

Test Your Concepts-I (Based on Thermometry and Thermal Expansion)

1. (a) When the temperature decreases, the length of the ruler also decreases through thermal contraction. Below 20 °C, each centimetre division is actually somewhat shorter than 1 cm, so the steel ruler gives readings that are too long.

- (b) At 40 °C, the increase in length of the ruler is

$$\Delta \ell = \ell \alpha \Delta T$$

$$\Rightarrow \Delta \ell = (20)(1.2 \times 10^{-5})(40 - 20)$$

$$\Rightarrow \Delta \ell = 0.48 \times 10^{-2} \text{ cm}$$

So, the actual length of the ruler is,

$$\ell' = \ell + \Delta \ell = 20.0048 \text{ cm}$$

2. (a) The 88.42 cm mark on aluminium rod is actually at a greater distance from the zero position than indicated because of the increase in temperature $\Delta T = 30$ °C. This increased length is

$$\Delta \ell = \alpha_{Al} \ell_{Al} \Delta T = (2.55 \times 10^{-5})(88.42)(30)$$

$$\Rightarrow \Delta \ell = 0.068 \text{ cm}$$

- (b) At 35 °C the measuring rod at 88.42 cm mark is actually $(88.42 + 0.068)$ cm = 88.49 cm long, as calculated from part (a). It thus measures 88.49 cm length of steel at 35 °C.

3. At 30 °C, the copper rod will be of length $\ell(1 + \alpha_c \Delta \theta)$, while adjacent 'centimetre' marks on the steel tape will be separated by a distance of $(1)(1 + \alpha_s \Delta \theta)$. Therefore, the number of centimetres reading on the tape is

$$\text{Reading (in cm)} = \frac{\ell(1 + \alpha_c \Delta \theta)}{(1)(1 + \alpha_s \Delta \theta)}$$

$$\Rightarrow \text{Reading (in cm)} = \frac{(90)(1 + 1.7 \times 10^{-5} \times 20)}{(1)(1 + 1.2 \times 10^{-5} \times 20)}$$

$$\Rightarrow \text{Reading (in cm)} = 90 \left(\frac{1 + 3.4 \times 10^{-4}}{1 + 2.4 \times 10^{-4}} \right)$$

Since $\frac{1}{1+x} = (1+x)^{-1} \cong 1 - x$ for $x \ll 1$, so we get

$$90 \left(\frac{1 + 3.4 \times 10^{-4}}{1 + 2.4 \times 10^{-4}} \right) = 90(1 + 3.4 \times 10^{-4})(1 - 2.4 \times 10^{-4})$$

So, reading (in cm) is $\approx 90(1 + 3.4 \times 10^{-4} - 2.4 \times 10^{-4})$

$$\Rightarrow \text{Reading (in cm)} = 90 + 0.009 \approx 90.01 \text{ cm}$$

So, the tape will read 90.01 cm

4. (i) Let V_C be the volume of the container and V_L be the volume of the liquid. According to the condition given, we have

$$\frac{V_L}{V_C} = \text{constant (i.e., independent of temperature)}$$

$$\Rightarrow \frac{\Delta V_L}{V_L} = \frac{\Delta V_C}{V_C}$$

$$\Rightarrow \gamma \Delta T = \gamma_C \Delta T = (3\alpha) \Delta T$$

For container, coefficient of volume expansion is $\gamma_C = 3\alpha$. Hence

$$\gamma = 3\alpha$$

- (ii) If one heating, H does not change, then

$$H = \frac{V}{A} = \text{constant}$$

$$\Rightarrow V \propto A$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{\Delta A}{A}$$

$$\Rightarrow \gamma \Delta T = \beta_C \Delta T$$

$$\Rightarrow \gamma = 2\alpha$$

$$\{\because \beta_C = 2\alpha\}$$

- (iii) This is exactly same as part (i), so $\gamma = 3\alpha$

5. At higher temperatures actual reading is more than the scale reading. The error in the reading is given by

$$\Delta \ell = (\text{scale reading}) (\alpha) (\Delta T)$$

$$\Rightarrow \Delta \ell = (60)(1.2 \times 10^{-5})(27 - 15) = 0.00864 \text{ cm}$$

6. Volume expansion of mercury with respect to bulb is

$$\Delta V = V_0 (\gamma - 3\alpha) t$$

Find area of cross section of capillary is

$$A_f = A_0 (1 + 2\alpha t)$$

Height to which mercury rises in capillary is

$$h = \frac{\Delta V}{A_f} = \frac{V_0 (\gamma - 3\alpha) t}{A_0 (1 + 2\alpha t)}$$

7. The time period of second's pendulum is 2 second. As the temperature increases, the length and hence time period increases and so the clock becomes slow and loses the time. The change in time period is given by

$$\Delta t = \frac{1}{2} t \alpha \Delta T = \left(\frac{1}{2} \right) (2) (1.2 \times 10^{-5}) (30 - 20)$$

$$\Rightarrow \Delta t = 1.2 \times 10^{-4} \text{ sec}$$

So, new time period is,

$$t' = t + \Delta t = (2 + 1.2 \times 10^{-4})$$

$$\Rightarrow t' = 2.00012 \text{ sec}$$

So, time lost in one week, i.e., 7 days is

$$\Delta t' = \frac{(1.2 \times 10^{-4})}{(2.00012)} (7 \times 24 \times 3600)$$

$$\Rightarrow \Delta t' = 36.28 \text{ s}$$

8. Due to drop in temperature, tension increment in the clamped wire is $F = YA\alpha\Delta T$

$$\Rightarrow F = 2.1 \times 10^{12} \times 0.5 \times 10^{-2} \times 1.1 \times 10^{-5} \times 20$$

$$\Rightarrow F = 2.31 \times 10^{-6} \text{ dyne}$$

H.76 JEE Advanced Physics: Waves and Thermodynamics

9. Since, thermal stress is given by

$$\begin{aligned} \frac{F}{A} &= Y\alpha\Delta T \\ \Rightarrow F &= YA\alpha\Delta T = (2 \times 10^{11})(0.8 \times 10^{-4})(10^{-5})(10) \\ \Rightarrow F &= 1.6 \times 10^3 \text{ N} \end{aligned}$$

10. Temperature on Kelvin scale are

$$T_K = T_C + 273$$

For $T_C = 4000^\circ\text{C}$, we have

$$T_K = 4000 + 273 = 4273 \text{ K}$$

and for $T_C = 1.5 \times 10^7^\circ\text{C}$, we have

$$T_K = (1.5 \times 10^7 + 273) \text{ K}$$

Percentage error is given by

$$\% \text{error} = \left(\frac{T_K - T_C}{T_C} \right) \times 100\% = \frac{273}{T_C} \times 100\%$$

For $T_C = 4000^\circ\text{C}$

$$\% \text{Error} = \frac{273}{4000} \times 100 = 6.825\%$$

For $T_C = 1.5 \times 10^7^\circ\text{C}$

$$\% \text{error} = \frac{273}{1.5 \times 10^7} \times 100 = 1.82 \times 10^{-3}\%$$

11. Angle $B'AC = \frac{1}{2} \left[\pi - \left(\frac{\pi}{3} - \theta \right) \right] = \frac{\pi}{3} + \frac{\theta}{2} = \beta$ (say)

Since, $AB' = L_0(1 + 100\alpha)$

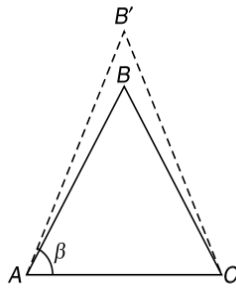
$$\text{Also, } AB' \cos \beta = \frac{L_0}{2} = \frac{AC}{2}$$

$$\Rightarrow L_0(1 + 100\alpha) \cos \left(\frac{\pi}{3} + \frac{\theta}{2} \right) = \frac{L_0}{2}$$

Using approximation, $\sin \left(\frac{\theta}{2} \right) \approx \frac{\theta}{2}$

and $\cos \left(\frac{\theta}{2} \right) \approx 1$, we get

$$\alpha = \frac{\sqrt{3}}{200} \theta$$



12. For an ideal gas, we have $PV = nRT$

$$\Rightarrow d(PV) = d(nRT)$$

Since, P is constant, we get $PdV = nRdT$

$$\Rightarrow \frac{dV}{dT} = \frac{nR}{P}$$

Now, by definition, $\gamma = \left(\frac{dV}{V} \right) \frac{1}{dT}$

$$\Rightarrow \gamma = \frac{nR}{PV} = \frac{nR}{nRT} = \frac{1}{T}$$

13. Since $\frac{T_C - 32}{212 - 32} = \frac{T_C - 0}{100 - 0}$... (1)

For same value on both the scales, we have

$$T_F = T_C = T_0$$

Substituting in equation (1), we get

$$T_0 - 32 = 1.8T_0$$

$$\Rightarrow -0.8T_0 = 32$$

$$\Rightarrow T_0 = -40^\circ\text{F}$$

14. (a) Since $L = I\omega = \left(\frac{1}{2}MR^2 \right) \omega$

$$\Rightarrow L = \frac{1}{2}(0.5)(0.030)^2(60) = 0.0135 \text{ kgm}^2\text{s}^{-1}$$

(b) When the cylinder is heated without any mechanical contact, then no external force and hence torque acts on the cylinder, so angular momentum is conserved. Hence,

$$I\omega = \text{constant}$$

$$\Rightarrow \Delta(I\omega) = 0$$

$$\Rightarrow \frac{\Delta\omega}{\omega} = -\frac{\Delta I}{I} = -\frac{2\Delta R}{R} \quad \left\{ \because I = \frac{1}{2}MR^2 \right\}$$

$$\text{Since, } \frac{\Delta R}{R} = \alpha\Delta T$$

$$\Rightarrow \frac{\Delta\omega}{\omega} = -2(\alpha\Delta T) = -2 \times 2 \times 10^{-5} \times 80$$

$$\Rightarrow \frac{\Delta\omega}{\omega} = -3.2 \times 10^{-3}$$

15. (a) Since, $\frac{T - 0}{100 - 0} = \frac{15.0 - 12.45}{21.30 - 12.45} = \frac{2.65}{8.85}$

$$\Rightarrow T = \frac{2.65}{8.85} \times 100 = 29.94^\circ\text{C}$$

(b) Since, $\frac{T - 0}{100 - 0} = \frac{22.95 - 12.45}{21.30 - 12.45} = \frac{10.5}{8.85}$

$$\Rightarrow T = \frac{10.5}{8.85} \times 100 = 118.64^\circ\text{C}$$

16. By definition, we have

$$\alpha(T) = \frac{dL}{L_0} \frac{1}{dT}$$

$$\Rightarrow dL = L_0\alpha(T)dT$$

$$\Rightarrow \int_{L_0}^L dL = L_0 \int_{T_0}^T \alpha(T)dT$$

$$\Rightarrow L - L_0 = L_0 \int_{T_0}^T \alpha(T)dT$$

$$\Rightarrow L = L_0 \left[1 + \int_{T_0}^T \alpha(T)dT \right]$$

17. Since, at all temperatures, we have $\ell_s - \ell_c = 5 \text{ cm}$... (1)

$$\Rightarrow \Delta\ell_s - \Delta\ell_c = 0$$

$$\Rightarrow \Delta\ell_s = \Delta\ell_c$$

By definition, we have $\Delta\ell_s = \ell_s\alpha_s\Delta T$ and $\Delta\ell_c = \ell_c\alpha_c\Delta T$

$$\Rightarrow \ell_s\alpha_s = \ell_c\alpha_c$$

$$\Rightarrow \frac{\ell_s}{\ell_c} = \frac{\alpha_c}{\alpha_s} = \frac{17}{11} \quad \dots (2)$$

So, from (1) and (2), we get

$$\ell_s = 14.16 \text{ cm and } \ell_c = 9.16 \text{ cm}$$

18. Since $\gamma = \frac{\Delta V}{V} \frac{1}{\Delta T} = \left(\frac{0.12}{100} \right) \frac{1}{20}$

$$\Rightarrow \gamma = 6 \times 10^{-5} (\text{°C})^{-1}$$

Further, we know that $\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}$

$$\Rightarrow \alpha = \frac{\gamma}{3} = 2 \times 10^{-5} (\text{°C})^{-1}$$

19. Let the distance of centre of mass from suspension point O be h , then $h = \text{constant}$

$$\Rightarrow h^2 = l_2^2 - \frac{l_1^2}{4} = \text{constant}$$

$$\Rightarrow \Delta(l_2^2) - \Delta\left(\frac{l_1^2}{4}\right) = 0$$

$$\Rightarrow 2l_2\Delta l_2 - \frac{1}{4}2l_1\Delta l_1 = 0$$

Since, by definition, $\Delta l = l\alpha\Delta T$

$$\Rightarrow \Delta l_2 = l_2\alpha_2\Delta T \text{ and } \Delta l_1 = l_1\alpha_1\Delta T$$

$$\Rightarrow 2l_2(l_2\alpha_2\Delta T) = \frac{1}{4}(2l_1)(l_1\alpha_1\Delta T)$$

$$\Rightarrow l_2^2\alpha_2 = \frac{1}{4}l_1^2\alpha_1$$

$$\Rightarrow \frac{l_1^2}{l_2^2} = 4\left(\frac{\alpha_2}{\alpha_1}\right)$$

$$\Rightarrow l_2 = \frac{l_1}{2} \sqrt{\frac{\alpha_1}{\alpha_2}} = \frac{5}{2} \sqrt{\frac{18}{9}} = 5\sqrt{2} \approx 7 \text{ cm}$$

20. Since, $D_T = D_0(1 + \alpha\Delta T)$

$$\Rightarrow D_T = 2.54(1 + 2.3 \times 10^{-5} \times 100) = 2.54(1.0023)$$

$$\Rightarrow D_T = 2.5458 \text{ cm}$$

21. Since, $T_K = T_C + 273$ and $T_F = \frac{9}{5}T_C + 32$

$$\Rightarrow T_C = T_K - 273.15 \text{ and } T_F = \frac{9}{5}T_C + 32$$

For Neon

$$T_C = T_K - 273.15$$

$$\Rightarrow T_C = 24.57 - 273.15 = -248.58 \text{ °C}$$

$$\text{and } T_F = \frac{9}{5}T_C + 32 = \frac{9}{5}(-248.58) + 32$$

$$\Rightarrow T_F = -447.44 + 32 = -415.44 \text{ °F}$$

For Carbon dioxide

$$T_C = T_K - 273.15 = 216.55 - 273.15$$

$$\Rightarrow T_C = -56.60 \text{ °C}$$

$$\text{and } T_F = \frac{9}{5}T_C + 32 = \frac{9}{5}(-56.60) + 32$$

$$\Rightarrow T_F = -101.88 + 32 = -69.88 \text{ °F}$$

22. Method-I

At any temperature t , the lengths of the two bars are

$$\ell'_1 = \ell_1(1 + \alpha_1 t), \ell'_2 = \ell_2(1 + \alpha_2 t) \quad \dots(1)$$

From the given conditions,

$$\ell'_2 - \ell'_1 = \ell \text{ and } \ell_2 - \ell_1 = \ell \quad \dots(2)$$

From equations (1) and (2),

$$\ell'_2 - \ell'_1 = (\ell_2 - \ell_1) + \ell_2\alpha_2 t - \ell_1\alpha_1 t$$

$$\ell_2\alpha_2 = \ell_1\alpha_1 \quad \dots(3)$$

$$\frac{\ell_2}{\ell_1} = \frac{\alpha_1}{\alpha_2} \quad \dots(4)$$

From equations (2) and (4),

$$\ell_1 = \frac{\ell\alpha_2}{\alpha_2 - \alpha_1}, \ell_2 = \frac{\ell\alpha_1}{\alpha_2 - \alpha_1}$$

Method-II

Since, the separation between the mirrors, ℓ , is constant. So,

$$\Delta\ell = \Delta(\ell_2 - \ell_1) = 0$$

$$\Rightarrow \Delta\ell_2 - \Delta\ell_1 = 0$$

$$\Rightarrow \Delta\ell_2 = \Delta\ell_1$$

$$\Rightarrow \ell_1\alpha_1\Delta T = \ell_2\alpha_2\Delta T$$

$$\Rightarrow \ell_1\alpha_1 = \ell_2\alpha_2 \quad \dots(5)$$

$$\text{Since, } \ell = \ell_2 - \ell_1 \quad \dots(6)$$

So, from (5) and (6), on solving, we get

$$\ell_1 = \frac{\ell\alpha_2}{\alpha_2 - \alpha_1} \text{ and } \ell_2 = \frac{\ell\alpha_1}{\alpha_2 - \alpha_1}$$

Test Your Concepts-II (Based on Calorimetry)

1. Let Q be the heat required to convert 200 g of water at 40 °C into 100 °C, then

$$Q = mc\Delta T = (200)(1)(100 - 40) = 12000 \text{ cal}$$

Now, suppose m_0 mass of steam converts into water to liberate this much amount of heat, then

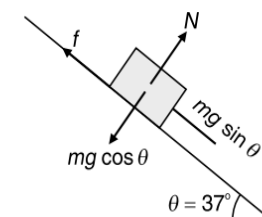
$$m_0 = \frac{Q}{L} = \frac{12000}{539} = 22.26 \text{ g}$$

Since this is less than 30 g, so the temperature of the mixture is 100 °C

Mass of steam in the mixture = 30 - 22.26 = 7.74 g

and mass of water in the mixture = 200 + 22.26 = 222.26 g

2. When the copper cube slides down a rough inclined plane at constant speed, then $f = mg \sin \theta = mg \sin(37^\circ)$



H.78 JEE Advanced Physics: Waves and Thermodynamics

(Heat gained) = (Work done against friction)

$$\Rightarrow mc\Delta T = f\Delta r$$

$$\Rightarrow mc\Delta T = mg \sin(37^\circ) \left(\frac{60}{100}\right)$$

$$\Rightarrow 420\Delta T = (10) \left(\frac{3}{5}\right) \left(\frac{3}{5}\right)$$

$$\Rightarrow \Delta T = 8.6 \times 10^{-3} \text{ }^\circ\text{C}$$

3. After time t , the mass of water inside pitcher is given by $m = (1000 - 0.1t)$ gram

In a further time dt , mass of water vapourised is $dm = (0.1)dt$ and let the temperature of water inside pitcher fall by dT .

So, we get $(1000 - 0.1t)(1)(-dT) = (0.1dt)(540)$

$$\Rightarrow \int_0^t \frac{54dt}{1000 - 0.1t} = - \int_{40}^{30} dT$$

$$\Rightarrow \int_0^t \frac{540dt}{10000 - t} = - \int_{40}^{30} dT$$

$$\Rightarrow -540 \ln(10000 - t) \Big|_0^t = 10$$

$$\Rightarrow \ln\left(\frac{10000 - t}{10000}\right) = -\frac{1}{54}$$

$$\Rightarrow 10000 - t = 10000e^{-1/54}$$

$$\Rightarrow t = 10000(1 - e^{-1/54}) \text{ s} = 10000(1 - 0.98) \text{ s}$$

$$\Rightarrow t = (10000)(0.02) = 200 \text{ s}$$

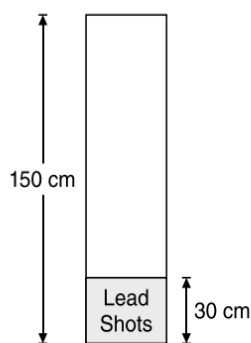
4. When inverted, each lead shot falls through an effective height of $(150 - 30)$ cm = 120 cm. So,

$$h = 120 \text{ cm} = \frac{120}{100} \text{ cm}$$

Let the tube has to be inverted n times, then

$$n(mgh) = mc_{\text{lead}}\Delta T$$

$$\Rightarrow n = \frac{c_{\text{lead}}\Delta T}{gh}$$



where $c_{\text{lead}} = (0.031)(4200) \text{ Jkg}^{-1}\text{K}^{-1}$ and $\Delta T = 2 \text{ }^\circ\text{C}$

$$\Rightarrow n = \frac{(0.031)(4200)(2)}{(9.8)\left(\frac{120}{100}\right)} = 22.1$$

So, the number of times the tube has to be inverted end to end is 23.

5. Since $\frac{1}{2}\left(\frac{1}{2}mv^2\right) = mc_{\text{bullet}}\Delta T$

$$\text{and } 1 \text{ calg}^{-1}(\text{ }^\circ\text{C})^{-1} = 4200 \text{ Jkg}^{-1}(\text{ }^\circ\text{C})^{-1} = 4200 \text{ Jkg}^{-1}\text{K}^{-1}$$

$$\Rightarrow \frac{(100)^2}{4} = 0.030 \times 4200 \times \Delta T$$

$$\Rightarrow \Delta T \cong 20 \text{ K or } \Delta T \cong 20 \text{ }^\circ\text{C}$$

6. Heat supplied is

$$Q = (22)(0.5)(8) + (22)(80) + (22)(1)(16)$$

$$\Rightarrow Q = 88 + 1760 + 352 = 2200 \text{ cal}$$

Heat capacity of body is

$$\text{Heat Capacity} = \frac{2200 \text{ cal}}{44 \text{ }^\circ\text{C}} = 50 \text{ cal}(\text{ }^\circ\text{C})^{-1}$$

Water equivalent of the body is

$$w = \frac{\text{Heat capacity of the body}}{\text{Specific heat of water}}$$

$$\Rightarrow w = \frac{50 \text{ cal}^\circ\text{C}^{-1}}{1 \text{ calg}^{-1}\text{ }^\circ\text{C}^{-1}} = 50 \text{ g}$$

7. $c = \frac{1}{m} \frac{\Delta Q}{\Delta T} = \left(\frac{1}{0.1}\right) \left(\frac{400}{20}\right) = 200 \text{ Jkg}^{-1}(\text{ }^\circ\text{C})^{-1}$

8. The mixture has 1 kg ice at $0 \text{ }^\circ\text{C}$ and 1 kg steam at $100 \text{ }^\circ\text{C}$. Heat required to completely melt ice is

$$Q_1 = 1 \times 3.36 \times 10^5 \text{ J}$$

Heat required to raise temperature of water from $0 \text{ }^\circ\text{C}$ to $100 \text{ }^\circ\text{C}$ is

$$Q_2 = 1 \times 4200 \times 100 = 4.2 \times 10^5 \text{ J}$$

So, total heat required to melt 1 kg ice at $0 \text{ }^\circ\text{C}$ to water at $0 \text{ }^\circ\text{C}$ and then take it to a temperature of $100 \text{ }^\circ\text{C}$ is

$$Q_{\text{required}} = Q_1 + Q_2 = 7.56 \times 10^5 \text{ J}$$

If this much entire heat is provided by condensation of steam, then mass m of steam condensed (to give water) would be

$$m = \frac{Q_1 + Q_2}{L_v} = \frac{7.56 \times 10^5}{2.27 \times 10^6} = 0.333 \text{ kg}$$

$$\Rightarrow m_{\text{water}} = 1333 \text{ g and } m_{\text{steam}} = 667 \text{ g}$$

Hence, final mixture is at $100 \text{ }^\circ\text{C}$. The final mixture has 667 g steam and 1333 g water.

9. Let $t \text{ }^\circ\text{C}$ be the temperature of the mixture. Then, by Law of Calorimetry, we have

$$\left(\text{Heat given by}\right) = \left(\text{Heat taken by}\right)$$

$$\left(10 \text{ g of water}\right) = \left(5 \text{ g of water}\right)$$

$$\Rightarrow m_1 c_{\text{water}} |\Delta t_1| = m_2 c_{\text{water}} |\Delta t_2|$$

$$\Rightarrow (10)(70 - t) = 5(t - 30)$$

$$\Rightarrow t = 36.67 \text{ }^\circ\text{C}$$

10. By Law of Calorimetry, heat supplied by the block equals heat gained by calorimeter and liquid. Assuming water equivalent of the calorimeter to be w , we get For 200 g of liquid, we have

$$(110)(0.1)(82) = (8)[w(1) + (200)c_L] \quad \dots(1)$$

For 400 g of liquid, we have

$$(110)(0.1)(85.5) = (4.5)[w(1) + (400)c_L] \quad \dots(2)$$

Multiply equation (2) by 8, equation (1) by 4.5 and subtracting, we get

$$110 \times 0.1(85.5 \times 8 - 82 \times 4.5) = c_L(400 \times 4.5 \times 8 - 200 \times 4.5 \times 8)$$

$$\Rightarrow c_L = \frac{3150}{7200} = 0.481 \text{ cal g}^{-1} \text{ } ^\circ\text{C}^{-1}$$

From equation (1), we get

$$w = \frac{902 - 769.6}{8} = 16.55 \text{ g}$$

11. Let s_A , s_B and s_C be the specific heats of liquid A, B and C. When A and B are mixed, then

$$\left(\begin{array}{c} \text{Heat Lost} \\ \text{by B} \end{array} \right) = \left(\begin{array}{c} \text{Heat Gained} \\ \text{by A} \end{array} \right)$$

$$\Rightarrow ms_B(19 - 16) = ms_A(16 - 12)$$

$$\Rightarrow 3s_B = 4s_A \quad \dots(1)$$

When B and C are mixed, then

$$\left(\begin{array}{c} \text{Heat Lost} \\ \text{by C} \end{array} \right) = \left(\begin{array}{c} \text{Heat Gained} \\ \text{by B} \end{array} \right)$$

$$\Rightarrow ms_C(28 - 23) = ms_B(23 - 19)$$

$$\Rightarrow 5s_C = 4s_B \quad \dots(2)$$

When A and C are mixed, let the equilibrium temperature be T , then

$$\left(\begin{array}{c} \text{Heat Lost} \\ \text{by C} \end{array} \right) = \left(\begin{array}{c} \text{Heat Gained} \\ \text{by A} \end{array} \right)$$

$$\Rightarrow ms_C(28 - T) = ms_A(T - 12) \quad \dots(3)$$

From (1) and (2), we get $s_C = \frac{16}{15}s_A$

$$\Rightarrow m \left(\frac{16}{15}s_A \right) (28 - T) = ms_A(T - 12)$$

$$\Rightarrow 16(28 - T) = 15(T - 12)$$

$$\Rightarrow 448 - 16T = 15T - 180$$

$$\Rightarrow 31T = 628$$

$$\Rightarrow T = 20.25 \text{ } ^\circ\text{C}$$

12. 75% of heat is retained by lead and due to this its temperature first rises from $27 \text{ } ^\circ\text{C}$ to $327 \text{ } ^\circ\text{C}$ and then at $327 \text{ } ^\circ\text{C}$ it melts. So, we have

$$\frac{3}{4} \left(\frac{1}{2}mv^2 \right) = mc_{\text{lead}}\Delta T + mL_{\text{lead}}$$

$$\Rightarrow \frac{3}{8}v^2 = (0.03)(4200)(300) + (6)(4200)$$

$$\Rightarrow v = 410 \text{ ms}^{-1}$$

13. Let x kg of water be frozen, then amount of heat released is

$$Q_1 = (x \times 4200 \times 50 + x \times 3.36 \times 10^5) \text{ J}$$

$$\Rightarrow Q_1 = (5.46 \times 10^5 x) \text{ J}$$

The heat required to vapourise $(1-x)$ kg of water from $50 \text{ } ^\circ\text{C}$ is

$$Q_2 = (1-x) \times 22.5 \times 10^5 \text{ J}$$

This heat Q_2 must be provided by the first part of water. So, we have $Q_2 = Q_1$

$$\Rightarrow (1-x)(22.5 \times 10^5) = 5.46 \times 10^5 x$$

$$\Rightarrow 22.5 - 22.5x = 5.46x$$

$$\Rightarrow x = \frac{22.5}{27.96} = 0.805 \text{ kg}$$

14. (a) Given $\frac{dT}{dt} = 0.5 \text{ } ^\circ\text{C s}^{-1}$

$$\text{Since, } dQ = mcdT \text{ i.e., } \frac{dQ}{dt} = mc \left(\frac{dT}{dt} \right)$$

$$\Rightarrow \frac{dQ}{dt} = \left(\frac{180}{1000} \right) (0.10)(4200)(0.5) \text{ Js}^{-1}$$

$$\Rightarrow \frac{dQ}{dt} = 37.8 \text{ W}$$

So, rate of working of drill in watt is 37.8

- (b) Since $P = \tau\omega$ and $1 \text{ rpm} = \frac{2\pi}{60} \text{ rads}^{-1}$

$$\Rightarrow 37.8 = \tau \left(180 \times \frac{2\pi}{60} \right) = 2 \text{ Nm}$$

15. Since $\frac{1}{2} \left(\frac{1}{2}mv^2 \right) = mc_{\text{bullet}}\Delta T$

$$\Rightarrow \frac{v^2}{4} = c_{\text{bullet}}\Delta T$$

where $c_{\text{bullet}} = (0.03)(4200) \text{ Jkg}^{-1}\text{K}^{-1}$

$$\Rightarrow \frac{(300)^2}{4} = (0.03)(4200)\Delta T$$

$$\Rightarrow \Delta T = 178 \text{ } ^\circ\text{C} = 178 \text{ K}$$

16. Heat supplied in 4 min is

$$Q = 100 \times 4 \times 60 = 24000 \text{ cal}$$

$$\Rightarrow 100 \times 0.215 \times (T + 20) + 200 \times 0.5 \times 20 +$$

$$200 \times 80 + 200 \times 1 \times T = 2400$$

$$\Rightarrow 221.5T = 2400 - 16000 - 2000 = 6000$$

$$\Rightarrow T = 27.08 \text{ } ^\circ\text{C}$$

17. $dQ = mcdT$

$$\Rightarrow \frac{dQ}{dt} = mc \left(\frac{dT}{dt} \right)$$

If heat lost per second to the surroundings is q , then

$$135 = \left(\frac{5}{1000} \right) c_w(5) + q \quad \dots(1)$$

$$\text{Also, } 240 = \left(\frac{10}{1000} \right) c_w(5) + q \quad \dots(2)$$

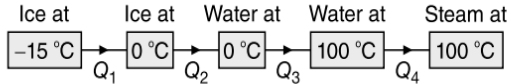
From (1) and (2), we get

$$105 = \left(\frac{25}{1000} \right) c_w$$

$$\Rightarrow c_w = 4200 \text{ Jkg}^{-1}\text{K}^{-1}$$

H.80 JEE Advanced Physics: Waves and Thermodynamics

18. $Q_1 = mc_{ice}(T_f - T_i) = (8)(0.5)[0 - (-15)] = 60 \text{ cal}$
 $Q_2 = mL_f = (8)(80) = 640 \text{ cal}$
 $Q_3 = mc_{water}(T_f - T_i) = (8)(1)(100 - 0) = 800 \text{ cal}$
 $Q_4 = mL_v = (8)(540) = 4320 \text{ cal}$



So, net heat required is

$$Q = Q_1 + Q_2 + Q_3 + Q_4 = 5820 \text{ cal}$$

19. Heat required to raise temperature of ice to 100 °C is

$$Q_1 = m_1L_1 + m_1c_1\Delta T_1 = 5 \times 80 + 5 \times 1 \times 100$$

$$\Rightarrow Q_1 = 400 + 500 + 900 = 1800 \text{ cal}$$

Heat liberated on condensation of steam is

$$Q_2 = m_2L_2 = 5 \times 536 = 2680 \text{ cal}$$

Since, $Q_2 > Q_1$

This simply means that the entire steam is not condensed and hence temperature of mixture will remain at 100 °C

20. Let calorimeter contains x gram of ice and $(200 - x)$ gram of water.

By Law of Calorimetry, the heat lost by $330 - (100 + 200) = 30$ g of steam equals the heat gained by 100 g calorimeter plus heat gained by x gram ice plus heat gained by $(200 - x)$ gram of water

$$\Rightarrow 30L_{steam} + 30s_{water}(100 - 50) = 100s_{Cu}(50) +$$

$$xL_{ice} + xs_{water}(50) + (200 - x)s_{water}(50)$$

$$\Rightarrow x = 88.4 \text{ g}$$

So, the desired ratio is

$$\frac{x}{200 - x} = \frac{88.4}{200 - 88.4} \approx 0.8$$

Test Your Concepts-III

(Based on Kinetic Theory of Gases and Ideal Gas Equation)

1. Initially let each bulb contain n moles of the gas, so two bulbs contains $2n$ moles. If the volume of each bulb is V then, at NTP, i.e., $P = 76$ cm of Hg and $T = 273$ K, we have

$$\frac{76 \times V}{273} = nR \quad \dots(1)$$

Further, $\frac{PV}{373} = n_1R$ and $\frac{PV}{273} = n_2R$

$$\Rightarrow PV \left(\frac{1}{373} + \frac{1}{273} \right) = (n_1 + n_2)R = 2nR \quad \dots(2)$$

From (1) and (2), we get

$$P = 87.76 \text{ cm of Hg}$$

2. We use $v_{rms} = \sqrt{\frac{3RT}{M}}$

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{T_1M_2}{T_2M_1}}$$

$$\Rightarrow v_2 = v_1 \sqrt{\frac{T_2M_1}{T_1M_2}} = 300 \sqrt{\frac{2T_1M_1}{T_1(M_1/2)}}$$

$$\Rightarrow v_2 = 600 \text{ ms}^{-1}$$

3. Each electron exerts an impulse of m_0v when it strikes and sticks to the surface. This times the number striking in unit time divided by area is pressure. Hence,

$$P = \frac{10^{15}}{(10^{-3})^2} m_0v = 10^{21} m_0v$$

Substituting the given values, we get

$$P = (9.1 \times 10^{-31})(8 \times 10^7)(10^{21}) = 0.073 \text{ Nm}^{-2}$$

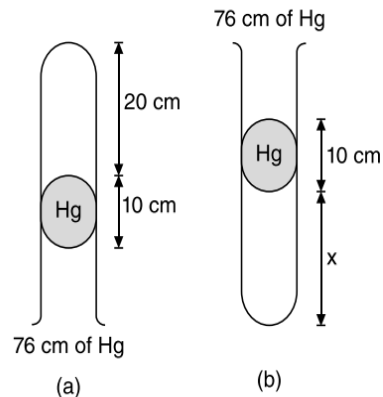
4. Since, initial number of moles equals the final number of moles of the gas, so we have

$$n_1 + n_2 = n$$

$$\Rightarrow \frac{p_1V}{T_1} + \frac{p_2V}{T_2} = \frac{p_f(2V)}{T}$$

$$\Rightarrow p_f = \frac{T}{2} \left[\frac{p_1}{T_1} + \frac{p_2}{T_2} \right]$$

5. The initial and final states of the tube are shown in Figure (a) and (b) respectively.



In the initial state, pressure just below the mercury pallet (at the bottom of pallet) is the atmospheric pressure i.e., 76 cm of Hg and at top of the pallet, 10 cm above the bottom is

$$76 - 10 = 66 \text{ cm of Hg}$$

Finally, when the tube is inverted with its open end upwards, then pressure at the top of pallet is atmospheric pressure i.e., 76 cm of Hg but at the bottom of the pallet, the pressure will be

$$76 + 10 = 86 \text{ cm of Hg}$$

If the length of air column is x , then applying Boyle's Law, we get

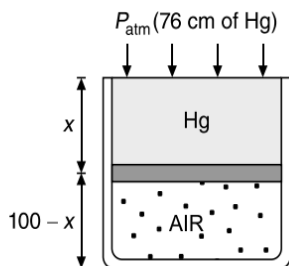
$$P_1V_1 = P_2V_2$$

$$\Rightarrow (66)(20A) = (86)(xA)$$

where, A is the cross-sectional area of tube

$$\Rightarrow x = 15.35 \text{ cm}$$

6. When mercury is poured on the top of the piston, due to increase in pressure, the volume of air will decrease according to Boyle's Law. Since atmospheric pressure is equivalent to the pressure due to a mercury column of height 76 cm, so if final mercury column of height x is poured on the piston, then gas pressure in equilibrium is given by



$$P_f = (76 + x) \text{ cm of Hg}$$

If A be the area of cross section of cylinder, then we have according to Boyle's Law

$$P_1 V_1 = P_2 V_2$$

$$\Rightarrow (76 \text{ cm})(100A) = (76 + x)(100 - x)A$$

$$\Rightarrow 7600 = 7600 + 24x - x^2$$

$$\Rightarrow x = 24 \text{ cm}$$

7. Moment of inertia, $I = 2(mr^2)$

$$\text{where } m = \frac{70 \times 10^{-3}}{2 \times 6.02 \times 10^{23}} = 5.81 \times 10^{-26} \text{ kg}$$

$$\text{and } r = \frac{2 \times 10^{-10}}{2} = 1 \times 10^{-10} \text{ m}$$

$$\Rightarrow I = 2(5.81 \times 10^{-26})(1 \times 10^{-10})^2 = 1.16 \times 10^{-45} \text{ kgm}^2$$

$$\text{Since, } K_R = \frac{1}{2} I \omega^2 = \frac{1}{2} \times (1.16 \times 10^{-45}) \times (2 \times 10^{12})^2$$

$$\Rightarrow K_R = 2.32 \times 10^{-21} \text{ J}$$



Conceptual Note(s)

At $T = 300 \text{ K}$, rotational K.E. should be equal to $\frac{1}{2} kT$

$$\Rightarrow K_R = \frac{1}{2} \times (1.38 \times 10^{-23}) \times (300) = 2.07 \times 10^{-21} \text{ J}$$

Approximately the same as calculated above.

8. Done in Theory.
9. Since, initial number of moles equals the final number of moles of the gas, so we have

$$n_1 = n_2$$

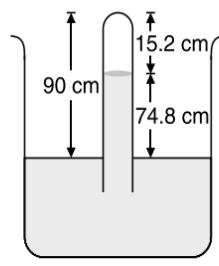
$$\Rightarrow \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\Rightarrow \frac{2 \times 10^5 \times V}{293} = \frac{p \times 1.02V}{313}$$

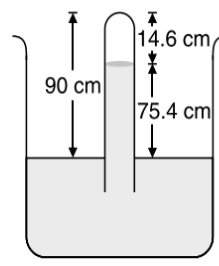
$$\Rightarrow p = 2.095 \times 10^5 \text{ Nm}^{-2}$$

10. Let A be the area of cross section of the tube at 30°C , volume of air be $V_1 = 15.2 \text{ A cm}^3$. The pressure of air will be $P_1 = 76 - 74.8 = 1.2 \text{ cm of Hg}$.

$$\text{At } 10^\circ \text{C}, V_2 = 14.6 \text{ A cm}^3$$



(a)



(b)

$$\text{and } P_2 = (P - 75.4) \text{ cm of Hg}$$

where, P is the true atmospheric pressure

$$\text{Since, } \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\Rightarrow P = 76.57 \text{ cm of Hg}$$

11. Friction force balances the force due to mercury pressure that corresponds to

$$76 - 5 - 69.47 = 1.53 \text{ cm of Hg}$$

$$\text{Hence, } f_r = PA = P(\pi r^2)$$

$$\Rightarrow f_r = (0.0153)(13600)(10) \left[3.14(2 \times 10^{-3})^2 \right]$$

$$\Rightarrow f_r = 0.026 \text{ N}$$

12. Let the initial pressure of the gas be P_1 and the final temperature be P_2 . Then from Ideal Gas Equation, we have

$$P_1 V = n_1 R T$$

$$P_2 V = n_2 R T \text{ where, } T = 273 \text{ K}$$

$$\Rightarrow (P_1 - P_2)V = (n_1 - n_2)RT = \left(\frac{m_1 - m_2}{M} \right) RT$$

$$\Rightarrow (\Delta P)V = \frac{\Delta m}{M} RT \quad \dots(1)$$

In the initial condition (at STP), i.e., at $T = 273 \text{ K}$, we have

$$\frac{RT}{M} = \frac{P_0}{\rho} \quad \dots(2)$$

From equations (1) and (2), we get

$$\Delta m = \frac{\rho V \Delta P}{P_0} = \frac{1.2 \times 0.4 \times 0.24}{1} = 0.1152 \text{ kg}$$

$$\Rightarrow \Delta m = 115.2 \text{ g}$$

13. Since, $P_{H_2} = \left(\frac{30}{2} \right) \times \frac{8.314 \times 300}{30 \times 10^{-3}}$

$$\Rightarrow P_{H_2} = 12.471 \times 10^5 \text{ Pa}$$

$$\text{Also, } P_{O_2} = \frac{160}{32} \times \frac{8.314 \times 300}{10 \times 10^{-3}}$$

$$\Rightarrow P_{O_2} = 12.471 \times 10^5 \text{ Pa}$$

$$\text{Similarly, } P_{N_2} = \frac{70}{28} \times \frac{8.314 \times 300}{20 \times 10^{-3}}$$

$$\Rightarrow P_{N_2} = 3.118 \times 10^5 \text{ Pa}$$

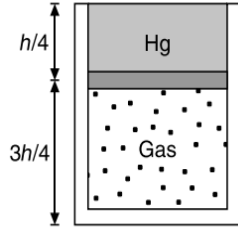
$$\text{Hence, } P_{\text{left}} = P_{H_2} = 12.471 \times 10^5 \text{ Pa}$$

$$P_{\text{middle}} = P_{H_2} + P_{O_2} + P_{N_2} = 28.06 \times 10^5 \text{ Pa}$$

$$\text{and } P_{\text{right}} = P_{H_2} + P_{N_2} = 15.589 \times 10^5 \text{ Pa}$$

H.82 JEE Advanced Physics: Waves and Thermodynamics

14. The final state of gas after half of mercury overflows is shown in Figure.



Let this temperature be T , then for ideal gas equation applied for initial and final state of gas, we get

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \dots(1)$$

The initial pressure, volume and temperature of gas are

$$P_1 = 76 + \frac{h}{2} = 76 + 76 = 152 \text{ cm of Hg}$$

$$V_1 = A \left(\frac{h}{2} \right) = 76A \text{ and } T_1 = 300 \text{ K}$$

Similarly after heating, the pressure, volume and temperature of gas are

$$P_2 = 76 + \frac{h}{4} = 76 + 38 = 114 \text{ cm of Hg}$$

$$V_2 = A \left(\frac{3h}{4} \right) = 114A \text{ and } T_2 = ?$$

$$\Rightarrow \frac{(152)(76A)}{300} = \frac{(114)(114A)}{T_2} \quad \text{\{from (1)\}}$$

$$\Rightarrow T_2 = 337.5 \text{ K}$$

15. Since, $P = P_0 e^{-\left(\frac{gM}{RT}\right)h}$ (when $T = \text{constant}$)

Since, $PV = RT$

$$\Rightarrow P \left(\frac{M}{\rho} \right) = RT$$

$$\Rightarrow \rho = \frac{PM}{RT}$$

$$\Rightarrow \rho \propto P$$

$$\Rightarrow \rho = \rho_0 e^{-\left(\frac{gM}{RT}\right)h}$$

$$\Rightarrow h = - \left(\frac{RT}{gM} \right) \log_e \left(\frac{\rho}{\rho_0} \right)$$

- (a) When, $\frac{\rho_0}{\rho} = e$ (∵ $\rho_0 > \rho$)

$$\text{then, } h = - \frac{RT}{Mg} \log_e (e^{-1}) = \frac{RT}{Mg}$$

$$\Rightarrow h = \frac{(8.31)(273)}{(28 \times 10^{-3})(9.8)} = 8.26 \times 10^3 \text{ m}$$

$$\Rightarrow h = 8.26 \text{ km}$$

- (b) When, $\frac{\rho}{\rho_0} = 0.98$

$$\Rightarrow h = - \frac{RT}{Mg} \log_e (0.98) \approx 0.09 \text{ km}$$

16. Centre of gravity of gas is given by $h_c = \frac{\int y dm}{\int dm}$

where dm is mass of a layer of width dy at a height y . If A be the base area of the vessel, then $dm = (A dy) \rho$

Since for uniform g and T , we have $\rho = \rho_0 e^{-(Mg/RT)y}$ where ρ_0 is density at base of the vessel

$$\Rightarrow h_c = \frac{\int_0^{\infty} A (\rho_0 e^{-(Mg/RT)y}) y dy}{\int_0^{\infty} A (\rho_0 e^{-(Mg/RT)y}) dy}$$

$$\Rightarrow h_c = \frac{\left[\frac{y e^{-(Mg/RT)y}}{(-Mg/RT)} - \frac{e^{-(Mg/RT)y}}{(-Mg/RT)^2} \right]_0^{\infty}}{\left[\frac{e^{-(Mg/RT)y}}{-Mg/RT} \right]_0^{\infty}}$$

$$\Rightarrow h_c = \frac{0 - (RT/Mg)^2}{(-RT/Mg)} = \frac{RT}{Mg}$$

17. Since, $\frac{v_{rms}}{v_{av}} = \sqrt{\frac{3\pi}{8}}$

$$\Rightarrow v_{rms} = \left(\sqrt{\frac{3\pi}{8}} \right) v_{av} = \left(\sqrt{\frac{3\pi}{8}} \right) (4.25 \times 10^2)$$

$$\Rightarrow v_{rms} = 4.613 \times 10^2 \text{ ms}^{-1}$$

Average kinetic energy is

$$\frac{1}{2} m v_{rms}^2 = \frac{1}{2} \times (5.28 \times 10^{-26}) (4.613 \times 10^2)^2$$

$$\Rightarrow \langle KE \rangle = 5.62 \times 10^{-21} \text{ J}$$

18. If vessel volume is V then partial pressures of N_2 and CO_2 are

$$P_{N_2} = \left(\frac{7}{28} \right) \frac{(8.314)(290)}{V}$$

$$\text{and } P_{CO_2} = \left(\frac{11}{44} \right) \frac{(8.314)(290)}{V}$$

Given that $P_1 + P_2 = 10^5$

$$\Rightarrow \frac{8.314}{V} \left(\frac{290}{4} + \frac{290}{4} \right) = 10^5$$

$$\Rightarrow V = 1.205 \times 10^{-2} \text{ m}^3$$

So, density of mixture is

$$\rho = \frac{m_1 + m_2}{V} = \frac{18 \times 10^{-3}}{1.205 \times 10^{-2}} = 1.494 \text{ kgm}^{-3}$$

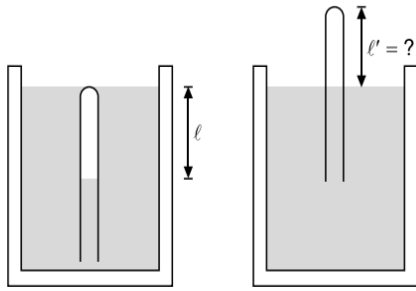
19. $P = P_{N_2} + P_N + P_{He}$

$$\Rightarrow P = \left(\frac{1.4 \times 0.7}{28} + \frac{1.4 \times 0.3}{14} + \frac{0.4}{4} \right) \frac{RT}{V}$$

$$\Rightarrow P = \frac{(0.035 + 0.03 + 0.1)(8.31)(1500)}{5 \times 10^{-3}}$$

$$\Rightarrow P = 4.1 \times 10^5 \text{ Nm}^{-2}$$

20. Since, $P_1V_1 = P_2V_2$



$$\Rightarrow (76 + 10) \times 10 = 76 \times l'$$

$$\Rightarrow l' = 11.315 \text{ cm}$$

$$\text{Mass of air is } m = \frac{PVM}{RT}$$

$$\Rightarrow m = \frac{(1.013 \times 10^5) \left(\frac{11.315}{100} \times 10^{-4} \right) (29 \times 10^{-3})}{8.314 \times 300}$$

$$\Rightarrow m = 13.32 \text{ mg}$$

21. Let v_i for $i = 1, 2, \dots, N$ represent the various speeds. Then,

$$v_{av} = \frac{1}{N} \sum_{i=1}^N v_i \text{ and } v_{rms}^2 = \frac{1}{N} \sum_{i=1}^N v_i^2$$

$$\text{Let us consider the expression } A = \frac{1}{N} \sum_{i=1}^N (v_i - v_{av})^2$$

where, A is inherently a non-negative quantity.

$$\Rightarrow A = \frac{1}{N} \sum_{i=1}^N v_i^2 - \frac{2v_{av}}{N} \sum_{i=1}^N v_i + \frac{1}{N} (Nv_{av}^2)$$

$$\Rightarrow A = v_{rms}^2 - 2v_{av}^2 + v_{av}^2$$

$$\Rightarrow A = v_{rms}^2 - v_{av}^2$$

Since, $A \geq 0$

$$\Rightarrow v_{rms}^2 \geq v_{av}^2$$

$$\Rightarrow v_{rms} \geq v_{av}$$

22. At a distance x from open end of the cylinder, we consider an elemental layer of gas width dx . If pressure difference across this layer is dp , then we have

$$dF = Adp = (dm)x\omega^2 = (\rho A dx)(x\omega^2)$$

$$\Rightarrow dp = \left(\frac{\rho M}{RT} \right) (x\omega^2) dx \quad \left\{ \because \rho = \frac{PM}{RT} \right\}$$

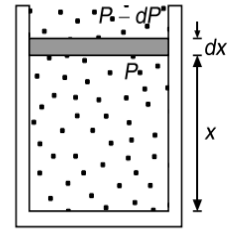
$$\Rightarrow \int_{p_0}^p \frac{dp}{p} = \frac{M\omega^2}{RT} \int_0^r x dx$$

$$\Rightarrow \ln \left(\frac{p}{p_0} \right) = \frac{M\omega^2 r^2}{2RT}$$

$$\Rightarrow p = p_0 e^{\frac{M\omega^2 r^2}{2RT}}$$

23. Consider a gas layer of width dx at a height x above the bottom of container as shown in Figure.

If P is the pressure just below this layer, then just above this layer, we can consider pressure to be $P - dP$, where dP is the pressure due to the gas layer of width dx and is given by



$$dP = -(dx)\rho g \quad \dots(1)$$

where, ρ is the density of gas which is constant.

$$\text{Since, } P = \frac{\rho RT}{M}$$

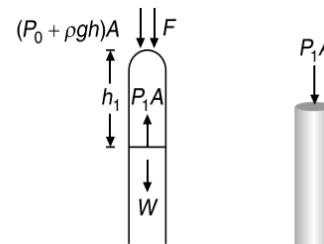
$$\Rightarrow dP = \frac{\rho R}{M} dT \quad \dots(2)$$

From equations (1) and (2), we get

$$\frac{\rho R}{M} dT = -(dx)\rho g$$

$$\Rightarrow \text{Temperature Gradient} = \frac{dT}{dx} = -\frac{Mg}{R}$$

24. In equilibrium, net downward force on the tube is F , $(P_0 + \rho gh)A$, W and upward force is P_1A , where P_1 is the pressure of air in the tube after submergence.



$$\Rightarrow F + (P_0 + \rho gh)A + W = P_1A$$

$$\Rightarrow F = P_1A - (P_0 + \rho gh)A - W \quad \dots(1)$$

Since temperature is constant, so according to Boyle's Law, we have

$$P_0(\ell A) = P_1(h_1 A)$$

$$\Rightarrow P_0\ell = P_1h_1 \quad \dots(2)$$

$$\text{Further, } P_1 = P_0 + \rho g(h + h_1) \quad \dots(3)$$

Solving the above three equations, for F , P_1 and h_1 , we get

$$F = 8.5 \times 10^{-2} \text{ N}$$

25. (a) For a layer of width dy at a height y , we use

$$dp = -(dy)\rho g = -(dy) \left(\frac{pM}{RT} \right) g$$

$$\Rightarrow \int_{p_0}^p \frac{dp}{p} = -\frac{Mg}{RT_0} \int_0^h \frac{dy}{1 - ay}$$

$$\Rightarrow \ln \frac{p}{p_0} = \frac{Mg}{RT_0} \ln(1 - ah)$$

$$\Rightarrow p = p_0 (1 - ah)^{Mg/aRT_0}$$

Above relation is valid if $h < \frac{1}{a}$

H.84 JEE Advanced Physics: Waves and Thermodynamics

(b) Similarly, we have

$$dp = -(dy)\rho g = -(dy)\left(\frac{pM}{RT}\right)g$$

$$\Rightarrow \int_{p_0}^p \frac{dp}{p} = -\frac{Mg}{RT_0} \int_0^h \frac{dy}{(1+ay)}$$

$$\Rightarrow \ln \frac{p}{p_0} = -\frac{Mg}{aRT_0} \ln(1+ah)$$

$$\Rightarrow p = \frac{p_0}{(1+ah)^{Mg/aRT_0}}$$

Test Your Concepts-IV (Based on Internal Energy, Degrees of Freedom and Molar Specific Heats for Ideal Gases)

1. Since $U_{\text{mixture}} = U_1 + U_2$

$$\Rightarrow (n_1 + n_2)\left(\frac{3}{2}RT\right) = n_1\left(\frac{3}{2}RT_1\right) + n_2\left(\frac{3}{2}RT_2\right)$$

$$\Rightarrow T = \frac{n_1T_1 + n_2T_2}{n_1 + n_2}$$

2. Let the gas have f degrees of freedom, then three degrees of freedom are associated with translational motion and the remaining $f - 3$ degrees of freedom are associated with rotational motion. So, we have

$$\frac{K_T}{K_R} = \frac{3(nRT/2)}{(f-3)(nRT/2)} = \frac{3}{2}$$

$$\Rightarrow \frac{3}{f-3} = \frac{3}{2}$$

$$\Rightarrow 6 = 3f - 9$$

$$\Rightarrow f = 5$$

$$\Rightarrow U = \left(\frac{nf}{2}\right)RT = \left(1 \times \frac{5}{2}\right)(8.3)(100) = 2075 \text{ J}$$

3. (a) Since, $TKE = U = n\left(\frac{3}{2}RT\right)$

where, $n = 2$ mol and $T = 293$ K, so

$$TKE = \frac{3}{2}(2)(8.31)(293) = 7.3 \times 10^3 \text{ J}$$

(b) The average kinetic energy per molecule is $\frac{3}{2}k_B T$

$$\Rightarrow \frac{1}{2}m\langle v^2 \rangle = \frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}k_B T$$

$$\Rightarrow \frac{1}{2}m\langle v^2 \rangle = \frac{3}{2}(1.38 \times 10^{-23})(293)$$

$$\Rightarrow \frac{1}{2}m\langle v^2 \rangle = 6.07 \times 10^{-21} \text{ J}$$

4. (a) An oxygen molecule has three translational degrees of freedom, thus the average translational kinetic energy of an oxygen molecule at 27°C is

$$E_T = \frac{3}{2}k_B T = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300$$

$$\Rightarrow E_T = 6.21 \times 10^{-21} \text{ Jmole}^{-1}$$

(b) An oxygen molecule has total five degrees of freedom, hence its total kinetic energy is

$$E_T = \frac{5}{2}k_B T = \frac{5}{2} \times 1.38 \times 10^{-23} \times 300$$

$$\Rightarrow E_T = 10.35 \times 10^{-21} \text{ Jmole}^{-1}$$

(c) Total kinetic energy of one mole of oxygen is its internal energy, which can be given as

$$U = \frac{5}{2}nRT = \frac{5}{2} \times 1 \times 8.314 \times 300$$

$$\Rightarrow U = 6235.5 \text{ Jmole}^{-1}$$

5. Helium is a monoatomic gas so, internal energy of n moles of the gas is $U = \frac{3}{2}nRT$

$$\Rightarrow \Delta U = \frac{3}{2}nR(\Delta T) = \left(\frac{3}{2}\right)(3)(8.31)(2) = 74.8 \text{ J}$$

6. Since, the number of moles remains constant, so, we have

$$\Sigma n_{\text{initial}} = \Sigma n_{\text{final}}$$

From ideal gas equation, we get

$$\frac{p_1V}{T_1} + \frac{p_2V}{T_2} = \frac{p'_1V}{T_f} + \frac{p'_2V}{T_f} \quad \dots(1)$$

Applying conservation of energy, we get

$$\frac{f}{2}p_1V + \frac{f}{2}p_2V = \frac{f}{2}p'_1V + \frac{f}{2}p'_2V \quad \dots(2)$$

$$\Rightarrow p'_1 + p'_2 = p_1 + p_2$$

From equations (1) and (2), we get

$$\frac{p_1}{T_1} + \frac{p_2}{T_2} = \frac{p_1 + p_2}{T_f}$$

$$\Rightarrow T_f = \frac{T_1T_2(p_1 + p_2)}{p_1T_2 + p_2T_1}$$

For left and right part of gas, we have

$$\frac{p_1}{T_1} = \frac{p'_1}{T_f} \text{ and } \frac{p_2}{T_2} = \frac{p'_2}{T_f}$$

$$\Rightarrow p'_1 = \frac{p_1T_2(p_1 + p_2)}{p_1T_2 + p_2T_1} \text{ and } p'_2 = \frac{p_2T_1(p_1 + p_2)}{p_1T_2 + p_2T_1}$$

Heat supplied from the left part is $Q = U_i - U_f$

$$\Rightarrow Q = \frac{f}{2}p_1V - \frac{f}{2}p'_1V = \frac{fV}{2} \left(p_1 - \frac{p_1T_2(p_1 + p_2)}{p_1T_2 + p_2T_1} \right)$$

$$\Rightarrow Q = \frac{fV}{2} \left(\frac{p_1p_2T_2 - p_1p_2T_2}{p_1T_2 + p_2T_1} \right)$$

Since the gas is monatomic, so $f = 3$

$$\Rightarrow Q = \frac{3p_1p_2V(T_2 - T_1)}{2(p_1T_2 + p_2T_1)}$$

7. Since, $U_{\text{mixture}} = U_1 + U_2$

$$\Rightarrow (n_1 + n_2)C_{V_{\text{mix}}}T = n_1C_{V_1}T + n_2C_{V_2}T$$

$$\Rightarrow C_{V_{\text{mix}}} = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2}$$

Since, for O_2 , $C_{V_1} = \frac{5R}{2}$ and for CO_2 , $C_{V_2} = \frac{7R}{2}$. So, we get

$$C_{V_{mix}} = \frac{(2)\left(\frac{5R}{2}\right) + 3\left(\frac{7R}{2}\right)}{2+3} = \left(\frac{31}{12}\right)R$$

Since, $C_{P_{mix}} - C_{V_{mix}} = R$

$$\Rightarrow C_{P_{mix}} = \left(\frac{31}{12} + 1\right)R = \left(\frac{43}{12}\right)R$$

$$\text{So, } \gamma_{mix} = \frac{C_{P_{mix}}}{C_{V_{mix}}} = \frac{43}{31} \approx 1.4$$

8. Rotational kinetic energy of a diatomic molecule is

$$E = 2\left(\frac{1}{2}k_B T\right) = \frac{1}{2}I\omega_{rms}^2$$

$$\Rightarrow \omega_{rms} = \sqrt{\frac{2k_B T}{I}}$$

$$\Rightarrow \omega_{rms} = \sqrt{\frac{2 \times 1.38 \times 10^{-23} \times 300}{2.1 \times 10^{-39}}} \approx 2 \times 10^9 \text{ rads}^{-1}$$

9. The disordered kinetic energy of the gas converts into the internal energy of its molecules, so

$$\frac{1}{2}mv^2 = nC_V \Delta T = \left(\frac{m}{M}\right)\left(\frac{R}{\gamma-1}\right)\Delta T \quad \left\{ \because C_V = \frac{R}{\gamma-1} \right\}$$

$$\Rightarrow v = \sqrt{\frac{2R\Delta T}{M(\gamma-1)}} \quad \dots(1)$$

Kinetic Energy of the system is $K = K_{gas} + K_{molecules}$

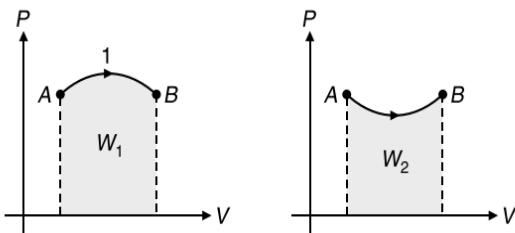
$$\Rightarrow K = \frac{1}{2}Mv^2 + \frac{1}{2}(N_A m)v^2 = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2$$

$$\Rightarrow K = \frac{1}{2}(2M)v^2 = Mv^2$$

$$\Rightarrow K = \frac{2R\Delta T}{\gamma-1} \quad \{\text{from (1)}\}$$

Test Your Concepts-V (Based on Work Done and First Law of Thermodynamics)

1. From FLTD, we have $Q_1 = W_1 + \Delta U_1$ and $Q_2 = W_2 + \Delta U_2$
Since, U is a state function so ΔU depends only on the initial and final positions. Therefore, $\Delta U_1 = \Delta U_2$



Now, since the area under 1 is greater than area under 2. So $W_1 > W_2$ and hence $Q_1 > Q_2$

2.
$$W = \int_{V_1}^{V_2} P dV = P(V_2 - V_1) = P\left(\frac{\alpha(2T_0)^2}{P} - \frac{\alpha(T_0)^2}{P}\right) = 3\alpha T_0^2$$

3. For a cyclic process, no change in internal energy takes place, so heat supplied is equal to the work done by the gas in one complete cycle. Hence

$$Q = \Delta U + W = 0 + W = W$$

Also, work done W by the gas equals the positive area enclosed by the ellipse, so

$$Q = \text{Area of ellipse} = \pi ab$$

where, a and b are semi major and semi-minor axis of the ellipse. From the figure, we have

$$2a = 2 \times 10^5 \text{ Nm}^{-2} \text{ and } 2b = 200 \times 10^{-6} \text{ m}^3$$

$$\Rightarrow Q = W = \pi \times 1.0 \times 10^5 \times 100 \times 10^{-6}$$

$$\Rightarrow Q = 3.14 \times 10 = 31.4 \text{ J}$$

4. Since work done equals the area enclosed between PV curve and the V axis, so we have

$$W_{AC} = \frac{1}{2}(540)(0.3) + (340)(1) = 421 \text{ kJ}$$

Similarly, we have

$$W_{AD} = 421 + \frac{1}{2}(440)(0.6) = 553 \text{ kJ}$$

5. According to FLTD, we have $Q = \Delta U + W$

where, $\Delta U = U_f - U_i$

For Process 1

$$Q = 35 \text{ J}, W = -15 \text{ J}$$

$$\Rightarrow 35 = \Delta U - 15$$

$$\Rightarrow \Delta U = U_f - U_i = 50 \text{ J}$$

$$\Rightarrow -10 - U_i = 50 \text{ J}$$

$$\Rightarrow U_i = -60 \text{ J}$$

For Process 2

$$Q = -15 \text{ J}, \Delta U = -20 \text{ J}$$

$$\Rightarrow -15 = -20 + W$$

$$\Rightarrow W = 5 \text{ J}$$

$$\text{Since, } \Delta U = U_f - U_i = -20 \text{ J}$$

$$\Rightarrow 60 - U_i = -20 \text{ J}$$

$$\Rightarrow U_i = 80 \text{ J}$$

For Process 3

$$W = -20 \text{ J}, \Delta U = 40 \text{ J}$$

$$\Rightarrow Q = 40 - 20 = 20 \text{ J}$$

$$\text{Since, } \Delta U = U_f - U_i = 40 \text{ J}$$

$$\Rightarrow U_f - 80 = 40 \text{ J}$$

$$\Rightarrow U_f = 120 \text{ J}$$

Process	Q	W	U_i	U_f	ΔU
1.	35	-15	-60	-10	50
2.	-15	5	80	60	-20
3.	20	-20	80	120	40

H.86 JEE Advanced Physics: Waves and Thermodynamics

6. According to FLTD, we have $Q = \Delta U + W = (U_F - U_I) + W$ where, U_I and U_F are the internal energies in the initial and the final state.

For path IAF, $Q = 55$ J and $W = 25$ J.

$$\Rightarrow \Delta U = U_F - U_I = Q - W = 55 - 25 = 30 \text{ J}$$

Now, we know that the internal energy is path independent i.e., it depends only on the initial and final states of the system. So, internal energy between I and F irrespective of the path followed by the system between I and F is 30 J

- (a) For path IBF, $Q = 35$ J and $\Delta U = 30$ J

$$\Rightarrow W = Q - \Delta U = 35 - 30 = 5 \text{ J}$$

i.e., work is being done by the system.

- (b) For path FI, we have

$$W = -15 \text{ J}, \Delta U = U_I - U_F = -30 \text{ J}$$

$$\Rightarrow Q = W + \Delta U = -15 - 30 = -45 \text{ J}$$

i.e., heat is lost by the system.

- (c) Given, $U_I = 10$ J and $\Delta U = U_F - U_I$

$$\Rightarrow U_F = \Delta U + U_I = 30 + 10 = 40 \text{ J}$$

- (d) The process BF is isochoric, i.e., the volume is constant, so $W = 0$.

$$\Rightarrow Q = \Delta U_{BF} = U_F - U_B = 40 - 20 = 20 \text{ J}$$

The process IB is isobaric (constant pressure). Therefore,

$$Q = Q_{IBF} - Q_{BF} = 35 - 20 = 15 \text{ J}$$

7. Change in internal energy of gas

$$\Delta U = \frac{3}{2}(P_B V_B - P_A V_A)$$

$$\Rightarrow \Delta U = \frac{3}{2}(8 \times 10^5 \times 0.8 - 4 \times 10^5 \times 0.5)$$

$$\Rightarrow \Delta U = 6.6 \times 10^5 \text{ J}$$

Work done by gas is equal to area under the PV diagram, so we have

$$W = \frac{1}{2}(12 \times 10^5)(0.3) = 1.8 \times 10^5 \text{ J}$$

By FLTD, we have $Q = \Delta U + W$

$$\Rightarrow Q = 8.4 \times 10^5 \text{ J}$$

8. Work done is given by

$$W = \underbrace{(\text{Area } 021)}_{\text{Clockwise}} - \underbrace{(\text{Area } 043)}_{\text{Counter Clockwise}}$$

$$W = \frac{1}{2}(V_2 - V_1)(P_0 - P_1) - \frac{1}{2}(V_4 - V_3)(P_2 - P_0)$$

Since by similarity, we have

$$\frac{V_4 - V_3}{V_2 - V_1} = \frac{P_2 - P_0}{P_0 - P_1}$$

$$\Rightarrow V_4 - V_3 = \left(\frac{P_2 - P_0}{P_0 - P_1} \right) (V_2 - V_1)$$

$$\Rightarrow V_4 - V_3 = \left(\frac{10^5}{2 \times 10^5} \right) (100 \times 10^{-3}) \text{ m}^3$$

$$\Rightarrow V_4 - V_3 = 50 \times 10^{-3} \text{ m}^3$$

$$W = \frac{1}{2}(100 \times 10^{-3})(2 \times 10^5) - \frac{1}{2}(50 \times 10^{-3})(10^5)$$

$$\Rightarrow W = 10^4 - 25 \times 10^2 = 7500 \text{ J}$$

9. Work done on the gas is the area bound by the semi-circle given by

$$W = \frac{\pi(1)(1)}{2} = \frac{\pi}{2} \text{ atm} \cdot \text{lt}$$

10. Work done are

$$W_{A \rightarrow B} = 8 \times (3 \times 10^5) \times 10^{-3} = 2400 \text{ J}$$

$$W_{B \rightarrow C} = 0$$

$$W_{C \rightarrow D} = -2 \times 3 \times 10^5 \times 10^{-3} = -600 \text{ J}$$

$$W_{D \rightarrow A} = 0$$

Total work is

$$W_{\text{cycle}} = 2400 - 600 = 1800 \text{ J}$$

Test Your Concepts-VI (Based on Isochoric, Isobaric and Isothermal Processes)

1. Since helium is a monatomic gas. Therefore,

$$C_V = \frac{3R}{2} \text{ and } C_P = \frac{5R}{2}$$

- (a) At constant volume, we have

$$Q = \Delta U = nC_V \Delta T = (2) \left(\frac{3R}{2} \right) (100)$$

$$\Rightarrow Q = 300R$$

- (b) At constant pressure,

$$Q = nC_P \Delta T = (2) \left(\frac{5R}{2} \right) (100)$$

$$\Rightarrow Q = 500R$$

- (c) At constant pressure, from FLTD, we have

$$W = Q - \Delta U = nC_P \Delta T - nC_V \Delta T$$

$$\Rightarrow W = nR \Delta T = (2)(R)(100)$$

$$\Rightarrow W = 200R$$

2. Process AB is an isothermal process with $T = \text{constant}$ and $P_B > P_A$.

For P-V graph: $P \propto \frac{1}{V}$ i.e., P-V graph is a hyperbola with $P_B > P_A$ and $V_B < V_A$.

