EXERCISE - O

SINGLE CORRECT TYPE QUESTIONS

1. The value of $\int_{\pi}^{2\pi}[2\cos x]dx$ where [.] represents the greatest integer function, is -

(A) $-\frac{5\pi}{6}$ (B) $-\frac{\pi}{2}$ (C) $-\pi$ (D) none

MDI001

2. The value of the definite integral $\int_{19}^{37}\left(\{x\}^2+3(\sin 2\pi x)\right)dx$ where { x } denotes the fractional part function.

(A) 0 (B) 6 (C) 9 (D) can't be determined

MDI002

3. $\int_{-1}^{1}\frac{x^3+|x|+1}{x^2+2|x|+1}dx = a\ell n2+b$ then -

(A) $a = 2;\ b = 1$ (B) $a = 2;\ b = 0$ (C) $a = 3;\ b = -2$ (D) $a = 4;\ b = -1$

MDI003

4. The value of the definite integral $\int_{2}^{4}\left(x(3-x)(4+x)(6-x)(10-x)+\sin x\right)dx$ equals -

(A) cos 2 + cos 4 (B) cos 2 – cos 4 (C) sin 2 + sin 4 (D) sin 2 – sin 4

MDI004

5. The value of the definite integral $\int_{0}^{\pi/2}\sin x \sin 2x \sin 3x\, dx$ is equal to :

(A) $\frac{1}{3}$ (B) $-\frac{2}{3}$ (C) $-\frac{1}{3}$ (D) $\frac{1}{6}$

MDI006

6. If $I = \int_{0}^{\pi/2}\ell n(\sin x)dx$ then $\int_{-\pi/4}^{\pi/4}\ell n(\sin x+\cos x)dx$

(A) $\frac{I}{2}$ (B) $\frac{I}{4}$ (C) $\frac{I}{\sqrt{2}}$ (D) I

MDI007

7. Let f be a positive function. Let $I_1 = \int_{1-k}^{k} xf(x(1-x))dx$; $I_2 = \int_{1-k}^{k} f(x(1-x))dx$, where $2k - 1 > 0$. Then $\frac{I_2}{I_1}$ is -

(A) k (B) 1/2 (C) 1 (D) 2

MDI011

8. Let f be a continuous function satisfying $f'(\ell n\,x) = \begin{bmatrix} 1 & \text{for} \quad 0 < x \le 1 \\ x & \text{for} \quad x > 1 \end{bmatrix}$ and $f(0) = 0$ then $f(x)$ can be defined as

(A) $f(x) = \begin{bmatrix} 1 & \text{if} \quad x \le 0 \\ 1 - e^x & \text{if} \quad x > 0 \end{bmatrix}$

(B) $f(x) = \begin{bmatrix} 1 & \text{if} \quad x \le 0 \\ e^x - 1 & \text{if} \quad x > 0 \end{bmatrix}$

(C) $f(x) = \begin{bmatrix} x & \text{if} \quad x < 0 \\ e^x & \text{if} \quad x > 0 \end{bmatrix}$

(D) $f(x) = \begin{bmatrix} x & \text{if} \quad x \le 0 \\ e^x - 1 & \text{if} \quad x > 0 \end{bmatrix}$

MDI013

9. $\int_0^\infty f\left(x + \frac{1}{x}\right) \cdot \frac{\ell n\,x}{x} dx$

(A) is equal to zero (B) is equal to one (C) is equal to $\frac{1}{2}$ (D) can not be evaluated

MDI014

10. If $\int_0^x f(t)dt = x + \int_x^1 t^2 \cdot f(t)dt + \frac{\pi}{4} - 1$, then the value of the integral $\int_{-1}^1 f(x)dx$ is equal to

(A) 0 (B) $\pi/4$ (C) $\pi/2$ (D) π

MDI015

11. Variable x and y are related by equation $x = \int_0^y \frac{dt}{\sqrt{1+t^2}}$. The value of $\frac{d^2y}{dx^2}$ is equal to

(A) $\frac{y}{\sqrt{1+y^2}}$ (B) y (C) $\frac{2y}{\sqrt{1+y^2}}$ (D) $4y$

MDI016

12. The true solution set of the inequality, $\sqrt{5x - 6 - x^2} + \left(\frac{\pi}{2}\int_0^x dz\right) > x\int_0^\pi \sin^2 x dx$ is :

(A) R (B) (1,6) (C) (–6,1) (D) (2,3)

MDI018

13. $\lim_{n\to\infty} \frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \ldots\ldots + \frac{1}{\sqrt{n}\sqrt{4n}}$ is equal to -

(A) 2 (B) 4 (C) $2\left(\sqrt{2} - 1\right)$ (D) $2\sqrt{2} - 1$

MDI019

MULTIPLE CORRECT TYPE QUESTIONS

14. Let $f(x)=\begin{cases} x+1, & 0\le x\le 1 \\ 2x^2-6x+6, & 1<x\le 2 \end{cases}$ and $g(t)=\int_{t-1}^{t} f(x)dx$ for $t \in [1,2]$

Which of the following hold(s) good ?

(A) $f(x)$ is continuous and differentiable in [0, 2]

(B) $g'(t)$ vanishes for $t = 3/2$ and 2

(C) $g(t)$ is maximum at $t = 3/2$

(D) $g(t)$ is minimum at $t = 1$

MDI053

15. Which one of the following functions is continuous on $(0,\pi)$?

(A) $f(x) = \cot x$

(B) $g(x)=\int_0^x t\sin\frac{1}{t}\,dt$

(C) $h(x)=\begin{cases} 1 & 0<x\le\frac{3\pi}{4} \\ 2\sin\frac{2}{9}x & \frac{3\pi}{4}<x<\pi \end{cases}$

(D) $l(x)=\begin{cases} x\sin x & 0<x\le\frac{\pi}{2} \\ \frac{\pi}{2}\sin(x+\pi) & \frac{\pi}{2}<x<\pi \end{cases}$

MDI055

16. Let $F(x)=\int_{\sin x}^{\cos x} e^{(1+\arcsin t)^2}dt$ on $\left[0,\frac{\pi}{2}\right]$ then

(A) $F''(c) = 0$ for all $c \in \left(0,\frac{\pi}{2}\right)$

(B) $F''(c) = 0$ for some $c \in \left(0,\frac{\pi}{2}\right)$

(C) $F'(c) \neq 0$ for all $c \in \left(0,\frac{\pi}{2}\right)$

(D) $F(c) = 0$ for some $c \in \left(0,\frac{\pi}{2}\right)$

MDI057

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 17 to 19

Let the function f satisfies

$f(x)\cdot f'(-x) = f(-x)\cdot f'(x)$ for all x and $f(0) = 3$.

