EXERCISE - O

SINGLE CORRECT TYPE QUESTIONS

1. Four points $A(+1, -1, 1)$; $B(1, 3, 1)$; $C(4, 3, 1)$ and $D(4, -1, 1)$ taken in order are the vertices of
(A) a parallelogram which is neither a rectangle nor a rhombus
(B) rhombus
(C) an isosceles trapezium
(D) a cyclic quadrilateral.

MVT097

2. Let $A(0, -1, 1)$, $B(0, 0, 1)$, $C(1, 0, 1)$ are the vertices of a ΔABC. If R and r denotes the circumradius and inradius of ΔABC, then $\frac{r}{R}$ has value equal to

(A) $\tan\frac{3\pi}{8}$ (B) $\cot\frac{3\pi}{8}$ (C) $\tan\frac{\pi}{12}$ (D) $\cot\frac{\pi}{12}$

MVT099

3. If the vector $6\hat{i} - 3\hat{j} - 6\hat{k}$ is decomposed into vectors parallel and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$ then the vectors are :

(A) $-(\hat{i} + \hat{j} + \hat{k})$ & $7\hat{i} - 2\hat{j} - 5\hat{k}$ (B) $-2(\hat{i} + \hat{j} + \hat{k})$ & $8\hat{i} - \hat{j} - 4\hat{k}$

(C) $+2(\hat{i} + \hat{j} + \hat{k})$ & $4\hat{i} - 5\hat{j} - 8\hat{k}$ (D) none

MVT101

4. A, B, C & D are four points in a plane with pv's $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ respectively such that $(\vec{a}-\vec{d})\cdot(\vec{b}-\vec{c})=(\vec{b}-\vec{d})\cdot(\vec{c}-\vec{a}) = 0$. Then for the triangle ABC, D is its

(A) incentre (B) circumcentre (C) orthocentre (D) centroid

MVT102

5. Let $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{a} + \hat{b} + \hat{c}$ is also a unit vector. If pairwise angles between $\hat{a}, \hat{b}, \hat{c}$ are θ_1, θ_2 and θ_3 respectively then $\cos\theta_1 + \cos\theta_2 + \cos\theta_3$ equals

(A) 3 (B) – 3 (C) 1 (D) – 1

MVT103

6. The vector equations of two lines L_1 and L_2 are respectively
$\vec{r} = 17\hat{i} - 9\hat{j} + 9\hat{k} + \lambda(3\hat{i} + \hat{j} + 5\hat{k})$ and $\vec{r} = 15\hat{i} - 8\hat{j} - \hat{k} + \mu(4\hat{i} + 3\hat{j})$

I L_1 and L_2 are skew lines
II $(11, -11, -1)$ is the point of intersection of L_1 and L_2
III $(-11, 11, 1)$ is the point of intersection of L_1 and L_2
IV $\cos^{-1}\left(3/\sqrt{35}\right)$ is the acute angle between L_1 and L_2

then , which of the following is true?
(A) II and IV (B) I and IV (C) IV only (D) III and IV

MVT106

7. Let $\vec{u},\vec{v},\vec{w}$ be such that $|\vec{u}|=1$, $|\vec{v}|=2$, $|\vec{w}|=3$. If the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and vectors $\vec{v}$, $\vec{w}$ are perpendicular to each other then $|\vec{u}-\vec{v}+\vec{w}|$ equals

(A) 2 (B) $\sqrt{7}$ (C) $\sqrt{14}$ (D) 14

MVT107

8. The vectors $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$; $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ & $\vec{c}=3\hat{i}+\hat{j}+4\hat{k}$ are so placed that the end point of one vector is the starting point of the next vector. Then the vectors are -

(A) not coplanar
(B) coplanar but cannot form a triangle
(C) coplanar but can form a triangle
(D) coplanar & can form a right angled triangle

MVT112

9. Given the vectors

$\vec{u}=2\hat{i}-\hat{j}-\hat{k}$

$\vec{v}=\hat{i}-\hat{j}+2\hat{k}$

$\vec{w}=\hat{i}-\hat{k}$

If the volume of the parallelopiped having $-c\vec{u}$, $\vec{v}$ and $c\vec{w}$ as concurrent edges, is 8 then 'c' can be equal to

(A) ± 2 (B) 4 (C) 8 (D) can not be determined

MVT113

10. Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$; $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$; $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ & $\vec{b}$. If the angle between $\vec{a}$ & $\vec{b}$ is $\frac{\pi}{6}$, then

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2 =$$

(A) 0
(B) 1
(C) $(a_1^2+a_2^2+a_3^2)\,(b_1^2+b_2^2+b_3^2)$
(D) $(a_1^2+a_2^2+a_3^2)\,(b_1^2+b_2^2+b_3^2)\,(c_1^2+c_2^2+c_3^2)$

MVT114

11. The altitude of a parallelopiped whose three coterminous edges are the vectors, $\vec{A}=\hat{i}+\hat{j}+\hat{k}$; $\vec{B}=2\hat{i}+4\hat{j}-\hat{k}$ & $\vec{C}=\hat{i}+\hat{j}+3\hat{k}$ with $\vec{A}$ and $\vec{B}$ as the sides of the base of the parallelopiped, is

(A) $2/\sqrt{19}$ (B) $4/\sqrt{19}$ (C) $2\sqrt{38}/19$ (D) none

MVT116

MULTIPLE CORRECT TYPE QUESTIONS

12. If $\vec{a}, \vec{b}, \vec{c}$ be three non zero vectors satisfying the condition $\vec{a} \times \vec{b} = \vec{c}$ & $\vec{b} \times \vec{c} = \vec{a}$ then which of the following always hold(s) good?

(A) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs

(B) $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \left|\vec{b}\right|$

(C) $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \left|\vec{c}\right|^2$

(D) $\left|\vec{b}\right| = \left|\vec{c}\right|$

MVT117

13. If $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ are four non zero vectors in the same plane no two of which are collinear then which of the following hold(s) good?

(A) $(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{D})=0$

(B) $(\vec{A}\times\vec{C})\cdot(\vec{B}\times\vec{D})\neq 0$

(C) $(\vec{A}\times\vec{B})\times(\vec{C}\times\vec{D})=\vec{0}$

(D) $(\vec{A}\times\vec{C})\times(\vec{B}\times\vec{D})\neq\vec{0}$

MVT118

14. If $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ are the pv's of the points A, B, C & D respectively in three dimensional space & satisfy the relation $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = \vec{0}$, then :

(A) A, B, C & D are coplanar

(B) the line joining the points B & D divides the line joining the point A & C in the ratio 2 : 1.

(C) the line joining the points A & C divides the line joining the points B & D in the ratio 1 : 1

(D) the four vectors $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ are linearly dependent.

MVT119

15. Let OAB be a regular triangle with side length unity (O being the origin). Also M, N are the points of trisection of AB, M being closer to A and N closer to B. Position vectors of A, B, M and N are $\vec{a}, \vec{b}, \vec{m}$ and $\vec{n}$ respectively. Which of the following hold(s) good ?

(A) $\vec{m} = x\vec{a} + y\vec{b} \Rightarrow \frac{2}{3}$ and $y = \frac{1}{3}$

(B) $\vec{m} = x\vec{a} + y\vec{b} \Rightarrow x = \frac{5}{6}$ and $y = \frac{1}{6}$

(C) $\vec{m}.\vec{n}$ equals $\frac{13}{18}$

(D) $\vec{m}.\vec{n}$ equals $\frac{15}{18}$

MVT122

16. Let $\vec{a}$ and $\vec{b}$ be two non-zero and non-collinear vectors then which of the following is/are always correct ?

(A) $\vec{a}\times\vec{b} = [\vec{a}\ \vec{b}\ \hat{i}]\hat{i} + [\vec{a}\ \vec{b}\ \hat{j}]\hat{j} + [\vec{a}\ \vec{b}\ \hat{k}]\hat{k}$

(B) $\vec{a}.\vec{b} = (\vec{a}.\hat{i})(\vec{b}.\hat{i}) + (\vec{a}.\hat{j}).(\vec{b}.\hat{j}) + (\vec{a}.\hat{k})(\vec{b}.\hat{k})$

(C) if $\vec{u} = \hat{a} - (\hat{a}.\hat{b})\hat{b}$ and $\vec{v} = \hat{a}\times\hat{b}$ then $|\vec{u}| = |\vec{v}|$

(D) if $\vec{c} = \vec{a}\times(\vec{a}\times\vec{b})$ and $\vec{d} = \vec{b}\times(\vec{a}\times\vec{b})$ then $\vec{c}+\vec{d}=\vec{0}$

MVT124

17. The value(s) of $\alpha \in [0, 2\pi]$ for which vector $\vec{a} = \hat{i} + 3\hat{j} + (\sin 2\alpha)\hat{k}$ makes an obtuse angle with the z-axis and the vectors $\vec{b} = (\tan\alpha)\hat{i} - \hat{j} + 2\sqrt{\sin\frac{\alpha}{2}}\,\hat{k}$ and $\vec{c} = (\tan\alpha)\hat{i} + (\tan\alpha)\hat{j} - 3\sqrt{\operatorname{cosec}\frac{\alpha}{2}}\,\hat{k}$ are orthogonal, is/are

(A) $\tan^{-1} 3$ (B) $\pi - \tan^{-1} 2$ (C) $\pi + \tan^{-1} 3$ (D) $2\pi - \tan^{-1} 2$

MVT125

18. Which of the followings is/are correct :

(A) The angle between the two straight lines $\vec{r} = 3\hat{i} - 2\hat{j} + 4\hat{k} + \lambda(-2\hat{i} + \hat{j} + 2\hat{k})$ and $\vec{r} = \hat{i} + 3\hat{j} - 2\hat{k} + \mu(3\hat{i} - 2\hat{j} + 6\hat{k})$ is $\cos^{-1}\left(\frac{4}{21}\right)$

(B) $(\vec{r}.\hat{i})(\hat{i} \times \vec{r}) + (\vec{r}.\hat{j})(\hat{j} \times \vec{r}) + (\vec{r}.\hat{k})(\hat{k} \times \vec{r}) = \vec{0}$

(C) The force determined by the vector $\vec{r}$ = (1, –8, –7) is resolved along three mutually perpendicular directions, one of which is in the direction of the vector $\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$. Then the vector component of the force $\vec{r}$ in the direction of the vector $\vec{a}$ is $-\frac{7}{3}(2\hat{i} + 2\hat{j} + \hat{k})$

(D) The cosine of the angle between any two diagonals of a cube is $\frac{1}{3}$.

