Parabola

SOLUTIONS

EXERCISE - O

1. **Ans. (B)**

$\Delta = -2; h^2 = ab \Rightarrow$ parabola

2. **Ans. (C)**

$SP = ePM$

$\Rightarrow e = 1$, possible when $k = \frac{1}{(\ell^2+m^2)}$

3. **Ans. (D)**

Eliminating t, we have

$x = \left(\frac{y+1}{2}\right)^2 + 3$

$\Rightarrow (y+1)^2 = 4(x-3)$

Vertex : $(3,-1) \Rightarrow$ focus : $(4,-1)$

4. **Ans. (C)**

$y^2 = 6\left(x - \frac{3}{2}\right)$

Equation of directrix $x - \frac{3}{2} = \frac{3}{2}$ i.e. $x = 0$

Let coordinate P be $\left(\frac{3}{2} + \frac{3}{2}t^2, 3t\right)$

$\therefore$ Coordinate of M are $(0, 3t)$

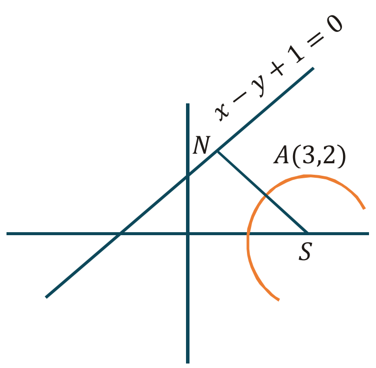
$\therefore MS = \sqrt{9 + 9t^2}$

$MP = \frac{3}{2} + \frac{3}{2}t^2$

$\therefore 9 + 9t^2 = \left(\frac{3}{2} + \frac{3}{2}t^2\right)^2 \Rightarrow 1 + t^2 = \frac{(1+t^2)^2}{4}$

$\therefore 4 = 1 + t^2 \Rightarrow$ MP = 6

5. **Ans. (A)**



N is foot of perpendicular from vertex on directrix.

$N(2,3)$

A is mid-point of NS

$S \rightarrow (4,1)$

6. **Ans. (D)**

a = distance between two parallel lines

$\therefore 4a = 8\sqrt{2}$

7. **Ans. (C)**

$$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & -2 & 1 \end{vmatrix} = \left|\frac{1}{2}(-4)\right| = 2$$

8. **Ans. (A)**

Vertex (1, 0), focus (1, 2)

Equation is $(x-1)^2 + (y-2)^2 = 4$

9. **Ans. (A)**

The two parabolas are

$y^2 = x, y^2 = -x$

if $(a, 2a)$ lies out side the parabola

$\Rightarrow 4a^2 - a > 0$

$4a^2 + a > 0$

$\Rightarrow a \in \left(-\infty, -\frac{1}{4}\right) \cup \left(\frac{1}{4}, \infty\right)$

10. **Ans. (A)**

Given line is focal chord for parabola and its slope is $\tan\alpha = \frac{-1}{\sqrt{3}}$

$\therefore$ Length = $4a\mathrm{cosec}^2\alpha = 4.\frac{1}{4}.(2)^2 = 4.$

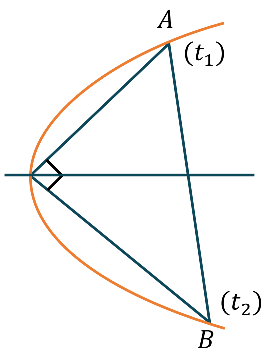
11. **Ans. (B)**

Focus will be (0,0) & directrix $6x + 8y = 5$

perpendicular distance of focus from directrix = $2a$

$\Rightarrow 2a = \left|\frac{0+0-5}{10}\right| = \frac{1}{2}$ $\quad\quad \because \ell(LR) = 4a = 1$

12. **Ans. (A)**



$t_1 t_2 = -4$

$A(3,6)$

$t_1 = 1$

$\Rightarrow t_2 = -4$

So $B \equiv (48, -24)$

$OB = 24\sqrt{5}$

13. **Ans. (A)**

The given equation is $\left(y-\frac{1}{2}\right)^2 = 4\left(x-\frac{3}{2}\right)$, whose focus is $\left(\frac{5}{2},\frac{1}{2}\right)$

The equation of circle is $\left(x-\frac{5}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=4$

and the axis $y=\frac{1}{2}$ cuts it at $\left(\frac{9}{2},\frac{1}{2}\right)$ and $\left(\frac{1}{2},\frac{1}{2}\right)$

length $=\sqrt{\left(\frac{9}{2}-\frac{1}{2}\right)^2+\left(\frac{1}{2}-\frac{1}{2}\right)^2}=\sqrt{4^2}=4$

14. **Ans. (A)**

Focus of the parabola $y^2=4x$ is $(1,0)$

So diagonals are focal chord

we known that $\frac{1}{SA}+\frac{1}{SC}=1$

$SA=\sqrt{(t^2-1)^2+4t^2}=1+t^2=a$ (say)

$\because \frac{1}{a}+\frac{1}{\frac{25}{4}-a}=1$

$\frac{25}{4}=\frac{25}{4}a-a^2$

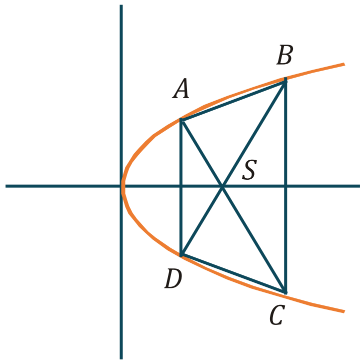
$4a^2-25a+25=0 \Rightarrow a=\frac{5}{4},5$

for $a=\frac{5}{4}, 1+t^2=\frac{5}{4} \Rightarrow t^2=\frac{1}{4} \Rightarrow t=\pm\frac{1}{2}$

$A\equiv\left(\frac{1}{4},1\right), B\equiv(4,4), C\equiv(4,-4)$ and $D\equiv\left(\frac{1}{4},-1\right)$

$AD=2$ and $BC=8$, distance between AD and $BC=\frac{15}{4}$

$\therefore$ Area of trapezium $ABCD=\frac{1}{2}(2+8)\times\frac{15}{4}=\frac{75}{4}$ sq. unit.



15. **Ans. (C)**

Using tangency condition $C=\frac{A}{M}$

We get $a=3$

16. **Ans. (D)**

End points of latus rectum $(8,\pm16)$

$T=0$

$\Rightarrow (\pm16)y=32\left(\frac{x+8}{2}\right) \Rightarrow \pm y=x+8$

$\Rightarrow x+y+8=0 \;\&\; x-y+8=0$

17. **Ans. (A)**

Let $P(h,k)$

Equation of tangent : $y=mx+\frac{4}{m}$

$\Rightarrow k=mh+\frac{4}{m} \Rightarrow m^2h-km+4=0 \begin{cases} m_1 \\ 4m_1 \end{cases}$

$5m_1=\frac{k}{h}; 4m_1^2=\frac{4}{h} \Rightarrow 4\left(\frac{k}{5h}\right)^2=\frac{4}{h}$

$\Rightarrow k^2=25h \Rightarrow y^2=25x \Rightarrow L_{LR}=25$

18. **Ans. (B)**

Any normal to $y^2 = 4ax$ at $(am^2, -2am)$

$y = mx - 2am - am^3$ if it passes through (h, k)

then $k = mh - 2am - am^3$

$\Rightarrow am^3 + m(2a - h) + k = 0 \Rightarrow \Sigma m_i = 0$

$y = \dfrac{-2a\Sigma m}{3} = 0$

19. **Ans. (D)**

Normal chord PQ subtends 90° at the vertex

$(p)\left(-p - \dfrac{2}{p}\right) = -4 \Rightarrow p^2 + 2 = 4 \Rightarrow p = \pm\sqrt{2}$

20. **Ans. (D)**

General point on $y^2 = 4x$ is $(t^2, 2t)$. Thus A corresponds to $t = 1$

$\Rightarrow$ B corresponds to $-t - \dfrac{2}{t} = -3$ and C corresponds to $3 + \dfrac{2}{3} = \dfrac{11}{3}$

Thus $C \equiv \left(\dfrac{121}{9}, \dfrac{22}{3}\right)$.

21. **Ans. (C)**

If normal is drawn at the point t, then its other end corresponds to the point $-t - \dfrac{2}{t}$, and slope of normal at $-t - \dfrac{2}{t}$ is $t + \dfrac{2}{t}$. If $t + \dfrac{2}{t}$ is negative then inclination is obtuse, and if $t + \dfrac{2}{t}$ is positive then $t + \dfrac{2}{t} \geq 2\sqrt{2}$ and hence inclination is more than 60° in each case.

22. **Ans. (C)**

The given line will be a normal, if

$\sin^3\theta = -2\sin\theta - \sin^3\theta \Rightarrow \sin\theta(\sin^2\theta + 1) = 0$

$\Rightarrow \sin\theta = 0 \Rightarrow$ given normal is $y = 0$

Thus no value of θ will make the given line a normal to the parabola $y^2 = 4x$, other than the x –axis.

23. **Ans. (C)**

Let $h = 2t_1^2$ & $12 = 4t_2$

We know $t_2 = -t_1 - \dfrac{2}{t_1}$

$\Rightarrow t_1 = -1$ or -2

$\Rightarrow h = 2$ or 8

24. **Ans. (A)**

Minimum distance will be along the common normal of parabola

$y^2 = 8x$ is $y = mx - 4m - 2m^3$ should passes through centre of circle

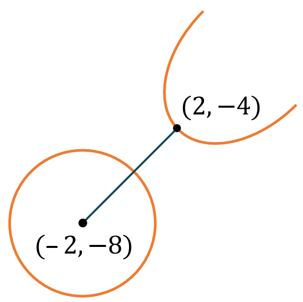
i.e. $(-2, -8)$

$\Rightarrow m = 1$

$\therefore$ foot of normal is $(am^2, -2am) = (2, -4)$

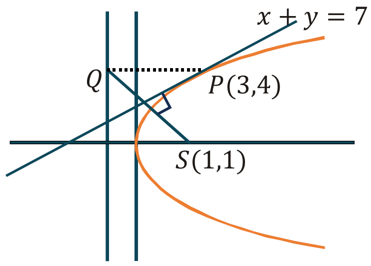
$\therefore$ Minimum distance between curves

$= 4\sqrt{2} - \sqrt{2} = 3\sqrt{2}$



25. **Ans. (A,B)**

Q lies on directrix, which is image of $(1,1)$ with respect to line $x+y=7 \Rightarrow Q \equiv (6,6)$



$PQ||$ axis

$m_{PQ} = \frac{2}{3} \Rightarrow$ axis $2x - 3y + 1 = 0$

Slope of the directrix is $-\frac{3}{2}$, which passes through $(6,6)$

$\Rightarrow$ Directrix is $3x + 2y = 30$

$LR = 2.\frac{25}{\sqrt{13}}$

26. **Ans. (A,B,D)**

normal, $y - 1 = m(x-1) - 2m - m^3$ put $x = 10, y = 7$

$m^3 – 7m + 6 = 0 \Rightarrow m = 1,2,-3$

Foot of normal is $(1+m^2, 1–2m)$

$\Rightarrow A(2,-1), B(5,-3)\ C(10,7)$

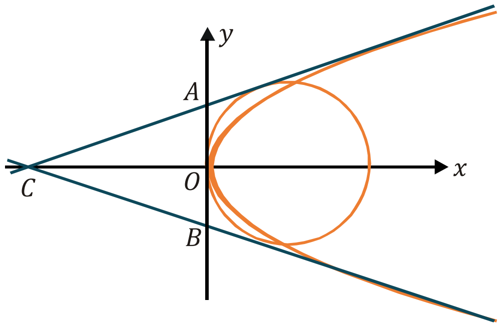
$\Delta = 20, G\left(\frac{17}{3}, 1\right)$

27. **Ans. (A,B,C,D)**

$y = mx + \frac{1}{m}$

$m^2x – my + 1 = 0$

$\frac{|3m^2+1|}{\sqrt{m^4+m^2}} = 3$



$\Rightarrow 9m^4 + 6m^2 + 1 = 9m^4 + 9m^2 \Rightarrow m = \infty, \pm\frac{1}{\sqrt{3}}$

