Quadratic Equation

SOLUTIONS

Exercise-I (JEE Main Pattern)

SECTION–A

1. **Ans. (4)**

$x^2 - 5x + 16 = 0$ (roots α, β)

$\alpha + \beta = 5$; $\alpha\beta = 16$

Now, when roots are $\alpha^2 + \beta^2$ and $\frac{\alpha\beta}{2}$

$\Rightarrow \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 5^2 - 2 \times 16 = -7$

$\frac{\alpha\beta}{2} = \frac{16}{2} = 8$

equation with these roots

$\Rightarrow x^2 - (-7 + 8)x - 7 \times 8 = 0 \qquad \Rightarrow x^2 - x - 56 = 0$

$p = -1, q = -56$

2. **Ans. (2)**

$x^2 + px + q = 0$ (roots 8, 2) $\qquad x^2 - rx + 5 = 0$ (roots 3, 3)

$-p = 8 + 2 \Rightarrow p = -10 \qquad -r = 3 + 3$

$q = (8)(2) \qquad S = (3)(3)$

$q = 16 \qquad S = 9$

Now the equation $x^2 + px + s = 0$ become

$\Rightarrow x^2 - 10x + 9 = 0$

$x = 1,9$

3. **Ans. (3)**

$(x - a)(x - b) - c = 0$ (roots α, β)

$\Rightarrow (x - a)(x - b) - c = (x - \alpha)(x - \beta)$

$\Rightarrow (x - \alpha)(x - \beta) + c = (x - a)(x - b) \qquad$ ...(i)

Roots of (i) will be $x = a, b$

4. **Ans. (1)**

$\alpha^2 - 5\alpha + 3 = 0 \qquad$ ...(i)

$\beta^2 - 5\beta + 3 = 0 \qquad$ ...(ii)

Form equation (i) and (ii) will can say that α and β are roots of $x^2 - 5x + 3 = 0$

$\alpha + \beta = 5$ and $\alpha\beta = 3$

Now, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$= \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{25 - 6}{3} = \frac{19}{3}$

5. Ans. (4)

$(a^2-5a+3)x^2+(3a-1)x+2=0 \; \begin{matrix} \alpha \\ 2\alpha \end{matrix}$

$\alpha+2\alpha=-\left(\frac{3a-1}{a^2-5a+3}\right) \Rightarrow \alpha=\frac{1}{3}\left(\frac{1-3a}{a^2-5a+3}\right)$ ...(1)

and $\alpha(2\alpha)=\frac{2}{a^2-5a+3} \Rightarrow \alpha^2=\frac{1}{a^2-5a+3}$ ...(2)

from (1) & (2)

$\Rightarrow \frac{1}{9}\left(\frac{(1-3a)^2}{(a^2-5a+3)^2}\right)=\frac{1}{(a^2-5a+3)}$

$\Rightarrow (1-3a)^2=9(a^2-5a+3)$

$\Rightarrow 1+9a^2-6a=9a^2-45a+27$

$\Rightarrow 39a=26 \Rightarrow a=\frac{2}{3}$

6. Ans. (1)

put $x=1-p$ in the equation $x^2+px+(1-p)=0$

$\Rightarrow (1-p)^2+p(1-p)+(1-p)=0$

$\Rightarrow (1-p)(1-p+p+1)=0$

$\Rightarrow (2)(1-p)=0 \Rightarrow p=1$

Now, $x^2+px+(1-p)=0$ become

$x^2+x+0=0$

$\Rightarrow x(x+1)=0$

$\Rightarrow x=0,-1$

7. Ans. (1)

$x^2-Ax+C-A=0$

$x_1+x_2=A$

$x_1x_2=C-A$

Now, $x_1^2+x_2^2+(2+A)x_1x_2$

$=(x_1+x_2)^2-2x_1x_2+(2+A)x_1x_2$

$=A^2-2(C-A)+(2+A)(C-A)$

$=A^2-2C+2A+2C-2A+AC-A^2$

$=AC$

8. Ans. (1)

Let α,β be roots

$a=-(\alpha+\beta)$

$b=\alpha\beta-1$

Now, $a^2+b^2=\alpha^2+\beta^2+\alpha^2\beta^2+1$

$=(1+\alpha^2)(1+\beta^2)$

a^2+b^2 can be split into two factor 17, 29, 53 cannot be split factors as they are primes

Hence a^2+b^2 can be equal to 50

9. **Ans. (2)**

$x^2 - 5x + 5 = 0$

$$x = \frac{5 \pm \sqrt{25 - 4(1)(5)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{5}}{2}$$

Irrational roots will occur in pair hence cubic will have two roots same as $\frac{5+\sqrt{5}}{2}$ and $\frac{5-\sqrt{5}}{2}$

$x^3 + ax^2 + bx + 5 = 0$

Product of roots

$$-(5) = \left(\frac{5+\sqrt{5}}{2}\right)\left(\frac{5-\sqrt{5}}{2}\right)(\alpha) \quad \text{(let 3rd root is } \alpha)$$

$\Rightarrow -(5) = (5)(\alpha) \Rightarrow \alpha = -1$

Sum of roots

$$\left(\frac{5+\sqrt{5}}{2}\right) + \left(\frac{5-\sqrt{5}}{2}\right) - 1 = -a$$

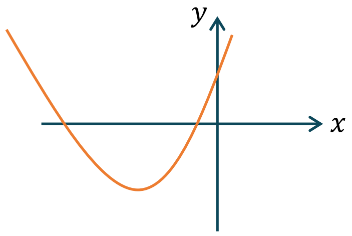
$\Rightarrow a = -4$

also, $\alpha = -1$ will satisfy cubic equation

$\Rightarrow -1 + a - b + 5 = 0 \quad \Rightarrow -1 - 4 - b + 5 = 0 \quad \Rightarrow b = 0 \Rightarrow a + b = -4$

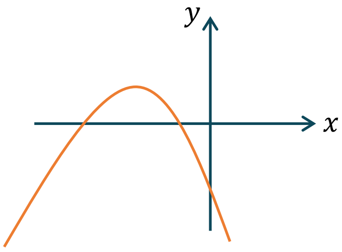
10. **Ans. (1)**

$ax^2 + bx + c = 0$



$c > 0, a > 0$

$-\frac{b}{2a} < 0 \Rightarrow b > 0$



$c < 0, a < 0$

$-\frac{b}{2a} < 0 \Rightarrow b < 0$

In both cases a, b, c are of same sign.

11. **Ans. (4)**

$D > 0$ (this equation will have two district roots)

$\Rightarrow (-(2-p))^2 - 4(1)(p-2) > 0 \quad \Rightarrow (p-2)^2 - 4(p-2) > 0$

$\Rightarrow (p-2)(p-2-4) > 0 \quad \Rightarrow (p-2)(p-6) > 0$

$\Rightarrow p \in (-\infty, 2) \cup (6, \infty)$

12. **Ans. (1)**

$x^2 + 2ax + 10 - 3a > 0$

$D < 0$

$\Rightarrow (2a)^2 - 4(1)(10-3a) < 0 \quad \Rightarrow a^2 - 10 + 3a < 0$

$\Rightarrow (a-2)(a+5) < 0 \quad \Rightarrow a \in (-5, 2)$

13. **Ans. (2)**

$g(x) = x^2 - (b+1)x + (b-1)$

$g(x) > -2$

$\Rightarrow x^2 - (b+1)x + (b-1) > -2 \quad \Rightarrow x^2 - (b+1)x + (b+1) > 0$

