Circle

SOLUTIONS

EXERCISE - O

1. **Ans. (D)**

Centre of first circle is $A(2,3)$

Centre of second circle is $B(-1,-2)$

Centre of third circle is $C(5,8)$

Now, $AB=\sqrt{3^2+5^2}=\sqrt{34}$

$BC=\sqrt{6^2+10^2}=\sqrt{136}=2\sqrt{34}$

$AC=\sqrt{3^2+5^2}=\sqrt{34}$

As $AB+AC=BC$. So, points A,B,C are collinear

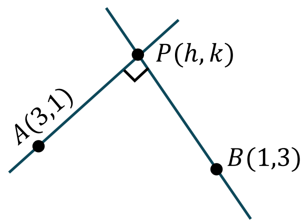
2. **Ans. (B)**

First family passes through $(3,1)$

and second family passes through $(1,3)$

Let point of intersection is $P(h,k)$

so, $m_{AP}.m_{BP}=-1$ (for locus)



$\Rightarrow\left(\frac{k-1}{h-3}\right).\left(\frac{k-3}{h-1}\right)=-1\Rightarrow(k-1)(k-3)=-(h-3)(h-1)$

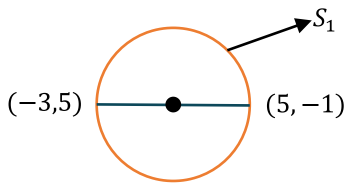
$\Rightarrow k^2-4k+3=-(h^2-4h+3)\Rightarrow h^2+k^2-4h-4k+6=0$

$\Rightarrow x^2+y^2-4x-4y+6=0$

3. **Ans. (A)**

$S_1:(x+3)(x-5)+(y-5)(y+1)=0$

$S_1:x^2+y^2-2x-4y-20=0$



$\Rightarrow\begin{cases}\text{centre } (1,2)\equiv(a,b)\\ \text{radius } r=\sqrt{1^2+2^2-(-20)}=\sqrt{25}=5\end{cases}$

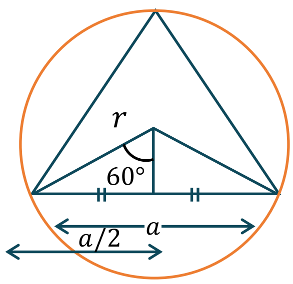
$\therefore a+b+r=1+2+5=8$

or, centre $(a,b)\equiv\left(-\frac{3+5}{2},\frac{5+(-1)}{2}\right)\equiv(1,2)$

$r=\frac{1}{2}\sqrt{(5+3)^2+(5+1)^2}=5$

4. **Ans. (A)**

radius of circle $=r=\sqrt{1^2+0^2-0}=1$



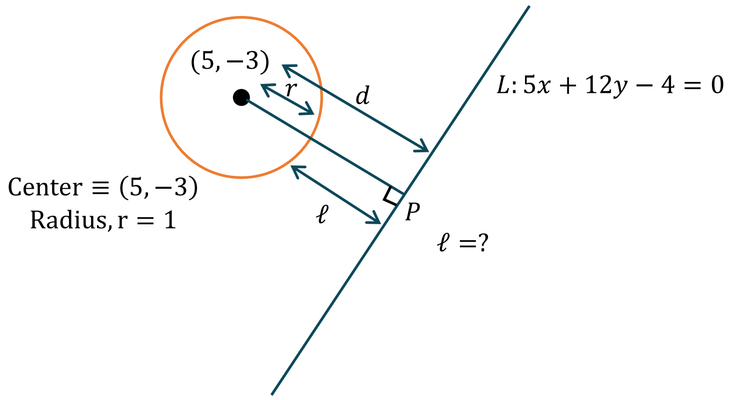
$\sin 60°=\frac{\frac{a}{2}}{r}$

$\Rightarrow\frac{\sqrt{3}}{2}=\frac{a}{2r}\Rightarrow a=\sqrt{3}$

Area $=\frac{\sqrt{3}}{4}a^2=\frac{3\sqrt{3}}{4}$

5. Ans. (B)

$$d=\left|\frac{5(5)+12(-3)-4}{\sqrt{5^2+12^2}}\right|=\left|\frac{25-36-4}{13}\right|=\frac{15}{13}$$

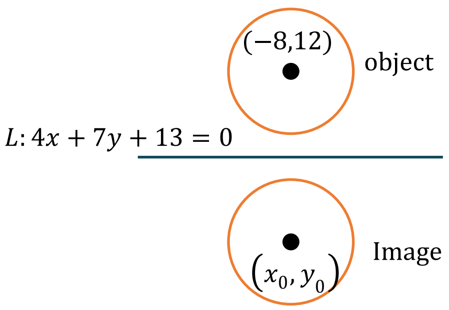


$\ell = |d - r|$ = minimum distance between line & circle

$$\Rightarrow \ell=\left|\frac{15}{13}-1\right|=\frac{2}{13}$$

6. Ans. (D)

Image of a circle in a line is a circle with same radius and centre as the reflection of centre of the given circle.



Let image is (x_0, y_0) of $(-8,12)$ in line $L: 4x + 7y + 13 = 0$

$$\text{so, } \frac{x_0-(-8)}{4}=\frac{y_0-12}{7}=(-2)\left(\frac{4\times(-8)+7\times12+13}{4^2+7^2}\right)$$

$$\Rightarrow \frac{x_0+8}{4}=\frac{y_0-12}{7}=-2$$

and radius, $r=\sqrt{(-8)^2+(12)^2-183}=5$

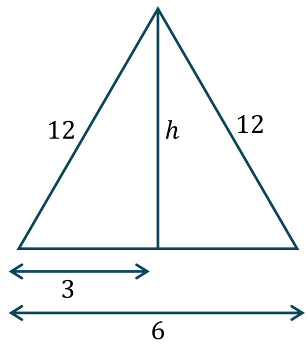
$\therefore$ equation of the circle is

$(x+16)^2+(y+2)^2=5^2$

$\Rightarrow x^2+y^2+32x+4y+235=0$

7. Ans. (A)

$$R=\frac{abc}{4\Delta}$$



$$\Delta=\frac{1}{2}\times6\times h=3h$$

$$=3\sqrt{12^2-3^2}=9\sqrt{15}$$

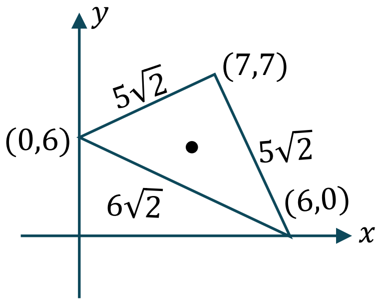
$$\therefore\ R=\frac{12\times12\times6}{4\times9\times\sqrt{15}}=\frac{24}{15}\times\sqrt{15}=\frac{8}{5}\sqrt{15}$$

8. Ans. (B)

For incircle we need to find incentre & inradius.

for incentre

$$I=\left(\frac{6\sqrt{2}\times7+5\sqrt{2}\times0+5\sqrt{2}\times6}{6\sqrt{2}+5\sqrt{2}+5\sqrt{2}},\frac{6\sqrt{2}\times7+5\sqrt{2}\times0+5\sqrt{2}\times6}{6\sqrt{2}+5\sqrt{2}+5\sqrt{2}}\right)\Rightarrow I=\left(\frac{9}{2},\frac{9}{2}\right)$$



and inradius, $r=\frac{\Delta}{s}=\dfrac{\frac{1}{2}\left\|\begin{matrix}7&7&1\\0&6&1\\6&0&1\end{matrix}\right\|}{\frac{6\sqrt{2}+5\sqrt{2}+5\sqrt{2}}{2}}=\frac{3}{\sqrt{2}}$

$\therefore$ Equation of circle,

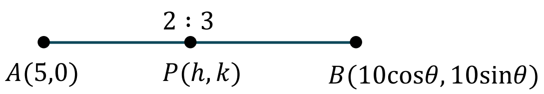
$$\left(x-\frac{9}{2}\right)^2+\left(y-\frac{9}{2}\right)^2=\left(\frac{3}{\sqrt{2}}\right)^2$$

$$\Rightarrow x^2+y^2-9x-9y+\frac{81}{4}+\frac{81}{4}-\frac{9}{2}=0$$

$$\Rightarrow x^2+y^2-9x-9y+\frac{81}{2}-\frac{9}{2}=0$$

$$\Rightarrow x^2+y^2-9x-9y+36=0$$

9. Ans. (B)



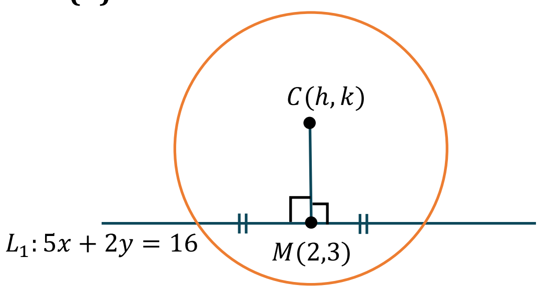
$$P\equiv\left(\frac{20\cos\theta+15}{2+3},\frac{20\sin\theta+3\times0}{2+3}\right)\equiv(3+4\cos\theta,4\sin\theta)$$

$\therefore h=3+4\cos\theta$ and $k=4\sin\theta$ (circle (think?))

as $\cos^2\theta+\sin^2\theta=1\Rightarrow\left(\frac{h-3}{4}\right)^2+\left(\frac{k}{4}\right)^2=1$

$\Rightarrow(h-3)^2+k^2=4^2$ (which is a circle)

10. Ans. (A)

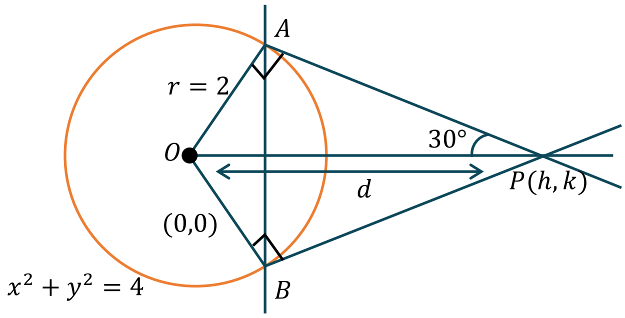


$$m_{CM}.m_{L_1}=-1\Rightarrow\left(\frac{k-3}{h-2}\right)\left(-\frac{5}{2}\right)=-1$$

$\Rightarrow 5k-15=2h-4$

$\Rightarrow 2x-5y+11=0$

11. Ans. (A)



$$\sin 30° = \frac{r}{d} \Rightarrow d = 4$$

$$\Rightarrow |OP| = 4 \Rightarrow h^2 + k^2 = 16$$

12. Ans. (B)

Equation of circle is $S: (x-3)(x+5) + (y-8)(y-2) = 0$

$\Rightarrow S: x^2 + 2x - 15 + y^2 - 10y + 16 = 0$

Put $x = k$ and $y = 10$ we get

$k^2 + 2k + 1 = 0 \Rightarrow k = -1$

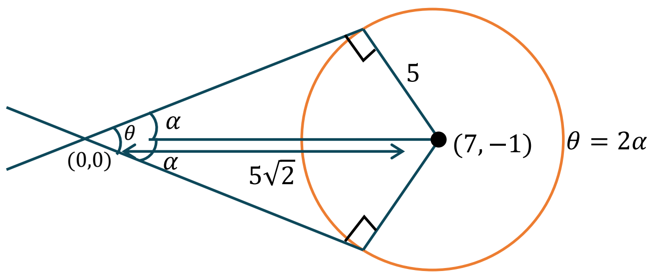
13. Ans. (D)

$S_P = 2^2 + 1^2 - 5.2 + 2.1 - 5 < 0$ ('P' is inside)

$S_Q = 0^2 + 0^2 - 5.0 + 2.0 - 5 < 0$ ('Q' is inside)

$S_R = 4^2 + (-3)^2 - 5(4) + 2(-3) - 5 < 0$ ('R' is inside)

14. Ans. (C)



$$\sin\alpha = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{2}$$

15. Ans. (B)

$S: x^2 + y^2 - 4x - 2y - 11 = 0$

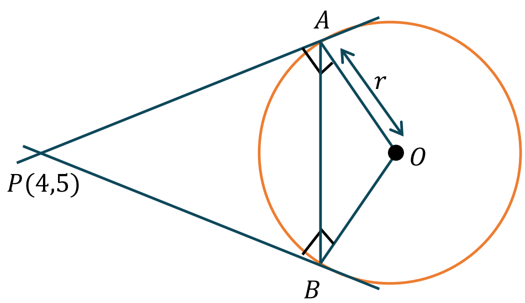
$r = \sqrt{(-2)^2 + (-1)^2 - (-11)}$

$r = 4$

$PA = PB = \sqrt{S_1} = \sqrt{4^2 + 5^2 - 4.4 - 2.5 - 11}$

$\Rightarrow PA = PB = 2$

$\therefore ar(PAOB) = 2ar(\Delta PAO) = 2\left(\frac{1}{2} PA \times OA\right) = 8$



16. Ans. (C)

Length of tangent from ext. pt. $=\sqrt{S_1}$

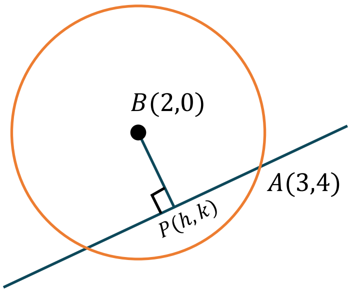
$L_1=\sqrt{0^2+25+2(0)-4}=\sqrt{21}$

$L_2=\sqrt{0+25-2(0)-5+1}=\sqrt{21}$

$\therefore L_1=L_2$

17. Ans. (A)

$m_{AP}.m_{BP}=-1$



$\Rightarrow\left(\frac{k-4}{h-3}\right)\left(\frac{k-0}{h-2}\right)=-1$

$\Rightarrow x^2+y^2-5x-4y+6=0$

18. Ans. (C)

$SS_1=T^2$ is formula for pair of tangents.

where $S=x^2+y^2+4x+6y+9$

$S_1=9$

$T=0.x+0.y+4\left(\frac{x+0}{2}\right)+6\left(\frac{y+0}{2}\right)+9=2x+3y+9$

so, $(x^2+y^2+4x+6y+9)(9)=(2x+3y+9)^2$

$\Rightarrow 9(x^2+y^2)+18(2x+3y)+81=(2x+3y)^2+18(2x+3y)+81$

$\Rightarrow 9(x^2+y^2)=(2x+3y)^2$

19. Ans. (C)

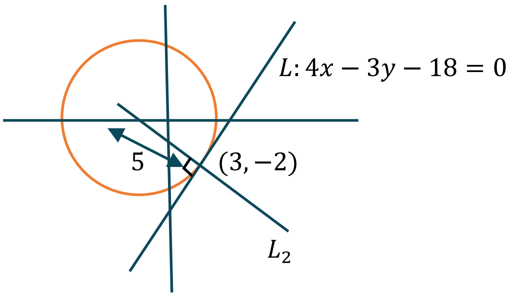
Use parametric form of line.

Slope of $L=\frac{4}{3}$

Slope of $\perp^r$ to $L=-\frac{3}{4}$

If inclination of line L_2 is 'θ'

then $\tan\theta=-\frac{3}{4}\Rightarrow\sin\theta=\frac{3}{5};\cos\theta=-\frac{4}{5}$ (Take $0<\theta<\pi$)



as centre is above the line L,

coordinate of centre at $\perp^r$ to L '5' units

from (3,–2) are $(3+5\cos\theta, -2+5\sin\theta)$

$\because$ centre $\equiv(-1,1)$ and radius = 5(given)

$\therefore$ circle is $(x+1)^2+(y-1)^2=5^2$

$\Rightarrow x^2+y^2+2x-2y-23=0$

20. **Ans. (C)**

$r_1 = \sqrt{10^2 - 64} = 6; r_2 = \sqrt{15^2 - 144} = 9$

Shortest common tangent is transverse common tangent $L_{TCT} = \sqrt{(C_1C_2)^2 - (r_1 + r_2)^2}$

$= \sqrt{25^2 - (6+9)^2} = 20$

C_1C_2 = distance between centres of the circles = 25

21. **Ans. (A,B)**

If 'θ' is constant then

$\tan\theta = \dfrac{y - y_1}{x - x_1}$ = constant

A straight line passing through a fixed point (x_1, y_1) and have a constant slope.

then $x - x_1 = r\cos\theta, y - y_1 = r\sin\theta$

$\Rightarrow (x - x_1)^2 + (y - y_1)^2 = r^2$

circle with a given centre (x_1, y_1) & radius 'r'.