For V-T graph: $T = \text{constant}$. Therefore, V-T graph is a straight line parallel to V-axis with $V_B < V_A$.

For ρ -T graph: Since $PV = RT$ where $V = \frac{M}{\rho}$

$$\Rightarrow \rho = \frac{PM}{RT}$$

$$\Rightarrow \rho \propto P$$

As T is constant. Therefore, ρ -T graph is a straight line parallel to ρ -axis with $\rho_B > \rho_A$ as $P_B > P_A$.

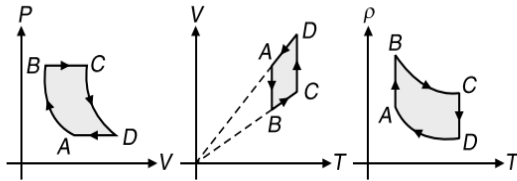
Process BC is an isobaric process with $P = \text{constant}$ and $T_C > T_B$.

For P-V graph: As P is constant. Therefore, P-V graph is a straight line parallel to V-axis with $V_C > V_B$ (because $V \propto T$ in an isobaric process).

For V-T graph: In isobaric process $V \propto T$, i.e., V-T graph is a straight line passing through origin, with $T_C > T_B$ and $V_C > V_B$.

For ρ -T graph: $\rho \propto \frac{1}{T}$ (when $P = \text{constant}$), i.e., ρ -T graph is a hyperbola with $T_C > T_B$ and $\rho_C < \rho_B$.

Similarly, we can repeat for the processes CD and DA. As they are opposite to AB and BC respectively. The corresponding three graphs are shown below.



3. One mole of gas occupies a volume of 22400 cm^3 at 0°C and at pressure of 1 atm. So, the initial volume of the gas is

$$V_1 = 22400 \text{ cm}^3 = 22400 \times 10^{-6} \text{ m}^3$$

$$\Rightarrow V_1 = 0.0224 \text{ m}^3$$

Since pressure is constant equal to atmospheric pressure $P_0 = 1 \text{ atm} = 10^5 \text{ Nm}^{-2}$, so the final volume V_2 is

$$V_2 = V_1 \left(\frac{T_2}{T_1} \right) \quad \left\{ \because \frac{V_1}{T_1} = \frac{V_2}{T_2} \right\}$$

where, $T_1 = 0^\circ \text{C} = 273 \text{ K}$, $T_2 = 100^\circ \text{C} = 373 \text{ K}$

$$\Rightarrow V_2 = 0.0224 \left(\frac{373}{273} \right) \text{ m}^3 = 0.0306 \text{ m}^3$$

Work done in the process is

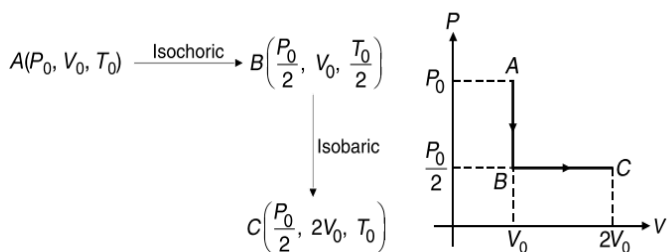
$$W = P(V_2 - V_1) \quad \{ \because P = \text{constant} \}$$

$$\Rightarrow W = 10^5 (0.0306 - 0.0224) = 10^5 (0.0082)$$

$$\Rightarrow W = 820 \text{ J}$$

i.e., work is done by the gas because the gas is expanding.

4. The most important thing to be observed here is that temperature of final state C (i.e., T_0) is equal to the temperature of initial state A (i.e., T_0). So, internal energy $\Delta U_{AC} = 0$



Now for the complete process $Q = \Delta U + W$

$$\Rightarrow Q_{\text{net}} = W_{\text{net}} = W_{\text{isochoric}} + W_{\text{isobaric}}$$

But $W_{\text{isochoric}} = 0$

$$\text{and } W_{\text{isobaric}} = \frac{P_0}{2} (\Delta V_0) = \frac{P_0 V_0}{2} = \frac{nRT_0}{2}$$

$$\Rightarrow W_{\text{net}} = \frac{(2)(8.3)(300)}{2} = 2490 \text{ J}$$

5. $W = P\Delta V = nR\Delta T = 1 \times 8.31 \times 72 \cong 600 \text{ J}$

$$\Delta U = Q - W = (500 \times 4.2 - 6 \times 10^2) = 1500 \text{ J}$$

$$\Rightarrow \Delta U = \frac{nR\Delta T}{\gamma - 1}$$

$$\Rightarrow 1500 = \frac{600}{\gamma - 1}$$

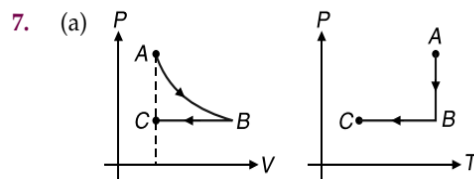
$$\Rightarrow \gamma = 1.4$$

6. At constant pressure work done is

$$W = nR\Delta T = 2 \text{ J}$$

So, heat supplied to diatomic gas is

$$\Delta Q = \frac{7}{2} nR\Delta T = \frac{7}{2} \times 2 = 7 \text{ J}$$



- (b) For the process AB (Isothermal),

$$\frac{P_0 V_0}{T_0} = \frac{P_B (2V_0)}{T_0}$$

$$\Rightarrow P_B = \frac{P_0}{2}$$

Since, for isothermal process, $\Delta U = 0$

$$\Rightarrow Q = W + \Delta U = nRT \log_e \left(\frac{V_B}{V_A} \right)$$

$$\Rightarrow Q = 3RT_0 \log_e (2)$$

For the process BC (Isobaric), $\frac{2V_0}{T_0} = \frac{V_0}{T_C}$

$$\Rightarrow T_C = \frac{T_0}{2}$$

For isobaric process,

$$W = nR\Delta T = 3R \left(\frac{T_0}{2} - T_0 \right) = -\frac{3}{2} RT_0$$

$$Q = nC_p \Delta T = -\frac{21}{4} RT_0$$

$$\Rightarrow W_{\text{Total}} = 3RT_0 \log_e (2) - \frac{3}{2} RT_0$$

$$(c) Q_{\text{Total}} = 3RT_0 \log_e (2) - \frac{21}{4} RT_0$$

8. Let spring be displaced by x towards left, then

$$P_0 (A\ell_0) = (P_0 + \Delta P) A(\ell_0 - x) \quad \{ \because P_1 V_1 = P_2 V_2 \}$$

$$\Rightarrow \Delta P = \frac{P_0 x}{\ell_0} \quad \{ \text{as } x \text{ is small} \}$$

When the spring is displaced by x , then spring force = $-kx$

Net, force against displacement = $-\frac{2P_0 x A}{\ell_0} - kx = -M\omega^2 x$

$$T = 2\pi \sqrt{\frac{M\ell_0}{2P_0 A + k\ell_0}}$$

H.88 JEE Advanced Physics: Waves and Thermodynamics

So, net restoring force is

$$F_{\text{net}} = \left(\begin{array}{c} \text{Force due to} \\ \text{Excess Pressure} \end{array} \right) + \left(\begin{array}{c} \text{Spring} \\ \text{Force} \end{array} \right)$$

$$\Rightarrow F_{\text{net}} = -(\Delta P)A - kx$$

$$\Rightarrow F_{\text{net}} = -\left(\frac{2P_0 x}{\ell_0}\right)A - kx$$

$$\Rightarrow M\ddot{x} + \left(\frac{2P_0 A}{\ell_0} + k\right)x = 0 \quad \{\because F_{\text{net}} = M\ddot{x}\}$$

Comparing with standard equation of SHM, i.e., $\ddot{x} + \omega^2 x = 0$, we get

$$\omega = \sqrt{\frac{1}{M} \left(\frac{2P_0 A}{\ell_0} + k \right)}$$

$$\Rightarrow T = 2\pi \sqrt{\frac{M\ell_0}{2P_0 A + k\ell_0}}$$

9. Given, mass of piston $M = 5 \text{ kg}$, cross-sectional area of piston $A = 5 \times 10^{-3} \text{ m}^2$, $g = 10 \text{ ms}^{-2}$ and atmospheric pressure $P_0 = 10^5 \text{ Nm}^{-2}$.

Initial pressure of gas in the cylinder is $P = P_0 + \frac{Mg}{A}$

$$\Rightarrow P = 10^5 + \frac{5 \times 10}{5 \times 10^{-6}} = 1.1 \times 10^5 \text{ Nm}^{-2}$$

When the piston rises by $x = 0.1 \text{ m}$ at constant pressure, then work done is $W = P_0 \Delta V = P_0 A \Delta x$

$$\Rightarrow W = (10^5)(5 \times 10^{-3})(0.1) = 50 \text{ J}$$

At constant pressure, heat supplied to gas is

$$Q = \Delta U + W = \Delta U + 50$$

Now the piston is clamped, so volume of the gas remains constant during cooling. Hence work done during cooling is zero and $\Delta U' = -\Delta U$ (because the gas is cooled back to initial temperature). So by FLTD, we have

$$Q' = \Delta U' + W' = -\Delta U + 0 = -\Delta U$$

i.e., the heat energy lost during the isochoric cooling process is $Q_{\text{lost}} = |Q'| = \Delta U$

The difference between heat energy added during heating and heat energy lost during cooling is

$$\Delta Q = Q - |Q'| = (\Delta U + 50) - \Delta U = 50 \text{ J}$$

10. Let initial pressure is P_0 and pressure after first stroke is P_1 . Then

$$P_1(V + \Delta V) = P_0 V$$

$$\Rightarrow P_1 = \frac{P_0}{\left(1 + \frac{\Delta V}{V}\right)}$$

Similarly, if P_n be the pressure after n strokes, then

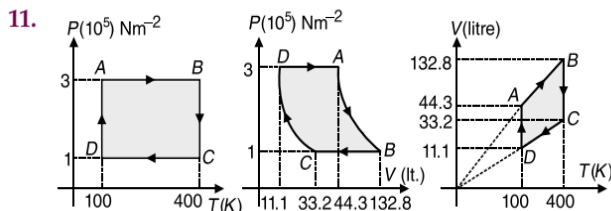
$$P_n = \frac{P_0}{\left(1 + \frac{\Delta V}{V}\right)^n}$$

Since, we want that $P_n = \frac{P_0}{x}$

$$\Rightarrow \frac{P_n}{P_0} = \frac{1}{x} = \frac{1}{\left(1 + \frac{\Delta V}{V}\right)^n}$$

Taking log both sides and solving, we get

$$n = \frac{\log_e(x)}{\log_e\left(1 + \frac{\Delta V}{V}\right)}$$



11. Heat absorbed in an isobaric process is

$$\Delta Q = nC_p \Delta T = \frac{n\gamma R}{\gamma - 1} \Delta T = \frac{\gamma P}{\gamma - 1} (V_2 - V_1)$$

$$\Rightarrow \Delta Q = \frac{1.67}{0.67} \times 10^5 \times 10^{-4} = 24.92 \text{ J}$$

12. Since, $(PV)_A = (PV)_C = 3P_0 V_0$

$$\Rightarrow T_A = T_C$$

$$\Rightarrow \Delta T = 0$$

$$\Rightarrow \Delta U_{ABC} = 0 \quad \{\because \Delta U = nC_V \Delta T\}$$

So, $Q_{ABC} = W_{ABC} = \text{Area under the graph}$

$$\Rightarrow Q_{ABC} = W_{ABC} = -2P_0 V_0 \quad \{\because \text{of compression}\}$$

So, heat is released during the process.

Since, $PV = nRT$

$$\Rightarrow \frac{PM}{\rho} = nRT$$

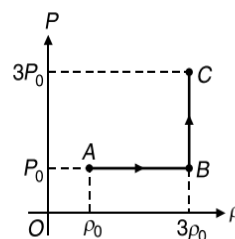
$$\Rightarrow P = \left(\frac{nRT}{M}\right)\rho$$

For AB, $P = P_0 = \text{constant}$ i.e., $V \propto T$

$$\Rightarrow \rho \propto \frac{1}{T}$$

Also, volume decreases from $3V_0$ to V_0 , so temperature decreases from $3T_0$ to T_0 and hence ρ increases from ρ_0 to $3\rho_0$.

For B to C, $V = \text{constant}$ and P increases from P_0 to $3P_0$.



14. As the piston is displaced slowly the change in kinetic energy is zero. From Work Energy Theorem, we have

$$W_{\text{atm}} + W_{\text{gas}} + W_{\text{ext}} = \Delta KE = 0 \quad \dots(1)$$

where, $W_{\text{atm}} = - \int_{V_0}^{nV_0} P_0 dV = -P_0 V_0 (n-1)$

Also, $W_{\text{gas}} = \int_{V_0}^{nV_0} PdV = RT \int_{V_0}^{nV_0} \frac{dV}{V}$ $\{\because PV = nRT\}$

$$\Rightarrow W_{\text{gas}} = RT \log_e n$$

From (1), we get $W_{\text{ext}} = -W_{\text{atm}} - W_{\text{gas}}$

$$\Rightarrow W_{\text{ext}} = -(-P_0 V_0 (n-1) + RT \log_e n)$$

$$\Rightarrow W_{\text{ext}} = RT((n-1) - \log_e n)$$

15. (a) At constant pressure $W = P\Delta V = nR\Delta T$ (as $PV = nRT$)

$$\Rightarrow W = nR\Delta T = 2 \times 2 \times (35 - 30) = 20 \text{ cal}$$

(b) $\Delta U = Q_p - W$

$$\Rightarrow \Delta U = 70 - 20 = 50 \text{ cal}$$

Test Your Concepts-VII (Based on Adiabatic Process)

1. In adiabatic process $\frac{dP}{dV} = -\gamma \left(\frac{P_0}{V_0} \right)$

$$\Rightarrow dP = -\gamma \left(\frac{P_0}{V_0} \right) dV$$

When the piston is displaced towards right by a small distance x , then $dV = Ax$

So, net restoring force is

$$F = (2dP)A = -2\gamma \left(\frac{P_0}{V_0} \right) A^2 x$$

Since, $F \propto -x$

Hence the motion of piston is simple harmonic in nature

So, acceleration a is given by

$$a = \frac{F}{m} = - \left(\frac{2\gamma P_0 A^2}{V_0 m} \right) x$$

$$\Rightarrow T = 2\pi \sqrt{\frac{|x|}{|a|}} = 2\pi \sqrt{\frac{mV_0}{2\gamma A^2 P_0}}$$

2. In case of isothermal expansion, the work done is given by

$$W_1 = W_{\text{isot}} = nRT \ln \left(\frac{V_2}{V_1} \right)$$

where, $T = 27^\circ\text{C} = 273 + 27 = 300^\circ\text{K}$, $n = 1$, $\frac{V_2}{V_1} = 2$ and $R = \frac{25}{3} \text{ Jmol}^{-1}\text{K}^{-1}$

$$\Rightarrow W_1 = (25/3) \times 300 \times \ln(2)$$

$$\Rightarrow W_1 = (1)(25/3)(300)(0.7) = 1750 \text{ J}$$

Now the gas is compressed adiabatically to its original volume. Initially at the beginning of adiabatic compression, the temperature of the gas is 300 K and at the end of adiabatic compression, the temperature becomes T_2 because the temperature is changed. Now, the initial volume of the gas is $2V_1$ and after compression it again becomes the original volume i.e., V_1 . For an adiabatic process

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\Rightarrow 300(2V_1)^{\gamma-1} = T_2 (V_1)^{\gamma-1}$$

$$\Rightarrow 300(2)^{1.4-1} = T_2 (1)^{1.4-1}$$

$$\Rightarrow T_2 = 300(2)^{0.4} = (300)(1.3) = 390 \text{ K}$$

Work done during adiabatic process is

$$W_2 = W_{\text{ad}} = \frac{nR}{1-\gamma} (T_2 - T_1)$$

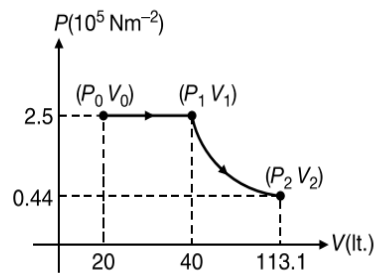
$$\Rightarrow W_2 = \frac{(1)(25/3)}{(-0.4)} (390 - 300) = -1875 \text{ J}$$

Total work done is given by

$$W = W_1 + W_2 = 1750 - 1875 = -125 \text{ J}$$

i.e., the work is done on the gas.

3. (a)



(b) $P_0 = \frac{nRT_0}{V_0} = \frac{2 \times 8.31 \times 300}{20 \times 10^{-3}}$

$$\Rightarrow P_0 = 2.5 \times 10^5 \text{ Nm}^{-2}$$

$$\Rightarrow P_1 = P_0 = 2.5 \times 10^5 \text{ Nm}^{-2}$$

$$\text{Also, } V_1 = 2V = 40 \times 10^{-3} \text{ m}^3$$

Process AB: $V \propto T$

$$\Rightarrow T_1 = 2T_0 = 600 \text{ K}$$

Process BC: Using $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$, we get

$$V_2 = 2\sqrt{2}V_1 = 113.1 \times 10^{-3} \text{ m}^3$$

$$\text{and } P_2 = \frac{nRT_2}{V_2} = \frac{(2)(8.31)(300)}{113.1 \times 10^{-3}}$$

$$P_2 = 0.44 \times 10^5 \text{ Nm}^{-2}$$

(c) $W_{\text{total}} = W_1 + W_2$

$$W_{\text{total}} = P_0(V_1 - V_0) + \frac{nR}{\gamma-1} (T_1 - T_2)$$

Substituting the values, we get

$$W_{\text{total}} = 12470 \text{ J}$$

4. Sudden compression implies adiabatic process, so

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\Rightarrow 300(V_1)^{0.5} = T_2 \left(\frac{V_1}{2} \right)^{0.5}$$

$$\Rightarrow T_2 = 300\sqrt{2} = 423 \text{ K}$$

At the end of adiabatic process, pressure is

$$P_2 = P_1 \left(\frac{V_1}{V_1/2} \right)^{1.5} = 2\sqrt{2}P_1 = 2\sqrt{2} \times 10^5 \text{ Pa}$$

H.90 JEE Advanced Physics: Waves and Thermodynamics

Now, for cooling the gas isobarically to 300 K from an initial temperature of $300\sqrt{2}$ K, we have

$$\frac{V_2}{T_2} = \frac{V_3}{T_3}$$

$$\Rightarrow V_3 = V_2 \left(\frac{T_3}{T_2} \right) = \left(\frac{V_1}{2} \right) \left(\frac{300}{300\sqrt{2}} \right) = \frac{V_1}{2\sqrt{2}}$$

Now the gas is isothermally expanded to its original volume, so we have $V_4 = V_1$

Since, $P_3V_3 = P_4V_4$

$$\Rightarrow (2\sqrt{2}P_1) \left(\frac{V_1}{2\sqrt{2}} \right) = P_4V_1$$

$$\Rightarrow P_4 = P_1$$

Work done by gas in first process is

$$W_{\text{adia}} = \frac{P_2V_2 - P_1V_1}{1-\gamma} = \frac{(2\sqrt{2}P_1) \left(\frac{V_1}{2} \right) - P_1V_1}{(-0.5)}$$

$$\Rightarrow W_{\text{adia}} = - \left(\frac{\sqrt{2}-1}{0.5} \right) (10^5)(10^{-3})$$

$$\Rightarrow W_{\text{adia}} = -82 \text{ J}$$

Also, $W_{\text{isob}} = P\Delta V = P_2(V_3 - V_2)$

$$\Rightarrow W_{\text{isob}} = (2\sqrt{2}P_1) \left(\frac{V_1}{2\sqrt{2}} - \frac{V_1}{2} \right)$$

$$\Rightarrow W_{\text{isob}} = P_1V_1(1 - \sqrt{2})$$

$$\Rightarrow W_{\text{isob}} = (1 - \sqrt{2})(10^5)(10^{-3}) = -41 \text{ J}$$

Now, $W_{\text{isot}} = nRT_3 \ln \left(\frac{V_4}{V_3} \right) = P_3V_3 \ln \left(\frac{V_4}{V_3} \right)$

$$\Rightarrow W_{\text{isot}} = (2\sqrt{2}P_1) \left(\frac{V_1}{2\sqrt{2}} \right) \ln \left(\frac{V_1}{V_1/2\sqrt{2}} \right)$$

$$\Rightarrow W_{\text{isot}} = P_1V_1 \ln(2\sqrt{2}) = P_1V_1 \ln(2)^{1.5}$$

$$\Rightarrow W_{\text{isot}} = 1.5(10^5)(10^{-3})(0.7) = 105 \text{ J}$$

Now, $W_{\text{total}} = W_{\text{adia}} + W_{\text{isob}} + W_{\text{isot}}$

$$\Rightarrow W_{\text{total}} = -82 - 41 + 105 = -18 \text{ J}$$

5. For Isothermal process, $\Delta U_1 = 0$

$$\Rightarrow Q_1 = W_1 = RT \log_e \left(\frac{V_f}{V_i} \right) = RT \log_e (2)$$

$$\Rightarrow Q_1 = 300R \log_e (2) \quad \dots(1)$$

For Isochoric process, $W_2 = 0$

$$\Rightarrow Q_2 = \Delta U_2 = C_V \Delta T = \frac{3}{2} R \Delta T \quad \dots(2)$$

Since it is given that $Q_1 = Q_2$

So, from (1) and (2), we get

$$\Delta T = \left(\frac{2}{3} \right) (300) (\log_e 2) = 138.6 \text{ K}$$

That is, final temperature of second vessel is

$$(300 + 138.6) \text{ K} = 438.6 \text{ K}$$

Let P_0 and T_0 be the final pressure and temperature of the mixture and since, same mass of the same gas is being mixed, so we have

$$T_0 = \frac{300 + 438.6}{2} = 369.3 \text{ K}$$

$$\Rightarrow P_0 = \frac{nRT_0}{V} = \frac{2 \times 8.3 \times 369.3}{3 \times 8.3 \times 10^{-3}} = 2.46 \times 10^5 \text{ Nm}^{-2}$$

6. For an adiabatic process, we have

$$P^{1-\gamma} T^\gamma = \text{constant}$$

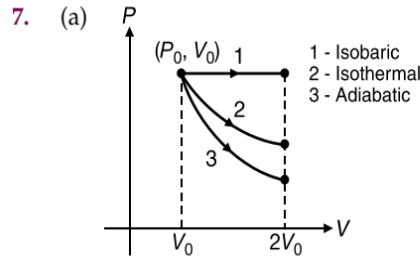
$$\Rightarrow T_2 = T_1 \left(\frac{P_1}{P_2} \right)^{\frac{1-\gamma}{\gamma}} = 300 \left(\frac{1}{10} \right)^{-\frac{2}{7}} = 300(10)^{2/7}$$

$$\Rightarrow T_2 = 300(1.4)^2 = 300(1.96) = 588 \text{ K}$$

Also, $W_{\text{ad}} = \frac{nR}{1-\gamma} (T_2 - T_1)$

$$\Rightarrow W_{\text{ad}} = \frac{(1)(8.3)}{1-1.4} (588 - 300) = -5976 \text{ J}$$

Negative sign implies that work is being done on the system.



Area under graph 3 is least. Therefore, work done is least in adiabatic process

(b) For 1, an Isobaric process, pressure remains constant, so $V \propto T$, so when V_0 becomes $2V_0$, T_0 also becomes $2T_0$.

For 2, an Isothermal process, T is constant while pressure goes from P_0 to $\frac{P_0}{2}$.

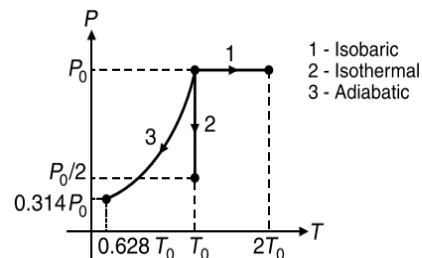
For 3, an Adiabatic equation, the P - T equation is given by

$$PT^{\left(\frac{\gamma}{1-\gamma} \right)} = \text{constant}$$

For a monatomic gas, $\gamma = \frac{5}{3}$

$$\Rightarrow P_0 V_0^{5/3} = P(2V_0)^{5/3}$$

$$\Rightarrow P = 0.314 P_0$$



Also, for an adiabatic process, we have

$$PV^\gamma = \text{constant and } TV^{\gamma-1} = \text{constant}$$

$$\Rightarrow \frac{T}{PV} = \text{constant}$$

$$\Rightarrow T \propto PV$$

$$\Rightarrow \frac{T}{T_0} = \left(\frac{P}{P_0}\right)\left(\frac{V}{V_0}\right)$$

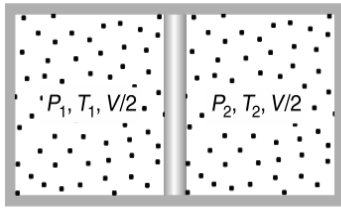
$$\Rightarrow T = (0.314)(2)T_0 = 0.628T_0$$

8. For an adiabatic process, $Q = 0$ i.e., $W = -\Delta U$

Given that $\Delta U = -100$ J, so $W = 100$ J

9. (a) Let T be the final temperature in both the compartments, then from First Law of Thermodynamics applied to the cylinder, we have

$$Q = \Delta U + W$$



Since the cylinder wall is adiabatic and fixed, so

$$Q = W = 0$$

$$\Rightarrow \Delta U = 0$$

So, there is no change in internal energy of the system

$$\Rightarrow n_1 C_V (T - T_1) = n_2 C_V (T_2 - T)$$

$$\Rightarrow \frac{P_1 V}{2RT_1} (T - T_1) = \frac{P_2 V}{2RT_2} (T_2 - T)$$

$$\text{Solving, we get } T = \frac{(P_1 + P_2)T_1 T_2}{P_1 T_2 + P_2 T_1}$$

$$\text{For the left compartment, } \frac{P_1}{T_1} = \frac{P'_1}{T}$$

where, P'_1 is the final pressure in the left compartment

$$\Rightarrow P'_1 = \frac{P_1(P_1 + P_2)T_2}{P_1 T_2 + P_2 T_1}$$

$$\text{Similarly, we get } P'_2 = \frac{P_2(P_1 + P_2)T_1}{P_1 T_2 + P_2 T_1}$$

- (b) Since, $P_2 T_1 > P_1 T_2$, it follows that $P'_2 > P'_1$. Therefore, the piston moves to the left. Let ΔV be the change in volume of any compartment, then on applying the Ideal Gas Equation, we get

$$\frac{P_f \left(\frac{V}{2} - \Delta V\right)}{T_f} = \frac{P_1 V}{2T_1} \quad \dots(1)$$

$$\text{and } \frac{P_f \left(\frac{V}{2} + \Delta V\right)}{T_f} = \frac{P_2 V}{2T_2} \quad \dots(2)$$

On dividing equation (2) by (1), we get

$$\frac{\left(\frac{V}{2} + \Delta V\right)}{\left(\frac{V}{2} - \Delta V\right)} = \frac{P_2 T_1}{P_1 T_2}$$

So, volume of left compartment is

$$V_L = \frac{V}{2} - \Delta V = V \left(\frac{P_1 T_2}{P_2 T_1 + P_1 T_2}\right)$$

and volume of right compartment is

$$V_R = \frac{V}{2} + \Delta V = V \left(\frac{P_2 T_1}{P_2 T_1 + P_1 T_2}\right)$$

10. Since, $P_1 V_1^\gamma = P_2 V_2^\gamma$

$$\Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma$$

$$\Rightarrow \frac{P_2}{P_1} = \left(\frac{16}{12}\right)^{3/2} = \left(\frac{4}{3}\right)^{3/2} = 1.533$$

$$\text{Also, } T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{16}{12}\right)^{1/2} = \left(\frac{4}{3}\right)^{1/2} = 1.153$$

11. For an adiabatic process, we have

$$TV^{\gamma-1} = \text{constant}$$

Differentiating with respect to T ,

$$V^{\gamma-1}(1) + (T)(\gamma-1)V^{\gamma-2} \frac{dV}{dT} = 0$$

$$\Rightarrow \frac{dV}{dT} = -\frac{V}{T(\gamma-1)}$$

But from the graph, we have slope = $\tan(\pi - \theta)$

$$\Rightarrow \left|\frac{dV}{dT}\right|_{T_0, V_0} = \tan(\pi - \theta) = -\tan \theta$$

$$\Rightarrow \frac{V_0}{T_0(\gamma-1)} = \tan \theta$$

$$\Rightarrow \gamma - 1 = \frac{V_0}{T_0 \tan \theta}$$

$$\Rightarrow \gamma = \frac{V_0}{T_0 \tan \theta} + 1$$

$$\Rightarrow C_p = \frac{\gamma R}{\gamma - 1} = \frac{(V_0 - T_0 \tan \theta)R}{(T_0 \tan \theta)V_0} T_0 \tan \theta$$

$$\Rightarrow C_p = \left(1 - \frac{T_0}{V_0} \tan \theta\right) R$$

$$\text{and } C_v = \frac{R}{\gamma - 1} = \frac{RT_0 \tan \theta}{V_0}$$

Test Your Concepts-VIII (Based on Polytropic Process)

1. Given equation is $VT^2 = \text{constant}$

Since, $PV = nRT$

$$\Rightarrow PV^{3/2} = \text{constant}$$

Comparing with $PV^x = \text{constant}$, we get $x = \frac{3}{2}$

Since, $C = C_v + \frac{R}{1-x} = \frac{R}{\gamma-1} + \frac{R}{1-x}$ and $\gamma = \frac{7}{5}$

H.92 JEE Advanced Physics: Waves and Thermodynamics

$$\Rightarrow C = \frac{R}{\frac{7}{5}-1} + \frac{R}{1-\frac{3}{2}} = \frac{5}{2}R - 2R = \frac{R}{2}$$

Now, $Q = nC\Delta T$

$$\Rightarrow Q = (3)\left(\frac{R}{2}\right)(2T - T) = \frac{3}{2}RT$$

2. (a) Heat supplied to raise the temperature of one mole of gas from T_0 to ηT_0

$$Q = \int C dT = \int_{T_0}^{\eta T_0} \frac{\alpha}{T} dT = \alpha \ln(\eta)$$

Increase in internal energy of gas is

$$\Delta U = C_V(\eta T_0 - T_0) = \frac{RT_0}{\gamma-1}(\eta-1)$$

From FLTD, work done is

$$W = Q - \Delta U = \alpha \ln(\eta) - \frac{RT_0(\eta-1)}{(\gamma-1)}$$

- (b) Since, $C = C_V + \frac{PdV}{ndT}$

$$\Rightarrow \frac{\alpha}{T} = \frac{R}{\gamma-1} + \frac{RT}{V} \frac{dV}{dT}$$

$$\Rightarrow \int \frac{dV}{V} = \int \frac{\alpha}{RT^2} dT - \frac{1}{\gamma-1} \int \frac{dT}{T}$$

$$\Rightarrow \ln V = -\frac{\alpha}{R} \frac{1}{T} - \frac{1}{\gamma-1} \ln T + C$$

$$\Rightarrow (\gamma-1)\ln V + \ln T = -\frac{\alpha(\gamma-1)}{RT} + C$$

$$\Rightarrow \ln(TV^{\gamma-1}) = -\frac{\alpha(\gamma-1)}{RT} + C$$

$$\Rightarrow TV^{\gamma-1} e^{\frac{\alpha(\gamma-1)}{RT}} = \text{constant}$$

$$\Rightarrow PV^\gamma e^{\frac{\alpha(\gamma-1)}{PV}} = \text{constant}$$

3. Equation of the process is $P = P_0 - \alpha V^2$

Using ideal gas equation $PV = RT$, we get

$$(P_0 - \alpha V^2)V = RT$$

$$\Rightarrow T = \frac{P_0 V}{R} - \frac{\alpha V^3}{R}$$

For maximum value of T , we have $\frac{dT}{dV} = 0$

$$\Rightarrow \frac{dT}{dV} = \frac{P_0}{R} - \frac{3\alpha V^2}{R} = 0$$

$$\Rightarrow V = \sqrt{\frac{P_0}{3\alpha}}$$

$$\Rightarrow T_{\max} = \frac{2P_0}{3R} \sqrt{\frac{P_0}{3\alpha}}$$

4. Equation for first thermodynamic process is

$$T = T_0 e^{\alpha V} \quad \dots(1)$$

Since, we know that for any process

$$C = \frac{1}{n} \left(\frac{dQ}{dT} \right) = \frac{1}{n} \left(\frac{dU + dW}{dT} \right)$$

$$\Rightarrow C = \frac{1}{n} \left(\frac{dU}{dT} \right) + \frac{P}{n} \left(\frac{dV}{dT} \right)$$

$$\Rightarrow C = C_V + \frac{P}{n} \left(\frac{dV}{dT} \right) \quad \dots(2)$$

From gas law, the pressure and volume relation is

$$P = \frac{nRT}{V} = \frac{nR(T_0 e^{\alpha V})}{V} \quad \dots(3)$$

Differentiating equation (1) w.r.t. V , we get

$$\frac{dT}{dV} = T_0 \alpha e^{\alpha V}$$

$$\Rightarrow \frac{dV}{dT} = \frac{1}{T_0 \alpha e^{\alpha V}} \quad \dots(4)$$

Substituting equations (3) and (4) in equation (2), we get

$$C = C_V + \left(\frac{nRT_0 e^{\alpha V}}{nV} \right) \left(\frac{1}{T_0 \alpha e^{\alpha V}} \right)$$

$$\Rightarrow C = C_V + \frac{R}{\alpha V}$$

Since, the molar specific heat of the gas undergoing this given process is not constant and is a function of volume, so this process is a non-polytropic process.

Equation for second thermodynamic process is

$$P = P_0 e^{\alpha V} \quad \dots(5)$$

$$\Rightarrow \frac{nRT}{V} = P_0 e^{\alpha V}$$

$$\Rightarrow T = \frac{P_0}{nR} (V e^{\alpha V}) \quad \dots(6)$$

Differentiate equation (6), w.r.t. V , we get

$$\frac{dT}{dV} = \frac{P_0}{nR} (e^{\alpha V} + V \alpha e^{\alpha V}) = \frac{P_0 e^{\alpha V}}{nR} (1 + \alpha V) \quad \dots(7)$$

Substituting equations (7) and (5) in equation (2), we get

$$C = C_V + \left(\frac{P_0 e^{\alpha V}}{n} \right) \left(\frac{nR}{P_0 e^{\alpha V} (1 + \alpha V)} \right)$$

$$\Rightarrow C = C_V + \frac{R}{1 + \alpha V}$$

Again, we note that the molar specific heat of the gas undergoing this given process is not constant and is a function of volume, so this process is a non-polytropic process.

5. Given that $TV = \text{constant}$

Compare with $TV^{x-1} = \text{constant}$, we get $x-1=1$

$$\Rightarrow x=2$$

$$\Rightarrow C = C_V + \frac{R}{1-x} = C_V + \frac{R}{1-2}$$

$$\Rightarrow C = C_V - R$$

Since, $Q = nC\Delta T = (1)C\Delta T$

$$\Rightarrow Q = (C_V - R)\Delta T = C_V \Delta T - R\Delta T$$

$$\Rightarrow Q = \frac{R}{\gamma-1} \Delta T - R \Delta T = R \Delta T \left(\frac{1}{\gamma-1} - 1 \right)$$

$$\Rightarrow Q = \frac{R \Delta T (1 - \gamma + 1)}{\gamma-1} = \frac{\gamma R \Delta T}{1-\gamma}$$

6. Since the rate of collisions with wall of vessel

$$N_C = \frac{1}{6} n_0 v_{rms} = \text{constant}$$

where, n_0 is the number density of molecules.

$$\Rightarrow \frac{1}{6} \left(\frac{N}{V} \right) \sqrt{\frac{3RT}{m}} = \text{constant}$$

$$\Rightarrow \frac{\sqrt{T}}{V} = \text{constant}$$

$$\Rightarrow T = kV^2$$

$$\Rightarrow \frac{dT}{dV} = 2kV$$

Molar specific of the gas is given by

$$C = C_V + \frac{PdV}{ndT}, \text{ where } C_V = \frac{5R}{2}$$

$$\Rightarrow C = \frac{5R}{2} + \left(\frac{RT}{V} \right) \frac{dV}{dT} = \frac{3R}{2} + \frac{R(kV^2)}{V} \frac{1}{2kV}$$

$$\Rightarrow C = \frac{5R}{2} + \frac{R}{2} = 3R$$

7. Let the initial volume of the gas is V and it is expanded to volume ηV . The work done in the process is given by

$$W = \int_V^{\eta V} PdV = \int_V^{\eta V} \alpha V dV = \frac{\alpha V^2}{2} (\eta^2 - 1)$$

The pressure of the gas as a function of volume is $P = \alpha V$. The initial and final values of pressure would be $V_i = \alpha V$ and $V_f = \eta \alpha V$ respectively.

From Ideal Gas Equation, we get

$$T_i = \frac{P_i V_i}{nR} = \frac{\alpha V^2}{nR} \text{ and } T_f = \frac{P_f V_f}{nR} = \frac{\eta^2 \alpha V^2}{nR}$$

$$\text{Since, } \Delta U = nC_V \Delta T = \frac{nR}{\gamma-1} \Delta T = \frac{nR}{\gamma-1} (T_f - T_i)$$

$$\Rightarrow \Delta U = \left(\frac{nR}{\gamma-1} \right) \left(\frac{\alpha V^2}{nR} \right) (\eta^2 - 1) = \frac{\alpha V^2}{\gamma-1} (\eta^2 - 1)$$

Since, $P = \alpha V$

$$\Rightarrow PV^{-1} = \text{constant}$$

Compare with $PV^x = \text{constant}$, we get $x = -1$

$$\Rightarrow C = C_V + \frac{R}{1-x} = C_V + \frac{R}{2}$$

8. For a given polytropic process $PV^x = \text{constant}$ and molar specific heat is given by

$$C = C_V + \frac{R}{1-x} = \frac{R}{\gamma-1} + \frac{R}{1-x} \quad \dots(1)$$

The given equation can be re-written as

$$PV^{-\alpha} = k = \text{constant}$$

So, comparison of the two equations gives

$$x = -\alpha$$

According to the problem, we should have $C = 0$

$$\Rightarrow C = \frac{R}{\gamma-1} + \frac{R}{1-(-\alpha)} = 0$$

$$\Rightarrow \frac{R}{1-\gamma} = \frac{R}{1+\alpha}$$

$$\Rightarrow \alpha = -\gamma$$

We could have given this answer directly, because for an adiabatic process, $Q = 0$ and hence $C = 0$.

9. (a) Given that $PT^{-1/2} = \text{constant} \quad \dots(1)$

For a polytropic process, we have

$$PT^{1-x} = \text{constant} \quad \dots(2)$$

Comparing equations (1) and (2), we get

$$\frac{x}{1-x} = -\frac{1}{2}$$

$$\Rightarrow -2x = 1-x$$

$$\Rightarrow x = -1$$

Work done by a gas in a polytropic process is

$$W_{\text{poly}} = \frac{nR}{1-x} (T_2 - T_1), \text{ where } x = -1$$

$$\Rightarrow W_{\text{poly}} = \frac{nR}{1-(-1)} (T_2 - T_1) = \frac{nR}{2} \Delta T$$

$$\Rightarrow W_{\text{poly}} = \frac{(2)(25/3)}{2} (60) = 500 \text{ J}$$

- (b) Since $C = C_V + \frac{R}{1-x}$ where, $(C_V)_{\text{monatomic}} = \frac{3R}{2}$

$$\Rightarrow C = \frac{3R}{2} + \frac{R}{1-(-1)} = 2R$$

Test Your Concepts-IX

(Based on Cyclic Process, Heat Engine and Refrigerator)

1. For process AB we use

$$PT = \text{constant}$$

$$\Rightarrow \left(\frac{RT}{V} \right) T = \text{constant}$$

$$\Rightarrow T^2 \propto V$$

$$\Rightarrow T = k\sqrt{V} \text{ OR } P\sqrt{V} = \text{constant}$$

$$\Rightarrow PV^{\frac{1}{2}} = \text{constant i.e., } x = \frac{1}{2}$$

Thus, molar specific heat of gas is

$$C = C_V + \frac{R}{1-x} = C_V + 2R$$

$$\Rightarrow C = \frac{3R}{2} + 2R = \frac{7R}{2}$$

Work done by gas in process AB is

$$W_{\text{poly}} = \frac{nR}{1-x} (T_2 - T_1)$$

H.94 JEE Advanced Physics: Waves and Thermodynamics

$$\Rightarrow W_{\text{poly}} = \frac{2R}{1 - (1/2)}(T_0 - 2T_0) = -4RT_0$$

$$\Rightarrow W_{\text{poly}} = -4RT_0 = -1200R$$

$$Q_{AB} = 2C(T_0 - 2T_0) = -2\left(\frac{7R}{2}\right)T_0$$

$$\Rightarrow Q_{AB} = -7RT_0 = -2100R$$

$$Q_{BC} = 2C_P(2T_0 - T_0) = 2\left(\frac{5R}{2}\right)T_0 = 1500R$$

$$Q_{CA} = 2R(2T_0)\ln(2)$$

$$\Rightarrow Q_{CA} = 1200R \times 0.693 = 831.6R$$

2. Done in Theory

3. Let P and V be the initial pressure and volume respectively, then for process $1 \rightarrow 2$

$$\frac{P_2}{T_2} = \frac{P}{T_1} \quad \{\because \text{volume remains constant}\}$$

$$\Rightarrow P_2 = P \frac{T_2}{T_1} \quad \dots(1)$$

For process $2 \rightarrow 3$

$$\frac{V_3}{T_3} = \frac{V}{T_2}$$

$$\Rightarrow V_3 = V \frac{T_3}{T_2} \quad \dots(2)$$

For process $3 \rightarrow 4$ (temperature at points 3 and 4 is the same), so

$$V_4 = V \frac{T_2}{T_1} \quad \dots(3)$$

Since work done by the gas is area of the trapezium 12341

$$\Rightarrow W = \frac{1}{2}[(V_3 - V_2) + (V_4 - V_1)](P_2 - P_1)$$

$$\Rightarrow W = \frac{1}{2}\left(V \frac{T_3}{T_2} + V \frac{T_2}{T_1} - 2V\right)\left(P \frac{T_2}{T_1} - P\right)$$

$$\Rightarrow W = \frac{1}{2}PV\left(\frac{T_3}{T_2} + \frac{T_2}{T_1} - 2\right)(T_2 - T_1)$$

But $PV = nRT_1$

$$\Rightarrow W = \frac{1}{2}nR\left(\frac{T_3}{T_2} + \frac{T_2}{T_1} - 2\right)(T_2 - T_1)$$

4. For $A \rightarrow B$, $V \propto T$

$$\Rightarrow P = \text{constant}$$

So, V and T both are increasing.

Hence we get a straight line parallel to V axis as P is constant.

For $B \rightarrow C$, V is constant

$$\Rightarrow P \propto T$$

So, T is decreasing and P is also decreasing.

Hence we get a straight line parallel to P axis, as V is constant.

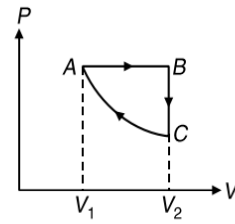
For $C \rightarrow A$, T is constant

$$\Rightarrow P \propto \frac{1}{V} \text{ So, if } V \text{ is decreasing, then } P \text{ should increase}$$

Hence, we get a rectangular hyperbola because

$$PV = \text{constant}$$

The corresponding graph is shown in figure given below.



5. For process CA , volume remains constant, so no work is done in this process.

For process AB , since $T \propto V$ so gas pressure remains constant and hence work done by the gas in this process is

$$W_{AB} = nR(T_B - T_A)$$

$$\Rightarrow W_{AB} = 2 \times 8.314 \times (500 - 300)$$

$$\Rightarrow W_{AB} = 2 \times 8.314 \times 200 = 3325.6 \text{ J}$$

For the cyclic process, we have $\Delta U = 0$, so

$$Q = W = W_{AB} + W_{BC} + W_{CA}$$

According to the problem a total 1200 J of heat is rejected by the gas, so we have

$$Q = -1200 \text{ J}$$

$$\Rightarrow -1200 = Q_{AB} + W_{BC} + W_{CA}$$

$$\Rightarrow -1200 = 3325.6 + W_{BC} + 0$$

$$\Rightarrow W_{BC} = -4525.6 \text{ J}$$

6. For isothermal process AB , we have

$$Q_{AB} = 2R(500)\ln(2)$$

For isothermal process CD , we have

$$Q_{CD} = -2R(300)\ln(2)$$

For isochoric process BC , we have

$$Q_{BC} = 2C_V(-200) = -400C_V$$

Similarly, for isochoric process DA , we have

$$Q_{DA} = 2C_V(200) = 400C_V$$

Total heat supplied to gas is

$$Q = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA}$$

$$\Rightarrow Q = 400R\ln 2 = (400)(8.314)(0.693)$$

$$\Rightarrow Q = 2304.64 \text{ J}$$

7. Since the process is cyclic, i.e., $\Delta U = 0$, so

$$Q = W_{AB} + W_{BC} + W_{CA} \quad \dots(1)$$

For process AB , volume of gas remains constant, so work done is zero.

$$\Rightarrow W_{AB} = 0$$

For process BC , temperature is constant at 500 K and volume of gas changes from $V_1 = 2 \text{ m}^3$ to $V_2 = 5 \text{ m}^3$, so we have

$$W_{BC} = nRT \ln\left(\frac{V_2}{V_1}\right) = (1)R(500)\ln\left(\frac{5}{2}\right)$$

For process CA , we observe that $V \propto T$ and the line passes through the origin, so the pressure is constant

$$W_{CA} = P\Delta V = nR\Delta T = nR(T_2 - T_1)$$

$$\Rightarrow W_{CA} = (1)R(300 - 500) = -200R$$

This is negative as gas is being compressed from volume 5 m^3 to 2 m^3 or work is done on the gas.

Substituting these values in equation (1), we get

$$Q = 0 + 500R \ln\left(\frac{5}{2}\right) - 200R$$

$$\Rightarrow Q = 8.3[500(0.9) - 200] = 2075 \text{ J}$$

8. $Q_1 = 0$ (adiabatic)

$Q_2 = C_p(T_3 - T_2)$ (isobaric)

Further, $\frac{V_2}{T_2} = \frac{V_3}{T_3}$

$$\Rightarrow \frac{nV}{T_2} = \frac{V}{T_3}$$

$$\Rightarrow T_3 = \frac{T_2}{n}$$

Since, $C_p = \left(\frac{\gamma}{\gamma-1}\right)R$

$$\Rightarrow Q_2 = \frac{R\gamma}{\gamma-1} \left(\frac{T_2}{n} - T_2\right) = \frac{R\gamma T_2}{n(\gamma-1)}(1-n)$$

So, heat rejected in isobaric process is

$$Q_2 = -\frac{R\gamma T_2}{n(\gamma-1)}(n-1)$$

At constant volume,

$$Q_3 = C_v(T_1 - T_3) = \frac{R}{\gamma-1}(T_1 - T_3)$$

$$\Rightarrow Q_3 = \frac{R}{\gamma-1} \left(T_1 - \frac{T_2}{n}\right)$$

Further, $T_1 V_1^{\gamma-1} = T_2 (nV_1)^{\gamma-1}$

$$\Rightarrow T_1 = T_2 n^{\gamma-1}$$

$$\Rightarrow Q_3 = \frac{RT_2}{\gamma-1} \left(n^{\gamma-1} - \frac{1}{n}\right) = \frac{RT_2}{n(\gamma-1)}(n^\gamma - 1)$$

So, $W_{\text{Total}} = Q_1 + Q_2 + Q_3$

$$W_{\text{Total}} = \frac{RT_2}{n(\gamma-1)} \left[(n^\gamma - 1) - \gamma(n-1) \right]$$

Now, efficiency η is $\eta = \frac{W_{\text{Total}}}{Q_{\text{+ive}}} = \frac{W_{\text{Total}}}{Q_3}$

$$\Rightarrow \eta = \frac{(n^\gamma - 1) - \gamma(n-1)}{(n^\gamma - 1)}$$

$$\Rightarrow \eta = 1 - \gamma \left(\frac{n-1}{n^\gamma - 1}\right)$$

9. Given that $U_A = 0$, $U_B = 50 \text{ J}$ and $Q_{BC} = 90 \text{ J}$.

Also, $P_A = P_B = 10 \text{ Nm}^{-2}$, $P_C = 30 \text{ Nm}^{-2}$

$V_A = 1 \text{ m}^3$ and $V_B = V_C = 3 \text{ m}^3$.

For process BC, volume of gas remains constant, so work done by gas in this process is zero i.e., $W_{BC} = 0$

Since heat absorbed by the gas is $Q_{BC} = 90 \text{ J}$, so from FLTD, we have

$$\Delta U_{BC} = U_C - U_B = Q_{BC} - W_{BC} = 90 \text{ J} - 0$$

$$\Rightarrow \Delta U_{BC} = U_C - U_B = 90 \text{ J}$$

$$\Rightarrow U_C = \Delta U_{BC} + U_B = 90 \text{ J} + 50 \text{ J} = 140 \text{ J}$$

For process AB, we have

$$\Delta U_{AB} = U_B - U_A = 50 - 0 = 50 \text{ J}$$

$$W_{AB} = P\Delta V = P_A(V_B - V_A) = 10(3 - 1) = 20 \text{ J}$$

Thus, heat absorbed by the system is

$$Q_{AB} = (\Delta U)_{AB} + W_{AB} = 50 + 20 = 70 \text{ J}$$

For process CA

$$\Delta U_{CA} = U_A - U_C = 0 - 140 = -140 \text{ J}$$

Work done is equal to the negative area under the curve CA (a compression process)

$$W_{CA} = -(\text{Area ACED})$$

$$\Rightarrow W_{CA} = -\left[\frac{1}{2}(10+30)(3-1)\right] = -40 \text{ J}$$

So, by FLTD, we have

$$Q_{CA} = \Delta U_{CA} + W_{CA} = -140 - 40 = -180 \text{ J}$$

Negative sign shows that heat is being rejected in the process CA.

Net work done in the counter clockwise cyclic process ABCA is equal to the negative of the area of triangle ABC i.e.,

$$W_{ABCA} = -(\text{Area } \Delta ABC)$$

$$\Rightarrow W_{ABCA} = -\frac{1}{2} \times 2 \times 20 = -20 \text{ J}$$

Negative sign indicates that the work is being done on the system.

10. If T is the temperature of state A we use

$$T_A = T; T_B = T_D = nT; T_C = n^2T$$

Total work done by gas in cycle for n_1 moles of gas

$$W = (P_2 - P_1)(V_2 - V_1)$$

$$n_1 R(n^2T - nT - nT + T) = n_1 RT(n-1)^2$$

Heat supplied to gas is $Q_{\text{supp}} = \Delta Q_{AB} + \Delta Q_{BC}$

$$\Rightarrow Q_{\text{supp}} = n_1 \left(\frac{R}{\gamma-1}\right) T(n-1) + n_1 \left(\frac{\gamma R}{\gamma-1}\right) T(n^2 - n)$$

$$\Rightarrow Q_{\text{supp}} = \frac{n_1 RT}{\gamma-1} (n-1 + \gamma n^2 - \gamma n)$$

$$\Rightarrow Q_{\text{supp}} = \frac{n_1 RT}{\gamma-1} (n-1)(1 + \gamma n)$$

Cycle efficiency is

$$\eta = \frac{W}{Q_{\text{supp}}} = \frac{n_1 RT(n-1)^2}{\frac{n_1 RT}{\gamma-1} (n-1)(1 + \gamma n)}$$

$$\Rightarrow \eta = \left(\frac{\gamma-1}{1 + \gamma n}\right)(n-1)$$

Test Your Concepts-X (Based on Conduction)

1. (a) Thermal resistance of aluminium cube $R_1 = \frac{\ell}{kA}$
 $\Rightarrow R_1 = \frac{(3 \times 10^{-2})}{(237)(3 \times 10^{-2})^2} = 0.14 \text{ KW}^{-1}$

and thermal resistance of copper cube $R_2 = \frac{\ell}{kA}$
 $\Rightarrow R_2 = \frac{(3 \times 10^{-2})}{(401)(3 \times 10^{-2})^2} = 0.08 \text{ KW}^{-1}$

As these two resistances are in parallel, their equivalent resistance is given by

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

$$\Rightarrow R = \frac{(0.14)(0.08)}{(0.14) + (0.08)}$$

$$\Rightarrow R = 0.05 \text{ KW}^{-1}$$

So, thermal current

$$H = \frac{\text{Temperature difference}}{\text{Thermal resistance}}$$

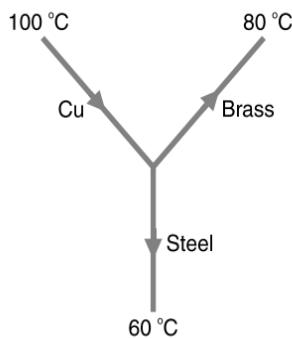
$$\Rightarrow H = \frac{(100 - 20)}{0.05}$$

$$\Rightarrow H = 1.6 \times 10^3 \text{ watt}$$

(b) In parallel thermal current distributes in the inverse ratio of resistance, so

$$\frac{H_{Cu}}{H_{Al}} = \frac{R_{Al}}{R_{Cu}} = \frac{R_1}{R_2} = \frac{0.14}{0.08} = 1.75$$

2. (a) Since $H_{Cu} = H_{Steel} + H_{Brass}$



If T be the temperature of the junction, then

$$\frac{k_{Cu}A(100 - T)}{46} = \frac{k_B A(T - 80)}{13} + \frac{k_S A(T - 60)}{12}$$

$$\Rightarrow \frac{0.92}{46}(100 - T) = \frac{0.26}{13}(T - 80) + \frac{0.12}{12}(T - 60)$$

$$\Rightarrow 200 - 2T = 2T - 160 + T - 60$$

$$\Rightarrow 5T = 420$$

$$\Rightarrow T = 84 \text{ }^\circ\text{C}$$

(b) $H_{Cu} = \frac{k_{Cu}A(100 - T)}{\ell_{Cu}}$
 $\Rightarrow H_{Cu} = \frac{(0.92)(4)(100 - 84)}{46}$
 $\Rightarrow H_{Cu} = 1.28 \text{ cal s}^{-1}$

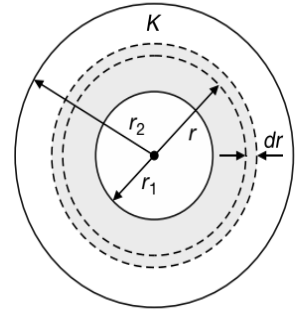
3. Consider a differential cylindrical element

$$H = \frac{dQ}{dt} = kA \frac{dT}{dr} = (2\pi k r \ell) \frac{dT}{dr}$$

$$\frac{H}{2\pi k \ell} \int_{r_1}^{r_2} \frac{dr}{r} = \int_0^{50} dT$$

$$\Rightarrow \frac{H}{2\pi k \ell} \log_e \left(\frac{r_2}{r_1} \right) = 50$$

$$\Rightarrow H = \frac{100\pi k \ell}{\log_e \left(\frac{r_2}{r_1} \right)}$$



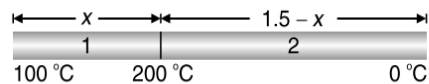
Since, $Q = Ht = mL$

$$\Rightarrow t = \frac{mL}{H} = \frac{mL \log_e \left(\frac{r_2}{r_1} \right)}{100\pi k \ell}$$

4. Since, $\frac{dQ}{dt} = kA \frac{\Delta T}{\ell}$

and $\frac{dQ}{dt} = L \left(\frac{dm}{dt} \right)$ { $\because dQ = (dm)L$ }

$$\Rightarrow \left(\frac{dm}{dt} \right) = \left(\frac{kA}{\ell} \right) \left(\frac{\Delta T}{L} \right)$$



Let the desired point be at a distance x from water at $100 \text{ }^\circ\text{C}$.
 Since

$$\left(\text{Rate of Melting of Ice} \right) = \left(\text{Rate of Production of Steam} \right)$$

$$\Rightarrow \left(\frac{dm}{dt} \right)_1 = \left(\frac{dm}{dt} \right)_2$$

$$\Rightarrow \left(\frac{\Delta T}{L \ell} \right)_1 = \left(\frac{\Delta T}{L \ell} \right)_2 \text{ \{ as } A_1 = A_2 \text{ and } k_1 = k_2 \}$$

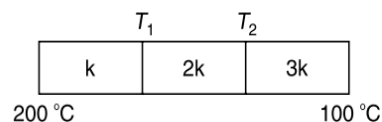
$$\Rightarrow \frac{200 - 100}{(540)(x)} = \frac{100 - 0}{(80)(1.5 - x)}$$

Solving this, we get

$$x = 0.1034 \text{ m} = 10.34 \text{ cm}$$

So, the temperature of $200 \text{ }^\circ\text{C}$ must be maintained at a distance 10.34 cm from water at $100 \text{ }^\circ\text{C}$.

5. Thermal resistance $\propto \frac{1}{k}$



$$\Rightarrow R_1 : R_2 : R_3 = \frac{1}{1} : \frac{1}{2} : \frac{1}{3} = 6 : 3 : 2$$

So if, $R_1 = 6R$, $R_2 = 3R$ and $R_3 = 2R$

$$\text{then, } H = \frac{\Delta T}{R_{\text{net}}} = \frac{200 - 100}{6R + 3R + 2R} = \frac{100}{11R}$$

Now, $200 - T_1 = HR_1$

$$\Rightarrow T_1 = 200 - \left(\frac{100}{11R}\right)(6R) = 145.5^\circ\text{C}$$

Also, $T_2 - 100 = HR_3$

$$\Rightarrow T_2 = 100 + HR_3$$

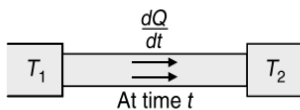
$$\Rightarrow T_2 = 100 + \left(\frac{100}{11R}\right)(2R)$$

$$\Rightarrow T_2 = 118.2^\circ\text{C}$$

6. Let T_1 and $T_2 (< T_1)$ be the temperatures of the two chunks at time t and T be their temperature difference. Then

$$T = T_1 - T_2$$

$$\Rightarrow \left(-\frac{dT}{dt}\right) = \left(-\frac{dT_1}{dt}\right) + \left(\frac{dT_2}{dt}\right) \quad \dots(1)$$



$$\text{Since, } \frac{dQ}{dt} = \frac{kAT}{\ell} = C_1 \left(-\frac{dT_1}{dt}\right) = C_2 \left(\frac{dT_2}{dt}\right) \quad \{\because T_1 - T_2 = T\}$$

$$\Rightarrow \left(-\frac{dT_1}{dt}\right) = \left(\frac{kA}{\ell C_1}\right)T$$

$$\text{and } \left(\frac{dT_2}{dt}\right) = \left(\frac{kA}{\ell C_2}\right)T$$

Substituting these values in Equation (1), we get

$$\left(-\frac{dT}{dt}\right) = \frac{kA}{\ell} \left(\frac{C_1 + C_2}{C_1 C_2}\right)T = \alpha T \text{ where } \alpha = \frac{kA(C_1 + C_2)}{\ell C_1 C_2}$$

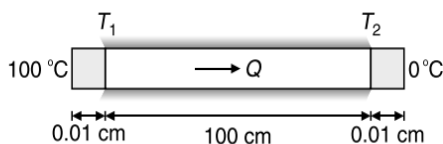
$$\Rightarrow \int_{(\Delta T)_0}^T -\frac{dT}{T} = \alpha \int_0^t dt$$

$$\Rightarrow T = (\Delta T)_0 e^{-\alpha t}$$

7. Since, $\frac{dQ}{dt} = \frac{0.0014 \times A \times (100 - T_1)}{0.01} \quad \dots(1)$

$$\frac{dQ}{dt} = \frac{1.04 \times A \times (T_1 - T_2)}{100} \quad \dots(2)$$

$$\text{and } \frac{dQ}{dt} = \frac{0.0014 \times A \times (T_1 - 0)}{0.01} \quad \dots(3)$$



From Equations (1) and (3), we get

$$100 - T_1 = T_2 \quad \dots(4)$$

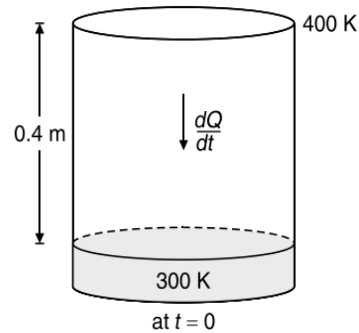
Solving above four equation, we get

$$T_1 = 93.5^\circ\text{C} \text{ and } T_2 = 6.5^\circ\text{C}$$

So, temperature gradient in the bar

$$\frac{\Delta T}{\ell} = \frac{T_1 - T_2}{100} = 0.87^\circ\text{C cm}^{-1}$$

8. Since, $\frac{dQ}{dt} = mc \frac{dT}{dt} = \frac{KA(T_1 - T_2)}{\ell}$ we have



$$mc \left(\frac{dT}{dt}\right) = \frac{KA(400 - T)}{0.4}$$

$$\Rightarrow (0.4)(600) \frac{dT}{dt} = \frac{(10)(0.04)(400 - T)}{0.4}$$

$$\Rightarrow \left(\frac{dT}{400 - T}\right) = \frac{1}{240} dt$$

$$\Rightarrow \int_0^t dt = 240 \int_{300}^{350} \frac{dT}{400 - T}$$

Solving this, we get $t \approx 166$ s

9. (a) Since, $Q = nC_p \Delta T \quad \{\because P = \text{constant}\}$

$$\Rightarrow \frac{kA(T_S - T)}{a} dt = \left(\frac{P_0 V_0}{RT_0}\right) \left(\frac{5R}{2}\right) dT$$

$$\Rightarrow \frac{kA(T_S - T)}{a} dt = \frac{5P_0 V_0}{2T_0} dT$$

$$\Rightarrow \frac{kA}{a} \int_0^t dt = \frac{5P_0 V_0}{2T_0} \int_{T_0}^T \frac{dT}{T_S - T}$$

$$\Rightarrow T = T_S + (T_S - T_0) e^{-\alpha t}$$

$$\text{where, } \alpha = \frac{2T_0 kA}{5P_0 V_0 a}$$

- (b) Since $P = \text{constant}$, so $\frac{V_i}{T_i} = \frac{V_f}{T_f}$

$$\Rightarrow \frac{h_0 A}{T_0} = \frac{hA}{T}$$

$$\Rightarrow h = \left(\frac{T}{T_0}\right) h_0$$

H.98 JEE Advanced Physics: Waves and Thermodynamics

10. Let θ be the temperature difference at time t , then

$$\frac{dQ}{dt} = \frac{kA\theta}{\ell} \quad \dots(1)$$

$$\text{Since, } \frac{dQ}{dt} = nC_{V_1} \left(-\frac{d\theta_1}{dt} \right) = nC_{V_2} \left(\frac{d\theta_2}{dt} \right) \quad \dots(2)$$

$$\text{where, } C_{V_1} = \frac{3R}{2} \text{ and } C_{V_2} = \frac{5R}{2}$$

$$\text{Since, } \left(-\frac{d\theta_1}{dt} \right) = \frac{2kA\theta}{3nR\ell}$$

$$\left(\frac{d\theta_2}{dt} \right) = \frac{2kA\theta}{5nR\ell}$$

$$\text{Also, } \theta = \theta_1 - \theta_2$$

$$\Rightarrow \left(-\frac{d\theta}{dt} \right) = \left(-\frac{d\theta_1}{dt} \right) + \left(\frac{d\theta_2}{dt} \right)$$

$$\Rightarrow \left(-\frac{d\theta}{dt} \right) = \frac{16kA\theta}{15nR\ell}$$

$$\Rightarrow dt = -\frac{15nR\ell}{16kA} \int_{T_1-T_2}^2 \frac{d\theta}{\theta}$$

$$\Rightarrow t = \frac{15nR\ell}{16kA} \log_e(2)$$

Test Your Concepts-XI (Based on Radiation)

1. Since, $\left(\frac{\text{Rate of Cooling}}{\text{Temperature}} \right) \propto \left(\frac{\text{Average Excess Temperature}}{\text{Temperature}} \right)$

$$\Rightarrow \frac{T_i - T_f}{t} = k \left(\frac{T_i + T_f}{2} - T_0 \right)$$

$$\Rightarrow \frac{80 - 50}{5} = k \left(\frac{80 + 50}{2} - 20 \right) \quad \dots(1)$$

Similarly

$$\frac{60 - 30}{t} = k \left(\frac{60 + 30}{2} - 20 \right) \quad \dots(2)$$

Dividing (1) by (2), we get

$$\Rightarrow \frac{t}{5} = \frac{65 - 20}{45 - 20}$$

$$\Rightarrow t = \frac{45}{25} \times 5 = 9 \text{ minute}$$

2. Since temperature of surrounding is very low as compared to that of heating element, so we can ignore the amount of radiant energy absorbed by the filament from its surroundings. So, according to Stefan's Law, we get

$$P = \sigma AT^4$$

$$\Rightarrow 1000 = 5.67 \times 10^{-8} \times 0.02 \times T^4$$

$$\Rightarrow T^4 = 8.82 \times 10^{11}$$

$$\Rightarrow T = 969.3 \text{ K}$$

3. Since the curved surface of the rod is insulated, so in steady state, the heat conducted from end A to end B of the rod is lost from the end B of rod by radiation.

The energy received through conduction at end B of rod per unit time is

$$u_1 = \left| \frac{dQ}{dt} \right| = \frac{kA(T_A - T_B)}{l} \quad \dots(1)$$

where, T_A, T_B are absolute temperatures of end A and B , l is the length and k is thermal conductivity of rod.

Energy radiated per unit time from end B is

$$u_2 = \left| \frac{dQ}{dt} \right| = \sigma A(T_B^4 - T_0^4) \quad \dots(2)$$

where T_0 is room temperature, σ is Stefan's constant.