$D < 0$

$\Rightarrow (b+1)^2 - 4(1)(b+1) < 0 \Rightarrow (b+1)(b+1-4) < 0$

$\Rightarrow (b+1)(b-3) < 0 \Rightarrow b \in (-1,3)$

$\Rightarrow b = 2$ largest natural number in the interval

14. **Ans. (4)**

$y = x^2 - 6x + 5, x \in [2,4]$

$\frac{-b}{2a} = -\frac{(-6)}{2\times1} = 3 \in [2,4]$

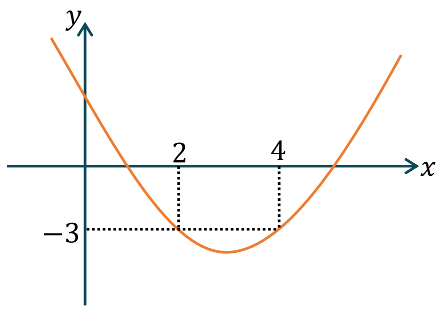
$-\frac{D}{4a} = \frac{-(36-20)}{4\times1} = -4$

$f(2) = (2)^2 - 6(2) + 5 = -3$

$f(4) = 16 - 24 + 5 = -3$

$y \in [-4, -3]$

greatest value of y will be -3



15. **Ans. (1)**

$y = \frac{16x^2 - 12x + 9}{16x^2 + 12x + 9}$

$\Rightarrow 16x^2y + 12xy + 9y - 16x^2 + 12x - 9 = 0$

$\Rightarrow x^2(16y - 16) + x(12y + 12) + (9y - 9) = 0$

$x \in R\,; D \geq 0$

$\Rightarrow (12y + 12)^2 - 4(16y - 16)(9y - 9) \geq 0$

$\Rightarrow (y+1)^2 - 4(y-1)(y-1) \geq 0 \quad \Rightarrow y^2 + 2y + 1 - 4(y^2 - 2y + 1) \geq 0$

$\Rightarrow 3y^2 - 10y + 3 \leq 0 \Rightarrow (3y-1)(y-3) \leq 0 \quad \Rightarrow y \in \left[\frac{1}{3}, 3\right]$

16. **Ans. (2)**

$(4p - p^2 - 5)x^2 - (2p-1)x + 3p = 0$

Given that, roots lies on different side of 1

So, $\quad af(1) < 0$

$\Rightarrow (4p - p^2 - 5)f(1) < 0$

$\Rightarrow (4p - p^2 - 5)\{4p - p^2 - 5 - 2p + 1 + 3p\} < 0$

$\Rightarrow (p^2 - 4p + 5)(p^2 - 5p + 4) < 0 \quad \{(p-2)^2 + 1$ is always positive$\}$

$\Rightarrow (p-1)(p-4) < 0$

$\Rightarrow p \in (1,4)$

Integer $p = 2,3$

17. **Ans. (4)**

$2^k x^2 - 4^k x + 2^k - 1 = 0$

Case – I

Exactly one root in $(0,1)$

$\Rightarrow f(0).f(1) < 0 \quad \Rightarrow (2^k - 1)(2^k - 4^k + 2^k - 1) < 0$

$\Rightarrow (2^k - 1)[(2^k)^2 - 2(2^k) + 1] > 0 \quad \Rightarrow (2^k - 1)^3 > 0$

$\Rightarrow 2^k > 1 \quad \Rightarrow k > 0$

Case – II

When one root is equal to 0, then other put $x = 0, 0 - 0 + 2^k - 1 = 0$

$\Rightarrow k = 0$

Now the equation is

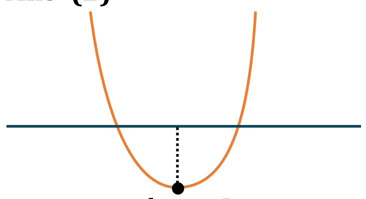
$x^2 - x = 0$

Hence, at $k = 0$, exactly one root in $[0,1)$

So, from case – I & case – II

$k \in [0, \infty)$

18. **Ans. (2)**



$\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$

$f(x) = 2x^2 + px + 1$

To have $f(x)$ negative integer for only one real we need y coordinate of vertex

$\frac{-D}{4a} = -1 \Rightarrow \frac{-(p^2 - 8)}{4(2)} = -1 \Rightarrow P = \pm 4$

19. **Ans. (2)**

$r_1, r_2 r_3$ roots of $x^3 - 2x^2 + 4x + 5074 = 0$

$r_1 + r_2 + r_3 = 2$

$r_1 r_2 + r_2 r_3 + r_3 r_1 = 4$

$r_1 r_2 r_3 = -5074$

Now,

$(r_1 + 2)(r_2 + 2)(r_3 + 2)$

$= r_1 r_2 r_3 + 4(r_1 + r_2 + r_3) + 2(r_1 r_2 + r_2 r_3 + r_3 r_1) + 8$

$= -5074 + 4(2) + 2(4) + 8$

$= -5050$

20. **Ans. (2)**

$f(x) = x^3 + x + 1$ (roots α, β, γ)

$p(x)$ with roots $\alpha^2, \beta^2, \gamma^2$

$x \to \sqrt{x}$ in $f(x)$

$\Rightarrow (\sqrt{x})^3 + \sqrt{x} + 1 = 0$

$\Rightarrow \sqrt{x}(x + 1) + 1 = 0$

$\Rightarrow \sqrt{x}(x + 1) = -1$

$\Rightarrow x(x^2 + 2x + 1) = 1$

$\Rightarrow x^3 + 2x^2 + x - 1 = 0$

Clearly $p(0) = -1$ & $p(9) = 729 + 162 + 9 - 1 = 899$

SECTION-B

1. Ans. (254)

$kx^2 - (k-1)x + 5 = 0$ with roots α, β

Given,

$\Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5} \quad \Rightarrow \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{4}{5} \Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{4}{5}$

$\Rightarrow \frac{\left(\frac{k-1}{k}\right)^2 - 2\left(\frac{5}{k}\right)}{\frac{5}{k}} = \frac{4}{5} \qquad \Rightarrow k^2 - 2k + 1 - 10k = 4k \quad \Rightarrow k^2 - 16k + 1 = 0$

Two roots of it are k_1, k_2

$\frac{k_1}{k_2} + \frac{k_2}{k_1} = \frac{(k_1+k_2)^2 - 2k_1k_2}{k_1k_2}$

$= \frac{(16)^2 - 2}{1} = 254$

2. Ans. (191)

$x^2 + 3x - k = 0$ with roots a, b, $\quad x^2 + 3x - 10$ with roots c, d

Given that $|a-b| = 2|c-d|$

$\Rightarrow (a+b)^2 - 4ab = 4\{(c+d)^2 - 4cd\} \qquad \Rightarrow 9 + 4k = 4(9+40)$

$\Rightarrow k = \frac{187}{4} = \frac{m}{n} \quad \Rightarrow m + n = 191$

3. Ans. (24)

$x^2 - 11x + m = 0$

$x^2 - 14x + 2m = 0$

For one roots common

$(c_1a_2 - c_2a_1)^2 = (a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1)$

$\Rightarrow (m-2m)^2 = (-14+11)(-22m+14m)$

$\Rightarrow m^2 = 24m \quad \Rightarrow m = 0, 24$

Sum = 24

4. Ans. (5)

$x^2 + 2(a+b)x + (a-b+8) = 0$

$D > 0$

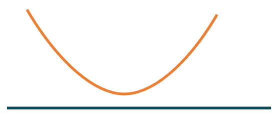
$4(a+b)^2 - 4(a-b+8) > 0$

$\Rightarrow a^2 + b^2 + 2ab - a + b - 8 > 0$

$\Rightarrow a^2 + (2b-1)a + b^2 + b - 8 > 0$

This is a quadratic polynomial in a,

Which is $+ve$ for all $a \in R$



$\Rightarrow D < 0 \qquad \Rightarrow (2b-1)^2 - 4(b^2+b-8) < 0$

$\Rightarrow -8b + 33 < 0 \qquad \Rightarrow b > \frac{33}{8} \qquad \Rightarrow b > 4.125$

