Tangent and Normal

SOLUTIONS

EXERCISE - O

1. **Ans. (D)**

$V=\frac{4}{3}\pi r^3 \Rightarrow \frac{dv}{dt}=4\pi r^2\frac{dr}{dt} \Rightarrow 35=4\pi(7)^2\frac{dr}{dt}$

$\Rightarrow 35 = 4.\frac{22}{7}.7.7\frac{dr}{dt} \Rightarrow \frac{dr}{dt}=\frac{5}{88}$ ...(1)

A = $4\pi r^2 \Rightarrow \frac{dA}{dt}=8\pi r\frac{dr}{dt} \Rightarrow \frac{dA}{dt}=8.\frac{22}{7}.7.\frac{5}{88}$ by (1)

$\Rightarrow \frac{dA}{dt}=10\text{cm}^2/\text{min}$

2. **Ans. (B)**

Slope = $\frac{dy}{dx}$ = cos$x$ – 2 cos$x$. sin$x$ = 0.

$\Rightarrow$ (cos$x$) (1–2 sin x) = 0 $\Rightarrow$ $\cos x=0$ or $\sin x=\frac{1}{2}$

$\cos x = 0 \downarrow x=\frac{\pi}{2}$; $\sin x=\frac{1}{2} \downarrow$ Not in option

3. **Ans. (A)**

ay^2 = x^3 $\Rightarrow$ 2ayy' = 3x^2 $\Rightarrow$ y'= $\frac{3x^2}{2ay}$

At $x = at^2, y = at^3$: $\frac{dy}{dx}=\frac{3(at^2)^2}{2a(at^3)} \Rightarrow \frac{dy}{dx}=\frac{3t}{2}$

$\therefore$ eq^n of tangent : $y - at^3 = \frac{3t}{2}(x - at^2)$

$\Rightarrow y - at^3 = \frac{3t}{2}x-\frac{3at^3}{2} \Rightarrow 3tx-2y=at^3$

4. **Ans. (A)**

Area of square $A = x^2 \Rightarrow \frac{dA}{dx}=2x$

Old area x_o^2 ; Now Area = $(x_o + h)^2 = x_o^2 + h^2 + 2hx_o$ (exact)

exact change in Area = $x_o^2 + h^2 + 2hx_o - x_o^2 = \boxed{h^2+2hx_0}$

Approximate change in Area = h. $\left.\frac{dA}{dx}\right|_{x=x_0}=2hx_0$

Difference = h^2

5. **Ans. (D)**

for tangent to be parallel to x axis : $\frac{dy}{dx}=0$

$\frac{dy}{dx}=1-\frac{8}{x^3}=0\Rightarrow x^3=8\Rightarrow x=2$

At $x=2 : y=2+\frac{4}{4}\Rightarrow y=3$ ∴ $P(2,3)$

eqⁿ of line of zero slope through (2,3) is $y=3$

6. **Ans. (B)**

$x^2+2xy-3y^2=0$

$\Rightarrow\ x^2+3xy-xy-3y^2=0$

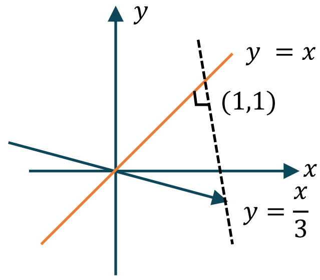
$\Rightarrow\ (x+3y)\,(x-y)=0$ (Pair of st. lines.)

(1, 1) lies on $y=x$ line

N at (1, 1) to line $y=x : y-1=-1(x-1)$

$\Rightarrow\ x+y=2$

Its pt. of int. with $x+3y=0$ is $(3,-1)$



Note : Answer can also be concluded from figure.

7. **Ans. (C)**

$$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{a(0-(-\sin\theta))}{a((1+\cos\theta))}=\frac{\sin\theta}{1+\cos\theta}$$

$$\frac{dy}{dx}=\tan\left(\frac{\pi}{4}\right)=\frac{\sin\theta}{1+\cos\theta}\Rightarrow\tan\left(\frac{\pi}{4}\right)=\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$$

$$\Rightarrow\ \tan\frac{\theta}{2}=\tan\frac{\pi}{4}\ \Rightarrow\ \boxed{\theta=\frac{\pi}{2}}$$

$$\therefore\ x=a\left(\frac{\pi}{2}+1\right),\ y=a(1-0)=a.$$

8. **Ans. (B)**

$$\frac{dy}{dt}=4t+3,\ \frac{dx}{dt}=3t^2-8t-3,\ \frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

for horizontal tangent : $\frac{dy}{dx}=0\Rightarrow\frac{dy}{dt}=0\ \Rightarrow 4t+3=0\Rightarrow t=\frac{-3}{4}$

for vertical tangent : $\frac{dx}{dy}=0\ \Rightarrow\frac{dx}{dt}=0$

$\Rightarrow\ 3t^2-8t-3=0\ \Rightarrow t=3,\ t=\frac{-1}{3}$

$\therefore\ \ H=1, V=2$

9. **Ans. (B)**

If $OA = a$; $OB = 2a \Rightarrow tan\theta = 2$

slope of the tangent is 2

$$\left(\frac{dy}{dx}\right)_{(x_1,y_1)} = 2$$

$\Rightarrow 3x_1^2 - 6x_1 - 7 = 2$

$\Rightarrow x_1^2 - 2x_1 - 3 = 0$

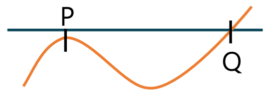
$\Rightarrow x_1 = 3$ or -1 (rejected)

$\Rightarrow (3,-15)$

10. **Ans. (B)**

$P\ (4t^2 + 3, 8t^3 - 1)$

$Q\left(4t_1^2 + 3, 8t_1^3 - 1\right), m_{PQ} = \left.\frac{dy}{dx}\right|_p$



$\Rightarrow \dfrac{8t_1^3 - 8t^3}{4t_1^2 - 4t^2} = \dfrac{24t^2}{8t} \Rightarrow \dfrac{2(t_1 - t)(t_1^2 + t^2 + tt_1)}{(t_1 - t)(t_1 + t)} = 3t \ (\because t_1 \neq t)$

$\Rightarrow 2t_1^2 + 2t^2 + 2tt_1 = 3tt_1 + 3t^2 \quad \Rightarrow t^2 + tt_1 - 2t_1^2 = 0 \Rightarrow (t - t_1)(t + 2t_1) = 0$

$\Rightarrow t = t_1$ (rejected) or $t_1 = \dfrac{-t}{2}$

put $t_1 = \dfrac{-t}{2}$, we get :

$$Q\left(\left(\frac{4.t^2}{4} + 3\right), \left(8\left(\frac{-t^3}{8}\right) - 1\right)\right)$$

$\Rightarrow Q\ ((t^2 + 3), (-t^3 - 1))$

11. **Ans. (C)**

$C_1 : x^2 - y^2 = 5$ (hyperbola) ; $C_2 : \dfrac{x^2}{18} + \dfrac{y^2}{8} = 1$ (ellipse)

Point of intersection : $x^2 - y^2 = 5 \Rightarrow x^2 = y^2 + 5$

put in ellipse : $\dfrac{y^2 + 5}{18} + \dfrac{y^2}{8} = 1 \Rightarrow y^2 = 4$

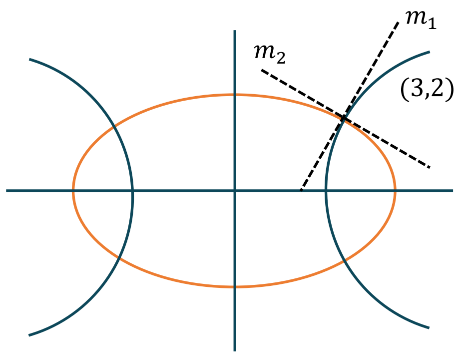
$\therefore x^2 = y^2 + 5 \Rightarrow x^2 = 4 + 5 \Rightarrow x = \pm 3.$

$\therefore$ four points of intersection $(\pm 3, \pm 2)$

By symmetry, angle of intersection at all points will be same.

$C_1 : x^2 - y^2 = 5$

$\Rightarrow x^2 - 2y\, y' = 0 \Rightarrow y' = \dfrac{x}{y}$



$\Rightarrow m_1 = \dfrac{3}{2}$

$C_2 : \dfrac{x^2}{18} + \dfrac{y^2}{8} = 1 \Rightarrow \dfrac{2x}{18} + \dfrac{2y}{8} y' = 0 \Rightarrow y' = \dfrac{-8x}{18y}$

$\therefore m_1\, m_2 = -1 \Rightarrow$ Angle of intersection $= \dfrac{\pi}{2}$

curves are orthogonal .

12. Ans. (D)

$C_1: y^3 - x^2 + 5y - 2x = 0 \Rightarrow 5y' - 2 = 0$

$C_2: x^4 - x^3y^2 + 5x + 2y = 0 \Rightarrow 5 + 2y' = 0$

$\Rightarrow m_1 = \frac{2}{5}, m_2 = -\frac{5}{2} \Rightarrow m_1m_2 = -1$

$\therefore\ \theta = \frac{\pi}{2}$

13. Ans. (D)

$$\left.\begin{array}{r} C_1: y^2 = 8x \Rightarrow 2yy' = 8 \Rightarrow y'\big|_{(2,4)} = 1 \\ C_2: x^2 = 4y - 12 \Rightarrow 2x = 4y' \Rightarrow y'\big|_{(2,4)} = 1 \end{array}\right\}\theta = 0°$$

the curves touch each other at this point.

14. Ans. (C)

$C_1: y^2 = 6x\ ; C_2: 9x^2 + by^2 = 16$

Let pt. of intersection be $P(x_1, y_1)$

P lies on C_1 : $y_1^2 = 6x_1$ ....(1)

$C_1: y^2 = 6x \Rightarrow 2yy' = 6 \Rightarrow y' = \frac{3}{y_1} = m_1$

$C_2: 9x^2 + by^2 = 16$

$\Rightarrow 18x + 2byy' = 0 \Rightarrow y' = \frac{9x_1}{by_1} = m_2$

$\because\ m_1m_2 = -1 \Rightarrow -\frac{27x_1}{by_1^2} = -1.$

by equation (1)

$\Rightarrow -\frac{27x_1}{b(6x_1)} = -1 \Rightarrow b = \frac{27}{6} \Rightarrow b = \frac{9}{2}$

15. Ans. (C)

$x^2 + xy + y^2 = 7 \Rightarrow 2x + xy' + y + 2yy' = 0$

At $x = 1, y = -3 : 2(1) + (1)y' + (-3) + 2(-3)y' = 0$

$\Rightarrow y' = \frac{-1}{5} \Rightarrow L_{ST} = \left|\frac{y}{y'}\right| = \left|\frac{-3}{-1/5}\right| = 15$

16. Ans. (D)

$L_{ST} \times L_{SN} = \left|\frac{y_1}{f'(x_1)}\right| \times |y_1 f'(x_1)| = (y_1)^2$

17. Ans. (A)

$$\left(\frac{LN}{LT}\right)^2 = \left(\frac{y\sqrt{1+(y')^2}}{\frac{y\sqrt{1+(y')^2}}{y'}}\right)^2 = (y')^2$$

Now check option

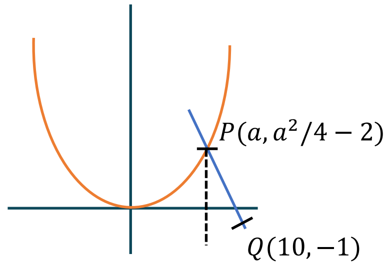
(A) $\frac{L_{SN}}{L_{ST}} = \frac{|yy'|}{|y/y'|} = (y')^2$

18. **Ans. (D)**

$y = \frac{x^2}{4} - 2$

$y' = \frac{2x}{4}$

m_N at $P = -\frac{2}{a}$



$\therefore \quad -\frac{2}{a} = m_{PQ} = \frac{\frac{a^2}{4} - 2 + 1}{a - 10}$

$\Rightarrow a^3 = -4a + 80$

$a^3 + 4a - 80 = 0 \Rightarrow a = 4$

$\therefore P(4,2)$.

