Vector Algebra

SOLUTIONS

EXERCISE - O

1. **Ans. (D)**

$\overrightarrow{AB}=4\hat{j}, \overrightarrow{BC}=3\hat{i}, \overrightarrow{CD}=-4\hat{j}, \overrightarrow{DA}=-3\hat{i}$

$\Rightarrow \overrightarrow{AB}$ II^{rl} $\overrightarrow{CD}$ and $\overrightarrow{BC}$ II^{rl} $\overrightarrow{DA}$, $\overrightarrow{AB} \perp^{r} \overrightarrow{BC}$

$\Rightarrow |\overrightarrow{AB}| \neq |\overrightarrow{BC}|$

$\Rightarrow ABCD$ is rectangle which is cyclic.

2. **Ans. (B)**

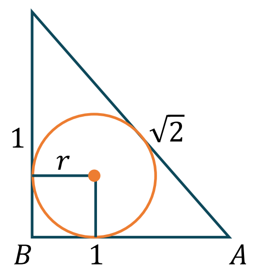
$AB = 1, BC = 1, AC = \sqrt{2}$

$\Rightarrow \Delta ABC$ is right angled

$\frac{r}{R}=\frac{\Delta / S}{abc/4\Delta}=\frac{4\Delta^2}{S.(abc)}$

$=\frac{8\Delta^2}{(a+b+c)(abc)}=\frac{8.\frac{1}{4}}{(2+\sqrt{2})\sqrt{2}}=\sqrt{2}-1$

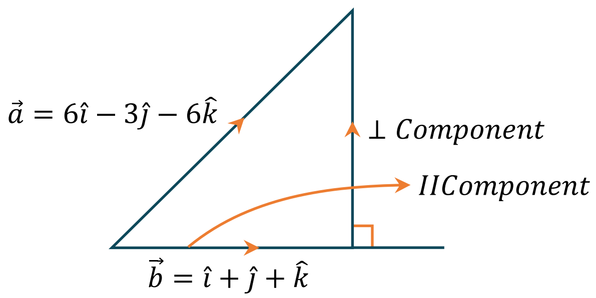
$=\cot\frac{3\pi}{8}$



3. **Ans. (A)**

The vector component $\vec{a}$ along $\vec{b}=(\vec{a}\cdot\hat{b})\hat{b}$

parallel component



$=\left[(6\hat{i}-3\hat{j}-6\hat{k})\cdot\frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}\right]\left(\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}\right)$

$=\left(\frac{6-3-6}{3}\right)(\hat{i}+\hat{j}+\hat{k})$

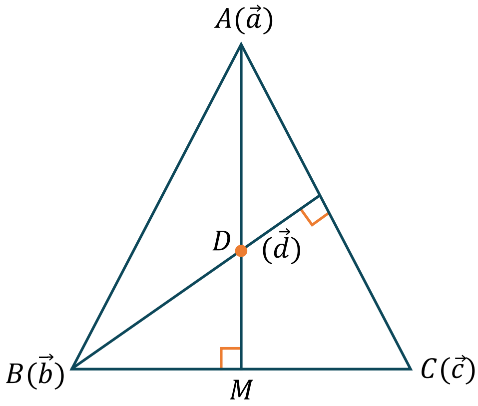
$=-(\hat{i}+\hat{j}+\hat{k})$

perpendicular component = $\vec{a}$ – (parallel component) (By triangle rule)

perpendicular component $=(6\hat{i}-3\hat{j}-6\hat{k})+\hat{i}+\hat{j}+\hat{k} = \boxed{7\hat{i}-2\hat{j}-5\hat{k}}$

4. **Ans. (C)**

$(\vec{a}-\vec{d})\cdot(\vec{b}-\vec{c})=0$



So $\overrightarrow{AD}$ and $\overrightarrow{BC}$ are mutually perpendicular and $(\vec{b}-\vec{d})\cdot(\vec{c}-\vec{a})=0$

So $\overrightarrow{BD}$ and $\overrightarrow{AC}$ is also mutually perpendicular

So D is orthocentre of ΔABC

5. **Ans. (D)**

$|\hat{a}+\hat{b}+\hat{c}|^2=1$

$|\hat{a}|^2+|\hat{b}|^2+|\hat{c}|^2+2(\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{c}+\hat{c}\cdot\hat{a})=1$

$1+1+1+2\ (\cos\theta_1+\cos\theta_2+\cos\theta_3)=1$

$\boxed{\cos\theta_1+\cos\theta_2+\cos\theta_3=-1}$

6. **Ans. (A)**

for point of intersection

$17\hat{i}-9\hat{j}+9\hat{k}+\lambda(3\hat{i}+\hat{j}+5\hat{k})=15\hat{i}-8\hat{j}-\hat{k}+\mu(4\hat{i}+3\hat{j})$, $(2+3\lambda-4\mu)\hat{i}+(-1+\lambda-3\mu)\hat{j}+(10+5\lambda)\hat{k}=\vec{0}$

$10+5\lambda=0$

$\boxed{\lambda=-2}$

$2+3\lambda-4\mu=0$

$\boxed{\mu=-1}$

$-1+\lambda-3\mu=0$ for $\lambda=-2$ and $\mu=-1$

$\vec{r}=15\hat{i}-8\hat{j}-\hat{k}+\mu(4\hat{i}+3\hat{j})$ for $\mu=-1$

for $\mu=-1$

point of intersection $\equiv(11,-11,-1)$

clearly lines are not skew because they are intersecting.

angle between line θ (Let)

$(3\hat{i}+\hat{j}+5\hat{k})\cdot(4\hat{i}+3\hat{j})=\sqrt{35}\cdot5\cos\theta$

$12+3=5\sqrt{35}\cos\theta$

$\cos\theta=\dfrac{3}{\sqrt{35}}$ $\quad\boxed{\theta=\cos^{-1}\left(\dfrac{3}{\sqrt{35}}\right)}$

7. Ans. (C)

Projection of $\vec{v}$ along $\vec{u} = \dfrac{\vec{v}.\vec{u}}{|\vec{u}|}$ ...(1)

Projection of $\vec{w}$ along $\vec{u} = \dfrac{\vec{w}.\vec{u}}{|\vec{u}|}$ ...(2)

$\because\ \vec{v} \perp \vec{w} \Rightarrow \vec{v}.\vec{w} = 0$ ...(3)

(1) = (2) (given in question)

$\Rightarrow \dfrac{\vec{v}\cdot\vec{u}}{|\vec{u}|} = \dfrac{\vec{w}\cdot\vec{u}}{|\vec{u}|} \Rightarrow \vec{v}.\vec{u} = \vec{w}.\vec{u}$ ...(4)

Now $|\vec{u}-\vec{v}+\vec{w}|^2 = \vec{u}^2+\vec{v}^2+\vec{w}^2-2\vec{u}.\vec{v}-2\vec{v}.\vec{w}+2\vec{u}.\vec{w}$ [from equtaion (3) & (4)]

= 1+ 4 + 9 – 0 = 14

$\boxed{|\vec{u}-\vec{v}+\vec{w}| = \sqrt{14}}$

8. Ans. (B)

$\vec{a} = \hat{i}+2\hat{j}+3\hat{k}$

$\vec{b} = 2\hat{i}-\hat{j}+\hat{k} \Rightarrow \vec{a}+\vec{b} = 3\hat{i}+\hat{j}+4\hat{k}$

$\vec{c} = 3\hat{i}+\hat{j}+4\hat{k}$

$\Rightarrow \vec{c} = \vec{a}+\vec{b} \Rightarrow$ vectors $\vec{a}, \vec{b}\ \&\ \vec{c}$ are coplanar

$\because\ \vec{a}+\vec{b}+\vec{c} \neq 0 \Rightarrow$ they can not form triangle

9. Ans. (A)

Volume = $[-c\vec{u}\ \ \vec{v}\ \ c\vec{w}] = 8$

= $-c^2\ [\vec{u}\ \ \vec{v}\ \ \vec{w}] = 8$

$$\Rightarrow -c^2 \begin{vmatrix} 2 & -1 & -1 \\ 1 & -1 & 2 \\ 1 & 0 & -1 \end{vmatrix} = 8$$

$(-c^2)$ [2(1 – 0) + 1(–1 – 2) – 1(0 + 1)] = 8

$2c^2 = 8 \Rightarrow \boxed{c = \pm 2}$

10. Ans. (C)

$\vec{a} \wedge \vec{b} = \dfrac{\pi}{6}\ \&\ (\vec{a}\times\vec{b}) \wedge \vec{c} = 0$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2 = \begin{vmatrix} a_1 & b_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$$

$= (\vec{a}\times\vec{b}\cdot\vec{c})^2$

$= \left(|a||b||c|\sin\dfrac{\pi}{6}\cos 0\right)^2$

$= \dfrac{1}{4}(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)$ $\because\ |c| = 1$

11. Ans. (C)

Altitude = projection of $\vec{c}$ on $\vec{A}\times\vec{B}$

$$= \left|\frac{(\vec{A}\times\vec{B})\cdot\vec{C}}{|A\times B|}\right|$$

$$\vec{A}\times\vec{B} = (\hat{i}+\hat{j}+\hat{k})\times(2\hat{i}+4\hat{j}-\hat{k})$$

$$= 4\hat{k}+\hat{j}-2\hat{k}-\hat{i}+2\hat{j}-4\hat{i}$$

$$= -5\hat{i}+3\hat{j}+2\hat{k}$$

Now $\dfrac{|(-5\hat{i}+3\hat{j}+2\hat{k}).(\hat{i}+\hat{j}+3\hat{k})|}{|-5\hat{i}+3\hat{j}+2\hat{k}|}$

$\therefore$ Altitude = $\dfrac{|-5+3+6|}{\sqrt{38}}$

$$= \frac{4}{\sqrt{38}}\times\frac{\sqrt{38}}{\sqrt{38}}$$

$$= \frac{4\times\sqrt{38}}{38}$$

$\therefore$ $\boxed{\dfrac{2\sqrt{38}}{19}}$

12. Ans. (A,C)

$\vec{a}\times\vec{b}=\vec{c}$ $\quad \vec{b}\times\vec{c}\times\vec{a}$

$\vec{a}\cdot\left(\vec{a}\times\vec{b}\right)=\vec{a}\cdot\vec{c}$ $\quad \vec{b}\cdot\left(\vec{b}\times\vec{c}\right)=\vec{b}\cdot\vec{a}$

$0=\vec{a}\cdot\vec{c}$ $\quad 0=\vec{b}\cdot\vec{a}$

$\vec{b}\cdot\left(\vec{a}\times\vec{b}\right)=\vec{b}\cdot\vec{c}$ $\quad \boxed{\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0}$

$\vec{a}\times\vec{b}=\vec{c}$

$\vec{c}.\left(\vec{a}\times\vec{b}\right)=\vec{c}\cdot\vec{c}$

$\left[\vec{c}\ \vec{a}\ \vec{b}\right]=|\vec{c}|^2$

$\boxed{\left[\vec{a}\ \vec{b}\ \vec{c}\right]=|\vec{c}|^2}$

$\vec{a}\times\vec{b}=\vec{c}$

$\vec{b}\times\vec{c}=\vec{a}\Rightarrow\left[\vec{a}\ \vec{b}\ \vec{c}\right]=|\vec{a}|^2$

$\vec{b}\times\left(\vec{a}\times\vec{b}\right)=\vec{a}$

$\left(\vec{b}\cdot\vec{b}\right)\vec{a}-\left(\vec{b}\cdot\vec{a}\right)\cdot\vec{b}=\vec{a}$

$|\vec{b}|^2\,\vec{a}=\vec{a}\Rightarrow|\vec{b}|^2=2\Rightarrow|\vec{b}|=1$

$\therefore\ \left(\vec{b}\times\vec{c}\right)\times\vec{b}=\vec{c}$

$\left(\vec{b}\cdot\vec{b}\right)\vec{c}=\left(\vec{b}\cdot\vec{c}\right)\vec{b}=\vec{c}$

$|\vec{a}|^2=|\vec{c}|^2\Rightarrow|\vec{a}|=|\vec{c}|$

$\therefore\ \vec{a}\times\vec{b}=\vec{c}$

$\left|\vec{a}\times\vec{b}\right|=|\vec{c}|$

$\left|\vec{a}\times\vec{b}\right|=|\vec{c}|$

$|\vec{a}|\left|\vec{b}\right|\ \sin\theta=|\vec{c}|\Rightarrow|\vec{a}|=|\vec{c}|$

$\therefore\ \vec{a},\vec{b},\vec{c}$ are orthogonal in pairs.

13. Ans. (B,C)

Let P be a plane containing four

vectors $\vec{A},\vec{B},\vec{C},\vec{D}$

$\vec{A}\times\vec{B}=$ is a vector perpendicular to plane P

$\vec{C}\times\vec{D}=$ is again a vector perpendicular to plane $\vec{P}$

Similarly $\vec{A}\times\vec{C}$ and $\vec{B}\times\vec{D}$

$\vec{A}\times\vec{C}=\lambda\vec{C}\times\vec{D}\ \therefore\left(\vec{A}\times\vec{B}\right)\times\left(\vec{C}\times\vec{D}\right)=0$

Hence option C is correct.

Also $\left(\vec{A}\times\vec{C}\right)\times\left(\vec{B}\times\vec{D}\right)=0$

But $\left(\vec{A}\times\vec{C}\right)\cdot\left(\vec{B}\times\vec{D}\right)\neq0$ $\{\vec{A},\vec{B},\vec{C},\vec{D}$ are Non-zero Non-collinear$\}$

Hence B is correct.

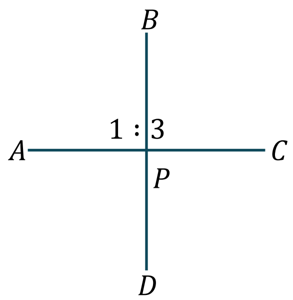
14. Ans. (A,C,D)

$3\vec{a}-2\vec{b}+\vec{c}-2\vec{d}=0$

$3\vec{a}-\vec{c}=2\vec{b}+2\vec{d}$

$\dfrac{3\vec{a}+1\cdot\vec{c}}{3+1}=\dfrac{2\vec{b}+2\vec{d}}{4}$

$\dfrac{3\vec{a}+1\cdot\vec{c}}{3+1}=\dfrac{\vec{b}+\vec{d}}{2}$



Thus we can say P is point of intersection of time AC and BD such that AC divides BD in ratio 1 : 1 and BD divides AC in ratio 1 : 3.

hence C is correct.

Also $x\vec{a}+y\vec{b}+g\vec{c}+w\vec{d}=0$ where $x+y+g+w=$

then $\vec{a},\vec{b},\vec{c},\vec{d}$ are representing four points A,B,C,D which are coplnanar and hence linearly dependent.

Thus A, C, D are correct.