Smallest natural $b = 5$

5. **Ans. (7)**

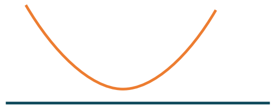
$f(x) = x^2 + 4ax + 5a^2 - 6a$

$f(x) > 0\ \forall x \in R$

$\Rightarrow D < 0 \qquad \Rightarrow 16a^2 - 4(5a^2 - 6a) < 0$

$\Rightarrow a^2 - 6a > 0 \Rightarrow a \in (-\infty, 0) \cup (6, \infty)$

Smallest positive integer $a = 7$



6. **Ans. (4)**

$\frac{2x^2+2x+3}{x^2+x+1} \le p$

Let $f(x) = \frac{2x^2+2x+3}{x^2+x+1}$

$= 2 + \frac{1}{x^2+x+1} \qquad \left\{\because x^2+x+1 \in \left[\frac{3}{4}, \infty\right)\right\}$

$f(x) \in \left(2, \frac{10}{3}\right]$

Hence, $\frac{2x^2+2x+3}{x^2+x+1} \le p$

$p \in \left[\frac{10}{3}, \infty\right) \Rightarrow p$ is a good no.

Smallest integral $p = 4$

7. **Ans. (4)**

$2x^2 - 2ax + a^2 - a - 6 = 0$

Roots of appositive sign $af(0) < 0$

$\Rightarrow 2(0 - 0 + a^2 - a - 6) < 0 \ \Rightarrow (a-3)(a+2) < 0$

$\Rightarrow a \in (-2, 3)$

8. **Ans. (3)**

$x^2 - (3k-1)x + 5k = 0$

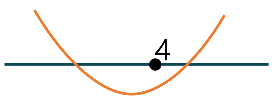
4 lies between the roots

$\Rightarrow af(4) < 0$

$\Rightarrow 1(16 - 4(3k-1) + 5k) < 0$

$\Rightarrow 7k - 20 > 0 \Rightarrow k > \frac{20}{3}$

Minimum value of $k = 3$



9. **Ans. (3)**

$x^3 - 3x^2 + 1 = 0$ with roots α, β, γ

Now, equation with roots $\alpha - 2, \beta - 2, \gamma - 2$ is

$x \to x + 2$

$\Rightarrow (x+2)^3 - 3(x+2)^2 + 1 = 0$

$\Rightarrow x^3 + 6x^2 + 12x + 8 - 3x^2 - 12x - 12 + 1 = 0 \qquad \Rightarrow x^3 + 3x^2 - 3 = 0$

Its roots are $\alpha - 2, \beta - 2, \gamma - 2$

Product of roots $(\alpha-2)(\beta-2)(\gamma-2) = 3$

10. Ans. (20)

$x^2 + 18x + 30 = 2\sqrt{x^2+18x+45}$

Let $x^2 + 18x = y \qquad (x^2 + 18x + 30 \geq 0)$

$\Rightarrow y^2 + 60y + 900 = 4y + 180 \qquad \Rightarrow y^2 + 56y + 720 = 0$

$\Rightarrow (y + 36)(y + 20) = 0$

$\Rightarrow x^2 + 18x = -36 \quad$ (not possible)

$\Rightarrow x^2 + 18x = -20 \qquad \Rightarrow x^2 + 18x + 20 = 0$

Product of roots = 20.

Exercise - II (JEE Main PYQs)

1. Ans. (1)

$P(x) = (a - a_1)x^2 + (b - b_1)x + (c - c_1) = 0$

only $x =- 1; P(x) = 0$

So roots are equal.

it means $D = 0$

$\Rightarrow (b - b_1)^2 = 4(a - a_1)(c - c_1) \Rightarrow q^2 = 4pr$

Let

$\Rightarrow a - a_1 = p \Rightarrow b - b_1 = q$

$\Rightarrow c - c_1 = r \Rightarrow P(-1) = 0$

$\Rightarrow p - q + r = 0 \qquad ...(1)$

$\Rightarrow 4p - 2q + r = 2 \qquad ...(2)$

$\Rightarrow 4p + 2q + r = ?$

From (1) $q = p + r$

$\Rightarrow (p + r)^2 - 4pr = 0$

$\Rightarrow (p - r)^2 = 0 \qquad \boxed{p = r}$

from eq. (1) $q = 2r$

So from eq. (2) $4r - 4r + r = 2 \qquad \Rightarrow r = 2$

So $4p + 2q + r = 4r + 4r + r = 9r = 18$

2. Ans. (2)

Sachin Rahul

Wrong eq^n. $x^2 - 7x + 12 = 0$

$\Rightarrow x^2 - 5x + 6 = 0$

Now correct eq^n. is

$\Rightarrow x^2 - 7x + 6 = 0 \qquad \Rightarrow (x - 6)(x - 1) = 0$

$x = 1, 6$

3. Ans. (1)

$x^2 + 2x + 3 = 0$

both roots are imaginary, since $a, b, c \in R$

If one root is common then both root is common

$\frac{a}{1} = \frac{b}{2} = \frac{c}{3} \Rightarrow a: b: c = 1: 2: 3.$

4. **Ans. (1)**

Given $x^2 - 6x - 2 = 0$

$\therefore a_{n+2} - 6a_{n+1} - 2a_n = 0 \quad \Rightarrow \frac{a_{n+2} - 2a_n}{2a_{n+1}} = 3$

Now, put $n = 8$

$\frac{a_{10} - 2a_8}{2a_9} = 3$

5. **Ans. (2)**

(i) if base = 1 $\Rightarrow x^2 - 5x + 5 = 1 \Rightarrow x = 1, 4$

(ii) if base = –1 $\Rightarrow x^2 - 5x + 5 = -1 \Rightarrow x = 2, 3$

for $x = 2$; $x^2 + 4x - 60 = 4 + 8 - 60 = -48$ (which is even)

for $x = 3$; $x^2 + 4x - 60 = 9 + 12 - 60 = -39$ (which is odd, rejected)

(iii) if power = 0 $\Rightarrow x^2 + 4x - 60 = 0 \Rightarrow x = -10, 6$

for $x = -10$; $x^2 - 5x + 5 = 100 + 50 + 5 \neq 0$ (which is valid)

for $x = 6$; $x^2 - 5x + 5 = 36 - 30 + 5 \neq 0$ (which is valid)

Sum of real values of $x = 1 + 4 + 2 - 10 + 6 = 3$

6. **Ans. (3)**

α and β are roots of $x^2 + 2x + 2 = 0$

$\Rightarrow \alpha + \beta = -2, \ \alpha \cdot \beta = 2$

Here α is a root of $x^2 + 2x + 2 = 0$

$\Rightarrow \alpha^2 + 2\alpha + 2 = 0 \Rightarrow \alpha^2 = -2\alpha - 2$

$\Rightarrow \alpha^3 = -2\alpha^2 - 2\alpha$

$= -2(-2\alpha - 2) - 2\alpha$

$= 2\alpha + 4$

$\Rightarrow \alpha^4 = 2\alpha^2 + 4\alpha$

$= 2(-2\alpha - 2) + 4\alpha$

$= -4$

Same way, $\beta^4 = -4$

Now, $\alpha^{15} + \beta^{15} = (\alpha^4)^3 \cdot \alpha^3 + (\beta)^3 \cdot \beta^3$

