Ellipse

SOLUTIONS

EXERCISE - O

1. **Ans. (D)**

$E \equiv \frac{x^2}{9} + \frac{y^2}{4} - 1 = 0$

$C \equiv x^2 + y^2 - 9 = 0$

$P(1,2)$ & $Q(2,1)$

$E_p > 0$ & $C_P < 0 \Rightarrow P$ lies inside C but outside E

$E_Q < 0$ & $C_Q < 0 \Rightarrow Q$ lies inside C & E

2. **Ans. (C)**

$2(x^2 - 4x) + 3(y^2 - 6y) + 35 = k$

$2(x-2)^2 + 3(y-3)^2 = k$

if $k > 0$, then locus is ellipse.

if $k < 0$, then no locus

if $k = 0$, then equation represents a point $(2, 3)$.

3. **Ans. (C)**

$\frac{x^2}{25} + \frac{y^2}{16} = 1$

$a = 5$

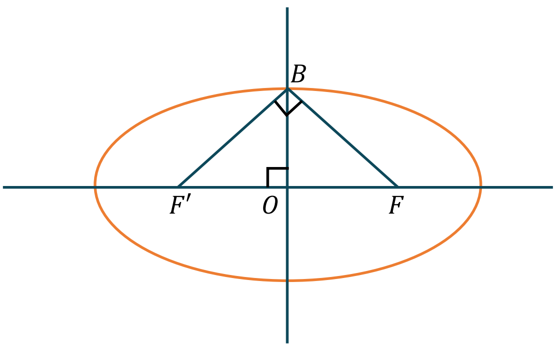
F_1 & F_2 are foci of ellipse

So $PF_1 + PF_2 = 2a = 10$

4. **Ans. (C)**

Area of $\Delta BFF'$

$\frac{1}{2} BF \times BF' = \frac{1}{2} \times b \times 2ae$



$BF \times BF' = 2abe$

$BF + BF' = 2a$

$\Rightarrow (BF)^2 + (BF')^2 + 2BF \cdot BF' = 4a^2$

$\Rightarrow (FF')^2 + 2(2abe) = 4a^2$

$\Rightarrow 4a^2e^2 + 4abe = 4a^2 \Rightarrow a^2e^2 + abe = a^2$

$\Rightarrow a^2 - b^2 + abe = a^2 \Rightarrow b^2 = abe$

$\Rightarrow \frac{b}{a} = e$

$\frac{b^2}{a^2} = e^2 = 1 - e^2$

$e = \frac{1}{\sqrt{2}}$

5. **Ans. (A)**

Directrix : $x = 0$

$e = 1/2$

Focus = (3, 0)

$\therefore \sqrt{(x-3)^2 + y^2} = \frac{1}{2} \cdot |x|$

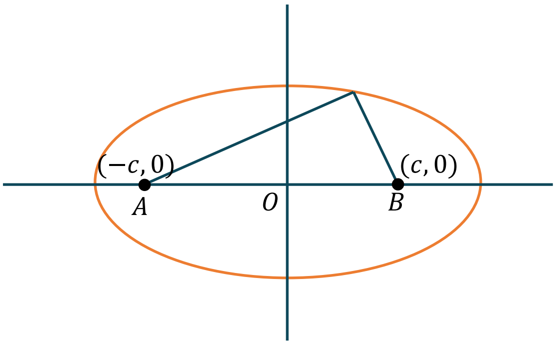
$\therefore (x-3)^2 + y^2 = \frac{1}{4} \cdot x^2 \Rightarrow 4(x-3)^2 + 4y^2 = x^2$

$\Rightarrow 3x^2 - 24x + 4y^2 + 36 = 0$

$\Rightarrow 3(x-4)^2 + 4y^2 = 12 \Rightarrow \frac{(x-4)^2}{4} + \frac{y^2}{3} = 1$ ....(1)

$\therefore a = 2 ; b = \sqrt{3}$; centre (4, 0) $\Rightarrow$ auxillary circle is $(x-4)^2 + y^2 = 4$.

6. **Ans. (B)**



$A = \pi ab$ 'a' is constant and b varies

$A^2 = \pi^2 a^2 (a^2 - c^2)$

for A to be maximum c must be minimum; A & $B \rightarrow$ centre

as $A \rightarrow B \Rightarrow c \rightarrow 0$

ellipse becomes circle

7. **Ans. (A)**

$3y = (x+2)^2 - 13$

$3y + 13 = (x+2)^2$

$\therefore (x+2)^2 = 3\left(y + \frac{13}{3}\right) \Rightarrow$ Latus Rectum = 3

The other conic is, $(x-3)^2 + 4(y^2 + 4y) = 24 + 9$

$(x-3)^2 + 4(y+2)^2 = 49$

$\frac{(x-3)^2}{7^2} + \frac{(y+2)^2}{(7/2)^2} = 1$ which is an ellipse

Latus Rectum = $\frac{2b^2}{a} = \frac{2 \cdot 49}{4 \cdot 7} = \frac{7}{2}$

$\therefore$ positive difference $\frac{7}{2} - 3 = \frac{1}{2}$ Ans.

8. Ans. (C)

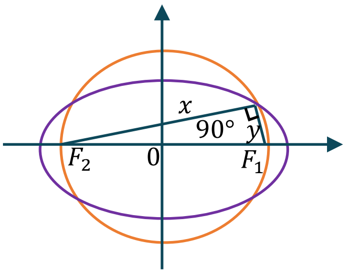
$2ae = 13\sqrt{2}$ = focal length ...(1)

$\because$ 2a = 26 $\Rightarrow a = 13$ (By focus-directrix property)

$\therefore$ On putting a = 13 in equation (1), we get

$2(13)e = 13\sqrt{2} \Rightarrow e = \frac{1}{\sqrt{2}}$

9. Ans. (C)



$x + y = 17$; $xy = 60$, To find $\sqrt{x^2 + y^2}$]

now, $x^2 + y^2 = (x + y)^2 - 2xy$

$= 289 - 120 = 169$

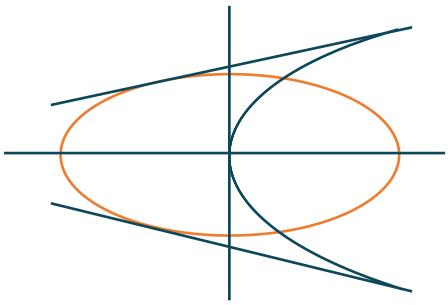
$\Rightarrow \sqrt{x^2 + y^2} = 13$

10. Ans. (A)

$y = x/2 + 2$ is tangent on the ellipse then 4 = 4.(1/4) + $b^2 \Rightarrow b^2 = 3$

parabola is, $y = mx + 1/m$

using condition of tangency, $\frac{1}{m^2} = 4m^2 + 3$



$4y^2 + 3y - 1 = 0$ (when $m^2 = y$)

$4y^2 + 4y - y - 1 = 0 \Rightarrow 4y(y + 1) - (y + 1) = 0$

$\Rightarrow y = 1/4$; $y = -1$

$m = \pm 1/2$

$y = \frac{x}{2} + 2$ or $y = -x/2 - 2$

$\Rightarrow 2y + x + 4 = 0$ (other tangent)

11. Ans. (A)

Tangent at (–8, 3) is $-\frac{8x}{100} + \frac{3y}{25} = 1$

put $x = 0$, $y = \frac{25}{3}$

12. Ans. (B)

$m = 4$

$c^2 = a^2m^2 + b^2$

$c^2 = 64 + 1 = 65$

$c = \pm\sqrt{65}$

two values of c.

13. Ans. (C)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad e=\sqrt{1-\frac{b^2}{a^2}} \qquad (a>b)$$

Let point P = ($a\cos\theta$, $b\sin\theta$)

Normal at $P \Rightarrow \frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2e^2$

$$x=0 \qquad y=-\frac{a^2e^2}{b}\sin\theta$$

$$y=0 \qquad x=\frac{a^2e^2\cos\theta}{a}$$

Let mid point $(h,k)=\left(\frac{a^2e^2\cos\theta}{2a},-\frac{a^2e^2\sin\theta}{2b}\right)$

$\cos^2\theta+\sin^2\theta=1$

$$\Rightarrow\left(\frac{2ah}{a^2e^2}\right)^2+\left(\frac{2bk}{-a^2e^2}\right)^2=1\Rightarrow\frac{h^2}{\left(\frac{a^4e^4}{4a^2}\right)}+\frac{k^2}{\left(\frac{a^4e^4}{4b^2}\right)}=1$$

eccentricity $\frac{a^4e^4}{4a^2}=\frac{a^4e^4}{4b^2}\left(1-(e')^2\right)$

$b^2=a^2\left(1-(e')^2\right)$

$e'=e$

14. Ans. (B)

$$(x-2)^2+9\left(y+\frac{1}{3}\right)^2=1$$

$$\frac{(x-2)^2}{1^2}+\frac{\left(y+\frac{1}{3}\right)^2}{\left(\frac{1}{3}\right)^2}=1$$

$x=2+\cos\theta$

$y=-\frac{1}{3}+\frac{\sin\theta}{3}$

$4x-9y=8+4\cos\theta+3-3\sin\theta$

$=11+4\cos\theta-3\sin\theta$

$=11+5=16$

15. Ans. (A)

by property $y_1y_2=b^2$

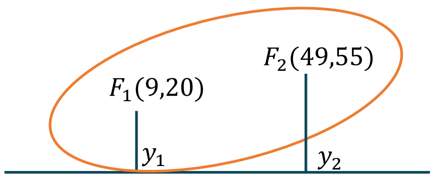
$b^2=20\times55=1100$

distance between foci

$f_1f_2=\ 2ae=\sqrt{2825}$

$4a^2=2825+4400$

$a=85$



16. **Ans. (A,B)**

When $a > b$

As given $2b = 2ae \Rightarrow b = ae$ ...(i)

Also $\frac{2b^2}{a} = 10 \Rightarrow b^2 = 5a$ ...(ii)

Now since $b^2 = a^2 - a^2e^2 \Rightarrow b^2 = a^2 - b^2$ [From (i)]

$\Rightarrow 2b^2 = a^2$ ...(iii)

(ii), (iii) $\Rightarrow a^2 = 100, b^2 = 50$

Hence equation of the ellipse will be $\frac{x^2}{100} + \frac{y^2}{50} = 1 \Rightarrow x^2 + 2y^2 = 100$

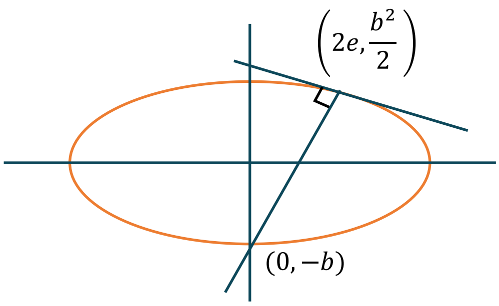
Similarly, when $a < b$ then required ellipse is $2x^2 + y^2 = 100$.

17. **Ans. (A,B,D)**

Equation of normal is

$\Rightarrow \frac{4x}{x_1} - \frac{b^2y}{y_1} = 4 - b^2$

$\Rightarrow \frac{4x}{2e} - \frac{2b^2y}{b^2} = 4 - b^2$



It passes through $(0, -b)$

$2b = 4 - b^2$

$\Rightarrow b^2 + 2b - 4 = 0$

$\Rightarrow b^2 = 6 - 2\sqrt{5}$

equation of director circle $x^2 + y^2 = 4 + 6 - 2\sqrt{5}$

length of latus rectum = $\frac{2b^2}{2} = b^2$

eccentricity $e = \sqrt{1 - \frac{(6 - 2\sqrt{5})}{4}}$

$= \sqrt{\frac{\sqrt{5} - 1}{2}}$

equation of auxiliary circle is $x^2 + y^2 = 4$

18. **Ans. (A,C)**

We have

Slope of AB = Slope of tangent at C

$$\Rightarrow \frac{b(\sin\beta - \sin\alpha)}{a(\cos\beta - \cos\alpha)} = \frac{-b\cos\theta}{a\sin\theta} \Rightarrow \frac{-\cos\left(\frac{\alpha+\beta}{2}\right)}{\sin\left(\frac{\alpha+\beta}{2}\right)} = \frac{-\cos\theta}{\sin\theta}$$

$$\Rightarrow \tan\left(\frac{\alpha+\beta}{2}\right) = \tan\theta \Rightarrow \theta = \frac{\alpha+\beta}{2} + n\pi (n \in I)$$

19. **Ans. (A,C)**

The tangent & normal at a point P on the ellipse bisect the external & internal angles between the focal distances of P.

