Trigonometric Ratios and Identities

SOLUTIONS

EXERCISE - O

1. **Ans. (C)**

We have, $\sin x + \sin^2 x = 1 \Rightarrow \sin x = 1 - \sin^2 x$

$\Rightarrow \cos^2 x = \sin x$ ...(i)

& $\cos^4 x = \sin^2 x$ ...(ii)

$\therefore$ Add (i) & (ii)

$\cos^2 x + \cos^4 x = \sin x + \sin^2 x = 1 \Rightarrow \cos^2 x + \cos^4 x = 1$

2. **Ans. (A)**

$\alpha = 10°;\ n = 36;\ \beta = 10°$

$$S = \frac{\sin\left(\frac{36°\times 10}{2}\right)\sin\left(10° + \frac{35\times 10}{2}\right)}{\sin 5°} = \frac{\sin 180°.\sin 185°}{\sin 5°} = 0$$

3. **Ans. (C)**

We have

$\cos x + \cos y = -\cos\alpha$

& $\sin x + \sin y = -\sin\alpha$

$\because\ \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$\Rightarrow 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = -\cos\alpha$ ...(1)

& $2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = -\sin\alpha$ ...(2)

(1)/(2) gives

$\cot\left(\frac{x+y}{2}\right) = \cot\alpha$

4. **Ans. (B)**

We know

$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$$\therefore \frac{2\sin\left(\frac{A-C+A+C}{2}\right)\cos\left(\frac{A-C-A-C}{2}\right) + 2\sin A}{2\sin\left(\frac{B-C+B+C}{2}\right)\cos\left(\frac{B-C-B-C}{2}\right) + 2\sin B}$$

$\Rightarrow \frac{\sin A\cos(-C) + \sin A}{\sin B\cos(-C) + \sin B}$ [$\because\ \cos(-\theta) = \cos\theta$]

$\Rightarrow \frac{\sin A(1+\cos C)}{\sin B(1+\cos C)} \Rightarrow \frac{\sin A}{\sin B}$

5. Ans. (B)

We have

$$\frac{2\times(\sin 8\theta\cos\theta-\sin 6\theta\cos 3\theta)}{2\times(\cos 2\theta\cos\theta-\sin 3\theta\sin 4\theta)}=\frac{\sin 9\theta+\sin 7\theta-\sin 9\theta-\sin 3\theta}{\cos 3\theta+\cos\theta-\cos\theta+\cos 7\theta}\Rightarrow\frac{(\sin 7\theta-\sin 3\theta)}{(\cos 3\theta+\cos 7\theta)}=\frac{2\cos 5\theta\sin 2\theta}{2\cos 5\theta\cos 2\theta}$$

Using formulae:

$2\sin A\cos B = \sin(A+B)+\sin(A-B)$

$2\cos A\cos B = \cos(A+B)+\cos(A-B)$

$2\sin A\sin B = \cos(A-B)-\cos(A+B)$

$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

6. Ans. (D)

$$\frac{1+\sin 2\theta+\cos 2\theta}{1+\sin 2\theta-\cos 2\theta}$$

$\because\ \cos 2\theta = 1-2\sin^2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\sin 2\theta = 2\sin\theta\cos\theta$

We have

$$\Rightarrow\frac{\sin 2\theta+1+2\cos^2\theta-1}{1-1+2\sin^2\theta+\sin 2\theta}\Rightarrow\frac{2\cos\theta[\sin\theta+\cos\theta]}{2\sin\theta[\sin\theta+\cos\theta]}=\cot\theta$$

7. Ans. (C)

We have

$$\frac{A}{B}=\frac{\tan 6^\circ\tan 42^\circ}{\cot 66^\circ\cot 78^\circ}$$

Multiply & divide by $\tan 54^\circ$

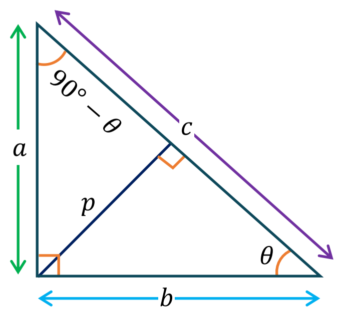
$$\Rightarrow \tan 6^\circ\tan 42^\circ\tan 66^\circ\tan 78^\circ\times\frac{\tan 54^\circ}{\tan 54^\circ}\Rightarrow\frac{\tan 6^\circ\tan(60^\circ-6^\circ)\tan(60^\circ+6^\circ)\tan 78^\circ\tan 42^\circ}{\tan 54^\circ}$$

$[\therefore\ \tan(60^\circ-\theta)\tan\theta\tan(60^\circ+\theta)=\tan 3\theta]$

$$\Rightarrow\frac{\tan 18^\circ\tan 42^\circ\tan 78^\circ}{\tan 54^\circ}\Rightarrow\frac{\tan 18^\circ\tan(60^\circ-18^\circ)\tan(60^\circ+18^\circ)}{\tan 54^\circ}$$

$$\Rightarrow\frac{\tan 54^\circ}{\tan 54^\circ}=1\Rightarrow\frac{A}{B}=1\Rightarrow A=B$$

8. Ans. (B)



$c^2 = a^2 + b^2$; $c=2\sqrt{2}p$

$a^2 + b^2 = 8p^2$ ...(1)

Area of $\Delta ABC = \frac{1}{2}ab = \frac{1}{2}pc$

$= ab = p(2\sqrt{2}\,p)$

$ab = 2\sqrt{2}\,p^2$ ...(2)

Divide (1) & (2)

$$\Rightarrow \frac{a^2+b^2}{ab}=\frac{8p^2}{2\sqrt{2}p^2}=2\sqrt{2}$$

$$\frac{a}{b}+\frac{b}{a}=2\sqrt{2}$$

$\tan\theta + \cot\theta = 2\sqrt{2}$

$$\frac{1}{\sin\theta\cos\theta}=2\sqrt{2}\Rightarrow \sin 2\theta=\frac{1}{\sqrt{2}}$$

$$2\theta=\frac{\pi}{4} \Rightarrow \theta=\frac{\pi}{8}$$

Hence other angle is $\frac{3\pi}{8}$

9. **Ans. (B)**

We have

$$\tan\alpha=(1+2^{-x})^{-1}=\left(\frac{2^x+1}{2^x}\right)^{-1}=\frac{1\times 2^x}{(1+2^x)}$$

$$\& \ \tan\beta=(1+2^{x+1})^{-1}=\frac{1}{(1+2^{x+1})}$$

$$\because \ \tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\therefore \ \tan(\alpha+\beta)=\frac{\frac{2^x}{(1+2^x)}+\frac{1}{(1+2^{x+1})}}{1-\frac{2^x}{(1+2^x)}\times\frac{1}{(1+2^{x+1})}}$$

$$\Rightarrow \frac{2^x(1+2^{x+1})+(1+2^x)}{(1+2^x)(1+2^{x+1})-2^x} \Rightarrow \frac{2^x+2^{2x+1}+1+2^x}{1+2^{x+1}+2^x+2^{2x+1}-2^x}$$

$$\Rightarrow \frac{(2^{x+1}+2^{2x+1}+1)}{(2^{x+1}+2^{2x+1}+1)}=1 \Rightarrow \tan(\alpha+\beta)=1$$

$\Rightarrow (\alpha + \beta) = \frac{\pi}{4}$ (from given options)

10. **Ans. (A)**

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$=\frac{\tan A+\frac{n\sin A\cos A}{1-n\cos^2 A}}{1-\tan A\cdot\frac{n\sin A\cos A}{1-n\cos^2 A}}=\frac{\sin A(1-n\cos^2 A)+n\sin A\cos^2 A}{\cos A(1-n\cos^2 A)-n\sin^2 A\cos A}$$

$$=\frac{\sin A-0}{\cos A(1-n\cos^2 A-n\sin^2 A)}=\frac{\sin A}{(1-n)\cos A}$$

11. Ans. (D)

$$\frac{1}{\sin\pi/18}-\frac{\sqrt{3}}{\cos\pi/18}$$

$$=\frac{2\left[\frac{1}{2}\cos\frac{\pi}{18}-\frac{\sqrt{3}}{2}\sin\frac{\pi}{18}\right]}{\frac{\sin(\pi/9)}{2}}=\frac{4\left[\sin\frac{\pi}{6}\cos\frac{\pi}{18}-\cos\frac{\pi}{6}\sin\frac{\pi}{18}\right]}{\sin\frac{\pi}{9}}=4$$

12. Ans. (D)

$$\cot x+\frac{\cos(60+x)}{\sin(60+x)}+\frac{\cos(x-60)}{\sin(x-60)}$$

$$=\frac{\cos x}{\sin x}+\frac{\sin(2x)}{\sin(x+60)\sin(x-60)}=\frac{\cos x}{\sin x}+\frac{8\sin x\cos x}{4\sin^2 x-3}$$

$$=\frac{4\sin^2 x\cos x-3\cos x+8\sin^2 x\cos x}{4\sin^3 x-3\sin x}=\frac{3[3\cos x-4\cos^3 x]}{-\sin 3x}=3\cot 3x$$

$$\Rightarrow\frac{3[1-3\tan^2 x]}{3\tan x-\tan^3 x}$$

13. Ans. (D)

On rationalizing ; we get

$$\frac{1-\sin x+1+\sin x+2|\cos x|}{1-\sin x-1-\sin x}=\frac{2(1+|\cos x|)}{-2(\sin x)}=\frac{1-\cos x}{-(\sin x)}$$

14. Ans. (B)

$$\left\{\frac{\cos\left(\frac{15}{2}\right)}{\sin\left(\frac{15}{2}\right)}-\frac{\sin\left(\frac{15}{2}\right)}{\cos\left(\frac{15}{2}\right)}\right\}+\left\{\frac{\sin\left(\frac{135}{2}\right)}{\cos\left(\frac{135}{2}\right)}-\frac{\cos\left(\frac{135}{2}\right)}{\sin\left(\frac{135}{2}\right)}\right\}$$

$$\Rightarrow\frac{2\left(\cos^2\left(\frac{15}{2}\right)-\sin^2\left(\frac{15}{2}\right)\right)}{2\sin\left(\frac{15}{2}\right)\cos\left(\frac{15}{2}\right)}-\frac{2\left(\cos^2\frac{135}{2}-\sin^2\frac{135}{2}\right)}{2\sin\left(\frac{135}{2}\right)\cos\left(\frac{135}{2}\right)}\Rightarrow\frac{2\cos 15}{\sin 15}-\frac{2\cos 135}{\sin 135}$$

$\Rightarrow 2\cot 15° - 2\cot 135° \Rightarrow 2\tan(90° - 15°) - 2\cot(90° + 45°)$

$\Rightarrow 2\tan 75° + 2\tan 45° \Rightarrow 2(2+\sqrt{3})+2(1)=6+2\sqrt{3}$ Irrational number

Aliter :

$\cot\theta - \tan\theta = 2\cot 2\theta$

$$\left(\cot\frac{15°}{2}-\tan\frac{15°}{2}\right)+\left(\tan\frac{135°}{2}-\cot\frac{135°}{2}\right)$$

$2\cot 15° - 2\cot 135°$

$$2(2+\sqrt{3})-2(-1)=4+2\sqrt{3}+2=6+2\sqrt{3}$$

15. Ans. (D)

On adding and subtracting

$x = \dfrac{3-\cos 4\theta+4\sin 2\theta}{2}$;

$y = \dfrac{3-\cos 4\theta-4\sin 2\theta}{2}$

$x = \dfrac{4(1+\sin 2\theta)-(1+\cos 4\theta)}{2}$;

$y = \dfrac{4(1-\sin 2\theta)-(1+\cos 4\theta)}{2}$

$x = 2\,(1 + \sin 2\theta\,) - \cos^2 2\theta$; $y = 2\,(1 - \sin 2\theta) - \cos^2 2\theta$

$x = 1 + 2\sin 2\theta + \sin^2 2\theta$; $y = 1 - 2\sin 2\theta + \sin^2 2\theta$

$x = (1 + \sin 2\theta)^2$; $y = (1 - \sin 2\theta)^2$

$\Rightarrow \sqrt{x}+\sqrt{y}=2$

[Alternate : Or put $\theta = \dfrac{\pi}{4}$ and verify]

16. Ans. (C)

$\tan A \tan B \tan C = \tan A + \tan B + \tan C$ if

$A + B + C = n\pi$.