$= (-4)^3 \cdot (2\alpha + 4) + (-4)^3 \cdot (2\beta + h)$

$= -128(\alpha + \beta) - 256 - 256$

$= -128(-2) - 512$

$= -256$

7. **Ans. (3)**

$6x^2 - 11x + \alpha = 0$

given roots are rational

$\Rightarrow D$ must be perfect square

$\Rightarrow 121 - 24\alpha = \lambda^2$

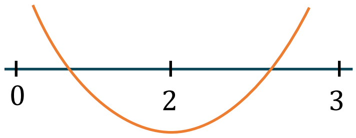
$\Rightarrow$ maximum value of α is 5

$\alpha = 1 \Rightarrow \lambda \notin I \quad \alpha = 2 \Rightarrow \lambda \notin I$

$\alpha = 3 \Rightarrow \lambda \in I \quad \Rightarrow 3$ integral values

$\alpha = 4 \Rightarrow \lambda \in I \quad \alpha = 5 \Rightarrow \lambda \in I$

8. Ans. (1)



Let $f(x) = (c-5)x^2 - 2cx + c - 4$

$\therefore f(0)f(2) < 0$ ...(1)

& $f(2)f(3) < 0$ ...(2)

from (1) & (2)

$(c-4)(c-24) < 0$

& $(c-24)(4c-49) < 0$ $\Rightarrow \frac{49}{4} < c < 24$

$\therefore s = \{13, 14, 15, \ldots\ldots 23\}$

Number of elements in set $S = 11$

9. Ans. (2)

$3m^2x^2 + m(m-4)x + 2 = 0$

$\lambda + \frac{1}{\lambda} = 1$, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1$, $\alpha^2 + \beta^2 = \alpha\beta$

$(\alpha + \beta)^2 = 3\alpha\beta$

$\left(-\frac{m(m-4)}{3m^2}\right)^2 = \frac{3(2)}{3m^2}$, $\frac{(m-4)^2}{9m^2} = \frac{6}{3m}$

$(m-4)^2 = 18, m = 4 \pm \sqrt{18}, 4 \pm 3\sqrt{2}$

10. Ans. (4)

$\alpha + \beta = 1, \alpha\beta = -1$

$P_k = \alpha^k + \beta^k$

$\alpha^2 - \alpha - 1 = 0$

$\Rightarrow \alpha^k - \alpha^{k-1} - \alpha^{k-2} = 0$ & $\beta^k - \beta^{k-1} - \beta^{k-2} = 0$

$\Rightarrow P_k = P_{k-1} + P_{k-2}$ $\Rightarrow P_1 = \alpha + \beta = 1$

$\Rightarrow P_2 = (\alpha + \beta)^2 - 2\alpha\beta = 1 + 2 = 3$

$\Rightarrow P_3 = 4$ $\Rightarrow P_4 = 7$ $\Rightarrow P_5 = 11$

11. Ans. (8.00)

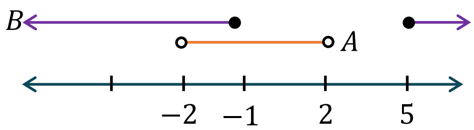
$D \geq 0 \Rightarrow (a-10)^2 - 4 \times 2 \times \left(\frac{33}{2} - 2a\right) \geq 0$

$\Rightarrow a^2 - 4a - 32 \geq 0$ $\Rightarrow a \in (-\infty, 4] \cup [8, \infty)$

12. Ans. (3)

$A : x \in (-2, 2); B : x \in (-\infty, -1] \cup [5, \infty)$

$\Rightarrow B - A = R - (-2, 5)$



13. Ans. (4)

Consider $(p^2+q^2)^2-2p^2q^2=272$

$\Rightarrow ((p+q)^2-2pq)^2-2p^2q^2=272 \Rightarrow 16-16pq+2p^2q^2=272$

$\Rightarrow (pq)^2-8pq-128=0$

$\Rightarrow (pq)^2-8pq-128=0 \quad \Rightarrow pq=\frac{8\pm 24}{2}=16,-8$

$\therefore pq=16$

$\therefore$ Required equation : $x^2-(2)x+16=0$

14. Ans. (1)

Let $x=3+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{3+...\infty}}}}$

So, $x=3+\cfrac{1}{4+\cfrac{1}{x}}=3+\cfrac{1}{\cfrac{4x+1}{x}}$

$\Rightarrow (x-3)=\frac{x}{(4x+1)} \quad \Rightarrow (4x+1)(x-3)=x$

$\Rightarrow 4x^2-12x+x-3=x \quad \Rightarrow 4x^2-12x-3=0$

$x=\frac{12\pm\sqrt{(12)^2+12\times 4}}{2\times 4}=\frac{12\pm\sqrt{12(16)}}{8}$

$=\frac{12\pm 4\times 2\sqrt{3}}{8}=\frac{3\pm 2\sqrt{3}}{2}$

$x=\frac{3}{2}\pm\sqrt{3}=1.5\pm\sqrt{3}$.

But only positive value is accepted

So, $x=1.5+\sqrt{3}$

15. Ans. (3)

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

let $f(a)=(3-a)^2-2(1-2a)$

$\Rightarrow f(a)=a^2-2a+7 \Rightarrow f(a)=(a-1)^2+6$

$(f(a))_{\text{min.}}=6$

16. Ans. (4)

$\frac{|x+3|-1}{|x|-2}\geq 0, x\in[-6,3]-\{-2,2\}$

Case-I

$\lvert x\rvert-2>0$ and	$\lvert x+3\rvert-1\geq 0$
$\lvert x\rvert>2$	$\lvert x+3\rvert\geq 1$
$x>2$ or $x<-2$	$x+3\geq 1$ or $x+3\leq -1$
	$x\geq -2$ or $x\leq -4$
	$\Rightarrow x\in[-6,-4]\cup(2,3]$

Case-II

$|x| - 2 < 0$ and $|x+3| - 1 \le 0$

$\Rightarrow |x| < 2$

$\Rightarrow -2 < x < 2$

$|x+3| \le 1$

$-1 \le x+3 \le 1$

$-4 \le x \le -2$

$\therefore$ No common solution exists

$\Rightarrow S = \{x \in [-6,-4] \cup (2,3]\}$ ...(1)

$T = \{x \in z: x^2 - 7|x| + 9 \le 0$

$x^2 - 7x + 9 \le 0$ for $x \ge 0 \Rightarrow x \in \{2,3,4,6\}$

& $x^2 + 7x + 9 \le 0$ for $x < 0 \Rightarrow x \in \{-5,-4,-3,-2\}$

$T = \{\pm 2, \pm 3, \pm 4, \pm 5\}$ ...(2)

from (1) & (2)

$S \cap T = \{-5,-4,3\}$

$\therefore S \cap T$ contains exactly 3 elements.

