CHAPTER 4: GRAVITATION AND SATELLITES

Test Your Concepts-I
(Based on Acceleration due to Gravity, Gravitational Field and Applications)

1. Escape velocity from the surface of moon is

$$v_e = \sqrt{\frac{2GM_m}{R_m}}$$

Substituting the values, we have

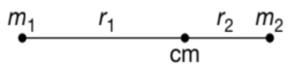
$$v_e = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.4 \times 10^{22}}{1.74 \times 10^6}}$$

$$\Rightarrow \quad v_e = 2.4 \times 10^3 \text{ ms}^{-1} \text{ or } 2.4 \text{ kms}^{-1}$$

2. Both the planet and the sun revolve around their centre of mass with same angular velocity (say ω)

$$r = r_1 + r_2 \qquad \ldots(1)$$

$$m_1 r_1 \omega^2 = m_2 r_2 \omega^2 = \frac{G m_1 m_2}{r^2} \qquad \ldots(2)$$



Solving these two equations, we get

$$r_1 = r\left(\frac{m_2}{m_1 + m_2}\right)$$

$$r_2 = r\left(\frac{m_1}{m_1 + m_2}\right)$$

and $\omega^2 = \dfrac{G(m_1 + m_2)}{r^3}$

Now, total energy of the system is

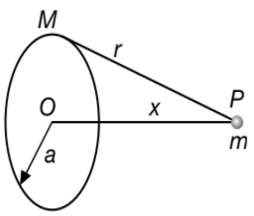
$$E = \text{P.E.} + \text{K.E.}$$

$$\Rightarrow \quad E = -\frac{G m_1 m_2}{r} + \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2$$

Substituting the values of r_1, r_2 and ω^2, we get

$$E = -\frac{G m_1 m_2}{2r}$$

3. (a) Gravitational potential at point P due to the ring.



$$V = -\frac{GM}{r} = -\frac{GM}{\sqrt{a^2 + x^2}}$$

$$\Rightarrow \quad U = mV = -\frac{GMm}{\sqrt{a^2 + x^2}}$$

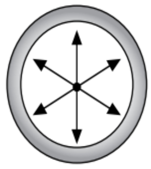
(b) When $x \quad a$, $a^2 + x^2 \approx x^2$

$$\Rightarrow \quad U = -\frac{GMm}{x}$$

{the potential energy of two point masses}

(c) $F_x = -\dfrac{dU}{dx} = -GMm \cdot \dfrac{d}{dx}\left(a^2 + x^2\right)^{-1/2}$

$$\Rightarrow \quad F_x = -\frac{GMm \cdot x}{\left(x^2 + a^2\right)^{3/2}}$$



(d) When $x \quad a$, $a^2 + x^2 \approx x^2$

and $F_x = -\dfrac{GMm}{x^2}$

{force between two point masses}

(e) At $x = 0$ {centre of the ring}

$F_x = 0$

As the particle is attracted equally from all the four sides.

4. Net force on M due to the pairs $2M$ and $2M$, $4M$ and $4M$, $5M$ and $5M$, $7M$ and $7M$, M and M is zero. So, the only left out mass is $3M$ and net force on M is due to $3M$, given by

$$F = \frac{G(3M)M}{d^2}$$

$$\Rightarrow \quad F = \frac{3GM^2}{d^2}, \text{ along } +x \text{ axis}$$

5. Given that $F = \dfrac{G m_A m_B}{r^2}$

Since acceleration of A is $a_A = \dfrac{F}{m_A}$

$$\Rightarrow \quad a = a_A = \frac{F}{m_A} = \frac{G m_B}{r^2} \qquad \ldots(1)$$

Now if, $F = \dfrac{G m_A m_B}{r^4}$, then

$$a'_A = \frac{F}{m_A} = \frac{G m_B}{r^4}$$

$$\Rightarrow \quad a'_A = \frac{a}{r^2} \qquad \{\because \text{ of } (1)\}$$

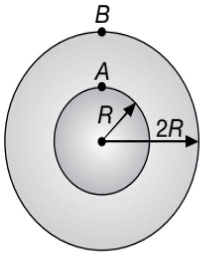
6. Since, $E=\dfrac{F}{m_0}$

$$\Rightarrow \quad E=\frac{4N}{20\times10^{-3}\text{ kg}}$$

$$\Rightarrow \quad E=200\text{ Nkg}^{-1}\text{, along }+x\text{ direction.}$$

7. Let m_1 be the mass of the core and m_2 the mass of outer shell, then

$$g_A=g_B$$



$$\Rightarrow \quad \frac{Gm_1}{R^2}=\frac{G(m_1+m_2)}{(2R)^2}$$

$$\Rightarrow \quad 4m_1=(m_1+m_2)$$

$$\Rightarrow \quad 4\left(\frac{4}{3}\pi R^3\rho_1\right)=\left(\frac{4}{3}\pi R^3\right)\rho_1+\left(\frac{4}{3}\pi(2R)^3-\frac{4}{3}\pi R^3\right)\rho_2$$

$$\Rightarrow \quad 4\rho_1=\rho_1+7\rho_2$$

$$\Rightarrow \quad \frac{\rho_1}{\rho_2}=\frac{7}{3}$$

8. $L=\pi R$

$$\Rightarrow \quad R=\frac{L}{\pi}$$

Gravitational field at the centre of a semicircular wire of radius R is

$$E=\frac{2G\lambda}{R}$$

where $\lambda=\dfrac{M}{L}$

$$\Rightarrow \quad E=\frac{2G\left(\frac{M}{L}\right)}{\left(\frac{L}{\pi}\right)}$$

$$\Rightarrow \quad E=\frac{2\pi GM}{L^2}$$

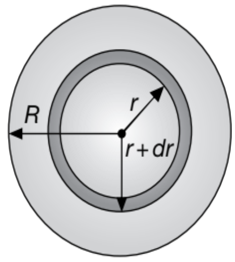
Since $F=mE$ $\quad\left\{\because E=\dfrac{F}{m}\right\}$

$$\Rightarrow \quad F=\frac{2\pi GMm}{L^2}$$

9. Since $r(=2R)$ lies outside the sphere, so

$$E=\frac{GM_{\text{total}}}{r^2}\text{ for }r\geq R$$

where M_{total} is the total mass of the sphere.



To calculate M_{total}, we consider an infinitesimal shell element of inner radius r and outer radius $r+dr$. If dm be the mass of this infinitesimal element, then

$$dm=(4\pi r^2dr)\rho$$

But, $\rho=\dfrac{\rho_0R}{r}$

$$\Rightarrow \quad dm=\left(\frac{\rho_0R}{r}\right)(4\pi r^2dr)$$

$$\Rightarrow \quad dm=(4\pi\rho_0R)(rdr)$$

$$\Rightarrow \quad M_{\text{total}}=\int dm=4\pi\rho_0R\int_0^R rdr$$

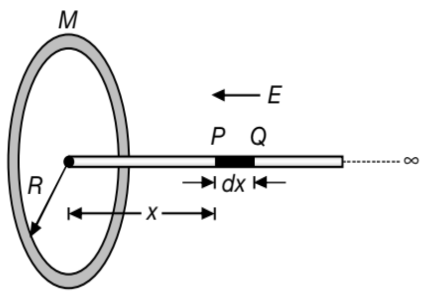
$$\Rightarrow \quad M_{\text{total}}=\frac{4\pi\rho_0R^3}{2}=2\pi\rho_0R^3$$

$$\Rightarrow \quad E(r)=\frac{GM}{r^2}$$

$$\Rightarrow \quad E=\frac{G(2\pi\rho_0R^3)}{4R^2}=\frac{\pi RG\rho_0}{2}$$

10. Field strength at the axis at distance x from the centre of the ring is,

$$E=\frac{GMx}{(R^2+x^2)^{3/2}}$$



Consider an infinitesimal mass element PQ of length dx, mass dm at a distance x from centre of ring. Then

$$dm=\lambda dx$$

Force on this element will be,

$$dF=Edm=\frac{GM\lambda x}{(R^2+x^2)^{3/2}}dx$$

$$\Rightarrow \quad F=\int_0^\infty dF=\int_0^\infty\frac{GM\lambda xdx}{(R^2+x^2)^{3/2}}$$

Since $\int \frac{dx}{\sqrt{R^2+x^2}} = -\frac{1}{\sqrt{R^2+x^2}}$

$$\Rightarrow \quad F = -GM\lambda\left(\frac{1}{\sqrt{R^2+x^2}}\bigg|_0^{\infty}\right)$$

$$\Rightarrow \quad F = -GM\lambda\left(\frac{1}{\infty}-\frac{1}{R}\right)$$

$$\Rightarrow \quad F = \frac{GM\lambda}{R}$$

11. At the equator, we have

$$g_e = g - R\omega^2$$

60% of his weight at the pole means

$$g_e = \frac{60}{100}g = \frac{3}{5}g$$

$$\Rightarrow \quad \frac{3}{5}g = g - R\omega^2$$

$$\Rightarrow \quad \frac{2}{5}g = R\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2}{5}\frac{g}{R}}$$

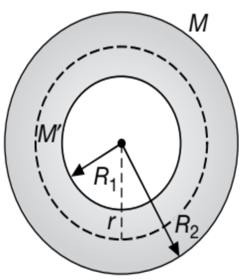
$$\Rightarrow \quad \omega = \sqrt{\frac{2\times 9.8}{5\times 6.4\times 10^6}}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{19.6}{32\times 10^6}}$$

$$\Rightarrow \quad \omega = 7.8\times 10^{-4}\ \text{rads}^{-1}$$

12. For $r < R_1$, we have $E_r = 0$

For $r > R_2$, we have, $E_r = \frac{GM}{r^2}$



Inside i.e., for $R_1 < r < R_2$, we have

$$E_r = \frac{GM'}{r^2}$$

where M' is mass of shell from R_1 to r. So,

$$M' = \frac{4}{3}\pi\left(r^3 - R_1^3\right)\rho$$

where $\rho = \dfrac{M}{\frac{4}{3}\pi\left(R_2^3 - R_1^3\right)}$ …(1)

$$\Rightarrow \quad E_r = \frac{G\left[\frac{4}{3}\pi\left(r^3-R_1^3\right)\rho\right]}{r^2}$$

From (1), we get

$$E_r = \frac{GM}{r^2}\left(\frac{r^3-R_1^3}{R_2^3-R_1^3}\right)$$

$$\text{So, } E_r = \begin{cases} 0 & \text{for } r < R_1 \\ \dfrac{GM}{r^2}\left(\dfrac{r^3-R_1^3}{R_2^3-R_1^3}\right) & \text{for } R_1 < r < R_2 \\ \dfrac{GM}{r^2} & \text{for } r > R_2 \end{cases}$$

13. Reduced by 36% means the value is 64% the original

$$\Rightarrow \quad g_h = \frac{64}{100}g$$

Since $g_h = \frac{gR^2}{(R+h)^2}$

$$\Rightarrow \quad \frac{64}{100} = \left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \quad \frac{R}{R+h} = \frac{8}{10}$$

$$\Rightarrow \quad 10R = 8R + 8h$$

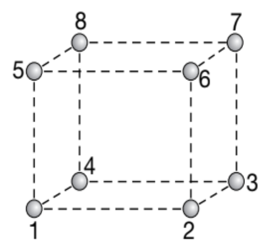
$$\Rightarrow \quad h = \frac{R}{4} = 1600\ \text{km}$$

Test Your Concepts-II (Based on Gravitational Potential, Potential Energy and Applications)

1. First of all let us calculate the total number of interactions between these eight particles. Since, total number of interactions (N) between n particles is

$$N = {}^nC_2 = \frac{n(n-1)}{2}$$

$$\Rightarrow \quad N = \frac{8(8-1)}{2} = 28$$



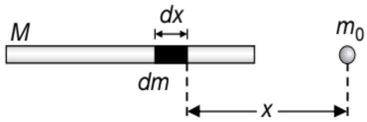
Out of these 28 interactions, 12 interactions are due to masses at separation a, 12 interactions are due to masses at separation $\sqrt{2}a$ (the face diagonal) and 4 interactions are due to masses at separation $\sqrt{3}a$ (the body diagonal). So,

$$U = -12\left(\frac{Gm^2}{a}\right) - 12\left(\frac{Gm^2}{\sqrt{2}a}\right) - 4\left(\frac{Gm^2}{\sqrt{3}a}\right)$$

$$\Rightarrow \quad U = -\frac{Gm^2}{a}\left(12 + \frac{12}{\sqrt{2}} + \frac{4}{\sqrt{3}}\right)$$

2. Consider an infinitesimal element of the rod of length dx at a distance x from m_0. If dm be the mass of the infinitesimal element, then

$$dm = \frac{M}{L}dx$$



The gravitational potential energy of the infinitesimal mass dm and the point mass m_0 is

$$dU = -\frac{Gm_0 dm}{x} = -\frac{Gm_0\left(\frac{M}{L}dx\right)}{x}$$

$$\Rightarrow \quad U = \int dU = -\frac{GMm_0}{L}\int_a^{a+L}\frac{dx}{x}$$

$$\Rightarrow \quad U = -\frac{GMm_0}{L}\log_e\left(\frac{a+L}{a}\right)$$

3. $U = -\dfrac{GMm}{R}$

So, the binding energy is $|U| = \dfrac{GMm}{R}$

i.e., this much energy is required to displace the particle from the centre of the ring to infinity.

4. Outside the sphere, we have

$$U = -\frac{GMm}{r}$$

as if the sphere were a point mass concentrated at its centre.

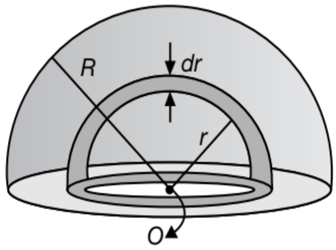
Even if the sphere is replaced by a thin shell, the gravitational potential energy of the particle-shell system will remain the same, because for the shell to the entire mass is concentrated at the centre, so

$$U = -\frac{GMm}{r}$$

5. Consider an elementary hemispherical shell of radius r and thickness dr. If dm is mass of this shell, then

$$dm = \frac{M}{\frac{1}{2}\left(\frac{4}{3}\pi R^3\right)}\frac{1}{2}4\pi r^2 dr$$

$$\Rightarrow \quad dm = \frac{3M}{R^3}r^2 dr$$



Initial potential energy at O due to this element and m is

$$U_i = \int dU_i = -\int\frac{Gm\,dm}{r} = -\frac{3GMm}{R^3}\int_0^R r\,dr$$

$$\Rightarrow \quad U_i = -\frac{3}{2}\frac{GMm}{R}$$

So, $W_{\text{external}} = U_f - U_i = 0 - \left(-\dfrac{3}{2}\dfrac{GMm}{R}\right) = \dfrac{3}{2}\dfrac{GMm}{R}$

So, the work performed in the process by gravitational force is $\left(-\dfrac{3}{2}\dfrac{GMm}{R}\right)$

6. Let gravitational field be zero at a point lying at distance x from M. Then,

$$\frac{GM}{x^2} = \frac{Gm}{(d-x)^2}$$

$$\Rightarrow \quad \frac{d-x}{x} = \sqrt{\frac{m}{M}}$$

$$\Rightarrow \quad \frac{d}{x} - 1 = \sqrt{\frac{m}{M}}$$

$$\Rightarrow \quad x = \left(\frac{\sqrt{M}}{\sqrt{M}+\sqrt{m}}\right)d \qquad \ldots(1)$$

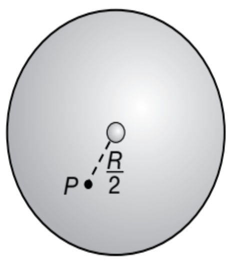
$$\Rightarrow \quad (d-x) = \left(\frac{\sqrt{m}}{\sqrt{M}+\sqrt{m}}\right)d \qquad \ldots(2)$$

Since, $V_P = -\dfrac{Gm}{d-x} - \dfrac{GM}{x}$ $\qquad \ldots(3)$

Substituting (1) and (2) in (3), we get

$$V_P = -\frac{G}{d}\left(\sqrt{m}+\sqrt{M}\right)^2$$

7. $V_P = \begin{pmatrix}\text{Potential due to} \\ \text{shell at P}\end{pmatrix} + \begin{pmatrix}\text{Potential due to} \\ \text{particle at P}\end{pmatrix}$



Since point P lies inside the shell, so the potential inside the shell is constant and equals the value at the surface i.e., $-\frac{GM}{R}$.

$$\Rightarrow \quad V_P = -\frac{GM}{R} - \frac{GM}{(R/2)}$$

$$\Rightarrow \quad V_P = -\frac{3M}{R}$$

8. (a) Since, $V(m) = -\frac{Gm}{R}$

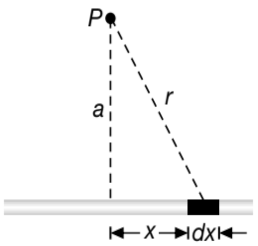
$$\Rightarrow \quad dW = V(dm) = -\left(\frac{Gm}{R}\right)dm$$

$$\Rightarrow \quad W = \int_0^M dW = -\frac{GM^2}{2R} = \text{self energy}$$

(b) See theory.

9. (a) Consider a mass element of length dx, mass dm at a distance x from the centre of rod. Then

$dm = \lambda dx$



The potential at the point P due to this infinitesimal element is

$$dV = -\frac{Gdm}{r} = -\frac{Gdm}{\sqrt{a^2+x^2}}$$

$$\Rightarrow \quad dV = -\frac{G\lambda dx}{\sqrt{a^2+x^2}}$$

$$\Rightarrow \quad V = \int dV = -G\lambda \int_{-\ell}^{\ell} \frac{dx}{\sqrt{a^2+x^2}}$$

Since $\int_{-\ell}^{\ell} f(x)dx = 2\int_0^{\ell} f(x)dx$, iff $f(x)$ is even

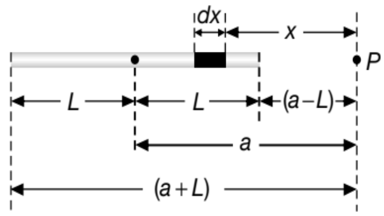
$$\Rightarrow \quad V = -2G\lambda \int_0^{\ell} \frac{dx}{\sqrt{a^2+x^2}}$$

Also, $\int \frac{dx}{\sqrt{a^2+x^2}} = \log_e\left(x+\sqrt{a^2+x^2}\right)$

$$V = -2G\lambda\left[\log_e\left(x+\sqrt{x^2+a^2}\right)\Big|_0^{\ell}\right]$$

$$\Rightarrow \quad V = -2G\lambda \log_e\left(\frac{\ell+\sqrt{\ell^2+a^2}}{a}\right)$$

(b) Again consider an infinitesimal element of length dx, mass dm at a distance x from P. Then $dm = \lambda dx$



If dV be the gravitational potential due to this element at the point P, then

$$dV = -\frac{Gdm}{x}$$

$$\Rightarrow \quad dV = -\frac{G(\lambda dx)}{x}$$

$$\Rightarrow \quad V = -G\lambda \int_{a-L}^{a+L} \frac{dx}{x}$$

$$\Rightarrow \quad V = -G\lambda\left(\log_e x\Big|_{a-L}^{a+L}\right)$$

$$\Rightarrow \quad V = -G\lambda \log_e\left(\frac{a+L}{a-L}\right)$$

Test Your Concepts-III (Based on Relation between Gravitational Field and Potential)

1. Since, $dV = -\vec{E}\cdot d\vec{\ell}$

$$\Rightarrow \quad \int_B^A dV = -\int_{(0,2,4)}^{(2,1,0)} \left(x\hat{i} - 2y\hat{j} + z\hat{k}\right)\cdot\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$\Rightarrow \quad V_A - V_B = -\int_{(0,2,4)}^{(2,1,0)} (xdx - 2ydy + zdz)$$

$$\Rightarrow \quad V_{AB} = -\left[\left(\frac{x^2}{2} - y^2 + \frac{z^2}{2}\right)\Bigg|_{(0,2,4)}^{(2,1,0)}\right]$$

$$\Rightarrow \quad V_{AB} = 3\ \text{Jkg}^{-1}$$

2. Since, $dV = -\vec{E}\cdot d\vec{\ell}$

$$\Rightarrow \quad dV = -a(ydx + axdy) + b(zdy + ydz)$$

$$\Rightarrow \quad dV = -\left[ad(xy) + bd(yz)\right]$$

Integrating we get,

$$V = -(axy + byz) + C$$

3. (a) $dV = -\vec{E}\cdot d\vec{\ell}$

$$\int_B^A dV = -\int_{(1,1,1)}^{(0,0,0)} \left(y\hat{i} + x\hat{j}\right)\cdot\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$\Rightarrow \quad V_A - V_B = -\int_{(1,1,1)}^{(0,0,0)} \left(ydx + xdy\right)$$

$$\Rightarrow \quad V_{AB} = -\int_{(1,1,1)}^{(0,0,0)} d(xy) \quad \{\text{as } y\,dx + xdy = d(xy)\}$$

$$\Rightarrow \quad V_{AB} = -\left[\left(xy\right)\Big|_{(1,1,1)}^{(0,0,0)}\right] = 1\ \text{Jkg}^{-1}$$

(b) $dV = -\vec{E}\cdot d\vec{\ell}$

$$\Rightarrow \quad \int_B^A dV = -\int_{(1,1,1)}^{(0,0,0)} \vec{E}\cdot d\vec{\ell}$$

$$\Rightarrow \quad V_A - V_B = -\int_{(1,1,1)}^{(0,0,0)} \left(3x^2 y dx + x^3 dy\right)$$

$$\Rightarrow \quad V_A - V_B = -\int_{(1,1,1)}^{(0,0,0)} d\left(x^3 y\right)$$

$$\Rightarrow \quad V_{AB} = -\left[\left(x^3 y\right)\Big|_{(1,1,1)}^{(0,0,0)}\right] = 1\ \text{Jkg}^{-1}$$

Since in both the cases, the line integral of the field is an exact differential and hence both the fields are conservative in nature.

4. Since $\vec{E} = -\left(\hat{i}\dfrac{\partial V}{\partial x} + \hat{j}\dfrac{\partial V}{\partial y} + \hat{k}\dfrac{\partial V}{\partial z}\right)$

where $\dfrac{\partial V}{\partial x} = 6xy$

$\dfrac{\partial V}{\partial y} = 3x^2 + 3y^2 z$

$\dfrac{\partial V}{\partial z} = y^3$

$$\Rightarrow \quad \vec{E} = -(6xy)\hat{i} - 3\left(x^2 + y^2 z\right)\hat{j} - y^3\hat{k}$$

5. (a) Given, $V = a\left(x^2 - y^2\right)$

So, $\vec{E} = -\vec{\nabla}V = -2a\left(x\hat{i} - y\hat{j}\right)$

(b) Since $V = axy$

So, $\vec{E} = -\vec{\nabla}V = -ay\hat{i} - ax\hat{j}$

6. $V = 5x - 3x^2 y + 2yz^2$

Since $\vec{E} = -\vec{\nabla}V = -\left(\dfrac{\partial V}{\partial x}\hat{i} + \dfrac{\partial V}{\partial y}\hat{j} + \dfrac{\partial V}{\partial z}\hat{k}\right)$

$$\Rightarrow \quad \vec{E} = \left(-5 + 6xy\right)\hat{i} + \left(3x^2 - 2z^2\right)\hat{j} - 4yz\hat{k}$$

$$\Rightarrow \quad \vec{E}\Big|_{(1,\,0,\,-2)} = -5\hat{i} - 5\hat{j}$$

$$\Rightarrow \quad E = \sqrt{E_x^2 + E_y^2 + E_z^2}$$

$$E = \sqrt{(-5)^2 + (-5)^2 + 0^2} = 5\sqrt{2}\ \text{Nkg}^{-1}$$

7. Since, $\vec{E}_g = -\left(\hat{i}\dfrac{\partial V}{\partial x} + \hat{j}\dfrac{\partial V}{\partial y}\right)$

where $\dfrac{\partial V}{\partial x} = 20$

and $\dfrac{\partial V}{\partial y} = 20$

$$\Rightarrow \quad \vec{E}_g = -20\left(\hat{i} + \hat{j}\right)$$

Since, $\vec{F} = m\vec{E}_g = -(0.5)(20)\left(\hat{i} + \hat{j}\right)$

$$\Rightarrow \quad \vec{F} = -10\left(\hat{i} + \hat{j}\right)$$

$$\Rightarrow \quad \left|\vec{F}\right| = 10\sqrt{2}\ \text{N}$$

8. Given, $V = a\left(x^2 + y^2\right) + bz^2$

Since, $\vec{E} = -\vec{\nabla}V$

$$\Rightarrow \quad \vec{E} = -\left(2ax\hat{i} + 2ay\hat{j} + 2bz\hat{k}\right)$$

Hence $\left|\vec{E}\right| = 2\sqrt{a^2\left(x^2 + y^2\right) + b^2 z^2}$

9. $\vec{E} = -\left(\dfrac{\partial V}{\partial x}\hat{i} + \dfrac{\partial V}{\partial y}\hat{j} + \dfrac{\partial V}{\partial z}\hat{k}\right)$

where, $\dfrac{\partial V}{\partial x} = \dfrac{\partial}{\partial x}\left(2x + 3y - z\right) = 2$

$\dfrac{\partial V}{\partial y} = \dfrac{\partial}{\partial y}\left(2x + 3y - z\right) = 3$

$\dfrac{\partial V}{\partial z} = \dfrac{\partial}{\partial z}\left(2x + 3y - z\right) = -1$

$$\Rightarrow \quad \vec{E} = -2\hat{i} - 3\hat{j} + \hat{k}$$

Test Your Concepts-IV (Based on Conservation Laws, Escape Velocity and Applications)

1. (a) Increase in potential energy is $\Delta U = U_f - U_i$

$$\Rightarrow \quad \Delta U = -\frac{GmM}{R+nR} - \left(-\frac{GmM}{R}\right)$$

$$\Rightarrow \quad \Delta U = \frac{GmM}{R}\left(1-\frac{1}{1+n}\right)$$

$$\Rightarrow \quad \Delta U = \left(\frac{n}{n+1}\right)\left(\frac{GM}{R^2}\right)mR$$

Since $g = \dfrac{GM}{R^2}$

$$\Rightarrow \quad \Delta U = \left(\frac{n}{n+1}\right)mgR$$

(b) By Law of Conservation of Energy,

$$\begin{pmatrix}\text{Gain in GPE}\\ \text{of } m\end{pmatrix} = \begin{pmatrix}\text{Loss in KE}\\ \text{of } m\end{pmatrix}$$

$$\Rightarrow \quad \left(\frac{n}{n+1}\right)mgR = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2ngR}{n+1}}$$

2. $(U+K)_{\text{at }\infty} = (U+K)_{\text{surface}}$

$$0+0 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2GM}{R}}$$

$$\Rightarrow \quad v = \sqrt{2\left(\frac{GM}{R^2}\right)R}$$

$$\Rightarrow \quad v = \sqrt{2gR} = 11.2 \text{ kms}^{-1}$$

3. By Law of Conservation of Energy, we have

$$(U+K)_{\text{at 1 m}} = (U+K)_{\text{at 0.5 m}}$$

$$-\frac{Gm_1m_2}{r_1} = -\frac{Gm_1m_2}{r_2} + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \ldots(1)$$

Further by Law of Conservation of Linear Momentum, we have

$$m_1(0) + m_2(0) = m_1v_1 + m_2(-v_2)$$

$$\Rightarrow \quad m_1v_1 = m_2v_2 \quad \ldots(2)$$

$$\Rightarrow \quad Gm_1m_2\left(\frac{1}{r_2}-\frac{1}{r_1}\right) = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2\left(\frac{m_1v_1}{m_2}\right)^2$$

$$\Rightarrow \quad Gm_1m_2\left(\frac{1}{r_2}-\frac{1}{r_1}\right) = \frac{1}{2}m_1v_1^2\left(1+\frac{m_1}{m_2}\right)$$

$$\Rightarrow \quad v_1 = \sqrt{\frac{2Gm_2^2}{m_1+m_2}\left(\frac{1}{r_2}-\frac{1}{r_1}\right)}$$

$$\Rightarrow \quad v_1 = \sqrt{\frac{2G(100)}{30}\left(\frac{1}{0.5}-\frac{1}{1}\right)}$$

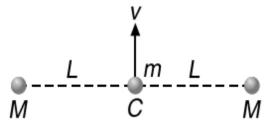
$$\Rightarrow \quad v_1 = \sqrt{\frac{20G}{3}}$$

$$\Rightarrow \quad v_1 = 2.1 \times 10^{-5} \text{ ms}^{-1}$$

From (2), we get

$$v_2 = 4.2 \times 10^{-5} \text{ ms}^{-1}$$

4. Let v is the minimum velocity, then by Law of Conservation of Energy, we have



$$(U+K)_C = (U+K)_\infty$$

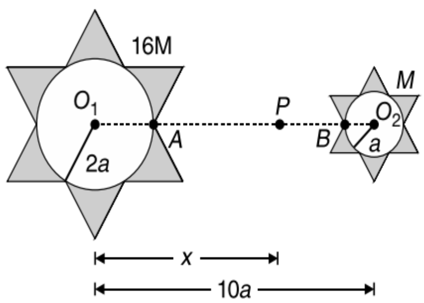
$$\Rightarrow \quad \left(-\frac{GMm}{L}\right)2 + \frac{1}{2}mv^2 = 0+0$$

$$\Rightarrow \quad v = 2\sqrt{\frac{GM}{L}}$$

5. Our first job is to find a point where the resultant field due to both is zero. Let the point P be at a distance x from centre of bigger star.

$$\Rightarrow \quad \frac{G(16M)}{x^2} = \frac{GM}{(10a-x)^2}$$

$$\Rightarrow \quad x = 8a \qquad \{\text{from } O_1\}$$



i.e. once the body reaches P the gravitational pull of attraction due to $16M$ vanishes and the gravitation pull due to M takes the lead to make m move towards it automatically.

i.e. a minimum K.E. or velocity has to be imparted to m from surface of $16M$ such that it is just able to overcome the gravitational pull of $16M$.

