

EXERCISE - O

SINGLE CORRECT TYPE QUESTIONS

1. Primitive of $\frac{3x^4 - 1}{(x^4 + x + 1)^2}$ w.r.t. x is -

- (A) $\frac{x}{x^4 + x + 1} + c$ (B) $-\frac{x}{x^4 + x + 1} + c$ (C) $\frac{x+1}{x^4 + x + 1} + c$ (D) $-\frac{x+1}{x^4 + x + 1} + c$

MII004

2. $\int \frac{x^2 + x}{(e^x + x + 1)^2} dx$ equals

- (A) $C - \ln(1 + (x+1)e^x) - \frac{1}{1 + (x+1)e^x}$ (B) $C - \ln(1 + (x+1)e^x) + \frac{1}{1 + (x-1)e^x}$
 (C) $C - \ln(1 - (x^2 - 1)e^{-x}) + \frac{1}{1 + (x+1)e^x}$ (D) $C - \ln(1 + (x+1)e^{-x}) - \frac{1}{1 + (x+1)e^{-x}}$

MII005

3. $\int x \cdot \frac{\ln(x + \sqrt{1+x^2})}{\sqrt{1+x^2}} dx$ equals -

- (A) $\sqrt{1+x^2} \ln(x + \sqrt{1+x^2}) - x + c$ (B) $\frac{x}{2} \cdot (x + \sqrt{1+x^2}) \ln^2 - \frac{x}{\sqrt{1+x^2}} + c$
 (C) $\frac{x}{2} \cdot \ln^2(x + \sqrt{1+x^2}) + \frac{x}{\sqrt{1+x^2}} + c$ (D) $\sqrt{1+x^2} \ln(x + \sqrt{1+x^2}) + x + c$

MII010

4. The integral $\int \sqrt{\cot x} e^{\sqrt{\sin x}} \sqrt{\cos x} dx$ equals

- (A) $\frac{\sqrt{\tan x} e^{\sqrt{\sin x}}}{\sqrt{\cos x}} + C$ (B) $2e^{\sqrt{\sin x}} + C$ (C) $-\frac{1}{2}e^{\sqrt{\sin x}} + C$ (D) $\frac{\sqrt{\cot x} e^{\sqrt{\sin x}}}{2\sqrt{\cos x}} + C$

MII012

5. $\int (\sin(101x) \cdot \sin^{99} x) dx$ equals

- (A) $\frac{\sin(100x)(\sin x)^{100}}{100} + C$ (B) $\frac{\cos(100x)(\sin x)^{100}}{100} + C$
 (C) $\frac{\cos(100x)(\cos x)^{100}}{100} + C$ (D) $\frac{\sin(100x)(\sin x)^{101}}{101} + C$

MII013

6. $\int \left[\sin \alpha \sin(x - \alpha) + \sin^2 \left(\frac{x}{2} - \alpha \right) \right] dx$ equals

- (A) $\frac{1}{2}(x + \sin x) + C$ (B) $\frac{1}{2}(x^2 - \sin x) + C$ (C) $\frac{1}{2}(x - \sin x) + C$ (D) $\frac{1}{2}(x - \cos x) + C$

MII015

7. $\int \frac{dx}{(x-\alpha)\sqrt{(x-\alpha)(x-\beta)}}$ equals
 (A) $\frac{2}{\alpha+\beta}\sqrt{\frac{x+\beta}{x+\alpha}}+C$ (B) $\frac{-2}{\alpha-\beta}\sqrt{\frac{x-\beta}{x-\alpha}}+C$ (C) $\frac{-2}{\alpha-\beta}\sqrt{\frac{x+\beta}{x+\alpha}}+C$ (D) $\frac{2}{\alpha+\beta}\sqrt{\frac{x-\beta}{x+\alpha}}+C$

MII016

8. $\int \frac{4x^5 - 7x^4 + 8x^3 - 2x^2 + 4x - 7}{x^2(x^2 + 1)^2} dx$ equals
 (A) $4\ln x - \frac{7}{x} - 6\tan^{-1}(x) - \frac{6x}{1-x^2} + C$ (B) $2\ln x + \frac{1}{x} + 6\tan^{-1}(x^2) + \frac{6x}{1-x^2} + C$
 (C) $\ln x - \frac{1}{x} + \tan^{-1}(x) + \frac{x}{1-x^2} + C$ (D) $4\ln x + \frac{7}{x} + 6\tan^{-1}(x) + \frac{6x}{1+x^2} + C$

MII018

9. $\int \frac{3x^2 + 1}{(x^2 - 1)^3} dx$ equals (where K is constant of integration)
 (A) $\frac{x}{(x^2 - 1)^2} + K$ (B) $K - \frac{x}{(x^2 + 1)^2}$ (C) $K - \frac{x}{(x^2 - 1)^2}$ (D) $K - \frac{x^2}{(x + 1)^2}$

MII019

10. $\int \frac{x^2 + 3}{x^6(x^2 + 1)} dx$ equals
 (A) $C - \frac{2}{x} + \frac{2}{3x^3} - \frac{3}{5x^5} - 2\tan^{-1} x$ (B) $C - \frac{2}{x^2} + \frac{2}{3x^3} - \frac{3}{5x^5} - 2\tan^{-1} x$
 (C) $C - \frac{2}{x} + \frac{2}{3x^2} - \frac{3}{5x^4} - 2\tan^{-1} x$ (D) $C - \frac{2}{x^2} + \frac{2}{3x^4} + \frac{3}{5x^6} + 2\tan^{-1} x$

