Permutations and Combinations

SOLUTIONS

EXERCISE - O

1. **Ans. (A)**

$^{50}C_4 + {}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{51}C_3 + {}^{50}C_3$

$= {}^{51}C_4 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3 = {}^{56}C_4$

2. **Ans. (A)**

$^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 = 385$

3. **Ans. (B)**

Number of ways $= \frac{12!}{(4!)^3.3!}\ 3! = \frac{12!}{(4!)^3}$

4. **Ans. (D)**

Number of 4 digit numbers greater than 6000 is $\underline{3} \times \underline{4} \times \underline{3} \times \underline{2} = 72$

Number of 5 digit numbers greater than 6000 is 5! = 120

So total number of numbers = 72 + 120 = 192.

5. **Ans. (B)**

The no. of ways to select 4 novels & 1 dictionary from 6 different novels & 3 different dictionary are $^6C_4 \times {}^3C_1$

and to arrange these things in shelf so that dictionary is always in middle _ _ D _ _ are 4!

Required No. of ways $^6C_4 \times {}^3C_1 \times 4! = 1080$

6. **Ans. (D)**

Urn $A \to$ 3 Red balls

Urn $B \to$ 9 Blue balls

So the number of ways = selection of 2 balls from urn A & B each.

$= {}^3C_2 \,.\, {}^9C_2 = 108$

7. **Ans. (D)**

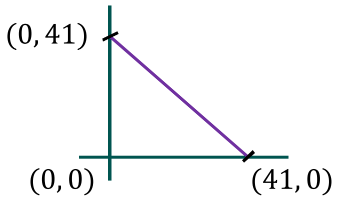
$A \to 5!$; $C \to 5!$; $H \to 5!$; $I \to 5!$; $N \to 5!$

SACHIN $\to 1$

Rank = 5 × 5! + 1 = 601

8. **Ans. (B)**

Let P be (x, y)



$x \geq 1$

$y \geq 1$

$\therefore x + y < 41$

$x + y \leq 40$

$x + y + t = 40, t \geq 0$

$(x-1) + (y-1) + t = 38$

$\therefore$ No. of points $= {}^{38+3-1}C_{3-1} = {}^{40}C_2 = 780$

9. **Ans. (B)**

$N = {}^{10}C_3 - {}^6C_3 = \frac{10\times9\times8}{3\times2\times1} - \frac{6\times5\times4}{3\times2\times1} = 120 - 20 = 100$

$N \leq 100$

10. **Ans. (B)**

$T_n = {}^nC_3 \Rightarrow {}^{n+1}C_3 - {}^nC_3 = 10$

$(n+1)n(n-1) - n(n-1)(n-2) = 60$

$n(n-1) = 20 \Rightarrow n = 5$

11. **Ans. (A)**

W^{10}, G^9, B^7

Selection of one or more balls

$= (10+1)(9+1)(7+1) - 1 = 11 \times 10 \times 8 - 1 = 879$

12. **Ans. (C)**

$(A,\ B)$

$\uparrow \quad \uparrow$

$2 \times 4 = 8$

${}^8C_3 + {}^8C_4 + \ldots.. + {}^8C_8$

$= 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2 = 256 - 37 = 219$

13. **Ans. (D)**

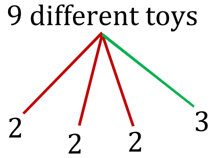
Case – I, digits used are 1,2,3,4,5 = 5! = 120

Case – II, when digits used are 0,1,4,5,2 = 5! – 4! = 96

$\therefore$ Total = 120 + 96 = 216

14. **Ans. (C)**

9 different toys



$= \dfrac{9!}{(2!)^3.3!.3!} \times 3!$

15. **Ans. (B)**

First we arrange n ladies by $(n-1)!$ ways.

Thus we get n gaps between them. Now arrange n gentelmen among them

So total $(n-1)! \times n!$ ways

16. **Ans. (A)**

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 10$

Giving one-one apples to each

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 4$

Here, $x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

So total ${}^9C_5 = 126$ ways

17. **Ans. (C)**

${}^9C_2 \times {}^7C_2 \times 2 = 1512$

$\uparrow$ Selection of male; $\uparrow$ Selection of female; $\uparrow$ Team formation

18. **Ans. (D)**

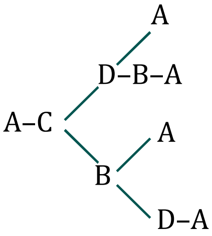
Case-I : $D, P, M, L.\boxed{EE}, \boxed{AA} \rightarrow 4! \times {}^5C_2 \times 2!$

Case-II : $D, P, M, L, \boxed{AE}\ \boxed{AE} \rightarrow 4! \times {}^5C_2 \times 2! \times 2! \times 1$

Total = 1440

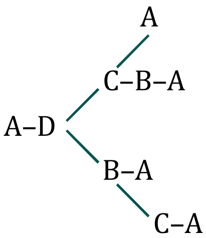
19. Ans. (B)

Paths are shown as :



4 paths

Similarly if we start from A towards D we get another 4 paths.



Similarly if we start from A towards B

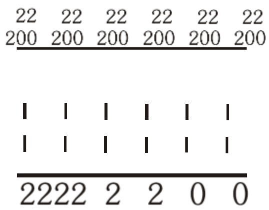
Again 4 paths

Total different paths = $4 \times 3 = 12$

II Method $\rightarrow {}^{3}C_1 \times {}^{2}C_1 \times {}^{2}C_1 = 12$

20. Ans. (A)

Number of numbers formed by 2,3,3,4,4, 4 are $= \frac{6!}{3!2!} = 60$

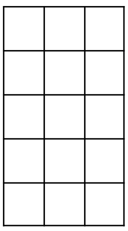


In each column 2,3,4 occurs 10, 20 & 30 times respectively.

So sum of each column is 2 × 10 + 3 × 20

$4 \times 30 = 200$

21. Ans. (A,B)



$m = 15 + 8 + 3 = 26$

$n = {}^{4}C_1 . {}^{6}C_4 = 24$

$\left.\begin{array}{l} m + n = 50 \\ m - n = 2 \end{array}\right\}$

22. **Ans. (A, B, D)**

In (C) number = ${}^{n+r}C_r$;

(D) ${}^{n}C_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$

23. **Ans. (B, C)**

$2^{10}-1; (A)2^{10}-2; (B)2^{10}-1; (C)2^{10}-1; (D)10$

24. **Ans. (C, D)**

$n = 2m$, arrange the numbers into disjoint sets

1, 3, 5, ... $(2m-1)$ m number

2, 4, 6, ... $2m$ m numbers

No. of $AP's = {}^{m}C_2 + {}^{m}C_2$

25. **Ans. (B, D)**

(A) $\frac{(n-1)!}{p!}$

(B) ${}^{n-1}C_p$

(C) $B_1 + B_2 + \ldots + B_P = n$

Put 1 ball in each box.

