CHAPTER 3: VECTORS

Test Your Concepts-I (Based on Addition, Subtraction and Resolution)

1. No, Yes

2. (a) $A_x = 4$ and $A_y = 6$ ...(1)

$$\vec{A}+\vec{B}=\left(A_x\hat{i}+A_y\hat{j}\right)+\left(B_x\hat{i}+B_y\hat{j}\right)$$

$$\Rightarrow \quad \vec{A}+\vec{B}=\left(A_x+B_x\right)\hat{i}+\left(A_y+B_y\right)\hat{j}$$

So, $A_x + B_x = 10$ and $A_y + B_y = 9$ ...(2)

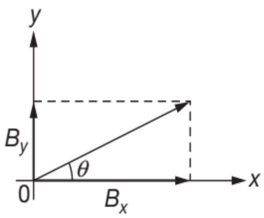
From equation sets (1) and (2), we get

$$B_x = 6 \text{ and } B_y = 3$$

So, $\vec{B} = 6\hat{i} + 3\hat{j}$

(b) $|\vec{B}| = \sqrt{6^2+3^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$ m

(c) Let $\vec{B}$ make an angle θ with x-axis, then from the diagram



$$\tan\theta = \frac{B_y}{B_x} = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{2}\right), \text{ with } x\text{-axis}$$

3. $\vec{A} = 4\hat{i} - 3\hat{j}$ and $\vec{B} = 6\hat{i} + 8\hat{j}$

$$|\vec{A}| = \sqrt{4^2+(-3)^2} = 5$$

$$|\vec{B}| = \sqrt{6^2+8^2} = 10$$

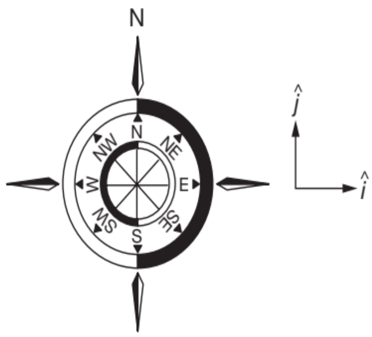
$$\vec{A}+\vec{B} = 10\hat{i}+5\hat{j}$$

$$\vec{A}-\vec{B} = -2\hat{i}-11\hat{j}$$

$$\vec{B}-\vec{A} = 2\hat{i}+11\hat{j}$$

	$\vec{A}$	$\vec{B}$	$\vec{A}+\vec{B}$	$\vec{A}-\vec{B}$	$\vec{B}-\vec{A}$
Magnitude	5	10	$5\sqrt{5}$	$5\sqrt{5}$	$5\sqrt{5}$
Direction (θ with x-axis)	$\tan^{-1}\left(\frac{3}{4}\right)$ Below x-axis	$\tan^{-1}\left(\frac{4}{3}\right)$ Above x-axis	$\tan^{-1}\left(\frac{1}{2}\right)$ Above x-axis	$\tan^{-1}\left(\frac{11}{2}\right)$ Below negative x-axis	$\tan^{-1}\left(\frac{11}{2}\right)$ Above x-axis

4. $\Delta\vec{r}_1 = 12\hat{i}$, $\Delta\vec{r}_2 = 5\hat{j}$ and $\Delta\vec{r}_3 = 6\hat{k}$



Please note that vertically upwards does not mean north, it actually means "vertically upwards in space towards you", which happens to be the +z direction.

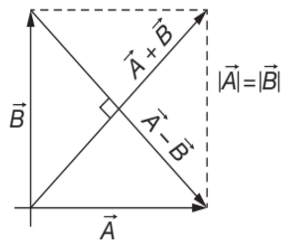
If $\Delta\vec{r}$ is the net displacement, then

$$\Delta\vec{r} = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3$$

$$\Rightarrow \quad \Delta\vec{r} = 12\hat{i} + 5\hat{j} + 6\hat{k}$$

$$\Rightarrow \quad \Delta r = \sqrt{144+25+36} = \sqrt{205} \text{ m}$$

5. $|\vec{A}| = |\vec{B}|$



Since $|\vec{A}+\vec{B}|$ and $|\vec{A}-\vec{B}|$ are the two diagonals of a parallelogram whose adjacent sides are equal in magnitude. So, this parallelogram can either be a square or a rhombus and in both the cases, the diagonals are perpendicular to each other.

6. Already discussed in theory.

7. $\vec{F}_1 = 3\sqrt{2}\ NE = 3\sqrt{2}\left[\cos(45°)\hat{i} + \sin(45°)\hat{j}\right]$

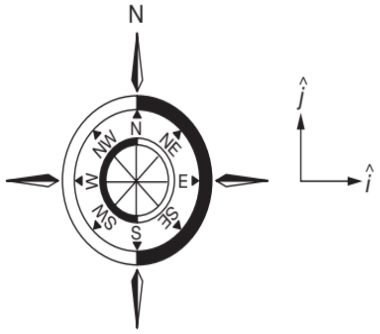
$$\Rightarrow \quad \vec{F}_1 = 3\hat{i} + 3\hat{j} \quad ...(1)$$

$$\vec{F}_2 = 6\sqrt{2}\ SE = 6\sqrt{2}\left[\cos(45°)\hat{i} - \sin(45°)\hat{j}\right]$$

$$\Rightarrow \quad \vec{F}_2 = 6\hat{i} - 6\hat{j} \quad ...(2)$$

$$\vec{F}_3 = \sqrt{2}\ NW = \sqrt{2}\left[-\cos(45°)\hat{i} + \sin(45°)\hat{j}\right]$$

$$\Rightarrow \quad \vec{F}_3 = -\hat{i} + \hat{j}$$



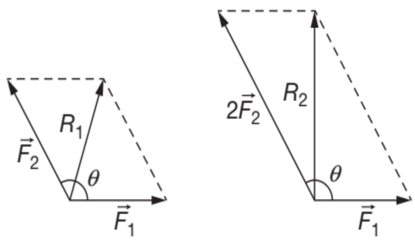
So, $\vec{F}=\vec{F}_1+\vec{F}_2+\vec{F}_3=8\hat{i}-2\hat{j}$

$\Rightarrow \quad |\vec{F}|=\sqrt{64+4}=\sqrt{68}=2\sqrt{17}$ kgf

8. $|\vec{a}+\vec{b}|=|\vec{a}|+|\vec{b}|$ just implies that $\vec{a}$ and $\vec{b}$ are parallel.

9. Let θ be the angle between $\vec{F}_1$ and $\vec{F}_2$, then

$$R_1^2=F_1^2+F_2^2+2F_1F_2\cos\theta \qquad \ldots(1)$$



When $\vec{F}_2$ is doubled i.e., made $2\vec{F}_2$, then too the angle between $\vec{F}_1$ and $2\vec{F}_2$ is still θ. But now the new resultant makes an angle of $90°$ with $\vec{F}_1$. So, using $\tan\beta=\dfrac{B\sin\theta}{A+B\cos\theta}$, we get

$$\tan\left(\frac{\pi}{2}\right)=\frac{2F_2\sin\theta}{F_1+2F_2\cos\theta}$$

$$\Rightarrow \quad \frac{2F_2\sin\theta}{F_1+2F_2\cos\theta}\to\infty \qquad \left\{\because \tan\left(\frac{\pi}{2}\right)\to\infty\right\}$$

$$\Rightarrow \quad F_1+2F_2\cos\theta=0$$

$$\Rightarrow \quad \cos\theta=-\frac{F_1}{2F_2} \qquad \ldots(2)$$

Negative sign gives an indication that θ is obtuse.
Substituting (2) in(1), we get

$$R_1^2=F_1^2+F_2^2+2F_1F_2\left(-\frac{F_1}{2F_2}\right)$$

$$\Rightarrow \quad R_1^2=\cancel{F_1^2}+F_2^2-\cancel{F_1^2}$$

$$\Rightarrow \quad R_1=F_2$$

10. Change in velocity $=\Delta\vec{v}=\vec{v}_B-\vec{v}_A$

Since $|\vec{v}_B-\vec{v}_A|=\sqrt{v_B^2+v_A^2-2v_Av_B\cos\theta}$

$$\Rightarrow \quad |\vec{v}_B-\vec{v}_A|=\sqrt{v^2+v^2-2v^2\cos(40°)}$$

$$\Rightarrow \quad |\vec{v}_B-\vec{v}_A|=\sqrt{2}v\sqrt{1-\cos(40°)}$$

Since $1-\cos(40°)=2\sin^2(20°)$

$$\Rightarrow \quad |\vec{v}_B-\vec{v}_A|=\sqrt{2}v\sqrt{2\sin^2(20°)}$$

$$\Rightarrow \quad |\vec{v}_B-\vec{v}_A|=2v\sin(20°)$$

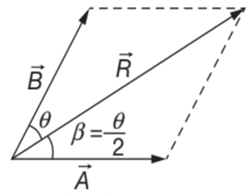
11. Given $|\vec{A}|=|\vec{B}|$

Let the resultant make an angle β with $\vec{A}$ and let the angle between $\vec{A}$ and $\vec{B}$ be θ. Then

$$\tan\beta=\frac{B\sin\theta}{A+B\cos\theta}$$

$$\Rightarrow \quad \tan\beta=\frac{A\sin\theta}{A+A\cos\theta} \qquad \{\because A=B\}$$

$$\Rightarrow \quad \tan\beta=\frac{\sin\theta}{1+\cos\theta}$$



$|\vec{A}|=|\vec{B}|$

Since $\sin\theta=2\sin\left(\dfrac{\theta}{2}\right)\cos\left(\dfrac{\theta}{2}\right)$

and $2\cos^2\left(\dfrac{\theta}{2}\right)=1+\cos\theta$

$$\Rightarrow \quad \tan\beta=\frac{\cancel{2}\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{\cancel{2}\cos^2\left(\frac{\theta}{2}\right)}$$

$$\Rightarrow \quad \tan\beta=\tan\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \quad \beta=\frac{\theta}{2}$$

Hence the resultant is equally inclined to $\vec{A}$ as well as $\vec{B}$.

12. (a) Since $|\vec{A}|=|\vec{B}|=1$

$$\Rightarrow \quad |\vec{A}+\vec{B}|=\sqrt{A^2+B^2+2AB\cos\theta}$$

$$\Rightarrow \quad |\vec{A}+\vec{B}|=\sqrt{1+1+2\cos\theta}=\sqrt{2}\sqrt{1+\cos\theta}$$

$$\Rightarrow \quad |\vec{A}+\vec{B}|=2\cos\left(\frac{\theta}{2}\right) \quad \left\{\because 1+\cos\theta=2\cos^2\left(\frac{\theta}{2}\right)\right\}$$

(b) Similarly,

$$|\vec{A}-\vec{B}|=\sqrt{1+1-2\cos\theta}=\sqrt{2}\sqrt{1-\cos\theta}$$

$$\Rightarrow \quad |\vec{A}-\vec{B}|=2\sin\left(\frac{\theta}{2}\right) \left\{\because 1-\cos\theta=2\sin^2\left(\frac{\theta}{2}\right)\right\}$$

13. Let the two vectors be $\vec{A}$ and $\vec{B}$ inclined at an angle θ, then

$$R^2=A^2+B^2+2AB\cos\theta$$

According to the question, we have been given

$$R=A=B$$

$$\Rightarrow \quad A^2=A^2+A^2+2A^2\cos\theta$$

$$\Rightarrow \quad \cos\theta=-\frac{1}{2}$$

$$\Rightarrow \quad \theta=120°$$

So, this is possible, when two vectors of equal magnitude are inclined to each other at an angle of $120°$.

14. Let the vector make an angle α with x-axis, β with y-axis and γ with z-axis. Then

$$\cos\alpha=\frac{x}{r}=\frac{1}{\sqrt{1+1+1}}=\frac{1}{\sqrt{3}}$$

$$\cos\beta=\frac{y}{r}=\frac{1}{\sqrt{1+1+1}}=\frac{1}{\sqrt{3}}$$

and $\cos\gamma=\dfrac{z}{r}=\dfrac{1}{\sqrt{1+1+1}}=\dfrac{1}{\sqrt{3}}$

So, we simply conclude that

$$\alpha=\beta=\gamma=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Hence the vector $\vec{r}=\hat{i}+\hat{j}+\hat{k}$ is equally inclined to all the three axis.

Test Your Concepts-II (Based on Dot Product)

1. Let $\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$ and

$\vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$, then

$$\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$$

2. $\vec{v}=v_x\hat{i}+v_y\hat{j}=\hat{i}+\sqrt{3}\hat{j}$, $|\vec{v}|=2$

$\vec{v}'=v_x'\hat{i}+v_y'\hat{j}=2\hat{i}+2\hat{j}$, $|\vec{v}|=2\sqrt{2}$

Let θ be the angle between $\vec{v}$ and $\vec{v}'$, then

$$\cos\theta=\frac{\vec{v}\cdot\vec{v}'}{|\vec{v}||\vec{v}'|}$$

$$\Rightarrow \quad \cos\theta=\frac{2+2\sqrt{3}}{2(2\sqrt{2})}$$

$$\Rightarrow \quad \cos\theta=\frac{1+\sqrt{3}}{2\sqrt{2}}=\frac{1}{2\sqrt{2}}+\frac{\sqrt{3}}{2\sqrt{2}} \quad \text{...(1)}$$

Since $\cos(A-B)=\cos A\cos B+\sin A\sin B$

$$\Rightarrow \quad \cos(15°)=\cos(45°-30°)=\frac{1}{2\sqrt{2}}+\frac{\sqrt{3}}{2\sqrt{2}} \quad \text{...(2)}$$

So, from (1) and (2), we get

$$\theta=15°$$

3. (a) Since $\vec{A}=4\hat{i}+6\hat{j}-3\hat{k}$ and $\vec{B}=-2\hat{i}-5\hat{j}+7\hat{k}$, so

$$|\vec{A}|=\sqrt{16+36+9}=\sqrt{61} \text{ and}$$

$$|\vec{B}|=\sqrt{4+25+49}=\sqrt{78}.$$

So, direction cosines for $\vec{A}$ are

$$(l_1, m_1, n_1)\equiv\left(\frac{4}{\sqrt{61}}, \frac{6}{\sqrt{61}}, -\frac{3}{\sqrt{61}}\right)$$

and that for $\vec{B}$ are

$$(l_2, m_2, n_2)\equiv\left(-\frac{2}{\sqrt{78}}, -\frac{5}{\sqrt{78}}, \frac{7}{\sqrt{78}}\right).$$

(b) Let θ, be the angle between $\vec{A}$ and $\vec{B}$, then

$$\cos\theta=\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$$

$$\Rightarrow \quad \cos\theta=\frac{-8-30-21}{\sqrt{61}\sqrt{78}}$$

$$\Rightarrow \quad \cos\theta=\frac{59}{\sqrt{61}\sqrt{78}}=\frac{59}{69}$$

$$\Rightarrow \quad \theta\cong 31.3°$$

4. Firstly we observe that $\vec{A}=\vec{B}+\vec{C}$, which means $\vec{A}$, $\vec{B}$ and $\vec{C}$ are representing the three sides of a triangle. Secondly, we observe that

$$\vec{A}\cdot\vec{C}=(3)(2)+(-2)(1)+(1)(-4)=0$$

$$\Rightarrow \quad \vec{A}\perp\vec{C}$$

Hence the three vectors form a right angled triangle.