In the steady state, we have $u_1 = u_2$

$$\Rightarrow \frac{k(T_A - T_B)}{l} = \sigma(T_B^4 - T_0^4)$$

$$\Rightarrow k = \frac{\sigma l(T_B^4 - T_0^4)}{T_A - T_B} \quad \dots(3)$$

Substituting the values, we get

$$k = 36.6 \text{ Wm}^{-1}\text{K}^{-1}$$

4. Since, $-\frac{dQ}{dt} = mc \left(-\frac{dT}{dt} \right) = \sigma AT^4$

$$\int_{800 \text{ K}}^{300 \text{ K}} -\frac{dT}{T^4} = \frac{\sigma A}{mc} \int_0^t dt$$

Substituting $A = 4\pi r^2$, $m = \frac{4}{3}\pi r^3 \rho$, so integrating the above equation, we get

$$t = 1.01 \times 10^5 \text{ s}$$

5. Since, $-\frac{dQ}{dt} = \sigma e(\pi d^2)T^4$

$$\Rightarrow \rho \left(\frac{4}{3}\pi \frac{d^3}{8} \right) c \frac{dT}{dt} = -\sigma e \pi d^2 T^4$$

$$\Rightarrow \frac{\rho dc}{6\sigma e} = \int_{T_0}^{T_0/\eta} \frac{dT}{T^4} = -\int_0^t dt$$

$$\Rightarrow t = \frac{\rho dc}{18\sigma e} \left(\frac{\eta^3}{T_0^3} - \frac{1}{T_0^3} \right) = \frac{\rho dc(\eta^3 - 1)}{18\sigma e T_0^3}$$

6. The rate of loss of heat by sphere is given by

$$\frac{dQ}{dt} = \sigma A(T^4 - T_0^4)$$

where, $A = 4\pi r^2$ is the surface area of the sphere of radius $r = 10 \text{ cm} = 0.1 \text{ m}$, $T = 327^\circ\text{C} = 600 \text{ K}$ and $T_0 = 27^\circ\text{C} = 300 \text{ K}$, So, we get

$$\frac{dQ}{dt} = (5.67 \times 10^{-8}) 4\pi (0.1)^2 [(600)^4 - (300)^4]$$

$$\Rightarrow \frac{dQ}{dt} = 866 \text{ Js}^{-1}$$

Since, $dQ = msdT$, where dT is the fall in temperature in time dt , hence $\frac{dQ}{dt} = ms \frac{dT}{dt}$

$$\Rightarrow 866 = 10 \times 420 \times \frac{dT}{dt}$$

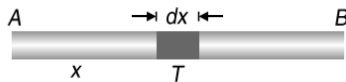
$$\Rightarrow \frac{dT}{dt} = \frac{866}{4200} = 0.206 \text{ } ^\circ\text{Cs}^{-1}$$

7. Since, $\lambda_m T_B = b$

$$\Rightarrow T_B = \frac{b}{\lambda_m}$$

$$\Rightarrow T_B = \frac{2.9 \times 10^{-3}}{14.5 \times 10^{-7}}$$

$$\Rightarrow T_B = 2000 \text{ K}$$



Since, $\frac{dQ}{dt} = P = -\frac{kAdT}{dx}$

$$\Rightarrow P = -\frac{k_0(1+ax)AdT}{dx}$$

$$\Rightarrow -\int_{T_0}^T \frac{dx}{1+ax} = \frac{k_0A}{P} \int_{T_0}^T dT$$

Solving this equation, we get

$$T = \frac{P}{k_0aA} \log_e \left(\frac{1+al_0}{1+ax} \right) + T_B$$

Substituting the values, we get

$$T = \frac{4560}{11.4 \times 0.2} \log_e \left(\frac{1+0.2 \times 5}{1+0.2x} \right) + 2000$$

$$\Rightarrow T = 2000 \left[\log_e \left(\frac{2}{1+0.2x} \right) + 1 \right]$$

$$\Rightarrow T = 2000 \left[\log_e \left(\frac{20}{10+2x} \right) + 1 \right]$$

$$\Rightarrow T = 2000 \left[\log_e \left(\frac{10}{5+x} \right) + 1 \right]$$

8. In this case, rate of heat lost by the block is

$$\frac{dQ}{dt} = \sigma A(T^4 - T_5^4)$$

where, A is the total surface area of cube given by $6a^2 = 150 \text{ cm}^2 = 150 \times 10^{-4} \text{ m}^2$

$$\left| \frac{dQ}{dt} \right| = (5.67 \times 10^{-8}) \left(\frac{150}{10^4} \right) ((500)^4 - (300)^4)$$

If $\frac{dT}{dt}$ is rate of cooling of block, then

$$\left| \frac{dQ}{dt} \right| = ms \frac{dT}{dt}$$

$$\Rightarrow \frac{dT}{dt} = \frac{1}{ms} \frac{dQ}{dt} = \left(\frac{1}{1 \times 400} \right) 38.556$$

$$\Rightarrow \frac{dT}{dt} = 0.115 \text{ } ^\circ\text{Cs}^{-1}$$

9. If emissivity of body is e , the power required equals the rate of emission of radiation.

$$P = e\sigma A(T^4 - T_5^4) \quad \dots(1)$$

For a black body

$$P_B = \sigma A(T^4 - T_5^4) \quad \dots(2)$$

Dividing, we get $\frac{P}{P_B} = e$

$$\Rightarrow e = \frac{210}{700} = 0.3$$

10. Since temperature difference of water with its surrounding is not large, so according to Newton's Law of Cooling, we have

$$\frac{T_i - T_f}{t} = k \left(\frac{T_i + T_f}{2} - T_0 \right), \text{ where } k = \frac{4\sigma AT_0^3}{mc}$$

$$\text{So, } k = \frac{4(5.67 \times 10^{-8})6(10/100)^2(300)^3}{1 \times 4200}$$

$$\Rightarrow k = 8.75 \times 10^{-5}$$

$$\Rightarrow \frac{30 - 29}{t} = 8.75 \times 10^{-5} (29.5 - 27)$$

$$\Rightarrow t = \frac{1}{(8.75 \times 10^{-5})(2.5)} = 4571.43 \text{ s} = 76.2 \text{ min}$$

11. Using average form of Newton's Law of Cooling, we use For water, we have

$$\frac{40 - 35}{5} = \frac{k}{0.1 \times 4200} \left(\frac{40 + 35}{2} - T_5 \right) \quad \dots(1)$$

For liquid, we have

$$\frac{40 - 35}{2} = \frac{k}{m \times 2100} \left(\frac{40 + 35}{2} - T_5 \right) \quad \dots(2)$$

Dividing (1) by (2), we get

$$\frac{2}{5} = \frac{m \times 2100}{0.1 \times 4200}$$

$$\Rightarrow m = \frac{2 \times 420}{5 \times 2100} = 0.08 \text{ kg} = 80 \text{ gm}$$

As the volume of liquid is same that of water 100 cm^3 , then density of liquid is

$$\rho = \frac{m}{V} = \frac{80 \times 10^{-3}}{100 \times 10^{-6}} = 800 \text{ kgm}^{-3}$$

12. In the bulb filament given, the energy radiated per sec per square metre of its surface area is given as

$$E = \frac{P}{A} = \frac{114.75}{0.25 \times 10^{-4}} = 459 \times 10^4 \text{ Js}^{-1}\text{m}^2$$

If T is the temperature of the filament then according to Stefan's law, we have

$$E = \sigma T^4$$

$$\Rightarrow 459 \times 10^4 = \left(\frac{17}{3} \times 10^{-8} \right) T^4$$

H.100 JEE Advanced Physics: Waves and Thermodynamics

$$\Rightarrow T^4 = \frac{459 \times 10^4}{(17/3) \times 10^{-8}} = 81 \times 10^{12}$$

$$\Rightarrow T = (81 \times 10^{12})^{\frac{1}{4}} = 3000 \text{ K}$$

Single Correct Choice Type Questions

1. If temperature of the source is increased by α K and temperature of the sink is decreased by the same amount i.e. α K then efficiency increases more in the second case.

$$\text{Since } \eta_0 = 1 - \frac{T_2}{T_1}$$

$$\text{According to the problem } \eta_1 = 1 - \left(\frac{T_2 - \alpha}{T_1} \right)$$

$$\Rightarrow \eta_1 = \left(1 - \frac{T_2}{T_1} \right) + \frac{\alpha}{T_1}$$

$$\Rightarrow \eta_1 = \eta_0 + \frac{\alpha}{T_1} = \frac{\eta_0 T_1 + \alpha}{T_1}$$

$$\Rightarrow \eta_1 > \eta_0$$

$$\text{Further, } \eta_2 = 1 - \frac{T_2}{T_1 + \alpha} = \frac{T_1 + \alpha - T_2}{T_1 + \alpha}$$

$$\Rightarrow \eta_2 = \frac{\eta_0 T_1 + \alpha}{T_1 + \alpha} \quad \{ \because T_1 - T_2 = \eta_0 T_1 \}$$

$$\Rightarrow \eta_2 < \eta_1$$

$$\Rightarrow \eta_1 > \eta_2 > \eta_0$$

Hence, the correct answer is (B).

2. $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$$\Rightarrow 300 = T_2 \left(\frac{8}{27} \right)^{\frac{5}{3}-1}$$

$$\Rightarrow T_2 = 300 \times \left(\frac{27}{8} \right)^{\frac{2}{3}} = 300 \times \frac{9}{4} = 675 \text{ K}$$

$$\Rightarrow \Delta T = 375 \text{ }^\circ\text{C}$$

Hence, the correct answer is (B).

3. At $V = V_0$, $P = \frac{P_0}{2}$

$$\Rightarrow T_i = \frac{PV}{nR} = \frac{\left(\frac{P_0}{2} \right) (V_0)}{R} = \frac{P_0 V_0}{2R} \quad \{ \because n = 1 \}$$

$$\text{and at } V = 2V_0, P = \frac{4P_0}{5}$$

$$\Rightarrow T_f = \frac{PV}{nR} = \frac{(2V_0) \left(\frac{4P_0}{5} \right)}{R} = \frac{8P_0 V_0}{5R}$$

$$\Rightarrow \Delta T = T_f - T_i = \left(\frac{8}{5} - \frac{1}{2} \right) \frac{P_0 V_0}{R} = \frac{11P_0 V_0}{10R}$$

Hence, the correct answer is (B).

4. $-mc \frac{dT}{dt} = \sigma T^4$

$$\Rightarrow -mc T^{-4} dT = \sigma dt$$

$$\Rightarrow - \int_{T_1}^{T_2} T^{-4} dT = \frac{\sigma}{mc} \int_0^t dt$$

$$\Rightarrow - \left[\frac{T^{-3}}{-3} \right]_{T_1}^{T_2} = \frac{\sigma}{mc} t$$

$$\Rightarrow t = \frac{mc}{3\sigma} \left(\frac{1}{T_2^3} - \frac{1}{T_1^3} \right)$$

Hence, the correct answer is (C).

5. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ and $v_{\text{sound}} = \sqrt{\frac{\gamma RT}{M}}$, $v_{\text{rms}} = \sqrt{2} v_{\text{sound}}$

$$\text{i.e., } \gamma = \frac{3}{2} = \text{ratio of } \frac{C_P}{C_V} \text{ for the mixture}$$

$$C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$$

$$\text{and } C_P = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 + n_2}$$

$$\Rightarrow \gamma = \frac{C_P}{C_V} = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 C_{V_1} + n_2 C_{V_2}}$$

$$\Rightarrow \frac{3}{2} = \frac{(2) \left(\frac{5}{2} R \right) + (n) \left(\frac{7}{2} R \right)}{2 \left(\frac{3}{2} R \right) + (n) \left(\frac{5}{2} R \right)}$$

$$\Rightarrow \frac{3}{2} = \frac{10 + 7n}{6 + 5n}$$

$$\Rightarrow n = 2$$

Hence, the correct answer is (C).

6. For gas A, we have

$$PV = n_A RT \quad \dots(1)$$

When another gas B is also added, then

$$3PV = (n_A + n_B) RT \quad \dots(2)$$

$$\Rightarrow n_A + n_B = 3n_A \quad \{\text{from (1) and (2)}\}$$

$$\Rightarrow n_B = 2n_A$$

Since masses of both the gases are same, so

$$n_B M_B = n_A M_A \quad \left\{ \because n = \frac{m}{M} \right\}$$

Hence, molecular weights are in the ratio

$$\frac{M_A}{M_B} = \frac{n_B}{n_A} = 2$$

Hence, the correct answer is (B).

7. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$$\Rightarrow v_{\text{rms}} \propto \sqrt{T}$$

v_{rms} is to reduce to time i.e., temperature of the gas will have no reduce four times or

$$\frac{T'}{T} = \frac{1}{4}$$

During adiabatic process

$$TV^{\gamma-1} = T'V'^{\gamma-1}$$

$$\Rightarrow \frac{V'}{V} = \left(\frac{T}{T'}\right)^{\frac{1}{\gamma-1}} = (4)^{\frac{1}{1.5-1}} = (4)^2 = 16$$

$$\Rightarrow V' = 16V$$

Hence, the correct answer is (B).

8. $-\frac{1}{A} \frac{dQ}{dt} = \sigma(T^4 - 0^4)$

$$\Rightarrow -mc \frac{dT}{dt} = \sigma AT^4$$

$$\Rightarrow T^{-4} dT = -\frac{\sigma A}{mc} dt$$

$$\Rightarrow \int_{200 \text{ K}}^{100 \text{ K}} T^{-4} dT = -\frac{\sigma A}{mc} \int_0^t dt$$

$$\Rightarrow \left. \frac{T^{-3}}{-3} \right|_{200}^{100} = -\frac{\sigma A}{mc} t$$

$$\Rightarrow \frac{1}{(100)^3} - \frac{1}{(200)^3} = \frac{\sigma(4\pi r^2)}{\frac{4}{3}\pi r^3 \rho c} t$$

$$\Rightarrow t = \frac{7}{72} \frac{r\rho c}{\sigma} \times 10^{-6} \text{ s}$$

$$\Rightarrow t = \frac{7}{72} \frac{r\rho c}{\sigma} \mu\text{s}$$

Hence, the correct answer is (B).

9. As two arms of U-tube are maintained at different temperatures, densities in two arms will be different but pressure at the bottom is same. So, we have

$$h_1 \left(\frac{\rho_0}{1 + \gamma T_1} \right) g = h_2 \left(\frac{\rho_0}{1 + \gamma T_2} \right) g$$

$$\Rightarrow 49 \left(\frac{\rho_0}{1 + 50\gamma} \right) = 50 \left(\frac{\rho_0}{1 + 60\gamma} \right)$$

$$\Rightarrow 49 + 49(60\gamma) = 50 + 50(50\gamma)$$

$$\Rightarrow \gamma(2940 - 2500) = 1$$

$$\Rightarrow \gamma = \frac{1}{440} \approx 2.3 \times 10^{-3} \text{ } ^\circ\text{C}^{-1}$$

Hence, the correct answer is (C).

10. Mass of P = 2(Mass of Q)

$$\Rightarrow \frac{4}{3}\pi r_p^3 \rho = 2 \left(\frac{4}{3}\pi r_Q^3 \rho \right)$$

$$\Rightarrow r_p = 2^{\frac{1}{3}} r_Q$$

$$\text{Since } \frac{1}{A} \frac{dQ}{dt} \propto (T - T_0)$$

$$\Rightarrow \frac{1}{A} mc \frac{dT}{dt} \propto (T - T_0)$$

$$\Rightarrow \frac{dT}{dt} \propto \frac{A}{m}$$

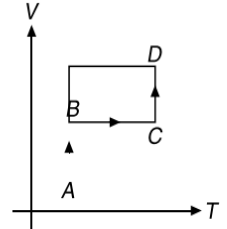
$$\Rightarrow \frac{dT}{dt} \propto \frac{4\pi r^2}{\frac{4}{3}\pi r^3 \rho}$$

$$\Rightarrow \frac{dT}{dt} \propto \frac{1}{r}$$

$$\Rightarrow \frac{\text{Rate of cooling of P}}{\text{Rate of cooling of Q}} = \frac{r_Q}{r_P} = \frac{1}{2^{1/3}}$$

Hence, the correct answer is (C).

11. Process AB is isothermal, so its V-T curve will remain a vertical straight line. Process BC is isochoric (because it is a straight line passing through origin) hence its V-T curve will be a horizontal straight line. Process CD is again isothermal, so it will be represented by a vertical straight line on V-T curve and hence the V-T curve is shown in Figure.



Hence, the correct answer is (D).

12. $\frac{mL}{t} = \frac{kA(T_1 - T_2)}{d}$

$$\Rightarrow T_1 - T_2 = \frac{mLd}{KA t}$$

$$\Rightarrow T_1 - T_2 = \left(\frac{1}{60} \right) \frac{(540)(0.2)}{(0.5)(300)}$$

$$\Rightarrow T_1 - T_2 = 0.012 \text{ } ^\circ\text{C}$$

Hence, the correct answer is (D).

13. From First Law of Thermodynamics,

$$dQ = dU + dW$$

$$\Rightarrow dQ = dU \text{ if } dW = 0$$

Since $dQ < 0$ therefore, $dU < 0$

$$\Rightarrow U_{\text{final}} < U_{\text{initial}}$$

\Rightarrow Temperature will decrease.



Conceptual Note(s)

Internal energy U of an ideal gas depends only on the temperature of the gas. Internal energy of n moles of an ideal gas is given by

$$U = \left(\frac{nf}{2} \right) RT$$

$$U \propto T$$

Here, f is the degree of freedom of the gas.

Hence, the correct answer is (A).

14. Heat released when 5 kg of water cools from $20 \text{ } ^\circ\text{C}$ to $0 \text{ } ^\circ\text{C}$ is

$$Q_1 = m_{\text{water}} c_{\text{water}} \Delta T = (5)(1)(20) = 100 \text{ kcal}$$

Heat required to raise temperature of 2 kg of ice from $-20 \text{ } ^\circ\text{C}$ to $0 \text{ } ^\circ\text{C}$ is

$$Q_2 = m_{\text{ice}} c_{\text{ice}} \Delta T = (2)(0.5)(20) = 20 \text{ kcal}$$

H.102 JEE Advanced Physics: Waves and Thermodynamics

Heat required to melt 2 kg of ice at 0 °C is

$$Q_3 = m_{\text{ice}}L_{\text{ice}} = (2)(80) = 160 \text{ kcal}$$

Since $Q_1 > Q_2$, so temperature of ice will reach 0 °C. However we observe that $Q_1 < Q_2 + Q_3$, therefore the complete ice will not melt and final mixture will have both ice and water. The amount of ice melted m is

$$m = \frac{\text{Available Heat}}{\text{Latent Heat}} = \frac{(100 - 20) \text{ kcal}}{80 \text{ cal g}^{-1}} = 1 \text{ kg}$$

So in equilibrium, the vessel will have water given by

$$m'_{\text{water}} = 5 + 1 = 6 \text{ kg}$$

Hence, the correct answer is (C).

15. Let the initial pressure and volume be P and V respectively.

$$\Rightarrow P_f = P + \frac{P}{100} = \frac{101}{100}P$$

For Boyle's Law

$$P_i V_i = P_f V_f$$

$$\Rightarrow V_f = \frac{100}{101}V$$

$$\Rightarrow \Delta V = \frac{1}{101}V$$

$$\Rightarrow \frac{\Delta V}{V} \times 100 = \frac{100}{101}\%$$

Hence, the correct answer is (B).

16. Work done (W) = $fx = \mu mg(vt)$

Since half the heat produced is absorbed by body, so

$$\frac{1}{2}\mu mg(vt) = mc\Delta T$$

$$\Rightarrow \frac{1}{2}\mu g(vt) = c\Delta T$$

$$\Rightarrow \frac{1}{2}(0.5)(9.8)(2000) = (0.1 \times 4200)\Delta T$$

$$\Rightarrow \Delta T = 11.6 \text{ }^\circ\text{C}$$

Hence, the correct answer is (D).

17. To double rms speed, temperature must be raised to four times the initial temperature, i.e.

$$T_f = 4 \times 300 = 1200 \text{ K.}$$

Heat required for doing this is

$$Q = \Delta U = n_1(C_V)_1 \Delta T + n_2(C_V)_2 \Delta T_2$$

$$\Rightarrow Q = [(1)(3R/2) + (1)(5R/2)](1200 - 300)$$

$$\Rightarrow Q = 3600R$$

Hence, the correct answer is (D).

18. $U = nC_v T = n\left(\frac{3}{2}R\right)T$

$$\Rightarrow \frac{U}{V} = \frac{3nRT}{2V} = \frac{3}{2}P \quad \left\{ \because \frac{nRT}{V} = P \right\}$$

Hence, the correct answer is (B).

19. Let M be the mass of the sphere and R its radius before increasing the temperature. Then From Conservation of Angular Momentum:

$$I\omega = I_0\omega_0$$

$$\Rightarrow \omega = \left(\frac{I_0}{I}\right)\omega_0$$

$$\Rightarrow \omega = \frac{\frac{2}{5}MR^2\omega_0}{\frac{2}{5}MR^2(1 + 2\alpha\Delta T)} = \frac{\omega_0}{1 + 2\alpha\Delta T}$$

$$\Rightarrow \omega = \frac{\omega_0}{1 + 2(20 \times 10^{-5})(100)} = 0.996\omega_0$$

Hence, the correct answer is (C).

20. Since $\beta = \frac{277}{300 - 277}$

$$\Rightarrow \beta = 12$$

$$\text{Further } \beta = \frac{Q_2}{W} = 12$$

$$\Rightarrow Q_2 = 12(1 \text{ J})$$

$$\Rightarrow Q_2 = 12 \text{ J}$$

Since by Law of Conservation of Energy, we have

$$Q_1 = Q_2 + W$$

$$\Rightarrow Q_1 = 12 + 1$$

$$\Rightarrow Q_1 = 13 \text{ J}$$

Hence, the correct answer is (D).

21. In a cyclic process, $\Delta U = 0$ i.e. $U = \text{constant}$.

Free expansion is an irreversible process.

In a cyclic process, $\Delta U = 0$, so $W = Q \neq 0$.

Efficiency of Carnot engine is 100% if temperature of sink is 0 K (not 0 °C).

Hence, the correct answer is (C).

22. Since $Q = \Delta U = U_f - U_i$, where

$$\Delta U = \left(U_{4 \text{ mole monatomic}} + U_{2 \text{ mole diatomic}} \right) - U_{4 \text{ mole diatomic}}$$

$$\Rightarrow \Delta U = \left(4 \times \frac{3}{2}RT + 2 \times \frac{5}{2}RT \right) - \left(4 \times \frac{5}{2}RT \right)$$

$$\Rightarrow \Delta U = (6RT + 5RT) - (10RT) = RT$$


Conceptual Note(s)

(a) Two moles of diatomic gas become 4 moles of a monatomic gas when get dissociated into atoms.

(b) Internal energy of n moles of an ideal gas of degrees of freedom f is given by

$$U = \left(\frac{nf}{2} \right) RT$$

$f = 3$ for a monatomic gas and $f = 5$ for diatomic gas

Hence, the correct answer is (B).

23. $\left(\begin{array}{c} \text{Heat} \\ \text{conducted} \\ \text{in at A} \\ \text{per second} \end{array} \right) + \left(\begin{array}{c} \text{Heat} \\ \text{generated} \\ \text{by source} \\ \text{per second} \end{array} \right) = \left(\begin{array}{c} \text{Heat} \\ \text{conducted} \\ \text{out at B} \\ \text{per second} \end{array} \right)$

$$\frac{(0.5)(12)(100 - T)}{8} + 36 = \frac{(0.5)(12)(T - 4)}{8}$$

Solving the above equation for T , we have

$$T = 76 \text{ }^\circ\text{C}$$

Hence, the correct answer is (C).

24. Since, $V \propto T$, the process is isobaric

$$\Rightarrow \frac{Q}{W} = \frac{nC_p \Delta T}{nR \Delta T} = \frac{C_p}{R} = \frac{5}{2}$$

Hence, the correct answer is (A).

25. $\frac{Q}{t} = kA \frac{\Delta T}{\ell}$

$$\Rightarrow k \propto \frac{1}{\left(\frac{\Delta T}{\ell}\right)}$$

$$\Rightarrow k \propto \frac{1}{\text{Temperature Gradient}}$$

Since, $K_C > K_M > K_G$

$$\Rightarrow X_C < X_M < X_G$$

Hence, the correct answer is (C).

26. Let initially n be the total number of moles of O_2 . Then after dissociation

$$C_v(\text{mixture}) = \frac{(0.6)(2)(n)C_{v_1} + (0.4)(n)C_{v_2}}{(0.6)(2)n + (0.4)n}$$

$$\Rightarrow (C_v)_{\text{mix}} = \frac{\left(1.2 \times \frac{3}{2} \times R\right) + \left(0.4 \times \frac{5}{2} \times R\right)}{1.6}$$

$$\Rightarrow (C_v)_{\text{mix}} = \frac{1.8R + R}{1.6} = \frac{7}{4}R$$

Hence, the correct answer is (C).

27. Since gas is undergoing a process given by

$$PT = \text{constant}$$

$$\Rightarrow P_0 T_0 = \frac{P_0}{2} T$$

$$\Rightarrow T = 2T_0$$

Applying Ideal Gas Equation, we get

$$P_0 V_0 = 2RT_0$$

So, change in internal energy of the gas is

$$\Delta U = nC_v \Delta T = 2 \left(\frac{3R}{2}\right) (T - T_0) = 3RT_0 = \frac{3P_0 V_0}{2}$$

Hence, the correct answer is (B).

28. $P = \frac{\mu RT}{V}$

where, μ is the number of molecules of the gas

$$R = k_B N_A$$

where N_A is Avogadro's Number.

$$\Rightarrow P = \frac{\mu k_B N_A T}{V} = \left(\frac{\mu N_A}{V}\right) k_B T$$

$$\frac{\mu N_A}{V} = \text{number of molecules per unit volume } (n)$$

$$\Rightarrow P = nk_B T$$

Hence, the correct answer is (B).

29. $H_1 + H_2 = \frac{KA(T_1 - T_2)}{3\ell} + \frac{KA(T_1 - T_2)}{\ell} = \frac{4}{3\ell} KA(T_1 - T_2)$

In later case

$$H_2 = 2H - H_1 = \frac{7KA}{3\ell} (T_1 - T_2) = \frac{K'A}{\ell} (T_1 - T_2)$$

$$\Rightarrow K' = \frac{7}{3}K$$

Hence, the correct answer is (A).

30. $Q = \int_0^{20} mS dT = \int_0^{20} m(AT^3) dT = 4 \times 10^4 \text{ mJ}$

Hence, the correct answer is (A).

31. Since the given graph is a straight line, so we have

$$P = P_0 + CT, \text{ where } C \text{ is a constant}$$

Applying Ideal Gas Equation $PV = nRT$, we get

$$V \propto \frac{T}{P_0 + CT}$$

$$\Rightarrow \frac{1}{V} \propto \left(\frac{P_0}{T} + C\right)$$

So, as T increases, $\frac{1}{V}$ decreases. Hence, volume increases.

Hence, the correct answer is (C).

32. Since, $kA \left(\frac{\sqrt{3}T - T_C}{l}\right) = kA \left(\frac{T_C - T}{l}\right)$

$$\Rightarrow \sqrt{3}T - T_C = T_C - T$$

$$\Rightarrow (\sqrt{3} + 1)T = 2T_C$$

$$\Rightarrow \frac{T_C}{T} = \frac{T_C}{T_B} = \frac{\sqrt{3} + 1}{2}$$

Hence, the correct answer is (A).

33. $PV = \text{constant}$

$$\Rightarrow T = \text{constant}$$

$$\text{Now, } \rho = \frac{PM}{RT}$$

$$\Rightarrow \rho \propto P \text{ for } T = \text{constant}$$

Hence, P - ρ graph is a straight line passing through origin.

Hence, the correct answer is (C).

34. By Laws of Calorimetry

$$\left(\begin{array}{c} \text{Total heat lost} \\ \text{by steam} \end{array}\right) = \left(\begin{array}{c} \text{Total heat gained} \\ \text{by water + ice} \end{array}\right)$$

Let m gram of steam be required for the purpose. Then,

$$Q_{\text{available}} = \underbrace{540m}_{\text{heat liberated due to condensation of } m \text{ gram steam at } 100^\circ\text{C to water}} + \underbrace{m(1)(100 - 5)}_{\text{heat liberated due to decrease in temperature from } 100^\circ\text{C to } 5^\circ\text{C}}$$

H.104 JEE Advanced Physics: Waves and Thermodynamics

$$\text{and } Q_{\text{required}} = \underbrace{100(1)(5-0)}_{\text{heat required to raise temperature of 100 g of water from } 0^\circ\text{C to } 5^\circ\text{C}} + \underbrace{10(80) + 10(1)(5-0)}_{\text{heat required to convert 10 g of ice to 10 g of water at } 0^\circ\text{C and then raise temperature of 10 g water from } 0^\circ\text{C to } 5^\circ\text{C}}$$

By Law of Calorimetry, $Q_{\text{available}} = Q_{\text{required}}$

$$\Rightarrow m = 2.1 \text{ g}$$

Hence, the correct answer is (A).

35. By FLTD, $Q = \Delta U + W$. Since ΔU is same for both as it depends upon initial and final state, so $\Delta U_1 = \Delta U_2$. Also, work done for process A is more than that for B because area under the P - V graph for the process A is more than that for process B and hence $Q_1 > Q_2$.

Hence, the correct answer is (D).

36. $\frac{\Delta T}{\ell} = \frac{80-0}{1} = 80^\circ\text{Cm}^{-1}$

$$\Delta T_1 = (0.60)(80) = 48^\circ\text{C}$$

Decrease in temperature after a distance of 60 cm is 48°C and hence actual temperature is $80 - 48 = 32^\circ\text{C}$

Hence, the correct answer is (B).

37. **On Mixing A and C**

Heat lost by A = Heat gained by C

$$\Rightarrow (4)s_A(60-55) = (3)s_C(55-50)$$

$$\Rightarrow 4s_A = 3s_C \quad \dots(1)$$

On Mixing A and B

Heat lost by A = Heat gained by B

$$2s_A(60-57) = 3s_B(57-55)$$

$$\Rightarrow 6s_A = 6s_B$$

$$\Rightarrow s_A = s_B \quad \dots(2)$$

On mixing equal masses of B and C , let equilibrium temperature be T .

Heat lost by B = Heat gained by C

$$ms_B(55-T) = ms_C(T-50) \quad \dots(3)$$

$$\Rightarrow \frac{s_B}{s_C} = \frac{3}{4} \quad \{\because \text{ of (1) and (2)}\}$$

So, equation (3) becomes $\frac{3}{4} = \frac{T-50}{55-T}$

$$\Rightarrow 165 - 3T = 4T - 200$$

$$\Rightarrow 7T = 365$$

$$\Rightarrow T = \frac{365}{7} = 52.1$$

Hence, the correct answer is (A).

38. For the given process $Q = \Delta U + W$

$$\Rightarrow 200 - 100 = \Delta U - 50$$

$$\Rightarrow \Delta U = 150 \text{ kJ}$$

So for the adiabatic process which restores original state, ΔU must be -150 kJ and hence work done in the process will be $+150 \text{ kJ}$

Hence, the correct answer is (C).

39. $W_{BA} = -30 \text{ J}$, $Q_{BA} = 0$

$$\Rightarrow \Delta U_{BA} = -W_{BA} = 30 \text{ J}$$

Now, $\Delta U_{AB} = -U_{BA} = -30 \text{ J}$

Hence, the correct answer is (B).

40. Since air bubble doubles in radius. Hence its volume becomes 8 times. Further since the bubble rises slowly, hence the rising process can be regarded as an isothermal process. So according to Boyle's Law.

$$(PV)_{\text{at depth } h} = (PV)_{\text{at surface}}$$

If P_a be the atmospheric pressure then

$$(P_a + h\rho g)V = P_a(8V) \quad \{\text{where } P_a = H\rho g\}$$

$$\Rightarrow P_a + h\rho g = 8P_a$$

$$\Rightarrow h\rho g = 7P_a = 7(H\rho g)$$

$$\Rightarrow h = 7H$$

Hence, the correct answer is (C).

41. Since, $PV = nRT$, so $T \propto PV$

$$\Rightarrow T_1 : T_2 : T_3 = 1 : 4 : 4$$

Also, average molecular speed i.e. $v_{av} = v \propto \sqrt{T}$

$$\Rightarrow v_1 : v_2 : v_3 = 1 : 2 : 2$$

Hence, the correct answer is (A).

42. $\frac{PV}{T} = nR = \left(\frac{m}{M}\right)R$

$$\Rightarrow \frac{PV}{T} = \left(\frac{R}{M}\right)m$$

i.e., $\frac{PV}{T}$ versus m graph is a straight line passing through origin with slope $\frac{R}{M}$ i.e., the slope depends on molecular

mass of the gas M and is different for different gases.

Hence, the correct answer is (C).

43. Rate of flow of heat $\frac{dQ}{dt}$ or H is equal throughout the rod.

Temperature difference is given by

$$\text{Since, } \Delta T = HR_{\text{Th}}$$

$$\text{where, } R_{\text{Th}} = \frac{\ell}{KA}$$

$$\Rightarrow R_{\text{Th}} \propto \frac{1}{A}$$

Area across CD is less. Therefore, temperature difference across CD will be more.

Hence, the correct answer is (C).

44. By FLTD, $Q = \Delta U + W$

Since U is a state function i.e. ΔU is same for all processes carried between same points.

For process $a \rightarrow c \rightarrow e$, net work done is negative

For process $a \rightarrow d \rightarrow e$, net work done is zero.

For process $a \rightarrow b \rightarrow e$, net work done is positive. Therefore Q is minimum for the process $a \rightarrow c \rightarrow e$.

Hence, the correct answer is (D).

45. $\eta = 1 - \left(\frac{T_2}{T_1}\right)$ and $\beta = \frac{T_2}{T_1 - T_2}$

$$\begin{aligned} \Rightarrow \eta\beta &= \frac{T_2}{T_1} \\ \Rightarrow \eta\beta &= 1 - \eta \\ \Rightarrow \eta(\beta + 1) &= 1 \\ \Rightarrow \eta &= \frac{1}{\beta + 1} \end{aligned}$$

Hence, the correct answer is (C).

46. η is maximum, when β is minimum i.e., 1

$$\Rightarrow \eta_{\max} = \frac{1}{2} = 50\%$$

Hence, the correct answer is (B).

47. Since, $PV = nRT$ i.e. $T \propto PV$

$$\Rightarrow T_1 : T_2 = 2 \times 2 : 1 \times 1 = 4 : 1$$

$$\text{Also, } v_{\text{rms}} \propto \sqrt{T}$$

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{4}{1}} = 2$$

Hence, the correct answer is (C).

48. For the two sheets if H is rate of heat transfer, then

$$\begin{aligned} H_1 &= H_2 \\ \Rightarrow \frac{\theta_1 - \theta}{R_1} &= \frac{\theta - \theta_2}{R_2} \end{aligned}$$

$$\text{Solving this we get, } \theta = \frac{\theta_1 R_2 + \theta_2 R_1}{R_1 + R_2}$$

Hence, the correct answer is (D).

49. Since $P_1V = n_1RT_1$ and $P_2V = n_2RT_2$

After joining the vessels we have

$$\begin{aligned} P(2V) &= (n_1 + n_2)RT \\ \Rightarrow P(2V) &= \left(\frac{P_1V}{RT_1} + \frac{P_2V}{RT_2} \right) RT \\ \Rightarrow P &= \frac{1}{2} \left(\frac{P_1}{T_1} + \frac{P_2}{T_2} \right) T \end{aligned}$$

Hence, the correct answer is (B).

50. Gas in cylinder A will undergo an isobaric process and gas in cylinder B will undergo an isochoric process. Since,

$$\begin{aligned} Q_A &= Q_B \\ \Rightarrow nC_p\Delta T_A &= nC_v\Delta T_B \\ \Rightarrow \left(\frac{5}{2}R \right) (15) &= \left(\frac{3}{2}R \right) \Delta T_B \\ \Rightarrow \Delta T_B &= 25 \text{ K} \end{aligned}$$

Hence, the correct answer is (B).

51. $\rho = \frac{PM}{RT}$ and $\rho \propto \frac{1}{V}$

During AB , ρ and hence V is constant.

Therefore, work done is zero.

During BC , $P \propto \rho$ i.e., T and hence, U is constant.

Hence, the correct answer is (D).

52. $P \propto AT^4$

Hence, the correct answer is (C).

53. $C = \frac{dQ}{dT} = 0$, so process is adiabatic, i.e., $n = \gamma = \frac{C_p}{C_v}$

Hence, the correct answer is (A).

54. Since the wall is conducting, temperature of both the gases is same. For rms speed of molecule of A to be equal to mean speed of molecules of B , we have

$$\sqrt{\frac{3RT}{M_A}} = \sqrt{\frac{8RT}{\pi M_B}} \Rightarrow \frac{M_A}{M_B} = \frac{3\pi}{8}$$

Hence, the correct answer is (D).

55. **Method I:** Heat transferred to the gas is

$$\begin{aligned} Q &= \Delta U + W, \text{ where } \Delta U = (1)C_v(T_2 - T_1) \\ \Rightarrow \Delta U &= (1)C_v[T_0 + \alpha V_2 - T_0 - \alpha V_1] \\ \Rightarrow \Delta U &= \alpha C_v(V_2 - V_1) \end{aligned} \quad \dots(1)$$

$$\text{Since, } dW = PdV = \frac{RT}{V}dV = R\left(\frac{T_0 + \alpha V}{V}\right)dV$$

$$\begin{aligned} \Rightarrow W &= \int dW = R \left[T_0 \int_{V_1}^{V_2} \frac{dV}{V} + \alpha \int_{V_1}^{V_2} dV \right] \\ \Rightarrow W &= RT_0 \ln \left(\frac{V_2}{V_1} \right) + \alpha R(V_2 - V_1) \end{aligned} \quad \dots(2)$$

$$\text{So, } Q = \alpha C_v(V_2 - V_1) + RT_0 \ln \left(\frac{V_2}{V_1} \right) + \alpha R(V_2 - V_1)$$

$$\Rightarrow Q = \alpha(V_2 - V_1)(C_v + R) + RT_0 \ln \left(\frac{V_2}{V_1} \right)$$

$$\Rightarrow Q = RT_0 \ln \left(\frac{V_2}{V_1} \right) + \alpha C_p(V_2 - V_1)$$

Method II: Since $T = T_0 + \alpha V$

<p>(a) Think $\alpha = 0$, then $T = T_0 = \text{constant}$ (Isothermal process) $\Rightarrow \Delta U = 0$ $\Rightarrow Q_1 = RT_0 \ln \left(\frac{V_2}{V_1} \right)$</p>	<p>(b) Think $T_0 = 0$, then $T = \alpha V$ where, α is a constant (Isobaric process) $\Rightarrow Q_2 = (1)C_p(T_2 - T_1)$ Since, $T_2 = T_0 + \alpha V_2$ and $T_1 = T_0 + \alpha V_1$ $\Rightarrow Q_2 = \alpha C_p(V_2 - V_1)$</p>
---	---

SUPERIMPOSING THE TWO, WE GET

$$Q = RT_0 \ln \left(\frac{V_2}{V_1} \right) + \alpha C_p(V_2 - V_1)$$

Hence, the correct answer is (C).

56. Process A is an isobaric expansion process, as V increases, T also increases.
 Process B is an isothermal process, so temperature is constant.
 Process C is an adiabatic expansion process, so as V increases, T decreases.
 Process D is an isochoric process in which P is decreasing, so T is also decreasing.

Hence, the correct answer is (C).

57. When the temperature of the gas is same as that of the wall there is no exchange of energy. Hence, molecules return with same average speed whether the collision is elastic or inelastic. Hence, the temperature and pressure would not change.
 Hence, the correct answer is (C).

58. Since $T = KV^2$, so $dT = 2KVdV$

$$\Rightarrow dV = \frac{dT}{2KV}$$

Further, $P = \frac{nRT}{V}$

$$\Rightarrow W = \int PdV = \int \left(\frac{nRT}{V} \right) \left(\frac{dT}{2KV} \right)$$

$$\Rightarrow W = \int_{T_0}^{4T_0} \frac{nR}{2} dT = \frac{3nRT_0}{2} \quad \left\{ \because KV^2 = T \right\}$$

Hence, the correct answer is (C).

59. In this problem two concepts are used:
 (i) When a solid floats in a liquid, then fraction of volume submerged is

$$k = \frac{\rho_{\text{solid}}}{\rho_{\text{liquid}}}$$

This result comes from the fact that

$$\text{Weight} = \text{Upthrust}$$

$$V\rho_{\text{solid}}g = V_{\text{submerged}}\rho_{\text{liquid}}g$$

$$\Rightarrow \frac{V_{\text{submerged}}}{V} = \frac{\rho_{\text{solid}}}{\rho_{\text{liquid}}}$$

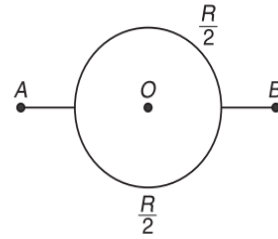
(ii) $\frac{\rho_T}{\rho_0} = \frac{1}{1 + \gamma T}$

$$k_1 = \left(\frac{\rho_{Fe}}{\rho_{Hg}} \right), k_2 = \left(\frac{\rho_{Fe}}{\rho_{Hg}} \right)_{60^\circ\text{C}} \quad \text{and } \rho = \text{density}$$

$$\Rightarrow \frac{k_1}{k_2} = \frac{(\rho_{Fe})_{0^\circ\text{C}}}{(\rho_{Hg})_{0^\circ\text{C}}} \times \left(\frac{\rho_{Hg}}{\rho_{Fe}} \right) = \frac{(1 + 60\gamma_{Fe})}{(1 + 60\gamma_{Hg})}$$

Hence, the correct answer is (A).

60. When $\theta = 180^\circ$, then thermal resistance of each branch is $\frac{R}{2}$.
 So, net thermal resistance is $\frac{R}{4}$. Further



$$\text{Thermal Current} = 1.2 = \frac{\Delta T}{R_{\text{net}}} = \frac{\Delta T}{\left(\frac{R}{4} \right)}$$

$$\Rightarrow \Delta T = 0.3R \quad \dots(1)$$

When $\theta = 90^\circ$, then thermal resistance of one branch is $\frac{R}{4}$ and that of other is $\frac{3R}{4}$ and both are in parallel.

$$\Rightarrow R_{\text{net}} = \frac{\left(\frac{R}{4} \right) \left(\frac{3R}{4} \right)}{\frac{R}{4} + \frac{3R}{4}} = \frac{3R}{16}$$

$$\Rightarrow I' = \frac{\Delta T}{\left(\frac{3R}{16} \right)} = \frac{(0.3R)16}{3R} = 1.6 \text{ W}$$

Hence, the correct answer is (C).

61. Net rate of absorption of heat by water is

$$\Rightarrow 840\Delta t = mc\Delta T = 2 \times 4200 \times (77 - 27)$$

$$\Rightarrow \Delta t = \frac{2 \times 4200 \times 50}{840} = 500 \text{ s} = 8 \text{ min } 20 \text{ s}$$

Hence, the correct answer is (A).

62. $C_p = \left(\frac{\gamma}{\gamma - 1} \right) R$

$$dQ = nC_p dT \quad \dots(1)$$

and $dU = nC_v dT$

Using FLTD, we get

$$dW = dQ - dU = n(C_p - C_v) dT = nRdT \quad \dots(2)$$

Given $dQ = 100 \text{ J}$

$$\Rightarrow ndT = \frac{100}{C_p} \quad \text{\{from equation (1)\}}$$

Substituting this in equation (2), we get

$$dW = R \left(\frac{100}{C_p} \right) = R \left(\frac{\gamma - 1}{\gamma R} \right) 100$$

$$\Rightarrow dW = \left(\frac{1.4 - 1}{1.4} \right) \times 100 = 28.57 \text{ J}$$

Hence, the correct answer is (D).

63. For the process shown, $32P_0V_0 = RT_A$ and $8P_0V_0 = RT_B$

$$\Rightarrow T_B - T_A = (8 - 32) \frac{P_0V_0}{R} = -\frac{24P_0V_0}{R}$$

$$\Rightarrow \Delta U = 1.5R(T_B - T_A) = -36P_0V_0$$

$$\text{Also } W = \frac{1}{2}(32P_0 + P_0)(8V_0 - V_0) = 115.5(P_0V_0)$$

Applying FLTD, we get

$$Q = \Delta U + W = -36P_0V_0 + 115.5(P_0V_0) = 79.5P_0V_0$$

The molar specific heat C of the gas is

$$C = \frac{Q}{\Delta T} = \frac{79.5(P_0V_0)}{-(24P_0V_0/R)} = -3.3R$$

Hence, the correct answer is (B).

64. In Ingen Hausz experiment

$$k \propto (\text{length})^2$$

$$\Rightarrow \frac{k_1}{k_2} = \left(\frac{5}{10}\right)^2 = \frac{1}{4}$$

Hence, the correct answer is (B).

65. Since, $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

$$\Rightarrow P + \frac{a}{V^2} = \frac{RT}{V - b}$$

$$\Rightarrow P = \frac{RT}{V - b} - \frac{a}{V^2}$$

$$\text{Further, } W = \int dW = \int_{V_1}^{V_2} PdV$$

$$\Rightarrow W = RT \int_{V_1}^{V_2} \frac{dV}{V - b} - a \int_{V_1}^{V_2} V^{-2} dV$$

$$\Rightarrow W = RT \ln \left(\frac{V_2 - b}{V_1 - b} \right) + a \left(\frac{1}{V_2} - \frac{1}{V_1} \right)$$

Hence, the correct answer is (D).

66. Thermal expansion for both is the same, but in the hollow sphere the air inside it also expands, thus exerting an extra thrust on the walls of the hollow sphere.

Hence, the correct answer is (C).

67. For Process 1, $W = 0$, so process is isochoric.

For Process 2, the positive work done increases linearly with temperature, so process is isobaric.

For Process 3, the gas heats a linearly with compression i.e. negative work is done and hence process is adiabatic.

Hence, the correct answer is (D).

68. We observe that W_{AB} is negative (volume is decreasing) and W_{BC} is positive (volume is increasing) and since $|W_{BC}| > |W_{AB}|$, so net work done is positive and area between semicircle which is equal to $\frac{\pi}{2}$ atm.l.

Hence, the correct answer is (B).

69. $W_{AB} = -P_0V_0$

$$W_{BC} = 0 \text{ and } W_{CD} = 4P_0V_0$$

$$\Rightarrow W_{ABCD} = -P_0V_0 + 0 + 4P_0V_0 = 3P_0V_0$$

Hence, the correct answer is (C).

70. Net rate of absorption of heat by water is

$$\frac{\Delta Q}{\Delta t} = 1000 - 160 = 840 \text{ Js}^{-1}$$

$$\Rightarrow 840\Delta t = mc\Delta T = 2 \times 4200 \times (77 - 27)$$

$$\Rightarrow \Delta t = \frac{2 \times 4200 \times 50}{840} = 500 \text{ s} = 8 \text{ min } 20 \text{ s}$$

Hence, the correct answer is (A).

71. Since, $\eta = \frac{W}{Q_1} = 1 - \frac{300}{600} = 0.5$

$$\Rightarrow \frac{800}{Q_1} = 0.5$$

$$\Rightarrow Q_1 = 1600 \text{ J}$$

Hence, the correct answer is (C).

72. Since, radius or volume of all the four spheres are equal, the ratio of their masses will be 2 : 3 : 5 : 1.

$$\text{Heat capacity} = (\text{mass}) \times (\text{specific heat})$$

So, ratio of heat capacities will be 6 : 18 : 10 : 4. The sphere having the maximum heat capacity will show the fastest rate of cooling.

Hence, the correct answer is (B).

73. Efficiency of heat engine,

$$\eta = 1 - \frac{\text{Total Heat Rejected}}{\text{Total Heat Absorbed}}$$

$$\Rightarrow \eta = 1 - \frac{100 + 50}{500 + 150} = \frac{500}{650} = 0.77$$

Hence, the correct answer is (A).

74. Internal energy of n moles of an ideal gas having f degrees of freedom at temperature T is given by $U = \frac{f}{2}nRT$

$$\text{Since, } U_1 = U_2$$

$$\Rightarrow f_1 n_1 T_1 = f_2 n_2 T_2$$

$$\Rightarrow \frac{n_1}{n_2} = \frac{f_2 T_2}{f_1 T_1} = \frac{(3)(2)}{(5)(1)} = \frac{6}{5}$$

where f_2 = degrees of freedom of He = 3

and f_1 = degrees of freedom of He = 5

Hence, the correct answer is (C).

75. For $A \rightarrow B$

$V \propto T$ (i.e., $P = \text{constant} = P_1$ (say)) i.e. A to B is Isobaric expansion, as T is increasing. Since $PV = RT$

$$\Rightarrow \frac{V}{T} = (\text{slope})_{AB} = \frac{R}{P_1} \quad \dots(1)$$

For $B \rightarrow C$

$T = \text{constant}$ (say T_1) and V is increasing i.e. P must be decreasing. So $B \rightarrow C$ is Isothermal expansion.

For $C \rightarrow D$

Again $V \propto T$ with greater slope for this process (i.e. $P = \text{constant} = P_2$ (say)) i.e. $C \rightarrow D$ is Isobaric compression as T is decreasing. Since $PV = RT$

$$\Rightarrow \frac{V}{T} = (\text{slope})_{CD} = \frac{R}{P_2} \quad \dots(2)$$

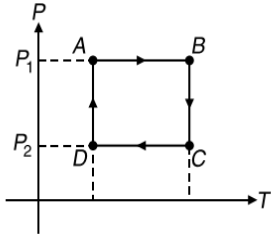
H.108 JEE Advanced Physics: Waves and Thermodynamics

For $D \rightarrow A$

$T = \text{constant}$ (say T_2) and V is decreasing, so P must be increasing. So $D \rightarrow A$ is Isothermal compression.

Also $(\text{slope})_{CD} > (\text{slope})_{AB}$

$$\Rightarrow P_1 > P_2$$



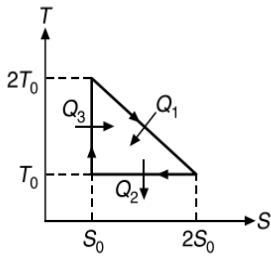
Hence, the correct answer is (A).

76. Change in entropy is given by $dS = \frac{dQ}{T}$

Therefore, heat exchange Q in a process is

$$Q = \int dQ = \int TdS = \text{Area under } T\text{-}S \text{ graph}$$

$$\Rightarrow Q_1 = T_0 S_0 + \frac{1}{2} T_0 S_0 = \frac{3}{2} T_0 S_0$$



$$\Rightarrow Q_2 = -T_0 S_0 \text{ and } Q_3 = 0$$

So, total work done in this cyclic process is

$$W = \Sigma Q = Q_1 + Q_2 + Q_3 \quad \left\{ \because \Delta U_{\text{cyclic}} = 0 \right\}$$

$$\Rightarrow W = \frac{3}{2} T_0 S_0 - T_0 S_0 = \frac{1}{2} T_0 S_0$$

$$\Rightarrow \eta = \frac{W}{Q_1} = \frac{T_0 S_0 / 2}{3 T_0 S_0 / 2} = \frac{1}{3}$$

Hence, the correct answer is (A).

77. Since $\gamma = 1 + \frac{2}{f}$, so for $f = 6$, we get

$$\gamma = \frac{4}{3}$$

Also, $dW = 25 \text{ J}$

By First Law of Thermodynamics $dQ = dU + dW$

where, $dQ = nC_p dT$, $dU = nC_v dT$, $dW = PdV = nRdT$

$$\text{Also, } \frac{dW}{dQ} = \frac{dQ - dU}{dQ} \quad \left\{ \text{Using First Law} \right\}$$

$$\Rightarrow \frac{dW}{dQ} = 1 - \frac{dU}{dQ}$$

$$\Rightarrow \frac{dW}{dQ} = 1 - \frac{nC_v dT}{nC_p dT}$$

$$\Rightarrow \frac{dW}{dQ} = 1 - \frac{C_v}{C_p} = \left(1 - \frac{1}{\gamma} \right)$$

$$\Rightarrow dQ = 25 \times 4 = 100 \text{ J}$$

Hence, the correct answer is (B).

78. Since, $\frac{(Q/t)_1}{4} = \frac{k_1 r_1^2 \ell_2}{k_2 r_2^2 \ell_1}$

$$\Rightarrow \left(\frac{Q}{t} \right)_1 = \left\{ \frac{1}{2} \left(\frac{1}{4} \right) 2 \right\} 4 = 1 \text{ cal s}^{-1}$$

Hence, the correct answer is (A).

79. Since, $\frac{50 - 40}{120 - 40} = \frac{Y - (-30)}{130 - (-30)}$

$$\Rightarrow \frac{10}{80} = \frac{Y + 30}{160}$$

$$\Rightarrow \frac{1}{8} = \frac{Y + 30}{160}$$

$$\Rightarrow Y + 30 = 20$$

$$\Rightarrow Y = -10^\circ$$

Hence, the correct answer is (C).

80. For AB , an isothermal process, pressure is increasing, so volume is decreasing and hence

$$Q = W = \text{NEGATIVE}$$

Also for AB , $\Delta U = 0$ and for BC , $\Delta U = \text{NEGATIVE}$

Hence, the correct answer is (B).

81. For adiabatic process $P_1 V_1^\gamma = P_2 V_2^\gamma$

$$\Rightarrow P_1 \left(\frac{m}{\rho_1} \right)^\gamma = P_2 \left(\frac{m}{\rho_2} \right)^\gamma$$

$$\Rightarrow \frac{P_1}{\rho_1^\gamma} = \frac{P_2}{\rho_2^\gamma}$$

$$\Rightarrow \frac{P_1}{P_2} = \left(\frac{\rho_1}{\rho_2} \right)^\gamma = \left(\frac{1}{32} \right)^{7/5}$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{1}{2^7} = \frac{1}{128}$$

Hence, the correct answer is (B).

82. $PV = nRT$ i.e., $n = \frac{PV}{RT}$

$$\Rightarrow n \propto \frac{PV}{T}$$

Number of molecules are directly proportional to the number of moles of the gas. (number of molecules = nN)

Here, P and T are doubled while volume is halved. Therefore, number of moles and hence, number of molecules will become half.

Hence, the correct answer is (B).

83. Condition for a balanced Wheatstone Bridge.

$$\frac{k_1}{k_2} = \frac{k_3}{k_4}$$

$$\Rightarrow k_1 k_4 = k_3 k_2$$

Hence, the correct answer is (A).

84. Coefficient of performance is $\beta = \frac{Q_2}{W} = 6$

$$\Rightarrow Q_2 = 6 \times 4 \times 10^6 = 24 \times 10^6 \text{ J}$$

So, mass of water converted to ice is

$$m = \frac{Q_2}{L} = \frac{24 \times 10^6}{80 \times 4.2 \times 1000} \approx 72 \text{ kg}$$

Hence, the correct answer is (D).

85. Heat rejected by 100 g of water at 80 °C when its temperature becomes 0 °C is

$$Q = ms\Delta\theta = (100)(1)(80) = 8000 \text{ cal}$$

But this heat can melt

$$m = \frac{Q}{L} = \frac{8000}{80} = 100 \text{ g of ice only}$$

Hence, the temperature of the mixture is 0 °C.

Hence, the correct answer is (A).

86. We have seen in the above question that only 100 gm ice melts. Therefore, mass of water in the mixture is

$$100 + 100 = 200 \text{ g}$$

and mass of ice = 120 - 100 = 20 g and temperature of the mixture is 0 °C.

Hence, the correct answer is (C).

87. Since, $\Delta Q = nC_v\Delta T$

$$\Rightarrow 25R = (1)(C_v)(310 - 300)$$

$$\Rightarrow C_v = \frac{5}{2}R \text{ i.e., gas is diatomic}$$

$$\Rightarrow \gamma = 1.4$$

Now work done in adiabatic process

$$W = \frac{nR(T_i - T_f)}{\gamma - 1} = \frac{(1)(R)(310 - 300)}{1.4 - 1} = 2.5R$$

Alternate Solution:

$$Q = nC_v\Delta T = C_v(310 - 300)$$

$$\Rightarrow Q = 10C_v \quad \dots(1)$$

In adiabatic process, $Q = 0$

$$\Rightarrow W = -\Delta U = -nC_v(T_f - T_i) = C_v(T_i - T_f)$$

$$\Rightarrow W = C_v(310 - 300)$$

$$\Rightarrow W = 10C_v \quad \dots(2)$$

From equations (1) and (2), we see that

$$W = Q = 25R$$

Hence, the correct answer is (C).

88. $\frac{PV}{T} = \frac{P}{\rho T} = \text{constant}$

$$\Rightarrow \frac{\rho_2}{\rho_1} = \left(\frac{T_1}{T_2}\right)\left(\frac{P_2}{P_1}\right) = \left(\frac{T_0}{3T_0}\right)\left(\frac{2.5P_0}{P_0}\right) = \frac{5}{6}$$

Hence, the correct answer is (A).

89. Since, $\lambda_m T = \text{constant}$, so $T = \frac{4}{3}T_0$

Also, $P \propto T^4$ (from Stefan's Law)

$$\Rightarrow P_{\text{new}} = \frac{256}{81}P$$

Hence, the correct answer is (A).

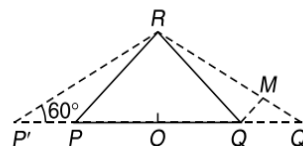
90. $W_{BCOB} = -\text{Area of triangle } BCO = -\frac{P_0V_0}{2}$

$$W_{AODA} = +\text{Area of triangle } AOD = +\frac{P_0V_0}{2}$$

$$\Rightarrow W_{\text{net}} = 0$$

Hence, the correct answer is (D).

91. Since distance OR remains unchanged, the expanded triangle on heating is shown above.



$$MQ' = QQ' \cos 60^\circ$$

$$MQ' \approx L\alpha_2\Delta T \text{ and } QQ' = 0.5L\alpha_1\Delta T$$

Therefore $L\alpha_2\Delta T = 0.5L\alpha_1\Delta T \cos 60^\circ$

$$\Rightarrow 4\alpha_2 = \alpha_1$$

Hence, the correct answer is (D).

92. $C = \frac{dQ}{ndT} = \frac{2dU}{ndT} = \frac{2nC_vdT}{ndT} = 2C_v = 3R$

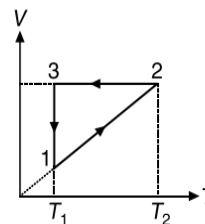
Hence, the correct answer is (B).

93. Due to thermal expansion, all x , r and d would increase.

Hence, the correct answer is (C).

94. Energy released in isochoric process 2 → 3 is

$$Q_{23} = nC_v(T_3 - T_2) = nC_v(T_1 - T_2)$$



Work done during isobaric process 1 → 2 is

$$W_{12} = nR(T_2 - T_1)$$

$$\text{Hence, } \frac{Q_{23}}{W_{12}} = -\frac{C_v}{R} = -\frac{5R/2}{R} = -2.5$$

Therefore, ratio of energy released in process 2 → 3 to work done in process 1 → 2 is 2.5

Hence, the correct answer is (B).

95. Since, $\alpha = \frac{\gamma}{3}$

$$\text{Energy Density} = \frac{(\text{Stress})^2}{2Y} = \frac{1}{2}Y(\text{Strain})^2$$

H.110 JEE Advanced Physics: Waves and Thermodynamics

$$\text{Strain} = \frac{\Delta \ell}{\ell} = \alpha \Delta T = \alpha T = \frac{\gamma T}{3}$$

$$\Rightarrow \text{Stress} = Y(\text{Strain}) = \frac{Y\gamma T}{3} \quad \left\{ \because Y = \frac{\text{stress}}{\text{strain}} \right\}$$

$$\Rightarrow U = \frac{1}{2} \frac{Y^2 \gamma^2 T^2}{9Y} = \frac{1}{18} \gamma^2 T^2 Y$$

Hence, the correct answer is (D).

96. Let, m gram of water whose temperature is T_0 ($> 30^\circ\text{C}$) and specific heat is $1 \text{ cal g}^{-1} \text{ }^\circ\text{C}$ added to 20 g of water at 30°C and let T be the final temperature of the mixture.

$$\left(\begin{array}{l} \text{Heat given by the} \\ m \text{ g of water} \end{array} \right) = \left(\begin{array}{l} \text{heat gained by} \\ 20 \text{ g of water} \end{array} \right)$$

$$\Rightarrow m(1)(T_0 - T) = (20)(1)(T - 30)$$

$$\text{Solving this we get, } T = \frac{600 + mT_0}{20 + m}$$

The right hand side is maximum for OPTION (D). Therefore, the correct answer is (D).

Hence, the correct answer is (D).

97. When temperature is increased all a , b , x and ℓ should increase.

Hence, the correct answer is (D).

$$98. \left(\frac{Q}{t} \right)_C = \left(\frac{Q}{t} \right)_A + \left(\frac{Q}{t} \right)_B$$

$$\Rightarrow k_C A \frac{\Delta T}{\ell} = k_A A \frac{\Delta T}{\ell} + k_B A \frac{\Delta T}{\ell}$$

$$\Rightarrow k_C = k_A + k_B$$

Hence, the correct answer is (A).

99. Let length of each portion of cylinder be L . Since final pressure on both sides of the movable piston must be same, therefore

$$\frac{V_A}{T_A} = \frac{V_B}{T_B}$$

$$\Rightarrow \frac{L+5}{373} = \frac{L-5}{273}$$

$$\Rightarrow L = 32.3 \text{ cm}$$

So, length of the cylinder is

$$L_{\text{cylinder}} = 2L = 64.6 \text{ cm}$$

Hence, the correct answer is (D).

$$100. k_1 A \frac{(T_1 - T)}{d_1} = k_2 A \frac{(T - T_2)}{d_2}$$

$$\Rightarrow T = \frac{k_1 T_1 d_2 + k_2 T_2 d_1}{k_1 d_2 + k_2 d_1}$$

Hence, the correct answer is (B).

$$101. \text{ Since, } W_1 = P_i(V_f - V_i) = P_i V_i \left(\frac{V_f}{V_i} - 1 \right)$$

$$\Rightarrow W_1 = nRT(2 - 1) = nRT$$

$$\text{and } W_2 = nRT \log_e \left(\frac{V_f}{V_i} \right) = nRT \log_e(2) = W_1 \log_e(2)$$

Hence, the correct answer is (A).

102. At 30°C , the copper rod will be of length $L_0(1 + \alpha_c \Delta\theta)$ while adjacent centimetre marks on the steel tape will be separated by a distance of $(1 \text{ cm})(1 + \alpha_s \Delta\theta)$. Therefore, the number of centimetres read on the tape will be

$$\frac{L_0(1 + \alpha_c \Delta\theta)}{(1 \text{ cm})(1 + \alpha_s \Delta\theta)} = \frac{(90)[1 + 1.7 \times 10^{-5}](20)}{(1)[1 + 1.2 \times 10^{-5}](20)} \approx 90.01 \text{ cm}$$

Hence, the correct answer is (D).

103. Heat exchanged, $\Delta Q = n C_V \Delta T = \frac{N}{N_A} C_V \Delta T$

where N is number of molecules of the gas and N_A is Avogadro's number.

In the given situation, let T be the final temperature of the mixture. Then, by Law of Calorimetry, we have

$$\text{Heat lost} = \text{Heat gained}$$

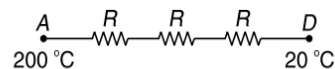
$$\Rightarrow \frac{N_1 C_V (T_1 - T)}{N_A} = \frac{N_2 C_V (T - T_2)}{N_A}$$

$$\Rightarrow N_1 T_1 - N_1 T = N_2 T - N_2 T_2$$

$$\Rightarrow T = \frac{N_1 T_1 + N_2 T_2}{N_1 + N_2}$$

Hence, the correct answer is (C).

104. Equivalent electrical circuit will be as shown in figure. Temperature difference between A and D is 180°C which is equally distributed in all the rods. Therefore, temperature difference between A and B will be 60°C or temperature of B should be 140°C .



Hence, the correct answer is (C).

105. From A to B , the process is isobaric i.e., $V \propto T$. From B to C and D to A , the process is isothermal but temperature from B to C is higher than that from D to A . From C to D , the process is isochoric.

Hence, the correct answer is (B).

106. Given that, $PT = \text{constant} \dots(1)$

According to Ideal Gas Equation, we have $PV = nRT$

$$\Rightarrow T = \frac{PV}{nR}$$

So, equation (1) becomes $P \left(\frac{PV}{nR} \right) = \text{constant}$

$$\Rightarrow P^2 V = \text{constant}$$

This is best represented by graph in OPTION (C).

Hence, the correct answer is (C).

107. Temperature decays exponentially with time.

Hence, the correct answer is (A).

108. 1 g of water equals 1 cc of water

$$\text{Volume of liquid is } V_\ell = 1 \text{ cc} = 10^{-6} \text{ m}^3$$

$$\text{Volume of vapours is } V_v = 1671 \text{ cc} = 1671 \times 10^{-6} \text{ m}^3$$

$$\Rightarrow \Delta V = V_v - V_\ell = 1670 \times 10^{-6} \text{ m}^3$$

$$\Rightarrow W = P \Delta V = 10^5 (1670 \times 10^{-6}) = 167 \text{ J}$$

$$\Rightarrow W = \frac{167}{4.18} \text{ cal} = 40 \text{ cal}$$

Further $Q = mL$

$$\Rightarrow Q = (1 \text{ g})(540 \text{ calg}^{-1}) = 540 \text{ cal}$$

According to First Law of Thermodynamics

$$Q = \Delta U + W$$

$$\Rightarrow 540 = \Delta U + 40$$

$$\Rightarrow \Delta U = 500 \text{ cal}$$

Hence, the correct answer is (B).

$$109. Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta L/L_0}$$

$$\Rightarrow Y = \frac{F/A}{\alpha \Delta T} \quad \left\{ \because \Delta L/L_0 = \alpha \Delta T \right\}$$

$$\Rightarrow F = YA\alpha \Delta T$$

Hence, the correct answer is (A).

110. Since, $PV = nRT$

For isobaric process, $V \propto T$, hence slope of the graph is given by

$$\frac{dV}{dT} = \frac{nR}{P} \propto \frac{1}{P} \text{ (if mass remains constant)}$$

Hence, the correct answer is (C).

111. Rate of melting of ice \propto rate of heat transfer $\left(\frac{dQ}{dt}\right)$

$$\text{Further } \frac{dQ}{dt} = \frac{\text{temperature difference}}{\left(\frac{\ell}{KA}\right)}$$

$$\Rightarrow \frac{dQ}{dt} \propto \frac{(\text{temperature difference})}{\ell} A$$

If temperature difference, A and ℓ are all doubled then $\frac{dQ}{dt}$ and hence, rate of melting of ice will be doubled.

Hence, the correct answer is (D).

112. Let the piston get compressed by x . Then for mechanical equilibrium, we have $P'A = P_0A + kx$

$$\Rightarrow P' = P_0 + \frac{kx}{A}$$

$$\Rightarrow 3 \times 10^5 = 1 \times 10^5 + \frac{10^4 x}{0.005}$$

$$\Rightarrow x = 0.1 \text{ m}$$

So, work done is

$$W = P_0 \Delta V + \frac{1}{2} kx^2$$

$$\Rightarrow W = (10^5)(0.005 \times 0.1) + \frac{1}{2}(10^4)(0.1)^2$$

$$\Rightarrow W = 50 + 50 = 100 \text{ J}$$

Hence, the correct answer is (A).

113. $\left(\text{Rate of Cooling}\right) \propto \left(\text{Average Excess Temperature}\right)$

$$\Rightarrow \frac{\Delta T}{\Delta t} \propto \left(\frac{T_i + T_f}{2} - T_{\text{surr}}\right)$$

Lets denote T_{surr} by T_0 , then

$$\frac{T_f - T_i}{t} \propto \left(\frac{T_i + T_f}{2} - T_{\text{surr}}\right)$$

$$\Rightarrow \frac{60 - 50}{10} \propto \left(\frac{60 + 50}{2} - T_0\right)$$

$$\Rightarrow 1 \propto 55 - T_0 \quad \dots(1)$$

$$\text{Also, } \frac{50 - 42}{10} \propto \left(\frac{50 + 42}{2} - T_0\right)$$

$$0.8 \propto 46 - T_0 \quad \dots(2)$$

$$\Rightarrow \frac{0.8}{1} = \frac{46 - T_0}{55 - T_0}$$

$$\Rightarrow \frac{4}{5} = \frac{46 - T_0}{55 - T_0}$$

$$\Rightarrow 220 - 4T_0 = 230 - 5T_0$$

$$\Rightarrow T_0 = 10 \text{ }^\circ\text{C}$$

Hence, the correct answer is (B).

$$114. r_0 \propto (600)^4 - (300)^4$$

$$r \propto (900)^4 - (300)^4$$

$$\Rightarrow \frac{r}{r_0} = \frac{(900)^4 - (300)^4}{(600)^4 - (300)^4}$$

$$\Rightarrow r = r_0 \left(\frac{9^4 - 3^4}{6^4 - 3^4}\right)$$

$$\Rightarrow r = r_0 \left(\frac{3^4 - 1}{2^4 - 1}\right)$$

$$\Rightarrow r = r_0 \left(\frac{80}{15}\right)$$

$$\Rightarrow r = \frac{16}{3} r_0$$

Hence, the correct answer is (A).

115. Molar heat capacity $C = \frac{\Delta Q}{\Delta T}$

$$\Rightarrow C = \frac{\Delta U + \Delta W}{dT}$$

ΔU is same in both the paths but

$$\Delta W_2 > \Delta W_1$$

$$\Rightarrow C_2 > C_1$$

$$\Rightarrow \frac{C_1}{C_2} < 1$$

Hence, the correct answer is (B).