17. Ans. (D)

We have $f(x)=\dfrac{\sin 3x}{sinx}$

$\because\ \sin 3x = 3\sin x - 4\sin^3 x \Rightarrow \dfrac{3\sin x-4\sin^3 x}{\sin x}$

$=\dfrac{\sin x(3-4\sin^2 x)}{\sin x}$ $\qquad (\because\ \sin x \neq 0)$

$\Rightarrow 3 - 4\sin^2 x$

Now $-1 \le \sin x \le 1$

$\Rightarrow 0 < \sin^2 x \le 1$ $\qquad (\because\ \sin x \neq 0)$

$-4 \le -4\sin^2 x < 0$

$3 - 4 \le 3 - 4\sin^2 x < 3 - 0$

$-1 \le 3 - 4\sin^2 x < 3$

$\therefore\ f(x) \in [-1, 3)$

18. Ans. (D)

$E=2\sin^2\theta-3\sin\theta+2$

$=2\left(\sin^2\theta-\dfrac{3}{2}\sin\theta\right)+2\ =2\left(\left(\sin\theta-\dfrac{3}{4}\right)^2-\dfrac{9}{16}\right)+2$

$E=2\left(\sin\theta-\dfrac{3}{4}\right)^2+\dfrac{7}{8}$

$E_{\min}=\dfrac{7}{8}$ at $\sin\theta=\dfrac{3}{4}$

$E_{\max}=7$ at $\sin\theta=-1$

19. Ans. (A)

$\sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{6}\right)$

$\Rightarrow E_{max}\sqrt{2}$ at $\theta+\frac{\pi}{6}=\frac{\pi}{4}\Rightarrow \theta=\frac{\pi}{12}$

20. Ans. (B)

$E = 1 + 2\sin^2\theta - 3\sin 2\theta + 2$

$=3+1-\cos 2\theta-3\sin 2\theta \Rightarrow 4-(\cos 2\theta+3\sin 2\theta)$

$E_{min} = 4-\sqrt{10}$

21. Ans. (B,C)

$\cos x = \tan x \Rightarrow \cos x=\frac{\sin x}{\cos x}$

$\Rightarrow \cos^2 x = \sin x \Rightarrow \cos^4 x = \sin^2 x \Rightarrow \cos^4 x = 1 - \cos^2 x$

$\Rightarrow \cos^4 x+\cos^2 x=1$ ...(C)

Now, $\sin^2 x + \sin x = 1$ (Use $\cos^2 x = \sin x$)

$\Rightarrow \sin x + 1 = \frac{1}{\sin x} \Rightarrow \cos^2 x+1=\frac{1}{\sin x}$

Therefore, $\frac{1}{\sin x} + \cos^4 x = \cos^2 x + 1 + \cos^4 x$

$= 1 + 1 = 2$ ...(B)

22. Ans. (B,C)

$\tan A.\tan B = 2 \Rightarrow \frac{\sin A.\sin B}{\cos A.\cos B}=\frac{2}{1}$

Applying componendo and dividendo, we have

$\frac{\sin A\sin B+\cos A\cos B}{\sin A\sin B-\cos A\cos B}=\frac{2+1}{2-1}$

$\Rightarrow \frac{\cos(A-B)}{-\cos(A+B)}=3 \Rightarrow \frac{3}{-5\cos(A+B)}=3$

$\Rightarrow \cos(A+B)=\frac{-1}{5}$ ...(C)

Now,

$\cos A.\cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$

$\Rightarrow \cos A.\cos B = \frac{1}{2}\left[\frac{-1}{5}+\frac{3}{5}\right]=\frac{1}{5}$

and $\sin A.\sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$

$\Rightarrow \sin A.\sin B = \frac{1}{2}\left[\frac{3}{5}-\left(\frac{-1}{5}\right)\right]=\frac{2}{5}$ ...(B)

Now, $\sin(A + B) = \pm\frac{\sqrt{24}}{5}$; $\sin(A - B) = \pm\frac{4}{5}$

So, (D) option is not correct

23. Ans. (B,C)

$$\sin t = \frac{2\tan\frac{t}{2}}{1+\tan^2\frac{t}{2}}; \quad \cos t = \frac{1-\tan^2\frac{t}{2}}{1+\tan^2\frac{t}{2}}$$

Let $\tan\frac{t}{2} = x$

$\therefore \frac{2x}{1+x^2} + \frac{1-x^2}{1+x^2} = \frac{1}{5} \Rightarrow 10x + 5 - 5x^2 = 1 + x^2$

$\Rightarrow 6x^2 - 10x - 4 = 0 \Rightarrow 3x^2 - 5x - 2 = 0 \Rightarrow 3x^2 - 6x + x - 2 = 0$

$\Rightarrow 3x(x-2) + 1(x-2) = 0 \Rightarrow (3x+1)(x-2) = 0$

$\Rightarrow x = \frac{-1}{3}$ or $x = 2$

$\therefore \tan\frac{t}{2} = \frac{-1}{3}$ or 2

24. Ans. (A,B)

$$\frac{\sin\beta}{\sin(2\alpha+\beta)} = \frac{1}{3}$$

Applying componendo and Dividendo,

we have, $\frac{\sin\beta + \sin(2\alpha+\beta)}{\sin\beta - \sin(2\alpha+\beta)} = \frac{1+3}{1-3}$

$\Rightarrow \frac{2\sin(\alpha+\beta).\cos\alpha}{-2\cos(\alpha+\beta).\sin\alpha} = \frac{4}{-2} \Rightarrow \frac{\tan(\alpha+\beta)}{\tan\alpha} = 2$

$\Rightarrow \boxed{\tan(\alpha+\beta) - 2\tan\alpha = 0}$

25. Ans. (C,D)

$\cos^2 x + \cos^2 y + \cos^2 z - 2\cos x.\cos y.\cos z$

$= \frac{1}{2}$ $[2\cos^2 x + 2\cos^2 y + 2\cos^2 z - 4\cos x.\cos y.\cos x]$

$= \frac{1}{2}$ $[1+\cos 2x+1+\cos 2y+2\cos^2 z - 2(2\cos x.\cos y)\cos z]$

$= \frac{1}{2}$ $[1+\cos 2x+1+\cos 2y+2\cos^2 z - 2(2\cos x.\cos y)\cos z]$

$= \frac{1}{2}$ $[2+\cos 2x+\cos 2y+2\cos^2 z - 2(\cos(x+y)+\cos(x-y))\cos z]$

$= \frac{1}{2}$ $[2+2\cos(x+y)\cos(x-y)+2\cos^2 z - 2(\cos z+\cos(x-y))\cos z]$

$=1+\cos z.\cos(x-y)+\cos^2 z-\cos^2 z-\cos(x-y)\cos z$

$= 1 = \cos(x+y-z)$

26. **Ans. (A,B,C,D)**

Method-I

$L = \frac{1}{2}$ [$2\cos^2 84° + 2\cos^2 36° + 2\cos 36°\cos 84°$]

$\Rightarrow L = \frac{1}{2}$ [$1 + \cos 168° + 1 + \cos 72° + \cos(36° + 84°) + \cos(36° - 84°)$]

$\Rightarrow L = [2+\cos 168°+\cos 72°+\cos 120°+\cos 48°]$

Use : $2\cos^2\theta = 1 + \cos 2\theta$;

$2\cos A.\cos B = \cos(A + B) + \cos(A - B)$

$$\Rightarrow L = \frac{1}{2}\left[2 + 2\cos\left(\frac{168° + 72°}{2}\right)\cos\left(\frac{168° - 72°}{2}\right) + \cos 120° + \cos 48°\right]$$

Use : $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

$$= \frac{1}{2}\left[2 + 2\cos 120°.\cos 48° - \frac{1}{2} + \cos 48°\right]$$

$$= \frac{1}{2}\left[2 + 2\times\left(\frac{-1}{2}\right)\cos 48° - \frac{1}{2} + \cos 48°\right] = \frac{3}{4} = L$$

Method-II

$L = \cos^2 84° + 1 - \sin^2 36° + \frac{1}{2}\ 2\cos 36°\cos 84°$

$=1 + \cos(84° + 36°)\cos(84° - 36°) + \frac{1}{2}\ [\cos 120° + \cos 48°]$

$= \frac{3}{4} + \cos 120°.\cos 48° + \frac{1}{2}\cos 48° = \frac{3}{4}$

$L = \frac{3}{4} < 1$

$M = \cot 73°.\cot 47°.\cot 13°$

$= \cot(60° + 13°)\cot(60° - 13°)\cot 13°$

$= \cot(3.13°) = \cot 39°$

as $\cot 45° < \cot 39° \Rightarrow L < M$

$\tan 2 < 0$

$\therefore M > \tan 2$

$N = 4\sin 156°\sin 84°\sin 36°$

$= 4\sin(180° - 156°)\sin 84°\sin 36°$

$= 4\sin 24°.\sin(60° + 24°).\sin(60° - 24°)$

$= 4 \times \frac{1}{4}\sin(3 \times 24°) = \sin 72° = \frac{\sqrt{5}-1}{4} > \sin 45°$

$N > \sin\frac{\pi}{4}$

And; $LMN > 0$

27. Ans. (A,B,C,D)

$P_n = \cos^n\theta + \sin^n\theta$

$P_{n-2} = \cos^{n-2}\theta + \sin^{n-2}\theta$

$P_n - P_{n-2} = \cos^n\theta + \sin^n\theta - (\cos^{n-2}\theta + \sin^{n-2}\theta)$

$= \cos^{n-2}\theta(\cos^2\theta - 1) + \sin^{n-2}\theta(\sin^2\theta - 1)$

$= -\sin^2\theta\cos^{n-2}\theta - \cos^2\theta\sin^{n-2}\theta = -\sin^2\theta\cos^2\theta\, P_{n-4}$

$P_4 = \cos^4\theta + \sin^4\theta$

$= (\cos^2\theta + \sin^2\theta)^2 - 2\sin^2\theta\cos^2\theta = 1 - 2\sin^2\theta\cos^2\theta$

$Q_n = \cos^n\theta - \sin^n\theta$

$Q_{n-2} = \cos^{n-2}\theta - \sin^{n-2}\theta$

$Q_n - Q_{n-2} = (\cos^n\theta - \sin^n\theta) - (\cos^{n-2}\theta - \sin^{n-2}\theta)$

$= \cos^{n-2}\theta(\cos^2\theta - 1) - \sin^{n-2}\theta(\sin^2\theta - 1)$

$= -\sin^2\theta\cos^{n-2}\theta + \cos^2\theta\sin^{n-2}\theta = -\sin^2\theta\cos^2\theta\, Q_{n-4}$

$Q_4 = \cos^4\theta - \sin^4\theta$

$= (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$= \cos^2\theta - \sin^2\theta$

28. Ans. (A,D)

$\alpha + \beta + \gamma = 2\pi \Rightarrow \frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2} = \pi$

$$\Rightarrow \tan\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) = \tan\left(\pi - \frac{\gamma}{2}\right) \Rightarrow \frac{\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}}{1 - \tan\frac{\alpha}{2}.\tan\frac{\beta}{2}} = -\tan\frac{\gamma}{2} \qquad \text{...(A)}$$

$$\& \ \frac{\alpha}{4} + \frac{\beta}{4} + \frac{\gamma}{4} = \frac{\pi}{2} \Rightarrow \frac{\alpha}{4} + \frac{\beta}{4} = \frac{\pi}{2} - \frac{\gamma}{4}$$

$$\Rightarrow \tan\left(\frac{\alpha}{4} + \frac{\beta}{4}\right) = \tan\left(\frac{\pi}{2} - \frac{\gamma}{4}\right) \Rightarrow \frac{\tan\frac{\alpha}{4} + \tan\frac{\beta}{4}}{1 - \tan\frac{\alpha}{4}\tan\frac{\beta}{4}} = \cot\frac{\gamma}{4} = \frac{1}{\tan\frac{\gamma}{4}} \qquad \text{...(D)}$$

29. Ans. (B,D)

$$y = \frac{\cos\left(\frac{x+7x}{2}\right)\frac{\sin\left(7\times\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)}}{\sin\left(\frac{x+7x}{2}\right)\frac{\sin\left(7\times\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)}} = \cot 4x$$

$$\cot\frac{\pi}{8} = \frac{1}{\sqrt{2}-1} = \sqrt{2}+1 \ ; \ \cot\frac{\pi}{2} = 0$$

$$\cot\frac{\pi}{12} = \frac{1}{2-\sqrt{3}} = 2+\sqrt{3}$$

30. **Ans. (C,D)**

$$f_n(\theta)=\sum_{r=0}^{n}\frac{1}{4^r}\sin^4\left(2^r\theta\right)$$

$$=\sum_{r=0}^{n}\frac{1}{4^r}\sin^2\left(2^r\theta\right)\left[1-\cos^2\left(2^r\theta\right)\right]$$

$$=\sum_{r=0}^{n}\left(\frac{\sin^2\left(2^r\theta\right)}{4^r}-\frac{\sin^2\left(2^r\theta\right)\cos^2\left(2^r\theta\right)}{4^r}\right)$$

$$=\sum_{r=0}^{n}\left(\frac{\sin^2\left(2^r\theta\right)}{4^r}-\frac{\sin^2\left(2^{r+1}\theta\right)}{4^{r+1}}\right)$$

$$=\left(\sin^2\theta-\frac{\sin^2 2\theta}{4}\right)+\left(\frac{\sin^2 2\theta}{4}-\frac{\sin^2 4\theta}{16}\right)+\left(\frac{\sin^2 4\theta}{16}-\frac{\sin^2 8\theta}{64}\right)\ldots\ldots\left(\frac{\sin^2\left(2^n\theta\right)}{4^n}-\frac{\sin^2\left(2^{n+1}\theta\right)}{4^{n+1}}\right)$$

$$\Rightarrow f_n(\theta)=\sin^2\theta-\frac{\sin^2\left(2^{n+1}\theta\right)}{4^{n+1}}$$

Now everything can be calculated by putting values as per options.