By Law of Conservation of Energy.

$$\begin{pmatrix}\text{Total Mechanical}\\ \text{Energy at } A\end{pmatrix} = \begin{pmatrix}\text{Total Mechanical}\\ \text{Energy at } P\end{pmatrix}$$

Total mechanical energy at A is

$$E_A = \frac{1}{2}mv_{\text{min}}^2 + \left[-\frac{G(16M)m}{2a} - \frac{GMm}{8a}\right]$$

$$\Rightarrow \quad E_A = \frac{1}{2}mv_{\text{min}}^2 - \frac{GMm}{2a}\left(16 + \frac{1}{4}\right)$$

$$\Rightarrow \quad E_A = \frac{1}{2}mv_{\text{min}}^2 - \frac{65GMm}{8a}$$

Total mechanical energy at B is

$$E_B = \left(-\frac{GMm}{2a} - \frac{G(16M)m}{8a}\right)$$

$$\Rightarrow \quad E_B = -\frac{GMm}{2a}(1+4) = -\frac{5GMm}{2a}$$

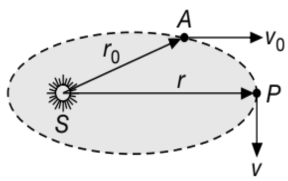
$$\Rightarrow \quad \frac{1}{2}mv_{\text{min}}^2 = \frac{GMm}{8a}(45)$$

$$\Rightarrow \quad v_{\text{min}} = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

6. At minimum and maximum distances, the velocity vector $(\vec{v})$ makes an angle of 90° with radius vector. Then, by Law of Conservation of Angular Momentum, we have

$$mv_0 r_0 \sin\phi = mvr \qquad \ldots(1)$$

where, m is the mass of the planet.



Further by Law of Conservation of Energy, we have

$$(U+K)_{\text{at } A} = (U+K)_P$$

$$\Rightarrow \quad \frac{mv_0^2}{2} - \frac{GMm}{r_0} = \frac{mv^2}{2} - \frac{GMm}{r} \qquad \ldots(2)$$

where, M is the mass of the sun.

Solving equations (1) and (2) for r using the concept of quadratic equations we get two values of r, one is r_{max} and another is r_{min}. So,

$$r_{\text{max}} = \frac{r_0}{2-K}\left(1 + \sqrt{1 - K(2-K)\sin^2\phi}\right)$$

and $$r_{\text{min}} = \frac{r_0}{2-K}\left(1 - \sqrt{1 - K(2-K)\sin^2\phi}\right)$$

where, $K = \dfrac{v_0^2 r_0^2}{GM}$

7. $E = U + K$

$$\Rightarrow \quad E = -\frac{GMm}{R} + \frac{1}{2}mv_e^2$$

But $v_e^2 = \dfrac{2GM}{R}$

$$\Rightarrow \quad E = -\frac{GMm}{R} + \frac{1}{2}m\left(\frac{2GM}{R}\right)$$

$$\Rightarrow \quad E = 0$$

8. Given that $v_e = 11.2 \text{ kms}^{-1} = \sqrt{\dfrac{2GM_e}{R_e}}$

By Law of Conservation of Energy, we have

$$K_i + U_i = K_f + U_f$$

$$\Rightarrow \quad \frac{1}{2}mv_s^2 - \frac{GM_s m}{r} - \frac{GM_e m}{R_e} = 0 + 0$$

where, r is the distance of rocket from sun

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2GM_s}{r}}$$

Since, $M_s = 3 \times 10^5 M_e$ and $r = 2.5 \times 10^4 R_e$

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2G(3\times 10^5 M_e)}{2.5\times 10^4 R_e}}$$

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e}\left(1 + \frac{3\times 10^5}{2.5\times 10^4}\right)}$$

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e} \times 13}$$

$$\Rightarrow \quad v_s \approx 42 \text{ kms}^{-1}$$

9. Since the gravitational potential energy between the disc of radius a, mass m_1 and a particle of mass m_2 placed at axis of disc at a distance ℓ from centre is

$$U_{\text{axis}} = -\frac{2Gm_1m_2}{a^2}\left(\sqrt{a^2+\ell^2} - \ell\right) \qquad \ldots(1)$$

$$\Rightarrow \quad U_{\text{axis}} = -\frac{2Gm_1m_2\ell}{a^2}\left[\left(1 + \frac{a^2}{\ell^2}\right)^{\frac{1}{2}} - 1\right]$$

Since $\dfrac{a}{\ell}$ 1, $\left(1+\dfrac{a^2}{\ell^2}\right)^{1/2}$ $1 + \dfrac{a^2}{2\ell^2}$

$$\Rightarrow \quad U_{\text{axis}} = -\left(\frac{2Gm_1m_2\ell}{a^2}\right)\left(\frac{a^2}{2\ell^2}\right)$$

$$\Rightarrow \quad U_{\text{axis}} = -\frac{Gm_1m_2}{\ell}$$

Further, when the particle collides with the disc (at its centre i.e., $\ell = 0$), then

$$U_{\text{centre}} = -\frac{2Gm_1m_2}{a} \qquad \{\because \text{ of } (1)\}$$

Applying Law of Conservation of Energy and using the concept of reduced mass we get

$$(U+K)_{\text{axis}} = (U+K)_{\text{centre}}$$

$$\Rightarrow \quad -\frac{Gm_1m_2}{\ell} + 0 = -2\left(\frac{Gm_1m_2}{a}\right) + \frac{1}{2}\mu v_r^2$$

where $\mu = \dfrac{m_1m_2}{m_1+m_2}$

$$\Rightarrow \quad Gm_1m_2\left(\frac{2}{a} - \frac{1}{\ell}\right) = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)v_r^2$$

$$\Rightarrow \quad v_r = \sqrt{2G(m_1+m_2)\left(\frac{2}{a} - \frac{1}{\ell}\right)}$$

10. $v_{es} = \sqrt{\dfrac{2GM}{R}} = \sqrt{\dfrac{2G\left(\frac{4}{3}\pi R^3\right)\rho}{R}} = \sqrt{\dfrac{4G\rho}{3}}R$

$$v_{es} \propto R$$

Surface area of 1 is $A = 4\pi R_1^2$

Surface area of 2 is $4A = 4\pi R_2^2$

$$\Rightarrow \quad R_2 = 2R_1$$

Given that mass of planet 3 is $M_3 = M_1 + M_2$

$$\Rightarrow \quad \left(\frac{4}{3}\pi R_3^3\right)\rho = \left(\frac{4}{3}\pi R_1^3\right)\rho + \left(\frac{4}{3}\pi R_2^3\right)\rho$$

$$\Rightarrow \quad R_3^3 = R_1^3 + R_2^3$$

$$\Rightarrow \quad R_3^3 = 9R_1^3$$

$$\Rightarrow \quad R_3 = 9^{1/3}R_1$$

$$\Rightarrow \quad R_3 > R_2 > R_1$$

$$\Rightarrow \quad v_3 > v_2 > v_1$$

$$\Rightarrow \quad \frac{v_3}{v_1} = 9^{1/3} \text{ and } \frac{v_2}{v_1} = 2$$

$$\Rightarrow \quad \frac{v_3^3}{v_1^3} = 9 \text{ and } \frac{v_2}{v_1} = 2$$

Test Your Concepts-V (Based on Satellites, Kepler's Laws and Applications)

1. $r_A = R + R = 2R$

and $r_B = R + 3R = 4R$

Since, kinetic energy $K = \dfrac{GMm}{2r}$

$$\Rightarrow \quad K \propto \frac{1}{r}$$

$$\Rightarrow \quad \frac{K_A}{K_B} = \frac{r_B}{r_A} = \frac{4R}{2R} = 2$$

Also, $U = -\dfrac{GMm}{r}$

$$\Rightarrow \quad |U| \propto \frac{1}{r}$$

$$\Rightarrow \quad \frac{U_A}{U_B} = \frac{r_B}{r_A} = 2$$

2. (a) Just before the explosion, the orbital velocity is $v_0 = \sqrt{\dfrac{GM}{R}}$ corresponding to which the total energy associated with the satellite in the orbit is NEGATIVE.

When a mass Δm is expelled very rapidly with a speed v, then for the satellite to still remain within the gravitational pull of the planet, the new total energy must still be negative. So

$$\frac{1}{2}(m-\Delta m)(v_0+v_r)^2 - \frac{GM(m-\Delta m)}{R} \le 0 \qquad \ldots(1)$$

$$\Rightarrow \quad (v_0+v_r) \le \frac{2GM}{R}$$

$$\Rightarrow \quad v_0 + v_r \le \sqrt{\frac{2GM}{R}}$$

$$\Rightarrow \quad \sqrt{\frac{GM}{R}} + v_r \le \sqrt{\frac{2GM}{R}} \qquad \left\{\because v_0 = \sqrt{\frac{GM}{R}}\right\}$$

$$\Rightarrow \quad v_r \le (\sqrt{2}-1)\sqrt{\frac{GM}{R}}$$

(b) Since the term $(m-\Delta m)$ cancels (see equation (1)), so, no modification is needed.

3. Since $T^2 = 4\pi^2\left(\dfrac{r^3}{GM}\right)$

When the satellite is revolving close to the planet, then $r \cong R$

$$\Rightarrow \quad T^2 = 4\pi^2\left(\frac{R^3}{GM}\right)$$

Since $M = \left(\dfrac{4}{3}\pi R^3\right)\rho$

$$\Rightarrow \quad T^2 = 4\pi^2\frac{R^3}{G\left(\frac{4}{3}\pi R^3\right)\rho}$$

$$\Rightarrow \quad \rho T^2 = \text{constant}$$

4. For the satellite to revolve in a circular orbit of radius r_0 the orbital velocity is given by

$$v_0 = \sqrt{\frac{GM}{r_0}} \qquad \ldots(1)$$

At maximum or minimum distances, velocity is perpendicular to the radius vector. So, on applying Conservation of Angular Momentum and Mechanical Energy, we get

$$mv_0r_0\cos\alpha = mvr \quad \ldots(2)$$

and $$-\frac{GMm}{2r_0} = -\frac{GMm}{r} + \frac{1}{2}mv^2 \quad \ldots(3)$$

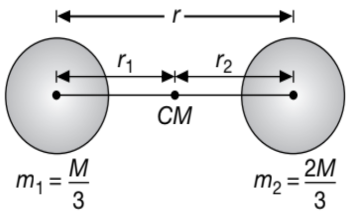
Solving these three equations, we get two values of r i.e., $(1+\sin\alpha)r_0$ and $(1-\sin\alpha)r_0$. Therefore,

$$r_{max} = (1+\sin\alpha)r_0 \text{ and } r_{min} = (1-\sin\alpha)r_0$$

5. Let M be the mass of the sun and r be the distance between the two stars, then

$$r_1 = \frac{m_2}{m_1+m_2}r = \frac{2}{3}r$$

and $$r_2 = \frac{m_1}{m_1+m_2}r = \frac{r}{3}$$



Centripetal force on m_2 is $\frac{Gm_1m_2}{r^2} = \frac{G\left(\frac{2}{9}\right)M^2}{r^2}$

$$\Rightarrow \frac{2}{9}\frac{GM^2}{r^2} = m_2r_2\omega^2 = \left(\frac{2}{3}M\right)\left(\frac{r}{3}\right)\omega^2$$

$$\Rightarrow \omega^2 = \frac{GM}{r^3}$$

$$\Rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{GM}{r^3}$$

$$\Rightarrow T^2 = \frac{4\pi^2r^3}{GM}$$

Since time period of earth around sun is

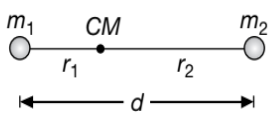
$$T^2 = \frac{4\pi^2R^3}{GM}$$

$$\Rightarrow r = R$$

6. (a) Let the origin be at m_1 and the centre of mass be at a distance r_1 from it. Then

$$r_1 = \frac{m_1(0)+m_2r}{m_1+m_2}$$

$$\Rightarrow r_1 = \left(\frac{m_2}{m_1+m_2}\right)r \text{ and}$$



The centripetal force is provided by gravitational force, so

$$m_1r_1\omega^2 = m_2r_2\omega^2 = \frac{Gm_1m_2}{d^2} \quad \ldots(1)$$

Solving these equations, we get

$$\omega = \sqrt{\frac{G(m_1+m_2)}{d^3}}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{d^3}{G(m_1+m_2)}}$$

(b) $$\frac{K_1}{K_2} = \frac{\frac{1}{2}I_1\omega^2}{\frac{1}{2}I_2\omega^2} = \frac{I_1}{I_2} = \frac{m_1r_1^2}{m_2r_2^2}$$

$$\Rightarrow \frac{K_1}{K_2} = \left(\frac{m_1}{m_2}\right)\left(\frac{r_1}{r_2}\right)^2$$

Since, $m_1r_1\omega^2 = m_2r_2\omega^2$

$$\Rightarrow m_1r_1 = m_2r_2$$

$$\Rightarrow \frac{K_1}{K_2} = \left(\frac{m_1}{m_2}\right)\left(\frac{m_2}{m_1}\right)^2 = \frac{m_2}{m_1}$$

(c) $$\frac{L_1}{L_2} = \frac{I_1\omega}{I_2\omega} = \frac{I_1}{I_2} = \frac{m_2}{m_1}$$

(d) $$L = L_1 + L_2 = (I_1+I_2)\omega$$

$$\Rightarrow L = (m_1r_1^2 + m_2r_2^2)\omega$$

$$\Rightarrow L = \left[\frac{m_1m_2^2d^2}{(m_1+m_2)^2} + \frac{m_2m_1^2d^2}{(m_1+m_2)^2}\right]\omega$$

$$\Rightarrow L = \mu\omega d^2$$

where $\mu = \frac{m_1m_2}{m_1+m_2}$ is the reduced mass

(e) $$K = \frac{1}{2}(I_1+I_2)\omega^2 = \frac{1}{2}\mu\omega^2d^2$$

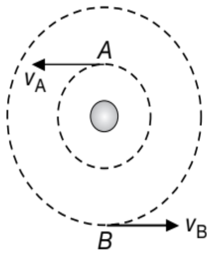
7. For two satellites revolving around same planet, we have

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

$$\Rightarrow T_2 = T_1\left(\frac{r_2}{r_1}\right)^{3/2} = 28\left(\frac{2\times10^4}{10^4}\right)^{3/2} = 56\sqrt{2} \text{ hour}$$

Since, $\omega = \frac{2\pi}{T}$

$$\Rightarrow \omega_1 = \frac{2\pi}{28} \text{ radhr}^{-1} \text{ and } \omega_2 = \frac{2\pi}{56\sqrt{2}} \text{ radhr}^{-1}$$



Positions corresponding to maximum separation are shown in figure, so we have

$$v_{BA} = \vec{v}_B - \vec{v}_A = |v_B| + |v_A| \quad \{\text{for shown position}\}$$

$$\Rightarrow \quad v_{BA} = \omega_2 r_2 + \omega_1 r_1$$

$$\Rightarrow \quad v_{BA} = \left[\frac{2\pi}{56\sqrt{2}} \times 2 \times 10^4 + \frac{2\pi}{28} \times 10^4\right] \text{kmhr}^{-1}$$

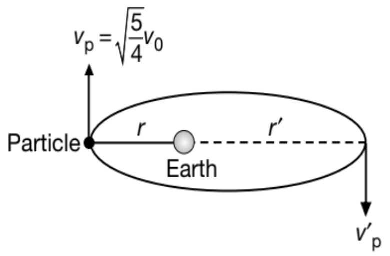
$$\Rightarrow \quad v_{BA} = (0.1587 + 0.2242) \times 10^4 \text{ kmhr}^{-1}$$

$$\Rightarrow \quad v_{BA} = 3829 \text{ kmhr}^{-1}$$

8. Orbital speed of the satellite is $v_0 = \sqrt{\frac{GM}{r}}$, where M is the mass of earth ...(1)

Absolute velocity of particle would be:

$$v_p = v + v_0 = \sqrt{\frac{5}{4}} v_0 = \sqrt{1.25} v_0 \quad ...(2)$$



Since, v_p lies between orbital velocity and escape velocity, path of the particle would be an ellipse with r being the minimum distance.

Let r' be the maximum distance and v'_p be the velocity at that moment.

Then applying the Law of Conservation of Angular Momentum and Conservation of Mechanical Energy, we get

$$mv_p r = mv'_p r' \quad ...(3)$$

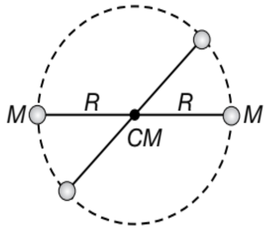
and $$\frac{1}{2} mv_p^2 - \frac{GMm}{r} = \frac{1}{2} mv_p'^2 - \frac{GMm}{r'} \quad ...(4)$$

Solving the above equations (1), (2), (3) and (4), we get

$$r' = \frac{5r}{3} \text{ and } r' = r$$

Hence, the maximum and minimum distances are $\frac{5r}{3}$ and r respectively.

9. (a) $F = \frac{G(M)(M)}{(2R)^2} = \frac{GM^2}{4R^2}$



(b) $F = MR\omega^2$

$$\Rightarrow \quad \frac{GM^2}{4R^2} = MR\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{F}{MR}} = \sqrt{\frac{GM}{4R^3}}$$

$$\Rightarrow \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{4R^3}{GM}} = 4\pi\sqrt{\frac{R^3}{GM}}$$

(c) $E = -\frac{G(M)(M)}{2R} + 2\left(\frac{1}{2} I\omega^2\right) = -\frac{GM^2}{2R} + I\omega^2$

$$\Rightarrow \quad E = -\frac{GM^2}{2R} + (MR^2)\left(\frac{GM}{4R^3}\right) = -\frac{GM^2}{4R}$$

So, binding energy is $|E| = \frac{GM^2}{4R}$

Hence, this much amount of energy will be required to separate the two stars to infinity.

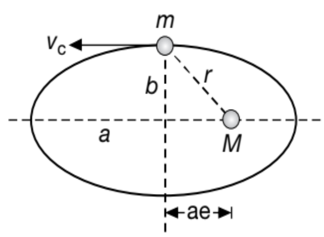
10. Since, we observe that

$$r^2 = a^2e^2 + b^2$$

But $b^2 = a^2(1 - e^2)$

$$\Rightarrow \quad r^2 = a^2e^2 + a^2 - a^2e^2$$

$$\Rightarrow \quad r = a$$



Total energy of a satellite in an elliptical orbit is,

$$E = -\frac{GMm}{2a}$$

$$\Rightarrow \quad E_C = \frac{1}{2} mv_c^2 - \frac{GMm}{r} = -\frac{GMm}{2a}$$

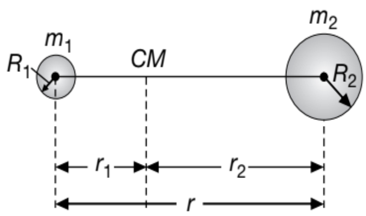
$$\Rightarrow \quad \frac{1}{2} mv_c^2 - \frac{GMm}{a} = -\frac{GMm}{2a} \quad \{r = a\}$$

CHAPTER 4

$$\Rightarrow \quad v_c = \sqrt{\frac{GM}{a}}$$

$$\Rightarrow \quad v_c = R\sqrt{\frac{g}{a}} \qquad \{\text{as } GM = gR^2\}$$

11. Both the stars rotate about their centre of mass.



For the position of CM let Origin be at m_1 and CM be at distance r_1 from origin. Then

$$r_1 = \frac{m_2 r}{m_1 + m_2} \text{ and } r_2 = \frac{m_1 r}{m_1 + m_2}$$

Also, $m_1 r_1 \omega^2 = \dfrac{Gm_1 m_2}{r^2}$ $\qquad \left\{\because \omega = \dfrac{2\pi}{T}\right\}$

Since, $r_1 = \dfrac{m_2 r}{m_1 + m_2}$

$$\Rightarrow \quad \omega^2 = \frac{G(m_1 + m_2)}{r^3}$$

$$\Rightarrow \quad r = \left[\frac{G(m_1 + m_2)}{\omega^2}\right]^{1/3} \qquad \ldots(1)$$

Applying Law of Conservation of Mechanical Energy, we get

$$-\frac{Gm_1 m_2}{r} = -\frac{Gm_1 m_2}{(R_1 + R_2)} + \frac{1}{2}\mu v_r^2 \qquad \ldots(2)$$

where, μ is the reduced mass given by $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$

and v_r is the relative velocity between the two stars

From equation (2), we get

$$v_r^2 = \frac{2Gm_1 m_2}{\mu}\left(\frac{1}{R_1 + R_2} - \frac{1}{r}\right)$$

$$\Rightarrow \quad v_r^2 = \frac{2Gm_1 m_2}{\left(\dfrac{m_1 m_2}{m_1 + m_2}\right)}\left(\frac{1}{R_1 + R_2} - \frac{1}{r}\right)$$

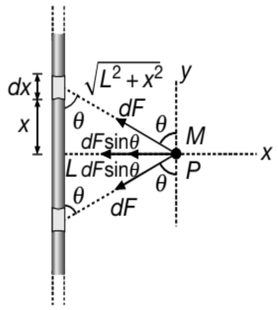
$$\Rightarrow \quad v_r^2 = 2G(m_1 + m_2)\left(\frac{1}{R_1 + R_2} - \frac{1}{r}\right)$$

Substituting the value of r from equation (1), we get

$$v_r = \sqrt{2G(m_1 + m_2)\left[\frac{1}{R_1 + R_2} - \left\{\frac{4\pi^2}{G(m_1 + m_2)T^2}\right\}^{1/3}\right]}$$

Single Correct Choice Type Questions

1.



Let the mass M be placed symmetrically.

$$\Rightarrow \quad F_{\text{net}} = \int_{-\infty}^{\infty} dF\sin\theta = \int_{-\infty}^{\infty} \frac{GM(\lambda\, dx)}{x^2 + L^2}\frac{L}{\sqrt{x^2 + L^2}}$$

$$\Rightarrow \quad F_{\text{net}} = GM\lambda L \int_{-\infty}^{\infty} \frac{dx}{(x^2 + L^2)^{3/2}}$$

$$\Rightarrow \quad F_{\text{net}} = \frac{GM\lambda L}{L^2}(2)$$

$$\Rightarrow \quad F_{\text{net}} = \frac{2GM\lambda}{L}$$

Hence, the correct answer is (C).

2. Let M be the mass of the planet and m the mass of satellite. Then

$$mr\omega^2 = \frac{GMm}{r^2}$$

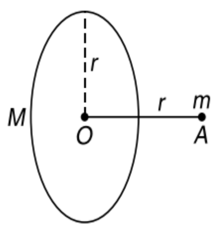
$$\Rightarrow \quad GM = r^2\omega^2$$

Now, $g = \dfrac{GM}{R^2}$

$$\Rightarrow \quad g = \frac{r^3\omega^2}{R^2}$$

Hence, the correct answer is (C).

3.



$$(U + K)_A = (U + K)_O$$

$$\Rightarrow \quad -\frac{GMm}{\sqrt{r^2 + r^2}} + 0 = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{GM}{r}\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad v=\sqrt{\frac{2GM}{r}\left(1-\frac{1}{\sqrt{2}}\right)}$$

Hence, the correct answer is (D).

4. $F=\int_h^{h+L}\frac{G\left(\frac{M}{L}dx\right)m}{x^2}=\frac{GMm}{L}\int_h^{h+L}x^{-2}dx$

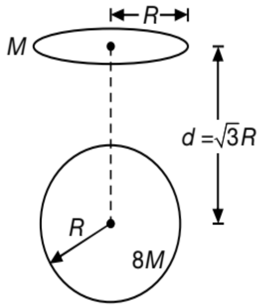
$$\Rightarrow \quad F=\frac{GMm}{L}\left(\frac{x^{-2+1}}{-2+1}\bigg|_h^{h+L}\right)$$

$$\Rightarrow \quad F=-\frac{GMm}{L}\left(\frac{1}{h+L}-\frac{1}{h}\right)$$

$$\Rightarrow \quad F=\frac{GMm}{h(h+L)}$$

Hence, the correct answer is (C).

5. Gravitational field due to the ring at a distance $d=\sqrt{3}R$ on its axis is



$$E=\frac{GMd}{(R^2+d^2)^{3/2}}=\frac{\sqrt{3}GM}{8R^2}$$

Force on sphere is $F=(8M)E=\frac{\sqrt{3}GM^2}{R^2}$

Hence, the correct answer is (D).

6. By Law of Conservation of Energy, we get

$$(U+K)_{\text{surface}}=(U+K)_{\text{centre}}$$

Now, for a solid sphere, we have

$$U_{surface}=-\frac{GMm}{R} \text{ and } U_{\text{centre}}=-\frac{3}{2}\frac{GMm}{R}$$

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{1}{2}m(0)^2=-\frac{3}{2}\frac{GMm}{R}+\frac{1}{2}mv^2$$

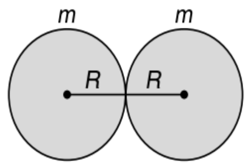
$$\Rightarrow \quad \frac{1}{2}mv^2=-\frac{GMm}{R}-\left(-\frac{3}{2}\frac{GMm}{R}\right)$$

$$\Rightarrow \quad \frac{1}{2}mv^2=\frac{1}{2}\frac{GMm}{R}$$

$$\Rightarrow \quad v=\sqrt{\frac{GM}{R}}=\frac{v_e}{\sqrt{2}}$$

Hence, the correct answer is (D).

7.



Since $F=\frac{Gm^2}{(2R)^2}$

If ρ be the density of each sphere, then

$$m=\left(\frac{4}{3}\pi R^3\right)\rho$$

$$\Rightarrow \quad F=\frac{G\left(\frac{4}{3}\pi R^3\rho\right)^2}{4R^2}$$

$$\Rightarrow \quad F=\frac{4}{9}G\pi^2\rho^2R^4$$

$$\Rightarrow \quad F\propto R^4$$

Hence, the correct answer is (C).

8. Since, $g=\frac{GM}{R^2}$ ...(1)

Also, $M=\left(\frac{4}{3}\pi R^3\right)\rho$

$$\Rightarrow \quad R=\left(\frac{3M}{4\pi\rho}\right)^{1/3}$$

Substituting in (1) we get,

$$g=GM\left(\frac{4\pi\rho}{3M}\right)^{2/3}$$

$$\Rightarrow \quad g\propto M^{1/3}\rho^{2/3}$$

$$\Rightarrow \quad g_{\text{planet}}=g(8)^{1/3}(8)^{2/3}=8g$$

Hence, the correct answer is (C).

9. $F_g\propto\frac{1}{r^2}$ and does not depend on the medium. Hence,

$$F_1'=\frac{F_1}{4}$$

$$F_c\propto\frac{1}{Kr^2}$$

and when air is sucked, force will slightly increase in vacuum, so

$$F_2'>\frac{F_2}{4}$$

Hence, the correct answer is (C).

