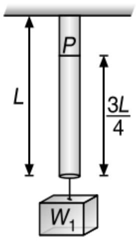
CHAPTER 1: MECHANICAL PROPERTIES OF MATTER

Test Your Concepts-I (Based on Young's Modulus, Longitudinal Stress and Strain)

1. Since the wire is uniform so the weight of wire below point P is $\frac{3W}{4}$

Total force at point P is $F = W_1 + \frac{3W}{4}$

At point P, Stress $= \frac{\text{Force}}{\text{Area}} = \frac{W_1 + \frac{3W}{4}}{S}$



2. $2T_C + T_S = mg$ ...(1)

$\Delta l_C = \Delta l_S$

$\Rightarrow \frac{T_C l}{AY_C} = \frac{T_S l}{AY_S}$

Since, $Y_S = 2Y_C$

$\Rightarrow \frac{T_C}{T_S} = \frac{1}{2}$ ...(2)

Solving these two equations, we get

$T_C = \frac{mg}{4}$ and $T_S = \frac{mg}{2}$

3. Increment in the wire of length AB is

$$\Delta L_{AB} = \frac{MgL_{AB}}{AY} = \frac{10\times10\times0.1}{10^{-7}\times2.5\times10^{10}}$$

$\Rightarrow \Delta L_{AB} = 4\times10^{-3}$ m = 4 mm

So, displacement of point B is

$\Delta L_B = \Delta L_{AB} = 4\times10^{-3}$ m = 4 mm

Increment in the wire of length BC is

$$\Delta L_{BC} = \frac{MgL_{BC}}{AY} = \frac{10\times10\times0.2}{10^{-7}\times4\times10^{10}}$$

$\Rightarrow \Delta L_{BC} = 5\times10^{-3}$ m = 5 mm

So, displacement of the point C is

$\Delta L_C = \Delta L_B + \Delta L_{BC} = 4$ mm + 5 mm = 9 mm

Increment in the wire of length CD is

$$\Delta L_{CD} = \frac{MgL_{CD}}{AY} = \frac{10\times10\times0.15}{10^{-7}\times4\times10^{10}}$$

$\Rightarrow \Delta L_{CD} = 15\times10^{-3}$ m = 15 mm

So, displacement of the point D is

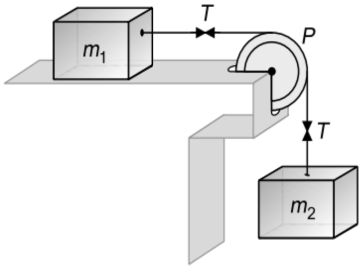
$\Delta L_D = \Delta L_C + \Delta L_{CD} = 9$ mm + 15 mm

$\Rightarrow \Delta L_D = 24$ mm

4. $\Delta l = \frac{Fl}{AY}$

$\Rightarrow \Delta l \propto \frac{1}{Y}$

5. If the system moves with acceleration a and T is the tension in the string W_2 then by comparing this condition from standard case $T = \frac{m_1 m_2}{m_1 + m_2} g$



In the given example

$m_1 = (m+m) = 2m$ and $m_2 = m$

$\Rightarrow$ Tension $= \frac{(m)(2m)g}{m+2m} = \frac{2}{3}mg$

$\Rightarrow$ Stress $= \frac{T}{a} = \frac{2}{3a}mg$

and Strain $= \frac{\text{Stress}}{\text{Young's modulus}} = \frac{2mg}{3aY}$

6. At first sight, it looks that rod B is stronger as it supports a larger load. But we cannot compare strengths without having a common basis of comparison. Since the rods have different cross-sectional area, therefore, the strengths can be compared by determining the load capacity per unit area.

For rod A

$$\sigma_1 = \frac{W_1}{A_1} = \frac{600\text{ N}}{10\text{ mm}^2} = \frac{600}{10\times10^{-6}}$$

$\sigma_1 = 60\times10^6$ Nm^{-2}

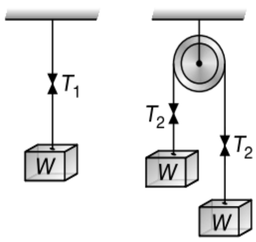
For rod B

$$\sigma_2 = \frac{W_2}{A_2} = \frac{6000\text{ N}}{1000\text{ mm}^2} = \frac{6000}{1000\times10^{-6}}$$

$\sigma_2 = 6\times10^6$ Nm^{-2}

Since $\sigma_1 > \sigma_2$, therefore, the material of rod A is stronger than that of rod B.

7. Since elongation in the wire is directly proportional to the tension in the wire



In first case $T_1 = W$ and in second case

$T_2 = \text{Thrust} = \frac{2W\times W}{W+W} = W$

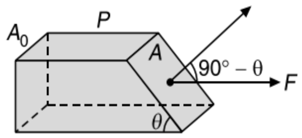
Since, $\frac{T_1}{T_2}=1$

$\Rightarrow \quad \frac{l_1}{l_2}=1$

$\Rightarrow \quad l_2 = l_1 = 1\text{ mm}$

8. Normal force $F_n = F\sin\theta$

Area $A=\frac{A_0}{\sin\theta}$



$$\Rightarrow \quad \sigma_n=\frac{F_n}{A}=\frac{F\sin\theta}{A_0/\sin\theta}=\frac{F}{A_0}\sin^2\theta$$

$$\Rightarrow \quad \frac{F_{max}\sin^2\theta}{A_0}=\sigma_0$$

$$\Rightarrow \quad F_{max}=\frac{\sigma_0 A_0}{\sin^2\theta}$$

9. Since, $\Delta l=\frac{FL}{AY}$, so for a given stretching force, we have

$$\Delta l \propto \frac{1}{AY}$$

Let three wires have young's moduli $2Y$, $2Y$, Y and cross sectional areas A, $2A$ and $3A$ respectively, so we have

$$\Delta l_1 : \Delta l_2 : \Delta l_3 = \frac{1}{A_1Y_1}:\frac{1}{A_2Y_2}:\frac{1}{A_3Y_3}$$

$$\Rightarrow \quad \Delta l_1 : \Delta l_2 : \Delta l_3 = \frac{1}{A(2Y)}:\frac{1}{(2A)(2Y)}:\frac{1}{(3A)Y}$$

$$\Rightarrow \quad \Delta l_1 : \Delta l_2 : \Delta l_3 = \frac{1}{2}:\frac{1}{4}:\frac{1}{3}$$

$$\Rightarrow \quad \Delta l_1 : \Delta l_2 : \Delta l_3 = 6:3:4$$

10. The maximum load that can be hung from a copper wire of diameter 0.42 mm is given by

$$F_{max}=(\sigma_{max})A$$

$$\Rightarrow \quad F_{max}=(3\times10^8)\left(\frac{\pi}{4}\right)(0.42\times10^{-3})^2$$

$$\Rightarrow \quad F_{max}=41.6\text{ N}$$

The percentage of length of wire that will stretch is given by

$$\frac{\Delta l}{l}\times100=\frac{F}{2YA}\times100$$

$$\Rightarrow \quad \frac{\Delta l}{l}\times100=\frac{41.6\times100}{2(1.1\times10^{11})\left(\frac{\pi}{4}\right)(0.42\times10^{-3})^2}$$

$$\Rightarrow \quad \frac{\Delta l}{l}\times100\% = 0.136\%$$

11. The cross-sectional area of column is

$$A=\pi r^2=\pi(0.045\text{ m})^2$$

$$\Rightarrow \quad A=6.36\times10^{-3}\text{ m}^2$$

Since, $Y=\left(\frac{F}{A}\right)\left(\frac{\Delta L}{L}\right)$

$$\Rightarrow \quad \Delta L=\frac{FL}{AY}=\frac{mgL}{AY}$$

$$\Rightarrow \quad \Delta L=\frac{(8\times10^4)(9.8\text{ N})(4\text{ m})}{(6.36\times10^{-3}\text{ m}^2)(1.9\times10^{11}\text{ Nm}^{-2})}$$

$$\Rightarrow \quad \Delta L=2.6\times10^{-3}\text{ m}=2.6\text{ mm}$$

12. $Y=\frac{Fl}{A\Delta l}=\frac{(5000)(4)}{(0.5\times10^{-4})(0.2\times10^{-2})}$

$$\Rightarrow \quad Y=2.0\times10^{11}\text{ Nm}^{-2}$$

13. Stress is defined as

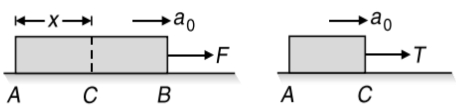
$$\sigma=\frac{F}{A}$$

Since both F and A are same, hence the ratio of stresses is 1. Strain is given by

$$\text{Strain}=\frac{\text{Stress}}{Y}\propto\frac{1}{Y}$$

$$\Rightarrow \quad \frac{(\text{Strain})_{steel}}{(\text{Strain})_{copper}}=\frac{Y_{copper}}{Y_{steel}}=\frac{13}{20}$$

14. Free body diagram of AB is shown in Figure.



Tension T at C is given by

$$T=(m_{AC})a_0=(xS\rho)a_0$$

$$\Rightarrow \quad \text{Stress}=\frac{T}{S}=\rho x a_0$$

At $x=\frac{L}{2}$, Stress $=\frac{\rho L a_0}{2}$

Total elongation is

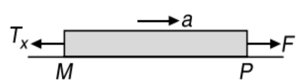
$$\Delta l=\int_0^L\frac{(\text{Stress})dx}{Y}$$

$$\Rightarrow \quad \Delta l=\int_0^L\frac{(\rho x a_0)dx}{Y}=\frac{\rho a_0 L^2}{2Y}$$

15. **(a)** Acceleration of the rod, $a=\frac{F}{m}$

$$F-T_x=(m_{PM})a=\left(\frac{m}{l}x\right)\left(\frac{F}{m}\right)$$

$$\Rightarrow \quad T_x=F\left(1-\frac{x}{l}\right)$$



(b) Stress $\sigma=\frac{F}{A}=\frac{T_x}{A}=\frac{F}{A}\left(1-\frac{x}{l}\right)$

(c) Change in length $\Delta l = \int_0^{\ell} \frac{T_x dx}{AY}$

$$\Rightarrow \Delta l = \int_0^1 \frac{F\left(1-\frac{x}{l}\right)dx}{AY} = \frac{Fl}{2AY}$$

(d) Strain, $\frac{\Delta l}{l} = \frac{F}{2AY}$

16. (a) $\sigma = \frac{F}{A} = \frac{8000\times 9.8}{\frac{\pi}{4}\times(15\times10^{-2})^2} = 4.4\times10^6 \text{ Nm}^{-2}$

(b) $\varepsilon = \frac{\sigma}{Y} = \frac{4.4\times10^6}{2.0\times10^{11}} = 2.2\times10^{-5}$

(c) $\Delta l = L\varepsilon = 3.0\times2.2\times10^{-5} = 6.6\times10^{-5}$ m

17. Increase in circumference of the ring is

$$\Delta l = 2\pi(R-r)$$

$$\Rightarrow \text{Strain} = \frac{2\pi(R-r)}{2\pi r} = \frac{R-r}{r}$$

Since, we know that

$$\text{Stress} = Y(\text{Strain}) = Y\left(\frac{R-r}{r}\right)$$

$$\Rightarrow \text{Tension} = (\text{Stress})A = Ybd\left(\frac{R-r}{r}\right)$$

Test Your Concepts-II (Based on Elastic Energy, Energy Density and Poisson's Ratio)

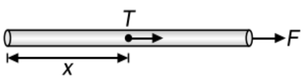
1. The elastic energy is given by

$$U = \frac{1}{2}\times Y\times(\text{strain})^2\times\text{volume}$$

$$\Rightarrow U = \frac{1}{2}\times2\times10^{11}\times(10^{-3})^2\times\frac{3.1}{7800}$$

$$\Rightarrow U = 39.74 \text{ J}$$

2. The tension T in the string at a distance x from its free end is given as



$$T = F\left(\frac{x}{l}\right)$$

Hence the stress is

$$\sigma = \frac{T}{A} = \frac{F}{Al}x$$

Since we know that

$$dU = \frac{(\text{Stress})^2}{2Y}dV$$

where $dV = Adx$

$$\Rightarrow U = \frac{1}{2Y}\int\sigma^2 dV = \frac{1}{2Y}\int\left(\frac{F}{Al}x\right)^2 Adx$$

$$\Rightarrow U = \frac{1}{2Y}\int_0^l \frac{F^2}{A^2l^2}x^2 Adx$$

$$\Rightarrow U = \frac{F^2 l}{6AY}$$

3. Energy stored

$$U = \frac{1}{2}\text{ (stress) (strain) (volume)}$$

$$\Rightarrow U = \frac{1}{2}\left(\frac{F}{A}\right)\left(\frac{\Delta l}{l}\right)(Al)$$

$$\Rightarrow U = \frac{1}{2}F\Delta l$$

$$\Rightarrow U = \frac{1}{2}(100)(0.3\times10^{-3})$$

$$\Rightarrow U = 0.015 \text{ J}$$

4. The stress in steel wire,

$$\sigma = \frac{F}{A} = \frac{36}{1.2\times10^{-7}} = 3\times10^8 \text{ Nm}^{-2}$$

As Young's modulus of elasticity, $Y = \frac{\text{Stress}}{\text{Strain}}$

$$\text{Strain} = \frac{\text{Stress}}{Y} = \frac{3\times10^8}{1.8\times10^{11}} = \frac{5}{3}\times10^{-3}$$

Increase of length is

$$\Delta l = l(\text{strain}) = 2(1.67\times10^{-3})$$

$$\Rightarrow \Delta l = 3.34\times10^{-3} \text{ m} = 3.34 \text{ mm}$$

Work done is given by

$$W = \frac{1}{2}(\text{stress})(\text{strain})(\text{volume})$$

$$\Rightarrow W = \frac{1}{2}(3\times10^8)\left(\frac{5}{3}\times10^{-3}\right)(2\times1.2\times10^{-7})$$

$$\Rightarrow W = 6\times10^{-2} \text{ J} = 60 \text{ mJ}$$

5. Work done = Potential energy stored

$$\Rightarrow W = \frac{1}{2}k(\Delta l)^2$$

Since, $k = \frac{YA}{l}$

$$\Rightarrow W = \frac{1}{2}\left(\frac{YA}{l}\right)(\Delta l)^2$$

Substituting the values, we get

$$W = \frac{1}{2}\frac{(2\times10^{11})(10^{-6})}{(2)}(0.1\times10^{-3})^2$$

$$\Rightarrow W = 5\times10^{-4} \text{ J}$$

6. By Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in}\\ \text{Gravitational}\\ \text{Potential Energy}\end{pmatrix} = \begin{pmatrix}\text{Gain in Elastic}\\ \text{Potential Energy}\\ \text{in Both Leg Bones}\end{pmatrix}$$

$$\Rightarrow mgh = 2\times\left(\frac{1}{2}\text{stress}\times\text{strain}\times\text{volume}\right)$$

Since we have, $m = 45$ kg, $h = 2$ m, $L = 0.50$ m and $A = 5\times10^{-4}$ m^2, so we get

$$mgh = (45)(10)(2) = 900\ \text{J}$$

$$\Rightarrow \quad 900 = 2\left[\frac{1}{2}(0.9\times10^8)\varepsilon(2.5\times10^{-4})\right]$$

$$\Rightarrow \quad \text{Strain} = \varepsilon = 0.04$$

$$\Rightarrow \quad Y = \frac{\text{Stress}}{\text{Strain}} = \frac{0.9\times10^8}{0.04} = 2.25\times10^9\ \text{Nm}^{-2}$$

7. A stretched catapult has elastic potential energy stored in it. So, elastic potential energy stored in both the rubber strings is

$$U = \left(\frac{1}{2}\frac{YAl^2}{L}\right)\times 2$$

This energy U, when imparted to the stone, will make it rise through a vertical height h so that the energy possessed by the stone is

$$U = mgh$$

By Law of Conservation of Energy, we get

$$\Rightarrow \quad \frac{YAl^2}{L} = mgh$$

$$\Rightarrow \quad Y = \frac{mghL}{Al^2} = \frac{100\times10^{-3}\times10\times25\times10^{-1}}{10\times10^{-6}\times(5\times10^{-2})^2}$$

$$\Rightarrow \quad Y = 1\times10^8\ \text{Nm}^{-2}$$

8. Given that, $l = 4.0\ \text{m}$, $\Delta l = 2\times10^{-3}\ \text{m}$, $A = 2.0\times10^{-6}\ \text{m}^2$, $Y = 2.0\times10^{11}\ \text{Nm}^{-2}$

The energy density of stretched wire

$$u_e = \frac{1}{2}\times\text{stress}\times\text{strain}$$

$$\Rightarrow \quad u_e = \frac{1}{2}\times Y\times(\text{strain})^2$$

$$\Rightarrow \quad u_e = \frac{1}{2}\times2.0\times10^{11}\times\left(\frac{(2\times10^{-3})}{4}\right)^2$$

$$\Rightarrow \quad u_e = 0.25\times10^5 = 2.5\times10^4\ \text{Jm}^{-3}$$

$$\text{Now,}\ \begin{pmatrix}\text{Elastic}\\ \text{potential}\\ \text{energy}\end{pmatrix} = \begin{pmatrix}\text{energy}\\ \text{density}\end{pmatrix}\times(\text{volume})$$

$$\Rightarrow \quad U = 2.5\times10^4\times(2.0\times10^{-6})\times4.0\ \text{J}$$

$$\Rightarrow \quad U = 20\times10^{-2} = 0.20\ \text{J}$$

Test Your Concepts-III (Based on Shear Modulus, Tangential Stress, Shear Strain)

1. $$\text{Shear Stress} = \frac{\text{Tangential Force}}{\text{Area of Face}}$$

$$\Rightarrow \quad \text{Shear Stress} = \frac{0.50\ \text{N}}{15\times10^{-4}\ \text{m}^2} = 333\ \text{Pa}$$

$$\begin{pmatrix}\text{Shear}\\ \text{Strain}\end{pmatrix} = \frac{\begin{pmatrix}\text{Displacement of Upper Face}\\ \text{wrt Lower Fixed Face}\end{pmatrix}}{(\text{Separation between Faces})}$$

$$\Rightarrow \quad \text{Shear Strain} = \frac{0.4\ \text{cm}}{3\ \text{cm}} = 0.133$$

Shear modulus is given by

$$\eta = \frac{\text{Tangential Stress}}{\text{Shear strain}}$$

$$\Rightarrow \quad \eta = \frac{333\ \text{Pa}}{0.133} = 2.5\ \text{kPa}$$

2. If the top face moves a distance Δx relative to the bottom face, then magnitude of the force required to produce this change in shape is given by

$$F = \eta A\frac{\Delta x}{L_0}$$

$$\Rightarrow \quad \eta = \frac{FL_0}{A\Delta x}$$

where, $A = (0.07)(0.07)\ \text{m}^2$ is the area of the top surface, and $L_0 = 0.03\ \text{m}$ is the separation between the top surface and bottom surface.

$$\Rightarrow \quad \eta = \frac{FL_0}{A\Delta x} = \frac{(0.49)(0.03)}{(0.07)(0.07)(6\times10^{-3})}$$

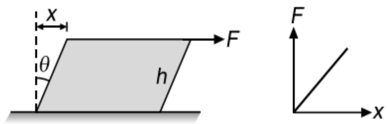
$$\Rightarrow \quad \eta = 500\ \text{Nm}^{-2}$$

This is a small value for modulus of rigidity and so the butter block can be deformed easily.

3. Since, $\text{Stress} = \eta(\text{Strain})$

$$\Rightarrow \quad \sigma = \frac{F}{A} = \eta\left(\frac{x}{h}\right)$$

$$\Rightarrow \quad x = \frac{\sigma h}{\eta}$$



Since force changes linearly with x, we can write its work done as

$$W = \frac{1}{2}Fx = \frac{1}{2}(\sigma h^2)\left(\frac{\sigma h}{\eta}\right)$$

$$\Rightarrow \quad W = \frac{\sigma^2 h^3}{2\eta}$$

4. The shear stress,

$$\frac{F}{A} = \frac{mg}{A} = \frac{(160)(10)}{3.2\times10^{-4}}$$

$$\Rightarrow \quad \frac{F}{A} = 5\times10^6\ \text{Nm}^{-2}$$

The vertical deflection Δl of the right end of the bar is

$$\Delta l = \left(\frac{F}{A}\right)\frac{L_0}{\eta}$$

$$\Rightarrow \quad \Delta l = \frac{(5\times10^6)(0.10)}{8\times10^{10}} = 6.25\times10^{-6}\ \text{m}$$

Test Your Concepts-IV (Based on Bulk's Modulus, Normal Stress and Volumetric Strain)

1. Volume of the spherical body $V = \frac{4}{3}\pi R^3$

$$\Rightarrow \quad \frac{\Delta V}{V} = 3\frac{\Delta R}{R}$$

$$\Rightarrow \quad \frac{\Delta R}{R} = \frac{1}{3}\frac{\Delta V}{V} \qquad \ldots(1)$$

Now Bulk modulus is defined as

$$B = -V\frac{\Delta P}{\Delta V}$$

Since $\Delta P = \frac{mg}{A}$

$$\Rightarrow \left|\frac{\Delta V}{V}\right| = \frac{\Delta P}{B} = \frac{mg}{AB} \quad \ldots(2)$$

Substituting the value of $\frac{\Delta V}{V}$ from equation (2) in equation (1) we get

$$\frac{\Delta R}{R} = \frac{mg}{3AB}$$

2. Since, $B = \frac{-dP}{dV/V} = \frac{1.8\times10^6}{\left(\frac{0.3}{800}\right)} = 4.8\times10^9\ \text{Nm}^{-2}$

Compressibility k is given by

$$k = \frac{1}{B} = \frac{1}{4.8\times10^9} = 2.1\times10^{-10}\ \text{m}^2\text{N}^{-1}$$

3. $B = \frac{\Delta p}{\frac{\Delta V}{V}} = \frac{\rho g h}{\left(\frac{\Delta V}{V}\right)}$

$$\Rightarrow B = \frac{(10^3)(9.8)(180)}{\left(\frac{0.1}{100}\right)}$$

$$\Rightarrow B = 1.76\times10^9\ \text{Nm}^{-2}$$

4. $B = \frac{-\Delta P}{\frac{\Delta V}{V}}$

$$\Rightarrow \left|\frac{\Delta V}{V}\times100\right| = \frac{\Delta P}{B}\times100 = \frac{1.01\times10^5}{4\times10^{10}}\times10^2$$

$$\Rightarrow \left|\frac{\Delta V}{V}\times100\right| = 2.525\times10^{-4}\%$$

5. Pressure increases with depth of a liquid. At a depth h below the water surface increase in pressure is given by

$$\Delta P = \rho g h$$

Using the equation, $\Delta\rho = \rho\frac{\Delta P}{B}$

we get $\Delta\rho = \frac{\rho(\rho g h)}{B} = \frac{\rho^2 g h}{B}$

Substituting the values, we have

$$\Delta\rho = \frac{(10^3)^2(10)(10^3)}{2.3\times10^9} = 4.33\ \text{kgm}^{-3}$$

6. The changed density is given by

$$\rho' = \frac{\rho}{1-\frac{dp}{B}}$$

Substituting the value, we have

$$\rho' = \frac{11.4}{1-\frac{2\times10^8}{8\times10^9}}$$

$$\Rightarrow \rho' = 11.69\ \text{gcm}^{-3}$$

7. Here, $B = 2.3\times10^9\ \text{Nm}^{-2}$

$$\Rightarrow B = \frac{2.3\times10^9}{1.01\times10^5}$$

$$\Rightarrow B = 2.27\times10^4\ \text{atm}$$

(a) Compressibility k is

$$k = \frac{1}{B} = \frac{1}{2.27\times10^4} = 4.4\times10^{-5}\ \text{atm}^{-1}$$

(b) Here, $\frac{\Delta V}{V} = -0.1\% = -0.001$

Required increase in pressure,

$$\Delta P = B\times\left(-\frac{\Delta V}{V}\right)$$

$$\Rightarrow \Delta P = 2.27\times10^4\times0.001$$

$$\Rightarrow \Delta P = 22.7\ \text{atm}$$

8. Since, $U = \frac{(\text{Stress})^2}{2B}$

$$\Rightarrow U = \frac{(\rho g h)^2}{2B}$$

Here, $\frac{1}{B}$ = compressibility $= 4.9\times10^{-10}\ \text{m}^2\text{N}^{-1}$

$$\Rightarrow U = \frac{(10^3\times9.8\times10^3)^2}{2}\times4.9\times10^{-10}$$

$$\Rightarrow U = 235\times10^2\ \text{Jm}^{-3} = 23.5\ \text{kJm}^{-3}$$

9. Since Bulk's modulus is

$$B = \frac{-\Delta P}{\left(\frac{\Delta V}{V}\right)}$$

Substituting the values, we get

$$B = \frac{(1.165-1.01)\times10^5}{\left(\frac{10}{100}\right)} = 1.55\times10^5\ \text{Nm}^{-2}$$

Test Your Concepts-V (Based on Fluid Properties, Pressure and Pascal's Law)

1. Density of oil in CGS units is numerically equal to the relative density. So, we have

$$\rho_{cgs} = (\text{RD})\ \text{gcm}^{-3} = 0.8\ \text{gcm}^{-3}$$

$$\Rightarrow \rho_{SI} = 800\ \text{kgm}^{-3}$$

2. Volume of fat in 1000 cm^3 of milk is

$$V = 4\% \times 1000\text{ cm}^3 = 40\text{ cm}^3$$

Mass of 40 cm^3 fat is

$$m = \rho V$$

$$\Rightarrow \quad m = \left(865\text{ kgm}^{-3}\right)\left(40\times10^{-6}\text{ m}^3\right)$$

$$\Rightarrow \quad m = 0.0346\text{ kg}$$

Density of skimmed milk is

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{(1.032-0.0346)\text{kg}}{(1000-40)\times10^{-6}\text{ m}^3}$$

$$\Rightarrow \quad \rho = 1039\text{ kgm}^{-3}$$

3. Since, $P = \dfrac{F}{A}$

$$\Rightarrow \quad F = PA = \left(1.01\times10^5\text{ Nm}^{-2}\right)\left(2\times10^{-4}\text{ m}^2\right)$$

$$\Rightarrow \quad F \approx 20\text{ N}$$

4. Pressure (and hence force) on the two equal base areas are identical. Since force is exerted by water on the sides of the vessels also, which has a non-zero vertical component when the sides of the vessel are not perfectly normal to the base. This net vertical component of force by water on the sides of the vessel is greater for the first vessel than the second. Hence, the vessels weigh different whereas force on the base is the same in the two cases.

5. The pressure at the surface is equal to the atmospheric pressure $P_0 = 10^5\text{ Nm}^{-2}$.
The pressure at the bottom of the beaker is
$P = P_0 + h\rho g$, where

$$h\rho g = (0.1)(13600)(10) = 13600\text{ Nm}^{-2}$$

$$\Rightarrow \quad P = 10^5\text{ Nm}^{-2} + 13600\text{ Nm}^{-2}$$

$$\Rightarrow \quad P \approx 1.14\times10^5\text{ Nm}^{-2}$$

So, force exerted by the mercury at the bottom of the beaker is

$$F = PA$$

where, $A = \pi\left(\dfrac{4}{100}\right)^2 \approx 5\times10^{-3}\text{ m}^2$

$$\Rightarrow \quad F = \left(1.14\times10^5\right)\left(5\times10^{-3}\right) = 570\text{ N}$$

6. Absolute pressure $P = P_a + \rho g h$
where $P_a = 1.01\times10^5\text{ Pa}$

$$\rho = 1.03\times10^3\text{ kgm}^{-3}$$

$$\Rightarrow \quad P = 1.01\times10^5\text{ Pa} + 1.03\times10^3\times10\times500\text{ Pa}$$

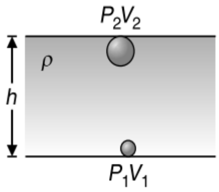
$$\Rightarrow \quad P = 1.01\times10^5\text{ Pa} + 51.5\times10^5\text{ Pa}$$

$$\Rightarrow \quad P = 52.5\times10^5\text{ Pa}$$

$$\Rightarrow \quad P \approx 52\text{ atm}$$

7. Since the bell is brought gradually to the surface of the lake, so the pressure volume relation for air inside the bell is isothermal. So according to Boyle's law, we have

$$P_1V_1 = P_2V_2$$



$$\Rightarrow \quad (P_0 + h\rho_w g)V_1 = P_0V_2$$

$$\Rightarrow \quad V_2 = \left(1 + \frac{h\rho_w g}{P_0}\right)V_1$$

Since $P_2 = P_0 = 70\text{ cm of Hg}$, so we have

$$P_2 = 70\times13.6\times1000\text{ Nm}^{-2}$$

$$\Rightarrow \quad V_2 = \left(1 + \frac{47.6\times10^2\times1\times1000}{70\times13.6\times1000}\right)V_1$$

$$\Rightarrow \quad V_2 = (1+5)50 = 300\text{ cm}^3$$

8. Force at the bottom of vessel equals the weight of liquid. So, we have

$$F_{\text{bottom}} = \left(\pi R^2\right)h\rho g$$

Force on side walls of vessel

$$F_{\text{curved}} = \left(\frac{h}{2}\rho g\right)(2\pi Rh)$$

According to the problem, we have

$$\pi R^2 \rho g = \left(\frac{h}{2}\rho g\right)(2\pi Rh)$$

$$\Rightarrow \quad h = R$$

9. Difference of pressure between sea level and the top of hill is

$$\Delta P = (h_1 - h_2)\times\rho_{Hg}\times g$$

$$\Rightarrow \quad \Delta P = (75-50)\times10^{-2}\times\rho_{Hg}\times g \quad \ldots(1)$$

and pressure difference due to h meter of air is

$$\Delta P = h\times\rho_{\text{air}}\times g \quad \ldots(2)$$

Equating (1) and (2), we get

$$h\times\rho_{\text{air}}\times g = (75-50)\times10^{-2}\times\rho_{Hg}\times g$$

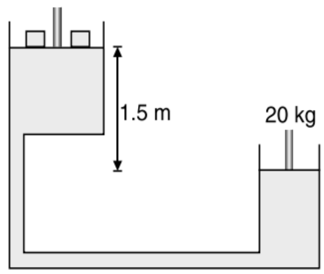
$$\Rightarrow \quad h = 25\times10^{-2}\left(\frac{\rho_{Hg}}{\rho_{\text{air}}}\right) = 25\times10^{-2}\times10^4$$

$$\Rightarrow \quad h = 2500\text{ m} = 2.5\text{ km}$$

10. Since pressure at same level in the liquid should be the same, so we must have

$$P_{\text{larger piston}} + h\rho g = P_{\text{smaller piston}} \quad \ldots(1)$$

where $h = 1.5\text{ m}$ and $\rho = 750\text{ kgm}^{-3}$



If P_{atm} be the atmospheric pressure, then pressure on the smaller piston is

$$P_{\substack{\text{smaller}\\\text{piston}}} = P_{\text{atm}} + \frac{20\times9.8}{\pi\times\left(5\times10^{-2}\right)^2}\text{ Nm}^{-2}$$

and pressure on the larger piston is

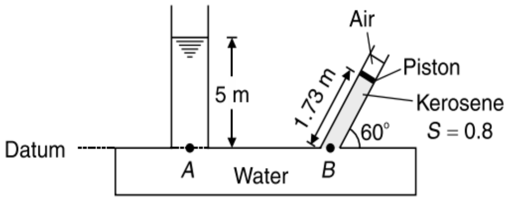
$$P_{\substack{\text{larger}\\ \text{piston}}} = P_{\text{atm}} + \frac{F}{\pi \times (17.5 \times 10^{-2})^2}\ \text{Nm}^{-2}$$

Substituting these values in equation (1), we get

$$\frac{20 \times 9.8}{\pi (5 \times 10^{-2})^2} - \frac{F}{\pi (17.5 \times 10^{-2})^2} = (1.5)(750)(9.8)$$

$$\Rightarrow \quad F = 1.3 \times 10^3\ \text{N} = 1.3\ \text{kN}$$

11. Let p_a be the air pressure above the piston and p_0 be the atmospheric pressure. Then by applying Pascal's Law at points A and B, we get

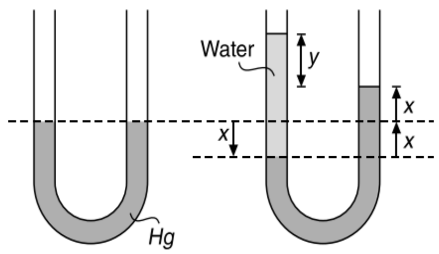


$$p_0 + \rho_w g(5) = p_a + \rho_k g(1.73)\sin 60^\circ$$

$$p_a = (10^3)(10)(5) + 10^5 - (800)(10)(1.73)\frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad p_a = 138\ \text{kPa}$$

12. The situation is shown in Figure.



If water depresses the mercury by x, the mercury in the other limb will rise by x above its initial level (as fluids are incompressible), so we have

$$(27.2)\rho_w g = (2x)\rho_{\text{Hg}} g$$

$$\Rightarrow \quad 27.2 = 2(13.6)x$$

$$\Rightarrow \quad x = 1\ \text{cm}$$

Also, from the diagram, we see that

$$y + 2x = 27.2\ \text{cm}$$

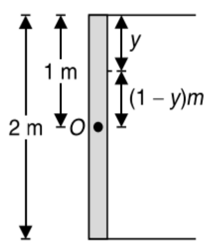
$$\Rightarrow \quad y = 25.2\ \text{cm}$$

13. The pressure at depth $y = \rho g y$

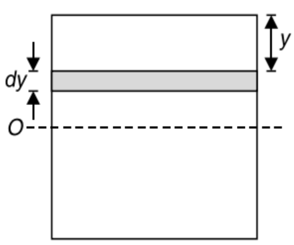
Force on small element step $= 3\rho g y\, dy$

Torque about O due to this force

$$d\tau = 3\rho g y\, dy(1 - y)$$



Side View



Front View

Total torque $\tau = \int d\tau = 3\rho g \int_0^2 (y - y^2)\, dy$

$$\Rightarrow \quad \tau = 3\rho g \left(\frac{y^2}{2} - \frac{y^3}{3} \right)\Bigg|_0^2$$

$$\Rightarrow \quad \tau = 3\rho g \left(2 - \frac{8}{3} \right) = -2\rho g = -2 \times 10^4\ \text{Nm}^{-1}$$

Negative sign indicates that gate tends to rotate in clockwise direction.

14. In equilibrium

$$\frac{600 \times 10}{800 \times 10^{-4}} = \frac{F}{25 \times 10^{-4}} + h\rho g$$

$$\Rightarrow \quad \frac{F}{25 \times 10^{-4}} = \frac{60}{8} \times 10^4 - 8 \times (0.75 \times 10^3) \times 10$$

$$\Rightarrow \quad \frac{F}{25 \times 10^{-4}} = 1.5 \times 10^4$$

$$\Rightarrow \quad F = 37.5\ \text{N}$$

15. Let P be the pressure in C. Then

$$P + h_1 \rho_w g = P + h_2 \rho g$$

$$\Rightarrow \quad \rho = \left(\frac{h_1}{h_2} \right)\rho_w = \left(\frac{10}{12} \right)(1.0) = 0.83\ \text{gcm}^{-3}$$

16. Pressure at A is

$$P_A = P_{\text{atm}} - \rho_1 g \ell \sin\theta \qquad \ldots(1)$$

Pressure at B is

$$P_B = P_{\text{atm}} + \rho_2 g h \qquad \ldots(2)$$

Also, we can say that pressure at B is

$$P_B = P_A + \rho_3 g \ell \sin\theta \qquad \ldots(3)$$

$$\Rightarrow \quad P_{\text{atm}} + \rho_2 g h = P_A + \rho_3 g \ell \sin\theta$$

$$\Rightarrow \quad P_{\text{atm}} + \rho_2 g h = P_{\text{atm}} - \rho_1 g \ell \sin\theta + \rho_3 g \ell \sin\theta$$

$$\Rightarrow \quad \sin\theta = \frac{\rho_2 h}{(\rho_3 - \rho_1)\ell}$$

17. Let the pressure of the liquid just below the piston be P. The forces acting on the piston are

(a) its weight, mg (downward)

(b) force due to the air above it, $P_0 A$ (downward)

(c) force due to the liquid below it, PA (upward).

For the piston to be in equilibrium, we have

$$PA = P_0 A + mg$$

$$\Rightarrow \quad P = P_0 + \frac{mg}{A}$$

18. The force exerted by liquid on the gate is equal to the product of the average pressure acting on the gate with the vertical projected area of the gate.

$$F_H = P_{\text{av}} \left(\begin{array}{c} \text{Vertical Projected} \\ \text{Area of the Gate} \end{array} \right)$$

$$\Rightarrow \quad F_H = \left(\frac{P_A + P_B}{2} \right)(2R\ell)$$

$$\Rightarrow \quad F_H = \left(\frac{R\rho g + 3R\rho g}{2} \right)(2R\ell) = 4\rho g \ell R^2$$

Test Your Concepts-VI (Based on Pressure in Accelerating Fluids)

1. For liquid not to spill out of the tube, we have

$$h\rho g = l\rho a$$

$$\Rightarrow \quad a = \frac{hg}{l}$$

2. Let us assume that the water does not spill when the cylinder rotates with an angular speed of 10 rad s^{-1}, so the rise (h) in the level of water given by

$$h = \frac{r^2\omega^2}{2g}$$

$$\Rightarrow \quad h = \frac{\omega^2 r^2}{2g} = \frac{(20)^2(10)^2}{2\times 1000} = 10 \text{ cm}$$

Therefore, rise in the liquid level at the periphery is 10 cm. For water to just spill over the sides, the maximum available height (h_{max}) is

$$h_{\text{max}} = (50-30) \text{ cm} = 20 \text{ cm}.$$

Since, $h_{\text{max}} = \dfrac{(\omega')^2 R^2}{2g}$

$$\Rightarrow \quad 20\times 10^{-2} = \frac{(\omega')^2(20\times 10^{-2})^2}{2\times 10}$$

$$\Rightarrow \quad (\omega')^2 = 100$$

$$\Rightarrow \quad \omega' = 10 \text{ rads}^{-1}$$

Hence, frequency of rotation is

$$f' = \frac{\omega'}{2\pi} = \frac{5}{\pi} \text{ s}^{-1}$$

3. Pressure difference due to horizontal length of liquid is

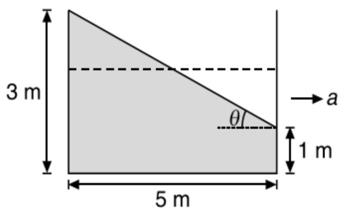
$$\Delta P = \int_0^L \rho\omega^2 x dx$$

Since the liquid is at rest w.r.t. the frame attached to the tube, so we have

$$\Delta P = \frac{\rho\omega^2 L^2}{2} = h\rho g$$

$$\Rightarrow \quad h = \frac{\omega^2 L^2}{2g}$$

4. The level of liquid should rise by 1 m on the left side and hence fall by 1 m on the right side.



$$\Rightarrow \quad \frac{a}{g} = \tan\theta = \frac{3-1}{5} = \frac{2}{5}$$

$$\Rightarrow \quad a = 0.4g$$

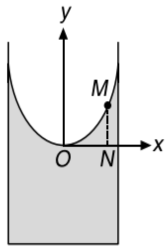
$$\Rightarrow \quad a = 4 \text{ ms}^{-2}$$

5. Since the tank is closed, so we do not need to take the atmospheric pressure into account inside the tank. Applying Pascal's equation, we get

$$P_A + h\rho g + l\rho a = P_B$$

$$\Rightarrow \quad P_B - P_A = (hg + la)\rho$$

6. In the figure shown, suppose the coordinates of point M are (x, y) with respect to the coordinate axes. Then, Points M and O are open to atmosphere.



$$\Rightarrow \quad p_M = p_O = p_{\text{atm}}$$

Now, $p_M - p_N = -\rho g y$

$$\Rightarrow \quad p_O - p_N = -\frac{\rho\omega^2 x^2}{2}$$

$$\Rightarrow \quad p_{\text{atm}} - p_N = -\frac{\rho\omega^2 x^2}{2} \qquad \ldots(1)$$

Also, due to rotation of the fluid we have

$$p_N - p_O = \frac{\rho\omega^2 x^2}{2}$$

$$\Rightarrow \quad p_N - p_{\text{atm}} = \frac{\rho\omega^2 x^2}{2} \qquad \ldots(2)$$

On adding equations (1) and (2), we get

$$0 = -\rho g y + \frac{\rho\omega^2 x^2}{2}$$

$$\Rightarrow \quad y = \frac{\omega^2 x^2}{2g}$$

This is the required equation of free surface of the liquid and we can see that this is an equation of a parabola.

Test Your Concepts-VII (Based on Archimedes' Principle and Buoyancy)

1. Let V be the total volume and V_i the volume of ice piece immersed in water. For equilibrium of ice piece, we have

$$\text{Weight} = \text{Upthrust}$$

$$\Rightarrow \quad V\rho_{\text{ice}} g = V_{\text{imm}}\rho_w g$$

Substituting in above equation, we get

$$\frac{V_{\text{imm}}}{V} = \frac{900}{1000} = 0.9$$

i.e., the fraction of volume outside the water,

$$f = 1 - 0.9 = 0.1$$

2. If V be the volume of the block, then

$$W = V\rho g \qquad \ldots(1)$$

When completely immersed in water, it weighs W', so we have

$$W' = W - U \qquad \ldots(2)$$

where, $U = V\rho_w g$

$$\Rightarrow \quad \frac{W'}{W} = \frac{W-U}{W} = \frac{V\rho g - V\rho_w g}{V\rho g} = 1 - \frac{\rho_w}{\rho}$$

$$\Rightarrow \quad \frac{\rho_w}{\rho} = 1 - \frac{W'}{W}$$

$$\Rightarrow \quad \frac{\rho_w}{\rho} = \frac{W - W'}{W}$$

$$\Rightarrow \quad \rho = \left(\frac{W}{W - W'}\right)\rho_w$$

3. Let the mass of the metal of specific gravity 11.4 be m, then the mass of the second metal of specific gravity 7.4 should be $(96 - m)$.

Volume of first metal is $V_1 = \frac{m}{11.4}\ \text{cm}^3$

Volume of second metal is $V_2 = \frac{96 - m}{7.4}\ \text{cm}^3$

Total volume of the alloy is $V = \frac{m}{11.4} + \frac{96 - m}{7.4}$

Buoyancy force on the alloy piece when immersed in water is

$$U = V_{\text{imm}}\rho_w g = \left(\frac{m}{11.4} + \frac{96 - m}{7.4}\right)\ \text{gwt}$$

Apparent weight in water is

$$W_{\text{app}} = W - U$$

$$\Rightarrow \quad W_{\text{app}} = 96 - \left(\frac{m}{11.4} + \frac{96 - m}{7.4}\right)$$

According to the given problem, $W_{\text{app}} = 86$ gwt

$$\Rightarrow \quad 96 - \left(\frac{m}{11.4} + \frac{96 - m}{7.4}\right) = 86$$

$$\Rightarrow \quad \frac{m}{11.4} + \frac{96 - m}{7.4} = 10$$

Solving we get

$$m = 62.7\ \text{g}$$

So, mass of second metal is

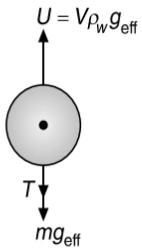
$$m' = 96 - m = 96 - 62.7 = 33.3\ \text{g}$$

4. The density of the sphere is

$$\rho = 0.5\rho_w = 500\ \text{kgm}^{-3}$$

If m be the mass of the sphere, V be the volume of the sphere immersed in water is

$$V = \frac{m}{(0.5)\rho_w}$$



The upthrust U acting on the sphere is

$$U = V\rho_w g_{\text{eff}} = \left[\frac{m}{(0.5)\rho_w}\right]\rho_w g_{\text{eff}} = 2mg_{\text{eff}}$$

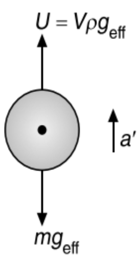
where, $g_{\text{eff}} = g + a$ and $a = 2\ \text{ms}^{-2}$.

$$\Rightarrow \quad U = 2m(g + a) = (2)(2)(10 + 2) = 48\ \text{N}$$

Since the arrangement is in equilibrium with respect to the tank, so we have

$$U = T + mg_{\text{eff}}$$

$$\Rightarrow \quad T = U - mg_{\text{eff}} = 48 - 2(10 + 2) = 24\ \text{N}$$



When the thread snaps, tension T disappears and let the sphere now starts accelerating upwards with an acceleration a' with respect to the tank as in the free body diagram shown in Figure.

$$\Rightarrow \quad U - mg_{\text{eff}} = ma'$$

$$\Rightarrow \quad 48 - 2(10 + 2) = 2a'$$

$$\Rightarrow \quad a' = 12\ \text{ms}^{-2}$$

5. Density of ball is $\rho_{\text{ball}} = \frac{10}{15} = \frac{2}{3}\ \text{gcm}^{-3}$

At a depth 10 m, acceleration of ball is

$$a = \frac{U - mg}{m} = \frac{V\rho_{\text{water}}g - V\rho_{\text{ball}}g}{V\rho_{\text{ball}}}$$

$$\Rightarrow \quad a = \left(\frac{\rho_{\text{water}} - \rho_{\text{ball}}}{\rho_{\text{ball}}}\right)g = \left(\frac{\rho_{\text{water}}}{\rho_{\text{ball}}} - 1\right)g$$

$$\Rightarrow \quad a = 10\left(\frac{3}{2} - 1\right) = 5\ \text{ms}^{-2}$$

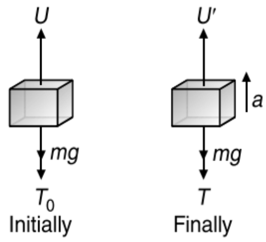
Time taken to reach the surface is

$$t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2 \times 10}{5}} = 2\ \text{s}$$

6. Let m be the mass of block. Initially for the equilibrium of block, we have

$$U = T_0 + mg \qquad \ldots(1)$$

Let U be the upthrust acting on the block. The free body diagrams for the initial and final situation of the block are shown in Figure.