17. Ans. (272)

$(px - q)^2 + (qx - r)^2 = 0$

$\Rightarrow x = \frac{q}{p} = \frac{r}{q} = -4$

$\Rightarrow \frac{q^2 + r^2}{p^2} = 272$

18. Ans. (16)

$P_n = \alpha^n - \beta^n$ $x^2 - x - 4 = 0$

$\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ ...(1)

As $P_n - P_{n-1} = (\alpha^n - \beta^n) - (\alpha^{n-1} - \beta^{n-1})$

$= \alpha^{n-2}(\alpha^2 - \alpha) - \beta^{n-2}(\beta^2 - \beta)$

$= 4(\alpha^{n-2} - \beta^{n-2})$

$P_n - P_{n-1} = 4 P_{n-2}$

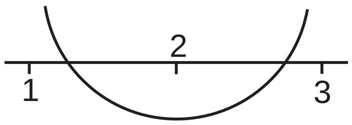
Hence Expression (1)

$\frac{P_{16}(P_{15} - P_{14}) - P_{15}(P_{15} - P_{14})}{P_{13}P_{14}}$

$= \frac{(P_{15} - P_{14})(P_{16} - P_{15})}{P_{13}P_{14}} = \frac{(4P_{13})(4P_{14})}{P_{13}P_{14}} = 16$

19. Ans. (3)

$2x^2 - 8x + k = 0$



$f(1) . f(2) < 0$ & $f(2) . f(3) < 0$

$(k - 6)(k - 8) < 0$ & $(k - 8)(k - 6) < 0$

$k \in (6, 8)$ $k \in (6, 8)$

integral value of $k = 7$

20. Ans. (25)

$$S=\left\{\alpha:\log_2\left(9^{2\alpha-4}+13\right)-\log_2\left(\frac{5}{2}3^{2\alpha-4}+1\right)=2\right\}$$

Now, $\log_2\left(\dfrac{9^{2\alpha-4}+13}{\frac{5}{2}\left(3^{2\alpha-4}\right)+1}\right)=2$ or $\dfrac{2\left(9^{2\alpha-4}+13\right)}{5\left(3^{2\alpha-4}\right)+2}=4$

$\Rightarrow 9^{2\alpha-4}+13=10\cdot(3)^{2\alpha-4}+4$ $\Rightarrow 9^{2(\alpha-2)}+13=10\cdot 9^{(\alpha-2)}+4$

Let $y=9^{\alpha-2}$

$\Rightarrow y^2+13=10y+4$ $\Rightarrow y^2-10y+9=0$

$\Rightarrow (y-9)(y-1)=0$ $\Rightarrow y=1,9$

when $y=9$	$y=1$
$9^{\alpha-2}=9^1$	$9^{\alpha-2}=9^0$
$\alpha-2=1$	$\alpha=2$
$\alpha=3$	

Now,

$$\Rightarrow x^2-2\left(\sum_{\alpha\in s}\alpha\right)^2x+\sum_{\alpha\in s}(\alpha+1)^2\beta=0$$

$\Rightarrow x^2-(2\times 25)x+25\beta=0$ $\Rightarrow x^2-50x+25\beta=0$

for real roots $D\geq 0$

$b^2-4ac\geq 0\Rightarrow 2500-4(25\beta)\geq 0$

$\beta\leq 25\Rightarrow \beta_{max}=25.$

21. Ans. (13)

Two equations have common root

$\therefore(4a)(26a)=(-6)^2=36$ $\Rightarrow a^2=\dfrac{9}{26}$ $\therefore a=\dfrac{3}{\sqrt{26}}\Rightarrow\beta=13$

22. Ans. (45)

$x^2+60^{\frac{1}{4}}x+a=0$ (roots α, β)

$\alpha+\beta=-60^{\frac{1}{4}}$ & $\alpha\beta=a$

Given $\alpha^4+\beta^4=-30$

$\Rightarrow\left(\alpha^2+\beta^2\right)^2-2\alpha^2\beta^2=-30$ $\Rightarrow\left\{(\alpha+\beta)^2-2\alpha\beta\right\}^2-2a^2=-30$

$\Rightarrow\left\{60^{\frac{1}{2}}-2a\right\}^2-2a^2=-30$ $\Rightarrow 60+4a^2-4a\times 60^{\frac{1}{2}}-2a^2=-30$

$\Rightarrow 2a^2-4.60^{\frac{1}{2}}a+90=0$

Product $=\dfrac{90}{2}=45$

Exercise - III (JEE Advanced Pattern)

SECTION-I

1. **Ans. (A,B,C)**

α, β will satisfy the quadratic equation

$\Rightarrow \alpha^2 - \alpha + 1 = 0$

$\alpha^2 = \alpha - 1$

On squaring

$(\alpha^2)^2 = (\alpha - 1)^2$

$\Rightarrow \alpha^4 = \alpha^2 - 2\alpha + 1$

$\Rightarrow (\alpha - 1) - 2\alpha + 1 \qquad \{\because \alpha^2 = \alpha - 1\}$

$= -\alpha$

$\alpha^4.\alpha^2 = -\alpha.\alpha^2$

$\Rightarrow \alpha^6 = -\alpha(\alpha - 1) = -\alpha^2 + \alpha$

$= -(\alpha - 1) + \alpha = 1$

$\alpha^{2009} = (\alpha^6)^{334}.\alpha^5 \quad \Rightarrow (1)^{334}.\alpha^4.\alpha \quad \Rightarrow -\alpha.\alpha \quad \Rightarrow -\alpha^2 = 1 - \alpha$

Similarly $\beta^{2009} = 1 - \beta$

$\alpha^{2009} + \beta^{2009} = 2 - (\alpha + \beta)$

$= 2 - (1) \qquad \{\because \alpha + \beta = 1\}$

$= 1$

2. **Ans. (C,D)**

For no real roots discriminant must be less than zero

$D = b^2 - 4ac < 0$

$a = 1 + m^2$

$b = -2(1 + 3m)$

$c = 1 + 8m$

$\Rightarrow [-2(1 + 3m)]^2 - 4(1 + m^2)(1 + 8m) < 0 \ \Rightarrow (1 + 3m)^2 - (1 + 8m + m^2 + 8m^3) < 0$

$\Rightarrow 1 + 9m^2 + 6m - 1 - 8m - m^2 - 8m^3 < 0 \ \Rightarrow 8m^2 - 8m^3 - 2m < 0$

$\Rightarrow -2m(4m^2 - 4m + 1) < 0 \quad \Rightarrow -2m(2m - 1)^2 < 0 \quad \Rightarrow m > 0$ and $m \neq \frac{1}{2}$

3. **Ans. (A,D)**

$x^2 - \alpha x + \alpha + 1 = 0$

$a = 1, b = -\alpha, c = \alpha + 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-\alpha) \pm \sqrt{(-\alpha)^2 - 4(1)(\alpha + 1)}}{2(1)}$$

$$= \frac{\alpha \pm \sqrt{\alpha^2 - 4\alpha - 4}}{2} = \frac{\alpha \pm \sqrt{(\alpha - 2)^2 - 8}}{2}$$

Now check options $\alpha = -1, 5$ satisfies for integer solutions

4. **Ans. (B,D)**

Let $p(x) = ax^2 + bx + c$

$p(0) = a(0)^2 + b(0) + c = 1 \quad \Rightarrow c = 1$

$p(1) = 4$

$\Rightarrow a(1)^2 + b(1) + 1 = 4$

$\Rightarrow a + b = 3$ ...(i)

$p(-1) = 6$

$\Rightarrow a(-1)^2 + b(-1) + 1 = 6$

$\Rightarrow a - b = 5$ ...(ii)

Solving both equations, we get,

$a = 4, b = -1$

$p(x) = 4x^2 - x + 1$

$P(2) = 4(2)^2 - 2 + 1 = 15$

$p(-2) = 4(-2)^2 - (-2) + 1 = 19$

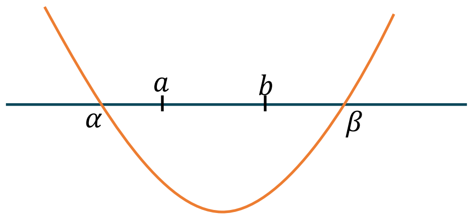
5. **Ans. (B,C)**

Let $p(x) = (x - a)(x - b) - 1$

$p(a) = (a - a)(a - b) - 1$

$= -1$

$p(b) = -1$



Possible graph for the above expression will be $p(a)$ and $p(b)$ both are negative and leading coefficient of quadratic is $+ve$ a and b both will be between both the roots

Looking at the graph

$\alpha \in (-\infty, a)$ and $\beta \in (b, \infty)$

6. **Ans. (A,B,D)**

Graph is upward opening so $a > 0$

Vertex. Of x-coordinate is $+ve$ i.e. $\frac{-b}{2a} > 0$

$\Rightarrow -b > 0 \qquad \Rightarrow b < 0$

Graph cutting y-axis on $+ve$ side so $c > 0$

So,

$ab^2c^3 > 0$

$ab^3c^2 < 0$

$ab^3c^5 < 0$

And graph is cutting x-axis at two distinct points so it has two distinct roots.

