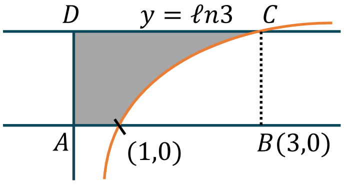
Area Under The Curve

SOLUTIONS

EXERCISE - O

1. **Ans. (C)**

$Ar(ABCD) = 3\ell n3$



shaded Area = $3 \log_e 3 - \int_1^3 \log_e x dx$

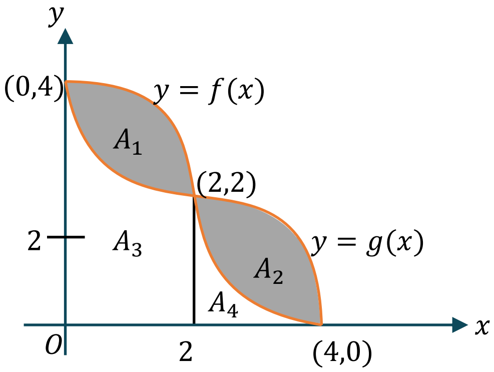
$Ar(ABCD)$

$3\log_e 3-\left(x\log_e x-x\right)_1^3$

$3\log_e 3-\left(3\log_e 3-2\right)=2$

2. **Ans. (C)**

Required area = $\int_0^2 f(x)-g(x)dx$



Now

$$\int_0^4 f(x)-g(x)dx=10$$

$$\int_0^2 f(x)-g(x)dx+\int_2^4 f(x)-g(x)dx=10$$

$$\int_0^2 f(x)-g(x)dx-\underbrace{\int_2^4 g(x)-f(x)dx}_{5}=10$$

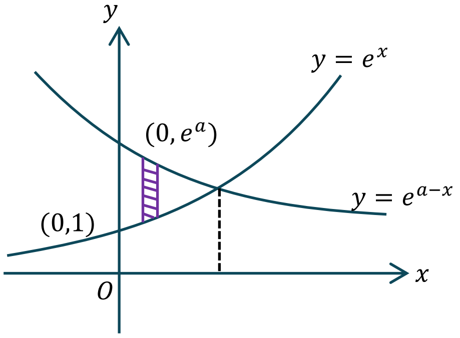
$$\Rightarrow \int_0^2 f(x)-g(x)dx=15$$

3. **Ans. (D)**

Solving $e^x = e^{a-x}$

we get $e^{2x} = e^a$

$\Rightarrow x=\frac{a}{2}$



$$=\int_0^{\frac{a}{2}}\left(e^{a-x}-e^x\right)dx=\left[-\left(e^{a-x}+e^x\right)\right]_0^{a/2}$$

$$= \left(e^a+1\right)-\left(e^{a/2}+e^{a/2}\right)$$

$$=e^a-2\cdot e^{a/2}+1=\left(e^{a/2}-1\right)^2$$

$$\lim_{a\to 0}\frac{S}{a^2}=\lim_{a\to 0}\left(\frac{e^{a/2}-1}{a}\right)^2=\frac{1}{4}$$

4. **Ans. (D)**

$x(x-3)^2 = x$

$\Rightarrow x = 0, 2, 4$

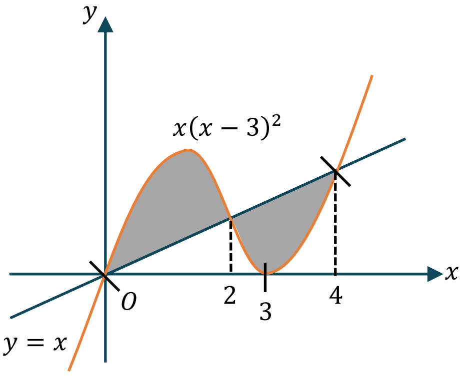
Required area

$$= \int_0^2 \left(x(x-3)^2 - x\right)dx + \int_2^4 \left(x - x(x-3)^2\right)dx$$

$$= \int_0^2 \left(x^3 - 6x^2 + 8x\right)dx - \int_2^4 \left(x^3 - 6x^2 + 8x\right)dx$$

$$= \left[\frac{x^4}{4} - 2x^3 + 4x^2\right]_0^2 - \left[\frac{x^4}{4} - 2x^3 + 4x^2\right]_2^4$$

$= 4 + 4 = 8$ sq units



5. **Ans. (A)**

$y = ax^2 + bx + c \rightarrow$ Passes through (1, 2)

$\Rightarrow 2 = a + b + c$ ...(1)

Also tangent at origin $\Rightarrow y = x$

$\Rightarrow$ Curve passes through origin $\Rightarrow c = 0$

$$\left.\frac{dy}{dx}\right|_{(0,0)} = 1 \Rightarrow 2ax + b\big|_{(0,0)} = 1$$

$\Rightarrow b = 1$

$\therefore$ by equation (1) : $a = 1$

$\Rightarrow$ Curve will be

$y = x^2 + x$

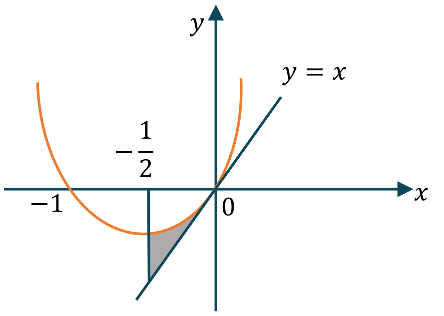
$\Rightarrow \frac{dy}{dx} = 0$

$\Rightarrow 2x + 1 = 0$

$\Rightarrow$ Minima at $x = -\frac{1}{2}$

$\therefore$ Required area = $\int_{-1/2}^{0} (x^2 + x - x)\,dx$

$= \frac{1}{24}$ sq. units



6. **Ans. (A)**

$y = xe^{-x}$

$y' = e^{-x} - xe^{-x}$

$y'' = -2e^{-x} + xe^{-x} = e^{-x}(x-2)$

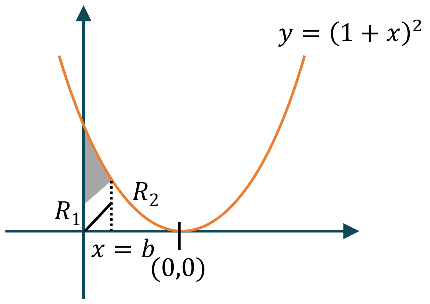
$y'' = 0 \Rightarrow x = 2$ is point of inflection

$\Rightarrow c = 2$

Area = $\int_0^2 \underset{I}{x}\,\underset{II}{e^{-x}}\,dx$

$= \left(-xe^{-x} - e^{-x}\right)_0^2 = 1 - 3e^{-2}$

7. **Ans. (B)**



$$R_1 = \int_0^b (1-x)^2 dx = \left.\frac{(x-1)^3}{3}\right|_0^b = \frac{(b-1)^3}{3} + \frac{1}{3} = \frac{1}{3}\left((b-1)^3 + 1\right)$$

$$R_2 = \int_b^1 (1-x)^2 dx = \left(\frac{(x-1)^3}{3}\right)_b^1 = 0 - \left(\frac{(b-1)^3}{3}\right)$$

$$R_1 - R_2 = \frac{1}{4} \Rightarrow \frac{1}{3}\left((b-1)^3 + 1 + (b-1)^3\right) = \frac{1}{4}$$

$$\Rightarrow 2(b-1)^3 = \frac{3}{4} - 1 = -\frac{1}{4} \Rightarrow (b-1)^3 = -\frac{1}{8} \Rightarrow b = \frac{1}{2}$$

8. **Ans. (A)**

$$x = \frac{1-t^2}{1+t^2},\ y = \frac{2t}{1+t^2}$$

$t = \tan\theta$

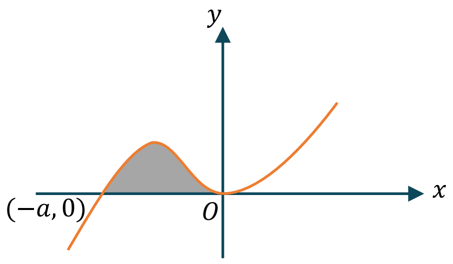
$$x = \frac{1-\tan^2\theta}{1+\tan^2\theta},\ y = \frac{2\tan\theta}{1+\tan^2\theta}$$

$x = \cos 2\theta$, $y = \sin 2\theta$

$x^2 + y^2 = 1$

Area = $\pi r^2 = \pi$ .... $\because r = 1$

9. **Ans. (D)**



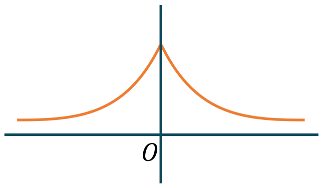
This curve is y = $\frac{x^2(x+a)}{a^2}$, which is a cubic polynomial.

Since $\frac{x^2(x+a)}{a^2}$ = 0 has repeated root $x = 0$,

it touches the x –axis at (0, 0) and intersects at $(-a, 0)$.

Required area = $\int_{-a}^{0} y dx = \int_{-a}^{0} \frac{x^2(x+a)}{a^2} dx = \frac{a^2}{12}$ sq.units

10. **Ans. (A)**



$y = \frac{a}{4+x^2}$ $(a > 0), y = c$

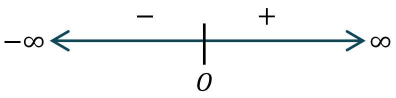
for area to be bounded $c = 0$

$$\Rightarrow 2\int_0^\infty \frac{a}{4+x^2} dx = 2 \Rightarrow 2\left(\frac{a}{2}\tan^{-1}\frac{x}{2}\right)\Bigg|_0^\infty = 2 \Rightarrow a\left(\frac{\pi}{2} - 0\right) = 2$$

$$a = \frac{4}{\pi}$$

$\Rightarrow [a] = 1 \Rightarrow [a] + [c] = 1$

11. **Ans. (A,C,D)**



$f : R \to [-1, 1]$

$$f(x) = \frac{x^2-1}{x^2+1} = 1 - \frac{2}{x^2+1}$$

$1 \le x^2 + 1 < \infty \Rightarrow -1 \le f(x) < 1$

$f(x)$ is even function $\Rightarrow$ Many one

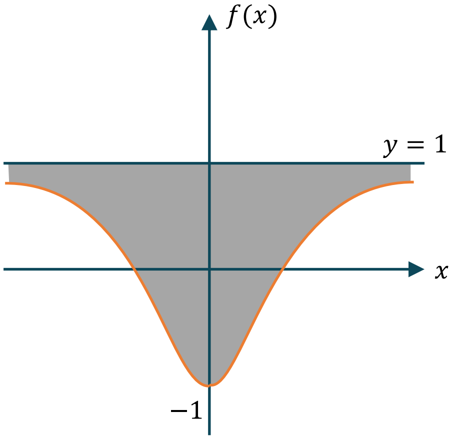
Codomain = [−1, 1]

Range = [−1, 1)

$f(x)$ is many one and into

$\therefore$ A is wrong

$$f'(x) = \frac{4x}{(x^2+1)^2}$$



Option B is correct.

Now For option D

Option 'C' is wrong.

Area between $y = f(x)$ & $y = 1$

$$A = \int_{-\infty}^{\infty} 1 - \left(\frac{x^2-1}{x^2+1}\right) dx = \int_{-\infty}^{\infty} \frac{2\,dx}{x^2+1}$$

$$A = 2 \cdot \int_{-\infty}^{\infty} \frac{dx}{1+x^2}$$

$$\Rightarrow A = 2\tan^{-1} x\Big|_{-\infty}^{\infty}$$

$$\Rightarrow A = 2\pi \text{ sq.units}$$

Option D is wrong

12. **Ans. (B,C,D)**

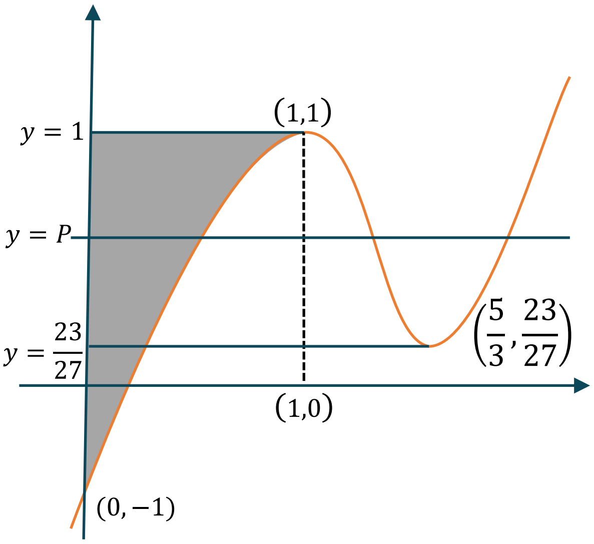
$f(x) = (x-1)^2(x-2) + 1, x \in [0, 2]$

$f(x) = (x-1)^3 - (x-1)^2 + 1$

$f'(x) = 3(x-1)^2 - 2(x-1)$

$f'(x) = (x-1)\,[3x-5]$

$f'(x) = 0$ at $x = 1, \frac{5}{3}$



$x = 0$ is y axis

$f(1) = 1$

$$f\left(\frac{5}{3}\right) = \frac{23}{27}$$

$$\int_0^1 (1 - f(x))dx = \int_0^1 \left[(x-1)^2 - (x-1)^3\right]dx$$

$$= \left[\frac{(x-1)^3}{3} - \frac{(x-1)^4}{4}\right]_0^1 = \frac{7}{12}$$

Range : [− 1, 1]