17. The value of $f(x)\cdot f(-x)$ for all x, is

(A) 4 (B) 9 (C) 12 (D) 16

MDI059

18. $\int_{-51}^{51}\frac{dx}{3+f(x)}$ has the value equal to

(A) 17 (B) 34 (C) 102 (D) 0

MDI060

19. Number of roots of $f(x) = 0$ in $[-2, 2]$ is

(A) 0 (B) 1 (C) 2 (D) 4

MDI061

MATCHING LIST TYPE QUESTION

20. **List-I** **List-II**

	List-I		List-II
(P)	$\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\left(1+\frac{3^2}{n^2}\right)....\left(1+\frac{n^2}{n^2}\right)\right]^{1/n}$	(1)	$3 - \ell n4$
(Q)	$\lim_{n\to\infty}\left[\frac{n!}{n^n}\right]^{1/n}$	(2)	$\tan^{-1}2$
(R)	$\lim_{n\to\infty}\frac{1}{n}\left[\frac{1}{n+1}+\frac{2}{n+2}+......+\frac{3n}{4n}\right]$	(3)	$2\,e^{(1/2)(\pi-4)}$
(S)	$\lim_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+...+\frac{1}{5n}\right)$ is equal to	(4)	$\frac{1}{e}$

(A) (P) $\to$ (3), (Q) $\to$ (4), (R) $\to$ (1), (S) $\to$ (2)

(B) (P) $\to$ (4), (Q) $\to$ (3), (R) $\to$ (2), (S) $\to$ (1)

(C) (P) $\to$ (1), (Q) $\to$ (2), (R) $\to$ (4), (S) $\to$ (3)

(D) (P) $\to$ (2), (Q) $\to$ (3), (R) $\to$ (1), (S) $\to$ (4)

MDI067

EXERCISE - S

1. $\lim_{n\to\infty} n^2 \int_{-1/n}^{1/n} (2010\sin x + 2012\cos x)|x|dx$

MDI021

2. Let $U_n = \int_0^{\frac{\pi}{2}} x\sin^n x dx$, then find the value of $\left(\frac{100U_{10}-1}{U_8}\right)$.

MDI023

3. Let $y = f(x)$ be a quadratic function with $f'(2) = 1$. Find the value of the integral $\int_{2-\pi}^{2+\pi} f(x).\sin\left(\frac{x-2}{2}\right)dx$

MDI024

4. If a_1, a_2 and a_3 are the three values of a which satisfy the equation $\int_0^{\pi/2} (\sin x + a\cos x)^3 dx - \frac{4a}{\pi-2}\int_0^{\pi/2} x\cos x dx = 2$ then find the value of $1000\left(a_1^2+a_2^2+a_3^2\right)$.

MDI025

5. $\int_1^2 \frac{(x^2-1)dx}{x^3.\sqrt{2x^4-2x^2+1}} = \frac{u}{v}$ where u and v are in their lowest form. Find the value of $\frac{(1000)u}{v}$.

MDI026

6. If $\int_0^{\pi} \sqrt{(\cos x+\cos 2x+\cos 3x)^2+(\sin x+\sin 2x+\sin 3x)^2}\,dx$ has the value equal to $\left(\frac{\pi}{k}+\sqrt{w}\right)$ where k and w are positive integers, find the value of (k^2+w^2).

MDI027

7. A continuous real function f satisfies $f(2x) = 3f(x)\ \forall\ x \in R$.If $\int_0^1 f(x)dx = 1$, then compute the value of definite integral $\int_1^2 f(x)dx$

MDI029

8. If $f(x) = x + \sin x$ and I denotes the value of integral $\int_{\pi}^{2\pi}\left(f^{-1}(x)+\sin x\right)dx$ then the value of $\left[\frac{2I}{3}\right]$ (where [.] denotes greatest integer function)

MDI030

9. $\lim_{n\to\infty} \frac{\int_{1/(n+1)}^{1/n} \tan^{-1}(nx)dx}{\int_{1/(n+1)}^{1/n} \sin^{-1}(nx)dx}$ is equal to

MDI069

10. Let $I_n = \int_{-n}^{n}\left(\{x+1\}.\{x^2+2\}+\{x^2+2\}\{x^3+4\}\right)dx$, where {.} denotes the fractional part of x. Find I_1.

MDI070

EXERCISE - JEE (Main) PYQ

1. If $\int_0^x f(t)dt = x^2 + \int_x^1 t^2 f(t)dt$, then $f'(1/2)$ is : **[JEE (Main) 2019]**

(1) $\frac{6}{25}$ (2) $\frac{24}{25}$ (3) $\frac{18}{25}$ (4) $\frac{4}{5}$

MDI038

2. Let $I = \int_a^b (x^4 - 2x^2)dx$. If I is minimum then the ordered pair (a, b) is : **[JEE (Main) 2019]**

(1) $(-\sqrt{2}, 0)$ (2) $(-\sqrt{2}, \sqrt{2})$ (3) $(0, \sqrt{2})$ (4) $(\sqrt{2}, -\sqrt{2})$

MDI039

3. The value of α for which $4\alpha \int_{-1}^{2} e^{-\alpha|x|}dx = 5$, is : **[JEE (Main) 2020]**

(1) $\log_e\left(\frac{3}{2}\right)$ (2) $\log_e\left(\frac{4}{3}\right)$ (3) $\log_e 2$ (4) $\log_e \sqrt{2}$

MDI041

4. If $I = \int_1^2 \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$, then : **[JEE (Main) 2020]**

(1) $\frac{1}{9} < I^2 < \frac{1}{8}$ (2) $\frac{1}{16} < I^2 < \frac{1}{9}$ (3) $\frac{1}{6} < I^2 < \frac{1}{2}$ (4) $\frac{1}{8} < I^2 < \frac{1}{4}$

MDI043

5. Let a be a positive real number such that $\int_0^a e^{x-[x]}dx = 10e - 9$ where $[x]$ is the greatest integer less than or equal to x. Then a is equal to : **[JEE (Main) 2021]**

(1) $10 - \log_e(1 + e)$ (2) $10 + \log_e 2$ (3) $10 + \log_e 3$ (4) $10 + \log_e(1 + e)$

MDI045

6. If $f(x) = \begin{cases} \int_0^x (5 + |1 - t|)dt & ; \quad x > 2 \\ 5x + 1 & ; \quad x \le 2 \end{cases}$, then **[JEE (Main) 2021]**

(1) $f(x)$ is not continuous at $x = 2$

(2) $f(x)$ is everywhere differentiable

(3) $f(x)$ is continuous but not differentiable at $x = 2$

(4) $f(x)$ is not differentiable at $x = 1$

MDI047

7. For $x > 0$, if $f(x) = \int_1^x \frac{\log_e t}{(1+t)}dt$, then $f(e) + f\left(\frac{1}{e}\right)$ is equal to **[JEE (Main) 2021]**

(1) 1 (2) –1 (3) $\frac{1}{2}$ (4) 0

MDI113

8. The value of $\int_0^{\pi} \frac{e^{\cos x}\sin x}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})}dx$ is equal to **[JEE (Main) 2022]**

(1) $\frac{\pi^2}{4}$ (2) $\frac{\pi^2}{2}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

MDI050

9. The value of the integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x+\frac{\pi}{4}}{2-\cos 2x}dx$ is : **[JEE (Main) 2022]**

(1) $\frac{\pi^2}{6}$ (2) $\frac{\pi^2}{12\sqrt{3}}$ (3) $\frac{\pi^2}{3\sqrt{3}}$ (4) $\frac{\pi^2}{6\sqrt{3}}$

MDI052

10. If $[t]$ denotes the greatest integer $\le t$, then the value of $\int_0^1 \left[2x-|3x^2-5x+2|+1\right]dx$ is:

[JEE (Main) 2022]

(1) $\frac{\sqrt{37}+\sqrt{13}-4}{6}$ (2) $\frac{\sqrt{37}-\sqrt{13}-4}{6}$ (3) $\frac{-\sqrt{37}-\sqrt{13}+4}{6}$ (4) $\frac{-\sqrt{37}+\sqrt{13}+4}{6}$

MDI114

11. The value of the integral $\int_{-\log_e 2}^{\log_e 2} e^x\left(\log_e\left(e^x+\sqrt{1+e^{2x}}\right)\right)dx$ is equal to **[JEE (Main) 2023]**

(1) $\log_e\left(\frac{2(2+\sqrt{5})}{\sqrt{1+\sqrt{5}}}\right)-\frac{\sqrt{5}}{2}$ (2) $\log_e\left(\frac{\sqrt{2}(3-\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right)+\frac{\sqrt{5}}{2}$

(3) $\log_e\left(\frac{(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right)+\frac{\sqrt{5}}{2}$ (4) $\log_e\left(\frac{\sqrt{2}(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right)-\frac{\sqrt{5}}{2}$