MVT127

19. $\hat{a}$ and $\hat{b}$ are two given unit vectors at right angle. The unit vector equally inclined with $\hat{a}$, $\hat{b}$ and $\hat{a} \times \hat{b}$ will be :

(A) $-\frac{1}{\sqrt{3}}(\hat{a} + \hat{b} + \hat{a} \times \hat{b})$ (B) $\frac{1}{\sqrt{3}}(\hat{a} + \hat{b} + \hat{a} \times \hat{b})$

(C) $\frac{1}{\sqrt{3}}(\hat{a} + \hat{b} - \hat{a} \times \hat{b})$ (D) $-\frac{1}{\sqrt{3}}(\hat{a} + \hat{b} - \hat{a} \times \hat{b})$

MVT128

20. Let $\bar{p}, \bar{q}, \bar{r}$ are $\bar{\alpha} = \bar{p} \times (\bar{q} \times \bar{r})$ non zero vectors and $\bar{\beta} = (\bar{p} \times \bar{q}) \times \bar{r}$ and $\bar{\alpha} = \bar{\beta}$. If then which of the following holds goods

(A) $\bar{p}$ and $\bar{q}$ are orthogonal. (B) $\bar{p}$ and $\bar{r}$ are collinear

(C) $\bar{q}$ and $\bar{r}$ are orthogonal (D) $\bar{q} = \lambda(\bar{p} \times \bar{r})$ where 'λ' is scalar

MVT130

21. If $|\vec{a}| = |\vec{b}| = |\vec{c}| = |\vec{d}| = 1$ such that $(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d}) = 1$ and $\vec{a} \cdot \vec{c} = \frac{1}{2}$, then which of the following can be correct ?

(A) $\vec{a} \cdot \vec{b} = 0$ (B) $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$ (C) $\vec{a} \cdot (\vec{b} \times \vec{c}) \neq 0$ (D) $|\vec{a} + \vec{b} + \vec{c}| = \sqrt{4 + \sqrt{3}}$

MVT131

COMPREHENSION TYPE QUESTIONS

Paragraph for Question 22 to 23

Vectors are essential tools for undrstanding and solving problems in physics, engineering and mathematics. One key aspect of working with vectors is finding the angle between them, which can provide valuable information about the relationship between the two vectors. The dot product, also known as the scalar product or inner product, is a crucual operation that allows us to determine the angle between two vectors.

The dot product of two vectors $\vec{A}$ and $\vec{B}$ is defined as :

$\vec{A}\cdot\vec{B}=|\vec{A}||\vec{B}|\cos(\theta)$

where and represent the magnitudes of the vectors A and B, respectively and θ is the angle between the vectors.

From the definition above, we can easily deduce the formula to find the angle between the two vectors :

$\theta=\arccos\left[(\vec{A}\cdot\vec{B})/(|\vec{A}||\vec{B}|)\right]$

22. In a quadrilateral ABCD, $\vec{AC}$ is the bisector of the $\left(\vec{AB}\ \hat{}\ \vec{AD}\right)$ which is $\frac{2\pi}{3}$,

$15\left|\vec{AC}\right|=3\left|\vec{AB}\right|=5\left|\vec{AD}\right|$ then $\cos\left(\vec{BA}\ \hat{}\ \vec{CD}\right)$ is

(A) $-\frac{\sqrt{14}}{7\sqrt{2}}$ (B) $-\frac{\sqrt{21}}{7\sqrt{3}}$ (C) $\frac{2}{\sqrt{7}}$ (D) $\frac{2\sqrt{7}}{14}$

MVT132

23. If the two adjacent sides of two rectangles are represented by the vectors $\vec{p}=5\vec{a}-3\vec{b}$; $\vec{q}=-\vec{a}-2\vec{b}$ and $\vec{r}=-4\vec{a}-\vec{b}$; $\vec{s}=-\vec{a}+\vec{b}$ respectively, then the angle between the vectors $\vec{x}=\frac{1}{3}(\vec{p}+\vec{r}+\vec{s})$ and $\vec{y}=\frac{1}{5}(\vec{r}+\vec{s})$

(A) is $-\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

(B) is $\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

(C) is $\pi-\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

(D) cannot be evaluated

MVT133

MATCHING LIST TYPE QUESTION

24. **List-I** **List-II**

(I) P is point in the plane of the triangle ABC. pv's of A, B and C are $\vec{a}, \vec{b}$ and $\vec{c}$ respectively with respect to P as the origin. If $(\vec{b}+\vec{c}).(\vec{b}-\vec{c})=0$ and $(\vec{c}+\vec{a}).(\vec{c}-\vec{a})=0$, then w.r.t. the triangle ABC, P is its — (P) centroid

(II) If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the three non collinear points A, B and C respectively such that the vector $\vec{V}=\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}$ is a null vector then w.r.t. the ΔABC, P is its — (Q) orthocentre

(III) If P is a point inside the ΔABC such that the vector $\vec{R}=(BC)\overrightarrow{PA}+(CA)(\overrightarrow{PB})+(AB)(\overrightarrow{PC})$ is a null vector then w.r.t. the ΔABC, P is its — (R) Incentre

(IV) If P is a point in the plane of the triangle ABC such that the scalar product $\overrightarrow{PA}.\overrightarrow{CB}$ and $\overrightarrow{PB}.\overrightarrow{AC}$ vanishes, then w.r.t. the ΔABC, P is its — (S) circumcentre

(A) I $\rightarrow$ P; II $\rightarrow$ Q; III $\rightarrow$ R; IV $\rightarrow$ S
(B) I $\rightarrow$ S; II $\rightarrow$ P; III $\rightarrow$ R; IV $\rightarrow$ Q
(C) I $\rightarrow$ R; II $\rightarrow$ P; III $\rightarrow$ Q; IV $\rightarrow$ S
(D) I $\rightarrow$ R; II $\rightarrow$ S; III $\rightarrow$ Q; IV $\rightarrow$ P

MVT140

MATRIX MATCH TYPE QUESTION

25. **Column-I** **Column-II**

(A) If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar unit vectors, such that $\vec{a}\times(\vec{b}\times\vec{c}) = \frac{\vec{b}+\vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ lies in — (P) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(B) Four vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are such that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \vec{0}$ Let P_1 and P_2 be planes determined by the pair of vectors $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ respectively, then angle between P_1 and P_2 lies in — (Q) $\left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$

(C) If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $2\vec{a}-\vec{b}$ and $4\vec{a}+5\vec{b}$ are perpendicular to each other, then angle between $\vec{a}$ and $\vec{b}$ lies in — (R) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

(D) If $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$ and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then angle between $\vec{a}$ and $\vec{b}$ lies in — (S) $\left(-\frac{\pi}{6}, \frac{7\pi}{12}\right)$

MVT141

EXERCISE - S

1. A rigid body rotates about an axis through the origin with an angular velocity $10\sqrt{3}$ radians/sec. If $\vec{\omega}$ points in the direction of $\hat{i}+\hat{j}+\hat{k}$ and the equation to the locus of the points having tangential speed 20 m/sec. is $x^2+y^2+z^2-axy-byz-czx-2=0$, then $(a+b+c)$ is equal to

MVT142

2. Let $\vec{a},\vec{b},\vec{c}$ be three non-zero vectors which are pairwise non-collinear. If $\vec{a}+3\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+2\vec{c}$ is colliner with $\vec{a}$, then $|\vec{a}+3\vec{b}+6\vec{c}|$ is :

MVT143

3. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a}=\dfrac{\hat{i}-2\hat{j}}{\sqrt{5}}$ and $\vec{b}=\dfrac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$, then the value of $(2\vec{a}+\vec{b}).\left[(\vec{a}\times\vec{b})\times(\vec{a}-2\vec{b})\right]$ is

MVT144

4. If $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=9$, then $|2\vec{a}+5\vec{b}+5\vec{c}|$ is

MVT145

5. Let $|\vec{a}|=2,|\vec{b}|=1$ and $|\vec{c}|=3$ such that $\vec{a}\wedge\vec{b}=\dfrac{\pi}{3},\vec{b}\wedge\vec{c}=\dfrac{\pi}{4}$ and $\vec{c}\wedge\vec{a}=\dfrac{\pi}{2}$. If $|\vec{a}-2\vec{b}+\vec{c}|$ can be expressed as $\sqrt{p+q\sqrt{2}}$ (where $p,q\in$ I), then $(p+q)$ is

MVT146

6. If $\vec{p}=3\vec{a}-5\vec{b}$; $\vec{q}=2\vec{a}+\vec{b}$; $\vec{r}=\vec{a}+4\vec{b}$; $\vec{s}=-\vec{a}+\vec{b}$ are four vectors such that $\sin(\vec{p}\wedge\vec{q})=1$ and $\sin(\vec{r}\wedge\vec{s})=1$ then $\sqrt{43}\cos(\vec{a}\wedge\vec{b})$ is :