19. **Ans. (C)**

$y = x^n, n \in N$

$\frac{dy}{dx} = n\,x^{n-1} = na^{n-1}$

slope of normal = $-\frac{1}{na^{n-1}}$

equation of normal $y - a^n = -\frac{1}{na^{n-1}}(x - a)$

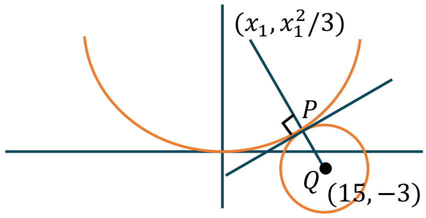
put $x = 0$ to get y-intercept

$y = \; a^n + \frac{1}{na^{n-2}}$; Hence $b = \; a^n + \frac{1}{na^{n-2}}$

20. **Ans. (D)**

Equation of normal is

$y - \frac{x_1^2}{3} = -\frac{3}{2x_1}(x - x_1)$



pass through (15,– 3)

$-3 - \frac{x_1^2}{3} = -\frac{3}{2x_1}(15 - x_1)$

solve for x_1, we get $x_1 = 3 \Rightarrow$ Radius = $PQ = \sqrt{180}$

21. **Ans. (A,D)**

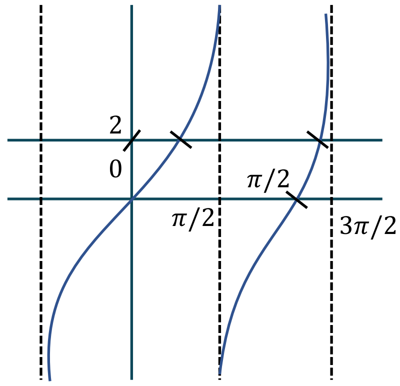
$xy = k^2$

$y + \frac{xdy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x} = -\frac{1}{t^2} = -\frac{b}{a}$

$\frac{b}{a} > 0$

b & a have same sign.

22. **Ans. (B,D)**



$y = e^{2x}\cos x$

At $x = 0 : y = 1$

$y' = 2e^{2x}\cos x + e^{2x}(-\sin x)$

At $x = 0 : y' = 2 \Rightarrow T : y - 1 = 2(x - 0)$

$\Rightarrow y = 2x + 1$

for tangent parallel to axis :

$y' = 0 \Rightarrow 2e^{2x}\cos x - e^{2x}\sin x = 0$

$\Rightarrow \tan x = 2$

$\therefore$ 2 solution in $x \in \left(-\frac{\pi}{2}, \frac{3\pi}{2}\right)$

23. **Ans. (B,C)**

Let $g(x) = ax + b = y \rightarrow m = a$

$\Rightarrow \; y = ax + b$

$\Rightarrow \; x = ay + b$ (for inverse function)

$\Rightarrow \; y = \frac{x-b}{a} = g^{-1}(x)$

$\therefore \; g(x) = ax + b;\; g^{-1}(x) = \frac{x}{a} - b$

Now check orthogonality using slopes

24. **Ans. (B,C)**

$y^2 = 4ax, \frac{dy}{dx} = \frac{2a}{y}$ ...(1)

and $xy = b, \frac{dy}{dx} = -\frac{y}{x}$ ...(2)

Also solving $y^2 = 4ax$ and $xy = b$

$\frac{b^2}{x^2} = 4ax \qquad \therefore \qquad x^3 = \frac{b^2}{4a}$

For curves to cut at right angles.

$\frac{2a}{y} \times \left(-\frac{y}{x}\right) = -1$

$2a = x$

$8a^3 = x^3 = \frac{b^2}{4a}$

$32a^4 = b^2$

25. **Ans. (A,B,C)**

$x = 2 \Rightarrow t^2 + 3t - 10 = 0 \Rightarrow t = 2, -5$

$y = -1 \Rightarrow t^2 - t - 2 = 0 \Rightarrow t = 2, -1$

$\Rightarrow t = 2$ (common value)

$\frac{dy}{dx} = \frac{4t-2}{2t+3}$

$$\left.\frac{dy}{dx}\right|_{t=2} = \frac{6}{7}$$

subtangent $= \dfrac{-1}{\frac{6}{7}} = \dfrac{-7}{6}$

tangent $= \dfrac{-1}{\frac{6}{7}}\sqrt{1+\dfrac{36}{49}} = -\dfrac{\sqrt{85}}{6}$

26. **Ans. (B,C)**

$$\left(\frac{x^2-y^2}{xy}\right)^3 = \frac{512}{27}$$

Since given equation is homogeneous equation

$\therefore\ \left.\dfrac{dy}{dx}\right|_{(3,1)} = \dfrac{1}{3}$ and $\left.\dfrac{d^2y}{dx^2}\right|_{(3,1)} = 0$

27. **Ans. (A,C)**

Equation of normal

$$\Rightarrow y-y_1 = \frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}}(x-x_1)$$

$x_1 = a$, $\left(\dfrac{y}{b}\right)^n = 1 \Rightarrow y_1^n = b^n$

If n is odd then $y_1 = b$ so point $(x_1, y_1) = (a, b)$

If n is even then $y_1 = \pm b$ so point (x_1, y_1)

$= (a, b)$ or $(a, -b)$

$$\left(\frac{dy}{dx}\right)_{(a,b)} = \frac{-b}{a}$$

normal at point (a, b)

$$\Rightarrow\ y-b = \frac{-1}{\left(\frac{dy}{dx}\right)_{(a,b)}}(x-a) \Rightarrow y - b = \frac{a}{b}(x-a)$$

$\Rightarrow ax - by = a^2 - b^2$ ...(1)

normal at point $(a, -b)$

$$\left(\frac{dy}{dx}\right)_{(a,-b)} = \frac{b}{a}$$

$$\Rightarrow y+b = \frac{-1}{\left(\frac{dy}{dx}\right)_{(a,-b)}}(x-a) \Rightarrow\ y+b = -\frac{a}{b}(x-a)$$

$ax + by = a^2 - b^2$ ...(2)

28. Ans. (A,B,C,D)

$f(1) = 0 \qquad g(1) = 0$

$1 + a + b = 0 \qquad c - 1 = 0$

$a + b = -1 \qquad c = 1$

$\left.\frac{dy}{dx} = 2x + a\right)_{x=1} \quad \frac{dy}{dx} = c - 2x)_{x=1} = c - 2$

$= 2 + a$

$M_1 = m$

$2 + a = c - 2$

$a = c - 4$

$= 1 - 4 = -3$

$b = 2$

29. Ans. (A)

Here, $m = \left.\frac{dy}{dx}\right|_{x=0}$

$\frac{dy}{dx} = 3x^2 + 6x + 4 \quad \Rightarrow m = 4$

and, $\quad k = y(0) \Rightarrow k = -1$

$\ell = |k|\sqrt{1 + \frac{1}{m^2}} \Rightarrow \ell = |(-1)|\sqrt{1 + \frac{1}{16}} = \frac{\sqrt{17}}{4}$

30. Ans. (C)

$|yy'| = |y/y'|$ at $(0, 1) \Rightarrow (y')$ at $(0, 1)$ equal ± 1

$\Rightarrow (pe^{px} + p)_{(0,1)} = \pm 1 \Rightarrow 2p = \pm 1 \Rightarrow p = \pm 1/2$

31. Ans. (D)

Length of subnormal $= \left|y\frac{dy}{dx}\right| = \left|-3\sin\left(\frac{-\pi}{4}\right)\frac{-3\cos\left(\frac{-\pi}{4}\right)}{\left(-\sqrt{2}\sin\left(\frac{-\pi}{4}\right)\right)}\right| = \frac{3}{\sqrt{2}}\frac{3}{\sqrt{2}} = \frac{9}{2}$.

32. Ans. (A)

33. Ans. (B)

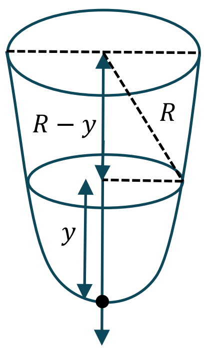
(i) $\quad R = 13\ ;\ \frac{dV}{dt} = -6\ ;\ y = 8$

$V = \frac{\pi}{3}y^2(3R - y)$

$\Rightarrow \frac{dV}{dt} = (2\pi Ry - \pi y^2)y' \Rightarrow 6 = \left(2\pi(13)(8) - \pi(8)^2\right)y'$

$\Rightarrow y' = \frac{-1}{24\pi}$

$\therefore \frac{dy}{dt} = -\frac{1}{24\pi}$ m/min.



$\frac{dV}{dt} = -6m^3 / \min$

(ii) $r=\sqrt{R^2-(R-y)^2} \Rightarrow \frac{dr}{dt}=\frac{+2(R-y)\cdot y'}{2\sqrt{R^2-(R-y)^2}}$

$\Rightarrow \frac{dr}{dt}=\frac{(13-8)}{\sqrt{169-(13-8)^2}}\left(\frac{-1}{24\pi}\right) \Rightarrow \frac{dr}{dt}=\frac{-5}{288\pi}m/\min.$

34. **Ans. (C)**

length of tangent, normal, subtangent, subnormal are

$|y\operatorname{cosec}\theta|,|y\sec\theta|,|y\cot\theta|,|y\tan\theta|$ Respectively

$\frac{dy}{dx}=4x\cdot e^{2x^2}+2\Big|_{x=0}$

$\frac{dy}{dx}=2=\tan\theta.$

35. **Ans. (A →p; B →r; C →q; D →s)**

(A) $4y\frac{dy}{dx}=2ax \Rightarrow -4\frac{dy}{dx}=2a$

$\Rightarrow \frac{dy}{dx}=\frac{-a}{2}=-1 \Rightarrow a=2$

$2y^2=ax^2+b$

$2=a+b$

$b=0$

$a-b=2-0=2$

(B) Slope of normal = −1

Slope of tangent $=1=\frac{dy}{dx}$

$18y\frac{dy}{dx}=3x^2$

$18b = 3a^2$

$b=\frac{a^2}{6}$ ...(i)

$9b^2=a^3$ ...(ii)

