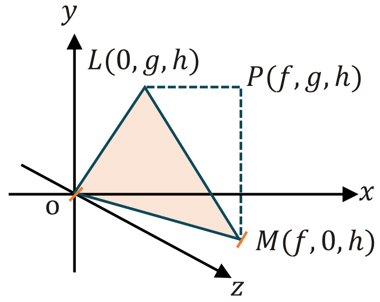
Three Dimensional Geometry

SOLUTIONS

EXERCISE - O

1. **Ans. (A)**

Equation of plane thro $\begin{cases}(0,0,0)\\(0,g,h)\\(f,0,h)\end{cases}$



$\perp$ vector : $(f\hat{i}+h\hat{k})\times(g\hat{j}+h\hat{k})$

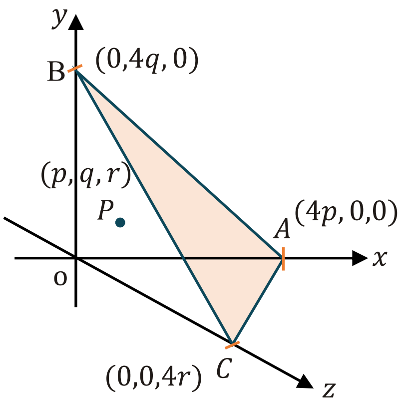
$= fg\hat{k}-fh\hat{j}-hg\hat{i}$

$\therefore$ equation : $-hg(x-0)-fh(y-0)+fg(z-0)=0$

Divide by fgh : $-\frac{x}{f}-\frac{y}{g}+\frac{z}{h}=0$

$\frac{x}{f}+\frac{y}{g}-\frac{z}{h}=0$

2. **Ans. (B)**



Let $P(p,q,r)$ be centroid of tetrahedron.

We get A, B, C as shown

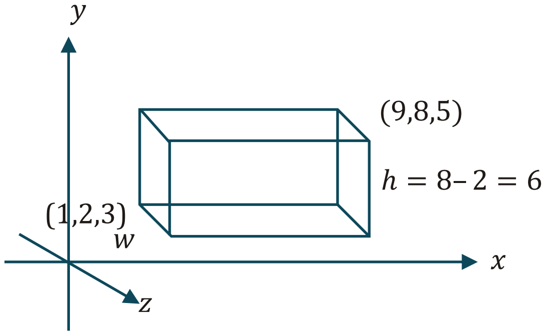
$\therefore$ coterminous edges of tetrahedron: $4p\hat{i}$; $4q\hat{j}$; $4r\hat{k}$

$v=\frac{1}{6}\left[4p\hat{i}\ \ 4q\hat{j}\ \ 4r\hat{k}\right]=64k^3$

$\Rightarrow pqr=6k^3$

Replace $\left.\begin{matrix}p\to x\\q\to y\\r\to z\end{matrix}\right\}xyz=6k^3$

3. **Ans. (B)**



$\ell = 9 - 1 = 8$

$h = 8 - 2 = 6$

$w = 5 - 3 = 2$

4. **Ans. (C)**

$M_1 \Rightarrow L_1 : r = 2\hat{i} + 9\hat{j} + 13\hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

Point A = $A = (2+\lambda, 9+2\lambda, 13+3\lambda)$ lie on L_1

$L_2 : r = -3\hat{i} + 7\hat{j} + P\hat{k} + \mu(-\hat{i} + 2\hat{j} - 3\hat{k})$

Point B = $(-3 - \mu, 7 + 2\mu, P - 3\mu)$ lie on L_2

Now, point A and point B are same (if possible)

Solve $2 + \lambda = -3 - \mu$ & $9 + 2\lambda = 7 + 2\mu \Rightarrow \mu = -2, \lambda = -3$

Satisfy $13 + 3\lambda = P - 3\mu \Rightarrow P = -2$

$\Rightarrow$ Point of intersection of lines $\equiv (2 + \lambda, 9 + 2\lambda, 13 + 3\lambda)$

$= (-1, 3, 4)$

OR

M_2 :

$\because L_1$ & L_2 are not parallel $\Rightarrow$ if lines are coplanar (lines will intersect)

lines coplanar if $\begin{vmatrix} 5 & 2 & 13-P \\ 1 & 2 & 3 \\ -1 & 2 & -3 \end{vmatrix} = 0 \Rightarrow P = -2$

$\Rightarrow$ line intersect if $P = -2$

5. **Ans. (C)**

$\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{a} = 0\hat{i} + \hat{j} + \hat{k}$

$\vec{b} = \hat{i} - \hat{j} + \hat{k}$

$\vec{c} = 2\hat{i} - \hat{j} + 0\hat{k}$

Equation of plane given by $\left[\vec{r}\ \vec{b}\ \vec{c}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right]$

$$\begin{vmatrix} x & y & z \\ 1 & -1 & 1 \\ 2 & -1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & -1 & 0 \end{vmatrix}$$

$x(0 + 1) - y(0 - 2) + z(-1 + 2) = 0 - 1(0 - 2) + 1(-1 + 2)$

$x + 2y + z = 3$ ...(1)

$\perp$ distance from (0, 0, 0) to (1) = $\left|\dfrac{-3}{\sqrt{1+4+1}}\right|$

$= \sqrt{\dfrac{3}{2}}$

6. Ans. (C)

$\dfrac{x-1}{1}=\dfrac{y-2}{2}=\dfrac{2-3}{3}=\lambda$ ...(1)

General point on line (1) $\equiv (1 + \lambda, 2 + 2\lambda, 3 + 3\lambda)$

(A) above point lies in plane $x - 2y + z = 0$

$\Rightarrow$ line (1) lies in plane $x - 2y + z = 0$

(B) $\dfrac{x-1}{1}=\dfrac{y-2}{2}=\dfrac{z-3}{3}$

$\Rightarrow \dfrac{x}{1}-1=\dfrac{y}{2}-1=\dfrac{z}{3}-1$

$\Rightarrow \dfrac{x}{1}=\dfrac{y}{2}=\dfrac{z}{3}$

(C) Point (2, 3, 5) does not lie on (1)

(D) Dr's of line $< 1, 2, 3 > = < \ell, m, n >$ (say)

and Dr's of normal of plane

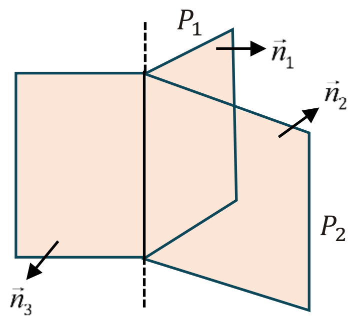
$x - 2y + z - 6 = 0 < 1, -2, 1 > = < \ell_2, m_2\, n_2 > \ell_1\,\ell_2 + m_1\,m_2 + n_1\,n_2 = 1 - 4 + 3 = 0$

line is parallel to plane $x - 2y + z - 6 = 0$

7. Ans. (B)

P_1, P_2 and P_3 pass through one line, if they here infinite solution.

Or {Normals of P_1, P_2, P_3 are coplanar}



$\Rightarrow \begin{vmatrix} -1 & c & b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$

$-1(1 - a^2) - c(-c - ab) + b(ac + b) = 0$

$a^2 + b^2 + c^2 + 2abc = 1$

8. Ans. (B)

Equation of plane containing L_1 is given by

$\Pi : a(x - 2) + b(y - 1) + c(z + 1) = 0$ ...(1)

where, $a\cdot 1 + b\cdot 0 + c\cdot 2 = 0 \Rightarrow a + 2c = 0$ ...(2)

$\because$ Plane is parallel to $L_2 : \vec{r}_2 = 3\hat{i} + \hat{j} + \mu(i + j - k)$

$\Rightarrow a\cdot 1 + b\cdot 1 + c\cdot(-1) = 0$ ...(3)

from equation (2) Put $a = -2c$ in equation (3)

$-2c + b - c = r \Rightarrow b = 3c$

Put $a = -2c, b = 3c$ in equation (1)

$\Pi : -2c(x-2) + 3c(y-1) + c(z+1) = 0$

$-2x + 4 + 3y - 3 + z + 1 = 0$

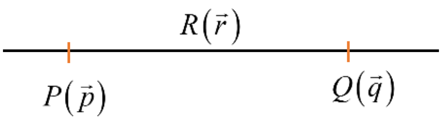
$\Pi : 2x - 3y - z - 2 = 0$

The distance of plane Π from origin.

$$= \left|\frac{2\times0-3\times0-0-2}{\sqrt{4+9+1}}\right| = \sqrt{\frac{2}{7}}$$

9. Ans. (C)

$\because$ Point P & Q are fixed & R is variable & $(\vec{r}-\vec{p})\times(\vec{r}-\vec{q})=0$ possible when point R lie on line PQ



$\Rightarrow$ Locus of R is a line passing through the points P and Q.

10. Ans. (C)

$L_1 : \frac{x}{2} = \frac{y}{3} = \frac{z}{5}$

$L_2 : \frac{x}{1} = \frac{y}{2} = \frac{z}{3}$

clearly L_1 and L_2 intersect at (0, 0, 0)

direction vector of angle bisectors is given by $\vec{a} = \frac{(2,3,5)}{\sqrt{38}} \pm \frac{(1,2,3)}{\sqrt{14}}$

L is a line perpendicular to L_1 and L_2 and passes through origin.

$$\therefore \text{ Dr's of } L = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ 1 & 2 & 3 \end{vmatrix} = (-1, -1, 1)$$

$\therefore L : \frac{x}{1} = \frac{y}{1} = \frac{z}{-1}$

11. Ans. (A)

A, B, C are points on axes, centroid is (α, β, γ)

So, coordinates of A, B, C will be $(3\alpha,0),(3\beta,0)(3\gamma,0)$

Now equation of plane will be $\frac{x}{3\alpha} + \frac{y}{3\beta} + \frac{z}{3\gamma} = 1$

$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$

12. **Ans. (A)**

$3r, 4r, 5r$ be the point that cuts the two plane.

$\therefore$ for plane 1, $2x + 6z - 3y - 12 = 0$

$\Rightarrow 6r + 30r - 12r - 12 = 0$

$\Rightarrow r = \frac{1}{2}$

for plane2,

$2x - 3y + 6z + 2 = 0$

$\Rightarrow 6r - 12r + 30r + 2 = 0$

$\Rightarrow r = \frac{-1}{2}$

point of intersection at plane 1 $\Rightarrow P\left(\frac{3}{2}, 2, \frac{5}{2}\right)$

point of intersection at plane 2 $\Rightarrow Q\left(\frac{-3}{12}, \frac{-1}{3}, \frac{-5}{12}\right)$

$PQ = \frac{35\sqrt{2}}{12}$

13. **Ans. (B)**

Consider a unit cube. $\overrightarrow{OG}$ direction is (1, 0, 1) on $x - z$ plane, when $x - z$ plane is rotated about z – axis by 90° it become $y - z$ plane and the line becomes

$\overrightarrow{OA}$ (0, 1, 1)

$\therefore$ angle between $\overrightarrow{OA}$ and $\overrightarrow{OG}$ be θ.

$\cos\theta = \frac{1}{\sqrt{2} \times \sqrt{2}} = \frac{1}{2}$

$\therefore \theta = 60°$

14. **Ans. (B)**

$L_1 : \frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2} = \lambda$

$(3\lambda + 1, \lambda + 2, 2\lambda + 3)$

$L_2 : \frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \mu$

$(\mu + 3, 2\mu + 1, 3\mu + 2)$

$3\lambda - \mu = 2, \lambda - 2\mu = -1, 2\lambda - 3\mu = -1$

$\therefore \lambda = 1, \mu = 1$

Point of intersection of L_1 and L_2 is (4, 3, 5) so plane which passes through POI of L_1 and L_2 and at a greatest distance from (0, 0, 0) is

$4(x - 4) + 3(y - 3) + 5(z - 5) = 0$

$4x + 3y + 5z = 50$

15. **Ans. (A,D)**

$AB = \sqrt{2}$

$AC = \sqrt{3}$

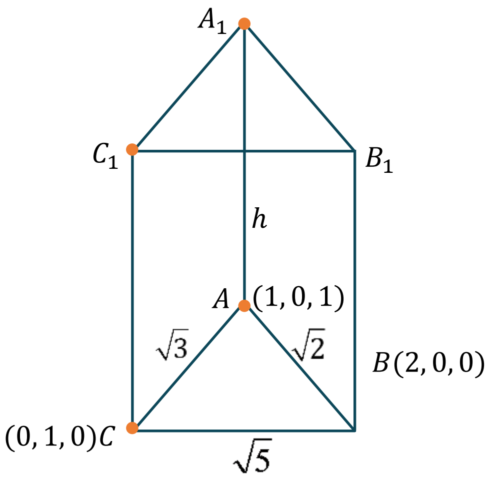
$BC = \sqrt{5}$

Area of $\Delta ABC = \dfrac{\sqrt{3}}{\sqrt{2}}$

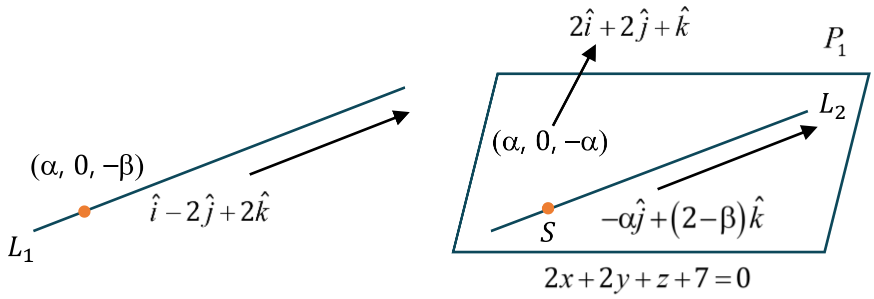
Volume $= \dfrac{\sqrt{3}}{\sqrt{2}} \cdot h = 3 \Rightarrow \ h = \sqrt{6}$

Vector perpendicular to plane ABC is

$$\overline{AC} \times \overline{BC} = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ -1 & 1 & -1 \\ 1 & 0 & -1 \end{vmatrix} = -\hat{i} - 2\hat{j} - \hat{k}$$



16. **Ans. (A,D)**



L_2 lies on $P_1 \Rightarrow (2, 2, 1) \cdot (0, -\alpha, 2-\beta) = 0$

$2(0) + 2(-\alpha) + 1(2-\beta) = 0$

& point satisfy the plane $2\alpha - \alpha + 7 = 0 \Rightarrow \boxed{\alpha = -7}$ $\boxed{\beta = 16}$

Basically L_1 is || to P_1 as $(1, -2, 2) \cdot (2, 2, 1) = 0$

minimum distance $\Rightarrow$ perpendicular distance

$$\left|\frac{2\alpha + 0 - \beta - 7}{3}\right| \Rightarrow \frac{23}{3}$$

17. **Ans. (A,B,C)**

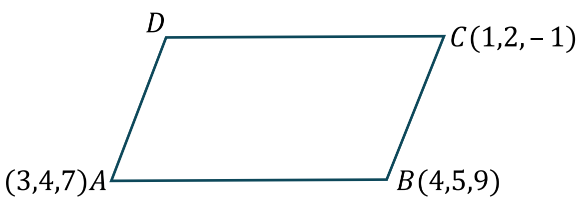
mid-point of AC & BD are same

$\Rightarrow D$ is $(0, 1, -3)$

$\overline{AB} = (4, 5, 9) + \lambda(1, 1, 2)$

$\overline{BC} = (4, 5, 9) + y(3, 3, 10)$

$$\equiv \frac{x-4}{3} = \frac{y-5}{3} = \frac{z-9}{10}$$

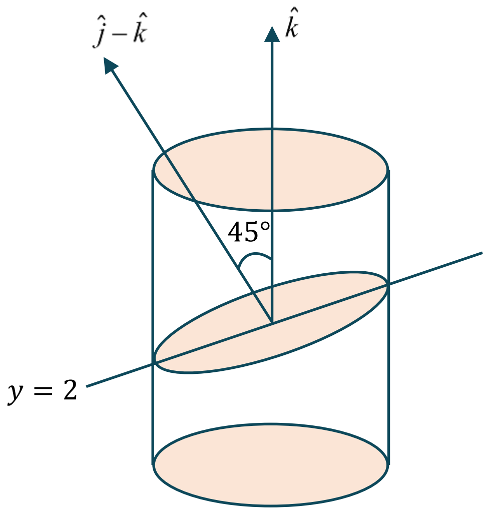


$\overline{AB} \cdot \overline{BC} = (1, 1, 2) \cdot (3, 3, 10) \neq 0 \Rightarrow$ not a rectangle

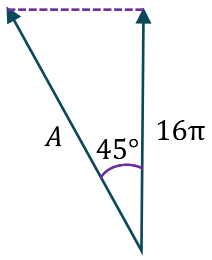
18. Ans. (A,C)

$x = 4\cos t\,, y = 4\sin t\,, z = 4\sin t$

$\Rightarrow y = z \Rightarrow$ locus must lies on a plane also satisfy $x^2 + y^2 = 16$ which is a cyclinder $\Rightarrow$ exact locus is intersection of $y = z$ & $x^2 + y^2 = 16 \Rightarrow$ locus in an ellipse



