Binomial Theorem

SOLUTIONS

EXERCISE - O

1. **Ans. (C)**

Coefficients of $x^7 = {}^nC_7(2)^{n-7}\left(\frac{1}{3}\right)^7$

Coefficients of $x^8 = {}^nC_8 2^{n-8}\left(\frac{1}{3}\right)^8$

$\because$ Given that: coeff of x^7 = coeff. of x^8

$\Rightarrow {}^nC_7\, 2^{n-7}\left(\frac{1}{3}\right)^7 = {}^nC_8 2^{n-8}\left(\frac{1}{3}\right)^8 \Rightarrow \frac{n!}{7!(n-7)!}\cdot\frac{2^{n-7}}{2^{n-8}} = \frac{n!}{8!(n-8)!}\times\frac{3^7}{3^8}$

$2 = \frac{7!\,(n-7)!}{8!\,(n-8)!}\times\frac{1}{3}$

$2 = \frac{1}{8}\times(n-7)\times\frac{1}{3}$

$\Rightarrow n - 7 = 48 \Rightarrow n = 55$

2. **Ans. (A)**

$T_2 = {}^nC_1\left(a^{\frac{1}{13}}\right)^{n-1}\left(a^{\frac{3}{2}}\right) = 14a^{\frac{5}{2}} \Rightarrow n = 14 \qquad \therefore \frac{{}^nC_3}{{}^nC_2} = 4$

3. **Ans. (B)**

$\left.\begin{array}{l} T_{2m+1} \Rightarrow {}^{10}C_{2m} \\ T_{4m+5} \Rightarrow {}^{10}C_{4m+4} \end{array}\right]$ equal

$2m + 4m + 4 = 10 \Rightarrow 6m + 4 = 10 \Rightarrow m = 1$

OR

$2m = 4m + 4 \Rightarrow m = -2$ (Which is not valid)

4. **Ans. (B)**

middle term $= T_5$

$T_5 = T_{4+1} = {}^8C_4 \cdot k^4 = 1120 \Rightarrow k = 2$

5. **Ans. (A)**

$\sum_{k=0}^{4}\frac{5^{4-k}}{(4-k)!}\frac{x^k}{k!} = \frac{1}{4!}\sum\frac{4!\ 5^{4-k}x^k}{(4-k)!k!}$

$\left(\because \frac{4!}{k!(4-k)!} = {}^4C_k\right) \Rightarrow \frac{1}{4!}\sum_{k=0}^{4}{}^4C_k 5^{4-k}x^k = \frac{8}{3}$

$\Rightarrow \frac{1}{4!}(x+5)^4 = \frac{8}{3}$

$\Rightarrow (x+5)^4 = 64 = 2^6 \Rightarrow x+5 = 2^{6/4}$

$\Rightarrow x+5 = 2^{3/2} \Rightarrow x = 2\sqrt{2}-5$

6. Ans. (B)

General term in the expansion $\left(\sqrt{2}+3^{1/4}\right)^{100}$

$$T_{r+1} = {}^{100}C_r\left(\sqrt{2}\right)^{100-r}\left(3^{1/4}\right)^r$$

$$= {}^{100}C_r\left(2^{1/2}\right)^{100-r}3^{r/4}$$

$$T_{r+1} = {}^{100}C_r\,2^{50-r/2}\ 3^{r/4}$$

$$\Rightarrow \text{Rational terms} : \frac{r}{4} = I$$

$\Rightarrow r = 0, 4, 8, 12 \ldots\ldots\ldots\ldots, 100$

$\Rightarrow$ Number of terms = 26

7. Ans. (A)

$$T_{r+1} = {}^{6561}C_r(7)^{\frac{6561-r}{3}}(11^{1/9})^r$$

Here r should be multiple of 9

$r = 0, 9, 18 \ldots\ldots 6561$

Number of terms = 730

8. Ans. (B)

$$\frac{1}{\lfloor n}\left(\frac{\lfloor n}{\lfloor n-1\lfloor 1}+\frac{\lfloor n}{\lfloor n-3\lfloor 3}+\ldots+\frac{\lfloor n}{\lfloor 1\lfloor n-1}\right)$$

$$\frac{1}{\lfloor n}\left({}^nC_1+{}^nC_3+{}^nC_5+\ldots\right)$$

$$=\frac{2^{n-1}}{\lfloor n}$$

9. Ans. (C)

${}^{18}C_{r-2} + 2\cdot {}^{18}C_{r-1} + {}^{18}Cr \geq {}^{20}C_{13}$

$\because$ We know that ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$$\Rightarrow \underbrace{{}^{18}C_{r-2}+{}^{18}C_{r-1}}_{\downarrow}+\underbrace{{}^{18}C_{r-1}+{}^{18}C_r}_{\downarrow}\geq {}^{20}C_{13}$$

$\Rightarrow {}^{19}C_{r-1} + {}^{19}C_r \geq {}^{20}C_{13}$

$\Rightarrow {}^{20}C_r \geq {}^{20}C_{13}$ ...(1)

$\because {}^{20}C_{13} = {}^{20}C_7$

Hence the values of r satisfying (1) are $r = 7\ 8, 9, 10, 11, 12, 13$

10. Ans. (D)

$${}^{99}C_{97}+{}^{98}C_{96}+{}^{97}C_{95}+\ldots\ldots\ldots+{}^3C_1+{}^2C_0={}^{99}C_{97}+{}^{98}C_{96}+{}^{97}C_{95}+\ldots\ldots\ldots+{}^3C_1+{}^3C_0$$

$\left(\because {}^2C_0={}^3C_0\right)$

$$={}^{99}C_{97}+{}^{98}C_{96}+{}^{97}C_{95}+\ldots\ldots\ldots+{}^4C_2+{}^4C_1 \qquad \left(\because {}^nC_r+{}^nC_{r-1}={}^{n+1}C_r\right)$$

$$={}^{99}C_{97}+{}^{98}C_{96}+{}^{97}C_{95}\ldots\ldots\ldots+{}^5C_2+{}^5C_1$$

$$={}^{99}C_{97}+{}^{99}C_{96}={}^{100}C_{97}$$

Hence option (D) is the answer.

11. Ans. (B)

$(2x^2-3x+1)^{11} = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots + a_{22}x^{22}$ ....(1)

Put $x = 1$ in equation (1) $\Rightarrow$

$(2-3+1)^{11} = a_0 + a_1 + a_2 + \ldots + a_{22}$ .....(2)

Now put $x = -1$ in equation (1)

$(2+3+1)^{11} = a_0 - a_1 + a_2 + \ldots + a_{22}$ .....(3)

Question is asking the sum of even powers in the expansion of (1) $\Rightarrow a_0 + a_2 + \ldots\ldots + a_{22}$

On adding (2) + (3)

$0 + 6^{11} = 2(a_0 + a_2 + a_4 + \ldots + a_{22})$

$\Rightarrow a_0 + a_2 + a_4 + \ldots\ldots + a_{22} = \frac{6^{11}}{2} = 3\times 6^{10}$

$\Rightarrow$ Answer is option (B)

12. Ans.(D)

$(1-x+2x^2)^{12}$

General term = $\frac{12!}{r_1!\,r_2!\,r_3!}(1)^{r_1}(-x)^{r_2}(2x^2)^{r_3}$

$r_2 + 2r_3 = 4 \Rightarrow r_3 = 0, r_2 = 4, r_1 = 8$

$r_3 = 1, r_2 = 2, r_1 = 9$

$r_3 = 2, r_2 = 0, r_1 = 10$

Co-efficient of $x^4 = \frac{12!}{4!\,8!} + \frac{12!}{2!\,10!}(2)^2 + \frac{12!}{2!\,9!}\times(2) = {}^{12}C_8 + 4.\,{}^{12}C_{10} + 6.\,{}^{12}C_9$

$= {}^{12}C_3 + 3\cdot{}^{13}C_3 + {}^{14}C_4$ (after solving)

13. Ans. (B)

$\sum_{r=1}^{10} r(r-1)\,{}^{10}C_r = k\times 2^9$

LHS

$\sum_{r=1}^{10} r(r-1)\,{}^{10}C_r$ $\left\{\text{using } {}^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-1}\right\}$

$= 90\sum_{r=2}^{10} {}^8C_{r-2} = 90\left[{}^8C_0 + {}^8C_1 \ldots\ldots + {}^8C_8\right]$

$= 90\left[2^8\right] = 45\times 2^9$

Thus $k = 45$

14. Ans. (D)

$\frac{{}^{11}C_0}{1} + \frac{{}^{11}C_1}{2} + \frac{{}^{11}C_2}{3} \ldots\ldots\ldots\ldots + \frac{{}^{11}C_{11}}{12}$

$\sum_{r=0}^{11} \frac{{}^{11}C_r}{r+1}$

$\sum_{r=0}^{11} \frac{1}{r+1}\cdot\frac{r+1}{12}\cdot{}^{12}C_{r+1}$ $\left\{{}^nC_r = \frac{r+1}{n+1}\cdot{}^{n+1}C_{r+1}\right\}$

$\frac{1}{12}\sum_{r=0}^{11} {}^{12}C_{r+1} = \frac{1}{12}\left[{}^{12}C_1 + {}^{12}C_2 \ldots\ldots\ldots + {}^{12}C_{12}\right]$

$\frac{1}{12}\left[2^{12}-1\right] = \frac{2^{12}-1}{12}$

15. Ans. (B)

$$\sum_{m=0}^{n}\sum_{p=0}^{m} {}^nC_m\, {}^mC_p = \sum_{m=0}^{n} {}^nC_m\, 2^m = 3^n$$

16. Ans. (A)

$\lfloor 10 = \lfloor 7 \times 8 \times 9 \times 10$

$5040 \times 8 \times 9 \times 10$

last three digits = 800

17. Ans. (A)

$$\frac{9+1}{\left|\frac{2}{3x}\right|+1} - 1 \le r \le \frac{9+1}{\left|\frac{2}{3x}\right|+1}$$

$$\Rightarrow \frac{10}{\frac{4}{9}+1} - 1 \le r \le \frac{10}{\frac{4}{9}+1} \Rightarrow \frac{77}{13} \le r \le \frac{90}{13}$$

$\Rightarrow 5 \times 9 \le r \le 6 \times 9$

$$\therefore T_7 = T_{6+1} = {}^9C_6 \left(2\right)^3 \left(3 \times \frac{3}{2}\right)^6 = {}^9C_6 \times 2^9 \times \left(\frac{3}{2}\right)^{12}$$

18. Ans. (B)

$(\sqrt{2}+1)^6 = I + f$

$(\sqrt{2}-1)^6 = f'$

$2[\,{}^6C_0 + {}^6C_2 \cdot 2 + {}^6C_4 (2)^2 + \cdots] = I + f + f'$

$f + f' = 1$ or $f' = 1 - f$

$I = 2[\,{}^6C_0 + {}^6C_2 \cdot 2 + {}^6C_4 \cdot 4 + {}^6C_6 \cdot 8] - 1$

$I = 2[1 + 30 + 60 + 8] - 1 = 197$

19. Ans. (A,B)

$\left(x + x^{\log_{10} x}\right)^5$

$T_3 = {}^5C_2 \left(x\right)^3 \left(x^{\log_{10} x}\right)^2 = 10^6$

$T_3 = 10x^3\ x^{2\log_{10} x} = 10^6$

$x^3 \cdot x^{2\log_{10} x} = 10^5$

Taking Logarithm both sides

$\log_{10} x^3 + \log_{10}\left(x^{2\log_{10} x}\right) = \log_{10} 10^5$

$3\log_{10} x + 2\log_{10} x\left(\log_{10} x\right) = 5$

$2\left(\log_{10} x\right)^2 + 3\log_{10} x - 5 = 0$

Put $\log_{10} x = y$ $\quad\therefore\ 2y^2 + 3y - 5 = 0$

$\Rightarrow (y-1)(2y+5) = 0 \Rightarrow y = 1,\ y = -\frac{5}{2}$

$\Rightarrow \log_{10} x = 1,\ \log_{10} x = -\frac{5}{2} \Rightarrow x = 10;\ x = 10^{-\frac{5}{2}}$

Hence A and B.