MDI115

12. Let the function $f:[0,2]\to R$ be defined as

$$f(x)=\begin{cases} e^{\min\{x^2,x-[x]\}}, & x\in[0,1) \\ e^{[x-\log_e x]}, & x\in[1,2] \end{cases}$$

where $[t]$ denotes the greatest integer less than or equal to t. Then the value of the integral $\int_0^2 xf(x)dx$ is **[JEE (Main) 2023]**

(1) $2e-1$ (2) $1+\frac{3e}{2}$ (3) $2e-\frac{1}{2}$ (4) $(e-1)\left(e^2+\frac{1}{2}\right)$

MDI116

13. If $\int_0^{\pi} \frac{5^{\cos x}\left(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\right)dx}{1+5^{\cos x}}=\frac{k\pi}{16}$, then k is equal to _____. **[JEE (Main) 2023]**

MDI118

14. Among :

(S1) : $\lim\limits_{n\to\infty}\frac{1}{n^2}\left(2+4+6+........+2n\right)=1$

(S2) : $\lim\limits_{n\to\infty}\frac{1}{n^{16}}\left(1^{15}+2^{15}+3^{15}+........+n^{15}\right)=\frac{1}{16}$ **[JEE (Main) 2023]**

(1) Both (S1) and (S2) are true
(2) Both (S1) and (S2) are false
(3) Only (S2) is true
(4) Only (S1) is true

MDI119

15. The minimum value of the function $f(x)=\int_0^2 e^{|x-t|}\,dt$ is **[JEE (Main) 2023]**

(1) $2(e-1)$
(2) $2e-1$
(3) 2
(4) $e(e-1)$

MDI120

EXERCISE - JEE (Advanced) PYQ

1. For $a \in R$ (the set of all real numbers), $a \neq -1$.

$$\lim_{n\to\infty}\frac{(1^a+2^a+......+n^a)}{(n+1)^{a-1}[(na+1)+(na+2)+......+(na+n)]}=\frac{1}{60}$$

Then $a =$ **[JEE (Advanced) 2013]**

(A) 5 (B) 7 (C) $\frac{-15}{2}$ (D) $\frac{-17}{2}$

MDI071

2. Let $f: [a, b] \to [1, \infty)$ be a continuous function and let $g: \mathbb{R} \to \mathbb{R}$ be defined as

$$g(x)=\begin{cases}0 & \text{if} \quad x<a \\ \int_a^x f(t)dt & \text{if} \quad a\le x\le b \\ \int_a^b f(t)dt & \text{if} \quad x>b\end{cases}.$$

Then **[JEE (Advanced) 2014]**

(A) $g(x)$ is continuous but not differentiable at a

(B) $g(x)$ is differentiable on $\mathbb{R}$

(C) $g(x)$ is continuous but not differentiable at b

(D) $g(x)$ is continuous and differentiable at either a or b but not both.

MDI072

3. The value of $\int_0^1 4x^3\left\{\frac{d^2}{dx^2}\left(1-x^2\right)^5\right\}dx$ is **[JEE (Advanced) 2014]**

MDI073

4. The following integral $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(2\operatorname{cosec} x)^{17}dx$ is equal to - **[JEE (Advanced) 2014]**

(A) $\int_0^{\log(1+\sqrt{2})}2(e^u+e^{-u})^{16}du$

(B) $\int_0^{\log(1+\sqrt{2})}(e^u+e^{-u})^{17}du$

(C) $\int_0^{\log(1+\sqrt{2})}(e^u-e^{-u})^{17}du$

(D) $\int_0^{\log(1+\sqrt{2})}2(e^u-e^{-u})^{16}du$

MDI074

5. Let $f : [0, 2] \to \mathbb{R}$ be a function which is continuous on [0, 2] and is differentiable on (0, 2) with $f(0) = 1$. Let $F(x)=\int_0^{x^2} f(\sqrt{t})dt$ for $x \in [0, 2]$. If $F'(x) = f'(x)$ for all $x \in (0, 2)$, then $F(2)$ equals -

[JEE (Advanced) 2014]

(A) $e^2 - 1$ (B) $e^4 - 1$ (C) $e - 1$ (D) e^4

MDI075

Paragraph For Questions 6 and 7

Given that for each $a \in (0,1)$, $\lim_{h\to 0^+} \int_h^{1-h} t^{-a}(1-t)^{a-1}\,dt$ exists. Let this limit be $g(a)$. In addition, it is given that the function $g(a)$ is differentiable on $(0,1)$.

6. The value of $g\left(\frac{1}{2}\right)$ is - **[JEE (Advanced) 2014]**

(A) π (B) 2π (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

MDI076

7. The value of $g'\left(\frac{1}{2}\right)$ is- **[JEE (Advanced) 2014]**

(A) $\frac{\pi}{2}$ (B) π (C) $-\frac{\pi}{2}$ (D) 0

MDI077

8. Let $f:(0,\infty)\to\mathbb{R}$ be given by $f(x)=\int_{\frac{1}{x}}^{x} e^{-\left(t+\frac{1}{t}\right)}\frac{dt}{t}$. Then **[JEE (Advanced) 2014]**

(A) $f(x)$ is monotonically increasing on $[1,\infty)$
(B) $f(x)$ is monotonically decreasing on $[0, 1)$
(C) $f(x)+f\left(\frac{1}{x}\right)=0$, for all $x \in (0,\infty)$
(D) $f(2^x)$ is an odd function of x on $\mathbb{R}$

MDI078

9.

	List-I		List-II
P.	The number of polynomials $f(x)$ with non-negative integer coefficients of degree ≤ 2, satisfying $f(0) = 0$ and $\int_0^1 f(x)dx=1$, is	**1.**	8
Q.	The number of points in the interval $\left[-\sqrt{13},\sqrt{13}\right]$ at which $f(x) = \sin(x^2) + \cos(x^2)$ attains its maximum value, is	**2.**	2
R.	$\int_{-2}^{2}\frac{3x^2}{(1+e^x)}dx$ equals	**3.**	4
S.	$\frac{\left(\int_{-\frac{1}{2}}^{\frac{1}{2}}\cos 2x\log\left(\frac{1+x}{1-x}\right)dx\right)}{\left(\int_{0}^{\frac{1}{2}}\cos 2x\log\left(\frac{1+x}{1-x}\right)dx\right)}$ equals	**4.**	0

Codes : **[JEE (Advanced) 2014]**

	P	Q	R	S
(A)	3	2	4	1
(B)	2	3	4	1
(C)	3	2	1	4
(D)	2	3	1	4

MDI079

10. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x)=\begin{cases}[x] & , \quad x\le 2 \\ 0 & , \quad x>2\end{cases}$,

where $[x]$ is the greatest integer less than or equal to x. If $I=\int_{-1}^{2}\frac{xf\left(x^2\right)}{2+f\left(x+1\right)}dx$, then the value of $(4I-1)$ is **[JEE (Advanced) 2015]**

MDI080

11. If $\alpha=\int_{0}^{1}\left(e^{9x+3\tan^{-1}x}\right)\left(\frac{12+9x^2}{1+x^2}\right)dx$, where $\tan^{-1}x$ takes only principal values, then the value of $\left(\log_e|1+\alpha|-\frac{3\pi}{4}\right)$ is **[JEE (Advanced) 2015]**

MDI081

12. Let $f:\mathbb{R}\to\mathbb{R}$ be a continuous odd function, which vanishes exactly at one point and $f(1)=\frac{1}{2}$.

Suppose that $F(x)=\int_{-1}^{x}f(t)dt$ for all $x\in[-1,2]$ and $G(x)=\int_{-1}^{x}t\left|f\left(f\left(t\right)\right)\right|dt$ for all $x\in[-1,2]$.