$\Rightarrow$ Tangents are $x = 0, y = \frac{1}{\sqrt{3}}x + \sqrt{3}; y = -\frac{1}{\sqrt{3}}x - \sqrt{3}$

Points of intersection are $A(0,\sqrt{3}), B(0,-\sqrt{3}), C(-3,0)$

28. **Ans. (A,B,C,D)**

General equation of normal is

$y = -tx + 2t + t^3$, put $y = 0$

$\Rightarrow \beta = 2 + t^2$

(A) $t^2 + 2 = 3 \Rightarrow t = \pm 1$

$\therefore$ Area of quadrilateral $PTQR = 8$

(B) If $PT = 4\sqrt{5} P(t^2, 2t), T = (-t^2, 0)$

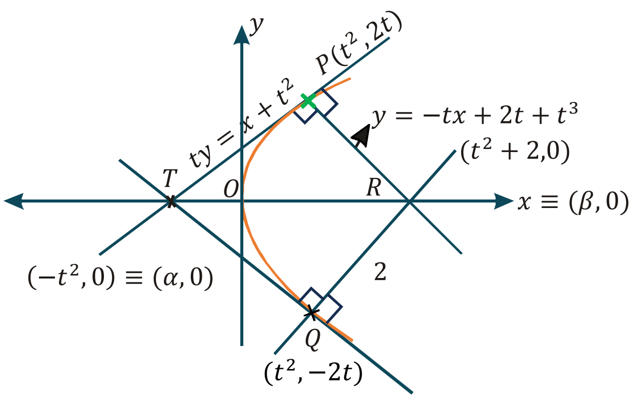
$\Rightarrow t = \pm 2$. So, $\beta = 6$

(C) $\beta > 2$

(D) $\beta = 4 \Rightarrow t^2 = 2 \Rightarrow T(-2,0), R(4,0)$

$\therefore TR$ is diameter, $TR = 6$

So, area of circle $= \frac{\pi}{4}(TR)^2 = 9\pi$



29. **Ans. (B)**

Right angle at origin $t_1 t_2 =-4$

equation of chord: $\left(t-\frac{4}{t}\right) y=2x-8$

$2(x-4)+\lambda y=0$

fixed point $(4,0)$

Normal passing through $(4,0) \Rightarrow 0=4m-2m-m^3$

$m^3-2m=0$

$m=0, \pm\sqrt{2}$

$A(0,0), B(2,2\sqrt{2}), C(2,-2\sqrt{2})$

Area of $\Delta ABC = 4\sqrt{2}$

30. **Ans. (A)**

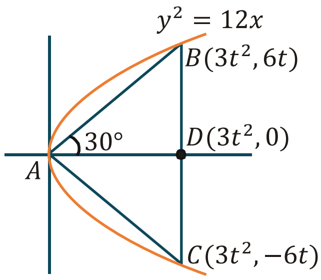
Length of normal chord PQ, which is normal at $P(t_1)$

$m=\sqrt{2} : t_1=-\sqrt{2}, P(2,-2\sqrt{2})$

$t_2=\sqrt{2}+\frac{2}{\sqrt{2}}=2\sqrt{2} : Q(8,4\sqrt{2})$

Length = $6\sqrt{3}$

31. **Ans. (B)**



$BC=12t$

$\tan 30° = \frac{1}{\sqrt{3}}=\frac{2}{t}$

$t=2\sqrt{3}$

$BC=24\sqrt{3}$

32. **Ans. (A)**

$x+y=1$

comparing with $yy_1 = 6(x + x_1)$

we get $x_1=-1, y_1=-6$

33. **Ans. (A)**

Figure is a square of side $2\sqrt{2}a$.

$\therefore$ Area = $(2\sqrt{2}a)^2=8$ ($\because$ $a=1$)

34. **Ans. (B)**

Centre is focus of parabola = $(1,0)$

EXERCISE - S

1. **Ans. (1)**

Equation of the normal is $y = mx - 2m - m^3$

If it passes through (21, 30) we have

$30 = 21m - 2m - m^3 \Rightarrow m^3 - 19m + 30 = 0$

Then $m = -5, 2, 3$

But if $m = 2$ or 3 then the point where the normal meets the curve will be $(am^2, -2am)$ where the curve does not exist.

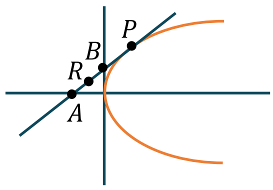
Therefore $m = -5$

$\therefore m + 6 = 1$

2. **Ans. (2)**

Let any tangent to parabola $y^2 = 4x$ at $P(t^2, 2t)$ is $yt = x + t^2$

and $A(-t^2, 0)$, $B(0, t)$



Let R is (h, k)

$2h = -t^2$ and $2k = t$

$\Rightarrow \frac{2h}{(2k)^2} = -1 \Rightarrow k^2 = -\frac{1}{2}h$

Locus is $y^2 = -\frac{x}{2}$

3. **Ans. (4)**

Equation of tangent of parabola $y^2 = 4x$ is

$y = mx + \frac{1}{m}$ ...(i)

Equation (i) is also a tangent of $x^2 = -32y$

Then $x^2 = -32\left(mx + \frac{1}{m}\right) \Rightarrow x^2 + 32mx + \frac{32}{m} = 0$

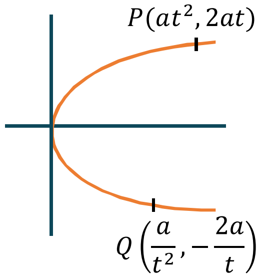
Condition of tangency ($D = 0$)

$m^3 = \frac{1}{8} \Rightarrow m = \frac{1}{2}$

From Equation (i), $x - 2y + 4 = 0$.

4. **Ans. (4)**

$PQ = \left(t + \frac{1}{t}\right)^2$



Equation of line PQ is

$2x - y\left(t - \frac{1}{t}\right) - 2a = 0$

$p = \frac{2a}{\left(t + \frac{1}{t}\right)} \Rightarrow t + \frac{1}{t} = \frac{2}{p}$

$PQ = \frac{4}{p^2} \Rightarrow \ell p^2 = 4$

5. Ans. (7)

$P(1,\ 2) \equiv (t_1^2, 2t_1) \Rightarrow t_1 = 1$

Let $Q(t_2^2, 2t_2)$ & $R(t_3^2, 2t_3)$

we know $t_2 = -t_1 - \frac{2}{t_1} \Rightarrow t_2 = -3 \Rightarrow Q(9,-6)$

Also $m_{PQ}.m_{QR} = -1 \Rightarrow t_3 = 5 \Rightarrow R(25,10)$

Area of $\Delta PQR = \frac{1}{2}\ PQ.QR\ = \frac{1}{2} 8\sqrt{2}.16\sqrt{2} = 2^7$

6. Ans. (8)

Given parabola is $x^2 + 4y - 6x + k = 0$

$\Rightarrow (x-3)^2 = -4\left(y + \frac{k-9}{4}\right)$

$\therefore$ equation of directrix is

$y + \frac{k-9}{4} = 1 \Rightarrow y = 1 - \left(\frac{k-9}{4}\right) = -1$

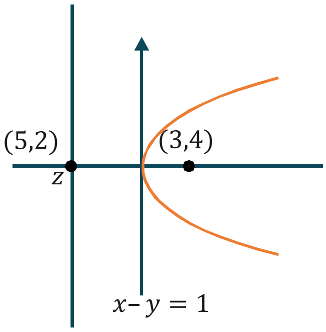
$\Rightarrow k = 17$

$\therefore$ Parabola (1) becomes

$(x-3)^2 = -4(y+2)$

$\therefore$ Vertex = (3, –2) and focus = (3, –3)

7. Ans. (4)



Reflection of $S(3,4)$ in tangent

is foot of directrix (5,2)

$\therefore$ Directrix is $x - y = 3$.

8. Ans. (6)

Equation of PQ $\qquad y = \frac{x}{\sqrt{3}}$

solve with parabola

$x^2 - 36x - 108 = 0$

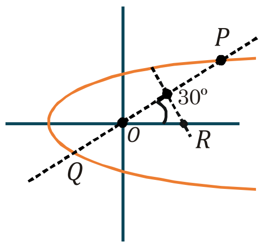
Mid-point of PQ $\left(18, 6\sqrt{3}\right)$

equation of perpendicular bisector of PQ

$y - 6\sqrt{3} = -\sqrt{3}(x - 18)$

$\Rightarrow R(24, 0)$

$S = (0,0) \Rightarrow RS = 24$



9. Ans. (3)

$A(-a, a(t_1 + t_2)), B(at_1^2, 2at_1), C(at_2^2, 2at_2)$

$h = \frac{a(t_1^2+t_2^2)-1}{3}$ & $k = a(t_1 + t_2)$

$\Rightarrow 3h = a((t_1 + t_2)^2 + 2) - a$

$\Rightarrow 3h = a\left(\frac{k^2}{a^2} + 1\right) \Rightarrow 3\text{h} = \frac{k^2}{a} + a$

$\Rightarrow k^2 = 3a(h - a/3) \Rightarrow \lambda = 3\text{a}$

10. Ans. (9)

Let equation of tangent is $y = mx + \frac{16}{m}$

It passes through (h, k)

then $k = mh + \frac{16}{m}$

$\Rightarrow m^2h - mk + 16 = 0$

It will have roots $m_1, 8m_1$

$m_1 + 8m_1 = \frac{k}{h}$ & $m_1.8m_1 = \frac{16}{h}$, eliminating m_1, we get

$\Rightarrow \frac{k^2}{18h} = 9$

11. Ans. (3)

Given parabola is $(x - 1)^2 =- (y - 3)$

$\therefore$ latus rectum= 1 $\Rightarrow$ SP, $\frac{1}{2}$, SQ are in H.P.

$\frac{1}{2} = \frac{2.SP.SQ}{SP + SQ} \Rightarrow SQ = \frac{1}{3} \Rightarrow (QS)^{-1} = 3$

12. Ans. (1)

$\Rightarrow y = m(x - 1) + \frac{a}{m} \Rightarrow -4 = -4m + \frac{1}{m}$

$4m^2 - 4m - 1 = 0 \Rightarrow m_1 + m_2 = 1$

13. Ans. (4)

$S(1,0)$; $Q(\alpha, 0)$

$\therefore$ Circle on SQ as diameter is

$(x - \alpha)(x - 1) + y^2 = 0$ ...(1)

As (1) touches $y^2 = 4x$, so equation $(x - \alpha)(x - 1) + 4x = 0$ has equal roots.

$\Rightarrow \alpha = 9$.

So, centre of circle is $(5, 0)$ and radius equals 4.

14. Ans. (6)

Equation of normal to the parabola $y^2 = 12x$ with slope -1 is $m = -1, a = 3$

$\Rightarrow y = -x - 2(3)(-1) + 3(-1)^3$

$x + y = 9$ i.e. $k = 9$

focus of parabola is $(3, 0)$

$p = \left|\frac{3 - 9}{\sqrt{2}}\right| \Rightarrow p^2 = 18 \Rightarrow p = 3\sqrt{2}$

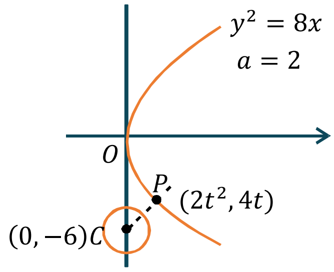
EXERCISE - JEE (Main) PYQ

1. Ans. (2)

Circle and parabola are as shown:

Minimum distance occurs along common normal.

