Differential Equations

SOLUTIONS

EXERCISE - O

1. **Ans. (B)**

$y = mx + c$

$\Rightarrow Dy = m$

$\Rightarrow D^2y = 0$

given solution of $D.E.$

$D^2y - 3Dy - 4y = -4x$

$\Rightarrow 0 - 3m - 4(mx + c) = -4x$

$\Rightarrow -3m - 4mx - 4c = -4x$

Equations

$-4m = -4$ & $-3m - 4c = 0$

$\Rightarrow m = 1$ $\quad \Rightarrow -3 = 4c$

$\Rightarrow c = \frac{-3}{4}$

2. **Ans. (A)**

Statement-1

$y = \sin kt$

$\Rightarrow y' = (\cos kt)k$

$\Rightarrow y'' = (-\sin kt)k^2$

$\Rightarrow y'' = -k^2y$

$y'' + k^2y = 0$

equations

$k^2 = 9$

$\Rightarrow k = \pm 3$

Statement-2

$y = e^{kt}$

$\Rightarrow y' = e^{kt} \cdot k$

$\Rightarrow y'' = e^{kt}k^2$

$\Rightarrow y'' + y' - 6y = 0$

$\Rightarrow e^{kt}k^2 + ke^{kt} - 6e^{kt} = 0$

$\Rightarrow e^{kt}(k^2 + k - 6) = 0$

$k^2 + k - 6 = 0 \qquad (\because e^{kt} \neq 0)$

$\Rightarrow (k + 3)(k - 2) = 0 \Rightarrow k = 2, -3.$

For $k = -3$ both statements are correct.

3. Ans. (A)

Equation of tangent

$y - y_1 = m(x - x_1)$

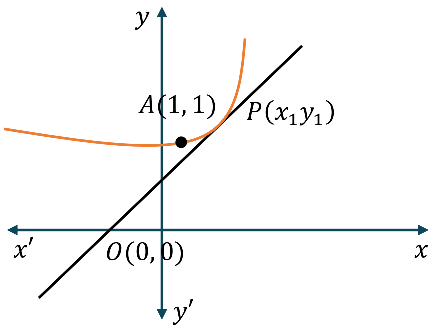
on x −axis, $y = 0$

$\Rightarrow -y_1 = (m(x - x_1)$

$\Rightarrow -\frac{y_1}{m} = x - x_1$

x −intercept = $x_1 - \frac{y_1}{m}$

A.T.Q $y_1 = x_1 - \frac{y_1}{m}$



$\Rightarrow \frac{y_1}{m} = x_1 - y_1$

$\Rightarrow m = \frac{y_1}{x_1 - y_1}$

$\therefore \frac{dy}{dx} = \frac{y}{x - y}$

Which is Homogeneous D.E.

$\therefore \frac{dy}{dx} = \frac{y/x}{1 - y/x}$

Put $\frac{y}{x} = v$

$\Rightarrow y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ D.E. $v + x\frac{dv}{dx} = \frac{v}{1-v}$

$\Rightarrow x\frac{dv}{dx} = \frac{v}{1-v} - v$

$\Rightarrow x\frac{dv}{dx} = \frac{v - v + v^2}{1-v}$

$\Rightarrow \frac{1-v}{v^2} dv = \frac{dx}{x}$

$\Rightarrow \int\left(\frac{1}{v^2} - \frac{1}{v}\right) dv = \int \frac{1}{x} dx$

$\Rightarrow -\frac{1}{v} - \log_e v = \log_e x + \log c$

$\Rightarrow -\frac{x}{y} - \log_e \frac{y}{x} = \log_e x + \log c$

$\Rightarrow -\frac{x}{y} = \log(cy)$

$\Rightarrow cy = e^{-\frac{x}{y}} \Rightarrow cye^{\frac{x}{y}} = 1 \Rightarrow y \cdot e^{\frac{x}{y}} = e$

4. Ans. (A)

$$\int_0^1 f(tx)dt = nf(x)$$

Put $tx = v$ upper limit

$xdt = 1 \cdot dv$ $t = 1 \Rightarrow v = x$

$dt = \frac{1}{x}dv$ lower limit

$t = 0 \Rightarrow v = 0$

$$\therefore \int_0^x f(v)\frac{dv}{x} = nf(x)$$

$$\int_0^x f(v)dv = xnf(x)$$

Diff. both side

$f(x) = n[xf'(x) + f(x)]$

$\Rightarrow (1-n)f(x) = nxf'(x)$

$$\Rightarrow \left(\frac{1-n}{n}\right)\cdot\frac{1}{x} = \frac{f'(x)}{f(x)}$$

Integration both side

$$\Rightarrow \int \frac{f'(x)}{f(x)}dx = \left(\frac{1-n}{n}\right)\int \frac{1}{x}dx$$

$$\Rightarrow \log_e f(x) = \left(\frac{1-n}{n}\right)\log_e x + \log_e c$$

$$\Rightarrow \log_e f(x) = \log_e cx^{\left(\frac{1-n}{n}\right)}$$

$$\Rightarrow f(x) = cx^{\frac{1-n}{n}}$$

5. Ans. (B)

$Dy = 100 - y$

$$\Rightarrow \frac{dy}{dx} = 100 - y$$

$$\Rightarrow \int \frac{dy}{100-y} = \int 1 \cdot dx$$

$\Rightarrow -\log_e(100-y) = x + c$

at $x = 0, y = 50$

$\Rightarrow -\log_e(100-50) = c$

$c = -\log_e 50$

$\therefore D.E. = -\log_e(100-y) = x - \log_e 50$

$-x = \log_e(100-y) - \log_e 50$

$$\Rightarrow -x = \log_e\left(\frac{100-y}{50}\right)$$

$$\Rightarrow e^{-x} = \frac{100-y}{50}$$

$\Rightarrow 50e^{-x} = 100 - y$

$\Rightarrow y = 100 - 50e^{-x}$

6. **Ans. (C)**

$y = u^m$ ...(1)

$\frac{dy}{dx} = mu^{m-1}\frac{du}{dx}$ ...(2)

given D.E.

$\Rightarrow 2x^4 y\frac{dy}{dx} + y^4 = 4x^6$

$\Rightarrow 2x^4 \cdot u^m \cdot mu^{m-1}\frac{du}{dx} + u^{4m} = 4x^6$

$2mx^4 u^{2m-1}\frac{du}{dx} = 4x^6 - u^{4m}$ (using (1) & (2))

$\Rightarrow 2mx^4 u^{2m-1} \cdot \frac{du}{dx} = 4x^6 - u^{4m}$

$\Rightarrow \frac{du}{dx} = \frac{4x^6 - u^{4m}}{2mx^4 u^{2m-1}}$

for homogeneous $4m = 6$

$\Rightarrow m = \frac{3}{2}$

7. **Ans. (B)**

$(x+1)f'(x) - 2(x^2+x)f(x) = \frac{e^{x^2}}{x+1}$

$\Rightarrow \frac{dy}{dx} - \left[\frac{2(x+1)x}{x+1}\right]y = \frac{e^{x^2}}{(x+1)^2}$

where $y = f(x)$

which is L.D.E.

here $p = -2x$ & $Q = \frac{e^{x^2}}{(x+1)^2}$

$\text{I.F.} = e^{\int -2x dx}$

$= e^{-x^2}$

Solution of L.D.E

$\Rightarrow y \cdot e^{-x^2} = \int e^{-x^2} \times \frac{e^{x^2}}{(x+1)^2} dx$

$\Rightarrow f(x) \times e^{-x^2} = \int \frac{1}{(x+1)^2} dx$

$\Rightarrow f(x) \cdot e^{-x^2} = \frac{-1}{x+1} + c$

$\because f(0) = 5 \Rightarrow 5 = -1 + c \Rightarrow c = 6$

$\therefore$ D.E. $f(x) \cdot e^{-x^2} = \frac{-1}{x+1} + 6$

$\Rightarrow f(x) \cdot e^{-x^2} = \frac{6x+5}{x+1}$

$\Rightarrow f(x) = e^{x^2}\left(\frac{6x+5}{x+1}\right)$

8. Ans. (A)

$$\int_a^x ty(t)\,dt = x^2 + y$$

Diff. both side

$$xy = 2x + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} - xy = -2x$$

Which is L.D.E.

here $P = -x, Q = -2x$

$$\text{I.F.} = e^{\int -xdx}$$

$$= e^{-\left(\frac{x^2}{2}\right)}$$

Solution of L.D.E.

$$y \cdot e^{-\left(\frac{x^2}{2}\right)} = 2\int e^{-\left(\frac{x^2}{2}\right)} \cdot (-xdx)$$

$$\Rightarrow y \cdot e^{-\left(\frac{x^2}{2}\right)} = 2e^{-\left(\frac{x^2}{2}\right)} + c$$

$$\Rightarrow y = 2 + ce^{\left(\frac{x^2}{2}\right)}$$

at $x = a, y = -a^2$

$$\Rightarrow -a^2 = 2 + ce^{-\left(\frac{a^2}{2}\right)} \Rightarrow c = -(a^2 + 2)e^{\left(\frac{a^2}{2}\right)}$$

Solution of L.D.E.

$$\Rightarrow y = 2 - (a^2 + 2)e^{\left(\frac{x^2 + a^2}{2}\right)}$$

9. Ans. (B)

$$\frac{dp}{dt} = 0.5p - 450$$

$$\Rightarrow \frac{dp}{dt} = \frac{1}{2}(p - 900)$$

$$\Rightarrow 2\int \frac{dp}{p - 900} = \int 1 \cdot dt$$

$$\Rightarrow 2\log_e |p - 900| = t + c$$

at $t = 0, p = 850 \Rightarrow 2\log_e |850 - 900| = c$

$\Rightarrow c = 2\log_e 50$

$\therefore$ D.E. $2\log_e |p - 900| = t + 2\log_e 50$

$$\Rightarrow 2\log_e \left|\frac{p - 900}{50}\right| = t$$

A.T.Q

$p = 0$ then $t = 2\log_e \left|\frac{0 - 900}{50}\right|$

$= 2\log_e(18)$

10. Ans. (D)

$$\frac{dV(t)}{dt}=-k(T-t)$$

$$\Rightarrow V(t)=\frac{k(T-t)^2}{2}+C$$

at $t=0, V(0)=I$

$$\Rightarrow I=\frac{kT^2}{2}+c\Rightarrow C=I-\frac{kT^2}{2}$$

$$\Rightarrow V(t)=\frac{k(T-t)^2}{2}+I-\frac{kT^2}{2}$$

$$\Rightarrow V(t)=\frac{k}{2}(t^2-2tT)+I$$

So, at $t=T$

$$V(T)=I-\frac{k}{2}T^2$$

11. Ans. (A)

$y=ax^2$

$$\frac{dy}{dx}=2ax$$

$$\therefore \frac{dy}{dx}=\frac{2y}{x}\rightarrow(1)$$

or $\frac{dy}{dx}=-\frac{x}{2y}$

$$\int 2y\ dy=\int -x\ dx$$

$$y^2=-\frac{x^2}{2}+c$$

or $y^2+\frac{x^2}{2}+c$

12. Ans. (C)

equation of tangent

$Y-y=m(X-x)$

For A $Y=0$

$$X=x-\frac{y}{m}$$

For B $X=0$

$Y=y-mx$

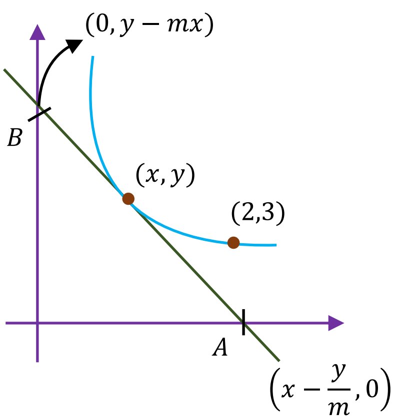
Now $x-\frac{y}{m}=2x\Rightarrow m=-\frac{y}{x}$

$$\Rightarrow\frac{dy}{dx}=-\frac{y}{x}\Rightarrow\int\frac{dx}{x}+\int\frac{dy}{y}=\int 0$$

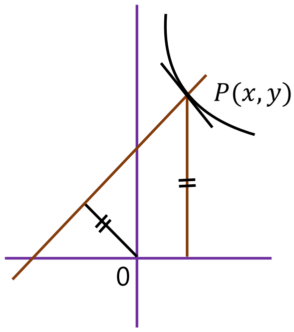
$$\Rightarrow \ell n|x|+\ell n|y|=\ell nk\Rightarrow xy=\pm k$$

it passes through $(2,3)\Rightarrow k=6$

curve $xy=6\Rightarrow y=\frac{6}{x}$



13. Ans. (A,B,D)



$$Y - y = -\frac{1}{m}(X - x)$$

$$-mY + my = X - x$$

$$X + mY - (x + my) = 0$$

perpendicular from (0, 0) = $\left|\frac{x+my}{\sqrt{1+m^2}}\right| = y$

$$x^2 + 2xym = y^2$$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \Rightarrow \text{homogeneous} \Rightarrow \text{(A)}$$

also $x \cdot 2y \cdot \frac{dy}{dx} + x^2 = y^2$ put $y^2 = t;\quad 2y\frac{dy}{dx} = \frac{dt}{dx}$

$$x \cdot \frac{dt}{dx} + x^2 = t$$

$\frac{dt}{dx} - \frac{1}{x}t = x$ which is linear differential equation $\Rightarrow$ B