$9.\frac{a^4}{36}=a^3 \Rightarrow a=4\,;\,b=\frac{16}{6}=\frac{8}{3}$

$a-b=4-\frac{8}{3}=\frac{4}{3}$

(C) (1, 2) satisfies $y=ax^2+bx+\frac{7}{2}$

$\Rightarrow 2=a+b+\frac{7}{2} \Rightarrow a+b=\frac{-3}{2}$

$\frac{dy}{dx}=2ax+b=2a+b$

for IInd curve $\frac{dy}{dx} = 2x + 6 = 2$

Slope of normal = $-\frac{1}{2}$

$2a + b = -\frac{1}{2}$

Solve for a & b

(D) Put, (1, 1) $\quad 1 + a + b = 0$ ...(i)

$\frac{dy}{dx} = 2$

$y + xy' + a + by' = 0$

$1 + 2 + a + 2b = 0$

$a + 2b = -3$ ...(ii)

get the values of a & b

EXERCISE - S

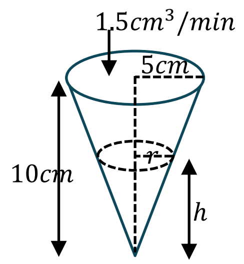
1. **Ans. (9.00)**

$V_t = \pi\,(3)^2 h$

$$\frac{dV_t}{dt} = 9\pi\frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{1}{9\pi}\text{m/min.}$$

2. **Ans. (8.00)**

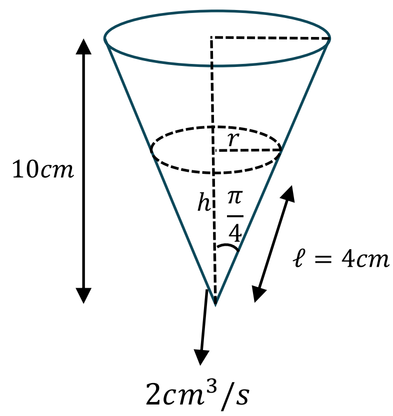


$$\frac{r}{h} = \frac{5}{10} \Rightarrow r = \frac{h}{2}$$

$$V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi\frac{h^2}{4}\cdot h \Rightarrow V = \frac{\pi}{12}\cdot h^3$$

$$\Rightarrow \frac{dV}{dt} = \frac{\pi}{4}\cdot h^2\frac{dh}{dt} \Rightarrow 1.5 = \frac{\pi}{4}(4)^2\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{3}{8\pi}\text{cm/min.}$$

3. **Ans. (4.00)**



$$\frac{r}{h} = \tan\frac{\pi}{4} \Rightarrow r = h$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi r^3$$

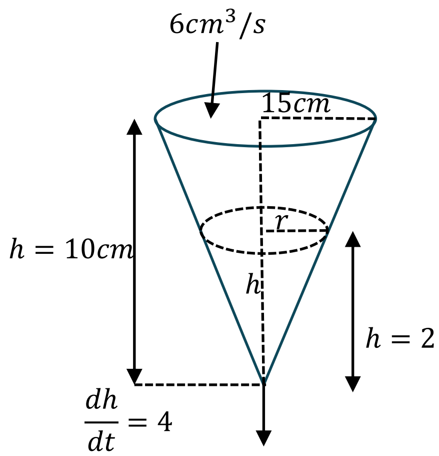
$$\left[\text{At } \ell = 4;\ h = \ell\cos\frac{\pi}{4} = \frac{\ell}{\sqrt{2}};\ r = \ell\sin\frac{\pi}{4} = \frac{\ell}{\sqrt{2}}\right]$$

$$\Rightarrow \frac{dV}{dt} = \pi\cdot r^2\cdot\frac{dr}{dt} \Rightarrow \frac{dV}{dt} = \pi\cdot\left(\frac{4}{\sqrt{2}}\right)^2\cdot\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{4\pi}$$

$$\text{Also, } r = \frac{\ell}{\sqrt{2}} \Rightarrow \frac{dr}{dt} = \frac{1}{\sqrt{2}}\frac{d\ell}{dt} \Rightarrow \frac{d\ell}{dt} = \frac{\sqrt{2}}{4\pi}\text{cm/s.}$$

4. **Ans. (37.00)**

$V = \frac{\pi r^2 h}{3}$ $\frac{r}{h} = \frac{15}{10}$ $\Rightarrow$ $r = \frac{3h}{2}$



$\Rightarrow$ $V = \frac{\pi h}{3}\left(\frac{3h}{2}\right)^2$ $\Rightarrow$ $V = \frac{3\pi h^3}{4}$

$\frac{dV}{dt} = \frac{3\pi}{4}\cdot(3h^2)\frac{dh}{dt}$, $\frac{dV}{dt} = C-1, \frac{dh}{dt} = 4$

$\Rightarrow$ $C-1 = \left(\frac{3\pi h^2}{4}\right)(3\cdot 4)$

$\Rightarrow 1 + 9\pi\ (2)^2 \Rightarrow C = 1 + 36\pi$

Note : For water level to rise, inlet rate (c cm³/&) must be more than outlet rate (1 cm³/s)

5. **Ans. (48.00)**

$\Rightarrow$ $h = \frac{r}{6}$; $V = \frac{1}{3}\pi r^2 h$

$\therefore$ $V = \frac{1}{3}\cdot\pi\cdot(36h^2)\cdot h \Rightarrow$ $V = 12\pi h^3 \Rightarrow$ $\frac{dV}{dt} = 36\pi h^2\frac{dh}{dt}$

$\Rightarrow$ $12 = 36\cdot\pi\cdot 16\cdot\frac{dh}{dt} \Rightarrow$ $\frac{dh}{dt} = \frac{1}{48\pi}$ cm/s

6. **Ans. (0.25)**

At, $t = 0$, $r = 0$ (blot starts), $\frac{dA}{dt} = 2 \rightarrow dA = 2dt$

$A = 2t$ $\quad$ ($\because$ $A = 0$ at $t = 0$)

At, $t = \frac{28}{11}$, $A = \frac{2.28}{11} = \frac{56}{11}cm^2$

$A = \pi r^2$

$A = \frac{56}{11} = \frac{22}{7}\cdot r^2 \Rightarrow r = \frac{14}{11}$; $\frac{dA}{dt} = 2$ (given)

$\Rightarrow$ $\frac{dA}{dt} = 2\pi r\frac{dr}{dt} \Rightarrow$ $2 = \frac{2.22}{7}\cdot\frac{14}{11}\frac{dr}{dt}$ $\Rightarrow$ $\frac{dr}{dt} = \frac{1}{4}$

7. **Ans. (6.00)**

(1, 2) lies on $y = ax^2 + bx + \frac{7}{2}$ $\Rightarrow$ $\boxed{a + b + \frac{3}{2} = 0}$

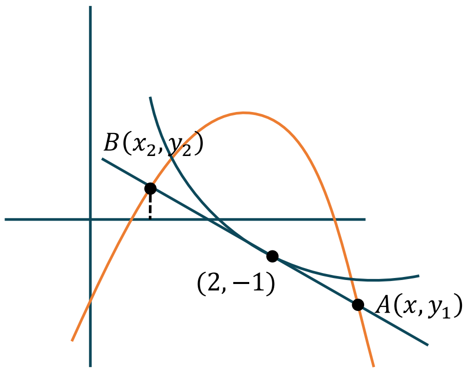
$y = ax^2 + bx + \frac{7}{2}$ $\Rightarrow y' = 2ax + b \Rightarrow y' = 2a + b$ at (1, 2)

$y = x^2 + 6x + 10$ $\Rightarrow$ $y' = 2x + b \Rightarrow y' = -4 + 6 = 2$ at (–2, 2)

$\Rightarrow$ $m_N = \frac{-1}{2}$

$\therefore$ $\boxed{2a + b = \frac{-1}{2}}$ solving, we get $a = 1$, $b = \frac{-5}{2}$

8. **Ans. (1.00)**



$y = \frac{1}{1-x}$ at $x = 2, y = -1$

$\frac{dy}{dx} = \frac{1}{(1-x)^2}$

$\left.\frac{dy}{dx}\right|_{(2,-1)} = 1$ ...(1)

equation of tangent

$y + 1 = 1\,(x - 2)$

$y = x - 3$ ...(2)

curve

$y = -a^2x^2 + 5ax - 4$

$x - 3 = -a^2x^2 + 5ax - 4$

$a^2x^2 + x(1 - 5a) + 1 = 0 <^{x_1}_{x_2}$ $\frac{x_1 + x_2}{2} = 2 \Rightarrow x_1 + x_2 = 4$

for a real chord root of the above quadratic must be real and distinct

$\therefore$ D of the above quadratic must be positive

$(5a - 1)^2 - 4a^2 > 0$

$21a^2 - 10a + 1 > 0$

$(7a - 1)(3a - 1) > 0$

$\therefore a > \frac{1}{3}$ or $a < \frac{1}{7}$

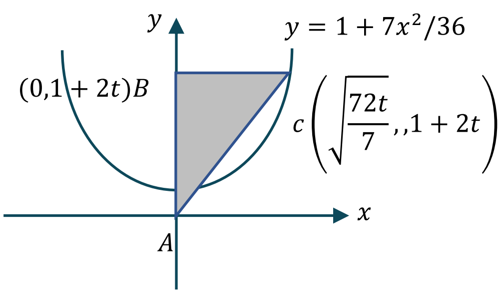
$x_1 + x_2 = \left(\frac{1-5a}{a^2}\right)$

$4 = \frac{5a-1}{a^2} \Rightarrow 4a^2 - 5a + 1 = 0$

$a = \frac{5 \pm \sqrt{25 - 4.4}}{2.4}$

$a = \frac{5 \pm 3}{8} \Rightarrow a = 1, \frac{1}{4}$ $\left(a = \frac{1}{4}\right.$ is rejected from $\left.D > 0\right)$ $\therefore a = 1$

9. **Ans. (9.42 or 9.43)**



$1 + 2t = 1 + \frac{7x^2}{36}$

$\Rightarrow x^2 = \left(\frac{72t}{7}\right)$

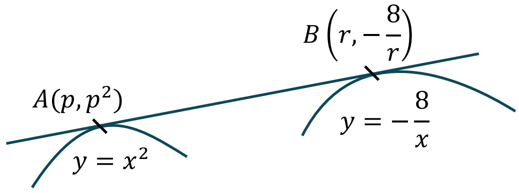
$\Rightarrow x = \sqrt{\frac{72t}{7}}$

$A = \frac{1}{2}(1+2t)\left(\sqrt{\frac{72t}{7}}\right)$

$\frac{dA}{dt} = \frac{1}{2}\left(2\cdot\sqrt{\frac{72t}{7}} + (1+2t)\sqrt{\frac{72}{7}}\cdot\frac{1}{2\sqrt{t}}\right)$

$\Rightarrow \frac{dA}{dt} = \frac{1}{2}\left(2.6 + \frac{4.12}{7}\right) = 6 + \frac{24}{7} = \frac{66}{7}$

10. Ans. (20.00)



where $q = p^2$, $s = -8/r$

$$\left.\frac{dy}{dx}\right|_A = 2x = 2p \qquad \text{....(i)}$$

Also, $\left.\frac{dy}{dx}\right|_B = \frac{8}{x^2} = \frac{8}{r^2}$

$$\therefore \ m_{AB} = 2p = \frac{8}{r^2} = \frac{p^2+\frac{8}{r}}{p-r}$$

$$\Rightarrow \ 2p^2 - 2pr = p^2 + \frac{8}{r} \ \Rightarrow \ p^2 = 2pr + \frac{8}{r}$$

$$\Rightarrow \ p^2 = \frac{16}{r} \ \Rightarrow \ \frac{16}{r^4} = \frac{16}{r} \ \Rightarrow r = 1$$

$$\left.\begin{array}{l}\therefore p = 4 \\ \therefore q = 16\end{array}\right\} p + q = 20$$