20. Ans. (B,C,D)

$$\left(x^3+3\cdot 2^{-\log_{\sqrt{2}}\sqrt{x^3}}\right)^{11}$$

Here, $2^{-\log_{\sqrt{2}}\sqrt{x^3}} = 2^{-\log_1 x^3} = \frac{1}{x^3}$

$$\Rightarrow\left(x^3+\frac{3}{x^3}\right)^{11} \Rightarrow T_{r+1}={}^{11}C_r\left(x^3\right)^{11-r}\cdot\left(\frac{3}{x^3}\right)^r = {}^{11}C_r x^{33-6r}3^r$$

For term containing x^2 ; $33-6r=2$

$r=\frac{31}{6}$

Hence no term containing x^2 [Hence B]

For term containing x^{-3} ; $33-6r=-3 \Rightarrow r=6$

(i) $T_7 = {}^{11}C_6 x^{-3}\cdot 3^6$

For term containing x^3 ; $33-6r=3\Rightarrow$ $r=5$

(ii) $T_6 = {}^{11}C_5 x^3\cdot 3^5$

Ratio of term containing x^3 to term containing x^{-3}

$$\frac{T_6}{T_7}=\frac{{}^{11}C_5\left(3^5\right)}{{}^{11}C_6\left(3^6\right)}=\frac{1}{3}$$ [Hence C and D]

21. Ans. (A,B)

$(1+x^2)(1+x)^n = A_0+A_1x+A_2x^2 \ldots$

Expanding $(1+x^2)^2(1+x)^n$

$(1+2x^2+x^4)({}^nC_0+{}^nC_1x+{}^nC_2x^2\ldots)$

$\Rightarrow {}^nC_0+{}^nC_1x+({}^nC_2+2\cdot{}^nC_0)x^2\ldots$

Thus on comparing,

$A_0={}^nC_0=1$

$A_1={}^nC_1=n$

$A_2={}^nC_2+2\cdot{}^nC_0=\frac{n(n-1)}{2}+2$

Given: A_0, A_1, A_2 in AP

$\Rightarrow 2n=1+\frac{n(n-1)}{2}+2 \Rightarrow 4n=2+n^2-n+4$

$\Rightarrow n^2-5n+6=0\Rightarrow(n-2)(n-3)=0$

$\Rightarrow n=2, n=3$

22. Ans. (A,B,C,D)

$$\left(\sqrt[3]{4}+\frac{1}{\sqrt[4]{6}}\right)^{20}$$

$$T_{r+1}={}^{20}C_r\left(4^{1/3}\right)^{20-r}\left(6^{-1/4}\right)^r$$

For rational terms

$20-r = 3k$ & $r = 4p$, where $k, p \in I \Rightarrow$ r = 20 & r = 8

$\therefore$ no. of rational terms = 2

$\therefore$ no. of irrational terms = 19

Middle term = T_{11} ; which is irrational

$T_9 = T_{8+1}$; which is rational

23. Ans. (A,C)

$7^9 + 9^7 = (8-1)^9 + (8+1)^7$

$= \left({}^9C_0(8)^9 - {}^9C_1(8)^8 + {}^9C_2(8)^7 + \ldots + {}^9C_8(8) - {}^9C_9\right) + \left({}^7C_0(8)^7 + \ldots + {}^7C_6(8) + {}^7C_7\right)$

= (72 – 1) + (56 + 1)= 128

This is divisible by 64 & 16

24. Ans. (A,C)

(A) $1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} \ldots\ldots\ldots\infty$

$\Rightarrow s = 1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} \ldots\ldots\ldots\infty$ .....(1)

$\Rightarrow \frac{1}{2}s = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} \ldots\ldots\ldots\infty$ .....(2)

subtract (2) from (1)

$\Rightarrow s - \frac{1}{2}s = 1 + \frac{1}{2} + \frac{1}{2^2} \ldots\ldots\ldots\infty$

$\Rightarrow \frac{1}{2}s = \frac{1}{1-\frac{1}{2}}$

$\Rightarrow s = 4$

so A is correct.

(B) $\left(9+4\sqrt{5}\right)^n + I + f$

$I + f = \left(9+4\sqrt{5}\right)^n = {}^nC_0(9)^n + {}^nC_1(9)^{n-1}\left(4\sqrt{5}\right) \ldots\ldots\ldots + {}^nC_n$ ...(1)

Consider

$0 < 9 - 4\sqrt{5} < 1$

$F = \left(9-4\sqrt{5}\right)^n = {}^nC_0(9)^n + {}^nC_1(9)^{n-1}\left(4\sqrt{5}\right) \ldots\ldots\ldots + {}^nC_n$ ....(2)

Add (1) and (2)

$(I + f) + F = 2\left[{}^nC_0(9)^n + {}^nC_2(9)^{n-2}\left(4\sqrt{5}\right)^2 \ldots\ldots\right]$

= 2 (integer)

$I + f + F$ = Even integer $\qquad \begin{cases} 0 < f < 1 \\ 0 < F < 1 \end{cases}$

$I + 1$ = Even integer $\qquad \Rightarrow 0 < f + F < 2$

$\therefore I$ = odd integer $\qquad \Rightarrow f + F = 1$

Option (B) is incorrect

(C) $({}^nC_0 + {}^nC_1 + {}^nC_2 \ldots + {}^nC_n)^2 = 1 + {}^{2n}C_1 + {}^{2n}C_2 + {}^{2n}C_{2n}$

Since ${}^nC_0 + {}^nC_1 + \ldots {}^nC_n = 2^n$ and ${}^{2n}C_0 + {}^{2n}C_1 + \ldots + {}^{2n}C_{2n} = 2^{2n}$

$\therefore \quad (2^n)^2 = 2^{2n}$

Hence C is correct.

(D) $\dfrac{1}{(3+2x)^2} = (3+2x)^{-2}$

$\Rightarrow 3^{-2}\left(1+\dfrac{2x}{3}\right)^{-2}$

$\left|\dfrac{2x}{3}\right| < 1$

$|x| < \dfrac{3}{2} \quad \Rightarrow$ Option D is wrong

25. Ans. (A,C)

$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots\ldots + {}^nC_n x^n$

Multiply it by x

$x(1+x)^n = {}^nC_0 x + {}^nC_1 x^2 + {}^nC_2 x^3 + \ldots\ldots + {}^nC_n x^{n+1}$

Differentiate w.r.t x and put $x = -3$

$n\,x(1+x)^{n-1} + (1+x)^n = {}^nC_0 + 2^n \cdot C_1 x + 3^n \cdot C_2 x^2 + 4^n \cdot C_3 x^3 + \ldots + (n+1)\,{}^nC_n x^n$

So answer $= -3n(-2)^{n-1} + (-2)^n$

$= (-2)^n\left(\dfrac{3n}{2} + 1\right)$

26. Ans. (A,C)

$$= a\sum_{r=1}^{n}(-1)^{r-1} \cdot {}^nC_r - \sum_{r=1}^{n} r \cdot {}^nC_r(-1)^{r-1} = [{}^nC_1 - {}^nC_2 + {}^nC_3 \ldots + (-1)^{n-1} \cdot {}^nC_n] - n\sum_{r=1}^{n}(-1)^{r-1} \cdot {}^{n-1}C_{r-1}$$

$$= a(1) - n\left[{}^{n-1}C_0 - {}^{n-1}C_1 + \ldots + (-1)^{(n-1)}\, {}^{n-1}C_{n-1}\right] = a - n(0) = a$$

27. Ans. (A,D)

$$P(n) = \sum_{r=0}^{n} \frac{(-1)^r r}{(r+1)} \cdot {}^nC_r$$

$$= \sum_{r=0}^{n}(-1)^r \cdot \left[\frac{r+1-1}{(r+1)}\right] \cdot {}^nC_r \quad = \sum_{r=0}^{n}(-1)^r \cdot {}^nC_r - \sum_{r=0}^{n}\frac{(-1)^r}{r+1} \cdot {}^nC_r$$

$$= \sum_{r=0}^{n}(-1)^r \cdot {}^nC_r - \sum_{r=0}^{n}\frac{(-1)^r}{r+1} \cdot \frac{n+1}{n+1} \cdot {}^{n+1}C_{r+1}$$

$$P(n) = \sum_{r=0}^{n}(-1)^r \cdot {}^nC_r - \frac{1}{n+1}\sum_{r=0}^{n}(-1)^r \cdot {}^{n+1}C_{r+1} \qquad \ldots(1)$$

Now, $\sum_{r=0}^{n}(-1)^r \cdot {}^nC_r = {}^nC_0 - {}^nC_1 \ldots + (-1)^n \cdot {}^nC_n = 0 \qquad \ldots(2)$

$$\sum_{r=0}^{n}(-1)^r \cdot {}^{n+1}C_{r+1} = {}^{n+1}C_1 - {}^{n+1}C_2 + {}^{n+1}C_3 \ldots + (-1)^n \cdot {}^{n+1}C_{n+1}$$

Now ${}^{n+1}C_0 - {}^{n+1}C_1 + {}^{n+1}C_2 \ldots + (-1)^{n+1}\, {}^{n+1}C_{n+1} = 0$

$\Rightarrow {}^{n+1}C_1 - {}^{n+1}C_2 \ldots + (-1)^{n+1} \cdot {}^{n+1}C_{n+1} = {}^{n+1}C_0 = 1$

Hence $\sum_{r=0}^{n}(-1)^r \cdot {}^{n+1}C_{r+1} = 1$ ...(3)

put (2) and (3) in (1)

$\therefore P(n) = 0 - \frac{1}{n+1}$

$|P_9| = \frac{1}{10}, |P_{10}| = \frac{1}{11}, |P_{11}| = \frac{1}{12}$

Thus $|P_{10}|$ is HM of $|P_9|$ and $|P_{11}|$

Hence (A)

$\sum_{r=5}^{10} P(r) \cdot P(r-1) = \sum_{r=5}^{10} \frac{1}{(r+1)} \cdot \frac{1}{r}$

$\Rightarrow \sum_{r=5}^{10}\left(\frac{1}{r} - \frac{1}{r+1}\right) = \left(\frac{1}{5} - \frac{1}{6}\right) + \left(\frac{1}{6} - \frac{1}{7}\right) + \ldots + \left(\frac{1}{10} - \frac{1}{11}\right)$

$= \frac{1}{5} - \frac{1}{11} = \frac{6}{55}$ Hence (D)

28. Ans. (B,D)

$(1+x)^m (1+x)^m = \left({}^nC_0 + {}^mC_1 x + {}^mC_2 x^2 + \ldots + {}^mC_m x^m\right)\left({}^nC_0 + {}^mC_1 x + {}^mC_2 x^2 + \ldots + {}^mC_m x^m\right)$

Coefficient of x^{m-2} in RHS is $C_0C_2 + C_1C_3 + C_2C_4 + C_3C_5 + C_4C_6 + \ldots + C_{m-2}C_m$

Coefficient of x^{m-2} in LHS is ${}^{2m}C_{m-2}$

So, ${}^{2m}C_{m-2} = C_0C_2 + C_1C_3 + C_2C_4 + C_3C_5 + C_4C_6 + \ldots + C_{m-2}C_m$

${}^{2m}C_{m-2} = A + C_0C_2$

$A < {}^{2m}C_{m-2}$ (Hence B)

Also ${}^mC_0^2 + {}^mC_1^2 + \ldots + m_{cm}^2 = {}^{2m}C_m$ and ${}^{2m}C_{m-2} < {}^{2m}C_m$

Obvious $A < {}^{2m}C_m$

$A < \left({}^mC_0\right)^2 + \left({}^mC_1\right)^2 + \ldots + \left({}^mC_m\right)^2$

29. Ans. (B,C)

$\left(1 + x + 2x^2\right)^{20} = a_0 + a_1 x + \ldots + a_{40} x^{40}$

$x = 1$, then $a_0 + a_1 + \ldots + a_{40} = 4^{20}$

$x = -1$, then $a_0 - a_1 + a_2 - \ldots + a_{40} = 2^{20}$

$2^{20} + 2^{40} = 2[a_0 + a_2 + \ldots + a_{38} + a_{40}]$

$\Rightarrow a_0 + a_2 + \ldots + a_{38} = 2^{19} + 2^{39} - 2^{20} = 2^{19}\left(2^{20} - 1\right)$ $\therefore a_{40} = a^{20}$

30. Ans. (A,B,C)

General term $= \frac{10!}{r_1!\, r_2!\, r_3!}(1)^{r_1}(2x)^{r_2}(3x^2)^{r_3}$

$a_1 = Coeff.\ of\ x$

$r_2 + 2r_3 = 1 \Rightarrow r_2 = 1,\ r_1 = 9,\ r_3 = 0$ $\therefore a_1 = \frac{10!}{1!9!}(2)^1 = 20$

$a_2 = Coeff.\ of\ x^2$

$r_2 + 2r_3 = 2 \Rightarrow r_2 = 2,\ r_1 = 8,\ r_3 = 0$

$r_2 = 0,\ r_1 = 9,\ r_3 = 1 \Rightarrow a_2 = 210$

also, $a_{20} = 3^{10}$

Now, for a_4

$r_2 + 2r_3 = 4$

r_1	r_2	r_3
6	4	0
7	2	1
8	0	2

$$a_4 = \frac{\lfloor 10}{\lfloor 6 \lfloor 4} 2^4\, 3^0 + \frac{\lfloor 10}{\lfloor 7 \lfloor 2 \lfloor 1} 2^2\, 3^1 + \frac{\lfloor 10}{\lfloor 8 \lfloor 2} 2^0\, 3^2$$