When the lift is accelerated upwards, then we have

$$U' - T - mg = ma \qquad \ldots(2)$$

where $U' = V\rho_w(g + a) = U\left(1 + \frac{a}{g}\right) \qquad \ldots(3)$

Solving equations (1), (2) and (3), we get

$$T = T_0\left(1 + \frac{a}{g}\right)$$

7. Average pressure at bottom of vessel is

$$P = P_{\text{atm}} + h\rho g + \frac{W_{\text{eff}}}{A}$$

where, $A = 16 \times 10^{-4}\ \text{m}^2$ and

$$W_{\text{eff}} = mg - U$$

$$\Rightarrow \quad W_{\text{eff}} = (0.1)(10) - \left(\frac{4}{100}\right)^3(1000)(10)$$

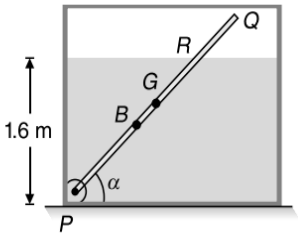
$$\Rightarrow \quad W_{\text{eff}} = 1 - 0.64 = 0.36\ \text{N}$$

$$\Rightarrow \quad P = 10^5 + (0.04)(1000)(10) + \frac{0.36}{16 \times 10^{-4}}$$

$$\Rightarrow \quad P = 10^5 + 400 + 225$$

$$\Rightarrow \quad P = 1.00625 \times 10^5\ \text{Pa}$$

8. Let G be the centre of gravity of the rod PQ, then G is the mid-point of PQ and B be the mid-point of the immersed portion PR of the rod. This point B is called the Centre of Buoyancy as shown in Figure.



Since the hinge is 1.6 m below the free surface of water, so we have

$$PR = 1.6\ \text{cosec}\alpha$$

Volume of PR is

$$V_{PR} = (1.6 \times 9.5 \times 10^{-4})\text{cosec}\,\alpha$$

Weight of water displaced by PR is

$$W_{PR} = (1.6)(9.5 \times 10^{-4})(10^3)(9.8)\text{cosec}\,\alpha$$

$$W_{PR} = 14.896\,\text{cosec}\,\alpha$$

Hence, the buoyant force is $14.896\,\text{cosec}\,\alpha$ acting vertically upwards at B.

The weight of the rod is 25 N acting vertically downwards at G. For equilibrium of the rod, net torque due to forces acting on the rod about the point A is zero. Hence, we have

$$(14.896\,\text{cosec}\,\alpha)(PB\,\cancel{\cos\alpha}) = (25)(PG\,\cancel{\cos\alpha})$$

Since, $PB = \dfrac{PR}{2}$, so we get

$$(14.896\,\text{cosec}\,\alpha)(0.8\,\text{cosec}\,\alpha) = (25)(1.5)$$

$\Rightarrow \quad \sin^2\alpha = 0.32$

$\Rightarrow \quad \sin\alpha = 0.56$

$\Rightarrow \quad \alpha = 34.3°$

Further, let N_y be the reaction at hinge in vertically downward direction (because no horizontal component of any force exists). Then considering the translatory equilibrium of rod in vertical direction, we get

$$N_y + W = U$$

$\Rightarrow \quad N_y = U - W$

$\Rightarrow \quad N_y = 14.896\,\text{cosec}(34.3°) - 25$

$\Rightarrow \quad N_y = 26.6 - 25$

$\Rightarrow \quad N_y = 1.6\text{ N}$ (downwards)

9. If cavity volume is V, then the loss in weight of the piece of copper must be equal to the upthrust experienced by it. So, we have

$$\left(\frac{264}{8.8} + V\right)\rho_w \cancel{g} = (264 - 221)\cancel{g} = 43\cancel{g}$$

$\Rightarrow \quad V = 43 - \dfrac{264}{8.8} = 13\text{ cm}^3$

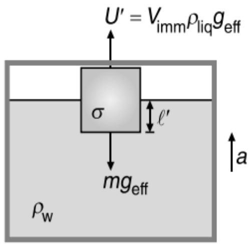
10. Assuming the density of water to be ρ_w and that of block to be σ. Initially, when beaker is at rest and when the block is floating with its length l inside water, then

$$U = mg$$

$\Rightarrow \quad (L^2 l)\rho_w g = (L^3\sigma)g$

$\Rightarrow \quad l = L\left(\dfrac{\sigma}{\rho_w}\right) \qquad \ldots(1)$

Now, when beaker is accelerating upwards with an acceleration a, then let the length of the cube inside water is l' as shown in Figure.



Consider the frame attached to the beaker, then in this frame the block is in equilibrium, so we have

$$U' = mg_{\text{eff}}$$

$\Rightarrow \quad (L^2 l')\rho_w \cancel{(g+a)} = (L^3)\sigma\cancel{(g+a)}$

$\Rightarrow \quad l' = L\left(\dfrac{\sigma}{\rho_w}\right)$

$\Rightarrow \quad \Delta l = l' - l = 0$

11. If container is at rest, let tension in string be T. Then, we have

$$T + mg = U$$

$\Rightarrow \quad T = U - mg = V(\rho_l - \rho_s)g$

$\Rightarrow \quad T = (1.2 - 0.8) \times 1000 \times 980$

$\Rightarrow \quad T = 3.92 \times 10^5\text{ dyne} = 3.92\text{ N}$

When the container accelerates upwards, let the tension in the thread be T' and the upthrust acting on the block be U'. In the frame attached to the beaker, the block is in equilibrium, so we have

$$T' + mg_{\text{eff}} = U'$$

where, $U' = V\rho_l g_{\text{eff}}$ and $g_{\text{eff}} = g + a$

$\Rightarrow \quad T' = V\rho_l(g+a) - m(g+a)$

$\Rightarrow \quad T' = V\rho_l(g+a) - V\rho_s(g+a)$

$\Rightarrow \quad T' = V(\rho_l - \rho_s)(g+a)$

$\Rightarrow \quad T = (1000)(1.2 - 0.8)(980 + 520)$

$\Rightarrow \quad T = 6 \times 10^5\text{ dyne} = 6\text{ N}$

Test Your Concepts-VIII (Based on Viscosity and Terminal Speed)

1. The velocity gradient in vertical direction is

$$\frac{dv}{dx} = \frac{18}{5} = 1\text{ s}^{-1}$$

The magnitude of the force of viscosity is

$$F = \eta A \frac{dv}{dx}$$

The shearing stress is

$$\frac{F}{A} = \eta \frac{dv}{dx} = \left(10^{-2} \text{ poise}\right)\left(1 \text{ s}^{-1}\right) = 10^{-3} \text{ Nm}^{-2}$$

2. If r be the radius of small rain drop, then the terminal speed of the small drop is

$$v_T \propto r^2 \quad \ldots(1)$$

If R be the radius of large drop, then equating the initial and the final volumes, we get

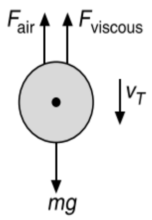
$$\frac{4}{3}\pi R^3 = 27\left(\frac{4}{3}\pi r^3\right)$$

$$\Rightarrow \quad R = 3r$$

$$\Rightarrow \quad \frac{v'_T}{v_T} = \left(\frac{R}{r}\right)^2 = 9$$

$$\Rightarrow \quad v'_T = 9 \text{ ms}^{-1}$$

3. The free body diagram of the ball is shown in Figure.
At $v = v_T$, the net force acting on the drop is zero.



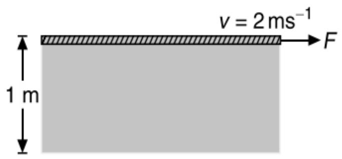
$$\Rightarrow \quad mg = F_{\text{air}} + F_{\text{viscous}}$$

$$\Rightarrow \quad \frac{4}{3}\pi R^3 dg = KRv_T + 6\pi\eta R v_T$$

$$\Rightarrow \quad v_t = \frac{4\pi R^2 gd}{3(6\pi\eta + K)}$$

4. The speed gradient is given by

$$\frac{\Delta v}{\Delta x} = \frac{2-0}{1-0} = 2 \text{ ms}^{-1}\text{m}^{-1}$$



From, Newton's law of viscous force,

$$|F| = \eta A \frac{\Delta v}{\Delta x}$$

$$\Rightarrow \quad |F| = \left(0.01 \times 10^{-1}\right)(2)(2)$$

$$\Rightarrow \quad |F| = 4 \times 10^{-3} \text{ N}$$

So, to keep the plate moving, a force of 4×10^{-3} N must be applied.

5. Terminal velocity of the largest particle which is just about to settle at the bottom of the vessel is

$$v_T = \frac{10 \times 10^{-2}}{3600} = \frac{1}{36000} \text{ ms}^{-1}$$

If r be the radius of that particle, then

$$v_T = \frac{2}{9}\frac{r^2}{\eta}(\rho - \sigma)g \quad \ldots(1)$$

Substituting $\rho = 4 \times 10^3 \text{ kgm}^{-3}$, $\eta = \frac{0.01}{10} \text{ Nsm}^{-3}$ and $v_T = \frac{1}{36000} \text{ ms}^{-1}$ in equation (1), we get

$$r = 2 \times 10^{-6} \text{ m}$$

6. Velocity of ball just before striking the water surface is

$$v = \sqrt{2gh} \quad \ldots(1)$$

Terminal velocity of ball inside water is

$$v = \frac{2}{9} r^2 g \frac{(\rho - 1)}{\eta} \quad \ldots(2)$$

Equating (1) and (2), we get

$$\sqrt{2gh} = \frac{2}{9}\frac{r^2 g}{\eta}(\rho - 1)$$

$$\Rightarrow \quad h = \frac{2}{81} r^4 \left(\frac{\rho - 1}{\eta}\right)^2 g$$

7. Area of each square metal plate is

$$A = (10 \text{ cm}) \times (10 \text{ cm}) = 100 \text{ cm}^2 = 0.01 \text{ m}^2$$

The viscous force is

$$F = 150 \text{ dyne} = 150 \times 10^{-5} \text{ N}$$

Relative speed between the plates is

$$\Delta v = 5 \text{ cms}^{-1} = 0.05 \text{ ms}^{-1}$$

Let the separation between the plates be Δx, then magnitude if the viscous force is

$$F = \eta A \frac{\Delta v}{\Delta x}, \text{ where } \eta = 0.001 \text{ Pl}$$

$$\Rightarrow \quad 150 \times 10^{-5} \text{ N} = \frac{0.001 \times 0.01 \times 0.05}{\Delta x}$$

$$\Rightarrow \quad \Delta x = \frac{0.05}{150} \text{ m} = \frac{1}{3000} \text{ m} = 0.033 \text{ cm}$$

8. Since, $R = \frac{\rho v d}{\eta}$

where, $v = 30 \text{ cms}^{-1} = 0.3 \text{ ms}^{-1}$, $\rho = 1060 \text{ kgm}^{-3}$

$$d = 2 \times 1.0 \text{ cm} = 0.02 \text{ m}$$

$$\eta = 4 \text{ mPas} = 4 \times 10^{-3} \text{ Pas} = 4 \times 10^{-3} \text{ Nsm}^{-2}$$

$$\Rightarrow \quad R = \frac{(1060)(0.3)(0.02)}{\left(4 \times 10^{-3}\right)} = 1590$$

Since $R < 2000$, so this flow will be laminar.

9. Since the rate of flow of liquid through the tubes is the same, so we have

$$Q = \frac{\pi r^4 P}{8\eta l} = \text{constant}$$

$$\Rightarrow \quad \frac{\pi (r/2)^4 P_2}{8\eta(2l)} = \frac{\pi (r/3)^4 P_3}{8\eta(l/2)}$$

$$\Rightarrow \quad \frac{P_2}{P_3} = \frac{64}{81}$$

10. According to Poiseuille equation, we have

$$Q=\frac{\pi R^4 \Delta P}{8\eta l}=\text{constant}$$

Also, according to the problem, the pressure gradient maintained is also constant, so we have

$$\frac{\Delta P}{l}=\text{constant}$$

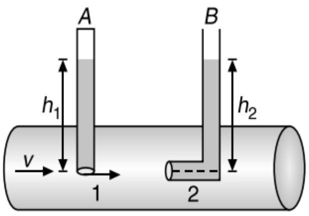
$$\Rightarrow \quad R^4 \propto \eta$$

$$\Rightarrow \quad \frac{R_2^4}{R_1^4}=\frac{\eta_2}{\eta_1}=\frac{12.96}{0.01}=1296$$

$$\Rightarrow \quad \frac{R_2}{R_1}=(1296)^{1/4}=6$$

Test Your Concepts-IX (Based on Equation of Continuity, Bernoulli's Theorem and Applications)

1. The liquid will not flow through the tubes. There is no speed of the liquid inside tubes. Let us take two points 1 (just outside the tube A) and 2 (just inside the tube B) as shown in Figure.



The speed of liquid at a point 2 is zero.
Using Bernoulli's equation between point 1 and 2 give

$$P_1+\frac{1}{2}\rho v_1^2=P_2+0$$

$$\Rightarrow \quad P_2-P_1=\frac{1}{2}\rho v^2 \qquad \{\because v_1=v \text{ and } v_2=0\}$$

$$\Rightarrow \quad \rho g h_2-\rho g h_1=\frac{1}{2}\rho v^2$$

$$\Rightarrow \quad h_2-h_1=\frac{v^2}{2g}$$

2. The maximum height to which the water rises will correspond to the height of the liquid when the inflow of water per second in the bucket equals the outflow of the water per second from the orifice.

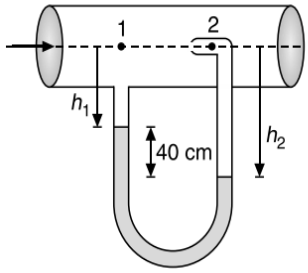
$$1.3\times 10^{-4}=av=a\sqrt{2gh_m}$$

$$\Rightarrow \quad h_m=\frac{(1.3\times 10^{-4})^2}{2a^2 g}=\frac{(1.3\times 10^{-4})^2}{2(10^{-4})^2(9.8)}$$

$$\Rightarrow \quad h_m=0.086 \text{ m}$$

$$\Rightarrow \quad h_m=8.6 \text{ cm}$$

3. Let P_1 be pressure of air at 1 and P_2 be the pressure of air at 2 as shown in Figure.



Since relative density of alcohol is 0.8, so density of alcohol is $0.8 \text{ gcm}^{-3}=800 \text{ kgm}^{-3}$. Also, we are given that $P_1-P_2=40 \text{ cm}$ of alcohol, so we have

$$P_2-P_1=(0.4 \text{ m})\rho_{\text{alcohol}}g$$

$$\Rightarrow \quad P_2-P_1=(0.4)(800)(10)=3200 \text{ Nm}^{-2}$$

If v_1 is velocity of air at the nozzle and v_2, at the other end of the tube, then according to Bernoulli's Theorem, we have

$$P_1+\frac{1}{2}\rho v_1^2=P_2+\frac{1}{2}\rho v_2^2$$

Just when the air enters the tube at 2, its velocity becomes zero, so $v_2=0$ and $v_1=v$.

$$\Rightarrow \quad P_2-P_1=\frac{1}{2}\rho v^2$$

$$\Rightarrow \quad \frac{1}{2}\rho v^2=3200$$

$$\Rightarrow \quad v^2=6400$$

$$\Rightarrow \quad v=80 \text{ ms}^{-1}$$

4. **CASE-1: For $h_1=4H$**

If h be the height of the free surface above the ground, then

$$h=h_1+6H=4H+6H=10H$$

Maximum range will be obtained when

$$h_2=\frac{h}{2}=5H$$

and this maximum range will be,

$$R_{\max}=h=10H$$

CASE-2: For $h_1=8H$

Here, $h=h_1+6H=8H+6H=14H$

Since, $\frac{h}{2}$ i.e. $7H$ lies inside the platform. So, maximum range will be obtained from the bottommost point lying in the liquid container.

$$\Rightarrow \quad h_2=6H$$

and this maximum range will be

$$R_{\max}=2\sqrt{h_2(h-h_2)}$$

$$\Rightarrow \quad R_{\max}=2\sqrt{6H\times 8H}=8\sqrt{3}H$$

5. Given that, $h_2-h_1=0.51 \text{ m}$

Since, we know that the thrust due to liquid of density ρ emerging with a speed v from an orifice of area A is

$$F=v\left(\frac{dm}{dt}\right)=v\left(\frac{A(dx)\rho}{dt}\right)=Av^2\rho$$

This thrust is opposite to the velocity of the liquid emerging from the orifice.

So, $F_1 = Av_1^2\rho = A(2gh_1)\rho$, opposite to v_1

and $F_2 = Av_2^2\rho = A(2gh_2)\rho$, opposite to v_2

$\Rightarrow \quad F_{net} = F_2 - F_1 = 2g\rho A(h_2 - h_1)$

$\Rightarrow \quad F_{net} = 2\times 9.8\times 10^3 \times 0.5\times 10^{-4}(0.51)$

$\Rightarrow \quad F_{net} = 0.5 \text{ N}$

6. The force of thrust acting on the slider is

$$F_t = v_r\left(\frac{dm}{dt}\right) = (v-V)^2(A\rho)$$

$\Rightarrow \quad F_t = (20-10)^2(0.005)(1000) = 500 \text{ N}$

So, acceleration of the slider is

$$a = \frac{F_t - \mu_k mg}{m}$$

$$\Rightarrow \quad a = \frac{500-(0.25)(40)(10)}{40} = 10 \text{ ms}^{-2}$$

At terminal velocity (v_T) of slider, we have

$a_{net} = 0$

$\Rightarrow \quad F_t = f_k = \mu_k mg$

$\Rightarrow \quad (v-v_T)^2(A\rho) = \mu_k mg$

$$\Rightarrow \quad v_T = v - \sqrt{\frac{\mu_k mg}{A\rho}} = 20 - \sqrt{\frac{(0.25)(40)(10)}{(0.005)(1000)}}$$

$\Rightarrow \quad v_T = 20 - 4.5 = 15.5 \text{ ms}^{-1}$

7. Applying Bernoulli's Theorem, we get

$$\frac{1}{2}\rho v^2 = \rho gh + \frac{mg}{A} \quad \text{...(1)}$$

where, $h = 1.0 - 0.5 = 0.5$ m and area of piston is $A = 0.5 \text{ m}^2$.

From equation (1), we get

$$v = \sqrt{2gh + \frac{2mg}{\rho A}}$$

$$\Rightarrow \quad v = \sqrt{2\times 10\times 0.5 + \frac{2\times 20\times 10}{10^3\times 0.5}}.$$

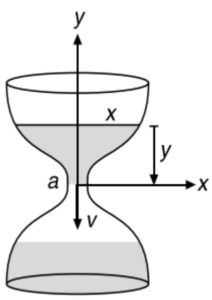
$\Rightarrow \quad v = 3.3 \text{ ms}^{-1}$

So, the speed with which it hits the surface is

$$v' = \sqrt{v^2 + 2gh'} = \sqrt{(3.25)^2 + (2\times 9.8\times 0.5)}$$

$\Rightarrow \quad v' = 4.51 \text{ ms}^{-1}$

8. At any instant t, let the level of liquid in the clock jug be y as shown in Figure. After time $t+dt$, the level of water in the jug be $y-dy$. Let the water exit the drain hole with a speed v, then from Torricelli's Theorem, we have



$$v = \sqrt{2gy}$$

Also, the rate of flow of liquid through the drain hole is

$$Q = \frac{dV}{dt} = av = a\sqrt{2gy}$$

where, $\dfrac{dV}{dt} = (\pi x^2)\left(-\dfrac{dy}{dt}\right)$

$$\Rightarrow \quad a\sqrt{2gy} = \pi x^2\left(-\frac{dy}{dt}\right) \quad \text{...(1)}$$

According to the problem, we have

$$-\frac{dy}{dt} = \frac{4\times 10^{-2}}{3600} = 1.11\times 10^{-5} \text{ ms}^{-1}$$

$$a = \pi r^2 = \pi(2\times 10^{-3})^2$$

$\Rightarrow \quad a = 1.26\times 10^{-8} \text{ m}^2$

Substituting these values in equation (1), we get

$$(1.26\times 10^{-5})\sqrt{2\times 9.8\times y} = \pi(1.11\times 10^{-5})\cdot x^2$$

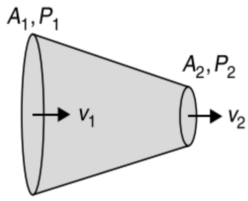
$\Rightarrow \quad y = 0.4x^4$

9. From continuity equation,

$$A_1 v_1 = A_2 v_2$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{A_2}{A_1} = \frac{\pi r_2^2}{\pi r_1^2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{0.04}{0.1}\right)^2 = \frac{4}{25} \quad \text{...(1)}$$

From Bernoulli's equation,



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow \quad v_2^2 - v_1^2 = \frac{2(P_1 - P_2)}{\rho}$$

$$\Rightarrow \quad v_2^2 - v_1^2 = \frac{2\times 10}{1.25\times 10^3} = 1.6\times 10^{-2} \text{ m}^2\text{s}^{-2} \quad \text{...(2)}$$

Solving equations (1) and (2), we get

$v_2 \approx 0.128 \text{ ms}^{-1}$

Rate of volume flow through the tube

$Q = A_2 v_2 = (\pi r_2^2)v_2$

$\Rightarrow \quad Q = \pi(0.04)^2(0.128)$

$\Rightarrow \quad Q = 6.43\times 10^{-4} \text{ m}^3\text{s}^{-1}$

10. Since, $Q = av = -A\left(\dfrac{dy}{dt}\right)$

$$\Rightarrow \quad a\sqrt{2gy} = -A\left(\frac{dy}{dt}\right)$$

$$\Rightarrow \quad dt = -\left(\frac{A}{a}\right)\frac{dy}{\sqrt{2gy}}$$

$$\Rightarrow \quad \int_0^t dt = -\left(\frac{A}{a}\right)\frac{dy}{\sqrt{2gy}}$$

$$\Rightarrow \quad t = \frac{400\times 2}{\sqrt{2\times 9.8}} \approx 180 \text{ s} \approx 3 \text{ minute}$$

When the water level in tank is maintained always at a level of 1 m above the orifice, then

$$t = \frac{\text{Total Volume}}{av} = \frac{Ah}{a\sqrt{2gh}}$$

$$\Rightarrow \quad t = \frac{A}{a}\sqrt{\frac{h}{2g}} = 400\sqrt{\frac{1.0}{2\times 9.8}}$$

$\Rightarrow \quad t \approx 90 \text{ s} = 1.5 \text{ minute}$

11. Applying Bernoulli's equation between points (1) and (2), we get

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Since, area of reservoir $\gg$ area of pipe, so we have

$$v_1 \approx 0$$

Also $P_1 = P_2 = P_{atm}$

$$\Rightarrow \quad v_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2 \times 10 \times 5} = 10 \text{ ms}^{-1}$$

The minimum pressure in the bend will be at A. Therefore, applying Bernoulli's equation between 1 and A, we get

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_A + \frac{1}{2}\rho v_A^2 + \rho g h_A$$

Since, $v_1 \approx 0$ and from equation of continuity, we have $v_A = v_2$.

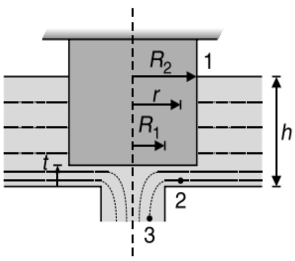
$$\Rightarrow \quad P_A = P_1 + \rho g(h_1 - h_A) - \frac{1}{2}\rho v_2^2$$

Substituting the values, we get

$$P_A = (10^5) + (1000)(10)(-1) - \frac{1}{2}(1000)(10)^2$$

$$\Rightarrow \quad P_A = 4 \times 10^4 \text{ Nm}^{-2}$$

12. Let v_2 be the speed of liquid at section 2 and v_3 at section 3 as shown in Figure.



Using Equation of Continuity at 2 and 3, we get

$$A_2 v_2 = A_3 v_3$$

$$\Rightarrow \quad (2\pi r t)v_2 = (2\pi R_1 t)v_3$$

$$\Rightarrow \quad \frac{v_2}{v_3} = \frac{R_1}{r} \qquad \ldots(1)$$

If t is the thickness of clearance, then on applying the Bernoulli's equation at 1, 2 and 3, we get

$$P_0 + 0 + \rho g h = P + \frac{1}{2}\rho v_2^2 + 0 = P_0 + \frac{1}{2}\rho v_3^2 + 0 \qquad \ldots(2)$$

From equation (1) and (2), we get

$$P = P_0 + \rho g h\left(1 - \frac{R_1^2}{r^2}\right)$$

13. Time taken for the level to fall from h to h' is

$$t = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left[\sqrt{h} - \sqrt{h'}\right]$$

So, the time taken for the level to fall from H to $\frac{H}{2}$ is

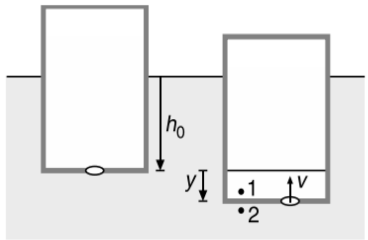
$$t_1 = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{\frac{H}{2}}\right]$$

and the time taken for the level to fall from $\frac{H}{2}$ to zero is

$$t_2 = \frac{A}{a}\sqrt{\frac{2}{g}}\left(\sqrt{\frac{H}{2}} - 0\right)$$

$$\Rightarrow \quad \frac{t_1}{t_2} = \frac{1 - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} - 0} = \sqrt{2} - 1$$

14. In this problem, we note that the level of water in the can will rise as much as the can sinks in water. At any instant, let the can sink in water through y as shown in Figure.



Just above the bottom of can, at the point 1, pressure is

$$P_1 = P_0 + \rho g y$$

Just below the bottom of can, at the point 2, pressure is

$$P_2 = P_0 + \rho g(h_0 + y)$$

$$\Rightarrow \quad \Delta P = P_2 - P_1 = \rho g h_0$$

Due to this difference in pressure, water will start flowing inside the can, such that

$$\Delta P = \frac{1}{2}\rho v^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2\Delta P}{\rho}} = \sqrt{2 g h_0}$$

So, the time taken to sink completely is

$$t = \frac{A(h - h_0)}{av} = \frac{A(h - h_0)}{a\sqrt{2 g h_0}}$$

15. $\frac{1}{2}\rho_a v^2 = \rho_w g h$

$$\Rightarrow \quad v = \sqrt{\frac{2 g h \rho_w}{\rho_a}} = \sqrt{\frac{2 \times 9.8 \times 10^{-2} \times 10^3}{1.3}}$$

$$\Rightarrow \quad v = 12.3 \text{ ms}^{-1}$$

16. Considering ZPEL at the ground, then on applying Bernoulli's Theorem, we get

$$\rho_2 g h + \rho_1 g(h - y) = \frac{1}{2}\rho_1 v^2$$

$$\Rightarrow \quad v = \sqrt{2g\left(\frac{\rho_2}{\rho_1}h + h - y\right)}$$

$$\Rightarrow \quad v = \sqrt{20\left[\left(\frac{600}{900}\right)(0.6) + 0.6 - 0.2\right]}$$

$$\Rightarrow \quad v = 4 \text{ ms}^{-1}$$

Due to this velocity, the thrust force acting on the tank is

$$F = a v^2 \rho_1$$

$$\Rightarrow \quad F = (900)(5 \times 10^{-4})(4)^2 = 7.2 \text{ N}$$

The normal reaction force is given by

$$N = mg = Ah(\rho_1 + \rho_2)g$$

$$\Rightarrow \quad N = (0.5)(0.6)(900 + 600)(10)$$

$$\Rightarrow \quad N = 4500 \text{ N}$$

So, the limiting friction is

$$f_L = \mu N = (0.01)(4500) = 45 \text{ N}$$

Since, $f_L > F$, so the minimum force required to keep the tank in equilibrium is

$$F_{\min} = 0$$

and the maximum force required to keep the tank in equilibrium is

$$F_{\max} = 45 + 7.2 = 52.2 \text{ N}$$

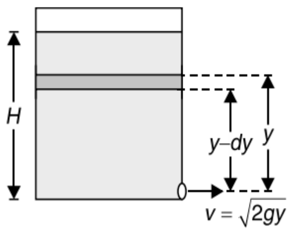
17. Now let at some time t the level of liquid inside the container be y. The speed of the liquid coming out of the hole will be $\sqrt{2gy}$

Applying equation of continuity

$$\Rightarrow \quad \left(\sqrt{2gy}\right)a = -A\frac{dy}{dt}$$

$$\Rightarrow \quad -\frac{a}{A}\int_0^t dt = \int_H^0 \frac{dy}{\sqrt{2gy}}$$

$$\Rightarrow \quad t = \frac{A}{a}\sqrt{\frac{2H}{g}}$$



18. At depth y, the velocity of efflux is $v = \sqrt{2gy}$

Force acting on small cross-sectional area $dA = bdy$ is

$$dF = (dA)v^2\rho = (bdy)(2gy)\rho$$

$$\Rightarrow \quad dF = \rho b(2gy)dy = 2bg\rho y dy$$

$$\Rightarrow \quad F = \int_{0.25}^{0.75} 2\rho g b y dy = 2\rho g b\left(\frac{y^2}{2}\right)\Bigg|_{0.25}^{0.75}$$

$$\Rightarrow \quad F = (10^3)(9.8)(10^{-3})\left[(0.75)^2 - (0.25)^2\right]$$

$$\Rightarrow \quad F = 4.9 \text{ N}$$

19. The volume flow rate in the line is

$$Q = \frac{dV}{dt} = Av = \pi(6.5\times10^{-3})^2(1.2)$$

$$\Rightarrow \quad Q = 1.6\times10^{-4} \text{ m}^3\text{s}^{-1}$$

When water flows out through the shower head with a speed v', then Q remains the same.

$$\Rightarrow \quad Q = \frac{dV}{dt} = nav'$$

$$\Rightarrow \quad v' = \frac{dV/dt}{na} = \frac{1.6\times10^{-4}}{12\times\pi\times(4.6\times10^{-4})^2}$$

$$\Rightarrow \quad v' = 20 \text{ ms}^{-1}$$

20. Applying Bernoulli's equation at points 1 and 2, we get

$$p_1 + \frac{1}{2}(2\rho)v_1^2 + (2\rho)gh_1 = p_2 + \frac{1}{2}(2\rho)v_2^2 + (2\rho)gh_2$$

where, $v_1 \approx 0$, $v_2 = v$, $h_1 = h_2$, $p_2 = p_0$

and $p_1 = p_0 + \rho gH + 2\rho gH = p_0 + 3\rho gH$

Substituting these values in the Bernoulli's equation, we get

$$p_0 + 3\rho gH = p_0 + \rho v^2$$

$$\Rightarrow \quad v = \sqrt{3gH}$$

Since this velocity is horizontal, so time taken by the liquid to fall to the ground is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2H}{g}} \qquad \{\because h = H\}$$

So, the required range of the liquid is

$$R = vt = \left(\sqrt{3gH}\right)\left(\sqrt{\frac{2H}{g}}\right)$$

$$\Rightarrow \quad R = \sqrt{6}H$$

21. The speed v with which liquid comes out of the orifice is

$$v = \sqrt{2g\begin{pmatrix}\text{depth of orifice below} \\ \text{free surface of liquid}\end{pmatrix}}$$

$$\Rightarrow \quad v = \sqrt{2gh_1}\text{, along the horizontal}$$

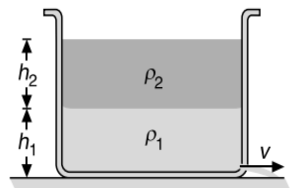
Let t be the time taken by the liquid to hit the ground, then since v is horizontal, so we have

$$t = \sqrt{\frac{2h_2}{g}}$$

The range is given by

$$R = vt = \left(\sqrt{2gh_1}\right)\left(\sqrt{\frac{2h_2}{g}}\right) = 2\sqrt{h_1h_2}$$

22. The arrangement discussed in problem is shown in Figure.



Assuming the atmospheric pressure to be P_a, then on applying the Bernoulli's Theorem, we get

$$P_a + \frac{1}{2}\rho_1 v^2 = P_a + \rho_1 gh_1 + \rho_2 gh_2$$

$$\Rightarrow \quad v = \sqrt{2g\left(h_1 + h_2\frac{\rho_2}{\rho_1}\right)}$$

$$\Rightarrow \quad v = \sqrt{2\times9.8\left(0.3 + \frac{0.2\times600}{1000}\right)}$$

$$\Rightarrow \quad v = 2.87 \text{ ms}^{-1}$$

23. Time taken to empty the complete tank is

$$t = \frac{A}{a}\sqrt{\frac{2H}{g}}$$

Since, $t_{H\to0} = t_{H\to\frac{H}{3}} + t_{\frac{H}{3}\to0}$

$$\Rightarrow \quad \frac{A}{a}\sqrt{\frac{2H}{g}} = t_0 + \frac{A}{a}\sqrt{\frac{2\left(\frac{H}{3}\right)}{g}}$$

$$\Rightarrow \quad \frac{A}{a}\sqrt{\frac{2\left(\frac{H}{3}\right)}{g}} = \frac{A}{a}\sqrt{\frac{2H}{g}} - t_0$$

$$\Rightarrow \quad t_{\frac{H}{3}\to 0} = \frac{A}{a}\sqrt{\frac{2H}{g}} - t_0$$

Test Your Concepts-X (Based on Surface Tension, Surface Energy, Excess Pressure and Capillarity)

1. Since, $h = \frac{2T\cos\theta}{r\rho g}$, so the surface tension of the liquid is

$$T = \frac{rh\rho g}{2} \qquad \{\because \ \theta = 0^\circ\}$$

$$\Rightarrow \quad T = \frac{(0.025)(3)(1.5)(980)}{2}$$

$$\Rightarrow \quad T = 55 \text{ dynecm}^{-1}$$

Hence excess pressure inside a spherical bubble is

$$\Delta p = \frac{4T}{R} = \frac{(4)(55)}{(0.5)} = 440 \text{ dynecm}^{-2}$$

2. When the ring is about to leave the water surface, surface tension force on it is

$$F_{ST} = 2\pi RT + 2\pi rT = 2\pi(R+r)T$$

Spring force $F_s = kx$

$$\Rightarrow \quad kx = 2\pi(R+r)T + mg$$

$$\Rightarrow \quad T = \frac{kx - mg}{2\pi(R+r)}$$

3. $h = \frac{2T\cos\theta}{r\rho g}$

Substituting the proper values, we have

$$h = \frac{(2)(7\times10^{-2})\cos 0^\circ}{(0.5\times10^{-3})(10^3)(9.8)} = 2.86\times10^{-2} \text{ m}$$

$$\Rightarrow \quad h = 2.86 \text{ cm}$$

4. In general, $2\pi T\cos\theta = (\pi r^2 h'\rho)g$

$$\Rightarrow \quad 2T\cos\theta = h' r\rho g$$

Now $r = R\cos\theta$

$$R = \frac{2T}{h'\rho g} = \frac{2\times0.07}{1\times10^{-2}\times1000\times9.8}$$

$$\Rightarrow \quad R = 1.4\times10^{-3} \text{ m=1.4 mm}$$

5. Excess pressure inside first bubble is $\Delta P_1 = \frac{4T}{r}$

Excess pressure inside second bubble is

$$\Delta P_2 = \frac{4T}{R}$$

So, excess of pressure on the two sides of the separating film is

$$\Delta P = \Delta P_1 - \Delta P_2 = 4T\left(\frac{1}{r} - \frac{1}{R}\right) \qquad \ldots(1)$$

If R' be the radius of the interface film, then we have

$$\Delta P = \frac{4T}{R'} \qquad \ldots(2)$$

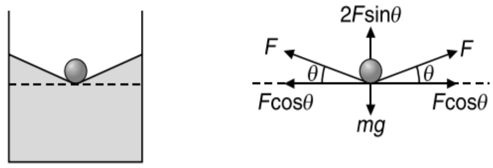
From equations (1) and (2), we get

$$\frac{4T}{R'} = 4T\left(\frac{1}{r} - \frac{1}{R}\right)$$

$$\Rightarrow \quad \frac{1}{R'} = \frac{1}{r} - \frac{1}{R}$$

$$\Rightarrow \quad R' = \frac{Rr}{R-r}$$

6. Let the mass of the needle be m. Since the liquid surface gets distorted, so the force due to surface tension acting on both sides of the needle makes an angle θ with the horizontal. The forces acting on the needle are F, F and mg as shown in Figure.



Cross-sectional View

On resolving the forces, the horizontal components cancel. However, for vertical equilibrium, we have

$$2F\sin\theta = mg \text{, where } F = Tl$$

$$\Rightarrow \quad m = \frac{2F\sin\theta}{g} = \frac{2Tl\sin\theta}{g}$$

For m to be maximum, $\sin\theta = \text{MAX} = 1$

$$\Rightarrow \quad m_{\max} = \frac{2Tl}{g}$$

7. $h = \frac{2T\cos\theta}{r\rho g}$

$$\Rightarrow \quad hr = \frac{2T\cos\theta}{\rho g} = \text{constant}$$

$$\Rightarrow \quad h_1 r_1 = h_2 r_2$$

$$\Rightarrow \quad h_2 = \frac{h_1 r_1}{r_2}$$

Substituting $\frac{r_2}{r_1} = \frac{1}{3}$, we get

$$h_2 = (2)(3) = 6 \text{ cm}$$

8. Let R be the radius of the big drop formed by the combination of 8 small droplets. Since volume of the big drop equals the total volume of 8 small droplets, so we have

$$\frac{4}{3}\pi R^3 = 8\left(\frac{4\pi}{3}r^3\right)$$

$$\Rightarrow \quad R^3 = 8r^3$$

$$\Rightarrow \quad R = 2r = 2(5\times10^{-4} \text{ m}) = 10^{-3} \text{ m}$$

Final surface area of the big drop is

$$A_f = 4\pi R^2 = 4\pi\left(10^{-3}\text{ m}\right)^2$$

$$\Rightarrow \quad A_f = 4\pi \times 10^{-6}\text{ m}^2$$

Total initial surface area of 8 small droplets is

$$A_i = 8\left(4\pi r^2\right) = 8\times 4\pi\left(5\times 10^{-4}\text{ m}\right)^2$$

$$\Rightarrow \quad A_i = 8\pi\times 10^{-6}\text{ m}^2$$

Decrease in surface area is

$$A_i - A_f = 8\pi\times 10^{-6}\text{ m}^2 - 4\pi\times 10^{-6}\text{ m}^2$$

$$\Rightarrow \quad A_i - A_f = |\Delta A| = 4\pi\times 10^{-6}\text{ m}^2$$

Energy evolved is

$$W = T|\Delta A| = \left(72\times 10^{-3}\right)\left(4\pi\times 10^{-6}\right)\text{ J}$$

$$\Rightarrow \quad W = 9\times 10^{-7}\text{ J}$$

9. Since pressure on concave side is greater than that on convex side, so pressure at A and B is

$$P_A = P_0 - \frac{2T}{r_1} \text{ and } P_B = P_0 - \frac{2T}{r_2}$$

So, the pressure difference between A and B is

$$\Delta P = 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

If this pressure difference corresponds to a height equal to h units of the liquid, then we have

$$2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = h\rho g$$

$$\Rightarrow \quad h = \frac{2T}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Rightarrow \quad h = \frac{2\times 0.07}{1000\times 9.8}\left(\frac{1}{1\times 10^{-3}} - \frac{1}{1.5\times 10^{-3}}\right)$$

$$\Rightarrow \quad h = 4.76\text{ mm}$$

10. Since a soap bubble has two free surfaces, so the initial surface area of the soap bubble is

$$A_{\text{initial}} = 2\left(4\pi r_1^2\right) = 2(4\pi)(0.1)^2\text{ m}^2$$

Final surface area of the soap bubble is

$$A_{\text{final}} = 2\left(4\pi r_2^2\right) = 2(4\pi)(0.2)^2\text{ m}^2$$

So, change in surface area of the soap bubble is

$$\Delta A = 8\pi\left[(0.2)^2 - (0.1)^2\right] = 0.24\pi\text{ m}^2$$

Hence the work done W is given by

$$W = T\Delta A = \left(25\times 10^{-3}\right)(0.24\pi)$$

$$\Rightarrow \quad W = 6\pi\times 10^{-3}\text{ J}$$

$$\Rightarrow \quad W \approx 19\text{ mJ}$$

11. Since a film has two free surfaces, so initial surface area of the film is

$$A_i = 2(l\times b) = 2(10\times 0.5)\text{ cm}^2 = 10^{-3}\text{ m}^2$$

Final surface area of the film is

$$A_f = 2[10\times(0.5+0.1)]\text{ cm}^2$$

$$\Rightarrow \quad A_f = 1.2\times 10^{-3}\text{ m}^2$$

So, the required work done is

$$W = T\Delta A = \left(72\times 10^{-3}\right)(1.2-1)\times 10^{-3}\text{ J}$$

$$\Rightarrow \quad W = 72\times 10^{-3}\times 2\times 10^{-4} = 144\times 10^{-7}\text{ J}$$

12. Let r_1 and r_2 be the radii of two given soap bubbles and r be that of the coalesced bubble.

Since a soap bubble has two free surfaces, so if W_1 and W_2 are the potential energies of the two given bubbles, then we have

$$W_1 = 2\left(4\pi r_1^2\right)T \text{ and } W_2 = 2\left(4\pi r_2^2\right)T$$

Potential energy of the bubble formed due to coalescing of the two bubbles is

$$W = 2\left(4\pi r^2\right)T$$

Since total energy of system is conserved, so we have

$$8\pi r^2 T = 8\pi r_1^2 T + 8\pi r_2^2 T$$

$$\Rightarrow \quad r^2 = r_1^2 + r_2^2 = (6)^2 + (8)^2 = 100$$

$$\Rightarrow \quad r = 10\text{ cm}$$

13. The depth of the water column, when the water just enters the hollow sphere through the hole is $h = 40$ cm. Let R be the radius of the hole in the hollow sphere, then on immersing the hollow sphere (having hole) in water, an air bubble is formed at the hole. The radius of this air bubble is approximately equal to the radius of the hole.

The pressure corresponding to water depth of 40 cm will try to push water into the hollow sphere, whereas the excess pressure inside the air bubble opposes the entry of water. So, water will enter the hollow sphere just when the excess of pressure inside the air bubble becomes equal to the pressure exerted by 40 cm column of water.

$$\Rightarrow \quad P_i - P_0 = \Delta P = \frac{2T}{R} \quad \ldots(1)$$

Also, $P_i - P_0 = h\rho g = (0.40)\left(10^3\right)(10)$

$$\Rightarrow \quad P_i - P_0 = \Delta P = 4\times 10^3\text{ Nm}^{-2} \quad \ldots(2)$$

Substituting in equation (2), we get

$$R = \frac{2T}{\Delta P} = \frac{2\left(75\times 10^{-3}\right)}{4\times 10^3} = 3.75\times 10^{-6}\text{ m}$$

14. Total initial surface energy of 1000 droplets is

$$U_i = (1000)\left[4\pi\left(0.5\times 10^{-3}\right)^2\right](0.475)$$

$$\Rightarrow \quad U_i \approx 1.5\times 10^{-3}\text{ J}$$

When the drops combine, then volume remains the same. So, we have

$$\Rightarrow \quad 1000\left[\frac{4\pi}{3}(0.5)^3\right] = \frac{4\pi}{3}R^3$$

$$\Rightarrow \quad R = 10(0.5\text{ mm}) = 5\text{ mm}$$

Final surface energy of the bigger drop is

$$U_f = 4\pi\left(5\times 10^{-3}\right)^2(0.475) = 0.15\times 10^{-3}\text{ J}$$

Loss in surface energy is

$$U_i - U_f = 1.5\times 10^{-3} - 0.15\times 10^{-3}$$

$$\Rightarrow \quad U_i - U_f = 1.35\times 10^{-3}\text{ J}$$

Single Correct Choice Type Questions

1. Equating the rate of flow of liquid

$$Q=\frac{\pi r^4 P}{8\eta\ell}$$

$$\Rightarrow\quad Q_1=Q_2$$

$$\Rightarrow\quad \frac{\pi(r)^4 P_1}{8\eta\ell}=\frac{\pi(2r)^4 P_2}{8\eta(2\ell)}$$

$$\Rightarrow\quad \frac{P_1}{P_2}=8$$

Hence, the correct answer is (C).

2. Increase in tension of wire is

$$F=YA\alpha\Delta\theta$$

$$\Rightarrow\quad F=8\times10^{-6}\times2.2\times10^{11}\times10^{-2}\times10^{-4}\times5=8.8\text{ N}$$

Hence, the correct answer is (D).

3. Let R_1 and R_2 are the radii of soap bubbles before and after collapsing. Then

$$V=2\left(\frac{4}{3}\pi R_1^3\right)-\frac{4}{3}\pi R_2^3 \qquad \ldots(1)$$

$$S=2\left(8\pi R_1^2\right)-8\pi R_2^2 \qquad \ldots(2)$$

Since, $P_1V_1=P_2V_2$

$$\Rightarrow\quad 2\left(\frac{4}{3}\pi R_1^3\right)\left(P+\frac{4T}{R_1}\right)=\left(P+\frac{4T}{R_2}\right)\left(\frac{4}{3}\pi R_2^3\right)$$

$$\Rightarrow\quad 2R_1^3\left(P+\frac{4T}{R_1}\right)=R_2^3\left(P+\frac{4T}{R_2}\right)$$

$$\Rightarrow\quad P\left(2R_1^3-R_2^3\right)=4T\left(R_2^2-2R_1^2\right) \qquad \ldots(3)$$

From equation (1), $4\pi\left(2R_1^3-R_2^3\right)=3V$

From equation (2), $8\pi\left(R_2^2-2R_1^2\right)=-S$

Substituting in equation (3), we get

$$3PV+4ST=0$$

Hence, the correct answer is (A).

