EXERCISE - O

SINGLE CORRECT TYPE QUESTIONS

1. If the function $f(x) = 2x^2 - kx + 5$ is strictly increasing in [1, 2], then complete set of values of k is

(A) $(-\infty, 4)$ (B) $(4, \infty)$ (C) $(-\infty, 8]$ (D) $(8, \infty)$

MMN002

2. If $f(x) = x^3 - 10x^2 + 200x - 10$, then $f(x)$ is-

(A) strictly decreasing in $(-\infty, 10]$ and strictly increasing in $[10, \infty)$

(B) strictly increasing in $(-\infty, 10]$ and strictly decreasing in $[10, \infty)$

(C) strictly increasing for every value of x

(D) strictly decreasing for every value of x

MMN004

3. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

	interval	function
(A)	$(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$
(B)	$[2, \infty)$	$2x^3 - 3x^2 - 12x + 6$
(C)	$\left(-\infty, \frac{1}{3}\right]$	$3x^2 - 2x + 1$
(D)	$(-\infty, -4)$	$x^3 + 6x^2 + 6$

MMN015

4. If function $f(x) = 2x^2 + 3x - m\ell nx$ is monotonic decreasing in the interval (0, 1), then the least value of the parameter m is -

(A) 7 (B) $\frac{15}{2}$ (C) $\frac{31}{4}$ (D) 8

MMN003

5. If $f:[1, 10] \to [1, 10]$ is a non-decreasing function and $g : [1, 10] \to [1, 10]$ is a non-increasing function. Let $h(x) = f(g(x))$ with $h(1) = 1$, then $h(2)$

(A) lies in (1, 2) (B) is more than 2 (C) is equal to 1 (D) is not defined

MMN014

6. If $2a + 3b + 6c = 0$, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval-

(A) (0, 1) (B) (1, 2) (C) (2, 3) (D) none

MMN005

7. The equation $\sin x + x \cos x = 0$ has at least one root in

(A) $\left(-\frac{\pi}{2}, 0\right)$ (B) $(0, \pi)$ (C) $\left(\pi, \frac{3\pi}{2}\right)$ (D) $\left(0, \frac{\pi}{2}\right)$

MMN011

8. Given: $f(x) = 4 - \left(\frac{1}{2} - x\right)^{2/3}$; $g(x) = \begin{cases} \frac{\tan[x]}{x} & , x \neq 0 \\ 1 & , x = 0 \end{cases}$

$h(x) = \{x\}$; $k(x) = 5^{\log_2(x+3)}$

then in [0, 1], Lagrange's Mean Value Theorem is NOT applicable to

(A) f, g, h (B) h, k (C) f, g (D) g, h, k

where $[x]$ and $\{x\}$ denotes the greatest integer and fractional part function.

MMN008

9. Given $f'(1) = 1$ and $\frac{d}{dx}(f(2x)) = f'(x)\ \forall\ x > 0$. If $f'(x)$ is differentiable then there exists a number $c \in (2, 4)$ such that $f''(c)$ equals

(A) – 1/4 (B) – 1/8 (C) 1/4 (D) 1/8

MMN009

10. If the function $f(x) = 2x^2 + 3x + 5$ satisfies LMVT at $x = 2$ on the closed interval $[1, a]$, then the value of 'a' is equal to

(A) 3 (B) 4 (C) 6 (D) 1

MMN010

MULTIPLE CORRECT TYPE QUESTIONS

11. If $f(x) = 2x + \cot^{-1}x + \ell n\left(\sqrt{1+x^2} - x\right)$, then $f(x)$:

(A) increases in $[0, \infty)$ (B) decreases in $[0, \infty)$

(C) neither increases nor decreases in $[0, \infty)$ (D) increases in $(-\infty, \infty)$

MMN038

12. Let $f(x) = \sin^2 x$ & $0 < A < B$, where $A, B \in \left(0, \frac{\pi}{4}\right)$, then

(A) $(B - A)\sin 2A < \sin^2 B - \sin^2 A < (B - A)\sin 2B$

(B) $(B - A)\sin 2A > \sin^2 B - \sin^2 A > (B - A)\sin 2B$

(C) $\cos 2B < \cos 2A$

(D) $B\cos 2A > A\cos 2B$

MMN042

13. Let $\phi(x) = (f(x))^3 - 3(f(x))^2 + 4f(x) + 5x + 3\sin x + 4\cos x\ \forall\ x \in R$, where $f(x)$ is a differentiable function $\forall\ x \in R$, then

(A) ϕ is increasing whenever f is increasing

(B) ϕ is increasing whenever f is decreasing

(C) ϕ is decreasing whenever f is decreasing

(D) ϕ is decreasing if $f'(x) = -11$

MMN040

14. Let $g(x) = 2f(x/2) + f(1-x)$ and $f''(x) < 0$ in $0 \le x \le 1$ then $g(x)$____

(A) decreases in $\left[0, \frac{2}{3}\right]$ (B) decreases $\left[\frac{2}{3}, 1\right]$

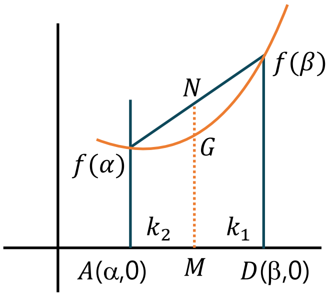
(C) increases in $\left[0, \frac{2}{3}\right]$ (D) increases in $\left[\frac{2}{3}, 1\right]$

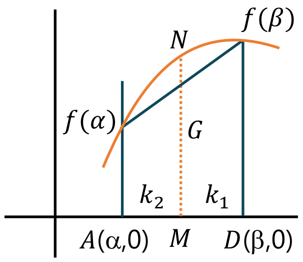
MMN039

COMPREHENSION TYPE QUESTIONS

Paragraph for Question 15 to 17

For a double differentiable function $f(x)$ if $f''(x) \ge 0$ then $f(x)$ is concave upward and if $f''(x) \le 0$ then $f(x)$ is concave downward