Integration factor = $e^{-\int \frac{1}{x}dx} = e^{-\ell nx} = \frac{1}{x}$

$$\Rightarrow \left(\frac{1}{x}\right).t = -\int 1dx = -x + c$$

$$\frac{1}{x} \cdot y^2 = -x + c$$

$x^2 + y^2 - cx = 0 \Rightarrow$ D

14. Ans. (A,B,D)

$$\frac{dy}{dx} - \frac{y\left(\tan\frac{1}{x}\right)}{x^2} = -\frac{\left(\sec\frac{1}{x}\right)}{x^2}$$

$$I.T. = e^{-\int \frac{\tan\frac{1}{x}}{x^2}dx} = \sec\left(\frac{1}{x}\right)$$

$$y \cdot \left(\sec\frac{1}{x}\right) = -\int \frac{\left(\sec^2\frac{1}{x}\right)}{x^2}dx$$

$$y\sec\left(\frac{1}{x}\right) = \tan\left(\frac{1}{x}\right) + c$$

$$y = \sin\frac{1}{x} + c \cdot \cos\left(\frac{1}{x}\right)$$

$$x \to \infty; y \to -1 \Rightarrow c = -1$$

So, function is

$$y = f(x) = \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)$$

$\Rightarrow$ A is correct

Now $\lim_{x\to\infty}(-f(x))^x = \lim_{x\to\infty}\left(\cos\frac{1}{x} - \sin\frac{1}{x}\right)^x$

Let $x = \frac{1}{t}$ as $x\to\infty$; $t\to 0$

$= \lim_{t\to 0}(\cos t - \sin t)^{\frac{1}{t}}$

$= \lim_{t\to 0} e^{\frac{(\cos t - \sin t - 1)}{t}} = \frac{1}{e}$

B is correct. For D but $f(x), f'(x)\ R f''(x)$

15. Ans. (A,C,D)

(A) $f(kx,ky) = k^2x^2e^{x/y} + \frac{k^3y^3}{kx} + k^2y^2\ln\left(\frac{y}{x}\right)$

$f(kx,ky) = k^2 f(x,y)$

degree 2.

(B) Not a homogeneous function.

(C) $f(x,y) = x\sin\left(\frac{y}{x}\right)dy + \left(y\sin\frac{y}{x} - x\right)dx = 0$

$f(kx,ky) = kx\sin\left(\frac{y}{x}\right)dy + \left(ky\sin\frac{y}{x} - kx\right)dx = 0$

$= kf(x,y)$

degree 1.

(D) $f(x,y) = e^{y/x} + \tan\frac{y}{x}$

$f(kx,ky) = e^{y/x} + \tan\frac{y}{x}$

$= k^\circ f(x,y)$

degree zero.

16. Ans. (A,C,D)

$x^2y_1^2 + xyy_1 - 6y^2 = 0$

$x^2y_1^2 + 3xyy_1 - 2xyy_1 - 6y^2 = 0$

$xy_1(xy_1 + 3y) - 2y(xy_1 + 3y) = 0$

$(xy_1 + 3y)(xy_1 - 2y) = 0$

$\therefore\ \frac{dy}{dx} = \frac{-3y}{x}$ or $\frac{2y}{x}$

$\therefore\ \frac{dy}{y} = \frac{2dx}{x}$ or $\frac{dy}{y} = -\frac{3dx}{x}$

$y = cx^2$ or $yx^3 = k$

17. Ans. (A,B)

$$\frac{dx}{dy}+y=ye^{(n-1)x}$$

$$\frac{dx}{dy}=y\left(e^{(n-1)x}-1\right)$$

$$\frac{e^{-(n-1)x}dx}{1-e^{-(n-1)x}}=ydy$$

$$\frac{\ell n\left(1-e^{-(n-1)x}\right)}{n-1}=\frac{y^2}{2}+c$$

18. Ans. (A,B,D)

$y=c\sin x$ ...(i)

$\therefore \frac{dy}{dx}=c\cos x$ ...(ii)

From (ii)

$\left(\frac{dy}{dx}\right)^2=c^2\cos^2 x$ ...(iii)

Putting $c=\frac{y}{\sin x}$ from (i), $\left(\frac{dy}{dx}\right)^2=y^2\cot^2 x$

Eliminating c from (i) and (ii), $\frac{dy}{dx}=y\cot x$

Squaring and adding (i) and (ii), $y^2+\left(\frac{dy}{dx}\right)^2=c^2$.

Putting the value of 'c' from (iii), $y^2+\left(\frac{dy}{dx}\right)^2=\left(\frac{dy}{dx}\sec x\right)^2$

19. Ans. (A,B,D)

$$\left(\frac{dy}{dx}\right)^2-\left(e^x+e^{-x}\right)\frac{dy}{dx}+1=0$$

$$\left(\frac{dy}{dx}-e^x\right)\left(\frac{dy}{dx}-e^{-x}\right)=0$$

$$\frac{dy}{dx}=e^x \text{ or } e^{-x}$$

$y=e^x+c_1$ or $y=-e^{-x}+c_2$

20. Ans. (B)

$2x^3dx+2y^3dy-(xy^2dx+x^2y\,dy)=0$

$$d\left(\frac{x^4}{2}\right)+d\left(\frac{y^4}{2}\right)-\frac{1}{2}d(x^2y^2)=0$$

$\Rightarrow d(x^4+y^4-x^2y^2)=0\Rightarrow x^4+y^4-x^2y^2=c$

21. Ans. (A)

$$\frac{xdy-ydx}{x^2+y^2}+dx=0\Rightarrow\frac{\frac{xdy-ydx}{x^2}}{1+\left(\frac{y}{x}\right)^2}+dx=0\Rightarrow\frac{d\left(\frac{y}{x}\right)}{1+\left(\frac{y}{x}\right)^2}+dx=0$$

$$\Rightarrow d\left(\tan^{-1}\left(\frac{y}{x}\right)\right)+dx=0\Rightarrow\tan^{-1}\left(\frac{y}{x}\right)+x=0$$

22. Ans. (A)

$e^y dx + xe^y dy - 2ydy = 0$

$d(xe^y) - d(y^2) = 0$

Solution is $xe^y - y^2 = c$

23. Ans. (B)

(I) $\frac{ydx - xdy}{y^2} = dx + \frac{dy}{y^2}$

$\Rightarrow \quad d\left(\frac{x}{y}\right) = dx + \frac{dy}{y^2} \quad \Rightarrow \frac{x}{y} = x - \frac{1}{y} + k$

$\Rightarrow x = xy - 1 + ky$

$\Rightarrow \quad (x+1)(1-y) = cy$

(II) $(2x - 10y^3)\frac{dy}{dx} + y = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y}{10y^3 - 2x}$

$\Rightarrow \frac{dx}{dy} = \frac{10y^3 - 2x}{y}$

$\frac{dx}{dy} = 10y^2 - 2\left(\frac{x}{y}\right)$

$\Rightarrow \frac{dx}{dy} + \frac{2}{y}x = 10y^2$

$\Rightarrow xy^2 = 10\frac{y^5}{5} + c$

$\Rightarrow xy^2 = 2y^5 + c$

(III) $\sec^2 y\frac{dy}{dx} + \tan y = 1$ put $\tan y = t$ $\quad \sec^2 y\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx} = 1 - t \Rightarrow \ell n(1-t) = -x + c$

$\Rightarrow 1 - t = e^{-x+c}$

$\Rightarrow t = 1 - e^{-x+c}$

$\Rightarrow \tan y = 1 - e^{-x}e^{c}$

$\Rightarrow \tan y = 1 - ce^{-x}$

$\Rightarrow \tan y = 1 + ce^{-x}$

(IV) $\sin y\frac{dy}{dx} = \cos y(1 - x\cos y)$

put $\cos y = t - \sin y\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow -\frac{dt}{dx} = t(1 - tx)$

$\Rightarrow \sec y = x + 1 + ce^{x}$

EXERCISE - S

1. Ans. (49)

$$\frac{dy}{dx}+\left(\frac{2}{x}\right)y=x$$

$\Rightarrow$ I.F. = x^2

$\therefore\ yx^2 = \frac{x^4}{4}+\frac{3}{4}$ (As, $y(1) = 1$)

$\therefore\ y\left(x=\frac{1}{2}\right) = \frac{49}{16}$

2. Ans. (4)

$f'(x)=7-\frac{3}{4}\frac{f(x)}{x}\qquad(x>0)$

Given $f(1) \neq 4\quad \lim\limits_{x\to 0^+} xf\left(\frac{1}{x}\right) = ?$

$\frac{dy}{dx}+\frac{3}{4}\frac{y}{x}=7$ (This is LDE)

IF = $e^{\int\frac{3}{4x}dx}=e^{\frac{3}{4}\ln|x|}=x^{\frac{3}{4}}$

$\Rightarrow\ y.x^{\frac{3}{4}}=\int 7.x^{\frac{3}{4}}\,dx \Rightarrow\ y.x^{\frac{3}{4}}=7.\frac{x^{\frac{7}{4}}}{\frac{7}{4}}+C$

$\Rightarrow f(x) = 4x+C.x^{-\frac{3}{4}} \Rightarrow\ f\left(\frac{1}{x}\right)=\frac{4}{x}+C.x^{\frac{3}{4}}$

$\Rightarrow \lim\limits_{x\to 0^+} xf\left(\frac{1}{x}\right)=\lim\limits_{x\to 0^+}\left(4+C.x^{\frac{7}{4}}\right)=4$

3. Ans. (2)

$$\frac{dy}{dx}+\left(\frac{2x}{x^2+1}\right)y=\frac{1}{(x^2+1)^2}$$

(Linear differential equation)

$\therefore$ I.F. = $e^{\ell n\left(x^2+1\right)}=(x^2+1)$

So, general solution is $y\cdot(x^2+1)=\tan^{-1}x+c$

As $y(0)=0 \Rightarrow c=0$

$\therefore\ y(x)=\frac{\tan^{-1}x}{x^2+1}$

As, $\sqrt{a}.\ y(1)=\frac{\pi}{32}$

$\Rightarrow \sqrt{a}=\frac{1}{4}\Rightarrow a=\frac{1}{16} \Rightarrow 32a = 2$

4. Ans. (2)

$\frac{dy}{dx} = (\tan x - y)\sec^2 x$

Now, put $\tan x = t \Rightarrow \frac{dt}{dx} = \sec^2 x$

So, $\frac{dy}{dt} + y = t$

On solving, we get $ye^t = e^t(t-1) + c$

$\Rightarrow y = (\tan x - 1) + ce^{-\tan x}$

$\Rightarrow y(0) = 0 \Rightarrow c = 1$

$\Rightarrow y = \tan x - 1 + e^{-\tan x}$

So $y\left(-\frac{\pi}{4}\right) = e - 2 = e - k \Rightarrow k = 2$

5. Ans. (2)

$\frac{2+\sin x}{y+1}\frac{dy}{dx} = -\cos x, y > 0$

$\Rightarrow \frac{dy}{y+1} = \frac{-\cos x}{2+\sin x}dx$

By integrating both sides :

$\ell n|y+1| = -\ell n|2+\sin x| + \ell nK$

$\Rightarrow y + 1 = \frac{K}{2+\sin x} \qquad (y+1>0)$

$\Rightarrow y(x) = \frac{K}{2+\sin x} - 1$

Given $y(0) = 1 \Rightarrow K = 4$

So, $y(x) = \frac{4}{2+\sin x} - 1$

$a = y(\pi) = 1$

$b = \left.\frac{dy}{dx}\right]_{x=\pi} = \left.\frac{-\cos x}{2+\sin x}(y(x)+1)\right]_{x=\pi} = 1$

$\Rightarrow a + b = 2$

6. Ans. (1)

$\frac{(5+e^x)}{2+y}\frac{dy}{dx} = -e^x$

$\Rightarrow \int\frac{dy}{2+y} = \int\frac{-e^x}{e^x+5}dx$

$\ell n(y+2) = -\ell n(e^x+5) + k$

$\Rightarrow (y+2)(e^x+5) = C$

$\because y(0) = 1$

$\Rightarrow C = 18$

$y + 2 = \dfrac{18}{e^x + 5}$

at $x = \ell n\, 13$

$y + 2 = \dfrac{18}{13+5} = -1$

$\Rightarrow |y(\log_e 13)| = 1$

7. **Ans. (1)**

$y = \left(\dfrac{2x}{\pi} - 1\right)\operatorname{cosec} x$ ...(1)

