

EXERCISE - O

SINGLE CORRECT TYPE QUESTIONS

1. The value of the constant 'm' and 'c' for which $y = mx + c$ is a solution of the differential equation $D^2y - 3Dy - 4y = -4x$.
 (A) is $m = -1; c = 3/4$ (B) is $m = 1; c = -3/4$
 (C) no such real m, c (D) is $m = 1; c = 3/4$

MDE001

2. Consider the two statements
 Statement-1: $y = \sin kt$ satisfies the differential equation $y'' + 9y = 0$.
 Statement-2: $y = e^{kt}$ satisfy the differential equation $y'' + y' - 6y = 0$
 The value of k for which both the statements are correct is

- (A) -3 (B) 0 (C) 2 (D) 3

MDE002

3. The x -intercept of the tangent to a curve is equal to the ordinate of the point of contact. The equation of the curve through the point (1, 1) is

- (A) $ye^{\frac{x}{y}} = e$ (B) $xe^{\frac{x}{y}} = e$ (C) $xe^{\frac{y}{x}} = e$ (D) $ye^{\frac{y}{x}} = e$

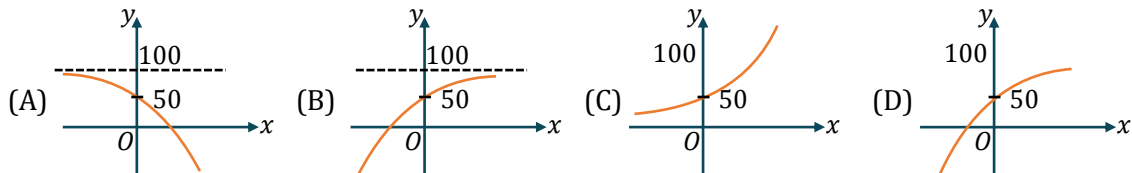
MDE006

4. A function $f(x)$ satisfying $\int_0^1 f(tx) dt = nf(x)$, where $x > 0$, is

- (A) $f(x) = c \cdot x^{\frac{1-n}{n}}$ (B) $f(x) = c \cdot x^{\frac{n}{n-1}}$ (C) $f(x) = c \cdot x^{\frac{1}{n}}$ (D) $f(x) = c \cdot x^{(1-n)}$

MDE007

5. Which one of the following curves represents the solution of the initial value problem $Dy = 100 - y$, where $y(0) = 50$



MDE008

6. The real value of m for which the substitution, $y = u^m$ will transform the differential equation, $2x^4y \frac{dy}{dx} + y^4 = 4x^6$ into a homogeneous equation is :

- (A) $m = 0$ (B) $m = 1$ (C) $m = 3/2$ (D) no value of m

MDE009

7. A function $y = f(x)$ satisfies $(x + 1) \cdot f'(x) - 2(x^2 + x) f(x) = \frac{e^{x^2}}{(x+1)} \forall x > -1$,

If $f(0) = 5$, then $f(x)$ is

- (A) $\left(\frac{3x+5}{x+1}\right) \cdot e^{x^2}$ (B) $\left(\frac{6x+5}{x+1}\right) \cdot e^{x^2}$ (C) $\left(\frac{6x+5}{(x+1)^2}\right) \cdot e^{x^2}$ (D) $\left(\frac{5-6x}{x+1}\right) \cdot e^{x^2}$

MDE011

8. If $\int_a^x t y(t) dt = x^2 + y(x)$ then y as a function of x is

- (A) $y = 2 - (2 + a^2) e^{\frac{x^2+a^2}{2}}$ (B) $y = 1 - (2 + a^2) e^{\frac{x^2-a^2}{2}}$
 (C) $y = 2 - (1 + a^2) e^{\frac{x^2-a^2}{2}}$ (D) none

MDE012

9. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5 p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is :

- (A) $\ln 18$ (B) $2 \ln 18$ (C) $\ln 9$ (D) $\frac{1}{2} \ln 18$

MDE013

10. Let I be the purchase value of an equipment and $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential equation $= -k(T - t)$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is :-

- (A) $I - \frac{k(T-t)^2}{2}$ (B) e^{-kT} (C) $T^2 - \frac{I}{k}$ (D) $I - \frac{kT^2}{2}$

MDE014

11. The equation to the orthogonal trajectories of the system of parabolas $y = ax^2$ is

- (A) $\frac{x^2}{2} + y^2 = c$ (B) $x^2 + \frac{y^2}{2} = c$ (C) $\frac{x^2}{2} - y^2 = c$ (D) $x^2 - \frac{y^2}{2} = c$

MDE051

12. The curve that passes through the point $(2, 3)$, and has the property that the segment of any tangent to it lying between the coordinate axes is bisected by the point of contact, is given by :

- (A) $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 2$ (B) $2y - 3x = 0$ (C) $y = \frac{6}{x}$ (D) $x^2 + y^2 = 13$

MDE053

MULTIPLE CORRECT TYPE QUESTIONS

13. A curve $y = f(x)$ has the property that the perpendicular distance of the origin from the normal at any point P of the curve is equal to the distance of the point P from the x -axis. Then the differential equation of the curve

- (A) is homogeneous.
 (B) can be converted into linear differential equation with some suitable substitution.
 (C) can corresponds to the family of circles touching the x -axis at the origin.
 (D) can corresponds to the family of circles touching the y -axis at the origin.

MDE057

14. If $y = f(x)$ is solution of the differential equation, $x^2 \frac{dy}{dx} \cdot \cos \frac{1}{x} - y \sin \frac{1}{x} = -1$, where $y \rightarrow -1$ as $x \rightarrow \infty$, then

(A) $f(x) = \sin \frac{1}{x} - \cos \frac{1}{x}$

(B) $\lim_{x \rightarrow \infty} (-f(x))^x = \frac{1}{e}$

(C) $\lim_{x \rightarrow \infty} (-f(x))^x = e$

(D) $f'(x) + 2xf'(x) + x^2 f''(x) = -\frac{2}{x^2} \sin \frac{1}{x}$

MDE058

15. Correct statement is/are

(A) $f(x, y) = x^2 e^{\frac{x}{y}} + \frac{y^3}{x} + y^2 \ln \left(\frac{y}{x} \right)$ is a homogenous function of degree two.

(B) $f(x, y) = \frac{\sin y + x}{\sin 2y + x \cos y}$ is homogenous function of degree one.

(C) $x \sin \left(\frac{y}{x} \right) dy + \left(y \sin \frac{y}{x} - x \right) dx = 0$ is a homogenous differential equation.

(D) $f(x, y) = e^{\frac{y}{x}} + \tan \frac{y}{x}$ is homogenous function of degree zero.