4. Since $Y=\dfrac{F/A}{x/L}$

$$\Rightarrow\quad Y=\frac{FL}{Ax}$$

For wires of same material Y is the same.

$$\Rightarrow\quad \frac{FL}{Ax}=\text{constant}$$

$$\Rightarrow\quad x\propto\frac{L}{A}$$

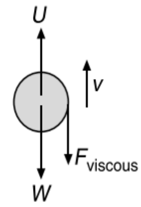
$$\Rightarrow\quad \frac{x_1}{x_2}=\frac{L_1A_2}{L_2A_1}$$

$$\Rightarrow\quad \frac{x_1}{x_2}=\frac{1}{2}\left(\frac{1}{4}\right)$$

$$\Rightarrow\quad \frac{x_1}{x_2}=\frac{1}{8}$$

Hence, the correct answer is (B).

5. Forces acting on the ball are



Upthrust, $U=2V\rho g$

Viscous drag, $F_{\text{viscous}}=6\pi\eta rv$

and weight of ball $W=V\rho g$

Net force acting on ball in the upward direction is

$$F_{\text{net}}=U-W-F_{\text{viscous}}$$

Since U and W are constants whereas F_{viscous} increases with increase in v. Hence, acceleration of the ball will go on decreasing.

Hence, the correct answer is (B).

6. For constant temperature,

$$P_iV_i=P_fV_f$$

$$\Rightarrow\quad \left(P_1+\frac{4T}{r}\right)\left(\frac{4}{3}\pi r^3\right)=\left(P_2+\frac{4T}{r/2}\right)\left[\frac{4}{3}\pi\left(\frac{r}{2}\right)^3\right]$$

$$\Rightarrow\quad P_2=8P_1+\frac{24T}{r}$$

Hence, the correct answer is (A).

7. Since, $W=\dfrac{1}{2}(\text{Tension})(\text{Extension})$

Further

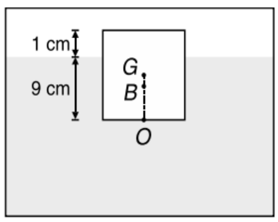
$$T=YA\alpha t \text{ and}$$

$$\Delta\ell=\ell\alpha t$$

$$\Rightarrow\quad W=\frac{1}{2}(YA\alpha t)(\ell\alpha t)$$

Hence, the correct answer is (B).

8. Relative density of ice is 0.9 i.e., 90% volume of ice is immersed in water. When ice melts completely level of water does not change.



$OB=4.5$ cm, $OG=5$ cm

Since, $GB=h=0.5$ cm

$$\Rightarrow\quad \Delta U=-mgh$$

$$\Rightarrow\quad \Delta U=-(0.1)^3(900)(10)\left(0.5\times10^{-2}\right)\text{ J}$$

$$\Rightarrow\quad \Delta U=-0.045\text{ J}$$

Hence, the correct answer is (D).

9. The pressure at the interface must be same, calculated via either tube. Since, both tubes are open to the atmosphere, we must have

$$h_1\rho_1 g=h_2\rho_2 g$$

$$\Rightarrow\quad h_1\rho_1=h_2\rho_2$$

$$\Rightarrow\quad h\rho=\text{constant}$$

$$\Rightarrow\quad h\propto\frac{1}{\rho}$$

Hence, the correct answer is (B).

10. $W = 2\pi(r_1 + r_2)T$

$$\Rightarrow T = \frac{W}{2\pi(r_1 + r_2)} = \frac{7.48 \times 10 \times 10^{-3}}{2\pi \times 17 \times 10^{-2}}$$

$$\Rightarrow T = 70 \times 10^{-3}\ \text{Nm}^{-1}$$

Hence, the correct answer is (D).

11. $\frac{1}{2}\left(\frac{YA}{L}\right)(\Delta\ell)^2 = \frac{1}{2}mv^2$

$$\Rightarrow Y = \frac{mv^2L}{A(\Delta\ell)^2} = \frac{0.02 \times 400 \times 0.42 \times 4}{\pi \times 36 \times 10^{-6} \times 0.04}$$

$$\Rightarrow Y = 2.3 \times 10^6\ \text{Nm}^{-2}$$

So, order is 10^6.

Hence, the correct answer is (D).

12. $2T\ell\cos\theta = mg = \pi r^2 \ell \rho g$

$$\Rightarrow r = \sqrt{\frac{2T}{\pi\rho g}}$$

Hence, the correct answer is (A).

13. $\frac{4T}{R} = \frac{4T}{r_1} - \frac{4T}{r_2} = \frac{4T}{r} - \frac{4T}{2r} = \frac{4T}{2r}$

$$\Rightarrow R = 2r$$

Hence, the correct answer is (C).

14. $\Delta L = \frac{FL}{YA}$

$$\Rightarrow \frac{L_A}{Y_A r_A^2} = \frac{L_B}{Y_B r_B^2}$$

$$\Rightarrow r_A = r_B\sqrt{\frac{L_A}{L_B}\frac{Y_B}{Y_A}}$$

$$\Rightarrow r_A = (2\ \text{mm})\sqrt{\frac{2 \times 2 \times 4}{3 \times 7}}$$

$$\Rightarrow r_A = \frac{4}{4.58} \times 2 = 1.7\ \text{mm}$$

Hence, the correct answer is (D).

15. Since $-\frac{Adh}{dt} = \frac{\pi(h\rho g)r^4}{8\eta\ell}$

$$\Rightarrow \int_0^t dt = \int_H^{H/2} -\frac{8\eta\ell A}{\pi\rho g r^4}$$

$$\Rightarrow t = \frac{8\eta\ell A}{\pi\rho g r^4}\ln(2)$$

Hence, the correct answer is (C).

16. $U = \frac{1}{2} \times \frac{(\text{stress})^2}{Y} \times \text{volume} = \frac{1}{2} \times \frac{F^2 \times A \times L}{A^2 \times Y}$

$$\Rightarrow U = \frac{1}{2} \times \frac{F^2L}{AY} = \frac{1}{2} \times \frac{(50)^2 \times 0.2}{1 \times 10^{-4} \times 1 \times 10^{11}}$$

$$\Rightarrow U = 2.5 \times 10^{-5}\ \text{J}$$

Hence, the correct answer is (B).

17. Given that, $h\rho g = 10^5$ Pa

$$\Rightarrow P_x = \left(h - \frac{h}{5}\right)\rho g = 0.8h\rho g$$

$$\Rightarrow P_x = 0.8 \times 10^5\ \text{Pa}$$

Hence, the correct answer is (B).

18. When ice melts into water its volume decreases. Hence, over all level should decrease. Now suppose m is the mass of ice, V_1 is volume immersed in water and V_2 the volume immersed in oil. For floating condition, we have

Weight = Upthrust

$$\Rightarrow mg = V_1\rho_w g + V_2\rho_0 g$$

$$\Rightarrow V_1 = \frac{m - V_2\rho_0}{\rho_w}$$

$$\Rightarrow V_1 = \frac{m}{\rho_w} - \frac{V_2\rho_0}{\rho_w} \quad \text{...(1)}$$

When ice melts, m mass of ice converts into m mass of water. Volume of water so formed is

$$V_3 = \frac{m}{\rho_w} \quad \text{...(2)}$$

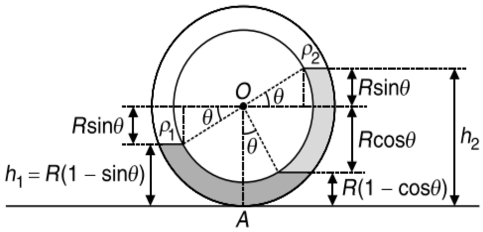
From equations (1) and (2), $V_3 > V_1$

So, the interface level will rise.

Hence, the correct answer is (A).

19. Since pressure at A due to the left column and the right column of the liquid should be the same, so we have

$$(P_A)_{\text{due to left column}} = (P_A)_{\text{due to right column}} \quad \text{...(1)}$$



Due to left column

Pressure at A is given by

$$(P_A)_{\text{left}} = R(1 - \sin\theta)\rho_1 g$$

Due to right column

Pressure at A is given by

$$(P_A)_{\text{right}} = R(\sin\theta + \cos\theta)\rho_2 g + (1 - \cos\theta)\rho_1 g$$

Substitution these both in equation (1), we get

$$\rho_1(1 - \cos\theta) + \rho_2(\sin\theta + \cos\theta) = \rho_1(1 - \sin\theta)$$

$$\Rightarrow \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{\rho_1}{\rho_2}$$

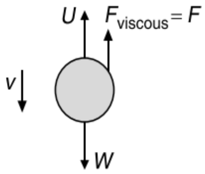
$$\Rightarrow \tan\theta = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$$

$$\Rightarrow \tan 30° = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$$

$$\Rightarrow \frac{\rho_1}{\rho_2} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

Hence, the correct answer is (D).

20. The free body diagram for the first case is shown in Figure.

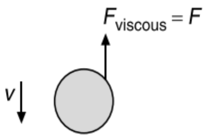


If U is the upthrust, F is the viscous force and W is the weight of the body, then at terminal velocity, we have

$$U + F = W$$

$$\Rightarrow \quad F = V(\rho - \sigma)g = V\sigma g \qquad \{\because \rho = 2\sigma\}$$

The free body diagram for the second case (when gravity is absent) is shown in Figure.



In this case, only the viscous drag F acts on the body. So, initial retardation of the ball is

$$a = \frac{F}{m} = \frac{V\sigma g}{2\sigma V} = \frac{g}{2}$$

This retardation with gradually decrease as the speed of the ball decreases and finally the ball will stop.

Hence, the correct answer is (D).

21. $E(\text{dissipated}) = \frac{1}{2}k\Delta x^2$

$$\Rightarrow \quad \frac{1}{2} \times 800 \times \left(\frac{2 \times 2}{100 \times 100}\right) = \frac{16}{100} J$$

$$\Rightarrow \quad \frac{16}{100} = \frac{1}{2} \times 400 \times \Delta T + 1 \times 4184 \times \Delta T$$

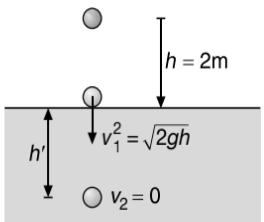
$$\Rightarrow \quad \frac{16}{100} = (200 + 4184)\Delta T = 4384\Delta T$$

$$\Rightarrow \quad \Delta T = \frac{16}{4384 \times 100} = 3.6 \times 10^{-5} \text{ K}$$

Hence, the correct answer is (A).

22. Retardation of the ball inside the water is

$$a = \frac{U - W}{m} = \frac{V(1)g - V(0.8)g}{V(0.8)} = \frac{g}{4}$$



Since, $v_2^2 = v_1^2 - 2ah'$

where, $v_2^2 = 0$ and $v_1^2 = 2gh$, so

$$\Rightarrow \quad h' = 8 \text{ m}$$

Hence, the correct answer is (A).

23. According to Pascal's Law, for an incompressible liquid confined to a closed rigid container, the excess pressure is transmitted equally in all the directions.

So, the increment in pressure at each point is $\Delta P = \frac{F}{A}$

Hence, the correct answer is (A).

24. $V_1 = \frac{\pi \text{Pr}^4}{8\eta\ell_1}$, $V_2 = \frac{\pi \text{Pr}^4}{8\eta\ell_2}$, $V_3 = \frac{\pi \text{Pr}^4}{8\eta\ell_3}$

and $V = \frac{\pi \text{Pr}^4}{8\eta\ell}$

Since, $V = V_1 + V_2 + V_3$

Substituting the values, we get

$$\ell = \frac{\ell_1\ell_2\ell_3}{\ell_1\ell_2 + \ell_1\ell_3 + \ell_2\ell_3}$$

$$\Rightarrow \quad \ell = \frac{(1)(2)(3)}{(1)(2) + (1)(3) + (2)(3)} = \frac{6}{11} \text{ m}$$

Hence, the correct answer is (A).

25. $\frac{4}{3}\pi R^3 = 10^6\left(\frac{4}{3}\pi r^3\right)$

$$\Rightarrow \quad R = (10^2 r)$$

Now, $U_i = 10^6(4\pi r^2)T$ and $U_f = (4\pi R^2)T$

$$\Rightarrow \quad \frac{U_f}{U_i} = \frac{1}{10^2}$$

Hence, the correct answer is (B).

26. Acceleration of container is given to be

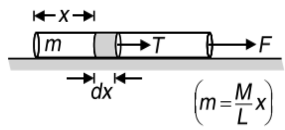
$$a = a_0(\hat{i} - \hat{j} + \hat{k})$$

Since there is no gravity, so the pressure difference will be only due to the acceleration of container. All points other than point H, are acted upon by a pseudo force. Hence, at point H pressure developed is zero.

Hence, the correct answer is (D).

27. $F = Ma$

$$\Rightarrow \quad a = \frac{F}{M}$$



$$\Rightarrow \quad T = ma = \left(\frac{M}{L}\right)x\frac{F}{M} = \frac{F}{L}x$$

Since, $Y = \frac{T/A}{d\ell/dx} = \frac{\left(\frac{F}{L}x\right)dx}{Ad\ell}$

$$\Rightarrow \quad \int_0^{\Delta\ell} d\ell = \frac{F}{ALY}\int_0^L x\,dx = \frac{FL^2}{2ALY} = \frac{FL}{2AY}$$

$$\Rightarrow \quad \frac{\Delta L}{L} = \frac{F}{2AY}$$

Hence, the correct answer is (C).

28. Since stress is shown on x-axis and strain on y-axis, so we have

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{1}{\tan\theta} = \frac{1}{\text{slope}}$$

So, elasticity of wire P is minimum and of wire R is maximum

Hence, the correct answer is (D).

29. Upthrust acting on cylinder is

$$U = \left(\frac{m}{\rho}\right)\sigma g$$

According to Newton's Third Law, force exerted by the cylinder on the liquid is also $\left(\frac{m}{\rho}\right)\sigma g$

So, increase in pressure is $\Delta P = \frac{m\sigma g}{\rho s}$

Hence, the correct answer is (C).

30. Stress $= \frac{400}{\pi r^2} \le 379 \times 10^6\ \text{Nm}^{-2}$

$\Rightarrow \quad r^2 \ge \frac{400}{379 \times 10^6 \pi}$

$\Rightarrow \quad 2r \ge 1.15$ mm

Hence, the correct answer is (B).

31. In steady flow the viscous force is balanced by the force due to the pressure difference at the back and front ends. So, we have

$$P_1\pi r^2 - P_2\pi r^2 = 4\pi\eta L v_m$$

$\Rightarrow \quad v_m = \frac{(\Delta P)r^2}{4\eta L} \qquad \{\because P_1 - P_2 = \Delta P\}$

Hence, the correct answer is (B).

32. $U = \frac{(\text{Normal Stress})^2}{2K}$

$\Rightarrow \quad U = \frac{1}{2}\beta(hdg)^2$

Hence, the correct answer is (A).

33. Mass of liquid in capillary is

$$m = \pi r^2 h\rho \text{, where } h = \frac{2T}{r\rho g}$$

$\Rightarrow$ Mass $\propto r$

When r is doubled, then mass of water in capillary will also become two times.

Hence, the correct answer is (C).

34. $\frac{\Delta V}{V} = (1 - 2\sigma)\frac{\Delta L}{L}$

$\Rightarrow \quad \frac{\Delta V}{V} = (1 - 0.4)2 \times 10^{-3}$

$\Rightarrow \quad \frac{\Delta V}{V} = 1.2 \times 10^{-3}$

So, percentage value is 0.12%

Hence, the correct answer is (A).

35. $F = YA\alpha\Delta t = (2 \times 10^{11})(3 \times 10^{-6})(10^{-5})(20 - 10)$

$\Rightarrow \quad F = 60$ N

Hence, the correct answer is (C).

36. Maximum stress on the wire will be at highest point (at point of suspension).

$\Rightarrow \quad \text{Stress} = \sigma = \frac{W}{A} = \frac{A\ell\rho g}{A}$

Hence, the correct answer is (D).

37. Since $Y = \frac{FL}{A|\Delta L|}$

$\Rightarrow \quad \Delta L = \frac{FL}{AY}$

Since $\Delta L = L\alpha T$

$\Rightarrow \quad \alpha = \frac{F}{AYT}$

$\Rightarrow \quad \gamma = 3\alpha = \frac{3F}{\pi r^2 YT}$

Hence, the correct answer is (B).

38. For a freely falling vessel, we have $g_{\text{eff}} = 0$

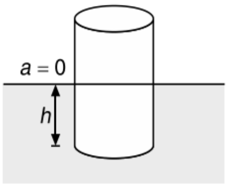
$\Rightarrow \quad$ Upthrust $= 0$

Therefore, the volume of block immersed in the liquid can be any arbitrary value.

Hence, the correct answer is (C).

39. Let h height of float submerged in water as shown in Figure.

For equilibrium, we have

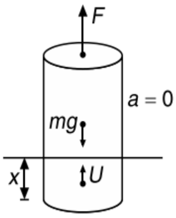


$$mg = U$$

$\Rightarrow \quad (AHd)g = (Ah\rho)g$

$\Rightarrow \quad h = \frac{Hd}{\rho}$

Now when force F is applied, then for minimum work, the float is slowly pulled out. When submerged length of float is x, then



$$(F + \rho Axg) - mg = 0$$

$\Rightarrow \quad F = mg - \rho Axg$

$\Rightarrow \quad W = \int F dx = \int (mg - \rho Axg)dx$

$\Rightarrow \quad W = mg\int dx - \rho Ag\int_0^h x dx$

$\Rightarrow \quad W = mgh - \frac{\rho Agh^2}{2}$

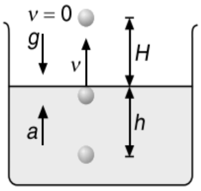
$\Rightarrow \quad W = (\rho Ah)gh - \frac{\rho Agh^2}{2} = \frac{\rho Agh^2}{2}$

$\Rightarrow \quad W = \frac{\rho Ag}{2}\left(\frac{Hd}{\rho}\right)^2 = \frac{Agd^2H^2}{2\rho}$

Hence, the correct answer is (C).

40. Volume of ball is $V = \frac{m}{\rho}$

Acceleration of ball inside the liquid is



$$a = \frac{F_{\text{net}}}{m} = \frac{\text{upthrust} - \text{weight}}{m}$$

$\Rightarrow \quad a = \frac{\left(\frac{m}{\rho}\right)(3\rho)(g) - mg}{m} = 2g$, upwards

Velocity of ball while reaching at surface is

$$v = \sqrt{2ah} = \sqrt{4gh}$$

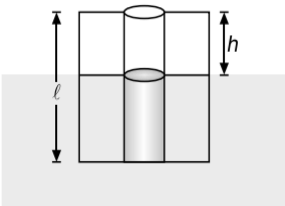
The ball will jump to a height, so we have

$$H=\frac{v^2}{2g}=\frac{4gh}{2g}=2h$$

Hence, the correct answer is (C).

41. Let area of ice-cuboid excluding hole be A, then for floating, we have

$$\begin{pmatrix}\text{Weight of Ice}\\ \text{Block}\end{pmatrix}=\begin{pmatrix}\text{Weight of Liquid}\\ \text{Displaced}\end{pmatrix}$$



$\Rightarrow$ $W_{\text{ice block}}=U$

$\Rightarrow$ $A\rho_{\text{ice}}lg=A\rho_{\omega}(l-h)g$

$\Rightarrow$ $\dfrac{9l}{10}=l-h$

$\Rightarrow$ $h=l-\dfrac{9l}{10}=\dfrac{l}{10}$

Hence, the correct answer is (D).

42. $\dfrac{\Delta P_1}{\Delta P_2}=\dfrac{0.01}{0.02}=\dfrac{1}{2}$

$\Rightarrow$ $\dfrac{4T/r_1}{4T/r_2}=\dfrac{1}{2}$

$\Rightarrow$ $\dfrac{r_1}{r_2}=2$

$\Rightarrow$ $\dfrac{V_1}{V_2}=\left(\dfrac{r_1}{r_2}\right)^3=8$

Hence, the correct answer is (A).

43. Let y be the height of liquid at some instant. Then

$$-\frac{dy}{dt}=\text{constant} \qquad \{\text{given}\}$$

From Equation of Continuity, we get

$$(\pi x^2)\left(-\frac{dy}{dt}\right)=a\sqrt{2gy} \qquad \ldots(1)$$

where, a is area of hole

Since, π, $\left(-\dfrac{dy}{dt}\right)$, a and g are constants. Hence, squaring equation (1), we get

$$y=kx^4$$

Hence, the correct answer is (A).

44. Let h be the height of water inside the capillary.
Total upward force tending to pull the water up supports the weight of the water.

$\Rightarrow$ $T(2\pi r_1+2\pi r_2)=h(\pi r_2^2-\pi r_1^2)\rho g$

$\Rightarrow$ $h=\dfrac{2T}{(r_2-r_1)\rho g}$

$\Rightarrow$ $h=\dfrac{2\times7\times10^{-2}}{(1\times10^{-3})(10^3)(10)}$

$\Rightarrow$ $h=14\times10^{-3}\text{ m}=1.4\text{ cm}$

Hence, the correct answer is (A).

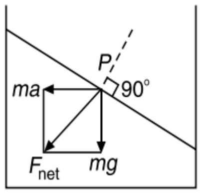
45. $(\Sigma Q)_{\text{inflow}}=(\Sigma Q)_{\text{outflow}}$

$\Rightarrow$ $(4\times10^{-6})+Q+(4\times10^{-6})=(8\times10^{-6})+(2\times10^{-6})+(5\times10^{-6})+(6\times10^{-4})$

$\Rightarrow$ $Q=13\times10^{-6}\text{ m}^3\text{s}^{-1}$

Hence, the correct answer is (C).

46. Net force on the free surface of the liquid in equilibrium (from accelerated frame) should be perpendicular to it. Forces on a water particle P on the free surfaces have been shown in Figure. In the figure ma is the pseudo force.



Hence, the correct answer is (C).

47. Area of wire is $A=\pi r^2$

Since $Y=\dfrac{\frac{F}{A}}{\frac{\Delta\ell}{\ell_0}}$

$\Rightarrow$ $\dfrac{Mg}{\pi r^2}=\dfrac{\Delta\ell}{\ell_0}Y$

$\Rightarrow$ $\dfrac{Mg}{\pi r^2}=\left(\dfrac{4\times10^{-3}}{2}\right)Y \qquad \ldots(1)$

Mass of load of volume V is given by

$$M=V(8\rho_w)$$

Now when load is immersed in liquid, then

$$\frac{8V\rho_w g-2V\rho_w g}{\pi r^2}=\frac{\Delta\ell'}{\ell_0}Y \qquad \ldots(2)$$

$\Rightarrow$ $\dfrac{6V\rho_w g}{\pi r^2}=\dfrac{\Delta\ell'}{\ell_0}Y$

$\Rightarrow$ $\dfrac{\Delta\ell'}{4\times10^{-3}}=\dfrac{6V\rho_w g}{8V\rho_w g}$

$\Rightarrow$ $\Delta\ell'=\dfrac{6}{8}\times4\times10^{-3}\text{ m}$

$\Rightarrow$ $\Delta\ell'=3\times10^{-3}\text{ m}=3\text{ mm}$

Hence, the correct answer is (D).

48. Let m_1 be the mass of concrete and m_2 be the mass of wood
From Law of Floatation, we have

$$W=U$$

$\Rightarrow$ $(m_1+m_2)g=\left(\dfrac{m_1}{2.5}+\dfrac{m_2}{0.5}\right)(1)(g)$

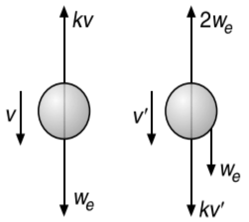
$\Rightarrow$ $\dfrac{m_1}{m_2}+1=\dfrac{2}{5}\left(\dfrac{m_1}{m_2}\right)+2$

$\Rightarrow$ $\dfrac{3}{5}\left(\dfrac{m_1}{m_2}\right)=1$

$\Rightarrow$ $\dfrac{m_1}{m_2}=\dfrac{5}{3}$

Hence, the correct answer is (A).

49. If W_e be the effective weight, then we have



$$kv = W_e \quad \ldots(1)$$

In equilibrium, we have

$$2W_e - W_e = kv' \quad \ldots(2)$$

From equations (1) and (2), we get

$$v' = v = 1 \text{ ms}^{-1}$$

Hence, the correct answer is (A).

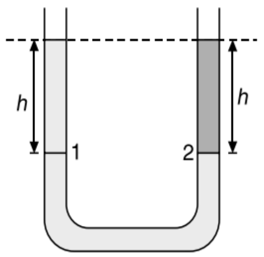
50. Since, $\Delta P = P_2 - P_1 = \rho g \Delta H$

$\Rightarrow \quad 3.03 \times 10^6 = 10^3 \times 10 \times \Delta H$

$\Rightarrow \quad \Delta H \simeq 300 \text{ m}$

Hence, the correct answer is (C).

51. Since $P_1 = P_2$



$\Rightarrow \quad P_0 + \rho_w g h = P_0 + \rho g h$

$\Rightarrow \quad \rho = \rho_w$

Hence, the correct answer is (A).

52. Energy of satellite at radius r is

$$E = -\frac{GMm}{2r}$$

Due to friction losses (due to air) energy of the satellite goes on decreasing i.e. r goes on increasing. Since, orbital speed

$$v_0 \propto \frac{1}{\sqrt{r}}$$

or it goes on increasing till it finally falls back on the earth.

Hence, the correct answer is (A).

53. $\text{Reading} = \begin{pmatrix} \text{weight of} \\ \text{bucket of} \\ \text{water} \end{pmatrix} + \begin{pmatrix} \text{magnitude of} \\ \text{upthrust of} \\ \text{block} \end{pmatrix}$

$$\text{Reading} = (10g) + \frac{1}{2}\left(\frac{7.2}{7.2\rho_w}\right)\rho_w g$$

$$\text{Reading} = 10.5g = 10.5 \text{ kg}$$

Hence, the correct answer is (B).

54. Volume flow rate is $Q = av = \dfrac{V}{t}$

$$\Rightarrow \quad v_1 = \frac{V}{A_1 t} = \frac{120 \times 10^{-3}}{5 \times 10^{-4} \times 2 \times 60}$$

Since, $A_1 v_1 = A_2 v_2$

and $A_2 = \dfrac{A_1}{5}$

$\Rightarrow \quad v_2 = 5v_1 = 5(2) = 10 \text{ ms}^{-1}$

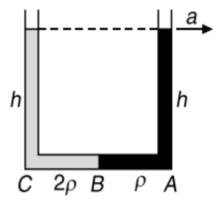
Now, $t_0 = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2 \times 1}{10}}$

$\Rightarrow \quad t_0 = 0.447 \text{ s}$

$\Rightarrow \quad R = v_2 t_0 = 4.47 \text{ m}$

Hence, the correct answer is (C).

55. For the given situation, liquid of density 2ρ should be behind that of ρ as both liquids are at same level as shown in Figure. Pressure in right limb at point A is given by



$$P_A = P_{\text{atm}} + \rho g h$$

Since, $P_B = P_A + \rho a \dfrac{l}{2} = P_{\text{atm}} + \rho g h + \rho a \dfrac{l}{2}$

Pressure in left limb at point C is given by

$$P_C = P_{\text{atm}} + (2\rho) g h \quad \ldots(1)$$

Also, $P_C = P_B + (2\rho) a \dfrac{l}{2}$

$$\Rightarrow \quad P_C = P_{\text{atm}} + \rho g h + \frac{3}{2}\rho a l \quad \ldots(2)$$

From (1) and (2), we get

$$P_{\text{atm}} + \rho g h + \frac{3}{2}\rho a l = P_{\text{atm}} + 2\rho g h$$

$$\Rightarrow \quad h = \left(\frac{3a}{2g}\right) l$$

Hence, the correct answer is (B).

56. The mercury will not come out from this hole as the pressure outside the tube is equal to atmospheric pressure which is greater than the pressure inside the tube which is $P_0 - \rho g H$.

Hence, the correct answer is (D).

57. Applying continuity equation, we get

$$v_A a_A = v_B a_B$$

Since $a_A = a_B$ so $v_A = v_B$

Applying Bernoulli's equation, we get

$$P_A + \rho g h_A + \frac{1}{2}\rho v_A^2 = P_B + \rho g h_B + \frac{1}{2}\rho v_B^2$$

Since $v_A = v_B$ and $h_A = h_B$, so we get

$$P_A = P_B$$

Hence, the correct answer is (C).

58. To measure the atmospheric pressure, same length of tubes containing mercury are required, no matter how many tubes are used as

$$P_{\text{atm}} = \rho_{Hg} g h$$

Hence, the correct answer is (D).

59. $W = 2(T)(\Delta A) = 2T\left(4\pi r^2\right)$

$\Rightarrow \quad W = 8 \times 3 \times 10^{-2} \times 3.14 \times \left(10^{-2}\right)^2$

$\Rightarrow \quad W = 75.36 \times 10^{-6} \text{ J} = 75.36 \text{ μJ}$

Hence, the correct answer is (C).

60. The required pressure should exceed the atmospheric pressure by an amount that can balance the hydrostatic pressure of the water column and the capillary pressure in the air bubble with a radius r.

$$\Rightarrow \quad P = \rho g h + \frac{2T}{r}$$

$$\Rightarrow \quad P = (10^3)(10)(2\times10^{-2}) + \frac{2\times7\times10^{-2}}{(00.5)(10^2)}$$

$$\Rightarrow \quad P = 200 + 280 = 480 \text{ Nm}^{-2}$$

Hence, the correct answer is (A).

61. Let L_0 be the unstretched length and L_3 be the length under a tension of 9 N.

$$\Rightarrow \quad Y = \frac{4L_0}{A(L_1 - L_0)} = \frac{5L_0}{A(L_2 - L_0)} = \frac{9L_0}{A(L_3 - L_0)} \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{4L_0}{A(L_1 - L_0)} = \frac{5L_0}{A(L_2 - L_0)}$$

$$\Rightarrow \quad L_0 = 5L_1 - 4L_2 \qquad \ldots(2)$$

Also, from (1), we get

$$\frac{4}{(L_1 - L_0)} = \frac{9}{(L_3 - L_0)}$$

$$\Rightarrow \quad 4L_3 - 4L_0 = 9L_1 - 9L_0$$

$$\Rightarrow \quad 4L_3 = 9L_1 - 5L_0$$

$$\Rightarrow \quad 4L_3 = 9L_1 - 25L_1 + 20L_2$$

$$\Rightarrow \quad L_3 = 5L_2 - 4L_1$$

Hence, the correct answer is (B).

62. Since, $\text{Strain} = 10^{-3}$

and $\text{Work done} = \frac{1}{2}(\text{Tension})(\text{Extension})$

$$\Rightarrow \quad W = \frac{1}{2}Fx$$

$$\Rightarrow \quad W = \frac{1}{2}\left(\frac{YAx}{L}\right)x$$

$$\Rightarrow \quad W = \frac{1}{2}\frac{(2\times10^{11})(10^{-6})(10^{-6})}{(1)}$$

$$\Rightarrow \quad W = 0.1 \text{ J}$$

Hence, the correct answer is (A).

63. $\rho = \dfrac{M}{V}$

$$\Rightarrow \quad \frac{\Delta\rho}{\rho} = -\frac{\Delta V}{V}$$

$$\Rightarrow \quad \frac{\Delta V}{V} = -0.1\%$$

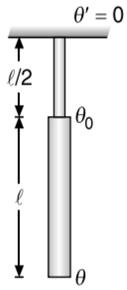
Since, $B = -\dfrac{\Delta P}{\left(\dfrac{\Delta V}{V}\right)}$

$$\Rightarrow \quad \Delta P = (2\times10^9)\left(\frac{0.1}{100}\right)$$

$$\Rightarrow \quad \Delta P = 2\times10^6 \text{ Nm}^{-2}$$

Hence, the correct answer is (C).

64. $\tau = C(\theta) = \dfrac{\pi\eta r^4\theta}{2L} = \text{Constant}$



$$\Rightarrow \quad \frac{\pi\eta r^4(\theta - \theta_0)}{2\ell} = \frac{\pi\eta\left(\frac{r}{2}\right)^4(\theta_0 - \theta')}{2\left(\frac{\ell}{2}\right)}$$

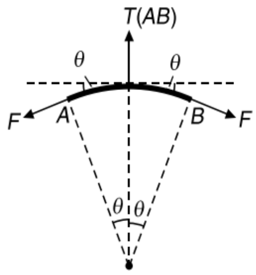
$$\Rightarrow \quad \frac{(\theta - \theta_0)}{2} = \frac{\theta_0}{16}$$

$$\Rightarrow \quad \theta_0 = \frac{8}{9}\theta$$

Hence, the correct answer is (D).

65. Let F be tension in the thread, then

$$2F\sin\theta = T(AB)$$



$$\Rightarrow \quad 2F\theta = T(2R\theta)$$

$$\Rightarrow \quad F = RT$$

Hence, the correct answer is (C).

66. Velocity of body just before touching the lake surface is,

$$v = \sqrt{2gh}$$

Retardation in the lake,

$$a = \frac{\text{upthrust} - \text{weight}}{\text{mass}}$$

$$\Rightarrow \quad a = \frac{V\rho g - V\rho g}{V\rho} = \left(\frac{\sigma - \rho}{\rho}\right)g$$

So, maximum depth is

$$d_{\max} = \frac{v^2}{2a} = \frac{h\rho}{\sigma - \rho}$$

Hence, the correct answer is (A).

67. From the ideal gas equation

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

For a gas the more the Bulk's modulus, the more is elasticity. Since

$$B \propto P$$

$$\Rightarrow \quad \frac{B_2}{B_1} = \frac{E_2}{E_1} = \frac{P_2}{P_1} = \frac{V_1}{V_2}\times\frac{T_2}{T_1} = \left(\frac{1}{4}\right)\times\left(\frac{400}{300}\right) = \frac{1}{3}$$

$$\Rightarrow \quad B_2 = \frac{B_1}{3}$$

So, elasticity will become $\dfrac{1}{3}$ times.

Hence, the correct answer is (D).

68. Inside the satellite, we have $g_{\text{eff}} = 0$

So, there will be no pressure difference inside the mercury. Hence, the mercury wil rise to full length of the tube, i.e., 90 cm because air pressure outside the tube is 76 cm of mercury, while pressure above the tube is zero.

Hence, the correct answer is (B).

69. When value of Poisson's ratio is 0.5, then $\Delta V = 0$ (always). Otherwise, this can be shown as follows.

$$\frac{\Delta V}{V} = \frac{\Delta(\pi r^2 L)}{\pi r^2 L}$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{\Delta L}{L} + \frac{2\Delta r}{r}$$

Since, $\sigma = -\frac{\Delta r/r}{\Delta L/L}$

$$\Rightarrow \frac{\Delta r}{r} = -\sigma\left(\frac{\Delta L}{L}\right)$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{\Delta L}{L}(1-2\sigma)$$

For $\sigma = 0.5$

$$\frac{\Delta V}{V} = 0$$

Hence, the correct answer is (D).

70. $U = \frac{1}{2}Y(\text{Strain})^2$

$$\Rightarrow U = \frac{1}{2}(2\times 10^{11})\left(\frac{0.5}{100}\right)^2$$

$$\Rightarrow U = 2.5\times 10^6 \text{ Jm}^{-3}$$

Hence, the correct answer is (A).

71. Force constant is $k = \frac{YA}{L}$

$$\Rightarrow k \propto Y$$

$$\Rightarrow \frac{k_A}{k_B} = \frac{Y_A}{Y_B} = 2$$

Hence, the correct answer is (B).

72. $\rho g h = \rho a L$

$$\Rightarrow h = \frac{aL}{g}$$

Hence, the correct answer is (B).

73. Flow rate of water $(Q) = 100 \text{ ltmin}^{-1}$

$$\Rightarrow Q = \frac{100\times 10^{-3}}{60} = \frac{5}{3}\times 10^{-3} \text{ m}^3$$

Since $Q = Av$

So, velocity of flow is $v = \frac{Q}{A} = \frac{5\times 10^{-3}}{3\times\pi\times(5\times 10^{-2})^2}$

$$\Rightarrow v = \frac{10}{15\pi} = \frac{2}{3\pi} \text{ ms}^{-1}$$

$$\Rightarrow v = 0.2 \text{ ms}^{-1}$$

Reynold's number $n_R = \frac{Dv\rho}{\eta}$

$$\Rightarrow n_R = \frac{(10\times 10^{-2})\times\frac{2}{3\pi}\times 1000}{1} \simeq 2\times 10^4$$

Order of n_R is 10^4

Hence, the correct answer is (B).

74. Liquid cannot be stretched as a wire. Therefore, young's modulus is not defined for a liquid. Similarly, the free surface of a liquid cannot sustain the shear stress. Hence, shear modulus is also not defined for a liquid. Only bulk modulus is defined for a liquid.

Hence, the correct answer is (A).

75. Volume Flow Rate $= \frac{0.74}{60} \text{ m}^3\text{s}^{-1}$

Speed of efflux $= \frac{0.74\times 10^4}{60\times\pi\times 4} \text{ ms}^{-1} = \sqrt{2gh}$

$$\Rightarrow 9.82 = \sqrt{2\times 10\times h}$$

$$\Rightarrow h = 4.8 \text{ m}$$

Hence, the correct answer is (C).

76. $\Delta\ell = \frac{FL}{AY}$

$$\Rightarrow \frac{\Delta\ell_S}{\Delta\ell_B} = \left(\frac{F_S}{F_B}\right)\left(\frac{L_S}{L_B}\right)\left(\frac{A_B}{A_S}\right)\left(\frac{Y_B}{Y_S}\right)$$

$$\Rightarrow \frac{\Delta\ell_S}{\Delta\ell_B} = \left(\frac{3M}{2M}\right)(a)\left(\frac{1}{b^2}\right)\left(\frac{1}{c}\right) = \frac{3a}{2b^2c}$$

Hence, the correct answer is (B).

77. According to Law of Floatation, $W = U$

$$\Rightarrow V\sigma g = \left(\frac{4V}{5}\right)\rho_\omega g \quad \text{...(1)}$$

Also, $V\sigma g = \frac{V}{2}\rho_\omega g + \frac{V}{2}\rho_{\text{oil}} g$ ...(2)

$$\Rightarrow \left(\frac{\rho_\omega}{2} + \frac{\rho_{\text{oil}}}{2}\right) = \frac{4}{5}\rho_\omega$$

$$\Rightarrow \frac{\rho_{\text{oil}}}{2} = \rho_\omega\left(\frac{4}{5} - \frac{1}{2}\right) = \frac{3}{10}\rho_\omega$$

$$\Rightarrow \rho_{\text{oil}} = \frac{3}{5}\rho_\omega = 0.6\rho_\omega$$

Hence, the correct answer is (D).

78. Since, decrease in weight equals the decrease in upthrust.

$$\Rightarrow mg = V\rho_w g$$

$$\Rightarrow m = V\rho_w$$

If A is area of the base, then

$$(0.2) = (2\times 10^{-2})(A)(10^3)$$

$$\Rightarrow A = 10^{-2} \text{ m}^2 = 100 \text{ cm}^2 = \ell^2$$

$$\Rightarrow \ell = 10 \text{ cm}$$

Hence, the correct answer is (D).

79. $\frac{4}{3}\pi R^3 = 27\left(\frac{4}{3}\pi r^3\right)$

$$\Rightarrow R^3 = 27r^3$$

$$\Rightarrow R = 3r$$

Since $v \propto r^2$

$\Rightarrow \quad \dfrac{v_1}{v_2} = \left(\dfrac{R}{r}\right)^2 = 9$

Hence, the correct answer is (B).

80. According to Law of Floatation, $W = U$

$\Rightarrow \quad (d_1 + d_2)LAg = \dfrac{3}{2}LAdg$

$\Rightarrow \quad (d_1 + d_2) = \dfrac{3}{2}d$

Since, $d_1 < d_2$

$\Rightarrow \quad d_1 < \dfrac{3}{4}d$

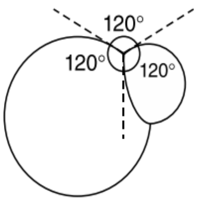
Hence, the correct answer is (A).

81. Upthrust is $U = V_{imm}\rho_{liq}g_{eff}$

Since value of g_{eff} will decrease, so upthrust will also decrease.

Hence, the correct answer is (C).

82. At any point of contact the forces of surface tension balance each other. Therefore, $\theta = 120°$.



Hence, the correct answer is (D).

83. Weight of rod is

$$W = (0.012)(1)(2 \times 10^3)(10) = 240 \text{ N}$$

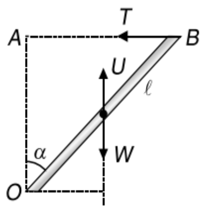
Buoyancy force on rod is

$$U = (0.012)(1)(10^3)(10) = 120 \text{ N}$$

For equilibrium, $\Sigma\tau_O = 0$

The free body diagram of rod is shown in Figure.

$\Rightarrow \quad \tau = (240 - 120)\left(\dfrac{\sin\alpha}{2}\right) = 45(\cos\alpha)$



$\Rightarrow \quad \tan\alpha = \dfrac{90}{120} = \dfrac{3}{4}$

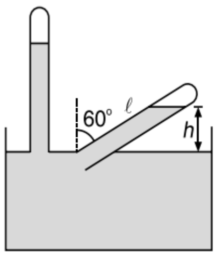
$\Rightarrow \quad \alpha = 37°$

Hence, the correct answer is (B).

84. Since, $\ell\cos 60° = h = 76$ cm

$\Rightarrow \quad \dfrac{\ell}{2} = 76$ cm

$\Rightarrow \quad \ell = 152$ cm



Hence, the correct answer is (A).

85. For floating, $W = U$

$\Rightarrow \quad W_{sphere} + W_{chain} = U$

$W = mg + (\lambda h)g$

Now $U = V_{sphere}\rho_{water}g + V_{chain}\rho_{water}g$

$\Rightarrow \quad U = V_{sphere}(3\rho_{sphere})g + V_{chain}\left(\dfrac{\rho_{chain}}{7}\right)g$

$\Rightarrow \quad U = 3mg + \dfrac{1}{7}(m_{chain}g)$

$\Rightarrow \quad U = 3mg + \dfrac{\lambda hg}{7}$

Since $W = U$

$\Rightarrow \quad mg + \lambda hg = 3mg + \dfrac{\lambda hg}{7}$

$\Rightarrow \quad 2mg = \dfrac{6\lambda hg}{7}$

$\Rightarrow \quad h = \dfrac{7m}{3\lambda}$

Hence, the correct answer is (B).

86. The centre of gravity of liquid in the first column is $\dfrac{h_1}{2}$ and that in the second column is $\dfrac{h_2}{2}$.

Let Initial Potential Energy be U_i

$\Rightarrow \quad U_i = (Ah_1\rho)g\left(\dfrac{h_1}{2}\right) + (Ah_2\rho)g\left(\dfrac{h_2}{2}\right)$

$\Rightarrow \quad U_i = \dfrac{1}{2}A\rho g(h_1^2 + h_2^2)$

When the water column in the two equavalises then equivalent height is $\dfrac{h_1 + h_2}{2}$ and the centre of gravity in both the columns is $\dfrac{h_1 + h_2}{4}$. If U_f be the final potential energy of liquid in both the columns then

$$U_f = 2\left[A\left(\dfrac{h_1 + h_2}{2}\right)\rho\left(\dfrac{h_1 + h_2}{4}\right)g\right]$$

$\Rightarrow \quad U_f = \dfrac{1}{4}A\rho g(h_1 + h_2)^2$

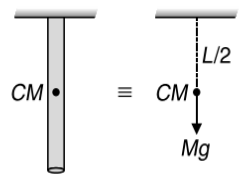
Since work done is equal to the decrease in potential energy, so

$$W = U_i - U_f$$

$\Rightarrow \quad W = A\rho g\left(\dfrac{h_1 - h_2}{2}\right)^2$

Hence, the correct answer is (B).

87. $Y = \dfrac{\text{Stress}}{\text{Strain}}$



$\text{Stress} = \dfrac{ALDg}{A} = LDg$

$\text{Strain} = \dfrac{\Delta L}{L_{eff}} = \dfrac{\ell}{(L/2)}$

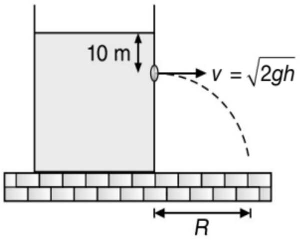
$\Rightarrow \quad Y = \dfrac{LDg}{(2\ell/L)}$

$$\Rightarrow \quad \ell = \frac{L^2 Dg}{2Y}$$

Hence, the correct answer is (B).

88. Area enclosed by hysterisis loop gives the energy loss in the process of stretching and unstretching of rubber band and this loss will appear in the form of heating.
Hence, the correct answer is (A).

89. Range becomes twice if speed of efflux becomes twice.
Since, $v = \sqrt{2gh}$
Therefore, h should become 4 times or 40 m
So, an extra pressure equivalent to 30 m of water should be applied.



Since, $1 \text{ atm} = 0.76 \times 13.6 \text{ m}$ of water
$\Rightarrow \quad 1 \text{ atm} = 10.336 \text{ m}$ of water
$\Rightarrow \quad 30 \text{ m}$ of water ≈ 3 atm
Hence, the correct answer is (D).

90. $B = \frac{\Delta P}{\Delta V/V} = \frac{h\rho g}{\Delta V/V} = \frac{200 \times 10^3 \times 10}{0.1/100} = 2 \times 10^9$

Hence, the correct answer is (D).

91. $F = 2000 \text{ N}$, $L = 6 \text{ m}$, $\ell = 0.5 \text{ cm}$, $A = 10^{-6} \text{ m}^2$

$$\Rightarrow \quad Y = \frac{FL}{A\ell} = \frac{2000 \times 6}{10^{-6} \times 0.5 \times 10^{-2}}$$

$$\Rightarrow \quad Y = 2.35 \times 10^{12} \text{ Nm}^{-2}$$

Hence, the correct answer is (A).

92. $U = \frac{1}{2} \times \frac{YA\ell^2}{L}$

$$\Rightarrow \quad U = \frac{1}{2} \times \frac{2 \times 10^{11} \times 3 \times 10^{-6} \times (1 \times 10^{-3})^2}{4}$$

$$\Rightarrow \quad U = 0.075 \text{ J}$$

Hence, the correct answer is (C).