MVT147

7. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c}=\hat{a}+2\hat{b}$ and $\vec{d}=5\hat{a}-4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is :

MVT148

8. Two adjacent sides of a parallelogram $ABCD$ are given by $\overrightarrow{AB}=2\hat{i}+10\hat{j}+11\hat{k}$ and $\overrightarrow{AD}=-\hat{i}+2\hat{j}+2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by -

MVT149

9. Let $\vec{r}=\sin x(\vec{a}\times\vec{b})+\cos y(\vec{b}\times\vec{c})+2(\vec{c}\times\vec{a})$, where $\vec{a},\vec{b}$ & $\vec{c}$ are non-zero and non-coplanar vectors. If $\vec{r}$ is orthogonal to $\vec{a}+\vec{b}+\vec{c}$, then minimum value of $\dfrac{4}{\pi^2}(x^2+y^2)$ is

MVT150

10. If three points $(2\vec{p}-\vec{q}+3\vec{r}),(\vec{p}-2\vec{q}+\alpha\vec{r})$ and $(\beta\vec{p}-5\vec{q})$ (where $\vec{p},\vec{q},\vec{r}$ are non-coplanar vectors) are collinear, then the value of $\dfrac{1}{\alpha+\beta}$ is

MVT151

EXERCISE - JEE (Main) PYQ

1. Let $\vec{\alpha}=(\lambda-2)\vec{a}+\vec{b}$ and $\vec{\beta}=(4\lambda-2)\vec{a}+3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is: **[JEE (Main) 2019]**

(1) –3 (2) 4 (3) 3 (4) –4

MVT026

2. Let $\vec{a}=3\hat{i}+2\hat{j}+x\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$, for some real x. Then $|\vec{a}\times\vec{b}|=r$ is possible if : **[JEE (Main) 2019]**

(1) $3\sqrt{\frac{3}{2}}<r<5\sqrt{\frac{3}{2}}$ (2) $0<r\le\sqrt{\frac{3}{2}}$

(3) $\sqrt{\frac{3}{2}}<r\le 3\sqrt{\frac{3}{2}}$ (4) $r\ge 5\sqrt{\frac{3}{2}}$

MVT028

3. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three units vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. If λ = $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ and $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$, then the ordered pair, $(\lambda,\vec{d})$ is equal to: **[JEE (Main) 2020]**

(1) $\left(-\frac{3}{2},3(\vec{a}\times\vec{b})\right)$ (2) $\left(-\frac{3}{2},3(\vec{c}\times\vec{b})\right)$ (3) $\left(\frac{3}{2},3(\vec{b}\times\vec{c})\right)$ (4) $\left(\frac{3}{2},3(\vec{a}\times\vec{c})\right)$

MVT030

4. Let x_0 be the point of local maxima of $f(x)=\vec{a}.(\vec{b}\times\vec{c})$, where $\vec{a}=x\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=-2\hat{i}+x\hat{j}-\hat{k}$ and $\vec{c}=7\hat{i}-2\hat{j}+x\hat{k}$. Then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ at $x=x_0$ is : **[JEE (Main) 2020]**

(1) –30 (2) 14 (3) –4 (4) –22

MVT032

5. If the volume of a parallelepiped, whose coterminous edges are given by the vectors $\vec{a}=\hat{i}+\hat{j}+n\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}-n\hat{k}$ and $\vec{c}=\hat{i}+n\hat{j}+3\hat{k}$ $(n\ge 0)$, is 158 cu. units, then : **[JEE (Main) 2020]**

(1) $\vec{a}\cdot\vec{c}=17$ (2) $\vec{b}\cdot\vec{c}=10$ (3) $n=7$ (4) $n=9$

MVT033

6. Let the position vectors of points 'A' and 'B' be $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio $\lambda : 1$ $(\lambda > 0)$. If O is the origin and $\overrightarrow{OB}\cdot\overrightarrow{OP}-3|\overrightarrow{OA}\times\overrightarrow{OP}|^2=6$, then λ is equal to. **[JEE (Main) 2020]**

MVT035

7. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$ is _. **[JEE (Main) 2020]**

MVT037

8. If $\vec{x}$ and $\vec{y}$ be two non-zero vectors such that $|\vec{x}+\vec{y}|=|\vec{x}|$ and $2\vec{x}+\lambda\vec{y}$ is perpendicular to $\vec{y}$, then the value of λ is ________. **[JEE (Main) 2020]**

MVT038

9. Let $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$. Let a vector $\vec{v}$ be in the plane containing $\vec{a}$ and $\vec{b}$. If $\vec{v}$ is perpendicular to the vector $3\hat{i}+2\hat{j}-\hat{k}$ and its projection on $\vec{a}$ is 19 units, then $|2\vec{v}|^2$ is equal to.

[JEE (Main) 2021]

MVT039

10. Let $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+5\hat{k}$. If $\vec{r}\times\vec{a}=\vec{b}\times\vec{r}$, $\vec{r}.\left(\alpha\hat{i}+2\hat{j}+\hat{k}\right)=3$ and $\vec{r}.\left(2\hat{i}+5\hat{j}-\alpha\hat{k}\right)=-1$, $\alpha\in$ R, then the value of $\alpha+\left|\vec{r}\right|^2$ is equal to : **[JEE (Main) 2021]**

(1) 9 (2) 15 (3) 13 (4) 11

MVT040

11. Let a vector $\alpha\hat{i}+\beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i}+\hat{j}$ by an angle 45° about the origin in counter clockwise direction in the first quadrant. Then the area of triangle having vertices (α, β), (0, β) and (0, 0) is equal to **[JEE (Main) 2021]**

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) $2\sqrt{2}$

MVT041

12. For $p>0$, a vector $\vec{v}_2=2\hat{i}+(p+1)\hat{j}$ is obtained by rotating the vector $\vec{v}_1=\sqrt{3}p\hat{i}+\hat{j}$ by an angle θ about origin in counter clockwise direction. If $\tan\theta=\frac{(\alpha\sqrt{3}-2)}{(4\sqrt{3}+3)}$, then the value of α is equal to.

[JEE (Main) 2021]

MVT044

13. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ, with the vector $\vec{a}+\vec{b}+\vec{c}$. Then $36\cos^2 2\theta$ is equal to ________. **[JEE (Main) 2021]**

MVT045

14. Let three vectors $\vec{a},\vec{b}$ and $\vec{c}$ be such that $\vec{a}\times\vec{b}=\vec{c},\vec{b}\times\vec{c}=\vec{a}$ and $|\vec{a}|=2$. Then which one of the following is **not** true ? **[JEE (Main) 2021]**

(1) $\vec{a}\times\left((\vec{b}+\vec{c})\times(\vec{b}-\vec{c})\right)=\vec{0}$ (2) Projection of $\vec{a}$ on $\left(\vec{b}\times\vec{c}\right)$ is 2

(3) $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]+\left[\vec{c}\ \ \vec{a}\ \ \vec{b}\right]=8$ (4) $\left|3\vec{a}+\vec{b}-2\vec{c}\right|^2=51$

MVT046

15. Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $a_i>0, i=1,2,3$ be a vector which makes equal angles with the coordinates axes OX, OY and OZ. Also, let the projection of $\vec{a}$ on the vector $3\hat{i}+4\hat{j}$ be 7. Let $\vec{b}$ be a vector obtained by rotating $\vec{a}$ with 90°. If $\vec{a},\vec{b}$ and x-axis are coplanar, then projection of a vector $\vec{b}$ on $3\hat{i}+4\hat{j}$ is equal to **[JEE (Main) 2022]**

(1) $\sqrt{7}$ (2) $\sqrt{2}$ (3) 2 (4) 7

MVT048

16. Let $\vec{a}=\hat{i}+\hat{j}+2\hat{k}, \vec{b}=2\hat{i}-3\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+\hat{k}$ be three given vectors. Let $\vec{v}$ be a vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{2}{\sqrt{3}}$. If $\vec{v}.\hat{j}=7$, then $\vec{v}.\left(\hat{i}+\hat{k}\right)$ is equal to : **[JEE (Main) 2022]**

(1) 6 (2) 7 (3) 8 (4) 9

MVT049

17. If $\vec{a}\cdot\vec{b}=1$, $\vec{b}\cdot\vec{c}=2$ and $\vec{c}\cdot\vec{a}=3$, then the value of $\left[\vec{a}\times\left(\vec{b}\times\vec{c}\right), \vec{b}\times\left(\vec{c}\times\vec{a}\right), \vec{c}\times\left(\vec{b}\times\vec{a}\right)\right]$ is :

[JEE (Main) 2022]

(1) 0 (2) $-6\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$ (3) $12\vec{c}\cdot\left(\vec{a}\times\vec{b}\right)$ (4) $-12\vec{b}\cdot\left(\vec{c}\times\vec{a}\right)$

MVT050

18. Let $\vec{a}$ and $\vec{b}$ be the vectors along the diagonal of a parallelogram having area $2\sqrt{2}$. Let the angle between $\vec{a}$ and $\vec{b}$ be acute. $|\vec{a}|=1$ and $\left|\vec{a}.\vec{b}\right|=\left|\vec{a}\times\vec{b}\right|$. If $\vec{c}=2\sqrt{2}\left(\vec{a}\times\vec{b}\right)-2\vec{b}$, then an angle between $\vec{b}$ and $\vec{c}$ is : **[JEE (Main) 2022]**

(1) $\frac{\pi}{4}$ (2) $-\frac{\pi}{4}$ (3) $\frac{5\pi}{6}$ (4) $\frac{3\pi}{4}$