10. $g\left(1-\frac{2h}{R}\right)$ $g\left(1-\frac{d}{R}\right)$

$$\Rightarrow \quad d=2h$$

Hence, the correct answer is (B).

11. Imagine an inverted hemispherical shell to be placed on this shell so as to complete the spherical shell inside which net gravitational field is zero. Net field can be zero only when field at P is directed along c.
Hence, the correct answer is (C).

12. $(U+K)_{\text{surface}} = (U+K)_{R+7R}$

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{1}{2}mv^2 = -\frac{GMm}{R+7R}+0$$

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{1}{2}mv^2 = -\frac{GMm}{8R}$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{GM}{R}-\frac{GM}{8R} \quad \Rightarrow \quad \frac{v^2}{2} = \frac{7}{8}\left(\frac{GM}{R}\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{7GM}{4R}}$$

Hence, the correct answer is (D).

13. Since, $g_h = g\left(1-\frac{2h}{R}\right)$ $\{\because h \quad R\}$

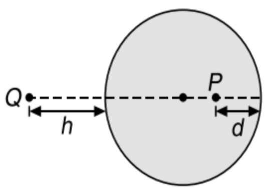
$$\Rightarrow \quad W_h = W\left(1-\frac{2h}{R}\right)$$

Rate of change of weight with height

$$\left|\frac{dW_h}{dh}\right| = \left|0-\frac{2W}{R}\right| = \frac{2mg}{R} \qquad \{\because W = mg\}$$

Hence, the correct answer is (B).

14.



Given that points P and Q located inside and outside the planet have same acceleration due to gravity equal to $\frac{g}{4}$. Thus

$$g\left(1-\frac{d}{R}\right) = \frac{gR^2}{(R+h)^2} = \frac{g}{4}$$

$$\Rightarrow \quad d = \frac{3R}{4} \text{ and } \frac{R}{R+h} = \frac{1}{2}$$

$$\Rightarrow \quad h = R$$

Maximum separation between P and Q will be when these are diametrically opposite. So, maximum separation is

$$r_{\max} = h+R+(R-d)$$

$$\Rightarrow \quad r_{\max} = R+R+\left(R-\frac{3R}{4}\right) = \frac{9R}{4}$$

Hence, the correct answer is (B).

15. $g' = g - R\omega^2\cos^2\phi$

At equator $\phi = 0°$

$$\Rightarrow \quad g' = g - R\omega^2$$

For g' to be zero

$$\omega = \sqrt{\frac{g}{R}} = \frac{1}{800}\ \text{rads}^{-1}$$

Hence, the correct answer is (C).

16. $g = \frac{GM}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\rho\right)}{R^2} = \frac{4}{3}\pi G\rho R$

$$\Rightarrow \quad g \propto \rho R$$

R is increased by a factor of 2 i.e., to keep the value of g to be the same, the value of ρ has to be changed by a factor of $\frac{1}{2}$.

Hence, the correct answer is (C).

17. $E = \frac{GM}{R^3}r$

i.e., Slope, $\frac{dE}{dr} = \frac{GM}{R^3}$

but $M = \frac{4}{3}\pi R^3\rho$

$$\Rightarrow \quad \frac{M}{R^3} = \frac{4\pi\rho}{3} \quad (\rho = \text{density})$$

$$\Rightarrow \quad \text{Slope} = \frac{4\pi G\rho}{3} = \text{constant}$$

Hence, the correct answer is (A).

18. $\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{i}+\frac{\partial U}{\partial y}\hat{j}\right)$

$$\Rightarrow \quad \vec{F} = -a\hat{i} - b\hat{j}$$

$$\Rightarrow \quad |\vec{F}| = \sqrt{a^2+b^2}$$

$$\Rightarrow \quad |\text{Acc}| = \frac{|\vec{F}|}{m} = \frac{\sqrt{a^2+b^2}}{m}$$

Hence, the correct answer is (C).

19. Let R be the radius of earth and g the acceleration due to gravity on earth's surface. Then the desired ratio (say x) is

$$x = \frac{g\left(1-\frac{h}{R}\right)}{\dfrac{g}{\left(1+\frac{h}{R}\right)^2}} = \left(1-\frac{h}{R}\right)\left(1+\frac{h}{R}\right)^2$$

Since $h \quad R$, so $\left(1+\frac{h}{R}\right)^2 \cong 1+\frac{2h}{R}$

$\Rightarrow \quad x=\left(1-\frac{h}{R}\right)\left(1+\frac{2h}{R}\right)$

$\Rightarrow \quad x \approx 1+\frac{h}{R}$

From this expression we see that x increases linearly with h.

Hence, the correct answer is (C).

20. $E(r)=\left(\frac{GM}{R^3}\right)r$ where $M=\left(\frac{4}{3}\pi R^3\right)\rho$,

$\Rightarrow \quad E(r)=\frac{Gr}{R^3}\left(\frac{4}{3}\pi R^3\rho\right)=\left(\frac{4}{3}\pi\rho G\right)r$

Hence, the correct answer is (A).

21. Energy required to raise a satellite to a height h is

$$\Delta E=U_f-U_i=-\frac{GMm}{R+h}-\left(-\frac{GMm}{R}\right)$$

$\Rightarrow \quad \Delta E=GMm\left(\frac{1}{R}-\frac{1}{R+h}\right)$

$\Rightarrow \quad \Delta E=\frac{GMmh}{R(R+h)}$

Further, if v_0 be the orbital velocity in the orbit, then

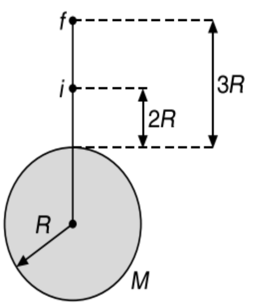
$$v_0=\sqrt{\frac{GM}{R+h}}$$

$\Rightarrow \quad KE=\frac{1}{2}mv_0^2=\frac{GMm}{2(R+h)}$

So, $\frac{\Delta E}{KE}=\frac{2h}{R}$

Hence, the correct answer is (C).

22.



Since $U=-\frac{GMm}{R+h}$

So, $U_i=-\frac{GMm}{2R+R}=-\frac{GMm}{3R}$

and $U_f=-\frac{GMm}{R+3R}=-\frac{GMm}{4R}$

Hence, external work done W is given by

$$W=\Delta U=U_f-U_i=\frac{GMm}{12R}=\frac{GM}{R^2}\times\frac{mR}{12}$$

$\Rightarrow \quad W=\frac{mgR}{12}$

Hence, the correct answer is (C).

23. Since, $F_1=\frac{GMm}{9R^2}$

Also, $F_2=\frac{GMm}{9R^2}-\frac{G(M/8)m}{(5R/2)^2}$

$\Rightarrow \quad F_2=\frac{GMm}{9R^2}-\frac{GMm}{50R^2}=\frac{41}{450}\frac{GMm}{R^2}$

$\Rightarrow \quad \frac{F_2}{F_1}=\frac{41}{50}$

Hence, the correct answer is (B).

24. By the principle of superposition of fields

$$\vec{E}=\vec{E}_1+\vec{E}_2$$

where, $\vec{E}$ is net field at the centre of hole due to entire mass, $\vec{E}_1$ is field due to remaining mass and $\vec{E}_2$ is field due to mass in hole $=0$.

Since, $\vec{E}_1=\vec{E}=\left(\frac{GM}{R^3}\right)r$ where $r=\frac{R}{2}$

$\Rightarrow \quad \vec{E}=\frac{GM}{2R^2}$

Hence, the correct answer is (C).

25. Initially field due to both is along positive x-axis. Due to the ring, field will first increase and then decrease to zero at centre. While field due to the solid sphere, will continuously increase in positive x-direction. On the other side of the ring field is now towards negative x-axis.

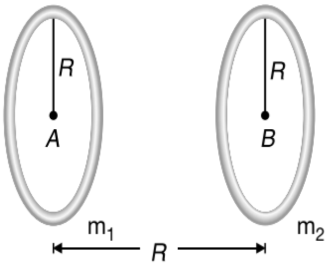
Hence, the correct answer is (B).

26. Potential due to particle at the surface is $V_1=-\frac{GM}{R}$ and potential due to shell at its own surface is $V_2=-\frac{G(3M)}{R}$

So, total potential is $V=V_1+V_2=-\frac{4GM}{R}$

Hence, the correct answer is (D).

27.



$$V_A = \begin{pmatrix}\text{Potential at} \\ A \text{ due to } A\end{pmatrix} + \begin{pmatrix}\text{Potential at} \\ A \text{ due to } B\end{pmatrix}$$

$$\Rightarrow \quad V_A = -\frac{Gm_1}{R} - \frac{Gm_2}{\sqrt{2}R} \text{ and}$$

Similarly,

$$V_B = \begin{pmatrix}\text{Potential at} \\ B \text{ due to } A\end{pmatrix} + \begin{pmatrix}\text{Potential at} \\ B \text{ due to } B\end{pmatrix}$$

$$\Rightarrow \quad V_B = -\frac{Gm_2}{R} - \frac{Gm_1}{\sqrt{2}R}$$

Since $W_{A \to B} = m(V_B - V_A)$

$$\Rightarrow \quad W_{A \to B} = \frac{Gm(m_1 - m_2)(\sqrt{2} - 1)}{\sqrt{2}R}$$

Hence, the correct answer is (B).

28. $E_g = G\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + \dots\right]$

$$\Rightarrow \quad E_g = G\left[1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots\right]$$

$$\Rightarrow \quad E_g = G\left(\frac{1}{1 - \frac{1}{4}}\right) \qquad \left\{S_\infty = \frac{\text{1st term}}{\text{1-Common Ratio}}\right\}$$

$$\Rightarrow \quad E_g = \frac{4G}{3}$$

Hence, the correct answer is (B).

29. For a satellite revolving near the surface of planet, we have

$$T \cong 2\pi\sqrt{\frac{R^3}{GM}} \qquad \{\because r \cong R\}$$

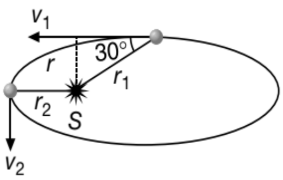
Since, $M = \left(\frac{4}{3}\pi R^3\right)\rho$

$$\Rightarrow \quad T = \sqrt{\frac{3\pi}{\rho G}}$$

i.e., T is independent of R.

Hence, the correct answer is (D).

30.



By Law of Conservation of Angular Momentum, we get

$$mv_1 r = mv_2 r_2$$

$$\Rightarrow \quad mv_1 r_1 \sin 30° = mv_2 r_2$$

$$\Rightarrow \quad v_1 r_1 = 2v_2 r_2$$

Hence, the correct answer is (C).

31. Kinetic energy of satellite is $K = \frac{GMm}{2r}$ and the magnitude of gravitational potential energy is

$$U = \left|-\frac{GMm}{r}\right| = \frac{GMm}{r}$$

So, $K = \frac{U}{2}$

Hence, the correct answer is (B).

32. Total mechanical energy, at earth's surface is

$$E = U + K = -\frac{GMm}{R} + \frac{1}{2}mv_e^2$$

Since $v_e = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \quad E = -\frac{GMm}{R} + \frac{1}{2}m\left(\frac{2GM}{R}\right) = 0$$

Hence, the correct answer is (D).

33. By Law of Conservation of Mechanical Energy, we get

$$-\frac{GmM}{2R} = -\frac{GmM}{\frac{R}{2}} + \frac{1}{2}k\left(\frac{R}{2}\right)^2$$

$$\Rightarrow \quad \frac{GMm}{R}\left(2 - \frac{1}{2}\right) = \frac{kR^2}{8}$$

$$\Rightarrow \quad k = \frac{12GMm}{R^3}$$

Hence, the correct answer is (D).

34. By Law of Conservation of Energy

$$(U + K)_{\text{surface}} = (U + K)_r$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{r} + 0$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}mk^2v_e^2 = -\frac{GMm}{r}$$

But $v_e = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \quad -\frac{GM}{R} + \frac{1}{2}k^2\left(\frac{2GM}{R}\right) = -\frac{GM}{r}$$

$$\Rightarrow \quad -\frac{1}{R} + \frac{k^2}{R} = -\frac{1}{r}$$

$\Rightarrow \quad \frac{1}{r} = \frac{1-k^2}{R}$

$\Rightarrow \quad r = \frac{R}{1-k^2}$

Hence, the correct answer is (B).

35. When kinetic energy $< E$, total energy will be negative, when kinetic energy equals E, total energy is zero and when kinetic energy $> E$, total energy is positive.

Hence, the correct answer is (C).

36. For $r \le r_1$

$$V = \frac{Gm_1}{r_1} - \frac{Gm_2}{r_2} = \text{constant}$$

For $r_1 \le r \le r_2$

$$V = -\frac{Gm_2}{r_2} - \frac{Gm_1}{r}$$

Slope of V-r graph $\frac{dV}{dr} = \frac{Gm_1}{r^2}$

For $V = -\frac{G(m_1+m_2)}{r}$, slope of V-r graph is $\frac{dV}{dr} = \frac{G(m_1+m_2)}{r^2}$

At the boundary of outer shell, slope of V-r graph changes from $\frac{Gm_1}{r_2^2}$ to $\frac{G(m_1+m_2)}{r_2^2}$ i.e., slope increases.

Hence, the correct answer is (D).

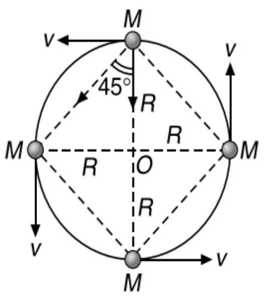
37. $c = \sqrt{\frac{2GM_{star}}{R}}$

$\Rightarrow \quad 3\times10^8 = \sqrt{\frac{2(6.67\times10^{-11})(3\times2\times10^{30})}{R}}$

$\Rightarrow \quad R \quad 9$ km

Hence, the correct answer is (D).

38. Gravitational force on each particle due to the other three particles will provide the necessary centripetal force.



$\Rightarrow \quad \sqrt{2}\frac{GM^2}{(\sqrt{2}R)^2}\cos 45^\circ + \frac{GM^2}{(2R)^2} = \frac{Mv^2}{R}$

$\Rightarrow \quad v = \sqrt{\frac{GM}{R}\left(\frac{2\sqrt{2}+1}{4}\right)}$

Hence, the correct answer is (D).

39. $dW = \vec{F}\cdot d\vec{\ell} = m_0(\vec{E}\cdot d\vec{\ell})$

$\Rightarrow \quad dW = m_0(4\hat{i}+\hat{j})\cdot(\hat{i}dx+\hat{j}dy)$

$\Rightarrow \quad dW = m_0(4dx+dy)$

Given that, $W = 0$

$\Rightarrow \quad \int dW = 0$

$\Rightarrow \quad \int(4dx+dy) = 0$

$\Rightarrow \quad \int d(4x+y) = 0$

$\Rightarrow \quad 4x+y = \text{constant}$

$\Rightarrow \quad y+4x = 2$, satisfies the condition.

Hence, the correct answer is (A).

40. Total energy of a planet of mass m, in an elliptical orbit is

$$E = -\frac{GMm}{2a} \qquad \{m = \text{mass of planet}\}$$

By Law of Conservation of Energy, we have

$$KE + PE = E$$

$\Rightarrow \quad \frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a}$

$\Rightarrow \quad v = \sqrt{GM\left(\frac{2}{r}-\frac{1}{a}\right)}$

Hence, the correct answer is (A).

41. Since $u = \frac{3}{4}v_e$

By Law of Conservation of Energy,

$$(U+K)_{\text{surface}} = (U+K)_r$$

$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}mu^2 = \frac{-GMm}{r} + 0$

$\Rightarrow \quad \frac{-GMm}{R} + \frac{1}{2}m\left(\frac{9}{16}v_e^2\right) = -\frac{GMm}{r}$

Since $v_e^2 = \frac{2GM}{R}$

$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}m\frac{9}{16}\left(\frac{2GM}{R}\right) = -\frac{GMm}{r}$

$\Rightarrow \quad -\frac{7}{16}\frac{GMm}{R} = -\frac{GMm}{r}$

$\Rightarrow \quad r = \frac{16}{7}R$

Hence, the correct answer is (B).

42. $\begin{pmatrix}\text{Work}\\\text{done}\end{pmatrix}=\begin{pmatrix}\text{Increase in Gravitational}\\\text{Potential Energy}\end{pmatrix}$

Since, $\Delta U=\dfrac{mgh}{1+\dfrac{h}{R}}$

$\Rightarrow \quad W_1=\dfrac{mgR}{1+\dfrac{R}{R}}=\dfrac{mgR}{2}$

and similarly, $W_2=\dfrac{mgh}{1+\dfrac{h}{R}}$

Since, $W_1=2W_2$

$\Rightarrow \quad \dfrac{mgR}{2}=\dfrac{2mgh}{1+\dfrac{h}{R}}$

$\Rightarrow \quad h=\dfrac{R}{3}$

Hence, the correct answer is (C).

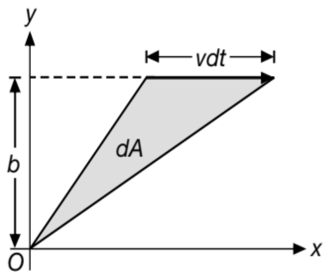
43. $\dfrac{dA}{dt}=\dfrac{1}{2}\times\dfrac{\text{base}\times\text{height}}{dt}$

$\Rightarrow \quad \dfrac{dA}{dt}=\dfrac{1}{2}\dfrac{(vdt)(b)}{dt}$

$\Rightarrow \quad \dfrac{dA}{dt}=\dfrac{bv}{2}$

Since, $v=at$, so

$\Rightarrow \quad \dfrac{dA}{dt}=\dfrac{abt}{2}$



Hence, the correct answer is (B).

44. $\vec{E}_g=-\vec{\nabla}V=-\left(\hat{i}\dfrac{\partial V}{\partial x}+\hat{j}\dfrac{\partial V}{\partial y}\right)=k\left(y\hat{i}+x\hat{j}\right)$

The work done by this field is independent of the path followed between any two states i.e.

$$W_{(0,0)\to(a,a)}=W_{(0,0)\to(0,a)\to(a,a)}=W_{(0,0)\to(a,0)\to(a,a)}$$

Since, $dW=m\vec{E}_g\cdot d\vec{r}$

where $d\vec{r}=\hat{i}dx+\hat{j}dy$

$\Rightarrow \quad dW=mk\left(y\,dx+x\,dy\right)$

$\Rightarrow \quad W=\displaystyle\int dW=mk\int_{(0,0)}^{(a,a)}d(xy)=mka^2$

$\Rightarrow \quad W=mka^2$ {where m is mass of particle}

$$W_{(0,0)\to(a,a)}=W_{(0,0)\to(0,a)\to(a,a)}=W_{(0,0)\to(a,0)\to(a,a)}=mka^2$$

Hence the field is conservative.

Hence, the correct answer is (B).

45. Let σ be the surface density, then

$$M_A=\sigma 4\pi R_A^2,\ m_B=\sigma 4\pi R_B^2$$

Since $V_A=\dfrac{-GM_A}{R_A}$ and $V_B=\dfrac{-GM_B}{R_B}$

$\Rightarrow \quad \dfrac{V_A}{V_B}=\dfrac{M_A}{M_B}\dfrac{R_B}{R_A}=\dfrac{\sigma\left(4\pi R_A^2\right)}{\sigma\left(4\pi R_B^2\right)}\dfrac{R_B}{R_A}=\dfrac{R_A}{R_B}$

Given that, $\dfrac{V_A}{V_B}=\dfrac{R_A}{R_B}=\dfrac{3}{4}$

$\Rightarrow \quad R_B=\dfrac{4}{3}R_A$

Since the new shell formed also has same surface mass density. So, we have

$$\sigma=\frac{M_A}{4\pi R_A^2}=\frac{M_B}{4\pi R_B^2}=\frac{M_A+M_B}{4\pi R^2} \quad \ldots(1)$$

From equation (1), we get

$$\frac{M_A}{M_B}=\frac{R_A^2}{R_B^2} \quad \ldots(2)$$

and $\dfrac{M_A+M_B}{M_A}=\dfrac{R^2}{R_A^2}$ ...(3)

From (2) and (3), we get

$$R^2=R_A^2+R_B^2$$

Now $\dfrac{V}{V_A}=\dfrac{M}{R}\dfrac{R_A}{M_A}=\left(\dfrac{M_A+M_B}{M_A}\right)\dfrac{R_A}{\sqrt{R_A^2+R_B^2}}$ ...(4)

From (3), we have $\dfrac{M_A+M_B}{M_A}=\dfrac{R^2}{R_A^2}$

Substituting in equation (4), we get

$$\frac{V}{V_A}=\frac{\sqrt{R_A^2+R_B^2}}{R_A}=\frac{5}{3}$$

Hence, the correct answer is (C).

46. Since, $T^2=kr^3$

$\Rightarrow \quad 2\dfrac{\Delta T}{T}=3\dfrac{\Delta r}{r}$

$\Rightarrow \quad \dfrac{\Delta T}{T}=\dfrac{3}{2}\dfrac{\Delta r}{r}$

Hence, the correct answer is (A).

47. Net mass, $M=\left[\dfrac{4}{3}\pi(2R)^3-\dfrac{4}{3}\pi R^3\right]\rho$

where ρ is the density of the material of sphere. So,

$$\rho=\frac{M}{\frac{4}{3}\pi\left[(2R)^3-(R)^3\right]}=\left(\frac{3M}{28\pi R^3}\right)$$

Now $V=V_{2R\text{ at centre}}-V_{R\text{ at centre}}$ ...(1)

$$V_{2R} = -\frac{GM_1}{2R} \text{ and } V_R = -\frac{GM_2}{2R}$$

where, $M_1 = \rho\left(\frac{4}{3}\pi\right)(2R)^3 = \frac{8}{7}M$

and $M_2 = \rho\left(\frac{4}{3}\pi\right)(R)^3 = \frac{M}{7}$

Substituting in equation (1), we get

$$V = -\frac{9GM}{14R}$$

Hence, the correct answer is (B).

48. By Law of Conservation of Energy

$$(U+K)_{\text{surface}} = (U+K)_h$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h} + 0$$

{$\because$ At maximum height velocity is zero}

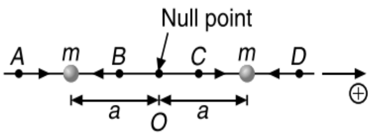
$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}mgR = -\frac{GMm}{R+h}$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}\frac{GMm}{R} = -\frac{GMm}{R+h} \qquad \left\{\because g = \frac{GM}{R^2}\right\}$$

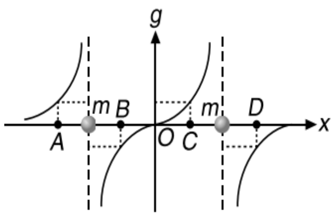
$$\Rightarrow \quad h = R$$

Hence, the correct answer is (B).

49.



The direction of net gravitational intensity at various points is shown above. Taking gravitational intensity towards right as positive, the graph will be



Hence, the correct answer is (A).

50. $\Delta U_1 = -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$

when taken from surface to h, we have

$$\Delta U_1 = -\frac{GMm}{R+h} + \frac{GMm}{R}$$

Now, when taken from h to infinity, we have

$$\Delta U_2 = 0 - \left(-\frac{GMm}{R+h}\right)$$

Since $\Delta U_1 = \Delta U_2$

$$\Rightarrow \quad -\frac{GMm}{R+h} + \frac{GMm}{R} = \frac{GMm}{R+h}$$

$$\Rightarrow \quad 2\left(\frac{GMm}{R+h}\right) = \frac{GMm}{R}$$

$$\Rightarrow \quad 2R = R+h$$

$$\Rightarrow \quad h = R$$

Hence, the correct answer is (A).

51. Since $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$

$$\Rightarrow \quad \frac{T_1^2}{T_2^2} = \left(\frac{R}{4R}\right)^3 = \frac{1}{64}$$

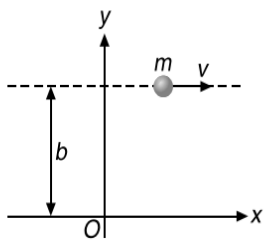
(Three earth radii from the surface means four radii from the centre).

$$\Rightarrow \quad \frac{T_1}{T_2} = \frac{1}{8}$$

$$\Rightarrow \quad T_2 = 8T_1 = 8(83) = 664 \text{ minute}$$

Hence, the correct answer is (C).

52.



Since angular momentum of the particle about origin O, $L = mvb =$ constant, therefore, areal velocity of the particle about origin O is constant. So, it sweeps equal areas in equal intervals of time. Thus, area swept in smaller time interval will also be smaller. Hence

$$A_1 > A_2 \text{ if } (t_4 - t_3) < (t_2 - t_1)$$

Hence, the correct answer is (B).

53. $v_0 = \sqrt{\frac{Gm}{5R}}$

After perfectly inelastic collision, velocity of combined mass will become zero. Therefore, if v be the desired speed, then by Law of Conservation of Mechanical Energy, we have

$$(U+K)_{r=5R} = (U+K)_{\text{surface}}$$

$$-\left(\frac{GmM}{5R}\right) - \left(\frac{GmM}{5R}\right) + 0 = -\frac{GM(2m)}{R} + \frac{1}{2}(2m)v^2$$

$$\Rightarrow \quad \frac{1}{2}(2m)v^2 = -\frac{GM(2m)}{5R} + \frac{GM(2m)}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{8GM}{5R}} = 2\sqrt{2}v_0$$

Hence, the correct answer is (A).

54. Since Areal velocity $= \dfrac{\text{Area Swept}}{\text{Time for one Revolution of Earth about the Sun}}$

Further Areal velocity $= \dfrac{L}{2M}$

$$\Rightarrow \quad \text{Area Swept} = \left(\frac{L}{2M}\right)\left(\begin{array}{c}\text{Time for one}\\ \text{Revolution of Earth}\\ \text{about the Sun}\end{array}\right)$$

$$\text{Area Swept} = \frac{1}{2}(4.4\times10^{15})(365\times24\times60\times60)$$

$$\Rightarrow \quad \text{Area Swept} \quad 7\times10^{22}\ \text{m}^2$$

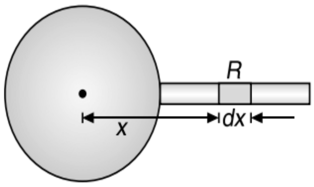
Hence, the correct answer is (D).

55. Force on element dx on rod is

$$dF = \frac{GM(dm)}{x^2}$$

where $dm = \dfrac{M}{2R}dx$

$$\Rightarrow \quad dF = \left(\frac{GM^2}{2R}\right)\frac{dx}{x^2}$$



Total force on the rod is

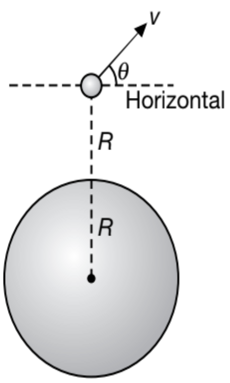
$$F = \int dF = \frac{GM^2}{2R}\int_R^{2R}\frac{1}{x^2}dx$$

$$\Rightarrow \quad F = \frac{GM^2}{2R}\left(-\frac{1}{x}\right)\Bigg|_R^{2R} = -\frac{GM^2}{2R}\left(\frac{1}{2R}-\frac{1}{R}\right)$$

$$\Rightarrow \quad F = \frac{GM^2}{4R^2}$$

Hence, the correct answer is (A).

56. At height $h = R$, the distance of the particle from centre of earth is $r = 2R$.
Let v be the velocity at that point. Then by Law of Conservation of Mechanical Energy, we have



$$(U+K)_{\text{surface}} = (U+K)_{\text{at } h}$$

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{1}{2}mv_e^2 = -\frac{GMm}{R+R}+\frac{1}{2}mv^2$$

Since, $v_e = \sqrt{\dfrac{2GM}{R}}$

$$\Rightarrow \quad \frac{1}{2}mv^2 = -\frac{GMm}{R}+\frac{GMm}{R}+\frac{GMm}{2R}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}}$$

Let θ be the angle of its velocity with horizontal, then by Law of Conservation of Angular Momentum about centre of earth, we have

$$mv_eR\cos(45^\circ) = mvr\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{v_eR\cos(45^\circ)}{vr}$$

$$\Rightarrow \quad \cos\theta = \frac{R\sqrt{\dfrac{2GM}{R}}\dfrac{1}{\sqrt{2}}}{(2R)\sqrt{\dfrac{GM}{R}}} = \frac{1}{2}$$

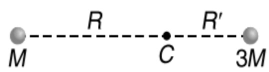
$$\Rightarrow \quad \theta = 60^\circ$$

Hence, the correct answer is (C).

