EXERCISE - O

SINGLE CORRECT TYPE QUESTIONS

1. If the eccentricity of the hyperbola $x^2 - y^2 \sec^2\alpha = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2 \sec^2\alpha + y^2 = 25$, then a value of α is :

(A) $\pi/6$ (B) $\pi/4$ (C) $\pi/3$ (D) $\pi/2$

MHB001

2. The foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide. Then the value of b^2 is-

(A) 5 (B) 7 (C) 9 (D) 4

MHB002

3. Distance between focus of the hyperbola $x^2 - 3y^2 - 4x - 6y - 11 = 0$, is -

(A) 4 (B) 6 (C) 8 (D) 10

MHB003

4. The equation $\frac{x^2}{29-p}+\frac{y^2}{4-p}=1$ $(p \neq 4, 29)$ represents -

(A) an ellipse if p is any constant greater than 4

(B) a hyperbola if p is any constant between 4 and 29.

(C) a rectangular hyperbola if p is any constant greater than 29.

(D) no real curve is p is less than 29.

MHB004

5. The magnitude of the gradient of the tangent at an extremity of latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is equal to (where e is the eccentricity of the hyperbola)

(A) be (B) e (C) ab (D) ae

MHB005

6. The number of possible tangents which can be drawn to the curve $4x^2 - 9y^2 = 36$, which are perpendicular to the straight line $5x + 2y - 10 = 0$ is :

(A) zero (B) 1 (C) 2 (D) 4

MHB006

7. Locus of the feet of the perpendiculars drawn from either foci on a variable tangent to the hyperbola $16y^2 - 9x^2 = 1$ is

(A) $x^2 + y^2 = 9$ (B) $x^2 + y^2 = 1/9$

(C) $x^2 + y^2 = 7/144$ (D) $x^2 + y^2 = 1/16$

MHB007

8. The asymptote of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}$ = 1 form with any tangent to the hyperbola a triangle whose area is $a^2 \tan\lambda$ in magnitude then its eccentricity is :

(A) $\sec\lambda$ (B) $\mathrm{cosec}\lambda$ (C) $\sec^2\lambda$ (D) $\mathrm{cosec}^2\lambda$

MHB008

9. If the normal to the rectangular hyperbola $xy = c^2$ at the point 't' meets the curve again at 't_1' then $t^3\ t_1$ has the value equal to

(A) 1 (B) – 1 (C) 0 (D) none

MHB009

10. AB is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that ΔAOB (where 'O' is the origin) is an equilateral triangle, then the eccentricity e of the hyperbola satisfies

(A) $e > \sqrt{3}$ (B) $1 < e < \frac{2}{\sqrt{3}}$ (C) $e = \frac{2}{\sqrt{3}}$ (D) $e > \frac{2}{\sqrt{3}}$

MHB010

11. P is a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, N is the foot of the perpendicular from P on the transverse axis. The tangent to the hyperbola at P meets the transverse axis at T . If O is the centre of the hyperbola, then $OT.ON$ is equal to :

(A) e^2 (B) a^2 (C) b^2 (D) b^2/a^2

MHB011

12. Let the major axis of a standard ellipse equals the transverse axis of a standard hyperbola and their director circles have radius equal to $2R$ and R respectively. If e_1 and e_2 are the eccentricities of the ellipse and hyperbola then the correct relation is

(A) $4e_1^2 - e_2^2 = 6$ (B) $e_1^2 - 4e_2^2 = 2$ (C) $4e_2^2 - e_1^2 = 6$ (D) $2e_1^2 - e_2^2 = 4$

MHB012

13. The tangent to the hyperbola $xy = c^2$ at the point P intersects the x-axis at T and the y-axis at T'. The normal to the hyperbola at P intersects the x-axis at N and the y-axis at N'. The areas of the triangles PNT and $PN'T'$ are Δ and Δ' respectively, then $\frac{1}{\Delta}+\frac{1}{\Delta'}$ is

(A) equal to 1 (B) depends on t (C) depends on c (D) equal to 2

MHB013

14. The locus of the foot of the perpendicular from the centre of the hyperbola $xy = c^2$ on a variable tangent is :

(A) $(x^2 - y^2)^2 = 4c^2xy$ (B) $(x^2 + y^2)^2 = 2c^2xy$

(C) $(x^2 + y^2) = 4c^2xy$ (D) $(x^2 + y^2)^2 = 4c^2xy$

MHB014

15. The equation to the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola $xy = c^2$ is :

(A) $\frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$ (B) $\frac{x}{x_1-x_2}+\frac{y}{y_1-y_2}=1$

(C) $\frac{x}{y_1+y_2}+\frac{y}{x_1+x_2}=1$ (D) $\frac{x}{y_1-y_2}+\frac{y}{x_1-x_2}=1$

MHB015

16. The locus of the point of intersection of the lines $\sqrt{3}\,x - y - 4\sqrt{3}\ t = 0$ & $\sqrt{3}\,tx + ty - 4\sqrt{3} = 0$ (where t is a parameter) is a hyperbola whose eccentricity is

(A) $\sqrt{3}$ (B) 2 (C) $\frac{2}{\sqrt{3}}$ (D) $\frac{4}{3}$

MHB042

17. Let F_1, F_2 are the foci of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ and F_3, F_4 are the foci of its conjugate hyperbola. If e_H and e_C are their eccentricities respectively then the statement which holds true is

(A) Their equations of the asymptotes are different.

(B) $e_H > e_C$

(C) Area of the quadrilateral formed by their foci is 50 sq. units.

(D) Their auxiliary circles will have the same equation.