$a_4 = 8085$

31. Ans. (A,C)

$(3x+2)^{-\frac{1}{2}}$ has infinite expansion when $\left|\frac{3x}{2}\right| < 1 \Rightarrow x \in \left(-\frac{2}{3}, \frac{2}{3}\right)$

32. Ans. (A,C,D)

$I + f = \left(9+\sqrt{80}\right)^n = {}^nC_0 (9)^n + {}^nC_1 (9)^{n-1}\left(\sqrt{80}\right) \ldots\ldots\ldots + \frac{n}{n}$ ....(1)

consider $0 < 9 - \sqrt{80} < 1$

$F = \left(9-\sqrt{80}\right)^n = {}^nC_0 (9)^n + {}^nC_1 (9)^{n-1}\left(\sqrt{80}\right) \ldots\ldots + {}^nC_n$ ....(2)

Add (1) and (2)

$I + f + F = 2\left[{}^nC_0 (5)^n + {}^nC_2 (5)^{n-2}\left(2\sqrt{6}\right)^2 \ldots\ldots\right]$ = 2 [Integer]

$\Rightarrow I + f + F =$ Even integer

since $f + F = 1$

$\therefore I + 1 =$ Even integer

$\therefore I$ is odd integer [*Hence A*]

Also

$(I+f)F = \left(9+\sqrt{80}\right)^n \cdot \left(9-\sqrt{80}\right)^n = 1$

$(I+f)(1-f) = 1$ $\{\because f + F = 1\}$ [Hence C]

Also $\left(9-\sqrt{80}\right)^n = F = 1 - f$

$\therefore 1 - f = \left(9-\sqrt{80}\right)^n$ [Hence D]

33. Ans. (C,D)

$(1+x)^2 (1-x)^{-2} = \left(1+x^2+2x\right)\ (1-x)^{-2}$

Co-efficient of $x^4 = {}^5C_4 + {}^3C_2 + 2\,{}^4C_3 = 16$

34. Ans. (A,B)

$\left(1+4x+4x^2\right)^n = a_0 + a_1 x + a_2 x^2 + \ldots\ldots\ldots + a_{2n} x^{2n}$ ...(1)

put $x = 1$

$\Rightarrow (1+4+4)^n = a_0 + a_1 x + a_2 + \ldots\ldots + a_{2n}$

$\Rightarrow a_0+a_1+a_2+.........+a_{2n}=9^n$ ...(2)

Now put $x = -1$

$\Rightarrow a_0-a_1+a_2-$ .... $+a_{2n}=1$ ...(3)

(2) + (3)

$=2(a_0+a_2+.........+a_{2n})=9^n+1$

$\Rightarrow$ unit place can be 0, 2

35. Ans. (A,D)

$(1+4x+4x^2)^n=a_0+a_1+a_2x^2+$ .... $+a_{2n}x^{2x}$ ...(1)

Put $x = 1$

$\Rightarrow(1+4+4)^n=a_0+a_1+a_2+.........+a_{2n}$

$\Rightarrow a_0+a_1+a_2+.........+a_{2n}=9^n$ ...(2)

Now put $x = -1$

$\Rightarrow(1-4+4)^n=a_0-a_1+a_2+..........+a_{2n}$

$\Rightarrow a_0-a_1+a_2+..........+a_{2n}=1$ ...(3)

$(2)-(3)\Rightarrow$

$\Rightarrow 2(a_1+a_3+a_5+.........+a_{2n-1})=9^n-1$

unit place can be 8, 0

36. Ans. (63.00)

$(1+x)^n={}^nC_0+{}^nC_1x+{}^nC_2x^2+...+{}^nC_nx^n=a_0+a_1x+a_2x^2+...+a_nx^n$

$a_1={}^nC_1,\ a_2={}^nC_2,\ a_3={}^nC_3$

as $a_1,\ a_2,\ a_3$ are in A.P.

$\Rightarrow\ 2.{}^nC_2={}^nC_1+{}^nC_3$

$\Rightarrow 2.\frac{n(n-1)}{2}=n+\frac{n(n-1)(n-2)}{6}\ \Rightarrow\ \frac{n}{6}\left[n^2-3n+2-6n+6+6\right]=0$

$\Rightarrow\ \frac{n}{6}\left[n^2-9n+14\right]=0\Rightarrow n(n-2)(n-7)=0\ \Rightarrow n=0,2,7$

but $n = 0, 2$ are not possible as it is given that $n > 2$

so $n = 7$

$\Rightarrow a_1+a_2+a_3={}^7C_1+{}^7C_2+{}^7C_3=7+21+35=63$

37. Ans. (128.00)

$(1+x)^n={}^nC_0+{}^nC_1x+{}^nC_2x^2+...+{}^nC_nx^n=a_0+a_1x+a_2x^2+...+a_nx^n$

$a_1={}^nC_1,\ a_2={}^nC_2,\ a_3={}^nC_3$

as $a_1,\ a_2,\ a_3$ are in A.P.

$\Rightarrow 2.{}^nC_2={}^nC_1+{}^nC_3$

$\Rightarrow 2.\frac{n(n-1)}{2}=n+\frac{n(n-1)(n-2)}{6}\ \Rightarrow\ \frac{n}{6}\left[n^2-3n+2-6n+6+6\right]=0$

$\Rightarrow \frac{n}{6}\left[n^2-9n+14\right]=0 \Rightarrow n(n-2)(n-7)=0 \Rightarrow n=0,2,7$

but $n=0,2$ are not possible as it is given that $n>2$

so $n=7$

$\sum_{r=0}^{n} a_r = a_0 + a_1 + ... + a_n = (1+1)^7 = 2^7 = 128$

38. Ans. (93.00)

$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ... + {}^nC_nx^n = a_0 + a_1x + a_2x^2 + ... + a_nx^n$

$a_1 = {}^nC_1,\ a_2 = {}^nC_2,\ a_3 = {}^nC_3$

as a_1, a_2, a_3 are in A.P.

$\Rightarrow 2.{}^nC_2 = {}^nC_1 + {}^nC_3$

$\Rightarrow 2.\frac{n(n-1)}{2} = n + \frac{n(n-1)(n-2)}{6} \Rightarrow \frac{n}{6}\left[n^2-3n+2-6n+6+6\right]=0$

$\Rightarrow \frac{n}{6}\left[n^2-9n+14\right]=0 \Rightarrow n(n-2)(n-7)=0$

$n=0,2,7$

but $n=0,2$ are not possible as it is given that $n>2$

so $n=7$

as $n=7$, so $A=\left(2^{1/3}+3^{1/4}\right)^{98}$

$T_{r+1} = {}^{98}C_r 2^{\frac{98-r}{3}} 3^{\frac{r}{4}}$

this term will be rational if $98-r$ is a multiple of 3 and r is a multiple of 4.

so $r = 8, 20, 32 ......92$

so $b_m = 93$

39. Ans. (A)

(A) $\sum_{r=0}^{10}(r+2^r)\ {}^{10}C_r = \sum_{r=0}^{10} r\cdot {}^{10}C_r + \sum_{r=0}^{10} 2^r\cdot {}^{10}C_r = S_1 + S_2$

$S_1 = 0\cdot {}^{10}C_0 + 1\cdot {}^{10}C_1 + ... + 10\cdot {}^{10}C_{10}$

$=(10)2^{10-1} = 10\cdot 2^9$

$S_2 = {}^{10}C_0 + 2^1\cdot {}^{10}C_1 + 2^2\cdot {}^{10}C_2 + ... + 2^{10}\cdot {}^{10}C_{10}$

$=(1+2)^{10} = 3^{10}$

$\therefore\ a+b=19$

(B) $T_{r+1} = {}^mC_r\cdot x^{m-r}\left((ax)^{-1}\right)^r = {}^mC_r \frac{x^{(m-2r)}}{a^r} = \frac{5}{2}$

RHS has no power of x so $m-2r=0$

$\Rightarrow m=2r$

we know, $r=3 \Rightarrow m=6$

$\therefore T_4 = \frac{{}^6C_3}{a^3} = \frac{5}{2} \Rightarrow \frac{20}{a^3} = \frac{5}{2} \Rightarrow a^3 = 8 \Rightarrow a=2$

$\Rightarrow ma = 12$

(C) $r=\dfrac{10+1}{1+\dfrac{1}{|5x|}}=\dfrac{55}{6}=9.1$

A.T.Q.

$T_{9+1}={}^{10}C_9(5)^9=\lambda$

$\Rightarrow \dfrac{\lambda}{{}^{10}C_9.5^7}=\dfrac{{}^{10}C_9(5)^9}{{}^{10}C_9.5^7}=25$

(D) ${}^nP_r=5040\left({}^nC_r\right)$

$\dfrac{n!}{(n-r)!}=5040\times\dfrac{n!}{r!(n-r)!}$

$r!=5040\Rightarrow r=7$

40. **Ans. (A) → (R); (B) → (S); (C) → (Q); (D) → (P)**

(A) $T_{r+1}={}^9C_r\left(\dfrac{x^2}{2}\right)^{9-r}\left(\dfrac{2}{x}\right)^r={}^9C_rx^{18-3r}2^{2r-9}$

so $18-3r=-9$ $\therefore 3r=27\Rightarrow r=9$

so $T_{10}=T_{9+1}={}^9C_9\left(\dfrac{2}{x}\right)^9=512\,x^{-9}$

so coefficient of x^{-9} is 512

(B) $T_{r+1}={}^6C_r(2r)^{6-r}\left(\dfrac{3}{x}\right)^r={}^6C_r2^{6-r}3^rx^{6-2r}$

$6-2r=0\Rightarrow r=3$

$T_4=T_{3+1}={}^6C_3\cdot 2^3\cdot 3^3=20\times8\times27=4320$

(C) $\left(1+x^2-x^3\right)^8=\left[\left(1+x^2\right)-x^3\right]^8$

$={}^8C_0\left(1+x^2\right)^8+{}^8C_1\left(1+x^2\right)^7\left(-x^3\right)+{}^8C_2\left(1+x^2\right)^6\left(-x^3\right)^2+{}^8C_3\left(1+x^2\right)^5+\left(-x^3\right)^3+\ldots$

so coefficient of $x^{10}={}^8C_5+{}^8C_2\times{}^6C_2=56+28\times15=56+420=476$

(D) coefficient of x^4 in $(3x+1)^7\left(1-2x+3x^2\right)$

$={}^7C_3\ .\ 3^4-{}^7C_4\ .\ 3^3(2)+{}^7C_53^2\ .\ 3$

$=35\times81-35\times54+21\times27=1512$

EXERCISE - S

1. **Ans. (0)**

Coefficient of x^3 in $(1+x+2x^2+3x^3)^4=a$

$\Rightarrow$ Coefficient of x^3 in $\left(f(x)\right)^4=a$

Now,

b = coefficient of x^3 in $(1+x+2x^2+3x^3+4x^4)^4$

= coefficient of x^3 in $\left(f(x)+4x^4\right)^4$ [where $f(x)=1+x+2x^2+3x^3$]

Now, b = coefficient of x^3 in $\left[\left(f(x)\right)^4 + {}^4C_1\left(f(x)\right)^3(4x^4)+....\right]$

= coefficient of x^3 in $\left(f(x)\right)^4 = a$

$\Rightarrow b - a = 0$

2. **Ans. (2)**

$a+1=4^{\frac{1}{401}}$

$b_n = {}^nC_1 + {}^nC_2 \cdot a + {}^nC_3 \cdot a^2 + \ldots + {}^nC_n \cdot a^{n-1}$

$\Rightarrow b_n \cdot a = {}^nC_1 a + {}^nC_2 a^2 + {}^nC_3 a^3 + \ldots + {}^nc_n a^n \Rightarrow b_n \cdot a + 1 = 1 + {}^nC_1 a + {}^nC_2 a^2 + {}^nC_3 a^3 + \ldots + {}^nC_n a^n$

$= {}^nC_0 + {}^nC_1 a + {}^nC_2 a^2 + \ldots + {}^nC_n a^n$

$\Rightarrow b_n a + 1 = (1+a)^n \Rightarrow \boxed{b_n = \dfrac{(1+a)^n - 1}{a}}$