93. Change in pressure $\Delta P = \rho gh$
By definition, Bulk modulus is

$$B = -\frac{\Delta P}{\Delta V/V}$$

But for a fixed mass, we have $\Delta m = 0$

$$\Rightarrow \quad 0 = \Delta m = \rho \Delta V + V \Delta \rho$$

$$\Rightarrow \quad \frac{\Delta V}{V} = -\frac{\Delta \rho}{\rho}$$

$$\Rightarrow \quad \Delta \rho = \frac{\rho \Delta P}{B} = \frac{\rho(\rho gh)}{B} = \frac{\rho^2 gh}{B}$$

Hence, the correct answer is (C).

94. Let area of hose pipe be A, then $F = Av^2\rho$
The thrust force will be due to the liquid whose momentum changes.

$$\Rightarrow \quad F = \left(\frac{\rho A}{4}\right)v^2 + \left(\frac{\rho A}{4}\right)(2v^2)$$

So, pressure P is given by

$$P = \frac{3\rho A v^2}{4A} = \frac{3\rho}{4}v^2$$

Hence, the correct answer is (A).

95. $\Delta P = \rho_{\text{oil}} g h_{\text{oil}} + \rho_w g h_w$

$$\Rightarrow \quad \Delta P = (600)(10)(10 \times 10^{-2}) + (1000)(10)(2 \times 10^{-2})$$

$$\Rightarrow \quad \Delta P = 800 \text{ Nm}^{-2} = 800 \text{ Pa}$$

Hence, the correct answer is (D).

96. Given $(50)^3 \times \frac{30}{100} \times (1) \times g = M_{\text{cube}} g$ ...(1)

Let m be the mass placed on block for just submerging it, then

$$(50)^3 \times (1) \times g = (M_{\text{cube}} + m)g \quad \text{...(2)}$$

Subtracting (1) from (2), we get

$$mg = (50)^3 \times g(1 - 0.3) = 125 \times 0.7 \times 10^3 \text{ g}$$

$$\Rightarrow \quad m = 87.5 \text{ kg}$$

Hence, the correct answer is (C).

97. Inside the water, weight is equal to upthrust.
So, apparent weight is zero.
Hence, the correct answer is (D).

98. Fraction of volume immersed is

$$f = \frac{\rho_s}{\rho_l}$$

$$\Rightarrow \quad f \propto \frac{1}{\rho_l}$$

$$\Rightarrow \quad \frac{f_1}{f_2} = \frac{\rho_{l_2}}{\rho_{l_1}}$$

$$\Rightarrow \quad \rho_{l_2} = \left(\frac{f_1}{f_2}\right)\rho_{l_1} = \frac{\left(V - \frac{V}{3}\right)}{\left(\frac{V}{3}\right)}\rho_{l_1}$$

$$\Rightarrow \quad \rho_{l_2} = 2\rho_{l_1}$$

Hence, the correct answer is (B).

99. $F = \eta A \frac{dv}{dy} = \frac{(2)(20 \times 10^{-4})(10^{-2})}{10^{-3}}$

$$\Rightarrow \quad F = 4 \times 10^{-2} \text{ N}$$

Hence, the correct answer is (D).

100. Since, $g_{\text{eff}} = 0$, so pressure inside and pressure outside the hole will be same.
Hence, the correct answer is (D).

101. Since $\Sigma F_{\text{upwards}} = \Sigma F_{\text{downwards}}$

$$\Rightarrow \quad N + kx = mg$$

$$\Rightarrow \quad N = 20 - 1 = 19 \text{ N}$$

Hence, the correct answer is (B).

CHAPTER 1

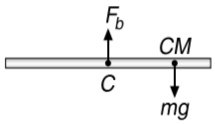
102. Since, $\Delta P = \frac{1}{2}\rho v^2$

$$\Rightarrow \quad v = \sqrt{\frac{2\Delta P}{\rho}} = \sqrt{\frac{2\times 5\times 10^5}{1000}} = 31.5 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

103. Torque about C of rod is

$$(mg)\frac{l}{4} = I\alpha$$

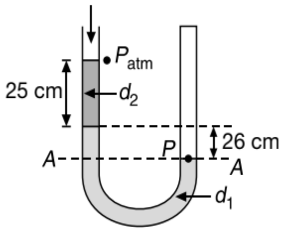


$$\Rightarrow \quad I\alpha = (\pi r^2)(l)(\rho)(g)\left(\frac{l}{4}\right)$$

$$\Rightarrow \quad \alpha = \frac{\pi r^2 l^2 g\rho}{4I}$$

Hence, the correct answer is (B).

104. Pressure in a liquid at same level is same, so using Pascal's equation from open end, we get



$$P_{atm} + yd_2g + xd_1g = P$$

In C.G.S. units, we have

$$P_{atm} + (13.6\times 2)(25)g + (13.6)(26)g = P$$

$$\Rightarrow \quad P_{atm} + 13.6g(50+26) = P$$

Since, $P_{atm} = 76\times 13.6\times g$

$$\Rightarrow \quad 2P_{atm} = P$$

$$\Rightarrow \quad P_{atm} = \frac{P}{2}$$

Hence, the correct answer is (C).

105. Weight of wire is $A\ell\rho g$

$$\Rightarrow \quad W = A\ell\rho g$$

Breaking Stress $= \frac{W}{A}$

$\Rightarrow$ Breaking Stress $= \ell\rho g$

$$\Rightarrow \quad \ell = \frac{10^6}{3\times 10^3(10)}$$

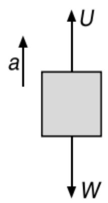
$$\Rightarrow \quad \ell = \frac{1}{3}\times 100$$

$$\Rightarrow \quad \ell \simeq 34 \text{ m}$$

Hence, the correct answer is (B).

106. Let m be the mass of block and a its acceleration in upward direction. Then

$$a = \frac{\text{upthrust} - \text{weight}}{m} = \frac{U - W}{m}$$



$$\Rightarrow \quad a = \frac{\left(\frac{m}{0.5\times 10^3}\right)(10^3)(g+a_0) - mg}{m}$$

$$\Rightarrow \quad a = 2\left(g + \frac{g}{2}\right) - g = 2g$$

Acceleration of block relative to water is

$$a_r = a - a_0 = 2g - \frac{g}{2} = \frac{3g}{2}$$

Since, $h = \frac{1}{2}a_r t^2$

$$\Rightarrow \quad t = \sqrt{\frac{2h}{a_r}} = \sqrt{\frac{2h}{\frac{3}{2}g}} = 2\sqrt{\frac{h}{3g}}$$

$$\Rightarrow \quad t = 2\sqrt{\frac{1.2}{3\times 10}} = 0.4 \text{ s}$$

Hence, the correct answer is (B).

107. In a freely falling system, $g_{eff} = 0$ and since

$$U = V_{imm}\rho_{liquid}g_{eff}$$

So, upthrust $U = 0$

Hence, the correct answer is (A).

108. $\ell = \frac{FL}{\pi r^2 Y}$

$\Rightarrow \quad r^2 \propto \frac{1}{Y}$ (F, L and ℓ are constant)

$$\Rightarrow \quad \frac{r_2}{r_1} = \left(\frac{Y_1}{Y_2}\right)^{1/2} = \left(\frac{7\times 10^{10}}{12\times 10^{10}}\right)^{1/2}$$

$$\Rightarrow \quad r_2 = 1.5\times\left(\frac{7}{12}\right)^{1/2} = 1.145 \text{ mm}$$

$\Rightarrow$ Diameter $= 2r_2 = 2.29$ mm

Hence, the correct answer is (C).

109. $B = \frac{\Delta P}{\Delta V / V}$

$$\Rightarrow \quad B = \frac{(1.165 - 1.01)\times 10^5}{10/100} = \frac{0.155\times 10^5}{1/10}$$

$$\Rightarrow \quad B = 1.55\times 10^5 \text{ Pa}$$

Hence, the correct answer is (D).

110. Shearing strain

$$\phi = \frac{x}{L} = \frac{0.02 \text{ cm}}{10 \text{ cm}}$$

$$\Rightarrow \quad \phi = 0.002$$

Hence, the correct answer is (D).

111. According to Newton's Second Law, we have

$$F = \frac{\Delta p}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right)(\Delta v)$$

$$\Rightarrow \quad F = \rho\left(\frac{\Delta V}{\Delta t}\right)(2v)$$

$$\Rightarrow \quad F = \rho(Av)(2v) = 2\rho Av^2$$

Hence, the correct answer is (C).

112. Since, $F = YA\alpha\Delta T$

$\Rightarrow \quad \dfrac{F_1}{F_2} = \dfrac{\alpha_1}{\alpha_2} = \dfrac{3}{2}$

Hence, the correct answer is (A).

113. $v_T = \dfrac{2r^2(\rho - \sigma)g}{9\eta}$

$\Rightarrow \quad v_T \propto r^2$

Let R be the radius of bigger drop, then by equating the volumes, we get

$$\frac{4}{3}\pi R^3 = 2\left(\frac{4}{3}\pi r^3\right)$$

$\Rightarrow \quad R = (2)^{1/3} r$

Hence, the terminal velocity of big drop

$$v' = (2)^{2/3} v = (4)^{1/3} v$$

Hence, the correct answer is (D).

114. For $\theta = 0°$, height h to which liquid rises in the capillary is

$$h = \frac{2T}{r\rho g}$$

$\Rightarrow \quad \Delta U = mg\dfrac{h}{2} = (\pi r^2 h\rho)g\dfrac{h}{2}$

$\Rightarrow \quad \Delta U = \dfrac{\pi r^2 \rho g}{2}\left(\dfrac{4T^2}{r^2\rho^2 g^2}\right) = \dfrac{2\pi T^2}{\rho g}$

Hence, the correct answer is (B).

115. Speed of efflux at a depth h is given by

$$v = \sqrt{2gh}$$

Since volume of water flowing out per second from both the holes are equal, so According to Equation of Continuity, we get

$$a_1 v_1 = a_2 v_2$$

$\Rightarrow \quad (L^2)\sqrt{2g(y)} = \pi R^2\sqrt{2g(4y)}$

$\Rightarrow \quad R = \dfrac{L}{\sqrt{2\pi}}$

Hence, the correct answer is (A).

116. $\Delta P_A = \rho gh = \rho a\ell$

Hence, the correct answer is (B).

117. Length of rod inside the water is

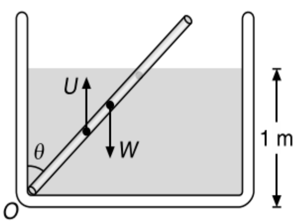
$$\ell = 1\sec\theta = \sec\theta$$

Upthrust $U = \left(\dfrac{2}{2}\right)(\sec\theta)\left(\dfrac{1}{500}\right)(1000)(10)$

$\Rightarrow \quad U = 20\sec\theta$

Weight of rod is $W = mg = 2\times 10 = 20$ N

For rotational equilibrium of rod net torque about O should be zero, i.e. $\Sigma\tau_O = 0$



$\Rightarrow \quad F\left(\dfrac{\sec\theta}{2}\right)(\sin\theta) = W(1\sin\theta)$

$\Rightarrow \quad \dfrac{20}{2}\sec^2\theta = 20$

$\Rightarrow \quad \theta = 45°$

$\Rightarrow \quad F = 20\sec 45° = 20\sqrt{2}$ N

For vertical equilibrium of rod, force exerted by the hinge on the rod will be $(20\sqrt{2} - 20)$ N downwards or $8.28\text{ N} \approx 8.3\text{ N}$ downwards.

Hence, the correct answer is (C).

118. For a capillary tube, $h \propto \dfrac{1}{r}$

Also, $M \propto \pi r^2 h$

$\Rightarrow \quad M \propto r$

Hence, the correct answer is (A).

119. Let ρ_s be the density of sugar solution, so

$$\rho_s > \rho_w$$

If m be the mass of ice, then for floating condition we have,

$$W = U$$

$\Rightarrow \quad mg = V_{\text{initial}}\rho_s g$

$\Rightarrow \quad V_{\text{initial}} = \dfrac{m}{\rho_{\text{sugar}}} = \dfrac{m}{\rho_s}$ …(1)

When ice melts, m mass of ice becomes m mass of water, so volume of this water formed is

$$V_{\text{final}} = \frac{m}{\rho_w} \quad \text{…(2)}$$

Since, $\rho_w < \rho_s$

$\Rightarrow \quad V_{\text{final}} > V_{\text{initial}}$. Hence, level will increase.

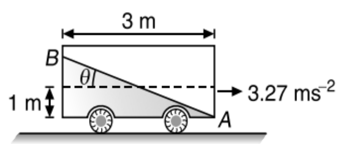
Hence, the correct answer is (A).

120. Density of liquid decrease with increase in temperature and level of liquid and hence, $h \propto \dfrac{1}{\rho}$

$\Rightarrow \quad h_2 > h_1 \qquad \{\because \rho_{100\,°C} < \rho_{50\,°C}\}$

Hence, the correct answer is (C).

121. The liquid is in equilibrium with respect to container, if θ be the angle made by liquid surface with horizontal as shown in Figure, then given as



$$\tan\theta = \frac{a}{g} = \frac{1}{3}$$

Depth of water at point A is

$$h_A = 1 - 1.5\tan\theta = 0.5 \text{ m}$$

Depth of water at point B is

$$h_B = 1 + 1.5\tan\theta = 1.5 \text{ m}$$

Hence, the correct answer is (C).

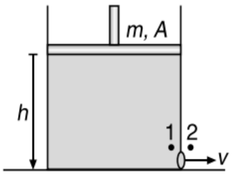
122. $F = YA\alpha\Delta T$

$\Rightarrow \quad F = (10^{11})(10^{-4})(10^{-5})(100)$

$\Rightarrow \quad F = 10^4$ N

Hence, the correct answer is (B).

123. Applying Bernoulli's theorem at 1 and 2, we get



$$\Rightarrow \quad \rho gh + \frac{mg}{A} = \frac{1}{2}\rho v^2$$

$$\Rightarrow \quad v = \sqrt{2gh + \frac{2mg}{\rho A}} = \sqrt{2\left(gh + \frac{mg}{\rho A}\right)}$$

Hence, the correct answer is (B).

124. If coefficient of volume expansion is α and rise in temperature is ΔT then $\Delta V = V\alpha\Delta T$

$$\Rightarrow \quad \frac{\Delta V}{V} = \alpha\Delta T$$

Volume elasticity i.e., Bulk's Modulus is defined as

$$\beta = \frac{P}{\Delta V/V}$$

$$\Rightarrow \quad \beta = \frac{P}{\alpha\Delta T}$$

$$\Rightarrow \quad \Delta T = \frac{P}{\alpha\beta}$$

Hence, the correct answer is (A).

125. $x \propto \frac{\ell}{d^2}$

$$\Rightarrow \quad x \propto \frac{\ell}{r^2}$$

$$\Rightarrow \quad \frac{x_2}{x_1} = \frac{r_1^2}{r_2^2}$$

$$\Rightarrow \quad x_2 = (3)\frac{1}{\left(\frac{1}{2}\right)^2}$$

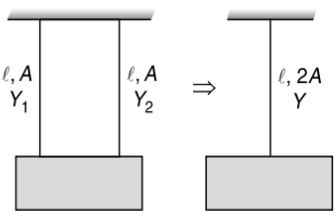
$$\Rightarrow \quad x_2 = 3 \times 4 = 12 \text{ mm}$$

Hence, the correct answer is (D).

126. Equivalent spring constant of a wire is given by

$$k = \frac{YA}{\ell}$$

Since, $k_{eq} = k_1 + k_2$



$$\Rightarrow \quad \frac{Y(2A)}{\ell} = \frac{Y_1 A}{\ell} + \frac{Y_2 A}{\ell}$$

$$\Rightarrow \quad Y = \frac{Y_1 + Y_2}{2}$$

Hence, the correct answer is (B).

127. According to Law of Floatation, we have

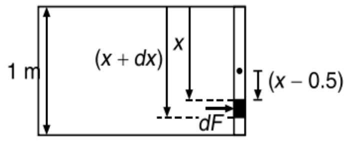
$$W = U$$

$$\Rightarrow \quad Mg = \left(\frac{3}{4}a^3\right)\rho g$$

$$\Rightarrow \quad a = \left(\frac{4M}{3\rho}\right)^{\frac{1}{3}}$$

Hence, the correct answer is (D).

128. For equilibrium, the net torque (τ) about hinge is zero. Let us calculate torque on the gate due to liquid. Consider a section of gate at distance x from water surface, having a cross-sectional width dx as shown in Figure. If dF be the infinitesimal force acting on this element, then



$$dF = PdA = (\rho gx)(1dx)$$

Torque on this element is

$$d\tau = (dF)r_{\perp} = (\rho gx)(x - 0.5)dx$$

So, net anticlockwise torque is

$$\tau = \int_0^1 d\tau = \left(\frac{\rho g}{12}\right)$$

Net clockwise torque due to external applied force is

$$\tau = F(0.5) = \frac{F}{2}$$

Equating the two torques, we get

$$F = \frac{\rho g}{6}$$

Hence, the correct answer is (C).

129. Work done against gravity

$$W_1 = mgh = (V\rho)gh$$

Work done against pressure difference is

$$W_2 = (\Delta P)V = (h\rho g)V$$

$\Rightarrow$ Total work done $W = W_1 + W_2 = 2h\rho gV$

$$\Rightarrow \quad P = \frac{W}{t} = \frac{2h\rho gV}{t}$$

$$\Rightarrow \quad P = \frac{(2)(6)(10^3)(10)(300 \times 10^{-3})}{60} = 600 \text{ W}$$

Hence, the correct answer is (A).

130. $Y = 10^4 \text{ Nm}^{-2}$, $A = 2 \times 10^{-4} \text{ m}^2$,

$F = 2 \times 10^5 \text{ dyne} = 2 \text{ N}$

Since, $\ell = \frac{FL}{AY} = \frac{2 \times L}{2 \times 10^{-4} \times 10^4} = L$

So, final length is the sum of initial length and increment i.e.

$$L_{\text{final}} = 2L$$

Hence, the correct answer is (C).

131. Let ρ be the density of liquid. Then

$$F_1 = (\Delta P)a = h\rho ga \quad \ldots(1)$$

In the second case, for the liquid striking the disc elastically, we have

$$F_2 = \frac{\Delta p}{\Delta t} = \frac{2(\Delta m)v}{\Delta t} = \frac{2(\rho a \Delta x)v}{\Delta t}$$

$$\Rightarrow \quad F_2 = 2av\rho\left(\frac{\Delta x}{\Delta t}\right) = 2av^2\rho$$

where, $v = \sqrt{2gh}$ {due to Torricelli's Theorem}

$$\Rightarrow \quad F_2 = 2av^2\rho = 2a\left(\sqrt{2gh}\right)^2 \rho = 4h\rho g a \quad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{F_1}{F_2} = \frac{1}{4}$$

Hence, the correct answer is (D).

132. By definition, Bulk's modulus is given by

$$K = \frac{dP}{\left(\frac{dV}{V}\right)}$$

$$\Rightarrow \quad K = \frac{mg/A}{\left(\frac{dV}{V}\right)}$$

$$\Rightarrow \quad \frac{dV}{V} = \frac{mg}{AK}$$

For a sphere

$$V = \frac{4\pi}{3}R^3$$

$$\Rightarrow \quad dV = \frac{4\pi}{3}d(R^3)$$

$$\Rightarrow \quad dV = \frac{4\pi}{3}(3R^2 dR)$$

$$\Rightarrow \quad \frac{dV}{V} = 3\left(\frac{dR}{R}\right)$$

$$\Rightarrow \quad 3\left(\frac{dR}{R}\right) = \frac{mg}{AK}$$

$$\Rightarrow \quad \frac{dR}{R} = \frac{mg}{3AK}$$

Hence, the correct answer is (B).

133. If length of the wire is doubled then strain $= 1$

$$\Rightarrow \quad Y = \text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{2\times10^5}{2} = 10^5 \text{ dynecm}^{-2}$$

Hence, the correct answer is (B).

134. $F = \frac{YA\ell}{L} = \frac{9\times10^{10}\times\pi\times4\times10^{-6}\times0.1}{100} = 360\pi \text{ N}$

Hence, the correct answer is (A).

135. Since, $\Delta\ell = \left(\frac{\ell}{YA}\right)W$

i.e. $\Delta\ell$ vs W graph is a straight line passing through origin (as shown in questions also). The slope of $\Delta\ell$ vs W graph is $\frac{\ell}{YA}$.

$$\Rightarrow \quad \text{Slope} = \frac{\ell}{YA}$$

$$\Rightarrow \quad Y = \left(\frac{\ell}{A}\right)\left(\frac{1}{\text{slope}}\right)$$

$$\Rightarrow \quad Y = \left(\frac{1}{10^{-6}}\right)\frac{(80-20)}{(4-1)\times10^{-4}} = 2\times10^{11} \text{ Nm}^{-2}$$

Hence, the correct answer is (A).

136. Increase in length due to own weight is

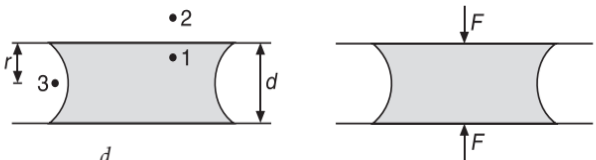
$$\Delta\ell = \int_0^L \frac{(L-x)mg\,dx}{LAY} = \frac{mgL}{2AY}$$

Since, $m = \lambda L$

$$\Rightarrow \quad Y = \frac{mgL}{2A\ell} = \frac{\lambda g L^2}{2A\ell}$$

Hence, the correct answer is (B).

137. For cylindrical surface $\Delta P = \frac{T}{r}$ $\left(\text{not } \frac{2T}{r}\right)$



Here, $r = \frac{d}{2}$

$$\Rightarrow \quad \Delta P = \frac{2T}{d}$$

$$\Rightarrow \quad P_2 - P_1 = \frac{2T}{d} = \Delta P$$

$$\Rightarrow \quad F = (\Delta P)A = (\Delta P)\left(\frac{V}{d}\right) = \frac{(\Delta P)m}{\rho d} \qquad \left\{\because A = \frac{V}{d}\right\}$$

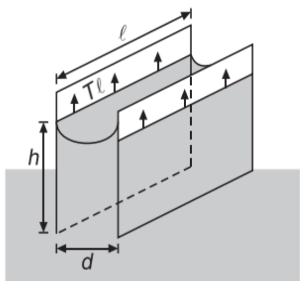
$$\Rightarrow \quad F = \frac{2Tm}{\rho d^2}$$

Hence, the correct answer is (C).

138. When two parallel plates with the spacing 'd' are placed in water reservoir, then liquid will rise till force due to surface tension balances the weight of liquid, so we have

$$2Tl\cos(0°) = (\rho l h d)g$$

$$\Rightarrow \quad h = \frac{2T}{\rho g d}$$



Hence, the correct answer is (B).

139. According to Newton's Second Law, we have

$$F = \frac{\Delta p}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right)(\Delta v)$$

$$\Rightarrow \quad F = \rho\left(\frac{\Delta V}{\Delta t}\right)(v_1 + v_2) = \rho V(v_1 + v_2)$$

Hence, the correct answer is (D).

140. $-\frac{dh}{dt} = \sqrt{2gh}$

$\Rightarrow \quad a\sqrt{2gh} = Q$

$\Rightarrow \quad 10^{-4}\sqrt{2gh} = 10^{-4}$

$\Rightarrow \quad h = \frac{1}{2g}$

$\Rightarrow \quad h = 5.1 \text{ cm}$

Hence, the correct answer is (A).

141. Since, $\frac{T_{Hg}}{T_{\text{water}}} = 7.5$, $\frac{\rho_{Hg}}{\rho_w} = 13.6$ and

$$\frac{\cos\theta_{Hg}}{\cos\theta_W} = \frac{\cos 135^\circ}{\cos 0^\circ} = \frac{1}{\sqrt{2}}$$

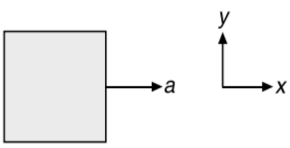
$$\Rightarrow \quad \frac{R_{Hg}}{R_{\text{water}}} = \left(\frac{S_{Hg}}{S_W}\right)\left(\frac{\rho_W}{\rho_{Hg}}\right)\left(\frac{\cos\theta_{Hg}}{\cos\theta_W}\right)$$

$$\Rightarrow \quad \frac{R_{Hg}}{R_{\text{water}}} = 7.5 \times \frac{1}{13.6} \times \frac{1}{\sqrt{2}} = 0.4 = \frac{2}{5}$$

Hence, the correct answer is (D).

142. When a fluid (gas or liquid) is accelerated in positive x direction, then pressure decreases in positive x direction. Change in pressure has differential relation given by

$$\frac{dP}{dx} = -\rho a$$



when ρ is the density of fluid.

Therefore, pressure is lower in front side.

Hence, the correct answer is (B).

143. According to Bernoulli's Theorem

$$\Delta P = \frac{1}{2}\rho v^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2\Delta P}{\rho}} = \sqrt{\frac{2 \times 0.5 \times 10^5}{10^3}} = 10 \text{ ms}^{-1}$$

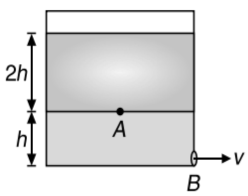
Hence, the correct answer is (D).

144. Pressure at point A is

$$P_A = P_{\text{atm}} + \rho g(2h)$$

Applying Bernoulli's theorem between points A and B, we get

$$P_{\text{atm}} + 2\rho gh + \rho g(2h) = P_{\text{atm}} + \frac{1}{2}(2\rho)v^2$$



$\Rightarrow \quad v = 2\sqrt{gh}$

Hence, the correct answer is (B).

145. $P = P_0 + h\rho g$

$\Rightarrow \quad h = \frac{P - P_0}{\rho g}$

i.e., h depends on P and ρ. So, we cannot conclude anything by seeing h only.

Hence, the correct answer is (D).

146. Since $P = \frac{W}{t}$, we have

$$\Rightarrow \quad P = \frac{\Delta K}{\Delta t} = \frac{\frac{1}{2}(\Delta m)v^2}{\Delta t}$$

$$\Rightarrow \quad P = \frac{1}{2}\left[A\left(\frac{\Delta x}{\Delta t}\right)\rho\right]v^2$$

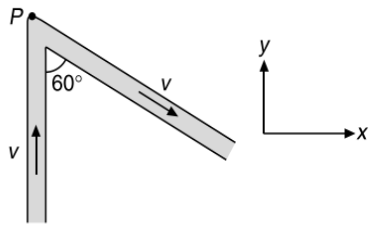
Since, $\frac{\Delta x}{\Delta t} = v$

$\Rightarrow \quad P \propto v^3$

Hence, the correct answer is (C).

147. Along x and y directions along in momentum of water is

$$|\Delta\vec{p}_x| = mv\sin 60^\circ = \frac{\sqrt{3}}{2}mv$$



$$|\Delta\vec{p}_y| = \frac{mv}{2} + mv = \frac{3}{2}mv$$

$$\Rightarrow \quad |\Delta\vec{p}_{\text{net}}| = \sqrt{\Delta p_x^2 + \Delta p_y^2} = mv\sqrt{\frac{9}{4} + \frac{3}{4}}$$

$$\Rightarrow \quad |\Delta\vec{p}_{\text{net}}| = \sqrt{3}mv$$

Force on bend is rate of change of momentum, so we get

$$|\vec{F}_{\text{net}}| = \sqrt{3}\left(\frac{dm}{dt}\right)v = \sqrt{3}\rho Av^2$$

Hence, the correct answer is (A).

148. Since, $v_T = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$

$$\Rightarrow \quad v_T = \frac{2 \times (0.003)^2(1260 \times 2 - 1260) \times 10}{9 \times 1.26}$$

$\Rightarrow \quad v_T = 0.02 \text{ ms}^{-1}$

$$\Rightarrow \quad t = \frac{d}{v_T} = \frac{10 \times 10^{-2}}{0.02} = 5 \text{ s}$$

Hence, the correct answer is (D).

149. $\frac{dV}{V} = (1 - 2\sigma)\frac{dL}{L}$

$$\Rightarrow \quad \frac{dV}{V} = 2 \times 2 \times 10^{-3} = 4 \times 10^{-3} \qquad \left\{\because \sigma = 0.5 = \frac{1}{2}\right\}$$

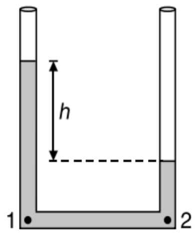
So, percentage change in volume is $4 \times 10^{-1} = 0.4\%$

Hence, the correct answer is (B).

150. Let P_1 and P_2 be the pressures at the bottom of the left end and right end of the tube respectively. Then

$$F = (P_1 - P_2)A = \rho ghA$$

when A is the cross section of tube



The mass of the liquid in the horizontal portion is

$$m = \rho LA$$

Since, $F = ma$

$\Rightarrow \quad \rho ghA = \rho LAa$

$\Rightarrow \quad h = \dfrac{aL}{g}$

Hence, the correct answer is (D).

151. Bulk's Modulus B is

$$B = \frac{(100)(10^3)(10)}{\frac{0.1}{100}}$$

$\Rightarrow \quad B = 10^9 \text{ Nm}^{-2}$

Hence, the correct answer is (B).

152. When the vessel is stationary, then

Weight = Upthrust

$\Rightarrow \quad V\rho_w g = V_i \rho_\ell g$

(ρ_w = density of wood and ρ_L = density of liquid)

$\Rightarrow \quad \dfrac{V_i}{V} = \dfrac{\rho_w}{\rho_L} \qquad \ldots(1)$

When the vessel moves upwards, then

$$(\text{upthrust}) - (\text{weight}) = (\text{mass})(\text{acceleration})$$

$\Rightarrow \quad U' - W = ma$

$\Rightarrow \quad V_i'\rho_L\left(g + \dfrac{g}{2}\right) - V\rho_w g = \dfrac{V\rho_w g}{2}$

$\Rightarrow \quad \dfrac{V_i'}{V} = \dfrac{\rho_w}{\rho_L} \qquad \ldots(2)$

From equation (1) and (2), we get

$$\frac{V_i}{V} = \frac{V_i'}{V}$$

i.e., fraction (or percentage) of volume immersed in liquid remains unchanged and is independent of value of g.

Hence, the correct answer is (C).

153. $F = \dfrac{YA\ell}{L}$

$\Rightarrow \quad F \propto r^2 \qquad \{\because Y, \ell \text{ and } L \text{ are constants}\}$

If diameter is made four times, then force required will become 16 times. i.e., 16×10^3 N

Hence, the correct answer is (B).

154. Let h be the height of the hill, then pressure difference equals the pressure due to h metre of air.

$\Rightarrow \quad (75 - 50)(10^{-2})\rho_{\mu g} g = h\rho_{\text{air}} g$

$\Rightarrow \quad h = (25 \times 10^{-2}) \times \dfrac{\rho_{Hg}}{\rho_{\text{air}}}$

$\Rightarrow \quad h = (25 \times 10^{-2})(10^4) \text{ m} = 2.5 \text{ km}$

Hence, the correct answer is (B).

155. Thermal Stress $= Y\alpha\Delta T$

$\Rightarrow \quad Y_1\alpha_1\Delta T = Y_2\alpha_2\Delta T$

$\Rightarrow \quad \dfrac{Y_1}{Y_2} = \dfrac{\alpha_2}{\alpha_1} = \dfrac{3}{2}$

Hence, the correct answer is (C).

156. The tension on heavier sphere is upwards and on lighter sphere is downwards. So, we have

$$(250 \times 10^{-6})(800)g + T = (250 \times 10^{-6})\rho_w g \qquad \ldots(1)$$

and $(250 \times 10^{-6})(1200)g + T = (250 \times 10^{-6})\rho_w g \qquad \ldots(2)$

From equations (1) and (2), we get

$$T = 0.5 \text{ N}$$

Hence, the correct answer is (B).

157. Torque due to hydrostatic force about centre of sphere is zero because hydrostatic force passes through the centre. So, horizontal force on the cylinder due to two liquids must cancel out.

$\Rightarrow \quad F_{\text{LHS}} = F_{\text{RHS}}$

If L be the length of cylinder, then

$$(P_{\text{avg}})_{\text{left}} hL = (P_{\text{avg}})_{\text{right}} RL$$

$\Rightarrow \quad \dfrac{1}{2}(2\rho gh)(hL) = \dfrac{1}{2}(3\rho gR)(RL)$

$\Rightarrow \quad h = \sqrt{\dfrac{3}{2}}R$

Hence, the correct answer is (B).

158. By Law of Conservation of Energy, we get

$$v_2^2 = v_1^2 + 2gh \qquad \ldots(1)$$

This can also be found by applying Bernoulli's theorem between 1 and 2 as shown in Figure.

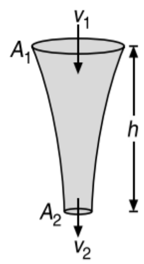
From continuity equation

$$A_1v_1 = A_2v_2$$

$$v_2 = \left(\frac{A_1}{A_2}\right)v_1 \qquad \ldots(2)$$

Substituting value of v_2 from equation (2) in equation (1), we get

$$\left(\frac{A_1^2}{A_2^2}\right)v_1^2 = v_1^2 + 2gh$$



$\Rightarrow \quad A_2^2 = \dfrac{A_1^2 v_1^2}{v_1^2 + 2gh}$

$\Rightarrow \quad A_2 = \dfrac{A_1 v_1}{\sqrt{v_1^2 + 2gh}}$

Substituting the given values, we get

$$A_2 = \frac{(10^{-4})(1)}{\sqrt{(1)^2 + 2(10)(0.15)}} = 5 \times 10^{-5} \text{ m}^2$$

Hence, the correct answer is (C).

159. Since $y = \dfrac{x^2\omega^2}{2g}$

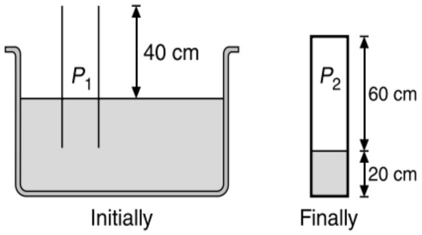
At $x = r$, we get $y = \dfrac{r^2\omega^2}{2g}$

Since $\omega = 4\pi$ rads^{-1}, $r = \frac{5}{100}$ m

$\Rightarrow \quad y = \frac{16\pi^2 \times 25 \times 10^{-4}}{2 \times 10} = 1.9 \text{ cm} \approx 2.0 \text{ cm}$

Hence, the correct answer is (D).

160. Initially $P_1 = P_0 = P_{atm}$



Since temperature is constant, so

$P_1V_1 = P_2V_2$

$\Rightarrow \quad P_2 = P_1\left(\frac{V_1}{V_2}\right) = P_0\left(\frac{40}{60}\right) = \frac{2}{3}P_0$

Finally, when top of tube is closed, then

$P_2 + (20 \text{ cm of Hg}) = P_0$

$\Rightarrow \quad \frac{2}{3}P_0 + (20 \text{ cm of Hg}) = P_0$

$\Rightarrow \quad \frac{P_0}{3} = (20 \text{ cm of Hg})$

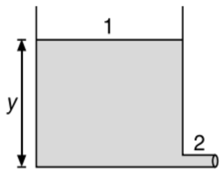
$\Rightarrow \quad P_0 = 60 \text{ cm of Hg}$

Hence, the correct answer is (C).

161. The velocity of fluid at the hole when the level of liquid in the container be $y(<h)$ at any instant is

$$v_2 = \sqrt{\frac{2gy}{1+\left(\frac{a^2}{A^2}\right)}} \quad \ldots(1)$$

Using continuity equation at the two cross-sections 1 and 2, we get



$\Rightarrow \quad Av_1 = av_2$

$\Rightarrow \quad v_1 = \left(\frac{a}{A}\right)v_2$

So, acceleration a_1 of top surface is

$\frac{dv_1}{dt} = \frac{dv_1}{dt} \times \frac{dy}{dy}$

$\Rightarrow \quad a_1 = \frac{dv_1}{dt} = v_1\frac{dv_1}{dy}$

$\Rightarrow \quad a_1 = \frac{a}{A}(v_2)\frac{d}{dy}\left(\frac{a}{A}v_2\right)$

$\Rightarrow \quad a_1 = \frac{a^2}{A^2}v_2\frac{d}{dy}(v_2)$

$\Rightarrow \quad a_1 = -\frac{a^2}{A^2}\sqrt{2gy}\sqrt{2g}\frac{1}{2\sqrt{y}}$

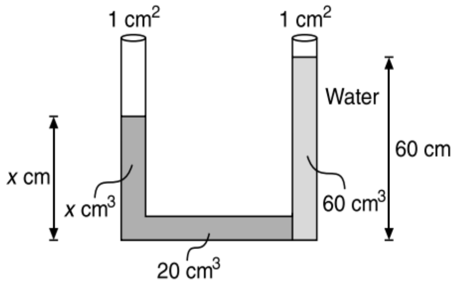
$\Rightarrow \quad a_1 = -\frac{ga^2}{A^2}$

Hence, the correct answer is (D).

162. Since area of cross-section is 1 cm^2, so height to which water should rise is 60 cm .

Hence, $h_1\rho_1 g = h_2\rho_2 g$

$\Rightarrow \quad x(4) = (60)(1)$



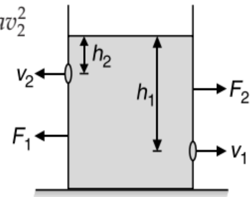
$\Rightarrow \quad x = 15 \text{ cm}$

So, total volume of liquid required is

$V = (20+15) \text{ cm}^3 = 35 \text{ cm}^3$

Hence, the correct answer is (D).

163. Thrust force $F = F_1 - F_2 = \rho a v_1^2 - \rho a v_2^2$



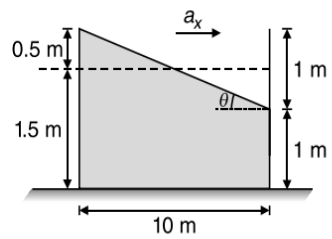
$\Rightarrow \quad F = \rho a(2gh_1) - \rho a(2gh_2)$

$\Rightarrow \quad F = 2\rho ag(h_1 - h_2)$

$\Rightarrow \quad F = 2\rho agh$

Hence, the correct answer is (C).

164. Figure shows the limiting state of oil surface in the tank at the instant when just spilling begins to start from rear end



Since, $\tan\theta = \frac{a_x}{g} = \frac{v}{gt}$

Also, $\tan\theta = \frac{1}{10}$

$\Rightarrow \quad t = \frac{10 \times 20}{10} = 20 \text{ s}$

Hence, the correct answer is (A).

165. Viscous force $F = mg\sin\theta$

$\Rightarrow \quad \eta(a^2)\frac{v}{t} = mg\sin(37°) = \frac{3}{5}mg$

$\Rightarrow \quad \eta a^2\left(\frac{v}{t}\right) = \frac{3}{5}(a^3\rho)g$

$\Rightarrow \quad \eta = \frac{3\rho agt}{5v}$

Hence, the correct answer is (A).

166. Total force at height $\frac{3L}{4}$ from its lower end is

$F = $ Weight suspended + Weight of $\frac{3}{4}$ of the chain

$\Rightarrow \quad F = W_1 + \left(\frac{3W}{4}\right)$

Hence, Stress $= \dfrac{F}{A} = \dfrac{W_1 + \left(\dfrac{3W}{4}\right)}{S}$

Hence, the correct answer is (C).

167. $Y = 3B(1 - 2\sigma)$

$\Rightarrow \quad \sigma = \dfrac{3B - Y}{6K}$

$\Rightarrow \quad \sigma = \dfrac{3 \times 11 \times 10^{10} - 7.25 \times 10^{10}}{6 \times 11 \times 10^{10}}$

$\Rightarrow \quad \sigma = 0.39$

Hence, the correct answer is (C).

168. Since $\eta = \dfrac{\dfrac{F}{A}}{\dfrac{x}{L}}$

$F = -\eta \dfrac{Ax}{L}$

(Negative sign indicates that restoring force is directed towards the mean position)

$\Rightarrow \quad M\ddot{x} = -\eta \dfrac{Ax}{L}$

$\Rightarrow \quad \ddot{x} + \dfrac{\eta A}{ML} x = 0$

Comparing with

$\ddot{x} + \omega^2 x = 0$

we get $\omega = \sqrt{\dfrac{\eta A}{ML}}$

$\Rightarrow \quad T = 2\pi \sqrt{\dfrac{ML}{\eta A}}$

Since $A = L^2$

$\Rightarrow \quad T = 2\pi \sqrt{\dfrac{M}{\eta L}}$

Hence, the correct answer is (D).

169. For points to lie on same stream line

$P + \dfrac{1}{2}\rho v^2 + \rho g h = \text{constant}$

Sum of all three terms are different at three points *A*, *B* and *C*.

Hence, the correct answer is (D).

170. $W = \dfrac{1}{2}\dfrac{YA\ell^2}{L}$

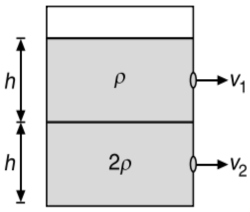
$\Rightarrow \quad 0.4 = \dfrac{1}{2} \times \dfrac{Y \times 10^{-6} \times \left(0.2 \times 10^{-2}\right)^2}{1}$

$\Rightarrow \quad Y = 2 \times 10^{11}\ \text{Nm}^{-2}$

Hence, the correct answer is (C).

171. Since, $v_1 = \sqrt{2g\left(\dfrac{h}{2}\right)} = \sqrt{gh}$ ...(1)

According to Bernoulli's theorem, we have



$\rho g h + 2\rho g\left(\dfrac{h}{2}\right) = \dfrac{1}{2}(2\rho)v_2^2$

$\Rightarrow \quad v_2 = \sqrt{2gh}$ ...(2)

$\Rightarrow \quad \dfrac{v_1}{v_2} = \dfrac{1}{\sqrt{2}}$

Hence, the correct answer is (D).

172. $T = 2\pi\sqrt{\dfrac{M}{K_{eq}}} = 2\pi\sqrt{\dfrac{M(YA + KL)}{YAK}}$

Hence, the correct answer is (B).

173. As the air is pumped out buoyancy due to air will become zero. Hence, $V_2 > V_1$

Hence, the correct answer is (C).

174. $P - P_0 = \dfrac{4T}{R}$

$\Rightarrow \quad P = \dfrac{4T}{R} + P_0$

$V = \dfrac{4}{3}\pi R^3 = kt$

$\Rightarrow \quad R = \left(\dfrac{3kt}{4\pi}\right)^{1/3}$

$\Rightarrow \quad P = P_0 + \dfrac{4T}{R} = P_0 + 4T\left(\dfrac{4\pi}{3kT}\right)^{1/3}$

$\Rightarrow \quad P = m\left(\dfrac{1}{t^{1/3}}\right) + c$

Hence, the correct answer is (D).

175. F_1 will decrease and F_2 will increase. So f_1 may or may not be greater than f_2.

Total weight of system in both conditions will remain same. Hence,

$f_1 + f_2 = F_1 + F_2$

Hence, the correct answer is (A).

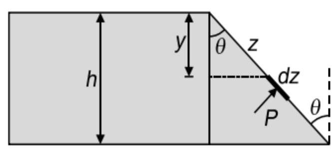
176. Since $F = PA = (\rho g h)A$

$\Rightarrow \quad F = (10^3)(10)(99 + 1) \times 10^{-2} \times 10^{-2}$

$\Rightarrow \quad F = 100\ \text{N}$

Hence, the correct answer is (D).

177. Since, $z = y\sec\theta$



$\Rightarrow \quad dz = dy\sec\theta$

Also, $P = \rho g y$ and

$$dA = bdz = bdy\sec\theta$$

$$\Rightarrow \quad dF = PdA = (\rho g b\sec\theta)ydy$$

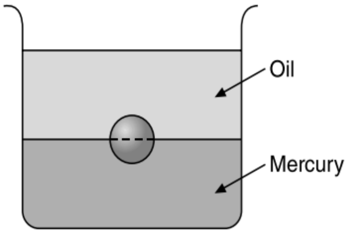
$$\Rightarrow \quad F = \int_0^h dF = \frac{1}{2}\rho b h^2 g\sec\theta$$

Hence, the correct answer is (C).

178. For floating of sphere, we have

$$W = U$$

$$\Rightarrow \quad V\rho_m g = \frac{V}{2}\rho_{Hg} g + \frac{V}{2}\rho_{\text{oil}}$$



$$\Rightarrow \quad \rho_m = \frac{\rho_{Hg} + \rho_{\text{oil}}}{2} = \frac{13.6 + 0.8}{2}$$

$$\Rightarrow \quad \rho_m = \frac{14.4}{2} = 7.2$$

Hence, the correct answer is (C).

179. Let $m_A = 2m$, $m_B = 3m$, density of liquid be ρ and density of block B to be 2ρ

Acceleration of system, when block B is inside the liquid is

$$a_1 = \frac{m_A g - (m_B g - \text{upthrust on } B)}{m_A + m_B}$$

$$\Rightarrow \quad a_1 = \frac{2mg - \left(3mg - \frac{3m}{2\rho}\cdot\frac{1}{2}\rho g\right)}{5\text{ m}}$$

$$\Rightarrow \quad a_1 = \frac{g}{10} = \text{constant}$$

However, when the block is outside the liquid, then the acceleration of system (in opposite direction) is

$$a_2 = \frac{m_B g - m_A g}{m_A + m_B} = \frac{3mg - 2mg}{5m} = \frac{g}{5}$$

Since, a_1 and a_2 are constants, motion is periodic but not simple harmonic.

Hence, the correct answer is (A).

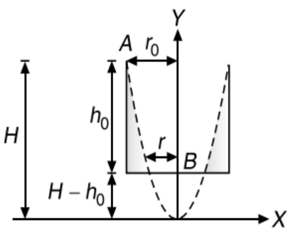
180. Area of bottom is $A_0 = \pi r_0^2$

If r is radius of the exposed bottom, then

$$A = \frac{A_0}{2}$$

$$\Rightarrow \quad \pi r^2 = \frac{1}{2}\pi r_0^2$$

$$\Rightarrow \quad r = \frac{r_0}{\sqrt{2}}$$



Due to rotation of cylinder, the liquid surface has an equation given by

$$y = \frac{\omega^2 x^2}{2g}$$

At $y = H$, $x = r_0$

and at $y = H - h_0$, $x = \frac{r_0}{\sqrt{2}}$

$$\Rightarrow \quad h_0 = \frac{\omega^2}{2g}\left(\frac{r_0^2}{2}\right)$$

$$\Rightarrow \quad \omega = 2\sqrt{\frac{gh_0}{r_0}}$$

$$\Rightarrow \quad \omega = \frac{2\sqrt{10\times 0.1}}{0.02}$$

$$\Rightarrow \quad \omega = 100\text{ rads}^{-1}$$

Hence, the correct answer is (C).