116. After first stroke, we have $PV = P_1(V+v)$

$$\Rightarrow P_1 = P \left(\frac{V}{V+v}\right)$$

After second stroke, we have $P_1V = P_2(V+v)$

$$\Rightarrow P_2 = P_1 \left(\frac{V}{V+v}\right) = P \left(\frac{V}{V+v}\right)^2$$

and so on for n strokes, we have

$$P_n = P_{n-1} \left(\frac{V}{V+v}\right) = P \left(\frac{V}{V+v}\right)^n$$

Hence, the correct answer is (D).

H.112 JEE Advanced Physics: Waves and Thermodynamics

117. Let A_1 be the cross sectional area of silica cylinder, A_2 be the cross-sectional area of the mercury in the glass cylinder, then $(A_1 + A_2)$ is cross sectional area of glass cylinder and h is the height of silica cylinder, then

$$h(A_1 + A_2)(2\alpha)\Delta T = hA_2\gamma\Delta T$$

$$\Rightarrow \frac{hA_1}{hA_2} = \frac{V_{\text{silica}}}{V_{\text{mercury}}} = \frac{\gamma}{2\alpha} - 1$$

Hence, the correct answer is (C).

118. $\frac{50 - 49.9}{5} \propto \left(\frac{50 + 49.9}{2} - 30\right)$ and ... (1)

$\frac{40 - 39.9}{t} \propto \left(\frac{40 + 39.9}{2} - 30\right)$... (2)

$$\Rightarrow \frac{t}{5} = \frac{99.9 - 60}{79.9 - 60} = \frac{39.9}{19.9} = 2$$

$$\Rightarrow t = 10 \text{ s}$$

Hence, the correct answer is (C).

119. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

Room temperature $T \approx 300 \text{ K}$

$$\Rightarrow 1930 = \sqrt{\frac{3 \times 8.31 \times 10^3 \times 300}{M}}$$

$$\Rightarrow M = 2 \text{ g mol}^{-1}$$

So, the gas is H_2

Hence, the correct answer is (A).

120. For processes ab and cd , $P \propto T$ so $V = \text{constant}$ and hence processes ab and cd are isochoric, work done in these processes is zero. So, net work done in cyclic process is

$$W = W_{bc} + W_{da}$$

$$\Rightarrow W = nR(T_c - T_b) + nR(T_a - T_d) \quad \{\text{isobaric}\}$$

$$\Rightarrow W = (2)(8.31)(1600 - 800) + (2)(8.31)(400 - 800)$$

$$\Rightarrow W = 6.6 \text{ kJ}$$

Hence, the correct answer is (C).

121. Since all P , V and T are changing simultaneously and the work is being done at the expense of the internal energy of the gas so this must be an adiabatic process because for an adiabatic process $dQ = 0$ and by First Law of Thermodynamics

$$dU = -dW$$

$$\text{Hence } C = \frac{dQ}{dT} = 0$$

Hence, the correct answer is (A).

122. $\frac{dQ}{dt} \propto A$

$$\Rightarrow \left(\frac{dQ}{dt}\right)_1 = \frac{R_1^2}{R_2^2}$$

Hence, the correct answer is (C).

123. $mc\left(\frac{dT}{dt}\right) \propto A$

$$\Rightarrow \frac{dT}{dt} \propto \frac{A}{mc}$$

$$\Rightarrow \left(\frac{dT}{dt}\right)_1 = \frac{A_1 m_2}{A_2 m_1} = \frac{4\pi R_1^2 \left(\frac{4}{3}\pi R_2^3 \rho\right)}{4\pi R_2^2 \left(\frac{4}{3}\pi R_1^3 \rho\right)} = \frac{R_2}{R_1}$$

Hence, the correct answer is (B).

124. When cooled isochorically (i.e. $V = \text{constant}$)

$$\frac{P}{T} = \frac{P/n}{T'}$$

$$\Rightarrow T' = \frac{T}{n}$$

Heat exchanged is $Q_1 = C_V \Delta T$

$$\Rightarrow Q_1 = C_V \left(\frac{T}{n} - T\right) = -C_V T \left(1 - \frac{1}{n}\right)$$

When heated isobarically (i.e. $P = \text{constant}$) heat exchanged is

$$Q_2 = C_P \left(T - \frac{T}{n}\right) = C_P T \left(1 - \frac{1}{n}\right)$$

Net heat exchanged is

$$Q = Q_2 + Q_1 = C_P T \left(1 - \frac{1}{n}\right) - C_V T \left(1 - \frac{1}{n}\right)$$

$$\Rightarrow Q = (C_P - C_V) T \left(1 - \frac{1}{n}\right) = RT \left(1 - \frac{1}{n}\right)$$

Hence, the correct answer is (A).

125. Let M be the molecular mass of the gas. Then

$$\text{So, } M(C_P - C_V) = R$$

$$\Rightarrow M(525 - 315) = 8.31$$

$$\Rightarrow M = 0.0392$$

Now let ρ be the density of the gas at NTP. Then it implies that So, mass of 22.4 L or $(22.4 \times 10^{-3}) m^3$ of gas should be $(\rho \times 22.4 \times 10^{-3}) \text{ kg}$ which should be equal to its molecular mass i.e.,

$$\rho \times 22.4 \times 10^{-3} = 0.0392$$

$$\Rightarrow \rho = 1.75 \text{ kgm}^{-3}$$

Hence, the correct answer is (D).

126. For equilibrium $F_1 - F_2 = 0$

$$\Rightarrow Y_1 \alpha_1 A \Delta T - Y_2 \alpha_2 A \Delta T = 0$$

$$\Rightarrow Y_1 \alpha_1 = Y_2 \alpha_2$$

Hence, the correct answer is (C).

127. Two molecules have a speed of 1.6 kms^{-1} , so most probable speed is also 1.6 kms^{-1} .

Hence, the correct answer is (C).

128. We use $(1 + \alpha_s) \times R = L(1 + \alpha_c)$

$$\Rightarrow R = L \left[\frac{1 + \alpha_c}{1 + \alpha_s} \right]$$

Hence, the correct answer is (C).

129. From First Law of Thermodynamics $\Delta U = Q - W$
 Here $Q = mL = L$ { $\because m = 1$ }

and $W = P_0(V_f - V_i) = P_0(V_2 - V_1)$
 $\Rightarrow \Delta U = L - P_0(V_2 - V_1)$

Hence, the correct answer is (C).

130. $C_V(\text{mixture}) = \frac{n_1 C_{V1} + n_2 C_{V2}}{n_1 + n_2}$

Here, $C_{V1} = \frac{5}{2}R, n_1 = \frac{10}{2} = 5$ {for H_2 }

$C_{V2} = \frac{3}{2}R, n_2 = \frac{40}{4} = 10$ {for He }

$\Rightarrow C_V(\text{mixture}) = \frac{11}{6}R$

$\Rightarrow Q = nC_V \Delta T = 2750 \text{ cal}$

Hence, the correct answer is (B).

131. $dQ = dU + dW$

$\Rightarrow dQ = dU - \frac{2dU}{3} = \frac{dU}{3} = \frac{1}{3}nC_v dT$

$\Rightarrow dQ = \frac{1}{3}n\left(\frac{3}{2}RdT\right) = \frac{nRdT}{2}$

$\Rightarrow C = \frac{1}{n} \frac{dQ}{dT} = \frac{R}{2}$

which is not equal to 0, infinity or $\frac{5R}{2}$

Hence, the correct answer is (D).

132. Given that $P\sqrt{T} = \text{constant}$...(1)

Also, for a polytropic process, we have

$P^{1-x}T^x = \text{constant}$

$\Rightarrow PT^{1-x} = \text{constant}$...(2)

Comparing equations (1) and (2), we get $\frac{x}{1-x} = \frac{1}{2}$

$\Rightarrow 2x = 1 - x$

$\Rightarrow x = \frac{1}{3}$

$\Rightarrow C = \frac{R}{\gamma-1} + \frac{R}{1-x} = \frac{R}{(5/3)-1} + \frac{R}{1-(1/3)} = 3R$

Hence, the correct answer is (C).

133. $PV = nRT$

$\Rightarrow PdV = nRdT$ { $\because P = \text{constant}$ }

$\Rightarrow dV = \left(\frac{nR}{P}\right)dT$

$\Rightarrow dV = \left(\frac{V}{T}\right)dT$...(1)

Further $dV = \gamma VdT$...(2)

From equations (1) and (2), we get $\gamma = \frac{1}{T}$

Hence, the correct answer is (C).

134. $PV^{3/2} = \text{constant}$

$\Rightarrow \frac{PV^{2/3}}{PV} = \frac{\text{constant}}{RT}$

$\Rightarrow \frac{1}{V^{1/3}} = \frac{\text{constant}}{RT}$

$\Rightarrow V \propto T^3$

Temperature increases with increase in volume.

Hence, the correct answer is (A).

135. $K_C A \left(\frac{\Delta T_{Cu}}{\ell}\right) = K_B A \left(\frac{\Delta T_B}{\ell}\right)$

$\Rightarrow \frac{\Delta T_{Cu}}{\Delta T_B} = \frac{K_B}{K_C} < 1$

$\Rightarrow \Delta T_{Cu} < \Delta T_B$

i.e. fall in temperature for copper is less than brass and hence temperature at junction is greater than 50°C .

Hence, the correct answer is (A).

136. For the isochoric process $ab, W = 0$, so we have

$Q = \Delta U = nC_V \Delta T = 5000$

$\Rightarrow n\left(\frac{5R}{2}\right)(800 - 300) = 5000$

$\Rightarrow nR = 4$

Now, for isobaric process ca , we have

$Q_{ca} = nC_P \Delta T = n\left(\frac{7R}{2}\right)(T_a - T_c)$

Since process bc is isothermal, so $T_b = T_c$ and hence $\Delta U = 0$.

For process $ca, P = \text{constant}$

$\Rightarrow Q_{ca} = n\left(\frac{7R}{2}\right)(T_a - T_b) = (4)\left(\frac{7}{2}\right)(300 - 800)$

$\Rightarrow Q_{ca} = -7000 \text{ J}$

Hence, the correct answer is (C).

137. $\gamma = \frac{\Delta V}{V \Delta T} = \frac{0.12}{100} \times \frac{1}{20} = 6 \times 10^{-5} (\text{ }^\circ\text{C})^{-1}$

$\Rightarrow \alpha = \frac{\gamma}{3} = 2 \times 10^{-5} (\text{ }^\circ\text{C})^{-1}$

Hence, the correct answer is (A).

138. Since, $\frac{E_2}{E_1} = \left(\frac{727 + 273}{227 + 273}\right)^4 = \left(\frac{1000}{500}\right)^4 = 16$

$\Rightarrow E_2 = 16 \times 5 = 80 \text{ cal cm}^{-2}$

Hence, the correct answer is (B).

139. $dQ = +35 \text{ J}$ and $dW = -15 \text{ J}$

By First Law, we have $dQ = dU + dW$

$\Rightarrow 35 = dU - 15$

$\Rightarrow dU = +50 \text{ J}$

Hence, the correct answer is (D).

140. Internal energy of n mole of gas having f degrees of free-

dom is $U = n\left(\frac{1}{2}RT\right)f$.

For 1 mole of helium at $600 \text{ K}, f = 3$

$\Rightarrow U_1 = (1)\left[\frac{1}{2} \times R \times 600\right](3) = 900R$

H.114 JEE Advanced Physics: Waves and Thermodynamics

For 56 g of nitrogen at 300 K, $f = 5$

$$\Rightarrow U_2 = \left(\frac{56}{28}\right) \left[\frac{1}{2} \times R \times 300\right] (5) = 1500R$$

For 32 g of oxygen at 300 K, $f = 5$ and

$$U_3 = \left(\frac{32}{32}\right) \left[\frac{1}{2} \times R \times 300\right] (5) = 750R$$

For 12×10^{23} molecules of argon at 450 K, $f = 3$

$$\Rightarrow U_4 = \left(\frac{12 \times 10^{23}}{6 \times 10^{23}}\right) \left[\frac{1}{2} R \times 450\right] (3) = 1350R$$

Hence U is maximum for 56 g of nitrogen at 300 K

Hence, the correct answer is (B).

141. Since, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$$\Rightarrow T_1 (AL_1)^{\gamma-1} = T_2 (AL_2)^{\gamma-1}$$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{\frac{2}{3}}$$

Hence, the correct answer is (D).

142. $\frac{PV}{kT} = \frac{nRT}{\left(\frac{R}{N}\right)T} = nN = (\text{Number of molecules in the gas})$

Hence, the correct answer is (C).

143. V - T graph is a straight line passing through origin

Hence, $V \propto T$ or $P = \text{constant}$

$$\Rightarrow dQ = nC_p dT \text{ and } dU = nC_v dT$$

$$\text{Since, } dW = dQ - dU = n(C_p - C_v) dT$$

$$\Rightarrow \frac{dQ}{dW} = \frac{nC_p dT}{n(C_p - C_v) dT}$$

$$\Rightarrow \frac{dQ}{dW} = \frac{C_p}{C_p - C_v} = \frac{1}{1 - \frac{C_v}{C_p}}$$

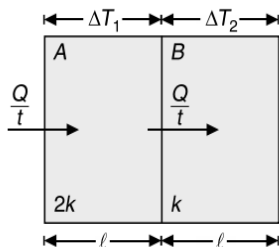
Since, $\frac{C_v}{C_p} = \frac{3}{5}$ for helium gas

$$\Rightarrow \frac{dQ}{dW} = \frac{1}{1 - 3/5} = \frac{5}{2}$$

Hence, the correct answer is (B).

144. $\Delta T_1 + \Delta T_2 = 36 \dots(1)$

Also, the slabs are in series. So, same thermal current $\left(\frac{Q}{t}\right)$



Flows through both. Hence

$$2kA \left(\frac{\Delta T_1}{l}\right) = kA \left(\frac{\Delta T_2}{l}\right)$$

$$\Rightarrow \Delta T_2 = 2\Delta T_1 \dots(2)$$

$$\Rightarrow \Delta T_1 = 12^\circ\text{C} \text{ and } \Delta T_2 = 24^\circ\text{C}$$

Hence, the correct answer is (B).

145. Apparent weight (w_a) = actual weight (w) - upthrust (F)

$$\text{Here, } F = V\rho_w g \quad \{\rho_w = \text{density of water}\}$$

$$\text{i.e., } F_{0^\circ\text{C}} = V_{0^\circ\text{C}} (\rho_w)_{50^\circ\text{C}} g$$

$$\text{and } F_{50^\circ\text{C}} = V_{50^\circ\text{C}} (\rho_w)_{50^\circ\text{C}} g$$

$$\Rightarrow \frac{F_{50^\circ\text{C}}}{F_{0^\circ\text{C}}} = \frac{V_{50^\circ\text{C}} (\rho_w)_{50^\circ\text{C}}}{V_{0^\circ\text{C}} (\rho_w)_{0^\circ\text{C}}} = \frac{(1 + \gamma_m \Delta T)}{(1 + \gamma_w \Delta T)} \quad \{\Delta T = 50^\circ\text{C}\}$$

Given that $\gamma_m < \gamma_w$, so $F_{50^\circ\text{C}} < F_{0^\circ\text{C}}$

Hence apparent weight at 50°C will be more.

Hence, the correct answer is (B).

146. Since $A_t = A_0 [1 + \beta \Delta T]$

Let diameter at 0°C is d_0 cm and at dry ice (-60°C) is d cm

$$\frac{\pi(10.02)^2}{4} = \frac{\pi d_0^2}{4} [1 + (4 \times 10^{-5})(40 - 0)] \dots(1)$$

$$\Rightarrow \frac{\pi d^2}{4} = \frac{\pi d_0^2}{4} [1 + (4 \times 10^{-5})(-60 - 0)] \dots(2)$$

Dividing (1) by (2), we get

$$\frac{(10.02)^2}{d^2} = \frac{1.0016}{0.9976}$$

$$\Rightarrow d = 9.99997 \text{ cm} \approx 10 \text{ cm}$$

Hence, the correct answer is (D).

147. By First Law of Thermodynamics

$$Q_1 + (-W_1) = Q_2 + (-W_2) = U_B - U_A$$

Hence, the correct answer is (D).

148. Cooling must lead to a decrease in volume of the system which is possible only if the piston moves to the right.

Hence, the correct answer is (B).

149. For both graphs, work done is same because area under the curve is same.

However, according to Ideal Gas Equation, $PV = nRT$, we see that the increase in temperature is more in CASE-1.

Therefore, increase in internal energy is also more in CASE-1. Hence heat added is more in CASE-1.

Hence, the correct answer is (A).

150. For a cyclic process $\Delta U = 0$, so $Q = W$

$$\Rightarrow (100 - 20) = 20 + W_2$$

$$\Rightarrow W_2 = 60 \text{ J}$$

Hence, the correct answer is (A).

151. Let h be the distance of the apex from the middle of the base. Further by Pythagoras theorem

$$L_2^2 - \frac{L_1^2}{4} = h^2 = \text{constant}$$

$$\Rightarrow \Delta(h^2) = \Delta(L_2^2) - \Delta\left(\frac{L_1^2}{4}\right)$$

$$\Rightarrow 0 = 2L_2\Delta L_2 - \frac{1}{4}(2L_1\Delta L_1)$$

$$\text{Since, } \alpha_1 = \frac{\Delta L_1}{L_1} \frac{1}{\Delta T} \text{ and } \alpha_2 = \frac{\Delta L_2}{L_2} \frac{1}{\Delta T}$$

$$\Rightarrow L_2(L_2\alpha_2\Delta T) = \frac{1}{4}L_1(L_1\alpha_1\Delta T)$$

$$\Rightarrow \frac{L_1}{L_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$$

Hence, the correct answer is (D).

152. 1 g of steam at 100 °C liberates 540 cal to convert to 1 g of water at 100 °C.

Further 1 g of ice at 0 °C melts and converts to water at 0 °C. For this purpose 80 cal of heat is required by it.

This 1 g of water at 0 °C requires 100 cal of heat to convert to 1 g of water at 100 °C.

i.e., a total of (80 + 100) cal = 180 cal of heat is required by ice at 0 °C to convert to water at 100 °C which is much less than the heat liberated by steam at 100 °C to convert to water at 100 °C i.e. 540 cal. Hence equilibrium temperature will just be 100 °C.

Hence, the correct answer is (D).

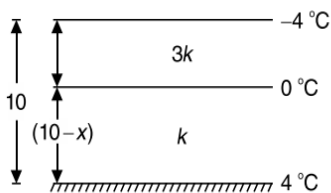
153. Even if 3 g of ice at 0 °C is taken then total heat required by 3 g of ice at 0 °C to convert to 3 g of water at 100 °C equals 3(180 cal) = 540 cal, which is the amount of heat liberated by steam at 100 °C to convert to water at 100 °C.

Hence, the correct answer is (C).

154. A refrigerator is reverse of heat engine and heat pump is same as refrigerator. The coefficient of performance of a refrigerator cannot be infinity and a heat engine cannot convert the heat input fully to work done.

Hence, the correct answer is (B).

$$155. kA\left(\frac{4-0}{10-x}\right) = 3kA\left[\frac{0-(-4)}{x}\right]$$



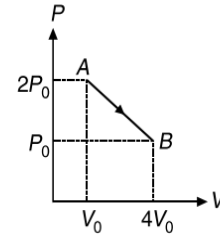
$$\Rightarrow x = 7.5 \text{ m}$$

Hence, the correct answer is (A).

156. Temperature of liquid oxygen will first increase in the same phase. Then, phase change (liquid to gas) will take place. During which temperature will remain constant. After that temperature of oxygen in gaseous state will further increase.

Hence, the correct answer is (C).

157. The process carried on an ideal monatomic gas is shown in Figure.



Applying Ideal Gas Equation at points A and B, we get

$$2P_0V_0 = nRT_A \text{ and } P_0(4V_0) = nRT_B$$

$$\Rightarrow \Delta U = n(1.5R)(T_B - T_A) = 3P_0V_0$$

$$\text{Also } W = \frac{1}{2}(2P_0 + P_0)(4V_0 - V_0) = 4.5P_0V_0$$

$$\Rightarrow Q = \Delta U + W = 3P_0V_0 + 4.5P_0V_0 = 7.5P_0V_0$$

$$\Rightarrow Q = 7.5 \times (0.5 \times 10^6) \times 0.1 = 375 \times 10^3 \text{ J}$$

Hence, the correct answer is (B).

158. For an adiabatic process $PV^\gamma = \text{constant}$... (1)

$$\text{Also, density } \rho = \frac{m}{V} \text{ i.e., } \rho \propto V^{-1}$$

So, equation (1) can be written as $P\rho^{-\gamma} = \text{constant}$

Hence, the correct answer is (B).

159. Volume of the gas is constant $V = \text{constant}$

$$\Rightarrow P \propto T$$

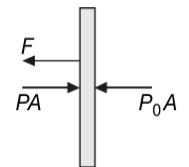
i.e., pressure will be doubled if temperature is doubled

$$\Rightarrow P = 2P_0$$

Now let F be the tension in the wire. Then equilibrium of any one piston gives

$$F = (P - P_0)A = (2P_0 - P_0)A = P_0A$$

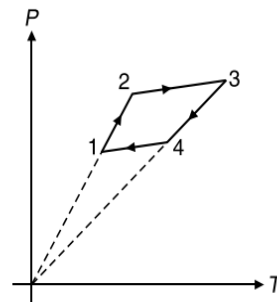
Hence, the correct answer is (B).



160. Processes 1 → 2 and 3 → 4 are isochoric, so work done is zero.

$$\Rightarrow W_{\text{total}} = W_{23} + W_{41}$$

$$\Rightarrow W_{\text{total}} = P_2(V_3 - V_2) + P_4(V_1 - V_4)$$



$$\Rightarrow W_{\text{total}} = nR(T_3 - T_2) + nR(T_1 - T_4)$$

$$\Rightarrow W_{\text{total}} = nR(T_3 - T_2 + T_1 - T_4)$$

$$\Rightarrow W_{\text{total}} = 800nR = 2400R$$

Hence, the correct answer is (C).

H.116 JEE Advanced Physics: Waves and Thermodynamics

161. $PV = nRT$, where $V = \frac{m}{\rho}$

$$\Rightarrow P\left(\frac{m}{\rho}\right) = \left(\frac{m}{M}\right)RT$$

$$\Rightarrow \rho = \left(\frac{M}{RT}\right)P$$

Therefore, slope of ρ - P graph is

$$\frac{M}{RT} \propto \frac{1}{T}$$

So, for lesser slope, temperature has to be more.

$$\Rightarrow T_B > T_A$$

Hence, the correct answer is (B).

162. Let T be the temperature of the mixture. Then

$$U = U_1 + U_2$$

$$\Rightarrow \frac{f}{2}(n_1 + n_2)RT = \frac{f}{2}(n_1)RT_0 + \frac{f}{2}(n_2)(R)(2T_0)$$

$$\Rightarrow (2+4)T = 2T_0 + 8T_0 \quad (n_1 = 2, n_2 = 4)$$

$$\Rightarrow T = \frac{5}{3}T_0$$

Hence, the correct answer is (A).

163. To 80 cm mark on the aluminium rod is really at a greater distance from the zero position than indicated because of the increase in temperature $\Delta T = 40^\circ\text{C}$. The increased length is

$$\Delta L = \alpha_{Al} L_{Al} \Delta T = (2.5 \times 10^{-5})(80)(40) = 0.08 \text{ cm}$$

So, the correct length of the line is

$$L = 80 + 0.08 = 80.08 \text{ cm}$$

Hence, the correct answer is (A).

164. Since, $\frac{(Q/t)_1}{(Q/t)_2} = \left(\frac{r_1}{r_2}\right)^2 \frac{\ell_2}{\ell_1}$

$$\Rightarrow \text{Ratio} = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

Hence, the correct answer is (C).

165. Due to Kirchhoff's Laws

Hence, the correct answer is (A).

166. Since, $dP = g\rho dy$

$$\Rightarrow \int_{P_1}^{P_2} \frac{dP}{P} = \frac{Mg}{RT}$$

Also, $H = \int_0^H dy$

$$\Rightarrow H = \frac{RT}{Mg} \ln \frac{P_1}{P_2}$$

Hence, the correct answer is (A).

167. Since $\frac{Q}{t} = kA \frac{\Delta T}{\ell}$, so $t \propto \frac{\ell}{A}$

$$\Rightarrow \frac{t_1}{t_2} = \frac{\ell_1 A_2}{\ell_2 A_1} = 4$$

$$\Rightarrow t_2 = \frac{t_1}{4} = \frac{4}{4} = 1 \text{ min}$$

Hence, the correct answer is (A).

168. $dU = C_v dT = \left(\frac{5}{2}R\right)dT$

$$\Rightarrow dT = \frac{2(dU)}{5R}$$

...(1)

From First Law of Thermodynamics, we have

$$dU = dQ - dW$$

$$\Rightarrow dU = Q - \frac{Q}{4} = \frac{3Q}{4}$$

Now molar heat capacity is $C = \frac{dQ}{dT} = \frac{Q}{2(dU)}$

$$\Rightarrow C = \frac{5RQ}{2\left(\frac{3Q}{4}\right)} = \frac{10}{3}R$$

Hence, the correct answer is (C).

169. Since volume is constant, so we have

$$\frac{P_2}{T_2} = \frac{P_1}{T_1}$$

When temperature of gas is doubled, then we get

$$P_2 = 2P_1 = 2P_0$$

Now, tension F in the wire is equal to net force on the piston given by

$$F = (2P_0 - P_0)A = (1.013 \times 10^5)(1) = 10^5 \text{ N}$$

Hence, the correct answer is (B).

170. Since, $\frac{dL}{L_0} = \alpha(T)dT$

$$\Rightarrow \int_{L_0}^L dL = L_0 \int_{T_0}^T \alpha(T)dT$$

$$\Rightarrow L - L_0 = L_0 \int_{T_0}^T \alpha(T)dT$$

$$\Rightarrow L = L_0 \left[1 - \int_T^{T_0} \alpha(T)dT \right]$$

Hence, the correct answer is (B).

171. Since $t \propto x_2^2 - x_1^2$

$$\Rightarrow t = \beta(x_2^2 - x_1^2)$$

where β is just a constant of proportionality

$$\Rightarrow \frac{t_2}{t_1} = \frac{4-1}{1-0}$$

$$\Rightarrow t_2 = 21 \text{ hour} > 14 \text{ hour}$$

Hence, the correct answer is (D).

172. Since all have same k (as they are made of same material)

So, the rod which has maximum $\frac{Q}{t}$ will conduct the most.
Here

$$\frac{Q}{t} \propto \frac{r^2}{\ell}$$

Hence, the correct answer is (A).

173. Since both the slabs are in series and hence $\frac{Q}{t}$ is same through both. Hence, required ratio is 1 : 1.

Hence, the correct answer is (D).

174. Average KE of each molecule is $E_{av} = E = \frac{3}{2}k_B T$

If N_1 is the number of molecules of hydrogen and N_2 is the number of molecules of oxygen, then

$$\text{Total KE of hydrogen is } E_1 = N_1 \left(\frac{3}{2} k_B T \right)$$

$$\text{Total KE of oxygen is } E_2 = N_2 \left(\frac{3}{2} k_B T \right)$$

$$\Rightarrow \frac{E_1}{E_2} = \frac{N_1}{N_2} = 2$$

Hence, the correct answer is (B).

175. Under steady state condition, heat released to the room = heat dissipated out of the room.

Let θ be the temperature of heater. Then

$$\theta - 20 = \alpha [20 - (-20)] \quad \dots(1)$$

$$\text{and } \theta - 10 = \alpha [10 - (-40)] \quad \dots(2)$$

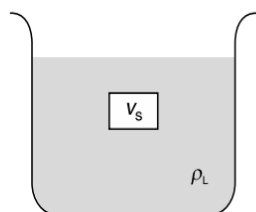
Solving equations (1) and (2), we get

$$\theta = 60^\circ \text{C}$$

Hence, the correct answer is (D).

176. Upthrust, $W_0 = V_s \rho_L g$

where, V_s is volume of solid at 0°C , ρ_L is density of liquid at 0°C . As the temperature increases, V_s increases and ρ_L decreases.



Since, $W_0 \propto V_s \rho_L$

$$\Rightarrow \frac{W}{W_0} = \frac{V_s' \rho_L'}{V_s \rho_L} = \frac{(V_s + \Delta V_s)}{V_s} \times \frac{\rho_L (1 + \gamma_L \Delta T)^{-1}}{\rho_L}$$

$$\Rightarrow \frac{W}{W_0} = (1 + \gamma_s \Delta T)(1 - \gamma_L \Delta T) \quad \text{(Using binomial)}$$

$$\Rightarrow W = W_0 (1 + \gamma_s \Delta T - \gamma_L \Delta T) = W_0 [1 + (\gamma_s - \gamma_L) \Delta T]$$

Hence, the correct answer is (B).

177. Since, $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

$$\Rightarrow Q_2 = \left(\frac{T_2}{T_1} \right) Q_1 = \left(\frac{300}{400} \right) 600 = 450 \text{ cal}$$

Hence, the correct answer is (D).

178. Since $PV = nRT$

$$\Rightarrow \text{Slope} = \frac{P}{T} = \frac{nR}{V}$$

The more the volume, the lesser the slope and vice-versa.

Hence, the correct answer is (B).

179. Given that, $\eta = 40\%$ and $T_2 = 47 + 273 = 320 \text{ K}$

$$\text{Since } \eta = 1 - \frac{T_2}{T_1}, \text{ so } \frac{40}{100} = 1 - \frac{320}{T_1}$$

$$\Rightarrow \frac{320}{T_1} = 0.6$$

$$\Rightarrow T_1 = \frac{3200}{6} = 533 \text{ K} = 260^\circ \text{C}$$

Hence, the correct answer is (A).

180. Since $P_1 = P_2$

$$\Rightarrow \sigma A_1 T_1^4 = \sigma A_2 T_2^4$$

$$\Rightarrow 4\pi r_1^2 T_1^4 = 4\pi r_2^2 T_2^4$$

$$\Rightarrow \frac{r_1}{r_2} = \left(\frac{T_2}{T_1} \right)^2$$

Hence, the correct answer is (B).

181. Since, $\Delta U = nC_v \Delta T$

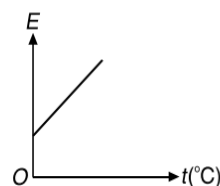
$$\Rightarrow \Delta U = n \left(\frac{R}{\gamma - 1} \right) (T_f - T_i) = \frac{nRT_f - nRT_i}{\gamma - 1}$$

$$\Rightarrow \Delta U = \frac{P(2V) - P(V)}{\gamma - 1} = \frac{PV}{\gamma - 1}$$

Hence, the correct answer is (C).

182. Mean KE of each molecule, $E_K = \frac{3}{2}k_B T = \frac{3}{2}k_B (t + 273)$

At 0°C or 273 K , the molecules will possess some kinetic energy. So, the graph does not start from origin.



Hence, the correct answer is (C).

183. Let R be radius of sphere, V its volume and ρ its density, then $\Delta R = R\alpha\Delta T$. So, percentage change is

$$\frac{\Delta R}{R} \times 100 = 100\alpha\Delta T \quad \dots(1)$$

Also, $\Delta V = V\gamma\Delta T = V(3\alpha)\Delta T$, so percentage change is

$$\frac{\Delta V}{V} \times 100 = 300\alpha\Delta T \quad \dots(2)$$

$$\text{Since, } \rho' = \frac{\rho}{1 + \gamma\Delta T} = \frac{\rho}{1 + 3\alpha\Delta T}$$

$$\Rightarrow \Delta\rho = \rho - \rho'$$

H.118 JEE Advanced Physics: Waves and Thermodynamics

$$\Rightarrow \Delta\rho = \rho \left(1 - \frac{1}{1+3\alpha\Delta T} \right) = \frac{(3\alpha\Delta T)\rho}{1+3\alpha\Delta T}$$

So, percentage change in density is $\frac{\Delta\rho}{\rho} \times 100$

$$\Rightarrow \frac{\Delta\rho}{\rho} \times 100 = \frac{300\alpha\Delta T}{1+3\alpha\Delta T} \quad \dots(3)$$

From equations (1), (2) and (3), we see that percentage change is maximum in volume.

Hence, the correct answer is (B).

184. Since, $dQ = nC_p dT$

$$\Rightarrow C_p = \frac{dQ}{ndT}$$

$$\Rightarrow C_p = \frac{70}{(2)(5)} = \frac{7}{2} (R)$$

i.e., the gas is diatomic or it may be H_2 .

Hence, the correct answer is (A).

185. We use $\Delta l = \alpha l \Delta T$

For rod A, $0.075 = 20\alpha_A(100 - 0)$

$$\Rightarrow \alpha_A = 3.75 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$$

For rod B, $0.045 = 20\alpha_B(100 - 0)$

$$\Rightarrow \alpha_B = 2.25 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$$

Let portion of metal A is l cm and that of metal B is $(20 - l)$ cm

$$\Delta l_A + \Delta l_B = 0.06 \text{ cm}$$

$$\Rightarrow \alpha_A l(100) + \alpha_B (20 - l)(100) = 0.06$$

$$\Rightarrow 3.75 \times 10^{-5} l + 4.5 \times 10^{-4} - 2.25 \times 10^{-5} l = 6 \times 10^{-4}$$

$$\Rightarrow 1.5 \times 10^{-5} l = 1.5 \times 10^{-4}$$

$$\Rightarrow l = 10 \text{ cm}$$

Hence, the correct answer is (B).

186. Since, $\frac{0.1}{t_1} \propto \left(\frac{50 + 49.9}{2} - 30 \right)$

and $\frac{0.1}{t_2} \propto \left(\frac{40 + 39.9}{2} - 30 \right)$

$$\Rightarrow t_2 = 10 \text{ s}$$

Hence, the correct answer is (B).

187. Let temperature of the junction be T , then in steady state, thermal current through steel equals the thermal current through copper, so we have

$$K_{\text{steel}}(2A) \left(\frac{300 - T}{15} \right) = K_{\text{copper}}(A) \left(\frac{T - 0}{10} \right)$$

$$(50.2)(2) \left(\frac{300 - T}{15} \right) = 385 \left(\frac{T}{10} \right)$$

$$\Rightarrow T \approx 44 \text{ } ^\circ\text{C}$$

Hence, the correct answer is (A).

188. Let m gram of steam be required for the purpose. By Law of Calorimetry

$$\left(\begin{array}{l} \text{Total heat lost} \\ \text{by steam to become} \\ \text{water at } 0 \text{ } ^\circ\text{C} \end{array} \right) = \left(\begin{array}{l} \text{Total heat gained} \\ \text{by ice to become} \\ \text{water at } 0 \text{ } ^\circ\text{C} \end{array} \right)$$

$$\Rightarrow 540m + m(1)(100 - 0) = 3200(0.5)(10) + 3200(80)$$

$$\Rightarrow 640m = 272000$$

$$\Rightarrow m = 425 \text{ g}$$

Hence, the correct answer is (C).

189. Energy received per second by the planet is $\frac{P}{4\pi d^2}(\pi R^2)$

where P is power radiated by the sun and R is the radius of the planet.

Further, energy radiated per second by the planet (according to Stefan's Law) is $\sigma(4\pi R^2)T_0^4$. For thermal equilibrium to exist, we get

$$\frac{P}{4\pi d^2}(\pi R^2) = \sigma(4\pi R^2)T_0^4$$

$$\Rightarrow T_0^4 \propto d^{-2}$$

$$\Rightarrow T_0 \propto d^{-1/2}$$

[OPTION (D)]

Hence, the correct answer is (D).

190. Since $V = kT^{2/3}$, so $TV^{-3/2} = \text{constant}$

For a polytropic process $TV^{x-1} = \text{constant}$

$$\Rightarrow x - 1 = -\frac{3}{2}$$

$$\Rightarrow x = -\frac{1}{2}$$

Work done in a polytropic process is

$$W = \frac{nR}{1-x}(T_2 - T_1), \text{ where } x = -\frac{1}{2}$$

$$\Rightarrow W = \frac{nR}{1+(1/2)}(30) = 20R$$

Hence, the correct answer is (B).

191. From Kinetic Theory of Gases, $p = \frac{2}{3} \left(\frac{E}{V} \right)$

$$\Rightarrow E = \frac{3}{2} pV$$

Hence, the correct answer is (D).

192. Efficiency of generator is,

$$\eta = 0.9 = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{out}}}{1000}$$

$$\Rightarrow P_{\text{out}} = 900 \text{ W}$$

$$\Rightarrow 900 = mc\Delta T + mL$$

$$\Rightarrow 900 = m(4200)(100 - 20) + m(2250 \times 10^3)$$

$$\Rightarrow m = \frac{900}{336000 + 2250000} = 0.00035 \text{ kg}$$

$$\Rightarrow m = 0.35 \text{ g}$$

Hence, the correct answer is (C).

193. Internal energy of an ideal gas depends on temperature only and change in internal energy.

$$\Delta U = nC_v \Delta T \text{ is applicable for all processes.}$$

Hence, the correct answer is (B).

$$194. W = \int PdV = a \int_{V_1}^{V_2} \frac{dV}{V} + b \int_{V_1}^{V_2} dV$$

$$\Rightarrow W = a \log_e \left(\frac{V_2}{V_1} \right) + b(V_2 - V_1)$$

Hence, the correct answer is (D).

195. According to *FLTD*, we have

$$\Delta U = Q - W$$

Also, work done W equals area under the curve

$$\Rightarrow W = \left(\begin{array}{c} \text{Area of Half} \\ \text{Ellipse i.e. } \pi ab/2 \end{array} \right) + \left(\begin{array}{c} \text{Area of} \\ \text{Rectangle} \end{array} \right)$$

$$\Rightarrow W = \left[\frac{\pi}{2} \times (7 - 4.2) \times 10^{-3} \times (500 - 300) \times 10^3 \right] + [(7 - 1.4) \times 10^{-3} \times 300 \times 10^3] \text{ J}$$

$$\Rightarrow W = 2.56 \times 10^3 \text{ J}$$

$$\Rightarrow \Delta U = 5.76 \times 10^3 - 2.56 \times 10^3 = 3.2 \times 10^3 \text{ J}$$

Hence, the correct answer is (A).

196. Since $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

T is doubled and M is halved. Therefore, r.m.s. speed will become two times or 600 ms^{-1} .

Hence, the correct answer is (C).

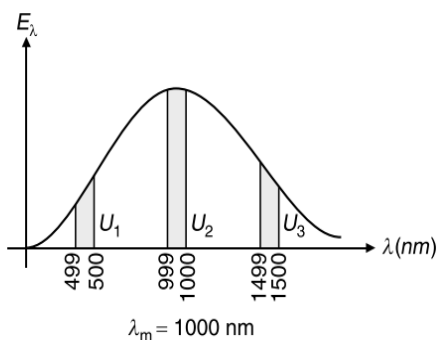
197. Wein's Displacement Law is

$$\lambda_m T = b \quad \{b = \text{Wein's constant}\}$$

$$\Rightarrow \lambda_m = \frac{b}{T} = \frac{2.88 \times 10^6 \text{ nm-K}}{2880 \text{ K}}$$

$$\Rightarrow \lambda = 1000 \text{ nm}$$

Energy distribution with wavelength will be as follows

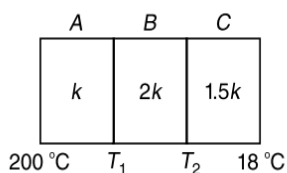


From the graph it is clear that

$$U_2 > U_1 \quad \{\text{In fact } U_2 \text{ is maximum}\}$$

Hence, the correct answer is (D).

$$198. \frac{kA(200 - T_1)}{\ell} = \frac{2kA(T_1 - T_2)}{\ell} = \frac{1.5kA(T_2 - 18)}{\ell}$$



$$\Rightarrow 200 - T_1 = 2T_1 - 2T_2$$

$$\Rightarrow 3T_1 - 2T_2 = 200 \quad \dots(1)$$

Further, $200 - T_1 = \frac{3}{2}(T_2 - 18)$

$$\Rightarrow 2T_1 + 3T_2 = 454 \quad \dots(2)$$

Solving (1) and (2), we get

$$T_1 = 116 \text{ }^\circ\text{C}$$

Hence, the correct answer is (D).

199. The change in internal energy for any process is

$$\Delta U = nC_V \Delta T$$

(Do not develop a MISCONCEPTION that $\Delta U = nC_V \Delta T$ only for an isochoric process)

Hence, the correct answer is (D).

200. Let the initial pressure of the three samples be P_A , P_B and P_C , then

$$P_A (V)^{3/2} = (2V)^{3/2} P$$

$$\Rightarrow P_B = P$$

$$P_C (V) = P(2V)$$

$$\Rightarrow P_A : P_B : P_C = (2)^{3/2} : 1 : 2 = 2\sqrt{2} : 1 : 2$$

Hence, the correct answer is (B).

201. Since, we know that $\frac{\Delta Q}{\Delta t} = \frac{KA(T_1 - T_2)}{L}$

In case of sphere A, we get

$$\frac{\frac{4}{3}\pi(20)^3 \times \rho_{\text{ice}} \times L_{\text{ice}}}{80 \times 60} = \frac{K_A \times 4\pi \times (20)^2 (T_1 - T_2)}{0.2} \quad \dots(1)$$

In case of sphere B, we get

$$\frac{\frac{4}{3}\pi(30)^3 \times \rho_{\text{ice}} \times L_{\text{ice}}}{t \times 60} = \frac{K_B \times 4\pi \times (30)^2 (\theta_1 - \theta_2)}{0.1} \quad \dots(2)$$

Dividing (1) by (2), we get

$$\frac{(20/80)}{(30/t)} = \frac{K_A}{2K_B} = \frac{1}{4}$$

$$\Rightarrow t = 30 \text{ minutes}$$

Hence, the correct answer is (A).

$$202. n = \frac{PV}{RT} = \frac{2 \times 10^5 \times 10^{-3}}{8.3 \times 300} = 0.080$$

$$\Rightarrow n = \frac{m}{M} = 0.080$$

$$\Rightarrow m = (0.080)(4 \text{ g}) = 0.32 \text{ g}$$

Hence, the correct answer is (B).

203. Since, $(3L)\alpha_{\text{eff}}\Delta\theta = L\alpha\Delta\theta + 2L(2\alpha)(\Delta\theta)$

$$\Rightarrow \alpha_{\text{eff}} = \frac{5}{3}\alpha$$

Hence, the correct answer is (C).

H.120 JEE Advanced Physics: Waves and Thermodynamics

204. By Law of Calorimetry, heat lost by steam must be equal to heat gained by water. Since, find temperature of water is 60°C , so we have

$$\begin{aligned} (8)L_{\text{steam}} + (8)c_{\text{water}}(100 - 60) &= (80)c_{\text{water}}(60 - 10) \\ \Rightarrow (8)L_{\text{steam}} + (8)(1)(100 - 60) &= (80)(1)(60 - 10) \\ \Rightarrow (8)L_{\text{steam}} &= 4000 - 320 = 3680 \\ \Rightarrow L_{\text{steam}} &= 460 \text{ calg}^{-1} \end{aligned}$$

Hence, the correct answer is (A).

205. Conceptual

Hence, the correct answer is (B).

206. $\lambda_m T = \text{constant}$

$$\begin{aligned} \Rightarrow 510T_1 &= 350T_2 \\ \Rightarrow \frac{T_1}{T_2} &= 0.69 \end{aligned}$$

Hence, the correct answer is (B).

207. $\% \text{age} = \frac{dU}{dQ} \times 100 = \frac{nC_V dT}{nC_P dT} \times 100$

$$\Rightarrow \% \text{age} = \frac{C_V}{C_P} \times 100 = \frac{3}{5} \times 100 = 60\%$$

Remaining $\% \text{age} = (100 - 60)\% = 40\%$

Hence, the correct answer is (C).

208. The ratio of rms speeds of molecules of gas A to that of molecules of gas B is

$$\frac{v_A}{v_B} = \sqrt{\frac{M_B}{M_A}} = \sqrt{3}$$

Since $v_A^2 = v_x^2 + v_y^2 + v_z^2$ and by postulates of KTG, we have $v_x = v_y = v_z$

$$\Rightarrow v_x^2 = v_y^2 = v_z^2 = \frac{v_A^2}{3} = \frac{1}{3}(3v_B^2)$$

$$\Rightarrow v_x^2 : v_B^2 = 1 : 1$$

$$\Rightarrow \frac{v^2}{V^2} = 1$$

Hence, the correct answer is (D).

209. Since, $P \propto V$ $\{\because P = aV\}$

Therefore, pressure and volume both are doubled or temperature becomes four times (as $T \propto PV$) or 1200 K. So, change in temperature is 900 K.

Hence, the correct answer is (A).

210. Heat released when 20 g of water cools from 20°C to 0°C is

$$Q_1 = m_{\text{water}} c_{\text{water}} \Delta T = (20)(1)(20) = 400 \text{ cal}$$

Heat required to raise temperature of 20 g of ice from -10°C to 0°C is

$$Q_2 = m_{\text{ice}} c_{\text{ice}} \Delta T = (20)(0.5)(10) = 100 \text{ cal}$$

Heat required to melt 20 g of ice at 0°C is

$$Q_3 = m_{\text{ice}} L_{\text{ice}} = (20)(80) = 1600 \text{ cal}$$

Since $Q_1 > Q_2$, so temperature of ice will reach 0°C . However, we observe that $Q_1 < Q_2 + Q_3$, therefore the

complete ice will not melt and final mixture will have both ice and water. The amount of ice melted m is

$$m = \frac{\text{Available Heat}}{\text{Latent Heat}} = \frac{400 - 100}{80} = 3.75 \text{ g}$$

So in equilibrium, the calorimeter will have the mass of ice and water given by

$$m'_{\text{ice}} = 20 - 3.75 = 16.25 \text{ g}$$

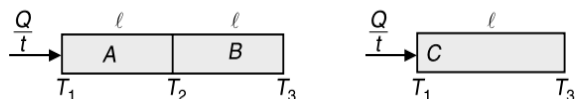
$$m'_{\text{water}} = 20 + 3.75 = 23.75 \text{ g}$$

Hence, the correct answer is (A).

211. $\eta = 1 - \frac{300}{600} = 0.5$

Hence, the correct answer is (C).

212. Since $\frac{Q}{t} = k_{\text{eq}} A \left(\frac{T_1 - T_3}{\ell} \right)$
- $$\Rightarrow \frac{Q}{t} = \frac{2k_A k_B}{k_A + k_B} A \left(\frac{T_1 - T_3}{2\ell} \right) \quad \dots(1)$$



$$\text{Also, } \frac{Q}{t} = k_C A \left(\frac{T_1 - T_3}{\ell} \right) \quad \dots(2)$$

From (1) and (2), we get $k_C = \frac{k_A k_B}{k_A + k_B}$

Hence, the correct answer is (B).

213. For Isobaric Process

$$W_{\text{isob}} = P(V_2 - V_1) = P(2V - V) = PV$$

For Isothermal Process

$$W_{\text{isot}} = nRT \ln \left(\frac{V_2}{V_1} \right) = nRT \ln(2) = PV \ln(2)$$

For Adiabatic Process

$$W_{\text{ad}} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

Since $PV^\gamma = P_2(2V)^\gamma$, so $P_2 = P/2^\gamma$

$$W_{\text{ad}} = \frac{PV - (P/2^\gamma)(2V)}{\gamma - 1} = \frac{PV(1 - 2^{1-\gamma})}{\gamma - 1}$$

Hence, the correct answer is (C).

214. Coefficient of performance

$$\beta = \frac{\text{Heat Extracted } (Q_2) \text{ From Sink}}{\text{External Work Input } (W)} = \frac{T_2}{T_1 - T_2}$$

$$\Rightarrow \frac{Q_2}{W} = \frac{(1 \text{ g})(80 \text{ calg}^{-1})}{27} = \frac{273}{27}$$

$$\Rightarrow W = 7.9 \text{ cal}$$

Hence, the correct answer is (A).

215. When the vessel is suddenly stopped the ordered motion of gas is converted into its disordered motion i.e., internal energy of the gas is increased. Hence,

$$\Delta U = \frac{1}{2}mv^2 \quad \{\because m = nM\}$$

$$\Rightarrow nC_v\Delta T = \frac{1}{2}(nM)v^2$$

where n is the number of moles of the gas.

$$\Rightarrow \Delta T = \frac{Mv^2}{2C_v}$$

$$\Rightarrow \Delta T = \frac{Mv^2}{2\left(\frac{R}{\gamma-1}\right)} \quad \left\{\because C_v = \frac{R}{\gamma-1}\right\}$$

$$\Rightarrow \Delta T = \frac{Mv^2(\gamma-1)}{2R}$$

Hence, the correct answer is (B).

216. Since $dQ = mc dT$

$$\Rightarrow Q = \int dQ = m\alpha \int_1^3 T^3 dT$$

$$\Rightarrow Q = (4\alpha) \frac{T^4}{4} \Big|_1^3 = \alpha(3^4 - 1^4) = 80\alpha$$

Hence, the correct answer is (C).

217. Given that, $U = a + bPV = a + b(nRT)$ $\{\because PV = nRT\}$

$$\text{Since, } C_V = \frac{1}{n} \frac{dU}{dT} = bR$$

$$\Rightarrow C_p = C_V + R = bR + R$$

$$\Rightarrow \gamma = \frac{C_p}{C_V} = \frac{b+1}{b}$$

Hence, the correct answer is (A).

218. Solar power received by earth is 1400 Wm^{-2}

Solar power received by 0.2 m^2 area of earth is

$$P = (1400 \text{ Wm}^{-2})(0.2 \text{ m}^2) = 280 \text{ W}$$

Mass of ice = $280 \text{ g} = 0.28 \text{ kg}$

Heat required to melt ice is

$$Q = (0.28)(3.3 \times 10^5) \quad \{\because Q = mL\}$$

$$\Rightarrow Q = 9.24 \times 10^4 \text{ J}$$

$$\Rightarrow t = \frac{9.24 \times 10^4}{280} = 331 \text{ s} \approx 330 \text{ s}$$

Hence, the correct answer is (B).

219. Since $C_p - C_V = R$, so $\frac{C_p}{C_V} - 1 = \frac{R}{C_V}$

$$\Rightarrow C_V = \frac{R}{\gamma-1} \text{ and } C_p = \frac{\gamma R}{\gamma-1}$$

Hence, the correct answer is (B).

220. At constant pressure, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$$\Rightarrow \frac{1}{400} = \frac{V_2}{300}$$

$$\Rightarrow V_2 = 0.75 \text{ L}$$

$$\Rightarrow \Delta V = 0.75 - 1 = -0.25 \text{ L} = -250 \text{ mL}$$

Hence, the correct answer is (D).

221. Since, $PV = nRT$

$$\Rightarrow (2P)V = n'RT$$

$$\Rightarrow n' = 2n$$

Hence, the correct answer is (B).

222. Since $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$, so $\frac{300}{150} = \frac{500}{T_2}$

$$\Rightarrow T_2 = 250 \text{ K}$$

Hence, the correct answer is (C).

223. Rate of flow of heat through the cork is

$$\frac{Q}{t} = \frac{KA(T_1 - T_2)}{L}$$

$$\Rightarrow \frac{Q}{t} = \frac{0.008 \times 100 \times (30 - 0)}{4} = 6 \text{ cal s}^{-1}$$

Therefore, time taken to melt 200 g of ice is

$$t = \frac{mL_{\text{ice}}}{Q/t} = \frac{(200)(80)}{6} = \frac{8000}{3} \text{ s}$$

$$\Rightarrow t \approx 45 \text{ min}$$

Hence, the correct answer is (A).

224. According to Newton's Law of Cooling Rate of Cooling $R \propto$ Temperature Difference

$$\text{Hence, } R = K(\theta - \theta_0)$$

This is an equation of a straight line.

Hence, the correct answer is (B).

225. Work done in the cycle is

$$W = (\text{Area of rectangle } ABCD)$$

$$\Rightarrow W = (2P_0 - P_0)(2V_0 - V_0) = P_0V_0$$

Heat added in the process A to B is

$$Q_{AB} = nC_V\Delta T = n\left(\frac{3R}{2}\right)\Delta T = \frac{3}{2}(2P_0V_0 - P_0V_0)$$

$$\Rightarrow Q_{AB} = \frac{3}{2}P_0V_0$$

Heat added in the process B to C is

$$Q_{BC} = nC_p\Delta T = n\left(\frac{5R}{2}\right)\Delta T = \frac{5}{2}(4P_0V_0 - 2P_0V_0)$$

$$\Rightarrow Q_{BC} = 5P_0V_0$$

Efficiency η is

$$\eta = \frac{W}{Q_{\text{input}}} = \frac{P_0V_0}{\frac{3}{2}P_0V_0 + 5P_0V_0} = \frac{2}{13}$$

$$\Rightarrow \% \eta = \frac{200}{13} = 15.4\%$$

Hence, the correct answer is (A).

226. $100^\circ \text{C} \equiv 150^\circ$ of the thermometer

$$\Rightarrow 1^\circ \text{C} = 1.5^\circ \text{ of the thermometer}$$

$$\Rightarrow 34^\circ \text{C} = 51^\circ \text{C of the thermometer}$$

H.122 JEE Advanced Physics: Waves and Thermodynamics

So, thermometer will read $-20^\circ + 51^\circ = 31^\circ$

Hence, the correct answer is (A).

227. Since $E = \sigma(673)^4$ and $2E = \sigma T^4$

$$\Rightarrow T = 2^{\frac{1}{4}}(673) = 1.18(673) \approx 800 \text{ K}$$

Hence, the correct answer is (D).

228. Since, the gas is expanding isobarically, so $Q = nC_p\Delta T$

$$\Rightarrow 3.5RT_0 = 10\left(\frac{7R}{2}\right)(T - T_0)$$

$$\Rightarrow T_0 = 10T - 10T_0$$

$$\Rightarrow T = 1.1T_0$$

According to Charle's Law, we have $\frac{V}{1.1T_0} = \frac{V_0}{T_0}$

$$\Rightarrow V = 1.1V_0$$

Hence, the correct answer is (A).

229. $P \propto T$ (at constant volume)

Hence, the correct answer is (D).

230. Since, $v_{O_2} = \frac{1}{2}v_{H_2}$

$$\Rightarrow \sqrt{\frac{3RT}{32}} = \frac{1}{2}\sqrt{\frac{3R(273)}{2}}$$

$$\Rightarrow \frac{T}{32} = \frac{273}{8}$$

$$\Rightarrow T = 4(273) = 1092 \text{ K} = 819^\circ \text{C}$$

Hence, the correct answer is (D).

231. A cooking pot must conduct more. Also, it must absorb less heat itself. Hence a good cooking pot has low specific heat and high conductivity.

Hence, the correct answer is (D).

232. Since $\frac{Q}{t} = kA\left(\frac{dT}{dx}\right)$, so $\frac{mL}{t} = kA\left(\frac{dT}{dx}\right)$

$$\Rightarrow k \propto \frac{1}{t} \text{ i.e., } \frac{k_1}{k_2} = \frac{t_2}{t_1}$$

Hence, the correct answer is (B).

233. Since $Q_{AB} = nC_p\Delta T = n\frac{5}{2}R(2T_0 - T_0)$ and

$$Q_{BC} = W_{BC} = NR \cdot 2T_0 \log_e 2$$

$$\Rightarrow \frac{Q_{AB}}{W_{BC}} = \frac{5}{4\log_e 2}$$

Hence, the correct answer is (C).

234. Let the length of the part of metal A be L (in cm). Then length of part of metal B will be $(20 - L)$ cm. Since the change in length is proportional to original length, so we have

$$\frac{0.02}{20}L + \frac{0.04}{20}(20 - L) = 0.025$$

$$\Rightarrow 0.02L + 0.8 - 0.04L = 0.5$$

$$\Rightarrow L = 15 \text{ cm}$$

Hence, the correct answer is (C).

235. The molar specific heat of an ideal gas $C = \frac{dQ}{dT}$ can have any value lying between $-\infty$ and ∞

Hence, the correct answer is (D).

236. Adiabatic constant for the mixture is

$$\gamma_{\text{mix}} = \frac{n_1(C_p)_1 + n_2(C_p)_2}{n_1(C_v)_1 + n_2(C_v)_2}$$

$$\Rightarrow \gamma_{\text{mix}} = \frac{(0.125)\left(\frac{7}{2}R\right) + (0.5)\left(\frac{5}{2}R\right)}{(0.125)\left(\frac{5}{2}R\right) + (0.5)\left(\frac{3}{2}R\right)}$$

$$\Rightarrow \gamma_{\text{mix}} = \frac{0.4375 + 1.25}{0.3125 + 0.75} = \frac{27}{17}$$

For an adiabatic process, we have

$$T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$$

$$\Rightarrow T_2 = 390\left(\frac{2}{3}\right)^{\frac{10}{17}} \text{ K}$$

Hence, the correct answer is (A).

237. Since the length of the liquid column does not alter in the tube, hence we have superficial expansion and so $\gamma = 2\alpha$, instead of $\gamma = 3\alpha$.

Hence, the correct answer is (B).

238. Process A-B is an isothermal process i.e., $P \propto \frac{1}{V}$ and since, $\rho \propto \frac{1}{V}$, ρ - V graph will be a rectangular hyperbola. Pressure is increasing. Therefore, volume will decrease and hence, density will increase.

Process B-C is an isochoric process. Therefore,

$V = \text{constant}$ and since $\rho = \frac{m}{V}$, density is also constant i.e., ρ - V graph is a dot.

Process C-D is inverse of A-B and D-A is inverse of B-C.

Hence, the correct answer is (B).

239. Let T_{faulty} be the reading of faulty thermometer and T_C be the correct reading on Centigrade scale, then

$$\frac{T_{\text{faulty}} - 10}{90 - 10} = \frac{T_C - 0}{100 - 0}$$

Given that $T_{\text{faulty}} = T_C = T$ (say)

$$\Rightarrow \frac{T - 10}{90 - 10} = \frac{T - 0}{100 - 0}$$

$$\Rightarrow 100(T - 10) = 80T$$

$$\Rightarrow 20T = 1000$$

$$\Rightarrow T = 50^\circ \text{C}$$

Hence, the correct answer is (C).

240. Using $\frac{T_1 - T_2}{t} = \alpha\left(\frac{T_1 + T_2}{2} - \theta_0\right)$

In the first case

$$\frac{75 - 65}{2} = \alpha\left(\frac{75 + 65}{2} - 30\right)$$

...(1)

In the second case

$$\frac{55-45}{t} = \alpha \left(\frac{55+45}{2} - 30 \right) \quad \dots(2)$$

Dividing equations (1) by (2), we get $\frac{t}{2} = 2$

$$\Rightarrow t = 4 \text{ min}$$

Hence, the correct answer is (C).

241. Since, $H = (1) \left(\frac{3R}{2} \right) (10) = 15R \quad \{ \because H = nC_V \Delta T \}$

Also, $H' = (2) \left(\frac{7R}{2} \right) (5) = 35R \quad \{ \because H' = nC_P \Delta T' \}$

$$\Rightarrow H' = \frac{7}{3} (15R) = \frac{7}{3} H$$

Hence, the correct answer is (D).

242. Let (P_1, V_1, T_1) be the initial pressure, volume and temperature of the gas. When the gas is first expanded isothermally, then V increases and P decreases, such that

$$(P_1, V_1, T_1) \xrightarrow{\text{isothermal}} (P_2, V_2, T_1)$$

When the gas is compressed adiabatically to its original volume, then V decreases and hence P and T both increase, such that

$$(P_2, V_2, T_1) \xrightarrow{\text{adiabatic}} (P_3, V_1, T_2)$$

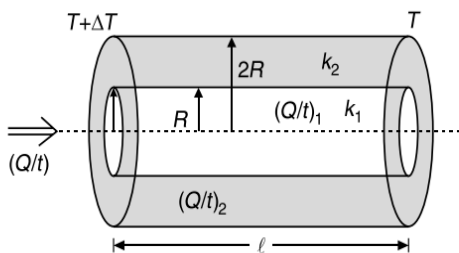
To restore the gas to its original state, the simplest operation is

$$(P_3, V_1, T_2) \xrightarrow{\text{isochoric}} (P_1, V_1, T_1)$$

This simplest operation should decrease both pressure and temperature and hence it can be an isochoric cooling process.

Hence, the correct answer is (B).

243. Since $\frac{Q}{t} = \left(\frac{Q}{t} \right)_1 + \left(\frac{Q}{t} \right)_2$



$$\Rightarrow k_{\text{eq}} \pi (2R)^2 \frac{\Delta T}{l} = k_1 (\pi R^2) \frac{\Delta T}{l} + k_2 [\pi (2R)^2 - \pi R^2] \frac{\Delta T}{l}$$

$$\Rightarrow 4k_{\text{eq}} = k_1 + 3k_2$$

$$\Rightarrow k_{\text{eq}} = \frac{k_1 + 3k_2}{4}$$

Hence, the correct answer is (C).

244. $dQ = mcdT = 0.1(100T + 500)dT$

$$\Rightarrow Q = \int_{20}^{400} (10T + 50)dT = \left[10 \left(\frac{T^2}{2} \right) + 50T \right]_{20}^{400}$$

$$\Rightarrow Q = 5(1600 - 400) + 50(40 - 20) = 7000 \text{ J}$$

Hence, the correct answer is (C).

245. P - V diagram of the gas is a straight line passing through origin. Hence, $P \propto V$

$$\Rightarrow PV^{-1} = \text{constant}$$

Molar heat capacity in process $PV^x = \text{constant}$ is

$$C = \frac{R}{\gamma-1} + \frac{R}{1-x}, \text{ where } \gamma = 1.4 \text{ for diatomic gas}$$

$$\Rightarrow C = \frac{R}{1.4-1} + \frac{R}{1+1} = 3R$$

Hence, the correct answer is (C).

246. Given that, $L_i - L_b = 0.1 \text{ m} \quad \dots(1)$

$$\Rightarrow \Delta L_b - \Delta L_i = 0$$

$$\Rightarrow L_b \alpha_b = L_i \alpha_i$$

$$\Rightarrow \frac{L_b}{L_i} = \frac{\alpha_i}{\alpha_b} = \frac{12}{18} = \frac{2}{3} \quad \dots(2)$$

Solving (1) and (2), we get

$$L_i = 0.3 \text{ m and } L_b = 0.2 \text{ m}$$

Hence, the correct answer is (C).

247. Since, $Q = nC\Delta T \quad \dots(1)$

By FLTD, we have

$$\Delta U = Q - W = Q - \frac{Q}{4} = \frac{3Q}{4}$$

$$\Rightarrow \frac{3Q}{4} = nC_V \Delta T \quad \dots(2)$$

Dividing (1) by (2), we get

$$C = \frac{4}{3} C_V = \frac{4}{3} \left(\frac{5R}{2} \right) = \frac{10R}{3}$$

Hence, the correct answer is (C).

248. Process 2 is an isothermal process

$$\text{Hence, } \Delta U_2 = 0$$

Process 1 is an isobaric ($P_1 = \text{constant}$) expansion

Hence, temperature of the gas will increase

$$\Rightarrow \Delta U_1 = \text{positive}$$

Process 3 is an adiabatic expansion. Hence, temperature will decrease

$$\Rightarrow \Delta U_3 = \text{negative}$$

Therefore, $\Delta U_1 > \Delta U_2 > \Delta U_3$ is the correct option

Hence, the correct answer is (A).

249. Conceptual

Hence, the correct answer is (C).

250. Since, $PV = nRT = \frac{m}{M} RT$, where m is mass of the gas of molar mass M . The slope of T - P graph is

$$\frac{dT}{dP} = \tan \theta = \frac{MV}{mR} \propto \frac{1}{m}$$

Hence, the correct answer is (B).

H.124 JEE Advanced Physics: Waves and Thermodynamics

251. A is free to move, therefore, heat will be supplied at constant pressure.

$$dQ_A = nC_P dT_A \quad \dots(1)$$

B is held fixed, therefore, heat will be supplied at constant volume.

$$dQ_B = nC_V dT_B \quad \dots(2)$$

But $dQ_A = dQ_B$ {given}

$$\Rightarrow nC_P dT_A = nC_V dT_B$$

$$\Rightarrow dT_B = \left(\frac{C_P}{C_V}\right) dT_A$$

$$\Rightarrow dT_B = \gamma(dT_A)$$

Since, $\gamma_{\text{diatomic}} = 1.4$ and $dT_A = 30$ K

$$\Rightarrow dT_B = (1.4)(30 \text{ K}) = 42 \text{ K}$$

Hence, the correct answer is (D).

252. Specific heat $C = \frac{\Delta Q}{\Delta T} = \frac{1}{\Delta T}(\Delta U + W) = C_V + \frac{W}{\Delta T}$

$$\text{For the given process, } W = 4V_0 \left(\frac{9P_0}{2}\right) = 18P_0V_0$$

$$\text{Also, } \Delta T = T_2 - T_1 = \frac{(6P_0)(5V_0)}{R} - \frac{(3P_0)V_0}{R} = \frac{27P_0V_0}{R}$$

$$\text{and } C_V = \frac{3}{2}R$$

$$\Rightarrow C = \frac{3R}{2} + \frac{2R}{3} = \frac{13R}{6}$$

Hence, the correct answer is (B).

253. Since work is being done at the expense of internal energy so it must be an adiabatic process i.e., $dQ = 0$

$$\Rightarrow n = \gamma$$

Hence, the correct answer is (C).

254. Since $V = \text{constant}$, so $W = 0$

$$\Rightarrow Q = nC_V \Delta T$$

$$\text{Now, } PV = nRT$$

$$\Rightarrow V \Delta P = nR \Delta T \quad \{V = \text{constant}\}$$

$$\Rightarrow \Delta T = \left(\frac{V}{nR}\right) \Delta P$$

For both processes ΔP and number of moles are same. Since V is more for process CD, hence ΔT is more for process CD. So, $Q_{CD} > Q_{AB}$

Hence, the correct answer is (C).

255. Since, no heat flows through AB, so

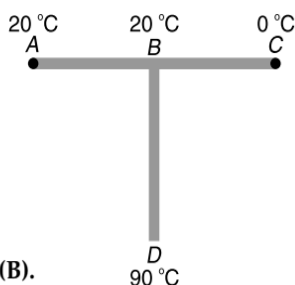
$$\theta_B = 20^\circ \text{C}$$

$$\text{Since, } H_{DB} = H_{BC}$$

$$\Rightarrow \frac{(90^\circ - 20^\circ)}{\ell_{BD}/KA} = \frac{(20^\circ - 0^\circ)}{\ell_{BC}/KA}$$

$$\Rightarrow \frac{\ell_{BD}}{\ell_{BC}} = \frac{7}{2}$$

Hence, the correct answer is (B).



256. Since $\rho = \frac{PM}{RT}$, so $\rho \propto \frac{P}{T}$

$$\text{Also, } \left(\frac{P}{T}\right)_A = \frac{P_0}{T_0} \text{ and } \left(\frac{P}{T}\right)_B = \left(\frac{3}{2}\right) \frac{P_0}{T_0}$$

$$\Rightarrow \left(\frac{P}{T}\right)_B = \frac{3}{2} \left(\frac{P}{T}\right)_A$$

$$\Rightarrow \rho_B = \frac{3}{2} \rho_A = \frac{3}{2} \rho_0$$

Hence, the correct answer is (B).

257. According to Ideal Gas Equation, we have $PV = nRT$

$$\Rightarrow n = \frac{PV}{RT}$$

Given that P, V, T are same, so, $n_1 = n_2 = n_3$

$$\text{Also } v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

Since, $M_{\text{neon}} < M_{\text{chlorine}} < M_{\text{uranium hexafluoride}}$

$$\Rightarrow v_1 > v_2 > v_3$$

Hence, the correct answer is (D).

258. Since, $\frac{\Delta L}{L} = \alpha \Delta T$

$$\Rightarrow -\frac{1.1 \times 10^{-3}}{10} = 11 \times 10^{-6} \Delta T$$

$$\Rightarrow \Delta T = T_f - T_i = -10$$

$$\Rightarrow T_f = 27 - 10 = 17^\circ \text{C}$$

Hence, the correct answer is (C).

