \frac{1}{r}$ . This is because without the information  $h \propto \frac{1}{r}$ , we will get four variables to be calculated from three equations which is just impossible. So, we shall be writing

$$h \propto r^{-1} S^a d^b g^c$$

$$\Rightarrow h = kr^{-1} S^a d^b g^c$$

where  $k$  is a dimensionless constant

$$\Rightarrow L = L^{-1} (MT^{-2})^a (ML^{-3})^b (LT^{-2})^c$$

$$\Rightarrow L = M^{a+b} L^{-1-3b+c} T^{-2a-2c}$$

Using Principle of Homogeneity, we get

$$a + b = 0 \quad \dots(1)$$

$$-1 - 3b + c = 1 \quad \dots(2)$$

$$-2a - 2c = 0 \quad \dots(3)$$

So, (2) + (3) gives

$$2b - 2c = 0 \quad \dots(4)$$

Again 2) + (4) gives

$$b = -1$$

$$\Rightarrow a = 1 \quad \{\because \text{of (1)}\}$$

$$\Rightarrow c = -1$$

So,  $a = 1, b = -1$  and  $c = -1$

$$\Rightarrow h = kr^{-1} S d^{-1} g^{-1}$$

$$\Rightarrow h = k \left( \frac{S}{rgd} \right)$$

8. To show that the planet obeys Kepler's Third Law of planetary motion, we have to prove that

$$T^2 \propto r^3$$

Now, according to the problem, we have

$$T \propto r^a M_S^b G^c$$

$$\Rightarrow T = kr^a M_S^b G^c$$

where  $k$  is a dimensionless constant.

$$\Rightarrow T = L^a M^b (M^{-1} L^3 T^{-2})^c$$

$$\Rightarrow T = M^{b-c} L^{a+3c} T^{-2c}$$

Using the Principle of Homogeneity, we get

$$b - c = 0 \quad \dots(1)$$

$$a + 3c = 0 \quad \dots(2)$$

$$-2c = 1 \quad \dots(3)$$

$$\Rightarrow c = -\frac{1}{2}, b = -\frac{1}{2}, a = \frac{3}{2}$$

So, we get

$$T = kr^{\frac{3}{2}} M_S^{-\frac{1}{2}} G^{-\frac{1}{2}}$$

$$\Rightarrow T^2 = \left( \frac{k^2}{M_S G} \right) r^3$$

Since the product  $M_S G$  is constant, so

$$T^2 \propto r^3$$

9. We have  $\rho = ML^{-3}, g = LT^{-2}, v = T^{-1}$

Solving for  $M, L$  and  $T$  in terms of  $\rho, g$  and  $v$ , we get

$$M = \rho g^3 v^6, L = g v^{-2}, T = v^{-1}$$

$$\Rightarrow [\text{Force}] = MLT^{-2} = (\rho g^3 v^6)(g v^{-2})(v^2) = \rho g^4 v^6$$

10. Let the quantity be  $Q$  then,

$$Q = f(v, F, T)$$

Assuming that the function is the product of power functions of  $v, F$  and  $T$ ,

$$Q = K v^x F^y T^z \quad \dots(1)$$

where  $K$  is a dimensionless constant of proportionality. The above equation dimensionally becomes

$$[Q] = (LT^{-1})^x (MLT^{-2})^y (T)^z$$

$$\text{i.e., } [Q] = [M^y][L^{x+y}T^{-x-2y+z}] \quad \dots(2)$$

(a)  $Q = \text{mass}$

$$\text{i.e., } [Q] = [M]$$

So Equation (2) becomes

$$M = M^y L^{x+y} T^{-x-2y+z}$$

its dimensional correctness requires

$$y = 1, x + y = 0 \text{ and } -x - 2y + z = 0$$

which on solving yields

$$x = -1; y = 1 \text{ and } z = 1$$

Substituting it in equation (1), we get

$$Q = Kv^{-1}FT$$

(b)  $Q = \text{energy i.e., } [Q] = ML^2T^{-2}$

So equation (2) becomes

$$ML^2T^{-2} = M^yL^{x+y}T^{-x-2y+z}$$

which in the light of principle of homogeneity yields

$$y = 1, x + y = 2 \text{ and } -x - 2y + z = -2$$

which on solving yields

$$x = y = z = 1$$

So equation (1) becomes

$$Q = KvFT$$

### Test Your Concepts-IV (Based on Errors, Significant Figures, Vernier Calliper and Screw Gauge)

1.  $R = \frac{V}{I} = \frac{8}{2} = 4 \Omega$

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$$

$$\Rightarrow \% \frac{\Delta R}{R} = \left( \frac{0.5}{8} + \frac{0.2}{2} \right) 100$$

$$\Rightarrow \% \frac{\Delta R}{R} = \left( \frac{50}{8} + \frac{20}{2} \right) = 16.25\%$$

$$\Rightarrow \frac{\Delta R}{R} = \frac{16.25}{100}$$

$$\Rightarrow \Delta R = \frac{16.25}{100} \times 4$$

$$\Rightarrow \Delta R = 0.65 \Omega$$

So,  $R = 4 \Omega \pm 16.25\%$  or  $R = (4 \pm 0.65) \Omega$

2.  $d = \frac{m}{lbh}$

Actual value of density is given by

$$d = \frac{39.3}{(5.12)(2.56)(0.37)} \text{ gcc}^{-1}$$

$$\Rightarrow d = 8.1 \text{ gcc}^{-1}$$

Now  $\frac{\Delta d}{d} = \frac{\Delta m}{m} + \frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta h}{h}$

$$\Rightarrow \frac{\Delta d}{d} = \frac{0.1}{39.3} + \frac{0.01}{5.12} + \frac{0.01}{2.56} + \frac{0.01}{0.37}$$

$$\Rightarrow \frac{\Delta d}{d} \times 100\% = \frac{10}{39.3} + \frac{1}{5.12} + \frac{1}{2.56} + \frac{1}{0.37}$$

$$\Rightarrow \frac{\Delta d}{d} \times 100\% = 0.25 + 0.19 + 0.39 + 2.7$$

$$\Rightarrow \frac{\Delta d}{d} \times 100\% = 3.6\%$$

So,  $d = 8.1 \text{ gcc}^{-1} \pm 3.6\%$

3. Least Count = 1 MSD - 1 VSD

Since  $N \text{ VSD} = (N - 1) \text{ MSD}$

$$\Rightarrow 1 \text{ VSD} = \left( \frac{N-1}{N} \right) \text{ MSD}$$

$$\Rightarrow \text{Least Count} = 1 \text{ MSD} - \left( \frac{N-1}{N} \right) \text{ MSD}$$

$$\Rightarrow \text{Least Count} = \left( \frac{N-N+1}{N} \right) \text{ MSD}$$

$$\Rightarrow \text{Least Count} = \frac{1}{N} \text{ MSD}$$

Since 1 MSD = 1 mm

So, least count =  $\frac{1}{N}$  mm

$$\Rightarrow \text{Least count} = \frac{1}{10N} \text{ cm}$$

4. Length of cylinder,

$$h = 4.54 \text{ cm} \quad \{3 \text{ significant figures}\}$$

Radius of cylinder,

$$r = 1.75 \text{ cm} \quad \{3 \text{ significant figures}\}$$

Volume of cylinder  $V$ ,

$$\Rightarrow V = \pi r^2 h = 3.14 \times (1.75)^2 \times 4.54 \text{ cm}^3$$

$$\Rightarrow V = 43.657775 \text{ cm}^3 = 43.7 \text{ cm}^3$$

{Rounded off upto 3 significant figures}

5.

Column A	Column B
17236	5
510 m	3
270	2
4.20	3
7042.6	5
0.017	2
$6.1 \times 10^{14}$	2

## H.10 JEE Advanced Physics: Mechanics - I

Measured Value	Rounded off value to Three Significant Figures
7.364	7.36
8.3251	8.33
9.445	9.44
15.75	15.8
7.367	7.37
9.4450	9.44
15.7500	15.8

7. (a)  $11.439 \approx 11.4$ , because the least number of significant figures after the decimal is 1.  
 (b)  $2.43 \approx 2.4$ , because the least number of significant figures after the decimal is 1.  
 (c)  $44.064 \approx 44$ , because the least number of significant figures is 2.  
 (d)  $107.843 \approx 108$ , because the least number of significant figures is 3 (not 2), because  $1100 \text{ ms}^{-1}$  has four significant figures.

8. (a) Least count

$$\text{L.C.} = \frac{\text{Smallest division on main scale}}{\text{Number of divisions on vernier scale}}$$

$$\Rightarrow \text{L.C.} = \frac{1}{10} \text{ mm} = 0.1 \text{ mm}$$

$$\Rightarrow \text{L.C.} = 0.01 \text{ cm}$$

- (b) So, the length,

$$L = N + n(\text{L.C.}) = (10.2 + 3 \times 0.01) \text{ cm}$$

$$\Rightarrow L = 10.23 \text{ cm}$$

9.  $\text{L.C.} = \frac{1}{100} = 0.01 \text{ mm}$

Linear scale reading = 6 (pitch) = 6 mm

Circular scale reading =  $n(\text{L.C.}) = 40 \times 0.01 = 0.4 \text{ mm}$

So, total reading =  $(6 + 0.4) = 6.4 \text{ mm}$

10. Here zero of vernier scale lies to the right of zero of main scale, hence, it has positive zero error.

Further,  $N = 0$ ,  $x = 5$

So, least count or Vernier constant is  $0.01 \text{ cm}$

Hence, Zero error =  $N + x \times \text{V.C.}$

$$\Rightarrow \text{Zero error} = 0 + 5 \times 0.01$$

$$\Rightarrow \text{Zero error} = 0.05 \text{ cm}$$

So, zero correction =  $-0.05 \text{ cm}$

Hence the actual length will be  $0.05 \text{ cm}$  less than the measured length.

## Single Correct Choice Type Questions

1. Maximum possible error

$$\left| \frac{\Delta H}{H} \times 100 \right|_{\text{max}} = 2 \frac{\Delta I}{I} \times 100 + \frac{\Delta R}{R} \times 100 + \frac{\Delta t}{t} \times 100$$

$$\left\{ \because H = I^2 R t \right\}$$

Hence, the correct answer is (D).

2. The MKS unit of  $\eta = \frac{\text{kg}}{\text{ms}}$  is  $\text{kgm}^{-1} \text{s}^{-1}$

The CGS of  $\eta = \frac{\text{g}}{\text{cms}}$  is  $\text{gcm}^{-1} \text{s}^{-1}$

$$\frac{\eta_{\text{MKS}}}{\eta_{\text{CGS}}} = \frac{\text{kg}}{\text{ms}} \times \frac{\text{cms}}{\text{g}}$$

$$\frac{\eta_{\text{MKS}}}{\eta_{\text{CGS}}} = \frac{10^3 \text{ g} \times \text{cm} \times \text{s}}{100 \text{ cm} \times \text{s} \times \text{g}} = 10$$

Hence, the correct answer is (A).

3.  $i = K \tan \theta$

Since  $\tan \theta$  is a dimensionless physical quantity and hence the unit of  $k$  is equivalent to that of current i.e. ampere

Hence, the correct answer is (B).

4. Impulse = change in momentum

So, the dimensions of impulse and momentum are the same.

Hence, the correct answer is (C).

5. Coefficient of viscosity

$$\eta = \frac{\text{tangential force}}{\text{contact area} \times \text{velocity gradient}}$$

$$\Rightarrow \eta = \frac{\text{newton}}{\text{m}^2 \times \text{m} / \frac{\text{s}}{\text{m}}} = \frac{\text{newton s}}{\text{m}^2}$$

$$\eta = \frac{\text{kg ms}}{\text{s}^2 \text{m}^2} = \frac{\text{kg}}{\text{ms}}$$

Hence, the correct answer is (B).

6. Pressure correction  $P' = \frac{an^2}{V^2}$

$$\Rightarrow a = \frac{P'V^2}{n^2} = \frac{(\text{Pressure correction})(\text{Volume})^2}{(\text{amount of gas})^2}$$

$$\Rightarrow a = \frac{(\text{Nm}^{-2})(\text{m}^3)^2}{\text{mol}^2}$$

$$\Rightarrow a = \frac{\text{Nm}^4}{\text{mol}^2} = \frac{\text{kgmm}^4}{\text{s}^2 \text{mol}^2} \quad \left\{ \because 1 \text{ N} = 1 \text{ kgms}^{-2} \right\}$$

$$\Rightarrow a = \frac{\text{kgm}^5}{\text{s}^2\text{mol}^2} = \frac{\text{kgm}^5}{\text{s}^2} \quad (\text{as mol is a number})$$

Hence, the correct answer is (C).

7.  $\frac{1}{\sqrt{\mu\epsilon}}$  is the velocity of electromagnetic waves in a medium. Hence its dimensions are equivalent to those of velocity.

Hence, the correct answer is (A).

8. There are 6 significant figures in the number 108.023 as all the figures define a particular measurement. In the number 00.19 there are only two significant figures as the zeroes on the left of decimal point and the zeroes on the right of non-zero numbers after the decimal point are not significant.

Hence, the correct answer is (A).

$$9. \quad n = \frac{1}{\text{area} \times \text{time}} = \frac{1}{L^2T}$$

$$[n_1 - n_2] = \frac{1}{\text{Volume}} = \frac{1}{L^3}$$

$$[x_2 - x_1] = L$$

$$\Rightarrow D = -\frac{n(x_2 - x_1)}{n_2 - n_1} = \frac{L^{-2}T^{-1}L}{L^{-3}} = L^2T^{-1}$$

Hence, the correct answer is (A).

10.  $\frac{L}{R}$  and  $RC$  are the time constants of  $LR$  and  $RC$  circuits respectively. Hence their dimensions are equal to those of time.

Hence, the correct answer is (A).

$$11. \quad V = d^a E^b$$

Writing dimensions on two sides of the equation

$$[LT^{-1}] = [ML^{-1}T^{-2}]^b [ML^{-3}]^a$$

$$\Rightarrow LT^{-1} = M^{a+b}L^{-3a-b}T^{-2b}$$

Comparing the dimensions on two sides, we get

$$a + b = 0$$

$$-3a - b = 1$$

$$-2b = -1$$

Solving for  $a$  and  $b$ , we get

$$b = \frac{1}{2} \quad \text{and} \quad a = -\frac{1}{2}$$

Hence, the correct answer is (A).

12. All the three represent the electrostatic potential energy stored in a condenser

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV$$

The dimensions of energy and work are the same

$$\text{So, } [U] = \left[\frac{1}{2}CV^2\right] = \left[\frac{1}{2}\frac{q^2}{C}\right] = \left[\frac{1}{2}qV\right] = ML^2T^{-2}$$

Hence, the correct answer is (B).

$$13. \quad \text{Mass} = \frac{\text{Force}}{\text{Acceleration}} = \frac{F}{LT^{-2}} = FL^{-1}T^2$$

Hence, the correct answer is (A).

$$14. \quad [C^2LR] = \left[C^2L^2\frac{R}{L}\right] = \left[(LC)^2\left(\frac{R}{L}\right)\right]$$

and we know that frequency of  $LC$  circuits is given by  $f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$ , i.e., the dimension of  $LC$  is equal to

$[T^2]$  and  $\left[\frac{L}{R}\right]$  gives the time constant of  $L$ - $R$  circuit so the dimension of  $\frac{L}{R}$  is equal to  $[T]$ . By substituting the above dimension in the give formula

$$\left[(LC)^2\left(\frac{R}{L}\right)\right] = [T^2]^2[T^{-1}] = [T^3].$$

Hence, the correct answer is (B).

15. From the principle of dimensional homogeneity  $[\alpha t] = \text{dimensionless}$

$$\Rightarrow [\alpha] = \left[\frac{1}{t}\right] = T^{-1}$$

$$\text{Similarly } [x] = \frac{[v_0]}{[\alpha]}$$

$$\Rightarrow [v_0] = [x][\alpha] = LT^{-1}$$

Hence, the correct answer is (A).

16. From the dimensional homogeneity  $[x^2] = [B]$

$$\Rightarrow [B] = L^2$$

$$\text{Also } [U] = \frac{[A][x^2]}{[x^2] + [B]}$$

$$\Rightarrow ML^2T^{-2} = \frac{[A]L^2}{L^2}$$

$$\Rightarrow [A] = ML^2T^{-2}$$

$$\text{Now } [AB] = \left(ML^2T^{-2}\right) \times L^2 = ML^4T^{-2}$$

Hence, the correct answer is (B).

17. Let  $x = \text{length}$ , then

$$[X] = [L] \quad \text{and} \quad [dx] = [L]$$

By principle of dimensional homogeneity

$$\left[\frac{x}{a}\right] = \text{dimensionless}$$

$$\Rightarrow [a] = [x] = [L]$$

## H.12 JEE Advanced Physics: Mechanics - I

By substituting dimensions of each quantity on both sides we get

$$\frac{L}{\sqrt{L^2}} = L^n$$

$$\Rightarrow n = 0$$

Hence, the correct answer is (C).

18. Here,  $\frac{2\pi ct}{\lambda}$  as well as  $\frac{2\pi x}{\lambda}$  are dimensionless (angle)

$$\text{i.e. } \left[ \frac{2\pi ct}{\lambda} \right] = \left[ \frac{2\pi x}{\lambda} \right] = M^0 L^0 T^0$$

So

(i) unit of  $ct$  is same as that of  $\lambda$

(ii) unit of  $x$  is same as that of  $\lambda$

$$\text{(iii) } \left[ \frac{2\pi c}{\lambda} \right] = \left[ \frac{2\pi x}{\lambda t} \right] \text{ and}$$

(iv)  $\frac{x}{\lambda}$  is unit less but this is not the case with  $\frac{c}{\lambda}$

Hence, the correct answer is (D).

19. Since, measurement of a physical  $P = nu = \text{constant}$

$$\Rightarrow n_1 u_1 = n_2 u_2 \text{ or } n \propto \frac{1}{u}$$

Hence, the correct answer is (D).