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2}{\pi}\operatorname{cosec} x - \left(\dfrac{2x}{\pi} - 1\right)\operatorname{cosec} x \cot x$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2\operatorname{cosec} x}{\pi} - y\cot x$

using equation (1)

$\Rightarrow \dfrac{dy}{dx} + y\cot x = \dfrac{2\operatorname{cosec} x}{\pi}$

$\Rightarrow \dfrac{dy}{dx} + p(x).y = \dfrac{2\operatorname{cosec} x}{\pi} \quad x \in \left(0, \dfrac{\pi}{2}\right)$

Compare: $p(x) = \cot x$

$\Rightarrow P\left(\dfrac{\pi}{4}\right) = 1$

8. **Ans. (4)**

$\dfrac{dy}{dx} = y + \int_0^1 y\, dx$

$\Rightarrow \dfrac{dy}{dx} = y + a \qquad$ let $\int_0^1 y\, dx = a$

$\Rightarrow \dfrac{dy}{dx} - y - a = 0$

I.F. $= e^{-\int 1.dx} = e^{-x}$

Now $e^{-x}\dfrac{dy}{dx} - ye^{-x} - ae^{-x} = 0$

$\Rightarrow ye^{-x} + ae^{-x} + c = 0$

$\Rightarrow y + a + ce^x = 0$

$a = \int_0^1 y\, dx = -\int_0^1 (a + ce^x)dx = -a(1-0) - c(e^x)\Big|_0^1 = -a - c(e-1)$

$\Rightarrow a = \dfrac{(1-e)c}{2}$

$y + \left(\dfrac{(1-e) + 2e^x}{2}\right)c = 0$

$\Rightarrow y = 1; x = 0 \Rightarrow c = \dfrac{2}{e-3}$

$\Rightarrow y(x) = \left(\frac{2e^x - e + 1}{3 - e}\right)$

$\Rightarrow y\left(\ell n\frac{11-3e}{2}\right) = \frac{11-3e-e+1}{3-e}$

$= \frac{4[3-e]}{3-e} = 4$

9. Ans. (2)

$\frac{dy}{dx} = \frac{1}{x\cos y + 2\sin y\cos y}$

$\therefore \frac{dx}{dy} = x\cos y + 2\sin y\cos y$

$\Rightarrow \frac{dx}{dy} + (-\cos y)\,x = 2\sin y\cos y$

$\therefore$ I.F. $= e^{-\int \cos y\,dy} = e^{-\sin y}$

$\therefore$ The solution is

$x \cdot e^{-\sin y} = 2\int e^{-\sin y} \cdot \sin y\cos y\,dy = -2\sin y\, e^{-\sin y} - 2\int(-e^{-\sin y})\cos y dx$

$= -2\sin y\, e^{-\sin y} + 2\int e^{-\sin y}\cos y\,dy = -2\sin y\, e^{-\sin y} - 2\,e^{-\sin y} + c$

i.e. $x = -2\sin y - 2 + c\,e^{\sin y} = ce^{\sin y} - 2\,(1 + \sin y)$

$\therefore k = 2$

10. Ans. (4)

$y\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y\frac{dy}{dx} - x = 0$

$\Rightarrow y\frac{dy}{dx}\left(\frac{dy}{dx} - 1\right) + x\left(\frac{dy}{dx} - 1\right) = 0$

$\Rightarrow \left(y\frac{dy}{dx} + x\right)\left(\frac{dy}{dx} - 1\right) = 0$

$\therefore$ either $ydy + xdx = 0$ or $dy - dx = 0$

since the curves pass through the point (3, 4)

$\therefore x^2 + y^2 = 25$ or $x - y + 1 = 0$

$\Rightarrow 2x - 2y + 2 = 0 \Rightarrow A = 2\ \&\ B = 2$

$\Rightarrow A - B = 4$

EXERCISE - JEE (Main) PYQ

1. Ans. (1)

$$\frac{dp(t)}{dt}=\frac{1}{2}p(t)-200$$

$$\Rightarrow \int_{100}^{p(t)}\frac{dp(t)}{p(t)-400}=\int_{0}^{t}\frac{dt}{2}$$

$$\Rightarrow \log\left|\frac{p(t)-400}{-300}\right|=\frac{t}{2}$$

$$\Rightarrow |P(t)-400| = 300\, e^{\frac{t}{2}}$$

$$\Rightarrow 400-P(t) = 300\, e^{\frac{t}{2}}$$

$$\therefore\ P(t) = 400-300e^{\frac{t}{2}}$$

2. Ans. (1)

$$(x\log x)\frac{dy}{dx}+y=2x\log x,(x\geq 1)$$

$$=\frac{dy}{dx}+\frac{y}{x\log x}=2$$

$$\text{I.F.} = e^{\int\frac{dx}{x\log x}} = e^{\log(\log x)} = logx$$

$$= y\log x = \int 2\log x\, dx$$

$y\log x = 2x(\log x-1)+C$ ...(1)

Put $x=1$ $\quad y.0=-2+C$

$\Rightarrow C=2$ ...(2)

Put $x=e$ in (1)

$y\log e = 2e\,(\log e-1)+C$

$y(e)=C=2$ .... from (2)

3. Ans. (1)

Given differential equation

$$ydx+xy^2dx=xdy$$

$$\Rightarrow \frac{xdy-ydx}{y^2}=xdx \Rightarrow -d\left(\frac{x}{y}\right)=d\left(\frac{x^2}{2}\right)$$

Integrating we get $-\frac{x}{y}=\frac{x^2}{2}+C$

$\because$ It passes through $(1,-1)$

$$\therefore 1=\frac{1}{2}+C\Rightarrow C=\frac{1}{2}$$

$$\therefore x^2+1+\frac{2x}{y}=0\Rightarrow y=\frac{-2x}{x^2+1}$$

$$\therefore f\left(-\frac{1}{2}\right)=\frac{4}{5}$$

4. Ans. (2)

$\sin x\, dy + y \cos x\, dx = 4xdx$

$\Rightarrow d(y.\sin x) = 4xdx$

Integrate we get

$\Rightarrow\ y.\sin x = 2x^2 + C$

$\Rightarrow$ passes through $\left(\frac{\pi}{2}, 0\right) \Rightarrow 0\ = \frac{\pi^2}{2} + C$

$\Rightarrow C = -\frac{\pi^2}{2}$

$\Rightarrow\ y \sin x = 2x^2 - \frac{\pi^2}{2}$ is the solution

$\Rightarrow\ y\left(\frac{\pi}{6}\right) = \left(2.\frac{\pi^2}{36} - \frac{\pi^2}{2}\right)2\ = -\ \frac{8\pi^2}{9}$

5. Ans. (2)

$f(xy) = f(x).f(y)$

$\Rightarrow f(0) = 1$ as $f(0) \neq 0\ \Rightarrow\ f(x) = 1$

$\Rightarrow \frac{dy}{dx} = f(x) = 1\ \Rightarrow\ \ y = x + c$

At, $x = 0, y = 1 \Rightarrow c = 1$

$y = x + 1$

$\Rightarrow y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right) = \frac{1}{4} + 1 + \frac{3}{4} + 1 = 3$

6. Ans. (1)

$\frac{dy}{dx} + 3\sec^2 x\,.\,y = \sec^2 x$

$\text{I. F.} = e^{3\int \sec^2 x dx} = e^{3\tan x}$

$or\ y.e^{3\tan x} = \int \sec^2 x\,.\,e^{3\tan x} dx$

or $y.e^{3\tan x} = \frac{1}{3}e^{3\tan x} + C$ ...(1)

Given $y\left(\frac{\pi}{4}\right) = \frac{4}{3}$

$\therefore \frac{4}{3}.e^3 = \frac{1}{3}e^3 + C \therefore \text{C} = e^3$

Now put $x = -\frac{\pi}{4}$ in equation (1)

$\therefore y.e^{-3} = \frac{1}{3}e^{-3} + e^3 \therefore y = \frac{1}{3} + e^6$

$\therefore y\left(-\frac{\pi}{4}\right) = \frac{1}{3} + e^6$

7. Ans. (2)

$(x^2 - y^2\ )dx + 2xydy = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Solving we get,

$$\int \frac{2v}{v^2+1}dv = \int -\frac{dx}{x}$$

$\Rightarrow \ell n(v^2+1) = -\ell n\, x + \ell nC$

$\Rightarrow (y^2+x^2) = Cx$

$\Rightarrow\ 1+1 = C \Rightarrow\ C = 2$

$\boxed{y^2+x^2 = 2x}$

8. Ans. (2)

$\frac{dy}{dx} = \frac{x^2-2y}{x}$ (Given)

$\Rightarrow \frac{dy}{dx} + 2\frac{y}{x} = x$

I.F $= e^{\int \frac{2}{x}dx} = x^2$

$\therefore y.x^2 = \int x.x^2 dx + C = \frac{x^4}{4} + C$

Curve passes through $(1, -2) \Rightarrow C = -\frac{9}{4}$

$\therefore yx^2 = \frac{x^4}{4} - \frac{9}{4}$

Now check option(s).

9. Ans. (3)

$(y^2-x)\frac{dy}{dx} = 1$

$\Rightarrow \frac{dx}{dy} + x = y^2$

I.F. $= e^{\int dy} = e^y$

Solution is given by

$xe^y = \int y^2 e^y dy + C$

$\Rightarrow xe^y = (y^2 - 2y + 2)e^y + \mathrm{C}$

$x = 0, y = 1$, gives $\mathrm{C} = -e$

If $y = 0$, then $x = 2 - e$

10. Ans. (3)

$(x+1)dy - y\,dx = ((x+1)^2 - 3)dx$

$\Rightarrow \frac{(x+1)dy - ydx}{(x+1)^2} = \left(1 - \frac{3}{(x+1)^2}\right)dx$

$\Rightarrow d\left(\frac{y}{(x+1)}\right) = \left(1 - \frac{3}{(x+1)^2}\right)dx$

integrating both sides

$\frac{y}{x+1} = x + \frac{3}{(x+1)} + C$

Given $y(2) = 0 \Rightarrow c = -3$

$\therefore y = (x+1)\left(x + \frac{3}{(x+1)} - 3\right)$

$\therefore\ y(3) = 3.00$

11. Ans. (2)

$$\frac{dy}{dx}+\frac{y}{x}=bx^3$$

I.F. = $e^{\int \frac{1}{x}dx}=x$

So, solution of D.E. is given by

$$y.x=\int b.x^3.x\,dx+c$$

$$\Rightarrow y=\frac{c}{x}+\frac{bx^4}{5}$$

Passes through (1, 2)

$$2=c+\frac{b}{5} \quad ...(1)$$

$$\int_1^2 f(x)dx=\frac{62}{5}$$

$$\Rightarrow \left[c\ln x+\frac{bx^5}{25}\right]_1^2=\frac{62}{5}$$

$$\Rightarrow c\ln 2+\frac{31b}{25}=\frac{62}{5} \quad ...(2)$$

By equation (1) & (2)

$c=0$ and $b=10$

12. Ans. (4)

Given

$y(0)=0$

& $\frac{dy}{dx}=\frac{(x-2)^2+y+4}{x-2}$

$$\Rightarrow \frac{dy}{dx}-\frac{y}{x-2}=(x-2)+\frac{4}{x-2}$$

$$\Rightarrow \text{I.F.} = e^{-\int\frac{1}{x-2}dx}=\frac{1}{x-2}$$

Solution of L.D.E.

$$\Rightarrow y\frac{1}{x-2}=\int\frac{1}{x-2}\left((x-2)+\frac{4}{x-2}\right).dx$$

$$\Rightarrow \frac{y}{x-2}=x-\frac{4}{x-2}+C$$

Now, at $x=0, y=0 \Rightarrow C=-2$

$y=x(x-2)-4-2(x-2)$

$\Rightarrow y=x^2-4x$

This curve passes through (5, 5)

