Statistics

SOLUTIONS

EXERCISE

SECTION–A

1. **Ans. (2)**

Mode + 2 mean = 3 median

(mean –3) + 2 mean = 3 median

3(mean – median) = 3

$\Rightarrow$ (mean – median) = 1

2. **Ans. (2)**

Variance $(x_i - 4) = \text{var}(x_i) = \frac{44}{11} - \left(\frac{11}{11}\right)^2 = 3$

3. **Ans. (3)**

$\bar{x}_{new} = \bar{x}_{old} - 5$

$\Rightarrow \bar{x}_{new} = 25 - 5 = 20$

4. **Ans. (3)**

Range = $21 - 12 = 9$

5. **Ans. (2)**

$\sum x_i = 63 \times 50 = 3150; \sum y_i = 40 \times 54 = 2160$

$\text{var}(x_i) = 81 = \frac{\sum {x_i}^2}{50} - (63)^2,\ \text{var}(y_i) = 36 = \frac{\sum {y_i}^2}{40} - (54)^2$

$\sum x_i^2 = 202500$

$\sum y_i^2 = 118080$

combined variance = $\frac{\left(\sum {x_i}^2 + \sum {y_i}^2\right)}{90} - \left(\frac{\sum x_i + \sum y_i}{90}\right)^2$

$= \frac{320580}{90} - (59)^2$

$= 3562 - 3481 = 81$

6. **Ans. (4)**

$\frac{(2 \times 1) + (14 \times 2) + (8 \times 5) + (32 \times 7)}{2 + 14 + 8 + 32} = \frac{294}{56} = 5.25$

7. **Ans. (4)**

Most frequent data = 3

8. **Ans. (1)**

$\frac{L - S}{L + S} = \frac{11 - 2}{11 + 2} = \frac{9}{13}$

9. **Ans. (3)**

Variance $(ax_i + b) = a^2\text{var}(x_i)$

$\Rightarrow$ Variance $(2x_i + 3) = 2^2\text{var}(x_i) = 4\lambda$

10. **Ans. (2)**

$\frac{\sigma}{\bar{x}} \times 100$ = coefficient of variation $\Rightarrow \sigma = 3$

11. **Ans. (2)**

Median = $\frac{24^{th}\text{term}+25^{th}\text{term}}{2} = \frac{76+77}{2}$ = 76.5

Mean deviation = $\frac{(23.5+22.5......0.5+0.5+.......23.5)}{48} = 12$

12. **Ans. (2)**

$\bar{x} = \frac{n(n+1)}{2n} = \frac{n+1}{2} \Rightarrow \sigma^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{\sum x_i}{n}\right)^2$

$= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{(n+1)}{2}\right)^2 = \frac{n^2-1}{12}$

13. **Ans. (3)**

Arrange marks in ascending order

(................), 17, 18, 19, 20, 21, 23, 24, 26, 27, 27, 28, 29

8 boys failed

median $= \frac{18 + 19}{2} = 18.5$

14. **Ans. (4)**

$\sum x_i = 20 \times 11 = 220$; $\sum y_i = 10 \times 8 = 80$

variance $(x_i) = \frac{\sum {x_i}^2}{20} - (121) \Rightarrow 2500 = \sum x_i^2$

variance $(y_i) = \frac{\sum {y_i}^2}{10} - 64 \Rightarrow 980 = \sum y_i^2$

$\sum x_i^2 + \sum y_i^2 = (x_i^2 + {y_i}^2) = 3480 \Rightarrow \sigma^2 = \frac{\sum({x_i}^2+{y_i}^2)}{30} - \left(\frac{\sum x_i+y_i}{30}\right)^2 = 116 - 100 = 16$

15. **Ans. (3)**

Actual data is more close to mean, therefore less variance.