MHB045

18. Tangents are drawn from any point on the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ to the circle $x^2+y^2=9$, then the locus of mid-point of the chord of contact is

(A) $\frac{x^2}{9}-\frac{y^2}{4}=\left(\frac{x^2+y^2}{9}\right)^2$ (B) $\frac{x^2}{9}+\frac{y^2}{4}=\left(\frac{x^2+y^2}{9}\right)^2$

(C) $\frac{x^2}{9}+\frac{y^2}{4}=\left(\frac{x^2-y^2}{9}\right)^2$ (D) $\frac{x^2}{9}-\frac{y^2}{4}=\left(\frac{x^2-y^2}{9}\right)^2$

MHB069

MULTIPLE CORRECT TYPE QUESTIONS

19. If θ is eliminated from the equations $a\sec\theta - x\tan\theta = y$ and $b\sec\theta + y\tan\theta = x$ (a and b are constant), then the eliminant denotes the equation of

(A) the director circle of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

(B) auxiliary circle of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

(C) Director circle of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

(D) Director circle of the circle $x^2+y^2=\frac{a^2+b^2}{2}$.

MHB046

20. If the circle $x^2+y^2=a^2$ intersects the hyperbola $xy=c^2$ in four points $P(x_1,y_1)$, $Q(x_2,y_2)$, $R(x_3,y_3)$, $S(x_4,y_4)$, then -

(A) $x_1+x_2+x_3+x_4=0$ (B) $y_1+y_2+y_3+y_4=0$

(C) $x_1x_2x_3x_4=c^4$ (D) $y_1y_2y_3y_4=c^4$

MHB047

21. Which of the following equations in parametric form can represent a hyperbolic profile, where 't' is a parameter.

(A) $x=\frac{a}{2}\left(t+\frac{1}{t}\right)$ & $y=\frac{b}{2}\left(t-\frac{1}{t}\right)$ (B) $\frac{tx}{a}-\frac{y}{b}+t=0$ & $\frac{x}{a}+\frac{ty}{b}-1=0$

(C) $x=e^t+e^{-t}$ & $y=e^t-e^{-t}$ (D) $x^2-6=2\cos t$ & $y^2+2=4\cos^2\frac{t}{2}$

MHB048

22. Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{3}=1$ of eccentricity e is confocal with the ellipse $\frac{x^2}{8}+\frac{y^2}{4}=1$. Let A, B, C & D are points of intersection of hyperbola & ellipse, then-

(A) $e=\frac{5}{2}$

(B) $e = 2$

(C) A, B, C, D are concyclic points

(D) Number of common tangents of hyperbola & ellipse is 2

MHB049

23. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then -

(A) the equation of the hyperbola is $\frac{x^2}{3}-\frac{y^2}{2}=1$

(B) a focus of the hyperbola is (2, 0)

(C) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(D) the equation of the hyperbola is $x^2 - 3y^2 = 3$

MHB050

24. If the equation of a hyperbola is given as $\left|\sqrt{(x-3)^2+(y-3)^2}-\sqrt{(x-1)^2+(y-1)^2}\right|=2$, then-

(A) The given hyperbola is a rectangular hyperbola.

(B) Distance between both directrices is $\sqrt{2}$

(C) Centre of hyperbola lies on line $y = x$

(D) Asymptotes are inclined at an angle of 45° with each other.

MHB052

25. If the ellipse $x^2 + 2y^2 = 4$ touches the hyperbola $xy = c$ at points P and Q, then -

(A) c is a rational number

(B) O, P and Q are collinear points (where 'O' is origin)

(C) length of PQ is $2\sqrt{3}$

(D) length of PQ is $2\sqrt{2}$

MHB053

26. If two distinct tangents can be drawn from the point (α, 2) on different branches of the hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1$, then the range of α is subset of

(A) $\left[-\frac{3}{2},\frac{3}{2}\right]$

(B) $[-2,2]$

(C) $[-1,1]$

(D) $\left(-\frac{1}{2},\frac{1}{2}\right)$

MHB070

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 27 to 28

Consider the conic $C : \frac{x^2}{16}+\frac{y^2}{12}=1$

27. Equation of circle touching C at one extremity of latus-rectum and passing through centre of C is/are-

(A) $8x^2 + 8y^2 - 19x - 22y = 0$

(B) $8x^2 + 8y^2 - 19x + 22y = 0$

(C) $8x^2 + 8y^2 + 19x - 22y = 0$

(D) $8x^2 + 8y^2 + 19x + 22y = 0$

MHB054

28. If a hyperbola passes through extremities of minor axis of conic C and its transverse and conjugate axis coincides with the minor and major axis of conic C respectively, and the product of eccentricity of hyperbola and conic C is 1, then -

(A) equation of hyperbola is $\frac{x^2}{36}-\frac{y^2}{12}=-1$.
(B) equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{3}=-1$.
(C) focus of hyperbola is $(0,2\sqrt{3})$
(D) focus of hyperbola is $(0,-4\sqrt{3})$

MHB055

Paragraph for Question No. 29 to 30

Equation of the transverse and conjugate axis of a hyperbola are respectively $x + 2y - 3 = 0$, $2x - y + 4 = 0$ and their respectively lengths are $\sqrt{2}$ and $\frac{2}{\sqrt{3}}$ then answer following :

29. If $x^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is equation of given hyperbola where h, b, g, f, c all are integers then the sum $h + b + g + f + c =$

(A) 3 (B) 4 (C) 5 (D) 6

MHB071

30. Equation of one of the directrix is

(A) $2x - y + 4 + \sqrt{\frac{3}{2}} = 0$
(B) $x + 2y + 4 - \sqrt{\frac{2}{3}} = 0$
(C) $2x - y = \sqrt{\frac{3}{2}}$
(D) $2x - y + 4 + \sqrt{\frac{3}{2}} = \sqrt{3}$

MHB072

MATRIX MATCH TYPE QUESTION

31.

	List-I		List-II
(P)	Value of c for which $3x^2 - 5xy - 2y^2 + 5x + 11y + c = 0$ are the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 5x + 11y - 8 = 0$	(1)	3
(Q)	If locus of a point, whose chord of contact with respect to the circle $x^2 + y^2 = 4$ is a tangent to the hyperbola $xy = 1$ is $xy = c^2$, then value of c^2 is	(2)	–4
(R)	If equation of a hyperbola whose conjugate axis is 5 and distance between its foci is 13, is $ax^2 - by^2 = c$ where a and b are coprime natural numbers, then	(3)	–12
	value of $\frac{ab}{c}$ is	(4)	4
(S)	If the vertex of a hyperbola bisects the distance between its centre and the corresponding focus, then ratio of square of its conjugate axis to the square of its transverse axis is	(5)	–6

Which of the following is correct option ?