31. **Ans. (A)**

$P(x) = (x - a)(x - b)(x - c)$

$= (x^2 - (a + b)x + ab)(x - c)$

$= x^3 - (a + b)x^2 + abx - cx^2 + (ac + bc)x - abc$

$P(x) = x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc$

coefficient of

$x^2 = -(a + b + c)$

$= -(\cos 36° + \cos 84° + \cos 156°)$

$$=-\left(\cos 36°+2\cos\left(\frac{84°+156°}{2}\right)\cos\left(\frac{156°-84°}{2}\right)\right)$$

$= -(\cos 36° + 2\cos 120°.\cos 36°)$

$= -(\cos 36° - 2 \times \frac{1}{2}\cos 36°) = 0$

32. **Ans. (C)**

Coefficient of $x = ab + bc + ca$

$= \cos 36°.\cos 84° + \cos 84°.\cos 156° + \cos 156°.\cos 36°$

$=\frac{1}{2}$ [$2\cos 36°.\cos 84°+2\cos 84°.\cos 156°+2\cos 156°.\cos 36°$]

$=\frac{1}{2}$ [$\cos 120°+\cos 48°+\cos 240°+\cos 72°+\cos 192°+\cos 120°$]

$=\frac{1}{2}$ [$-\frac{1}{2}\times 3 + \cos 48° + \cos 72° + \cos 192°$] $= \frac{1}{2}\left[\frac{-3}{2}+2\cos\left(\frac{48°+72°}{2}\right)\cos\left(\frac{72°-48°}{2}\right)-\cos 12°\right]$

$$=\frac{1}{2}\left[\frac{-3}{2}+2\cos 60°\cos 12°-\cos 12°\right]=\frac{-3}{4}$$

33. **Ans. (B)**

Absolute term = $-abc$

= –cos36°.cos84°.cos156°

= –cos36°.cos84°.cos(180° – 24°)

= cos24°.cos36°.cos84°

= cos24°.cos(60°–24°).cos(60°+24°)

$= \frac{1}{4}\cos(3\times24°) = \frac{1}{4}\cos72°$

$= \frac{1}{4}\left(\frac{\sqrt{5}-1}{4}\right) = \frac{\sqrt{5}-1}{16}$

34. **Ans. (A)**

(i) taking L.H.S. = tan 20° tan 40° tan 60° tan 80°

= (tan 20° tan 40° tan 80°) tan 60°

we know tan20° tan(60° – 20°) tan(60° + 20°) = tan60°

So $\tan^2 60° = \left(\sqrt{3}\right)^2 = 3$

(ii) $4\left(\sin^4\frac{\pi}{16}+\cos^4\frac{\pi}{16}+\sin^4\frac{3\pi}{16}+\cos^4\frac{3\pi}{16}\right)$

$4\left(1-2\sin^2\frac{\pi}{16}\cos^2\frac{\pi}{16}+1-2\sin^2\frac{3\pi}{16}\cos^2\frac{3\pi}{16}\right)$

$4\left(2-\frac{1}{2}\left\{\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}\right\}\right)$

$4\left(2-\frac{1}{2}\left\{\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}\right\}\right)$

= 6

(iii) $\cot\alpha - \tan\alpha = 2\cot2\alpha$

$\cot\alpha - \cot\alpha + \tan\alpha + 2\tan2\alpha + 4\tan4\alpha + 8\cot8\alpha$

$= \cot\alpha - 2\cot2\alpha + 2\tan2\alpha + 4\tan4\alpha + 8\cot8\alpha$

$= \cot\alpha - 4\cot4\alpha + 4\tan4\alpha + 8\cot8\alpha$

$= \cot\alpha - 8\cot8\alpha + 8\cot8\alpha$

$= \cot\alpha$

$\boxed{\lambda = 1}$

(iv) $(4\cos^2 9° - 3)(4\cos^2 27° - 3)$

$= \frac{\cos27°}{\cos9°}(4\cos^2 27° - 3)$

$= \frac{4\cos^3 27° - 3\cos27°}{\cos9°}$

(using $\cos 3A = 4\cos^3 A - 3\cos A$)

$= \frac{\cos81°}{\cos9°} = \frac{\sin9°}{\cos9°} = \tan9°$

35. Ans. (A→S; B→R; C→Q; D→P)

(A) Ans. (S)

$E = |2.(4\cos^3\theta) - 4\cos^2\theta - 4\cos\theta|$

We know that, $4\cos^3\theta = \cos 3\theta + 3\cos\theta$
& $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

$\Rightarrow E = |2(\cos 3\theta + 3\cos\theta) - 2(1 + \cos 2\theta) - 4\cos\theta|$

$= 2|\cos 3\theta + \cos\theta - 1 - \cos 2\theta| = 2|2\cos 2\theta.\cos\theta - 2\cos^2\theta|$

$= 4|\cos\theta(\cos 2\theta - \cos\theta)|$

$= 4\left|\cos\theta.2\sin\frac{3\theta}{2}.\sin\left(\frac{-\theta}{2}\right)\right|$ We know $\left\{\begin{array}{l}\sin\frac{\pi}{14} = \cos\left(\frac{\pi}{2}-\frac{\pi}{14}\right) = \cos\left(\frac{3\pi}{7}\right) \\ \sin\frac{3\pi}{14} = \cos\left(\frac{\pi}{2}-\frac{3\pi}{14}\right) = \cos\left(\frac{2\pi}{7}\right)\end{array}\right\}$

$= 8\left|\sin\frac{\pi}{14}.\sin\frac{3\pi}{14}.\cos\frac{\pi}{7}\right| = 8\left|\cos\frac{3\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{\pi}{7}\right|$

$= 8\left|-\cos\frac{4\pi}{7}\cos\frac{2\pi}{7}\cos\frac{\pi}{7}\right| = 8\left|\frac{-\sin\frac{8\pi}{7}}{2^3\sin\frac{\pi}{7}}\right| = 1$

(B) Ans. (R)

$A + B + C = \pi;\ \cos A = \cos B.\cos C$

$\Rightarrow \cos(\pi - (B + C)) = \cos B.\cos C \Rightarrow -\cos(B + C) = \cos B.\cos C$

$\Rightarrow -(\cos B.\cos C - \sin B.\sin C) = \cos B.\cos C \Rightarrow \sin B\sin C = 2\cos B.\cos C$

$\Rightarrow \tan B.\tan C = 2$

(C) Ans. (Q)

$$4\left(\frac{\cos 20° + 8\sin 70°\sin 50°\sin 10°}{\sin^2 80°}\right) = 4\left(\frac{\cos 20° + 8\times\frac{1}{4}\sin 30°}{\frac{1-\cos 160°}{2}}\right) = \frac{8(\cos 20°+1)}{1-\cos 160°} = 8$$

(D) Ans. (P)

$f(\theta) = 12\sin\theta - 9\sin^2\theta$

$= -9(\sin^2\theta - \frac{4}{3}\sin\theta) = -9\left(\left(\sin\theta - \frac{2}{3}\right)^2 - \frac{4}{9}\right) = 4 - 9\left(\sin\theta - \frac{2}{3}\right)^2$

$\sin\theta \in [-1, 1] \Rightarrow \sin\theta - \frac{2}{3} \in \left[\frac{-5}{3}, \frac{1}{3}\right]$

$\left(\sin\theta - \frac{2}{3}\right)^2 \in \left[0, \frac{25}{9}\right] \Rightarrow 9\left(\sin\theta - \frac{2}{3}\right)^2 \in [0, 25]$

$-9\left(\sin\theta - \frac{2}{3}\right)^2 \in [-25, 0]$

$4 - 9\left(\sin\theta - \frac{2}{3}\right)^2 \in [-21, 4]$

EXERCISE - S

1. **Ans. (0)**

Using the formulae

$\because\ (A + B)^3 = A^3 + B^3 + 3AB(A + B)$

$\Rightarrow A^3 + B^3 = (A + B)^3 - 3AB(A + B)$ &

$\because\ (A + B)^2 = A^2 + B^2 + 2AB$

$\Rightarrow A^2 + B^2 = (A + B)^2 - 2AB$

$\sin^2\theta + \cos^2\theta = 1$

We have

$2[(\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)]$

$-3[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta] + 1$

$\Rightarrow 2 - 6\sin^2\theta\cos^2\theta - 3 + 6\sin^2\theta\cos^2\theta + 1 \Rightarrow 2 - 3 + 1 = 0$

2. **Ans. (0)**

We have

$x = y\cos\frac{2\pi}{3} = z\cos\frac{4\pi}{3}$

$x = y\cos120° = z\cos240°$

$\Rightarrow x = y\cos(90° + 30°) = z\cos(180° + 60°)$

$\Rightarrow x = y(-\sin30°) = z(-\cos60°) \Rightarrow x = \frac{-y}{2} = \frac{-z}{2}$

$\therefore\ xy + yz + xz = y\left(\frac{-y}{2}\right) + y.y + y\left(\frac{-y}{2}\right)$

$\Rightarrow \frac{-y^2}{2} + y^2 - \frac{y^2}{2} = 0$

3. **Ans. (1)**

$\cos\alpha\cos\beta - \sin\alpha\sin\beta + \sin\alpha\cos\beta - \cos\alpha\sin\beta = 0$

$(\cos\beta)(\cos\alpha + \sin\alpha) - (\sin\beta)(\sin\alpha + \cos\alpha) = 0$

$(\cos\alpha + \sin\alpha)(\cos\beta - \sin\beta) = 0$

$\Rightarrow \cos\alpha + \sin\alpha = 0$ or $\cos\beta = \sin\beta \Rightarrow \tan\alpha = -1$ (Ans.)

or $\tan\beta = 1$ (reject as $\tan\beta = \frac{-1}{2010}$ (given))

4. **Ans. (150)**

$\because\ \cot x + \cot y = \frac{1}{\tan x} + \frac{1}{\tan y} = 30$

$\frac{\tan x + \tan y}{\tan x.\tan y} = 30$

$\tan x.\tan y = \frac{\tan x + \tan y}{30} = \frac{25}{30} = \frac{5}{6}$

$\therefore\ \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{25}{\left(1 - \frac{5}{6}\right)} = 150$

5. Ans. (4)

We have

$3\sin\alpha = 5\sin\beta \Rightarrow \frac{\sin\alpha}{\sin\beta} = \frac{5}{3}$

(Applying componendo & dividendo rule)

$\Rightarrow \frac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta} = \frac{5+3}{5-3} = 4$

$\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

& $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\Rightarrow \frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)} = 4 \Rightarrow \frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)} = 4$

6. Ans. (3)

$\frac{1}{\cos\alpha} = \frac{2-\cos\beta}{2\cos\beta-1}$ (Applying componendo & dividendo rule)

$\Rightarrow \frac{1-\cos\alpha}{1+\cos\alpha} = \frac{3(1-\cos\beta)}{1+\cos\beta} \Rightarrow \tan^2\frac{\alpha}{2} = 3\tan^2\frac{\beta}{2}$