Let normal to parabola be



$y + tx = 2.2.t + 2t^3$

pass through $(0, -6)$:

$-6 = 4t + 2t^3 \Rightarrow t^3 + 2t + 3 = 0$

$\Rightarrow t = -1$(only real value)

$\therefore P(2, -4)$

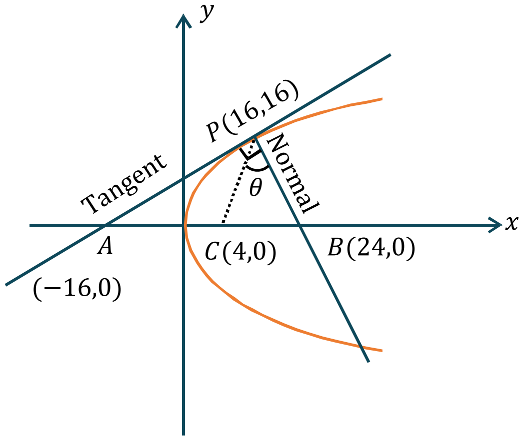
$\therefore CP = \sqrt{4+4} = 2\sqrt{2}$

$\therefore$ equation of circle

$(x-2)^2 + (y+4)^2 = \left(2\sqrt{2}\right)^2$

$\Rightarrow x^2 + y^2 - 4x + 8y + 12 = 0$

2. Ans. (1)



Equation of tangent at $P(16, 16)$

is $x - 2y + 6 = 0$

$m_{PC} = \frac{4}{3}$

$m_{PB} = -2$

Hence, $\tan\theta = \left|\frac{m_{PC} - m_{PB}}{1 + m_{PC} \cdot m_{PB}}\right|$

$\tan\theta = 2$

3. Ans. (3)

Given $y^2 = 4x$ ...(i)

and $x^2 + y^2 = 5$ ...(ii)

by (i) and (ii)

$\Rightarrow x = 1$ and $y = 2$

Equation of tangent at $(1, 2)$ to $y^2 = 4x$ is $y = x + 1$

4. **Ans. (3)**

$T : y(\beta) = \frac{1}{2}(x + \beta^2)$

$2y\beta = x + \beta^2$

$y = \left(\frac{1}{2\beta}\right)x + \frac{\beta}{2}$

$m = \frac{1}{2\beta}; C = \frac{\beta}{2}$

$\frac{\beta}{2} = \pm\sqrt{\frac{1}{4\beta^2} + \frac{1}{2}}$

$\frac{\beta^2}{4} = \frac{1}{4\beta^2} + \frac{1}{2}$

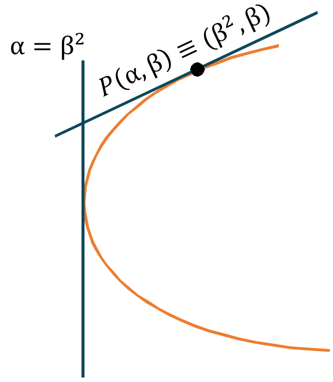
$\frac{\beta^2}{4} = \frac{1 + 2\beta^2}{4\beta^2}$

$\Rightarrow \beta^4 - 2\beta^2 - 1 = 0$

$(\beta^2 - 1)^2 = 2$

$\beta^2 - 1 = \sqrt{2}$

$\beta^2 = \sqrt{2} + 1$



5. **Ans. (2)**

Equation of circle is

$(x - 1)^2 + (y - 2)^2 + \lambda(x - y + 1) = 0$

$\Rightarrow x^2 + y^2 + x(\lambda - 2) + y(-4 - \lambda) + (5 + \lambda) = 0$

As circle touches x axis then $g^2 - c = 0$

$\frac{(\lambda - 2)^2}{4} = (5 + \lambda)$

$\lambda^2 + 4 - 4\lambda = 20 + 4\lambda$

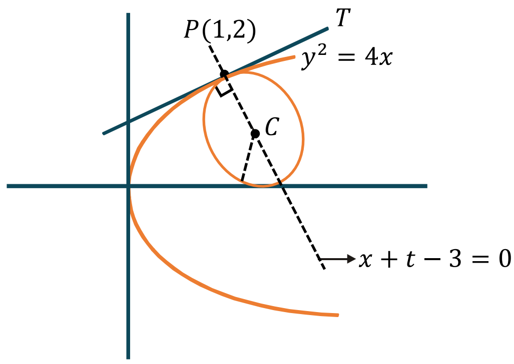
$\lambda^2 - 8\lambda - 16 = 0$

$\lambda = \frac{8 \pm \sqrt{128}}{2}$

$\lambda = 4 \pm 4\sqrt{2}$

Radius $= \left|\frac{(-4-\lambda)}{2}\right|$

Put λ and get least radius.



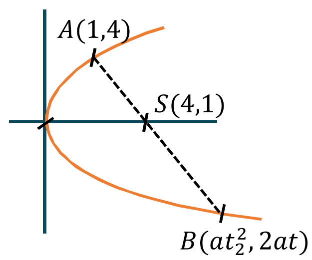
6. **Ans. (1)**

$y^2 = 4ax = 16x \Rightarrow a = 4$

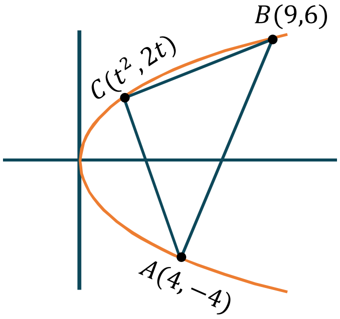
$A(1, 4) \Rightarrow 2.4.t_1 = 4 \Rightarrow t_1 = \frac{1}{2}$

$\therefore$ length of focal chord $= a\left(t + \frac{1}{t}\right)^2$

$= 4\left(\frac{1}{2} + 2\right)^2 = 4.\frac{25}{4} = 25$



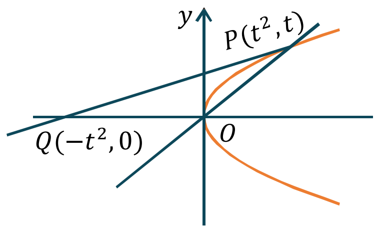
7. **Ans. (4)**



Area $= 5|t^2 - t - 6| = 5\left|\left(t - \frac{1}{2}\right)^2 - \frac{25}{4}\right|$

is maximum if $t = \frac{1}{2}$

8. **Ans. (0.50)**



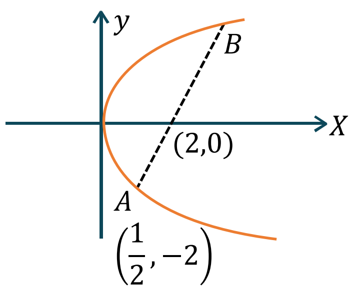
$\Delta OPQ = 4$

$$\frac{1}{2}\left\|\begin{matrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{matrix}\right\| = 4$$

t = 2 (∵ t > 0)

$\therefore\ m = \frac{1}{2}$ or 0.50

9. **Ans. (2)**



$y^2 = 8x$

$4t_1 = -2 \Rightarrow t_1 = -\frac{1}{2},$

$t_1 . t_2 = -1$

$t_2 = -\frac{1}{t_1}$

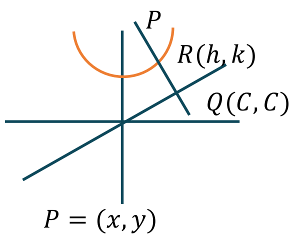
$\Rightarrow t_2 = 2$

So coordinate of B is (8, 8)

$\therefore$ Equation of tangent at B is

$8y = 4(x + 8) \Rightarrow 2y = x + 8$

10. **Ans. (2)**



$\frac{K - C}{h - C} = -1$

$C = \frac{h + K}{2}\quad P(x, y)$

$R = \left(\frac{x + C}{2}, \frac{y + C}{2}\right)$

$R = \left(\frac{x}{2} + \frac{h}{4} + \frac{K}{4}, \frac{y}{2} + \frac{h}{4} + \frac{k}{4}\right)$

$h = \frac{x}{2} + \frac{h}{4} + \frac{K}{4}$

$K = \frac{y}{2} + \frac{h}{4} + \frac{K}{4}$

$\Rightarrow x = \frac{3h}{2} - \frac{K}{2}, y = \frac{3K}{2} - \frac{h}{2}$

$Y = 4x^2 + 1$

$\left(\frac{3k-h}{2}\right) = 4\left(\frac{3h-k}{2}\right)^2 + 1$

11. **Ans. (9)**

$P \equiv \left(\frac{3}{2}t^2, 3t\right)$

Normal at point P

$tx + y = 3t + \frac{3}{2}t^3$

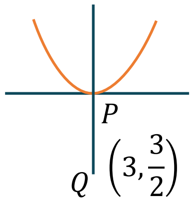
Passes through $\left(3, \frac{3}{2}\right)$

$\Rightarrow 3t + \frac{3}{2} = 3t + \frac{3}{2}t^3$

$P \equiv \left(\frac{3}{2}, 3\right) = (\alpha, \beta)$

$\Rightarrow t^3 = 1 \Rightarrow t = 1$

$2(\alpha + \beta) = 2\left(\frac{3}{2} + 3\right) = 9$



12. **Ans. (1)**

Tangent at $P : y(2) = 2(1/2)(x + 2)$

$\Rightarrow 2y = x + 2$

$\therefore\ Q = (-2, 0)$

Normal at $P : y - 2 = -\frac{(2)}{2.\frac{1}{2}}(x - 2)$

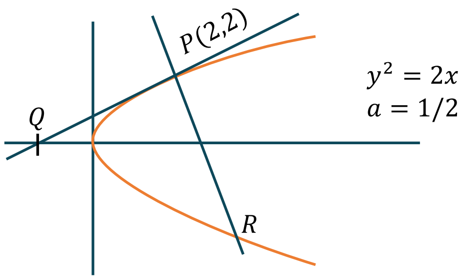
$\Rightarrow y - 2 = -2(x - 2)$

$\Rightarrow y = 6 - 2x$

$\therefore$ Solving with $y^2 = 2x \Rightarrow R\left(\frac{9}{2} - 3\right)$

$\therefore \text{Ar}(\Delta PQR) = \frac{1}{2}\begin{vmatrix} 2 & 2 & 1 \\ -2 & 1 & 1 \\ \frac{9}{2} & 3- & 1 \end{vmatrix}$

$= \frac{25}{2}$ sq. units



13. **Ans. (2)**

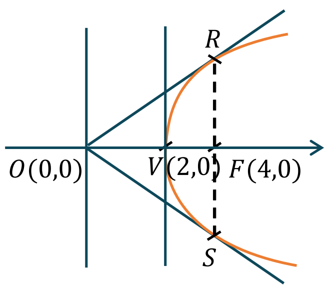
Clearly RS is latus-rectum

$\because VF = 2 = a$

$\therefore RS = 4a = 8$

Now $OF = 2a = 4$

$\Rightarrow$ Area of triangle $ORS = 16$



14. **Ans. (2)**

$\left(y-\frac{3}{4}\right)=\left(x-\frac{1}{2}\right)^2$ ...(1)

For $x=-\frac{1}{2}$

$y-\frac{3}{4}=1\Rightarrow y=\frac{7}{4}\Rightarrow P\left(-\frac{1}{2},\frac{7}{4}\right)$

Now $y'=2\left(x-\frac{1}{2}\right)$ At $x=-\frac{1}{2}$

$\Rightarrow m_T=-2, m_N=\frac{1}{2}$

Equation of Normal is

$y-\frac{7}{4}=\frac{1}{2}\left(x+\frac{1}{2}\right)$

$y=\frac{x}{2}+2$

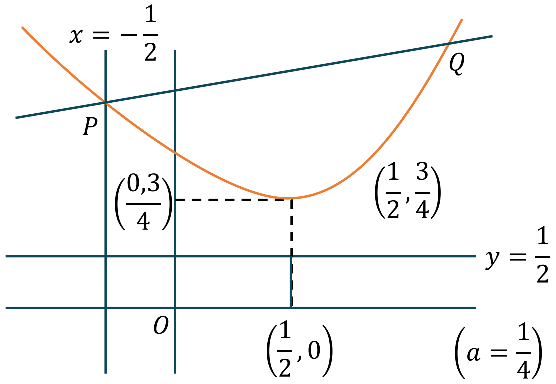
Now put y in equation (1)