(A) P $\to$ 2; Q $\to$ 1; R $\to$ 4; S $\to$ 1
(B) P $\to$ 3; Q $\to$ 4; R $\to$ 1; S $\to$ 5
(C) P $\to$ 3; Q $\to$ 4; R $\to$ 4; S $\to$ 1
(D) P $\to$ 1; Q $\to$ 4; R $\to$ 5; S $\to$ 3

MHB056

EXERCISE - S

1. The hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ $(a, b > 0)$ passes through the point of intersection of the lines, $7x + 13y - 87 = 0$ & $5x - 8y + 7 = 0$ and the latus rectum is $\frac{32\sqrt{2}}{5}$, then value of b is equal to

MHB016

2. The line $2x + y = 1$ is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

MHB017

3. Tangents to the hyperbola $x^2 - 9y^2 = 9$ that are drawn from (3, 2). Find the area of the triangle that these tangents form with their chord of contact.

MHB018

4. For the hyperbola $\frac{x^2}{100} - \frac{y^2}{25} = 1$, the value of $\sqrt{SA \cdot S'A}$ is equal to, (where S & S' are the foci & A is the vertex)

MHB019

5. If tangents drawn from the point $(a, 2)$ to the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ are perpendicular, then the value of a^2 is

MHB020

6. From the centre C of the hyperbola $x^2 - y^2 = 9$, CM is drawn perpendicular to the tangent at any point of the curve, meeting the tangent at M and the curve at N. Find the value of the product $(CM)(CN)$.

MHB057

7. If the lines $x + y + 1 = 0$ and $2x - y + 2 = 0$ are the asymptotes of a hyperbola. If the line $x - 2 = 0$ touches the hyperbola then the equation of the hyperbola is $4(x + y + 1)(2x - y + 2) = \lambda$. Find the value of λ.

MHB058

8. Tangent and normal are drawn at the upper end (x_1, y_1) of the latus rectum P with $x_1 > 0$ and $y_1 > 0$, of the hyperbola $\frac{x^2}{4}-\frac{y^2}{12}=1$, intersecting the transverse axis at T and G respectively. Find the area of the triangle PTG.

MHB059

9. If $7x^2 + pxy + qy^2 + rx - sy + t = 0$ is the eqaution of the hyperbola whose one focus is $(-1, 1)$, eccentricity = 3 and the equation of the corresponding directrix is $x - y + 3 = 0$, then the value of 't' is:

MHB073

10. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the point P and Q. If these tangents intersect at the point $T(0, 3)$ then the area (in sq. units) of ΔPTQ is -

MHB074

EXERCISE - JEE (Main) PYQ

1. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval : **[JEE (Main) 2019]**

(1) (2, 3] (2) $(3, \infty)$ (3) (3/2, 2] (4) (1, 3/2]

MHB029

2. The equation of a tangent to the hyperbola $4x^2 - 5y^2 = 20$ parallel to the line $x - y = 2$ is : **[JEE (Main) 2019]**

(1) $x - y + 9 = 0$ (2) $x - y + 7 = 0$ (3) $x - y + 1 = 0$ (4) $x - y - 3 = 0$

MHB030

3. Let $S = \left\{(x, y) \in R^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1\right\}$, where $r \neq \pm 1$. Then S represents: **[JEE (Main) 2019]**

(1) A hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, where $0 < r < 1$.

(2) An ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, where $r > 1$

(3) A hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when $0 < r < 1$.

(4) An ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when $r > 1$

MHB031

4. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is :- **[JEE (Main) 2019]**

(1) 2 (2) $\frac{13}{6}$ (3) $\frac{13}{8}$ (4) $\frac{13}{12}$

MHB032

5. If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$, then a value of m is **[JEE (Main) 2019]**

(1) $\frac{\sqrt{5}}{2}$ (2) $\frac{3}{\sqrt{5}}$ (3) $\frac{2}{\sqrt{5}}$ (4) $\frac{\sqrt{15}}{2}$

MHB033

6. If a hyperbola passes through the point $P(10,16)$ and it has vertices at $(\pm 6, 0)$, then the equation of the normal to it at P is **[JEE (Main) 2020]**

(1) $x + 2y = 42$ (2) $3x + 4y = 94$ (3) $2x + 5y = 100$ (4) $x + 3y = 58$

MHB034

7. A line parallel to the straight line $2x - y = 0$ is tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1). Then $x_1^2 + 5y_1^2$ is equal to : **[JEE (Main) 2020]**

(1) 5 (2) 6 (3) 8 (4) 10

MHB035

8. A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2 + 4y^2 = 12$, then this hyperbola does not pass through which of the following points? **[JEE (Main) 2020]**

(1) $\left(1, -\frac{1}{\sqrt{2}}\right)$ (2) $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$ (3) $\left(\frac{1}{\sqrt{2}}, 0\right)$ (4) $\left(-\sqrt{\frac{3}{2}}, 1\right)$

MHB036

9. If the line $y = mx + c$ is a common tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{64} = 1$ and the circle $x^2 + y^2 = 36$, then which one of the following is true? **[JEE (Main) 2020]**

(1) $5m = 4$ (2) $4c^2 = 369$

(3) $c^2 = 369$ (4) $8m + 5 = 0$

MHB037

10. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25} + \frac{y^2}{b^2} = 1 (b < 5)$ and the hyperbola, $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$ respectively satisfying $e_1 e_2 = 1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to : **[JEE (Main) 2020]**

(1) (8, 10) (2) (8, 12) (3) $\left(\frac{20}{3}, 12\right)$ (4) $\left(\frac{24}{5}, 10\right)$

MHB075

11. Let a line $L : 2x + y = k, k > 0$ be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, then α is equal to : **[JEE (Main) 2021]**

(1) 12 (2) –12 (3) 24 (4) –24

MHB038

12. The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, and its foci is : **[JEE (Main) 2021]**

(1) $16x^2 - 9y^2 + 32x + 36y - 36 = 0$

(2) $9x^2 - 16y^2 + 36x + 32y - 144 = 0$

(3) $16x^2 - 9y^2 + 32x + 36y - 144 = 0$

(4) $9x^2 - 16y^2 + 36x + 32y - 36 = 0$

MHB039

13. A hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities in one, then the equation of the hyperbola is : **[JEE (Main) 2021]**