$\Rightarrow \tan^2\frac{\alpha}{2}\cdot\cot^2\frac{\beta}{2} = 3$

7. Ans. (2)

$A^2 + B^2 = 3 + 2\left[\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}\right]$

$= 3 + 2\left(-\frac{1}{2}\right) = 2 \Rightarrow A^2+B^2 = 2$

8. Ans. (9)

$S = \sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}$

$= \sin\frac{\pi}{14}\cos\frac{2\pi}{14}\cos\frac{4\pi}{14}$

Multiply divide by $8\cos\frac{\pi}{14}$,

we get $S = \frac{\sin\frac{8\pi}{14}}{8\cos\frac{\pi}{14}} = \frac{1}{8}$

9. Ans. (24)

$$\frac{96\sin 80^\circ \sin 65^\circ.\sin 35^\circ}{\sin 20^\circ + \sin 50^\circ + \sin 110^\circ} = \frac{N^r}{D^r}$$

$\therefore D^r = \sin 20^\circ + \sin 50^\circ + \sin 110^\circ$

$= 2\sin 35^\circ.\cos 15^\circ + 2\sin 55^\circ.\cos 55^\circ$

$\because \cos 55^\circ = \cos(90^\circ - 35^\circ) = \sin 35^\circ$

$= 2\sin 35^\circ.\cos 15^\circ + 2\sin 55^\circ.\sin 35^\circ$

$= 2\sin 35^\circ.(\cos 15^\circ + \sin 55^\circ)$

$\because \cos 15^\circ = \cos(90^\circ - 75^\circ) = \sin 75^\circ$

$= 2\sin 35^\circ(\sin 75^\circ + \sin 55^\circ)$

$=2\sin 35^\circ(2\sin 65^\circ.\cos 10^\circ)$

$\because \cos 10^\circ = \cos(90^\circ - 80^\circ) = \sin 80^\circ$

$= 4\sin 35^\circ.\sin 65^\circ \sin 80^\circ$

$$\therefore \frac{N^r}{D^r} = \frac{96}{4} = 24$$

10. Ans. (11)

$\cos 0 + \cos 2\theta + \cos 4\theta + \cos 6\theta + \cos 8\theta + \cos 10\theta$

$\alpha = 0;\ b = 2\theta;\ n = 6$

$$S = \frac{\cos\left(\alpha + \left(\frac{n-1}{2}\right)\beta\right)\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} = \frac{\cos 5\theta.\sin 6\theta}{\sin\theta}$$

$m = 5$ & $n = 6$

Hence $(m + n) = 11$

11. Ans. (5)

$$\left|\frac{3\sec B - 4\operatorname{cosec} B}{2}\right| = \left|\frac{\frac{3}{\cos B} - \frac{4}{\sin B}}{2}\right|$$

$$= \left|\frac{3\sin B - 4\cos B}{2\sin B\cos B}\right| = 5\left|\frac{\frac{3}{5}\sin B - \frac{4}{5}\cos B}{\sin 2B}\right| = 5\left|\frac{\cos A\sin B - \sin A\cos B}{\sin 2B}\right|$$

$$= 5\left|\frac{\sin(B-A)}{\sin 2B}\right| = 5\left|\frac{\sin(B-3B)}{\sin 2B}\right| = 5\left|\frac{\sin(-2B)}{\sin 2B}\right| = 5\left|\frac{-\sin 2B}{\sin 2B}\right| = 5|-1| = 5$$

12. Ans. (2)

$$\frac{\sin\theta}{\cos 3\theta} = \frac{\sin\theta\cos\theta}{\cos 3\theta\cos\theta} = \frac{1}{2}\frac{\sin 2\theta}{\cos 3\theta.\cos\theta}$$

$$= \frac{1}{2}\left[\frac{\sin(3\theta - \theta)}{\cos 3\theta.\cos\theta}\right] = \frac{1}{2}\left[\frac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\cos 3\theta.\cos\theta}\right]$$

$$\frac{\sin\theta}{\cos 3\theta} = \frac{1}{2}[\tan 3\theta - \tan\theta]$$

Similarly,

$$\frac{\sin 3\theta}{\cos 9\theta}=\frac{\sin 3\theta \cos 3\theta}{\cos 9\theta \cos 3\theta}=\frac{1}{2}\frac{\sin(9\theta-3\theta)}{\cos 9\theta.\cos 3\theta}=\frac{1}{2}[\tan 9\theta-\tan 3\theta]$$

and $\frac{\sin 9\theta}{\cos 27\theta}=\frac{1}{2}[\tan 27\theta-\tan 9\theta]$

adding all,

$$\frac{\sin\theta}{\cos 3\theta}+\frac{\sin 3\theta}{\cos 9\theta}+\frac{\sin 9\theta}{\cos 27\theta}=\frac{1}{2}[\tan 27\theta-\tan\theta]$$

$\Rightarrow k_2=\frac{k_1}{2} \Rightarrow \frac{k_1}{k_2}=2$

13. Ans. (1.33)

sin 25°.sin 35°.sin 85°

sin25° sin(60° – 25°) sin(60° + 25°)

$=\frac{1}{4}\sin 75°=\frac{\sqrt{3}+1}{8\sqrt{2}}$

$=\frac{\sqrt{6}+\sqrt{2}}{16}\to a+b+c=6+2+16=24$

14. Ans. (5.75)

$A + B = 45° \to \tan A = \tan(45° - B) = \frac{1-\tan B}{1+\tan B}$

$\tan A + \tan B + \tan A \tan B = 1$

$(1 + \tan A)(1+ \tan B) = 2$,

$(1 + \tan 1°)(1 + \tan 44°) = 2$

$2^{22}.2 = 2^{23} = 2^n \to n = 23$

15. Ans. (1.73)

$$\frac{2\cos 40°-\cos 20°}{\sin 20°}$$

$$=\frac{2\cos(60°-20°)-\cos 20°}{\sin 20°}=\frac{2\cos 60°\cos 20°+2\sin 60°\sin 20°-\cos 20°}{\sin 20°}$$

$=\frac{2\sin 20°}{\sin 20°}\times\frac{\sqrt{3}}{2}=\sqrt{3}$ = 1.73

EXERCISE - JEE (Main) PYQ

1. Ans. (1)

We have,

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$

$= 3(1 - \sin 2\theta)^2 + 6(1 + \sin 2\theta) + 4\sin^6\theta = 3(1 - 2\sin 2\theta + \sin^2 2\theta) + 6 + 6\sin 2\theta + 4\sin^6\theta$

$= 9 + 12\sin^2\theta \cdot \cos^2\theta + 4(1 - \cos^2\theta)^3 = 13 - 4\cos^6\theta$

2. Ans. (3)

$$2\sin\frac{\pi}{2^{10}}\cos\frac{\pi}{2^{10}}\ldots\ldots\cos\frac{\pi}{2^2}$$

$$\frac{1}{2^9}\sin\frac{\pi}{2}=\frac{1}{512}$$

3. Ans. (4)

$f_4(x) - f_6(x)$

$$=\frac{1}{4}(\sin^4 x+\cos^4 x)-\frac{1}{6}(\sin^6 x+\cos^6 x)$$

$$=\frac{1}{4}\left(1-\frac{1}{2}\sin^2 2x\right)-\frac{1}{6}\left(1-\frac{3}{4}\sin^2 2x\right)=\frac{1}{12}$$

4. Ans. (1)

$$y=3\cos\theta+5\left(\sin\theta\frac{\sqrt{3}}{2}-\cos\theta\frac{1}{2}\right)$$

$$\frac{5\sqrt{3}}{2}\sin\theta+\frac{1}{2}\cos\theta$$

$$y_{\max}=\sqrt{\frac{75}{4}+\frac{1}{4}}=\sqrt{19}$$

5. Ans. (2)

$$\tan\alpha + \tan\beta = \frac{\lambda\sqrt{2}}{k+1}$$

$$\tan\alpha.\ \tan\beta = \frac{k-1}{k+1}$$

$$\tan(\alpha+\beta) = \frac{\frac{\lambda\sqrt{2}}{k+1}}{1-\frac{k-1}{k+1}}=\frac{\lambda\sqrt{2}}{2}=\frac{\lambda}{\sqrt{2}}$$

$$\Rightarrow \frac{\lambda^2}{2}=50\Rightarrow\lambda=10\ \&\ -10$$

6. Ans. (3)

$$\sin^3\left(\frac{3\pi}{8}\right)\cos\left(\frac{3\pi}{8}\right)+\cos^3\left(\frac{3\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)$$

$$\Rightarrow \sin\left(\frac{3\pi}{8}\right)\cos\left(\frac{3\pi}{8}\right)\left[\sin^2\frac{3\pi}{8}+\cos^2\frac{3\pi}{8}\right]=\frac{\sin\left(\frac{3\pi}{4}\right)}{2}=\frac{1}{2\sqrt{2}}$$

7. Ans. (4)

$15\sin^4\alpha + 10\cos^4\alpha = 6$

$15\sin^4\alpha + 10\cos^4\alpha = 6(\sin^2\alpha + \cos^2\alpha)^2$

$(3\sin^2\alpha - 2\cos^2\alpha)^2 = 0$

$$\tan^2\alpha=\frac{2}{3}.\ \ \cot^2\alpha=\frac{3}{2}$$

$\Rightarrow 27\sec^6\alpha + 8\operatorname{cosec}^6\alpha = 27(\sec^2\alpha)^3 + 8(\operatorname{cosec}^2\alpha)^3$

$= 27(1 + \tan^2\alpha)^3 + 8(1 + \cot^2\alpha)^3$

$= 250$

8. Ans. (2)

16 sin20° sin 40° sin 80°

= 16 sin 40° sin 20° sin 80° = 4(4 sin (60 – 20) sin (20) sin (60 + 20))

= 4 × sin (3 × 20°) [∵ sin 3θ = 4 sin(60 – θ) × sin θ × sin (60 + θ)]

$= 4 \times \sin 60° = 4\times\frac{\sqrt{3}}{2} = 2\sqrt{3}$

9. Ans. (1)

$\cot\alpha = 1, \sec\beta = \frac{-5}{3}, \cos\beta = \frac{-3}{5}, \tan\beta = \frac{-4}{3}$

$$\tan(\alpha+\beta) = \frac{1-\frac{4}{3}}{1+\frac{4}{3}\times 1} = \frac{-1}{7}$$

10. Ans. (2)

$\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$

$$= \frac{\sin\left(3\times\frac{\pi}{7}\right)}{\sin\frac{\pi}{7}} \times \cos\left(\frac{\frac{2\pi}{7}+\frac{6\pi}{7}}{2}\right) = \frac{2\sin\left(\frac{3\pi}{7}\right)}{2\sin\frac{\pi}{7}} \times \cos\left(\frac{4\pi}{7}\right)$$

$$= \frac{\sin\left(\frac{7\pi}{7}\right) + \sin\left(\frac{-\pi}{7}\right)}{2\sin\frac{\pi}{7}} = \frac{-\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}} = -\frac{1}{2}$$

11. Ans. (80)

$$\sin 10°\left(\frac{1}{2}.2\sin 20°\sin 40°\right).\sin 10°\sin(60°-10°)\sin(60°+10°)$$

$$\sin 10°\frac{1}{2}(\cos 20° - \cos 60°).\frac{1}{4}\sin 30°$$

$$\frac{1}{2}.\frac{1}{4}.\frac{1}{2}.\sin 10°\left(\cos 20° - \frac{1}{2}\right)$$

$$= \frac{1}{32}(2\sin 10°\cos 20° - \sin 10°) = \frac{1}{32}(\sin 30° - \sin 10° - \sin 10°)$$

$$= \frac{1}{32}\left(\frac{1}{2} - 2\sin 10°\right) = \frac{1}{64}(1 - 4\sin 10°) = \frac{1}{64} - \frac{1}{16}\sin 10°$$