16. **Ans. (3)**

Variations = $\frac{2^2+4^2+6^2+8^2+10^2}{5} - \left(\frac{2+4+6+8+10}{5}\right)^2 = 44 - 36 = 8$

17. **Ans. (3)**

$\text{Var}(3x_i + 4) = 9\sigma^2$

18. **Ans. (3)**

Arranging the data in ascending order 34, 38, 42, 44, 46, 48, 54, 55, 63, 70

median = $\frac{46+48}{2} = 47$

$\sum|x_i - 47| = 13 + 9 + 5 + 3 + 1 + 1 + 7 + 8 + 16 + 23 = 86.$

Mean deviation = $\frac{\sum|x_i-47|}{10} = 8.6$

19. **Ans. (2)**

variance $(ax_i) = a^2\text{var}(x_i)$

S.D. $(ax_i) = |a|\sqrt{var(x_i)}$

20. **Ans. (2)**

$$\frac{\sum x_i}{7} = 7$$

$2 + 4 + 7 + 11 + 10 + a + b = 49 \Rightarrow a + b = 15$

$$\frac{\sum (x_i - \bar{x})^2}{7} = \frac{100}{7}$$

$\Rightarrow 25 + 9 + 0 + 16 + 9 + (7 - a)^2 + (7 - b)^2 = 100$

$\Rightarrow (7 - a)^2 + (7 - b)^2 = 41 \ \Rightarrow a = 3$

$b = 12$

SECTION–B

1. **Ans. (23)**

Here N = $\sum f_i = 15, \sum f_i x_i = 2 \times 5 + 4 \times 15 + 5 \times 25 + 3 \times 35 + 1 \times 45 = 345$

$\therefore \bar{x} = \frac{\sum f_i x_i}{N} = \frac{345}{15} = 23$

2. **Ans. (7)**

$\sum (x_i - 4) = 30$

$\Rightarrow \sum x_i - 4n = 30$ ...(i)

and $\sum (x_i - 3) = 40$

$\Rightarrow \sum x_i - 3n = 40$ ...(ii)

Solving (i) and (ii)

$\Rightarrow \sum x_i = 70, n = 10 \Rightarrow$ Mean = $\frac{\sum x_i}{n} = \frac{70}{10} = 7$

3. **Ans. (89)**

Arranged the values 76, 78, 80, 86, 92, 94, 96, 99

Here $n = 8$ (even)

$\therefore$ Median = $\frac{\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}}{2}$

$= \frac{4^{\text{th}}\text{term} + 5^{\text{th}}\text{term}}{2}$

$= \frac{86 + 92}{2}$

$= \frac{178}{2} = 89$

4. **Ans. (155)**

x_i	f_i	C.f.
150	8	8
152	4	12
154	3	15
155	7	22
156	3	25
160	12	37
161	4	41

Here $N = 41$ (odd)

Median $=\left(\frac{N+1}{2}\right)^{th}$ term

$= 21^{st}$ term

= 155

5. Ans. (11)

Here n = 8, $\sum x_i = 72$

$\therefore$ Mean $(\bar{x}) = \frac{\sum x_i}{n} = \frac{72}{8} = 9$

$\because \sum|x_i - \bar{x}| = 3 + 2 + 1 + 3 + 4 + 5 + 1 + 3 = 22$

$\therefore$ M.D. about mean (λ) = $\frac{\sum|x_i-\bar{x}|}{n} = \frac{22}{8} = 2.75 \Rightarrow 4\lambda = 4(2.75) = 11.$

6. Ans. (43)

Here $n = 10$ (even)

Median $(M) = \frac{\left(\frac{n}{2}\right)^{th}\text{term}+\left(\frac{n}{2}+1\right)^{th}\text{term}}{2} = \frac{5^{th}\text{term}+6^{th}\text{term}}{2}$

$= \frac{46 + 48}{2} = 47$

$\because \sum|x_i - M| = 13 + 9 + 5 + 3 + 1 + 1 + 7 + 8 + 16 + 23 = 86$

$\therefore$ M.D. (λ) = $\frac{\sum|x_i-M|}{n} = \frac{86}{10} = 8.6 \Rightarrow 5\lambda = 5(8.6) = 43.$

7. Ans. (933)

M=25

Class	f_i	c.f.	x_i	$f_i\lvert x_i - M\rvert$
0 – 10	2	2	5	2·20 = 40
10 – 20	8	10	15	8·10 = 80
20 – 30	10	20	25	10·0 = 0
30 – 40	4	24	35	4·10 = 40
40 – 50	6	30	45	6·20 =120
	N = 30			$\sum f_i\lvert x_i - M\rvert = 280$