MVT051

19. Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ respectively such that $\frac{QA}{AR}=\frac{RB}{BP}=\frac{PC}{CQ}=\frac{1}{2}$. Then $\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)}$ is equal to **[JEE (Main) 2023]**

(1) 4 (2) 3 (3) 2 (4) $\frac{5}{2}$

MVT152

20. For any vector $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, with $10|a_i|<1, i=1,2,3$, consider the following statements:

(A) : max $\{|a_1|,|a_2|,|a_3|\}\leq|\vec{a}|$

(B) : $|\vec{a}|\leq 3$ max $\{|a_1|,|a_2|,|a_3|\}$ **[JEE (Main) 2023]**

(1) Only (B) is true (2) Only (A) is true

(3) Neither (A) nor (B) is true (4) Both (A) and (B) are true

MVT155

EXERCISE - JEE (Advanced) PYQ

1. Let $\overrightarrow{PR}=3\hat{i}+\hat{j}-2\hat{k}$ and $\overrightarrow{SQ}=\hat{i}-3\hat{j}-4\hat{k}$ determine diagonals of a parallelogram $PQRS$ and $\overrightarrow{PT}=\hat{i}+2\hat{j}+3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT}$, $\overrightarrow{PQ}$ and $\overrightarrow{PS}$ is **[JEE (Advanced) 2013]**

(A) 5 (B) 20 (C) 10 (D) 30

MVT074

2. Perpendicular are drawn from points on the line $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z}{3}$ to the plane $x+y+z=3$. The feet of perpendiculars lie on the line **[JEE (Advanced) 2013]**

(A) $\frac{x}{5}=\frac{y-1}{8}=\frac{z-2}{-13}$ (B) $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{-5}$ (C) $\frac{x}{4}=\frac{y-1}{3}=\frac{z-2}{-7}$ (D) $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

MVT075

3. A line l passing through the origin is perpendicular to the lines

$l_1 : (3+t)\hat{i}+(-1+2t)\hat{j}+(4+2t)\hat{k}, -\infty < t < \infty$

$l_2 : (3+2s)\hat{i}+(3+2s)\hat{j}+(2+s)\hat{k}, -\infty < s < \infty$

Then, the coordinate(s) of the point(s) on l_2 at a distance of $\sqrt{17}$ from the point of intersection of l and l_1 is(are) **[JEE (Advanced) 2013]**

(A) $\left(\frac{7}{3},\frac{7}{3},\frac{5}{3}\right)$ (B) $(-1,-1,0)$ (C) $(1,1,1)$ (D) $\left(\frac{7}{9},\frac{7}{9},\frac{8}{9}\right)$

MVT076

4. Consider the set of eight vectors $V=\left\{a\hat{i}+b\hat{j}+c\hat{k} : a,b,c \in\{-1,1\}\right\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is **[JEE (Advanced) 2013]**

MVT077

5. Match **List I** with **List II** and select the correct answer using the code given below the lists :

List - I	List - II
(P) Volume of parallelepiped determined by vectors $\vec{a},\vec{b}$ and $\vec{c}$ is 2. Then the volume of the parallelepiped determined by vectors $2(\vec{a}\times\vec{b}),3(\vec{b}\times\vec{c})$ and $(\vec{c}\times\vec{a})$ is	(1) 100
(Q) Volume of parallelepiped determined by vectors $\vec{a},\vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepiped determined by vectors $3(\vec{a}+\vec{b}),(\vec{b}+\vec{c})$ and $2(\vec{c}+\vec{a})$ is	(2) 30
(R) Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $(2\vec{a}+3\vec{b})$ and $(\vec{a}-\vec{b})$ is	(3) 24
(S) Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $(\vec{a}+\vec{b})$ and $\vec{a}$ is	(4) 60

[JEE (Advanced) 2013]

Codes :

	P	Q	R	S
(A)	4	2	3	1
(B)	2	3	1	4
(C)	3	4	1	2
(D)	1	4	3	2

MVT078

6. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a nonzero vector perpendicular to $\vec{x}$ and $\vec{y}\times\vec{z}$ and $\vec{b}$ is a nonzero vector perpendicular to $\vec{y}$ and $\vec{z}\times\vec{x}$, then **[JEE (Advanced) 2014]**

(A) $\vec{b}=(\vec{b}.\vec{z})(\vec{z}-\vec{x})$ (B) $\vec{a}=(\vec{a}.\vec{y})(\vec{y}-\vec{z})$ (C) $\vec{a}.\vec{b}=-(\vec{a}.\vec{y})(\vec{b}.\vec{z})$ (D) $\vec{a}=(\vec{a}.\vec{y})(\vec{z}-\vec{y})$

MVT079

7. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$, where p, q and r are scalars, then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is

[JEE (Advanced) 2014]

MVT080

8. **List I** **List II**

(P) Let $y(x)=\cos\left(3\cos^{-1}x\right)$, $x \in [-1, 1]$, $x \neq \pm\frac{\sqrt{3}}{2}$. Then $\frac{1}{y(x)}\left\{\left(x^2-1\right)\frac{d^2y(x)}{dx^2}+x\frac{dy(x)}{dx}\right\}$ equals — (1) 1

(Q) Let $A_1, A_2, ..., A_n$ $(n>2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\vec{a}_k$ be the position vector of the point $A_k, k=1,2,....,n$. If $\left|\sum_{k=1}^{n-1}(\vec{a}_k\times\vec{a}_{k+1})\right|=\left|\sum_{k=1}^{n-1}(\vec{a}_k.\vec{a}_{k+1})\right|$, then the minimum value of n is — (2) 2

(R) If the normal from the point $P(h,1)$ on the ellipse $\frac{x^2}{6}+\frac{y^2}{3}=1$ is perpendicular to the line $x+y=8$, then the value of h is — (3) 8

(S) Number of positive solutions satisfying the equation is $\tan^{-1}\left(\frac{1}{2x+1}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$ — (4) 9

Code: **[JEE (Advanced) 2014]**

	P	Q	R	S
(A)	4	3	2	1
(B)	2	4	3	1
(C)	4	3	1	2
(D)	2	4	1	3

MVT081

9. Let ΔPQR be a triangle. Let $\vec{a}=\overrightarrow{QR}, \vec{b}=\overrightarrow{RP}$ and $\vec{c}=\overrightarrow{PQ}$. If $|\vec{a}|$ = 12, $|\vec{b}|=4\sqrt{3}$ and $\vec{b}.\vec{c}=24$, then which of the following is(are) true? **[JEE (Advanced) 2015]**

(A) $\frac{|\vec{c}|^2}{2}-|\vec{a}|=12$ (B) $\frac{|\vec{c}|^2}{2}+|\vec{a}|=30$

(C) $|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}|=48\sqrt{3}$ (D) $\vec{a}.\vec{b}=-72$

MVT082

10.

Column- I	**Column-II**
(A) In a triangle ΔXYZ, let a,b and c be the lengths of the sides opposite to the angles X,Y and Z, respectively. If $2\left(a^2-b^2\right)=c^2$ and $\lambda=\frac{\sin(X-Y)}{\sin Z}$, then possible values of n for which $\cos\left(n\pi\lambda\right)=0$ is (are)	(P) 1
(B) In a triangle ΔXYZ, let a,b and c be the lengths of the sides opposite to the angles X,Y and Z, respectively. If $1+\cos 2X-2\cos 2Y=2\sin X\sin Y$, then possible value(s) of $\frac{a}{b}$ is (are)	(Q) 2
(C) In R^2, let $\sqrt{3}\hat{i}+\hat{j}$, $\hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ be the position vectors of X,Y and Z with respect to the origin O, respectively. If the distance of Z from the bisector of the acute angle of $\overrightarrow{OX}$ and $\overrightarrow{OY}$ is, $\frac{3}{\sqrt{2}}$ then possible value(s) of $\|\beta\|$ is (are)	(R) 3
(D) Suppose that $F\left(\alpha\right)$ denotes the area of the region bounded by $x=0$, $x=2$, $y^2=4x$ and $y=\left\|\alpha x-1\right\|+\left\|\alpha x-2\right\|+\alpha x$, where $\alpha\in\{0,1\}$. Then the value(s) of $F\left(\alpha\right)+\frac{8}{3}\sqrt{2}$, when α = 0 and α = 1, is (are)	(S) 5

[JEE (Advanced) 2015]

MVT083

11. Suppose that $\vec{p},\vec{q}$ and $\vec{r}$ are three non-coplanar vectors in R^3. Let the components of a vector $\vec{s}$ along $\vec{p},\vec{q}$ and $\vec{r}$ be 4, 3 and 5, respectively. If the components of this vector $\vec{s}$ along $(-\vec{p}+\vec{q}+\vec{r}),(\vec{p}-\vec{q}+\vec{r})$ and $(-\vec{p}-\vec{q}+\vec{r})$ are x, y and z, respectively, then the value of $2x+y+z$ is **[JEE (Advanced) 2015]**

MVT084

12. Let $\hat{u}=u_1\hat{i}+u_2\hat{j}+u_3\hat{k}$ be a unit vector in $\mathbb{R}^2$ and $\hat{w}=\frac{1}{\sqrt{6}}(\hat{i}+\hat{j}+2\hat{k})$. Given that there exists a vector $\vec{v}$ in $\mathbb{R}^3$ such that $|\hat{u}\times\vec{v}|=1$ and $\hat{w}.(\hat{u}\times\vec{v})=1$. Which of the following statement(s) is(are) correct? **[JEE(Advanced)-2016]**

(A) There is exactly one choice for such $\vec{v}$
(B) There are infinitely many choice for such $\vec{v}$
(C) If $\hat{u}$ lies in the xy-plane then $|u_1|=|u_2|$
(D) If $\hat{u}$ lies in the xz-plane then $2|u_1|=|u_3|$

MVT085

13. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that $\overrightarrow{OP}.\overrightarrow{OQ}+\overrightarrow{OR}.\overrightarrow{OS}=\overrightarrow{OR}.\overrightarrow{OP}+\overrightarrow{OQ}.\overrightarrow{OS}=\overrightarrow{OQ}.\overrightarrow{OR}+\overrightarrow{OP}.\overrightarrow{OS}$. Then the triangle PQR has S as its **[JEE(Advanced) 2017]**

(A) incentre (B) orthocenter (C) circumcenter (D) centroid

MVT086

Paragraph : (Q.14 to 15)

Let O be the origin, and $\overrightarrow{OX},\overrightarrow{OY},\overrightarrow{OZ}$ be three-unit vectors in the directions of the sides $\overrightarrow{QR},\overrightarrow{RP},\overrightarrow{PQ}$, respectively, of a triangle PQR.