13. Ans. (1)

Let Point $P(x,y)$

$Y-y=y'(X-x)$

$$\Rightarrow Y=0 \Rightarrow X=x-\frac{y}{y'}$$

$$Q\left(x-\frac{y}{y'},0\right)$$

Mid Point of PQ lies on y-axis

$$x-\frac{y}{y'}+x=0$$

$$y'=\frac{y}{2.x} \Rightarrow 2\frac{dy}{y}=\frac{dx}{x}$$

$\Rightarrow\ 2\ell ny = \ell nx + \ell nk$

$\Rightarrow\ y^2 = kx$

It passes through (3, 3) $\Rightarrow k = 3$

curve $c \Rightarrow y^2 = 3x$

Length of L.R. = 3

Focus = $\left(\frac{3}{4},0\right)$

14. Ans. (4)

Slope of normal = $\frac{-dx}{dy}=\frac{x^2}{xy-x^2y^2-1}$

$\Rightarrow\ x^2y^2dx + dx - xydx = x^{2dy}$

$\Rightarrow\ x^2y^2dx + dx = x^2dy + xydx$

$\Rightarrow\ x^2y^2dx + dx = x(xdy + ydx)$

$\Rightarrow\ x^2y^2dx + dx = xd(xy)$

$$\Rightarrow \frac{dx}{x}=\frac{d(xy)}{1+x^2y^2}$$

ln kx = $\tan^{-1}(xy)$ ...(i)

passes though (1, 1)

ln k = $\frac{\pi}{4} \Rightarrow k = e^{\frac{\pi}{4}}$

equation (i) be becomes

$$\frac{\pi}{4}+\ell n\,x=\tan^{-1}(xy)$$

$$\Rightarrow xy = \tan\left(\frac{\pi}{4}+\ell n\,x\right)$$

$$\Rightarrow xy = \left(\frac{1+\tan(\ell n\,x)}{1-\tan(\ell n\,x)}\right) \quad ...(ii)$$

put $x = e$ in (ii)

$$\therefore\ ey\,(e) = \frac{1+\tan 1}{1-\tan 1}$$

15. Ans. (2)

$(x\cos x)dy + (xy\sin x + y\cos x - l)dx = 0,\ 0<x<\frac{\pi}{2}$

$$\frac{dy}{dx}+\left(\frac{x\sin x+\cos x}{x\cos x}\right)y=\frac{1}{x\cos x}$$

IF = $x\sec x$

$y.x\sec x=\int\frac{x\sec x}{x\cos x}dx=\tan x+c$

Since $y\left(\frac{\pi}{3}\right)=\frac{3\sqrt{3}}{\pi}$

Hence $c=\sqrt{3}$

Hence $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right)+y'\left(\frac{\pi}{6}\right)\right|=|-2|=2$

16. Ans. (12)

$\left(\log_e(\cos y)\right)^2\cos y dx-\left(1+3x\log_e(\cos y)\right)\sin y dy=0$

$\Rightarrow\frac{dx}{dy}-\frac{3\sin y}{\cos y(\log_e\cos y)}x=\frac{\sin y}{(\log_e\cos y)^2\cdot\cos y}$

I.F = $e^{\int\frac{-3\sin y}{\cos y(\log_e\cos y)}dy}$

Put $\ell n(\cos y)=t$

I.F = $e^{\int\frac{3}{t}dt}=(\ell n\cos y)^3$

$x.(\log_e\cos y)^3$ = $\int(\log_e\cos y)^3\cdot\frac{\sin y}{(\log_e\cos y)^2\times\cos y}dy$

$\Rightarrow x.(\log_e\cos y)^3$ = $\frac{-(\log_e\cos y)^2}{2}+c$

Given, $x\left(\frac{\pi}{3}\right)=\frac{1}{2\log_e 2}$

$\Rightarrow$ $c = 0$

$\Rightarrow x=\frac{-1}{2\ell n(\cos y)}$

$\Rightarrow x\left(\frac{\pi}{6}\right)=\frac{1}{\ell n4-\ell n3}$

$\Rightarrow$ $m = 4, n = 3$

Hence, $m.n = 12$

17. Ans. (1)

Differentiate the given equation

$\Rightarrow 2xf(x)+x^2f'(x)-1=4x\,f(x)$

$\Rightarrow x^2\frac{dy}{dx}-2xy=1$

$\Rightarrow\frac{dy}{dx}+\left(-\frac{2}{x}\right)y=\frac{1}{x^2}$

$I.F.=e^{\int-\frac{2}{x}\ell nx}=\frac{1}{x^2}$

$\therefore y\left(\frac{1}{x^2}\right)=\int\frac{1}{x^4}dx$

$$\Rightarrow \frac{y}{x^2} = \frac{-1}{3x^3} + c$$

$$\Rightarrow y = -\frac{1}{3x^3} + c$$

$$\Rightarrow y = -\frac{1}{3x} + cx^2$$

$$\because f(1) = \frac{2}{3} = -\frac{1}{3} + c \Rightarrow c = 1$$

$$\Rightarrow \quad f(x) = -\frac{1}{3x} + x^2$$

$$\Rightarrow \quad 18f(3) = 160$$

EXERCISE - JEE (Advanced) PYQ

1. **Ans. (D)**

$f'(x) - 2f(x) < 0$

$$\frac{d}{dx}(e^{-2x} f(x)) < 0$$

$\Rightarrow e^{-2x} f(x)$ is decreasing

$\Rightarrow x > 1/2$

$e^{-2x} f(x) < 1/e$

$\Rightarrow f(x) < e^{2x-1}$

$$\Rightarrow 0 < \int_{1/2}^{1} f(x)\,dx < \int_{1/2}^{1} \left(e^{2x-1}\right)dx \quad \Rightarrow \quad 0 < \int_{1/2}^{1} f(x)dx < \frac{e-1}{2}$$

2. **Ans. (A)**

Given slope at (x, y) is

$$\frac{dy}{dx} = \frac{y}{x} + \sec(y/x)$$

let $\frac{y}{x} = t \Rightarrow y = xt \quad \Rightarrow \quad \frac{dy}{dx} = t + x\frac{dt}{dx}$

$$t + x\frac{dt}{dx} = t + \sec(t)$$

$$\int \cos t\, dt = \int \frac{1}{x} dx$$

$\sin t = \ell n\, x + c$

$\sin(y/x) = \ell nx + c$

This curve passes through $(1, \pi/6)$

$\sin(\pi/6) = \ell n(1) + c \Rightarrow c = 1/2$

$\sin(y/x) = \ell nx + 1/2$

3. **Ans. (C)**

$e^{-x}\left(f''(x)-2f'(x)+f(x)\right)\geq 1$

$D\left(\left(f'(x)-f(x)\right)e^{-x}\right)\geq 1$

$\Rightarrow D((f'(x)-f(x)e^{-x})\geq 0$

$\Rightarrow (f'(x)-f(x))e^{-x}$ is an increasing function.

As we know that $e^{-x}f(x)$ has local minima at $x=\frac{1}{4}$

$e^{-x}(f'(x)-f(x))=0$ at $x=\frac{1}{4}$

Let $F(x)=e^{-x}(f'(x)-f(x))$

$F(x)<0$ in $\left(0,\frac{1}{4}\right)$

$e^{-x}(f'(x)-f(x)<0$ in $\left(0,\frac{1}{4}\right)$

$f'(x)<f(x)$ in $\left(0,\frac{1}{4}\right)$

4. **Ans. (D)**

$f''(x)-2f'(x)+f(x)\geq e^{x}$

$f''(x)\,e^{-x}-f'(x)\,e^{-x}-f'(x)\,e^{-x}+f(x)e^{-x}\geq 1$

$\frac{d}{dx}\left(f'(x)e^{-x}\right)-\frac{d}{dx}\left(f(x)e^{-x}\right)\geq 1$

$\frac{d}{dx}\left(f'(x)e^{-x}-f(x)e^{-x}\right)\geq 1$

$\Rightarrow\frac{d^2}{dx^2}\left(e^{-x}f(x)\right)\geq 1 \qquad \forall x\in[0,1]$

Let $\phi(x)=e^{-x}f(x)$

$\Rightarrow\phi(x)$ is concave upward

$f(0)=f(1)=0$

$\Rightarrow\phi(0)=0=\phi(1)$

$\Rightarrow\phi(x)<0\Rightarrow f(x)<0$

$\Rightarrow\phi'(x)<0, x\in(0,1/4)$ and $\phi'(x)>0, x\in(1/4,1)$

$\Rightarrow e^{-x}f'(x)-e^{-x}f(x)<0, x\in(0,1/4)$

$f'(x)<f(x), 0<x<1/4$

5. **Ans. (B)**

$\frac{dy}{dx}+\frac{x}{x^2-1}y=\frac{x^4+2x}{\sqrt{1-x^2}}$

This is a linear differential equation

I.F. = $e^{\int\frac{x}{x^2-1}dx}=e^{\frac{1}{2}In\left|x^2-1\right|}=\sqrt{1-x^2}$

$\Rightarrow$ solution is

$$y\sqrt{1-x^2}=\int\frac{x\left(x^3+2\right)}{\sqrt{1-x^2}}\sqrt{1-x^2}\,dx$$

or $y\sqrt{1-x^2}=\int\left(x^4+2x\right)dx=\frac{x^5}{5}+x^2+c$

$f(0)=0\Rightarrow c=0$

$\Rightarrow f(x)\sqrt{1-x^2}=\frac{x^5}{5}+x^2$

Now, $\int_{-\sqrt{3}/2}^{\sqrt{3}/2} f(x)dx=\int_{-\sqrt{3}/2}^{\sqrt{3}/2}\frac{x^2}{\sqrt{1-x^2}}dx$ (Using property)

$=2\int_0^{\sqrt{3}/2}\frac{x^2}{\sqrt{1-x^2}}dx=2\int_0^{\pi/3}\frac{\sin^2\theta}{\cos\theta}\cos\theta\,d\theta$(Taking $x=\sin\theta$)

$=2\int_0^{\pi/3}\sin^2\theta\,d\theta=2\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^{\pi/3}=2\left(\frac{\pi}{6}\right)-2\left(\frac{\sqrt{3}}{8}\right)=\frac{\pi}{3}-\frac{\sqrt{3}}{4}$

6. Ans. (A,C)

$\frac{dy}{dx}+\frac{ye^x}{1+e^x}=\frac{1}{e^x+1}$

I.F. $=e^{\int\frac{e^x}{1+e^x}dx}=e^{In\left(1+e^x\right)}=1+e^x$

$\Rightarrow y(1+e^x)=\int 1dx$

$y(1+e^x)=x+c$

$y=\frac{x+c}{1+e^x}$

$y(0)=2\Rightarrow c=4$

$\Rightarrow\ y=\frac{x+4}{1+e^x}$

$y=(-4)=0$

$\Rightarrow y'=\frac{\left(1+e^x\right)-\left(x+4\right)e^x}{\left(1+e^x\right)^2}=0$

Let $g(x)=\frac{\left(1+e^x\right)-\left(x+4\right)e^x}{\left(1+e^x\right)^2}$

$g(0)=\frac{2-4}{2^2}<0$

$g(-1)=\frac{\left(1+\frac{1}{e}\right)-\frac{3}{e}}{\left(1+\frac{1}{e}\right)^2}=\frac{1-\frac{2}{e}}{\left(1+\frac{1}{e}\right)^2}>0$

$g(0)\,g(-1)<0$ Hence $g(x)$ has a root in between $(-1, 0)$

7. **Ans. (B,C)**

Let Circle

$x^2 + y^2 - 2ax - 2ay + c = 0$

On differentiation

$2x + 2yy' - 2a - 2ay' = 0$

$\Rightarrow x + yy' - a(1 + y') = 0$

$\Rightarrow a = \dfrac{x + yy'}{1 + y'}$

again differentiation

$$\frac{(1+(y')^2 + yy'')(1+y') - (x+yy')(y'')}{(1+y')^2} = 0$$

$\Rightarrow 1 + y'((y')^2 + y' + 1) + y''(y - x) = 0$

$\therefore P = y - x$

$Q = 1 + y' + (y')^2$

8. **Ans. (A,D)**

$(x^2 + xy + 4x + 2y + 4)\dfrac{dy}{dx} - y^2 = 0$

$((x+2)^2 + y(x+2))\dfrac{dy}{dx} = y^2$

Let $x + 2 = x, y = y$

$(x)(x + y)\dfrac{dy}{dx} = y^2$

$-x^2\, d\ y = xydy - y^2\, d\, x \Rightarrow -x^2\, d\, y = y(xdy - ydx)$

$-\dfrac{dy}{y} = \dfrac{xdy - ydx}{x^2} \Rightarrow -\ell n|y| = \left(\dfrac{y}{x}\right) + C$

$-\ell n|y| = \dfrac{y}{x+2} + C$

$\because$ it is passing through $(1, 3)$

$-\ell n3 = 1 + C$

$C = -1 - \ell n3$

$\therefore$ curve is $\dfrac{y}{x+2} + \ell n|y| - 1 - \ell n3 = 0, x > 0$ ...(i)

put $y = x + 2$ in equation (i)

then $\dfrac{x+2}{x+2} + \ell n|x + 2| - 1 - \ell n3 = 0$

$x = 1, -5$ (reject) (as $x > 0$)

$\therefore$ curve intersect $y = x + 2$ at point $(1, 3)$

for option (C), put $y = (x + 2)^2$, we will get

$x + 2 + 2\ell n(x + 2) = 1 + \ell n3$

Clearly left-hand side is an increasing function

Hence, it is always greater than $2 + 2\ell n2$ therefore no solution

for option (D) put $y = (x + 3)^2$ in equation (i)

$\dfrac{(x+3)^2}{x+2} + \ell n(x+3)^2 - 1 - \ell n3 = 0$

$\dfrac{(x+3)^2}{x+2} + \ell n\dfrac{(x+3)^2}{3} - 1 = 0$

$\because x > 0 \Rightarrow x + 3 > x + 2$

and $x+3>3$

So $\frac{(x+3)^2}{x+2}+\ell n\frac{(x+3)^2}{3}>1$

$\therefore \frac{(x+3)^2}{x+2}+\ell n\frac{(x+3)^2}{3}-1=0$ has no solution

$\Rightarrow$ curve $y=(x+3)^2$ does not intersect with the given curve.