$\frac{x}{2}+2-\frac{3}{4}=\left(x-\frac{1}{2}\right)^2$

$\Rightarrow x=2\ \&\ -\frac{1}{2}$

$\Rightarrow Q(2,3)$

Now $(PQ)^2=\frac{125}{16}$



15. **Ans. (4)**

Equation of tangent to $y^2=30x$

$y=mx+\frac{30}{4m}$

Pass thru $(-30,0)$: $a=-30m+\frac{30}{4m}\Rightarrow m^2=1/4$

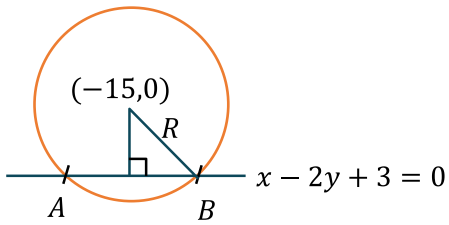
$\Rightarrow m=\frac{1}{2}$ or $m=-\frac{1}{2}$

At $m=\frac{1}{2}$: $y=\frac{x}{2}+15\Rightarrow x-2y+30=0$

$P=\frac{15}{\sqrt{5}}$

$\ell_{AB}=2\sqrt{R^2-P^2}=2\sqrt{\frac{225}{4}-\frac{225}{5}}$

$\Rightarrow \ell_{AB}=30.\sqrt{\frac{1}{20}}=\frac{15}{\sqrt{5}}=3\sqrt{5}$



16. **Ans. (2)**

Locus is directrix of parabola

$x-3+4=0\Rightarrow x+1=0.$

17. **Ans. (1)**

Equation of tangent at $(2,-4)(T = 0)$

$-4y = 4(x+2)$

$x + y + 2 = 0$ ...(1)

equation of normal

$x - y + \lambda = 0$

$\downarrow (2,-4)$

$\lambda = -6$

thus $x - y = 6$ ...(2)

equation of normal

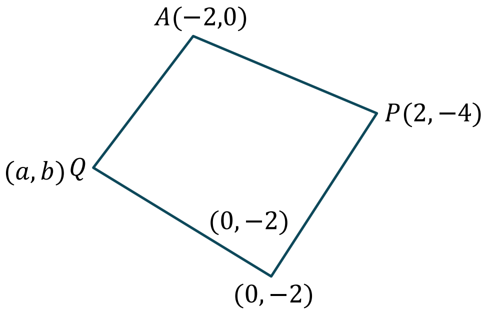
POI of (1) & $x = -2$ is $A(-2,0)$

POI of (2) & $x = -2$ is $B(-2,-8)$

Given $AQBP$ is a sq.

$\Rightarrow m_{AQ}.m_{AP} = -1$

$\Rightarrow \left(\frac{b}{a+2}\right)\left(\frac{4}{-4}\right) = -1 \Rightarrow a + 2 = b$ ...(3)



Also PQ must be parallel to x-axis thus

$\Rightarrow b = -4$

$\therefore a = -6$

Thus $2a + b = -16$

18. **Ans. (2)**

Tangent of $y^2 = 8x$ is $y = mx + \frac{2}{m}$

$P(2,-4) \Rightarrow -4 = 2m + \frac{2}{m}$

$\Rightarrow m + \frac{1}{m} = -2 \Rightarrow m = -1$

$\therefore$ tangent is $y = -x - 2$

$\Rightarrow x + y + 2 = 0$ ...(1)

(1) is also tangent to $x^2 + y^2 = a$

So $\frac{2}{\sqrt{2}} = \sqrt{a} \Rightarrow \sqrt{a} = \sqrt{2}$

$\Rightarrow a = 2$

19. **Ans. (3)**

$V \rightarrow$ Vertex

$F \rightarrow$ focus

$VF = S - R$

So latus rectum = $4(S - R)$

20. **Ans. (4)**

Both point $A(1, 3), B(1, -1)$ lies on the parabola $y^2 - 2y - 2x - 1 = 0$

Equation of tangent aty $A(1, 3)$ is $T = 0$

$x - 2y + 5 = 0$

and equation of tangent at $B(1, -1)$ is $T = 0$

$x + 2y + 1 = 0$

So point P is (–3, 1)

$$\Rightarrow A=\frac{1}{2}\begin{vmatrix}1 & 3 & 1\\ 1 & -1 & 1\\ -3 & 1 & 1\end{vmatrix}=8$$

21. Ans. (2)

Equation of tangent of $y = x^2$ be

$tx = y + at^2$ ...(1)

$y = tx - \frac{t^2}{4}$

Solve with $y = -(x-2)^2$

$tx - \frac{t^2}{4} = -(x-2)^2$

$x^2 + x(t-4) - \frac{t^2}{4} + 4 = 0$

$D = 0$

$(t-4)^2 - 4\cdot\left(4-\frac{t^2}{4}\right) = 0$

$t^2 - 4t = 0$

$t = 0$ or $t = 4$

From eq. (1), required common tangent is

$y = 4(x-1)$

22. Ans. (10)

P_1 : Directorix :

$x + 2y = k$

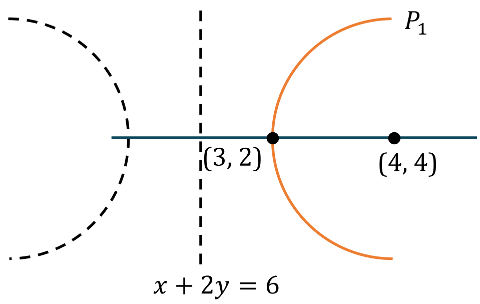
$x + 2y - k = 0$

$\left|\frac{3+4-K}{\sqrt{5}}\right|=\sqrt{5}$

$|7-k|=5$

$7 - K = 5$	$7 - K = -5$
$\boxed{k = 2}$	$\boxed{k = 12}$
Accepted	Rejected

Passes through focus



$\left.\begin{array}{l} D_1 = x+2y=2 \\ \ell = x+2y=6 \end{array}\right] \Rightarrow d$

$\left.\begin{array}{l} \ell = x+2y=6 \\ D_2 = x+2y=C \end{array}\right] \Rightarrow d$

$\Rightarrow \boxed{c = 10}$

23. Ans. (1)

$x^2+y^2+Ax+By+C=0$ is passing through (0,6)

$\Rightarrow 6B + C = -36$

The tangent of the parabola $y = x^2$ at (2, 4) is

$4x - y - 4 = 0$ ...(1)

The tangent of circle $x^2+y^2+Ax+By+C=0$ at (2, 4) is

$(4 + A)\,x + (8 + B)y + 2A + 4B + 2C = 0$ ...(2)

From Equation (1) and (2)

$$\frac{4+A}{4}=\frac{8+B}{-1}=\frac{2A+4B+2C}{-4}$$

$A + 4B = -36$ ...(3)

$3A + 4B + 2C = -4$ ...(4)

From equation (3) and (4)

$A + C = 16$

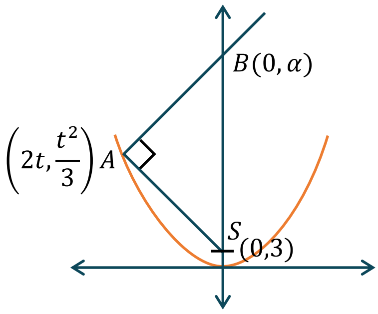
24. **Ans. (4)**

parabola $x^2 = 12y$

$SA \perp SB$

so, $m_{AS} \cdot m_{AB} = -1$

$$\frac{\left(3-\frac{t^2}{3}\right)}{(0-2t)}\cdot\frac{\left(\alpha-\frac{t^2}{3}\right)}{(0-2t)}=-1$$



by solving

$$3\alpha=\frac{27t^2+t^4}{t^2-9}$$

ordinate of centroid of $\Delta SAB = K = \dfrac{\alpha+\frac{t^2}{3}+3}{3}$

$$k=\frac{9+3\alpha+t^2}{9}$$

$$\lim_{t\to 1} k=\lim_{t\to 1}\frac{1}{9}\left(9+t^2+\frac{27t^2+t^4}{(t^2-9)}\right)=\frac{13}{18}$$

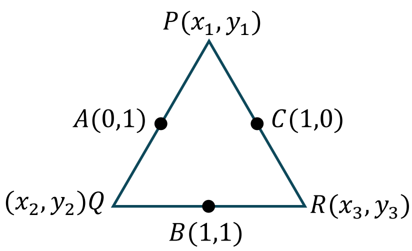
25. **Ans. (2)**

By mid point theorem, we get

$x_1 = 0, x_2 = 0, x_3 = 2\,;\ y_1 = 0, y_2 = 2, y_3 = 0$

Incentre of ΔPQR (PQ = 2, QR = $2\sqrt{2}$, PR = 2)

is $Ds\left(\dfrac{4}{4+2\sqrt{2}},\dfrac{4}{4+2\sqrt{2}}\right)$



parabola $y^2=4ax$ passes through D

we get a = $\dfrac{1}{4+2\sqrt{2}}=\dfrac{1}{2}-\dfrac{\sqrt{2}}{4}$ = $\left(\alpha+\beta\sqrt{2},0\right)$ (Given)

$\alpha=\dfrac{1}{2}$ and $\beta=-\dfrac{1}{4}$

26. Ans. (16)

Given parabola : $y^2 = 12x$

Let $P : (3t_1^2, 6t_1)$ & $Q : (3t_2^2, 6t_2)$

$\frac{t_1}{t_2} = 3 \quad \Rightarrow t_1 = 3t_2$

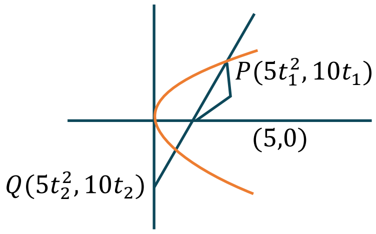
Point of intersection of tangent (α, β)

$\alpha = 3t_1 \cdot t_2 = 9t_2^2$

$\beta = 3(t_1 + t_2) = 12t_2$

Now, $\frac{\beta^2}{\alpha} = \frac{144t_2^2}{9t_2^2} = 16$

27. Ans. (1)



$10t_1 + 10t_2 = 30$

$\Rightarrow m = \frac{2}{t_1 + t_2} = \frac{2}{3}$

$C = m + 6 = \frac{20}{3}$

$PQ = \frac{4\sqrt{a^2 - amc}\sqrt{1+m^2}}{m^2} = \sqrt{325}$

28. Ans. (1)

$9\left(t + \frac{1}{t}\right)^2 = 100$

$t = 3 \Rightarrow P(81, 54)$ & $Q(1, -6)$

$M(21, 9) \Rightarrow L$ is $(y - 9) = \frac{-4}{3}(x - 21)$

$3y - 27 = -4x + 84$

$4x + 3y = 111$

29. Ans. (16)

$y^2 = 8x + 4y + 4$

$(y-2)^2 = 8(x+1)$

$y^2 = 4ax$

$a = 2, X = x + 1, Y = y - 2$

focus $(1, 2)$

$y - 2 = m\,(x - 1)$

Put $(3, 0)$ in the above line

$m = -1$

Length of focal chord = 16

30. **Ans. (3)**

$y^2 = 6x$ & $y^2 = 4ax$

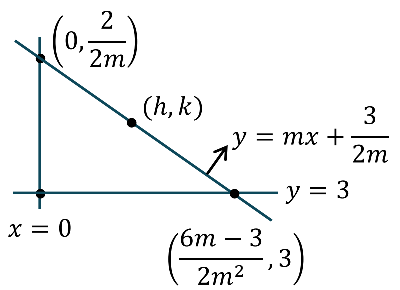
$\Rightarrow 4a = 6 \Rightarrow a = \frac{3}{2}$

$y = mx + \frac{3}{2m}$; $(m \neq 0)$

$h = \frac{6m-3}{4m^2}$, $k = \frac{6m+3}{4m}$, Now eliminating m and we get

$\Rightarrow 3h = 2(-2k^2 + 9k - 9)$

$\Rightarrow 4y^2 - 18y + 3x + 18 = 0$



EXERCISE - JEE (Advanced) PYQ

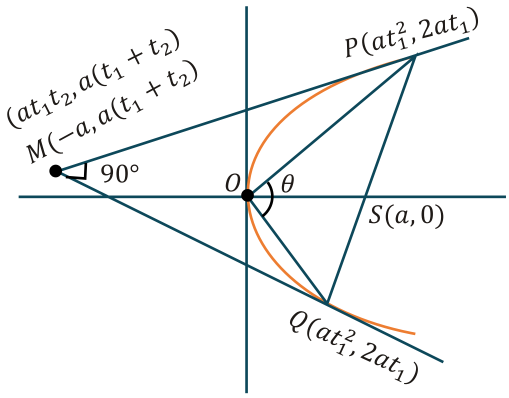
1. **Ans. (D)**

Single tangent at the extrimities of a focal chord will intersect on directrix.