MII020

MULTIPLE CORRECT TYPE QUESTIONS

11. The value of $\int \frac{\ln\left(\frac{x-1}{x+1}\right)}{x^2 - 1} dx$ is equal to
 (A) $\frac{1}{2}\ln^2 \frac{x-1}{x+1} + C$ (B) $\frac{1}{4}\ln^2 \frac{x-1}{x+1} + C$ (C) $\frac{1}{2}\ln^2 \frac{x+1}{x-1} + C$ (D) $\frac{1}{4}\ln^2 \frac{x+1}{x-1} + C$

MII045

12. If $\int \frac{(x-1) dx}{x^2 \sqrt{2x^2 - 2x + 1}} = \frac{\sqrt{f(x)}}{g(x)} + C$, where $f(x)$ is a quadratic expression and $g(x)$ is a monic linear expression.
 (A) $f(x) = 2x^2 - 2x + 1$ (B) $g(x) = x + 1$
 (C) $g(x) = x$ (D) $f(x) = 2x^2 - 2x$

MII046

13. If $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}} dx = Ax + B \ln |9e^{2x} - 4| + C$, then
 (A) $A + 18B = 16$ (B) $18B - A = 19$ (C) $A - 18B = 17$ (D) $A + 18B = 32$

MII047

14. $\int \frac{x+x^{2/3}+x^{1/6}}{x(1+x^{1/3})} dx = px^{2/3} + q \tan^{-1}(x^{1/6}) + c$, then :-
 (A) $p < q$ (B) $p = q$ (C) $pq = 9$ (D) $p > q$

MII048

MATCHING LIST TYPE QUESTION

- | 15. | Column-I | | Column-II |
|-----|---|-----|------------|
| (A) | Let $f(x) = \int x^{\sin x} (1+x \cos x \cdot \ln x + \sin x) dx$ and $f\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4}$
then the value of $f(\pi)$ is | (P) | rational |
| (B) | Let $g(x) = \int \frac{1+2\cos x}{(\cos x+2)^2} dx$ and $g(0) = 0$
then the value of $g\left(\frac{\pi}{2}\right)$ is | (Q) | irrational |
| (C) | Let $k(x) = \int \frac{(x^2+1)dx}{\sqrt[3]{x^3+3x+6}}$ and $k(-1) = \frac{1}{\sqrt[3]{2}}$ then the value
of $k(-2)$ is | (R) | integral |
| (D) | If $\int \frac{\cos x - \sin x + 1 - x}{e^x + \sin x + x} dx = \ln(f(x)) + g(x) + C$
(where C is the constant of integration and $f(x)$ is positive),
then $f(0) + g(0)$ is | (S) | prime |

MII052

EXERCISE - S

1. A function $y = f(x)$ satisfies $f''(x) = -\frac{1}{x^2} - \pi^2 \sin(\pi x)$; $f'(2) = \pi + \frac{1}{2}$ and $f(1) = 0$. The value of $\left[f\left(\frac{1}{2}\right) \right]$ (where $[.]$ represents greatest integer function) is - MII021
2. If $\int \frac{(2x+3)dx}{x(x+1)(x+2)(x+3)+1} = C - \frac{1}{f(x)}$, where $f(x)$ is of the form of $ax^2 + bx + c$ then $(a + b + c)$ equals MII022
3. $\int \frac{dx}{\sqrt[3]{x^{5/2}(x+1)^{7/2}}}$ is equal to $K\left(\frac{x+1}{x}\right)^{-1/6} + C$ (where 'C' is integration constant) then $K =$ MII024
4. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}(x + \sqrt{x})dx$ is equal to $be^{\sqrt{x}}[x - \sqrt{x} + 1] + C$ (where 'C' is integration constant) then $b =$ MII025
5. If the value $\int \frac{1 - (\cot x)^{2008}}{\tan x + (\cot x)^{2009}} dx = \frac{1}{k} \ln |\sin^k x + \cos^k x| + C$, then find k . MII053
6. Let $f(x)$ is a quadratic function such that $f(0) = 1$ and $\int \frac{f(x)dx}{x^2(x+1)^3}$ is a rational function, find the value of $f'(0)$ MII054
7. If the integral $\int \frac{5 \tan x}{\tan x - 2} dx = x + a \ln |\sin x - 2 \cos x| + k$ then a is equal to : MII055
8. If $\int \frac{x^4 - 2x^2 - 3}{x^4 \sqrt{x(1+x^{-2})^2 + 1}} dx = 2(a + bx + cx^{-1} + dx^{-3})^{\frac{1}{2}} + K$, where K is the integration constant, then the value of $a + b + c + d$ is MII056
9. Let F is anti-derivative of f defined by $f(x) = 5(x^4 - 1)$ where $F(1) = 20$, then value of $F(2)$ is MII057
10. $\int \frac{dx}{16+x^2} = f(x)$ where $f(4) = \frac{\pi}{16}$ and $(\pi = 3.14)$, then value of $f(2) + f\left(\frac{4}{3}\right)$ is - MII058

EXERCISE - JEE (Main) PYQ

1. If $\int \frac{\cos x dx}{\sin^3 x (1 + \sin^6 x)^{2/3}} = f(x)(1 + \sin^6 x)^{1/\lambda} + c$

where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to

[JEE (Main) 2020]