5. Projection of $\vec{A}$ along $\vec{r}$ is $A\cos\theta$. Since $\vec{A}\cdot\vec{r}=Ar\cos\theta$

$$\Rightarrow \quad A\cos\theta=\frac{\vec{A}\cdot\vec{r}}{|\vec{r}|}=\frac{50+48-54}{\sqrt{25+36+81}}=\frac{44}{\sqrt{142}}$$

6. $\vec{A}=6\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{B}=2\hat{i}-9\hat{j}+6\hat{k}$

$$|\vec{A}|=\sqrt{36+4+9}=7$$

and $\vec{B} = \sqrt{4+81+36} = 11$

Now $\vec{A}\cdot\vec{B} = 12 - 18 + 18 = 12$

If θ is the angle between $\vec{A}$ and $\vec{B}$, then

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{12}{(7)(11)} = \frac{12}{77}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{12}{77}\right)$$

7. $\vec{u} = \text{constant}$

$$\Rightarrow \quad u^2 = \text{constant}$$

$$\Rightarrow \quad \vec{u}\cdot\vec{u} = \text{constant} \qquad \ldots(1)$$

Taking derivative of both sides of (1), we get

$$\frac{d}{dt}(\vec{u}\cdot\vec{u}) = \vec{u}\cdot\frac{d\vec{u}}{dt} + \vec{u}\cdot\frac{d\vec{u}}{dt} = \frac{d}{dt}(\text{Constant})$$

$$\Rightarrow \quad 2\vec{u}\cdot\frac{d\vec{u}}{dt} = 0$$

$$\Rightarrow \quad \vec{u}\cdot\frac{d\vec{u}}{dt} = 0$$

Also, we can say that for a vector of constant magnitude $\vec{u} \perp \frac{d\vec{u}}{dt}$.

8. Let us reconstruct the question, for our convenience. According to the statement,

A force $F_1 = 5\text{ N}$ acts along $\vec{A} = 6\hat{i} + 2\hat{j} + 3\hat{k}$

So,

$$\vec{F}_1 = 5(\hat{A}) = 5\left(\frac{6\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{6^2+2^2+3^2}}\right) = \frac{5}{7}(6\hat{i}+2\hat{j}+3\hat{k})$$

A force $F_2 = 3\text{ N}$ acts along $\vec{B} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

So, $\vec{F}_2 = 3(\hat{B}) = 3\left(\frac{3\hat{i}-2\hat{j}+6\hat{k}}{\sqrt{9+4+36}}\right) = \frac{3}{7}(3\hat{i}-2\hat{j}+6\hat{k})$

A force $F_3 = 1\text{ N}$ acts along $\vec{C} = 2\hat{i} - 3\hat{j} - 6\hat{k}$

So, $\vec{F}_3 = 1(\hat{C}) = 1\left(\frac{2\hat{i}-3\hat{j}-6\hat{k}}{\sqrt{4+9+36}}\right) = \frac{1}{7}(2\hat{i}-3\hat{j}-6\hat{k})$

Now, the resultant of these three forces is

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\Rightarrow \quad \vec{F} = \frac{1}{7}(30\hat{i}+10\hat{j}+15\hat{k}+9\hat{i}-6\hat{j}+18\hat{k}+2\hat{i}-3\hat{j}-6\hat{k})$$

$$\Rightarrow \quad \vec{F} = \frac{1}{7}(41\hat{i}+\hat{j}+27\hat{k}) \qquad \ldots(1)$$

Also $\Delta\vec{r} = \vec{r}_{\text{final}} - \vec{r}_{\text{initial}} = 3\hat{i} + 0\hat{j} + 2\hat{k} = 3\hat{i} + 2\hat{k}$

So,

$$W = \vec{F}\cdot\Delta\vec{r} = \frac{1}{7}[(41)(3)+0+(27)(2)] = \frac{177}{7}\text{ J.}$$

Test Your Concepts-III (Based on Cross Product, Scalar and Vector Triple Product)

1. (a) $\vec{\tau} = \vec{r}\times\vec{F}$

where $\vec{r} = (7\hat{i}-2\hat{j}+5\hat{k}) - (0\hat{i}+0\hat{j}+0\hat{k})$

$$\Rightarrow \quad \vec{r} = 7\hat{i} - 2\hat{j} + 5\hat{k}$$

$$\text{So, } \vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -2 & 5 \\ 3 & 5 & -2 \end{vmatrix}$$

$$\Rightarrow \quad \vec{\tau} = -21\hat{i} + 29\hat{j} + 41\hat{k}$$

(b) In this case, we have

$$\vec{r} = (7\hat{i}-2\hat{j}+5\hat{k}) - (0\hat{i}-\hat{j}+0\hat{k})$$

$$\Rightarrow \quad \vec{r} = 7\hat{i} - \hat{j} + 5\hat{k}$$

$$\Rightarrow \quad \vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -1 & 5 \\ 3 & 5 & -2 \end{vmatrix}$$

$$\Rightarrow \quad \vec{\tau} = -23\hat{i} + 29j + 38\hat{k}$$

2. $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$$\Rightarrow \quad \vec{A}\times\vec{B} = \hat{i}(A_yB_z - A_zB_y) - \hat{j}(A_xB_z - A_zB_x) + \hat{k}(A_xB_y - A_yB_x)$$

3. $|\vec{a}\times\vec{b}| = ab\sin\theta$

$$\Rightarrow \quad |\vec{a}\times\vec{b}|^2 = a^2b^2\sin^2\theta$$

$$\Rightarrow \quad |\vec{a}\times\vec{b}|^2 = a^2b^2(1-\cos^2\theta)$$

$$\Rightarrow \quad |\vec{a}\times\vec{b}| = \sqrt{a^2b^2 - a^2b^2\cos\theta}$$

Since $ab\cos\theta = \vec{a}\cdot\vec{b}$

$$\Rightarrow \quad |\vec{a}\times\vec{b}| = \sqrt{a^2b^2 - (\vec{a}\cdot\vec{b})^2}$$

4. $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = \vec{a}\times\vec{a} - \vec{a}\times\vec{b} + \vec{b}\times\vec{a} - \vec{b}\times\vec{b}$

Since $\vec{a}\times\vec{a} = \vec{b}\times\vec{b} = \vec{0}$ and $\vec{b}\times\vec{a} = -\vec{a}\times\vec{b}$, so

$$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=-\vec{a}\times\vec{b}-\vec{a}\times\vec{b}$$

$$\Rightarrow\quad (\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=-2(\vec{a}\times\vec{b})=2(\vec{b}\times\vec{a})$$

Since area of a parallelogram with diagonals

$\vec{d}_1=\hat{i}-2\hat{j}-3\hat{k}$ and $\vec{d}_2=2\hat{i}-3\hat{j}+2\hat{k}$ is

Area $=\dfrac{1}{2}|\vec{d}_1\times\vec{d}_2|$

Now $\vec{d}_1\times\vec{d}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & -3\\ 2 & -3 & 2\end{vmatrix}$

$$\Rightarrow\quad \vec{d}_1\times\vec{d}_2=-13\hat{i}-8\hat{j}+\hat{k}$$

$$\Rightarrow\quad |\vec{d}_1\times\vec{d}_2|=\sqrt{(-13)^2+(-8)^2+(1)^2}=\sqrt{234}$$

So, Area $=\dfrac{1}{2}\sqrt{234}$ sq units

5. Area, with direction as the outward normal.

6. $\vec{A}\times\vec{B}=\vec{0}$ and $\vec{A}\cdot\vec{B}=0$

$$\Rightarrow\quad AB\sin\theta=0 \text{ and } AB\cos\theta=0$$

Since $\sin\theta$ and $\cos\theta$ can never be zero simultaneously, so we have either $A=0$ or $B=0$. Hence for both the conditions to be met simultaneously either of the two vectors has to be a null vector.

7. Since $\vec{L}=\vec{r}\times\vec{p}=m(\vec{r}\times\vec{v})$ $\{\because \vec{p}=m\vec{v}\}$

$$\Rightarrow\quad \vec{L}=m\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \dfrac{0.7v_0^2}{g} & \dfrac{0.2v_0^2}{g} & 0\\ 0.7v_0 & -0.3v_0 & 0\end{vmatrix}$$

$$\Rightarrow\quad \vec{L}=m\left[\hat{i}(0-0)-\hat{j}(0-0)+\hat{k}((0.7)(-0.3)-(0.7)(0.2))\right]$$

$$\Rightarrow\quad \vec{L}=-\frac{mv_0^3}{g}(0.35)\hat{k}$$

$$\Rightarrow\quad \vec{L}=-\left(\frac{7mv_0^3}{20g}\right)\hat{k}$$

$$\Rightarrow\quad |\vec{L}|=\frac{7\ mv_0^2}{20\ g}$$

8. Since, $\vec{F}=q(\vec{E}+\vec{v}\times\vec{B})$

For the particle to pass undeviated, $\vec{F}=\vec{0}$

$$\Rightarrow\quad \vec{E}=-\vec{v}\times\vec{B}$$

$$\Rightarrow\quad \vec{E}=\vec{B}\times\vec{v}$$

Let θ be the angle between $\vec{B}$ and $\vec{v}$, then

$E=Bv\sin\theta$ {in magnitude}

$$\Rightarrow\quad v=\frac{E}{B\sin\theta}$$

For velocity to be minimum, we must have

$\sin\theta=\text{MAXIMUM}=1$

$$\Rightarrow\quad \theta=90^\circ$$

Hence angle between B and v must be 90°, i.e., the $\vec{v}$ must be directed either along $+x$ axis or along $-x$ axis. However, for $\vec{E}=\vec{B}\times\vec{v}$ to be valid and $\vec{E}$ to be along $+y$ axis, we must have $\vec{v}$ along $+x$ axis, because $\hat{k}\times\hat{i}=\hat{j}$. So, $v_{\min}=\dfrac{E}{B}$.

9. Moment of force is actually the torque, so

$\vec{\tau}=\vec{r}\times\vec{F}$

where $\vec{r}=(-2\hat{i}+3\hat{j}+4\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})$

$$\Rightarrow\quad \vec{r}=-3\hat{i}+\hat{j}+\hat{k}$$

So, $\vec{\tau}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -3 & 1 & 1\\ 1 & 1 & 1\end{vmatrix}$

$$\Rightarrow\quad \vec{\tau}=4\hat{j}-4\hat{k}$$

$$\Rightarrow\quad \vec{\tau}=4(\hat{j}-\hat{k})$$

10. $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$ ...(1)

$\vec{b}\times(\vec{c}\times\vec{a})=(\vec{b}\cdot\vec{a})\vec{c}-(\vec{b}\cdot\vec{c})\vec{a}$ ...(2)

$\vec{c}\times(\vec{a}\times\vec{b})=(\vec{c}\cdot\vec{b})\vec{a}-(\vec{c}\cdot\vec{a})\vec{b}$ ...(3)

On adding (1), (2) and (3), we get

$$\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})=\vec{0}$$

Single Correct Choice Type Questions

1. $\vec{r}=\dfrac{1}{2}\vec{a}t^2$

$$\Rightarrow\quad \vec{r}=\frac{1}{2}(2\hat{i}+8\hat{j}-6\hat{k})(25)$$

$$\Rightarrow\quad \vec{r}=(\hat{i}+4\hat{j}-3\hat{k})25$$

$\Rightarrow \quad \vec{r} = 25\hat{i} + 100\hat{j} - 75\hat{k}$

Hence, the correct answer is (A).

2. $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$

$\Rightarrow \quad 3\hat{i} + 2\hat{j} - \hat{k} + 3\hat{i} + 4\hat{j} - 5\hat{k} + A\hat{i} + A\hat{j} - A\hat{k} = \vec{0}$

$\Rightarrow \quad \hat{i}(6+A) + \hat{j}(6+A) + \hat{k}(-6-A) = \vec{0}$

$\Rightarrow \quad (\hat{i} + \hat{j} - \hat{k})(6+A) = \vec{0}$

$\Rightarrow \quad 6 + A = 0$

$\Rightarrow \quad A = -6$

Hence, the correct answer is (A).

3. $|\vec{A} \times \vec{B}| = 0$

$\Rightarrow \quad AB\sin\theta = 0$

$\Rightarrow \quad \theta = 0°$

Further

$\vec{A} \cdot (\vec{A} + \vec{B}) = \vec{A} \cdot \vec{A} + \vec{A} \cdot \vec{B}$

$\Rightarrow \quad \vec{A} \cdot (\vec{A} + \vec{B}) = A^2 + AB\cos 0$

$\Rightarrow \quad \vec{A} \cdot (\vec{A} + \vec{B}) = A^2 + AB$

Hence, the correct answer is (D).

4. $R = A = B$

Since

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$\Rightarrow \quad A^2 = A^2 + A^2 + 2A^2\cos\theta$

$\Rightarrow \quad \cos\theta = -\dfrac{1}{2}$

$\Rightarrow \quad \theta = 120°$

$\Rightarrow \quad \theta = \dfrac{2\pi}{3}$

Hence, the correct answer is (B).

5. For $\vec{a} + \vec{b}$ parallel to $\vec{a} - \vec{b}$

$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \vec{0}$

$\Rightarrow \quad \vec{a} \times \vec{a} - \vec{b} \times \vec{b} + 2(\vec{b} \times \vec{a}) = \vec{0}$

$\Rightarrow \quad \vec{b} \times \vec{a} = \vec{0}$

$\Rightarrow \quad \vec{a} \parallel \vec{b}$

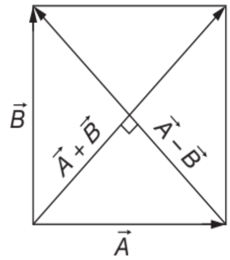
Hence, the correct answer is (D).

6. For $\vec{A} \parallel \vec{B}$

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} = +k = 3(k > 0)$$

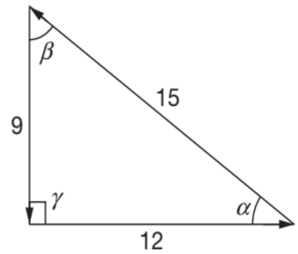
Hence, the correct answer is (A).