Hence $\alpha = \frac{1}{64}$

$16 + \alpha^{-1} = 80$

12. Ans. (4)

As we know

$$4\cos^2\theta-1=\frac{\sin 3\theta}{\sin\theta}$$

Value of the above expression will be

$$= 36\,.\ \frac{\sin 27^\circ}{\sin 9^\circ}\cdot\frac{\sin 81^\circ}{\sin 27^\circ}\cdot\frac{\sin 243^\circ}{\sin 81^\circ}\cdot\frac{\sin 729^\circ}{\sin 243^\circ}$$

$$= 36\cdot\frac{\sin 729^\circ}{\sin 9^\circ}=36$$

13. Ans. (1)

Option (1)

$$\tan 15^\circ=2-\sqrt{3}$$

$$\frac{1}{\tan 75^\circ}=\cot 75^\circ=2-\sqrt{3}$$

$$\frac{1}{\tan 105^\circ}=\cot(105^\circ)=-\cot 75^\circ=\sqrt{3}-2$$

$$\tan 195^\circ=\tan 15^\circ=2-\sqrt{3}$$

$$\therefore\ 2(2-\sqrt{3})=2a\ \Rightarrow\ a=2-\sqrt{3}$$

$$\Rightarrow\ a+\frac{1}{a}=4$$

14. Ans. (1)

$$P=96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$$

$$2P\times\sin\frac{\pi}{33}=96\times 2\sin\frac{\pi}{33}\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$$

$$2P\times\sin\frac{\pi}{33}=6\times\sin\frac{32\pi}{33}=6\sin\frac{\pi}{33}$$

$$P=3$$

15. Ans. (1)

Since,

$-\sqrt{2}\le \sin x-\cos x\le \sqrt{2}$

$\therefore\ -2\le \sqrt{2}\ (\sin x-\cos x)\le 2$

(Assume $\sqrt{2}\ (\sin x-\cos x)=k$)

$-2\le k\le 2$ ...(i)

$f(x)=log_{\sqrt{m}}(k+m-2)$

Given,

$0\le f(x)\le 2$

$0\le log_{\sqrt{m}}(k+m-2)\le 2$

$1\le k+m-2\le m$

$-m+3\le k\le 2$ ...(ii)

From eq. (i) & (ii), we get $-m+3=-2$

$\Rightarrow m=5$

EXERCISE - JEE (Advanced) PYQ

1. Ans. (B)

$\because\ \theta \in \left(0, \frac{\pi}{4}\right) \Rightarrow \tan\theta < 1\ \&\ \cot\theta > 1$

let $\tan\theta = 1 - x$ and $\cot\theta = 1 + y$

$0 < x < 1, y > 0$

$\therefore\ t_1 = (1-x)^{1-x}, t_2 = (1-x)^{1+y}$

$t_3 = (1+y)^{1-x}\ t_4 = (1+y)^{1+y}$

clearly $t_4 > t_3\ \&\ t_1 > t_2$

also $t_3 > t_1$

Then $t_4 > t_3 > t_1 > t_2$

2. Ans. (A,B)

$$\frac{\tan^4 x}{2} + \frac{1}{3} = \frac{\sec^4 x}{5}$$

put $\tan^2 x = t \Rightarrow t = 2/3 \Rightarrow \sin^2 x = \frac{2}{5} \Rightarrow \cos^2 x = \frac{3}{5}$.

3. Ans. (C,D)

$$\sqrt{2}\sum_{m=1}^{6} \frac{\sin\left[\left(\theta+\frac{m\pi}{4}\right)-\left(\theta+\frac{m\pi}{4}-\frac{\pi}{4}\right)\right]}{\sin\left(\theta+\frac{m\pi}{4}-\frac{\pi}{4}\right)\sin\left(\theta+\frac{m\pi}{4}\right)}$$

$$\sqrt{2}\sum_{m=1}^{6}\left[\cot\left(\theta+\frac{(m-1)\pi}{4}\right)-\cot\left(\theta+\frac{m\pi}{4}\right)\right]$$

$\Rightarrow \cot\theta + \tan\theta = 4$

$$\frac{1}{\sin\theta\cos\theta} = 4 \Rightarrow \sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\Rightarrow \theta = \theta = \frac{\pi}{12}, \frac{5\pi}{12}$$

4. Ans. (2)

$$y = \frac{1}{\frac{(1-\cos 2\theta)}{2} + \frac{3\sin 2\theta}{2} + 5\frac{(1+\cos 2\theta)}{2}} = \frac{1}{3 + \left(2\cos 2\theta + \frac{3\sin 2\theta}{2}\right)}$$

$$y = \frac{2}{6 + (4\cos 2\theta + 3\sin 2\theta)}$$

$$y_{\max} = \frac{2}{6-5} = 2$$

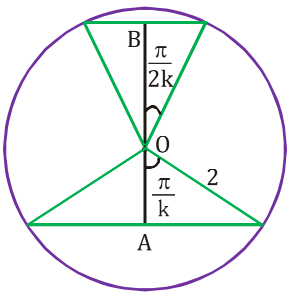
(Since $(-5 \le 4\cos 2\theta + 3\sin 2\theta \le 5)$)

5. Ans. (3)

$$OA = 2\cos\frac{\pi}{k}$$

$$OB = 2\cos\frac{\pi}{2k}$$

$$2\cos\frac{\pi}{k} + 2\cos\frac{\pi}{2k} = \sqrt{3}+1$$



$$2\cos^2\frac{\pi}{2k}-1+\cos\frac{\pi}{2k}=\frac{\sqrt{3}+1}{2}$$

Let $\cos\frac{\pi}{2k}=t$

$2t^2 + t - 1 - \frac{\sqrt{3}+1}{2}=0 \Rightarrow 4t^2 + 2t - (3+\sqrt{3}) = 0$

$\Rightarrow t = \frac{\sqrt{3}}{2}, -\frac{1+\sqrt{3}}{2}\ \left(t=-\frac{1+\sqrt{3}}{2}(\text{Not Possible})\right)$

$t=\frac{\sqrt{3}}{2}=\cos 30^\circ=\cos\frac{\pi}{6} \Rightarrow \cos\frac{\pi}{2k}=\cos\frac{\pi}{6}$

$k = 3$

6. **Ans. (D)**

$P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$

$\Rightarrow \tan\theta = \sqrt{2}+1$ ...(i)

$Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$

$\Rightarrow \tan\theta=\frac{1}{\sqrt{2}-1}=\sqrt{2}+1$ ...(ii)

from (i) & (ii)

$\Rightarrow P = Q$

7. **Ans. (C)**

We have,

$$=2.\sum_{k=1}^{13}\frac{\sin\left(\left(\frac{k\pi}{6}+\frac{\pi}{4}\right)-\left((k-1)\frac{\pi}{6}+\frac{\pi}{4}\right)\right)}{\sin\left(\frac{\pi}{4}+(k-1)\frac{\pi}{6}\right).\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$$

$$=2\sum_{k=1}^{13}\left(\cot\left((k-1)\frac{\pi}{6}+\frac{\pi}{4}\right)-\cot\left(\frac{k\pi}{6}+\frac{\pi}{4}\right)\right)$$

$$=2\left[\cot\frac{\pi}{4}-\cot\left(\frac{13\pi}{6}+\frac{\pi}{4}\right)\right]\ =2\left(1-\cot\left(\frac{5\pi}{12}\right)\right)$$

$$=2\left(1-\left(2-\sqrt{3}\right)\right)=2\left(\sqrt{3}-1\right)$$

8. **Ans. (Bonus)**

$2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$

$\cos\beta=\frac{1+2\cos\alpha}{2+\cos\alpha}$

$$\Rightarrow \frac{1-\tan^2\left(\frac{\beta}{2}\right)}{1+\tan^2\left(\frac{\beta}{2}\right)}=\frac{3-\tan^2\left(\frac{\alpha}{2}\right)}{3+\tan^2\left(\frac{\alpha}{2}\right)} \Rightarrow \tan^2\left(\frac{\beta}{2}\right)=\frac{\tan^2\left(\frac{\alpha}{2}\right)}{3}$$

(Componendo-Dividendo)

$\Rightarrow \tan\left(\frac{\alpha}{2}\right)=\pm\sqrt{3}\tan\left(\frac{\beta}{2}\right)$

If, $\alpha = \beta = \pi$, given equation is satisfy but $\tan\frac{\alpha}{2}$ and $\tan\frac{\beta}{2}$ are undefined so question is Bonus.

9. Ans. (1)

$$\alpha \in \left(0,\frac{\pi}{4}\right),\beta \in \left(-\frac{\pi}{4},0\right) \Rightarrow \alpha+\beta \in \left(-\frac{\pi}{4},\frac{\pi}{4}\right)$$

$$\sin(\alpha+\beta)=\frac{1}{3},\cos(\alpha-\beta)=\frac{2}{3}$$

$$\left(\frac{\sin\alpha}{\cos\beta}+\frac{\cos\alpha}{\sin\beta}+\frac{\cos\beta}{\sin\alpha}+\frac{\sin\beta}{\cos\alpha}\right)^2$$

$$\left(\frac{\cos(\alpha-\beta)}{\cos\beta\sin\beta}+\frac{\cos(\beta-\alpha)}{\sin\alpha\cos\alpha}\right)^2$$

$$=4\cos^2(\alpha-\beta)\left(\frac{1}{\sin 2\beta}+\frac{1}{\sin 2\alpha}\right)^2$$

$$=4\cos^2(\alpha-\beta)\left(\frac{2\sin(\alpha+\beta)\cos(\alpha-\beta)}{\sin 2\alpha\sin 2\beta}\right)^2 \qquad ...(1)$$

$$=\frac{16\cos^4(\alpha-\beta)\sin^2(\alpha+\beta)\times 4}{\left(\cos 2(\alpha-\beta)-\cos 2(\alpha+\beta)\right)^2}=\frac{64\cos^4(\alpha-\beta)\sin^2(\alpha+\beta)}{\left(2\cos^2(\alpha-\beta)-1-1+2\sin^2(\alpha+\beta)\right)^2}$$

$$=64\times\frac{16}{81}\times\frac{1}{9}\frac{1}{\left(2\times\frac{4}{9}-1-1+\frac{2}{9}\right)^2}=\frac{64\times 16}{81\times 9}\cdot\frac{81}{64}=\frac{16}{9}\Rightarrow\left[\frac{16}{9}\right]=1 \quad \text{Ans.}$$

JEE (Main) Practice Paper

SECTION–A

1. Ans. (1)

$$\cos x+\sin x=\frac{1}{2}$$

Divide both sides by $\cos x$

$$1+\tan x=\frac{1}{2\cos x}$$

Square both sides

$$(1+\tan x)^2=\frac{\sec^2 x}{4}$$

$(1+\tan^2 x+2\tan x)4=\sec^2 x=1+\tan^2 x$

After simplification

$3\tan^2 x+8\tan x+3=0$

$D=d^2=b^2-4ac=64-36=28$

$d=\pm 2\sqrt{7}$

These are 2 real roots

$$\tan x=\frac{-b}{2a}\pm\frac{d}{2a}=\frac{-8}{6}\pm\frac{2\sqrt{7}}{6}$$

$$=\frac{-4\pm\sqrt{7}}{3}.$$

2. Ans. (1)

$$y=4+\frac{1}{2}\sin^2 2x-2\cos^4 x$$

$$y=4+\frac{1}{2}(1-\cos^2 2x)-2\left(\frac{1+\cos 2x}{2}\right)^2$$

$$y=4-\cos^2 2x-\cos 2x$$

$$y=\frac{17}{4}-\left(\cos 2x+\frac{1}{2}\right)^2$$

$$M=\frac{17}{4}\ \&\ m=2$$

$$M-m=\frac{17}{4}-2=\frac{9}{4}$$

3. Ans. (1)

$$5\left[\frac{1-t}{t}-t\right]=2(2t-1)+9\ \{\text{Let } \cos^2 x=t\}$$

$$\Rightarrow 5(1-t-t^2)=t(4t+7)\Rightarrow 9t^2+12t-5=0$$

$$\Rightarrow 9t^2+15t-3t-5=0\Rightarrow (3t-1)(3t+5)=0$$

$$\Rightarrow\ t=\frac{1}{3}\ \text{as } t\neq-\frac{5}{3}.$$

$$\cos 2x=2\left(\frac{1}{3}\right)-1=-\frac{1}{3}$$

$$\cos 4x=2\left(-\frac{1}{3}\right)^2-1=-\frac{7}{9}$$

4. Ans. (2)

$$\left\{-\left(\frac{1-\tan^2\left(\frac{\alpha-\pi}{4}\right)}{1+\tan^2\left(\frac{\alpha-\pi}{4}\right)}\right)+\cos\frac{\alpha}{2}\cot 4\alpha\right\}\sec\frac{9\alpha}{2}$$

$$=\left\{-\cos\left(\frac{\alpha-\pi}{2}\right)+\cos\frac{\alpha}{2}\cot 4\alpha\right\}\sec\frac{9\alpha}{2}=\left\{-\sin\frac{\alpha}{2}+\frac{\cos\frac{\alpha}{2}\cos 4\alpha}{\sin 4\alpha}\right\}\sec\frac{9\alpha}{2}$$

$$=\frac{1}{\sin 4\alpha}\left[\cos 4\alpha\cos\frac{\alpha}{2}-\sin 4\alpha\sin\frac{\alpha}{2}\right]\sec\frac{9\alpha}{2}=\frac{1}{\sin 4\alpha}\times\cos\frac{9\alpha}{2}\ .\sec\frac{9\alpha}{2}=\text{cosec } 4\alpha$$