Now, $b_{2006} - b_{2005} = \left[\dfrac{(1+a)^{2006}-1}{a} - \dfrac{(1+a)^{2005}-1}{a}\right]$

$$= \frac{4^{\frac{2006}{401}} - 4^{\frac{2005}{401}}}{a} = 4^{\frac{2005}{401}} \frac{\left[4^{\frac{1}{401}} - 1\right]}{\left[4^{\frac{1}{401}} - 1\right]} = 4^5 = 2^{10}.$$

3. **Ans. (6)**

${}^{n+5}C_{r-1}, {}^{n+5}C_r, {}^{n+5}C_{r+1}$

are 3 consecutive co-efficients now $\dfrac{{}^{n+5}C_{r-1}}{{}^{n+5}C_r} = \dfrac{5}{10}$

$\Rightarrow \dfrac{1}{2}$

$\Rightarrow 2r = n + 6 - r$

$\Rightarrow 3r = n + 6$ ...(1)

$\dfrac{{}^{n+5}C_r}{{}^{n+5}C_{r+1}} = \dfrac{10}{14}$

$\Rightarrow 5n - 12r + 18 = 0$ ...(2)

on solving (1) & (2)

$\Rightarrow n = 6$

4. **Ans. (12.00)**

If we want to discuss about the coefficients then $x = 1$

Now, for $x = 1$ $|T_9| > |T_8|$ and $|T_9| > |T_{10}|$

$\therefore \quad (1) \Rightarrow \left|{}^nC_8\left(\dfrac{1}{5}\right)^{n-8}\left(\dfrac{2}{5}\right)^8\right| > \left|{}^nC_7\left(\dfrac{1}{5}\right)^{n-7}\left(\dfrac{2}{5}\right)^7\right|$

$\Rightarrow \dfrac{{}^nC_8}{{}^nC_7}\left(\dfrac{2}{5}\right) > \left(\dfrac{1}{5}\right) \Rightarrow \dfrac{n-7}{8} > \dfrac{1}{2} \Rightarrow n = 11$

and (2) $\Rightarrow \left|{}^nC_8\left(\frac{1}{5}\right)^{n-8}\left(\frac{2}{5}\right)^8\right| > \left|{}^nC_9\left(\frac{1}{5}\right)^{n-9}\left(\frac{2}{5}\right)^9\right|$

$\Rightarrow \frac{{}^nC_8}{{}^nC_7}\left(\frac{2}{5}\right) > \left(\frac{1}{5}\right) \Rightarrow \frac{n-7}{8} > \frac{1}{2} \Rightarrow n > 11$

and (2) $\Rightarrow \left|{}^nC_8\left(\frac{1}{5}\right)^{n-8}\left(\frac{2}{5}\right)^8\right| > \left|{}^nC_9\left(\frac{1}{5}\right)^{n-9}\left(\frac{2}{5}\right)^9\right|$

$\Rightarrow \left(\frac{1}{5}\right) > \frac{2}{5}\frac{{}^nC_9}{{}^nC_8} \Rightarrow 1 > \left(\frac{n-8}{9}\right)^2$

$\Rightarrow n < 12.5$

$\therefore \quad 11 < n < 12.5 \Rightarrow n = 12$

Aliter: If 9^{th} term has numerically <u>greatest coefficieint</u> means $x = 1$ for numerically greatest term method.

& only 9^{th} term means $r = 8$ If T_{r+1} is NGT

$k = \frac{n+1}{1+\left|\frac{x}{a}\right|}$ for $(x+a)^n$

Pot = $x = x_0$
T_{r+1} is NGT

$k \in I$ → $r = k, k-1$

$r = [k]$ → $k < r < k+1$

Here $r = 8 = [k]$

$\Rightarrow 8 < k < 9 \Rightarrow k = \left.\frac{n+1}{1+\left|\frac{x}{2}\right|}\right|_{x=1} \quad \Rightarrow \quad 8 < \frac{n+1}{1+\frac{1}{2}} < 9$

$\Rightarrow \quad 12 < n+1 < 13.5 \Rightarrow 11 < n < 12.5 \Rightarrow n = 12$

5. **Ans. (500.00)**

$x^{2001} + (-1)^{2001}\left(x - \frac{1}{2}\right)^{2001} = 0$

$\Rightarrow \quad x^{2001} = \left({}^{2001}C_0x^{2001} - {}^{2001}C_1x^{2000}\frac{1}{2} + {}^{2001}C_2x^{1999}\left(\frac{1}{2}\right)^2 + \ldots\right)$

$\Rightarrow \quad {}^{2001}C_1x^{2000}\frac{1}{2} - {}^{2001}C_2x^{1999}\left(\frac{1}{2}\right)^2 + \ldots\ldots = 0$

sum of roots = $-\frac{\text{coefficient of } x^{1999}}{\text{coefficient of } x^{2000}}$

$= -\frac{\left(-{}^{2001}C_2\left(\frac{1}{2}\right)^2\right)}{{}^{2001}C_1\left(\frac{1}{2}\right)} = \frac{2001\times2000}{2\times2\times2001} = 500$

6. Ans. (–22100.00)

We know that, $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-(n-1)}{r} = \frac{n-r+1}{r}$

Further, $r^2 \cdot \frac{{}^{50}C_r}{{}^{50}C_{r-1}} = r(50-r+1) = r(51-r)$

for a polynomial if expressed as:

$x^{50} + a_1x^{49} + a_2x^{48} + ... + a_{49}x^o = (x-a_1)(x-a_2)...(x-a_{50})$

Coefficient of $x^{49} = a_1 = -(\text{sum of zeroes})$

$\Rightarrow \quad S = -[a_1 + a_2 + ... + a_{50}] \Rightarrow S = -\left[\sum_{r=1}^{50} r(51-r)\right]$

$\Rightarrow \quad S = -\left[51 \times \frac{50\times51}{2} - \frac{50\times51\times101}{6}\right] \Rightarrow S = -\frac{50\times51}{6}[153-101] = -25\times17\times52 = -22100$

7. Ans. (–144.00)

$\Rightarrow \quad (1-x)^6(1-x^2)^6$

coeff. of x^7 can be found by expanding and then multiplying above binomial expressions.

Thus. $\quad (1-x)^6 \times (1-x^2)^6$

Coeff of x^1 × coeff of x^6 + coeff of x^3 × coeff of x^4 + coeff. of x^5 × coeff of x^2

$\Rightarrow \quad -{}^6C_1 \times -{}^6C_3 + (-{}^6C_3)\times({}^6C_2) + (-{}^6C_5)\times(-{}^6C_1)$

$\Rightarrow \quad 120 - 300 + 36 \Rightarrow -144$ **Ans.**

8. Ans. (816.00)

$1 - x + x^2 - x^3 + ... + x^{16} - x^{17}$

$= \frac{1.(1-(-x)^{18})}{1-(-x)} = \frac{1-x^{18}}{1+x}$

$a_0 + a_1(1+x) + a_2(1+x)^2 + ... + a_{17}(1+x)^{17}$

Let $(1+x) = t \Rightarrow -x = 1-t$

$a_0 + a_1 + a_2t^2 + ... + a_{17}t^{17}$

$= \frac{1-(1-t)^{18}}{t}$

a_2 = coeff of t^2 in $\frac{1-(1-t)^{18}}{t}$

= coeff of t^3 in $1-(1-t)^{18} = 1 - ({}^{18}C_0 - {}^{18}C_1(t) + {}^{18}C_2(t)^2 - {}^{18}C_3(t)^3 + ...)$

$= 1 - 1 + {}^{18}C_1(t) - {}^{18}C_2(t)^2 + {}^{18}C_3(t)^3 ......$

$\therefore a_2$ = coeff of $t^3 = {}^{18}C_3 = 816$

9. Ans. (2)

$P = (2+\sqrt{3})^5 = [P] + f \,;\, 0 < f < 1$

observe, $0 < 2-\sqrt{3} < 1$

So, let $(2-\sqrt{3})^5 = f'$ where $0 < f' < 1$

Now, Add,

$$(2+\sqrt{3})^5 = {}^5C_0 2^5 + {}^5C_1 2^4(\sqrt{3})^1 + {}^5C_2 2^3(\sqrt{3})^2 + {}^5C_3 2^2(\sqrt{3})^3 + {}^5C_4 2^1(\sqrt{3})^4 + {}^5C_5(\sqrt{3})^5$$

$$(2+\sqrt{3})^5 = {}^5C_0 2^5 - {}^5C_1 2^4(\sqrt{3})^1 + {}^5C_2 2^3(\sqrt{3})^2 - {}^5C_3 2^2(\sqrt{3})^3 + {}^5C_4 2^1(\sqrt{3})^4 - {}^5C_5(\sqrt{3})^5$$

$$(2+\sqrt{3})^5 + (2-\sqrt{3})^5 = 2({}^5C_0 2^5 + {}^5C_2 2^3(\sqrt{3})^2 + {}^5C_4 2^1(\sqrt{3})^4)$$

$$= 2(32 + 10\times 8\times 3 + 5\times 2\times 9)$$

$$[P] + f + f' = 2\times \text{Integer} = 2\times[32+240+90] = 724$$

Observe, $0 < f + f' < 2$

& $f + f' = 2 \times \text{Integer} - [P] = \text{integer}$

So, $f + f' = 1 \Rightarrow f' = 1 - f \Rightarrow [P] + 1 = 724 \Rightarrow [P] = 723$

∴ Note: $f' \cdot P = (2-\sqrt{3})^5\,(2+\sqrt{3})^5 = 1$

$$\Rightarrow (1-f)([P]+f) = 1 \Rightarrow (1-f)[P] + f - f^2 = 1 \Rightarrow (1-f)[P] = 1 - f + f^2 \Rightarrow [P] = 1 + \frac{f^2}{1-f}$$

$$\therefore \frac{2}{1-f} = [P] - 1 = (723) - 1 = 722$$

10. Ans. (0)

Observe $(7-4\sqrt{3})(7+4\sqrt{3}) = 1$

$0 < 7 - 4\sqrt{3} < 1$

Let $(7-4\sqrt{3})^n = \beta'$, where, $0 < \beta' < 1, 0 < \beta < 1$

As $(7+4\sqrt{3})^n = p + \beta$

Now,

$$(7+4\sqrt{3})^n = p + \beta = {}^nC_0 7^n + {}^nC_1 7^{n-1}(\sqrt{3})^1 + {}^nC_2 7^{n-2}(\sqrt{3})^2 + ...$$

$$(7-4\sqrt{3})^n = \beta' = {}^nC_0 7^n - {}^nC_1 7^{n-1}(\sqrt{3})^1 + {}^nC_2 7^{n-2}(\sqrt{3})^2 + ...$$

Add $p + \beta + \beta' = 2 \times \text{Integer}$ [as $p \in I$]

$\Rightarrow \beta + \beta' \Rightarrow$ {as $0 < \beta + \beta' < 2$}

$\therefore 1 - \beta = \beta'$

So, $\beta'.(p+\beta) = 1 = (1-\beta)(p+\beta)$

$(p+\beta)(1-\beta) = (7+4\sqrt{3})^n (7-4\sqrt{3})^n$

$(p+\beta)(1-\beta) = 1$

$(p+\beta)(1-\beta) - 1 = 0$

EXERCISE - JEE (Main) PYQ

1. Ans. (4)

$T_{2+1} = {}^5C_2\left(x^{\log_2 x}\right)^2 = 2560$

$\Rightarrow x^{\log_2 x} = \sqrt{256}$

Let $\log_2 x = t \Rightarrow x = 2^t$

$\Rightarrow (2^t)^t = \sqrt{256} \Rightarrow t^2 = 4 \Rightarrow t = \pm 2$

$\Rightarrow x = 2^2$ or $2^{-2} = \frac{1}{4}$

2. Ans. (2)

$x^2\left({}^{10}C_r(\sqrt{x})^{10-r}\left(\frac{\lambda}{x^2}\right)^r\right)$

$x^2\left[{}^{10}C_r(x)^{\frac{10-r}{2}}(\lambda)^r(x)^{-2r}\right]$

$x^2\left[{}^{10}C_r\lambda^r x^{\frac{10-5r}{2}}\right]$

$\therefore\ r = 2$

Hence, ${}^{10}C_2\ \lambda^2 = 720$

$\lambda^2 = 16$

$\lambda = \pm 4$

3. Ans. (3)

$\sum_{r=0}^{25}({}^{50}C_r.{}^{50-r}C_{25-r})$

${}^{50}C_{25}\sum_{r=0}^{25}\frac{\lfloor 25}{\lfloor 25-r\lfloor r} = {}^{50}C_{25}\sum_{r=0}^{25}{}^{25}C_r = {}^{50}C_{25}.2^{25} \Rightarrow k = 2^{25}$