If $\lim_{x\to1}\frac{F(x)}{G(x)}=\frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is **[JEE (Advanced) 2015]**

MDI082

13. The option(s) with the values of a and L that satisfy the following equation is(are)

$$\frac{\int_{0}^{4\pi}e^t(\sin^6 at+\cos^4 at)dt}{\int_{0}^{\pi}e^t(\sin^6 at+\cos^4 at)dt}=L\ ?$$ **[JEE (Advanced) 2015]**

(A) $a=2,\ L=\frac{e^{4\pi}-1}{e^{\pi}-1}$ (B) $a=2,\ L=\frac{e^{4\pi}+1}{e^{\pi}+1}$ (C) $a=4,\ L=\frac{e^{4\pi}-1}{e^{\pi}-1}$ (D) $a=4,\ L=\frac{e^{4\pi}+1}{e^{\pi}+1}$

MDI083

14. Let $f(x)=7\tan^8x+7\tan^6x-3\tan^4x-3\tan^2x$ for all $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

Then the correct expression(s) is(are) **[JEE (Advanced) 2015]**

(A) $\int_{0}^{\pi/4}xf(x)dx=\frac{1}{12}$ (B) $\int_{0}^{\pi/4}f(x)dx=0$

(C) $\int_{0}^{\pi/4}xf(x)dx=\frac{1}{6}$ (D) $\int_{0}^{\pi/4}f(x)dx=1$

MDI084

15. Let $f'(x)=\frac{192x^3}{2+\sin^4\pi x}$ for all $x\in\mathbb{R}$ with $f\left(\frac{1}{2}\right)=0$. If $m\le\int_{1/2}^{1}f(x)dx\le M$, then the possible values of m and M are **[JEE (Advanced) 2015]**

(A) $m=13, M=24$ (B) $m=\frac{1}{4}, M=\frac{1}{2}$

(C) $m=-11, M=0$ (D) $m=1, M=12$

MDI085

Paragraph For Questions 16 and 17

Let $F:\mathbb{R}\to\mathbb{R}$ be a thrice differentiable function. Suppose that $F(1)=0, F(3)=-4, F'(x)<0$ for all $x\in(1/2,3)$. Let $f(x)=xF(x)$ for all $x\in\mathbb{R}$.

16. The correct statement(s) is(are) **[JEE (Advanced) 2015]**

(A) $f'(1)<0$ (B) $f(2)<0$

(C) $f'(x)\neq 0$ for any $x\in(1,3)$ (D) $f'(x)=0$ for some $x\in(1,3)$

MDI086

17. If $\int_1^3 x^2F'(x)dx=-12$ and $\int_1^3 x^3F''(x)dx=40$, then the correct expression(s) is(are) **[JEE (Advanced) 2015]**

(A) $9f'(3)+f'(1)-32=0$ (B) $\int_1^3 f(x)dx=12$

(C) $9f'(3)-f'(1)+32=0$ (D) $\int_1^3 f(x)dx=-12$

MDI087

18. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{x^2\cos x}{1+e^x}dx$ is equal to **[JEE (Advanced) 2016]**

(A) $\frac{\pi^2}{4}-2$ (B) $\frac{\pi^2}{4}+2$ (C) $\pi^2-e^{\frac{\pi}{2}}$ (D) $\pi^2+e^{\frac{\pi}{2}}$

MDI088

19. The total number of distinct $x\in[0,1]$ for which $\int_0^x\frac{t^2}{1+t^4}dt=2x-1$ is **[JEE (Advanced) 2016]**

MDI089

20. Let $f(x)=\lim_{n\to\infty}\left(\frac{n^n(x+n)\left(x+\frac{n}{2}\right).....\left(x+\frac{n}{n}\right)}{n!(x^2+n^2)\left(x^2+\frac{n^2}{4}\right).....\left(x^2+\frac{n^2}{n^2}\right)}\right)^{x/n}$, for all $x>0$. Then **[JEE (Advanced) 2016]**

(A) $f\left(\frac{1}{2}\right)\geq f(1)$ (B) $f\left(\frac{1}{3}\right)\leq f\left(\frac{2}{3}\right)$ (C) $f'(2)\leq 0$ (D) $\frac{f'(3)}{f(3)}\geq\frac{f'(2)}{f(2)}$

MDI090

21. Let $f:R\to R$ be a differentiable function such that $f(0)=0, f\left(\frac{\pi}{2}\right)=3$ and $f'(0)=1$. If

$g(x)=\int_x^{\frac{\pi}{2}}\left[f'(t)\operatorname{cosec} t-\cot t\operatorname{cosec} t\, f(t)\right]dt$ for $x\in\left(0,\frac{\pi}{2}\right]$, then $\lim_{x\to 0}g(x)=$

[JEE (Advanced) 2017]

MDI091

22. If $I=\sum_{k=1}^{98}\int_{k}^{k+1}\frac{k+1}{x(x+1)}dx$, then **[JEE (Advanced) 2017]**

(A) $I<\frac{49}{50}$ (B) $I<\log_e 99$ (C) $I>\frac{49}{50}$ (D) $I>\log_e 99$

MDI092

23. If $g(x)=\int_{\sin x}^{\sin(2x)}\sin^{-1}(t)\,dt$, then **[JEE (Advanced) 2017]**

(A) $g'\left(\frac{\pi}{2}\right)=-2\pi$ (B) $g'\left(-\frac{\pi}{2}\right)=2\pi$ (C) $g'\left(\frac{\pi}{2}\right)=2\pi$ (D) $g'\left(-\frac{\pi}{2}\right)=-2\pi$

MDI093

24. Let $f:\mathbb{R}\to(0,1)$ be a continuous function. Then, which of the following function(s) has(have) the value zero at some point in the interval (0, 1) ? **[JEE (Advanced) 2017]**

(A) $e^x-\int_0^x f(t)\sin t\,dt$ (B) $x^9-f(x)$

(C) $f(x)+\int_0^{\frac{\pi}{2}} f(t)\sin t\,dt$ (D) $x-\int_0^{\frac{\pi}{2}-x} f(t)\cos t\,dt$

MDI094

25. For each positive integer n, let $y_n=\frac{1}{n}(n+1)(n+2)\ldots(n+n)^{1/n}$ For $x\in\mathbb{R}$, let $[x]$ be the greatest integer less than or equal to x. If $\lim_{n\to\infty}y_n=L$, then the value of $[L]$ is _____. **[JEE (Advanced) 2018]**

MDI095

26. The value of the integral $\int_0^{\frac{1}{2}}\frac{1+\sqrt{3}}{\left((x+1)^2(1-x)^6\right)^{\frac{1}{4}}}dx$ is _____ . **[JEE (Advanced) 2018]**

MDI096

27. Let $f:\mathbb{R}\to\mathbb{R}$ be given by $f(x)=(x-1)(x-2)(x-5)$. Define $F(x)=\int_0^x f(t)dt$, $x>0$. Then which of the following options is/are correct ? **[JEE (Advanced) 2019]**

(A) F has a local minimum at $x=1$

(B) F has a local maximum at $x=2$

(C) $F(x)\neq 0$ for all $x\in(0,5)$

(D) F has two local maxima and one local minimum in $(0,\infty)$

MDI097

28. If $I=\frac{2}{\pi}\int_{-\pi/4}^{\pi/4}\frac{dx}{(1+e^{\sin x})(2-\cos 2x)}$ then $27I^2$ equals ____ **[JEE (Advanced) 2019]**

MDI098

29. For $a\in\mathbb{R}$, $|a|>1$, let $\lim_{n\to\infty}\left(\frac{1+\sqrt[3]{2}+.....+\sqrt[3]{n}}{n^{7/3}\left(\frac{1}{(an+1)^2}+\frac{1}{(an+2)^2}+.......+\frac{1}{(an+n)^2}\right)}\right)=54$. Then the possible value(s) of a is/are : **[JEE (Advanced) 2019]**

(A) 8 (B) –9 (C) –6 (D) 7

MDI099

30. The value of the integral $\int\limits_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5} d\theta$ equals **[JEE (Advanced) 2019]**

MDI100

31. Which of the following inequalities is/are TRUE ? **[JEE (Advanced) 2020]**

(A) $\int\limits_0^1 x\cos x dx \geq \frac{3}{8}$ (B) $\int\limits_0^1 x\sin x dx \geq \frac{3}{10}$ (C) $\int\limits_0^1 x^2 \cos x\, dx \geq \frac{1}{2}$ (D) $\int\limits_0^1 x^2 \sin x\, dx \geq \frac{2}{9}$

MDI101

32. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that its derivative f' is continuous and $f(\pi) = -6$.