259. In isochoric process $V = \text{constant}$ and $P \propto T$

Therefore, P - T graph is a straight line passing through origin. But since

$$P = \left(\frac{nR}{V}\right) T$$

Slope of the straight line $\propto \frac{1}{V}$

$$(\text{Slope})_{12} < (\text{Slope})_{34}$$

$$\Rightarrow V_2 > V_3$$

$$\text{Also } V_1 = V_2 \text{ and } V_3 = V_4$$

Hence, the correct answer is (A).

260. **METHOD I**

Let resistance at 0°C be R_0 . Then

$$2 = R_0(1 + \alpha(20)) \quad \dots(1)$$

$$4 = R_0(1 + \alpha(100)) \quad \dots(2)$$

Dividing (2) by (1), we get

$$2 = \frac{1 + 100\alpha}{1 + 20\alpha}$$

$$\Rightarrow 2 + 40\alpha = 1 + 100\alpha$$

$$\Rightarrow \alpha = \frac{1}{60}^\circ \text{C}^{-1}$$

If resistance is 2.2Ω at $T^\circ \text{C}$, then we have

$$2.2 = R_0(1 + \alpha T) \quad \dots(3)$$

Dividing equation (3) by (1), we get

$$\frac{2.2}{2} = \frac{1 + \alpha T}{1 + 20T} = \frac{1 + \frac{T}{60}}{1 + \frac{20}{60}}$$

$$\Rightarrow (1.1) \left(\frac{4}{3} \right) = 1 + \frac{T}{60}$$

$$\Rightarrow T = 60(1.463 - 1) \approx 28 \text{ }^\circ\text{C}$$

METHOD II

$$\frac{T - 20}{100 - 20} = \frac{R_T - R_{20}}{R_{100} - R_{20}} = \frac{2.2 - 2}{4 - 2}$$

$$\Rightarrow T = \left(\frac{0.2}{2} \times 80 \right) + 20 = 28 \text{ }^\circ\text{C}$$

Hence, the correct answer is (D).

261. Conceptual

Hence, the correct answer is (B).

262. (Rate of Loss of Heat) \propto (Temperature Difference)

$$\Rightarrow \frac{dQ}{dt} \propto T - T_0$$

$$\Rightarrow 10 = \beta(50 - 20)$$

$$\Rightarrow \beta = \frac{1}{3} \quad (\text{where } \beta \text{ is just a constant of proportionality})$$

At average temperature of $35 \text{ }^\circ\text{C}$ we have

$$\frac{dQ}{dt} = \beta(35 - 20) = \frac{1}{3}(15) = 5 \text{ Js}^{-1}$$

So, heat lost in a time of one minute (= 60 second) is

$$Q_{\text{total}} = 5 \times 60 = 300 \text{ J}$$

By definition, heat capacity is

$$c = \frac{Q_{\text{total}}}{\Delta T} = \frac{300}{0.2} = 1500 \text{ J}(\text{ }^\circ\text{C})^{-1}$$

Hence, the correct answer is (B).

263. $\text{KE} = n \left(\frac{3}{2} RT \right)$

$$\Rightarrow n_1 \left(\frac{3}{2} RT_1 \right) = n_2 \left(\frac{3}{2} RT_2 \right)$$

Since $n_2 = 2n_1$, so $T_1 = 2T_2$

Also $PV = nRT$ and since, $V = \text{constant}$, so $P \propto nT$

$$\Rightarrow \frac{P_1}{P_2} = \left(\frac{n_1}{n_2} \right) \left(\frac{T_1}{T_2} \right) = \left(\frac{1}{2} \right) (2) = 1$$

Hence, the correct answer is (B).

264. Since $dS = \frac{dQ}{T}$

$$\Rightarrow dS = \frac{dU}{T} + \frac{PdV}{T}$$

$$\Rightarrow dS = nC_V \frac{dT}{T} + \frac{nRT}{V} \frac{dV}{T}$$

$$\Rightarrow dS = nC_V \frac{dT}{T} + nR \frac{dV}{V}$$

$$\Rightarrow \int_{S_1}^{S_2} dS = nC_V \int_{T_1}^{T_2} \frac{dT}{T} + nR \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\Rightarrow S_2 - S_1 = \Delta S = nC_V \ln \left(\frac{T_2}{T_1} \right) + nR \ln \left(\frac{V_2}{V_1} \right)$$

Hence, the correct answer is (A).

265. Since $dU = 0$, so by First Law of Thermodynamics

$$dQ_{\text{Cyclic}} = dW_{\text{Cyclic}}$$

Process $B \rightarrow C$ is isochoric, so

$$\Rightarrow dW_{B \rightarrow C} = 0 \quad \{\because dV = 0\}$$

$$\Rightarrow 5 = dW_{A \rightarrow B} + dW_{B \rightarrow C} + dW_{C \rightarrow A}$$

$$\Rightarrow 5 = 10(2 - 1) + 0 + dW_{C \rightarrow A}$$

$$\Rightarrow dW_{C \rightarrow A} = -5 \text{ J}$$

Hence, the correct answer is (A).

266. The volume of the liquid that overflows is $\Delta V = V \gamma_a \Delta T$

$$\Rightarrow \frac{V}{100} = V \gamma_a (80)$$

$$\Rightarrow \gamma_a = \frac{1}{8000} = 1.25 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$$

Hence, the correct answer is (A).

267. $W_{AB} = 0$ as $V = \text{constant}$

$$\Rightarrow Q_{AB} = \Delta U_{AB} = 50 \text{ J} \quad \{\text{given}\}$$

$$U_A = 1500 \text{ J}$$

$$\Rightarrow U_B = (1500 + 50) \text{ J} = 1550 \text{ J}$$

$$W_{BC} = -\Delta U_{BC} = -40 \text{ J} \quad \{\text{given}\}$$

$$\Rightarrow \Delta U_{BC} = 40 \text{ J}$$

$$\Rightarrow U_C = (1550 + 40) \text{ J} = 1590 \text{ J}$$

Hence, the correct answer is (A).

268. Since, $Q = mc_{\text{body}} \Delta T$, where $c_{\text{body}} = 1 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$ and

$$\Delta T = (103 - 98.4) \text{ }^\circ\text{F} = 4.6 \text{ }^\circ\text{F}$$

$$\text{Also we know that } \Delta C = \frac{5}{9} \Delta F$$

So, a temperature increment of $4.6 \text{ }^\circ\text{F}$ equals a temperature

$$\text{increment of } (4.6) \left(\frac{5}{9} \text{ }^\circ\text{C} \right) = 2.56 \text{ }^\circ\text{C}$$

$$\Rightarrow Q = (60000 \text{ g}) (1 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}) (2.56 \text{ }^\circ\text{C})$$

$$\Rightarrow Q = 153600 \text{ cal} \approx 154 \text{ kcal}$$

Hence, the correct answer is (B).

269. Given that, $\left(\frac{C_P}{C_V} \right)_{\text{mix}} = \frac{3}{2}$

$$\text{Since } C_{P_{\text{mix}}} = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 + n_2} \text{ and } C_{V_{\text{mix}}} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$$

H.126 JEE Advanced Physics: Waves and Thermodynamics

$$\begin{aligned} \Rightarrow \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 C_{V_1} + n_2 C_{V_2}} &= \frac{3}{2} \\ \Rightarrow \frac{n_1(5R/2) + n_2(7R/2)}{n_1(3R/2) + n_2(5R/2)} &= \frac{3}{2} \\ \Rightarrow 10n_1 + 14n_2 &= 9n_1 + 15n_2 \\ \Rightarrow n_1 &= n_2 \end{aligned}$$

Hence, the correct answer is (C).

270. Since $Q = nC_p\Delta T = \frac{7}{2}nR\Delta T$ $\left\{ \because C_p = \frac{7}{2}R \right\}$

and $\Delta U = nC_v\Delta T = \frac{5}{2}nR\Delta T$ $\left\{ \because C_v = \frac{5}{2}R \right\}$

$$\Rightarrow W = Q - \Delta U = nR\Delta T$$

$$\Rightarrow Q : \Delta U : \Delta W = 7 : 5 : 2$$

Hence, the correct answer is (C).

271. For the first process, we have $Q_1 = \Delta U + W_1$

$$\Rightarrow 8 \times 10^5 = \Delta U + 6.5 \times 10^5$$

$$\Rightarrow \Delta U = 1.5 \times 10^5 \text{ J}$$

For the second process, ΔU is same, therefore,

$$Q_2 = \Delta U_2 + W_2 = \Delta U + W_2$$

$$\Rightarrow 10^5 = 1.5 \times 10^5 + W_2$$

$$\Rightarrow W_2 = -0.5 \times 10^5 \text{ J}$$

Hence, the correct answer is (A).

272. Since, $\rho = \frac{PM}{RT}$

Density ρ remains constant, when $\frac{P}{T}$ or volume remains constant

In graph (i) volume is decreasing hence, density is increasing while in graphs (ii) and (iii) volume is increasing hence, density is decreasing. Note that volume would had been constant in case the straight line in graph (iii) had passed through origin.

Hence, the correct answer is (C).

273. Since $C_p = \frac{3}{2}R$

$$\Rightarrow C_v = \frac{1}{2}R \quad \left\{ \because C_p - C_v = R \right\}$$

$$\Rightarrow \gamma = 3$$

$$\text{Since } \gamma = 1 + \frac{2}{f}$$

$$\Rightarrow f = 1$$

Possible for a monatomic gas molecule moving in a straight fixed line (one degrees of freedom).

Hence, the correct answer is (B).

274. Thermal Resistance = $R = \frac{\text{Temperature Difference}}{\text{Thermal Current}}$

$$\Rightarrow [\text{Thermal Resistance}] = \frac{K}{\left(\frac{ML^2T^{-2}}{T} \right)}$$

$$\Rightarrow [R] = M^{-1}L^{-2}T^3K$$

Hence, the correct answer is (B).

275. Efficiency of an ideal heat engine is $\eta = 1 - \frac{T_2}{T_1}$

$$\Rightarrow \frac{60}{100} = 1 - \frac{300}{T_2}$$

$$\Rightarrow T_2 = 750 \text{ K} = 477^\circ \text{C}$$

Hence, the correct answer is (C).

276. Since, $P = \frac{\alpha T^2}{V}$ ($P = \text{constant}$)

$$\Rightarrow V = \frac{\alpha T^2}{P}$$

$$\Rightarrow dV = \left(\frac{2\alpha T}{P} \right) dT$$

$$\Rightarrow W = \int_{T_0}^{2T_0} PdV = \int_{T_0}^{2T_0} (P) \left(\frac{2\alpha T}{P} \right) dT = 3\alpha T_0^2$$

Hence, the correct answer is (D).

277. For an adiabatic process

$$dQ = 0 = dU + dW$$

$$\Rightarrow 0 = dU + PdV$$

From the given equation

$$dU = 3(PdV + VdP)$$

$$\Rightarrow 0 = 3(PdV + VdP) + PdV$$

$$\Rightarrow 4P(dV) + 3V(dP) = 0$$

$$\Rightarrow 4 \left(\frac{dV}{V} \right) = -3 \left(\frac{dP}{P} \right)$$

On integrating, we get

$$\ln(V^4) + \ln(P^3) = \text{constant}$$

$$\Rightarrow PV^{4/3} = \text{constant}$$

$$\Rightarrow \gamma = \frac{4}{3}$$

i.e., gas is polyatomic.

Hence, the correct answer is (C).

278. For a cyclic process, $\Delta U = 0$ and hence $Q = W$

$$\Rightarrow Q_{ab} + Q_{bc} + Q_{ca} = W_{ab} + W_{bc} + W_{ca}$$

Since the process ab is isochoric, so $W_{ab} = 0$

Also, it is given that $W_{bc} = -50 \text{ J}$

$$\Rightarrow 100 + 0 + Q_{ca} = 0 + (-50) + W_{ca}$$

$$\Rightarrow Q_{ca} - W_{ca} = -150 \text{ J}$$

$$\Rightarrow \Delta U_{ca} = Q_{ca} - W_{ca} = -150 \text{ J}$$

Hence, the correct answer is (B).

279. Volume of the gas is constant

$$\Rightarrow \frac{T_2}{T_1} = \frac{P_2}{P_1}$$

$$\Rightarrow \left(\frac{T_1 + 1}{T_1} \right) = \frac{1.01P_1}{P_1}$$

$$\Rightarrow 0.01T_1 = 1$$

$$\Rightarrow T_1 = 100 \text{ K}$$

Hence, the correct answer is (A).

280. Since, $P_1V = n_1RT_1$ and $P_2V = n_2RT_2$

$$\Rightarrow \frac{n_1R}{V} = \frac{P_1}{T_1} \text{ and } \frac{n_2R}{V} = \frac{P_2}{T_2}$$

When the vessels are joined, then we have

$$P(V+V) = (n_1 + n_2)RT$$

$$\Rightarrow \frac{P}{T} = \frac{(n_1 + n_2)R}{2V} = \frac{1}{2} \left(\frac{P_1}{T_1} + \frac{P_2}{T_2} \right)$$

Hence, the correct answer is (A).

281. $\frac{W}{Q} = \frac{Q - dU}{Q}$ $\{\because \text{From First Law } dQ = dU + dW\}$

$$\Rightarrow \frac{W}{Q} = 1 - \frac{dU}{dQ} = 1 - \frac{nC_V dT}{nC_P dT} = 1 - \frac{C_V}{C_P}$$

$$\Rightarrow \frac{dW}{dQ} = 1 - \frac{1}{\gamma}$$

Hence, the correct answer is (B).

282. Since, $P = \frac{1}{3} \frac{mn}{V} v_{\text{rms}}^2$... (1)

$$\Rightarrow P \propto m \propto v_{\text{rms}}^2$$

Now, m is halved and v_{rms} is doubled, so P will become two times.

In equation (1) m is the mass of one gas molecule and n the total number of gas molecules.

Hence, the correct answer is (B).

283. Pressure is exerted on the wall due to collisions of molecules with the walls of the container. It is independent of the velocity of container and the frame of reference.

Hence, the correct answer is (A).

284. Since, $U - 0 = nC_V(T - 0)$

$$\Rightarrow U = nC_V T$$

$$\Rightarrow U = \frac{nRT}{\gamma - 1} = \frac{PV}{\gamma - 1} \quad \{\because PV = nRT\}$$

Hence, the correct answer is (C).

285. According to Stefan's Law $E = e\sigma T^4$

where e is the emissivity of body and $0 < e < 1$

$$\Rightarrow E = \frac{300}{60 \times 5 \times 10^{-4}} = 10000 \text{ Wm}^{-2}$$

$$\Rightarrow 10^4 = e(5.67 \times 10^{-8})(1000)^4$$

$$\Rightarrow 10^{-8} = e(5.67)$$

$$\Rightarrow e = \frac{1}{5.67} \approx 0.18$$

Hence, the correct answer is (B).

286. In process ABC, work done is

$$W = P_0(2V_0 - V_0) = P_0V_0$$

According to Ideal Gas Equation, we have

$$\frac{P_A V_A}{T_A} = \frac{P_C V_C}{T_C}$$

$$\Rightarrow \frac{P_0 V_0}{T_0} = \frac{(0.5P_0)(2V_0)}{T_C}$$

$$\Rightarrow T_C = T_A = T_0$$

$$\Rightarrow \Delta U = nC_V \Delta T = (1) \left(\frac{3R}{2} \right) (T_C - T_A) = 0$$

$$\text{and } Q = \Delta U + W = 0 + P_0 V_0 = RT_0$$

Hence, the correct answer is (C).

287. For an Isotherm $T = \text{constant}$

$$\Rightarrow PV = \text{constant}$$

$$\Rightarrow PV \text{ vs } V \text{ is a straight line.}$$

{OPTION (A)}

Similarly, S vs T is a straight line.

{OPTION (B)}

V and P plotted together must give a rectangular hyperbola ($PV = \text{constant}$). P vs T will again be a straight line.

{OPTION (D)}

Hence, the correct answer is (C).

288. $C_p - C_V = 2 \text{ cal mol}^{-1} (\text{°C})^{-1}$

Hence, the correct answer is (A).

289. For ideal monatomic gas $C_p = \frac{5}{2}R$

$$\Rightarrow R = \frac{2}{5}C_p = 0.4C_p$$

$$\Rightarrow n = 0.4$$

Hence, the correct answer is (C).

290. $v_{\text{mps}} = \sqrt{\frac{2RT}{M}}$

$$\Rightarrow 1600 = \sqrt{\frac{2 \times 8.31 \times 300}{M}}$$

$$\Rightarrow M \approx 2 \times 10^{-3} \text{ kg} = 2 \text{ g}$$

Hence the gas is H_2

Hence, the correct answer is (C).

291. When, external work done equals heat supplied, then

$$dU = 0$$

$$\Rightarrow T = \text{constant i.e., Isothermal process}$$

Hence, the correct answer is (C).

292. $F = YA\alpha\Delta T = E(1)(\alpha)(1) = E\alpha$

Hence, the correct answer is (A).

293. Initial and final temperatures are same in the both the graphs. Therefore, ΔU is same. Also area under P - V graphs is same. Therefore, work done is same and hence heat exchanged is also same.

Hence, the correct answer is (A).

294. Area of the shell is $A = 4\pi R^2$

$$\text{Rate of flow of heat is } P = k(4\pi R^2) \frac{T}{d}$$

H.128 JEE Advanced Physics: Waves and Thermodynamics

$$\Rightarrow d = \frac{4\pi R^2 T k}{P}$$

Hence, the correct answer is (B).

295. $\frac{E_2}{E_1} = \frac{327 + 273}{27 + 273} = 2$

Hence, the correct answer is (D).

Multiple Correct Choice Type Questions

1. Potential energy of an ideal gas is zero because intermolecular forces between them is zero. Degree of freedom of an ideal diatomic gas is 5. In this 2 is rotational and 3 is translational. According to Law of Equipartition of Energy internal energy is equally distributed in all degrees of freedom.

Therefore, rotational kinetic energy = $\left(\frac{2}{2+3}\right) \times 100 = 40$ J

and translational kinetic energy = $\left(\frac{3}{2+3}\right) \times 100 = 60$ J.

Hence, (A), (B) and (C) are correct.

2. Internal energy (U) depends only on the initial and final status. Hence, ΔU will be same in all the three paths. In all the three paths work done by the gas is positive (volume is increasing) and the product PV or temperature. T is increasing. Therefore, internal energy is also increasing so, from First Law of Thermodynamics heat will be absorbed by the gas. Further area under P-V graph is maximum in path 1 while ΔU is same for all three paths. Therefore, heat absorbed by the gas is maximum in path 1. For temperature of the gas we can see the product PV which first increases in path 1 but whether it is decreasing or increasing later on we cannot say anything about it unless the exact values are known to us.

Hence, (A), (B) and (C) are correct.

3. $P(\text{in watt}) = \frac{dQ}{dt}$ (in joule per second)

$$\Rightarrow P(\text{in watt}) = mc \left(\frac{dT}{dt} \right)$$

$$\Rightarrow P = \left(\frac{180}{1000} \right) (0.10) (4200) (0.5)$$

$$\Rightarrow P = 37.8 \text{ watt}$$

Further $P = \tau \omega$

$$\Rightarrow 37.8 = \tau \left(180 \times \frac{2\pi}{60} \right)$$

$$\Rightarrow \tau = 2 \text{ Nm}$$

Hence, (A) and (C) are correct.

4. Since $C_V = \frac{dU}{dT}$ {at constant volume}

$$C_P = \left(\frac{dQ}{dT} \right)_P \quad \text{{at constant pressure}}$$

$$C = \frac{dU + dW}{dT} \quad \left\{ \begin{array}{l} \text{for any process other} \\ \text{than an adiabatic process} \end{array} \right\}$$

$$\Rightarrow C = \frac{dU}{dT} + \frac{PdV}{dT} \quad \left\{ \because \text{work done } dW = PdV \right\}$$

Hence, (A), (B) and (C) are correct.

5. In 1 mole of all the gases there are N (Avagadro's number) number of molecules which are independent of the nature of gas. Hence, OPTION (A) is correct. The translational kinetic energy of 1 mole of any gas at temperature T is $\frac{3}{2RT}$. Therefore, OPTION (C) is also correct.

Hence, (A) and (C) are correct.

6. Let C_A and C_B be their heat capacities. Both the spheres have the same surface area. At any given temperature θ they lose heat at the same rate.

$$\Rightarrow C_A \left(-\frac{d\theta}{dt} \right)_A = C_B \left(-\frac{d\theta}{dt} \right)_B$$

$$\Rightarrow \frac{\left(-\frac{d\theta}{dt} \right)_A}{\left(-\frac{d\theta}{dt} \right)_B} = \frac{C_B}{C_A} = \text{constant}$$

i.e., rate of cooling of B is fast, but their ratio is constant.

Hence, (C) and (D) are correct.

7. Speed of a gas molecules, $v \propto \sqrt{T}$
Change in momentum of a molecule on hitting a wall and then rebounding is $\Delta p \propto v \propto \sqrt{T}$
Since, pressure of a gas is equal to change in momentum of a molecule colliding with the wall (Δp) multiplied by number of collisions per unit area per second (n). So, for
OPTION (A): If temperature and volume are doubled, pressure remains same. Therefore, n decreases since Δp increases. Thus, OPTION (A) is wrong.

OPTION (B): If temperature and volume are halved, n increases since Δp decreases. Therefore OPTION (B) is correct.

OPTION (C): If pressure and temperature are doubled, n increases because pressure doubles whereas Δp increases only $\sqrt{2}$ times.

Hence, (B) and (C) are correct.

8. $\frac{Q}{t} = KA \left(\frac{T_S - T_R}{\ell} \right)$

$$T_S = \text{Temperature of steam}$$

$$T_R = \text{Temperature of room}$$

T_0 increase $\frac{Q}{t}$ either A is increased or ℓ is decreased or T_R is decreased.

Hence, (A) and (D) are correct.

9. $H = \text{rate of heat flow} = \frac{900}{\frac{\ell_1}{K_i A} + \frac{\ell_0}{K_0 A}}$

$$\text{Now } 1000 - T = H \left(\frac{\ell_1}{K_i A} \right)$$

$$\Rightarrow T = 1000 - \left(\frac{900}{\frac{\ell_1}{K_i A} + \frac{\ell_0}{K_0 A}} \right) \frac{\ell_1}{K_i A}$$

$$\Rightarrow T = 1000 - \frac{900}{1 + \frac{\ell_0 K_i}{K_0 \ell_1}}$$

We also observe that T can be decreased by increasing thermal conductivity of outer layer (K_0) and thickness of inner layer (ℓ_1).

Hence, (A) and (D) are correct.

10. Since, 1 kg ice at 0°C needs 80000 cal to get converted to water at 0°C and 1.5 kg water at 45°C gives out 67500 cal to get converted to water at 0°C .

So, entire ice cannot be converted to water and the equilibrium temperature will be 0°C . So mixture contains both ice and water. Amount of ice left unmelted due to insufficiency of available heat is

$$m_{\text{ice}} = \frac{80000 - 67500}{80} = 156.25 \text{ g}$$

Hence, (A) and (B) are correct.

11. $L_1 - L_2 = \text{constant}$

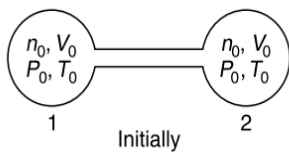
$$\Rightarrow \Delta L_1 - \Delta L_2 = 0$$

$$\Rightarrow \Delta L_2 = \Delta L_1$$

$$\Rightarrow L_2 \alpha_2 = L_1 \alpha_1$$

Hence, (B) and (C) are correct.

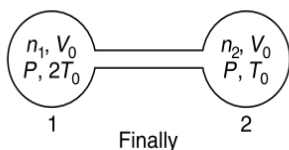
12. $P_0 V_0 = n_0 R T_0$ (for container 1) ... (1)



$$P_0 V_0 = n_0 R T_0 \text{ (for container 2)} \quad \dots (2)$$

$$\text{Total number of moles} = N = n_0 + n_0 = 2n_0$$

Since, even on heating the total number of moles is conserved, hence



$$n_1 + n_2 = 2n_0 \quad \dots (3)$$

Further if P be the common pressure, then

$$PV_0 = n_1 R (2T_0) \quad \text{\{in container 1\}}$$

$$\Rightarrow n_1 = \frac{PV_0}{2RT_0} \quad \dots (4)$$

$$\text{Also, } PV_0 = n_2 RT_0 \quad \text{\{in container 2\}}$$

$$\Rightarrow n_2 = \frac{PV_0}{RT_0} \quad \dots (5)$$

Since, $n_1 + n_2 = 2n_0$

$$\Rightarrow \frac{PV_0}{2RT_0} + \frac{PV_0}{RT_0} = 2 \frac{P_0 V_0}{RT_0} \quad \text{\{ \because of (1) \}}$$

$$\Rightarrow P = \frac{4}{3} P_0$$

$$\text{Also, } n_1 = \frac{4}{3} \frac{P_0 V_0}{RT_0} = \frac{2}{3} \frac{P_0 V_0}{RT_0}$$

Hence, (B) and (C) are correct.

13. During expansion an isotherm lies above an adiabat, so greater work is done in isothermal process than in adiabatic process. {OPTION (A)}

Let both expand to a final volume V , then

$$P_0 V_0 = P_{\text{iso}} V \quad \text{and} \quad P_0 V_0^\gamma = P_{\text{adi}} V^\gamma$$

$$\Rightarrow P_{\text{iso}} = P_0 \left(\frac{V_0}{V} \right) \quad \text{and} \quad P_{\text{adi}} = P_0 \left(\frac{V_0}{V} \right)^\gamma$$

Since $\frac{V_0}{V} < 1$ and $\gamma > 1$

$$\Rightarrow P_{\text{iso}} > P_{\text{adi}} \quad \text{\{OPTION (B)\}}$$

Finally, since we know that adiabatic expansion always leads to cooling, so we have final temperature of adiabatic process less than the constant temperature of isothermal process. {OPTION (C)}

Hence, (A), (B) and (C) are correct.

14. Refer to SOLUTION of PROBLEM 13.

Hence, (A), (B) and (C) are correct.

15. Since, $Q_1 = W + Q_2$

$$\Rightarrow W = Q_1 - Q_2 > 0$$

$$\Rightarrow Q_1 > Q_2 > 0 \text{ OR } Q_1 < Q_2 < 0$$

Hence, (A) and (C) are correct.

16. Process DA is isothermal

$$\Rightarrow T_D = T_A$$

Further $T \propto PV$

From the graph we can see that $(PV)_C > (PV)_D$

$$\Rightarrow T_C > T_D$$

Further process BC is an adiabatic expansion. So, $T_D > T_C$. Therefore, maximum temperature is at B and minimum temperature at D or A . Hence,

$$T_B = 4T_0 \text{ and } T_D = T_A = T_0$$

In process CD :

$$W = 0$$

$$\Rightarrow Q = \Delta U \quad \text{\{ \because of FLTD \}}$$

Volume of the gas is constant and pressure is decreasing. Therefore, temperature and hence, internal energy will decrease i.e., Q is negative or heat is released by the gas in the process CD .

In process AB :

$$Q = n C_p \Delta T \quad \text{\{process is isobaric\}}$$

Pressure is constant and volume is increasing. Therefore, temperature will also increase or Q is positive. Thus, heat is supplied to the gas only in process AB

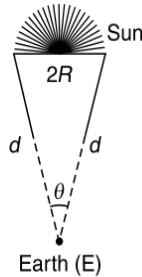
$$\text{and } Q = n \left(\frac{5R}{2} \right) (T_B - T_A)$$

H.130 JEE Advanced Physics: Waves and Thermodynamics

$$\Rightarrow Q = n \left(\frac{5R}{2} \right) (4T_0 - T_0) = \frac{15}{2} nRT_0$$

Hence, (A), (B) and (C) are correct.

17. Power radiated by the sun is $P = (4\pi R^2) \sigma T^4$



Further,

$$\text{Solar constant} = \left(\frac{\text{Energy received per second per unit area}} \right) = \frac{P}{4\pi d^2} = S$$

$$\Rightarrow P = 4\pi d^2 S$$

$$\text{For equilibrium } 4\pi R^2 \sigma T^4 = 4\pi d^2 S$$

$$\Rightarrow S = \frac{1}{4} \sigma T^4 \left(\frac{2R}{d} \right)^2 = \frac{1}{4} \sigma T^4 \theta^2 \quad \left\{ \because \frac{2R}{d} = \theta \right\}$$

$$\Rightarrow S \propto T^4$$

$$\Rightarrow S \propto \theta^2$$

Hence, (B) and (D) are correct.

18. Conceptual

Hence, (B) and (D) are correct.

19. A refrigerator is reverse of heat engine and heat pump is same as refrigerator. The coefficient of performance of a refrigerator cannot be infinity and a heat engine cannot convert the heat input fully to work done.

Hence, (C) and (D) are correct.

20. Water equivalent (w) of a body of mass m , gram specific heat c is just the product of m and c i.e., $w = mc$. Similarly, heat capacity of a body of mass m , gram specific heat c is also product of m and c . The major difference between both is that w is expressed in the units of mass and heat capacity is expressed in the units of energy (calorie or joule).

Hence, (A) and (C) are correct.

21. Since, $p^2V = \text{constant}$

$$\Rightarrow \left(\frac{RT}{V} \right)^2 V = \text{constant}$$

$$\Rightarrow T^2 \propto V$$

So, when volume becomes $3V_0$, then temperature also becomes $\sqrt{3}T_0$. Since temperature has increased, so internal energy of the gas will increase.

Hence, (A) and (B) are correct.

22. P - T graph is a straight line passing through origin. Therefore,

$$V = \text{constant}$$

So, work done on the gas is zero.

Further density of the gas $\rho = \frac{m}{V} \propto \frac{1}{V}$

Volume of the gas is constant. Therefore, density of gas is also constant.

$$PV = nRT$$

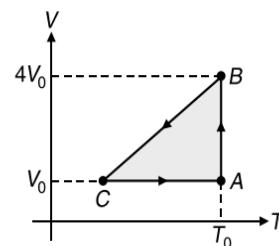
$$\Rightarrow P = \left(\frac{nR}{V} \right) T$$

i.e., slope of P - T line $\propto n$

Hence, (A), (B) and (D) are correct.

23. For n moles of gas with f degrees of freedom at temperature T , we have

$$U = \frac{f}{2} nRT$$



Since, temperature T is same at A and B , so

$$U_A = U_B$$

$$\text{Also, } W_{AB} = nRT_0 \log_e \left(\frac{V_f}{V_i} \right) = nRT_0 \log_e \left(\frac{4V_0}{V_0} \right)$$

$$W_{AB} = nRT_0 \log_e (4) = P_0 V_0 \log_e (4)$$

Hence, (A) and (B) are correct.

24. Conceptual

Hence, (A), (B), (C) and (D) are correct.

25. Since work done is area under the graph, OPTION (A) is incorrect.

Since net work done in the cyclic clockwise process is positive and for cyclic process $Q = W$, so OPTION (B) is correct. Since $PV = nRT$, so temperature is maximum at C and hence OPTION (C) is incorrect.

Process C to A is isochoric and pressure is falling. Therefore, temperature also falls i.e. ΔU and Q are negative. Therefore OPTION (D) is correct.

Hence, (B) and (D) are correct.

26. Vibrational kinetic energy of a monatomic gas = 0 at all temperatures. So, $C_V = \frac{3}{2}R$ for a monatomic gas even at high temperatures also.

In case of a diatomic gas

$$C_V = \frac{5}{2}R \text{ at low temperatures, while}$$

$$C_V > \frac{5}{2}R \text{ at high temperature due to vibrational kinetic energy}$$

Hence, (A) and (D) are correct.

27. Equilibrium of piston gives

$$PS = P_a S + mg + kx_0$$

where, P is the final pressure of the gas

$$\Rightarrow P = P_a + \frac{mg}{S} + \frac{kx_0}{S}$$

Work done by the gas (W) equals the sum of work done against atmospheric pressure (W_{pr}), elastic potential energy storied in the spring (ΔU_{sp}) and the increase in gravitational potential energy (ΔU_g) of the piston.

$$\Rightarrow W = W_{pr} + \Delta U_{sp} + \Delta U_g$$

where, $W_{pr} = P_0 \Delta V$, $\Delta U_{sp} = \frac{1}{2} kx_0^2$, $\Delta U_g = mvx_0$

$$\Rightarrow W = P_0 \Delta V + \frac{1}{2} kx_0^2 + mgx_0$$

$$\Rightarrow W = P_a Sx_0 + \frac{1}{2} kx_0^2 + mgx_0$$

This is also the decrease in internal energy of the gas. Because the gas is thermally insulated and this work is done in the expense of internal energy of the gas.

Hence, (A) and (C) are correct.

28. Equilibrium of piston gives

$$PS = kx_0$$

$$\Rightarrow P = \frac{kx_0}{S}$$

Since, the chamber is thermally insulated

$$\Delta Q = 0$$

$$\Rightarrow \left(\begin{array}{c} \text{Elastic Potential Energy} \\ \text{of Spring} \end{array} \right) = \left(\begin{array}{c} \text{Work Done} \\ \text{by Gas} \end{array} \right)$$

$$\Rightarrow \text{Work done by gas} = \frac{1}{2} kx_0^2$$

This work is done in the expense of internal energy of the gas. Therefore, internal energy of the gas is decreased by

$$\frac{1}{2} kx_0^2.$$

Internal energy of an ideal gas depends on its temperature only. Internal energy of the gas is decreasing. Therefore, temperature of the gas will decrease.

Hence, (A), (B), (C) and (D) are correct.

29. Since $TP^{-2/5} = \text{constant}$, so comparing it with equation of a polytropic process in which

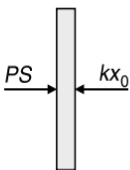
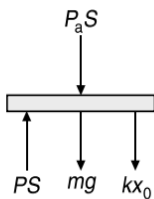
$$TP^{\frac{1-x}{x}} = \text{constant}$$

$$\Rightarrow \frac{1-x}{x} = -\frac{2}{5}$$

$$\Rightarrow 5 - 5x = -2x$$

$$\Rightarrow x = \frac{5}{3} = \gamma_{\text{monatomic}}$$

So, this represents an adiabatic process, for which $Q = 0$ i.e., $C = 0$

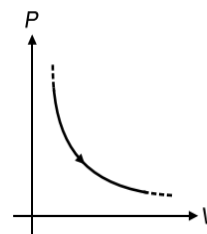


$$\text{So, } \Delta U = nC_V \Delta T = (2) \left(\frac{3R}{2} \right) (2T) = 6RT$$

For an adiabatic process, $Q = -\Delta U = -6RT$

Hence, (B), (C) and (D) are correct.

30. (A) It is incorrect because isothermal process is graphically represented like as shown.



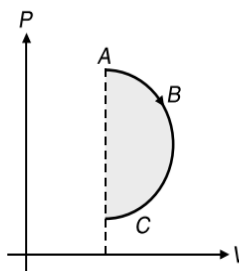
- (B) For $B \rightarrow C \rightarrow D$, $\Delta U = -ve$ and $W = -ve$

$$\Rightarrow Q = -ve$$

- (C) $W = \text{shaded area} = +ve$

- (D) Since the P - V cycle is clockwise, so

$$W = +ve$$



Hence, (B) and (D) are correct.

31. Since $U_c - U_a = (1) \left(\frac{3R}{2} \right) (T_c - T_a)$

$$\Rightarrow U_c - U_a = \left(\frac{3R}{2} \right) \left(\frac{8P_0V_0}{R} - \frac{P_0V_0}{R} \right) = \left(\frac{21}{2} \right) P_0V_0$$

$$\Rightarrow U_c - U_a = 10.5RT_0$$

and similarly it can be done for others.

Hence, (A), (B), (C) and (D) are correct.

32. Internal energy of n moles of an ideal gas is

$$U = \left(\frac{f}{2} \right) (nRT) \quad f = \text{degrees of freedom}$$

i.e., U depends on number of moles n and temperature T .

Similarly, $PV = nRT$

i.e., PV depends on n and T

$$\text{But } \frac{P}{\rho} = \frac{RT}{M}$$

i.e., the ratio $\frac{P}{\rho}$ depends on T only

$$\text{Similarly, } v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

i.e., v_{rms} also depends on T only.

Hence, (C) and (D) are correct.

H.132 JEE Advanced Physics: Waves and Thermodynamics

33. Heat required to raise the temperature of a solid to its melting point is

$$Q_1 = mc\Delta T = (0.1)(0.5 \times 10^{-3})(100 - 0) = 5000 \text{ cal}$$

Therefore, rate of supply of heat is $5000 \text{ cal min}^{-1}$

Since the solid takes one minute to melt, heat required for making the solid is $Q_2 = 5000 \times 1 = 5000 \text{ cal}$

Heat required to raise the temperature of liquid formed to boiling point is $Q_3 = 5000 \times 4 = 20000 \text{ cal}$

If c is the specific heat of the liquid, then

$$20000 = (0.1)c(300 - 100)$$

$$\Rightarrow c = 1000 \text{ cal kg}^{-1} \text{ } ^\circ\text{C}^{-1} = 1 \text{ kcal kg}^{-1} \text{ } ^\circ\text{C}^{-1}$$

Hence, (A), (B) and (C) are correct.

34. Since $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$

$$\Rightarrow \frac{2}{f} = \frac{C_p - C_v}{C_v}$$

$$\Rightarrow f = \frac{2C_v}{R} \quad \left\{ \because C_p - C_v = R \right\}$$

$$\Rightarrow f = 2 \left(\frac{C_p}{R} - 1 \right)$$

Hence, (A) and (B) are correct.

35. When heat is supplied to the gas under Isobaric, Isochoric, Adiabatic conditions then

$$\Delta U = nC_v(T_2 - T_1) \quad \text{\{OPTION (A)\}}$$

For an Adiabatic process $dQ = 0$

$$\Rightarrow dW = -dU$$

Work is being done at the expense of internal energy and both have equal value. \{OPTION (B)\}

For an Isothermal process $\Delta T = 0$

$$\Rightarrow \Delta U = 0 \quad \text{\{OPTION (C)\}}$$

For an Adiabatic process, since $dQ = 0$

$$\Rightarrow C = \frac{1}{n} \frac{dQ}{dT} = 0 \quad \text{\{OPTION (D)\}}$$

Hence, (A), (B), (C) and (D) are correct.

36. $C = 0$

$$\Rightarrow dQ = 0$$

\(\Rightarrow\) Adiabatic process

Hence, (A) and (B) are correct.

37. $T = \text{constant}$

$$\Rightarrow PV = \text{constant} \quad \text{\{Boyle's Law\}}$$

$$\Rightarrow P \propto \frac{1}{V}$$

Pressure of the gas is increasing. Therefore, volume should decrease. Work done by the gas is negative or work done on the gas will be positive. Further temperature of the gas is constant. Therefore, internal energy will remain constant.

Hence, (A), (C) and (D) are correct.

38. **Process AB:** $P = \text{constant}$

$$\Rightarrow V \propto T$$

$$\Rightarrow T_B = 2T_A = 2T_0$$

$$\Rightarrow W_{AB} = P_0 V_0 = RT_0$$

$$\text{Since, } Q_{AB} = C_p \Delta T = \left(\frac{5}{2} R \right) (2T_0 - T_0)$$

$$\Rightarrow Q_{AB} = \frac{5}{2} RT_0$$

So, from *FLTD*, we have

$$Q_{AB} = \Delta U_{AB} + W_{AB}$$

$$\Rightarrow \Delta U_{AB} = Q_{AB} - W_{AB} = \frac{3}{2} RT_0$$

Process BC: $V = \text{constant}$

$$\Rightarrow P \propto T$$

$$\Rightarrow P_C = 2P_B$$

$$\Rightarrow T_C = 2T_B = 4T_0$$

Since, $W_{BC} = 0$

$$\Rightarrow Q_{BC} = \Delta U_{BC} = C_v \Delta T$$

$$\Rightarrow Q_{BC} = \left(\frac{3}{2} R \right) (4T_0 - 2T_0) = 3RT_0$$

$$\Rightarrow W_{\text{net}} = RT_0, Q_{\text{net}} = \frac{11}{2} RT_0$$

$$\text{and } \Delta U_{\text{net}} = \frac{9}{2} RT_0$$

Hence, (A) and (C) are correct.

39. Conceptual

Hence, (A) and (B) are correct.

40. For isobaric process, we have

$$W = nR\Delta T \text{ and } Q = nC_p\Delta T$$

From the graph, we see that

$$\text{Slope} = \frac{W}{Q} = \frac{R}{C_p}$$

For Graph 1

$$\frac{W}{Q} = \frac{20}{80} = \frac{1}{4} = \frac{R}{C_p}$$

$$\Rightarrow C_p = 4R$$

Therefore, gas is polyatomic

For Graph 2

$$\frac{W}{Q} = \frac{32}{80} = \frac{2}{5} = \frac{R}{C_p}$$

$$\Rightarrow C_p = \frac{5R}{2}$$

Therefore, gas is monatomic

For Graph 3

$$\frac{W}{Q} = \frac{80}{80} = 1 = \frac{R}{C_p}$$

$$\Rightarrow C_p = R$$

which is not possible because $C_V \neq 0$ for isobaric process

Hence, the correct answer is (D).

$$41. P \propto AT^4 \quad \left\{ \because T_0 \ll T \right\}$$

$$\text{i.e., } P \propto r^2 \quad \left\{ \because A = 4\pi r^2 \right\}$$

$$\text{Rate of cooling} \propto \frac{1}{r}$$

Hence, (B), (C) and (D) are correct.

$$42. \text{Rate of cooling} = R = -\frac{dT}{dt}$$

Since, $\frac{1}{A} \frac{dQ}{dt} = \sigma(T^4 - T_0^4)$ (By Stefan's Law)

$$\Rightarrow -\frac{mc}{A} \frac{dT}{dt} = \sigma(T^4 - T_0^4) \quad \left\{ \because dQ = mc dT \right\}$$

$$\Rightarrow -\frac{dT}{dt} = R = \frac{\sigma A}{mc} (T^4 - T_0^4)$$

For T comparable to T_0 we have

$$T^4 - T_0^4 = 4T_0^3(T - T_0)$$

$$\Rightarrow R = \frac{4\sigma AT_0^3}{mc} (T - T_0)$$

$$\Rightarrow R \propto \frac{A}{m} \propto \frac{1}{r} \quad \left\{ \because A = 4\pi r^2 \text{ and } m = \frac{4}{3}\pi r^3 \rho \right\}$$

and $R \propto (T - T_0)$

Hence, (A) and (C) are correct.

$$43. \frac{dQ}{dt} = \text{Thermal Current } I = kA \left(\frac{dT}{dl} \right)$$

Hence, (A) and (B) are correct.

$$44. \text{Thermal Stress} = \frac{\text{Tension}}{\text{Area}} = Y\alpha\Delta T$$

$$\Rightarrow \text{Tension} = YA\alpha\Delta T$$

Hence, (C) and (D) are correct.

45. During process A to B, pressure and volume both are decreasing. Therefore, temperature and hence, internal energy of the gas will decrease ($T \propto PV$) or $\Delta U_{A \rightarrow B}$ = negative. Further $\Delta W_{A \rightarrow B}$ is also negative as the volume is increasing. Hence, temperature should increase or $\Delta U_{B \rightarrow C}$ = positive. During C to A volume is constant while pressure is increasing. Therefore, temperature and hence, internal energy of the gas should increase or $\Delta U_{C \rightarrow A}$ = positive. During process CAB volume of the gas is decreasing. hence, work done by the gas is negative.

Hence, (A), (B) and (D) are correct.

$$46. m = nM$$

n = number of moles, M = molecular mass of gas

$$\Rightarrow \frac{mkT}{V} = \frac{nMkT}{V} = \frac{nM \left(\frac{R}{N} \right) T}{V} \quad \left\{ \because k = \frac{R}{N} \right\}$$

Since, $PV = nRT$

$$\Rightarrow \frac{mkT}{V} = \frac{PVM}{NV} = \frac{PM}{N}$$

i.e., the given quantity depends on P and M of the gas.

Hence, (C) and (D) are correct.

$$47. \text{For adiabat } bc \quad T_1 V_b^{\gamma-1} = T_2 V_c^{\gamma-1} \quad \dots(1)$$

For adiabat da

$$T_2 V_d^{\gamma-1} = T_1 V_a^{\gamma-1} \quad \dots(2)$$

Multiplying both (1) and (2)

$$\Rightarrow T_1 T_2 (V_b V_d)^{\gamma-1} = T_1 T_2 (V_a V_c)^{\gamma-1}$$

$$\Rightarrow V_b V_d = V_a V_c$$

Since adiabatic expansion leads to cooling,

$$\text{so } T_1 > T_2$$

Hence, (B) and (C) are correct.

48. Since, net change in internal energy in one cycle of a cyclic process is zero, therefore,

$$\Delta U_1 + \Delta U_2 = 0$$

Also, for the complete cycle, $Q = W$

Hence, (A) and (C) are correct.

$$49. \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{2}{3} = \frac{1}{3} = 0.33$$

$$\text{Further } \eta = \frac{W}{Q_1}$$

$$\Rightarrow \frac{1}{3} = \frac{W}{1000 \text{ cal}}$$

$$\Rightarrow W = \frac{1000 \times 4.2}{3}$$

$$\Rightarrow W = 1400 \text{ J}$$

Hence, (B) and (D) are correct.

50. Conceptual
Hence, (C) and (D) are correct.

51. Since for any cyclic process $dU = 0$
 $\Rightarrow U = \text{constant}$ (OPTION (A))

For an Isothermal process $dT = 0$ i.e., $dU = 0$

$\Rightarrow U = \text{constant}$ (OPTION (B))

Further according to First Law of Thermodynamics

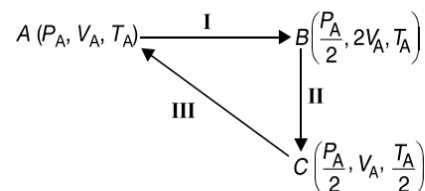
$$dQ = dU + dW$$

For constant U , $dU = 0$

$\Rightarrow dQ = dW$ (OPTION (D))

Hence, (A), (B) and (D) are correct.

52. Let the process start from initial pressure P_A , volume V_A and temperature T_A .



H.134 JEE Advanced Physics: Waves and Thermodynamics

I: Isothermal expansion ($PV = \text{constant}$) at temperature T_A to twice the initial volume V_A

II: Compression at constant pressure $\frac{P_A}{2}$ to original volume V_A (i.e. $V \propto T$)

III: Isochoric process (at volume V_A) to initial conditions (i.e. $P \propto T$)

Hence, (A) and (C) are correct.

53. Since work done is area under the graph, OPTION (A) is incorrect.

Since net work done in the cyclic clockwise process is positive and for cyclic process $Q = W$, so OPTION (B) is correct. Since $PV = nRT$, so temperature is maximum at C and hence OPTION (C) is incorrect.

Process C to A is isochoric and pressure is falling. Therefore, temperature also falls i.e. ΔU and Q are negative. Therefore OPTION (D) is correct.

Hence, (B) and (D) are correct.

54. On heating, a bimetallic strip always bends towards the metal with low α and the reverse process takes place when the bimetallic strip is cooled.

Hence, (B) and (C) are correct.

55. Container is thermally insulated, so we have

$$Q = 0$$

The gas expands against vacuum, then

$$W = 0$$

From First Law of Thermodynamics, we have

$$Q = \Delta U + W$$

$$\Rightarrow \Delta U = 0$$

i.e., internal energy and hence, temperature of the gas is constant

$$\Rightarrow P \propto \frac{1}{V} \quad \{\text{Boyle's Law}\}$$

Hence, (A), (B), (C) and (D) are correct.

56. Since $T = KV^2$, so we have $TV^{-2} = \text{constant}$

Comparing this with equation of a polytropic process i.e., $TV^{x-1} = \text{constant}$, we get

$$x - 1 = -2$$

$$\Rightarrow x = -1$$

So, molar specific heat of the gas is

$$C = C_V + \frac{R}{1-x} = \frac{3R}{2} + \frac{R}{2} = 2R$$

$$\Rightarrow Q = nC\Delta T = n(2R)(3T_0) = 6nRT_0$$

$$\Rightarrow W = \frac{nR\Delta T}{1-x} = \frac{nR(3T_0)}{2} = \frac{3}{2}nRT_0$$

Hence, (B) and (C) are correct.

57. Since no mechanical contact is there, so angular momentum is conserved.

$$\Rightarrow I\omega = \text{constant}$$

$$\Rightarrow \Delta(I\omega) = 0$$

$$\Rightarrow I\Delta\omega + \omega\Delta I = 0$$

$$\Rightarrow \frac{\Delta\omega}{\omega} = -\frac{\Delta I}{I}$$

$$\Rightarrow \frac{\Delta\omega}{\omega_0} = -\frac{\Delta I}{I} = -2\alpha\Delta T \quad \left\{ \because \frac{\Delta I}{I} = \frac{2\Delta R}{R} = 2\alpha\Delta T \right\}$$

Hence, (A), (B) and (C) are correct.

58. For isothermal process

$$T = \text{constant}$$

$$\text{Since, } v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$\Rightarrow v_{\text{rms}} \text{ is also constant}$$

Hence, (B) and (C) are correct.

59. During free expansion, gas is allowed to expand in vacuum. This happens so quickly that there is no heat transferred. Since the gas does not displace anything, so no work is also done by the gas and hence by FLTD, change in internal energy of the gas is also zero. So, there is no change in temperature of the gas.

Hence, (A) and (C) are correct.

60. $W_{\text{cyclic}} = nR(T_1 - T_2)\log_e \alpha$

$$\eta = \frac{W}{Q_1} = 1 - \left(\frac{T_2}{T_1} \right)$$

$$\Delta U_{\text{cyclic}} = 0$$

Hence, (A), (B) and (C) are correct.

61. During expansion an isotherm lies above an adiabat

$$\text{Since, } (\text{Slope})_{\text{adiabat}} = \gamma (\text{Slope})_{\text{isotherm}}$$

$$\Rightarrow m_2 = \frac{C_p}{C_v} (m_1)$$

$$\Rightarrow m_2 C_v = m_1 C_p$$

$$\text{Since } \gamma > 1$$

$$\Rightarrow m_2 > m_1$$

Hence, (C) and (D) are correct.

Reasoning Based Questions

1. Since, for an adiabatic process, $TV^{\gamma-1} = \text{constant}$

$$\Rightarrow d(TV^{\gamma-1}) = 0$$

$$\Rightarrow T(\gamma-1)V^\gamma dv + V^{\gamma-1}dT = 0$$

$$\Rightarrow \frac{T}{V}(\gamma-1)dV + dT = 0$$

$$\Rightarrow \frac{dT}{dV} = -\frac{T}{V}(\gamma-1)$$

$$\text{Slope is negative, so } \frac{dT}{dV} < 0$$

Hence, the correct answer is (C).

2. Since, $C = \frac{\Delta Q}{n\Delta T}$

$$\text{In adiabatic process, } \Delta Q = 0$$

$$\Rightarrow C = 0$$

In isothermal process, $\Delta T = 0$

$$\Rightarrow C \rightarrow \infty$$

Hence, the correct answer is (A).

3. Statement-1 is true and Statement-2 is false.

We can change the temperature of a body without giving (or taking) heat to (or from) it. For example, in an adiabatic compression temperature rises and in an adiabatic expansion temperature falls, although no heat is given (or taken) from the system in the respective changes.

Hence, the correct answer is (C).

4. Coefficient of thermal conductivity is the property of the material and changes only when the material changes.

Hence, the correct answer is (D).

5. Since, V - T graph is a rectangular hyperbola, so

$$VT = \text{constant}$$

$$\Rightarrow PV^2 = \text{constant}$$

$$\Rightarrow \frac{P}{T^2} = \text{constant}$$

Hence, the correct answer is (B).

6. A gas have may values of specific heat capacity depending on the process used.

Hence, the correct answer is (D).

7. Statement-1 is incorrect, because two isothermal curves can never intersect each other but Statement-2 is correct.

Hence, the correct answer is (D).

8. In a constant pressure process, the heat supplied has two jobs to do

- (i) increase the internal energy of the gas by ΔU and
(ii) to push the piston to maintain the pressure.

$$\text{Hence } C_p > C_v$$

Hence, the correct answer is (A).

9. If V increases temperature of gas decreases.

Hence, the correct answer is (A).

10. Coefficient of volume expansion $\delta = \frac{1}{V} \frac{dV}{dT}$

$$\text{Since, } PV = nRT$$

$$\Rightarrow V = \left(\frac{nR}{P}\right)T$$

$$\Rightarrow \frac{dV}{dT} = \frac{nR}{P}$$

$$\Rightarrow \delta = \frac{1}{V} \frac{dV}{dT} = \frac{nR}{PV} = \frac{nR}{nRT} = \frac{1}{T}$$

Hence, the correct answer is (D).

11. Conceptual and discussed in Theory.

Hence, the correct answer is (A).

12. Upon increasing pressure, the ice melts so as to decrease the volume and thereby pressure. To freeze the melted ice one needs to decrease the temperature down and hence the melting point decreases.

Hence, the correct answer is (A).

13. Internal energy increases in the melting process.

Linked Comprehension

Hence, the correct answer is (D).

Linked Comprehension Type Questions

1. Since $W = \frac{Q}{2}$

From $FLTD$, we have $dQ = dU + dW$

$$\Rightarrow dQ = dU + \frac{Q}{2}$$

$$\Rightarrow dU = dW = \frac{Q}{2}$$

$$\Rightarrow C_v dT = PdV$$

$$\Rightarrow \left(\frac{R}{\gamma-1}\right)dT = PdV$$

Since, for a monatomic gas $\gamma = \frac{5}{3}$, so on integrating, we get

$$P^3V = \text{constant}$$

Hence, the correct answer is (C).

2. Also, $C = C_v + \frac{PdV}{dT} = 3R$

Hence, the correct answer is (C).

3. Since, $PV = nRT$

$$\left(\frac{nRT}{V}\right)^3 V = \text{constant}$$

$$\Rightarrow T^3 \propto V^2$$

$$\Rightarrow \frac{T_0^3}{T^3} = \frac{V_0^3}{V^3}$$

$$\Rightarrow V = 8V_0 \quad \left\{ \because T = 4T_0 \right\}$$

Hence, the correct answer is (B).

4. AB is an isobaric process. In $B \rightarrow C$ magnitude of $\frac{dT}{dv}$ increases and $O \rightarrow A$ is an isochoric process.

Hence (C) OPTION is correct.

Hence, the correct answer is (C).

5. For a cyclic process, we have

$$\Delta U = 0$$

$$\Rightarrow W_{\text{net}} = Q_{\text{net}} = -1000 \text{ J}$$

$$\Rightarrow W_{AB} + W_{BC} + W_{CA} = -1000 \text{ J}$$

$$\Rightarrow (4)(R)(200) + W_{BC} + 0 = -1000 \text{ J}$$

$$\Rightarrow W_{BC} = -7640 \text{ J}$$

Hence, the correct answer is (C).

6. The volume of gas at state C is given by

$$V_C = V_A = \frac{nRT_A}{P_A}$$

$$\Rightarrow V_C = 0.0332 \text{ m}^3$$

Hence, the correct answer is (A).

7. The force due to pressure of liquid is Buoyant force. Besides this, gravity and viscous force act on the bubble.

Hence, the correct answer is (D).

H.136 JEE Advanced Physics: Waves and Thermodynamics

8. Since, no heat exchange takes place, so the gas expands adiabatically.

$$P^{1-\gamma}T^\gamma = \text{constant}$$

$$\Rightarrow T_2 = T_1 \left(\frac{P_1}{P_2} \right)^{\frac{1-\gamma}{\gamma}}$$

where, $T_1 = T_0$, $P_1 = P_0 + \rho_l g H$, $P_2 = P_0 + \rho_l g (H - y)$ and $\gamma = \frac{5}{3}$

$$\Rightarrow T_2 = T_0 \left(\frac{P_0 + \rho_l g H}{P_0 + \rho_l g (H - y)} \right)^{\frac{1-\frac{5}{3}}{\frac{5}{3}}}$$

$$\Rightarrow T_2 = T_0 \left(\frac{P_0 + \rho_l g (H - y)}{P_0 + \rho_l g H} \right)^{\frac{2}{5}}$$

Hence, the correct answer is (B).

9. Buoyant Force $F_B = V\rho_l g$

$$\text{Since, } V = \frac{nRT}{P} = \frac{nRT_2}{(P_0 + \rho_l g (H - y))}$$

Substituting the value of T_2 , V , we get

$$F_B = \frac{\rho_l n R g T_0}{(P_0 + \rho_l g H)^{\frac{2}{5}} (P_0 + \rho_l g (H - y))^{\frac{3}{5}}}$$

Hence, the correct answer is (B).

10. Since, $\alpha_t > \alpha_b$, therefore, tube tries to expand more than the bolt when assembly is heated. But the tube is tightened by the bolt, therefore, its expansion cannot be more than that of the bolt. Due to this a compressive stress is developed in the tube and a tensile stress in bolt.

Let initial length of the assembly be ℓ and let its elongation be $\Delta\ell$. Elongation of tube if it were free to expand would be $\ell\alpha_t\Delta T$. But its actual elongation is $\Delta\ell$, therefore elongation prevented in it is equal to $(\ell\alpha_t\Delta T - \Delta\ell)$

So, compressive strain in tube is

$$E_t = \frac{(\ell\alpha_t\Delta T - \Delta\ell)}{\ell} = 5 \times 10^{-5}$$

Hence, the correct answer is (B).

11. Compressive stress in tube

$$\sigma_t = E_t Y_t = 5 \times 10^6 \text{ Nm}^{-2} \quad \left\{ \because Y = \frac{\text{Stress}}{\text{Strain}} \right\}$$

Hence, the correct answer is (A).

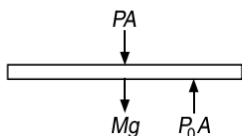
12. Tensile stress in bolt $\sigma_b = \frac{Y_b(\Delta\ell - \ell\alpha_b\Delta T)}{\ell} = 10^7 \text{ Nm}^{-2}$

Hence, the correct answer is (B).

13. Since it is open from top, pressure will be P_0 .

Hence, the correct answer is (A).

14. Let P be the pressure in equilibrium.



Then, $PA = P_0 A - Mg$

$$\Rightarrow P = P_0 - \frac{Mg}{A} = P_0 - \frac{Mg}{\pi R^2} \quad \dots(1)$$

Applying $P_1 V_1 = P_2 V_2$

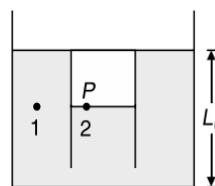
$$\Rightarrow P_0 (2AL) = (P)(AL')$$

$$\Rightarrow L' = \frac{2P_0 L}{P} = \left(\frac{P_0}{P_0 - \frac{Mg}{\pi R^2}} \right) (2L)$$

$$L' = \left(\frac{P_0 \pi R^2}{\pi R^2 P_0 - Mg} \right) (2L)$$

Hence, the correct answer is (D).

15. Since we observe, $P_1 = P_2$



$$\Rightarrow P_0 + \rho g (L_0 - H) = P \quad \dots(1)$$

Now, applying $P_1 V_1 = P_2 V_2$ for the air inside the cylinder, we have

$$P_0 (L_0) = P (L_0 - H)$$

$$\Rightarrow P = \frac{P_0 L_0}{L_0 - H}$$

Substituting in equation (1), we have

$$P_0 + \rho g (L_0 - H) = \frac{P_0 L_0}{L_0 - H}$$

$$\Rightarrow \rho g (L_0 - H)^2 + P_0 (L_0 - H) - P_0 L_0 = 0$$

Hence, the correct answer is (C).

16. The given network forms a Wheatstone's bridge. There will be no heat flow through bd , if the bridge is balanced.

$$\Rightarrow \frac{R_{ab}}{R_{bc}} = \frac{R_{ad}}{R_{dc}}$$

Since, $R \propto \ell$

$$\Rightarrow \frac{\ell}{3\ell} = \frac{2\ell}{x}$$

$$\Rightarrow x = 6\ell$$

Hence, the correct answer is (C).

17. Since, $H_{bd} = 0$

$$\Rightarrow H_1 = H_2$$

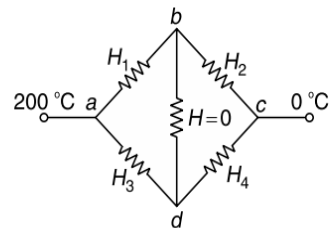
$$\Rightarrow H_3 = H_4$$

$$\Rightarrow \frac{200 - T_b}{\left(\frac{\ell}{KA} \right)} = \frac{T_b - 0}{\left(\frac{3\ell}{KA} \right)}$$

Solving this equation, we get

$$T_b = 150^\circ \text{C}$$

Hence, the correct answer is (D).



18. The correct answer is (D).

19. The correct answer is (C).

Combined solution to 18 and 19

Let m be the mass of ice and q be the rate by which the heat is supplied to it. If T_i and T_f be the initial and final temperature of the ice, then

$$Q_1 = q \times 1 = m \times 0.5 \times (-T_i)$$

$$Q_2 = q \times 4 = m \times 80$$

$$\text{and } Q_3 = q \times 2 = m \times 1 \times T_f$$

Solving these three equations, we get

$$T_f = 40^\circ\text{C}$$

$$\text{and } T_i = -40^\circ\text{C}$$

20. Since, $C_p - C_v = R$

$$\Rightarrow C_v = \frac{R}{\gamma - 1}$$

$$\text{So, } C_{V_A} = \frac{R}{\left(\frac{5}{3} - 1\right)} = \frac{3R}{2}$$

$$\Rightarrow C_{P_A} = \frac{5R}{2}$$

$$\text{Similarly, } C_{V_B} = \frac{R}{\left(\frac{7}{5} - 1\right)} = \frac{5R}{2}$$

$$\Rightarrow C_{P_B} = \frac{7R}{2}$$

$$\text{Also, } \gamma_{\text{mixture}} = \frac{19}{13} = \frac{n_A C_{P_A} + n_B C_{P_B}}{n_A C_{V_A} + n_B C_{V_B}}$$

where $n_A = 1$

$$\Rightarrow n_B = 2 \text{ mole}$$

Hence, the correct answer is (D).

21. Since, Bulk Modulus of the gas under adiabatic conditions is

$$B = \gamma P$$

$$\text{Compressibility, } K = \frac{1}{B} = \frac{1}{\gamma P}$$

So, change in adiabatic compressibility is

$$\Delta K = K_2 - K_1$$

$$\Rightarrow \Delta K = \frac{1}{\gamma P_2} - \frac{1}{\gamma P_1} = \frac{1}{\gamma} \left(\frac{1}{P_2} - \frac{1}{P_1} \right)$$

Since, the process is adiabatic, so we have

$$P_2 V_2^\gamma = P_1 V_1^\gamma$$

$$\Rightarrow P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma = P_1 \left(\frac{V_1}{V_1/5} \right)^\gamma = P_1 5^\gamma$$

$$\Rightarrow \Delta K = \frac{1}{\gamma} \left(\frac{1}{P_1 5^\gamma} - \frac{1}{P_1} \right) = \frac{1}{\gamma P_1} \left(\frac{1}{5^\gamma} - 1 \right)$$

$$\text{where, } P_1 = \frac{(n_A + n_B)RT}{V}$$

$$\Rightarrow P_1 = \frac{(1+2) \times 8.31 \times T}{V} = \frac{24.93T}{V}$$

$$\Rightarrow \Delta K = \frac{1}{\left(\frac{19}{13}\right)(24.93) \left(\frac{T}{V}\right) \left(\frac{1}{5^{13}} - 1\right)}$$

$$\Rightarrow \Delta K = -(0.025) \frac{V}{T}$$

Hence, the correct answer is (A).

22. The correct answer is (B).

23. The correct answer is (A).

24. The correct answer is (B).

Combined solution to 22, 23 and 24

Let P_1 and P_2 be the initial pressure in lower chamber of gas and upper chamber of gas, then

$$P_2 = P_0 + \frac{mg}{A} = 2P_0 \text{ and } V_2 = A \times 12 \times 10^{-2} \text{ m}^3$$

If P'_2 and V'_2 are final pressure and volume in upper chamber, then we have

$$V'_2 = A \times (28 - l) \times 10^{-2} \text{ m}^3$$

Since, $P_2 V_2 = P'_2 V'_2$

$$\Rightarrow P'_2 = \frac{P_2 V_2}{V'_2} = \frac{24P_0}{28 - l}$$

New consider lower chamber, then

$$P_1 = P_0 + \frac{2mg}{A} = 3P_0 \text{ and } V_1 = A \times 8 \times 10^{-2} \text{ m}^3$$

$$\text{Also, } P'_1 = P'_2 + \frac{mg}{A} = P_0 \left(\frac{52 - l}{28 - l} \right)$$

$$\text{and } V'_1 = A \times (8 + l) \times 10^{-2} \text{ m}^3$$

Since, $P_1 V_1 = P'_1 V'_1$

$$\Rightarrow (3P_0 A)(8 \times 10^{-2}) = P_0 \left(\frac{52 - l}{28 - l} \right) A [(8 + l) \times 10^{-2}]$$

$$\Rightarrow 24 = \left(\frac{52 - l}{28 - l} \right) (8 + l)$$

$$\Rightarrow l = 4 \text{ cm}$$

$$\text{So, } P'_1 = 2P_0 = 2 \times 10^5 \text{ Am}^{-2}$$

$$\text{and } P'_2 = \frac{24P_0}{28 - l} = P_0 = 1 \times 10^5 \text{ Nm}^{-2}$$

$$\Rightarrow \frac{V'_2}{V'_1} = \frac{28 - l}{8 + l} = \frac{24}{12} = 2$$

25. $W = \text{Area under the curve} = \frac{3}{2} P_1 V_1$

Since, $P_1 V_1 = nRT_1$

$$\Rightarrow \frac{W}{nRT_1} = \frac{\frac{3}{2} P_1 V_1}{P_1 V_1} = \frac{3}{2}$$

Hence, the correct answer is (A).

26. From FLTD, we have

$$Q = dU + W$$

H.138 JEE Advanced Physics: Waves and Thermodynamics

Since, $dU = nC_v dT$

For final state, we have

$$P_2 V_2 = (2P_1)(2V_1) = 4P_1 V_1 = nR(4T_1)$$

Hence final temperature is $4T_1$

$$\Rightarrow dU = n \left(\frac{3R}{2} \right) (3T_1) = \frac{9}{2} nRT_1$$

$$\Rightarrow Q = \frac{3}{2} nRT_1 + \frac{9}{2} nRT_1 = 6nRT_1$$

$$\Rightarrow \frac{Q}{nRT_1} = 6$$

Hence, the correct answer is (D).

27. Since, $Q = 6nRT_1$... (1)

Also, we know that

$$Q = nC\Delta T$$

$$\Rightarrow Q = nC(3T_1) \quad \dots (2)$$

Equating (1) and (2), we get

$$6nRT_1 = nC(3T_1)$$

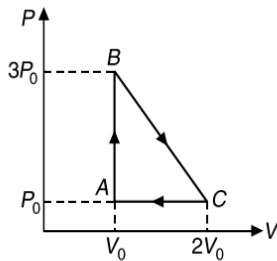
$$\Rightarrow \frac{C}{R} = 2$$

Hence, the correct answer is (B).

28. ABCA is a clockwise cyclic process, so work done by the gas is

$$W = \text{Area of triangle } ABC = \frac{1}{2}(\text{base})(\text{height})$$

$$\Rightarrow W = \frac{1}{2}(2V_0 - V_0)(3P_0 - P_0) = P_0 V_0$$



Hence, the correct answer is (B).