So answer are (A) and (C)

20. Ans. (A,B)

y-intercept of the tangent is positive.

Equation of tangent to circle $y = mx + 4\sqrt{1+m^2}$

Equation of tangent to ellipse $y = mx + \sqrt{25m^2+4}$

Both represents the same line so

$4\sqrt{1+m^2} = \sqrt{25m^2+4} \Rightarrow m = \pm\frac{2}{\sqrt{3}}$

The tangent is at the point in first quadrant so $m < 0 \Rightarrow m = -\frac{2}{\sqrt{3}}$

So common tangent is $y = -\frac{2}{\sqrt{3}}x + 4\sqrt{\frac{7}{3}}$

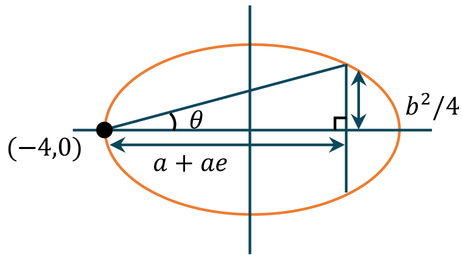
It meets the co-ordinate axis at $A(2\sqrt{7},0)$ and $B(0,4\sqrt{7/3})$

So, $AB = \lambda_1 = 14/\sqrt{3}$

21. Ans. (A,C,D)

Farthest vertex is (–4, 0) for L.R. Through $S(4e, 0)$

$\tan\theta = \frac{b^2/a}{a(1+e)} = \frac{b^2}{16(1+e)} = \frac{16(1-e^2)}{16(1+e)} = 1-e$



Given cosecθ = $\sqrt{5} \Rightarrow \tan\theta = \frac{1}{2}$

So $e = \frac{1}{2}$ & $b = a\sqrt{1-e^2} = 4\sqrt{1-\frac{1}{4}} = 2\sqrt{3}$

Area formed by latus rectum with nearest vertex is $\frac{1}{2}\times\frac{2b^2}{a}\times(a-ae) = 6.$

22. Ans. (A,C,D)

Given slope of common tangent $m = \frac{1}{2}$.

Equation of general tangent to $y^2 = 4x$ is

$y = mx + \frac{1}{m}$ ...(i)

$\Rightarrow y = \frac{1}{2}x + 2$ [$\because m = \frac{1}{2}$ in given equation]

On comparing with given equation, we get $k = 4$

Equation of tangent of $\frac{x^2}{a^2} + \frac{y^2}{3} = 1$ is

$y = mx \pm \sqrt{a^2m^2+3}$ ...(ii)

On comparing (i) & (ii)

$\frac{1}{m} = \pm\sqrt{a^2m^2+3}$ ...(iii)

Put $m = \frac{1}{2}$

$\Rightarrow a^2 = 4 \Rightarrow a = \pm 2$ ...(iv)

Using (iii) & (iv) we get $m = \pm \frac{1}{2}$.

So equation of other common tangent is

$x + 2y + 4 = 0$.

23. **Ans. (C,D)**

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{9}{a^2}+\frac{1}{b^2}=1 \qquad ...(1)$$

Case-I when $a > b$

$b^2 = a^2(1-e^2)$

$b^2 = a^2(1-2/5)$

$5b^2 = 3a^2 \qquad ...(2)$

from (1) & (2)

$$\frac{9\times3}{5b^2}+\frac{1}{b^2}=1 \Rightarrow b^2=\frac{32}{5}$$

$$\therefore\ a^2=\frac{32}{3}$$

$\Rightarrow$ equation is $3x^2 + 5y^2 - 32 = 0$

Case-II when $b > a$

$a^2 = b^2(1-e^2) = \frac{3}{5}b^2 \qquad ...(3)$

from (1) & (3)

$a^2=\frac{48}{5}$, $b^2 = 16$

$$\therefore\ \frac{5x^2}{48}+\frac{y^2}{16}=1$$

$\Rightarrow 5x^2 + 3y^2 - 48 = 0$

24. **Ans. (A,D)**

$\therefore (3a-c)(3a-2c) < 0$

$\Rightarrow 9a^2 - 9ac + 2c^2 < 0$

$\Rightarrow \boxed{9ac-9a^2-2c^2>0}$

Now we have

$c < 3a < 2c \qquad ...(1)$

$c^2 < 9a^2 < 4c^2 \qquad ...(2)$

now multiply 1 by $2c$

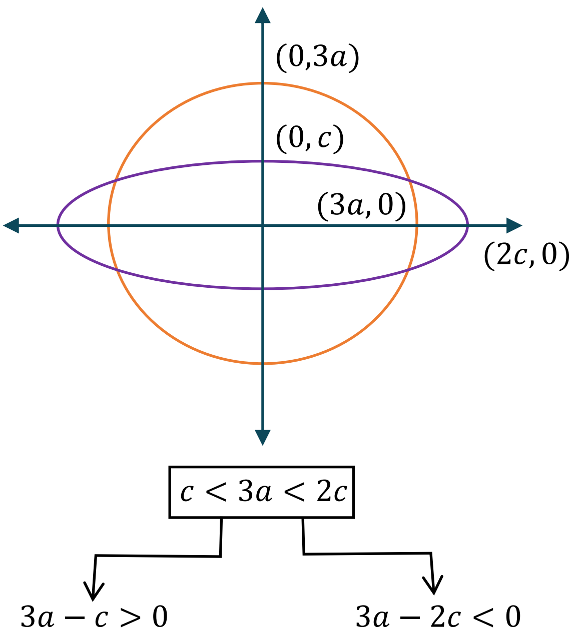
$2c^2 < 6ac < 4c^2 \qquad ...(3)$

Now by (2) + (3)

$3c^2 < 9a^2 + 6ac < 8c^2$

$\Rightarrow c^2 < 9a^2 + 6ac - 2c^2 < 6c^2$

$\Rightarrow 9a^2 + 6ac - 2c^2 > 0$



25. Ans. (B,D)

$\because\ e_1 \cdot e_2 = \frac{1}{2}$ Given $\qquad \because\ e_1^2 \cdot e_2^2 = \left(1-\frac{2}{3}\right)\left(1-\frac{b^2}{16}\right)$

$\Rightarrow\ \frac{1}{4} = \frac{1}{3}\cdot\left(1-\frac{b^2}{16}\right)\ \Rightarrow\ \frac{3}{4} = 1-\frac{b^2}{16}\ \Rightarrow\ \frac{b^2}{16} = 1-\frac{3}{4} \Rightarrow b^2 = 4$

$\Rightarrow b = 2$

length of minor axis of ellipse $E_2 = 2b = 4$

Now second possibility is

$e_1^2 \cdot e_2^2 = \left(1-\frac{2}{3}\right)\left(1-\frac{16}{b^2}\right) = \frac{1}{4} = \frac{1}{3}\left(1-\frac{16}{b^2}\right) = 1-\frac{16}{b^2} = \frac{3}{4}$

$\Rightarrow \frac{16}{b^2} = \frac{1}{4}$

$b^2 = 64 \Rightarrow b = 8$

length of minor axis of ellipse

$E_2 = 2b = 16.$

26. Ans. (A,B)

Tangent to parabola is

$y = mx + \frac{1}{m}$ ...I

for ellipse

$y = mx \pm \sqrt{a^2m^2+b^2} = mx \pm \sqrt{4m^2+3}$ ...II

comparing both $4m^2 + 3 = \frac{1}{m^2}$

$4m^3 + 3m^2 - 1 = 0$

$4m^4 + 4m^2 - m^2 - 1 = 0$

$4m^2(m^2+1) - (m^2+1) = 0$

$(4m^2-1)\ (m^2+1) = 0$

$m = \frac{1}{2}; m = -\frac{1}{2}$

put in equation (I)

$\boxed{x-2y+4=0\ \&\ x+2y+4=0}$

27. Ans. (A,B)

Equation of normal at the point $(a\cos\theta, b\sin\theta)$ is $ax.\sec\theta$ – by $\text{cosec}\theta = a^2 - b^2$

$\frac{4x}{\cos\theta} - \frac{5y}{\sin\theta} + 9 = 0$

$\ell = \frac{9}{\sqrt{16\sec^2\theta + 25\text{cosec}^2\theta}}$

$a^2\sec^2\theta + b^2\text{cosec}^2\theta = (a+b)_{\min}$

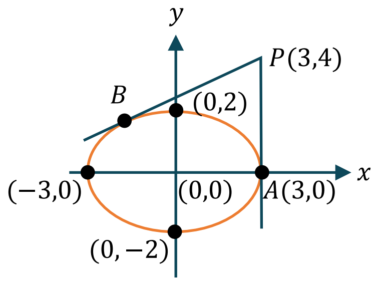
$\ell_{\max} = \frac{9}{(4+5)} = 1$

so only A & B are possible

28. Ans. (D)

As shown in figure, one of the point of contact is (3,0)

Let equation of other tangent,



$y = mx + \sqrt{9m^2 + 4}$ as $c > 0$

It passes through (3,4), so

$4 = 3m + \sqrt{9m^2 + 4}$

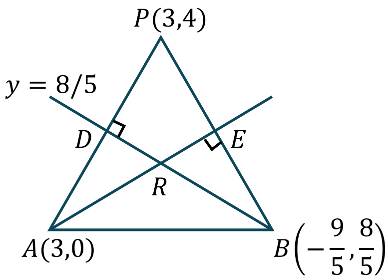
$(4 - 3m)^2 = 9m^2 + 4$

Solving, $m = \frac{1}{2}$

As we know that point of contact for the tangent given by $\left(-\frac{a^2m}{\sqrt{a^2m^2+b^2}}, \frac{b^2}{\sqrt{a^2m^2+b^2}}\right)$

$\therefore$ Point of contact is $\left(-\frac{9}{5}, \frac{8}{5}\right)$

29. Ans. (C)



Equation of line BD: $y = \frac{8}{5}$

Equation of line AE: $2x + y = 6$

Now orthocentre R of ΔPAB will be intersection of line BD and line AE.