20.  $n_1 = 100$ ,  $M_1 = 1 \text{ g}$ ,  $L_1 = 1 \text{ cm}$ ,  $T_1 = 1 \text{ sec}$ , and  $M_2 = 1 \text{ kg}$ ,  
 $L_2 = 1 \text{ metre}$ ,  $T_2 = 1 \text{ minute}$

By substituting these values in the following conversion formula

$$n_2 = n_1 \left( \frac{M_1}{M_2} \right)^a \left( \frac{L_1}{L_2} \right)^b \left( \frac{T_1}{T_2} \right)^c$$

where  $a = 1$ ,  $b = 1$ ,  $c = -2$

$$n_2 = 100 \left( \frac{1 \text{ g}}{1 \text{ kg}} \right)^1 \left( \frac{1 \text{ cm}}{1 \text{ meter}} \right)^1 \left( \frac{1 \text{ s}}{1 \text{ minute}} \right)^{-2}$$

$$n_2 = 100 \left( \frac{1 \text{ g}}{10^3 \text{ g}} \right)^1 \left( \frac{1 \text{ cm}}{10^2 \text{ cm}} \right)^1 \left( \frac{1 \text{ s}}{60 \text{ s}} \right)^{-2} = 3.6$$

Hence, the correct answer is (C).

21. Relation between centigrade and Fahrenheit

$$\frac{K - 273}{5} = \frac{F - 32}{9}$$

According to problem  $\frac{X - 273}{5} = \frac{X - 32}{9}$

$$\Rightarrow X = 574.25$$

Hence, the correct answer is (B).

22. Because 1 newton =  $10^5$  dyne

Hence, the correct answer is (D).

23. We know that the dimensional formula of Young's modulus is  $ML^{-1}T^{-2}$  and its C.G.S. unit is  $\text{gcm}^{-1}\text{sec}^{-2}$  and M.K.S. unit is  $\text{kgm}^{-1}\text{sec}^{-2}$ .

$$\text{Since, } n_2 = n_1 \left( \frac{M_1}{M_2} \right)^1 \left( \frac{L_1}{L_2} \right)^{-1} \left( \frac{T_1}{T_2} \right)^{-2}$$

$$\Rightarrow n^2 = \left( \frac{1 \text{ g}}{1 \text{ kg}} \right)^1 \left( \frac{1 \text{ cm}}{1 \text{ m}} \right)^{-1} \left( \frac{1 \text{ s}}{1 \text{ s}} \right)^{-2}$$

So, conversion factor

$$\frac{n_2}{n_1} = \left[ \frac{1 \text{ g}}{10^{-3} \text{ g}} \right]^1 \left[ \frac{1 \text{ cm}}{10^2 \text{ cm}} \right]^{-1} \left[ \frac{1 \text{ s}}{1 \text{ s}} \right]^{-2} = \frac{1}{10} = 0.1$$

Hence, the correct answer is (C).

24.  $[P] = [ML^2T^{-3}]$

Using the relation

$$n_2 = n_1 \left( \frac{M_1}{M_2} \right)^a \left( \frac{L_1}{L_2} \right)^b \left( \frac{T_1}{T_2} \right)^c$$

$$n_2 = (1 \times 10^6) \left( \frac{1 \text{ kg}}{10 \text{ kg}} \right)^1 \left( \frac{1 \text{ m}}{1 \text{ dm}} \right)^2 \left( \frac{1 \text{ s}}{1 \text{ min}} \right)^{-3}$$

{As  $1 \text{ MW} = 10^6 \text{ W}$ }

$$\Rightarrow n_2 = 10^6 \left( \frac{1 \text{ kg}}{10 \text{ kg}} \right) \left( \frac{10 \text{ dm}}{1 \text{ dm}} \right)^2 \left( \frac{1 \text{ s}}{60 \text{ s}} \right)^{-3} = 2.16 \times 10^{12}$$

Hence, the correct answer is (A).

25.  $v_2 = v_1 \frac{\alpha^2}{\beta}$

$$\Rightarrow [L_2 T_2^{-1}] = [L_1 T_1^{-1}] \frac{\alpha^2}{\beta} \quad \dots(1)$$

$$a_2 = a_1 \alpha \beta$$

$$\Rightarrow [L_2 T_2^{-2}] = [L_1 T_1^{-2}] \alpha \beta \quad \dots(2)$$

$$\text{and } F_2 = \frac{F_1}{\alpha \beta}$$

$$\Rightarrow [M_2 L_2 T_2^{-2}] = [M_1 L_1 T_1^{-2}] \times \frac{1}{\alpha \beta} \quad \dots(3)$$

Dividing equation (3) by equation (2) we get

$$M_2 = \frac{M_1}{(\alpha \beta) \alpha \beta} = \frac{M_1}{\alpha^2 \beta^2}$$

Squaring equation (1) and dividing by equation (2) we get

$$L_2 = L_1 \frac{\alpha^3}{\beta^3}$$

Dividing equation (1) by equation (2) we get

$$T_2 = T_1 \frac{\alpha}{\beta^2}$$

Hence, the correct answer is (B).

26. Let the new system be designated as 2 and the old system be designated as 1. Then

$$\frac{n_2}{n_1} = \left(\frac{M_2}{M_1}\right)^a \left(\frac{L_2}{L_1}\right)^b \left(\frac{T_2}{T_1}\right)^c$$

**For velocity**

$$\frac{n_2}{n_1} = \left(\frac{M_2}{M_1}\right)^0 \left(\frac{L_2}{L_1}\right)^1 \left(\frac{T_2}{T_1}\right)^{-1}$$

$$\frac{n_2}{n_1} = (100)(100)^{-1} = 1$$

So, no change for velocity and hence, OPTION (A) is incorrect

**For Force**

$$\frac{n_2}{n_1} = \left(\frac{M_2}{M_1}\right)^1 \left(\frac{L_2}{L_1}\right)^1 \left(\frac{T_2}{T_1}\right)^{-2}$$

$$\Rightarrow \frac{n_2}{n_1} = \left(\frac{1}{10}\right)(100)(100)^{-2}$$

$$\Rightarrow \frac{n_2}{n_1} = \frac{1}{1000}$$

So, OPTION (B) is correct

**For Energy**

$$\frac{n_2}{n_1} = \left(\frac{M_2}{M_1}\right)^1 \left(\frac{L_2}{L_1}\right)^2 \left(\frac{T_2}{T_1}\right)^{-2}$$

$$\Rightarrow \frac{n_2}{n_1} = \left(\frac{1}{10}\right)(100)^2(100)^{-2}$$

$$\Rightarrow \frac{n_2}{n_1} = \frac{1}{10}$$

So, OPTION (C) is incorrect

**For Pressure**

$$\frac{n_2}{n_1} = \left(\frac{M_2}{M_1}\right)^1 \left(\frac{L_2}{L_1}\right)^{-1} \left(\frac{T_2}{T_1}\right)^{-2}$$

$$\Rightarrow \frac{n_2}{n_1} = \left(\frac{1}{10}\right)(100)^{-1}(100)^{-2}$$

$$\Rightarrow \frac{n_2}{n_1} = \frac{1}{10^7}$$

So, OPTION (D) is also incorrect.

Hence, the correct answer is (B).

27.  $[E] = [ML^2T^{-2}]$

$$1 \text{ eluoj} = [100 \text{ kg}] \times [1 \text{ km}]^2 \times [100 \text{ sec}]^{-2}$$

$$\Rightarrow 1 \text{ eluoj} = 100 \text{ kg} \times 10^6 \text{ m}^2 \times 10^{-4} \text{ sec}^{-2}$$

$$\Rightarrow 1 \text{ eluoj} = 10^4 \text{ kg m}^2 \times \text{sec}^{-2} = 10^4 \text{ Joule}$$

Hence, the correct answer is (A).

28.  $1 \text{ gcms}^{-1} = 10^{-3} \text{ kg} \times 10^{-2} \text{ m} \times \text{s}^{-1}$

$$\Rightarrow 1 \text{ gmcs}^{-1} = 10^{-5} \text{ kg} \times \text{m} \times \text{s}^{-1} = 10^{-5} \text{ Ns}$$

Hence, the correct answer is (D).

29. Given  $m = \text{mass} = [M]$ ,  $\eta = \text{coefficient of rigidity} = [ML^{-1}T^{-2}]$ ,  $L = \text{length} = [L]$ .

By substituting the dimensions of these quantities we can check the accuracy of the given formulae

$$[T] = 2\pi \left( \frac{[M]}{[\eta][L]} \right)^{1/2} = \left[ \frac{M}{ML^{-1}T^{-2}L} \right]^{1/2} = T$$

Hence, the correct answer is (D).

30. Let  $m \propto c^x G^y h^z$

$$\Rightarrow m = Kc^x G^y h^z$$

By substituting the dimension of each quantity in both sides and using the Principle of Homogeneity

$$[M^1 L^0 T^0] = K [LT^{-1}]^x [M^{-1} L^3 T^{-2}]^y [ML^2 T^{-1}]^z$$

$$[M^1 L^0 T^0] = [M^{-y+z} L^{x+3y+2z} T^{-x-2y-z}]$$

By equating the power of M, L and T on both sides, we get

$$-y + z = 1, \quad x + 3y + 2z = 0 \quad \text{and} \quad -x - 2y - z = 0$$

Solving above three equations we get,

$$x = \frac{1}{2}, \quad y = -\frac{1}{2} \quad \text{and} \quad z = \frac{1}{2}$$

$$\Rightarrow m \propto c^{1/2} G^{-1/2} h^{1/2}$$

Hence, the correct answer is (B).

31.  $[P^x Q^y C^z] = M^0 L^0 T^0$

By substituting the dimension of each quantity in the given expression

$$[ML^{-1}T^{-2}]^x [MT^{-3}]^y [LT^{-1}]^z = M^0 L^0 T^0$$

$$\Rightarrow [M^{x+y} L^{-x+z} T^{-2x-3y-z}] = M^0 L^0 T^0$$

Equating the power of M, L and T in both sides, we get

$$x + y = 0, \quad -x + z = 0 \quad \text{and} \quad -2x - 3y - z = 0$$

Solving the above equations, we get

$$x = 1, \quad y = -1, \quad z = 1$$

Hence, the correct answer is (B).

32. Writing dimensions of both sides

$$[L^3] = [L^2]^\alpha [LT^{-1}]^\beta [T]^\gamma$$

## H.14 JEE Advanced Physics: Mechanics - I

$$\Rightarrow [L^3 T^0] = [L^{2\alpha + \beta} T^{\gamma - \beta}]$$

By comparing powers of both sides  $2\alpha + \beta = 3$  and  $\gamma - \beta = 0$

which gives  $\beta = \gamma$  and  $\alpha = \frac{1}{2}(3 - \beta)$ ,

i.e.  $\alpha \neq \beta = \gamma$

Hence, the correct answer is (B).

33. Let  $A = \frac{KA_0 V \lambda^x}{r}$

By substituting the dimension of each quantity in both sides

$$\Rightarrow [L] = \frac{[L] \cdot [L^3][L^x]}{[L]}$$

$$\Rightarrow L^{-2} = L^x$$

$$\Rightarrow x = -2$$

$$\Rightarrow A \propto \lambda^{-2}$$

$$\Rightarrow A \propto \frac{1}{\lambda^2}$$

Hence, the correct answer is (B).

34. Volume =  $a^3 = (7.023)^3 = 373.715 \text{ m}^3$

In significant figures, the volume of cube will be  $373.7 \text{ m}^3$  because the side of the cube has four significant figures.

Hence, the correct answer is (C).

35. Unit of luminous intensity is candela.

Hence, the correct answer is (C).

36. Assume the thickness of window pane to be 3 mm. If  $t$  is the time taken by light to penetrate it, then

$$t = \frac{\text{Thickness of Window Pane}}{\text{Speed of Light}}$$

$$\Rightarrow t = \frac{3 \times 10^{-3}}{3 \times 10^8} = 10^{-11} \text{ s}$$

Hence, the correct answer is (C).

37. Latent Heat =  $\frac{\text{Heat energy}}{\text{Mass}}$

So, unit of latent heat is  $\text{J kg}^{-1}$ .

Hence, the correct answer is (C).

38.  $E = \sigma T^4$

$$\Rightarrow [E] = [\sigma][T^4]$$

$$\Rightarrow \frac{ML^2 T^{-2}}{L^2 T} = [\sigma] K^4$$

$$\Rightarrow [\sigma] = MT^{-3} K^{-4}$$

Hence, the correct answer is (B).

39.  $F = (\text{mass})(\text{acceleration})$

$$\Rightarrow F = MLT^{-2}$$

$$\Rightarrow T^2 = \frac{ML}{F}$$

$$\Rightarrow T = M^{\frac{1}{2}} L^{\frac{1}{2}} F^{-\frac{1}{2}}$$

Hence, the correct answer is (A).

40.  $70 \text{ dyne cm}^{-1} = 70 \frac{\text{dyne}}{\text{cm}} = \frac{70 \times 10^{-5} \text{ N}}{10^{-2} \text{ m}}$

$$\Rightarrow 70 \text{ dyne cm}^{-1} = 7 \times 10^{-2} \text{ Nm}^{-1}$$

Hence, the correct answer is (B).

41. Temperature is a fundamental quantity independent of all expressed.

Hence, the correct answer is (D).

42.  $[\text{Density}] = ML^{-3}$

$$\text{Since } n_2 = n_1 \left( \frac{M_1}{M_2} \right)^a \left( \frac{L_1}{L_2} \right)^b \left( \frac{T_1}{T_2} \right)^c$$

where  $a = 1$ ,  $b = -3$ ,  $c = 0$  and  $n_1 = 8$ ,  $n_2 = ?$

$$\Rightarrow n_2 = 8 \left( \frac{1 \text{ g}}{20 \text{ g}} \right) \left( \frac{1 \text{ cm}}{5 \text{ cm}} \right)^{-3}$$

$$\Rightarrow n_2 = 8 \times \frac{1}{20} \times 5 \times 5 \times 5$$

$$\Rightarrow n_2 = 50$$

So, New Density is 50 new units

Hence, the correct answer is (B).

43.  $[\text{ergm}^{-1}] = [\text{force}]$

Hence, the correct answer is (A).

44.  $[\text{Pressure}] = \left[ \frac{\text{Force}}{\text{Area}} \right] = ML^{-1} T^{-2}$

Hence, the correct answer is (D).

45.  $[F] = MLT^{-2}$

$$\Rightarrow [F] = (10 \text{ g})(10 \text{ cm})(0.1 \text{ s})^{-2}$$

$$\Rightarrow [F] = 10^4 \text{ gcms}^{-2} = 0.1 \text{ N} \quad \{ \because 1 \text{ N} = 10^5 \text{ dyne} \}$$

Hence, the correct answer is (A).

46.  $R = \frac{\rho \ell}{A}$

$$\Rightarrow \rho = \frac{RA}{l} = \frac{VA}{Il} = \frac{WA}{q(Il)} = \frac{WAt}{q^2 l}$$

$$\Rightarrow [\rho] = \frac{(ML^2 T^{-2})(L^2)(T)}{(Q^2)(L)} = ML^3 T^{-1} Q^{-2}$$

Hence, the correct answer is (A).

47. SI unit of pressure is pascal.  
Hence, the correct answer is (C).

$$48. \left[ \frac{\pi}{3}(a^2 - b^2)h \right] = L^3 = \text{Volume}$$

Hence, the correct answer is (D).

$$49. [\text{Angular momentum}] = [mvr] = ML^2T^{-1}$$

$$[\text{joulesecond}] = ML^2T^{-1}$$

Hence, the correct answer is (D).

$$50. x = at + bt^2 + c$$

$$\Rightarrow [x] = [at] = [bt^2] = [c]$$

$$\Rightarrow [a] = mhr^{-1}, [b] = mhr^{-2}, [c] = m$$

Hence, the correct answer is (B).

$$51. \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c = \text{Velocity of Light in vacuum.}$$

Hence, the correct answer is (A).

52.  $109.832 \times 0.6107$  must have 4 significant figures.

Hence, the correct answer is (C).

$$53. [\tau] = [\text{Energy}] = ML^2T^{-2}$$

Hence, the correct answer is (A).

$$54. ML^2T^{-3} = \frac{ML^2T^{-2}}{T} = \frac{[\text{Work}]}{[\text{Time}]} = [\text{Power}]$$

Hence, the correct answer is (D).

$$55. \left[ \frac{L}{p} \right] = \frac{ML^2T^{-1}}{MLT^{-1}} = M^0L^1T^0$$

Hence, the correct answer is (A).

$$56. e \propto \frac{1}{\sqrt{n}}$$

$$\Rightarrow \frac{e_1}{e_2} = \sqrt{\frac{n_2}{n_1}} = \sqrt{4} = 2$$

$$\Rightarrow e_2 = \frac{e_1}{2} = \frac{x}{2}$$

Hence, the correct answer is (D).

$$57. [v] = [at] = \left[ \frac{b}{t+c} \right] \text{ and } [t] = [c]$$

$$\Rightarrow [c] = T$$

$$\Rightarrow [t+c] = T$$

$$\Rightarrow \frac{[b]}{T} = LT^{-1}$$

$$\Rightarrow [b] = L \text{ and } [a] = LT^{-2}$$

$$\text{So, } [a] = LT^{-2}, [b] = L, [c] = T$$

Hence, the correct answer is (C).

$$58. [mc^2] = [\text{Energy}] = ML^2T^{-2}$$

Hence, the correct answer is (C).

59.  $\text{kgms}^{-1}$  is the unit of momentum.

Hence, the correct answer is (B).

60. 1 micron =  $10^{-6}$  m

Hence, the correct answer is (C).

$$61. \sigma = \frac{E}{T^4}$$

where  $E$  is energy per second per unit area. So,

$$\sigma = \text{watt } m^{-2}K^{-4}$$

Hence, the correct answer is (C).