181. Rate of flow $Q = \dfrac{\Delta P}{\left(\dfrac{8\eta L}{\pi R^4}\right)}$

$$\Rightarrow \quad Q \propto (\Delta P)(R^4)$$

Since, ΔP is increased two times, whereas radius is reduced to half, so rate of flow of liquid will become $\frac{1}{8}$ times.

Hence, the correct answer is (D).

182. For wires of same material, we have

$$\frac{F\ell}{Ax} = \text{constant}$$

For same tension applied on all, we have

$$x \propto \frac{\ell}{A}$$

$$\Rightarrow \quad x \propto \frac{\ell}{d^2}$$

Extension produced x is maximum for dimensions given in OPTION **(A)**.

Hence, the correct answer is (C).

183. Acceleration a of body is

$$a = \frac{\text{upthrust} - \text{weight}}{\text{mass}}, \text{ upwards}$$

$$\Rightarrow \quad a = \frac{V\sigma g - V\rho g}{V\rho} = \left(\frac{\sigma}{\rho} - 1\right)g, \text{ upwards}$$

Hence, the correct answer is (A).

184. $B = \dfrac{\Delta P}{\Delta V/V} = \dfrac{h\rho g}{0.1/100} = \dfrac{200\times 10^3\times 9.8}{1/1000}$

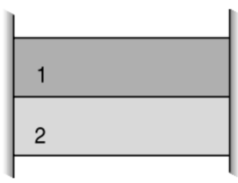
$$\Rightarrow \quad B = 19.6\times 10^8\text{ Nm}^{-2}$$

Hence, the correct answer is (A).

185. Thermal stress $\sigma = \dfrac{F}{A} Y\alpha\Delta T$

Given, $\sigma_1 = \sigma_2$

$$\Rightarrow \quad Y_1\alpha_1\Delta T = Y_2\alpha_2\Delta T$$



$\Rightarrow \quad \frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2}$

Hence, the correct answer is (C).

186. Since $P = P_0 + \rho gh + \frac{2T}{r}$

$\Rightarrow \quad P = (10^5) + (10^3)(10)(10) + \frac{2\times7\times10^{-2}}{10^{-3}}$

$\Rightarrow \quad P = 2.0014\times10^5 \text{ Nm}^{-2}$

Hence, the correct answer is (D).

187. According to Archimedes Principle, we have

Loss in Weight = Upthrust.

$\Rightarrow \quad (38.2-36.2)g = (V_{gold} + V_{cavity})\rho_w g$

$\Rightarrow \quad 2 = \left(\frac{38.2}{19.3}\right) + V_{cavity} \qquad \{\because \rho_w = 1 \text{ gcc}^{-1}\}$

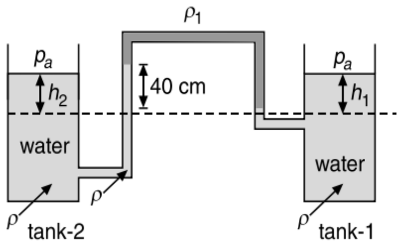
$\Rightarrow \quad V_{cavity} = 0.02 \text{ cm}^3$

Hence, the correct answer is (C).

188. Using Pascal's equation from first to second tank surface, we get

$P_a + h_1\rho g - 40\rho_1 g + 40\rho g = P_a + h_2\rho g$

$\Rightarrow \quad h_2\rho g - h_1\rho g = 40\rho g - 40\rho_1 g$



as $\rho_1 = 0.9\rho$

$(h_2 - h_1)\rho g = 40\rho g - 36\rho g$

$\Rightarrow \quad h_2 - h_1 = 4 \text{ cm}$

Hence, the correct answer is (B).

189. ℓ will decrease because the block moves up, h will decrease because the coin will displace the volume of water (V_1) equal to its own volume, when it is in the water whereas when it is on the block it will displace the volume of water (V_2), whose weight is equal to weight of coin and since, density of coin is greater than the density of water $V_1 < V_2$.

Hence, the correct answer is (D).

190. $\frac{Y_A}{Y_B} = \frac{\tan\theta_A}{\tan\theta_B} = \frac{\tan 60}{\tan 30} = \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = 3$

$\Rightarrow \quad Y_A = 3Y_B$

Hence, the correct answer is (D).

191. $\eta = \frac{F}{A\theta}$

$\Rightarrow \quad \eta = \frac{5\times10^5}{\left(\frac{10}{100}\right)^2 0.001}$

$\Rightarrow \quad \eta = 5\times10^{10} \text{ Nm}^{-2}$

Hence, the correct answer is (C).

192. For maximum range, orifice should be at the middle of ground and free surface of liquid i.e.,

$$x = \frac{2H+H}{2} = 1.5H$$

Since, this point lies in the tank. So, hole should be made at this point.

Hence, the correct answer is (C).

193. $\ell = \frac{FL}{AY}$

$\Rightarrow \quad \ell \propto \frac{L}{r^2}$ (F and Y are constant)

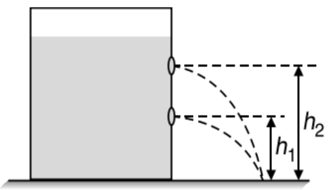
$\Rightarrow \quad \frac{\ell_2}{\ell_1} = \frac{L_2}{L_1}\times\left(\frac{r_1}{r_2}\right)^2 = 2\times\left(\frac{1}{2}\right)^2 = \frac{1}{2}$

$\Rightarrow \quad \ell_2 = \frac{\ell_1}{2}$

i.e., the change in the length of other wire is $\frac{\ell}{2}$

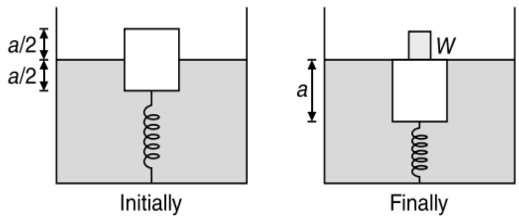
Hence, the correct answer is (C).

194. If H is the height of the liquid surface, then for same range, we have $h_2 = H - h_1$ and for maximum range, we have $h = \frac{H}{2} = \frac{h_1 + h_2}{2}$



Hence, the correct answer is (D).

195. Since, $\rho_{block} = \frac{1}{2}\rho_{water}$, so 50% of its volume is immersed in water



When weight is put over the block, then half of the volume of block is further immersed in water. Therefore,

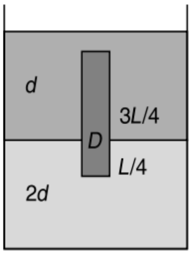
$W = (\text{Extra Upthrust}) + (\text{Spring Force})$

$\Rightarrow \quad W = (a)(a)\left(\frac{a}{2}\right)(2\rho)(g) + k\left(\frac{a}{2}\right)$

$\Rightarrow \quad W = a\left(a^2\rho g + \frac{k}{2}\right)$

Hence, the correct answer is (D).

196. Considering vertical equilibrium of cylinder, we get



$$\begin{pmatrix}\text{Weight}\\\text{of}\\\text{cylinder}\end{pmatrix}=\begin{pmatrix}\text{Upthrust due}\\\text{to upper}\\\text{liquid}\end{pmatrix}+\begin{pmatrix}\text{Upthrust due}\\\text{to lower}\\\text{liquid}\end{pmatrix}$$

$$\Rightarrow \left(\frac{A}{5}\right)(L)Dg=\left(\frac{A}{5}\right)\left(\frac{3L}{4}\right)(d)g+\left(\frac{A}{5}\right)\left(\frac{L}{4}\right)(2d)(g)$$

$$\Rightarrow D=\left(\frac{3}{4}\right)d+\left(\frac{1}{4}\right)(2d)$$

$$\Rightarrow D=\frac{5}{4}d$$

Hence, the correct answer is (A).

197. Since, $W=U$

$$\Rightarrow (A\times 0.5\times 900)g+(100)g=A\times 0.5\times 1000\times g$$

$$\Rightarrow A=2\text{ m}^2$$

Hence, the correct answer is (D).

198. When the vessel is at rest, then for equilibrium of block, we have

$$U+kx_1=mg \qquad \ldots(1)$$

where, x_1 is the compression in the spring. Also, if l_1 is the length of the spring in the first case, then

$$x_1=l_0-l_1 \qquad \ldots(2)$$

When the vessel accelerates vertically downwards with an acceleration a, then from the reference frame attached to the vessel, the block is in equilibrium, so

$$U+kx_2=m(g-a) \qquad \ldots(3)$$

where, x_2 is the new extension in the spring. Also, if l_2 is the length of the spring in the second case, then

$$x_2=l_0-l_2 \qquad \ldots(4)$$

From equations (1) and (3), we conclude that

$$x_2<x_1$$

So, from equations (2) and (4), we get

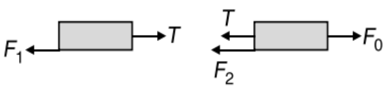
$$l_2>l_1$$

Hence the spring length will increase.

Hence, the correct answer is (D).

199. Since, viscous force F is proportional to area A, so let $F=kA$

$$v=\text{constant (i.e. } a=0)$$



So, $T=F_1=kA_1 \qquad \ldots(1)$

$$F_0=T+F_2=kA_1+kA_2$$

$$\Rightarrow F_0=k(A_1+A_2) \qquad \ldots(2)$$

Dividing equation (2) by equation (1), we get

$$\frac{T}{F_0}=\frac{A_1}{A_1+A_2}$$

$$\Rightarrow T=\frac{F_0A_1}{A_1+A_2}$$

Hence, the correct answer is (C).

200. Buoyant force is

$$U=V_{imm}\rho_{liq}g$$

Since, $V_{imm}\rho_{liq}$ and g all are same w.r.t. O_1 and O_2, so U is same.

Hence, the correct answer is (A).

201. $\rho=\dfrac{M}{V} \qquad \ldots(1)$

$$\rho+\Delta\rho=\frac{M}{V-\Delta V} \qquad \ldots(2)$$

$$\Rightarrow \Delta\rho=\frac{M}{V-\Delta V}-\frac{M}{V}$$

$$\Rightarrow \Delta\rho=\frac{M}{V}\left[\frac{1}{1-\frac{\Delta V}{V}}-1\right]$$

$$\Rightarrow \Delta\rho=\rho\left[\frac{1}{\left(1-\frac{P}{K}\right)}-1\right] \qquad \left\{\because K=\frac{P}{\left(\frac{\Delta V}{V}\right)}\right\}$$

$$\Rightarrow \Delta\rho=\frac{\rho P}{K-P}$$

Hence, the correct answer is (C).

202. Since mercury meniscus is convex, so the pressure just inside the hole will be less than the outside pressure by $\dfrac{2T}{r}$

$$\Rightarrow h\rho g=\frac{2T}{r}$$

$$\Rightarrow h=\frac{2T}{r\rho g}$$

Hence, the correct answer is (C).

203. Since, $W=\dfrac{1}{2}(\text{Tension})(\text{Extension})$

$$\Rightarrow W=\frac{1}{2}Fx$$

$$\Rightarrow \frac{W_1}{W_2}=\frac{F_1}{F_2}$$

$$\Rightarrow \frac{W_1}{W_2}=\frac{YA_1x/L_1}{YA_2x/L_2}$$

$$\Rightarrow \frac{W_1}{W_2}=\frac{A_1L_2}{A_2L_2}$$

$$\Rightarrow W_2=(4)(2)(2)$$

$$\Rightarrow W_2=16\text{ J}$$

Hence, the correct answer is (A).

204. For an isothermal process

$PV = \text{constant}$

$\Rightarrow PdV + VdP = 0$

$\Rightarrow \frac{dP}{dV} = -\left(\frac{P}{V}\right)$

Since bulk modulus is

$B = -\left(-\frac{dP}{dV/V}\right) = -\left(\frac{dP}{dV}\right)V$

$\Rightarrow B = -\left[\left(-\frac{P}{V}\right)V\right] = P$

$\Rightarrow B = P$

Similarly, adiabatic bulk modulus is given by $B = \gamma P$

Hence, the correct answer is (B).

205. Let x be the fraction of its volume which is hollow. Then weight of shell is

$W = (V - xV)(5\times 1000)g$

Weight of water displaced is equal to upthrust i.e.

$V \times 1000 \times g$

Since, $\begin{pmatrix}\text{Weight of water}\\ \text{displaced}\end{pmatrix} = \begin{pmatrix}\text{Loss in}\\ \text{weight}\end{pmatrix}$

$\Rightarrow V(1000)g = \frac{1}{2}(V - xV)(5\times 1000)g$

$\Rightarrow x = \frac{3}{5}$

Hence, the correct answer is (A).

206. $\Delta P = h\rho g = 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = 2T\left(\frac{r_2 - r_1}{r_1 r_2}\right)$

$\Rightarrow T = \frac{\rho g h r_1 r_2}{2(r_2 - r_1)}$

Hence, the correct answer is (A).

207. Let the spring constant be k. When the piece is hanging in air, then for equilibrium, we have

$kx = mg$

$\Rightarrow k(0.01) = (0.01)(10)$

$\Rightarrow k = 10\ \text{Nm}^{-1}$

The volume of the copper piece is

$V_{Cu} = \frac{0.01\ \text{kg}}{9000\ \text{kgm}^{-3}} = \frac{1}{9}\times 10^{-5}\ \text{m}^3$

This is also the volume of water displaced when the piece is immersed in water. Since the force of buoyancy is equal to the weight of the liquid displaced. So,

$U = \left(\frac{1}{9}\times 10^{-5}\right)(1000)(10) = 0.011\ \text{N}$

If the elongation of the spring is x' when the piece is immersed in water, then for equilibrium condition of the piece, we have

$kx' = 0.1\ \text{N} - 0.011\ \text{N} = 0.089\ \text{N}$

Since $k = 10\ \text{Nm}^{-1}$

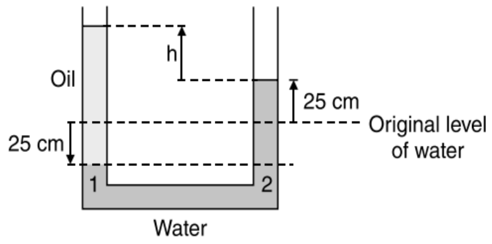
$\Rightarrow x = \frac{0.089}{10}\ \text{m} = 0.0089\ \text{m} = 0.89\ \text{cm}$

Hence, the correct answer is (B).

208. $P_1 = P_2$

$\Rightarrow P_0 + \rho_0 g h_0 = P_0 + \rho_w g h_w$

$\Rightarrow \rho_0 h_0 = \rho_w h_w$



$\Rightarrow (0.8)(h+50) = (1)(50)$

$\Rightarrow h = 12.5\ \text{cm}$

Hence, the correct answer is (B).

209. Given that compressibility k is 5×10^{-5} per unit atmospheric pressure

$\Rightarrow k = \frac{5\times 10^{-5}}{10^5} = 5\times 10^{-10}\ \text{N}^{-1}\text{m}^2$

$\Rightarrow B = \frac{1}{k} = \frac{1}{5}\times 10^{10}\ \text{Nm}^{-2}$

Since, $\Delta V = \frac{V\Delta P}{B}$

$\Rightarrow \Delta V = \frac{(100)(100\times 10^5)}{\frac{1}{5}\times 10^{10}} = 0.5\ \text{cm}^3$

Hence, the correct answer is (A).

210. $\Delta P = h\rho g = \frac{2T}{r}$

$\Rightarrow h = \frac{2T}{r\rho g}$

Hence, the correct answer is (C).

211. $\ell = \frac{L^2 dg}{2Y} = \frac{(10)^2\times 1500\times 10}{2\times 5\times 10^8} = 15\times 10^{-4}\ \text{m}$

Hence, the correct answer is (A).

212. Let a be the length of each side of the cube, then

$200\times g = (2)\times(a^2)\times 1\times g$

$\Rightarrow a = 10\ \text{cm}$

Hence, the correct answer is (C).

213. At terminal speed, net force on the ball is zero, so $F = 0$.

$\Rightarrow (\text{Weight}) = (\text{Upthrust}) + (\text{Viscous force})$

$\Rightarrow W = U + F$

$\Rightarrow \frac{4}{3}\pi r^3 \rho_1 g = \frac{4}{3}\pi r^3 \rho_2 g + krv_T$

$\Rightarrow v_T = \frac{4\pi g r^2}{3k}(\rho_1 - \rho_2)$

Hence, the correct answer is (A).

214. Let density of material of sphere (in gcm^{-3}) be ρ

Applying the condition of floatation, we get

Weight = Upthrust

$\Rightarrow V\rho g = \frac{V}{2}\rho_{oil} g + \frac{V}{2}\rho_{Hg} g$

$$\Rightarrow \quad \rho = \frac{\rho_{oil}}{2} + \frac{\rho_{Hg}}{2} = \frac{0.8}{2} + \frac{13.6}{2} = 7.2 \text{ gcm}^{-3}$$

Hence, the correct answer is (C).

215. $\text{Stress} = \frac{F}{A} = \frac{4 \times 3.1\pi}{\pi \times (2 \times 10^{-3})^2}$

$$\Rightarrow \quad \text{Stress} = 3.1 \times 10^6 \text{ Nm}^{-2}$$

Hence, the correct answer is (B).

216. Since, $h = \frac{2T\cos\theta}{r\rho g}$

$$\Rightarrow \quad 10 = \frac{2T_1 \cos 0°}{r\rho_1 g} \quad \ldots(1)$$

$$\Rightarrow \quad -3.42 = \frac{2T_2 \cos 135°}{r\rho_2 g} \quad \ldots(2)$$

From these two equations, we get

$$\frac{T_1}{T_2} = \frac{10(\cos 135°)}{(-3.42)(\cos 0°)\rho_2} = \frac{(10)\left(-\frac{1}{\sqrt{2}}\right)(1)}{(-3.42)(1)(13.6)}$$

$$\Rightarrow \quad \frac{T_1}{T_2} = 1:6.5$$

Hence, the correct answer is (D).

217. For equilibrium, we have

Surface Tension Force + Upthrust = Weight

$$\Rightarrow \quad (2\pi rT) + \left(\frac{2}{3}\pi r^3\right)\rho_w g = \left(\frac{4}{3}\pi r^3\right)\rho_s g$$

Substituting the values, we get

$$r \approx 1.2 \text{ mm}$$

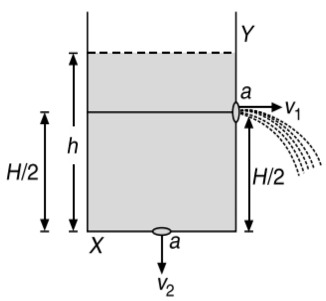
Hence, the correct answer is (C).

218. Efflux speed at holes X and Y respectively is

$$v_1 = \sqrt{2g\left(\frac{h-H}{2}\right)} \text{ and } v_2 = \sqrt{2gh}$$

Applying equation of continuity at top surface and holes, we get

$$A\left(-\frac{dh}{dt}\right) = a(v_1 + v_2)$$



$$\Rightarrow \quad A\left(-\frac{dh}{dt}\right) = a\left[\sqrt{2g\left(\frac{h-H}{2}\right)} + \sqrt{2gh}\right]$$

$$\Rightarrow \quad -\frac{A}{a\sqrt{2g}} \int_H^{\frac{H}{2}} \frac{dh}{\sqrt{h} + \sqrt{h - \frac{H}{2}}} = \int_0^t dt$$

$$\Rightarrow \quad t = \frac{2A}{3a}\left(\sqrt{2} - 1\right)\sqrt{\frac{H}{g}}$$

Hence, the correct answer is (B).

219. When the water reaches the lower end of the tube a convex meniscus will be formed such that

$$\Delta P = 2 \times \frac{2T}{r} = \frac{4T}{r}$$

$$\Rightarrow \quad \frac{2T}{r} = \rho g h$$

Also, $\Delta P = \rho g h'$

$$\Rightarrow \quad h' = 2h$$

Hence, the correct answer is (D).

220. Since, $F = YA\left(\frac{\Delta L}{L}\right) = YA\alpha\Delta T \qquad \left\{\because \frac{\Delta L}{L} = \alpha\Delta T\right\}$

$$\Rightarrow \quad F = (2 \times 10^{11})(10^{-6})(1.1 \times 10^{-5})(20)$$

$$\Rightarrow \quad F = 44 \text{ N}$$

Hence, the correct answer is (D).

221. Let tension in the thread be F

Surface tension force is acting radially outwards.

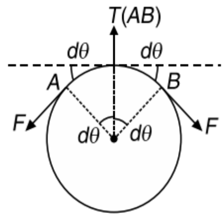
On section AB, we have

$$2F\sin(d\theta) = T(AB) = T(2Rd\theta)$$

Since, $\sin d\theta \approx d\theta$

$$\Rightarrow \quad 2Fd\theta = 2TRd\theta$$

$$\Rightarrow \quad F = 2TR$$

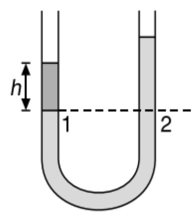


Hence, the correct answer is (D).

222. Surface area decreases and hence, the surface energy also decreases.

Hence, the correct answer is (B).

223. Since, $P_1 = P_2$

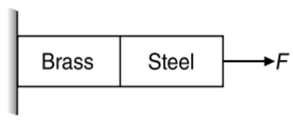


$$\Rightarrow \quad P_0 + \rho_1 g h = P_0 + \rho_2 g h$$

$$\Rightarrow \quad \rho_1 = \rho_2$$

Hence, the correct answer is (B).

224.



Corresponding to the stress (σ), total elongation is

$$\Delta\ell_{net} = \frac{\sigma L_1}{Y_1} + \frac{\sigma L_2}{Y_2}$$

$$\Rightarrow \quad \sigma = \Delta\ell\left(\frac{Y_1 Y_2}{Y_1 + Y_2}\right)$$

$$\Rightarrow \quad \sigma = 0.2 \times 10^{-3} \times \left(\frac{120 \times 60}{180}\right) \times 10^9$$

$\Rightarrow \quad \sigma = 8 \times 10^6 \text{ Nm}^{-2}$

Hence, the correct answer is (A).

225. We know that, $h = \dfrac{2T\cos\theta}{R\rho g}$

$\Rightarrow \quad h \propto \dfrac{1}{R}$

Since, $M = (\pi R^2 h)\rho$

$\Rightarrow \quad M \propto R^2 h$

$\Rightarrow \quad M \propto R$

Radius is doubled, so mass in the capillary tube will also become two times.

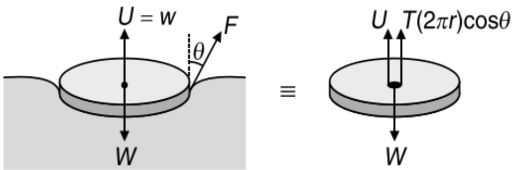
Hence, the correct answer is (A).

226. Since, $h = \dfrac{2T\cos\theta}{r\rho g}$

For a freely falling lift $g_{\text{eff}} = 0$, so $h \to \infty$ or it will fill the entire length of the tube.

Hence, the correct answer is (B).

227. Surface tension force on liquid is downwards, but on the disc it is upwards as shown in Figure.



For equilibrium of disc, we have

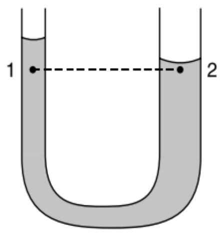
$$W = U + F = w + 2\pi Tr\cos\theta$$

Hence, the correct answer is (C).

228. The average velocity in the first half of the distance $< v$, while in the second half the average velocity is v. Therefore, $t_1 > t_2$. The work done against gravity in both halves is $\dfrac{mg\ell}{2}$.

Hence, the correct answer is (D).

229. Since $P_1 = P_2$



$\Rightarrow \quad \left(P_0 - \dfrac{2T}{r_1}\right) + \rho g h = \left(P_0 - \dfrac{2T}{r_2}\right)$

$\Rightarrow \quad T = \dfrac{\rho g h r_1 r_2}{2(r_2 - r_1)}$

Hence, the correct answer is (A).

230. $\ell = \dfrac{FL}{AY} = \dfrac{FL^2}{(AL)Y} = \dfrac{FL^2}{VY}$

If volume is fixed then $\ell \propto L^2$

Hence, the correct answer is (C).

231. Since loss in weight equals the upthrust, so we get

$$(w - w_1) = V\rho_w g \qquad \ldots(1)$$

$$(w - w_2) = V\rho_L g \qquad \ldots(2)$$

Dividing equation (2) by (1), we get

$$\frac{\rho_L}{\rho_w} = \frac{w - w_2}{w - w_1} = \text{Relative Density of liquid}$$

Hence, the correct answer is (B).

232. Since, $W_{\text{app}} = W_{\text{actual}} - U = W - V_S \rho_L g$

At higher temperature, we have

$U' = V_S' \rho_L' g$

$\Rightarrow \quad \dfrac{U'}{U} = \left(\dfrac{V_S'}{V_S}\right)\left(\dfrac{\rho_L'}{\rho_L}\right)$

$\Rightarrow \quad \dfrac{U'}{U} = \dfrac{(1 + \gamma_S \Delta T)}{(1 + \gamma_L \Delta T)}$

Since, $\gamma_S < \gamma_L$

$\Rightarrow \quad U' < U$

$\Rightarrow \quad W'_{\text{app}} > W_{\text{app}}$

Hence, the correct answer is (C).

233. $\dfrac{Mg\ell}{A\Delta\ell} = Y$

$\Rightarrow \quad \Delta\ell_{\text{Mechanical}} = \dfrac{Mg\ell}{AY}$

Since $\Delta\ell_{\text{Thermal}} = \ell\alpha\Delta T = \ell\alpha \times 20$

$\Rightarrow \quad \dfrac{Mg\ell}{AY} = 20\alpha\ell$

$\Rightarrow \quad M = \dfrac{20 \times 10^{-5} \times \pi \times 1 \times 10^{-6} \times 10^{11}}{10}$

$\Rightarrow \quad M = 6.28 \text{ kg}$

Hence, the correct answer is (C).

234. Since $\Delta P = \dfrac{4T}{r}$ and $r_A > r_B$

$\Rightarrow \quad P_A < P_B$

So, the air rushes from B to A

Hence, the correct answer is (A).

235. Speed of efflux is given by

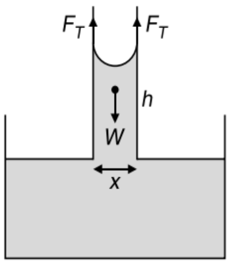
$v = \sqrt{2gh}$

$\Rightarrow \quad \dfrac{dV}{dt} = av = a\sqrt{2gh}$

This is independent of ρ_{liquid}.

Hence, the correct answer is (C).

236. Let L be the width of plates (perpendicular to paper inwards), then surface tension force in upward direction equals weight of liquid that rises to a height h



$2F_T = W$

$\Rightarrow \quad 2(TL\cos\theta) = V\rho g$

$\Rightarrow \quad 2TL\cos\theta = (Lxh)\rho g$

$\Rightarrow \quad h = \dfrac{2T\cos\theta}{x\rho g}$

Hence, the correct answer is (B).

237. Let the velocity at A be v_A and that at B be v_B, then by Equation of Continuity, we get

$$\frac{v_B}{v_A} = \frac{30}{15} = 2$$

Applying Bernoulli's equation, we get

$$P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2$$

$$\Rightarrow \quad P_A - P_B = \frac{1}{2}\rho(2v_A)^2 - \frac{1}{2}\rho v_A^2 = \frac{3}{2}\rho v_A^2$$

$$\Rightarrow \quad 600 = \frac{3}{2}(1000)v_A^2$$

$$\Rightarrow \quad v_A = \sqrt{0.4} = 0.63 \text{ ms}^{-1}$$

The rate of flow is

$$Q = (30)(0.63) = 1800 \text{ cm}^3\text{s}^{-1}$$

Hence, the correct answer is (C).

Multiple Correct Choice Type Questions

1. Speed gradient is $\frac{\Delta v}{\Delta x} = \frac{2 \text{ ms}^{-1}}{1 \text{ m}} = 2 \text{ s}^{-1}$

and $F = \eta A \frac{\Delta v}{\Delta x} = (10^{-3})(10)(2) = 0.02 \text{ N}$

$\Rightarrow \quad F = 0.02 \text{ N}$

Hence, (A) and (C) are correct.

2. $(RD)_{\text{metal}} = \frac{W_{\text{air}}}{W_{\text{air}} - W_{\text{water}}} = \frac{210}{210-180} = 7$

Since, loss in weight of the body immersed in liquid is equal to the upthrust, so we get

$$(210-120) = \left(\frac{210}{7}\right)(RD)_{\text{liquid}}$$

$$\Rightarrow \quad (RD)_{\text{liquid}} = 3$$

Hence, (B) and (C) are correct.

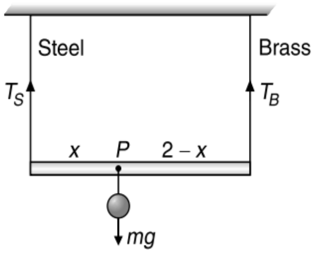
3. For a tube with uniform cross-sectional area, speed will remain same and hence pressure will be the same if both the points lie at the same horizontal level.

However, if the cross-sectional area is different, then too pressure may be same at points lying on different heights (i.e. points not on the same level).

Hence, (B) and (D) are correct.

4. When stress in both wires are equal, then $\frac{T_S}{A_S} = \frac{T_B}{A_B}$

$$\Rightarrow \quad \frac{T_S}{T_B} = \frac{A_S}{A_B} = \frac{10^{-3}}{2 \times 10^{-3}} = \frac{1}{2} \quad \ldots(1)$$



For, system to be in equilibrium, conserving moments about P, we get

$$T_S x = T_B(2-x)$$

$$\Rightarrow \quad \frac{T_S}{T_B} = \frac{2-x}{x} \quad \ldots(2)$$

From Equations (1) and (2), we get

$$x = 1.33 \text{ m}$$

So $x = 1.33$ m, if stress is equal in both wires.

Since, Strain $= \frac{\text{Stress}}{Y}$

If strain in both wires is equal, then $\frac{T_S/A_S}{Y_S} = \frac{T_B/A_B}{Y_B}$

$$\Rightarrow \quad \frac{T_S}{T_B} = \frac{A_S Y_S}{A_B Y_B} = \frac{(10^{-3})(2 \times 10^{11})}{(2 \times 10^{-3})(10^{11})} = 1 \quad \ldots(3)$$

From Equations (2) and (3), we get

$$x = 1 \text{ m}$$

So $x = 1$ m, if strain is equal in both wires.

Hence, (A) and (C) are correct.

5. When the second ball having $\rho < \rho_L$ falls from the same height and hits the liquid surface, then we see that the buoyant force acting on the ball is more than its weight due to which the ball experiences a constant retardation up to a certain depth and then starts rising. So, it will return to its original position, i.e. at the same height, in a time $t_2 > t_1$.

When $\rho_L = \rho$, then net force on the ball inside the liquid is zero and hence it continues to move with the constant velocity independent of its depth.

The acceleration of the ball inside and outside the liquid is constant, so the motion of the ball cannot be simple harmonic.

Hence, (B), (C) and (D) are correct.

6. According to continuity equation, $Av =$ constant

Since $A_2 < A_1$, so $\quad v_2 > v_1$

According to Bernoulli's equation, we have

$$P + \frac{1}{2}\rho v^2 = \text{constant} \quad \{\because h = \text{constant}\}$$

Since $v_2 > v_2$, so $P_2 < P_1$

The volume of liquid flowing per second or the mass of liquid flowing per second through both the sections of tube is constant.

Hence, (A), (C) and (D) are correct.

7. The more is the modulus of elasticity, the more is the resistance offered to external deforming forces.

Hence, (B) and (C) are correct.

8. Area of cross-section is $A = \frac{\pi d^2}{4}$

Since, $\Delta L = \frac{FL}{AY} = \frac{FL}{(\pi d^2/4)Y}$

$$\Rightarrow \quad \Delta L \propto \frac{1}{d^2}$$

$$\Rightarrow \quad \frac{\Delta L_B}{\Delta L_A} = \frac{(2d)^2}{(d)^2}$$

$$\Rightarrow \quad \Delta L_B = 4\Delta L_A$$

Also, Strain $= \frac{\Delta L}{L} = \frac{4F}{\pi d^2 Y}$

$\Rightarrow$ Strain $\propto \frac{1}{d^2}$

$\Rightarrow$ $(\text{Strain})_B = 4(\text{Strain})_A$

Hence, (B) and (D) are correct.

9. For the siphon to work, the pressure inside the lower face of the tube should be more than the atmospheric pressure for which the condition $h_2 > h_1$ must be obeyed. The height h_1 should be less than the height of corresponding liquid barometer, because otherwise the liquid will not rise to that level.
Hence, (A) and (D) are correct.

10. According to Equation of Continuity, the rate of flow of liquid or volume of liquid flowing per second through both the sections of the tube is the same, i.e.

$$A_1 v_1 = A_2 v_2$$

According to Bernoulli's Theorem, we have

$$\frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \rho g h$$

$\Rightarrow$ $v_2^2 - v_1^2 = 2gh$

Also, as a consequence of Bernoulli's Theorem, the energy per unit mass of the liquid is the same in both sections of the tube.
Hence, (A), (B) and (D) are correct.

11. The speed of efflux of the liquid coming out of the orifice is

$$v = \sqrt{2gy}$$

The range x of the liquid is

$$R = x = 2\sqrt{y(H-y)}$$

So, as y is increased, x increases, becomes maximum and then decreases.

x is maximum for $y = \frac{H}{2}$ and $x_{max} = H$

Hence, (B), (C) and (D) are correct.

12. Since the elastic potential energy is $U = \frac{1}{2}kx^2$.

So U-x graph is a parabola symmetric about U axis.
From the graph, we see that at $x = 0.2$ mm, $U = 0.2$ J

$\Rightarrow$ $0.2 = \frac{1}{2}k\left(2\times10^{-4}\right)^2$

$\Rightarrow$ $k = 10^7\ \text{Nm}^{-1}$

Also, the spring constant of a wire is $k = \frac{YA}{L}$

$$\Rightarrow \frac{A}{L} = \frac{k}{Y} = \frac{10^7}{2\times10^{11}} = 5\times10^{-5} \quad \ldots(1)$$

Given that the volume of the wire is

$$AL = 200\times10^{-6}\ \text{m}^3 \quad \ldots(2)$$

Solving Equations (1) and (2), we get

$$A = 10^{-4}\ \text{m}^2 \text{ and } L = 2\ \text{m}$$

Hence, (B) and (C) are correct.

13. At the level of interface between ρ_2 and ρ_3, pressures will be equal from both sides. Hence,

$$\rho_1 g h + \rho_3 g\left(\frac{h}{2}\right) = \rho_2 g h$$

$\Rightarrow$ $\rho_3 = 2(\rho_2 - \rho_1)$

From this expression, we conclude that $\rho_2 > \rho_1$.
Hence, (B) and (D) are correct.

14. The fraction of volume of body immersed inside liquid is given by

$$f = \frac{\rho_s}{\rho_l}$$

Since ρ_s and ρ_l are same, so

$$f_1 = f_2 = f_3$$

Also, we note that base area in the third case is uniform, so h_3 is minimum.
Hence, (B), (C) and (D) are correct.

15. The work done in slowly stretching the wire through l is $W = mgl$.
So, loss in gravitational potential energy of the wire is mgl.

$$\text{Since, } l = \frac{mgl}{AY} \quad \ldots(1)$$

$$\text{and } U_{\text{elastic}} = \frac{1}{2}Kl^2 = \frac{1}{2}\left(\frac{YA}{L}\right)l^2 \quad \ldots(2)$$

From Equations (1) and (2), we get

$$U_{\text{elastic}} = \frac{1}{2}mgl$$

$$\Rightarrow \Delta H = W - U_{\text{elastic}} = \frac{mgl}{2}$$

So, half of work done is stored as elastic energy and the remaining is lost as heat.
Hence, (B), (C) and (D) are correct.

16. The reaction force exerted by the liquid on the tube is

$$F = Av^2\rho$$

Hence, (A), (B) and (C) are correct.

17. As we move up from the base of the vessels, the vessel C is narrowing and hence the height of liquid in vessel C is maximum. So, pressure at base of vessel C is maximum. Therefore, force on the base of vessel C is maximum and is given by

$$F_3 = \left(P_0 + h_3\rho g\right)A.$$

In equilibrium, net force on all the three vessels equals the weight of liquid, which is same for all the three vessels.
Hence, (B) and (C) are correct.

18. Viscosity of a liquid decreases with increase in temperature, whereas the viscosity of a gas increases with increase in temperature. The surface tension of the liquid decreases with an increase in temperature and for a contact angle $\theta = 90°$ the liquid neither rises nor falls.
Hence, (B) and (C) are correct.

19. Elastic forces are not always conservative. Elastic forces are conservative as long as the loading and deloading curves are coincident even if the curves are not linear.
Hence, elastic forces may be conservative even when Hooke's law is not obeyed if loading and deloading curves are coincident.
Hence, (B) and (D) are correct.

20. Area of steel rod, $A_S = 16\text{ cm}^2$

Collective area of two brass rods is

$$A_B = 2\times 10 = 20\text{ cm}^2$$

Also, $F = 5000$ kgf

If σ_S be the stress in steel and σ_B be the stress in brass, then

$$\begin{pmatrix}\text{Decrease in length}\\ \text{of steel rod}\end{pmatrix} = \begin{pmatrix}\text{Decrease in length}\\ \text{of brass rod}\end{pmatrix}$$

$$\Rightarrow \left(\frac{\sigma_S}{Y_S}\right)L_S = \left(\frac{\sigma_B}{Y_B}\right)L_B$$

$$\Rightarrow \sigma_S = \left(\frac{Y_S}{Y_B}\right)\left(\frac{L_B}{L_S}\right)\sigma_B = \left(\frac{2\times 10^6}{10^6}\right)\left(\frac{20}{30}\right)\sigma_B$$

$$\Rightarrow \sigma_S = \frac{4}{3}\sigma_B \quad \text{...(1)}$$

Since, $F = \sigma_S A_S + \sigma_B A_B$

$$\Rightarrow 5000 = 16\sigma_S + 20\sigma_B \quad \text{...(2)}$$

From Equations (1) and (2), we get

$$\sigma_B = 120.9\text{ kgfcm}^{-2} \text{ and } \sigma_S = 161.2\text{ kgfcm}^{-2}$$

Hence, (C) and (D) are correct.

21. Since, $\Delta L = \dfrac{WL}{AY}$

So, ΔL can be increased by making W two times or by making L two times or by making A half.

Hence, (A), (B) and (C) are correct.

22. The pressure at the base of the vessel is $P_{\text{base}} = (2h)\rho g$ and so the force exerted by the liquid at the base of container is $F_{\text{base}} = P_{\text{base}}A_2 = 2h\rho g A_2$.

The weight is in the beaker is

$$W = (A_1 + A_2)h\rho g < 2h\rho g A_2$$

The force on the vertical walls of the vessel cancel. So, for equilibrium, we have

$$F_X + W = F_{\text{base}}$$

$$\Rightarrow F_X = F_{\text{base}} - W = 2hA_2\rho g - (A_1 + A_2)h\rho g$$

$$\Rightarrow F_X = (A_2 - A_1)h\rho g \text{, downwards}$$

Hence the force on the wall at X is downwards.

Hence, (A), (B), (C) and (D) are correct.

23. Since extension $\Delta x = \dfrac{Fl}{AY}$

$$\Rightarrow F = \left(\frac{AY}{L}\right)\Delta x$$

i.e. F versus Δx graph is a straight line of slope $\dfrac{YA}{L}$ passing through the origin.

Since $(\text{Slope})_B > (\text{Slope})_A$

$$\Rightarrow \left(\frac{YA}{L}\right)_B > \left(\frac{YA}{L}\right)_A$$

$$\Rightarrow (A)_B > (A)_A$$

Since the wires are of same material, so

$$Y_A = Y_B$$

Hence, the correct answer is (C).

24. Since $F = 6\pi\eta r v$

So, $F \propto v$ and $F \propto r$ or $F \propto A^{1/2}$ or $F \propto V^{1/3}$

Hence, (B), (C) and (D) are correct.

25. Since the liquid in which the block is immersed applies an upthrust on the block, so an equal force will be exerted (from Newton's Third Law) on the liquid. Hence, A will read less than 2 kg and B more than 5 kg.

Hence, (B) and (C) are correct.

26. Since, loss in weight equals the upthrust, so we have

$$(27-18)g = V_{\text{imm}}\rho_{\text{liq}}g = V\rho_w g$$

$$\Rightarrow V = \frac{9}{1000}\text{ m}^3 = 9000\text{ cm}^3$$

Let V_C be the volume of the cavity inside the iron casting, then

$$(27000)\cancel{g} = (V - V_C)\rho_{\text{iron}}\cancel{g}$$

$$\Rightarrow V_C = 9000 - \frac{27000}{7.8} = 5538\text{ cm}^3$$

Hence, (A) and (C) are correct.

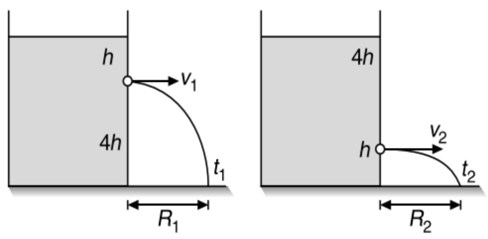
27. In an accelerated container, weight remains unchanged (apparent weight changes) while pressure and upthrust increases.

If m is mass of block and a be its acceleration, then the equation motion of block in new situation is

$$U' - W = ma$$

Hence, (B) and (C) are correct.

28. The situation discussed in the problem is shown in Figure.



Since $v = \sqrt{2gh}$, where h is the depth of the orifice below the free surface of the liquid. So, we have

$$v_1 = \sqrt{2gh} \text{ and } v_2 = \sqrt{2g(4h)} = 2\sqrt{2gh}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{1}{2}$$

Also, $t = \sqrt{\dfrac{2h'}{g}}$, where h' is the height of the orifice.

$$\Rightarrow \frac{t_1}{t_2} = \sqrt{\frac{h_1'}{h_2'}} = \sqrt{\frac{4h}{h}} = 2$$

The range of the liquid is

$$R = vt = 2\sqrt{h\times h'}$$

$$\Rightarrow \frac{R_1}{R_2} = \sqrt{\frac{(h)(4h)}{(4h)(h)}} = 1$$

Hence, (A), (C) and (D) are correct.

29. Since the liquid will be in equilibrium, so the side walls of the conical flask will exert a downward force on the liquid due to which net force on the base of the conical flask will increase and hence $W_1 > W_2$.
If F be the force exerted by the liquid on the walls of the flask, then for the flask we have

$$F + W_2 = W_1$$

$$\Rightarrow \quad F = W_1 - W_2$$

Hence, (B) and (D) are correct.

30. When $F = \frac{mg}{3}$, then tension in the wire Q and P respectively are

$$T_Q = \frac{mg}{3} \text{ and } T_P = mg + \frac{mg}{3} = \frac{4mg}{3}$$

Since stress is $\sigma = \frac{T}{\pi r^2}$

$$\Rightarrow \quad \frac{\sigma_P}{\sigma_Q} = \left(\frac{T_P}{T_Q}\right)\left(\frac{r_Q}{r_P}\right)^2$$

When $r_P = r_Q$, then $\sigma_P = 4\sigma_Q$ and hence P breaks.
When $r_P < 2r_Q$, then $\sigma_P > \sigma_Q$ and hence P breaks.
When $r_P = 2r_Q$, then $\sigma_A = \sigma_B$ and hence either P or Q may break.

Hence, (A), (B) and (C) are correct.

31. The level of water in beaker will fall if initially the impurity pieces plus ice system was floating and on the melting of ice the impurities sink.
The level of water in beaker will remain unchanged if initially the impurities plus ice system was floating and on melting the impurities also float.
Hence, (C) and (D) are correct.

32. In air, $a_1 = g$, downwards

In liquid, $a_2 = \frac{\text{upthrust} - \text{weight}}{\text{mass}} = \frac{U - W}{m}$

$$\Rightarrow \quad a_2 = \frac{(V)(2\rho)(g) - (V)(\rho)(g)}{(V\rho)}$$

$$\Rightarrow \quad a_2 = g \text{, upwards}$$

So, $a_1 \neq a_2$ because the balls have accelerations of same value but different directions.
On entering the non-viscous liquid, the ball will be retarded to zero velocity and then will be accelerated upwards towards the surface. It comes out of the surface with the same acceleration and will rise to the same height from which it had started falling and in this process the motion becomes periodic but not harmonic (because acceleration is constant).
Hence, (A), (C) and (D) are correct.

33. Work done against the elastic force is

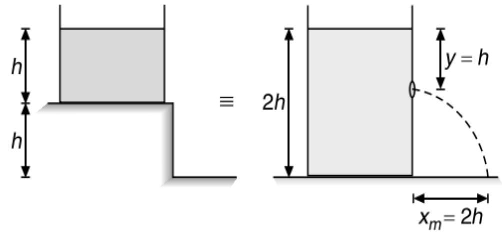
$$W = \frac{1}{2}k(\Delta l)^2 = \frac{1}{2}kl^2 = \frac{1}{2}\left(\frac{YA}{L}\right)l^2 = \frac{YAl^2}{2L}$$

This work done against the elastic force equals the elastic potential energy stored in the wire, so

$$U_{\text{elastic}} = \frac{YAl^2}{2L}$$

Hence, (A) and (C) are correct.

34. This is similar to the case as if a tank is filled with a liquid up to a height of $2h$.



In that case, range becomes maximum when hole is punched at the middle of the tank i.e. at $y = h$ and the maximum range is equal to the level of liquid in the tank.
Hence, (A) and (C) are correct.