57. $$U(r) = \begin{cases} -\dfrac{GMm}{r}, & r \ge R \\ -\dfrac{GMm}{R}, & r < R \end{cases}$$

Hence, the correct answer is (C).

58. In a double star system, both stars revolve around centre of mass of system (C).



From the definition of centre of mass

$$MR = 3MR'$$

$$\Rightarrow \quad R' = \frac{R}{3}$$

Hence, distance between two stars is

$$r = R + \frac{R}{3} = \frac{4R}{3}$$

The gravitational force between them will provide the centripetal force to revolve in the circle. Therefore, for the smaller star, we have

$$\frac{GM\times3M}{\left(\dfrac{4R}{3}\right)^2} = MR\omega^2$$

$\Rightarrow \quad \omega = \sqrt{\frac{27GM}{16R^3}}$

$\Rightarrow \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{16R^3}{27GM}}$

Hence, the correct answer is (C).

59. By Law of Conservation of Energy, we have

$$(U+K)_{surface} = (U+K)_h$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}m\left(\frac{v_e}{2}\right)^2 = -\frac{GMm}{R+h} = U_h$$

$$\Rightarrow \quad U_h = -\frac{GMm}{R} + \frac{1}{8}mv_e^2 = -\frac{GMm}{R} + \frac{1}{8}m\left(\frac{2GM}{R}\right)$$

$$\Rightarrow \quad U_h = -\frac{3}{4}\frac{GMm}{R}$$

Hence, the correct answer is (C).

60. $E_i = -\frac{GMm}{2r}$, $E_f = -\frac{GMm}{2(3r)} = -\frac{GMm}{6r}$

Energy Required is

$$\Delta E = E_f - E_i = \frac{GMm}{3r}$$

Hence, the correct answer is (C).

61. Since, $W = U_f - U_i$

Where, $U_f = -\frac{3Gm^2}{r_f}$ and $U_i = -\frac{3Gm^2}{r_i}$

$$\Rightarrow \quad W = U_f - U_i = 3Gm^2\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

where, $G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}, $m = 0.1$ kg, $r_f = 0.4$ m and $r_i = 0.2$ m

Substituting the values, we get

$$W = 5 \times 10^{-12} \text{ J}$$

Hence, the correct answer is (C).

62. The whole space can be divided into three regions

(i) $0 < r < R_1$, we have $F(r) = 0$

(ii) $R_1 < r < R_2$, we have $F(r) = \frac{4}{3}\pi G\rho m\left(r - \frac{R_1^3}{r^2}\right)$

(iii) $R_2 < r < \infty$, we have $F(r) = \frac{4}{3}\pi G\rho m\left(\frac{R_2^3 - R_1^3}{r^2}\right)$

where, ρ is the density of material of the sphere.

Hence, the correct answer is (C).

63. The weight of the traveler first decreases, then becomes zero and after that again increases.

Hence, the correct answer is (C).

64. Since $E = -\frac{dV}{dx}$

$\Rightarrow \quad -dV = Edx$

$\Rightarrow \quad -dV = Kx^{-3}dx$

$\Rightarrow \quad -\int dV = K\int x^{-3}\,dx$

$\Rightarrow \quad -V = K\left(\frac{x^{-3+1}}{-3+1}\right)$

$\Rightarrow \quad V = \frac{K}{2x^2}$

Hence, the correct answer is (D).

65. Total number of interactions $= {}^nC_2 = \frac{n(n-1)}{2}$

Hence, the correct answer is (D).

66. Gravitational field due to element dx at origin is

$$dE_g = \frac{G(a+bx^2)dx}{x^2}$$

Net field at O is

$$E_g = \int dE_g = G\int_{\alpha}^{l+\alpha}\left(\frac{a}{x^2} + b\right)dx$$

$$\Rightarrow \quad E_g = \left(-\frac{Ga}{x}\right)\Bigg|_{\alpha}^{l+\alpha} + Gbl$$

$$\Rightarrow \quad E_g = Ga\left(\frac{1}{\alpha} - \frac{1}{l+\alpha}\right) + Gbl$$

Hence force on mass m is given by

$$F = mE_g = Gm\left[a\left(\frac{1}{\alpha} - \frac{1}{l+\alpha}\right) + bl\right]$$

Hence, the correct answer is (A).

67. Net gravitational field inside a shell is zero. Hence, net force on the particle will be zero i.e., the particle stays at rest in its original position.

Hence, the correct answer is (D).

68. Angular momentum of earth remains constant even after the hit because the meteor is coming radially inward. However, the MOI increases by mR^2 as meteorite strikes the earth at equator.

$I\omega = \text{constant}$

$\Rightarrow \quad Id\omega + \omega dI = 0$

$\Rightarrow \quad \frac{d\omega}{\omega} = -\frac{dI}{I}$

Since $\omega = \frac{2\pi}{T}$, we get $\frac{dT}{T} = \frac{dI}{I}$

Hence $dT = \frac{dI}{I}T = \frac{mR^2}{\frac{2}{5}MR^2}T = \frac{5mT}{2M}$

Hence, the correct answer is (A).

69. The gravitational potential at the centre of the ring is

$$V=-\frac{GM}{R}$$

irrespective of the distribution of mass

Since, $W_{C\to\infty}=m\left(V_{\infty}-V_C\right)$

$$\Rightarrow \quad W_{C\to\infty}=m\left[0-\left(-\frac{GM}{R}\right)\right]$$

$$\Rightarrow \quad W_{C\to\infty}=\frac{GMm}{R}$$

Hence, the correct answer is (B).

70. When mass of the shell is m, the potential is $V=-\frac{Gm}{R}$

To add an elementary mass dm, the work done is $dW=Vdm=-\frac{Gm}{R}dm$

This, net work done equals the gravitational self energy, so

$$U=-\int_0^M \frac{Gm\,dm}{R}=-\frac{GM^2}{2R}$$

Hence, the correct answer is (D).

71. Given that $v_0=\frac{v_e}{2}$

$$\Rightarrow \quad \sqrt{\frac{gR^2}{R+h}}=\frac{1}{2}\sqrt{2gR}$$

$$\Rightarrow \quad \frac{R}{R+h}=\frac{1}{2}$$

$$\Rightarrow \quad h=R$$

Since the satellite now stops and falls from this height, using conservation of mechanical energy $U_i+K_i=U_f+K_f$

$$\Rightarrow \quad -\frac{GMm}{R+R}+0=-\frac{GMm}{R}+\frac{1}{2}mv^2$$

$$\Rightarrow \quad v=\sqrt{\frac{GM}{R}}=\sqrt{gR}$$

Hence, the correct answer is (A).

72. Potential energy at a height of $2R$ from surface of earth is

$$U=-\frac{GMm}{R+2R}=-\frac{GMm}{3R}$$

and total energy of a satellite at a height h is $E=-\frac{GMm}{2(R+h)}$

Given that $U=E$

$$\Rightarrow \quad -\frac{GMm}{3R}=-\frac{GMm}{2(R+h)}$$

$$\Rightarrow \quad h=\frac{R}{2}$$

Hence, the correct answer is (B).

73. **CONCEPT OF REDUCED MASS** (μ)

Let m_1 be at rest and think that m_2 has been replaced by μ and is moving with velocity v. Then by Law of Conservation of Energy

$$\frac{1}{2}m_1(0)^2+\frac{1}{2}\mu v^2=-\frac{Gm_1m_2}{r}+0$$

where, μ = reduced mass of system = $\frac{m_1m_2}{m_1+m_2}$

$$\Rightarrow \quad v=\sqrt{\frac{2G(m_1+m_2)}{r}}$$

Hence, the correct answer is (B).

74. Since $U=v_O=\sqrt{\frac{GM}{R}}$

By Law of Conservation of Energy, we have

$$(U+K)_{\text{surface}}=(U+K)_{at\ \text{h}}$$

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{1}{2}mu^2=-\frac{GMm}{R+h}+\frac{1}{2}m(0)^2$$

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{GMm}{2R}=-\frac{GMm}{R+h}$$

$$\Rightarrow \quad h=R$$

Hence, the correct answer is (C).

75. Gravitational force between balls is

$$F=\frac{Gm^2}{(a-x)^2}$$

For equilibrium of each ball, we have

$$T\sin\theta=\frac{Gm^2}{(a-x)^2} \qquad \ldots(1)$$

$$T\cos\theta=mg \qquad \ldots(2)$$

$$\Rightarrow \quad \tan\theta=\frac{Gm}{(a-x)^2g}$$

$$\Rightarrow \quad \theta=\tan^{-1}\left[\frac{Gm}{(a-x)^2g}\right]$$

Hence, the correct answer is (B).

76. Since the torque on the planet due to gravitational pull about the sun is zero, so angular momentum of the planet about the sun is constant.

$$\Rightarrow \quad mvr_{\perp}=\text{constant}$$

Hence, the correct answer is (B).

77. By Law of Conservation of Energy

$$(U+K)_{\text{surface}} = (U+K)_{\infty}$$

$$\Rightarrow \frac{-GMm}{R} + \frac{1}{2}m(3v_e)^2 = 0 + \frac{1}{2}mv^2$$

$$\Rightarrow -\frac{GM}{R} + \frac{9v_e^2}{2} = \frac{1}{2}v^2$$

Since $v_e^2 = \frac{2GM}{R}$

$$\Rightarrow -\frac{v_e^2}{2} + \frac{9v_e^2}{2} = \frac{1}{2}v^2$$

$$\Rightarrow v^2 = 8v_e^2$$

$$\Rightarrow v = 2\sqrt{2}v_e$$

Hence, the correct answer is (D).

78. A lighter body inside a denser medium behaves like a negative mass so far the gravitational force is concerned even the air bubbles with negative masses will attract each other.

Hence, the correct answer is (B).

79. According to Kepler's Laws the planets sweep equal areas in equal intervals of time. So, for

$$\Delta A_1 = \Delta A_2$$

$$t_1 = t_2$$

Hence, the correct answer is (C).

80. By Kepler's Second Law, we have

$$\frac{\Delta A}{\Delta t} = \frac{L}{2M} = \frac{\pi ab}{T}$$

$$\Rightarrow \frac{L}{2M} = \frac{\pi ab}{T}$$

where a is semi major and b is semi minor axis of elliptical orbit.

$$\Rightarrow L = \frac{m\pi(r_a + r_p)\sqrt{r_a r_p}}{T}$$

Hence, the correct answer is (A).

81. Areal velocity $\frac{dA}{dt} = \frac{L}{2m} = \text{constant}$

Since, $L = mvr\sin\theta$

$$\Rightarrow \frac{dA}{dt} = \frac{vr\sin\theta}{2}$$

$$\Rightarrow \frac{dA}{dt} \propto m^0$$

Hence, the correct answer is (D).

82. Since, $T = \frac{2\pi r}{v}$

$$\Rightarrow \frac{2\pi R}{v_0} = 2 \text{ and } \frac{2\pi R'}{v_0'} = 16$$

This is possible when $R' = 4R$ and $v_0' = \frac{v_0}{2}$

Hence, the correct answer is (D).

83. Distance of second satellite from the centre of earth becomes four times.

Since, according to Kepler's Laws we have

$$T^2 \propto r^3$$

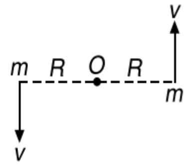
$$\Rightarrow T \propto r^{3/2}$$

So, time period of second satellite must become $(4)^{3/2} = 8$ times

Hence, $T' = 8T = (8)(90) = 720$ min

Hence, the correct answer is (D).

84. The gravitational force provides the necessary centripetal force for circular motion



$$\Rightarrow \frac{Gm^2}{(2R)^2} = \frac{mv^2}{R}$$

$$\Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$$

Hence, the correct answer is (C).

85. Energy of the satellite at the surface of earth is $E_i = -\frac{GMm}{R}$ and energy of satellite at a distance $r = (2R+R) = 3R$ from the centre of earth is

$$E_f = -\frac{GMm}{2(3R)} = -\frac{GMm}{6R}$$

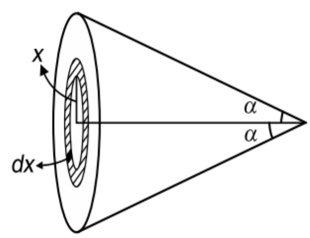
So, the energy required to launch the satellite is

$$E = E_f - E_i = \frac{5GMm}{6R}$$

Hence, the correct answer is (C).

86. $E = -2\pi\sigma G\int_0^{\alpha}\sin\theta\, d\theta$

$$\Rightarrow E = -2\pi\sigma G(1-\cos\alpha)$$



$$\Rightarrow \quad E=-2\pi\sigma G\left(1-\frac{r}{\sqrt{r^2+R^2}}\right)$$

$$V=-2\pi\sigma rG\left(1-\frac{r}{\sqrt{r^2+R^2}}\right)$$

Hence, the correct answer is (C).

87. For all points lying in the plane of base of a hemispherical shell, gravitational field is normal to the base and hence this surface is equipotential. So, $V_A=V_B=V_C$
Hence, the correct answer is (D).

88. Since, Total energy $=-$ Kinetic energy $=-E$
So, energy E will have to be supplied to escape the electron to infinity.
Hence, the correct answer is (C).

89. $U_h=-\dfrac{GMm}{R+h}$

$$\Rightarrow \quad U_h=-\frac{GMm}{R}\left(1+\frac{h}{R}\right)^{-1}=-\frac{GMm}{R}\left(1-\frac{h}{R}\right)$$

$$\{\because h \quad R\}$$

$$\Rightarrow \quad U_h=-\frac{GMm}{R}+mgh \qquad \left\{\because g=\frac{GM}{R^2}\right\}$$

Hence, the correct answer is (B).

90. Since $W=\int \vec{F}\cdot d\vec{l}$

where $\vec{F}=m_0\vec{I}$ and $d\vec{l}=dx\hat{i}+dy\hat{j}$

$$\Rightarrow \quad W=m_0\int_{(1,\,1)}^{\left(2,\,\frac{1}{3}\right)} 2dx+3dy$$

Since $3y+2x=5$

$$\Rightarrow \quad 3dy+2dx=0$$

$$\Rightarrow \quad W=0$$

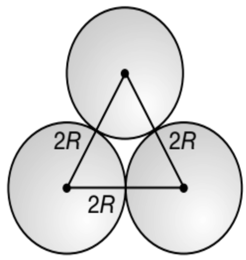
Hence, the correct answer is (A).

91. By Law of Conservation of Energy, we have

$$(U+K)_{\text{initial}}=(U+K)_{\text{final}}$$

$$\Rightarrow \quad -\frac{3Gm^2}{d}+3\left(\frac{1}{2}m(0)^2\right)=-\frac{3Gm^2}{2R}+3\left(\frac{1}{2}mv^2\right)$$

$$\Rightarrow \quad 3\left(\frac{1}{2}mv^2\right)=3\left(\frac{Gm^2}{2R}-\frac{Gm^2}{d}\right)$$



$$\Rightarrow \quad v^2=Gm\left(\frac{1}{R}-\frac{2}{d}\right)$$

$$\Rightarrow \quad v=\sqrt{Gm\left(\frac{1}{R}-\frac{2}{d}\right)}$$

Hence, the correct answer is (D).

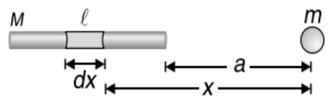
92. $(U+K)_{\text{initial}}=(U+K)_{\infty}$

$$-\frac{GM_em}{\frac{r}{2}}-\frac{GM_mm}{\frac{r}{2}}+\frac{1}{2}mv^2=0$$

$$\Rightarrow \quad v=\sqrt{\frac{4G}{r}(M_e+M_m)}$$

Hence, the correct answer is (A).

93. $dU=-\dfrac{Gm\left(\frac{M}{\ell}dx\right)}{x}$



$$\Rightarrow \quad U=\int dU=-\frac{GmM}{\ell}\int_a^{a+\ell}\frac{dx}{x}$$

$$\Rightarrow \quad U=-\frac{GMm}{\ell}\log_e\left(\frac{a+\ell}{a}\right)$$

Hence, the correct answer is (C).

94. By Law of Conservation of Mechanical Energy

$$U_i+K_i=U_f+K_f$$

$$-\frac{GMm}{R+h_i}+0=-\frac{GMm}{R+h_f}+\frac{1}{2}mv^2$$

$$-\frac{GM}{R+R}+0=-\frac{GM}{R+0.5R}+\frac{1}{2}v^2$$

$$\Rightarrow \quad -\frac{GM}{2R}+\frac{2GM}{3R}=\frac{1}{2}v^2$$

$$\Rightarrow \frac{GM}{6R} = \frac{1}{2}v^2$$

$$\Rightarrow v = \sqrt{\frac{GM}{3R}}$$

Hence, the correct answer is (D).

95. Given that $\frac{U}{m} = E$. By Law of Conservation of Energy, we have

$$(U+K)_{\text{surface}} = (U+K)_\infty$$

$$-mE + \frac{1}{2}mv_e^2 = 0 \qquad \left\{\because U = -\frac{GmM}{R}\right\}$$

$$\Rightarrow v_e = \sqrt{2E}$$

Hence, the correct answer is (A).

96. Total initial energy is $E_i = -\frac{GMm}{R}$

Total final energy is $E_f = -\frac{GMm}{2(2R+R)}$

Energy required $= E_f - E_i$

$$\Rightarrow \Delta E = -\frac{GMm}{6R} + \frac{GMm}{R} = \frac{5}{6}\frac{GMm}{R}$$

$$\Rightarrow \Delta E = \frac{5}{6}mgR$$

Hence, the correct answer is (D).

97. Time period of a satellite above the earth's surface is

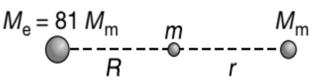
$$T^2 = \frac{4\pi^2 R^3}{GM} = \frac{3\pi}{G\left(\frac{M}{\frac{4}{3}\pi R^3}\right)} = \frac{3\pi}{G\rho}$$

$$\Rightarrow \rho T^2 = \frac{3\pi}{G} = \text{a universal constant}$$

Hence, the correct answer is (A).

98. $\int_S \vec{E}_g \cdot d\vec{A} = -4\pi m_0 G$ (Gauss Theorem for Gravitation)

Hence, the correct answer is (B).

99. 

$$U = -Gm\left(\frac{M_e}{R} + \frac{M_m}{r}\right)$$

$$\Rightarrow U = -GmM_m\left(\frac{81}{R} + \frac{1}{r}\right)$$

Hence, the correct answer is (C).

100. Since, we know that from Work – Energy Theorem,

$$W_{\text{ext}} = \Delta U + \Delta K$$

$$\Rightarrow W_{A\to B} = (U_B - U_A) + (K_B - K_A)$$

$$\Rightarrow W_{A\to B} = m(V_B - V_A) + (K_B - K_A)$$

Substituting the values, we get

$$-10 = 2(V_B - V_A) + 4$$

$$\Rightarrow V_B - V_A = -7\ \text{Jkg}^{-1}$$

Hence, the correct answer is (D).

101. Gravitational pull on satellite revolving in an orbit of radius $\frac{3R}{2}$ is

$$F = \frac{GMm}{\left(\frac{3R}{2}\right)^2} = \frac{4}{9}\frac{GMm}{R^2}$$

As weight of 1 kg body on the surface of earth is

$$\frac{GM \times 1}{R^2} = 10\ \text{N} \qquad \text{(given)}$$

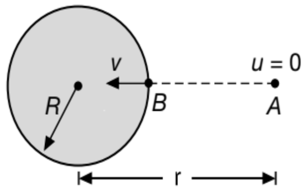
$$\Rightarrow F = \frac{4}{9}\frac{GMm}{R^2} = \frac{4}{9} \times 10 \times 200 = \frac{8000}{9}$$

$$\Rightarrow F = 889\ \text{N}$$

Hence, the correct answer is (B).

102. Let r be the radius of the satellite. Then

$$v_0^2 = \frac{GM}{r}$$



By Law of Conservation of Mechanical Energy,

$$(U+K)_A = (U+K)_B$$

$$\Rightarrow -\frac{GMm}{r} + \frac{1}{2}m(0)^2 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow v^2 = \frac{2GM}{R} - \frac{2GM}{r} = v_e^2 - 2v_0^2$$

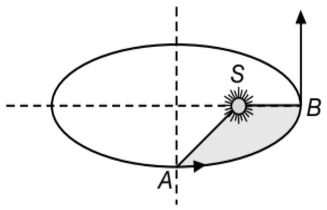
$$\Rightarrow v = \sqrt{v_e^2 - 2v_0^2}$$

Hence, the correct answer is (D).

103. L is conserved both in magnitude and direction.

Hence, the correct answer is (C).

104. Since, we know that areal velocity of planet is constant. So, we have



$$\frac{\text{Area of Ellipse}}{\text{Period of Revolution}} = \frac{\text{Area } SAB}{t_{AB}}$$

$$\Rightarrow \quad t_{AB} = \left(\frac{\text{Area } SAB}{\text{Area of ellipse}}\right)T$$

$$\Rightarrow \quad t_{AB} = \frac{T\left(\frac{\pi ab}{4} - \frac{1}{2}(b)(ea)\right)}{\pi ab}$$

$$\Rightarrow \quad t_{AB} = T\left(\frac{1}{4} - \frac{e}{2\pi}\right)$$

Hence, the correct answer is (B).

105. Gravitational field inside the cavity is

$$\vec{E} = \frac{4}{3}\pi\rho G\vec{r}$$

where ρ is mass density and $\vec{r}$ is separation between centre of sphere and centre of cavity. Escape velocity from point A is calculated by applying Law of Conservation of Energy according to which

$$(U+K)_A = (U+K)_\infty$$

$$\frac{1}{2}mv_{\text{esc}}^2 - \frac{GMm}{R} + \frac{G\left(\frac{M}{9}\right)m}{\left(\frac{5R}{3}\right)} = 0$$

$$\Rightarrow \quad v_{\text{esc}} = \sqrt{\frac{28GM}{15R}}$$

Hence, the correct answer is (A).

106. $L = mvr$ ...(1)

Also, $\dfrac{mv^2}{r} = \dfrac{GMm}{r^2}$ ...(2)

From equations (1) and (2), we get

$$L = m\sqrt{GMr}$$

$$\Rightarrow \quad L \propto r^{1/2}$$

Hence, the correct answer is (D).

107. Let us consider the shell when a mass m is already piled on it by the agency. If V is the potential on the shell, then

$$V = -\frac{Gm}{R}$$

To add a mass dm further, we have

$$dW = Vdm$$

$$\Rightarrow \quad dW = -\frac{Gm}{R}dm$$

$$\Rightarrow \quad W = -\frac{G}{R}\int_0^M m\,dm$$

$$\Rightarrow \quad W = -\frac{1}{2}\frac{GM^2}{R}$$

$$\Rightarrow \quad W = \text{Potential Energy of Interaction}$$

Hence, the correct answer is (B).

108. Total energy of satellite in the first case is $E_i = -\dfrac{GMm}{2r}$

and in the second case is $E_f = \dfrac{GMm}{2\left(\frac{3r}{2}\right)} = -\dfrac{GMm}{3r}$

Energy increased $\Delta E = \dfrac{GMm}{r}\left(\dfrac{1}{2} - \dfrac{1}{3}\right) = \dfrac{GMm}{6r}$

So, percentage increase $= \dfrac{\left(\frac{GMm}{6r}\right)}{\left(\frac{GMm}{2r}\right)} \times 100\% = 33.33\%$

Hence, the correct answer is (C).

109. $\Delta E = U_f - U_i = -\dfrac{GM^2}{2(2R)} + \dfrac{GM^2}{2R} = \dfrac{GM^2}{4R}$

Hence, the correct answer is (C).

110. With respect to earth, satellite appears after every 6 hours over same place. Since it is revolving in opposite direction to that of earth, angular speed of satellite relative to earth is

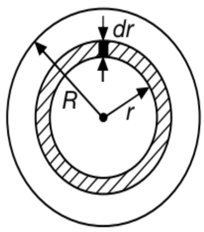
$$\omega_{\text{relative}} = \omega_{\text{satellite}} + \omega_{\text{earth}} = \frac{2\pi}{T} = \frac{2\pi}{6}$$

$$\Rightarrow \quad \frac{2\pi}{24} + \omega_{\text{satellite}} = \frac{2\pi}{6}$$

$$\Rightarrow \quad \omega_{\text{satellite}} = \frac{2\pi}{6} - \frac{2\pi}{24} = \frac{\pi}{4}\ \text{radhr}^{-1}$$

Hence, the correct answer is (C).

111. $dU = -\dfrac{G\,m\,dm}{r}$



$$\Rightarrow \quad dU = -\frac{G\left(\frac{4}{3}\pi r^3\rho\right)\left(4\pi r^2 dr\rho\right)}{r}$$

$$\Rightarrow \quad dU = -\frac{16\pi^2 G\rho^2}{3}r^4 dr$$

$$\Rightarrow \quad U = -\frac{16}{3}\pi^2 G\left(\frac{M}{\frac{4}{3}\pi R^3}\right)^2 \int_0^R r^4 dr$$

$$\Rightarrow \quad U = \left(-\frac{16}{3}\pi^2 G\right)\left(\frac{M^2}{\frac{16}{9}\pi^2 R^6}\right)\left(\frac{R^5}{5}\right)$$

$$\Rightarrow \quad U = -\frac{3}{5}\frac{GM^2}{R}$$

Hence, the correct answer is (C).

112. $\frac{dA}{dt} = \frac{A}{T} = \frac{L}{2m}$

$$\Rightarrow \quad L = \frac{2mA}{T}$$

Hence, the correct answer is (C).

113. $\text{K.E.} = \frac{GMm}{2r} = -E_0$, and $\text{P.E.} = -\frac{GMm}{r} = 2E_0$

$$\Rightarrow \quad \text{T.E.} = \text{K.E.} + \text{P.E.} = -\frac{GMm}{2r} = E_0$$

Hence, the correct answer is (C).

114. Since the satellite appears after every 8 hours above the same place on earth, therefore its angular speed relative to earth is $\frac{2\pi}{8}$. Hence

$$\frac{2\pi}{8} = \omega_{\text{satellite}} - \omega_{\text{earth}}$$

$$\Rightarrow \quad \frac{2\pi}{8} = \frac{2\pi}{T_{\text{satellite}}} - \frac{2\pi}{24}$$

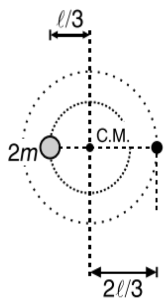
$$\Rightarrow \quad \frac{2\pi}{T_{\text{satellite}}} = \frac{2\pi}{8} + \frac{2\pi}{24} = \frac{2\pi}{6}$$

$$\Rightarrow \quad T_{\text{satellite}} = 6 \text{ hr}$$

Hence, the correct answer is (C).

115. The system will revolve/rotate about an axis passing through the centre of mass of the combined system. Considering origin at the particle of mass $2m$, we have the centre of mass at a distance $\frac{\ell}{3}$ from $2m$ and $\frac{2\ell}{3}$ from m.

The gravitational force of attraction between $2m$ and m provides the necessary centripetal force to the mass to revolve in a circle of radius $\frac{2\ell}{3}$ for m or $\frac{\ell}{3}$ for $2m$.



$$\Rightarrow \quad m\left(\frac{2\ell}{3}\right)\omega^2 = \frac{Gm(2m)}{\ell^2}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3Gm}{\ell^3}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\ell^3}{3Gm}}$$

Hence, the correct answer is (C).

116. $W' = \frac{3}{4}$

$$\Rightarrow \quad g_{\text{eff}} = \frac{3}{4}g$$

At equator, we have

$$g_{\text{eff}} = g - R\omega^2 = \frac{3}{4}g$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{4R}} = \frac{1}{2}\left(\sqrt{\frac{g}{R}}\right) = \frac{1}{2} \times \frac{1}{800}$$

$$\Rightarrow \quad \omega = \frac{1}{1600} \text{ rads}^{-1}$$

Hence, the correct answer is (C).