9. Ans. (A)

Let $y=f(x) \Rightarrow \frac{dy}{dx}+\frac{y}{x}=2$

(linear differential equation)

$\therefore y.e^{\int\frac{dx}{x}}=2\int e^{\int\frac{dx}{x}}dx$

$\Rightarrow yx=2\int xdx \qquad \therefore \quad yx=x^2+c$

$\Rightarrow f(x)=x+\frac{c}{x}$; As $f(1)\neq 1 \Rightarrow c\neq 0$

$\Rightarrow f'(x)=1-\frac{c}{x^2}, c\neq 0$

(A) $\lim\limits_{x\to 0^+} f'\left(\frac{1}{x}\right)=\lim\limits_{x\to 0^+}(1-cx^2)=1$

(B) $\lim\limits_{x\to 0^+} xf\left(\frac{1}{x}\right)=\lim\limits_{x\to 0^+} x\left(\frac{1}{x}+cx\right)=\lim\limits_{x\to 0^+}(1+cx^2)=1$

(C) $\lim\limits_{x\to 0^+} x^2f'(x)=\lim\limits_{x\to 0^+} x^2\left(1-\frac{c}{x^2}\right)=\lim\limits_{x\to 0^+}(x^2-c)=-c$

(D) $f(x)=x+\frac{c}{x}, c\neq 0$ for $c>0$

$\therefore$ for $c>0$, $\lim\limits_{x\to 0^+} f(x)=\infty$

$\Rightarrow$ function is not bounded in (0,2).

10. Ans. (B)

$dy=\frac{1}{8}\int\frac{dx}{\sqrt{4+\sqrt{9+x}}.\sqrt{x}.\sqrt{9+\sqrt{x}}}$

put $\sqrt{9+\sqrt{x}}=t \Rightarrow \frac{dx}{\sqrt{x}.\sqrt{9+\sqrt{x}}}=4dt$

$\therefore \quad y=\frac{4}{8}\int\frac{dt}{\sqrt{4+t}}$

$\Rightarrow y=\sqrt{4+t}+C$

$\Rightarrow y(x)=\sqrt{4+\sqrt{9+\sqrt{x}}}+C$

at $x=0$: $y(0)=\sqrt{7} \Rightarrow C=0$

$\therefore \quad y(x)=\sqrt{4+\sqrt{9+\sqrt{x}}}$

$\Rightarrow y(256)=3$

11. Ans. (A,C)

Given that,

$f'(x) > 2f(x) \ \forall\, x\in R$

$\Rightarrow f'(x)-2f(x)>0 \quad \forall\, x\in R$

$\therefore\ e^{-2x}\,(f'(x) - 2f(x)) > 0\ \forall\ x \in R$

$\Rightarrow \dfrac{d}{dx}\left(e^{-2x} f(x)\right) > 0\ \forall\ x \in R$

Let $g(x) = e^{-2x} f(x)$

Now, $g'(x) > 0\ \forall\ x \in R$

$\Rightarrow g(x)$ is strictly increasing $\forall\ x \in R$

Also, $g(0) = 1$

$\therefore\ \forall\ x > 0$

$g(x) > g(0) = 1$

$\therefore\ e^{-2x}.f(x) > 1\ \forall\ x \in (0,\infty) \Rightarrow f(x) > e^{2x}\ \forall\ x \in (0,\infty)$

$\therefore$ option (A) is correct

As, $f'(x) > 2f(x) > 2e^{2x} > 2\ \forall\ x \in (0,\infty)$

$\Rightarrow f(x)$ is strictly increasing on $x \in (0,\infty)$

$\Rightarrow$ option (C) is correct

As, we have proved above that

$f'(x) > 2.e^{2x}\ \forall\ x \in (0,\infty)$

$\Rightarrow$ option (D) is incorrect

$\therefore$ options (A) and (C) are correct.

12. **Ans. (B,C)**

$f'(x) = e^{(f(x)-g(x))}\, g'(x)\ \forall\ x \in \mathbb{R}$

$\Rightarrow e^{-f(x)}.f'(x) - e^{-g(x)} g'(x) = 0$

$\Rightarrow \int\left(e^{-f(x)} f'(x) - e^{-g(x)}.g'(x)\right)dx = C$

$\Rightarrow -e^{-f(x)} + e^{-g(x)} = C \Rightarrow -e^{-f(1)} + e^{-g(1)} = -e^{-f(2)} + e^{-g(2)}$

$\Rightarrow -\dfrac{1}{e} + e^{-g(1)} = -e^{-f(2)} + \dfrac{1}{e} \Rightarrow e^{-f(2)} + e^{-g(1)} = \dfrac{2}{e}$

$\therefore\ e^{-f(2)} < \dfrac{2}{e}$ and $e^{-g(1)} < \dfrac{2}{e}$

$\Rightarrow -f(2) < \ell n2 - 1$ and $-g(1) < \ell n2 - 1$

$\Rightarrow f(2) > 1 - \ell n2$ and $g(1) > 1 - \ell n2$

13. **Ans. (B,C,D)**

$\lim\limits_{t\to x} \dfrac{f(x)\sin t - f(t)\sin x}{t - x} = \sin^2 x$

by using L'Hopital

$\lim\limits_{t\to x} \dfrac{f(x)\cos t - f'(t)\sin x}{1} = \sin^2 x$

$\Rightarrow f(x)\cos x - f'(x)\sin x = \sin^2 x$

$\Rightarrow -\left(\dfrac{f'(x)\sin x - f(x)\cos x}{\sin^2 x}\right) = 1$

$\Rightarrow -d\left(\dfrac{f(x)}{\sin x}\right) = 1 \Rightarrow \dfrac{f(x)}{\sin x} = -x + c$

Put $x = \dfrac{\pi}{6}$ & $f\left(\dfrac{\pi}{6}\right) = -\dfrac{\pi}{12}$

$\therefore\ c = 0 \Rightarrow f(x) = -x\sin x$

(A) $f\left(\frac{\pi}{4}\right) = \frac{-\pi}{4}\frac{1}{\sqrt{2}}$

(B) $f(x) = -x\sin x$

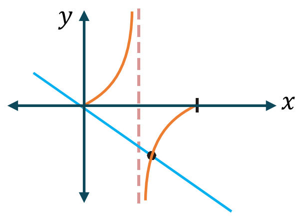
For $x > 0$ $\sin x > x - \frac{x^3}{6} \Rightarrow -\sin x < -x + \frac{x^3}{6} \Rightarrow -x\sin x < -x^2 + \frac{x^4}{6}$

$\therefore\ f(x) < -x^2 + \frac{x^4}{6}\ \forall x \in (0, \pi)$

(C) $f'(x) = -\sin x - x\cos x$

$f'(x) = 0 \Rightarrow \tan x = -x$

$\Rightarrow$ there exist $\alpha \in (0, \pi)$ for which $f'(\alpha) = 0$



(D) $f''(x) = -2\cos x + x\sin x$

$f''\left(\frac{\pi}{2}\right) = \frac{\pi}{2}, f\left(\frac{\pi}{2}\right) = -\frac{\pi}{2}$

$f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$

14. Ans. (0.4)

$\frac{dy}{dx} = 25y^2 - 4$

So, $\frac{dy}{25y^2 - 4} = dx$

Integrating, $\frac{1}{25} \times \frac{1}{2 \times \frac{2}{5}} \ln\left|\frac{y - \frac{2}{5}}{y + \frac{2}{5}}\right| = x + c$

$\Rightarrow \ell n\left|\frac{5y - 2}{5y + 2}\right| = 20(x + c)$

Now, $c = 0$ as $f(0) = 0$

Hence $\left|\frac{5y - 2}{5y + 2}\right| = e^{(20x)}$

$\lim_{x\to-\infty}\left|\frac{5f(x) - 2}{5f(x) + 2}\right| = \lim_{x\to-\infty} e^{(20x)}$

Now, RHS = 0 $\Rightarrow \lim_{x\to-\infty}(5f(x) - 2) = 0$

$\Rightarrow \lim_{x\to-\infty} f(x) = \frac{2}{5}$

15. Ans. (A,D)

$Y - y = y'(X - x)$

So, $Y_P = (0, y - xy')$

Now, $PY_P = \sqrt{x^2 + x^2(y')^2} = 1$

So, $x^2 + (xy')^2 = 1 \Rightarrow \frac{dy}{dx} = -\frac{\sqrt{1-x^2}}{x}$

[$\frac{dy}{dx}$ cannot be positive i.e. $f(x)$ can not be increasing in first quadrant, for $x \in (0, 1)$]

Hence, $\int dy = -\int \frac{\sqrt{1-x^2}}{x}dx$

$\Rightarrow y = -\int \frac{\cos^2\theta\, d\theta}{\sin\theta}$; put $x = \sin\theta$

$\Rightarrow y = -\int \text{cosec}\theta d\theta + \int \sin\theta\, d\theta$

$\Rightarrow y = \ell n(\text{cosec}\,\theta + \cot\theta) - \cos\theta + C$

$\Rightarrow y = \ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2} + C$

$\Rightarrow y = \ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$ (as $y(1) = 0$)

16. Ans. (A,C)

$f'(x) = \frac{f(x)}{b^2+x^2}$

$\int \frac{f'(x)}{f(x)}dx = \int \frac{dx}{x^2+b^2}$

$\Rightarrow \ell n|f(x)| = \frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right) + c$

Now $f(0) = 1$

$\therefore c = 0$

$\therefore |f(x)| = e^{\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)}$

$\Rightarrow f(x) = \pm e^{\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)}$

since $f(0) = 1 \therefore f(x) = e^{\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)}$

$x \to -x$

$f(-x) = e^{-\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)} \neq f(x)$

$\therefore f(x).f(-x) = e^0 = 1$ (option C)

$f(x) = e^{\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)} > 0 \Rightarrow f'(x) = \frac{f(x)}{b^2+x^2} > 0$

$\Rightarrow f(x)$ is increasing for all $x \in$ R (option A)

17. Ans. (A,C)

$\frac{dy}{dx} + \alpha y = xe^{\beta x}$

$d(e^{\alpha x}y) = xe^{\alpha x}\ e^{\beta x}$

$d(e^{\alpha x}y) = xe^{(\alpha+\beta)x}$

Case – I : $\alpha + \beta \neq 0$

$d(e^{\alpha x}y) = xe^{(\alpha+\beta)x}$

$e^{\alpha x}y = \frac{xe^{(\alpha+\beta)x}}{(\alpha+\beta)} - \frac{e^{(\alpha+\beta)x}}{(\alpha+\beta)^2} + C$

$y = \frac{xe^{\beta x}}{(\alpha+\beta)} - \frac{e^{\beta x}}{(\alpha+\beta)^2} + Ce^{-\alpha x}$

$\alpha = 1, \beta = 1 \Rightarrow y = \frac{xe^x}{2} - \frac{e^x}{4} + Ce^{-x}$

as $y(1) = 1 \Rightarrow C = e\left(1 - \frac{e}{4}\right)$

$y(x) = \frac{xe^x}{2} - \frac{e^x}{4} + \left(e - \frac{e^2}{4}\right)e^{-x}$

Case – II : $\alpha + \beta = 0$

$$\Rightarrow \frac{dy}{dx} - \beta y = xe^{\beta x}$$

$$d\left(e^{-\beta x} y\right) = x$$

$$e^{-\beta x} y = \frac{x^2}{2} + C$$

$$y = \frac{e^{\beta x} x^2}{2} + Ce^{\beta x}$$

$$y(1) = 1$$

$$\Rightarrow C = \left(1 - \frac{e}{2}\right)\frac{1}{e} \Rightarrow y = e^{\beta x}\frac{x^2}{2} + \left(1 - \frac{e}{2}\right)\frac{1}{e}e^{\beta x}$$

Take $\beta = -1 \Rightarrow y = \frac{x^2}{2}e^{-x} + \left(1 - \frac{e}{2}\right)e^{-x}$

18. Ans. (8)

$$xdy - (y^2 - 4y)dx = 0, x > 0$$

$$\int \frac{dy}{y^2 - 4y} = \int \frac{dx}{x}$$

$$\int \left(\frac{1}{y-4} - \frac{1}{y}\right)dy = 4\int \frac{dx}{x}$$

$$\log_e |y-4| - \log_e |y| = 4\log_e x + \log_e c$$

$$\frac{|y-4|}{|y|} = cx^4 \xrightarrow{(1,2)} c = 1$$

$$|y-4| = |y|x^4$$

Hence two curves are

$y - 4 = yx^4$ $\qquad$ $y - 4 = -yx^4$

$y = \frac{4}{1-x^4}$ $\qquad$ $y = \frac{4}{1+x^4}$

$y(1) =$ not defined (rejected) $\quad y(1) = 2$

19. Ans. (C)

$$\frac{dy}{dx} + 12y = \cos\left(\frac{\pi}{12}x\right)$$

Linear D.E.