MDE059

16. The solution of $x^2 y_1^2 + xy y^1 - 6y^2 = 0$ are (here $y_1 = dy/dx$)

(A) $y = Cx^2$

(B) $x^2 y = C$

(C) $\frac{1}{2} \ln y = C + \log x$

(D) $x^3 y = C$

MDE060

17. The solution of $\frac{dx}{dy} + y = ye^{(n-1)x}$, ($n \neq 1$)

(A) $\frac{1}{n-1} \ln \left(\frac{e^{(n-1)x} - 1}{e^{(n-1)x}} \right) = \frac{y^2}{2} + C$

(B) $e^{(1-n)x} = 1 + ce^{\frac{(n-1)y^2}{2}}$

(C) $\ln \left(1 + ce^{\frac{(n-1)y^2}{2}} \right) + nx + 1 = 0$

(D) $e^{(n-1)x} = ce^{\frac{(n-1)y^2}{2}} + 1$

MDE063

18. The differential equation for the family of curves $y = c \sin x$ can be given by

(A) $\left(\frac{dy}{dx} \right)^2 = y^2 \cot^2 x$

(B) $\left(\frac{dy}{dx} \right)^2 - \left(\sec x \frac{dy}{dx} \right)^2 + y^2 = 0$

(C) $\left(\frac{dy}{dx} \right)^2 = \tan^2 x$

(D) $\frac{dy}{dx} = y \cot x$

MDE064

19. The solution of differential equation $\left(\frac{dy}{dx} \right)^2 - \frac{dy}{dx} (e^x + e^{-x}) + 1 = 0$ is

(A) $y = e^x + c$

(B) $y = -e^{-x} + c$

(C) $y = 2e^x + 3e^{-x} + c$

(D) $ye^x + 1 = ce^x$

MDE066

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 20 to 22

Consider the polynomial $P(x) = (x - \cos 36^\circ)(x - \cos 84^\circ)(x - \cos 156^\circ)$

Differential equations are solved by reducing them to the exact differential of an expression in x & y i.e., they are reduced to the form $d(f(x, y)) = 0$

e.g. : $\frac{xdx + ydy}{\sqrt{x^2 + y^2}} = \frac{ydx - xdy}{x^2}$

$\Rightarrow \frac{1}{2} \frac{2xdx + 2ydy}{\sqrt{x^2 + y^2}} = -\frac{xdy - ydx}{x^2} \Rightarrow d(\sqrt{x^2 + y^2}) = -d\left(\frac{y}{x}\right)$

$\Rightarrow d\left(\sqrt{x^2 + y^2} + \frac{y}{x}\right) = 0$

\therefore solution is $\sqrt{x^2 + y^2} + \frac{y}{x} = c$.

Use the above method to answer the following question :

20. The general solution of $(2x^3 - xy^2)dx + (2y^3 - x^2y)dy = 0$ is
 (A) $x^4 + x^2y^2 - y^4 = c$ (B) $x^4 - x^2y^2 + y^4 = c$
 (C) $x^4 - x^2y^2 - y^4 = c$ (D) $x^4 + x^2y^2 + y^4 = c$

MDE067

21. General solution of the differential equation $\frac{xdy}{x^2 + y^2} + \left(1 - \frac{y}{x^2 + y^2}\right)dx = 0$ is
 (A) $x + \tan^{-1}\left(\frac{y}{x}\right) = c$ (B) $x + \tan^{-1}\frac{x}{y} = c$
 (C) $x - \tan^{-1}\left(\frac{y}{x}\right) = c$ (D) none of these

MDE068

22. General solution of the differential equation $e^y dx + (xe^y - 2y) dy = 0$ is
 (A) $xe^y - y^2 = c$ (B) $ye^x - x^2 = c$ (C) $ye^y + x = c$ (D) $xe^y - 1 = cy^2$

MDE069

MATRIX MATCH TYPE QUESTION

23. Match the following -

List-I	List-II
(I) Solution of $y - \frac{xdy}{dx} = y^2 + \frac{dy}{dx}$ is	(P) $xy^2 = 2y^5 + c$
(II) Solution of $(2x - 10y^3) \frac{dy}{dx} + y = 0$ is	(Q) $\sec y = x + 1 + ce^x$
(III) Solution of $\sec^2 y dy + \tan y dx = dx$ is	(R) $(x + 1)(1 - y) = cy$
(IV) Solution of $\sin y \frac{dy}{dx} = \cos y(1 - x \cos y)$ is	(S) $\tan y = 1 + ce^{-x}$
(A) I \rightarrow P; II \rightarrow Q; III \rightarrow R; IV \rightarrow S	(B) I \rightarrow R; II \rightarrow P; III \rightarrow S; IV \rightarrow Q
(C) I \rightarrow P; II \rightarrow R; III \rightarrow S; IV \rightarrow Q	(D) I \rightarrow R; II \rightarrow S; III \rightarrow P; IV \rightarrow Q

MDE070

EXERCISE - S

1. If $y = y(x)$ is the solution of the differential equation, $x \frac{dy}{dx} + 2y = x^2$ satisfying $y(1) = 1$, if $y\left(\frac{1}{2}\right) = k$, then find the value of $16k$: **MDE016**
2. Let f be a differentiable function such that $f'(x) = 7 - \frac{3f(x)}{x}$, ($x > 0$) and $f(1) \neq 4$. Then the value of $\lim_{x \rightarrow 0^+} x f\left(\frac{1}{x}\right)$ is : **MDE017**
3. Let $y = y(x)$ be the solution of the differential equation, $(x^2 + 1)^2 \frac{dy}{dx} + 2x(x^2 + 1)y = 1$ such that $y(0) = 0$. If $\sqrt{a}y(1) = \frac{\pi}{32}$, then the value of $'32a'$ is : **MDE018**
4. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} = (\tan x - y)\sec^2 x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that $y(0) = 0$, then $y\left(-\frac{\pi}{4}\right)$ is equal to $(e - k)$, find k **MDE019**
5. Let $y = y(x)$ be the solution of the differential equation, $\frac{2 + \sin x}{y + 1} \cdot \frac{dy}{dx} = -\cos x$, $y > 0$, $y(0) = 1$. If $y(\pi) = a$ and $\frac{dy}{dx}$ at $x = \pi$ is b , then the value of $(a + b)$ is **MDE020**
6. If $y = y(x)$ is the solution of the differential equation $\frac{5 + e^x}{2 + y} \cdot \frac{dy}{dx} + e^x = 0$ satisfying $y(0) = 1$, then the value of $|y(\log_e 13)|$ is **MDE021**
7. If $y = \left(\frac{2}{\pi}x - 1\right) \operatorname{cosec} x$ is the solution of the differential equation, $\frac{dy}{dx} + p(x)y = \frac{2}{\pi} \operatorname{cosec} x$, $0 < x < \frac{\pi}{2}$, then find the value of $P\left(\frac{\pi}{4}\right)$. **MDE022**
8. If $y(x)$ satisfies the equation $y'(x) = y(x) + \int_0^1 y dx$ & $y(0) = 1$ then value of $y\left(\ln \frac{11 - 3e}{2}\right)$ **MDE023**
9. If solution of the differential equation $\frac{dy}{dx} = \frac{1}{x \cos y + \sin 2y}$ is $x = ce^{\sin y} - k(1 + \sin y)$, then $k =$ **MDE024**
10. If the equation of curve passing through $(3, 4)$ and satisfying the differential equation $y\left(\frac{dy}{dx}\right)^2 + (x - y)\frac{dy}{dx} - x = 0$ is $Ax + By + 2 = 0$ then value of $(A - B)$ is **MDE025**