4. Ans. (4)

$(10+x)^{50}+(10-x)^{50}$

$\Rightarrow a_2 = 2.\ {}^{50}C_2 10^{48}, a_0 = 2.10^{50}$

$\frac{a_2}{a_0} = \frac{{}^{50}C_2}{10^2} = 12.25$

5. Ans. (2)

$2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + 11.{}^{20}C_3 + \ldots + 62.{}^{20}C_{20}$

$= \sum_{r=0}^{20}(3r+2){}^{20}C_r = 3\sum_{r=0}^{20}r.{}^{20}C_r + 2\sum_{r=0}^{20}{}^{20}C_r$

$= 3\sum_{r=0}^{20}r\left(\frac{20}{r}\right){}^{19}C_{r-1} + 2.2^{20} = 60.2^{19} + 2.2^{20} = 2^{25}$

6. Ans. (1)

Coefficient of $x^2 = {}^{15}C_2 \times 9 - 3a(15C_1) + b = 0$

$\Rightarrow -45a + b + {}^{15}C_2 \times 9 = 0$ ...(i)

Also, $-27\ {}^{15}C_3 + 9a\ {}^{15}C_2 - 3b\ {}^{15}C_1 = 0$

$\Rightarrow 9 \times {}^{15}C_2\ a - 45b - 27 \times {}^{15}C_3 = 0$

$\Rightarrow 21a - b - 273 = 0$ ...(ii)

(i) + (ii)

$-24a + 672 = 0$

$\Rightarrow a = 28$ So, $b = 315$

7. Ans. (3)

$6 \times {}^{35}C_r = (k^2-3){}^{36}C_{r+1}$

$k^2 - 3 > 0 \Rightarrow k^2 > 3$

$k^2 - 3 = \frac{6 \times {}^{35}C_r}{{}^{36}C_{r+1}} = \frac{r+1}{6}$

Possible values of r for integral values of k, are

$r = 5, 35$

number of ordered pairs are 4

$(5, 2), (5, -2), (35, 3), (35, -3)$

8. Ans. (4)

$a = {}^{19}C_{10}, b = {}^{20}C_{10}$ and $c = {}^{21}C_{10}$

$\Rightarrow a = {}^{19}C_9, b = 2({}^{19}C_9)$ and $c = \frac{20}{11}\left({}^{20}C_{10}\right)$

$\Rightarrow b = 2a$ and $c = \frac{21}{11}b = \frac{42a}{11} \Rightarrow a : b : c = a : 2a : \frac{42a}{11}$

$= 11 : 22 : 42$

9. Ans. (3)

$2[{}^6C_0.x^6 + {}^6C_2x^4(x^2-1) + {}^6C_4x^2(x^2-1)^2 + {}^6C_6(x^2-1)^3]$

$\alpha = -96$ & $\beta = 36$

$\therefore \alpha - \beta = -132$

10. Ans. (2)

$(-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + \ldots\ldots - 15.{}^{15}C_{15}) + ({}^{14}C_1 + {}^{14}C_3 + \ldots\ldots + {}^{14}C_{11})$

$= \sum_{r=1}^{15}(-1)^r.r^{15}C_r + ({}^{14}C_1 + {}^{14}C_3 + \ldots + {}^{14}C_{11} + {}^{14}C_{13}) - {}^{14}C_{13}$

$= \sum_{r=1}^{15}(-1)^r 15.{}^{14}C_{r-1} + 2^{13} - 14 = 15(-{}^{14}C_0 + {}^{14}C_1 \ldots\ldots - {}^{14}C_{14}) + 2^{13} - 14$

$= 2^{13} - 14$

11. Ans. (2)

${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + {}^4C_2 + \ldots\ldots + {}^nC_2)$

${}^{n+1}C_2 + 2({}^3C_3 + {}^3C_2 + {}^4C_2 + \ldots\ldots + {}^nC_2)$

$\{use\ {}^nC_{r+1} + {}^nC_r = {}^{n+1}C_r\}$

$= {}^{n+1}C_2 + 2({}^4C_3 + {}^4C_2 + {}^5C_3 + \ldots\ldots + {}^nC_2)$

$= {}^{n+1}C_2 + 2({}^5C_3 + {}^5C_2 + \ldots\ldots + {}^nC_2)$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$= {}^{n+1}C_2 + 2({}^nC_3 + {}^nC_2)$

$= {}^{n+1}C_2 + 2.{}^{n+1}C_3$

$= \frac{(n+1)n}{2} + 2.\frac{(n+1)(n)(n-1)}{2.3} = \frac{n(n+1)(2n+1)}{6}$

12. Ans. (96)

$11^n > 10^n + 9^n$

$\Rightarrow 11^n - 9^n > 10^n \Rightarrow (10+1)^n - (10-1)^n > 10^n \Rightarrow \{{}^nC_1 \cdot 10^{n-1} + {}^nC_3 10^{n-0} + {}^nC_5 10^{n-5} + \cdots\cdots\} > 10^n$

$\Rightarrow 2n \cdot 10^{n-1} + 2\{{}^nC_3 10^{n-3} + {}^nC_5 10^{n-5} + \cdots\cdots\} > 10^n$ ..... (1)

For $n = 5$

$10^5 + 2\{{}^5C_3 10^2 + {}^5C_5\} > 10^5$ (True)

For $n = 6, 7, 8, \ldots.. 100$

$2n10^{n-1} > 10^n$

$\Rightarrow 2n10^{n-1} + 2\left\{{}^nC_3 10^{n-3} + {}^nC_5 10^{n-5} + \cdots\right\} > 10^n$

$\Rightarrow 11^n - 9^n > 10^n$ For n = 5, 6, 7, ... 100

For n = 4, Inequality (1) is not satisfied

$\Rightarrow$ Inequality does not hold good for

N = 1, 2, 3, 4

So, required number of elements

= 96

13. **Ans. (1)**

Coeff. of middle term in $(1 + x)^{20} = {}^{20}C_{10}$

& Sum of Coeff. of two middle terms in

$(1 + x)^{19} = {}^{19}C_9 + {}^{19}C_{10}$

So required ratio = $\dfrac{{}^{20}C_{10}}{{}^{19}C_9 + {}^{19}C_{10}} = \dfrac{{}^{20}C_{10}}{{}^{20}C_{10}} = 1$

14. **Ans. (3)**

$(1 - x + x^3)^n = \sum_{j=0}^{3n} a_j x^j$

$\left(1 - x + x^3\right)^n = a_0 + a_1 x + a_2 x^2 .... + a_{3n} x^{3n}$

$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j}$ = Sum of $a_0 + a_2 + a_4$ ....

$\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j} + 1$ = Sum of $a_1 + a_3 + a_5$ ....

put x = 1

$1 = a_0 + a_1 + a_2 + a_3 .... + a_{3n}$ ...(A)

Put x = –1

$1 = a_0 - a_1 + a_2 - a_3 .... + (-1)^{3n} a_{3n}$ ...(B)

Solving (A) and (B)

$a_0 + a_2 + a_4 ..... = 1$

$a_1 + a_3 + a_5 .... = 0$

$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4 \sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} = 1$

15. **Ans. (2)**

$\left(1 + x + 2x^2\right)^{20} = a_0 + a_1 x + ... + a_{40} x^{40}$ put x = 1, – 1

$\Rightarrow a_0 + a_1 + a_2 + ... + a_{40} = 2^{20}$

$a_0 - a_1 + a_2 + ... + a_{40} = 2^{20} \Rightarrow a_1 + a_3 + ... + a_{39} = \dfrac{4^{20} - 2^{20}}{2}$

$\Rightarrow a_1 + a_3 + ... + a_{37} = 2^{39} - 2^{19} - a_{39}$

here $a_{39} = \dfrac{20!(2)^{19} \times 1}{19!} = 20 \times 2^{19}$

$\Rightarrow a_1 + a_3 + ... + a_{37} = 2^{19}\left(2^{20} - 1 - 20\right)$

$= 2^{19}(2^{20} - 21)$

16. Ans. (1)

$(5+x)^{500} + x(5+x)^{499} + x^2(5+x)^{498} + ... + x^{500}$

$= \frac{(5+x)^{501} - x^{501}}{(5+x) - x} = \frac{(5+x)^{501} - x^{501}}{5}$

$\Rightarrow$ coefficient x^{101} in given expression

$= \frac{^{501}C_{101}5^{400}}{5} = {}^{501}C_{101}5^{399}$

17. Ans. (57)

coefficients and there cumulative sum are :

Coefficient	Commulative sum
$x^{7n} \to {}^7C_0$	1
$x^{6n-5} \to 2 \cdot {}^7C_1$	$1+14$
$x^{5n-10} \to 2^2 \cdot {}^7C_2$	$1+14+84$
$x^{4n-15} \to 2^3 \cdot {}^7C_3$	$1+14+84+280$
$x^{3n-20} \to 2^4 \cdot {}^7C_4$	$1+4+84+280+560=939$
$x^{2n-25} \to 2^5 \cdot {}^7C_5$	

$3n - 20 \geq 0 \cap 2n - 25 < 0 \cap n \in I$

$\therefore 7 \leq n \leq 12$

Sum = 7 + 8 + 9 + 10 + 11 + 12 = 57

18. Ans. (3)

$\alpha = \frac{(1+8)^n - 8n - 1}{64} = {}^nC_2 + {}^nC_3 8 + {}^nC_4 8^2 + ...$

$\beta = {}^nC_2 + {}^nC_3 5 + {}^nC_4 5^2 + ...$

option (3) will be the answer.

19. Ans. (3)

$(2021)^{2023} = (7\lambda - 2)^{2023}$

$= {}^{2023}C_0(7A)^{2023} - .....\ {}^{2023}C_{2023}2^{2023} = 7t - 2^{2023}$

$\therefore -2^{2023} = -2 \times 2^{2022}$

$= -2 \times (2^3)^{674} = -2(7+1)^{674} = -2(7\mu + 2) = 7\mu - 2$

$\Rightarrow$ remainder $= -2$ or + 5

20. Ans. (1)

$(2021)^{2022} + (2022)^{2021}$

$= (2023 - 2)^{2022} + (2023 - 1)^{2021} = 7n_1 + 2^{2022} + 7n_2 - 1$

$= 7(n_1 + n_2) + 8^{674} - 1 = 7(n_1 + n_2) + (7-1)^{674} - 1 = 7(n_1 + n_2) + 7n_3 + 1 - 1$

$= 7(n_1 + n_2 + n_3)$

$\therefore$ Given number is divisible by 7 hence remainder is zero

21. Ans. (102)

$^{40}C_0 + ^{41}C_1 + ^{42}C_1 + ... + ^{59}C_{19} + ^{60}C_{20}$

$\left(\frac{1}{41}+1\right)^{41}C_1 + ^{42}C_2 + ...$

$\left[\frac{42}{41}\left(\frac{2}{42}\right)+1\right]^{42}C_2 + ^{43}C_3 + ...$

$\left(\frac{2}{41}+1\right)^{42}C_2 + ^{43}C_3 + ...$

$\left(\frac{43}{41}\times\frac{3}{43}+1\right)^{43}C_3 + ^{44}C_4 + ...$

$\frac{3+41}{41}.^{43}C_3 + ...$

Similarly :

$\frac{20+41}{41}$

$\Rightarrow m = 61 ; n = 41$

$m + n = 102$

22. Ans. (1)

$\sum_{R=1}^{31} {}^{31}C_R . {}^{31}C_{R-1}$

$= {}^{31}C_1 \cdot {}^{31}C_0 + {}^{31}C_2 \cdot {}^{31}C_1 + ... + {}^{31}C_{31} \cdot {}^{31}C_{30}$

$= {}^{31}C_0 \cdot {}^{31}C_{30} + {}^{31}C_1 \cdot {}^{31}C_{29} + ... + {}^{31}C_{30} \cdot {}^{31}C_0 = {}^{62}C_{30}$.