15. Ans. (A,C)

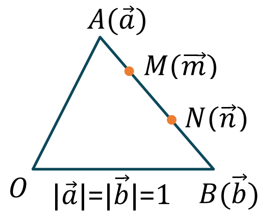
M and N are point of trisection of AB

$AM = MN = NB$

Thus $AM : MB = 1 : 2$

$\therefore \ \vec{m} = \dfrac{2\vec{a} + \vec{b}}{3}$

similarly, $\vec{n} = \dfrac{\vec{a} + 2\vec{b}}{3}$



triangle is equilateral

Now $\vec{m} = x\vec{a} + y\vec{b} \quad \therefore \quad x = \dfrac{2}{3} ; y = \dfrac{1}{3}$

$$\vec{m} \cdot \vec{n} = \left(\frac{2\vec{a} + \vec{b}}{3}\right) \cdot \left(\frac{\vec{a} + 2\vec{b}}{3}\right) = \frac{2|\vec{a}|^2 + 5\vec{a} \cdot \vec{b} + 2|\vec{b}|^2}{9}$$

$$\vec{m} \cdot \vec{n} = \frac{2(1) + 5|\vec{a}|\,|\vec{b}|\cos 60^\circ + 2}{9} = \frac{4 + \frac{5}{2}}{9} = \frac{13}{18}$$

16. Ans. (A,B,C)

$\vec{a}$ and $\vec{b}$ are two non-zero vectors and non-collinear

since we know that

$\vec{a} = (\vec{a} - \hat{i})\hat{i} + (\vec{a} \cdot \hat{j})\hat{j} + (\vec{a} \cdot \hat{k})\hat{k}$ ...(1)

Replace $\vec{a}$ by $\vec{a} \times \vec{b}$

$\vec{a} \times \vec{b} = ((\vec{a} \times \vec{b}) \cdot \hat{i})\hat{i} + ((\vec{a} \times \vec{b}) \cdot \hat{j})\hat{j} + ((\vec{a} \times \vec{b}) \cdot \hat{k})\hat{k}$

$\vec{a} \times \vec{b} = [\vec{a}\,\vec{b}\,\hat{i}]\hat{i} + [\vec{a}\,\vec{b}\,\hat{j}]\hat{j} + [\vec{a}\,\vec{b}\,\hat{k}]\hat{k}$ [Option A]

From (1) $\vec{a} \cdot \vec{b} = (\vec{a} \cdot \hat{i})(\vec{b} \cdot \hat{i}) + (\vec{a} \cdot \hat{j})(\vec{b} \cdot \hat{j}) + (\vec{a} \cdot \hat{k})(\vec{b} \cdot \hat{k})$

(Hence option B)

(C) $\vec{u} = \hat{a} - (\hat{a} \cdot \hat{b})\hat{b}$

$|\vec{u}|^2 = (\hat{a} - (\hat{a} \cdot \hat{b})\hat{b})^2$

$= |\hat{a}|^2 + |(\hat{a} \cdot \hat{b})\hat{b}|^2 - 2(\hat{a} \cdot \hat{b})^2$

$|\vec{u}|^2 = 1 + (\hat{a} \cdot \hat{b})^2 - 2(\hat{a} \cdot \hat{b})^2 = 1 - (\hat{a} \cdot \hat{b})^2$

Also $|\vec{v}|^2 = |\hat{a} \times \hat{b}|^2 = |\hat{a}|^2 |\hat{b}|^2 \sin^2\theta$

$= |\hat{a}|^2 |\hat{b}|^2 (1 - \cos^2\theta)$

$= |\hat{a}|^2 |\hat{b}|^2 - (\hat{a} \cdot \hat{b})^2$

$|\vec{v}|^2 = 1 - (\hat{a} \cdot \hat{b})^2$

Thus $|\vec{u}|^2 = |\vec{v}|^2 \quad \Rightarrow \quad |\vec{u}| = |\vec{v}|$

(D) $\vec{c} = \vec{a} \times (\vec{a} \times \vec{b}) = (\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b}$

$\vec{d} = \vec{b} \times (\vec{a} \times \vec{b}) = (\vec{b} \cdot \vec{b})\vec{a} - (\vec{b} \cdot \vec{a})\vec{b}$

$\vec{c} + \vec{d} \neq 0$

17. **Ans. (B,D)**

$\vec{a} = \hat{i} + 3\hat{j} + (\sin 2\alpha)\hat{k}$

$\vec{b} = (\tan\alpha)\,\hat{i} - \hat{j} + 2\sqrt{\sin\frac{\alpha}{2}}\,\hat{k}$

$\vec{c} = (\tan\alpha)\,\hat{i} + (\tan\theta)\hat{j} - 3\sqrt{\operatorname{cosec}\frac{\alpha}{2}}\,\hat{k}$

$\vec{b}\,.\,\vec{c} = 0$ and $\vec{a}\,.\,\hat{k} < 0$

$\vec{b}\,.\,\vec{c} = \tan^2\alpha - \tan\alpha - b = 0$

$(\tan\alpha - 3)\,(\tan\alpha + 2) = 0$

$\tan\alpha = 3, -2$

$\vec{a}\,.\,\hat{k} = \sin 2\alpha < 0$

$$\sin 2\alpha = \frac{2\tan\alpha}{1 + \tan^2\alpha} < 0$$

$\therefore\ \tan\alpha < 0 \quad \Rightarrow \quad \boxed{\tan\alpha = -2}$

$\alpha \in [0, 2\pi]$

$\Rightarrow\ \alpha = \pi - \tan^{-1} 2$ or $2\pi - \tan^{-1}2$

18. **Ans. (A,B,C,D)**

(A) $\vec{a} = -2\hat{i} + \hat{j} + 2\hat{k}$

$\vec{b} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

$$\cos\theta = \frac{\vec{a}\,.\,\vec{b}}{|\vec{a}|\,|\vec{b}|} = \frac{-6 - 2 + 12}{\sqrt{9}\,.\,\sqrt{49}} = \frac{4}{21}$$

$$\theta = \cos^{-1}\left(\frac{4}{21}\right)$$

(B) $\vec{r} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{r}\,.\,\hat{i} = a_1,\ \hat{i} \times \vec{r} = a_2\hat{k} - a_3\hat{j}$

$(\vec{r}\,.\,\hat{i}) = (\hat{i} \times \vec{r}) = a_1a_2\hat{k} - a_1a_3\hat{j}$

$\vec{r}\,.\,\hat{j} = a_2,\ \hat{j} \times \vec{r} = -a_1\hat{k} + a_3\hat{i}$

$(\vec{r}\,.\,\hat{j})\,(\hat{j} \times \vec{r}) = -a_1a_2\hat{k} + a_2a_3\hat{i}$

$\vec{r}\,.\,\hat{k} = a_3,\ \hat{k} \times \vec{r} = a_1\hat{j} - a_2\hat{i}$

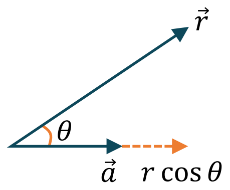
$(\vec{r}\,.\,\hat{k})\,(\hat{k} \times \vec{r}) = a_1a_2\hat{j} - a_2a_3\hat{i}$

$\therefore\ (\vec{r}\,.\,\hat{i})\,(\hat{i} \times \vec{r}) + (\vec{r}\,.\,\hat{j})\,(\hat{j} \times \vec{r}) + (\vec{r}\,.\,\hat{k})\,(\hat{k} \times \vec{r})$

$= a_1a_2\hat{k} - a_1a_3\hat{j} + a_1a_3\hat{j} - a_2a_3\hat{i} + a_1a_3\hat{j} - a_2a_3\hat{i}$

$= \vec{0}$

(C) $\vec{r}$ comp. mag. along $\vec{a}$ = rcosθ



$$= r \times \frac{(\vec{r}\ .\ \vec{a})}{|\vec{r}|\ |\vec{a}|}$$

$$= r \times \frac{\vec{r}\ .\ \vec{a}}{r\ .\ a}$$

$$= \frac{2 - 16 - 7}{\sqrt{9}} = -7$$

direction $\hat{a} = \dfrac{2\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{4 + 4 + 1}} = \dfrac{2}{3}\hat{i} + \dfrac{2}{3}\hat{j} + \dfrac{1}{3}\hat{k}$

$$(r\cos\theta)\hat{a} = \frac{1 - 14}{3}\hat{i} - \frac{14}{3}\hat{j} - \frac{7}{3}\hat{k}$$

(D) Let a cube with side length a be placed at origin with one vertex and one face along x – y plane
let vertex at origin be A(0, 0, 0) so opposite vertex will be B(a, a, a)
Let other vertex be C(a, 0, 0) opposite vertex will be D(0, a, a)
Direction ratios of the two lines are (a, a, a) and (–a, a, a)

$$\cos\theta = \frac{-a \times a + a \times a + a \times a}{\sqrt{3} \times \sqrt{3a^2}} = \frac{1}{3}$$

19. **Ans. (A,B)**

Let the required vector $\vec{r}$ be such that $\vec{r} = x_1\vec{a} + x_2\vec{b} + x_3\vec{a} \times \vec{b}$

We must have $\vec{r} = \vec{a} = \vec{r}\ .\ \vec{b} = \vec{r}\ .\ (\vec{a} \times \vec{b})$ (as $\vec{r}, \vec{a}, \vec{b}$ and $\vec{a} \times \vec{b}$ are unit vectors and $\vec{r}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{a} \times \vec{b}$)

Now $\vec{r}\ .\ \vec{a} = x_1\ .\ \vec{b} = x_2, \vec{r}\ .\ (\vec{a} \times \vec{b}) = x_3$

$\Rightarrow \quad \vec{r} = \lambda(\vec{a} + \vec{b} + (\vec{a} \times \vec{b}))$

Also $\vec{r}\ .\ \vec{r} = 1$

$\Rightarrow \quad \lambda^2(\vec{a} + \vec{b} + \vec{a} \times \vec{b})\ .\ (\vec{a} + \vec{b} + (\vec{a} \times \vec{b})) = 1$

$\Rightarrow \quad \lambda^2(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{a} \times \vec{b}|^2) = 1$

$\Rightarrow \quad \lambda^2 = \dfrac{1}{3}$

$\Rightarrow \quad \lambda = \pm\dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad \vec{r} = \pm\dfrac{1}{\sqrt{3}}(\vec{a} + \vec{b} + \vec{a} \times \vec{b})$

20. **Ans. (B,D)**

$\vec{\alpha} = \vec{p}.\vec{r}\vec{q} - \vec{p}.\vec{q}\vec{r}$ and $\vec{\beta} = \vec{p}.\vec{r}\vec{q} - \vec{q}.\vec{r}\vec{p}$

$\vec{\alpha} = \vec{\beta} \Rightarrow \vec{p}.\vec{q}\vec{r} = \vec{q}.\vec{r}\vec{p}$

21. **Ans. (A,B,D)**

It is possible $|\vec{a}\times\vec{b}|=1=|\vec{c}\times\vec{d}|$ & $\vec{a}\times\vec{b}\,||\,\vec{c}\times\vec{d}$

$\therefore\ \vec{a}\perp\vec{b},\ \vec{c}\perp\vec{d}$ & $(\vec{a}\times\vec{b})||(\vec{c}\times\vec{d})$

$\vec{a}\cdot(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\cdot\vec{c}\ =k(\vec{c}\times\vec{d})\cdot\vec{c}=0$

$\Rightarrow\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$

22. **Ans. (C)**

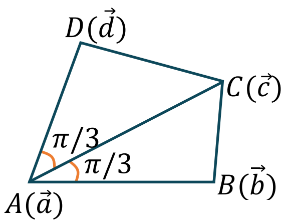
$15|\overline{AC}| = 3|\overline{AB}| = 5|\overline{AD}|$

$15c = 3b = 5d$

$b = 5c,\ d = 3c$

$\overrightarrow{BA} = -\vec{b}$

$\overrightarrow{CD} = \vec{d} - \vec{c}$



$$\cos\theta=\frac{\overrightarrow{BA}\,.\,\overrightarrow{CD}}{|\overrightarrow{BA}|\,|\overrightarrow{CD}|}\Rightarrow\frac{-\vec{b}\,.\,(\vec{d}-\vec{c})}{|-\vec{b}|\,|\vec{d}-\vec{c}|}$$

$$\frac{-\vec{b}\,.\,\vec{d}+\vec{b}\,.\,\vec{c}}{|\vec{b}|\,.\,\sqrt{|\vec{d}|^2+|\vec{c}|^2-2\vec{c}\,.\,\vec{d}}}$$

$$=\frac{-b\,.\,d(-1/2)+b\,.\,c\,.\,1/2}{b\sqrt{d^2+c^2-2c\,1/2d}}$$

$$=\frac{b(d+c)}{2b\sqrt{9c^2+c^2-3c^2}}$$

$$\cos\theta=\frac{3c+c}{2\sqrt{7c^2}}\ \Rightarrow\ \frac{2c}{\sqrt{7c^2}}\ \Rightarrow\ \frac{2}{\sqrt{7}}$$

$$\therefore\ \boxed{\theta=\cos^{-1}\frac{2}{\sqrt{7}}}$$

23. **Ans. (B)**

$$\because\ \ \vec{x}=\frac{1}{3}(\vec{p}+\vec{r}+\vec{s})$$

$$\vec{x}=\frac{1}{3}(5\vec{a}-3\vec{b}+(-4\vec{a})-\vec{b}-\vec{a}+\vec{b})$$

$$\vec{x}=\frac{1}{3}(-3\vec{b})\Rightarrow\vec{x}=-\vec{b}$$

$$\because\ \ \vec{y}=\frac{1}{5}(\vec{r}+\vec{s})$$

$$\vec{y}=\frac{1}{5}(-4\vec{a}-\vec{b}-\vec{a}+\vec{b})$$

$$\vec{y}=\frac{1}{5}(-5\vec{a})\qquad\Rightarrow\qquad\vec{y}=-\vec{a}$$

$$\cos\theta=\frac{(-\vec{b})\,.\,(-\vec{a})}{|-\vec{b}|\,|-\vec{a}|}\qquad\Rightarrow\qquad\frac{\vec{a}\,.\,\vec{b}}{|a|\,|b|}$$

$\vec{p}$ & $\vec{q}$, $\vec{r}$ & $\vec{s}$ are adjacent sides

$\vec{p}\,.\,\vec{q} = 0 \qquad \vec{r}\,.\,\vec{s} = 0$

$-5|\vec{a}|^2 - 7\vec{a}\,.\,\vec{b} + 6|\vec{b}|^2 = 0$ ...(1)

$4|\vec{a}|^2 - 3\vec{a}\,.\,\vec{b} - |\vec{b}|^2 = 0$ ...(2)

$25|\vec{b}|^2 = 43|\vec{a}|^2$

$|\vec{a}| = \dfrac{5|\vec{b}|}{5\sqrt{43}}$

$\boxed{\cos\theta = \dfrac{19}{5\sqrt{43}}}$

24. Ans. (B)

(A) P is in plane of ΔABC as origin

Now $(\vec{b} + \vec{c})\,.\,(\vec{b} - \vec{c}) = 0$

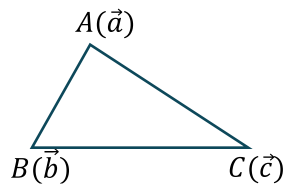
$\vec{b}^2 - \vec{c}^2 = 0 \qquad \therefore \qquad |\vec{b}| = |\vec{c}|$

Also $(\vec{c} + \vec{a})\,.\,(\vec{c} - \vec{a}) = 0$

$\vec{c} - \vec{a}^2 = 0 \qquad \therefore \qquad |\vec{c}| = |\vec{a}|$

$\Rightarrow \qquad |\vec{a}| = |\vec{b}| = |\vec{c}|$

$|\overrightarrow{OA}| = |\overrightarrow{OB}| = |\overrightarrow{OC}| =$ constant



Hence P is circum centre [S]

(B) $\vec{v} = \overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = 0 \;\therefore\; \vec{a} - \vec{p} + \vec{b} - \vec{p} + c - \vec{p} = 0$

$\vec{a} + \vec{b} + \vec{c} = 0 + 3\vec{p}$ or $\vec{p} = \dfrac{\vec{a} + \vec{b} + \vec{c}}{3}$

then P is centroid. [P]

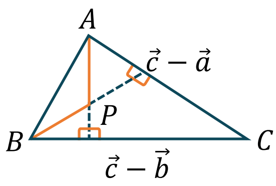
(C) $BC(\overrightarrow{PA}) + CA(\overrightarrow{PB}) + AB(\overrightarrow{PC}) = 0$

$BC(\vec{a} - \vec{p}) + CA(\vec{b} - \vec{p}) + AB(\vec{c} - \vec{p}) = 0$

$\vec{P} = \dfrac{BC\,\vec{a} + CA\vec{b} + AB\,\vec{c}}{BC + CA + AB}$

Thus P is in centre. [R]

(D) $\overrightarrow{PA}\,.\,\overrightarrow{CB}$ and $\overrightarrow{PB}\,.\,\overrightarrow{AC}$ vanishes



$\overrightarrow{PA}\,.\,\overrightarrow{CB} = (\vec{a} - \vec{p})\,.\,(\vec{b}\,.\,\vec{c}) = 0 \qquad \Rightarrow \qquad PA \perp CB$

$\overrightarrow{PB}\,.\,\overrightarrow{AC} = (\vec{b} - \vec{p})\,.\,(\vec{c} - \vec{a}) = 0 \qquad \Rightarrow \qquad PB \perp AC$

Thus P is orthocentre, i.e. point of intersection of altitudes. (Q)

25. Ans. (A) → (R); (B) → (Q,S); (C) → (Q,R); (D) → (P,Q,R,S)

(A) $(\vec{a}.\vec{c}) - (\vec{a}.\vec{b})\vec{c} = \dfrac{\vec{b}+\vec{c}}{\sqrt{2}}$

$\because$ $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non coplanar

$\therefore$ they are linearly independent

$\Rightarrow$ $\vec{a}.\vec{c} = \dfrac{1}{\sqrt{2}}$ and $\vec{a}.\vec{b} = -\dfrac{1}{\sqrt{2}}$ $\Rightarrow \vec{a} \wedge \vec{b} = \dfrac{3\pi}{4}$

(B) Let $\vec{n}_1$ be normal to plane P_1 and $\vec{n}_2$ be normal to plane P_2

$\Rightarrow \vec{n}_1 = \vec{a}\times\vec{b}$ and $\vec{n}_2 = \vec{c}\times\vec{d}$

$\because$ $(\vec{a}\times\vec{b}) \times (\vec{c}\times\vec{d}) = 0$

$\therefore$ $\vec{n}_1 \times \vec{n}_2 = 0$

$\Rightarrow$ P_1 and P_2 are parallel angle between them is zero.