$\therefore M(-a, a(t_1 + t_2))$

lies on $y = 2x + a$

$a(t_1 + t_2) = -2a + a \quad \Rightarrow t_1 + t_2 = -1$ & $t_1t_2 = -1$



$$\tan\theta = \left(\frac{\frac{2}{t_1} - \frac{2}{t_2}}{1 + \frac{4}{t_1t_2}}\right) = \left(\frac{2(t_2 - t_1)}{3}\right)$$

$\because (t_2 - t_1)^2 = (t_2 + t_1)^2 - 4t_1t_2 = 5$

$t_2 - t_1 = \pm\sqrt{5}$

$\therefore \tan\theta = \pm\frac{2\sqrt{5}}{3}$

but θ is obtuse because O is the interior point of the circle for which PQ is diameter.

$\therefore \tan\theta = \frac{-2\sqrt{5}}{3}$

2. **Ans. (B)**

Length of focal chord $PQ = a(t_1 - t_2)^2$

$= a[(t_1 + t^2)^2 - 4t_1t_2]$

$= a[1 + 4] = 5a$

3. **Ans. (D)**

$y = mx + \frac{2}{m}$

$$\frac{\left|0 - 0 + \frac{2}{m}\right|}{\sqrt{1 + m^2}} = \sqrt{2}$$

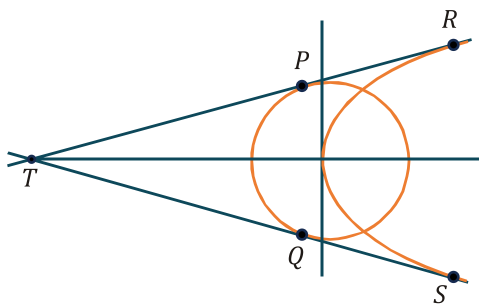
$\Rightarrow 2 = m^2(1 + m^2) \Rightarrow m = \pm 1$

$TP : -x + y = 2$

So $P(-1, 1)$ & $Q(-1, -1)$

& $R\left(\frac{2}{m}, \frac{4}{m}\right) \equiv R(2, 4)$ & $S(2, -4)$

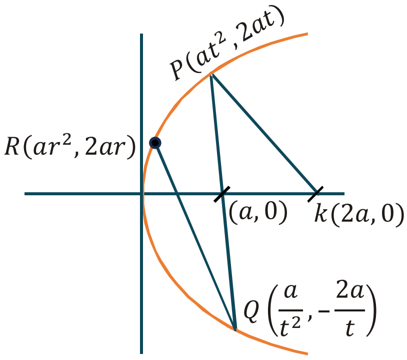
So $\Delta = \frac{1}{2} \times 10 \times 3 = 15$



4. **Ans. (D)**

$\because$ PQ is a focal chord

$\therefore$ co-ordinates of point Q are $=\left(\frac{a}{t^2},-\frac{2a}{t}\right)$



$$m_{QR}=\frac{2a\left(r+\frac{1}{t}\right)}{a\left(r^2-\frac{1}{t^2}\right)}=\frac{2}{\left(r-\frac{1}{t}\right)}$$

$$m_{PK}=\frac{2at-0}{a\left(t^2-2\right)}=\frac{2t}{t^2-2}$$

Given $m_{QR}=m_{PK}$

$$\Rightarrow \frac{2}{r-\frac{1}{t}}=\frac{2t}{t^2-2} \Rightarrow r=\frac{t^2-2}{t}+\frac{1}{t}$$

$$\Rightarrow r=t-\frac{2}{t}+\frac{1}{t}\Rightarrow r=\frac{t^2-1}{t}$$

5. **Ans. (B)**

Equation of tangent at point P is $ty = x + at^2$ ...(i)

Equation of normal at point S is $\frac{1}{t}x+y=\frac{2a}{t}+\frac{a}{t^3}$

$\Rightarrow$ $t^2x + t^3y = 2at^2 + a$ ...(ii)

Multiply equation (i) by t^2 and then add to equation (ii),

we get,

$2t^{3y} = 2at^2 + at^4 + a$

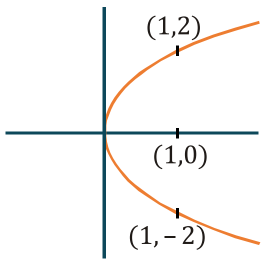
$\Rightarrow 2t^3y = a(1 + t^4 + 2t^2)$

$$\Rightarrow \boxed{y=\frac{a(1+t^2)^2}{2t^3}}$$

6. **Ans. (2)**

The co-ordinates of latus rectum are (1,2) and (1, −2)

clearly slope of tangent is given by $\frac{dy}{dx}=\frac{2}{y}$



$\therefore$ At $y = 2$ slope of normal = −1

and At $y = -2$ slope of normal = 1

$\therefore$ Equation of normal at (1,2)

$(y-2) = -1(x-1) \Rightarrow x + y = 3$

Now, this line is tangent to circle

$(x-3)^2 + (y+2)^2 = r^2$

$\therefore$ perpendicular distance from centre to line = Radius of circle

$$\therefore \frac{|3-2-3|}{\sqrt{2}}=r\Rightarrow r^2=2$$

$\therefore$ Ans. is 2.

7. **Ans. (4)**

Let there be a point $(t^2, 2t)$ on $y^2 = 4x$

Clearly its reflection in $x + y + 4 = 0$ is given by

$$\frac{x-t^2}{1} = \frac{y-2t}{1} = \frac{-2(t^2+2t+4)}{2}$$

$\therefore x = -(2t+4)$ & $y = -(t^2+4)$

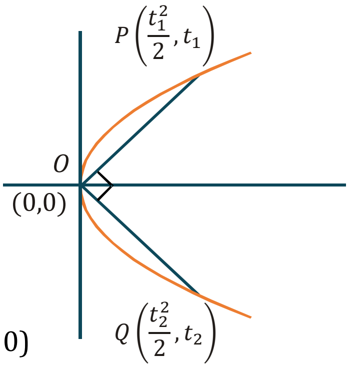
Now, $y = -5 \Rightarrow t = \pm 1$

$\therefore x = -6$ or $x = -2$

$\therefore$ Distance between A & $B = 4$

8. **Ans. (A, D)**

$\because \angle POQ = \frac{\pi}{2} \Rightarrow t_1 t_2 = -4$



$$\therefore \left| \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ \frac{t_1^2}{2} & t_1 & 1 \\ \frac{t_2^2}{2} & t_2 & 1 \end{vmatrix} \right| = 3\sqrt{2} \Rightarrow \left| \frac{t_1^2 t_2 - t_1 t_2^2}{2} \right| = 6\sqrt{2}$$

$$\Rightarrow |t_1 - t_2| = 3\sqrt{2} \Rightarrow t_1 + \frac{4}{t_1} = 3\sqrt{2} \qquad (\because t_1 > 0)$$

We get $t_1 = 2\sqrt{2}, \sqrt{2}$

$P(4, 2\sqrt{2})$ or $(1, \sqrt{2})$

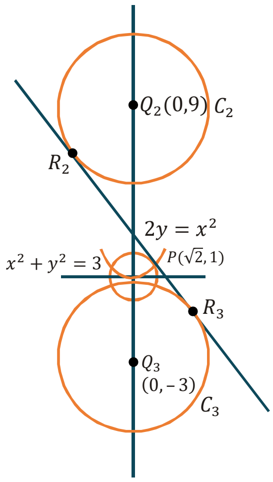
9. **Ans. (A, B, C)**

On solving $x^2 + y^2 = 3$ and $x^2 = 2y$ we get point $P(\sqrt{2}, 1)$

Equation of tangent at P

$\sqrt{2}.x + y = 3$

Let Q_2 be $(0, k)$ and radius is $2\sqrt{3}$



$$\therefore \left| \frac{\sqrt{2}(0) + k - 3}{\sqrt{2+1}} \right| = 2\sqrt{3}$$

$\therefore k = 9, -3$

$Q_2(0,9)$ and $Q_3(0,-3)$

hence $Q_2Q_3 = 12$

R_2R_3 is internal common tangent of circle C_2 and C_3

$\therefore R_2R_3 = \sqrt{(Q_2Q_3)^2 - \left(2\sqrt{3} + 2\sqrt{3}\right)^2}$

$= \sqrt{12^2 - 48} = \sqrt{96} = 4\sqrt{6}$

Perpendicular distance of origin O from R_2R_3 is equal to radius of circle $C_1 = \sqrt{3}$

Hence area of $\Delta OR_2R_3 = \frac{1}{2} \times (R_2R_3)\sqrt{3} = \frac{1}{2}.4\sqrt{6}.\sqrt{3} = 6\sqrt{2}$

Perpendicular Distance of P from $Q_2Q_3 = \sqrt{2}$

$\therefore$ Area of $\Delta PQ_2Q_3 = \frac{1}{2} \times 12 \times \sqrt{2} = 6\sqrt{2}$

10. **Ans. (A,C,D)**

$y^2 = 4x$

point P lies on normal to parabola passing through centre of circle

$y + tx = 2t + t^3$ ...(i)

$8 + 2t = 2t + t^3$

$t = 2$

$P(4,4)$

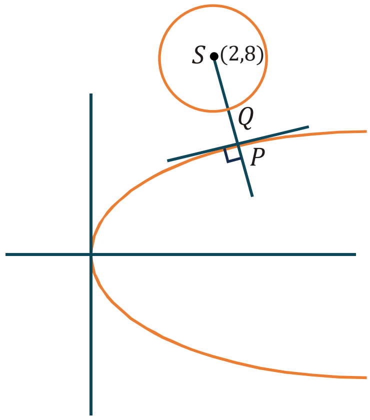
$SP = \sqrt{(4-2)^2 + (4-8)^2}$

$SP = 2\sqrt{5}$

$SQ = 2$

$\Rightarrow PQ = 2\sqrt{5} - 2$

$\frac{SQ}{QP} = \frac{1}{\sqrt{5}-1} = \frac{\sqrt{5}+1}{4}$



To find x intercept

put $y = 0$ in (i)

$\Rightarrow x = 2 + t^2$

$x = 6$

$\because$ Slope of common normal $= -t = -2$

$\therefore$ Slope of tangent $= \frac{1}{2}$

11. **Ans. (D)**

Equation of chord with mid point (h, k) :

$k.y - 16\left(\frac{x+h}{2}\right) = k^2 - 16h$

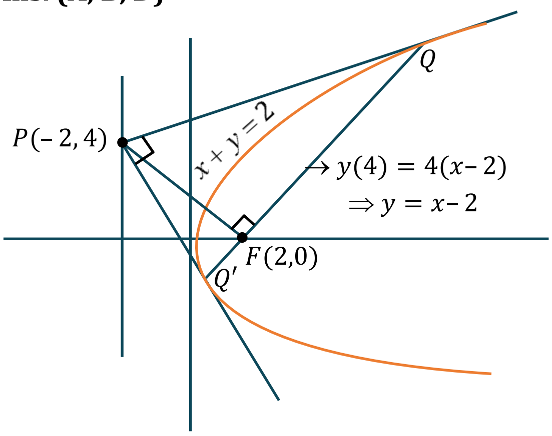
$\Rightarrow 8x - ky + k^2 - 8h = 0$

Comparing with $2x + y - p = 0$, we get

$k = -4;\ 2h - p = 4$

only (D) satisfies above relation.