- (1) -2 (2) $-\frac{9}{8}$ (3) 2 (4) $\frac{9}{8}$

MII035

2. If $\int \frac{d\theta}{\cos^2 \theta (\tan 2\theta + \sec 2\theta)} = \lambda \tan \theta + 2 \log_e |f(\theta)| + C$ where C is a constant of integration, then

the ordered pair $(\lambda, f(\theta))$ is equal to :

[JEE (Main) 2020]

- (1) $(-1, 1 + \tan \theta)$ (2) $(-1, 1 - \tan \theta)$ (3) $(1, 1 - \tan \theta)$ (4) $(1, 1 + \tan \theta)$

MII036

3. The integral $\int \frac{dx}{(x+4)^7 (x-3)^7}$ is equal to :

(where C is a constant of integration)

[JEE (Main) 2020]

- (1) $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$ (2) $-\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$ (3) $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{\frac{3}{7}} + C$ (4) $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{\frac{13}{7}} + C$

MII037

4. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \sin^{-1} \left(\frac{\sin x + \cos x}{b} \right) + c$, where c is a constant of integration, then the ordered

pair (a, b) is equal to :

[JEE (Main) 2021]

- (1) $(-1, 3)$ (2) $(3, 1)$ (3) $(1, 3)$ (4) $(1, -3)$

MII038

5. $\int \frac{(x^2 + 1)e^x}{(x+1)^2} dx = f(x)e^x + C$, Where C is a constant, then $\frac{d^3 f}{dx^3}$ at $x = 1$ is equal to :

[JEE (Main) 2022]

- (1) $-\frac{3}{4}$ (2) $\frac{3}{4}$ (3) $-\frac{3}{2}$ (4) $\frac{3}{2}$

MII039

6. For $I(x) = \int \frac{\sec^2 x - 2022}{\sin^{2022} x} dx$ if $I\left(\frac{\pi}{4}\right) = 2^{1011}$, then

[JEE (Main) 2022]

- (1) $3^{1010} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$ (2) $3^{1010} I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$
 (3) $3^{1011} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$ (4) $3^{1011} I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$

MII040

7. For, $\alpha, \beta, \gamma, \delta \in \mathbb{N}$, if $\int \left(\left(\frac{x}{e} \right)^{2x} + \left(\frac{e}{x} \right)^{2x} \right) \log_e x dx = \frac{1}{\alpha} \left(\frac{x}{e} \right)^{\beta x} - \frac{1}{\gamma} \left(\frac{e}{x} \right)^{\delta x} + C$,

Where $e = \sum_{n=0}^{\infty} \frac{1}{n!}$ and C is constant of integration, then $\alpha + 2\beta + 3\gamma - 4\delta$ is equal to:

[JEE (Main) 2023]

- (1) 1 (2) -4 (3) -8 (4) 4

MII064

8. Let $f(x) = \int \frac{2x}{(x^2+1)(x^2+3)} dx$.

If $f(3) = \frac{1}{2}(\log_e 5 - \log_e 6)$, then $f(4)$ is equal to

[JEE (Main) 2023]

- (1) $\frac{1}{2}(\log_e 17 - \log_e 19)$ (2) $\log_e 17 - \log_e 18$
 (3) $\frac{1}{2}(\log_e 19 - \log_e 17)$ (4) $\log_e 19 - \log_e 20$

MII065

9. Let $I(x) = \int \sqrt{\frac{x+7}{x}} dx$ and $I(9) = 12 + 7 \log_e 7$. If $I(1) = \alpha + 7 \log_e (1+2\sqrt{2})$, then α^4 is equal to _____.

[JEE (Main) 2023]

MII066

10. If $\int \sqrt{\sec 2x - 1} dx = \alpha \log_e \left| \cos 2x + \beta + \sqrt{\cos 2x \left(1 + \cos \frac{1}{\beta} x \right)} \right| + \text{constant}$, then $\beta - \alpha$ is equal to ____.

[JEE (Main) 2023]

MII067

11. Let $I(x) = \int \frac{x^2(x \sec^2 x + \tan x)}{(x \tan x + 1)^2} dx$. If $I(0) = 0$ the $I\left(\frac{\pi}{4}\right)$ is equal to

[JEE (Main) 2023]

- (1) $\log_e \frac{(x+4)^2}{16} - \frac{\pi^2}{4(\pi+4)}$ (2) $\log_e \frac{(x+4)^2}{16} + \frac{\pi^2}{4(\pi+4)}$
 (3) $\log_e \frac{(x+4)^2}{32} - \frac{\pi^2}{4(\pi+4)}$ (4) $\log_e \frac{(x+4)^2}{32} + \frac{\pi^2}{4(\pi+4)}$

MII068

EXERCISE - JEE (Advanced) PYQ

1. $\int \frac{x^2 - 1}{x^3 \sqrt{2x^4 - 2x^2 + 1}} dx$ is equal to -

[JEE 2006]

(A) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^2} + c$

(B) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^3} + c$

(C) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x} + c$

(D) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + c$

MII059

2. Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for $n \geq 2$ and $g(x) = \underbrace{(f \circ f \circ \dots \circ f)}_{f \text{ occurs } n \text{ times}}(x)$. Then $\int x^{n-2} g(x) dx$ equals.

(A) $\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}} + K$

(B) $\frac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}} + K$

[JEE 2007]

(C) $\frac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}} + K$

(D) $\frac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}} + K$

MII060

3. Let $F(x)$ be an indefinite integral of $\sin^2 x$.

[JEE 2007]

Statement-1 : The function $F(x)$ satisfies $F(x + \pi) = F(x)$ for all real x .

because

Statement-2 : $\sin^2(x + \pi) = \sin^2 x$ for all real x .