EXERCISE - JEE (Main) PYQ

1. Ans. (1)

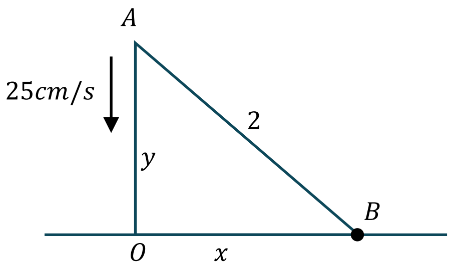
$$y = xe^{x^2} \Rightarrow y' = e^{x^2} + x.e^{x^2}(2x)$$

At $x = 1$: $y' = e + 1.e.(2) \Rightarrow y' = 3e$

Equation of tangent : $y - e = 3e\,(x - 1)$

2. Ans. (3)

$$x^2 + y^2 = 4 \left(\frac{dy}{dt} = -25\right)$$



$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

$$100\sqrt{3}\frac{dx}{dt} - 100(25) = 0$$

$$\frac{dx}{dt} = \frac{25}{\sqrt{3}}\ cm/\sec$$

3. Ans. (2)

$x^2 + 2xy - 3y^2 = 0 \Rightarrow (x + 3y)(x - y) = 0$

(2, 2) lies on $x - y = 0$

Normal at (2, 2) to the line $y = x$ is $x + y = 4$

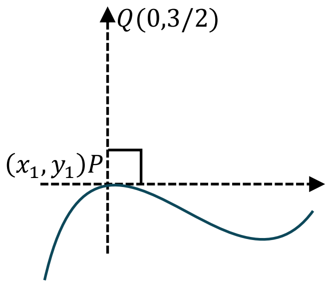
Distance from (0, 0) is $\left|\frac{0+0-4}{\sqrt{2}}\right| = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

4. Ans. (4.00)

$y^2 - 3x^2 + y + 10 = 0$

$\Rightarrow y' = \frac{6x}{2y+1}$

Let $P(x_1, y_1) \Rightarrow m_N = -\left(\frac{2y_1+1}{6x_1}\right)$



$m_N = m_{PQ} \Rightarrow -\left(\frac{2y_1+1}{6x_1}\right) = \frac{\frac{3}{2} - y_1}{0 - x_1}$

$\Rightarrow 2y_1 + 1 = \frac{18}{2} - 6y_1 \Rightarrow y_1 = 1$

Put $y_1 = 1$ in given curve : $1 - 3x_1^2 + 1 + 10 = 0$

$\Rightarrow x_1 = \pm 2$

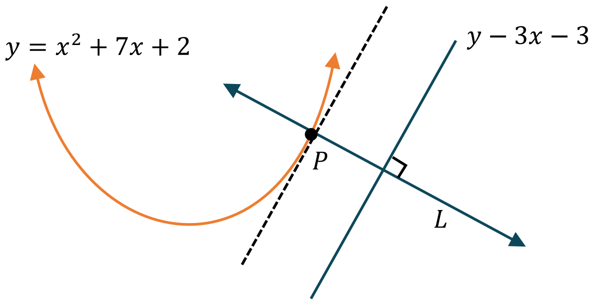
$\therefore |y'| = \left|\frac{6(\pm 2)}{2(1)+1}\right| = 4$

5. Ans. (4)

Let L be the common normal to parabola $y = x^2 + 7x + 2$ and line $y = 3x - 3$

$\Rightarrow$ slope of tangent of $y = x^2 + 7x + 2$ at $P = 3$

$\Rightarrow \left.\frac{dy}{dx}\right]_{For\ P} = 3$



$\Rightarrow 2x + 7 = 3 \Rightarrow x = -2 \Rightarrow y = -8$

So $P(-2, -8)$

Normal at $P : x + 3y + C = 0$

$\Rightarrow C = 26$ (P satisfies the line)

$\boxed{\text{Normal} : x + 3y + 26 = 0}$

6. Ans. (2)

Given equation of curve

$y = (1 + x)^{2y} + \cos^2(\sin^{-1}x)$

at $x = 0$

$y = (1 + 0)^{2y} + \cos^2(\sin^{-1}0)$

$y = 1 + 1 = 2$

So, we have to find the normal at (0, 2)

Now $y = e^{2y\ell n(1+x)} + \cos^2\left(\cos^{-1}\sqrt{1-x^2}\right)$

$y = e^{2y\ell n(1+x)} + \left(\sqrt{1-x^2}\right)^2$

$y = e^{2y\ell n(1+x)} + (1 - x^2)$ ...(1)

Now differentiate w.r.t. x

$$y' = e^{2y\ell n(1+x)}\left[2y\cdot\left(\frac{1}{1+x}\right) + \ell n(1+x).2y'\right] - 2x$$

Put $x = 0$ & $y = 2$

$$y' = e^{2\times 2\ell n1}\left[2\times 2\left(\frac{1}{1+0}\right) + \ell n(1+0).2y'\right] - 2\times 0$$

$y' = e^0\,[4 + 0] - 0$

$y' = 4$ = slope of tangent to the curve

so slope of normal to the curve = $-\dfrac{1}{4}$ $\{m_1m_2 = -1\}$

Hence equation of normal at (0, 2) is

$$y - 2 = -\frac{1}{4}(x - 0) \Rightarrow 4y - 8 = -x \Rightarrow x + 4y = 8$$

7. Ans. (2)

$x^4e^y + 2\sqrt{y+1} = 3$

d.w.r. to x: $x^4\, e^y\, y' + e^y\, 4x^3 + \dfrac{2y'}{2\sqrt{y+1}} = 0$

at $P(1, 0)$

$y'_P + 4 + y'_P = 0 \Rightarrow y'_P = -2$

Tangent at $P(1, 0)$ is

$y - 0 = -2\,(x - 1) \Rightarrow 2x + y = 2$

(–2, 6) lies on it

8. Ans. (3)

$f(x) = x\log_e x$

$$f'(x)\big|_{(c,f(c))} = \frac{e-0}{e-1}$$

$f'(x) = 1 + \log_e x$

$$f'(x)\big|_{(c,f(c))} = 1 + \log_e c = \frac{e}{e-1}$$

$$\log_e c = \frac{e-(e-1)}{e-1} = \frac{1}{e-1} \Rightarrow c = e^{\frac{1}{e-1}}$$

9. Ans. (3)

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1,\ x^2 + y^2 = ab$

$$\frac{2x_1}{a^2} + \frac{2y_1y'}{b^2} = 0$$

$$\Rightarrow y_1' = \frac{-x_1}{a^2}\frac{b^2}{y_1} \quad ...(1)$$

$\therefore\ 2x_1 + 2y_1\,y' = 0$

$$\Rightarrow y_2' = \frac{-x_1}{y_1} \quad ...(2)$$

Here $(x_1 y_1)$ is point of intersection of both curves

$$\therefore\ x_1^2 = \frac{a^2 b}{a+b},\ y_1^2 = \frac{ab^2}{a+b}$$

$$\therefore\ \tan\theta = \left|\frac{y_1' - y_2'}{1 + y_1' y_2'}\right| = \left|\frac{\dfrac{-x_1 b^2}{a^2 y_1} + \dfrac{x_1}{y_1}}{1 + \dfrac{x_1^2 b^2}{a^2 y_1^2}}\right| \Rightarrow \tan\theta = \left|\frac{-b^2 x_1 y_1 + a^2 x_1 y_1}{a^2 y_1^2 + b^2 x_1^2}\right|$$

$$\tan\theta = \left|\frac{a-b}{\sqrt{ab}}\right|$$

10. **Ans. (3)**

$a + b + c = 2$ ...(1)

and $\left.\dfrac{dy}{dx}\right|_{(0,0)} = 1$

$\left. 2ax + b \right|_{(0,0)} = 1$

$b = 1$

Curve passes through origin

So, $c = 0$

and $a = 1$

11. **Ans. (195)**

$y = 5x^2 + 2x - 25$ P(2, –1)

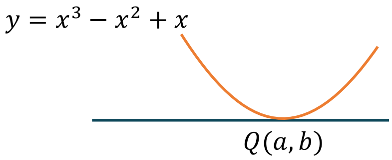
$y' = 10x + 2$

$y'_P = 22$

$\therefore$ tangent to curve at P

$y + 1 = 22(x - 2)$

$y = 22x - 45$



$\left.\dfrac{dy}{dx}\right|_{C_2} = 3x^2 - 2x + 1$

$\left.\dfrac{dy}{dx}\right|_{Q} = 3a^2 - 2a + 1$

Hence $3a^2 - 2a + 1 = 22$

$\therefore\ 3a^2 - 2a - 21 = 0$

$3a^2 - 9a + 7a - 21 = 0$

$(3a + 7)(a - 3) = 0$

from curve $b = a^3 - a^2 + a$

$a = 3$

$b = 21$ $|2a + 9b| = 195$

at $a = -7/3$ tangent will be parallel

Hence it is rejected

12. Ans. (1)

Slope of tangent at $P(t, t^3)$ = $\left.\frac{dy}{dx}\right]_{(t,t^3)}$

= $(3x^2)_{x=t} = 3t^2$

So equation tangent at $P(t, t^3)$:

$y - t^3 = 3t^2(x - t)$

for point of intersection with $y = x^3$

$x^3 - t^3 = 3t^2x - 3t^3 \Rightarrow (x - t)(x^2 + xt + t^2) = 3t^2(x - t)$

for $x \neq t$

$x^2 + xt + t^2 = 3t^2 \Rightarrow x^2 + xt - 2t^2 = 0 \Rightarrow (x - t)(x + 2t) = 0$

So for $Q : x = -2t, Q(-2t, -8t^3)$

ordinate of required point : $\frac{2t^3 - 8t^3}{2+1} = -2t^3$

13. Ans. (1)

Let r be the radius of spherical balloon

S = Surface area

$S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r \times \frac{dr}{dt} = k$ (constant)

$4\pi r^2 = kt + C$ (C is constant of integration)

For $t = 0$, $r = 3 \Rightarrow 36\pi = C$

For $t = 5$, $r = 7 \Rightarrow K = 32\pi$

$4\pi r^2 = 32\pi t + 36\pi$

$r^2 = 8t + 9$

for $t = 9$

$r^2 = 81 \Rightarrow r = 9.$

14. Ans. (4)

The tangent at (x_1, y_1) to the curve

$y = x^3 + 3x^2 + 5$

$y - y_1 = (3x_1^2 + 6x_1)(x - x_1)$ passing through origin

$-y_1 = (3x_1^3 + 6x_1)(-x_1)$

$y_1 = (3x_1^3 + 6x_1^2)$ ...(1)

And (x_1, y_1) lies on the curve

$y = x^3 + 3x^2 + 5$

$y_1 = x_1^3 + 3x_1^2 + 5$ ...(2)

From equation (1) and (2)