(1) $\frac{x^2}{9} - \frac{y^2}{25} = 1$ (2) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(3) $x^2 - y^2 = 9$ (4) $\frac{x^2}{9} - \frac{y^2}{4} = 1$

MHB076

14. The locus of the point of intersection of the lines $(\sqrt{3})kx + ky - 4\sqrt{3} = 0$ and $\sqrt{3}x - y - 4(\sqrt{3})k = 0$ is a conic, whose eccentricity is. **[JEE (Main) 2021]**

MHB077

15. Consider a hyperbola $H: x^2 - 2y^2 = 4$. Let the tangent at a point $P(4, \sqrt{6})$ meet the x –axis at Q and latus rectum at $R(x_1, y_1), x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to **[JEE (Main) 2021]**

(1) $4\sqrt{6}$ (2) $\sqrt{6}-1$ (3) $\frac{7}{\sqrt{6}}-2$ (4) $4\sqrt{6}-1$

MHB078

16. The normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{9}=1$ at the point $(8, 3\sqrt{3})$ on it passes through the point : **[JEE (Main) 2022]**

(1) $(15, -2\sqrt{3})$ (2) $(9, 2\sqrt{3})$ (3) $(-1, 9\sqrt{3})$ (4) $(-1, 6\sqrt{3})$

MHB040

17. Let the tangent drawn to the parabola $y^2 = 24x$ at the point (α, β) is perpendicular to the line $2x + 2y = 5$. Then the normal to the hyperbola $\frac{x^2}{\alpha^2}-\frac{y^2}{\beta^2}=1$ at the point $(\alpha+4, \beta+4)$ does NOT pass through the point : **[JEE (Main) 2022]**

(1) (25, 10) (2) (20, 12) (3) (30, 8) (4) (15, 13)

MHB041

18. Let the hyperbola $H : \frac{x^2}{a^2}-y^2=1$ and the ellipse $E : 3x^2 + 4y^2 = 12$ be such that the length of latus rectum of H is equal to the length of latus rectum of E. If e_H and e_E are the eccentricities of H and E respectively, then the value of $12\left(e_H^2+e_E^2\right)$ is equal to ____. **[JEE (Main) 2022]**

MHB079

19. Let $a > 0, b > 0$. Let e and ℓ respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. Let e' and ℓ' respectively the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2=\frac{11}{14}\ell$ and $(e')^2=\frac{11}{8}\ell'$, then the value of $77a + 44b$ is equal to **[JEE (Main) 2022]**

(1) 100 (2) 110 (3) 120 (4) 130

MHB080

20. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $\frac{5}{4}$. If the equation of the normal at the point $\left(\frac{8}{\sqrt{5}}, \frac{12}{5}\right)$ on the hyperbola is $8\sqrt{5}x+\beta y=\lambda$, then $\lambda-\beta$ is equal to **[JEE (Main) 2022]**

MHB081

21. Let $H_n = \frac{x^2}{1+n}-\frac{y^2}{3+n}=1, n \in N$. Let k be the smallest even value of n such that the eccentricity of H_k is a rational number. If l is length of the latus return of H_k, then $21l$ is equal to. **[JEE (Main) 2023]**

MHB082

22. The foci of a hyperbola are $(\pm 2,0)$ and its eccentricity is $\frac{3}{2}$. A tangent, perpendicular to the line $2x + 3y = 6$, is drawn at a point in the first quadrant on the hyperbola. If the intercepts made by the tangent on the $x-$ and $y-$ axes are a and b respectively, then $|6a| + |5b|$ is equal to.

[JEE (Main) 2023]

MHB083

23. Let $P(x_0, y_0)$ be the point on the hyperbola $3x^2 - 4y^2 = 36$, which is nearest to the line $3x + 2y = 1$. Then $\sqrt{2}\ (y_0 - x_0)$ is equal to : **[JEE (Main) 2023]**

(1) –3 (2) 9 (3) –9 (4) 3

MHB084

24. Let the eccentricity of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ is reciprocal to that of the hyperbola $2x^2 - 2y^2 = 1$. If the ellipse intersects the hyperbola at right angles, then square of length of the latus-rectum of the ellipse is. **[JEE (Main) 2023]**

MHB085

25. The vertices of a hyperbola H are $(\pm 6,0)$ and its eccentricity is $\frac{\sqrt{5}}{2}$. Let N be the normal to H at a point in the first quadrant and parallel to the line $\sqrt{2}x+y=2\sqrt{2}$. If d is the length of the line segment of N between H and the y-axis then d^2 is equal to. **[JEE (Main) 2023]**

MHB086

EXERCISE - JEE (Advanced) PYQ

1. Let $P(6,3)$ be a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal at the point P intersects the x-axis at $(9,0)$, then the eccentricity of the hyperbola is - **[JEE (Advanced) 2011]**

(A) $\sqrt{\frac{5}{2}}$ (B) $\sqrt{\frac{3}{2}}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

MHB060

2. Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$, parallel to the straight line $2x-y=1$. The points of contact of the tangents on the hyperbola are **[JEE (Advanced) 2012]**

(A) $\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{9}{2\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$ (C) $(3\sqrt{3},-2\sqrt{2})$ (D) $(-3\sqrt{3},2\sqrt{2})$

MHB061

3. Consider the hyperbola $H:\ x^2-y^2=1$ and a circle S with centre $N(x_2,0)$. Suppose that H and S touch each other at a point $P(x_1,y_1)$ with $x_1>1$ and $y_1>0$. The common tangent to H and S at P intersects the x-axis at point M. If (l,m) is the centroid of the triangle ΔPMN, then the correct expression(s) is(are) **[JEE (Advanced) 2015]**

(A) $\frac{dl}{dx_1}=1-\frac{1}{3x_1^2}$ for $x_1>1$

(B) $\frac{dm}{dx_1}=\frac{x_1}{3\left(\sqrt{x_1^2-1}\right)}$ for $x_1>1$

(C) $\frac{dl}{dx_1}=1+\frac{1}{3x_1^2}$ for $x_1>1$

(D) $\frac{dm}{dy_1}=\frac{1}{3}$ for $y_1>0$

MHB062

Triple Match : (Q. No. 4 to 6)

Column 1, 2 and 3 contain conics, equation of tangents to the conics and points of contact, respectively.