7. $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ always lie in the plane of $\vec{A}$ and $\vec{B}$. Further $\vec{A} \times \vec{B}$ is a vector normal to plane containing $\vec{A}$ and $\vec{B}$ i.e. $\vec{A} \times \vec{B}$ is perpendicular to $\vec{A}$ as well as $\vec{B}$. Hence $\vec{A} \times \vec{B}$ is perpendicular to $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$. For $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ to be mutually perpendicular. They must be the diagonals of a rhombus with sides $\vec{A}$ and $\vec{B}$ (diagonals of a rhombus are perpendicular) and then the sides must have equal magnitude. So, $(\vec{A} + \vec{B}) \perp (\vec{A} - \vec{B})$ when $|\vec{A}| = |\vec{B}|$.



Hence, the correct answer is (C).

8. From Lami's Theorem



$$\frac{\sin\alpha}{9} = \frac{\sin\beta}{12} = \frac{\sin\gamma}{15}$$

$\Rightarrow \quad \dfrac{\sin\alpha}{9} = \dfrac{\sin\beta}{12} = \dfrac{1}{15}$

$\Rightarrow \quad \sin\beta = \dfrac{4}{5}$

$\Rightarrow \quad \cos\beta = \dfrac{3}{5}$

$\Rightarrow \quad \theta = \pi - \cos^{-1}\left(\dfrac{3}{5}\right)$

Hence, the correct answer is (C).

9. $(\vec{a} \cdot \vec{b})^2 = a^2b^2$

$\Rightarrow \quad a^2b^2\cos^2\theta = a^2b^2$

$\Rightarrow \quad \cos\theta = 1$

$\Rightarrow \quad \theta = 0°$

$\Rightarrow \quad \vec{a} \parallel \vec{b}$

Hence, the correct answer is (A).

10. $a:b:c::2000:1732:1000$

$$\Rightarrow \quad a:b:c::1:\frac{1.732}{2}:\frac{1}{2}$$

$$\Rightarrow \quad a:b:c::1:\frac{\sqrt{3}}{2}:\frac{1}{2}$$

so according to law of sines the angles of the triangle in degrees are $90°$, $60°$ and $30°$.

Hence, the correct answer is (C).

11. $\vec{F} = \left(-3\hat{i} + \hat{j} + 5\hat{k}\right)\text{N}$

$$\Delta\vec{r} = 7\hat{i} + 3\hat{j} + \hat{k} - 0\hat{i} - 0\hat{j} - 0\hat{k}$$

$$\Rightarrow \quad \Delta\vec{r} = \left(7\hat{i} + 3\hat{j} + \hat{k}\right)$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\Rightarrow \quad \vec{\tau} = \left(7\hat{i} + 3\hat{j} + \hat{k}\right) \times \left(-3\hat{i} + \hat{j} + 5\hat{k}\right)$$

$$\Rightarrow \quad \vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix}$$

$$\Rightarrow \quad \vec{\tau} = \hat{i}(15-1) - \hat{j}(35+3) + \hat{k}(7+9)$$

$$\Rightarrow \quad \vec{\tau} = 14\hat{i} - 38\hat{j} + 16\hat{k}$$

Hence, the correct answer is (A).

12. $\vec{C} = \vec{A} - \vec{B}$

$$\left|\vec{C}\right| = A + B$$

$$\Rightarrow \quad (A+B)^2 = A^2 + B^2 - 2AB\cos\theta$$

$$\Rightarrow \quad A^2 + B^2 + 2AB = A^2 + B^2 - 2AB\cos\theta$$

$$\Rightarrow \quad \cos\theta = -1$$

$$\Rightarrow \quad \theta = 180°$$

$$\Rightarrow \quad \theta = \pi$$

Hence, the correct answer is (C).

13. A vector perpendicular to two vectors $\vec{A}$ and $\vec{B}$ is $\vec{A} \times \vec{B}$. If $\hat{n}$ is the unit vector. Then $\hat{n} = \pm\dfrac{\vec{A} \times \vec{B}}{\left|\vec{A} \times \vec{B}\right|}$

So, a vector perpendicular to $\vec{A}$ and $\vec{B}$ of magnitude R is $R\hat{n}$.

$$\Rightarrow \quad \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 2 & -2 & 3 \end{vmatrix}$$

$$\Rightarrow \quad \vec{A} \times \vec{B} = 8\hat{i} - 4\hat{j} - 8\hat{k}$$

$$\Rightarrow \quad \hat{n} = \pm\frac{1}{12}\left(8\hat{i} - 4\hat{j} - 8\hat{k}\right)$$

$$\Rightarrow \quad \vec{R} = R\hat{n} = \pm\frac{3}{12}\left(8\hat{i} - 4\hat{j} - 8\hat{k}\right)$$

$$\Rightarrow \quad \vec{R} = \pm\left(2\hat{i} - \hat{j} - 2\hat{k}\right)$$

Hence, the correct answer is (A).

14. $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$\Rightarrow \quad \vec{a} + \vec{b} = -\vec{c}$

$\Rightarrow \quad \vec{c}$ is in the plane of $\hat{i}$ and $\hat{j}$.

Hence, the correct answer is (A).

15. The forces must form a regular polygon of 12 sides. So angle between any two adjacent forces is $\dfrac{360}{12} = 30°$

Hence, the correct answer is (B).

16. $\left|\vec{F}_1 \times \vec{F}_2\right| = \vec{F}_1 \cdot \vec{F}_2$

$$\Rightarrow \quad F_1 F \sin\theta = F_1 F_2 \cos\theta$$

$$\Rightarrow \quad \tan\theta = 1$$

$$\Rightarrow \quad \theta = 45°$$

$$\left|\vec{F}_1 + \vec{F}_2\right| = \sqrt{F_1^2 + F_2^2 + 2F_1 F \cos(45°)}$$

$$\Rightarrow \quad \left|\vec{F}_1 + \vec{F}_2\right| = \sqrt{F_1^2 + F_2^2 + \sqrt{2}F_1 F_2}$$

Hence, the correct answer is (D).

17. For coplanarity $\text{STP} = 0$

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & p & 5 \end{vmatrix} = 0$$

$$2(10+3p) + 1(5+9) + 1(p-6) = 0$$

$$20 + 6p + 5 + 9 + p - 6 = 0$$

$$7p = 28$$

$$\Rightarrow \quad p = \frac{28}{7} = 4$$

Hence, the correct answer is (C).

18. $\vec{R} = \vec{BA} + \vec{BC} + \vec{CD} + \vec{DA}$

$$\Rightarrow \quad \vec{R} = \vec{BA} + \vec{BA} \qquad \left\{\because \vec{BC} + \vec{CD} + \vec{DA} = \vec{BA}\right\}$$

$$\Rightarrow \quad \vec{R} = 2\vec{BA}$$

Hence, the correct answer is (D).

19. No acceleration implies no net force on the body and hence resultant must be zero vector.

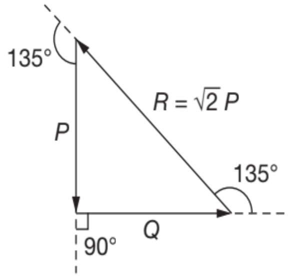
Hence, the correct answer is (A).

20. $A-B \le |\vec{A}+\vec{B}| \le A+B$

Hence, the correct answer is (C).

21. Since $\vec{P}+\vec{Q}+\vec{R}=\vec{0}$

$\Rightarrow$ $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are in same order forming a closed polygon/triangle (see figure)



We are not asked about the angles of triangle, but we are to find the angle between vectors. While finding angle between two vectors we must make sure that either their heads are coinciding or their tails are coinciding.

Hence, the correct answer is (A).

22. We know $\hat{A}\cdot\hat{A}=1$, where $\hat{A}$ is any unit vector.

$$\Rightarrow \frac{d}{dt}(\hat{A}\cdot\hat{A})=0$$

$$\Rightarrow \hat{A}\cdot\frac{d\hat{A}}{dt}+\hat{A}\cdot\frac{d\hat{A}}{dt}=0$$

$$\Rightarrow 2\left(\hat{A}\cdot\frac{d\hat{A}}{dt}\right)=0$$

$$\Rightarrow \hat{A}\cdot\frac{d\hat{A}}{dt}=0$$

Hence, the correct answer is (A).

23. $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ ...(1)

$\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$ ...(2)

Subtracting (2) from (1), we get

$$(\vec{a}\times\vec{b})-(\vec{a}\times\vec{c})=(\vec{c}\times\vec{d})-(\vec{b}\times\vec{d})$$

$$\Rightarrow \vec{a}\times(\vec{b}-\vec{c})=(\vec{c}-\vec{b})\times\vec{d}$$

$$\Rightarrow \vec{a}\times(\vec{b}-\vec{c})=-(\vec{b}-\vec{c})\times\vec{d}$$

$$\Rightarrow \vec{a}\times(\vec{b}-\vec{c})+(\vec{b}-\vec{c})\times\vec{d}=\vec{0}$$

$$\Rightarrow \vec{a}\times(\vec{b}-\vec{c})-\vec{d}\times(\vec{b}-\vec{c})=\vec{0}$$

$$\Rightarrow (\vec{a}-\vec{d})\times(\vec{b}-\vec{c})=\vec{0}$$

So, $(\vec{a}-\vec{d})$ and $(\vec{b}-\vec{c})$ are parallel

Hence, the correct answer is (B).

24. $(\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B})=\vec{A}\cdot\vec{A}-\vec{B}\cdot\vec{B}=A^2-B^2$

$$\Rightarrow (\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B})=0$$

$$\Rightarrow (\vec{A}+\vec{B})\perp(\vec{A}-\vec{B})$$

Hence, the correct answer is (C).

25. $W=\vec{F}\cdot\Delta\vec{r}$

$$\Rightarrow W=\vec{F}\cdot(\vec{r}_2-\vec{r}_1)$$

$$\Rightarrow W=(-5\hat{i}+3\hat{j}+2\hat{k})\cdot(3\hat{i}-5\hat{j}+0\hat{k})$$

$$\Rightarrow W=-15-15$$

$$\Rightarrow W=-30 \text{ unit}$$

Hence, the correct answer is (C).

26. For perpendicular vectors $\vec{a}$ and $\vec{b}$

$$\vec{a}\cdot\vec{b}=0$$

$$\Rightarrow (2\hat{i}+4\hat{j}-\hat{k})\cdot(3\hat{i}-2\hat{j}+x\hat{k})=0$$

$$\Rightarrow 6-8-x=0$$

$$\Rightarrow -x=2$$

$$\Rightarrow x=-2$$

Hence, the correct answer is (B).

27. $\vec{r}_f=4\hat{i}+4\hat{j}+12\hat{k}$

$\vec{r}_i=\hat{i}$

$$\Rightarrow \Delta\vec{r}=\vec{r}_f-\vec{r}_i$$

$$\Rightarrow \Delta\vec{r}=3\hat{i}+4\hat{j}+12\hat{k}$$

$$\Rightarrow |\Delta\vec{r}|=\sqrt{3^2+4^2+12^2}$$

$$\Rightarrow |\Delta\vec{r}|=13$$

Since $|\vec{v}|=\left|\frac{\Delta\vec{r}}{\Delta t}\right|$

$$\Rightarrow 65=\frac{|\Delta\vec{r}|}{\Delta t}$$

$$\Rightarrow \Delta t=\frac{13}{65}=\frac{1}{5}\text{ s}$$

Velocity vector is $\vec{v}=\frac{\Delta\vec{r}}{\Delta t}$

$$\Rightarrow \quad \vec{v} = \frac{3\hat{i}+4\hat{j}+12\hat{k}}{\left(\frac{1}{5}\right)}$$

$$\Rightarrow \quad \vec{v} = 15\hat{i}+20\hat{j}+60\hat{k}$$

Since particle has uniform velocity, so

$$\vec{r} = \vec{r}_0 + \vec{v}t$$

$$\Rightarrow \quad \vec{r} = \vec{i} + \left(15\hat{i}+20\hat{j}+60\hat{k}\right)2$$

$$\Rightarrow \quad \vec{r} = 31\hat{i}+40\hat{j}+120\hat{k}$$

Hence, the correct answer is (D).

28. $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 0$

$$\Rightarrow \quad a^2 - b^2 = 0$$

$$\Rightarrow \quad |\vec{a}| = |\vec{b}|$$

Hence, the correct answer is (C).

29. Let $AC = x$ and the length of string between B and m_2 be y. Since A and B are fixed, so AO and BO are constants.

Let $AO = BO = l$

Since length of string is a constant

$$\Rightarrow \quad AC + CB + y = \text{constant}$$

$$\Rightarrow \quad 2\sqrt{l^2+x^2} + y = \text{constant}$$

$$\Rightarrow \quad 2\frac{1}{2}(l^2+x^2)^{\frac{1}{2}-1}\frac{d}{dt}(l^2+x^2) + \frac{dy}{dt} = 0$$

$$\Rightarrow \quad \frac{2x}{\sqrt{l^2+x^2}}\left(\frac{dx}{dt}\right) + \frac{dy}{dt} = 0$$

$$\Rightarrow \quad \frac{2x}{\sqrt{l^2+x^2}}v_1 + v_2 = 0$$

Since $\cos\theta = \dfrac{x}{\sqrt{l^2+x^2}}$

$$\Rightarrow \quad 2\cos\theta v_1 + v_2 = 0$$

$$\Rightarrow \quad \cos\theta = -\frac{v_2}{2v_1}$$

Negative sign indicates that v_1 and v_2 are directed opposite to each other.

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{v_2}{2v_1}\right)$$

Hence, the correct answer is (B).

30. Let $\vec{P} = \hat{i} - \hat{j} - 2\hat{k}$

$$\overrightarrow{PS} = 0\hat{i} + \hat{j} + 3\hat{k} - \left(\hat{i} - \hat{j} - 2\hat{k}\right)$$

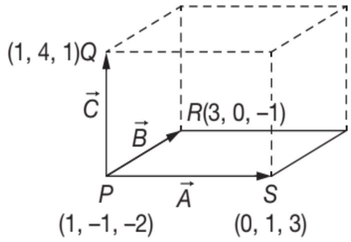
$$\Rightarrow \quad \overrightarrow{PS} = \vec{A} = -\hat{i} + 2\hat{j} + 5\hat{k}$$

$$\overrightarrow{PR} = \left(3\hat{i} + 0\hat{j} - \hat{k}\right) - \left(\hat{i} - \hat{j} - 2\hat{k}\right)$$

$$\Rightarrow \quad \overrightarrow{PR} = \vec{B} = 2\hat{i} + \hat{j} + \hat{k}$$

$$\overrightarrow{PQ} = \left(\hat{i} + 4\hat{j} + \hat{k}\right) - \left(\hat{i} + \hat{j} + 2\hat{k}\right)$$

$$\Rightarrow \quad \overrightarrow{PQ} = \vec{C} = 5\hat{j} + 3\hat{k}$$



Volume of parallelepiped $= (\vec{A}\times\vec{B})\cdot\vec{C}$

$$\Rightarrow \quad \text{Volume} = \begin{vmatrix} -1 & 2 & 5 \\ 2 & 1 & 1 \\ 0 & 5 & 3 \end{vmatrix}$$

$$\Rightarrow \quad \text{Volume} = (-1)(3-5) - 2(6-0) + 5(10-0)$$

$$\Rightarrow \quad \text{Volume} = 2 - 12 + 50$$

$$\Rightarrow \quad \text{Volume} = 40 \text{ unit}$$

Hence, the correct answer is (C).