Similarly

$\sum_{R=1}^{30} \left({}^{30}C_R . {}^{30}C_{R-1}\right) = {}^{60}C_{29}$

$^{62}C_{30} - {}^{60}C_{29} = \frac{62!}{30!32!} - \frac{60!}{29!31!} = \frac{60!}{29!\,31!}\left\{\frac{62\cdot 61}{30\cdot 32} - 1\right\}$

$= \frac{60!}{30!31!}\left(\frac{2822}{32}\right)$

$\therefore\ 16\alpha = 16\times\frac{2822}{32} = 1411$

23. Ans. (5)

$T_{r+1} = (-1)^r . {}^{15}C_r . 2^{15-r} x^{\frac{15-2r}{5}}$

$m = {}^{15}C_{10} 2^5$

$n = -1$

so $mn^2 = {}^{15}C_5 2^5$

24. **Ans. (1)**

$$\sum_{r=0}^{22} {}^{22}C_r \, . \, {}^{23}C_r = \sum_{r=0}^{22} {}^{22}C_r \, . \, {}^{23}C_{23-r} = {}^{45}C_{23}$$

25. **Ans. (1080)**

General term is $\sum \frac{5!(2x)^{n_1}(x^{-7})^{n_2}(3x^2)^{n_3}}{n_1!\,n_2!\,n_3!}$

For constant term,

$n_1 + 2n_3 = 7n_2$

& $n_1 + n_2 + n_3 = 5$

Only possibility $n_1 = 1$, $n_2 = 1$, $n_3 = 3$

$\Rightarrow$ constant term = 1080

26. **Ans. (2)**

Option (2)

Coefficient of x^{15} in $\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$

$$T_{r+1} = {}^{15}C_r \left(ax^3\right)^{15-r}\left(\frac{1}{bx^{1/3}}\right)^r$$

$$45 - 3r - \frac{r}{3} = 15$$

$$30 = \frac{10r}{3}$$

$r = 9$

Coefficient of $x^{15} = {}^{15}C_9 \, a^6 \, b^{-9}$

Coefficient of x^{-15} in $\left(ax^{1/3} - \frac{1}{bx^3}\right)^{15}$

$$T_{r+1} = {}^{15}C_r \left(ax^{1/3}\right)^{15-r}\left(-\frac{1}{bx^3}\right)^r$$

$$5 - \frac{r}{3} - 3r = -15$$

$$\frac{10r}{3} = 20$$

$r = 6$

Coefficient $= {}^{15}C_6 \, a^9 \times b^{-6}$

$$\Rightarrow \frac{a^9}{b^6} = \frac{a^6}{b^9} \Rightarrow a^3b^3 = 1 \Rightarrow ab = 1$$

27. **Ans. (2)**

$$T_{r+1} = {}^{13}C_r \left(ax\right)^{13-r}\left(-\frac{1}{bx^2}\right)^r$$

$$= {}^{13}C_r \left(a\right)^{13-r}\left(-\frac{1}{b}\right)^r x^{13-3r}$$

$13 - 3r = 7 \Rightarrow r = 2$

Coefficient of $x^7 = {}^{13}C_2 (a)^{11} . \frac{1}{b^2}$

In the other expansion $T_{r+1} = {}^{13}C_r (ax)^{13-r} \left(\frac{1}{bx^2}\right)^r$

13 – 3r = –5 ⇒ r = 6

Coefficient of $x^{-5} = {}^{13}C_6 (a)^7 . \frac{1}{b^6}$

$${}^{13}C_2 \frac{a^{11}}{b^2} = {}^{13}C_6 \frac{a^7}{b^6}$$

$$a^4b^4 = \frac{{}^{13}C_6}{{}^{13}C_2} = 22$$

28. **Ans. (98)**

In, $\left(\frac{x^{\frac{5}{2}}}{2} - \frac{4}{x^\ell}\right)^9$

$$T_{r+1} = {}^9C_r \frac{\left(x^{5/2}\right)^{9-r}}{2^{9-r}} \left(\frac{-4}{x^\ell}\right)^r$$

$$= (-1)^r \frac{{}^9C_r}{2^{9-r}} 4^r x^{\frac{45}{2} - \frac{5r}{2} - lr}$$

$$= 45 - 5r - 2lr = 0$$

$$r = \frac{45}{5+2l} \qquad \text{......(1)}$$

Now, according to the question, $(-1)^r \frac{{}^9C_r}{2^{9-r}} 4^r = -84$

= $(-1)^r \; {}^9C_r 2^{3r-9} = 21 \times 4$

Only natural value of r possible if $3r - 9 = 0$

$r = 3$ and ${}^9C_3 = 84$

$\therefore \; l = 5$ from equation (1)

Now, coefficient of $x^{-3l} = x^{\frac{45}{2} - \frac{5r}{2} - lr}$ at l = 5, gives $r = 5$

$\therefore \; {}^9c_5 (-1) \frac{4^5}{2^4} = 2^\alpha \times \beta = -63 \times 2^7 \Rightarrow \alpha = 7, \beta = -63$

$\therefore$ value of $|\alpha\ell - \beta| = 98$

29. **Ans. (4)**

$$T_6 = {}^mC_5 \left(10 - 3^x\right)^{\frac{m-5}{2}} . \left(3^{x-2}\right) = 21 \qquad \text{....(1)}$$

${}^mC_1, {}^mC_2, {}^mC_3$ are in A.P.

$2. {}^mC_2 = {}^mC_1 + {}^mC_3$

Solving for m, we get

$m = 2$(rejected), 7

Put in equation (1)

$$21\,.\left(10-3^x\right)\frac{3^x}{9}=21$$

$$3^x = 3^0, 3^2$$

$x = 0, 2$

Sum of the squares of all possible values of $x = 4$

30. **Ans. (2)**

$(22)^{2022} + (2022)^{22}$

divided by 3

$(21 + 1)^{2022} + (2022)^{22}$

$= 3k + 1$ $(\alpha = 1)$

Divided by 7

$(21 + 1)^{2022} + (2023 - 1)^{22}$

$7k + 1 + 1$ $(\beta = 2)$

$7k + 2$

So $\alpha^2 + \beta^2 \Rightarrow 5$

EXERCISE - JEE (Advanced) PYQ

1. **Ans. (D)**

$A_r = {}^{10}C_r,\ B_r = {}^{20}C_r, C_r = {}^{30}C_r$

$$\sum_{r=1}^{10}\left({}^{20}C_{10}\ {}^{10}C_r\ {}^{20}C_r - {}^{30}C_{10}\left({}^{10}C_r\right)^2\right)$$

$$= {}^{20}C_{10}\left({}^{10}C_1\ {}^{20}C_1 + {}^{10}C_2\ {}^{20}C_2 + ... + {}^{10}C_{10}\ {}^{20}C_{10}\right) - {}^{30}C_{10}\left({}^{10}C_1^2 + {}^{10}C_2^2 + ... + {}^{10}C_{10}^2\right)$$

$$= {}^{20}C_{10}\left({}^{30}C_{10} - 1\right) - {}^{30}C_{10}\left({}^{20}C_{10} - 1\right) = {}^{30}C_{10} - {}^{20}C_{10} = C_{10} - B_{10}$$

2. **Ans. (6)**

Let the three consecutive terms be

${}^{n+5}C_{r-1}, {}^{n+5}C_r, {}^{n+5}Cr_{+1}$

$$\therefore \frac{{}^{n+5}C_{r-1}}{{}^{n+5}C_r} = \frac{1}{2} \Rightarrow \frac{r}{n-r+6} = \frac{1}{2}$$

$\Rightarrow n = 3r - 6$ ....(1)

Also, $\dfrac{{}^{n+5}C_r}{{}^{n+5}C_{r+1}} = \dfrac{5}{7} \Rightarrow \dfrac{r+1}{n-r+5} = \dfrac{5}{7}$

$$\Rightarrow \frac{{}^{n+5}C_r}{{}^{n+5}C_{r+1}} = \frac{5}{7} \Rightarrow \frac{r+1}{n-r+5} = \frac{5}{7}$$

$\Rightarrow 12r = 5n + 18$ ....(2)

Solving (1) and (2), we get $n = 6$

3. **Ans. (C)**

Coefficient of x^{11} in

$(1+x^2)^4(1+x^3)^7(1+x^4)^{12}$

$$= {}^4C_0.{}^7C_1.{}^{12}C_2 + {}^4C_1.{}^7C_3.{}^{12}C_0 + {}^4C_2.{}^7C_1.{}^{12}C_1 + {}^4C_4.{}^7C_1.{}^{12}C_0$$

$= 462 + 140 + 504 + 7 = 1113$

4. **Ans. (5)**

Coefficient of x^2 in the expansion of

$(1+x)^2 + (1+x)^3 + \ldots\ldots (1+x)^{49} + (1+mx)^{50}$ is

$^{2}C_2 + {}^{3}C_2 + \ldots\ldots + {}^{49}C_2 + {}^{50}C_2 m^2 = (3n+1)\, {}^{51}C_3$

$^{3}C_3 + {}^{3}C_2 + \ldots\ldots\ {}^{49}C_2 + {}^{50}C_2 m^2 = (3n+1)\, {}^{51}C_3$

$^{50}C_3 + {}^{50}C_2 m^2 = (3n+1)\, {}^{51}C_3$

$$\frac{50.49.48}{6} + \frac{50.49}{2} m^2 = (3n+1)\frac{51.50.49}{6}$$

$m^2 = 51n + 1$

must be a perfect square

$\Rightarrow n = 5$ and $m = 16$

Ans. $\Rightarrow 5$

5. **Ans. (646)**

$$X = \sum_{r=0}^{n} r.({}^{n}C_r)^2\,; n = 10$$

$$X = n.\sum_{r=0}^{n} {}^{n}C_r . {}^{n-1}C_{r-1}$$

$$X = n.\sum_{r=1}^{n} {}^{n}C_{n-r} . {}^{n-1}C_{r-1}$$

$X = n. {}^{2n-1}C_{n-1}\,; n = 10$

$X = 10. {}^{19}C_9$

$$\frac{X}{1430} = \frac{1}{143}. {}^{19}C_9$$

$= 646$

6. **Ans. (6.20)**

Suppose

$$\begin{vmatrix} \frac{n(n+1)}{2} & n(n-1).2^{n-2} + n.2^{n-1} \\ n.2^{n-1} & 4^n \end{vmatrix} = 0$$

$$\frac{n(n+1)}{2}.4^n - n^2(n-1).2^{2n-3} - n^2 2^{2n-2} = 0$$

$$\frac{n(n+1)}{2} - \frac{n^2(n-1)}{8} - \frac{n^2}{4} = 0$$

$n^2 - 3n - 4 = 0$

$n = 4$

$$\text{Now } \sum_{k=0}^{4} \frac{{}^{4}C_k}{k+1} = \sum_{k=0.}^{4} \frac{k+1}{5}. {}^{5}C_{k+1} \frac{1}{k+1}$$

$$= \frac{1}{5}.[{}^{5}C_1 + {}^{5}C_2 + {}^{5}C_3 + {}^{5}C_4 + {}^{5}C_5]$$

$$= \frac{1}{5}[2^5 - 1] = \frac{31}{5} = 6.20$$

7. **Ans. (A, B, D)**

Solving

$f(m,n,p)=\sum_{i=0}^{p}{}^{m}C_i{}^{n+i}C_p\,.{}^{p+n}C_{p-i}$

${}^{m}C_i.{}^{n+i}C_p.{}^{p+n}C_{p-i}$

${}^{m}C_i.\frac{(n+i)!}{p!\,(n-p+i)!}\times\frac{(n+p)!}{(p-i)!\,(n+i)!}$

$l\,{}^{m}C_i\times\frac{(n+p)!}{p!}\times\frac{1}{(n-p+i)!\,(p-i)!}$

${}^{m}C_i\times\frac{(n+p)!}{p!\,n!}\times\frac{n!}{(n-p+i)!\,(p-i)!}$

${}^{m}C_i.{}^{n+p}C_p.{}^{n}C_{p-i}\{{}^{m}C_i.{}^{n}C_{p-i}={}^{m+n}C_p\}$

$f(m,n,p)={}^{n+p}C_p.\,{}^{m+n}C_p$

$\frac{f(m,n,p)}{{}^{n+p}C_p}={}^{m+n}C_p$

Now

$g(m,n)=\sum_{p=0}^{m+n}\frac{f(m,n,p)}{{}^{n+p}C_p}$

$g(m,n)=\sum_{p=0}^{m+n}{}^{m+n}C_p$

$g(m,n)=2^{m+n}$

(A) $g(m,n)=q(n,m)$

(B) $g(m,n+1)=2^{m+n+1}\quad g(m+n,n)=2^{m+1+n}$

(D) $g(2m,2n)=2^{2m+2n}$

$=(2^{m+n})^2$

$=\left(g(m,n)\right)^2$

8. **Ans. (3)**

$T_{r+1}={}^{4}C_r\left(a.x^2\right)^{4-r}.\left(\frac{70}{27bx}\right)^r$

$={}^{4}C_r.a^{4-r}.\frac{70^r}{(27b)^r}.x^{8-3r}$

here $8-3r=5$

$8-5=3r\Rightarrow r=1$

$\therefore$ coeff. $=4.a^3.\frac{70}{27b}$

$T_{r+1}={}^{7}C_r(ax)^{7-r}\left(\frac{-1}{bx^2}\right)^r$

$={}^{7}C_r.a^{7-r}\left(\frac{-1}{b}\right)^r.x^{7-3r}$

$7-3r=-5\Rightarrow 12=3r\Rightarrow r=4$

coeff. : ${}^{7}C_4.a^3.\left(\frac{-1}{b}\right)^4 = \frac{35a^3}{b^4}$

now $\frac{35a^3}{b^4} = \frac{280a^3}{27b}$

$b^3 = \frac{35\times 27}{280} = b = \frac{3}{2} \Rightarrow 2b = 3$

JEE (Main) Practice Paper

SECTION–A

1. Ans. (3)

$T_{r+1} = {}^{10}C_r(2x^2)^{10-r}\left(\frac{1}{3x^2}\right)^r$

$\therefore\ T_{5+1} = T_6 = {}^{10}C_5(2x^2)^5\left(\frac{1}{3x^2}\right)^5 = \frac{896}{27} = \frac{a}{b} \quad \therefore a + b = 923$