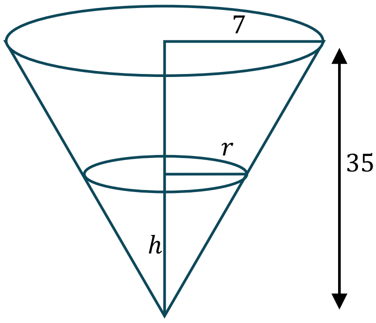
$2y_1 = 3x_1^2 + 15$

Hence the equation of curve $y=\frac{3}{2}x^2+\frac{15}{2}$

This curve does not intersect $\frac{x}{3}-y^2=2$

15. Ans. (3)

From figure $\frac{r}{h}=\frac{7}{35}\Rightarrow h=5r$



Given $\frac{dV}{dt}=1\Rightarrow\frac{d}{dt}\left(\frac{\pi r^2h}{3}\right)=1$

$\Rightarrow\frac{d}{dt}\left(\frac{5\pi}{3}r^3\right)=1\Rightarrow r^2\frac{dr}{dt}=\frac{1}{5\pi}$

Let wet conical surface area = S

$=\pi r\ell=\pi r\sqrt{h^2+r^2}$ $=\sqrt{26}\pi r^2\Rightarrow\frac{dS}{dt}=2\sqrt{26}\pi r\frac{dr}{dt}$

When h = 10 then r = 2 $\Rightarrow\frac{dS}{dt}=\frac{2\sqrt{26}}{10}$

16. Ans. (3)

$\frac{dy}{dx}=\frac{2(1+\sin t)\times\cos t}{1+\cos 2t}$

$\Rightarrow\frac{2(1+\sin t)\cos t}{2\cos^2 t}=\sqrt{3}$ $\Rightarrow t=\frac{\pi}{6},y_0=27$

17. Ans. (5)

$\tan\theta=\frac{3}{4}=\frac{r}{h}$

$\frac{dV}{dt}=6$

$V=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi h^3\tan^2\theta=\frac{9\pi}{48}h^3=\frac{3\pi}{16}h^3$

$\Rightarrow\frac{dV}{dt}=\frac{3\pi}{16}.3h^2.\frac{dh}{dt}=6\Rightarrow\left(\frac{dh}{dt}\right)_{h=4}=\frac{2}{3\pi}m/hr$

Now, S = $\pi r\ell$ = $\frac{15}{16}\pi h^2$

$\Rightarrow\frac{dS}{dt}=\frac{15\pi}{16}.2h\frac{dh}{dt}\Rightarrow\left(\frac{dS}{dt}\right)_{h=4}=5m^2/hr$

18. Ans. (4)

y = $\frac{x-a}{(x+b)(x-2)}$

At point (1, –3),

$-3=\frac{1-a}{(1+b)(1-2)}$

$\Rightarrow$ 1 – a = 3(1 + b) .....(1)

Now, y = $\frac{x-a}{(x+b)(x-2)}$ $\Rightarrow\frac{dy}{dx}=\frac{(x+b)(x-2)\times(1)-(x-a)(2x+b-2)}{(x+b)^2(x-2)^2}$

At (1, – 3) slope of normal is $\frac{1}{4}$ hence $\frac{dy}{dx}$ = –4,

So, $-4=\frac{(1+b)(-1)-(1-a)b}{(1+b)^2(-1)^2}$

Using equation (1)

$\Rightarrow -4=\frac{(1+b)(-1)-3(b+1)b}{(1+b)^2} \Rightarrow -4=\frac{(-1)-3b}{(1+b)}\ (b\neq -1)$

$\Rightarrow b = -3$ So, $a = 7$

Hence, $a + b = 7 - 3 = 4$

19. **Ans. (3)**

Normal of line is parallel to line $x + 90y + 2 = 0$

$m_N=-\frac{1}{90}$

$-\left(\frac{dx}{dy}\right)_{(x_1y_1)}=-\frac{1}{90} \quad \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1y_1)}=90$

Now,

$\frac{dy}{dx}=270x^4-540x^3-210x^2+360x+210=90$

$\Rightarrow x=1,\ 2,\ \frac{-2}{3},\ \frac{-1}{3}$

(4) normal

20. **Ans. (11)**

$f(x) = (x + 1)(ax + b)$

$1 = 2a + 2b$ ...(1)

$f'(x) = (ax + b) + a(x + 1)$

$1 = (3a + b)$ ...(2)

$\Rightarrow b = 1/4, a = 1/4$

$f(x)=\frac{(x+1)^2}{4}$

$\alpha+1=\frac{(\alpha+1)^2}{4}$, $\alpha > -1$

$\alpha + 1 = 4$

$\alpha = 3$

$f'(x)=\frac{x}{2}+\frac{1}{2}$

$f'(3) = \frac{3}{2}+\frac{1}{2} = 2$ = slope of tangent

Slope of normal = $-\frac{1}{2}$

normal at (3, 4)

$y-4=-\frac{1}{2}(x-3)$

for x-intercept

$y = 0$; $\quad x = 8 + 3 = 11$

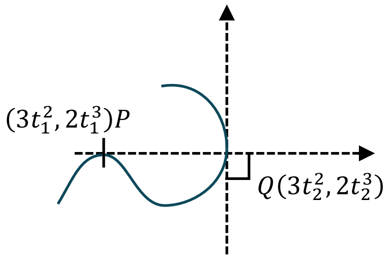
EXERCISE - JEE (Advanced) PYQ

1. **Ans. ($\sqrt{2}\,x - y - 2\sqrt{2} = 0$)**

Tangent at P is normal at Q.

$$\left.\frac{dy}{dx}\right|_P = -\left.\frac{dx}{dy}\right|_Q = m_{PQ} \Rightarrow \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_P = \left.\frac{-\frac{dx}{dt}}{\frac{dy}{dt}}\right|_Q = m_{PQ}$$

$$\Rightarrow \frac{6t_1^2}{6t_1} = -\frac{6t_2}{6t_2^2} = \frac{2t_2^3 - 2t_1^3}{3t_2^2 - 3t_1^2}$$



$$\Rightarrow t_1 = -\frac{1}{t_2} = \frac{2}{3}\frac{(t_1^2 + t_2^2 + t_1t_2)}{(t_1 + t_2)}$$

$t_1t_2 = -1$ ....(1)

and $3t_1^2 + 3t_1t_2 = 2t_1^2 + 2t_2^2 + 2t_1t_2$ (by I and III term)

$\Rightarrow t_1^2 + t_1t_2 - 2t_2^2 = 0 \Rightarrow (t_1 - t_2)(t_1 + 2t_2) = 0 \Rightarrow t_1 + 2t_2 = 0$ or $t_1 = t_2$ (reject)

$\Rightarrow t_1 = -2t_2 \rightarrow$ Put in (1)

$\Rightarrow -2t_2^2 = -1 \Rightarrow \quad t_1 = -\sqrt{2}\,;\, t_1 = \sqrt{2}$

$$t_2 = \frac{1}{\sqrt{2}} \text{ or } t_2 = -\frac{1}{\sqrt{2}}$$

At $t_1 = -\sqrt{2}\,;\, t_2 = \dfrac{1}{\sqrt{2}} \Rightarrow P(6, -4\sqrt{2});\ Q\left(\dfrac{3}{2}, \dfrac{1}{\sqrt{2}}\right)$

$\Rightarrow$ Equation of PQ : $\sqrt{2}\,x + y - 2\sqrt{2} = 0$ (Ans.)

At $t_1 = \sqrt{2}\,;\, t_2 = -\dfrac{1}{\sqrt{2}} \Rightarrow P(6,\ 4\sqrt{2});\ Q\left(\dfrac{3}{2}, -\dfrac{1}{\sqrt{2}}\right)$

$\Rightarrow$ Equation of PQ : $\sqrt{2}\,x - y - 2\sqrt{2} = 0$ (Ans.)

2. **Ans. (D)**

$$m_N = \tan\left(\frac{3\pi}{4}\right)$$

$\Rightarrow m_N = -1 \Rightarrow \quad m_T = 1 = f'(3)$

3. **Ans. (D)**

$y^3 + 3x^2 = 12y$

$\Rightarrow y' = \dfrac{6x}{12 - 3y^2} \Rightarrow y' = \dfrac{2x}{4 - y^2}$ = DNE at $y = \pm 2$

$\therefore$ At $\quad y = 2 : 8 + 3x^2 = 24$

$$\Rightarrow x = \pm\frac{4}{\sqrt{3}}$$

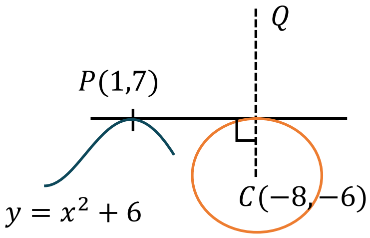
At $y = -2 : -8 + 3x^2 = -24$

$\Rightarrow 3x^2 = -16$ (not possible)

$$\therefore \left(\pm\frac{4}{\sqrt{3}}, 2\right)$$

4. Ans. (D)

Q is point of intersection of tangent at P and line $\perp$ to this tangent and passing through $C(-8, -6)$



$y = x^2 + 6$

$\Rightarrow y' = 2x|_P$

$\Rightarrow y' = 2 = m_T$ at P.

$\therefore$ Equation of tangent : $y - 7 = 2(x - 1)$

$\Rightarrow 2x - y + 5 = 0$ ...(1)

Also $m_{line\ CQ} = -\dfrac{1}{2}$

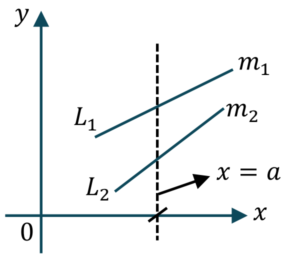
$\Rightarrow$ Equation of CQ : $y + 6 = \left(-\dfrac{1}{2}\right)(x + 8)$

$\Rightarrow x + 2y + 20 = 0$ ....(2)

Solving (1) and (2), we get $Q(-6, -7)$.

