EXERCISE - O

SINGLE CORRECT TYPE QUESTIONS

1. A spherical balloon is being inflated at the rate of 35 cc/min. The rate of increase in the surface area (in cm^2/min.) of the balloon when its diameter is 14 cm, is : (Use $\pi = \frac{22}{7}$)

(A) $\sqrt{10}$ (B) $10\sqrt{10}$ (C) 100 (D) 10

MTN001

2. The slope of the curve $y = \sin x + \cos^2 x$ is zero at the point, where-

(A) $x = \frac{\pi}{4}$ (B) $x = \frac{\pi}{2}$ (C) $x = \pi$ (D) No where

MTN002

3. The equation of tangent at the point (at^2, at^3) on the curve $ay^2 = x^3$ is-

(A) $3tx - 2y = at^3$ (B) $tx - 3y = at^3$ (C) $3tx + 2y = at^3$ (D) None of these

MTN003

4. Let S be a square with sides of length x. If we approximate the change in size of the area of S by $h \cdot \frac{dA}{dx}\Big|_{x=x_0}$, when the sides are changed from x_0 to $x_0 + h$, then the absolute value of the error in our approximation, is

(A) h^2 (B) $2hx_0$ (C) x_0^2 (D) h

MTN051

5. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is :-

(A) $y = 0$ (B) $y = 1$ (C) $y = 2$ (D) $y = 3$

MTN052

6. The normal to the curve, $x^2 + 2xy - 3y^2 = 0$, at (1, 1) :

(A) meets the curve again in the third quadrant

(B) meets the curve again in the fourth quadrant

(C) does not meet the curve again

(D) meets the curve again in the second quadrant

MTN053

7. The coordinates of the points on the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, where tangent is inclined an angle $\pi/4$ to the x-axis are -

(A) (a, a) (B) $\left(a\left(\frac{\pi}{2}-1\right), a\right)$ (C) $\left(a\left(\frac{\pi}{2}+1\right), a\right)$ (D) $\left(a, a\left(\frac{\pi}{2}+1\right)\right)$

MTN004

8. Consider the curve represented parametrically by the equation

$x = t^3 - 4t^2 - 3t$ and $y = 2t^2 + 3t - 5$ where $t \in R$.

If H denotes the number of point on the curve where the tangent is horizontal and V the number of point where the tangent is vertical then

(A) $H = 2$ and $V = 1$ (B) $H = 1$ and $V = 2$ (C) $H = 2$ and $V = 2$ (D) $H = 1$ and $V = 1$

MTN054

9. The point(s) at each of which the tangents to the curve $y = x^3 - 3x^2 - 7x + 6$ cut off on the positive semi axis OX a line segment half that on the negative semi axis OY then the co-ordinates of the point(s) is/are given by :

(A) $(-1, 9)$ (B) $(3, -15)$ (C) $(1, -3)$ (D) none

MTN055

10. If the tangent at a point P, with parameter t, on the curve $x = 4t^2 + 3, y = 8t^3 - 1, t \in R$, meets the curve again at a point Q, then the coordinates of Q are :

(A) $(t^2 + 3, t^3 - 1)$ (B) $(t^2 + 3, -t^3 - 1)$

(C) $(16t^2 + 3, -64t^3 - 1)$ (D) $(4t^2 + 3, -8t^3 - 1)$

MTN056

11. The curve $x^2 - y^2 = 5$ and $\frac{x^2}{18} + \frac{y^2}{8} = 1$ cut each other at any common point at an angle-

(A) $\pi/4$ (B) $\pi/3$ (C) $\pi/2$ (D) None of these

MTN006

12. The lines tangent to the curve $y^3 - x^2 + 5y - 2x = 0$ and $x^4 - x^3y^2 + 5x + 2y = 0$ at the origin intersect at an angle θ equal to -

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

MTN057

13. The angle of intersection between the curves $y^2 = 8x$ and $x^2 = 4y - 12$ at (2, 4) is-

(A) 90° (B) 60° (C) 45° (D) 0°

MTN007

14. If the curves $y^2 = 6x, 9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is :

(A) $\frac{7}{2}$ (B) 4 (C) $\frac{9}{2}$ (D) 6

MTN058

15. The length of subtangent to the curve $x^2 + xy + y^2 = 7$ at the point (1, –3) is -

(A) 3 (B) 5 (C) 15 (D) 3/5

MTN008

16. At any point of a curve (subtangent) × (subnormal) is equal to the square of the-

(A) slope of the tangent at that point (B) slope of the normal at that point

(C) abscissa of that point (D) ordinate of that point

MTN009

17. For a curve $\frac{(\text{length of normal})^2}{(\text{length of tangent})^2}$ is equal to -

(A) (subnormal)/(subtangent) (B) (subtangent)/(subnormal)

(C) (subtangent × subnormal) (D) constant

MTN059

18. A line L is perpendicular to the curve $y = \frac{x^2}{4} - 2$ at its point P and passes through (10, –1). The coordinates of the point P are

(A) (2, –1) (B) (6, 7) (C) (0, –2) (D) (4, 2)

MTN010

19. At the point $P(a, a^n)$ on the graph of $y = x^n$ $(n \in N)$ in the first quadrant a normal is drawn. The normal intersects the y-axis at the point $(0, b)$. If $\lim_{a\to 0} b = \frac{1}{2}$, then n equals