35. Initial rate of flow of volume through holes 1 and 2 respectively is

$$Q_1 = \frac{dV_1}{dt} = a_1 v_1 = \pi(2R)^2\sqrt{2gh} \quad \ldots(1)$$

$$Q_2 = \frac{dV_2}{dt} = a_2 v_2 = \pi R^2\sqrt{2g(16h)} \quad \ldots(2)$$

From Equations (1) and (2), we can see that

$$Q_1 = Q_2$$

After some time, v_1 and v_2 both will decrease, but decrease in the value of v_1 will be more rapid compared to v_2. So,

$$Q_1 < Q_2$$

Hence, (B) and (D) are correct.

36. Since $v = \sqrt{2gh}$, where h is the depth of the orifice from the free surface. Since $h_2 = \frac{h_1}{4}$, so $v_2 = \frac{v_1}{2}$

$$\Rightarrow \quad v_1 = 2v_2$$

Further $Q = Av$
Given that $A_1 = 2A_2$ and $v_1 = 2v_2$

$$\Rightarrow \quad A_1 v_1 = A_2 v_2$$

$$\Rightarrow \quad Q_1 = Q_2$$

Hence, (B) and (C) are correct.

37. If ice floating on water melts, then the level of liquid (if the liquid is water) will remain the same. However, if the liquid is denser than water then the level of liquid will rise on melting of ice and vice versa.
Hence, (A), (C) and (D) are correct.

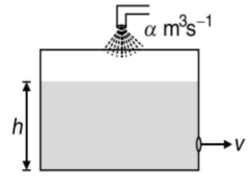
38. Since, pressure increases with depth in vertical direction and in horizontal direction it increases in the direction opposite to acceleration. So, pressure is maximum at point D and minimum at B.
Hence, (B) and (D) are correct.

39. Level of water in the vessel will be maximum when the rate of inflow of water in the vessel equals the rate of outflow of water from the vessel

$$\Rightarrow \quad \alpha = av = a\sqrt{2gh}$$

$$\Rightarrow \quad h = \frac{\alpha^2}{2ga^2}$$

So, $h \propto \alpha^2$ and $h \propto a^{-2}$



Hence, (B) and (C) are correct.

40. The liquid is incompressible and the area of cross-section of the tube is same at both the points, so

$v_A = v_B$

However, the pressure at the point B will be more than the pressure at the point A.

Hence, (B) and (C) are correct.

41. The siphon will work when the atmospheric pressure helps the liquid to flow out of the vessel.

Hence, (B) and (C) are correct.

42. Since viscous force equals the weight, so

$$6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g \quad \ldots(1)$$

Substituting the values, we can find r. Also

$v \propto r^2$

Hence, (B) and (C) are correct.

43. Since the terminal velocity is

$$v_T = \frac{2}{9}\left(\frac{r^2}{\eta}\right)(\rho_{\text{oil}} - \rho_{\text{air}})g \approx \frac{2}{9}\left(\frac{r^2}{\eta}\right)\rho_{\text{oil}}$$

$$\Rightarrow \quad 5\times10^{-4} = \frac{2}{9}\left(\frac{r^2}{1.8\times10^{-5}}\right)(900)$$

$$\Rightarrow \quad r = 6.7\times10^{-6}\ \text{m}$$

When r is halved, terminal velocity will become one fourth the initial value.

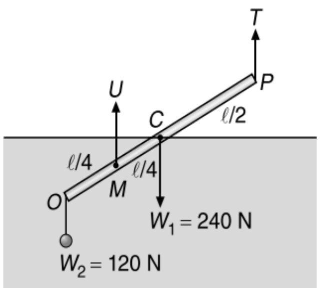
Hence, (A) and (C) are correct.

44. For translational equilibrium of the rod, we have

$$T + U = W_1 + W_2 = 360\ \text{N} \quad \ldots(1)$$

Since, $U = \left(\frac{V}{2}\right)\rho_w g = \left(\frac{V}{2}\right)(10^3)(10)$

$$\Rightarrow \quad U = 0.5\times10^4 V \quad \ldots(2)$$



For rotational equilibrium of the rod, we have

$$\Sigma\tau_{\text{about } M} = 0$$

$$\Rightarrow \quad 120\left(\frac{l}{4}\right) + T\left(\frac{3l}{4}\right) = 240\left(\frac{l}{4}\right)$$

$$\Rightarrow \quad T = \frac{240-120}{3} = 40\ \text{N}$$

So, from equations (1) and (2), we get

$$V = 6.4\times10^{-2}\ \text{m}^3$$

The point of application of the buoyant force is passing through the centre of the immersed part of the rod.

Hence, (A), (B) and (C) are correct.

45. $U = V_{\text{immersed}}\rho_{\text{liquid}} g$

Since, g is different on moon and on the earth.

Hence only (C) is a correct statement and all others incorrect.

Hence, (A), (B) and (D) are correct.

46. Radius of meniscus is $r = \frac{R}{\cos\theta}$

Due to spherical surface $\Delta P = \frac{2\sigma}{r}$

$$\Rightarrow \quad \Delta P = \frac{2\sigma\cos\theta}{R}$$

$$\Rightarrow \quad P_0 - P_A = \Delta P = \frac{2\sigma\cos\theta}{R}$$

$$\Rightarrow \quad P_A = P_0 - \frac{2\sigma\cos\theta}{R}$$

Since $P_B = P_0$, so $P_A = P_B - \frac{2\sigma\cos\theta}{R}$

Also, $P_A = P_B - h\rho g = P_0 - h\rho g$

Hence, (A), (B) and (C) are correct.

Reasoning Based Questions

1. When a rain drop falls in air (viscous medium) then after falling through same height, the viscous drag balances the weight of the drop and then through rest height its velocity is constant or it attains a terminal velocity.

Hence, the correct answer is (A).

2. From the formula, excess pressure = $\frac{2T}{r}$ Here, T is surface tension, r is the radius of liquid drop. Hence, excess pressure is inversely proportional to radius and hence, the surface area. Therefore, the excess pressure inside a smaller drop is large as compared to the larger drop due to which smaller drop of liquid resists deforming forces better than a larger drop.

Hence, the correct answer is (C).

3. The barometer has to measure the atmospheric pressure which has a constant value, so when it accelerates upwards, then $g_{\text{eff}}(= g + a)$ increases and hence h decreases. Also, due to the increase in value of g, upthrust U also increases. So, both are true but Statement-2 is not the reason to Statement-1.

Hence, the correct answer is (B).

4. Usually the air will not strike the wings of the aeroplane with large velocity, so to get the lift, the aeroplane runs for some distances on the runway before taking off. Due to the special shape of wings, the velocity of the layers of air above the wings increases and hence pressure decreases. Due to this aeroplane gets an uplift.

Hence, the correct answer is (A).

5. As we go deeper in the lake, the density of water increases due to pressure.

Hence, the correct answer is (A).

6. Since, the particle of dust is like spheres of very small radii and when it acquires the terminal velocity they begin to settle down. As from the definition the terminal velocity of dust particles is directly proportional to the square of its radii. Thus, the terminal velocity of dust particle is very small and so, they settle down in a closed room after sometime.

Hence, the correct answer is (C).

7. Area of cross-section is different. So, heights are different. In pressure height is more important.
Hence, the correct answer is (D).

8. A body much displace as much amount of liquid greater than the actual weight of the body to float in it. In case of floating, no net downward force acts on the body.
Hence, the correct answer is (B).

9. When the ball enters the liquid it may accelerate or retard depending on the density of ball and the density of liquid, because if $\rho_{\text{ball}} > \rho_{\text{liquid}}$, then $W > U$ and the ball will accelerate and if $\rho_{\text{ball}} < \rho_{\text{liquid}}$, then $W < U$ and the ball will decelerate.
Hence, the correct answer is (A).

10. Bulk modulus of elasticity measures how good the body is to regain its original volume on being compressed. Therefore, it represents incompressibility of the material.

Bulk modulus $B = -\dfrac{\Delta P}{\Delta V / V} = -\dfrac{(\Delta P)V}{\Delta V}$

Where ΔP = change in pressure, ΔV = change in volume
Here, negative sign implies that when the pressure increases, volume decreases and vice-versa.
Hence, the correct answer is (B).

11. Speed will also depend on h.
Hence, the correct answer is (D).

12. With increase in depth the pressure increases from the formula $P \propto h$. Therefore the force perpendicular to the walls of dam increases. Hence, the dam must have greater strength at base than at top. Due to this dam are made thicker at the base than at the top.
Hence, the correct answer is (A).

13. If length is doubled, Δl will also become double but Y will remain the same as it is the property of the material, i.e. it changes only when the material changes.
Hence, the correct answer is (D).

14. A hydraulic lift is an arrangement used to multiply the force. When a force is applied hydraulic pressure is transmitted in all directions. This is the working principle of Pascal's laws.
Hence, the correct answer is (B).

15. When a needle is placed carefully on the surface of water, if floats on the surface of water due to surface tension of water which does not allow the needle to sink. On the other hand, if the ball is placed on the surface of water, the surface tension of water is not sufficient to keep it floating so it sinks down.
Hence, the correct answer is (C).

16. Excess pressure is inversely proportional to radius of the bubble.
Hence, the correct answer is (A).

17. In a small drop, the force due to surface tension is very large as compared to its weight and hence it is spherical shape. A big drop becomes oval in shape due to its large weight. The surface tension of liquid decreases with increase of temperature.
Hence, the correct answer is (B).

18. Bulk modulus is related to volume change and volume change is possible in all three states, however Young's modulus is related to length change, which is possible only in solids.
Hence, the correct answer is (B).

19. From definition, elasticity is the measure of tendency of the body to regain its original configuration. Since, steel is deformed less than rubber hence, steel is more elastic.
Hence, the correct answer is (A).

20. Force of buoyancy,

$U = V\rho_{\text{air}}g$

Since, ρ_{air} is negligible. Hence, U is negligible
Hence, the correct answer is (C).

21. Stress is the internal force per unit area of body. If the same force is applied to the rubber and steel, then strain in rubber is more. It means the steel is more elastic than rubber.
Hence, the correct answer is (C).

22. $p_1 = p_0 + \rho g h$

$p_2 = p_0 + \rho g(2h) \neq 2p_1$

Hence, the correct answer is (D).

23. Breaking load depends on the area of cross-section and is independent of length of rod. Hence,

$$\text{Breaking load} = \begin{pmatrix}\text{breaking}\\ \text{stress}\end{pmatrix} \times \begin{pmatrix}\text{cross-sectional}\\ \text{area}\end{pmatrix}$$

Hence, the correct answer is (C).

24. Since there is no atmosphere on moon, so atmospheric pressure is zero. Hence barometer height is zero.
Hence, the correct answer is (D).

25. On applying load on lead and rubber wires of same cross-sectional area, the strain produced in lead is much less than rubber wires. Now from the relation, $Y =$ Young's modulus

$\Rightarrow \quad Y = \dfrac{\text{Stress}}{\text{Strain}}$

The Young's modulus of elasticity is greater for lead wires. Hence, lead is elastic than rubber.
Hence, the correct answer is (A).

27. According to the ascent formula $h = \dfrac{2T\cos\theta}{r\rho g}$

$\Rightarrow \quad h \propto \dfrac{1}{r}$

From the relation if radius is less then, height h to which the liquid will rise is greater.
Hence, the correct answer is (A).

28. Pressure at P is less. As liquid is flowing at P while liquid is at rest at Q.
Hence, the correct answer is (D).

29. As the temperature decreases in winter. So, on decreasing temperature, the coefficient of viscosity of engine oil and the lubricants increases due to which the machine parts get jammed in winter.
Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. Mass of water = (Volume)(density)

$\Rightarrow \quad m_o = (AH)\rho$

$\Rightarrow \quad H = \dfrac{m_o}{A\rho} \qquad \ldots(1)$

Velocity of efflux

$$V = \sqrt{2gH} = \sqrt{2g\frac{m_o}{A\rho}} = \sqrt{\frac{2m_o g}{A\rho}}$$

Hence, the correct answer is (D).

2. Thrust force on the container due to draining out of liquid from the bottom is given by,

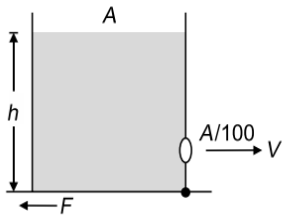
$$F = \begin{pmatrix}\text{density of}\\ \text{liquid}\end{pmatrix}\begin{pmatrix}\text{area of}\\ \text{hole}\end{pmatrix}\begin{pmatrix}\text{velocity}\\ \text{of efflux}\end{pmatrix}^2$$

$\Rightarrow \quad F = \rho a V^2$

$\Rightarrow \quad F = \rho\left(\dfrac{A}{100}\right)V^2 = \rho\left(\dfrac{A}{100}\right)\left(\dfrac{2m_o g}{A\rho}\right)$

$\Rightarrow \quad F = \dfrac{m_o g}{50}$

So, acceleration of the container is



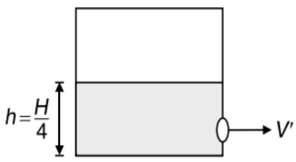
$$a = \frac{F}{m_o} = \frac{g}{50}$$

$\Rightarrow \quad a = \dfrac{g}{50}$

Hence, the correct answer is (B).

3. Velocity of efflux when 75% liquid has been drained out i.e., height of liquid,

$$h = \frac{H}{4} = \frac{m_o}{4A\rho}$$



$\Rightarrow \quad V' = \sqrt{2gh} = \sqrt{2g\left(\dfrac{m_o}{4A\rho}\right)}$

$\Rightarrow \quad V' = \sqrt{\dfrac{m_o g}{2A\rho}}$

Hence, the correct answer is (B).

4. Unless the upthrust becomes equal to weight of the cube, it will be in contact with vessel and normal reaction between the two will be

$$N = mg - U$$

As liquid starts collecting in the vessel at a constant rate, U increases linearly with time and hence N decreases linearly with time. Once, the upthrust equals the weight of the cube, then $N = 0$.

Hence, the correct answer is (A).

5. When contact between the cube and the vessel breaks, then

$$W = \text{Upthrust}$$

$\Rightarrow \quad (10\times10\times10)\times0.5\times g = 10\times10\times h\times1\times g$

$\Rightarrow \quad h = 5\text{ cm}$

Now, $Q\times t = (20\times20 - 10\times10)\times h$

$\Rightarrow \quad t = \dfrac{300\times5}{50} = 30\text{ s}$

Hence, the correct answer is (B).

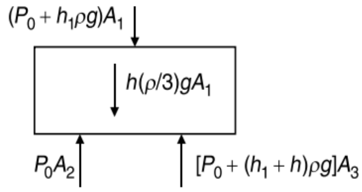
6. Let A_1 = Area of cross-section of cylinder $= 4\pi r^2$

A_2 = Area of base of cylinder in air $= \pi r^2$

and A_3 = Area of base of cylinder in water

$\Rightarrow \quad A_3 = A_1 - A_2 = 3\pi r^2$

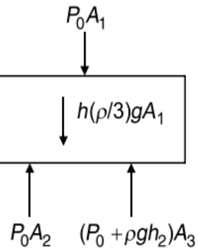
Drawing free body diagram of cylinder



Equating the net downward forces and net upward forces, we get $h_1 = \dfrac{5}{3}h$

Hence, the correct answer is (C).

7. Again, equating the forces, we get



$$h_2 = \frac{4h}{9}$$

Hence, the correct answer is (A).

8. For $h_2 < \dfrac{4h}{9}$, buoyant force will further decrease.

Hence the cylinder remains at its original position.

Hence, the correct answer is (A).

9. For the orifice (i.e. hole) at a depth h, we have

$$\Delta P_1 = \frac{1}{2}\rho v_1^2$$

$\Rightarrow \quad h\rho g = \dfrac{1}{2}\rho v_1^2$

$\Rightarrow \quad v_1 = \sqrt{2gh}$

Similarly, for the orifice at a depth $2h$, we have

$$\Delta P_2 = \frac{1}{2}(2\rho)v_2^2$$

$\Rightarrow \quad (\rho gh + 2\rho gh) = \rho v_2^2$

$\Rightarrow \quad v_2 = \sqrt{3gh}$

$\Rightarrow \quad \dfrac{v_1}{v_2} = \sqrt{\dfrac{2}{3}}$

Hence, the correct answer is (C).

10. For the orifice at a depth h, the speed v_1 will not change, because speed of efflux does not depend on the density of liquid below the orifice. When the density of lower liquid is increased and made $2n\rho$, where $n > 1$, then for orifice at a depth $2h$, we have

$$\Delta P_2 = \frac{1}{2}(2n\rho)v_2^2$$

$$\Rightarrow \quad \rho gh + 2(n\rho)gh = n\rho v_2^2$$

$$\Rightarrow \quad v_2 = \sqrt{\frac{gh + 2ngh}{n}} = \sqrt{2gh + \frac{gh}{n}}$$

Since, $n > 1$

$$\Rightarrow \quad v_2 < \sqrt{3gh}$$

Hence, the correct answer is (B).

11. Considering vertical equilibrium of cylinder:

$$\begin{pmatrix}\text{Weight}\\ \text{of}\\ \text{cylinder}\end{pmatrix} = \begin{pmatrix}\text{upthrust due}\\ \text{to upper}\\ \text{liquid}\end{pmatrix} + \begin{pmatrix}\text{upthrust due}\\ \text{to lower}\\ \text{liquid}\end{pmatrix}$$

Note that h_1 and $h_2 \neq \frac{H}{2}$

$$\Rightarrow \quad \left(\frac{A}{5}\right)(L)D.g = \left(\frac{A}{5}\right)\left(\frac{3L}{4}\right)(d)g + \left(\frac{A}{5}\right)\left(\frac{L}{4}\right)(2d)(g)$$

$$\Rightarrow \quad D = \left(\frac{3}{4}\right)d + \left(\frac{1}{4}\right)(2d)$$

$$\Rightarrow \quad D = \frac{5}{4}d$$

Hence, the correct answer is (C).

12. Considering vertical equilibrium of two liquids and the cylinder.

$$(P - P_o)A = \begin{pmatrix}\text{weight of}\\ \text{two liquids}\end{pmatrix} + \begin{pmatrix}\text{weight of}\\ \text{cylinder}\end{pmatrix}$$

$$\Rightarrow \quad P = P_o + \frac{\begin{pmatrix}\text{weight of}\\ \text{two liquids}\end{pmatrix} + \begin{pmatrix}\text{weight of}\\ \text{cylinder}\end{pmatrix}}{A} \qquad \ldots(1)$$

Now, weight of cylinder

$$W = \left(\frac{A}{5}\right)(L)(D)(g) = \left(\frac{A}{5}Lg\right)\left(\frac{5}{4}d\right) = \frac{ALdg}{4}$$

Weight of upper liquid $= \left(\frac{H}{2}Adg\right)$ and

Weight of lower liquid $= \frac{H}{2}A(2d)g = HAgd$

Total weight of two liquids $= \frac{3}{2}HAgd$

From equation (1) pressure at the bottom of the container will be

$$P = P_o + \frac{\left(\frac{3}{2}\right)HAgd + \frac{ALgd}{4}}{A}$$

$$\Rightarrow \quad P = P_o + \frac{gd}{4}(L + 6H)$$

Hence, the correct answer is (B).

13. Young's modulus is property of material of the wire, so the ratio of the Young's moduli is 1

Hence, the correct answer is (C).

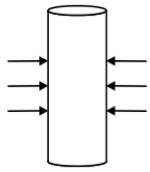
14. On crossing the yield region, the material will experience the breaking stress and further elongation causes reduction in stress and breaking of the wire.

Hence, the correct answer is (C).

15. Since the stress-strain curve flattens on crossing the elastic region, so $x > y$

Hence, the correct answer is (D).

16. Liquid A is applying the hydrostatic force on cylinder from all the sides. So, net force is zero.



Hence, the correct answer is (A).

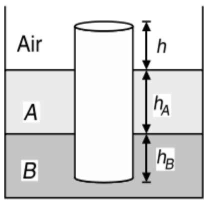
17. In equilibrium:

Weight of cylinder = Net upthrust on the cylinder.

Let s be the area of cross-section of the cylinder, then

$$weight = (s)(h + h_A + h_B)\rho_{\text{cylinder}}g$$

and upthrust on the cylinder = upthrust due to liquid A + upthrust due to liquid $B = sh_A\rho_A g + sh_B\rho_B g$



Equating these two

$$s(h + h_A + h_B)\rho_{\text{cylinder}}g = sh_{A}\rho_{A}g = h_A\rho_A + h_B\rho_B$$

$$\Rightarrow \quad (h + h_A + h_B)\rho_{\text{cylinder}} = h_A\rho_A + h_B\rho_B$$

Substituting,

$h_A = 1.02$ cm , $h_B = 0.8$ cm and $\rho_A = 0.7\ \text{gcm}^{-3}$

$\rho_B = 1.02\ \text{gcm}^{-3}$ and $\rho_{\text{cylinder}} = 0.8\ \text{gcm}^{-3}$

In the above equation, we get

$h = 0.25$ cm

Hence, the correct answer is (C).

18. Net upward force = Extra Upthrust $= (sh)\rho_B g$

$$\Rightarrow \quad \text{Net acceleration } a = \frac{\text{Force}}{\text{mass of cylinder}}$$

$$\Rightarrow \quad a = \frac{sh\rho_B g}{s(h + h_A + h_B)\rho_{\text{cylinder}}}$$

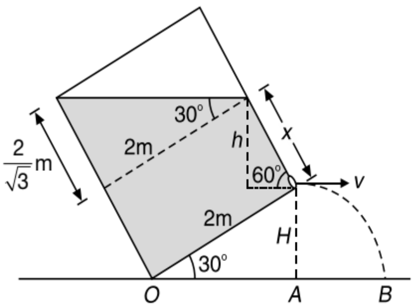
$$\Rightarrow \quad a = \frac{h\rho_B g}{(h + h_A + h_B)\rho_{\text{cylinder}}}$$

Substituting the values of h, h_A, h_B, ρ_B and ρ_{cylinder}, we get $a = \frac{g}{6}$ (upwards)

Hence, the correct answer is (B).

19. When the vessel is tilted such that no liquid spills, then the volume of liquid should remain unchanged in both the configurations of the vessel. Hence, we have

$$V_1 = V_2$$



From the figures, we observe that

$$(2)(2)(2)=(2)\left[\frac{1}{2}\left(x+\left(x+\frac{2}{\sqrt{3}}\right)\right)(2)\right]$$

$\Rightarrow \quad x \approx 1.42$ m

Since, $h = x\sin(60°) = 1.23$ m

$\Rightarrow \quad v=\sqrt{2gh}=\sqrt{2\times10\times1.23}=4.96\approx 5\text{ ms}^{-1}$

Hence, the correct answer is (C).

20. Since, $H = 2\sin 30° = 1$ m

$\Rightarrow \quad t=\sqrt{\frac{2H}{g}}=\frac{1}{\sqrt{5}}$ s

Hence, the correct answer is (C).

21. Since $OA = 2\cos 30° = \sqrt{3}$ m

and $AB = vt = (5)\left(\frac{1}{\sqrt{5}}\right)=\sqrt{5}$ m

$\Rightarrow \quad OB = OA + AB = \left(\sqrt{3}+\sqrt{5}\right)$ m

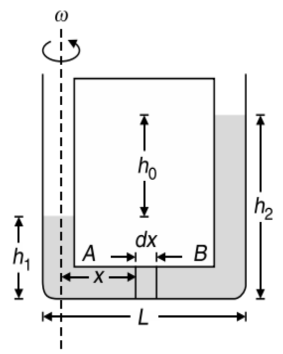
Hence, the correct answer is (C).

22. Force on small element of length dx

$dF = dm x\omega^2$

where, dm is the mass of the small element given by

$dm = (Adx\rho)x\omega^2$



Total force, $F=\int_0^L A\rho\omega^2 x dx = A\rho\omega^2\int_0^L x\,dx$

$$F = A\rho\omega^2\left|\frac{x^2}{2}\right|_0^2=\frac{A\rho\omega^2L^2}{2}$$

Hence, the correct answer is (C).

23. From the left side, point B receives atmospheric pressure, pressure from column of height h_1 and pressure from centrifugal force.

Hence, the correct answer is (D).

24. $P_0 + h_2\rho g = P_0 + h_1\rho g + \frac{\rho\omega^2L^2}{2}$

$\Rightarrow \quad (h_2 - h_1)\rho g = \frac{\rho\omega^2L^2}{2}$

$\Rightarrow \quad h_0 = \frac{\omega^2L^2}{2g}$

Hence, the correct answer is (B).

25. $v=\sqrt{2gh}=\sqrt{2\times10\times(4+1)}=10\text{ ms}^{-1}$

Hence, the correct answer is (C).

26. Applying Bernoulli's theorem at A and B, we get

$$P_0 + 0 + 0 = P_B + \frac{1}{2}\rho v^2 + \rho g(1.5)$$

$\Rightarrow \quad P_B = 1.01\times10^5 - \frac{1}{2}(900)(10)^2 - (900)(10)(1.5)$

$\Rightarrow \quad P_B = 4.25\times10^4\text{ Nm}^{-2}$

Hence, the correct answer is (C).

27. Applying Bernoulli's theorem at B and C

$$P_B + \frac{1}{2}\rho g(2.5) = P_C + \frac{1}{2}\rho v^2$$

$\Rightarrow \quad P_C = 4.25\times10^4 + (900)(10)(2.5)$

$\Rightarrow \quad P_C = 6.5\times10^4\text{ Nm}^{-2}$

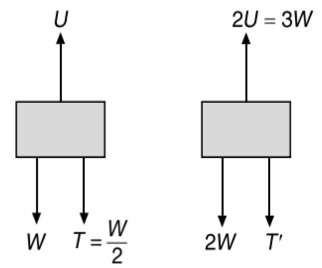
Hence, the correct answer is (B).

28. When the string is cut, tension becomes zero i.e., net upward force on the block becomes $\frac{W}{2}$, i.e. net upward acceleration experienced by the block $\frac{g}{2}$ or 5 ms^{-2}.

$\Rightarrow \quad t=\sqrt{\frac{2s}{a}}=\sqrt{\frac{2\times2}{5}}=\frac{2}{\sqrt{5}}$ s

Hence, the correct answer is (D).

29. If weight is doubled, then upthrust will also be doubled because weight can be increased only by doubling the volume of the block.



Before, $U = W + \frac{W}{2} = \frac{3W}{2}$

After, $2U = 3W = 2W + T'$

$\Rightarrow \quad T' = W = 2T$

When string is cut in the second case, then net upward acceleration will be

$$a' = \frac{F_{net}}{m'} = \frac{3W-2W}{(2W/g)} = \frac{g}{2} = a$$

So, time taken by block to reach the liquid surface will be same in both the cases.

Hence, $x = 2$ and $y = 1$

$\Rightarrow \quad \frac{x}{y} = 2$

Hence, the correct answer is (B).

30. Given: $A_1 = 4\times10^{-3}\ \text{m}^2$, $A_2 = 8\times10^{-3}\,\text{m}^2$,

$h_1 = 2\ \text{m}$, $h_2 = 5\ \text{m}$

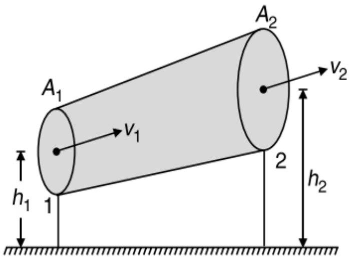
$v_1 = 1\ \text{ms}^{-1}$ and $\rho = 10^3\ \text{kgm}^{-3}$

From continuity equation, we have

$$A_1v_1 = A_2v_2$$

$$\Rightarrow \quad v_2 = \left(\frac{A_1}{A_2}\right)v_1$$

$$\Rightarrow \quad v_2 = \left(\frac{4\times10^{-3}}{8\times10^{-3}}\right)\left(1\ \text{ms}^{-1}\right)$$



$$\Rightarrow \quad v_2 = \frac{1}{2}\ \text{ms}^{-1} = 0.5\ \text{ms}^{-1}$$

Applying Bernoulli's equation at section 1 and 2

Hence, the correct answer is (A).

31. $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$ ...(1)

$\Rightarrow \quad P_1 - P_2 = \rho g\left(h_2 - h_1\right) + \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$ ...(2)

Work done per unit volume by the pressure as the fluid flows from P to Q

$$W_1 = P_1 - P_2 = \rho g\left(h_2 - h_1\right) + \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

{from equation (1)}

$$\Rightarrow \quad W_1 = \left\{\left(10^3\right)(9.8)(5-2) + \frac{1}{2}\left(10^3\right)\left(\frac{1}{4}-1\right)\right\}\ \text{Jm}^{-3}$$

$$\Rightarrow \quad W_1 = (29400 - 375)\ \text{Jm}^{-3} = 29025\ \text{Jm}^{-3}$$

Hence, the correct answer is (C).

32. Work done per unit volume by the gravity as the fluid flows from P to Q

$$W_2 = \rho g\left(h_2 - h_1\right) = \left[\left(10^3\right)(9.8)(5-2)\right]\ \text{Jm}^{-3}$$

$$\Rightarrow \quad W_2 = 29400\ \text{Jm}^{-3}$$

Hence, the correct answer is (B).

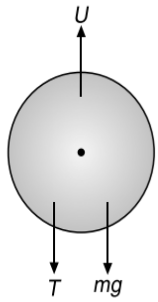
33. Volume of liquid displaced equals the volume of the ball, i.e. V , so mass of liquid that overflows is ρV.

Hence, the correct answer is (A).

34. When the weight mg is added, the weight $V\rho g$ of liquid is overflowed.

Hence, the correct answer is (C).

35. The free body diagram of the forces acting on the sphere is shown in Figure.



For, equilibrium, we have

$$U = T + mg$$

$$\Rightarrow \quad V\rho g = T + mg$$

$$\Rightarrow \quad T = \left(V\rho - m\right)g$$

Hence, the correct answer is (A).

36. In this case too, when the weight mg is added, the weight $V\rho g$ of liquid is overflowed.

Hence, the correct answer is (C).

37. Applying Bernoulli's theorem at 1 and 2

$$P_o + dg\left(\frac{H}{2}\right) + 2dg\left(\frac{H}{2} - h\right) = P_o + \frac{1}{2}(2d)v^2$$

Here, v is velocity of efflux at 2.

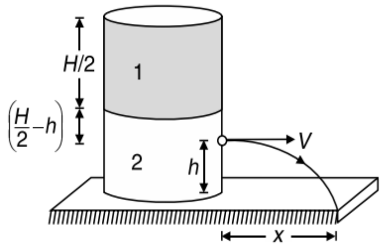
Solving this, we get

$$v = \sqrt{(3H - 4h)\frac{g}{2}} = \sqrt{\left(\frac{3H}{2} - 2h\right)g}$$

Hence, the correct answer is (A).

38. Time taken to reach the liquid to the bottom will be

$$t = \sqrt{\frac{2h}{g}}$$



Horizontal distance x travelled by the liquid is

$$\Rightarrow \quad x = vt = \sqrt{(3H - 4h)\frac{g}{2}}\left(\sqrt{\frac{2h}{g}}\right)$$

$$\Rightarrow \quad x = \sqrt{h(3H - 4h)}$$

Hence, the correct answer is (B).

39. For x to be maximum

$$\frac{dx}{dh} = 0$$

$$\Rightarrow \quad \frac{1}{2\sqrt{h(3H-4h)}}(3H - 8h) = 0$$

$$\Rightarrow \quad h = \frac{3H}{8}$$

Therefore, x will be maximum at $h = \frac{3H}{8}$.

Hence, the correct answer is (B).

40. The maximum value of x will be

$$x_m = \sqrt{\left(\frac{3H}{8}\right)\left[3H - 4\left(\frac{3H}{8}\right)\right]}$$

$$x_m = \frac{3}{4}H$$

Hence, the correct answer is (C).

41. Initially, $\Delta L = \frac{FL}{AY}$

$$\Rightarrow \quad \Delta L = \frac{10 \times 1}{10^{-3} \times 2 \times 10^5}$$

$$\Rightarrow \quad \Delta L = 0.05 \text{ m} = 5 \text{ cm}$$

Hence, the correct answer is (B).

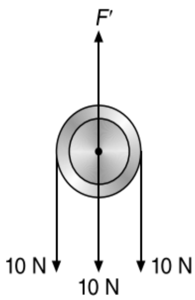
42. Force constant of string is

$$k = \frac{\text{Force}}{\text{Elongation}} = \frac{F}{\frac{FL}{AY}}$$

$$\Rightarrow \quad k = \frac{YA}{L} = \frac{1 \times 10^5 \times 10^{-3}}{1} = 200 \text{ Nm}^{-1}$$

Initial elastic potential energy of string

$$U_i = \frac{1}{2}k(0.05)^2 = \frac{1}{2}(200)(25 \times 10^{-4}) = 25 \times 10^{-2} \text{ J}$$



Let after force $F = 10$ N is applied extra elongation is x, then

$$0.25 + 30x = \frac{1}{2}(200)(x^2 + 25 \times 10^{-4} + 0.1x)$$

$$0.25 + 30x = 100x^2 + 0.25 + 10x$$

$$\Rightarrow \quad 100x^2 = 20x$$

$$\Rightarrow \quad x = \frac{20}{100} = \frac{1}{5} \text{ m} = 0.2 \text{ m}$$

$$\Rightarrow \quad x = 20 \text{ cm}$$

$$\Rightarrow \quad x_{\max} = 20 + 5 = 25 \text{ cm}$$

Hence, the correct answer is (C).

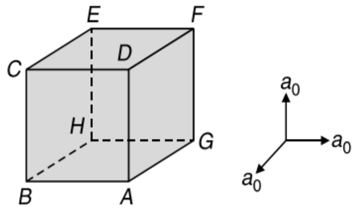
43. When the displacement of the pulley is 25 cm, the string gets loosened on both the sides. So, point A moves down by 50 cm.
Hence, the correct answer is (D).

Matrix Match/Column Match Type Questions

1. A $\rightarrow$ (p, t)
B $\rightarrow$ (q)
C $\rightarrow$ (r)
D $\rightarrow$ (s)

Since $\vec{a} = a_0\hat{i} + a_0\hat{j} + a_0\hat{k}$, so

$$a_x = a_0,\ a_y = a_0,\ a_z = a_0$$



Assuming the cube to be of side length ℓ, we observe that in the gravity free space, we get

$$P_A - P_D = \ell\rho a_y = \ell\rho a_0$$
$$P_G - P_A = \ell\rho a_z = \ell\rho a_0$$

Similarly,

$$P_B - P_A = \ell\rho a_0,\ P_C - P_D = \ell\rho a_0$$
$$P_F - P_D = \ell\rho a_0,\ P_H - P_E = \ell\rho a_0$$
$$P_E - P_C = \ell\rho a_0,\ P_E - P_F = \ell\rho a_0$$
$$P_H - P_B = \ell\rho a_0,\ P_H - P_G = \ell\rho a_0$$
$$P_G - P_F = \ell\rho a_0$$

From these equations, we can also, see that

$$P_A = P_F,\ P_E = P_B$$

2. A $\rightarrow$ (t, r)
B $\rightarrow$ (p, r)
C $\rightarrow$ (q, r)
D $\rightarrow$ (s)

From, $Q = A_1v_1 = A_2v_2$ or $v \propto \frac{1}{A}$

At a point where area of cross-section is less, volume of liquid flowing per second, i.e. the rate of flow of liquid is same but speed is more and vice-versa. Also, when area is less, then v is more and consequently due to Bernoulli's Theorem pressure is less and hence $P_{narrow} < P_{broad}$, so $\Delta P < 0$. The rate of flow of liquid and the speed are also positive.

3. A $\rightarrow$ (q)
B $\rightarrow$ (r)
C $\rightarrow$ (s)
D $\rightarrow$ (p)
Stoke's law gives viscous force acting on a spherical body falling through a liquid column of infinite length.
Equation of continuity i.e. $a_1v_1 = a_2v_2$ is based on Law of Conservation of Mass
Bernoulli's Theorem is based on Law of Conservation of Energy
Surface tension is the phenomenon in which the free surface of the liquid behaves like a stretched elastic membrane.

4. A $\rightarrow$ (p, q, r)
B $\rightarrow$ (q, s)
C $\rightarrow$ (q)
D $\rightarrow$ (q, s)

In case of solid, all three elastic constants are found. In case of liquid and gas only bulk modulus is defined. The concept of surface tension is also applicable for liquids.

5. A $\rightarrow$ (p, r, s, t)
B $\rightarrow$ (p, t)
C $\rightarrow$ (p, q, r, s)
D $\rightarrow$ (p, r, s, t)

Since $k = \frac{YA}{L}$ and $T = 2\pi\sqrt{\frac{m}{k}}$

Also, work done $W = \frac{1}{2}kx^2$

$$\Rightarrow \quad W = \frac{1}{2}\left(\frac{YA}{L}\right)x^2$$

Thermal stress $= Y\alpha\Delta T$
i.e. thermal stress depends upon coefficient of linear expansion, Young's modulus of wire and rise in temperature ΔT.
Stress $= Y(\text{strain}) = Y\alpha\Delta T$

6. A $\rightarrow$ (s)
B $\rightarrow$ (p)
C $\rightarrow$ (p)
D $\rightarrow$ (p)
Contact angle between silver and water is $90°$, so $h = 0$

Capillary rise in glass tube is $h = \frac{2T}{r\rho g}$

Capillary rise in parallel glass plate with separation Δt is

$$h = \frac{2T\ell}{(\ell\rho g)\Delta t}$$

$$\Rightarrow \quad h = \frac{2T}{r\rho g}$$

The capillary rise in co-axial cylinder is

$$h = \frac{2T}{\rho g(r_2 - r_1)} = \frac{2T}{r\rho g}$$

7. A $\rightarrow$ (r)
B $\rightarrow$ (q)
C $\rightarrow$ (p)
D $\rightarrow$ (s)
Conceptual

8. A $\rightarrow$ (p, q, s)
B $\rightarrow$ (p, r, s)
C $\rightarrow$ (p, q, s)
D $\rightarrow$ (p)
For a liquid drop and air bubble, the excess pressure is

$$\Delta P = \frac{2T}{R}$$

For a soap bubble, the excess pressure is

$$\Delta P = \frac{4T}{R}$$

Also, surface tension decreases with increase in temperature

9. A $\rightarrow$ (r)
B $\rightarrow$ (t)
C $\rightarrow$ (p)
D $\rightarrow$ (p)
E $\rightarrow$ (q)

The pressure per unit speed gradient equals the viscosity η, so we have

$$[\eta] = ML^{-1}T^{-1}$$
$$[\text{Surface Tension}] = MT^{-2}$$
$$[E] = ML^{-1}T^{-2}$$
$$[u] = ML^{-1}T^{-2}$$
$$[n_R] = M^0L^0T^0$$

10. A $\rightarrow$ (q)
B $\rightarrow$ (p, q, r, s)
C $\rightarrow$ (q)
D $\rightarrow$ (p, q, r, s)
Conceptual

11. A $\rightarrow$ (p)
B $\rightarrow$ (q, r)
C $\rightarrow$ (s)
D $\rightarrow$ (s)

$$F_2 - F_1 = U = W$$

For liquid-1, the horizontal pair of forces cancel out.
So, net force is zero.

12. A $\rightarrow$ (r)
B $\rightarrow$ (s)
C $\rightarrow$ (q)
D $\rightarrow$ (p)
Conceptual

13. A $\rightarrow$ (p)
B $\rightarrow$ (q, r, s)
C $\rightarrow$ (p, s)
D $\rightarrow$ (q, r)
When cohesive forces are greater than adhesive forces, then shape of meniscus is concave from the liquid side and pressure is greater on concave side due to surface tension.

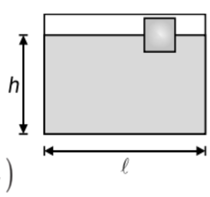
14. A $\rightarrow$ (r)
B $\rightarrow$ (p)
C $\rightarrow$ (r)
D $\rightarrow$ (p)
According to Torricelli's Theorem, we have

$$v = \sqrt{2gh_1} \quad \text{and} \quad R = 2\sqrt{h_1h_2}$$

where, h_1 is the depth of the orifice from the free surface and h_2 is the height of the orifice from the ground.

15. A $\rightarrow$ (r)
B $\rightarrow$ (s)
C $\rightarrow$ (p)
D $\rightarrow$ (q)
Conceptual

16. A $\rightarrow$ (p)
B $\rightarrow$ (p, s)
C $\rightarrow$ (q, s)
D $\rightarrow$ (p, r)



Since, $U = V_{\text{imm}}\rho_{\text{liq}}g_{\text{eff}} = V_{\text{imm}}\rho_{\text{liq}}(g + a_y)$

$$\Delta P_y = h\rho(g + a_y) \qquad \ldots(1)$$

$$\Delta P_x = \ell \rho a_x \quad \ldots(2)$$

In both equations (1) and (2), pressure increases opposite to the acceleration of the container.

17. A → (q)
B → (p)
C → (r)
D → (t)

If the orifice be at a depth h below the free surface of the liquid filled to a height H, then we have

$$v = \sqrt{2gh}$$

$$t = \sqrt{\frac{2h}{g}} \text{ and}$$

$$R = 2\sqrt{h(H-h)}$$

At the mountain, value of g will be less than its value at the surface of the earth, so v will decrease, t will increase and R will remain the same.
Range will be maximum when the orifice is at the middle of the liquid level that fills the beaker and not at the middle of the beaker.

18. A → (q)
B → (p)
C → (r)
D → (q)

Let the cube be of length L, density ρ_s be floating in a liquid of density ρ_ℓ
Since the cube is floating, so weight of cube is balanced by upthrust. Hence

$$\rho_s ALg = \rho_\ell Axg$$

$$\Rightarrow \quad x = \left(\frac{\rho_s}{\rho_\ell}\right)L$$

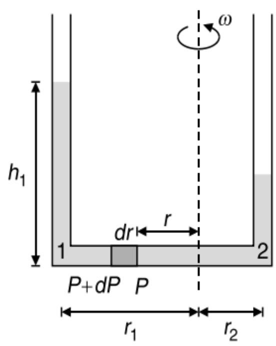
So, if ρ_ℓ is increased, x decreases and vice-versa.
When base area and density of cube remain same, then on increasing the length of cube, x also increases
Since x is independent of g, so when the whole system is accelerated upwards, then value of x will remain the same.
Also, when cube is replaced by another cube of same size but lesser density, then x decreases.