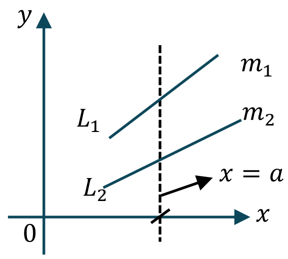
5. Ans. (A)

Concept:



Here $m_1 < m_2$

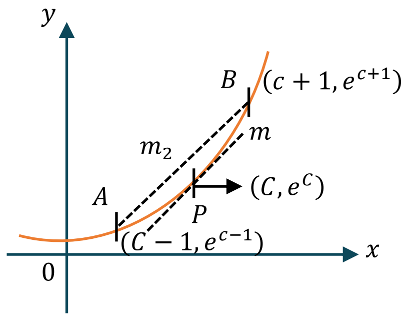
L_1 and L_2 intersect behind $x = a$



Here $m_1 > m_2$

L_1 and L_2 intersect beyond $x = a$

$y = e^x \Rightarrow y' = e^x$



$m_1 = e^C$

$m_2 = \dfrac{e^{C+1} - e^{C-1}}{C+1-(C-1)}$

$\Rightarrow m_2 = e^C \dfrac{\left(e - \dfrac{1}{e}\right)}{2}$

Put $e \approx 2.71$

$\Rightarrow m_2 > m_1$

$\Rightarrow$ lines intersect to left of C (As case I above)

6. Ans. (8)

$\because (y - x^5)^2 = x(1 + x^2)^2$

Differenciate w.r.t. (x)

$\Rightarrow 2(y - x^5) \cdot [y' - 5x^4] = 1 \cdot (1 + x^2)^2 + x \cdot 2(1 + x^2) \cdot [2x]$

Now, put $x = 1, y = 3$

$\Rightarrow 2(3 - 1) \cdot [y' - 5] = 1 \cdot (1 + 1)^2 + 1 \cdot 2(1 + 1) \cdot [2]$

$\Rightarrow 4 \cdot (y' - 5) = 4 + 8 \Rightarrow y' - 5 = 3 \Rightarrow y' = 8 \Rightarrow \left.\dfrac{dy}{dx}\right)_{(1,3)} = 8$

JEE (Main) Practice Paper

SECTION–A

1. Ans. (1)

$\frac{dy}{dx} = -\frac{1}{2\sqrt{x}} \quad \Rightarrow \quad \left.\frac{dy}{dx}\right|_{(1,\,1)} = -\frac{1}{2}$

Slope of normal = 2

Equation of normal is $2x - y = 1$

2. Ans. (3)

$\left.\frac{dy}{dx}\right]_{(3,0)}$ is 4 $\quad \Rightarrow \quad \theta = \tan^{-1} 4$

3. Ans. (3)

$\frac{dy}{dx} = 5x^4$

Equation of tangent at (h, k) is $\frac{y-k}{x-h} = 5\,h^4$ Which passes through (2,2)

$\Rightarrow (2 - k) = 5h^4\,(2 - h) \Rightarrow -h^5 = 10h^4 - 5h^5 \quad \Rightarrow 4h^5 = 10h^4 \quad \Rightarrow h = 0, \frac{5}{2}$

$\Rightarrow$ Equation of tangents are $y - 2 = 0$ and $16(y - 2) = 5^5\,(x - 2)$

4. Ans. (2)

Equation of tangent is

$y - 4/h = -4/h^2\,(x - h)$

It passes through (0, 1) $\quad \Rightarrow 1 - 4/h^2 = 4h/h^2$

$\Rightarrow h - 4 = 4 \Rightarrow \quad h = 8 \quad\quad \Rightarrow$ tangent is $y - 1/2 = -1/16\,(x - 8)$.

5. Ans. (3)

$y - e^{xy} + x = 0$

Differentiating w.r.t. to y

$1 - e^{xy}\left(\frac{dx}{dy}.\;\; y + x\right) + \frac{dx}{dy} = 0$

$\frac{dx}{dy} = 0$

$1 - xe^{xy} = 0$

$xe^{xy} = 1 \quad\quad\quad x = 1, y = 0$

Point is (1, 0)

6. Ans. (4)

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a(-\sin\theta)}{a(1+\cos\theta)}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \frac{-\sqrt{3}}{3} = -\frac{1}{\sqrt{3}}$$

$\tan\alpha = -\frac{1}{\sqrt{3}} \quad \Rightarrow \quad \alpha = \pi - \frac{\pi}{6}$

$\alpha = \frac{5\pi}{6}$

7. Ans. (2)

$$\left.\frac{dy}{dx}\right]_{(3t,4/t)} = \left.\frac{-12}{x^2}\right]_{(3t,4/t)} = \frac{-4}{3t^2}$$

equation of normal $\frac{y-4/t}{x-3t} = \frac{3t^2}{4}$ which passes through $(3t_1, 4/t_1)$

$$\Rightarrow \frac{4}{t_1} - \frac{4}{t} = \frac{3t^2}{4}(3t_1 - 3t) \Rightarrow \frac{4}{tt_1} = \frac{-9t^2}{4} \qquad \Rightarrow t_1 = \frac{-16}{9t^3}$$

8. Ans. (1)

$\frac{dy}{dx}$ for $y = x^2 + \frac{1}{x}$ is $2x - \frac{1}{x^2}$

$\frac{dy}{dx}$ at (h, k) is $2h - \frac{1}{h^2}$

equation of tangent at (h, k) is $\frac{y-(h^2+1/h)}{x-h} = 2h - \frac{1}{h^2}$

$$y = \left(2h - \frac{1}{h^2}\right)x - 2h^2 + \frac{1}{h} + h^2 + \frac{1}{h}$$

$$y = \left(2h - \frac{1}{h^2}\right)x + \frac{2}{h} - h^2 \Rightarrow \frac{2}{h} - h^2 = \frac{1}{2h - 1/h^2} \qquad \Rightarrow \frac{2-h^3}{h} = \frac{h^2}{2h^3-1}$$

$\Rightarrow h^6 - 2h^3 + 1 = 0 \qquad \Rightarrow (h^3 -1)^2 = 0 \Rightarrow (h -1)^2 (h^2 + h +1)^2 = 0 \Rightarrow h =1$

$\Rightarrow$ equation of tangent is $y = x + 1$

9. Ans. (1)

$1 = 1 + b + c \qquad \Rightarrow c = -b$

$\frac{dy}{dx} = 2x + b = 2 + b$ at $(1, 1)$

equation of tangent is $y -1 = (2 + b)(x -1) \Rightarrow x$ intercept $= \frac{b+1}{b+2}$

y intercept $= -(b + 1) \Rightarrow \left|\frac{1}{2} \times \left(\frac{b+1}{b+2}\right) \times (b+1)\right| = 2$

$b^2 + 2b + 1 = 4b + 8$ or $-4b - 8$

$b^2 - 2b - 7 = 0$ or $b^2 + 6b + 9 = 0 \Rightarrow$ integral $b = -3$

10. Ans. (2)

For C_1, $\left.\frac{dy}{dx}\right|_{(0,1)} = a^x \ell na\Big|_{(0,1)} = \ell na$

For C_2, $\left.\frac{dy}{dx}\right|_{(0,1)} = b^x \ell nb\Big|_{(0,1)} = \ell nb$

$\Rightarrow$ angle between curve is $\left|\frac{\log a - \log b}{1 + \log a \log b}\right| = \left|\frac{\log(a/b)}{1 + \log a \log b}\right|$

11. Ans. (4)

Both curves are confocal, where focus of both curves is $(\sqrt{24},0)$ and $(-\sqrt{24},0)$

$\Rightarrow$ angle between curves $x^2 + 4y^2 = 32$ and $x^2 - y^2 = 12$ is $\frac{\pi}{2}$

12. Ans. (4)

$$\left.\frac{dy}{dx}\right]_{C_1} = \frac{x^2-y^2}{2xy}, \left.\frac{dy}{dx}\right]_{C_2} = \frac{-2xy}{x^2-y^2}$$

$$\Rightarrow \left.\frac{dy}{dx}\right]_{C_1} \times \left.\frac{dy}{dx}\right]_{C_2} = -1$$

13. Ans. (3)

For C_1, $\frac{dy}{dx} = \frac{-4x}{a^2 y}$ and For C_2, $\frac{dy}{dx} = \frac{16}{3y^2}$

$$\Rightarrow \frac{-4x}{a^2 y}\times\frac{16}{3y^2} = -1 \qquad \Rightarrow \left(\frac{4}{3a^2}\right)\times\left(\frac{16x}{y^3}\right) = -1 \qquad \Rightarrow a^2 = 4/3$$

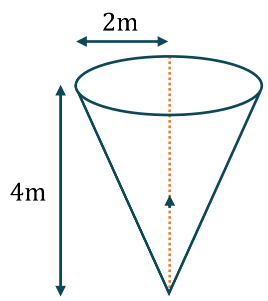
14. Ans. (2)

$$V = \frac{1}{3}\pi r^2 h \qquad \left(\because \quad \frac{r}{h} = \frac{2}{4} = \frac{1}{2}\right)$$

$$V = \frac{1}{3}\pi\frac{h^3}{4} = \frac{\pi}{12}h^3$$

$$77 \times 10^3 = \frac{22}{7}\times\frac{1}{4} \times 70 \times 70 \times \frac{dh}{dt} \qquad (\because 1 \text{ litre} = 10^3\, c.c.)$$

$$\therefore \frac{dh}{dt} = 20 \text{ cm/min.}$$



15. Ans. (3)

$x^3 = 12y$

$$3x^2 \frac{dx}{dt} = 12\frac{dy}{dt} \Rightarrow \frac{dx}{dt} > \frac{dy}{dt}$$

$$\Rightarrow 12\frac{dy}{dt}\cdot\frac{1}{3x^2} > \frac{dy}{dt}$$

$$\Rightarrow x^2 < 4 \Rightarrow x \in (-2, 2)$$

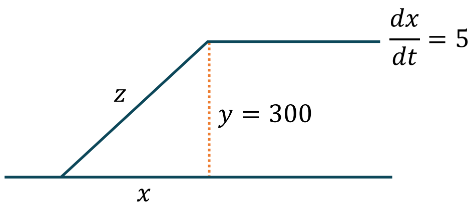
16. Ans. (1)

From figure $z^2 = x^2 + y^2$

$$z\frac{dz}{dt} = x\frac{dx}{dt}$$

If $z = 500$ then $x = 400$

$$\Rightarrow 500\frac{dz}{dt} = 400(5) \Rightarrow \frac{dz}{dt} = 4$$



Figure

17. Ans. (2)

Let $y = \tan x$

$\Delta y = \tan(x + \Delta x) - \tan x \Rightarrow \frac{dy}{dx}\Delta x = \tan(x + \Delta x) - \tan x \Rightarrow (\sec^2 x)\Delta x = \tan(x + \Delta x) - \tan x$

put $x = 45°, \Delta x = 1° \Rightarrow \frac{2\pi}{180} = \tan 46° - 1 \Rightarrow \tan 46° = 1 + \frac{\pi}{90}$

18. Ans. (3)

$V = 4\frac{\pi}{3}(10+r)^3,\ 0 \le r \le 15$

$\because \frac{dV}{dt} = -50.$

$4\pi(10+r)^2\frac{dr}{dt} = -50 \Rightarrow \frac{dr}{dt} = \frac{-1}{18\pi}$ (where $r = 5$)

19. Ans. (1)

$f'(0) = \lim_{x\to 0}\frac{\frac{\sin x^2}{x} - 0}{x - 0} = 1$ (slope of tangent)

slope of normal is – 1

Equation of normal is $y - 0 = -(x - 0)$

20. Ans. (4)

$\frac{y}{b} = 1 - \frac{x}{a}$

$\frac{y}{b} = e^{-x/a} \Rightarrow e^{-x/a} = 1 - \frac{x}{a}$

put $t = -\frac{x}{a}$

$e^t = 1 + t$

Draw graph of $y = e^t, y = 1 + t$

It is clear that $t = 0$ is the only Solution

$\Rightarrow x = 0 \Rightarrow y = b$ $(0, b)$

SECTION–B

1. Ans. (1)

For C_1, $\left.\frac{dy}{dx}\right|_{(1,0)} = \left(\frac{2^x}{x} + \ell nx.2^x.\ell n2\right)_{(1,0)} = 2$

For C_2, $\left.\frac{dy}{dx}\right|_{(1,0)} = \left(x^{2x}.\ell nx \times 2 + 2x(x)^{2x-1}\right)_{(1,0)} = 2$ $\Rightarrow \theta = 0 \Rightarrow \cos\theta = 1$

2. Ans. (90)

Given curves are $y^2 = 4x + 4$ and $y^2 = 36(9 - x)$ ......(i)

On solving, we get the point (8,6) and (8,–6)