(A) 1 (B) 3 (C) 2 (D) 4

MTN060

20. A circle with centre at (15, –3) is tangent to $y = \frac{x^2}{3}$ at a point in the first quadrant. The radius of the circle is equal to

(A) $5\sqrt{6}$ (B) $8\sqrt{3}$ (C) $9\sqrt{2}$ (D) $6\sqrt{5}$

MTN061

MULTIPLE CORRECT TYPE QUESTIONS

21. If $\frac{x}{a} + \frac{y}{b} = 1$ is a tangent to the curve $x = Kt, y = \frac{K}{t}, K > 0$ then :

(A) $a > 0,\ b > 0$ (B) $a > 0,\ b < 0$ (C) $a < 0, b > 0$ (D) $a < 0, b < 0$

MTN029

22. For the curve $C : y = e^{2x}\cos x$, which of the following statement(s) is/are true ?

(A) equation of the tangent where C crosses y-axis is $y = 3x + 1$

(B) equation of the tangent where C crosses y-axis is $y = 2x + 1$

(C) number of points in $\left(-\frac{\pi}{2}, \frac{3\pi}{2}\right)$ where tangent on the curve C is parallel to x-axis is 4.

(D) number of points in $\left(-\frac{\pi}{2}, \frac{3\pi}{2}\right)$ where tangent on the curve C is parallel to x-axis is 2.

MTN030

23. Given that $g(x)$ is a non constant linear function defined on R-

(A) $y = g(x)$ and $y = g^{-1}(x)$ are orthogonal (B) $y = g(x)$ and $y = g^{-1}(-x)$ are orthogonal

(C) $y = g(-x)$ and $y = g^{-1}(x)$ are orthogonal (D) $y = g(-x)$ and $y = g^{-1}(-x)$ are orthogonal

MTN062

24. If the curve $y^2 = 4ax, a > 0$ cuts the curve $xy = b, b > 0$ at right angle, then -

(A) $a = \frac{1}{\sqrt{2}}, b = \sqrt{2}$ (B) $a = \frac{1}{2}, b = \sqrt{2}$ (C) $a = \frac{1}{4}, b = \frac{1}{2\sqrt{2}}$ (D) $a = \frac{1}{4}, b = \sqrt{2}$

MTN063

25. For the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$, at point (2, –1)

(A) length of subtangent is 7/6 (B) slope of tangent = 6/7

(C) length of tangent = $\sqrt{(85)}/6$ (D) none of these

MTN031

26. Given the curve $\left(\frac{x^2 - y^2}{xy}\right)^3 = \frac{512}{27}$, then

(A) the slope of normal drawn to the curve at (3,1) is $\frac{1}{3}$

(B) the value of $\left(2\frac{dy}{dx} - 5\frac{d^2y}{dx^2}\right)_{(3,1)} = \frac{2}{3}$

(C) the slope of normal drawn to the curve at (3,1) is –3.

(D) the equation of tangent at (3,1) to given curve is $3y + x = 0$.

MTN032

27. The equation of normal to curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$, where $n \in N$, at the point with abscissa equal to 'a' can be -

(A) $ax + by = a^2 - b^2$ (B) $ax + by = a^2 + b^2$

(C) $ax - by = a^2 - b^2$ (D) $bx + ay = a^2 - b^2$

MTN033

28. If curves $f(x) = x^2 + ax + b$ and $g(x) = cx - x^2$ touches each other at point (1, 0), then

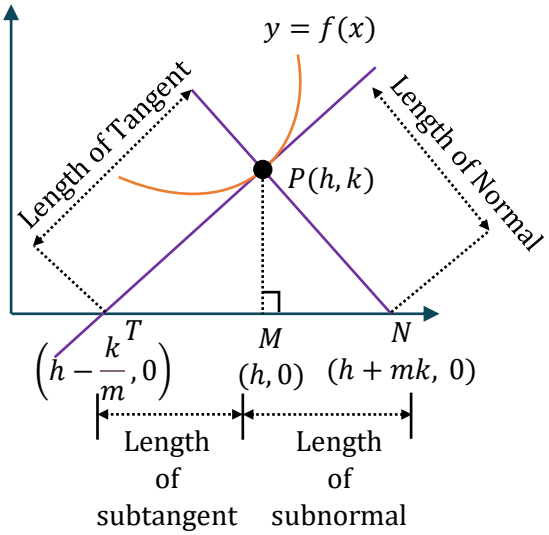
(A) $a = -3$ (B) $b = 2$ (C) $c = 1$ (D) $abc = -6$

MTN034

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 29 to 31

Let $P(h, k)$ be any point on curve $y = f(x)$. Let tangent drawn at point P meets x-axis at T & normal at point P meets x-axis at N (as shown in figure) and $m = \left.\frac{dy}{dx}\right]_{(h,k)}$ = slope of tangent.