Column 1	**Column 2**	**Column 3**
(I) $x^2+y^2=a^2$	(i) $my=m^2x+a$	(P) $\left(\frac{a}{m^2},\frac{2a}{m}\right)$
(II) $x^2+a^2y^2=a^2$	(ii) $y=mx+a\sqrt{m^2+1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}},\frac{a}{\sqrt{m^2+1}}\right)$
(III) $y^2=4ax$	(iii) $y=mx+\sqrt{a^2m^2-1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}},\frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV) $x^2-a^2y^2=a^2$	(iv) $y=mx+\sqrt{a^2m^2+1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}},\frac{-1}{\sqrt{a^2m^2-1}}\right)$

4. The tangent to a suitable conic (Column 1) at $\left(\sqrt{3},\frac{1}{2}\right)$ is found to be $\sqrt{3}x+2y=4$, then which of the following options is the only **CORRECT** combination? **[JEE (Advanced) 2017]**

(A) (II) (iii) (R) (B) (IV) (iv) (S) (C) (IV) (iii) (S) (D) (II) (iv) (R)

MHB063

5. If a tangent to a suitable conic (Column 1) is found to be $y = x + 8$ and its point of contact is (8,16), then which of the following options is the only **CORRECT** combination? **[JEE (Advanced) 2017]**

(A) (III) (i) (P) (B) (III) (ii) (Q) (C) (II) (iv) (R) (D) (I) (ii) (Q)

MHB064

6. For $a=\sqrt{2}$, if a tangent is drawn to a suitable conic (Column 1) at the point of contact (– 1,1), then which of the following options is the only **CORRECT** combination for obtaining its equation?

[JEE (Advanced) 2017]

(A) (II) (ii) (Q) (B) (III) (i) (P) (C) (I) (i) (P) (D) (I) (ii) (Q)

MHB065

7. Let H: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, where $a > b > 0$, be a hyperbola in the xy-plane whose conjugate axis LM subtends an angle of 60° at one of its vertices N. Let the area of the triangle LMN be $4\sqrt{3}$.

LIST-I	LIST-II
P. The length of the conjugate axis of H is	1. 8
Q. The eccentricity of H is	2. $\frac{4}{\sqrt{3}}$
R. The distance between the foci of H is	3. $\frac{2}{\sqrt{3}}$
S. The length of the latus rectum of H is	4. 4

The correct option is : **[JEE (Advanced) 2018]**

(A) $P \to 4$; $Q \to 2$, $R \to 1$; $S \to 3$ (B) $P \to 4$; $Q \to 3$; $R \to 1$; $S \to 2$

(C) $P \to 4$; $Q \to 1$, $R \to 3$; $S \to 2$ (D) $P \to 3$; $Q \to 4$; $R \to 2$; $S \to 1$

MHB066

8. Let a and b be positive real numbers such that $a > 1$ and $b < a$. Let P be a point in the first quadrant that lies on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. Suppose the tangent to the hyperbola at P passes through the point (1,0), and suppose the normal to the hyperbola at P cuts off equal intercepts on the coordinate axes. Let Δ denote the area of the triangle formed by the tangent at P, the normal at P and the x-axis. If e denotes the eccentricity of the hyperbola, then which of the following statements is/are TRUE? **[JEE (Advanced) 2020]**

(A) $1 < e < \sqrt{2}$ (B) $\sqrt{2} < e < 2$ (C) $\Delta = a^4$ (D) $\Delta = b^4$

MHB067

9. Consider the hyperbola $\frac{x^2}{100}-\frac{y^2}{64}=1$ with foci at S and S_1, where S lies on the positive x-axis. Let P be a point on the hyperbola, in the first quadrant. Let $\angle SPS_1 = \alpha$, with $\alpha < \frac{\pi}{2}$. The straight line passing through the point S and having the same slope as that of the tangent at P to the hyperbola, intersects the straight line S_1P at P_1. Let δ be the distance of P from the straight line SP_1, and $\beta = S_1P$. Then the greatest integer less than or equal to $\frac{\beta\delta}{9}\sin\frac{\alpha}{2}$ is_____. **[JEE (Advanced) 2022]**

MHB068

JEE (Main) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION–A

- This section contains **TWENTY** questions.
- Each question has **FOUR** options (1), (2), (3) and (4). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +4, if only the bubble corresponding to the correct option is darkened.
Zero Marks : 0, if none of the bubbles is darkened.
Negative Marks : –1 in all other cases.

1. Which of the following pair, may represent the eccentricities of two conjugate hyperbolas, for all $\alpha \in (0, \pi/2)$?

(1) $\sin\alpha, \cos\alpha$ (2) $\tan\alpha, \cot\alpha$ (3) $\sec\alpha, \operatorname{cosec}\alpha$ (4) $1 + \sin\alpha, 1 + \cos\alpha$

MHB087

2. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the focii, is :

(1) $\frac{4}{3}$ (2) $\frac{4}{\sqrt{3}}$ (3) $\frac{2}{\sqrt{3}}$ (4) none of these

MHB088

3. The equation of the hyperbola whose conjugate axis is 5 and the distance between the focii is 13, is

(1) $25x^2 - 144y^2 = 900$ (2) $144x^2 - 25y^2 = 900$

(3) $144x^2 + 25y^2 = 990$ (4) $25x^2 + 144y^2 = 900$

MHB089

4. The vertices of a hyperbola are at (0, 0) and (10, 0) and one of its foci is at (18, 0). The equation of the hyperbola is

(1) $\frac{x^2}{25} - \frac{y^2}{144} = 1$ (2) $\frac{(x-5)^2}{25} - \frac{y^2}{144} = 1$ (3) $\frac{x^2}{25} - \frac{(y-5)^2}{144} = 1$ (4) $\frac{(y-5)^2}{25} - \frac{x^2}{144} = 1$