(C) $(2\vec{a}-\vec{b}).(4\vec{a}+5\vec{b}) = 0$

$\Rightarrow$ $8 - 5 + 6\vec{a}.\vec{b} = 0 \Rightarrow \vec{a}.\vec{b} = -\dfrac{1}{2} \Rightarrow \vec{a}\wedge\vec{b} = \dfrac{2\pi}{3}$

(D) $\because$ $\vec{a}+\vec{b}+\vec{c} = \vec{0}$ $\Rightarrow |\vec{a}+\vec{b}|^2 = |\vec{c}|^2$

$\Rightarrow$ $|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = |\vec{c}|^2$

$9 + 25 + 2.3.5\cos(\vec{a}\wedge\vec{b}) = 49$

$\cos(\vec{a}\wedge\vec{b}) = \dfrac{1}{2} \Rightarrow \vec{a}\wedge\vec{b} = \dfrac{\pi}{3}$

EXERCISE - S

1. Ans. (3)

$N = r\omega$

$r = \dfrac{20}{10\sqrt{3}}$

Let $A(\lambda\hat{i}+\lambda\hat{j}+\lambda\hat{k})$

$\overrightarrow{AP} = (h-\lambda)\hat{i} + (k-\lambda)\hat{j} + (l-\lambda)\hat{k}$

$\overrightarrow{AP}.\vec{m} = 0$

$h - \lambda + k - \lambda + \ell - \lambda = 0$

$\lambda = \dfrac{h+k+\ell}{3}$

$\overrightarrow{AP} = \dfrac{2}{\sqrt{3}}$

$$\sqrt{\left(h-\frac{h+k+\ell}{3}\right)^2 + \left(k-\frac{h+k+\ell}{3}\right)^2 + \left(\ell-\frac{h+k+\ell}{3}\right)^2} = \frac{2}{\sqrt{3}}$$

$$\sqrt{(2h-k-\ell)^2 + (2k-h-\ell)^2 + (2\ell-h-k)^2} = 2\sqrt{3}$$

$4h^2 + k^2 + \ell^2 - 4hk + 2k\ell - 4h\ell + 4k^2 + h^2 + \ell^2 - 4kh + 2h\ell$

$-4k\ell + 4\ell^2 + h^2 + k^2 - 4h\ell + 2hk - 4k\ell = 12$

$6h^2 + 6k^2 + 6\ell^2 - 6hk - 6k\ell - 6\ell h = 12$

$h^2 + k^2 + \ell^2 - hk - k\ell - \ell h = 2$

$\therefore$ Locus $\boxed{x^2+y^2+z^2-xy-yz-7x-2=0}$

2. **Ans. (0)**

* $\vec{a}+3\vec{b}$ is collinear with $\vec{c}$

$\therefore\ \vec{a}+3\vec{b}=\lambda\vec{c}$ ...(1)

* $\vec{b}+2\vec{c}$ is colliner with $\vec{a}$

$\therefore\ \vec{b}+2\vec{c}=\mu\vec{c}$

$\Rightarrow 3\vec{b}+6\vec{c}=3\mu\vec{a}$ ...(2)

(1) - (2) $\Rightarrow \vec{a}-6\vec{c}=\lambda\vec{c}-3\mu\vec{a}$

$\Rightarrow (1+3\mu)\vec{a}=(6+\lambda)\vec{c}$

$\because$ Given $\vec{a}\ \&\ \vec{c}$ are Non-collinear

So, $1 + 3\mu = 6 + \lambda = 0$

$\Rightarrow \mu=-\frac{1}{3}$ & $\lambda = -6$ put in (1)

$\therefore\ \vec{a}+3\vec{b}=-b\vec{c}$

$\Rightarrow \boxed{\vec{a}+3\vec{b}+6\vec{c}=\vec{0}}$

3. **Ans. (5)**

$E=(2\vec{a}+\vec{b})\cdot[2|\vec{b}|^2\,\vec{a}-2(\vec{a}\cdot\vec{b})\vec{b}-(\vec{a}\cdot\vec{b})\vec{a}+|\vec{a}|^2\,\vec{b}]$

$\vec{a}\cdot\vec{b}=\frac{2-2}{\sqrt{70}}=0$

$|\vec{a}|=1$

$|\vec{b}|=1$

$\vec{a}\times\vec{b}=0$

$E=(2\vec{a}+\vec{b})\cdot[2|\vec{b}|^2\,\vec{a}+|\vec{a}|^2\,\vec{b}]$

$= 4|\vec{a}|^2|\vec{b}|^2+|\vec{a}|^2(\vec{a}\cdot\vec{b})+2|\vec{b}|^2(\vec{b}\cdot\vec{a})+|\vec{a}|^2|\vec{b}|^2$

$= 5|\vec{a}|^2|\vec{b}|^2=5$

4. **Ans. (3)**

As, $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=3\left(|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2\right)-|\vec{a}+\vec{b}+\vec{c}|^2$

$\Rightarrow 3\times 3-|\vec{a}+\vec{b}+\vec{c}|^2=9 \Rightarrow |\vec{a}+\vec{b}+\vec{c}|=0 \Rightarrow \vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow \vec{b}+\vec{c}=-\vec{a} \Rightarrow |2\vec{a}+5(\vec{b}+\vec{c})|=|-3\vec{a}|=3|\vec{a}|=3.$

5. **Ans. (7)**

$\left|\vec{a}-2\vec{b}+\vec{c}\right|=\sqrt{a^2+4b^2+c^2-4\vec{a}.\vec{b}+2\vec{a}.\vec{c}-4\vec{b}.\vec{c}}$

$=\sqrt{13-6\sqrt{2}}$

$\therefore p + q = 7$

6. **Ans. (3.80)**

$\vec{p} \perp^r \vec{q}$

$\vec{p}\cdot\vec{q}=0$

$(3\vec{a}-5\vec{b})\,(2\vec{a}+\vec{b})=0$

$6|\vec{a}|^2-10\vec{a}\cdot\vec{b}+3\vec{a}\cdot\vec{b}-5|\vec{b}|^2=0$

$6|\vec{a}|^2-5|\vec{b}|^2-7(\vec{a}\cdot\vec{b})=0$ ...(1)

$\vec{r} \perp^r \vec{s}$

$\vec{r}\cdot\vec{s}=0$

$(\vec{a}+4\vec{b})\cdot(-\vec{a}+\vec{b})=0$

$\therefore\ |\vec{a}|^2-4\vec{a}\vec{b}+\vec{a}.\vec{b}+4|\vec{b}|^2=0$

$-|\vec{a}|^2+4|\vec{b}|^2-3\vec{a}\cdot\vec{b}=0$ ...(2)

equation (1) (2) we get

$$\frac{6|\vec{a}|^2-5|\vec{b}|^2}{-|\vec{a}|^2+4|\vec{b}|^2}=\frac{7}{3}$$

$18|\vec{a}|^2-15|\vec{b}|^2=-7|\vec{a}|^2+28|\vec{b}|^2$

$25|\vec{a}|^2=43|\vec{b}|^2$

$$|\vec{b}|=\frac{\sqrt{25}}{\sqrt{43}}|\vec{a}|=\frac{5\cdot|\vec{a}|}{\sqrt{43}}$$

from equation (1) $6|\vec{a}|^2-5\cdot\frac{25|\vec{a}|^2}{43}-7|\vec{a}|\frac{5|\vec{a}|}{\sqrt{43}}\cos\theta=0$

$$6-\frac{125}{43}-\frac{35}{\sqrt{43}}\cos\theta=0$$

$$\frac{13^3}{43}=\frac{35}{\sqrt{43}}\cos\theta$$

$$\boxed{\cos\theta=\frac{19}{5\sqrt{43}}}$$

7. **Ans. (1.04 or 1.05)**

$\vec{c}\cdot\vec{d}=\vec{0}\Rightarrow\ 5|\vec{a}|^2+6\vec{a}\cdot\vec{b}-8|\vec{b}|^2=0$

$\Rightarrow\ 6\vec{a}\cdot\vec{b}=3\Rightarrow\ \vec{a}\cdot\vec{b}=\frac{1}{2}\Rightarrow(\vec{a}\cdot\vec{b})=\frac{\pi}{3}$

8. **Ans. (0.45 or 0.46)**

$\overrightarrow{AD}=\overrightarrow{AB}\times(\overrightarrow{AB}\times\overrightarrow{AD})=5(61\hat{i}-10\hat{j}-21\hat{k})\Rightarrow\cos\alpha=\frac{|\overrightarrow{AD'}\cdot\overrightarrow{AD}|}{|\overrightarrow{AD'}||\overrightarrow{AD}|}=\frac{\sqrt{17}}{9}.$

9. **Ans. (5.00)**

$\vec{r}.\left(\vec{a}+\vec{b}+\vec{c}\right)=0$

$\Rightarrow$ (cosx + cosy + 2) $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$ $\Rightarrow$ cosx + cosy + 2 = 0 $\Rightarrow$ sinx = cosy = –1

$\Rightarrow$ we take $x=-\frac{\pi}{2}$, $y=\pi$ so that x^2+y^2 obtain its minimum value.

$\therefore$ Minimum value of $\frac{4}{\pi^2}\left(x^2+y^2\right)=5$

10. Ans. (4.00)

$\overrightarrow{BA} = \vec{p} + \vec{q} + (3-\alpha)\vec{r}$

$\overrightarrow{CB} = |\beta - 1|\, p - 3q - \alpha\vec{r}$

$\overrightarrow{BA} = \lambda\overrightarrow{CB}$

$\Rightarrow \dfrac{\beta-1}{1} = -\dfrac{3}{1} = -\dfrac{\alpha}{3-\alpha} \Rightarrow \beta = -2 \;\; \& \;\; \alpha = \dfrac{9}{4}$

$\Rightarrow \dfrac{1}{\alpha+\beta} = 4$

EXERCISE - JEE (Main) PYQ

1. Ans. (4)

$\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$

$\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$

$\dfrac{\lambda-2}{4\lambda-2} = \dfrac{1}{3}$

$3\lambda - 6 = 4\lambda - 2$

$\boxed{\lambda = -4}$

$\therefore$ Option (4)

2. Ans. (4)

$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix}$

$= (2+x)\hat{i} + (x-3)\hat{j} - 5k$

$\left|\vec{a}\times\vec{b}\right| = \sqrt{4 + x^2 + 4x + x^2 + 9 - 6x + 25}$

$= \sqrt{2x^2 - 2x + 38}$

$\Rightarrow \left|\vec{a}\times\vec{b}\right| \geq \sqrt{\dfrac{75}{2}}$

$\Rightarrow \left|\vec{a}\times\vec{b}\right| \geq 5\sqrt{\dfrac{3}{2}}$

3. Ans. (1)

$\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b}) + 2(\vec{b}\cdot\vec{c}) + 2(\vec{c}\cdot\vec{a}) = 0$

$\lambda = \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = \dfrac{-3}{2}$

$\vec{d} = \vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}$

Using $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$\Rightarrow \vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$

$\Rightarrow \vec{d} = 3(\vec{a}\times\vec{b})$

4. Ans. (4)

$$f(x)=\vec{a}.\left(\vec{b}\times\vec{c}\right)=\begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix}=x^3-27x+26$$

$$f'(x)=3x^2-27=0 \;\Rightarrow\; x=\pm 3$$

and $f''(-3)<0$

$\Rightarrow$ local maxima at $x=x_0=-3$

Thus, $\vec{a}=-3\hat{i}-2\hat{j}+3\hat{k}$,

$\vec{b}=-2\hat{i}-3\hat{j}-\hat{k}$,

and $\vec{c}=7\hat{i}-2\hat{j}-3\hat{k}$

$\Rightarrow \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=9-5-26=-22$

5. Ans. (2)

v = $\left[\vec{a}\;\vec{b}\;\vec{c}\right]$

$$158=\begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix},\; n\geq 0$$

$$158=1\left(12+n^2\right)-\left(6+n\right)+n\left(2n-4\right)$$

$$158=n^2+12-6-n+2n^2-4n$$

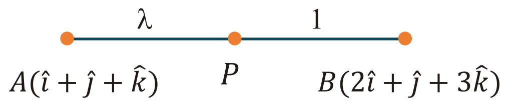
$$3n^2-5n-152=0$$

$$n=8,\,-\frac{38}{6}\text{ (rejected)}$$

$\vec{a}\cdot\vec{c}$ = 1 + $n+3n=1+4n=33$

$\vec{b}\cdot\vec{c}=2+4n-3n=2+n=10$

6. Ans. (0.8)



Using section formula we get

$$\overline{OP}=\frac{2\lambda+1}{\lambda+1}\hat{i}+\frac{\lambda+1}{\lambda+1}\hat{j}+\frac{3\lambda+1}{\lambda+1}\hat{k}$$

Now $\overline{OB}\cdot\overline{OP}=\dfrac{4\lambda+2+\lambda+1+9\lambda+3}{\lambda+1}$

$$=\frac{14\lambda+6}{\lambda+1}$$

$$\overline{OA}\times\overline{OP}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ \frac{2\lambda+1}{\lambda+1} & 1 & \frac{3\lambda+1}{\lambda+1} \end{vmatrix}$$

$= \frac{2\lambda}{\lambda+1}\hat{i} + \frac{-\lambda}{\lambda+1}\hat{j} + \frac{-\lambda}{\lambda+1}\hat{k}$

$|\overline{OA} \times \overline{OP}|^2 = \frac{(2\lambda)^2 + \lambda^2 + \lambda^2}{(\lambda+1)^2}$

$= \frac{6\lambda^2}{(\lambda+1)^2}$

$\Rightarrow \frac{14\lambda+6}{\lambda+1} - 3 \times \frac{6\lambda^2}{(\lambda+1)^2} = 6$

$\Rightarrow 10\lambda^2 - 8\lambda = 0$

$\Rightarrow \lambda = 0, \frac{8}{10} = 0.8$

$\Rightarrow \lambda = 0.8$

7. Ans. (4)

$\sqrt{3}|\vec{a}+\vec{b}| + |\vec{a}-\vec{b}|$

$= \sqrt{3}\left(\sqrt{2+2\cos\theta}\right) + \sqrt{2-2\cos\theta}$

$= \sqrt{6}\left(\sqrt{1+\cos\theta}\right) + \sqrt{2}\left(\sqrt{1-\cos\theta}\right)$

$= 2\sqrt{3}\left|\cos\frac{\theta}{2}\right| + 2\left|\sin\frac{\theta}{2}\right|$

$\le \sqrt{\left(2\sqrt{3}\right)^2 + (2)^2} = 4$

8. Ans. (1)

$|\vec{x}+\vec{y}| = |\vec{x}|$

$\sqrt{|\vec{x}|^2 + |\vec{y}|^2 + 2\vec{x}.\vec{y}} = |\vec{x}|$

$|\vec{y}|^2 + 2\vec{x}.\vec{y} = 0$ ...(1)

Now $(2\vec{x} + \lambda\vec{y}).\vec{y} = 0$

$2\vec{x}.\vec{y} + \lambda|\vec{y}|^2 = 0$

from (1)

$-|\vec{y}|^2 + \lambda|\vec{y}|^2 = 0$

$(\lambda-1)|\vec{y}|^2 = 0$

given $|\vec{y}| \neq 0 \Rightarrow \lambda = 1$

9. Ans. (1494)

$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$

$\vec{c} = 3\hat{i} + 2\hat{j} - \hat{k}$

$\vec{v} = x\vec{a} + y\vec{b}$ $\quad\quad \vec{v}.\left(3\hat{i} + 2\hat{j} - k\right) = 0$

$\vec{v}\cdot\hat{a}=19$

$\vec{v}=\lambda\left[\vec{c}\times\left(\vec{a}\times\vec{b}\right)\right]$

$\vec{v}=\lambda\left[\left(\vec{c}.\vec{b}\right)\vec{a}-\left(\vec{c}\cdot\vec{a}\right)\vec{b}\right]$

$=\lambda[(3+4+1)\left(2\hat{i}-\hat{j}+2\hat{k}\right)-(6-2-2)\left(\hat{i}+2\hat{j}+\hat{k}\right)]$

$=\lambda[16\hat{i}-8\hat{j}+16\hat{k}-2\hat{i}-4\hat{j}+2\hat{k}]$

$\vec{v}=\lambda\left[14\hat{i}-12\hat{j}+18\hat{k}\right]$

$\lambda[14\hat{i}-12\hat{j}+18\hat{k}]\cdot\dfrac{\left(2\hat{i}-\hat{j}+2\hat{k}\right)}{\sqrt{4+1+4}}=19$

$\lambda\dfrac{[28+12+36]}{3}=19$

$\lambda\left(\dfrac{76}{3}\right)=19$

$4\lambda=3\Rightarrow\lambda=\dfrac{3}{4}$

$|2\vec{v}|^2=\left|2\times\dfrac{3}{4}\left(14\hat{i}-12\hat{j}+18\hat{k}\right)\right|^2$

$\dfrac{9}{4}\times4\left(7\hat{i}-6\hat{j}+9\hat{k}\right)^2$

$=9\,(49+36+81)$

$=9\,(166)=1494$

10. Ans (2)

$\vec{r}\times\vec{a}=\vec{b}\times\vec{r}\ \Rightarrow\ \vec{r}\times(\vec{a}+\vec{b})=0$

$\vec{r}=\vec{\lambda}(\vec{a}+\vec{b})\Rightarrow\ \vec{r}=\vec{\lambda}(\hat{i}+2\hat{j}-3\hat{k}+2\hat{i}-3\hat{j}+5\hat{k})$

$\vec{r}=\vec{\lambda}(3\hat{i}-\hat{j}+2\hat{k})$ ...(1)

$\vec{r}\cdot(\alpha\,\hat{i}+2\hat{j}+\hat{k})=3$

Put $\vec{r}$ from (1) $\alpha\lambda=1$ ...(2)

$\vec{r}\cdot(2\hat{i}+5\hat{j}-\alpha\,\hat{k})=-1$

Put $\vec{r}$ from (1) $2\lambda\alpha-\lambda=1$ ...(3)