EXERCISE - JEE (Main) PYQ

- Let the population of rabbits surviving at a time t be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If $p(0) = 100$, then $p(t)$ equals: [JEE (Main) 2014]

(1) $400 - 300 e^{\frac{t}{2}}$ (2) $300 - 200 e^{-\frac{t}{2}}$ (3) $600 - 500 e^{\frac{t}{2}}$ (4) $400 - 300 e^{-\frac{t}{2}}$

MDE028
- Let $y(x)$ be the solution of the differential equation $(x \log x) \frac{dy}{dx} + y = 2x \log x, (x \geq 1)$. Then $y(e)$ is equal to: [JEE (Main) 2015]

(1) 2 (2) $2e$ (3) e (4) 0

MDE029
- If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation, $y(1 + xy) dx = x dy$, then $f\left(-\frac{1}{2}\right)$ is equal to : [JEE (Main) 2016]

(1) $\frac{4}{5}$ (2) $-\frac{2}{5}$ (3) $-\frac{4}{5}$ (4) $\frac{2}{5}$

MDE030
- Let $y = y(x)$ be the solution of the differential equation $\sin x \frac{dy}{dx} + y \cos x = 4x, x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to: [JEE (Main) 2018]

(1) $\frac{-8}{9\sqrt{3}} = 0 \pi^2$ (2) $-\frac{8}{9} \pi^2$ (3) $-\frac{4}{9} \pi^2$ (4) $\frac{4}{9\sqrt{3}} \pi^2$

MDE032
- Let $f : [0, 1] \rightarrow R$ be such that $f(xy) = f(x) \cdot f(y)$ for all $x, y, \in [0, 1]$, and $f(0) \neq 0$. If $y = y(x)$ satisfies the differential equation, $\frac{dy}{dx} = f(x)$ with $y(0) = 1$, then $y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right)$ is equal to [JEE (Main) 2019]

(1) 4 (2) 3 (3) 5 (4) 2

MDE033
- If $\frac{dy}{dx} + \frac{3}{\cos^2 x} y = \frac{1}{\cos^2 x}, x \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$ and $y\left(\frac{\pi}{4}\right) = \frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals : [JEE (Main) 2019]

(1) $\frac{1}{3} + e^6$ (2) $\frac{1}{3}$ (3) $-\frac{4}{3}$ (4) $\frac{1}{3} + e^3$

MDE034
- The curve amongst the family of curves, represented by the differential equation, $(x^2 - y^2)dx + 2xy dy = 0$ which passes through $(1, 1)$ is : [JEE (Main) 2019]

(1) A circle with centre on the y -axis
 (2) A circle with centre on the x -axis
 (3) An ellipse with major axis along the y -axis
 (4) A hyperbola with transverse axis along the x -axis

MDE035

8. If a curve passes through the point $(1, -2)$ and has slope of the tangent at any point (x, y) on it as $\frac{x^2 - 2y}{x}$, then the curve also passes through the point : [JEE (Main) 2019]

- (1) $(-\sqrt{2}, 1)$ (2) $(\sqrt{3}, 0)$ (3) $(-1, 2)$ (4) $(3, 0)$

MDE037

9. Let $y = y(x)$ be the solution curve of the differential equation, $(y^2 - x)\frac{dy}{dx} = 1$, satisfying $y(0) = 1$. This curve intersects the x -axis at a point whose abscissa is : [JEE (Main) 2020]

- (1) $2 + e$ (2) 2 (3) $2 - e$ (4) $-e$

MDE038

10. If for $x \geq 0$, $y = y(x)$ is the solution of the differential equation $(x + 1)dy = ((x + 1)^2 + y - 3)dx$, $y(2) = 0$, then $y(3)$ is equal to . [JEE (Main) 2020]

MDE039

11. If a curve $y = f(x)$ passes through the point $(1, 2)$ and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value

of b , $\int_1^2 f(x)dx = \frac{62}{5}$? [JEE (Main) 2021]

- (1) 5 (2) 10 (3) $\frac{62}{5}$ (4) $\frac{31}{5}$

MDE040

12. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2 - 4x + y + 8}{x - 2}$, then this curve also passes through the point : [JEE (Main) 2021]

- (1) $(5, 4)$ (2) $(4, 5)$ (3) $(4, 4)$ (4) $(5, 5)$

MDE042

13. A particle is moving in the xy -plane along a curve C passing through the point $(3, 3)$. The tangent to the curve C at the point P meets the x -axis at Q . If the y -axis bisects the segment PQ , then C is a parabola with [JEE (Main) 2022]

- (1) length of latus rectum 3 (2) length of latus rectum 6
 (3) focus $\left(\frac{4}{3}, 0\right)$ (4) focus $\left(0, \frac{3}{4}\right)$

MDE043

14. The slope of normal at any point (x, y) , $x > 0, y > 0$ on the curve $y = y(x)$ is given by $\frac{x^2}{xy - x^2y^2 - 1}$.

If the curve passes through the point $(1, 1)$, then $e \cdot y(e)$ is equal to [JEE (Main) 2022]

- (1) $\frac{1 - \tan(1)}{1 + \tan(1)}$ (2) $\tan(1)$ (3) 1 (4) $\frac{1 + \tan(1)}{1 - \tan(1)}$

MDE044

15. Let $y = y(x)$ be a solution of the differential equation $(x \cos x)dy + (xy \sin x + y \cos x - l)dx = 0$, $0 < x < \frac{\pi}{2}$. If $\frac{\pi}{3}y\left(\frac{\pi}{3}\right) = \sqrt{3}$, then $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right)\right|$ is equal to _____ **[JEE (Main) 2023]**

MDE047

16. Let the solution curve $x = x(y)$, $0 < y < \frac{\pi}{2}$, of the differential equation $(\log_e(\cos y))^2 \cos y dx - (1 + 3x \log_e(\cos y)) \sin y dy = 0$ satisfy $x\left(\frac{\pi}{3}\right) = \frac{1}{2 \log_e 2}$. If $x\left(\frac{\pi}{6}\right) = \frac{1}{\log_e m - \log_e n}$, where m and n are co-prime, then mn is equal to