Here $M\left(\frac{k_1\alpha + k_2\beta}{k_1 + k_2}, 0\right)$

If $f(x)$ is a concave upward in $[a, b]$ and $\alpha, \beta \in [a, b]$ then $\frac{k_1 f(\alpha) + k_2 f(\beta)}{k_1 + k_2} \ge f\left(\frac{k_1\alpha + k_2\beta}{k_1 + k_2}\right)$,

where $k_1, k_2 \in R^+$

If $f(x)$ is a concave downward in $[a, b]$ and $\alpha, \beta \in [a, b]$ then $\frac{k_1 f(\alpha) + k_2 f(\beta)}{k_1 + k_2} \le f\left(\frac{k_1\alpha + k_2\beta}{k_1 + k_2}\right)$,

where $k_1, k_2 \in R^+$

then answer the following :

15. Which of the following is true

(A) $\frac{\sin\alpha + \sin\beta}{2} > \sin\left(\frac{\alpha+\beta}{2}\right)$; $\alpha, \beta \in (0, \pi)$ (B) $\frac{\sin\alpha + \sin\beta}{2} < \sin\left(\frac{\alpha+\beta}{2}\right)$; $\alpha, \beta \in (\pi, 2\pi)$

(C) $\frac{\sin\alpha + \sin\beta}{2} < \sin\left(\frac{\alpha+\beta}{2}\right)$; $\alpha, \beta \in (0, \pi)$ (D) None of these

MMN043

16. Which of the following is true

(A) $\frac{2^{\alpha} + 2^{\beta+1}}{3} \le 2^{\frac{\alpha+2\beta}{3}}$ (B) $\frac{2\ell n\ \alpha + \ell n\beta}{3} \ge \ell n\left(\frac{2\alpha+\beta}{3}\right)$

(C) $\frac{\tan^{-1}\alpha + \tan^{-1}\beta}{2} \le \tan^{-1}\left(\frac{\alpha+\beta}{2}\right)$; $a, b \in R^-$ (D) $\frac{e^{\alpha} + 2e^{\beta}}{3} \ge e^{\frac{\alpha+2\beta}{3}}$

MMN044

17. Let α, β and γ are three distinct real numbers and $f''(x) < 0$. Also $f(x)$ is increasing function and let $A = \dfrac{f^{-1}(\alpha) + f^{-1}(\beta) + f^{-1}(\gamma)}{3}$ and $B = f^{-1}\left(\dfrac{\alpha+\beta+\gamma}{3}\right)$, then order relation between A and B is ?

(A) $A > B$ (B) $A < B$ (C) $A = B$ (D) none of these

MMN045

MATCHING LIST TYPE QUESTION

18. Match List-I with List-II and select the correct answer using the code given below the list.

	List - I		List - II
(P)	$f(x) = 2.\ e^{x^2-4x}$	(1)	Strictly increasing in $(2, \infty)$ & Strictly decreasing in $(-\infty, 2)$
(Q)	$f(x) = e^x/x$	(2)	Strictly increasing in $(0, 2)$ & Strictly decreasing in $(-\infty, 0) \cup (2, \infty)$
(R)	$f(x) = x^2\, e^{-x}$	(3)	Strictly increasing in $(1, \infty)$ & Strictly decreasing in $(-\infty, 0) \cup (0, 1)$
(S)	$f(x) = 2x^2 - ln\|x\|$	(4)	Strictly increasing in $\left(-\dfrac{1}{2}, 0\right) \cup \left(\dfrac{1}{2}, \infty\right)$ & Strictly decreasing in $\left(-\infty, -\dfrac{1}{2}\right) \cup \left(0, \dfrac{1}{2}\right)$

(A) P $\rightarrow$ 3; Q $\rightarrow$ 2; R $\rightarrow$ 4; S $\rightarrow$ 1
(B) P $\rightarrow$ 1; Q $\rightarrow$ 3; R $\rightarrow$ 2; S $\rightarrow$ 4
(C) P $\rightarrow$ 2; Q $\rightarrow$ 3; R $\rightarrow$ 4; S $\rightarrow$ 1
(D) P $\rightarrow$ 4; Q $\rightarrow$ 1; R $\rightarrow$ 3; S $\rightarrow$ 2

MMN049

EXERCISE - S

1. If $f(x) = 2e^x - ae^{-x} + (2a+1)x - 3$ increases for every $x \in R$ then the range of values of 'a' is $[k, \infty)$ find k.

MMN016

2. Find the sum of value of a, m and b if the function $f(x) = \begin{cases} 3 & x = 0 \\ -x^2 + 3x + a & 0 < x < 1 \\ mx + b & 1 \le x \le 2 \end{cases}$

satisfy the hypothesis of the mean value theorem for the interval [0, 2].

MMN017

3. Let $f(x)$ be a strictly increasing function defined on $(0, \infty)$. If $f(2a^2 + a + 1) > f(3a^2 - 4a + 1)$. The range of a is $\left(0, \frac{1}{p}\right) \cup (q, r)$. Find $(p + q + r)$.

MMN019

4. Find the number of integral values of 'a' for which the function,

$f(x) = \left(1 - \frac{\sqrt{21 - 4a - a^2}}{a+1}\right)x^3 + 5x + \sqrt{7}$ is strictly increasing at every point of its domain.

MMN051

5. Find the minimum value of the function $f(x) = x^{3/2} + x^{-3/2} - 4\left(x + \frac{1}{x}\right)$ for all permissible real x.