2. Ans. (4)

$T_{25} = T_{26} \Rightarrow {}^{44}C_{24}(-x)^{24} = {}^{44}C_{25}(-x)^{25}$

$\Rightarrow x = -\frac{{}^{44}C_{24}}{{}^{44}C_{25}} = -\frac{25}{44-25+1} \Rightarrow x = -\frac{25}{20} = -\frac{5}{4}$

3. Ans. (4)

Given ${}^{m}C_0 + {}^{m}C_1 + {}^{m}C_2 = 46$

$\Rightarrow 1 + m + \frac{m(m-1)}{2} = 46$

$\Rightarrow m^2 + m - 90 = 0 \Rightarrow m = 9$

$\therefore T_{r+1} = {}^{9}C_r(x^2)^{9-r}\left(\frac{1}{x}\right)^r = {}^{9}C_r x^{18-3r}$

For constant term, $18-3r = 0 \Rightarrow r = 6$

$\therefore$ constant term $= {}^{9}C_6 = \frac{9.8.7}{3.2.1} = 84$

4. Ans. (1)

$T_{r+1} = {}^{n}C_r\left(\sqrt[3]{2}\right)^{n-r}\left(\frac{1}{\sqrt[3]{3}}\right)^r$

$\therefore T_{6+1} = {}^{n}C_6\left(\sqrt[3]{2}\right)^{n-6}\left(\frac{1}{\sqrt[3]{3}}\right)^6 = T_7$ from beginning

7th term from the end is:

$T_7' = {}^{n}C_6\left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}\left(\sqrt[3]{2}\right)^6$

A.T.Q.

$6\cdot {}^{n}C_6\cdot\left(\sqrt[3]{2}\right)^{n-6}\left(\frac{1}{\sqrt[3]{3}}\right)^6 = {}^{n}C_6\left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}\left(\sqrt[3]{2}\right)^6$

$\Rightarrow 6\cdot 2^{\frac{n-6}{3}}\cdot 3^{\frac{-6}{3}} = 3^{-\frac{n-6}{3}}\cdot 2^{\frac{6}{3}} \Rightarrow 2\times 3\cdot 2^{\frac{n-6}{3}}\cdot 3^{-2} = 3^{-\frac{n-6}{3}}2^2$

$\Rightarrow 2^{\frac{n-6}{3}-1} = 3^{-\frac{n-6}{3}+1} \Rightarrow 2^{\frac{n-9}{3}} = 3^{\frac{9-n}{3}} \Rightarrow n = 9.$

5. Ans. (2)

Here $T_{r+1} = {}^{15}C_r\, x^r$

$\therefore T_{2r-1} = T_{(2r-2)+1} = {}^{15}C_{2r-2}\, x^{2r-2}$ & $T_{r+3} = T_{(r+2)+1} = {}^{15}C_{r+2}\, x^{r+2}$

given ${}^{15}C_{2r-2} = {}^{15}C_{r+2}$

$\Rightarrow 2r - 2 = r + 2$ or $2r - 2 + r + 2 = 15$

$\Rightarrow r = 4$ or $3r = 15 \Rightarrow r = 4$ or 5

6. Ans. (3)

General term of $(ab + bc + ca)^6$ = General term of $a^6\, b^6\, c^6\, (a^{-1} + b^{-1} + c^{-1})^6$

$= a^6\, b^6\, c^6 \dfrac{6!\,(a^{-1})^{k_1}.(b^{-1})^{k_2}.(c^{-1})^{k_3}}{k_1!\,k_2!\,k_3!}$

$\therefore k_1 = 3, k_2 = 2, k_3 = 1$

$\therefore$ Coefficient of $a^3\, b^4\, c^5$ is $\dfrac{6!}{3!2!1!} = 60$

7. Ans. (4)

Let $S = \dfrac{1}{1!\,(n-1)!} + \dfrac{1}{3!\,(n-3)!} + \dfrac{1}{5!\,(n-5)!} + \ldots\ldots.$

$= \dfrac{1}{n!}\left(\dfrac{n!}{1!\,(n-1)!} + \dfrac{n!}{3!\,(n-3)!} + \dfrac{1}{5!\,(n-5)!} + \ldots..\right) = \dfrac{1}{n!}\left({}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots..\right)$

$= \dfrac{1}{n!}.\dfrac{2^n}{2} = \dfrac{2^{n-1}}{n!}$

8. Ans. (4)

Sum of coefficient is zero

$\therefore a^3 - 2a^2 + 1 = 0$

$\Rightarrow a^3 - a^2 - a^2 + a - a + 1 = 0 \Rightarrow a^2\,(a-1) - a(a-1) - 1(a-1) = 0$

$\Rightarrow (a-1)\,(a^2 - a - 1) = 0 \Rightarrow a = 1, a = \dfrac{1 \pm \sqrt{1+4}}{2}$

9. Ans. (2)

n is even here

$\therefore$ middle term $= {}^nC_{\frac{n}{2}}(x^2)^{\frac{n}{2}} \cdot \left(\frac{1}{x}\right)^{\frac{n}{2}} = 924x^6 \Rightarrow {}^nC_{n/2} \cdot x^{n/2} = 924x^6$

$\therefore \dfrac{n}{2} = 6 \Rightarrow n = 12$

10. Ans. (1)

Here $(1 - 2 + 3)^n = 128$

$\Rightarrow 2^n = 128 \Rightarrow n = 7$

$\therefore$ greatest coefficient of $(1+x)^{14}$ is ${}^{14}C_7$

11. Ans. (1)

Let ${}^nC_r = 165$, ${}^nC_{r+1} = 330$ and ${}^nC_{r+2} = 462$

$\therefore \dfrac{{}^nC_{r+1}}{{}^nC_r} = \dfrac{330}{165} = 2 \Rightarrow r = \dfrac{1}{3}(n-2)$

and $\frac{{}^nC_{r+2}}{{}^nC_{r+1}} = \frac{n-r-1}{r+2} = \frac{231}{165} \Rightarrow 165\,n - 627 = 396\,r$

$\therefore 165n - 627 = 132(n-2) \Rightarrow n = 11.$

12. Ans. (2)

$17^{256} = (17^2)^{128} = (290-1)^{128}$

$= 1000\,m + {}^{128}C_2(290)^2 - {}^{128}C_1(290) + 1 = 1000(m + 683527) + 681$

$\therefore$ last two digits = 81

13. Ans. (1)

${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots. + {}^{20}C_{20} = 0$

$\Rightarrow 2({}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots. - {}^{20}C_9) + {}^{20}C_{10} = 0$

$\Rightarrow {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots. - {}^{20}C_9 + {}^{20}C_{10}$

$= -\frac{1}{2}\,{}^{20}C_{10} + {}^{20}C_{10} = \frac{1}{2}\,{}^{20}C_{10}$

14. Ans. (1)

Given

$$\sum_{k=0}^{4} \frac{3^{4-k}}{(4-k)!}\cdot\frac{x^k}{k!}\cdot\frac{4!}{4!} = \sum_{k=0}^{4} \frac{{}^4C_k 3^{4-k}.x^k}{4!} = \frac{(3+x)^4}{4!} = \frac{32}{3}$$

$\Rightarrow x = 1$

15. Ans. (4)

Coefficient of x^5 in $(1+x^2)^5(1+x)^4$

$= {}^4C_1.{}^5C_2 + {}^4C_3.{}^5C_1 = 40 + 20 = 60$

16. Ans. (2)

$\left(\sqrt{5}+1\right)^5 - \left(\sqrt{5}-1\right)^5 = 2\{T_2 + T_4 + T_6\}$

$= 2\left\{{}^5C_1\left(\sqrt{5}\right)^4 + {}^5C_3\left(\sqrt{5}\right)^2 + {}^5C_5\left(\sqrt{5}\right)^0\right\}$

$= 2\{125 + 50 + 1\} = 352.$

17. Ans. (2)

$${}^{95}C_4 + \sum_{j=1}^{5} {}^{100-j}C_3$$

$= {}^{95}C_4 + {}^{99}C_3 + {}^{98}C_3 + {}^{97}C_3 + {}^{96}C_3 + {}^{95}C_3$

$= ({}^{95}C_3 + {}^{95}C_4) + {}^{96}C_3 + {}^{97}C_3 + {}^{98}C_3 + {}^{99}C_3$

$= ({}^{96}C_4 + {}^{96}C_3) + {}^{97}C_3 + {}^{98}C_3 + {}^{99}C_3$

$= ({}^{97}C_4 + {}^{97}C_3) + {}^{98}C_3 + {}^{99}C_3$

$= ({}^{98}C_4 + {}^{98}C_3) + {}^{99}C_3 = {}^{99}C_4 + {}^{99}C_3 = {}^{100}C_4$

18. Ans. (1)

${}^nC_3 + {}^nC_4 > {}^{n+1}C_3$

$\Rightarrow {}^{n+1}C_4 > {}^{n+1}C_3 \Rightarrow \frac{(n+1)!}{4!\,(n-3)!} > \frac{(n+1)!}{3!\,(n-2)!}$

$\Rightarrow \frac{1}{4} > \frac{1}{n-2} \Rightarrow n > 6.$

19. **Ans. (4)**

Exp. = $(1+x)^n(1+x^2)^n = (1+{}^nC_1x+{}^nC_2x^2+{}^nC_3x^3+{}^nC_4x^4+.....+x^n)$

$(1+{}^nC_1x^2+{}^nC_2x^4+...+x^{2n})$

Coefficient of $x^4 = {}^nC_4+{}^nC_2.{}^nC_1+{}^nC_2$

20. **Ans. (3)**

Putting $x = 1$ and $x = -1$ in the given expansion, we get

$a_0+a_1+a_2+a_3+a_4+....=0$

$a_0-a_1+a_2-a_3+a_4-....=2^{2n}$

Adding $2(a_0+a_2+a_4+.....)=2^{2n}$

$a_0+a_2+a_4+...=2^{2n-1}$

SECTION–B

1. **Ans. (11)**

$$\frac{1}{11!}\left[\frac{11!}{1!\,10!}+\frac{11!}{2!\,9!}+\frac{11!}{3!\,10!}+\cdots ...+\frac{11!}{1!\,10!}\right]$$

$$=\frac{1}{11!}[{}^{11}C_1+{}^{11}C_2+......+{}^{11}C_{10}]$$

$$=\frac{1}{11!}[2^{11}-2]=\frac{2}{11!}[2^{10}-1]\Rightarrow k=11$$

2. **Ans. (5)**

$T_2={}^nC_1\,(x)^{n-1}.a=240$ ...(i)

$T_3={}^nC_2\,(x)^{n-2}\,a^2=720$ ...(ii)

$T_4={}^nC_3\,(x)^{n-3}\,a^3=1080$ ...(iii)

From (i) and (ii)

Here $\dfrac{{}^nC_1(x)^{n-1}a}{{}^nC_2x^{n-2}a^2}=\dfrac{2x}{(n-1)a}=\dfrac{240}{720}=\dfrac{1}{3}$

$\Rightarrow\ 6x=(n-1)a$

From (ii) and (iii)

$9x=2(n-2)a$

On dividing $\dfrac{3}{2}=\dfrac{2(n-2)}{(n-1)}\Rightarrow 3n-3=4n-8\Rightarrow n=5$

3. **Ans. (4)**

We get the sum of the coefficients of terms by putting $x=1$ in the polynomial

$(1+x-3x^2)^{2143}$

$(1+1-3)^{2143}=(-1)^{2143}=-1=\lambda$

4. **Ans. (8)**

$$\frac{2^{403}}{15}=\frac{2^3.(2^4)^{100}}{15}=\frac{8(15+1)^{100}}{15}$$

$\Rightarrow$ Fractional part $=\dfrac{8(1)^{100}}{15}=\dfrac{8}{15}\Rightarrow k=8$

5. **Ans. (20)**

Given expression is ${}^{40}C_r$

Hence max. value at $r = 20$.