- (A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
 (B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
 (C) Statement-1 is True, Statement-2 is False.
 (D) Statement-1 is False, Statement-2 is True.

MII061

4. Let $I = \int \frac{e^x}{e^{4x} + e^{2x} + 1} dx$, $J = \int \frac{e^{-x}}{e^{-4x} + e^{-2x} + 1} dx$. Then, for an arbitrary constant c , the value of

$J - I$ equals

[JEE 2008]

(A) $\frac{1}{2} \ln \left(\frac{e^{4x} - e^{2x} + 1}{e^{4x} + e^{2x} + 1} \right) + c$

(B) $\frac{1}{2} \ln \left(\frac{e^{2x} + e^x + 1}{e^{2x} - e^x + 1} \right) + c$

(C) $\frac{1}{2} \ln \left(\frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1} \right) + c$

(D) $\frac{1}{2} \ln \left(\frac{e^{4x} + e^{2x} + 1}{e^{4x} - e^{2x} + 1} \right) + c$

MII062

5. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$ equals (for some arbitrary constant K) [JEE (Advanced) 2012]

(A) $-\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} - \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$

(B) $\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} - \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$

(C) $-\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} + \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$

(D) $\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} + \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$

MII063

6. $\int [1 + \tan x \cdot \tan(x + \alpha)] dx$ is equal to

(1) $\cos \alpha \cdot \ln \left| \frac{\sin x}{\sin(x + \alpha)} \right| + C$

(2) $\tan \alpha \cdot \ln \left| \frac{\sin x}{\sin(x + \alpha)} \right| + C$

(3) $\cot \alpha \cdot \ln \left| \frac{\sec(x + \alpha)}{\sec x} \right| + C$

(4) $\cot \alpha \cdot \ln \left| \frac{\cos(x + \alpha)}{\cos x} \right| + C$

MII074

7. $\int \frac{x^2 + \cos^2 x}{1 + x^2} \operatorname{cosec}^2 x dx$ is equal to:

(1) $-\tan^{-1} x + \cot x + c$

(2) $2\tan^{-1} x + c$

(3) $-\tan^{-1} x - \frac{\operatorname{cosec} x}{\sec x} + c$

(4) none of these

MII075

8. $\int \sqrt{\frac{x-1}{x+1}} \cdot \frac{1}{x^2} dx$ is equal to

(1) $\sin^{-1} \frac{1}{x} + \frac{\sqrt{x^2-1}}{x} + c$

(2) $\frac{\sqrt{x^2-1}}{x} + \cos^{-1} \frac{1}{x} + c$

(3) $\sec^{-1} x - \frac{\sqrt{x^2-1}}{x} + c$

(4) $\tan^{-1} \sqrt{x^2+1} - \frac{\sqrt{x^2-1}}{x} + c$

MII076

9. If $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}} dx = Ax + B \ln |9e^{2x} - 4| + C$, then

(1) $A + 18B = 16$

(2) $18B - A = 20$

(3) $A - 18B = 17$

(4) $A + 18B = 32$

MII077

10. If $0 < x < \pi$, then $\int \frac{dx}{\sqrt{\sin^3 x \sin(x - \alpha)}}$ is equal to

(1) $\sqrt{\cos \alpha + \sin \alpha \cot x} + c$

(2) $2 \operatorname{cosec} \alpha \sqrt{\cos \alpha - \sin \alpha \cot x} + c$

(3) $-2 \operatorname{cosec} \alpha \sqrt{\cos \alpha + \sin \alpha \cot x} + c$

(4) None of these

MII078

11. If $\int \frac{\cos 4x + 1}{\cot x - \tan x} dx = A \cos 4x + B$; where A & B are constants, then

(1) $A = -1/4$ & B may have any value

(2) $A = -1/8$ & B may have any value

(3) $A = -1/2$ & $B = -1/4$

(4) none of these

MII079

Indefinite Integration

12. The value of $\int \cos^6 x dx$ is

- (1) $\frac{5}{16} [x + \cos x \sin x] - \frac{5}{24} \cos^3 x \sin x + \frac{1}{6} \cos^5 x \sin x + C$
- (2) $\frac{5}{16} [x + \cos x \sin x] + \frac{5}{24} \cos^3 x \sin x - \frac{1}{6} \cos^5 x \sin x + C$
- (3) $\frac{5}{8} [x + \cos x \sin x] + \frac{5}{12} \cos^3 x \sin x + \frac{1}{3} \cos^5 x \sin x + C$
- (4) $\frac{5}{16} [x + \cos x \sin x] + \frac{5}{24} \cos^3 x \sin x + \frac{1}{6} \cos^5 x \sin x + C$

MII080

13. If $\int \sqrt{\frac{\cos^3 x}{\sin^{11} x}} dx = -2 \left(A \tan^{\frac{-9}{2}} x + B \tan^{\frac{-5}{2}} x \right) + C$, then

- (1) $A = \frac{1}{9}, B = \frac{-1}{5}$
- (2) $A = \frac{1}{9}, B = \frac{1}{5}$
- (3) $A = -\frac{1}{9}, B = \frac{1}{5}$
- (4) $A = -\frac{1}{9}, B = -\frac{1}{5}$