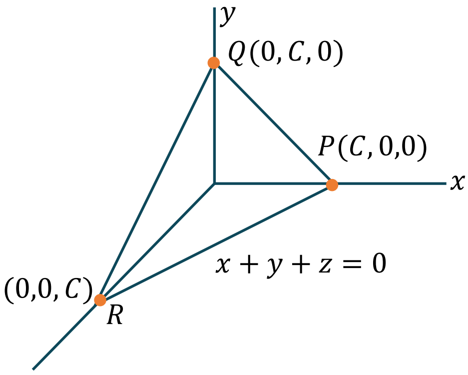
Area of ellipse: area of ellipse is along vector $(\hat{j}-\hat{k})$, area of circle is along vector $(\hat{k})$. Projection of vector area of ellipse on $(\hat{k})$ is area of circle A cos 45° = 16π



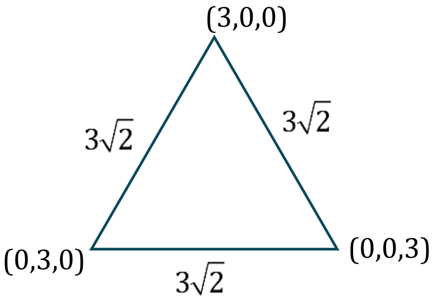
$\boxed{A = 16\sqrt{2}\pi}$

19. Ans. (A,B,C)

angle between plane & x-axis, sinα = $\dfrac{(111)\cdot(100)}{\sqrt{3}}$



$\sin\alpha = \dfrac{1}{\sqrt{3}}$



$\sin\beta = \frac{1}{\sqrt{3}}, \sin\gamma = \frac{1}{\sqrt{3}}$

$\Rightarrow$ equally inclined

$\Rightarrow \sum \sin^2\alpha = 1$

$\Rightarrow \sum \cos^2\alpha = 2$

area: $\frac{\sqrt{3}}{4} \cdot 18 \Rightarrow \frac{9\sqrt{3}}{2}$

20. Ans. (B,C,D)

L-makes equal angles with coordinate axes $\Rightarrow \frac{B+C}{2}$ dr's of L are 1, 1, 1 L-bisects BC

$\Rightarrow L$ passes through mid point of BC i.e. = (3, 4, 6)

$\Rightarrow$ Equation of L is $L: \frac{x-3}{1} = \frac{y-4}{1} = \frac{z-6}{1}$

Since $A(\lambda, 3, \mu)$ lies on L

$\lambda - 3 = 3 \times 4 = \mu - 6$

$\Rightarrow \lambda = 2, \mu = 5$

$\Rightarrow A = (2, 3, 5)$

$B = (-1, 3, 2)$

$C = (7, 5, 10)$

$\Rightarrow \overline{AB} = -3\hat{i} - 3\hat{k}$

$\overline{AC} = 5\hat{i} + 2\hat{j} + 5\hat{k}$

$$\overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 0 & -3 \\ 5 & 2 & 5 \end{vmatrix}$$

$= 6\hat{i} - 6\hat{k}$

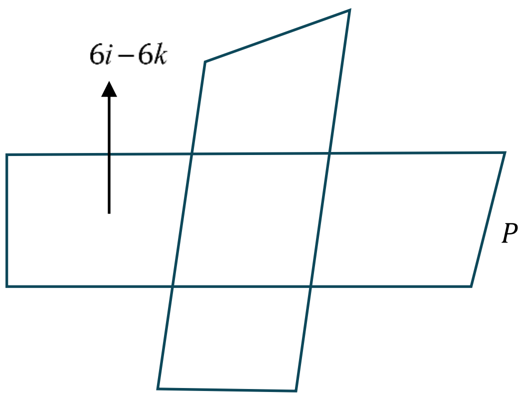
Area of ABC = $\frac{1}{2}|\overline{AB} \times \overline{AC}|$

$= \frac{1}{2}\sqrt{6^2 + 6^2}$

ar(ΔABC) = $3\sqrt{2}$

$\Rightarrow L_1$ is || to L, passing through origin.

$\Rightarrow L_1 : x = y = z$



Jar top

$\Rightarrow L_1 : \frac{x-1}{1} = \frac{y-1}{1} = z-1$

($\because$ (1, 1, 1) satisfies x = y = z)

$\overline{AB} \times \overline{AC}$ is

$\perp^{ar}$ to plane

Now Let us solve.

Option (C)

The required plane contains L_1 and is || to vector $6\hat{i} - 6\hat{k}$

Th equation of L_1 is

$$\begin{vmatrix} x & y & z \\ 6 & 0 & -6 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow 6x - 12y + 6z = 0$

$\Rightarrow x - 2y + z = 0$

21. **Ans. (B,C)**

Given lines $\frac{x-1}{\ell} = \frac{y+1}{m} = \frac{z}{n}$ and $\frac{x+1}{m} = \frac{y-3}{n} = \frac{z-1}{\ell}$,

Angle between lines is given by, $\cos\theta = \frac{\ell m + mn + n\ell}{\ell^2 + m^2 + n^2}$ ...(i)

Now, ℓ, m, n are the roots of $x^3 + x^2 - 4x - 4 = 0$

Thus, $\ell + m + n = -1$ and $\ell m + mn + \ell n = -4$

$\Rightarrow \left(\ell + m + n\right)^2 = \ell^2 + m^2 + n^2 + 2\left(\ell m + mn + \ell n\right)$

$\Rightarrow \left(-1\right)^2 = \ell^2 + m^2 + n^2 + 2\left(-4\right)$

$\Rightarrow \ell^2 + m^2 + n^2 = 9$

So, $\cos\theta = -\frac{4}{9}$

$\therefore$ Acute angle between the lines is $\cos^{-1}\frac{4}{9}$

22. **Ans. (A,B)**

$\frac{x+6}{5} = \frac{y+10}{3} = \frac{z+14}{8} = \lambda$ (say)

General point on above line is $(5\lambda - 6, 3\lambda - 10, 8\lambda - 14)$.

Let $B \equiv (5\lambda - 6, 3\lambda - 10, 8\lambda - 14)$

Dr's of line AB are $< (5\lambda - 6 - 7), (3\lambda - 10 - 2), (8\lambda - 14 - 4) >$ i.e., $< 5\lambda - 13, 3\lambda - 12, 8\lambda - 18 >$

Also, dr's of line BC are $< 5, 3, 8 >$

Since angle between AB and BC is $\frac{\pi}{4}$

$\Rightarrow \cos\frac{\pi}{4} = \frac{|(5\lambda - 13)5 + 3(3\lambda - 12) + 8(8\lambda - 18)|}{\sqrt{5^2 + 3^2 + 8^2}\sqrt{(5\lambda - 13)^2 + (3\lambda - 12)^2 + (8\lambda - 18)^2}}$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{|25\lambda + 9\lambda + 64\lambda - 65 - 36 - 144|}{\sqrt{98}\sqrt{(5\lambda - 13)^2 + (3\lambda + 12)^2 + (8\lambda - 18)^2}}$$

Solving above equation, we get $\lambda = 3, 2$

Hence, equation of line are

$$\frac{x-7}{2} = \frac{y-2}{-3} = \frac{z-4}{6} \text{ and } \frac{x-7}{3} = \frac{y-2}{6} = \frac{z-4}{2}$$

23. Ans. (C)

Let L_2 intersect L_1 at P then $P(2 + 4\lambda, \lambda - 1, 1 - \lambda)$, $A(2, -2, 1)$

direction vector of L_2

$= (4\lambda, \lambda + 1, -\lambda)$

$\because$ L_2 and plane π are perpendicular we get $1(4\lambda) + (-1)(\lambda + 1) + 2(-\lambda) = 0$

$\lambda = 1 \rightarrow P(6, 0, 0)$

$\overrightarrow{PA} = 4\hat{i} + 2\hat{j} - \hat{k}$

equation of L_2 : $6\hat{i} + \alpha(4\hat{i} + 2\hat{j} - \hat{k})$

$\alpha \in R$

$L_2 : ((4\alpha + 6)\hat{i} + 2\alpha\hat{j} - \alpha\hat{k})$

24. Ans. (A)

$O(0, 0, 0)$, $A(2, -2, 1)$

$\pi : x - y + 2z = 5$

$L_1 : (2 + 4\lambda)\ \hat{i} + (\lambda - 1)\hat{j} + (1 - \lambda)\hat{k}$

For Q

$2 + 4\lambda - (\lambda - 1) + 2(1 - \lambda) = 5$

$\lambda = 0$

$Q(2, -1, 1)$

for R

$1 - \lambda = 0 \rightarrow \lambda = 1$

$R(6, 0, 0)$

$V = \frac{1}{6}[\overrightarrow{OA}\ \ \overrightarrow{OQ}\ \ \overrightarrow{OR}]$

$$V = \frac{1}{6}\left\|\begin{matrix} 2 & -2 & 1 \\ 2 & -1 & 1 \\ 6 & 0 & 0 \end{matrix}\right\| = 1$$

25. Ans. (A,B,C)

dr's of line passing through P and Q $= (x, y, z - 2)$

dr's of z-axis = $(0, 0, 1)$

$$\cos\theta = \frac{(x, y, z-2)\cdot(0, 0, 1)}{\sqrt{x^2 + y^2 + (z-2)^2}\cdot\sqrt{0^2 + 0^2 + 1}}$$

$$\sqrt{x^2 + y^2 + (z-2)^2}\cos\theta = z - 2$$

(A) if $\theta = 0$,

$x^2 + y^2 + (z-2)^2 = (z-2)^2$

$x^2 + y^2 = 0,\ z \in R \rightarrow x = y = 0, z \in R$

$\therefore$ Locus of P is z-axis

(B) if $\theta = 90°$,

$z = 2$ is the locus of P

(C) if $\theta = 60°$

$\sqrt{x^2 + y^2 + (z-2)^2} = 2(z-2)$

$x^2 + y^2 - 3(z-2)^2 = 0$

is the locus of P

26. Ans. (A,B,C)

(A) $x^2 + y^2 - 3(z-2)^2 = 0$

$x + y + z = 1 \rightarrow z - 2 = -1 - x - y$

$x^2 + y^2 = 3(x + y + 1)^2$

$(x-0)^2 + (y-0)^2 = 6\left(\frac{x+y+1}{\sqrt{2}}\right)^2$

$PS = e\,PM \rightarrow e = \sqrt{6} > 1$

$\therefore$ hyperbola

(B) $x^2 + y^2 - 3(z-2)^2 = 0$ and $x + y + z = 2$

$x^2 + y^2 = 3(x + y)^2$

$x^2 + y^2 + xy = 0$

pair of imaginary st. lines.

(C) $x^2 + y^2 - 3(z-2)^2$ and $2x + y + z = 2 \rightarrow z - 2 = -(2x + y)$

$x^2 + y^2 = 3(2x + y)^2$

$11x^2 + 2y^2 + 12xy = 0$

Pair of straight lines

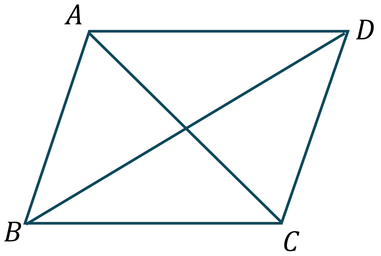
27. Ans. (1.41 or 1.42)

Given planes are $y + z = 0$ ...(i)

$x + z = 0$ ...(ii)

$x + y = 0$ ...(iii)

$x + y + z = \sqrt{3}$ ...(iv)



Clearly planes (i), (ii) and (iii) meet at O(0, 0, 0)

Let the tetrahedron be $OABC$ O(0, 0, 0) (0, 0, 3)

Let the equation to one of the pair of opposite edges OA and BC be

$y + z = 0, x + z = 0$ ...(1)

$x + y = 0, x + y + z = 0$ ...(2)

Equation (1) and (2) can be expressed in symmetrical form as

$\frac{x+0}{1} = \frac{y-0}{1} = \frac{z-0}{-1}$ ...(3)

and, $\frac{x-0}{1} = \frac{y-0}{-1} = \frac{z-\sqrt{3}}{0}$ ...(4)

d.r. of OA and BC are (1, –1) and (1, – 1, 0)

Let PQ be the shortest distance between OA and BC having direction cosine (ℓ, m, n)

$\therefore\ \ell + m - n = 0$ and $\ell - m = 0$ Solving (5) and (6), we get,

$\frac{\ell}{1} = \frac{m}{1} = \frac{n}{2} = k$ (say)

Also, $\ell^2 + m^2 + n^2 = 1$

$\therefore k^2 + k^2 + 4k^2 = 1$

$\Rightarrow k = \frac{1}{\sqrt{6}}$

$\therefore\ \ell = \frac{1}{\sqrt{6}}, m = \frac{1}{\sqrt{6}}, n = \frac{2}{\sqrt{6}}$

Shortest distance between OA and BC

i.e. PQ = The length of projection of OC on PQ

– $|(x_2 - x_1)\ell + (y_2 - y_1)m + (z_2 - z_1)n|$

= $\left|0 \cdot \frac{1}{\sqrt{6}} + 0 . \frac{1}{\sqrt{6}} + \sqrt{3} \cdot \frac{2}{\sqrt{6}}\right| = \sqrt{2}$

28. Ans. (0.66 or 0.67)

Find the vertices, which are the planes intersections i.e.,

$(\pi_1) x + y = 0; (\pi_2) y + z = 0; (\pi_3) z + x = 0 \rightarrow A(0, 0, 0)$

$(\pi_1)\ x + y = 0; (\pi_2)\ y + z = 0; (\pi_4)\ x + y + z - 1 = 0 \rightarrow B(1, -1, 1)$

$(\pi_1)\ x + y = 0; (\pi_3)\ z + x = 0; (\pi_4)\ x + y + z - 1 = 0 \rightarrow C\ (-1, 1, 1)$

$(\pi_2)\ y + z = 0; (\pi_3)\ z + x = 0; (\pi_4)\ x + y + z - 1 = 0 \rightarrow D(1, 1, -1)$

Thus, you have the tetrahedron edge vectors

$\overrightarrow{AB} = (1,-1,1); \overrightarrow{AC} = (-1,1,1); \overrightarrow{AD} = (1,1,-1)$

The volume is given by the triple product

$$V = \frac{1}{6}|\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD})| = \frac{1}{6}\begin{vmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = \frac{|-4|}{6} = \frac{2}{3}$$

29. Ans. (D)

(A) $L_1 : x = 1 + t, y = t, z = 2 - 5t$

$\frac{x-1}{1} = \frac{y-0}{1} = \frac{z-2}{-5}$

$L_2 : \frac{x-2}{2} = \frac{y-1}{2} = \frac{z+3}{-10}$

L_1 Passing through $(x_1, y_1, z_1) \equiv (1, 0, 2)$

& Dr's $< a, b, c > \equiv < 1, 1, -5 >$

L_2 : passing through $(x_2, y_2, z_2) \equiv (2, 1, -3)$ and passing through $(x_2, y_2, z_2) \equiv (2, 1, -3)$ and Dr's $< a', b', c' > \equiv < 2, 2, -10 >$

Here $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$ and L_1 passing through point (2, 1, –3) $\Rightarrow L_1$ & L_2 lines are coincident line

$\Rightarrow$ infinite planes possible passing through both L_1 & L_2

$\because$ Lines are coincident $\Rightarrow$ they intersect at infinite points

$\Rightarrow$ R, S

(B) $L_1 : \frac{x-1}{2} = \frac{y-3}{2} = \frac{z-2}{-1} = \gamma$

$L_2 : \frac{x-2}{1} = \frac{y-6}{-1} = \frac{z+2}{3} = \mu$

Now Point on line L_1 = $(1 + 2\lambda, 3 + 2\lambda, 2 - \lambda)$

Point on line L_2 = $(2 + \mu, 6 - \mu, -2 + 3\mu)$

$$\underbrace{1 + 2\lambda = 2 + u \quad | \quad 3 + 2\lambda = 6 - \mu}_{\text{solve}}$$

$\lambda = 1$ which satisfy the z co-ordinate

$\mu = 1$ $2 - \lambda = -2 + 3\mu$

$\Rightarrow$ L_1 & L_2 interest at point (3, 5, 1) and lines are not parallel

$\Rightarrow$ lines lie in an unique plane. (Q)

(C) $L_1 : x = -6t, y = 1 + 9t, z = -3t$

$L_2 : x = 1 + 2s, y = 4 - 3s, z = s$

$L_1 : \frac{x}{-6} = \frac{y-1}{9} = \frac{z}{-3} = t$

$L_2 : \frac{x-1}{2} = \frac{y-4}{-3} = \frac{z}{1} = s$

L_1 & L_2 are parallel lines but not coincident

($\because$ Point (0, 1, 0) does not lie on L_2)

$\Rightarrow \boxed{Q, S}$

(D) $L_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

$L_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$ lines are not parallel.