Solve (2) & (3)

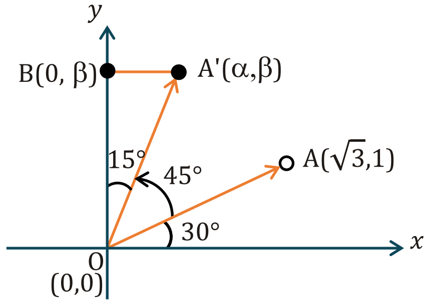
$\alpha=1,\ \lambda=1$

$\Rightarrow\ \vec{r}=3\hat{i}-\hat{j}+2\hat{k}$

$|\vec{r}|^2=14$ & $\alpha=1$

$\alpha+|\vec{r}|^2=15$

11. Ans. (1)



Area of $\Delta(OA'B)$ = $\frac{1}{2}\, OA'\cos 15^\circ \times OA'\sin 15^\circ$

$=\frac{1}{2}(OA')^2 \frac{\sin 30^\circ}{2}$

$=(3+1)\times\frac{1}{8}=\frac{1}{2}$

12. Ans. (6)

$|\vec{V_1}|=|\vec{V_2}|$

$3P^2+1=4+(P+1)^2$

$2P^2-2P-4=0 \Rightarrow P^2-P-2=0$

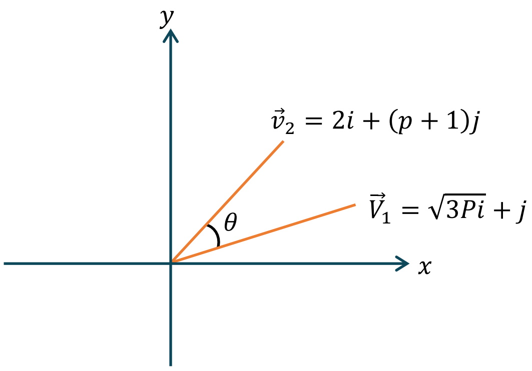
$P=2,-1$ (rejected)

$\cos\theta=\frac{\vec{V_1}\cdot\vec{V_2}}{|\vec{V_1}||\vec{V_2}|}=\frac{2\sqrt{3}P+(P+1)}{\sqrt{(P+1)^2+4}\sqrt{3P^2+1}}$

$\cos\theta=\frac{4\sqrt{3}+3}{\sqrt{13}\sqrt{13}}=\frac{4\sqrt{3}+3}{13}$

$\tan\theta = \frac{\sqrt{112-24\sqrt{3}}}{4\sqrt{3}+3}=\frac{6\sqrt{3}-2}{4\sqrt{3}+3}=\frac{\alpha\sqrt{3}-2}{4\sqrt{3}+3}$

$\Rightarrow \alpha = 6$



13. Ans. (4)

$|\vec{a}+\vec{b}+\vec{c}|^2=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{a}.\vec{c}+\vec{b}.\vec{c})$

= 3

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|=\sqrt{3}$

$\vec{a}\cdot(\vec{a}+\vec{b}+\vec{c})=|\vec{a}|\cdot|\vec{a}+\vec{b}+\vec{c}|\cos\theta$

$\Rightarrow 1 = \sqrt{3}\cos\theta$

$\Rightarrow \cos 2\theta=-\frac{1}{3}$

$\Rightarrow 36\cos^2 2\theta = \boxed{4}$

14. Ans. (4)

(1) $\vec{a}\times((\vec{b}+\vec{c})\times(\vec{b}-\vec{c}))$

$=\vec{a}(-\vec{b}\times\vec{c}+\vec{c}\times\vec{b})=-2(\vec{a}\times(\vec{b}\times\vec{c}))$

$=-2(\vec{a}\times\vec{a})=\vec{0}$

(2) Projection of $\vec{a}$ on $\vec{b}\times\vec{c}$

$$=\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|}=\frac{\vec{a}\cdot\vec{a}}{|\vec{a}|}=|\vec{a}|=2$$

(3) $[\vec{a}\ \vec{b}\ \vec{c}]+[\vec{c}\ \vec{a}\ \vec{b}]=2[\vec{a}\ \vec{b}\ \vec{c}]=2\vec{a}\cdot(\vec{b}\times\vec{c})$

$=2\vec{a}.\vec{a}=2|\vec{a}|^2=8$

(4) $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{b}\times\vec{c}=\vec{a}$

$\Rightarrow \vec{a},\vec{b},\vec{c}$ are mutually $\perp$ vectors.

$\therefore\ |\vec{a}\times\vec{b}|=|\vec{c}|\Rightarrow|\vec{a}||\vec{b}|=|\vec{c}|\Rightarrow|\vec{b}|=|\vec{c}|/2$

Also, $|\vec{b}\times\vec{c}|=|\vec{a}|\Rightarrow|\vec{b}||\vec{c}|=2\ \Rightarrow\ |\vec{c}|=2\ \&\ |\vec{b}|=1$

$|3\vec{a}+\vec{b}-2\vec{c}|^2 = (3\vec{a}+\vec{b}-2\vec{c})\cdot(3\vec{a}+\vec{b}-2\vec{c})$

$=9|\vec{a}|^2+|\vec{b}|^2+4|\vec{c}|^2$

$=(9\times4)+1+(4\times4)$

$=36+1+16=53$

15. Ans. (2)

$\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

$$\vec{a}=\lambda\left(\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}\right)=\frac{\lambda}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$$

Now projection of $\vec{a}$ *on* $\vec{b}=7$

$$\Rightarrow \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=7$$

$$\frac{\lambda}{\sqrt{3}}\frac{(\hat{i}+\hat{j}+\hat{k})\cdot(3\hat{i}+4\hat{j})}{5}=7$$

$\lambda=5\sqrt{3}$

$\vec{a}=5(\hat{i}+\hat{j}+\hat{k})$

now $\vec{b}=5\alpha(\hat{i}+\hat{j}+\hat{k})+\beta(\hat{i})$

$\vec{a}\cdot\vec{b}=0$

$\Rightarrow 25\alpha(3)+5\beta=0$

$\Rightarrow 15\alpha+\beta=0$

$\Rightarrow \beta=-15\alpha$

$\vec{b}=5\alpha(-2\hat{i}+\hat{j}+\hat{k})$

$\left|\vec{b}\right| = 5\sqrt{3}$

$\Rightarrow \alpha = \pm \dfrac{1}{\sqrt{2}}$

$\vec{b} = \pm \dfrac{5}{\sqrt{2}}(-2\hat{i} + \hat{j} + \hat{k})$

Projection of $\vec{b}$ on $3\hat{i} + 4\hat{j}$ is

$\dfrac{\vec{b}\cdot(3\hat{i}+4\hat{j})}{5} = \pm\dfrac{5}{\sqrt{2}}\left(\dfrac{-6+4}{5}\right) = \pm\sqrt{2}$

16. Ans. (4)

$\vec{v} = \lambda\,\vec{a} + \mu\,\vec{b}$

$\vec{v} = \lambda(1,1,2) + \mu(2,-3,1)$

$\vec{v} = (\lambda+2\mu, \lambda-3\mu, 2\lambda+\mu)$

$\vec{v}.\hat{j} = 7$ and $\vec{v}.\dfrac{\vec{c}}{|\vec{c}|} = \dfrac{2}{\sqrt{3}}$

$\lambda - 3\mu = 7$ and $\vec{v}.\vec{c} = 2$

$\therefore \lambda + 2\mu - \lambda + 3\,\mu + 2\lambda + \mu = 2$

$2\lambda + 6\mu = 2$

$\lambda + 3\mu = 1$ and $\lambda - 3\mu = 7$

$\Rightarrow 2\lambda = 8$

$\Rightarrow \lambda = 4,\quad \mu = -1$

We get $\vec{v} = (2,7,7)$

17. Ans. (1)

$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = 3\vec{b} - \vec{c}$

$\vec{b}\times(\vec{c}\times\vec{a}) = (\vec{b}.\vec{a})\vec{c} - (\vec{b}.\vec{c})\vec{a} = \vec{c} - 2\vec{a}$

$\vec{c}\times(\vec{b}\times\vec{a}) = (\vec{c}.\vec{a})\vec{b} - (\vec{c}.\vec{b})\vec{a} = 3\vec{b} - 2\vec{a}$

$\left[3\vec{b}-\vec{c}, \vec{c}-2\vec{a}, 3\vec{b}-2\vec{a}\right]$

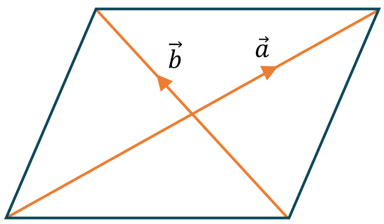
$(3\vec{b}-\vec{c}).\left[(\vec{c}-2\vec{a})\times(3\vec{b}-2\vec{a})\right]$

$(3\vec{b}-\vec{c}).\left[3(\vec{c}\times\vec{b}) - 2(\vec{c}\times\vec{a}) - 6(\vec{a}\times\vec{b})\right]$

$= -6\left[\vec{b}\ \vec{c}\ \vec{a}\right] + 6\left[\vec{c}\ \vec{a}\ \vec{b}\right] = 0$

18. Ans. (4)

Area = $\dfrac{1}{2}\left|\vec{a}\times\vec{b}\right| = 2\sqrt{2} \Rightarrow \left|\vec{a}\times\vec{b}\right| = 4\sqrt{2}$



$|\vec{a}| = 1$ and $\left|\vec{a}.\vec{b}\right| = \left|\vec{a}\times\vec{b}\right|$

$\Rightarrow \cos\theta = \sin\theta$

$\Rightarrow \theta = \dfrac{\pi}{4}$

$\therefore \left|\vec{a}\times\vec{b}\right| = 4\sqrt{2} \Rightarrow |\vec{a}||\vec{b}|\sin\frac{\pi}{4} = 4\sqrt{2}$

$\Rightarrow \left|\vec{b}\right| = 8$

Now, $\vec{c} = 2\sqrt{2}\left(\vec{a}\times\vec{b}\right) - 2\vec{b}$

$|\vec{c}| = \sqrt{(2\sqrt{2})^2\left|\vec{a}\times\vec{b}\right|^2 + \left(2\left|\vec{b}\right|\right)^2} = 16\sqrt{2}$

Now, $\vec{b}.\vec{c} = -2\left|\vec{b}\right|^2$

$\Rightarrow 8\times16\sqrt{2}\times\cos\alpha = -2.64$

$\Rightarrow \cos\alpha = -\frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{3\pi}{4}$

19. Ans. (2)

Let P is $\vec{0}$, Q is $\vec{q}$ and R is $\vec{r}$

A is $\frac{2\vec{q}+\vec{r}}{3}$, B is $\frac{2\vec{r}}{3}$ and C is $\frac{\vec{q}}{3}$

Area of ΔPQR is = $\frac{1}{2}\left|\vec{q}\times\vec{r}\right|$

Area of ΔABC is $\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$

$\overrightarrow{AB} = \frac{\vec{r}-2\vec{q}}{3}, \overrightarrow{AC} = \frac{-\vec{r}-\vec{q}}{3}$

Area of ΔABC = $\frac{1}{6}\left|\vec{q}\times\vec{r}\right|$

$\frac{\text{Area}\left(\Delta PQR\right)}{\text{Area}\left(\Delta ABC\right)} = 3$

20. Ans. (4)

Without loss of generality

Let $|a_1| \le |a_2| \le |a_3|$

$|\vec{a}|^2 = |a_1|^2 + |a_2|^2 + |a_3|^2 \ge (a_3)^2$

$\Rightarrow |\vec{a}| \ge |a_3| = \max\{|a_1|,|a_2|,|a_3|\}$

A is true

$|\vec{a}|^2 = |a_1|^2 + |a_2|^2 + |a_3|^2 \le |a_3|^2 + |a_3|^2 + |a_3|^2$

$\Rightarrow |\vec{a}|^2 \le 3|a_3|^2$

$\Rightarrow |\vec{a}| \le \sqrt{3}|a_3| = \sqrt{3}\max\{|a_1|,|a_2|,|a_3|\}$

$\le 3\max\{|a_1|,|a_2|,|a_3|\}$

(2) is true.

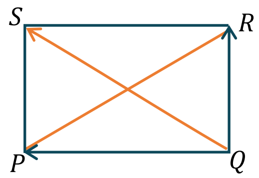
EXERCISE - JEE (Advanced) PYQ

1. Ans. (C)

$\overrightarrow{PR}=\overrightarrow{PQ}+\overrightarrow{PS}$

$\overrightarrow{SQ}=\overrightarrow{PQ}-\overrightarrow{PS}$

$\overrightarrow{PS}=\dfrac{\overrightarrow{PR}-\overrightarrow{SQ}}{2}$



$V=\left|\left[\overrightarrow{PQ}\ \ \overrightarrow{PS}\ \ \overrightarrow{PT}\right]\right|$

$V=\dfrac{1}{4}\left|\left[\overrightarrow{PR}\ +\overrightarrow{SQ}\ \ \overrightarrow{PR}-\overrightarrow{SQ}\ \ \overrightarrow{PT}\right]\right|$

$V=\dfrac{1}{2}\left|\left[\overrightarrow{PR}\ \ \overrightarrow{SQ}\ \ \overrightarrow{PT}\right]\right|\Rightarrow\dfrac{1}{2}\begin{vmatrix}3 & 1 & -2\\ 1 & -3 & -4\\ 1 & 2 & 3\end{vmatrix}$

$\Rightarrow\dfrac{1}{2}(-3-7-10)=10$

2. Ans.(D)

Any point on line $\dfrac{x+2}{2}=\dfrac{y+1}{-1}=\dfrac{z}{3}=\lambda$

Let any two points on this line are

$A(-2,-1,0),B(0,-2,3)$ Put $(\lambda=0,1)$

Let foot of perpendicular from $A(-2,-1,0)$ on plane is (α, β, γ)

$\Rightarrow\dfrac{\alpha+2}{1}=\dfrac{\beta+1}{1}=\dfrac{\gamma-0}{1}=\mu$ (say)

Also, $\alpha+\beta+\gamma=3$

$\Rightarrow\mu-2+\mu-1+\mu=3\Rightarrow\mu=2$

$\Rightarrow M\ (0, 1, 2)$

Similarly foot of perpendicular from $B\ (0, -2, 3)$ on plane is $N\left(\dfrac{2}{3},\dfrac{-4}{3},\dfrac{11}{3}\right)$

So, equation of MN is $\dfrac{x-0}{\frac{2}{3}}=\dfrac{y-1}{\frac{-7}{3}}=\dfrac{z-2}{\frac{5}{3}}$.

3. Ans. (B, D)

Let equation of line ℓ is

$\ell:\dfrac{x-0}{a}=\dfrac{y-0}{b}=\dfrac{z-0}{c}=k$

This line ℓ is perpendicular to given line ℓ_1 and ℓ_2.

Hence

$a+2b+2c=0$

$2a+2b+c=0$

$\dfrac{a}{-2}=\dfrac{b}{3}=\dfrac{c}{-2}$

Hence equation of ℓ is $\frac{x}{-2} = \frac{y}{3} = \frac{z}{-2} = k_1, k_2$

↓ ↓

for ℓ_1 for ℓ_2

Now $A(-2k_1, 3k_1, -2k_1)$ $B(-2k_2, 3k_2, -2k_2)$

Point A satisfied ℓ_1

$-2k_1\hat{i} + 3k_1\hat{j} - 2k_1\hat{k} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$

$3 + t = -2k_1$ ...(1)

$-1 + 2t = 3k_1$ ...(2)

$4 + 2t = -2k_1$ ...(3)

(2) & (3) $-5 = 5k_1 \Rightarrow k_1 = -1 \Rightarrow A(2, -3, 2)$

Let any point on ℓ_2 $(3+2S, 3+2S, 2+S)$

Given $\sqrt{(1+2S)^2 + (6+2S)^2 + (S)^2} = \sqrt{17}$

$9S^2 + 28S + 37 = 17$

$9S^2 + 28S + 20 = 0$

$9S^2 + 18S + 10S + 20 = 0$

$9S(S+2) + 10(S+2) = 0$

$S = -2, -10/9$

Hence $(-1, -1, 0), (7/9, 7/9, 8/9)$

Ans. (B) & (D)

4. **Ans. (32)**

Among set of eight vectors four vectors form body diagonals of a cube, remaining four will be parallel (unlike) vectors.