31. $|\vec{A}\times\vec{B}|^2 + |\vec{A}\cdot\vec{B}|^2$

$$= A^2B^2\sin^2\theta + A^2B^2\cos^2\theta$$

$$= A^2B^2\left(\sin^2\theta + \cos^2\theta\right)$$

$$= A^2B^2$$

Hence, the correct answer is (D).

33. Displacement $(\vec{s}) = \left(-2\hat{i} + 3\hat{j} + 4\hat{k}\right) - \left(\hat{i} + 2.5\hat{k}\right)$

$$\Rightarrow \quad \vec{s} = \left(-3\hat{i} + 3\hat{j} + 1.5\hat{k}\right)\text{m}$$

$$\vec{F} = \left(\hat{i} + 4\hat{k}\right)N$$

Work done $= \vec{F}\cdot\vec{s}$

$$\Rightarrow \quad W = \left(-3\hat{i} + 3\hat{j} + 1.5\hat{k}\right)\cdot\left(\hat{i} + 4\hat{k}\right)$$

$$\Rightarrow \quad W = -3 + 0 + 6 = 3 \text{ J}$$

Hence, the correct answer is (C).

34. Let the required vector be $\vec{r}$.

$\Rightarrow \quad \vec{r} = \hat{i} - \hat{i} + 5\hat{j} - 2\hat{k} - 3\hat{i} - 6\hat{j} + 7\hat{k}$

$\Rightarrow \quad \vec{r} = -\hat{j} - 3\hat{i} + 5\hat{k}$

$\Rightarrow \quad \vec{r} = -3\hat{i} - \hat{j} + 5\hat{k}$

Hence, the correct answer is (C).

35. Direction of resultant will be given by $\tan\theta$, where θ is the angle which resultant makes with x-axis.

$\Rightarrow \quad \tan\theta = \dfrac{y}{x} = \dfrac{1}{2}$

$\Rightarrow \quad 2y - x = 0$

Hence, the correct answer is (B).

36. Moment of Force $= \vec{\tau} = \vec{r} \times \vec{F}$

$$\Rightarrow \quad \vec{\tau} = \left(\hat{i} - 2\hat{j} + 3\hat{k}\right) \times \left(4\hat{i} + 5\hat{j}\right) = \begin{vmatrix} \overset{\oplus}{\hat{i}} & \overset{\ominus}{\hat{j}} & \overset{\oplus}{\hat{k}} \\ 1 & -2 & 3 \\ 4 & 5 & 0 \end{vmatrix}$$

$\Rightarrow \quad \vec{\tau} = \hat{i}(0-15) - \hat{j}(0-12) + \hat{k}(5+8)$

$\Rightarrow \quad \vec{\tau} = -15\hat{i} + 12\hat{j} + 13\hat{k}$

Hence, the correct answer is (A).

37. Area of $\Delta = \dfrac{1}{2}\left|\left(\vec{b} \times \vec{c}\right)\right| = \dfrac{1}{2}\left|\vec{a} \times \vec{b}\right| = \dfrac{1}{2}\left|\vec{c} \times \vec{a}\right|$

$\Rightarrow \quad 3\Delta = \dfrac{1}{2}\left|\vec{b} \times \vec{c}\right| + \dfrac{1}{2}\left|\vec{a} \times \vec{b}\right| + \dfrac{1}{2}\left|\vec{c} \times \vec{a}\right|$

$\Rightarrow \quad 3\Delta = \dfrac{1}{2}\left|\vec{b} \times \vec{c} + \vec{a} \times \vec{b} + \vec{c} \times \vec{a}\right|$

$\Rightarrow \quad \Delta = \dfrac{1}{6}\left|\left(\vec{b} \times \vec{c}\right) + \left(\vec{a} \times \vec{b}\right) + \left(\vec{c} \times \vec{a}\right)\right|$

Hence, the correct answer is (C).

38. $\vec{a} \cdot \vec{b} = ab\cos\theta$

$\Rightarrow \quad 6 - 3 - 2 = \sqrt{(4+9+1)(9+1+4)}\cos\theta$

$\Rightarrow \quad \cos\theta = \dfrac{1}{14}$

Hence, the correct answer is (D).

39. $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

$|\Delta\vec{v}| = \sqrt{v^2 + v^2 - 2v^2\cos(40^\circ)}$

$\Rightarrow \quad |\Delta\vec{v}| = v\sqrt{2 - 2\cos(40^\circ)}$

$\Rightarrow \quad |\Delta\vec{v}| = \sqrt{2}v\sqrt{1 - \cos(40^\circ)}$

$\Rightarrow \quad |\Delta\vec{v}| = \sqrt{2}v\sqrt{2\sin^2(20^\circ)} \quad \left\{\because 1 - \cos(2\theta) = 2\sin^2\theta\right\}$

$\Rightarrow \quad |\Delta\vec{v}| = 2v\sin(20^\circ)$

Hence, the correct answer is (D).

40. The zero resultant can only be produced by three vectors when they form a closed triangle whose sides are in the same order. For a triangle to be formed the sum of any two sides must be greater than the third side.

Hence, the correct answer is (D).

41. $\dfrac{d}{dt}\left(\vec{A} \times \vec{B}\right) = \dfrac{d\vec{A}}{dt} \times \vec{B} + \vec{A} \times \dfrac{d\vec{B}}{dt}$

Hence, the correct answer is (B).

44. $\vec{A} \times \vec{B} = \vec{0}$

$\Rightarrow \quad \vec{A} \parallel \vec{B}$

$\vec{B} \times \vec{C} = \vec{0}$

$\Rightarrow \quad \vec{B} \parallel \vec{C}$

$\Rightarrow \quad \vec{A} \times \vec{C} = \vec{0}$

Hence, the correct answer is (C).

46. Scalar Triple product gives the volume of parallelopiped.

Hence, the correct answer is (B).

48. Impulse, $\vec{I} = \vec{F}t$

Hence, the correct answer is (B).

49. Electrostatic potential (i.e., the work done per unit charge) is a scalar quantity.

Hence, the correct answer is (B).

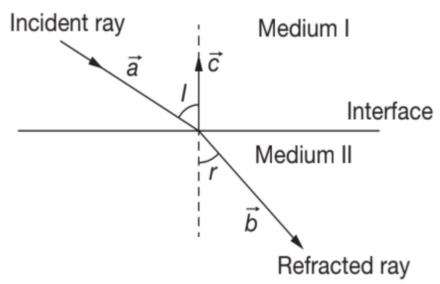
50. Kinetic Energy Gained $=$ Work Done $= \vec{F} \cdot \Delta\vec{r}$

$\Rightarrow \quad K.E. = \left(2\hat{i} + 5\hat{j} + 7\hat{k}\right) \cdot \left(10\hat{j}\right) = 50\ \text{J}$

Hence, the correct answer is (A).

51. According to the Law of Refraction i.e., Snell's Law, we have

$1\sin i = x\sin r$



where i and r are the angle of incidence and the angle of refraction respectively.

$$\sin i = \frac{\vec{a}\times\vec{c}}{|\vec{a}||\vec{c}|} = \vec{a}\times\vec{c} \qquad \{\because |\vec{a}| = |\vec{c}| = 1\}$$

Also, $$\sin r = \frac{\vec{b}\times\vec{c}}{|\vec{b}||\vec{c}|} = \vec{b}\times\vec{c} \qquad \{\because |\vec{b}| = |\vec{c}| = 1\}$$

Since both $\vec{a}\times\vec{c}$ and $\vec{b}\times\vec{c}$ have the same direction, so we have $\vec{a}\times\vec{c} = x(\vec{b}\times\vec{c})$

So, (C) is correct and not (B).

Hence, the correct answer is (C).

52. $A + B = 17$

$$A - B = 7$$

So, $A = 12$ and $B = 5$

Since $\vec{A}\perp\vec{B}$, so we have

$$R = \sqrt{A^2 + B^2} = 13$$

Hence, the correct answer is (D).

53. $|\vec{A}\times\vec{B}| = AB\sin\theta = (5\sqrt{3})(10)\sin(30^\circ)$

$\Rightarrow \quad |\vec{A}\times\vec{B}| = 25\sqrt{3}$

Hence, the correct answer is (C).

54. Since, $\vec{v} = \vec{\omega}\times\vec{r}$

$$\Rightarrow \quad \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix}$$

$\Rightarrow \quad \vec{v} = \hat{i}(-24+6) - \hat{j}(18-5) + \hat{k}(-18+20)$

$\Rightarrow \quad \vec{v} = -18\hat{i} - 13\hat{j} + 2\hat{k}$

Hence, the correct answer is (D).

55. All others are scalars.

Hence, the correct answer is (D).

56. $\vec{v} = \dfrac{d\vec{r}}{dt} = a\omega\left(-\sin(\omega t)\hat{i} + \cos(\omega t)\hat{j}\right)$

$$\vec{v}\cdot\vec{r} = a^2\omega(-\sin(\omega t)\cos(\omega t)) + \cos(\omega t)\sin(\omega t)$$

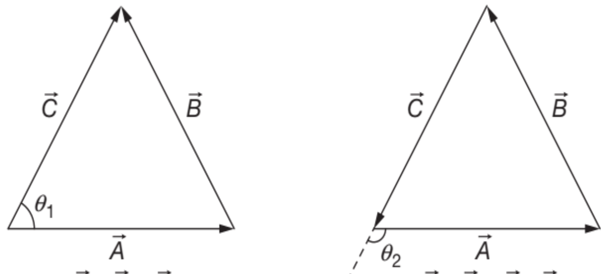
$\Rightarrow \quad \vec{v}\cdot\vec{r} = 0$

$\Rightarrow \quad \vec{v}\perp\vec{r}$

$\{\because$ dot product of $\perp$ vectors is always zero$\}$

Hence, the correct answer is (B).

57. Since $|\vec{A}| = |\vec{B}| = |\vec{C}|$, so $\theta_1 = \dfrac{\theta_2}{2}$



$\vec{A}+\vec{B}=\vec{C}$ $\qquad$ $\vec{A}+\vec{B}+\vec{C}=\vec{0}$

Hence, the correct answer is (B).

58. $P_x = 7$ and $P_y = 6$

$$P_x + Q_x = 11 \text{ and } P_y + Q_y = 9$$

$\Rightarrow \quad Q_x = 4$ and $Q_y = 3$

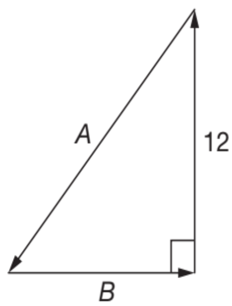
So, $|\vec{Q}| = \sqrt{Q_x^2 + Q_y^2} = 5$

Hence, the correct answer is (A).

60. Let the two vectors be $\vec{A}$ and $\vec{B}$, then

$$A + B = 18 \qquad \ldots(1)$$

Also, given that $R = 12$, and is perpendicular to one of these vectors. So, we can draw the situation as, where $\vec{B}\perp\vec{R}$.



From Pythagora's theorem, we get

$$A^2 - B^2 = 144$$

$\Rightarrow \quad (A+B)(A-B) = 144$

$\Rightarrow \quad A - B = 8 \qquad \ldots(2)$

So, $A = 13$, $B = 5$ OR $A = 5$, $B = 13$

Hence, the correct answer is (A).

61. Angular Momentum is always directed along the axis of rotation.

Hence, the correct answer is (C).

62. $\hat{p} = 2\left[(\cos t)\hat{i} + (\sin t)\hat{j}\right]$

Since $\vec{F} = \dfrac{d\vec{p}}{dt}$

$\Rightarrow \quad \vec{F} = 2\left[(-\sin t)\hat{i} + (\cos t)\hat{j}\right]$

Since, $\vec{F} \cdot \vec{p} = 0$

So, $\vec{F} \perp \vec{p}$

Hence angle between $\vec{F}$ and $\vec{p}$ is 90°.

Hence, the correct answer is (B).

63. $P^2 = (30)^2 + (60)^2 + 2(30)(60)\cos(60°)$

$\Rightarrow \quad P = \sqrt{900 + 3600 + 1800}$

$\Rightarrow \quad P = \sqrt{6300}$

$\Rightarrow \quad P = 30\sqrt{7}$ N

Hence, the correct answer is (D).

64. $\vec{a} = \left[2\hat{i} + (6t)\hat{j} + \dfrac{2\pi^2}{9}\cos\left(\dfrac{\pi t}{3}\right)\hat{k}\right] \text{ms}^{-2}$

$$\Rightarrow \quad \frac{d\vec{v}}{dt} = 2\hat{i} + (6t)\hat{j} + \frac{2\pi^2}{9}\cos\left(\frac{\pi t}{3}\right)\hat{k}$$

$$\Rightarrow \quad \int_{\vec{v}(0)}^{\vec{v}} d\vec{v} = 2\hat{i}\int_0^t dt + 6\hat{j}\int_0^t t\,dt + \frac{2\pi^2}{9}\hat{k}\int_0^t \cos\left(\frac{\pi t}{3}\right)dt$$

$$\Rightarrow \quad \vec{v} - \vec{v}(0) = (2t)\hat{i} + (3t^2)\hat{j} + \frac{2\pi}{3}\sin\left(\frac{\pi t}{3}\right)\hat{k}$$

$$\Rightarrow \quad \vec{v} = (2t)\hat{i} + (3t^2)\hat{j} + \frac{2\pi}{3}\sin\left(\frac{\pi t}{3}\right)\hat{k} + (2\hat{i} + \hat{j})$$

$$\Rightarrow \quad \vec{v} = 2(t+1)\hat{i} + (3t^2+1)\hat{j} + \frac{2\pi}{3}\sin\left(\frac{\pi t}{3}\right)\hat{k}$$

$$\Rightarrow \quad \frac{d\vec{r}}{dt} = (2t+2)\hat{i} + (3t^2+1)\hat{j} + \frac{2\pi}{3}\sin\left(\frac{\pi t}{3}\right)\hat{k}$$

$$\Rightarrow \quad \int_{\vec{r}(0)}^{\vec{r}} d\vec{r} = \hat{i}\int_0^2 (2t+2)dt + \hat{j}\int_0^2 (3t^2+1)dt + \frac{2\pi}{3}\hat{k}\int_0^2 \sin\left(\frac{\pi t}{3}\right)dt$$

$$\Rightarrow \quad \vec{r} - \vec{r}(0) = \hat{i}(t^2+2t)\Big|_0^2 + \hat{j}(t^3+t)\Big|_0^2 - 2\hat{k}\left[\cos\left(\frac{\pi t}{3}\right)\right]_0^2$$

Since $\vec{r}(0) = \vec{0} = 0\hat{i} + 0\hat{j} + 0\hat{k}$, so

$$\Rightarrow \quad \vec{r} = 8\hat{i} + 10\hat{j} - 2\hat{k}\left(\cos\left(\frac{2\pi}{3}\right) - \cos 0\right)$$

$$\Rightarrow \quad \vec{r} = 8\hat{i} + 10\hat{j} - 2\hat{k}\left(-\frac{1}{2} - 1\right)$$

$$\Rightarrow \quad \vec{r} = 8\hat{i} + 10\hat{j} + 3\hat{k}$$

Hence, the correct answer is (B).