117. Let gravitational field be zero at a point lying at distance x from M. Then,

$$\frac{GM}{x^2} = \frac{Gm}{(d-x)^2}$$

$$\Rightarrow \quad \frac{d-x}{x} = \sqrt{\frac{m}{M}}$$

$$\Rightarrow \quad \frac{d}{x} - 1 = \sqrt{\frac{m}{M}}$$

$$\Rightarrow \quad x = \left(\frac{\sqrt{M}}{\sqrt{M}+\sqrt{m}}\right)d \qquad \ldots(1)$$

$$\Rightarrow \quad (d-x) = \left(\frac{\sqrt{m}}{\sqrt{M}+\sqrt{m}}\right)d \qquad \ldots(2)$$

Since, $V_P = -\frac{Gm}{d-x} - \frac{GM}{x}$ $\qquad \ldots(3)$

Substituting (1) and (2) in (3), we get

$$V_P = -\frac{G}{d}\left(\sqrt{m}+\sqrt{M}\right)^2$$

Hence, the correct answer is (D).

118. $\left(\begin{matrix}\text{Total}\\ \text{Mechanical Energy}\end{matrix}\right)_P = \left(\begin{matrix}\text{Total}\\ \text{Mechanical Energy}\end{matrix}\right)_O$

$$\Rightarrow \quad \frac{1}{2}m(0)^2 - \frac{GMm}{\sqrt{\left(\sqrt{3}R\right)^2 + R^2}} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$\Rightarrow \quad -\frac{GMm}{2R} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

CHAPTER 4

$$\Rightarrow \quad v=\sqrt{\frac{GM}{R}}$$

Hence, the correct answer is (A).

119. By Law of Conservation of Energy, we have

$$(U+K)_{axis}=(U+K)_C$$

If m_0 be the mass of the particle, then

$$-\frac{Gmm_0}{\sqrt{r^2+r^2}}+\frac{1}{2}m_0(0)^2=-\frac{Gmm_0}{r}+\frac{1}{2}m_0v^2$$

$$\Rightarrow \quad \frac{1}{2}m_0v^2=\frac{Gmm_0}{r}\left(1-\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad v=\sqrt{\frac{2Gm}{r}\left(1-\frac{1}{\sqrt{2}}\right)}$$

Hence, the correct answer is (C).

120. $U_i=-\dfrac{GMm}{R}$ and $U_f=0$

$$\Rightarrow \quad W=\Delta U=\frac{GMm}{R}$$

Hence, the correct answer is (A).

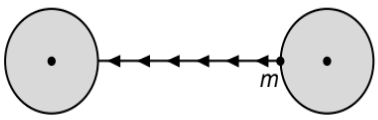
121. Use Gauss Theorem for Gravitation.

Hence, the correct answer is (C).

122. $E=-\dfrac{dV}{dr}$

Hence, the correct answer is (C).

123. Centre point is the unstable equilibrium position where potential energy is maximum.



Hence, the correct answer is (C).

124. Since, $g=\dfrac{GM}{R^2}=GMR^{-2}$

Fractional change in g is

$$\frac{\Delta g}{g}=\frac{\Delta M}{M}-\frac{2\Delta R}{R}$$

$$\Rightarrow \quad \frac{\Delta g}{g}=0.5\%-2(0.5\%)=-0.5\%$$

So, g will decrease by 0.5%

$$v_e=\sqrt{\frac{2GM}{R}}$$

Fractional change in escape velocity is

$$\frac{\Delta v_e}{v_e}=\frac{1}{2}\frac{\Delta M}{M}-\frac{1}{2}\frac{\Delta R}{R}$$

Percentage change in escape velocity is

$$\frac{\Delta v_e}{v_e}=\frac{1}{2}(0.5-0.5)=0\%$$

So, escape velocity will remain same.

Potential energy of an object of mass m on the surface of earth is

$$U=-\frac{GMm}{R}$$

Fractional change in potential energy is

$$\frac{\Delta U}{U}=-\frac{\Delta M}{M}+\frac{\Delta R}{R}$$

Percentage change in PE is

$$\frac{\Delta U}{U}-0.5+0.5=0\%$$

So, potential energy will remain same.

Hence, the correct answer is (C).

Multiple Correct Choice Type Questions

1. $T^2=\dfrac{4\pi^2}{GM}(R_e+h)^3$ and $v_0=\sqrt{\dfrac{GM}{R_e+h}}$

If $h \quad R_e$, then

$$T=2\pi\sqrt{\frac{R_e}{g}}$$

$$\Rightarrow \quad T=84.4 \text{ minute}$$

Hence, (B) and (C) are correct.

2. $U_i=\dfrac{-GMm}{R}=U$ (given)

$$\Delta U=U_f-U_i=\frac{-GMm}{(R+R)}+\frac{GMm}{R}$$

$$\Rightarrow \quad \Delta U=\frac{GMm}{2R}=-\frac{U}{2}$$

Same is the case with potential.

Hence, (A) and (D) are correct.

3. At a point P lying outside both the shells, the field will be due to both the shells as if they were point masses with total mass (m_1+m_2) concentrated at the centre. So, $E=G\left(\dfrac{m_1+m_2}{r^2}\right)$ for $r>r_1$ and $r>r_2$.

If P lies inside one shell and outside the other. Then field at point P due to the shell enclosing it is zero and is $\dfrac{Gm_2}{r^2}$ due to the other shell.

If P lies inside both the shells, then $E_p=0$.

Hence, (A), (B) and (C) are correct.

4. Similarly, as in SOLUTION to PROBLEM 3,

$$V_p = -\frac{G(m_1+m_2)}{r} \text{ for } r > R_1,\ r > R_2$$

Which satisfies OPTION **(A)**

For the point P lying inside shell 1 and outside shell 2, the total potential at P equals the potential at P due to shell 1 plus potential at P due to shell 2.

$$V_p = V_{p1} + V_{p2}$$

$$\Rightarrow\ V_p = \frac{-Gm_1}{r_1} + \frac{-Gm_2}{r}$$

$$\Rightarrow\ V_p = -G\left(\frac{m_1}{r_1} + \frac{m_2}{r}\right)$$

Which satisfies OPTION **(C)**

(Since field inside shell 1 is zero, so potential at a point lying inside shell 1 is a constant and that equals the value of the potential at the surface of shell 1 i.e. $-\frac{Gm_1}{r_1}$).

Similarly, for point P lying inside both the shells, we have

$$V = -\frac{Gm_1}{r_1} - \frac{Gm_2}{r_2}$$

$$V = -G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2}\right) \text{ for } r < r_2$$

Which satisfies OPTION **(D)**

Hence, (A), (C) and (D) are correct.

5. **Conceptual (A), (B), (C) and (D) are correct.**

6. Inside the inner sphere field is zero but potential is constant.

Between two, field is due to inner sphere and potential is due to both but it is constant due to outer shell.

Outside the outer shell, field and potential are due to both and it decreases in both the cases.

Hence, (A) and (C) are correct.

7.

Velocity of Satellite	Nature of Path
$v = v_0$	Circular path around the earth.
$v < v_0$	Elliptical path and body returns to earth.
$v > v_0$ but $< v_e$	Elliptical path around the earth and will not escape.
$v = v_e$	Parabolic path and it escapes from the earth.
$v > v_e$	Hyperbolic path and escapes from earth.

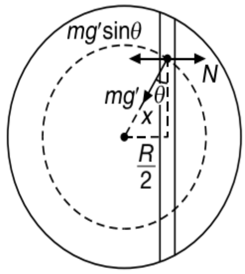
Hence, (A), (B), (C) and (D) are correct.

8. Net force towards centre of earth is mg', where

$$g' = g\left(\frac{x}{R}\right)$$

$$\Rightarrow\ mg' = \frac{mgx}{R}$$

Normal force $N = mg'\sin\theta$



Since $\sin\theta = \frac{\frac{R}{2}}{x}$

$$\Rightarrow\ \sin\theta = \frac{R}{2x}$$

$$\Rightarrow\ N = \left(\frac{mgx}{R}\right)\left(\frac{R}{2x}\right)$$

$$\Rightarrow\ N = \frac{mg}{2}$$

Constant and independent of x

Tangential force is given by

$$F = ma = mg'\cos\theta$$

$$\Rightarrow\ a = g'\cos\theta$$

Since $\cos\theta = \frac{\sqrt{\frac{R^2}{4} - x^2}}{x}$

$$\Rightarrow\ a = \frac{gx}{R}\sqrt{R^2 - 4x^2}$$

So, curve is parabolic and at $x = \frac{R}{2}$ we have $a = 0$

Hence, (B) and (C) are correct.

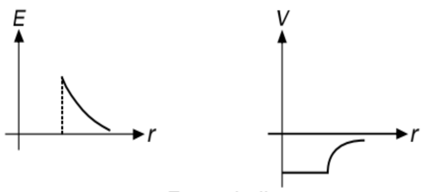
9. $V = -\frac{GM}{R}$

$$\Rightarrow\ V = -\frac{GM}{R^2}R$$

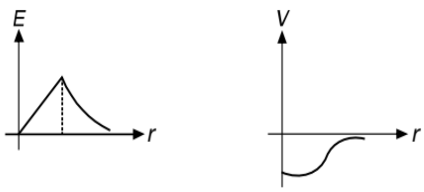
$$\Rightarrow\ V = -gR$$

Hence, (C) and (D) are correct.

10. E-r and V-r graphs for a spherical shell and a solid sphere are shown here.



For a shell



For a solid sphere

Hence, (A), (B) and (C) are correct.

11. $E = -\dfrac{GMm}{2a}$

So, $E \propto m$

$E \propto m_s$

$E \propto \dfrac{1}{a}$

Hence, (A), (B) and (C) are correct.

12. $T^2 = \dfrac{4\pi^2}{GM}\left(\dfrac{r_A + r_P}{2}\right)^3$ $\left\{\because r = \dfrac{r_A + r_P}{2}\right\}$

$\Rightarrow T^2 = \dfrac{\pi^2}{2GM}(r_A + r_P)^3$

By Law of Conservation of Angular Momentum

$mv_A r_A = mv_P r_P$

$\Rightarrow v_A r_A = v_P r_P$

Hence, (B), (C) and (D) are correct.

13. Time period in both the cases is

$$T_1 = T_2 = 2\pi\sqrt{\frac{R^3}{GM}} \approx 84.6 \text{ min}$$

However, $v_1 > v_2$, because the difference in potential energy between the extreme position and mean position will be more in the first case.
Hence, (B) and (D) are correct.

14. $E_g = -\dfrac{dV}{dx}$

If $E_g = 0$, then $V = \text{constant}$ and this constant may also be zero.
Hence, (A) and (C) are correct.

15. At two positions, when the planet is closest to the sun (perigee) and when it is farthest from the sun (apogee), velocity vector is perpendicular to force vector i.e., work done is zero. In one complete revolution work done is zero.
Hence, (A) and (D) are correct.

16. The field inside the shell is zero and so potential inside the shell is constant equal to the value that exists at the surface i.e. $-\dfrac{GM}{a}$.
Hence, (A) and (D) are correct.

17. **Hence, (B) and (D) are correct.**

18. By Law of Conservation of Mechanical Energy, we get

$(U + K)_\infty = (U + K)_r$

$\Rightarrow 0 + 0 = -\dfrac{Gm(4m)}{r} + \dfrac{1}{2}\mu v_r^2$

$\Rightarrow \dfrac{G(m)(4m)}{r} = \dfrac{1}{2}\mu v_r^2$ ...(1)

where, $\mu = \text{reduced mass} = \dfrac{(m)(4m)}{m + 4m} = \dfrac{4m}{5}$ and

$v_r =$ relative velocity of approach

Substituting in equation (1), we get

$\Rightarrow v_r = \sqrt{\dfrac{10Gm}{r}}$

From equation (1), the total kinetic energy is

$K = \dfrac{G(m)(4m)}{r}$

$\Rightarrow K = \dfrac{4Gm^2}{r}$

Net torque of two equal and opposite forces acting on two objects is zero. Therefore, angular momentum will remain conserved. Initially both the objects were stationary i.e., angular momentum about any point was zero. Hence, angular momentum of both the particles about any point will be zero at all instants.
Hence, (A), (B), (C) and (D) are correct.

19. When total force is zero, $g_{\text{eff}} = 0$

$V = -\dfrac{Gm}{r}$

For a shell of radius R,

$V_{\substack{\text{inside}\\ \text{shell}}} = V_{\text{surface}} = -\dfrac{GM}{R}$

$I_{\text{inside}} = 0$ whereas

$I_{\text{outside}} = \dfrac{GM}{r^2}$

So, plot of V vs r is continuous whereas plot of I vs r is discontinuous
Hence, (A), (B) and (C) are correct.

20. Orbital radius of first satellite is $2R$ and that of second satellite is $8R$.
For a satellite

$K.E. = \dfrac{GMm}{2r}$

$P.E. = -\dfrac{GMm}{r}$

$T.E. = -\dfrac{GMm}{2r}$

Hence, (A), (B) and (C) are correct.

21. **Conceptual (A), (B) and (C) are correct.**

22. According to Kepler's Laws Areal Speed is constant. Also, by Law of Conservation of Angular Momentum L must be constant.
Hence, (B) and (C) are correct.

23. According to Right Hand Thumb Rule curl the fingers of right hand in the direction of rotation then thumb gives the direction of the areal velocity/angular momentum.
Hence, (A) and (C) are correct.

24. At centre, $V = -\frac{3}{2}\frac{GM}{R}$

and $E = 0$

Hence, (A) and (D) are correct.

25. Force on P is $\frac{Gm_1m_3}{r^2}$, attractive i.e. towards O.

Hence, (A), (B) and (D) are correct.

26. $v_0 = \sqrt{\frac{GM}{r}}$

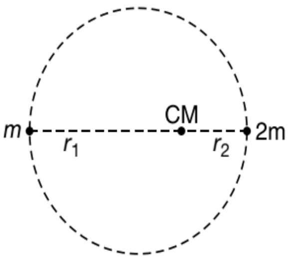
$T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$

$U = \frac{-GMm}{2r}$

$K = \frac{GMm}{2r}$

Hence, (A) and (D) are correct.

27.



Let CM of system be at a distance r_1 from m and r_2 from $2m$. Then, $mr_1 = 2m(r - r_1)$

$\Rightarrow \quad r_1 = \frac{r}{3}$ and $r_2 = r - r_1 = \frac{2r}{3}$

Since, $T_2^2 = \frac{4\pi^2 r_2^3}{Gm}$

$\Rightarrow \quad T_2^2 = \frac{32\pi^2 r^3}{27Gm}$

$\Rightarrow \quad T_2 \propto r^{\frac{3}{2}}$ and $T_2 \propto m^{-\frac{1}{2}}$

Hence, (A) and (D) are correct.

28. Conceptual (A), (B) and (D) are correct.

29. Distance from centre of sun and hence the kinetic energy and potential energy keep changing.
Hence, (C) and (D) are correct.

30. $U_i = -\frac{GMm}{R}$

$\Rightarrow \quad U_f = -\frac{GMm}{R+h}$

$(\text{Work done}) = (\text{Change in G.P.E.})$

$\Rightarrow \quad W = U_f - U_i = -GMm\left(\frac{1}{R+h} - \frac{1}{R}\right)$

$\Rightarrow \quad W = -\frac{GMm}{R}\left[\left(1+\frac{h}{R}\right)^{-1} - 1\right]$

$\Rightarrow \quad W = -\frac{GMm}{R}\left[1 - \frac{h}{R} - 1\right]$

$\Rightarrow \quad W \approx \frac{GMmh}{R^2}$

$\Rightarrow \quad W \approx mgh$ for $h \quad R$

Also, $U_i = -\frac{GMm}{R}$, and

$U_f(h = R) = -\frac{GMm}{R+R} = -\frac{GMm}{2R}$

$\Rightarrow \quad W = U_f - U_i = \frac{GMm}{2R}$

$\Rightarrow \quad W = \frac{1}{2}mgR \qquad \left\{\because g = \frac{GM}{R^2}\right\}$

Hence, (A) and (D) are correct.

31. $T^2 = \frac{4\pi^2}{GM}(R+h)^3$

Since $h \quad R$

$\Rightarrow \quad T^2 = \text{MIN} = \frac{4\pi^2 R^3}{GM}$

Also, $v = \frac{2\pi R}{T}$

$\Rightarrow \quad v$ is MAXIMUM

and total energy is $E = -\frac{GMm}{2R} = \text{MINIMUM}$

Hence, (A), (B) and (C) are correct.

32. Since, $t = nT$

$\Rightarrow \quad T = \frac{t}{n} = \frac{40}{20}$

$\Rightarrow \quad T = 2\text{ s}$

Now, $\Delta T = n\Delta T$

$\Rightarrow \quad \frac{\Delta t}{t} = \frac{\Delta T}{T}$

So, $\frac{1}{40} = \frac{\Delta T}{2}$

$\Rightarrow \quad \Delta T = 0.05$

Since, time period, $T = 2\pi\sqrt{\frac{\ell}{g}}$

$\Rightarrow \quad \frac{\Delta T}{T} = -\frac{1}{2}\frac{\Delta g}{g}$

$\Rightarrow \quad -\frac{\Delta g}{g} = 2\frac{\Delta T}{T}$

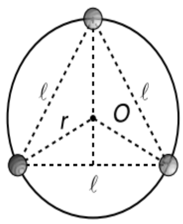
So, percentage error in g is $\frac{\Delta g}{g} \times 100\%$

% Error $= -2\frac{\Delta T}{T} \times 100\% = -2 \times \frac{0.05}{2} \times 100\%$

$\Rightarrow \quad$ % Error $= 5\%$

Hence, (A) and (C) are correct.

33. The force of attraction between any two point masses is responsible for providing the necessary centripetal force to a mass to revolve in a circle of radius r. Using trigonometry, we get



$$\cos 30 = \frac{\ell/2}{r} = \frac{\ell}{2r}$$

$\Rightarrow \quad r = \frac{\ell}{2\cos 30}$

$\Rightarrow \quad r = \frac{\ell}{\sqrt{3}}$

$\Rightarrow \quad mr\omega^2 = \frac{Gmm}{\ell^2}$

$\Rightarrow \quad m\frac{\ell}{\sqrt{3}}\omega^2 = \frac{Gm^2}{\ell^2}$

$\Rightarrow \quad \omega = \sqrt{\frac{\sqrt{3}Gm}{\ell^3}}$

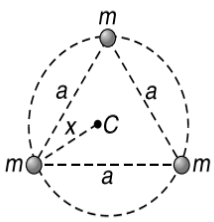
$\Rightarrow \quad T = 2\pi\sqrt{\frac{\ell^3}{\sqrt{3}Gm}}$

$\Rightarrow \quad T \propto \ell^{\frac{3}{2}}$ and {OPTION **(A)**}

$T \propto m^{-\frac{1}{2}}$ {OPTION **(D)**}

Hence, (A) and (D) are correct.

34.



Distance of any mass from centre is

$$r = \frac{a}{\sqrt{3}}$$

So, radius of circular path followed is

$$r = \frac{a}{\sqrt{3}}$$

Mass is moving in circular path of radius $r = \frac{a}{\sqrt{3}}$

Such that gravitational force on the particle due to other two provides the necessary centripetal force.

$\Rightarrow \quad \frac{mv^2}{\left(\frac{a}{\sqrt{3}}\right)} = \frac{\sqrt{3}Gm^2}{a}$

$\Rightarrow \quad v = \sqrt{\frac{Gm}{a}}$

$\Rightarrow \quad T = \frac{2\pi\left(\frac{a}{\sqrt{3}}\right)}{v} = \sqrt{\frac{2\pi a^3}{3Gm}}$

Total kinetic energy of the particles is

$$K = 3\left(\frac{1}{2}mv^2\right) = \frac{3}{2}\frac{Gm^2}{a}$$

Potential energy U of the system is

$$U = -3\left(\frac{Gm^2}{a}\right)$$

$\Rightarrow \quad$ Total energy E is

$$E = U + K = -\frac{3}{2}\left(\frac{Gm^2}{a}\right)$$

$\Rightarrow \quad$ Binding Energy $= \frac{3}{2}\left(\frac{Gm^2}{a}\right)$

Hence, (B) and (C) are correct.

35. Kinetic energy, $KE = \frac{GMm}{2r}$

Potential energy, $PE = -\frac{GMm}{r}$

and the total energy, $E = -\frac{GMm}{2r}$

Kinetic energy is always positive and $KE \propto \frac{1}{r}$

Potential energy is negative and $|PE| \propto \frac{1}{r}$

Similarly, total energy is also negative and $|E| \propto \frac{1}{r}$

Also $|E| < |PE|$, so from the graph we observe that A is kinetic energy, B is potential energy and C is total energy of the satellite.

Hence, (A), (B) and (D) are correct.

36. At all the places potential will increase by $\frac{GM}{R}$, so $V_\infty = \frac{GM}{R}$ and $V_{centre} = -\frac{3GM}{2R} + \frac{GM}{R} = -\frac{GM}{2R}$

Hence, (C) and (D) are correct.

37. **Conceptual (A), (B), (C) and (D) are correct.**

38. Both the stars will revolve about their centre of mass. So, if the centre of mass be at a distance x from $2m$, then

$$x = \frac{2m(0)+mr}{3m} = \frac{r}{3}$$

So, $r_1 = \frac{2r}{3}$ and $r_2 = \frac{r}{3}$

ω and T will be same for both the stars, so

$$K_1 = \frac{1}{2}I_1\omega^2 \text{ and } K_2 = \frac{1}{2}I_2\omega^2$$

$$\Rightarrow \frac{K_1}{K_2} = \frac{I_1}{I_2} = \frac{m\left(\frac{2r}{3}\right)^2}{2m\left(\frac{r}{3}\right)^2} = 2$$

$$L_1 = I_1\omega \text{ and } L_2 = I_2\omega$$

$$\Rightarrow \frac{L_1}{L_2} = \frac{I_1}{I_2} = 2$$

Hence, (A), (B) and (C) are correct.

Reasoning Based Questions

1. $\frac{d\vec{A}}{dt} = \frac{\vec{L}}{2m}$

Since $\vec{L}$ = constant

$$\Rightarrow \frac{d\vec{A}}{dt} = \text{constant}$$

Also, $\frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega$

$$\Rightarrow \frac{dA}{dt} = \frac{mr^2\omega}{2m} = \frac{L}{2m} = \text{constant}$$

Hence, the correct answer is (A).

2. The torque on a body is given by $\vec{\tau} = \frac{d\vec{L}}{dt}$. In case of planet orbiting around Sun no torque is acting on it. So,

$$\frac{d\vec{L}}{dt} = 0$$

$$\Rightarrow \vec{L} = \text{constant}$$

Hence, the correct answer is (A).

3. Using energy conservation $\frac{1}{2}mv_e^2 - \frac{GMm}{R} = 0$

$$\Rightarrow v_e = \sqrt{\frac{2Gm}{R}} = \sqrt{2gR}$$

Hence, the correct answer is (A).

4. Although no gravitational field is produced inside a symmetric shell, it produces a field at points outside of shell.

Hence, the correct answer is (D).

5. Work will be done only in bringing the unit mass from infinity upto the surface of shell because inside shell there is no gravitational field and in moving inside the shell no work will be done.

Hence, the correct answer is (D).

6. $g' = g\left(1 - \frac{2h}{R}\right)$

$$\Rightarrow \Delta g = g - g' = g\left(\frac{2h}{R}\right)$$

$$\Rightarrow \frac{dW}{dh} = \frac{d}{dh}(m\Delta g) = m\frac{d}{dh}(\Delta g) = m\frac{d}{dh}\left(\frac{2gh}{R}\right)$$

$$\Rightarrow \frac{dW}{dh} = \frac{2mg}{R} = \text{constant}$$

Hence, the correct answer is (B).

7. $V_{in} = \frac{GM}{2R^3}(3R^2 - r^2)$

At surface, $r = R$, so $V_s = \frac{GM}{R}$

At centre, $r = 0$, so $V_{centre} = \frac{3}{2}\frac{GM}{R}$

$$\Rightarrow V_{centre} = \frac{3}{2}V_S$$

$$\Rightarrow V_{in} > V_S$$

V is not same everywhere as indicated by V_{in}.

Hence, the correct answer is (C).

8. The time period of satellite which is very near to earth is given by $T = 2\pi\sqrt{\frac{R}{g}} = 84 \text{ min} = 1 \text{ hr } 24 \text{ min}$

Hence, the correct answer is (A).

9. Work done in conservative field in cyclic process is zero.

Hence, the correct answer is (A).

10. Acceleration due to gravity is given by $g = \frac{GM}{r^2}$, so it does not depend on mass of body on which it is acting. Also, it is not a constant quantity because it changes with the change in value of both M and r (distance between two bodies).

Hence, the correct answer is (C).

Linked Comprehension Type Questions

1. According to Kepler's Law, we have

$$T^2 \propto r^3$$

$$\Rightarrow \frac{T_m^2}{T_e^2} = \frac{r_m^3}{r_e^3}$$

$$\Rightarrow T_m = \left(\frac{r_m}{r_e}\right)^{3/2} T_e$$

Since $T_e = 1$ yr

$$\Rightarrow T_m = \left(\frac{6\times10^{10}}{1.5\times10^{11}}\right)^{3/2}(1\text{ yr})$$

$$\Rightarrow T_m = \left(\frac{6\times10^{10}}{15\times10^{10}}\right)^{3/2}\text{ yr}$$

$$\Rightarrow T_m = \left(\frac{2}{5}\right)^{3/2}\text{ yr}$$

Hence, the correct answer is (D).

2. Since orbital velocity v_o is given by

$$v_o = \sqrt{\frac{GM}{r}}$$

$$\Rightarrow (v_o)_m = \sqrt{\frac{GM_{sun}}{r_m}}$$

$$\Rightarrow (v_o)_e = \sqrt{\frac{GM_{sun}}{r_e}}$$

$$\Rightarrow \frac{(v_o)_m}{(v_o)_e} = \sqrt{\frac{r_e}{r_m}}$$

$$\Rightarrow \frac{(v_o)_m}{(v_o)_e} = \sqrt{\frac{1.5\times10^{11}}{6\times10^{10}}} = \sqrt{\frac{15}{6}}$$

$$\Rightarrow \frac{(v_o)_m}{(v_o)_e} = \sqrt{\frac{5}{2}}$$

Hence, the correct answer is (A).

3. $$U_i = -\frac{GMm}{R} = -\frac{GMm}{R+\frac{R}{4}}$$

$$\Rightarrow \Delta U = U_f - U_i = \frac{1}{5}\frac{GMm}{R}$$

$$\Rightarrow \Delta U = \frac{mgR}{5} \qquad \left\{\because g = \frac{GM}{R^2}\right\}$$

Hence, the correct answer is (C).

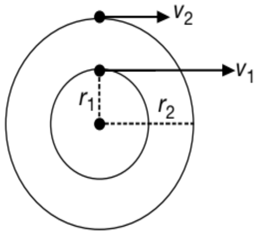
4. By Law of Conservation of Energy

$$\Delta U = \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{\frac{2}{5}gR}$$

Hence, the correct answer is (D).

5. $$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$



$$\Rightarrow \frac{1}{64} = \frac{r_1^3}{r_2^3}$$

$$\Rightarrow r_2 = 4r_1$$

$$\Rightarrow r_2 = 4\times10^4\text{ km}$$

$$v_1 = \frac{2\pi r_1}{T_1} \text{ and } v_2 = \frac{2\pi r_2}{T_2}$$

$$\Rightarrow v_1 = \frac{2\pi(10^4)}{1}\text{ kmh}^{-1}$$

and $$v_2 = \frac{2\pi(4\times10^4)}{8} = \pi\times10^4\text{ kmh}^{-1}$$

$\Rightarrow$ Speed of S_2 relative to S_1 is v_{21}

$$v_{21} = -\pi\times10^4\text{ kmh}^{-1}$$

Hence, the correct answer is (A).

6. $$\omega = \frac{v_2 - v_1}{r_2 - r_1}$$

Hence, the correct answer is (B).

7. $$F = \begin{cases}\frac{GMm}{r^2}, & r \ge R\\ 0, & r < R\end{cases}$$

Hence, the correct answer is (A).

8. $$F(r) = \begin{cases}\frac{GMm}{r^2}, & r \ge R\\ \frac{4\pi G\rho r m}{3}, & r < R\end{cases}$$

where ρ is density of sphere

Hence, the correct answer is (C).

9. Given that $v_o = \frac{v_e}{2}$, where $v_e = \sqrt{\frac{2GM}{R}}$

Since $v_o = \sqrt{\frac{GM}{R+h}} = \frac{1}{2}\sqrt{\frac{2GM}{R}}$

$$\Rightarrow R + h = 2R$$

$$\Rightarrow h = R$$

Hence, the correct answer is (C).