Here $\frac{N}{2} = 15$, Median class is 20 – 30

$\therefore$ Median = $\ell + \frac{\left(\frac{N}{2}-F\right)}{f} \times h = 20 + \frac{(15-10)}{10} \times 10 = 25$

$\therefore$ M.D. (λ) about Median = $\frac{\sum f_i|x_i-M|}{N} = \frac{280}{30} = 9.33 \Rightarrow 100\lambda = 933.$

8. Ans. (244)

Here $n = 5, \sum x_i = 605$

$\therefore \bar{x} = \frac{\sum x_i}{n} = \frac{605}{5} = 121$

$\because \sum(x_i - \bar{x})^2 = 81 + 25 + 1 + 16 + 121 = 244$

$\therefore \sigma^2\ (\lambda) = \frac{\sum(x_i-\bar{x})^2}{n} = \frac{244}{5} = 48.8 \Rightarrow 5\lambda = 5(48.8) = 244.$

9. **Ans. (2)**

$x_i \rightarrow \underbrace{a, a, \ldots\ldots. a}_{n}, \underbrace{-a, -a, \ldots\ldots a}_{n}$

$\bar{x} = \frac{\sum x_1}{n} = 0$

Variance $(x_i) = \frac{\sum x_i^2}{n} - (\bar{x})^2$

$4 = \frac{a^2 + a^2 + \ldots. + 2n \text{ times}}{2n} - 0$

$4 = \frac{2na^2}{2n}$

$a^2 = 4$

$\therefore a = \pm 2$

So, $|a| = 2$.

10. **Ans. (5)**

Variance (4, 8, 12, ..., 40)

$= 4^2$ Variance (1, 2, 3, ..., 10)

$= 16 \times \frac{25}{16}$

$\therefore$ S.D. = 5

EXERCISE - JEE (Main) PYQ

1. **Ans. (2)**

$\sum (x_i + 1)^2 = 9n$ ...(i)

$\sum (x_i - 1)^2 = 5n$ ...(ii)

(i) + (ii) $\Rightarrow \sum \left(x_i^2 + 1\right) = 7n$

$\Rightarrow \frac{\sum x_i^2}{n} = 6$

(i) – (ii) $\Rightarrow 4\Sigma x_i = 4n$

$\Rightarrow \Sigma x_i = n$

$\Rightarrow \frac{\Sigma x_i}{n} = 1$

$\Rightarrow$ variance = $6 - 1 = 5$

$\Rightarrow$ Standard deviation $= \sqrt{5}$

2. **Ans. (1)**

Let two observations are x_1 & x_2

mean $= \frac{\sum x_i}{5} = 5 \Rightarrow 1 + 3 + 8 + x_1 + x_2 = 25$

$\Rightarrow x_1 + x_2 = 13$ ...(i)

variance $(\sigma^2) = \frac{\sum x_i^2}{5} - 25 = 9.20$

$\Rightarrow \sum x_i^2 = 171$

$\Rightarrow x_1^2 + x_2^2 = 97$ ...(ii) by (i) & (ii)

$(x_1 + x_2)^2 - 2x_1x_2 = 97$

or $x_1x_2 = 36$

$\therefore x_1 : x_2 = 4 : 9$

3. Ans. (2)

$\bar{x} = 10 \Rightarrow \sum_{i=1}^{5} x_i = 50$

$$S.D. = \sqrt{\frac{\sum_{i=1}^{5} x_i^2}{5} - (\bar{x})^2} = 3$$

$$\Rightarrow \sum_{i=1}^{5} (x_i)^2 = 545$$

$$\text{variance} = \frac{\sum_{i=1}^{5}(x_i)^2 + (-50)^2}{6} - \left(\frac{\sum_{i=1}^{5} x_i - 50}{6}\right)^2$$

= 507.5

4. Ans. (4)

Variance is independent of origin. So we shift the given data by $\frac{1}{2}$.