Numbers of ways of selecting three vectors will be $^4C_3 \times 2 \times 2 \times 2 = 2^5$

Hence $p = 5$

Alternative

Eight vectors

$\vec{x} \equiv \hat{i} + \hat{j} + \hat{k}$

$\vec{y} \equiv \hat{i} + \hat{j} - \hat{k}$

$\vec{z} \equiv \hat{i} - \hat{j} + \hat{k}$

$\vec{\omega} = \hat{i} - \hat{j} - \hat{k}$

$\vec{x}' = -\hat{i} - \hat{j} - \hat{k}$

$\vec{y}' = -\hat{i} - \hat{j} + \hat{k}$

$\vec{z}' = -\hat{i} + \hat{j} - \hat{k}$

$\vec{\omega}' = -\hat{i} + \hat{j} + \hat{k}$

If we take $\vec{x}, \vec{x}'$ and any one of remaining six vectors will always be coplaner

$\therefore$ No. of coplanar vectors = 6

similarly on taking $\vec{y}, \vec{y}' = 6$

$z,\ \vec{z}' = 6$

$\omega, \vec{\omega}' = 6$

$\therefore$ No. of set of coplanar vectors = 24

5. **Ans. (C)**

(P) $[\vec{a}\ \ \vec{b}\ \ \vec{c}] = 2$

$2\left(\vec{a}\times\vec{b}\right),\ 3\left(\vec{b}\times\vec{c}\right),\ \left(\vec{c}\times\vec{a}\right)$

$6\ [\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\ ] = 6\ [\vec{a}\ \vec{b}\ \vec{c}]^2 = 6\times 4 = 24$

$P \to 3$

(Q) $[\vec{a}\ \ \vec{b}\ \ \vec{c}] = 5$

$[3\ (\vec{a}+\vec{b})\ \ (\vec{b}+\vec{c})\ \ 2(\vec{c}+\vec{a})\ ] = 6 \times 2[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

$= 12 \times 5 = 60$

$Q \to 4$

(R) $\dfrac{1}{2}|\vec{a}\times\vec{b}| = 20$

$\Delta_1 = \dfrac{1}{2}\ |(2\vec{a}+3\vec{b})\times(\vec{a}-\vec{b})|$

$= |-2\vec{a}\times\vec{b} - 3(\vec{a}\times\vec{b})| = \dfrac{5}{2}\ |\vec{a}\times\vec{b}| = 5 \times 20 = 100$

$R \to 1$

(S) $|\vec{a}\times\vec{b}| = 30$

$|(\vec{a}+\vec{b})\times\ \vec{a}\ | = |\vec{b}\times\vec{a}| = 30$

$S \to 2$

6. **Ans. (A,B,C)**

$|\vec{x}| = |\vec{y}| = |\vec{z}| = \sqrt{2}$

$\theta = \dfrac{\pi}{3}$

$\vec{a} = \lambda\vec{x}\times\left(\vec{y}\times\vec{z}\right)$

$\vec{b} = \mu\vec{y}\times\left(\vec{z}\times\vec{x}\right)$

$\vec{a} = \left(\left(\vec{x}.\vec{z}\right)\vec{y} - \left(\vec{x}.\vec{y}\right)\vec{z}\right)$

$\vec{a} = \lambda\left(2\times\dfrac{1}{2}\vec{y} - 2\times\dfrac{1}{2}\vec{z}\right)$

$\vec{a} = \lambda\left(\vec{y}-\vec{z}\right)$

$\vec{b} = \mu\left(\vec{z}-\vec{x}\right)$

Similarly

$\vec{a}.\vec{y} = \lambda\left(2 - 2\times\dfrac{1}{2}\right) = \lambda$

$\vec{a}=(\vec{a}.\vec{y})(\vec{y}-\vec{z}) \Rightarrow$ (B)

$\vec{b}.\vec{z}=\mu\left(2-2\times\frac{1}{2}\right)$

$\mu = \vec{b}.\vec{z}$

$\therefore\ \vec{b}=(\vec{b}.\vec{z})(\vec{z}-\vec{x}) \Rightarrow$ (A)

$\vec{a}.\vec{b}=(\vec{a}.\vec{y})(\vec{y}-\vec{z}).(\vec{b}.\vec{y})(\vec{z}-\vec{x})$

$= (\vec{a}.\vec{y})(\vec{b}.\vec{z})(\vec{y}\cdot\vec{z}-\vec{y}\cdot\vec{x}-2+\vec{x}\cdot\vec{z}) = (\vec{a}.\vec{y})(\vec{b}.\vec{z})(-1)$

$= -\ (\vec{a}.\vec{y})(\vec{b}.\vec{z}) \Rightarrow$ (C)

7. Ans. (4)

$p + q + r = a\times b+b\times c$

Taking dot product with $\vec{a},\vec{b},\vec{c}$ we get

$p+\frac{q}{2}+\frac{r}{2}=[abc]$ ...(1)

$\frac{p}{2}+q+\frac{r}{2}=0$ ...(2)

$\frac{p}{2}+\frac{q}{2}+r=[abc]$ ...(3)

(1) & (3) $\Rightarrow p=r \ \& q=-p$

$\frac{p^2+2q^2+r^2}{q^2} = \frac{p^2+2p^2+p^2}{p^2}=4$ Ans.

8. Ans. (A)

(P) $y=4x^3-3x$ where $\cos\theta=x$

$\frac{dy}{dx}=12x^2-3$

$(x^2-1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=(x^2-1).24x+x(12x^2-3)$

$=36x^3-27x=9(4x^3-3x)=9y$

Hence $\frac{1}{y}\left\{(x^2-1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}\right\}=9$

(Q) $|\vec{a}_1\times\vec{a}_2+\vec{a}_2\times\vec{a}_3+...+\vec{a}_{n-1}\times\vec{a}_n|$

$= |\vec{a}_1\ .\ \vec{a}_2+\vec{a}_2\ .\ \vec{a}_3+...+\vec{a}_{n-1}.\vec{a}_n|$

Let $|\vec{a}_1|=|\vec{a}_2|\ .....=|\vec{a}_n| = \lambda$ (as centre is origin)

More over angle between 2 consecutive $\vec{a}_i$'s is $\frac{2\pi}{n}$

Hence given equation reduces to

$(n-1)\lambda^2\sin\left(\frac{2\pi}{n}\right) = (n-1)\lambda^2\cos\left(\frac{2\pi}{n}\right)$

$\Rightarrow \tan\left(\frac{2\pi}{n}\right) = 1 \Rightarrow \frac{2\pi}{n} = \frac{\pi}{4} \Rightarrow n = 8$

(R) Equation of normal $\frac{6x}{h} - \frac{3y}{1} = 3$ $\left(\text{Equation of normal is } \frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2\right)$

slope = $\frac{6}{3h} = 1$ (as it is perpendicular to $x + y = 8$) $\Rightarrow h = 2$

(S) $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$

$\Rightarrow \frac{\frac{1}{2x+1} + \frac{1}{4x+1}}{1 - \frac{1}{(2x+1)(4x+1)}} = \frac{2}{x^2} \Rightarrow \frac{6x+2}{8x^2+6x} = \frac{2}{x^2}$

$\Rightarrow 3x^3 + x^2 = 8x^2 + 6x \Rightarrow 3x^3 - 7x^2 - 6x = 0$

$\Rightarrow 3x^2 - 7x - 6 = 0$ (as $x \neq 0$)

$\Rightarrow (x-3)(3x+2) = 0 \Rightarrow x = -\frac{2}{3}, 3$ $\left(-\frac{2}{3} \text{ is rejected}\right)$

9. Ans. (A,C,D)

$\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \vec{b} + \vec{c} = -\vec{a}$

$\Rightarrow 48 + |c|^2 + 48 = 144$

$\Rightarrow \vec{c}^2 = 48$

$\Rightarrow |\vec{c}| = 4\sqrt{3}$

$\therefore \frac{|\vec{c}|^2}{2} - |\vec{a}| = 24 - 12 = 12$ **Ans. (A)**

Further;

$\vec{a} + \vec{b} = -\vec{c}$

$\Rightarrow 144 + 48 + 2\vec{a}.\vec{b}$

= 48

$\Rightarrow \vec{a}.\vec{b} = -72$ **Ans. (D)**

$\because \vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = 0$

$\therefore |\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 2.\sqrt{144.48 - (72)^2} = 48\sqrt{3}$ **Ans. (C)**

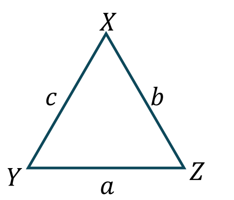
10. Ans. (A → P,R,S; B → P; C → P,Q ; D → S, T)

(A) Given $2(a^2 - b^2) = c^2$

$\Rightarrow 2(\sin^2 X - \sin^2 Y) = \sin^2 Z$

$\Rightarrow 2\sin(X+Y)\sin(X-Y) = \sin^2 Z$

$\Rightarrow 2\sin(\pi - Z)\sin(X-Y) = \sin^2 Z$



$\Rightarrow \sin(X-Y) = \dfrac{\sin Z}{2}$ ...(i)

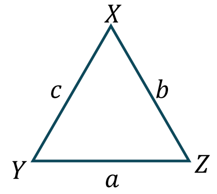
also given,

$\lambda = \dfrac{\sin(X-Y)}{\sin Z} = \dfrac{1}{2}$

Now, $\cos(n\pi\lambda) = 0$

$\Rightarrow \cos\left(\dfrac{n\pi}{2}\right) = 0$

$\therefore\ n = 1,3,5$ $\therefore$ **(A → P,R,S)**



(B) $1 + \cos 2X - 2\cos 2Y = 2\sin X \sin Y$

$2\cos^2 X - 2\cos 2Y = 2\sin X \sin Y$

$1 - \sin^2 X - 1 + 2\sin^2 Y = \sin X \sin Y$

$\sin^2 X + \sin X \sin Y = 2\sin^2 Y$

$\sin X(\sin X + \sin Y) = 2\sin^2 Y$

$\sin X = ak, \sin Y = bk$

$a(a+b) = 2b^2$

$a^2 + ab - 2b^2 = 0$

$\left(\dfrac{a}{b}\right)^2 + \dfrac{a}{b} - 2 = 0$

$\dfrac{a}{b} = -2, 1$

$\dfrac{a}{b} = 1$ **(B → P)**

Hence equation of acute angle bisector of OX and OY is $y = x$

Hence $x - y = 0$

Now, distance of $\beta\hat{i} + (1-\beta)\hat{j} \equiv z(\beta, 1-\beta)$ from $x-y$ is $\left|\dfrac{\beta-(1-\beta)}{\sqrt{2}}\right| = \dfrac{3}{\sqrt{2}}$

$|2\beta - 1| = 3$

$2\beta - 1 = \pm 3$

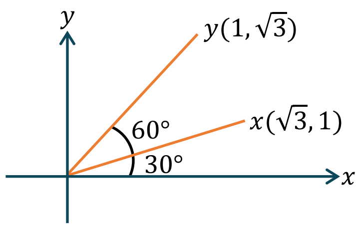
$2\beta = 4, -2$

$\beta = 2, -1$

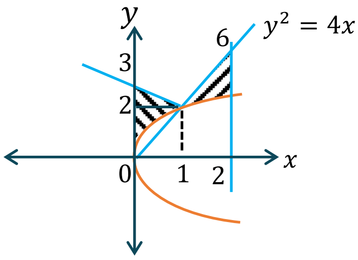
$|\beta| = 2, 1$ **Ans. (P,Q)**

(D) For $\alpha = 1$

$y = |x-1| + |x-2| + x = \begin{cases} 3-x & ; \quad x<1 \\ 1+x & ; \quad 1 \le x < 2 \\ 3x-3 & ; \quad x \ge 2 \end{cases}$



$A = \dfrac{1}{2}(2+3)\times 1 + \dfrac{1}{2}(2+3)\times 1 - \int_0^2 2\sqrt{x}\,dx$



$$A=5-\frac{8}{3}\sqrt{2}$$

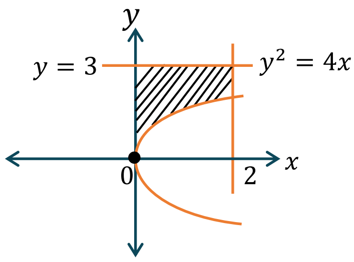
$$\therefore\ F(1)+\frac{8}{3}\sqrt{2}=5$$

For $\alpha = 0$, $y = |-1| + |-2| = 3$

$$A=6-\int_0^2 2\sqrt{x}dx \Rightarrow A=6-\frac{8}{3}\sqrt{2}$$

$$\therefore\ F(0)+\frac{8}{3}\sqrt{2}=6$$

$\therefore$ **(D → S, T)**



11. Ans. (BONUS)

This question in seem to be wrong but examiner may think like this

$$\vec{S}=4\vec{p}+3\vec{q}+5\vec{r}$$

$$\vec{S}=x(-\vec{p}+\vec{q}+\vec{r})+y(\vec{p}-\vec{q}+\vec{r})+z(-\vec{p}-\vec{q}+\vec{r})$$

$-x+y-z=4$ ...(1)

$x-y-z=3$ ...(2)

$x+y+z=5$ ...(3)

add (1) and (2)

$$-2z=7 \Rightarrow z=-\frac{7}{2}$$

$$2x=8 \Rightarrow x=4$$

$$y+z=1$$

$$2x+y+z=2(4)+1=9$$

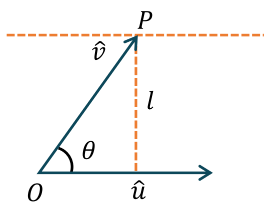
12. Ans. (B,C)

$|\hat{w}||\hat{u}\times\hat{v}|\cos\phi=1 \Rightarrow \phi=0$

$\Rightarrow \hat{u}\times\vec{v}=\hat{w}$ also $|\vec{v}|\sin\theta=1$

$\Rightarrow$ there may be infinite vectors $\vec{v}=\overrightarrow{OP}$

such that P is always 1 unit dist. from $\hat{u}$



For option (C) : $\hat{u}\times\vec{v}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ u_1 & u_2 & 0\\ v_1 & v_2 & v_3\end{vmatrix}$

$\hat{w}=(u_2v_3)\hat{i}-(u_1v_3)\hat{j}+(u_1v_2-u_2v_1)\hat{k}$

$u_2v_3=\frac{1}{\sqrt{6}}, -u_1v_3=\frac{1}{\sqrt{6}} \Rightarrow |u_1|=|u_2|$

for option (D) : $\hat{u}\times\vec{v}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ u_1 & 0 & u_3\\ v_1 & v_2 & v_3\end{vmatrix}$

$\hat{w}=(-v_2u_3)\hat{i}-(u_1v_3-u_3v_1)\hat{j}+(u_1v_2)\hat{k}$

$-v_2u_3=\frac{1}{\sqrt{6}}, u_1v_2=\frac{2}{\sqrt{6}}$

$\Rightarrow 2|u_3|=|u_1|$ So (D) is wrong

13. **Ans. (B)**

$\overrightarrow{OP}.(\overrightarrow{OQ}-\overrightarrow{OR})=\overrightarrow{OS}.(\overrightarrow{OQ}-\overrightarrow{OR})$

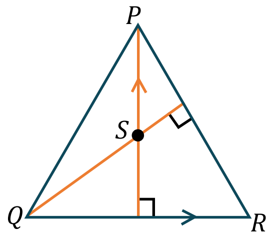
$\Rightarrow (\overrightarrow{OQ}-\overrightarrow{OR}).(\overrightarrow{OP}-\overrightarrow{OS})=0$

$\Rightarrow \overrightarrow{RQ}.\overrightarrow{SP}=0$

$\Rightarrow \overrightarrow{SP}\perp\overrightarrow{RQ}$

Similarly, $\overrightarrow{SQ}\perp\overrightarrow{PR}$

$\therefore$ S is orthocenter



14. **Ans. (D)**

$\overrightarrow{OX}=\frac{\overrightarrow{QR}}{QR}$

$\overrightarrow{OY}=\frac{\overrightarrow{RP}}{RP}$

$\left|\overrightarrow{OX}\times\overrightarrow{OY}\right|=\sin R=\sin(P+Q)$

15. **Ans. (B)**

$-(\cos P+\cos Q+\cos R) \geq -\frac{3}{2}$ as we know

$\cos P+\cos Q+\cos R$ will take its maximum value when $P=Q=R=\frac{\pi}{3}$

16. **Ans. (3)**

$\vec{c}=x\vec{a}+y\vec{b}+\vec{a}\times\vec{b}$

$\vec{c}.\vec{a}=x$ and $x=2\cos\alpha$

$\vec{c}.\vec{b}=y$ and $y=2\cos\alpha$

Also, $|\vec{a}\times\vec{b}|=1$

$\therefore$ $\vec{c}=2\cos\alpha(\vec{a}+\vec{b})+\vec{a}\times\vec{b}$

$\vec{c}^2=4\cos^2\alpha(\vec{a}+\vec{b})^2+(\vec{a}\times\vec{b})^2$ + $2\cos\alpha.(\vec{a}+\vec{b}).(\vec{a}\times\vec{b})$

4 = $8\cos^2\alpha+1 \Rightarrow 8\cos^2\alpha=3$

17. **Ans. (C, D)**

Let $P(\lambda,0,0)$, $Q(0,\mu,1)$, $R(1,1,\nu)$ be points. L_1, L_2 and L_3 respectively

Since P,Q,R are collinear, $\overrightarrow{PQ}$ is collinear with $\overrightarrow{QR}$

Hence $\frac{-\lambda}{1}=\frac{\mu}{1-\mu}=\frac{1}{\nu-1}$

For every $\mu \in R$ – {0, 1} there exist unique $\lambda,\ \nu \in R$

Hence Q cannot have coordinates (0, 1, 1) and (0, 0, 1).