Solving for R, we get $R \equiv \left(\frac{11}{5}, \frac{8}{5}\right)$

30 Ans. (A)

Equation of line AB is $x + 3y = 3$

Now let the point be (h, k)

According to question,

$$\left|\frac{h+3k-3}{\sqrt{1^2+3^2}}\right| = \sqrt{(h-3)^2 + (4-k)^2}$$

After solving, we get

$9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

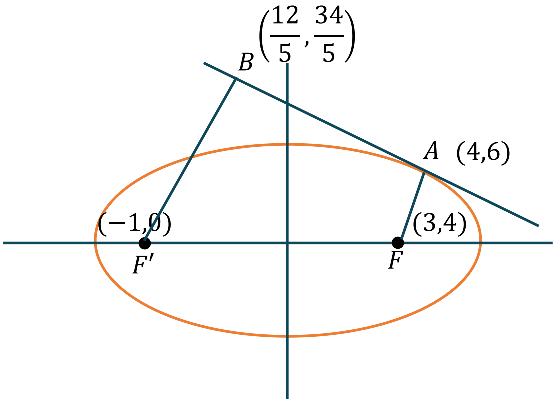
31. Ans. (A)

Equation tangent is $(y-6) = -\left(\frac{4-3}{6-4}\right)(x-4)$ i.e., $x+2y-16=0$

So, $\frac{\alpha-(-1)}{1} = \frac{\beta-0}{2} = -\frac{[1(-1)+2(0)-16]}{(1)^2+(2)^2}$

$\Rightarrow (\alpha,\beta) \equiv \left(\frac{12}{5},\frac{34}{5}\right)$

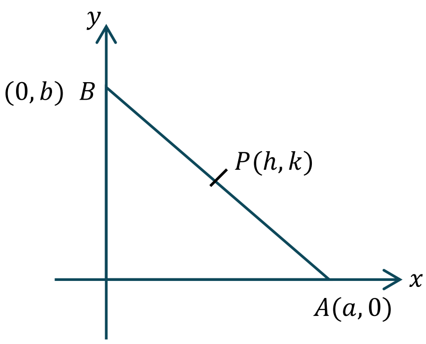
32. Ans. (D)



$FA \cdot F'B = b^2$

$\Rightarrow \sqrt{5}\cdot\frac{\sqrt{289}}{\sqrt{5}} = b^2 \Rightarrow 17 = b^2 \Rightarrow \sqrt{17} = b$

33. Ans. (A→S;B→R;C→P;D→Q)



(A) $2x-3y=6$ intersects x –axis at (3, 0) and $4x+5y=20$ intersects y –axis at (0, 4)

equation of ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$

$e=\sqrt{1-\frac{9}{16}}=\frac{\sqrt{7}}{4}$

(B) $AB = 20$

$a^2+b^2=400$ ...(1)

P divides AB in ratio 2 : 3

$(h,k)=\left(\frac{3a}{5},\frac{2b}{5}\right)$

$a=\frac{5h}{3}, b=\frac{5k}{2}$

from (1), $\frac{h^2}{\left(\frac{9}{25}\right)}+\frac{k^2}{\left(\frac{4}{25}\right)}=1$

Locus is

$\frac{x^2}{\left(\frac{9}{25}\right)}+\frac{y^2}{\left(\frac{4}{25}\right)}=1$

$e=\sqrt{1-\frac{4}{9}}=\frac{\sqrt{5}}{3}$

(C) $S(ae, 0)$ $S'(-ae, 0)$ $B(0, b)$

$SB = S'B = SS'$ and $b^2 = a^2(1-e^2)$

$\sqrt{a^2e^2+b^2}=2ae$

$a = 2ae = e=\frac{1}{2}$

(D) The given distance is clearly the length of semi major axis

Thus, $\sqrt{\frac{a^2+2b^2}{2}} = a \Rightarrow 2b^2 = a^2 \Rightarrow 2a^2(1-e^2) = a^2$

$\Rightarrow e^2 = \frac{1}{2} \Rightarrow e = \frac{1}{\sqrt{2}}$

EXERCISE - S

1. **Ans. (5.00)**

$a(x^2 + y^2 + 2y + 1) = (x - 2y + 3)^2$

$\Rightarrow a(x^2 + (y + 1)^2) = (x - 2y + 3)^2 \Rightarrow \sqrt{x^2+(y+1)^2}=\frac{\sqrt{5}}{\sqrt{a}}\frac{|x-2y+3|}{\sqrt{5}}$

Above is an ellipse equation whose focus is $(0, -1)$ and directrix $x - 2y + 3 = 0$

This represents ellipse if $e=\frac{\sqrt{5}}{\sqrt{a}}<1.$

This gives $a \in (5, \infty)$.

Hence, $b = 5$

2. **Ans. (64.00)**

Let the equation be

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(1)

It pass through $(-3, 1)$ & $(2, -2)$

So $\frac{9}{a^2}+\frac{1}{b^2}=1$ ...(2)

& $\frac{4}{a^2}+\frac{4}{b^2}=1$ ...(3)

Solve (2) & (3) to get

$a^2=\frac{32}{3}$ & $b^2=\frac{32}{5}$

$10 \times b^2 = 64$

3. **Ans. (2)**

$2ae = 10$ ...(1)

& $\frac{a}{e} - ae = 15$ ...(2)

Solving we get divide (1) by (2)

$$\frac{ae}{a\left(\frac{1}{e}-e\right)} = \frac{5}{15} \Rightarrow 3e^2 = 1 - e^2$$

$$e = \frac{1}{2}$$

4. **Ans. (8.00)**

Let $x - 4 = 2\cos\theta$, i.e., $x = 2\cos\theta + 4$ and $y = 3\sin\theta$. Now,

$$E = \frac{x^2}{4} + \frac{y^2}{9} = \frac{(2\cos\theta + 4)^2}{4} + \sin^2\theta$$

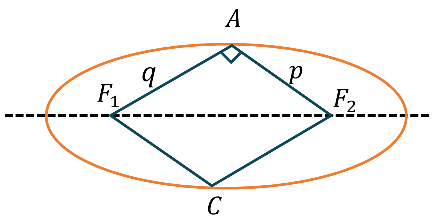
$$= \frac{4\cos^2\theta + 16 + 16\cos\theta + 4\sin^2\theta}{4} = \frac{20 + 16\cos\theta}{4} = 5 + 4\cos\theta$$

$E_{max} = 5 + 4 = 9$

$E_{min} = 5 - 4 = 1$

Hence, $E_{max} - E_{min} = 9 - 1 = 8$.

5. **Ans. (80)**



$pq = 200$ & $\pi ab = 200\pi$

$F_1F_2 = 2ae$

$p^2 + q^2 = (F_1F_2)^2$

$p^2 + q^2 = 4(ae)^2 = 4(a^2 - b^2)$

$p + q = 2a$

$p^2 + q^2 + 2pq = 4a^2$

$4(a^2 - b^2) + 400 = 4a^2$

$a^2 - b^2 + 100 = a^2 \Rightarrow b = 10$

$ab = 200 \Rightarrow a = 20$

$p + q = 40$

Perimeter $= 2(p + q) = 80$

6. **Ans. (2)**

Equation of ellipse is $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Equal intercepts $\Rightarrow m = -1$

Let equation of tangent be

$x + y = \pm\sqrt{a^2m^2 + b^2}$

$x + y = \pm 5$

$$25\left(\frac{l^2 + m^2}{c^2}\right) = 25\left(\frac{1+1}{25}\right) = 2$$

7. **Ans. (3)**

Let point is $P(2\cos\theta, \sin\theta)$.

Equation of tangent is $\frac{x}{2}\cos\theta + \frac{y\sin\theta}{1} = 1$

Equation of normal is $2x\sec\theta - y\,\text{cosec}\theta = 3$

Now tangent and normal meet major axis at

$Q\left(\frac{2}{\cos\theta}, 0\right)$ and $R\left(\frac{3}{2}\cos\theta, 0\right)$ respectively

Given $QR = 2 \Rightarrow \left|\frac{2}{\cos\theta} - \frac{3}{2}\cos\theta\right| = 2 \Rightarrow 3|\cos\theta|^2 + 4|\cos\theta| - 4 = 0$

$\Rightarrow |\cos\theta| = \frac{2}{3}, -2(reject)$

$\Rightarrow \cos\theta = \pm\left(\frac{2}{3}\right) \Rightarrow \frac{a}{b} = \frac{2}{3}$

8. **Ans. (1.05 or 1.06)**

$b^2 = a^2(1 - e^2)$

$b = \frac{9a^2}{25}$ ...(1)

Let $A(a\cos\theta, b\sin\theta)$

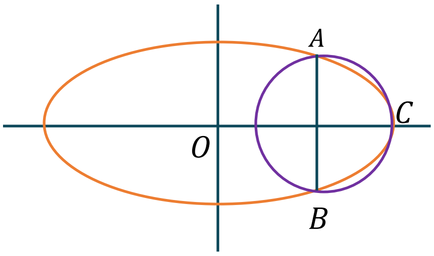
$B(a\cos\theta - b\sin\theta)$

$C(a, 0)$

centre of circle is $(a\cos\theta, 0)$

radius = $a - a\cos\theta = b\sin\theta$

$a^2(1 - \cos\theta)^2 = b^2\sin^2\theta$



use (1) $\Rightarrow 25\left(2\sin^2\frac{\theta}{2}\right)^2 = 9\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^2$

$\tan^2\frac{\theta}{2} = \frac{9}{25}$

$\cos\theta = \frac{1 - \frac{9}{25}}{1 + \frac{9}{25}} = \frac{16}{34} = \frac{8}{17}$

radius $= a\left(1 - \frac{8}{17}\right) = \frac{9a}{17}$

9. **Ans. (60)**

Given $a = 20$

$(SP_i)(S'P_i') = b^2$

$\therefore \sum_{i=1}^{10}(SP_i)(S'P_i') = 10b^2$

$\Rightarrow 10b^2 = 2560 \Rightarrow b^2 = 256$

$b^2 = a^2(1 - e^2)$

$\Rightarrow 256 = 400(1 - e^2)$

$\Rightarrow e = \frac{3}{5} \Rightarrow 100e = 60$

10. Ans. (0.50)

Here $2a = 10$

perimeter of triangle $PS_1S_2 = PS_1 + PS_2 + S_1S_2 = 15$

$= 2a + 2ae = 15$

$\Rightarrow 2ae = 5 \Rightarrow e = \frac{1}{2}$

EXERCISE - JEE (Main) PYQ

1. Ans. (2)

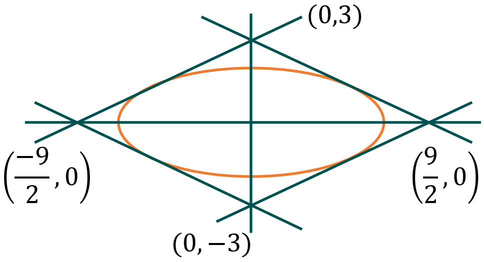
$\frac{x^2}{9} + \frac{y^2}{5} = 1$

$a = 3, b = \sqrt{5}$

$5 = 9(1 - e^2)$

$e = \frac{2}{3}$

Equation of tangent at $(2, \frac{5}{3})$



$\frac{x \cdot 2}{9} + \frac{y}{5} \cdot \left(\frac{5}{3}\right) = 1$

$\frac{2x}{9} + \frac{y}{3} = 1$

$\therefore$ (0, 3) and $\left(\frac{9}{2}, 0\right)$ are the points of intersection of above line with co-ordinate axes.

$4 \cdot \frac{1}{2} \cdot 3 \cdot \frac{9}{2} = 27$

2. Ans. (3)

Eccentricity of ellipse = $\frac{1}{2}$

Now, $-\frac{a}{e} = -4 \Rightarrow a = 4 \times \frac{1}{2} = 2$

$\therefore b^2 = a^2(1 - e^2) = a^2\left(1 - \frac{1}{4}\right) = 3$

$\therefore$ Equation of ellipse

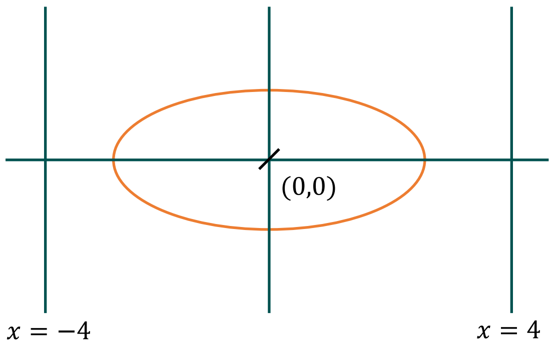
$\frac{x^2}{4} + \frac{y^2}{3} = 1$

$\Rightarrow \frac{x}{2} + \frac{2y}{3} \times y' = 0 \Rightarrow y' = -\frac{3x}{4y}$

$y'|_{(1,3/2)} = -\frac{3}{4} \times \frac{2}{3} = -\frac{1}{2}$

$\therefore$ Equation of normal at $\left(1, \frac{3}{2}\right)$

$y - \frac{3}{2} = 2(x - 1) \Rightarrow 2y - 3 = 4x - 4$



3. Ans. (3)

Equation of general tangent on ellipse

$$\frac{x}{a\sec\theta}+\frac{y}{b\operatorname{cosec}\theta}=1$$

$$a=\sqrt{2}, b=1 \Rightarrow \frac{x}{\sqrt{2}\sec\theta}+\frac{y}{\operatorname{cosec}\theta}=1$$