So, $\frac{10d^2 + 10 \times 0^2 + 10d^2}{30} - (0)^2 = \frac{4}{3}$

$\Rightarrow d^2 = 2 \Rightarrow |d| = \sqrt{2}$

5. Ans. (4)

$$\sum_{i=1}^{50} (x_i - 30) = 50$$

$\Sigma x_i - 50 \times 30 = 50$

$\Sigma x_i = 1550$

Mean = $\bar{x} = \frac{\Sigma x_i}{n} = \frac{50\times30+50}{50} = 30 + 1 = 31$

6. Ans. (3)

Mean $\bar{x} = 4, \sigma^2 = 5.2, n = 5, . x_1 = 3, x_2 = 4 = x_3$

$\sum x_i = 20$

$x_4 + x_5 = 9$ ...(i)

$\frac{\sum x_i^2}{5} - (\bar{x})^2 = \sigma^2 \Rightarrow \sum x_i^2 = 106$

$x_4^2 + x_5^2 = 65$ ...(ii)

Using (i) and (ii) $(x_4 - x_5)^2 = 49$

$|x_4 - x_5| = 7$

7. Ans. (3)

Let 7 observations be $x_1, x_2, x_3, x_4, x_5, x_6, x_7$

$$\bar{x} = 8 \Rightarrow \sum_{i=1}^{7} x_i = 56 \qquad \text{...(i)}$$

Also $\sigma^2 = 16$

$$\Rightarrow 16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - (\bar{x})^2 \Rightarrow 16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - 64$$

$$\Rightarrow \left(\sum_{i=1}^{7} x_i^2\right) = 560 \qquad \text{...(ii)}$$

Now, $x_1 = 2, x_2 = 4, x_3 = 10, x_4 = 12, x_5 = 14$

$\Rightarrow x_6 + x_7 = 14$ (from (i))

$\& \, x_6^2 + x_7^2 = 100$ (from (ii))

$\therefore x_6^2 + x_7^2 = (x_6 + x_7)^2 - 2x_6.x_7 \Rightarrow x_6.x_7 = 48$

8. Ans. (1)

Let x be the 6^{th} observation

$\Rightarrow 45 + 54 + 41 + 57 + 43 + x = 48 \times 6 = 288$

$\Rightarrow x = 48$

$$\text{variance } = \left(\frac{\Sigma x_i^2}{6} - (\bar{x})^2\right)$$

$$\Rightarrow \text{variance } = \frac{14024}{6} - (48)^2 = \frac{100}{3}$$

$\Rightarrow$ standard deviation = $\dfrac{10}{\sqrt{3}}$

9. Ans. (2)

$$\text{S.D.} = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}$$

$$\bar{x} = \frac{\Sigma x}{4} = \frac{-1 + 0 + 1 + k}{4} = \frac{k}{4}$$

$$\text{Now } \sqrt{5} = \sqrt{\frac{\left(-1 - \frac{k}{4}\right)^2 + \left(0 - \frac{k}{4}\right)^2 + \left(1 - \frac{k}{4}\right)^2 + \left(k - \frac{k}{4}\right)^2}{4}}$$

$$\Rightarrow 18 = \frac{3k^2}{4} \Rightarrow k^2 = 24 \Rightarrow k = 2\sqrt{6}$$

10. Ans. (1)

$$\frac{34 + x}{2} = 35$$

$x = 36$

$$42 = \frac{10 + 22 + 26 + 29 + 34 + 36 + 42 + 67 + 70 + y}{10}$$

$420 - 336 = y \Rightarrow y = 84$

$$\frac{y}{x} = \frac{84}{36} = \frac{7}{3}$$

11. Ans. (4)

Mean (μ) = $\frac{\sum x_i}{50} = 16$

Standard deviation (σ) = $\sqrt{\frac{\sum x_i^2}{50} - (\mu)^2} = 16$

$\Rightarrow (256) \times 2 = \frac{\sum x_i^2}{50}$

⇒ New mean

$= \frac{\sum (x_i - 4)^2}{50} = \frac{\sum x_i^2 + 16 \times 50 - 8\sum x_i}{50}$

$= (256) \times 2 + 16 - 8 \times 16 = 400$

12. Ans. (1)

$\frac{\sum x_i}{20} = 10 \Rightarrow \Sigma x_i = 200$ ...(i)

$\frac{\sum {x_i}^2}{20} - 100 = 4 \Rightarrow \Sigma x_i^2 = 2080$ ...(ii)

Actual mean = $\frac{200-9+11}{20} = \frac{202}{20}$

Variance = $\frac{2080-81+121}{20} - \left(\frac{202}{20}\right)^2 = 3.99$

13. Ans. (1)

$20p - q = 10$ ...(i)

and $2|p| = 1 \Rightarrow p = \pm \frac{1}{2}$ ...(ii)