10. Since the satellite is stopped suddenly, so its kinetic energy in this orbit becomes zero, however its gravitational potential energy in this orbit is

$$U = -\frac{GMm}{R+h} = -\frac{GMm}{2R}$$

Since $(U+K)_{\text{initial}} = (U+K)_{\text{final}}$

$$\Rightarrow \quad -\frac{GMm}{2R} + \frac{1}{2}m(0)^2 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}v^2 = \frac{GM}{2R}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}}$$

Also, we know that $g = \dfrac{GM}{R^2}$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

Hence, the correct answer is (A).

11. $T = 2\pi\sqrt{\dfrac{R}{g}}$, substituting $g = \dfrac{GM}{R^2}$ we get,

$$T = 2\pi\sqrt{\frac{R^3}{GM}}$$

Hence, (A), (B) and (C) are correct.

12. Maximum speed is attained by the particle at centre. Applying Law of Conservation of Mechanical Energy (from surface to centre), we get

$$(U+K)_{\text{surface}} = (U+K)_{\text{centre}}$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}m(0)^2 = -\frac{3}{2}\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$v = \sqrt{2\left(-\frac{GM}{R} + \frac{3GM}{2R}\right)} = \sqrt{\frac{GM}{R}}$$

Hence, the correct answer is (B).

13. $U = -\dfrac{GmM}{\sqrt{a^2 + x^2}}$

Hence, the correct answer is (A).

14. Since $F = \dfrac{GMmx}{\left(a^2 + x^2\right)^{3/2}}$

When $x = 0$, $F = 0$

Hence, the correct answer is (D).

15. For small x i.e., x a, the force acting on the particle is

$$F \approx -\left(\frac{GMm}{a^3}\right)x$$

The negative sign showing that this force is directed towards the mean position (i.e. position where $F = 0$). Hence the particle will perform oscillations about the mean position i.e. the centre of the ring.

Hence, the correct answer is (B).

16. Given that $r = 3.6 \times 10^7$ m, $m = 5000$ kg,

$v = 4000\ \text{ms}^{-1}$ and $\phi = 30°$

Angular momentum of satellite

$$L = mvr\sin\phi$$

$$\Rightarrow \quad L = 5000 \times 4000 \times 3.6 \times 10^7 \times \sin 30°$$

$$\Rightarrow \quad L = 3.6 \times 10^{14}\ \text{Js}$$

Hence, the correct answer is (D).

17. Energy of the satellite is

$$E = K + U = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(5000)(4000)^2$$

$$\Rightarrow \quad K = 4 \times 10^{10}\ \text{J}$$

$$\Rightarrow \quad U = -\frac{\left(6.67 \times 10^{-11}\right) \times \left(5.97 \times 10^{24}\right) \times 5000}{3.6 \times 10^7}$$

$$\Rightarrow \quad U = -5.5 \times 10^{10}\ \text{J}$$

$$\Rightarrow \quad E = K + U = -1.5 \times 10^{10}\ \text{J}$$

Hence, the correct answer is (D).

18. Since $E = -\dfrac{GMm}{2a}$

So, semi-major axis a is given by

$$a = \frac{-GMm}{2E}$$

$$\Rightarrow \quad a = -\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 5000}{2 \times \left(-1.5 \times 10^{10}\right)}$$

$$\Rightarrow \quad a = 6.6 \times 10^7\ \text{m}$$

Hence, the correct answer is (A).

19. Eccentricity (e) of the orbit is given by

$$e = \left(1 + \frac{2EL^2}{G^2M^2m^3}\right)$$

$$\Rightarrow \quad e = 1 + \frac{2 \times \left(-1.5 \times 10^{10}\right) \times \left(3.6 \times 10^{14}\right)^2}{\left(6.67 \times 10^{-11}\right)^2 \times \left(5.97 \times 10^{24}\right)^2 \times (5000)^3}$$

$$\Rightarrow \quad e = 0.804$$

Since, semi-minor axis b is given by

$$b = a\sqrt{1 - e^2}$$

$$\Rightarrow \quad b = 6.6 \times 10^7 \sqrt{1 - (0.804)^2}$$

$$\Rightarrow \quad b = 3.92 \times 10^7\ \text{m}$$

Hence, the correct answer is (B).

20. Minimum distance r_{min} is

$$r_{\text{min}} = a(1 - e)$$

$\Rightarrow \quad r_{min} = 6.6\times10^7 \times(1-0.804)$

$\Rightarrow \quad r_{min} = 1.29\times10^7 \text{ m}$

Hence, the correct answer is (C).

21. Maximum distance r_{max} is

$$r_{max} = a(1+e) = 6.6\times10^7\times(1+0.804)$$

$\Rightarrow \quad r_{max} = 11.9\times10^7 \text{ m}$

Hence, the correct answer is (C).

22. $F_1 = \dfrac{GMm}{(2R)^2} = \dfrac{GMm}{4R^2}$

Hence, the correct answer is (D).

23. Mass of complete sphere,

$$M = \frac{4}{3}\pi R^3 \rho$$

Mass removed $M' = \dfrac{4}{3}\pi\left[\dfrac{R}{2}\right]^3\rho = \dfrac{1}{8}M$

Force due to hollow sphere F_2 at P equals force due to solid sphere at P minus force due to removed mass at P.

$\Rightarrow \quad F_2 = \dfrac{GMm}{4R^2} - \dfrac{G(M/8)m}{(3R/2)^2}$

$\Rightarrow \quad F_2 = \dfrac{GMm}{R^2}\left[\dfrac{1}{4} - \dfrac{4}{9\times8}\right]$

$\Rightarrow \quad F_2 = \dfrac{GMm}{R^2}\left[\dfrac{1}{4} - \dfrac{1}{18}\right]$

$\Rightarrow \quad F_2 = \dfrac{GMm}{R^2}\left[\dfrac{9-2}{36}\right]$

$\Rightarrow \quad F_2 = \dfrac{7}{36}\dfrac{GMm}{R^2}$

Hence, the correct answer is (D).

24. $\dfrac{F_1}{F_2} = \dfrac{GMm}{4R^2}\times\dfrac{36R^2}{7GMm} = \dfrac{9}{7}$

Hence, the correct answer is (A).

25. The system must revolve about the center of mass.

Hence, the correct answer is (B).

26. If we consider the origin at the heavier mass, then the center of mass is at a distance $\dfrac{\ell}{3}$ from the origin. So, the heavier star revolves in an orbit of radius $\ell/3$.

Hence, the correct answer is (C).

27. For m,

$$\frac{Gm(2m)}{\ell^2} = m\left(\frac{2\ell}{3}\right)\omega^2 \qquad \ldots(1)$$

For $2m$,

$$\frac{Gm(2m)}{\ell^2} = (2m)\left(\frac{\ell}{3}\right)\omega^2 \qquad \ldots(2)$$

Adding (1) and (2), we get

$$\frac{4Gm^2}{\ell^2} = \left(\frac{4m\ell}{3}\right)\omega^2$$

$\Rightarrow \quad \omega = \sqrt{\dfrac{3Gm}{\ell^3}}$

Hence, the correct answer is (C).

28. $(\text{K.E.})_{light} = K_L = \dfrac{1}{2}\text{m}\left(\dfrac{2\ell\omega}{3}\right)^2 = \left(\dfrac{2}{9}m\ell\right)\omega^2$

$(\text{K.E.})_{heavy} = K_H = \dfrac{1}{2}(2m)\left(\dfrac{\ell\omega}{3}\right)^2 = \dfrac{m\ell^2\omega^2}{9}$

$\Rightarrow \quad K_L + K_H = \dfrac{1}{3}m\ell^2\omega^2$

Hence, the correct answer is (A).

29. Acceleration due to gravity near the surface of shell can be assumed to be uniform $(h \quad R)$.

$$g = \frac{G(2M)}{(2R)^2} = \frac{GM}{2R^2}$$

Since $h = \dfrac{1}{2}gt^2$

$\Rightarrow \quad t = \sqrt{\dfrac{2h}{g}} = 2\sqrt{\dfrac{hR^2}{GM}}$

Hence, the correct answer is (A).

30. $U_A = \sqrt{2gh} = \sqrt{2\left(\dfrac{Gm}{2R^2}\right)h} = \sqrt{\dfrac{Gmh}{R^2}}$

From A to B, field due to shell is zero, but field due to sphere is non-zero.

Hence, $t_{AB} < \dfrac{R}{V_A}$

$\Rightarrow \quad t_{AB} < \dfrac{R^2}{\sqrt{GMh}}$

Hence, the correct answer is (C).

31. Since $h \quad R$, so we have $K_A \approx 0$

Potential between A and B due to shell is constant. Applying the Law of Conservation of Energy, we get

$$K_A + U_A = K_B + U_B$$

$\Rightarrow \quad K_B = U_A - U_B = m(V_A - V_B)$

$\Rightarrow \quad \dfrac{1}{2}mv_B^2 = m(V_A - V_B)$

$\Rightarrow \quad v_B = \sqrt{2(V_A - V_B)}$

$\Rightarrow \quad v_B = \sqrt{2\left[-\frac{GM}{2R} + \frac{GM}{R}\right]}$

$\Rightarrow \quad v_B = \sqrt{\frac{GM}{R}}$

Hence, the correct answer is (D).

32-33. The correct answer is 32(C) and 33(B).

Combined solution to 32 and 33

If m be the mass of satellite, then by Law of Conservation of Angular Momentum, we have

$$mv_1(2R) = mv_2(4R)$$

$\Rightarrow \quad v_1 = 2v_2 \quad \ldots(1)$

Also, by Law of Conservation of Mechanical Energy, we have

$$(U+K)_{2R} = (U+K)_{4R}$$

$\Rightarrow \quad -\frac{GmM}{2R} + \frac{1}{2}mv_1^2 = -\frac{GmM}{4R} + \frac{1}{2}mv_2^2$

$$\frac{1}{2}(v_1^2 - v_2^2) = \frac{GM}{R}\left(\frac{1}{2} - \frac{1}{4}\right)$$

$\Rightarrow \quad 4v_2^2 - v_2^2 = \frac{GM}{2R}$

$\Rightarrow \quad v_2 = \sqrt{\frac{GM}{6R}}$

Since, $v_1 = 2v_2$

$\Rightarrow \quad v_1 = \sqrt{\frac{2GM}{3R}}$

34. $\frac{mv_1^2}{r_1} = \frac{GmM}{(2R)^2}$

$\Rightarrow \quad r_1 = \frac{4R^2}{GM}v_1^2 = \left(\frac{4R^2}{GM}\right)\left(\frac{2GM}{3R}\right)$

$\Rightarrow \quad r_1 = \frac{8R}{3}$

Hence, the correct answer is (A).

35. Let M be the mass of the planet, m_1 be the mass of the moon 1, m_2 be the mass of moon 2 and m_3 be the mass of moon 3.

$$\frac{GM}{R_2^2} = \omega^2 R^2$$

$\Rightarrow \quad M = \frac{\omega^2 R_2^3}{G}$

$\Rightarrow \quad M = \left(\frac{2\pi}{3\times 10^5}\right)^2 \times (9\times 10^7)^3 \times \frac{1}{6.67\times 10^{-11}}$

$\Rightarrow \quad M = 4\pi^2 \times 81\times 10^{11} \times \frac{10^{11}}{6.67}$

$\Rightarrow \quad M = 4.8\times 10^{24}$

Hence, the correct answer is (D).

36. $\frac{Gm_3}{R_3^2} = 0.2$

$\Rightarrow \quad \frac{6.67\times 10^{-11} m_3}{(2\times 10^5)^2} = 0.2$

$\Rightarrow \quad m_3 = \frac{0.2\times 4\times 10^{10}}{6.67\times 10^{-11}}$

$\Rightarrow \quad m_3 \approx 0.12\times 10^{21} = 1.2\times 10^{20}$ kg

Hence, the correct answer is (C).

37. Centripetal Acceleration of Moon II is

$$a_C = \left(\frac{2\pi}{3\times 10^5}\right)^2 \times 9\times 10^7$$

$\Rightarrow \quad a_C = 4\pi^2 \times 0 \times 10^{-3}$

$\Rightarrow \quad a_C \approx 0.04 \text{ ms}^{-2}$

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. A $\rightarrow$ (p, q, r); B $\rightarrow$ (r); C $\rightarrow$ (t); D $\rightarrow$ (p, q, r, s)

Conceptual

2. A $\rightarrow$ (q); B $\rightarrow$ (p); C $\rightarrow$ (p); D $\rightarrow$ (q)

$$E = \frac{Gm}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\right)\rho}{R^2}$$

$\Rightarrow \quad E \propto \rho R$

At surface, $V = -\frac{Gm}{R} = -\frac{G\left(\frac{4}{3}\pi R^3 \rho\right)}{R}$

$\Rightarrow \quad V \propto \rho R^2$

$$V_{\text{centre}} = -\frac{3}{2}\frac{GM}{R}$$

At centre of a solid sphere field strength is zero.

3. A $\rightarrow$ (q); B $\rightarrow$ (r); C $\rightarrow$ (q); D $\rightarrow$ (p); E $\rightarrow$ (s)

At perihelion position planet is nearest to sun.

4. A $\rightarrow$ (p, q, r, s); B $\rightarrow$ (p, q, r, s); C $\rightarrow$ (q, r, s); D $\rightarrow$ (p, r)

$T = 2\pi\sqrt{\frac{r^3}{GM}}$, $v_0 = \sqrt{\frac{GM}{r}}$ and $E = -\frac{GMm}{2r}$

5. A $\rightarrow$ (p); B $\rightarrow$ (q); C $\rightarrow$ (q); D $\rightarrow$ (q)

Due to rotation of earth, variation in value of g is given by

$$g' = g - R\omega^2\cos^2\phi$$

At pole $\phi = 90°$, so there is no effect of rotation of earth.

At equator g' will increase if ω decreases.

Further, T will increase with decrease in ω

From Kepler's Third Law $T^2 \propto r^3$, so r should also increase.

Since, $E = -\dfrac{GMm}{2r}$, so with increase in r, E also increases.

6. A → (r, t); B → (s); C → (p); D → (q)
Conceptual

7. A → (p); B → (q); C → (s); D → (t)

$$E = \frac{GM}{R^2},\ V = -\frac{GM}{R}$$

Gravitational field of the earth is actually the measure of acceleration due to gravity.

At height $h = R$:

$$E' = \frac{GM}{(R+h)^2} = \frac{GM}{4R^2} = \frac{E}{4}$$

$$V' = -\frac{GM}{2R}$$

i.e., E' decreases by a factor $\dfrac{1}{4}$ and V' increases by a factor of 2.

At depth $d = \dfrac{R}{2}$:

$$E' = E\left(\phi - \frac{d}{R}\right) = \frac{E}{2}$$

$$V' = -\frac{GM}{2R^3}\left(3R^2 - r^2\right)$$

$$V' = -\frac{GM}{2R^3}\left(3R^2 - \frac{R^2}{4}\right) = -\frac{11}{8}\frac{GM}{R}$$

i.e., E' decreases by a factor $\dfrac{1}{2}$ and V' also decreases by a factor $\dfrac{11}{8}$.

8. A → (q); B → (s); C → (p); D → (r)
Conceptual

9. A → (r, q); B → (s); C → (p); D → (r)
Conceptual

10. A → (s); B → (r); C → (p); D → (p)
Conceptual

11. A → (r); B → (q); C → (p); D → (r)

For planet revolving around sun in an elliptical orbit, we have from Kepler's Second Law

$$\frac{\Delta A}{\Delta t} = \frac{L}{2m}$$

$$\Rightarrow \quad \frac{\pi ab}{T} = \frac{L}{2m}$$

$$\Rightarrow \quad \frac{2\pi mab}{T} = L = \text{constant}$$

Integer/Numerical Answer Type Questions

1. Area that cannot be covered by the satellite moving in an orbit of radius r as shown in figure is

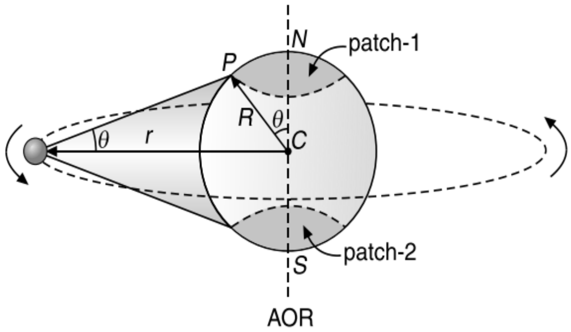
$$A = 4\pi R^2 - \frac{4\pi R^2\sqrt{r^2 - R^2}}{r}$$

$$\Rightarrow \quad A = 2\Omega R^2$$

where, $\Omega = 2\pi(1 - \cos\theta)$

$$\Rightarrow \quad \Omega = 2\pi\left(1 - \frac{\sqrt{r^2 - R^2}}{r}\right)$$

$$\Rightarrow \quad A = 4\pi R^2\left(1 - \frac{\sqrt{r^2 - R^2}}{r}\right)$$



According to the problem, $A = 0.25 \times 4\pi R^2$

$$\Rightarrow \quad 1 - \frac{\sqrt{r^2 - R^2}}{r} = 0.25$$

$$\Rightarrow \quad \frac{\sqrt{r^2 - R^2}}{r} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{r^2 - R^2}{r^2} = \frac{9}{16}$$

$$\Rightarrow \quad 16r^2 - 16R^2 = 9r^2$$

$$\Rightarrow \quad r = \frac{4}{\sqrt{7}}R = 1.515R$$

$$\Rightarrow \quad \frac{r}{R} = \frac{4}{\sqrt{7}} = 1.51$$

2. Since, $dW = \vec{F} \cdot d\vec{\ell}$

$$\Rightarrow \quad dW = m\vec{E}_g \cdot d\vec{\ell}$$

$$\Rightarrow \quad dW = m\left(2\hat{i} + 3\hat{j}\right)\cdot\left(\hat{i}dx + \hat{j}dy\right)$$

$$\Rightarrow \quad dW = m\left(2dx + 3dy\right) \qquad \ldots(1)$$

Since the particle is moved along the line

$$3y + 2x = 5$$

$$\Rightarrow \quad d(3y + 2x) = 0$$

$$\Rightarrow \quad 3dy + 2dx = 0$$

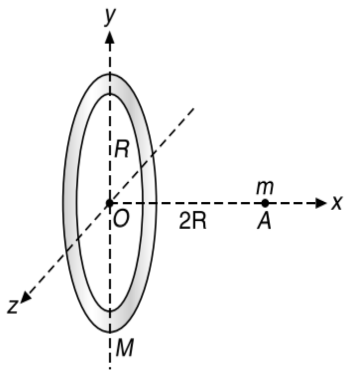
$$\Rightarrow \quad 3dy = -2dx$$

$$\Rightarrow \quad dW = 0 \qquad \{\because \text{ of } (1)\}$$

$$\Rightarrow \quad W = 0$$

3. Applying Law of Conservation of Energy, we get

$$(U+K)_A = (U+K)_O$$



$$U_A = -\frac{GMm}{\sqrt{R^2+4R^2}} = -\frac{GMm}{\sqrt{5}R}$$

$$K_A = 0$$

Finally, when m passes through O, we have

$$U_O = -\frac{GMm}{R} \text{ and } K_O = \frac{1}{2}mv^2$$

Since $(U+K)_A = (U+K)_O$

$$\Rightarrow \quad -\frac{GMm}{\sqrt{5}R} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{2(\sqrt{5}-1)GM}{\sqrt{5}R}}$$

$$\Rightarrow \quad v = \left[2\left(1-\frac{1}{\sqrt{5}}\right)\frac{GM}{R}\right]^{\frac{1}{2}}$$

$$\Rightarrow \quad x = 2,\ y = \sqrt{5} \text{ and } z = 2$$

$$\Rightarrow \quad \frac{y^2}{xz} = \frac{(\sqrt{5})^2}{(2)(2)} = \frac{5}{4} = 1.25$$

4. By Law of Conservation of Energy, we have

$$(U+K)_{surface} = (U+K)_{\text{at }\infty}$$

$$\Rightarrow \quad -\frac{GmM}{R} + \frac{1}{2}mu^2 = 0 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad -\frac{2GM}{R} + u^2 = v^2$$

Since $v_e = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \quad -v_e^2 + u^2 = v^2$$

$$\Rightarrow \quad v^2 = -(11.2)^2 + (15)^2 = -125 + 225$$

$$\Rightarrow \quad v = 10 \text{ kms}^{-1}$$

5. By Law of Conservation of Angular Momentum, we have

$$m(v_0 \cos 60°)4R = mvR$$

$$\Rightarrow \quad \frac{v}{v_0} = 2$$

Applying Law of Conservation of Energy, we get

$$-\frac{GMm}{4R} + \frac{1}{2}mv_0^2 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}v^2 - \frac{1}{2}v_0^2 = \frac{3}{4}\frac{GM}{R}$$

$$\Rightarrow \quad v_0 = \frac{1}{\sqrt{2}}\sqrt{64\times10^6} = \frac{8000}{\sqrt{2}} \text{ ms}^{-1}$$

$$\Rightarrow \quad v_0 = 4000\sqrt{2} \text{ ms}^{-1}$$

$$\Rightarrow \quad v_0 = 5656 \text{ ms}^{-1}$$

6. (a) Let x be the displacement of ring, then the displacement of the particle is $(3-x)$ m. Since, no external force is acting on the system and particles are initially at rest, so the centre of mass will not move. Hence,

$$(5.4\times10^9)x = (6\times10^8)(3-x)$$

$$\Rightarrow \quad x = 0.3 \text{ m} = 30 \text{ cm}$$

(b) By Law of Conservation of Linear Momentum, we have

$$0 = 5.4\times10^9 v_1 - 6\times10^8 v_2$$

$$\Rightarrow \quad v_2 = 9v_1 \qquad \ldots(1)$$

By Law of Conservation of Energy, we have

$$(U+K)_{\text{initial}} = (U+K)_{\text{final}}$$

$$\Rightarrow \quad -\frac{Gm_1m_2}{\sqrt{4^2+3^2}} + 0 = -\frac{Gm_1m_2}{4} + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\Rightarrow \quad Gm_1m_2\left(\frac{1}{4}-\frac{1}{5}\right) = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2(81v_1^2)$$

$$\Rightarrow \quad \frac{G(9m_2^2)}{20} = \frac{1}{2}(90m_2)v_1^2$$

$$\Rightarrow \quad v_1 = \sqrt{\frac{Gm_2}{100}} = \sqrt{\frac{6.67\times10^{-11}\times6\times10^8}{100}}$$

$$\Rightarrow \quad v_1 = 0.02 \text{ ms}^{-1} = 2 \text{ cms}^{-1}$$

$$\Rightarrow \quad v_2 = 9v_1 = 18 \text{ cms}^{-1}$$

7. Potential at origin is

$$V_0 = -\frac{2GM}{a}$$

Potential at point $(0,\ 0,\ 2\sqrt{3}a)$ is

$$V_P = -\frac{2GM}{\sqrt{a^2+12a^2}} = -\frac{2}{\sqrt{13}}\frac{GM}{a}$$

By Law of Conservation of Energy, we get

$$(U+K)_O = (U+K)_P$$

$$\Rightarrow \frac{1}{2}\left(\frac{M}{2}\right)v^2 + \left(\frac{M}{2}\right)V_0 = \left(\frac{M}{2}\right)V_P$$

$$\Rightarrow v = \sqrt{2(V_P - V_0)}$$

$$\Rightarrow v = \sqrt{2\left(\frac{2GM}{a} - \frac{2}{\sqrt{13}}\frac{GM}{a}\right)}$$

$$\Rightarrow v = 2\sqrt{\frac{GM}{a}\left(1-\frac{1}{\sqrt{13}}\right)}$$

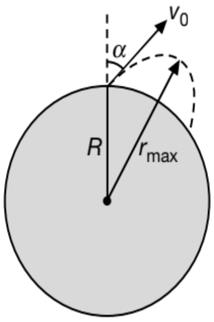
$$\Rightarrow v = \sqrt{\frac{4GM}{a}\left(1-\frac{1}{\sqrt{13}}\right)}$$

$$\Rightarrow \alpha = 4 \text{ and } \beta = 13$$

8. Let v be the speed of the projectile at highest point and r_{max} its distance from the centre of the earth. By Law of Conservation of Angular Momentum and Law of Conservation of Mechanical Energy, we have

$$mv_0 \sin\alpha = mvr_{max} \quad \ldots(1)$$

$$\frac{1}{2}mv_0^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 - \frac{GMm}{r_{max}} \quad \ldots(2)$$



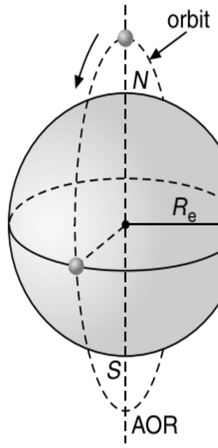
Solving these two equation, we get

$$r_{max} = \frac{3R}{2}$$

So, the maximum height is $h_{max} = r_{max} - R = \frac{R}{2}$

$$\Rightarrow * = 2$$

9. A satellite which rotates with angular speed equal to earth's rotation has angular speed of revolution is given by



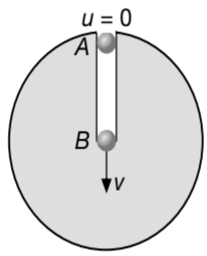
$$\omega = \frac{2\pi}{T}, \text{ where } T = 24 \text{ hr}$$

When satellite moves from a point above north pole to a point above equator, it traverses an angle $\frac{\pi}{2}$. So, this time taken is given by

$$T' = \frac{\pi/2}{(2\pi/T)} = \frac{T}{4} = 6 \text{ hr}$$

10. By Law of Conservation of Energy, we have

$$(U+K)_{surface} = (U+K)_{centre}$$



$$\Rightarrow -\frac{GMm}{R} + \frac{1}{2}m(0)^2 = -\frac{3}{2}\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{2R}$$

where m is the mass of the ball

$$\Rightarrow v = \sqrt{\frac{GM}{R}}$$

Velocity of ball just after collision

$$v' = ev = \frac{1}{2}\sqrt{\frac{GM}{R}}$$

Let r be the distance from the centre upto the point P, where the ball reaches after collision. Then by Law of Conservation of Energy, we have

$$(U+K)_{centre} = (U+K)_P$$

where $U_P = -\frac{GMm}{2R^3}(3R^2 - r^2)$

$$\Rightarrow \quad -\frac{3}{2}\frac{GMm}{R}+\frac{1}{2}mv'^2=-\frac{GMm}{2R^3}(3R^2-r^2)+0$$

But $v'=\frac{1}{2}\sqrt{\frac{GM}{R}}$

$$\Rightarrow \quad -\frac{3}{2}\frac{GMm}{R}+\frac{1}{8}\frac{GMm}{R}=-\frac{GMm}{2R^3}(3R^2-r^2)$$

$$\Rightarrow \quad -\frac{11}{8}=-\left(\frac{3R^2-r^2}{2R^2}\right)$$

$$\Rightarrow \quad 22R^2=24R^2-8r^2$$

$$\Rightarrow \quad 8r^2=2R^2$$

$$\Rightarrow \quad r^2=\frac{R^2}{4}$$

$$\Rightarrow \quad r=\frac{R}{2}$$

So, the desired distance, is

$$s=R+\frac{R}{2}+\frac{R}{2}=2R$$

$$\Rightarrow \quad x=2$$

11. At a point P at a distance r from Moon, we have net gravitational field equal to zero

$$\Rightarrow \quad g=\frac{GM}{r^2}-\frac{G(81M)}{(D-r)^2}=0$$

$$\Rightarrow \quad D-r=9r$$

$$\Rightarrow \quad 10r=D$$

So, $r=\frac{D}{10}$ from the Moon and $\frac{9D}{10}$ from the earth.

$$\Rightarrow \quad x=9 \text{ and } y=10$$

$$\Rightarrow \quad y-x=1$$

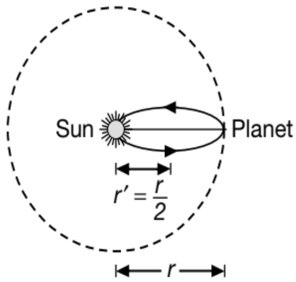
12. Consider an imaginary planet moving along a strongly extended flat ellipse, the extreme points of which are located on the planet's orbit and at the centre of the sun, the semi-major axis of the orbit of such a planet would apparently be half the semi-major axis of the planet's orbit. So, the time period of the imaginary planet T' according to Kepler's Law will be given by:

$$\left(\frac{T'}{T}\right)=\left(\frac{r'}{r}\right)^{3/2}$$

$$\Rightarrow \quad T'=T\left(\frac{1}{2}\right)^{3/2} \qquad \left\{\because r'=\frac{r}{2}\right\}$$