65. Electric field, magnetic moment and average velocity, all are vectors.
Hence, the correct answer is (A).

67. $\vec{P} \perp \vec{Q}$

$\Rightarrow \quad \vec{P} \cdot \vec{Q} = 0$

$\Rightarrow \quad a^2 - 2a - 3 = 0$

$\Rightarrow \quad a^2 - 3a + a - 3 = 0$

$\Rightarrow \quad a(a-3) + 1(a-3) = 0$

$\Rightarrow \quad (a+1)(a-3) = 0$

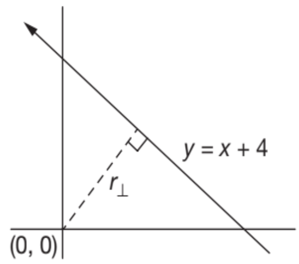
$\Rightarrow \quad a = -1$ OR $a = 3$

So, the positive value of a is 3

Hence, the correct answer is (A).

68. $L = mvr_\perp$

$v = 3\sqrt{2}$ unit along $y = x + 4$



Using the concept of Coordinate Geometry, we get

$$r_\perp = \frac{|0 - 0 + 4|}{\sqrt{1+1}}$$

$$\Rightarrow \quad r_\perp = \frac{4}{\sqrt{2}} \text{ units}$$

So, $|\vec{L}| = (5)(3\sqrt{2})\left(\dfrac{4}{\sqrt{2}}\right)$ unit

$\Rightarrow \quad |\vec{L}| = 60$ unit

Hence, the correct answer is (A).

69. Since (B) and (C) will purely depend upon the choice of origin and the axis selected, so we have (A) as the correct option.
Hence, the correct answer is (A).

70. Force is parallel to a line $y=\frac{3}{2}x+c$

The equation of given line can be written as

$$y=-\frac{k}{3}x+\frac{5}{3}$$

Work done will be zero, when force is perpendicular to the displacement i.e., the above two lines are perpendicular or

$$m_1m_2=-1$$

$$\Rightarrow \left(\frac{3}{2}\right)\left(-\frac{k}{3}\right)=-1$$

$$\Rightarrow k=2$$

Hence, the correct answer is (A).

74. Given that $(\vec{F}_1+\vec{F}_2)\cdot\vec{F}_1=0$

where $F_1<F_2$

$$\Rightarrow F_1^2+F_1F_2\cos\theta=0$$

Given that, $F_2=2F_1$

$$\Rightarrow F_1^2+2F_1^2\cos\theta=0$$

$$\Rightarrow \cos\theta=-\frac{1}{2}$$

$$\Rightarrow \theta=\frac{2\pi}{3}\text{ radian}=120^\circ$$

Hence, the correct answer is (A).

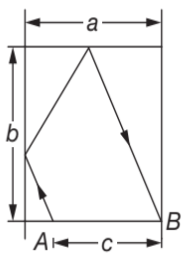
75. If $|\vec{R}|=|\vec{a}+\vec{b}|$ then, $(a-b)\le R\le(a+b)$

Hence, the correct answer is (C).

79. Component of $\vec{A}$ along $\vec{B}=\frac{\vec{A}\cdot\vec{B}}{|\vec{B}|}\cdot\vec{B}$

Hence, the correct answer is (A).

80. Let us resolve the velocity v imparted to the ball in components parallel with the sides of the table and consider the path of a ball as shown.



As seen from the diagram, we obtain the following two equations i.e.,

$$\frac{2a-c}{t}=v\cos\alpha \text{ and } \frac{2b}{t}=v\sin\alpha$$

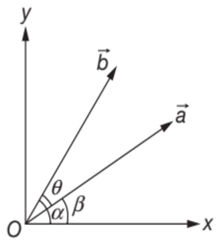
From these equations, we get

$$\cot\alpha=\frac{2a-c}{2b}$$

So, we get the angle α, at which the ball must be struck. It is important to note that the value for the velocity v which is imparted to the ball plays no part at all.

Hence, the correct answer is (C).

81.



$$\tan\beta=\frac{1}{\sqrt{3}}$$

$$\Rightarrow \beta=30^\circ$$

$$\Rightarrow \alpha=\theta+\beta=53^\circ$$

$$\Rightarrow \vec{b}=10\cos53\hat{i}+10\sin53\hat{j}=6\hat{i}+8\hat{j}$$

$$\Rightarrow \vec{a}+\vec{b}=(6+\sqrt{3})\hat{i}+9\hat{j}$$

Hence, the correct answer is (A).

82. Let $\vec{A}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and

$$\vec{B}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$$

Given that $\vec{A}+\vec{B}$ is perpendicular to $\vec{A}-\vec{B}$, so

$$(\vec{A}+\vec{B})(\vec{A}-\vec{B})=0$$

$$\Rightarrow (a_1+b_1)(a_1-b_1)+(a_2+b_2)(a_2-b_2)+$$
$$(a_3+b_3)(a_3-b_3)=0$$

$$\Rightarrow a_1^2+a_2^2+a_3^2=b_1^2+b_2^2+b_3^2$$

$$\Rightarrow |\vec{A}|=|\vec{B}|$$

Cross product of $\vec{A}$ and $\vec{B}$ is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$ or $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$.

Hence, the correct answer is (D).

83. $x=2t$

$$\Rightarrow v_x=\frac{dx}{dt}=2$$

$$y=2t^2$$

$$\Rightarrow v_y=\frac{dy}{dt}=4t$$

CHAPTER 3

$\Rightarrow \quad \tan\theta = \frac{v_y}{v_x} = \frac{41}{2} = 2t$

Differentiating with respect to time we get,

$$(\sec^2\theta)\frac{d\theta}{dt} = 2$$

$$\Rightarrow \quad (1+\tan^2\theta)\frac{d\theta}{dt} = 2$$

$$\Rightarrow \quad (1+4t^2)\frac{d\theta}{dt} = 2$$

$$\Rightarrow \quad \frac{d\theta}{dt} = \frac{2}{1+4t^2}$$

So, $\frac{d\theta}{dt}$ at $t = 2$ s is given by

$$\left.\frac{d\theta}{dt}\right|_{t=2\text{ s}} = \frac{2}{1+4(2)^2} = \frac{2}{17}\text{ rads}^{-1}$$

Hence, the correct answer is (A).

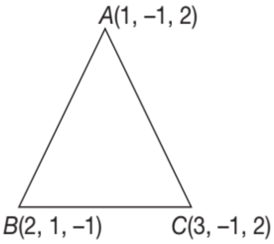
90. Vector in the direction of $\vec{A}$ and having same magnitude as B is $\vec{C} = B\hat{A}$

$$\vec{C} = B\left(\frac{\vec{A}}{A}\right)$$

$$\Rightarrow \quad \vec{C} = \frac{7}{3}(\hat{i}+2\hat{j}+2\hat{k})$$

Hence, the correct answer is (A).

91.



$$\overrightarrow{AB} = \hat{i}+2\hat{j}-3\hat{k}$$

$$\overrightarrow{BC} = \hat{i}-2\hat{j}+3\hat{k}$$

$$\overrightarrow{AB}\times\overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 1 & -2 & 3 \end{vmatrix} = -6\hat{j}-4\hat{k}$$

$$\Rightarrow \quad |\overrightarrow{AB}\times\overrightarrow{BC}| = \sqrt{(-6)^2+(-4)^2} = \sqrt{52}\text{ unit}$$

$$\Rightarrow \quad \text{Area of } \Delta = \frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{BC}| = \sqrt{13}\text{ unit}$$

Hence, the correct answer is (C).

92. Since, we know that

$$P = \vec{F}\cdot\vec{v} = (20\hat{i}-3\hat{j}+5\hat{k})\cdot(6\hat{i}+20\hat{j}-3\hat{k})$$

$$\Rightarrow \quad P = 120-60-15 = 45\text{ W}$$

Hence, the correct answer is (A).

93. $A+B=17$

and $A-B=7$

$\Rightarrow \quad A=12$

$\Rightarrow \quad B=5$

Since $\theta = 90°$

$$\Rightarrow \quad R = \sqrt{A^2+B^2} = \sqrt{12^2+5^2} = 13\text{ unit}$$

Hence, the correct answer is (D).

94. Since $\vec{R}$ is perpendicular to $\vec{a}$, therefore

$$\cos\theta = \frac{-a}{b}$$

$$\Rightarrow \quad R = \sqrt{b^2-a^2} \text{ and } S = \sqrt{3a^2+b^2}$$

Since, $2|\vec{R}| = |\vec{S}|$

$$\Rightarrow \quad 4b^2-4a^2 = 3a^2+b^2$$

$$\Rightarrow \quad 3b^2 = 7a^2$$

Hence, the correct answer is (A).

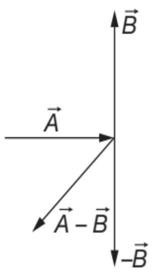
96. $\vec{v} = \frac{d\vec{r}}{dt} = 2bt\hat{i}+3ct^2\hat{j}$

$$\Rightarrow \quad \tan(60°) = \frac{v_x}{v_y} = \frac{2bt}{3ct^2}$$

$$\Rightarrow \quad t = \frac{2b}{3\sqrt{3}c}$$

Hence, the correct answer is (B).

97.



Hence, the correct answer is (D).

98. $2|\vec{A}| = |\vec{B}|$

$$\Rightarrow \quad |\vec{A}| = \frac{|\vec{B}|}{2}$$

Hence, the correct answer is (C).

99. $\dfrac{|\vec{R}|_{\min}}{|R|_{\max}} = \dfrac{1}{4} = \dfrac{|A-B|}{|A+B|}$

$\Rightarrow \quad A+B = 4|A-B|$

If $A > B$, then we have

$A+B = 4(A-B)$

$\Rightarrow \quad 3A = 5B$

$\Rightarrow \quad \dfrac{A}{B} = \dfrac{5}{3}$

If $B > A$, then we have

$A+B = 4(B-A)$

$\Rightarrow \quad \dfrac{A}{B} = \dfrac{3}{5}$

Hence, the correct answer is (C).

100. $\vec{a}\cdot\vec{b} = 0$ when $\vec{a} \perp \vec{b}$

$\Rightarrow \quad \left(2\hat{i}+3\hat{j}+8\hat{k}\right)\cdot\left(-4\hat{i}+4\hat{j}+\alpha\hat{k}\right) = 0$

$\Rightarrow \quad -8+12+8\alpha = 0$

$\Rightarrow \quad \alpha = \dfrac{-1}{2}$

Hence, the correct answer is (C).

101. $\vec{C} = \vec{A}+\vec{B} = \left(3\hat{i}+6\hat{j}-2\hat{k}\right)$

$\Rightarrow \quad \hat{C} = \dfrac{\vec{C}}{|C|} = \dfrac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{3^2+6^2+(-2)^2}} = \dfrac{3\hat{i}+6\hat{j}-2\hat{k}}{7}$

Hence, the correct answer is (A).

102. Resultant can be zero if

$|F_1 - F_2| \le F_3 \le F_1 + F_2$

Hence, the correct answer is (D).

103. $\vec{F}_1 = F_1\hat{i}$ and $F_1\hat{i}\times\left(-4\hat{i}\right) = 0$

Hence, the correct answer is (D).

104. Component of $\vec{A}$ along $\vec{B}$ is

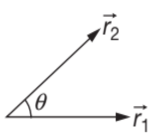
$\dfrac{\vec{A}\cdot\vec{B}}{B} = \dfrac{3+4}{\sqrt{2}} = \dfrac{7}{\sqrt{2}}$

In vector form, component of $\vec{A}$ along $\vec{B}$ is

$\dfrac{7}{\sqrt{2}}\hat{B} = \dfrac{7}{\sqrt{2}}\left(\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \dfrac{7}{2}\left(\hat{i}+\hat{j}\right)$

Hence, the correct answer is (D).

105. Given that $|\vec{r}_1| = |\vec{r}_2| = l$

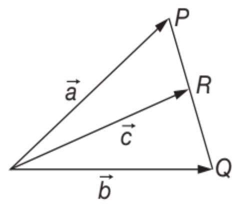


$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$\Rightarrow \quad |\Delta\vec{r}| = \sqrt{r_2^2 + r_1^2 - 2r_1r_2\cos\theta} = 2l\sin\left(\dfrac{\theta}{2}\right)$

Hence, the correct answer is (B).

106.



$\vec{a} + \overrightarrow{PR} = \vec{c}$ ...(1)

and $\vec{b} + \overrightarrow{QR} = \vec{c}$ ...(2)

Adding (1) and (2), we get

$\vec{a} + \vec{b} + \overrightarrow{QR} + \overrightarrow{PR} = 2\vec{c}$

Since, we have $\overrightarrow{PR} = -\overrightarrow{QR}$

$\Rightarrow \quad \vec{a} + \vec{b} = 2\vec{c}$

Hence, the correct answer is (A).

107. $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -2\hat{i} + 2\hat{k}$

Here, $\vec{a}\times\vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, so unit vector $\hat{n}$ along $\vec{a}\times\vec{b}$ is

$\hat{n} = \dfrac{-2\hat{i}+2\hat{k}}{\sqrt{(-2)^2+2^2}} = \dfrac{-\hat{i}+\hat{k}}{\sqrt{2}}$

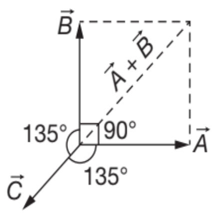
Hence, the correct answer is (B).

108. $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 1 & 4 & 0 \end{vmatrix} = \hat{k}(8-3) = 5\hat{k}$

Hence, the correct answer is (A).