[JEE (Main) 2023]

MDE048

17. Let f be a differentiable function such that $x^2 f(x) - x = 4 \int_0^x t f(t) dt$, $f(1) = \frac{2}{3}$. Then $18 f(3)$ is equal to **[JEE (Main) 2023]**
- (1) 160 (2) 210 (3) 180 (4) 150

MDE050

EXERCISE - JEE (Advanced) PYQ

1. Let $f: \left[\frac{1}{2}, 1\right] \rightarrow R$ (the set of all real numbers) be a positive, non-constant and differentiable function such that $f'(x) < 2f(x)$ and $f\left(\frac{1}{2}\right) = 1$. Then the value of $\int_{1/2}^1 f(x)dx$ lies in the interval

[JEE (Advanced) 2013]

- (A) $(2e - 1, 2e)$ (B) $(e - 1, 2e - 1)$
 (C) $\left(\frac{e-1}{2}, e-1\right)$ (D) $\left(0, \frac{e-1}{2}\right)$

MDE071

2. A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right)$, $x > 0$. Then the equation of the curve is

[JEE (Advanced) 2013]

- (A) $\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$ (B) $\operatorname{cosec}\left(\frac{y}{x}\right) = \log x + 2$
 (C) $\sec\left(\frac{2y}{x}\right) = \log x + 2$ (D) $\cos\left(\frac{2y}{x}\right) = \log x + \frac{1}{2}$

MDE072

Paragraph for Question 3 and 4

Let $f: [0,1] \rightarrow IR$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, $f(0) = f(1) = 0$ and satisfies $f''(x) - 2f'(x) + f(x) \geq e^x$, $x \in [0,1]$.

3. If the function $e^{-x}f(x)$ assumes its minimum in the interval $[0,1]$ at $x = \frac{1}{4}$, which of the following is true?

[JEE (Advanced) 2013]

- (A) $f'(x) < f(x)$, $\frac{1}{4} < x < \frac{3}{4}$ (B) $f'(x) > f(x)$, $0 < x < \frac{1}{4}$
 (C) $f'(x) < f(x)$, $0 < x < \frac{1}{4}$ (D) $f'(x) < f(x)$, $\frac{3}{4} < x < 1$

MDE073

4. Which of the following is true for $0 < x < 1$?

[JEE (Advanced) 2013]

- (A) $0 < f(x) < \infty$ (B) $-\frac{1}{2} < f(x) < \frac{1}{2}$ (C) $-\frac{1}{4} < f(x) < 1$ (D) $-\infty < f(x) < 0$

MDE074

5. The function $y = f(x)$ is the solution of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^4 + 2x}{\sqrt{1 - x^2}}$ in $(-1, 1)$

satisfying $f(0) = 0$. Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ is

[JEE (Advanced) 2014]

- (A) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ (B) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$ (C) $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$ (D) $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

MDE075

6. Let $y(x)$ be a solution of the differential equation $(1 + e^x)y' + ye^x = 1$. If $y(0) = 2$, then which of the following statements is(are) true ? **[JEE (Advanced) 2015]**
 (A) $y(-4) = 0$
 (B) $y(-2) = 0$
 (C) $y(x)$ has a critical point in the interval $(-1,0)$
 (D) $y(x)$ has no critical point in the interval $(-1,0)$

MDE076

7. Consider the family of all circles whose centres lie on the straight line $y = x$. If this family of circles is represented by the differential equation $P y'' + Q y' + 1 = 0$, where P, Q are functions of x, y and y' (here $y' = \frac{dy}{dx}$, $y'' = \frac{d^2y}{dx^2}$), then which of the following statements is (are) true ?

[JEE (Advanced) 2015]

- (A) $P = y + x$ (B) $P = y - x$
 (C) $P + Q = 1 - x + y + y' + (y')^2$ (D) $P - Q = x + y - y' - (y')^2$

MDE077

8. A solution curve of the differential equation $(x^2 + xy + 4x + 2y + 4) \frac{dy}{dx} - y^2 = 0, x > 0$, passes through the point $(1, 3)$. The solution curve- **[JEE (Advanced) 2016]**
 (A) intersects $y = x + 2$ exactly at one point (B) intersects $y = x + 2$ exactly at two points
 (C) intersects $y = (x + 2)^2$ (D) does NOT intersect $y = (x + 3)^2$

MDE078

9. Let $f: (0, \infty) \rightarrow \mathbb{R}$ be a differentiable function such that $f'(x) = 2 - \frac{f(x)}{x}$ for all $x \in (0, \infty)$ and $f(1) \neq 1$. Then **[JEE (Advanced) 2016]**

- (A) $\lim_{x \rightarrow 0^+} f' \left(\frac{1}{x} \right) = 1$ (B) $\lim_{x \rightarrow 0^+} x f' \left(\frac{1}{x} \right) = 2$
 (C) $\lim_{x \rightarrow 0^+} x^2 f'(x) = 0$ (D) $|f(x)| \leq 2$ for all $x \in (0, 2)$

MDE079

10. If $y = y(x)$ satisfies the differential equation $8\sqrt{x}(\sqrt{9+\sqrt{x}}) dy = (\sqrt{4+\sqrt{9+\sqrt{x}}})^{-1} dx, x > 0$ and $y(0) = \sqrt{7}$, then $y(256) =$ **[JEE (Advanced) 2017]**
 (A) 80 (B) 3 (C) 16 (D) 9

MDE080

11. If $f: \mathbb{R} \rightarrow \mathbb{R}$ is a differentiable function such that $f'(x) > 2f(x)$ for all $x \in \mathbb{R}$, and $f(0) = 1$, then **[JEE (Advanced) 2017]**
 (A) $f(x) > e^{2x}$ in $(0, \infty)$ (B) $f(x)$ is decreasing in $(0, \infty)$
 (C) $f(x)$ is increasing in $(0, \infty)$ (D) $f'(x) < e^{2x}$ in $(0, \infty)$

MDE081

12. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be two non-constant differentiable functions. If $f'(x) = (e^{f(x)-g(x)})g'(x)$ for all $x \in \mathbb{R}$, and $f(1) = g(2) = 1$, then which of the following statement(s) is (are) TRUE ?

[JEE (Advanced) 2018]

- (A) $f(2) < 1 - \log_e 2$ (B) $f(2) > 1 - \log_e 2$ (C) $g(1) > 1 - \log_e 2$ (D) $g(1) < 1 - \log_e 2$

MDE082

13. Let $f:(0, \pi) \rightarrow \mathbb{R}$ be a twice differentiable function such that $\lim_{t \rightarrow x} \frac{f(x)\sin t - f(t)\sin x}{t-x} = \sin^2 x$ for all $x \in (0, \pi)$. If $f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12}$, then which of the following statement(s) is (are) TRUE ?