12. **Ans. (A, B, D)**



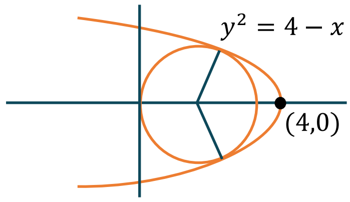
Note that P lies on directrix so triangle PQQ' is right angled, hence QQ' passes through focus F.

$PF = 4\sqrt{2}$

Equation of QF is $y = x - 2$ & PF is $x + y = 2$

Hence A,B,D.

13. Ans. (1.50)



Let the circle be

$x^2 + y^2 + \lambda x = 0$

For point of intersection of circle & parabola $y^2 = 4 - x$.

$x^2 + 4 - x + \lambda x = 0 \Rightarrow x^2 + x(\lambda - 1) + 4 = 0$

For tangency: $\Delta = 0 \Rightarrow (\lambda - 1)^2 - 16 = 0$

$\Rightarrow \lambda = 5$ (rejected) or $\lambda = -3$

Circle: $x^2 + y^2 - 3x = 0$

Radius $= \dfrac{3}{2} = 1.5$

14. Ans. (2.00)

For point of intersection:

$x^2 - 4x + 4 = 0 \Rightarrow x = 2$ so $\alpha = 2$

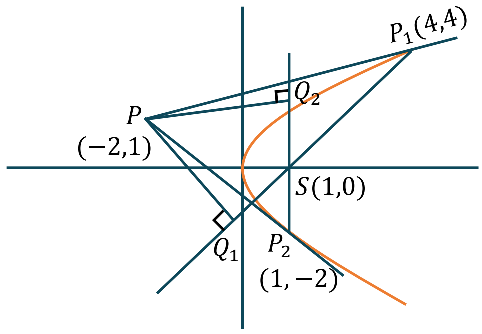
15. Ans. (B,C,D)

Let equation of tangent with slope 'm' be

$T : y = mx + \dfrac{1}{m}$

T : passes through $(-2, 1)$ so

$1 = -2m + \dfrac{1}{m} \Rightarrow m = -1$ or $m = \dfrac{1}{2}$



Points are given by $\left(\dfrac{a}{m^2}, \dfrac{2a}{m}\right)$

So, one point will be $(1, -2)$ & $(4, 4)$

Let $P_1(4, 4)$ & $P_2(1, -2)$

$P_1S : 4x - 3y - 4 = 0$

$P_2S : x - 1 = 0$

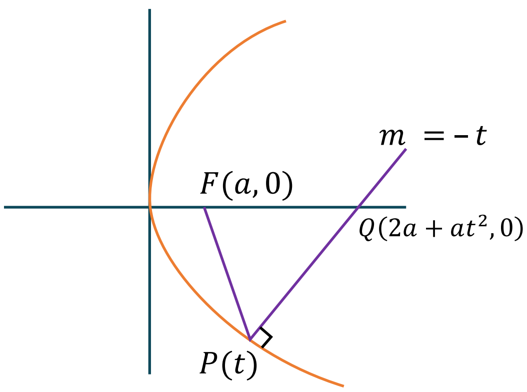
$PQ_1 = \left|\dfrac{4(-2) - 3(1) - 4}{5}\right| = 3$

$SP = \sqrt{10}$; $PQ_2 = 3$; $SQ_1 = 1 = SQ_2$

$\dfrac{1}{2}\left(\dfrac{Q_1Q_2}{2}\right) \times \sqrt{10} = \dfrac{1}{2} \times 3 \times 1$ (comparing Areas)

$\Rightarrow Q_1Q_2 = \dfrac{2 \times 3}{\sqrt{10}} = \dfrac{3\sqrt{10}}{5}$

16. Ans. (A)



Let point $P(at^2, 2at)$

normal at P is $y = -tx + 2at + at^3$

$y = 0$, $x = 2a + at^2$

$Q(2a + at^2, 0)$

Area of ΔPFQ = $\left|\dfrac{1}{2}(a + at^2)(2at)\right| = 120$

$\because m = -t$

$\because a^2 [1 + m^2] m = 120$

$(a, m) = (2, 3)$ will satisfy

JEE (Main) Practice Paper

SECTION–A

1. **Ans. (3)**

$y^2 = k(x - 8/k)$, Directrix will be = $x - \frac{8}{K} = -\frac{k}{4}$

$\Rightarrow \frac{8}{K} - \frac{k}{4} = 1 \Rightarrow k^2 + 4k - 32 = 0 \Rightarrow k = -8, 4$

2. **Ans. (1)**

$x = 0 \Rightarrow y^2 - 5y + 6 = 0 \Rightarrow y = 2, 3$

3. **Ans. (4)**

Centre of variable circle remains equidistant from given line and centre of given circle. that's why locus will be Parabola.

4. **Ans. (1)**

Let AB is double oxidinate

$A\left(\frac{\ell t^2}{4}, \frac{-\ell}{2}t\right); B\left(\frac{\ell t^2}{4}, \frac{\ell}{2}t\right)$

$P\left(\frac{\ell t^2}{4}, \frac{\ell t}{6}\right)$ Locus of P will be $y^2 = \frac{\ell}{9}x$

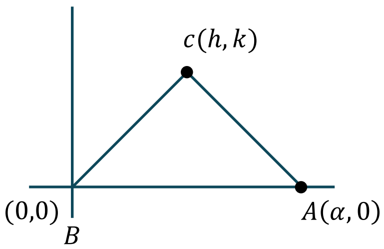
5. **Ans. (4)**

$m^2(x - a)^2 - 4ax = 0 \quad (D > 0)$

$m^2x^2 - 2a(m^2 + 2)x + m^2a^2 = 0$

$4a^2(m^2 + 2)^2 - 4a^2m^4 > 0 \Rightarrow m \in R - \{0\}$

6. **Ans. (2)**



$\tan A + \tan B = \lambda$

$-\frac{K}{h - \alpha} + \frac{K}{h} = \lambda$

Locus of Point C is $\lambda x(x - \alpha) = -\alpha y$ (Parabola)

7. **Ans. (4)**

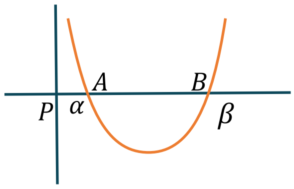
Centre (1, 2) and circle passes through (1, 0)

$\therefore$ Equation $(x - 1)^2 + (y - 2)^2 = 4$

8. **Ans. (3)**

Length of focal Chord = $4a \operatorname{cosec}^2\alpha$

9. Ans. (4)



Length of tangent = $PT = \sqrt{PA.PB} = \sqrt{\alpha\beta} = \frac{c}{a}$

10. Ans. (1)

$(at^2, 2at)$ one end then $\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$ another end.

$16t = -8 \Rightarrow t = -1/2$

$\therefore$ Another end (32, 32)

11. Ans. (4)

If slope of focal chord is $\tan\alpha$ then it's equation $y = \tan\alpha(x - a) \Rightarrow b = a\sin\alpha$ and $C = 4a\ \text{cosec}^2\alpha$

$\Rightarrow 4a^3 = b^2c$

12. Ans. (3)

If PQ is double ordinate, then $P(2bt, bt^2)$ & $Q(-2bt, bt^2)$ Point of trisection $R \equiv \left(\frac{2bt}{3}, bt^2\right)$

Locus $x^2 = \frac{4b^2t^2}{9} = \frac{4by}{9}$

13. Ans. (1)

$y = Ax^2$ $\qquad\qquad$ $y^2 + 3 = x^2 + 4y$

Solving both $y^2 + 3 = \frac{y}{A} + 4y$

$Ay^2 - y - 4Ay + 3A = 0$

$Ay^2 - y(1 + 4A) + 3A = 0$

$$y = \frac{(1+4A) \pm \sqrt{(1+4A)^2 - 4A.3A}}{2A}$$

Both value of y are possitive so there are four point of intersection

14. Ans. (3)

$y = mx + \frac{1}{m} \Rightarrow m_2h - mK + 1 = 0, (m_1, m_2)$ where $P(h, k)$ and $\tan\theta_1 = m_1$, $\tan\theta_2 = m_2$

$\theta_1 + \theta_2 = \pi/4 \Rightarrow \frac{m_1 + m_2}{1 - m_1m_2} = 1 \Rightarrow x - y - 1 = 0$

15. Ans. (2)

Tangent to $y^2 = 32x$ is $y = mx + \frac{8}{m}$ and tangent to $x^2 = 108y$ is $y = mx - 27m^2$ for common tangent $\frac{8}{m} = -27m^2 \Rightarrow m = -\frac{2}{3}$

16. Ans. (3)

$x - y + 3 = 0$ ...(1)

polar and required point (x_1, y_1) be pole then $yy_1 = 4(x + x_1)$ ...(2)

Comparing (1) & (2) $(x_1, y_1) \equiv (3, 4)$.

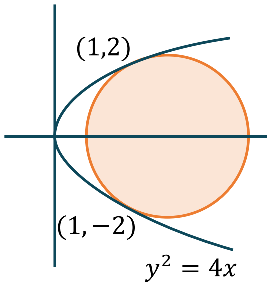
17. Ans. (2)

$C_1 : \left(y-\sqrt{3}\right)^2 = 4\left(x-\sqrt{2}\right)$

$C_2 : x^2 + y^2 = \left(6+2\sqrt{2}\right)x + 2\sqrt{3}y - 6\left(1+\sqrt{2}\right)$

$x^2 - 2\left(3+\sqrt{2}\right)x + y^2 - 2\sqrt{3}y + 3 + \left(3+\sqrt{2}\right)^2 = 8$

$\left(x-\left(3+\sqrt{2}\right)\right)^2 + \left(y-\sqrt{3}\right)^2 = \left(2\sqrt{2}\right)^2$



Shifting both carve to origin

$C_1 : y^2 = 4x$

$C_2 : (x-3)^2 + y^2 = \left(2\sqrt{2}\right)^2$

solving both

$(x-3)^2 + 4x = \left(2\sqrt{2}\right)^2$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$

$y = \pm 2$

Both curve touch at two points.