Let the midpoint be (h, k)

$$h=\frac{\sqrt{2}\sec\theta}{2} \Rightarrow \cos\theta=\frac{1}{\sqrt{2}h}$$

and $k=\frac{\operatorname{cosec}\theta}{2} \Rightarrow \sin\theta=\frac{1}{2k}$

$\because \sin^2\theta+\cos^2\theta=1$

$$\Rightarrow \frac{1}{2h^2}+\frac{1}{4k^2}=1 \Rightarrow \frac{1}{2x^2}+\frac{1}{4y^2}=1$$

4. Ans. (3)

$m_{SB}\,.\,m'_{SB}=-1$

$b^2=a^2e^2$ ...(i)

$\frac{1}{2}S'B.SB=8$

$S'B.SB=16$

$a^2e^2+b^2=16$ ...(ii)

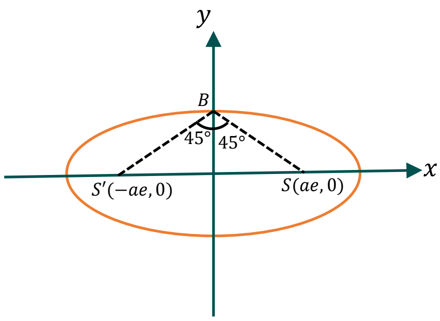
$b^2=a^2(1-e^2)$ ...(iii)

using (i),(ii), (iii) $a=4$

$b=2\sqrt{2}$

$e=\frac{1}{\sqrt{2}}$

$\therefore \ell$ (L.R) = $\frac{2b^2}{a}=4$



5. Ans. (1)

Tangent at $\left(3,-\frac{9}{2}\right)$

$$\frac{3x}{a^2}-\frac{9y}{2b^2}=1$$

Comparing this with $x-2y=12$

$$\frac{3}{a^2}=\frac{9}{4b^2}=\frac{1}{12}$$

we get $a=6$ and $b=3\sqrt{3}$

$$L(LR)=\frac{2b^2}{a}=9$$

6. Ans. (4)

$3x^2 + 4y^2 = 12$

$\frac{x^2}{4}+\frac{y^2}{3}=1$

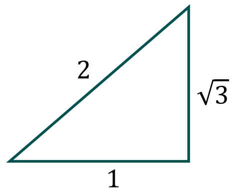
$x = 2\cos\theta, y = \sqrt{3}\sin\theta$

Let $P(2\cos\theta, \sqrt{3\sin\theta})$

Equation of normal is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$

$2x\sin\theta-\sqrt{3}\cos\theta y=\sin\theta\cos\theta$

Slope $\frac{2}{\sqrt{3}}\tan\theta=-2 \quad \therefore \tan\theta=-\sqrt{3}$



Equation of tangent is

it passes through (4, 4)

$3x\cos\theta + \sqrt{3}\, 2\sin\theta\, y = 6$

$12\cos\theta + \sqrt{3}\, 8\sin\theta = 6$

$\cos\theta=-\frac{1}{2}, \sin\theta=\frac{\sqrt{3}}{2} \quad \therefore \theta = 120°$

Hence point P is $(2\cos 120°, \sqrt{3}\sin 120°)$

$P\left(-1, \frac{3}{2}\right), Q(4, 4)$

$PQ=\frac{5\sqrt{5}}{2}$

7. Ans. (2)

$3x + 4y = 12\sqrt{2}$ is tangent to $\frac{x^2}{a^2}+\frac{y^2}{9}=1$

$c^2 = m^2a^2 + b^2$

$\Rightarrow a^2 = 16$

$e = \sqrt{1-\frac{9}{16}}=\frac{\sqrt{7}}{4}$

Distance between focii = $2ae = 2\sqrt{7}$

8. Ans. (3)

Given $2ae = 6 \Rightarrow ae=3$ ...(1)

and $\frac{2a}{e}=12\Rightarrow a=6e$ ...(2)

from (1) and (2)

$6e^2 = 3 \Rightarrow e=\frac{1}{\sqrt{2}}$

$\Rightarrow a=3\sqrt{2}$

Now, $b^2 = a^2(1-e^2)$

$\Rightarrow b^2 = 18\left(1-\frac{1}{2}\right)=9$

Length of L.R = $\frac{2(9)}{3\sqrt{2}}=3\sqrt{2}$

9. Ans. (4)

Any normal to the ellipse is

$$\frac{x\sec\theta}{\sqrt{2}} - y\operatorname{cosec}\theta = -\frac{1}{2}$$

$$\Rightarrow \frac{x}{\left(\frac{-\cos\theta}{\sqrt{2}}\right)} + \frac{y}{\left(\frac{\sin\theta}{2}\right)} = 1$$

$$\Rightarrow \frac{\cos\theta}{\sqrt{2}} = \frac{1}{3\sqrt{2}} \text{ and } \frac{\sin\theta}{2} = \beta$$

$$\Rightarrow \beta = \frac{\sqrt{2}}{3}$$

10. Ans. (2)

Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1;\ a > b;$

$$2b = \frac{4}{\sqrt{3}} \Rightarrow b = \frac{2}{\sqrt{3}} \Rightarrow b^2 = \frac{4}{3}$$

tangent $y = \frac{-x}{6} + \frac{4}{3}$ compare with $y = mx \pm \sqrt{a^2m^2 + b^2}$

$$\Rightarrow m = \frac{-1}{6} \Rightarrow \sqrt{\frac{a^2}{36} + \frac{4}{3}} = \frac{4}{3} \Rightarrow a = 4$$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{2}\sqrt{\frac{11}{3}}$$

11. Ans. (1)

$y^2 = 3x^2$

and $x^2 + y^2 = 4b$

Solve both we get

so $x^2 = b$

$$\frac{x^2}{16} + \frac{3x^2}{b^2} = 1$$

$$\frac{b}{16} + \frac{3}{b} = 1$$

$b^2 - 16b + 48 = 0$

$(b - 12)\ (b - 4) = 0$

$b = 12, b > 4$

12. Ans. (2)

Tangent to parabola

$2y = 2(x + 6) - 20$

$\Rightarrow y = x - 4$

Condition of tangency for ellipse.

$c^2 = a^2m^2 + b^2$

$\Rightarrow 16 = 2(1)^2 + b$

$\Rightarrow b = 14$

13. Ans. (2)

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

equation of tangent to the ellipse is

$$y=mx\pm\sqrt{a^2m^2+b^2}$$

$$y=mx\pm\sqrt{16m^2+9} \quad ...(i)$$

$x^2+y^2=12$

equation of tangent to the circle is

$$y=mx\pm\sqrt{12}\sqrt{1+m^2} \quad ...(ii)$$

for common tangent equate eq. (i) and (ii)

$\Rightarrow 16m^2+9=12(1+m^2)$

$16m^2-12m^2=3$

$4m^2=3$

$12m^2=9$

14. Ans. (3)

$P(4,3)$ — D — $Q(2cos\theta,\sqrt{2}sin\theta)$

$$\frac{x^2}{4}+\frac{y^2}{2}=1$$

Coordinate of D is

$$\left(\frac{2\cos\theta+4}{2}\cdot\frac{\sqrt{2}\sin\theta+3}{2}\right)=(h,k)$$

$$\frac{2h-4}{2}=\cos\theta \quad ...(i)$$

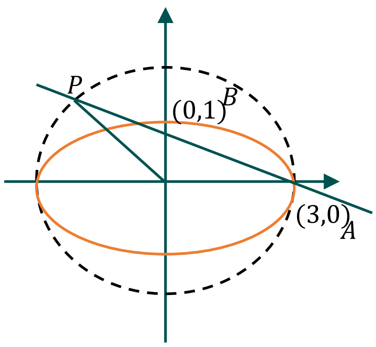
$$\frac{2k-3}{\sqrt{2}}=\sin\theta \quad ...(ii)$$

$$\left(\frac{2h-4}{2}\right)^2+\left(\frac{2k-3}{\sqrt{2}}\right)^2=1 \Rightarrow \frac{(x-2)^2}{1}+\frac{\left(y-\frac{3}{2}\right)^2}{\left(\frac{1}{2}\right)}=1$$

$\therefore$ Required eccentricity is $e=\sqrt{1-\frac{1}{2}}=\frac{1}{\sqrt{2}}$

15. Ans. (3)

For line AB $x+3y=3$ and circle is $x^2+y^2=9$



$(3-3y)^2+y^2=9$

$\Rightarrow 10y^2-18y=0$

$\Rightarrow y=0,\frac{9}{5}$

$\therefore$ Area $=\frac{1}{2}\times3\times\frac{9}{5}=\frac{27}{10}$

$m-n=17$

16. Ans. (1)

$Kx^2 + K^2y^2 = 1$

$\frac{x^2}{1/K} + \frac{y^2}{1/K^2} = 1$

Now equation of

$A_1B_2; \frac{x}{1/\sqrt{K}} + \frac{y}{1/K} = 1 \Rightarrow \sqrt{K}x + Ky = 1$

$r_K = \perp r$ distance of (0,0) from line A_1B_1

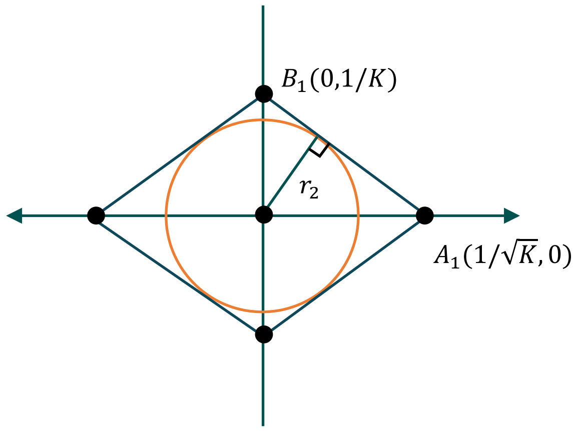
$r_k = \left|\frac{(0+0-1)}{\sqrt{K+K^2}}\right| = \frac{1}{\sqrt{K+K^2}}$

$\frac{1}{r_K^2} = K + K^2 \Rightarrow \sum_{k=1}^{20} \frac{1}{r_K^2} = \sum_{K=1}^{20}(K+K^2)$

$= \sum_{K=1}^{20} K + \sum_{K=1}^{20} K^2 = \frac{20\times 21}{2} + \frac{20.21.41}{6}$

$= 210 + 10\times 7\times 41 = 210 + 2870$

$= 3080$



EXERCISE - JEE (Advanced) PYQ

1. Ans. (4)

$\therefore P_1$ is $y^2 = 8x$

P_2 is $y^2 = -16x$

Let equation of tangent at $(2t^2, 4t)$

$\therefore y = m_1x + \frac{2}{m_1}$

If passes through (–4,0)

$\therefore -4m_1 + \frac{2}{m_1} = 0$

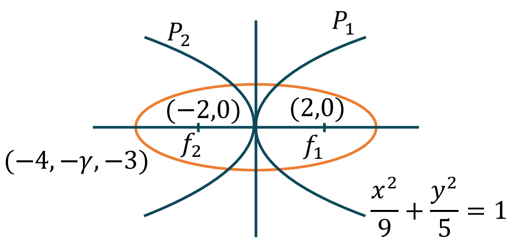
$\therefore m_1^2 = \frac{1}{2}$

equation of tangent to P_2

$y = m_2x + \frac{(-4)}{m_2}$

It passes through (2,0),

$2m_2 - \frac{4}{m_2} = 0 \Rightarrow m_2^2 = 2 \therefore \frac{1}{m_1^2} + m_2^2 = 4$



2. Ans. (A,B)

Let $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b)$

& $E_2 : \frac{x^2}{c^2} + \frac{y^2}{d^2} = 1 \quad (c < d)$

& $S : x^2 + (y-1)^2 = 2$

& tangent to E_1, E_2 & S is $x + y = 3$

Now, point of contact of S & tangent is (x_1, y_1)

Let $x = X$ & $y - 1 = Y$

$\therefore X^2 + Y^2 = 2$

& $X + Y = 2$

Let (X_1, Y_1) be point of contact.