[JEE (Advanced) 2018]

- (A) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}}$ (B) for all $x \in (0, \pi)$
 (C) There exists $\alpha \in (0, \pi)$ such that $f'(\alpha) = 0$ (D) $f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$

MDE083

14. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function with $f(0) = 0$. If $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} = (2+5y)(5y-2)$, then the value of $\lim_{x \rightarrow -\infty} f(x)$ is ____.

[JEE (Advanced) 2018]

MDE084

15. Let Γ denote a curve $y = y(x)$ which is in the first quadrant and let the point $(1, 0)$ lie on it. Let the tangent to Γ at a point P intersect the y -axis at Y_P . If PY_P has length 1 for each point P on Γ , then which of the following options is/are correct ?

[JEE (Advanced) 2019]

- (A) $y = \log_e \left(\frac{1 + \sqrt{1-x^2}}{x} \right) - \sqrt{1-x^2}$ (B) $xy' - \sqrt{1-x^2} = 0$
 (C) $y = -\log_e \left(\frac{1 + \sqrt{1-x^2}}{x} \right) + \sqrt{1-x^2}$ (D) $xy' + \sqrt{1-x^2} = 0$

MDE085

16. Let b be a nonzero real number. Suppose $f: \mathbb{R} \rightarrow \mathbb{R}$ is a differentiable function such that $f(0) = 1$. If the derivative f' of f satisfies the equation $f'(x) = \frac{f(x)}{b^2 + x^2}$

for all $x \in \mathbb{R}$, then which of the following statements is/are TRUE ? [JEE (Advanced) 2020]

- (A) If $b > 0$, then f is an increasing function (B) If $b < 0$, then f is a decreasing function
 (C) $f(x)f(-x) = 1$ for all $x \in \mathbb{R}$ (D) $f(x) - f(-x) = 0$ for all $x \in \mathbb{R}$

MDE086

17. For any real numbers α and β , let $y_{\alpha, \beta}(x)$, $x \in \mathbb{R}$, be the solution of the differential equation

$$\frac{dy}{dx} + \alpha y = x e^{\beta x}, y(1) = 1$$

[JEE (Advanced) 2021]

Let $S = \{y_{\alpha, \beta}(x) : \alpha, \beta \in \mathbb{R}\}$. Then which of the following functions belong(s) to the set S ?

- (A) $f(x) = \frac{x^2}{2} e^{-x} + \left(e - \frac{1}{2}\right) e^{-x}$ (B) $f(x) = -\frac{x^2}{2} e^{-x} + \left(e + \frac{1}{2}\right) e^{-x}$
 (C) $f(x) = \frac{e^x}{2} \left(x - \frac{1}{2}\right) + \left(e - \frac{e^2}{4}\right) e^{-x}$ (D) $f(x) = \frac{e^x}{2} \left(\frac{1}{2} - x\right) + \left(e + \frac{e^2}{4}\right) e^{-x}$

MDE087

18. If $y(x)$ is the solution of the differential equation $xdy - (y^2 - 4y)dx = 0$ for $x > 0, y(1) = 2$, and the slope of the curve $y = y(x)$ is never zero, then the value of $10y(\sqrt{2})$ is ____.

[JEE (Advanced) 2022]
MDE088

19. For $x \in \mathbb{R}$, let the function $y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} + 12y = \cos\left(\frac{\pi}{12}x\right), y(0) = 0.$$

Then, which of the following statements is/are TRUE ?

[JEE (Advanced) 2022]

- (A) $y(x)$ is an increasing function
 (B) $y(x)$ is a decreasing function
 (C) There exists a real number β such that the line $y = \beta$ intersects the curve $y = y(x)$ at infinitely many points
 (D) $y(x)$ is a periodic function

MDE089

20. Let $f : [(1, \infty) \rightarrow \mathbb{R}$ be a differentiable function such that $f(1) = \frac{1}{3}$ and

$3 \int_1^x f(t) dt = xf(x) - \frac{x^3}{3}, x \in [1, \infty)$. Let e denote the base of the natural logarithm. Then the value of $f(e)$ is

[JEE (Advanced) 2023]

- (A) $\frac{e^2 + 4}{3}$ (B) $\frac{\log_e 4 + e}{3}$ (C) $\frac{4e^2}{3}$ (D) $\frac{e^2 - 4}{3}$

MDE090

21. Let S be the set of all twice differentiable functions f from \mathbb{R} to \mathbb{R} such that $\frac{d^2f}{dx^2}(x) > 0$ for all $x \in (-1, 1)$. For $f \in S$, let X_f be the number of points $x \in (-1, 1)$ for which $f(x) = x$. Then which of the following statements is(are) true?

[JEE (Advanced) 2023]

- (A) There exists a function $f \in S$ such that $X_f = 0$
 (B) For every function $f \in S$, we have $X_f \leq 2$
 (C) There exists a function $f \in S$ such that $X_f = 2$
 (D) There does **NOT** exist any function f in S such that $X_f = 1$

MDE091

22. For $x \in \mathbb{R}$, let $y(x)$ be a solution of the differential equation $(x^2 - 5) \frac{dy}{dx} - 2xy = -2x(x^2 - 5)^2$ such that $y(2) = 7$. Then the maximum value of the function $y(x)$ is

[JEE (Advanced) 2023]
MDE092

JEE (Main) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION-A

- This section contains **TWENTY** questions.
- Each question has **FOUR** options (1), (2), (3) and (4). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +4, if only the bubble corresponding to the correct option is darkened.

Zero Marks : 0, if none of the bubbles is darkened.

Negative Marks : -1 in all other cases.

1. The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where c_1 and c_2 are arbitrary constants, is :-
 (1) $yy'' = y'$ (2) $yy'' = (y')^2$ (3) $y' = y^2$ (4) $y'' = y'y$

MDE093

2. Let I be the purchase value of an equipment and $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T - t)$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is :-

(1) $I - \frac{k(T-t)^2}{2}$ (2) e^{-kT} (3) $T^2 - \frac{I}{k}$ (4) $I - \frac{kT^2}{2}$

MDE094

3. The curve that passes through the point (2, 3), and has the property that the segment of any tangent to it lying between the coordinate axes is bisected by the point of contact, is given by:

(1) $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 2$ (2) $2y - 3x = 0$ (3) $y = \frac{6}{x}$ (4) $x^2 + y^2 = 13$

MDE095

4. Consider the differential equation $y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$. If $y(1) = 1$, then x is given by :

(1) $1 - \frac{1}{y} + \frac{1}{e}$ (2) $4 - \frac{2}{y} - \frac{1}{e}$ (3) $3 - \frac{1}{y} + \frac{1}{e}$ (4) $1 + \frac{1}{y} - \frac{1}{e}$