MMN052

6. If $b > a$ and the minimum value of $|(x-a)^3| + |(x-b)^3|$, $x \in R$, is $\frac{(b-a)^p}{k}$, then the value of $(k - p)$ is

MMN053

7. If the set of values of x for which the inequality $\ln(1+x) > x/(1+x)$ is valid is $(a, b) \cup (b, \infty)$, then the value of $(2a - b)$ is

MMN054

EXERCISE - JEE (Main) PYQ

1. If the function f given by $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$, for some $a \in R$ is increasing in (0, 1] and decreasing in [1, 5), then a root of the equation, $\frac{f(x)-14}{(x-1)^2} = 0 \ (x \neq 1)$ is : **[JEE (Main) 2019]**

(1) 6 (2) 5 (3) 7 (4) –7

MMN021

2. Let $f(x) = e^x - x$ and $g(x) = x^2 - x$, $\forall\ x \in$ R. Then the set of all $x \in$ R, where the function $h(x) = (fog)(x)$ is increasing, is : **[JEE (Main) 2019]**

(1) $\left[-1,\frac{-1}{2}\right] \cup \left[\frac{1}{2},\infty\right)$ (2) $\left[0,\frac{1}{2}\right] \cup [1,\infty)$ (3) $\left[\frac{-1}{2},0\right] \cup [1,\infty)$ (4) $[0,\infty)$

MMN022

3. If m is the minimum value of k for which the function $f(x) = x\sqrt{kx - x^2}$ is increasing in the interval [0, 3] and M is the maximum value of f in [0, 3] when $k = m$, then the ordered pair (m, M) is equal to :

[JEE (Main) 2019]

(1) $(4,\ 3\sqrt{2})$ (2) $(4,\ 3\sqrt{3})$ (3) $(3,\ 3\sqrt{3})$ (4) $(5,\ 3\sqrt{6})$

MMN023

4. The value of c in the Lagrange's mean value theorem for the function $f(x) = x^3 - 4x^2 + 8x + 11$, when $x \in [0, 1]$ is : **[JEE (Main) 2020]**

(1) $\frac{2}{3}$ (2) $\frac{\sqrt{7}-2}{3}$ (3) $\frac{4-\sqrt{5}}{3}$ (4) $\frac{4-\sqrt{7}}{3}$

MMN024

5. Let the function, $f : [-7, 0] \rightarrow R$ be continuous on [–7, 0] and differentiable on (–7, 0). If $f(-7) = -3$ and $f'(x) \leq 2$, for all $x \in (-7, 0)$, then for all such functions f, $f(-1) + f(0)$ lies in the interval : **[JEE (Main) 2020]**

(1) [–6, 20] (2) $(-\infty, 20]$ (3) $(-\infty, 11]$ (4) [–3, 11]

MMN025

6. Let $f(x) = x\cos^{-1}(-\sin|x|)$, $x \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, then which of the following is true? **[JEE (Main) 2020]**

(1) f' is decreasing in $\left(-\frac{\pi}{2},0\right)$ and increasing in $\left(0,\frac{\pi}{2}\right)$

(2) f is not differentiable at $x = 0$

(3) $f'(0) = -\frac{\pi}{2}$

(4) f' is increasing in $\left(-\frac{\pi}{2},0\right)$ and decreasing in $\left(0,\frac{\pi}{2}\right)$

MMN026

7. Let f be any function continuous on $[a, b]$ and twice differentiable on (a, b). If for all $x \in (a,\ b), f'(x) > 0$ and $f''(x) < 0$, then for any $c \in (a, b)$, $\dfrac{f(c)-f(a)}{f(b)-f(c)}$ is greater than:

[JEE (Main) 2020]

(1) $\dfrac{b+a}{b-a}$ (2) $\dfrac{b-c}{c-a}$ (3) $\dfrac{c-a}{b-c}$ (4) 1

MMN027

8. Let $f : \mathbf{R} \to \mathbf{R}$ be defined as,

$$f(x)=\begin{cases}-55x, & \text{if } x<-5\\ 2x^3-3x^2-120x, & \text{if } -5\le x\le 4\\ 2x^3-3x^2-36x-336, & \text{if } x>4,\end{cases}$$

Let $A = \{\boldsymbol{x} \in \boldsymbol{R} : \text{f is increasing}\}$. Then A is equal to :

[JEE (Main) 2021]

(1) $(-\infty, -5) \cup (4, \infty)$ (2) $(-5, \infty)$

(3) $(-\infty, -5) \cup (-4, \infty)$ (4) $(-5, -4) \cup (4, \infty)$

MMN028

9. If Rolle's theorem holds for the function $f(x) = x^3 - ax^2 + bx - 4, x \in [1, 2]$ with $f'\left(\dfrac{4}{3}\right)=0$, then ordered pair (a, b) is equal to : **[JEE (Main) 2021]**

(1) (5, 8) (2) (–5, 8) (3) (5, –8) (4) (–5, –8)

MMN029

10. Consider the function $f : R \to R$ defined by $f(x)=\begin{cases}\left(2-\sin\left(\dfrac{1}{x}\right)\right)|x|, & x\neq 0\\ 0, & x=0\end{cases}$. Then f is :

[JEE (Main) 2021]

(1) monotonic on $(-\infty, 0) \cup (0, \infty)$

(2) not monotonic on $(-\infty, 0)$ and $(0, \infty)$

(3) monotonic on $(0, \infty)$ only

(4) monotonic on $(-\infty, 0)$ only

MMN030

11. Let $f : R \to R$ and $g : R \to \mathrm{R}$ be two functions defined by $f(x) = \log_e(x^2 + 1) - e^{-x} + 1$ and $g(x) = \dfrac{1-2e^{2x}}{e^x}$. Then, for which of the following range of α, the inequality $f\left(g\left(\dfrac{(\alpha-1)^2}{3}\right)\right) > f\left(g\left(\alpha-\dfrac{5}{3}\right)\right)$ holds ? **[JEE (Main) 2022]**

(1) (2, 3) (2) (–2, –1) (3) (1, 2) (4) (–1, 1)

MMN031

12. The number of real solutions of $x^7 + 5x^3 + 3x + 1 = 0$ is equal to ______. **[JEE (Main) 2022]**

(1) 0 (2) 1 (3) 3 (4) 5

MMN032

13. The function $f(x)=xe^{x(1-x)}, x\in R$, is **[JEE (Main) 2022]**

(1) increasing in $\left(-\frac{1}{2},1\right)$ (2) decreasing in $\left(\frac{1}{2},2\right)$

(3) increasing in $\left(-1,-\frac{1}{2}\right)$ (4) decreasing in $\left(-\frac{1}{2},\frac{1}{2}\right)$