$\therefore XX_1 + YY_1 = 2$

$\therefore X_1 = 1$ & $Y_1 = 1$

$\therefore x_1 = 1$ & $y_1 = 2$

Now, parametric equation of $x + y = 3$

is $\dfrac{x-1}{-\frac{1}{\sqrt{2}}} = \dfrac{y-2}{\frac{1}{\sqrt{2}}} = \pm\dfrac{2\sqrt{2}}{3} \Rightarrow x = \dfrac{5}{3},\ y = \dfrac{4}{3}$

$\Rightarrow x = \dfrac{1}{3}$ & $y = \dfrac{8}{3}$

$\therefore P \equiv (1, 2),\ Q \equiv \left(\dfrac{5}{3}, \dfrac{4}{3}\right)$ & $R \equiv \left(\dfrac{1}{3}, \dfrac{8}{3}\right)$

Now, equation tangent at Q on ellipse E_1

$\dfrac{x.5}{3a^2} + \dfrac{y.4}{3b^2} = 1$

Comparing it with $x + y = 3$

$\therefore a^2 = 5$ & $b^2 = 4$

Now, $e_1^2 = 1 - \dfrac{4}{5} = \dfrac{1}{5}$

Similarly, $e_2^2 = \dfrac{7}{8}$

$\therefore e_1^2 e_2^2 = \dfrac{7}{40} \Rightarrow e_1 e_2 = \dfrac{\sqrt{7}}{2\sqrt{10}}$

$e_1^2 + e_2^2 = \dfrac{1}{5} + \dfrac{7}{8} = \dfrac{43}{40}$; $|e_1^2 - e_2^2| = \left|\dfrac{1}{5} - \dfrac{7}{8}\right| = \dfrac{27}{40}$

$\therefore$ (A) & (B)

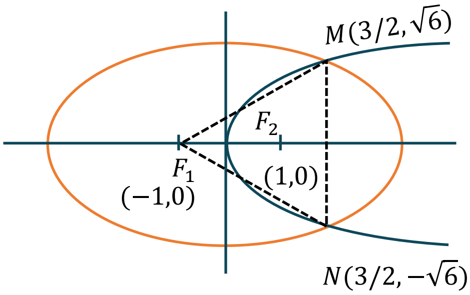
3. **Ans. (A)**

Orthocentre lies on x –axis

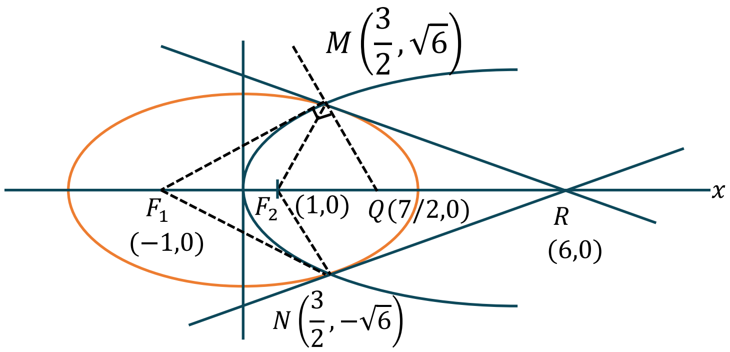
Equation of altitude through M : $y - \sqrt{6} = \dfrac{5}{2\sqrt{6}}\left(x - \dfrac{3}{2}\right)$

Equation of altitude through F_1: $y = 0$

solving, we get orthocentre $\left(-\dfrac{9}{10}, 0\right)$



4. **Ans. (C)**



Normal to parabola at M: $y-\sqrt{6}=-\frac{\sqrt{6}}{2.1}\left(x-\frac{3}{2}\right)$

Solving it with $y = 0$, we get $Q\equiv\left(\frac{7}{2},0\right)$

Tangent to ellipse at M: $\frac{x.\frac{3}{2}}{9}+\frac{y(\sqrt{6})}{8}=1$

Solving it with $y = 0$, we get $R \equiv (6, 0)$

$\therefore$ Area of triangle $MQR=\frac{1}{2}.\left(6-\frac{7}{2}\right).\sqrt{6}=\frac{5\sqrt{6}}{4}$

Area of quadrilateral $MF_1NF_2=2.\frac{1}{2}.(1-(-1)).\sqrt{6}=2\sqrt{6}$

Required ratio = 5 : 8

5. **Ans. (A,C)**

$y = mx + \frac{1}{m}$

$y=mx\pm\frac{1}{\sqrt{2}}\sqrt{1+m^2}\Rightarrow\frac{2}{m^2}=I+m^2$

$\Rightarrow (m^2+2)(m^2-1)=0 \Rightarrow m = \pm 1$

$\Rightarrow y = x + 1$ & $y = -x - 1 \Rightarrow Q \equiv (-1, 0)$

$e=\sqrt{1-\frac{1/2}{1}}=\frac{1}{\sqrt{2}}$

L.R. = $\frac{2b^2}{a}=\frac{2\times1/2}{1}=1$

Ellipse = $\frac{x^2}{1}+\frac{y^2}{1/2}=1\Rightarrow y=-\frac{1}{\sqrt{2}}\sqrt{1-x^2}$

Area of region = $2\int_{1/\sqrt{2}}^{1}\frac{1}{\sqrt{2}}\sqrt{1-x^2}\,dx$

= $\sqrt{2}\left[\left|\frac{2}{x}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right|_{1/\sqrt{2}}^{1}\right]$

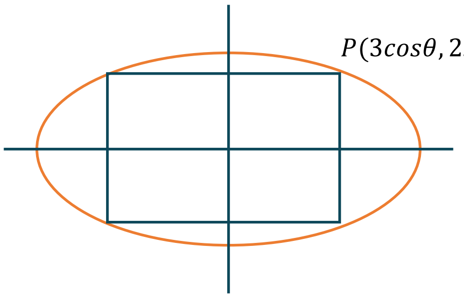
= $\sqrt{2}\left(-\frac{1}{2\sqrt{2}}\frac{1}{\sqrt{2}}+\frac{\pi}{4}-\frac{\pi}{8}\right)$ = $\sqrt{2}\left(\frac{\pi}{8}-\frac{1}{4}\right)=\frac{(\pi-2)}{4\sqrt{2}}$

6. **Ans. (C,D)**

Area of $R_1 = 3\sin 2\theta$; for this to be maximum

$\Rightarrow \theta = \frac{\pi}{4} \Rightarrow \left(\frac{3}{\sqrt{2}}, \frac{2}{\sqrt{2}}\right)$

Hence for subsequent areas of rectangles R_n to be maximum the coordinates will be in GP with common ratio $r = \frac{1}{\sqrt{2}} \Rightarrow a_n = \frac{3}{(\sqrt{2})^{n-1}}$; $b_n = \frac{3}{(\sqrt{2})^{n-1}}$



Eccentricity of all the ellipses will be same

Distance of a focus from the centre in

$E_9 = a_9 e_9 = \sqrt{a_9^2 - b_9^2} = \frac{\sqrt{5}}{16}$

Length of latus rectum of $E_9 = \frac{2b_9^2}{a_9} = \frac{1}{6}$

$\because \sum_{n=1}^{\infty} \text{Area of } R_n = 12 + \frac{12}{2} + \frac{12}{4} + ...\infty = 24$

$\Rightarrow \sum_{n=1}^{N} (area \text{ of } R_n) < 24$, for each positive integer N

7. **Ans. (A)**

$y^2 = 4\lambda x, P(\lambda, 2\lambda)$

Slope of the tangent to the parabola at point P

$\frac{dy}{dx} = \frac{4\lambda}{2y} = \frac{4\lambda}{2\times 2\lambda} = 1$

Slope of the tangent to the ellipse at P

$\frac{2x}{a^2} + \frac{2yy'}{b^2} = 0$

As tangents are perpendicular $y' = -1$

$\Rightarrow \frac{2\lambda}{a^2} - \frac{4\lambda}{b^2} = 0 \Rightarrow \frac{a^2}{b^2} = \frac{1}{2}$

$2a^2 = b^2$

$\Rightarrow b^2 > a^2 \Rightarrow b > a$

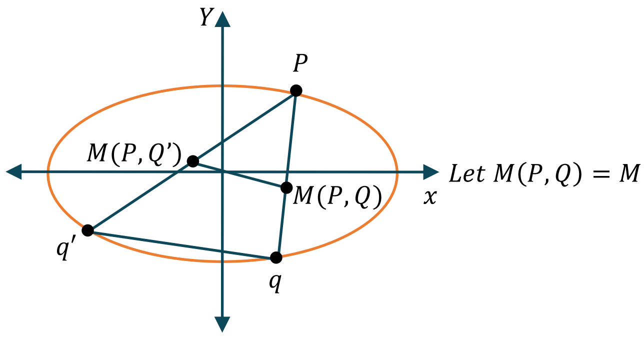
$\therefore e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{1}{2}} = \frac{1}{\sqrt{2}}$

8. **Ans. (4)**

and $M(P, Q') = M'$

By mid point theorem:

$MM' = \frac{1}{2}(QQ')$



$\Rightarrow \max(MM') = \frac{1}{2} \times \max(QQ')$

maximum of QQ' is possible when $QQ' =$ major axis.

$\therefore QQ' = 8$

$\Rightarrow \max(MM') = \frac{1}{2} \times 8 = 4$

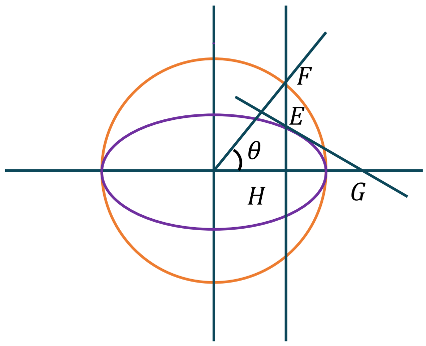
9. Ans. (C)

Equation of auxiliary circle $x^2 + y^2 = 4$

$\therefore$ Let f be $(2\cos\theta, 2\sin\theta)$

$\therefore$ E is $(2\cos\theta, \sqrt{3}\sin\theta)$

Equation of tangent at E, $\frac{x\cos\theta}{2}+\frac{y\sin\theta}{\sqrt{3}}=1$



It cuts x –axis at $(2\sec, \theta, 0)$

$\therefore$ G is $(2\sec 0, 0)$

H is $(2\cos\theta, 0)$ and $F(2\cos\theta, 2\sin\theta)$

$\therefore$ Area of ΔFGH is $\frac{1}{2}\times 2\sin\theta(2\sec\theta-2\cos\theta)$

$= 2\sin\theta(\sec\theta - \cos\theta)$

If $\theta=\frac{\pi}{4}$, area $=2\times\frac{1}{\sqrt{2}}\left(\sqrt{2}-\frac{1}{\sqrt{2}}\right)=1$

If $\theta=\frac{\pi}{3}$, area $= 2\times\frac{\sqrt{3}}{2}\left(2-\frac{1}{2}\right)=\frac{3\sqrt{3}}{2}$

If $\theta=\frac{\pi}{6}$, area $=2\times\frac{1}{2}\left(\frac{2}{\sqrt{3}}-\frac{\sqrt{3}}{2}\right)=\frac{1}{2\sqrt{3}}$

If $\theta=\frac{\pi}{12}$, area $=2\times\frac{\sqrt{3}-1}{2\sqrt{2}}\left(\frac{2\sqrt{2}}{\sqrt{3}+1}-\frac{\sqrt{3}+1}{2\sqrt{2}}\right) = \left(\frac{\sqrt{3}-1}{8}\right)^4$