14. $\left|\overrightarrow{OX}\times\overrightarrow{OY}\right|=$ **[JEE (Advanced) 2017]**

(A) $\sin(Q+R)$ (B) $\sin(P+R)$ (C) $\sin 2R$ (D) $\sin(P+Q)$

MVT087

15. If the triangle PQR varies, then the minimum value of $\cos(P+Q)+\cos(Q+R)+\cos(R+P)$ is **[JEE (Advanced) 2017]**

(A) $\frac{3}{2}$ (B) $-\frac{3}{2}$ (C) $\frac{5}{3}$ (D) $-\frac{5}{3}$

MVT088

16. Let $\vec{a}$ and $\vec{b}$ be two-unit vectors such that $\vec{a}.\vec{b}=0$. For some $x,y\in\mathbb{R}$, let $\vec{c}=x\vec{a}+y\vec{b}+(\vec{a}\times\vec{b})$. If $|\vec{c}|=2$ and the vector $|\vec{c}|$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2\alpha$ is. **[JEE (Advanced) 2018]**

MVT089

17. Three lines

$L_1: \vec{r}=\lambda\hat{i},\ \lambda\in\mathbb{R}$,

$L_2: \vec{r}=\hat{k}+\mu\hat{j},\ \mu\in\mathbb{R}$ and

$L_3: \vec{r}=\hat{i}+\hat{j}+\nu\hat{k},\ \nu\in\mathbb{R}$

are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P,Q and R are collinear **[JEE (Advanced) 2019]**

(A) $\hat{k}+\hat{j}$ (B) $\hat{k}$ (C) $\hat{k}+\frac{1}{2}\hat{j}$ (D) $\hat{k}-\frac{1}{2}\hat{j}$

MVT090

18. Let $\vec{a}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$ be two vectors. Consider a vector $\vec{c}=\alpha\vec{a}+\beta\vec{b}$, $\alpha,\beta\in\mathbb{R}$. If the projection of $\vec{c}$ on the vector $(\vec{a}+\vec{b})$ is $3\sqrt{2}$, then the minimum value of $\left(\vec{c}-(\vec{a}\times\vec{b})\right).\vec{c}$ equals

[JEE (Advanced) 2019]

MVT091

19. In a triangle PQR, let $\vec{a}=\overrightarrow{QR},\vec{b}=\overrightarrow{RP}$ *and* $\vec{c}=\overrightarrow{PQ}$. If $|\vec{a}|$ = 3, $|\vec{b}|$ = 4 and $\dfrac{\vec{a}.(\vec{c}-\vec{b})}{\vec{c}.(\vec{a}-\vec{b})}=\dfrac{|\vec{a}|}{|\vec{a}|+|\vec{b}|}$, then the value of $|\vec{a}\times\vec{b}|^2$ is.

[JEE (Advanced) 2020]

MVT092

20. Let a and b be positive real numbers. Suppose $\overrightarrow{PQ}=a\hat{i}+b\hat{j}$ and $\overrightarrow{PS}=a\hat{i}-b\hat{j}$ are adjacent sides of a parallelogram $PQRS$. Let $\vec{u}$ and $\vec{v}$ be the projection vectors of $\vec{w}=\hat{i}+\hat{j}$ along $\overrightarrow{PQ}$ and $\overrightarrow{PS}$, respectively. If $|\vec{u}|+|\vec{v}|=|\vec{w}|$ and if the area of the parallelogram $PQRS$ is 8, then which of the following statements is/are TRUE ?

[JEE (Advanced) 2020]

(A) $a+b=4$

(B) $a-b=2$

(C) The length of the diagonal PR of the parallelogram $PQRS$ is 4

(D) $\vec{w}$ is an angle bisector of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PS}$

MVT093

21. Let $\vec{u},\vec{v}$ and $\vec{w}$ be vectors in three-dimensional space, where $\vec{u}$ and $\vec{v}$ are unit vectors which are not perpendicular to each other and $\vec{u}.\vec{w}=1$, $\vec{v}.\vec{w}=1$, $\vec{w}.\vec{w}=4$. If the volume of the parallelepiped, whose adjacent sides are represented by the vectors $\vec{u},\vec{v}$ and $\vec{w}$, is $\sqrt{2}$, then the value of $|3\vec{u}+5\vec{v}|$ is.

[JEE (Advanced) 2021]

MVT094

22. Let O be the origin and $\overrightarrow{OA}=2\hat{i}+2\hat{j}+\hat{k}$, $\overrightarrow{OB}=\hat{i}-2\hat{j}+2\hat{k}$ and $\overrightarrow{OC}=\frac{1}{2}(\overrightarrow{OB}-\lambda\overrightarrow{OA})$ for some $\lambda>0$. If $|\overrightarrow{OB}\times\overrightarrow{OC}|=\frac{9}{2}$, then which of the following statements is (are) **TRUE**?

[JEE (Advanced) 2021]

(A) Projection of $\overrightarrow{OC}$ on $\overrightarrow{OA}$ is $-\frac{3}{2}$

(B) Area of the triangle OAB is $\frac{9}{2}$

(C) Area of the triangle ABC is $\frac{9}{2}$

(D) The acute angle between the diagonals of the parallelogram with adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is $\frac{\pi}{3}$

MVT095

23. Let $\hat{i}, \hat{j}$ and $\hat{k}$ be the unit vectors along the three positive coordinate axes. Let

$\vec{a} = 3\hat{i} + \hat{j} - \hat{k}$,

$\vec{b} = \hat{i} + b_2\hat{j} + b_3\hat{k}$, $\quad b_2, b_3 \in \mathbb{R}$,

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$, $\quad c_1, c_2, c_3 \in \mathbb{R}$

be three vectors such that $b_2b_3 > 0$, $\vec{a}\cdot\vec{b} = 0$ and

$$\begin{pmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 3-c_1 \\ 1-c_2 \\ -1-c_3 \end{pmatrix}.$$

Then, which of the following is/are TRUE ? **[JEE (Advanced) 2022]**

(A) $\vec{a}\cdot\vec{c} = 0$ (B) $\vec{b}\cdot\vec{c} = 0$ (C) $|\vec{b}| > \sqrt{10}$ (D) $|\vec{c}| \le \sqrt{11}$

MVT096

24. Let P be the plane $\sqrt{3}x + 2y + 3z = 16$ and let

$S = \left\{\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} : \alpha^2 + \beta^2 + \gamma^2 = 1 \text{ and the distance of } (\alpha,\beta,\gamma) \text{ from the plane } P \text{ is } \frac{7}{2}\right\}$.

Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three distinct vectors in S such that $|\vec{u}-\vec{v}| = |\vec{v}-\vec{w}| = |\vec{w}-\vec{u}|$. Let V be the volume of the parallelepiped determined by vectors $\vec{u}, \vec{v}$ and $\vec{w}$. Then the value of $\frac{80}{\sqrt{3}}V$ is

[JEE (Advanced) 2023]

MVT156

25. Let ℓ_1 and ℓ_2 be the lines $\vec{r}_1 = \lambda(\hat{i} + \hat{j} + \hat{k})$ and $\vec{r}_2 = (\hat{j} - \hat{k}) + \mu(\hat{i} + \hat{k})$, respectively. Let X be the set of all the planes H that contain the line ℓ_1. For a plane H, let $d(H)$ denote the smallest possible distance between the points of ℓ_2 and H. Let H_0 be plane in X for which $d(H_0)$ is the maximum value of $d(H)$ as H varies over all planes in X.

Match each entry in **List-I** to the correct entries in **List-II.**

List-I	List-II
(P) The value of $d(H_0)$ is	(1) $\sqrt{3}$
(Q) The distance of the point (0,1,2) from H_0 is	(2) $\frac{1}{\sqrt{3}}$
(R) The distance of origin from H_0 is	(3) 0
(S) The distance of origin from the point of intersection	(4) $\sqrt{2}$
of planes $y = z$, $x = 1$ and H_0 is	(5) $\frac{1}{\sqrt{2}}$

The correct option is : **[JEE (Advanced) 2023]**

(A) (P)→ (2) (Q)→ (4) (R)→ (5) (S)→ (1) (B) (P)→ (5) (Q)→ (4) (R)→ (3) (S)→ (1)

(C) (P)→ (2) (Q)→ (1) (R)→ (3) (S)→ (2) (D) (P)→ (5) (Q)→ (1) (R)→ (4) (S)→ (2)

MVT157

26. Let the position vectors of the points P, Q, R and S be $\vec{a}=\hat{i}+2\hat{j}-5\hat{k}$, $\vec{b}=3\hat{i}+6\hat{j}+3\hat{k}$, $\vec{c}=\frac{17}{5}\hat{i}+\frac{16}{5}\hat{j}+7\hat{k}$ and $\vec{d}=2\hat{i}+\hat{j}+\hat{k}$, respectively. Then which of the following statements is true?