(i) Length of Tangent = $PT = |k|\sqrt{1+\frac{1}{m^2}}$

(ii) Length of Normal = $PN = |k|\sqrt{1+m^2}$

(iii) Length of subtangent = Projection of segment PT on x-axis = $TM = \left|\frac{k}{m}\right|$

(iv) Length of subnormal = projection of line segment PN on x axis = $MN = |km|$

29. The product of length of tangent and length of normal for the curve $y = x^3 + 3x^2 + 4x - 1$ at point $x = 0$ is

(A) $\frac{17}{4}$ (B) $\frac{\sqrt{15}}{4}$ (C) 17 (D) $\frac{4}{\sqrt{17}}$

MTN035

30. Value of 'p' such that the length of the subtangent and subnormal is equal for the curve $y = e^{px} + px$ at the point (0, 1)

(A) ±1 (B) ±2 (C) $\pm\frac{1}{2}$ (D) $\pm\frac{1}{4}$

MTN036

31. The length of subnormal to $x = \sqrt{2}\cos t, y = -3\sin t$ at $t = \frac{-\pi}{4}$.

(A) $\frac{2}{9}$ (B) 1 (C) $\frac{7}{2}$ (D) $\frac{9}{2}$

MTN037

Paragraph for Question No. 32 to 33

Water is flowing out at the rate of 6 m^3/min from a reservoir shaped like a hemispherical bowl of radius $R = 13m$. The volume of water in the hemispherical bowl is given by $V = \frac{\pi}{3}\cdot y^2(3R - y)$ when the water is y meter deep. Find

32. At what rate is the water level changing when the water is $8m$ deep.

(A) $\frac{-1}{24\pi}m/min.$ (B) $\frac{-5}{288\pi}m/min.$ (C) $\frac{-1}{12\pi}m/min.$ (D) $\frac{-5}{144\pi}m/min.$

MTN038

33. At what rate is the radius of the water surface changing when the water is $8m$ deep.

(A) $\frac{-1}{24\pi}m/min.$ (B) $\frac{-5}{288\pi}m/min.$ (C) $\frac{-1}{12\pi}m/min.$ (D) $\frac{-5}{144\pi}m/min.$

MTN039

MATCHING LIST TYPE QUESTION

34. Consider the curve $y = e^{2x^2} + 2x$, then match List-I with List-II and select the correct answer using the code given below the list.

	List-I		**List-II**
(P)	Length of tangent at $x = 0$ is	(1)	$\sqrt{5}$
(Q)	Length of normal at $x = 0$ is	(2)	$\frac{1}{2}$
(R)	Length of subtangent at $x = 0$ is	(3)	$\frac{\sqrt{5}}{2}$
(S)	Length of subnormal at $x = 0$ is	(4)	2

(A) P $\to$ 1; Q $\to$ 3; R $\to$ 2; S $\to$ 4 (B) P $\to$ 4; Q $\to$ 3; R $\to$ 1; S $\to$ 2

(C) P $\to$ 3; Q $\to$ 1; R $\to$ 2; S $\to$ 4 (D) P $\to$ 3; Q $\to$ 1; R $\to$ 4; S $\to$ 2

MTN040

MATRIX MATCH TYPE QUESTION

35.

	Column-I		**Column-II**
(A)	The slope of the curve $2y^2 = ax^2 + b$ at $(1, -1)$ is -1, then	(p)	$a - b = 2$
(B)	If (a, b) be the point on the curve $9y^2 = x^3$ where normal to the curve makes equal intercepts with the axes, then	(q)	$a - b = 7/2$
(C)	If the tangent at a point $(1, 2)$ on the curve $y = ax^2 + bx + \frac{7}{2}$ be parallel to the normal at $(-2, 2)$ on the curve $y = x^2 + 6x + 10$, then	(r)	$a - b = 4/3$
(D)	If the tangent to the curve $xy + ax + by = 0$ at $(1, 1)$ is inclined at an angle $\tan^{-1} 2$ with x-axis, then	(s)	$a - b = 3$

MTN041

EXERCISE - S

1. Water is being poured on to a cylindrical vessel at the rate of 1 m^3/min and the vessel has a circular base of radius 3 m. If the rate at which the level of water is rising in the vessel is $\frac{1}{k\pi}$ m/min, then the value of 'k' is

MTN011

2. An inverted cone has a depth of 10 cm & a base of radius 5 cm. Water is poured into it at the rate of 1.5 cm^3/min. If the rate at which level of water in the cone is rising, when the depth of water is 4 cm is $\frac{3}{k\pi}$ cm/min, then the value of 'k' is

MTN064

3. Water is dripping out from a conical funnel of semi vertical angle $\pi/4$, at the uniform rate of 2 cm^3/sec through a tiny hole at the vertex at the bottom. When the slant height of the water is 4 cm, then the rate of decrease of the slant height of the water is given by $\frac{\sqrt{2}}{k\pi}$ cm/sec, then the value of 'k' is

MTN042

4. A water tank has the shape of a right circular cone with its vertex down. Its altitude is 10 cm and the radius of the base is 15 cm. Water leaks out of the bottom at a constant rate of $1cu.cm/sec$. Water is poured into the tank at a constant rate of C $cu.cm/sec$. Compute C so that the water level will be rising at the rate of 4 cm/sec at the instant when the water is 2 cm deep. If C is of the form $p + q\pi$, then the value of $p + q$ is

MTN065

5. Sand is pouring from a pipe at the rate of 12 cc/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always $1/6^{th}$ of the radius of the base. If the height of the sand cone increase at a rate $\frac{1}{p\pi}$ when the height is 4 cm, then the value of 'p' is