62. Ns is unit of impulse. Though  $1 \text{ Ns} = 1 \text{ kgms}^{-1}$  still it is not a unit of momentum.

Hence, the correct answer is (B).

$$64. F = \frac{Gm_1m_2}{r^2}$$

$$\Rightarrow [G] = \left[ \frac{Fr^2}{m_1m_2} \right] = M^{-1}L^3T^{-2}$$

Hence, the correct answer is (C).

$$65. [KE] = ML^2T^{-2}$$

$$\Rightarrow [KE]_{\text{new}} = (2M)(2L)^2T^{-2}$$

$$\Rightarrow [KE]_{\text{new}} = 8(ML^2T^{-2}) = 8(KE)_{\text{old}}$$

Hence, the correct answer is (A).

66. See drawbacks of dimensional analysis.

Hence, the correct answer is (A).

$$67. Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\Rightarrow [Y] = \frac{[\text{Stress}]}{[\text{Strain}]} = \frac{[\text{Force}]}{[\text{Area}]}$$

$$M^0L^0T^0$$

$$\Rightarrow [Y] = FA^{-1}D^0$$

Hence, the correct answer is (B).

68. 1 decapoise = 10 poise

$$MKS = 10(CG S)$$

Hence, the correct answer is (B).

69. [Light year] = L

Hence, the correct answer is (D).

71. Torque and work have same dimensions.

Hence, the correct answer is (B).

$$72. \left[ \frac{t}{p} \right] = M^0L^0T^0$$

## H.16 JEE Advanced Physics: Mechanics - I

$$\Rightarrow [t] = [p]$$

So, unit of  $t$  and  $p$  are the same.

Hence, the correct answer is (D).

73.  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Hence, the correct answer is (C).

74.  $F \propto D^a V^b A^c$

$$MLT^{-2} = (ML^{-3})^a (LT^{-1})^b (L^2)^c$$

$$\Rightarrow MLT^{-2} = M^a L^{-3a+b+2c} T^{-b}$$

$$\Rightarrow a = 1, -3a + b + 2c = 1, -b = -2$$

$$\Rightarrow -3 + 2 + 2c = 1$$

$$\Rightarrow c = 1$$

So,  $F = DV^2 A^1$

$$\Rightarrow F = AV^2 D$$

Hence, the correct answer is (A).

75.  $X = \epsilon_0 L \left( \frac{\Delta V}{\Delta t} \right)$

$$X = \left( \frac{C}{L} \right) (L) \left( \frac{\Delta V}{\Delta t} \right) \quad \left\{ \because \epsilon_0 = \frac{\text{Capacitance}}{\text{Length}} \right\}$$

$$\Rightarrow X = C \left( \frac{\Delta V}{\Delta t} \right)$$

$$\Rightarrow X = \frac{C \Delta V}{\Delta t} = \frac{\Delta Q}{\Delta t} = \text{Current}$$

So,  $X$  happens to be the current.

Hence, the correct answer is (D).

76.  $\left[ \frac{L}{R} \right] = [\text{Time}]$

Hence, the correct answer is (D).

77.  $l \propto c^\alpha g^\beta p^\gamma$

$$\Rightarrow L = (LT^{-1})^\alpha (LT^{-2})^\beta (ML^{-1}T^{-2})^\gamma$$

$$\Rightarrow L = M^\gamma L^{\alpha+\beta-\gamma} T^{-\alpha-2\beta-2\gamma}$$

$$\Rightarrow \gamma = 0, \alpha + \beta = 1 \text{ and } -\alpha - 2\beta = 0$$

$$\Rightarrow -\beta = 1$$

$$\Rightarrow \beta = -1 \text{ and } \alpha = 2$$

So,  $l = c^2 g^{-1} p^0$

Also, we could have done this by taking dimensional formulae for velocity and acceleration and eliminating  $t$ .

Hence, the correct answer is (A).

79.  $v = G^a M^b R^c$

$$\Rightarrow LT^{-1} = (M^{-1}L^3T^{-2})^a (M)^b (L)^c$$

$$\Rightarrow LT^{-1} = M^{-a+b} L^{3a+c} T^{-2a}$$

$$\Rightarrow -a + b = 0, 3a + c = 1, -2a = -1$$

$$\Rightarrow a = b, 3a + c = 1, a = \frac{1}{2}$$

$$\Rightarrow b = \frac{1}{2}, c = -\frac{1}{2}, a = \frac{1}{2}$$

$$\Rightarrow v = \frac{G^{\frac{1}{2}} M^{\frac{1}{2}}}{R^{\frac{1}{2}}} = \sqrt{\frac{GM}{R}}$$

Hence, the correct answer is (B).

80.  $T = 2\pi \sqrt{\frac{l}{g}}$

$$\Rightarrow T^2 = 4\pi^2 \left( \frac{l}{g} \right)$$

$$\Rightarrow g = 4\pi^2 \left( \frac{l}{T^2} \right)$$

$$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta l}{l} + 2 \left( \frac{\Delta T}{T} \right)$$

$$\Rightarrow \frac{\Delta g}{g} \times 100 = 1\% + 2(3\%) = 7\%$$

Hence, the correct answer is (A).

83.  $v = \frac{1}{2l} \sqrt{\frac{T}{m}}$

$$\Rightarrow T^{-1} = \frac{1}{L} \sqrt{\frac{MLT^{-2}}{[m]}}$$

$$\Rightarrow T^{-2} = \frac{1}{L^2} \frac{MLT^{-2}}{[m]}$$

$$\Rightarrow [m] = ML^{-1}T^0$$

Hence, the correct answer is (A).

86.  $g = \frac{GM}{r^2}$

$$M = \frac{4}{3} \pi r^3 \rho$$

$$\Rightarrow g = \frac{G \left( \frac{4}{3} \pi r^3 \rho \right)}{r^2}$$

$$\Rightarrow \rho = \frac{3g}{4\pi Gr}$$

Hence, the correct answer is (C).

87.  $\left[ \frac{1}{2} \epsilon_0 E^2 \right] = \left[ \frac{B^2}{2\mu_0} \right] = [\text{Pressure}] = ML^{-1}T^{-2}$

Hence, the correct answer is (B).

88. (A), (B) and (C) all have units same as energy per unit volume.  
Hence, the correct answer is (D).
89.  $[\alpha t^2] = M^0 L^0 T^0$   
 $\Rightarrow [\alpha] = M^0 L^0 T^{-2}$   
 Hence, the correct answer is (B).
90. Magnetic Moment = (Current) (Area)  
 $\Rightarrow [\text{Magnetic moment}] = AL^2$   
 Hence, the correct answer is (C).
91. Surface Tension  $\propto E^a V^b T^c$   
 $\Rightarrow MT^{-2} = (ML^2T^{-2})^a (LT^{-1})^b (T)^c$   
 $\Rightarrow MT^{-2} = M^a L^{2a+b} T^{-2a-b+c}$   
 $\Rightarrow a = 1, 2a + b = 0, -2a - b + c = -1$   
 $\Rightarrow b = -2$  and  $-2 + 2 + c = -2$   
 $\Rightarrow a = 1, b = -2, c = -2$   
 So, the desired relation is  $EV^{-2}T^{-2}$  OR  

$$\text{Surface tension} = \frac{\text{Work Done}}{\text{Change in Surface Area}}$$
  
 $\Rightarrow \sigma = \frac{E}{l^2} = \frac{E}{(VT)^2} = EV^{-2}T^{-2}$   
 Hence, the correct answer is (A).
92.  $[Y] = \left[ \frac{X}{Z^2} \right] = \left[ \frac{\text{Capacitance}}{(\text{Magnetic Induction})^2} \right]$   
 $\Rightarrow [Y] = \frac{M^{-1}L^{-2}Q^2T^2}{M^2Q^{-2}T^{-2}} = M^{-3}L^{-2}Q^4T^4$   
 Hence, the correct answer is (B).
95.  $[\lambda t] = M^0 L^0 T^0$   
 $\Rightarrow [\lambda] = M^0 L^0 T^{-1}$   
 Hence, the correct answer is (B).
97.  $[\text{Mobility}] = \left[ \frac{\text{Velocity}}{\text{Electric Field}} \right]$   
 $\Rightarrow [\mu] = \frac{ms^{-1}}{NC^{-1}}$   
 But  $1 \text{ N} = 1 \text{ kgms}^{-2}$   
 $\Rightarrow [\mu] = \frac{ms^{-1}}{\text{kgms}^{-2}\text{C}^{-1}} = \text{Cskg}^{-1}$   
 Hence, the correct answer is (A).
100.  $\tan \theta = \frac{v^2}{rg}$  is both numerically and dimensionally correct.  
 Hence, the correct answer is (A).
102. Amplification factor is the ratio of identical physical quantities, so it has no units.  
 Hence, the correct answer is (D).
103.  $\frac{\Delta Y}{Y} = a \frac{\Delta M}{M} + b \frac{\Delta L}{L} + c \frac{\Delta T}{T}$   
 $\Rightarrow \frac{\Delta Y}{Y} (\%) = (\alpha a + \beta b + \gamma c) \%$   
 Hence, the correct answer is (C).
104. Since  $T = 2\pi \sqrt{\frac{l}{g}}$   
 $\Rightarrow T^2 = 4\pi^2 \left( \frac{l}{g} \right)$   
 So, the relation  $T^2 = \frac{l}{g}$  is just dimensionally correct, but numerically incorrect.  
 Hence, the correct answer is (C).
106.  $1 \text{ MSD} = \frac{1}{10} (1 \text{ cm}) = 1 \text{ mm}$   
 $10 \text{ VSD} = 8 \text{ MSD}$   
 $1 \text{ VSD} = \frac{8}{10} \text{ MSD}$   
 Least count =  $1 \text{ MSD} - 1 \text{ VSD}$   
 $\Rightarrow \text{LC} = 1 \text{ mm} - \frac{8}{10} \text{ mm}$   
 $\Rightarrow \text{LC} = \frac{2}{10} \text{ mm}$   
 $\Rightarrow \text{LC} = \frac{2}{100} \text{ cm} = 0.02 \text{ cm}$   
 Hence, the correct answer is (B).
107.  $50 \text{ VSD} = 49 \text{ MSD}$   
 $1 \text{ VSD} = \frac{49}{50} \text{ MSD}$   
 $\Rightarrow \text{VC} = 1 \text{ MSD} - 1 \text{ VSD}$   
 $\Rightarrow \text{VC} = 1 \text{ MSD} - \frac{49}{50} \text{ MSD}$   
 $\Rightarrow \text{VC} = \frac{1}{50} \text{ MSD}$   
 $\Rightarrow 1 \text{ MSD} = 50 (\text{VC}) = 50 (0.001 \text{ cm})$   
 $\Rightarrow 1 \text{ MSD} = 0.05 \text{ cm} = 0.5 \text{ mm}$   
 $\Rightarrow 1 \text{ MSD} = 0.5 \text{ mm}$   
 Hence, the correct answer is (B).

## H.18 JEE Advanced Physics: Mechanics - I

108.

Device	Least Count
1. Metre Scale	1 mm
2. Vernier Calliper	0.1 mm
3. Screw Gauge	0.01 mm
4. Spherometer	0.01 mm

Hence, the correct answer is (D).

109.  $LC = \frac{1}{N} \text{MSD}$

$$0.005 \text{ cm} = \frac{1}{N} (0.1 \text{ cm})$$

$$\Rightarrow N = \frac{0.1}{0.005} = \frac{1000}{50}$$

$$\Rightarrow N = 20$$

Hence, the correct answer is (B).

110.  $LC = 1 \text{MSD} - 1 \text{VSD}$

$$\frac{1}{10N} = \frac{1}{10} - 1 \text{VSD} \quad \left\{ \because 1 \text{MSD} = \frac{1}{10} \text{cm} \right\}$$

$$\Rightarrow 1 \text{VSD} = \frac{1}{10} - \frac{1}{10N}$$

$$\Rightarrow 1 \text{VSD} = \frac{1}{10} \left( 1 - \frac{1}{N} \right)$$

$$\Rightarrow 1 \text{VSD} = 1 \text{MSD} \left( 1 - \frac{1}{N} \right)$$

$$\Rightarrow N \text{VSD} = (N-1) \text{MSD}$$

Hence, the correct answer is (C).

111.  $100 \text{VSD} = 99 \text{MSD}$

$$\Rightarrow 1 \text{VSD} = \frac{99}{100} \text{MSD}$$

$$LC = 1 \text{MSD} - 1 \text{VSD}$$

$$\Rightarrow LC = 1 \text{MSD} - \frac{99}{100} \text{MSD}$$

$$\Rightarrow LC = 0.01 \text{MSD}$$

$$\Rightarrow LC = 0.01 (1 \text{mm}) = 0.01 \text{mm}$$

Hence, the correct answer is (C).

112. Error can be, at the maximum equal to the least count of the device used.

Hence, the correct answer is (D).

113.  $\text{Error} \propto \frac{1}{\sqrt{\text{Number of readings}}}$

$$\Rightarrow \frac{E_2}{E_1} = \sqrt{\frac{100}{400}} = \frac{1}{2}$$

Hence, the correct answer is (B).

114. Vernier constant is actually the least count of the device.

Hence, the correct answer is (D).

115. The fraction of a degree of an angle is measured with the help of angular vernier, which is provided in spherometers and sextants used to measure angular displacements.

Hence, the correct answer is (C).

131. As  $x = at^2 + b$  and therefore  $[a] = [LT^{-2}]$  and  $[b] = [L]$ .

$$\Rightarrow [ab] = [L^2T^{-2}]$$

Hence, the correct answer is (A).

136. Let time period depends on mass ( $m$ ), amplitude ( $A$ ) and constant ( $k$ ) as  $T \propto m^\alpha a^\beta k^\gamma$ .

$$M^0 L^0 [T] \propto M^\alpha L^\beta \left[ \frac{ML^2 T^{-2}}{L^3} \right]^\gamma$$

$$\Rightarrow M^0 L^0 [T] \propto M^{\alpha+\gamma} L^{\beta-\gamma} T^{-2\gamma}$$

Equating dimensions both sides

$$\alpha + \gamma = 0$$

$$\Rightarrow \alpha = -\gamma$$

$$\Rightarrow \alpha = \frac{1}{2}$$

$$\beta - \gamma = 0$$

$$\Rightarrow \beta = \gamma$$

$$\Rightarrow \beta = -\frac{1}{2}$$

$$\Rightarrow -2\gamma = 1$$

$$\Rightarrow \gamma = -\frac{1}{2}$$

$$\Rightarrow t \propto m^{1/2} a^{-1/2} k^{1/2}$$

$$\Rightarrow t \propto \sqrt{\frac{m}{ak}}$$

$$\Rightarrow t \propto \frac{1}{\sqrt{a}}$$

Hence, the correct answer is (A).

137. Dimensionally

$$F = MLT^{-2}$$

In C.G.S. system

$$1 \text{ dyne} = 1 \text{ g } 1 \text{ cm } (1 \text{ s})^{-2}$$

In new system

$$1x = (10 \text{ g})(10 \text{ cm})(0.1 \text{ s})^{-2}$$

$$\Rightarrow \frac{1 \text{ dyne}}{1x} = \frac{1 \text{ g}}{10 \text{ g}} \times \frac{1 \text{ cm}}{10 \text{ cm}} \left( \frac{10 \text{ s}}{1 \text{ s}} \right)^{-2}$$

$$\Rightarrow 1 \text{ dyne} = \frac{1}{10,000} \times 1x$$

$$\Rightarrow 10^4 \text{ dyne} = 1x$$

$$\Rightarrow 10x = 10^5 \text{ dyne} = 1 \text{ N}$$

$$\Rightarrow x = \frac{1}{10} \text{ N}$$

Hence, the correct answer is (A).

138.  $[F] = \frac{[A-x]}{[B][t]}$

$$\Rightarrow [B] = \frac{[L]}{[MLT^{-2}][T]} = [M^{-1}T^1]$$

Hence, the correct answer is (C).

139. Dimension of  $[\eta] \equiv \left[ \frac{Fr}{Av} \right] \equiv \frac{[MLT^{-2}][L]}{[L^2][LT^{-1}]}$

$$[\eta] \equiv [ML^{-1}T^{-1}]$$

Dimensions of  $\mu = Fr^n$

$$[\mu] = [MLT^{-2}]L^n$$

$$\Rightarrow [\mu] = ML^{n+1}T^{-2}$$

Let  $\eta$  depend on mass  $m$  mean speed  $v$  and constant  $\mu$  as

$$\eta \propto m^a v^b \mu^c$$

$$\Rightarrow [ML^{-1}T^{-1}] \propto M^a [LT^{-1}]^b [ML^{n+1}T^{-2}]^c$$

$$\Rightarrow [ML^{-1}T^{-1}] \propto M^{a+c} L^{b+c(n+1)} T^{-b-2c}$$

Equating dimensions on both sides, we get

$$a + c = 1$$

$$\Rightarrow c = 1 - a$$

$$\Rightarrow b + c(n+1) = -1$$

$$\Rightarrow b = -[1 + c(n+1)]$$

$$\Rightarrow -(b + 2c) = -1$$

$$\Rightarrow b = 1 - 2c$$

$$\Rightarrow 1 - 2c = -[1 + c(n+1)]$$

$$\Rightarrow 2c - 1 = 1 + c(n+1)$$

$$\Rightarrow 2c - c(n+1) = 2$$

$$\Rightarrow c[2 - n - 1] = 2$$

$$\Rightarrow c[1 - n] = 2$$

$$\Rightarrow c = -\frac{2}{n-1}$$

Since,  $a = 1 - c$

$$\Rightarrow a = 1 + \frac{2}{n-1} = \frac{n-1+2}{n-1}$$

$$\Rightarrow a = \frac{n+1}{n-1}$$

and  $b = 1 - 2c = 1 + \frac{4}{n-1} = \frac{n-1+4}{n-1}$

$$\Rightarrow b = \frac{n+3}{n-1}$$

$$\Rightarrow \eta \propto m^{\frac{n+1}{n-1}} v^{\frac{n+3}{n-1}} \mu^{-\frac{2}{n-1}}$$

Hence, the correct answer is (B).

140. If number of observations are large, then average deviation

$$\Rightarrow \text{Possible error} \propto \frac{1}{\text{No. of observation}}$$

Hence, the correct answer is (A).