Integer/Numerical Answer Type Questions

1. Consider a small element of length dr at a distance r from the axis of rotation. Considering the equilibrium of this element.

$$(P+dP) - p = \rho\omega^2 rdr$$

$$\Rightarrow \quad dP = \rho\omega^2 rdr$$



On integrating between 1 and 2, we get

$$P_1 - P_2 = \rho\omega^2 \int_{-r_2}^{r_1} rdr$$

$$\Rightarrow \quad P_1 - P_2 = \frac{\rho\omega^2}{2}\left(r_1^2 - r_2^2\right)$$

Also, we note that $P_1 - P_2 = (h_1 - h_2)\rho g$

$$\Rightarrow \quad h_1 - h_2 = \frac{\omega^2}{2g}\left(r_1^2 - r_2^2\right)$$

$$\Rightarrow \quad h_1 - h_2 = \frac{(2\pi)^2}{2(10)}\left[(0.5)^2 - (0.25)^2\right]$$

$$\Rightarrow \quad h_1 - h_2 = 0.37 \text{ m} = 37 \text{ cm}$$

2. We know change in pressure,

$$\Delta P = -B\frac{\Delta V}{V_0}$$

$$\Rightarrow \quad \Delta P = -\left(2.6\times10^{10}\ \text{Nm}^{-2}\right)\frac{-1.0\times10^{-10}\ \text{m}^3}{1.0\times10^{-6}\ \text{m}^3}$$

$$\Rightarrow \quad \Delta P = 2.6\times10^{6}\ \text{Nm}^{-2}$$

where we have expressed the volume V_0 of the cube at the ocean's surface as

$$V_0 = \left(1.0\times10^{-2}\ \text{m}\right)^3 = 1.0\times10^{-6}\ \text{m}^3.$$

Since the pressure increases by $1.0\times10^4\ \text{Nm}^{-2}$ per meter of depth, the depth is

$$h = \frac{\Delta P}{\text{Pressure Gradient}} = \frac{2.6\times10^6\ \text{Nm}^{-2}}{1.0\times10^4\ \frac{\text{Nm}^{-2}}{\text{m}}}$$

$$\Rightarrow \quad h = 260 \text{ m}$$

3. Since, $\Delta l = \dfrac{mgl}{2AY} = \dfrac{\rho(\pi r^2 l)gl}{2(\pi r^2)Y}$

$$\Rightarrow \quad \Delta l = \frac{gl^2\rho}{2Y} = \frac{(9.8)(5)^2(8000)}{2\times2\times10^{11}}$$

$$\Rightarrow \quad \Delta l = 4.9\times10^{-6}\ \text{m} = 4.9\ \mu\text{m}$$

$$\Rightarrow \quad U = \frac{1}{2}k(\Delta l)^2 = \frac{1}{2}\frac{YA}{l}(\Delta l)^2$$

$$\Rightarrow \quad U = \frac{1}{2}\times\frac{Y\times\pi r^2}{l}(\Delta l)^2$$

$$\Rightarrow \quad U = \frac{\left(2\times10^{11}\right)(3.14)\left(6\times10^{-3}\right)^2\left(4.9\times10^{-6}\right)^2}{2\times5}$$

$$\Rightarrow \quad U = 5.43\times10^{-5}\ \text{J} = 54.3\ \mu\text{J}$$

4. Work done in stretching the wire from its initial length through $0.61\ \text{mm}\left(=0.61\times10^{-3}\ \text{m}\right)$ when the stretching force is $3\ \text{kgwt}(=3\times9.8\ \text{N})$, i.e.,

$$W_1 = \frac{1}{2}\times\text{Stretching force}\times\text{extension}$$

$$\Rightarrow \quad W_1 = \frac{1}{2}\times(3\times9.8)\times\left(0.61\times10^{-3}\right) = 0.9\times10^{-2}\ \text{J}$$

Work done in stretching the wire from its initial length through $1.02\text{ mm}(=1.02\times10^{-3}\text{ m})$ when the stretching force is $5\text{ kgwt}(=5\times9.8\text{ N})$, i.e.,

$$W_2=\frac{1}{2}\times(5\times9.8)\times(1.02\times10^{-3})=2.5\times10^{-2}\text{ J}$$

Work done in stretching the wire from 0.61 mm to 1.02 mm is

$$W=(W_2-W_1)=2.5\times10^{-2}\text{ J}-0.9\times10^{-2}\text{ J}$$

$$\Rightarrow\quad W=1.6\times10^{-2}\text{ J}=16\text{ mJ}$$

5. Velocity of flow is

$$v=\sqrt{2gh}=\sqrt{2\times10\times3.6}=6\sqrt{2}=8.5\text{ ms}^{-1}$$

Rate of flow is

$$Q=Av=\pi\left(\frac{4\times10^{-2}}{\sqrt{\pi}}\right)^2(8.5)$$

$$\Rightarrow\quad Q=(16\times10^{-4})(8.5)=13.6\times10^{-3}\text{ m}^3\text{s}^{-1}$$

$$\Rightarrow\quad Q=13.6\times10^3\text{ cm}^3\text{s}^{-1}=13600\text{ cm}^3\text{s}^{-1}$$

Applying Bernoulli's theorem between surface and A, we get

$$P_{\text{atm}}=P+\frac{1}{2}\rho v^2+\rho gh$$

$$\Rightarrow\quad P=P_{\text{atm}}-\frac{1}{2}\rho v^2-\rho gh$$

$$\Rightarrow\quad P=10^5-\frac{1}{2}(10^3)(6\sqrt{2})^2-(10^3)(10)(1.8)$$

$$\Rightarrow\quad P=4.6\times10^4\text{ Nm}^{-2}=46\text{ kNm}^{-2}$$

6. The excess of pressure inside the air bubble blown at the lower end of the tube is

$$P_i-P_0=\frac{2T}{R}$$

Given that, $T_{Hg}=0.465\text{ Nm}^{-1}$, $\rho_{Hg}=13600\text{ kgm}^{-3}$ and the radius of the tube is

$$R=\frac{1}{2}\text{ mm}=0.5\text{ mm}=5\times10^{-4}\text{ m}$$

$$\Rightarrow\quad P_i-P_0=\frac{2T}{R}=\frac{2\times0.465}{5\times10^{-4}}=1860\text{ Nm}^{-2}$$

$$\Rightarrow\quad P_i-P_0=\frac{1860}{13600\times9.8}=0.014\text{ m of Hg}$$

$$\Rightarrow\quad P_i-P_0=14\text{ mm of Hg}$$

7. Writing equation of motion for the block

$$T-mg\sin30^\circ=ma \qquad \ldots(1)$$

For the sphere

$$\text{Weight}-\text{Buoyant force}-T=ma \qquad \ldots(2)$$

$$\Rightarrow\quad mg-\frac{mg}{2}-T=ma$$

Solving, we get

$$a=0$$

8. **For steel wire:**

Total force on steel wire $F_1=(4+6)\times10=100\text{ N}$

Radius of steel wire $r_1=\left(\frac{0.25}{2}\right)\text{ cm}=0.125\times10^{-2}\text{ m}$

Young's modulus of steel $Y_1=2.0\times10^{11}\text{ Pa}$

Since, $Y_1=\frac{F_1\times l_1}{A_1\times\Delta l_1}=\frac{F_1\times l_1}{\pi r_1^2\times\Delta l_1}$

Elongation of the steel wire is $\Delta l_1=\frac{F_1\times l_1}{\pi r_1^2\times Y_1}$

$$\Rightarrow\quad \Delta\ell_1=\frac{100\times1.5\times7}{22\times(0.125\times10^{-2})^2\times2\times10^{11}}$$

$$\Rightarrow\quad \Delta\ell_1=1.49\times10^{-4}\text{ m}\approx0.15\text{ mm}$$

For brass wire:

Total force on brass wire $F_2=6\times10=60\text{ N}$

Radius of brass wire $r_2=\left(\frac{0.25}{2}\right)\text{ cm}=0.125\times10^{-2}\text{ m}$

$$Y_2=9\times10^{11}\text{ Pa},\ l_2=1.0\text{ m},\ \Delta l_2=?$$

Elongation of the brass wire is $\Delta l_2=\frac{F_2\times l_2}{\pi r_2^2\times Y_2}$

$$\Rightarrow\quad \Delta\ell_2=\frac{60\times1.0\times7}{22\times(0.125\times10^{-2})^2\times(0.9\times10^{11})}$$

$$\Rightarrow\quad \Delta\ell_2=1.3\times10^{-4}\text{ m}=0.13\text{ mm}$$

9. According to Bernoulli's Theorem, we have

$$P_1+\frac{1}{2}\rho v_1^2=P_2+\frac{1}{2}\rho v_2^2$$

$$\Rightarrow\quad P_1-P_2=\frac{1}{2}\rho(v_2^2-v_1^2)$$

$$\Rightarrow\quad P_1-P_2=\frac{1}{2}(1.3)(120^2-90^2)$$

$$\Rightarrow\quad P_1-P_2=4095\text{ Nm}^{-2}\approx4.1\text{ kPa}$$

Lift force is given by

$$F=A(P_1-P_2)$$

$$\Rightarrow\quad F=(4.1\times10^3)(10\times2)=8.2\times10^4\text{ N}$$

$$\Rightarrow\quad F=82\text{ kN}$$

10. The distance Δx that the top surface of the disc moves relative to the bottom surface is given by

$$\Delta x=\frac{FL_0}{\eta A}$$

where F is the magnitude of the shearing force, L_0 is the thickness of the cartilage, A is the cross-sectional area and η is the shear modulus. Since the cross-section is circular, so $A=\pi r^2$, where r is the radius.

$$\Delta x=\frac{(11\text{ N})(7.0\times10^{-3}\text{ m})}{(1.2\times10^7\text{ Nm}^{-2})\pi(3.0\times10^{-2}\text{ m})^2}$$

$$\Rightarrow\quad \Delta x=2.3\times10^{-6}\text{ m}=2.3\ \mu\text{m}$$

11. Since, $\Delta l=\frac{Fl}{AY}=\frac{Fl}{(\pi d^2/4)Y}$

$$\Rightarrow\quad d=\sqrt{\frac{4Fl}{\pi(\Delta l)Y}}=\sqrt{\frac{4\times400\times3}{3.14\times0.2\times10^{-2}\times2.1\times10^{11}}}$$

$$\Rightarrow\quad d=1.91\times10^{-3}\text{ m}=1.91\text{ mm}$$

12. Change in pressure, $\Delta P=501\text{ atm}-1\text{ atm}$

$$\Rightarrow\quad \Delta P=500\text{ atm}=500\times1.01\times10^5\text{ Pa}$$

Let ρ' be the density of water at the given depth, then

$$\rho' = \frac{\rho}{1-(\Delta V/V)} = \frac{\rho}{1-(\Delta P/B)}$$

$$\Rightarrow \rho' = \frac{1.0\times10^3}{1-2.525\times10^{-2}} = 1025\ \text{kgm}^{-3}$$

13. The horizontal reaction force on vessel due to ejection of liquid is

$F = Av^2\rho$, where $v^2 = 2gh$

For the vessel to slide, we must have

$$F \ge f_{\text{limiting}}$$

$$\Rightarrow Av^2\rho \ge \mu mg$$

$$\Rightarrow A \ge \frac{\mu mg}{\rho(2gh)}$$

$$\Rightarrow A \ge \frac{(0.4)(100)}{2(2)(1000)} = 0.01\ \text{m}^2$$

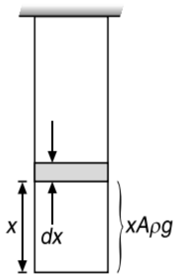
If d is the diameter of the hole, then

$$A = \frac{\pi d^2}{4}$$

$$\Rightarrow d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(0.01)}{3.14}} = 0.1128\ \text{m}$$

$$\Rightarrow d = 11.3\ \text{cm}$$

14. Consider an element as shown in the figure.



Stress in the element $= \frac{\text{Force}}{\text{Area}} = \frac{xA\rho g}{A} = x\rho g$

Now, elastic potential energy stored in the wire is

$$dU = \frac{1}{2}(\text{Stress})(\text{Strain})(\text{Volume})$$

$$\Rightarrow dU = \frac{1}{2}\frac{(\text{Stress})^2}{Y}(\text{Volume})$$

$$\Rightarrow dU = \frac{1}{2}\frac{(x\rho g)^2}{Y}Adx = \frac{1}{2}\frac{\rho^2 g^2 A}{Y}x^2 dx$$

Total elastic potential energy is $U = \frac{1}{2}\frac{\rho^2 g^2 A}{Y}\int_0^L x^2 dx$

$$U = \frac{\rho^2 g^2 A L^3}{6Y}$$

$$\Rightarrow p = 6$$

15. $\Delta l = \frac{Fl}{AY} = \frac{1000\times100}{4\times2\times10^6}$

$$\Rightarrow \Delta\ell = 0.0125\ \text{cm} = 0.125\ \text{mm}$$

16. Since, the terminal velocity is given by

$$v_T = \frac{2r^2}{9}\frac{(\rho-\sigma)g}{\eta}$$

Also, we know $v = \frac{s}{t}$

$$\Rightarrow \frac{s}{t} = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

$$\Rightarrow r^2 = \frac{9s}{2t}\frac{\eta}{(\rho-\sigma)g} \quad \ldots(1)$$

Given that, $s = 2\times10^{-2}$ m, $t = 1\ \text{h} = 3600$ s

$$\eta = 1\times10^{-2}\ \text{poise} = 1\times10^{-3}\ \text{kgm}^{-1}\text{s}^{-1}$$

Substituting these given values, we get

$$r^2 = \frac{9}{2}\left(\frac{2\times10^{-2}}{3600}\right)\frac{1\times10^{-3}}{(1.8\times10^3 - 1\times10^3)\times10}$$

$$\Rightarrow r^2 = \frac{9}{36}\times\frac{1}{8}\times10^{-10} = \frac{1}{32}\times10^{-10}$$

$$\Rightarrow r = \sqrt{\frac{100}{32}}\times10^{-6}\ \text{m} = 1.77\ \mu\text{m}$$

So, the diameter is

$$D = 2r = 2\times1.77\ \mu\text{m} = 3.54\ \mu\text{m}$$

17. Maximum stress on the upper string is

$$\sigma_{\max} = \frac{(m_1+m_2+m)g}{0.006\times10^{-4}}$$

$$\Rightarrow 8\times10^8 = \frac{(10+20+m)\times10}{0.006\times10^{-4}}$$

$$\Rightarrow m = 18\ \text{kg}$$

Maximum stress on the upper string is

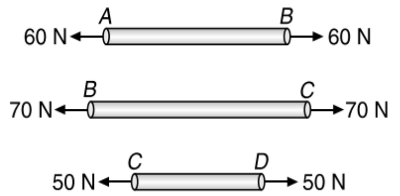
$$\sigma_{\max} = \frac{(m_1+m')g}{0.003\times10^{-4}}$$

$$\Rightarrow 8\times10^8 = \frac{(10+m')\times10}{0.003\times10^{-4}}$$

$$\Rightarrow m' = 14\ \text{kg}$$

So, answer is 14 kg and lower string will break earlier.

18. The action of forces on each part of rod is shown in Figure.



We know that the extension due to external force F is given by,

$$\Delta L = \frac{FL}{AY}$$

$$\Delta L_{AB} = \frac{(60\times10^3)\times1.5}{1\times2\times10^{11}} = 4.5\times10^{-7}\ \text{m}$$

$$\Delta L_{BC} = \frac{(70\times10^3)X_1}{1\times2\times10^{11}} = 3.5\times10^{-7}\ \text{m and}$$

$$\Delta L_{CD} = \frac{(50\times10^3)X_2}{1\times2\times10^{11}} = 5.0\times10^{-7}\ \text{m}$$

The total extension is given by

$$\Delta L = \Delta L_{AB} + \Delta L_{BC} + \Delta L_{CD}$$

$$\Rightarrow \Delta L = 4.5\times10^{-7} + 3.5\times10^{-7} + 5.0\times10^{-7}$$

$$\Rightarrow \Delta L = 13\times10^{-7}\text{ m} = 1.3\ \mu\text{m}$$

19. $\dfrac{F}{A_{\min}} = \sigma_m$

$$\Rightarrow \frac{F}{\left(\frac{\pi d_{\min}^2}{4}\right)} = \sigma_m$$

$$\Rightarrow d_{\min} = \sqrt{\frac{4F}{\pi\sigma_m}}$$

$$\Rightarrow d_{\min} = \sqrt{\frac{4\times10\times9.8}{3.14\times1.5\times10^8}} = 9.1\times10^{-4}\text{ m} = 0.91\text{ mm}$$

20. Using equation of continuity, $A_1v_1 = A_2v_2$

$$v_2 = \left(\frac{A_1}{A_2}\right)v_1 \text{ or } v_2 = \left(\frac{\pi r_1^2}{\pi r_2^2}\right)v_1 = \left(\frac{r_1}{r_2}\right)^2 v_1$$

Substituting the values, we get

$$v_2 = \left(\frac{1.0\times10^{-2}}{2.0\times10^{-2}}\right)^2(2)$$

$$\Rightarrow v_2 = 0.5\text{ ms}^{-1} = 50\text{ cms}^{-1}$$

21. Taking the subscript G for ground floor and F for first floor, then according to Bernoulli's theorem, we have

$$P_G + \rho g h_G + \frac{1}{2}\rho v_G^2 = P_F + \rho g h + \frac{1}{2}\rho v_F^2$$

Assuming zero potential energy level (ZPEL) at the ground floor, then $h_G = 0$.

$$\Rightarrow P_G + \frac{1}{2}\rho v_G^2 = P_F + \rho g h + \frac{1}{2}\rho v_F^2 \quad \ldots(1)$$

According to Equation of Continuity, we have

$$A_F v_F = A_G v_G$$

$$\Rightarrow v_F = \frac{A_G v_G}{A_F} = \frac{\pi(2.5)^2\times3}{\pi(1.25)^2} = 12\text{ ms}^{-1}$$

From equation (1), we get

$$P_F = P_G - \frac{1}{2}\rho\left(v_F^2 - v_G^2\right) - \rho g h$$

$$\Rightarrow P_F = 317600 - (10^3)\left(\frac{144-9}{2} + (10)(25)\right)$$

$$\Rightarrow P_F = 317600 - 317500 = 100\text{ Nm}^{-2}$$

22. The stress should not exceed the elastic limit otherwise the wire will suffer permanent deformation. So,

$$\frac{F}{A} = 2.48\times10^8\text{ Pa}$$

Area of cross section of the wire is

$$A = \pi r^2 = 3.14\times(1\times10^{-3})^2 = 3.14\times10^{-6}\text{ m}^2$$

$$\Rightarrow F = 779\text{ N}$$

Also, $\text{Strain} = \dfrac{\text{Stress}}{Y} = \dfrac{2.48\times10^8}{2\times10^{11}} = 1.24\times10^{-3}$

$$\Rightarrow \frac{\Delta L}{L} = 1.24\times10^{-3}$$

$$\Rightarrow \Delta L = 1.24\times10^{-3}\times10\text{ m} = 1.24\text{ cm}$$

The stress should not exceed the ultimate strength.

$$\frac{F}{A} = 4.89\times10^8\text{ Pa} \qquad \{\text{Take } b \ll r\}$$

$$\Rightarrow F = 4.89\times10^8\times3.14\times10^{-6} = 1535\text{ N} \approx 1.5\text{ kN}$$

23. $\rho' = \dfrac{\rho}{1-\dfrac{dP}{B}} \approx \rho\left(1+\dfrac{dP}{B}\right)$

$$\Rightarrow \Delta\rho = \rho' - \rho = \frac{\rho(dP)}{B}$$

$$\Rightarrow \Delta\rho = \frac{\rho(\rho g h)}{B}$$

$$\Rightarrow \Delta\rho = \frac{(1030)^2\times9.8\times400}{2\times10^9}$$

$$\Rightarrow \Delta\rho = 2.0\text{ kgm}^{-3}$$

24. Let the atmospheric pressure be P_0. Then, pressure at A is

$$P_A = P_0 + h(1000)g \quad \ldots(1)$$

Similarly, pressure at B is

$$P_B = P_0 + (0.02\text{ m})(13600)g \quad \ldots(2)$$

Since these pressures are equal as A and B because these points are at the same horizontal level, so equating (1) and (2), we get

$$h = (0.02\text{ m})(13.6) \approx 27\text{ cm}$$

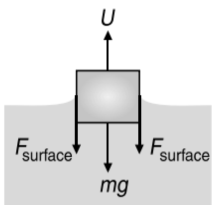
25. If surface tension is ignored, then according to the Laws of Floatation, the upthrust acting on the cube must balance its weight. So, if x be the length of the cube immersed inside the water, then

$$W = U$$

$$\Rightarrow (800\times10^{-3})g = \left[(0.1)^2 x\right]\rho_w g$$

$$\Rightarrow x = 0.08\text{ m} \quad \ldots(1)$$

Now, we know that since water wets the cube and the angle of contact is zero, so the force due to surface tension acts vertically downwards as shown in Figure and hence the cube is buoyed down due to surface tension.



Let x' be the new immersed length at which the cube is in equilibrium, then we have

$$mg + F = U' \quad \ldots(2)$$

where, F is the force due to surface tension acting on the four edges of the cube. So

$$F = 4TL = 4(0.07)(0.1)\text{ N} = 0.028\text{ N}$$

So, from equation (2) we get

$$(800\times10^{-3})g + 0.028 = \left[(0.1)^2 x'\right]\rho_w g$$

$$\Rightarrow \quad x' = \frac{8+0.028}{100} = 0.08028 \text{ m}$$

Therefore, the additional distance is

$$x' - x = 0.00028 \text{ m} = 280\ \mu\text{m}$$

26. Force constant of the wire is

$$k = \frac{F}{\Delta L} = \frac{F}{\frac{FL}{AY}} = \frac{YA}{L}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{YA}{Lm}}$$

$$\Rightarrow \quad \sqrt{\frac{YA}{Lm}} = 140$$

$$\Rightarrow \quad \frac{Y \times 4.9 \times 10^{-7}}{1 \times 0.1} = 140 \times 140$$

$$\Rightarrow \quad Y = \frac{140 \times 140}{49 \times 10^{-7}} = 4 \times 10^{9}$$

$$\Rightarrow \quad x = 4$$

27. The stress in either cable is

$$\text{Stress} = \frac{F}{A} = \frac{F}{\pi r^2} \qquad \ldots(1)$$

Now, for equal stresses in the cables, we have

$$\frac{F_2}{\pi r_2^2} = \frac{F_1}{\pi r_1^2}$$

$$\Rightarrow \quad F_2 = \frac{F_1 r_2^2}{r_1^2} = \frac{(270 \text{ N})(5.1 \times 10^{-3} \text{ m})^2}{(3.5 \times 10^{-3} \text{ m})^2} = 570 \text{ N}$$

28. The velocity attained by the sphere in falling freely from a height h is

$$v = \sqrt{2gh} \qquad \ldots(1)$$

This is the terminal velocity of the sphere in water. Hence by Stokes law, we get

$$v = v_T = \frac{2}{9}\left(\frac{r^2}{\eta}\right)(\rho - \sigma)g$$

$$\Rightarrow \quad v_T = \frac{2}{9}\frac{(1.0 \times 10^{-3})^2(1.0 \times 10^4 - 1.0 \times 10^3) \times 10}{1.0 \times 10^{-3}}$$

$$\Rightarrow \quad v_T = 20 \text{ ms}^{-1}$$

So, from Equation (1), we get

$$h = \frac{v^2}{2g} = \frac{v_T^2}{2g} = \frac{(20)^2}{2(10)} = 20 \text{ m}$$

29. $\sigma = \frac{F}{A} = \frac{F}{\pi(R^2 - r^2)}$

$$\Rightarrow \quad R = \sqrt{\frac{F}{\sigma\pi} + r^2}$$

$$\Rightarrow \quad R = \sqrt{\frac{1.6 \times 10^6}{90 \times 10^6 \times 3.14} + (0.1)^2}$$

$$\Rightarrow \quad R = 0.1251 \text{ m} = 125.1 \text{ mm}$$

So, diameter $d = 2R = 250.2$ mm

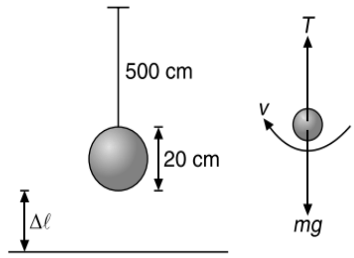
30. According to the Equation of Continuity, we have

$$a_1 v_1 + a_2 v_2 = av$$

$$\Rightarrow \quad (12)(20) + (8)(16) = (16)v$$

$$\Rightarrow \quad v = \frac{240 + 128}{16} = 23 \text{ kmh}^{-1}$$

31. $\Delta l = 521 - 500 - 20 = 1 \text{ cm} = 0.01 \text{ m}$



$$T - mg = \frac{mv^2}{R}$$

$$\Rightarrow \quad T = m\left(g + \frac{v^2}{R}\right) = m\left(g + \frac{v^2}{l}\right)$$

$$\Rightarrow \quad \Delta l = \frac{Tl}{AY} = \frac{m\left(g + \frac{v^2}{l}\right)l}{\left(\frac{\pi d^2}{4}\right)Y}$$

$$\Rightarrow \quad \Delta l = \frac{mgl + mv^2}{\left(\frac{\pi d^2}{4}\right)Y}$$

$$\Rightarrow \quad v = \sqrt{\frac{\pi d^2 \Delta l Y}{4m} - gl}$$

$$\Rightarrow \quad v = \sqrt{\frac{(3.14)(4 \times 10^{-3})^2(0.01)(2 \times 10^{11})}{4 \times 25} - 9.8 \times 5}$$

$$\Rightarrow \quad v \approx 31 \text{ ms}^{-1}$$

32. We are given that

$$(\text{Stress})_{\text{lim}} = 0.9 \times 10^8 \text{ Nm}^{-2}, \quad Y = 1.4 \times 10^{10} \text{ Nm}^{-2}$$

Length of two bones in a leg is

$$L = 2 \times 50 \text{ cm} = 100 \text{ cm} = 1.0 \text{ m}$$

Area of cross-section of the bone is

$$A = 5 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2$$

Since, $\text{Stress} = \frac{F}{A}$, so $F = (\text{Stress})A$

$$\Rightarrow \quad F = (0.9 \times 10^8)(5 \times 10^{-4}) = 4.50 \times 10^4 \text{ N}$$

Further, $Y = \frac{F/A}{\Delta L/L}$

$$\Rightarrow \quad \Delta L = \left(\frac{\text{Stress}}{Y}\right)L$$

$$\Rightarrow \quad \Delta L = \left(\frac{0.9 \times 10^8}{1.4 \times 10^{10}}\right)(1) = 6.43 \times 10^{-3} \text{ m}$$

Elastic potential energy is $U = \frac{1}{2}F\Delta L$

$$\Rightarrow \quad U = \frac{1}{2}(4.50 \times 10^4)(6.43 \times 10^{-3}) = 145 \text{ J}$$

33. $\frac{1}{2}\rho v^2 = \rho gh + \frac{mg}{A}$

where, $h = 1.0 - 0.5 = 0.5$ m,

A = Area of piston $= 0.5\ \text{m}^2$

$\Rightarrow v = \sqrt{2gh + \frac{2mg}{\rho A}}$

$\Rightarrow v = \sqrt{2\times 9.8\times 0.5 + \frac{2\times 20\times 9.8}{10^3\times 0.5}} = 3.25\ \text{ms}^{-1}$

Speed with which it hits the surface is

$\Rightarrow v' = \sqrt{v^2 + 2gh'}$

$\Rightarrow v' = \sqrt{(3.25)^2 + (2\times 9.8\times 0.5)}$

$\Rightarrow v' = 4.51\ \text{ms}^{-1}$

34. Applying Bernoulli's theorem for a horizontal pipe, we get

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$\Rightarrow P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$

So, kinetic energy per unit mass is

$\frac{\text{K.E.}}{\text{Mass}} = \frac{1}{2}\left(v_2^2 - v_1^2\right) = \frac{P_1 - P_2}{\rho} = \frac{8}{800}\ \text{Jkg}^{-1}$

$\Rightarrow \frac{1}{2}\left(v_2^2 - v_1^2\right) = \frac{1}{100}\ \text{Jkg}^{-1} = 10\ \text{mJkg}^{-1}$

35. Since, $T - mg = ml\omega^2 = ml(2\pi f)^2$

$\Rightarrow T = mg + 4\pi^2 mlf^2$

$\Rightarrow T = 6\times 9.8 + 4\pi^2(6)(0.6)(2)^2$

$\Rightarrow T = 628$ N

Now, $\Delta l = \frac{Tl}{AY}$

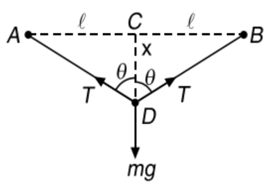
$\Rightarrow \Delta l = \frac{(628)(0.6)}{(0.05\times 10^{-4})(2\times 10^{11})}$

$\Rightarrow \Delta l = 3.8\times 10^{-4}\ \text{m} = 0.38\ \text{mm}$

36. Given that $m = 100\ \text{g} = 0.100\ \text{kg}$

Refer figure, let x be the depression at the mid-point i.e., $CD = x$

In figure, $AC = CB = \ell = 0.5$ m



$AD = BD = (l^2 + x^2)^{\frac{1}{2}}$

Increase in length, $\Delta l = AD + DB - AB = 2AD - AB$

$\Rightarrow \Delta l = 2(l^2 + x^2)^{\frac{1}{2}} - 2l = 2l\left(1 + \frac{x^2}{l^2}\right)^{\frac{1}{2}} - 2l$

$\Rightarrow \Delta l = 2l\left(1 + \frac{x^2}{2l^2}\right) - 2l = \frac{x^2}{l}$

Strain $= \frac{x^2}{2l^2}$

If T is the tension in the wire, then

$2T\cos\theta = mg$

$\Rightarrow T = \frac{mg}{2\cos\theta}$

Here, $\cos\theta = \frac{x}{(l^2 + x^2)^{\frac{1}{2}}} = \frac{x}{l\left(1 + \frac{x^2}{l^2}\right)^{\frac{1}{2}}} = \frac{x}{l\left(1 + \frac{1}{2}\frac{x^2}{l^2}\right)}$

Since $x \ll l$, so $\frac{x^2}{2l^2} \ll 1$ and $1 + \frac{x^2}{2l^2} \approx 1$

$\Rightarrow \cos\theta = \frac{x}{l}$

Hence, $T = \frac{mg}{2\left(\frac{x}{l}\right)} = \frac{mgl}{2x}$

Stress $= \frac{T}{A} = \frac{mgl}{2Ax}$

$Y = \frac{\text{stress}}{\text{strain}} = \frac{mgl}{2Ax}\times\frac{2l^2}{x^2} = \frac{mgl^3}{Ax^3}$

$\Rightarrow x = l\left(\frac{mg}{YA}\right)^{\frac{1}{3}} = 0.5\left(\frac{0.1\times 10}{2\times 10^{11}\times 0.5\times 10^{-6}}\right)^{\frac{1}{3}}$

$\Rightarrow x = 1.077\times 10^{-2}\ \text{m} = 10.8\ \text{mm}$

37. Given that, $\frac{m(g+a)}{A} = \frac{1}{3}\sigma_{\max}$

$\Rightarrow a = \frac{\sigma_{\max}A}{3m} - g$

$\Rightarrow a = \frac{(3\times 10^8)(4\times 10^{-4})}{3\times 900} - 9.8$

$\Rightarrow a = 34.64\ \text{ms}^{-2}$

38. According to Equation of Continuity, we have

$A_1 v_1 = A_2 v_2$

$\Rightarrow \pi\left(\frac{4}{2}\right)^2 v_1 = \pi\left(\frac{3}{2}\right)^2 v_2$

$\Rightarrow v_2 = \frac{16}{9}v_1$

The pressure at the cross-sections are

$P_1 = 20\times 1\times 980 = 19600\ \text{dynecm}^{-2}$

and $P_2 = 15\times 1\times 980 = 14700\ \text{dynecm}^{-2}$

Using Bernoulli's theorem, we get

$v_2^2 - v_1^2 = \frac{2}{\rho}(P_1 - P_2)$

$\Rightarrow \left(\frac{16v_1}{9}\right) - v_1^2 = \frac{2}{1}(19600 - 14700)$

$\Rightarrow v_1 = 67.4\ \text{cms}^{-1}$

Now discharge rate is

$Q = A_1 v_1 = \pi(2)^2(67.4) \approx 847\ \text{cm}^3\text{s}^{-1}$

$\Rightarrow Q = 847\ \text{cm}^3\text{s}^{-1} = 847\ \text{cm}^3\text{s}^{-1}$

39. Since, $\Delta l = \dfrac{Fl}{AY}$

Here, $F = \text{Upthrust} = V_i \rho_l g$, $A = \dfrac{\pi d^2}{4}$

$\Rightarrow \quad \Delta l = \dfrac{4V_i \rho_l g l}{\pi d^2 Y}$

$\Rightarrow \quad \Delta l = \dfrac{(4)(10^{-3})(800)(9.8)(3)}{(3.14)(0.4\times10^{-3})^2(8\times10^{10})}$

$\Rightarrow \quad \Delta \ell = 2.34\times10^{-3}\ \text{m} = 2.34\ \text{mm}$

40. The volume of the blood given to the patient per second is given by Poiseuille Equation according to which

$$Q = \frac{V}{t} = \frac{\pi}{8}\left(\frac{r^4}{\eta}\right)\left(\frac{\Delta P}{l}\right) \qquad \ldots(1)$$

Given that, length of the needle is $l = 5$ cm , radius of the needle is $r = 0.03$ cm , viscosity of blood is, $\eta = 0.017$ poise , density of blood is $\rho = 1.02\ \text{gcm}^{-3}$ and height at which blood bag hangs is $h = 85$ cm . So, the pressure difference is

$$\Delta P = h\rho g = (85)(1.02)(980)\ \text{dynecm}^{-2}$$

$\Rightarrow \quad Q = \dfrac{V}{t} = \dfrac{\pi}{8}\left(\dfrac{(0.03)^4}{0.017}\right)\left(\dfrac{85\times1.02\times980}{5}\right)$

$\Rightarrow \quad Q = 0.32\ \text{cm}^3\text{s}^{-1}$

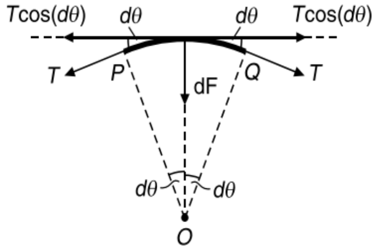
So, time for which transfusion takes place is

$$t = \frac{\text{Total Volume of Blood}}{\text{Rate of Flow of Blood}} = \frac{V}{Q} = \frac{500\ \text{cm}^3}{0.32\ \text{cm}^3\text{s}^{-1}}$$

$\Rightarrow \quad t = 1562.5\ \text{s} \approx 26\ \text{min}$

41. The component of tension force radially inwards provides the necessary centripetal force dF to the element to revolve in a circle, so

$$2T\sin d\theta = (dm)R\omega^2$$



For small angles, $\sin d\theta \approx d\theta$

$\Rightarrow \quad 2T(d\theta) = (2Rd\theta)(\pi r^2)(\rho)(R)(2\pi f)$

$\Rightarrow \quad T = (4\pi^3 f^2 R^2 r^2 \rho)$

Now $\Delta l = \dfrac{Tl}{AY}$

$\Rightarrow \quad \dfrac{\Delta l}{l} = \dfrac{T}{AY} = \dfrac{T}{\pi r^2 Y}$

Also, $l = 2\pi R$

$\Rightarrow \quad \Delta l = 2\pi(\Delta R)$

$\Rightarrow \quad \dfrac{\Delta l}{l} = \dfrac{\Delta R}{R} = \dfrac{T}{\pi r^2 Y}$

$\Rightarrow \quad \dfrac{\Delta l}{l} = \dfrac{4\pi^3 f^2 R^2 r^2 \rho}{\pi r^2 Y} = 4\pi^2\left(\dfrac{f^2 R^2 \rho}{Y}\right)$

$\Rightarrow \quad \dfrac{\Delta l}{l} = 40\left(\dfrac{f^2 R^2 \rho}{Y}\right)$

$\Rightarrow \quad a = 40$

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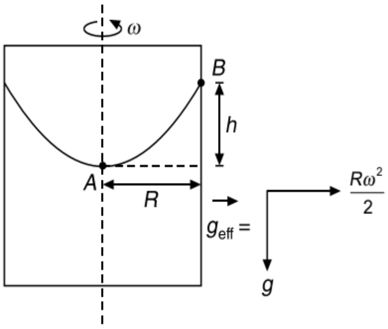
1. Applying pressure equation between A and B, we get



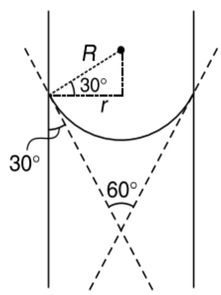
$$P_0 + \rho\frac{R\omega^2}{2}R - \rho g h = P_0$$

$\Rightarrow \quad \dfrac{\rho R^2\omega^2}{2} = \rho g h$

$\Rightarrow \quad h = \dfrac{R^2\omega^2}{2g} = (5)^2\dfrac{\omega^2}{2g} = \dfrac{25\omega^2}{2g}$

Hence, the correct answer is (A).

2. Let r be radius of capillary and R be radius of meniscus



From figure, we see that angle of contact $\theta = 30°$

$\Rightarrow \quad \dfrac{r}{R} = \cos 30°$

Height of liquid in capillary is

$$h = \frac{2T\cos\theta}{r\rho g}$$

$\Rightarrow \quad h = \dfrac{2\times0.05\times(\sqrt{3}/2)}{0.15\times10^{-3}\times667\times10}$

$\Rightarrow \quad h = 0.087$ m

Hence, the correct answer is (C).

3. Since, $\Delta P_1 = 0.01 = \dfrac{4T}{R_1}$ and $\Delta P_2 = 0.02 = \dfrac{4T}{R_2}$

$$\Rightarrow \quad \frac{R_1}{R_2} = 2$$

$$\Rightarrow \quad \frac{V_1}{V_2} = \frac{R_1^3}{R_2^3} = \frac{8R^3}{R_2^3} = 8$$

Hence, the correct answer is (A).

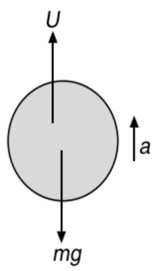
4. Since, $V = \dfrac{4\pi}{3}r^3 = \dfrac{4\pi}{3} \times (1)^3 = 4.19 \text{ cm}^3$

$a = 9.8 \text{ cms}^{-2}$

Since, $U - mg = ma$

$$\Rightarrow \quad m = \frac{U}{g+a} = \frac{(V\rho_\omega g)}{g+a} = \frac{V\rho_\omega}{1+\dfrac{a}{g}}$$

$$\Rightarrow \quad m = \frac{(4.19)\times 1}{1+\dfrac{9.8}{980}} = \frac{4.19}{1.01} = 4.15 \text{ g}$$



Hence, the correct answer is (C).

5. Since, $B = -\dfrac{\Delta P}{\Delta V / V}$

$$\Rightarrow \quad \left|\frac{\Delta V}{V}\right| = \frac{\Delta P}{B} = \frac{4\times 10^9}{8\times 10^{10}} = \frac{1}{20}$$

$$\Rightarrow \quad \frac{\Delta l}{l} = \frac{1}{3} \times \frac{\Delta V}{V} = \frac{1}{60}$$

Percentage change is $\dfrac{\Delta l}{l} \times 100\% = \dfrac{100}{60}\% = 1.67\%$

Hence, the correct answer is (B).

6. Since, $\dfrac{4}{3}\pi(R^3 - r^3)\rho_m g = \dfrac{4}{3}\pi R^3 \rho_w g$

$$\Rightarrow \quad 1 - \left(\frac{r}{R}\right)^3 = \frac{8}{27}$$

$$\Rightarrow \quad \frac{r}{R} = \left(\frac{19}{27}\right)^{1/3} = \frac{19^{1/3}}{3} = 0.88 \approx \frac{8}{9}$$

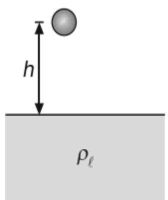
Hence, the correct answer is (B).

7. After falling through h, the velocity be equal to terminal velocity, so

$$\sqrt{2gh} = \frac{2}{9}\frac{r^2 g}{\eta}(\rho_l - \rho)$$

$$\Rightarrow \quad h = \frac{2}{81}\frac{r^4 g(\rho_l - \rho)^2}{\eta^2}$$

$$\Rightarrow \quad h \propto r^4$$



Hence, the correct answer is (B).

8. Applying Bernoulli's Equation, we get

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$\Rightarrow \quad P + \frac{1}{2}\rho v^2 = \frac{P}{2} + \frac{1}{2}\rho V^2$$

$$\Rightarrow \quad \frac{P}{\rho} + v^2 = V^2$$

$$\Rightarrow \quad V = \sqrt{\frac{P}{\rho} + v^2}$$

Hence, the correct answer is (B).

9. Capillary rise

$$h = \frac{2T\cos\theta}{r\rho g}$$

$$\Rightarrow \quad T = \frac{\rho g r h}{2\cos\theta}$$

$$\Rightarrow \quad T = \frac{(900)(10)(15\times 10^{-5})(15\times 10^{-2})}{2}$$

$$\Rightarrow \quad T = 1012.5 \times 10^{-4} \text{ Nm}^{-1}$$

$$\Rightarrow \quad T = 101.25 \times 10^{-3} = 101.25 \text{ mNm}^{-1}$$

In question closest integer is asked

So closest integer is 101.00

Hence, the correct answer is 101.

10. Since, $A_1 v_1 = A_2 v_2$

$$\Rightarrow \quad \frac{v_{\min}}{v_{\max}} = \frac{A_{\min}}{A_{\max}} = \left(\frac{4.8}{6.4}\right)^2 = \frac{9}{16}$$

Hence, the correct answer is (D).

11. In case of minimum density of liquid, sphere will be floating while completely submerged, so

$$mg = U$$

$$\Rightarrow \quad mg = \left(\int_0^R \rho(4\pi r^2 dr)\right) g = U$$

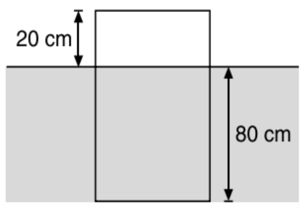
$$\rho_0 g \int_0^R \left(1 - \frac{r^2}{R^2}\right) 4\pi r^2 dr = \frac{4}{3}\pi R^3 \rho_l g$$

$$\Rightarrow \quad \rho_l = \frac{2\rho_0}{5}$$

Hence, the correct answer is (D).

12. For equilibrium, $mg = U$

$\Rightarrow \quad m = \rho_0 A(80)$ ...(1)

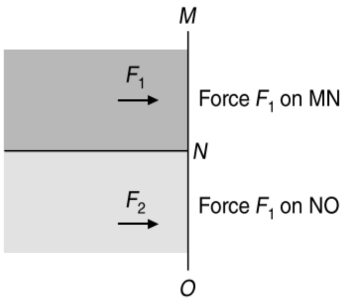


Similarly, $m = \rho A(79)$ ...(2)

Dividing, we get $\dfrac{\rho_0}{\rho} = \dfrac{80}{79} = 1.01$

Hence, the correct answer is (A).

13. Since, $F_1 = \left(\frac{\rho g h}{2}\right)A$



$\Rightarrow \quad F_2 = \left(\rho g h + \frac{2\rho g h}{2}\right)A$

$\Rightarrow \quad \frac{F_1}{F_2} = \frac{1}{4}$

Hence, the correct answer is (A).

14. Rate of flow of water is $Q = A_A V_A = A_B V_B$

$\Rightarrow \quad (40)V_A = (20)V_B$

$\Rightarrow \quad V_B = 2V_A$ ...(1)

According to Bernoulli's theorem, we have

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

$\Rightarrow \quad P_A - P_B = \frac{1}{2}\rho\left(V_B^2 - V_A^2\right)$

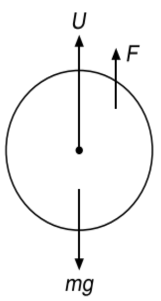
$\Rightarrow \quad 700 = \frac{1}{2}\times 1000\left(4V_A^2 - V_A^2\right)$

$\Rightarrow \quad V_A = 0.68\ \text{ms}^{-1} = 68\ \text{cms}^{-1}$

Rate of flow $Q = A_A V_A = (40)(68) = 2720\ \text{cm}^3\text{s}^{-1}$

Hence, the correct answer is (C).

15. FBD of droplet is shown in Figure.



Since, $U + F = mg$

where, $U = \left(\frac{2}{3}\pi R^3\right)\rho g$, $F = T(2\pi R)$, $m = d\left(\frac{4}{3}\pi R^3\right)$

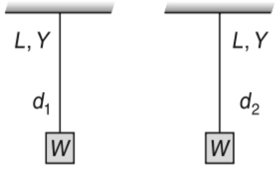
$\Rightarrow \quad \left(\frac{2}{3}\pi R^3\right)\rho g + T(2\pi R) = d\left(\frac{4}{3}\pi R^3\right)g$

$\Rightarrow \quad T(2\pi R) = \left(\frac{2}{3}\pi R^3\right)g(2d - \rho)$

$\Rightarrow \quad R = \sqrt{\frac{3T}{(2d-\rho)g}}$

Hence, the correct answer is (B).

16. Since, $\frac{\text{Energy stored}}{\text{Volume}} = \frac{1}{2}\frac{(\text{Stress})^2}{Y}$



$\Rightarrow \quad \frac{u_1}{u_2} = \frac{1}{4}$

$\Rightarrow \quad 4u_1 = u_2$

$\Rightarrow \quad 4\frac{1}{2Y}\left(\frac{W4}{\pi d_1^2}\right)^2 = \frac{1}{2Y}\left(\frac{W4}{\pi d_2^2}\right)^2$

$\Rightarrow \quad 4 = \left(\frac{d_1}{d_2}\right)^4$

$\Rightarrow \quad \frac{d_1}{d_2} = \sqrt{2} : 1$

Hence, the correct answer is (D).

17. Since, $T = m\omega^2 l$

Breaking stress is $\frac{T}{A} = \frac{m\omega^2 l}{A}$

$\Rightarrow \quad \omega^2 = \frac{4.8\times 10^7 \times \left(10^{-2}\times 10^{-4}\right)}{10\times 0.3} = 16$

$\Rightarrow \quad \omega = 4$

Hence, the correct answer is 4.00.

18. Flow rate of water $(Q) = 100$ lit/min

$\Rightarrow \quad Q = \frac{100\times 10^{-3}}{60} = \frac{5}{3}\times 10^{-3}\ \text{m}^3$

Since $Q = Av$

So, velocity of flow is $v = \frac{Q}{A} = \frac{5\times 10^{-3}}{3\times\pi\times\left(5\times 10^{-2}\right)^2}$

$\Rightarrow \quad v = \frac{10}{15\pi} = \frac{2}{3\pi}\ \text{ms}^{-1} = 0.2\ \text{ms}^{-1}$

Reynold's number $n_R = \frac{Dv\rho}{\eta}$

$\Rightarrow \quad n_R = \frac{\left(10\times 10^{-2}\right)\times\frac{2}{3\pi}\times 1000}{1} \simeq 2\times 10^4$

Order of n_R is 10^4

Hence, the correct answer is (B).

19. $h \propto \frac{1}{r}$ and $M \propto \pi r^2 h$

So, $M \propto r$

Hence, the correct answer is (A).

20. Since, $V\sigma g = \frac{4}{5}v\rho_\omega g$ ...(1)

When oil is poured in it, then

$$V\sigma g = \frac{v}{2}\rho_\omega g + \frac{v}{2}\rho_0 g$$

$\Rightarrow \quad \left(\frac{\rho_\omega}{2} + \frac{\rho_{oil}}{2}\right) = \frac{4}{5}\rho_\omega$

$\Rightarrow \quad \frac{\rho_{oil}}{2} = \rho_\omega\left(\frac{4}{5} - \frac{1}{2}\right) = \frac{3}{10}\rho_\omega$

$\Rightarrow \quad \rho_{oil} = \frac{3}{5}\rho_{\omega} = 0.6\rho_{\omega}$

Hence, the correct answer is (D).

21. Since, $h = \frac{2T\cos\theta}{r\rho g}$

$\Rightarrow \quad r = \frac{2T\cos\theta}{h\rho g}$

$\Rightarrow \quad \frac{r_{Hg}}{r_{water}} = \frac{r_1}{r_2} = \left(\frac{T_{Hg}}{T_W}\right)\left(\frac{\rho_W}{\rho_{Hg}}\right)\left(\frac{\cos\theta_{Hg}}{\cos\theta_W}\right)$

$\Rightarrow \quad \frac{r_1}{r_2} = 7.5 \times \frac{1}{13.6} \times \frac{1}{\sqrt{2}} = 0.4 = \frac{2}{5}$

Hence, the correct answer is (D).

22. Given $(50)^3 \times \frac{30}{100} \times (1) \times g = M_{cube}g$ ...(1)

Let m mass should be placed

Hence $(50)^3 \times (1) \times g = (M_{cube} + m)g$ ...(2)

From (1) and (2), we get

$\Rightarrow \quad mg = (50)^3 \times g(1 - 0.3) = 125 \times 0.7 \times 10^3$ g

$\Rightarrow \quad m = 87.5$ kg

Hence, the correct answer is (C).

23. Using Bernoullie's equation, we get

$v_2 = \sqrt{v_1^2 + 2gh}$

According to equation of continuity, we have

$A_1V_1 = A_2V_2$

$\Rightarrow \quad (1)(1) = (A_2)\left(\sqrt{(1)^2 + 2 \times 10 \times \frac{15}{100}}\right)$

$\Rightarrow \quad A_2 = \frac{1}{2}\text{ cm}^2 = 5 \times 10^{-5}\text{ m}^2$

Hence, the correct answer is (B).