If F: $[0, \pi] \to \mathbb{R}$ is defined by $F(x) = \int_0^x f(t)dt$, and if $\int\limits_0^{\pi} (f'(x)+F(x))\cos x\, dx = 2$, then the value of $f(0)$ is ____ **[JEE (Advanced) 2020]**

MDI102

33. Let $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to \mathbb{R}$ be a continuous function such that $f(0) = 1$ and $\int_0^{\frac{\pi}{3}} f(t)\, dt = 0$

Then which of the following statements is (are) **TRUE** ? **[JEE (Advanced) 2021]**

(A) The equation $f(x) - 3\cos 3x = 0$ has at least one solution in $\left(0, \frac{\pi}{3}\right)$

(B) The equation $f(x) - 3\sin 3x = -\frac{6}{\pi}$ has at least one solution in $\left(0, \frac{\pi}{3}\right)$

(C) $\lim\limits_{x\to 0} \frac{x\int_0^x f(t)dt}{1-e^{x^2}} = -1$

(D) $\lim\limits_{x\to 0} \frac{\sin x\int_0^x f(t)dt}{x^2} = -1$

MDI103

Question Stem for Questions Nos. 34 and 35

Question Stem

Let $g_i : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to \mathbb{R}$, $i = 1, 2$, and $f : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to \mathbb{R}$ be functions such that

$g_1(x) = 1$, $g_2(x) = |4x - \pi|$ and $f(x) = \sin^2 x$, for all $x \in \left[\frac{\pi}{8}, \frac{3\pi}{8}\right]$

Define $S_i = \int\limits_{\frac{\pi}{8}}^{\frac{3\pi}{8}} f(x)\cdot g_i(x) dx$, $i = 1, 2$

34. The value of $\frac{16S_1}{\pi}$ is ____. **[JEE (Advanced) 2021]**

MDI104

35. The value of $\frac{48S_2}{\pi^2}$ is ____. **[JEE (Advanced) 2021]**

MDI105

Paragraph

Let $\psi_1 : [0, \infty) \to \mathbb{R}$, $\psi_2 : [0, \infty) \to \mathbb{R}$, $f : [0, \infty) \to \mathbb{R}$ and $g: [0, \infty) \to \mathbb{R}$ be functions such that

$f(0) = g(0) = 0$,

$\psi_1(x) = e^{-x} + x, x \geq 0$,

$\psi_2(x) = x^2 - 2x - 2e^{-x} + 2, x \geq 0$,

$f(x) = \int_{-x}^{x} (|t| - t^2) e^{-t^2}\, dt, x > 0$

and

$g(x) = \int_0^{x^2} \sqrt{t}\, e^{-t}\, dt,\ x > 0$

36. Which of the following statements is **TRUE** ? **[JEE (Advanced) 2021]**

(A) $f(\sqrt{\ell n 3}) + g(\sqrt{\ell n 3}) = \frac{1}{3}$

(B) For every $x > 1$, there exists an $\alpha \in (1, x)$ such that $\psi_1(x) = 1 + \alpha x$

(C) For every $x > 0$, there exists *a* $\beta \in (0, x)$ such that $\psi_2(x) = 2x(\psi_1(\beta) - 1)$

(D) f is an increasing function on the interval $\left[0, \frac{3}{2}\right]$

MDI106

37. Which of the following statements is **TRUE** ? **[JEE (Advanced) 2021]**

(A) $\psi_1(x) \leq 1$, for all $x > 0$

(B) $\psi_2(x) \leq 0$, for all $x > 0$

(C) $f(x) \geq 1 - e^{-x^2} - \frac{2}{3}x^3 + \frac{2}{5}x^5$, for all $x \in \left(0, \frac{1}{2}\right)$

(D) $g(x) \leq \frac{2}{3}x^3 - \frac{2}{5}x^5 + \frac{1}{7}x^7$, for all $x \in \left(0, \frac{1}{2}\right)$

MDI107

38. For any real number x, let $[x]$ denote the largest integer less than or equal to x. If $I = \int_0^{10} \left[\sqrt{\frac{10x}{x+1}}\right] dx$, then the value of $9I$ is ____. **[JEE (Advanced) 2021]**

MDI108

39. Consider the equation $\int_1^e \frac{(\log_e x)^{1/2}}{x\left(a - (\log_e x)^{3/2}\right)^2} dx = 1$, $a \in (-\infty, 0) \cup (1, \infty)$.

Which of the following statements is/are TRUE ? **[JEE (Advanced) 2022]**

(A) **No** a satisfies the above equation

(B) An integer a satisfies the above equation

(C) An irrational number a satisfies the above equation

(D) More than one a satisfy the above equation

MDI109

40. The greatest integer less than or equal to **[JEE (Advanced) 2022]**

$$\int_{1}^{2}\log_2(x^3+1)dx+\int_{1}^{\log_2 9}\left(2^x-1\right)^{\frac{1}{3}}dx$$

is_____.

MDI110

41. For positive integer n, define

$$f(n)=n+\frac{16+5n-3n^2}{4n+3n^2}+\frac{32+n-3n^2}{8n+3n^2}+\frac{48-3n-3n^2}{12n+3n^2}+\ldots+\frac{25n-7n^2}{7n^2}.$$

Then, the value of $\lim_{n\to\infty} f(n)$ is equal to **[JEE (Advanced) 2022]**

(A) $3+\frac{4}{3}\log_e 7$ (B) $4-\frac{3}{4}\log_e\left(\frac{7}{3}\right)$ (C) $4-\frac{4}{3}\log_e\left(\frac{7}{3}\right)$ (D) $3+\frac{3}{4}\log_e 7$

MDI111

42. Let $f:[(1,\infty)\to\mathbb{R}$ be a differentiable function such that $f(1)=\frac{1}{3}$ and

$3\int_1^x f(t)dt=xf(x)-\frac{x^3}{3}, x\in[1,\infty)$. Let e denote the base of the natural logarithm. Then the value of $f(e)$ is **[JEE (Advanced) 2023]**

(A) $\frac{e^2+4}{3}$ (B) $\frac{\log_e 4+e}{3}$ (C) $\frac{4e^2}{3}$ (D) $\frac{e^2-4}{3}$

MDI121

43. For $x\in\mathbb{R}$, let $\tan^{-1}(x)\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Then the minimum value of the function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=\int_0^{x\tan^{-1}x}\frac{e^{(t-\cos t)}}{1+t^{2023}}dt$ is **[JEE (Advanced) 2023]**

MDI122

JEE (Main) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION–A

- This section contains **TWENTY** questions.
- Each question has **FOUR** options (1), (2), (3) and (4). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +4, if only the bubble corresponding to the correct option is darkened.

Zero Marks : 0, if none of the bubbles is darkened.

Negative Marks : –1 in all other cases.