141. According to Faraday's law, electric potential is

$$V \equiv \frac{d\phi}{dt} \quad \dots(1)$$

and according to photo-electric effect, we have

$$eV = hv$$

$$\Rightarrow V = \left( \frac{h}{e} \right) v \quad \dots(2)$$

From equations (1) and (2) we can see that magnetic flux ( $\phi$ ) and  $\frac{h}{e}$  have the same dimensions.

Hence, the correct answer is (A).

142.  $\left[ \frac{E^2}{\mu_0} \right] = \left[ \frac{\epsilon_0 E^2}{\epsilon_0 \mu_0} \right] = \left[ \frac{\text{energy/volume}}{(1/\text{speed of light})^2} \right]$

$$\Rightarrow \left[ \frac{E^2}{\mu_0} \right] = \left[ \frac{\text{energy (speed)}^2}{\text{volume}} \right]$$

$$\Rightarrow \left[ \frac{E^2}{\mu_0} \right] = \left[ \frac{ML^2T^{-2}L^2T^{-2}}{L^3} \right] = [MLT^{-4}]$$

Hence, the correct answer is (B).

143. Since, according to Wien's Law,

$$\lambda_m T = b$$

$$\Rightarrow b^4 = \lambda_m^4 T^4 \text{ and } \frac{\text{Energy}}{\text{Area} \times \text{Time}} = \sigma T^4$$

$$\Rightarrow \sigma = \frac{\text{Energy}}{(\text{Area} \times \text{Time}) T^4}$$

$$\Rightarrow \sigma b^4 = \left( \frac{\text{Energy}}{\text{Area} \times \text{Time}} \right) \lambda_m^4$$

## H.20 JEE Advanced Physics: Mechanics - I

$$\Rightarrow [\sigma b^4] = \frac{[ML^2T^{-2}]}{[L^2][T]} [L^4] = [ML^4T^{-3}]$$

Hence, the correct answer is (B).

150. Because constant involved (G) is not dimensionless.

Hence, the correct answer is (D).

151.  $[\alpha x] = M^0L^0T^0$

$$\Rightarrow [\alpha] = M^0L^{-1}T^0$$

$$\Rightarrow [Bx] = [T]$$

$$\Rightarrow [B] = M^0L^{-1}T^1$$

$$\Rightarrow [\beta t] = M^0L^0T^0$$

$$\Rightarrow [\beta] = M^0L^0T^{-1}$$

Hence, the correct answer is (D).

152. Since, we have  $[P] = ML^2T^{-3}$

$$\text{So, unit of } P = 20 \text{ kg} \times (10 \text{ m})^2 \times (5 \text{ sec})^{-3}$$

$$\Rightarrow P = 16 \text{ W}$$

Hence, the correct answer is (A).

153.  $\left[\frac{ma}{K}\right] = [l] = b$

Hence, the correct answer is (C).

154.  $\frac{\Delta Y}{Y} = \frac{2\Delta D}{D} + \frac{\Delta l}{l}$

$$\Rightarrow \frac{\Delta Y}{Y} = 2\left(\frac{0.01}{0.4}\right) + \left(\frac{0.05}{0.8}\right)$$

$$\Rightarrow \frac{\Delta Y}{Y} = 2 \times 0.025 + 0.0625$$

$$\Rightarrow \frac{\Delta Y}{Y} = 0.05 + 0.0625 = 0.1125$$

$$\Rightarrow \Delta Y = 2 \times 10^{11} \times 0.1125 = 0.225 \times 10^{11}$$

$$\text{So, } (2 \pm 0.2) \times 10^{11} \text{ Nm}^{-2}$$

Hence, the correct answer is (B).

155.  $[-\alpha t^2 + \beta t + \gamma] = M^0L^0T^0$

$$\Rightarrow [\alpha] = T^{-2}, [\beta] = T^{-1}, [\gamma] = 1$$

Hence, the correct answer is (A).

156.  $T = (ML^{-1}T^{-2})^\alpha (ML^{-3})^\beta (MT^{-2})^\gamma$

$$\Rightarrow \alpha + \beta + \gamma = 0 \quad \dots(1)$$

$$\Rightarrow -\alpha - 3\beta = 0 \quad \dots(2)$$

$$\Rightarrow -2\alpha - 2\gamma = 1 \quad \dots(3)$$

Hence, the correct answer is (A).

157.  $T^2 = 4\pi^2 \frac{l}{g}$

$$\Rightarrow g = \frac{4\pi^2 l}{T^2}$$

$$\Rightarrow \frac{\Delta g}{g} \times 100 = \frac{\Delta l}{l} \times 100 + 2 \frac{\Delta T}{T} \times 100$$

$$\Rightarrow \frac{\Delta g}{g} = y + 2x$$

Hence, the correct answer is (C).

158. Since,  $F = \frac{q^2}{4\pi\epsilon_0 r^2}$

$$\Rightarrow ke^2 = Fr^2$$

$$\text{Energy } E = \frac{hc}{\lambda}$$

$$\Rightarrow hc = E\lambda$$

$$\text{So, } \frac{e^2}{4\pi\epsilon_0 hc} = \frac{Fr^2}{hc} = \frac{Fr^2}{E\lambda} = [M^0L^0T^0]$$

Hence, the correct answer is (A).

159.  $\frac{GIM^2}{E^2} = \frac{[M^{-1}L^3T^{-2}][M^1L^1T^{-1}][M^2]}{[M^1L^2T^{-2}]^2} = [T]$

Hence, the correct answer is (A).

160.  $\frac{\Delta f_{net}}{f_{net}^2} = \frac{\Delta f_1}{f_1^2} + \frac{\Delta f_2}{f_2^2}$

Hence, the correct answer is (C).

161.  $\frac{d^2y}{dx^2} = \frac{d^2l}{dt^2} = \text{Acceleration}$

Hence, the correct answer is (D).

### Multiple Correct Choice Type Questions

1. For light travelling in a medium, we have

$$\mu = \frac{\text{Velocity of light in vacuum}}{\text{Velocity of light in medium}}$$

where  $\mu$  is the refractive index of the medium. So, we have that the given relation in question is dimensionally correct but numerically incorrect.

Hence, (A) and (C) are correct.

2.  $[y] = [At^2] = [Bt^3]$

$$\Rightarrow [A] = LT^{-2} \text{ and } [B] = LT^{-3}$$

$$\text{Now, } [A^3] = L^3T^{-6} \text{ and } [B^2] = L^2T^{-6}$$

$$\Rightarrow \frac{[A^3]}{[B^2]} = L$$

Hence, (A), (B) and (C) are correct.

3. All are the drawbacks of dimensional analysis.  
Hence, (A), (B), (C) and (D) are correct.

4. Refractive index, poisson's ratio and specific gravity are dimensionless.

Hence, (A), (B) and (D) are correct.

5.  $\left[\frac{2\pi ct}{\lambda}\right] = M^0 L^0 T^0 \quad \dots(1)$

$\Rightarrow [ct] = [\lambda] \quad \{(A) \text{ is correct}\}$

$\left[\frac{2\pi x}{\lambda}\right] = M^0 L^0 T^0 \quad \dots(2)$

$\Rightarrow [x] = [\lambda] \quad \{(B) \text{ is correct}\}$

From (1) and (2)

$\left[\frac{2\pi ct}{\lambda}\right] = \left[\frac{2\pi x}{\lambda}\right]$

$\Rightarrow \left[\frac{2\pi c}{\lambda}\right] = \left[\frac{2\pi x}{\lambda t}\right]$

So, (C) is also correct and the above result also shows that (D) is incorrect.

Hence, the correct answer is (C).

6. Dyne  $\text{cm}^{-2}$  is the unit of pressure, stress and Young's Modulus.

Hence, (A), (B) and (C) are correct.

8. [Renold's number] = [Coefficient of friction] =  $M^0 L^0 T^0$

[Curie] = [Frequency] =  $M^0 L^0 T^{-1}$

[Latent heat] = [Gravitational potential] =  $M^0 L^2 T^{-2}$

Hence, (A), (B) and (C) are correct.

9.  $L = G^x c^y h^z$

$\Rightarrow L = (M^{-1} L^3 T^{-2})^x (L T^{-1})^y (M L^2 T^{-1})^z$

$\Rightarrow L = M^{-x+z} L^{3x+y+2z} T^{-2x-y-z}$

Using the Principle of Homogeneity, we get

$-x + z = 0 \quad \dots(1)$

$3x + y + 2z = 1 \quad \dots(2)$

$-2x - y - z = 0 \quad \dots(3)$

Solving equations (1), (2) and (3), we get

$x = \frac{1}{2}, y = -\frac{3}{2}, z = \frac{1}{2}$

Hence, (B) and (C) are correct.

10. Please remember that dimensional correctness does not guarantee numerical correctness of a physical relation.

Hence, (A), (B) and (C) are correct.

11. Electrical unit of energy is kWhr. So (A) is correct Also

$(\text{volt})^2 (\text{sec})(\text{ohm})^{-1} \equiv \left(\frac{V^2}{R}\right)t$

where  $R$  is the resistance,  $t$  is the time and  $V$  is the electric potential difference. Now since we know that

$W = \left(\frac{V^2}{R}\right)t$ . So, we get (B) as the another correct option.

We have (pascal) (foot)<sup>2</sup> as the unit of force, so (C) is incorrect

Finally, we have

$(\text{weber})(\text{ampere}) \equiv (\phi_B)(I) = LI^2 \quad \{\because \phi_B = LI\}$

Now since  $\frac{1}{2}LI^2 = \text{Energy}$  associated with the magnetic field of a solenoid, so (D) is also correct.

Hence, (A), (B) and (D) are correct.

12.  $x = \left[\frac{E}{B}\right] = [\text{velocity}] = \left[\frac{1}{\sqrt{\mu_0 \epsilon_0}}\right] = \left[\frac{\ell}{CR}\right]$

So, all  $x$ ,  $y$  and  $z$  have dimensional formula same as that of velocity.

Hence, (A), (B) and (C) are correct.

13. [Light year] = [Wavelength] = [Radius of gyration] =  $L$   
Hence, (B) and (C) are correct.

14. The units parsec, light year and micron represent length.  
Hence, (A), (B) and (C) are correct.

15. [Impulse] = [Linear momentum] =  $MLT^{-1}$   
[Planck's constant] = [Angular momentum] =  $ML^2T^{-1}$   
[Young's modulus] = [Pressure] =  $ML^{-1}T^{-2}$   
Hence, (A), (B) and (D) are correct.

16. Energy per unit volume represents pressure or stress or modulus of elasticity.

Hence, (B), (C) and (D) are correct.

17.  $\left[\frac{B^2}{2\mu_0}\right] = \left[\begin{matrix} \text{Magnetic} \\ \text{Energy} \\ \text{Density} \end{matrix}\right] = [\text{Pressure}] = ML^{-1}T^{-2}$

Hence, (A) and (B) are correct.

25. Since  $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$\Rightarrow f^{-1} = v^{-1} + u^{-1}$

$\Rightarrow \Delta(f^{-1}) = \Delta(u^{-1}) + \Delta(v^{-1})$

$\Rightarrow (-1)f^{-2}\Delta f = (-1)u^{-2}\Delta u + (-1)v^{-2}\Delta v$

## H.22 JEE Advanced Physics: Mechanics - I

$$\Rightarrow \frac{\Delta f}{f^2} = \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2} \quad \dots(1)$$

So, (A) is correct.

$$\text{From (1), } \frac{\Delta f}{f} = f \left( \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2} \right)$$

$$\Rightarrow \frac{\Delta f}{f} = \left( \frac{uv}{u+v} \right) \left( \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2} \right)$$

So, (C) is also correct

**Hence, (A) and (C) are correct.**

30.  $F' = \left( \frac{1}{\alpha\beta} \right) F$

$$\Rightarrow m'a' = \left( \frac{1}{\alpha\beta} \right) ma$$

$$\Rightarrow m' = \left( \frac{1}{\alpha\beta} \right) \left( \frac{a}{a'} \right) m$$

But  $\frac{a}{a'} = \frac{1}{\alpha\beta}$

$$\Rightarrow m' = \left( \frac{1}{\alpha^2\beta^2} \right) m$$

So, (B) is correct

Further

$$a' = (\alpha\beta)a$$

$$\Rightarrow \frac{v'}{t'} = (\alpha\beta) \left( \frac{v}{t} \right)$$

$$\Rightarrow t' = \left( \frac{1}{\alpha\beta} \right) \left( \frac{v'}{v} \right) t$$

$$\Rightarrow t' = \left( \frac{1}{\alpha\beta} \right) \left( \frac{\alpha^2}{\beta} \right) t$$

$$\Rightarrow t' = \left( \frac{\alpha}{\beta^2} \right) t$$

So, (C) is correct

Further

$$v' = \left( \frac{\alpha^2}{\beta} \right) v$$

$$\Rightarrow \frac{l'}{t'} = \left( \frac{\alpha^2}{\beta} \right) \left( \frac{l}{t} \right)$$

$$\Rightarrow l = \left( \frac{\alpha^2}{\beta} \right) \left( \frac{t'}{t} \right) l$$

$$\Rightarrow l' = \left( \frac{\alpha^2}{\beta} \right) \left( \frac{\alpha}{\beta^2} \right) l$$

$$\Rightarrow l' = \left( \frac{\alpha^3}{\beta^3} \right) l$$

So, (A) is also correct

Finally, we have

$$p' = m'v'$$

$$\Rightarrow p' = \left( \frac{1}{\alpha^2\beta^2} \right) m \left( \frac{\alpha^2}{\beta} \right) v$$

$$\Rightarrow p' = \left( \frac{1}{\beta^3} \right) mv = \left( \frac{1}{\beta^3} \right) p$$

So, (D) is also correct.

**Hence, (A), (B), (C) and (D) are correct.**

### Reasoning Based Questions

1. Light year is distance travelled by light in vacuum in 1 year.

The wavelength is the distance between two consecutive crests or troughs of a wave.

The dimensions of both light year and wavelength is  $[M^0LT^0]$  so, both represent distances.

**Hence, the correct answer is (A).**

2.  $[c] = LT^{-1} = 3 \times 10^8 \text{ ms}^{-1}$

$$\text{and } [g] = LT^{-2} = 10 \text{ ms}^{-2}$$

$$\Rightarrow \frac{c}{g} = \frac{LT^{-1}}{LT^{-2}} = T = \frac{3 \times 10^8}{10} = 3 \times 10^7 \text{ s}$$

$$\Rightarrow T = 3 \times 10^7 \text{ s}$$

**Hence, the correct answer is (B).**

3. Surface energy is defined as the work done per unit surface area.

So, dimensional formula of surface energy is

$$\frac{ML^2T^{-2}}{L^2} \text{ i.e., } MT^{-2}$$

**Hence, the correct answer is (D).**

4.  $g = \frac{GM}{R_e^2} = \frac{G}{R_e^2} \times \frac{4}{3} \pi R_e^3 \rho$   $\left\{ \because M = \frac{4}{3} \pi R_e^3 \rho \right\}$

$$\Rightarrow g = \frac{4G\pi R_e \rho}{3}$$

$$\Rightarrow \rho = \frac{3g}{4G\pi R_e}$$

**Hence, the correct answer is (A).**

5. 1 micron =  $10^{-6} \text{ m}$

This measurement is used to measure the microscopic distance (length of biological cell) by the microscope.

**Hence, the correct answer is (A).**

6. Electrostatic potential is given by

$$V = \frac{\text{Work done}}{\text{Charge}} \quad \dots(1)$$

$\Rightarrow V \propto$  work done  
Hence, Statement 1 is true.  
From Equation (1),

$$[V] = \frac{[\text{Work done}]}{[\text{Charge}]}$$

$$\Rightarrow [V] = \frac{ML^2T^{-2}}{AT} = ML^2T^{-3}A^{-1}$$

Hence, statement 2 is true. But statement 2 is not the correct explanation to statement 1.

**Hence, the correct answer is (B).**

7. The distance travelled in a particular second has dimensions same as that of velocity and not as distance. Statement-2 is an obvious correct statement.  
**Hence, the correct answer is (D).**

8. Since  $E = \frac{1}{2}mv^2$   
$$\Rightarrow \frac{\Delta E}{E} = \frac{\Delta m}{m} + \frac{2\Delta v}{v} = 1\% + 2(2\%) = 5\%$$

**Hence, the correct answer is (A).**

9. Dimensions of  $[\epsilon_0] = M^{-1}L^{-3}T^4A^2$

$$\text{Dimensions of } [\mu_0] = MLT^{-2}A^{-2}$$

$$\Rightarrow \left[ \frac{1}{\sqrt{\mu_0\epsilon_0}} \right] = \frac{1}{\sqrt{(M^{-1}L^{-3}T^4A^2)(MLT^{-2}A^{-2})}}$$

$$\Rightarrow \left[ \frac{1}{\sqrt{\mu_0\epsilon_0}} \right] = \frac{1}{\sqrt{L^{-2}T^2}} = \frac{L}{T} = \text{Velocity}$$

$$\text{Also, } \frac{1}{\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{\frac{\mu_0}{4\pi} \times 4\pi\epsilon_0}} = \sqrt{\frac{9 \times 10^9}{10^{-7}}}$$

$$\Rightarrow \frac{1}{\sqrt{\mu_0\epsilon_0}} = \sqrt{9 \times 10^{16}} = 3 \times 10^8 \text{ ms}^{-1} = c$$

**Hence, the correct answer is (A).**

10. The percentage of moisture in air is measured by hygrometer and the degree of hotness of a body is measured by the thermometer. Hence, statement 1 is false and statement 2 is true.  
**Hence, the correct answer is (D).**

11.  $A = 4\pi r^2$

$$\text{Fractional error } \frac{\Delta A}{A} = 2 \frac{\Delta r}{r}$$

(Error will not be involved in constant  $4\pi$ )

$$\Rightarrow \frac{\Delta A}{A} \times 100 = 2 \times 0.3\% = 0.6\%$$

$$\text{So, } \frac{\Delta A}{A} = 4 \frac{\Delta r}{r} \text{ is false}$$

**Hence, the correct answer is (C).**

12. Pressure gradient  $= \frac{dP}{dx} = \frac{[ML^{-1}T^{-2}]}{[L]} = [ML^{-2}T^{-2}]$

Which is different with the dimension of pressure. Since, like quantities can be added or subtracted, so, we cannot subtract the pressure from pressure gradient.