So, discriminant is greater than zero.

$b^2 - 4ac > 0 \quad \Rightarrow b^2 > 4ac$

7. **Ans. (A,B,D)**

$x^2 - |x+2| + x > 0$

Case – I

$x \geq -2 \Rightarrow a \in [-2, \infty)$

$x^2 - (x+2) + x > 0$

$x^2 - 2 > 0 \qquad \Rightarrow (x+\sqrt{2})(x-\sqrt{2}) > 0$

$x \in (\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

$x \in \{(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)\} \cap [-2, \infty)$

$x \in [-2, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

Case – II

$x < -2 \; or \; x \in (-\infty, -2)$

$\Rightarrow x^2 - (-(x+2)) + x > 0 \qquad \Rightarrow x^2 + x + 2 + x > 0$

$\Rightarrow x^2 + 2x + 2 > 0$

It is always greater than zero as

$D = (2)^2 - 4(2) < 0$

$x \in (-\infty, -2)$ is the solution form case II final answer (case – I)$\cup$ (case – II)

$\Rightarrow (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

Subsets of this set is in option A,B,D.

8. **Ans. (A,B,D)**

$|x|^2 + |x| - 6 = 0$

Case – I

$x \geq 0$

$x^2 + x - 6 = 0$

$\Rightarrow (x+3)(x-2) \Rightarrow x = -3, 2$

But $x \geq 0$ so $x = 2$

Case – II

$x < 0$

$(-x)^2 - x - 6 = 0 \qquad \Rightarrow x^2 - x - 6 = 0$

$\Rightarrow (x+2)(x-3) = 0 \;\Rightarrow x = -2, 3$

But $x < 0$

So $x = -2$

Roots are $2, -2$

Sum of roots = $2 - 2 = 0$

Product of roots = $2(-2) = -4$

Only two real solutions.

9. **Ans. (B,D)**

$ax^2 + bx + c = 0$

α, β are roots

So,

$\alpha + \beta = \dfrac{-b}{a}$ ...(i)

$\alpha\beta=\frac{c}{a}$ ...(ii)

$(\alpha-\beta)^2=\alpha^2+\beta^2-2\alpha\beta$

$\Rightarrow (\alpha+\beta)^2-4\alpha\beta \Rightarrow \left(\frac{-b}{a}\right)^2-4\frac{c}{a}$

$\Rightarrow \frac{b^2-4ac}{a^2}$ ...(iii)

$px^2+qx+r=0$

$\alpha+h, \beta+h$ are roots

$\Rightarrow (\alpha+h)+(\beta+h)=\frac{-q}{p}, (\alpha+h)(\beta+h)=\frac{r}{p}$

$\Rightarrow \alpha+\beta+2h=\frac{-q}{p} \quad \Rightarrow \frac{-b}{a}+2h=\frac{-q}{p} \Rightarrow h=\frac{1}{2}\left(\frac{b}{a}-\frac{q}{p}\right)$

$\Rightarrow \left[(\alpha+h)-(\beta+h)\right]^2=\left((\alpha+h)+(\beta+h)\right)^2-4(\alpha+h)(\beta+h)$

$\Rightarrow (\alpha-\beta)^2=\left(\frac{-q}{p}\right)^2-4\frac{r}{p}=\frac{q^2-4pr}{p^2}$ ...(iv)

Divide (iii) and (iv)

$\Rightarrow \frac{(\alpha-\beta)^2}{(\alpha-\beta)^2}=\frac{\left(\frac{b^2-4ac}{a^2}\right)}{\left(\frac{q^2-4pr}{p^2}\right)} \Rightarrow \frac{q^2-4pr}{p^2}=\frac{b^2-4ac}{a^2}$

10. **Ans. (B,C,D)**

$x^2+ax+b=0$

$\alpha+\beta=-a$

$\alpha\beta=b$

(A) If α^2, β^2 are roots

$\Rightarrow$ sum of roots = $\alpha^2+\beta^2 \quad \Rightarrow (\alpha+\beta)^2-2\alpha\beta$

$\Rightarrow (-a)^2-2b \quad \Rightarrow a^2-2b$

Product of roots = $\alpha^2\beta^2$

= $(\alpha\beta)^2=b^2$

Quadratic equation

$\Rightarrow x^2-$(sum of roots)x + product of roots = 0

$\Rightarrow x^2-(a^2-2b)x+b^2=0$

(B) sum of roots = $\frac{1}{\alpha}+\frac{1}{\beta}$

$=\frac{\beta+\alpha}{\alpha\,\beta}=\frac{-a}{b}$

Product of roots = $\frac{1}{\alpha}\times\frac{1}{\beta}=\frac{1}{\alpha\beta}=\frac{1}{b}$

Quadratic equation

$\Rightarrow x^2-\left(-\frac{a}{b}\right)x+\frac{1}{b}=0 \quad \Rightarrow bx^2+ax+1=0$

(C) sum of roots $\Rightarrow \frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

$\Rightarrow \frac{\alpha^2+\beta^2}{\alpha\ \beta} \quad \Rightarrow \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$\Rightarrow \frac{(-a)^2-2b}{b} \quad \Rightarrow \frac{a^2-2b}{b}$

Product or roots = $\frac{\alpha}{\beta}\times\frac{\beta}{\alpha}=1$

Quadratic equation

$\Rightarrow x^2-\left(\frac{a^2-2b}{b}\right)x+1=0 \qquad \Rightarrow bx^2+(2b-a^2)x+b=0$

(D) sum of roots = $(\alpha-1)+(\beta-1)$

$\Rightarrow \alpha+\beta-2 \quad \Rightarrow -a-2=-(a+2)$

Product of roots = $(\alpha-1)(\beta-1)$

$\Rightarrow \alpha\beta-(\alpha+\beta)+1 \quad \Rightarrow b+a+1$

Quadratic equation

$\Rightarrow x^2-(-(a+2))x+b+a+1=0$

$\Rightarrow x^2+(a+2)x+b+a+1=0$

11. **Ans. (A,B,D)**

$y=ax^2+bx+c$

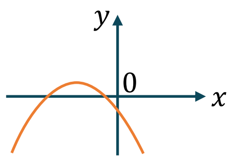
Here graph is downward, so $a<0$.

Clearly, y has two distinct roots.

So, $D>0 \Rightarrow b^2-4ac>0$

for $x=0$; $f(0)<0$

$\therefore c<0$



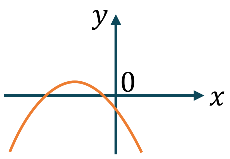
vertex is at $-\frac{b}{2a}$, which is negative.

For $a<0$ and $-\frac{b}{2a}$ to be negative, b must be negative.