MMN033

14. Let $f(x) = 2x + \tan^{-1}x$ and $g(x) = \log_e(\sqrt{1+x^2}+x)$, $x \in [0, 3]$. Then **[JEE (Main) 2023]**

(1) There exists $x\in[0,3]$ such that $f'(x)<g'(x)$

(2) $\max f(x) > \max g(x)$

(3) There exist $0 < x_1 < x_2 < 3$ such that $f(x) < g(x), \forall x\in(x_1,x_2)$

(4) $\min f'(x) = 1 + \max g'(x)$

MMN034

15. Let $f:(0,1)\to\mathbb{R}$ be a function defined by

$f(x)=\frac{1}{1-e^{-x}}$, and

$g(x)=(f(-x)-f(x))$. Consider two statements

(I) g is an increasing function in (0, 1)

(II) g is one-one in (0, 1)

Then, **[JEE (Main) 2023]**

(1) Only (I) is true

(2) Only (II) is true

(3) Neither (I) nor (II) is true

(4) Both (I) and (II) are true

MMN035

16. Let $g(x)=f(x)+f(1-x)$ and $f''(x)>0, x\in(0,1)$. If g is decreasing in the interval $(0, \alpha)$ and increasing in the interval $(\alpha, 1)$, then $\tan^{1}(2\alpha)+\tan^{-1}\left(\frac{1}{\alpha}\right)+\tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$ is equal to :

[JEE (Main) 2023]

(1) $\frac{3\pi}{2}$ (2) π (3) $\frac{5\pi}{4}$ (4) $\frac{3\pi}{4}$

MMN036

EXERCISE - JEE (Advanced) PYQ

1. Let $f:(0,1)\to R$ be defined by $f(x)=\dfrac{b-x}{1-bx}$, where b is a constant such that $0 < b < 1$. Then

[JEE (Advanced) 2011]

(A) f is not invertible on $(0,1)$

(B) $f \neq f^{-1}$ on $(0,1)$ and $f'(b)=\dfrac{1}{f'(0)}$

(C) $f = f^{-1}$ on $(0,1)$ and $f'(b)=\dfrac{1}{f'(0)}$

(D) f^{-1} is differentiable on $(0,1)$

MMN055

2. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is **[JEE (Advanced) 2011]**

MMN056

3. The number of points in $(-\infty, \infty)$, for which $x^2 - x\sin x - \cos x = 0$, is **[JEE (Advanced) 2013]**

(A) 6 (B) 4 (C) 2 (D) 0

MMN057

4. Let $f(x) = x\sin \pi x$, $x > 0$. Then for all natural numbers n, $f'(x)$ vanishes at -

[JEE (Advanced) 2013]

(A) a unique point in the interval $\left(n, n+\dfrac{1}{2}\right)$

(B) a unique point in the interval $\left(n+\dfrac{1}{2}, n+1\right)$

(C) a unique point in the interval $(n, n+1)$

(D) two points in the interval $(n, n+1)$

MMN058

5. Let $a \in \mathbb{R}$ and let $f:\mathbb{R}\to\mathbb{R}$ be given by $f(x) = x^5 - 5x + a$. Then **[JEE (Advanced) 2014]**

(A) $f(x)$ has three real roots if $a > 4$

(B) $f(x)$ has only one real roots if $a > 4$

(C) $f(x)$ has three real roots if $a < -4$

(D) $f(x)$ has three real roots if $-4 < a < 4$

MMN059

6. Let $f:\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)\to\mathbb{R}$ be given by $f(x) = (\log(\sec x + \tan x))^3$ Then :-

[JEE (Advanced) 2014]

(A) $f(x)$ is an odd function

(B) $f(x)$ is a one-one function

(C) $f(x)$ is an onto function

(D) $f(x)$ is an even function

MMN060

7. Let $f, g : [-1, 2]\to\mathbb{R}$ be continuous function which are twice differentiable on the interval $(-1, 2)$. Let the values of f and g at the points -1, 0 and 2 be as given in the following table :

	$x=-1$	$x=0$	$x=2$
$f(x)$	3	6	0
$g(x)$	0	1	-1

In each of the intervals $(-1, 0)$ and $(0, 2)$ the function $(f - 3g)''$ never vanishes. Then the correct statement(s) is(are) **[JEE (Advanced) 2015]**

(A) $f'(x) - 3g'(x) = 0$ has exactly three solutions in $(-1, 0) \cup (0, 2)$

(B) $f'(x) - 3g'(x) = 0$ has exactly one solution in $(-1, 0)$

(C) $f'(x) - 3g'(x) = 0$ has exactly one solutions in $(0, 2)$

(D) $f'(x) - 3g'(x) = 0$ has exactly two solutions in $(-1, 0)$ and exactly two solutions in $(0, 2)$

MMN061

Answer Q.8, Q.9 and Q.10 by appropriately matching the information given in the three columns of the following table. [JEE (Advanced) 2017]

Let $f(x) = x + \log_e x - x\log_e x,\ x \in (0, \infty)$.

* Column 1 contains information about zeros of $f(x)$, $f'(x)$ and $f''(x)$.

* Column 2 contains information about the limiting behavior of $f(x)$, $f'(x)$ and $f''(x)$ at infinity.

* Column 3 contains information about increasing/decreasing nature of $f(x)$ and $f'(x)$.