5. Ans. (4)

Let $y=\cos x\,.\cos\left(\frac{2\pi}{3}+x\right)\ \cos\left(\frac{2\pi}{3}-x\right)$

$$y=\frac{1}{2}\cos x\left[\cos\frac{4\pi}{3}+\cos 2x\right]\Rightarrow y=\frac{1}{2}\cos x\left[\frac{-1+2\cos 2x}{2}\right]$$

$$y=\frac{1}{4}[2\cos 2x\cos x-\cos x]\Rightarrow y=\frac{1}{4}[\cos 3x+\cos x-\cos x]$$

$y = \frac{1}{4}\cos 3x \qquad \because -1 \le \cos 3x \le 1$

$y_{min} = -\frac{1}{4}$ and $y_{max} = \frac{1}{4}$

6. Ans. (3)

$y = \cos^2\left(\frac{\pi}{4}+x\right) + (\sin x - \cos x)^2 = \cos^2\left(\frac{\pi}{4}+x\right) + 2\left(\cos^2\left(\frac{\pi}{4}+x\right)\right)$

$y = 3\cos^2\left(\frac{\pi}{4}+x\right) \qquad \because\ 0 \le \cos^2\theta \le 1 \Rightarrow y_{max} = 3.1 = 3 \Rightarrow y_{min} = 0$

7. Ans. (3)

$0° < x < 90° \,\&\, \cos x = \frac{3}{\sqrt{10}} \Rightarrow \log_{10}\sin x + \log_{10}\cos x + \log_{10}\tan x$

$= \log_{10}(\sin x\ \cos x\ \tan x) = \log_{10}(1-\cos^2 x) = \log_{10}(1-9/10) = \log_{10}\left(\frac{1}{10}\right) = -1$

8. Ans. (2)

$\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{\pi}{10}\right) = \left(1-\cos^2\frac{\pi}{10}\right)\left(1-\cos^2\frac{3\pi}{10}\right)$

$\sin^2\frac{\pi}{10} \cdot \sin^2\frac{3\pi}{10} = \left(\frac{\sqrt{5}-1}{4} \cdot \frac{\sqrt{5}+1}{4}\right)^2 = \left(\frac{4}{16}\right)^2 = \frac{1}{16}$

9. Ans. (1)

$\tan A + \tan B = a \Rightarrow \tan A \tan B = b \Rightarrow \tan(A+B) = \frac{a}{1-b}$

$\therefore \sin^2(A+B) = \left[\frac{|a|}{\sqrt{a^2+(1-b)^2}}\right]^2 = \frac{a^2}{a^2+(1-b)^2}$

10. Ans. (1)

$\frac{\tan(180°-25°)-\tan(90°+25°)}{1+(\tan(180°-25°)\ \tan(90°+25°))} = \frac{-\tan 25° + \frac{1}{\tan 25°}}{2} = \frac{1-x^2}{2\ x}$

11. Ans. (3)

$\cos A = \frac{3}{4} \Rightarrow 16\cos^2\frac{A}{2} - 32\sin\frac{A}{2}\sin\frac{5A}{2} = \frac{16(1+\cos A)}{2} - 16\ (\cos 2A - \cos 3A)$

$= \frac{16(1+\cos A)}{2} - 16\ \{(2\cos^2 A - 1) - (4\cos^3 A - 3\cos A)\}$

$= 8\left(1+\frac{3}{4}\right) - 16\left\{2\times\frac{9}{16} - 1 - 4\times\frac{27}{64} + 3\times\frac{3}{4}\right\} = 3$

12. Ans. (4)

$\tan^2\theta = 2\tan^2\phi + 1 \qquad$ ...(i)

$\cos 2\theta + \sin^2\phi = \frac{1-\tan^2\theta}{1+\tan^2\theta} + \sin^2\phi = \frac{1-2\tan^2\phi-1}{1+2\tan^2\phi+1} + \sin^2\phi = \frac{-2\tan^2\phi}{2\ (1+\tan^2\phi)} + \sin^2\phi$

$= -\sin^2\phi + \sin^2\phi = 0.$ which is independent of ϕ

13. Ans. (2)

Add & subtract $\cot\alpha$.

$(\tan\alpha - \cot\alpha) + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha + \cot\alpha$

$= -2\cot 2\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha + \cot\alpha$

$= -4\cot 4\alpha + 4\tan 4\alpha + 8\cot 8\alpha + \cot\alpha$

$= -8\cot 8\alpha + 8\cot 8\alpha + \cot\alpha = \cot\alpha$

14. Ans. (2)

$\cos\frac{\pi}{10}\cos\frac{2\pi}{10}\cos\frac{4\pi}{10}\cos\frac{8\pi}{10}\cos\frac{16\pi}{10}$

$= \dfrac{\sin 2^5\frac{\pi}{10}}{2^5\sin\frac{\pi}{10}} = \frac{1}{32}\dfrac{\sin\frac{32\pi}{10}}{\sin\frac{\pi}{10}} = \frac{1}{32}\dfrac{\sin\left(3\pi+\frac{2\pi}{10}\right)}{\sin\left(\frac{\pi}{10}\right)} = -\frac{1}{32}\cdot\dfrac{2\sin\frac{\pi}{10}\cos\frac{\pi}{10}}{\sin\frac{\pi}{10}} = -\frac{1}{16}\cos\frac{\pi}{10}$

$= -\frac{1}{64}\sqrt{10+2\sqrt{5}}$

15. Ans. (1)

$= \sec^4 A\,(1+\sin^2 A)(1-\sin^2 A) - 2\tan^2 A$

$= \sec^2 A + \sec^2 A\sin^2 A - 2\tan^2 A$

$= 1 + \tan^2 A + \tan^2 A - 2\tan^2 A = 1$

16. Ans. (1)

The given expression is equal to

$\sqrt{\sin^4 x + 4(1-\sin^2 x)} - \sqrt{\cos^4 x + 4(1-\cos^2 x)}$

$= \sqrt{(2-\sin^2 x)^2} - \sqrt{(2-\cos^2 x)^2} = (2-\sin^2 x) - (2-\cos^2 x)$

$= \cos^2 x - \sin^2 x = \cos 2x$

17. Ans. (4)

The given expression is equal to

$(\sin 47° + \sin 61°) - (\sin 11° + \sin 25°)$

$= 2\sin 54°\cos 7° - 2\sin 18°\cos 7°$

$= 2\cos 7°(\sin 54° - \sin 18°)$

$= 2\cos 7°\left(\frac{\sqrt{5}+1}{4} - \frac{\sqrt{5}-1}{4}\right) = \cos 7°$

18. Ans. (2)

$\Rightarrow \sec(\theta+\phi) + \sec(\theta-\phi) = 2\sec\theta$

$\Rightarrow \dfrac{\cos(\theta+\phi)+\cos(\theta-\phi)}{\cos(\theta+\phi)\cos(\theta-\phi)} = \dfrac{2}{\cos\theta} \Rightarrow \dfrac{2\cos\theta\cos\phi}{\cos 2\theta\cos 2\phi} = \dfrac{1}{\cos\theta}$

$\Rightarrow \cos^2\theta\cos\phi = \cos^2\theta + \cos^2\phi - 1$

$\Rightarrow \cos^2\theta = \cos\phi + 1 \Rightarrow \cos^2\theta = 2\cos^2\frac{\phi}{2}$

$\Rightarrow \cos\theta = \left|\sqrt{2}\cos\frac{\phi}{2}\right|$

$k = \pm\sqrt{2}$

19. **Ans. (3)**

If $a = \sin(\theta + \alpha)$, $b = \sin(\theta + \beta)$

$\therefore\ 2ab = 2\sin(\theta + \alpha)\sin(\theta + \beta)$

$2ab = \cos(\alpha - \beta) - \cos(2\theta + \alpha + \beta)$

Multiply both sides by $2\cos(\alpha - \beta)$

$\Rightarrow 4ab\cos(\alpha - \beta) = 2\cos^2(\alpha - \beta) - 2\cos(2\theta + \alpha + \beta)\,.\,\cos(\alpha - \beta)$

$= 1 + \cos2(\alpha - \beta) - \cos2(\theta + \alpha) - \cos2(\theta + \beta)$

$\Rightarrow \cos2(\alpha - \beta) - 4ab\cos(\alpha - \beta) = \cos2(\theta + \alpha) + \cos2(\theta + \beta) - 1$

$= 1 - 2\sin^2(\theta + \alpha) + 1 - 2\sin^2(\theta + \beta) - 1$

$= 1 - 2a^2 - 2b^2$

20. **Ans. (1)**

Let P denotes the desired product, and let

$Q = \sin a \sin 2a \sin 3a ....\sin 999a$.

Then

$2^{999} PQ = (2\sin a\cos a)(2\sin 2a\cos 2a).........(2\sin 999a\cos 999a)$

$= \sin 2a \sin 4a .....\sin 1998a$

$= (\sin 2a \sin 4a ....\sin 998a)\,[-\sin(2\pi - 1000a)].\,[-\sin(2\pi - 1002a)]........[-\sin(2\pi - 1998a)]$

$= \sin 2a \sin 4a ....\sin 998a \sin 999a \sin 997a ....\sin a = Q$.

It is easy to see that $Q \neq 0$. Hence the desired product is $P = \dfrac{1}{2^{999}}$.

SECTION–B

1. **Ans. (130)**

Given $\sin\alpha + \sin\beta = -\dfrac{21}{65}$ and $\cos\alpha + \cos\beta = -\dfrac{27}{65}$

Squaring and adding, we get

$$2(1 + \cos\alpha\cos\beta + \sin\alpha\sin\beta) = \frac{1170}{(65)^2}$$

or $2.2\cos^2\dfrac{\alpha-\beta}{2} = \dfrac{18}{65} = \dfrac{36}{130}$

$\therefore\ \cos\dfrac{\alpha-\beta}{2} = \pm\dfrac{3}{\sqrt{130}}$

Since $\pi < \alpha - \beta < 3\pi \Rightarrow \dfrac{\pi}{2} < \dfrac{\alpha-\beta}{2} < \dfrac{3\pi}{2}$

in the above interval $\cos\dfrac{\alpha-\beta}{2}$ is -ive

$\therefore\ \cos\dfrac{\alpha-\beta}{2} = -\dfrac{3}{\sqrt{130}}$

2. **Ans. (3)**

$$\left(\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}\right)+\cos\frac{7\pi}{7}$$

$$=\cos\left(\frac{\frac{2\pi}{7}+\frac{6\pi}{7}}{2}\right).\frac{\sin\left(\frac{3\left(\frac{2\pi}{7}\right)}{2}\right)}{\sin\left(\frac{\frac{2\pi}{7}}{2}\right)}+\cos\pi=\frac{\cos\left(\frac{4\pi}{7}\right)\sin\left(\frac{3\pi}{7}\right)}{\sin\left(\frac{\pi}{7}\right)}+(-1)$$

$$=\frac{2\cos\left(\frac{4\pi}{7}\right)\sin\left(\frac{3\pi}{7}\right)}{2\sin\left(\frac{\pi}{7}\right)}+(-1)=\frac{\sin\pi+\sin\left(-\frac{\pi}{7}\right)}{2\sin\frac{\pi}{7}}-1=-\frac{1}{2}-1$$

$$=\frac{-3}{2}\Rightarrow \ell=3$$

3. **Ans. (3)**

$\cot(\theta-\alpha)+\cot(\theta+\alpha)=2.3\cot\theta$

$$\Rightarrow \frac{\cos(\theta-\alpha)}{\sin(\theta-\alpha)}+\frac{\cos(\theta+\alpha)}{\sin(\theta+\alpha)}=6\cot\theta$$

$$\Rightarrow \frac{\sin((\theta+\alpha)+(\theta-\alpha))}{\sin(\theta-\alpha).\sin(\theta+\alpha)}=6\cot\theta$$

$$\Rightarrow \frac{\sin 2\theta}{\sin^2\theta-\sin^2\alpha}=\frac{6\cos\theta}{\sin\theta}$$

$$\Rightarrow \frac{2\sin\theta\cos\theta}{\sin^2\theta-\sin^2\alpha}=\frac{6\cos\theta}{\sin\theta}$$