29. Number of moles $n = 1$ and gas is monatomic, therefore

$$C_V = \frac{3}{2}R \text{ and } C_P = \frac{5}{2}R$$

$$\Rightarrow \frac{C_V}{R} = \frac{3}{2} \text{ and } \frac{C_P}{R} = \frac{5}{2}$$

Heat rejected in path CA: (process is isobaric)

$\Rightarrow C \rightarrow D$:

$$\Rightarrow dQ_{CA} = \frac{C_P}{R}(P_f V_f - P_i V_i)$$

Substituting the values

$$dQ_{CA} = \frac{5}{2}(P_0 V_0 - 2P_0 V_0) = -\frac{5}{2}P_0 V_0$$

Hence, the correct answer is (C).

30. Heat rejected in the process AB (process is isochoric) is

$$dQ_{AB} = C_V dT = C_V (T_f - T_i)$$

Also, $Q_{CD} = -900R$ (released)

$$\Rightarrow dU = 0$$

$$dQ_{AB} = 3P_0 V_0$$

Heat absorbed in the process AB is $3P_0 V_0$

Hence, the correct answer is (B).

31. Let dQ_{BC} be the heat absorbed in the process BC :

Total heat absorbed,

$$dQ = dQ_{CA} + dQ_{AB} + dQ_{BC}$$

$$dQ = \left(-\frac{5}{2}P_0 V_0 \right) + 3(P_0 V_0) + dQ_{BC}$$

$$dQ = dQ_{BC} + \frac{P_0 V_0}{2}$$

Change in internal energy, $dU = 0$

$$\Rightarrow dQ = dW$$

$$\Rightarrow dQ_{BC} + \frac{P_0 V_0}{2} = P_0 V_0$$

$$\Rightarrow dQ_{BC} = \frac{P_0 V_0}{2}$$

Heat absorbed in the process BC is $\frac{P_0 V_0}{2}$

Hence, the correct answer is (C).

32. Maximum temperature of the gas will be somewhere between B and C. Line BC is a straight line. Therefore, P - V equation for the process BC can be written as

$$P = -mV + c \quad (y = mx + c)$$

Here, $m = \frac{2P_0}{V_0}$ and $c = 5P_0$

$$\Rightarrow P = -\left(\frac{2P_0}{V_0} \right)V + 5P_0$$

Multiplying the equation by V ,

$$PV = -\left(\frac{2P_0}{V_0} \right)V^2 + 5P_0 V \quad (PV = RT \text{ for } n = 1)$$

$$RT = -\left(\frac{2P_0}{V_0} \right)V^2 + 5P_0 V$$

$$\text{or } T = \frac{1}{R} \left[5P_0 V - \frac{2P_0}{V_0} V^2 \right] \quad \dots (1)$$

For T to be maximum, $\frac{dT}{dV} = 0$

$$\Rightarrow 5P_0 - \frac{4P_0}{V_0} V = 0$$

$$\Rightarrow V = \frac{5V_0}{4}$$

i.e. at $V = \frac{5V_0}{4}$, (on line BC), temperature of the gas is maximum. From equation (1) this maximum temperature will be

$$T_{\max} = \frac{1}{R} \left[5P_0 \left(\frac{5V_0}{4} \right) - \frac{2P_0}{V_0} \left(\frac{5V_0}{4} \right)^2 \right]$$

$$\Rightarrow T_{\max} = \frac{25 P_0 V_0}{8 R}$$

Hence, the correct answer is (A).

33. $W = \int P dV = \alpha \int_{V_0}^{nV_0} V dV = \frac{\alpha V_0^2}{2} (n^2 - 1)$

Hence, the correct answer is (A).

34. Since, $dU = C_V dT = \frac{R}{\gamma - 1} dT$

$$\Rightarrow \Delta U = \frac{R}{\gamma - 1} \int dT$$

Since, $PV = RT = \alpha V^2$

$$\Rightarrow R dT = 2\alpha V dV$$

$$\Rightarrow \Delta U = \frac{2\alpha}{(\gamma - 1)} \int_{V_0}^{nV_0} V dV = \frac{\alpha}{\gamma - 1} V_0^2 (n^2 - 1)$$

Hence, the correct answer is (B).

35. Since, $Q = \Delta U + W$

and $Q = (1)C\Delta T$

$$\Rightarrow C = \left(\frac{\gamma + 1}{\gamma - 1} \right) \frac{R}{2}$$

Hence, the correct answer is (B).

36. At constant pressure, we have

$$W = P\Delta V = P_0(2V_0 - V_0) = P_0V_0$$

$$\Rightarrow \Delta U = 3P_0(2V_0) - 3P_0(V_0) = 3P_0V_0$$

So, from FLTD, we get

$$Q = \Delta U + W = 4P_0V_0$$

Hence, the correct answer is (D).

37. Also, for n moles of gas with f degrees of freedom, at temperature T , we have

$$U = \frac{f}{2} nRT = \frac{f}{2} (PV)$$

Since $U = 3PV$

So, comparing, we get

$$\frac{f}{2} = 3$$

$$\Rightarrow f = 6$$

Since, $\gamma = 1 + \frac{2}{f}$

$$\Rightarrow \gamma = 1 + \frac{2}{6}$$

$$\Rightarrow \gamma = \frac{4}{3} = 1.33$$

Hence, the correct answer is (A).

38. Process AB : $PT = \text{constant}$

Since, $PV = (1)RT$

$$\Rightarrow \left(\frac{RT}{V} \right) T = \text{constant}$$

$$\Rightarrow \frac{RT^2}{V} = \text{constant} \quad \dots(1)$$

Since, $W = \int_A^B P dV = \int_A^B \frac{\text{constant}}{T} dV$

Also, $\frac{P_A}{P_B} = \frac{T_B}{T_A} \quad \{ \because PT = \text{constant} \}$

$$\Rightarrow \frac{1}{3} = \frac{T_B}{T_A}$$

$$\Rightarrow T_B = \frac{300}{3} = 100$$

From (1), we get

$$\frac{dV}{dT} = \frac{2RT}{\text{constant}}$$

$$\Rightarrow W = \int_{300}^{100} \frac{(\text{constant})}{T} \frac{2RT}{(\text{constant})} dt$$

$$\Rightarrow W = 2R(100 - 300)$$

$$\Rightarrow W_{AB} = -400 R$$

Hence, the correct answer is (B).

39. Since, the process CA is isochoric

So, $\frac{P}{T} = \text{constant}$

$$\Rightarrow \frac{T_A}{T_C} = \frac{P_A}{P_C}$$

$$\Rightarrow \frac{T_A}{T_C} = \frac{P_A}{P_B} = \frac{1}{3}$$

$$\Rightarrow T_C = 3T_A = 900 \text{ K}$$

Since, $\Delta U = nC_V \Delta T$

$$\Rightarrow \Delta U = (1) \frac{3}{2} R \times (T_A - T_C)$$

$$\Rightarrow \Delta U = \frac{3}{2} R \times (300 - 900)$$

$$\Rightarrow \Delta U = -\frac{3}{2} R \times 600 = -900R$$

Hence, the correct answer is (D).

40. Since, process BC is isobaric, so we have

$$Q = nC_P \Delta T$$

$$\Rightarrow Q = (1) \frac{5}{2} R (T_C - T_B) = \frac{5}{2} R (900 - 100)$$

H.140 JEE Advanced Physics: Waves and Thermodynamics

$$\Rightarrow Q = \frac{5}{2}R(800) = 2000R$$

Hence, the correct answer is (C).

41. Let $T_B = T_0$ and $V_C = V_D = V_0$

Then, $T_D = T_A = 4T_0$ and $V_A = (8\sqrt{2})V_0$

Now, $V_B = \left(\frac{V_A}{T_A}\right)T_B = 2\sqrt{2}V_0$

and $T_C = T_B \left(\frac{V_B}{V_C}\right)^{\gamma-1} = 2T_0$

$$\Rightarrow T_C = 2T_B$$

Hence, the correct answer is (B).

42. Since, we have

$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$\Rightarrow W = R(T_B - T_A) + nC_V(T_B - T_C) + 0 + RT_D \log_e \left(\frac{V_A}{V_D}\right)$$

$$\Rightarrow W = -3RT_0 - \frac{3}{2}RT_0 + 0 + 14RT_0 \log_e 2$$

$$\Rightarrow W = 5.2RT_0$$

Hence, the correct answer is (A).

43. $Q_{\text{absorbed}} = Q_{CD} + Q_{DA}$

$$\Rightarrow Q_{\text{absorbed}} = 3RT_0 + 14RT_0 \log_e 2 = 12.7T_0$$

So, efficiency, $\eta = \frac{W}{Q_{\text{absorbed}}} \times 100 = 41\%$

Hence, the correct answer is (B).

44. $W_{ab} = W_{cd} = 0$ as $V = \text{constant}$

$$W_{da} = nRT_a \log_e \left(\frac{V_a}{V_d}\right)$$

$$W_{bc} = nRT_b \log_e \left(\frac{V_c}{V_b}\right) = -nRT_b \log_e \left(\frac{V_a}{V_d}\right)$$

$$\Rightarrow W_{\text{net}} = nR \log_e \left(\frac{V_a}{V_d}\right) (T_a - T_b)$$

Since $T_b - T_a > 0$ and $\frac{V_a}{V_d} > 1$

$$\Rightarrow W_{\text{net}} = \text{NEGATIVE}$$

Hence, the correct answer is (C).

45. $d \rightarrow a: T = \text{constant}$

$$\Rightarrow \Delta U = 0$$

$$\Rightarrow Q = W$$

Since, volume of the gas is increasing. Hence, W is positive, so Q is positive.

$a \rightarrow b: V = \text{constant}$

$$\Rightarrow W = 0$$

$$\Rightarrow Q = \Delta U$$

Since, temperature is increasing. Therefore, ΔU will increase, so Q is positive.

Hence, in processes $d \rightarrow a$ and $a \rightarrow b$ heat is supplied to the gas while in processes $b \rightarrow c$ and $c \rightarrow d$ heat is taken from the gas.

Hence, the correct answer is (B).

46. From ideal gas equation, we have

$$PV = nRT$$

$$\Rightarrow P = \frac{nRT}{V} = \frac{nR(T_0 + aV^3)}{V}$$

$$\Rightarrow P = \frac{nRT_0}{V} + nRaV^2$$

For P to be minimum, we have $\frac{dP}{dV} = 0$

$$\Rightarrow -\frac{nRT_0}{V^2} + 2anRV = 0$$

$$\Rightarrow \frac{T_0}{V^2} = 2aV$$

$$\Rightarrow V^3 = \frac{T_0}{2a}$$

$$\Rightarrow V = \left(\frac{T_0}{2a}\right)^{\frac{1}{3}}$$

Hence, the correct answer is (A).

47. From ideal gas equation, we get

$$P = \frac{nRT}{V} = \frac{nR(T_0 + aV^3)}{V}$$

$$\Rightarrow P = \frac{nRT_0}{V} + anRV^2$$

$$\Rightarrow P = \frac{nRT_0}{T_0^{\frac{1}{3}}(2a)^{\frac{1}{3}}} + anR \frac{T_0^{\frac{2}{3}}}{(2a)^{\frac{2}{3}}}$$

For $n = 1$, we get

$$\Rightarrow P_{\text{min}} = RT_0^{\frac{2}{3}}(2)^{\frac{1}{3}}(a)^{\frac{1}{3}} + \frac{1}{2} \frac{a^{\frac{1}{3}}RT_0^{\frac{2}{3}}}{2^{\frac{2}{3}}}$$

$$\Rightarrow P_{\text{min}} = \frac{3RT_0^{\frac{2}{3}}a^{\frac{1}{3}}}{2^{\frac{2}{3}}} = \frac{3}{2} \left(a^{\frac{1}{3}}RT_0^{\frac{2}{3}} \right) (2)^{\frac{1}{3}}$$

$$\Rightarrow P_{\text{min}} = \frac{3R}{2} (2aT_0^2)^{1/3}$$

Hence, the correct answer is (D).

48. In adiabatic process $TV^{\gamma-1} = \text{constant}$

$$\Rightarrow TV^{\gamma-1} = \left(\frac{T}{2}\right)(5.66V)^{\gamma-1}$$

$$\Rightarrow (5.66V)^{\gamma-1} = 2$$

$$\Rightarrow (\gamma-1)\ln(5.66) = \ln(2)$$

$$\Rightarrow \gamma-1 = 0.4$$

$$\Rightarrow \gamma = 1.4$$

$$\text{Since, } \gamma = 1 + \frac{2}{f}$$

$$\Rightarrow f = 5 \quad \{\text{for } \gamma = 1.4\}$$

Hence, the correct answer is (D).

49. Using $PV^\gamma = \text{constant}$

$$\Rightarrow PV^{1.4} = P_f(5.66V)^{1.4}$$

$$\Rightarrow P_f = 0.09P$$

Since, work done in adiabatic process is given by

$$W = \frac{P_i V_i - P_f V_f}{\gamma - 1} = \frac{(PV) - (0.09P)(5.66V)}{1.4 - 1}$$

$$\Rightarrow W = 1.23PV$$

Hence, the correct answer is (B).

50. Since, $U = U_0 + 2PV$

$$\Rightarrow \Delta U = 2\Delta(PV) = 2R\Delta T$$

$$\Rightarrow C_V = 2R$$

$$\Rightarrow C_P = 3R$$

Hence, the correct answer is (B).

$$51. W = RT \log_e \left(\frac{2V_0}{V_0} \right) = PV \log_e (2) = \frac{U_1 - U_0}{2} \log_e (2)$$

Hence, the correct answer is (D).

52. $C_V = 2R$, which lies between C_V value of a monatomic and a diatomic gas.

Hence, the correct answer is (C).

53. Let at any instant t temperature is T . The net rate at which heat is absorbed by the gas is

$$\frac{dQ}{dt} = q - \frac{KA(T - T_0)}{\ell} \quad \dots(1)$$

$$\text{Since, } dQ = nC_P dT = n \left(\frac{7R}{2} \right) dT \quad \dots(2)$$

$$\Rightarrow n \left(\frac{7R}{2} \right) \frac{dT}{dt} = q - \frac{KA(T - T_0)}{\ell}$$

$$\Rightarrow \int_{T_0}^T \frac{dT}{q\ell - KA(T - T_0)} = \frac{2}{7nR\ell} \int_0^t dt$$

Solving, we get

$$T = T_0 + \frac{q\ell}{KA} \left(1 - e^{-\frac{2KA t}{7nR\ell}} \right)$$

Hence, the correct answer is (A).

54. At maximum temperature, we have

$$\frac{dQ}{dt} = 0$$

So, from (1), we get

$$q = \frac{KA}{\ell} (T_{\max} - T_0)$$

$$\Rightarrow T_{\max} = T_0 + \frac{q\ell}{KA}$$

Hence, the correct answer is (B).

55. Since, the process is isobaric, so we have

$$\frac{V_0}{T_0} = \frac{V_{\max}}{T_{\max}}$$

$$\Rightarrow \frac{V_{\max}}{V_0} = \frac{T_{\max}}{T_0}$$

Hence, the correct answer is (D).

56. In process $a \rightarrow c$, we observe that

$$V \propto T \text{ i.e., } P = \text{constant}$$

$$\Rightarrow Q = nC_p \Delta T = 2 \left(\frac{5}{2} R \right) (600 - 300) = 1500R$$

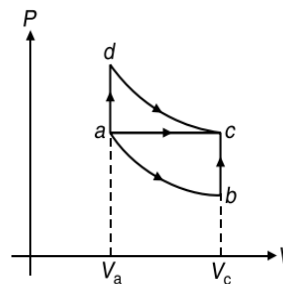
$$\Delta U = nC_V \Delta T = 2 \left(\frac{3}{2} R \right) (600 - 300) = 900R$$

From FLTD, we have $Q = \Delta U + W$

$$\Rightarrow W = Q - \Delta U = 600R$$

Hence, the correct answer is (C).

57. The corresponding P - V graph is shown in figure.



Since, area under the P - V graph gives the work done.

$$\text{Hence, } W_{adc} > W_{ac} > W_{abc}$$

$$\Rightarrow W_3 > W_2 > W_1$$

Hence, the correct answer is (D).

58. The correct answer is (C).

59. The correct answer is (B).

60. The correct answer is (D).

Combined solution to 58, 59 and 60

For AB : $P = \text{constant}$

$$\Rightarrow V \propto T$$

So, if V is doubled, T also becomes two times.

$$T_A = 300 \text{ K}$$

$$\Rightarrow T_B = 600 \text{ K}$$

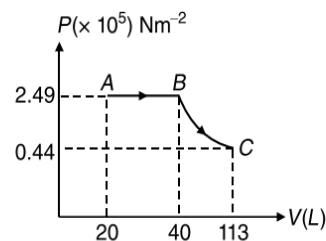
$$V_A = 20 \text{ L}$$

$$\Rightarrow V_B = 40 \text{ L}$$

$$P_A = \frac{nRT_A}{V_A} = \frac{(2)(8.31)(300)}{20 \times 10^{-3}} = 2.49 \times 10^5 \text{ Nm}^{-2}$$

Process 2: Process is adiabatic. So, applying $T^\gamma P^{1-\gamma} = \text{constant}$.

$$\Rightarrow \left(\frac{600}{300} \right)^{5/3} = \left(\frac{2.49 \times 10^5}{P_C} \right)^{5-1} = \left(\frac{2.49 \times 10^5}{P_C} \right)^{2/3}$$



H.142 JEE Advanced Physics: Waves and Thermodynamics

$$\Rightarrow P_C = (2.49 \times 10^5) \left(\frac{300}{600} \right)^{5/2}$$

$$\Rightarrow P_C = 0.44 \times 10^5 \text{ Nm}^{-2} = 4.4 \times 10^5 \text{ Nm}^{-2}$$

Similarly using $PV^\gamma = \text{constant}$ we can find that $V_C = 113$ litre

W_{AB} = Area under P - V graph

$$\Rightarrow W_{AB} = (2.49 \times 10^5) (40 - 20) \times 10^{-3} = 4980 \text{ J}$$

Also, $W_{BC} = -\Delta U = nC_V(T_B - T_C)$

$$\Rightarrow W_{BC} = (2) \left(\frac{3}{2} \times 8.31 \right) (600 - 300) = 7479 \text{ J}$$

$$\Rightarrow W_{\text{total}} = 12459 \text{ J}$$

61. Since, $P_1A + Kx = P_2A$

$$\Rightarrow \frac{nRT}{(L-x)} + Kx = \frac{nR(T + \Delta T)}{L+x}$$

$$\Rightarrow \frac{nRT}{\left(\frac{3L}{4}\right)} + K \frac{L}{4} = \frac{nR(T + \Delta T)}{\left(\frac{5L}{4}\right)}$$

$$\Rightarrow \Delta T = \frac{2T}{3} + \frac{5}{16} \frac{KL^2}{nR}$$

Hence, the correct answer is (C).

62. Since, $\Delta U = nC_V \Delta T$

$$\Rightarrow \Delta U = n \left(\frac{3R}{2} \right) \left(\frac{2T}{3} + \frac{5}{16} \frac{KL^2}{nR} \right)$$

$$\Rightarrow \Delta U = nRT + \left(\frac{15}{32} \right) KL^2$$

Hence, the correct answer is (A).

63. Using $FLTD$, we get

$$Q = \Delta U + W$$

$$\text{where } W = \frac{1}{2} Kx^2 = \frac{1}{2} K \left(\frac{L}{4} \right)^2$$

$$\Rightarrow W = \frac{1}{2} \frac{KL^2}{16} = \frac{KL^2}{32}$$

$$\Rightarrow Q = \Delta U + W = nRT + \frac{KL^2}{2}$$

Hence, the correct answer is (B).

64. Since, $T = T_0 + \alpha V^3$

For an Ideal Gas, we have

$$PV = nRT$$

$$\Rightarrow T = \frac{PV}{nR}$$

$$\Rightarrow \frac{PV}{nR} = T_0 + \alpha V^3$$

$$\Rightarrow P = \frac{nRT_0}{V} + (nR\alpha)V^2$$

Now P is minimum, when $\frac{dP}{dV} = 0$

$$\Rightarrow -\frac{nRT_0}{V^2} + nR\alpha(2V) = 0$$

$$\Rightarrow \frac{T_0}{V^2} = 2\alpha V$$

$$\Rightarrow V^3 = \frac{T_0}{2\alpha}$$

$$\Rightarrow V = \left(\frac{T_0}{2\alpha} \right)^{1/3}$$

Hence, the correct answer is (C).

65. Since, $C = C_V + \frac{P}{n} \left(\frac{dV}{dT} \right)$

$$\Rightarrow C = C_V + \frac{P}{n(3\alpha V^2)}$$

$$\left\{ \because \frac{dV}{dT} = 3\alpha V^2 \right\}$$

$$\text{But } P = \frac{nRT_0}{V} + (nR\alpha)V^2$$

$$\Rightarrow C = C_V + \left(\frac{nRT_0}{V} + (nR\alpha)V^2 \right) \frac{1}{3n\alpha V^2}$$

$$\Rightarrow C = C_V + \frac{nRT_0}{V(3n\alpha V^2)} + \frac{nR\alpha V^2}{3n\alpha V^2}$$

$$\Rightarrow C = C_V + \frac{R}{3} + \frac{RT_0}{3\alpha V^3}$$

Hence, the correct answer is (A).

66. Since, $P = \frac{nRT_0}{V} + (nR\alpha)V^2$

This P is minimum, when

$$V = \left(\frac{T_0}{2\alpha} \right)^{1/3}$$

$$\Rightarrow P_{\min} = \frac{nRT_0}{\left(\frac{T_0}{2\alpha} \right)^{1/3}} + nR\alpha \left(\frac{T_0}{2\alpha} \right)^{2/3}$$

$$\Rightarrow P_{\min} = 2^{1/3} nRT_0^{2/3} \alpha^{1/3} + 2^{2/3} nRT_0^{2/3} \alpha^{1/3}$$

$$\Rightarrow P_{\min} = 2^{1/3} nRT_0^{2/3} \alpha^{1/3} \left(1 + 2^{1/3} \right)$$

$$\Rightarrow \frac{P_{\min}}{nRT_0^{2/3} \alpha^{1/3}} = 2^{1/3} \left(1 + 2^{1/3} \right)$$

$$\Rightarrow \frac{P_{\min}}{nRT_0^{2/3} \alpha^{1/3}} = \left(2^{1/3} + 2^{2/3} \right)$$

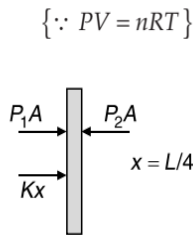
Hence, the correct answer is (B).

$$67. E = \int_0^\infty E_\lambda d\lambda$$

The area under the curve represents the total intensity (energy per unit area per second) radiated at a particular temperature.

Hence, the correct answer is (C).

68. According to the Stefan's Law, the intensity of each wavelength increases by increasing the temperature and the area under the curve increases in fourth power of temperature.



According to Wein's Law,

$$\lambda_m \propto \frac{1}{T}$$

Hence, the correct answer is (D).

69. While answering, we must keep in mind that the curve at higher temperature

- (i) lies above the curve at lower temperature.
- (ii) never intersects with the curve at lower temperature.
- (iii) the maximum occurs at a lower wavelength.

Hence, the correct answer is (B).

70. Initially, we have $U_i = n_1 \left(\frac{3}{2} k_B T_1 \right) + n_2 \left(\frac{3}{2} k_B T_2 \right)$

$$\Rightarrow U_i = \frac{3}{2} k_B (n_1 T_1 + n_2 T_2)$$

Hence, the correct answer is (B).

71. Finally, we have $U_i = \frac{3}{2} k_B T (n_1 + n_2)$

Hence, the correct answer is (B).

72. Heat lost by one equals the heat gained by the other gas, so we have

$$\frac{n_1}{N_A} \left(\frac{3R}{2} \right) (T_1 - T) = \frac{n_2}{N_A} \left(\frac{3R}{2} \right) (T - T_2)$$

$$\Rightarrow T = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}$$

Hence, the correct answer is (D).

73. $C = \frac{R}{\gamma-1} + \frac{R}{1-x} = C_V + \frac{R}{1-x}$

Since it is given that $P \propto V$

$$\Rightarrow PV^{-1} = \text{constant}$$

$$\Rightarrow x = -1$$

$$\Rightarrow C = C_V + \frac{R}{2} = \frac{5R}{2} + \frac{R}{2} = 3R$$

Hence, the correct answer is (A).

74. $v_{\text{rms}} \propto \sqrt{T}$

Since, rms speed has doubled. Therefore, temperature must have become four times.

$$\Rightarrow Q = nC\Delta T$$

$$\Rightarrow Q = nC(4T_0 - T_0)$$

$$\Rightarrow Q = 3n(3RT_0)$$

$$\Rightarrow Q = 9P_0V_0 \quad \left\{ \because nRT_0 = P_0V_0 \right\}$$

Hence, the correct answer is (C).

75. Since, $T_i = T_f$, therefore net change in internal energy, $dU = 0$.

Hence, the correct answer is (C).

76. Given number of moles $n = 2$

Process AB and CD are isobaric.

$$\text{Hence, } Q_{AB} = -Q_{CD}$$

$$\left[\text{Because } (\Delta T)_{AB} = +100 \text{ K} \right]$$

$$\text{whereas } (\Delta T)_{CD} = -100 \text{ K and } Q_{\text{isobaric}} = nC_P \Delta T]$$

$$\text{or } Q_{AB} + Q_{CD} = 0$$

Process BC is isothermal ($dU = 0$)

$$\Rightarrow Q_{BC} = W_{BC} = nRT_B \ln \left(\frac{P_B}{P_C} \right)$$

$$V_A = 10 \text{ m}^3, Q_{BC} = 4608 \text{ J}$$

Similarly, process DA is also isothermal hence

$$Q_{DA} = W_{DA} = nRT_D \ln \left(\frac{P_D}{P_A} \right)$$

$$\Rightarrow Q_{DA} = (2)(8.31)(300) \ln \left(\frac{1}{2} \right)$$

$$Q_{DA} = 3456 \text{ J}$$

- (a) Net heat in the process

$$Q = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA} = (4608 - 3456) \text{ J}$$

$$Q = 1152 \text{ J}$$

- (b) From First Law of Thermodynamics, $dU = 0$ (in complete cycle) $dQ = dW$

Hence, net work done in the cycle

$$W = Q = 1152 \text{ J}$$

$$W = 1152 \text{ J}$$

Hence, the correct answer is (D).

77. Since entropy is a state function, so $S_f = S_i$ and hence dS for one complete cycle is zero.

Hence, the correct answer is (D).

Matrix Match/Column Match Type Questions

1. A \rightarrow (q, t); B \rightarrow (r); C \rightarrow (s); D \rightarrow (p)

For a cyclic process, $\Delta U = 0$

By FLTD, we get $Q = W$

For Isobaric process, $W = nR\Delta T$

For Isochoric process $W = 0$, so $Q = \Delta U$

Since, adiabatic expansion is accompanied by decrease in temperature, so a decrease in internal energy.

2. A \rightarrow (p, s); B \rightarrow (p, r); C \rightarrow (q, s); D \rightarrow (q, r)

Process $a \rightarrow b$:

$$T = \text{constant}$$

$$\Rightarrow \Delta T = 0$$

$$\Rightarrow \Delta U = 0$$

$$\Rightarrow P \propto \frac{1}{V}$$

ρ is increasing. Therefore, V is decreasing. Since, $V \propto \frac{1}{\rho}$ or P will increase.

Process $b \rightarrow c$:

$$\rho = \text{constant}$$

$$\Rightarrow V = \text{constant}$$

Therefore, $P \propto T$

Since, temperature is increasing. Hence pressure should also increase.

H.144 JEE Advanced Physics: Waves and Thermodynamics

3. A → (p, s); B → (q, r); C → (p, r, s); D → (p, s)

4. A → (r); B → (s); C → (p); D → (q)
Since, Isothermal Bulk modulus is

$$B_{\text{iso}} = P = \frac{RT}{V} \text{ and}$$

Adiabatic Bulk modulus is

$$B_{\text{ad}} = \gamma P = \frac{5RT}{3V} \quad \left\{ \because \gamma_{\text{monatomic}} = \frac{5}{3} \right\}$$

$$(\text{Slope})_{\text{isot}} = \left(\frac{dP}{dV} \right)_{\text{isot}} = -\frac{P}{V} = -\frac{RT}{V^2}$$

$$(\text{Slope})_{\text{ad}} = \left(\frac{dP}{dV} \right)_{\text{ad}} = -\gamma \left(\frac{P}{V} \right) = -\frac{5P}{3V}$$

5. A → (r); B → (p); C → (s); D → (q)

6. A → (p, s); B → (r, s); C → (p, t); D → (q, t)

M → N is an Isochoric Process

$$\Rightarrow V = \text{constant i.e., } P \propto T$$

$$\Rightarrow W_{\text{isoc}} = 0$$

Since, P is increasing, so T is also increasing and hence $\Delta U > 0$

N → O is an Isobaric Process

$$\Rightarrow P = \text{constant i.e., } V \propto T$$

Since, V is increasing, so T is also increasing and hence $\Delta U > 0$

Also, $W > 0$ for isobaric expansion.

O → P is an Isochoric Process

$$\Rightarrow V = \text{constant i.e., } P \propto T$$

$$\Rightarrow W_{\text{isoc}} = 0$$

Since, P is decreasing, so T is also decreasing and hence $\Delta U < 0$

P → M is an Isobaric Process

$$\Rightarrow P = \text{constant i.e., } V \propto T$$

Since, V is decreasing, so T is also decreasing and hence $\Delta U < 0$

Furthermore, for isobaric compression $W < 0$

7. A → (u); B → (p, q, t); C → (s); D → (r)

$$\text{Since } W_{\text{ad}} = \frac{R}{1-\gamma} (T_f - T_i) \text{ for 1 mole of gas}$$

$$W_{\text{isochoric}} = 0$$

$$\Delta U_{\text{isothermal}} = 0$$

$$C_{\text{adiabatic}} = 0$$

$$W_{\text{isobaric}} = (1)R\Delta T = R(T_f - T_i)$$

$$\text{Since, } U = nC_V T = \frac{nRT}{\gamma-1} \quad \left\{ \because C_V = \frac{R}{\gamma-1} \right\}$$

$$\Rightarrow U = \frac{PV}{\gamma-1}$$

8. Conceptual

A → (s); B → (r); C → (q); D → (p)

9. A → (s); B → (r); C → (q); D → (p)

For an ideal monatomic gas $PV^\gamma = \text{constant}$, where $\gamma = \frac{5}{3}$

$$\Rightarrow PV^{5/3} = \text{constant (for an Adiabatic process)}$$

$$\Rightarrow C_{\text{adiabatic}} = 0$$

Since, for a polytropic process, $TV^{x-1} = \text{constant}$

$$\Rightarrow x-1 = 2$$

$$\Rightarrow x = 3$$

$$\text{Also, } C = C_V + \frac{R}{1-x}$$

$$\Rightarrow C = C_V - \frac{R}{2}$$

$$\Rightarrow C = \frac{3R}{2} - \frac{R}{2}$$

$$\left\{ \because C_V = \frac{3R}{2} \right\}$$

$$\Rightarrow C = R$$

$$\text{When } V = \text{constant, then } C = C_V = \frac{3R}{2}$$

$$\text{When } P = \text{constant, then } C = C_p = \frac{5R}{2}$$

10. Conceptual

A → (p, q, r); B → (p, r, s); C → (p, q, r); D → (p, q, r)

11. A → (s); B → (r); C → (q); D → (t)

When A and B are mixed

$$(m)(c)(T-20) = (2m)(c)(40-T)$$

$$\Rightarrow T = \frac{100}{3} = 33.3^\circ\text{C}$$

When A and C are mixed

$$(m)(c)(T-20) = (3m)(c)(60-T)$$

$$\Rightarrow T = 50^\circ\text{C}$$

When B and C are mixed

$$(2m)(c)(T-40) = (3m)(c)(60-T)$$

$$\Rightarrow T = 52^\circ\text{C}$$

When A, B and C are mixed

$$\left(\text{Heat Gained by } \begin{matrix} A+B \\ \end{matrix} \right) = \left(\text{Heat Lost by } \begin{matrix} C \\ \end{matrix} \right)$$

$$\Rightarrow mc(T-20) + (2m)c(T-40) = (3m)c(60-T)$$

$$\Rightarrow (T-20) + 2(T-40) = 3(60-T)$$

$$\Rightarrow 3T - 100 = 180 - 3T$$

$$\Rightarrow 6T = 280$$

$$\Rightarrow T = 46.67^\circ\text{C}$$

Integer/Numerical Answer Type Questions

1. Let m be the mass of the container

Initial temperature of container is

$$T_i = (227 + 273) = 500 \text{ K}$$

And final temperature of container is

$$T_f = (27 + 273) = 300 \text{ K}$$

Now, $\left(\text{Heat Gained by the Ice Cube} \right) = \left(\text{Heat Lost by the Container} \right)$

$$m_{\text{ice}}L_{\text{ice}} + m_{\text{ice}}c_{\text{water}}\Delta T = -m \int_{T_i}^{T_f} cdT$$

$$\Rightarrow (0.1)(8 \times 10^4) + (0.1)(10^3)(27)$$

$$= -m \int_{500}^{300} (A + BT) dT$$

$$\Rightarrow 10700 = -m \left(AT + \frac{BT^2}{2} \right) \Big|_{500}^{300}$$

After substituting the values of A and B and the proper limits, we get

$$m = 0.495 \text{ kg}$$

$$\Rightarrow m = 495 \text{ g}$$

2. Thermal resistance $R = \frac{\ell}{kA}$

$$\Rightarrow \frac{R_x}{R_y} = \frac{k_y}{k_x} \quad \left\{ \because \ell_x = \ell_y \text{ and } A_x = A_y \right\}$$

$$\Rightarrow \frac{R_x}{R_y} = \frac{0.46}{0.92} = \frac{1}{2}$$

If $R_x = R$ then $R_y = 2R$

CEDB forms a balanced Wheatstone bridge, i.e., $T_C = T_D$ and no heat flows through CD , so

$$\frac{1}{R_{BE}} = \frac{1}{R+R} + \frac{1}{2R+2R}$$

$$\Rightarrow R_{BE} = \frac{4}{3}R$$

The total resistance between A and E will be,

$$R_{AE} = R_{AB} + R_{BE} = 2R + \frac{4}{3}R = \frac{10}{3}R$$

Now, heat current between A and E is

$$H = \frac{(\Delta T)_{AE}}{R_{AE}} = \frac{(60-10)}{\left(\frac{10}{3}\right)R} = \frac{15}{R}$$

If T_B is the temperature at B , then

$$H_{AB} = \frac{(\Delta T)_{AB}}{R_{AB}}$$

$$\Rightarrow \frac{15}{R} = \frac{60 - T_B}{2R}$$

$$\Rightarrow T_B = 30 \text{ }^\circ\text{C}$$

Further, $H_{AB} = H_{BC} + H_{BD}$

$$\Rightarrow \frac{15}{R} = \frac{30 - T_C}{R} + \frac{30 - T_D}{2R} \quad \left\{ T_C = T_D = T \text{ (say)} \right\}$$

$$\Rightarrow 15 = (30 - T) + \frac{(30 - T)}{2}$$

Solving this, we get

$$T = 20 \text{ }^\circ\text{C}$$

$$\Rightarrow T_C = T_D = 20 \text{ }^\circ\text{C}$$

3. For an adiabatic process, we have

$$P_f = \left(\frac{V_i}{V_f} \right)^\gamma P_i \quad \dots(1)$$

$$\text{Further, } C_V = \frac{3R}{2}$$

$$\Rightarrow C_P = C_V + R = \frac{5R}{2}$$

$$\Rightarrow \gamma = \frac{C_P}{C_V} = \frac{5}{3}$$

$$\Rightarrow P_f = \left(\frac{6}{2} \right)^{5/3} (10)^5 = 6.24 \times 10^5 \text{ Nm}^{-2}$$

Now, work done in adiabatic process is given by

$$W = \frac{P_i V_i - P_f V_f}{\gamma - 1}$$

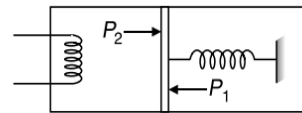
$$\Rightarrow W = \frac{10^5 \times 6 \times 10^{-3} - 6.24 \times 10^5 \times 2 \times 10^{-3}}{\left(\frac{5}{3}\right) - 1}$$

$$\Rightarrow W = -972 \text{ J}$$

\Rightarrow Work done on the gas is 972 J

4. (a) Since, $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$$\Rightarrow T_2 = \left(\frac{P_2 V_2 T_1}{P_1 V_1} \right) \quad \dots(1)$$



Here, $P_1 = 1 \times 10^5 \text{ Nm}^{-2}$, $V_1 = 2.4 \times 10^{-3} \text{ m}^3$, $T_1 = 300 \text{ K}$

Since the piston is light and moves out slowly, so we have

$$P_2 A = P_1 A + kx$$

$$\Rightarrow P_2 = P_1 + \frac{kx}{A} = (1 \times 10^5) + \left(\frac{8000 \times 0.1}{8 \times 10^{-3}} \right)$$

$$\Rightarrow P_2 = 2 \times 10^5 \text{ Nm}^{-2}$$

and $V_2 = V_1 + Ax = 2.4 \times 10^{-3} + 8 \times 10^{-3} \times 0.1$

$$\Rightarrow V_2 = 3.2 \times 10^{-3} \text{ m}^3$$

Substituting in equation (1), we get

$$T_2 = 800 \text{ K}$$

(b) Heat supplied by the heater, using the FLTD is

$$Q = W + \Delta U$$

$$\text{where, } \Delta U = nC_V \Delta T = \left(\frac{P_1 V_1}{RT_1} \right) \left(\frac{3}{2} R \right) (800 - 300)$$

$$\Rightarrow \Delta U = \frac{(1 \times 10^5)(2.4 \times 10^{-3})(1.5)(500)}{(300)}$$

H.146 JEE Advanced Physics: Waves and Thermodynamics

$$\Rightarrow \Delta U = 600 \text{ J}$$

$$\text{Further, } W = \frac{1}{2}kx^2 + P_1\Delta V$$

$$\Rightarrow W = \frac{1}{2} \times (8000)(0.1)^2 + (1 \times 10^5)(0.1)(8 \times 10^{-3})$$

$$\Rightarrow W = (40 + 80) \text{ J}$$

$$\Rightarrow W = 120 \text{ J}$$

$$\Rightarrow Q = 600 + 120 = 720 \text{ J}$$

5. Initially when only one sphere is there

$$P_{\text{source}} = \sigma AT_0^4 \quad \left\{ \because (T_0 = 1000 \text{ K}) \right\}$$

After enveloping, let temperature of the inner sphere is T_1 and that of the outer sphere is T_2

$$\sigma AT_1^4 = P_{\text{source}} = \sigma AT_0^4$$

$$\therefore T_1 = T_0 = 1000 \text{ K}$$

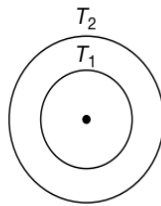
$$\text{Further, } \sigma AT_2^4 - \sigma AT_1^4 = P_{\text{source}} = \sigma AT_0^4$$

$$\Rightarrow T_2^4 = 2T_0^4$$

$$\Rightarrow T_2 = (2)^{1/4}(1000)$$

$$\Rightarrow T_2 = (1.19)(1000)$$

$$\Rightarrow T_2 = 1190 \text{ K}$$



6. For the given case $\Delta T = 30 \text{ }^\circ\text{C}$

Expansion of girders is $\Delta l = l\alpha\Delta T$

$$\Rightarrow \Delta l = 12 \times 1.1 \times 10^{-5} \times 30 = 0.00396 \text{ m}$$

$$\Rightarrow \Delta l = 3.96 \text{ mm}$$

7. For a cyclic process, $\Delta U = 0$

$$\Rightarrow Q_{\text{cycle}} = W_{\text{cycle}} \quad \dots(1)$$

$$\text{where, } W_{\text{cycle}} = W_{A \rightarrow B} + W_{B \rightarrow C} + W_{C \rightarrow A} \quad \dots(2)$$

Process $C \rightarrow A$ is isochoric, hence $W_{C \rightarrow A} = 0$

Process $A \rightarrow B$ is isobaric, as its T - V graph is a straight line passing through origin.

In an isobaric process, work done is

$$W_{A \rightarrow B} = P(V_B - V_A) = nR(T_B - T_A)$$

$$\Rightarrow W_{A \rightarrow B} = 1 \times 8.3(400 - 300) = 830 \text{ J} \quad \dots(3)$$

$W_{A \rightarrow B}$ is positive, so expansion of gas takes place.

From equations (1), (2) and (3), we get

$$-1000 = 830 + W_{B \rightarrow C} + 0$$

$$\Rightarrow W_{B \rightarrow C} = -1830 \text{ J}$$

Negative work done implies that compression of the gas takes place or work is done on the system/gas.

8. Since $t = 2\pi\sqrt{\frac{\ell}{g}}$

For a pendulum to give correct time, its period must be 2 s.

$$\Rightarrow 2 = 2\pi\sqrt{\frac{\ell_0}{9.8}} \quad \dots(1)$$

$$\Rightarrow 2 = 2\pi\sqrt{\frac{\ell_0 [1 + 12 \times 10^{-6}(T - 32)]}{9.788}} \quad \dots(2)$$

Equating (1) and (2), we get

$$\frac{\ell_0}{9.8} = \frac{\ell_0}{9.788} [1 + 12 \times 10^{-6}(T - 32)]$$

$$\Rightarrow T - 32 = -102$$

$$\Rightarrow T = -102 + 32$$

$$\Rightarrow T = -70 \text{ }^\circ\text{C}$$

9. Since process is a cyclic process, so we have

So, we have

$$\Delta U = 0 \quad \text{\{Answer to Part (c)\}}$$

Applying FLTD, we get

$$Q_{\text{net}} = W_{\text{net}} = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

Now $W_{AB} = nR\Delta T = (2)R(100) = 400 \text{ cal}$

$$W_{BC} = (2)R(400)\log_e(2) = 1120 \text{ cal}$$

$$W_{CD} = nR\Delta T = (2)R(-100) = -400 \text{ cal}$$

$$W_{DA} = (2)R(300)\log_e\left(\frac{1}{2}\right) = -840 \text{ cal}$$

So, $W_{\text{net}} = 400 + 1120 - 400 - 840$

$$\Rightarrow W_{\text{net}} = 280 \text{ cal}$$

\{Answer to Part (b)\}

$$\Rightarrow Q_{\text{net}} = 280 \text{ cal}$$

\{Answer to Part (a)\}

10. Thermal expansion in rod is $\Delta l = l\alpha\Delta T$

Due to clamps, the elastic strain developed in the rod is

$$\left| \frac{\Delta l}{l} \right| = \alpha\Delta T = (1.2 \times 10^{-5})(30) = 36 \times 10^{-5}$$

11. Since, in steady state, rate of heat flow is

$$H = \frac{4\pi Kr_1 r_2 \Delta T}{r_2 - r_1} = \frac{(4)(\pi)(0.002)(11)(9)(100 - 0)}{(11 - 9)}$$

$$\Rightarrow H = \frac{dQ}{dt} = 124.4 \text{ cal s}^{-1}$$

This rate should be equal to $L\left(\frac{dm}{dt}\right)$

$$\Rightarrow \left(\frac{dm}{dt}\right) = \frac{dQ}{L} = \frac{124.4}{80} = 1.555 \text{ gs}^{-1}$$

Total mass of ice is $m = \rho_{\text{ice}}(4\pi r_1^2)$

$$\Rightarrow m = (0.9)(4)(\pi)(9)^2 = 916 \text{ g}$$

So, time taken for the ice to melt completely is

$$t = \frac{m}{\left(\frac{dm}{dt}\right)} = \frac{916}{1.555} = 589 \text{ sec.}$$

12. (a) $P = \alpha\sqrt{T}$

$$\Rightarrow PT^{-\frac{1}{2}} = \text{constant}$$

$$\Rightarrow P^2V^{-\frac{1}{2}} = \text{constant}$$

$$\Rightarrow PV^{-1} = \text{constant}$$

Compare with $PV^x = \text{constant}$, we get

$$x = -1$$

$$\Rightarrow W = \left(\frac{R}{1-x} \right) (\Delta T)$$

$$\Rightarrow W = \frac{R}{2} (50) = 25R$$

$$\Rightarrow * = 25$$

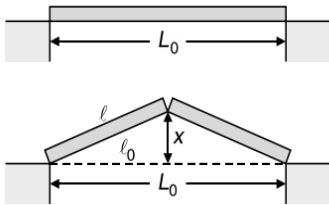
(b) Since, $C = C_V + \frac{R}{1-x} = \frac{3}{2}R + \frac{R}{2} = 2R$

$$\Rightarrow C = 2R$$

$$\Rightarrow * = 2$$

13. Consider one-half of the bar, with original length

$$\ell_0 = \frac{L_0}{2}$$



The length after temperature increase is

$$\ell = \ell_0 + \alpha \ell_0 \Delta T$$

From Pythagoras theorem, we have

$$x^2 = \ell^2 - \ell_0^2 = \ell_0^2 (1 + \alpha \Delta T)^2 - \ell_0^2$$

$$\Rightarrow x^2 = \ell_0^2 (1 + 2\alpha \Delta T) - \ell_0^2 = 2\ell_0^2 \alpha \Delta T$$

$$\Rightarrow x = \ell_0 \sqrt{2\alpha \Delta T}$$

$$\Rightarrow x = \frac{4}{2} \sqrt{2(25 \times 10^{-6})(32)}$$

$$\Rightarrow x = 2 \times 40 \times 10^{-3}$$

$$\Rightarrow x = 80 \text{ mm}$$

14. Since it is given that, initially the separator is in equilibrium, so pressures on both sides of gas are equal say P_i . If A be the area of cross-section of cylinder, then number of moles of gas in left and right part are n_1 and n_2 given by

$$n_1 = \frac{P_i(10A)}{R(100)} \text{ and } n_2 = \frac{P_i(20A)}{R(400)}$$

Finally, when separator is displaced to right through a distance x , then we have

$$n_1 = \frac{P_f(10+x)A}{RT_f} \text{ and } n_2 = \frac{P_f(20-x)A}{RT_f}$$

where, P_f and T_f be the final pressure and temperature on both sides after a long time.

Equating the ratio of number of moles $\frac{n_1}{n_2}$ in initial and final state, we get

$$\frac{n_1}{n_2} = \frac{(10A/100)}{(20A/400)} = \frac{(10+x)A}{(20-x)A}$$

$$\Rightarrow 2(20-x) = 10+x$$

$$\Rightarrow x = 10 \text{ cm}$$

Hence in final state, when gases in both parts are in thermal equilibrium, the piston is displaced 10 cm to the right from its initial position.

15. When the temperature is increased, volume of the cube will increase while density of liquid will decrease. The depth upto which the cube is submerged in the liquid remains the same, hence the upthrust will not change.

$$F = F'$$

$$\Rightarrow V_i \rho_L g = V_i' \rho_L' g \quad \{\text{where } V_i \text{ is the volume immersed}\}$$

$$\Rightarrow (Ah_i)(\rho_L)(g) = A(1+2\alpha_s \Delta T)(h_i) \left(\frac{\rho_L}{1+\gamma_L \Delta T} \right) g$$

Solving this equation, we get $\gamma_L = 2\alpha_s$

$$\Rightarrow \frac{\gamma_L}{\alpha_s} = 2$$

16. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = v_e = 11.2 \text{ kms}^{-1} = 11.2 \times 10^3 \text{ ms}^{-1}$

$$\Rightarrow \sqrt{\frac{3RT}{M}} = 11.2 \times 10^3$$

$$\Rightarrow \frac{3(8.314)T}{\left(\frac{2}{1000}\right)} = (11.2 \times 10^3)^2$$

$$\Rightarrow T \approx 20000 \text{ K}$$

17. $n = 1, \gamma = \frac{7}{5}$

$$\Rightarrow \gamma - 1 = \frac{2}{5}$$

$$Q_{AB} = Q_{CD} = 0$$

Let $T_A = T_1$ and $T_D = T_2$ with $T_2 > T_1$

For the process AB, we have

$$TV^{\gamma-1} = \text{constant}$$

$$\Rightarrow TV^{\frac{2}{5}} = \text{constant}$$

$$\Rightarrow T_A (10V_0)^{\frac{2}{5}} = T_B (V_0)^{\frac{2}{5}}$$

$$\Rightarrow T_B = T_1 (10)^{0.4} \approx 2.5T_1$$

Similarly, $T_C = 2.5T_2$

Now, $Q_{BC} = C_V \Delta T$

$$\Rightarrow Q_{BC} = \left(\frac{5}{2}R \right) (T_C - T_B) = (6.25)(T_2 - T_1) \text{ and}$$

$$Q_{DA} = -2.5(T_2 - T_1)$$

So, $W_{\text{net}} = Q_{\text{net}} = 3.75(T_2 - T_1) \quad \{\because \Delta U = 0\}$

$$\Rightarrow \eta = \frac{W_{\text{net}}}{\text{heat absorbed}} \times 100 = \frac{3.75}{6.25} \times 100 = 60\%$$

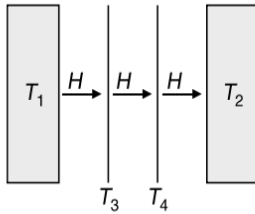
18. Under steady state conditions the net rate of heat flow per unit area is the same everywhere.

$$H = \sigma(T_1^4 - T_3^4)$$

H.148 JEE Advanced Physics: Waves and Thermodynamics

$$H = \sigma(T_3^4 - T_4^4)$$

$$H = \sigma(T_4^4 - T_2^4)$$



Adding these three equations, we get

$$3H = \sigma(T_1^4 - T_2^4) = H_0$$

Here $H_0 = \sigma(T_1^4 - T_2^4)$ is the rate of heat flow per unit area in the absence of the shields.

So, the desired fraction is

$$f = \frac{H}{H_0} = \frac{1}{3}$$

$$\Rightarrow x = 3.$$

19. Piston A is free to move, so gas will expand isobarically and piston B is fixed, so the process is isochoric.

Given that, $Q_A = Q_B$

$$\Rightarrow nC_p \Delta T_A = nC_v \Delta T_B$$

$$\Rightarrow \left(\frac{7R}{2}\right)(30) = \left(\frac{5R}{2}\right)\Delta T_B$$

$$\Rightarrow \Delta T_B = 42 \text{ K}$$

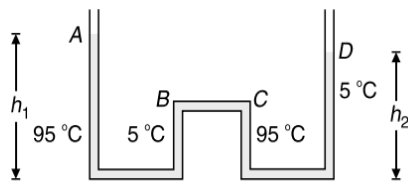
20. Density of a liquid varies with temperature as

$$\rho_{t \text{ } ^\circ\text{C}} = \left(\frac{\rho_0 \text{ } ^\circ\text{C}}{1 + \gamma t}\right)$$

where, γ is the coefficient of volume expansion of temperature.

In the figure,

$$h_1 = 52.8 \text{ cm}, h = 49 \text{ cm}, h_2 = 51 \text{ cm}$$



Now, (Pressure at B) = (Pressure at C)

$$\Rightarrow P_0 + h_1 \rho_{95^\circ} g - h \rho_{5^\circ} g = P_0 + h_2 \rho_{5^\circ} g - h \rho_{95^\circ} g$$

$$\Rightarrow \rho_{95^\circ} (h_1 + h) = \rho_{5^\circ} (h_2 + h)$$

$$\Rightarrow \frac{\rho_{95^\circ}}{\rho_{5^\circ}} = \frac{h_2 + h}{h_1 + h}$$

$$\Rightarrow \frac{\left(\frac{\rho_0}{1 + 95\gamma}\right)}{\left(\frac{\rho_0}{1 + 5\gamma}\right)} = \frac{h_2 + h}{h_1 + h}$$

$$\Rightarrow \frac{1 + 5\gamma}{1 + 95\gamma} = \frac{51 + 49}{52.8 + 49} = \frac{100}{101.8}$$

Solving this equation, we get

$$\gamma = 2 \times 10^{-4} (\text{ } ^\circ\text{C})^{-1}$$

$$\Rightarrow x = 2$$

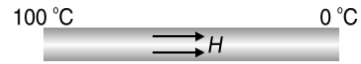
21. For an isobaric process, we have

$$W_{\text{isob}} = P \Delta V = nR \Delta T$$

$$\Rightarrow W_{\text{isob}} = (0.2)(8.3)(100) = 166 \text{ J}$$

22. Thermal resistance of the rod,

$$R = \frac{\ell}{kA} = \frac{1}{(401)(4 \times 10^{-4})} = 6.23 \text{ KW}^{-1}$$



$$\Rightarrow \text{Heat current } H = \frac{\text{Temperature difference}}{\text{Thermal resistance}}$$

$$\Rightarrow H = \frac{(100 - 0)}{6.23} = 16 \text{ watt}$$

$$\text{Heat transferred in 1 hr. is } Q = Ht \quad \left\{ \because H = \frac{Q}{t} \right\}$$

$$\Rightarrow Q = (16)(3600) = 57600 \text{ J}$$

Now, if m mass of ice melts in 1 hr., then $Q = mL$

$$m = \frac{Q}{L} = \frac{57600}{3.35 \times 10^5}$$

$$\Rightarrow m = 0.172 \text{ kg} = 172 \text{ g}$$

23. Force by rod on wall is $F = YA\alpha\Delta T$

$$\Rightarrow F = (2 \times 10^{11})(2 \times 10^{-6})(1.2 \times 10^{-5})(80) = 384 \text{ N}$$

24. In both the cases, the weight of the body is balanced by the buoyant force acting on it. At $t_0 = 0 \text{ } ^\circ\text{C}$, we have buoyant force given by

$$F_b = (\delta V_0) \rho_0 g \quad \dots(1)$$

where δ is the fraction of volume submerged in the liquid, V_0 is the volume of body and ρ_0 is the density of liquid at $t_0 = 0 \text{ } ^\circ\text{C}$.

At $t = 50 \text{ } ^\circ\text{C}$, the volume of the body now becomes $V = V_0(1 + \gamma_s t)$

Further, the density of the liquid at $t = 50 \text{ } ^\circ\text{C}$ is $\rho_1 = \frac{\rho_0}{1 + \gamma_l t}$

So, the buoyant force is now given by

$$F_b = \frac{V_0 \rho_0 g (1 + \gamma_s t)}{1 + \gamma_l t} \quad \dots(2)$$

From (1) and (2), we get $\% \delta = 96\%$

25. For an adiabatic process $TV^{\gamma-1} = \text{constant}$

$$\Rightarrow TV^{\gamma-1} = \left(\frac{T}{2}\right)(5.66V)^{\gamma-1}$$

$$\Rightarrow (5.66V)^{\gamma-1} = 2$$

$$\Rightarrow (\gamma - 1) \log_e(5.66) = \log_e(2)$$

$$\Rightarrow \gamma - 1 = 0.4$$

$$\Rightarrow \gamma = 1.4$$

$$\text{Since, } \gamma = 1 + \frac{2}{f} = 1.4$$

$$\Rightarrow \frac{2}{f} = 0.4$$

$$\Rightarrow f = \frac{2}{0.4} = 5$$

26. $W = \text{Area Enclosed} = (10)(4-1)(3-1) \times 10^5 \text{ Nm}^{-2}$

$$\Rightarrow W = 6 \times 10^6 \text{ J} = 6 \text{ MJ}$$

If the net work is done by the gas, then direction of traversal around the cycle is clockwise.

27. Let m be the mass of the steam required to raise the temperature of 100 g of water from 24 °C to 90 °C. Then

$$\left(\begin{array}{c} \text{Heat Lost} \\ \text{by Steam} \end{array} \right) = \left(\begin{array}{c} \text{Heat Gained} \\ \text{by Water} \end{array} \right)$$

$$\Rightarrow m(L + c\Delta T_1) = 100c\Delta T_2$$

$$\Rightarrow m = \frac{(100)(c)(\Delta T_2)}{L + c(\Delta T_1)}$$

where, $c = \text{specific heat of water} = 1 \text{ cal g}^{-1} \text{ }^\circ\text{C}$

$L = \text{Latent heat of vaporization} = 540 \text{ cal g}^{-1}$

$$\Delta T_1 = (100 - 90) = 10 \text{ }^\circ\text{C}$$

and $\Delta T_2 = (90 - 24) = 66 \text{ }^\circ\text{C}$

Substituting the values, we get

$$m = \frac{(100)(1)(66)}{(540) + (1)(10)} = 12 \text{ g}$$

$$\Rightarrow m = 12 \text{ g}$$

28. Since the boiling process is an isobaric process, so

$$W_{\text{isob}} = P\Delta V = (10^5)(1671 - 1) \times 10^{-6} \text{ Nm}$$

$$\Rightarrow W_{\text{isob}} = 167 \text{ J} \cong 40 \text{ cal}$$

Now, according to FLTD we have $Q = \Delta U + W$

where $Q = mL = (1)(540) = 540 \text{ cal}$

$$\Rightarrow 540 = \Delta U + 40$$

$$\Rightarrow \Delta U = 500 \text{ cal}$$

29. A constant pressure, $V \propto T$

$$\Rightarrow \frac{V_2}{V_1} = \frac{T_2}{T_1}$$

$$\Rightarrow \frac{Ah_2}{Ah_1} = \frac{T_2}{T_1}$$

$$\Rightarrow h_2 = h_1 \left(\frac{T_2}{T_1} \right) = (1) \left(\frac{400}{300} \right) m = \frac{4}{3} m$$

As there is no heat loss, process is adiabatic.

For adiabatic process,

$$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1}$$

$$\Rightarrow T_f = T_i \left(\frac{V_i}{V_f} \right)^{\gamma-1} = (400) \left(\frac{h_i}{h_f} \right)^{1.4-1} = 400 \left(\frac{4}{3} \right)^{0.4}$$

$$\Rightarrow T_f = (400)(1.12) = 448 \text{ K}$$

30. If after time t , temperature of water inside the pitcher drops to 15 °C, then by law of calorimetry, we have

$$\left(10 - \frac{0.2}{1000} t \right) (4200)(5) = \left(\frac{0.2t}{1000} \right) (2.27 \times 10^6)$$

Here $\frac{0.2t}{1000} \ll 10$, so we have $210000 = 454t$

$$\Rightarrow t \approx 462 \text{ s}$$

31. (a) $U \propto \sqrt{V}$

Since, $U \propto T$

$$\Rightarrow T \propto V^{\frac{1}{2}}$$

$$\Rightarrow TV^{\frac{1}{2}} = \text{constant}$$

$$\Rightarrow PV^{\frac{1}{2}} = \text{constant}$$

Comparing with $PV^x = \text{constant}$, we get $x = \frac{1}{2}$

Since, molar specific heat is

$$\Rightarrow C = \frac{R}{\gamma-1} + \frac{R}{1-x} = \frac{R}{\frac{7}{5}-1} + \frac{R}{1-\frac{1}{2}}$$

$$\Rightarrow C = \frac{5}{2}R + 2R = \frac{9R}{2}$$

$$\Rightarrow * = 9$$

(b) Further $Q = nC\Delta T$ and $\Delta U = nC_V\Delta T$

$$\Rightarrow W = Q - \Delta U = n(C - C_V)\Delta T$$

$$\Rightarrow \frac{W}{\Delta U} = \frac{C - C_V}{C_V}$$

$$\Rightarrow W = \left(\frac{C - C_V}{C_V} \right) \Delta U = \left(\frac{\frac{9}{2} - \frac{5}{2}}{\frac{5}{2}} \right) (100) = 80 \text{ J}$$

32. According to Newton's Law of Cooling

$$\left(\frac{T_1 - T_2}{t} \right) = \alpha \left[\left(\frac{T_1 + T_2}{2} \right) - T_0 \right]$$

For the given conditions,

$$\frac{60 - 40}{10} = \alpha \left(\frac{60 + 40}{2} - 10 \right) \quad \dots(1)$$

Let T be the temperature after next 10 minutes. Then,

$$\frac{40 - T}{10} = \alpha \left(\frac{40 + T}{2} - 10 \right) \quad \dots(2)$$

Solving equations (1) and (2), we get

$$T = 28 \text{ }^\circ\text{C}$$

33. Number of moles (n) is $n = \frac{m}{M}$

Since the masses of two gases is same, therefore the ratio of number of moles is

$$\frac{n_1}{n_2} = \frac{M_2}{M_1} = \frac{28}{32} = \frac{7}{8}$$

H.150 JEE Advanced Physics: Waves and Thermodynamics

Also, pressure and temperature of both gases is same, so from Ideal Gas Equation, we get $V \propto n$ therefore their volumes are in the ratio.

$$\Rightarrow \frac{V_1}{V_2} = \frac{n_1}{n_2} = \frac{7}{8} = \frac{360 - \theta}{\theta}$$

$$\Rightarrow \theta = 192^\circ$$

34. Since, $mgh = mc\Delta T + mL$

$$\Rightarrow L = gh - c\Delta T$$

$$\Rightarrow L = (10)(6 \times 10^3) - (125)(200 - 40)$$

$$\Rightarrow L = 60000 - 20000$$

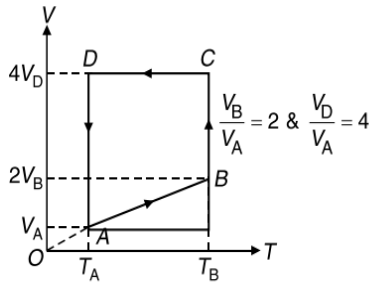
$$\Rightarrow L = 40000 \text{ Jkg}^{-1}$$

$$\Rightarrow L = 40 \text{ kJkg}^{-1}$$

35. Number of moles, $n = 2$

$$C_V = \frac{3}{2}R \text{ and } C_P = \frac{5}{2}R \quad (\text{Monatomic})$$

$$T_A = 27^\circ\text{C} = 300 \text{ K}$$



Let $V_A = V_0$ then $V_B = 2V_0$ and $V_D = V_C = 4V_0$.

(a) Process $A \rightarrow B$, we have

$$V \propto T$$

$$\Rightarrow \frac{T_B}{T_A} = \frac{V_B}{V_A}$$

$$\Rightarrow T_B = T_A \left(\frac{V_B}{V_A} \right) = (300)(2) = 600 \text{ K}$$

$$\Rightarrow T_B = 600 \text{ K}$$

(b) Process $A \rightarrow B$, we have $V \propto T$

$$\Rightarrow P = \text{constant}$$

$$\Rightarrow Q_{AB} = nC_P dT = nC_P (T_B - T_A)$$

$$\Rightarrow Q_{AB} = (2) \left(\frac{5}{2}R \right) (600 - 300) = 1500R, \text{ absorbed}$$

$$\Rightarrow Q_{AB} = 3000 \text{ cal, absorbed}$$

Process $B \rightarrow C$, we have $T = \text{constant}$

$$\Rightarrow dU = 0$$

$$\Rightarrow Q_{BC} = W_{BC} = nRT_B \log_e \left(\frac{V_C}{V_B} \right)$$

$$\Rightarrow Q_{BC} = (2)(R)(600) \log_e \left(\frac{4V_0}{2V_0} \right)$$

$$\Rightarrow Q_{BC} = (1200R) \log_e (2) = (1200)(2)(0.7)$$

$$\Rightarrow Q_{BC} \approx 1680 \text{ cal, absorbed}$$

Process $C \rightarrow D$, we have $V = \text{constant}$

$$\Rightarrow Q_{CD} = nC_V dT = nC_V (T_D - T_C)$$

$$\Rightarrow Q_{CD} = n \left(\frac{3}{2}R \right) (T_A - T_B) \quad \{ \because T_D = T_A \text{ and } T_C = T_B \}$$

$$\Rightarrow Q_{CD} = (2) \left(\frac{3}{2}R \right) (300 - 600)$$

$$\Rightarrow Q_{CD} = -900R \text{ (released)}$$

$$\Rightarrow Q_{CD} = -1800 \text{ cal, released}$$

Process $D \rightarrow A$, we have $T = \text{constant}$

$$\Rightarrow dU = 0$$

$$\Rightarrow Q_{DA} = W_{DA} = nRT_D \log_e \left(\frac{V_A}{V_D} \right)$$

$$\Rightarrow Q_{DA} = (2)(R)(300) \log_e \left(\frac{V_0}{4V_0} \right)$$

$$\text{Since, } \log_e \left(\frac{1}{4} \right) = -2 \log_e (2)$$

$$\Rightarrow Q_{DA} = 600R \log_e \left(\frac{1}{4} \right)$$

$$\Rightarrow Q_{DA} \approx -1680 \text{ cal, released}$$

(c) In the complete cycle, $dU = 0$

Therefore, from $FLTD$, we have

$$W_{\text{net}} = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA}$$

$$\Rightarrow W_{\text{net}} = 3000 + 1680 - 1800 - 1680$$

$$\Rightarrow W_{\text{net}} = W_{\text{total}} = 1200 \text{ cal}$$

ARCHIVE: JEE MAIN

1. Since, $U = \frac{n_1 f_1 RT}{2} + \frac{n_2 f_2 RT}{2}$

$$\Rightarrow U = \frac{(3)(5)}{2} RT + \frac{(5)(3)}{2} RT = 15RT$$

Hence, the correct answer is (B).

2. The mean free path of molecules of an ideal gas is

$$\lambda = \frac{V}{\sqrt{2} N d^2} = \frac{RT}{\sqrt{2} \pi N P d^2}$$

where, V is the volume of container, N is number of molecules and d is the diameter of the molecule.

Since volume of container does not change (closed container)

i.e., $\frac{T}{P} = \text{constant}$, so mean free path is unchanged. Hence

with increasing temp, mean free path does not change if gas is confined to a closed container. Also, average collision time is

Average collision time

$$\tau = \frac{\text{Mean Free Path}}{\text{Average Speed}} = \frac{\lambda}{v_{av}}$$

Since, average speed $v_{av} \propto \sqrt{T}$

So, average collision time $\tau \propto \frac{1}{\sqrt{T}}$

Hence with increase in temperature the average collision time decreases.

Hence, the correct answer is (D).

3. Given that, $\frac{\Delta L}{L} = 0.02\%$

Since, $\Delta L = L\alpha\Delta T$

$$\Rightarrow \frac{\Delta L}{L} = \alpha\Delta T = 0.02\%$$

Also, $\beta = 2\alpha$

$$\Rightarrow \beta\Delta T = 2\alpha\Delta T = 0.04\%$$

Since, $\rho = \frac{M}{V} = \frac{M}{AL}$

$$\Rightarrow \frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} - \frac{\Delta A}{A} - \frac{\Delta L}{L}$$

Due to heating, mass remains constant, so we have

$$\left| \frac{\Delta\rho}{\rho} \right| = \frac{\Delta A}{A} + \frac{\Delta L}{L} = \beta\Delta T + \alpha\Delta T$$

$$\Rightarrow \left| \frac{\Delta\rho}{\rho} \right| = 0.04\% + 0.02\% = 0.06\%$$

Hence, the correct answer is (B).

4. Unfilled volume V of the beaker is constant, so

$$V = V_0 - V_m = \text{constant}$$

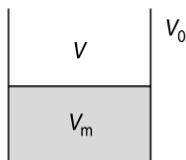
After increasing temperature, V remains same.

$$\Rightarrow \Delta V = 0$$

$$\Rightarrow \Delta V_0 - \Delta V_m = 0$$

$$\Rightarrow V_0\gamma_b\Delta T = V_m\gamma_m\Delta T$$

$$\Rightarrow V_m = \frac{V_0\gamma_b}{\gamma_m} = \frac{(500)(6 \times 10^{-6})}{(1.5 \times 10^{-4})} = 20 \text{ cc}$$



Hence, the correct answer is 20.

5. At constant pressure, $nC_p(50) = 160$

At constant volume, $nC_v(100) = 240$

$$\Rightarrow \frac{C_p}{2C_v} = \frac{160}{240} = \frac{\gamma}{2}$$

$$\Rightarrow \gamma = 1 + \frac{2}{f} = \frac{4}{3}$$

$$\Rightarrow f = \frac{2}{\gamma - 1} = 6$$

Hence, the correct answer is (C).