MTN066

6. A circular ink blot grows at the rate of 2 cm^2 per second. Then the rate at which the radius is increasing after $2\frac{6}{11}$ seconds is (Use $\pi = \frac{22}{7}$)

MTN043

7. The tangent to $y = ax^2 + bx + \frac{7}{2}$ at (1, 2) is parallel to the normal at the point (–2, 2) on the curve $y = x^2 + 6x + 10$. Then value of $a - 2b$ is

MTN012

8. The chord of the parabola $y = -a^2x^2 + 5ax - 4$ touches the curve $y = \frac{1}{1-x}$ at the point $x = 2$ and is bisected by that point, then the value of 'a' is

MTN013

9. A variable ΔABC in the xy plane has its orthocentre at vertex 'B' , a fixed vertex 'A' at the origin and the third vertex 'C' restricted to lie on the parabola $y = 1 + \frac{7x^2}{36}$. The point B starts at the point $(0, 1)$ at time $t = 0$ and moves upward along the y axis at a constant velocity of 2 cm/sec, then how fast is the area of the triangle increasing when $t = \frac{7}{2} sec$.

MTN044

10. There is a point (p, q) on the graph of $f(x) = x^2$ and a point (r, s) on the graph of $g(x) = -8/x$ where $p > 0$ and $r > 0$. If the line through (p, q) and (r, s) is also tangent to both the curves at these points respectively then the value of $(p + q)$ is

MTN045

EXERCISE - JEE (Main) PYQ

1. The tangent to the curve, $y = xe^{x^2}$ passing through the point $(1, e)$ also passes through the point : **[JEE (Main) 2019]**

(1) $\left(\frac{4}{3}, 2e\right)$ (2) $(2, 3e)$ (3) $\left(\frac{5}{3}, 2e\right)$ (4) $(3, 6e)$

MTN014

2. A $2m$ ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25 $cm/sec.$, then the rate (in $cm/sec.$) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is $1m$ above the ground is : **[JEE (Main) 2019]**

(1) $25\sqrt{3}$ (2) 25 (3) $\frac{25}{\sqrt{3}}$ (4) $\frac{25}{3}$

MTN015

3. The length of the perpendicular from the origin, on the normal to the curve, $x^2 + 2xy - 3y^2 = 0$ at the point $(2, 2)$ is **[JEE (Main) 2020]**

(1) $4\sqrt{2}$ (2) $2\sqrt{2}$ (3) 2 (4) $\sqrt{2}$

MTN016

4. Let the normal at a point P on the curve $y^2 - 3x^2 + y + 10 = 0$ intersect the y-axis at $\left(0, \frac{3}{2}\right)$. If m is the slope of the tangent at P to the curve, then $|m|$ is equal to **[JEE (Main) 2020]**

MTN017

5. Let $P(h, k)$ be a point on the curve $y = x^2 + 7x + 2$, nearest to the line, $y = 3x - 3$. Then the equation of the normal to the curve at P is : **[JEE (Main) 2020]**

(1) $x + 3y - 62 = 0$ (2) $x - 3y - 11 = 0$ (3) $x - 3y + 22 = 0$ (4) $x + 3y + 26 = 0$

MTN018

6. The equation of the normal to the curve $y = (1 + x)^{2y} + \cos^2(\sin^{-1}x)$ at $x = 0$ is :**[JEE (Main) 2020]**

(1) $y = 4x + 2$ (2) $x + 4y = 8$ (3) $y + 4x = 2$ (4) $2y + x = 4$

MTN019

7. Which of the following points lies on the tangent to the curve $x^4e^y + 2\sqrt{y+1} = 3$ at the point $(1, 0)$? **[JEE (Main) 2020]**

(1) $(2, 2)$ (2) $(-2, 6)$ (3) $(-2, 4)$ (4) $(2, 6)$

MTN020

8. If the tangent to the curve, $y = f(x) = x \log_e x$, $(x > 0)$ at a point $(c, f(c))$ is parallel to the line - segement joining the points $(1, 0)$ and (e, e), then c is equal to : **[JEE (Main) 2021]**

(1) $\frac{1}{e-1}$ (2) $e^{\left(\frac{1}{1-e}\right)}$ (3) $e^{\left(\frac{1}{e-1}\right)}$ (4) $\frac{e-1}{e}$

MTN021

9. An angle of intersection of the curves, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $x^2+y^2=ab$, $a > b$, is : **[JEE (Main) 2021]**

(1) $\tan^{-1}\left(\frac{a+b}{\sqrt{ab}}\right)$ (2) $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$ (3) $\tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$ (4) $\tan^{-1}\left(2\sqrt{ab}\right)$

MTN022

10. If the curve $y = ax^2 + bx + c$, $x \in R$, passes through the point (1,2) and the tangent line to this curve at origin is $y = x$, then the possible values of a, b, c are : **[JEE (Main) 2021]**

(1) $a = \frac{1}{2}, b = \frac{1}{2}, c = 1$ (2) $a = 1, b = 0, c = 1$

(3) $a = 1, b = 1, c = 0$ (4) $a = -1, b = 1, c = 1$

MTN067

11. If the tangent to the curve $y = x^3 - x^2 + x$ at the point (a , b) is also tangent to the curve $y = 5x^2 + 2x - 25$ at the point $(2, -1)$, then $|2a + 9b|$ is equal to ____. **[JEE (Main) 2022]**