Column 1	Column 2	Column 3
(I) $f(x) = 0$ for some $x \in (1, e^2)$	(i) $\lim\limits_{x\to\infty} f(x) = 0$	(P) f is increasing in (0,1)
(II) $f'(x) = 0$ for some $x \in (1, e)$	(ii) $\lim\limits_{x\to\infty} f(x) = -\infty$	(Q) f is decreasing in (e, e^2)
(III) $f'(x) = 0$ for some $x \in (0, 1)$	(iii) $\lim\limits_{x\to\infty} f'(x) = -\infty$	(R) f' is increasing in (0,1)
(IV) $f''(x) = 0$ for some $x \in (1, e)$	(iv) $\lim\limits_{x\to\infty} f''(x) = 0$	(S) f' is decreasing in (e, e^2)

8. Which of the following options is the only **CORRECT** combination ?

(A) (IV) (i) (S) (B) (I) (ii) (R) (C) (III) (iv) (P) (D) (II) (iii) (S)

MMN062

9. Which of the following options is the only **CORRECT** combination ?

(A) (III) (iii) (R) (B) (I) (i) (P) (C) (IV) (iv) (S) (D) (II) (ii) (Q)

MMN063

10. Which of the following options is the only **INCORRECT** combination ?

(A) (II) (iii) (P) (B) (II) (iv) (Q) (C) (I) (iii) (P) (D) (III) (i) (R)

MMN064

11. If $f : \mathbb{R} \to \mathbb{R}$ is a twice differentiable function such that $f''(x) > 0$ for all $x \in \mathbb{R}$, and $f\left(\frac{1}{2}\right)=\frac{1}{2}$, $f(1)=1$, then [JEE (Advanced) 2017]

(A) $0 < f'(1) \le \frac{1}{2}$ (B) $f'(1) \le 0$ (C) $f'(1) > 1$ (D) $\frac{1}{2} < f'(1) \le 1$

MMN065

12. For a polynomial $g(x)$ with real coefficient, let m_g denote the number of distinct real roots of $g(x)$. Suppose S is the set of polynomials with real coefficients defined by

$S = \{(x^2 - 1)^2 (a_0 + a_1x + a_2x^2 + a_3x^3) : a_0, a_1, a_2, a_3 \in R\}$.

For a polynomial f, let f' and f'' denote its first and second order derivatives, respectively. Then the minimum possible value of $(m_{f'} + m_{f''})$, where $f \in S$, is ____ [JEE (Advanced) 2020]

MMN066

13. $f : R \to R$ is defined by $f(x) = |x|(x - \sin x)$, then which of the following statements is **TRUE** ? [JEE (Advanced) 2020]

(A) f is one-one, but **NOT** onto (B) f is onto, but **NOT** one-one

(C) f is **BOTH** one-one and onto (D) f is **NEITHER** one-one **NOR** onto

MMN067

14. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \frac{x^2 - 3x - 6}{x^2 + 2x + 4}.$$

Then which of the following statements is (are) TRUE ? [JEE (Advanced) 2021]

(A) f is decreasing in the interval $(-2, -1)$ (B) f is increasing in the interval (1, 2)

(C) f is onto (D) Range of f is $\left[-\frac{3}{2}, 2\right]$

MMN068

JEE (Main) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION–A

- This section contains **TWENTY** questions.
- Each question has **FOUR** options (1), (2), (3) and (4). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

  Full Marks : +4, if only the bubble corresponding to the correct option is darkened.
  Zero Marks : 0, if none of the bubbles is darkened.
  Negative Marks : –1 in all other cases.

1. Let $f(x)$ and $g(x)$ be two continuous functions defined from $R \to R$, such that $f(x_1) > f(x_2)$ and $g(x_1) < g(x_2)$, $\forall\ x_1 > x_2$, then solution set of $f\left(g(\alpha^2 - 2\alpha)\right) > f\left(g(3\alpha - 4)\right)$ is

(1) R (2) ϕ (3) (1, 4) (4) R – [1, 4]

MMN069

2. When $0 \le x \le 1$, $f(x) = |x| + |x-1|$ is -

(1) strictly increasing (2) strictly decreasing
(3) constant (4) None of these

MMN070

3. Function $f(x) = x^2(x-2)^2$ is -

(1) increasing in [0, 1]; [2, ∞) (2) decreasing in [0, 1]; [2, ∞]
(3) decreasing function (4) increasing function

MMN071

4. If f and g are two strictly decreasing function such that fog is defined, then fog will be -

(1) increasing function (2) decreasing function
(3) neither increasing nor decreasing (4) None of these

MMN072

5. The function x^x strictly decreases on the interval -

(1) $(0, e)$ (2) (0, 1) (3) $\left(0, \frac{1}{e}\right)$ (4) None of these

MMN073

6. Which one of the following statements does not hold good for the function $f(x) = cos^{-1}\left(2x^2 - 1\right)$?

(1) f is not differentiable at $x = 0$ (2) f is monotonic
(3) f is even (4) f has an extremum

MMN074

7. The function $f(x) = tan^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is -

(1) strictly increasing in its domain
(2) strictly decreasing in its domain
(3) strictly decreasing in (– ∞, 0) and strictly increasing in (0, ∞)
(4) strictly increasing in (– ∞, 0) and strictly decreasing in (0, ∞)

MMN075

8. The function $f : [a, \infty) \rightarrow R$ where R denotes the range corresponding to the given domain, with rule $f(x) = 2x^3 - 3x^2 + 6$, will have an inverse provided

(1) $a \geq 1$ (2) $a \geq 0$ (3) $a \leq 0$ (4) $a \leq 1$

MMN076

9. If the equation $a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x = 0$ has a positive root $x = \alpha$, then the equation $na_n x^{n-1} + (n-1)\, a_{n-1}x^{n-2} + \dots + a_1 = 0$ has a positive root, which is-

(1) Smaller than α (2) Greater than α
(3) Equal to α (4) Greater than or equal to α

MMN077

10. The value of c in Lagrange's theorem for the function $f(x) = \begin{cases} x\cos\left(\dfrac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ in the interval $[-1, 1]$ is-

(1) 0 (2) $\dfrac{1}{2}$
(3) $-\dfrac{1}{2}$ (4) Non existent in the interval

MMN078

11. If $f : R \rightarrow R^+$ be a differentiable function and $g(x) = e^{2x}\left(2f(x) - 3(f(x))^2 + 2(f(x))^3\right)\ \forall\, x \in R$, then which of the following is/are always correct -

(1) $g(x)$ is strictly increasing wherever $f(x)$ is strictly increasing
(2) $g(x)$ is strictly increasing wherever $f(x)$ is strictly decreasing
(3) $g(x)$ is strictly decreasing wherever $f(x)$ is strictly decreasing
(4) $g(x)$ is strictly decreasing wherever $f(x)$ is strictly increasing