On differentiating equation (i) , we get

$2y\frac{dy}{dx} = 4$ and $2y\frac{dy}{dx} = -36 \Rightarrow \frac{dy}{dx} = \frac{2}{y}$ and $\frac{dy}{dx} = \frac{-18}{y}$

At point (8,6), $m_1 = \frac{dy}{dx} = \frac{1}{3}$ and $m_2 = \frac{dy}{dx} = -3$

$m_1 m_2 = -1$

3. **Ans. (2)**

Let P be perimeter

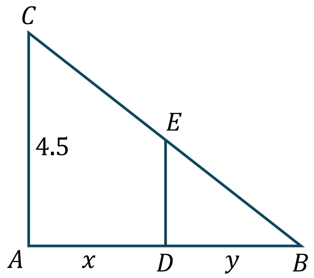
$P = 2x + 2y$

$$\frac{dP}{dt} = 2\frac{dx}{dt} + 2\frac{dy}{dt}$$

$$\frac{dA}{dt} = \frac{dx}{dt}y + x\frac{dy}{dt} - 6 + 4 = -2$$

4. **Ans. (2)**

Let AC be pole, DE be man and B be farther end of shadow as shown in figure



From triangles ABC and DBE

$$\frac{4.5}{x+y} = \frac{1.5}{y}$$

$3y = 1.5\,x$

$$\frac{dy}{dt} = 2$$

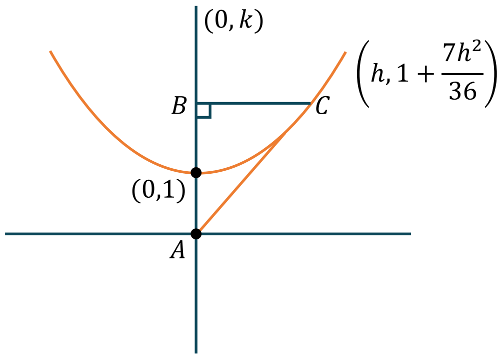
5. **Ans. (66)**

at $t = 0$; $x = 0$, $y = 1$

$$\frac{dy}{dt} = 2\text{cm/sec}$$

$$A = \frac{1}{2}\times h\times\left(1+\frac{7h^2}{36}\right)$$

$$\frac{dA}{dt} = \left(\frac{1}{2}\left(1+\frac{7h^2}{36}\right)+\frac{h}{2}\left(\frac{14h}{36}\right)\right)\frac{dh}{dt}$$;



$$\frac{dA}{dt} = \left(\frac{1}{2}\times 8 + 3\times\frac{14\times 6}{36}\right)\times\frac{6}{7}$$

(At, $t = 7/2$ sec, change in y-co-ordinate = 7 hence, pt. C has

y-co-ordinate = 8 and x- co-ordinate = 6 at $t = 7/2$ sec.) $= (4 + 7)\times\frac{6}{7}\times 2 = \frac{132}{7}$ cm^2/sec

6. **Ans. (1)**

Both curves passes through (1, 2) hence

$2 = 1 - c$ and $2 = 1 + a + b \Rightarrow c = -1$ and $a + b = 1$

Also $3x^2\big|_{x=1} = 2x + a\big|_{x=1}$ $3 = a + 2 \Rightarrow a = 1$ and $b = 0$

Hence $|a| + |b| + |c| = 2$ and $|a + b - c| = 2$

7. Ans. (3)

$x^2 = -4y \Rightarrow 2x = -4\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{-x}{2} \Rightarrow \left(\frac{dy}{dx}\right)_{(-4,-4)} = 2$

We know that equation of tangent is,

$(y - y_1) = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} (x - x_1)$

$\Rightarrow y + 4 = 2(x + 4) \Rightarrow 2x - y + 4 = 0$

8. Ans. (3)

$x = t^2$ and $y = 2t \Rightarrow$ At $t = , x = 1$ and $y = 2$

Now $\left(\frac{dy}{dx}\right) = \frac{dy/dt}{dx/dt} = \frac{2}{2t} = \frac{1}{t} \Rightarrow \left(\frac{dy}{dx}\right)_{t=1} = 1$

$\therefore$ Equation of the normal at (1,2) is $y - 2 = \frac{-1}{\frac{dy}{dx}}(x - 1)$

$\Rightarrow y - 2 = -1\ (x - 1) \Rightarrow x + y - 3 = 0$

9. Ans. (1)

Given curve is $y^4 = ax^3 \Rightarrow 4y^3 = \frac{dy}{dx}3ax^2$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(a,a)} = \frac{3a^3}{4a^3} = \frac{3}{4}$

$\therefore$ Equation of normal at point (a, a) is

$y - a = -\frac{4}{3}\ (x - a) \Rightarrow 4x + 3y = 7a$

10. Ans. (1)

Slope of tangent to the curve $y = x + \sin y$

at (a, b) is $\frac{2-\frac{3}{2}}{\frac{1}{2}-0} = 1$

$\Rightarrow \left.\frac{dy}{dx}\right]_{x=a} = 1$

$\frac{dy}{dx} = 1 + \cos y.\frac{dy}{dx}$ (from equation of curve)

$\left.\frac{dy}{dx}\right]_{x=a} = 1 + \cos b.\left.\frac{dy}{dx}\right]_{x=a}$

$\Rightarrow \cos b = 0$

$\Rightarrow \sin b = \pm 1$

Now, from curve $y = x + \sin y$

$b = a + \sin b$

$\Rightarrow |b - a| = |\sin b| = 1$

JEE (Advanced) Practice Paper

1. Ans. (A)

$x^3 + y^3 = 8xy,\ y^2 = 4x \Rightarrow x^3 + 8x^{3/2} = 8x\ 2x^{1/2}$

$x^3 - 8x^{3/2} = 0 \Rightarrow x^6 - 64x^3 = 0 \Rightarrow x^3 = 0$ or $x^3 = 64$

$x = 4,\ y = 4$. Point of intersection (4, 4)

$3x^2 + 3y^2\ y' = 8y + 8xy'$

$y' = -1$

slope of normal is 1

Equation of normal is $y - 4 = 1\ (x - 4) \Rightarrow x = y$

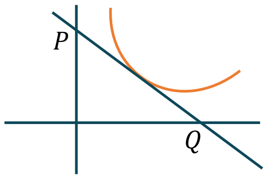
2. Ans. (B)

Here $x^{2/3} + y^{2/3} = a^{2/3}$

Differentiating w.r.t. to x $\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$

$\left.\frac{dy}{dx}\right|_{(h,\ k)} = -\left(\frac{k}{h}\right)^{1/3}$

Equation $(y - k) = -\left(\frac{k}{h}\right)^{1/3}(x - h)$



$P(0,\ k^{1/3}\ (h^{2/3} + k^{2/3}))$ or $P(0,\ a^{2/3}\ k^{1/3})$

And, $\quad Q(h^{1/3}\ a^{2/3}, 0)$

$PQ = \sqrt{h^{2/3}a^{4/3} + a^{4/3}k^{2/3}} = |a| =$ constant.

3. Ans. (C)

The tangent at $(x_1, \sin x_1)$ is $y - \sin x_1 = \cos x_1\ (x - x_1)$

It passes through the origin.

$\sin x_1 = x_1 \cos x_1 = x_1\sqrt{1 - \sin^2 x_1}$

$y_1^2 = \sin^2 x_1 = x_1^2(1 - y_1^2) \Rightarrow \quad (x_1 y_1)$ lies on the curve

$y^2 = x^2(1 - y^2) \qquad \Rightarrow \qquad x^2 - y^2 = x^2y^2$

4. Ans. (B)

$y = e^{\{x\}} = e^{x-a}$ in $x \in [a, a + 1)$

$\frac{dy}{dx} = e^{x-a} = e^{\{x\}}$

equation of tangent $\quad (Y - y) = \frac{dy}{dx}(X - x)$

passing through (–1/2, 0)

$(0 - y) = e^{\{x\}}\ (-1/2 - x)$

$\Rightarrow -1 = -\frac{1}{2} - x \Rightarrow x = \frac{1}{2}$

$\therefore$ point $\left(\frac{1}{2},\ e^{1/2}\right)$

Number of tangent = 1

5. Ans. (B)

Let $y = mx + c$ be tangent touching both branches.

$f(x) = -x^2,\ y = mx + c,\quad x < 0$

$x^2 + mx + c = 0, \quad m > 0 \quad (\because\ x < 0)$ (negative roots)

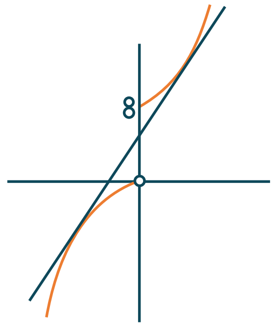
$D = 0 \Rightarrow m^2 = 4c$

$f(x) = x^2 + 8, y = mx + c, x > 0$

$x^2 - mx + 8 - c = 0, \quad m > 0$ (positive roots)

$D = 0 \Rightarrow m^2 = 32 - 4c$

$\Rightarrow c = 4, m^2 = 16 \Rightarrow c = 4, m = 4$



6. Ans. (C)

Let thickness of ice be 'h'.

Vol. of ice = $v = \frac{4\pi}{3}\left((10+h)^3 - 10^3\right)$

$\frac{dv}{dt} = \frac{4\pi}{3}\left(3(10+h)^2\right).\frac{dh}{dt}$

Given $\frac{dv}{dt} = 50cm^3/\text{min}$ and $h = 5$cm

$\Rightarrow 50 = \frac{4\pi}{3}\left(3(10+5)^2\right)\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{50}{4\pi\times 15^2} = \frac{1}{18\pi}\ cm/\text{min}$

7. Ans. (C,D)

$2y^3 = ax^2 + x^3$

$6y^2\ \frac{dy}{dx} = 2ax + 3x^2$

$\left.\frac{dy}{dx}\right|_{(a,\,a)} = \frac{5a^2}{6a^2} = \frac{5}{6}$

Tangent at (a, a) is $5x - 6y = -a$

$(a,a)5x - 6y = -a$

$\alpha = \frac{-a}{5},\quad \beta = \frac{a}{6}$

$\alpha^2 + \beta^2 = 61 \Rightarrow\ \frac{a^2}{25} + \frac{a^2}{36} = 61$

$a^2 = 25.36$

$a = \pm\ 30$

8. Ans. (A,B,C)

$x = 2 \quad \Rightarrow \quad t^2 + 3t - 10 = 0 \quad \Rightarrow \quad t = 2, -5$

$y = -1 \quad \Rightarrow \quad t^2 - t - 2 = 0 \quad \Rightarrow \quad t = 2, -1$

$\Rightarrow\ t = 2$ (common value)

$\frac{dy}{dx} = \frac{4t-2}{2t+3}$

$.\ \left.\frac{dy}{dx}\right|_{t=2} = \frac{6}{7}$

length of subtangent = $\left|\dfrac{-1}{\frac{6}{7}}\right| = \dfrac{7}{6}$

length of tangent = $\left|\dfrac{-1}{\frac{6}{7}}\sqrt{1+\dfrac{36}{49}}\right| = \dfrac{\sqrt{85}}{6}$

9. **Ans. (B,C)**

Since both C_1 and C_2 are symmetric about x-axis. if they have a common point (x_0, y_0), then $\left(x_0, -y_0\right)$ will also be common to them, so they meet at even number of points as they do not meet on x-axis.