$\Rightarrow \sin^2\theta=3(\sin^2\theta-\sin^2\alpha)$

$$\Rightarrow \frac{2\sin^2\theta}{\sin^2\alpha}=3$$

4. **Ans. (1)**

$$\frac{1-4(\sin 10^\circ\sin 70^\circ)}{2\sin 10^\circ}$$

$$=\frac{1-2(\cos 60^\circ-\cos 80^\circ)}{2\sin 10^\circ}=\frac{1-2\left(\frac{1}{2}-\cos 80^\circ\right)}{2\sin 10^\circ}$$

$$=\frac{2\cos 80^\circ}{2\sin 10^\circ}=\frac{\sin 10^\circ}{\sin 10^\circ}=1$$

5. **Ans. (56)**

$\cos(\alpha+\beta)=4/5$ ...(1)

$\therefore \tan(\alpha+\beta)=\frac{\sqrt{5^2-4^2}}{4}=\frac{3}{4}$ ...(2)

$\sin(\alpha-\beta)=5/13$ ...(3)

$\therefore \tan(\alpha-\beta)=\frac{5}{\sqrt{13^2-5^2}}=\frac{5}{12}$ ...(4)

Now $\tan 2\alpha = \tan\{(\alpha+\beta)+(\alpha-\beta)\}$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)} = \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}.\frac{5}{12}} = \frac{56}{33}$$

6. Ans. (4)

K= tan 9° – tan 27° – tan 63° + tan 81°

= tan 9° – tan 27° – tan (90° – 27°) + tan (90° – 9°)

= tan 9° – tan 27° – cot 27° + cot 9° = (tan 9° + cot 9°) – (tan 27° + cot 27°)

$$= \left(\frac{\sin^2 9°+\cos^2 9°}{\sin 9°.\cos 9°}\right) - \left(\frac{\sin^2 27°+\cos^2 27°}{\sin 27°.\cos 27°}\right) = \frac{1}{\sin 9°.\cos 9°} - \frac{1}{\sin 27°.\cos 27°}$$

$$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°} = 2\left[\frac{4}{\sqrt{(5)}-1} - \frac{4}{\sqrt{(5)}+1}\right]$$

$$= 4 \Rightarrow \frac{k}{100} = 4$$

7. Ans. (8)

$2\cos x + \sin x = 1$ ...(1)

$4\cos^2 x = (1-\sin x)^2 \Rightarrow 4 - 4\sin^2 x = 1 + \sin^2 x - 2\sin x$

$5\sin^2 x - 2\sin x - 3 = 0 \Rightarrow (\sin x - 1)(5\sin x + 3) = 0$

$\Rightarrow \sin x = 1, \sin x = -\frac{3}{5}$

$\because \cos x = \frac{1-\sin x}{2}$ (from equation (1)) $\therefore$ when $\sin x = 1$

$7\cos x + 6\sin x = 7\left(\frac{1-\sin x}{2}\right) + 6\sin x = 7\left(\frac{1-1}{2}\right) + 6 \times 1 = 6$ Ans.

and when $\sin x = \frac{-3}{5}$ then $7\cos x + 6\sin x = 7\left(\frac{1+\frac{3}{5}}{2}\right) - \frac{6\times 3}{5} = \frac{28-18}{5} = 2$ Ans.

8. Ans. (2)

$4\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right) + \sqrt{4\sin^4 x + 4\sin^2 x\cos^2 x} = 4\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right) + |2\sin x| = 4\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right) - 2\sin x$

$= 2\left(1+\cos\left(\frac{\pi}{2}-x\right)\right) - 2\sin x = 2$

9. Ans. (16)

$y = 1 + 2\sin x + 3\cos^2 x \Rightarrow y = 1 + 2\sin x + 3 - 3\sin^2 x$

$y = 1 - \left(3\sin^2 x - 2\sin x - 3\right) \Rightarrow y = 1 - 3\left(\sin^2 x - \frac{2}{3}\sin x + \frac{1}{9} - \frac{1}{9} - 1\right)$

$y = 1 - 3\left[\left(\sin x - \frac{1}{3}\right)^2 - \frac{10}{9}\right] = -3\left(\sin x - \frac{1}{3}\right)^2 + \frac{13}{3}$

$y_{max} = \frac{13}{3}$, $y_{min} = -3\left(\frac{16}{9}\right) + \frac{13}{3} = -1$

10. Ans. (10)

$y = 3\cos\left(\theta+\frac{\pi}{3}\right) + 5\cos\theta + 3 \Rightarrow y = 3\cos\theta \cdot \frac{1}{2} - 3\frac{\sqrt{3}}{2}\sin\theta + 5\cos\theta + 3$

$y = \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 5\cos\theta + 3 \Rightarrow y = \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

$y_{max} = \sqrt{\frac{169}{4}+\frac{27}{4}} + 3 = 7 + 3 = 10$

JEE (Advanced) Practice Paper

1. Ans. (A)

$5 - 12\tan\theta = 11\sec\theta \Rightarrow 25 + 144\tan^2\theta - 120\tan\theta = 121 + 121\tan^2\theta$

$23\tan^2\theta - 120\tan\theta - 96 = 0 \Rightarrow \tan\alpha + \tan\beta = \frac{120}{23} \Rightarrow \tan\alpha\tan\beta = -\frac{96}{23}$

$\tan(\alpha+\beta) = \frac{\frac{120}{23}}{1+\frac{96}{23}} = \frac{120}{119} \Rightarrow \sin(\alpha+\beta) = -\frac{120}{169} = -\frac{5k}{169}$.

$\Rightarrow K = 24$

2. Ans. (D)

$\frac{\sin A}{\sin B} = \frac{\sqrt{3}}{2}, \frac{\cos A}{\cos B} = \frac{\sqrt{5}}{2}, 0 < A, B < \pi/2$

$\Rightarrow \tan A = \frac{\sqrt{3}}{\sqrt{5}}\frac{\sin B}{\cos B}$

$\tan A = \frac{\sqrt{3}}{\sqrt{5}}\tan B$ ...(1)

$\frac{\sin A\cos A}{\sin B\cos B} = \frac{\sqrt{15}}{4} \Rightarrow \frac{\tan A\ .\sec^2 B}{\tan B\ .\sec^2 A} = \frac{\sqrt{15}}{4}$

from (1)

$\Rightarrow \frac{\sqrt{3}}{\sqrt{5}}\frac{(1+\tan^2 B)}{(1+\tan^2 A)} = \frac{\sqrt{15}}{4} \Rightarrow 4 + 4\tan^2 B = 5 + 5\tan^2 A$

$\Rightarrow -1 + 4\tan^2 B = 5 \times \frac{3}{5}\tan^2 B \Rightarrow \tan B = \pm 1$

$\Rightarrow \tan B = +1 \qquad (\because 0 < B < \frac{\pi}{2})$

Now $\tan A + \tan B = \frac{\sqrt{3}}{\sqrt{5}} + 1 = \frac{\sqrt{3}+\sqrt{5}}{\sqrt{5}}$

3. Ans. (D)

$\tan 142\frac{1}{2}^\circ = -\cot 52\frac{1}{2}^\circ = \frac{-1}{\tan 52\frac{1}{2}^\circ} = \frac{-1}{\tan\left(45+7\frac{1}{2}^\circ\right)}$

$$= -\frac{1-\tan 7\frac{1}{2}^{o}}{1+\tan 7\frac{1}{2}^{o}} = -\frac{\cos 7\frac{1}{2}^{o} - \sin 7\frac{1}{2}^{o}}{\cos 7\frac{1}{2}^{o} + \sin 7\frac{1}{2}^{o}}$$

$$= -\frac{\left(\cos 7\frac{1}{2}^{o} - \sin 7\frac{1}{2}^{o}\right)^2}{\cos 15°} = -\frac{1-\sin 15°}{\cos 15°} = -\left(\frac{1-\frac{\sqrt{3}-1}{2\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}}\right) = -\frac{(2\sqrt{2}-\sqrt{3}+1)(\sqrt{3}-1)}{2}$$

$$= -\frac{[2\sqrt{2}\ (\sqrt{3}-1)-(\sqrt{3}-1)^2]}{2} = -\frac{[2\sqrt{2}\ (\sqrt{3}-1)-(4-2\sqrt{3})]}{2}$$

$$= -[\sqrt{2}(\sqrt{3}-1)-(2-\sqrt{3})] = -\sqrt{6}+\sqrt{2}+2-\sqrt{3} = 2+\sqrt{2}-\sqrt{3}-\sqrt{6}$$

4. And. (A)

Because $3a + 4a = \pi$, it follows that $\tan 3a + \tan 4a = 0$.

$$\Rightarrow \frac{\tan a + \tan 2a}{1-\tan a \tan 2a} + \frac{2\tan 2a}{1-\tan^2 2a} = 0$$

or $\tan a + 3 \tan 2a - 3 \tan a \tan^2 2a - \tan^3 2a = 0$

Let $\tan a = x$. Then $\tan 2a = \frac{2\tan a}{1-\tan^2 a} = \frac{2x}{1-x^2}$. Hence

$$x + \frac{6x}{1-x^2} - \frac{12x^3}{(1-x^2)^2} - \frac{8x^3}{(1-x^2)^3} = 0$$

or $(1 - x^2)^3 + 6(1 - x^2)^2 - 12x^2(1 - x^2) - 8x^2 = 0$

Expanding the left-hand side of the above equation gives

$x^6 - 21x^4 + 35x^2 - 7 = 0$ ...(i)

Thus tan a is a root of the above equation. Note that $6a + 8a = 2\pi$ and $9a + 12a = 3\pi$, and so $\tan [3(2a)] + \tan[4(2a)] = 0$ and $\tan [3(3a)] + \tan [4(3a)] = 0$. Hence $\tan 2a$ and $\tan 3a$ are also the roots of equation (i).

putting $x^2 = t$ in (i), we get $t^3 - 21t^2 + 35t - 7 = 0$. ...(ii)

Therefore $\tan^2 ka, k = 1,2,3$ are the distinct roots of the cubic equation (ii)

$\therefore \tan^2 a + \tan^2 2a + \tan^2 3a = 21$

$\tan^2 a \tan^2 2a + \tan^2 2a \tan^2 3a + \tan^2 a \tan^2 3a = 35$

5. Ans. (B, C)

$$\sin t + \cos t = \frac{1}{5} \Rightarrow \frac{2\ \tan\frac{t}{2}+1-\tan^2\frac{t}{2}}{1+\tan^2\frac{t}{2}} = \frac{1}{5} \Rightarrow 10 \tan\frac{t}{2} + 5 - 5\tan^2\frac{t}{2} = 1 + \tan^2\frac{t}{2}$$

$$\Rightarrow 6\tan^2\frac{t}{2} - 10\tan\frac{t}{2} - 4 = 0 \Rightarrow 3\tan^2\frac{t}{2} - 6\tan\frac{t}{2} + \tan\frac{t}{2} - 2 = 0$$

$$\Rightarrow 3\tan\frac{t}{2}\left(\tan\frac{t}{2}-2\right) + 1\left(\tan\frac{t}{2}-2\right) = 0 \Rightarrow \tan\frac{t}{2} = 2,\ \tan\frac{t}{2} = -\frac{1}{3}$$

6. Ans. (B, D)

$$\frac{\sin x+\cos x}{\cos^3 x} = \tan x \sec^2 x + \sec^2 x = \sec^2 x\,(1+\tan x) = (1+\tan^2 x)(1+\tan x)$$

7. Ans. (A, B, C)

$\sin x + \sin y = a$ ...(1)

$\cos x + \cos y = b$ ...(2)

$$\frac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)} = \frac{a}{b} \Rightarrow \tan\left(\frac{x+y}{2}\right) = \frac{a}{b}$$

$$\Rightarrow \sin\left(\frac{x+y}{2}\right) = \frac{a}{\sqrt{a^2+b^2}},\ \cos\left(\frac{x+y}{2}\right) = \frac{b}{\sqrt{a^2+b^2}}$$

$$\Rightarrow \sin(x+y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x+y}{2}\right) = \frac{2ab}{a^2+b^2}$$