22. **Ans. (A,B,C,D)**

Lines which cut equal intercepts on the circle are equidistant from the centres of the circle. Here centre is (1,–2)

(A) $d_1 = \left|\dfrac{3(1) - (-2)}{\sqrt{3^2 + (-1)^2}}\right| = \dfrac{5}{\sqrt{10}}$

(B) $d_2 = \left|\dfrac{1 + 3(-2)}{\sqrt{1^2 + 3^2}}\right| = \dfrac{5}{\sqrt{10}}$

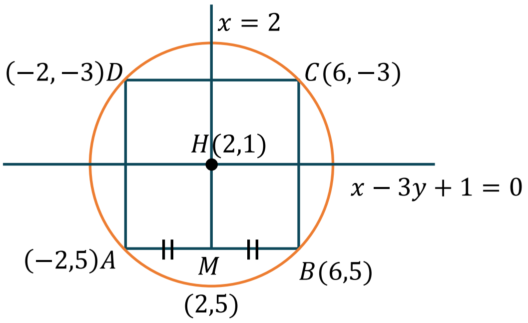
(C) $d_3 = \dfrac{|1 + 3(-2) + 10|}{\sqrt{1^2 + 3^2}} = \dfrac{5}{\sqrt{10}}$

(D) $d_4 = \left|\dfrac{3(1) - (-2) - 10}{\sqrt{3^2 + 1^2}}\right| = \dfrac{5}{\sqrt{10}}$

As $d_1 = d_2 = d_3 = d_4$ so, all the options are correct.

23. **Ans. (A,B,C)**

Area of rectangle = (8)(8) = 64 sq. units



Let $D(x, y)$

$\therefore \dfrac{x+6}{2} = 2$ and $\dfrac{x+5}{2} = 1$

$\therefore D(-2, -3) \quad C(6, -3)$

24. Ans. (C,D)

Let 'd' be the common difference

$\therefore$ the radii of the three circles be $1-2d, 1-d, 1$

$\therefore$ equation of smallest circle is $x^2+y^2=(1-2d)^2$ ...(i)

$\because$ $y=x+1$ intersect (i) at real and distinct points

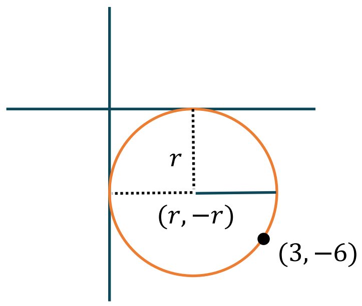
$\therefore$ $x^2+x+2d-2d^2=0$ ...(ii)

$D>0 \Rightarrow 8d^2-8d+1>0 \Rightarrow d>\frac{2+\sqrt{2}}{4}$ or $d<\frac{2-\sqrt{2}}{4}$

but d cannot be greater than $\frac{2+\sqrt{2}}{4}$

$\therefore$ $d \in \left(0, \frac{2-\sqrt{2}}{4}\right)$

25. Ans. (A,D)



Now

$(r-3)^2+(-r+6)^2=r^2$

$r^2-18r+45=0 \Rightarrow r=3, 15$

Hence circle

$(x-3)^2+(y+3)^2=3^2 \Rightarrow x^2+y^2-6x+6y+9=0$

$(x-15)^2+(y+15)^2=(15)^2$

$\Rightarrow x^2+y^2-30x+30y+225=0$

26. Ans. (A,C)

$OP=5\sqrt{2}\sec\theta$, $OP_1=5\sqrt{2}\operatorname{cosec}\theta$

area $(\Delta PP_1P_2)=\frac{100}{\sin 2\theta}$, area $(\Delta PP_1P_2)_{\min}=100$

$\Rightarrow \theta=\pi/4 \Rightarrow OP=10 \Rightarrow P=(10, 0), (-10, 0)$

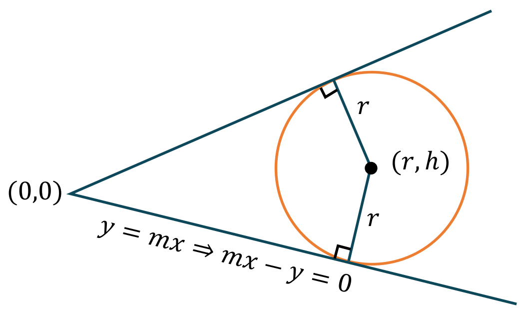
Hence (A), (C) are correct

27. Ans. (A,C)

Clearly, radius of circle is 'r' and centre is (r, h)

Let tangent is $y=mx$ from origin

Applying condition of tangency



|distance from centre| = |radius| $\Rightarrow \left|\frac{m.r-h}{\sqrt{m^2+1}}\right|=r$

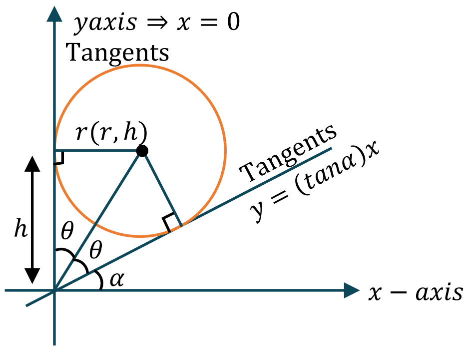
$\Rightarrow m^2r^2+h^2-2mrh=r^2m^2+r^2$

$\Rightarrow om^2-2mrh+h^2-r^2=0$

m is not defined or $m=\frac{h^2-r^2}{2rh}$

$\Rightarrow x=0$ or $y=\left(\frac{h^2-r^2}{2rh}\right)x$

M-II



$\tan\theta = \frac{r}{h}$

$\alpha = 90° - 2\theta$

$\Rightarrow \tan\alpha = \cot 2\theta$

$= \frac{1-\tan^2\theta}{2\tan\theta}$

$= \frac{h^2 - r^2}{2rh}$

28. Ans. (A,C,D)

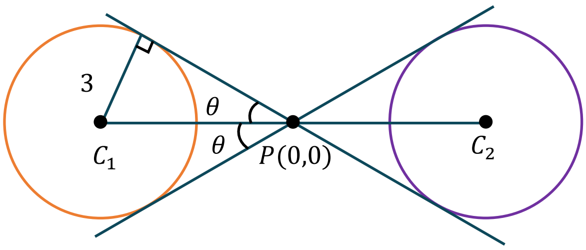
$S_1: x^2 + y^2 - 6x - 6y + 9 = 0;$

$S_2: x^2 + y^2 + 6x + 6y + 9 = 0$

	S_1	S_2
Centre	$C_1(3,3)$	$C_2(-3,-3)$
Radius	$r_1 = 3$	$r_2 = 3$

$C_1C_2 = 6\sqrt{2};\ r_1 + r_2 = 6;\ r_1 - r_2 = 0$

$C_1C_2 > r_1 + r_2 \Rightarrow$ circles don't intersect



$PC_1 = 3\sqrt{2} \Rightarrow \theta = 45°$

so, interior common tangents are perpendicular.

Note: Intersection point of interior common tangent divides the line joing centre of the circles internally in the ratio of radii.

29. Ans. (A,C)

We have $(x-5)^2 + (y+8)^2 = 25 + 64 + r^2 - 89$

and $(x+3)^2 + (y-7)^2 = 49 + 9 - 42 = 16$

$\Rightarrow (x-5)^2 + (y+8)^2 = r^2$

And $(x+3)^2 + (y-7)^2 = (4)^2$

$\underline{(5,-8)\qquad(-3,7)} \Rightarrow \sqrt{64+225} = \sqrt{289} = 17$ distance between their centres

Now, $|r-4| < 17 < r+4$

$\Rightarrow r + 4 > 17 \Rightarrow r > 13$

And $-17 < r - 4 < 17$

$\Rightarrow -13 < r < 21$

Hence $13 < r < 21$

$\therefore$ Possible values of 'r' can be 14, 15, 16, 17, 18, 19, 20

30. Ans. (A,C,D)

Clearly $PC_1^2 + PC_2^2 = (C_1C_2)^2$

$\Rightarrow$ Two circles intersect orthogonally.

Equation of common chord is $S_1 - S_2 = 0$

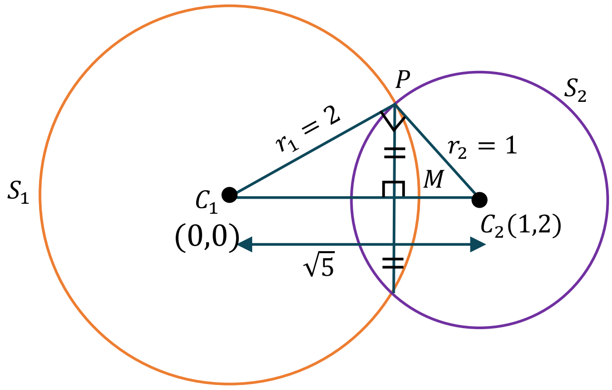
$\Rightarrow x + 2y - 4 = 0$

Now $C_1M = \dfrac{4}{\sqrt{5}}$

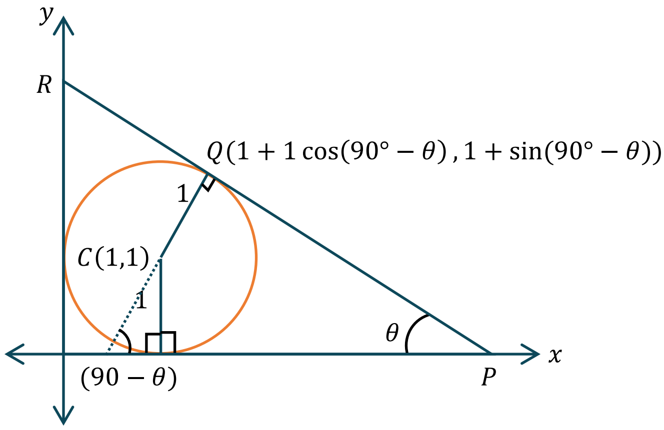
$\therefore$ Length of common chord = $2\sqrt{4 - \dfrac{16}{5}} = \dfrac{4}{\sqrt{5}}$

Clearly $S_1(2,3) > 0$ and $S_2(2,3) > 0$

So, the point (2, 3) lies outside the circles S_1 & S_2



31. Ans. (D)



So, $Q \equiv (1 + \sin\theta, 1 + \cos\theta)$

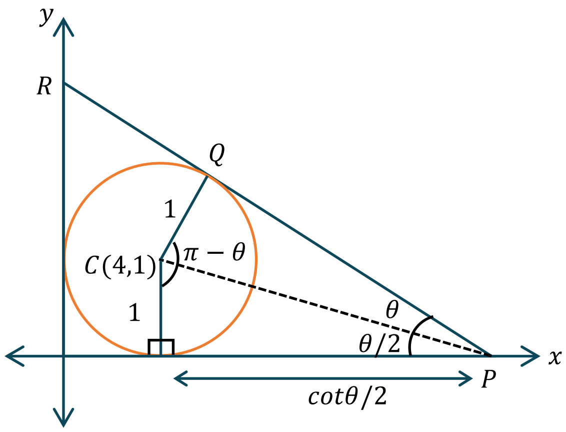
32. Ans. (C)

$PR: (y - (1 + \cos\theta)) = \tan(180° - \theta)\,(x - (1 + \sin\theta)$

$\Rightarrow (y - 1 - \cos\theta)\cos\theta = -\sin\theta(x - 1 - \sin\theta)$

$\Rightarrow x\sin\theta + y\cos\theta = \cos\theta + \sin\theta + 1$

33. Ans. (A)



$$A(\theta) = \left(\frac{1}{2} \times 1 \times \cot\frac{\theta}{2}\right) \times 2 - \frac{1}{2} \times 1^2 \times (\pi - \theta) \Rightarrow A(\theta) = \cot\left(\frac{\theta}{2}\right) - \left(\frac{\pi - \theta}{2}\right)$$

$$\therefore A\left(\frac{\pi}{4}\right) = \cot\frac{\pi}{8} - \frac{\left(\pi - \frac{\pi}{4}\right)}{2} = \sqrt{2} + 1 - \frac{3\pi}{8}$$

34. Ans. (B)

Point from which length of tangents to these circle is same is radical centre

$S_1 - S_2 = 0 \Rightarrow 4x - 4y - 4 = 0 \Rightarrow x - y - 1 = 0$

$S_2 - S_3 = 0 \Rightarrow -6x + 14y - 10 = 0 \Rightarrow -3x + 7y - 5 = 0$

$3x - 3y - 3 = 0$

$4y - 8 = 0 \Rightarrow y = 2 \quad x = 3$

35. Ans. (C)

If circle be drawn taking radical centre as centre and length of tangents from radical centre to any circle as radius will cut all the three circles orthogonally.

Length of tangent = $\sqrt{9+4+9+4+1} = \sqrt{S_1} = \sqrt{27}$

Equation of circle = $(x-3)^2 + (y-2)^2 = 27 \Rightarrow S_4 : x^2 + y^2 - 6x - 4y - 14 = 0$

36. Ans. (A)

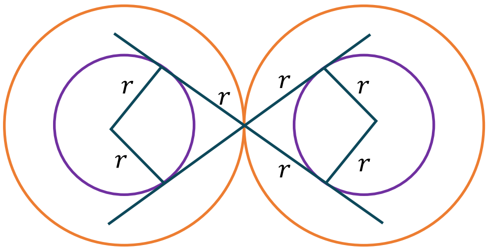
$S_1 - S_2 = 0 \Rightarrow x - y - 1 = 0$

$S_1 - S_4 = 0 \Rightarrow 9x + 6y + 15 = 0 \Rightarrow 3x + 2y + 5 = 0 \Rightarrow 3x - 3y - 3 = 0$

$5y + 8 = 0 \Rightarrow y = -8/5; \quad x = -3/5$

37. Ans. (A→S;B→R;C→Q;D→P)

(A) Length of internal common tangent equals to 2r



(B) $\frac{1}{2} r_1 r_2 = \frac{1}{2} \times \left(\frac{\ell}{2}\right) \times \sqrt{r_1^2 + r_2^2}$ {where ℓ is length of common chord}

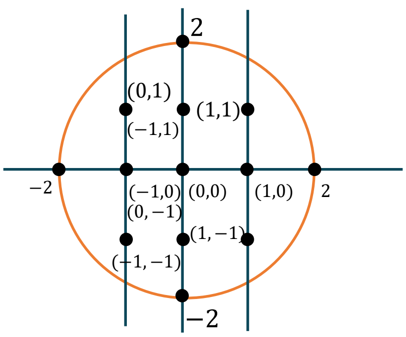
$\Rightarrow \ell = \sqrt{2} \sqrt{\frac{2r_1^2 r_2^2}{r_1^2 + r_2^2}} \Rightarrow k = \sqrt{2} \Rightarrow k^2 = 2$

(C) $x^2 + y^2 - 5x + 2y - 5 = 0 \Rightarrow \left(x - \frac{5}{2}\right)^2 + (y+1)^2 - 5 - \frac{25}{4} - 1 = 0$

$\Rightarrow \left(x - \frac{5}{2}\right)^2 + (y+1)^2 = \frac{49}{4} \Rightarrow$ So the axes are shifted to $\left(\frac{5}{2}, -1\right)$

New equation of circle must be $x^2 + y^2 = \frac{49}{4}$

(D)



38. Ans. (A→S;B→R;C→Q;D→P)

C_1 of radius 'a' touching coordinate axes

$\Rightarrow$ centre = (a, a)

$C_1 : (x - a)^2 + (y - a)^2 = a^2$

$\Rightarrow x^2 + y^2 - 2ax - 2ay + a^2 = 0$

$\Rightarrow C_2 : x^2 + y^2 - 2bx - 2by + b^2 = 0$

centre = (b, b) radius = b

(A) C_1, C_2 touch each other

$\Rightarrow C_1, C_2$ touch each other

$\Rightarrow \sqrt{(a-b)^2 + (a-b)^2} = a + b$

[Here they can't touch internally as both touch coordinate axes]

$\Rightarrow 2(a - b)^2 = (a + b)^2$

$\Rightarrow b^2 - 6ab + a^2 = 0$

$\Rightarrow \left(\frac{b}{a}\right)^2 - 6\left(\frac{b}{a}\right) + 1 = 0 \Rightarrow \frac{b}{a} = \frac{6 \pm \sqrt{36-4}}{2}$

$\frac{b}{a} = 3 + 2\sqrt{2}$ ($\because b > a$ $3 - \sqrt{2}$ is rejected)

(B) $C_1 C_2$ are orthogonal

$\Rightarrow 2g_1g_2 + 2f_1f_2 = C_1 + C_2$

$\Rightarrow 2(-a)(-b) + 2(-a)(-b) = a^2 + b^2$

$\Rightarrow \left(\frac{b}{a}\right)^2 - 4\left(\frac{b}{a}\right) + 1 = 0 \Rightarrow \frac{b}{a} = \frac{4 \pm \sqrt{16-4}}{2}$