13. **Ans. (B,C)**

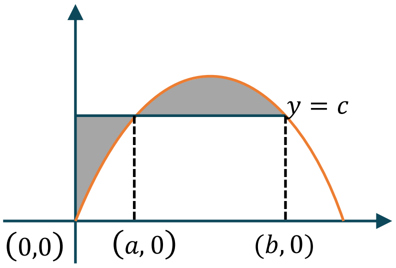
Let $y = c$ line also intersects at $x = a$

According to question

$$= \int_0^a \{c - (8x - 27x^3)\}\,dx = \int_a^b (8x - 27x^3 - c)\,dx$$

$$= \left[cx - 4x^2 - \frac{27x^4}{4}\right]_0^a = \left[4x^2 - \frac{27x^4}{4} - cx\right]_a^b$$

$$0 = 4b^2 - \frac{27b^4}{4} - cb$$



$27b^4 - 16b^2 + 4bc = 0$

$27b^3 - 16b + 4c = 0$ ...(i)

Also (b, c) lies on $y = 8x - 27x^3$

$c = 8b - 27b^3$ ...(ii)

from (i) & (ii)

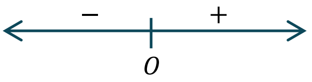
$b = \frac{4}{9}$, $c = \frac{32}{27}$

14. **Ans. (B,C)**

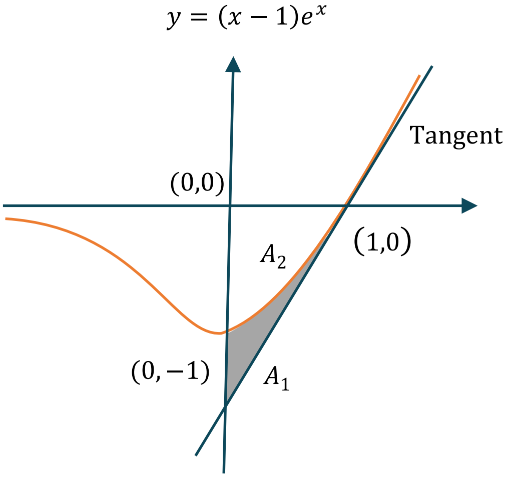
$C_1 : y = (x - 1)e^x$; $(x \to \infty, y \to \infty)$

$y = x\,e^x - e^x$; $(x \to \infty, y \to 0)$

$y' = x\,e^x$



$y" = (x + 1)e^x$



$$A_2 = \left|\int_0^1 \left(xe^x - e^x\right)dx\right|$$

$$A_2 = \left|\left(xe^x - 2e^x\right)_0^1\right| = e - 2$$

tangent to C_1 at $(1, 0)$ is

$y - 0 = e(x - 1) \Rightarrow y = ex - e$

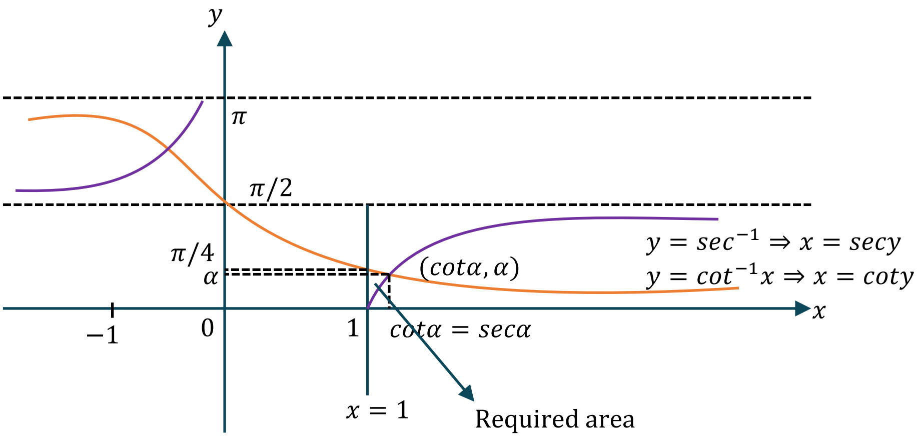
$$A_1 = \int_0^1 \{(xe^x - e^x) - (ex - e)\}\,dx$$

$$A_1 = \int_0^1 xe^x dx - \int_0^1 e^x dx - \left(e\frac{x^2}{2}\right) + (ex)_0^1$$

$$A_1 = 2 - \frac{e}{2} \Rightarrow 2A_1 = 4 - e$$

$A_2 > A_1$

15. **Ans. (A,B)**



For point of intersection

$\sec^{-1}x = \cot^{-1}x = \alpha$ (let)

$x = \cot\alpha = \sec\alpha$

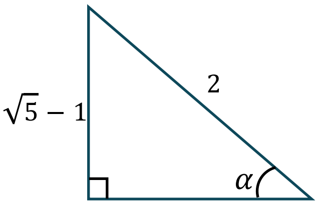
$\cos^2\alpha = \sin\alpha$

$1 - \sin^2\alpha = \sin\alpha$

$\sin^2\alpha + \sin\alpha - 1 = 0$

$\sin\alpha = \frac{-1\pm\sqrt{5}}{2} \Rightarrow \sin\alpha = \frac{\sqrt{5}-1}{2}$

$\cot\alpha = \sqrt{\frac{1+\sqrt{5}}{2}} = \sec\alpha$



Area w.r.t to x axis

$$\int_{1}^{\cot\alpha}(\cot^{-1}x-\sec^{-1}x)dx$$

Area w.r.t to y axis

$$\int_{0}^{\alpha}\sec y\,dx+\int_{\alpha}^{\pi/4}\cot y\,dx-\frac{\pi}{4}$$

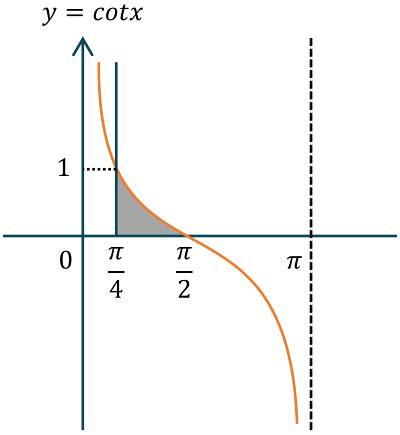
16. **Ans. (A,B)**

Area w.r. to y axis

$$=\int_{0}^{1}(\cot^{-1}y)dy-\frac{\pi}{4}=\int_{0}^{1}\left(\frac{\pi}{2}-\tan^{-1}y\right)dy-\frac{\pi}{4}$$

$$=\frac{\pi}{2}-\int_{0}^{1}\tan^{-1}dy-\frac{\pi}{4}$$

$$=\frac{\pi}{4}-\int_{0}^{1}\tan^{-1}dy$$ .... option (B)



Area w.r. to x axis

$$=\int_{\pi/4}^{\pi/2}\cot x\,dx$$

Let $x=\frac{\pi}{2}-t$

$$=\int_{\pi/4}^{0}-\cot\left(\frac{\pi}{2}-t\right)dt$$

$$=\int_{0}^{\pi/4}\tan t\,dt$$

use $t\rightarrow\frac{\pi}{4}-t$

$$=\int_{0}^{\pi/4}\tan\left(\frac{\pi}{4}-t\right)dt$$ .... option (A)

17. **Ans. (A,D)**

Since the curve $y = ax^{\frac{1}{2}} + bx$ passes through the point (1, 2)

$\therefore\ 2 = a + b$ ...(1)

By observation the curve also passes through (0, 0)

Therefore, the area enclosed by the curve, x −axis and $x = 4$ is given by

$$A=\int_{0}^{4}(ax^{1/2}+bx)dx=8\ \Rightarrow\frac{2a}{3}\times 8+\frac{b}{2}\times 16=8$$

$$\Rightarrow\frac{2a}{3}+b=1 \qquad\qquad ...(2)$$

Solving (1), (2) we get $a = 3, b = 1$

18. **Ans. (A,B)**

$y = \sin 2x$

Let A be the area between $y = \sin 2x,\ x=\frac{\pi}{6}, x = c, y = 0$

$\therefore$ given that $A=\frac{1}{2}$

we have two possible cases for c i.e.

(i) $c>\frac{\pi}{6}$

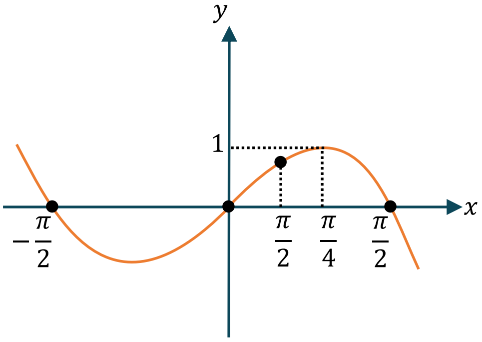
now area between $y = \sin 2x, y = 0,\ x=\frac{\pi}{6}\ \&\ x=\frac{\pi}{2}$ is greater than $\frac{1}{2}$

$\therefore\ c\in\left(\frac{\pi}{6},\frac{\pi}{2}\right)$

$$\therefore\ A=\int_{\pi/6}^{c}\sin 2x\,dx=-\frac{\cos 2x}{2}\Big|_{\pi/6}^{c}=\frac{1}{2}$$

$$-\frac{\cos 2c}{2}+\frac{1}{4}=\frac{1}{2}\Rightarrow\cos 2c=-\frac{1}{2}$$

$\therefore\ c=\frac{\pi}{3}$



(ii) $c<\frac{\pi}{6}$

now area between $y = \sin 2x, y\ =\ 0,\ x=\frac{\pi}{6}\ \&\ x = 0$ is $\frac{1}{4}$

$\therefore$ by symmetry $c=-\frac{\pi}{6}$

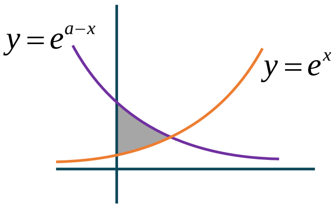
19. **Ans. (A,C)**

$$\left.\begin{array}{l}\because\ \ f(x)>0\\ \&\ \ g(x)<0\end{array}\right\}\text{for}\ \ x\in(a,b)$$

$\Rightarrow |f(x)| = f(x)$ and $|g(x)| = -g(x)$

20. **Ans. (A,B,C,D)**

$$S=\int_{0}^{a/2}(e^{a-x}-e^{x})dx$$

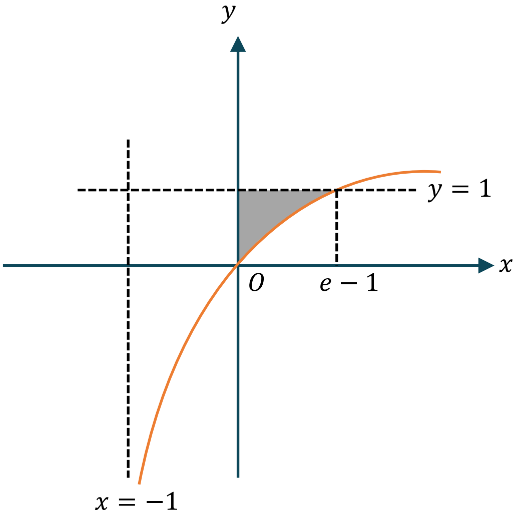


$$= - [2e^{\frac{a}{2}} - (e^{a} + 1)] \Rightarrow \left(e^{\frac{a}{2}}-1\right)^{2}$$

$$\text{Now}\lim_{a\to 0}\frac{e^{a}-2e^{a/2}+1}{a^{2}}$$

$$=\lim_{a\to 0}\left(\frac{e^{a/2}-1}{a/2}\right)^{2}\frac{1}{4}=\frac{1}{4}$$

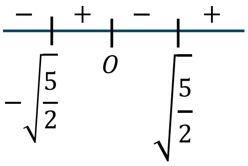
21. **Ans. (A,C,D)**

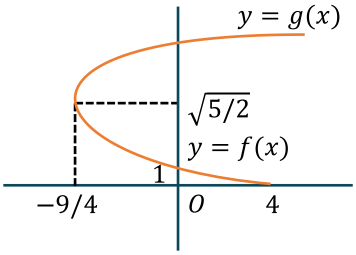


$$A=\int_0^1 (e^y-1)dy=e-2$$

22. **Ans. (A)**

$x = y^4 - 5y^2 + 4$



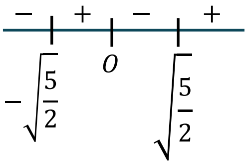


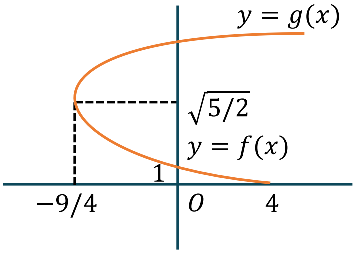
$$\frac{dx}{dy}=2y(2y^2-5)$$

$$\int_0^1 x dy=\int_0^1 (y^4-5y^2+4)dy=\frac{38}{15}$$

23. **Ans. (B)**

$x = y^4 - 5y^2 + 4$





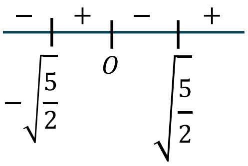
$$\frac{dx}{dy}=2y(2y^2-5)$$

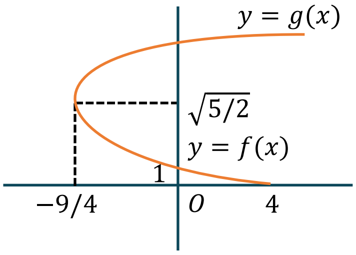
$$\int_7^9 ydx=yx\Big|_7^9-\int_7^9 y'x=9g(9)-7f(7)-\int_7^9\frac{x}{2g(x)(2g^2(x)-5)}\text{dx}$$

$$\{\because\ y'=\frac{1}{2y(2y^2-5)}\}$$

24. **Ans. (D)**

$x = y^4 - 5y^2 + 4$





$$\frac{dx}{dy} = 2y(2y^2 - 5)$$

$$\frac{dx}{dy} = 4y^3 - 10y$$

$$\left.\frac{dx}{dy}\right|_{(-2,\sqrt{2})} = 8\sqrt{2} - 10\sqrt{2} = -2\sqrt{2}$$