1. If $\int_{\ell n 2}^{x} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}$, then x is equal to

(1) 4 (2) $\ell n\ 8$ (3) $\ell n\ 4$ (4) $\ell n\ 2$

MDI123

2. $\int_0^{\infty} \frac{x^2+1}{x^4+7x^2+1} dx =$

(1) π (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{6}$

MDI124

3. The value of $\int_0^{[x]} \{x\} dx$ (where [.] and { . } denotes greatest integer and fraction part function respectively) is

(1) $\frac{1}{2}[x]$ (2) $2[x]$ (3) $\frac{1}{2[x]}$ (4) $[x]$

MDI125

4. If $\int_0^{11} \frac{11^x}{11^{[x]}} dx = \frac{k}{\ln 11}$, (where [ ] denotes greatest integer function) then value of k is

(1) 11 (2) 101 (3) 110 (4)121

MDI126

5. The value of $\int_0^{\pi/2} \ell n|\tan x + \cot x|\ dx$ is equal to :

(1) $\pi \ell n\ 2$ (2) $-\pi\ \ell n\ 2$ (3) $\frac{\pi}{2}\ \ell n\ 2$ (4) $-\frac{\pi}{2}\ \ell n\ 2$

MDI127

6. $f(x) = \int_1^x \frac{\sin x \cos y}{y^2+y+1} dy$, then

(1) $f'(x) = 0 \ \forall \ x = \frac{n\pi}{2}, n \in Z$ (2) $f'(x) = 0 \ \forall \ x = (2n+1)\frac{\pi}{2}, n \in Z$

(3) $f'(x) = 0 \ \forall \ x = n\pi, n \in Z$ (4) $f'(x) \neq 0 \ \forall \ x \in R$

MDI128

7. $\int_0^1 x^2(1-x)^3 dx$ is equal to

(1) $\frac{1}{60}$ (2) $\frac{1}{30}$ (3) $\frac{2}{15}$ (4) $\frac{\pi}{120}$

MDI129

8. Let $I = \int_1^3 \sqrt{x^4+x^2}\, dx$, then

(1) $I > 6\sqrt{10}$ (2) $I < 2\sqrt{2}$

(3) $2\sqrt{2} < I < 6\sqrt{10}$ (4) $I < 1$

MDI130

9. $\lim_{n\to\infty} \frac{\pi}{n}\left[\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+.....+\sin\frac{(n-1)\pi}{n}\right]$ is equals to:

(1) 0 (2) π (3) 2 (4) 3

MDI131

10. Let $I_n = \int_0^1 (1-x^3)^n dx,\ (n \in N)$ then

(1) $3nI_n = (3n-1)I_{n-1} \ \forall \ n \geq 2$ (2) $(3n-1)I_n = 3nI_{n-1} \forall n \geq 2$

(3) $(3n-1)I_n = (3n+1)I_{n-1} \forall n \geq 2$ (4) $(3n+1)I_n = 3nI_{n-1} \forall n \geq 2$

MDI132

11. Let $A = \int_0^1 \frac{e^t}{1+t} dt$, then $\int_{a-1}^{a} \frac{e^{-t}}{t-a-1} dt$ has the value :

(1) Ae^{-a} (2) $-Ae^{-a}$ (3) $-ae^{-a}$ (4) Ae^{a}

MDI133

12. $\int_1^2 x^{2x^2+1}(1+2\ell n\, x)dx$ is equal to

(1) 256 (2) 255 (3) $\frac{255}{2}$ (4) 128

MDI134

13. If $f(x)$ is a function satisfying $f\left(\frac{1}{x}\right) + x^2 f(x) = 0$ for all non-zero x, then $\int_{\sin\theta}^{\mathrm{cosec}\theta} f(x)\, dx$ equals to:

(1) $\sin\theta + \mathrm{cosec}\theta$ (2) $\sin^2\theta$ (3) $\mathrm{cosec}^2\theta$ (4) none of these

MDI135

14. If $\frac{C_0}{1}+\frac{C_1}{2}+\frac{C_2}{3}=0$, where C_0, C_1, C_2 are all real, the equation $C_2x^2 + C_1x + C_0 = 0$ has:

(1) atleast one root in $(0, 1)$
(2) one root in $(1, 2)$ & other in $(3, 4)$
(3) one root in $(-1, 1)$ & the other in $(-5, -2)$
(4) both roots imaginary

MDI136

15. If $f(x) = \int_0^x (2\cos^2 3t + 3\sin^2 3t)\,dt$, $f(x + \pi)$ is equal to :

(1) $f(x) + 2f(\pi)$
(2) $f(x) + 2f\left(\frac{\pi}{2}\right)$
(3) $f(x) + 4f\left(\frac{\pi}{4}\right)$
(4) $2f(x)$

MDI137

16. Let $f(x) = \int_0^x \frac{dt}{\sqrt{1+t^3}}$ and $g(x)$ be the inverse of $f(x)$, then which one of the following holds good?

(1) $2g'' = g^2$
(2) $2g'' = 3g^2$
(3) $3g'' = 2g^2$
(4) $3g'' = g^2$

MDI138

17. Let $f(x)$ is differentiable function satisfying $2\int_1^2 f(tx)dt = x + 2$, $\forall\ x \in R$.

Then $\int_0^1 (8f(8x) - f(x) - 21x)dx$ equals to

(1) 3
(2) 5
(3) 7
(4) 9

MDI139

18. Let $I_n = \int_0^1 x^n(\tan^{-1}x)dx$, $n \in N$, then

(1) $(n + 1)I_n + (n - 1)I_{n-2} = \frac{\pi}{4}+\frac{1}{n}\ \forall\ n \geq 3$

(2) $(n + 1)I_n + (n - 1)I_{n-2} = \frac{\pi}{2}-\frac{1}{n}\ \forall\ n \geq 3$

(3) $(n + 1)I_n - (n - 1)\,I_{n-2} = \frac{\pi}{4}+\frac{1}{n}\ \forall\ n \geq 3$

(4) $(n + 1)I_n - (n - 1)I_{n-2} = \frac{\pi}{2}-\frac{1}{n}\ \forall\ n \geq 3$

MDI140

19. The value of $\int_{1/e}^{\tan x} \frac{t}{1+t^2}dt + \int_{1/e}^{\cot x} \frac{1}{t(1+t^2)}dt$, where $x \in (\pi/6, \pi/3)$, is equal to :

(1) 0
(2) 2
(3) 1
(4) cannot be determined

MDI141

20. Let $A_1 = \int_0^x \left(\int_0^u f(t)\,dt\right) du$ and $A_2 = \int_0^x f(u).(x-u)\,du$ then $\frac{A_1}{A_2}$ is equal to :

(1) $\frac{1}{2}$
(2) 1
(3) 2
(4) -1

MDI142

SECTION-B

- This section will have **TEN** questions. Candidate can choose to attempt any 5 question out of these 10 questions. In case if candidate attempts more than 5 questions, first 5 attempted questions will be considered for marking.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (Answer should be rounded off to the nearest integer).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4, if only correct answer is given.
Zero Marks	:	0, if no answer is given.
Negative Marks	:	-1 for incorrect answer

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1. $\int_2^4 \frac{3x^2+1}{(x^2-1)^3}dx = \frac{\lambda}{n^2}$ where $\lambda, n \in N$ and $gcd(\lambda, n) = 1$, then find the value of $\lambda + n$

MDI143

2. Let $U = \int_{\pi/6}^{\pi/2} \min.(\sqrt{3}\sin x, \cos x)\,dx$ and $V = \int_{-3}^{5} x^2\ sgn(x-1)dx$. If $V = \lambda U$, then find the value of λ.

[**Note :** sgn k denotes the signum function of k.]