Now for $\tan\left(\frac{x-y}{2}\right) \Rightarrow (1)^2 + (2)^2 \Rightarrow 1 + 1 + 2\cos(x-y) = a^2 + b^2$

$$\cos(x-y) = \frac{a^2+b^2-2}{2}$$

$$\because \tan^2\left(\frac{x-y}{2}\right) = \frac{1-\cos(x-y)}{1+\cos(x-y)}$$

$$\Rightarrow \tan^2\left(\frac{x-y}{2}\right) = \frac{1-\left(\frac{a^2+b^2-2}{2}\right)}{1+\frac{a^2+b^2-2}{2}} \Rightarrow \tan\left(\frac{x-y}{2}\right) = \pm\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$$

8. Ans. (B, C, D)

$\tan^2\alpha + 2\tan\alpha \,.\, \tan 2\beta = \tan^2\beta + 2\tan\beta \,.\, \tan 2\alpha$

$$\Rightarrow (\tan^2\alpha - \tan^2\beta) + 4\tan\alpha\tan\beta\left(\frac{1}{1-\tan^2\beta} - \frac{1}{1-\tan^2\alpha}\right) = 0$$

$$\Rightarrow (\tan^2\alpha - \tan^2\beta) + 4\tan\alpha\tan\beta\,\frac{(\tan^2\beta-\tan^2\alpha)}{(1-\tan^2\alpha)(1-\tan^2\beta)} = 0$$

$$\Rightarrow (\tan^2\alpha - \tan^2\beta)\left\{1 - \frac{4\tan\alpha\tan\beta}{(1-\tan^2\alpha)(1-\tan^2\beta)}\right\} = 0$$

$$\Rightarrow (\tan^2\alpha - \tan^2\beta)(1 - \tan 2\alpha \,.\, \tan 2\beta) = 0$$

$$\Rightarrow \tan^2\alpha = \tan^2\beta \text{ or } \tan 2\alpha \,.\, \tan 2\beta = 1$$

$$\text{L.H.S.} = \tan^2\alpha + 2\tan\alpha\,\frac{1}{\tan 2\alpha} . = \tan^2\alpha + \frac{2\tan\alpha}{2\tan\alpha}\,.\,(1-\tan^2\alpha) = 1$$

$$\text{R.H.S.} = \tan^2\beta + 2\tan\beta\,.\,\frac{1}{\tan 2\beta} = \tan^2\beta + \frac{2\tan\beta}{2\tan\beta}\,.\,(1-\tan^2\beta) = 1$$

9. Ans. (A,B)

Let $(\sec A+\tan A)(\sec B+\tan B)(\sec C+\tan C)=x$ ...(i)

$\therefore$ $(\sec A-\tan A)(\sec B-\tan B)(\sec C-\tan C)=x$ ...(ii)

Multiply equation (i) and (ii)

$(\sec^2 A-\tan^2 A)(\sec^2 B-\tan^2 B)(\sec^2 C-\tan^2 c)=1$

$x^2=1\Rightarrow x=\pm 1$

10. Ans. (C)

$\cos\alpha + \cos\beta = a$ ...(i)

$\sin\alpha + \sin\beta = b$ ...(ii)

$$\frac{2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}}{2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}} = \frac{a}{b}\Rightarrow \tan\left(\frac{\alpha+\beta}{2}\right)=\frac{b}{a}\Rightarrow \tan\theta=\frac{b}{a}$$

Now, $\sin2\theta + \cos2\theta = \frac{2\tan\theta}{1+\tan^2\theta} + \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{2\frac{b}{a}}{1+\frac{b^2}{a^2}} + \frac{1-\frac{b^2}{a^2}}{1+\frac{b^2}{a^2}} = \frac{2ab+a^2-b^2}{a^2+b^2}$

$\sin2\theta + \cos2\theta = 1+\frac{2b(a-b)}{a^2+b^2}$

$\therefore n = 2$

11. Ans. (A)

$\sin^2 A = x$

$\sin A \sin 2A \sin 3A \sin 4A = \sin A\,(2\sin A\cos A)\,(3\sin A-4\sin^3 A)\,(2\sin 2A\cos 2A)$

$= 2\sin^2 A.\cos A.\sin A(3-4\sin^2 A)\,(4\sin A\cos A).(1-2\sin^2 A)$

$= 2\sin^2 A.4\sin^2 A\,(1-\sin^2 A)\,(3-4\sin^2 A)\,(1-2\sin^2 A)$

$= 8x^2(1-x)\,(3-4x)\,(1-2x)$

$\therefore$ degree of polynomial is 5.

12. Ans. (B)

$p = 5$

$\sin\theta, \cos\theta, \tan\theta$ are in G.P.

$\cos^2\theta = \sin\theta.\,\frac{\sin\theta}{\cos\theta}$

$\Rightarrow \cos^3\theta = \sin^2\theta \Rightarrow \cos^3\theta = 1 - \cos^2\theta$

$\Rightarrow \cos^9\theta = 1 - 3\cos^2\theta + 3\cos^4\theta - \cos^6\theta$

Now $\cos^9\theta + \cos^6\theta + 3\cos^5\theta - 1$

$= 3\cos^5\theta + 3\cos^4\theta - 3\cos^2\theta = 3\cos^2\theta\,(\cos^3\theta + \cos^2\theta - 1) = 0$

13. Ans. (A,B,C,D)

$a + b + c = \sin\alpha + \sin\left(\alpha+\frac{2\pi}{3}\right)+\sin\left(\alpha+\frac{4\pi}{3}\right)$

$= \sin\alpha + 2\sin(\alpha + \pi)\cos\left(\frac{\pi}{3}\right) = 0$

$ab + bc + ca$

$$=\sin\alpha.\sin\left(\alpha+\frac{2\pi}{3}\right)+\sin\left(\alpha+\frac{2\pi}{3}\right)\sin\left(\alpha+\frac{4\pi}{3}\right)+\sin\left(\alpha+\frac{4\pi}{3}\right)\sin\alpha$$

$$=\frac{1}{2}\left[2\sin\alpha.\sin\left(\alpha+\frac{2\pi}{3}\right)+2\sin\left(\alpha+\frac{2\pi}{3}\right)\sin\left(\alpha+\frac{4\pi}{3}\right)+2\sin\left(\alpha+\frac{4\pi}{3}\right)\sin\alpha\right]$$

$$=\frac{1}{2}\left[\cos\frac{2\pi}{3}-\cos\left(2\alpha+\frac{2\pi}{3}\right)+\cos\frac{2\pi}{3}-\cos(2\alpha+2\pi)+\cos\frac{4\pi}{3}-\cos\left(2\alpha+\frac{4\pi}{3}\right)\right]$$

$$=\frac{1}{2}\left[\frac{-1}{2}-\frac{1}{2}-\frac{1}{2}-\left\{\cos\left(2\alpha+\frac{2\pi}{3}\right)+\cos(2\alpha+2\pi)+\cos\left(2\alpha+\frac{4\pi}{3}\right)\right\}\right]$$

$$=\frac{1}{2}\left[\frac{-3}{2}-\left\{\cos\left(2\alpha+\frac{2\pi}{3}\right)+\cos2\alpha+\cos\left(2\alpha+\frac{4\pi}{3}\right)\right\}\right]$$

$$=\frac{1}{2}\left[\frac{-3}{2}-\left\{2\cos(2\alpha+\pi).\cos\frac{\pi}{3}+\cos2\alpha\right\}\right]=\frac{-3}{4}$$

So, $a + b + c + ab + bc + ca = \frac{-3}{4}$

14. Ans. (A,C)

$qc - rb$

$$=\cos\left(\alpha+\frac{2\pi}{3}\right)\sin\left(\alpha+\frac{4\pi}{3}\right)-\cos\left(\alpha+\frac{4\pi}{3}\right)\sin\left(\alpha+\frac{2\pi}{3}\right)$$

$$=\sin\left(\left(\alpha+\frac{4\pi}{3}\right)-\left(\alpha+\frac{2\pi}{3}\right)\right)=\sin\frac{2\pi}{3}=\frac{\sqrt{3}}{2}$$

15. Ans. (6)

[Express L.H.S. in multiple of x and compare with R.H.S]

Given $\sin^3\sin3x=\sum_{m=0}^{n}c_m\cos mx$

Or, $\left(\frac{3\sin x-\sin3x}{4}\right).\sin3x=\sum_{m=0}^{n}c_m\cos mx$

Or, $\frac{3}{8}.(2\sin3x.\sin x)-\frac{1}{8}.[1-\cos6x]=\sum_{m=0}^{n}c_m\cos mx$

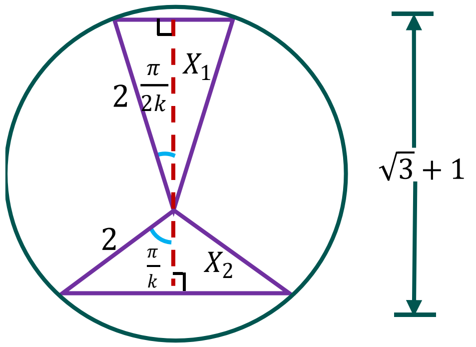
Or $\frac{3}{8}[\cos2x-\cos4x]-\frac{1}{8}[1-\cos6x]=\sum_{m=0}^{n}c_m\cos mx$

Or, $-\frac{1}{8}+\frac{3}{8}\cos2x-\frac{3}{8}\cos4x+\frac{1}{8}\cos6x=\sum_{m=0}^{n}\cos mx$

Comparing, we get $n = 6$

[$\because$ Highest multiple of angle on L.H.S. is $6x$ and on R.H.S. in nx]

16. **Ans. (3)**



Distance between chords is more than radius = chord will be on opposite sides of center

$\because\ x_1 + x_2 = \sqrt{3}+1$

$\therefore\ 2\cos\frac{\pi}{2k} + 2\cos\frac{\pi}{k} = \sqrt{3}+1$

$\cos\frac{\pi}{2k} + \cos\frac{\pi}{k} = \frac{\sqrt{3}+1}{2}$

Let $\frac{\pi}{k} = \theta$, $\cos\theta + \cos\frac{\theta}{2} = \frac{\sqrt{3}+1}{2} \Rightarrow 2\cos^2\frac{\theta}{2} - 1 + \cos\frac{\theta}{2} = \frac{\sqrt{3}+1}{2}$

$\cos\frac{\theta}{2} = t$ $\qquad 2t^2 + t - \frac{\sqrt{3}+3}{2} = 0$

$t = \frac{-1 \pm \sqrt{1+4\left(3+\sqrt{3}\right)}}{4} = \frac{-1 \pm \left(2\sqrt{3}+1\right)}{4} = \frac{-2-2\sqrt{3}}{4}, \frac{\sqrt{3}}{2}$ $\qquad \because t \in [-1,1], \cos\frac{\theta}{2} = \frac{\sqrt{3}}{2}$

$\frac{\theta}{2} = \frac{\pi}{6} \Rightarrow k = 3$

17. **Ans. (5.00)**

$(1+\tan\theta)\left[1+\tan(45^\circ-\theta)\right] = (1+\tan\theta)\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right)$

$(1+\tan\theta)\left(\frac{2}{1+\tan\theta}\right) = 2$

Hence L.H.S. is equal to

$2(1+\tan 5^\circ)\,(1+\tan 40^\circ)\,(1+\tan 10^\circ)\,(1+\tan 35^\circ)\,(1+\tan 15^\circ)$

$(1+\tan 30^\circ)\,(1+\tan 20^\circ)\,(1+\tan 25^\circ)$

$= 2 \times 2^4 = 2^5$

18. **Ans. (2)**

$\sin\frac{7\pi}{8} = \sin\left(\pi - \frac{\pi}{8}\right) = \sin\frac{\pi}{8}$; $\sin\frac{5\pi}{8} = \sin\left(\pi - \frac{3\pi}{8}\right) = \sin\frac{3\pi}{8}$

Therefore, the given value = $2\left[\sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8}\right] = 2\left[\sin^2\frac{\pi}{8} + \cos^2\frac{\pi}{8}\right]$

$= 2(1) = 2$ $\left[\because \sin\frac{3\pi}{8} = \sin\left(\frac{\pi}{2} - \frac{\pi}{8}\right) = \cos\frac{\pi}{8}\right]$