Slope of tangent $= -\dfrac{1}{2\sqrt{2}}$

$$(y - \sqrt{2}) = -\frac{1}{2\sqrt{2}}(x + 2)$$

$T : x + 2\sqrt{2}y = 2$

$$\Delta = \frac{1}{2}.2.\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

25. **Ans. (B)**

(I) Area $= \displaystyle\int_0^1 (3y^2 - 9)dy$

$= \left|[y^3 - 9y]_0^1\right| = |1 - 9| = 8$ square units

(II) Given equation of curve is $y = a\sqrt{x} + bx$

this curve is passing through (1, 2)

$2 = a + b$ ...(i)

and $\displaystyle\int_0^4 (a\sqrt{x} + bx)dx = 8$

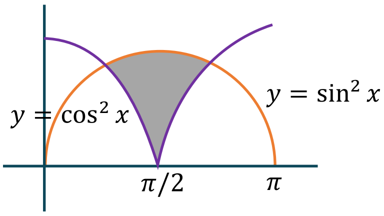
$$\frac{2a}{3}\left[x^{3/2}\right]_0^4 + \frac{b}{2}\left[x^2\right]_0^4 = 8 \Rightarrow \frac{2a}{3}.8 + 8b = 8$$

$\Rightarrow 2a + 3b = 3$ ...(ii)

Solving (i) & (ii) $a = 3, b = -1$

then $2a + b = 3 \times 2 - 1 = 5$

(III) Area $= \displaystyle\int_{\pi/4}^{3\pi/4} (\sin^2 x - \cos^2 x)dx$



$$= -\int_{\pi/4}^{3\pi/4} \cos 2x dx = -\left[\frac{\sin 2x}{2}\right]_{\pi/4}^{3\pi/4}$$

$= -\dfrac{1}{2}(-1 - 1) = 1$ square units

(IV) $\int_{0}^{16/m^2} (\sqrt{16x} - mx)dx = \frac{2}{3} \Rightarrow \left[4 \times \frac{2}{3}x^{3/2} - m\frac{x^2}{2}\right]_0^{16/m^2} = \frac{2}{3}$

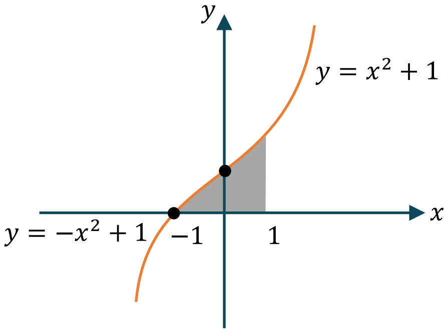
$\Rightarrow \frac{8}{3} \times \frac{64}{m^3} - \frac{m}{2} \times \frac{256}{m^4} = \frac{2}{3} \Rightarrow \frac{1}{m^3}\left[\frac{512}{3} - 128\right] = \frac{2}{3}$

$\Rightarrow m^3 = 64 \Rightarrow m = 4$

EXERCISE - S

1. **Ans. (2)**

The graph is a follows



$$\int_{-1}^{0} \left(-x^2 + 1\right)dx + \int_{0}^{1} \left(x^2 + 1\right)dx = 2$$

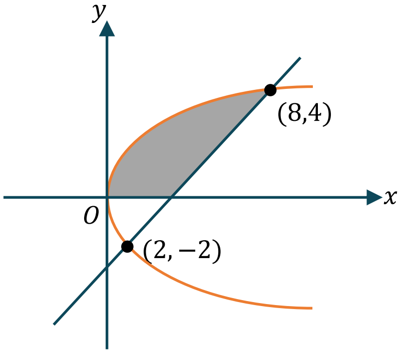
2. **Ans. (18)**

$y^2 = 2x$

$x - y - 4 = 0$

$(x - 4)^2 = 2x$

$x^2 + 16 - 8x - 2x = 0$



$x^2 - 10x + 16 = 0$

$x = 8, 2$

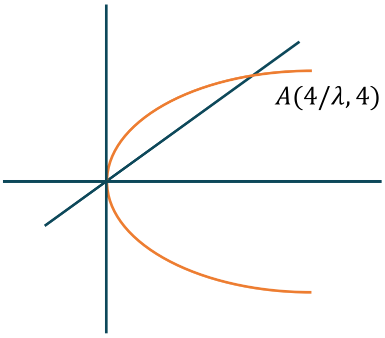
$y = 4, -2$

$$A = \int_{-2}^{4} \left(y + 4 - \frac{y^2}{2}\right)dy$$

$$= \left.\frac{y^2}{2}\right|_{-2}^{4} + 4y\Big|_{-2}^{4} - \left.\frac{y^3}{6}\right|_{-2}^{4} = (8-2) + 4(6) - \frac{1}{6}(64+8)$$

$= 6 + 24 - 12 = 18$

3. **Ans. (24)**



$$\text{Area } = \frac{1}{9} = \int_0^{\frac{4}{\lambda}}\left(\sqrt{4\lambda x} - \lambda x\right)dx$$

$$\Rightarrow \lambda = 24$$

4. **Ans. (2)**

$$x = \ell n\frac{1}{y} \ \& \ y = \ell n(x+e)$$

$$\boxed{e^{-x} = \ell n(x+e)}$$

$x = 0 \ \& \ y = 1$

Area enclosed

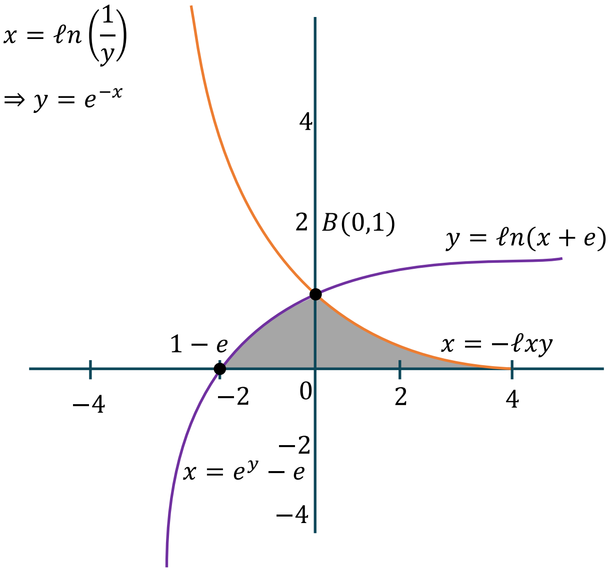
$$= \int_{1-e}^{0} \ell n(x+e)dx + \int_0^{\infty} e^{-x}dx$$

$$= \left[(x+e)\ell n(x+e) - x\right]_{1-e}^{0} + \left[-e^{-x}\right]_0^{\infty}$$

$$= [e + (1-e) + 1] = \boxed{2}$$

OR

Using horizontal strip



$$Area = \int_0^1\left((-\ell ny) - (e^y - e)\right)dy = -\left[(y\ell ny - y)\right]_0^1 - \left[e^y - ey\right]_0^1$$

$= 1 - [(e - e) - 1] = \boxed{2}$ sq. unit.

5. **Ans. (104)**

$y = x^2$ ...(i)

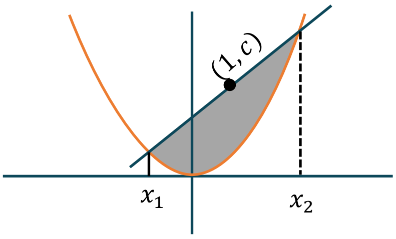
$y = c + m(x-1)$ ...(ii)

Solving (i) and (ii)

$x^2 - mx + m - c = 0$ ...(iii)

x_1 and x_2 are roots of above equation

$$\Rightarrow x_1 + x_2 = m \, ; x_1x_2 = m - c \, ; x_2 - x_1 = \sqrt{m^2 - 4(m-c)}$$



$$Area = \int_{x_1}^{x_2}\left((c-m) + mx - x^2\right)dx$$

$$= (c-m)(x_2 - x_1) + \frac{m}{2}\left(x_2^2 - x_1^2\right) - \frac{1}{3}\left(x_2^3 - x_1^3\right)$$

$$=(x_2-x_1)\left[(c-m)+\frac{m}{2}(x_2+x_1)-\frac{1}{3}\left(x_2^2+x_1x_2+x_1^2\right)\right]$$

$$=\sqrt{m^2-4m+4c}\left[c-m+\frac{m^2}{2}-\frac{1}{3}\left(m^2-(m-c)\right)\right]$$

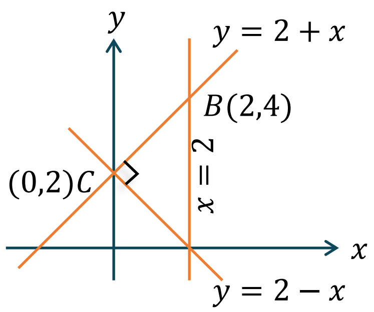
$$=\frac{\left(m^2-4m+3c\right)^{3/2}}{6}=\frac{\left((m-2)^2+4(c-1)\right)^{3/2}}{6}$$

Area will be minimum when $m = 2$

$$\therefore \frac{\left(4(c-1)\right)^{3/2}}{6}=36 \Rightarrow c=10$$

$$\therefore \boxed{c^2+m^2=104}$$

6. **Ans. (4)**



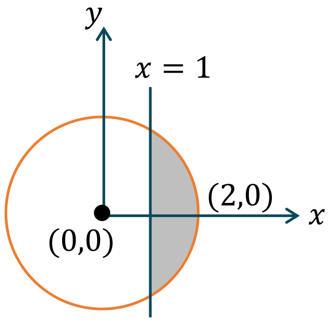
Obviously, triangle ACB is right angled at C.

$\therefore$ Required area $= \frac{1}{2} \times AC \times BC$

$= \frac{1}{2} \times 2\sqrt{2} \times 2\sqrt{2}$

$= 4$ *sq. unit.*

7. **Ans.** $\left(\frac{4\pi}{3} - \sqrt{3}\right)$



Area of smaller part $= 2\int_1^2 \sqrt{4-x^2}\,dx$

$$= 2\left[\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}\right]_1^2 = 2\left[2.\frac{\pi}{2}-\left[\frac{\sqrt{3}}{2}+2.\frac{\pi}{6}\right]\right]$$

$$= 2\left[\pi-\left[\frac{\sqrt{3}}{2}+\frac{\pi}{3}\right]\right] = \frac{4\pi}{3}-\sqrt{3}$$

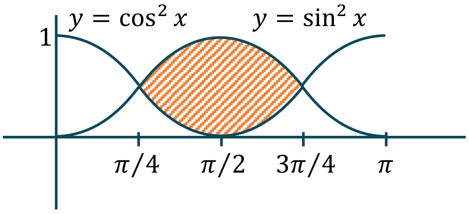
8. **Ans. (1)**

$y = \sin^2 x$ and $y = \cos^2 x$

Solving $\sin^2 x = \cos^2 x$

$\therefore x = \frac{\pi}{4}, \frac{3\pi}{4}$

Graph of functions is as shown in the following figure.



From the figure, the required area

$$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (sin^2 x - cos^2 x)dx$$

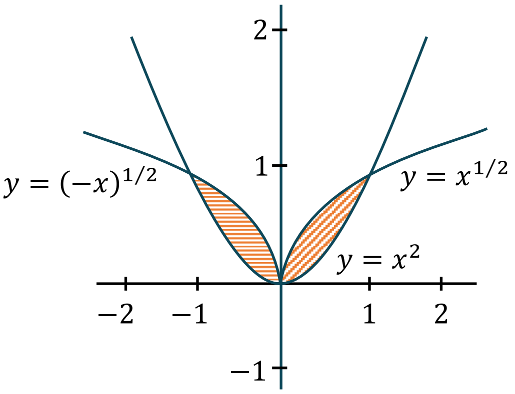
$$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} cos\,2\,xdx = 1$$

9. **Ans. (0.67)**

$y = x^2$ is upward parabola

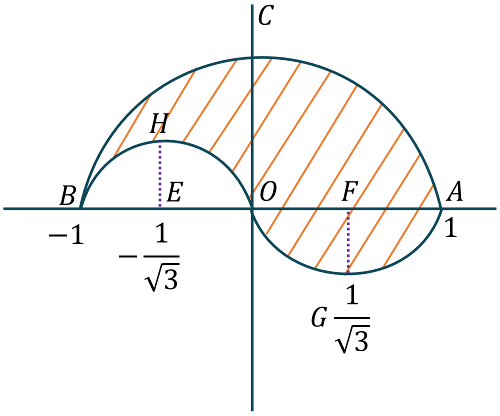
$$y=\sqrt{|x|}=\begin{cases}\sqrt{x}, & x \geq 0\\ \sqrt{-x}, & x<0\end{cases}$$

The graph of the functions is as shown in the following figure.