$\Rightarrow B_1' + B_2' + \ldots + B_p' = n - p$

using beggars method

number of ways = ${}^{n-p+(p-1)}C_{p-1} = {}^{n-1}C_{p-1}$

(D) balls of same colour are identical

$\Rightarrow$ arrange white balls $\rightarrow$ no of ways = 1

gaps in white balls = $(n-1)$

Now p black balls put in gaps

$\Rightarrow$ No of ways (D) = ${}^{(n-1)}C_p$

26. **Ans. (A,B,C,D)**

Say there are $(x)a's, (2n-x)b$'s

so no permutations $\frac{(2n)!}{x!(2n-x)!} = {}^{2n}C_x$

= ${}^{2n}C_x$ is max if $x = n$

so no of max permutation = ${}^{2n}C_n$

$${}^{2n}C_n = \frac{(2n)!}{n!n!} = \frac{1.2.3.4...2n}{n!n!}$$

$$= \frac{[1.3.5\ldots(2n-1)][2.4.6\ldots2n]}{n!\,n!} = \frac{[1.3.5\ldots.(2n-1)]2^n[1.2.3\ldots n]}{n!\,n!}$$

$${}^{2n}C_n = \frac{1.2.3\ldots n(n+1)(n+2)\ldots(2n)}{n!\,n!}$$

$${}^{2n}C_n = \frac{n+1}{1}.\frac{n+2}{2}\ldots\frac{2n}{n}$$

$${}^{2n}C_n = \frac{(1.2.3\ldots2n)}{n!\,n!} = \frac{[1.3.5\ldots.(2n-3)(2n-1)][1.2.3\ldots n]}{n!\,n!}$$

$$= \frac{2^n[1.3.5\ldots.(2n-3)(2n-1)][1.2.3\ldots n]}{n!\,n!} = \frac{\left[(2.1)(2.3)(2.5)\ldots\left(2(2n-3)\left(2(2n-1)\right)\right)\right]}{n!}$$

$${}^{2n}C_n = \frac{2}{1}.\frac{6}{2}.\frac{10}{3}\ldots.\frac{4n-6}{n-1}.\frac{4n-2}{n}$$

27. Ans. (A)

Exponent of 7 in 100!

$\left[\frac{100}{7}\right]+\left[\frac{14}{7}\right] = 14+2=16$

exponent of 7 in 50!

$\left[\frac{50}{7}\right]+\left[\frac{7}{7}\right] = 8$

Exponent of 7 in ${}^{100}C_{50} = \frac{100!}{50!\,50!} = \frac{7^{16}}{7^8 7^8} = 7^{\circ}$

$\therefore$ exponent of 7 will be 0.

28. Ans. (C)

Product of 5's & 2's constitute 0's at the end of a number $\Rightarrow$ No. of 0's in 108!

= exponent of 5 in 108!

(Note that exponent of 2 will be more than exponent of 5 in 108 !)

$\Rightarrow \left[\frac{108}{5}\right]+\left[\frac{21}{5}\right] = 21+4=25$

29. Ans. (B)

As $12 = 2^2.3$, here we have to calculate exponent of 2 and exponent of 3 in 100!

exponent of 2

$= \left[\frac{100}{2}\right]+\left[\frac{50}{2}\right]+\left[\frac{25}{2}\right]+\left[\frac{12}{2}\right]+\left[\frac{6}{2}\right]+\left[\frac{3}{2}\right] = 97$

exponent of 3 $= \left[\frac{100}{3}\right]+\left[\frac{33}{3}\right]+\left[\frac{11}{3}\right]+\left[\frac{3}{3}\right] = 48$

Now, 12 = 2 × 2 × 3

we require two 2's & one 3

$\therefore$ exponent of 3 will give us the exponent of 12 in 100! i.e. 48

30. Ans. (B)

31. Ans. (C)

32. Ans. (C)

Solution for Q.30 to Q.32

Selection	**Arrangement**	**Correct combinations**
Dream ${}^5C_3 = 10$	${}^5C_3 \times 3! = 60$	(I) (ii) (P)
Dedication ${}^2C_1{}^7C_1 + {}^8C_3 = 70$	${}^2C_1{}^7C_1 \times \frac{3!}{2!} + {}^8C_3 \times 3! = 378$	(II) (i) (S)
Powerful ${}^8C_3 = 56$	${}^8C_3 \times 3! = 336$	(III) (iv) (R)
Combination ${}^3C_1{}^7C_1 + {}^8C_3 = 77$	${}^3C_1{}^7C_1 \times \frac{3!}{2!} + {}^8C_3 \times 3! = 399$	(IV) (iii) (Q)

EXERCISE - S

1. Ans. (420)

A	B
5	3
4	4
3	5

= ${}^7C_5 \times {}^5C_3 + {}^7C_4 \times {}^5C_4 + {}^7C_3 \times {}^5C_5 = 420$

2. Ans. (9)

Step 1^{st} : Arrange 5 boys in 5! ways

Step 2^{nd} : Select 2 gaps from 6 gaps for 4 girls (2girls for each gap) in ${}^6C_2$ ways.

Step 3^{rd} : Select 2 girls to sit in one of the gaps and other 2 in remaining selected gaps

= ${}^4C_2$ ways

Step 4 : Arrange 1^{st}, 2 girls in 2! and other 2 in 2! Ways

Hence, total ways $(N) = 5! \times {}^6C_2 \times {}^4C_2 \times 2 \times 2 = 43200$

Sum of digits of $N = 4 + 3 + 2 + 0 + 0 = 9$

3. Ans. (169)

Given: ${}^nC_2 \times 2 = {}^nC_1 \times {}^2C_1 \times 2 + 66$

where n = number of men

${}^nC_2 - 2.{}^nC_1 = 33$

$n = 11$

So total no. of participants $(M) = 11 + 2 = 13$

Total no. of games played (N)

= ${}^{11}C_2 \times 2 + {}^2C_1 \times {}^{11}C_1 \times 2 + 2 = 156$

4. Ans. (420)

${}^{20}C_r$ is maximum when $r = \frac{20}{2} = 10$

${}^{25}C_r$ is maximum when $r = \frac{25-1}{2}$ or $\frac{25+1}{2}$

$r = 12$ or 13

So $\frac{{}^{20}C_{10}}{{}^{25}C_{12}} = 420$

5. Ans. (8)

$P_n = {}^{n-3+1}C_3 = {}^{n-2}C_3$

Now $P_{n+1} - P_n = 15$

$\Rightarrow {}^{(n-1)}C_3 - {}^{(n-2)}C_3 = 15 \Rightarrow \frac{(n-1)!}{(n-4)!} - \frac{(n-2)!}{(n-5)!} = 90$

$\Rightarrow \frac{3}{(n-4)} \cdot (n-2)(n-3)(n-4) = 90$

$\Rightarrow (n-2)(n-3) = 30 \Rightarrow n^2 - 5n - 24 = 0$

$n = 8$

6. Ans. (54)

$x_1x_2x_3 = 60$

$x_1.x_2.x_3 = 2^2 \times 3 \times 5$

$x_1 = 2^{a_1} \cdot 3^{a_2} \cdot 5^{a_3}$

$x_2 = 2^{b_1} \cdot 3^{b_2} \cdot 5^{b_3}$

$x_3 = 2^{c_1} \cdot 3^{c_2} \cdot 5^{c_3}$

$a_1 + b_1 + c_1 = 2 \rightarrow {}^4C_2 = 6$

$a_2 + b_2 + c_2 = 1 \rightarrow {}^3C_2 = 3$

$a_3 + b_3 + c_3 = 1 \rightarrow {}^3C_2 = 3$

Total solutions = $6 \times 3 \times 3 = 54$

7. Ans. (108)

$N = 2079000$

$N = 2^3 \times 3^3 \times 5^3 \times 7 \times 11$

Total no. of even divisions which are divisible by 15.

$= 3 \times 3 \times 3 \times 2 \times 2 = 108$

8. Ans. (36)

5 different digits can be arranged in 5! ways

so each digit will appear every place = 24 times

Sum of all digits at unit place = $24(0 + 1 + 2 + 4 + 5) = 288$

Sum of all digits = $288 \times 11111 = 3199968$

When 0 is at first place 4 digit number will be formed

Each number will appear 6 times at every place.