[JEE (Advanced) 2023]

(A) The points P, Q, R and S are **NOT** coplanar

(B) $\frac{\vec{b}+2\vec{d}}{3}$ is the position vector of a point which divides PR internally in the ratio 5 : 4

(C) $\frac{\vec{b}+2\vec{d}}{3}$ is the position vector of a point which divides PR externally in the ratio 5 : 4

(D) The square of the magnitude of the vector $\vec{b}\times\vec{d}$ is 95

MVT158

JEE (Main) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION–A

- This section contains **TWENTY** questions.
- Each question has **FOUR** options (1), (2), (3) and (4). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +4, if only the bubble corresponding to the correct option is darkened.
Zero Marks : 0, if none of the bubbles is darkened.
Negative Marks : –1 in all other cases.

1. The perimeter of the triangle whose vertices have the position vectors $(\hat{i}+\hat{j}+\hat{k})$, $(5\hat{i}+3\hat{j}-3\hat{k})$ and $(2\hat{i}+5\hat{j}+9\hat{k})$, is given by

(1) $15+\sqrt{157}$ (2) $15-\sqrt{157}$ (3) $\sqrt{15}-\sqrt{157}$ (4) $\sqrt{15}+\sqrt{157}$

MVT001

2. If the position vectors of three consecutive vertices of any parallelogram are respectively $\hat{i}+\hat{j}+\hat{k}, \hat{i}+3\hat{j}+5\hat{k}, 7\hat{i}+9\hat{j}+11\hat{k}$ then the position vector of its fourth vertex is -

(1) $6(\hat{i}+\hat{j}+\hat{k})$ (2) $7(\hat{i}+\hat{j}+\hat{k})$ (3) $2\hat{j}-4\hat{k}$ (4) $6\hat{i}+8\hat{j}+10\hat{k}$

MVT002

3. The vectors $\hat{i}+2\hat{j}+3\hat{k}$, $2\hat{i}-\hat{j}+\hat{k}$ and $3\hat{i}+\hat{j}+4\hat{k}$ are so placed that the end point of one vector is the starting point of the next vector. Then the vectors are:

(1) not coplanar (2) coplanar but cannot form a triangle
(3) coplanar but can form a triangle (4) coplanar & can form a right-angled triangle

MVT003

4. The vertices of a triangle are $A(1,1,2)$, $B(4,3,1)$ and $C(2,3,5)$. A vector representing the internal bisector of the angle A is:

(1) $\hat{i}+\hat{j}+2\hat{k}$ (2) $2\hat{i}-2\hat{j}+\hat{k}$ (3) $2\hat{i}+2\hat{j}-\hat{k}$ (4) $2\hat{i}+2\hat{j}+\hat{k}$

MVT004

5. If θ be the angle between vectors $\hat{i}+2\hat{j}+3\hat{k}$ and $3\hat{i}+2\hat{j}+\hat{k}$, then the value of sinθ is

(1) $\sqrt{\frac{6}{7}}$ (2) $\frac{2\sqrt{6}}{7}$ (3) $\frac{1}{7}$ (4) $2\sqrt{\frac{6}{7}}$

MVT005

6. If $\vec{x}$ and $\vec{y}$ are two-unit vectors and θ is the angle between them, then $\frac{1}{2}|\vec{x}-\vec{y}|$ is equal to

(1) $\frac{\pi}{2}$ (2) 0 (3) $\left|\cos\frac{\theta}{2}\right|$ (4) $\left|\sin\frac{\theta}{2}\right|$

MVT006

7. Angle between diagonals of a parallelogram whose side are represented by $\vec{a}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}-\hat{k}$ is

(1) $\cos^{-1}\left(\frac{1}{3}\right)$ (2) $\cos^{-1}\left(\frac{1}{2}\right)$ (3) $\cos^{-1}\left(\frac{4}{9}\right)$ (4) $\cos^{-1}\left(\frac{5}{9}\right)$

MVT007

8. If $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$ then the angle between $\vec{a}$ and $\vec{b}$ is -

(1) $\frac{\pi}{6}$ (2) $\frac{2\pi}{3}$ (3) $\frac{5\pi}{3}$ (4) $\frac{\pi}{3}$

MVT008

9. Let $\vec{a}, \vec{b}, \vec{c}$ be vectors of length 3, 4, 5 respectively. Let $\vec{a}$ be perpendicular to $\vec{b}+\vec{c}$, $\vec{b}$ perpendicular and to $\vec{c}+\vec{a}$ and $\vec{c}$ perpendicular to $\vec{a}+\vec{b}$. Then $\left|\vec{a}+\vec{b}+\vec{c}\right|$ is equal to:

(1) $2\sqrt{5}$ (2) $2\sqrt{2}$ (3) $10\sqrt{5}$ (4) $5\sqrt{2}$

MVT009

10. If $\vec{a}=2\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=3\hat{i}-4\hat{j}+2\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+2\hat{k}$ then the projection of $\vec{a}+\vec{b}$ on $\vec{c}$ is -

(1) $\frac{17}{3}$ (2) $\frac{5}{3}$ (3) $\frac{4}{3}$ (4) $\frac{17}{\sqrt{43}}$

MVT010

11. If $A(6,3,2)$, $B(5,1,4)$, $C(3,-4,7)$, $D(0,2,5)$ are four points, then projection of CD on AB is

(1) $-\frac{13}{3}$ (2) $-\frac{13}{7}$ (3) $-\frac{3}{13}$ (4) $-\frac{7}{13}$

MVT011

12. Let $\vec{b}=3\hat{j}+4\hat{k}$, $\vec{a}=\hat{i}+\hat{j}$ and let $\vec{b}_1$ and $\vec{b}_2$ be component of vector $\vec{b}$ parallel and perpendicular to $\vec{a}$. If $\vec{b}_1=\frac{3}{2}\hat{i}+\frac{3}{2}\hat{j}$, then $\vec{b}_2$ is equal to

(1) $-\frac{3}{2}\hat{i}+\frac{3}{2}\hat{j}$ (2) $\frac{3}{2}\hat{i}+\frac{3}{2}\hat{j}+4\hat{k}$ (3) $-\frac{3}{2}\hat{i}+\frac{3}{2}\hat{j}+4\hat{k}$ (4) $\frac{3}{2}\hat{i}+\frac{3}{2}\hat{j}-4\hat{k}$

MVT012

13. The vector $\frac{1}{3}(2\hat{i}-2\hat{j}+\hat{k})$ is:

(1) a unit vectors (2) parallel to the vector $-\hat{i}+\hat{j}-\frac{1}{2}\hat{k}$

(3) perpendicular to the vector $3\hat{i}+2\hat{j}-2\hat{k}$ (4) all of these

MVT013

14. For a non-zero vector $\vec{A}$ if the equations $\vec{A}.\vec{B}=\vec{A}.\vec{C}$ and $\vec{A}\times\vec{B}=\vec{A}\times\vec{C}$ hold simultaneously, then:

(1) $\vec{A}$ is perpendicular to $\vec{B}-\vec{C}$ (2) $\vec{A}=\vec{B}$

(3) $\vec{B}=\vec{C}$ (4) $\vec{C}=\vec{A}$

MVT014

15. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, then the vectors $\vec{a}-\vec{d}$ and $\vec{b}-\vec{c}$ are:

(1) null vectors (2) linearly independent

(3) perpendicular (4) parallel

MVT015

16. The value of $\left[\left(\vec{a}+2\vec{b}-\vec{c}\right)\left(\vec{a}-\vec{b}\right)\left(\vec{a}-\vec{b}-\vec{c}\right)\right]$ is equal to

(1) $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ (2) $2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ (3) $3\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ (4) $4\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

MVT016

17. Vector of length 3 unit which is perpendicular to $\hat{i}+\hat{j}+\hat{k}$ and lies in the plane of $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}-3\hat{j}$, is

(1) $\frac{3}{\sqrt{6}}(\hat{i}-2\hat{j}+\hat{k})$ (2) $\frac{3}{\sqrt{6}}(2\hat{i}-\hat{j}-\hat{k})$ (3) $\frac{3}{\sqrt{114}}(8\hat{i}-7\hat{j}-\hat{k})$ (4) $\frac{3}{\sqrt{114}}(-7\hat{i}+8\hat{j}-\hat{k})$

MVT017

18. If $\vec{a}=\hat{i}-2\hat{j}-2\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+3\hat{j}-\hat{k}$ then $\vec{a}\times(\vec{b}\times\vec{c})$ is equal to

(1) $-8\hat{i}-3\hat{j}-k$ (2) $20\hat{i}-3\hat{j}-7k$ (3) $20\hat{i}+3\hat{j}-7k$ (4) $8\hat{i}-3\hat{j}+k$

MVT018

19. In $\Delta OBC, O$ is origin and position vector of B and C are $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$ respectively D and E divides OC and BC in $2:1$ and $1:2$ respectively. Also, OE and BD intersect at P, then

(1) P divides BD in $3:4$ (2) P divides BD in $4:3$

(3) P divides OE in $1:5$ (4) P divides OE in $6:1$

MVT019

20. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}$, then the vectors $(\vec{a}\cdot\hat{i})\hat{i}+(\vec{a}\cdot\hat{j})\hat{j}+(\vec{a}\cdot\hat{k})\hat{k}$, $(\vec{b}\cdot\hat{i})\hat{i}+(\vec{b}\cdot\hat{j})\hat{j}+(\vec{b}\cdot\hat{k})\hat{k}$ and $\hat{i}+\hat{j}-2\hat{k}$

(1) are mutually perpendicular (2) are coplanar

(3) form a parallelepiped of volume 3 units (4) None of these

MVT020

SECTION-B

- This section contains **TEN** Questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30, if answer is 11.36777..... then both 11.36 and 11.37 will be correct).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4, if ONLY the correct numerical value is entered as answer.