$\Rightarrow \frac{b}{a} = 2 + \sqrt{3}$

(C) common chord of C_1, C_2 is

$C_1 - C_2 = 0$

$\Rightarrow 2(b - a)x + 2(b - a)y + (a - b)(a + b) = 0$

$\Rightarrow 2x + 2y - (a + b) = 0$

common chord is largest

It can be a maximum of the diameter of smaller circle

$\Rightarrow$ it passes through (a, a)

$\Rightarrow 2a + 2a - a - b = 0$

$\Rightarrow \frac{b}{a} = 3$

(D) C_2 passes through centre of $C = (a, a)$

$C_1 = (a, a)$

$\Rightarrow a^2 + a^2 - 2ba - 2ba + b^2 = 0$

$\Rightarrow \left(\frac{b}{a}\right)^2 - 4\left(\frac{b}{a}\right) + 2 = 0$

$\Rightarrow \frac{b}{a} = \frac{4 \pm \sqrt{16-8}}{2} \Rightarrow \frac{b}{a} = 2 + \sqrt{2}$

EXERCISE - S

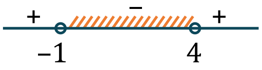
1. **Ans. (4)**

If point $P(\lambda,-\lambda)$ lie inside the circle $x^2 + y^2 - 4x + 2y - 8 = 0$

$\therefore$ Put $P(\lambda,-\lambda)$ in the equation of circle < 0

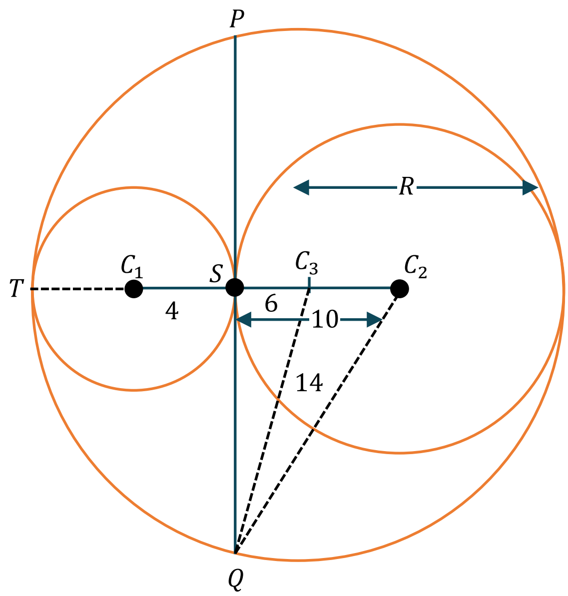
$\Rightarrow \lambda^2 + \lambda^2 - 4\lambda - 2\lambda - 8 < 0$

$\Rightarrow \lambda^2 - 3\lambda - 4 < 0 \Rightarrow (\lambda - 4)(\lambda + 1) < 0$



$\therefore \lambda \in (-1, 4)$

2. **Ans. (19)**



Let the radius of bigger circle is R.

Then distance $C_2C_3 = R - 10$ ...(1)

distance $C_1C_3 = R - 4$ ...(2)

now (1) + (2) $\Rightarrow C_1C_3 + C_2C_3 = 2R - 14$

or $C_1C_2 = 2R - 14$

or $(4 + 10) = 2R - 14$

since $R = 14$ or $\boxed{R=14}$

therefore, $TC_3 = 14$

or $TC_1 + C_1S + SC_3 = 14$

or $4 + 4 + SC_3 = 14 \Rightarrow \boxed{SC_3 = 6}$

$\boxed{C_2C_3 = 14 - 10 = 4}$

from $\Delta\, QSC_3$, $(SQ)^2 = (QC_3)^2 - (SC_3)^2$

$= (14)^2 - (6)^2 = 160$

or $\boxed{SQ = \sqrt{160} = 4\sqrt{10}}$

Now length of chord $PQ = 2SQ$

$= 2 \times 4\sqrt{10}$

or $PQ = 8\sqrt{10}$

or $\boxed{PQ = \dfrac{8\sqrt{10}}{1} = \dfrac{m\sqrt{n}}{p}}$

here, $m = 8, n = 10, p = 1$

then $m + n + p = 8 + 10 + 1 = 19$ **Ans.**

3. **Ans. (63)**

$\sin\theta = \frac{1}{2}$

or $\theta = \dfrac{5\pi}{6} \Rightarrow y - y_1 = m(x - x_1)$

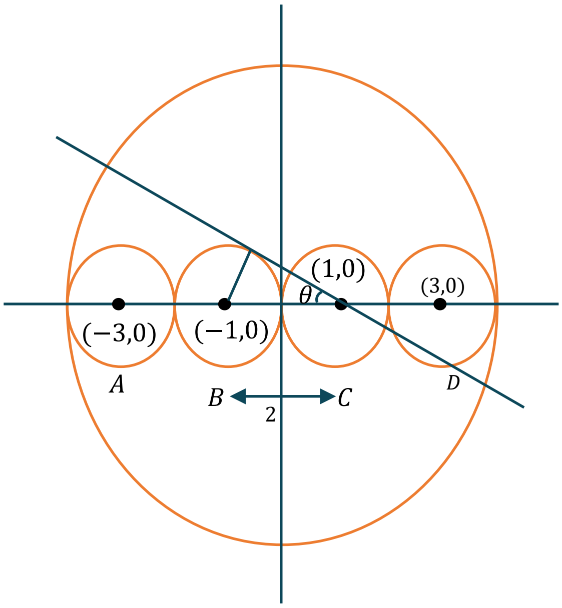
$m = \tan\dfrac{5\pi}{6} = -\dfrac{1}{\sqrt{3}}$

this line passes through (1, 0)

$\therefore\ y - 0 = -\dfrac{1}{\sqrt{3}}(x-1)$

or $\sqrt{3}y = -x+1$

or $\boxed{x+\sqrt{3}y-1=0}$



$\perp$ from (0, 0) on $x+\sqrt{3}y-1=0$

$p = \left|\dfrac{-1}{\sqrt{1+3}}\right| = \left|-\dfrac{1}{2}\right|$

$\boxed{p^2 = \dfrac{1}{4}}$

length of chord $\Rightarrow \ell = 2\sqrt{r^2 - p^2}$

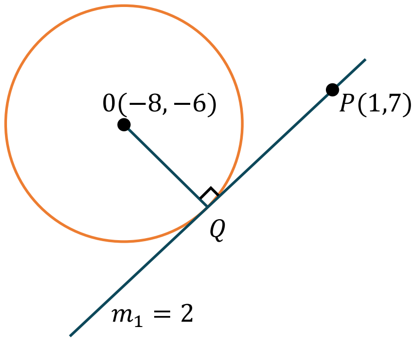
$\Rightarrow \ell = 2\sqrt{(4)^2 - \dfrac{1}{4}}$

$= 2\sqrt{16 - \dfrac{1}{4}} = \dfrac{2\sqrt{63}}{2}$

or $\boxed{\ell = \sqrt{63}}$

$x = 63$ **Ans**

4. **Ans. (4)**



$PQ: (y-7) = 2(x-1) \Rightarrow PQ: 2x - y + 5 = 0$ ....(1)

$OQ = r = \sqrt{g^2+f^2-c} = \left|\dfrac{2(-8)-(-6)+5}{\sqrt{2^2+1^2}}\right|$

$\Rightarrow \sqrt{8^2+6^2-c} = \sqrt{5} \Rightarrow c = 95$

equation $OQ: x + 2y = (-8) + 2(-6)$

$\Rightarrow x + 2y + 20 = 0$ ...(2)

solving (1) & (2) we get $Q \equiv (-6,-7) \equiv (a, b)$

so, $7a + 7b + c = -42 - 49 + 95 = 4$

5. **Ans. (215)**

Power of point (5, 2) w.r.t $x^2 + y^2 = 25$ is = 4

i.e. $PA_i \cdot PB_i = 4\ \forall\ i \in \{1, 2, 3, 4, 5\}$

$PA_1^2 + PB_1^2 + 2\,PA_1PB_1 = 25$

$PA_2^2 + PB_2^2 + 2\,PA_2\,PB_2 = 36$

$PA_3^2 + PB_3^2 + 2\,PA_3\,PB_3 = 49$

$PA_4^2 + PB_4^2 + 2\,PA_4\,PB_4 = 64$

$PA_5^2 + PB_5^2 + 2\,PA_5\,PB_5 = 81$

$\sum_{i=1}^{5} PA_i^2 + \sum_{i=1}^{5} PB_i^2 = (5^2+6^2+7^2+8^2+9^2) - 2\cdot4\cdot5$

$= 255 - 40 = 215$

6. **Ans. (6)**

$\because$ Equation of circle $(x-2)^2+(y+2)^2+\lambda(x+y)=0$ ....(i)

$\because$ Centre lies on the x-axis

$\therefore$ $\lambda=-4$ put in (i)

$\therefore$ equation of circle is $x^2+y^2-8x+8=0$

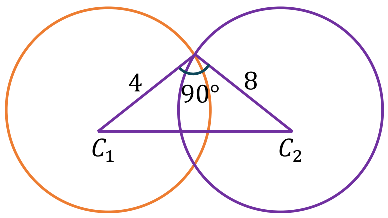
(α,β) lies on it $\Rightarrow \beta^2=-\alpha^2+8\alpha-8\geq 0$

$\therefore$ greatest value of 'α' is $4+2\sqrt{2}$

7. **Ans. (16)**

$C_1C_2=\sqrt{80}$

Area $=\frac{1}{2}\times 4\times 8=\frac{1}{2}\times\sqrt{80}\times\frac{\ell}{2}$



$\ell=\frac{64}{\sqrt{80}}=\frac{16}{\sqrt{5}}$

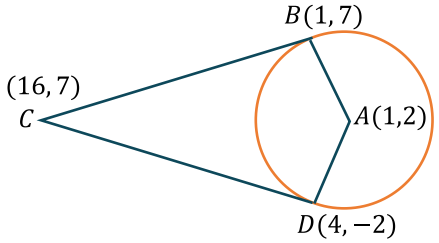
8. **Ans. (75)**

Given circle $x^2+y^2-2x-4y-20=0$

Tangents at $B(1,7)$ is

$x+7y-(x+1)-2(y+7)-20=0$

$5y-35=0\Rightarrow y=7$



at $D(4,-2)$

$4x-2y-(x+4)-2(y-2)-20=0$

$3x-4y=20$

Hence $C(16,7)$

Area of quadrilateral $ABCD=AB\times BC=5\times 15=75$ square units.

9. **Ans. (0)**

Let $S_1: x^2+y^2+2ax+cy+a=0$

$S_2: x^2+y^2-3ax+dy-1=0$

common chord $S_1-S_2=0\Rightarrow 5ax+y(c-d)+(a+1)=0$

given line is $5x+by-a=0$

compare both $\underset{\text{(i)}}{\frac{5a}{5}}=\underset{\text{(ii)}}{\frac{c-d}{b}}=\underset{\text{(iii)}}{\frac{a+1}{-a}}\Rightarrow a=\frac{c-d}{b}=-1-\frac{1}{a}$

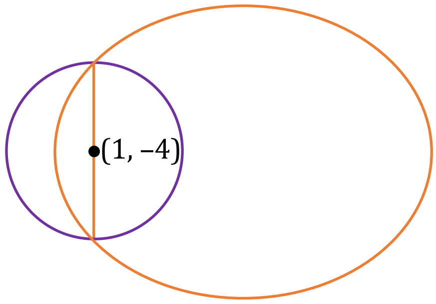
From (i) & (iii) $a^2+a+1=0\Rightarrow a=\omega,\omega^2\Rightarrow$ no real value of a.

10. **Ans. (10)**

Common chord of given circle is $S_1 - S_2 = 0$

$\Rightarrow 6x + 4y + (p + q) = 0$

This is diameter of $x^2 + y^2 - 2x + 8y - q = 0$



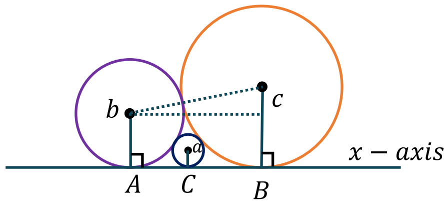
$\because$ centre (1, –4)

$\Rightarrow 6 - 16 + (p + q) = 0 \Rightarrow p + q = 10$

EXERCISE - JEE (Main) PYQ

1. **Ans. (1)**

$AB = AC + CB$



$\sqrt{(b+c)^2-(b-c)^2}$

$=\sqrt{(b+a)^2-(b-a)^2}+\sqrt{(a+c)^2-(a-c)^2}$

$\sqrt{bc}=\sqrt{ab}+\sqrt{ac}$

$\frac{1}{\sqrt{a}}=\frac{1}{\sqrt{c}}+\frac{1}{\sqrt{b}}$

2. **Ans. (4)**

$x^2 + y^2 + 4x - 6y - 12 = 0$

Equation of tangent at (1, –1)

$x - y + 2(x + 1) - 3(y - 1) - 12 = 0$

$3x - 4y - 7 = 0$

$\therefore$ Equation of circle is

$(x^2 + y^2 + 4x - 6y - 12) + \lambda(3x - 4y - 7) = 0$

It passes through (4, 0):

$(16 + 16 - 12) + \lambda(12 - 7) = 0$

$\Rightarrow 20 + \lambda(5) = 0$

$\Rightarrow \lambda = -4$

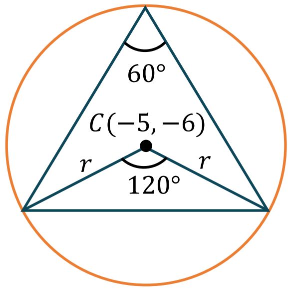
$\therefore (x^2 + y^2 + 4x - 6y - 12) - 4(3x - 4y - 7) = 0$

or $x^2 + y^2 - 8x + 10y + 16 = 0$

Radius = $\sqrt{16+25-16}=5$

3. **Ans. (2)**

$$3\left(\frac{1}{2}r^2.\sin 120°\right)=27\sqrt{3}$$



$$\Rightarrow \frac{r^2}{2}\frac{\sqrt{3}}{2}=\frac{27\sqrt{3}}{3}$$

$$\Rightarrow r^2 = \frac{108}{3}=36$$

Radius = $\sqrt{25+36-C}=\sqrt{36}$

$\boxed{C=25}$

$\therefore$ Option (2)

4. **Ans. (4)**

Radius of circle = $\sqrt{9+16+103}$

$=\sqrt{128}\ =8\sqrt{2}$

diagonal = $16\sqrt{2}=a\sqrt{2}$

side of square = 16

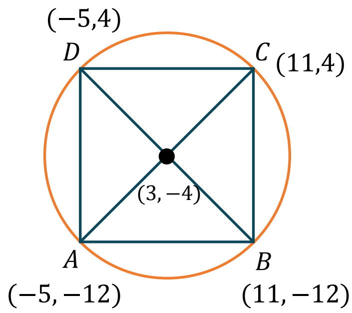
vertices of square

$A(3-8,-4-8)=(-5,-12)$

$B(3+8,-4-8)=(11,-12)$

$C=(3+8,8-4)=(11,4)$

$D(3-8,8-4)=(-5,4)$



$R=\sqrt{9+16+103}=8\sqrt{2}$

$OA=13$

$OB=\sqrt{265}$

$OC=\sqrt{137}$

$OD=\sqrt{41}$

5. **Ans. (2)**

Let equation of circle is

$x^2+y^2+2fx+2fy+e=0$, it passes through $(0,2b)$

$\Rightarrow 0+4b^2+2g\times 0+4f+c=0$

$\Rightarrow 4b^2+4f+c=0$ ...(i)

$2\sqrt{g^2-c}=4a$ ...(ii)

$g^2-c=4a^2 \Rightarrow c=\left(g^2-4a^2\right)$

Putting in equation (i)

$\Rightarrow 4b^2+4f+g^2-4a^2=0$

$\Rightarrow x^2+4y+4(b^2-a^2)=0$, it represent a parabola.