109. Let $|\vec{A}| = |\vec{B}| = a$ and $|\vec{C}| = \sqrt{2}a$ then,

$\vec{A} + \vec{B} + \vec{C} = 0$



$\Rightarrow \quad \vec{C} = -\left(\vec{A}+\vec{B}\right)$

$\Rightarrow \quad C^2 = A^2 + B^2 + 2AB\cos\theta$

$\Rightarrow \quad 2a^2 = a^2 + a^2 + 2a^2\cos\theta$

$\Rightarrow \quad \cos\theta = 0$

$\Rightarrow \quad \theta = 90°$

Hence, the correct answer is (D).

110. $\vec{r} = b\cos(\omega t)\hat{i} + a\sin(\omega t)\hat{j}$

$$\Rightarrow \quad \frac{d\vec{r}}{dt} = -b\omega\sin(\omega t)\hat{i} + a\omega\cos(\omega t)\hat{j}$$

$$\Rightarrow \quad \frac{d^2\vec{r}}{dt^2} = -b\omega^2\cos(\omega t)\hat{i} - a\omega^2\sin(\omega t)\hat{j}$$

$$\Rightarrow \quad a = -\omega^2\left(b\cos(\omega t)\hat{i} + a\sin(\omega t)\hat{j}\right)$$

$\Rightarrow \quad -\omega^2\vec{r}$

Hence, the correct answer is (C).

111. $P = \sqrt{(30)^2 + (60)^2 + 2(30)(60)\cos(60°)}$

$\Rightarrow \quad P = \sqrt{900 + 3600 + 1800}$

$\Rightarrow \quad P = \sqrt{6300} = 30\sqrt{7}$

Hence, the correct answer is (D).

112. $$\cos\gamma = \frac{5}{\sqrt{4^2 + 3^2 + 5^2}} = \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \gamma = \frac{\pi}{4}$$

Hence, the correct answer is (B).

113. $(\vec{A}_1 + 2\vec{A}_2)\cdot(3\vec{A}_1 - 4\vec{A}_2)$

$$= 3|\vec{A}_1|^2 + 2|\vec{A}_1||\vec{A}_2|\cos\theta - 8|\vec{A}_2|^2$$

$$= \left(|\vec{A}_1|^2 + 2|\vec{A}_1||\vec{A}_2|\cos\theta + |\vec{A}_2|^2\right) + 2|\vec{A}_1|^2 - 9|\vec{A}_2|^2$$

$$= 3^2 + 2\times 2^2 - 9\times 3^2 = 9 + 8 - 81 = -64$$

Hence, the correct answer is (A).

114. Let the components of $\vec{A}$ make angles α, β and γ with x, y and z-axis respectively, then $\alpha = \beta = \gamma$.

Since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \quad 3\cos^2\alpha = 1$

$$\Rightarrow \quad \cos\alpha = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad A_x = A_y = A_z = A\cos\alpha = \frac{A}{\sqrt{3}}$$

Hence, the correct answer is (A).

115. $\vec{A} = \hat{i} + \hat{j}$

$$\cos\alpha = \frac{A_x}{|A|} = \frac{1}{\sqrt{2}} = \cos 45°$$

$\Rightarrow \quad \alpha = 45°$

Hence, the correct answer is (B).

116. Magnitude of unit vector $= 1$

$\Rightarrow \quad \sqrt{(0.5)^2 + (0.8)^2 + c^2} = 1$

$\Rightarrow \quad c = \sqrt{0.11}$

Hence, the correct answer is (B).

117. $$\frac{\vec{A}\cdot\vec{B}}{|\hat{i} + \hat{j}|} = \frac{(2\hat{i} + 3\hat{j})\cdot(\hat{i} + \hat{j})}{\sqrt{2}} = \frac{2+3}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

Hence, the correct answer is (A).

118. For 17 N both the vector should be parallel i.e. angle between them should be zero.

For 7 N both the vectors should be antiparallel i.e. angle between them should be $180°$.

For 13 N both the vectors should be perpendicular to each other i.e. angle between them should be $90°$.

Hence, the correct answer is (A).

119. $\vec{A} + \vec{B} = 4\hat{i} - 3\hat{j} + 6\hat{i} + 8\hat{j} = 10\hat{i} + 5\hat{j}$

$\Rightarrow \quad |\vec{A} + \vec{B}| = \sqrt{(10)^2 + (5)^2} = 5\sqrt{5}$

$$\Rightarrow \quad \tan\theta = \frac{5}{10} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{2}\right)$$

Hence, the correct answer is (B).

120. $A = 3\ N$, $B = 2\ N$ then

$\Rightarrow \quad R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\Rightarrow \quad R = \sqrt{9 + 4 + 12\cos\theta}$ ...(1)

Now $A = 6\ N$, $B = 2\ N$ then

$2R = \sqrt{36 + 4 + 24\cos\theta}$ ...(2)

From (1) and (2), we get

$$\cos\theta = -\frac{1}{2}$$

$\Rightarrow \quad \theta = 120°$

Hence, the correct answer is (D).

121. From the property of vector product, $\vec{C}$ is perpendicular to $\vec{A}$ as well as $\vec{B}$. Also $(\vec{A} + \vec{B})$ vector must lie in the plane formed by vector $\vec{A}$ and $\vec{B}$. Thus $\vec{C}$

must be perpendicular to $(\vec{A}+\vec{B})$ also but the cross product $(\vec{A}\times\vec{B})$ gives a vector $\vec{C}$ which cannot be perpendicular to itself. So, the last statement is wrong.

Hence, the correct answer is (D).

122. Component of $\vec{v}$ along $\vec{a}$ is

$$\frac{\vec{v}\cdot\vec{a}}{|\vec{a}|}=\left(6\hat{i}+2\hat{j}-2\hat{k}\right)\cdot\frac{\left(\hat{i}+\hat{j}+\hat{k}\right)}{\sqrt{3}}=\frac{6+2-2}{\sqrt{3}}$$

$$\Rightarrow \frac{\vec{v}\cdot\vec{a}}{|\vec{a}|}=\frac{6}{\sqrt{3}}=2\sqrt{3}$$

In vector form, the component is $(2\sqrt{3})\hat{a}$.

$$\Rightarrow (2\sqrt{3})\hat{a}=2\sqrt{3}\frac{\left(\hat{i}+\hat{j}+\hat{k}\right)}{\sqrt{3}}=2\left(\hat{i}+\hat{j}+\hat{k}\right)$$

Hence, the correct answer is (B).

123. $|7-5|\le F\le 7+5$

$\Rightarrow 2\le F\le 12$

$\Rightarrow F_{\text{min}}=2$ newton

Hence, the correct answer is (D).

Multiple Correct Choice Type Question

1. A scalar, a vector and its magnitude do not depend on the orientation of the frame of reference.

Hence, (A), (B) and (C) are correct.

2. Think of $\hat{i}$, $\hat{j}$ and $\hat{k}$ to give an appropriate answer.

Hence, (C) and (D) are correct.

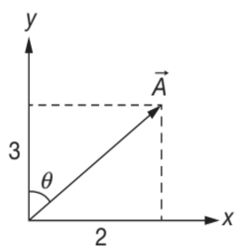
9. $\left|3\hat{i}+4\hat{j}\right|=\sqrt{3^2+4^2}=5$

Area of parallelogram $=|\vec{a}\times\vec{b}|$

Hence, (A) and (C) are correct.

10. $\tan\theta=\dfrac{2}{3}$

$$\Rightarrow \cos\theta=\frac{3}{\sqrt{13}}$$



Hence, (B) and (C) are correct.

11. $\vec{p}=2\left[(\sin t)\hat{i}-(\cos t)\hat{j}\right]$

$\Rightarrow |\vec{p}|=2\sqrt{\sin^2 t+\cos^2 t}=2=\text{Constant}$

Also, $\vec{F}=\dfrac{d\vec{p}}{dt}=2\left(\cos t\hat{i}+\sin t\hat{j}\right)$

$\Rightarrow \vec{F}\cdot\vec{p}=4(\sin t\cos t-\cos t\sin t)$

$\Rightarrow \vec{F}\cdot\vec{p}=0$

$\Rightarrow \vec{F}\perp\vec{p}$

Hence, (A) and (C) are correct.

Reasoning Based Questions

1. Cross product of two vectors is perpendicular to the plane containing both the vectors.

Hence, the correct answer is (A).

2. $\cos\theta=\dfrac{\left(\hat{i}+\hat{j}\right)\cdot\left(\hat{i}\right)}{\left|\hat{i}+\hat{j}\right|\left|\hat{i}\right|}=\dfrac{1}{\sqrt{2}}$. Hence $\theta=45°$

Hence, the correct answer is (A).

3. $\dfrac{|\vec{A}\times\vec{B}|}{\vec{A}\cdot\vec{B}}=\dfrac{AB\sin\theta}{AB\cos\theta}=\tan\theta$

Also, $\vec{A}\times\vec{B}$ is normal to the plane containing $\vec{A}$ as well as $\vec{B}$. So, $\vec{A}\times\vec{B}\perp(\vec{A}+\vec{B})$.

Hence, the correct answer is (B).

4. $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$

$\Rightarrow A^2+B^2+2AB\cos\theta=A^2+B^2-2AB\cos\theta$

$\Rightarrow 4AB\cos\theta=0$

$\Rightarrow \cos\theta=0$

$\Rightarrow \theta=90°$

Also vector addition is commutative

Hence $\vec{A}+\vec{B}=\vec{B}+\vec{A}$.

Hence, the correct answer is (B).

5. $\vec{v}=\vec{\omega}\times\vec{r}$ is the correct expression.

The expression $\vec{\omega}=\vec{v}\times\vec{r}$ is wrong.

Hence, the correct answer is (C).

6. For giving a zero resultant, it should be possible to represent the given vectors along the sides of a closed polygon and minimum number of sides of a polygon is three. If $\vec{A}+\vec{B}+\vec{C}=\vec{0}$, then this represents the sides of a triangle taken in same order, hence they must lie in one plane.

Hence, the correct answer is (B).

7. Vector addition of two vectors is commutative i.e.,

$\vec{A}+\vec{B}=\vec{B}+\vec{A}$.

Hence, the correct answer is (B).

9. Cross product of two vectors is anticommutative i.e.,

$\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$.

Hence, the correct answer is (C).

11. If a vector quantity has zero magnitude then it is called a null vector. That quantity may have some direction even if its magnitude is zero.

Hence, the correct answer is (D).

12. Let $\vec{P}$ and $\vec{Q}$ are two vectors in opposite direction, then their sum $\vec{P}+(-\vec{Q})=\vec{P}-\vec{Q}$.

If $\vec{P}=\vec{Q}$ then sum equal to zero.

Hence, the correct answer is (A).

13. If two vectors are in opposite direction, then they cannot be like vectors.

Hence, the correct answer is (C).

14. If θ be the angle between two vectors $\vec{A}$ and $\vec{B}$, then their scalar product, $\vec{A}\cdot\vec{B}=AB\cos\theta$.

If $\theta=90°$, then $\vec{A}\cdot\vec{B}=0$,

i.e., if $\vec{A}$ and $\vec{B}$ are perpendicular to each other then their scalar product will be zero.

Hence, the correct answer is (A).

15. We can multiply any vector by any scalar and the result is a vector, however dot product of a scalar with a vector is meaningless.

Hence, the correct answer is (B).

17. $\vec{A}\cdot\vec{B}=|\vec{A}||\vec{B}|\cos\theta=0$

$\vec{A}\times\vec{B}=|\vec{A}||\vec{B}|\sin\theta=0$

If $\vec{A}$ and $\vec{B}$ are not null vectors then it follows that $\sin\theta$ and $\cos\theta$ both should be zero simultaneously. But it cannot be possible so it is essential that one of the vector must be null vector.

Hence, the correct answer is (B).

19. The resultant of two vectors of unequal magnitude given by $R=\sqrt{A^2+B^2+2AB\cos\theta}$ cannot be zero for any value of θ.

Hence, the correct answer is (C).

20. $\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{C}$

$\Rightarrow\quad AB\cos\theta_1=BC\cos\theta_2$

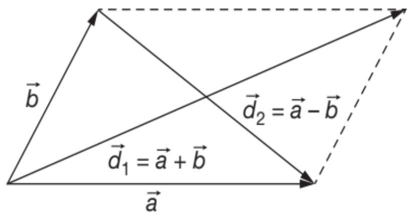
So, $A=C$, only when $\theta_1=\theta_2$

Hence, when angle between $\vec{A}$ and $\vec{B}$ is equal to angle between $\vec{B}$ and $\vec{C}$ only then $\vec{A}$ equal to $\vec{C}$.

Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. Let $\vec{d}_1$ and $\vec{d}_2$ be the two diagonals of the parallelogram with sides $\vec{a}$ and $\vec{b}$, then



So, $\vec{d}_1=\vec{a}+\vec{b}=(2\vec{m}+\vec{n})+(\vec{m}-2\vec{n})$

$\Rightarrow\quad \vec{d}_1=3\vec{m}-\vec{n}$

Hence, the correct answer is (C).

2. Now $\vec{d}_2=\vec{a}-\vec{b}=(2\vec{m}+\vec{n})-(\vec{m}-2\vec{n})$

$\Rightarrow\quad \vec{d}_2=\vec{m}+3\vec{n}$

Hence, the correct answer is (B).

3. Since the dot product of a vector with itself is equal to the square of its magnitude, so we have

$d_1^2=\vec{d}_1\cdot\vec{d}_1$

$\Rightarrow\quad d_1^2=(3\vec{m}-\vec{n})\cdot(3\vec{m}-\vec{n})$

$\Rightarrow\quad d_1^2=9(\vec{m}\cdot\vec{m})+(\vec{n}\cdot\vec{n})-6(\vec{m}\cdot\vec{n})$

$\Rightarrow\quad d_1^2=9|\vec{m}|^2+|\vec{n}|^2-6|\vec{m}||\vec{n}|\cos(60°)$

Since $|\vec{m}|=|\vec{n}|=1$

$\Rightarrow\quad d_1^2=9+1-6(1)(1)\left(\frac{1}{2}\right)=7$

$\Rightarrow\quad d_1=\sqrt{7}$ units

Hence, the correct answer is (A).

4. Similarly, $d_2^2=\vec{d}_2\cdot\vec{d}_2$

$\Rightarrow\quad d_2^2=(\vec{m}+3\vec{n})\cdot(\vec{m}+3\vec{n})$

$\Rightarrow\quad d_2^2=|\vec{m}|^2+9|\vec{n}|^2+6(\vec{m}\cdot\vec{n})$

$\Rightarrow\quad d_2^2=1+9+6(1)(1)\left(\frac{1}{2}\right)=13$

$\Rightarrow\quad d_2=\sqrt{13}$ units

Hence, the correct answer is (D).