MTN068

12. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is : **[JEE (Main) 2022]**

(1) $-2t^3$ (2) 0 (3) $-t^3$ (4) $2t^3$

MTN023

13. The surface area of a balloon of spherical shape being inflated, increases at a constant rate. If initially, the radius of balloon is 3 units and after 5 seconds,, it becomes 7 units, then its radius after 9 seconds is : **[JEE (Main) 2022]**

(1) 9 (2) 10 (3) 11 (4) 12

MTN024

14. The tangent at the point (x_1 , y_1) on the curve $y = x^3 + 3x^2 + 5$ passes through the origin, then (x_1, y_1) does NOT lie on the curve : **[JEE (Main) 2022]**

(1) $x^2+\frac{y^2}{81}=2$ (2) $\frac{y^2}{9}-x^2=8$ (3) $y=4x^2+5$ (4) $\frac{x}{3}-y^2=2$

MTN025

15. Water is being filled at the rate of 1 cm^3 / sec in a right circular conical vessel (vertex downwards) of height 35 cm and diameter 14 cm. When the height of the water level is 10 cm, the rate (in cm^2 / sec) at which the wet conical surface area of the vessel increases is **[JEE (Main) 2022]**

(1) 5 (2) $\frac{\sqrt{21}}{5}$ (3) $\frac{\sqrt{26}}{5}$ (4) $\frac{\sqrt{26}}{10}$

MTN026

16. If the angle made by the tangent at the point (x_0, y_0) on the curve $x = 12(t + \sin t \cos t)$, $y=12(1+\sin t)^2, 0<t<\frac{\pi}{2}$, with the positive x-axis is $\frac{\pi}{3}$, then y_0 is equal to **[JEE (Main) 2022]**

(1) $6\left(3+2\sqrt{2}\right)$ (2) $3\left(7+4\sqrt{3}\right)$ (3) 27 (4) 48

MTN027

17. A water tank has the shape of a right circular cone with axis vertical and vertex downwards. Its semi-vertical angle is $\tan^{-1}\frac{3}{4}$. Water is poured in it at a constant rate of 6 cubic meter per hour. The rate (in square meter per hour), at which the wet curved surface area of the tank is increasing, when the depth of water in the tank is 4 meters, is _____. **[JEE (Main) 2022]**

MTN028

18. If the equation of the normal to the curve $y=\frac{x-a}{(x+b)(x-2)}$ at the point (1, –3) is $x - 4y = 13$, then the value of $a + b$ is equal to. **[JEE (Main) 2023]**

MTN069

19. The number of points on the curve $y = 54x^5 - 135x^4 - 70x^3 + 180x^2 + 210x$ at which the normal lines are parallel to $x + 90y + 2 = 0$ is : **[JEE (Main) 2023]**

(1) 2 (2) 3 (3) 4 (4) 0

MTN070

20. Let quadratic curve passing through the point (–1, 0) and touching the line $y = x$ at (1, 1) be $y = f(x)$. Then the x-intercept of the normal to the curve at the point $(\alpha, \alpha + 1)$ in the first quadrant is. **[JEE (Main) 2023]**

MTN071

EXERCISE - JEE (Advanced) PYQ

1. Find the equation of the straight line which is tangent at one point and normal at another point of the curve, $x = 3t^2$, $y = 2t^3$. **[JEE (Advanced) 2000]**

MTN046

2. If the normal to the curve , $y = f(x)$ at the point (3, 4) makes an angle $\frac{3\pi}{4}$ with the positive x–axis. Then $f'(3) =$ **[JEE (Advanced) 2000]**

(A) – 1 (B) $-\frac{3}{4}$ (C) $\frac{4}{3}$ (D) 1

MTN047

3. The point(s) on the curve $y^3 + 3x^2 = 12y$ where the tangent is vertical, is(are) **[JEE (Advanced) 2002]**

(A) $\left(\pm\frac{4}{\sqrt{3}}, -2\right)$ (B) $\left(\pm\sqrt{\frac{11}{3}}, 1\right)$

(C) (0, 0) (D) $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

MTN048

4. Tangent to the curve $y = x^2 + 6$ at a point $P(1, 7)$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at a point Q. Then the coordinates of Q are **[JEE (Advanced) 2005]**

(A) (– 6, –11) (B) (–9, –13) (C) (– 10, – 15) (D) (–6, –7)

MTN049

5. The tangent to the curve $y = e^x$ drawn at the point (c, e^c) intersects the line joining the points $(c - 1, e^{c-1})$ and $(c + 1, e^{c+1})$ **[JEE (Advanced) 2007]**

(A) on the left of $x = c$ (B) on the right of $x = c$

(C) at no point (D) at all points

MTN050

6. The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point (1,3) is **[JEE (Advanced) 2014]**

MDF078

JEE (Main) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION–A

- This section contains **TWENTY** questions.
- Each question has **FOUR** options (1), (2), (3) and (4). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +4, if only the bubble corresponding to the correct option is darkened.
Zero Marks : 0, if none of the bubbles is darkened.
Negative Marks : –1 in all other cases.