6. **Ans. (1)**

Centre of circles are opposite side of line

$(3+4-\lambda)(27+4-\lambda)<0$

$(\lambda-7)(\lambda-31)<0$

$\lambda\in(7,31)$

distance from S_1

$\left|\frac{3+4-\lambda}{5}\right| \ge 1 \Rightarrow \lambda \in (-\infty, 2] \cup [(12,\infty]$

distance from S_2

$\left|\frac{27+4-\lambda}{5}\right| \ge 2 \Rightarrow \lambda \in (-\infty, 21] \cup [41, \infty)$

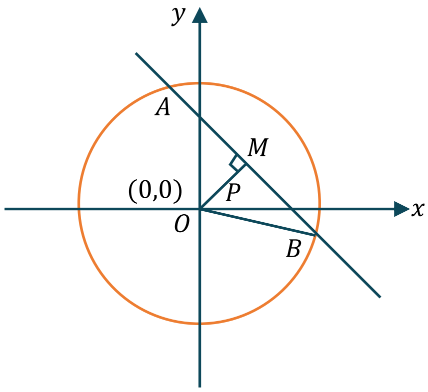
so $\lambda \in [12, 21]$

7. **Ans. (4)**

$p = \frac{n}{\sqrt{2}}$, but $\frac{n}{\sqrt{2}} < 4 \Rightarrow n = 1, 2, 3, 4, 5.$

Length of chord $AB = 2\sqrt{16 - \frac{n^2}{2}}$

$= \sqrt{64 - 2n^2} = \ell(say)$



For $n = 1, \ell^2 = 62$

$n = 2, \ell^2 = 56$

$n = 3, \ell^2 = 46$

$n = 4, \ell^2 = 32$

$n = 5, \ell^2 = 14$

$\therefore$ Required sum = 62 + 56 + 46 + 32 + 14 = 210

8. **Ans. (3)**

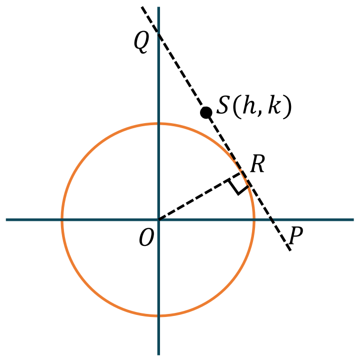
Let the mid point be $S(h, k)$

$\therefore P(2h, 0)$ and $Q(0,2k)$

equation of PQ: $\frac{x}{2h} + \frac{y}{2k} = 1$

$\because PQ$ is tangent to circle at R(say)

$\therefore OR = 1 \Rightarrow \left|\frac{-1}{\sqrt{\left(\frac{1}{2h}\right)^2 + \left(\frac{1}{2k}\right)^2}}\right| = 1$



$\Rightarrow \frac{1}{4h^2} + \frac{1}{4k^2} = 1 \Rightarrow x^2 + y^2 - 4x^2y^2 = 0$

Aliter:

tangent to circle

$x\cos\theta + y\sin\theta = 1$

P: $(\sec\theta, 0)$

Q: $(0, \text{cosec}\theta)$

$2h = \sec\theta \Rightarrow \cos\theta = \frac{1}{2h}$ & $\sin\theta = \frac{1}{2k}$

$\frac{1}{(2x)^2} + \frac{1}{(2y)^2} = 1$

9. **Ans. (2)**

Circle touches internally

$C_1(0,0); r_1 = 2$

$C_2: (-3,-4);\ r_2 = 7$

$C_1C_2 = |r_1 - r_2|$

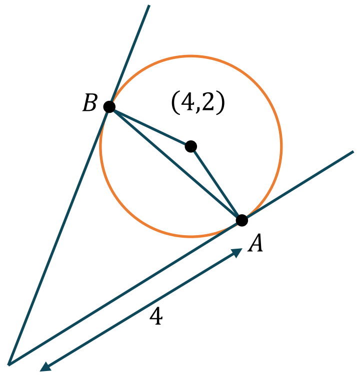
$S_1 - S_2 = 0 \Rightarrow$ eqn. of common tangent

$6x + 8y - 20 = 0$

$3x + 4y = 10$

$(6, -2)$ satisfy it

10. **Ans. (4)**



$R = \sqrt{16+4-16} = 2$

$L = \sqrt{S_1} = 4$

$AB(\text{Chord of contact}) = \dfrac{2LR}{\sqrt{L^2+R^2}} = \dfrac{8}{\sqrt{5}}$

$(AB)^2 = \dfrac{64}{5}$

11. **Ans. (2)**

Slope of tangent to $x^2 + y^2 = 1$ at $P\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$

$2x + 2yy' = 0 \Rightarrow\ m_{TP} = -1$

$y = mx + c$ is tangent to $(x-3)^2 + y^2 = 1$, is perpendicular to L_1

$\Rightarrow y = x + c$ is tangent to $(x-3)^2 + y^2 = 1$

$\left|\dfrac{c+3}{\sqrt{2}}\right| = 1 \Rightarrow c^2 + 6c + 7 = 0$

(2) Option

12. **Ans. (36)**

Common tangent is $S_1 - S_2 = 0$

$\Rightarrow -6x + 8y - 8 + k = 0$ ...(1)

L_1 is tangent so perpendicular distance from $(3, 0)$ is equal to radius of circle

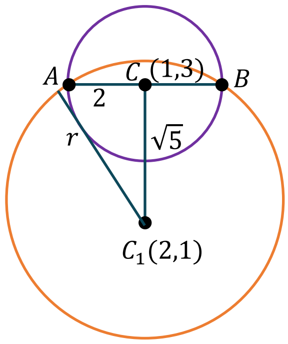
Use $p = r$ for Ist circle

$\Rightarrow \dfrac{|-18-8+k|}{10} = 1$

$\Rightarrow k = 36$ or $16 \Rightarrow\ k_{\max} = 36$

13. Ans. (3)

$x^2 + y^2 - 2x - 6y + 6 = 0$



center (1, 3)

radius = 2

distance between (1, 3) and (2, 1) is $\sqrt{5}$

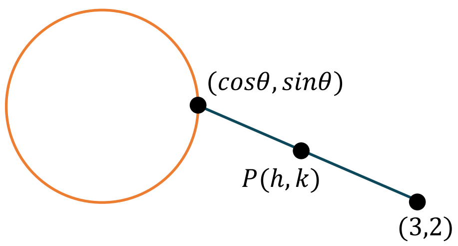
$\therefore \left(\sqrt{5}\right)^2 + (2)^2 = r^2$

$\Rightarrow r = 3$

14. Ans. (2)

$h = \frac{\cos\theta + 3}{2} \Rightarrow h - \frac{3}{2} = \frac{\cos\theta}{2}$

$k = \frac{\sin\theta + 2}{2} \Rightarrow k - 1 = \frac{\sin\theta}{2}$



Square & add

$\Rightarrow \left(h - \frac{3}{2}\right)^2 + (k-1)^2 = \frac{1}{4}$

$\Rightarrow r = \frac{1}{2}$

15. Ans. (9)

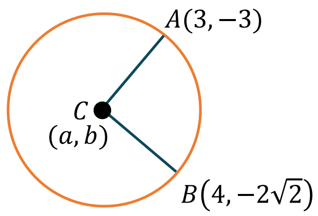
All normals of circle passes through centre

Radius = $CA = CB$

$CA^2 = CB^2$

$(a-3)^2 + (b+3)^2$

= $(a-4)^2 + \left(b + 2\sqrt{2}\right)^2$



= $a + (3 - 2\sqrt{2})b = 3$

= $a - 2\sqrt{2}\,b + 3b = 3$ ...(1)

given that $a - 2\sqrt{2}\,b = 3$ ...(2)

from (1) & (2) $\Rightarrow a = 3, b = 0$

$a^2 + b^2 + ab = 9$

16. Ans. (3)

$x^2 + y^2 + ax + 2ay + c = 0$

$2\sqrt{g^2 - c} = 2\sqrt{\frac{a^2}{4} - c} = 2\sqrt{2}$

$\Rightarrow \frac{a^2}{4} - c = 2$ ...(1)

& $2\sqrt{f^2 - c} = 2\sqrt{a^2 - c} = 2\sqrt{5}$

$\Rightarrow a^2 - c = 5$ ...(2)

(1) & (2)

$\frac{3a^2}{4} = 3 \Rightarrow a = -2 \quad (\because\ a < 0)$

$\therefore c = -1$

Circle $\Rightarrow x^2 + y^2 - 2x - 4y - 1 = 0$

$\Rightarrow (x-1)^2 + (y-2)^2 = 6$

Given $x + 2y = 0 \Rightarrow m = -\frac{1}{2}$

$m_{\text{tangent}} = 2$

Equation of tangent

$\Rightarrow (y-2) = 2(x-1) \pm \sqrt{6}\sqrt{1+4}$

$\Rightarrow 2x - y \pm \sqrt{30} = 0$

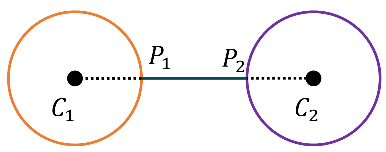
Perpendicular distance from (0, 0) = $\left|\frac{\pm\sqrt{30}}{\sqrt{4+1}}\right| = \sqrt{6}$

17. Ans. (1)

Given $C_1(5,5), r_1 = 3$ and $C_2(12,5), r_2 = 3$

Now, $C_1C_2 > r_1 + r_2$

Thus, $(P_1P_2)_{\min} = 7 - 6 = 1$



18. Ans. (7)

$k = r$

$h = 1$

$OP = r, PR = 1$

$OR = \left|\frac{r+1}{\sqrt{2}}\right|$

$r^2 = 1 + \frac{(r+1)^2}{2}$

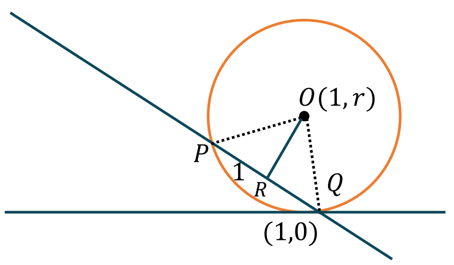
$2r^2 = 2 + r^2 + 1 + 2r$

$r^2 - 2r - 3 = 0$

$(r-3)(r+1) = 0$

$\boxed{r=3}$, -1

$h + k + r = 1 + 3 + 3 = 7$



19. Ans. (4)

$C: 4x^2 + 4y^2 - 12x + 8y + k = 0$

$\Rightarrow x^2 + y^2 - 3x + 2y + \left(\frac{k}{4}\right) = 0$

Centre $\left(\frac{3}{2}, -1\right)$; $r = \sqrt{\frac{13-k}{2}} \Rightarrow k \le 13$ ...(1)

(i) Point $\left(1, \frac{-1}{3}\right)$ lies on or inside circle C

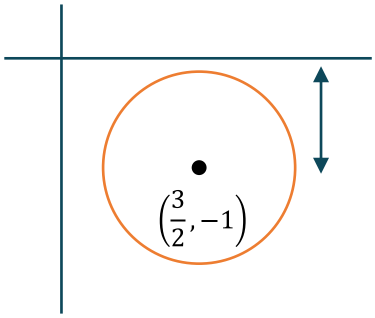
$\Rightarrow S_1 \le 0 \Rightarrow k \le \frac{92}{9}$ ...(2)

(ii) C lies in 4th quadrant

$r < 1$

$\Rightarrow \frac{\sqrt{13-k}}{2} < 1 \Rightarrow k > 9$ ....(3)

Hence (1) $\cap$ (2) $\cap$ (3) $\Rightarrow k \in \left(9, \frac{92}{9}\right]$



20. Ans. (16)

Eq. of line AB is $y = 2x$

Slope of $AB = 2$

Slope of given diameter = 2

So the diameter is parallel to AB

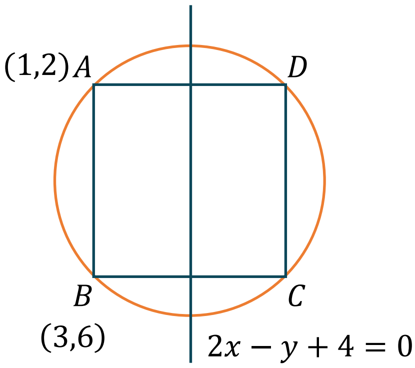
Distance between diameter and line AB

$= \left(\frac{4}{\sqrt{2^2 + 1^2}}\right) = \frac{4}{\sqrt{5}}$

Thus $BC = 2 \times \frac{4}{\sqrt{5}} = \frac{8}{\sqrt{5}}$

$AB = \sqrt{(1-3)^2 + (2-6)^2} = \sqrt{20} = 2\sqrt{5}$

Area = $AB \times BC = \frac{8}{\sqrt{5}} \times 2\sqrt{5} = 16$ **Ans**



21. Ans. (3)

$OP = \left|\frac{2-3+2}{\sqrt{2}}\right|$

$OP = \frac{3}{\sqrt{2}}$

$AP = \sqrt{OA^2 - OP^2}$

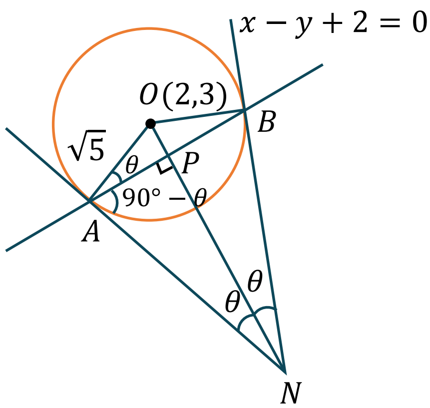
$= \frac{1}{\sqrt{2}}$

$\tan\theta = 3$

$\sin\theta = \frac{3}{\sqrt{10}} = \frac{AP}{AN}$

$\Rightarrow AN = \frac{\sqrt{5}}{3} = BN$

Area of $\Delta ANB = \frac{1}{2}.\left(AN^2\right)\sin 2\theta = \frac{1}{6}$

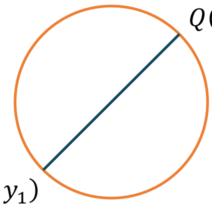


22. **Ans. (1)**

Equation of circle in diameter form

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

(where x_1, x_2 are the roots of $x^2 - 4x - 6 = 0$ and y_1, y_2 are the roots of $y^2 + 2y - 7 = 0$)



$x^2 + y^2 - 4x + 2y - 13 = 0$

Now,

Compare it with the given equation, we get

$a = -2, b = 1, c = -13$

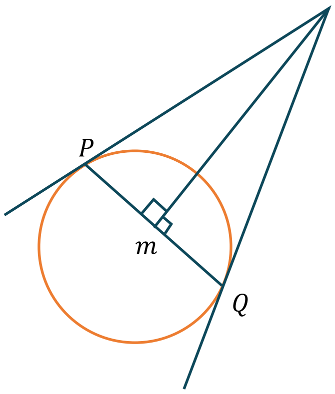
Now

$a + b - c = 12$

23. **Ans. (4)**

Equation of circle is $x^2 + y^2 - 2x + y - 5 = 0$

$R = \frac{5}{2}$



Length of $PR = QR = \sqrt{S_1}$

$= \sqrt{\frac{81}{16} + 4 - \frac{2\times 9}{4} + 2 - 5} = \frac{5}{4}$

Area of triangle PQR $\frac{R \times L^3}{R^2 + L^2} = \frac{\frac{5}{2}\cdot\frac{125}{64}}{\frac{25}{4} + \frac{25}{16}} = \frac{5}{8}$

EXERCISE - JEE (Advanced) PYQ

1. **Ans. (A,C)**

As per figure,

$R^2 = 3^2 + \left(\sqrt{7}\right)^2$

$\Rightarrow R = 4$

$\therefore$ centre $\equiv (3,4)$

radius 4

$\therefore$ equation $x^2 + y^2 - 6x - 8y + 9 = 0$

such a circle can lie in all 4 quadrants as shown in figure.

$\therefore$ equation can be $x^2 + y^2 \pm 6x \pm 8y + 9 = 0$

2. **Ans. (B,C)**

Let circle is $x^2+y^2+2gx+2fy+c=0$

Put (0,1); $1+2f+c=0$ ...(1)

orthogonal with $x^2+y^2-2x-15=0$

$2g(-1)=c-15 \Rightarrow c=15-2g$ ...(2)

orthogonal with $x^2+y^2-1=0$

$c=1$ ...(3)

$\Rightarrow g=7 \;\&\; f=-1$

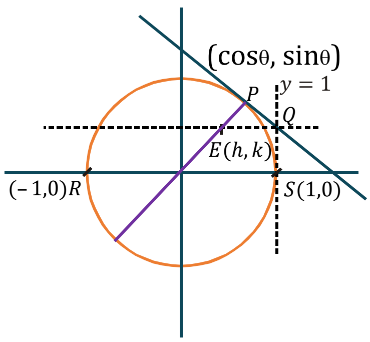
centre is $(-g,-f) \equiv (-7,1)$

radius $=\sqrt{g^2+f^2-c}=\sqrt{49+1-1}=7$

3. **Ans. (A, C)**

Tangent at $P: x\cos\theta+y\sin\theta=1$ ...(i)

Tangent at $S: x=1$ ...(ii)



By (i) & (ii) : $Q\left(1,\dfrac{1-\cos\theta}{\sin\theta}\right)$

Line through Q parallel to RS :

$y=\dfrac{1-\cos\theta}{\sin\theta} \Rightarrow y=\tan\dfrac{\theta}{2}$ ...(iii)

Normal at P: $y=\dfrac{\sin\theta}{\cos\theta}x \Rightarrow y=(\tan\theta).x$ ...(iv)

Point of intersection of equation (iii) and (iv),

$E\;:\; h=\dfrac{1-\tan^2\dfrac{\theta}{2}}{2};k=\tan\dfrac{\theta}{2}$

eliminating: $h=\dfrac{1-k^2}{2} \Rightarrow y^2=1-2x$

Options (A) and (C) satisfies the locus.