24. Since, $\Delta P = P_2 - P_1 = \rho g \Delta H$

$\Rightarrow \quad 3.03 \times 10^6 = 10^3 \times 10 \times \Delta H$

$\Rightarrow \quad \Delta H \approx 300$ m

Hence, the correct answer is (C).

25. $\frac{4}{3}\pi R^3 = 27\left(\frac{4}{3}\pi r^3\right)$

$\Rightarrow \quad R^3 = 27r^3$

$\Rightarrow \quad R = 3r$

Since $v \propto r^2$

$\Rightarrow \quad \frac{v_1}{v_2} = \left(\frac{R}{r}\right)^2 = 9$

Hence, the correct answer is (B).

26. Volume Flow Rate $Q = \frac{0.74}{60}\text{ m}^3\text{s}^{-1}$

Speed of efflux $v = \frac{0.74 \times 10^4}{60 \times \pi \times 4}\text{ ms}^{-1} = \sqrt{2gh}$

$\Rightarrow \quad 9.82 = \sqrt{2 \times 10 \times h}$

$\Rightarrow \quad h = 4.8$ m

Hence, the correct answer is (C).

27. $-\frac{dh}{dt} = \sqrt{2gh}$

$\Rightarrow \quad Q = a\sqrt{2gh} = Q$

$\Rightarrow \quad 10^{-4} = 10^{-4}\sqrt{2gh} = 10^{-4}$

$\Rightarrow \quad h = \frac{1}{2g}$

$\Rightarrow \quad h = 5.1$ cm

Hence, the correct answer is (A).

28. If area be A, then thrust force is $F = Av^2\rho$

$\Rightarrow \quad F = \frac{1}{2}\rho A v^2 + \frac{1}{4}\rho A v^2$

So, pressure P is given by

$P = \frac{3\rho A v^2}{4A} = \frac{3\rho}{4}v^2$

Hence, the correct answer is (A).

29. Area of wire is $A = \pi r^2$

Since $Y = \frac{F/A}{\Delta\ell/\ell_0}$

$\Rightarrow \quad \frac{Mg}{\pi r^2} = \frac{\Delta\ell}{\ell_0}Y$

$\Rightarrow \quad \frac{Mg}{\pi r^2} = \left(\frac{4 \times 10^{-3}}{2}\right)Y$ ...(1)

Mass of load of volume V is given by

$M = V(8\rho_w)$

Now when load is immersed in liquid, then

$\frac{8V\rho_w g - 2V\rho_w g}{\pi r^2} = \frac{\Delta\ell'}{\ell_0}Y$ ...(2)

$\Rightarrow \quad \frac{6V\rho_w g}{\pi r^2} = \frac{\Delta\ell'}{\ell_0}Y$

$\Rightarrow \quad \frac{\Delta\ell'}{4 \times 10^{-3}} = \frac{6V\rho_w g}{8V\rho_w g}$

$\Rightarrow \quad \Delta\ell' = \frac{6}{8} \times 4 \times 10^{-3}$ m

$\Rightarrow \quad \Delta\ell' = 3 \times 10^{-3}$ m = 3 mm

Hence, the correct answer is (D).

30. $P - P_0 = \frac{4S}{R}$ when S is surface tension

$\Rightarrow \quad P = \frac{4S}{R} + P_0$

$\Rightarrow \quad P = 4S\left(\frac{4\pi}{3v}\right)^{\frac{1}{3}} + P_0$

$\Rightarrow \quad P = 4S\left(\frac{4\pi}{3kt}\right)^{\frac{1}{3}} + P_0$

Also, $V = \frac{4}{3}\pi R^3$

$\Rightarrow \quad R = \left(\frac{3V}{4\pi}\right)^{1/3} = R$

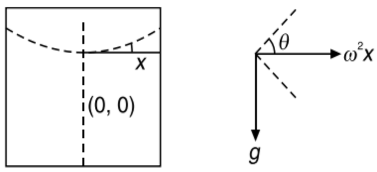
Given that, $V = kt$

So, correct form is $P = m\left(\frac{1}{t^{1/3}}\right) + c$

*No given option is correct.

31. Given that $\omega = 4\pi$ rads^{-1}

Since, $\tan\theta = \frac{dy}{dx} = \frac{\omega^2 x}{g}$



$\Rightarrow \quad y = \int_0^h dy = \int_0^x \frac{\omega^2 x}{g} dx = \frac{\omega^2 x^2}{2g}\Bigg|_0^{5\times10^{-2}}$

$\Rightarrow \quad y = \frac{16\pi^2 \times 25 \times 10^{-4}}{2\times 10} = 1.9 \text{ cm} \approx 2.0 \text{ cm}$

Hence, the correct answer is (D).

32. Bulk modulus $= \frac{\text{Volumetric stress}}{\text{Volumetric strain}}$

$K = \left|\frac{\Delta P}{\frac{\Delta V}{V}}\right| = \frac{\frac{F}{a}}{\frac{dV}{V}}$

Here, $F = mg$, $\frac{dV}{V} = 3\frac{dr}{r}$

$\Rightarrow \quad K = \frac{\frac{mg}{a}}{3\frac{dr}{r}}$

$\Rightarrow \quad \frac{dr}{r} = \frac{mg}{3Ka}$

Hence, the correct answer is (C).

33. At depth h, $\Delta P = \frac{4T}{r}$

$\Rightarrow \quad P_1 = P_0 + \rho g h + \frac{4T}{r}$

At the surface of lake, $\Delta P' = \frac{4T}{5r/4} = \frac{16T}{5r}$

$\Rightarrow \quad P_2 = P_0 + \frac{16T}{5r}$

Also, $P_1V_1 = P_2V_2$

$\Rightarrow \quad \frac{P_1}{P_2} = \frac{r_2^3}{r_1^3}$

$\Rightarrow \quad \frac{P_0 + \rho g h + (4T/r)}{P_0 + (16T/5r)} = \frac{(5r/4)^3}{r^3}$

$\Rightarrow \quad \frac{125}{64} = \frac{10+h}{10}$

$\Rightarrow \quad h = 9.5$ cm

(Excess pressure is very small so we can neglect it).

Hence, the correct answer is (C).

34. For a given material, shear modulus is constant, so

$\left(\frac{F}{A_1}\right)\left(\frac{L_1}{\Delta x_1}\right) = \left(\frac{F}{A_2}\right)\left(\frac{L_2}{\Delta x_2}\right)$

Since, $L_2 = 2L_1$, $A_2 = L_2^2 = 4L_1^2 = 4A_1$, $\Delta x_1 = 0.5$ cm

$\Rightarrow \quad \frac{1}{A_1} \times \frac{L_1}{0.5} = \frac{1}{4A_1} \times \frac{2L_1}{\Delta x_2}$

$\Rightarrow \quad \Delta x_2 = 0.25$ cm

Hence, the correct answer is (A).

35. Excess pressure inside the inner bubble,

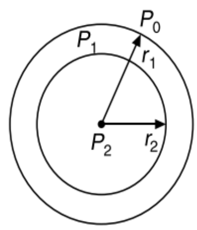
$P_2 - P_1 = \frac{4T}{r_2}$ ...(1)

Excess pressure inside the outer bubble,

$P_1 - P_0 = \frac{4T}{r_1}$ ...(2)

From equation (1) and (2), we get

$P_2 - P_0 = 4T\left(\frac{1}{r_2} + \frac{1}{r_1}\right) = \frac{4T}{r}$



where, r is required radius of a soap bubble. So

$r = \frac{r_2 r_1}{r_1 + r_2} = \frac{4\times 6}{4+6} = \frac{24}{10} = 2.4$ cm

Hence, the correct answer is (C).

36. Given that, $\frac{V_f}{V_i} = 9^3$ $\{\because V \propto \ell^3\}$

Since density remains same, so

mass $\propto$ volume

$\Rightarrow \quad \frac{m_f}{m_i} = 9^3$

$\Rightarrow \quad \frac{(\text{Area})_f}{(\text{Area})_i} = 9^2$

$\Rightarrow \quad \text{Stress} = \frac{\text{Weight of man}}{\text{Area}}$

$\Rightarrow \quad \frac{\sigma_2}{\sigma_1} = \left(\frac{m_f}{m_i}\right)\left(\frac{A_i}{A_f}\right)$

$\Rightarrow \quad \frac{\sigma_2}{\sigma_1} = \frac{9^3}{9^2} = 9$

Hence, the correct answer is (A).

37. Rate of flow of liquid (Q) through narrow tube is

$$Q = \frac{dv}{dt} = \frac{\pi \Pr^4}{8\eta\ell}$$

Since both the given tubes are connected in series so rate of flow of liquid is same.

$$\Rightarrow \quad \frac{\pi P_1 r_1^4}{8\eta\ell_1} = \frac{\pi P_2 r_2^4}{8\eta\ell_2}$$

$$\Rightarrow \quad r_2^4 = \left(\frac{P_1}{P_2}\right)\left(\frac{\ell_2}{\ell_1}\right) r_1^4$$

Given $P_2 = 4P_1$, $\ell_2 = \frac{\ell_1}{4}$

$$\Rightarrow \quad r_2^4 = \left(\frac{P_1}{4P_1}\right)\left(\frac{\ell_1}{4\ell_1}\right) r_1^4 = \frac{r_1^4}{16} = \left(\frac{r_1}{2}\right)^4$$

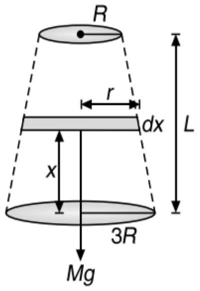
$$\Rightarrow \quad r_2 = \frac{r_1}{2}$$

Hence, the correct answer is (B).

38. Consider a uniform cross-section of wire of length dx and radius r at a vertical distance of x from the lower end. Here,

$$r = 3R - \frac{2R}{L}x$$

Extension in wire of length dx is



$$d\ell = \frac{Fdx}{AY} = \frac{Mgdx}{\pi\left(3R - \frac{2R}{L}x\right)^2 Y}$$

Hence total extension in wire is

$$\Delta L = \int d\ell = \int_0^L \frac{Mgdx}{\pi\left(3R - \frac{2R}{L}x\right)^2 Y}$$

$$\Rightarrow \quad \Delta L = \frac{Mg}{\pi Y}\int_0^L \frac{dx}{\left(3R - \frac{2R}{L}x\right)^2} = \frac{MgL}{3\pi R^2 Y}$$

So, extended length of wire is

$$L_f = L + \frac{MgL}{3\pi R^2 Y} = L\left(1 + \frac{Mg}{3\pi R^2 Y}\right)$$

Hence, the correct answer is (C).

39. Let v_1 and v_2 be the velocities of water when it leaks out through the hole and when it hits the ground respectively. Then, applying Bernoulli's theorem, we get

$$v_1^2 + 2gh = v_2^2$$

Also, $v_1 = \sqrt{2gH}$ ...(1)

$$\Rightarrow \quad 2gH + 2gh = v_2^2 \qquad ...(2)$$

According to continuity Equation, $a_1 v_1 = a_2 v_2$

$$\Rightarrow \quad \pi r^2\sqrt{2gH} = \pi x^2\sqrt{2g(H+h)} \qquad \{\text{using (1) and (2)}\}$$

$$\Rightarrow \quad x^2 = r^2\sqrt{\frac{H}{H+h}}$$

$$\Rightarrow \quad x = r\left(\frac{H}{H+h}\right)^{\frac{1}{4}}$$

Hence, the correct answer is (A).

40. Equation of motion for the point mass

$$ma = mg - kv \qquad ...(1)$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{mg - kv}{m}$$

$$\Rightarrow \quad \frac{dv}{mg - kv} = \frac{dt}{m}$$

Integrating $\int_0^v \frac{dv}{mg - kv} = \frac{1}{m}\int_0^t dt$

$$\Rightarrow \quad -\frac{1}{k}\left[\ell n(mg - kv)\right]_0^v = \frac{t}{m}$$

$$\Rightarrow \quad \ell n\left(\frac{mg - kv}{mg}\right) = \frac{-k}{m}t$$

$$\Rightarrow \quad 1 - \frac{kv}{mg} = e^{\frac{-kt}{m}}$$

$$\Rightarrow \quad \frac{kv}{mg} = 1 - e^{-\frac{kt}{m}}$$

$$\Rightarrow \quad v = \frac{mg}{k}\left(1 - e^{\frac{-kt}{m}}\right) \qquad ...(2)$$

Substituting (2) in (1), we get

$$ma = mg - k \times \frac{mg}{k}\left(1 - e^{-\frac{kt}{m}}\right)$$

$$\Rightarrow \quad a = ge^{\frac{-kt}{m}}$$

Hence, the correct answer is (C).

41. Bulk modulus, $B = \frac{\text{Normal stress}}{\text{Volumetric strain}}$

Pressure, $P = \frac{N}{A} = \frac{N}{(2\pi a)b}$

Volumetric strain $\frac{\Delta V}{V} = \frac{2\pi a\Delta a \times b}{\pi a^2 \times b} = \frac{2\Delta a}{a}$

$$\Rightarrow \quad B = \frac{N}{2\pi ab} \times \frac{a}{2\Delta a}$$

$$\Rightarrow \quad N = (4\pi b\Delta a)B$$

So, required force equals the frictional force

$$\Rightarrow \quad F_{\text{required}} = f = \mu N = (4\pi\mu Bb)\Delta a$$

Hence, the correct answer is (D).

42. Given that time $t = 5 \text{ min} = 300 \text{ s}$, volume, $V = 15 \text{ ltr} = 15 \times 10^{-3} \text{ m}^3$ and diameter, $D = \frac{2}{\sqrt{\pi}}$ cm

So, cross sectional area of tap is

$$A = \pi\left(\frac{1}{\sqrt{\pi}} \times 10^{-2}\right)^2 = 10^{-4} \text{m}^2$$

Velocity of water is

$$v=\frac{Q}{A}=\frac{V}{At}=\frac{15\times10^{-3}}{1\times10^{-4}\times300}=0.5\ \text{ms}^{-1}$$

$$\Rightarrow\quad R_w=\frac{\rho v D}{\eta}=\frac{10^3\times0.5\times\frac{2}{\sqrt{\pi}}\times10^{-2}}{10^{-3}}=5642\approx5500$$

Hence, the correct answer is (A).

43. For cylindrical shape, excess pressure is given by $\Delta P=\frac{T}{R}$

*None of the given options is correct.

44. For air trapped in the tube, we have

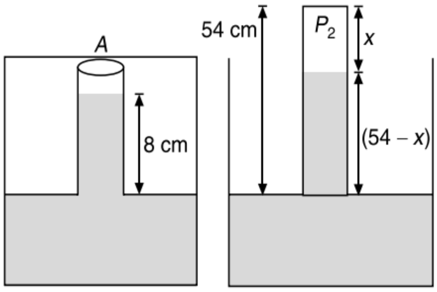
$$P_1V_1=P_2V_2$$

where, $P_1=P_{\text{atm}}=76(\rho_m g)$ and

$$P_2=P_{\text{atm}}-(54-x)\rho_m g$$

$$\Rightarrow\quad P_2=76\rho_m g-54\rho_m g+x\rho_m g=(22+x)\rho_m g$$

where, ρ_m =Density of mercury



Also, $V_1=8A$ and $V_2=Ax$

$$\Rightarrow\quad(76\rho_m g)(8A)=(22+x)\rho_m g Ax$$

$$\Rightarrow\quad 608=22x+x^2$$

$$\Rightarrow\quad x^2+22x-608=0$$

$$\Rightarrow\quad x^2+38x-16x-608=0$$

$$\Rightarrow\quad x(x+38)-16(x+38)=0$$

$$\Rightarrow\quad (x-16)(x+38)=0$$

$$\Rightarrow\quad x=16\ \text{cm}$$

Hence, the correct answer is (A).

45. $(R\sin\alpha)d_2+(R\cos\alpha)d_2+R(1-\cos\alpha)d_1=R(1-\sin\alpha)d_1$

$$\Rightarrow\quad(\sin\alpha+\cos\alpha)d_2=(\cos\alpha-\sin\alpha)d_1$$

$$\Rightarrow\quad\frac{d_1}{d_2}=\frac{\sin\alpha+\cos\alpha}{\cos\alpha-\sin\alpha}$$

$$\Rightarrow\quad\frac{d_1}{d_2}=\frac{1+\tan\alpha}{1-\tan\alpha}$$

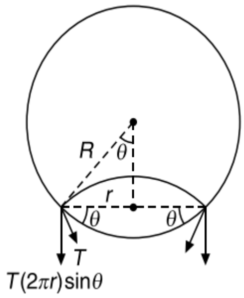
Hence, the correct answer is (C).

46. The bubble will detach when

(Buoyant Force) ≥ (Surface Tension Force)

$$\Rightarrow\quad\left(\frac{4}{3}\pi R^3\right)\rho_w g\ge T(2\pi r)\sin\theta$$

Since, $\sin\theta=\frac{r}{R}$



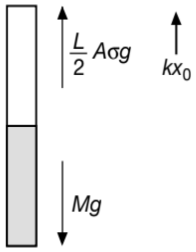
$$\Rightarrow\quad\left(\frac{4}{3}\pi R^3\right)\rho_w g\ge\frac{2\pi r^2 T}{R}$$

$$\Rightarrow\quad r^2=\frac{2R^4\rho_w g}{3T}$$

$$\Rightarrow\quad r=R^2\sqrt{\frac{2\rho_w g}{3T}}$$

Hence, the correct answer is (A).

47. For equilibrium, we have



$$Mg=\frac{L}{2}A\sigma g+kx_0$$

$$\Rightarrow\quad x_0=\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$$

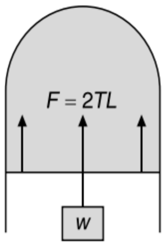
Hence, the correct answer is (C).

48. The force due to the surface tension will balance the weight, so

$$F=w$$

$$\Rightarrow\quad 2TL=w$$

$$\Rightarrow\quad T=\frac{w}{2L}$$



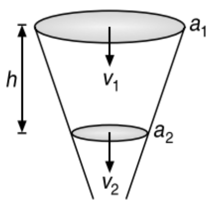
Substituting the given values, we get

$$T=\frac{1.5\times10^{-2}\ \text{N}}{2\times30\times10^{-2}\ \text{m}}=0.025\ \text{Nm}^{-1}$$

Hence, the correct answer is (C).

49. According to equation of motion,

$$v_2=\sqrt{v_1^2+2gh}=\sqrt{(0.4)^2+2\times10\times0.2}\approx2\ \text{ms}^{-1}$$



According to equation of continuity

$$a_1v_1=a_2v_2$$

$$\pi\times\left(\frac{8\times10^{-3}}{2}\right)^2\times0.4=\pi\times\left(\frac{d_2}{2}\right)^2\times2$$

$$d_2=3.6\times10^{-3}\ \text{m}$$

Hence, the correct answer is (D).

50. Here, surface tension, $T = 0.03\ \text{Nm}^{-1}$

$$r_1 = 3\ \text{cm} = 3\times10^{-2}\ \text{m},\ r_2 = 5\ \text{cm} = 5\times10^{-2}\ \text{m}$$

Since bubble has two surfaces, so, initial surface area of the bubble

$$A_i = 2\times4\pi r_1^2 = 2\times4\pi\times(3\times10^{-2})^2 = 72\pi\times10^{-4}\ \text{m}^2$$

Final surface area of the bubble is

$$A_f = 2\times4\pi r_2^2 = 2\times4\pi(5\times10^{-2})^2 = 200\pi\times10^{-4}\ \text{m}^2$$

Increase in surface is

$$\Delta A = 200\pi\times10^{-4} - 72\pi\times10^{-4} = 128\pi\times10^{-4}\ \text{m}^2$$

Since work done W is

$$W = T\Delta A$$

$$\Rightarrow\ W = 0.03\times128\times\pi\times10^{-4} = 3.84\pi\times10^{-4}$$

$$\Rightarrow\ W = 4\pi\times10^{-4}\ \text{J} = 0.4\pi\ \text{mJ}$$

Hence, the correct answer is (D).

51. Given that $U = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$

So, force $F = -\dfrac{dU}{dx} = -\dfrac{d}{dx}\left(\dfrac{a}{x^{12}} - \dfrac{b}{x^6}\right)$

$$\Rightarrow\ F = -\left[\frac{-12a}{x^{13}} + \frac{6b}{x^7}\right] = \left[\frac{12a}{x^{13}} - \frac{6b}{x^7}\right]$$

At equilibrium, $F = 0$

$$\Rightarrow\ \frac{12a}{x^{13}} - \frac{6b}{x^7} = 0$$

$$\Rightarrow\ x^6 = \frac{2a}{b}$$

$$U_{\text{at equilibrium}} = U_{eq} = \frac{a}{(2a/b)^2} - \frac{b}{(2a/b)}$$

$$\Rightarrow\ U_{eq} = \frac{ab^2}{4a^2} - \frac{b^2}{2a} = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$$

Since, $U_\infty = U|_{x\to\infty} = 0$

$$\Rightarrow\ D = [U_\infty - U_{eq}] = \left[0 - \left(-\frac{b^2}{4a}\right)\right] = \frac{b^2}{4a}$$

Hence, the correct answer is (D).

52. Since $\rho_{oil} < \rho_{water}$, so oil should be above the water. Also, $\rho > \rho_{oil}$, hence the ball will sink in the oil but $\rho < \rho_{water}$ so ball will float in the water.

Hence, the correct answer is (C).

53. For the same material, Young's modulus is the same and it is given that the volume is the same and the area of cross-section for the wire ℓ_1 is A and that of ℓ_2 is $3A$.

$$V = V_1 = V_2$$

$$\Rightarrow\ V = A\times\ell_1 = 3A\times\ell_2$$

$$\Rightarrow\ \ell_2 = \frac{\ell_1}{3}$$

$$Y = \frac{F/A}{\Delta\ell/\ell}$$

So, $F_1 = YA\dfrac{\Delta\ell_1}{\ell_1}$ and $F_2 = Y(3A)\dfrac{\Delta\ell_2}{\ell_2}$

For the same extension $\Delta\ell_1 = \Delta\ell_2 = \Delta x$

$$\Rightarrow\ F_2 = Y(3A)\frac{\Delta x}{\ell_1/3} = 9\left(\frac{YA\Delta x}{\ell_1}\right) = 9F_1$$

$$\Rightarrow\ F_2 = 9F$$

Hence, the correct answer is (D).

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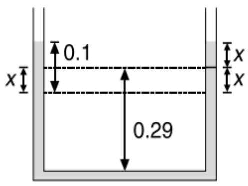
Single Correct Choice Type Problems

1. Let water level fall by x m due to oil, so we have

$$(800)(g)(0.1) = (1000)(g)(2x)$$

$$\Rightarrow\ x = \frac{8}{200} = \frac{1}{25} = 0.04\ \text{m}$$



So, $h_2 = 0.04 + 0.29 = 0.33$ m and

$$h_1 = 0.29 - 0.04 + 0.1 = 0.35$$

$$\Rightarrow\ \frac{h_1}{h_2} = \frac{35}{33}$$

Hence, the correct answer is (B).

2. Using geometry, we get

$$\frac{b}{R} = \cos\left(\theta + \frac{\alpha}{2}\right)$$

$$\Rightarrow\ R = \frac{b}{\cos\left(\theta + \dfrac{\alpha}{2}\right)}$$

Applying pressure equation along path $MNTK$

$$p_0 - \frac{2S}{R} + h\rho g = p_0$$

Substituting the value of R, we get

$$h = \frac{2S}{R\rho g} = \frac{2S}{b\rho g}\cos\left(\theta + \frac{\alpha}{2}\right)$$

Hence, the correct answer is (D).

3. Since, $\Delta l = \dfrac{FL}{AY} = \dfrac{FL}{(\pi r^2)Y}$

$$\Rightarrow\ \Delta l \propto \frac{L}{r^2}$$

$$\Rightarrow\ \frac{\Delta l_1}{\Delta l_2} = \frac{L/R^2}{2L/(2R)^2} = 2$$

Hence, the correct answer is (C).

4. Let V_1 be the total material volume of shell, V_2 be the total inside volume of shell and x be the fraction of V_2 volume filled with water.

For floating condition, we have

Total weight = Upthrust

$\Rightarrow \quad V_1\rho_c g+(xV_2)(1)g=\left(\frac{V_1+V_2}{2}\right)(1)g$

$\Rightarrow \quad x=0.5+(0.5-\rho_c)\frac{V_1}{V_2}$

From here we can see that

$x>0.5$ if $\rho_c<0.5$

Hence, the correct answer is (A).

5. Since, $\Delta p_1=\frac{4T}{r_1}$ and $\Delta p_2=\frac{4T}{r_2}$

$\Rightarrow \quad r_1<r_2$

$\Rightarrow \quad \Delta p_1>\Delta p_2$

So, air will flow from 1 to 2 and volume of bubble at end 1 will decrease.
Hence, the correct answer is (B).

6. Force from right hand side liquid on left hand side liquid
(i) due to surface tension is $F_1=2RT$, towards right
(ii) due to liquid pressure is

$$F_2=\int_{x=0}^{x=h}(p_0+\rho gh)(2Rx)dx$$

$\Rightarrow \quad F_2=(2p_0Rh+R\rho gh^2)$, towards left

So, net force is

$$|F_2-F_1|=|2p_0Rh+R\rho gh^2-2RT|$$

Hence, the correct answer is (B).

7. Applying continuity equation at 1 and 2, we get

$A_1v_1=A_2v_2 \quad \ldots(1)$

Further applying Bernoulli's equation at these two points, we get

$P_0+\rho gh+\frac{1}{2}\rho v_1^2=P_0+0+\frac{1}{2}\rho v_2^2 \quad \ldots(2)$

Solving equations (1) and (2), we have

$$v_2^2=\frac{2gh}{1-(A_2^2/A_1^2)}$$

Substituting the values, we have

$$v_2^2=\frac{2\times10\times2.475}{1-(0.1)^2}=50\text{ m}^2\text{s}^{-2}$$

Hence, the correct answer is (A).

8. From the definition of bulk modulus,

Since, $B=\left|\frac{dP}{dV/V}\right|$

Substituting the values, we get

$$B=\frac{(1.165-1.01)\times10^5}{(10/100)}\text{Pa}=1.55\times10^5\text{ Pa}$$

Hence, the correct answer is (D).

9. From the graph $\ell=10^{-4}$ m, $F=20$ N

$A=10^{-6}$ m^2, $L=1$ m

$\Rightarrow \quad Y=\frac{FL}{A\ell}$

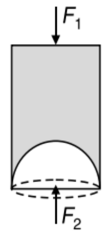
$\Rightarrow \quad Y=\frac{20\times1}{10^{-6}\times10^{-4}}=20\times10^{10}$

$\Rightarrow \quad Y=2\times10^{11}\text{ Nm}^{-2}$

Hence, the correct answer is (A).

10. As the block moves up with the fall of coin, ℓ decreases, similarly h will also decrease because when the coin is in water, it displaces water equal to its own volume only.
Hence, the correct answer is (D).

11. Net force F_2-F_1 equals the upthrust (U)



$\Rightarrow \quad F_2-F_1=U$

$\Rightarrow \quad F_2=F_1+U$

$\Rightarrow \quad F_2=\rho gh(\pi R^2)+V\rho g$

$\Rightarrow \quad F_2=\rho g(V+\pi R^2h)$

Hence, the correct answer is (D).

12. Velocity of efflux when the hole is at depth h, $v=\sqrt{2gh}$
Rate of flow of water from square hole

$Q_1=a_1v_1=L^2\sqrt{2gy}$

Rate of flow of water from circular hole

$Q_2=a_2v_2=\pi R^2\sqrt{2g(4y)}$

According to problem $Q_1=Q_2$

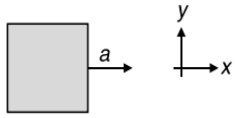
$\Rightarrow \quad L^2\sqrt{2gy}=\pi R^2\sqrt{2g(4y)}$

$\Rightarrow \quad R=\frac{L}{\sqrt{2\pi}}$

Hence, the correct answer is (A).

13. When a fluid (gas or liquid) is accelerated along positive x-direction, then pressure decreases along positive x-direction. Change in pressure has following differential equation.

$\frac{dP}{dx}=-\rho a$



where, ρ is the density of the fluid.
Therefore, pressure is lower in front side.
Hence, the correct answer is (B).

14. $a_1v_1=a_2v_2$

where $v_2^2=v_1^2+2gh$

$v_2^2=1+2\times10\times0.15$

$\Rightarrow \quad v_2=2\text{ ms}^{-1}$

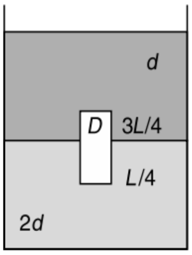
$\Rightarrow \quad 10^{-4}\times1=a_2\times2$

$\Rightarrow \quad a_2=5\times10^{-5}\text{ m}^{-2}$

Hence, the correct answer is (C).

15. Considering vertical equilibrium of cylinder, we get

$W = U_{total}$



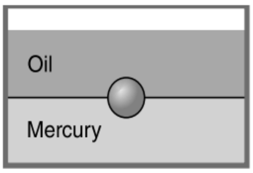
$\Rightarrow \left(\frac{A}{5}\right)(L)gD = \left(\frac{A}{5}\right)\left(\frac{3L}{4}\right)gd + \left(\frac{A}{5}\right)\left(\frac{L}{4}\right)g(2d)$

$\Rightarrow D = \left(\frac{3}{4}\right)d + \left(\frac{1}{4}\right)(2d)$

$\Rightarrow D = \frac{5}{4}d$

Hence, the correct answer is (A).

16. As the sphere floats in the liquid. Therefore, its weight will be equal to the upthrust force on it.



Weight of sphere is

$W = \frac{4}{3}\pi R^3 \rho g$ ...(1)

Upthrust due to oil and mercury is

$\frac{2}{3}\pi R^3 \times \sigma_{oil} g + \frac{2}{3}\pi R^3 \sigma_{Hg} g$ ...(2)

Equating (1) and (2)

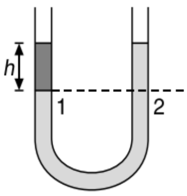
$\frac{4}{3}\pi R^3 \rho g = \frac{2}{3}\pi R^3 0.8 g + \frac{2}{3}\pi R^3 \times 13.6 g$

$\Rightarrow 2\rho = 0.8 + 13.6 = 14.4$

$\Rightarrow \rho = 7.2$

Hence, the correct answer is (C).

17. Since, $P_1 = P_2$



$\Rightarrow P_0 + \rho_I gh = P_0 + \rho_{II} gh$

$\Rightarrow \rho_I = \rho_{II}$

Hence, the correct answer is (B).

18. For a freely falling system $g_{eff} = 0$ and since, upthrust U is

$U = V_{imm}\rho_{liq} g_{eff}$

$\Rightarrow U = 0$

Hence, the correct answer is (A).

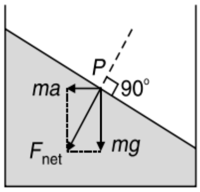
19. Since $\Delta\ell = \frac{F\ell}{AY} = \frac{F\ell}{(\pi d^2/4)Y}$

$\Rightarrow \Delta\ell \propto \frac{\ell}{d^2}$

$\frac{\ell}{d^2}$ is maximum in (A)

Hence, the correct answer is (A).

20. Net force on the free surface of the liquid in equilibrium (from accelerated frame) should be perpendicular to it. Forces on a water particle P on the free surfaces have been shown in the figure. In the figure ma is the pseudo force.



Hence, the correct answer is (C).

Multiple Correct Choice Type Problems

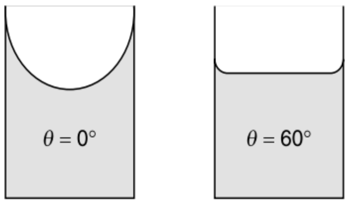
1. For A, we have $h = \frac{2T\cos(60°)}{\rho g r}$

$\Rightarrow h = \frac{2 \times 0.075 \times 1}{10^3 \times 10 \times 2 \times 10^{-4} \times 2} \times 100 \text{ cm} = 3.75 \text{ cm}$

For B, we have $h = \frac{2T\cos(0°)}{\rho g r} = 7.5 \text{ cm}$

For C, angle of contact will adjust so as to make $h = 5$ cm

For D, the shape of meniscus will be different in the two cases



So, correction is different for both cases.

Hence, (A), (B) and (D) are correct.

2. For floating, $W = U$

$\Rightarrow (\rho_1 + \rho_2)Vg = (\sigma_1 + \sigma_2)Vg$

Since string is taut, so, $\rho_1 < \sigma_1$ and $\rho_2 > \sigma_2$

$|\vec{v}_P| = \frac{2r^2 g}{2\eta_2}(\sigma_2 - \rho_1)$, upward terminal velocity

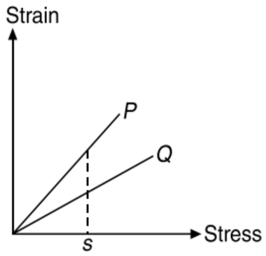
$|\vec{v}_Q| = \frac{2r^2 g}{9\eta_1}(\rho_2 - \sigma_1)$, downward terminal velocity

$\Rightarrow \left|\frac{\vec{v}_P}{\vec{v}_Q}\right| = \frac{\eta_1}{\eta_2}$

Further, $\vec{v}_P \cdot \vec{v}_Q$ will be negative as they are opposite to each other.

Hence, (A) and (D) are correct.

3. Since, $Y = \frac{\text{Stress}}{\text{Strain}}$



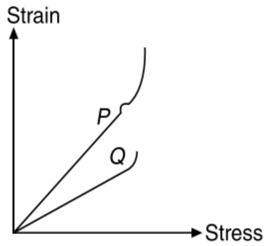
$\Rightarrow \quad Y \propto \frac{1}{\text{Strain}}$

For same stress, say σ

$(\text{Strain})_Q < (\text{Strain})_P$

$\Rightarrow \quad Y_Q > Y_P$

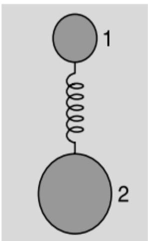
So, P is more ductile than Q. Further, from the given figure we can also see that breaking stress of P is more than Q. So, it has more tensile strength.



Hence, (A) and (B) are correct.

4. On small sphere i.e., sphere 1

$$\frac{4}{3}\pi R^3(\rho)g + kx = \frac{4}{3}\pi R^3(2\rho)g \quad \ldots(1)$$



On large sphere i.e., sphere 2

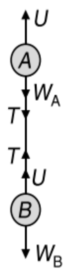
$$\frac{4}{3}\pi R^3(3\rho)g = \frac{4}{3}\pi R^3(2\rho)g + kx \quad \ldots(2)$$

Solving equations (1) and (2), we get

$$x = \frac{4\pi R^3 \rho g}{3k}$$

Hence, (A) and (D) are correct.

5. Upthrust $U = Vd_F g$



For equilibrium of A, $U = T + W_A$

$\Rightarrow \quad Vd_F g = T + W_A$

$\Rightarrow \quad Vd_F g = T + Vd_A g \quad \ldots(1)$

For equilibrium of B, $T + U = W_B$

$T + Vd_F g = Vd_B g \quad \ldots(2)$

Adding equations (1) and (2), we get

$2d_F = d_A + d_B$

From equation (1), we see that $d_F > d_A$ $\{\because T > 0\}$

From equation (2), we see that, $d_B > d_F$

Hence, (A), (B) and (D) are correct.

6. Liquid will apply an upthrust on m. An equal force will be exerted (from Newton's third law) on the liquid. Hence, A will read less than 2 kg and B more than 5 kg.

Hence, (B) and (C) are correct.

Reasoning Based Questions

1. From continuity equation, $Av = \text{constant}$

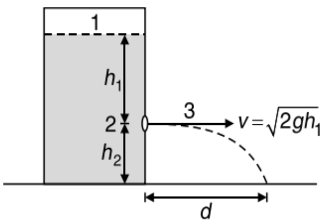
$\Rightarrow \quad A \propto \frac{1}{v}$

Hence, the correct answer is (A).

Matrix Match/Column Match Type Questions

1. A $\rightarrow$ (p); B $\rightarrow$ (p); C $\rightarrow$ (p); D $\rightarrow$ (s)

$d = 2\sqrt{h_1 h_2} = \sqrt{4h_1 h_2}$



This is independent of the value of g.

For A, $g_{\text{eff}} > g$ $d = \sqrt{4h_1h_2} = 1.2$ m

For B, $g_{\text{eff}} < g$ $d = \sqrt{4h_1h_2} = 1.2$ m

For C, $g_{\text{eff}} = g$ $d = \sqrt{4h_1h_2} = 1.2$ m

For D, $g_{\text{eff}} = 0$

No water leaks out of jar. As there will be no pressure difference between top of the container and any other point.

$p_1 = p_2 = p_3 = p_0$

Linked Comprehension Type Questions

1. From continuity equation, we have

$A_1 v_1 = A_2 v_2$

where, $A_1 = 400A_2$

Also, $r_1 = 20r_2$ and $A = \pi r^2$

$\Rightarrow \quad v_2 = \left(\frac{A_1}{A_2}\right)v_1 = 400v_1$

$\Rightarrow \quad v_2 = 400(5)\ \text{mms}^{-1} = 2000\ \text{mms}^{-1} = 2\ \text{ms}^{-1}$

Hence, the correct answer is (C).

2. According to Bernoulli equation, we have

$$p_1 - p_2 = \frac{1}{2}\rho_a v_a^2 \quad \ldots(1)$$

$$p_3 - p_2 = \frac{1}{2}\rho_l v_l^2 \quad \ldots(2)$$

Since, $p_3 = p_1$

$$\Rightarrow \frac{1}{2}\rho_l v_l^2 = \frac{1}{2}\rho_a v_a^2$$

$$\Rightarrow v_l = \sqrt{\frac{\rho_a}{\rho_l}}v_a$$

$$\Rightarrow \text{Volume flow rate} \propto \sqrt{\frac{\rho_a}{\rho_l}}$$

Hence, the correct answer is (A).

3. Vertical force due to surface tension

$$F_V = F\sin\theta = [(T)(2\pi r)]\left(\frac{r}{R}\right)$$

$$\Rightarrow F_V = \frac{2\pi r^2 T}{R}$$

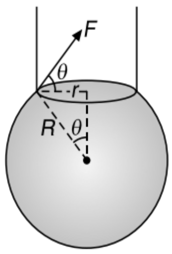
Hence, the correct answer is (C).

4. Since, $F_V = \dfrac{2\pi r^2 T}{R} = mg = \dfrac{4}{3}\pi R^3 \rho g$

$$\Rightarrow R^4 = \frac{3r^2T}{2\rho g} = \frac{3\times(5\times10^{-4})^2(0.11)}{2\times10^3\times10}$$

$$\Rightarrow R^4 = 4.125\times10^{-12}\ \text{m}^4$$

$$\Rightarrow R = 1.425\times10^{-3}\ \text{m} \approx 1.4\times10^{-3}\ \text{m}$$



Hence, the correct answer is (A).

5. Surface energy is $E = (4\pi R^2)T$

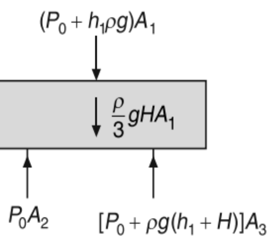
$$\Rightarrow E = (4\pi)(1.4\times10^{-3})^2(0.11)$$

$$\Rightarrow E = 2.7\times10^{-6}\ \text{J}$$

Hence, the correct answer is (B).

6. Let A_1 = Area of cross-section of cylinder = $4\pi r^2$

A_2 = Area of base of cylinder in air = πr^2



and A_3 = Area of base of cylinder in water = $A_1 - A_2 = 3\pi r^2$

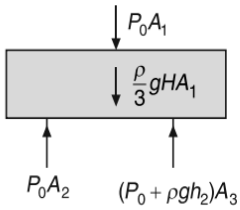
Drawing free body diagram of cylinder:

Equating the net downward forces and net upward forces, we get

$$h_1 = \frac{5}{3}H$$

Hence, the correct answer is (C).

7. Again, equating the forces, we get



Hence, the correct answer is (A).

8. For $h_2 < \dfrac{4h}{9}$, buoyant force will further decrease.

Hence, the cylinder remains at its original position.

No solution is required.

Hence, the correct answer is (A).

Integer/Numerical Answer Type Questions

1. Taking the velocity w.r.t. train, from equation of continuity, we get

$$4S_t v_t = (4S_t - S_t)v$$

$$\Rightarrow v = \frac{4}{3}v_t$$

According to Bernoulli's equation, we get

$$\Rightarrow p_0 + \frac{1}{2}\rho v_t^2 = p + \frac{1}{2}\rho\left(\frac{16}{9}v_t^2\right)$$

$$\Rightarrow p_0 - p = \frac{7\rho v_t^2}{18} = \frac{7}{2(9)}\rho v_t^2$$

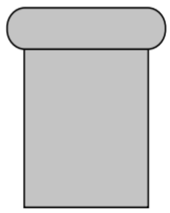
$$\Rightarrow N = 9$$

2. Water surface will break, when

$$P_0 + h\rho g \geq P_0 + \frac{T}{h/2}$$

$$\Rightarrow h^2 \geq \frac{2T}{\rho g}$$

$$\Rightarrow h \geq \sqrt{\frac{2T}{\rho g}}$$



3. Since $\sin\theta$, $T_1\sin(30°) = T_2\sin(60°)$

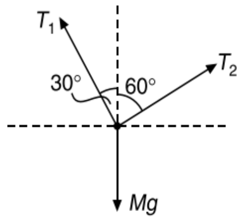
$$\Rightarrow \frac{T_1}{2} = \frac{T_2\sqrt{3}}{2}$$

$$\Rightarrow T_1 = \sqrt{3}T_2$$

Since $\Delta\ell = \dfrac{F\ell}{AY}$

$$\Rightarrow \frac{\Delta\ell_2}{\Delta\ell_1} = \left(\frac{T_2\ell_2}{A_2Y_2}\right)\left(\frac{A_1Y_1}{T_1\ell_1}\right)$$

$$\Rightarrow \frac{\Delta\ell_2}{\Delta\ell_1} = \frac{\sqrt{3}T_2}{T_1}\times\frac{2}{1} = 2$$



4. From mass conservation, we have

$$\frac{4}{3}\pi R^3\rho = K\frac{4}{3}\pi r^3\rho$$

$\Rightarrow \quad R = K^{\frac{1}{3}} r$

$\Rightarrow \quad \Delta U = T\Delta A = T\left(K4\pi r^2 - 4\pi R^2\right)$

$\Rightarrow \quad \Delta U = T\left(K4\pi R^2 K^{-\frac{2}{3}} - 4\pi R^2\right)$

$\Rightarrow \quad \Delta U = T\left[4\pi R^2 T\left(K^{\frac{1}{3}} - 1\right)\right]$

Putting the values, we get

$$10^{-3} = \frac{10^{-1}}{4\pi} \times 4\pi \times 10^{-4}\left[K^{\frac{1}{3}} - 1\right]$$

$\Rightarrow \quad 100 = K^{\frac{1}{3}} - 1$

$\Rightarrow \quad K^{\frac{1}{3}} \cong 100 = 10^2$

Given that $K = 10^{\alpha}$

$\Rightarrow \quad 10^{\frac{\alpha}{3}} = 10^2$

$\Rightarrow \quad \frac{\alpha}{3} = 2$

$\Rightarrow \quad \alpha = 6$

5. Terminal velocity is given by

$$v_T = \frac{2}{9}\frac{r^2}{\eta}(d - \rho)g$$

$\Rightarrow \quad \frac{v_P}{v_Q} = \frac{r_P^2}{r_Q^2} \times \frac{\eta_Q}{\eta_P} \times \frac{(d - \rho_P)}{(d - \rho_Q)}$

$\Rightarrow \quad \frac{v_P}{v_Q} = \left(\frac{1}{0.5}\right) \times \left(\frac{2}{3}\right) \times \frac{(8 - 0.8)}{(8 - 1.6)}$

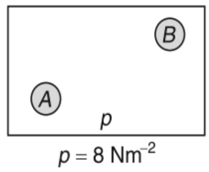
$\Rightarrow \quad \frac{v_P}{v_Q} = 4 \times \frac{2}{3} \times \frac{7.2}{6.4} = 3$

6. Although not given in the question, but we will have to assume that temperatures of A and B are same.

$$\frac{n_B}{n_A} = \frac{p_B V_B/RT}{p_A V_A/RT} = \frac{p_B V_B}{p_A V_A}$$

If S be the surface tension of bubble, then

$$p_A = p + \frac{45}{r_A},\ p_B = p + \frac{4S}{r}$$



$\Rightarrow \quad V_A = \frac{4}{3}\pi r_A^3,\ V_B = \frac{4}{3}\pi r_B^3$

$\Rightarrow \quad \frac{n_B}{n_A} = \frac{\left(p = \frac{4S}{r_B}\right)\left(\frac{4}{3}\pi r_B^3\right)}{\left(p + \frac{4S}{r_A}\right)\left(\frac{4}{3}\pi r_A^3\right)}$

Substituting the values, we get

$$\frac{n_B}{n_A} = 6$$

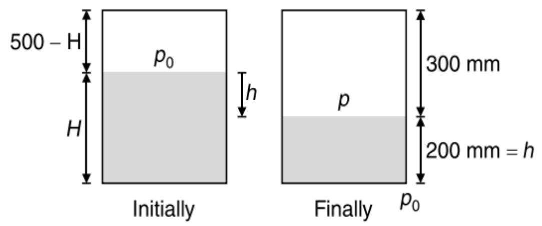
7. Assuming that the temperature of enclosed air is constant i.e., pV = constant

$\Rightarrow \quad p_0 V_0 = pV$

$p = p_0 - \rho g h \qquad \ldots(1)$

$p_0\left[A(500 - H)\right] = p\left[A(300)\right] \qquad \ldots(2)$

where, $p = p_0 - \rho g h$ and $h = H - 200$



Solving these two equations, we get $H = 206$ mm

So, level fall is

$$h = H - 200 = (206 - 200)\text{ mm} = 6\text{ mm}$$