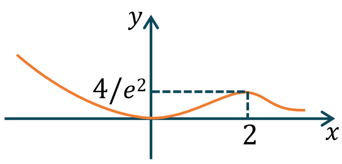
MMN079

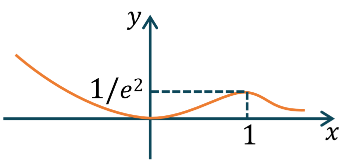
12. Let a, b, c, d are non-zero real numbers such that $6a + 4b + 3c + 3d = 0$, then the equation $ax^3 + bx^2 + cx + d = 0$ has

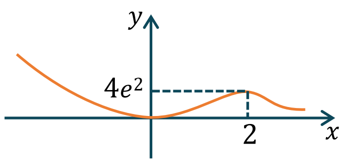
(1) atleast one root in [–2, 0] (2) atleast one root in [0, 2]
(3) atleast two roots in [–2, 2] (4) no root in [–2, 2]

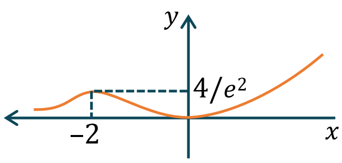
MMN080

13. Draw graph of $y = x^2 e^{-2x}$.

(1) 

(2) 

(3) 

(4) 

MMN081

14. If $f(x)=xe^{x(1-x)}$, $x \in R$, then $f(x)$ is :-

(1) increasing on R
(2) Increasing on $[-1/2, 1]$
(3) Decreasing on R
(4) Decreasing on $[-1/2, 1]$

MMN082

15. Let f and g be two differentiable functions on $\boldsymbol{R}$ such that $f'(x) > 0$ and $g'(x) < 0$, for all $x \in \boldsymbol{R}$. Then for all x :-

(1) $g(f(x)) > g(f(x-1))$
(2) $f(g(x)) > f(g(x-1))$
(3) $g(f(x)) < g(f(x+1))$
(4) $f(g(x)) > f(g(x+1))$

MMN083

16. The function f defined by $f(x)=x^3-3x^2+5x+7$, is :

(1) decreasing in $(0, \infty)$ and increasing in $(-\infty, 0)$
(2) decreasing in R
(3) increasing in R
(4) increasing in $(0, \infty)$ and decreasing in $(-\infty, 0)$

MMN084

17. Let $f(x)=\dfrac{x}{\sqrt{a^2+x^2}}-\dfrac{d-x}{\sqrt{b^2+(d-x)^2}}$, $x \in R$, where a, b and d are non-zero real constants.

Then :-

(1) f is a decreasing function of x
(2) f is neither increasing nor decreasing function of x
(3) f' is not a continuous function of x
(4) f is an increasing function of x

MMN085

18. Let $f : [0, 2] \to R$ be a twice differentiable function such that $f''(x) > 0$, for all $x \in (0, 2)$.
If $\phi(x) = f(x) + f(2-x)$, then ϕ is :

(1) decreasing on $(0, 2)$
(2) decreasing on $(0, 1)$ and increasing on $(1, 2)$
(3) increasing on $(0, 2)$
(4) increasing on $(0, 1)$ and decreasing on $(1, 2)$

MMN086

19. The value of c in the Lagrange's mean value theorem for the function $f(x)=x^3-4x^2+8x+11$, when $x \in [0, 1]$ is :

(1) $\dfrac{2}{3}$
(2) $\dfrac{\sqrt{7}-2}{3}$
(3) $\dfrac{4-\sqrt{5}}{3}$
(4) $\dfrac{4-\sqrt{7}}{3}$

MMN087

20. For all x in $[1, 2]$
Let $f''(x)$ of a non-constant function $f(x)$ exist and satisfy $|f''(x)| \leq 2$. If $f(1) = f(2)$, then

(1) There exist some $a \in (1, 2)$ such that $f'(a) = 0$
(2) $f(x)$ is strictly increasing in $(1, 2)$
(3) There exist no $c \in (1, 2)$ such that $f'(c) > 0$
(4) $|f'(x)| > 2 \ \forall x \in [1,2]]$

MMN088

SECTION-B

- This section will have **TEN** questions. Candidate can choose to attempt any 5 question out of these 10 questions. In case if candidate attempts more than 5 questions, first 5 attempted questions will be considered for marking.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (Answer should be rounded off to the nearest integer).
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +4, if only correct answer is given.
Zero Marks : 0, if no answer is given.
Negative Marks : -1 for incorrect answer

---

1. Consider the function for $x \in [-2, 3]$

$f(x)=\begin{cases} -6 & ; \quad x=1 \\ \dfrac{x^3-2x^2-5x+6}{x-1} & ; \quad x\neq 1 \end{cases}$. The value of $\dfrac{1}{c}$ obtained by applying Rolle's theorem for which $f'(c) = 0$ is

MMN089

2. The function $f(x)=x(x+3)e^{-x/2}$ satisfies all the conditions of Rolle's theorem on [–3, 0]. The value of c which verifies Rolle's theorem, then c^2

MMN090

3. Let $f(x)=\begin{cases} x & x\in[1,2] \\ 5-x & x\in(2,4) \\ 2 & x=4 \\ 7-x & x\in(4,6] \end{cases}$ then Function is not increasing at how many integral values of x when $x \in [1, 6)$?