4. **Ans. (A, B, C)**

On solving $x^2+y^2=3$ and $x^2=2y$ we get point $P(\sqrt{2},1)$

Equation of tangent at P

$\sqrt{2}.x+y=3$

Let Q_2 be $(0,k)$ and

radius is $2\sqrt{3}$

$\left|\dfrac{\sqrt{2}(0)+k-3}{\sqrt{2+1}}\right|=2\sqrt{3}$

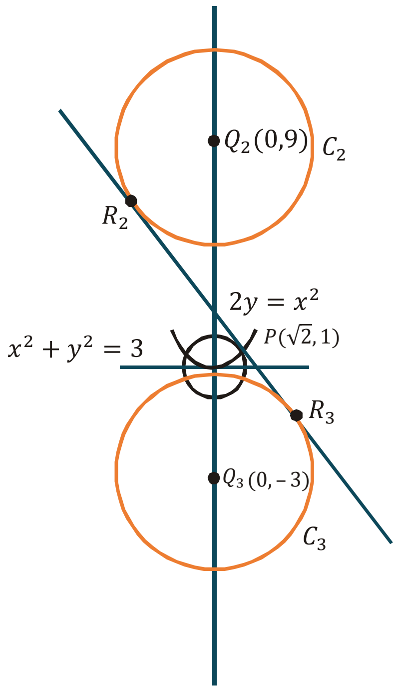
$k=9,-3$

$Q_2(0,9)$ and $Q_3(0,-3)$

hence $Q_2Q_3=12$

R_2R_3 is internal common tangent of circle C_2 and C_3

$R_2R_3=\sqrt{(Q_2Q_3)^2-\left(2\sqrt{3}+2\sqrt{3}\right)^2}.$



$= \sqrt{12^2 - 48} = \sqrt{96} = 4\sqrt{6}$

Perpendicular distance of origin O from R_2R_3 is equal to radius of circle $C_1 = \sqrt{3}$

Hence area of $OR_2R_3 = \frac{1}{2} \times (R_2R_3)\sqrt{3} = \frac{1}{2}.4\sqrt{6}.\sqrt{3} = 6\sqrt{2}$

Perpendicular Distance of P from $Q_2Q_3 = \sqrt{2}$

Area of $PQ_2Q_3 = \frac{1}{2} \times 12 \times \sqrt{2} = 6\sqrt{2}$

5. **Ans. (2.00)**

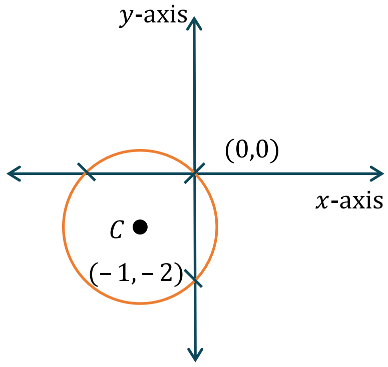
We shall consider 3 cases.

Case I : When $p = 0$

(i.e. circle passes through origin)

Now, equation of circle becomes

$x^2 + y^2 + 2x + 4y = 0$



Case II : When circle intersects x-axis at 2 distinct points and touches y-axis

Now $(g^2 - c) > 0$ & $f^2 - c = 0$

$\Rightarrow 1 - (-p) > 0$ & $4 - (-p) = 0 \Rightarrow p = -4 \Rightarrow p > -1$

$\therefore$ Not possible.

Case III : When circle intersects y-axis at 2 distinct points & touches x-axis.

Now, $g^2 - c = 0$ & $f^2 - c > 0$

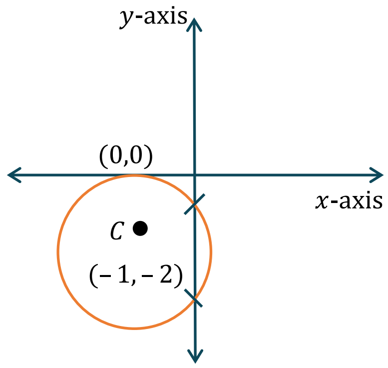
$\Rightarrow 1 - (-p) = 0$ & $4 - (-p) > 0$

$\Rightarrow p = -1 \Rightarrow p > -4$

$\therefore p = -1$ is possible.

$\therefore$ Finally we conclude that $p = 0, -1$

$\Rightarrow$ Two possible values of p.



6. **Ans. (A)**

co-ordinates of E_1 and E_2 are obtained by solving $y = 1$ and $x^2 + y^2 = 4$

$\therefore E_1(-\sqrt{3}, 1)$ and $E_2(\sqrt{3}, 1)$

co-ordinates of F_1 and F_2 are obtained by solving $x = 1$ and $x^2 + y^2 = 4$

$F_1(1, \sqrt{3})$ and $F_2(1, -\sqrt{3})$

Tangent at $E_1: -\sqrt{3}x + y = 4$

Tangent at $E2: \sqrt{3}x + y = 4$

$\therefore E_3\ (0, 4)$

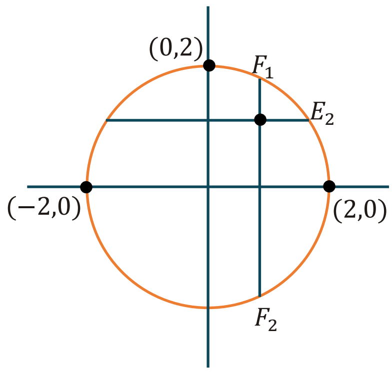
Tangent at $F_1: x + \sqrt{3}y = 4$

Tangent at $F_2: x - \sqrt{3}y = 4$

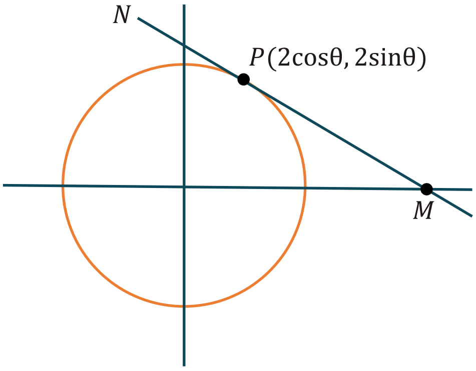
$\therefore F_3(4, 0)$

and similarly $G_3(2, 2)$

$(0, 4), (4, 0)$ and $(2, 2)$ lies on $x + y = 4$.



7. Ans. (D)



Tangent at $P(2\cos\theta, 2\sin\theta)$ is $x\cos\theta + y\sin\theta = 2$

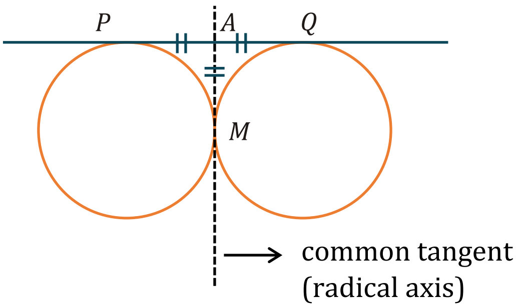
$M(2\sec\theta, 0)$ and $N(0, 2\operatorname{cosec}\theta)$

Let midpoint be (h, k)

$h = \sec\theta, k = \operatorname{cosec}\theta$

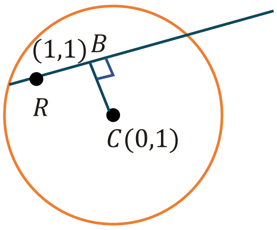
$\frac{1}{h^2}+\frac{1}{k^2}=1 \Rightarrow \frac{1}{x^2}+\frac{1}{y^2}=1$

8. Ans. (B, D)



$AP = AQ = AM$

Locus of M is a circle having PQ as its diameter



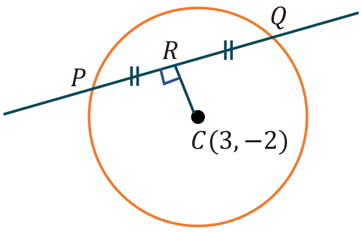
Hence, $E_1: (x-2)(x+2) + (y-7)(y+5) = 0$ and $x \neq \pm 2$

Locus of B (midpoint) is a circle having RC as its diameter

$E_2: x(x-1) + (y-1)^2 = 0$

Now, after checking the options, we get (D)

9. Ans. (B)

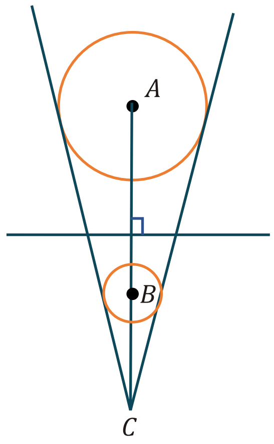


$R \equiv \left(-\frac{3}{5}, \frac{-3m}{5}+1\right)$

So, $m\left(\frac{-\frac{3m}{5}+3}{-\frac{3}{5}-3}\right)=-1$

$\Rightarrow m^2 - 5m + 6 = 0 \Rightarrow m = 2, 3$

10. Ans. (10.00)



Distance of point A from given line $= \frac{5}{2}$

$\frac{CA}{CB}=\frac{2}{1}$

$\Rightarrow \frac{AC}{AB}=\frac{2}{1}$

$\Rightarrow AC = 2\times 5 = 10$

11. Ans. (A)

12. Ans. (D)

(Solution of Q.11 & Q.12)

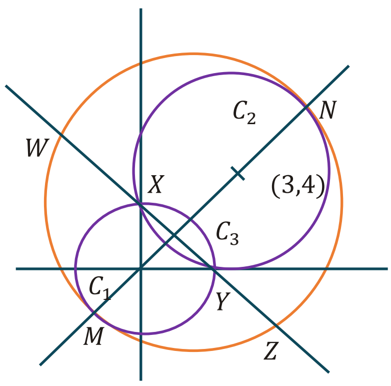
$MC_1 + C_1C_2 + C_2N = 2r$

$\Rightarrow 3 + 5 + 4 = 2r \Rightarrow r = 6 \Rightarrow$ Radius of $C_3 = 6$

Suppose centre of C_3 be

$(0+r_4\cos\theta, 0+r_4\sin\theta)$, $\left\{\begin{matrix} r_4 = C_1C_3 = 3 \\ \tan\theta = \frac{4}{3} \end{matrix}\right\}$

$C_3 = \left(\frac{9}{5}, \frac{12}{5}\right) = (h, k) \Rightarrow 2h + k = 6$



Equation of ZW and XY is $3x + 4y - 9 = 0$

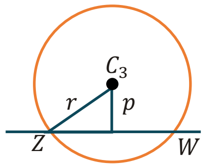
(Common chord of circle $C_1 = 0$ and $C_2 = 0$)

$ZW = 2\sqrt{r^2 - p^2} = \frac{24\sqrt{6}}{5}$

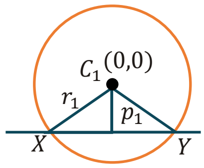
(where $r = 6$ and $p = \frac{6}{5}$)

$XY = 2\sqrt{r_1^2 - p_1^2}$

(where $r_1 = 3$ and $p_1 = \frac{9}{5}$)



$$\frac{\text{Length of } ZW}{\text{Length of } XY} = \sqrt{6}$$



Let length of perpendicular from M to ZW be λ,

$$\lambda = 3 + \frac{9}{5} = \frac{24}{5}$$

$$\frac{\text{Area of } \Delta MZN}{\text{Area of } \Delta ZMW} = \frac{\frac{1}{2}(MN) \times \frac{1}{2}(ZW)}{\frac{1}{2} \times ZW \times \lambda} = \frac{1}{2}\frac{MN}{\lambda} = \frac{5}{4}$$

$$C_3 : \left(x - \frac{9}{5}\right)^2 + \left(y - \frac{12}{5}\right)^2 = 6^2$$

$C_1 : x^2 + y^2 - 9 = 0$

common tangent to C_1 and C_3 is common chord of C_1 and C_3 is $3x + 4y + 15 = 0$.

Now $3x + 4y + 15 = 0$ is tangent to parabola $x^2 = 8\alpha y$.

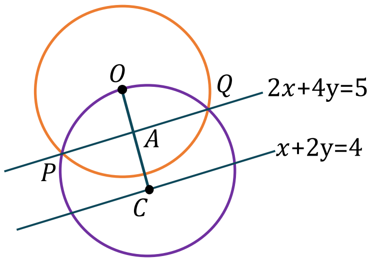
$$x^2 = 8\alpha\left(\frac{-3x - 15}{4}\right) \Rightarrow 4x^2 + 24\alpha x + 120\alpha = 0$$

$$D = 0 \Rightarrow \alpha = \frac{10}{3}$$

13. **Ans. (2.00)**

M-I

$OA = \frac{\sqrt{5}}{2}$ $\qquad$ $OC = \frac{4}{\sqrt{5}}$



$CQ = OC = \frac{4}{\sqrt{5}}$ and $CA = \frac{3}{2\sqrt{5}}$

$$\therefore\ OQ = \sqrt{OA^2 + AQ^2} = \sqrt{OA^2 + (CQ^2 - CA^2)}$$

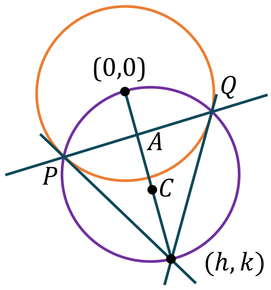
$$\Rightarrow \sqrt{\frac{5}{4} + \frac{16}{5} - \frac{9}{20}} = \sqrt{4}$$

$\Rightarrow 2 = r$

M-II

$PQ : hx + ky = r^2$

Given $PQ :\ 2x + 4y = 5$



$$\Rightarrow \frac{h}{2} = \frac{k}{4} = \frac{r^2}{5} \Rightarrow h = \frac{2r^2}{5} \quad k = \frac{4r^2}{5}$$

$$\therefore\ C = \left(\frac{r^2}{5}, \frac{2r^2}{5}\right)$$

$\therefore$ C lies on $x + 2y = 4$ $\Rightarrow$ $\frac{r^2}{5} + 2\left(\frac{2r^2}{5}\right) = 4$

$\Rightarrow r^2 = 4 \Rightarrow r = 2$

14. Ans. (B)

one of the vertex is intersection of x-axis

and $x+y+1=0 \Rightarrow A(-1,0)$

Let vertex B be $(\alpha,-\alpha-1)$

Line $AC \perp BH \Rightarrow \alpha=1 \Rightarrow B(1,-2)$

Let vertex C be$(\beta,0)$

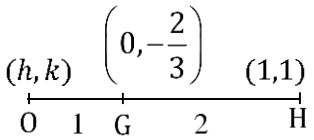
Line $AH \perp BC$

$m_{AH}.m_{BC}=-1$

$\frac{1}{2}.\frac{2}{\beta-1}=-1 \Rightarrow \beta=0$

Centroid of ΔABC is $\left(0,-\frac{2}{3}\right)$

Now G(centroid) divides line joining circumcentre (O) and orthocentre (H) in the ratio 1: 2

$\Rightarrow$ 

$2h+1=0 \quad 2k+1=-z$

$h=-\frac{1}{2} \quad k=-\frac{3}{2}$

$\Rightarrow$ circumcenter is $\left(-\frac{1}{2},-\frac{3}{2}\right)$

Equation of circumcircle is (passing through $C(0,0)$) is $x^2+y^2+x+3y=0$

15. Ans. (D)

$S_n=1+\frac{1}{2}+\frac{1}{2^2}+...+\frac{1}{2^{n-1}}$

$=2\left(1-\frac{1}{2^n}\right)=2-\frac{1}{2^{n-1}}$

Centre of C_n is $\left(2-\frac{1}{2^{n-2}},0\right)$

and radius of C_n is $\frac{1}{2^{n-1}}$

when $r=\frac{1025}{513}<2$

C_n will lie inside m

when $2-\frac{1}{2^{n-2}}+\frac{1}{2^{n-1}}<\frac{1025}{513}$

$\Rightarrow k=10$

Also $\ell=5$

$3k+2\ell=30+10=40$

Ans. (D)