MDE096

5. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5 p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is:

(1) $\ln 18$ (2) $2 \ln 18$ (3) $\ln 9$ (4) $\frac{1}{2} \ln 18$

MDE097

6. A spherical balloon is filled with 4500π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is :
- (1) $9/2$ (2) $9/7$ (3) $7/9$ (4) $2/9$

MDE098

7. The differential equation for all the straight lines which are at a unit distance from the origin is

(1) $\left(y - x \frac{dy}{dx}\right)^2 = 1 - \left(\frac{dy}{dx}\right)^2$ (2) $\left(y + x \frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$
 (3) $\left(y - x \frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$ (4) $\left(y + x \frac{dy}{dx}\right)^2 = 1 - \left(\frac{dy}{dx}\right)^2$

MDE099

8. If the differential equation representing the family of all circles touching x-axis at the origin is $(x^2 - y^2) \frac{dy}{dx} = g(x) y$, then $g(x)$ equals

(1) $2x^2$ (2) $\frac{1}{2}x^2$ (3) $\frac{1}{2}x$ (4) $2x$

MDE100

9. If $\phi(x) = \phi'(x)$ and $\phi(1) = 2$, then $\phi(3)$ equals

(1) e^2 (2) $2e^2$ (3) $3e^2$ (4) $2e^3$

MDE101

10. If $\frac{dy}{dx} = 1 + x + y + xy$ and $y(-1) = 0$, then function y is

(1) $e^{(1-x)^2/2}$ (2) $e^{(1+x)^2/2} - 1$ (3) $\ln(1+x) - 1$ (4) $1+x$

MDE102

11. The solution of the differential equation $\frac{dy}{dx} - ky = 0, y(0) = 1$, approaches zero when $x \rightarrow \infty$, if

(1) $k = 0$ (2) $k > 0$ (3) $k < 0$ (4) $k \geq 0$

MDE103

12. The general solution of the differential equation $ydy + \sqrt{1+y^2} dx = 0$ represents a family of :

(1) circles (2) ellipses other than circles
 (3) hyperbolas (4) parabolas

MDE104

13. The conic whose differential equation is $(1 + y^2)dx - xydy = 0$ and which passess through $(1, 0)$ is

(1) $x^2 - y^2 = 1$ (2) $2x^2 - 3y^2 = 1$ (3) $x^2 + y^2 = 1$ (4) $x^2 - y^2 = 2$

MDE105

14. The general solution of the differential equation $\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$ is

- (1) $\ln\left(\cot\frac{y}{2}\right) + 2\sin x = C$ (2) $\ln\left(\cot\frac{y}{4}\right) + 2\sin\frac{x}{2} = C$
 (3) $\ln\left(\tan\frac{y}{2}\right) + 2\sin x = C$ (4) $\ln\left(\tan\frac{y}{4}\right) + 2\sin\frac{x}{2} = C$

MDE106

15. The solution of the differential equation $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$ is -

- (1) $\ln\left|\tan\left(\frac{x+y}{2}\right)+1\right| = x+c$ (2) $\ln(\tan x) = y+c$
 (3) $\ln \tan\left(\frac{x+y}{2}\right) = x+c$ (4) $\ln(\cos x) = y+c$

MDE107

16. The solution of differential equation $\frac{dy}{dx} = \frac{4x+6y+5}{3y+2x+4}$ is

- (1) $x^2 + y^2 - xy + x - y = c$ (2) $y - 2x + \frac{3}{8} \ln(24y + 16x + 23) = c$
 (3) $4xy + 3(x^2 + y^2) - 10(x + y) = c$ (4) $y + 2x + \frac{3}{8} \ln(24y + 16x + 23) = c$

MDE108

17. The equation of the curve satisfying $\frac{dy}{dx} = \frac{y^2 - 2xy - x^2}{y^2 + 2xy - x^2}$ and passing through (1, -1) is.

- (1) $x = y + c$ (2) $y = 2x + c$ (3) $y = x$ (4) $y = -x$

MDE109

18. The solution of differential equation $\left[x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right)\right] y - \left[y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right)\right] x \frac{dy}{dx} = 0$ is

- (1) $y \cos\left(\frac{y}{x}\right) = c$ (2) $xy \sin\left(\frac{y}{x}\right) = c$ (3) $xy \cos\left(\frac{y}{x}\right) = c$ (4) $xy \tan\left(\frac{y}{x}\right) = c$

MDE110

19. The solution of the equation $\frac{dv}{dt} + \frac{k}{m} v = -g$ is

- (1) $v = ce^{\frac{k}{m}t} - \frac{mg}{k}$ (2) $v = c - \frac{mg}{k} e^{-\frac{k}{m}t}$ (3) $v e^{\frac{k}{m}t} = c - \frac{mg}{k}$ (4) $v e^{\frac{k}{m}t} = c - \frac{mg}{k}$

MDE111

20. The solution of the differential equation $\frac{dy}{dx} = y \tan x - 2\sin x$ is

- (1) $y = \cos x + c \sec x$ (2) $y = \cos x + c$
 (3) $y = \sin x + c$ (4) $y = \operatorname{cosec} x + c$

MDE112

SECTION-B

- This section will have **TEN** questions. Candidate can choose to attempt any 5 question out of these 10 questions. In case if candidate attempts more than 5 questions, first 5 attempted questions will be considered for marking.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (Answer should be rounded off to the nearest integer).
- Answer to each question will be evaluated according to the following marking scheme:
 Full Marks : +4, if only correct answer is given.
 Zero Marks : 0, if no answer is given.
 Negative Marks : -1 for incorrect answer

1. If $y(x)$ is the solution of the differential equation $(x + 2) \frac{dy}{dx} = x^2 + 4x - 9$, $x \neq -2$ and $y(0) = 0$, then $y(-4)$ is equal to: **MDE113**
2. The value of $\lim_{x \rightarrow \infty} y(x)$ obtained from the differential equation $\frac{dy}{dx} = y - y^2$, where $y(0) = 2$ is **MDE114**
3. Consider the differential equation, $ydx - (x + y^2)dy = 0$. If for $y = 1$, x takes value 1, then value of x when $y = 4$ is : **MDE115**
4. If $\frac{dy}{dx} + y \tan x = \sin 2x$ and $y(0) = 1$, then $|y(\pi)|$ is equal to : **MDE116**
5. The solution of the differential equation $ydx - (x + 2y^2)dy = 0$ is $x = f(y)$. If $f(-1) = 1$, then $f(1)$ is equal to : **MDE117**
6. If the solution of $\frac{dy}{dx} = \frac{ax+3}{2y+1}$ represents a circle, then the value of $|a|$ is **MDE118**
7. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $|y'(1)|$ equals :- **MDE119**
8. If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then $y(\ln 2)$ is equal to :- **MDE120**
9. If $y(x)$ is a solution of the differential equation $\frac{dy}{dx} + 3y = 2$, then $\lim_{x \rightarrow \infty} y(x)$ is equal to $\frac{k}{3}$ then value of k - **MDE121**
10. If differential equation of family of curves $y \ln|cx| = x$, where c is an arbitrary constant, is $y' = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$, for some function ϕ , then $\frac{1}{|\phi(2)|}$ is equal to : **MDE122**