I.F. $= e^{\int 12dx} = e^{12x}$

Solution of DE

$$y.e^{12x} = \int e^{12x}\cos\left(\frac{\pi}{12}x\right)dx$$

$$y.e^{12x} = \frac{e^{12x}}{(12)^2 + \left(\frac{\pi}{12}\right)^2}\left(12\cos\frac{\pi}{12}x + \frac{\pi}{12}\sin\frac{\pi}{12}x\right) + C$$

$$\Rightarrow y = \frac{(12)}{(12)^4 + \pi^2}\left((12)^2\cos\left(\frac{\pi x}{12}\right) + \pi\sin\left(\frac{\pi x}{12}\right)\right) + \frac{C}{e^{12x}}$$

Given $y(0) = 0$

$$\Rightarrow 0 = \frac{12}{12^4 + \pi^2}\left(12^2 + 0\right) + C \Rightarrow C = \frac{-12^3}{12^4 + \pi^2}$$

$$\therefore y = \frac{12}{12^4 + \pi^2}\left[(12)^2\cos\left(\frac{\pi x}{12}\right) + \pi\sin\left(\frac{\pi x}{12}\right) - 12^2 e^{-12x}\right]$$

$$\text{Now } \frac{dy}{dx} = \frac{12}{12^4 + \pi^2}\left[\underbrace{-12\pi\sin\left(\frac{\pi x}{12}\right) + \frac{\pi^2}{12}\cos\left(\frac{\pi x}{12}\right) + 12^3 e^{-12x}}_{\min.value}\right]$$

$$\left(-\sqrt{144\pi^2 + \frac{\pi^4}{144}} = -12\pi\sqrt{1 + \frac{\pi^2}{12^4}}\right)$$

$\Rightarrow \frac{dy}{dx} > 0 \; \forall \; x \le 0$ & may be negative/positive for $x > 0$

So, $f(x)$ is neither increasing nor decreasing

For some $\beta \in R, y = \beta$ intersects $y = f(x)$ at infinitely many points

20. Ans. (C)

Diff. wr.t 'x'

$3f(x) = f(x) + xf'(x) - x^2$

$$\frac{dy}{dx} - \left(\frac{2}{x}\right)y = x$$

$$\text{IF} = e^{-2\ell nx} = \frac{1}{x^2}$$

$$y\left(\frac{1}{x^2}\right) = \int x.\frac{1}{x^2}dx$$

$y = x^2\ell nx + cx^2$

$$\therefore y(1) = \frac{1}{3} \Rightarrow c = \frac{1}{3}$$

$$y(e) = \frac{4e^2}{3}$$

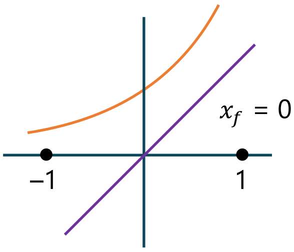
21. **Ans. (A,B,C)**

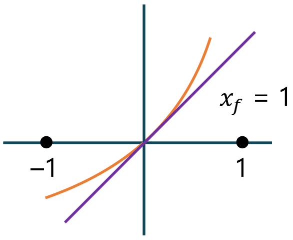
S = Set of all twice differentiable functions $f : R \to R$

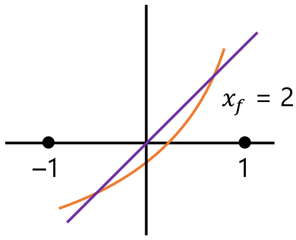
$\frac{d^2 f}{dx^2} > 0$ in $(-1, 1)$

Graph 'f' is Concave upward.

Number of solutions of $f(x) = x \to x_f$

(1)


(2)


(3)


$\Rightarrow$ Graph of $y = f(x)$ can intersect graph of $y = x$ at atmost two points $\Rightarrow 0 \le x_f \le 2$

Aliter

$\frac{d^2 f(x)}{dx^2} > 0$

Let $\phi(x) = f(x) - x$

$\phi''(x) > 0$

$\therefore$ $\phi'(x) = 0$ has atmost 1 root in $x \in (-1, 1)$

$\therefore$ $\phi(x) = 0$ has atmost 2 roots in $x \in (-1, 1)$

$\therefore$ $x_f \le 2$

22. **Ans. (16)**

$$\frac{dy}{dx} - \frac{2x}{x^2-5} y = -2x(x^2-5)$$

$$\text{IF} = e^{-\int \frac{2x}{x^2-5} dx} = \frac{1}{(x^2-5)}$$

$$y.\frac{1}{x^2-5} = \int -2x.dx + c$$

$$\Rightarrow \frac{y}{x^2-5} = -x^2 + c$$

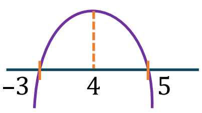
$x = 2$, $y = 7$

$$\frac{7}{-1} = -4 + c \Rightarrow c = -3$$

$y = -(x^2 - 5)(x^2 + 3)$

put $x^2 = t > 0$

$y = -(t - 5)(t + 3)$



y_{max} 16 when $x^2 = 1$

y_{max} 16

JEE (Main) Practice Paper

SECTION–A

1. **Ans. (2)**

$y = c_1 e^{c_2 X}$ ...(1)

$y' = c_1 c_2 e^{c_2 x}$ ...(2)

$y'' = c_1 c_2^2 e^{c_2 x}$

$y'' = c_2 y'$ ...(3)

Now $\frac{(2)}{(1)}$

$\frac{y'}{y} = c_2$

$\Rightarrow$ Put in (3)

$y'' = \frac{y'}{y}.y' \Rightarrow y''y = (y')^2$

2. **Ans. (4)**

$\frac{dV}{dt} = -k(T-t)$

$dV = \int -K(T-t)dt$

$\int V = -K\left[Tt - \frac{t^2}{2}\right] + C$

At $t = 0$ $V = I \Rightarrow C = I$

$V = -Kt\left(T - \frac{t}{2}\right) + I$

$V(T) = -KT\left(T - \frac{T}{2}\right) + I$

$= \frac{-KT^2}{2} + I$

3. **Ans. (3)**

Equation of tangent at (x_1, y_1) is –

$y - y_1 = \frac{dy_1}{dx_1}(x - x_1)$

$x-$ intercept = $x_1 - y_1\frac{dx_1}{dy_1}$

According to question

$x_1 = \frac{x_1 - y_1\frac{dx_1}{dy_1}}{2}$

$\Rightarrow x_1 = -y_1\frac{dx_1}{dy_1}$

$$\int \frac{dy}{y} = \int -\frac{dx}{x}$$

$\Rightarrow \ell ny = -\ \ell nx + \ell nc$

$\Rightarrow y = \frac{c}{x} \Rightarrow xy = c$

Now at $x = 2, y = 3$

$\Rightarrow c = 6$

$\therefore\ xy = 6 \Rightarrow y = \frac{6}{x}$

4. **Ans. (4)**

$y^2dx + \left(x - \frac{1}{y}\right)dy = 0$

$\Rightarrow y^2\frac{dx}{dy} + x = \frac{1}{y}$

$\Rightarrow \frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$

$\therefore$ Integrating factor (I.F.) = $e^{\int \frac{1}{y^2}dy} = e^{-\frac{1}{y}}$

$\therefore$ General solution is –

$x\ .\ e^{-1/y} = \int \frac{1}{y^3}e^{-1/y}dy\ +\ c$

Let $I_1 = \int \frac{1}{y^3}e^{-1/y}dy$

Put $\frac{-1}{y} = t$

$y^{-2}dy = dt$

$\therefore\ I_1 = -\int te^t dt$

$= -e^t(t-1)$

$= e^t(1-t)$

$\therefore$ General solution is

$xe^{-\frac{1}{y}} = e^{-\frac{1}{y}}\left(1 + \frac{1}{y}\right) + C$

$\Rightarrow x = 1 + \frac{1}{y} + Ce^{\frac{1}{y}}$

Put $x = 1, y = 1$

$\therefore\ 1 = 1 + \frac{1}{1} + Ce^{\frac{1}{1}}$

$\Rightarrow C = -1/e$

$\therefore\ x = 1 + \frac{1}{y} - \frac{e^{1/y}}{e}$

5. Ans. (2)

$\frac{dP(t)}{dt} = \frac{1}{2}P(t) - 450$

integrate

$\int \frac{dP}{P-900} = \int \frac{1}{2} dt$

$\ell n|(P - 900)| = \frac{1}{2}t + C$ ...(1)

given $t = 0 \to P = 850$

$\therefore C = \ell n\ 50$

from (1)

$ln|(P - 900)| = \frac{1}{2}t + \ell n 50$

$\frac{1}{2}t = \ell n\left|\left(\frac{P-900}{50}\right)\right|$

$t = 2\ \ell n\left|\left(\frac{P-900}{50}\right)\right|$

At $P = 0$

$t = 2\ell n \frac{900}{50}$

$t = 2\ell n 18$

6. Ans. (4)

$V = \frac{4}{3}\pi r^3$

Initially $r = 4500\ \pi$, $r = r_0$

$4500\ \pi = \frac{4}{3}\ \pi\ r_0^3 \Rightarrow \boxed{r_0 = 15m}$

Now $\frac{dV}{dt} = \frac{4}{3}\ \pi\ (3r^2)\ \frac{dr}{dt}$

$-72\pi = 4\pi r^2\ \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{-18}{r^2}$ ...(i)

$\int r^2 dr = -\int 18\ dt \Rightarrow \frac{r^3}{3} = -18t + C$

At $t = 0,\ r = 15m$

So, $\frac{(15)^3}{3} = -18(0) + C \Rightarrow C = 1125$

$\Rightarrow r^3 = -54t + 3375$ ...(ii)

At time $t = 49$ min $r = 9$ m

from eq. (i)

$\left(\frac{dr}{dt}\right)_{t=49} = \frac{-18}{(9)^2} = -2/9$

(Negative sign shows decrement in radii)

7. Ans. (3)

Let equation of St. Line

$Y - y = m(X - x)$

Distance from origin $\Rightarrow \left|\dfrac{mx-y}{\sqrt{1+m^2}}\right| = 1$

$\therefore (mx - y)^2 = 1 + m^2$

$\left(y - \dfrac{dy}{dx}x\right)^2 = 1 + \left(\dfrac{dy}{dx}\right)^2$

8. Ans. (4)

$(x-0)^2 + (y-\lambda)^2 = \lambda^2$

$x^2 + y^2 - 2\lambda y = 0$

$2x + 2y\dfrac{dy}{dx} - 2\lambda\dfrac{dy}{dx} = 0$

$\lambda = \dfrac{x + y\dfrac{dy}{dx}}{\dfrac{dy}{dx}}$

so differential equation is $\dfrac{dy}{dx}(x^2 + y^2) = 2y\left(x + y\dfrac{dy}{dx}\right)$

$\dfrac{dy}{dx}(x^2 - y^2) = 2xy$

so $g(x) = 2x$

9. Ans. (2)

$\phi(x) = \phi'(x)$ $\qquad\qquad \phi(1) = 2$

$\dfrac{d\phi}{dx} = \phi(x)$

$\ell n\, \phi(x) = x + c$

$\ell n\, 2 = 1 + c \Rightarrow c = \ell n 2 - 1$

$\ell n \phi(3) = 3 + c = 2 + \ell n 2$

$\Rightarrow \phi(3) = 2e^2$

10. Ans. (2)

$\dfrac{dy}{dx} = 1 + x + y + xy = (1 + x)(1 + y)$

$\Rightarrow \displaystyle\int \dfrac{dy}{1+y} = \int (1+x)\, dx$

$\Rightarrow \ell n(1 + y) = x + \dfrac{x^2}{2} + c$

$y(-1) = 0 \Rightarrow c = \dfrac{1}{2}$

$\ell n(1 + y) = x + \dfrac{x^2}{2} + \dfrac{1}{2} = \dfrac{(1+x)^2}{2}$

$\Rightarrow y = e^{\frac{(1+x)^2}{2}} - 1$

11. **Ans. (3)**

$\frac{dy}{dx} - ky = 0,\ \frac{dy}{y} = kdx$

$\ell ny = kx + c$

at $x = 0, y = 1$ $\therefore c = 0$

Now $\ell ny = kx$

$y = e^{kx}$

$\lim\limits_{x \to \infty} y = \lim\limits_{x \to \infty} e^{kx} = 0$

$\therefore k < 0$

12. **Ans. (3)**

$ydy + \sqrt{1+y^2}\,dx = 0$

$\frac{y}{\sqrt{1+y^2}}dy + dx = 0$

$\int \frac{y}{\sqrt{1+y^2}}dy + \int dx = 0$

$\Rightarrow \sqrt{1+y^2} + x = c \Rightarrow (c-x)^2 = (1+y^2)$

$\Rightarrow (x-c)^2 - y^2 = 1$

hyperbola

13. **Ans. (1)**

$\int \frac{dx}{x} = \int \frac{y}{1+y^2}dy$

$\ell nx = \frac{1}{2}\ell n(1+y^2) + c$

$\ell n\left(\frac{x^2}{1+y^2}\right) = k_1$

$\Rightarrow x^2 = k_2(1+y^2)$

at $(1, 0)$ $k_2 = 1$

$\Rightarrow x^2 = 1 + y^2$

14. **Ans. (4)**

$\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right) \Rightarrow \frac{dy}{dx} = -2\cos\frac{x}{2}\sin\frac{y}{2}$