$$A = 2\int_0^1\left(\sqrt{x}-\frac{x^2}{2}\right)dx=\frac{2}{3}$$

10. **Ans. (2)**



$y=\sqrt{1-x^2}$

$y=x^3-x$

$y=0$ in $x=0,1,-1$

Required area = Area of region $BCAGOHB$

= Area of semi-circle $BCAOB$

$=\frac{\pi}{2}$ ($\because$ area of $BHOEB$ = area of $OFAGO$)

$$\frac{\pi}{A}=\frac{\pi}{\left(\frac{\pi}{2}\right)}=2$$

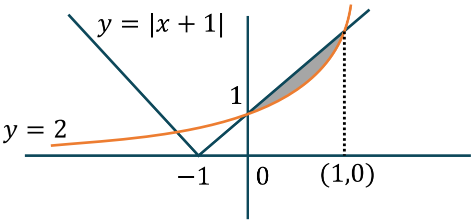
EXERCISE - JEE (Main) PYQ

1. **Ans. (1)**

$2^x = x + 1$

Hit & trial at

$x = 0,1$



$$A=\int_0^1\left(x+1-2^x\right)dx=\left(\frac{x^2}{2}+x-\frac{2^x}{\log_e 2}\right)_0^1$$

$$A=\left(\frac{3}{2}-\frac{2}{\log_e 2}\right)-\left(\frac{-1}{\log_e 2}\right)=\frac{3}{2}-\frac{1}{\log_e 2}$$

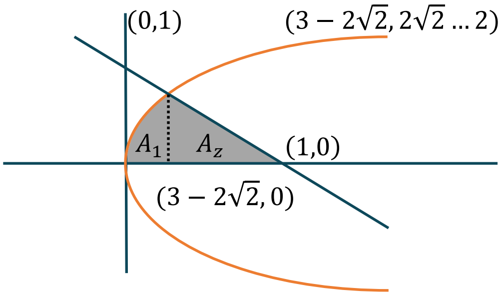
2. **Ans. (3)**

$y^2 = 4x$ and $x + y = 1$

$x^2 - 2x + 1 = 4x$

$x^2 - 6x + 1 = 0$

$x = 3 \pm 2\sqrt{2}$

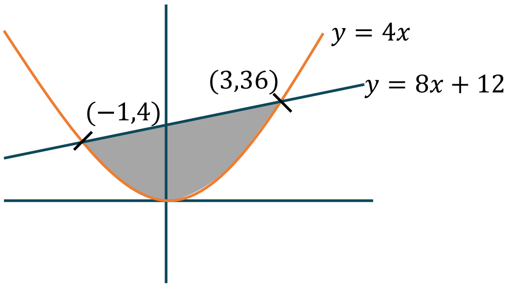


$$\int_0^{3-2\sqrt{2}} 2\sqrt{x}\,dx+\frac{1}{2}\left(2\sqrt{2}-2\right)\left(2\sqrt{2}-2\right)$$

$$A=\frac{4}{3}\left(x^{\frac{3}{2}}\right)_0^{3-2\sqrt{2}}+\left(2-\sqrt{2}\right)^2\Rightarrow\frac{4}{3}\left(\sqrt{2}-1\right)^3+2\left(\sqrt{2}-1\right)^2$$

$$=\left(3-2\sqrt{2}\right)\left(\frac{4\sqrt{2}}{3}-\frac{4}{3}+2\right)=\frac{8\sqrt{2}-10}{3}$$

3. **Ans. (4)**



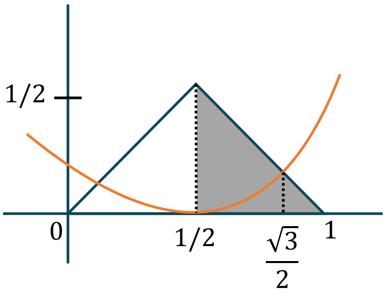
Point intersection , $y = 4x^2$ & $y = 8x + 12$

$(x, y) \equiv (3, 36)$ & $(-1, 4)$

$$A=\int_{-1}^{3}\left(8x+12-4x^2\right)dx=\frac{128}{3}$$

4. **Ans. (2)**

Point of intersection of $y=\left(x-\frac{1}{2}\right)^2$ and $y = 1 - x$ is $x=\frac{\sqrt{3}}{2}$



$$A=\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}\left\{(1-x)-\left(x-\frac{1}{2}\right)^2\right\}dx=\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}\left(\frac{3}{4}-x^2\right)dx$$

$$A=\left(\frac{3}{4}x-\frac{x^3}{3}\right)_{\frac{1}{2}}^{\frac{\sqrt{3}}{1}}=\frac{\sqrt{3}}{4}-\frac{1}{3}$$

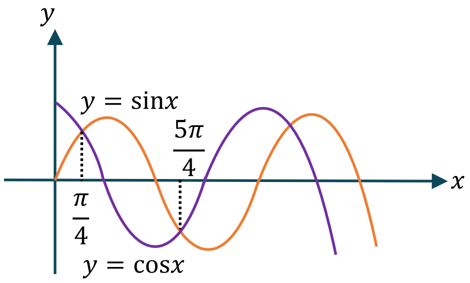
5. **Ans. (2)**

Required area = $2\int_0^{\sqrt{3}}(2x^2+9-5x^2)\,dx$

= $2\left[9x-x^3\right]_0^{\sqrt{3}}$

$=2\left[9\sqrt{3}-3\sqrt{3}\right]$ = $12\sqrt{3}$

6. **Ans. (64)**



$A=\int_{\pi/4}^{5\pi/4}(\sin x-\cos x)\,dx$

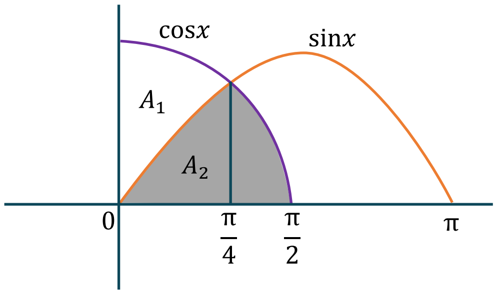
$=(-\cos x-\sin x)\Big|_{\pi/4}^{5\pi/4}$

$=\left(-\left(\frac{-1}{\sqrt{2}}\right)-\left(\frac{-1}{\sqrt{2}}\right)\right)-\left(-\left(\frac{1}{\sqrt{2}}\right)-\left(\frac{1}{\sqrt{2}}\right)\right)$

$\Rightarrow A = \frac{2}{\sqrt{2}}+\frac{2}{\sqrt{2}}=2\sqrt{2}$

$\Rightarrow A^4=\left(2\sqrt{2}\right)^4$ = 16 × 4 = 64

7. **Ans. (1)**



$A_1=\int_0^{\pi/4}\left(\cos x-\sin x\right)dx$

$A_1=\left(\sin x+\cos x\right)_0^{\pi/4}=\sqrt{2}-1$

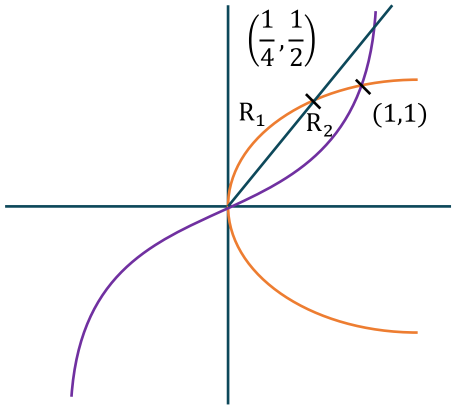
$A_2=\int_0^{\pi/4}\sin x\,dx+\int_{\pi/4}^{\pi/2}\cos x\,dx$

$=\left(-\cos x\right)_0^{\pi/4}+\left(\sin x\right)_{\pi/4}^{\pi/2}$

$A_2=\sqrt{2}\left(\sqrt{2}-1\right)$

$A_1 : A_2 = 1 : \sqrt{2}, A_1 + A_2 = 1$

8. **Ans. (19)**



$S=\int_0^1\sqrt{x}-x^3=\left[\frac{2x^{3/2}}{3}-\frac{x^4}{4}\right]_1^0=\frac{5}{12}$

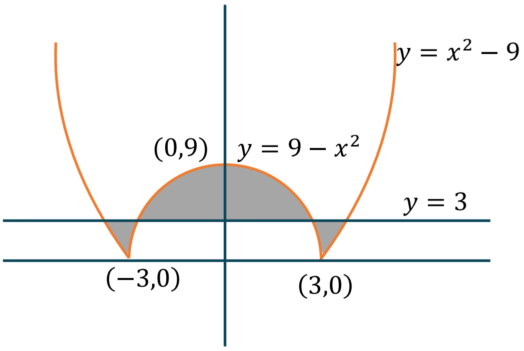
$R_1=\int_0^{1/4}(\sqrt{x}-2x)dx=\left[\frac{2x^{3/2}}{3}-x^2\right]_0^{1/4}=\frac{1}{48}$

$\therefore\ R_2=\frac{19}{48}$

So, $\frac{R_2}{R_1}=19$

9. **Ans. (4)**

Area of shaded region



$$=2\int_0^3\left(\sqrt{9+y}-\sqrt{9-y}\right)dy+2\int_3^9\left(\sqrt{9-y}\right)dy$$

$$=2\left[\int_0^3(9+y)^{1/2}dy-\int_0^3(9-y)^{1/2}dy+\int_3^9(9-y)^{1/2}dy\right]$$

$$=2\left[\frac{2}{3}\left[(9+y)^{3/2}\right]_0^3+\frac{2}{3}\left[(9-y)^{3/2}\right]_0^3-\frac{2}{3}\left[(9-y)^{3/2}\right]_3^9\right]$$

$$=\frac{4}{3}\left[12\sqrt{12}-27+6\sqrt{6}-27-\left(0-6\sqrt{6}\right)\right]$$

$$=\frac{4}{3}\left[24\sqrt{3}+12\sqrt{6}-54\right]$$

$$=8\left(4\sqrt{3}+2\sqrt{6}-9\right)$$

10. **Ans. (62)**

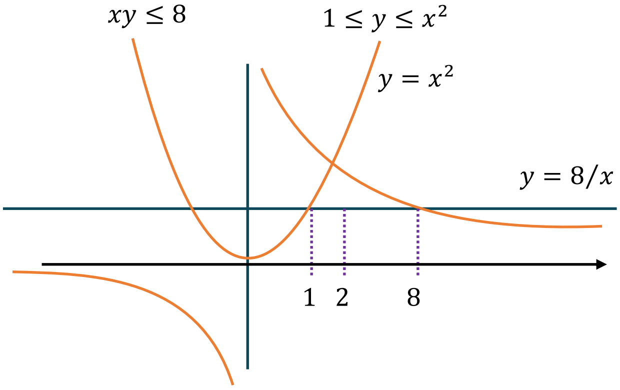
$$A=\int_{-1}^{0}(x^2-3x)dx+\int_0^2(3x-x^2)\,dx$$

$$\Rightarrow\ A=\left.\frac{x^3}{3}-\frac{3x^2}{2}\right|_{-1}^{0}+\left.\frac{3x^2}{2}-\frac{x^3}{3}\right|_0^2$$

$$\Rightarrow\ A=\frac{11}{6}+\frac{10}{3}=\frac{31}{6}$$

$\therefore$ $12A = 62$

11. **Ans. (2)**



$$\text{Area} = \int_1^2\left(x^2-1\right)dx+\int_2^8\left(\frac{8}{x}-1\right)dx$$

$$=\left(\frac{x^3}{3}\right)_1^2+8\left(\ell n\,x\right)_2^8-\left(x\right)_1^8$$

$$=\frac{7}{3}+8\left(2\ell n2\right)-7$$

$$= 16\ \ell n2 - \frac{14}{3}$$

12. **Ans. (36)**

$y^2 - 2y = -x$

$\Rightarrow y^2 - 2y + 1 = -x + 1$

$(y - 1)^2 = -(x - 1)$

$y = -x$

Points of intersection

$x^2 + 2x = -x$

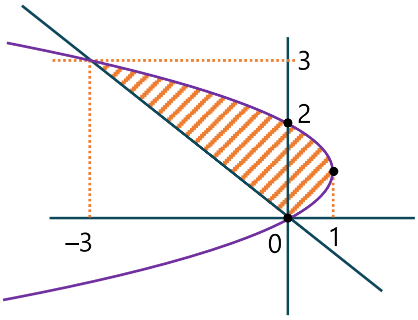
$x^2 + 3x = 0$

$x = 0, -3$

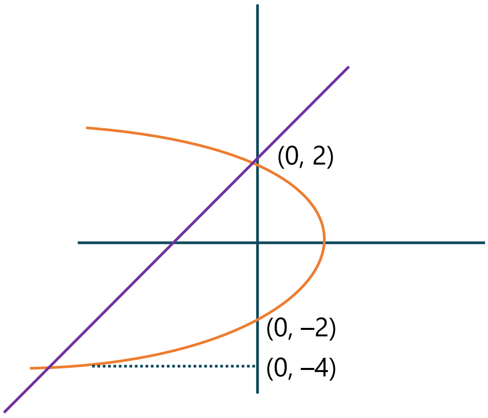
$$A = \int_0^3 (-y^2 + 2y + y)dy$$

$$= \frac{3y^2}{2} - \frac{y^3}{3}\Big|_0^3 = \frac{9}{2}$$

$8A = 36$



13. **Ans. (3)**



$y^2 + 4x = 4$

$y^2 = -4(x - 1)$

$$A = \int_{-4}^{2} \left(\frac{4 - y^2}{4} - \frac{y - 2}{2}\right) dy = 9$$

14. **Ans. (2)**

$16(x^2 + 4x) - (y^2 - 4y) + 44 = 0$

$16(x + 2)^2 - 64 - (y - 2)^2 + 4 + 44 = 0$

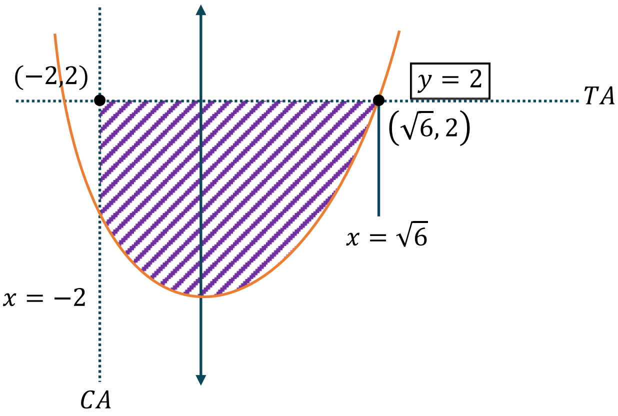
$16(x + 2)^2 - (y - 2)^2 = 16$

$$\frac{(x+2)^2}{1} - \frac{(y-2)^2}{16} = 1$$

$$A = \int_{-2}^{\sqrt{6}} \left(2 - \left(x^2 - 4\right)\right) dx$$

$$A = \int_{-2}^{\sqrt{6}} \left(6 - x^2\right) dx = \left(6x - \frac{x^3}{3}\right)_{-2}^{\sqrt{6}}$$