5. Finally, we know that area of the parallelogram with adjacent sides $\vec{a}$ and $\vec{b}$ is

Area $= |\vec{a}\times\vec{b}|$

Now $\vec{a}\times\vec{b} = (2\vec{m}+\vec{n})\times(\vec{m}-2\vec{n})$

$\Rightarrow \quad \vec{a}\times\vec{b} = 2(\vec{m}\times\vec{m}) - 2(\vec{n}\times\vec{n}) - 4(\vec{m}\times\vec{n}) + (\vec{n}\times\vec{m})$

$\Rightarrow \quad \vec{a}\times\vec{b} = 5(\vec{n}\times\vec{m}) \qquad \{\because\ \vec{m}\times\vec{n} = -\vec{n}\times\vec{m}\}$

$\Rightarrow \quad |\vec{a}\times\vec{b}| = 5|\vec{n}\times\vec{m}|$

$\Rightarrow \quad |\vec{a}\times\vec{b}| = 5|\vec{n}||\vec{m}|\sin(60^\circ)$

$\Rightarrow \quad |\vec{a}\times\vec{b}| = \dfrac{5\sqrt{3}}{2}$

$\Rightarrow \quad$ Area $= |\vec{a}\times\vec{b}| = \dfrac{5\sqrt{3}}{2}$ sq. units

Hence, the correct answer is (C).

6. If θ be the angle between $\vec{d}_1$ and $\vec{d}_2$, then

$$\cos\theta = \frac{\vec{d}_1\cdot\vec{d}_2}{|\vec{d}_1||\vec{d}_2|}$$

Now $\vec{d}_1\cdot\vec{d}_2 = (3\vec{m}-\vec{n})\cdot(\vec{m}+3\vec{n})$

$\Rightarrow \quad \vec{d}_1\cdot\vec{d}_2 = 3(\vec{m}\cdot\vec{m}) - 3(\vec{n}\cdot\vec{n}) - (\vec{n}\cdot\vec{m}) + 9(\vec{m}\cdot\vec{n})$

$\Rightarrow \quad \vec{d}_1\cdot\vec{d}_2 = 3|\vec{m}|^2 - 3|\vec{n}|^2 + 8|\vec{m}||\vec{n}|\cos(60^\circ)$

$\Rightarrow \quad \vec{d}_1\cdot\vec{d}_2 = 3 - 3 + 8(1)(1)\left(\dfrac{1}{2}\right) = 4$

$\Rightarrow \quad \vec{d}_1\cdot\vec{d}_2 = 4$

$\Rightarrow \quad \cos\theta = \dfrac{4}{\sqrt{7}\sqrt{13}}$

$\Rightarrow \quad \cos\theta = \dfrac{4}{\sqrt{91}}$

Hence, the correct answer is (D).

9. The diagonal are perpendicular either for a square or a rhombus and both have sides of equal magnitude.
Hence, the correct answer is (A).

10. $\vec{a}+\vec{b}$ lies in the plane containing $\vec{a}$ and $\vec{b}$, and $\vec{a}\times\vec{b}$ is a new vector perpendicular to $\vec{a}$ and $\vec{b}$. Hence angle between $\vec{a}+\vec{b}$ and $\vec{a}\times\vec{b}$ is 90°.
Hence, the correct answer is (D).

11. x-component of $\vec{F}_2$ is $(F_2)_x = -10\cos(60^\circ)$

$\Rightarrow \quad (F_2)_x = -5$ N

Hence, the correct answer is (B).

12. y-component of $\vec{F}_3$ is $(F_3)_y = -20\sin(60^\circ)$

$\Rightarrow \quad (F_3)_y = -10\sqrt{3}$ N

Hence, the correct answer is (B).

13. $\vec{F}_1 = 5\sqrt{2}\left(\cos 45\hat{i} + \sin 45\hat{j}\right)$

$\Rightarrow \quad \vec{F}_1 = \left(5\hat{i} + 5\hat{j}\right)$ N

Hence, the correct answer is (C).

14. Since $B_x = -10\cos(30^\circ) = -5\sqrt{3}$ m

and $D_x = 10\cos(30^\circ) = 5\sqrt{3}$ m

$\Rightarrow \quad B_x + D_x = 0$

Hence, the correct answer is (C).

15. Since $A_x = 5\sqrt{2}\cos(45^\circ) = 5$ m

and $C_x = -10\cos(80^\circ) = -5$ m

$\Rightarrow \quad A_x + C_x = 0$

Hence, the correct answer is (A).

16. Since $A_y = 5\sqrt{2}\sin(45^\circ) = 5$ m

$B_y = 10\sin(30^\circ) = 5$ m

$C_y = -10\sin(60^\circ) = -5\sqrt{3}$ m

$D_y = -10\sin(30^\circ) = -5$ m

$\Rightarrow \quad A_y + B_y + C_y + D_y = 5\left(1-\sqrt{3}\right)$ m

Hence, the correct answer is (A).

17. $\Sigma F_x = F_1 - F_3\cos 60^\circ$

$\Rightarrow \quad \Sigma F_x = 10 - 20\left(\dfrac{1}{2}\right) = 0$

$\Sigma F_y = F_2 + F_3\sin(60^\circ)$

$\Rightarrow \quad \Sigma F_y = 10\sqrt{3} + 20\dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad \Sigma F_y = 20\sqrt{3}$ N (along $+y$)

So, for ΣF_y to be zero, a force of $20\sqrt{3}$ must act along $-y$ direction.

Hence, the correct answer is (B).

18. Since $\vec{F}_1 = 10\hat{i}$, $\vec{F}_2 = 10\sqrt{3}\hat{j}$

and $\vec{F}_3 = 20\left(-\cos(60^\circ)\hat{i} + \sin(60^\circ)\hat{j}\right)$

$\Rightarrow \quad \vec{F}_3 = -10\hat{i} + 10\sqrt{3}\hat{j}$

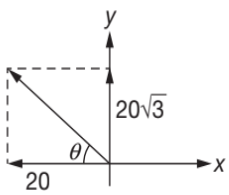
$\Rightarrow \quad \vec{F}_1 + \vec{F}_2 - \vec{F}_3 = 10\hat{i} + 10\sqrt{3}\hat{j} + 10\hat{i} - 10\sqrt{3}\hat{j}$

$\Rightarrow \quad \vec{F}_1 + \vec{F}_2 - \vec{F}_3 = 20\hat{i}$

Hence, the correct answer is (A).

19. $\vec{F}_2 + \vec{F}_3 - \vec{F}_1 = -20\hat{i} + 20\sqrt{3}\hat{j}$

$\Rightarrow \quad \left|\vec{F}_2 + \vec{F}_3 - \vec{F}_1\right| = \sqrt{20^2 + \left(20\sqrt{3}\right)^2} = 40 \text{ N}$



From diagram, $\tan\theta = \dfrac{20\sqrt{3}}{20}$

$\Rightarrow \quad \tan\theta = \sqrt{3}$

$\Rightarrow \quad \theta = 60^\circ$

So, $\left(\vec{F}_2 + \vec{F}_3 - \vec{F}_1\right)$ makes an angle of 120° with x-axis.

Hence, the correct answer is (B).

20. Since $\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \quad \vec{a} + \vec{b} - \vec{c} = 0 = -2\vec{c}$

Hence, the correct answer is (C).

21. Since $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$ as well as $\vec{b}$, so $\left(\vec{a} \times \vec{b}\right) \cdot \vec{c} = 0$

Hence, the correct answer is (D).

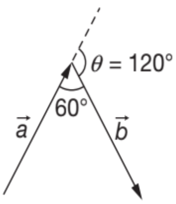
22. Since $\vec{b} = -\left(\vec{a} + \vec{c}\right)$

$\Rightarrow \quad \vec{b} \cdot \left(\vec{a} + \vec{c}\right) = -\vec{b} \cdot \vec{b}$

$\Rightarrow \quad \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} = -b^2$

Hence, the correct answer is (A).

23.



So, angle between $\vec{a}$ and $\vec{b}$ is 120° (and not 60°)

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. A $\rightarrow$ (p, r)
B $\rightarrow$ (p, s)
C $\rightarrow$ (q)
D $\rightarrow$ (p, s)

$\vec{A} \cdot \vec{B}$ is defined and is a scalar

$\left(\vec{A} \cdot \vec{B}\right)\vec{C}$ is defined and is a vector

$\left(\vec{A} \cdot \vec{B}\right) \cdot \vec{C}$ is not defined, because dot product is always defined/taken between two vectors but not between a scalar and a vector

$\left(\vec{A} \times \vec{B}\right) \times \vec{C}$ is defined and is a vector.

2. A $\rightarrow$ (q, s)
B $\rightarrow$ (p)
C $\rightarrow$ (r)
D $\rightarrow$ (r)

$\vec{A} \times \vec{B}$ is a new vector perpendicular to $\vec{A}$ as well as $\vec{B}$.

So, $\left(\vec{A} \times \vec{B}\right) \cdot \vec{A} = 0$

Also, $\vec{A} \times \vec{A} = \vec{0}$ and $\vec{0} \times \vec{A} = \vec{0}$

3. A $\rightarrow$ (r)
B $\rightarrow$ (t)
C $\rightarrow$ (s)
D $\rightarrow$ (p)
E $\rightarrow$ (q)

For parallel vectors, $\vec{A} \times \vec{B} = \vec{0}$ and for perpendicular vectors $\vec{A} \cdot \vec{B} = 0$. So (A) $\rightarrow$ (r), (B) $\rightarrow$ (t)

$\vec{A} \times \vec{B} + \vec{C} \times \vec{A} = \vec{0}$

$\Rightarrow \quad \vec{A} \times \vec{B} - \vec{A} \times \vec{C} = \vec{0}$

$\Rightarrow \quad \vec{A} \times \left(\vec{B} - \vec{C}\right) = \vec{0}$

$\Rightarrow \quad \vec{B} - \vec{C} = k\vec{A} \qquad \{\because \text{ Vectors are parallel}\}$

$\Rightarrow \quad \vec{B} = \vec{C} + k\vec{A}$

So, (C) $\rightarrow$ (s)

Now area of parallelogram is $\left|\vec{A} \times \vec{B}\right|$,

so (D) $\rightarrow$ (p)

Further component vector of $\vec{A}$ along $\vec{B}$ is $\left(A\cos\theta\right)\hat{B}$.

$\Rightarrow \quad \left(A\cos\theta\right)\hat{B} = \left(\dfrac{\vec{A} \cdot \vec{B}}{B}\right)\hat{B} = \left(\dfrac{\vec{A} \cdot \vec{B}}{B^2}\right)\vec{B}$

So, (E) $\rightarrow$ (q)

4. A $\rightarrow$ (r, s)
B $\rightarrow$ (s)
C $\rightarrow$ (t)
D $\rightarrow$ (p)
E $\rightarrow$ (q)
Resultant of two ordered vectors is given by triangle law of vector addition and is commutative in nature.
Dot product of two vectors is a scalar and is also commutative in nature
So, we have (A) $\rightarrow$ (r, s) and (B) $\rightarrow$ (s)
Further $\vec{A}\cdot\vec{B}=0$ and $|\vec{A}\times\vec{B}|=0$ is satisfied only when either of the two vectors is a null vector. So, (C) $\rightarrow$ (t).
As done already,

$$\vec{A}\times(\vec{B}\times\vec{C})+\vec{B}\times(\vec{A}\times\vec{C})+\vec{C}\times(\vec{A}\times\vec{B})=\vec{0}$$

So, (D) $\rightarrow$ (p)
Now for three vectors to form a closed polygon, we must have

$$\vec{A}+\vec{B}+\vec{C}=\vec{0}$$

$$\Rightarrow \vec{A}+\vec{B}=-\vec{C}$$

$$\Rightarrow \vec{A}\cdot\vec{C}+\vec{B}\cdot\vec{C}=-\vec{C}\cdot\vec{C}$$

$$\Rightarrow \vec{A}\cdot\vec{C}+\vec{B}\cdot\vec{C}=-C^2 \quad \ldots(1)$$

Similarly $\vec{A}+\vec{C}=-\vec{B}$

$$\Rightarrow \vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{C}=-B^2 \quad \ldots(2)$$

Again $\vec{B}+\vec{C}=-\vec{A}$

$$\Rightarrow \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}=-A^2 \quad \ldots(3)$$

Adding (1), (2) and (3), we get

$$2(\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{C}+\vec{C}\cdot\vec{A})=-(A^2+B^2+C^2)$$

$$\Rightarrow \vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{C}+\vec{C}\cdot\vec{A}=-\frac{1}{2}(A^2+B^2+C^2)$$

Hence $\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{C}+\vec{C}\cdot\vec{A}=$ Negative

So, (E) $\rightarrow$ (q)

5. A $\rightarrow$ (s, t)
B $\rightarrow$ (r)
C $\rightarrow$ (q)
D $\rightarrow$ (p)

$$|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$$

$\Rightarrow \vec{A}\perp\vec{B}$, so (A) $\rightarrow$ (s, t)

$$\vec{A}-\vec{B}=\vec{A}+\vec{B}$$

$\Rightarrow \vec{B}=\vec{0}$, so (B) $\rightarrow$ (r)

$(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})=2(\vec{B}\times\vec{A})$,

so (C) $\rightarrow$ (q)

$$(\vec{A}+\vec{B})\perp(\vec{A}-\vec{B})$$

$\Rightarrow |\vec{A}|=|\vec{B}|$, so (D) $\rightarrow$ (p)

10. A $\rightarrow$ (r)
B $\rightarrow$ (s)
C $\rightarrow$ (p)
D $\rightarrow$ (q)

$A:\vec{a}+\vec{b}+\vec{c}=0$ (polygon law)

$B:\vec{a}+\vec{b}=\vec{c}$ (triangle law)

$C:\vec{c}+\vec{b}=\vec{a}$ (triangle law)

$D:\vec{c}+\vec{a}=\vec{b}$

11. A $\rightarrow$ (r)
B $\rightarrow$ (q)
C $\rightarrow$ (s)
D $\rightarrow$ (p)

(A) $\vec{A}+\vec{B}=\vec{A}-\vec{B}$

$$\Rightarrow B=0$$

(B) $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$

$$\Rightarrow \theta=\frac{\pi}{2}$$

(C) $|\vec{A}\times\vec{B}|=|\vec{A}\cdot\vec{C}|$

$$\Rightarrow \theta=\frac{\pi}{4}$$

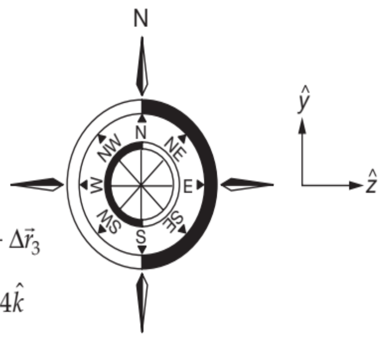
(D) $\vec{A}\times\vec{B}=\vec{A}\cdot\vec{C}$ (Not possible) because $\vec{A}\times\vec{B}$ is a vector and $\vec{A}\cdot\vec{C}$ is a scalar quantity.