So, $p = -\frac{1}{2}$ and $q = -20$

14. Ans. (4)

$$\sum_{i=1}^{10} (x_i - 5) = 10$$

⇒ Mean of observation $x_i - 5 = \frac{1}{10}\sum_{i=1}^{10}(x_i - 5) = 1$

⇒ μ = mean of observation $(x_i - 3)$

= (mean of observation $(x_i - 5)) + 2$

$= 1 + 2 = 3$

Variance of observation

$x_i - 5 = \frac{1}{10}\sum_{i=1}^{10}(x_i - 5)^2 - (\text{Mean of } (x_i - 5))^2 = 3$

$\Rightarrow \lambda =$ variance of observation $(x_i - 3)$

= variance of observation $(x_i - 5) = 3$

$\therefore (\mu, \lambda) = (3, 3)$

15. Ans. (68)

Let number be $a_1, a_2, a_3, \ldots\ldots a_{2n}, b_1, b_2, b_3 \ldots. b_n$

$$\sigma^2 = \frac{\sum a^2 + \sum b^2}{3n} - (5)^2 \Rightarrow \sum a^2 + \sum b^2 = 87n$$

Now, distribution becomes

$a_1 + 1, a_2 + 1, a_3 + 1, \ldots\ldots. a_{2n} + 1, b_1 - 1, b_2 - 1 \ldots.. b_n - 1$

Variance

$$= \frac{\sum (a+1)^2 + \sum (b-1)^2}{3n} - \left(\frac{12n+2n+3n-n}{3n}\right)^2 = \frac{\left(\sum a^2 + 2n + 2\sum a\right) + \left(\sum b^2 + n - 2\sum b\right)}{3n}$$

$$= \frac{\left(\sum a^2 + 2n + 2\sum a\right) + \left(\sum b^2 + n - 2\sum b\right)}{3n} - \left(\frac{16}{3}\right)^2 = \frac{\left(\sum a^2 + 2n + 2\sum a\right) + \left(\sum b^2 + n - 2\sum b\right)}{3n} - \left(\frac{16}{3}\right)^2$$

$$\Rightarrow k = \frac{108}{3} - \left(\frac{16}{5}\right)^2 \Rightarrow 9k = 3(108) - (16)^2 = 324 - 256 = 68$$

Ans. 68.00

16. Ans. (4)

Class	Frequency	x_i	$f_i x_i$
0–6	a	3	$3a$
6–12	b	9	$9b$
12–18	12	15	180
18–24	9	21	189
24–30	5	27	135
	$N = (26 + a + b)$		$(504 + 3a + 9b)$

Mean = $\frac{3a + 9b + 180 + 189 + 135}{a + b + 26} = \frac{309}{22}$

$\Rightarrow 66a + 198b + 11088 = 309a + 309b + 8034$

$\Rightarrow 243a + 111b = 3054$

$\Rightarrow \boxed{81a + 37b = 1018}$ ...(1)

Now, Median = $12 + \frac{\frac{a+b+26}{2} - (a+b)}{12} \times 6 = 14$

$$\Rightarrow \frac{13}{2} - \left(\frac{a+b}{4}\right) = 2$$

$$\Rightarrow \frac{a+b}{4} = \frac{9}{2}$$

$\Rightarrow \boxed{a + b = 18}$ ...(2)

From equation (1) & (2)

$a = 8, b = 10$

$\therefore (a - b)^2 = (8 - 10)^2$

17. Ans. (4)

$$\sum_{i=1}^{18}(x_i-\alpha)=36,\sum_{i=1}^{18}(x_i-\beta)^2=90$$

$$\Rightarrow \sum_{i=1}^{18}x_i=18(\alpha+2),\sum_{i=1}^{18}x_i^2-2\beta\sum_{i=1}^{18}x_i+18\beta^2=90$$

Hence $\sum x_i^2=90-18\beta^2+36\beta(\alpha+2)$

Given $\frac{\sum x_i^2}{18}-\left(\frac{\sum x_i}{18}\right)^2=1$

$\Rightarrow 90-18\beta^2+36\beta(\alpha+2)-18(\alpha+2)^2=18$

$\Rightarrow 5-\beta^2+2\alpha\beta+4\beta-\alpha^2-4\alpha-4=1$

$\Rightarrow (\alpha-\beta)^2+4(\alpha-\beta)=0 \Rightarrow |\alpha-\beta|=0$ or 4