$\Rightarrow \int \text{cosec}\frac{y}{2}dy = \int -2\cos\frac{x}{2}dx \Rightarrow 2\,\ell n\left(\tan\frac{y}{4}\right) = -4\sin\frac{x}{2} + c'$

$\Rightarrow \ell n\left(\tan\frac{y}{4}\right) + 2\sin\frac{x}{2} = c$

15. Ans. (1)

$$\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$$

$$x + y = u$$

$$1 + \frac{dy}{dx} = \frac{du}{dx}$$

$$\frac{du}{dx} - 1 = \sin u + \cos u$$

$$\int \frac{du}{1+\sin u+\cos u} = \int dx$$

$$\Rightarrow \ell n\left| \tan\left(\frac{x+y}{2}\right) + 1 \right| = x + c$$

16. Ans. (2)

$$3y + 2x = v \qquad \therefore\ 3\frac{dy}{dx} + 2 = \frac{dv}{dx}$$

$$\left(\frac{dv}{dx} - 2\right) = \frac{2v+5}{v+4}$$

$$\frac{dv}{dx} = \frac{6v+15}{v+4} + 2 = \frac{8v+23}{v+4}$$

$$\frac{8v+32}{8v+23}\, dv = 8dx$$

$$\int\left(1+\frac{9}{8v+23}\right) dv = \int\left(1+\frac{9}{8v+23}\right)$$

$$v + \frac{9}{8}\ell n(8v+23) = 8x + c$$

$$y - 2x + \frac{3}{8}\ell n(24y + 16x + 23) = k$$

17. Ans. (4)

$$\frac{dy}{dx} = \frac{y^2 - 2xy - x^2}{y^2 + 2xy - x^2}$$

Put $y = tx$

$$\frac{dy}{dx} = t + x\frac{dt}{dx}$$

$$t + \frac{xdt}{dx} = \frac{t^2 - 2t - 1}{t^2 + 2t - 1}$$

$$\frac{xdt}{dx} = \frac{t^2 - 2t - 1 - t^3 - 2t^2 + t}{t^2 + 2t - 1} \Rightarrow \int \frac{t^2 + 2t - 1}{-t^3 - t^2 - t - 1}\, dt = \int \frac{dx}{x}$$

$$\int \frac{t^2 + 2t - 1}{(t+1)\ (t^2+1)}\, dt = -\ell nx + c$$

$$= \int\left(\frac{2t}{t^2+1} - \frac{1}{t+1}\right) dt = -\ell nx + c$$

$$\ell n\frac{t^2+1}{t+1} = c - \ell nx$$

$$\frac{y^2+x^2}{y+x}=\frac{1}{k}$$

$x + y = k(x^2 + y^2)$

$k = 0$

$\therefore x + y = 0$

18. **Ans. (3)**

$$\frac{dy}{dx} = \frac{\cos\frac{y}{x}+\frac{y}{x}\sin\frac{y}{x}}{\sin\frac{y}{x}-\frac{x}{y}\cos\frac{y}{x}}$$

put $y = tx$

$$t + x\frac{dt}{dx} = \frac{\cos t+t\sin t}{\sin t-\frac{\cos t}{t}} \Rightarrow \int\frac{t\sin t-\cos t}{2t\cos t}dt = \int\frac{dx}{x}$$

$$-\frac{1}{2}\ell n(t \cos t) = \ell nx + c$$

$$-\frac{1}{2}\ell n\left(\frac{y}{x}\cos\frac{y}{x}\right) = \ell n\, x + c$$

19. **Ans. (1)**

$$\frac{dv}{dt}+\frac{k}{m}v = -g$$

Integrating factor (I.F.) = $e^{\int\frac{k}{m}dt} = e^{\frac{k}{m}t}$

$$\therefore v\, e^{\frac{k}{m}t} = -\int g\, e^{k\cdot t/m}\, dt$$

$$v\, e^{\frac{k}{m}t} = \frac{-gm}{k}e^{\frac{k}{m}t} + c$$

$$v = c\cdot e^{\frac{-k}{m}t} - \frac{mg}{k}$$

20. **Ans. (1)**

$$\frac{dy}{dx} = y\tan x - 2\sin x$$

$$\frac{dy}{dx} - y\tan x = -2\sin x$$

I.F. = $e^{-\int\tan x\, dx} = |\cos x|$

$$y\cos x = \frac{\cos 2x}{2} + k$$

$$\Rightarrow y = \frac{\cos 2x}{2\cos x} + k\sec x$$

$$\Rightarrow y = \cos x + c\sec x$$

SECTION-B

1. Ans. (0)

$(x+2)\dfrac{dy}{dx} = (x+2)^2 - 13$ $\dfrac{dy}{dx} = (x+2) - \dfrac{13}{x+2}$

$y = \dfrac{x^2}{2} + 2x - 13\,\ell n(x+2) + C$ at $x = 0, y = 0 \Rightarrow c = 13\,\ell n2$

$y = \dfrac{x^2}{2} + 2x - 13\,\ell n|x+2| + 13\,\ell n2$

Now $y(-4) = 8 - 8 - 13\ell n|-4+2| + 13\ell n2 = 0$

2. Ans. (1)

$\dfrac{dy}{dx} = y - y^2 \Rightarrow \int \dfrac{dy}{y-y^2} = \int dx$

$\int \dfrac{1}{1-y} + \dfrac{1}{y}dy = x + c \Rightarrow \ell n\dfrac{y}{1-y} = x + c$

$\dfrac{y}{1-y} = ke^x \Rightarrow y = ke^x - kye^x \Rightarrow y = \dfrac{ke^x}{1+ke^x}$

$x = 0, y = 2;\ 2 = \dfrac{k}{1+k} \Rightarrow 2 + 2k = k$

$\Rightarrow k = -2, y = \dfrac{-2e^x}{1-2e^x} \Rightarrow y = \dfrac{-2}{e^{-x}-2}$

$\lim\limits_{x\to\infty}(y(x)) = \lim\limits_{x\to\infty}\dfrac{-2}{e^{-x}-2} = 1$

3. Ans. (16)

$ydx = (x+y^2)\,dy$

$\dfrac{dx}{dy} - \dfrac{x}{y} = y$

I.F. $= e^{\int -\frac{1}{y}dy} = e^{-\ell n\, y} = \dfrac{1}{y}$

so $x\left(\dfrac{1}{y}\right) = \int y\left(\dfrac{1}{y}\right)dy$

$\Rightarrow \dfrac{x}{y} = y + c$

Put $y = 1, x = 1 \Rightarrow c = 0$

$\Rightarrow x = y^2$

Now at $y = 4, x = 4^2 = 16$

4. Ans. (5)

$\dfrac{dy}{dx} + y\tan x = \sin 2x$

IF $= e^{\int \tan x dx} = e^{\ell n|\sec x|} = |\sec x|$

$\therefore$ solution is

$y(\sec x) = \int \sec x.\sin 2x dx$

$y\sec x = \int 2\sin x dx$

$y\sec x = -2\cos x + c$

At $x = 0, y = 1$

$1 = -2 + c$

$c = 3$

So $y\sec x = -2\cos x + 3$

$y = -2\cos^2 x + 3\cos x$

at $x = \pi$

$y = -2 - 3 = -5$

5. **Ans. (3)**

$\frac{ydx - xdy}{y^2} = 2dy \Rightarrow d\left(\frac{x}{y}\right) = 2dy$

$\frac{x}{y} = 2y + c \Rightarrow c = 1 \Rightarrow \frac{x}{y} = 2y + 1$

put $y = 1$

$f(1) = 3$

6. **Ans. (2)**

$\frac{dy}{dx} = \frac{ax+3}{2y+1}$

i.e. $(2y + 1)\, dy = (ax + 3)\, dx$

$\therefore y^2 + y = \frac{ax^2}{2} + 3x + c$

$\therefore a = -2$

7. **Ans. (1)**

$(x^x)^2 - 2\cot y\, x^x - 1 = 0$

$x^x = \frac{2\cot y \pm \sqrt{4\cot^2 y + 4}}{2}$

$= \cot y \pm \operatorname{cosec} y$

$x^x = \cot y + \operatorname{cosec} y$

$\left\{\begin{array}{l} at\ x = 1, \\ 1 = \cot y + \operatorname{cosec} y \\ \Rightarrow y = \frac{\pi}{2} \end{array}\right.$

diff. w.r. to x

$x^x(1 + \log x) = [-\operatorname{cosec}^2 y - \operatorname{cosec} y \cot y]\frac{dy}{dx}$

$1 = -\operatorname{cosec} y\,[\operatorname{cosec} y + \cot y]\frac{dy}{dx}$

$\frac{dy}{dx} = -1$

8. **Ans. (7)**

$\frac{dy}{dx} = y + 3 > 0 \quad y(0) = 2, y(\log 2) = ?$

$\int \frac{dy}{y+3} = \int dx$

$\log|y + 3| = x + c$

$y(0) = 2$

$\log|2 + 3| = 0 + c \Rightarrow c = \log 5.$

$y.(\log 2) = ?$

$\log|y + 3| = \log 2 + \log 5$

$\log|y + 3| = \log 10$

$y + 3 = 10$

$y = 7$

9. Ans. (2)

$\frac{dy}{dx} + 3y = 2 \Rightarrow \int \frac{dy}{2-3y} = \int dx$

$\Rightarrow \frac{-\ell n(2-3y)}{3} = x + c \Rightarrow \ell n(2-3y) = -3x - c$

$\Rightarrow 2 - 3y = e^{-3x} . e^{-c} \quad \Rightarrow y = \frac{2-e^{-3x} . \ e^{-c}}{3}$

$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{2-e^{-3x} . \ e^{-c}}{3} = \frac{2}{3}$

10. Ans. (4)

$y\ell n|cx| = x$

$y(\ell n|c| + \ell n|x|) = x$

$yk + y\ell n|x| = x$ ...(i)

$k\frac{dy}{dx} + \frac{dy}{dx}\ell n|x| + \frac{y}{x} = 1$

$k = \frac{1-\frac{y}{x}-\ell n|x|\frac{dy}{dx}}{\frac{dy}{dx}}$ ...(ii)

Now $y.\left(\frac{1-\frac{y}{x}-\ell n|x|\frac{dy}{dx}}{\frac{dy}{dx}}\right) + y\ell n|x| = x$

$y - \frac{y^2}{x} - y\ell n|x|\frac{dy}{dx} + y\ell n|x|\frac{dy}{dx} = x\frac{dy}{dx}$

$x\frac{dy}{dx} = y - \frac{y^2}{x}$

$\frac{dy}{dx} = \frac{y}{x} - \frac{y^2}{x^2}$

so $\phi\left(\frac{x}{y}\right) = -\left(\frac{x}{y}\right)^{-2}$

$\phi(2) = -(2)^{-2} = -\frac{1}{4}$

JEE (Advanced) Practice Paper

1. Ans. (A)

$(x^3 \cos y \sin^2 y - 2y \sin x)\, dy - (y^2 \cos x - x^2 \sin^3 y)\, dx = 0$

$\left(\frac{x^3}{3} d \sin^3 y - \sin x\, dy^2\right) + \sin^3 y d\left(\frac{x^3}{3}\right) - y^2 d \sin x = 0$

$\frac{x^3}{3}\, d \sin^3 y + \sin^3 y d\left(\frac{x^3}{3}\right) - (\sin x\, dy^2 + y^2\, d \sin x)$

$d\left(\frac{x^3}{3} \sin^3 y\right) - d(y^2 \sin x) = 0$

$\frac{x^3}{3} \sin^3 y - y^2 \sin x = c$

2. Ans. (B)

By rearranging the terms we have

$x^{m-1} y^{n-1}\ (my\, dx + nx\, dy) = \frac{xdy + ydx}{x^2 y^2}$

$\Rightarrow d\left(x^m y^n\right) = d\ -\frac{1}{xy} + c \Rightarrow x^m y^n = -\frac{1}{xy} + c$

$\Rightarrow x^{m+1}.y^{n+1} + 1 = cxy$.