18. **Ans. (18)**

$\vec{c}=(2\alpha+\beta)\hat{i}+\hat{j}(\alpha+2\beta)+\hat{k}(\beta-\alpha)$

$\frac{\vec{c}.(\vec{a}+\vec{b})}{|\vec{a}+\vec{b}|}=3\sqrt{2} \Rightarrow \alpha+\beta=2$ ...(1)

$\left(\vec{c}-\left(\vec{a}\times\vec{b}\right)\right).\vec{c}=|\vec{c}|^2-(\vec{a}\times\vec{b}).\vec{c}$

$\because$ is coplanar with $\vec{a}$ and $\vec{b}$

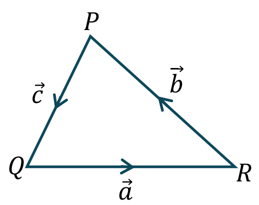
$\therefore\ (\vec{a}\times\vec{b}).\vec{c}=0$

$\left(\vec{c}-\left(\vec{a}\times\vec{b}\right)\right).\vec{c}=|\vec{c}|^2=\alpha^2|\vec{a}|^2+\beta^2|b|^2+2\alpha\beta(\vec{a}.\vec{b})$

$=6\left(\alpha^2+\beta^2+\alpha\beta\right)=6\left(\alpha^2+(2-\alpha)^2+\alpha(2-\alpha)\right)$

$=6\left((\alpha-1)^2+3\right) \Rightarrow$ Min. value = 18

19. **Ans. (108)**



We have $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow \vec{c}=-\vec{a}-\vec{b}$

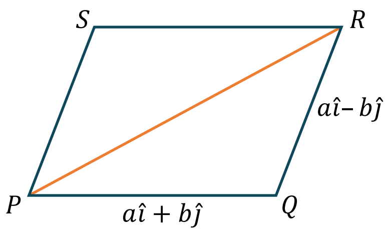
Now, $\frac{\vec{a}.(-\vec{a}-2\vec{b})}{(-\vec{a}-\vec{b}).(\vec{a}-\vec{b})}=\frac{3}{7}$

$\Rightarrow \frac{9+2\vec{a}.\vec{b}}{9-16}=\frac{3}{7}$

$\Rightarrow\ \vec{a}.\vec{b}=-6$

$\Rightarrow |\vec{a}\times\vec{b}|^2=a^2b^2-(\vec{a}.\vec{b})^2=9\times16-36=108$

20. **Ans. (A, C)**



$\vec{u}=\frac{\left(\left(\hat{i}+\hat{j}\right).\overrightarrow{PQ}\right)}{|\overrightarrow{PQ}|}$

$\vec{u}=\left|\left(\hat{i}+\hat{j}\right).\widehat{PQ}\right|$

$|\vec{u}|=\left|\left(\hat{i}+\hat{j}\right).\frac{\left(a\hat{i}+b\hat{j}\right)}{\sqrt{a^2+b^2}}\right|=\frac{a+b}{a^2+b^2}$

$\vec{v}=\left(\hat{i}+\hat{j}\right).\widehat{PS}$

$$|\vec{v}| = \left|\frac{(\hat{i}+\hat{j}).(a\hat{i}-b\hat{j})}{\sqrt{a^2+b^2}}\right| = \frac{a-b}{\sqrt{a^2+b^2}}$$

$$|\vec{u}|+|\vec{v}|=|\vec{w}|$$

$$\frac{|(a+b)|+|(a-b)|}{\sqrt{a^2+b^2}}=\sqrt{2}$$

For $a>b$

$$2a=\sqrt{2}.\sqrt{a^2+b^2}$$

$$4a^2=2a^2+2b^2$$

$a^2=b^2 \;\therefore\; a=b$ ...(1)

$(a>0, b>0)$

similarly, for $a<b$ we will get $a=b$

Now area of parallelogram $=\left|(a\hat{i}+b\hat{j})\times(a\hat{i}-b\hat{j})\right|$

$= 2ab$

$\therefore\; 2ab=8$

$ab=4$ ...(2)

from (1) and (2)

$a=2,\; b=2$ $\quad\therefore\; a+b=4$ $\quad$ option (A)

length of diagonal is

$$|2a\hat{i}|=|4\hat{i}|=4$$

So option (C)

21. Ans. (7)

Given, $|\vec{u}|=1$; $|\vec{v}|=1$; $\vec{u}.\vec{v}\neq 0$; $\vec{u}.\vec{w}=1$; $\vec{v}.\vec{w}=1$;

$\vec{w}.\vec{w}=|\vec{w}|^2=4 \Rightarrow |\vec{w}|=2$; $[\vec{u}\;\;\vec{v}\;\;\vec{w}]=\sqrt{2}$

and $$[\vec{u}\;\;\vec{v}\;\;\vec{w}]^2=\begin{vmatrix}\vec{u}.\vec{u} & \vec{u}.\vec{v} & \vec{u}.\vec{w}\\ \vec{v}.\vec{u} & \vec{v}.\vec{v} & \vec{v}.\vec{w}\\ \vec{w}.\vec{u} & \vec{w}.\vec{v} & \vec{w}.\vec{w}\end{vmatrix}=2$$

$$\Rightarrow \begin{vmatrix}1 & \vec{u}.\vec{v} & 1\\ \vec{u}.\vec{v} & 1 & 1\\ 1 & 1 & 4\end{vmatrix}=2 \Rightarrow \vec{u}.\vec{v}=\frac{1}{2}$$

So, $|3\vec{u}+5\vec{v}|=\sqrt{9|\vec{u}|^2+25|\vec{v}|^2+2.3.5\vec{u}.\vec{v}}$

$$=\sqrt{9+25+30\left(\frac{1}{2}\right)}=\sqrt{49}=7$$

22. Ans. (A,B,C)

$\overrightarrow{OB}\times\overrightarrow{OC}=\frac{1}{2}\overrightarrow{OB}\times(\overrightarrow{OB}-\lambda\overrightarrow{OA})$

$=\frac{\lambda}{2}(\overrightarrow{OA}\times\overrightarrow{OB})$

$|\overrightarrow{OB}\times\overrightarrow{OC}|=\frac{\lambda}{2}|\overrightarrow{OA}|\times|\overrightarrow{OB}|$ (Note $\overrightarrow{OA}$ & $\overrightarrow{OB}$ are perpendicular)

$\Rightarrow \frac{9\lambda}{2}=\frac{9}{2} \Rightarrow \lambda = 1$ (given $\lambda > 0$)

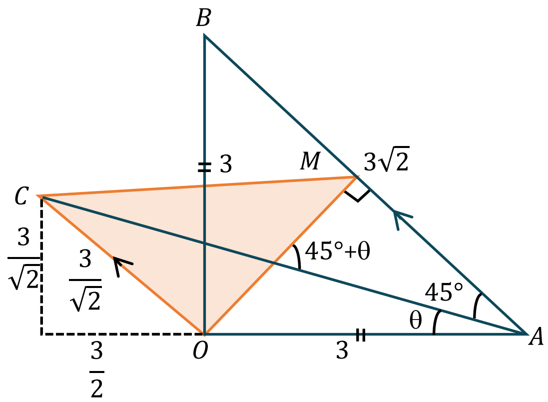
So $\overrightarrow{OC}=\frac{\overrightarrow{OB}-\overrightarrow{OA}}{2}=\frac{\overrightarrow{AB}}{2}$

M is mid point of AB

Note projection of $\overrightarrow{OC}$ on $\overrightarrow{OA}=-\frac{3}{2}$

$\tan\theta = \frac{1}{3}$

Area of $\Delta ABC = \frac{9}{2}$



Acute angle between diagonals is

$\tan^{-1}\left(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}\right)=\tan^{-1}2$

23. Ans. (B,C,D)

$\vec{a}=3\hat{i}+\hat{j}-\hat{k}$

$\vec{b}=\hat{i}+b_2\hat{j}+b_3\hat{k}$

$\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$

$$\begin{pmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ b_2 \\ b_3 \end{pmatrix}=\begin{pmatrix} 3-c_1 \\ 1-c_2 \\ -1-c_3 \end{pmatrix}$$

multiply & compare

$\Rightarrow b_2c_3-b_3c_2=c_1-3$ ...(1)

$\Rightarrow c_3-b_3c_1=1-c_2$ ...(2)

$\Rightarrow c_2-b_2c_1=1+c_3$ ...(3)

$(1)\hat{i}-(2)\hat{j}+(3)\hat{k}$

$\Rightarrow \hat{i}\ (b_2c_3-b_3c_2)-\hat{j}\ (c_3-b_3c_1)+\hat{k}\ (c_2-b_2c_1)$

$= c_1\hat{i}+c_2\hat{j}+c_3\hat{k}-3\hat{i}-\hat{j}+\hat{k} \Rightarrow \vec{b}\times\vec{c}=\vec{c}-\vec{a}$

Take dot product with $\vec{b}$

$\Rightarrow 0=\vec{c}.\vec{b}-\vec{a}.\vec{b} \Rightarrow \vec{b}.\vec{c}=0 \Rightarrow \vec{b}\perp\vec{c} \Rightarrow \vec{b}\hat{}\vec{c}=90°$

Take dot product with $\vec{c}$

$\Rightarrow 0=|\vec{c}|^2-\vec{a}.\vec{c} \Rightarrow \vec{a}.\vec{c}=|\vec{c}|^2 \Rightarrow \vec{a}.\vec{c}\neq 0 \Rightarrow \vec{b}\times\vec{c}=\vec{c}-\vec{a}$

Squaring

$\Rightarrow |\vec{b}|^2|\vec{c}|^2=|\vec{c}|^2+|\vec{a}|^2-2\vec{c}.\vec{a} \Rightarrow |\vec{b}|^2|\vec{c}|^2=|\vec{c}|^2+11-2|\vec{c}|^2 \Rightarrow |\vec{b}|^2|\vec{c}|^2=11-|\vec{c}|^2$

$\Rightarrow |\vec{c}|^2\left(|\vec{b}|^2+1\right)=11 \Rightarrow |\vec{c}|^2=\dfrac{11}{|\vec{b}|^2+1} \Rightarrow |\vec{c}|\leq\sqrt{11}$

given $\vec{a}.\vec{b}=0$

$\Rightarrow b_2-b_3=-3$ also

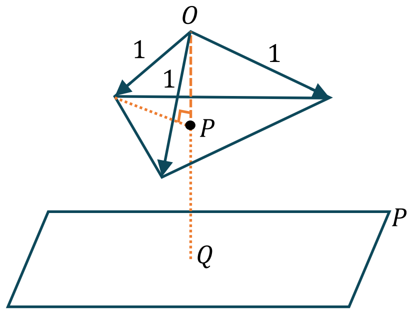
$\Rightarrow b_2^2+b_3^2-2b_2b_3=9 \qquad b_2b_3>0$

$\Rightarrow b_2^2+b_3^2=9+2b_2b_3 \Rightarrow b_2^2+b_3^2=9+2b_2b_3>9 \Rightarrow b_2^2+b_3^2>9$

$\Rightarrow |\vec{b}|=\sqrt{1+b_2^2+b_3^2} \Rightarrow |\vec{b}|>\sqrt{10}$

24. Ans. (45)

Given $|\vec{u}-\vec{v}|=|\vec{v}-\vec{w}|=|\vec{w}-\vec{u}|$



$\Rightarrow \Delta UVW$ is an equilateral Δ

Now distances of U,V,W from $P=\dfrac{7}{2}$

$\Rightarrow PQ=\dfrac{7}{2}$

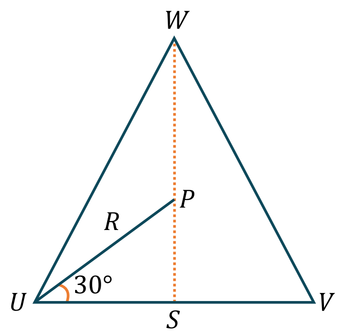
Also, Distance of plane P from origin

$\Rightarrow OQ = 4$

$\therefore OP = OQ - PQ \Rightarrow OP=\dfrac{1}{2}$

Hence, $PU=\sqrt{OU^2-OP^2} \Rightarrow PU=\dfrac{\sqrt{3}}{2}=R$

Also, for ΔUVW, P is circumcentre



$\therefore$ for ΔUVW : $US = R\cos 30°$

$\Rightarrow UV = 2R\cos 30°$

$\Rightarrow UV=\dfrac{3}{2}$

$\therefore Ar(\Delta UVW)=\dfrac{\sqrt{3}}{4}\left(\dfrac{3}{2}\right)^2=\dfrac{9\sqrt{3}}{16}$

$\therefore$ Volume of tetrahedron with coterminous edges $\vec{u},\vec{v},\vec{w}$

$=\dfrac{1}{3}(Ar.\Delta UVW)\times OP=\dfrac{1}{3}\times\dfrac{9\sqrt{3}}{16}\times\dfrac{1}{2}=\dfrac{3\sqrt{3}}{32}$

$\therefore$ parallelopiped with coterminous edges

$\vec{u},\vec{v},\vec{w}=6\times\dfrac{3\sqrt{3}}{32}=\dfrac{9\sqrt{3}}{16}=V$

$\therefore \dfrac{80}{\sqrt{3}}V=45$

25. **Ans. (B)**

$L_1 : \vec{r}_1 = \lambda(\hat{i} + \hat{j} + \hat{k})$

$L_2 : \vec{r}_2 = \hat{j} - \hat{k} + \mu(\hat{i} + \hat{k})$

Let system of planes are

$ax + by + cz = 0$ ...(1)

$\because$ It contain L_1

$\therefore a + b + c = 0$ ...(2)

For largest possible distance between plane (1) and L_2 the line L_2 must be parallel to plane (1)

$\therefore a + c = 0$ ...(3)

$\Rightarrow \boxed{b = 0}$

$\therefore$ Plane H_0 : $\boxed{x - z = 0}$

Now d (H_0) = $\perp$ distance from point (0, 1, –1) on L_2 to plane.

$\Rightarrow d(H_0) = \left|\frac{0+1}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$

$\therefore P \to 5$

for 'Q' distance = $\left|\frac{2}{\sqrt{2}}\right| = \sqrt{2}$

$\therefore Q \to 4$

$\therefore$ (0, 0, 0) lies on plane

$\therefore R \to 3$

for 'S' $x = z$; $y = z$; $x = 1$

$\therefore$ point of intersection p (1, 1, 1).

$\therefore OP = \sqrt{1+1+1} = \sqrt{3}$

$\therefore S \to 2$

$\therefore$ option [B] is correct

26. **Ans. (B)**

$P(\hat{i} + 2\hat{j} - 5\hat{k}) = P(\vec{a})$

$Q(3\hat{i} + 6\hat{j} + 3\hat{k}) = Q(\vec{b})$

$R\left(\frac{17}{5}\hat{i} + \frac{16}{5}\hat{j} + 7\hat{k}\right) = R(\vec{c})$

$S(2\hat{i} + \hat{j} + \hat{k}) = S(\vec{d})$

$\frac{\vec{b} + 2\vec{d}}{3} = \frac{7\hat{i} + 8\hat{j} + 5\hat{k}}{3}$

$\frac{5\vec{c} + 4\vec{a}}{9} = \frac{21\hat{i} + 24\hat{j} + 15\hat{k}}{9}$

$\Rightarrow \frac{\vec{b} + 2\vec{d}}{3} = \frac{5\vec{c} + 4\vec{a}}{9}$

so [B] is correct.

option –D

$\left|\vec{b}\times\vec{d}\right|^2 = \left|\vec{b}\right|\left|\vec{d}\right|^2 - \left(\vec{b}.\vec{d}\right)^2 = (9 + 36 + 9)(4 + 1 + 1) - (6 + 6 + 3)^2$

$= 54 \times 6 - (15)^2 = 324 - 225 = 99$

JEE (Main) Practice Paper

SECTION–A

1. Ans. (1)

Let sides of ΔABC

$\Rightarrow \vec{a} = -3\hat{i} + 2\hat{j} + 12\hat{k} \Rightarrow |\vec{a}| = \sqrt{144+4+9} = \sqrt{157}$

$\Rightarrow \vec{b} = -\hat{i} - 4\hat{j} - 8\hat{k} \Rightarrow |\vec{b}| = \sqrt{64+16+1} = 9$

$\Rightarrow \vec{c} = 4\hat{i} + 2\hat{j} - 4\hat{k} \Rightarrow |\vec{c}| = \sqrt{16+16+4} = 6$

Hence perimeter is $15 + \sqrt{157}$.

2. Ans. (2)

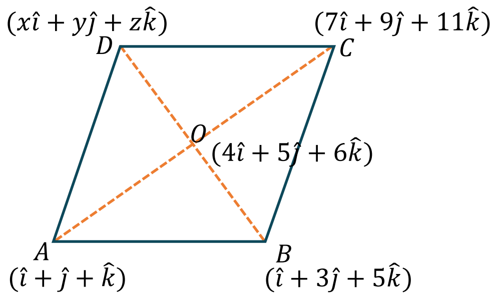
Using section formula

$\Rightarrow x + 1 = 8$

$\Rightarrow y + 3 = 10$

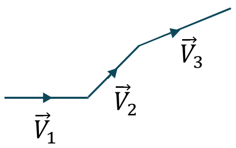
$\Rightarrow z + 5 = 12$

$\Rightarrow \ (7\hat{i} + 7\hat{j} + 7\hat{k}D)$



3. Ans. (2)

Clearly Δ is not possible.