**Hence, the correct answer is (D).**

13. Trigonometrical ratio have no dimensions. So, method of finding dimensions cannot be utilised for deriving formulae which are dependent on trigonometrical ratios.

**Hence, the correct answer is (A).**

14. The physical quantities which do not depend on the other quantities are called fundamental quantities. Also, length, mass and time are not derived quantities but are fundamental quantities.

**Hence, the correct answer is (C).**

15. According, the Principle of Homogeneity of dimensions, an equation representing the physical quantity is true, only when the dimensions on LHS and RHS must be the same. The dimensions of a physical quantity are the powers to which the fundamental units are raised to get the required physical quantity.

**Hence, the correct answer is (B).**

16. Nuclear cross section is measured in unit called barn. In SI system the value of 1 barn =  $10^{-28} \text{ m}^2$ . Therefore, Statement-1 is true and Statement-2 is false.

**Hence, the correct answer is (C).**

17. The given relation is

$$f = \frac{1}{2l} \sqrt{\frac{T}{m}}$$

$$\Rightarrow f^2 = \frac{T}{4l^2m}, T \text{ being tension.}$$

$$\Rightarrow [m] = \left[ \frac{T}{4l^2f^2} \right] = \frac{[MLT^{-2}]}{\left[ L^2 \frac{1}{T^2} \right]} = \frac{[MLT^{-2}]}{[L^2T^{-2}]}$$

$$= \frac{[M]}{[L]} = \frac{\text{Mass}}{\text{Length}} = \text{linear mass density.}$$

**Hence, the correct answer is (C).**

## H.24 JEE Advanced Physics: Mechanics - I

18. Avogadro number is the number of atoms/molecules/ions/nuclei/particles present per mole of a sample. So, it has unit  $\text{mol}^{-1}$  and hence is not dimensionless. Hence, the correct answer is (D).

19. Statement 1 is true because it is an obvious statement known to us.

However Statement 2 is incorrect because dimensional correctness may or may not establish numerical correctness of a physical relation.

Hence, the correct answer is (C).

20. AU is an astronomical unit. It is the mean distance between earth and sun.

$$1 \text{ AU} = 1.496 \times 10^{11} \text{ m} = 1.5 \times 10^{11} \text{ m}$$

$$\text{\AA} \text{ is angstrom unit } 1 \text{ \AA} = 10^{-10} \text{ m}$$

Hence, the correct answer is (A).

21. Suppose  $m = kv^a \rho^b g^c$

$$\Rightarrow ML^0T^0 = (LT^{-1})^a (ML^{-3})^b (LT^{-2})^c$$

$$\Rightarrow ML^0T^0 = M^b L^{a-3b+c} T^{-a-2c}$$

According to Principle of Homogeneity, we get

$$b = 1 \quad \dots(1)$$

$$a - 3b + c = 0 \quad \dots(2)$$

$$-a - 2c = 0 \quad \dots(3)$$

$$\Rightarrow c = -3 \text{ and } a = 6$$

So,  $a = 6$ ,  $b = 1$ ,  $c = -3$

$$\Rightarrow m = k \frac{v^6 \rho}{g^3}$$

Hence, the correct answer is (B).

22. Pressure is given by

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{\text{Force} \times \text{Distance}}{\text{Area} \times \text{Distance}}$$

$$\Rightarrow \text{Pressure} = \frac{\text{Energy}}{\text{Volume}} = \text{Energy density}$$

Thus, pressure has the dimensions of energy density.

Hence, the correct answer is (A).

23. Since, the graph is a straight line therefore  $P \propto Q$ .

$$\Rightarrow P = \text{constant} \times Q$$

$$\Rightarrow \frac{P}{Q} = \text{constant} = \text{slope of the straight line.}$$

Hence, the correct answer is (A).

### Linked Comprehension Type Questions

1. Please keep in mind that acceleration due to gravity just represents the gravitational field or gravitational intensity.

Hence, (A) and (C) are correct.

2. Torque and moment of force have same dimensional formula.

Hence, the correct answer is (B).

3. Remember that actually, force is negative energy gradient.

Hence, the correct answer is (C).

4. Surface tension, surface energy and spring constant all have same dimensional formula.

Hence, the correct answer is (B).

5. Pressure, stress, energy density, modulus of elasticity all have same dimensional formula.

Hence, the correct answer is (A).

$$6. [V] = [b]$$

Hence, the correct answer is (B).

$$7. [p] = \left[ \frac{a}{V^2} \right]$$

$$\Rightarrow [a] = [pV^2]$$

Hence, the correct answer is (C).

8. All  $pV$ ,  $pb$  and  $\frac{ab}{V^2}$  has dimensional formula same as that of  $RT$ .

Hence, the correct answer is (C).

9. As discussed already,  $\left[ \frac{ab}{V^2} \right] = [RT]$

$$\Rightarrow \left[ \frac{ab}{RT} \right] = [V^2] = L^6$$

Hence, the correct answer is (D).

$$10. [RT] = [pV] = [\text{Energy}]$$

Hence, the correct answer is (A).

$$11. [c] = c = LT^{-1} \quad \dots(1)$$

$$[G] = G = M^{-1}L^3T^{-2} \quad \dots(2)$$

$$[h] = h = ML^2T^{-1} \quad \dots(3)$$

$$\Rightarrow \frac{hc}{G} = M^2$$

$$\Rightarrow M = c^{\frac{1}{2}} G^{-\frac{1}{2}} h^{\frac{1}{2}}$$

Hence, the correct answer is (A).

$$12. \frac{Gh}{c^3} = \frac{L^5T^{-3}}{L^3T^{-3}}$$

$$\Rightarrow L^2 = Ghc^{-3}$$

$$\Rightarrow L = c^{-\frac{3}{2}} G^{\frac{1}{2}} h^{\frac{1}{2}}$$

Hence, the correct answer is (B).

13. Put value of  $L$  (in terms of  $c$ ,  $G$  and  $h$ ) in (1), we get

$$T = c^{-\frac{5}{2}} G^{\frac{1}{2}} h^{\frac{1}{2}}$$

Hence, the correct answer is (C).

14. Least count of metre stick = 1 mm = 0.1 cm

$$\% \text{age error} = \frac{\Delta L}{L} \times 100\% = \frac{0.1}{50} \times 100\% = 0.2\%$$

Hence, the correct answer is (B).

15.  $\frac{\Delta m}{m} \times 100\% = \frac{0.1 \times 10^{-3}}{1} \times 100\% = 0.01\%$

Hence, the correct answer is (D).

16.  $\frac{\Delta T}{T} \times 100\% = \frac{0.2}{4 \times 60} \times 100\% = 0.083\%$

Hence, the correct answer is (C).

17. Angular Momentum, Planck's constant and energy per unit frequency have same dimensional formula. Hence, the correct answer is (C).

18. Frequency, angular frequency, angular speed and speed gradient all have same dimensional formula. Hence, the correct answer is (A).

19. Electric field strength and electric potential gradient have same dimensional formula. Gravitational field strength and gravitational potential gradient have same dimensional formula. Hence, (A) and (D) are correct.

20. Latent heat, gravitational potential, kinetic energy per unit mass, gravitational potential energy per unit mass all have same dimensional formula. Hence, (A), (B) and (C) are correct.

21. Angle, Strain and Fine structure constant all are dimensionless. Hence, (A) and (C) are correct.

22.  $[D] = [T^2]$ ,  $\frac{[C][T]}{[T^2]} = [F]$

$$\Rightarrow C = [MLT^{-1}]$$

Hence, the correct answer is (B).

23.  $\frac{[B]}{[T]} = [F]$

$$\Rightarrow [B] = [MLT^{-1}]$$

Hence, the correct answer is (C).

24.  $\frac{1}{\sqrt{D}} = \frac{1}{[T^2]^{1/2}} = \frac{1}{[T]}$

Hence, the correct answer is (A).

25. The correct answer is (A).

26. The correct answer is (B).

27. The correct answer is (D).

Combined solution to 25, 26, 27

$$\text{Let } \omega \propto V^a$$

$$\Rightarrow \omega \propto R^b$$

$$\Rightarrow \omega \propto M^c$$

$$\Rightarrow \omega \propto l^d$$

$$\Rightarrow \omega = kV^a R^b m^c l^d$$

Equating power of same base in dimensional formula of L.H.S. and R.H.S and solving.

$$a = \frac{2}{3}, b = -\frac{1}{3}, c = -\frac{1}{3}, d = -\frac{1}{3}$$

$$\text{So, } \omega \propto V^{2/3}$$

$$\Rightarrow \omega \propto R^{-1/3}$$

31. The correct answer is (B).

32. The correct answer is (D).

33. The correct answer is (A).

Combined solution to 31, 32, 33

$$\begin{array}{ccc} E & V & F \\ ML^2T^{-2} & LT^{-1} & MLT^{-2} \end{array}$$

$$\Rightarrow [S] = MT^{-2} = \frac{(MLT^{-2})^2}{(ML^2T^{-2})} = \frac{F^2}{E}$$

$$\Rightarrow [L] = ML^2T^{-1} = \frac{(ML^2T^{-2})^2}{(LT^{-1})(MLT^{-2})} = \frac{E}{VF}$$

$$\Rightarrow [t] = T = \frac{ML^2T^{-2}}{(LT^{-1})(MLT^{-2})} = \frac{E}{VF}$$

34. The correct answer is (A).

35. The correct answer is (B).

36. The correct answer is (C).

Combined solution to 34, 35, 36

$$T = kp^a d^b E^c$$

$$\Rightarrow T = 1(ML^{-1}T^{-2})^a (ML^{-3})^b (ML^2T^{-2})^c$$

$$\Rightarrow a + b + c = 0 \quad \dots(1)$$

$$\Rightarrow -a - 3b + 2c = 0 \quad \dots(2)$$

$$\Rightarrow -2a - 2c = 1 \quad \dots(3)$$

40. Quantities having same dimensions can only be added and in  $\ln(x)$  and  $\sin(x)$ ,  $x$  is always dimensionless. Hence, the correct answer is (B).

## H.26 JEE Advanced Physics: Mechanics - I

43. Quantities having same dimensions can only be added & in  $\ln(x)$  and  $\sin(x)$ ,  $x$  is always dimensionless.  
Hence, the correct answer is (C).

44. The correct answer is (A).

45. The correct answer is (B).

46. The correct answer is (B).

**Combined solution to 44, 45, 46**

$$[a] = [P][V]^2 = ML^4T^{-2}$$

$$[b] = [V] = L^3$$

$$\Rightarrow [R] = \frac{[P][V]}{[T]} = ML^2T^{-2}K^{-1}$$

$$[\text{Boltzmann's Constant}] = ML^2T^{-2}K^{-1}$$

$$[\text{Wein's Constant}] = LK$$

$$[\text{Stefan's Constant}] = MT^{-3}K^{-4}$$

4. A  $\rightarrow$  (q, r)

B  $\rightarrow$  (q)

C  $\rightarrow$  (p, s, t)

D  $\rightarrow$  (q)

$$[\text{Impulse}] = MLT^{-1} = [\text{Linear Momentum}]$$

$$[\text{Planck's Constant}] = ML^2T^{-1}$$

$$[\text{Angular Momentum}] = ML^2T^{-1}$$

$$[\text{Energy per unit frequency}] = \left[ \frac{E}{\nu} \right] = [h]$$

### Matrix Match/Column Match Type Questions

1. A  $\rightarrow$  (q)

B  $\rightarrow$  (p)

C  $\rightarrow$  (r)

D  $\rightarrow$  (t)

$$\left[ \frac{L}{R} \right] = [\text{Inductive Time Constant}] = T$$

$$[CR] = [\text{Capacitive Time Constant}] = T$$

$$\left[ \frac{E}{B} \right] = [\text{Velocity}] = LT^{-1}$$

$$\left[ \frac{1}{\mu_0 \epsilon_0} \right] = [(\text{Velocity})^2] = L^2T^{-2}$$

2. A  $\rightarrow$  (p, q)

B  $\rightarrow$  (r, t)

C  $\rightarrow$  (p, q)

D  $\rightarrow$  (s)

Stress, pressure and energy density have same dimensional formula  $ML^{-1}T^{-2}$ . So,

(A)  $\rightarrow$  (p, q)

Strain, Angle, Fine structure constant are dimensionless physical quantities. So,

(B)  $\rightarrow$  (r, t)

Modulus of elasticity has the dimensional formula same as that of stress, pressure and energy density. So,

(C)  $\rightarrow$  (p, q)

Torque has dimensional formula, same as that of energy i.e.,  $ML^2T^{-2}$ .

3. A  $\rightarrow$  (s)

B  $\rightarrow$  (p)

C  $\rightarrow$  (r)

D  $\rightarrow$  (q)

$$[\text{Specific Heat}] = L^2T^{-2}K^{-1}$$

5. A  $\rightarrow$  (r, s)

B  $\rightarrow$  (p, q)

C  $\rightarrow$  (t)

D  $\rightarrow$  (p, q)

Stress, bulk modulus have unit  $Nm^{-2}$ .

Force constant, surface tension and surface energy have unit  $Nm^{-1}$  and  $1Nm^{-1} = 1kgs^{-2}$ .

Torque has a unit  $Nm$

6. A  $\rightarrow$  (p, t)

B  $\rightarrow$  (r)

C  $\rightarrow$  (q)

D  $\rightarrow$  (r)

Angle and fine structure constant is dimensionless

7. A  $\rightarrow$  (r)

B  $\rightarrow$  (s)

C  $\rightarrow$  (p, q, r)

D  $\rightarrow$  (t)

$$[\text{Latent Heat}] = \left[ \frac{\text{Energy}}{\text{Mass}} \right] = L^2T^{-2}$$

$$[\text{Mean Square Velocity}] = \left[ \frac{k_B T}{m} \right] = L^2T^{-2}$$

However unit of latent heat is  $Jkg^{-1}$ .

$$[\text{Specific Heat}] = \left[ \frac{1}{m} \left( \frac{\Delta Q}{\Delta T} \right) \right] = L^2T^{-2}K^{-1}$$

9. A  $\rightarrow$  (p, q)

B  $\rightarrow$  (r, s)

C  $\rightarrow$  (r, s)

D  $\rightarrow$  (r, s)

$$\left[ \frac{GM_e M_s}{r} \right] = \text{joule}$$

$$\Rightarrow [GM_e M_s] = (\text{joule})(\text{metre})$$

Since joule = (coulomb)(volt)

$$\Rightarrow [GM_e M_s] = (\text{volt})(\text{coulomb})(\text{metre})$$

Also,  $[GM_e M_s] = ML^3 T^{-2}$

$$\Rightarrow [GM_e M_s] = (\text{kilogram})(\text{metre})^3 (\text{second})^{-2}$$

Now,  $\left[\frac{3RT}{M}\right] = \left[\frac{F^2}{q^2 B^2}\right] = \left[\frac{GM_e}{R_e}\right] = v^2$

Also  $v^2 = \left[\frac{\text{Energy}}{\text{Mass}}\right] = \left[\frac{CV^2}{\text{Mass}}\right] = (\text{farad})(\text{volt})^2 (\text{kg})^{-1}$

10. A → (r)  
 B → (p)  
 C → (p)  
 D → (q)

(A)  $y = \frac{F \ell}{A \Delta \ell}$

(B)  $V = -\frac{Gm}{r}$

(C)  $L = \frac{Q}{m}$

(D)  $G = \frac{Fr^2}{Gm_1 m_2}$

17. A → (r)  
 B → (p, q)  
 C → (p)  
 D → (s)

$$\eta = \frac{F}{6\pi r v}$$

$$\Rightarrow [\eta] = ML^{-1}T^{-1}$$

$$[\text{Angle}] = [\text{Strain}] = 1$$

$$[\text{Stress}] = \frac{[F]}{[A]} = ML^{-1}T^{-2}$$

### Integer/Numerical Answer Type Questions

1.  $\left[\frac{EJ^2}{M^5 G^2}\right] = \frac{(ML^2 T^{-2})(ML^2 T^{-1})^2}{(M^5)(M^{-1}L^3 T^{-2})^2} = M^0 L^0 T^0$

2.  $\left[\frac{e^2}{2h\epsilon_0 c}\right] = M^0 L^0 T^0$

$$\Rightarrow x = 2$$

3. Since we know that  $V = \frac{\pi r^4}{8 \eta} p$ , so we get  $x = 4$ ,  $y = 1$  and  $z = 1$ .

4.  $F = 20 \text{ N}$ ,  $E = 200 \text{ J}$ ,  $v = 5 \text{ ms}^{-1}$

Since  $E = FL$

$$\Rightarrow L = \frac{E}{F} = \frac{200}{20} = 10 \text{ m}$$

Since  $v = LT^{-1}$

$$\Rightarrow 5 = 10 T^{-1}$$

$$\Rightarrow T = 2 \text{ s}$$

Further  $F = MLT^{-2}$

$$\Rightarrow 20 = M(10)(2)^{-2}$$

$$\Rightarrow M = \frac{80}{10} = 8 \text{ kg}$$

5.  $n_2 = n_1 \left(\frac{M_1}{M_2}\right)^1 \left(\frac{L_1}{L_2}\right)^2 \left(\frac{T_1}{T_2}\right)^{-2}$

$$\Rightarrow n_2 = 4.18 \left(\frac{1 \text{ kg}}{\alpha \text{ kg}}\right) \left(\frac{1 \text{ m}}{\beta \text{ m}}\right)^2 \left(\frac{1 \text{ s}}{\gamma \text{ s}}\right)^{-2}$$

$$\Rightarrow n_2 = 4.18 \alpha^{-1} \beta^{-2} \gamma^2$$

So,  $4.18 \text{ J} = 4.18 \alpha^{-1} \beta^{-2} \gamma^2$  new units

6. To show that the planet obeys Kepler's Third Law of planetary motion, we have to prove that

$$T^2 \propto r^3$$

Now, according to the problem, we have

$$T \propto r^a M_s^b G^c$$

$$\Rightarrow T = kr^a M_s^b G^c$$

where  $k$  is a dimensionless constant.

$$\Rightarrow T = L^a M^b (M^{-1}L^3 T^{-2})^c$$

$$\Rightarrow T = M^{b-c} L^{a+3c} T^{-2c}$$

Using the Principle of Homogeneity, we get

$$b - c = 0 \quad \dots(1)$$

$$a + 3c = 0 \quad \dots(2)$$

$$-2c = 1 \quad \dots(3)$$

$$\Rightarrow c = -\frac{1}{2}, b = -\frac{1}{2}, a = \frac{3}{2}$$

So, we get

$$T = kr^{\frac{3}{2}} M_s^{-\frac{1}{2}} G^{-\frac{1}{2}}$$

By experiments, we know that

$$k = 2\pi$$

$$\Rightarrow T^2 = 4\pi^2 \left(\frac{r^3}{GM}\right)$$

So,  $a = 2$ ,  $b = 3$ ,  $c = 1$ ,  $d = 1$

## ARCHIVE: JEE MAIN

1. 
$$\left[ \frac{\epsilon_0}{\sqrt{\mu_0}} \right] = \left[ \frac{\epsilon_0}{\sqrt{\mu_0 \epsilon_0}} \right] = [LT^{-1}] \times [\epsilon_0]$$

Since,  $F = \frac{q^2}{4\pi\epsilon_0 r^2}$

$$\Rightarrow [\epsilon_0] = \frac{[AT]^2}{[MLT^{-2}] \times [L^2]}$$

$$\Rightarrow \left[ \frac{\epsilon_0}{\sqrt{\mu_0}} \right] = [LT^{-1}] \times [A^2 M^{-1} L^{-3} T^4] = [M^{-1} L^{-2} T^3 A^2]$$

Hence, the correct answer is (D).