1. Equation of the normal to the curve $y = -\sqrt{x} + 2$ at the point (1, 1)

(1) $2x - y - 1 = 0$ (2) $2x - y + 1 = 0$ (3) $2x + y - 3 = 0$ (4) none of these

MTN072

2. The angle between x-axis and tangent of the curve $y = (x + 1)(x - 3)$ at the point (3, 0)

(1) $\tan^{-1}\left(\frac{8}{15}\right)$ (2) $\tan^{-1}\left(\frac{15}{8}\right)$ (3) $\tan^{-1} 4$ (4) none of these

MTN073

3. The numbers of tangent to the curve $y - 2 = x^5$ which are drawn from point (2,2) is / are

(1) 3 (2) 1 (3) 2 (4) 5

MTN074

4. The equation of tangent drawn to the curve $xy = 4$ from point (0, 1) is

(1) $y - \frac{1}{2} = -\frac{1}{16}(x + 8)$ (2) $y - \frac{1}{2} = -\frac{1}{16}(x - 8)$

(3) $y + \frac{1}{2} = -\frac{1}{16}(x - 8)$ (4) $y - 8 = -\frac{1}{16}\left(x - \frac{1}{2}\right)$

MTN075

5. The curve $y - e^{xy} + x = 0$ has a vertical tangent at point

(1) (1, 1) (2) (0, 1) (3) (1, 0) (4) no point

MTN076

6. If the tangent to the curve $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ at $\theta = \frac{\pi}{3}$ has angle of inclination α, then α is

(1) $\frac{\pi}{3}$ (2) $\frac{2\pi}{3}$ (3) $\frac{\pi}{6}$ (4) $\frac{5\pi}{6}$

MTN077

7. If the normal at the point $(3t, 4/t)$ of the curve $xy = 12$ cuts the curve again at $(3t_1, 4/t_1)$ then find 't_1' in terms of 't'

(1) $\frac{-9}{16t^3}$ (2) $\frac{-16}{9t^3}$ (3) $\frac{9}{16t^3}$ (4) $\frac{16}{9t^3}$

MTN078

8. The common tangent of the curves $y = x^2 + \frac{1}{x}$ and $y^2 = 4x$ is

(1) $y = x + 1$ (2) $y = x - 1$ (3) $y = -x + 1$ (4) $y = -x - 1$ **MTN079**

9. The area of triangle formed by tangent at (1,1) on $y = x^2 + bx + c$ with coordinate axis is equal to 2 then the integral value of b is

(1) –3 (2) 3 (3) 2 (4) –2

MTN080

10. The angle of intersection of $y = a^x$ and $y = b^x$ is given by

(1) $\tan\theta = \left|\frac{\log(ab)}{1-\log(ab)}\right|$ (2) $\left|\frac{\log(a/b)}{1+\log a \log b}\right|$ (3) $\left|\frac{\log(a/b)}{1-\log(a/b)}\right|$ (4) None

MTN081

11. The angle between curves $x^2 + 4y^2 = 32$ and $x^2 - y^2 = 12$ is

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{2}$

MTN082

12. Find the angle at which two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$ intersect

(1) 0 (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

MTN083

13. The value of a^2 if the curves $\frac{x^2}{a^2}+\frac{y^2}{4} = 1$ and $y^3 = 16x$ cut orthogonally is

(1) 3/4 (2) 1 (3) 4/3 (4) 4

MTN084

14. Water is poured into an inverted conical vessel of which the radius of the base is 2 m and height 4 m, at the rate of 77 litre/minute. The rate at which the water level is rising at the instant when the depth is 70 cm is (use $\pi = 22/7$)

(1) 10 cm/min (2) 20 cm/min (3) 40 cm/min (4) 30 cm/min

MTN085

15. On the curve $x^3 = 12y$. The interval in which abscissa changes at a faster rate then its ordinate

(1) (–3, 0) (2) $(-\infty, -2) \cup (2, \infty)$ (3) (–2, 2) (4) (–3, 3)

MTN086

16. A kite is 300 m high and there are 500 m of cord out. If the wind moves the kite horizontally at the rate of 5 km/hr. directly away from the person who is flying it, find the rate at which the cord is being paid?

(1) 4 (2) 8 (3) 3 (4) cannot be determined

[**Hint:** $\ell^2 = x^2 + y^2 \Rightarrow 2\ell\frac{d\ell}{dt} = 2x\frac{dx}{dt}\left(\frac{dy}{dt} = 0\right)$]

MTN087

17. The approximate value of tan 46° is (take $\pi = 22/7$) :

(1) 3 (2) 1.035 (3) 1.033 (4) 1.135

MTN088

18. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is-

(1) $\frac{5}{6\pi}$ cm/min (2) $\frac{1}{54\pi}$ cm/min (3) $\frac{1}{18\pi}$ cm/min (4) $\frac{1}{36\pi}$ cm/min

MTN089

19. Equation of normal drawn to the graph of the function defined as $f(x) = \frac{\sin x^2}{x}$, $x \neq 0$ and $f(0) = 0$ at the origin is

(1) $x + y = 0$ (2) $x - y = 0$ (3) $y = 0$ (4) $x = 0$

MTN090

20. The line $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y = be^{-x/a}$ at the point

(1) $(-a, 2b)$ (2) $\left(\frac{a}{2}, \frac{b}{2}\right)$ (3) $\left(a, \frac{b}{e}\right)$ (4) $(0, b)$