$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda \Rightarrow P = (\lambda, 1 + 2\lambda, 2 + 3\lambda)$

$\frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2} = \mu \Rightarrow Q \equiv (3 - 4\mu, 2 - 3u, 1 + 2\mu)$

$$\Rightarrow \underbrace{\lambda = 3 - 4\mu \quad \& \quad 1 + 2\lambda = 2 - 3\mu}_{\text{solve}}$$

$$\mu = 1$$
$$\lambda = -1$$

$\therefore$ $2 + 3\lambda \neq 1 + 2\mu$ at $\lambda = -1$

$\mu = 1$

$\Rightarrow L_1$ & L_2 are skew lines

$\Rightarrow \boxed{P, S}$

30. **Ans. (A→R; B→Q; C→P; D→S)**

dr's of L_1 are (3 – 0, 7 – 3, –1 + 2)

= (3, 4, 1)

$\because$ L_1 passes through P

$$\Rightarrow L_1 : \frac{x}{3} = \frac{y \cdot 3}{4} = \frac{z+2}{1}$$

$$\Rightarrow L_2 : \frac{x-1}{1} = \frac{y+3}{0} = \frac{z+1}{1}$$

(A) Let arbitary point K on L_2 be

$K = (1 + r, -3, -1 + r)$

$PK \perp_{ar} L_2$

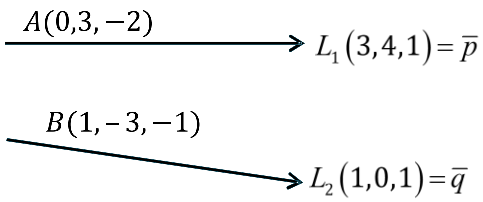
$$\Rightarrow \left[(1+r)\hat{i} - 6\hat{j} + (1+r)\hat{k}\right] \cdot \left[\hat{i} + \hat{k}\right] = 0$$

$$\Rightarrow 2(1+r) = 0 \ \Rightarrow r = -1$$

$$\Rightarrow K = (0, -3, -2)$$

$$\Rightarrow PK = \sqrt{0^2 + 6^2 + 0^2} = 6$$

(B)



Shortest distance

$$d = \frac{\overline{AB} \cdot (\bar{p} \times \bar{q})}{|\bar{p} \times \bar{q}|}$$

$$\bar{p} \times \bar{q} = \begin{vmatrix} i & j & k \\ 3 & 4 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$$\bar{p} \times \bar{q} = 4\hat{i} - 2\hat{j} - 4\hat{k}$$

$$\Rightarrow |\bar{p} \times \bar{q}| = \sqrt{4^2 + 2^2 + 4^2} = 6$$

$$\overline{AB} = \hat{i} - 6\hat{j} + \hat{k}$$

$$\Rightarrow d = \frac{1 \times 4 + (-6) \times (-2) + 1 \times (-4)}{6}$$

$$d = \frac{4 + 12 - 4}{6} = 2$$

(C) P = (0, 3, –2) Q = (3, 7, –1)

R(1, –3, –1)

$$\overline{PQ} \times \overline{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 1 \\ 1 & -6 & 1 \end{vmatrix}$$

$\overline{PQ}\times\overline{PR}=10\hat{i}-2\hat{j}-22\hat{k}$

Area of $\Delta PQR=\frac{1}{2}\left|\overline{PQ}\times\overline{PR}\right|$

$=\frac{1}{2}\sqrt{10^2+2^2+22^2}$

$=7\sqrt{3}$

(D) Normal to plane PQR is

$\overline{PQ}\times\overline{PR}=10\hat{i}-2\hat{j}-22\hat{k}$

$\Rightarrow$ Equation of PQR is

$10x-2y-22z=k_1$

$\Rightarrow 5x-y-11z=k$

$\Rightarrow 5x-y-112=5(0)-3-11(-2)$

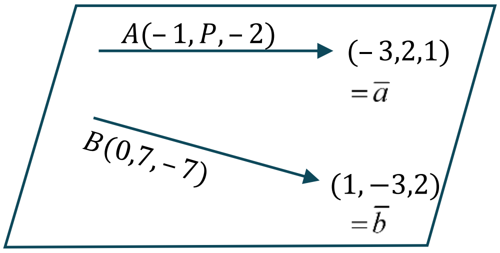
$\Rightarrow 5x-y-112=19$

$\perp_{ar}$ distance from O to PQR is

$\frac{|19|}{\sqrt{5^2+1^2+11^2}}=\frac{19}{\sqrt{147}}$

EXERCISE - S

1. **Ans. (3)**



For the lines to be in same plane, $\overline{AB},\bar{a},\bar{b}$ should be is same plane

$\Rightarrow\left[\overline{AB}\ \ \bar{a}\ \ \bar{b}\right]=0$

$\Rightarrow\begin{vmatrix}1 & 7-P & -5\\ -3 & 2 & 1\\ 1 & -3 & 2\end{vmatrix}=0$

$\Rightarrow 7+(7-P)(7)+(-5)(7)=0$

$\Rightarrow 21=7p$

$\Rightarrow P=3$

2. **Ans. (17)**

$\overrightarrow{PQ}=4\hat{i}+2\hat{j}+2\hat{k}$

$=2\left(2\hat{i}+\hat{j}+\hat{k}\right)$

normal of plane is proportional to $\overrightarrow{PQ}$

$\frac{a}{4}=\frac{b}{2}=\frac{c}{2}\Rightarrow\frac{a}{2}=\frac{b}{1}=\frac{c}{1}$

$\therefore a = 2, b = 1, c = 1$ (least & natural number)

mid-point of PQ will lie on the plane

$$M\left(\frac{\overrightarrow{PQ}}{2}\right)=(3,3,4)$$

M satisfy plane

$2x + y + z = d$

$\therefore 3(3) + 3(1) + 4(1) = d$

$d = 13$

$a + b + c + d \Rightarrow 2 + 1 + 1 + 13$

$\Rightarrow 17$

3. Ans. (6)

Let the line through origin is $\frac{x}{\lambda}=\frac{y}{\mu}=\frac{z}{1}$

$\Rightarrow x = \lambda z, y = \mu z$

$$\frac{x-1}{1}=\frac{y}{2}=\frac{z+1}{2} \quad ...(1)$$

To find point of intersection of line (1) and line

we have $\frac{\lambda z-1}{1}=\frac{\mu z}{2}=\frac{z+1}{2}$ ...(2)

$$\Rightarrow z=\frac{3}{2\lambda-1}=\frac{1}{\mu-1}$$

$\Rightarrow -2\lambda + 3\mu = 2$ ...(3)

To find point of intersection of line (1) and line $\frac{x}{2}=\frac{y-1}{3}=\frac{z}{1}$ ...(4)

we have $\frac{\lambda z}{2}=\frac{\mu z-1}{3}=\frac{z}{1}$

$\Rightarrow \lambda = 2$ ...(5)

Solving (3) and (5), $\lambda=2$ and $\mu = 2$

$\therefore z = 1, x = 2, y = 2$ for point A

and $z = -1, x = -2, y = -2$ for point B

$\therefore AB^2=4^2+4^2+2^2=36$

$\therefore AB = 6$

4. Ans. (9)

Let the variable point be $P(x, y, z)$

given, $\sqrt{(x-1)^2+(y-1)^2+(z-1)^2}=2\sqrt{y^2+z^2}$

$\therefore x^2 - 3y^2 - 3z^2 - 2x - 2y - 2z + 3 = 0$

$\therefore \lambda_1 = 3, \lambda_2 = 3, \lambda_3 = 3$

5. **Ans. (6)**

Equation of plane is $\begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}=0$

$x-2y+z=0$ ...(1)

$Ax-2y+z=d$ ...(2)

Compare $\frac{A}{1}=\frac{-2}{-2}=\frac{1}{1}\Rightarrow A=1$

Distance between planes is $\left|\frac{d}{\sqrt{1+1+4}}\right|=\sqrt{6}$

$\Rightarrow |d|=6$

6. **Ans. (3)**

If $\lambda=-7$, then planes will be parallel & distance

between them will be $\frac{3}{\sqrt{633}}\Rightarrow k=3$

But if $\lambda\neq-7$, then planes will be intersecting & distance between them will be 0.

7. **Ans. (4.50)**

Any point on $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}=t$ is $(2t+1,3t-1,4t+1)$

And any point on $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}=s$ is $(s+3,2s+k,s)$

Given lines are intersecting.

$\Rightarrow t=-\frac{3}{2}$ and $s=-5$

$\therefore\ k=\frac{9}{2}$

8. **Ans. (3.50)**

$2x+y+2z-8=0$ .... (P_1)

$2x+y+2z+\frac{5}{2}=0$ .... (P_2)

Distance between P_1 and P_2 = $\left|\frac{-8-\frac{5}{2}}{\sqrt{2^2+1^2+2^2}}\right|=\frac{7}{2}$

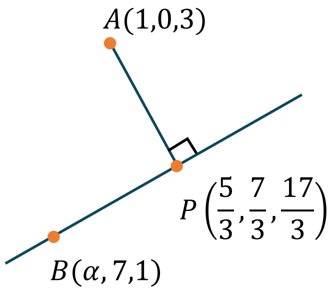
9. **Ans. (4.00)**

D.R. of BP = $<\frac{5}{3}-\alpha,\frac{7}{3}-7,\frac{17}{3}-1>$

D.R of AP = $<\frac{5}{3}-1,\frac{7}{3}-0,\frac{17}{3}-3>$

$BP\perp^r AP\Rightarrow\left(\frac{5}{3}-\alpha\right)\cdot\frac{2}{3}+\left(-\frac{14}{3}\right)\cdot\frac{7}{3}+\frac{14}{3}\cdot\frac{8}{3}=0$

$\Rightarrow\alpha=4$



10. Ans. (2.44 or 2.45)

Let the line through origin is $\frac{x}{\lambda}=\frac{y}{\mu}=\frac{z}{1}$

$\Rightarrow x = \lambda z, y = \mu z$ ...(1)

To find point of intersection of line (1) and line $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ ...(2)

we have $\frac{\lambda z-2}{1}=\frac{\mu z-1}{-2}=z+1$

$\Rightarrow z=\frac{3}{\lambda-1}=\frac{-1}{\mu+2}$

$\Rightarrow \lambda + 3\mu + 5 = 0$ ...(3)

To find point of intersection of line (1) and line $\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ ....(4)

we have $\frac{\lambda z-\frac{8}{3}}{2}=\frac{\mu z+3}{-1}=\frac{z-1}{1}$

$\Rightarrow z=\frac{2}{3(\lambda-2)}=\frac{-2}{\mu+1}$

$\Rightarrow 3\lambda + \mu = 5$ ...(5)

Solving (3) and (5), $\lambda=\frac{5}{2}$ and $\mu=-\frac{5}{2}$

$\therefore z = 2, x = 5, y = -5$ for point P

and $z=\frac{4}{3}$, $x=\frac{10}{3}$, $y=\frac{10}{3}$ for point Q

$\therefore PQ^2=\frac{4}{9}+\frac{25}{9}+\frac{25}{9}=6$

EXERCISE - JEE (Main) PYQ

1. Ans. (2)

Line $x = ay + b, z = cy + d \Rightarrow \frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}$

Line $x = a'z + b', y = c'z + d'$

$\Rightarrow \frac{x-b'}{a'}=\frac{y-d'}{c'}=\frac{z}{1}$

Given both the lines are perpendicular.

$\Rightarrow aa' + c' + c = 0$

2. Ans. (2)

equation of bisector of angle

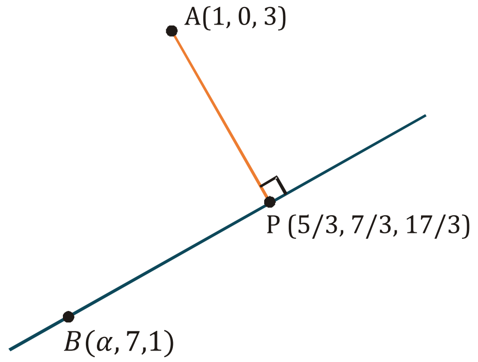
$\frac{2x-y+2z-4}{3}=\pm\frac{x+2y+2z-2}{3}$

(+) gives $x - 3y = 2$

(−) gives $3x + y + 4z = 6$

therefore option (ii) satisfy

3. Ans. (4.00)



D.R. of BP = $<\frac{5}{3}-\alpha, \frac{7}{3}-7, \frac{17}{3}-1>$

D.R. of AP = $<\frac{5}{3}-1, \frac{7}{3}-0, \frac{17}{3}-3>$

$BP \perp^r AP$

$\Rightarrow \alpha = 4$

4. Ans. (2)

Shortest distance $= \dfrac{\left\|\begin{matrix} 6 & 15 & -3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{matrix}\right\|}{\sqrt{36+225+9}} = \dfrac{270}{\sqrt{270}}$

$= \sqrt{270} = 3\sqrt{30}$

5. Ans. (1)

For planes to intersect on a line

$\Rightarrow$ there should be infinite solution of the given system of equations

for infinite solutions

$\Delta = \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0 \Rightarrow 3\alpha + 9 = 0 \Rightarrow \alpha = -3$

$\Delta_z = \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0 \Rightarrow 13 - \beta = 0 \Rightarrow \beta = 13$

Also for $\alpha = -3$ and $\beta = 13$ $\Delta_x = \Delta_y = 0$

$\therefore \alpha + \beta = -3 + 13 = 10$

6. Ans. (3)

Point C is

$\left(\frac{2k-3}{k+1}, \frac{4k-6}{k+1}, \frac{-3k+1}{k+1}\right)$

$\frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3}$

Plane $lx + my + nz = 0$

$l(-1)+m(2)+n(3)=0$

$-l+2m+3n=0$ ...(1)

It also satisfy point $(1,-4,-2)$

$l-4m-2n=0$ ...(2)

Solving (1) and (2)

$2m+3n=4m+2n$

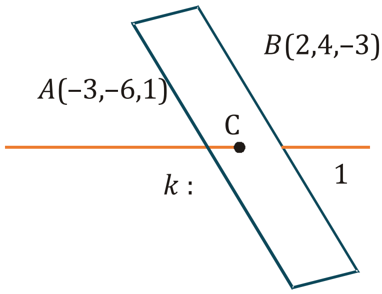
$n=2m$

$l-4m-4m=0$

$l=8m$

$\frac{l}{8}=\frac{m}{1}=\frac{n}{2}$

$l:m:n=8:1:2$



Plane is $8x+y+2z=0$

It will satisfy point C

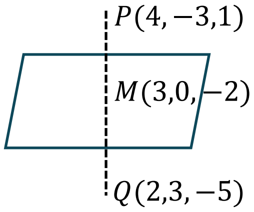
$8\left(\frac{2k-3}{k+1}\right)+\left(\frac{4k-6}{k+1}\right)+2\left(\frac{-3k+1}{k+1}\right)=0$

$16k-24+4k-6-6k+2=0$

$14k=28$

$\therefore k=2$

7. Ans. (28)



Plane is $1(x-3)-3(y-0)+3(z+2)=0$

$x-3y+3z+3=0$

$(a^2+b^2+c^2+d^2)_{\min}=28$

8. Ans. (2)

$P_1: x+5y+7z=3,$

$P_2: x-3y-z=5$

$P_3: x+5y+7z=\frac{5}{2}$

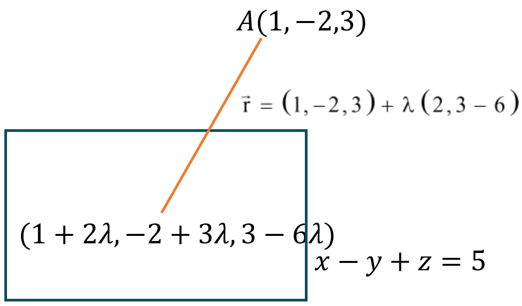
so P_1 and P_3 are parallel.