As α and β are distinct $|\alpha-\beta|=4$

18. Ans. (238)

Wrong mean = $\mu_1=30$

Wrong S.D = $\sigma_1=5$

$$\frac{\sum x_i}{40}=30 \Rightarrow \sum x_i=1200$$

$$\sigma_1^2=25 \Rightarrow \frac{\sum x_i^2}{40}-30^2=25$$

$$\Rightarrow \sum x_i^2=925\times 40=37000$$

New sum = $\sum x_i'=1200-10-12=1178$

New mean = $\mu_1'=\frac{1178}{38}=31$

New $\sum x_i^2=37000-(10)^2-(12)^2=36756$

New S.D, $\sigma_1'=\sqrt{\frac{36756}{38}-(31)^2}=\sigma$

$36756-(31)^2\times 38=38\sigma^2$

$\Rightarrow 38\sigma^2=238$

19. Ans. (142)

$$\sum x_0^1=\frac{3\left(1-\left(\frac{1}{2}\right)\right)^{20}}{1-\frac{1}{2}}=6\left(1-\frac{1}{2^{20}}\right)$$

$$=\sum_{i=1}^{20}(x_i-i)^2=\sum_{i=1}^{20}(x_i)^2+(i)^2-2x_i i$$

$$\text{Now }=\sum_{i=1}^{20}(x_i)^2=\frac{9\left(1-\left(\frac{1}{4}\right)\right)^{20}}{1-\frac{1}{4}}=12\left(1-\frac{1}{2^{40}}\right)$$

$$\sum_{i=1}^{20} i^2 = \frac{1}{6}\times 20\times 21\times 41 = 2870$$

$$\sum_{i=1}^{20} x_i\ i = s = 3 + 2.3\frac{1}{2} + 3.3\frac{1}{2^2} + 4.3\frac{1}{2^3} + ...AGP$$

$$= 6\left(2 - \frac{22}{2^{20}}\right)$$

$$\bar{x} = \frac{12 - \frac{12}{2^{40}} + 2870 - 12\left(2 - \frac{22}{2^{20}}\right)}{20}$$

$$\bar{x} = \frac{2858}{20} + \left(\frac{-12}{2^{40}} + \frac{22}{2^{20}}\right)\times\frac{1}{20}$$

$$\left[\bar{x}\right] = 142$$

20. **Ans. (4)**

$$\sum x_1 = 15\times 20 = 300 \qquad ...(i)$$

$$\frac{\sum x_1^2}{20} - (15)^2 = 9 \qquad ...(ii)$$

$$\sum x_1^2 = 234\times 20 = 4680$$

$$\frac{\sum (x_1+\alpha)^2}{20} = 178 \Rightarrow \sum (x_1+\alpha)^2 = 3560 \ \Rightarrow \sum x_1^2 + 2\alpha\sum x_1 + \sum \alpha^2 = 3560$$

$$4680 + 600\alpha + 20\alpha^2 = 3560$$

$$\Rightarrow \alpha^2 + 30\alpha + 56 = 0 \ \Rightarrow (\alpha+28)(\alpha+2) = 0$$

$$\alpha = -2, -28$$

Square of maximum value of α is 4

21. **Ans. (3)**

let a_1 be any natural number

$a_1, a_1+1, a_1+2, ..., a_1+99$ are values of a_i'S

$$\bar{x} = \frac{a_1 + (a_1+1) + (a_1+2) + ... + a_1 + 99}{100}$$

$$= \frac{100a_1 + (1+2+.....+99)}{100} = a_1 + \frac{99\times 100}{2\times 100} \ = a_1 + \frac{99}{2}$$

Mean deviation about mean $= \dfrac{\sum_{1=1}^{100}|x_i - \bar{x}|}{100}$

$$= \frac{2\left(\frac{99}{2} + \frac{97}{2} + \frac{95}{2} + ... + \frac{1}{2}\right)}{100} = \frac{1+3+.....+99}{100} = \frac{\frac{50}{2}[1+99]}{100} = 25$$