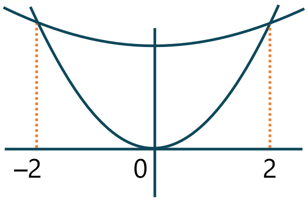
$$A = \left(6\sqrt{6} - \frac{6\sqrt{6}}{3}\right) - \left(-12 + \frac{8}{3}\right)$$



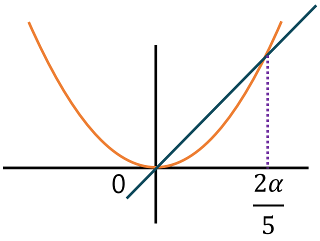
$$A = \frac{12\sqrt{6}}{3}+\frac{28}{3}$$

$$A = 4\sqrt{6}+\frac{28}{3}$$

15. **Ans. (600)**



Abscissa of point of intersection of $2y=5x^2$ and $y = x^2 + 6$ is ± 2



$$\text{Area} = 2\int_0^2\left(x^2+6-\frac{5x^2}{2}\right)dx=\int_0^{\frac{2\alpha}{5}}\left(\alpha x-\frac{5x^2}{2}\right)dx$$

$$\Rightarrow\int_0^{\frac{2\alpha}{5}}\left(\alpha x-\frac{5x^2}{2}\right)dx=16$$

$$\Rightarrow\alpha^3=600$$

16. **Ans. (4)**

$$|\cos x-\sin x|\le y\le\sin x$$

Intersection point of $\cos x - \sin x = \sin x$

$$\Rightarrow\ \tan x=\frac{1}{2}$$

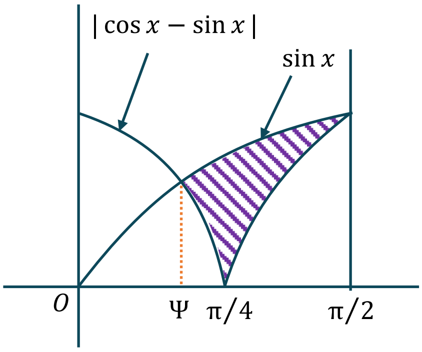
$$\text{Let } \psi=\tan^{-1}\frac{1}{2}$$

$$\text{So, } \tan\psi=\frac{1}{2},\ \sin\psi=\frac{1}{\sqrt{5}},\ \cos\psi=\frac{2}{\sqrt{5}}$$

$$\text{Area} = \int_{\psi}^{\pi/2}(\sin x-|\cos x-\sin x|)dx$$

$$=\int_{\psi}^{\pi/4}(\sin x-(\cos x-\sin x))dx+\int_{\pi/4}^{\pi/2}(\sin x-(\sin x-\cos x))dx$$

$$=\int_{\psi}^{\pi/4}(2\sin x-\cos x)dx+\int_{\pi/4}^{\pi/2}\cos x\ dx$$



$$=\left[-2\cos x-\sin x\right]_{\psi}^{\pi/4}+\left[\sin x\right]_{\pi/4}^{\pi/2}$$

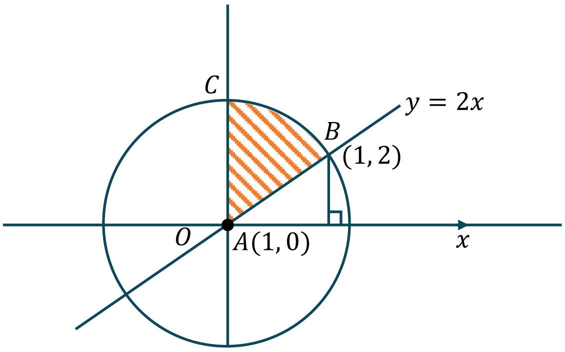
$$=-\sqrt{2}-\frac{1}{\sqrt{2}}+2\cos\psi+\sin\psi+\left(1-\frac{1}{\sqrt{2}}\right)$$

$$=-\sqrt{2}-\frac{1}{\sqrt{2}}+2\left(\frac{2}{\sqrt{5}}\right)+\left(\frac{1}{\sqrt{5}}\right)+1-\frac{1}{\sqrt{2}}$$

$$=\sqrt{5}-2\sqrt{2}+1$$

17. **Ans. (1)**

$$y^2+(x-1)^2=4$$

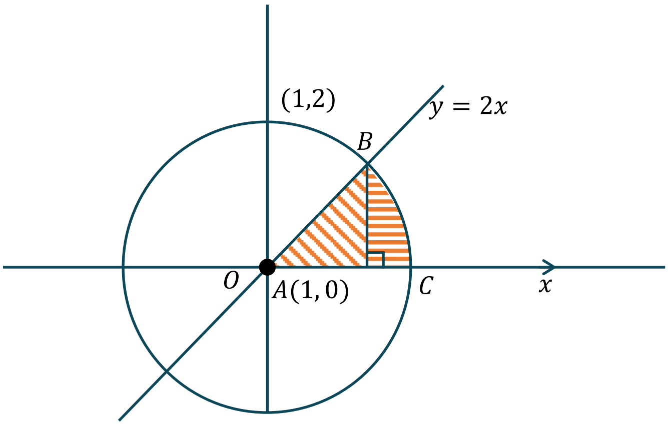


shaded portion = circular $(OABC)$

$-Ar(\Delta OAB)$

$$=\frac{\pi(4)}{4}-\frac{1}{2}(2)(1)$$

$$A=(\pi-1)$$

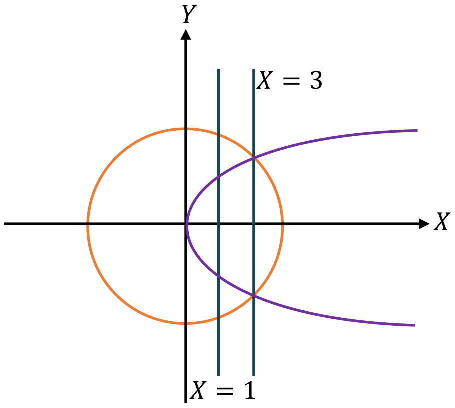


Area B = Ar $(\Delta AOB)+$ Area of arc of circle (ABC)

$$=\frac{1}{2}(1)(2)+\frac{\pi(2)^2}{4}=\pi+1$$

$$\frac{A}{B}=\frac{\pi-1}{\pi+1}$$

18. **Ans. (4)**



Area $2\int_{1}^{3} 2\sqrt{x}\,dx + 2\int_{3}^{\sqrt{21}} \sqrt{21-x^2dx}$

$$\Delta = \frac{8}{3}\left(3\sqrt{3}-1\right)+21\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)-6\sqrt{3}$$

$$\frac{1}{2}\left(\Delta-21\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)\right)=\frac{2\sqrt{3}-\frac{8}{3}}{2}$$

$$=\sqrt{3}-\frac{4}{3}$$

EXERCISE - JEE (Advanced) PYQ

1. **Ans. (B)**

$$y=\left|\cos x-\sin x\right|=\begin{cases}\cos x-\sin x, & x\in\left[0,\frac{\pi}{4}\right]\\ \sin x-\cos x, & x\in\left[\frac{\pi}{4},\frac{\pi}{2}\right]\end{cases}$$

So, required area

$$=\int_{0}^{\pi/4}\left(\sin x+\cos x\right)-\left(\cos x-\sin x\right)dx+\int_{\pi/4}^{\pi/2}\left(\sin x+\cos x\right)-\left(\sin x-\cos x\right)\cdot dx$$

$$=\int_{0}^{\pi/4}2\sin x\,dx+\int_{\pi/4}^{\pi/2}2\cos x\cdot dx=2\left(-\cos x\right)_{0}^{\pi/4}+2\left(\sin x\right)_{\pi/4}^{\pi/2}$$

$$=-2\left(\frac{1}{\sqrt{2}}-1\right)+2\left(1-\frac{1}{\sqrt{2}}\right)=2\left(2-\sqrt{2}\right)=2\sqrt{2}\left(\sqrt{2}-1\right)$$

2. **Ans. (3)**

$$F'(x)=2\cos^2\left(x^2+\frac{\pi}{6}\right)2x-2\cos^2 x$$

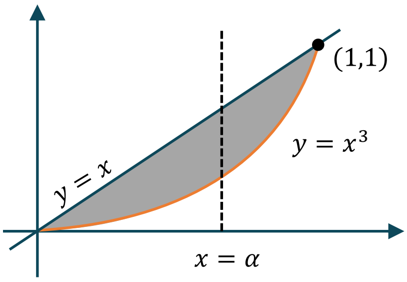
$$F'(a)+2=4a\cos^2\left(a^2+\frac{\pi}{6}\right)-2\cos^2 a+2=\int_0^a f(x)\cdot dx$$

On differentiation we get

$$f(a)=(4a)(2)\cos\left(a^2+\frac{\pi}{6}\right)\left(-\sin\left(a^2+\frac{\pi}{6}\right)\right)(2a)+4\cos^2\left(a^2+\frac{\pi}{6}\right)+4\cos a\sin a$$

$$\Rightarrow f(0)=4\cos^2\frac{\pi}{6}=4\cdot\frac{3}{4}=3 \text{ Ans.}$$

3. **Ans. (A,D)**



$$\frac{1}{2}\int_0^1\left(x-x^3\right)dx=\int_0^{\alpha}\left(x-x^3\right)dx$$

$$\Rightarrow \frac{1}{2}\left(\frac{x^2}{2}-\frac{x^4}{4}\right)_0^1=\left(\frac{x^2}{2}-\frac{x^4}{4}\right)_0^{\alpha}$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{2}-\frac{1}{4}\right)=\left(\frac{\alpha^2}{2}-\frac{\alpha^4}{4}\right)\Rightarrow \frac{1}{8}=\frac{\alpha^2}{2}-\frac{\alpha^4}{4}$$

$\Rightarrow 2\alpha^4 - 4\alpha^2 + 1 = 0 \Rightarrow$ Option (D)

Let $f(\alpha) = 2\alpha^4 - 4\alpha^2 + 1$

$$f\left(\frac{1}{2}\right)=2\cdot\frac{1}{16}-4\times\frac{1}{4}+1=\frac{1}{8}$$

$f(1) = -1$ & $f(0) = 1$

so, $\alpha\in\left(\frac{1}{2},1\right)$

4. **Ans. (B,C)**

$$f\left(x\right)=1-2x+e^x\cdot\int\limits_0^x e^{-t}f\left(t\right)dt$$

$$\Rightarrow f\left(x\right)\cdot e^{-x}=\left(1-2x\right)e^{-x}+\int\limits_0^x e^{-t}f\left(t\right)dt$$

Differentiate and simplify :

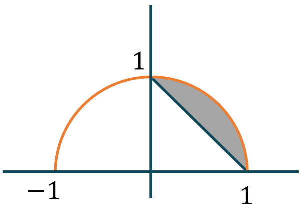
$f'(x) - f(x) = -(1 - 2x) - 2 + f(x)$

$\Rightarrow f'(x) = 2f(x) + 2x - 3$

If $y = f(x), f'(x) = dy/dx$

$$\Rightarrow \frac{dy}{dx}=2y+2x-3\Rightarrow \frac{dy}{dx}=2\left(x+y\right)-3$$

Put $x+y=t\Rightarrow 1+\frac{dy}{dx}=\frac{dt}{dx}$



$$\Rightarrow \frac{dt}{dx}-1=2t-3\Rightarrow \frac{dt}{dx}=2\left(t-1\right)\Rightarrow \frac{dt}{t-1}=2dx\Rightarrow \ell n\left(t-1\right)=2x+c$$

$\Rightarrow \ell n\ (x + y - 1) = 2x + c$

$\Rightarrow x + y - 1 = e^{2x+c} = e^{2x}\cdot e^{c} = k\cdot e^{2x}$

$\Rightarrow y = k\cdot e^{2x} - x + 1$ ...(1)

$$\because f(x) = 1 - 2x + e^x\ .\int\limits_0^x e^{-t}f\left(t\right)dt$$

At $x = 0$: $f(0) = 1 - 0 + 0 = 1 \Rightarrow$ At $x = 0, y = 1$

Put in (1) : $1 = k - 0 + 1 \Rightarrow k = 0$

$\therefore y = 1 - x = f(x) \Rightarrow$ B is correct.