9. Ans. (4)

$(1+2\lambda)+2-3\lambda+3-6\lambda=5$

$\Rightarrow 6-7\lambda=5\Rightarrow\lambda=\frac{1}{7}$

So, $P=\left(\frac{9}{7},-\frac{11}{7},\frac{15}{7}\right)$



$AP=\sqrt{\left(1-\frac{9}{7}\right)^2+\left(-2+\frac{11}{7}\right)^2+\left(3-\frac{15}{7}\right)^2}$

$AP=\sqrt{\left(\frac{4}{49}\right)+\frac{9}{49}+\frac{36}{49}}=1$

10. Ans. (4)

Required equation of plane

$P_1 + \lambda P_2 = 0$

$(x - y - z - 1) + \lambda(2x + y - 3z + 4) = 0$

$(1 + 2\lambda)x + (-1 + \lambda)y + (-1 - 3\lambda)z + (-1 + 4\lambda) = 0$

Given that its dist. From origin is $\frac{\sqrt{2}}{\sqrt{21}}$

Thus $\frac{|4\lambda - 1|}{\sqrt{(2\lambda+1)^2 + (\lambda-1)^2 + (-3\lambda-1)^2}} = \frac{\sqrt{2}}{\sqrt{21}}$

$\Rightarrow 21(4\lambda - 1)^2 = 2(14\lambda^2 + 8\lambda + 3)$

$\Rightarrow 336\lambda^2 - 168\lambda + 21 = 28\lambda^2 + 16\lambda + 6$

$\Rightarrow 308\lambda^2 - 184\lambda + 15 = 0$

$\Rightarrow 308\lambda^2 - 154\lambda - 30\lambda + 15 = 0$

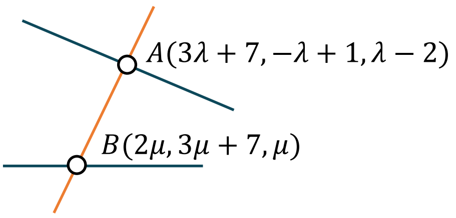
$\Rightarrow (2\lambda - 1)(154\lambda - 15) = 0$

$\Rightarrow \lambda = \frac{1}{2} \text{ or } \frac{15}{154}$

for $\lambda = \frac{1}{2}$ reqd. plane is

$4x - y - 5z + 2 = 0$

11. Ans. (84)



DR's of AB

$(3\lambda - 2\mu + 7, -\lambda - 3\mu - 6, \lambda - \mu - 2)$

$\frac{3\lambda - 2\mu + 7}{1} = \frac{-\lambda - 3\mu - 6}{-4} = \frac{\lambda - \mu - 2}{2}$

Taking first (2) $-12\lambda + 8\mu - 28 = -\lambda - 3\mu - 6$

$\lambda - \mu + 2 = 0$

Taking second & third

$-2\lambda - 6\mu - 12 = -4\lambda + 4\mu + 8$

$\lambda - 5\mu - 10 = 0$

After solving above two equation $\lambda = -5, \mu = -3$

$A = (-8, 6, -7)$

$B = (-6, -2, -3)$

$(AB)^2 = 4 + 64 + 16 = 84$

12. Ans. (1)

Let equation of rotated plane be :

$(2x + 3y + z + 20) + \lambda(x - 3y + 5z - 8) = 0$

$(2 + \lambda)x + (3 - 3\lambda)y + (1 + 5\lambda)z + 20 - 8\lambda = 0$

Above plane is perpendicular to $2x + 3y + z + 20 = 0$

So, $(2 + \lambda).2 + (3 - 3\lambda).3 + (1 + 5\lambda).1 = 0 \Rightarrow \lambda = 7$

$\Rightarrow$ Equation of rotated plane : $x - 2y + 4z - 4 = 0$

Mirror image of $A\left(2,\frac{-1}{2},2\right)$ in rotated plane is $B(a,b,c)$

Equation of AB : $\frac{x-2}{1}=\frac{y+1/2}{-2}=\frac{z-2}{4}=k$

Let coordinate of B be $(2 + k, \frac{-1}{2}-2k, 2 + 4k)$ midpoint of AB is $\left(2+\frac{k}{2},\frac{-1}{2}-k,2+2k\right)$ which will lie on the plane $x - 2y + 4z - 4 = 0$

Hence $k=\frac{-2}{3}$

Therefore B is $\left(\frac{4}{3},\frac{5}{6},\frac{-2}{3}\right)\equiv\left(\frac{8}{6},\frac{5}{6},\frac{-4}{6}\right)$

So, $\frac{a}{8}=\frac{b}{5}=\frac{c}{-4}$

13. Ans. (1)

$l + m - n = 0$

$3l^2 + m^2 + cl(l + m) = 0$

$n = l + m$

$3l^2 + m^2 + cl^2 + clm = 0$

$(3 + c)l^2 + clm + m^2 = 0$

$(3+c)\left(\frac{l}{m}\right)^2+c\left(\frac{l}{m}\right)+1=0$ ...(1)

$\because$ lies are parallel.

Roots of (1) must be equal

$\Rightarrow D=0$

$c^2 - 4(3 + c) = 0$

$c^2 - 4c - 12 = 0$

$(c - 6)(c + 2) = 0$

$c = 6$ or $c = -2$

+ve value of $c = 6$

14. Ans. (1)

Equation of plane passing through the intersection of planes

$5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$ is

$5x+8y+3z-29+\lambda(8x-7y+z-20)=0$ ...(i)

if it is passing through (2,1,3) then $\lambda=\frac{7}{2}$

and if it is passing through (0,1,2) then $\lambda=\frac{1}{5}$

$$5x+8y+3z-29+\frac{7}{2}(8x-7y+z-20)=0$$

$\Rightarrow 2x-y+z=6$

Similarly, P_2 : Equation of plane through intersection of

$5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$ and the point (0,1,2) is

$\Rightarrow x+y+2z=5$

Angle between planes $=\theta=\cos^{-1}\left(\frac{3}{\sqrt{6}\sqrt{6}}\right)=\frac{\pi}{3}$

15. Ans. (2)

$$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a & b & 0\\ a & b & c\end{vmatrix}$$

$= bc\hat{i}-ac\hat{j}$

Direction ratios of line are $b, -a, 0$

Direction ratios of normal of the plane are 0, 1, – 1

$$\cos 60^\circ=\left|\frac{-a}{\left|\sqrt{2}\right|\sqrt{b^2+a^2}}\right|=\frac{1}{2}$$

$$\Rightarrow \left|\frac{a}{\sqrt{a^2+b^2}}\right|=\frac{1}{\sqrt{2}}$$

$\Rightarrow b = \pm a$

So, D.R.'s can be $(\pm a, -a, 0)$

$\therefore$ D.C.'s can be $\pm\left(\frac{\pm 1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$

16. Ans. (4)

Given, $L_1:\frac{x-1}{\lambda}=\frac{y-2}{1}=\frac{z-3}{2}$

and $L_2:\frac{x+26}{-2}=\frac{y+18}{3}=\frac{z+28}{\lambda}$

are coplanar

$$\Rightarrow\begin{vmatrix}27 & 20 & 31\\ \lambda & 1 & 2\\ -2 & 3 & \lambda\end{vmatrix}=0$$

$\Rightarrow \lambda=3$

Now, normal of plane P, which contains L_1 and L_2

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & 2\\ -2 & 3 & 3\end{vmatrix}$$

$=-3\hat{i}-13\hat{j}+11\hat{k}$

$\Rightarrow$ Equation of required plane P :

$3(x-1)+13(y-2)-11(z-3)=0$

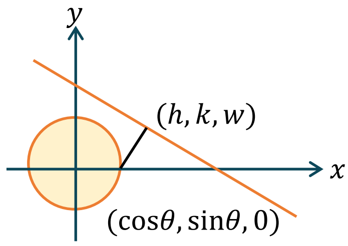
$3x+13y-11z+4=0$

$(0, 4, 5)$ does not lie on plane P.

17. **Ans. (2)**

$$\frac{h-\cos\theta}{2}=\frac{k-\sin\theta}{3}=\frac{w-0}{1}$$

$$=\frac{-1(2\cos\theta+3\sin\theta-6)}{14}$$



$$h=\cos\theta+\frac{-2(2\cos\theta+3\sin\theta-6)}{14}$$

$$=\frac{10\cos\theta-6\sin\theta+12}{14}$$

$$k=\sin\theta-\frac{3}{14}(2\cos\theta+3\sin\theta-6)$$

$$k=\frac{5\sin\theta-6\cos\theta+18}{14}$$

Elementary $\sin\theta$ and $\cos\theta$

$(5h+6k-12)^2+4(3h+5k-9)^2=1$

18. **Ans. (4)**

$A(2, 3, 9)$; $B(5, 2, 1)$; $C(1, \lambda, 8)$; $D(\lambda, 2, 3)$

$$\left[\overrightarrow{AB}\ \ \overrightarrow{AC}\ \ \overrightarrow{AD}\right]=0$$

$$\begin{vmatrix} 3 & -1 & -8 \\ -1 & \lambda-3 & -1 \\ \lambda-2 & -1 & -6 \end{vmatrix}=0$$

$\Rightarrow 3[-6(\lambda-3)-1]+(6+(\lambda-2)-8(1-(\lambda-3)(\lambda-2))=0$

$3(-6\lambda+17)+(\lambda+4)-8(-\lambda^2+5\lambda-5)=0$

$8\lambda^2-57\lambda+95=0$

$$\lambda_1\lambda_2=\frac{95}{8}$$

19. **Ans. (28)**

$P_1:\vec{r}.(2\hat{i}+\hat{j}-3k)=4$

$P_1: 2x+y-3z=4$

$$P_2\ \begin{vmatrix} x-2 & y+3 & z-2 \\ 0 & 1 & -5 \\ -1 & -1 & 0 \end{vmatrix}=0$$

$\Rightarrow -5x+5y+z+23=0$

Let a, b, c be the d’rs of line of intersection

Then $a=\dfrac{16\lambda}{15}$; $b=\dfrac{13\lambda}{15}$; $c=\dfrac{15\lambda}{15}$

$\therefore \alpha=13; \beta=15$

20. Ans. (2)

$$l_1: \frac{x-2}{3}=\frac{y+1}{-2}=\frac{z-2}{0}$$

$$l_2: \frac{x-1}{1}=\frac{y+3/2}{\alpha/2}=\frac{z+5}{2}$$

$$l_3: \frac{x-1}{-3}=\frac{y-1/2}{-2}=\frac{z-0}{4}$$

$l_1 \perp l_2 \Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow\ 3\cdot 1-2\times\frac{\alpha}{2}+0\times 2=0$

$\Rightarrow \alpha = 3$

angle between l_2 & l_3

$$\cos\theta=\frac{|1\times(-3)+(-2)(\alpha/2)+2\times 4|}{\sqrt{1+4+\frac{\alpha^2}{4}}\sqrt{9+16+4}}$$

$$\cos\theta=\frac{|-3-\alpha+8|}{\sqrt{5+\frac{\alpha^2}{4}}\sqrt{29}}$$

put $\alpha = 3$

$$\cos\theta=\frac{2}{\sqrt{\frac{29}{4}}\sqrt{29}}=\frac{4}{29}$$

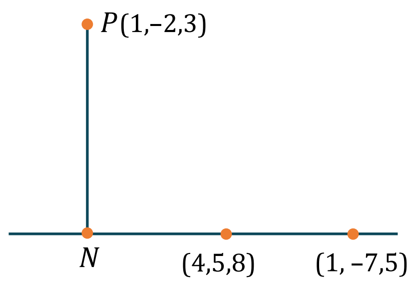
$$\theta=\cos^{-1}\left(\frac{4}{29}\right) \Rightarrow \theta=\sec^{-1}\left(\frac{29}{4}\right)$$

21. Ans. (3)

Equation of line

$$\frac{x-4}{4-1}=\frac{y-5}{5-(-7)}=\frac{z-8}{8-5}$$

$$\frac{x-4}{3}=\frac{y-5}{12}=\frac{z-8}{3}$$



Let point $N(3\lambda + 4, 12\lambda + 5, 3\lambda + 8)$

$\overrightarrow{PN}=(3\lambda+4-1)\hat{i}+(12\lambda+5-(-2))\hat{j}\ +\ (3\lambda+8-3)\hat{k}$

$\overrightarrow{PN}=(3\lambda+3)\hat{i}+(12\lambda+7)\hat{j}+(3\lambda+5)\hat{k}$

And parallel vector to line (say $\vec{a}=3\hat{i}+12\hat{j}+3\hat{k}$)

Now, $\overrightarrow{PN}\cdot\vec{a}=0$

$(3\lambda + 3)3 + (12\lambda + 7)12 + (3\lambda + 5)3 = 0$

$162\lambda + 108 = 0 \Rightarrow \lambda = \frac{-108}{162}=\frac{-2}{3}$

So, point N is $(2, -3, 6)$

Now distance is = $\left|\frac{2(2)-2(-3)+6+5}{\sqrt{4+4+1}}\right|$ = 7

22. Ans. (4)

Direction ratio of line $=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 1\\ 0 & 3 & -1\end{vmatrix}$

$=\hat{i}(-5)-\hat{j}(-1)+\hat{k}(3)$

$=-5\hat{i}+\hat{j}+3\hat{k}$

$M(-5\lambda+3,\ \lambda+2,\ 3\lambda+1)$

$\overrightarrow{PM}\perp(-5\hat{i}+\hat{j}+3\hat{k})$

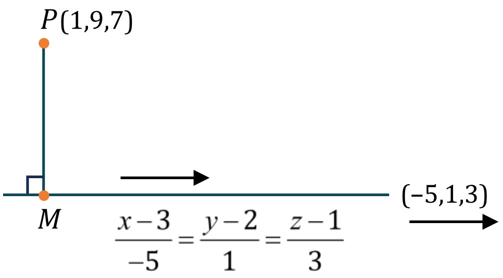
$-5(-5\lambda+2)+(\lambda-7)+3(3\lambda-6)=0$

$\Rightarrow 25\lambda+\lambda+9\lambda-10-7-18=0$

$\Rightarrow \lambda=1$

Point M = (–2, 3, 4) = (α, β, γ)

$\alpha+\beta+\gamma=5$



23. Ans. (4)

Equation of OP is $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$

$a_1=(0,0,0)$ $\quad a_2=(3,0,5)$

$b_1=(3,4,5)$ $\quad b_2=(0,0,1)$

Equation of edge parallel to z axis

$\frac{x-3}{0}=\frac{y-0}{0}=\frac{z-5}{1}$

$S.D=\frac{(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}$

$$\frac{\begin{vmatrix}3 & 0 & 5\\ 3 & 4 & 5\\ 0 & 0 & 1\end{vmatrix}}{\left\|\begin{matrix}\hat{i} & j & k\\ 3 & 4 & 5\\ 0 & 0 & 1\end{matrix}\right\|}=\frac{3(4)}{|4\hat{i}-3j|}=\frac{12}{5}$$

24. Ans. (4)

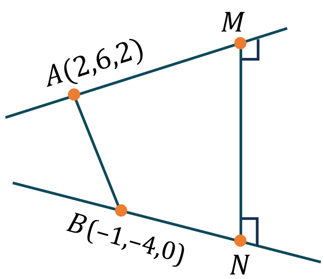
Line l, is given by

$L_1:\frac{x-2}{2}=\frac{y-6}{1}=\frac{z-2}{-2}$

Given,

$L_2:\frac{x+1}{2}=\frac{y+4}{-3}=\frac{z}{2}$

Shortest distance = $\left|\frac{\overrightarrow{AB}\cdot\overrightarrow{MN}}{MN}\right|$



$$\overrightarrow{AB}=3\hat{i}+10\hat{j}+2\hat{k}$$

$$\overrightarrow{MN}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -2\\ 2 & -3 & 2\end{vmatrix}=-4\hat{i}-8\hat{j}-8\hat{k}$$

$$MN=\sqrt{16+64+64}=12$$

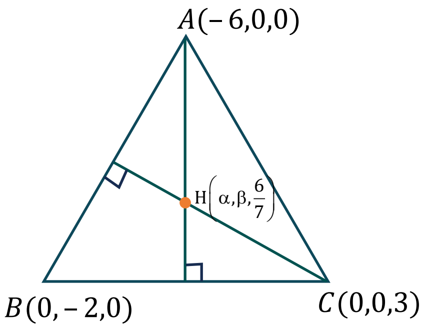
$$\therefore \text{ Shortest distance } =\left|\frac{-12-80-16}{12}\right|=9$$

$\therefore$ Option (4) is correct.

25. Ans. (288)

A (– 6, 0, 0)   B (0, – 2, 0)   C = (0, 0, 3)

$\overrightarrow{AB} = 6-2\hat{j}$, $\overrightarrow{BC} = 2\hat{j}+3\hat{k}$, $\overrightarrow{AC} = 6\hat{i}+3\hat{k}$



$$\overrightarrow{AH}\cdot\overrightarrow{BC}=0$$

$$\left(\alpha+6,\beta,\frac{6}{7}\right)\cdot(0,2,3)=0$$

$$\boxed{\beta=\frac{-9}{7}}$$

$$\overrightarrow{CH}\cdot\overrightarrow{AB}=0$$

$$\left(\alpha,\beta,\frac{-15}{7}\right)\cdot(6,-2,0)=0$$

$$6\alpha-2\beta=0$$

$$\alpha=\frac{-3}{7}$$

$$98(\alpha+\beta)^2=(98)\frac{(144)}{49}=288$$

26. Ans. (315)

$$P_1=\vec{r}\cdot\left(3\hat{i}-5j+k\right)=7$$

$$P_2=\vec{r}\cdot\left(\lambda\hat{i}+j-3k\right)=9$$

$$\theta=\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$$

$$\Rightarrow \sin\theta=\frac{2\sqrt{6}}{5}$$

$$\therefore \cos\theta=\frac{1}{5}\cdot$$

$$\cos\theta=\frac{\vec{r}\cdot\vec{r}}{|\vec{r}_1||\vec{r}_2|}$$

$$=\frac{(3i-5j+K)(\lambda i+j-3K)}{\sqrt{35}\cdot\sqrt{\lambda^2+10}}$$

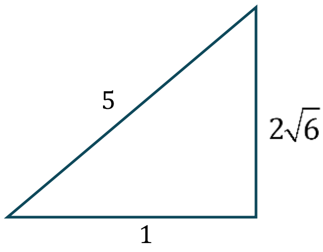
$$\frac{1}{5}=\left|\frac{3\lambda-8}{\sqrt{35}\cdot\sqrt{\lambda^2+10}}\right|$$

Square $\Rightarrow \frac{1}{25}=\frac{9\lambda^2+64-48\lambda}{35\left(\lambda^2+10\right)}$

$\Rightarrow 19\lambda^2-120\lambda+125=0$

$\Rightarrow 19\lambda^2-95\lambda-25\lambda+125=0$

$\Rightarrow x=5,\frac{25}{19}$



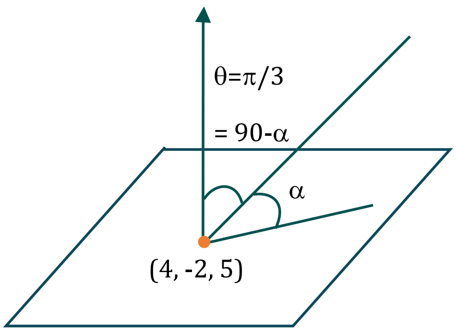
Perpendicular distance of point

$\left(38\lambda_1,10\lambda_2,2\right)\equiv\left(50,50,2\right)$ from plane P_1

$=\frac{|3\times50-5\times50+2-7|}{\sqrt{35}}=\frac{105}{\sqrt{35}}$

Square $=\frac{105\times105}{35}=315$

27. Ans. (9)



$\cos\theta=\frac{\left(\hat{i}+\hat{j}+2\hat{k}\right).\left(2\hat{i}-\hat{j}+\hat{k}\right)}{6}=\frac{2-1+2}{6}=\frac{1}{2}$

$\theta=\pi/3$ $\qquad\qquad$ $\alpha=\pi/6$

$\left(\tan^2\theta\right)\left(\cot^2\alpha\right)$

(3) (3) = 9

28. Ans. (158)

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 3 & -2\\ 1 & -1 & 2\end{vmatrix}=4\hat{i}-4\hat{j}-4\hat{k}$$

$\therefore$ Equation of line is $\frac{x-2}{1}=\frac{y-3}{-1}=\frac{z-1}{-1}$

Let Q be (5, 3, 8) and foot of $\perp$ from Q on this line be R.