MTN091

SECTION-B

- This section will have **TEN** questions. Candidate can choose to attempt any 5 question out of these 10 questions. In case if candidate attempts more than 5 questions, first 5 attempted questions will be considered for marking.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (Answer should be rounded off to the nearest integer).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4, if only correct answer is given.
  Zero Marks : 0, if no answer is given.
  Negative Marks : -1 for incorrect answer

1. Find the cosine of angle of intersection of curves $y = 2^x \ell nx$ and $y = x^{2x} - 1$ at $(1, 0)$.

MTN092

2. Find the angle between the curves $y^2 = 4x + 4$ and $y^2 = 36(9 - x)$ in degrees.

MTN093

3. The length x of rectangle is decreasing at a rate of 3 cm/min and width y is increasing at a rate of 2 cm/min. When $x = 10$ cm and $y = 6$ cm, then the rate by which the perimeter is decreasing

MTN094

4. A man 1.5 m tall walks away from a lamp post 4.5 m high at a rate of 4 km/hr. How fast is his shadow lengthening?

MTN095

5. A variable ΔABC in the xy plane has its orthocentre at vertex 'B', a fixed vertex 'A' at the origin and the third vertex 'C' restricted to lie on the parabola $y = 1 + \frac{7x^2}{36}$. The point B starts at the point $(0, 1)$ at time $t = 0$ and moves upward along the y axis at a constant velocity of 2 cm/sec. If the area of the triangle is increasing at the rate of 'p' cm^2/sec when $t = \frac{7}{2}$ sec, then $7p$ is. **MTN096**

6. If $y = x^3 - c$ and $y = x^2 + ax + b$ touch each other at the point $(1, 2)$ then $\frac{|a|+|b|+|c|}{|a+b-c|}$ is equal to-

MTN097

7. The equation of tangent at $(-4, -4)$ on the curve $x^2 = -4y$ is $2x + \alpha y + \beta = 0$, then $(\alpha + \beta)$ equals

MTN098

8. If $x = t^2$ and $y = 2t$, then equation of the normal at $t = 1$ is $x + y = \alpha$, then α is **MTN099**

9. The equation of the normal to the curve $y^4 = ax^3$ at (a, a) is $\alpha x + \beta y = \gamma a$, then $\frac{\alpha+\beta}{\gamma}$ is

MTN100

10. If the tangent to the curve $y = x + \sin y$ at a point (a, b) is parallel to the line joining $\left(0, \frac{3}{2}\right)$ and $\left(\frac{1}{2}, 2\right)$, then $|b - a|$ is **MTN101**

JEE (Advanced) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION–I

- This section contains **SIX** questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3, if only the bubble corresponding to the correct option is darkened.
Zero Marks : 0, if none of the bubbles is darkened.
Negative Marks : –1, in all other cases

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1. The equation of normal to the curve $x^3 + y^3 = 8xy$ at point where it is meet by the curve $y^2 = 4x$, other than origin is

(A) $y = x$ (B) $y = -x + 4$ (C) $y = 2x$ (D) $y = -2x$

MTN102

2. The length of segment of all tangents to curve $x^{2/3} + y^{2/3} = a^{2/3}$ intercepted between coordinate axes is

(A) $2|a|$ (B) $|a|$ (C) $\frac{|a|}{2}$ (D) $\frac{3|a|}{2}$

MTN103

3. If tangents are drawn from the origin to the curve $y = \sin x$, then their points of contact lie on the curve

(A) $x - y = xy$ (B) $x + y = xy$ (C) $x^2 - y^2 = x^2y^2$ (D) $x^2 + y^2 = x^2y^2$

MTN104

4. Number of tangents drawn from the point $(-1/2, 0)$ to the curve $y = e^{\{x\}}$. (Here $\{\}$ denotes fractional part function).

(A) 2 (B) 1 (C) 3 (D) 4

MTN105

5. Let $f(x) = \begin{cases} -x^2 & , \ x < 0 \\ x^2 + 8 & , \ x \geq 0 \end{cases}$ Equation of tangent line touching both branches of $y = f(x)$ is

(A) $y = 4x + 1$ (B) $y = 4x + 4$ (C) $y = x + 4$ (D) $y = x + 1$

MTN106

6. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness the melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate (in cm/min.) at which of the thickness of ice decreases, is :

(A) $\frac{1}{36\pi}$ (B) $\frac{5}{6\pi}$ (C) $\frac{1}{18\pi}$ (D) $\frac{1}{54\pi}$

MTN107

SECTION-II

- This section contains **FIVE** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	If only (all) the correct option(s) is (are) chosen.
Partial Marks	: +3	If all the four options are correct but ONLY three options are chosen.
Partial Marks	: +2	If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks	: +1	If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	: –2	In all other cases.