Also, Region : $f\left(x\right)\le y\le\sqrt{1-x^2}$

$$A=\frac{\pi\left(1\right)^2}{4}-\frac{1}{2}\cdot1\cdot1$$

$$=\frac{\pi-2}{4}\text{sq.units}$$

$\Rightarrow$ (C) is correct

5. **Ans. (4)**

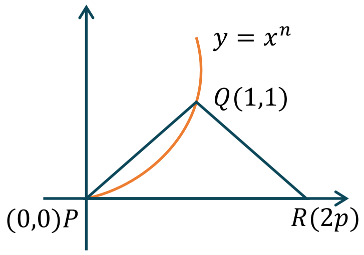
Area of $\Delta PQR = \frac{1}{2} \times 2 \times 1 = 1$

So, $\int_0^1 \left(x - x^n\right)dx = \frac{3}{10}$

$\left(\frac{x^2}{2} - \frac{x^{n+1}}{n+1}\right)_0^1 = \frac{3}{10}$

$\Rightarrow \frac{1}{2} - \frac{1}{n+1} = \frac{3}{10} \Rightarrow \frac{1}{n+1} = \frac{1}{2} - \frac{3}{10} = \frac{1}{5}$

$\Rightarrow n = 4$ Ans.



6. **Ans. (B)**

Required area

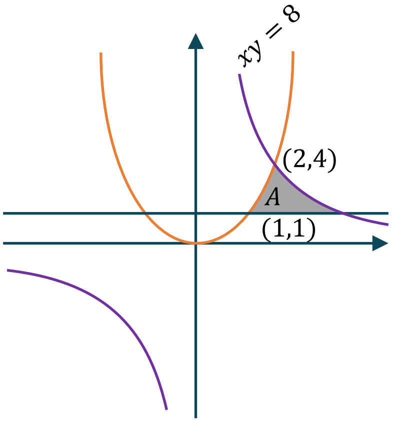
$A = \int_1^4 \left(\frac{8}{y} - \sqrt{y}\right)dy$

$= \left(8\ell ny - \frac{2}{3}y^{3/2}\right)_1^4$

$= \left(8\ell n4 - \frac{2}{3}(8)\right) - \left(0 - \frac{2}{3}\right)$

$= 8\ell n4 - \frac{16}{3} + \frac{2}{3}$

$= 8\ell n4 - \frac{14}{3} = 16\ell n2 - \frac{14}{3}$



7. **Ans. (A)**

Here,

$$\mathrm{f}(x) = \begin{cases} 0 & x \le 1 \\ e^{x-1} - e^{1-x} & x \ge 1 \end{cases}$$

& $g(x) = \frac{1}{2}\left(e^{x-1} + e^{1-x}\right)$

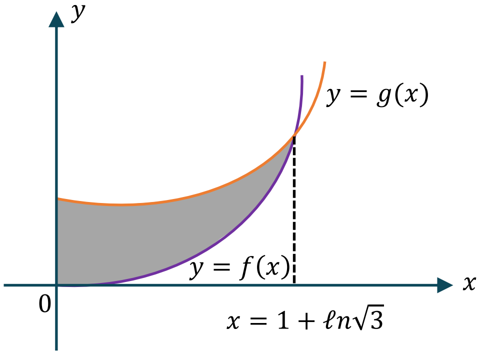
Solve $f(x)$ & $g(x) \Rightarrow x = 1 + \ell n\sqrt{3}$

So bounded area $\int_0^1 \frac{1}{2}\left(e^{x-1} + e^{1-x}\right)dx$

$+\int_1^{1+\ell n\sqrt{3}} \frac{1}{2}\left(e^{x-1} + e^{1-x}\right) - \left(e^{x-1} + e^{1-x}\right)dx$

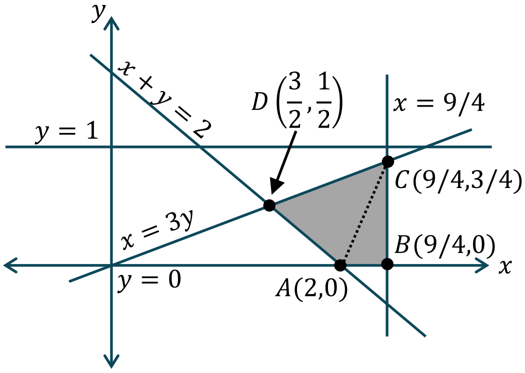
$= \frac{1}{2}\left[e^{x-1} - e^{1-x}\right]_0^1 + \left[-\frac{1}{2}e^{x-1} - \frac{3}{2}e^{1-x}\right]_1^{1+\ell n\sqrt{3}}$

$= \frac{1}{2}\left[e - \frac{1}{e}\right] + \left[\left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right) + 2\right] = 2 - \sqrt{3} + \frac{1}{2}\left(e - \frac{1}{e}\right)$



8. **Ans. (A)**

Rough sketch of required region is



∴ Required area is

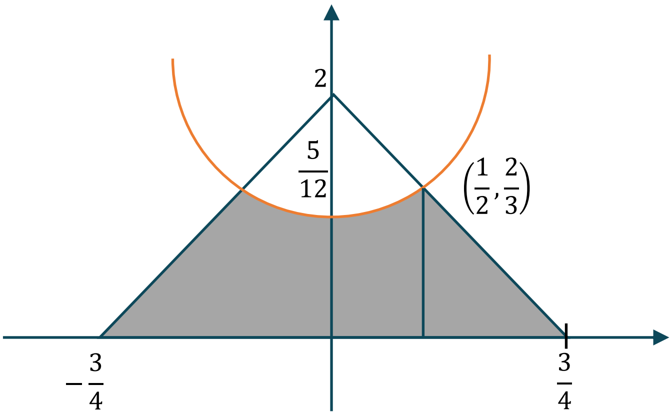
Area of ΔACD + Area of ΔABC

$$\left|\frac{1}{2}\begin{vmatrix}2 & 0 & 1\\ 9/4 & 3/4 & 1\\ 3/2 & 1/2 & 1\end{vmatrix}\right|+\frac{1}{2}\times\left(\frac{9}{4}-2\right)\times\frac{3}{4}$$

$$=\frac{1}{2}\left|2\left(\frac{3}{4}-\frac{1}{2}\right)+1\left(\frac{9}{8}-\frac{9}{8}\right)\right|+\frac{3}{32}$$

$$=\frac{1}{4}+\frac{3}{32}=\frac{11}{32}\text{sq.units}$$

9. **Ans. (6)**



$$9\alpha=9\cdot2\left[\int_0^{\frac{1}{2}}\left(x^2+\frac{5}{12}\right)dx+\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{1}{4}\right]=18\left(\frac{1}{24}+\frac{5}{24}+\frac{2}{24}\right)=6$$

10. **Ans. (B,C,D)**

$$f(x)=\frac{x^3}{3}-x^2+\frac{5x}{9}+\frac{17}{36}$$

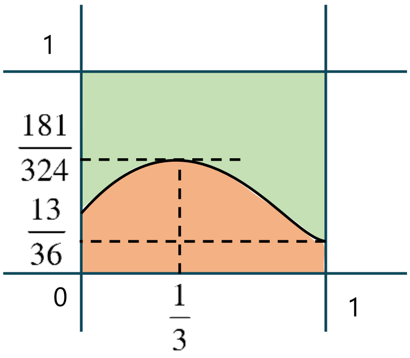
$$f'(x)=x^2-2x+\frac{5}{9}$$

$f'(x) = 0$ at $x = \frac{1}{3}$ in [0, 1]

A_R = Area of Red region

A_G = Area of Green region

$$A_R=\int_0^1 f(x)dx=\frac{1}{2}$$



Total area = 1

$\Rightarrow A_G = \frac{1}{2}$

$$\int_0^1 f(x)dx=\frac{1}{2}$$

$A_G = A_R$

$f(0)=\frac{17}{36}$

$f(1)=\frac{13}{36}\approx 0.36$

$f\left(\frac{1}{3}\right)=\frac{181}{324}\approx 0.558$

(A) Correct when $h = \frac{3}{4}$ but $h\in\left[\frac{1}{4},\frac{2}{3}\right]$

$\Rightarrow$ (A) is incorrect

(B) Correct when $h=\frac{1}{4}$

$\Rightarrow$ (B) is correct

(C) When $h=\frac{181}{324}, A_R=\frac{1}{2}, A_G<\frac{1}{2}$

$h=\frac{13}{36}, A_R<\frac{1}{2}, A_G=\frac{1}{2}$

$\Rightarrow A_R = A_G$ for some $h\in\left(\frac{13}{36},\frac{181}{324}\right)$

$\Rightarrow$ (C) is correct

(D) Option (D) is remaining coloured part of option (C), hence option (D) is also correct.

11. **Ans. (8)**

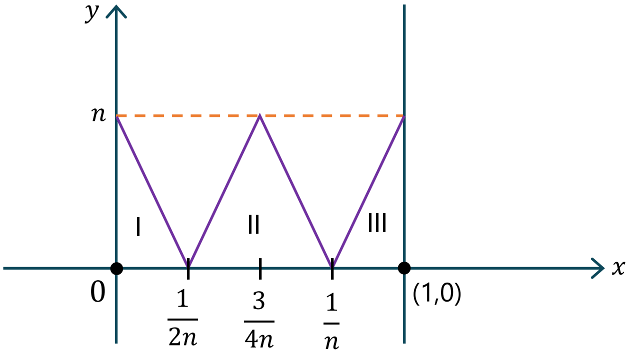
Area = Area of (I + II + III) = 4

$=\frac{1}{2}\times\frac{1}{2n}\times n+\frac{1}{2}\times\frac{1}{2n}\times n+\frac{1}{2}\left(1-\frac{1}{n}\right)\times n$

$=\frac{1}{4}+\frac{1}{4}+\frac{n-1}{2}=4$

$\boxed{n=8}$

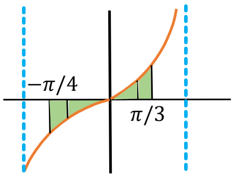
$\therefore$ maximum value of $f(x)$ = 8



JEE (Main) Practice Paper

SECTION–A

1. **Ans. (4)**



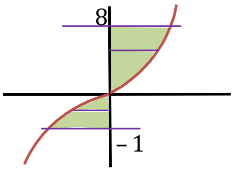
$$A = \int_{\frac{-\pi}{4}}^{0}(0-\tan x)dx + \int_{0}^{\frac{\pi}{3}}(\tan x - 0)\,dx$$

$$= -[\ell n\sec x]_{\frac{-\pi}{4}}^{0} + [\ell n\sec x]_{0}^{\frac{\pi}{3}}$$

$$= -\left[\ell n\sec(0) - \ell n\sec\left(-\frac{\pi}{4}\right)\right] + \left[\ell n\sec\left(\frac{\pi}{3}\right) - \ell n\sec(0)\right]$$

$$= -0 + \ell n\sqrt{2} + \ell n2 - 0 = \frac{3}{2}\ \ell n2$$

2. **Ans. (3)**

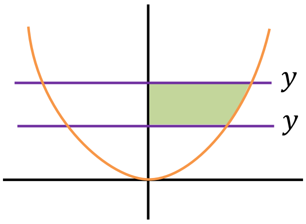


$$A = \int_{-1}^{0}\left(0 - y^{\frac{1}{3}}\right)dy + \int_{0}^{8} y^{\frac{1}{3}}\,dy$$

$$= -\left[\frac{y^{4/3}}{4/3}\right]_{-1}^{0} + \left[\frac{y^{4/3}}{4/3}\right]_{0}^{8} = -\frac{3}{4}(0-1) + \frac{3}{4}(16-0)$$

$$= \frac{3}{4}(16+1) = \frac{51}{4}$$

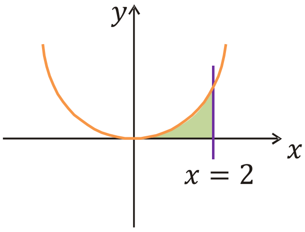
3. **Ans. (3)**



Graph of $y = 4x^2$

$$\text{Area} = \int_{1}^{4}\frac{1}{2}\sqrt{y}dy = \frac{7}{3}$$

4. **Ans. (2)**



$$y = \frac{x^2}{4}$$

$$A = 1\int_{0}^{2}\frac{x^2}{4}dx$$

$$A = \frac{1}{4}\left[\frac{x^3}{3}\right]_{0}^{2}$$

$$A = \frac{1}{4}\left(\frac{8}{3}\right)$$

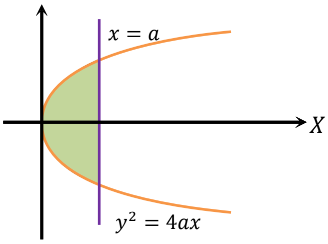
$$A = \frac{2}{3}$$

5. **Ans. (1)**

$$\int_0^2 (x^2-4x)dx = \left[\frac{x^3}{3}-\frac{4x^2}{2}\right]_0^2 = \frac{16}{3} \text{ sq. unit.}$$

6. **Ans. (2)**

Required area $=2\int_0^a \sqrt{4ax}dx$



$$= 4\sqrt{a}\times\frac{2}{3}[x^{3/2}]_0^a = \frac{8\sqrt{a}}{3}.a\sqrt{a} = \frac{8}{3}a^2$$

7. **Ans. (2)**

Required area is $\int_0^a ydx = \int_0^a xe^{x^2}dx$

We put $x^2 = t \Rightarrow dx = \frac{dt}{2x}$ as $x = 0 \Rightarrow t = 0$ and $x = a \Rightarrow t = a^2$, then it reduces to

$$\frac{1}{2}\int_0^{a^2} e^t dt = \frac{1}{2}[e^t]_0^{a^2} = \frac{e^{a^2}-1}{2} \text{ sq. unit}$$

8. **Ans. (2)**

Let the ordinate at $x = a$ divide the area into two equal parts

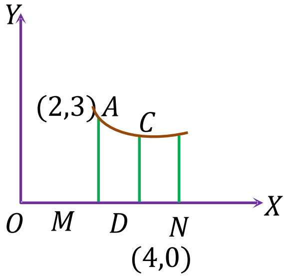
Area of $AMN = \int_2^4 \left(1+\frac{8}{x^2}\right) dx = \left[x-\frac{8}{x}\right]_2^4 = 4$

Area of ACDM $= \int_2^a \left(1+\frac{8}{x^2}\right) dx = 2$

$$\Rightarrow \left[x-\frac{8}{x}\right]_2^a = 2 \Rightarrow a-\frac{8}{a}-[2-4] = 2$$

$$\Rightarrow a-\frac{8}{a} = 0 \Rightarrow a^2-8 = 0$$

$$\Rightarrow a = 2\sqrt{2}$$



9. **Ans. (4)**

Required area $= \int_1^4 x^3 dx = \left[\frac{x^4}{4}\right]_1^4 = \frac{255}{4}$ sq. unit.