MHB090

5. If e and e' are the eccentricities of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, then the point $\left(\frac{1}{e}, \frac{1}{e'}\right)$ lies on the circle :

(1) $x^2 + y^2 = 1$ (2) $x^2 + y^2 = 2$ (3) $x^2 + y^2 = 3$ (4) $x^2 + y^2 = 4$

MHB091

6. If $P(\sqrt{2}\sec\theta, \sqrt{2}\tan\theta)$ is a point on the hyperbola whose distance from the origin is $\sqrt{6}$ where P is in the first quadrant then θ =

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{6}$ (4) None of these

MHB092

7. If the normal at $\left(ct, \frac{c}{t}\right)$ on the curve $xy = c^2$ meets the curve again at t', then

(1) $t' = -\frac{1}{t^3}$ (2) $t' = \frac{1}{t}$ (3) $t' = \frac{1}{t^2}$ (4) $t'^2 = -\frac{1}{t^2}$

MHB093

8. The line $x + y = a$ touches the hyperbola $x^2 - 2y^2 = 18$, if a is equal to ± b, then value of $|b|$ is

(1) 3 (2) 4 (3) 12 (4) 8

MHB094

9. Equation of a tangent passing through (2, 8) to the hyperbola $5x^2 - y^2 = 5$ is :

(1) $3x - y + 2 = 0$ (2) $3x + y - 14 = 0$

(3) $23x + 3y - 22 = 0$ (4) $3x - 23y + 178 = 0$

MHB095

10. Tangent at any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ cut the axes at A and B respectively. If the rectangle $OAPB$ (where O is origin) is completed then locus of point P is given by

(1) $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$ (2) $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$ (3) $\frac{a^2}{y^2} - \frac{b^2}{x^2} = 1$ (4) none of these

MHB096

11. The number of possible tangents which can be drawn to the curve $4x^2 - 9y^2 = 36$, which are perpendicular to the straight line $5x + 2y - 10 = 0$ is :

(1) zero (2) 1 (3) 2 (4) 4

MHB097

12. An equation of a tangent to the hyperbola, $16x^2 - 25y^2 - 96x + 100y - 356 = 0$ which makes an angle $\frac{\pi}{4}$ with the transverse axis is $y = x + \lambda$, $(\lambda > 0)$, then 2λ is

(1) 16 (2) 4 (3) 3 (4) 9

MHB098

13. If $(a \sec \theta, b \tan \theta)$ and $(a \sec\phi, b \tan \phi)$ are the ends of a focal chord of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $\tan \frac{\theta}{2} \tan \frac{\phi}{2}$ equals to

(1) $\frac{e-1}{e+1}$ (2) $\frac{e+1}{e-1}$ (3) $\frac{1+e}{1-e}$ (4) none of these

MHB099

14. The locus of the middle points of chords of hyperbola $3x^2 - 2y^2 + 4x - 6y = 0$ parallel to $y = 2x$ is

(1) $3x - 4y = 4$ (2) $3y - 4x + 4 = 0$ (3) $4x - 4y = 3$ (4) $3x - 4y = 2$

MHB100

15. The chords passing through $L(2, 1)$ intersects the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ at P and Q. If the tangents at P and Q intersects at R then Locus of R is

(1) $x - y = 1$ (2) $9x - 8y = 72$ (3) $x + y = 3$ (4) None of these

MHB101

16. The number of points from where a pair of perpendicular tangents can be drawn to the hyperbola, $x^2 \sec^2 \alpha - y^2 \operatorname{cosec}^2 \alpha = 1, \alpha \in (0, \pi/4)$, is :

(1) 0 (2) 1 (3) 2 (4) infinite

MHB102

17. The product of the lengths of the perpendiculars from the two focii on any tangent to the hyperbola $\frac{x^2}{25} - \frac{y^2}{3} = 1$ is $\sqrt{k}$, then k is

(1) 16 (2) 4 (3) 3 (4) 9

MHB103

18. From Point $P(2,3)$ two tangents PA and PB are drawn to the hyperbola $x^2 - y^2 - 4x + 4y + 16 = 0$. The equation of line AB is

(1) $y = 3$ (2) $y = 2$ (3) $x = 1$ (4) $x = 3$

MHB104

19. The line $y = x$ intersects the hyperbola $\frac{x^2}{9} - \frac{y^2}{25} = 1$ at the points P and Q. The eccentricity of ellipse with PQ as major axis and minor axis of length $\frac{5}{\sqrt{2}}$ is

(1) $\frac{\sqrt{5}}{3}$ (2) $\frac{5}{\sqrt{3}}$ (3) $\frac{5}{9}$ (4) $\frac{2\sqrt{2}}{3}$

MHB105

20. The equation of common tangent to the parabola $y^2 = 8x$ and hyperbola $3x^2 - y^2 = 3$ is

(1) $2x \pm y + 1 = 0$ (2) $2x \pm y - 1 = 0$ (3) $x \pm 2y + 1 = 0$ (4) $x \pm 2y - 1 = 0$

MHB106

SECTION-B

- This section will have **TEN** questions. Candidate can choose to attempt any 5 question out of these 10 questions. In case if candidate attempts more than 5 questions, first 5 attempted questions will be considered for marking.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (Answer should be rounded off to the nearest integer).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4, if only correct answer is given.
Zero Marks : 0, if no answer is given.
Negative Marks : -1 for incorrect answer

1. If two points P & Q on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ whose centre is C be such that CP is perpendicular to CQ & $a < b$, then $\frac{1}{CP^2} + \frac{1}{CQ^2} = \lambda\left(\frac{1}{a^2} - \frac{1}{b^2}\right)$ where λ is :

MHB107

2. An ellipse and a hyperbola have the same centre origin, the same foci and the minor-axis of the one is the same as the conjugate axis of the other. If e_1, e_2 be their eccentricities respectively, then $\frac{1}{e_1^2} + \frac{1}{e_2^2} =$

MHB108

3. If $(\lambda, 4)$ is the orthocentre of the triangle whose vertices lie on the rectangular hyperbola $xy = 16$, then λ is equal to

MHB109

4. If m_1 and m_2 are slopes of the tangents to the hyperbola $\frac{x^2}{25}-\frac{y^2}{16}=1$ which passes through the point of contact of $3x - 4y = 5$ and $x^2 - 4y^2 = 5$ then $32(m_1 + m_2 - m_1m_2)$ = ...