So, it is true for every natural no. 'a_1'

22. Ans. (2)

$9 = x_1 < x_2 < ....< x_7$

$9, 9 + d, 9 + 2d, ...9 + 6d$

$0, d, 2d, ...6d$

$$\bar{x}_{new} = \frac{21d}{7} = 3d$$

$$16 = \frac{1}{7}\left(0^2 + 1^2 + ... + 6^2\right)d^2 - 9d^2$$

$$= \frac{1}{7}\left(\frac{6 \times 7 \times 13}{6}\right)d^2 - 9d^2$$

$16 = 4d^2$

$d^2 = 4$

$d = 2$

$\bar{x} + x_6 = 6 + 9 + 10 + 9$

23. Ans. (1)

$$\frac{1+3+5+a+b}{5} = 5$$

$a + b = 16$ ...(1)

$$\sigma^2 = \frac{\sum x_1^2}{5} - \left(\frac{\sum x}{5}\right)^2$$

$$8 = \frac{1^2 + 3^2 + 5^2 + a^2 + b^2}{5} - 25$$

$a^2 + b^2 = 130$ ...(2)

by (1), (2)

$a = 7, b = 9$

or $a = 9, b = 7$

24. Ans. (37)

$$\frac{x_1 + x_2 + ... + x_7}{7} = 8$$

$$\frac{x_1 + x_2 + x_3 .... + x_6 + 14}{7} = 8 \Rightarrow x_1 + x_2 + ... + x_6 = 42$$

$$\therefore \frac{x_1 + x_2 .... + x_6}{6} = \frac{42}{6} = 7 = a$$

$$\frac{\Sigma x_i^2}{7} - 8^2 = 16$$

$$\Sigma xi^2 = 560 \Rightarrow x_1^2 + x_2^2 + ... + x_6^2 = 364$$

$$b = \frac{x_1^2 + x_2^2 + ... + x_6^2}{6} - 7^2 = \frac{364}{6} - 49$$

$$b = \frac{70}{6}$$

$$a + 3b - 5 = 7 + 3 \times \frac{70}{6} - 5 = 37$$

25. Ans. (3)

Given mean $(\bar{x}) = \frac{9}{2}$

$$\bar{x}_{new} = \frac{12\times\frac{9}{2}+7+14-9-10}{12} = \frac{14}{3} \qquad \text{...(i)}$$

Given, $\sigma^2 = 4$

$$\sigma^2 = \frac{\sum x_i^2}{12} - \left(\frac{9}{2}\right)^2$$

$$4 = \frac{\sum x_i^2}{12} - \frac{81}{4}$$

$$\frac{\sum x_i^2}{12} = \frac{97}{4}$$

$$\sum x_i^2 = 291$$

Now,

$$\sum\left(x_i^2\right)_{new} = 291 - 9^2 - 10^2 + 7^2 + 14^2 = 355$$

$$\therefore \sigma^2_{new} = \frac{\sum\left(x_i^2\right)_{new}}{12} - \left(\bar{x}_{new}\right)^2$$

$$\sigma^2_{new} = \frac{355}{12} - \left(\frac{14}{3}\right)^2 = \frac{281}{36} \quad \text{(from eq.(i))}$$

26. Ans. (5)

x_i	f_i	$d_i = x_i - 5$	$f_i d_i^2$	$f_i d_i$
2	3	-3	27	-9
3	6	-2	24	-12
4	16	-1	16	-16
5	α	0	0	0
6	9	1	9	9
7	5	2	20	10
8	6	3	54	18

$$\sigma_x^2 = \sigma_d^2 = \frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2 = \frac{150}{45+\alpha} - 0 = 3 \Rightarrow 150 = 135 + 3\alpha$$

$$\Rightarrow 3\alpha = 15 \Rightarrow \alpha = 5$$

27. Ans. (25)

x_i	f_i	$f_i x_i$	$f_i x_i^2$
2	4	8	16
4	4	16	64
6	α	6α	36α
8	15	120	960
10	8	80	800
12	β	12β	144β
14	4	56	784
16	5	80	1280

$$N = \sum f_i = 40 + \alpha + \beta$$

$$\sum f_i x_i = 360 + 6\alpha + 12\beta$$

$$\sum f_i x_i^2 = 3904 + 36\alpha + 144\beta$$

$$\text{Mean}(\bar{x}) = \frac{\sum f_i x_i}{\sum f_i} = 9$$

$\Rightarrow 360+6\alpha+12\beta=9(40+\alpha+\beta)$

$3\alpha=3\beta\Rightarrow\alpha=\beta$

$\sigma^2=\frac{\sum f_i x_1^2}{\sum f_i}-\left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$