6. **Ans. (21)**

In the given expansion the coefficients of $5^{th}, 6^{th}$ and 7^{th} terms are ${}^nC_4, {}^nC_5, {}^nC_6$ respectively

$\therefore\ {}^nC_4+{}^nC_6=2\cdot{}^nC_5$

$\Rightarrow \dfrac{n!}{4!(n-4)!}+\dfrac{n!}{6!(n-6)!}=2\dfrac{n!}{(n-5)!5!} \Rightarrow 30+(n-5)(n-4)=2\cdot 6(n-4)$

$\Rightarrow n^2-21n+98=0 \Rightarrow n=7,14$

$\therefore$ Sum of values of $n = 21$

7. **Ans. (54)**

Total Rational terms $=\left(\frac{60}{10}\right)+1=7$

$\Rightarrow$ Total irrational terms

$= 61 - 7 = 54$

8. **Ans. (401)**

Put $x^3 = t$

$\left(t+\frac{1}{t}+1\right)^{200} = \frac{1}{t^{200}}(t^2+t+1)^{200}$

Max. power in $(t^2+t+1)^{200}$ is 400

$\Rightarrow$ total terms = 400 + 1 = 401

9. **Ans. (1)**

$3^{100}=9^{50}=(10-1)^{50}$

$= {}^{50}C_0\, 10^{50} - {}^{50}C_1\, 10^{49} + {}^{50}C_2\, 10^{48} + \ldots + {}^{50}C_{48}\, 10^2 - {}^{50}C_{49}\, 10^1 + {}^{50}C_{50}$

Last three terms are ${}^{50}C_2 \times 100 - {}^{50}C_1 \times 10 + 1$

= 122500 – 500 + 1 = 122001

Hence sum of last three digits = 1

10. **Ans. (0)**

$(x+x^2+x^4)^{20}$

$= x^{20}\left(1+(x+x^3)\right)^{20}$

$\Rightarrow$ Coefficient of x^{59} in $\left(1+(x+x^3)\right)^{20}$

${}^{20}C_0 + {}^{20}C_1(x+x^3) + \ldots\ldots + {}^{20}C_{19}(x+x^3)^{19} + {}^{20}C_{20}(x+x^3)^{20}$

Now Coefficient of x^{59} in $(x+x^3)^{20}$

or in $x^{20}(1+x^2)^{20}$

$\Rightarrow$ Coefficient of x^{39} in $(1+x^2)^{20}=0$.

JEE (Advanced) Practice Paper

1. **Ans. (A)**

Given : $\sum_{r=0}^{n} {}^{n+r}C_r = {}^{n}C_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + \ldots\ldots + {}^{2n}C_n$

$\sum_{r=0}^{n} {}^{n+r}C_r = {}^{n}C_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + \ldots\ldots + {}^{2n}C_n$

$= {}^{n+1}C_{n+1} + {}^{n+1}C_n + {}^{n+2}C_n + \ldots\ldots + {}^{2n}C_n$

$= {}^{2n+1}C_{n+1}$ (by pascal rule ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$)

2. **Ans. (C)**

Here $T_{r+1} = {}^{100}C_r\left(5^{\frac{1}{6}}\right)^r \cdot \left(2^{\frac{1}{8}}\right)^{100-r} = {}^{100}C_r 5^{\frac{r}{6}}\, 2^{\frac{100-r}{8}}$

For rational terms,

$r = 6k_1, k_1 \in I$

and $100 - r = 8k_2, k_2 \in I$

So, possible values of $r = 12,36,60,84$

$\therefore$ No. of irrational terms = 101 – 4 = 97

3. **Ans. (A)**

Here $(1+x)^{101}(1-x+x^2)^{100} = (1+x)(1+x)^{100}(1-x+x^2)^{100}$

$= (1+x)(1+x^3)^{100} = (1+x^3)^{100} + x(1+x^3)^{100}$

$\therefore$ coefficient of $x^{50} = 0 + 0 = 0$

4. **Ans. (B)**

Given $6^{83} + 8^{83} = (7-1)^{83} + (7+1)^{83} = (7+1)^{83} - (1-7)^{83}$

= 2.7.83 + 49I, I is integer

$= 49I + 23 \times 49 + 35 \therefore R = 35 \therefore \frac{R}{5} = 7$

5. **Ans. (D)**

General term $= (1+x+2x^2)\,{}^4C_r(3x^2)^{4-r}\left(\frac{-1}{3x^2}\right)^r = (1+x+2x^2)\,{}^4C_r\,3^{4-r}\left(\frac{-1}{3}\right)^r x^{8-4r}$

$= {}^4C_r\,3^{4-r}\left(\frac{-1}{3}\right)^r x^{8-4r} + {}^4C_r\;3^{4-r}\left(\frac{-1}{3}\right)^r x^{9-4r} + {}^4C_r\,3^{4-r}\,2\left(\frac{-1}{3}\right)^r x^{10-4r}$

For independent term of x

$8 - 4r = 0 \Rightarrow r = 2$

and $9 - 4r = 0 \Rightarrow r = \frac{9}{4}$ Not possible

and $10 - 4r = 0$

$r = \frac{5}{2}$ Not possible

Coefficient of required term $= {}^4C_2 \cdot 3^2 \times \frac{1}{3^2} = 6$

6. **Ans. (B)**

$T_6 = {}^8C_5\left(\frac{1}{x^{8/3}}\right)^3 (x^2\log_{10}x)^5 = 5600 \Rightarrow \frac{1}{x^8}x^{10}(\log_{10}x)^5 = 100 \Rightarrow x = 10$

7. **Ans. (A, B, C, D)**

$\left(\sqrt[3]{4} + \frac{1}{\sqrt[4]{6}}\right)^{20}$

$T_{r+1} = {}^{20}C_r\left(4^{\frac{1}{3}}\right)^{20-r}\left(6^{-\frac{1}{4}}\right)^r$

For rational terms

$20 - r = 3k$ & $r = 4p$, where $k, p \in I \Rightarrow r = 20$ & $r = 8$

$\therefore$ No. of rational terms = 2

$\therefore$ No. of irrational terms = 19

8. **Ans. (A, C)**

$= a\sum_{r=1}^{n}(-1)^{r-1}.{}^nC_r - \sum_{r=1}^{n} r.{}^nC_r(-1)^{r-1}$

$= a[{}^nC_1 - {}^nC_2 + {}^nC_3 \ldots\ldots + (-1)^{n-1}.{}^nC_n] - n$

$= a[{}^nC_1 - {}^nC_2 + {}^nC_3 \ldots\ldots + (-1)^{n-1}.{}^nC_n] - n\sum_{r=1}^{n}(-1)^{r-1}\,{}^{n-1}C_{r-1}$

$= a(1) - n[{}^{n-1}C_0 - {}^{n-1}C_1 + \ldots\ldots + (-1)^{(n-1)}\,{}^{n-1}C_{n-1}] = a - n(0) = a$

9. **Ans. (C, D)**

$a_n = \dfrac{(1000)(1000)\ldots\ldots\ldots\ldots(1000)}{1.2\ldots\ldots\ldots n}$

$a_{999} = a_{1000}$

a_n is maximum for $n = 999$ and $n = 1000$

10. **Ans. (A, B)**

General term of $(x+y+z)^{25} = \frac{25!}{r_1!r_2!r_3!}x^{r_1}y^{r_2}z^{r_3}$

Putting $r_3 = k, r_2 = r - k$ and $r_1 = 25 - r$

$= \dfrac{25!}{(25-r)!\,(r-k)!\,(k)!} \times \dfrac{r!}{r!} \times x^{25-r}\,y^{r-k}\,z^k = {}^{25}C_r.\,{}^rC_k.x^{25-r}\,y^{r-k}\,z^k$

$r_1 + r_2 + r_3 = 25$

$\therefore$ coefficient of $x^8\,y^9\,z^9$ is 0

11. **Ans. (A, B, C)**

Let $(1 + x + x^2 + \cdots + x^{2n})(1 - x + x^2 - x^3 + \cdots + x^{2n})$

$= a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \cdots + a_{4n}x^{4n}$

So,

$a_0 + a_1 + a_2 + \cdots + a_{4n} = 2n + 1$ ...(1)

$a_0 - a_1 + a_2 - a_3 + \cdots + a_{4n} = 2n + 1$ ...(2)

Adding equation (1) & (2), we get

$\Rightarrow a_0 + a_2 + a_4 + \cdots + a_{4n} = 2n + 1$

$\Rightarrow 2n + 1 = 61 \Rightarrow n = 30$

12. **Ans. (C, D)**

$T_{r+1} = {}^{100}C_r\left(\sqrt{2}\right)^{100-r}\left(\sqrt[4]{3}\right)^r$

$= {}^{100}C_r 2^{\frac{100-r}{2}} 3^{\frac{r}{4}}$

Term will rational if $\dfrac{100-r}{2}$ & $\dfrac{r}{4}$ both are integers

$\Rightarrow r = 0, 4, 8, 12 \ldots..100$

$\Rightarrow$ No. of terms = 26

13. **Ans. (2.00)**

${}^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$

$= {}^{39}C_{3r-1} + {}^{39}C_{3r} = {}^{39}C_{r^2} + {}^{39}C_{r^2-1} = {}^{40}C_{3r} = {}^{40}C_{r^2} \quad \Rightarrow r^2 = 3r \text{ or } r^2 + 3r = 40$

$\Rightarrow r = 0, 3, 5, -8$ where $r = 0$ and $r = -8$ will be rejected.

Hence, number of values of r is 2.

14. **Ans. (12.00)**

$$\sum_{k=1}^{n} k^3\left(\frac{n-k+1}{k}\right)^2 = \sum_{k=1}^{n} k(n-k+1)^2 = \sum_{k=1}^{n}(n^2k + k^3 + k - 2nk^2 + 2nk - 2k^2)$$

$$= \frac{(n+1)^2.n(n+1)}{2} + \left[\frac{n(n+1)}{2}\right]^2 - \frac{2(n+1)n(n+1)(2n+1)}{6} = \frac{n(n+1)^2(n+2)}{12} = 12$$

15. Ans. (1.00)

$$\left(\sum_{r=0}^{10} {}^{10}C_r\right)\left(\sum_{k=0}^{10}(-1)^k \frac{{}^{10}C_k}{2^k}\right)$$

$$= ({}^{10}C_0 + \ldots\ldots + {}^{10}C_{10})\left({}^{10}C_0 - \frac{{}^{10}C_1}{2} + \frac{{}^{10}C_2}{2^2} \ldots\ldots + \frac{{}^{10}C_{10}}{2^{10}}\right) = 2^{10} \times \left(1 - \frac{1}{2}\right)^{10} = 1$$

16. Ans. (3.00)

$$\sum_{m=p}^{n} {}^{n}C_m . {}^{m}C_p = \sum_{m=p}^{n} \frac{n!}{m!\,(n-m)!} \times \frac{m!}{p!\,(m-p)!} = \sum_{m=p}^{n} {}^{n}C_p . {}^{n-p}C_{m-p}$$

$= {}^{n}C_p\,[{}^{n-p}C_0 + {}^{n-p}C_1 + \ldots\ldots + {}^{n-p}C_{n-p}] = {}^{n}C_p\, 2^{n-p}$; where $n = 100$ and $p = 97$.

17. Ans. (2.00)

$$T_4 = {}^{8}C_3\left(5^{\frac{1}{5}log_5(4^x+44)}\right)^5\left(\frac{1}{5^{\frac{1}{3}log_5(2^{x-1}+7)}}\right)^3$$

$$\Rightarrow\ {}^{8}C_3(4^x + 44)\left(\frac{1}{2^{x-1}+7}\right) = 336$$

$$\Rightarrow \frac{4^x+44}{2^{x-1}+7} = 6 \Rightarrow 4^x + 44 = 3.2^x + 42 \Rightarrow (2^x)^2 - 3.2^x + 2 = 0$$

$$\Rightarrow\ (2^x - 1)(2^x - 2) = 0 \Rightarrow x = 0 \ \&\ 1$$

18. Ans. (2.00)

$$\sum_{r=0}^{n} \frac{2r+3}{r+1} . {}^{n}C_r = \sum_{r=0}^{n} 2 . {}^{n}C_r + \sum_{r=0}^{n} \frac{1}{r+1} . {}^{n}C_r$$

$$= 2 . 2^n + \frac{1}{n+1} . \sum_{r=0}^{n} . {}^{n+1}C_{r+1}$$

$$= 2^{n+1} + \frac{1}{n+1} . (2^{n+1} - 1) = \frac{(n+2) . 2^{n+1} - 1}{n+1} = 2.$$