Sum of all 4 digit number at unit place

$= 6(1 + 2 + 4 + 5) = 72$

Hence sum of all four digit number

$= 72 \times 1111 = 79992$

So required sum = $3199968 - 79992 = 3119976$

$\therefore N = 36$

9. Ans. (30)

$M's = 2, U's = 2, N's = 2$

Total $= \dfrac{6!}{2! \cdot 2! \cdot 2!} = 90$

Let $n(E_1)$ = When one alike letter is together.

$n(E_2)$ = When two alike letters are together.

$n(E_3)$ = When three alike letters are together.

$n(E_3) = MM\ \ UU\ \ NN = 3! = 6$

$n(E_2) = 3[n(A \cap B) - n(A \cap B \cap C)] = 3\left[\dfrac{4!}{2!} - 6\right] = 18$

$n(E_1) = 3[n(A) - \{n(A \cap B) + n(B \cap C)\} + n(A \cap B \cap C)]$

$= 3\left[\dfrac{5!}{2!2!} - (12.2 - 6)\right] = 3[30 - 18] = 36$

Hence required number of ways

$= 90 - [6 + 18 + 36] = 30$

10. Ans. (26)

$x + y + z + w = 29$

when $x > 0, y > 1, z > 2,\ w \geq 0$

$x \geq 1, y \geq 2, z \geq 3,\ w \geq 0$

Now put

$x = 1 + t, y = 2 + t_2, z = 3 + t_3, w = 0 + t_4$

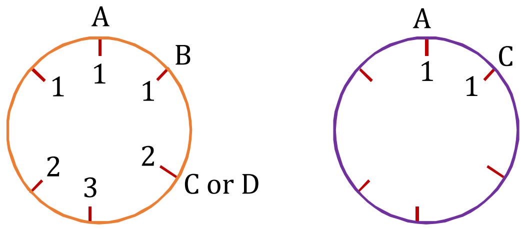
where $t_1, t_2, t_3, t_4 \geq 0$

$t_1 + t_2 + t_3 + t_4 = 23$

So no. of integral solutions = ${}^{23+4-1}C_{4-1} = {}^{26}C_3 = 2600 = N$

$\frac{N}{100} = 26$

11. **Ans. (18)**



Case-I: 2 × 3 × 2 = 12 **Case-II:** ${}^3C_1 \times 2 \times 1 = 6$

Total = 18

12. **Ans. (135)**

Method-I

Total number of handshakes possible ${}^{20}C_2$

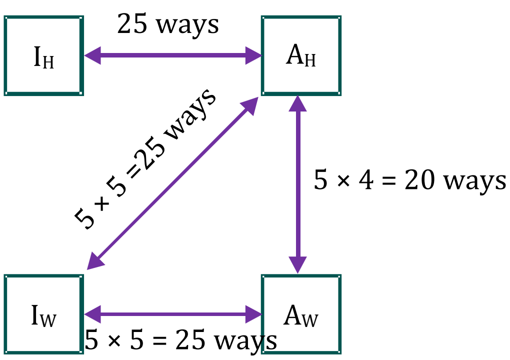
undesirable handshakes : ${}^{10}C_2 + {}^{10}C_1$

Hence, Desired ways = ${}^{20}C_2 - ({}^{10}C_2 + {}^{10}C_1)$

Method-II

$I_H \rightarrow$ Indian husband, $I_W \rightarrow$ Indian wife

$A_H \rightarrow$ American husband, $A_W \rightarrow$ American wife



Case 1 : Hand shaking occurring between same nationals and same genders (M/F)

$I_H - I_H \rightarrow {}^5C_2 = 10$ ways

Similarly for $I_W - I_W, A_W - A_W, A_H - A_H$

Total ways $10 \times 4 = 40$.

Case 2 : All other possible hand shakes

Hence total number of handshakes

= (25 × 3 + 40) + (20) = 135

13. **Ans. (124)**

Case 1 : Mr. B & Miss C are in committee and Mr. A is excluded

$\Rightarrow {}^4C_2 \times {}^4C_1 = 24$ ways

(Men) (Women)

Case 2 : Mr. B not there : ${}^5C_3 \times {}^5C_2 = 100$ ways

(Men) (Women)

Total ways = $24 + 100 = 124$

EXERCISE - JEE (Main) PYQ

1. **Ans. (4)**

(1) The number of four-digit numbers Starting with 5 is equal to $6^3 = 216$

(2) Starting with 44 and 45 is equal to $36 \times 2 = 72$

(3) Starting with 433, 434 and 435 is equal to $6 \times 3 = 18$

(4) Remaining numbers are 4322, 4323, 4324, 4325 is equal to 4

So total numbers are 216 + 72 + 18 + 4 = 310

2. **Ans. (1)**

Since there are 8 males and 5 females. Out of these 13, if we select 11 persons, then there will be at least 6 males and atleast 3 females in the selection.

$$m = n = \binom{13}{11} = \binom{13}{2} = \frac{13 \times 12}{2} = 78$$

3. **Ans. (1)**

$$\frac{n(n+1)}{2} + 99 = (n-2)^2$$

$n^2 + n + 198 = 2(n^2 + 4 - 4n)$

$n^2 - 9n - 190 = 0$

$n^2 - 19n + 10 - 190 = 0$

$n(n - 19) + 10(n - 19) = 0$

$n = 19$

4. **Ans. (2)**

a_1	a_2	a_3

Number of numbers = $5^3 - 1$

a_4	a_1	a_2	a_3

2 ways for a_4

Number of numbers = 2×5^3

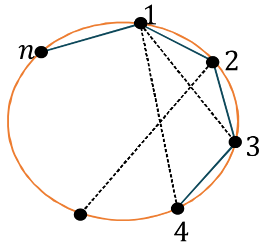
Required number = $5^3 + 2 \times 5^3 - 1$

= 374

5. **Ans. (3)**

Number of blue lines = Number of sides = n

Number of red lines = Number of diagonals = ${}^nC_2 - n$



$${}^nC_2 - n = 99\,n \Rightarrow \frac{n(n-1)}{2} - n = 99\,n$$

$$\frac{n-1}{2} - 1 = 99 \Rightarrow n = 201$$

6. **Ans. (4)**

ABC

5	5	5

1	2	2
2	1	2
2	2	1
1	1	3
1	3	1
3	1	1

Total number of selection

$= ({}^5C_1\,{}^5C_2\,{}^5C_2) \times 3 + ({}^5C_1\,{}^5C_1\,{}^5C_3) \times 3$

$= 5 \times 10 \times 10 \times 3 + 5 \times 5 \times 10 \times 3 = 2250$

7. Ans. (1)

_ _ _ 2 _

No. of five digits numbers =

No. of ways of filling remaining 4 places

= 8 × 8 × 7 × 6

$\therefore\ k = \dfrac{8\times8\times7\times6}{336} = 8$

8. Ans. (309)

MOTHER

$1 \to$ E

$2 \to$ H

$3 \to$ M

$4 \to$ O

$5 \to$ R

$6 \to$ T

So position of word MOTHER in dictionary

2 × 5! + 2 × 4! + 3 × 3! + 2! + 1

= 240 + 48 + 18 + 2 + 1

$= \boxed{309}$

9. Ans. (96)

2,4,6,8				
4	4	3	2	1

= 4 × 4 × 3 × 2 = 96

10. Ans. (238)

Class	10^{th}	11^{th}	12^{th}	
Total student	5	6	8	
	2	3	5	$\Rightarrow {}^5C_2 \times {}^6C_3 \times {}^8C_5$
Number of selection	2	2	6	$\Rightarrow {}^5C_2 \times {}^6C_2 \times {}^8C_6$
	3	2	5	$\Rightarrow {}^5C_3 \times {}^6C_2 \times {}^8C_5$

$\Rightarrow$ Total number of ways = 23800

According to question

100 K = 23800

$\Rightarrow K$ = 238

11. Ans. (7744)

209, 220, 231, ..., 495

Sum = $\dfrac{27}{2}(209+495) = 9504$

Number containing 1 at unit place: $\underline{2}\ \underline{3}\ \underline{1}$, $\underline{3}\ \underline{4}\ \underline{1}$, $\underline{4}\ \underline{5}\ \underline{1}$

Number containing 1 at 10^{th} place: $\underline{3}\ \underline{1}\ \underline{9}$, $\underline{4}\ \underline{1}\ \underline{8}$

Required = 9501 – (231 + 341 + 451 + 319 + 418) = 7744

12. **Ans. (1)**

Digits are 1, 2, 2, 3

total distinct numbers $\frac{4!}{2!} = 12$.

total numbers when 1 at unit place is 3.