16. Ans. (B)

Center of D_n is (S_{n-1}, S_{n-1})

$$r = \frac{1}{2^{n-1}}$$

D_n will lie inside

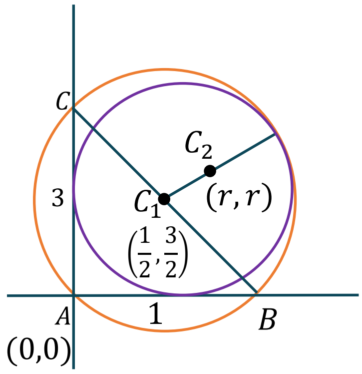
when $\sqrt{2}\left(S_{n-1}\right) < \frac{2^{199}-1}{2^{198}}\sqrt{2}$

$$\Rightarrow \frac{\sqrt{2}}{2^{n-2}} > \frac{\sqrt{2}}{2^{198}} + \frac{1}{2^{n-1}}$$

$\Rightarrow n = 199$

17. Ans. (0.83 or 0.84)

$4-\sqrt{10} = 0.83 \text{ or } 0.84$



$C_1\left(\frac{1}{2},\frac{3}{2}\right)$ and $r_1 = \frac{\sqrt{10}}{2}$

$C_2 = (r, r)$

$\therefore$ circle C_2 touches C_1 internally

$$\Rightarrow C_1C_2 = \left|r - \frac{\sqrt{10}}{2}\right|$$

$$\Rightarrow \left(r-\frac{1}{2}\right)^2 + \left(r-\frac{3}{2}\right)^2 = \left(r-\frac{\sqrt{10}}{2}\right)^2$$

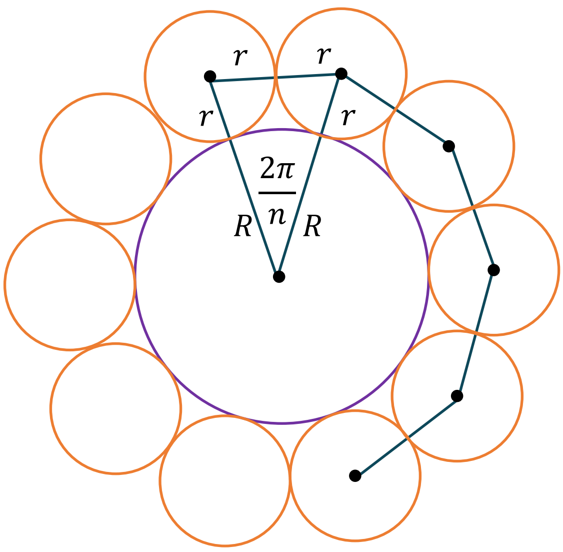
$$r^2 - 4r + \sqrt{10}r = 0$$

$r = 0$ (reject) or $r = 4-\sqrt{10}$

18. Ans. (C,D)

$$2(R+r)\sin\frac{\pi}{n} = 2r$$

$$\frac{R+r}{r} = \operatorname{cosec}\frac{\pi}{n}$$



(A) $n = 4, R + r = \sqrt{2}r$

(B) $n = 5, \frac{R+r}{r} = \operatorname{cosec}\frac{\pi}{5} < \operatorname{cosec}\frac{\pi}{6}$

$R + r < 2r \Rightarrow r > R$

(C) $n = 8, \frac{R+r}{r} = \operatorname{cosec}\frac{\pi}{8} > \operatorname{cosec}\frac{\pi}{4}$

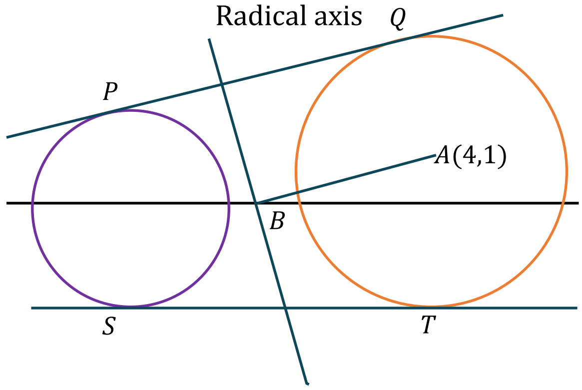
$R + r > \sqrt{2}r$

(D) $n = 12, \frac{R+r}{r} = \operatorname{cosec}\frac{\pi}{12} = \sqrt{2}\left(\sqrt{3}+1\right)$

$R + r = \sqrt{2}\left(\sqrt{3}+1\right)r$

$\sqrt{2}\left(\sqrt{3}+1\right)r > R$

19. Ans. (2.00)



Let $C_2 (x-4)^2 + (y-1)^2 = r^2$

radical axis: $8x + 2y - 17 = 1 - r^2$

$8x + 2y = 18 - r^2$

$B\left(\frac{18-r^2}{8}, 0\right)$; $A(4,1)$

Given, $AB = \sqrt{5}$

$\Rightarrow \sqrt{\left(\frac{18-r^2}{8} - 4\right)^2 + 1} = \sqrt{5}$

$\Rightarrow r^2 = 2$

JEE (Main) Practice Paper

SECTION–A

1. Ans. (3)

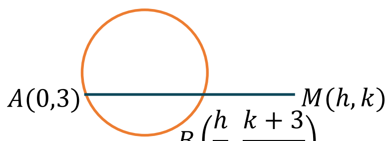
Point $\left(t, \frac{1}{t}\right)$ lies on $x^2 + y^2 = 16 \Rightarrow t^2 + \frac{1}{t^2} = 16$

$\Rightarrow t^4 - 16t^2 + 1 = 0$ ...(i)

If roots are t_1, t_2, t_3, t_4 then

$t_1 t_2 t_3 t_4 = 1$ ...(ii)

2. Ans. (2)



$B\left(\frac{h}{2}, \frac{k+3}{2}\right)$

B lies on circle $\left(\frac{h}{2}\right)^2 + 4\left(\frac{h}{2}\right) + \left(\frac{k+3}{2} - 3\right)^2 = 0 \Rightarrow \frac{h^2}{4} + 2h + \frac{(k-3)^2}{4} = 0$

Hence locus of (h, k) $x^2 + 8x + (y-3)^2 = 0$.

3. **Ans. (4)**

Tangent at (1, 2) to the circle $x^2 + y^2 = 5$

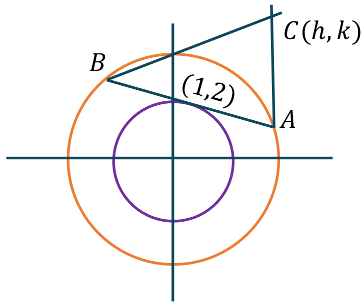
$x + 2y - 5 = 0$

chord of contact from $C(h, k)$ to $x^2 + y^2 = 9$

$hx + ky - 9 = 0$

compare both equations $\frac{h}{1} = \frac{k}{2} = \frac{9}{5}$

$(h, k) \equiv \left(\frac{9}{5}, \frac{18}{5}\right)$



4. **Ans. (1)**

$(x^2 - 2x + 1) - y^2 = 0 \Rightarrow (x + y - 1) = 0 \Rightarrow x - y - 1 = 0$

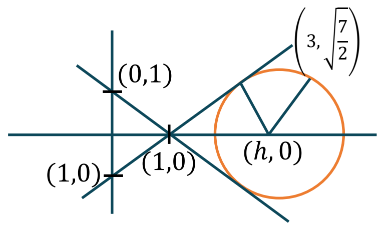
$\left|\frac{h-0-1}{\sqrt{2}}\right| = \sqrt{(h-3)^2 + \frac{7}{2}}$

$h^2 + 1 - 2h = 2\left(h^2 + 9 - 6h + \frac{7}{2}\right) \Rightarrow h^2 - 10h + 24 = 0$

$\Rightarrow h = 6, 4$

But centre lies inside the circle $x^2 + y^2 - 8x + 10y + 15 = 0$

Hence required point (4, 0).



5. **Ans. (1)**

Let any point $P(x_1, y_1)$ to the circle $x^2 + y^2 - \frac{16x}{5} + \frac{64y}{15} = 0 \Rightarrow x_1^2 + y_1^2 - \frac{16x}{5}x_1 + \frac{64y}{15}y_1 = 0$

Length of tangent from $P(x_1, y_1)$ to the circle are in ration

$$\frac{\sqrt{S_1}}{\sqrt{S_2}} = \frac{\sqrt{x_1^2 + y_1^2 - \frac{24}{5}x_1 + \frac{32}{5}y_1 + 15}}{\sqrt{x_1^2 + y_1^2 - \frac{48}{5}x_1 + \frac{64}{5}y_1 + 60}} = \sqrt{\frac{\frac{16}{5}x_1 - \frac{64}{15}y_1 - \frac{24}{5}x_1 + \frac{32}{5}y_1 + 15}{\frac{16}{5}x_1 - \frac{64}{15}y_1 - \frac{48}{5}x_1 + \frac{64}{5}y_1 + 60}}$$

$$= \sqrt{\frac{-24x_1 + 32y_1 + 225}{-96x_1 + 128y_1 + 900}} = \sqrt{\frac{-24x_1 + 32y_1 + 225}{4(-24x_1 + 32y_1 + 225)}} = \frac{1}{2}$$

6. **Ans. (3)**

Equation of chords of contact from (0, 0) is $gx + fy + c = 0$

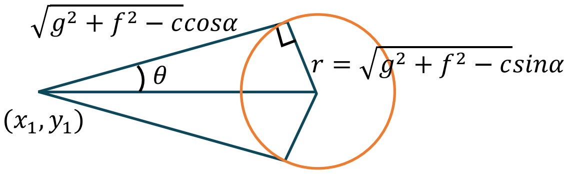
Equation of chords of contact from (g, f) is

$\Rightarrow gx + fy + g(x + g) + f(y + f) + c = 0 \Rightarrow gx + fy + \frac{(g^2 + f^2 + c)}{2} = 0$

Distance between these parallel lines = $\left|\frac{g^2 + f^2 - c}{2\sqrt{g^2 + f^2}}\right|$

7. **Ans. (2)**

tanθ = tan α ⇒ θ = α



angle = 2α

8. Ans. (2)

If two circles touch each other, then

$C_1C_2 = r_1 + r_2$

$\sqrt{(-g_1+g_2)^2+(-f_1+f_2)^2} = \sqrt{g_1^2+f_1^2} + \sqrt{g_2^2+f_2^2}$ squaring both sides

$-2g_1g_2 - 2f_1f_2 = 2\sqrt{\left(g_1^2+f_1^2\right)\left(g_2^2+f_2^2\right)} \Rightarrow (g_1f_2)^2 + (g_2f_1)^2 - 2g_1g_2f_1f_2 = 0 \Rightarrow \frac{g_1}{g_2} = \frac{f_1}{f_2}$

9. Ans. (1)

Let required equation of circle is $x^2 + y^2 + 2gx + 2gx + 2fy + c = 0$

it cuts the circle $x^2 + y^2 - 9 = 0$ orthogonally

$\therefore\ 2g(0) + 2f(0) = c - 9 \Rightarrow c = 9$

It also touches straight line $\ell x + my + n = 0$

$\therefore \left|\frac{\ell(-g)+m(-f)+n}{\sqrt{\ell^2+m^2}}\right| = \sqrt{g^2+f^2-9}$

Locus of centre $(-g, -f)$ is $(\ell x + my + n)^2 = (x^2 + y^2 - 9)(\ell^2 + m^2)$

10. Ans. (1)

Common chord of given circle

$2x + 3y - 1 = 0$

family of circle passing through point of intersection of given circle

$(x^2 + y^2 + 2x + 3y - 5) + \lambda(x^2 + y^2 - 4) = 0 \Rightarrow (\lambda + 1)x^2 + (\lambda + 1)y^2 + 2x + 3y - (4\lambda + 5) = 0$

$x^2 + y^2 + \frac{2x}{\lambda+1} + \frac{3}{\lambda+1}y - \frac{(4\lambda+5)}{\lambda+1} = 0$

centre $\left(-\frac{1}{\lambda+1}, \frac{-3}{2(\lambda+1)}\right)$

This centre lies on AB

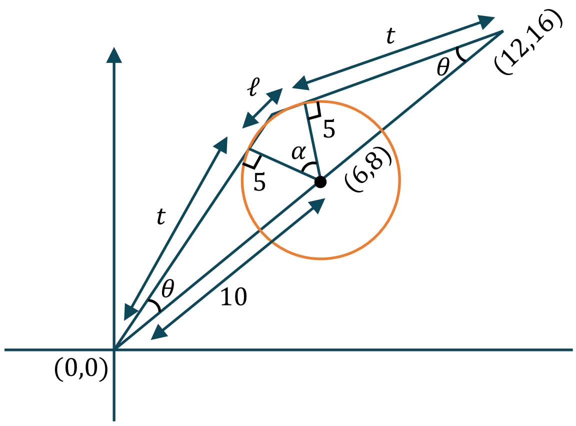
$2\left(-\frac{1}{\lambda+1}\right) + 3\left(\frac{-3}{2(\lambda+1)}\right) - 1 = 0 \Rightarrow -4 - 9 - 2\lambda - 2 = 0 \Rightarrow 2\lambda = -15 \Rightarrow \lambda = -15/2$

$\left(-\frac{15}{2}+1\right)x^2 + \left(-\frac{15}{2}+1\right)y^2 + 2x + 3y - \left(-4\times\frac{15}{2}+5\right) = 0 \Rightarrow -\frac{13x^2}{2} - \frac{13y^2}{2} + 2x + 3y + 25 = 0$

$\Rightarrow 13(x^2 + y^2) - 4x - 6y - 50 = 0.$

11. Ans. (3)

Note : (0,0), (6,8), (12,16) are collinear and (6,8) is mid-point of (0,0) and (12,16)



$\sin\theta = \frac{5}{10} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

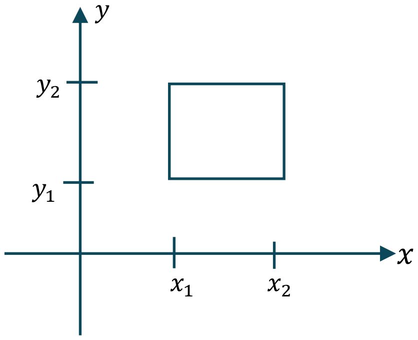
so, $\alpha = \pi - \left(\frac{\pi}{2} - \theta\right) - \left(\frac{\pi}{2} - \theta\right)$

$\alpha = 2\theta = \frac{\pi}{3} = \frac{\ell}{r} \Rightarrow \ell = \frac{5\pi}{3}$

$t = \sqrt{10^2 - 5^2} = 5\sqrt{3}$

$\therefore$ Required length $= t + \ell + t = \frac{5\pi}{3} + 10\sqrt{3}$

12. **Ans. (2)**



Let $x_1 < x_2$
and $y_1 < y_2$
forming a rectangle hence, concyclic.

13. **Ans. (1)**

Rearrange the given equation as :

$x^3 + y^3 + (-1)^3 = 3(x)(y)(-1)$

$(\because a^3 + b^3 + c^3 = 3abc \to a + b + c = 0 \text{ or } a = b = c)$

$\Rightarrow x + y + 1(-1) = 0 \text{ or } x = y = -1$

$\Rightarrow$ a line or an isolated point.

14. **Ans. (1)**

Let $A(\alpha,0)$ and $B(0,\beta)$

$\angle BOA = 90° \Rightarrow BA$ is dimeter of the circle.