So, time taken by the planet to fall onto the sun is

$$t=\frac{T'}{2}=\frac{T}{2}\left(\frac{1}{2}\right)^{3/2}$$



$$\Rightarrow \quad t=\frac{\sqrt{2}}{8}T=\sqrt{\frac{2}{64}}T=\sqrt{\frac{1}{32}}=\frac{1}{\sqrt{2^5}}$$

$$\Rightarrow \quad x=5$$

13. For point mass at distance $r=3\ell$

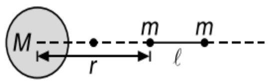
$$\frac{GMm}{(3\ell)^2}-\frac{Gm^2}{\ell^2}=ma \qquad \ldots(1)$$

For point mass at distance $r=4\ell$

$$\frac{GMm}{(4\ell)^2}+\frac{Gm^2}{\ell^2}=ma \qquad \ldots(2)$$

Equating the two equations, we have

$$\frac{GMm}{9\ell^2}-\frac{Gm^2}{\ell^2}=\frac{GMm}{16\ell^2}+\frac{Gm^2}{\ell^2}$$



$$\frac{7GMm}{144}=\frac{2Gm^2}{\ell^2}$$

$$m=\frac{7M}{288}$$

$$\Rightarrow \quad k=7$$

14. Since, $g_p=\frac{GM_p}{R_p^2}=\frac{4}{3}G\pi R_p\rho_p$

$$\Rightarrow \quad \frac{g_p}{g_e}=\frac{R_p\rho_p}{R_e\rho_e}$$

Also, $v_e=\sqrt{2gR}$

$$\Rightarrow \quad \frac{v_p}{v_e}=\sqrt{\frac{g_pR_p}{g_eR_e}}=\left(\frac{g_p}{g_e}\right)\sqrt{\frac{\rho_e}{\rho_p}}=\frac{\sqrt{12}}{11}\times\sqrt{\frac{3}{4}}$$

$$\Rightarrow \quad \frac{v_p}{v_e}=\frac{2\sqrt{3}}{11}\times\frac{\sqrt{3}}{2}=\frac{3}{11}$$

$$\Rightarrow \quad v_p=3\text{ kms}^{-1}$$

15. The gravitational field due to cylinder at a distance x from its axis is $E_g=\frac{2G\lambda}{x}$

CHAPTER 4

$$\Rightarrow \quad m\left(\frac{2G\lambda}{x}\right)=\frac{mv^2}{x}$$

where λ is the linear mass density given by

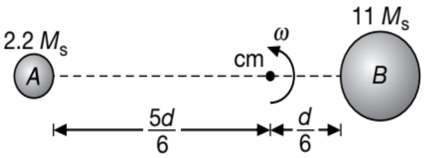
$$\lambda=\rho\left(\pi R^2\right)$$

$$\Rightarrow \quad 2G\rho\pi R^2=v^2$$

$$\Rightarrow \quad v=R\sqrt{2G\rho\pi}$$

$$\Rightarrow \quad v=R\left(2G\rho\pi\right)^{\frac{1}{2}}$$

$$\Rightarrow \quad *=2$$

16. 

$$\frac{\text{Total angular momentum about cm}}{\text{Angular momentum of B about cm}}=\frac{L}{L_B}$$

$$\Rightarrow \quad \frac{L}{L_B}=\frac{\left(2.2M_S\right)\left(\frac{5\omega d}{6}\right)\left(\frac{5d}{6}\right)+\left(11M_S\right)\left(\frac{\omega d}{6}\right)\left(\frac{d}{6}\right)}{\left(11M_S\right)\left(\frac{\omega d}{6}\right)\left(\frac{d}{6}\right)}$$

$$\Rightarrow \quad \frac{L}{L_B}=6$$

17. The time period T of the satellite is

$$T=2\pi\sqrt{\frac{r^3}{GM}}=2\pi\sqrt{\frac{r^3}{gR^2}}$$

$$\Rightarrow \quad T=2\sqrt{\frac{\pi^2 r^3}{gR^2}}=2\sqrt{\frac{r^3}{R^2}}$$

where, $r=6400+1600=800\text{ km}=8000\times10^3\text{ m}$

and $R=6400\times10^3\text{ m}$

Substituting these values, we get

$$T=2\sqrt{\frac{\left(8000\times10^3\right)^3}{\left(6400\times10^3\right)^2}}=7071\text{ s}$$

Further, orbital speed,

$$v=\sqrt{\frac{GM}{r}}=\sqrt{\frac{gR^2}{r}}$$

$$\Rightarrow \quad v=\sqrt{\left(\frac{9.8}{8000\times10^3}\right)}\times\left(6400\times10^3\right)$$

$$\Rightarrow \quad v=7155\text{ ms}^{-1}$$

Let t be the time interval between two successive moments at which the satellite is overhead to an observer at a fixed position on the equator. Since both satellite and earth are moving in the same sense (i.e. from west to east) with angular speed ω_S and ω_E respectively so, we can write the time of separation (t) as

$$t=\frac{2\pi}{\omega_S-\omega_E}$$

where $\omega_S=\frac{2\pi}{7071}$ and $\omega_E=\frac{2\pi}{86400}$

$$\Rightarrow \quad t=\frac{86400\times7071}{86400-7071}$$

$$\Rightarrow \quad t=\frac{611\times10^6}{79329}$$

$$\Rightarrow \quad t=7702\text{ s}$$

18. Applying conservation of mechanical energy, we have

$$(U+K)_{\text{at }B}=(U+K)_{\text{at }A}$$

$$\Rightarrow \quad U_B+K_B=U_A+K_A$$

$$\Rightarrow \quad U_B+0=U_A+\frac{1}{2}mv_A^2$$

where $U_B=mV_B$ and $U_A=mV_A$, so we have

$$\frac{1}{2}mv_A^2=U_B-U_A=m\left(V_B-V_A\right)$$

$$\Rightarrow \quad v_A=\sqrt{2\left(V_B-V_A\right)} \qquad \ldots(1)$$

Potential at A

$$V_A=\begin{pmatrix}\text{Potential Due}\\ \text{to the Complete}\\ \text{Sphere at }A\end{pmatrix}-\begin{pmatrix}\text{Potential Due}\\ \text{to the Spherical}\\ \text{Cavity at }A\end{pmatrix}$$

$$V_A=-\frac{3}{2}\frac{GM}{R}-\left(-\frac{GM'}{r}\right)=\frac{GM'}{r}-\frac{3}{2}\frac{GM}{R}$$

where

$$M=\frac{4}{3}\pi R^3\rho,\ r=\frac{R}{2}\text{ and }M'=\frac{4}{3}\pi r^3\rho=\frac{\pi\rho R^3}{6}$$

Substituting the values, we get

$$V_A=\frac{G}{R}\left(\frac{\pi\rho R^3}{3}-2\pi\rho R^3\right)=-\frac{5}{3}\pi G\rho R^2$$

Potential at B

$$V_B=\begin{pmatrix}\text{Potential Due}\\ \text{to the Complete}\\ \text{Sphere at }B\end{pmatrix}-\begin{pmatrix}\text{Potential Due}\\ \text{to the Spherical}\\ \text{Cavity at }B\end{pmatrix}$$

$$\Rightarrow \quad V_B=-\frac{GM}{2R^3}\left(3R^2-r^2\right)-\left(-\frac{3}{2}\frac{GM'}{r}\right)$$

where

$$M=\frac{4}{3}\pi R^3\rho,\ r=\frac{R}{2}\text{ and }M'=\frac{4}{3}\pi r^3\rho=\frac{\pi\rho R^3}{6}$$

$\Rightarrow \quad V_B = -\frac{11}{8}\frac{GM}{R} + \frac{3GM'}{R}$

$\Rightarrow \quad V_B = \frac{G}{R}\left(\frac{\pi\rho R^3}{2} - \frac{11\pi\rho R^3}{6}\right) = -\frac{4}{3}\pi G\rho R^2$

$\Rightarrow \quad V_B - V_A = \frac{1}{3}\pi G\rho R^2$

So, from equation (1)

$v = \sqrt{\frac{2}{3}\pi G\rho R^2}$

Also, $v_e = \sqrt{\frac{2GM_{\text{net}}}{R}}$

where $M_{\text{net}} = M - M' = \frac{7}{6}\pi R^3\rho$

$\Rightarrow \quad v_e = \sqrt{\frac{2G\left(\frac{7}{6}\pi R^3\right)\rho}{R}}$

$\Rightarrow \quad v_e = \sqrt{\frac{7}{3}\pi G\rho R^2}$

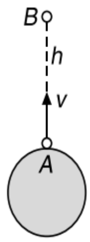
$\Rightarrow \quad \frac{v_e}{v} = \sqrt{\frac{7}{2}}$

$\Rightarrow \quad \frac{v_e^2}{v^2} = \frac{7}{2}$

$\Rightarrow \quad \frac{2v_e^2}{v^2} = 7$

19. At height h, we have

$g_h = \frac{gR^2}{(R+h)^2}$ ...(1)



Given that $g_h = \frac{g}{9}$

Substituting in equation (1) we get,

$\frac{1}{9} = \left(\frac{R}{R+h}\right)^2$

$\Rightarrow \quad h = 2R$

Applying Law of Conservation of Energy, from A to B, we get

$(U+K)_A = (U+K)_B$

$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h} + 0$

Since $h = 2R$, so we have

$\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{3R}$

$\Rightarrow \quad \frac{v^2}{2} = \frac{2GM}{3R}$

$\Rightarrow \quad v = \sqrt{\frac{4GM}{3R}}$

Since $v_e = \sqrt{\frac{2GM}{R}}$

Given that $v_e = v\sqrt{N}$

$\Rightarrow \quad \sqrt{\frac{2GM}{R}} = \sqrt{\frac{4GM}{3R}}\sqrt{N}$

$\Rightarrow \quad N = \frac{3}{2} = 1.5$

20. Let E be the gravitational field at x due to the complete sphere.

If E_1 be the field due to hole and E_2 be the field due to the remaining portion, then we have

$E = E_1 + E_2$

$\Rightarrow \quad E_2 = E - E_1$

$\Rightarrow \quad E_2 = \frac{GM}{x^2} - \frac{Gm}{\left(x - \frac{R}{2}\right)^2}$ ...(1)

where, $M = \frac{4}{3}\pi R^3\rho_0$ and $m = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3\rho_0$

Substituting the values in equation (1), we get

$E_2 = -\left(\frac{\pi G\rho_0 R^3}{6}\right)\left[\frac{1}{\left(x - \frac{R}{2}\right)^2} - \frac{8}{x^2}\right]$

$\Rightarrow \quad E_2 = -\left(\frac{\pi G\rho_0 R^3}{6}\right)\left[\frac{1}{\left(2R - \frac{R}{2}\right)^2} - \frac{8}{(2R)^2}\right]$

$\Rightarrow \quad E_2 = -\frac{\pi G\rho_0 R^3}{6}\left(\frac{4}{9R^2} - \frac{2}{R^2}\right)$

$\Rightarrow \quad E_2 = -\frac{\pi G\rho_0 R}{6}\left(\frac{4-18}{9}\right)$

$\Rightarrow \quad E_2 = \frac{14}{54}\pi G\rho_0 R$

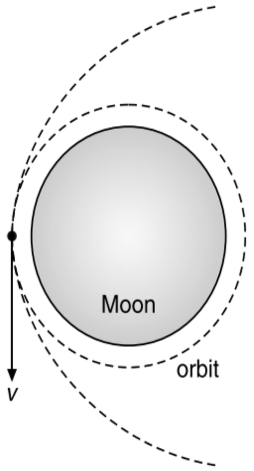
$\Rightarrow \quad E_2 = \left(\frac{7}{27}\right)\pi G\rho_0 R$

Since, $E = \left(\frac{a}{a+20}\right)\pi G\rho_0 R$

$\Rightarrow \quad \frac{a}{a+20} = \frac{7}{27}$

$\Rightarrow \quad a = 7$

21. Figure shows the corresponding situation



Since the spaceship follows a parabolic trajectory tangential to the moon. When at the surface of moon it has speed equal to escape speed

$$v_e = \sqrt{\frac{2GM}{R}}$$

Now to transform it into a circular orbit, its speed should be decreased to orbital speed v_0 given by

$$v_0 = \sqrt{\frac{GM}{R}}$$

So, change in speed is

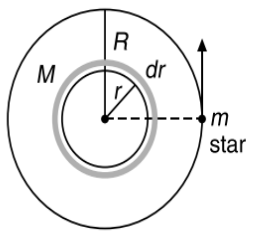
$$\Delta v = v_e - v_0$$

$$\Rightarrow \quad \Delta v = \left(\sqrt{2} - 1\right)\sqrt{\frac{GM}{R}}$$

$$\Rightarrow \quad x = 2$$

ARCHIVE: JEE MAIN

1. Since $dm = \rho dV$, where $dV = 4\pi r^2 dr$



$$\Rightarrow \quad dm = \left(\frac{k}{r}\right)\left(4\pi r^2 dr\right) = 4\pi krdr$$

$$\Rightarrow \quad M = \int_0^R dm = \int_0^R 4\pi krdr = 4\pi k \left.\frac{r^2}{2}\right|_0^R$$

$$\Rightarrow \quad M = 2\pi k\left(R^2 - 0\right) = 2\pi kR^2$$

For circular motion, gravitational force will provide the centripetal force so that

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$\Rightarrow \quad \frac{G\left(2\pi kR^2\right)m}{R^2} = \frac{mv^2}{R}$$

$$\Rightarrow \quad v = \sqrt{2\pi GkR}$$

Time period T is given by

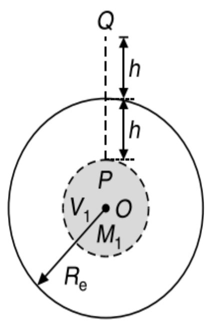
$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{2\pi GkR}} \propto \sqrt{R}$$

$$\Rightarrow \quad T^2 \propto R$$

Hence, the correct answer is (C).

2. Let M be the mass of earth, M_1 be the mass of shaded portion and R be the radius of earth.

At height h, $g_h = \dfrac{GM}{(R+h)^2}$



At depth h, we have

$$M_1 = \rho V_1 = \frac{M}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi(R-h)^3 = \frac{M(R-h)^3}{R^3}$$

Weight of body is same at P and Q, so

$$mg_P = mg_Q$$

$$\Rightarrow \quad g_P = g_Q$$

$$\Rightarrow \quad \frac{GM_1}{(R-h)^2} = \frac{GM}{(R+h)^2}$$

$$\Rightarrow \quad \frac{GM(R-h)^3}{(R-h)^2 R^3} = \frac{GM}{(R+h)^2}$$

$$\Rightarrow \quad (R-h)(R+h)^2 = R^3$$

$$\Rightarrow \quad R^3 - hR^2 - h^2R - h^3 + 2R^2h - 2Rh^2 = R^3$$

$$\Rightarrow \quad R^2 - Rh^2 - h^3 = 0$$

$$\Rightarrow \quad R^2 - Rh - h^2 = 0$$

$$\Rightarrow \quad h^2 + Rh - R^2 = 0$$

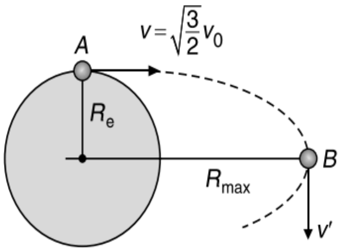
$$\Rightarrow \quad h = \frac{-R \pm \sqrt{R^2 + 4R^2}}{2}$$

$$\Rightarrow \quad h = \frac{-R + \sqrt{5}R}{2} = \left(\frac{\sqrt{5}-1}{2}\right)R$$

Hence, the correct answer is (A).

3. Since, $v_0 = \sqrt{\frac{GM}{R_e}}$

$$(U+K)_A = (U+K)_B$$



$$\Rightarrow \quad \frac{-GMm}{R_e} + \frac{1}{2}mv^2 = \frac{-GMm}{R_{max}} + \frac{1}{2}mv'^2 \qquad \ldots(1)$$

Also, $vR_e = v'R_{max}$ ...(2)

Solving equations (1) and (2), we get

$$R_{max} = 3R_e$$

Hence, the correct answer is (B).

4. According to Gauss Law for gravitation, we have

$$E(4\pi r^2) = \int \rho_0 4\pi r^2 dr$$

$$\Rightarrow \quad Er^2 = 4\pi G \int_0^r \rho_0 \left(1 - \frac{r^2}{R^2}\right) r^2 dr$$

$$\Rightarrow \quad |E| = E = 4\pi G\rho_0 \left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)$$

For E to be maximum, $\frac{dE}{dr} = 0$

$$\Rightarrow \quad r = \sqrt{\frac{5}{9}}R$$

Hence, the correct answer is (B).

5. Given that $E_G = \frac{Ax}{(x^2+a^2)^{3/2}}$ and $V_\infty = 0$

Since, $\int_{V_\infty}^{V_x} dV = -\int_\infty^x \vec{E}_G \cdot d\vec{x}$

$$\Rightarrow \quad V_x - V_\infty = -\int_\infty^x \frac{Ax}{(x^2+a^2)^{3/2}} dx$$

Substitute $x^2 + a^2 = z$ i.e., $2xdx = dz$

$$\Rightarrow \quad V_x - 0 = -\int_\infty^x \frac{Adz}{2(z)^{3/2}} = \left(\frac{A}{z^{1/2}}\right)\Bigg|_\infty^x = \left(\frac{A}{(x^2+a^2)^{1/2}}\right)\Bigg|_\infty^x$$

$$\Rightarrow \quad V_x = \frac{A}{(x^2+a^2)^{1/2}} - 0 = \frac{A}{(x^2+a^2)^{1/2}}$$

Hence, the correct answer is (A).

6. For orbit close to the planet, $R + h \approx R$

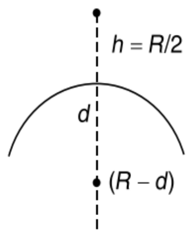
$$v_{orbit} = \sqrt{\frac{GM}{R}}$$

Since, $v_{escape} = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \quad \frac{v_{orbit}}{v_{escape}} = \frac{1}{\sqrt{2}}$$

Hence, the correct answer is (C).

7. Since, $g_1 = \frac{GM}{(R+h)^2} = \frac{GM}{(3R/2)^2}$ ...(1)



Also, $g_2 = \frac{GM(R-d)}{R^3}$ ...(2)

Given that, $g_1 = g_2$

$$\Rightarrow \quad \frac{GM}{(3R/2)^2} = \frac{GM(R-d)}{R^3}$$

$$\Rightarrow \quad \frac{4}{9} = \frac{(R-d)}{R}$$

$$\Rightarrow \quad 4R = 9R - 9d$$

$$\Rightarrow \quad 5R = 9d$$

$$\Rightarrow \quad \frac{d}{R} = \frac{5}{9}$$

Hence, the correct answer is (D).

8. At equator, we have $g_e = g - R\omega^2$

For $h \quad R$, we have

$$g_2 \approx g\left(1 - \frac{2h}{R}\right) = g - \frac{2gh}{R}$$

CHAPTER 4

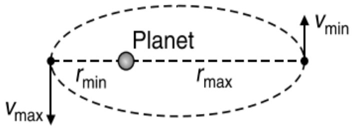
Since, $g_1 = g_2$

$$\Rightarrow \quad R\omega^2 = \frac{2gh}{R}$$

$$\Rightarrow \quad h = \frac{R^2\omega^2}{2g}$$

Hence, the correct answer is (D).

9. Applying conservation of angular momentum, we get



$$r_{min}v_{max} = r_{max}v_{min} \quad \ldots(1)$$

Given that, $v_{min} = \dfrac{v_{max}}{6}$

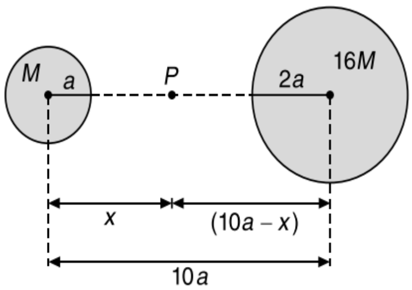
From equation (1), we get

$$\frac{r_{min}}{r_{max}} = \frac{v_{min}}{v_{max}} = \frac{1}{6}$$

Hence, the correct answer is (A).

10. The minimum speed will take the body to the neutral point P after which the bigger planet will automatically attract the body towards itself. So,

$$\frac{GM}{x^2} = \frac{G(16M)}{(10a-x)^2}$$



$$\Rightarrow \quad \frac{1}{x} = \frac{4}{(10a-x)}$$

$$\Rightarrow \quad 4x = 10a - x$$

$$\Rightarrow \quad x = 2a \quad \ldots(1)$$

Applying law of conservation of mechanical energy, we get

$$-\frac{GMm}{8a} - \frac{G(16M)m}{2a} + KE = -\frac{GMm}{2a} - \frac{G(16M)m}{8a}$$

$$\Rightarrow \quad KE = GMm\left(\frac{1}{8a} + \frac{16}{2a} - \frac{1}{2a} - \frac{16}{8a}\right)$$

$$\Rightarrow \quad KE = GMm\left(\frac{1+64-4-16}{8a}\right)$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = GMm\left(\frac{45}{8a}\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{90GM}{8a}} = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

Hence, the correct answer is (B).

11. $W = 196 - mR\omega^2$

Hence, the correct answer is (D).

12. Gravitational field on the surface of a solid sphere

$$I_g = \frac{GM}{R^2}$$

From graph, $\dfrac{GM_1}{(1)^2} = 2$ and $\dfrac{GM_2}{(2)^2} = 3$

$$\Rightarrow \quad \frac{M_1}{M_2} = \frac{1}{6}$$

Hence, the correct answer is (D).

13. Initially, the body of mass m is moving in a circular orbit of radius R. So, it must be moving with orbital speed given by

$$v_0 = \sqrt{\frac{GM}{R}}$$

After collision, let the combined mass moves with speed v_1, then by conservation of momentum, we get

$$mv_0 + \left(\frac{m}{2}\right)\left(\frac{v_0}{2}\right) = \left(\frac{3m}{2}\right)v_1$$

$$\Rightarrow \quad v_1 = \frac{5v_0}{6}$$

Since after collision, the speed is not equal to orbitial speed at the point. So, motion cannot be circular. Also, velocity will remain tangential, so it cannot fall vertically towards the planet. Their speed after collision is less than escape speed $\sqrt{2}v_0$, so they cannot escape gravitational field. Hence their motion will be elliptical around the planet.

Hence, the correct answer is (A).

14. The escape velocity is

$$v_e = \sqrt{\frac{2GM}{R}}$$

So, $v_A = \sqrt{\dfrac{2GM}{R}}$ and $v_B = \sqrt{\dfrac{2G(M/2)}{R/2}} = \sqrt{\dfrac{2GM}{R}}$

$$\Rightarrow \quad \frac{v_A}{v_B} = 1 = \frac{n}{4}$$

$$\Rightarrow \quad n = 4$$

Hence, the correct answer is (A).

15. Since, $U_1 + K_1 = U_2 + K_2$

$$\Rightarrow \quad -\frac{GM_em}{10R} + \frac{1}{2}mv_0^2 = -\frac{GM_em}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{9}{10}\left(\frac{GM_e m}{R}\right)+\frac{1}{2}mv_0^2=\frac{1}{2}mv^2$$

$$\Rightarrow \frac{9}{10}\left(\frac{1}{2}mv_e^2\right)+\frac{1}{2}mv_0^2=\frac{1}{2}mv^2$$

$$\Rightarrow v^2=\frac{9}{10}v_e^2+v_0^2$$

$$\Rightarrow v^2=\frac{9}{10}\times(11.2)^2+(12)^2$$

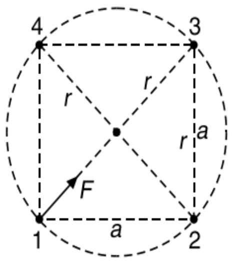
$$\Rightarrow v^2=112.896+144$$

$$\Rightarrow v=16.027\approx 16\ \text{kms}^{-1}$$

Hence, the correct answer is 16.

16. Net force on 1 acting towards the centre of circle is

$$F=\frac{GM^2}{a^2}(\sqrt{2})+\frac{GM^2}{2a^2}=\frac{Mv^2}{r}$$



$$\Rightarrow \frac{Mv^2}{\left(\frac{a}{\sqrt{2}}\right)}=\frac{GM^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right) \qquad \left\{\because r=\frac{a}{\sqrt{2}}\right\}$$

$$\Rightarrow v^2=\frac{GM}{a}\left(1+\frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow v=\sqrt{\frac{GM}{a}\left(1+\frac{1}{2\sqrt{2}}\right)}=1.16\sqrt{\frac{GM}{a}}$$

Hence, the correct answer is (B).

17. Let m_0 be the mass of rocket, then

$$E=\frac{GM_e m_0}{R_e} \text{ and } E'=\frac{GM_m m_0}{R_m}$$

Since $V_e=64V_m$

$$\rho R_e^3=64\rho R_m^3$$

$$\Rightarrow R_e=4R_m$$

$$\Rightarrow \frac{E'}{E}=\left(\frac{M_m}{M_e}\right)\left(\frac{R_e}{R_m}\right)=\left(\frac{1}{64}\right)(4)=\frac{1}{16}$$

$$\Rightarrow E'=\frac{E}{16}$$

Hence, the correct answer is (C).

18. $E=\frac{GM}{(3a)^2}+\frac{2GM}{(3a)^2}$

$$\Rightarrow E=\frac{GM}{3a^2}$$

Hence, the correct answer is (C).

19. $M=\int_0^R (4\pi r^2 dr)\frac{K}{r^2}$

$$\Rightarrow M=4\pi KR$$

$$\Rightarrow G\left(\frac{4\pi KR}{R^2}\right)=\frac{v_0^2}{R}$$

$$\Rightarrow v_0=\sqrt{4\pi GK}$$

Since $T=\frac{2\pi R}{v_0}=\frac{2\pi R}{\sqrt{4\pi GK}}$

$$\Rightarrow \frac{T}{R}=\text{constant}$$

Hence, the correct answer is (A).

20. Given that $g_h=\frac{g}{2}$

Since $g_h=\frac{gR^2}{(R+h)^2}$

$$\Rightarrow \frac{g}{2}=g\left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \frac{R}{R+h}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow h=(\sqrt{2}-1)R$$

$$\Rightarrow h=6400\times 0.414$$

$$\Rightarrow h=2649.6\ \text{km}$$

$$\Rightarrow h=2.6\times 10^6\ \text{m}$$

Hence, the correct answer is (D).

21. Since $T=\frac{2\pi r}{v_0}$ and $v_0=\sqrt{\frac{GM}{r}}$

$$\Rightarrow T=2\pi r\sqrt{\frac{r}{GM}}=2\pi\sqrt{\frac{r^3}{GM}}$$

$$\Rightarrow T=2\pi\sqrt{\frac{(202)^3\times 10^{12}}{6.67\times 10^{-11}\times 8\times 10^{22}}}\ \text{s}$$

$$\Rightarrow T=7812.2\ \text{s}$$

$$\Rightarrow T\quad 2.17\ \text{hr}$$

So, number of revolutions is

$$N=\frac{24}{T}\approx 11$$

Hence, the correct answer is (C).

22. Given that $\frac{g_e}{g_p} = \frac{9}{4}$

Since $g = \frac{GM}{R^2}$

$\Rightarrow \quad \frac{M_e \times R_p^2}{R_e^2 \times M_p} = \frac{9}{4}$

$\Rightarrow \quad 9 \times \left(\frac{R_p}{R_e}\right)^2 = \frac{9}{4}$

$\Rightarrow \quad R_p = \frac{R_e}{2} = \frac{R}{2}$

Hence, the correct answer is (D).

23. Areal velocity is given by

$$\frac{\Delta A}{\Delta t} = \frac{L}{2m}$$

Hence, the correct answer is (C).

24. $E_1 = U_f - U_i$

$\Rightarrow \quad E_1 = \frac{GMm}{R} - \frac{GMm}{(R+h)}$

$\Rightarrow \quad E_1 = \frac{GMmh}{R(R+h)}$

KE of satellite in this orbit is

$$KE = E_2 = \frac{GMm}{2r} = \frac{GMm}{2(R+h)}$$

$\Rightarrow \quad E_2 = \frac{1}{2}\frac{GMm}{(R+h)}$

Given that $E_1 = E_2$

$\Rightarrow \quad \frac{h}{R} = \frac{1}{2}$

$\Rightarrow \quad h = \frac{R}{2}$

Hence, the correct answer is (A).