solving them we get $x^3+3x-1=0$

Let $g(x)=x^3+3x-1$ and $g(0) = -1$ and $g(1) = 3$

and $g'(x)=3\left(x^2+1\right)$ so $g(x) = 0$ has exactly one real solution in (0, 1).

for C_1 $\dfrac{dy}{dx} = \dfrac{2}{y}$ and for C_2 $\dfrac{dy}{dx} = \dfrac{3x^3+7}{2y}$

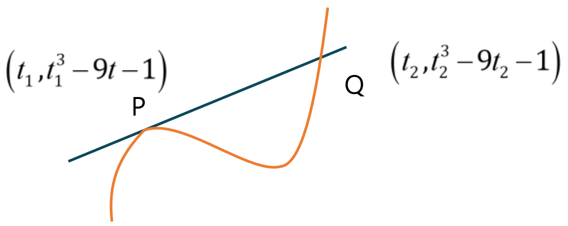
It $\left(x_0, y_0\right)$ is a common point

$\dfrac{2}{y_0} = \dfrac{3x^2+7}{2y_0} \Rightarrow 3x^2+7=4 \Rightarrow$ no real root

10. **Ans. (A,C)**

$f'(x)=3x^2-9$

$\Rightarrow \dfrac{\left(t_2^3-9t_2-1\right)-\left(t_1^3-9t-1\right)}{\left(t_2-t_1\right)}=3t_1^2-9$



$\Rightarrow \dfrac{\left(t_2^3-t_1^3\right)-9\left(t_2-t_1\right)}{\left(t_2-t_1\right)}=3t_1^2-9$

$\Rightarrow t_1^2+t_2^2+t_1t_2-9=3t_1^2-9 \quad \Rightarrow 2t_1^2-t_2^2-t_1t_2=0$

$\Rightarrow 2t_1^2-2t_1t_2+t_1t_2-t_2^2=0 \quad \Rightarrow -2t_1\left(-t_1+t_2\right)-t_2\left(t_2-t_1\right)=0$

$\Rightarrow 2t_1+t_2=0 \Rightarrow t_2=-2t_1$

$m_Q=3t_2^2-9,\ m_p=3t_1^2-9$

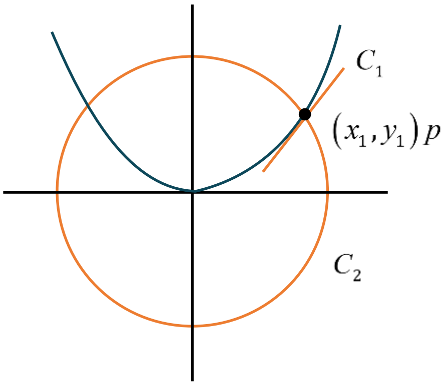
$m_Q-4m_p=3t_2^2-9-12t_1^2+36=27$

$\dfrac{m_{OQ}}{m_{OP}}=\dfrac{\left(t_2^3-9t_2-1\right)}{t_2}\times\dfrac{t_1}{\left(t_1^3-9t_1-1\right)} = \dfrac{\left(-8t_1^3+18t_1-1\right)\times t_1}{\left(-2t_1\right)\left(t_1^3-9t_1-1\right)} = \dfrac{(-8+18-1)}{(-2)(1-9-1)} = \dfrac{9}{(-2)(-9)} = \dfrac{1}{2}$

$\Rightarrow \dfrac{m_{OP}}{m_{OQ}}=2$

11. Ans. (A,B,C)

$C_1 : y = \frac{2x^2}{(x^2+1)}$, $C_2 : x^2 + y^2 = r^2$



$y_1 = \frac{2x_1^2}{1+x_1^2}$

$x_1^2 + y_1^2 = r_1^2$

$\frac{dy}{dx} = \frac{4x(x^2+1) - 2x^2(2x)}{(x^2+1)^2}$

$\frac{dy}{dx} = \left.\frac{4x}{(x^2+1)^2}\right|_P$

Tangent at $P : (y - y_1) = \frac{4x_1}{(x_1^2+1)^2}(x - x_1)$

Passes through (0, 0)

$\Rightarrow -y_1 = \frac{4x_1}{(x_1^2+1)^2}(-x_1) \quad \Rightarrow y_1(x_1^2+1)^2 = 4x_1^2$

$\Rightarrow \frac{2x_1^2}{(1+x_1^2)}(1+x_1^2)^2 = 4x_1^2 \quad \Rightarrow 1 + x_1^2 = 2$ or $x_1 = 0$

$x_1 = 0$ or $x_1^2 = 1$

$x_1 = \pm 1$

If $x_1 = 0 \quad y_1 = 0$

$\Rightarrow x_1^2 + y_1^2 = r^2 \Rightarrow r = 0$ (rejected)

If $x_1^2 = 1 \Rightarrow y_1 = \frac{2x_1^2}{1+x_1^2}$

$y_1 = 1$

$x_1^2 + y_1^2 = r^2$

$\Rightarrow r^2 = 2 \Rightarrow r = \sqrt{2}$

12. Ans. (7.5 m³)

$V = a^3 \quad \Rightarrow \quad \Delta V = \frac{dV}{da}\Delta a$

$= 3a^2\Delta a = 3a^2 \times \frac{2a}{100} = \frac{6\times 5^3}{100} = 7.5\ m^3$

13. Ans. (2)

Parametric form of curve is $x = 3t^2, y = 2t^3$

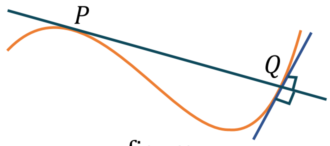
$\frac{dy}{dx} = t$

Let $P(3t_1^2,\ 2t_1^3), Q(3t_2^2,\ 2t_2^3)$

Conditions are

(i) $\left(\left.\frac{dy}{dx}\right|_P\right)\left(\left.\frac{dy}{dx}\right|_Q\right) = -1$

(ii) $\left.\frac{dy}{dx}\right|_P$ = Slope of line segment PQ



figure

$t_1 t_2 = -1$ ...(i) ; $t_1 = \frac{2}{3}\frac{t_2^2+t_1t_2+t_1^2}{t_2+t_1}$ ...(ii)

$\Rightarrow 3(-1 + t_1^2) = 2\left(t_2^2-1+t_1^2\right)$

$t_1^2 = \frac{2}{t_1^2} + 1$

$t_1^2 = 2, -1$

$t_1^2 = 2 \Rightarrow t_1 = \pm\sqrt{2} \Rightarrow t_2 = \mp\frac{1}{\sqrt{2}}$

If $t_1 = \sqrt{2}$, $t_2 = -\frac{1}{\sqrt{2}} \Rightarrow P(6, 4\sqrt{2}), Q\left(\frac{3}{2}, -\frac{1}{\sqrt{2}}\right) \Rightarrow$ Required line is $y = \sqrt{2}x - 2\sqrt{2}$

If $t_1 = -\sqrt{2}$, $t_2 = \frac{1}{\sqrt{2}} \Rightarrow P(6, -4\sqrt{2}), Q\left(\frac{3}{2}, \frac{1}{\sqrt{2}}\right) \Rightarrow$ Required line is $y = -\sqrt{2}x + 2\sqrt{2}$

14. **Ans. (24)**

Any point on $y = x^2 + 4x + 8$ is $P(h, h^2 + 4h + 8)$.

$\left.\frac{dy}{dx}\right|_p = 2h + 4$

Tangent at P is $(2h + 4)x - y = h^2 - 8$.

It is sufficient to have only one solution for equations $y = x^2 + 8x + 4$, $y = (2h + 4)x + 8 - h^2$

$\Rightarrow$ $x^2 + (4 - 2h)x + h^2 - 4 = 0 \Rightarrow$ $D = 0$

$(4 - 2h)^2 - 4(h^2 - 4) = 0 \Rightarrow$ $h = 2$

$8x - y + 4 = 0$

coordinates of point of contact are (2, 20) and (0, 4)

15. **Ans. (3)**

$|\ell nx| = px$

It is sufficient to find values of p for which $y = |\ell nx|$ and $y = px$ has three points in common.

If $y = px$ is passing through points O, A, B then we obtain three roots.

Let us consider line $y = px$. When it is passing through points O, P. (tangent)

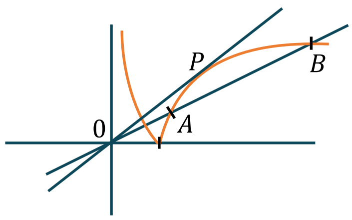
Let $P(\alpha, p\alpha)$ $(\alpha > 1)$

$\Rightarrow$ $p\alpha = |\ell n\alpha|$

$p\alpha = \ell n\alpha$ .....(1)

$\frac{dy}{dx} = \frac{1}{x}$

$p = \frac{1}{\alpha}$ .....(2)



Figure

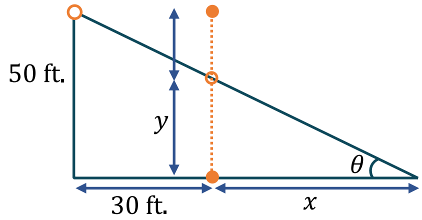
(1), (2) $\Rightarrow \alpha = e$

Slope of tangent (OP) is $\frac{1}{e}$.

For three roots condition is $0 < p < \frac{1}{e}$.

16. Ans. (15)

$y = 50 - 16t^2$ So, $\frac{dy}{dt} = -32t$



Figure

$\tan\theta = \frac{y}{x} = \frac{50}{30+x} \Rightarrow y = \left(\frac{50}{30+x}\right) . x$

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$= \frac{1500}{(30+x)^2} \cdot \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = -16\frac{(375)^2}{1500}$

$= -1500$ ft/sec. $= 100\lambda$ ft/sec $\Rightarrow \lambda = -15 \Rightarrow |\lambda| = 15$

17. Ans. (4)

$x = y^4$ $xy = k$

for intersection $y^5 = k$ ...(1)

Also $x = y^4 \Rightarrow 1 = 4y^3\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{4y^3}$

for $xy = k \Rightarrow x = \frac{k}{y} \Rightarrow 1 = -\frac{k}{y^2} \cdot \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{-y^2}{k}$

$\because$ Curve cut orthogonally

$\Rightarrow \frac{1}{4y^3} \times \left(\frac{-y^2}{k}\right) = -1 \Rightarrow y = \frac{1}{4k}$

$\therefore$ from (1) $y^5 = k$

$\Rightarrow \frac{1}{(4k)^5} = k \Rightarrow 4 = (4k)^6$

18. Ans. (0.50)

$y = x^2 - 3x + 2$

At x-axis $y = 0 = x^2 - 3x + 2$

$x = 1, 2$

$\frac{dy}{dx} = 2x - 3$

$A(1, 0)$ $B(2, 0)$

$\left(\frac{dy}{dx}\right)_{x=1} = -1$ and $\left(\frac{dy}{dx}\right)_{x=2} = 1$

$x + y = a \Rightarrow \frac{dy}{dx} = -1$ So $A(1, 0)$ lies on it

$\Rightarrow 1 + 0 = a \Rightarrow \boxed{a=1}$

$x - y = b \Rightarrow \frac{dy}{dx} = 1$ So $B(2, 0)$ lies on it

$2 - 0 = b \Rightarrow \boxed{b=2}$

$\frac{a}{b} = 0.50$