2.  $[p] = MLT^{-1} = [I^x h^y S^z]$

$$\Rightarrow MLT^{-1} = M^x L^{2x} (ML^2 T^{-1})^y (MT^{-2})^z$$

$$\Rightarrow MLT^{-1} = M^{x+y+z} L^{2x+2y} T^{-y-2z}$$

Applying Principle of Homogeneity, we get

$x + y + z = 1$

$2(x + y) = 1$

$$\Rightarrow x + y = \frac{1}{2}$$

$$\Rightarrow z = \frac{1}{2}$$

$y + 2z = 1$

$$\Rightarrow y = 0$$

$$\Rightarrow x = \frac{1}{2}$$

$$\Rightarrow [p] = I^{\frac{1}{2}} h^0 S^{\frac{1}{2}}$$

Hence, the correct answer is (D).

3.  $T = 2\pi \sqrt{\frac{l}{g}}$

$$\Rightarrow g = 4\pi^2 \frac{l}{T^2}$$

$$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T} = \left( \frac{0.1}{55} + \frac{2 \times 1}{30} \right) \times 100$$

$$\Rightarrow \frac{\Delta g}{g} = 6.8\%$$

Hence, the correct answer is (A).

4. Since  $\rho = \frac{m}{V}$

So, maximum percentage error in  $\rho$  will be given by

$$\frac{\Delta \rho}{\rho} \times 100\% = \left( \frac{\Delta m}{m} \right) \times 100\% + 3 \left( \frac{\Delta L}{L} \right) \times 100\% \dots (1)$$

This is only possible when error is small, which is not the case in this question.

However, if we apply equation (1), then we get

$$\Delta \rho = 3100 \text{ kgm}^{-3}$$

Let us now calculate the error, without using the approximation, by calculating the minimum and maximum values of the density. Now,

$$\rho_{\min} = \frac{m_{\min}}{V_{\max}} = \frac{9.9}{(0.11)^3} = 7438 \text{ kgm}^{-3} \text{ and}$$

$$\rho_{\max} = \frac{m_{\max}}{V_{\min}} = \frac{10.1}{(0.09)^3} = 13854.6 \text{ kgm}^{-3}$$

$$\Rightarrow \Delta \rho = (\rho_{\max} - \rho_{\min}) = 6416.6 \text{ kgm}^{-3}$$

So, correct answer should be (C) but not (B).

Hence, the correct answer is (C).

5.  $5.29 \times 7 = 37.0 \text{ cm}^2$

Answer should be in 3 significant digits.

Hence, the correct answer is (B).

6.  $X = 5YZ^2$

$$\Rightarrow Y \propto \frac{X}{Z^2} \dots (1)$$

Since  $X = C = \frac{Q^2}{E} = \frac{[A^2 T^2]}{[ML^2 T^{-2}]}$

$$\Rightarrow X = [M^{-1} L^{-2} T^4 A^2]$$

Also,  $Z = B = \frac{F}{IL}$

$$\Rightarrow Z = [MT^{-2} A^{-1}]$$

So, from equation (1), we get

$$Y = \frac{[M^{-1} L^{-2} T^4 A^2]}{[MT^{-2} A^{-1}]^2}$$

$$\Rightarrow Y = [M^{-3} L^{-2} T^8 A^4]$$

Hence, the correct answer is (D).

7.  $[R] = \sqrt{\frac{\mu_0}{\epsilon_0}}$

Hence, the correct answer is (C).

8.  $LC = \frac{0.5}{100} = 0.005 \text{ mm}$

Zero error,  $e = -3 \times 0.005 = -0.015 \text{ mm}$

Thickness =  $(5.5 + 48 \times 0.005 + 0.015) \text{ mm}$

$\Rightarrow$  Thickness =  $5.755 \text{ mm}$

Hence, the correct answer is (C).

9.  $[T] = [G]^a [h]^b [c]^c$

$\Rightarrow T' = [M^{-1}L^3T^{-2}]^a [ML^2T^{-1}]^b [LT^{-1}]^c$

Applying Principle of Homogeneity, we get

$-a + b = 0$

$\Rightarrow a = b$

$3a + 2b + c = 0$

$\Rightarrow 5a + c = 0$

$-2a - b - c = 1$

$\Rightarrow 3a + c = -1$

... (1)

... (2)

... (3)

From (2) and (3), we get

$a = \frac{1}{2}, b = \frac{1}{2}, c = -\frac{5}{2}$

$\Rightarrow [T] = \sqrt{\frac{Gh}{c^5}}$

Hence, the correct answer is (A).

10. Since  $\rho = 128 \text{ kgm}^{-3}$

$\Rightarrow \rho = \frac{128 \text{ kg}}{1 \text{ m}^3}$

$\Rightarrow \rho = \frac{128}{\left(\frac{100}{25}\right)^3} \frac{1000}{50} = \frac{128}{4^3} \times 20 = 40$

Hence, the correct answer is (C).

11. Volume of cylinder is given by

$V = \frac{\pi D^2 h}{4}$

$\Rightarrow V = \frac{\pi}{4} \times 34.2 \times (12.6)^2 = 4264.39 \text{ cm}^3$

Since,  $\frac{\Delta V}{V} = \frac{2\Delta D}{D} + \frac{\Delta h}{h}$

$\Rightarrow \frac{\Delta V}{V} = \frac{2 \times 0.1}{12.6} + \frac{0.1}{34.2}$

$\Rightarrow \Delta V = 80.157$

Reducing the answers to appropriate significant figures, then

$V = 4260 \pm 80 \text{ cm}^3$

Hence, the correct answer is (C).

12. Since  $\left[\frac{x^2}{\alpha k T}\right] = M^0 L^0 T^0$

$\Rightarrow [x^2] = [\alpha k T]$

$\Rightarrow [\alpha M L^2 T^{-2}] = L^2$

$\Rightarrow [\alpha] = M^{-1} T^2$

Also  $[\alpha \beta] = M L T^{-2}$

$\Rightarrow M^{-1} T^{+2} [\beta] = M L T^{-2}$

$\Rightarrow [\beta] = M^2 L T^{-4}$

Hence, the correct answer is (B).

13.  $[V] = L^1 T^{-1}$

$[A] = L^1 T^{-2}$

$[F] = M^1 L^1 T^{-2}$

Since  $Y = \frac{\text{Force}}{\text{Area}}$

$\Rightarrow [Y] = M^1 L^{-1} T^{-2}$

$\Rightarrow M^1 L^{-1} T^{-2} = [F]^\alpha [A]^\beta [V]^\gamma$

$\Rightarrow M^1 L^{-1} T^{-2} = M^\alpha L^{\alpha+\beta+\gamma} T^{-2\alpha-2\beta-\gamma}$

By Principle of Homogeneity, we get

$\alpha = 1, \beta = 2, \gamma = -4$

Hence, the correct answer is (C).

14. Since the least count is given by

$\text{L.C.} = \frac{\text{Pitch}}{\text{Number of division on circular scale}}$

$\Rightarrow 5 \times 10^{-6} = \frac{10^{-3}}{N}$

$\Rightarrow N = 200$

Hence, the correct answer is (A).

**H.30 JEE Advanced Physics: Mechanics - I**

15.  $\left[\frac{l}{rcv}\right] = \left[\frac{l}{TV}\right] = ?$

Since  $W = \frac{1}{2}Il^2$

$$\Rightarrow [ML^2T^{-2}] = [IA^2]$$

$$\Rightarrow [I] = [ML^2T^{-2}A^{-2}]$$

Since  $V = \frac{W}{q}$

$$\Rightarrow [v] = \frac{ML^2T^{-2}}{AT} = ML^2T^{-3}A^{-1}$$

$$\Rightarrow \left[\frac{l}{rcv}\right] = \frac{ML^2T^{-2}A^{-2}}{TML^2T^{-3}A^{-1}} = [A^{-1}]$$

Hence, the correct answer is (A).

16. Density of a material is given by

$$\rho = \frac{m}{V} = \frac{m}{l^3}$$

For maximum error in  $\rho$

$$\frac{d\rho}{\rho} = \frac{dm}{m} + 3\frac{dl}{l}$$

$$\Rightarrow \frac{d\rho}{\rho} \times 100 = \frac{dm}{m} \times 100 + 3\frac{dl}{l} \times 100 = 1.5 + (3 \times 1) = 4.5\%$$

Hence, the correct answer is (C).

17. Least count =  $\frac{0.25}{5 \times 100}$  cm =  $5 \times 10^{-4}$  cm

Thickness of wire is

$$D = 4 \times \frac{0.25}{5} \text{ cm} + 30 \times \text{L.C.}$$

$$\Rightarrow D = 4 \times 0.05 \text{ cm} + 30 \times 5 \times 10^{-4} \text{ cm}$$

$$\Rightarrow D = 0.20 \text{ cm} + 0.0150 \text{ cm} = 0.2150 \text{ cm}$$

Hence, the correct answer is (B).

18. As we know  $\frac{\Delta S}{S} = 2 \times \frac{\Delta r}{r}$  and  $\frac{\Delta V}{V} = 3 \times \frac{\Delta r}{r} = \frac{3}{2} \times \frac{\Delta S}{S}$

$$\Rightarrow \frac{\Delta V}{V} = \frac{3}{2} \alpha$$

Hence, the correct answer is (A).

19. For planck length,  $l = kG^p \hbar^q c^r$

$$[M^0LT^0] = [M^{-1}L^3T^{-2}]^p [ML^2T^{-1}]^q [LT^{-1}]^r$$

$$[M^0LT^0] = [M^{-p+q}L^{3p+2q+r}T^{-(2p+q+r)}]$$

On comparing powers of  $M$ ,  $L$  and  $T$  from both sides, we get

$$-p+q=0, 3p+2q+r=1, -(2p+q+r)=0$$

On solving these equations,  $p=q=\frac{1}{2}$ ,  $r=\frac{-3}{2}$

$$\Rightarrow l = \left(\frac{G\hbar}{c^3}\right)^{1/2} \quad \{\text{Take, } k=1\}$$

Hence, the correct answer is (C).

20. Relative error in  $A$  is given by

$$\frac{\Delta A}{A} = \frac{3\Delta P}{P} + \frac{2\Delta Q}{Q} + \frac{1}{2} \frac{\Delta R}{R} + \frac{\Delta S}{S}$$

The maximum percentage error in the value of  $A$  will be

$$\frac{\Delta A}{A} \times 100 = 3 \times 0.5 + 2 \times 1 + \frac{1}{2} \times 3 + 1.5 = 6.5\%$$

Hence, the correct answer is (A).

21. From Kepler's law

$$T^2 = \frac{4\pi^2}{GM} r^3$$

$$\Rightarrow M = \left(\frac{4\pi^2}{G}\right) \frac{r^3}{T^2}$$

$$\Rightarrow \frac{\Delta M}{M} = 2 \frac{\Delta T}{T} + 3 \frac{\Delta r}{r}$$

Since,  $\frac{\Delta r}{r} \approx 0$

$$\Rightarrow \left|\frac{\Delta M}{M}\right| = 2 \frac{\Delta T}{T} = 2 \times 10^{-2}$$

Hence, the correct answer is (C).

22. Let  $m = kT^x C^y h^z$  where  $k$  is a dimensionless constant.

$$\Rightarrow [ML^0T^0] = [T]^x [LT^{-1}]^y [ML^2T^{-1}]^z$$

$$\Rightarrow [ML^0T^0] = [M^z L^{y+2z} T^{x-y-z}]$$

$$\Rightarrow z=1, y+2z=0 \text{ and } x-y-z=0$$

Solving, we get,  $x=-1$ ,  $y=-2$ ,  $z=1$ ; on putting values we get

$$[M] = [T^{-1}C^{-2}h]$$

Hence, the correct answer is (C).

23. Here,  $P = a^{1/2}b^2c^3d^{-4}$

$$\frac{\Delta P}{P} = \frac{1}{2} \frac{\Delta a}{a} + 2 \frac{\Delta b}{b} + 3 \frac{\Delta c}{c} + 4 \frac{\Delta d}{d}$$

$$\Rightarrow \left( \frac{\Delta P}{P} \times 100 \right) \% = \left( \frac{1}{2} \frac{\Delta a}{a} + 2 \frac{\Delta b}{b} + 3 \frac{\Delta c}{c} + 4 \frac{\Delta d}{d} \right) \times 100\%$$

So, percentage relative error in the measurement of  $P$  is

$$\frac{\Delta P}{P} \times 100\% = \left( \frac{1}{2} \times 2 + 2 \times 1 + 3 \times 3 + 4 \times 5 \right) \% = 32\%$$

Hence, the correct answer is (D).

24. Here,  $t_1 = 90$  s,  $t_2 = 91$  s,  $T_3 = 95$  s,  $T_4 = 92$  s

L.C. = 1 s

Mean of the measurements,  $\langle t \rangle = \frac{\sum t_i}{N}$

$$\langle t \rangle = \frac{90 + 91 + 95 + 92}{4} = 92 \text{ s}$$

$$\text{Mean deviation} = \frac{\sum |\bar{t} - t_i|}{N} = \frac{2 + 1 + 3 + 0}{4} = 1.5 \text{ s}$$

Since the least count of the instrument is 1 s, so reported mean time =  $(92 \pm 2)$  s.

Hence, the correct answer is (A).

25. Electrical conductivity =  $[M^{-1}L^{-3}T^3I^2]$

Hence, the correct answer is (B).

26. Given,  $A$ ,  $B$ ,  $C$  and  $D$  have different dimensions.

Also,  $AD = C \ln(BD)$

log is the dimensionless, so  $[B] = \frac{1}{[D]}$

Also,  $[AD] = [C]$

$$(A) \left[ \frac{C}{BD} \right] = \frac{[C]}{1} = [C]$$

$$\left[ \frac{AD^2}{C} \right] = \frac{[AD][D]}{[C]} = [D]$$

So,  $\frac{C}{BD} - \frac{AD^2}{C} = C - D$  which is not meaningful.

$$(B) [B^2C^2] = [B^2][A^2D^2] = A^2[BD]^2 = [A^2]$$

$\Rightarrow (A^2 - B^2C^2)$  is meaningful.

$$(C) \left[ \frac{A}{B} \right] = [AD] = [C]$$

$\Rightarrow \left( \frac{A}{B} - C \right)$  is meaningful.

(D)  $\left( \frac{A-C}{D} \right)$  is not meaningful as  $A$  and  $C$  both have different dimensions.

Hence, (A) and (D) are correct.

27. Here, capacitance  $C = ke^x a_0^y h^z c^a$

$$[C] = [M^{-1}L^{-2}A^2T^4]$$

$$[e] = [AT], [a_0] = [L]$$

$$[c] = [LT^{-1}], [h] = [M^1L^2T^{-1}]$$

$$\Rightarrow [M^{-1}L^{-2}A^2T^4] = [AT]^x [L]^y [M^1L^2T^{-1}]^z [LT^{-1}]^a$$

Comparing both sides, we get

$$x = 2, z = -1, y + 2z + a = -2, x - z - a = 4$$

On solving these equations, we get

$$x = 2, y = 1, z = -1, a = -1$$

Also,  $[C] = u$  so  $u = \frac{e^2 a_0}{hc}$

Hence, the correct answer is (C).

28.  $[e] = [IT]$ ,  $[m] = [M]$ ,  $[c] = [LT^{-1}]$

$$[h] = [ML^2T^{-1}], [\mu_0] = [MLI^{-2}T^{-2}]$$

If  $\mu_0 = ke^a m^b c^h d$ , then

$$[MLI^{-2}T^{-2}] = [IT]^a [M]^b [LT^{-1}]^c [ML^2T^{-1}]^d$$

By equating powers, we get  $a = -2$ ,  $b + d = 1$

$$c + 2d = 1, a - c - d = -2$$

Solving these equations, we get

$$a = -2; b = 0; c = -1; d = 1$$

$$\Rightarrow [\mu_0] = \left[ \frac{h}{ce^2} \right]$$

Hence, the correct answer is (C).