MII081

14. $\int \frac{dx}{(x+1)\sqrt{x-2}}$ is equal to

- (1) $\frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{x-2}{3} \right) + c$
- (2) $\frac{2}{3} \tan^{-1} \left(\frac{\sqrt{x-2}}{3} \right) + c$
- (3) $\frac{2}{\sqrt{3}} \tan^{-1} \left(\sqrt{\frac{x-2}{3}} \right) + c$
- (4) None of these

MII082

15. $\int \frac{1}{x^2(x^4+1)^{3/4}} dx$ is equal to

- (1) $\left(1 + \frac{1}{x^4} \right)^{1/4} + c$
- (2) $(x^4 + 1)^{1/4} + c$
- (3) $\left(1 - \frac{1}{x^4} \right)^{1/4} + c$
- (4) $-\left(1 + \frac{1}{x^4} \right)^{1/4} + c$

MII083

16. $\int \frac{1-x^7}{x(1+x^7)} dx$ is equal to

- (1) $\ln|x| + \frac{2}{7} \ln|1+x^7| + c$
- (2) $\ln|x| - \frac{2}{7} \ln|1-x^7| + c$
- (3) $\ln|x| - \frac{2}{7} \ln|1+x^7| + c$
- (4) $\ln|x| + \frac{2}{7} \ln|1-x^7| + c$

MII084

17. The value of $\int \frac{dx}{x\sqrt{1-x^3}}$ is equal to

- (1) $\frac{1}{3} \ln \left| \frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1} \right| + C$
- (2) $\frac{1}{3} \ln \left| \frac{\sqrt{1-x^2}+1}{\sqrt{1-x^2}-1} \right| + C$
- (3) $\frac{1}{3} \ln \left| \frac{1}{\sqrt{1-x^3}} \right| + C$
- (4) $\frac{1}{3} \ln |1-x^3| + C$

MII085

18. The value of $\int \sqrt{\frac{e^x-1}{e^x+1}} dx$ is equal to

- (1) $\ln(e^x + \sqrt{e^{2x}-1}) - \sec^{-1}(e^x) + C$ (2) $\ln(e^x + \sqrt{e^{2x}-1}) + \sec^{-1}(e^x) + C$
 (3) $\ln(e^x - \sqrt{e^{2x}-1}) - \sec^{-1}(e^x) + C$ (4) None of these

MII086

19. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}(x + \sqrt{x}) dx$ is equal to :

- (1) $2e^{\sqrt{x}}[\sqrt{x} - x + 1] + c$ (2) $2e^{\sqrt{x}}[x - 2\sqrt{x} + 1] + c$
 (3) $2e^{\sqrt{x}}[x - \sqrt{x} + 1] + c$ (4) $2e^{\sqrt{x}}(x + \sqrt{x} + 1) + c$

MII087

20. Let $F(x) = \int e^{\sin^{-1}x} \left(1 - \frac{x}{\sqrt{1-x^2}}\right) dx$ and $F(0) = 1$, If $F(1/2) = \frac{k\sqrt{3}e^{\pi/6}}{\pi}$, then the value of k is

- (1) π (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

MII088

SECTION-B

- This section will have **TEN** questions. Candidate can choose to attempt any 5 question out of these 10 questions. In case if candidate attempts more than 5 questions, first 5 attempted questions will be considered for marking.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (Answer should be rounded off to the nearest integer).
- Answer to each question will be evaluated according to the following marking scheme:
 Full Marks : +4, if only correct answer is given.
 Zero Marks : 0, if no answer is given.
 Negative Marks : -1 for incorrect answer

1. If $f(x) = \int \frac{2\sin x - \sin 2x}{x^3} dx$, where $x \neq 0$, then $\lim_{x \rightarrow 0} f'(x)$ has the value

MII089

2. If $\int \sin^4 x \cos^4 x dx = \frac{1}{128} \left[ax - \sin 4x + \frac{1}{8} \sin 8x \right] + C$ then value of 'a' equal to :

MII090

3. Let $f(x)$ be the primitive of $\frac{3x+2}{\sqrt{x-9}}$ w.r. to x . If $f(10) = 60$ then twice of sum of digits of the value of $f(13)$ is.

MII091

4. If $\int \frac{\sqrt{4+x^2}}{x^6} dx = \frac{(a+x^2)^{3/2} \cdot (x^2-b)}{120x^5} + C$ then $a + b$ equals to:

MII092

5. If $\int \sqrt{\frac{x}{a^3-x^3}} dx = \frac{d}{b} \sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right) + C$, (where b & d are coprime integer) then $b + d$ equals to.

MII093

6. If $\int \frac{x dx}{\sqrt{1+x^2} + \sqrt{(1+x^2)^3}} = k \sqrt{1 + \sqrt{1+x^2}} + C$ then k equals to :

MII094

7. If $\int e^{\sin x} \cdot \frac{x \cos^3 x - \sin x}{\cos^2 x} dx = e^{\sin x} f(x) + C$ such that $f(0) = -1$ then $\frac{\pi}{3} - f\left(\frac{\pi}{3}\right)$ is equal to:

MII095

8. Let $g(x) = \int \frac{1+2\cos x}{(\cos x+2)^2} dx$ and $g(0) = 0$ then value of $32 g\left(\frac{\pi}{2}\right)$ is.

MII096

9. If $f(x) = \sqrt{x-1}$; $g(x) = e^x$ and $\int f \circ g(x) dx = A f \circ g(x) + B \tan^{-1}(f \circ g(x)) + C$ then $A^3 + B^2$ equals

MII097

10. If $\int \frac{2 \sin 2 \phi - \cos \phi}{6 - \cos^2 \phi - 4 \sin \phi} d\phi = p \ln |\sin^2 \phi - 4 \sin \phi + 5| + q \tan^{-1}(\sin \phi - r) + C$ then $p + q + r$ equal to:

MII098

JEE (Advanced) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION-I

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:
Full Marks : +3 If only the bubble corresponding to the correct option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : -1 In all other cases.