JEE (Advanced) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION-I

- This section contains **SIX** questions.
 - Each question has FOUR options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
 - For each question, darken the bubble corresponding to the correct option in the ORS.
 - For each question, marks will be awarded in one of the following categories:
 Full Marks : +3, if only the bubble corresponding to the correct option is darkened.
 Zero Marks : 0, if none of the bubbles is darkened.
 Negative Marks : -1, in all other cases
-
1. The solution of the differential equation $(x^2 \sin^3 y - y^2 \cos x)dx + (x^3 \cos y \sin^2 y - 2y \sin x)dy = 0$ is
 (A) $x^3 \sin^3 y = 3y^2 \sin x + C$ (B) $x^3 \sin^3 y + 3y^2 \sin x = C$
 (C) $x^2 \sin^3 y + y^3 \sin x = C$ (D) $2x^2 \sin y + y^2 \sin x = C$ **MDE123**
2. Solution of differential equation $xy(my dx + nx dy) = \frac{xdy + ydx}{x^m y^n}$, given $m + n = 1$, is
 (A) $x^{m+1} \cdot y^{n+1} + 1 = c(x/y)$ (B) $x^{m+1} \cdot y^{n+1} + 1 = cxy$
 (C) $x^{m+1} \cdot y^{n+1} - 1 = cxy$ (D) $x^m \cdot y^n + 1 = cxy$. **MDE124**
3. $f(x)$ is a continuous and differentiable function defined in $\in [0, \infty)$. If $f(0) = 1$ and $f'(x) > 3f(x) \forall x \geq 0$ then
 (A) $f(x) \leq e^{3x} \forall x \geq 0$ (B) $f(x) \leq e^{-3x} \forall x \geq 0$
 (C) $f(x) > e^{3x} \forall x \geq 0$ (D) $f(x) \geq e^{3x} \forall x \geq 0$ **MDE125**
4. Find the curve which passes through the point (2, 0) such that the segment of the tangent between the point of tangency & the y-axis has a constant length equal to 2.
 (A) $y = \pm \left[\sqrt{4-x^2} - 2 \ln \frac{2-\sqrt{4-x^2}}{x} \right]$ (B) $y = \pm \left[\sqrt{4-x^2} + 2 \ln \frac{2-\sqrt{4-x^2}}{x} \right]$
 (C) $y = \pm \left[\sqrt{4-x^2} + 2 \ln \frac{2+\sqrt{4-x^2}}{x} \right]$ (D) $y = \pm \left[\sqrt{4-x^2} - 2 \ln \frac{2+\sqrt{4-x^2}}{x} \right]$ **MDE126**
5. Given $y = f(x)$ is a solution of differential equation $xdy + xdx = ydy - ydx$, also $f(0) = 0$, $f(1) = \tan \frac{3\pi}{8}$, then the area bounded by $y = f(x)$ on x-axis & $x = 0$ to $x = \tan \frac{\pi}{8}$ is equal to
 (A) $\frac{1}{2}(\sqrt{2}-1)$ (B) $\frac{1}{2}(\sqrt{2}+1)^3$ (C) $\frac{1}{2}(\sqrt{2}-1)^3$ (D) $\frac{1}{2}(\sqrt{2}+1)^2$ **MDE127**

6. Solution of differential equation $\frac{dy}{dx} = \frac{1}{xy(x^2 \sin y^2 + 1)}$ is

(where C is integral constant)

(A) $e^{y^2} \left(\frac{1}{x^2} - \frac{\cos y^2}{2} + \frac{\sin y^2}{2} \right) = C$

(B) $e^{y^2} \left(\frac{1}{x^2} + \frac{\cos y^2}{2} - \frac{\sin y^2}{2} \right) = C$

(C) $e^{y^2} \left(\frac{1}{x^2} - \frac{\cos y^2}{2} + \frac{\sin^2 y}{2} \right) = C$

(D) $e^{y^2} \left(\frac{1}{x^2} - \frac{\cos y}{2} + \frac{\sin y}{2} \right) = C$

MDE128

SECTION-II

- This section contains **FIVE** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 if only (all) the correct option(s) is (are) chosen.

Partial Marks : +3 if all the four options are correct but **ONLY** three options are chosen.

Partial Marks : +2 if three or more options are correct but **ONLY** two options are chosen, both of which are correct options.

Partial Marks : +1 if two or more options are correct but **ONLY** one option is chosen and it is a correct option.

Zero Marks : 0 if none of the options is chosen (i.e. the question is unanswered).

Negative Marks : -2 in all other cases.

For Example : If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks.

7. A function $f : R \rightarrow R$ satisfies the differential equation $2xy + (1 + x^2)y' = 1$, where $f(0) = 0$, then
 (A) $f(x)$ is odd (B) $f(x) = 1$ has no real solution
 (C) $f(x) = \frac{1}{2}$ has exactly two distinct solution (D) $f(x) = -1$ has no real solution

MDE129

8. A conic passing through (2,2), has the property that slope of the tangent at any point P , on the curve is equal to reciprocal of the ordinate of point P then -

(A) focus of the conic is $\left(\frac{1}{2}, 0 \right)$

(B) latus rectum is equal to a prime number

(C) it passes through (8, -4)

(D) $\lim_{x \rightarrow \infty} \frac{y^2}{x} = 2$, where (x, y) lies on the curve

MDE130

9. Let graph of $f(x)$ passes through $(0, 2)$ and tangent at any point (x, y) of $f(x)$ intersects x -axis at $(x + 2, 0)$, then -
 (A) $\lim_{x \rightarrow \infty} f(x) = 0$
 (B) $\lim_{x \rightarrow \infty} f(x) = 2$
 (C) equation of normal at $(0, 2)$ is $x - y + 2 = 0$
 (D) equation of normal at $(0, 2)$ is $x + y - 2 = 0$

MDE131

10. Solution of differential equation $f(x) \frac{dy}{dx} = f^2(x) + f(x)y + f'(x)y$ is
 (A) $y = f(x) + ce^x$ (B) $y = -f(x) + ce^x$ (C) $y = -f(x) + ce^x f(x)$ (D) $y = cf(x) + e^x$

MDE132

11. A differentiable function satisfies equation $f(x) = \int_0^x (f(t) \cos t - \cos(t-x)) dt$ then
 (A) $f''\left(\frac{\pi}{2}\right) = e$ (B) $\lim_{x \rightarrow -\infty} f(x) = 1$
 (C) $f(x)$ has minimum value $1 - e^{-1}$ (D) $f'(0) = -1$

MDE133

SECTION-III

- This section contains **ONE** paragraph.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D) **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks	:	+3	if only the bubble corresponding to the correct answer is darkened.
Zero Marks	:	0	in all other cases.