2 at unit place is 6

3 at unit place is 3.

So, sum = $(3 + 12 + 9)(10^3 + 10^2 + 10 + 1)$

= $(1111) \times 24$

= 26664

13. **Ans. (1492)**

M	A	N	K	I	N	D

$$\left(\frac{4\times 6!}{2!}\right) + (5!\times 0) + \left(\frac{4!\times 3}{2!}\right) + (3!\times 2) + (2!\times 1) + (1!\times 1) + (0!\times 0) + 1 = 1492$$

14. **Ans. (1120)**

$n(B) = 10$

$n(a) = 5$

The number of ways of forming a group of 3 girls of 3 boys.

$= {}^{10}C_3 \times {}^{5}C_3$

$= \frac{10\times 9\times 8}{3\times 2} \times \frac{5\times 4}{2} = 1200$

The number of ways when two particular boys B_1 of B_2 be the member of group together

$= {}^{8}C_1 \times {}^{5}C_3 = 8 \times 10 = 80$

Number of ways when boys B_1 of B_2 hot in the same group together

$= 1200 \times 80 = 1120$

15. **Ans. (7073)**

Required no. = Total – no character from {1, 2, 3, 4, 5}

$= (10^6 - 5^6) + (10^7 - 5^7) + (10^8 - 5^8) = 10^6(1 + 10 + 100) - 5^6(1 + 5 + 25)$

$= 10^6 \times 111 - 5^6 \times 31 = 2^6 \times 5^6 \times 111 - 5^6 \times 31$

$= 5^6(2^6 \times 111 - 31) = 5^6 \times \underbrace{7073}_{\alpha}$

$\therefore \alpha = 7073$

16. **Ans. (1086)**

Let the number is $abcd$, where a, b, c are divisible by d.

	No. of such numbers
$d = 1$,	$9 \times 10 \times 10 = 900$
$d = 2$	$4 \times 5 \times 5 = 100$
$d = 3$	$3 \times 4 \times 4 = 48$
$d = 4$	$2 \times 3 \times 3 = 18$
$d = 5$	$1 \times 2 \times 2 = 4$
$d = 6, 7, 8, 9$	$4 \times 4 = 16$
	1086

17. **Ans. (1)**

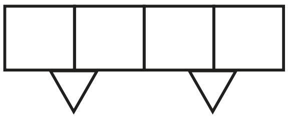
7 boys can be seated in 6! ways

now girls will be placed in gaps

$\therefore$ total ways $= 6! \times {}^7C_5 \times 5!$

$= 126\ (5!)^2$

18. **Ans. (72)**



Sum of first two digits

Sum of last two digits = α

Case-I : $\alpha = 7$

$2 \times 12 = 24$ ways.

7 0	
0 7	1 6
	2 5
	3 4
	4 3
	5 2
	6 1

Case-II : $\alpha = 8$

17	26
71	62
	35
	53

2×8 ways

= 16 ways

Case-III : $\alpha = 9$

27	36
72	63
	45
	54

2×8 ways

= 16 ways

Case-IV : $\alpha = 10$

37	46
73	64

2×4 ways

8 ways

Case-V : $\alpha = 11$

47	56
74	65

2 × 4 ways

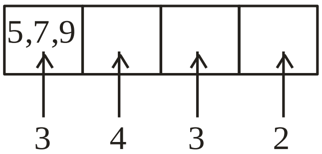
= 8 ways

Ans. 24 + 16 + 16 + 8 + 8 = 72

19. **Ans. (4)**

Numbers between 5000 & 10000

Using digits 1, 3, 5, 7, 9



Total Numbers = 3 × 4 × 3 × 2 = 72

20. **Ans. (4)**

/ M / A / T / H / E / M / A / T / I /

Arrange remaining 9 letters and put C and S in any 2 gaps out of 10 gaps.

i.e. $\frac{9!}{2!\times 2!\times 2!}\times {}^{10}C_2 \times 2!$ = (6!) k (Given)

k = 5670

EXERCISE - JEE (Advanced) PYQ

1. **Ans. (5)**

When 1 and 2 are removed from numbers 1 to n then

we get maximum possible sum of remaining numbers and when n −1, n are removed then we get minimum possible sum of remaining numbers.

$$\Rightarrow \frac{n(n+1)}{2}-(2n-1)\le 1224\le \frac{n(n+1)}{2}-3 \Rightarrow \begin{cases} n^2+n-2454\ge 0 \\ n^2-3n-2446\le 0 \end{cases}$$

$$\Rightarrow \begin{cases} n\ge 50 \\ n\le 50 \end{cases} \Rightarrow n=50$$

Now let x and x + 1 be two consecutive numbers

$$\Rightarrow \frac{50(50+1)}{2}-x-x-1=1224 \Rightarrow x = 25$$

⇒ 25th and 26th cards are removed from pack

⇒ k = 25 ⇒ k – 20 = 5

2. **Ans. (7)**

as $n_1\ge 1,\ n_2\ge 2,\ n_3\ge 3,\ n_4\ge 4,\ n_5\ge 5$

Let $n_1-1=x_1\ge 0,\ n_2-2=x_2\ge 0$ .... $n_5-5=x_5\ge 0$

⇒ New equation will be

$x_1+1+x_2+2+\ldots\ldots x_5+5=20$

$\Rightarrow x_1+x_2+x_3+x_4+x_5=20-15=5$

Now $x_1\le x_2\le x_3\le x_4\le x_5$

x_1	x_2	x_3	x_4	x_5
0	0	0	0	5
0	0	0	1	4
0	0	0	2	3
0	0	1	1	3
0	0	1	2	2
0	1	1	1	2
1	1	1	1	1

So 7 possible cases will be there.

3. **Ans. (5)**

Numbr of red line segments = ${}^{n}C_2 - n$

Number of blue line segments = n

$\therefore\ {}^{n}C_2 - n = n$

$\frac{n(n-1)}{2} = 2n \Rightarrow n = 5$ Ans.