$\Rightarrow\frac{3904+36\alpha+144\beta}{40+\alpha+\beta}-(\bar{x})^2=15.08$

$\Rightarrow\frac{3904+180\alpha}{40+2\alpha}-(9)^2=15.08$

$\Rightarrow\alpha=5$

Now, $\alpha^2+\beta^2-\alpha\beta=\alpha^2=25$

28. Ans. (1)

$\sum f_i=62$

$\Rightarrow 3k^2+16k-12k-64=0$

$\Rightarrow k=4$ or $-\frac{16}{3}$ (rejected)

$\mu=\frac{\sum f_i x_i}{\sum f_i}$

$\mu=\frac{8+2(15)+3(15)+4(17)+5}{62}=\frac{156}{62}$

$\sigma^2=\sum f_i x_i^2-\left(\sum f_i x_i\right)^2$

$=\frac{8\times1^2+15\times13+17\times16+25}{62}-\left(\frac{156}{62}\right)^2$

$\sigma^2=\frac{500}{62}-\left(\frac{156}{62}\right)^2$

$\sigma^2+\mu^2=\frac{500}{62}$

$\left[\sigma^2+\mu^2\right]=8$

29. Ans. (603)

$\bar{x}=\frac{\sum_{i=11}^{41} i}{31}=\frac{11+41}{2}=26$ (31 elements)

$\bar{y}=\frac{\sum_{j=61}^{91} j}{31}=\frac{61+91}{2}=76$ (31 elements)

Combined mean, $\mu=\frac{31\times26+31\times76}{31+31}=\frac{26+76}{2}=51$

$\sigma^2=\frac{1}{62}\times\left(\sum_{i=1}^{31}(x_i-\mu)^2+\sum_{i=1}^{31}(y_i-\mu)^2\right)=705$

Since, $x_i\in X$ are in A.P. with 31 elements & common difference 1, same is $y_i\in y$, when written in increasing order.

$\therefore\ \sum_{i=1}^{31}(x_i-\mu)^2=\sum_{i=1}^{31}(y_i-\mu)^2$

$=10^2+11^2+.....+40^2$

$= \dfrac{40\times41\times81}{6} - \dfrac{9\times10\times19}{6} = 21855$

$\therefore \left|\bar{x}+\bar{y}-\sigma^2\right| = |26+76-705| = 603$

30. **Ans. (1)**

A	***B***	***A + B***
$\bar{x}_1 = 40$	$\bar{x}_2 = 55$	$\bar{x} = 50$
$\sigma_1 = \alpha$	$\sigma_2 = 30-\alpha$	$\sigma^2 = 350$
$n_1 = 100$	$n_2 = n$	$100+n$

$\bar{x} = \dfrac{100\times40+55n}{100+n}$

$5000+50n = 4000+55n$

$1000 = 5n$

$n = 200$

$\sigma_1^{\,2} = \dfrac{\sum x_i^{\,2}}{100} - 40^2$

$\sigma_2^{\,2} = \dfrac{\sum x_j^{\,2}}{100} - 55^2$

$350 = \sigma^2 = \dfrac{\sum x_i^{\,2} + \sum x_j^{\,2}}{300} - \left(\bar{x}\right)^2$

$350 = \dfrac{\left(1600+\alpha^2\right)\times100 + \left[(30-\alpha)^2+3025\right]\times200}{300} - (50)^2$

$2850\times3 = \alpha^2 + 2(30-\alpha)^2 + 1600 + 6050$

$8550 = \alpha^2 + 2(30-\alpha)^2 + 7650$

$\alpha^2 + 2(30-\alpha)^2 = 900$

$\alpha^2 - 40\alpha + 300 = 0$

$\alpha = 10, 30$

$\sigma_1^{\,2} + \sigma_2^{\,2} = 10^2 + 20^2 = 500$

Important Notes

Important Notes