29. Let  $\left( \frac{\dot{Q}}{A} \right)$  is derived quantity which is derived from three fundamental quantities  $\eta$ ,  $\left( \frac{S\Delta\theta}{h} \right)$  and  $\left( \frac{1}{\rho g} \right)$

By using Principle of Homogeneity of dimensions, we get

$$\left[ \frac{\dot{Q}}{A} \right] = [\eta]^x \left[ \frac{S\Delta\theta}{h} \right]^y \left[ \frac{1}{\rho g} \right]^z$$

$$\Rightarrow \left[ \frac{\dot{Q}}{A} \right] = [M^1T^{-3}]; [\eta] = [M^1L^{-1}T^{-1}]$$

$$\Rightarrow \left[ \frac{S\Delta\theta}{h} \right] = [L^1T^{-2}]; \left[ \frac{1}{\rho g} \right] = [M^{-1}L^2T^2]$$

## H.32 JEE Advanced Physics: Mechanics - I

$$\Rightarrow [M^{-1}L^0T^{-3}] = [M^1L^{-1}T^{-1}]^x [M^0L^1T^{-2}]^y [M^{-1}L^2T^2]^z$$

Comparing both sides, we get

$$x + 0 - z = 1, -x + y + 2z = 0 \text{ and } -x - 2y + 2z = -3$$

Solving these equations, we get

$$x = 1, y = 1, z = 0$$

$$\Rightarrow \frac{\dot{Q}}{A} = \eta \frac{S\Delta\theta}{h}$$

Hence, the correct answer is (A).

30. Measured value of the length of rod = 3.50 cm

So, least count of the measuring instrument must be 0.01 cm

$$\Rightarrow \text{L.C.} = 0.1 \text{ mm}$$

For, vernier scale, 10 MSD = 1 cm = 10 mm

$$\Rightarrow 1 \text{ MSD} = 1 \text{ mm}$$

Also, 9 MSD = 10 VSD

$$\text{L.C.} = 1 \text{ MSD} - 1 \text{ VSD} = (1 - 0.9) \text{ mm} = 0.1 \text{ mm}$$

Hence, the correct answer is (C).

31. According to Coulomb's law

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r^2}$$

$$\Rightarrow \epsilon_0 = \frac{1}{4\pi} \frac{q_1q_2}{Fr^2}$$

$$\Rightarrow [\epsilon_0] = \frac{[AT][AT]}{[MLT^{-2}][L]^2} = [M^{-1}L^{-3}T^4A^2]$$

Hence, the correct answer is (C).

$$32. R = \frac{V}{I}$$

$$\Rightarrow \frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$$

The percentage error in  $R$  is

$$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100 = 3\% + 3\% = 6\%$$

Hence, the correct answer is (D).

33. (i) All the non-zero digits are significant  
 (ii) All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all.  
 (iii) If the number is less than 1, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant.  
 (iv) The power of 10 is irrelevant to the determination of significant figures.

According to the above rules

23.023 has 5 significant figures

0.0003 has 1 significant figures

$2.1 \times 10^{-3}$  has 2 significant figures

Hence, the correct answer is (B).

## ARCHIVE: JEE ADVANCED

### Single Correct Choice Type Problems

$$1. m = \left(\frac{4\pi R^3}{3}\right)\rho$$

Taking log both sides, we get

$$\ln(m) = \ln\left(\frac{4\pi}{3}\right) + \ln(\rho) + 3\ln(R)$$

Differentiating with respect to time, we get

$$0 = 0 + \frac{1}{\rho} \frac{d\rho}{dt} + \frac{3}{R} \frac{dR}{dt}$$

$$\Rightarrow \left(\frac{dR}{dt}\right) = -R \times \frac{1}{\rho} \left(\frac{d\rho}{dt}\right)$$

$$\Rightarrow \frac{dR}{dt} = v$$

$$\Rightarrow v = -R \propto \frac{1}{\rho} \left(\frac{d\rho}{dt}\right)$$

$$\Rightarrow v \propto R$$

Hence, the correct answer is (A).

2. Distance covered by the stone

$$L = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times t_1^2$$

$$\Rightarrow t_1 = \sqrt{\frac{L}{5}}$$

Time taken by sound to reach the top is

$$t_2 = \frac{L}{v} = \frac{L}{300}$$

So total time taken is

$$T = \sqrt{\frac{L}{5}} + \frac{L}{300}$$

Time interval between dropping a stone and receiving the sound of impact is

$$dT = \frac{1}{\sqrt{5}} \frac{1}{2} L^{-1/2} dL + \left( \frac{1}{300} dL \right)$$

$$\Rightarrow dT = \frac{1}{2\sqrt{5}} \frac{1}{\sqrt{20}} dL + \frac{dL}{300} = 0.01$$

$$\Rightarrow dL \left( \frac{1}{20} + \frac{1}{300} \right) = 0.01$$

$$\Rightarrow dL \left( \frac{15}{300} \right) = 0.01$$

$$\Rightarrow dL = \frac{3}{16}$$

$$\Rightarrow \frac{dL}{L} \times 100 = \frac{3}{16} \times \frac{1}{20} \times 100 = \frac{15}{16} \approx 1\%$$

Hence, the correct answer is (A).

3. For  $C_1$ : 10 VSD = 9 MSD = 9 mm

$$\Rightarrow 1 \text{ VSD} = 0.9 \text{ mm}$$

Since least count for vernier calliper 1 is

$$(LC)_1 = 1 \text{ MSD} - 1 \text{ VSD}$$

$$\Rightarrow (LC)_1 = 0.1 \text{ mm} = 0.01 \text{ cm}$$

$$\Rightarrow \left( \text{Reading of } C_1 \right) = \left( \begin{array}{c} \text{Main} \\ \text{Scale} \\ \text{Reading} \end{array} \right) + \left( \begin{array}{c} \text{Coinciding} \\ \text{Divisions} \end{array} \right) \left( \begin{array}{c} \text{Least} \\ \text{Count} \end{array} \right)$$

$$\Rightarrow (\text{Reading of } C_1) = 2.8 \text{ cm} + (7)(0.01) \text{ cm} = 2.87 \text{ cm}$$

For  $C_2$ : 10 VSD = 11 MSD = 11 mm

$$\Rightarrow 1 \text{ VSD} = 1.1 \text{ mm}$$

Least count for vernier calliper 2 is

$$(LC)_2 = 1 \text{ MSD} - 1 \text{ VSD} = -0.1 \text{ mm} = -0.01 \text{ cm}$$

$$\Rightarrow (\text{Reading of } C_2) = (2.8 + 0.1 - 0.07) \text{ cm} = 2.83 \text{ cm}$$

Hence, the correct answer is (C).

4. 1 MSD = 5.15 cm - 5.10 cm = 0.05 cm

$$1 \text{ VSD} = \frac{2.45 \text{ cm}}{50} = 0.049 \text{ cm}$$

$$\Rightarrow LC = 1 \text{ MSD} - 1 \text{ VSD} = 0.01 \text{ cm}$$

Since

$$\left( \begin{array}{c} \text{Diameter} \\ \text{of} \\ \text{Cylinder} \end{array} \right) = \left( \begin{array}{c} \text{Main} \\ \text{Scale} \\ \text{Reading} \end{array} \right) + \left( \begin{array}{c} \text{Vernier} \\ \text{Scale} \\ \text{Reading} \end{array} \right) \left( \begin{array}{c} \text{Least} \\ \text{Count} \end{array} \right)$$

$$\Rightarrow D = 5.10 + (24)(0.001) = 5.124 \text{ cm}$$

Hence, the correct answer is (B).

5.  $\Delta d = \Delta l = \frac{0.5}{100} \text{ mm} = 0.005 \text{ mm}$

$$\text{Since, } Y = \frac{4MLg}{\pi l d^2}$$

$$\Rightarrow \left( \frac{\Delta Y}{Y} \right)_{\text{max}} = \left( \frac{\Delta l}{l} \right) + 2 \left( \frac{\Delta d}{d} \right)$$

$$\Rightarrow \left( \frac{\Delta l}{l} \right) = \frac{0.5/100}{0.25} = 0.02 \text{ and } \frac{2\Delta d}{d} = \frac{(2)(0.5/100)}{0.5} = 0.02$$

$$\Rightarrow \frac{\Delta l}{l} = 2 \frac{\Delta d}{d}$$

Hence, the correct answer is (C).

6. Least count of screw gauge =  $\frac{0.5}{50} = 0.01 \text{ mm} = \Delta r$

$$\text{Diameter, } r = 2.5 \text{ mm} + 20 \times \frac{0.5}{50} = 2.70 \text{ mm}$$

$$\Rightarrow \frac{\Delta r}{r} = \frac{0.01}{2.70}$$

$$\Rightarrow \frac{\Delta r}{r} \times 100 = \frac{1}{2.7}$$

$$\text{Now, density, } d = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi \left( \frac{r}{2} \right)^3}$$

where  $r$  is the diameter of sphere

$$\Rightarrow \frac{\Delta d}{d} \times 100 = \left\{ \frac{\Delta m}{m} + 3 \left( \frac{\Delta r}{r} \right) \right\} \times 100$$

$$\Rightarrow \frac{\Delta d}{d} = \frac{\Delta m}{m} \times 100 + 3 \times \left( \frac{\Delta r}{r} \right) \times 100$$

$$\Rightarrow \frac{\Delta d}{d} = 2\% + 3 \times \frac{1}{2.7} = 3.11\%$$

Hence, the correct answer is (C).

7. Least count of vernier callipers

$$LC = 1 \text{ MSD} - 1 \text{ VSD}$$

$$\Rightarrow LC = \frac{\text{Smallest division on main scale}}{\text{Number of divisions on vernier scale}}$$

**H.34 JEE Advanced Physics: Mechanics - I**

20 divisions of vernier scale = 16 divisions of main scale

$$\Rightarrow 1 \text{ VSD} = \frac{16}{20} \text{ mm} = 0.8 \text{ mm}$$

$$\Rightarrow LC = 1 \text{ MSD} - 1 \text{ VSD} = 1 \text{ mm} - 0.8 \text{ mm} = 0.2 \text{ mm}$$

Hence, the correct answer is (D).

8.  $T = 2\pi\sqrt{\frac{l}{g}}$ , where  $T = \frac{t}{n}$

$$\Rightarrow \frac{t}{n} = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow g = \frac{(4\pi^2)(n^2)l}{t^2}$$

% error in  $g$  is

$$\frac{\Delta g}{g} \times 100 = \left( \frac{\Delta l}{l} + \frac{2\Delta t}{t} \right) \times 100\%$$

$$\Rightarrow E_I = \left( \frac{0.1}{64} + \frac{2 \times 0.1}{128} \right) \times 100 = 0.3125\%$$

$$\Rightarrow E_{II} = \left( \frac{0.1}{64} + \frac{2 \times 0.1}{64} \right) \times 100 = 0.46875\%$$

$$\Rightarrow E_{III} = \left( \frac{0.1}{20} + \frac{2 \times 0.1}{36} \right) \times 100 = 1.055\%$$

Hence  $E_I$  is minimum

Hence, the correct answer is (B).

9.  $Y = \frac{FL}{Al} = \frac{4FL}{\pi d^2 l} = \frac{(4)(1 \times 9.8)(2)}{\pi(0.4 \times 10^{-3})^2(0.8 \times 10^{-3})}$

$$\Rightarrow Y = 2 \times 10^{11} \text{ Nm}^{-2}$$

Further  $\frac{\Delta Y}{Y} = 2 \left( \frac{\Delta d}{d} \right) + \left( \frac{\Delta l}{l} \right)$

$$\Rightarrow \Delta Y = \left\{ 2 \left( \frac{\Delta d}{d} \right) + \left( \frac{\Delta l}{l} \right) \right\} Y$$

$$\Rightarrow \Delta Y = \left\{ 2 \times \frac{0.01}{0.4} + \frac{0.05}{0.8} \right\} \times 2 \times 10^{11}$$

$$\Rightarrow \Delta Y = 0.225 \times 10^{11} \text{ Nm}^{-2}$$

$$\Rightarrow \Delta Y = 0.2 \times 10^{11} \text{ Nm}^{-2} \quad \text{\{by rounding off\}}$$

$$\Rightarrow (Y + \Delta Y) = (2 + 0.2) \times 10^{11} \text{ Nm}^{-2}$$

Hence, the correct answer is (B).

10. Length of air column in resonance is odd integer multiple of  $\frac{\lambda}{4}$ .

Hence, the correct answer is (A).

11. Least count  $LC = \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$

$$\Rightarrow LC = \frac{0.5}{50} = 0.01 \text{ mm}$$

Now, diameter of ball is

$$D = (2 \times 0.5 \text{ mm}) + (25 - 5)(0.01) = 1.2 \text{ mm}$$

Hence, the correct answer is (A).

12.  $\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2 \frac{\Delta T}{n}$

In answer (D) error in  $\Delta g$  is minimum and number of observations made are maximum. Hence, in this case error in  $g$  will be minimum.

Hence, the correct answer is (D).

13. Dipole moment = (charge)  $\times$  (distance)

Electric flux = (electric field)  $\times$  (area)

Hence, the correct answer is (D).

14. Density,  $\rho = \frac{M}{V} = \frac{M}{\pi r^2 L}$

$$\Rightarrow \frac{\Delta \rho}{\rho} = \frac{\Delta M}{M} + 2 \frac{\Delta r}{r} + \frac{\Delta L}{L}$$

$$\Rightarrow \frac{\Delta \rho}{\rho} = \frac{0.003}{0.3} + 2 \times \frac{0.005}{0.5} + \frac{0.06}{6}$$

$$\Rightarrow \frac{\Delta \rho}{\rho} = 0.01 + 0.02 + 0.01 = 0.04$$

$$\Rightarrow \%RE = \frac{\Delta \rho}{\rho} \times 100 = 0.04 \times 100 = 4\%$$

Hence, the correct answer is (D).

15.  $\left[ \frac{\alpha Z}{k\theta} \right] = [M^0 L^0 T^0]$

$$[\alpha] = \left[ \frac{k\theta}{Z} \right]$$

Further  $[P] = \left[ \frac{\alpha}{\beta} \right]$

$$\Rightarrow [\beta] = \left[ \frac{\alpha}{P} \right] = \left[ \frac{k\theta}{ZP} \right]$$

Dimensions of  $k\theta$  are that of energy. Hence,

$$[\beta] = \left[ \frac{ML^2T^{-2}}{LML^{-1}T^{-2}} \right] = [M^0L^2T^0]$$

Hence, the correct answer is (A).

16.  $V = l^3 = (1.2 \times 10^{-2} \text{ m})^3 = 1.728 \times 10^{-6} \text{ m}^3$

Since the length ( $l$ ) has two significant figures, the volume ( $V$ ) will also have two significant figures.

Hence, the correct answer is (A).

17.  $X = \epsilon_0 L \left( \frac{\Delta V}{\Delta t} \right)$

$$X = \left( \frac{C}{L} \right) (L) \left( \frac{\Delta V}{\Delta t} \right) \quad \left\{ \because \epsilon_0 = \frac{\text{Capacitance}}{\text{Length}} \right\}$$

$$\Rightarrow X = C \left( \frac{\Delta V}{\Delta t} \right)$$

$$\Rightarrow X = \frac{C\Delta V}{\Delta t} = \frac{\Delta Q}{\Delta t} = \text{Current}$$

So,  $X$  happens to be the current.

Hence, the correct answer is (D).

18.  $\left[ \frac{1}{2} \epsilon_0 E^2 \right] = \left[ \frac{B^2}{2\mu_0} \right] = [\text{Pressure}] = ML^{-1}T^{-2}$

Hence, the correct answer is (B).

19.  $[Y] = \left[ \frac{X}{Z^2} \right] = \left[ \frac{\text{Capacitance}}{(\text{Magnetic Induction})^2} \right]$

$$\Rightarrow [Y] = \frac{M^{-1}L^{-2}Q^2T^2}{M^2Q^{-2}T^{-2}} = M^{-3}L^{-2}Q^4T^4$$

Hence, the correct answer is (B).

20. Substituting the dimensions of mass [ $M$ ], length [ $L$ ] and coefficient of rigidity [ $ML^{-1}T^{-2}$ ] we get

$T = 2\pi \sqrt{\frac{M}{\eta L}}$  is the right formula for time period of oscillations.

Hence, the correct answer is (D).

21.  $RC$  has dimensions of time and  $V$  has the dimensions of  $L \frac{dI}{dt}$

$$\text{Hence, } \left[ \frac{L}{RCV} \right] = \left[ \frac{1}{T} \times \frac{T}{A} \right] = \frac{1}{\text{current}}$$

Hence, the correct answer is (B).

22.  $\left[ \frac{EJ^2}{M^5G^2} \right] = \frac{[ML^2T^{-2}] \times [ML^2T^{-1}]^2}{[M^5] \times [M^{-1}L^3T^{-2}]^2} = [M^0L^0T^0]$

Hence, the correct answer is (B).

### Multiple Correct Choice Type Problems

1. Angular momentum  $J = mvr$

$$\Rightarrow [J] = \frac{ML^2}{T}$$

Since  $m$  and  $J$  are dimensionless, so

$$T = L^2$$

$$\text{Since, } p = mv = \frac{ML}{T}$$

Since  $M$  is dimensionless, so

$$[p] = \left[ \frac{ML}{T} \right] = \frac{L}{L^2} = L^{-1}$$

$$\text{Similarly, (Energy)} = \frac{ML^2}{T^2} = \frac{L^2}{L^2L^2} = L^{-2}$$

$$(\text{Power}) = \frac{ML^2}{T^2T} = \frac{L^2}{L^2L^2L^2} = L^{-4}$$

$$(\text{Force}) = \frac{ML}{T^2} = \frac{L}{L^2L^2} = L^{-3}$$

Hence, (A), (C) and (D) are correct.

2.  $[k_B T] = [\text{Energy}] = [FL]$

$$[\epsilon] = \left[ \frac{Q^2}{FL^2} \right] \text{ and } [n] = L^{-3}$$

where  $Q$  is charge,  $F$  is force,  $L$  is length

Hence, (B) and (D) are correct.