MMN091

4. The function $\dfrac{|x-1|}{x^2}$ is at the how many integral points monotonically increasing

MMN092

5. If the Rolle's theorem holds for the function $f(x)=2x^3+ax^2+bx$ in the interval [–1, 1] for the point $c = \dfrac{1}{2}$, then the value of $|2a + b|$ is:

MMN093

6. Let the function, f: [–7, 0] → R be continuous on [–7, 0] and differentiable on (–7, 0). If $f(-7) = -3$ and $f'(x) \leq 2$, for all $x \in (-7, 0)$, then for all such functions f, Maximum value of $\dfrac{f(-1)+f(0)}{5} =$

MMN094

7. If c is a point at which Rolle's theorem holds for the function, $f(x)=\log_e\left(\dfrac{x^2+\alpha}{7x}\right)$ in the interval [3,4], where $\alpha \in R$, then $48.f''(c)$ is equal to

MMN095

8. If $f(x) = (x-4)(x-5)(x-6)(x-7)$ then, minimum number of roots of $f(x) = 0$ in $(4, 7)$.

MMN096

9. Let $f : [1, 2] \to [1, 4]$ and $g : [1, 2] \to [2, 7]$ be two continuous bijective functions such that $f(1) = 4$ & $g(2) = 7$. The number of solutions of the equation $f(x) = g(x)$ in $(1, 2)$, is :

MMN097

10. If $[a, b], (b < 1)$ is largest interval in which $f(x) = 3x^4 + 8x^3 - 6x^2 - 24x + 19$ is strictly increasing then $\frac{a}{b}$ is ....

MMN098

JEE (Advanced) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION-I

- This section contains **EIGHT** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If only (all) the correct option(s) is (are) chosen.
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen.
Partial Marks	:	+2	If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	:	–2	In all other cases.

For Example : If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

1. Which of the following is/are correct ?

(A) Between any two roots of $e^x cosx = 1$, there exists atleast one root of $tanx = 1$.

(B) Between any two roots of $e^x sinx = 1$, there exists atleast one root of $tanx = -1$.

(C) Between any two roots of $e^x cosx = 1$, there exists atleast one root of $e^x sinx = 1$.

(D) Between any two roots of $e^x sinx = 1$, there exists atleast one root of $e^x cosx = -1$.

MMN099

2. Let $f(x) = \sin^2 x$ & $0 < A < B$, where $A, B \in \left(0, \frac{\pi}{4}\right)$, then

(A) $(B-A)\sin 2A < \sin^2 B - \sin^2 A < (B-A)\sin 2B$

(B) $(B-A)\sin 2A > \sin^2 B - \sin^2 A > (B-A)\sin 2B$

(C) $\cos 2B < \cos 2A$

(D) $B\cos 2A > A\cos 2B$

MMN100

3. For the equation $\frac{e^{-x}}{x+1}=a$; which of the following statement(s) is/are correct ?

(A) If $a \in (0, \infty)$, then equation has 2 real and distinct roots
(B) If $a \in (-\infty, -e^2)$, then equation has 2 real & distinct roots.
(C) If $a \in (0, \infty)$, then equation has 1 real root
(D) If $a \in (-e, 0)$, then equation has no real root.

MMN101

4. The number of roots of the equation $x^2-2x-log_2|1-x|=3$ is

(A) 4 (B) 2 (C) 1 (D) 0

MMN102

5. Let $f(x)=x^{m/n}$ for $x \in R$ where m and n are integers, m even and n odd and $0 < m < n$. Then

(A) $f(x)$ decreases on $(-\infty, 0]$
(B) $f(x)$ increases on $[0, \infty)$
(C) $f(x)$ increases on $(-\infty, 0]$
(D) $f(x)$ decreases on $[0, \infty)$

MMN103

6. If p, q, r be real, then the intervals in which, $f(x) = \begin{vmatrix} x+p^2 & pq & pr \\ pq & x+q^2 & qr \\ pr & qr & x+r^2 \end{vmatrix}$,

(A) increase is $x < \frac{2}{3}\ -(p^2+q^2+r^2), x > 0$
(B) decrease is $\left(-\frac{2}{3}(p^2+q^2+r^2),\ 0\right)$
(C) decrease is $x < -\ \frac{2}{3}\ (p^2+q^2+r^2), x > 0$
(D) increase is $\left(-\frac{2}{3}(p^2+q^2+r^2),\ 0\right)$

MMN104

7. If $f(x) = \frac{x^2}{2-2\cos x}$; $g(x) = \frac{x^2}{6x-6\sin x}$ where $0 < x < 1$, then

(A) $'f'$ is increasing function
(B) $'g'$ is decreasing function
(C) $\frac{f(x)}{g(x)}$ is increasing function
(D) $g(f(x))$ is decreasing function

MMN105

8. For the function $f(x)=x cot^{-1}x, x \geq 0$

(A) there is atleast one $x \in (0, 1)$ for which $cot^{-1}x=\frac{x}{1+x^2}$
(B) for atleast one x in the interval $(0, \infty)$, $f\left(x+\frac{2}{\pi}\right)-f(x) < 1$
(C) number of solution of the equation $f(x) = sec\ x$ is 1
(D) $f'(x)$ is strictly decreasing in the interval $(0, \infty)$

MMN106

SECTION–II

- This section contains **TWO** paragraphs.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D) **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :
  Full Marks : +3 if only the bubble corresponding to the correct answer is darkened.
  Zero Marks : 0 in all other cases.

Comprehension # 1 (Q. No. 9 & 10)

Consider a function f defined by $f(x)=\sin^{-1}\sin\left(\frac{x+\sin\ x}{2}\right)$, $\forall\ x\in[0,\pi]$, which satisfies

$f(x)+f(2\pi-x)=\pi,\ \ \forall\ x\in[\pi,\ 2\pi]$ and $f(x)=f(4\pi-x)$ for all $x\in[2\pi,\ 4\pi]$, then

9. If α is the length of the largest interval on which $f(x)$ is increasing, then $\alpha=$

(A) $\frac{\pi}{2}$ (B) π (C) 2π (D) 4π

MMN107

10. If $f(x)$ is symmetric about $x=\beta$, then $\beta=$

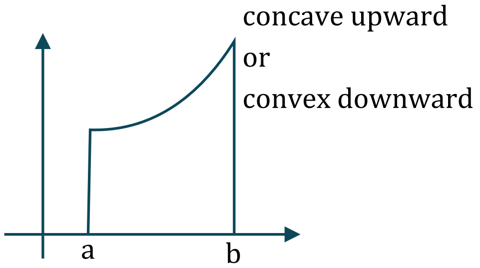
(A) $\frac{\alpha}{2}$ (B) α (C) $\frac{\alpha}{4}$ (D) 2α