For Example : If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

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7. If tangent to curve $2y^3 = ax^2 + x^3$ at point (a, a) cuts off intercepts α, β on co-ordinate axes, where $\alpha^2 + \beta^2 = 61$, then the value of 'a' is equal to

(A) 20 (B) 25 (C) 30 (D) – 30

MTN108

8. For the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$, at point (2, – 1)

(A) length of subtangent is 7/6. (B) slope of tangent = 6/7

(C) length of tangent = $\sqrt{(85)}/6$ (D) none of these

MTN109

9. Consider the curves $C_1 : y^2 = 4x$ and $C_2 : y^2 = x^3 + 7x - 1$. Then

(A) C_1 and C_2 have exactly one point in common

(B) C_1 and C_2 have exactly two points in common

(C) C_1 and C_2 do not touch each other at any-point

(D) C_1 and C_2 touch each other at exactly one point

MTN110

10. If tangent drawn to the curve $f(x) = x^3 - 9x - 1$ at $P(x_0, f(x_0))$ meets the curve again at Q, m_A denotes the slope o the tantgent at A and m_{OB} denotes the slope of the line joining 'O'origin and a point B on the curve, then

(A) $m_Q - 4m_p = 27$ (B) $m_Q - 4m_p = 9$

(C) $\frac{m_{OP}}{m_{OQ}} = 2$, where $x_0 = 1$ (D) $\frac{m_{OP}}{m_{OQ}} = \frac{1}{2}$, *where* $x_0 = 1$

MTN111

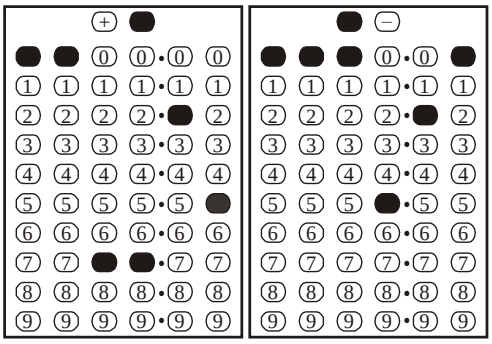
11. Let r be a positive constant. If two curves $C_1 : y = \frac{2x^2}{x^2+1}$ and $C_2 : y = \sqrt{r^2 - x^2}$ intersect orthogonally, then r cannot be :

(A) $\frac{2}{3}$ (B) $\sqrt{3}$ (C) $\sqrt{\frac{2}{3}}$ (D) $\sqrt{2}$

MTN112

SECTION-III

- This section contains **SEVEN** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30, if answer is 11.36777..... then both 11.36 and 11.37 will be correct) by darken the corresponding bubbles in the ORS.
For Example : If answer is –77.25, 5.2 then fill the bubbles as follows.



- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +4 If ONLY the correct numerical value is entered as answer.
Zero Marks : 0 In all other cases.

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12. Find the approximate change in volume V of a cube of side $5m$ caused by increasing its side length by 2%. **MTN113**

13. The number of distinct line(s) which is/are tangent at a point on curve $4x^3 = 27\,y^2$ and normal at other point, is:

MTN114

14. The sum of the ordinates of point of contacts of the common tangent to the parabolas $y = x^2 + 4x + 8$ and $y = x^2 + 8x + 4$, is

MTN115

15. If $p \in (0, 1/e)$ then the number of the distinct roots of the equation $|\ell n\, x| - px = 0$ is:

MTN116

16. A light shine from the top of a pole 50 ft. high. A ball is dropped from the same height from a point 30 ft. away from the light. If the shadow of the ball moving at the rate of 100λ ft/sec along the ground 1/2 sec. later [Assume the ball falls a distance $s = 16\,t^2$ ft. in 't' sec.], then |λ| is:

MTN117

17. If the curves $x = y^4$ and $xy = k$ cut at right angles, then $(4k)^6$ is equal to _____.

MTN118

18. If the lines $x + y = a$ and $x - y = b$ touch the curve $y = x^2 - 3x + 2$ at the points where the curve intersects the x-axis, then $\frac{a}{b}$ is equal to. **MTN119**

ANSWER KEY

EXERCISE - O

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	B	A	A	D	B	C	B	B	B
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	C	D	D	C	C	D	A	D	C	D
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	A,D	B,D	B,C	B,C	A,B,C	B,C	A,C	A,B,C,D	A	C
Que.	31	32	33	34	35					
Ans.	D	A	B	C	A $\rightarrow$p; B $\rightarrow$r; C $\rightarrow$q; D $\rightarrow$s					

EXERCISE - S

1.	9.00	**2.**	8.00	**3.**	4.00	**4.**	37.00	**5.**	48.00
6.	0.25	**7.**	6.00	**8.**	1.00	**9.**	9.42 or 9.43	**10.**	20.00

EXERCISE - JEE (Main) PYQ

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1	3	2	4.00	4	2	2	3	3	3
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	195	1	1	4	3	3	5	4	3	11

EXERCISE - JEE (Advanced) PYQ

Que.	1	2	3	4	5	6
Ans.	$\sqrt{2}x - y - 2\sqrt{2} = 0$	D	D	D	A	8

JEE (Main) Practice Paper

Section-A	Q.	1	2	3	4	5	6	7	8	9	10
	A.	1	3	3	2	3	4	2	1	1	2
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	4	4	3	2	3	1	2	3	1	4
Section-B	Q.	1	2	3	4	5	6	7	8	9	10
	A.	1	90	2	2	66	1	3	3	1	1

JEE (Advanced) Practice Paper

Section-I	**Q.**	**1**	**2**	**3**	**4**	**5**	**6**		
	A.	A	B	C	B	B	C		
Section-II	**Q.**	**7**	**8**	**9**	**10**	**11**			
	A.	C,D	A,B,C	B,C	A,C	A,B,C			
Section-III	**Q.**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	
	A.	7.5	2	24	3	15	4	0.50	