10. **Ans. (4)**

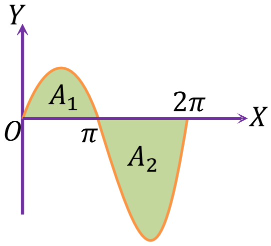
Required area is $A_1 + A_2 = \int_0^{\pi} y\,dx + \left|\int_{\pi}^{2\pi} y\,dx\right|$

$$A_1 + A_2 = \int_0^{\pi} x\sin x\,dx + \left|\int_{\pi}^{2\pi} x\sin x\,dx\right|$$

$$= [-x\cos x + \sin x]_0^{\pi} + |[-x\cos x + \sin x]_{\pi}^{2\pi}|$$

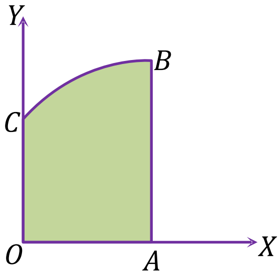
$$= \pi + 0 - [0+0] + |(-2\pi + 0) - (\pi + 0)|$$

$$= \pi + 3\pi = 4\pi$$



11. **Ans. (4)**

$$\text{Area} = \int_0^4 \sqrt{3x+4}\,dx = \left|\frac{(3x+4)^{3/2}}{3.(3/2)}\right|_0^4$$



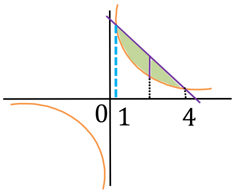
$$= \frac{2}{9} \times 56 = \frac{112}{9} sq.unit.$$

12. **Ans. (2)**

$$\int_0^2 2^{kx}dx = \frac{3}{\log 2} \Rightarrow 2^{2k} - 1 = 3k.$$

Now check from options, only (2) satisfies the above condition.

13. **Ans. (3)**

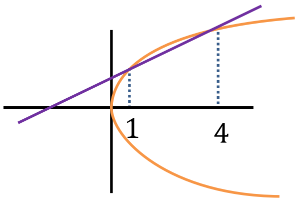


$$\text{A} = \int_1^4 \left(5 - x - \frac{4}{x}\right) dx$$

$$= \left[5x - \frac{x^2}{2} - 4\ell nx\right]_1^4 = 20 - 8 - 4\,\ell n4 - 5 + \tfrac{1}{2} + 0$$

$$= 7 + \tfrac{1}{2} - 4\,\ell n4 = \frac{15}{2} - 4\ell n4$$

14. **Ans. (1)**



Solving $x = 1,4$

From graph it is clear that required

$$\text{area } = \int_1^4 \left(2\sqrt{x} - \frac{1}{3}(2x+4)\right) dx = \frac{1}{3}$$

15. **Ans. (4)**

$$\int_1^b f(x)dx = \sqrt{b^2+1} - \sqrt{2} = \sqrt{b^2+1} - \sqrt{1+1} = [\sqrt{x^2+1}]_1^b$$

$$\therefore\ f(x) = \frac{d}{dx}\sqrt{x^2+1} = \frac{2x}{2\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}.$$

16. **Ans. (2)**

Given that, $\int_{\pi/4}^{\beta} f\ (x)dx = \beta\, sin\,\beta + \frac{\pi}{4} cos\,\beta + \sqrt{2}\beta$

Differentiating w.r.t. β, we get

$\therefore f(\beta) = sin\,\beta + \beta\, cos\,\beta - \frac{\pi}{4} sin\,\beta + \sqrt{2},$

Hence,$f\ \left(\frac{\pi}{2}\right) = \left(1 - \frac{\pi}{4} + \sqrt{2}\right)$

17. **Ans. (3)**

Given curve is $y(x-2) = 3x + 10 \Rightarrow y = \frac{3x+10}{x-2}$

Required area is $\int_3^4 ydx = \int_3^4 \frac{3x+10}{x-2}dx$

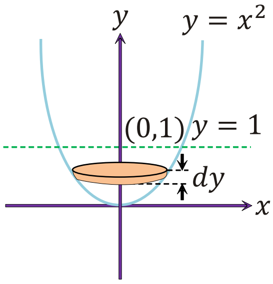
$=[3x + 16\, log(\,x-2)]_3^4 = 3 + 16\, log\ 2$ sq. unit.

18. **Ans. (2)**

Surface area of the solid formed by rotating the area enclosed between the curve $y = x^2$ and line $y = 1$ will be

$\int_0^1 2\pi x dy = 2\int_0^1 \pi\sqrt{y}dy$

$= \frac{4\pi}{3}[y^{3/2}]_0^1 = \frac{4\pi}{3}$



19. **Ans. (1)**

In both cases area will be same, hence ratio A : B is 1 : 1.

20. **Ans. (3)**

$f'\,(x) = x^2 - 2x = x\,(x-2) = 0$

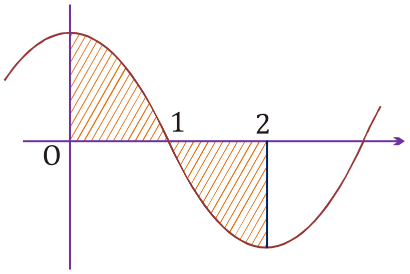
(note that $f(x)$ is monotonic in $(0, 2)$)

Hence for the minimum and $f(x)$

must cross the x – axis are $\frac{0+2}{2} = 1$

Hence $f(1) = \frac{1}{3} - 1 + a = 0$

$\Rightarrow a = \frac{2}{3}$



SECTION-B

1. **Ans. (1)**

$y = mx + 2 \Rightarrow x = \left(\frac{y-2}{m}\right)$ ...(1)

$x = 2y - y^2$ ...(2)

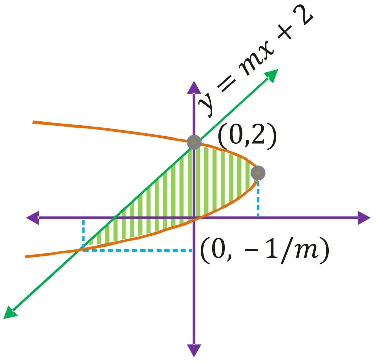
$(y-1)^2 =-(x-1)$ vertex $(1, 1)$

From (1) and (2) $\frac{y-2}{m} = 2y - y^2$

$\Rightarrow my^2 + (1-2m)y - 2 = 0\alpha\beta = -\frac{2}{m}$

$\alpha = 2,\ \beta = -\frac{1}{m}$

Area = $\int_{-1/m}^{2}\left[\left(2y-y^2\right) - \frac{(y-2)}{m}\right] dy$



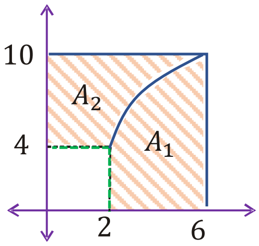
$$\frac{9}{2}=\left[\frac{2y^2}{2}-\frac{y^3}{3}-\frac{1}{m}\frac{y^2}{2}+\frac{2y}{m}\right]_{-1/m}^{2}$$

$$\Rightarrow \frac{9}{2}=\left(\frac{4}{3}+\frac{2}{m}+\frac{1}{6m^3}+\frac{1}{m^2}\right)$$

$m = 1$ satisfy the equation $\Rightarrow\ m = 1$

2. **Ans. (22)**

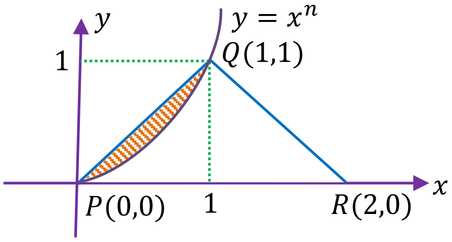
Now as per the above information.



$\int_4^{10} f^{-1}(x)dx = A_2,$ $A_1=\int_2^6 f(x)dx$

Now $A_2 + A_1\ = 60-8 = 52 \Rightarrow A_2 = 22$ (as $A_1 = 30$)

3. **Ans. (4)**



$$\int_0^1 (x-x^n)dx=\frac{3}{10}\left(\frac{1}{2}\times2\times1\right)\Rightarrow\left|\frac{x^2}{2}-\frac{x^{n+1}}{n+1}\right|_0^1=\frac{3}{10}\Rightarrow\frac{1}{2}-\frac{1}{n+1}=\frac{3}{10}$$

$$\Rightarrow \frac{1}{n+1}=\frac{1}{2}-\frac{3}{10}=\frac{1}{5}\ \Rightarrow\ n = 4$$

4. **Ans. (1)**

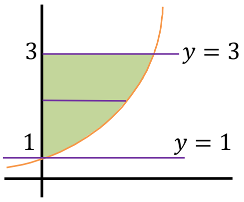
$A = \int_1^3 xdy$

$A = \int_1^3 \ell nydy$

$= [y\ell ny - y]_1^3$

$= 3\ell n3 - 3 - 0 + 1$

$= 3\ell n3 - 2$



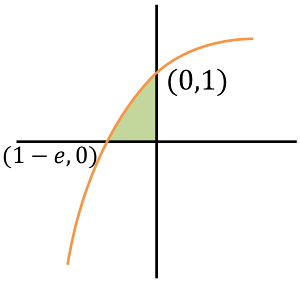
5. **Ans. (4)**

$y^2 = x$ and $2y = x \Rightarrow y^2 = 2y \Rightarrow y = 0,2$

$\therefore$ Required area= $\int_0^2 (y^2 - 2y)dy = \left(\frac{y^3}{3} - y^2\right)_0^2 = \frac{4}{3}$ sq. unit.

6. **Ans. (1)**

Required area = $\int_{1-e}^{0} log_e(x+e)\ dx$



$= \int_{1}^{e} log\ \ tdt = [t\,log\,t - t]_1^e = 1$ sq. unit, (Put $x + e = t$).

7. **Ans. (2)**

The two curves $y^2 = 4ax$ and $y = mx$ intersect at $\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$ and the area enclosed by the two curves is given by $\int_0^{4a/m^2}(\sqrt{4ax} - mx)\,dx$.

$$\therefore \int_0^{4a/m^2} (\sqrt{4ax} - mx)\,dx = \frac{a^2}{3} \Rightarrow \frac{8\,a^2}{3\,m^3} = \frac{a^2}{3} \Rightarrow m^3 = 8 \Rightarrow m = 2$$

8. **Ans. (1)**

Required area $= \int_0^{\pi/4}(sin\,2\,x + cos\,2\,x)dx$

$$= \left[-\frac{cos\,2\,x}{2} + \frac{sin\,2\,x}{2}\right]_0^{\pi/4}$$

$$= \frac{1}{2}\left[-cos\frac{\pi}{2} + sin\frac{\pi}{2} + cos\,0 - sin\,0\right] = 1 \text{ sq. unit.}$$

9. **Ans. (2)**

Given curve $y = a\sqrt{x} + bx$. This curve passes through $(1, 2), \therefore 2 = a + b$ ....(i)

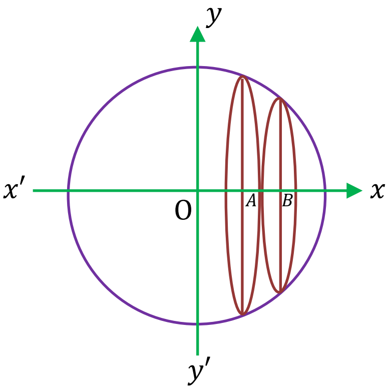
and area bounded by this curve and line $x = 4$ and $x -$ axis is 8 sq. unit, then $\int_0^4(a\sqrt{x} + bx)\,dx = 8$

$$\Rightarrow \frac{2a}{3}[x^{\frac{3}{2}}]_0^4 + \frac{b}{2}[x^2]_0^4 = 8, \frac{2a}{3}.8 + 8b = 8$$

$\Rightarrow\ 2a + 3b = 3$ ....(ii)

From equation (i) and (ii), we get $a = 3, b = -1$.