Let centroid is $G(p, q)$

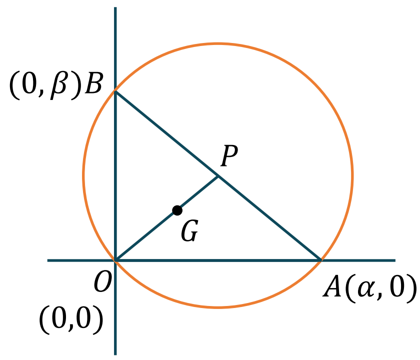
So, $p = \dfrac{\alpha+0+0}{3}$ and $q = \dfrac{0+\beta+0}{3}$

$\Rightarrow \alpha = 3p$ and $\beta = 3q$

Note : $AB = 2 \times$ radius $= 6k$

$\Rightarrow \sqrt{\alpha^2+\beta^2} = 6k \Rightarrow (3p)^2 + (3p)^2 = 36k^2$

$\Rightarrow p^2 + q^2 = 4k^2 \Rightarrow x^2 + y^2 = 4k^2$



M-II: OP is median of ΔOAB from 'O'. where 'P' is the centre of the circle.

so, $\dfrac{OG}{GP} = \dfrac{2}{1}$ ($G \equiv$ centroid)

$\Rightarrow OG = \dfrac{2}{3}(OP) \Rightarrow OG = 2K$

so, $\sqrt{p^2+q^2} = 2k \Rightarrow p^2 + q^2 = (2k)^2$

$\Rightarrow x^2 + y^2 = (2k)^2$

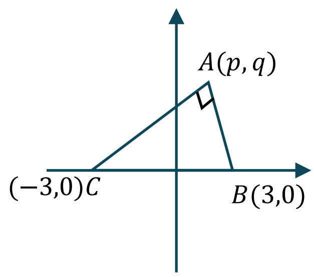
15. **Ans. (1)**

Let $A(p, q)$

$\angle BAC = 90°$

$\Rightarrow BC$ is diameter of the circumcircle of ΔABC so locus of 'A' is

$(p + 3)(p - 3) + (q - 0)(q - 0) \Rightarrow p^2 + q^2 = 9$



Now, let centroid of ΔABC is $G(h,k)$

$\therefore\ h=\frac{p}{3}$ and $k=\frac{q}{3}$

as $p^2+q^2=9 \Rightarrow h^2+k^2=1 \Rightarrow x^2+y^2=1$

16. Ans. (2)

$r=\sqrt{1^2+1^2-(-7)}=3$

$AB=2h$

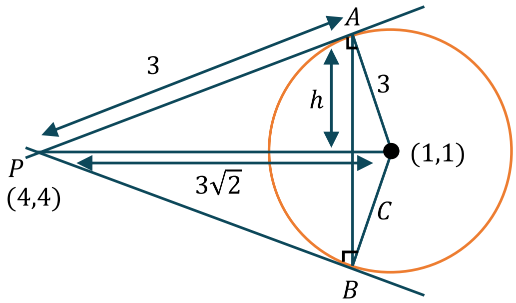
$PC=3\sqrt{2}$

$AP=3$

$\text{ar}(\Delta APC)=\frac{1}{2}(3\times 3)=\frac{1}{2}\left(3\sqrt{2}\right)\times h$

$\Rightarrow h=\frac{3}{\sqrt{2}}$

$\therefore A=2h=3\sqrt{2}$



17. Ans. (3)

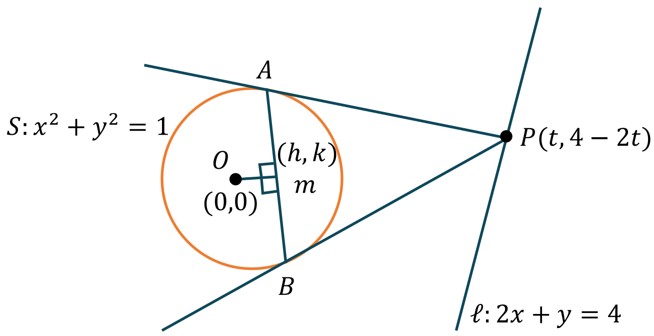
Consider a variable point on the line $L: 2x+y=4$ as $P(t,4-2t)$

AB is a chord of contact of 'P' to circle 'S'.

So, $AB: T=0 \Rightarrow x(t)+y(4-2t)=1$

$\Rightarrow (4y-1)+t(x-2y)=0$

This is family of concurrent lines of the from $L_1+\lambda L_2=0$



As chord AB is of the form $L_1+\lambda L_2=0$,

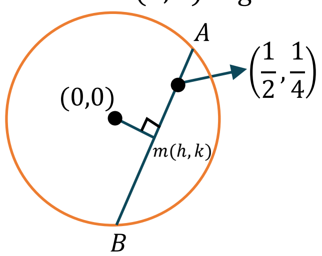
it means AB will pass through intersection point of $L_1=0$ and $L_2=0$.

i.e. $(4y-1=0$ and $x-2y=0)$

That intersection point is $\left(\frac{1}{2},\frac{1}{4}\right)$

so, chord AB is always passing through a fixed point (see figure)

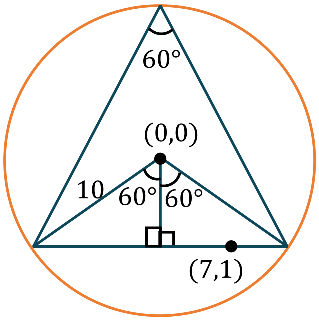
locus of $m(h,k)$ is given as

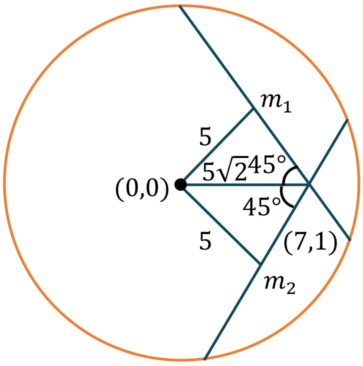


$$\left(\frac{k-0}{h-0}\right)\left(\frac{k-\frac{1}{4}}{h-\frac{1}{2}}\right)=-1$$

$$\Rightarrow x^2+y^2-\frac{x}{2}-\frac{y}{4}=0 \Rightarrow 4\left(x^2+y^2\right)=2x+y$$

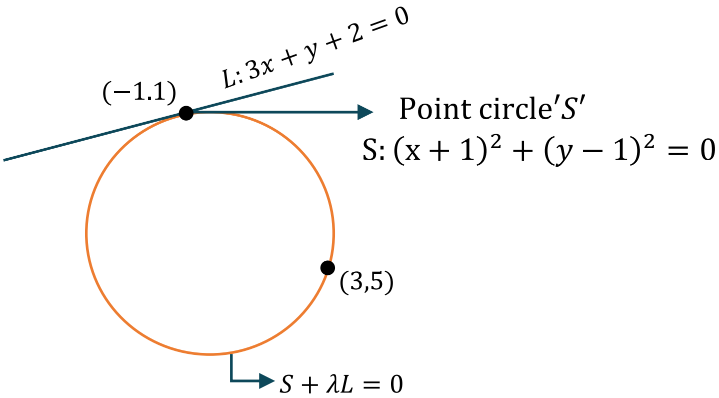
18. **Ans. (1)**





clearly, $m_1 . m_2 = -1$ options

19. **Ans. (3)**



from family of circles the equation of required circle is given by $S + \lambda L = 0$

$S_1 : (x+1)^2 + (y-1)^2 + \lambda(3x + y + 2) = 0$

which passes through (3,5)

so, $(3+1)^2 + (5-1)^2 + \lambda(3.3 + 5 + 2) = 0$

$\Rightarrow \lambda = -2$

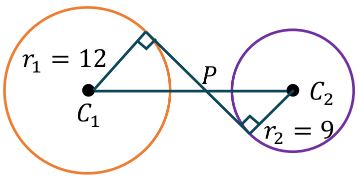
$\therefore S_1 : (x+1)^2 + (y-1)^2 - 2(3x + y + 2) = 0$

$\Rightarrow S_1 : x^2 + y^2 - 4x - 4y - 2 = 0$

centre $\equiv (2, 2)$

20. **Ans. (2)**

$$\frac{C_1P}{PC_2} = \frac{r_1}{r_2} = \frac{12}{9} = \frac{4}{3}$$



$$\Rightarrow \frac{C_1P}{4} = \frac{PC_2}{3} = k$$

$C_1P = 4k, C_2P = 3k \rightarrow C_1C_2 = C_1P + C_2P$

$\therefore C_1C_2 = 35 = 4k + 3k$

$\Rightarrow k = 5 \Rightarrow C_1P = 4k = 20$

SECTION–B

1. **Ans. (7)**

Let r be the radius of new circle $C_1C_2 = 4\sqrt{5}$.

So $r = 2(\sqrt{5}-1)$

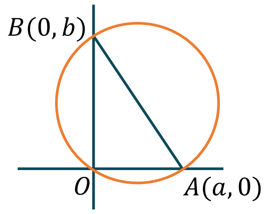
Slope of line joining C_1 and C_2 i.e. $\tan\theta = 2$

$\therefore$ Equation of line joining C_1 and C_2 is

$$\frac{x-0}{\cos\theta} = \frac{y-1}{\sin\theta} = 2 + 2(\sqrt{5}-1) = 2\sqrt{5}$$

$x = 2$ and $y = 5$ $\therefore$ Centre (2, 5)

2. **Ans. (2)**



Equation of circum circle of triangle OAB $x^2 + y^2 - ax - by = 0$.

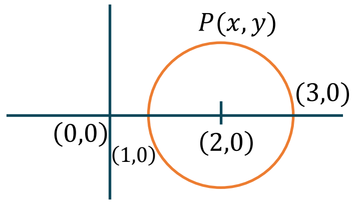
Equation of tangent at origin $ax + by = 0$.

$$d_1 = \frac{|a^2|}{\sqrt{a^2+b^2}} \text{ and } d_2 = \frac{|b^2|}{\sqrt{a^2+b^2}} \Rightarrow d_1 + d_2 = \sqrt{a^2+b^2} = \text{diameter}$$

3. **Ans. (10)**

$x^2 + y^2 - 4x + 3 = 0$

$\sqrt{x^2+y^2}$ represents distance of p from origin

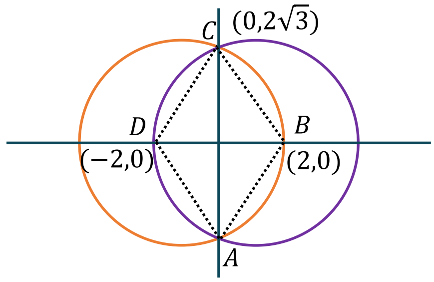


Hence $M = 3^2 + 0^2$

$M = 1^2 + 0^2$

$M - m = 10$.

4. **Ans. (8)**



Area of $ABCD = 4\left(\frac{1}{2}.2.2\sqrt{3}\right)$.

5. **Ans. (1)**

Point $(2a, a+1)$ lies inside circle $x^2 + y^2 - 2x - 2y - 8 = 0$

$4a^2 + (a+1)^2 - 2(2a) - 2(a+1) - 8 < 0 \Rightarrow 5a^2 - 4a - 9 < 0$

$5a^2 - 9a + 5a - 9 < 0 \Rightarrow a(5a-9) + 1(5a-9) < 0$

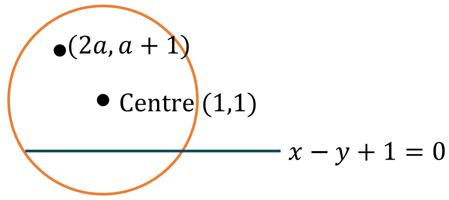
$(5a-9)(a+1) < 0 \Rightarrow a \in (-1, 9/5)$

centre & $(2a, a+1)$ lies on same side w.r.t. line

$x - y + 1 = 0$

$2a - (a+1) + 1 > 0 \Rightarrow a > 0$

Hence $a \in (0, 9/5)$



6. **Ans. (10)**

Distance between line L_1 & L_2

[Since it touch y-axis; diameter is $2h$ = distance between parallel line]

$2h = \dfrac{|10+6|}{\sqrt{2}} \Rightarrow h = \dfrac{8}{\sqrt{2}}$

$h = 4\sqrt{2}$ [radius]

Also radius = $\perp^r$ distance from centre of circle to tangent an it

radius $\Rightarrow \dfrac{|k - 4\sqrt{2} + 6|}{\sqrt{2}} = 4\sqrt{2}$

$\Rightarrow k + 6 - 4\sqrt{2} = 8 \Rightarrow k = 2 + 4\sqrt{2}$

$h + k = 4\sqrt{2} + 2 + 4\sqrt{2}$

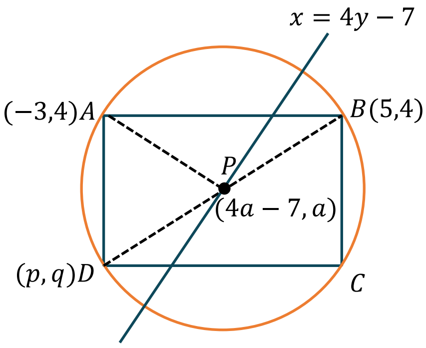
$= 2 + 8\sqrt{2} = a + b\sqrt{a}$

$\therefore a = 2$

$b = 8$

$a + b = 10$

7. **Ans. (32)**



$\Rightarrow \sqrt{(4a-12)^2 + (a-4)^2} = \sqrt{(4a-4)^2 + (a-4)^2}$

$\Rightarrow a = 2 \therefore P(1,2)$

$\because P(1,2)$ is mid point $B(5,4)$ and $D(p,q)$

$\therefore \dfrac{p+5}{2} = 1$ and $\dfrac{q+4}{2} = 2 \Rightarrow (p,q) \equiv (-3,0)$

Now, area of rectangle $ABCD$

$= AB \times AD$

$= \sqrt{64+0} \times \sqrt{0+16} = 8 \times 4$

= 32 sq. unit

8. Ans. (4)

$x^2 + y^2 = 1$

$z = \dfrac{4-y}{7-x}$

Observe that z is the slope of line joining $P(7,4)$ and (x, y) and (x, y) lies on the circle.

So minimum value of z is the slope of the tangent T_2 and maximum value is the slope of tangent T_1.

Let the tangent of slope m to the circle is $y = mx + c$

Perpendicular distance from centre = radius

$\Rightarrow \dfrac{|c|}{\sqrt{m^2+1}} = 1 \Rightarrow c = \pm\sqrt{m^2+1}$

$\Rightarrow y = mx \pm \sqrt{m^2+1}$ passes through (7,4)

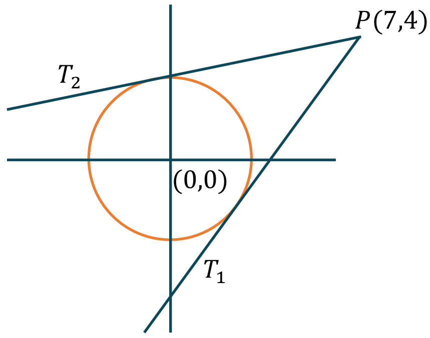
$\Rightarrow 4 = 7m \pm \sqrt{m^2+1}$

$\Rightarrow (7m-4)^2 = m^2+1$

$\Rightarrow 48m^2 - 56m + 15 = 0$

$\Rightarrow m = \dfrac{36}{48}, \dfrac{20}{48} = \dfrac{3}{4}, \dfrac{5}{12} \Rightarrow m = \dfrac{5}{12}, M = \dfrac{3}{4}$

$2M + 6m = \dfrac{3}{2} + \dfrac{5}{2} = 4$



9. Ans. (3)

Equation of circles will be

$\left(x - \dfrac{r}{t}\right)^2 + (y-r)^2 = r^2$

$x^2 + \dfrac{r^2}{t^2} - \dfrac{2xr}{t} + y^2 + r^2 - 2yr = r^2$

passes through (6,4)

$3s + \dfrac{r^2}{t^2} - \dfrac{12r}{t} + 16 - 8r = 0$

$\dfrac{r^2}{t^2} - r\left(8 + \dfrac{12}{t}\right) + 52 = 0$ < r_1, r_2

It has two roots (r_1 & r_2)

we are given

$r_1 r_2 = \dfrac{52}{3}$

$\dfrac{52}{1/t^2} = \dfrac{52}{3}$

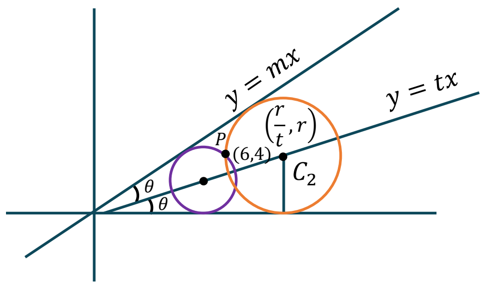
$t^2 = \dfrac{1}{3}$

$t = \dfrac{1}{\sqrt{3}}$

$\theta = 30°$

$\boxed{2\theta = 60°}$

$m = \sqrt{3}$



10. **Ans. (625)**

Circle bisects the circumference hence centre lies on radical axis

Radical axis = $s_1 - s_2 = 0$

$6x + 14y + a + b = 0$

$(1, -4)$ lies on it $6 - 56 + a + b = 0$

$a + b = 50$

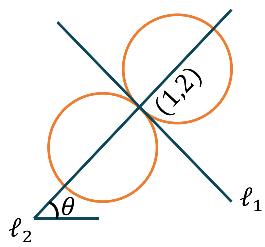
$AM \geq GM$

$\frac{a+b}{2} \geq \sqrt{ab}$

$(ab)_{max} = 625$

JEE (Advanced) Practice Paper

1. **Ans. (A,B)**



$\ell_1 \equiv 4x + 3y = 10; \ell_2 \equiv 3x - 4y = -5$

Let θ be the inclination of ℓ_2

$\therefore \tan\theta = \frac{3}{4}$

$\therefore$ equation of ℓ_2 in parametric form $\frac{x-1}{4/5} = \frac{y-2}{3/5} = \pm 5$

co-ordinates of centres are (5, 5), (–3, –1)

2. **Ans. (B,D)**

Given circles are

$x^2 + y^2 = a^2$ ...(1)

and $(x - 2a)^2 + y^2 = a^2$ ...(2)

Let A and B be the centres and r_1 and r_2 the radii of the circles (1) and (2) respectively. Then

$A \equiv (0, 0), B \equiv (2a, 0), r_1 = a, r_2 = a$

Now $AB = \sqrt{(0-2a)^2 + 0^2} = 2a = r_1 + r_2$

Hence the two circles touch each other extenally.