4. **Ans. (C)**

Total number of dearrangement

$6!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!}\right]$

= 360 – 120 + 30 – 6 + 1

= 240 + 25 = 265

There are equal chances that card 1 goes into any envelope from 2 to 6

$\therefore \frac{1}{5}(265) = 53$

5. **Ans. (5)**

$n = 5!6!$

$m = 5!\ {}^{6}C_2.{}^{5}C_4.{}^{2}C_1.4!$

$\therefore \frac{m}{n} = 5$

6. **Ans. (A)**

$({}^{6}C_4 + {}^{6}C_3.\ {}^{4}C_1).\ {}^{4}C_1 = 380$

7. **Ans. (5)**

$x = 10!$

$y = {}^{10}C_1\ {}^{9}C_8\ \frac{10!}{2!}$

$\frac{y}{9x} = \frac{5.9.10!}{9.10!} = 5$

8. **Ans. (D)**

$N_1 + N_2 + N_3 + N_4 + N_5 =$ Total ways – {when no odd}

Total ways = ${}^{9}C_5$

Number of ways when no odd, is zero

($\because$ only available even are 2, 4, 6, 8)

$\therefore\ {}^{9}C_5 -$ zero = 126

9. **Ans. (625)**

Option for last two digits are (12), (24), (32), (44) are (52).

$\therefore$ Total No. of digits = 5 × 5 × 5 × 5 = 625

10. **Ans. (119)**

$n(X) = 5$

$n(Y) = 7$

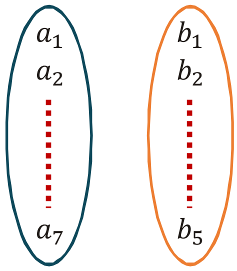
α → Number of one-one function = ${}^{7}C_5 \times 5!$

β → Number of onto function Y to X

1, 1, 1, 1, 3 1, 1, 1, 2, 2

$$\frac{7!}{3!\,4!} \times 5! + \frac{7!}{(2!)^3 3!} \times 5! = ({}^{7}C_3 + 3.{}^{7}C_3)5! = 4 \times {}^{7}C_3 \times 5!$$

$$\frac{\beta - \alpha}{5!} = 4 \times {}^{7}C_3 - {}^{7}C_5 = 4 \times 35 - 21 = 119$$



11. **Ans. (C)**

(1) $\alpha_1 = \binom{6}{3}\binom{5}{2} = 200$

So $P \to 4$

(2) $\alpha_2 = \binom{6}{1}\binom{5}{1} + \binom{6}{2}\binom{5}{2} + \binom{6}{3}\binom{5}{3} + \binom{6}{4}\binom{5}{4} + \binom{6}{5}\binom{5}{5}$

$= \binom{11}{5} - 1 = 46!$

So $Q \to 6$

(3) $\alpha_3 = \binom{5}{2}\binom{6}{3} + \binom{5}{3}\binom{6}{2} + \binom{5}{4}\binom{6}{1} + \binom{5}{5}\binom{6}{0}$

$= \binom{11}{5} - \binom{5}{0}\binom{6}{5} - \binom{5}{1}\binom{6}{4} = 381$

So $R \to 5$

(4) $\alpha_2 = \binom{5}{2}\binom{6}{2} - \binom{4}{1}\binom{5}{1} + \binom{5}{3}\binom{6}{1} - \binom{4}{2}\binom{1}{1} + \binom{5}{4}$

= 189

So $S \to 2$

12. **Ans. (30)**

When $1R, 2B, 2G$

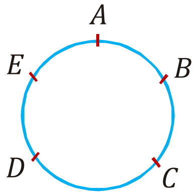
${}^{5}C_1 \times 2 = 10$

Other possibilities

$1B, 2R, 2G$

Or $1G, 2R, 2B$

So total no. of ways = $3 \times 10 = 30$



13. **Ans. (495)**

Selection of 4 days out of 15 days such that no two of them are consecutive

$$= {}^{15-4+1}C_4 = {}^{12}C_4 = \frac{12\times11\times10\times9}{4\times3\times2} = 11 \times 5 \times 9 = 495$$

14. **Ans. (1080)**

Required ways $= \dfrac{6!}{2!2!1!1!2!2!} \times 4! = 1080$

15. Ans. (A,B,D)

(A) $n_1 = 10 \times 10 \times 10 = 1000$

(B) As per given condition $1 \le i < j + 2 \le 10 \Rightarrow j \le 8$ & $i \ge 1$

for $i = 1, 2,$ $j = 1, 2, 3, ..., 8$ $\rightarrow (8 + 8)$ possibilities

for $i = 3,$ $j = 2, 3, ..., 8$ $\rightarrow 7$ possibilities

$i = 4,$ $j = 3, ..., 8$ $\rightarrow 6$ possibilities

$i = 9,$ $j = 1$ $\rightarrow 1$ possibility

So $n_2 = (1 + 2 + 3 + ...+ 8) + 8 = 44$

(C) $n_3 = {}^{10}C_4$ (Choose any four)

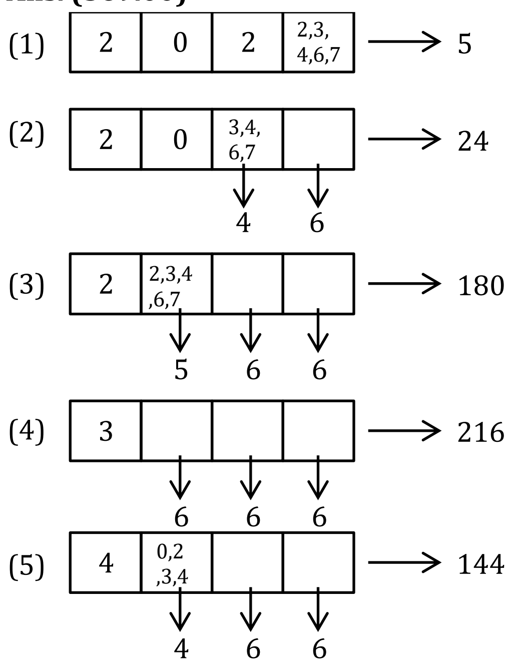
$= 210$

(D) $n_4 = {}^{10}C_4 . 4! = (210)(24)$

$\Rightarrow \frac{n_4}{12} = 420$

So correct Ans. (A), (B), (D)

16. Ans. (569.00)



Number of 4 digit integers in [2022,4482]

$= 5 + 24 + 180 + 216 + 144 = 569$

17. Ans. (A)

$3R$ / $2B$	$3R$ / $2B$	$3R$ / $2B$	$3R$ / $2B$
B-1	B-2	B-3	B-4

Case-I : when exactly one box provides four balls ($3R$ $1B$ or $2R$ $2B$)

Number of ways in this case ${}^5C_4\,({}^3C_1 \times {}^2C_1)^3 \times 4$

Case-II : when exactly two boxes provide three balls ($2R$ $1B$ or $1R$ $2B$) each

Number of ways in this case $({}^5C_3 - 1)^2\,({}^3C_1 \times {}^2C_1)^2 \times 6$

Required number of ways = 21816

Language ambiguity : If we consider at least one red ball and exactly one blue ball, then required number of ways is 9504. None of the option is correct.