25. Since $U = -2\left(\frac{1}{2}mv^2\right)$

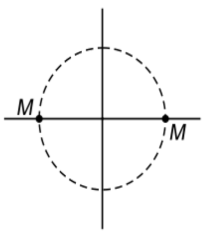
In order to escape, we have $U + K = 0$

$\Rightarrow \quad K = mv^2$

Hence, the correct answer is (B).

26. $2R = d = 2 \times 10^{11}$ m

$\Rightarrow \quad R = 10^{11}$ m



$\Rightarrow$ Let v_0 be the minimum speed of meteorite at O, then by Law of Conservation of Energy, we have

$$\frac{1}{2}mv_0^2 - \frac{2GMm}{R} = 0$$

$\Rightarrow \quad v_0 = \sqrt{\frac{4GM}{R}} = \sqrt{\frac{4 \times 6.67 \times 10^{-11} \times 3 \times 10^{31}}{10^{11}}}$

$\Rightarrow \quad v_0 = 2.83 \times 10^5 \text{ ms}^{-1} \approx 2.8 \times 10^5 \text{ ms}^{-1}$

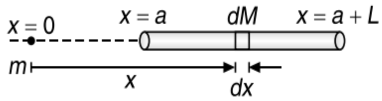
Hence, the correct answer is (A).

27. Since, $v_0 = \sqrt{2gR}$, $v_e = \sqrt{2gR}$

$\Rightarrow \quad \Delta v = \sqrt{gR}\left(\sqrt{2} - 1\right)$

Hence, the correct answer is (B).

28.



$$dF = \frac{GmdM}{x^2}, \text{ where } dM = \lambda dx$$

$\Rightarrow \quad dM = \left(A + Bx^2\right)dx$

So, $\quad dF = -Gm\int_a^{L+a}\frac{\left(A+Bx^2\right)dx}{x^2}$

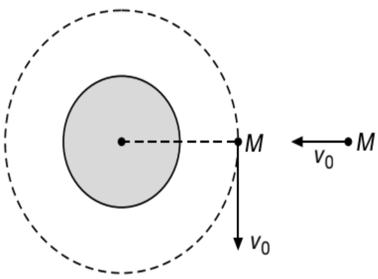
$$F = -Gm\left[-A\left(\frac{1}{L+a} - \frac{1}{a}\right) + BL\right]$$

$\Rightarrow \quad F = -Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$

Hence, the correct answer is (C).

29. Orbital velocity v_0 is given by

$$v_0 = \sqrt{\frac{GM}{R}}$$



After collision, we have

$$mv_0\left(-\hat{j}\right) + mv_0\left(-\hat{i}\right) = 2m\vec{v}$$

$\Rightarrow \quad \vec{v} = -\frac{v_0}{2}\hat{i} - \frac{v_0}{2}\hat{j}$

$\Rightarrow \quad |\vec{v}| = \frac{v_0}{\sqrt{2}} = 0.7v_0$

$\Rightarrow \quad v < v_0$

So, the path will be elliptical.

Hence, the correct answer is (C).

30. Since $\frac{v^2}{r}=\frac{GM}{r^2}$

$$\Rightarrow \quad v^2=\frac{GM}{r}$$

$$\Rightarrow \quad \frac{T_A}{T_B}=\frac{\frac{1}{2}mv_A^2}{\frac{1}{2}2mv_B^2}=\frac{1}{2}\left(\frac{v_A}{v_B}\right)^2$$

$$\Rightarrow \quad \frac{T_A}{T_B}=\frac{1}{2}\frac{R_B}{R_A}=1$$

Hence, the correct answer is (A).

31. Since $U=-\frac{k}{2r^2}$

Force acting on the particle is $F=-\frac{dU}{dr}=\frac{k}{r^3}$

This force provides necessary centripetal force, so

$$\frac{mv^2}{r}=\frac{k}{r^3}$$

$$\Rightarrow \quad mv^2=\frac{k}{r^2}$$

Kinetic energy of particle, $K=\frac{1}{2}mv^2=\frac{k}{2r^2}$

Total energy of the particle $=U+K=-\frac{k}{2r^2}+\frac{k}{2r^2}=0$

Hence, the correct answer is (C).

32. Since, central force is given by

$$F_c \propto \frac{1}{R^n}$$

$$\Rightarrow \quad F_c=k\left(\frac{1}{R^n}\right)$$

$$\Rightarrow \quad m\omega^2R=k\frac{1}{R^n}$$

$$\Rightarrow \quad m\frac{(2\pi)^2}{T^2}=k\frac{1}{R^{n+1}}$$

$$\Rightarrow \quad T^2 \propto R^{n+1}$$

$$\Rightarrow \quad T \propto R^{\frac{(n+1)}{2}}$$

Hence, the correct answer is (C).

33. Initially, total energy is $E_i=-\frac{GMm}{2R}$

Final total energy is $E_f=-\frac{GM\left(\frac{m}{2}\right)}{2\left(\frac{R}{2}\right)}-\frac{GM\left(\frac{m}{2}\right)}{2\left(\frac{3R}{2}\right)}$

$$\Rightarrow \quad E_f=-\frac{2GMm}{3R}$$

Required difference in energies is $\Delta E=E_f-E_i$

$$\Rightarrow \quad \Delta E=-\frac{GMm}{R}\left(\frac{2}{3}-\frac{1}{2}\right)=-\frac{GMm}{6R}$$

Hence, the correct answer is (D).

34. $F_1=\frac{GM_eM_m}{r_1^2}$ and $F_2=\frac{GM_eM_s}{r_2^2}$

$$\Rightarrow \quad \Delta F_1=-\frac{2GM_eM_m}{r_1^3}\Delta r_1 \text{ and } \Delta F_2=-\frac{2GM_eM_s}{r_2^3}\Delta r_2$$

$$\Rightarrow \quad \frac{\Delta F_1}{\Delta F_2}=\frac{M_m\Delta r_1}{r_1^3}\frac{r_2^3}{M_s\Delta r_2}=\left(\frac{M_m}{M_s}\right)\left(\frac{r_2^3}{r_1^3}\right)\left(\frac{\Delta r_1}{\Delta r_2}\right)$$

Given that

$\Delta r_1=\Delta r_2=2R_{\text{earth}}$, $M_m=8\times10^{22}$ kg

$M_s=2\times10^{30}$ kg, $r_1=0.4\times10^6$ km,

$r_2=150\times10^6$ km.

$$\Rightarrow \quad \frac{\Delta F_1}{\Delta F_2}=2$$

Hence, the correct answer is (A).

35. Effect of rotation of earth on acceleration due to gravity is given by $g'=g-R\omega^2\cos^2\phi$
where ϕ is the latitude angle. There will be no change in gravity at poles because $\phi=90°$ at the poles and at all other points as ω increases, g' will decrease.
Hence, the correct answer is (C).

36. This force provides the centripetal force to the particle to move in a circular orbit.

$$\Rightarrow \quad \frac{mv^2}{r}=\frac{16}{r}+r^3$$

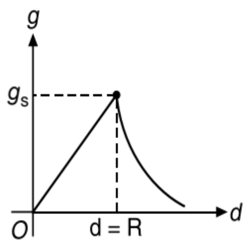
Kinetic energy, $K=\frac{1}{2}mv^2=\frac{1}{2}(16+r^4)$

$$\Rightarrow \quad \frac{K_1}{K_2}=\frac{\frac{16+1}{2}}{\frac{16+256}{2}}=\frac{17}{272}=0.0625$$

$$\Rightarrow \quad \frac{K_1}{K_2}\quad 6\times10^{-2}$$

Hence, the correct answer is (A).

37. Variation of g inside earth surface



For $d < R$, $g = \left(\frac{Gm}{R^2}\right)d$

For $d = R$, $g_s = \frac{Gm}{R^2}$

For $d > R$, $g = \frac{Gm}{d^2}$

Hence, the correct answer is (D).

38. Here, the weight of person on the equator is W. If the earth rotates about its axis, then weight is $\frac{3W}{4}$

Radius of the earth $= 6400$ km

The acceleration due to gravity at the equator is

$$g_e = g - R\omega^2$$

$$\Rightarrow \frac{3}{4}g = g - R\omega^2$$

$$\Rightarrow R\omega^2 = \frac{g}{4}$$

$$\Rightarrow \omega = \sqrt{\frac{g}{4R}} = \sqrt{\frac{10}{4 \times 6400 \times 10^3}} = 0.625 \times 10^{-3} \text{ rads}^{-1}$$

$$\Rightarrow \omega \approx 0.63 \times 10^{-3} \text{ rads}^{-1}$$

Hence, the correct answer is (A).

39. The correct answer is (A).

40. $v_0 = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

So, increase in velocity is

$$\Delta v = \sqrt{gR}\left(\sqrt{2} - 1\right)$$

Hence, the correct answer is (A).

41. Let the area of the ellipse be A. According to Kepler's Second Law, areal velocity of a planet around the sun is constant, i.e., $\frac{dA}{dt} = \text{constant}$, so we have

$$\frac{t_1}{t_2} = \frac{\text{Area of } abcsa}{\text{Area of } adcsa} = \frac{\frac{A}{2} + \frac{A}{4}}{\frac{A}{2} - \frac{A}{4}} = \frac{\frac{3A}{4}}{\frac{A}{4}} = 3$$

$$\Rightarrow t_1 = 3t_2$$

Hence, the correct answer is (C).

42. Gravitational pull on the astronaut $F_G = \frac{GMm}{(R+h)^2}$

Net force on the astronaut is zero.

Hence, the correct answer is (C).

43. $V_P = V_{\text{sphere at } P} - V_{\text{cavity at } P}$

Since, $V = -\frac{GM}{2R^3}\left(3R^2 - r^2\right)$

$$\Rightarrow V_{\text{sphere at P}} = -\frac{GM}{2R^3}\left(3R^2 - \left(\frac{R}{2}\right)^2\right) = -\frac{11GM}{8R}$$

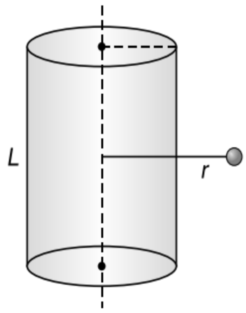
Now, $V_{\text{cavity at P}} = -\frac{3}{2}\frac{G\left(\frac{M}{8}\right)}{\left(\frac{R}{2}\right)} = -\frac{3GM}{8R}$

$$\Rightarrow V_P = -\frac{11GM}{8R} - \left(-\frac{3GM}{8R}\right) = -\frac{GM}{R}$$

Hence, the correct answer is (B).

44. Centripetal force is provided by the gravitational force which is proportional to $\frac{1}{r}$. So, $F \propto \frac{1}{r}$

$$\Rightarrow \frac{mv_0^2}{r} \propto \frac{1}{r}$$



$$\Rightarrow v_0 = \text{constant}$$

Since $T = \frac{2\pi r}{v}$

$$\Rightarrow T \propto r$$

Hence, the correct answer is (C).

45. Potential $V(r)$ due to a large planet of radius R is given by

$$V_0(r) = -\frac{GM}{r} \text{ for } r > R$$

$$V(r) = \frac{-GM}{R} \text{ for } r = R$$

$$V_{in} = -\frac{3}{2}\frac{GM}{R}\left(1 - \frac{r^2}{3R^2}\right) \text{ for } r < R$$

Hence, the correct answer is (B).

46. $F = F_{\text{gravitational force on M}} = \frac{Mv^2}{R}$

$$\Rightarrow F = 2\left(\frac{GM^2}{\left(\sqrt{2}R\right)^2}\frac{1}{\sqrt{2}}\right) + \frac{GM^2}{(2R)^2} = \frac{Mv^2}{R}$$

$$\Rightarrow \frac{GM^2}{\sqrt{2}R^2} + \frac{GM^2}{4R^2} = \frac{Mv^2}{R}$$

$$\Rightarrow v = \frac{1}{2}\sqrt{\frac{GM}{R}\left(1 + 2\sqrt{2}\right)}$$

Hence, the correct answer is (D).

47. At surface, $E_i = -\frac{GMm}{R}$

In orbit, $E_f = -\frac{GMm}{2(3R)} = -\frac{GMm}{6R}$

$\Rightarrow$ Required energy is

$\Rightarrow \quad \Delta E = E_f - E_i = \frac{5GMm}{6R}$

Hence, the correct answer is (A).

48. Energy required to make the spaceship reach the free space is

$$\Delta E = \frac{GMm}{R}$$

Since $g = \frac{GM}{R^2}$

$\Rightarrow \quad \Delta E = gR^2 \times \frac{m}{R}$

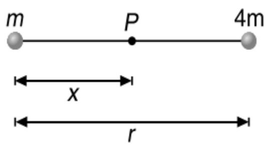
$\Rightarrow \quad \Delta E = mgR$

$\Rightarrow \quad \Delta E = 1000 \times 10 \times 6400 \times 10^3 = 64 \times 10^9$ J

$\Rightarrow \quad \Delta E = 6.4 \times 10^{10}$ J

Hence, the correct answer is (C).

49. Let x be the distance of the point P from the mass m where gravitational field is zero.



$\Rightarrow \quad \frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2}$

$\Rightarrow \quad \left(\frac{x}{(r-x)}\right)^2 = \frac{1}{4}$

$\Rightarrow \quad x = \frac{r}{3}$ ...(1)

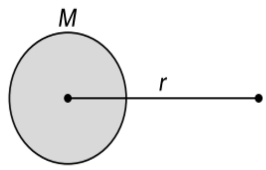
Gravitational potential at a point P i.e. at $x = \frac{r}{3}$ is

$$V = -\frac{Gm}{x} - \frac{G(4m)}{(r-x)} = -\frac{Gm}{\left(\frac{r}{3}\right)} - \frac{G(4m)}{\left(r - \frac{r}{3}\right)}$$

$\Rightarrow \quad V = -\frac{3Gm}{r} - \frac{3G(4m)}{2r} = -9\frac{Gm}{r}$

Hence, the correct answer is (D).

50. The acceleration due to gravity at a height h from the ground is given as $\frac{g}{9}$.



$$\frac{GM}{r^2} = \left(\frac{GM}{R^2}\right)\frac{1}{9}$$

$\Rightarrow \quad r = 3R$

The height above the ground is $2R$

Hence, the correct answer is (A).

ARCHIVE: JEE ADVANCED

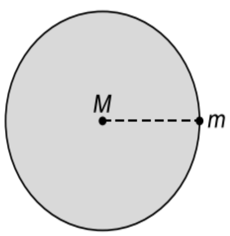
Single Correct Choice Type Problems

1. For a particle revolving in a circular orbit of radius r due to the gravitational attraction of inner cloud of mass M, we have

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$\Rightarrow \quad M = \frac{v^2 r}{G} = \frac{2mv^2 r}{2Gm}$

Since $K = \frac{1}{2}mv^2 =$ constant



$\Rightarrow \quad mv^2 = 2K$

$\Rightarrow \quad M = \frac{2Kr}{Gm}$

$\Rightarrow \quad dM = \frac{2Kdr}{Gm}$ ...(1)

Also, we know that

$$dM = \rho(r)dV$$

$\Rightarrow \quad dM = \rho(r)4\pi r^2 dr$

So, equation (1) becomes,

$$\rho(r)4\pi r^2 dr = \frac{2Kdr}{Gm}$$

$\Rightarrow \quad \frac{\rho(r)}{m} = n(r) = \frac{K}{2\pi Gm^2 r^2}$

Hence, the correct answer is (B).

2. Given that $v_e = 11.2 \text{ kms}^{-1} = \sqrt{\dfrac{2GM_e}{R_e}}$

By Law of Conservation of Energy, we have

$$K_i + U_i = K_f + U_f$$

$$\Rightarrow \quad \frac{1}{2}mv_s^2 - \frac{GM_s m}{r} - \frac{GM_e m}{R_e} = 0 + 0$$

where, r is the distance of rocket from sun

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2GM_s}{r}}$$

Since, $M_s = 3\times10^5 M_e$ and $r = 2.5\times10^4 R_e$

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2G3\times10^5 M_e}{2.5\times10^4 R_e}}$$

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e}\left(1 + \frac{3\times10^5}{2.5\times10^4}\right)}$$

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e}\times 13}$$

$$\Rightarrow \quad v_s \quad 42 \text{ kms}^{-1}$$

Hence, the correct answer is (C).

3. Given, $R_{\text{planet}} = R = \dfrac{R_{\text{earth}}}{10}$

Since, density $\rho = \dfrac{M_{\text{earth}}}{\frac{4}{3}\pi R_{\text{earth}}^3}$

Also, $\rho = \dfrac{M_{\text{planet}}}{\frac{4}{3}\pi R_{\text{planet}}^3}$

$$\Rightarrow \quad M_{\text{planet}} = \frac{M_{\text{earth}}}{10^3} = \frac{M_e}{1000}$$

Let the acceleration due to gravity at surface of planet and at the surface of earth be g_p and g_e respectively. Then

$$g_p = \frac{GM_{\text{planet}}}{R_{\text{planet}}^2} = \frac{GM_e(10)^2}{(10)^3 R_e^2} = \frac{GM_e}{10R_e^2}$$

$$\Rightarrow \quad g_p = \frac{g_e}{10}$$

The value of g inside the planet at a distance x from centre of the planet is

$$g_{\text{inside}} = g_{\text{surface of planet}}\left(\frac{x}{R}\right) = g_p\left(\frac{x}{R}\right)$$

So, total force acting on wire is

$$F = \int_{\frac{4R}{5}}^{R} (\lambda dx) g_p\left(\frac{x}{R}\right)$$

$$\Rightarrow \quad F = \frac{\lambda g_p}{R}\left(\frac{x^2}{2}\right)\Bigg|_{4R/5}^{R}$$

Substituting the given values, we get

$$F = 108 \text{ N}$$

Hence, the correct answer is (B).

4. In circular orbit of a satellite, potential energy U is

$$U = -2\times(\text{kinetic energy}) = -2\times\frac{1}{2}mv^2 = -mv^2$$

Just to escape from the gravitational pull, its total mechanical energy should be zero. Therefore, its kinetic energy should be $+mv^2$.

Hence, the correct answer is (B).

5. For annular disc, gravitational potential at the point P lying on the axis at a distance x from centre is

$$V_P = -\frac{2GM}{(R_2^2 - R_1^2)}\left(\sqrt{R_2^2 + x^2} - \sqrt{R_1^2 + x^2}\right)$$

where $R_2 = 4R$, $R_1 = 3R$ and $x = 4R$

$$\Rightarrow \quad V_P = -\frac{2GM}{7R^2}\left(4\sqrt{2}R - 5R\right)$$

Also, $V_\infty = 0$

Since $W_{P\to\infty} = m_0(V_\infty - V_P)$, where $m_0 = 1$ unit

$$\Rightarrow \quad W_{P\to\infty} = \frac{2GM}{7R}\left(4\sqrt{2} - 5\right)$$

Hence, the correct answer is (A).

6. For $r \le R$, $\dfrac{mv^2}{r} = \dfrac{GmM}{r^2}$ …(1)

Here, $M = \left(\dfrac{4}{3}\pi r^3\right)\rho_0$

Substituting in Equation (1), we get $v \propto r$

i.e. v-r graph is a straight line passing through origin.

For $r > R$, we have

$$\frac{mv^2}{r} = \frac{Gm\left(\frac{4}{3}\pi R^3\right)\rho_0}{r^2}$$

$$\Rightarrow \quad v \propto \frac{1}{\sqrt{r}}$$

The corresponding v-r graph will be as shown in option (C).

Hence, the correct answer is (C).

7. In case of binary star system, angular velocity and hence, the time period of both the stars are equal.

Hence, the correct answer is (D).

8. Time period of a satellite very close to earth's surface is 84.6 min. Time period increases as the distance of the satellite from the surface of earth increases. So, time period of spy satellite orbiting a few 100 km above

the earth's surface should be slightly greater than 84.6 min. Therefore, the most appropriate option is (C) or 2 h

Hence, the correct answer is (C).

9. Since $T_1 = 2\pi\sqrt{\frac{\ell}{g}}$

$$T_2 = 2\pi\sqrt{\frac{\ell}{g_h}}$$

Now $g_h = \frac{gR^2}{(R+h)^2}$

$$\Rightarrow \quad g_h = \frac{g}{4} \qquad \{\because h = R\}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \sqrt{\frac{g}{g/4}} = 2$$

Hence, the correct answer is (D).

10. Force on satellite is always towards earth, therefore, acceleration of satellite S is always directed towards centre of the earth due to which net torque of this gravitational force F about centre of earth is zero. Therefore, angular momentum (both in magnitude and direction) of S about centre of earth is constant throughout. Since, the force F is conservative in nature, therefore mechanical energy of satellite remains constant. Speed of S is maximum when it is nearest to earth and minimum when it is farthest.

Hence, the correct answer is (A).

11. From Kepler's Third Law, we have

$$T^2 \propto r^3$$

$$\Rightarrow \quad T \propto (r)^{3/2}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

$$\Rightarrow \quad T_2 = T_1\left(\frac{r_2}{r_1}\right)^{3/2} = (365)\left(\frac{1}{2}\right)^{3/2}$$

$$\Rightarrow \quad T_2 \approx 129 \text{ days}$$

Hence, the correct answer is (B).

12. $F \propto R^{-5/2}$

This gravitational force of attraction provides necessary centripetal force to planet to revolve about the massive star.

$$\Rightarrow \quad mR\omega^2 \propto R^{-5/2}$$

$$\Rightarrow \quad \omega^2 \propto R^{-7/2}$$

$$\Rightarrow \quad \frac{4\pi^2}{T^2} \propto R^{-7/2}$$

$$\Rightarrow \quad T^2 \propto R^{7/2}$$

Hence, the correct answer is (B).

13. $\Delta U = -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$

$$\Rightarrow \quad \Delta U = -\frac{GMm}{2R} + \frac{GMm}{R} \qquad \{\because h = R\}$$

$$\Rightarrow \quad \Delta U = \frac{GMm}{2R} = \frac{1}{2}m\left(\frac{GM}{R^2}\right)R = \frac{mgR}{2}$$

Hence, the correct answer is (A).

14. Since, $g = \frac{GM}{R^2}$

$$\Rightarrow \quad g \propto \frac{1}{R^2}$$

$$\Rightarrow \quad \frac{\Delta g}{g} = -2\frac{\Delta R}{R}$$

So, g will increase, if R decreases

$$\Rightarrow \quad \frac{\Delta g}{g} = -2(-1\%) = 2\%$$

Hence, the correct answer is (C).

Multiple Correct Choice Type Problems

1. Gravitational field at a distance r due to mass $m\left(=\frac{4}{3}\pi r^3\rho\right)$ is

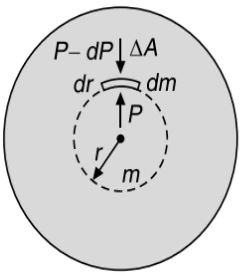
$$E = \frac{G\rho\left(\frac{4}{3}\pi r^3\right)}{r^2} = \frac{4G\rho\pi r}{3}$$

Consider a small element of width dr and area ΔA at a distance r from the centre. Pressure force on this element is due to the gravitational force on dm from m inwards towards the centre.

$$\Rightarrow \quad (dP)\Delta A = E(dm)$$

where $dm = (\Delta A)(dr)\rho$ and $m = \left(\frac{4}{3}\pi r^3\right)\rho$

$$\Rightarrow \quad -dP\Delta A = \left(\frac{4}{3}G\pi\rho r\right)(\rho\Delta A dr)$$



$$\Rightarrow \quad -\int_0^P dP = \int_R^r \left(\frac{4G\rho^2\pi}{3}\right) r\,dr$$

$$\Rightarrow \quad -P = \frac{4G\rho^2\pi}{3\times 2}(r^2 - R^2)$$

CHAPTER 4

$$\Rightarrow \quad P = \frac{2G\rho^2\pi}{3}(R^2 - r^2)$$

$$\Rightarrow \quad P = k(R^2 - r^2), \text{ where } k = \frac{2G\rho^2\pi}{3} = \text{constant}$$

For, $r = \frac{3R}{4}$, $P_1 = k\left(R^2 - \frac{9R^2}{16}\right) = k\left(\frac{7R^2}{16}\right)$

For, $r = \frac{2R}{3}$, $P_2 = k\left(R^2 - \frac{4R^2}{9}\right) = k\left(\frac{5R^2}{9}\right)$

$$\Rightarrow \quad \frac{P_1}{P_2} = \frac{63}{80}$$

For, $r = \frac{3R}{5}$, $P_3 = k\left(R^2 - \frac{9}{25}R^2\right) = k\left(\frac{16R^2}{25}\right)$

For, $r = \frac{2R}{5}$, $P_4 = k\left(R^2 - \frac{4R^2}{25}\right) = k\left(\frac{21R^2}{25}\right)$

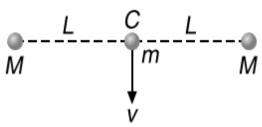
$$\Rightarrow \quad \frac{P_3}{P_4} = \frac{16}{21}$$

For $r = \frac{R}{2}$, $P_5 = k\left(R^2 - \frac{R^2}{4}\right) = k\left(\frac{3R^2}{4}\right)$

For $r = \frac{R}{3}$, $P_6 = k\left(R^2 - \frac{R^2}{9}\right) = k\left(\frac{8R^2}{9}\right)$

$$\Rightarrow \quad \frac{P_5}{P_6} = \frac{27}{32}$$

Hence, (B) and (C) are correct.

2. 

Let v is the minimum velocity, then by Law of Conservation of Energy, we have

$$(U + K)_C = (U + K)_\infty$$

$$\Rightarrow \quad \left(-\frac{GMm}{L}\right)2 + \frac{1}{2}mv^2 = 0 + 0$$

$$\Rightarrow \quad v = 2\sqrt{\frac{GM}{L}}$$

Hence, (B) and (D) are correct.

3. $$v_{es} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G\left(\frac{4}{3}\pi R^3\right)\rho}{R}} = \sqrt{\frac{4G\rho}{3}}R$$

$$v_{es} \propto R$$

Surface area of P is $A = 4\pi R_P^2$

Surface area of Q is $4A = 4\pi R_Q^2$

$$\Rightarrow \quad R_Q = 2R_P$$

Mass of R is $M_R = M_P + M_Q$

$$\Rightarrow \quad \left(\frac{4}{3}\pi R_R^3\right)\rho = \left(\frac{4}{3}\pi R_P^3\right)\rho + \left(\frac{4}{3}\pi R_Q^3\right)\rho$$

$$\Rightarrow \quad R_R^3 = R_P^3 + R_Q^3$$

$$\Rightarrow \quad R_R^3 = 9R_P^3$$

$$\Rightarrow \quad R_R = 9^{1/3}R_P$$

$$\Rightarrow \quad R_R > R_Q > R_P$$

$$\Rightarrow \quad V_R > V_Q > V_P$$

Also, $\frac{V_R}{V_P} = 9^{1/3}$ and $\frac{V_P}{V_Q} = \frac{1}{2}$

Hence, (B) and (D) are correct.

4. Gravitational field is the acceleration due to gravity.

So, $g = \begin{cases} \frac{GM}{r^2} & r \ge R \text{ (Outside)} \\ \frac{4}{3}\pi G\rho r & r < R \text{ (Inside)} \end{cases}$

$$\Rightarrow \quad \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2} (\text{Outside})$$

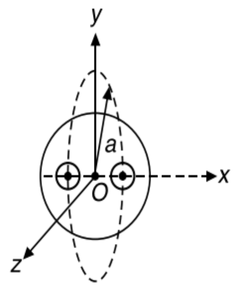
where $r_1 > R$ and $r_2 > R$

and $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ (Inside)

where $r_1 < R$ and $r_2 < R$

Hence, (A) and (B) are correct.

5. The spherical cavities can be assumed to be negative masses placed symmetrically about origin O. So gravitational force due to this object at origin is zero.



The dotted circle is lying in the yz plane and is an equipotential surface (as we can see that two cavities are lying symmetrically on each side of the EPS (Equi-Potential Surface) so gravitational potential is same at all the points lying on the circle $y^2 + z^2 = a^2$ (where a^2 = constant).

Hence, (A), (C) and (D) are correct.