10. Ans. (A,C)

$y=mx+\frac{3}{m}$

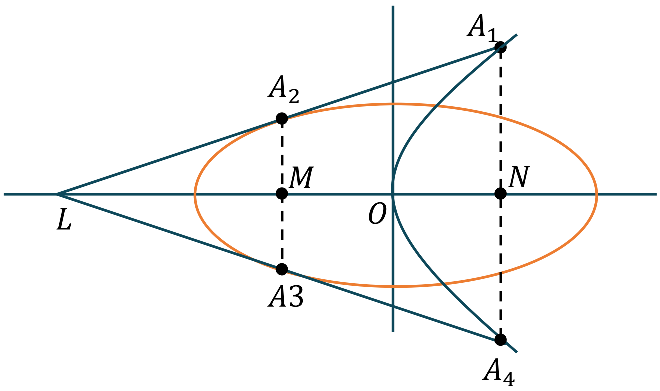
$C^2 = a^2m^2 + b^2$

$\frac{9}{m^2}=6m^2+3 \Rightarrow m^2 = 1$

T_1 & T_2

$y = x + 3, y = -x - 3$

Cuts x-axis at $(-3, 0)$



$A_1(3, 6)$ $A_4(3, -6)$

$A_2(-2, 1)$ $A_3(-2, -1)$

$A_1A_4 = 12, A_2A_3 = 2, MN = 5$

Area $= \frac{1}{2}(12+2)\times 5=35\,sq.unit$

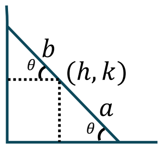
JEE (Main) Practice Paper

SECTION–A

1. Ans. (1)

Let the fixed lines are co-ordinate axes

from diagram $h = b\cos\theta \Rightarrow k = a\sin\theta$



$\Rightarrow \frac{h^2}{b^2} + \frac{k^2}{a^2} = 1 \rightarrow$ which is ellipse

2. Ans. (1)

$4\tan\frac{B}{2}\tan\frac{C}{2} = 4\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}\sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = 1 \Rightarrow 4\frac{(s-a)}{s} = 1 \Rightarrow s = \frac{4a}{3} = 4\times\frac{6}{3} = 8$

but $2s = a + b + c = 16 \Rightarrow b + c = 10$

Hence locus is an ellipse having center $\equiv (5, 0)$

$2ae = 6$ and $2a = 10$

$b^2 = a^2 - a^2e^2 = 25 - 9 = 16$

$\therefore$ Equation of ellipse

$$\frac{(x-5)^2}{25} + \frac{y^2}{16} = 1$$

3. Ans. (4)

Point of intersection of tangent at point having eccentric angle 'α' & 'β' is

$$h = \frac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}$$

$$k = \frac{b\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)} \qquad \because \alpha + \beta = \text{constant (let } k\text{)}$$

hence $\frac{h}{k} = \frac{a}{b\tan k}$

hence locus is straight line

4. Ans. (4)

Let the director circle is $(x - h)^2 + (y - k)^2 = a^2 + b^2$

origin lies on director circle

$\Rightarrow h^2 + k^2 = a^2 + b^2 = 5$

Locus is $x^2 + y^2 = 5$

5. **Ans. (3)**

Let focus of ellipse are (α, β) and (h, k)

$\because$ Product of length of perpendicular dropped from focus on any tangent is b^2

$\therefore h\alpha = 1 \Rightarrow \alpha = \frac{1}{h}$

$k\beta = 1 \Rightarrow \beta = \frac{1}{k}$

Now distance between foci is 2 ae

$\therefore (h-\alpha)^2 + (k-\beta)^2 = \left(2\times 2\times \frac{\sqrt{3}}{2}\right)^2 = 12$

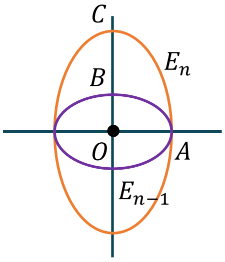
$\left(h - \frac{1}{h}\right)^2 + \left(k - \frac{1}{k}\right)^2 = 12$

After simplification $(h^2 + k^2)(1 + h^2k^2) = 16h^2k^2$

Hence the locus $(x^2 + y^2)(1 + x^2y^2) = 16x^2y^2$

6. **Ans. (3)**

The figure shows two ellipses E_{n-1} and E_n.



The eccentricity is given to be independent of n, implies that the ratio of minor axis to the major axis, is same for all the ellipses.

For ellipse E_{n-1}, let minor axis = b, major axis = a

For ellipse E_n, we have

minor axis = a, major axis = $\frac{OB}{e} = \frac{b}{e}$ [$\because$ B is the focus of E_n]

assuming e to be the eccentricity. Thus, we have

$\frac{b}{a} = \frac{a}{b/e} \Rightarrow e = \frac{b^2}{a^2} = 1 - e^2 \Rightarrow e^2 + e - 1 = 0$

Gives $e = \frac{\sqrt{5}-1}{2}$ [$\because$ e must be $+ve$]

7. **Ans. (3)**

$y = x$ is angle bisector of $y = x/2$ and $y = 2x$

8. **Ans. (1)**

$16 + 9 = r^2 + r^2 \Rightarrow r = \frac{5}{\sqrt{2}}$

Area of square is $= \frac{10}{\sqrt{2}} \times \frac{10}{\sqrt{2}} = 50$

9. Ans. (1)

$$\frac{2}{3}=\frac{\pi a^2-\pi ab}{\pi a^2}=1-\frac{b}{a}=1-\sqrt{1-e^2}\Rightarrow e^2=\frac{8}{9}\Rightarrow e=\frac{2\sqrt{2}}{3}$$

10. Ans. (4)

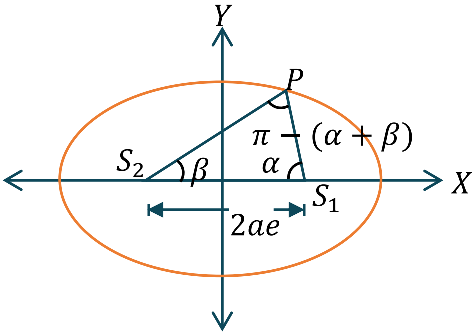
$$\frac{x}{a}\cos\frac{\alpha+\beta}{2}+\frac{y}{b}\sin\frac{\alpha+\beta}{2}=\cos\frac{\alpha-\beta}{2};\ \cos\frac{\alpha+\beta}{2}=\cos\frac{\alpha-\beta}{2}$$

$$\Rightarrow e=\frac{\cos\frac{\alpha-\beta}{2}}{\cos\frac{\alpha+\beta}{2}}\cdot\frac{2\sin\frac{\alpha+\beta}{2}}{2\sin\frac{\alpha+\beta}{2}}=\frac{\sin\alpha+\sin\beta}{\sin(\alpha+\beta)}$$

11. Ans. (1)

By sine rule in ΔPS_1S_2, we get $\frac{2ae}{\sin(\alpha+\beta)}=\frac{S_1P}{\sin\beta}=\frac{S_2P}{\sin\alpha}=\frac{2a}{\sin\alpha+\sin\beta}$

$$\Rightarrow e=\frac{\sin(\alpha+\beta)}{\sin\alpha+\sin\beta}\Rightarrow\frac{e}{1}=\frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)}{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}$$



Now $\frac{1-e}{1+e}=\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{1-\frac{1}{2}}{1+\frac{1}{2}}=\frac{\frac{1}{2}}{\frac{3}{2}}=\frac{1}{3}$

$\therefore \tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{1}{3}$ ...(1)

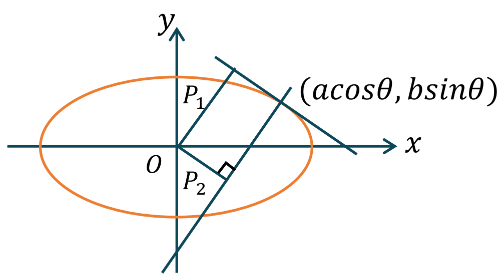
Also we know that

$$\cot\frac{\alpha}{2}+\cot\frac{\beta}{2}+\cot\frac{\gamma}{2}=\cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$$

$\Rightarrow 2\cot\frac{\gamma}{2}=\cot\frac{\alpha}{2}+\cot\frac{\beta}{2}\Rightarrow\cot\frac{\alpha}{2},\cot\frac{\gamma}{2},\cot\frac{\beta}{2}$ are in A.P.

12. Ans. (1)

$T:\ \frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$



$p_1=\left|\frac{ab}{\sqrt{b^2\cos^2\theta+a^2\sin^2\theta}}\right|$ ...(1)

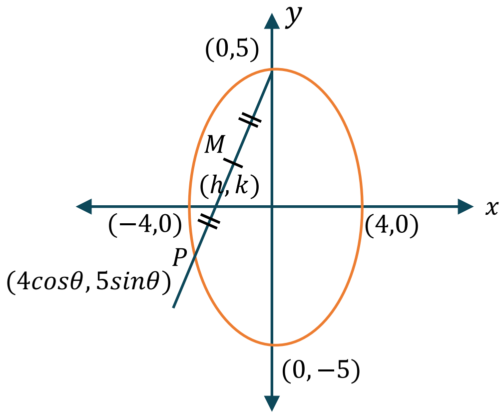
$N_1:\ \frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$

$p_2=\left|\frac{(a^2-b^2)\sin\theta\cos\theta}{\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}}\right|$ ...(2)

$p_1p_2=\frac{ab(a^2-b^2)}{2\left(\frac{a^2}{2}+\frac{b^2}{2}\right)}$ when $\theta=\pi/4$; $p_1p_2==\frac{ab(a^2-b^2)}{a^2+b^2}$

13. Ans. (3)

We have $4\cos\theta = 2h$ and $5(1+\sin\theta) = 2k$



As $\cos^2\theta + \sin^2\theta = 1$

$$\Rightarrow \frac{h^2}{4} + \left(\frac{2k}{5}-1\right)^2 = 1 \Rightarrow \frac{x^2}{4} + \frac{4y^2}{25} - \frac{4y}{5} = 0$$

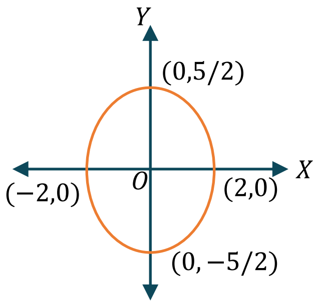
$$\Rightarrow \frac{x^2}{4} + \frac{4}{25}(y^2 - 5y) = 0$$

$$\Rightarrow \frac{x^2}{4} + \frac{4}{25}\left[\left(y-\frac{5}{2}\right)^2 \frac{-25}{4}\right] = 0$$

$$\Rightarrow \frac{x^2}{4} + \frac{\left(y-\frac{5}{2}\right)^2}{\frac{25}{4}} = 1 \qquad \text{...(1)}$$

Put $X = x, y - \frac{5}{2} = Y$

$\therefore$ Equation (1) becomes



$$\frac{X^2}{4} + \frac{Y^2}{\frac{25}{4}} = 1 \qquad \text{(Ellipse)}$$

$$e^2 = 1 - \frac{4\cdot 4}{25} = \frac{9}{25} \Rightarrow e = \frac{3}{5}$$

$$\therefore F_1 = \left(0, \frac{3}{2}\right), F_2 = \left(0, \frac{-3}{2}\right)$$

Hence in $x - y$ system, foci are (0, 4), (0, 1)

14. Ans. (1)

$$\frac{x^2}{1} + \frac{y^2}{4} = 1$$

Equation of director circle = $x^2 + y^2 = 5$

$(-\sqrt{3}, \sqrt{2})$ lies on director circle so tangents are perpendicular.

15. Ans. (3)

equation of tangent

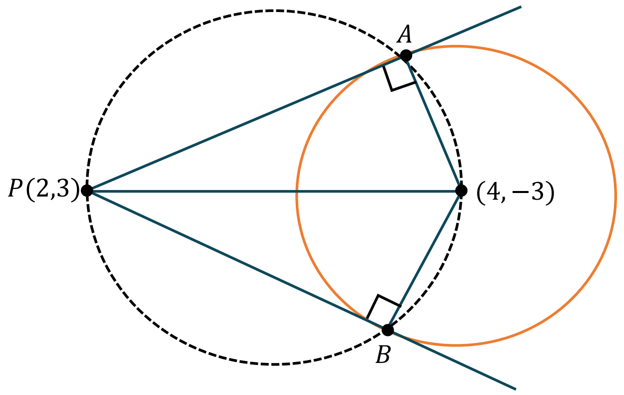
$$y = -\frac{4x}{3} \pm \sqrt{32+32} \qquad \left[\therefore y = mx \pm \sqrt{a^2m^2 + b^2}\right]$$

$$\Rightarrow \frac{x}{6} + \frac{y}{8} = \pm 1 \quad \Rightarrow x \text{ intercept} = \pm 6$$

y intercept = ± 8

$\Rightarrow$ area of $\Delta AOB = \frac{1}{2} \times 6 \times 8 = 24$ Sq. units.