Zero Marks : 0 in all other cases.

1. If $(\vec{a}\times\vec{b})^2+(\vec{a}.\vec{b})^2=144$ and $|\vec{a}|=4$, then $|\vec{b}|$ equals

MVT021

2. The volume of the parallelepiped constructed on the diagonals of the faces of the given rectangular parallelepiped is m times the volume of the given parallelepiped. Then m is equal to

MVT022

3. Let $\vec{p}$ is the position vector of the orthocenter and $\vec{g}$ is the position vector of the centroid of the triangle ABC, where circumcenter is the origin. If $\vec{p} = K\vec{g}$, then K is equal to:

MVT023

4. If the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = 4\hat{i} - 2\hat{j} - 6\hat{k}$ constitute the sides of a ΔABC. If length of the median bisecting the vector $\vec{c}$ is λ, then λ^2

MVT024

5. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{c} = 3\hat{i} + \hat{j}$ and $\vec{a} + P\vec{b}$ is normal to $\vec{c}$, then P is equal

MVT025

6. If A is the area of triangle formed by the lines L_1, L_2, L_3 where $L_1 : \vec{r} = \hat{i} - \hat{j} + \lambda(\hat{i} - 2\hat{j} + \hat{k})$, $L_2 : \vec{r} = \hat{i} - \hat{j} + \mu(\hat{i} + \hat{j} + \hat{k})$ & $L_3 : \vec{r} = 2\hat{i} + \hat{k} + t\hat{j}$ then $(4A^2)$ is equal to

MVT159

7. A fixed vector $\vec{d}$ is perpendicular to three non-zero vectors $\vec{a}, \vec{b}, \vec{c}$ whose magnitudes are 1,2,3 respectively. It is given that $\vec{a} \wedge \vec{b} = \alpha$, $\vec{b} \wedge \vec{c} = \beta$, $\vec{c} \wedge \vec{a} = \gamma$ then $\begin{vmatrix} 1 & 2\cos\alpha & 3\cos\gamma \\ 2\cos\alpha & 4 & 6\cos\beta \\ 3\cos\gamma & 6\cos\beta & 9 \end{vmatrix}$ is equal to

MVT160

8. Let $\hat{a}, \hat{b}, \hat{c}$ be unit vectors such that $\hat{b} \wedge \hat{c} = 60^\circ$ & $(\hat{a} \times (\hat{b} \times \hat{c})) \times \hat{a} = \vec{0}$ $([\hat{a}\ \hat{b}\ \hat{c}] \neq 0)$ and if volume of the parallelepiped formed by $\hat{a} + \hat{b}, \hat{b} + \hat{c}, \hat{c} + \hat{a}$ is k, then (k^2) is equal to

MVT161

9. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors having magnitude 1,1,2 respectively. If $\vec{b} + (\vec{a}.\vec{c})\vec{a} = (\vec{a}.\vec{a})\vec{c}$, then cosecant of acute angle between $\vec{a}$ and $\vec{c}$ is

MVT162

10. Let $\vec{a} = 5\hat{i} + p\hat{j} + 12\hat{k}$ & $\vec{b} = \hat{i} + 13\hat{j} + 2\sqrt{q}\hat{k}$ where $p, q \in N$. It is given that $|\vec{a}| = |\vec{b}|$ & $p, q \in [1,100]$. If total number of ordered pairs of (p, q) is λ, then $\dfrac{(\lambda + 1)}{16}$ is

MVT163

JEE (Advanced) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION–I

- This section contains **SIX** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

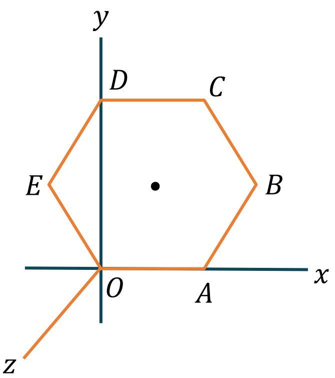
Full Marks	:	+3	If only the bubble corresponding to the correct option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	−1	In all other cases

1. If the vector $\vec{b}$ is collinear with the vector $\vec{a}=\left(2\sqrt{2},\ -1,\ 4\right)$ and $\left|\vec{b}\right|=10$, then:

(A) $\vec{a}\pm\vec{b}=0$ (B) $\vec{a}\pm2\vec{b}=0$ (C) $2\vec{a}\pm b=0$ (D) $3\vec{a}\pm\vec{b}=0$

MVT056

2. $OABCDE$ is a regular hexagon of side 2 units in the XY –plane as shown in figure. O being the origin and OA taken along the X –axis. A point P is taken on a line parallel to Z –axis through the center of the hexagon at a distance of 3 units from O in the positive Z direction. Then vector $\overrightarrow{AP}$ is :



(A) $-\hat{i}+3\hat{j}+\sqrt{5}\hat{k}$ (B) $\hat{i}-\sqrt{3}\hat{j}+5\hat{k}$ (C) $-\hat{i}+\sqrt{3}\hat{j}+\sqrt{5}\hat{k}$ (D) $\hat{i}+\sqrt{3}\hat{j}+\sqrt{5}\hat{k}$

MVT057

3. If $|\vec{a}|=5$, $|\vec{a}-\vec{b}|=8$ and $|\vec{a}+\vec{b}|=10$, then $|\vec{b}|$ is equal to :

(A) 1 (B) $\sqrt{57}$ (C) 3 (D) 57

MVT058

4. If $\vec{a}\times\vec{b}=\vec{c}$, $\vec{b}\times\vec{c}=\vec{a}$, then find value of $|3\vec{a}+4\vec{b}+12\vec{c}|$ if $\vec{a},\vec{b},\vec{c}$ are vectors of same magnitude.

(A) 11 (B) 12 (C) 13 (D) 14

MVT059

5. Angle between diagonals of a parallelogram whose side are represented by $\vec{a}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}-\hat{k}$

(A) $\cos^{-1}$ (B) $\cos^{-1}$ (C) $\sin^{-1}$ (D) $\tan^{-1}$

MVT060

6. A,B,C & D are four points in a plane with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ & $\vec{d}$ respectively such that $\left(\vec{a}-\vec{d}\right).\left(\vec{b}-\vec{c}\right)=\left(\vec{b}-\vec{d}\right).\left(\vec{c}-\vec{a}\right)$ = 0. Then for the triangle ABC, D is its:

(A) incentre (B) circumcenter (C) orthocenter (D) centroid

MVT061

SECTION-II

- This section contains **FOUR** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If only (all) the correct option(s) is (are) chosen.
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen.
Partial Marks	:	+2	If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	:	– 2	In all other cases.

For Example: If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

7. A vector $\vec{a}$ has components $2p$ and 1 with respect to a rectangular cartesian system. The system is rotated through a certain angle about the origin in the counter clockwise sense. If with respect to the new system, $\vec{a}$ has components $p+1$ and 1, then

(A) $p=-\frac{1}{3}$ (B) $p=1$ (C) $p=-1$ (D) $p=\frac{1}{3}$

MVT062

8. The value(s) of $\alpha \in [0, 2\pi]$ for which vector $\vec{a}=\hat{i}+3\hat{j}+(\sin 2\alpha)\hat{k}$ makes an obtuse angle with the z-axis and the vectors $\vec{b}=(\tan\alpha)\ \hat{i}-\hat{j}+2\sqrt{\sin\frac{\alpha}{2}}\ \hat{k}$ and $\vec{c}=(\tan\alpha)\hat{i}+(\tan\alpha)\hat{j}-3\sqrt{\operatorname{cosec}\frac{\alpha}{2}}\ \hat{k}$ are orthogonal, is/are

(A) $\tan^{-1}3$ (B) $\pi-\tan^{-1}2$ (C) $\pi+\tan^{-1}3$ (D) $2\pi-\tan^{-1}2$

MVT063

9. The vertices of a triangle are $A(1,1,2)$, $B(4,3,1)$ and $C(2,3,5)$. A vector representing the bisector of the angle A is:

(A) $2\hat{i}-4\hat{k}$ (B) $-2\hat{i}+4\hat{k}$ (C) $-2\hat{i}-2\hat{j}-\hat{k}$ (D) $2\hat{i}+2\hat{j}+\hat{k}$

MVT064

10. The vector $\vec{c}$, parallel to the internal bisector of the angle between the vectors $\vec{a}=7\hat{i}-4\hat{j}-4\hat{k}$ and $\vec{b}=-2\hat{i}-\hat{j}+2\hat{k}$ with $|\vec{c}|=5\sqrt{6}$, is :

(A) $\frac{5}{3}\left(\hat{i}-7\hat{j}+2\hat{k}\right)$ (B) $\frac{5}{3}\left(\hat{i}+7\hat{j}-2\hat{k}\right)$ (C) $\frac{5}{3}\left(-\hat{i}+7\hat{j}-2\hat{k}\right)$ (D) $\frac{5}{3}\left(-\hat{i}-7\hat{j}+2\hat{k}\right)$

MVT065

SECTION–III

- This section contains **ONE** paragraph.
- Based on each paragraph, there are **THREE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D) **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.

Zero Marks : 0 In all other cases.