MDI144

3. Let $f(x)$ be a function satisfying $f(x) = f\left(\frac{100}{x}\right)\ \forall x > 0$. If $\int_1^{10} \frac{f(x)}{x}dx = 5$ then find the value of $\int_1^{100} \frac{f(x)}{x}dx$

MDI145

4. Evaluate $\dfrac{2005 \int_0^{1002} \frac{dx}{\sqrt{1002^2-x^2}+\sqrt{1003^2-x^2}} + \int_{1002}^{1003} \sqrt{1003^2-x^2}\,dx}{\int_0^1 \sqrt{1-x^2}\,dx} = k$, then find the sum of squares of digits of natural number k.

MDI146

5. If $\int_0^{\pi/2} \sqrt{\sin 2\theta}.\sin\theta\, d\theta = \frac{\pi}{n}$ then find n

MDI147

6. Let $I_1 = \int_0^{\pi/4} (1+\tan x)^2 dx$, $I_2 = \int_0^1 \frac{dx}{(1+x)^2(1+x^2)}$

then find the value of $\frac{I_1}{I_2}$

MDI148

7. Find the value of $\ell n\left(\int_{0}^{1} e^{t^2+t}(2t^2+t+1)dt\right)$

MDI149

8. If $\int_{-1}^{0} \frac{x}{x+1+e^x}dx$ is equal to $-\ell nk$, then find the value of k.

MDI150

9. If f, g, h be continuous functions on $[0, a]$ such that $f(a-x) = f(x)$, $g(a-x) = -g(x)$ and $3h(x) - 4h(a-x) = 5$, then find the value of $\int_{0}^{a} f(x)\, g(x)\, h(x)dx$

MDI151

10. If $f(x) = \frac{\sin x}{x}\ \forall\ x \in (0, \pi]$, If $\frac{\pi}{k}\int_{0}^{\pi/2} f(x)\ f\left(\frac{\pi}{2}-x\right)dx = \int_{0}^{\pi} f(x)\ dx$ then find the value of k.

MDI152

JEE (Advanced) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION–I

- This section contains **SIX** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : –1 In all other cases.

1. The value of $\int_0^1 (\{2x\}-1)(\{3x\}-1)\,dx$, (where {.} denotes fractional part of x) is equal to :

(A) $\frac{19}{36}$ (B) $\frac{19}{144}$ (C) $\frac{19}{72}$ (D) $\frac{19}{18}$

MDI153

2. If $\int_0^{100} f(x)\,dx = a$, then $\sum_{r=1}^{100}\left(\int_0^1 f(r-1+x)\,dx\right) =$

(A) 100 a (B) a (C) 0 (D) 10a

MDI154

3. $\lim_{t\to\left(\frac{\pi}{2}\right)^-} \int_0^t \tan\theta\sqrt{\cos\theta}\,\ell n(\cos\theta)d\theta$ is equal to

(A) –4 (B) 4 (C) –2 (D) Does not exists

MDI155

4. If $f(x) = \begin{cases} 0, & \text{where } x = \frac{n}{n+1},\ n=1, 2, 3..... \\ 1, & \text{else where} \end{cases}$, then the value of $\int_0^2 f(x)\,dx$.

(A) 1 (B) 0 (C) 2 (D) ∞

MDI156

5. If $\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$, then $\int_0^\infty e^{-ax^2}\,dx$ where $a > 0$ is:

(A) $\frac{\sqrt{\pi}}{2}$ (B) $\frac{\sqrt{\pi}}{2a}$ (C) $2\frac{\sqrt{\pi}}{a}$ (D) $\frac{1}{2}\sqrt{\frac{\pi}{a}}$

MDI157

6. If $\sum_{i=1}^{4}\left(\sin^{-1}x_i + \cos^{-1}y_i\right) = 6\pi$, then $\int_{\sum_{i=1}^{4}x_i}^{\sum_{i=1}^{4}y_i} x\ln(1+x^2)\left(\frac{e^x}{1+e^{2x}}\right)dx$ is equal to

(A) 0 (B) $e^4 + e^{-4}$ (C) $ln\left(\frac{17}{12}\right)$ (D) $e^4 - e^{-4}$

MDI158

SECTION-II

- This section contains **SEVEN** questions.
- Each question has **FOUR** options for correct answer(*s*). **ONE OR MORE THAN ONE** of these four option(*s*) is (are) correct option(*s*).
- For each question, choose the correct option(*s*) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 if only (all) the correct option(*s*) is (are) chosen.
Partial Marks : +3 if all the four options are correct but ONLY three options are chosen.
Partial Marks : +2 if three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks : +1 if two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks : 0 if none of the options is chosen (i.e. the question is unanswered).
Negative Marks : –2 in all other cases.

For Example : If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(*s*) (second option in this case), with or without selection of any correct option(*s*) will result in –2 marks.

7. If $I_n = \int_0^1 \frac{dx}{\left(1+x^2\right)^n}$; $n \in N$, then which of the following statements hold good?

(A) $2n\, I_{n+1} = 2^{-n} + (2n-1)\, I_n$
(B) $I_2 = \frac{\pi}{8} + \frac{1}{4}$
(C) $I_2 = \frac{\pi}{8} - \frac{1}{4}$
(D) $I_3 = \frac{\pi}{16} - \frac{5}{48}$

MDI159

8. Given f is an odd function defined everywhere, periodic with period 2 and integrable on every interval. Let $g(x) = \int_0^x f(t)\, dt$. Then :

(A) $g(2n) = 0$ for every integer n
(B) $g(x)$ is an even function
(C) $g(x)$ and $f(x)$ have the same period
(D) $g(x)$ is an odd function

MDI160

9. Let $f : R \to R$ be defined as $f(x) = \int_{-1}^{e^x} \frac{dt}{1+t^2} + \int_{1}^{e^{-x}} \frac{dt}{1+t^2}$, then

(A) $f(x)$ is periodic
(B) $f(f(x)) = f(x)\ \forall\ x \in R$
(C) $f(1) = f'(1) = \frac{\pi}{2}$
(D) $f(x)$ is unbounded

MDI161

10. If $a, b \in R^{+}$ then $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{n}{(k+an)(k+bn)}$ is equal to

(A) $\dfrac{1}{a-b}\ \ln\dfrac{b(b+1)}{a(a+1)}$ if $a \neq b$

(B) $\dfrac{1}{a-b}\ \ln\dfrac{a(b+1)}{b(a+1)}$ if $a \neq b$

(C) non existent if $a = b$

(D) $\dfrac{1}{a(1+a)}$ if $a = b$

MDI162

11. Let $f(x) = \int\limits_{x}^{x+\frac{\pi}{3}} |\sin\theta|\,d\theta\ \ (x \in [0,\pi])$

(A) $f(x)$ is strictly increasing in this interval

(B) $f(x)$ is differentiable in this interval

(C) Range of $f(x)$ is $\left[2-\sqrt{3}, 1\right]$

(D) $f(x)$ has a maxima at $x = \dfrac{\pi}{3}$

MDI163

12. Let $I_n = \int\limits_{0}^{\pi}\dfrac{\sin^2(nx)}{\sin^2 x}\,dx$, $n \in N$, then

(A) $I_{n+2} + I_n = 2I_{n+1}$

(B) $I_n = I_{n+1}$

(C) $I_n = n\pi$

(D) $I_1, I_2, I_3, ...I_n$ are in Harmonic progression

MDI164

13. $\lim\limits_{n\to\infty}\dfrac{(1^k+2^k+3^k+.....+n^k)}{(1^2+2^2+.....+n^2)(1^3+2^3+........+n^3)} = F(k)$, then $(k \in N)$

(A) $F(k)$ is finite for $k \leq 6$

(B) $F(5) = 0$

(C) $F(6) = \dfrac{12}{7}$

(D) $F(6) = \dfrac{5}{7}$

MDI165

SECTION–III

- This section contains **ONE** paragraph.
- Based on each paragraph, there are **THREE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D) **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +3 if only the bubble corresponding to the correct answer is darkened.