Now, $R\equiv\left(k+2,-k+3,-k+1\right)$

DR of QR are $(k-3,-k,-k-7)$

$\therefore(1)(k-3)+(-1)(-k)+(-1)(-k-7)=0$

$\Rightarrow k=-\frac{4}{3}$

$\therefore \alpha^2=\left(\frac{13}{3}\right)^2+\left(\frac{4}{3}\right)^2+\left(\frac{17}{3}\right)^2=\frac{474}{9}$

$\therefore 3\alpha^2=158$

29. **Ans. (1)**

P: $8x+\alpha_1 y+\alpha_2 z+12=0$

L: $\frac{x+2}{2}=\frac{y-3}{3}=\frac{z+4}{5}$

$\because$ P is parallel to L

$\Rightarrow 8(2)+\alpha_1(3)+5(\alpha_2)=0$

$\Rightarrow 3\alpha_1+5(\alpha_2)=-16$

Also, y-intercept of plane P is 1

$\Rightarrow \alpha_1=-12$

And $\alpha_2=4$

$\Rightarrow$ Equation of plane P is $2x-3y+z+3=0$

$\Rightarrow$ Distance of line L from Plane P is

$=\left|\frac{0-3(6)+1+3}{\sqrt{4+9+1}}\right|$

$=\sqrt{14}$

30. **Ans. (1)**

$P \equiv P_1+\lambda P_2=0$

$(2+\lambda)x+(3+2\lambda)y+(3\lambda-1)z-2-6\lambda=0$

Plane P is perpendicular to P_3 $\therefore$ $\vec{n}.\vec{n}_3=0$

$2(\lambda+2)+(2\lambda+3)-(3\lambda-1)=0$

$\lambda=-8$

$P \equiv -6x-13y-25z+46=0$

$6x+13y+25z-46=0$

Dist. from $(-7, 1, 1)$

$d=\left|\frac{-42+13+25-46}{\sqrt{36+169+625}}\right|=\frac{50}{\sqrt{830}}$

$d^2=\frac{50\times 50}{830}=\frac{250}{83}$

EXERCISE - JEE (Advanced) PYQ

1. **Ans. (D)**

$$\frac{\alpha-2t+2}{1}=\frac{\beta+t+1}{1}=\frac{\gamma-3t}{1}=k$$

$\alpha = k + 2t - 2$

$\beta = k - t - 1$

$\gamma = k + 3t$

$\alpha + \beta + \gamma = 3$

$$k=\frac{6-4t}{3}$$

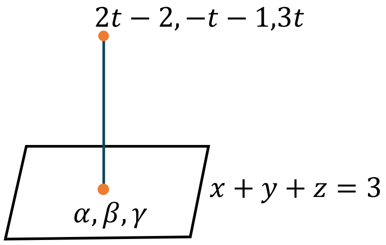
$$\alpha=\frac{6-4t}{3}+2t-2=\frac{2t}{3}$$

$$\beta=\frac{6-4t}{3}-t-1=\frac{3-7t}{3}$$

$$\gamma=\frac{6-4t}{3}+3t=\frac{5t+6}{3}$$

$$\Rightarrow \frac{3\alpha}{2}=\frac{3\beta-3}{-7}=\frac{3\gamma-6}{5}$$

$$\Rightarrow \frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$$



2. **Ans. (B,D)**

$\ell_1: \vec{r} = (3, -1, 4) + (1, 2, 2)t$

$\ell_2: \vec{r} = (3, 3, 2) + (2, 2, 1)s$

vector perpendicular to ℓ_1 and ℓ_2:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -2\hat{i}+3\hat{j}-2\hat{k}$$

$\therefore$ Equation of line ℓ:

$\vec{r}=0+(-2,3,-2)\lambda$

Point of intersection of ℓ_1 and ℓ:

$3 + t = -2\lambda$

$-1 + 2t = 3\lambda.$

$4 + 2t = -2\lambda.$

On solving we get $\lambda = -1$, $t = -1$

$\therefore$ Point of intersection of ℓ_1 & ℓ : $P(2, -3, 2)$

A point on ℓ_2 at distance of $\sqrt{17}$ from P :

$\Rightarrow (1 + 2s)^2 + (6 + 2s)^2 + s^2 = 17$

$\Rightarrow s = -\frac{10}{9}$; $s = -2$

for above s, point will be (B), (D)

3. Ans. (A,D)

$$L_1 : \frac{x-5}{0} = \frac{y}{3-\alpha} = \frac{z}{-2}$$

$$L_2 : \frac{x-\alpha}{0} = \frac{y}{-1} = \frac{z}{2-\alpha}$$

for lines to be coplanar

$$\begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$$

$\Rightarrow (5-\alpha)\,((3-\alpha)\,(2-\alpha) - 2) = 0 \Rightarrow (5-\alpha)(\alpha^2 - 5\alpha + 4) = 0$

$\Rightarrow \alpha = 1, 4, 5$

4. Ans. (A)

For point of intersection of L_1 and L_2

$$\begin{cases} 2\lambda + 1 = \mu + 4 \\ -\lambda = \mu - 3 \\ \lambda - 3 = 2\mu - 3 \end{cases} \Rightarrow \mu = 1 \Rightarrow \text{point of intersection is } (5, -2, -1)$$

Now, vector normal to the plane is $\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix}$

$= -16(\hat{i} - 3\hat{j} - 2\hat{k})$

Let equation of required plane be $x - 3y - 2z = \alpha$

$\because$ it passes through $(5, -2, -1)$

$\therefore \alpha = 13 \Rightarrow$ equation of plane is $x - 3y - 2z = 13$

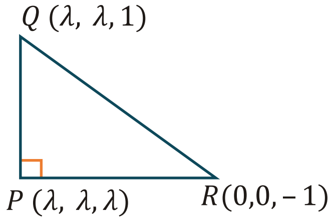
5. Ans. (C)

Line L_1 given by $y = x$; $z = 1$ can be expressed as

$$L_1 : \frac{x}{1} = \frac{y}{1} = \frac{z-1}{0}$$

Similarly, L_2 ($y = -x$; $z = -1$) can be expressed as

$$L_2 : \frac{x}{1} = \frac{y}{-1} = \frac{z+1}{0}$$



Let any point $Q(\alpha,\alpha,1)$ on L_1 and $R(\beta,-\beta,-1)$ on L_2

Given that PQ is perpendicular to L_1

$\Rightarrow (\lambda - \alpha).1 + (\lambda - \alpha).1 + (\lambda - 1).0 = 0 \Rightarrow \lambda = \alpha$

$\therefore Q(\lambda,\lambda,1)$

Similarly, PR is perpendicular to L_2

$(\lambda - \beta).1 + (\lambda + \beta)(-1) + (\lambda + 1).0 = 0 \Rightarrow \beta = 0$

$\therefore R(0,0,-1)$

Now as given

$\Rightarrow \overrightarrow{PR}.\overrightarrow{PQ} = 0$

$0.\lambda + 0.\lambda + (\lambda - 1)\,(\lambda + 1) = 0$

$\lambda \neq 1$ as P & Q are different points $\Rightarrow \lambda = -1$

6. Ans. (B,D)

Let $P_3 : (x + z - 1) + \lambda y = 0$

$x + \lambda y + z - 1 = 0$ ...(i)

distance of (0,1,0) from P_3 is 1

$\Rightarrow \dfrac{|\lambda-1|}{\sqrt{2+\lambda^2}} = 1 \Rightarrow (\lambda - 1)^2 = 2 + \lambda^2 \Rightarrow \lambda = -\dfrac{1}{2}$

$\therefore$ P_3 is $2x - y + 2z - 2 = 0$

distance from (α,β,γ) is $\left|\dfrac{2\alpha-\beta+2\gamma-2}{\sqrt{9}}\right| = 2$

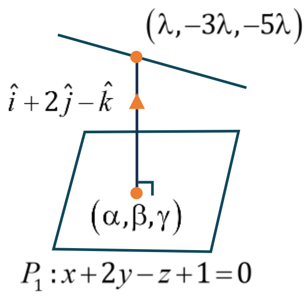
$\therefore$ $2\alpha - \beta + 2\gamma - 2 = 6$ or $2\alpha - \beta + 2\gamma - 2 = -6$

$2\alpha - \beta + 2\gamma - 8 = 0$ or $2\alpha - \beta + 2\gamma + 4 = 0$

7. Ans. (A,B)

Straight line 'L' is parallel to line of intersection of plane P_1 & plane P_2.

$\therefore$ Equation of line 'L' is



$\dfrac{x}{1} = \dfrac{y}{-3} = \dfrac{z}{-5} = \lambda$

$\dfrac{\alpha-\lambda}{1} = \dfrac{\beta+3\lambda}{2} = \dfrac{\gamma+5\lambda}{-1} = k$

$$\left.\begin{aligned} \alpha &= k+\lambda \\ \beta &= 2k-3\lambda \\ y &= -k-5\lambda \end{aligned}\right\} \quad ...(1)$$

satisfying in plane P_1

$k + \lambda + 4k - 6\lambda + k + 5\lambda + 1 = 0$

$6k = -1$

putting in (1) required locus is

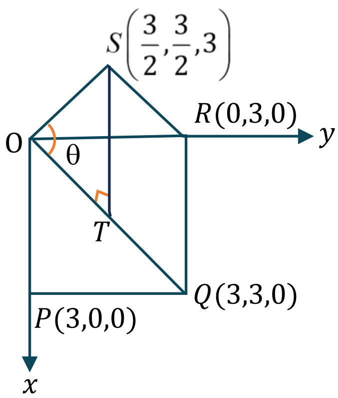
$x = -\dfrac{1}{6} + \lambda$

$y = -\dfrac{1}{3} - 3\lambda$

$z = \dfrac{1}{6} - 5\lambda$

Now check the options.

8. Ans. (B,C,D)



Given $OP = OR = 3$ and $OPQR$ is a square

$\Rightarrow OQ = 3\sqrt{2} \Rightarrow OT = \dfrac{3}{\sqrt{2}}$ and $ST = 3$

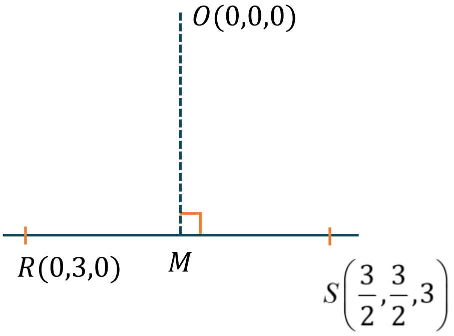
using ΔSOT, $\tan\theta = \dfrac{ST}{OT} = \sqrt{2} \Rightarrow \theta = \tan^{-1}\sqrt{2}$

clearly, equation of plane containing triangle OQS is $Y - X = 0$

Also, length of perpendicular from P to the plane containing the triangle OQS is $PT = \frac{3}{\sqrt{2}}$

Also equation of RS is $\bar{r} = 3\hat{j} + t\left(\frac{3}{2}\hat{i} - \frac{3}{2}\hat{j} + 3\hat{k}\right)$

$= \left(\frac{3t}{2}, 3 - \frac{3t}{2}, 3t\right)$



Let co-ordinates of $M = \left(\frac{3t}{2}, 3 - \frac{3t}{2}, 3t\right)$

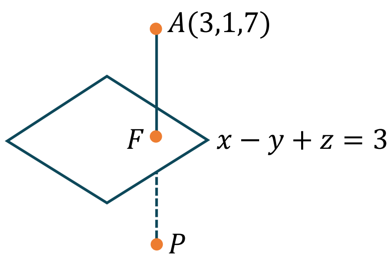
$\because\ \overrightarrow{OM}.\overrightarrow{RS} = 0$

$\Rightarrow \frac{9}{4}t - \frac{3}{2}\left(3 - \frac{3t}{2}\right) + 9t = 0 \Rightarrow \frac{9t}{2} + 9t = \frac{9}{2} \Rightarrow t = \frac{1}{3}$

$\therefore\ M = \left(\frac{1}{2}, \frac{5}{2}, 1\right) \Rightarrow OM = \sqrt{\frac{1}{4} + \frac{25}{4} + 1} = \sqrt{\frac{30}{4}} = \sqrt{\frac{15}{2}}$

9. Ans. (C)

Line AP : $\frac{x-3}{1} = \frac{y-1}{-1} = \frac{z-7}{1} = \lambda$



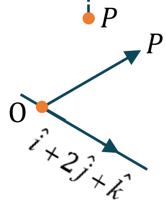
$\Rightarrow F(3 + \lambda, 1 - \lambda, \lambda + 7)$ lies in the plane

$\therefore\ 3 + \lambda - (1 - \lambda) + \lambda + 7 = 3$

$3\lambda = -6 \Rightarrow \lambda = -2$

$\Rightarrow F(1,3,5) \Rightarrow P(-1,5,3)$

so required plane is $\begin{vmatrix} x-0 & y-0 & z-0 \\ 1 & 2 & 1 \\ -1 & 5 & 3 \end{vmatrix} = 0$



$\therefore\ x - 4y + 7z = 0$

10. Ans. (A)

The normal vector of required plane is parallel to vector

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix} = -14\hat{i} - 2\hat{j} - 15\hat{k}$

$\therefore$ The equation of required plane passing through (1, 1, 1) will be

$-14(x - 1) - 2(y - 1) - 15(z - 1) = 0$

$\Rightarrow \boxed{14x + 2y + 15z = 31}$

$\therefore$ Option (A) is correct

11. Ans. (C,D)

$D.C.$ of line of intersection (a, b, c)

$\Rightarrow 2a + b - c = 0$

$a + 2b + c = 0$

$\frac{a}{1+2} = \frac{b}{-1-2} = \frac{c}{4-1}$

$\therefore\ D.C.$ is (1, –1, 1)

(B) $\dfrac{3x-4}{9}=\dfrac{1-3y}{9}=\dfrac{z}{3}$

$\Rightarrow \dfrac{x-4/3}{3}=\dfrac{y-1/3}{-3}=\dfrac{z}{3}$

$\Rightarrow$ lines are parallel.