18. Ans. (31)

No. of elements in X which are multiple of 5

$$\left.\begin{array}{l}
\underbrace{_\,_\,_\,_}_{1,2,2,2}\ \underset{\text{fixed}}{0} \to \dfrac{\lfloor 4}{\lfloor 3} = 4 \\
\underbrace{_\,_\,_\,_}_{1,4,2,2}\ \underset{\text{fixed}}{0} \to \dfrac{\lfloor 4}{\lfloor 2} = 12 \\
\underbrace{_\,_\,_\,_}_{4,2,2,2}\ \underset{\text{fixed}}{0} \to \dfrac{\lfloor 4}{\lfloor 3} = 4 \\
\underbrace{_\,_\,_\,_}_{2,2,4,4}\ \underset{\text{fixed}}{0} \to \dfrac{\lfloor 4}{\lfloor 2\lfloor 2} = 6 \\
\underbrace{_\,_\,_\,_}_{1,2,4,4}\ \underset{\text{fixed}}{0} \to \dfrac{\lfloor 4}{\lfloor 2} = 12
\end{array}\right\}\text{Total} = 38$$

$$\left.\begin{array}{l}
\underbrace{_\,_\,_\,_}_{1,2,2,2}\ \underset{fixed}{0} \to \dfrac{\lfloor 4}{\lfloor 3} = 4 \\
\underbrace{_\,_\,_\,_}_{1,4,2,2}\ \underset{fixed}{0} \to \dfrac{\lfloor 4}{\lfloor 2} = 12 \\
\underbrace{_\,_\,_\,_}_{4,2,2,2}\ \underset{fixed}{0} \to \dfrac{\lfloor 4}{\lfloor 3} = 4 \\
\underbrace{_\,_\,_\,_}_{2,2,4,4}\ \underset{fixed}{0} \to \dfrac{\lfloor 4}{\lfloor 2\lfloor 2} = 6 \\
\underbrace{_\,_\,_\,_}_{1,2,4,4}\ \underset{fixed}{0} \to \dfrac{\lfloor 4}{\lfloor 2} = 12
\end{array}\right\}Total = 38$$

Among these 38 elements, let us calculate when element is not divisible by 20

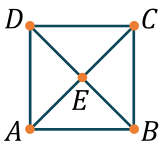
$$\left.\begin{array}{l}
\underset{2,2,2}{_\,_\,_}\ \underset{fixed}{1\ \ 0} \to \dfrac{\lfloor 3}{\lfloor 3} = 1 \\
\underset{2,2,4}{_\,_\,_}\ \underset{fixed}{1\ \ 0} \to \dfrac{\lfloor 3}{\lfloor 2} = 3 \\
\underset{2,4,4}{_\,_\,_}\ \underset{fixed}{1\ \ 0} \to \dfrac{\lfloor 3}{\lfloor 2} = 3
\end{array}\right\}Total = 7$$

$\therefore\ p = \dfrac{38-7}{38}$ $\quad\therefore\ 38p = 31$

JEE (Main) Practice Paper

SECTION–A

1. Ans. (3)



A, E, C are collinear

& B, E, D are collinear

$\Rightarrow {}^5C_3 - 2.{}^3C_3 = 8$

2. Ans. (2)

Teacher's visits = ${}^{25}C_5$ (different sets of all kids) A kid's visit = ${}^{24}C_4$ (different sets of other kids)

Difference = ${}^{25}C_5 - {}^{24}C_4$.

$= {}^{24}C_5\,\{{}^{n+1}C_r - {}^nC_{r-1} = {}^nC_r\}$

3. Ans. (3)

${}^nC_3 - [n + n(n-4)] = 30$

$n = 9$ satisfies it

4. Ans. (3)

A .... = 4! = 24

N ....= 4! = 24

R .... = 4! = 24

U .... = 4! = 24

VAN .... = 2! = 2

VARN .... = 1

VARUN ....= 1

VARUN = 100

5. Ans. (2)

If last digit is 5

				5

= 2688

8 × 8 × 7 × 6

If Last digit is 0 choice of places for 5 = 4 $\Rightarrow$

				0

= 1344

4 × 8 × 7 × 6

Total = 4032 $\Rightarrow k = 168$

6. Ans. (4)

Select any 4 pairs and one shoe from each pair

$= {}^5C_4 . {}^2C_1 . {}^2C_1 {}^2C_1 {}^2C_1 = 80$

7. Ans. (2)

Consider A & B as one. C & D are not to be arranged.

Total = $\dfrac{7! \times 2!}{2!}$ $\to$ To arrange A & B; $\to$ For not arranging C & D

= 5040

8. Ans. (3)

1, 2, 3, 4, 5, 0

A number is divisible by 25 if the last two digits are 25 or 50

(i) Now, if 5 is not taken then number of such numbers = 0

(ii) If 0 is not taken

			2	5

= 6

(iii) if 2 is not taken then

			5	0

= 6 and

(iv) If 1 | 3 | 4 is rejected

then in each case we have,

			5	0

= 6

			2	5

= 4

2.2 1

if 3, 4, or 1 is not taken then 10 number for each case.

Total = 30

Hence, Total = 30 + 6 + 6 = 42

9. Ans. (1)

Last match is won by India.

0	match won by Pakistan	$= {}^4C_0 = 1$
1	match won by Pakistan	$= {}^5C_1 = 5$
2	matches won by Pakistan	$= {}^6C_2 = 15$
3	matches won by Pakistan	$= {}^7C_3 = 35$
4	matches won by Pakistan	$= {}^8C_4 = 70$
	Total	$= 126$

10. Ans. (1)

1, 2, will be arranged in only one ways and similarly 3, 4, 5, 6 will be arranged in only one way.

Number of permutations $\frac{9!}{2!2!2!} = 9.7!$

11. Ans. (3)

5 in car I & 3 in car II = ${}^8C_5$

4 in car I & 4 in car II = ${}^8C_4$

Total = 56 + 70 = 126

12. Ans. (3)

$$L = \frac{\lfloor p+q}{\lfloor p \lfloor q} \times 1$$

x gets p books and y gets q books

$$M = \frac{\lfloor p+q}{\lfloor p \lfloor q} \times \lfloor 2$$

$$N = \frac{\lfloor p+q}{\lfloor p \lfloor q}$$

clearly $2L = M = 2N$

13. Ans. (4)

(1 3 4) or (2 2 4) or (2 3 3)

$$= \left(\frac{8!}{1!\,3!\,4!} + \frac{8!}{2!\,2!\,4!\,2!} + \frac{8!}{2!\,3!\,3!\,2!}\right) \times 3! = (280 + 210 + 280) \times 3!$$

$$k = \frac{770 \times 6}{7 \times 6 \times 5} = 22$$

14. Ans. (2)

Any one subjective is allotted two periods

$$= {}^5C_1 \times \frac{6!}{2!} = 1800$$

15. Ans. (3)

$(x_1 + x_2 + x_3)\,(y_1 + y_2) = 11 \cdot 7$ or $7 \cdot 11$

In the first case $(x_1 + x_2 + x_3) = 11$ and $(y_1 + y_2) = 7$, which have ${}^{10}C_2 \cdot {}^6C_1$ solutions (using beggar)

In the second case $(x_1 + x_2 + x_3) = 7$ & $(y_1 + y_2) = 11$, which have ${}^6C_2 \cdot {}^{10}C_1$ solutions (using beggar)

$\therefore$ total number of solutions $= {}^{10}C_2 \cdot {}^6C_1 + {}^6C_2 \cdot {}^{10}C_1 = 270 + 150 = 420$ Ans.