MHB110

5. Tangents are drawn from the point $(\alpha, 2)$ to the hyperbola $3x^2 - 2y^2 = 6$ and are inclined at angles θ & ϕ to the x-axis . If $\tan\theta \cdot \tan\phi = 2$, then the value of $2\alpha^2 - 7$ is

MHB111

6. C be the centre of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$. The tangents at any point P on this hyperbola meets the straight lines $4x - 3y = 0$ and $4x + 3y = 0$ in the points Q and R respectively. Then $CQ.CR$ =

MHB112

7. If radii of director circles of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are 2r and r respectively and e_e and e_h be the eccentricities of the ellipse and the hyperbola respectively then $4e_h^2 - e_e^2$ is equal to

MHB113

8. The length of that focal chord of the hyperbola $xy = 8$ which touches the circle $x^2 + y^2 = 8$ is.

MHB114

9. The sum of lengths of perpendiculars drawn from focii to any real tangent to the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ is always greater than a, then find maximum value of a.

MHB115

10. Let $x^2 + y^2 = r^2$ and $xy = 1$ intersect at A & B in first quadrant, If $AB = \sqrt{14}$ then find the value of r.

MHB116

JEE (Advanced) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION–I

- This section contains **FIVE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	: +3	If only the bubble corresponding to the correct option is darkened.
Zero Marks	: 0	If none of the bubbles is darkened.
Negative Marks	: –1	In all other cases.

1. The transverse axis of a hyperbola is of length 2a and a vertex divides the segment of the axis between the centre and the corresponding focus in the ratio 2 : 1. Find the equation of the hyperbola.

(A) $5x^2-4y^2=5a^2$ (B) $4x^2-5y^2=4a^2$ (C) $4x^2-5y^2=5a^2$ (D) $5x^2-4y^2=4a^2$

MHB117

2. If x cos α + y sin α = p, a variable chord of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{2a^2}=1$ subtends a right angle at the centre of the hyperbola, then the chords touch a fixed circle, find the radius of the circle.

(A) $2\sqrt{2}\,a$ (B) $\sqrt{3}\,a$ (C) $\sqrt{2}\,a$ (D) $2\sqrt{3}\,a$

MHB118

3. Find an equation of the hyperbola whose directrix is the normal to circle $x^2+y^2-4x-6y+9=0$ having slope is 2 and eccentricity is equal to radius of given circle where focus of hyperbola is point of contact of given circle with y-axis.

(A) $11x^2-y^2-16xy+16x-38y-41=0$ (B) $11x^2-y^2+16xy-16x-38y+41=0$

(C) $11x^2-y^2-16xy-16x+38y-41=0$ (D) $11x^2-y^2+16xy+16x-38y+41=0$

MHB119

4. Chords of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are tangents to the circle drawn on the line joining the foci as diameter . Find the locus of the point of intersection of tangents at the extremities of the chords.

(A) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{a^2+b^2}$ (B) $\frac{x^2}{a^4}-\frac{y^2}{b^4}=\frac{1}{a^2+b^2}$

(C) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=\frac{1}{a^2+b^2}$ (D) $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{a^2+b^2}$

MHB120

5. From any point on the hyperbola $H_1 : \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ tangents are drawn to the hyperbola $H_2 : \frac{x^2}{a^2}-\frac{y^2}{b^2}=2$. Then find the area cut-off by the chord of contact on the asymptotes of H_2.

(A) $2ab$ (B) $4ab$ (C) $\frac{ab}{2}$ (D) $\frac{ab}{4}$

MHB121

SECTION-II

- This section contains **FIVE** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 if only (all) the correct option(s) is (are) chosen.
Partial Marks : +3 if all the four options are correct but ONLY three options are chosen.
Partial Marks : +2 if three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks : +1 if two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks : 0 if none of the options is chosen (i.e. the question is unanswered).
Negative Marks : –2 in all other cases.

For Example : If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

6. If $(3\sin\alpha, 2\cos\alpha)$ lies on the same side as that of origin w.r.t conic $2x^2 - 3y^2 = 6$, then $\sin\alpha$ may be

(A) $-\sqrt{\frac{4}{5}}$ (B) $\sqrt{\frac{2}{5}}$ (C) $\frac{1}{\sqrt{5}}$ (D) $\frac{2}{15}$

MHB122

7. A rectangular hyperbola whose centre is C is cut by any circle of radius r in four points P, Q, R and S. Then $CP^2 + CQ^2 + CR^2 + CS^2 =$

(A) 16 if $r = \sqrt{2}$ (B) 16 if $r = 2$ (C) 2 if $r = 1$ (D) 4 if $r = 1$

MHB123

8. If the chord joining the points whose eccentric angles are 'α' and 'β' on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is a focal chord then

(A) $\pm e\cos\left(\frac{\alpha-\beta}{2}\right) = \cos\left(\frac{\alpha+\beta}{2}\right)$

(B) $\pm e\cos\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)$

(C) $\tan(\alpha/2)\tan(\beta/2) + \left(\frac{ke-1}{ke+1}\right) = 0$ where $k = \pm 1$

(D) $\tan(\alpha/2)\tan(\beta/2) + \left(\frac{ke+1}{ke-1}\right) = 0$ where $k = \pm 1$

MHB124

9. Circles are drawn on chords of rectangular hyperbola $xy = c^2$ parallel to the line $y = x$ as diameters. All such circles pass through two fixed points whose co-ordinates are :

(A) (c, c) (B) $(c, -c)$ (C) $(-c, c)$ (D) $(-c, -c)$

MHB125

10. If a circle and the rectangular hyperbola $xy = c^2$ meet in the four points t_1, t_2, t_3 & t_4, then

(A) $t_1\, t_2\, t_3\, t_4 = 1$

(B) The arthmetic mean of the four points bisects the distance between the centres of the two curves.