3. Error in  $T$

$$T_{\text{mean}} = \frac{0.52 + 0.56 + 0.57 + 0.54 + 0.59}{5} = 0.556 \approx 0.56 \text{ s}$$

## H.36 JEE Advanced Physics: Mechanics - I

and  $\Delta T_{\text{mean}} = \frac{0.04 + 0.00 + 0.01 + 0.02 + 0.03}{5} = 0.02 \text{ s}$

Percentage error in  $T$  is given by

$$\frac{\Delta T}{T} \times 100\% = \frac{0.02}{0.56} \times 100 = 3.57\%$$

Percentage Error in  $r$  is given by

$$\frac{\Delta r}{r} \times 100\% = \frac{1}{10} \times 100 = 10\%$$

Since,  $T = 2\pi \sqrt{\frac{7(R-r)}{5g}}$

$$\Rightarrow T^2 = 4\pi^2 \frac{7}{5} \left( \frac{R-r}{g} \right)$$

$$\Rightarrow g = \frac{28\pi^2}{5} \left( \frac{R-r}{T^2} \right)$$

So, error in measurement of  $g$  is

$$\frac{\Delta g}{g} = \left( \frac{\Delta R + \Delta r}{R-r} \right) + 2 \frac{\Delta T}{T} = \frac{2}{50} + 2 \times 0.0357$$

$$\Rightarrow \frac{\Delta g}{g} \times 100 \approx 11\%$$

**Hence, (A), (B) and (D) are correct.**

4.  $M \propto h^a c^b G^c$

$$M^1 \propto (ML^2T^{-1})^a (LT^{-1})^b (M^{-1}L^3T^{-2})^c$$

$$\propto M^{a-c} L^{2a+b+3c} T^{-a-b-2c}$$

$$a - c = 1 \quad \dots(1)$$

$$2a + b + 3c = 0 \quad \dots(2)$$

$$a + b + 2c = 0 \quad \dots(3)$$

On solving (1), (2), (3)

$$a = \frac{1}{2}, b = +\frac{1}{2}, c = -\frac{1}{2}$$

$$\Rightarrow M \propto \sqrt{c}$$

In the same way we can find that,

$$L \propto h^{1/2} c^{-3/2} G^{1/2}$$

$$L \propto \sqrt{h}, L \propto \sqrt{G}$$

**Hence, (A), (C) and (D) are correct.**

5. (A) Energy of inductor

$$\frac{1}{2} LI^2 = \frac{1}{2} \frac{\mu_0 N^2 A}{l} I^2$$

Energy of capacitor is

$$\frac{1}{2} CV^2 = \frac{1}{2} \epsilon_0 \left( \frac{A}{d} \right) V^2$$

So,  $\mu_0 \frac{A}{l} I^2$  and  $\epsilon_0 \frac{A}{d} V^2$  have same dimensions

$$\Rightarrow \mu_0 I^2 \text{ and } \epsilon_0 V^2 \text{ have same dimension}$$

$$(C) I = [\epsilon_0 CV] = \left[ \left( \frac{C}{l} \right) \left( \frac{l}{t} \right) V \right] = \left[ \frac{CV}{t} \right] = \left[ \frac{q}{t} \right] = I$$

$$\left\{ \because C = 4\pi\epsilon_0 r, CV = q, c = \frac{l}{t} \right\}$$

**Hence, (A) and (C) are correct.**

6. For Vernier Callipers

$$1 \text{ MSD} = \frac{1}{8} \text{ cm}$$

$$5 \text{ VSD} = 4 \text{ MSD}$$

$$\Rightarrow 1 \text{ VSD} = \frac{4}{5} \text{ MSD} = \frac{4}{5} \times \frac{1}{8} = \frac{1}{10} \text{ cm}$$

Least count of vernier calliper = 1 MSD - 1 VSD

$$LC = \frac{1}{8} \text{ cm} - \frac{1}{10} \text{ cm} = 0.025 \text{ cm}$$

(a) and (b)

$$\text{Pitch of screw gauge} = 2 \times 0.025 = 0.05 \text{ cm}$$

$$\text{Least count of screw gauge} = \frac{0.05}{100} \text{ cm} = 0.005 \text{ mm}$$

(c) and (d)

$$\text{Least count of linear scale of screw gauge} = 0.05$$

$$\text{Pitch} = 0.05 \times 2 = 0.1 \text{ cm}$$

$$\text{Least count of screw gauge} = \frac{0.1}{100} \text{ cm} = 0.01 \text{ mm}$$

**Hence, (B) and (C) are correct.**

7.  $T = \frac{40 \text{ s}}{20} = 2 \text{ s}$

$$\text{Further, } t = nT = 20T$$

$$\Rightarrow \Delta t = 20\Delta T$$

$$\Rightarrow \frac{\Delta t}{t} = \frac{\Delta T}{T}$$

$$\Rightarrow \Delta T = \frac{T}{t} \Delta t = \left( \frac{2}{40} \right) (1) = 0.05 \text{ s}$$

$$\text{Further, } T = 2\pi \sqrt{\frac{l}{g}}$$

$$\Rightarrow T \propto g^{-1/2}$$

$$\Rightarrow \frac{\Delta T}{T} \times 100 = -\frac{1}{2} \times \frac{\Delta g}{g} \times 100$$

% error in determination of  $g$  is

$$\frac{\Delta g}{g} \times 100 = -200 \times \frac{\Delta T}{T}$$

$$\frac{\Delta g}{g} = -\frac{200 \times 0.05}{2} = -5\%$$

Hence, (A) and (C) are correct.

9. (A)  $L = \frac{\phi}{i}$  or henry =  $\frac{\text{weber}}{\text{ampere}}$

(B)  $e = -L \left( \frac{di}{dt} \right)$

$$\Rightarrow L = -\frac{e}{(di/dt)} \text{ or henry} = \frac{\text{volt-second}}{\text{ampere}}$$

(C)  $U = \frac{1}{2} Li^2$

$$\Rightarrow L = \frac{2U}{i^2} \text{ or henry} = \frac{\text{joule}}{(\text{ampere})^2}$$

(D)  $U = \frac{1}{2} Li^2 = i^2 Rt$

$$\Rightarrow L = Rt \text{ or henry} = \text{ohm-second}$$

Hence, (A), (B), (C) and (D) are correct.

10. [Renold's Number] = [Coefficient of friction] =  $M^0 L^0 T^0$

[Curie] = [Frequency] =  $M^0 L^0 T^{-1}$

[Latent Heat] = [Gravitational Potential] =  $M^0 L^2 T^{-2}$

Hence, (A), (B) and (C) are correct.

### Linked Comprehension Type Problems

1. In terms of dimension,  $F_e = F_m$

$$\Rightarrow qE = qvB$$

$$\Rightarrow E = vB$$

$$[E] = [B][LT^{-1}]$$

Hence, the correct answer is (C).

2.  $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$$\Rightarrow c^2 = \frac{1}{\mu_0 \epsilon_0}$$

$$\Rightarrow \mu_0 = \epsilon_0^{-1} c^{-2}$$

$$\Rightarrow [\mu_0] = [\epsilon_0]^{-1} [L^{-2} T^2]$$

Hence, the correct answer is (D).

3.  $r = \frac{1-a}{1+a}$

$$\ln r = \ln(1-a) - \ln(1+a)$$

Differentiating, we get

$$\frac{dr}{r} = -\frac{da}{1-a} - \frac{da}{1+a}$$

or, we can write

$$\frac{\Delta r}{r} = -\left[ \frac{\Delta a}{1-a} + \frac{\Delta a}{1+a} \right]$$

$$\Rightarrow \frac{\Delta r}{r} = \frac{-2\Delta a}{1-a^2}$$

$$\Rightarrow \Delta r = -\left( \frac{2\Delta a}{1-a^2} \right) (r) = \frac{-2\Delta a}{(1+a)^2}$$

Hence, the correct answer is (B).

4.  $N = N_0 e^{-\lambda t}$

$$\ln N = \ln N_0 - \lambda t$$

Differentiating w.r.t.  $\lambda$ , we get

$$\frac{1}{N} \frac{dN}{d\lambda} = 0 - t$$

$$\Rightarrow |d\lambda| = \frac{dN}{Nt} = \frac{40}{2000 \times 1} = 0.02$$

Hence, the correct answer is (C).

5.  $N$  = Number of electrons per unit volume

$$\Rightarrow [N] = [L^{-3}], [e] = [q] = [It] = [AT]$$

$$[\epsilon_0] = [M^{-1} L^{-3} T^4 A^2]$$

Substituting the dimensions, we can see that

$$\left[ \sqrt{\frac{Ne^2}{m\epsilon_0}} \right] = [T^{-1}]$$

Angular frequency has also the dimension  $[T^{-1}]$

Hence, the correct answer is (C).

6.  $\omega = 2\pi f = \frac{2\pi c}{\lambda}$

$$\Rightarrow \lambda = \frac{2\pi c}{\omega} = \frac{2\pi c}{\sqrt{\frac{Ne^2}{m\epsilon_0}}}$$

Substituting the values, we get  $\lambda \approx 600 \text{ nm}$

Hence, the correct answer is (B).

### Matrix Match/Column Match Type Questions

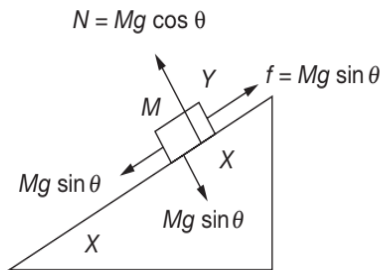
1. A → (p, q, s, t)  
 B → (q)  
 C → (s)  
 D → (s)

**For (t)**

A time varying magnetic field will produce a non-conservative electric field. Due to this electric field, a current starts flowing in the resistive loop and heat is produced.

2. A → (p, t)  
 B → (q, s, t)  
 C → (p, r, t)  
 D → (q)

**For (p)**



Net force on Y due to

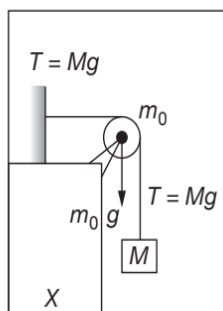
$$X = \sqrt{(Mg \cos \theta)^2 + (Mg \sin \theta)^2} = Mg$$

- (B) As the inclined is fixed. So, gravitational P.E. of X is constant.  
 (C) As K.E. is constant and P.E. of Y is decreasing. So mechanical energy of (X + Y) is decreasing.

**For (q)**

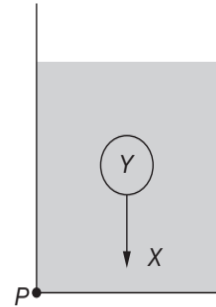
- (A) Force on Y due to X will be greater than Mg which is equal to (Mg + repulsion force)  
 (B) As the system is moving up, P.E. of X is increasing  
 (C) Mechanical energy of (X + Y) is increasing  
 (D) Torque of the weight of Y about point P = 0

**For (r)**



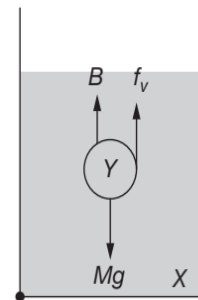
- (A) Force on Y due to X =  $\sqrt{[(M + m_0)g]^2 + (Mg)^2}$   
 (B) As the system moves down, gravitational P.E. of X decreases  
 (C) As the system moves down, total mechanical energy of (X + Y) also decreases  
 (D)  $\tau_P \neq 0$

**For (s)**



- (A) Force on Y due to X = Buoyancy force which is less than mg  
 (B) As the sphere moves down, that volume of water comes up, so gravitational P.E. of X increases.  
 (C) As there is no non-conservative force, so total mechanical energy of X + Y remains conserved.  
 (D)  $\tau_P \neq 0$

**For (t)**



- (A) As the sphere is moving with constant velocity  

$$B + f_v = Mg$$
 so force on Y due to X is  $B + f_v = Mg$   
 (B) As the sphere moves down, that volume of water comes up, so gravitational P.E. of X will increase  
 (C) Increase in mechanical energy is the work done by friction force which is negative.  

$$\Rightarrow \Delta U = W_{fr} = -ve$$
  
 (D)  $\tau_P = 0$

3. A → (p, q)  
 B → (r, s)  
 C → (r, s)  
 D → (r, s)

$$\left[ \frac{GM_e M_s}{r} \right] = \text{joule}$$

$$\Rightarrow [GM_e M_s] = (\text{joule})(\text{metre})$$

Since joule = (coulomb)(volt)

$$\Rightarrow [GM_e M_s] = (\text{volt})(\text{coulomb})(\text{metre})$$

$$\text{Also, } [GM_e M_s] = ML^3 T^{-2}$$

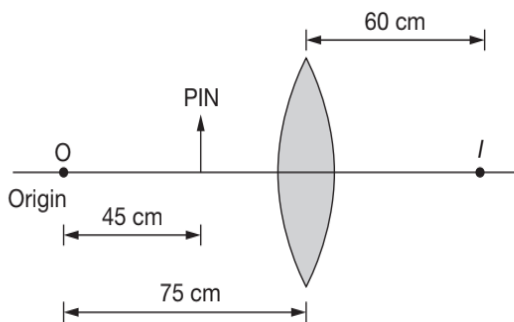
$$\Rightarrow [GM_e M_s] = (\text{kilogram})(\text{metre})^3 (\text{second})^{-2}$$

$$\text{Now, } \left[ \frac{3RT}{M} \right] = \left[ \frac{F^2}{q^2 B^2} \right] = \left[ \frac{GM_e}{R_e} \right] = v^2$$

$$\text{Also } v^2 = \left[ \frac{\text{Energy}}{\text{Mass}} \right] = \left[ \frac{CV^2}{\text{Mass}} \right] = (\text{farad})(\text{volt})^2 (\text{kg})^{-1}$$

### Integer/Numerical Answer Type Questions

1. Since there are four equal divisions in each cm, so least count is  $\frac{1}{4}$  cm



Object distance i.e., distance of pin from lens is

$$u = -(75 - 45) = -30 \text{ cm}$$

$$\Rightarrow \Delta u = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \text{ cm}$$

Similarly, image distance is

$$v = +(135 - 75) = +60 \text{ cm}$$

$$\Delta v = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \text{ cm}$$

Using Lens formula  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ , we get

$$\frac{1}{60} - \frac{1}{(-30)} = \frac{1}{f}$$

$$\Rightarrow f = 20 \text{ cm}$$

Applying Error Analysis on the expression

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \text{ we get}$$

$$\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2} = \frac{\Delta f}{f^2}$$

$$\Rightarrow \frac{\Delta f}{f} \times 100 = \left( \frac{\Delta v}{v} + \frac{\Delta u}{u} \right) f \times 100\%$$

$\Rightarrow$  %age Error in  $f$  is

$$\frac{\Delta f}{f} \times 100 = \left[ \frac{0.5}{(60)^2} + \frac{0.5}{(30)^2} \right] \times 20 \times 100$$

$$\Rightarrow \frac{\Delta f}{f} \times 100 = 1.39\%$$

2. Given,  $d = 0.5 \text{ mm}$

$$Y = 2 \times 10^{11} \text{ Nm}^{-2}$$

$$\Rightarrow l = 1 \text{ m}$$

$$\Rightarrow \Delta l = \frac{Fl}{AY} = \frac{mgl}{\frac{\pi d^2}{4} Y}$$

$$\Rightarrow \Delta l = \frac{1.2 \times 10 \times 1}{\frac{\pi}{4} \times (5 \times 10^{-4})^2 \times 2 \times 10^{11}} = 0.3 \text{ mm}$$

$$\text{LC of vernier} = \left( 1 - \frac{9}{10} \right) \text{ mm} = 0.1 \text{ mm}$$

So, 3<sup>rd</sup> division of vernier scale will coincide with main scale.

3.  $E(t) = A^2 e^{-\alpha t}$  ... (1)

$$\alpha = 0.2 \text{ s}^{-1}$$

$$\left( \frac{dA}{A} \right) \times 100 = 1.25\%$$

$$\left( \frac{dt}{t} \right) \times 100 = 1.50\%$$

$$\Rightarrow (dt \times 100) = 1.5t$$

$$\Rightarrow (dt \times 100) = 1.5 \times 5 = 7.5$$

$$\Rightarrow \left( \frac{dE}{E} \right) \times 100 = \pm 2 \left( \frac{dA}{A} \right) \times 100 \pm \alpha (dt \times 100)$$

Taking log on both sides of equation (1), we get

$$\log E = 2 \log A - \alpha t$$

$$\Rightarrow \frac{dE}{E} = \pm 2 \frac{dA}{A} \pm \alpha dt$$

#### H.40 JEE Advanced Physics: Mechanics - I

$$\Rightarrow \left(\frac{dE}{E}\right) \times 100 = \pm 2 \left(\frac{dA}{A}\right) \times 100 \pm \alpha(dt \times 100)$$

$$\Rightarrow \left(\frac{dE}{E}\right) \times 100 = \pm 2(1.25) \pm 0.2(7.5)$$

Since, during propagation, errors are always added up, so

$$\Rightarrow \left(\frac{dE}{E}\right) \times 100 = \pm 2.5 \pm 1.5 = \pm 4\%$$

4. Let  $d = k(\rho)^a (S)^b (f)^c$

where,  $k$  is a dimensionless constant. Then,

$$[L] = \left[\frac{M}{L^3}\right]^a \left[\frac{ML^2T^{-2}}{L^2T}\right]^b \left[\frac{1}{T}\right]^c$$

Equating the powers of  $M$  and  $L$ , we get

$$0 = a + b \quad \dots(1)$$

$$1 = -3a \quad \dots(2)$$

Solving these two equations, we get

$$b = \frac{1}{3}$$

$$\Rightarrow n = 3$$

5.  $Y = \frac{F/A}{\Delta l/l}$ , where  $\Delta l = 25 \times 10^{-50}$  m

$$\Rightarrow Y = \frac{F}{A} \left(\frac{l}{\Delta l}\right)$$

$$\Rightarrow \frac{\Delta Y}{Y} \times 100 = \frac{10^{-5}}{25 \times 10^{-5}} \times 100 = 4\%$$