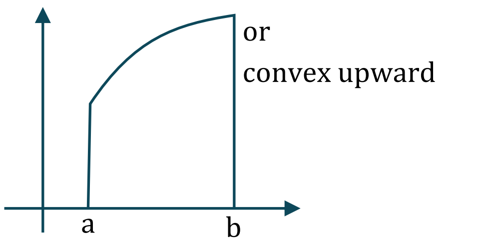
MMN108

Comprehension # 2 (Q. No. 11 & 12)

Concavity and Convexity:

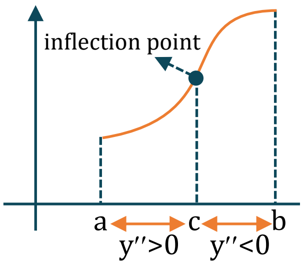
If $f''(x)>0\ \forall\ x\in(a,b)$, then the curve $y=f(x)$ is concave up (or convex down) in (a,b) and
If $f''(x)<0\ \forall\ x\in(a,b)$ then the curve $y=f(x)$ is concave down (or convex up) in (a,b).





Inflection point :

The point where concavity of the curve changes is known as point of inflection (at inflection point $f''(x)$ is equal to 0 or undefined).



11. Number of point of inflection for $f(x)=(x-1)^3(x-2)^2$, is

(A) 1 (B) 2 (C) 3 (D) 4

MMN109

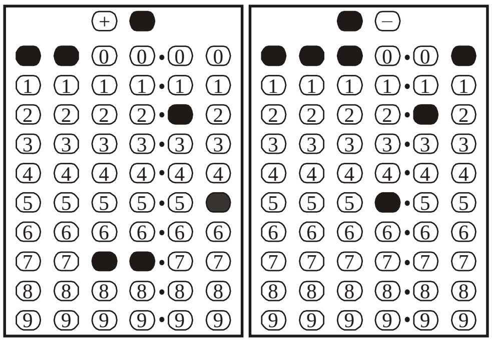
12. Exhaustive set of values of 'a' for which the function $f(x)=x^4+ax^3+\frac{3x^2}{2}+1$ will be concave upward along the entire real line, is :

(A) [−1,1] (B) [−2,2] (C) [0,2] (D) [0,4]

MMN110

SECTION-III

- This section contains **SIX** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30, if answer is 11.36777..... then both 11.36 and 11.37 will be correct) by darken the corresponding bubbles in the ORS.
  For Example : If answer is –77.25, 5.2 then fill the bubbles as follows.



- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If ONLY the correct numerical value is entered as answer.
  Zero Marks : 0 In all other cases.

13. If the set of all values of the parameter 'a' for which the function $f(x) = \sin 2x - 8(a+1)\sin x + (4a^2 + 8a - 14)\, x$ increases for all $x \in R$ and has no critical points for all $x \in R$, is $(-\infty,\ -m - \sqrt{n}\ ) \cup (\sqrt{n}, \infty)$ then $(m+n)$ is (where m, n are prime numbers):

MMN111

14. Let $f(x)$ = Max. $\{x^2, (1-x)^2, 2x(1-x)\}$ where $x \in [0, 1]$ If Rolle's theorem is applicable for $f(x)$ on largest possible interval $[a,\ b]$ then the value of $2(a+b+c)$ when $c \in (a, b)$ such that $f'(c) = 0$, is ___________.

MMN112

15. If Rolle's theorem is applicable to the function $f(x) = \dfrac{\ell nx}{x}$, $(x > 0)$ over the interval $[a, b]$ where $a \in I, b \in I$, then the value of smallest value of $a + b$ can be

MMN113

16. Find C of the LMVT for fraction $f(x) = 3x^2 + 5x + 7$ in interval [1,3]

MMN114

17. If $f(x) = \dfrac{49-7}{3}x^3 + (a-3)x^2 + x + 5$ is monotonic for all $x \in R$ for $a \in (m, M)$ the $M - m =$

MMN115

18. $S_1 \to \tan^{-1} x > \dfrac{x}{1+\dfrac{x^2}{3}}$ $x \in (0, \infty)$

$S_2 \to 1 + x\ell n\left(x + \sqrt{x^2+1}\right) > \sqrt{1+x^2}$ $\quad x \ge 0$

$S_3 \to x - \dfrac{x^3}{6} < \sin x < x$ $\quad 0 < x \le \dfrac{\pi}{2}$

Total numbers correct statements=

MMN116

ANSWER KEY

EXERCISE - O

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	C	C	A	C	A	B	A	B	A
Que.	11	12	13	14	15	16	17	18		
Ans.	A,D	A,C,D	A,D	B,C	C	D	A	B		

EXERCISE - S

1. 0 **2.** 8 **3.** 9 **4.** 8 **5.** −10
6. 1 **7.** −2

EXERCISE - JEE (Main) PYQ

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	2	2	4	2	1	3	4	1	2
Que.	11	12	13	14	15	16				
Ans.	1	2	1	2	4	2				

EXERCISE - JEE (Advanced) PYQ

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	2	C	B,C	B,D	A,B,C	B,C	D	D	D
Que.	11	12	13	14						
Ans.	C	5	C	A,B						

JEE (Main) Practice Paper

Section-A	Q.	1	2	3	4	5	6	7	8	9	10
	A.	3	3	1	1	3	2	4	1	1	4
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	1	2	2	2	4	3	4	2	4	1
Section-B	Q.	1	2	3	4	5	6	7	8	9	10
	A.	2	4	2	0	1	4	4	3	1	2

JEE (Advanced) Practice Paper

Section-I	Q.	1	2	3	4	5	6	7	8
	A.	A,B,C,D	A,C,D	B,C,D	A	A,B	A,B	A,B,C	B,D
Section-II	Q.	9	10	11	12				
	A.	C	B	C	B				
Section-III	Q.	13	14	15	16	17	18		
	A.	7	3	6	2	6	3		