(C) The geometrical mean of the four points bisects the distance between the centres of the two curves.

(D) The centre of the circle through the points t_1, t_2 & t_3 is :

$$\left\{\frac{c}{2}\left(t_1 + t_2 + t_3 + \frac{1}{t_1\, t_2\, t_3}\right), \frac{c}{2}\left(\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + t_1\, t_2 t_3\right)\right\}$$

MHB126

SECTION–III

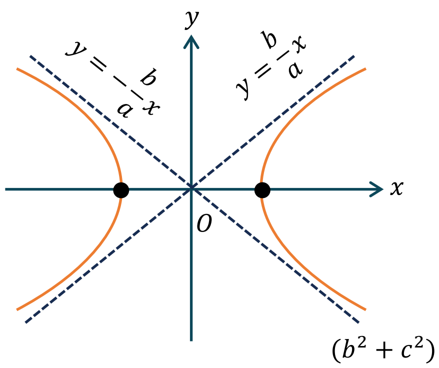
- This section contains **ONE** paragraph.
- Based on each paragraph, there are **THREE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D) **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in <u>one of the following categories</u> :

Full Marks : +3 if only the bubble corresponding to the correct answer is darkened.

Zero Marks : 0 in all other cases.

Comprehension (Q. No. 11 - 13)

Asymptotes are lines whose distance from the curve at infinity tends to zero Let $y = mx + c$ is asymptote of H $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$. Solving the two equations, we have $\left(b^2 - a^2m^2\right)x^2 - 2a^2mcx - a^2\left(b^2 + c^2\right) = 0$. Both roots of this equation must be infinite so $m = \pm\frac{b}{a}$ and $c = 0$ which implies that $y = \pm\frac{b}{a}x$ are asymptotes of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Note that no real tangent can be drawn to the hyperbola from its centre and only one real tangent can be drawn from a point lying on its asymptote other than centre. Further combined equation of asymptotes is $A = \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ and conjugate hyperbola $C = \frac{x^2}{a^2} - \frac{y^2}{b^2} + 1 = 0$. Hence $2A = H + C$, as we can see, equation of A, H and C vary only by a constant, for asymptotes which can be evaluated by applying condition of pair of lines.



11. The points of contact of tangents drawn to the hyperbola $\frac{x^2}{3}-\frac{y^2}{2}=1$ from point (2, 1) are

(A) (3, 2), (1, 5) (B) (3, 2), $\left(\frac{9}{5},\frac{2}{5}\right)$ (C) (1, 2), (3, 4) (D) (3, 2), (3, 4)

MHB127

12. The number of real distinct tangents drawn to hyperbola $4x^2 - y^2 = 4$ from point (1, 2) is

(A) 1 (B) 2 (C) 3 (D) 4

MHB128

13. The asymptotes of $xy - 3y - 2x = 0$ is

(A) $x + 2 = 0$ and $y + 3 = 0$ (B) $x - 2 = 0$ and $y - 3 = 0$

(C) $x - 3 = 0$ and $y - 2 = 0$ (D) $x + 3 = 0$ and $y + 2 = 0$

MHB129

SECTION-IV

- This section contains **FIVE** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the correct numerical value is entered;

Zero Marks : 0 In all other cases.

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14. If λ_1 and λ_2 are the two value of l for which the line $y = 3x + \lambda$ touch the hyperbola $9x^2 - 5y^2 = 45$, then find the value of $|\lambda_1|+|\lambda_2|$?

MHB130

15. If the straight line $2x + \sqrt{2}\,y + n = 0$ touches the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$, then find the sum of all the values of n.

MHB131

16. The curve $xy = c(c > 0)$ and the circle $x^2 + y^2 = 25$ touch at two points, then find the distance between the points of contact.

MHB132

17. If the tangent on the point $(3 \sec \phi, 4 \tan \phi)$ (which is in first quadrant) of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is perpendicular to $3x + 8y - 12 = 0$, then find the value of ϕ is (in degree).

MHB133

18. If P is any point common to the hyperbola $\frac{x^2}{16}-\frac{y^2}{25}=1$ and the circle having line segment joining its focii as diameter and sum of focal distances of point P is $\sqrt{a}$, then the value of a is .

MHB134

ANSWER KEY

EXERCISE - O

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	B	C	B	B	A	D	A	B	D
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B	C	C	D	A	B	C	A	C,D	A,B,C,D
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	A,C,D	B,C	B,D	A,B,C	A,B,C	A,B	A,B,C,D	A,D	A	A
Que.	31									
Ans.	C									

EXERCISE - S

1. 4 **2.** 2 **3.** 8 **4.** 5 **5.** 3

6. 9 **7.** 81.00 **8.** 45.00 **9.** 77 **10.** $45\sqrt{5}$

EXERCISE - JEE (Main) PYQ

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	2	3	4	4	3	3	2	2	2	1
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	4	1	2	2	3	3	4	42	4	85
Que.	21	22	23	24	25					
Ans.	306	12	3	2	216					

EXERCISE - JEE (Advanced) PYQ

Que.	1	2	3	4	5	6	7	8	9
Ans.	B	A,B	A,B,D	D	A	D	B	A,D	7

JEE (Main) Practice Paper

Section-A	Q.	1	2	3	4	5	6	7	8	9	10
	A.	3	3	1	2	1	1	1	1	1	1
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	1	2	3	1	2	4	4	2	4	1
Section-B	Q.	1	2	3	4	5	6	7	8	9	10
	A.	1	2	4	22	4	25	6	16	6	3

JEE (Advanced) Practice Paper

Section-I	**Q.**	**1**	**2**	**3**	**4**	**5**
	A.	A	C	C	D	B
Section-II	**Q.**	**6**	**7**	**8**	**9**	**10**
	A.	B,C,D	B,D	A,C,D	A,D	A,B,D
Section-III	**Q.**	**11**	**12**	**13**		
	A.	B	A	**C**		
Section-IV	**Q.**	**14**	**15**	**16**	**17**	**18**
	A.	12	0	10	30	264

Important Notes