(C) Acute angle between P_1 and P_2 = $\cos^{-1}\left(\dfrac{2\times1+1\times2-1\times1}{\sqrt{6}\sqrt{6}}\right)$

$=\cos^{-1}\left(\dfrac{3}{6}\right)=\cos^{-1}\left(\dfrac{1}{2}\right)=60°$

(D) Plane is given by $(x-4)-(y-2)+(z+2)=0$

$\Rightarrow x-y+z=0$

Distance of (2, 1, 1) from plane = $\dfrac{2-1+1}{\sqrt{3}}=\dfrac{2}{\sqrt{3}}$

12. Ans. (8)

Let $P(\alpha,\beta,\gamma)$

$Q(0,0,\gamma)$ &

$R(\alpha,\beta,-\gamma)$

Now, $\overline{PQ}\,||\,\hat{i}+\hat{j} \Rightarrow (\alpha\hat{i}+\beta\hat{j})\,||\,(\hat{i}+\hat{j}) \Rightarrow \alpha=\beta$

Also, mid point of PQ lies on the plane $\Rightarrow \dfrac{\alpha}{2}+\dfrac{\beta}{2}=3 \Rightarrow \alpha+\beta=6 \Rightarrow \alpha=3$

Now, distance of point P from X-axis is $\sqrt{\beta^2+\gamma^2}=5$

$\Rightarrow \beta^2+\gamma^2=25 \Rightarrow \gamma^2=16$

as $\beta=\alpha=3$

as $\gamma=4$

Hence, $PR=2\gamma=8$

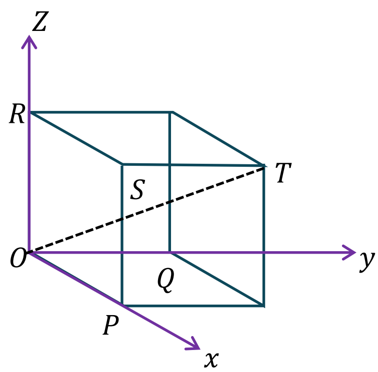
13. Ans. (0.5)

$\vec{p}=\overrightarrow{SP}=\left(\dfrac{1}{2},-\dfrac{1}{2},-\dfrac{1}{2}\right)=\dfrac{1}{2}(\hat{i}-\hat{j}-\hat{k})$

$\vec{q}=\overrightarrow{SQ}=\left(-\dfrac{1}{2},\dfrac{1}{2},-\dfrac{1}{2}\right)=\dfrac{1}{2}(-\hat{i}+\hat{j}-\hat{k})$

$\vec{r}=\overrightarrow{SR}=\left(-\dfrac{1}{2},-\dfrac{1}{2},\dfrac{1}{2}\right)=\dfrac{1}{2}(-\hat{i}-\hat{j}+\hat{k})$

$\vec{t}=\overrightarrow{ST}=\left(\dfrac{1}{2},\dfrac{1}{2},\dfrac{1}{2}\right)=\dfrac{1}{2}(\hat{i}+\hat{j}+\hat{k})$



$|(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t})|=\dfrac{1}{4}\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-1&-1\\-1&1&-1\end{vmatrix}\times\dfrac{1}{4}\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-1&-1&1\\1&1&1\end{vmatrix}$

$=\dfrac{1}{16}\left|(2\hat{i}+2\hat{j})\times(-2\hat{i}+2\hat{j})\right|$ = $\left|\dfrac{\hat{k}}{2}\right|=\dfrac{1}{2}$

14. Ans. (A,B,D)

Points on L_1 and L_2 are respectively $A(1-\lambda, 2\lambda, 2\lambda)$ and $B(2\mu, -\mu, 2\mu)$

So, $\overrightarrow{AB} = (2\mu+\lambda-1)\hat{i} + (-\mu-2\lambda)\hat{j} + (2\mu-2\lambda)\hat{k}$

and vector along their shortest distance = $2\hat{i}+2\hat{j}-\hat{k}$.

Hence, $\dfrac{2\mu+\lambda-1}{2} = \dfrac{-\mu-2\lambda}{2} = \dfrac{2\mu-2\lambda}{-1} \Rightarrow \lambda = \dfrac{1}{9}$ & $\mu = \dfrac{2}{9}$

Hence, $A \equiv \left(\dfrac{8}{9}, \dfrac{2}{9}, \dfrac{2}{9}\right)$ and $B \equiv \left(\dfrac{4}{9}, -\dfrac{2}{9}, \dfrac{4}{9}\right)$

$\Rightarrow$ Mid point of $AB \equiv \left(\dfrac{2}{3}, 0, \dfrac{1}{3}\right)$

15. Ans. (0.75)

$A(1, 0, 0)$, $B\left(\dfrac{1}{2}, \dfrac{1}{2}, 0\right)$ & $C\left(\dfrac{1}{3}, \dfrac{1}{3}, \dfrac{1}{3}\right)$

Hence, $\overrightarrow{AB} = -\dfrac{1}{2}\hat{i} + \dfrac{1}{2}\hat{j}$ & $\overrightarrow{AC} = -\dfrac{2}{3}\hat{i} + \dfrac{1}{3}\hat{j} + \dfrac{1}{3}\hat{k}$

So, $\Delta = \dfrac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \dfrac{1}{2}\sqrt{\dfrac{1}{2} \times \dfrac{2}{3} - \dfrac{1}{4}} = \dfrac{1}{2 \times 2\sqrt{3}}$

$\Rightarrow (6\Delta)^2 = \dfrac{3}{4} = 0.75$

16. Ans. (A,B)

Point of intersection of L_1 & L_2 is $(1, 0, 1)$

Line L passes through $(1, 0, 1)$

$\dfrac{1-\alpha}{\ell} = -\dfrac{1}{m} = \dfrac{1-\gamma}{-2}$ ...(1)

acute angle bisector of L_1 & L_2

$\vec{r} = \hat{i} + \hat{k} + \lambda\left(\dfrac{\hat{i} - \hat{j} + 3\hat{k} - 3\hat{i} - \hat{j} + \hat{k}}{\sqrt{11}}\right)$

$\vec{r} = \hat{i} + \hat{k} + t\left(\hat{i} + \hat{j} - 2\hat{k}\right) \Rightarrow \dfrac{\ell}{1} = \dfrac{m}{1} = \dfrac{-2}{-2} \Rightarrow \ell = m = 1$

From (1) $\dfrac{1-\alpha}{1} = -1 \Rightarrow \alpha = 2$

& $\dfrac{1-\gamma}{-2} = -1 \Rightarrow \gamma = -1$

17. Ans. (A,B,C)

R is mid point of PQ

$\therefore$ $R(2,1,-1)$ and it lies on plane

equation of plane is $\alpha a + \beta y + \gamma z = \delta$

$\therefore 2\alpha + \beta - \gamma = \delta$ ...(1)

Normal vector to plane is

$\vec{n} = 2i + 2j$

$\therefore \dfrac{\alpha}{2} = \dfrac{\beta}{2} = \dfrac{\gamma}{0} = k$

$\therefore \alpha = 2k, \beta = 2k, \gamma = 0$ ...(2)

and $\alpha + \gamma = 1$ (given) ...(3)

from (2) and (3)

$\therefore \alpha = 1, \beta = 1, \gamma = 0$

and from (1)

$2(1) + 1 - 0 = \delta$

$\delta = 3$

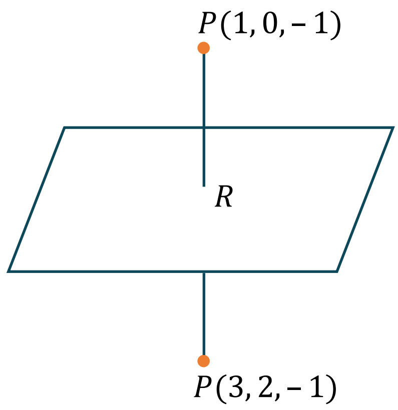
Now :

$\alpha + \beta = 2$

$\delta - \gamma = 3$

$\delta + \beta = 4$

so, A,B,C are correct.



18. Ans. (1.00)

19. Ans. (1.50)

Sol. 18 & 19

$7x + 8y + 9z - (\gamma - 1) = A(4x + 5y + 6z - \beta) + B(x + 2y + 3z - \alpha)$

$x : 7 = 4A + B$

$y : 8 = 5A + 2B$

$A = 2, B = -1$

const. term : $-(\gamma - 1) = -A\beta - \alpha B \Rightarrow -(\gamma - 1) \equiv 2\beta + \alpha$

$\alpha - 2\beta + \gamma = 1$

$$M = \begin{pmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \Rightarrow |M| = \alpha - 2\beta + \gamma = 1$$

Plane $P : x - 2y + z = 1$

Perpendicular distance $= \left|\frac{3}{\sqrt{6}}\right| = P \Rightarrow D = P^2 = \frac{9}{6} = 1.5$

20. Ans. (A,B,D)

line of intersection is $\frac{x}{0} = \frac{y-4}{-4} = \frac{z}{5}$

(1) Any skew line with the line of intersection of given planes can be edge of tetrahedron.

(2) any intersecting line with line of intersection of given planes must lie either in plane P_1 or P_2 can be edge of tetrahedron.

21. Ans. (A)

DR'S of OG = 1, 1, 1

DR'S of AF = –1, 1, 1

DR'S of CE = 1, 1, –1

DR'S of BD = 1, –1, 1

Equation of $OG \Rightarrow \frac{x}{1} = \frac{y}{1} = \frac{z}{1}$

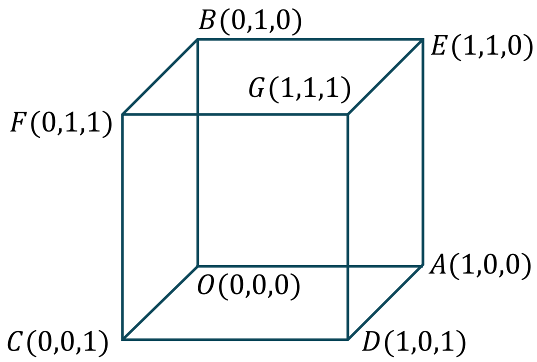
Equation of $AB \Rightarrow \frac{x-1}{1}=\frac{y}{-1}=\frac{z}{0}$

Normal to both the line's

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & -1 & 0\end{vmatrix}=\hat{i}+\hat{j}-2\hat{k}$$

$\overrightarrow{OA}=\hat{i}$

S.D. $=\frac{\left|\hat{i}.(\hat{i}+\hat{j}-2\hat{k})\right|}{\left|\hat{i}+\hat{j}-2\hat{k}\right|}=\frac{1}{\sqrt{6}}$



22. Ans. (45)

Given $|\vec{u}-\vec{v}|=|\vec{v}-\vec{w}|=|\vec{w}-\vec{u}|$

$\Rightarrow \Delta UVW$ is an equilateral Δ

Now distances of U, V, W from $P=\frac{7}{2}$

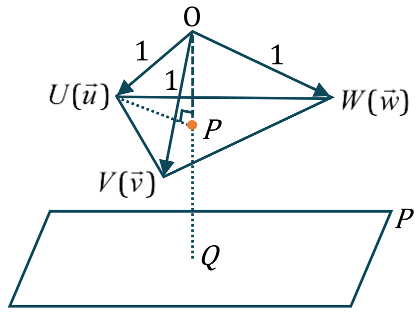
$\Rightarrow PQ=\frac{7}{2}$

Also, Distance of plane P from origin

$\Rightarrow OQ = 4$

$\therefore OP = OQ - PQ \Rightarrow OP=\frac{1}{2}$

Hence, $PU=\sqrt{OU^2-OP^2} \Rightarrow PU=\frac{\sqrt{3}}{2}=R$

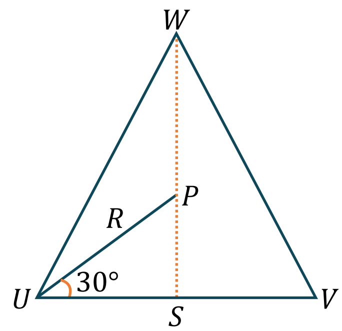


Also, for $\Delta UVW, P$ is circumcenter

$\therefore$ for ΔUVW : $US = R\cos 30°$

$\Rightarrow$ UV = 2Rcos30° $\Rightarrow UV=\frac{3}{2}$

$\therefore Ar(\Delta UVW)=\frac{\sqrt{3}}{4}\left(\frac{3}{2}\right)^2=\frac{9\sqrt{3}}{16}$



$\therefore$ Volume of tetrahedron with coterminous edges $\vec{u}, \vec{v}, \vec{w}$

$=\frac{1}{3}(Ar.\Delta UVW)\times OP=\frac{1}{3}\times\frac{9\sqrt{3}}{16}\times\frac{1}{2}=\frac{3\sqrt{3}}{32}$

$\therefore$ parallelopiped with coterminous edges

$\vec{u}, \vec{v}, \vec{w}=6\times\frac{3\sqrt{3}}{32}=\frac{9\sqrt{3}}{16}=V$

$\therefore \frac{80}{\sqrt{3}}V=45$

23. Ans. (B)

$L_1 : \vec{r}_1 = \lambda(\hat{i} + \hat{j} + \hat{k})$

$L_2 : \vec{r}_2 = \hat{j} - \hat{k} + \mu(\hat{i} + \hat{k})$

Let system of planes are

$ax + by + cz = 0$ ...(1)

$\because$ It contain L_1

$\therefore a + b + c = 0$ ...(2)

For largest possible distance between plane (1) and L_2 the line L_2 must be parallel to plane (1)

$\therefore a + c = 0$ ...(3)

$\Rightarrow \boxed{b=0}$

$\therefore$ Plane H_0 : $\boxed{x - z = 0}$

Now $d(H_0)$ = $\perp$ distance from point (0, 1, –1) on L_2 to plane.

$\Rightarrow d(H_0) = \left|\frac{0+1}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$

$\therefore P \to 5$

for 'Q' distance = $\left|\frac{2}{\sqrt{2}}\right| = \sqrt{2}$

$\therefore Q \to 4$

$\therefore$ (0, 0, 0) lies on plane

$\therefore R \to 3$

for $'S'x = z\,;y = z\,;x = 1$

$\therefore$ point of intersection $p(1, 1, 1)$.

$\therefore\ OP = \sqrt{1+1+1} = \sqrt{3}$

$\therefore S \to 2$

$\therefore$ option [B] is correct.

JEE (Main) Practice Paper

SECTION–A

1. Ans. (1)

$n = -(\ell + m)$ ...(1)

$2mn + 2ml - nl = 0$ ...(2)

$-2m(\ell + m) + 2m\ell + (\ell + m)\ell = 0$

$\Rightarrow -2m\ell - 2m^2 + 2m\ell + \ell^2 + m\ell = 0 \Rightarrow \ \ell^2 + m\ell - 2m^2 = 0 \Rightarrow \frac{\ell}{m} = 1,\ \ -2$

Case-I: We have $\frac{\ell}{m} = 1 \Rightarrow \ell = m$ So, $n = -2\ell$ So $\ell : m : n = 1 : 1 : -2$.

$\Rightarrow$ direction cosines are $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$ or $-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$

Case-II: We have $\frac{\ell}{m} = -2 \Rightarrow \ell = -2m \Rightarrow \ell : m : n = -2 : 1 : 1$

Hence direction cosines are $\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$

2. Ans. (2)

$(x, y, 0)$ divides line segment joining $(3, 2, 5)$ and $(-2, 3, -5)$ in the ratio $1 : 1$ internally.

$\Rightarrow x = \frac{3-2}{2} = \frac{1}{2},\ y = \frac{2+3}{2} = \frac{5}{2},\ z = 0$

3. Ans. (4)

$\because \vec{r} = 2\hat{i} - \hat{j} + 4\hat{k} + \lambda(\hat{i} + \hat{j} - 2\hat{k})$

Let $(x\hat{i} + y\hat{j} + z\hat{k})$ be a point lying on the line

$\therefore x\hat{i} + y\hat{j} + z\hat{k} = \hat{i}(2+\lambda) + \hat{j}(\lambda-1) + \hat{k}(4-2\lambda) \Rightarrow x - 2 = \lambda; y + 1 = \lambda;$

$\therefore$ equation of line is $\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}$

4. Ans. (1)

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = r$ (Let) ...(1)

$\Rightarrow (2r + 1, 3r + 2, 4r + 3)$ represents any point on (1)

$\frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1}$ ...(2)

To find point of intersection of (1) and (2)

$\frac{2r+1-4}{5} = \frac{3r+2-1}{2} = \frac{4r+3}{1} \Rightarrow \frac{2r-3}{5} = \frac{3r+1}{2} = \frac{4r+3}{1}$

$\Rightarrow 4r - 6 = 15r + 5 \Rightarrow 11r = -11 \Rightarrow r = -1$

$\therefore$ point of intersection of (1) and (2) is $(-1, -1, -1)$

$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ ...(1)

$\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ ...(2)

For their point of intersection

$3\lambda + 1 = 4 + 2\mu \Rightarrow 3\lambda - 2\mu - 3 = 0$ ...(3)

$1 - \lambda = 0 \Rightarrow \lambda = 1$ ...(4)

and $-1 = -1 + 3\mu \Rightarrow \mu = 0$

$\therefore$ point of intersection is $(4, 0, -1)$

$\therefore$ required distance $= \sqrt{(4+1)^2 + 1 + 0} = \sqrt{25+1} = \sqrt{26}$

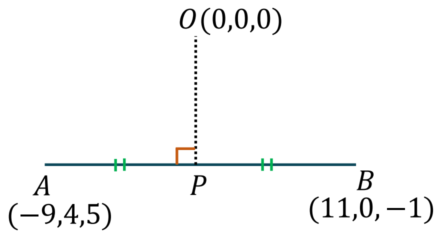
5. Ans. (1)

$\because P$ (mid – point of AB) is $(1, 2, 2)$

$\because dr's$ of OP are : $1, 2, 2$ and $dr's$ of AB are : $20, -4, -6$

$\because 1 \times 20 + 2 \times (-4) + 2 \times (-6) = 20 - 12 - 8 = 0$

$\therefore OP \perp AB$



As the midpoint of $(0, 0, 0)$ and $(2, 4, 4)$ is $(1, 2, 2)$ so the distance $= \sqrt{1 + 2^2 + 2^2} = 3$

6. Ans. (3)

Any point on the given line is $(2r+3, r+3, r)$

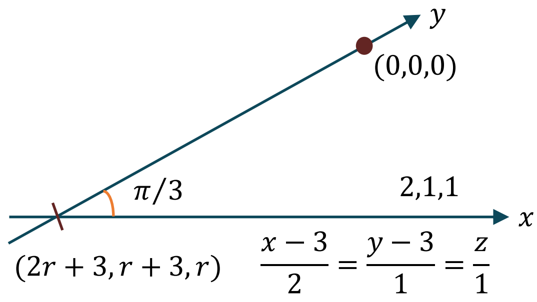
Dr's of new line are $\equiv 2r+3, r+3, r$

$$\cos\frac{\pi}{3}=\frac{2(2r+3)+1(r+3)+1.r}{\sqrt{6}\sqrt{(2r+3)^2+(r+3)^2+r^2}}$$

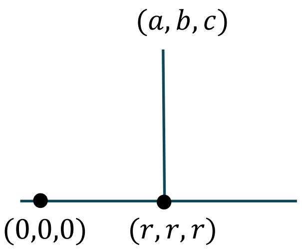
Solving above we get $r=-1,-2$

Dr's are $1,2,-1$ or $-1,1,-2$

Equation of the line is $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}$ or $\frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$



7. Ans. (2)



$x=y=z=r$

$(r-a)1+(r-b)1+(r-c)1=0$

$a+b+c=3r$

8. Ans. (3)

$$SD=\frac{(\hat{i}-\hat{j}+2\hat{k}-4\hat{i}+\hat{j})\cdot[(\hat{i}+2\hat{j}-3\hat{k})\times(2\hat{i}+4\hat{j}-5\hat{k})]}{|(\hat{i}+2\hat{j}-3\hat{k})\times(2\hat{i}+4\hat{j}-5\hat{k})|}=\left|\frac{(-3\hat{i}+2\hat{k})\ .\ (2\hat{i}-\hat{j})}{|2\hat{i}-\hat{j}|}\right|=\frac{6}{\sqrt{5}}$$

9. Ans. (1)

Let the co-ordinates of A be $(3\lambda+3, 8-\lambda, \lambda+3)$

and the co-ordinates of B be $(-3\mu-3, 2\mu-7, 4\mu+6)$.