16. **Ans. (4)**



it clearly shows that (2, 3) & (4, –3) will be ends of diameter of circle PAB

So equation of circle is $(x-2)(x-4)+(y-3)(y+3)=0$

$\Rightarrow x^2+y^2-6x-1=0$

as it intersects the circle $(x+5)^2+(y-3)^2=9+b^2$

originally then $(2g_1g_2+2f_1f_2=c_1+c_2)$

$2(-3)(5)+2(0)(-3)=-1+25-b^2$

$\Rightarrow b^2=54$

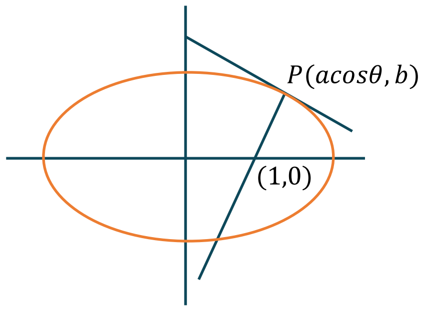
17. **Ans. (2)**

$\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=12$

$a=4, b=2$

$\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=12$

$\frac{4x}{\cos\theta}-\frac{2y}{\sin\theta}=12$



It passes through (1, 0)

$2x\sec\theta - y\operatorname{cosec}\theta = 6 \Rightarrow 2\sec\theta = 6 \Rightarrow \sec\theta = 3$

$\cos\theta=\frac{1}{3}$, $\sin^2\theta=1-\frac{1}{9}=\frac{8}{9}$

$\sin\theta=\frac{2\sqrt{2}}{3}$

$r^2=(a\cos\theta-1)^2+b^2\sin^2\theta$

$=a^2\cos^2\theta+b^2\sin^2\theta+1-2a\cos\theta$

$=\ 16\cdot\frac{1}{9}+4\cdot\frac{8}{9}+1-\frac{8}{3}$

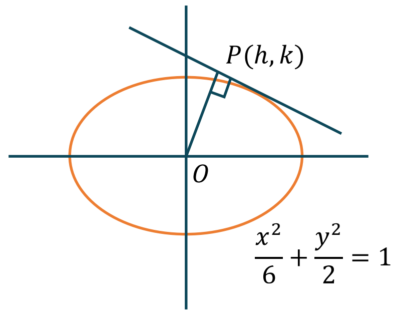
$=\ \frac{16}{9}+\frac{32}{9}+1$

$=\frac{48}{9}+1-\frac{8}{3}=\frac{59}{9}-\frac{24}{9}=\frac{33}{9}=\frac{11}{3}$

Hence circle in $(x-1)^2+y^2=\frac{11}{3}$

18. Ans. (3)

Let the foot of perpendicular be (h, k)



then $m_{op} = \frac{k}{h}$

equation of tangent is

$y = mx \pm \sqrt{a^2m^2+b^2}$

$y = mx \pm \sqrt{6m^2+2}$

satisfied by (h, k) and $m = -\frac{1}{m_{op}} = -\frac{h}{k}$

$$\left(k+\frac{h^2}{k}\right)^2 = \frac{6h^2}{k^2}+2$$

multiply by k^2

$(k^2+h^2)^2 = 6h^2+2k^2$

$\Rightarrow (x^2+y^2)^2 = 6x^2+2y^2$

19. Ans. (3)

$e^2 = 1-\frac{\tan^2\alpha}{\sec^2\alpha} = \cos^2\alpha$ (as $\sec^2\alpha > \tan^2\alpha$)

hence $e = \cos\alpha$; vertex $(0, \pm\sec\alpha)$

foci = $(0, 1)$; $l(\text{LR}) = \frac{2b^2}{a} = \frac{2\tan^2\alpha}{\sec\alpha} = 2\sin\alpha\cdot\tan\alpha$

20. Ans. (4)

$A_1 = \pi\sqrt{ab}$ $x^2+y^2 = a+b$ (director circle)

$A_2 = \pi(a+2)$

$\frac{a+b}{2} \geq \sqrt{ab}$ (By $AM \geq CM$)

$\pi(a+b) \geq 2\pi\sqrt{ab}$

$\boxed{A_2 \geq 2A_1}$

If $a = 4b$ then $A_1 = 2\pi b$

$A_2 = 5\pi b$

$\Rightarrow \boxed{A_1 = \frac{2}{5}A_2}$

SECTION–B

1. Ans. (2)

$$A = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ a\cos\theta & b\sin\theta & 1 \\ a\cos(\theta+\alpha) & b\sin(\theta+\alpha) & 1 \end{vmatrix} = \frac{1}{2}ab[\cos\theta\sin(\theta+\alpha)-\sin\theta\cos\theta(\theta+\alpha)]$$

$= \frac{ab}{2}\sin[\theta+\alpha-\theta] = \frac{ab}{2}\sin\alpha$

as here $a = 4$, $b = 2$ and $\alpha = \pi/6$ so $A = 2$

2. Ans. (2)

$PS = e\left(\frac{a}{e} - a\cos\theta\right) = a - ae\cos\theta$; $PS' = e\left(\frac{a}{e} + a\cos\theta\right) = a + ae\cos\theta$ and $SS^1 = 2ae$

Let incentre be (h, k)

$$h = \frac{-ae(a - ae\cos\theta) + ae(a + ae\cos\theta) + 2a^2e\cos\theta}{2a(1+e)} \Rightarrow h = \frac{2a^2e^2\cos\theta + 2a^2e\cos\theta}{2a(1+e)}$$

$$= \frac{ae^2\cos\theta + ae\cos\theta}{(1+e)} = ae\cos\theta$$

$k = \frac{b\sin\theta \times 2ae}{2a(1+e)} = \frac{be\sin\theta}{1+e}$ equation of ellipse $\left(\frac{h}{ae}\right)^2 + \left(\frac{k(1+e)}{be}\right)^2 = 1 \Rightarrow \frac{x^2}{a^2e^2} + \frac{y^2(1+e)^2}{b^2e^2} = 1$

$$e' = \sqrt{1 - \frac{b^2e^2}{(1+e)^2a^2e^2}} = \sqrt{1 - \frac{1-e}{1+e}} = \sqrt{\frac{2e}{1+e}} \Rightarrow \left(1 + \frac{1}{e_1}\right)e_2^2 = 2$$

3. Ans. (3)

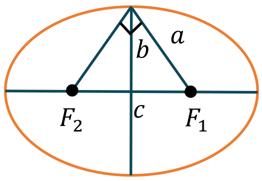
Let $P = \left(0, 3+\sqrt{5}\right)$ and foci $S = (2, 3)$, $S' = (-2, 3)$.

P is a point one the ellipse if $PS + PS' = 2a$

Now, $PS = \sqrt{(-2)^2 + \left(\sqrt{5}^2\right)} = \sqrt{9} = 3$, $PS' = \sqrt{2^2 + \left(\sqrt{5}\right)^2} = 3 \Rightarrow PS + PS' = 6 = 2a$

$\Rightarrow a = 3$

4. Ans. (1)



$a^2 = b^2 + c^2$ ...(i)

given $a^2 + a^2 = (2c)^2$

$2a^2 = 4c^2$

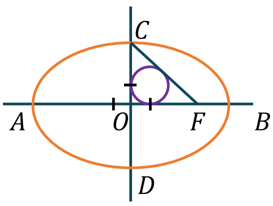
$a^2 = 2c^2$ ...(ii)

$c = ae$

$\Rightarrow \frac{c^2}{a^2} = e^2 \Rightarrow \frac{c^2}{2c^2} = e^2 \Rightarrow e^2 = \frac{1}{2}$

5. Ans. (65)

$ae = 6 \Rightarrow b^2 + 36 = (b+4)^2 \Rightarrow 36 = 16 + 8b$



$b = \frac{5}{2} \Rightarrow a^2 = a^2e^2 + b^2 = 36 + \frac{25}{4} = \frac{169}{4}$

$a = \frac{13}{2} \Rightarrow (2a)(2b) = 65$

6. Ans. (21)

Largest circle will be that circle which touches the ellipse at any point (5cosθ, 3sinθ)

⇒ Line joining points (5cosθ, 3sinθ) and (3, 0) will be perpendicular to tangent of ellipse at point (5cosθ, 3sinθ)

so that $\frac{3\sin\theta}{5\cos\theta-3}\times\frac{-3}{5}\frac{\cos\theta}{\sin\theta}=-1$

or $\cos\theta=\frac{15}{16}$ and $\sin\theta=\frac{\sqrt{31}}{16}$

so that point on ellipse is $\left(\frac{75}{16},\frac{3\sqrt{31}}{16}\right)$.

Hence radius of circle is $\frac{3\sqrt{7}}{4}$.

7. Ans. (15)

Equation of a tangent to the given ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(1)

can be chosen as $y = mx + \sqrt{a^2m^2+b^2}$ ...(2)

The coordinates of the intersection points of this tangent with the coordinate axes are

$\left(0,\sqrt{a^2m^2+b^2}\right)$ and $\left(\frac{-\sqrt{a^2m^2+b^2}}{m},0\right)$

Length of the intercept made by the coordinate axes on the tangent is given by the equation

$\ell^2=\left(1+\frac{1}{m^2}\right)(a^2m^2+b^2)$

The value of m for which ℓ is minimum (by calculus), is given by

$\frac{d(\ell^2)}{dm}=-\left(\frac{2}{m^3}\right)(a^2m^2+b^2)+2a^2m\left(1+\frac{1}{m^2}\right)=0$ ...(3)

$\Rightarrow -\frac{2a^2}{m}-\frac{2b^2}{m^3}+2a^2m+\frac{2a^2}{m}=0$

$2m(a^2-\frac{b^2}{m^4})=0 \Rightarrow m=0$, not possible

$m^4=\frac{b^2}{a^2}\Rightarrow m^2=+\frac{b}{a}$

Putting this value of m^2 in equation (3), gives the minimum value of $\ell=a+b$.

8. Ans. (9)

$\frac{\left(\frac{x-2y+1}{\sqrt{5}}\right)^2}{\left(\frac{\sqrt{5}}{2}\right)^2}+\frac{\left(\frac{2x+y+2}{\sqrt{5}}\right)^2}{\left(\frac{\sqrt{5}}{3}\right)^2}=1\Rightarrow a=\frac{\sqrt{5}}{2}, b=\frac{\sqrt{5}}{3}$,

equation of major axis is $2x+y+2=0$, equation of minor exis is $x-2y+1=0$.

⇒ centre is (–1, 0) and $e=\frac{\sqrt{5}}{3}$

as given $\sqrt{2}=\lambda.\frac{\sqrt{5}}{3}\Rightarrow 5\lambda^2=9$

9. Ans. (14)

Equation of circle

$x - 1 = r\cos\theta$

$y - 0 = r\sin\theta$

lies in ellipse $\therefore S < 0$

$\Rightarrow (1 + r\cos\theta)^2 + 4\,(r\sin\theta)^2 - 16 < 0$

$\Rightarrow 3r^2\cos^2\theta - 2r\cos\theta + 15 - 4r^2 > 0$

$\because a = 3r^2 > 0$

$\therefore f(\theta) > 0 \quad \forall \theta \in R$

$\therefore D < 0$

$4r^2 - 4 \times 3r^2(15 - 4r^2) < 0$

$4r^2[3r^2 - 11] < 0$

$4r^2[3r^2 - 11] < 0$

$\because r^2 > 0$

$\therefore 3r^2 - 11 < 0$

$r^2 < \frac{11}{3}$

Hence equation of circle

$(x - 1)^2 + y^2 = \frac{11}{3}$

10. Ans. (55)

$y^2 = 4x,\ \frac{x^2}{16} + \frac{y^2}{6} = 1$

For common tangent

$\frac{1}{m} = \pm\sqrt{16m^2 + 6}$

$\Rightarrow 16m^4 + 6m^2 - 1 = 0$

$\Rightarrow m = \pm\frac{1}{2\sqrt{2}}$

$\therefore$ Equation of tangent

$y = \frac{1}{2\sqrt{2}}x + 2\sqrt{2}$ given $\frac{xx_1}{16} + \frac{yy_1}{6} = 1$

$yy_1 = 2a(x + x_1)$

$\Rightarrow x_1 = 8,\ y_1 = 4\sqrt{2}$ $x_3 = -2, y_3 = \frac{3}{\sqrt{2}}$

$\therefore x_2 = 8, y_1 = -4\sqrt{2}$ $x_4 = -2, y_4 = -\frac{3}{\sqrt{2}}$

$\therefore A = \frac{1}{2} \times \left[8\sqrt{2} + \frac{6}{\sqrt{2}}\right] \times (10)$

$= 55\sqrt{2}$

JEE (Advanced) Practice Paper

1. **Ans. (A,B,C,D)**

$3(x-3)^2+4(y+2)^2=C$

if $C=0$ a point

if $C>0$ ellipse

if $C<0$ no locus.