12. **Ans. (A,B,C,D)**

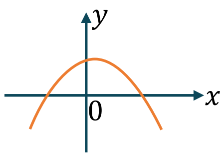
$y=ax^2+bx+c$

(A)



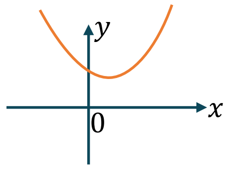
$a<0, b<0<c<0 \quad \Rightarrow abc<0$

(B)



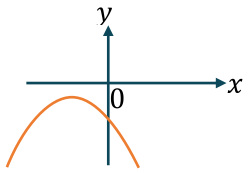
$a<0, b>0, c>0 \qquad \Rightarrow abc<0$

(C)



$a > 0, b < 0, c > 0 \quad \Rightarrow abc < 0$

(D)



$a < 0, b < 0, c < 0 \quad \Rightarrow abc < 0.$

13. **Ans. (A,D)**

$x^2 + x + 1$ is a factor of $ax^3 + bx^2 + cx + d = 0$

So, let $ax^3 + bx^2 + cx + d = (ax + d)(x^2 + x + 1)$

$\Rightarrow (ax + d)(x^2 + x + 1) = 0$

$\Rightarrow x = -\frac{d}{a}$ or $x = \frac{-1 \pm \sqrt{3}\, i}{2} = w, w^2$

Now, sum of roots ; $-\frac{d}{a} + w + w^2 = -\frac{b}{a}$

$\Rightarrow -\frac{d}{a} - 1 = -\frac{b}{a} \qquad (\because w + w^2 + 1 = 0)$

$\Rightarrow -\frac{d}{a} = 1 - \frac{b}{a} = \frac{a-b}{a}$

SECTION–II

14. **Ans. (C)**

$x^2 + 2ax + 10 - 3a > 0$ for every real value of x,

So, discriminant $D < 0$

$\Rightarrow (2a)^2 - 4 \cdot (10 - 3a) < 0 \quad \Rightarrow 4a^2 - 4(10 - 3a) < 0$

$\Rightarrow a^2 + 3a - 10 < 0 \quad \Rightarrow (a + 5)(a - 2) < 0$

$\Rightarrow -5 < a < 2.$

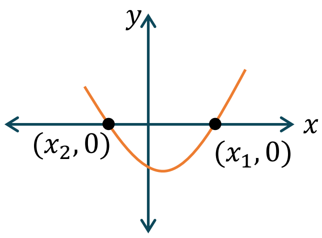
15. **Ans. (D)**

$x^2 + 2bx + c > 0$ then $D < 0$

$\Rightarrow (2b)^2 - 4c < 0 \quad \Rightarrow 4b^2 - 4c < 0 \quad \Rightarrow b^2 < c.$

16. **Ans. (B)**

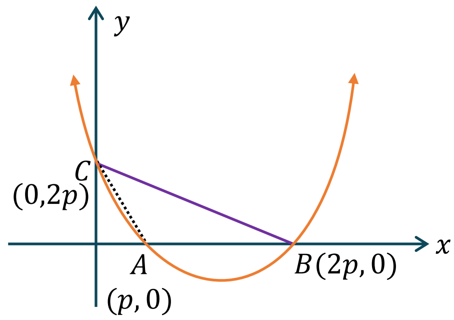
$y = ax^2 + bx + c$



Here, $a > 0, D > 0 \qquad (\because$ two real roots)

for $x = 0; y = c$ which is negative.

17. Ans. (3.00)



ΔOBC is isosceles triangle and AC is medium.

So, let $A(p, 0)$, $B(2p, 0)$ and $C(0, 2p)$.

for equation of curve; $y = x^2 + bx + c$;

If $x = 0$ then, $y = c = 2p$ ...(1)

For $y = 0$; $x^2 + bx + c = 0$ and $x = p, 2p$ will be the solutions.

$\Rightarrow$ product of roots; $p \cdot 2p = c$ ...(2)

From equation (1) and (2);

$2p^2 = 2p \Rightarrow p = 1$ $(\because\ p = 0$ not possible$)$

Now sum of roots of equation $x^2 + bx + c = 0$ is $p + 2p = 1 + 2 = 3$.

18. Ans. (2.00)

Roots of previous equation are 1, 2

So, $\alpha = 2, \beta = 1$ $(\alpha > \beta)$

$\Rightarrow (\alpha + \beta) = 2 + 1 = 3, (\alpha - \beta) = 2 - 1 = 1$

expression having 3 and 1 as roots

$y = (x - 3)(x - 1)$

$\Rightarrow y = x^2 - 4x + 3$

For minimum value of $x^2 - 4x + 3$;

$x^2 - 4x + 3 = x^2 - 4x + 4 - 1$

$= (x - 2)^2 - 1$

So, minimum value occurs at $x = 2$.

SECTION–III

19. Ans. (A)

(P) $\alpha, \alpha + 4$ are two roots of $x^2 - 8x + k = 0$,

$\Rightarrow$ Sum of roots; $\alpha + \alpha + 4 = 8$

$\Rightarrow \alpha = 2$

Now, product of roots; $k = \alpha \cdot (\alpha + 4)$

$\Rightarrow k = 2(2 + 4)$

$\Rightarrow k = 12$

(Q) $x^2 - 5|x| + 6 = 0$

$\Rightarrow (|x|)^2 - 5|x| + 6 = 0$

$\Rightarrow (|x| - 3)(|x| - 2) = 0$

$\Rightarrow |x| = 3, |x| = 2$

$\Rightarrow x = \pm 3, x = \pm 2$

So, $n = 4 \Rightarrow \frac{n}{2} = 2$.

(R) $x^2 + ax + b = 0$

As $3 - i$ is a root, $3 + i$ is also a root.

$\Rightarrow$ sum of roots; $-a = 6 \Rightarrow a = -6$

$\Rightarrow$ product of roots; $b = (3 - i)(3 + i)$

$\Rightarrow b = 9 + 1 = 10$

(S) Both roots of $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5,

So, sum of roots $2k < 5 + 5 \Rightarrow k < 5$ ...(1)

Discriminant, $D \geq 0$

$\Rightarrow (-2k)^2 - 4(k^2 + k - 5) \geq 0$

$\Rightarrow 4k^2 - 4(k^2 + k - 5) \geq 0$

$\Rightarrow -k + 5 \geq 0$

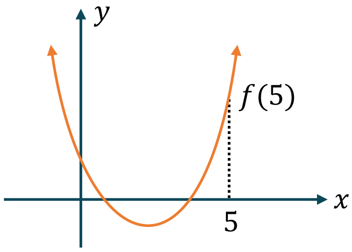
$\Rightarrow k \leq 5$ ...(2)

$f(5) > 0 \Rightarrow (5)^2 - 2k(5) + k^2 + k - 5 > 0$

$\Rightarrow k^2 - 9k + 20 > 0$

$\Rightarrow (k-4)(k-5) > 0$

$\Rightarrow k < 4, k > 5$ ...(3)



From (1), (2) and (3);

$k < 4$

So, $k = 3$ possible, $k = 2$ possible.