Then direction ratios of AB are $3\lambda+3\mu+6, -\lambda-2\mu+15, \lambda-4\mu-3$ $AB\perp L_1$ so

$3(3\lambda+3\mu+6)-(-\lambda-2\mu+15)+(\lambda-4\mu-3)=0$

i.e. $11\lambda+7\mu=0$

and $AB\perp L_2$ so $-3(3\lambda+3\mu+6)+2(-\lambda-2\mu+15)+4(\lambda-4\mu-3)=0$

i.e. $-7\lambda-29\mu=0$

$\Rightarrow \lambda=\mu=0$

so, the point A is $(3,8,3)$ and the point B is $(-3,-7,6)$

$\therefore AB=\sqrt{36+225+9}=\sqrt{270}=3\sqrt{30}$

10. Ans. (4)

$$\begin{vmatrix} x & y-1 & z \\ 0 & 1 & -1 \\ -1 & -1 & -3 \end{vmatrix}=0 \Rightarrow x(-3-1)-(y-1)(0-1)+z(1)=0$$

$-4x+y-1+z=0 \Rightarrow 4x-y-z+1=0$

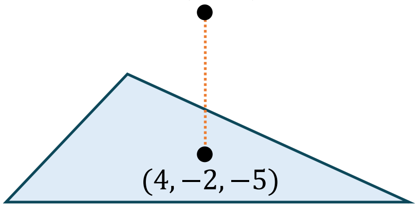
11. **Ans. (1)**

Let point is (α, β, γ)

$$\left|\frac{\alpha+\beta+\gamma}{\sqrt{3}}\right|^2+\left|\frac{\alpha-\gamma}{\sqrt{2}}\right|^2+\left|\frac{\alpha-2\beta+\gamma}{\sqrt{6}}\right|^2=9 \Rightarrow 2(\alpha+\beta+\gamma)^2+3(\alpha-\gamma)^2+(\alpha-2\beta+\gamma)^2=54$$

$6\alpha^2+6\beta^2+6\gamma^2=54 \Rightarrow$ Locus is $x^2+y^2+z^2=9$

12. **Ans. (4)**



Equation of plane is

$$\left\{\vec{r}-(4\hat{i}-2\hat{j}-5\hat{k})\right\}\cdot\left\{4\hat{i}-2\hat{j}-5\hat{k}\right\}=0$$

$$\Rightarrow \vec{r}\cdot\left\{4\hat{i}-2\hat{j}-5\hat{k}\right\}=45$$

13. **Ans. (2)**

$\therefore$ $3.1-2.4+5\times1=0$, line is parallel to the plane

$\therefore$ reflection of line will also have same direction ratios i.e. 3, 4, 5

Also mirror image of (1,2,3) will be on required line.

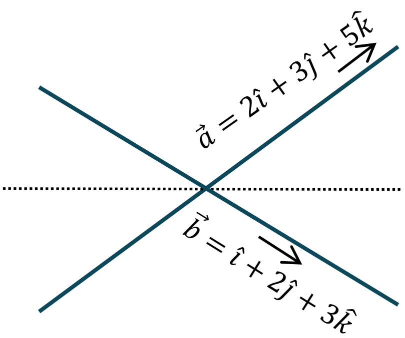
$$\frac{x-1}{1}=\frac{y-2}{-2}=\frac{z-3}{1}=-2\left(\frac{1-4+3-6}{1^2+1^2+(-2)^2}\right)\Rightarrow (x,y,z)=(3,-2,5)$$

$\therefore$ equation of straight line $\dfrac{x-3}{3}=\dfrac{y+2}{4}=\dfrac{z-5}{5}$

14. **Ans. (3)**

$\dfrac{x}{2}=\dfrac{y}{3}=\dfrac{z}{5}$ ...(i)

$\dfrac{x}{1}=\dfrac{y}{2}=\dfrac{z}{3}$ ...(ii)



$$\hat{a}+\hat{b}=\frac{2\hat{i}+3\hat{j}+5\hat{k}}{\sqrt{38}}+\frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{14}}$$

Option (1) and (2) will be incorrect

Let the dr's of line $\perp$ to (1) and (2) be a, b, c

$\Rightarrow 2a+3b+5c=0$ ...(iii) and

$a+2b+3c=0$ ...(iv)

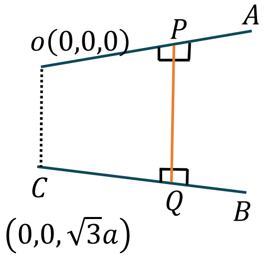
$$\therefore \frac{a}{9-10}=\frac{b}{5-6}=\frac{c}{4-3}\Rightarrow\frac{a}{-1}=\frac{b}{-1}=\frac{c}{1}\Rightarrow\frac{a}{1}=\frac{b}{1}=\frac{c}{-1}$$

$\therefore$ equation of line passing through (0, 0, 0) and is $\perp r$ to the lines (1) and (2) is

$$\frac{x}{1}=\frac{y}{1}=\frac{z}{-1}$$

15. Ans. (4)

Let the equation to one of the pair of opposite edges OA and BC be



$y + z = 0, x + z = 0$ ...(1)

and

$x + y = 0, x + y + z = \sqrt{3}a$ ...(2)

equation (1) and (2) can be expressed in symmetrical form as

$\frac{x-0}{1}=\frac{y-0}{1}=\frac{z-0}{-1}$ ...(3) and

$\frac{x-0}{1}=\frac{y-0}{-1}=\frac{z-\sqrt{3}\,a}{0}$ ...(4)

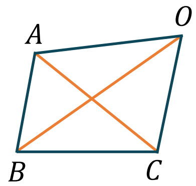
d. r. of OA and BC are respectively $(1, 1, -1)$ and $(1, -1, 0)$.

Let PQ be the shortest distance between OA and BC having direction cosines (ℓ, m, n)

$\therefore$ PQ is perpendicular to both OA and BC.

$\therefore\ \ell + m - n = 0$ ...(5) and

$\ell - m = 0$ ...(6)

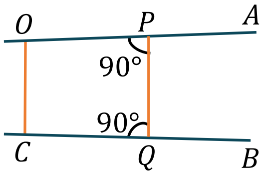


Solving (5) and (6), we get, $\frac{\ell}{1}=\frac{m}{1}=\frac{n}{2}=k$ (say)

also, $\ell^2 + m^2 + n^2 = 1$

$\therefore\ k^2 + k^2 + 4k^2 = 1 \Rightarrow k = \pm\frac{1}{\sqrt{6}}$

$\therefore\ \ell = \pm\frac{1}{\sqrt{6}}, m = \pm\frac{1}{\sqrt{6}}, n = \pm\frac{2}{\sqrt{6}}$



Shortest distance between OA and BC

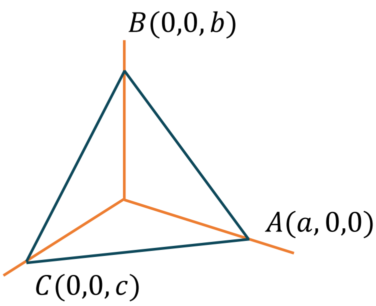
i.e. PQ =The length of projection of OC on PQ

$= \left|(x_2-x_1)\ell+(y_2-y_1)m+(z_2-z_1)n\right|$

$= \left|0.\frac{1}{\sqrt{6}}+0.\frac{1}{\sqrt{6}}+\sqrt{3}a.\frac{2}{\sqrt{6}}\right| = \sqrt{2}a.$

16. Ans. (1)

Let the equation of plane be



$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

as (α, β, γ) is centroid

$\because\ \alpha=\frac{a}{3};\ \beta=\frac{b}{3}$ and $\gamma=\frac{c}{3}$

$\therefore$ equation of plane $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$

17. Ans. (2)

$\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}=r$ ...(1)

$\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ ...(2)

$\therefore$ Coordinates of any point P on line (1)

$\therefore P(3r+1, r+2, 2r+3)$ for point of intersection of (1) and (2)

$\frac{3r+1-3}{1}=\frac{r+2-1}{2}=\frac{2r+3-2}{3} \Rightarrow \frac{3r-2}{1}=\frac{r+1}{2}=\frac{2r+1}{3}$

$\therefore r=1$

$\therefore$ point of intersection is $(4,3,5)$

$\therefore$ the equation of required plane

$\Rightarrow 4(x-4)+3(y-3)+5(z-5)=0 \Rightarrow 4x+3y+5z=50$

18. Ans. (1)

$2x-y+3z+4=0=ax+y-z+2$ ...(1)

$\because$ equation of plane through (1) is $(2x-y+3z+4)+\lambda(ax+y-z+2)=0$

$x(2+a\lambda)+y(\lambda-1)+z(3-\lambda)+(4+2\lambda)=0$ ...(2)

$x-3y+z=0=x+2y+z+1$ ...(3)

$\because$ equation of plane passing through (3) is

$(x-3y+z)+\mu(x+2y+z+1)=0$

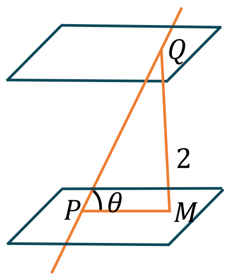
$\Rightarrow x(1+\mu)+y(2\mu-3)+z(\mu+1)+\mu=0$ ...(4)

if lines (1) and (3) are coplanar, then $\frac{2+a\lambda}{\mu+1}=\frac{\lambda-1}{2\mu-3}=\frac{3-\lambda}{\mu+1}=\frac{4+2\lambda}{\mu}$

Solving this we get $\lambda=-1, \mu=1$

19. Ans. (1)

Distance between plane = $\left|\frac{2+12}{7}\right|=2$



Angle between line and plane = θ

$\sin\theta = \frac{6-12+30}{5\sqrt{2}.7} = \frac{24}{35\sqrt{2}}$

$\sin\theta = \frac{2}{PQ} \Rightarrow PQ = 2\times\frac{35\sqrt{2}}{24} = \frac{35\sqrt{2}}{12}$

20. Ans. (1)

$L_1 : 1,0,1 \Rightarrow \alpha=45°, \beta=90°, \gamma=45° \Rightarrow \ell_1=\frac{1}{\sqrt{2}}, m_1=0, n_1=\frac{1}{\sqrt{2}}$

$L_2 : \alpha=90°, \beta=45°, \gamma=45°$

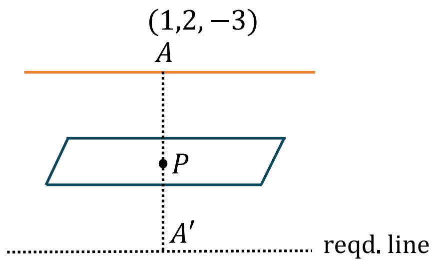
So $\ell_1=0, m_1=\frac{1}{\sqrt{2}}, n_1=\frac{1}{\sqrt{2}}$

Let angle between L_1 & L_2 be θ.

so $\cos\theta=\ell_1\ell_2+m_1m_2+n_1n_2 \Rightarrow \cos\theta=\frac{1}{2}, \theta=60°$

SECTION–B

1. Ans. (9)



$$\frac{x-1}{9}=\frac{y-2}{-1}=\frac{z+3}{-3}$$

$$3x-3y+10z-26=0$$

$\because$ A' is mirror image of A w.r.t. the given plane

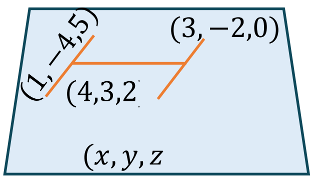
$$\therefore \frac{x-1}{3}=\frac{y-2}{-3}=\frac{z+3}{10}=-2\left(\frac{3-6-30-26}{9+9+100}\right)$$

$$\frac{x-1}{3}=\frac{y-2}{-3}=\frac{z+3}{10}=-2\left(-\frac{59}{118}\right)$$

$\therefore A'(4,-1,7)$

$\therefore$ Equation of required line $\frac{x-4}{9}=\frac{y+1}{-1}=\frac{z-7}{-3}$

2. Ans. (5)



$$\frac{x-4}{1}=\frac{y-3}{-4}=\frac{z-2}{5} \quad \text{...(1)}$$

$$\frac{x-3}{1}=\frac{y+2}{1/\lambda}=\frac{z-0}{1/\mu} \quad \text{...(2)}$$

Equation of the plane is

$$\begin{vmatrix} x-3 & y+2 & z \\ 1 & 5 & 2 \\ 1 & -4 & 5 \end{vmatrix}=0 \Rightarrow (x-3)(25+8)-(y+2)(5-2)+z(-4-5)=0$$

$$33x-99-3y-6-9z=0 \Rightarrow 33x-3y-9z-105=0 \Rightarrow 11x-y-3z=35$$

3. Ans. (3)

$\therefore$ Equation of plane through (3,4,1) is

$$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$$

$$\Rightarrow a(x-3)+b(y-4)+c(z-1)=0 \quad \text{...(i)}$$

$\because$ plane (i) passes through (0,1,0)

$$\therefore\ -3a-3b-c=0$$

$$\Rightarrow 3a+3b+c=0 \quad \text{...(ii)}$$

Also, the plane (i) is parallel to the line $\frac{x+3}{2}=\frac{y-3}{7}=\frac{z-2}{5}$

$$\therefore\ a_1a_2+b_1b_2+c_1c_2=0$$

$$\Rightarrow\ 2a+7b+5c=0 \quad \text{...(iii)}$$

Eliminating a, b, c from (i), (ii) and (iii), we get

$$\begin{vmatrix} x-3 & y-4 & z-1 \\ 3 & 3 & 1 \\ 2 & 7 & 5 \end{vmatrix}=0 \Rightarrow (x-3)(15-7)-(y-4)(15-2)+(z-1)(21-6)=0$$

$$8(x-3)-13(y-4)+15(z-1)=0\ \Rightarrow\ 8x-24-13y+52+15z-15=0$$

4. Ans. (1)

$$\sin\alpha = \left|\frac{\hat{i}.\left(2\hat{i}-3\hat{j}+6\hat{k}\right)}{\sqrt{4+9+36}}\right|$$

$$\Rightarrow \sin\alpha = \frac{2}{7} = k$$

5. Ans. (6)

Let $P(p,q,r)$ be centroid of tetrahedron

Edges of tetrahedron

$4p\hat{i}, 4q\hat{j}, 4r\hat{k}$

$$V = \frac{1}{6}\left[4p\hat{i}\ 4q\hat{j}\ 4r\hat{k}\right] = 64k^3 \Rightarrow pqr = 6k^3$$

$$\Rightarrow xyz = 6k^3$$

6. Ans. (5151)

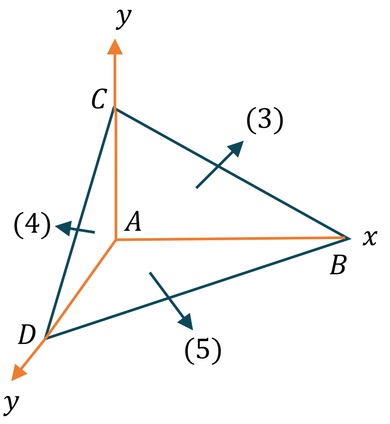
$P_1 : 4x + 6y - 7z = 1$

$P_2 : 4x + 6y - 7z = 2$

$$\text{Distance} = \left|\frac{2-1}{\sqrt{16+36+49}}\right| = \frac{1}{\sqrt{101}}$$

$$\therefore N = 101 \Rightarrow \frac{N(N+1)}{2} = 5151$$

7. Ans. (7.07)



by concept of projection area :

$$\left(\text{Ar.}\left(\Delta BCD\right)\right)^2 = 3^2 + 4^2 + 5^2$$

$$\Rightarrow \text{Ar.}\Delta BCD = \sqrt{50} = 5\sqrt{2} \text{ sq. units}$$

8. Ans. (23)

$\because$ Line intersect the plane at point

$P \equiv (2 + \lambda, -2 - 3\lambda, 5 + 2\lambda)$ (P is general point on line)