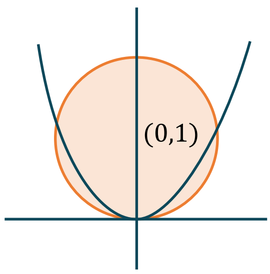
18. Ans. (4)

Centre of circle $(0,1) r = 1$

circle $(x-0)^2 + (y-1)^2 = 1$

point on circle $(\cos\theta, 1 + \sin\theta)$

this point lies on parabola $y = ax^2$



$1 + \sin\theta = a \cos^2\theta$

$\frac{1+\sin\theta}{\cos^2\theta} = a$

$\frac{1+\sin\theta}{1-\sin^2\theta} = a$

$\frac{1}{1-\sin\theta} = a$

$\sin\theta \; \varepsilon \; [-1, 1]$

$-\sin\theta \; \varepsilon \; [-1, 1]$

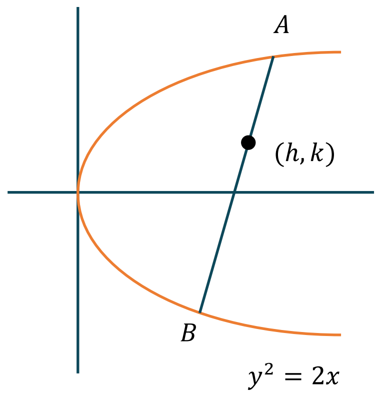
$1-\sin\theta \; \varepsilon \; [0, 2]$

$\frac{1}{1-\sin\theta} \varepsilon \left[\frac{1}{2}, \infty\right)$

$a \, \varepsilon \left[\frac{1}{2}, \infty\right)$

19. Ans. (3)

parabola $y^2 = 2x$



let midpoint of chord (h, k)

equation of chord $T = s_1$

$yk - (x + h) = k^2 - 2h$

$yk - x + h - k^2 = 0$

This chord touches the circle

$x^2 + y^2 - 2x - 4 = 0$

centre $(1,0)$ $r = \sqrt{5}$

$$\left|\frac{0.k - 1 + h - k^2}{\sqrt{k^2 + 1}}\right| = \sqrt{5}$$

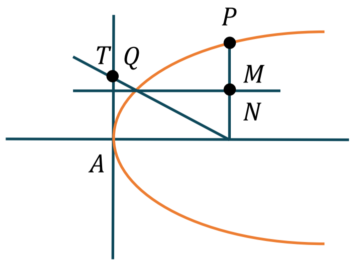
$(x - y^2 - 1)^2 = 5(y^2 + 1)$

$(y^2 + 1 - x)^2 = 5(y^2 + 1)$

$\lambda = 5$

20. Ans. (2)

$P(at^2, 2at), N(at^2, 0)$



Equation of MQ. is $y = at \Rightarrow Q = \left(\frac{at^2}{4}, at\right)$

Equation of QM $y = \frac{-4}{3t}(x - at^2)$

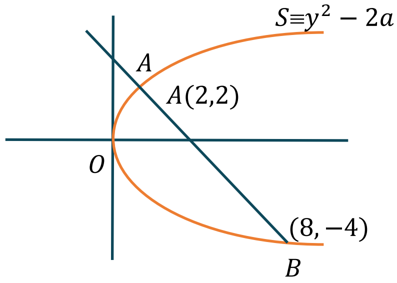
$\Rightarrow T = \left(0, \frac{4}{3}at\right)$

SECTION–B

1. Ans. (1)

I.F.$(a^2, a - 2)$

$S \equiv y^2 - 2x$



Equation of line AB

$y - 2 = \frac{-6}{6}(x - 2)$

$y - 2 = -x + 2$

$L \equiv x + y - 4 = 0$

$S_1 \equiv (a - 2)^2 - 2a^2 < 0$

$a^2 + 4 - 4a - 2a^2 < 0 \Rightarrow$	$a^2 + 4a - 4 > 0$
$-4a - a^2 + 4 < 0$	$a^2 + 4a + 4 > 8$
$L_1 < 0$	$(a + 2)^2 > 8$
$a^2 + a - 6 < 0$	$a + 2 > 2\sqrt{2}$ $a + 2 < -2\sqrt{2}$
$-3 < a < 2$	$a > -2 + 2\sqrt{2}$ $a < -2\sqrt{2} - 2$ $\Rightarrow$ $-2 + 2\sqrt{2} < a < 2$

so integral value of a is equal to 1 only.

2. **Ans. (0)**

The point $P(-2a, a+1)$ will be an interior point of both the circle $x^2+y^2-4=0$ and the parabola $y^2-4x=0$.

$\therefore (-2a)^2+(a+1)^2-4<0$

i.e. $5a^2+2a-3<0$ ...(i)

and $(a+1)^2-4(-2a)<0$

i.e. $a^2+10a+1<0$ ...(ii)

The required values of a will satisfy both (i) and (ii)

From (i), $(5a-3)(a+1)<0$

$\therefore$ by sign scheme we get $-1<a<3/5$ ...(iii)

Solving (ii), the corresponding equation is

$a^2+10a+1\ \ 0$

or $a=\dfrac{-10\pm\sqrt{100-4}}{2}=-5\pm2\sqrt{6}$

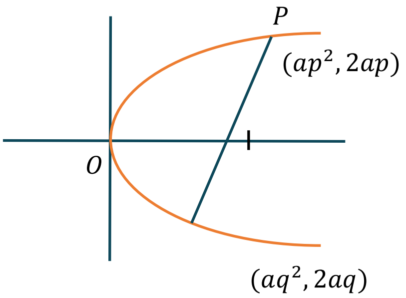
$\therefore$ by sign scheme for (ii)

$-5-2\sqrt{6}<a<-5+2\sqrt{6}$ ...(iv)

The set of values of a satisfying (iii) and (iv) is

$-1<a<-5+2\sqrt{6}$

3. **Ans. (2)**



slope of $PQ = \dfrac{2a(p-q)}{a(p-q)(p+q)}=1$

4. **Ans. (9)**

As the axis is parallel to the y-axis, it will be $x-\alpha=0$ for some α and the tangent to the vertex (which is perpendicular to the axis) will be $y-\beta=0$ for some β.

Hence the equation of the parabola will be of the form

$(x-\alpha)^2=4a(y-\beta)$ ...(i)

when α, β, a are unknown constants, $4a$ being latus rectum.

(1) passes through (0, 4), (1, 9) and (–2, 6) so

$(0-\alpha)^2=4a(4-\beta)$,

i.e. $\alpha^2=4a(4-\beta)$ ...(ii)

and $(1-\alpha)^2=4a(9-\beta)$

i.e. $1-2\alpha+\alpha^2=4a(9-\beta)$ ...(iii)

and $(-2-\alpha)^2=4a(6-\beta)$

i.e. $4 + 4\alpha + \alpha^2 = 4a(6 - \beta)$ ...(iv)

$\therefore\ \alpha = -\frac{3}{4}$

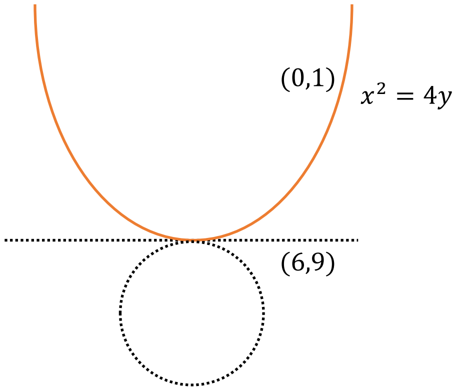
$\therefore\ a = \frac{5}{40} = \frac{1}{8}$ or $\beta = \frac{23}{8}$

$\therefore$ from (i), the equation of the parabola is

$\left(x+\frac{3}{4}\right)^2 = 4\cdot\frac{1}{8}\cdot\left(y-\frac{23}{8}\right)$ or $x^2+\frac{3}{2}x+\frac{9}{16}=\frac{1}{2}y-\frac{23}{16}$ or $x^2+\frac{3}{2}x-\frac{1}{2}y+2=0$

$\therefore\ 2x^2 + 3x - y + 4 = 0 \Rightarrow y = 2x^2 + 3x + 4 \Rightarrow \alpha = 2 \times 2^2 + 3 \times 2 + 4 = 18$

5. **Ans. (5)**



$xx_1 = 2(y + y_1)$

$6x = 2(y + 9)$

$3x = y + 9$ $\qquad 3x - y - 9 = 0$

from equation of family circle is $S + \lambda L = 0$

$S \equiv (x-6)^2 + (y-91)^2 + k(3x - y - 9) = 0$

is passes through (0, 1)

$36 + 64 + k(-10) = 0 \qquad k = 10$

$x^2 + 36 - 12x + y^2 + 81 - 18y + 30x - 30y - 90 = 0$

$x^2 + y^2 + 18x - 28y + 27 = 0$

6. **Ans. (2)**

Here $h^2 - ab = (-12)^2 - 9\cdot 16 = 144 - 144 = 0$ Also $\Delta \neq 0$

$\therefore$ the equation represents a parabola

Now, the equation is $(3x - 4y)^2 = 5(4x + 3y + 12)$

Clearly, the lines $3x - 4y = 0$, $4x + 3y + 12 = 0$ are perpendicular to each other. So let

$\frac{3x-4y}{\sqrt{3^2+(-4)^2}} = Y, \frac{4x+3y+12}{\sqrt{4^2+3^2}} = X$ ...(i)

The equation of the parabola becomes $Y^2 = X = 4\cdot\frac{1}{4}X$

$\therefore$ Here $a = 1/4$ in the standard equation

as $\ell = 2a = 1/2 \Rightarrow 4\ell = 2$

7. **Ans. (2)**

Equation of parabola is $y^2 = 4ax$ ..(1)

Let $A\equiv(at_1^2, 2at_1)$, $B\equiv(at_2^2, 2at_2)$, $C\equiv(at_3^2, 2at_3)$

Equation of the tangents to parabola (1) at A, B, C are

$yt_1 = x + at_1^2$ ...(2)

$yt_2 = x + at_2^2$ ...(3)

and $yt_3 = x + at_3^2$ ...(4)

Let the points of intersection of lines (2), (3) be P; (3), (4) be Q and (2), (4) be R.

Then $P \equiv (at_1t_2, a(t_1 + t_2))$, $Q \equiv (at_2t_3, a(t_2 + t_3))$, $R \equiv (at_1t_3, a(t_1 + t_3))$

Now area of ΔABC,

$$\Delta_1 = \text{modulus of } \frac{1}{2}\begin{vmatrix} at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \\ at_3^2 & 2at_3 & 1 \end{vmatrix}$$

$$= \text{modulus of } \frac{1}{2}\cdot a\cdot 2a\begin{vmatrix} t_1^2 & t_1 & 1 \\ t_2^2 & t_2 & 1 \\ t_3^2 & t_3 & 1 \end{vmatrix}$$

$= a^2|(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)|$

Area of ΔPQR

$$\Delta_2 = \text{modulus of } \frac{1}{2}\begin{vmatrix} at_1t_2 & a(t_1+t_2) & 1 \\ at_2t_3 & a(t_2+t_3) & 1 \\ at_3t_1 & a(t_3+t_1) & 1 \end{vmatrix}$$

$$= \text{modulus of } \frac{a^2}{2}\begin{vmatrix} t_1t_2 & t_1+t_2 & 1 \\ t_2t_3 & t_2+t_3 & 1 \\ t_3t_1 & t_3+t_1 & 1 \end{vmatrix}$$

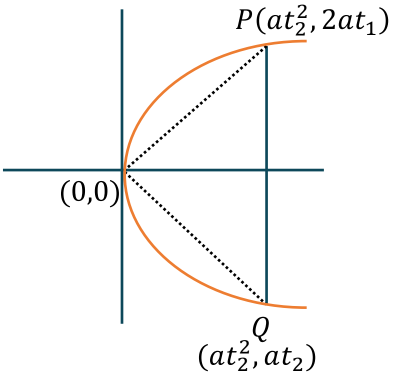
$$= \text{modulus of } \frac{a^2}{2}\begin{vmatrix} t_2(t_1-t_3) & t_1-t_3 & 0 \\ t_3(t_2-t_1) & t_2-t_1 & 0 \\ t_3t_1 & t_3+t_1 & 1 \end{vmatrix} \; [R_1 \to R_1 - R_2, \to R_2 - R_3]$$

= modulus of $(t_1 - t_3)(t_2 - t_1)(t_2 - t_3)$

$$= \frac{a^2}{2}|(t_1-t_2)(t_2-t_3)(t_3-t_1)|$$

Clearly $\frac{\Delta_1}{\Delta_2} = \frac{2}{1}$

8. **Ans. (9)**



Equation of parabola

$y^2 = 4ax$

$OQ = \sqrt{a^2t_2^4 + 4a^2t_2^2} = at_2\sqrt{t_2^2+4}$

$QQ \geq 2\sqrt{2}a.2\sqrt{3} \geq 4\sqrt{6}a$

as $t_2 = t_1 - \frac{2}{t_1}$

9. **Ans. (3)**

$y = mx - 2am - am^3$ Here $a = 1$

$0 = cm - 2m - m^3$

$m^3 + (2 - c)m = 0$

$m = 0$ $m^2 = c - 2 \Rightarrow c > 2$

sum $m_1 + m_2 + m_3 = 0$

$\Sigma m_1 m_2 = \dfrac{2a - h}{a}$

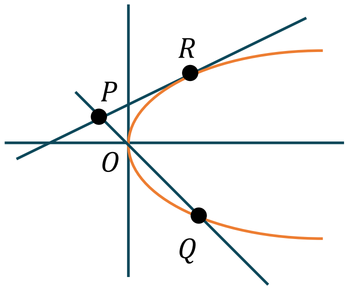
$m_1 m_2 m_3 = \dfrac{-k}{a}$

$m_1 m_2 = 2 - c$

$-1 = 2 - c \Rightarrow c = 3$

10. **Ans. (4)**

$y = mx + \dfrac{a}{m}$ ...(i)



equation of OP is

$y = -\dfrac{1}{m}x$ ...(ii)