2. **Ans. (A,B,C)**

(A) Director circle $x^2+y^2=a^2+b^2=9+5=14$

(B) By definition $2.b=12$

(C) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\sqrt{\frac{(s-2ae)(s-b)}{s(s-a)}}\sqrt{\frac{(s-2ae)(s-a)}{s(s-b)}}$

$\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{s-2ae}{s}=\frac{a+ae-2ae}{a+ae}=\frac{a-ae}{a+ae}=\frac{1-e}{1+e}$

3. **Ans. (A,C)**

We have

Slope of AB = Slope of tangent at C

$$\Rightarrow \frac{b(\sin\beta-\sin\alpha)}{a(\cos\beta-\cos\alpha)}=\frac{-b\cos\theta}{a\sin\theta}\Rightarrow\frac{-\cos\left(\frac{\alpha+\beta}{2}\right)}{\sin\left(\frac{\alpha+\beta}{2}\right)}=\frac{-\cos\theta}{\sin\theta}$$

$$\Rightarrow\tan\left(\frac{\alpha+\beta}{2}\right)=\tan\theta\Rightarrow\theta=\frac{\alpha+\beta}{2}+n\pi\ (n\in I).$$

4. **Ans. (A,C)**

The tangent & normal at a point P on the ellipse bisect the external & internal angles between the focal distances of P.

So answer are (A) and (C)

5. **Ans. (A,B)**

Equation of tangent to circle $y=mx+4\sqrt{1+m^2}$

Equation of tangent to ellipse $y=mx+\sqrt{25m^2+4}$

Both represent the same line so

$4\sqrt{1+m^2}=\sqrt{25m^2+4}\Rightarrow m=\pm\frac{2}{\sqrt{3}}$

The tangent is at the point in first quadrant so $m<0\Rightarrow m=-\frac{2}{\sqrt{3}}$

So common tangent is $y=-\frac{2}{\sqrt{3}}x+4\sqrt{\frac{7}{3}}$

It meets the co-ordinate axis at $A(2\sqrt{7},0)$ and $B(0,4\sqrt{7/3})$

So $AB=14/\sqrt{3}$

6. Ans. (A,D)

Let $P(h,k)$ be the point of intersection of E_1 and E_2

$$\Rightarrow \frac{h^2}{a^2}+k^2=1 \Rightarrow h^2 = a^2(1-k^2) \quad ...(1)$$

$$\text{and } \frac{h^2}{1}+\frac{k^2}{a^2}=1 \Rightarrow k^2 = a^2(1-h^2) \quad ...(2)$$

Eliminating a from (1) and (2), we get

$$\frac{h^2}{1-k^2}=\frac{k^2}{1-h^2} \Rightarrow h^2(1-h^2) = k^2(1-k^2)$$

$$\Rightarrow (h-k)(h+k)(h^2+k^2-1) = 0$$

Hence the locus is a set of curves consisting of the straight lines

$y = x, y = -x$ and circle $x^2 + y^2 = 1$.

7. Ans. (C)

$$y = mx \pm \sqrt{a^2m^2+b^2}$$

$$k = mh \pm \sqrt{a^2m^2+b^2}$$

$$(k-mh)^2 = a^2m^2 + b^2$$

$$m^2(h^2-a^2) - 2mhk + k^2 - b^2 = 0 \quad \because (m_1 = \tan\theta_1 , m_2 = \tan\theta_2)$$

$$m_1m_2 = \frac{k^2-b^2}{h^2-a^2} = \tan\theta_1 \tan\theta_2 = 4$$

$$\Rightarrow \frac{y^2-b^2}{x^2-a^2}=4 \Rightarrow \left(\frac{y-b}{x-a}\right)=4\left(\frac{x+a}{y+b}\right)$$

8. Ans. (B)

$\because \angle QAP = \angle PBQ = 90°$

hence a circle drawn taking 'PQ' as diameter will pass through B, A, P, Q

$\therefore$ center will be mid point of PQ

9. Ans. (A)

$$m_1 + m_2 = \frac{2hk}{h^2-a^2}$$

and $\cot\theta_1 + \cot\theta_2 = \lambda$

$$\Rightarrow \frac{1}{\tan\theta_1}+\frac{1}{\tan\theta_2}=\lambda$$

$$\Rightarrow \frac{\tan\theta_1+\tan\theta_1}{\tan\theta_1 \tan\theta_2} = \lambda \quad \Rightarrow \frac{\frac{2hk}{h^2-a^2}}{\frac{k^2-b^2}{h^2-a^2}}=\lambda$$

$$\Rightarrow 2hk = \lambda\,(k^2 - b^2)$$

$$2xy = \lambda\,(y^2 - b^2)$$

Paragraph for question nos. 10 to 12

Solving the curves $y^2 = 2x$ and $\frac{x^2}{9}+\frac{y^2}{4} = 1$ for the points of intersection, we have

$4x^2 + 18x - 36 = 0 \Rightarrow x = \frac{3}{2}, -6$

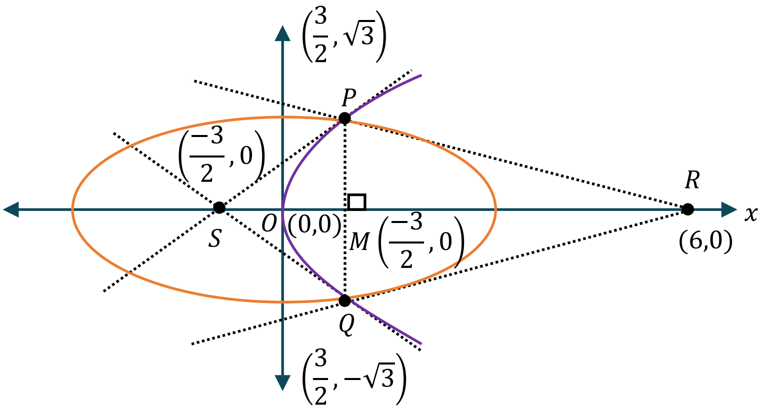
But from $y^2 = 2x$ we have $x > 0$

$\therefore x = \frac{3}{2}$

at which $y^2 = 2 \cdot \frac{3}{2}$

$\Rightarrow y = \pm\sqrt{3}$

$\therefore P\left(\frac{3}{2}, \sqrt{3}\right)$ and $Q\left(\frac{3}{2}, -\sqrt{3}\right)$



Now equation of tangents at P and Q to ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ is $\frac{x}{9}\left(\frac{3}{2}\right)+\frac{y}{4}\left(\pm\sqrt{3}\right)=1$ which intersect at $R(6, 0)$

Equation of tangents at P and Q to parabola $y^2 = 2x$ will be $y\left(\pm\sqrt{3}\right) = x + \frac{3}{2}$ which cut x-axis $S\left(\frac{-3}{2},0\right)$

10. **Ans. (C)**

$$\therefore \frac{Area\,\Delta PQS}{Area\,\Delta PQR} = \frac{\frac{1}{2}PQ\cdot MS}{\frac{1}{2}PQ\cdot MR} = \frac{MS}{MR} = \frac{\frac{3}{2}-\left(\frac{-3}{2}\right)}{6-\frac{3}{2}} = \frac{3}{\frac{9}{2}} = \frac{2}{3}$$

11. **Ans. (B)**

Area of quadrilateral $PRQS = \frac{1}{2}PQ(MS + MR) = \frac{1}{2}\cdot 2\sqrt{3}\,(6-(-3/2)) = \frac{15\sqrt{3}}{2}$

12. **Ans. (D)**

Clearly upper end of latus rectum of parabola is $\left(\frac{1}{2},1\right)$.

And equation of tangent at $\left(\frac{1}{2},1\right)$ to

$y^2 = 2x$ is $y = x + \frac{1}{2}$

$\therefore$ The equation of circle is

$\left(x-\frac{1}{2}\right)^2+(y-1)^2+\lambda\left(y-x-\frac{1}{2}\right) = 0$

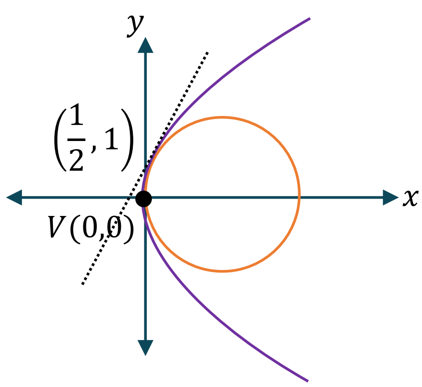
As above circle passes through $V(0,0)$, so

$\frac{1}{4}+1-\frac{\lambda}{2} = 0 \quad \Rightarrow \lambda = \frac{5}{2}$

$\Rightarrow$ The equation of required circle is

$\left(x-\frac{1}{2}\right)^2+(y-1)^2+\frac{5}{2}\left(y-x-\frac{1}{2}\right) = 0$

$\Rightarrow 2x^2 + 2y^2 - 7x + y = 0$



13. Ans. (7)

$x^2 + y^2 = r^2 \Rightarrow y = mx \pm r\sqrt{1+m^2}$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow y = mx \pm \sqrt{a^2m^2 + b^2}$

For common tangent

$r^2(1 + m^2) = a^2 m^2 + b^2$

$r^2 + r^2m^2 = a^2m^2 + b^2$

$\frac{r^2 - b^2}{a^2 - r^2} = m^2$

$m = \pm\sqrt{\frac{r^2 - b^2}{a^2 - r^2}}$

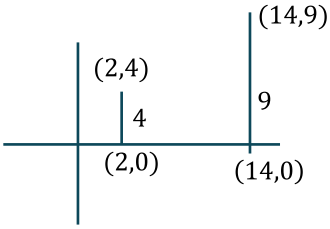
14. Ans. (13)

$2ae = \sqrt{(12)^2 + 5^2} = 13$

$b^2 = 36 \Rightarrow ae = \frac{13}{2}$

$b = 6$

$a^2 = \frac{169}{4} + 36 \Rightarrow a = \frac{\sqrt{313}}{2}$ $e = \frac{13}{\sqrt{313}}$



15. Ans. (9)

Standard result

16. Ans. (6)

$\frac{x^2}{r^2 - r - 6} + \frac{y^2}{r^2 - 6r + 5} = 1$

(i) $r^2 - r - 6 > 0$

$(r - 3)(r + 2) > 0$

$(-\infty, -2) \cup (3, \infty)$

(ii) $r^2 - 6r + 5 > 0$

$(r - 1)(r - 5) > 0$

$(-\infty, 1) \cup (5, \infty)$

(iii) Since in ellipse $e^2 = 1 - \frac{b^2}{a^2}$

$e < 1$

$\therefore 0 < 1 - \frac{r^2 - 6r + 5}{r^2 - r - 6} < 1$

$r \in (5, \infty)$

$\therefore$ for above equation to be ellipse

$\therefore r \in (5, \infty)$

17. Ans. (1)

Centre is intersection point of axes

$$\begin{array}{l} x+y-1=0 \\ \underline{x-y+2=0} \\ x=-\frac{1}{2}, y=\frac{3}{2} \end{array}$$

18. Ans. (3)

$\Rightarrow$ Length of semi minor axis is = 2

Length of semi major axis is 4

then equation of ellipse is

$$\frac{x^2}{16}+\frac{y^2}{4}=1$$

$$x^2+4y^2=16$$