Zero Marks : 0 in all other cases.

Comprehension (Q. No. 14 - 16)

If $y = \int\limits_{u(x)}^{v(x)} f(t)dt$, let us define $\dfrac{dy}{dx}$ in a different manner as $\dfrac{dy}{dx} = v'(x)\,f^2(v(x)) - u'(x)\,f^2(u(x))$ and the equation of the tangent at (a, b) as $y - b = \left(\dfrac{dy}{dx}\right)_{(a,b)}(x - a)$

14. If $y = \int_{x}^{x^2} t^2 \, dt$, then equation of tangent at $x = 1$ is

(A) $y = x + 1$ (B) $x + y = 1$ (C) $y = x - 1$ (D) $y = x$

MDI166

15. If $F(x) = \int_{1}^{x} e^{t^2/2} (1 - t^2)\, dt$, then $\frac{d}{dx}F(x)$ at $x = 1$ is

(A) 0 (B) 1 (C) 2 (D) – 1

MDI167

16. If $y = \int_{x^3}^{x^4} \ell nt \, dt$, then $\lim_{x \to 0^+} \frac{dy}{dx}$ is

(A) 0 (B) 1 (C) 2 (D) – 1

MDI168

SECTION–IV

- This section contains **ONE** question.
- Each question contains two columns, **Column-I** and **Column-II.**
- **Column-I** has **four** entries (A), (B), (C) and (D).
- **Column-II** has **five** entries (P), (Q), (R), (S) and (T).
- Match the entries in **Column-I** with the entries in **column-II.**
- One or more entries in **Column-I** may match with one or more entries in **Column-II.**
- The ORS contains a 4 × 5 matrix whose layout will be similar to the one shown below:

(A)	(P)	(Q)	(R)	(S)	(T)
(B)	(P)	(Q)	(R)	(S)	(T)
(C)	(P)	(Q)	(R)	(S)	(T)
(D)	(P)	(Q)	(R)	(S)	(T)

- For each entry in **column-I,** darken the bubbles of all the matching entries. For example, if entry (A) in **Column-I** matches with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).
- For each question, marks will be awarded in one of the following categories :
  For each entry in **Column-I**

Full Marks	:	+2	if only the bubble(s) corresponding to all the correct match(es) is (are) darkened.
Zero Marks	:	0	if none of the bubbles is darkened.
Negative Marks	:	−1	in all other cases.

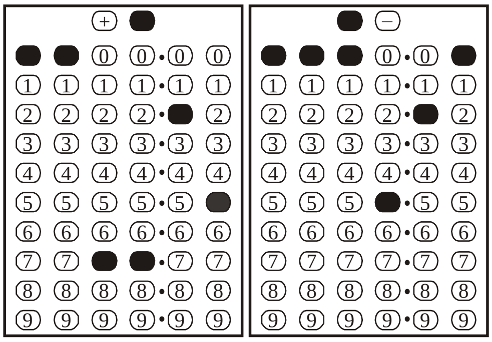
17. Let $\lim_{T \to \infty} \frac{1}{T}\int_{0}^{T} (\sin x + \sin ax)^2 dx = L$ then

	Column - I		**Column- II**
(A)	for $a = 0$, the value of L is	(p)	0
(B)	for $a = 1$ the value of L is	(q)	1/2
(C)	for $a = -1$ the value of L is	(r)	3/2
(D)	$\forall\, a \in R - \{-1, 0, 1\}$ the value of L is	(s)	2
		(t)	1

MDI169

SECTION-V

- This section contains **SEVEN** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30, if answer is 11.36777..... then both 11.36 and 11.37 will be correct) by darken the corresponding bubbles in the ORS.
  For Example : If answer is –77.25, 5.2 then fill the bubbles as follows.



- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If ONLY the correct numerical value is entered as answer.
  Zero Marks : 0 In all other cases.

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18. Let a be a real number in the interval [0, 314] such that $\int_{-\pi+a}^{3\pi+a} |x-a-\pi|\sin\left(\frac{x}{2}\right)dx = -16$, then determine number of such values of a.

MDI170

19. $\sum_{n=1}^{\infty}\left(\frac{1}{4n-3}-\frac{1}{4n-1}\right)=\frac{\pi}{n}$, find '$n$' $\left(\text{Note that } \tan^{-1}x+c=\int\frac{1}{1+x^2}dx\right)$

MDI171

20. If $f(x) = x + \int_0^1 t(x+t)\ f(t)\ dt$, then the value of the definite integral $\int_0^1 f(x)\ dx$ can be expressed in the form of rational as $\frac{p}{q}$ (where p and q are coprime). Find $(p + q)$. **MDI172**

21. If $f(x) = (ax + b)e^x$ satisfies the equation : $f(x)=\int_0^x e^{x-y} f'(y)dy-(x^2-x+1)e^x$, find $(a^2 + b^2)$

MDI173

22. If the minimum of the following function $f(x)$ defined at $0 < x < \frac{\pi}{2}$. $f(x) = \int_0^x \frac{d\theta}{\cos\theta}+\int_x^{\frac{\pi}{2}}\frac{d\theta}{\sin\theta}$ is equal to $\ell n(a+\sqrt{b})$ where $a, b \in N$ and b is not a perfect square then find the value of $(a + b)$

MDI174

23. If $f(\pi) = 2$ and $\int_0^{\pi}(f(x)+f''(x))\ \sin x\ dx = 5$, then find the value of $f(0)$
(it is given that $f(x)$ is continuous in $[0, \pi]$)

MDI175

24. If $f(x) = 2x^3 - 15x^2 + 24x$ and $g(x) = \int_0^x f(t)\ dt\ + \int_0^{5-x} f(t)\ dt$ $(0 < x < 5)$. Find the number of integers for which $g(x)$ is increasing. **MDI176**

ANSWER KEY

EXERCISE - O

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	B	B	B	D	A	D	D	A	C
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B	D	A	B,C,D	A,B,C	B,C,D	B	A	A	A

EXERCISE - S

1. 2012 **2.** 90 **3.** 8 **4.** 5250 **5.** 125
6. 153 **7.** 5 **8.** 9 **9.** 0.5
10. 0.66 or 0.67

EXERCISE - JEE (Main) PYQ

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	2	2	3	1	2	3	3	3	4	1
Que.	11	12	13	14	15					
Ans.	4	3	13	1	1					

EXERCISE - JEE (Advanced) PYQ

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	A,C	2	A	B	A	D	A,C,D	D	0
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	9	7	A,C	A,B	D	A,B,C	C,D	A	1	B,C
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	2	B,C	Bonus	B,D	1	2	A,B,C	4.00	A,B	0.50
Que.	31	32	33	34	35	36	37	38	39	40
Ans.	A,B,D	4	A,B,C	2.00	1.50	C	D	182	C,D	5
Que.	41	42	43							
Ans.	B	C	0							

JEE (Main) Practice Paper

Section-A	Q.	1	2	3	4	5	6	7	8	9	10
	A.	3	3	1	3	1	2	1	3	3	4
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	2	3	4	1	2	2	3	2	3	2
Section-B	Q.	1	2	3	4	5	6	7	8	9	10
	A.	61	64	10	29	4	4	2	2	0	2

JEE (Advanced) Practice Paper

Section-I	**Q.**	**1**	**2**	**3**	**4**	**5**	**6**	
	A.	C	B	A	C	D	A	
Section-II	**Q.**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
	A.	A,B	A,B,C	A,B	B,D	B,C,D	A,C	A,B,C
Section-III	**Q.**	**14**	**15**	**16**				
	A.	C	A	A				
Section-IV	**Q.**	**17**						
	A.	$A \to q; B \to s; C \to p; D \to t$						
Section-V	**Q.**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
	A.	25	4	65	5	11	3	2