Integer/Numerical Answer Type Questions

1. Given $\Delta\vec{r}_1=12\hat{i}$

$\Delta\vec{r}_2=3\hat{j}$ and

$\Delta\vec{r}_3=4\hat{k}$



$$\Rightarrow \Delta\vec{r}=\Delta\vec{r}_1+\Delta\vec{r}_2+\Delta\vec{r}_3$$

$$\Rightarrow \Delta\vec{r}=12\hat{i}+3\hat{j}+4\hat{k}$$

$$\Rightarrow |\Delta\vec{r}|=\sqrt{144+9+16}=\sqrt{169}=13 \text{ m}$$

$$\Rightarrow |\Delta\vec{r}|=1300 \text{ cm}$$

2. Since $R^2=F_1^2+F_2^2+2F_1F\cos\theta$

$$\Rightarrow 3A^2+B^2=(A+B)^2+(A-B)^2+2(A^2-B^2)\cos\theta$$

CHAPTER 3

$\Rightarrow \quad 3A^2 + B^2 = 2A^2 + 2B^2 + 2(A^2 - B^2)\cos\theta$

$\Rightarrow \quad A^2 - B^2 = 2(A^2 - B^2)\cos\theta$

Since $A > B$, so we can cancel the common factor $A^2 - B^2$ from both the sides

$\Rightarrow \quad \cos\theta = \dfrac{1}{2}$

$\Rightarrow \quad \theta = 60°$

3. Let the vectors be $\vec{A}$ and $\vec{B}$ inclined at an angle θ. Since $|\vec{A}| = |\vec{B}|$ and $R^2 = 3A^2$, so we get

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

$\Rightarrow \quad 3A^2 = A^2 + A^2 + 2A^2\cos\theta$

$\Rightarrow \quad A^2 = 2A^2\cos\theta$

$\Rightarrow \quad \cos\theta = \dfrac{1}{2}$

$\Rightarrow \quad \theta = 60°$

4. $\vec{A} + \vec{B} = \vec{C}$

If θ is angle between $\vec{A}$ and $\vec{B}$, then

$\Rightarrow \quad C^2 = A^2 + B^2 + 2AB\cos\theta$

$\Rightarrow \quad A^2 + B^2 = A^2 + B^2 + 2AB\cos\theta$

$\Rightarrow \quad \cos\theta = 0$

$\Rightarrow \quad \theta = 90°$

5. If θ be the angle between $\vec{v}_1$ and $\vec{v}_2$, then

$$v_1^2 + v_2^2 + 2v_1v_2\cos\theta = v_1^2 + v_2^2 - 2v_1v_2\cos\theta$$

$\Rightarrow \quad 4v_1v_2\cos\theta = 0$

$\Rightarrow \quad \cos\theta = 0$

$\Rightarrow \quad \theta = 90°$

6. Let θ be angle between $\vec{F}_1$ and $\vec{F}_2$, then

$$P^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta \qquad \ldots(1)$$

Now, when $\vec{F}_2$ is reversed, angle between $\vec{F}_1$ and $-\vec{F}_2$ becomes $(180 - \theta)$. So,

$$Q^2 = F_1^2 + F_2^2 + 2F_1F_2\cos(180 - \theta)$$

$\Rightarrow \quad Q^2 = F_1^2 + F_2^2 - 2F_1F_2\cos\theta \qquad \ldots(2)$

Adding (1) and (2), we get

$$P^2 + Q^2 = 2(F_1^2 + F_2^2)$$

$\Rightarrow \quad \dfrac{P^2 + Q^2}{F_1^2 + F_2^2} = 2$

7. Let the new vector be $\vec{r} = x\hat{i} + y\hat{j}$. Magnitude of $\vec{A}$ is given by

$$|\vec{A}| = \sqrt{49 + 576} = \sqrt{625} = 25$$

Now a vector having a magnitude of 25 units and parallel to $\vec{B} = 3\hat{i} + 4\hat{j}$ is

$$\vec{r} = \pm 25\hat{B} = \pm 25\left(\frac{\vec{B}}{|\vec{B}|}\right)$$

$\Rightarrow \quad \vec{r} = \pm 25\left(\dfrac{3\hat{i} + 4\hat{j}}{5}\right)$

$\Rightarrow \quad \vec{r} = \pm(15\hat{i} + 20\hat{j})$

So, for the vector to be in the first quadrant, both x and y must be positive. Hence

$x = 15$ and $y = 20$.

8. For vectors to be collinear, we must have

$$\frac{x}{2} = \frac{4}{y} = -\frac{8}{4}$$

$\Rightarrow \quad \dfrac{x}{2} = \dfrac{4}{y} = -2$

$\Rightarrow \quad x = -4$ and $y = -2$

So, $y - x = -2 - (-4) = 2$

9. Since we know that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \quad (1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$

$\Rightarrow \quad \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

10. Let, $\vec{P} = \hat{i} - \hat{j} - 2\hat{k}$

$\Rightarrow \quad \overrightarrow{PS} = (0\hat{i} + \hat{j} + 3\hat{k}) - (\hat{i} - \hat{j} - 2\hat{k})$

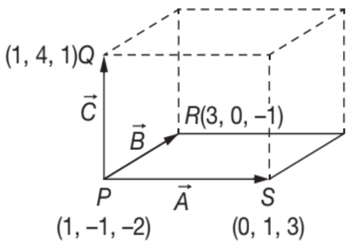
$\Rightarrow \quad \overrightarrow{PS} = \vec{A} = -\hat{i} + 2\hat{j} + 5\hat{k}$

$$\overrightarrow{PR} = (3\hat{i} + 0\hat{j} - \hat{k}) - (\hat{i} - \hat{j} - 2\hat{k})$$

$\Rightarrow \quad \overrightarrow{PR} = \vec{B} = 2\hat{i} + \hat{j} + \hat{k}$

$$\overrightarrow{PQ} = (\hat{i} + 4\hat{j} + \hat{k}) - (\hat{i} + \hat{j} + 2\hat{k})$$

$$\Rightarrow \quad \overrightarrow{PQ} = \vec{C} = 5\hat{j} + 3\hat{k}$$



Volume of parallelopiped $= (\vec{A} \times \vec{B}) \cdot \vec{C}$

$$\Rightarrow \quad \text{Volume} = \begin{vmatrix} -1 & 2 & 5 \\ 2 & 1 & 1 \\ 0 & 5 & 3 \end{vmatrix}$$

$$\Rightarrow \quad \text{Volume} = (-1)(3-5) - 2(6-0) + 5(10-0)$$

$$\Rightarrow \quad \text{Volume} = 2 - 12 + 50$$

$$\Rightarrow \quad \text{Volume} = 40 \text{ unit}$$

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1. Let $P = (2\vec{A}_1 + 3\vec{A}_2) \cdot (3\vec{A}_1 - 2\vec{A}_2)$

$$\Rightarrow \quad P = 6|\vec{A}_1|^2 + 5\vec{A}_1 \cdot \vec{A}_2 - 6|\vec{A}_2|^2$$

$$\Rightarrow \quad P = (6 \times 9) + 5\vec{A}_1 \cdot \vec{A}_2 - (6 \times 25) \qquad \ldots(1)$$

Since $25 = 9 + 25 + 2\vec{A}_1 \cdot \vec{A}_2$

$$\Rightarrow \quad 2\vec{A}_1 \cdot \vec{A}_2 = -9 \qquad \ldots(2)$$

From (1) and (2), we get

$$P = 54 - 22.5 - 150$$

$$\Rightarrow \quad P = -118.5$$

Hence, the correct answer is (B).

2. Position vector of point G is

$$\overrightarrow{OG} = \frac{a}{2}(\hat{i} + \hat{k})$$

Position vector of point H is

$$\overrightarrow{OH} = \frac{a}{2}(\hat{j} + \hat{k})$$

$$\Rightarrow \quad \overrightarrow{GH} = \overrightarrow{OH} - \overrightarrow{OG} = \frac{a}{2}(\hat{j} - \hat{i})$$

Hence, the correct answer is (A).

3. Clearly, $|\vec{A} + \vec{B}|^2 = A^2 + B^2 + 2AB\cos\theta$

$$\Rightarrow \quad |\vec{A} + \vec{B}|^2 = 2A^2(1 + \cos\theta) \qquad \ldots(1)$$

and $|\vec{A} - \vec{B}|^2 = A^2 + B^2 - 2AB\cos\theta$

$$\Rightarrow \quad |\vec{A} - \vec{B}|^2 = 2A^2(1 - \cos\theta) \qquad \ldots(2)$$

Since, $|\vec{A} + \vec{B}|^2 = n^2|\vec{A} - \vec{B}|^2$

$$\Rightarrow \quad \cancel{2A^2}(1 + \cos\theta) = n^2 \cancel{2A^2}(1 - \cos\theta)$$

$$\Rightarrow \quad 1 + \cos\theta = n^2 - n^2\cos\theta$$

$$\Rightarrow \quad (1 + n^2)\cos\theta = n^2 - 1$$

$$\Rightarrow \quad \cos\theta = \frac{n^2 - 1}{n^2 + 1}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{n^2 - 1}{n^2 + 1}\right)$$

Hence, the correct answer is (B).

4. Given that $|\vec{P}| = 2F$, $|\vec{Q}| = 3F$

Let θ be the angle between $\vec{P}$ and $\vec{Q}$, then

$$R^2 = |\vec{P} + \vec{Q}|^2 = P^2 + Q^2 + 2PQ\cos\theta$$

where $\vec{R}$ is the resultant of $\vec{P}$ and $\vec{Q}$

$$\Rightarrow \quad |\vec{P} + \vec{Q}|^2 = 4F^2 + 9F^2 + 12F^2\cos\theta = R^2 \qquad \ldots(1)$$

Now, when $\vec{Q}$ is doubled, then angle between $\vec{P}$ and $2\vec{Q}$ is still θ, but new resultant of $\vec{P}$ and $2\vec{Q}$ (say $\vec{R}'$) has a value twice the previous resultant. So,

$$|\vec{R}'| = 2R$$

$$\Rightarrow \quad R'^2 = 4R^2 = P^2 + (2Q)^2 + 2P(2Q)\cos\theta$$

$$\Rightarrow \quad 4R^2 = (2F)^2 + (6F)^2 + 24F^2\cos\theta \qquad \ldots(2)$$

From (1) and (2), we get

$$F^2 + 9F^2 + 6F^2\cos\theta = 4F^2 + 9F^2 + 12F^2\cos\theta$$

$$\Rightarrow \quad -3F^2 = 6F^2\cos\theta$$

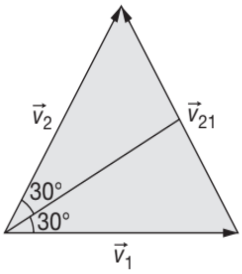
$$\Rightarrow \quad \cos\theta = \left(-\frac{1}{2}\right)$$

$$\Rightarrow \quad \theta = 120^\circ$$

Hence, the correct answer is (D).

5. From figure, $\vec{v}_1 + \vec{v}_{21} = \vec{v}_2$

$$\vec{v}_{21} = \vec{v}_2 - \vec{v}_1$$



$$|\Delta\vec{v}| = |\vec{v}_{21}| = 2v\sin(30°)$$

$$\Rightarrow \quad |\Delta\vec{v}| = 2v \times \frac{1}{2} = v$$

Hence, the correct answer is (A).

6. If $\vec{C} = a\hat{i} + b\hat{j}$ then

$$\vec{A}\cdot\vec{C} = \vec{A}\cdot\vec{B}$$

$$\Rightarrow \quad a + b = 1 \qquad \ldots(1)$$

$$\vec{B}\cdot\vec{C} = \vec{A}\cdot\vec{B}$$

$$\Rightarrow \quad 2a - b = 1 \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$a = \frac{1}{3},\ b = \frac{2}{3}$$

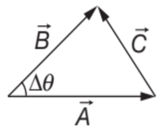
$$\Rightarrow \quad |\vec{C}| = \sqrt{\frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{5}{9}}$$

Hence, the correct answer is (B).

7. By triangle rule

$$\vec{A} + \vec{C} = \vec{B}$$

$$\Rightarrow \quad \vec{B} - \vec{A} = \vec{C}$$



$$\Rightarrow \quad |\vec{B} - \vec{A}| = |\vec{C}| = |\vec{B}|\sin\Delta\theta \qquad \{\because\ \Delta\theta \ll 1\}$$

$$\Rightarrow \quad |\vec{B} - \vec{A}| = |\vec{B}|\Delta\theta \qquad \{\because \text{ for small } \Delta\theta,\ \sin\Delta\theta \simeq \Delta\theta\}$$

Again $|\vec{B}|\cos\Delta\theta = |\vec{A}|$

$$\Rightarrow \quad |\vec{B}| = |\vec{A}| \qquad \{\because \text{ for small } \Delta\theta,\ \cos\Delta\theta \simeq 1\}$$

$$\Rightarrow \quad |\vec{B} - \vec{A}| = |\vec{B}|\Delta\theta = |\vec{A}|\Delta\theta$$

Hence, the correct answer is (C).

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Integer/Numerical Answer Type Questions

1. $\vec{A} = a\hat{i}$ and $\vec{B} = a\cos\omega t\hat{i} + a\sin\omega t\hat{j}$

$$\Rightarrow \quad \vec{A} + \vec{B} = (a + a\cos\omega t)\hat{i} + a\sin\omega t\hat{j} \text{ and}$$

$$\vec{A} - \vec{B} = (a - a\cos\omega t)\hat{i} + a\sin\omega t\hat{j}$$

Since, $|\vec{A} + \vec{B}| = \sqrt{3}|\vec{A} - \vec{B}|$

$$\Rightarrow \quad \sqrt{(a + a\cos\omega t)^2 + (a\sin\omega t)^2} = \sqrt{3}\sqrt{(a - a\cos\omega t)^2 + (a\sin\omega t)^2}$$

$$\Rightarrow \quad 2\cos\left(\frac{\omega t}{2}\right) = \pm\sqrt{3} \times 2\sin\left(\frac{\omega t}{2}\right)$$

$$\Rightarrow \quad \tan\left(\frac{\omega t}{2}\right) = \pm\frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{\omega t}{2} = n\pi \pm \frac{\pi}{6}$$

$$\Rightarrow \quad \frac{\pi}{12}t = n\pi \pm \frac{\pi}{6}$$

$$\Rightarrow \quad t = (12n \pm 2)\ \text{s}$$

$$\Rightarrow \quad t = 2\ \text{s},\ 10\ \text{s},\ 14\ \text{s and so on.}$$