Since $\vec{v}_3 = \vec{v}_1 + \vec{v}_2$

Hence $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are coplanar

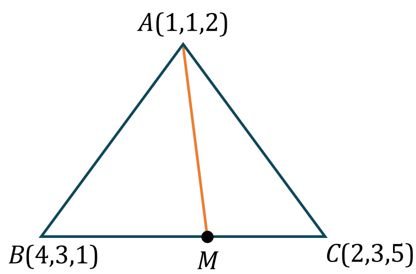
4. Ans. (4)

$\Rightarrow AB = \sqrt{9+4+1} = \sqrt{14}$

$\Rightarrow AC = \sqrt{1+4+9} = \sqrt{14}$

$\Rightarrow M \equiv (3, 3, 3)$

$\Rightarrow \overrightarrow{AM} = 2\hat{i} + 2\hat{j} + \hat{k}$



5. Ans. (2)

$$\cos\theta = \frac{(\hat{i} + 2\hat{j} + 3\hat{k}).(3\hat{i} + 2\hat{j} + \hat{k})}{\sqrt{9+5}\sqrt{9+5}} = \frac{3+4+3}{14} = \frac{5}{7}$$

$$\Rightarrow \sin\theta = \frac{2\sqrt{6}}{7}$$

6. Ans. (4)

$|\vec{x}|=|\vec{y}|=1$

$\Rightarrow |\vec{x}-\vec{y}|^2 = |\vec{x}|^2+|\vec{y}|^2 - 2|\vec{x}||\vec{y}|\cos\theta$

$=2\left(2\sin^2\frac{\theta}{2}\right) \Rightarrow \frac{1}{2}|\vec{x}-\vec{y}|=\left|\sin\frac{\theta}{2}\right|$

7. Ans. (1)

diagonals are $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$

$\Rightarrow \cos\theta=\frac{(\vec{a}+\vec{b}).(\vec{a}-\vec{b})}{|\vec{a}+\vec{b}|\ |\vec{a}-\vec{b}|}=\frac{1}{3}$

$\therefore\ \theta=\cos^{-1}\left(\frac{1}{3}\right)$

8. Ans. (4)

$\Rightarrow \vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow \vec{a}+\vec{b}=-\vec{c}$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2\vec{a}.\vec{b}=|\vec{c}|^2$ (taking mode and then square both sides)

$\Rightarrow 9+25+2\times3\times5\cos\theta=49$

$\Rightarrow \cos\theta=\frac{1}{2}$

$\Rightarrow \theta=\frac{\pi}{3}$

9. Ans. (4)

$|\vec{a}|=3, |\vec{b}|=4, |\vec{c}|=5$

$$\left.\begin{array}{l}\vec{a}.\vec{b}+\vec{a}.\vec{c}=0\\ \vec{b}.\vec{c}+\vec{a}.\vec{b}=0\\ \vec{c}.\vec{a}+\vec{b}.\vec{c}=0\end{array}\right]\Rightarrow \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{a}.\vec{c}=0$$

$\vec{a}+\vec{b}+\vec{c}=\sqrt{9+16+25}=5\sqrt{2}$

10. Ans. (1)

$\vec{a}+\vec{b} = 5\hat{i}-3\hat{j}+3\hat{k}$

Projection of $\vec{a}+\vec{b}$ on $\vec{c}$

$$\frac{(\vec{a}+\vec{b}).\vec{c}}{|\vec{c}|} = (5\hat{i}-3\hat{j}+3\hat{k}).\frac{(\hat{i}-2\hat{j}+2\hat{k})}{3} = \frac{5+6+6}{3} = \frac{17}{3}$$

11. Ans. (1)

$\overrightarrow{AB}=(-\hat{i}-2\hat{j}+2\hat{k})$

$\overrightarrow{CD}=(-3\hat{i}+6\hat{j}-2\hat{k})$

Projection = $\frac{\overrightarrow{CD}\ .\ \overrightarrow{AB}}{\left|\overrightarrow{AB}\right|}=\frac{3-12-4}{3}=-\frac{13}{3}$

12. Ans. (3)

$\vec{a} = \hat{i} + \hat{j}$

$\vec{b} = 3\hat{i} + 4\hat{k}$

$\vec{b}_2 = \vec{b} - \vec{b}_1 = \frac{-3\hat{i}}{2} + \frac{3\hat{j}}{2} + 4\hat{k}$

13. Ans. (4)

(1) $\left|\frac{2\hat{i} - 2\hat{j} + \hat{k}}{3}\right| = 1$

(2) $\frac{1}{3}(2\hat{i} - 2\hat{j} + \hat{k}) = -\frac{2}{3}\left(-\hat{i} + \hat{j} - \frac{\hat{k}}{2}\right)$

(3) $\frac{1}{3}(2\hat{i} - 2\hat{j} + \hat{k}) \ . \ (3\hat{i} + 2\hat{j} - 2\hat{k}) = 0$

14. Ans. (3)

$\vec{A}.\vec{B} = \vec{A}.\vec{C} \Rightarrow \vec{A}.\vec{B} - \vec{A}.\vec{C} = 0$

Or $\vec{A}.(\vec{B} - \vec{C}) = 0 \Rightarrow \vec{A} \perp (\vec{B} - \vec{C})$ ...(i)

$\vec{A} \times \vec{B} = \vec{A} \times \vec{C} \Rightarrow \vec{A} \times \vec{B} - \vec{A} \times \vec{C} = \vec{0}$

Or $\vec{A} \times (\vec{B} - \vec{C}) = \vec{0} \Rightarrow \vec{A} \,||\, (\vec{B} - \vec{C})$ ...(ii)

(i) & (ii) both possible if $\vec{B} - \vec{C} = \vec{0}$

i.e. $\vec{B} = \vec{C}$

15. Ans. (4)

$\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ ...(i)

& $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ ...(ii)

(i) – (ii)

$\Rightarrow \vec{a} \times \vec{b} - \vec{a} \times \vec{c} = \vec{c} \times \vec{d} - \vec{b} \times \vec{d}$

$\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = (\vec{c} - \vec{b}) \times \vec{d} \Rightarrow (\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = 0$

16. Ans. (3)

$\left[(\vec{a} + 2\vec{b} - \vec{c})(\vec{a} - \vec{b})(\vec{a} - \vec{b} - \vec{c})\right]$

$$= \begin{vmatrix} 1 & 2 & -1 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{vmatrix} \left[\begin{matrix} \vec{a} & \vec{b} & \vec{c} \end{matrix}\right] = 3\left[\vec{a}\ \vec{b}\ \vec{c}\right]$$

17. Ans. (4)

Let $\vec{v} = \lambda\left[(\hat{i} + \hat{j} + \hat{k}) \times \left\{(\hat{i} + \hat{j} + \hat{k}) \times (2\hat{i} - 3\hat{j})\right\}\right]$

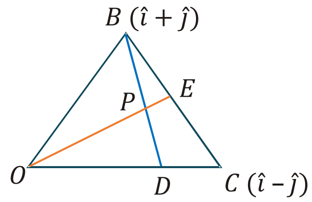
$\vec{v} = -7\hat{i} + 8\hat{j} - \hat{k}$

required vector is $3\hat{v} = \frac{3}{\sqrt{114}}\left(-7\hat{i} + 8\hat{j} - \hat{k}\right)$

18. Ans. (1)

$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = -8\hat{i} - 3\hat{j} - \hat{k}$

19. Ans. (1)



Here $D\left(\dfrac{2\hat{i}-2\hat{j}}{3}\right)$ and $E\left(\dfrac{\hat{i}-\hat{j}+2\hat{i}+2\hat{j}}{3}\right)$ or $E\left(\dfrac{3i+j}{3}\right)$

Now, let P divides $\overrightarrow{BD}$ in $\lambda : 1$ and P divides $\overrightarrow{OE}$ in $1:\mu$.

So, $p\left(\dfrac{\lambda\dfrac{(2\hat{i}-2\hat{j})}{3}+\hat{i}+\hat{j}}{\lambda+1}\right) = \dfrac{3\hat{i}+\hat{j}}{3(\mu+1)}$

Solving we get $\lambda = 3 : 4$

$\mu = 1 : 6$.

20. Ans. (1)

Let $\vec{a}=(\vec{a}\cdot\hat{i})\hat{i}+(\vec{a}\cdot\hat{j})\hat{j}+(\vec{a}\cdot\hat{k})\hat{k} = (1)\hat{i}+(1)\hat{j}+(1)\hat{k}=\hat{i}+\hat{j}+\hat{k}$

$\vec{b}=(\vec{b}.\hat{i})\hat{i}+(\vec{b}.\hat{j})\hat{j}+(\vec{b}.\hat{k})\hat{k}=(1)\hat{i}+(-1)\hat{j}+0\hat{k}=\hat{i}-\hat{j}$

and $\vec{c}=\hat{i}+\hat{j}-2\hat{k}$

Then, $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$

$\therefore$ $\vec{a},\vec{b}$ and $\vec{c}$ are mutually perpendicular.

Also, $\vec{a}\cdot(\vec{b}\times\vec{c}) = 6$

$\therefore$ $\vec{a},\vec{b},\vec{c}$, form the parallelepiped of volume 6 units.

SECTION–B

1. Ans. (3)

$(\vec{a}\times\vec{b})^2+(\vec{a}\cdot\vec{b})^2=144$

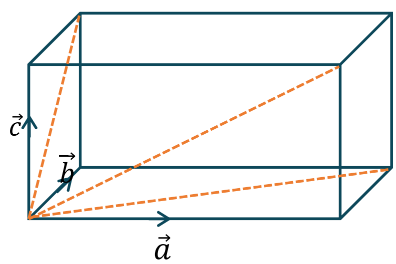
$= |\vec{a}|^2|\vec{b}|^2(\sin^2\theta+\cos^2\theta)=144$

$\Rightarrow |\vec{a}||\vec{b}|=12$

$\Rightarrow |\vec{b}|=\dfrac{12}{4}=3$

2. Ans. (2)

Diagonals of faces of given parallelepiped are $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$

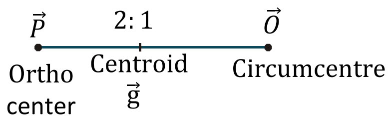


Volume of parallelepiped using these vectors

$=[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}\ ]=\begin{vmatrix}1&1&0\\0&1&1\\1&0&1\end{vmatrix}[\vec{a}\,\vec{b}\,\vec{c}]$

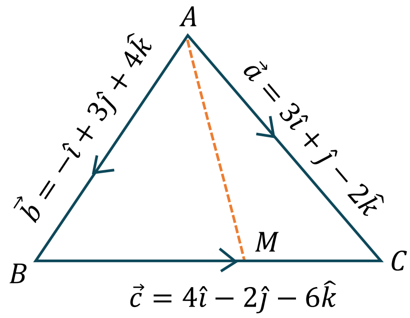
$=2[\vec{a}\,\vec{b}\,\vec{c}]$

3. Ans. (3)



$$\vec{g}=\frac{2.\vec{0}+1.\vec{p}}{2+1} \Rightarrow \vec{p}=3\vec{g} \qquad \therefore\ k=3$$

4. Ans. (6)



Here $\vec{a}=\vec{b}+\vec{c}$

$$\overrightarrow{AM}=\frac{1}{2}(\vec{a}+\vec{b})=\frac{1}{2}[2\hat{i}+4\hat{j}+2\hat{k}]=\hat{i}+2\hat{j}+\hat{k} \Rightarrow \lambda=\sqrt{6}$$

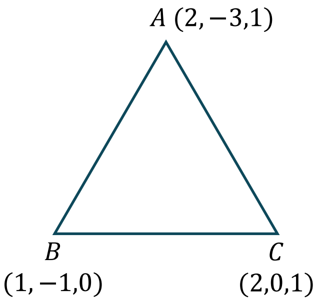
5. Ans. (5)

$$(\vec{a}+P\vec{b}).\vec{c}=0 \Rightarrow P=-\frac{\vec{a}.\vec{c}}{\vec{b}.\vec{c}}=5$$

6. Ans. (18)

Solving the lines we get,

$$A=\frac{1}{2}\left|\overrightarrow{BC}\times\overrightarrow{AC}\right|=\frac{3}{\sqrt{2}}$$



$\Rightarrow 4A^2 = 18$

7. Ans. (0)

$\because\ \vec{d}$ is $\perp$ to $\vec{a},\vec{b},\vec{c}$

$\Rightarrow \vec{a},\vec{b},\vec{c}$ are coplanar $\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$

$$\Rightarrow \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}=0$$

8. **Ans. (3)**

$(\hat{a}\times(\hat{b}\times\hat{c}))\times\hat{a}=\vec{0}$

$\Rightarrow \hat{b}\times\hat{c}-[\hat{a}\ \hat{b}\ \hat{c}]\hat{a}=\vec{0} \Rightarrow \hat{b}\times\hat{c}=[\hat{a}\ \hat{b}\ \hat{c}]\hat{a}$

squaring

$\Rightarrow 1-(\hat{b}.\hat{c})^2=[\hat{a}\hat{b}\hat{c}]^2 \Rightarrow |\hat{a}\ \hat{b}\ \hat{c}|=\frac{3}{4}$

$V=\left|[\hat{a}+\hat{b}\ \hat{b}+\hat{c}\ \hat{c}+\hat{a}]\right|=|2[\hat{a}\ \hat{b}\ \hat{c}]|^2 = 3$

then volume of the parallelepiped formed by $\hat{a}+\hat{b},\hat{b}+\hat{c},\hat{c}+\hat{a}$ is equal to

9. **Ans. (2)**

$|a| = 1 = |b|, |c| = 2$

$\vec{b}+(\vec{a}.\vec{c})\vec{a}=(\vec{a}.\vec{a})\vec{c}$

Let angle between $\vec{a}$ & $\vec{c}$ be θ.

$\vec{a}.\vec{c}=|a||c|\cos\theta=2\cos\theta$

$\vec{b}+2\cos\theta\vec{a}=\vec{c} \Rightarrow \vec{b}=\vec{c}-2\cos\theta\vec{a}$

squaring

$b^2=c^2+4\cos^2\theta a^2-4\cos\theta\vec{a}.\vec{c}$

$\Rightarrow \cos^2\theta=1-\frac{1}{4}=\frac{3}{4} \Rightarrow \cos\theta=\pm\frac{\sqrt{3}}{2}$

$\Rightarrow$ cosecθ = 2

10. **Ans. (2)**

$|\vec{a}|=|\vec{b}| \Rightarrow 25 + p^2 + 144 = 1 + 169 + 4q$

$\Rightarrow p^2 = 4q + 1$

p is odd integer where $p = (2k + 1)$

$\Rightarrow (2k + 1)^2 = 4q + 1 \Rightarrow q = k(k + 1)$

$\therefore 1\le k(k+1)\le 100$

Number of possible values of k is 31

$\frac{(k+1)}{16}=\frac{(31+1)}{16}=2$

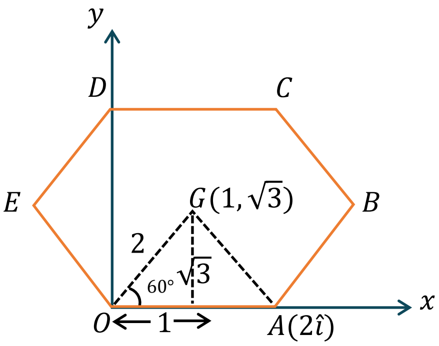
JEE (Advanced) Practice Paper

1. **Ans. (C)**

$\vec{b}=\lambda(2\sqrt{2}\ \hat{i}-\hat{j}+4\hat{k})$; $|\vec{b}|=10$

$\Rightarrow |\lambda|\ \sqrt{8+1+16}=10 \Rightarrow \lambda=\pm 2 \Rightarrow \vec{b}=\pm\ 2\vec{a}$

2. **Ans. (C)**



$G \equiv (\hat{i}+\sqrt{3}\,\hat{j})$

Let Position vector of P is $\vec{p}$

$\because\ \overrightarrow{GP}\,||\,\hat{k}$ then $\vec{p}-(\hat{i}+\sqrt{3}\hat{j}) = \lambda\hat{k}$

$\Rightarrow\ \vec{p} = \hat{i}+\sqrt{3}\,\hat{j}\ +\ \lambda\hat{k}$ also $\left|\overrightarrow{OP}\right| = 3 \Rightarrow \sqrt{1+3+\lambda^2}=3$

$\Rightarrow \lambda^2 = 5 \Rightarrow \lambda=\pm\ \sqrt{5}\ \Rightarrow\ \vec{p}\ =\ \hat{i}+\sqrt{3}\,\hat{j}\pm\sqrt{5}\,\hat{k}$

For positive Z-axis $\vec{p}=\hat{i}+\sqrt{3}\,\hat{j}+\sqrt{5}\,\hat{k}$.