Comprehension # 1 (Q. No. 12 - 13)

Consider a curve C passing through the point $(1, 1)$ and has the property that the perpendicular distance of the normal at any point P on the curve from origin is equal to the perpendicular distance of P from x -axis.

On the basis of above information, answer the following questions :

12. Area of the smaller region bounded by curve ' C ' and $x^2 - 3y^2 - 2x + 1 = 0$
 (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{8}$
13. Maximum distance of curve ' C ' from any point in set $A = \{(x, y) : y^2 + 4x - 4 = 0; x \in [0, 1]\}$
 (A) $\sqrt{5} + 1$ (B) $2\sqrt{2}$ (C) $\sqrt{5} - 1$ (D) $2\sqrt{2} - 1$

MDE134

MDE135

SECTION-IV

- This section contains **TWO** questions.
- **Each question has matching lists.** The codes for the lists have choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**
- For each question, marks will be awarded in one of the following categories :
Full Marks : +3 If only correct answer is given.
Zero Marks : 0 If no answer is given.
Negative Marks : -1 For incorrect answer

14. If $y = f(x)$ is solution of the differential equation $\frac{dy}{dx} - y = -e^x \tan^2 x$; $y(0) = 0$ and $g(x) = e^x$,
 $h(x) = e^x(1 - \sec^2 x)$.

Match List-I with List-II and select the correct answer using the code given below the list.

List-I	List-II
(P) The number of points of inflection of $f(x)$ in $\left(0, \frac{\pi}{2}\right)$ is (are)	(1) 3
(Q) If $\left \lim_{x \rightarrow 0} \frac{f(x)}{x^3 g(x)} \right = \frac{a}{b}$, where a and b are coprime, then $(b - a)$ is equal to	(2) 0
(R) If $\int_0^{\pi} (f(x) + h(x)) dx = ke^{\pi}$, then $[k]$ is (where $[.]$ denotes greatest integer function)	(3) 2
(S) The number of solution(s) of the equation $f(x) = 0$ in $[0, \pi)$ is (are)	(4) 1

Codes :

	P	Q	R	S
(A)	1	3	4	1
(B)	3	3	4	1
(C)	2	3	1	4
(D)	3	1	2	4

MDE136

15. Match List-I with List-II and select the correct answer using the code given below the list.

- | List-I | List-II |
|---|--------------------------------------|
| (P) Solution of differential equation
$y(2x^4 + y)\frac{dy}{dx} = (1 - 4xy^2)x^2$ is | (1) $y^2 = (\ln x)^2 + C$ |
| (Q) Solution of differential equation
$\sum_{n=1}^{\infty} \frac{x^{2n-1}}{(2n-1)!} = \frac{dx-dy}{dx+dy}, n \in N$ is | (2) $3(x^2y)^2 + y^3 - x^3 = C$ |
| (R) Solution of differential equation
$x = \sum_{r=0}^{\infty} \frac{(xy)^r}{r!} \left(\frac{dy}{dx}\right)^r$ is | (3) $e^{x^2}(y^2 - 1) + e^{y^2} = C$ |
| (S) Solution of differential equation
$(e^{x^2} + e^{y^2})y\frac{dy}{dx} + e^{x^2}(xy^2 - x) = 0$ is | (4) $2ye^{2x} = ce^{2x} - 1$ |

Codes :

	P	Q	R	S
(A)	2	1	4	3
(B)	1	4	2	3
(C)	2	4	1	3
(D)	3	2	4	1

MDE137

SECTION-V

- This section contains **THREE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 TO 9, **BOTH INCLUSIVE**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +3 If ONLY the correct integer is entered;
Zero Marks : 0 If the question is unanswered;
Negative Marks : -1 In all other cases.

16. If $y(x)$ satisfies the differential equation; $\cos^2 x (dy/dx) - (\tan 2x)y = \cos^4 x, |x| < \frac{\pi}{4}$, and $y(0) = 0$

then $\frac{64y\left(\frac{\pi}{6}\right)}{3\sqrt{3}}$ is equal to

MDE138

Differential Equations

17. If $y = f(x)$ be a curve passing through (e, e^e) and which satisfy the differential equation $(2ny + xy \log x)dx - x \log x dy = 0$, value of $\int_{1/e}^e g(x) dx$ where $g(x) = \lim_{n \rightarrow \infty} f(x)$, is :

MDE139

18. If $y = f(x)$ ($y > 0$) satisfies the differential equation $x \frac{dy}{dx} - y = \frac{x^4}{y}$ such that $y(1) = 2$, then the value of $f(-1)$ is

MDE140

ANSWER KEY

EXERCISE - O

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	A	A	A	B	C	B	A	B	D
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A	C	A,B,D	A,B,D	A,C,D	A,C,D	A,B	A,B,D	A,B,D	B
Que.	21	22	23							
Ans.	A	A	B							

EXERCISE - S

1.	49	2.	4	3.	2	4.	2	5.	2
6.	1	7.	1	8.	4	9.	2	10.	4

EXERCISE - JEE (Main) PYQ

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1	1	1	2	2	1	2	2	3	3
Que.	11	12	13	14	15	16	17			
Ans.	2	4	1	4	2	12	1			

EXERCISE - JEE (Advanced) PYQ

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	A	C	D	B	A,C	B,C	A,D	A	B
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A,C	B,C	B,C,D	0.4	A,D	A,C	A,C	8	C	C
Que.	21	22								
Ans.	A,B,C	16								

JEE (Main) Practice Paper

Section-A	Q.	1	2	3	4	5	6	7	8	9	10
	A.	2	4	3	4	2	4	3	4	2	2
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	3	3	1	4	1	2	4	3	1	1
Section-B	Q.	1	2	3	4	5	6	7	8	9	10
	A.	0	1	16	5	3	2	1	7	2	4

JEE (Advanced) Practice Paper

Section-I	Q.	1	2	3	4	5	6	
	A.	A	B	D	B	A	A	
Section-II	Q.	7	8	9	10	11		
	A.	A,B,D	A,B,C,D	A,C	C	A,D		
Section-III	Q.	12	13					
	A.	C	A					
Section-IV	Q.	14	15					
	A.	C	C					
Section-V	Q.	16	17	18				
	A.	8	0	2				