10. **Ans. (10)**



Equation of circle is, $x^2 + y^2 = 5^2$

$\therefore y^2 = 25 - x^2, 2y\,\frac{dy}{dx} = -2x, \therefore\ \frac{dy}{dx} = -\ \frac{x}{y}$

Curved surface of frustum $= 2\pi\int_2^3 yds = 2\pi\int_2^3 y.\frac{5}{y}dx$

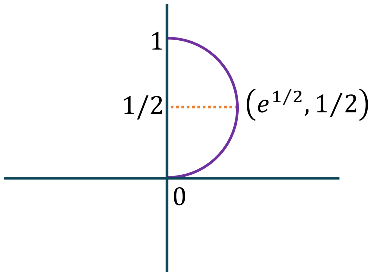
$$= 2\pi\int_2^3 5dx$$

$= 2\pi[5x]_2^3$

$= 10\,\pi cm^2$

JEE (Advanced) Practice Paper

1. **Ans. (A)**



Figure

Area = $\int_0^1 e^y \sin(\pi y)\ dy$

$= \frac{e^y}{1+\pi^2}(\sin\ \pi y - \pi\ \cos \pi y)\Big]_0^1$

$= \frac{(e+1)\pi}{1+\pi^2}$

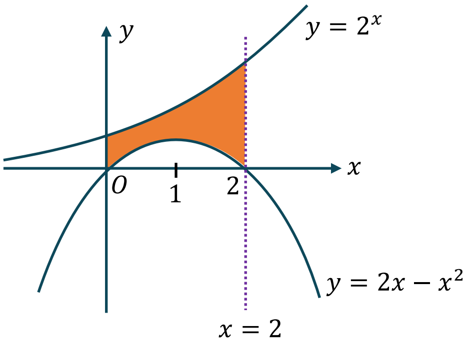
2. **Ans. (C)**

$0 < x < \frac{1}{2}$ $\qquad \{x\} = x$

$A = \int_0^{1/2} x\ .dx = \left(\frac{x^2}{2}\right)_0^{1/2} = \frac{1}{8}$

3. **Ans. (B)**

Area = $\int_0^2 (2^x - 2x + x^2)\ \ dx$



$= \left(\frac{2^x}{\ell n2} - x^2 + \frac{x^3}{3}\right)_0^2 = \frac{4}{\ell n2} - 4 + \frac{8}{3} - \frac{1}{\ell n2} = \frac{3}{\ell n2} - \frac{4}{3}$

4. **Ans. (B)**

Let A be area.

$A = \int_0^c (f(c) - f(x))dx + \int_c^a (f(x) - f(c))dx$

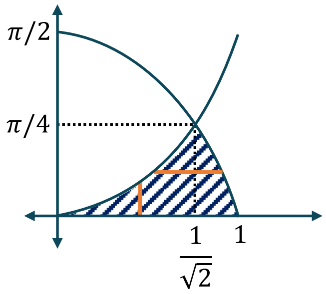
$\frac{dA}{dc} = (2c - a)\ \frac{\sec^2 c}{2\sqrt{\tan c}}$

$\frac{dA}{dc} = 0 \Rightarrow c = \frac{a}{2}$

At $c = \frac{a}{2}$, $\frac{dA}{dc}$ changes sign from negative to positive. Hence A is minimum when $c = \frac{a}{2}$

5. **Ans. (A, B, C, D)**

Shaded area can be expressed by any one of A, B, C & it equals $\sqrt{2}-1$. Hence (A), (B), (C) & (D) are correct option.



6. **Ans. (A,D)**

$$A = \int_0^x f(t)dt = (x^2 + x^3) \ \forall\ x \geq 0 \Rightarrow f(x) = 2x + 3x^2 \ \forall\ x \geq 0$$

as $f(x)$ is even

$f(x) = 3x^2 - 2x \ \forall\ x \geq 0$

now $f'(x) = 6x + 2 \ \ \forall\ x \geq 0$

$$\sum_{r=1}^{n} f'(r) = 6.\frac{n(n+1)}{2} + 2n \ = 3n^2 + 5\text{n}$$

7. **Ans. (A,B)**

$$\int_0^c [\tan^{-1} x]dx = \int_0^c [\cot^{-1} x]dx$$

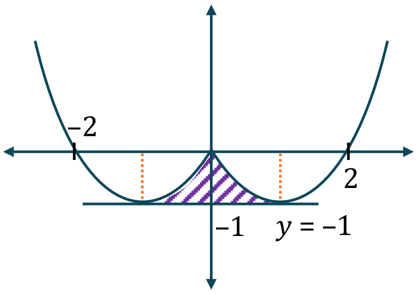
Now clearly $c > \tan 1$ as $[\tan^{-1}x] = 0 \ \ \forall\ x \in [0, \tan 1)$

Hence $\int_0^{\tan 1} 0dx + \int_{\tan 1}^{c} 1dx = \int_0^{\cot 1} dx + \int_{\cot 1}^{c} 0dx \ \Rightarrow 0 + c - \tan 1 = \cot 1 \Rightarrow \tan 1 + \cot 1 = 2 \text{ cosec} 2$

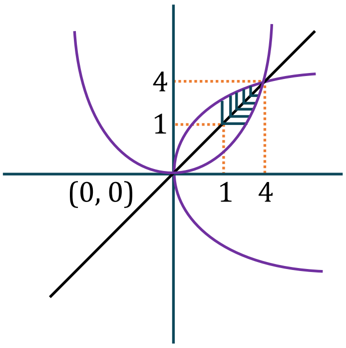
8. **Ans. (B,D)**

$$\text{Area } 2 - 2\int_0^1 (2x - x^2)dx = 2 - 2\left(x^2 - \frac{x^3}{3}\right)\Bigg|_0^1$$

$$= 2 - \frac{4}{3} = \frac{2}{3}$$



9. **Ans. (A)**



$1 \le x < 4$

$1 \le \sqrt{x} < 2$

$[\sqrt{x}] = 1$

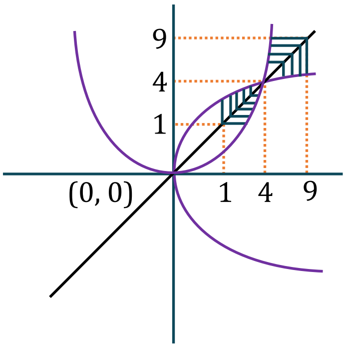
$x^2 \le 4y\ ;\ y^2 \le 4x$

$$\text{Area} = 2\int_1^4 \left(2\sqrt{x} - x\right)\ dx\ = 2\left(\frac{4}{3}x^{3/2} - \frac{x^2}{2}\right)\Bigg|_1^4 = \frac{11}{3}$$

10. **Ans. (C)**

For $2 \le \sqrt{x} \le \sqrt{8}\ ,\ 2 \le \sqrt{y} \le \sqrt{8}$

we have $x^2 \le 8y$ and $y^2 \le 8x$



for $\sqrt{8} \le \sqrt{x} < 3,\ \ \sqrt{8} \le \sqrt{y} < 3$

we have $x^2 \ge 8y$ and $y^2 \ge 8x$

$$\text{Area} = 2\int_4^8\left(\sqrt{8x} - x\right)\ dx + 2\int_8^9\left(x - \sqrt{8x}\right)dx + 2\int_8^9\left(x - \sqrt{8x}\right)\ dx = 65 - \frac{280\sqrt{2}}{3}$$

11. **Ans. (1)**

$y = mx + 2 \Rightarrow x = \left(\dfrac{y-2}{m}\right)$ ...(1)

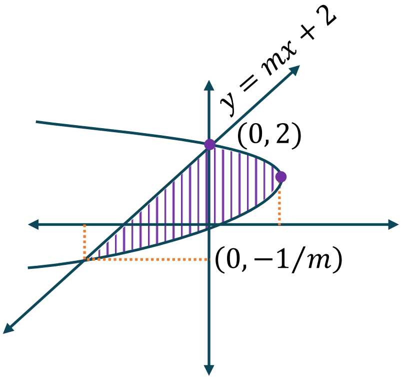
$x = 2y - y^2$ ...(2)

$(y - 1)^2 = -(x - 1)$ vertex $(1, 1)$

From (1) and (2) $\dfrac{y-2}{m} = 2y - y^2$

$\Rightarrow my^2 + (1 - 2m)y - 2 = 0 \qquad \alpha\beta = -\dfrac{2}{m}$

$\alpha = 2,\ \beta = -\dfrac{1}{m}$



Figure

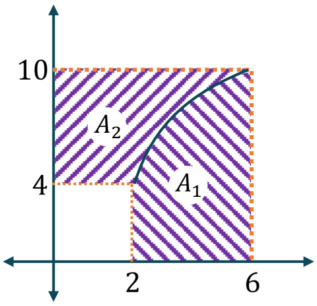
$$\text{Area} = \int_{-1/m}^{2}\left[\left(2y - y^2\right) - \frac{(y-2)}{m}\right]\ dy$$

$$\frac{9}{2} = \left[\frac{2y^2}{2} - \frac{y^3}{3} - \frac{1}{m}\frac{y^2}{2} + \frac{2y}{m}\right]_{-1/m}^{2} \Rightarrow \frac{9}{2} = \left(\frac{4}{3} + \frac{2}{m} + \frac{1}{6m^3} + \frac{1}{m^2}\right)$$

$m = 1$ satisfy the equation $\Rightarrow m = 1$

12. **Ans. (22)**

Now as per the above information.



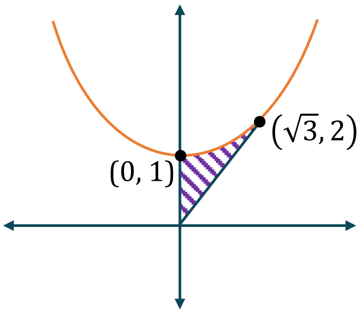
$$\int_4^{10} f^{-1}(x)dx = A_2,\ A_1 = \int_2^6 f(x)dx$$

Now $A_2 + A_1 = 60 - 8 = 52 \Rightarrow A_2 = 22$ (as $A_1 = 30$)

13. **Ans. (55)**

$x = \tan t$ & $y = \sec t$ $\left(t \in \left(0, \frac{\pi}{2}\right)\right) \Rightarrow y^2 - x^2 = 1$ $(x > 0, y > 1)$

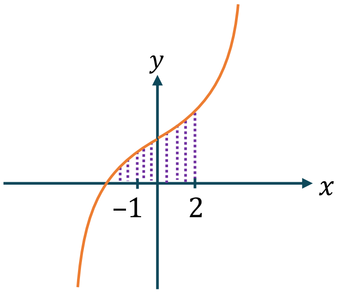
solving $y = \frac{2x}{\sqrt{3}}$ and $(\tan t, \sec t)$, we get $t = \pi/3$



Area = $\int_0^1 \frac{\sqrt{3}y}{2}dt + \int_0^2\left(\frac{\sqrt{3}y}{2} - \sqrt{y^2-1}\right)dy = \frac{\sqrt{3}}{4}.y^2\Big|_0^2 - \left(\frac{y}{2}\sqrt{y^2-1} - \frac{1}{2}\ell n(y+\sqrt{y^2-1})\right)\Big|_1^2$

$= \sqrt{3} - \left\{\sqrt{3} - \frac{1}{2}\ell n(2+\sqrt{3})\right\} = \frac{1}{4}\ell n(7+\sqrt{48}) \Rightarrow a + b = 55$

14. **Ans. (12.75)**



Graph of $y = x^3 + 3$

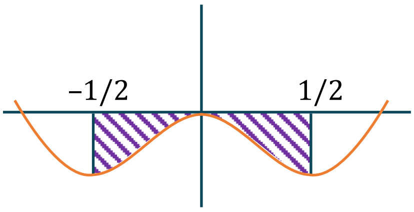
Area = $\int_{-1}^{2}(x^3+3)dx = \left(\frac{16}{4}+3.2\right) - \left(\frac{1}{4}+3(-1)\right) = \frac{51}{4}$

15. **Ans. (7)**

$$\frac{dy}{dx} = 8x^3 - 2x.\ \frac{dy}{dx} = 0$$

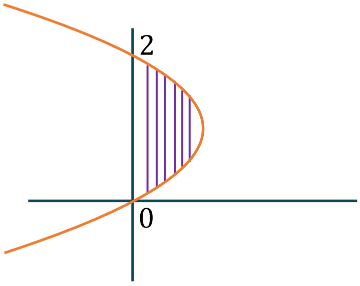
$$\Rightarrow (4x^2 - 1)\,x = 0 \Rightarrow x = -\frac{1}{2}, 0, \frac{1}{2}$$

$$\text{Required area} = -2\int_0^{1/2}(2x^4 - x^2)\ dx = \frac{7}{120}$$



Figure

16. **Ans. (1.33)**



Graph of $x = 2y - y^3$

$$\text{Area} = \int_0^2 (2y - y^3)dy = \frac{4}{3}$$

17. **Ans. (4)**

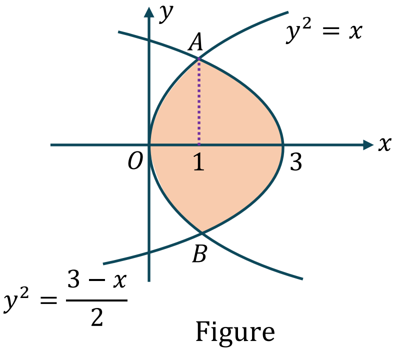
$y^2 = x$

$y^2 = \frac{3-x}{2}$ for A and B

$x = \frac{3-x}{2}$ $x = 1,\ y = \pm 1$

$A(1, 1)$ and $B(1, -1)$

$$A = \int_{-1}^{1}(3 - 2y^2 - y^2)\ dy = 2\left(3y - \frac{3y^3}{3}\right)_0^1 = 4$$



Figure

18. **Ans. (5.33)**

$x^2 + 2x - 4ky + 3 = 0$

$2x + 2 - 4k\frac{dy}{dx} = 0$

put $x = 3$, $\frac{dy}{dx} = -2$

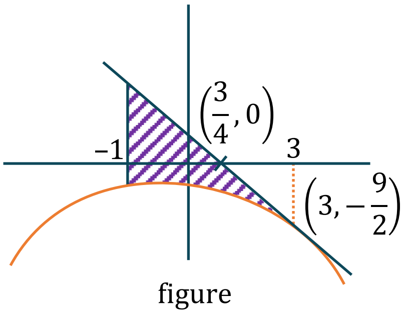
$6 + 2 + 8k = 0$

$k = -1$

$y = -\frac{1}{4}(x^2 + 2x + 3)$

Tangent is $4x + 2y - 3 = 0$

$$\text{Area} = \int_{-1}^{3}\left(\left(\frac{3-4x}{2}\right) - \left(-\frac{1}{4}(x^2 + 2x + 3)\right)\right)\ dx = \frac{16}{3}$$



figure

Important Notes