Let the equation of the circle having same radius 'a' and touching the circles (1) and (2) be

$(x - \alpha)^2 + (y - \beta)^2 = a^2$ ...(3)

Its centre C is (α, β) and radius $r_3 = a$

Since circle (3) touches the circle (1),

$AC = r_1 + r_3 = 2a.$ [Here $AC \neq |r_1 - r_3|$ as $r_1 - r_3 = a - a = 0$]

$\Rightarrow AC^2 = 4a^2 \Rightarrow \alpha^2 + \beta^2 = 4a^2$ ...(4)

Again since circle (3) touches the circle (2)

$BC = r_2 + r_3 \Rightarrow BC^2 = (r_2)$

$\Rightarrow (2a - \alpha)^2 + \beta^2 = (a + a)^2 \Rightarrow \alpha^2 + \beta^2 - 4a\alpha = 0$

$\Rightarrow 4a^2 - 4a\alpha = 0$ [from (4)]

$\Rightarrow \alpha = a$ and from (4), we have $\beta = \pm\sqrt{3}\,a.$

Hence, the required circles are

$(x - a)^2 + (y \mp a\sqrt{3})^2 = a^2$

or $x^2 + y^2 - 2ax \mp 2\sqrt{3}\,ay + 3a^2 = 0.$

3. **Ans. (A,D)**

Two fixed pts. are point of intersection of

$x^2 + y^2 - 2x - 2 = 0$ & $y = 0$

Point $x^2 - 2x - 2 = 0$

$(x-1)^2 - 3 = 0 \Rightarrow x - 1 = \sqrt{3}$, $x - 1 = -\sqrt{3} \Rightarrow (1+\sqrt{3}, 0)\ (1-\sqrt{3}, 0)$

4. **Ans. (B,C,D)**

Equation of a curve passing through the intersection points of the given curves

$ax^2 + 2hxy + by^2 - 2gx - 2fy + c = 0$ ...(1)

and $a'x^2 - 2hxy + (a' + a - b)y^2 - 2g'x - 2f'y + c = 0$ ...(2)

can be written as $\{a'x^2 - 2hxy + (a' + a - b)y^2 - 2g'x - 2f'y + c\}$

$+ \lambda\{ax^2 + 2hxy + by^2 - 2gx - 2fy + c\} = 0$

i.e. $(a' + \lambda a)x^2 + 2h(\lambda - 1)xy + (a' + a - b + \lambda b)y^2$

$- 2(g' + \lambda g)x - 2(f' + \lambda f)y + (1 + \lambda)\, c = 0$ ...(3)

According to the given condition equation (3) must represent a circle, therefore, we have

coeff. of x^2 = coeff. of y^2

i.e. $a' + \lambda a = a' + a - b + \lambda b$

i.e. $\lambda(a - b) = a - b$

gives $\lambda = 1$ and coeff. of $xy = 0$ i.e. $\lambda - 1 = 0$ gives $\lambda = 1$.

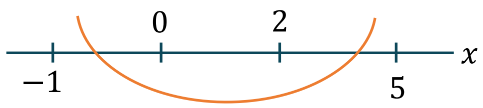
The identical values prove that the curve is a circle.

Putting the above value of λ in equation (3) gives the equation of the circle passing through the intersection points of the curves represented by equations (1) and (2) as $(a'+a)(x^2 + y^2) - 2(g' + g)x - 2(f' + f)y + 2c = 0$

which has its centre at the point $\left(\dfrac{g'+g}{a'+a}, \dfrac{f'+f}{a'+a}\right)$

We can see that the coordinates of the given point P is the same as the centre of the circle passing through the points A, B, C and D. Therefore, we have $PA^2 = PB^2 = PC^2 = PD^2$ = radius of the circle which gives the desired result $PA^2 + PB^2 + PC^2 = 3PD^2$.

5. **Ans. (A,B,C,D)**



Since $(a, 0)$ is a point on the diameter of the circle $x^2 + y^2 = 4$,

So maximum value of a^2 is 4

Let $f(x) = x^2 - 4x - a^2$

clearly $f(-1) = 5 - a^2$ is 4

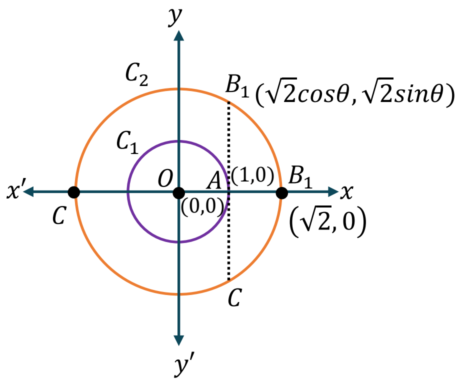
$f(2) = -(a^2 + 4) < 0$

$f(0) = -a^2 < 0$ and $f(5) = 5 - a^2 > 0$

so graph of $f(x)$ will be as shown

Hence (a), (b), (c), (d) are the correct answer.

6. **Ans. (A,C,D)**



We have maximum $BC = 2\sqrt{2}$ and minimum $BC = 2$

$\therefore OA^2 + OB^2 + BC^2 \in [7, 11]$

Let M be the midpoint of AB.

$\Rightarrow M \equiv \left(\frac{1+\sqrt{2}\cos\theta}{2}, \frac{\sqrt{2}\sin\theta}{2}\right) = (h, k)$

$\therefore \sin\theta = \frac{2k}{\sqrt{2}}$, $\cos\theta = \frac{2h-1}{\sqrt{2}}$

$\therefore$ Now on squaring & adding, we get

$\Rightarrow 4k^2 + (2h-1)^2 = 2$

$\therefore$ Locus of $M(h,k)$ is $\left(x-\frac{1}{2}\right)^2 + y^2 = \left(\frac{1}{\sqrt{2}}\right)^2$ **Ans.**

7. **Ans. (B)**

Point from which length of tangents to these circle is same is radical centre

$S_1 - S_2 = 0 \Rightarrow 4x - 4y - 4 = 0 \Rightarrow x - y - 1 = 0$ ...(1)

$S_2 - S_3 = 0 \Rightarrow -6x + 14y - 10 = 0 \Rightarrow -3x + 7y - 5 = 0$ ...(2)

From eq. (1) & (2)

$4y - 8 = 0 \Rightarrow y = 2 \quad x = 3$

8. **Ans. (D)**

If circle be drawn taking radical centre as centre and length of tangents from radial centre to any circle as radius will cut all the three circles orthogonally

Length of tangent = $\sqrt{9+4+9+4+1} = \sqrt{S_1} = \sqrt{27}$

Equation of circle $(x-3)^2 + (y-2)^2 = 27$

$\Rightarrow S_4 : x^2 + y^2 - 6x - 4y - 14 = 0$

9. **Ans. (A)**

$S_1 - S_2 = 0 \Rightarrow x - y - 1 = 0$ ...(1)

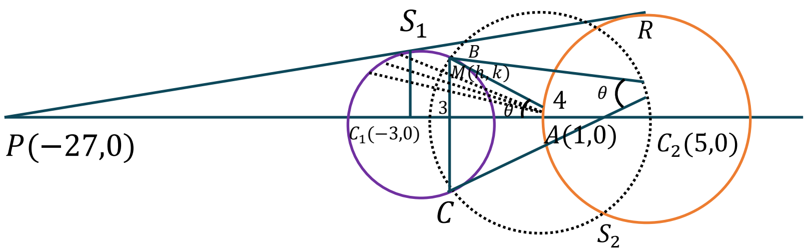
$S_1 - S_4 = 0 \Rightarrow 9x + 6y + 15 = 0 \Rightarrow 3x + 2y + 5 = 0 \Rightarrow 3x - 3y - 3 = 0$ ...(2)

From eq. (1) & (2)

$5y + 8 = 0 \Rightarrow y = -8/5 ; x = -3/5$

10. **Ans. (B)**

ΔPQC_1 and ΔPRC_2 are similar



$\frac{\text{Area of } \Delta PQC_1}{\text{Area of } \Delta PRC_2} = \frac{r_1^2}{r_2^2} = \frac{9}{16}$

11. Ans. (C)

Let mid point $m(h,k)$. Now equation of chord

$T = S_1$

$hx + ky + 3(x+h) = h^2 + k^2 + 6h$

it passes through (1, 0)

$h + 3(1+h) = h^2 + k^2 + 6h$

locus $x^2 + y^2 + 2x - 3 = 0$

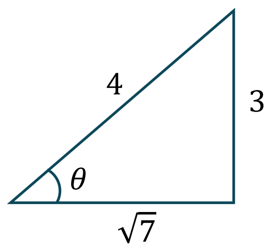
But clear from Geometry it will be arc of BC

12. Ans. (A)

Common chord of S_1 & answer of 5

$4x + 3 = 0 \Rightarrow x = -3/4$

at $x = -3/4$ $\quad \left(-\frac{3}{4}+3\right)^2 + y^2 = 9 \Rightarrow y^2 = 9 - \frac{81}{16}$



$y^2 = \frac{63}{16} \Rightarrow y = \pm\frac{3\sqrt{7}}{4}$

Hence $\tan\theta = \frac{\frac{3\sqrt{7}}{4}}{(1+3/4)} = \frac{3\sqrt{7}}{7} \Rightarrow \tan\theta = \frac{3}{\sqrt{7}}$

13. Ans. (A → q; B → p; C → r; D → s)

(A) $S_1 - S_2 = 0$ is the required common chord i.e $2x = a$

Make homogeneous, we get $x^2 + y^2 - 8.4\frac{x^2}{a^2} = 0$

As pair of lines subtending angle of 90° at origin

$\therefore$ coefficient of x^2 + coefficient of $y^2 = 0$

$\therefore a = \pm 4$

(B) Three lines are parallel so not any circle is possible

(C) Equation of common chord is $4x - 3y + 2 = 0$.

End points of common chord are (1,2) & (4,6)

Length of common chord is 5

(D) C_1 (1, 0), $r_1 = 1$ and C_2 (–3, 3), $r_2 = 4$

distance between centres C_1 and $C_2 = d = 5$

$d = r_1 + r_2 = 5 \Rightarrow 3$ common tangents

14. Ans. (11)

$\because \tan 60° = \frac{OA}{1} = \sqrt{3}$

$\therefore A(\sqrt{3},0)$ and $C(-\sqrt{3},0)$

$\because \sin 60° = \frac{r}{1} = \frac{\sqrt{3}}{2}$

Let coordinates of any point P on the circle be $P \equiv (r\cos\theta, r\sin\theta)$

$\therefore PA^2 = (\sqrt{3} - r\cos\theta)^2 + (r\sin\theta)^2$

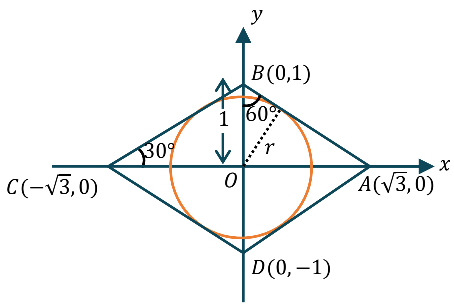
$PB^2 = (r\cos\theta)^2 + (1 - r\sin\theta)^2$

$PC^2 = (r\cos\theta + \sqrt{3})^2 + (r\sin\theta)^2$

and $PD^2 = (r\cos\theta)^2 + (r\sin\theta + 1)^2$

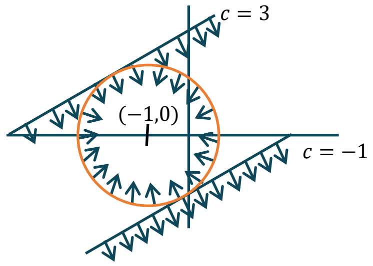
$\therefore PA^2 + PB^2 + PC^2 + PD^2 = 4r^2 + 8 = 11$

$\because \; r = \sqrt{3}/2$



15. **Ans. (1)**

$$\left|\frac{-1-0+c}{\sqrt{2}}\right| = \sqrt{2} \Rightarrow c - 1 = \pm 2 \Rightarrow c = -1, 3$$



But $c = -1$ common point is one

$c = 3$ common point is infinite

Hence $c = -1$ is Answer.

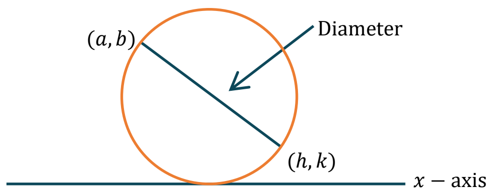
16. **Ans. (4)**

Equation of circle whose diameter's end points are (a, b) and (h, k)

$(x - a)(x - h) + (y - b)(y - k) = 0$

$x^2 + y^2 - x(a + h) - y(b + k) + ah + bk = 0$

it touches x-axis.



Hence $g^2 = c \Rightarrow \left(\frac{a+h}{2}\right)^2 = ah + bk \Rightarrow (h - a)^2 = 4bk$

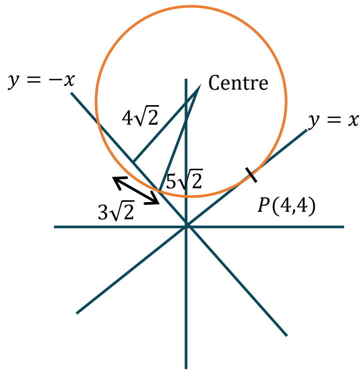
$\therefore$ Locus of (h, k) is $(x - a)^2 = 4by$.

17. Ans. (4)

By family of circle equation of circle touching $y = x$ at $p(4, 4)$

$(x-4)^2 + (y-4)^2 + \lambda(x-y) = 0 \Rightarrow x^2 + y^2 + x(\lambda - 8) - y(\lambda + 8) + 32 = 0$

radius $= \sqrt{\left(\frac{\lambda-8}{2}\right)^2 + \left(\frac{\lambda+8}{2}\right)^2 - 32} = 5\sqrt{2}$



$2\lambda^2 + 128 - 128 = 200 \Rightarrow \lambda = \pm 10$

$\lambda = 10 \qquad x^2 + y^2 + 2x - 18y + 32 = 0$

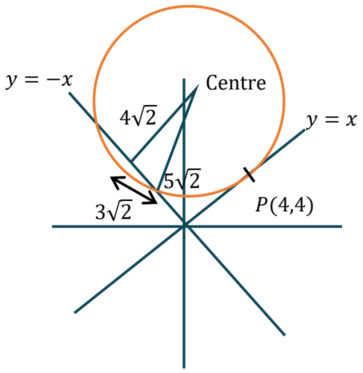
$\lambda = -10 \qquad x^2 + y^2 - 18x + 2y + 32 = 0$

By family of circle equation of circle touching $y = x$ at $p(-4, -4)$

$(x+4)^2 + (y+4)^2 + \lambda(x-y) = 0$

$\Rightarrow x^2 + y^2 + x(\lambda + 8) + y(8 - \lambda) + 32 = 0$

radius $= \sqrt{\left(\frac{\lambda+8}{2}\right)^2 + \left(\frac{\lambda-8}{2}\right)^2 - 32} = 5\sqrt{2} \Rightarrow \lambda^2 = 100 \Rightarrow \lambda = \pm 10$



Hence $x^2 + y^2 + 18x - 2y + 32 = 0 \Rightarrow x^2 + y^2 - 2x - 18y + 32 = 0$

as $(-10, 2)$ lies inside $x^2 + y^2 + 18x - 2y + 32 = 0$

18. Ans. (1)

Let equation of circle is $(x - \sqrt{2})^2 + (y - \sqrt{3}) = r^2$, (x_1, y_1) & (x_2, y_2) are integer points on circle

$(x_1 - \sqrt{2})^2 + (y_1 - \sqrt{3})^2 = (x_2 - \sqrt{2})^2 + (y_2 - \sqrt{3})^2 = r^2$

$(x_2 - x_1)(x_2 + x_1 - 2\sqrt{2}) + (y_2 - y_1)(y_2 + y_1 - 2\sqrt{3}) = 0$

$(x_2^2 - x_1^2) + (y_2^2 - y_1^2) = 2\sqrt{3}(y_2 - y_1) + 2\sqrt{2}(x_2 - x_1)$

$A = \sqrt{3}\,B + \sqrt{2}\,C$

Therefore $A = B = C = 0$

$x_1 = x_2$ & $y_1 = y_2$

So, no distinct points are possible.