6. Initially, we have

$$\underbrace{\text{Calorimeter}}_{20 \text{ g, } 25^\circ\text{C}}, \underbrace{\text{Water}}_{20 \text{ g, } 25^\circ\text{C}}, \underbrace{\text{Steam}}_{m, 100^\circ\text{C}}$$

Finally, due to condensation of steam, we have

$$\underbrace{\text{Calorimeter}}_{20 \text{ g, } 31^\circ\text{C}}, \underbrace{\text{Water}}_{20 \text{ g, } 31^\circ\text{C}}, \underbrace{\text{Water}}_{m, 100^\circ\text{C}}$$

By law of calorimetry, we have

$$(180 + 20)(1)(31 - 25) = 540m + m(1)(100 - 31)$$

$$\Rightarrow m = 1.97 \text{ g} \approx 2 \text{ g}$$

Hence, the correct answer is (B).

7. Since, $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$

where, f the is degrees of freedom of a gas.

(A) For monatomic gas molecule, we have

$$f = 3, \gamma = 1 + \frac{2}{3} = \frac{5}{3}$$

(B) For diatomic rigid molecules, we have

$$f = 5, \gamma = 1 + \frac{2}{5} = \frac{7}{5}$$

(C) For diatomic non-rigid molecules, we have

$$f = 7, \gamma = 1 + \frac{2}{7} = \frac{9}{7}$$

(D) For triatomic rigid molecules, we have

$$f = 6, \gamma = 1 + \frac{2}{6} = \frac{4}{3}$$

Hence, the correct answer is (A).

8. Here the water will provide heat for ice to melt therefore

$$m_w c_w \Delta T = m_{\text{ice}} L_{\text{ice}}$$

$$\Rightarrow m_{\text{ice}} = \frac{0.2 \times 4200 \times 25}{3.4 \times 10^5} = 0.0617 \text{ kg}$$

$$\Rightarrow m_{\text{ice}} = 61.7 \text{ g}$$

Remaining ice will remain unmelted

Hence, the correct answer is (A).

9. As work done on gas and heat supplied to the gas are zero, total internal energy of gases remains same

$$U_1 + U_2 = U'_1 + U'_2$$

$$\Rightarrow (0.1)C_v(200) + (0.05)C_v(400) = (0.1 + 0.05)C_v T$$

$$\Rightarrow T = \frac{800}{3} \text{ K} = 266.67 \text{ K}$$

Or, we could have simply said that internal energy lost by one gas equals the internal energy gained by other i.e.,

$$(0.05)C_v(400 - T) = (0.1)C_v(T - 200)$$

$$\Rightarrow 400 - T = 2(T - 200)$$

$$\Rightarrow 3T = 800$$

$$\Rightarrow T = \frac{800}{3} = 266.67 \text{ K}$$

Hence, the correct answer is 266.67.

10. For an ideal gas, we have $PV = nRT$

$$\Rightarrow \Delta(PV) = \Delta(nRT)$$

Since volume is constant, so we have

$$V\Delta P = nR\Delta T$$

$$\Rightarrow \frac{\Delta T}{\Delta P} = \frac{V}{nR} = \frac{T}{P}$$

$$\{\because PV = nRT\}$$

H.152 JEE Advanced Physics: Waves and Thermodynamics

$$\Rightarrow \left| \frac{\Delta T}{\Delta P} \right| = \left| \frac{300}{2} \right| = 150$$

Hence, the correct answer is 150.

11. Since, $\frac{1}{2} \left(\frac{1}{2} m v^2 \right) = mc\Delta T$

$$\Rightarrow \Delta T = \frac{v^2}{4c} = \frac{(210)^2}{4(0.030 \times 4200)} = 87.5 \text{ }^\circ\text{C}$$

Hence, the correct answer is (B).

12. Mean kinetic energy of a molecule at T is

$$\frac{3}{2} k_B T = 4 \times 10^{-14} \quad \dots(1)$$

Since $n = \frac{PV}{RT}$ and if N be the number of molecules. then we

$$\text{have } n = \frac{N}{N_A}$$

$$\Rightarrow N = \left(\frac{PV}{RT} \right) N_A = \frac{PV}{k_B T} = \frac{2 \times 13.6 \times 980 \times 4}{(8/3) \times 10^{-14}}$$

$$\Rightarrow N = 3.99 \times 10^{18}$$

Hence, the correct answer is (C).

13. Since, $L = L_1 + L_2$

$$\Rightarrow \Delta L = \Delta L_1 + \Delta L_2$$

$$\Rightarrow (L_1 + L_2) \alpha_{\text{eq}} \Delta T = L_1 \alpha_1 \Delta T + L_2 \alpha_2 \Delta T$$

$$\Rightarrow \alpha_{\text{eq}} = \frac{L_1 \alpha_1 + L_2 \alpha_2}{L_1 + L_2}$$

Hence, the correct answer is (D).

14. Since $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ and $v_{N_2} = v_{H_2}$

$$\Rightarrow \sqrt{\frac{3RT_{N_2}}{M_{N_2}}} = \sqrt{\frac{3RT_{H_2}}{M_{H_2}}}$$

$$\Rightarrow \frac{573}{28} = \frac{T_{H_2}}{2}$$

$$\Rightarrow T_{H_2} = 40.93 \text{ K} \approx 41 \text{ K}$$

Hence, the correct answer is 41.

15. Total degree of freedom = $3 + 2 = 5$

$$\Rightarrow U = \frac{nfRT}{2} = \frac{5RT}{2} \quad \{\because n = 1\}$$

$$\Rightarrow \gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f} = 1 + \frac{2}{5} = \frac{7}{5}$$

Hence, the correct answer is (D).

16. For an ideal gas, we have $PV = nRT$

$$\text{So, } P_1 V_1 = nR(250) \quad \dots(1)$$

$$\text{and } P_2 (2V_1) = \left(\frac{5n}{4} \right) R(2000) \quad \dots(2)$$

Dividing equation (2) by (1), we get

$$\frac{2P_2}{P_1} = \frac{5 \times 2000}{4 \times 250}$$

$$\Rightarrow \frac{P_2}{P_1} = 5$$

Hence, the correct answer is (5).

17. Average collision time

$$\tau = \frac{\text{Mean Free Path}}{\text{Average Speed}} = \frac{\lambda}{v_{\text{av}}}$$

Since, average speed $v_{\text{av}} \propto \sqrt{T}$

So, average collision time $\tau \propto \frac{1}{\sqrt{T}}$

Hence, the correct answer is (C).

18. Since, $\eta = \frac{\text{Work done}}{\text{Heat Input}} = \frac{W}{\Sigma Q_{\oplus}}$

$$\Rightarrow \frac{1}{2} = \eta = \frac{1915 - 40 + 125 - Q}{1915 + 125}$$

$$\Rightarrow \frac{1}{2} = \frac{2000 - Q}{2040}$$

$$\Rightarrow 2040 = 4000 - 2Q$$

$$\Rightarrow 2Q = 1960$$

$$\Rightarrow Q = 980 \text{ J}$$

Hence, the correct answer is (C).

19. Bursting of helium balloon is an irreversible and adiabatic process.

Hence, the correct answer is (B).

20. Since $\frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$, where, $Q_2 = mL_{\text{ice}} = 8000 \text{ cal}$

Also, $T_2 = 273 \text{ K}$ and $T_1 = 300 \text{ K}$

$$\Rightarrow \frac{8000}{W} = \frac{273}{300 - 273} = \frac{273}{27}$$

$$\Rightarrow W = 791.2 \text{ cal}$$

Also $Q_1 = Q_2 + W$, so we get

$$Q_1 = 791.2 + 8000 = 8791.2 \text{ cal} \approx 8791 \text{ cal}$$

Hence, the correct answer is 8791.

21. For an adiabatic process,

$$\Delta Q = 0$$

For an isothermal process, carried on an ideal gas

$$\Delta U = 0$$

For an isochoric process volume remains constant, so

$$W = 0$$

For an isobaric process, $\Delta U \neq 0$, $\Delta W \neq 0$, $\Delta Q \neq 0$

Hence, the correct answer is (A).

22. Since $\Delta U = nC_V \Delta T$, so internal energy change remains the same in all the cases. Along path AB volume is increasing, so $W > 0$. Along path AD volume is decreasing, so $W < 0$ and along AC volume is constant, so $W = 0$

Hence, the correct answer is (A).

23. In adiabatic process

$$PV^\gamma = \text{constant}$$

$$\Rightarrow P\left(\frac{m}{\rho}\right)^\gamma = \text{constant}$$

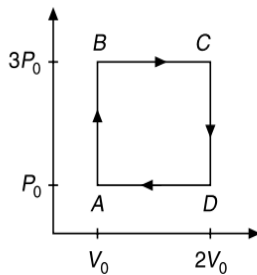
As mass is constant

$$P \propto \rho^\gamma$$

$$\Rightarrow \frac{P_f}{P_i} = \left(\frac{\rho_f}{\rho_i}\right)^\gamma = (32)^{\frac{7}{5}} = 2^7 = 128$$

Hence, the correct answer is (D).

24. Since, $W_{ABCD} = 2P_0V_0$



$$\text{Also, } Q_{\text{in}} = Q_{AB} + Q_{BC}$$

Along path AB, volume is constant, so

$$Q_{AB} = nC_V(T_B - T_A) = n\left(\frac{3R}{2}\right)(T_B - T_A)$$

$$\Rightarrow Q_{AB} = \frac{3}{2}(P_B V_B - P_A V_A)$$

$$\Rightarrow Q_{AB} = \frac{3}{2}(3P_0 V_0 - P_0 V_0) = 3P_0 V_0$$

Along path BC, pressure is constant, so

$$Q_{BC} = nC_P(T_C - T_B) = n\left(\frac{5R}{2}\right)(T_C - T_B)$$

$$\Rightarrow Q_{BC} = \frac{5}{2}(P_C V_C - P_B V_B)$$

$$\Rightarrow Q_{BC} = \frac{5}{2}(6P_0 V_0 - 3P_0 V_0) = \frac{15}{2}P_0 V_0$$

The percentage efficiency is given by

$$\eta = \frac{W}{Q_{\text{in}}} \times 100 = \frac{2P_0 V_0}{3P_0 V_0 + \frac{15}{2}P_0 V_0} \times 100$$

$$\Rightarrow \eta = \frac{400}{21} = 19.04 \approx 19$$

Hence, the correct answer is 19.

25. According to Newton's Law of Cooling, we have the rate of cooling to be proportional to the average excess temperature, so

$$\frac{50 - 40}{300} = \beta \left(\frac{50 + 40}{2} - 20 \right) \quad \dots(1)$$

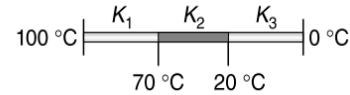
$$\text{Similarly, } \frac{40 - T}{300} = \beta \left(\frac{40 + T}{2} - 20 \right) \quad \dots(2)$$

Solving equations (1) and (2), we get

$$T = \frac{100}{3} \text{ }^\circ\text{C} \approx 33 \text{ }^\circ\text{C}$$

Hence, the correct answer is (A).

26. Rods are identical have same length (l) and area of cross section (A), so for series combination of rods heat current is same, hence



$$\left(\frac{Q}{t}\right)_{AB} = \left(\frac{Q}{t}\right)_{BC} = \left(\frac{Q}{t}\right)_{CD} = \text{Heat current}$$

$$\Rightarrow \frac{(100 - 70)K_1 A}{l} = \frac{(70 - 20)K_2 A}{l} = \frac{(20 - 0)K_3 A}{l}$$

$$\Rightarrow 30K_1 = 50K_2 = 20K_3$$

$$\text{So, } 3K_1 = 2K_3$$

$$\Rightarrow \frac{K_1}{K_3} = \frac{2}{3} = 2:3$$

$$\text{Also, } 5K_2 = 2K_3$$

$$\Rightarrow \frac{K_2}{K_3} = \frac{2}{5} = 2:5$$

Hence, the correct answer is (A).

27. Since, $\eta = \frac{1}{1 + \beta}$

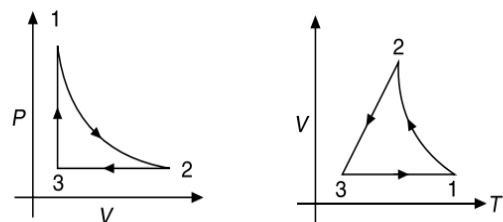
$$\Rightarrow \frac{1}{10} = \frac{1}{1 + \beta}$$

$$\Rightarrow \beta = \frac{Q_2}{W} = 9$$

$$\Rightarrow Q_2 = 9W = 90 \text{ J}$$

Hence, the correct answer is (C).

28. In process 2 to 3 pressure is constant and in process 3 to 1 volume is constant which is correct only in option 4. So, correct graph is



Hence, the correct answer is (D).

29. Since $PV^\gamma = \text{constant}$

$$\Rightarrow T^{\gamma-1} = C$$

$$\Rightarrow 300 \times V^{\frac{7}{5}-1} = T_2 \left(\frac{V}{16}\right)^{\frac{7}{5}-1}$$

$$\Rightarrow T_2 = 300 \times 2^{4 \times \frac{2}{5}}$$

H.154 JEE Advanced Physics: Waves and Thermodynamics

In an isobaric process, we have

$$V = \frac{nRT}{P}$$

$$\Rightarrow V_2 = kT_2 \quad \dots(1)$$

$$\Rightarrow 2V_2 = kT_f \quad \dots(2)$$

$$\Rightarrow T_f = 2 \times 300 \times 2^{\frac{8}{5}} = 1818.85$$

Hence, the correct answer is 1818.15.

30. Since, $T_i = T = 273 + 20 = 293 \text{ K}$

$$\text{If } V_i = V, \text{ then } V_f = \frac{V}{10}$$

For adiabatic process, we have $TV^{\gamma-1} = \text{constant}$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\Rightarrow T V^{\frac{7}{5}-1} = T_2 \left(\frac{V}{10} \right)^{\frac{7}{5}-1}$$

$$\Rightarrow T_2 = T (10)^{\frac{2}{5}} = 293 (10)^{\frac{2}{5}} \text{ K}$$

$$\Rightarrow \Delta U = n C_V \Delta T = 5 \left(\frac{5R}{2} \right) \left(10^{\frac{2}{5}} - 1 \right) T$$

$$\Rightarrow \Delta U = \frac{25(8.314)(293)(2.5-1)}{2} \approx 46 \text{ kJ}$$

Hence, the correct answer is 46.

31. Since, $(C_V)_{\text{mix}} = \frac{n_1 C_{V1} + n_2 C_{V2}}{n_1 + n_2} = \frac{2(3R/2) + 3(3R)}{2+3}$

$$\Rightarrow (C_V)_{\text{mix}} = \frac{12R}{5}$$

$$\Rightarrow (C_P)_{\text{mix}} = (C_V)_{\text{mix}} + R = \frac{12R}{5} + R = \frac{17R}{5}$$

$$\Rightarrow \gamma_{\text{mix}} = \frac{(C_P)_{\text{mix}}}{(C_V)_{\text{mix}}} = \frac{17}{12} \approx 1.42$$

Hence, the correct answer is (B).

32. Since, $\gamma = \alpha_x + \alpha_y + \alpha_z$

$$\Rightarrow \gamma = 5 \times 10^{-5} + 5 \times 10^{-6} + 5 \times 10^{-6}$$

$$\Rightarrow \gamma = (50 + 5 + 5) \times 10^{-6} = 60 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$$

$$\Rightarrow C = 60$$

Hence, the correct answer is 60.

33. Applying Law of Calorimetry, we get

$$540M + M(1)60 = (200)(80) + 200(1)(40 - 0)$$

$$\Rightarrow M = 40 \text{ g}$$

Hence, the correct answer is 40.

34. Since, $(C_V)_{\text{mix}} = \frac{n_1 C_{V1} + n_2 C_{V2}}{n_1 + n_2} = \frac{n(3R/2) + 2n(5R/2)}{n+2n}$

$$\Rightarrow (C_V)_{\text{mix}} = \frac{13R}{6}$$

$$\Rightarrow (C_P)_{\text{mix}} = (C_V)_{\text{mix}} + R = \frac{13R}{6} + R = \frac{19R}{6}$$

$$\Rightarrow \gamma_{\text{mix}} = \frac{(C_P)_{\text{mix}}}{(C_V)_{\text{mix}}} = \frac{19}{13}$$

Hence, the correct answer is (B).

35. According to table and applying law of calorimetry

$$1T_1 + 2T_2 = (1+2)60^\circ = 180 \quad \dots(1)$$

$$1T_2 + 2T_3 = (1+2)30^\circ = 90 \quad \dots(2)$$

$$2T_1 + 1T_3 = (1+2)60 = 180 \quad \dots(3)$$

Adding (1)+(2)+(3)

$$3(T_1 + T_2 + T_3) = 450$$

$$\Rightarrow T_1 + T_2 + T_3 = 150^\circ$$

Also, we have

$$1T_1 + 1T_2 + 1T_3 = (1+1+1)T$$

$$\Rightarrow 150 = 3T$$

$$\Rightarrow T = \theta = 50 \text{ } ^\circ\text{C}$$

Hence, the correct answer is 50.

36. Degree of freedom of a diatomic molecule if vibration is absent is 5

Degree of freedom of a diatomic molecule if vibration is present is 7

$$\Rightarrow \frac{C_V^A}{C_V^B} = \frac{f_A R/2}{f_B R/2} = \frac{f_A}{f_B} = \frac{5}{7}$$

Hence, the correct answer is (B).

37. Since, $\lambda = \frac{1}{\sqrt{2\pi n_0 d^2}}$, where $n_0 = \frac{N}{V}$

$$\Rightarrow \tau = \frac{\lambda}{v_{\text{rms}}} = \frac{1}{\sqrt{2\pi n_0 d^2} v_{\text{rms}}} = \frac{1}{\sqrt{2\pi n_0 d^2}} \sqrt{\frac{M}{3RT}}$$

$$\Rightarrow \frac{\tau_1}{\tau_2} = \sqrt{\frac{M_1 d_2^2}{M_2 d_1^2}}$$

$$\Rightarrow \frac{\tau_1}{\tau_2} = \sqrt{\frac{40 (0.1)^2}{140 (0.07)^2}}$$

$$\Rightarrow \frac{\tau_1}{\tau_2} = 1.09$$

Nearest possible answer is 1.83

Hence, the correct answer is (C).

38. Let amount of water evaporated be m gram.

$$\Rightarrow mL_v = (150 - m)L_s$$

$$\Rightarrow m(2.1 \times 10^6) = (150 - m)(3.36 \times 10^5)$$

$$\Rightarrow m = \frac{150}{7.25} \approx 20 \text{ g}$$

Hence, the correct answer is (C).

39. Between the isothermal and the adiabatic processes, P - V graph for adiabatic is steeper.

Hence, the correct answer is (C).

40. Since, $v \propto \sqrt{T}$

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\Rightarrow \frac{200}{v_2} = \sqrt{\frac{400}{500}}$$

$$\Rightarrow v_2 = 200\sqrt{\frac{5}{4}} = 100\sqrt{5} \text{ ms}^{-1}$$

Hence, the correct answer is (A).

41. Since, $\Delta W_A > \Delta W_B$ and $\Delta U_A = \Delta U_B = U_f - U_i$
So from FLTD, we have

$$\Delta Q = \Delta U + \Delta W$$

$$\Rightarrow \Delta Q_A > \Delta Q_B$$

Hence, the correct answer is (A).

42. Since, $v_{\text{rms}} = \bar{v} = \sqrt{\frac{3RT}{M}}$

$$\Rightarrow \bar{v} = \sqrt{\frac{3RT}{mN_A}} = \sqrt{\frac{3k_B T}{m}}$$

$$\Rightarrow T = \frac{m\bar{v}^2}{3k_B}$$

Hence, the correct answer is (D).

43. According to Stefan's Law, we have

$$mc \left(-\frac{dT}{dt} \right) = e\sigma A (T^4 - T_0^4)$$

$$\Rightarrow -\frac{dT}{dt} = \frac{e\sigma A (T^4 - T_0^4)}{\rho V c} \quad \{ \because m = \rho V \}$$

$$\Rightarrow -\frac{dT}{dt} \propto \frac{1}{\rho c}$$

Since, $\rho_A c_A < \rho_B c_B$

$$\Rightarrow \left(-\frac{dT}{dt} \right)_A > \left(-\frac{dT}{dt} \right)_B$$

So, A cools down at faster rate

Hence, the correct answer is (B).

44. For gas A, $C_p = 29$, $C_v = 22$

For gas B, $C_p = 30$, $C_v = 21$

$$\Rightarrow \gamma_A = \frac{C_{pA}}{C_{vA}} = \frac{29}{22} = 1.31$$

When A has vibrational degree of freedom, then

$$\gamma_A = \frac{9}{7} = 1.29$$

$$\gamma_B = \frac{C_{pB}}{C_{vB}} = \frac{30}{21} = 1.42$$

So, B has no vibrational degree of freedom

Hence, the correct answer is (B).

45. $H = \frac{3KA}{d}(\theta_2 - \theta) = \frac{KA}{3d}(\theta - \theta_1)$

$$\Rightarrow \theta = \frac{9\theta_2 + \theta_1}{10}$$

Hence, the correct answer is (B).

46. For isobaric process, work done $W = nR\Delta T$ and heat given $Q = nC_p\Delta T$, so we have

$$\frac{W}{Q} = \frac{R}{C_p} = \frac{R}{C_v + R}$$

47. Number of moles of He at STP is $n = \frac{67.2}{22.4} = 3$

Since the volume is constant, so process is isochoric and hence, we have

$$Q = nC_v\Delta T = 3 \left(\frac{3R}{2} \right) (20) = 90R = 90(8.31)$$

$$\Rightarrow Q \approx 748 \text{ J}$$

Hence, the correct answer is (D).

48. When $V_1 = V_0$, then $P_1 = P_0 \left(1 - \frac{1}{2} \right) = \frac{P_0}{2}$

$$\text{When } V_2 = 2V_0, \text{ then } P_2 = P_0 \left[1 - \frac{1}{2} \left(\frac{1}{4} \right) \right] = \frac{7P_0}{8}$$

$$\text{Since, } T = \frac{PV}{nR}$$

$$\Rightarrow \Delta T = \left| \frac{P_1 V_1}{nR} - \frac{P_2 V_2}{nR} \right|$$

$$\Rightarrow \Delta T = \left| \left(\frac{P_1 V_1 - P_2 V_2}{nR} \right) \right| = \frac{1}{nR} \left| \frac{P_0 V_0}{2} - \frac{7P_0 V_0}{4} \right|$$

$$\Rightarrow \Delta T = \frac{5P_0 V_0}{4nR}$$

Hence, the correct answer is (B).

49. Heat supplied at constant volume $Q = nC_v\Delta T$ and heat supplied at constant pressure is $Q_1 = nC_p\Delta T$

$$\Rightarrow \frac{Q_1}{Q} = \frac{C_p}{C_v}$$

$$\Rightarrow Q_1 = \gamma Q = \frac{7Q}{5}$$

Hence, the correct answer is (C).

50. For the process ca, $\Delta U_{ca} = -180 \text{ J}$

For process bc, isochoric $W_{bc} = 0$

$$\Rightarrow \Delta U = 60 \text{ J}$$

Heat absorbed along ab is $Q_{ab} = 250 \text{ J}$

Since $\Delta U_{\text{cycle}} = 0$, so $\Delta U_{ab} = 120 \text{ J}$

So by FLTD, $W_{a \rightarrow b} = 130 \text{ J}$

Total work done from (a \rightarrow b \rightarrow c)

$$W = W_{ab} + W_{bc} = 130 \text{ J}$$

Hence, the correct answer is (A).

51. Applying law of calorimetry, we get

$$5M_1 + M_1 L = 50M_2$$

H.156 JEE Advanced Physics: Waves and Thermodynamics

$$\Rightarrow L = \frac{50M_2}{M_1} - 5$$

Hence, the correct answer is (A).

52. Since, $(2+3)C_V = 2\left(\frac{3R}{2}\right) + 3\left(\frac{5R}{2}\right)$

$$\Rightarrow C_V = \frac{21R}{10}$$

Hence, the correct answer is (D).

53. Since, $\eta = \frac{1}{6} = 1 - \frac{T_C}{T_H}$

$$\Rightarrow \frac{T_C}{T_H} = \frac{5}{6} \quad \dots(1)$$

Now $\eta' = 2\eta = \frac{1}{3} = 1 - \frac{(T_C - 62)}{T_H}$

$$\Rightarrow \frac{T_C - 62}{T_H} = \frac{2}{3} \quad \dots(2)$$

Substituting equation (1) in (2), we get

$$\frac{T_C - 62}{T_C} = \frac{2 \times 6}{3 \times 5} = \frac{4}{5}$$

$$\Rightarrow T_C = 310 \text{ K} = 37^\circ \text{C} \text{ and } T_H = 372 \text{ K} = 99^\circ \text{C}$$

Hence, the correct answer is (A).

54. Since, $W_{\text{isob}} = nR\Delta T = 10 \text{ J}$

and $Q = nC_P\Delta T = n\left(\frac{7R}{2}\right)\Delta T = \frac{7}{2}(nR\Delta T)$

$$\Rightarrow \Delta Q = \frac{7}{2}(10) = 35 \text{ J}$$

Hence, the correct answer is (B).

55. Since, $v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = 11.2 \times 10^3 \text{ ms}^{-1}$

$$\Rightarrow T = \frac{M}{3R} \times (11.2 \times 10^3)^2 = \frac{2 \times 10^{-3}}{3 \times 8.3} \times 125.44 \times 10^6$$

$$\Rightarrow T \approx 10^4 \text{ K}$$

Hence, the correct answer is (A).

56. Given that $V = 25 \times 10^{-3} \text{ m}^3$, $n = 1$ mole of O_2

$$T = 300 \text{ K}, v_{\text{rms}} = 200 \text{ ms}^{-1}, d = 0.3 \text{ nm}$$

Since, $\lambda = \frac{1}{\sqrt{2\pi n_0 d^2}}$, where $n_0 = \frac{N}{V} = \frac{N_A}{V}$

$$\Rightarrow \tau = \frac{\lambda}{v_{\text{rms}}} = \frac{1}{\sqrt{2\pi n_0 d^2} v_{\text{rms}}}$$

The average collision rate (per second) is

$$\frac{1}{\tau} = \frac{v_{\text{rms}}}{\lambda} = \sqrt{2\pi} \left(\frac{N_A}{V}\right) d^2 v_{\text{rms}}$$

$$\Rightarrow \frac{1}{\tau} = \sqrt{2} (3.14) \left(\frac{6.023 \times 10^{23}}{25 \times 10^{-3}}\right) (0.09 \times 10^{-18}) (200)$$

Average number of collisions $\approx 10^{10}$

Hence, the correct answer is (B).

57. Since, $n = n_0 e^{-\alpha r^4}$

$$\Rightarrow N = \int dN = \int_0^\infty (n_0 e^{-\alpha r^4}) 4\pi r^2 dr$$

$$\Rightarrow N = 4\pi n_0 \int_0^\infty r^2 e^{-\alpha r^4} dr$$

Substitute $\sqrt{\alpha} r^2 = z$, we get

$$\sqrt{\alpha} (2r dr) = dz$$

$$\Rightarrow N = \frac{4\pi n_0}{2\sqrt{\alpha}} \int_0^\infty \frac{z^{1/2} e^{-z^2}}{\alpha^{1/4}} dz = \frac{4\pi n_0}{2\alpha^{3/4}} \int_0^\infty z^{1/2} e^{-z^2} dz$$

$$\Rightarrow N \propto n_0 \alpha^{-\frac{3}{4}}$$

Hence, the correct answer is (A).

58. For path ACB, applying FLTD, we get

$$Q_{ACB} = W_{ACB} + U_{ACB}$$

$$\Rightarrow 60 = 30 + U_{ACB}$$

$$\Rightarrow U_{ACB} = U_{ADB} = 30 \text{ J}$$

Similarly, for path ADB, we get from FLTD

$$Q_{ADB} = W_{ADB} + U_{ADB} = (10 + 30) \text{ J}$$

$$\Rightarrow Q_{ADB} = 40 \text{ J}$$

Hence, the correct answer is (D).

59. Applying Kirchhoff's Junction Law (KJL) for heat currents at P and Q respectively, we get

At P, $\frac{T_A - T_P}{L/2} = \frac{T_P - T_Q}{L} + \frac{T_P - T_Q}{3L/2}$

$$\Rightarrow 2(T_A - T_P) = \frac{5}{3}(T_P - T_Q) \quad \dots(1)$$

At Q, $\frac{T_P - T_Q}{L} + \frac{T_P - T_Q}{3L/2} = \frac{T_Q - T_B}{L/2}$

$$\Rightarrow 2(T_Q - T_B) = \frac{5}{3}(T_P - T_Q) \quad \dots(2)$$

From equations (1) and (2), we get

$$2(T_A - T_B) = 2(T_P - T_Q) + \frac{10}{3}(T_P - T_Q)$$

$$\Rightarrow 2 \times 120 = \frac{16}{3}(T_P - T_Q)$$

$$\Rightarrow T_P - T_Q = \frac{2 \times 120 \times 3}{16} = 45^\circ \text{C}$$

Hence, the correct answer is (B).

60. $\frac{(v_{\text{rms}})_{\text{He}}}{(v_{\text{rms}})_{\text{Ar}}} = \sqrt{\frac{M_{\text{Ar}}}{M_{\text{He}}}} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3.16$

Hence, the correct answer is (C).

61. For no change in length of the rod, we have

$$\Delta L_{\text{Thermal}} - \Delta L_{\text{mechanical}} = 0$$

$$\Rightarrow L_0 \alpha \Delta T = \frac{FL_0}{AY}$$

$$\Rightarrow Y = \frac{F}{A\alpha\Delta T}$$

Hence, the correct answer is (B).

62. Since, $\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{Q_3}{T_3}$

Also, $W_1 = Q_1 - Q_2$ and $W_2 = Q_2 - Q_3$

Given $W_1 = W_2$

$$\Rightarrow Q_1 - Q_2 = Q_2 - Q_3$$

$$\Rightarrow 2Q_2 = Q_1 + Q_3$$

$$\Rightarrow 2T_2 = T_1 + T_3$$

$$\Rightarrow T_2 = \frac{T_1 + T_3}{2} = 500 \text{ K}$$

Hence, the correct answer is (D).

63. Since, $\frac{v_2}{v_1} = 2 = \sqrt{\frac{T_2}{T_1}}$

$$\Rightarrow T_2 = 1200 \text{ K}$$

For a closed vessel, volume is constant, so

$$Q = nC_V(T_2 - T_1) = \left(\frac{15}{28}\right)\left(\frac{5R}{2}\right)(900) = 10 \text{ kJ}$$

Hence, the correct answer is (C).

64. Since, $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_3}$

$$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} \quad \dots(1)$$

$$\Rightarrow T_2 = \sqrt{T_1 T_3} \text{ and } T_3 = \sqrt{T_2 T_4}$$

Substituting value of T_2 in equation (1), we get

$$\Rightarrow \frac{\sqrt{T_1 T_3}}{T_1} = \frac{T_4}{T_3}$$

$$\Rightarrow \frac{T_1 T_3}{T_1^2} = \frac{T_4^2}{T_3^2}$$

$$\Rightarrow T_3^3 = T_1 T_4^2$$

$$\Rightarrow T_3 = (T_1 T_4^2)^{1/3}$$

Similarly, we get $T_2 = (T_1^2 T_4)^{1/3}$

Hence, the correct answer is (D).

65. Since, $W_{\text{isob}} = P\Delta V = nR\Delta T$

$$\Rightarrow W_{\text{isob}} = \frac{1}{2}(8.31)(70) \approx 291 \text{ J}$$

Hence, the correct answer is (A).

66. Applying law of calorimetry, we get

$$192c(100 - 21.5) = (128)(0.394)(21.5 - 8.4) +$$

$$(240)(4.18)(21.5 - 8.4)$$

$$\Rightarrow c = \frac{660.65 + 13142}{15072} \approx 916 \text{ Jkg}^{-1}\text{K}^{-1}$$

Hence, the correct answer is (A).

67. Since, $V = \frac{m}{\rho} = \frac{2}{8} = \frac{1}{4} \text{ m}^3$

From ideal gas equation, we have $PV = nRT$

$$\Rightarrow 4 \times 10^4 \times \frac{1}{4} = nRT$$

$$\Rightarrow nRT = 10^4$$

$$\text{Internal Energy } U = \frac{3}{2}nRT = \frac{3}{2} \times 10^4 = 1.5 \times 10^4 \text{ J}$$

So internal energy is of order of 10^4 J

Hence, the correct answer is (C).

68. $\frac{Q}{A} = \frac{K(T_1 - T_2)}{L} = \frac{0.1 \times 900}{1} = 90 \text{ Wm}^{-2}$

Hence, the correct answer is (D).

69. $U = 3\left(\frac{5R}{2}\right)T + 5\left(\frac{3R}{2}\right)T$

$$\Rightarrow U = 15RT$$

Hence, the correct answer is (C).

70. For adiabatic process, $TV^{\gamma-1} = \text{constant}$

$$\Rightarrow x = \gamma - 1$$

$$\Rightarrow x = \frac{2}{5}$$

Hence, the correct answer is (A).

71. Let m gram of ice be added, then heat lost by water is

$$Q_{\text{lost}} = 50(1)(40 - 0) = 2000 \text{ cal}$$

Similarly, heat gained by ice is

$$Q_{\text{gained}} = m(0.5)(20) + (m - 20)(80)$$

By law of calorimetry, we have

$$Q_{\text{lost}} = Q_{\text{gained}}$$

$$\Rightarrow m(0.5)(20) + (m - 20)(80) = 2000$$

$$\Rightarrow 90m = 3600$$

$$\Rightarrow m = 40 \text{ g}$$

Hence, the correct answer is (B).

72. Let final temperature be T . Since, heat lost equals the heat gained, so we have

$$(0.1)(500 - T)(400) = (800 + 0.5 \times 4200)(T - 30)$$

$$\Rightarrow (500 - T) = \frac{2900}{40}(T - 30)$$

$$\Rightarrow T = 36.39 \text{ }^\circ\text{C}$$

$$\% \text{ increase} = \frac{36.39 - 30}{30} \approx 20\%$$

Hence, the correct answer is (B).

73. Since, $100c_1(100 - 90) = 50c_2(90 - 75)$

$$\Rightarrow 20S_1 = 15S_2$$

$$\Rightarrow 4S_1 = 3S_2$$

Let final temperature be T , then we have

$$100c_1(100 - T) = 50c_2(T - 50)$$

$$\Rightarrow 75c_2(100 - T) = 50c_2(T - 50)$$

$$\Rightarrow 3(100 - T) = 2T - 100$$

H.158 JEE Advanced Physics: Waves and Thermodynamics

$$\Rightarrow T = 80 \text{ }^\circ\text{C}$$

Hence, the correct answer is (B).

74. Since, $\Delta l = \ell \alpha \Delta T$

$$\Rightarrow \alpha_A (180 - 30) = \alpha_B (T - 30)$$

$$\Rightarrow 4(180 - 30) = 3(T - 30)$$

$$\Rightarrow T = 230 \text{ }^\circ\text{C}$$

Hence, the correct answer is (B).

75. Since, for a polytropic process, $TV^{x-1} = \text{constant}$

$$\Rightarrow x - 1 = 1$$

$$\Rightarrow x = 2$$

$$\Rightarrow C = C_V + \frac{R}{1-x} = \frac{3R}{2} + \frac{R}{1-2} = \frac{R}{2}$$

$$\text{So, } Q = nC\Delta T = \frac{R}{2}\Delta T$$

Hence, the correct answer is (B).

76. Since, $\frac{(x_0/2) - (x_0/3)}{x_0 - (x_0/3)} = \frac{T - 0}{100}$

$$\Rightarrow \frac{x_0/6}{2x_0/3} = \frac{T}{100}$$

$$\Rightarrow \frac{T}{100} = \frac{1}{4}$$

$$\Rightarrow T = 25 \text{ }^\circ\text{C}$$

Hence, the correct answer is (D).

77. Work done, $W = \text{Area under } PV \text{ graph}$

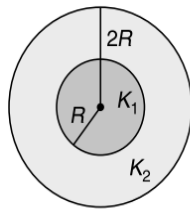
$$\Rightarrow W = \frac{1}{2} \times 4 \times 5 = 10 \text{ J}$$

Hence, the correct answer is (B).

78. Since $K_{\text{eq}} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$

$$\Rightarrow K_{\text{eq}} = \frac{K_1 \pi R^2 + K_2 3\pi R^2}{4\pi R^2}$$

$$\Rightarrow K_{\text{eq}} = \frac{K_1 + 3K_2}{4}$$



Hence, the correct answer is (B).

79. Since $U = \frac{3}{2}nRT$, for a monatomic gas

$$\Rightarrow U = \frac{3}{2}PV = \frac{3}{2} \times 3 \times 10^6 \times 2$$

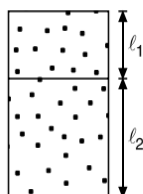
$$\Rightarrow U = 9 \times 10^6 \text{ J}$$

Hence, the correct answer is (B).

80. Since, $(P_2 - P_1)A = mg$

$$\Rightarrow \left(\frac{nRT}{Al_2} - \frac{nRT}{Al_1} \right) A = mg$$

$$\Rightarrow m = \frac{nRT}{g} \left(\frac{\ell_1 - \ell_2}{\ell_1 \ell_2} \right)$$



Hence, the correct answer is (A).

81. Average collision time

$$\tau = \frac{\text{Mean Free Path}}{\text{Average Speed}} = \frac{\lambda}{v_{av}}$$

The mean free path of molecules of an ideal gas is

$$\lambda = \frac{1}{\sqrt{2}n_0 d^2} = \frac{V}{\sqrt{2}\pi N d^2} = \frac{RT}{\sqrt{2}\pi N P d^2}$$

where, V is the volume of container, N is number of molecules and d is the diameter of the molecule.

$$\tau = \frac{\lambda}{v_{av}} = \frac{RT}{\sqrt{2}\pi N P d^2 v_{av}}$$

Since, average speed $v_{av} \propto \sqrt{T}$

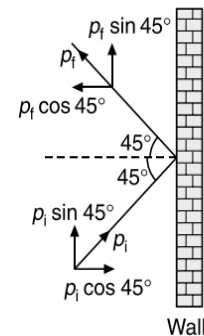
$$\Rightarrow \tau \propto \frac{\sqrt{T}}{P}$$

$$\Rightarrow \frac{\tau_1}{\tau_2} = \frac{\sqrt{5}}{3} \times \frac{1}{2} \times 6 \times 10^{-8} = 3.87 \times 10^{-8} \text{ s}$$

$$\Rightarrow \frac{\tau_1}{\tau_2} = 4 \times 10^{-8} \text{ s}$$

Hence, the correct answer is (D).

82. As $p_i = p_f$



Net force on the wall,

$$F = \frac{dp}{dt} = 2np_f \cos 45^\circ = 2nmv \cos 45^\circ$$

Here, n is the number of hydrogen molecules striking per second.

$$\text{Pressure } P = \frac{F}{A} = \frac{2nmv \cos 45^\circ}{\text{Area}}$$

$$\Rightarrow P = \frac{2 \times 10^{23} \times 3.32 \times 10^{-27} \times 10^3 \times \left(\frac{1}{\sqrt{2}}\right)}{2 \times 10^{-4}}$$

$$\Rightarrow P = 2.35 \times 10^3 \text{ Nm}^{-2}$$

Hence, the correct answer is (A).

83. Since, $\frac{3}{2}k_B T = \frac{1}{2}mv^2$

$$\Rightarrow v = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3 \times 1.4 \times 10^{-23} \times 300}{7 \times 10^{-27}}}$$

$$\Rightarrow v = \sqrt{1.8} \times 10^3 \text{ ms}^{-1} \approx 1.3 \times 10^3 \text{ ms}^{-1}$$

Hence, the correct answer is (A).

84. Given $\gamma_{\text{mix}} = \frac{3}{2}$, so $(C_V)_{\text{mix}} = \frac{R}{\gamma_{\text{mix}} - 1} = 2R$

$$\Rightarrow (C_V)_{\text{mix}} = \frac{n_1 C_{V1} + n_2 C_{V2}}{n_1 + n_2}$$

$$\Rightarrow 2R = \frac{2(3R/2) + n(5R/2)}{2 + n}$$

$$\Rightarrow 8 + 4n = 6 + 5n$$

$$\Rightarrow n = 2$$

Hence, the correct answer is (C).

85. For an adiabatic process $TV^{\gamma-1} = \text{constant}$

$$\Rightarrow T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

Here, $T_1 = 27^\circ\text{C} = 300\text{ K}$, $V_1 = V$, $V_2 = 2V$, $\gamma = \frac{5}{3}$

$$\Rightarrow T_2 = 300 \left(\frac{V}{2V} \right)^{\left(\frac{5}{3}-1\right)} = 300 \left(\frac{1}{2} \right)^{\frac{2}{3}} \approx 189\text{ K}$$

Change in internal energy, $\Delta U = nC_V \Delta T$

$$\Rightarrow \Delta U = n \left(\frac{fR}{2} \right) (T_2 - T_1) = 2 \times \frac{3}{2} \times \frac{25}{3} (189 - 300)$$

$$\Rightarrow \Delta U = -25 \times 111 = -2775\text{ J} = -2.7\text{ kJ}$$

Hence, the correct answer is (C).

86. Work done on gas $W_{\text{isot}} = nRT \ln \left(\frac{P_f}{P_i} \right)$

$$\Rightarrow W_{\text{isot}} = R(300) \ell n(2) = 300R \ell n 2$$

Hence, the correct answer is (B).

87. For a refrigerator, $\frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$

$$\Rightarrow \frac{500}{W} = \frac{250}{50}$$

$$\Rightarrow W = 100\text{ cal} = (100)(4.2)\text{ J} = 420\text{ J}$$

Hence, the correct answer is (D).

88. Assuming equal works to be done by both the engines (should have been mentioned in the question for solving it), we have

$$T = \frac{T_1 + T_2}{2} = \frac{600 + 100}{2} = 350\text{ K}$$

$$\Rightarrow \frac{\eta_A}{\eta_B} = \frac{1 - \frac{350}{600}}{1 - \frac{100}{350}} = \frac{250/600}{250/350} = \frac{350}{600} = \frac{7}{12}$$

Hence, the correct answer is (C).

89. Equation of line BC is given by

$$P = P_0 - \frac{2P_0}{V_0}(V - 2V_0)$$

$$\Rightarrow PV = P_0V - \frac{2P_0}{V_0}(V - 2V_0)V$$

$$\Rightarrow T = \frac{P_0V - \frac{2P_0V^2}{V_0} + 4P_0V}{1 \times R} \quad \{\because PV = nRT\}$$

$$\Rightarrow T = \frac{P_0}{R} \left(5V - \frac{2V^2}{V_0} \right)$$

For maximum value of T , $\frac{dT}{dV} = 0$

$$\Rightarrow 5 - \frac{4V}{V_0} = 0$$

$$\Rightarrow V = \frac{5}{4}V_0$$

$$\Rightarrow T_{\text{max}} = \frac{P_0}{R} \left[5 \left(\frac{5V_0}{4} \right) - \frac{2}{V_0} \left(\frac{25}{16} V_0^2 \right) \right] = \frac{25}{8} \frac{P_0 V_0}{R}$$

Hence, the correct answer is (B).

90. Let molar heat capacity at constant pressure be X_p and molar heat capacity at constant volume be X_v

$$X_p - X_v = R$$

$$\Rightarrow MC_p - MC_v = R$$

$$\Rightarrow C_p - C_v = \frac{R}{M}$$

For hydrogen, we have $a = \frac{R}{2}$

For N_2 , we have $b = \frac{R}{28}$

$$\Rightarrow \frac{a}{b} = 14$$

$$\Rightarrow a = 14b$$

Hence, the correct answer is (C).

91. An ideal gas has molecules with 5 degrees of freedom, then

$$C_v = \frac{5}{2}R \text{ and } C_p = \frac{7}{2}R$$

$$\Rightarrow \frac{C_p}{C_v} = \frac{7R/2}{5R/2} = \frac{7}{5}$$

Hence, the correct answer is (D).

92. Initial kinetic energy of the system $K_i = \frac{5}{2}RTN$

Final kinetic energy of the system

$$K_f = \frac{5}{2}RT(N - n) + \frac{3}{2}RT(2n)$$

$$\Rightarrow \Delta K = K_f - K_i = nRT \left(3 - \frac{5}{2} \right) = \frac{1}{2}nRT$$

Hence, the correct answer is (C).

93. Let n_1 be initial number of moles, so

$$n_1 = \frac{P_1 V_1}{RT_1} = \frac{10^5 \times 30}{8.3 \times 290} \approx 1.24 \times 10^3$$

Let n_2 be final number of moles, so

$$n_2 = \frac{P_2 V_2}{RT_2} = \frac{10^5 \times 30}{8.3 \times 300} \approx 1.20 \times 10^3$$

H.160 JEE Advanced Physics: Waves and Thermodynamics

Change of number of molecules is

$$\Delta n = n_f - n_i = (n_2 - n_1) \times 6.023 \times 10^{23}$$

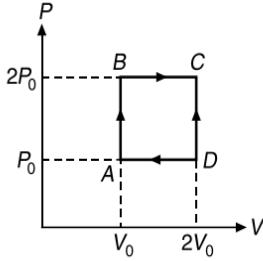
$$\Rightarrow \Delta n \approx -2.5 \times 10^{25}$$

Hence, the correct answer is (D).

94. Work done by engine equals the area enclosed by the PV diagram, so $W = P_0 V_0$

Heat given to the system is

$$Q = Q_{AB} + Q_{BC} = nC_V \Delta T_{AB} + nC_P \Delta T_{BC}$$



$$\Rightarrow Q = \frac{3}{2}(nRT_B - nRT_A) + \frac{5}{2}(nRT_C - nRT_B)$$

$$\Rightarrow Q = \frac{3}{2}(2P_0 V_0 - P_0 V_0) + \frac{5}{2}(4P_0 V_0 - 2P_0 V_0)$$

$$\Rightarrow Q = \frac{13}{2} P_0 V_0$$

$$\text{Thermal efficiency, } \eta = \frac{W}{\Sigma Q_{\oplus}} = \frac{W}{Q_{\text{input}}} = \frac{P_0 V_0}{13 P_0 V_0 / 2}$$

$$\Rightarrow \eta = \frac{2}{13} \approx 0.15$$

Hence, the correct answer is (A).

95. Here, $PV = \text{constant}$, so given process is isothermal i.e., temperature is constant. Pressure at point 1 is higher than that at point 2. So, correct OPTION is (C).

Hence, the correct answer is (C).

96. Ideal gas equation, $PV = nRT$

As temperature is constant.

$$PV = \text{constant}$$

$$\Rightarrow P \left(\frac{m}{\rho} \right) = \text{constant}$$

$$\Rightarrow P \propto \rho \text{ (for given } m)$$

Hence, the correct answer is (D).

97. Specific heat of a polytropic process is

$$C = C_V + \frac{R}{1-n}$$

$$\Rightarrow \frac{R}{1-n} + C_V = C$$

$$\Rightarrow \frac{R}{1-n} = C - C_V$$

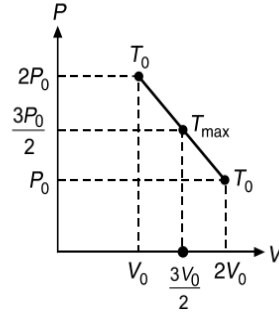
$$\Rightarrow \frac{R}{C - C_V} = 1 - n$$

Since, $R = C_P - C_V$

$$\Rightarrow n = \frac{C - C_P}{C - C_V}$$

Hence, the correct answer is (C).

98. Method-I



Since initial and final temperature are equal hence maximum temperature is at middle of line.

$$PV = nRT$$

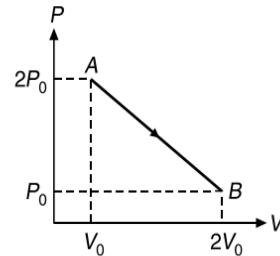
$$\Rightarrow T_{\text{max}} = \frac{(3P_0/2)(3V_0/2)}{nR} = \frac{9P_0 V_0}{4nR}$$

Method-II

Equation of line

$$P = -\left(\frac{P_0}{V_0}\right)V + 3P_0$$

$$\Rightarrow PV = -\left(\frac{P_0}{V_0}\right)V^2 + 3P_0 V = x \text{ (say)}$$



For temperature to be maximum, the product $PV = x$ should

also be the maximum, so we have $\frac{dx}{dV} = 0$.

$$\Rightarrow V = \frac{3V_0}{2} \text{ and } P = \frac{3P_0}{2}$$

$$\Rightarrow T = \frac{PV}{nR} = \frac{9}{4} \frac{P_0 V_0}{nR}$$

Hence, the correct answer is (B).

99. For an ideal gas in an isobaric process,

Heat supplied, $Q = nC_P \Delta T$

Work done, $W = P \Delta V = nR \Delta T$

$$\Rightarrow \frac{W}{Q} = \frac{nR \Delta T}{nC_P \Delta T} = \frac{R}{5R/2} = \frac{2}{5}$$

Hence, the correct answer is (A).

100. For isochoric process, $\Delta U = Q = mc \Delta T$

where, $m = 200 \text{ g} = 0.2 \text{ kg}$, $c = 4184 \text{ J kg}^{-1} \text{ K}^{-1}$

$$\Delta T = 60 - 40 = 20 \text{ }^\circ\text{C} = 20 \text{ K}$$

$$\Rightarrow \Delta U = 0.2 \times 4184 \times 20 = 16736 \text{ J} = 16.7 \text{ kJ}$$

Hence, the correct answer is (D).

101. For a refrigerator, we have

$$\frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$

So, energy consumed by the freezer is given by

$$W = Q_2 \left(\frac{T_1}{T_2} - 1 \right)$$

where, $T_1 = 27 \text{ }^\circ\text{C} = 300 \text{ K}$, $T_2 = 0 \text{ }^\circ\text{C} = 273 \text{ K}$ and

$$Q_2 = mL = 5 \times 336 \times 10^3 \text{ J}$$

$$\Rightarrow W = 5 \times 336 \times 10^3 \left(\frac{300}{273} - 1 \right) = 1.67 \times 10^5 \text{ J}$$

Hence, the correct answer is (D).

102. Since, average time (τ) is given by

$$\tau = \frac{1}{\sqrt{2} \pi n_0 d^2 v_{\text{rms}}}, \text{ where } n_0 = \frac{N}{V}$$

Now, since $n_0 \propto \frac{1}{V}$ and $v_{\text{rms}} \propto \sqrt{T}$

$$\Rightarrow \tau \propto \frac{V}{\sqrt{T}} \quad \dots(1)$$

Now, for an adiabatic process,

$$TV^{\gamma-1} = \text{constant} \quad \dots(2)$$

So, from (1) and (2), we get

$$\tau \propto V^{\frac{\gamma+1}{2}}$$

Hence, the correct answer is (C).

103. Average force applied on the walls by a molecule,

$$F = \frac{2mv}{t}$$

$$\Rightarrow t = \frac{2\ell}{v}$$

$$\Rightarrow t \propto \frac{\ell}{v}$$

$$\Rightarrow F \propto v^2 \quad \dots(1)$$

$$\text{Since, } v^2 \propto T \quad \dots(2)$$

From (1) and (2), we get

$$F \propto T$$

Hence, the correct answer is (B).

104. For metals, there is no free motion but rather oscillation about mean position. Thus, these have kinetic energy and potential energy, which are almost equal.

$$(\text{P.E.})_{\text{avg}} = (\text{K.E.})_{\text{avg}} = \frac{3}{2} k_B T$$

Energy per molecule is $E = \text{K.E.} + \text{P.E.} = 3k_B T$

Energy per mole is $E = 3RT$

Also, $E = MCT$

$$\Rightarrow 3RT = MCT$$

$$\Rightarrow C = \frac{3k_B}{m} = \frac{3R}{M}$$

$$\Rightarrow C = \frac{3 \times 8.314}{27 \times 10^{-3}} \approx 925 \text{ Jkg}^{-1} \text{ K}^{-1}$$

Hence, the correct answer is (C).

105. Since, entropy is a state function, so it does not depend upon the path and only depends on the initial and final state of the system. So, we have

$$dS = \frac{dQ}{T} = mc \left(\frac{dT}{T} \right)$$

$$\Rightarrow \Delta S = \int ds = mc \int_{T_i}^{T_f} \frac{dT}{T}$$

$$\Rightarrow \Delta S = mc \log_e \left(\frac{T_f}{T_i} \right)$$

Now, $mc = 1 \text{ J}^\circ\text{C}^{-1}$

$$\Rightarrow \Delta S = \log_e \left(\frac{527 + 273}{127 + 273} \right) = \log_e (2)$$

Hence, the correct answer is (B).

106. Since, $p = \frac{1}{3} \left(\frac{U}{V} \right)$ and $pV = nRT$

$$\Rightarrow \frac{nRT}{V} \propto \frac{T^4}{3} \quad \left\{ \because \frac{U}{V} \propto T^4 \right\}$$

$$\Rightarrow VT^3 = \text{constant}$$

$$\Rightarrow \left(\frac{4}{3} \pi R^3 \right) T^3 = \text{constant}$$

$$\Rightarrow TR = \text{constant}$$

$$\Rightarrow T \propto \frac{1}{R}$$

Hence, the correct answer is (C).

107. For a cyclic process, $\Delta U = 0$

So, OPTION (A) is incorrect.

$$\text{For process CA, } \Delta U = (1) \left(\frac{5R}{2} \right) (-200)$$

$$\Rightarrow \Delta U_{CA} = -500R$$

So, OPTION (B) is incorrect.

$$\text{For process AB, } \Delta U = (1) \left(\frac{5R}{2} \right) (400)$$

$$\Rightarrow \Delta U_{AB} = 1000R$$

So, OPTION (C) is incorrect.

$$\text{For process BC, } \Delta U = (1) \left(\frac{5R}{2} \right) (-200)$$

$$\Rightarrow \Delta U_{BC} = -500R$$

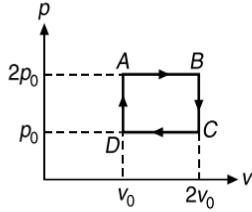
So, OPTION (D) is correct.

Hence, the correct answer is (D).

108. Heat is extracted from source in AB and DA

$$\text{In AB, } Q_{AB} = nC_p \Delta T = n \left(\frac{5R}{2} \right) \Delta T$$

$$\Rightarrow Q_{AB} = \frac{5}{2} (4p_0v_0 - 2p_0v_0) = 5p_0v_0$$



In DA, $Q_{DA} = nC_V\Delta T = n\left(\frac{3R}{2}\right)\Delta T = \frac{3}{2}p_0v_0$

Total heat extracted from source i.e., absorbed is

$$Q = Q_{AB} + Q_{DA} = \frac{13}{2}p_0v_0$$

Hence, the correct answer is (B).

109. Efficiency of Carnot engine, $\eta = 1 - \frac{T_2}{T_1}$

where T_1 is the temperature of the source and T_2 is the temperature of the sink?

For 1st case $\eta = 40\%$, $T_1 = 500$ K

$$\Rightarrow \frac{40}{100} = 1 - \frac{T_2}{500}$$

$$\Rightarrow \frac{T_2}{500} = 1 - \frac{40}{100} = \frac{3}{5}$$

$$T_2 = \frac{3}{5} \times 500 = 300 \text{ K}$$

For 2nd case $\eta = 60\%$, $T_2 = 300$ K

$$\Rightarrow \frac{60}{100} = 1 - \frac{300}{T_1}$$

$$\Rightarrow \frac{300}{T_1} = 1 - \frac{60}{100} = \frac{2}{5}$$

$$T_1 = \frac{5}{2} \times 300 = 750 \text{ K}$$

Hence, the correct answer is (B).

110. In case of a cyclic process, work done is equal to the area under the cycle and is taken to be positive if the cycle is clockwise, so work done is

$$W = \text{Area of Rectangle } ABCD = P_0V_0$$

Since, helium gas is a monatomic gas, so

$$C_V = \frac{3R}{2} \text{ and } C_P = \frac{5R}{2}$$

Along the path AB, heat is supplied to the gas at constant volume, so

$$\Delta Q_{AB} = nC_V\Delta T = n\left(\frac{3R}{2}\right)\Delta T = \frac{3}{2}P_0V_0$$

Along the path BC, heat is supplied to the gas at constant pressure, so

$$\Delta Q_{BC} = nC_P\Delta T = n\left(\frac{5R}{2}\right)\Delta T = \frac{5}{2}(2P_0V_0) = 5P_0V_0$$

Along the path CD and DA, heat is rejected by the gas, so efficiency of the cycle is

$$\eta = \frac{\text{Work done by the gas}}{\text{Heat supplied to the gas}} \times 100\%$$

$$\Rightarrow \eta = \frac{P_0V_0}{\frac{3P_0V_0}{2} + 5P_0V_0} \times 100 = \frac{200}{13} = 15.4\%$$

Hence, the correct answer is (D).

111. Kinetic energy of vessel $K = \frac{1}{2}mv^2$

$$\text{Increase in internal energy is } \Delta U = nC_V\Delta T$$

where, n is the number of moles of the gas in vessel. When the vessel is stopped suddenly, its kinetic energy is used to increase temperature of the gas, so

$$\frac{1}{2}mv^2 = \Delta U = nC_V\Delta T = \frac{m}{M}C_V\Delta T \quad \left\{ \because n = \frac{m}{M} \right\}$$

$$\Rightarrow \Delta T = \frac{Mv^2}{2C_V}$$

$$\Rightarrow \Delta T = \frac{Mv^2(\gamma-1)}{2R} \quad \left\{ \because C_V = \frac{R}{(\gamma-1)} \right\}$$

Hence, the correct answer is (D).

112. The efficiency of Carnot engine, $\eta = 1 - \frac{T_2}{T_1}$

$$\Rightarrow \frac{1}{6} = 1 - \frac{T_2}{T_1} \quad \left(\text{Given, } \eta = \frac{1}{6} \right)$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{5}{6}$$

$$\Rightarrow T_1 = \frac{6T_2}{5} \quad \dots(1)$$

As per question, when T_2 is lowered by 62 K, then its efficiency becomes $2\eta = 2\left(\frac{1}{6}\right) = \frac{1}{3}$

$$\Rightarrow \frac{1}{3} = 1 - \frac{T_2 - 62}{T_1}$$

$$\Rightarrow \frac{T_2 - 62}{T_1} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\Rightarrow \frac{T_2 - 62}{6T_2/5} = \frac{2}{3} \quad \left\{ \text{using equation (1)} \right\}$$

$$\Rightarrow \frac{5(T_2 - 62)}{6T_2} = \frac{2}{3}$$

$$\Rightarrow 5T_2 - 310 = 4T_2$$

$$\Rightarrow T_1 = \frac{6 \times 310}{5} = 372 \text{ K}$$

Hence, the correct answer is (A).

113. Since $Q = mc\Delta T$

$$\text{where, } m = 100 \text{ g} = 100 \times 10^{-3} \text{ kg}$$

$$c = 4184 \text{ Jkg}^{-1}\text{K}^{-1} \text{ and } \Delta T = (50 - 30) = 20 \text{ }^\circ\text{C}$$

$$\Rightarrow Q = 100 \times 10^{-3} \times 4184 \times 20 = 8.4 \times 10^3 \text{ J}$$

By FLTD, we have $Q = \Delta U + W$

Since we are ignoring the slight expansion of water, so we have $W \approx 0$. Hence change in internal energy is

$$U = Q = 8.4 \times 10^3 \text{ J} = 8.4 \text{ kJ}$$

Hence, the correct answer is (B).

114. For an adiabatic process $TV^{\gamma-1} = \text{constant}$

$$\Rightarrow T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}, \text{ where } \gamma = \frac{7}{5}$$

$$\Rightarrow T_1 = T_2 \left(\frac{V_2}{V_1} \right)^{\frac{2}{5}} = T_2 \left(\frac{32V}{V} \right)^{\frac{2}{5}} = T_2 (2^5)^{\frac{2}{5}} = 4T_2$$

$$\text{Efficiency of the engine, } \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{1}{4}$$

$$\Rightarrow \eta = \frac{3}{4} = 0.75$$

Hence, the correct answer is (C).

115. For path AB, pressure is constant and $\Delta T = 200 \text{ K}$

$$\Rightarrow W_{\text{isob}} = P\Delta V = nR\Delta T = 2R(200) = 400R$$

Hence, the correct answer is (C).

116. For path DA, temperature is constant

$$\Rightarrow W_{\text{isot}} = nRT_D \ln \left(\frac{V_A}{V_D} \right) = nRT_D \ln \left(\frac{P_D}{P_A} \right)$$

$$\Rightarrow W = 2R(300) \ln(1/2) = -600R(0.69) = -414R$$

So work done on the gas is $W = +414R$

Hence, the correct answer is (B).

117. Total work done on the gas when taking from A to B is $400R$ and from C to D is equal and opposite, so they cancel each other.

Since $W_{D \rightarrow A} = -414R$ and

$$W_{B \rightarrow C} = (2)R(500) \ln(2) = 690R$$

$$\Rightarrow W = -414R + 690R = 276R$$

Hence, the correct answer is (B).

118. The internal energy is the energy due to thermal motion, so we have

$$U = \left(\frac{nf}{2} \right) RT = \left(\frac{5n}{2} \right) RT$$

Since $PV = nRT$, $V = \frac{m}{\rho} = \frac{1 \text{ kg}}{4 \text{ kgm}^{-3}} = \frac{1}{4} \text{ m}^3$ and

$P = 8 \times 10^4 \text{ Nm}^{-2}$, so we get

$$U = \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4} = 5 \times 10^4 \text{ J}$$

Hence, the correct answer is (B).

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Questions

1. Rate of heat transfer through metal rod is

$$\frac{dQ}{dt} = C \frac{dT}{dt} = P(\text{constant})$$

...(1)

Since temperature variation is given by

$$T = T_0 \left(1 + \beta t^{\frac{1}{4}} \right)$$

...(2)

$$\Rightarrow \frac{dT}{dt} = \frac{T_0 \beta}{4} t^{-\frac{3}{4}}$$

So from equation (1), we get

$$C = \frac{P}{dT/dt} = \frac{4P}{\beta T_0} t^{\frac{3}{4}}$$

Substituting the value of t from equation (2), we get

$$C = \frac{4P(T - T_0)^3}{(\beta T_0)^4}$$

Hence, the correct answer is (C).

2. Heat generated in device in 3 h is

$$W = Pt = 3 \times 3600 \times 3 \times 10^3 = 324 \times 10^5 \text{ J}$$

Heat used to heat water

$$Q = mc\Delta T = 120 \times 1 \times 4.2 \times 10^3 \times 20 \text{ J}$$

$$\Rightarrow Q = 100.8 \times 10^5 \text{ J}$$

So, heat absorbed by coolant is

$$H = 324 \times 10^5 - 100.8 \times 10^5 = 223.2 \times 10^5 \text{ J}$$

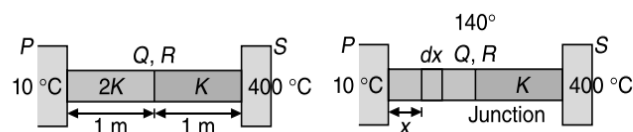
$$\Rightarrow H = P't = (325 - 100.8) \times 10^5 \text{ J}$$

$$\Rightarrow P' = \frac{223.2 \times 10^8}{3600} = 2067 \text{ W}$$

Hence, the correct answer is (B).

3. Rate of heat flow from P to Q is

$$\frac{dQ}{dt} = \frac{2KA(T - 10)}{1}$$



Rate of heat flow from Q to S is

$$\frac{dQ}{dt} = \frac{KA(400 - T)}{1}$$

At steady state, rate of heat flow is same

$$\frac{2KA(T - 10)}{1} = \frac{KA(400 - T)}{1}$$

$$\Rightarrow 2T - 20 = 400 - T$$

$$\Rightarrow 3T = 420$$

$$\Rightarrow T = 140^\circ$$

Temperature of junction is 140°C

Temperature at a distance x from end P is

$$T_x = (130x + 10^\circ)$$

Change in length of dx is supposed to be dy , then

$$dy = \alpha dx (T_x - 10) \quad \{ \because \Delta l = l\alpha \Delta T \}$$

$$\Rightarrow \int_0^{\Delta y} dy = \int_0^1 \alpha dx (130x + 10 - 10)$$

$$\Rightarrow \Delta y = 130 \left(\frac{\alpha x^2}{2} \right) \Big|_0^1 = (65)(1.2 \times 10^{-5}) = 78.0 \times 10^{-5} \text{ m}$$

$$\Rightarrow \Delta y = 0.78 \text{ mm}$$

Hence, the correct answer is (A).

H.164 JEE Advanced Physics: Waves and Thermodynamics

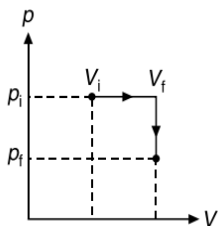
4. For the first process, $p_i V_i^\gamma = p_f V_f^\gamma$

$$\Rightarrow \frac{p_i}{p_f} = \left(\frac{V_f}{V_i} \right)^\gamma$$

$$\Rightarrow 32 = 8^\gamma$$

$$\Rightarrow 2^5 = 2^{3\gamma}$$

$$\Rightarrow \gamma = \frac{5}{3}$$



...(1)

For the two-step process, we have

$$W = p_i(V_f - V_i) = 10^5(7 \times 10^{-3})$$

$$\Rightarrow W = 7 \times 10^2 \text{ J}$$

$$\text{Also, } \Delta U = \frac{f}{2}(p_f V_f - p_i V_i)$$

$$\Rightarrow \Delta U = \frac{1}{\gamma-1} \left(\frac{1}{4} \times 10^2 - 10^2 \right) = -\frac{9}{8} \times 10^2 \text{ J}$$

By FLTD, we have $Q = \Delta U + W$

$$\Rightarrow Q = 7 \times 10^2 - \frac{9}{8} \times 10^2 = \frac{47}{8} \times 10^2 \text{ J} = 588 \text{ J}$$

Hence, the correct answer is (C).

5. In CASE-1, $R_I = R_1 + R_2 = \left(\frac{l}{KA} \right) + \left(\frac{l}{2KA} \right) = \frac{3}{2} \left(\frac{l}{KA} \right)$

$$\text{In CASE-2, } \frac{1}{R_{II}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{KA}{l} + \frac{2KA}{l}$$

$$\Rightarrow R_{II} = \frac{l}{3KA} = \frac{R_I}{4.5}$$

Since thermal resistance R_{II} in 4.5 times less than thermal resistance R_I .

$$\Rightarrow t_{II} = \frac{t_I}{4.5} = \frac{9}{4.5} \text{ s} = 2 \text{ s}$$

Hence, the correct answer is (A).

6. From ideal gas equation, we have $\rho = \frac{pM}{RT}$

$$\Rightarrow \rho \propto pM$$

$$\Rightarrow \frac{\rho_1}{\rho_2} = \left(\frac{p_1}{p_2} \right) \left(\frac{M_1}{M_2} \right) = \left(\frac{2}{3} \right) \left(\frac{4}{3} \right) = \frac{8}{9}$$

Hence, the correct answer is (D).

7. $Q = nC_p \Delta T = 208 \text{ J}$

Hence, the correct answer is (D).

8. At equilibrium, (Heat Absorbed) = (Heat Radiated)

$$\Rightarrow A(3T)^4 + A(2T)^4 = 2AT_0^4$$

$$\Rightarrow T_0 = \left(\frac{97}{2} \right)^{1/4} T$$

Hence, the correct answer is (C).

9. $\frac{v_{He}}{v_{Ar}} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{10} = 3.16$

Hence, the correct answer is (D).

10. Since, $T_1 V_1^{(\gamma-1)} = T_2 V_2^{(\gamma-1)}$

$$\Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

$$\Rightarrow T_2 = 4T_1$$

$$n = \text{Number of moles of gas} = \frac{5.6 \text{ lt}}{22.4 \text{ lt}} = \frac{1}{4}$$

$$\Rightarrow W_{\text{ext}} = \frac{nR}{1-\gamma} (T_2 - T_1)$$

$$\Rightarrow W_{\text{ext}} = +\frac{9}{8} RT_1$$

Hence, the correct answer is (A).