16. **Ans. (3)**

Let i appears on a_i dice i = 1, 2, 3, 4, 5, 6

so no. of out comes is equal to no. of solution of $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = n = {}^{(n+5)}C_5$

17. **Ans. (1)**

Required number of ways ${}^{36}C_9 \,.\, {}^{27}C_9 \,.\, {}^{18}C_9 \,.\, {}^{9}C_9 \,.\, 4\,! = \dfrac{36!}{(9!)^4} \times 4!$

18. **Ans. (2)**

If P_3 sits on C_1

$4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right) = 4.3 - 4 + 1 = 9$

If P_3 does not sit on C_1

$= 5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right) = 44$

total number of ways = 44 + 9 = 53

$$\begin{cases} P_1 & C_1 \\ P_2 & C_2 \quad \to P_1 \\ P_3 & C_3 \quad \to P_2 \\ P_4 & C_4 \\ P_5 & C_5 \\ P_6 & C_6 \\ P_7 & C_7 \end{cases}$$

19. **Ans. (1)**

First we select 3 length from the given 6 length so the no. of ways = ${}^6C_3$

But these some pair i.e. (2, 3, 7), (2, 3, 6), (2, 3, 5) (2, 4, 6), (2, 4, 7), (2, 5, 7), (3, 4, 7) are not form a triangle so that total no. of ways is

${}^6C_3 - 7$ ways

20. **Ans. (1)**

m, n both product of element of one of the subset of $\{2^{22}, 3^{10}, 5^6, 7^3, 11^2, 13, 17, 19, 23\}$ such that

$mn = 25! \Rightarrow$ number of ways of selecting 'm' is 2^9 ways (here n is automatically fixed according to m)

$\Rightarrow$ Total number of required ways = $\dfrac{2^9}{2} = 256$

{because in half of the ways $\dfrac{m}{n} > 1$ and in half of the ways $0 < \dfrac{m}{n} < 1$}

SECTION–B

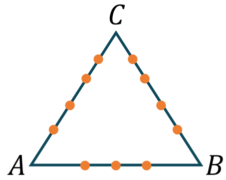
1. **Ans. (93)**

Number divisible by 3 if sum of digits divisible

case-I	If 1 + 2 + 3 + 4 + 8 = 18	Number of ways = 120
case-II	If 1 + 2 + 3 + 7 + 8 = 21	Number of ways = 120
case-III	If 2 + 3 + 4 + 7 + 8 = 24	Number of ways = 120
case-IV	If 1 + 2 + 0 + 4 + 8 = 15	Number of ways = 96
case-V	If 1 + 2 + 0 + 7 + 8 = 18	Number of ways = 96
case-VI	If 2 + 0 + 4 + 7 + 8 = 21	Number of ways = 96
case-VII	If 0 + 1 + 3 + 4 + 7 = 15	Number of ways = 96
	total number	744

2. Ans. (41)

Total number of triangle = Two points taken from AB and one point either BC or CA + similarly BC + similarly $\angle A$ + one point each sides.



= ${}^3C_2\,[{}^4C_1 + {}^5C_1] + {}^4C_2\,[{}^5C_1 + {}^3C_1] + {}^5C_2\,[{}^3C_1 + {}^4C_1] + {}^3C_1{}^4C_1{}^5C_1 = 205$

Total – (collinear points used)

= ${}^{12}C_3 - ({}^3C_3 + {}^4C_3 + {}^5C_3) = 220 - 15 = 205$

Alternate

Total – Collinear points used = ${}^{12}C_3 - ({}^3C_3 + {}^4C_3 + {}^5C_3) = 220 - 15 = 205$

3. Ans. (63)

Coin dividing in any are possible i.e.

1, 2, 4

so the number of ways is

${}^7C_1 \cdot {}^6C_2 \cdot {}^4C_4 \cdot 3! = 7 \times 15 \times 6 = 630$

4. Ans. (62)

Let number be $x_1\ x_2\ x_3\ x_4\ x_5\ x_6$

But Here $x_1 + x_2 + \ldots . x_6 = 12$

So coefficient of x^{12} in expansion $\left(1+x+x^2+\ldots .+x^9\right)^6 = \left(1-x^{10}\right)^6 . \left(1-x\right)^{-6}$

$\Rightarrow\ {}^{17}C_{12} - {}^6C_1 . {}^7C_2 = 6188 - 126 = 6062$

5. Ans. (10)

Let team X wins 'm' matches, if it wins $(m + r)th$ match and wins $m - 1$ match from the first $m + r - 1$ matchs, so total no. of ways = $\sum_{r=0}^{m} {}^{m+r-1}C_{m-1} = \frac{{}^{20}C_m}{2}$ hence $m = 10$

6. Ans. (42)

SERIES

S - 2, E - 2 , R, I

case-I when all letter distinct is

${}^4C_3 \times 3! = 4 \times 6 = 24$

case-II when 2 letters are same the

${}^2C_1 . {}^3C_1 \times \frac{3!}{2!} = 2 . 3 . 3 = 18$

total number is 24 + 18 = 42

7. Ans. (31)

Case -I If all are different then no. of ways is = ${}^6C_3 = 20$

Case-II If three each of two colours, then combination is

3 0 $\rightarrow 2!$

2 1 $\rightarrow 2!$ = 2! + 2! = 4 ways

Case-III If two each of three colours, then combination is

2 1 0 $\rightarrow 3!$

1 1 1 $\rightarrow 1!$ = 3! + 1! = 7ways

Hence required no.is = 20 + 7 + 4 = 31

8. Ans. (13)

Using multinomial theorem total number of required selection is ${}^{8+3}C_8 = {}^{11}C_8 = {}^{11}C_3 = 165$

9. Ans. (28)

$P_n = {}^{n-2}C_3$

$P_{n+1} - P_n = {}^{n-1}C_3 - {}^{n-2}C_3 = {}^{n-2}C_2 = 15$

$\Rightarrow (n - 2)(n - 3) = 30 \Rightarrow n = 8$

10. Ans. (1)

$^{2002}C_{1001} = \frac{(2002)!}{(1001)!(1001)!}$

no. of zeros in (2002)! are

400 + 80 + 16 + 3 = 499

no. of zeroes in $(1001\,!)^2 = 2(200 + 40 + 8 + 1) = 498$

Hence no. of zeroes is $\frac{(2002)!}{(1001!)^2} = 1$

JEE (Advanced) Practice Paper

1. Ans. (C)

$^{11}C_3 + {}^{11}C_4 + \ldots\ldots + {}^{11}C_{11} = 2^{11} - {}^{11}C_0 - {}^{11}C_1 - {}^{11}C_2 = 1981$

2. Ans. (B)

$\beta - \alpha$

$= y_1 y_2 y_3$

$- x_1 x_2 x_3$

Number of pairs = $(10 + 9 + 8 + \ldots + 1)^2 . (9 + 8 + \ldots\ldots + 1) = (55)^2 .45$

3. Ans. (D)

Total n-digit numbers using 1, 2 or 3 = 3^n

total n-digit numbers using any two digits out of 1, 2 or 3 = $^3C_2 \times 2^n - 6 = 3 \times 2^n - 6$

total n-digit numbers using only one digit of 1, 2 or 3 = 3

$\therefore$ the numbers containing all three of the digits

1, 2 and 3 at least once = $3^n - (3 \times 2^n - 6) - 3 = 3^n - 3 . 2^n + 3$

4. Ans. (A)

Ordered pair = total – $(A \cup B = X) = 4^n - 3^n$

Subsets of $X = 2^n$ will not repeat in both but here the whole set X has not been taken

So subsets of x which are not repeated $\left(2^n - 1\right)$

Hence unordered pair = $\frac{(4^n - 3^n) - (2^n - 1)}{2} + (2^n - 1)$

5. Ans. (C)

Using multinomial theorem

Total no. of ways = $^{15+3-1}C_{15} \times {}^{10+3-1}C_{10} = {}^{17}C_{15} \times {}^{12}C_{10} = \frac{17 \times 16}{2} \times \frac{12 \times 11}{2} = 8976$

6. Ans. (D)

A	*B*	*A*	*B*	*A*	*B*

= total number of required possible is

B	*A*	*B*	*A*	*B*	A

$^{12}C_6 \times 6\,! \times 6! \times 2! = \frac{12!}{6! \times 6!} \times 6\,! \times 6\,! \times 2! = 2 \times 12!$