$OP = \dfrac{a/m}{\sqrt{1+m^2}}$

equation (ii) meets the parabola at Q

$\dfrac{1}{m^2}x^2 = 4ax \Rightarrow x = 4am^2, y = -4am$

$\therefore OQ = 4am\sqrt{1+m^2}$, $OP \cdot OQ = 4a^2$

JEE (Advanced) Practice Paper

1. **Ans. (A,B,C,D)**

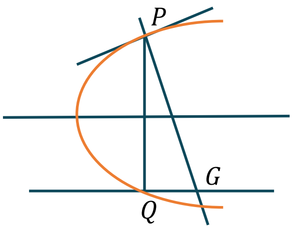
$P(2t^2, 4t)$: $5(2, 0)$

Mid-point $(t^2 + 1, 2t) \Rightarrow$ Locus will be $y^2 = 4(x - 1)$

2. **Ans. (A, B, C)**

Put $y = 1 - x$ and check for $D = 0$

3. **Ans. (A,B,C,D)**



$P(at^2, 2at)$ & $Q(at^2, -2at)$; Equation of normal at P is $y = -tx + 2at + at^3$ and Equation of Line QG is $y = -2at$

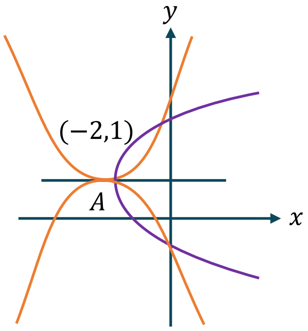
$\therefore$ Locus of G $y = \dfrac{y}{2a}x - y - \dfrac{y^3}{8a^2}$

$y^2 = 4a(x - 4a)$

4. **Ans. (A, B)**

$y^2 - 2y = 4x + 7 \Rightarrow (y-1)^2 = 4x + 8$

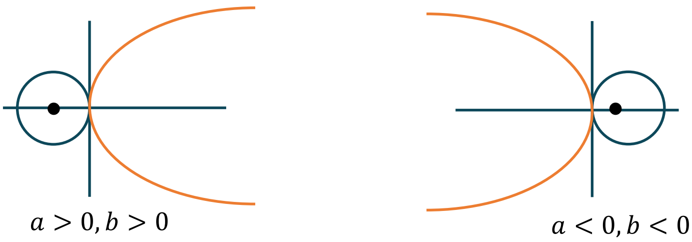
$\Rightarrow (y-1)^2 = 4(x+2)$



Equation of required parabolas is

$(x+2)^2 = 8(y-1)$ & $(x+2)^2 = -8(y-1)$

5. **Ans. (A, D)**

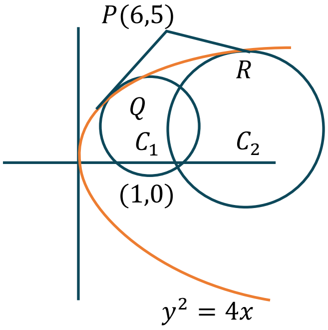


6. **Ans. (B, C)**

$$t_1t_2 = -4 \Rightarrow P \equiv \left(\frac{at_1^2}{2}, at_1\right) \& Q \equiv \left(\frac{at_2^2}{2}, at_2\right) \Rightarrow h = \frac{a}{4}\left(t_1^2 + t_2^2\right) \& k = \frac{a}{2}(t_1 + t_2)$$

$$\Rightarrow k^2 = \frac{a^2}{4}\left(t_1^2 + t_2^2 + 2t_1t_2\right) \Rightarrow k^2 = a\left(\frac{a}{4}\right)\left(t_1^2 + t_2^2\right) - 2a^2 \Rightarrow k^2 + 2a^2 = ah \Rightarrow k^2 = a(h - 2a)$$

7. **Ans. (A)**



Parabola $y^2 = 4x$

$P(6, 5)$

area of triangle formed by pair of tangent & chord of contact is

$$\frac{(y_1^2 - 4ax_1)^{3/2}}{2a}$$

$$= \frac{(25-24)^{3/2}}{2} = \frac{1}{2}$$

8. Ans. (B)

Let tangent $y = mx + \frac{1}{m}$

it passes through (6,5)

$5 = 6m + \frac{1}{m}$

$6m^2 - 5m + 1 = 0$

$6m^2 - 3m - 2m + 1 = 0$

$3m(2m-1) - 1(2m-1) = 0$

$(2m-1)(3m-1) = 0$

$m = \frac{1}{2}\ \ m = \frac{1}{3}$

tangent $y = \frac{x}{2} + 2$ & $y = \frac{x}{3} + 3$

point of contact Q (4, 4)

Circle C_1

$(x - 4)^2 + (y - 4)^2 + \lambda(x - 2y + 4) = 0$

$9 + 16 + \lambda 5 = 0$

$\lambda = -5$

$C_1 : x^2 - 8x + 16 + y^2 - 8y + 16 - 5x + 10y - 20 = 0$

$x^2 + y^2 - 13x + 2y + 12 = 0$

radius $\sqrt{\frac{169}{4} + 1 - 12}$

$\frac{5}{2}\sqrt{5}\ 5\sqrt{10}$

R (9,6)

Circle C_2

$(x - 9)^2 + (y - 6)^2 + \mu(x - 3y + 9) = 0$

passes through (1, 0)

$64 + 36 + \mu 10 = 0$

$\mu = -10$

$x^2 + y^2 - 18x - 12y + 81 + 36 - 10x + 30y - 90 = 0$

$x^2 + y^2 - 28x + 18y + 27 = 0$

$r = \sqrt{196 + 81 - 27} = \sqrt{250}$

9. Ans. (C)

Common chord $S_1 - S_2 = 0$

$15x - 16y - 15 = 0$

centroid $PQR\ \left(\frac{19}{3}, 5\right)$

$15\frac{19}{3} - 16.5 - 15$

$96 - 95 = 0$

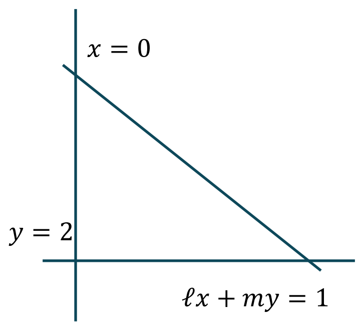
Common chord passes through centroid of ΔPQR:-

10. Ans. (A)

11. Ans. (B)

12. Ans. (C)

Sol. (10 to 12)



Clearly triangle formed by given line is right triangles, so circumcentre of triangle is midpoint of hypotenuses which is $\left(\frac{1-2m}{2\ell}, \frac{1+2m}{2\ell}\right)$

$\therefore$ Now let $P(h, k)$ be the circumecentre

$\therefore h=\frac{1-2m}{2\ell}, k=\frac{1+2m}{2m} \Rightarrow m=\frac{1}{2k-2}, \ell=\frac{k-2}{2h(k-1)}$

$\therefore$ (ℓ, m) lies on $y^2 = 4x$

$\therefore$ $m^2 = 4\ell \Rightarrow \left(\frac{1}{2k-2}\right)^2 = 4\left(\frac{k-2}{2h(k-1)}\right) \Rightarrow \left(k-\frac{3}{2}\right)^2 = \frac{1}{8}(h+2)$

$\Rightarrow \left(y-\frac{3}{2}\right)^2 = \frac{1}{8}(x+2)......(i)$

$\therefore$ $V\left(-2,\frac{3}{2}\right)$ length of L.R. $4a=\frac{1}{8}$

Required area of triangle = $\frac{1}{2}4a \times a = 2a^2 = \frac{1}{2^9}$ (unit)^2

also, parametric form of (i) is $\left(-2+\frac{t^2}{32}, \frac{3}{2}+\frac{t}{16}\right)$

13. Ans. (2)

Given parabolas $y^2 = 4ax$ and $y^2 = 4c(x-b)$ have common normals. Then equation of normal in terms of slopes are $y = mx - 2am - am^3$ and $y = m(x-b) - 2cm - cm^3$ respectively then normal must be identical, compare the co-efficient.

$1=\frac{2am+am^3}{mb+2cm+cm^3} \Rightarrow m[(c-a)m^2 + (b+2c-2a)] = 0, m \neq 0$ ($\because$ other than axis)

And $m^2 = \frac{2a-2c-b}{c-a}$, $m = \pm\sqrt{\frac{2(a-c)-b}{c-a}}$

Or $m=\pm\sqrt{\left(-2-\frac{b}{c-a}\right)}$

$\therefore -2-\frac{b}{c-a}>0$

or $-2+\frac{b}{a-c}>0 \Rightarrow \frac{b}{a-c}>2$

14. Ans. (2)

The equation of the tangent at (−3, 2) to the parabola $y^2 + 4x + 4y = 0$ is

$2y + 2(x-3) + 2(y+2) = 0$

or $2x + 4y - 2 = 0 \Rightarrow x + 2y - 1 = 0$

Since the tangent at one end of the focal chord is parallel to the normal at the other end, the slope of the normal at the other end of the focal chord is $-\frac{1}{2}$.

15. Ans. (3)

Let (h, k) be point of intersection of tangents then chord of contact is

$yk = 4(x+h)$

$4x - yk + 4h = 0$ ...(i)

But given line is

$x - y - 1 = 0$ ...(ii)

Comparing (i) and (ii)

$\therefore \frac{4}{1}=\frac{-k}{-1}=\frac{4h}{-1} \Rightarrow h=-1, k=4$

$\therefore$ point $\equiv (-1, 4)$

16. Ans. (8)

Let the slope of the tangent be m

$\therefore \tan 45° = \left|\frac{3-m}{1+3m}\right| \Rightarrow 1+3m = \pm(3-m)$

$\therefore m = -2 \text{ or } \frac{1}{2}$

As we know that equation of tangent of slope m to the parabola $y^2 = 4ax$ is $y = mx \ \frac{a}{m}$ and point of contact is $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

for $m = -2$, equation of tangent is $y = -2x - 1$ and point of contact is $\left(\frac{1}{2}, -2\right)$

for $m = \frac{1}{2}$, equation of tangent is $y = \frac{1}{2}x + 4$ and point of contact is (8, 8).

17. Ans. (0)

Putting value of y from the line in the parabola -

$(3x + \lambda)^2 = 4x$

$\Rightarrow 9x^2 + (6\lambda - 4)x + \lambda^2 = 0$

$\because$ line cuts the parabola at two distinct points

$\therefore D > 0$

$\Rightarrow 4(3\lambda - 2)^2 - 4.9\lambda^2 > 0$

$\Rightarrow 9\lambda^2 - 12\lambda + 4 - 9\lambda^2 > 0$

$\Rightarrow \lambda < 1/3$

Hence, $\lambda \in (-\infty, 1/3)$

18. Ans. (4)

The given parabola is $y^2 = 4x$ ...(i)

Let $P \equiv (t_1^2, 2t_1)$, $Q \equiv (t_2^2, 2t_2)$

Slope of $OP = \frac{2t_1}{t_1^2} = \frac{2}{t_1}$ and slope of $OQ = \frac{2}{t_2}$

Since $OP \perp OQ$, $\frac{4}{t_1 t_2} = -1$ or $t_1 t_2 = -4$ ...(ii)

The equation of PQ is $y(t_1 + t_2) = 2(x + t_1 t_2)$

$\Rightarrow y\left(t_1 - \frac{4}{t_1}\right) = 2(x - 4)$ [from (ii)]

$\Rightarrow 2(x - 4) - y\left(t_1 - \frac{4}{t_1}\right) = 0 \Rightarrow L_1 + \lambda L_2 = 0$

$\therefore$ variable line PQ passes through a fixed point which is point of intersection of $L_1 = 0$ & $L_2 = 0$ i.e. (4, 0)