1. Value of $\int \frac{1}{\sin(x-a)\cos(x-b)} dx$ is equal to

- (A) $\frac{1}{\cos(a-b)} \ln \left| \frac{\sin(x-a)}{\cos(x-b)} \right| + C$ (B) $\frac{1}{\cos(a-b)} \ln \left| \frac{\cos(x-b)}{\sin(x-a)} \right| + C$
 (C) $\frac{1}{\sin(a-b)} \ln \left| \frac{\sin(x-a)}{\cos(x-b)} \right| + C$ (D) $\frac{1}{\sin(a+b)} \ln \left| \frac{\cos(x-a)}{\sin(x-b)} \right| + C$

MII099

2. $\int x \sqrt{\frac{a^2-x^2}{a^2+x^2}} dx =$

- (A) $\frac{1}{2} a^2 \cos^{-1} \left(\frac{x^2}{a^2} \right) + \frac{1}{2} \sqrt{a^4+x^4} + C$ (B) $\frac{1}{2} \sin^{-1} \left(\frac{x^2}{a^2} \right) + \sqrt{a^4+x^4} + C$
 (C) $\frac{1}{2} a^2 \sin^{-1} \left(\frac{x^2}{a^2} \right) + \frac{1}{2} \sqrt{a^4-x^4} + C$ (D) $\frac{1}{2} \cos^{-1} \left(\frac{x^2}{a^2} \right) + \frac{1}{2} \sqrt{a^4-x^4} + C$

MII100

3. The value of $\int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}} dx$ is equal to

- (A) $\frac{4}{3} \left(\frac{x-1}{x+2} \right)^{1/4} + C$ (B) $\frac{4}{3} \left(\frac{x+2}{x-1} \right)^{1/4} + C$ (C) $\frac{1}{3} \left(\frac{x-1}{x+2} \right)^{1/4} + C$ (D) $\frac{1}{3} \left(\frac{x+1}{x-1} \right)^{1/4} + C$

MII101

4. $\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$ is equal to

- (A) $(a+x) \arctan \sqrt{\frac{x}{a}} - \sqrt{ax} + C$ (B) $(a+x) \arctan \sqrt{\frac{x}{a}} + \sqrt{ax} + C$
 (C) $(a-x) \arctan \sqrt{\frac{x}{a}} - \sqrt{ax} + C$ (D) $(a+x) \operatorname{arccot} \sqrt{\frac{x}{a}} - \sqrt{ax} + C$

MII102

Indefinite Integration

5. If $I = \int \frac{2}{x}(x^{\ln x})(\ln x)^3 dx = Ax^{\ln x}(\ln x)^2 - Bx^{\ln x} + C$, then $\frac{A}{B}$ is equal to :
 (A) 1 (B) -1 (C) 2 (D) -2 MII103
6. The value of $\int e^{\tan \theta} (\sec \theta - \sin \theta) d\theta$ is equal to
 (A) $-e^{\tan \theta} \sin \theta + C$ (B) $e^{\tan \theta} \sin \theta + C$ (C) $e^{\tan \theta} \sec \theta + C$ (D) $e^{\tan \theta} \cos \theta + C$ MII104
7. The value of $\int \left\{ \ln(1 + \sin x) + x \tan \left(\frac{\pi}{4} - \frac{x}{2} \right) \right\} dx$ is equal to:
 (A) $x \ln(1 + \sin x) + C$ (B) $\ln(1 + \sin x) + C$
 (C) $-x \ln(1 + \sin x) + C$ (D) $\ln(1 - \sin x) + C$ MII105
8. The value of $\int x \cdot \frac{\ln(x + \sqrt{1+x^2})}{\sqrt{1+x^2}} dx$ equals:
 (A) $\sqrt{1+x^2} \ln(x + \sqrt{1+x^2}) - x + C$ (B) $\frac{x}{2} \cdot \ln^2(x + \sqrt{1+x^2}) - \frac{x}{\sqrt{1+x^2}} + C$
 (C) $\frac{x}{2} \cdot \ln^2(x + \sqrt{1+x^2}) + \frac{x}{\sqrt{1+x^2}} + C$ (D) $\sqrt{1+x^2} \ln(x + \sqrt{1+x^2}) + x + C$ MII106

SECTION-II

- This section contains **SIX** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

<i>Full Marks</i>	: +4	if only (all) the correct option(s) is (are) chosen.
<i>Partial Marks</i>	: +3	if all the four options are correct but ONLY three options are chosen.
<i>Partial Marks</i>	: +2	if three or more options are correct but ONLY two options are chosen, both of which are correct options.
<i>Partial Marks</i>	: +1	if two or more options are correct but ONLY one option is chosen and it is a correct option.
<i>Zero Marks</i>	: 0	if none of the options is chosen (i.e. the question is unanswered).
<i>Negative Marks</i>	: -2	in all other cases.

For Example : If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks.