20. **Ans. A → p, r, s ; B → q, s ; C → q, s ; D → p, r, s**

$$f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6} = \frac{(x^2 - 5x + 6) - (x+1)}{x^2 - 5x + 6}$$

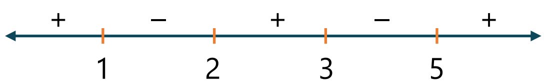
$$= 1 - \frac{x+1}{(x-2)(x-3)}$$

for $-1 < x < 1, 1 < x < 2, 3 < x < 5$ and $x > 5$;

$$\frac{x+1}{(x-2)(x-3)} > 0 \Rightarrow 1 - \frac{x+1}{(x-2)(x-3)} < 1$$

$\Rightarrow f(x) < 1$

Now, $$f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6} = \frac{(x-1)(x-5)}{(x-2)(x-3)}$$



(A) If $-1 < x < 1$; $f(x) > 0$

Hence, also $0 < f(x) < 1$

(B) If $1 < x < 2$; $f(x) < 0$

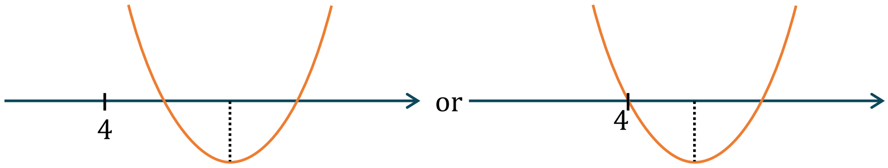
(C) If $3 < x < 5$; $f(x) < 0$

(D) If $x > 5$; $f(x) > 0$

Hence, also, $0 < f(x) < 1$.

Exercise - IV (JEE Advanced PYQs)

1. **Ans. (2)**



$f(x) = x^2 - 8kx + 16(k^2 - k + 1) = 0$

$\because$ roots are real, $D > 0$

$64k^2 - 64(k^2 - k + 1) > 0$

$k > 1 \Rightarrow k \in (1, \infty)$

at both the roots ≥ 4

$$\Rightarrow \begin{cases} f(4) \geq 0 \\ \frac{-b}{2a} > 4 \end{cases} \Rightarrow \begin{cases} k^2 - 3k + 2 \geq 0 \\ k > 1 \end{cases}$$

$\Rightarrow k \in [2, \infty)$ least value of $k = 2$

2. **Ans. (B)**

$\alpha^3 + \beta^3 = q$

$\Rightarrow (\alpha + \beta)^3 - 3\alpha\ \beta(\alpha + \beta) = q$

$$\Rightarrow \alpha\ \beta = \frac{p^3 + q}{3p}$$

$$\text{sum of the roots} = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\ \beta}$$

$$= \frac{p^2 - 2\left(\frac{p^3 + q}{3p}\right)}{\frac{p^3 + q}{3p}} = \frac{p^3 - 2q}{p^3 + q}$$

Product of the roots = 1.

Required equation is

$(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

3. **Ans. (C)**

α, β are roots of $x^2 - 6x - 2 = 0$

$\Rightarrow \alpha^2 - 6\alpha - 2 = 0$ & $\beta^2 - 6\beta - 2 = 0$

$$\Rightarrow \frac{a_{10} - 2a_8}{2a_9} = \frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8.6\alpha - \beta^8.6\beta}{2(\alpha^9 - \beta^9)} = 3$$

4. **Ans. (B)**

$$\frac{x^2}{b^2+1}=\frac{-x}{b+1}=\frac{1}{1-b}$$

$$\Rightarrow x=\frac{b+1}{b-1} \quad \text{...(i)}$$

$$\& \ x^2=\frac{b^2+1}{1-b} \quad \text{...(ii)}$$

from (i) & (ii)

$$\left(\frac{b+1}{b-1}\right)^2=\frac{b^2+1}{1-b}$$

$\Rightarrow (b^2+1)(1-b)=(b+1)^2$

$\Rightarrow -b^3+1+b^2-b=b^2+1+2b$

$\Rightarrow -b^3-3b=0 \Rightarrow b(b^2+3)=0$

$\Rightarrow b=0,\ b=\pm\sqrt{3}i$

5. **Ans. (A,D)**

$\alpha x^2-x+\alpha=0$

$D=1-4\alpha^2$

distinct real roots $D>0$

$$\Rightarrow \alpha\in\left(-\frac{1}{2},\frac{1}{2}\right) \quad \text{...(i)}$$

given $|x_1-x_2|<1$

$$\Rightarrow \frac{\sqrt{1-4\alpha^2}}{1\alpha 1}<1$$

$\Rightarrow 1-4\alpha^2<\alpha^2$

$$\Rightarrow \alpha\in\left(-\infty,\frac{-1}{\sqrt{5}}\right)\cup\left(\frac{1}{\sqrt{5}},\infty\right) \quad \text{...(ii)}$$

from (i) & (ii)

$$\alpha\in\left(\frac{-1}{2},\frac{-1}{\sqrt{5}}\right)\cup\left(\frac{1}{\sqrt{5}},\frac{1}{2}\right)$$

6. **Ans. (C)**

$\alpha^2=\alpha+1\Rightarrow\alpha^4=3\alpha+2$

$\therefore a_4=28\Rightarrow p\alpha^4+q\beta^4=p(3\alpha+2)+q(3\beta+2)=28$

$\Rightarrow p(3\alpha+2)+q(3-3\alpha+2)=28$

$\Rightarrow \alpha(3p-3q)+2p+5q=28 \quad (\text{as } \alpha\in Q^c)$

$\Rightarrow p=q,\ 2p+5q=28 \Rightarrow p=q=4$

$\therefore p+2q=12$ **Ans : C**

7. **Ans. (C)**

$\alpha^2=\alpha+1\Rightarrow\alpha^n=\alpha^{n-1}+\alpha^{n-2}$

$\Rightarrow p\alpha^n+q\beta^n=p(\alpha^{n-1}+\alpha^{n-2})+q(\beta^{n-1}+\beta^{n-2})$

$a_n=a_{n-1}+a_{n-2}\Rightarrow a_{12}=a_{11}+a_{10}$

8. Ans. (D)

$x^2 + 20x - 2020 = 0$ has two roots $a, b \in R$

$x^2 - 20x + 2020 = 0$ has two roots $c, d \in$ complex

$ac\,(a - c) + ad\,(a - d) + bc\,(b - c) + bd\,(b - d)$

$= a^2c - ac^2 + a^2d - ad^2 + b^2c - bc^2 + b^2d - bd^2$

$= a^2(c + d) + b^2(c + d) - c^2(a + b) - d^2(a + b)$

$= (c + d)(a^2 + b^2) - (a + b)(c^2 + d^2)$

$= (c + d)((a + b)^2 - 2ab) - (a + b)((c + d)^2 - 2cd)$

$= 20\,[(20)^2 + 4040] + 20[(20)^2 - 4040]$

$= 20\,[(20)^2 + 4040 + (20)^2 - 4040]$

$= 20 \times 800 = 16000$

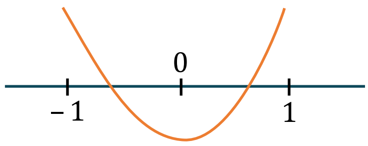
9. Ans. (4)

$3x^2 + x - 1 = 4|x^2 - 1|$

If $x \in [-1, 1]$,

$3x^2 + x - 1 = -4x^2 + 4 \Rightarrow 7x^2 + x - 5 = 0$

say $f(x) = 7x^2 + x - 5$



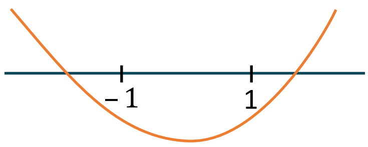
$f(1) = 3;\ f(-1) = 1;\ f(0) = -1$

[Two Roots]

If $x \in (-\infty, -1] \cup [1, \infty)$

$3x^2 + x - 1 = 4x^2 - 4 \Rightarrow x^2 - x - 3 = 0$

Say $g(x) = x^2 - x - 3$



$g(-1) = -1;\ g(1) = -3$

[Two Roots]

So total 4 roots.