3. Ans. (D)

$f'(x) - 3f(x) > 0$

$\Rightarrow \frac{d}{dx}(e^{-3x}.f(x)) > 0\ \forall\ x \geq 0$

$\Rightarrow e^{-3x}\ f(x)) \geq f(0)\ \forall\ x \geq 0$

$\Rightarrow f(x)) \geq e^{3x}\ \forall\ x \geq 0$

4. Ans. (B)

$PA = 2$

$x\ \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ \sec\theta = 2$

$x = 2$

$\left(\frac{dy}{dx}\right)^2 + 1 = \frac{4}{x^2}$

$\frac{dy}{dx} = \pm\sqrt{\frac{4 - x^2}{x^2}}$

$\int dy = \pm\int \frac{\sqrt{4 - x^2}}{x}\, dx$

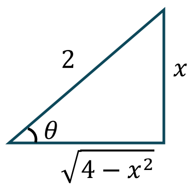
Put $x = 2\sin\theta$

$\int \frac{2\cos\theta}{2\sin\theta} \times 2\cos\theta\ d\theta = 2\int \frac{1 - \sin^2\theta}{\sin\theta} d\theta$

$= 2\int (\operatorname{cosec}\theta - \sin\theta)\, d\theta$

$= 2\log|\operatorname{cosec}\theta - \cot\theta| + 2\cos\theta + c$

$y = 2\log\left|\frac{2}{x} - \frac{\sqrt{4 - x^2}}{x}\right| + \sqrt{4 - x^2} + c$



5. Ans. (A)

$xdy + ydx = ydy - xdx$

$\Rightarrow d(xy) = ydy - xdx$

$\Rightarrow 2xy = y^2 - x^2 + c$ ...(1)

Put $x = 0$, $y = 0$, We get $c = 0$

$\therefore$ (1) $\Rightarrow y^2 - 2xy - x^2 = 0$

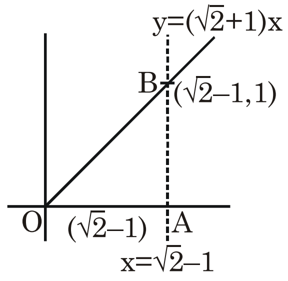
$\Rightarrow y = 2x \pm \sqrt{4x^2 + 4x^2}$

$y = x + \sqrt{2}|x|$

or $y = x - \sqrt{2}|x|$

but $y(1) = \tan\frac{3\pi}{8} \Rightarrow y = x + \sqrt{2}|x|$

$\therefore$ Required area = $\frac{1}{2}\left(\sqrt{2}-1\right) = \frac{1}{2}\left(\sqrt{2}-1\right)$



6. Ans. (A)

Let $y^2 = t \Rightarrow 2y\frac{dy}{dx} = \frac{dt}{dx}$

$2\frac{dt}{dx} = \frac{1}{x(x^2\sin t + 1)}$

$\frac{dx}{dt} = \frac{x^3 \sin t}{2} + \frac{x}{2}$

$x^{-3}\frac{dx}{dt} = \frac{\sin t}{2} + \frac{x^{-2}}{2}$

Let $x^{-2} = z \Rightarrow x^{-3}\frac{dx}{dt} = -\frac{dz}{2dt}$

$-\frac{1}{2}\frac{dz}{dt} = \frac{\sin t}{2} + \frac{z}{2}$

$\frac{dz}{dt} = -\sin t - z$

$\frac{dz}{dt} + z = -\sin t$

$ze^t = -\int e^t \sin t\, dt$

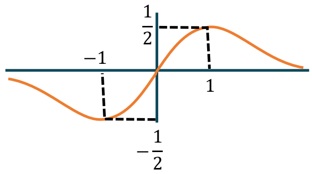
$ze^t = -\frac{(-e^t\cos t + e^t \sin t)}{2} + C$

$\frac{e^{y^2}}{x^2} = e^t\frac{(\cos t - \sin t)}{2} + C$

$e^{y^2}\left(\frac{1}{x^2} - \frac{\cos t}{2} + \frac{\sin t}{2}\right) \pm C$

$e^{y^2}\left(\frac{1}{x^2} - \frac{\cos y^2}{2} + \frac{\sin y^2}{2}\right) = C$

7. Ans. (A,B,D)



$d((1+x^2)y) = 1$

$(1+x^2)y = x + C$

$\Rightarrow y = \frac{x}{1+x^2}\{\because f(0)=0\}$

8. Ans. (A,B,C,D)

$\frac{dy}{dx} = \frac{1}{y} \Rightarrow ydy = dx \Rightarrow y^2 = 2x + c$

It passes through (2, 2) $\Rightarrow c = 0$

$y^2 = 2x$ is a parabola with focus $\left(\frac{1}{2},0\right)$ and latus rectum 2

(8, –4) satisfies the curve

$\frac{y^2}{x} = 2$, Hence $\lim_{x\to\infty}\frac{y^2}{x} = 2$

9. Ans. (A,C)

Slope of tangent $= -\frac{y}{2}$

$\frac{dy}{dx} = -\frac{y}{2} \Rightarrow y = 2e^{-\frac{x}{2}}$

10. Ans. (C)

Given DE can be written as $\frac{dy}{dx} - \left(1+\frac{f'(x)}{f(x)}\right)y = f(x)$

Which is linear differential equation

I.F. = $e^{-x-\ell n f(x)} = \frac{e^{-x}}{f(x)}$

General solution $y\frac{e^{-x}}{f(x)} = \int f(x)\frac{e^{-x}}{f(x)}dx + c = -e^{-x} + c$

$\Rightarrow y = -f(x) + ce^x f(x)$

11. Ans. (A,D)

$f(x) = \int_0^x f(t)\cos t - \int_0^x \cos(t-x)dt$

$f(x) = \int_0^x f(t)\cos t - \int_0^x \cos t\ dt$

$f'(x) = f(x)\cos x - \cos x$

$\Rightarrow \frac{dy}{dx} = y\cos x - \cos x$ $\quad (y = f(x))$

$\Rightarrow \frac{dy}{dx} - y\cos x = -\cos x$

I.F. $= e^{\int -\cos x\,dx} = e^{-\sin x}$

$\Rightarrow y.e^{-\sin x} = \int -e^{-\sin x}.\cos x\,dx$

$y.e^{-\sin x} = e^{-\sin x} + c$

$\Rightarrow y = ce^{\sin x} + 1$

$y = 0$ when $x = 0$

$\Rightarrow c = -1 \Rightarrow f(x) = 1 - e^{\sin x}$

$f'(0) = f(0) - 1 = -1$

$f''(x) = f(x)(-\sin x) + \cos x . f'(x) + \sin x$

$f''\left(\frac{\pi}{2}\right) = -f\left(\frac{\pi}{2}\right) + 1 = -1 + e + 1 = e$

12. **Ans. (C)**

13. **Ans. (A)**

Sol. for Q. 12 to 13

$$Y - y = \frac{-1}{\frac{dy}{dx}}(X - x)$$

$$\therefore\ x + Y\frac{dy}{dx} - \left(y.\frac{dy}{dx} + x\right) = 0$$

According to given condition

$$\left|\frac{0 - \left(y\frac{dy}{dx} + x\right)}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}\right| = y$$

$$\therefore\ \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

$2xydy - y^2dx = -x^2.dx$

$$d\left(\frac{y^2}{x}\right) = -dx$$

$$\frac{y^2}{x} = -x + C$$

It passes through (1, 1)

Hence curve is $C : x^2 + y^2 - 2x = 0$

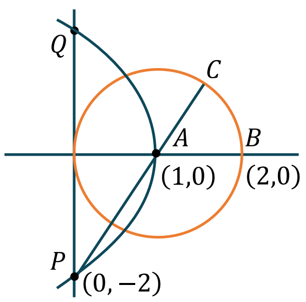
Curve C is $(x - 1)^2 + y^2 = 1$

and $x^2 - 3y^2 - 2x + 1 = 0$ represents

lines $x - \sqrt{3}y - 1 = 0$ and $x + \sqrt{3}y - 1 = 0$

passing through centre of circle (1, 0)

Hence area of smaller region $\frac{1}{2}r^2\theta = \frac{1}{2}\left(\frac{\pi}{3}\right) = \frac{\pi}{6}$ **Ans. (C)**



Maximum distance is along the line through P passing from centre 'A' and is equal to $\sqrt{5} + 1$

Ans. (A)

14. Ans. (C)

$$\frac{dy}{dx} - y = -e^x \tan^2 x \text{ (This is LDE)}$$

IF = e^{-x}

on solving, $f(x) = e^x(x - \tan x)$.

15. Ans. (C)

(P) $2x^4 y dy + y^2 dy = x^2 dx - 4x^3 y^2 dx$

$x^4(2y)dy + y^2(4x^3 dx) + y^2 dy - x^2 dx = 0$

$$d(x^4.y^2) + \frac{1}{3}d(y^3) - \frac{1}{3}d(x^3) = 0$$

$$x^4 y^2 + \frac{y^3}{3} - \frac{x^3}{3} = c$$

$$\boxed{3x^4 y^2 + y^3 - x^3 = 3c}$$

(Q) Apply componendo & dividendo

$$\frac{e^x}{e^{-x}} = \frac{dx}{dy} \Rightarrow dy = e^{-2x} dx$$

$$y = \frac{-e^{-2x}}{2} + c$$

$2ye^{2x} = -1 + 2ce^{2x}$

(R) $$y = \frac{-e^{-2xx=1+xy\frac{dy}{dx}+\frac{\left(xy\frac{dy}{dx}\right)^2}{\lfloor 2}+.....}}{2} + c$$

$$x = e^{xy\frac{dy}{dx}}$$

$$\log_e x = xy\frac{dy}{dx}$$

$$\int \frac{\log x}{x} dx = \int y dy$$

$$\frac{\log^2 x}{2} = \frac{y^2}{2} + c$$

$y^2 = \log^2 x + c$

(S) $e^{x^2} y dy + e^{y^2} y dy + e^{x^2} x(y^2 - 1) dx = 0$

$$e^{x^2} 2y dy + (y^2 - 1)e^{x^2} 2x dx + e^{y^2} 2y dy = 0$$

$$d\left(e^{x^2}(y^2 - 1)\right) + d\left(e^{y^2}\right) = 0$$

$$e^{x^2}(y^2 - 1) + e^{y^2} = c$$

16. Ans. (8)

$\cos^2 x \dfrac{dy}{dx} - (\tan 2x)\, y = \cos^4 x$

$R = e^{\int -\frac{2\tan x}{1-\tan^2 x}\sec^2 x\, dx} = (1 - \tan^2 x)$

After solving differential equation $2y\,(1 - \tan^2 x) = \sin(2x) + c$

put $x = 0$

$\Rightarrow c = 0 \Rightarrow y = \dfrac{1}{2}\tan 2x.\cos^2 x$

$\Rightarrow y\left(\dfrac{\pi}{6}\right) = \dfrac{1}{2} \cdot \sqrt{3} \cdot \dfrac{3}{4} = \dfrac{3\sqrt{3}}{8}$

17. Ans. (0)

$(2ny + xy \log x)dx = x \log x\, dy \Rightarrow \dfrac{dy}{y} = \left(\dfrac{2n}{x\log x} + 1\right)dx$

$\Rightarrow \log(y) = 2n \log|\log x| + x + c$ (Since the curve passes through (e, e^e) we get $c = 0$)

$\Rightarrow y = e^{x+\log(\log x)^{2n}} \Rightarrow y = e^x \cdot e^{\log(\log x)^{2n}}$

$\Rightarrow y = e^x (\log x)^{2n} \Rightarrow f(x) = e^x (\log x)^{2n}$

$$\Rightarrow g(x) = \lim_{n\to\infty} f(x) = \begin{cases} h \to \infty & , \quad x \le 1/e \qquad \because (\log x)^{2n} \to \text{infinity} \\ 0 & , \quad 1/e < x < e \qquad (\log x)^{2n} \to 0 \\ k \to \infty & , \quad x \ge e \qquad (\log x)^{2n} \to \text{infinity} \end{cases}$$

$\Rightarrow \displaystyle\int_{1/e}^{e} g(x)dx = 0.$

18. Ans. (2)

$xdy - ydx = \dfrac{x^4}{y}dx$

$\dfrac{y}{x}\left(\dfrac{xdy - ydx}{x^2}\right) = xdx$

$\Rightarrow \displaystyle\int \frac{y}{x} d\left(\frac{y}{x}\right) = \int xdx$

$\left(\dfrac{y}{x}\right)^2 = x^2 + 2c$

$\Rightarrow y = \sqrt{x^4 + 2cx^2}$

put $x = 1$ to get $c = \dfrac{3}{2}$

$\therefore f(x) = \sqrt{x^4 + 3x^2}$

$f(-1) = 2$