$\Rightarrow$ Point P lie on plane $2x - 3y + 42 = 163$

$2(2 + \lambda) - 3(-2 - 3\lambda) + 4(5 + 2\lambda) = 163$

$19\lambda = 133 \Rightarrow \lambda = 7 \Rightarrow P \equiv (9, -23, 19)$

$\because$ line intersect the yz plane at Q

$\Rightarrow Q \equiv (2+\lambda, -2-3\lambda, 5+2\lambda)$

$2+\lambda = 0 \Rightarrow \lambda = -2$ ($\because$ in yz plane $x = 0$)

$\Rightarrow Q \equiv (0, 4, 1) \Rightarrow PQ = \sqrt{(9-0)^2+(-23-4)^2+(19-1)^2} = \sqrt{1134}$

$= 9\sqrt{14} = a\sqrt{b} \Rightarrow a = 9,\ b = 14$

$a + b = 23$

9. Ans. (13)

$$\frac{x-2}{2} = \frac{y+1}{4} = \frac{z-2}{12} = \lambda \quad \text{(Let)} \quad ....(1)$$

General point on line (1) $P \equiv (2+2\lambda, -1+4\lambda, 2+12\lambda)$

$\because$ line (1) intersects the plane $x - y + z = 5$

$\Rightarrow$ point P lie in the plane $x - y + z = 5$

$\Rightarrow (2+2\lambda) - (-1+4\lambda) + (2+12\lambda) = 5$

$10\lambda = 0 \Rightarrow \lambda = 0$

$\Rightarrow P \equiv (2, -1, 2)$ & $Q \equiv (-1, -5, -10)$ given

$\Rightarrow$ Distance between point P & Q is

$= \sqrt{(2+1)^2+(-1+5)^2+(2+10)^2} = 13$

10. Ans. (21)

$$\frac{|c_1+c_2|}{\sqrt{3^2+4^2+5^2}} = 2\sqrt{2},\ |c_1+c_2| = 5\sqrt{2}\times 2\sqrt{2}$$

$|c_1+c_2| = 20,\ c_1, c_2 \in W$

$(c_1, c_2) \equiv (0, 20), (1, 19), (2, 18)$, -------, $(19, 1), (20, 0) \rightarrow 21$ pairs

JEE (Advanced) Practice Paper

1. Ans. (B)

$\cos\theta = \ell_1\ell_2 + m_1m_2 + n_1n_2$; $Dc's$ of β_1 (bisector)

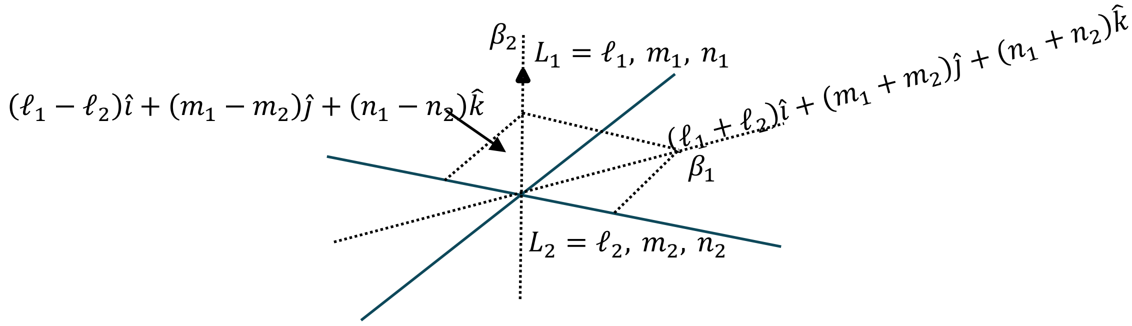
$$\frac{\ell_1+\ell_2}{\sqrt{(\ell_1+\ell_2)^2+(m_1+m_2)^2+(n_1+n_2)^2}} = \frac{\ell_1+\ell_2}{\sqrt{2+2(\ell_1\ell_2+m_1m_2+n_1n_2)}} = \frac{\ell_1+\ell_2}{\sqrt{2+2\cos\theta}} = \frac{\ell_1+\ell_2}{2\cos\theta/2}$$

Similarly, $\dfrac{m_1+m_2}{2\cos\theta/2}, \dfrac{n_1+n_2}{2\cos\theta/2}$

Similarly, Dc's for bisector β_2

$$\frac{\ell_1-\ell_2}{\sqrt{(\ell_1-\ell_2)^2+(m_1-m_2)^2+(n_1-n_2)^2}} = \frac{\ell_1-\ell_2}{\sqrt{2-2(\ell_1\ell_2+m_1m_2+n_1n_2)}} = \frac{\ell_1-\ell_2}{\sqrt{2-2\cos\theta}} = \frac{\ell_1-\ell_2}{2\sin\theta/2}$$

$$\frac{\ell_1-\ell_2}{2\sin\frac{\theta}{2}}, \frac{m_1-m_2}{2\sin\frac{\theta}{2}}, \frac{n_1-n_2}{2\sin\frac{\theta}{2}}$$



2. Ans. (C)

$$\cos\theta = \frac{\ell m + mn + n\ell}{\ell^2 + m^2 + n^2} \quad \text{...(i)}$$

$x^3 + x^2 - 4x - 4 = 0 \Rightarrow \ell + m + n = -1 \Rightarrow \ell m + mn + n\ell = -4$

$(\ell + m + n)^2 = \ell^2 + m^2 + n^2 + 2(-4) \Rightarrow \ell^2 + m^2 + n^2 = 1 + 8 = 9$

$\therefore \cos\theta = -\frac{4}{9}$

$\therefore$ acute angle between the lines is $\cos^{-1}\frac{4}{9}$

3. Ans. (B)

Any pt. on line is $(3\lambda + 2, 2\lambda - 1, 1 - \lambda)$

but it lies on the curve $xy = c^2$ & $z = 0 \Rightarrow (3\lambda + 2)(2\lambda - 1) = c^2$ & $1 - \lambda = 0$

$\Rightarrow (3\lambda + 2)(2\lambda - 1) = c^2$ & $\lambda = 1 \Rightarrow c^2 = 5 \Rightarrow c = \pm\sqrt{5}$

4. Ans. (B)

First two lines pass through the point (1,2,3) the third line must pass through the same point therefore $\lambda^2 = 1 \Rightarrow \lambda = \pm 1$. But λ cannot be 1 as the third line coincide with the first.

5. Ans. (A)

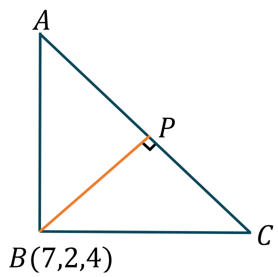
$P = (5k - 6, 3k - 10, 8k - 14)$

Now $\overrightarrow{BP}.\overrightarrow{AC} = 0$

$5(5k - 13) + 3(3k - 12) + 8(8k - 18) = 0$

$k = \frac{245}{98} = \frac{5}{2}$

$P = \left(\frac{13}{2}, -\frac{5}{2}, 6\right)$, Now length $BP = \frac{\sqrt{98}}{2}$



Now $A, C = \left(\frac{13}{2} \pm \frac{5}{\sqrt{98}} \cdot \frac{\sqrt{98}}{2}, -\frac{5}{2} \pm \frac{3}{\sqrt{98}} \cdot \frac{\sqrt{98}}{2}, 6 \pm \frac{8}{\sqrt{98}} \cdot \frac{\sqrt{98}}{2}\right)$

$A = (9, -1, 10)$

$C = (4, -4, 2)$

Equation of BC $\frac{x-7}{3} = \frac{y-2}{6} = \frac{z-4}{2}$

Equation of AC $\frac{x-7}{2} = \frac{y-2}{-3} = \frac{z-4}{6}$

6. **Ans. (D)**

Line: $\frac{x-2}{2}=\frac{y+1}{4}=\frac{z-2}{12}\Rightarrow\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-2}{6}=r$

$\Rightarrow x=r+2\Rightarrow y=2r-1\Rightarrow z=6r+2$

Plane : $x-y+z=5\Rightarrow r+2-(2r-1)+6r+2=5\Rightarrow 5r=0\Rightarrow r=0$

$\therefore$ point of intersection is $(2,-1,2)$

$\Rightarrow(\alpha-2)^2+(5\alpha+1)^2+(10\alpha-2)^2=169$

$126\alpha^2-34\alpha-160=0$

$\Rightarrow 63\alpha^2-17\alpha-80=0$

$\Rightarrow\alpha=-1,\frac{80}{63}$.

7. **Ans. (A,B,C,D)**

If x,y,z are not all zero, then consistency of given equations

$$\Rightarrow\begin{vmatrix}a&b&c\\b&c&a\\c&a&b\end{vmatrix}=0$$

i.e. $a^2+b^2+c^2-3abc=0$

$(a+b+c)(a^2+b^2+c^2-ab-bc-ca)=0$

adding given equations $\Rightarrow(a+b+c)(x+y+z)=0$

(A) $a+b+c\neq 0$ and $a=b=c$

$\Rightarrow x+y+z=0$ represents identical planes

(B) $a+b+c=0$ and a,b,c are all distinct

$\Rightarrow$ equation have infinitely many solutions.

$ax+by-(a+b)z=0$; $bx+cy-(b+c)z=0$

$(\because a+b+c=0)$

eliminating x, we get $y=z\Rightarrow x=y=z$

(C) $a+b+c\neq 0$ and a,b,c are all distinct

$\Rightarrow$ unique solutions, represents all planes having only unique point of intersection.

(D) $a+b+c=0$ and $a=b=c$

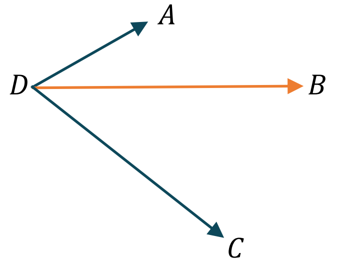
$\Rightarrow a=b=c=0$ i.e. $0.x+0.y+0.z=0$

8. **Ans. (A,B)**

$A(2-x,2,2),B(2,2-y,2),C(2,2,2-z),D(1,1,1)$

$\overrightarrow{DA}=(1-x)\ \hat{i}+\hat{j}+\hat{k}$; $\overrightarrow{DB}=\ \hat{i}+(1-y)\ \hat{j}+\hat{k}$; $\overrightarrow{DC}=\ \hat{i}+\ \hat{j}+(1-z)\ \hat{k}$

If four points are coplanar then $[\overrightarrow{DA},\ \overrightarrow{DB},\ \overrightarrow{DC}]=0$



$$\Rightarrow\begin{vmatrix}1-x&1&1\\1&1-y&1\\1&1&1-z\end{vmatrix}=0$$

$c_1\to c_1-c_2$

$$\Rightarrow\begin{vmatrix}-x&1&1\\y&1-y&1\\0&1&1-z\end{vmatrix}=0$$

$-x(-y+yz-z)+1(+yz)=0 \Rightarrow xy-xyz+xz+yz=0$

$xy+yz+zx=xyz$

$\therefore \dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}=1$.

Also, $\dfrac{x-1}{x}+\dfrac{y-1}{y}+\dfrac{z-1}{z}=2$

9. Ans. (B,C,D)

$(4,2,k^2)$ lies on $2x-4y+z=1 \Rightarrow k=\pm 1$

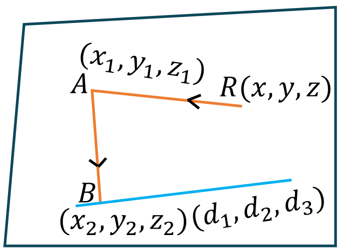
Also $(k\hat{i}+\hat{j}+2\hat{k}).(2\hat{i}-4\hat{j}+\hat{k})=0 \Rightarrow k=1$

10. Ans. (A,B)

Vectors $\overrightarrow{AR}$, $\overrightarrow{AB}$ & $\vec{C}$ are coplanar

Equation of the required plane $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ d_1 & d_2 & d_3 \end{vmatrix}=0$

or $\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ d_1 & d_2 & d_3 \end{vmatrix}=0$



$\vec{C}=d_1\hat{\imath}+d_2\hat{\jmath}+d_3\hat{k}$

11. Ans. (A,B,C,D)

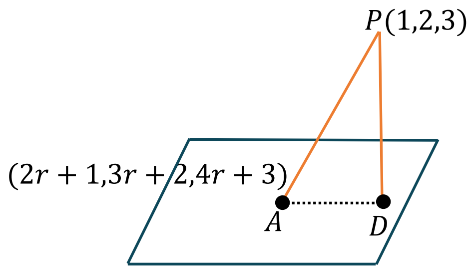
$2r+1-(3r+2)+2(4r+3)+2=0$

$\Rightarrow 7r+7=0 \Rightarrow r=-1$

$\therefore A(-1,-1,-1)$

Required line will be projection of given line in the plane

Foot of $\perp$ of P will be on D



$\dfrac{x-1}{1}=\dfrac{y-2}{-1}=\dfrac{z-3}{2}=-\left(\dfrac{1-2+2.3+2}{1^2+(-1)^2+2^2}\right)$;

$\dfrac{x-1}{1}=\dfrac{y-2}{-1}=\dfrac{z-3}{2}=\dfrac{-7}{6}$

$x=\dfrac{-1}{6}$; $y=\dfrac{19}{6}$; $z=\dfrac{4}{6} \Rightarrow \dfrac{x+1}{1}=\dfrac{y+1}{5}=\dfrac{z+1}{2} \Rightarrow \dfrac{x+1}{1}+1=\dfrac{y+1}{5}+1=\dfrac{z+1}{2}+1$

Also $\dfrac{x+1}{1}=\dfrac{y+1}{5}=\dfrac{z+1}{2} \Rightarrow \dfrac{x+1}{1}=\dfrac{y+1}{5}$ and $\dfrac{y+1}{5}=\dfrac{z+1}{2}$

$\Rightarrow 5x-y+4=0=2y-5z-3$

Also $(5x-y+4)+(2y-5z-3)=0=2y-5z-3 \Rightarrow 5x+y-5z+1=0=2y-5z-3$

12. Ans. (A, D)

Let the plane is

$(2x+3y-z)+1+\lambda(x+y-2z+3)=0$ ...(1)

$(2+\lambda)x+(3+\lambda)y-(1+2\lambda)z+1+3\lambda=0$

$\Rightarrow 3(2+\lambda)-(3+\lambda)+2(1+2\lambda)=0$

$+6\lambda+5=0 \Rightarrow \lambda=-5/6$

Putting value of λ in (1)

$7x + 13y + 4z - 9 = 0 \Rightarrow \alpha = 9$

Now image of (1,1,1) in plane π is

$$\frac{x-1}{7} = \frac{y-1}{13} = \frac{z-1}{4} = -2\left(\frac{7+13+4-9}{49+169+16}\right) ; \frac{x-1}{7} = \frac{y-1}{13} = \frac{z-1}{4} = -\frac{15}{117}$$

$$x = \frac{12}{117}, \quad y = \frac{-78}{117}, \quad z = \frac{57}{117} \Rightarrow \beta = 117.$$

13. Ans. (B)

Equation of the second plane is $-x + 2y - 3z + 5 = 0 \Rightarrow 2(-1) + 3.2 + (-4)(-3) > 0$

$\therefore O$ lies in obtuse angle $(2 \times 1 + 3(-2) - 4 \times 3 + 7)(-1 + 2(-2) - 3 \times 3 + 5)$

$= (2 - 6 - 12 + 7)(-1 - 4 - 9 + 5) > 0 \;\therefore\; P$ lies in obtuse angle.

14. Ans. (C)

$1 \times 2 + 2 \times 1 - 3 \times 3 < 0 \therefore O$ lies in acute angle.

Also $(2 + 2(-1) - 3(2) + 5)(2 \times 2 - 1 + 3 \times 2 + 1) = (-1)(10) < 0$

$\therefore P$ lies in obtuse angle.

15. Ans. (A)

$1 - 4 - 9 < 0$ $\therefore$ O lies in acute angle.

Further $(1 + 4 - 6 + 2)(1 - 4 + 6 + 7) > 0$

$\therefore$ The point P lies in acute angle.

16. Ans. (A)

Volume of pyramid $= \frac{1}{3} \times$ base area $\times$ height $= \frac{1}{3} \times 16 \times 6 = 32$

17. Ans. (C)

Co-ordinates of centroid of faces EAB, EBC, ECD & EDA are $(2,2,2)$ $\left(\frac{10}{3}, 2, \frac{10}{3}\right)$, $\left(2, 2, \frac{14}{3}\right)$, $\left(\frac{2}{3}, 2, \frac{10}{3}\right)$. y co-ordinate of each point is 2.

Hence these points are co-planar & plane is parallel to base plane.

18. Ans. (C)

Ortho-centre of ΔABD is (0,0,0) equation of plane EBC is $\begin{vmatrix} x-4 & y & z \\ -2 & 6 & 6 \\ 0 & 0 & 4 \end{vmatrix} = 0$

$4.\left((x-4).6 + 2y\right) = 0 \Rightarrow 6x - 24 + 2y = 0 \Rightarrow 3x + y - 12 = 0$

$$d = \left|\frac{0+0+12}{\sqrt{3^2+1}}\right| = \frac{12}{\sqrt{10}}$$

Important Notes