9. If $f\left(\frac{1-x}{1+x}\right) = x$ and $g(x) = \int f(x) dx$ then
- (A) $g(x)$ is continuous in domain (B) $g(x)$ is discontinuous at two points in its domain
- (C) $\lim_{x \rightarrow \infty} g'(x) = -1$ (D) $\int g(x) dx = -\frac{x^2}{2} + (2x+1)\lambda n\left(\frac{1+x}{e}\right) + C$

MII107

10. The value of $\int \frac{\ln\left(\frac{x-1}{x+1}\right)}{x^2-1} dx$ is equal to
- (A) $\frac{1}{2} \ln^2 \frac{x-1}{x+1} + C$ (B) $\frac{1}{4} \ln^2 \frac{x-1}{x+1} + C$ (C) $\frac{1}{2} \ln^2 \frac{x+1}{x-1} + C$ (D) $\frac{1}{4} \ln^2 \frac{x+1}{x-1} + C$

MII108

11. The value of $\int \frac{\cos^3 x}{\sin^2 x + \sin x} dx$ is equal to :
- (A) $\ln |\sin x| + \sin x + C$ (B) $\ln |\sin x| - \sin x + C$
- (C) $-\ln |\operatorname{cosec} x| - \sin x + C$ (D) $-\ln |\sin x| + \sin x + C$

MII109

12. If $\int \frac{(x-1) dx}{x^2 \sqrt{2x^2 - 2x + 1}} = \frac{\sqrt{f(x)}}{g(x)} + C$, where $f(x)$ is a quadratic expression and $g(x)$ is a monic linear expression.
- (A) $f(x) = 2x^2 - 2x + 1$ (B) $g(x) = x + 1$
- (C) $g(x) = x$ (D) $f(x) = 2x^2 - 2x$

MII110

13. $I = \int \frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} dx$ equals to
- (A) $-x + \frac{2}{\pi} (2x-1) \sin^{-1} \sqrt{x} + \frac{2}{\pi} \sqrt{x-x^2} + C$
- (B) $x - \frac{4x}{\pi} \cos^{-1} \sqrt{x} - \frac{2}{\pi} \sin^{-1} \sqrt{x} + \frac{2}{\pi} \sqrt{x} \sqrt{1-x} + C$
- (C) $-x + \frac{2}{\pi} (2x+1) \cos^{-1} \sqrt{x} + \frac{2}{\pi} \sqrt{x} \sqrt{1-x} + C$
- (D) $x - \frac{4x}{\pi} \sin^{-1} \sqrt{x} + C$

MII111

14. If $\int \frac{x^2-x+1}{(1+x^2)^{3/2}} e^x dx = e^x f(x) + C$ then
- (A) $f(x)$ is a an even function (B) $f(x)$ is a bounded function
- (C) Range of $f(x)$ is $(0, 1]$ (D) $f(x)$ has two points of extrema.

MII112

SECTION-III

- This section contains **ONE** paragraph.
- Based on each paragraph, there are **THREE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D) **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks	:	+3	if only the bubble corresponding to the correct answer is darkened.
Zero Marks	:	0	in all other cases.

Comprehension # 1 (Q. No. 15 - 17)

Let $I_{n,m} = \int \sin^n x \cos^m x dx$. Then we can relate $I_{n,m}$ with each of the following

- | | | |
|------------------|-------------------|--------------------|
| (i) $I_{n-2,m}$ | (ii) $I_{n+2,m}$ | (iii) $I_{n,m-2}$ |
| (iv) $I_{n,m+2}$ | (v) $I_{n-2,m+2}$ | (vi) $I_{n+2,m-2}$ |

Suppose we want to establish a relation between $I_{n,m}$ and $I_{n,m-2}$, then we set

$$P(x) = \sin^{n+1} x \cos^{m-1} x \quad \dots(1)$$

In $I_{n,m}$ and $I_{n,m-2}$ the exponent of $\cos x$ is m and $m-2$ respectively, the minimum of the two is $m-2$, adding 1 to the minimum we get $m-2+1 = m-1$. Now choose the exponent $m-1$ of $\cos x$ in $P(x)$. Similarly choose the exponent of $\sin x$ for $P(x)$

Now differentiating both sides of (1), we get

$$\begin{aligned} P'(x) &= (n+1) \sin^n x \cos^m x - (m-1) \sin^{n+2} x \cos^{m-2} x \\ &= (n+1) \sin^n x \cos^m x - (m-1) \sin^n x (1 - \cos^2 x) \cos^{m-2} x \\ &= (n+1) \sin^n x \cos^m x - (m-1) \sin^n x \cos^{m-2} x + (m-1) \sin^n x \cos^m x \\ &= (n+m) \sin^n x \cos^m x - (m-1) \sin^n x \cos^{m-2} x \end{aligned}$$

Now integrating both sides, we get

$$\sin^{n+1} x \cos^{m-1} x = (n+m) I_{n,m} - (m-1) I_{n,m-2}$$

Similarly we can establish the other relations.