Comprehension # 1 for Question 11 & 13

In a parallelogram $OABC$, vectors $\vec{a},\vec{b},\vec{c}$ are respectively the position vectors of vertices A,B,C with reference to O as origin. A point E is taken on the side BC which divides it in the ratio of $2:1$ internally. Also, the line segment AE intersect the line bisecting the angle O internally in point P. If CP, when extended meets AB in point F. Then

11. The position vector of point P, is

(A) $\frac{3|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left\{\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{c}}{|\vec{c}|}\right\}$ (B) $\frac{|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left\{\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{c}}{|\vec{c}|}\right\}$

(C) $\frac{2|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left\{\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{c}}{|\vec{c}|}\right\}$ (D) $\frac{3|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left\{\frac{\vec{a}}{|\vec{a}|}-\frac{\vec{c}}{|\vec{c}|}\right\}$

MVT066

12. The position vector of point F, is

(A) $\vec{a}+\frac{1}{3}\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$ (B) $\vec{a}+\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$ (C) $\vec{a}+\frac{2|\vec{a}|}{|\vec{c}|}\vec{c}$ (D) $\vec{a}-\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$

MVT067

13. The vector $\overrightarrow{AF}$, is given by

(A) $-\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$ (B) $\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$ (C) $\frac{2|\vec{a}|}{|\vec{c}|}\vec{c}$ (D) $\frac{1}{3}\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$

MVT068

SECTION–IV

- This section contains **TWO** questions.
- **Each question has matching lists.** The codes for the lists have choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**
- For each question, marks will be awarded in one of the following categories :

Full Marks	:	+3	If only the bubble corresponding to the correct option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	–1	In all other cases

14. **List–I** **List–II**

(P) If the vectors $\vec{a}=3\hat{i}+\hat{j}-2\hat{k}$, $\vec{b}=-\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{c}=4\hat{i}-2\hat{j}-6\hat{k}$ constitute the sides of a ΔABC and length of the median bisecting the vector $\vec{c}$ is λ, then λ^2 — (1) 2

(Q) Let $\vec{p}$ is the position vector of the orthocenter and $\vec{g}$ is the position vector of the centroid of the triangle ABC, where circumcenter is the origin. If $\vec{p}=K\vec{g}$, then K is equal to: — (2) 3

(R) Twice of the area of the parallelogram constructed on the vectors $\vec{a}=\vec{p}+2\vec{q}$ and $\vec{b}=2\vec{p}+\vec{q}$, where $\vec{p}$ and $\vec{q}$ are unit vectors containing an angle of $30°$, is: — (3) 6

(S) Let $\vec{u},\vec{v}$ and $\vec{w}$ are vector such that $\vec{u}+\vec{v}+\vec{w}=\vec{0}$. If $|\vec{u}|=3$, $|\vec{v}|=4$, $|\vec{w}|=5$ then $\sqrt{|\vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{w}.\vec{u}|}$ is — (4) 5

The correct option is :

(A) (P) $\to$ (1), (Q) $\to$ (2), (R) $\to$ (3), (S) $\to$ (4)

(B) (P) $\to$ (3), (Q) $\to$ (2), (R) $\to$ (2), (S) $\to$ (4)

(C) (P) $\to$ (3), (Q) $\to$ (2), (R) $\to$ (1), (S) $\to$ (4)

(D) (P) $\to$ (4), (Q) $\to$ (1), (R) $\to$ (2), (S) $\to$ (3)

MVT069

15. **List–I** **List–II**

(P) The volume of the parallelepiped constructed on the diagonals of the faces of the given rectangular parallelepiped is m times the volume of the given parallelepiped. Then m is equal to — (1) –3

(Q) If $\vec{x}$ satisfying the conditions $\vec{b}.\vec{x}=\beta$ & $\vec{b}\times\vec{x}=\vec{a}$ is $\vec{x}=\dfrac{(\beta^2-12)\vec{b}}{|\vec{b}|^2}+\dfrac{\vec{a}\times\vec{b}}{|\vec{b}|^2}$ then β can be — (2) 2

(R) The points (0, –1, –1), (4, 5, 1), (3, 9, 4) and (–4, 4, k) are coplanar, then k = — (3) 4

(S) In ΔABC the mid points of the sides AB, BC and CA are respectively $(\ell, 0, 0)$ $(0, m, 0)$ and $(0, 0, n)$. Then $\dfrac{AB^2+BC^2+CA^2}{\ell^2+m^2+n^2}$ is equal to — (4) 8

The correct option is :

(A) (P) $\rightarrow$ (4), (Q) $\rightarrow$ (1, 3), (R) $\rightarrow$ (3), (S) $\rightarrow$ (1)

(B) (P) $\rightarrow$ (3), (Q) $\rightarrow$ (1), (R) $\rightarrow$ (2), (S) $\rightarrow$ (4)

(C) (P) $\rightarrow$ (2), (Q) $\rightarrow$ (1), (R) $\rightarrow$ (3), (S) $\rightarrow$ (4)

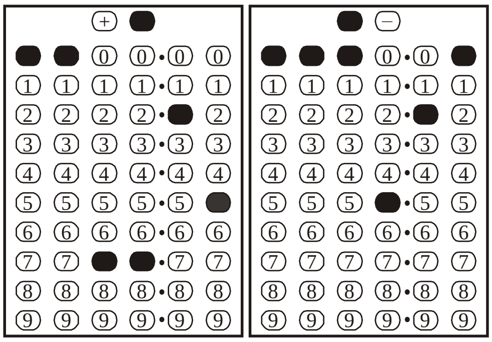
(D) (P) $\rightarrow$ (2), (Q) $\rightarrow$ (1, 3), (R) $\rightarrow$ (3), (S) $\rightarrow$ (4)

MVT070

SECTION-V

- This section contains **THREE** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/ rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30, if answer is 11.36777..... then both 11.36 and 11.37 will be correct) by darken the corresponding bubbles in the ORS.

For Example : If answer is –77.25, 5.2 then fill the bubbles as follows.



- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct numerical value is entered as answer.

Zero Marks : 0 In all other cases.

16. Let $\vec{A}=2\hat{i}+\hat{k}$, $\vec{B}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{C}=4\hat{i}-3\hat{j}+7\hat{k}$. If a vector $\vec{R}=\alpha\hat{i}-\beta\hat{j}+\gamma\hat{k}$ satisfies $\vec{R}\times\vec{B}=\vec{C}\times\vec{B}$ and $\vec{R}.\vec{A}=0$ then $\alpha+\beta+\gamma$ is.

MVT071

17. Line L_1 is parallel to vector $\vec{\alpha}=-3\hat{i}+2\hat{j}+4\hat{k}$ and passes through a point $A(7,6,2)$ and line L_2 is parallel to a vector $\vec{\beta}=2\hat{i}+\hat{j}+3\hat{k}$ and passes through a point $B(5,3,4)$. Now a line L_3 parallel to a vector $\vec{r}=2\hat{i}-2\hat{j}-\hat{k}$ intersects the lines L_1 and L_2 at points C and D respectively, then $|\overrightarrow{CD}|$ is equal to:

MVT072

18. Let $\vec{a},\vec{b},\vec{c}$ be vectors of length 3, 4, 5 respectively. Let $\vec{a}$ be perpendicular to $\vec{b}+\vec{c}$, $\vec{b}$ to $\vec{c}+\vec{a}$ and $\vec{c}$ to $\vec{a}+\vec{b}$, and $|\vec{a}+\vec{b}+\vec{c}|=5\sqrt{m}$ find m :

MVT073

ANSWER KEY

EXERCISE - O

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	B	A	C	D	A	C	B	A	C
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	C	A,C	B,C	A,C,D	A,C	A,B,C	B,D	A,B,C,D	A,B	B,D
Que.	21	22	23	24	25					
Ans.	A,B,D	C	B	B	(A) $\rightarrow$ (R); (B) $\rightarrow$ (Q,S); (C) $\rightarrow$ (Q,R); (D) $\rightarrow$ (P,Q,R,S)					

EXERCISE - S

1. 3 **2.** 0 **3.** 5 **4.** 3 **5.** 7
6. 3.80 **7.** 1.04 or 1.05 **8.** 0.45 or 0.46 **9.** 5.00 **10.** 4.00

EXERCISE - JEE (Main) PYQ

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	4	1	4	2	0.8	4	1	1494	2
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	1	6	4	4	2	4	1	4	2	4

EXERCISE - JEE (Advanced) PYQ

Que.	1	2	3	4	5	6	7	8	9	
Ans.	C	D	B,D	32	C	A,B,C	4	A	A,C,D	
Que.	10			11	12	13	14	15	16	17
Ans.	(A $\rightarrow$ P,R,S; B $\rightarrow$ P; C $\rightarrow$ P,Q ; D $\rightarrow$ S, T)			Bonus	B,C	B	D	B	3	C,D
Que.	18	19	20	21	22	23	24	25	26	
Ans.	18	108	A,C	7	A,B,C	B,C,D	45	B	B	

JEE (Main) Practice Paper

Section-A	Q.	1	2	3	4	5	6	7	8	9	10
	A.	1	2	2	4	2	4	1	4	4	1
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	1	3	4	3	4	3	4	1	1	1
Section-B	Q.	1	2	3	4	5	6	7	8	9	10
	A.	3	2	3	6	5	18	0	3	2	2

JEE (Advanced) Practice Paper

Section-I	**Q.**	**1**	**2**	**3**	**4**	**5**	**6**
	A.	C	C	B	C	C	C
Section-II	**Q.**	**7**	**8**	**9**	**10**		
	A.	A,B	B,D	C,D	A,C		
Section-III	**Q.**	**11**	**12**	**13**			
	A.	A	A	D			
Section-IV	**Q.**	**14**	**15**				
	A.	B	D				
Section-V	**Q.**	**16**	**17**	**18**			
	A.	9	9	2			

Important Notes