So $\overrightarrow{AP}\ =\ \vec{p}-2\hat{i}\ = -\hat{i}+\sqrt{3}\hat{j}+\sqrt{5}\,\hat{k}$

3. **Ans. (B)**

$|\vec{a}+\vec{b}|^2=100\ =\ \vec{a}^2+\vec{b}^2+2\vec{a}\cdot\vec{b}$

$|\vec{a}-\vec{b}|^2=64\ =\ \vec{a}^2+\vec{b}^2-2\vec{a}\ \cdot\ \vec{b}$

$\Rightarrow 164 = 2(\vec{a}^2+\vec{b}^2) \Rightarrow\ \vec{b}^2=82-25=57$

4. **Ans. (C)**

$\vec{c}.\vec{a}=\vec{a}.(\vec{a}\times\vec{b})\ \Rightarrow\ \vec{c}.\vec{a}=0=\vec{c}.\vec{b}=\vec{a}.\vec{b}$

Also $|\vec{a}\times\vec{b}|=|\vec{c}|\ \Rightarrow\ |\vec{a}||\vec{b}|\sin 90^\circ=|\vec{c}|$

$|\vec{a}|^2=|\vec{a}|\Rightarrow\ |\vec{a}|=|\vec{b}|=|\vec{c}|=1$

$|3\vec{a}+4\vec{b}+12\vec{c}|=\sqrt{9a^2+16b^2+144c^2}=13 \qquad \{\because|\vec{a}|=|\vec{b}|=|\vec{c}|=1\}$

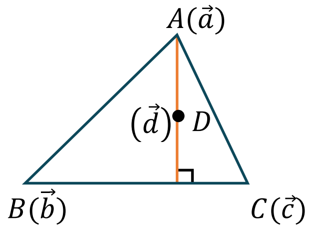
5. **Ans. (C)**

diagonals are $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$

$\because$ and $\vec{a}-\vec{b}=\hat{i}+2\hat{j}+2\hat{k}$

$$\Rightarrow\theta=\cos^{-1}\frac{(\vec{a}+\vec{b}).(\vec{a}-\vec{b})}{|(\vec{a}+\vec{b})||(\vec{a}-\vec{b})|}=\cos^{-1}\left(\frac{3}{3\times 3}\right)\Rightarrow\theta=\cos^{-1}\frac{1}{3}=\sin^{-1}\sqrt{\frac{8}{9}}$$

6. **Ans. (C)**



Given

$(\vec{a}-\vec{d})\cdot(\vec{b}-\vec{c})=(\vec{b}-\vec{d})\ .\ (\vec{c}-\vec{a})=0$

$\Rightarrow\overrightarrow{DA}.\overrightarrow{CB}=0\ \ \&\ \overrightarrow{DB}.\overrightarrow{AC}=0$

$AD\perp BC\ \&\ BD\perp AC$

Hence D is orthocentre.

7. **Ans. (A,B)**

Before rotation $\vec{a}=2p\hat{i}+\hat{j}$ after rotation $\vec{a}=(p+1)\hat{i}'+\hat{j}'$. Since length of vector remains unaltered

$\sqrt{4p^2+1} = \sqrt{(p+1)^2+1}$

$\Rightarrow 4p^2=(p+1)^2 \Rightarrow p+1=\pm 2p \Rightarrow p=1$ or $-\frac{1}{3}$

8. **Ans. (B,D)**

Since $\vec{a}$ makes obtuse angle with z-axis

$\therefore \frac{\sin 2\alpha}{\sqrt{1+9+\sin^2 2\alpha}}<0$ i.e. $\sin 2\alpha < 0$

$\therefore$ either $\frac{\pi}{2} < \alpha < \pi$ or $\frac{3\pi}{2} < \alpha < 2\pi$ ...(i)

since $\vec{b}$ and $\vec{c}$ are orthogonal

$\therefore \tan^2\alpha - \tan\alpha - 6 = 0$ i.e. $\tan\alpha = 3, -2$ ...(ii)

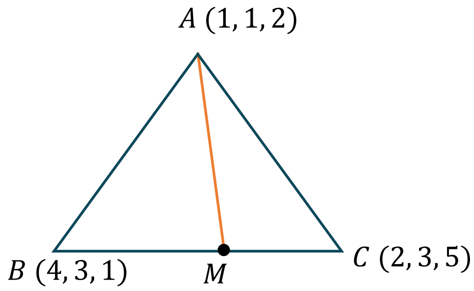
from (i) and (ii), we get

$\tan\alpha = -2$

$\therefore \alpha = \pi - \tan^{-1}2$ or $\alpha = 2\pi - \tan^{-1}2$

9. **Ans. (C,D)**

$AB = \sqrt{9+4+1}=\sqrt{14}$; $AC = \sqrt{1+4+9}=\sqrt{14}$



$M \equiv (3, 3, 3)$; $\overrightarrow{AM} = 2\hat{i}+2\hat{j}+\hat{k}$

10. **Ans. (A,C)**

any such vector $\vec{c} = \lambda\ (\hat{a}+\hat{b}) = \lambda \left(\frac{7\hat{i}-4\hat{j}-4\hat{k}}{9}+\frac{-2\hat{i}-\hat{j}+2\hat{k}}{3}\right) = \frac{\lambda}{9}\left[7\hat{i}-4\hat{j}-4\hat{k}+3(-2\hat{i}-\hat{j}+2\hat{k})\right]$

$=\frac{\lambda}{9}\left[\hat{i}-7\hat{j}+2\hat{k}\right] \Rightarrow |\vec{c}|=5\sqrt{6} \Rightarrow \left|\frac{\lambda}{9}\sqrt{1+49+4}\right|=5\sqrt{6}$

$\Rightarrow \left|\frac{\lambda}{9}\sqrt{54}\right|=5\sqrt{6} \Rightarrow \lambda = \pm\frac{9\times 5\sqrt{6}}{\sqrt{54}} = \pm 15$

$\Rightarrow \vec{c} = \pm\frac{15}{9}(\hat{i}-7\hat{j}+2\hat{k}) = \pm\frac{5}{3}(\hat{i}-7\hat{j}+2\hat{k})$

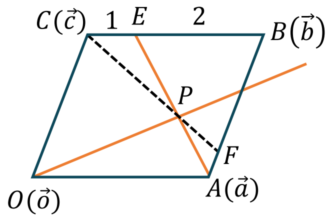
11. **Ans. (A)**

12. **Ans. (A)**

13. **Ans (D)**

Sol. for 11 to 13

$E=\dfrac{2\vec{c}+\vec{b}}{3}$



equation of OP $\vec{r}=\lambda\left(\dfrac{\vec{a}}{|\vec{a}|}+\dfrac{\vec{c}}{|\vec{c}|}\right)$ ...(1)

Let P divide EA in $\mu:1 \Rightarrow P\left[\dfrac{\mu\vec{a}+\dfrac{2\vec{c}+\vec{b}}{3}}{\mu+1}\right]$

P lies on (1)

$$\frac{\mu\vec{a}+\dfrac{2\vec{c}+\vec{b}}{3}}{\mu+1}=\lambda\left(\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{c}}{|\vec{c}|}\right) \Rightarrow \vec{a}+\vec{c}=\vec{b} \Rightarrow \frac{\mu\vec{a}+\dfrac{3\vec{c}+\vec{a}}{3}}{\mu+1}=\lambda\left(\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{c}}{|\vec{c}|}\right)$$

Comparing coefficient of $\vec{a}$ and $\vec{c}$

$\dfrac{\mu+\dfrac{1}{3}}{\mu+1}=\dfrac{\lambda}{|\vec{a}|}$ ...(2)

and $\dfrac{1}{\mu+1}=\dfrac{\lambda}{|\vec{c}|}$ ...(3)

divided (2) by (3)

$\mu+\dfrac{1}{3}=\dfrac{|\vec{c}|}{|\vec{a}|} \Rightarrow \mu=\dfrac{|\vec{c}|}{|\vec{a}|}-\dfrac{1}{3}$

Put in (3) $\Rightarrow \dfrac{1}{\dfrac{|\vec{c}|}{|\vec{a}|}+\dfrac{2}{3}}=\dfrac{\lambda}{|\vec{c}|} \Rightarrow \lambda=\dfrac{3|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}$

So position vector of P $\vec{r}=\dfrac{3|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left(\dfrac{\vec{a}}{|\vec{a}|}+\dfrac{\vec{c}}{|\vec{c}|}\right)$. Now for solution of 4 equation of AB

$\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})=\vec{a}+\lambda(\vec{c})$ ...(4)

equation of $CP \Rightarrow \vec{r}=\vec{c}+\mu\left(\dfrac{3|\vec{c}|\vec{a}}{3|\vec{c}|+2|\vec{a}|}+\dfrac{3|\vec{a}|\vec{c}}{3|\vec{c}|+2|\vec{a}|}-\vec{c}\right)$

$\vec{r}=\vec{c}+\mu\left[\dfrac{3|\vec{c}|\vec{a}+3|\vec{a}|\vec{c}-3|\vec{c}|\vec{c}-2|\vec{a}|\vec{c}}{3|\vec{c}|+2|\vec{a}|}\right]$

$\Rightarrow r=\vec{c}+\mu\left[\dfrac{3|\vec{c}|\vec{a}+|\vec{a}|\vec{c}-3|\vec{c}|\vec{c}}{3|\vec{c}|+2|\vec{a}|}\right]$ ...(5)

Comparing (4) and (5)

$\lambda = 1 + \dfrac{\mu|\vec{a}| - 3\mu|\vec{c}|}{3|\vec{c}| + 2|\vec{a}|}$

$\Rightarrow \lambda = \dfrac{3|\vec{c}| + 2|\vec{a}| + \mu|\vec{a}| - 3\mu|\vec{c}|}{3|\vec{c}| + 2|\vec{a}|}$ ...(6)

$\mu = \dfrac{3|\vec{c}| + 2|\vec{a}|}{3|\vec{c}|}$

Put value of μ in equation 6

$\lambda = 1 + \dfrac{\mu(|\vec{a}| - 3|\vec{c}|)}{3|\vec{c}| + 2|\vec{a}|} \Rightarrow \lambda = 1 + \dfrac{|\vec{a}| - 3|\vec{c}|}{3|\vec{c}|} = \dfrac{1}{3}\dfrac{|\vec{a}|}{|\vec{c}|}$

So position vector of F is $= \vec{a} + \dfrac{1}{3}\dfrac{|\vec{a}|}{|\vec{c}|}\vec{c}$

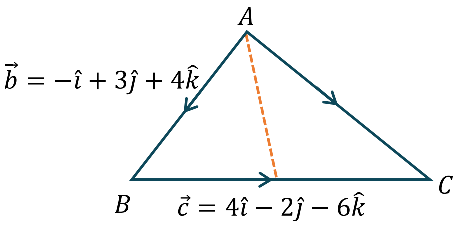
$\Rightarrow \overrightarrow{AF}$ = p.v. of F – p.v. of A $= \vec{a} + \dfrac{1}{3}\dfrac{|\vec{a}|}{|\vec{c}|}\vec{c} - \vec{a} = \dfrac{1}{3}\dfrac{|\vec{a}|}{|\vec{c}|}\vec{c}$

14. Ans. (B)

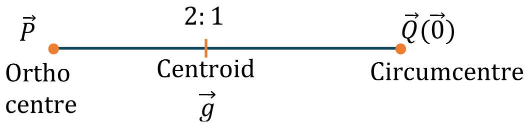
(P) here $\vec{a} = \vec{b} + \vec{c}$

$\overrightarrow{AM} = \dfrac{1}{2}(\vec{a} + \vec{b}) = \dfrac{1}{2}[2\hat{i} + 4\hat{j} + 2\hat{k}] = \hat{i} + 2\hat{j} + \hat{k}$

$\Rightarrow \lambda = \sqrt{6}$



(Q)



$\dfrac{\vec{g} = 2\times\vec{0} + 1\times\vec{p}}{3} \Rightarrow \vec{p} = 3\vec{g}$

(R) Area $= |\vec{a}\times\vec{b}| = |(\vec{p} + 2\vec{q})\times(2\vec{p} + \vec{q})|$

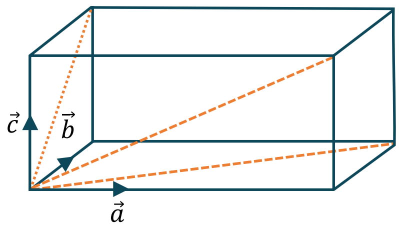
$= |\vec{p}\times\vec{q} + 4\vec{q}\times\vec{p}| = |3\vec{p}\times\vec{q}| = 3\times\dfrac{1}{2} = \dfrac{3}{2}$

(S) $\vec{u} + \vec{v} + \vec{w} = 0 \Rightarrow |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 + 2(\vec{u}.\vec{v}) + 2(\vec{v}.\vec{w}) + 2(\vec{w}.\vec{u}) = 0$

$\Rightarrow 9 + 16 + 25 + 2[\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}] = 0 \Rightarrow \sqrt{|\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}|} = 5$

15. Ans. (D)

(P) Diagonals of faces of given parallelepiped are $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$, $\vec{c} + \vec{a}$



Volume of parallelepiped using these vectors = $[\vec{a} + \vec{b} \;\; \vec{b} + \vec{c} \;\; \vec{c} + \vec{a}] = 2[\vec{a} \;\; \vec{b} \;\; \vec{c}]$

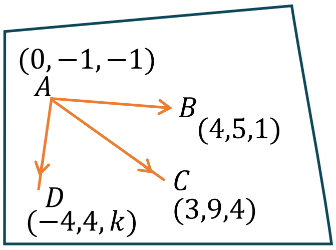
(Q) $\vec{b}\times(\vec{b}\times\vec{x})=\vec{b}\times\vec{a} \Rightarrow \beta\vec{b}-|\vec{b}|^2\,\vec{x}=\vec{b}\times\vec{a}$

$\vec{x}=\frac{\beta\vec{b}}{|\vec{b}|^2}+\frac{\vec{a}\times\vec{b}}{|\vec{b}|^2}$

$\beta^2-12=\beta$

$\Rightarrow\beta=4,-3$

(R) $\because\left[\overrightarrow{AD}\ \overrightarrow{AC}\ \overrightarrow{AB}\right]=0$



$\therefore\begin{vmatrix}-4 & 5 & k+1\\ 3 & 10 & 5\\ 4 & 6 & 2\end{vmatrix}=0$

$-4(20-30)-5(6-20)+(k+1)(18-40)=0$

$\Rightarrow 40+70-22(k+1)=0\Rightarrow k=4$

(S) $AB=2EF=2\sqrt{m^2+n^2}$

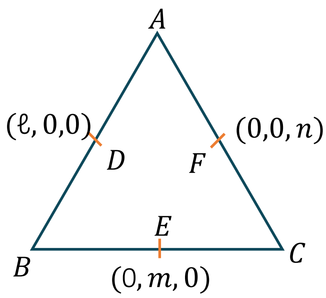
Similarly

$BC=2DF=2\sqrt{\ell^2+n^2}$

$CA=2\sqrt{\ell^2+m^2}$

$AB^2+BC^2+CA^2=8(\ell^2+m^2+n^2)$

$\therefore\frac{AB^2+BC^2+CA^2}{\ell^2+m^2+n^2}=8$



16. Ans. (9)

$(\vec{R}-\vec{C})\times\vec{B}=\vec{O} \Rightarrow \vec{R}=\vec{C}+\lambda\vec{B} \Rightarrow A.C+\lambda\,A.B=0 \Rightarrow 15+3\lambda=0$

$\Rightarrow\lambda=-5\Rightarrow\vec{R}=-\hat{i}-8\hat{j}+2\hat{k}$

17. Ans. (9)

Equation of line L_1 is $7\hat{i}+6\hat{j}+2\hat{k}+\lambda(-3\hat{i}+2\hat{j}+4\hat{k})$

Equation of line L_2 is $5\hat{i}+3\hat{j}+4\hat{k}+\mu(2\hat{i}+\hat{j}+3\hat{k})$

$\overrightarrow{CD}=2\hat{i}+3\hat{j}-2\hat{k}+\lambda(-3\hat{i}+2\hat{j}+4\hat{k})-\mu(2\hat{i}+\hat{j}+3\hat{k})$. since it is parallel to $2\hat{i}-2\hat{j}-\hat{k}$

$\therefore\frac{2-3\lambda-2\mu}{2}=\frac{3+2\lambda-\mu}{-2}=\frac{-2+4\lambda-3\mu}{-1}$ $\quad\therefore\lambda=2,\mu=1$

$\overrightarrow{CD}=-6\hat{i}+6\hat{j}+3\hat{k}$

$\therefore|\overrightarrow{CD}|=9$

18. Ans. (2)

$\vec{a}.(\vec{b}+\vec{c})=0$, $\vec{b}.(\vec{c}+\vec{a})=0$, $\vec{c}.(\vec{a}+\vec{b})=0$

$\Rightarrow\vec{a}.\vec{b}+\vec{a}.\vec{c}=0,\ \vec{b}.\vec{c}+\vec{b}.\vec{a}=0,\ \vec{c}.\vec{a}+\vec{c}.\vec{b}=0\Rightarrow\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$

$|\vec{a}+\vec{b}+\vec{c}|=\sqrt{\vec{a}^2+\vec{b}^2+\vec{c}^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})}=\sqrt{9+16+25}=\sqrt{50}$

$\Rightarrow m=2$