11. Since $ML = Pt$

$$\Rightarrow L = \frac{Pt}{M}$$

Hence, the correct answer is (C).

12. A real gas behaves like an ideal gas at low pressure and high temperature.

Hence, the correct answer is (D).

13. It cracks due to low thermal conductivity.

Hence, the correct answer is (A).

14. $PT^2 = \text{constant}$

Since, $PV = nRT$

$$\Rightarrow \left(\frac{T}{V} \right) T^2 = \text{constant}$$

$$\Rightarrow V \propto T^3$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{3\Delta T}{T}$$

$$\Rightarrow \frac{\Delta V}{V\Delta T} = \frac{3}{T}$$

Hence, the correct answer is (C).

15. 1 calorie is the heat required to raise the temperature of 1 g of water from 14.5 °C to 15.5 °C at 760 mm of Hg.

Hence, the correct answer is (A).

16. $\lambda_m T = \text{constant}$

From the graph $T_3 > T_2 > T_1$

Temperature of sun will be maximum. Therefore, (C) is the correct OPTION.

Hence, the correct answer is (C).

17. Glass of bulb heats due to filament by radiation.

Hence, the correct answer is (C).

18. Energy gained by water (in 1 s) is

$$\frac{dQ}{dt} = 1000 - 160 = 840 \text{ W}$$

Total heat required to raise the temperature of water from 27 °C to 77 °C is $mc\Delta T$.

Hence, the required time

$$t = \frac{mc\Delta T}{\text{Rate at which energy is gained by water}}$$

$$\left\{ \because \gamma = \frac{5}{3} \right\}$$

$$\Rightarrow t = \frac{(2)(4.2 \times 10^3)(50)}{840} = 500 \text{ s} = 8 \text{ min } 20 \text{ s}$$

Hence, the correct answer is (A).

19. $Q \propto AT^4$ and $\lambda_m T = \text{constant}$. Hence,

$$Q \propto \frac{A}{(\lambda_m)^4}$$

$$\Rightarrow Q \propto \frac{r^2}{(\lambda_m)^4}$$

$$\Rightarrow Q_A : Q_B : Q_C = \frac{(2)^2}{(3)^4} : \frac{(4)^2}{(4)^4} : \frac{(6)^2}{(5)^4}$$

$$\Rightarrow Q_A : Q_B : Q_C = \frac{4}{81} : \frac{1}{16} : \frac{36}{625}$$

$$\Rightarrow Q_A : Q_B : Q_C = 0.05 : 0.0625 : 0.0576$$

$$\Rightarrow Q_B \text{ is maximum.}$$

Hence, the correct answer is (B).

20. $\frac{dQ}{dt} = L \left(\frac{dm}{dt} \right)$

$$\Rightarrow \frac{\text{Temperature difference}}{\text{Thermal resistance}} = L \left(\frac{dm}{dt} \right)$$

$$\Rightarrow \frac{dm}{dt} \propto \frac{1}{\text{Thermal resistance}}$$

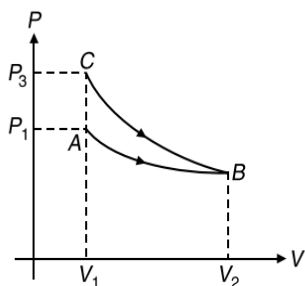
$$q \propto \frac{1}{R}$$

In the first case rods are in parallel and thermal resistance is $\frac{R}{2}$ while in second case rods are in series and thermal resistance is $2R$.

$$\frac{q_1}{q_2} = \frac{2R}{R/2} = \frac{4}{1}$$

Hence, the correct answer is (C).

21. Slope of adiabatic process at a given state (P, V, T) is more than the slope of isothermal process. The corresponding P-V graph for the two processes is as shown in figure.



In the graph, AB is isothermal and BC is adiabatic.

$$W_{AB} = \text{positive (as volume is increasing)}$$

and $W_{BC} = \text{negative (as volume is decreasing)}$ plus, $|W_{BC}| > |W_{AB}|$, as area under P-V graph gives the work done.

$$\text{Hence, } W_{AB} + W_{BC} = W < 0$$

From the graph itself, it is clear that $P_3 > P_1$.

Conceptual Note(s)

At point B, slope of adiabatic (process BC) is greater than the slope of isothermal (process AB).

Hence, the correct answer is (C).

22. Temperature of liquid oxygen will first increase in the same phase. Then, phase change (liquid to gas) will take place. During which temperature will remain constant. After that temperature of oxygen in gaseous state will further increase. Hence, the correct answer is (C).

23. Out of the alternative provided, none appears completely correct.

AB is an isothermal process.

$\left(T = \text{constant}, p \propto \frac{1}{V} \right)$. So, p - V graph should be rectangular hyperbola with p decreasing and V increasing.

BC is an isobaric process.

($p = \text{constant}, V \propto T$). Temperature is increasing. Hence, volume should also increase.

CA is an adiabatic process ($pV^\gamma = \text{constant}$). Pressure is increasing. So, volume should decrease. At point A, an isotherm AB and an adiabatic curve AC are meeting. We know that

(slope of an adiabatic graph in p - V diagram) $= \gamma$ (slope of an isothermal graph in the same diagram) with $\gamma > 1$ or (Slope)_{adiabatic} $>$ (Slope)_{isothermal}

None of the given examples fulfill all the above requirements.

Hence, the correct answer is (None).

24. Heat released by 5 kg of water when its temperature falls from 20°C to 0°C is,

$$Q_1 = mc\Delta\theta = (5)(10^3)(20 - 0) = 10^5 \text{ cal}$$

when 2 kg ice at -20°C comes to a temperature of 0°C , it takes an energy.

$$Q_2 = mc\Delta\theta = (2)(500)(20) = 0.2 \times 10^5 \text{ cal}$$

The remaining heat

$$Q = Q_1 - Q_2 = 0.8 \times 10^5 \text{ cal}$$

will melt a mass m of the ice, where

$$m = \frac{Q}{L} = \frac{0.8 \times 10^5}{80 \times 10^3} = 1 \text{ kg}$$

So, the temperature of the mixture will be 0°C , mass of water in it is $5 + 1 = 6$ kg and mass of ice is $2 - 1 = 1$ kg.

Hence, the correct answer is (B).

25. Given $\Delta\ell_1 = \Delta\ell_2$

$$\Rightarrow \ell_1 \alpha_a t = \ell_2 \alpha_s t$$

$$\Rightarrow \frac{\ell_1}{\ell_2} = \frac{\alpha_s}{\alpha_a}$$

$$\Rightarrow \frac{\ell_1}{\ell_1 + \ell_2} = \frac{\alpha_s}{\alpha_a + \alpha_s}$$

Hence, the correct answer is (C).

H.166 JEE Advanced Physics: Waves and Thermodynamics

26. Rate of cooling $\left(-\frac{dT}{dt}\right) \propto \text{emissivity } (E)$

From the graph, $\left(-\frac{dT}{dt}\right)_x > \left(-\frac{dT}{dt}\right)_y$

$\Rightarrow E_x > E_y$

Further emissivity $(E) \propto$ absorptive power (a) (good absorbers are good emitters also)

$\Rightarrow a_x > a_y$

Conceptual Note(s)

Emissivity is a pure ratio (dimensionless) while the emissive power has a unit J s^{-1} or watt.

Hence, the correct answer is (C).

27. Black body radiates maximum number of wavelength and maximum energy if all other conditions (e.g., temperature surface area etc.) are same. So, when the temperature of black body becomes equal to the temperature of the furnace, the black body will radiate maximum energy and it will be brightest of all. Initially it will absorb all the radiant energy incident on it, so, it is the darkest one.

Hence, the correct answer is (A).

28. $\beta = -\frac{dV/dP}{V} = \text{compressibility of gas}$

$\Rightarrow \beta = \frac{1}{\text{Bulk modulus of elasticity}}$

$\Rightarrow \beta = \frac{1}{P}$ under isothermal conditions

Thus, β versus P graph will be a rectangular hyperbola.

Hence, the correct answer is (A).

29. $\Delta W_{AB} = P\Delta V = (10)(20-1) = 10 \text{ J}$

$\Delta W_{BC} = 0$

From First Law of Thermodynamics

$\Delta Q = \Delta W + \Delta U$

$\Delta U = 0$ $\{\because \text{process } ABCA \text{ is cyclic}\}$

$\Rightarrow \Delta Q = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$

$\Rightarrow \Delta W_{CA} = \Delta Q - \Delta W_{AB} - \Delta W_{BC} = 5 - 10 - 0 = -5 \text{ J}$

Hence, the correct answer is (A).

30. In adiabatic process

Slope of P - V graph, $\frac{dP}{dV} = -\gamma \frac{P}{V}$

Slope $\propto \gamma$ $\{\text{with negative sign}\}$

From the given graph, $(\text{slope})_2 > (\text{slope})_1$

$\Rightarrow \gamma_2 > \gamma_1$

Therefore, 1 should correspond to O_2 ($\gamma = 1.4$) and 2 should correspond to He ($\gamma = 1.67$).

Hence, the correct answer is (B).

31. $dW = 0$

\Rightarrow Isochoric process

$\Rightarrow dQ = dU$ (by First Law of Thermodynamics)

Since $dQ < 0$

$\Rightarrow dU < 0$

There must be a fall in temperature.

Hence, the correct answer is (A).

32. Let θ be the temperature of the junction (say B). Thermal resistance of all the three rods is equal.

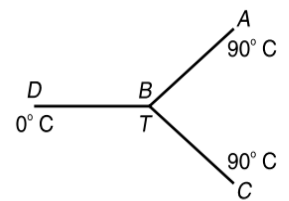
$$\left(\begin{array}{c} \text{Rate of} \\ \text{heat flow} \\ \text{through} \\ AB \end{array}\right) + \left(\begin{array}{c} \text{Rate of} \\ \text{heat flow} \\ \text{through} \\ CB \end{array}\right) = \left(\begin{array}{c} \text{Rate of} \\ \text{heat flow} \\ \text{through} \\ BD \end{array}\right)$$

$\Rightarrow \frac{90-T}{R} + \frac{90-T}{R} = \frac{T-0}{R}$

Here, $R = \text{Thermal resistance}$

$\Rightarrow 3T = 180^\circ$

$\Rightarrow T = 60^\circ \text{C}$



Conceptual Note(s)

Rate of heat flow

$$(H) = \frac{\text{Temperature difference (TD)}}{\text{Thermal resistance (R)}}$$

where $R = \frac{\ell}{KA}$

$K = \text{Thermal conductivity of the rod.}$

This is similar to the current flow through a resistance (R) where current (i) = Rate of flow of charge

$$= \frac{\text{Potential difference (PD)}}{\text{Electrical resistance (R)}}$$

Here, $R = \frac{\ell}{\sigma A}$ where $\sigma = \text{Electrical conductivity}$

Hence, the correct answer is (B).

33. $v \propto \frac{1}{\sqrt{m}}$

Hence, the correct answer is (B).

34. At constant pressure

$V = kT$

$\Rightarrow \Delta V = k\Delta T$

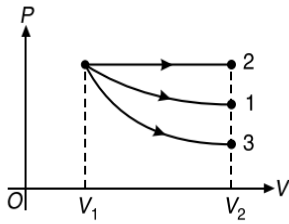
$\Rightarrow \frac{\Delta V}{V} = \frac{\Delta T}{T}$

$\Rightarrow \delta = \frac{\Delta V}{V\Delta T} = \frac{1}{T}$

\Rightarrow Plot of δ vs T must represent a rectangular hyperbola.

Hence, the correct answer is (C).

35. The work done equals area under the curve



$$\Rightarrow W_2 > W_1 > W_3$$

Hence, the correct answer is (A).

36. Following steps are involved in this process

- Ice at -10°C will convert to ice at 0°C and will absorb heat in accordance with formula $dQ = mc_{\text{ice}}dT$ i.e. Q vs T must be a straight line with slope $\frac{1}{mc_{\text{ice}}} = s_1$ (say) (for T on y-axis and Q on x-axis)
- Ice at 0°C will melt to water at 0°C (at constant temperature) by absorbing a heat $Q_2 = mL_{\text{ice}}$
- Water at 0°C will convert to water at 100°C and will again absorb heat in accordance with formula $dQ = mc_{\text{water}}dT$ i.e. Q vs T is again a straight line with slope $\frac{1}{mc_{\text{water}}} = s_2$ (say) ($s_2 < s_1$)
- Water at 100°C will vapourise to steam at 100°C (at constant temperature) by absorbing heat $Q_4 = mL_{\text{steam}}$

Hence, the correct answer is (A).

37. According to Wien's Displacement Law

$$\lambda_m T = \text{constant}$$

$$\text{Since, } \lambda_1 < \lambda_3 < \lambda_2$$

$$\Rightarrow T_1 > T_3 > T_2$$

Hence, the correct answer is (B).

38. $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$$\Rightarrow T_1 (AL_1)^{\frac{5}{3}-1} = T_2 (AL_2)^{\frac{5}{3}-1}$$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{\frac{2}{3}}$$

Hence, the correct answer is (D).

39. $U = U_{\text{Oxygen}} + U_{\text{Argon}}$

$$\Rightarrow U = (2) \left(\frac{5R}{2}\right) T + 4 \left(\frac{3R}{2}\right) T$$

$$\Rightarrow U = 11RT$$

Hence, the correct answer is (D).

40. Speed of sound in an ideal gas is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$\Rightarrow v \propto \sqrt{\frac{\gamma}{M}} \quad \{T \text{ is same for both the gases}\}$$

$$\Rightarrow \frac{v_{N_2}}{v_{He}} = \sqrt{\frac{\gamma_{N_2} \cdot M_{He}}{\gamma_{He} \cdot M_{N_2}}} = \sqrt{\frac{7/5 \left(\frac{4}{28}\right)}{5/3}}$$

$$\text{Since, } \gamma_{N_2} = \frac{7}{5} \quad \{\text{Diatomic}\}$$

$$\text{and } \gamma_{He} = \frac{5}{3} \quad \{\text{Monatomic}\}$$

$$\Rightarrow v = \frac{\sqrt{3}}{5} \text{ ms}^{-1}$$

Hence, the correct answer is (C).

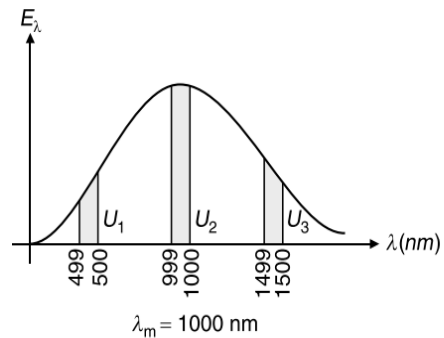
41. Wein's Displacement Law is

$$\lambda_m T = b \quad \{b = \text{Wein's constant}\}$$

$$\Rightarrow \lambda_m = \frac{b}{T} = \frac{2.88 \times 10^6 \text{ nm-K}}{2880 \text{ K}}$$

$$\Rightarrow \lambda = 1000 \text{ nm}$$

Energy distribution with wavelength will be as follows



From the graph it is clear that

$$U_2 > U_1 \quad \{\text{In fact } U_2 \text{ is maximum}\}$$

Hence, the correct answer is (D).

42. $P = \frac{nRT}{V} = \frac{mRT}{MV} \quad \left\{ \because n = \frac{m}{M} \right\}$

So, at constant volume pressure versus temperature graph

is a straight line passing through origin with slope $\frac{mR}{MV}$.

As the mass is doubled and volume is halved slope becomes four times. Therefore, pressure versus temperature graph will be shown by the line B.

Hence, the correct answer is (B).

43. For gas in container A

$$\Delta P = (P_A)_{\text{final}} - (P_A)_{\text{initial}} = \frac{n_A RT}{2V} - \frac{n_A RT}{V}$$

$$\Delta P = -\frac{n_A RT}{2V} \quad \dots(1)$$

For gas in container B

$$1.5\Delta P = (P_B)_{\text{final}} - (P_B)_{\text{initial}} = \frac{n_B RT}{2V} - \frac{n_B RT}{V}$$

$$1.5\Delta P = -\frac{n_B RT}{2V} \quad \dots(2)$$

From (1) and (2), we get

$$n_B = 1.5 n_A$$

H.168 JEE Advanced Physics: Waves and Thermodynamics

$$\Rightarrow 2n_B = 3n_A$$

$$\Rightarrow 2m_B = 3m_A \quad \text{\{OPTION (C)\}}$$

Hence, the correct answer is (C).

44. A diatomic molecule has 5 degrees of freedom out of which 3 are translational degrees of freedom and remaining 2 are rotational degrees of freedom. According to Theorem of Equipartition of Energy, the energy associated with each degree of freedom per molecule of a gas in thermal equilibrium is $\frac{1}{2}k_B T$, where k_B is the Boltzmann constant. So, the average rotational kinetic energy associated with each O_2 molecule is $2\left(\frac{1}{2}k_B T\right) = k_B T$. Similarly, average rotational kinetic energy associated with each N_2 molecule is $2\left(\frac{1}{2}k_B T\right) = k_B T$. So, the ratio amounts to 1:1.

\{OPTION (A)\}

However, if we are asked to compare the total rotational kinetic energy associated with 1 mole O_2 gas to that with 2 mole of N_2 gas, then our answer would have been 1:2.

Hence, the correct answer is (A).

45. Power radiated \propto (surface area) $(T)^4$. The radius is halved, hence, surface area will become $\frac{1}{4}$ times. Temperature is doubled, therefore, T^4 becomes 16 times. New power = $(450)\left(\frac{1}{4}\right)(16) = 1800$ W.

Hence, the correct answer is (D).

46. $PV = nRT$

$$\Rightarrow P = \frac{nRT}{V}$$

$$\frac{P_2}{P_1} = \frac{n_2 T_2}{n_1 T_1}$$

$$\frac{P_2}{P_1} = \frac{1}{1} \times \frac{2}{1}$$

$$\Rightarrow P_2 = 2P$$

Hence, the correct answer is (C).

47. Average translational kinetic energy of an ideal gas molecule is $\frac{3}{2} kT$ which depends on temperature only. Therefore, if temperature is same, translational kinetic energy of O_2 and N_2 both will be equal.

Hence, the correct answer is (C).

48. From Wein's Displacement Law

$$\lambda_m T = \text{constant}$$

$$\Rightarrow T = \frac{1}{\lambda_m}$$

$$\Rightarrow \frac{T_{\text{sun}}}{T_{\text{north star}}} = \frac{(\lambda_m)_{\text{north star}}}{(\lambda_m)}$$

$$\Rightarrow \frac{T_{\text{sun}}}{T_{\text{north star}}} = \frac{350}{510} \approx 0.69$$

Hence, the correct answer is (B).

49. The average translational KE = $\frac{3}{2} kT$ which is directly proportional to T , while rms speed molecules is given by

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \text{ i.e., } v_{\text{rms}} \propto \sqrt{T}$$

When temperature of gas is increased from 300 K to 600 K (i.e., 2 times), the average translational KE will increase to 2 times and rms speed to $\sqrt{2}$ or 1.414 times.

So, average translational KE = $2 \times 6.21 \times 10^{-21}$ J

$$(KE)_{\text{av}} = 12.42 \times 10^{-21} \text{ J and}$$

$$v_{\text{rms}} = (1.414)(484) \text{ ms}^{-1}$$

$$\Rightarrow v_{\text{rms}} \approx 684 \text{ ms}^{-1}$$

Hence, the correct answer is (D).

50. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$$\text{i.e., } v_{\text{rms}} \propto \sqrt{T}$$

When temperature is increased from 120 K to 480 K (i.e., four times), the root mean square speed will become $\sqrt{4}$ or 2 times i.e., $2v$.

Hence, the correct answer is (B).

51. The rate at which energy leaves the object is $\frac{\Delta Q}{\Delta t} = e\sigma AT^4$

Since, $\Delta Q = mC\Delta T$, we get

$$\frac{\Delta T}{\Delta t} = \frac{e\sigma AT^4}{mC}$$

Also, since $m = \frac{4}{3}\pi r^3 \rho$ for a sphere, we get

$$A = \pi r^2 = \pi \left(\frac{3m}{4\pi\rho}\right)^{2/3}$$

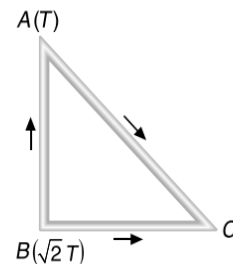
$$\text{Hence, } \frac{\Delta T}{\Delta t} = \frac{e\sigma T^4}{mC} \left[\pi \left(\frac{3m}{4\pi\rho}\right)^{2/3}\right] = K \left(\frac{1}{m}\right)^{1/3}$$

For the given two bodies

$$\frac{(\Delta T/\Delta t)_1}{(\Delta T/\Delta t)_2} = \left(\frac{m_2}{m_1}\right)^{1/3} = \left(\frac{1}{3}\right)^{1/3}$$

Hence, the correct answer is (D).

52. As $T_B > T_A$, heat flows from B to A through both paths



Rate of heat flow in BC = Rate of heat flow in CA

$$\Rightarrow \frac{KA(\sqrt{2}T - T_C)}{\ell} = \frac{KA(T_C - T)}{\sqrt{2}\ell}$$

Solving this we get,

$$T_C = \frac{3T}{\sqrt{2}+1}$$

Hence, the correct answer is (A).

53. The average speed of a gas molecule is $\langle v \rangle = \sqrt{\frac{8k_B T}{\pi m}}$, where m is the mass of the molecule. Since, all the three vessels have same temperature, so average speed of O_2 molecule in the vessel C is also v_1 .

Hence, the correct answer is (B).

54. The desired fraction is

$$f = \frac{\Delta U}{\Delta Q} = \frac{nC_V \Delta T}{nC_P \Delta T} = \frac{C_V}{C_P} = \frac{1}{\gamma}$$

$$\Rightarrow f = \frac{5}{7}$$

$$\left\{ \text{as } \gamma = \frac{7}{5} \right\}$$

Hence, the correct answer is (D).

55. In the process $PV^x = \text{constant}$
Molar heat capacity is given by

$$C = \frac{R}{\gamma-1} + \frac{R}{1-x}$$

Here the process is $PV^{-1} = \text{constant}$

i.e., $x = -1$

and gas is monatomic. Therefore, $\gamma = \frac{5}{3}$

$$\Rightarrow C = \frac{R}{\frac{5}{3}-1} + \frac{R}{1-(-1)} = 2R$$

$$\Rightarrow \Delta Q = nC\Delta T$$

$$\left\{ \because n = 1 \right\}$$

$$\Rightarrow \Delta Q = (1)(2R)(2T_0 - T_0)$$

$$\Rightarrow \Delta Q = 2RT_0$$

Hence, the correct answer is (A).

56.
$$C_{V_{\text{mixture}}} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$$

$$C_{P_{\text{mixture}}} = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 + n_2}$$

Hence, the correct answer is (B).

57. Heat required $Q = (1.1 + 0.02) \times 10^2 \times 1 \times (80 - 15)$

$$\Rightarrow Q = 72800 \text{ cal}$$

Therefore, mass of steam condensed (in kg)

$$m = \frac{Q}{L} = \frac{72800}{540} \times 10^{-2} = 0.135 \text{ kg}$$

Hence, the correct answer is (D).

58. Since $70 = (2)C_p(45 - 40)$

$$\Rightarrow C_p = 7 \text{ cal mol}^{-1} \text{K}^{-1}$$

$$\Rightarrow C_v = 5 \text{ cal mol}^{-1} \text{K}^{-1} \quad \left\{ \text{Since } C_p - C_v = 2 \text{ cal mol}^{-1} \text{K}^{-1} \right\}$$

$$\Rightarrow Q = (2)(5)(5) = 50 \text{ cal}$$

Hence, the correct answer is (C).

59.
$$v_{\text{rms}}^2 = \frac{3RT}{M}$$

$$\Rightarrow M = \frac{3RT}{v_{\text{rms}}^2} = \frac{3(8.3)(300)}{(1930)^2} \approx 2 \text{ g} \quad (\text{for } H_2 \text{ gas})$$

Hence, the correct answer is (A).

60. Since, $\left(\begin{array}{l} \text{Work done in a} \\ \text{cyclic process} \end{array} \right) = \left(\begin{array}{l} \text{Positive area enclosed} \\ \text{by the cycle} \end{array} \right)$

$$\Rightarrow W = AB \times BC = (2P - P) \times (2V - V) = PV$$

Hence, the correct answer is (A).

Multiple Correct Choice Type Questions

1. Since, $P = \sigma AT^4$

$$\Rightarrow P = 5.67 \times 10^{-8} \times 64 \times 10^{-6} \times (2500)^4$$

$$\Rightarrow P = 141.75 \text{ W}$$

$$\text{Also, } I = \frac{P}{4\pi r^2} = \frac{141.75}{4\pi \times (100)^2} = 1.13 \times 10^{-3} \text{ W m}^{-2}$$

$$\Rightarrow \Delta E_{\text{eye}} = I \times (\pi r_e^2)$$

$$\Rightarrow \Delta E_{\text{eye}} = I \times (\pi \times 9 \times 10^{-6})$$

$$\Rightarrow \Delta E_{\text{eye}} = 3.2 \times 10^{-8} \text{ W}$$

According to Wein's Law, we have

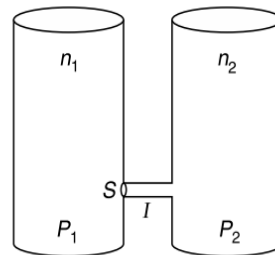
$$\lambda_m = \frac{b}{T} = \frac{2.90 \times 10^{-3}}{2500} = 1160 \text{ nm}$$

$$\Rightarrow N_{\text{photons}} = \frac{\Delta E}{\left(\frac{hc}{\lambda} \right)} = \frac{3.2 \times 10^{-8} \times 1740 \times 10^{-9}}{6.63 \times 10^{-34} \times 3 \times 10^8}$$

$$\Rightarrow N_{\text{photons}} = 2.8 \times 10^{11}$$

Hence, (B), (C) and (D) are correct.

2. Since, $P_1 = n_1 k_B T$ and $P_2 = n_2 k_B T$



$$\Rightarrow \Delta F = (P_1 - P_2)S$$

$$\Rightarrow \Delta F = (n_1 - n_2)k_B T S = \Delta n k_B T S \quad \dots(1)$$

For balancing of force, we have

$$\beta v \times (S\ell) n_1 = \Delta n k_B T S$$

$$\Rightarrow n_1 \beta v \ell = \Delta n k_B T \quad \dots(2)$$

$$\Rightarrow v = \frac{\Delta n k_B T}{\beta n_1 \ell}$$

Number of molecules per second is

$$\frac{\Delta N}{\Delta t} = \frac{(v dt) \times S \times n_1}{dt} = v S n_1$$

H.170 JEE Advanced Physics: Waves and Thermodynamics

$$\Rightarrow \frac{\Delta N}{\Delta t} = \frac{\Delta n k_B T}{\beta n_1 I} \times S n_1 = \left(\frac{\Delta n}{I} \right) \left(\frac{k_B T}{\beta} \right) S \quad \dots(3)$$

So, Δn decreases with time, so rate of transfer decreases with time.

Hence, (A), (B) and (C) are correct.

3. Since, $\gamma_{\text{mixture}} = \frac{5(5R/2) + 1(7R/2)}{5(3R/2) + 1(5R/2)} = \frac{8}{5} = 1.6$

For adiabatic process, $PV^\gamma = \text{constant}$

$$\Rightarrow P = P_0 \left(\frac{V_0}{V} \right)^\gamma = (4)^{\frac{8}{5}} P_0 = 9.2 P_0$$

$$\Rightarrow W = \frac{P_0 V_0 - 9.2 P_0 (V_0/4)}{\gamma - 1} = -\frac{13 P_0 V_0}{6}$$

From ideal gas equation, we have $P_0 V_0 = 6 R T_0$

$$\Rightarrow W = -\frac{13 P_0 V_0}{6} = -13 R T_0$$

Final temperature after compression is

$$T = T_0 \left(\frac{PV}{P_0 V_0} \right) = 2.3 T_0$$

$$\text{Also, } (C_V)_{\text{mixture}} = \frac{n_1 C_{V1} + n_2 C_{V2}}{n_1 + n_2} = \frac{5R}{3}$$

So, average kinetic energy of gas is

$$U_f = n C_V T = 6 \left(\frac{5R}{3} \right) (2.3 T_0) = 23 R T_0$$

Hence, (A), (B) and (C) are correct.

4. $|Q_{1 \rightarrow 2}| = n C_p dT = n \left(\frac{5R}{2} \right) T_0 = \frac{5RT_0}{2}$

$$|Q_{3 \rightarrow 4}| = n C_p dT = n \left(\frac{5R}{2} \right) \left(\frac{T_0}{2} \right) = \frac{5RT_0}{4}$$

$$\Rightarrow \frac{|Q_{1 \rightarrow 2}|}{|Q_{3 \rightarrow 4}|} = 2$$

$$\text{Also } |Q_{2 \rightarrow 3}| = \frac{3}{2} R T_0$$

$$\frac{Q_{1 \rightarrow 2}}{Q_{2 \rightarrow 3}} = \frac{5RT_0/2}{3RT_0/2} = \frac{5}{3}$$

Since, $W = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + W_{3 \rightarrow 4} + W_{4 \rightarrow 1}$

$$\Rightarrow W = (1)R(2T_0 - T_0) + 0 + (1)R \left(\frac{T_0}{2} - T_0 \right) + 0$$

$$\Rightarrow W = \frac{RT_0}{2}$$

Hence, (C) and (D) are correct.

5. In Process II, T is constant, so V is increasing. Hence Q is positive i.e., gas absorbs heat. Similarly, in process IV, gas releases heat. For an isobaric process, $V \propto T$.

Hence, (B), (C) and (D) are correct.

6. Assumption $e = 1$ i.e., body is a black body, so

$$P = \sigma A (T^4 - T_0^4)$$

$$\Rightarrow P_{\text{rad}} = \sigma A T^4 = \sigma 1 (T_0 + 10)^4$$

$$\Rightarrow P_{\text{rad}} = \sigma T_0^4 \left(1 + \frac{10}{T_0} \right)^4 \quad [T_0 = 300 \text{ K given}]$$

$$\Rightarrow P_{\text{rad}} = \sigma (300)^4 \left(1 + \frac{40}{300} \right) \approx 460 \times \frac{17}{15} \approx 520 \text{ J}$$

$$\Rightarrow P_{\text{net}} = 520 - 460 \approx 60 \text{ W}$$

So, energy radiated in 1 s = 60 J

$$\text{Since, } P = \sigma A (T^4 - T_0^4)$$

$$\Rightarrow dP = \sigma A (0 - 4T_0^3 dT) \text{ and } dT = -\Delta T$$

$$\Rightarrow dP = 4\sigma A T_0^3 \Delta T$$

If surface area decreases, then energy radiation also decreases.


Conceptual Note(s)

While giving answer (B) and (C) it is assumed that energy radiated refers the net radiation. If energy radiated is taken as only emission, then (B) and (C) will not include in answer.

Hence, (B), (C) and (D) are correct.

7. (A) Total internal energy $U = \frac{f_1}{2} nRT + \frac{f_2}{2} nRT$

$$(U_{\text{ave}})_{\text{per mole}} = \frac{U}{2n} = \frac{1}{4} [5RT + 3RT] = 2RT$$

(B) $\gamma_{\text{mix}} = \frac{n_1 C_{p1} + n_2 C_{p2}}{n_2 C_{V1} + n_2 C_{V2}} = \frac{(1) \frac{7R}{2} + (1) \frac{5R}{2}}{(1) \frac{5R}{2} + (1) \frac{3R}{2}} = \frac{3}{2}$

$$\Rightarrow M_{\text{mix}} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2} = \frac{M_1 + M_2}{2} = \frac{2 + 4}{2} = 3$$

$$\text{Speed of sound } v = \sqrt{\frac{\gamma RT}{M}}$$

$$\Rightarrow v \propto \sqrt{\frac{\gamma}{M}}$$

$$\Rightarrow \frac{v_{\text{mix}}}{v_{\text{He}}} = \frac{\sqrt{\gamma_{\text{mix}} \times \frac{M_{\text{He}}}{M_{\text{mix}}}}}{\sqrt{\gamma_{\text{He}}}} = \sqrt{\frac{3/2}{5/3} \times \frac{4}{3}} = \sqrt{\frac{6}{5}}$$

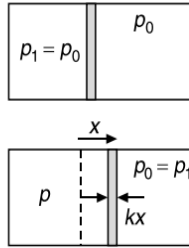
(D) $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$$\Rightarrow v_{\text{rms}} \propto \frac{1}{\sqrt{M}}$$

$$\Rightarrow \frac{v_{\text{He}}}{v_{\text{H}}} = \frac{\sqrt{\frac{M_{\text{H}}}{M_{\text{He}}}}}{\sqrt{4}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

Hence, (A), (B) and (D) are correct.

8. Please note that this question can be solved if right hand side chamber is assumed open, so that its pressure remains constant even if the piston shifts towards right.



(A) Since $pV = nRT$, so $p \propto \frac{T}{V}$

When temperature is made three times and volume is doubled, then

$$p_2 = \frac{3}{2}p_1$$

$$\text{Further } x = \frac{\Delta V}{A} = \frac{V_2 - V_1}{A} = \frac{2V_1 - V_1}{A} = \frac{V_1}{A}$$

$$\Rightarrow p_2 = \frac{3p_1}{2} = p_1 + \frac{kx}{A}$$

$$\Rightarrow kx = \frac{p_1 A}{2}$$

So, energy stored in the spring is

$$\frac{1}{2}kx^2 = \frac{1}{2}(kx)x = \frac{p_1 A}{4}x = \frac{p_1 V_1}{4}$$

(B) Since, $\Delta U = nC_V \Delta T = n\left(\frac{3R}{2}\right)\Delta T$

$$\Rightarrow \Delta U = \frac{3}{2}(p_2 V_2 - p_1 V_1) = \frac{3}{2}\left[\left(\frac{3}{2}p_1\right)(2V_1) - p_1 V_1\right]$$

$$\Rightarrow \Delta U = 3p_1 V_1$$

(C) When $V_2 = 3V_1$, $T_2 = 4T_1$, then $p_2 = \frac{4p_1}{3}$

$$\Rightarrow p_2 = \frac{4}{3}p_1 = p_1 + \frac{kx}{A}$$

$$\Rightarrow kx = \frac{p_1 A}{3}$$

$$\Rightarrow x = \frac{\Delta V}{A} = \frac{2V_1}{A}$$

$$\text{Since, } W_{\text{gas}} = (p_0 \Delta V + W_{\text{spring}}) = \left(p_1 A x + \frac{1}{2}(kx)x\right)$$

$$\Rightarrow W = +\left(p_1 A \frac{2V_1}{A} + \frac{1}{2} \frac{p_1 A}{3} \frac{2V_1}{A}\right)$$

$$\Rightarrow W = 2p_1 V_1 + \frac{p_1 V_1}{3} = \frac{7p_1 V_1}{3}$$

(D) By FLTD, we have $Q = \Delta U + W$

$$\Rightarrow Q = \frac{7p_1 V_1}{3} + \frac{3}{2}(p_2 V_2 - p_1 V_1)$$

$$\Rightarrow Q = \frac{7p_1 V_1}{3} + \frac{3}{2}\left(\frac{4}{3}p_1 3V_1 - p_1 V_1\right)$$

$$\Rightarrow Q = \frac{7p_1 V_1}{3} + \frac{9}{2}p_1 V_1 = \frac{41p_1 V_1}{6}$$

Hence, (A), (B) and (C) are correct.

9. $dQ = mCdT$

$$\Rightarrow \frac{dQ}{dT} = mC$$

For $0 < T < 100$ graph is approximately linear

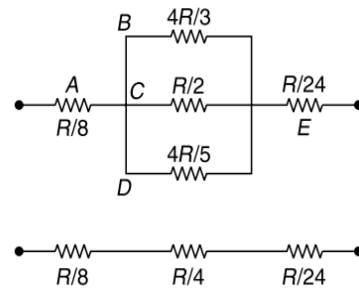
$$\Delta Q = m \int C dT = m(\text{area under } C-T \text{ graph})$$

For 400-500 K area is more than for 0-100 K, so

$\frac{dQ}{dT}$ increases for $200 < T < 300$ K as C increases in this region.

Hence, (A), (B), (C) and (D) are correct.

10. In steady state: (Heat In) = (Heat Out). So, A is true



OPTION (B) is also true because total heat is flowing through E.

$$(\text{Heat current}) = Q = \frac{\Delta T}{R}$$

Since, heat current (Q) is the same and

R_E is minimum. So, ΔT is minimum

Hence, OPTION (C) is true

$$\text{So, } Q_B = \frac{\Delta T}{\frac{4R}{3}}, Q_C = \frac{\Delta T}{\frac{4R}{2}} \text{ and } Q_D = \frac{\Delta T}{\frac{4R}{5}}$$

$$\Rightarrow Q_B + Q_D = Q_C$$

So, (D) is also true.

Hence, (A), (B), (C) and (D) are correct.

11. For monatomic gas, $C_p = \frac{5}{2}R$, $C_v = \frac{3}{2}R$, $C_p - C_v = R$

$$\text{For diatomic gas } C_p = \frac{7}{2}R, C_v = \frac{5}{2}R, C_p - C_v = R$$

So, $(C_p - C_v)$ is same for both

$$C_p + C_v = 6R \quad \{\text{for diatomic}\}$$

$$C_p + C_v = 4R \quad \{\text{for monatomic}\}$$

$$\text{So, } (C_p + C_v)_{\text{dia}} > (C_p + C_v)_{\text{mono}}$$

$$\frac{C_p}{C_v} = \frac{7}{5} = 1.4 \quad \{\text{for diatomic}\}$$

$$\frac{C_p}{C_v} = \frac{5}{3} = 1.66 \quad \{\text{for monatomic}\}$$

$$(C_p)(C_v) = \frac{35}{4}R^2 = 8.75R^2 \quad \{\text{for diatomic}\}$$

H.172 JEE Advanced Physics: Waves and Thermodynamics

$$(C_P)(C_V) = \frac{15}{4}R^2 \quad \text{\{for monatomic\}}$$

$$\text{So, } (C_P \cdot C_V)_{\text{diatomic}} > (C_P \cdot C_V)_{\text{monatomic}}$$

Hence, (B) and (D) are correct.

12. Since, the temperature of black body is constant, total heat absorbed = total heat radiated.
Hence, the correct answer is (D).

13. Let ℓ_0 be the initial length of each strip before heating.

Length after heating will be

$$\ell_B = \ell_0(1 + \alpha_B \Delta T) = (R + d)\theta$$

$$\text{and } \ell_C = \ell_0(1 + \alpha_C \Delta T) = R\theta$$

$$\Rightarrow \frac{R+d}{R} = \frac{(1 + \alpha_B \Delta T)}{(1 + \alpha_C \Delta T)}$$

$$\Rightarrow 1 + \frac{d}{R} = 1 + (\alpha_B - \alpha_C)\Delta T \quad \text{\{From binomial expansion\}}$$

$$\Rightarrow R = \frac{d}{(\alpha_B - \alpha_C)\Delta T}$$

$$\Rightarrow R \propto \frac{1}{\Delta T} \text{ and } \propto \frac{1}{|\alpha_B - \alpha_C|}$$

Hence, (B) and (D) are correct.

14. There is a decrease in volume during melting on an ice slab at 273 K. Therefore, negative work is done by ice-water system on the atmosphere or positive work is done on the ice-water system by the atmosphere. Hence, OPTION (B) is correct. Secondly heat is absorbed during melting (i.e., dQ is positive) and as we have seen, work done by ice-water system is negative (dW is negative). Therefore, From First Law of Thermodynamics

$$dU = dQ - dW$$

Change in internal energy of ice-water system, dU will be positive or internal energy will increase.

Hence, (B) and (C) are correct.

15. Average speed = $\langle v \rangle = \sqrt{\frac{8RT}{\pi M}}$,

$$\text{RMS speed} = v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \text{ and}$$

$$\text{Most probable speed} = v_p = \sqrt{\frac{2RT}{M}}$$

From these we have

$$v_p < \langle v \rangle < v_{\text{rms}}$$

Also, the average kinetic energy of the gaseous molecules is

$$\langle E \rangle = \frac{1}{2}mv_{\text{rms}}^2 = \frac{1}{2}m\left(\frac{3}{2}v_p^2\right) = \frac{3}{4}mv_p^2$$

Hence, (C) and (D) are correct.

16. For one mole of an ideal gas $PV = RT$
The coefficient of volume expansion at constant pressure is given by

$$\left(\frac{\Delta V}{\Delta T}\right)_P = \frac{R}{P} = \text{constant} \quad \text{\{OPTION (A)\}}$$

The average translational kinetic energy per molecule is $\frac{3}{2}kT$ and not $3kT$. With decrease of pressure, collisions between molecules will be less frequent, hence mean free path will increase.

The average translational kinetic energy of the molecules is independent of their nature, so each component of the gaseous mixture will have the same value of average translational kinetic energy.

Hence, (A) and (C) are correct.

17. Since both bodies emit total radiant power at the same rate.

$$\Rightarrow e_A \sigma (T_A^4) = e_B \sigma (T_B^4)$$

$$\Rightarrow 0.01(T_A^4) = 0.81(T_B^4)$$

$$\Rightarrow T_A = 3T_B$$

$$\Rightarrow T_B = \frac{1}{3}T_A = \frac{1}{3}(5802 \text{ K})$$

$$\Rightarrow T_B = 1934 \text{ K}$$

$$\lambda_A = \frac{1}{3}\lambda_B \text{ (due to Wien's Law)}$$

Since, $\lambda_B - \lambda_A = 1 \mu\text{m}$

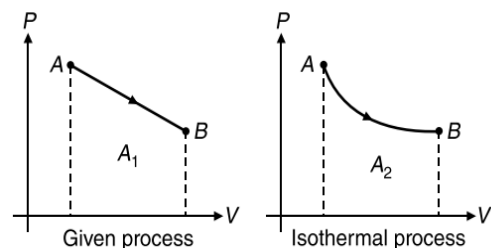
$$\Rightarrow \lambda_B - \frac{\lambda_B}{3} = 1 \mu\text{m}$$

$$\Rightarrow \frac{2}{3}\lambda_B = 1 \mu\text{m}$$

$$\Rightarrow \lambda_B = 1.5 \mu\text{m}$$

Hence, (A) and (B) are correct.

18. (a) Work done = Area under P - V graph



- (b) In the given process P - V equation will be a straight line with negative slope and positive intercept i.e., $P = -\alpha V + \beta$ (Here, α and β are positive constant)

$$\Rightarrow PV = -\alpha V^2 + \beta V$$

$$\Rightarrow nRT = -\alpha V^2 + \beta V$$

$$\Rightarrow T = \frac{1}{nR}(-\alpha V^2 + \beta V) \quad \dots(1)$$

This is an equation of parabola in T and V .

(c) $\frac{dT}{dV} = 0 = \beta - 2\alpha V$

$$\Rightarrow V = \frac{\beta}{2\alpha}$$

$$\text{Now, } \frac{d^2T}{dV^2} = -2\alpha = -ve$$

i.e., T has some maximum value.

Now, $T \propto PV$

and $(PA)_A = (PV)_B$

$$\Rightarrow T_A = T_B$$

We conclude that temperatures are same at A and B and temperature has a maximum value. Therefore, in going from A to B , T will first increase to a maximum value and then decrease.

Hence, (A), (B) and (D) are correct.

19. (a) $\Delta U = nC_V\Delta T = nC_V(T_2 - T_1)$ in all processes
 (b) For an adiabatic process $Q = 0$
 $\Rightarrow \Delta U = -W$
 $\Rightarrow |\Delta U| = |W|$
 (c) In isothermal process $\Delta T = 0$
 $\Rightarrow \Delta U = 0$ $\{\because \Delta U = nC_V\Delta T\}$
 (d) In adiabatic process $Q = 0$
 Hence, all options are correct.
Hence, (A), (B), (C) and (D) are correct.

Reasoning Based Questions

1. Total translational kinetic energy

$$K_T = \frac{3}{2}nRT = \frac{3}{2}pV = 1.5pV$$

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. **A** \rightarrow (q, r, s, u); **B** \rightarrow (q, s, r, u); **C** \rightarrow (s, r, q, t); **D** \rightarrow (q, r, p, u)

(A) $W_{1 \rightarrow 2 \rightarrow 3} = P_0V_0 = \frac{1}{3}RT_0$

(B) $\Delta U_{1 \rightarrow 2 \rightarrow 3} = \frac{3}{2} \left[\left(\frac{3P_0}{2} \right) (2V_0) - P_0V_0 \right] = RT_0$

(C) $Q_{1 \rightarrow 2 \rightarrow 3} = \Delta U + W = \frac{4}{3}RT_0$

(D) $Q_{1 \rightarrow 2} = \frac{1}{3}RT_0 + \frac{3}{2}(2P_0V_0 - P_0V_0) = \frac{5}{6}RT_0$

2. **A** \rightarrow (p, r, t, s); **B** \rightarrow (p, t, q, t); **C** \rightarrow (p, r, t, p); **D** \rightarrow (s, t, q, u)

Since, $W_{1 \rightarrow 2 \rightarrow 3} = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} = R \left(\frac{T_0}{3} \right) \ln \left(\frac{2V_0}{V_0} \right) + 0$

$$\Rightarrow W_{1 \rightarrow 2 \rightarrow 3} = \frac{RT_0}{3} \ln 2$$

$$\Delta U_{1 \rightarrow 2 \rightarrow 3} = \Delta U_{1 \rightarrow 2} + \Delta U_{2 \rightarrow 3} = 0 + nC_V \left(T_0 - \frac{T_0}{3} \right)$$

$$\Rightarrow \Delta U_{1 \rightarrow 2 \rightarrow 3} = (1) \left(\frac{3R}{2} \right) \left(\frac{2T_0}{3} \right) = RT_0$$

Along the path $1 \rightarrow 2$, we have $dT = 0$, so

$$Q_{1 \rightarrow 2} = \int TdX = \int T \left(\frac{3R}{2T} dT + \frac{R}{2} dV \right)$$

$$Q_{1 \rightarrow 2} = \int_{V_0}^{2V_0} \frac{RT}{V} dV = 0 + RT \ln 2$$

Similarly, along the path $2 \rightarrow 3$, we have $dV = 0$, so

$$Q_{2 \rightarrow 3} = \int TdX = \frac{3R}{2} \int_{\frac{T_0}{3}}^{T_0} dT = \left(\frac{3R}{2} \right) \left(\frac{2T_0}{3} \right) + 0$$

$$\Rightarrow Q_{2 \rightarrow 3} = RT_0$$

$$\Rightarrow Q_{1 \rightarrow 2 \rightarrow 3} = RT \ln 2 + RT_0 = \frac{1}{3}RT_0 (3 + \ln 2)$$

3. **A** \rightarrow (r); **B** \rightarrow (s); **C** \rightarrow (p); **D** \rightarrow (q)

Since $\left(\frac{dP}{dV} \right)_{\text{adia}} = \gamma \left(\frac{dP}{dV} \right)_{\text{isot}}$, so process I is adiabatic and

hence $Q = 0$

Process II is isobaric, so work done is

$$W_{\text{isob}} = P\Delta V = 3P_0(3V_0 - V_0) = 6P_0V_0$$

Process III is isochoric, so $W_{\text{isoc}} = 0$

Process IV is isothermal, so temperature is constant.

4. Since, $v_{\text{sound}} = \sqrt{\frac{\gamma RT}{M}}$. As the sound wave propagates, the air in a chamber undergoes compression and rarefaction very fastly, hence undergo a adiabatic process. So, curves are steeper than isothermal.

$$\left(\frac{dp}{dV} \right)_{\text{Adi}} = -\gamma \left(\frac{p}{V} \right) \quad \dots(1)$$

$$\left(\frac{dp}{dV} \right)_{\text{Iso}} = - \left(\frac{p}{V} \right) \quad \dots(2)$$

Graph Q satisfied Equation (1)

Hence, the correct answer is (C).

5. By FLTD, we have $Q = \Delta U + p\Delta V$

Since, $\Delta U \neq 0$, $W \neq 0$, $\Delta Q \neq 0$. The process represents, isobaric process, so

$$W_{\text{gas}} = -p\Delta V = -p(V_2 - V_1) = -pV_2 + pV_1$$

Graph P satisfies isochoric process.

Hence, the correct answer is (B).

6. Work done in isochoric process is zero.

$$W_{12} = 0 \text{ as } \Delta V = 0$$

Graph S represents isochoric process.

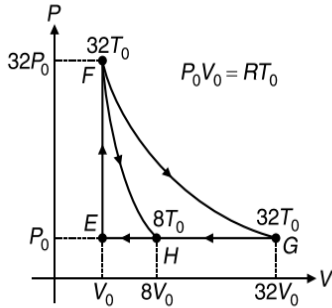
Hence, the correct answer is (B).

7. **A** \rightarrow (s); **B** \rightarrow (r); **C** \rightarrow (q); **D** \rightarrow (p)

FG is isothermal and FH is adiabatic, so we have

$$P_0V_G^{\frac{5}{3}} = 32P_0V_0^{\frac{5}{3}} \quad \{\because \gamma_{\text{monatomic}} = 5/3\}$$

$$\Rightarrow V_G = (32)^{\frac{3}{5}} V_0 = 8V_0$$

H.174 JEE Advanced Physics: Waves and Thermodynamics


$$\text{So, } W_{GE} = P_0(V_0 - 32V_0) = -31P_0V_0$$

$$W_{GH} = P_0(8V_0 - 32V_0) = -24P_0V_0$$

$$W_{FH} = \frac{P_0(8V_0) - 32P_0V_0}{1 - (5/3)} = \frac{-24P_0V_0}{-2/3} = 36P_0V_0$$

$$W_{FG} = 32RT_0 \log_e 32 = 32RT_0 \log_e 2^5$$

$$\text{Hence, } W_{FG} = 160RT_0 \log_e 2$$

8. **A** → (p, r, t); **B** → (p, r); **C** → (q, s); **D** → (r, t)

In AB temperature and volume are decreasing

In BC temperature decreases, volume does not change

In CD temperature and volume increase

In DA final temperature equals initial temperature. Also, volume decreases

For all processes, we have

$$\Delta U = nC_V \Delta T, W = \int PdV, Q = \Delta U + W$$

9. **A** → (q); **B** → (p, r); **C** → (p, s); **D** → (q, r)

(A) in free expansion, we have

$$W = \Delta U = Q = 0$$

For a polytropic process, we have

$$PV^x = \text{constant if } x < \gamma, Q > 0$$

$$x > \gamma, Q < 0$$

$$x = \gamma, Q = 0$$

$$x = 1, T = \text{constant}$$

$$x > 1, T = \text{decreases}$$

$$x < 1, T = \text{increases}$$

10. Conceptual

A → (s); **B** → (q); **C** → (p, q); **D** → (q, r)

11. **A** → (s); **B** → (p, r); **C** → (p); **D** → (q, s)

In process J → K V is constant whereas p is decreasing. Therefore, T should also decrease.

$$\Rightarrow W = 0, \Delta U = -ve \text{ and } Q < 0$$

In process K → L p is constant while V is increasing. Therefore, temperature should also increase.

$$\Rightarrow W > 0, \Delta U > 0 \text{ and } Q > 0$$

In process L → M This is inverse of process J → K.

$$\Rightarrow W = 0, \Delta U > 0 \text{ and } Q > 0$$

In process M → J

V is decreasing. Therefore, W < 0

$$(pV)_J < (pV)_M$$

$$\Rightarrow T_J < T_M$$

$$\Rightarrow \Delta U < 0$$

Therefore, Q < 0

Linked Comprehension Type Questions

1. Let final equilibrium temperature of gases is T
Heat rejected by gas in lower compartment is

$$Q_{\text{lower}} = nC_V \Delta T = 2 \times \frac{3}{2} R(700 - T)$$

Heat received by the gas in above compartment

$$Q_{\text{upper}} = nC_p \Delta T = 2 \times \frac{7}{2} R(T - 400)$$

Equating the two, we get

$$2100 - 3T = 7T - 2800$$

$$\Rightarrow T = 490 \text{ K}$$

Hence, the correct answer is (D).

2. By FLTD, we get

$$Q_1 = \Delta U_1 + W_1 \quad \dots(1)$$

$$\text{Also, } Q_2 = \Delta U_2 + W_2 \quad \dots(2)$$

$$\text{Since, } Q_1 + Q_2 = 0$$

$$\Rightarrow (nC_p \Delta T)_1 + (nC_p \Delta T)_2 = 0$$

$$\text{Since, } n_1 = n_2 = 2$$

$$\Rightarrow \frac{5}{2} R(T - 700) + \frac{7}{2} R(T - 400) = 0$$

Solving, we get T = 525 K

From equations (1) and (2), we get

$$\Delta W_1 + W_2 = -\Delta U_1 - \Delta U_2 \quad \{\because \Delta Q_1 + \Delta Q_2 = 0\}$$

$$\Rightarrow W_1 + W_2 = -[(nC_V \Delta T)_1 + (nC_V \Delta T)_2]$$

$$\Rightarrow W = -\left[2\left(\frac{3R}{2}\right)(525 - 700) + 2\left(\frac{5R}{2}\right)(525 - 400)\right]$$

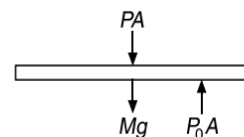
$$\Rightarrow W = -100R$$

Hence, the correct answer is (D).

3. Since it is open from top, pressure will be P₀.

Hence, the correct answer is (A).

4. Let P be the pressure in equilibrium



$$\text{Then } PA = P_0A - Mg$$

$$P = P_0 - \frac{Mg}{A} = P_0 - \frac{Mg}{\pi R^2} \quad \dots(1)$$

Applying $P_1V_1 = P_2V_2$

$$\Rightarrow P_0(2AL) = (P)(AL')$$

$$\Rightarrow L' = \frac{2P_0L}{P} = \left(\frac{P_0}{P_0 - \frac{Mg}{\pi R^2}} \right) (2L)$$

$$L' = \left(\frac{P_0\pi R^2}{\pi R^2 P_0 - Mg} \right) (2L)$$

Hence, the correct answer is (D).

5. Since we observe, $P_1 = P_2$

$$\Rightarrow P_0 + \rho g(L_0 - H) = P \quad \dots(1)$$

Now, applying $P_1V_1 = P_2V_2$ for the air inside the cylinder, we have

$$P_0(L_0) = P(L_0 - H)$$

$$\Rightarrow P = \frac{P_0L_0}{L_0 - H}$$

Substituting in equation (1), we have

$$P_0 + \rho g(L_0 - H) = \frac{P_0L_0}{L_0 - H}$$

$$\Rightarrow \rho g(L_0 - H)^2 + P_0(L_0 - H) - P_0L_0 = 0$$

Hence, the correct answer is (C).

Integer/Numerical Answer Type Questions

1. In an isothermal process, work done is

$$W_{\text{isot}} = RT \ln 4$$

In an adiabatic process, we have

$$T(4V_1)^{\frac{2}{3}} = T'(32V_1)^{\frac{2}{3}}$$

$$\Rightarrow T' = \frac{T}{4}$$

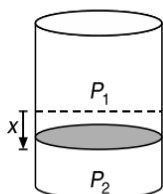
$$\text{So, } W_{\text{adia}} = \frac{2R \left(T - \frac{T}{4} \right)}{3} = \left(\frac{9RT}{8} \right)$$

$$\Rightarrow \frac{W_{\text{isot}}}{W_{\text{ad}}} = \frac{8}{9} \ln 4 = \frac{16}{9} \ln 2$$

$$\Rightarrow f = \frac{16}{9} = 1.78$$

2. Assuming the gas to be an ideal monoatomic gas

$$\Rightarrow C_V = \frac{3}{2}R$$



Let area of cross section of the cylinder be A .

Initially, we have $PV = nRT$

$$\Rightarrow P_0(4A) = 0.1R \times 300 \quad \text{(For both)}$$

Let partition shifts by x meter and final temperature be T , then we have (for lower portion)

$$P_2A(4-x) = 0.1RT$$

$$\Rightarrow P_2A = \frac{0.1RT}{4-x} = \frac{RT}{10(4-x)}$$

Similarly, for upper portion, we have

$$P_1A = \frac{RT}{10(4+x)}$$

But finally, in equilibrium, we have

$$P_1A + mg = P_2A$$

$$\Rightarrow P_2A - P_1A = mg = 83$$

$$\frac{RT}{10} \left(\frac{1}{4-x} - \frac{1}{4+x} \right) = 83$$

$$\Rightarrow \frac{RT}{10} \left(\frac{2x}{16-x^2} \right) = 83 \quad \dots(1)$$

Also, we have

$$0.2C_V \times 300 + mgx = 0.2C_V T$$

$$\Rightarrow \left(\frac{2}{10} \right) \left(\frac{3}{2}R \right) (300) + 83x = \left(\frac{2}{10} \right) \left(\frac{3}{2}R \right) T$$

$$\Rightarrow \frac{900R}{10} + 83x = 3 \left(\frac{RT}{10} \right)$$

$$\frac{300R}{10} + \frac{83}{3}x = \frac{RT}{10}$$

Using equation (1), we get

$$\left(30R + \frac{83x}{3} \right) \left(\frac{2x}{16-x^2} \right) = 83$$

$$\Rightarrow 83 \left(3 + \frac{x}{3} \right) \left(\frac{2x}{16-x^2} \right) = 83$$

$$\Rightarrow \frac{(9+x)(2x)}{3(16-x^2)} = 1$$

$$\Rightarrow 18x + 2x^2 = 48 - 3x^2$$

$$\Rightarrow 5x^2 + 18x - 48 = 0$$

$$\Rightarrow x = 1.78 \approx 2$$

So, the distance from top is 6 m.

3. For an adiabatic process, we have

$$\Delta P = - \left(\frac{rP}{V} \right) \Delta V$$

where, $P = p_0$ and $\Delta V = -(4\pi R^2)a$

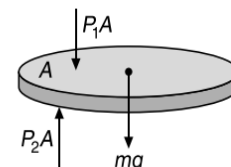
$$\Rightarrow \Delta P = \left(\frac{rp_0}{\frac{4}{3}\pi R^3} \right) (4\pi R^2)a = \frac{3rp_0a}{R}$$

Work done in the process is

$$W = -(\Delta P)_{av} \Delta V$$

Since the change is small, so $(\Delta P)_{av}$ equals the arithmetic

mean and hence $(\Delta P)_{av} = \frac{\Delta P}{2}$



H.176 JEE Advanced Physics: Waves and Thermodynamics

$$\Rightarrow W = -\left(\frac{\Delta P}{2}\right)(-4\pi R^2 a) = \left(\frac{3rp_0 a}{2R}\right)(4\pi R^2 a)$$

$$\Rightarrow W = (4\pi p_0 R a^2) \left(\frac{3r}{2}\right)$$

$$\Rightarrow W = (4\pi p_0 R a^2) \left(\frac{3}{2} \times \frac{41}{30}\right)$$

$$\Rightarrow W = (4\pi p_0 R a^2)(2.05)$$

$$\Rightarrow X = 2.05$$

4. In steady state, heat lost per second equals the heat gained per second.

$$\left(\frac{dQ}{dt}\right)_{\text{lost}} = \left(\frac{dQ}{dt}\right)_{\text{gained}} \quad \dots(1)$$

Since, $\left(\frac{dQ}{dt}\right)_{\text{gained}} = (700)(0.05) \text{ W}$

Also, according to Newton's Law of cooling, we have

$$\frac{1}{A} \frac{dQ}{dt} = (4\sigma e T_0^3) \Delta T$$

$$\Rightarrow \left(\frac{dQ}{dt}\right)_{\text{lost}} = (4\sigma e T_0^3) A \Delta T \quad \dots(2)$$

Given that the constant of proportionality for Newton's Law of cooling is 0.001 s^{-1}

Also, $-\frac{dT}{dt} = \left(\frac{4\sigma e A}{mc}\right) T_0^3 \Delta T$

So, constant of proportionality for Newton's Law of cooling

is $k = \left(\frac{4\sigma e A}{mc}\right) T_0^3$

$$\Rightarrow 4\sigma e T_0^3 A = k(mc)$$

Substituting in (2), we get

$$\left(\frac{dQ}{dt}\right)_{\text{lost}} = k(mc) \Delta T$$

$$\Rightarrow 700 \times 0.05 = (0.001)(4200) \Delta T$$

$$\Rightarrow \Delta T = \frac{25}{3} \text{ K}$$

$$\Rightarrow \Delta T = 8.33 \text{ K}$$

5. CASE-1: $5c(50) + 5L = c_2(30) \quad \dots(1)$

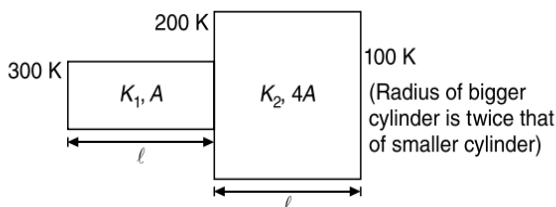
CASE-2: $80c(50 - 30) = c_2(80 - 50) \quad \dots(2)$

From equations (1) and (2), we get

$$1600c = 250c + 5L$$

$$\Rightarrow \frac{L}{c} = \frac{1350}{5} = 270 \text{ }^\circ\text{C}$$

6. In steady state, the rate of flow of heat through both the conducting cylinders will be equal. So, we have



$$\frac{K_1 A (300 - 200)}{\ell} = \frac{K_2 4A (200 - 100)}{\ell}$$

$$\Rightarrow \frac{K_1}{K_2} = 4 = 4.00$$

7. For an adiabatic process, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

where $\gamma = \gamma_{\text{monatomic}} = \frac{5}{3}$

$$\Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 100 \left(\frac{1}{8}\right)^{\frac{2}{3}}$$

$$\Rightarrow T_2 = 25 \text{ K}$$

Since, $\Delta U = nC_V \Delta T = 1 \left(\frac{3R}{2}\right) (T_2 - T_1)$

$$\Delta U = 1 \left(\frac{3}{2} \times 8\right) (25 - 100) = -900 \text{ J}$$

Hence, decrease in internal energy is 900 J

8. For reading of sensor to be 1, we have

$$\Rightarrow \log_2 \frac{P_1}{P_0} = 1$$

$$\Rightarrow \frac{P_1}{P_0} = 2$$

According to Stefan's law, we have

$$P \propto T^2$$

$$\Rightarrow \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{2767 + 273}{487 + 273}\right)^4 = 4^4$$

$$\Rightarrow \frac{P_2}{P_1} = \frac{P_2}{2P_0} = 4^4$$

$$\Rightarrow \frac{P_2}{P_0} = 2 \times 4^4$$

So, new reading is

$$\log_2 \left(\frac{P_2}{P_0}\right) = \log_2 2 + \log_2 4^4$$

$$\log_2 \left(\frac{P_2}{P_0}\right) = 1 + \log_2 (2^8) = 9$$

9. Power, $P_{\text{radiated}} = P = \sigma T^4 A = \sigma T^4 (4\pi R^2)$

$$\Rightarrow P \propto T^4 R^2 \quad \dots(1)$$

If λ is the wavelength at which peak occurs, then according to Wien's law, we have

$$\lambda \propto \frac{1}{T}$$

So, equation (1) becomes

$$P \propto \frac{R^2}{\lambda^4}$$

$$\Rightarrow \lambda \propto \left(\frac{R^2}{P}\right)^{\frac{1}{4}}$$

$$\Rightarrow \frac{\lambda_A}{\lambda_B} = \left(\frac{R_A}{R_B}\right)^{\frac{1}{2}} \left(\frac{P_B}{P_A}\right)^{\frac{1}{4}} = (400)^{\frac{1}{2}} \left(\frac{1}{10^4}\right)^{\frac{1}{4}} = 2$$

10. Since, $W_{ibf} = W_{ib} + W_{bf} = 50 \text{ J} + 100 \text{ J} = 150 \text{ J}$

Similarly, $W_{iaf} = W_{ia} + W_{af} = 0 + 200 \text{ J} = 200 \text{ J}$

Also, $Q_{iaf} = 500 \text{ J}$

So, by FLTD, we have $\Delta U_{iaf} = Q_{iaf} - W_{iaf}$

$$\Rightarrow \Delta U_{iaf} = 500 \text{ J} - 200 \text{ J} = 300 \text{ J}$$

$$\Rightarrow U_f - U_i = 300 \text{ J}$$

$$\Rightarrow U_f = U_{iaf} + U_i = 300 \text{ J} + 100 \text{ J} = 400 \text{ J}$$

Similarly, $\Delta U_{ib} = U_b - U_i = 200 \text{ J} - 100 \text{ J} = 100 \text{ J}$

By FLTD, we have $Q_{ib} = \Delta U_{ib} + W_{ib}$

$$\Rightarrow Q_{ib} = 100 \text{ J} + 50 \text{ J} = 150 \text{ J}$$

Similarly by FLTD, we have

$$Q_{ibf} = \Delta U_{ibf} + W_{ibf} = \Delta U_{iaf} + W_{ibf}$$

$$Q_{ibf} = 300 \text{ J} + 150 \text{ J} = 450 \text{ J}$$

So, the required ratio is

$$\frac{Q_{bf}}{Q_{ib}} = \frac{Q_{ibf} - Q_{ib}}{Q_{ib}} = \frac{450 - 150}{150} = 2$$

11. Increase in length is $\Delta l_1 = \frac{FL}{AY} = \frac{mgL}{\pi r^2 Y}$

On cooling, the decrease in length is $\Delta l_2 = L\alpha\Delta T$

To regain its original length, we must have $\Delta l_1 = \Delta l_2$

$$\Rightarrow \frac{mgL}{\pi r^2 Y} = L\alpha\Delta T$$

$$\Rightarrow m = \frac{r^2 Y \alpha \Delta T}{g}$$

Substituting the values, we get

$$m \approx 3 \text{ kg}$$

12. $S = 2100 \text{ Jkg}^{-1} \text{ }^\circ\text{C}^{-1}$

$$L = 3.36 \times 10^5 \text{ Jkg}^{-1}$$

$$420 = mc\Delta T + (1 \times 10^{-3})L$$

$$\Rightarrow 420 = mc(5) + 3.36 \times 10^2$$

$$\Rightarrow 420 - 336 = m(2100)(5)$$

$$\Rightarrow m = \frac{1000}{125} = 8 \text{ g}$$

13. Given that, $(\lambda_m)_B = 3(\lambda_m)_A$

$$\Rightarrow T_A = 3T_B$$

$$\text{So, } E_1 = 4\pi(6)^2 \sigma T_A^4 = 4\pi(6)^2 (3T_B)^4$$

$$\text{and } E_2 = 4\pi(18)^2 \sigma T_B^4$$

$$\Rightarrow \frac{E_1}{E_2} = 9$$

14. For adiabatic process,

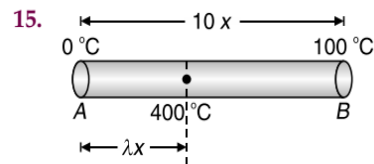
$$TV^{\gamma-1} = \text{constant}$$

$$\Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

$$\Rightarrow T_2 = T_1 (32)^{\frac{7}{5}-1}$$

$$\Rightarrow T_2 = 4T_1$$

$$\Rightarrow a = 4$$



$$\frac{400 - 0}{\frac{\lambda x}{KA}} = \frac{dm_{\ell}}{dt} \times L_{\text{ice}}$$

$$\text{Also, } \frac{400 - 100}{\frac{10x - \lambda x}{KA}} = \frac{dm_w}{dt} \times L_{\text{steam}}$$

$$\Rightarrow \frac{400(10 - \lambda)}{\lambda \times 300} = \frac{L_{\text{ice}}}{L_{\text{Steam}}} = \frac{80}{540} = \frac{4}{27}$$

$$\Rightarrow 9(10 - \lambda) = \lambda$$

$$\Rightarrow 90 = 10\lambda$$

$$\Rightarrow \lambda = 9$$