15. The relation between $I_{4,2}$ and $I_{2,2}$ is

- | | |
|---|---|
| (A) $I_{4,2} = \frac{1}{6} (-\sin^3 x \cos^3 x + 3I_{2,2})$ | (B) $I_{4,2} = \frac{1}{6} (\sin^3 x \cos^3 x + 3I_{2,2})$ |
| (C) $I_{4,2} = \frac{1}{6} (\sin^3 x \cos^3 x - 3I_{2,2})$ | (D) $I_{4,2} = \frac{1}{6} (-\sin^3 x \cos^3 x + 2I_{2,2})$ |

MII113

16. The relation between $I_{4,2}$ and $I_{6,2}$ is

- | | |
|--|---|
| (A) $I_{4,2} = \frac{1}{5} (\sin^5 x \cos^3 x + 8I_{6,2})$ | (B) $I_{4,2} = \frac{1}{5} (-\sin^5 x \cos^3 x + 8I_{6,2})$ |
| (C) $I_{4,2} = \frac{1}{5} (\sin^5 x \cos^3 x - 8I_{6,2})$ | (D) $I_{4,2} = \frac{1}{5} (\sin^5 x \cos^3 x + 8I_{6,2})$ |

MII114

17. The relation between $I_{4,2}$ and $I_{4,4}$ is

- (A) $I_{4,2} = \frac{1}{3} (\sin^5 x \cos^3 x + 8 I_{4,4})$ (B) $I_{4,2} = \frac{1}{3} (-\sin^5 x \cos^3 x + 8 I_{4,4})$
 (C) $I_{4,2} = \frac{1}{3} (\sin^5 x \cos^3 x - 8 I_{4,4})$ (D) $I_{4,2} = \frac{1}{3} (\sin^5 x \cos^3 x + 6 I_{4,4})$

MII115

SECTION-IV

- This section contains **SIX** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30, if answer is 11.36777.... then both 11.36 and 11.37 will be correct) by darkening the corresponding bubbles in the ORS.

For Example : If answer is -77.25, 5.2 then fill the bubbles as follows.

(+) ● ● ● (0) (0) (0) (0) (1) (1) (1) (1) (1) (1) (2) (2) (2) (2) ● (2) (3) (3) (3) (3) (3) (3) (4) (4) (4) (4) (4) (4) (5) (5) (5) (5) (5) ● (6) (6) (6) (6) (6) (6) (7) (7) ● ● (7) (7) (8) (8) (8) (8) (8) (8) (9) (9) (9) (9) (9) (9)	● (-) ● ● ● (0) (0) ● (1) (1) (1) (1) (1) (1) (2) (2) (2) (2) ● (2) (3) (3) (3) (3) (3) (3) (4) (4) (4) (4) (4) (4) (5) (5) (5) ● (5) (5) (6) (6) (6) (6) (6) (6) (7) (7) (7) (7) (7) (7) (8) (8) (8) (8) (8) (8) (9) (9) (9) (9) (9) (9)
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- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +4 If **ONLY** the correct numerical value is entered as answer.
Zero Marks : 0 In all other cases.

18. If $\int \frac{(x-1)^2}{x^4+x^2+1} dx = \frac{1}{\sqrt{a}} \tan^{-1} \left(\frac{x^2-1}{x\sqrt{3}} \right) - \frac{b}{\sqrt{a}} \tan^{-1} \left(\frac{2x^2+1}{\sqrt{3}} \right) + C$ then $a^2 + b^2$ equals to :

MII116

19. If $\int \frac{1+x \cos x}{x(1-x^2 e^{2 \sin x})} dx = k \ln \sqrt{\frac{x^2 e^{2 \sin x}}{1-x^2 e^{2 \sin x}}} + C$ then k is equal to :

MII117

20. If $\int \frac{x^4+1}{x(x^2+1)^2} dx = A \ln|x| + \frac{B}{1+x^2} + C$, then $A+B$ equals to :

MII118

21. If $\int \frac{1}{1-\sin^4 x} dx = \frac{1}{a\sqrt{b}} \tan^{-1}(\sqrt{a} \tan x) + \frac{1}{b} \tan x + C$ then $\frac{a}{b}$ is equal to :

MII119

22. If $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} dx = p \sin x - \frac{q}{\sin x} - r \tan^{-1}(\sin x) + C$ then $p + 2q + r$ is equal to :

MII120

23. If $\int \frac{dx}{\sqrt{\sin^3 x \cos^5 x}} = a \sqrt{\cot x} + b \sqrt{\tan^3 x} + C$, where C is an arbitrary constant of integration, then the values of $a^2 + 9b$ equals to :

MII121

ANSWER KEY

EXERCISE - 0

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	D	A	B	A	C	B	D	C	A
Que.	11	12	13	14	15					
Ans.	B,D	A,C	A,B	A,C	A→Q, B→P, C→P,R,S, D→P,R					

EXERCISE - S

1. 0 2. 5 3. 6 4. 2 5. 2010
 6. 3 7. 2 8. 5 9. 46
 10. 0.19 or 0.20

EXERCISE - JEE (Main) PYQ

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1	1	1	3	2	1	4	1	64	1
Que.	11									
Ans.	3									

EXERCISE - JEE (Advanced) PYQ

Que.	1	2	3	4	5					
Ans.	D	A	D	C	C					

JEE (Main) Practice Paper

Section-A	Q.	1	2	3	4	5	6	7	8	9	10
	A.	2	1	2	3	1	3	3	3	1	2
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	2	4	2	3	4	3	1	1	3	4
Section-B	Q.	1	2	3	4	5	6	7	8	9	10
	A.	1	3	12	10	5	2	2	16	12	11

JEE (Advanced) Practice Paper

Section-I	Q.	1	2	3	4	5	6	7	8
	A.	A	C	A	A	A	D	A	A
Section-II	Q.	9	10	11	12	13	14		
	A.	A,C	B,D	B,C	A,C	A,B	A,B,C		
Section-III	Q.	15	16	17					
	A.	A	A	B					
Section-IV	Q.	18	19	20	21	22	23		
	A.	13	1	2	1	11	10		