7. Ans. (B,C)

Total required number of teams is

$= {}^{10}C_4 \cdot {}^{6}C_3 \cdot {}^{3}C_3 \cdot \frac{1}{2!} 2100 = {}^{10}C_4 \cdot {}^{5}C_2 = 2100$

8. Ans. (C,D)

SAMANVAYA

Number of circular permutations is = $\frac{(9-1)!}{4!} = \frac{8!}{4!} = \begin{bmatrix} \frac{8!}{0!4!} \\ \frac{8!}{1!4!} \end{bmatrix}$

$x^2 + y^2 - n = \begin{bmatrix} 0+16-8=8 \\ 1+16-8=9 \end{bmatrix}$

9. Ans. (A,B,D)

$\left.\begin{matrix} ....08 & \\ ....60 & \rightarrow \\ ....80 & \rightarrow \end{matrix}\right] 3\times2\times1\times3=18 \; + = 30$

$\left.\begin{matrix} ....16 & \rightarrow \\ ....36 & \rightarrow \\ ....68 & \rightarrow \end{matrix}\right] 2\times2\times1\times3=12$

Similarly B & D are also correct

10. Ans. (B,D)

Number of 7 digit numbers with starting digit 1 will be 6! = 720. But these also contain numbers ending at the digit 5, which become divisible by 5. So, fixing starting digit 1 and ending digit 5, we get 5! = 120 numbers. Thus, number of 7 digit numbers starting with 1 and not divisible by 5 will be 720 – 120 = 600.

Similarly, number of 7 digit numbers starting with 2 and not divisible by 5 will be 600. Similarly, number of 7 digit numbers starting with 3 and not divisible by 5 will also be 600.

Thus, when put in required increasing order, 1801^{th} number will be the smallest numbers starting with 4 and not divisible by 5 which is 4123567.

Similarly we can check other options also

11. Ans. (B,C)

$a^2 - b^2 = c \Rightarrow (a-b)(a+b) = c$

$\Rightarrow a - b = 1 \Rightarrow a = 3, b = 2, c = 5 \;\&\; d = 7$

and sum of all possible numbers

= 3!(1111)(2 + 3 + 5 + 7) = 113322

12. Ans. (A,B,C)

Exponent of 2 in 31! = $\left[\frac{31}{2}\right] + \left[\frac{31}{2^2}\right] + \left[\frac{31}{2^3}\right] + \left[\frac{31}{2^4}\right]$

= 15 + 7 + 3 + 1 = 26

Exponent of 2 in 33! = 31

Exponent of 2 in 35! = 32

So $31! = 2^{26} \lambda_1$

$33! = 2^{31}\lambda_2$

$35! = 2^{32}\lambda_3$

$\Rightarrow$ maximum value of m is 89

13. **Ans. (C)**

$16 \to P_1, P_2, P_3, P_4$; 12 others

12 others can be divided into 4 equal groups in each of 3 person $\frac{(12)!}{(3!)^4} = \frac{(12)\cdot(11)!}{6\cdot6\cdot6\cdot6} = \frac{(11)!}{108}$

14. **Ans. (D)**

$16 \to P_1, P_2, P_3, P_4$; P_6 ; $P_7 - P_{16}(10)$; $10 \to 3, 7 = \frac{10!}{3!\,7!}$

0 0 0 $+P_6$ — 4 players ; $x\,x\,x\,x\,x\,x\,x$ P_1 to P_5 — 12 players

Now, $12 \to 4, 4, 4 = \frac{12!}{(4!)^3\cdot 3!}$

Total ways = $\frac{(10)!}{3!\,7!}\cdot\frac{(12)!}{(4!)^3\cdot 3!} = \left(\frac{12!}{(4!)^3}\right).20$

$\Rightarrow$ $k = 20$

15. **Ans. (C)**

$N = 2910600$

$N = 2^3.3^3.5^2.7^2.11$

Divisible by (15)

$N = (5.(3))(2^3.3^2.5.7^2.11)$

$\downarrow$

(15)

$\frac{N}{15} = \underset{(3+1)}{(2^3)} \; \underset{(2+1)}{(3^2)} \; \underset{(1+1)}{(5)} \; \underset{(2+1)}{(7^2)} \; \underset{(1+1)}{(11)}$

$\Rightarrow 144$

Divisible by 180 $\rightarrow$ (L.C.M. of 15 and 36)

$N = (2^2.3^2.5)\,(2)\,(3)\,(5)\,(7^2)\,(11)$

$\frac{N}{180} = \underset{(1+1)}{(2)} \; \underset{(1+1)}{(3)} \; \underset{(1+1)}{(5)} \; \underset{(2+1)}{(7)} \; \underset{(1+1)}{(11)}$

$\Rightarrow 48$

Divisible by 15 not by 36

$\Rightarrow 144 - 48 = 96$

16. **Ans. (C)**

$N = 2^3 3^3 5^2 7^2 11$

for H.C.F. = 2

$7^* \to 2$ choice

$11^* \to 2$ choice

$\Rightarrow 2 \times 2 \times 2 \times 2 = 16$ ways

same for H.C.F. = 3 = 16 ways

but for H.C.F. = 5 or 7

same for H.C.F. = 3 = 16 ways

But for H.C.F. = 5 or 7

for first pocket (3*, 2*, 7* or 11*) we only have one choice and for rest we have 2 choice

$\Rightarrow 2 \times 2 \times 2 \times (1) = 8$

Total $\Rightarrow 16 + 16 + 8 + 8 \Rightarrow 48$

(HCF 11 not possible)

17. **Ans. (B)**

$(A_1, A_2), (B_1, B_2), (C_1, C_2), D, E, F, G$

we select 2 pairs and keep them together and arrange the remaining pair within gaps after arranging together pair and singles

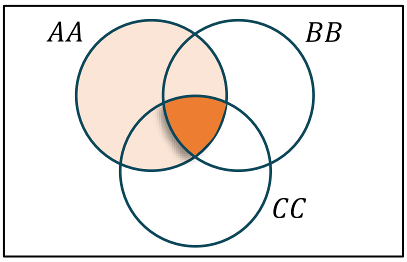
$$= \underbrace{{}^3C_2}_{\text{2 pairs chosen}} \times \underbrace{(6-1)!}_{\text{Circular arrangement of 2 pairs and singles}} \times \underbrace{{}^6C_2}_{\text{2 gaps out of six for}} \times \underbrace{2\times2\times2}_{\text{Arranging pairs at respective places.}} = 60\times6!$$

18. **Ans. (A)**

$\boxed{A_1A_2}\ B_1B_2C_1C_2DEFG$

$$\underbrace{\text{Only } A_1, A_2 = \text{Together \& other pairs separat}}_{a} \qquad \underbrace{\text{ways in}-\text{which } A_1, A_2 \text{ together}}_{s}$$

$$\left[\underbrace{A_1A_2\text{together } B_1B_2\text{together but } C_1C_2 \text{ separate}}_{b}\right] - \left[\underbrace{A_1A_2\text{together}, C_1C_2\text{together \& } B_1B_2 \text{ separate}}_{c}\right] + \left[\underbrace{A_1A_2\text{together}, B_1B_2\text{together } C_1C_2 \text{ together}}_{d}\right]$$



$s = 8!\ 2!$

$b = 7! \times 2!\ 2! = c$

$d = 6!\ 2!\ 2! \times 2!$

$\therefore\ a = 8!2! - (7! \times 2! \times 2! \times 2) + 6!2!2!2!$

$= 6!\ (2!(8 \times 7 \times 7 \times 2 \times 2 + 4)$

$\therefore\ a = (64)\ 6! \Rightarrow q = 64$